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# Reporte Técnico RT 12-04

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2012

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Risso, Claudio; Robledo, Franco
ISSN 0797-6410
Reporte Técnico RT 12-04
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# Optimal Design of a Multi-Layer Network An IP/MPLS over DWDM application case

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**Abstract:** In this paper we study a network design problem arising from the deployment of an IP/MPLS network over an existing transport infrastructure. The goal is to find a minimum cost installation of links such that traffic demands can resiliently be accomplished. We present an integer programming formulation for our problem and metaheuristics to find good quality solutions. This work is based on a real application case for a telecommunications company.

**Keywords:** telecommunications network; multi-layer network design; GRASP.

**Reference** to this paper should be made as follows: Risso and Robledo "Optimal Design of a Multi-Layer Network. An IP/MPLS over DWDM application case."

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## 1 Introduction

Some decades ago the increasing importance of the telephony service pushed most telecommunications companies (TELCOs) to deploy optical fiber networks. In order to guarantee appropriate service availability, these networks were designed in such a way that several independent paths were available between each pair of nodes, and in order to optimize these large capital investments several models and algorithms were developed. Already the optimal design of a single layer network is a challenging task that has been considered by many research groups, see for instance the references: Okamura and Seymour (1981), Stoer (1998) and Kerivin and Mahjoub (2005). Throughout this work this optical network is referred to as the *physical layer*.

Some years afterwards, the exponential growth of Internet traffic volume demanded for higher capacity networks. This demand led to the deployment of dense wavelength division multiplexing (DWDM) technology. This technology allows multiplexing several connections -lightpath connections- over one single cable of optical fiber using different wavelengths, and rapidly became very popular with telecommunications companies because it allowed them to expand the capacity of their networks without laying more fiber. Today, DWDM has turned out to be the dominant network technology in high-capacity optical backbone networks. Repeaters and amplifiers must be placed at regular intervals for compensating the loss in optical power while the signal travels along the fiber; hence the cost of a lighpath is proportional to its length over the physical layer. DWDM supports a set of standard high-capacity interfaces (e.g. 1, 2.5, 10 or 40 Gbps). The cost of a connection also depends of the capacity but not proportionally. For economies of scale reasons, the higher the bit-rate the lower the per-bandwidth-cost. The client nodes together with these lightpath connections form a so-called *logical layer* on top of the physical one.

The increasing number of per-physical-link connections -intrinsic to DWDM- may cause multiple logical link failures from a single physical link failure (e.g., fiber cut). This issue led to the development of new multi-layer models aware of the stack of network layers. Most of these models share in common the 1+1 protection mechanism, i.e., for every demand two independent lightpaths must be routed such that in case of any single physical link -or even node- failure, at least one of them survive. The following references: Orlowski et al. (2007) and Koster et al. (2008) are good examples of this kind of models. The combined optimization of two layers significantly increases the complexity of the planning task, especially if we attempt to optimize both layers simultaneously as is the case for the referenced articles.

Those multi-layer models are suitable for certain families of logical layer technologies such as: synchronous optical networking (SONET) or synchronous digital hierarchy (SDH) since both standards have 1+1 protection as their native protection mechanism. During many years the connections of IP networks were implemented over SONET/SDH. As a consequence the IP layer -the third and top layer of this stack- rarely suffered unplanned topology changes. Most recently: multiprotocol label switching (MPLS), traffic engineering extensions for dynamic routing protocols (e.g. OSPF-TE, ISIS-TE), fast reroute algorithms (FRR) and other new features were added to the traditional IP routers. This new *technology bundle* known as IP/MPLS, opens a competitive alternative against traditional protection mechanisms based on SONET/SDH.

