EDF vehicle charging under deadline uncertainty

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ABSTRACT

In this paper, we analyze the performance of the EDF scheduling policy for charging electrical vehicles when the exact deadlines are not known by the scheduler. Instead, they are declared by users. We quantify the effect of this uncertainty in a mean field regime, and show that incentives appear for users to under-report their sojourn time. We characterize the average gain for a given uncertainty model and devise a policy to curtail strategic users.

Keywords

EV charging, Scheduling, Deadlines

1. INTRODUCTION

In recent years, the development of Electrical Vehicles (EVs) has brought up the problem of deploying a charging infrastructure. A typical scenario [2] is of a parking lot at a certain corporate or school site, where EVs stay for a considerable time and may thus be conveniently recharged. In this setting, individual chargers may be provided across the facility, but power provisioning need not cover the peak load due to its expense, so not all chargers may be active at the same time. Therefore, the system behaves as a many server queue where we can use the time flexibility of users to *schedule* the sharing of this limited capacity [3].

In a previous paper [5], we analyzed the performance of several deadline-based scheduling policies resorting to a mean field analysis. Among others, the Earliest-Deadline-First (EDF) policy was considered, which fills the capacity with EVs closest to their departure.

One of the problems with such deadline-based policies is that they require advance knowledge of the exact sojourn time of customers. In practice, these may not be known exactly or may be incorrectly reported by users. Furthermore, smart users may game the system by under-reporting their departure time in order to achieve priority. This situation leaves the EDF policy working with *uncertain* deadlines.

In this paper, we extend the framework of [5] to take into account the uncertainty, and analyze how the performance is affected in such situation. We also provide a simple variant of the policy that provides incentives to users to truthfully reveal their deadlines. These results based on mean field assumptions are validated through simulations.

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2. SYSTEM MODEL

Consider a system where EVs arrive as a Poisson process of intensity λ. Each EV has three main characteristics: its *service time* S (or requested energy, normalized by the charger nominal power), its *sojourn time* T and its *declared sojourn time* T'. We assume that these magnitudes are random and independent for each vehicle, and that $S \leqslant T$, i.e. the vehicle demand is feasible. The parking lot can activate at most C chargers simultaneously.

We assume that the system *is unaware* of the real sojourn time, and serves the EVs following the EDF algorithm using the *declared* deadline (but vehicles depart according to their true deadlines). If an EV is still in the system, it can still be charged after their declared deadline has expired. The *load* of the system is $\rho := \lambda E[S]$ and we would like to understand the performance of the system *in overload*, i.e. $\rho > C$.

We first recall briefly the mean field analysis of EDF from [5]. A typical charging profile is as follows: the EV begins receiving service when its remaining sojourn time τ satisfies $\tau < \tau_0^*$. The threshold value τ_0^* is fixed in the mean field limit. As a consequence, the total service time is bounded by τ_0^* and the *attained service* S_a^0 is:

$$S_a^0 = \min\{S, \tau_0^*\}. \tag{1}$$

The threshold τ_0^* follows from the capacity condition:

$$\lambda E[S_a^0] = \lambda E[\min\{S, \tau_0^*\}] = C. \tag{2}$$

We now analyze the case where EDF has to cope with the perturbed sojourn time T'. Let τ' denote the remaining *declared* sojourn time of the vehicle. In the mean field limit, a threshold τ^* emerges and the vehicle becomes prioritized whenever $\tau' < \tau^*$, i.e. at time $t + T' - \tau^*$ since arrival. Its service ends whenever it reaches full charge or departs at time $t + T$. Therefore in this case its service time is $\min\{S, T - T' + \tau^*\}$, as depicted in Fig. 1. If $T - T' + \tau^* < 0$ it departs before getting any service. We have the following:

PROPOSITION 1. *Consider an EDF charging system working under deadline uncertainty. Assume that the system is in overload, i.e. $\rho > C$. Then in the mean field limit the attained service of a given vehicle satisfies:*

$$S_a = \min\{S, (T - T' + \tau^*)^+\}, \tag{3}$$

where the threshold τ^* satisfies the fixed point equation:

$$\lambda E[\min\{S, (T - T' + \tau^*)^+\}] = C. \tag{4}$$

Note that eq. (4) always has a unique solution when $\rho > C$ since $E[\min\{S, (T - T' + \tau^*)^+\}] \to E[S]$ monotonically whenever $\tau^* \uparrow \infty$.