Since IP/MPLS allows recovering from a failure in about 50ms, capital savings may come from the elimination of the intermediate SONET/SDH layer -for simplicity we will only mention SDH from now on-. Another improvement of this technology is that the number of paths to route demands between nodes is not pre-bounded; so it might exist in fact a feasible different configuration for most failure scenarios. Moreover, there is no need to pre-establish all of these paths explicitly: if the appropriate information is fed to the routing protocols and the network is designed with care, the dynamic routing algorithms usually construct solutions of very good quality. Since IP/MPLS allows the elimination of an intermediate layer, manages Internet traffic natively, and makes possible a much easier and cheaper operation for virtual private network (VPN) services, it is gaining relative importance every day.

In this paper we address the problem of finding the optimal -minimum cost-configuration of a logical topology over a fixed physical layer. The input data set is constituted by: the physical layer topology -DWDM network-, the client nodes of the logical layer -IP/MPLS nodes- and the potential links between them, as well as the traffic demand to satisfy between each pair of nodes and the per-distance cost in the physical network associated with the bitrates of the lightpaths to deploy over it. The decision variables are: what logical links do we have to implement, which bitrate must be assigned to each of them and what path do these lightpaths have to follow in the physical layer. For being a feasible solution a configuration must be capable of routing every traffic demand over the remaining active links of the logical layer for every single physical link failure scenario.

Unlike the referenced SDH over DWDM models, where both layers are optimized simultaneously, in our case we assume that the physical layer is already installed and cannot be changed. This is a consequence of the application case, because the TELCO we developed this work for did not have any intention to modify its optical fiber network. Furthermore, important portions of this physical layer are rented to international carriers making some physical changes impossible.

The most remarkable differences between the models for SDH and IP/MPLS client networks come from the technology itself. Setting aside technical details, the IP/MPLS technology does not fit well with two natural features of the SDH technology. The first one is the need of SDH to keep different demands between the same nodes. In IP/MPLS networks all the traffic from one node to another follows the same path in the network referred to as *IP/MPLS tunnel*. Although possible, splitting the traffic between a pair of nodes into more than one tunnel requires complex configurations. The second remarkable difference is how these technologies handle the existence of parallel links in the logical layer. In SDH the existence of parallel links is typical but in IP/MPLS, parallel links may conflict with some applications. IP/MPLS allows easily setting the path for a tunnel through a node by node sequence; but when there is more than one link to be used for a node-to-node hop, it is much harder to force a particular one.

Because of the changes in the technology this model is significantly different from those previously referred, so are the algorithms to find solutions.

This paper is organized as follows. The mixed-integer programming model will be presented in Section-2. In Section-3 we will show some exact solutions found with CPLEX for small/simple but illustrative problems; we will also analyze the intrinsic complexity of the problem. For finding solutions to real size problems we developed a metaheuristic based on GRASP that is presented in Section-4. Finally, in Section-5 we will show the solutions found through the previous metaheuristic.

## 2 Mathematical model

We will now introduce the basic mixed-integer programming model that arises from the detailed interaction of technologies.

**Parameters** The physical network is represented by an undirected graph $(V, P)$, and the logical network is represented by another undirected graph $(V, L)$. Both layers share the same set of nodes. The links of the logical layer are potential -admissible logical links- while the links of the physical layer are definite. In both graphs the edges are simple since multigraphs are not allowed in this model.

For every different pair of nodes $\forall p, q \in V$ is known the traffic volume $d_{pq}$ to fulfill along the unique path this traffic follows throughout a logical layer configuration. These paths are unique at every moment, but in case of link failures they may change to follow an alternate route. For simplicity we assume that the traffic volume is symmetric (i.e. $d_{pq} = d_{qp}$).

Let $\hat{B} = \{b_1, \ldots, b_{\bar{B}}\}$ be the set of possible bitrate capacities for the lightpaths on the physical layer and therefore for the links of the logical one. Every capacity $b \in \hat{B}$ has a known per-distance cost $c_b$. For economies of scale reasons it holds that if $b' < b''$ then $(c_{b'}/b') > (c_{b''}/b'')$.

Since both graphs of this model are simple and undirected, we will express links as pairs of nodes. For every physical link $(ij)$ is known its length $l_{ij}$.

**Variables** This model comprises three classes of variables. The first class is composed of the logical link capacity variables. We will use boolean variables $\tau_{pq}^{b}$ to indicate whether or not the logical link $(pq) \in L$ has been assigned with the capacity $b \in \hat{B}$. As a consequence the capacity of the link $(pq)$ could be computed as: $\sum_{b \in \hat{B}} b \cdot \tau_{pq}^{b}$.

The second class of variables determines how are going to be routed the logical links over the physical network. If $\sum_{b \in \hat{B}} \tau_{pq}^{b} = 1$ then the logical link $(pq) \in L$ was assigned with a capacity, it is going to be used in the logical network and requires a lightpath in the physical one. $y_{pq}^{ij}$ is a boolean variable that indicates whether or not the physical link $(ij) \in P$ is being used to implement the lightpath of $(pq)$.

Since lightpaths cannot automatically recover from a link failure, whenever a physical link $(ij)$ fails all the logical links $(pq)$ such that $y_{pq}^{ij} = 1$ do fail as well. The only protection available in this model is that of the logical layer. For demands being protected against single physical link failures, it is necessary to have a feasible route through the remaining active logical links. The third and final class of variables is that that determines how the IP/MPLS tunnels are going to be routed against a failure in a physical link. ${}^{rs}x_{pq}^{ij}$ is a boolean variable that indicates whether the logical link $(pq) \in L$ is going to be used or not, to route traffic demand $d_{rs} > 0$, under a fault condition in the physical link $(ij) \in P$.

NOTE: To keep the nomenclature of the variables as easy as possible we always placed: logical links subindexes at bottom right position, physical links subindexes at top right position and demands subindexes at top left position.

**Constraints** This problem comprises three groups of constraints. The first group of constraints establishes the rules that the routes of the lightpaths must follow to be feasible.

$$
\left\{
\begin{array}{lll}
\sum\limits_{b\in\hat{B}} \tau_{pq}^{b} \leq 1 & \forall (pq)\in L. & (1)\\
\sum\limits_{j/(pj)\in P} y_{pq}^{pj} = \sum\limits_{b\in\hat{B}} \tau_{pq}^{b} & \forall (pq)\in L. & (2)\\
\sum\limits_{i/(iq)\in P} y_{pq}^{iq} = \sum\limits_{b\in\hat{B}} \tau_{pq}^{b} & \forall (pq)\in L. & (3)\\
\sum\limits_{j/(ij)\in P} y_{pq}^{ij} = 2\hat{\theta}_{pq}^{i} & \begin{array}{l}\forall (pq)\in L, \forall i\in V,\\ i\neq p, i\neq q.\end{array} & (4)\\
\tilde{\theta}_{pq}^{i} + \hat{\theta}_{pq}^{i} = 1 & \begin{array}{l}\forall (pq)\in L, \forall i\in V,\\ i\neq p, i\neq q.\end{array} & (5)\\
y_{pq}^{ij} - y_{pq}^{ji} = 0 & \forall (pq)\in L, \forall (ij)\in P. & (6)\\
\tau_{pq}^{b},\, y_{pq}^{ij}, \hat{\theta}_{pq}^{i}, \tilde{\theta}_{pq}^{i} \in \{0,1\} & \begin{array}{l}\forall (pq)\in L, \forall (ij)\in P\\ \forall b\in\hat{B}, \forall i\in V.\end{array} & (7)
\end{array}
\right.
\qquad \text{(A)}
$$

The meaning of constraints in group (A) is the following: (1) establishes that the number of capacities assigned to every logical link is at most 1 -it could be 0 if the link is not going to be used-; (2) and (3) guarantee that if any particular link $(pq) \in L$ was assigned with a capacity ($\sum_{b\in\hat{B}} \tau_{pq}^{b} = 1$) then there must exist one and only one outgoing -or incoming- physical link used for its lightpath.