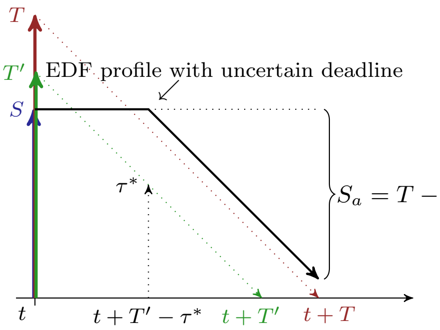

Figure 1: Charging profiles for EDF under uncertain deadlines.

3. THE EFFECT OF UNCERTAINTY

In order to quantify the effect of having random uncertainty in the deadlines, we now solve this fixed point equation under some distributional assumptions on S, T, T'. Assume that the charging time $S \sim \exp(\mu)$, $T = S + L$ where L is a positive random variable representing the *laxity* the user has on arrival. Finally, assume that:

$$T' = T + U \quad \text{where } U \sim \text{Uniform}[-\theta, \theta].$$

Here θ acts as an uncertainty parameter.

A useful Lemma is stated first:

Lemma 1. *If S is an exponential random variable with parameter μ and $x \geqslant 0$ then:*

$$E[\min\{S, x\}] = \int_0^x e^{-\mu x} dx = \frac{1}{\mu}(1 - e^{-\mu x}).$$

Let $X = (T - T' + \tau^*) \sim \text{Uniform}[\tau^* - \theta, \tau^* + \theta]$. Assuming $\tau^* \geq \theta$ so $X \geq 0$ $a.s.$[1], we can compute:

$$E[\min\{S, (T - T' + \tau^*)^+\}] = E[E[\min\{S, X\} \mid X]] =$$

$$= E\left[\frac{1}{\mu}(1 - e^{-\mu X})\right] = \frac{1}{\mu}\left(1 - e^{-\mu\tau^*}\frac{\sinh(\mu\theta)}{\mu\theta}\right).$$

With the above formula at hand, we can compute the threshold for a given λ, C by solving (4) to yield:

$$\tau^* = -\frac{1}{\mu}\log\left(\frac{\mu\theta}{\sinh(\mu\theta)}\left(1 - \frac{C}{\rho}\right)\right). \tag{5}$$

As $\theta \to 0$, the uncertainty disappears, $\frac{\mu\theta}{\sinh(\mu\theta)} \to 1$ and we have the expression $\tau_0^* = -\frac{1}{\mu}\log\left(1 - \frac{C}{\rho}\right)$ for the EDF threshold studied in [5].

To better understand the effect of the uncertainty, we focus on individual EV performance and compute:

$$E[S_a \mid U] = E[\min\{S, (T - T' + \tau^*)^+\} \mid U],$$

the expected service attained by vehicles that misreport their deadline by U. Since $T - T' = -U$ and τ^* is fixed, the second random variable in the min is measurable with respect to U, so we can resort to Lemma 1 to obtain:

Proposition 2. *In an EDF system in overload, working with uncertain deadlines and exponential service times, the attained service for a given deadline uncertainty satisfies:*

$$E[S_a \mid U] = \frac{1 - e^{-\mu(\tau^* - U)^+}}{\mu}, \tag{6}$$

where τ^* satisfies the fixed point equation (4).

[1] The alternate case $\tau^* < \theta$ can be handled by a corrected calculation.