Before going any further we have to introduce a set of auxiliary variables: $\tilde{\theta}_{pq}^{i}$ and $\hat{\theta}_{pq}^{i}$. These variables are defined for every combination of logical links $(pq) \in L$ and physical nodes $i \in V$. (5) guarantees that exactly one of the following conditions must meet: $(\tilde{\theta}, \hat{\theta}) = (1, 0)$ or $(\tilde{\theta}, \hat{\theta}) = (0, 1)$. Hence, (4) guarantees flow balance for routing the lightpaths through the remaining -not terminal- nodes.

Finally (6) guarantees that the lightpaths go back and forth through the same path, while (7) stands the integrity of the variables.

The second group of constraints establishes the rules that the routes of the IP/MPLS tunnels must follow in the logical layer.

The meaning of the constraints in (B) is similar to those of (A) except for (1). The inequality in (1) were added to guarantee that whatever the failure scenario is ($\forall (ij) \in P$), its associated routing configuration over the logical network keeps the aggregated traffic load below the link capacity for every data link ($\forall (pq) \in L$).

Constrains (2) and (3) from (A) and (B) are equivalent, except for the fact that in the latter the existence of a tunnel relies on the existence of demand and this is known in advance.

Another remarkable point is that (B) has as many possible routing scenarios as arcs in $P$, so the number of variables is much greater than those of (A).

$$\left\{\begin{array}{lll}
\sum_{rs:d_{rs}>0} d_{rs}\cdot {}^{rs}x^{ij}_{pq} \leq \sum_{b\in\hat{B}} b\cdot\tau^{b}_{pq} & \forall(pq)\in L, \forall(ij)\in P. & (1)\\
\sum_{q/(rq)\in L} {}^{rs}x^{ij}_{rq} = 1 & \forall d_{rs}>0, \forall(ij)\in P. & (2)\\
\sum_{p/(ps)\in L} {}^{rs}x^{ij}_{ps} = 1 & \forall d_{rs}>0, \forall(ij)\in P. & (3)\\
\sum_{q/(pq)\in L} {}^{rs}x^{ij}_{pq} = 2\cdot {}^{rs}\hat{\mu}^{ij}_{p} & \begin{array}{l}\forall d_{rs}>0, \forall(ij)\in P,\\ \forall p\in V, p\neq r, p\neq s.\end{array} & (4)\\
{}^{rs}\tilde{\mu}^{ij}_{p} + {}^{rs}\hat{\mu}^{ij}_{p} = 1 & \begin{array}{l}\forall d_{rs}>0, \forall(ij)\in P,\\ \forall p\in V, p\neq r, p\neq s.\end{array} & (5)\\
{}^{rs}x^{ij}_{pq} - {}^{rs}x^{ij}_{qp} = 0 & \begin{array}{l}\forall d_{rs}>0, \forall(pq)\in L,\\ \forall(ij)\in P.\end{array} & (6)\\
{}^{rs}x^{ij}_{pq}, {}^{rs}\tilde{\mu}^{ij}_{p}, {}^{rs}\hat{\mu}^{ij}_{p} \in \{0,1\} & \begin{array}{l}\forall d_{rs}>0, \forall(pq)\in L,\\ \forall(ij)\in P, \forall p\in V.\end{array} & (7)
\end{array}\right. \qquad (B)$$

Variables sets ${}^{rs}\tilde{\mu}^{ij}_{p}$ and ${}^{rs}\hat{\mu}^{ij}_{p}$ are homologous to $\hat{\theta}^{i}_{pq}$ and $\tilde{\theta}^{i}_{pq}$; so are constraints from (4) to (7).

Before proceeding any further we must notice that (A) and (B) are not independent. Many logical links may not be available for routing after a physical link failure. Which logical links are in this condition, relies on how the lightpaths were routed in the physical layer. Specifically, if some logical link $(pq)$ uses a physical link $(ij)$ for its lightpath implementation then this logical link cannot be used to route any tunnel under $(ij)$ failure scenario.

$${}^{rs}x^{ij}_{pq} \leq 1 - y^{ij}_{pq} \quad \forall rs{:}d_{rs}>0, \forall(pq)\in L, \forall(ij)\in P. \qquad (C)$$

The group of constrains (C) prevents from using $(pq)$ to route any traffic (${}^{rs}x^{ij}_{pq} = 0, \forall rs : d_{rs} > 0$) in any failure scenario that affects the link (when $y^{ij}_{pq} = 1$).

**Objective** The function to minimize is the sum of the cost of every logical link. According on what capacity was assigned to a logical link there is an associated per-distance-cost ($c_b$), and according on how the corresponding lightpath was routed over the physical layer it has an associated length ($\sum_{(ij)\in P} l_{ij}y^{ij}_{pq}$).

The product of both terms is the cost of a particular logical link and the sum of these products for all the logical links is the total cost of the solution. The direct arithmetic expression for the previous statement would be: $\sum_{(pq)\in L}(\sum_{b\in\hat{B}} c_b\tau^{b}_{pq})(\sum_{(ij)\in P} l_{ij}y^{ij}_{pq}) = \sum_{(pq)\in L,(ij)\in P,b\in\hat{B}} c_b l_{ij}\cdot\tau^{b}_{pq}y^{ij}_{pq}$.

Although straightforward, this approximation is inappropriate because it is non-linear. The sub-problem (D) expresses the objective value with an equivalent linear expression. We used the real variable ${}^{b}\eta^{ij}_{pq}$ instead of $\tau^{b}_{pq}y^{ij}_{pq}$ and added some extra constraints to guarantee the consistency. This consistency arises from the following observations: the result of $\tau^b_{pq} y^{ij}_{pq}$ is also a boolean variable, and since ${}^b\eta^{ij}_{pq}$ is being multiplied by a positive constant in a minimization problem it will take its lowest value (zero) whenever this is possible.

$$\left\{\begin{array}{lll} \min \sum\limits_{\substack{(pq)\in L\\ (ij)\in P\\ b\in\hat{B}}} c_b l_{ij} \cdot {}^b\eta^{ij}_{pq} & & (1) \\ {}^b\eta^{ij}_{pq} \geq \tau^b_{pq} + y^{ij}_{pq} - 1 & \substack{\forall (pq)\in L, \forall (ij)\in P,\\ \forall b\in\hat{B}.} & (2) \\ {}^b\eta^{ij}_{pq} \geq 0 & \substack{\forall (pq)\in L, \forall (ij)\in P,\\ \forall b\in\hat{B}.} & (3) \end{array}\right. \qquad \text{(D)}$$

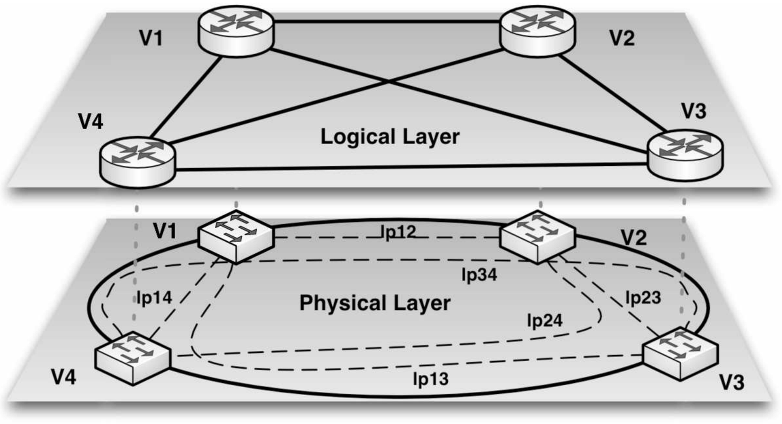
The only exception is when the values of $\tau^b_{pq}$ and $y^{ij}_{pq}$ are both 1, in which case the value of ${}^b\eta^{ij}_{pq}$ should be 1 as well to keep consistency. This is guaranteed by constrain (2) of (D).