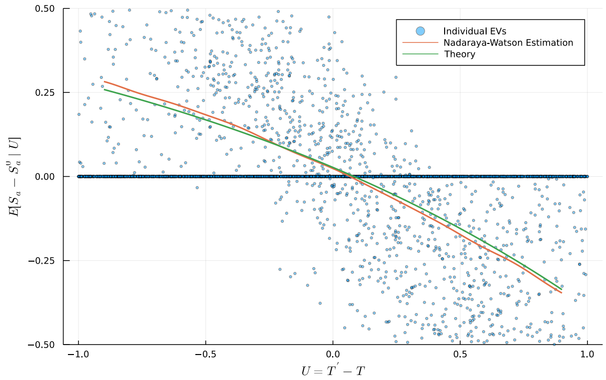

Figure 2: Attained energy difference between the uncertain and perfect information case.

Going back now to our specific parametric model and under the assumption $\tau^* \geq \theta$, we can combine (6) with the threshold in (5) to quantify the uncertainty impact in that case. Specifically, we compute the gain in attained service for a given uncertainty level with respect to the perfect information case, normalized by mean service demanded:

$$\frac{E[S_a - S_a^0 \mid U]}{E[S]} = \frac{E[S_a \mid U] - E[S_a^0]}{E[S]}.$$

From eq. (2) we know that $E[S_a^0] = C/\lambda$ in the mean field limit, and $E[S_a \mid U]$ follows from eq. (6). For the uniform uncertainty case, we solve this explicitly to yield:

$$\frac{E[S_a - S_a^0 \mid U]}{E[S]} = \left(1 - \frac{C}{\rho}\right)\left(1 - \frac{\mu\theta}{\sinh(\mu\theta)}e^{\mu U}\right). \tag{7}$$

In Fig. 2, we show the results of a simulation experiment performed using the Julia library `EVQueues.jl` [1]. The parameters are $\lambda = 30$, $C = 40$, $\mu = 0.5$ and $\theta = 1$. The normalized attained energy difference is plotted for each vehicle as a function of $U = T' - T$. Note there are multiple points in the 0 level. In solid lines, the average gain is estimated via the Nadaraya-Watson kernel regression estimator for the conditional expectation [4], and compared with the theoretical mean field expression in (7), showing good fit.

We observe that EVs under-reporting deadlines tend to receive more service. This is true regardless of the uncertainty model because the expression (6) is *decreasing* in U.

4. ENFORCING INCENTIVES

Given the preceding observation, strategic users may get an advantage from the previous policy by under-reporting their deadlines. In order to to provide incentive compatibility, a simple variant of the policy is: serve EVs only up to departure or declared departure time, whichever happens first. We now analyze this alternative.

A representative trajectory is depicted in Fig. 3. A vehicle arriving at time t will receive service up to $\min\{t+T, t+T'\}$,

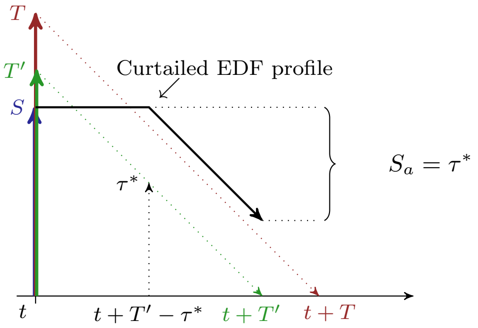

Figure 3: Charging profiles under a curtailed EDF policy.

and its service time will be either S or:

$$\min\{t+T, t+T'\} - (t+T'-\tau^*) - t =$$
$$= \min\{T, T'\} - T' + \tau^* = (T-T')\mathbf{1}_{\{T<T'\}} + \tau^*,$$

provided that the above term is positive. Otherwise it will not receive service. We call this the *curtailed EDF policy*, which is characterized in the following:

Proposition 3. *Consider an EDF charging system working under deadline uncertainty and curtailing users when their declared deadline expires. In the mean field limit with the system in overload ($\rho > C$) the attained service of a given vehicle satisfies:*

$$S_a = \min\{S, ((T-T')\mathbf{1}_{\{T<T'\}} + \tau^*)^+\}, \qquad (8)$$

where the threshold comes from the fixed point equation:

$$\lambda E[\min\{S, ((T-T')\mathbf{1}_{\{T<T'\}} + \tau^*)^+\}] = C. \qquad (9)$$

Note that the indicator term in (8) only becomes active when an EV over-reports its deadline.

We now solve the above equations for the parametric case we have been considering, i.e. uniform deadline uncertainty. The attained work can be written as (for $\tau^* > \theta$):

$$S_a = \min\{S, \tau^* - U\mathbf{1}_{\{U>0\}}\}.$$

Invoking again Lemma 1 we arrive at:

$$E[S_a] = E[E[S_a \mid U]] = \int_{-\theta}^{\theta} \frac{1 - e^{-\mu(\tau^* - u\mathbf{1}_{\{u>0\}})}}{\mu} \frac{1}{2\theta} du$$
$$= \frac{1}{\mu}\left[1 - e^{-\mu\tau^*}\left(\frac{1}{2} + \frac{e^{\mu\theta}-1}{2\mu\theta}\right)\right],$$

and we can solve for the threshold in the same way as before:

$$\tau^* = -\frac{1}{\mu}\log\left[\left(\frac{1}{2} + \frac{e^{\mu\theta}-1}{2\mu\theta}\right)^{-1}\left(1 - \frac{C}{\rho}\right)\right]. \qquad (10)$$

Analogous to Proposition 2, we find an expression for the conditional expectation of attained service with respect to the misreport in deadline, for exponential service times:

Proposition 4. *In a curtailed EDF system in overload, working with uncertain deadlines and exponential service times, the attained service for a given uncertainty satisfies:*

$$E[S_a \mid U] = \frac{1 - e^{-\mu(\tau^* - U\mathbf{1}_{\{U>0\}})^+}}{\mu}, \qquad (11)$$

where τ^* satisfies the fixed point equation (9).

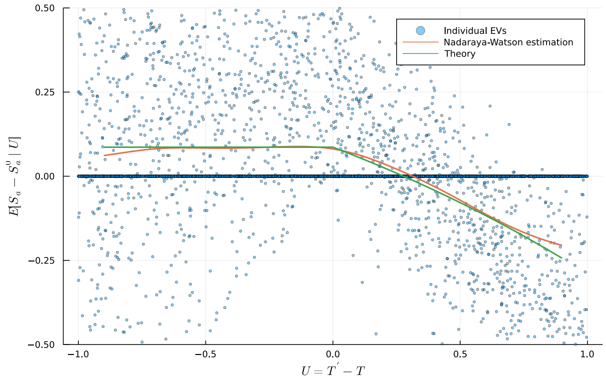

Figure 4: Attained energy difference between the uncertain and perfect information case.

The expression in (11) is again non-increasing in U, but it is now *constant* whenever $U < 0$, i.e. $T' < T$. This curbs the incentive to under-report the deadline since no gain is obtained on average with respect to $T' = T$.

For the parametric model of U under consideration, we obtain from (10) a formula for the conditional gain in service:

$$\frac{E[S_a - S_a^0 \mid U]}{E[S]} = \left(1 - \frac{C}{\rho}\right)\left(1 - \frac{e^{\mu U\mathbf{1}_{\{U>0\}}}}{\frac{1}{2} + \frac{e^{\mu\theta}-1}{2\mu\theta}}\right). \qquad (12)$$

In Fig. 4, we show the results of a simulation experiment under the same setting than the previous Section but with the new curtailed policy in use. We can see that the average gain for a given uncertainty level is curbed whenever $U < 0$, and coincides with the theoretical expression in (12).

5. CONCLUSIONS

We analyzed the performance of the EDF scheduling policy for charging electrical vehicles when deadlines are uncertain, quantifying the effect of this uncertainty, and provided a policy to curtail users that under-report their deadlines with provable performance.

Acknowledgments

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