The complete MIP is the result of merging: (A), (B), (C) and (D).

## 3 Finding exact solutions

We will start by showing particular solutions for some simple example cases. The first example has four nodes $V = \{v_1, v_2, v_3, v_4\}$, the physical layer is the cycle ($\mathcal{C}^4$) while the logical layer is the clique ($\mathcal{K}^4$). The remaining parameters are: $B = \{3\}$, $d_{pq} = 1, \forall 1 \leq p < q \leq 4$ and $l_{ij} = 1, \forall (ij) \in P$. $c_b$ is irrelevant in this case because there is only one bitrate available. The optimal solution found for this case uses all of the logical links. Figure-1 shows with dashed lines the route in that solution followed for each lightpath over the physical cycle.

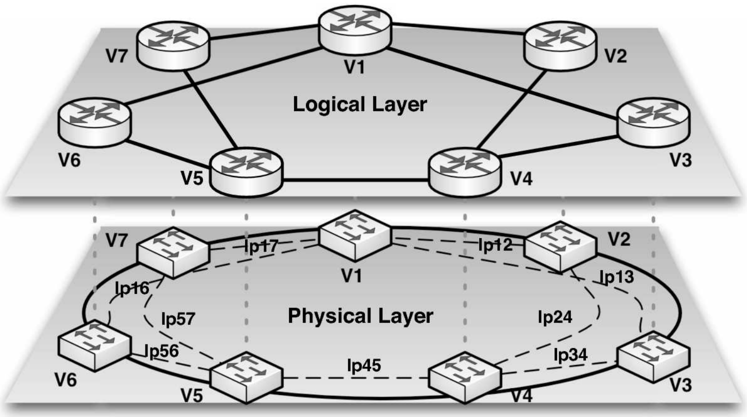
**Figure 1** Optimal solution found for $\mathcal{K}^4$ over $\mathcal{C}^4$, with $d_{pq} = 1$ and $B = \{3\}$



This is an example where lightpaths routes are not intuitive, even for a very simple input data set.

The following example comprises seven nodes and explores again the clique-over-cycle case. The remaining parameters are analogous: $B = \{3\}$, $l_{ij} = 1, \forall (ij) \in P$, except for demands that in this case are to/from one single node ($d_{1q} = 1, \forall 1 < q \leq 7$). Unlike the previous example, the optimal solution in this case (sketched in Figure-2) does not make use of all the logical links.

**Figure 2** Optimal solution found for $\mathcal{K}^7$ over $\mathcal{C}^7$, with $d_{1q} = 1$ and $B = \{3\}$



Although the route followed by each lightpath looks more natural in this example, it is not immediate why this set of logical links ought to be the appropriate to construct the optimal solution. Through these two examples we attempted to show that solutions are not intuitive even for very simple cases.

To find optimal solutions we used ILOG CPLEX v12.1. All computations were performed on a Linux machine with an INTEL CORE i3 Processor and 4GB of DDR3 RAM.

**Table 1** Overall results for some particular cases

| $\lvert V \rvert$ | $b_1$ | *#variables* | *#constrains* | *elapsed time* |
|---|---|---|---|---|
| | *range* | | | *(hh:mm:ss)* |
| 5 | 2 - 6 | 1230 | 1640 | 00:00:00 - 000:00:11 |
| 6 | 3 - 9 | 3390 | 4035 | 00:00:02 - 000:19:31 |
| 7 | 2 - 12 | 7896 | 8652 | 00:00:05 - 087:19:05(*) |
| 8 | 3 - 16 | 16296 | 16772 | 00:00:02 - 100:10:17(*) |

*(*)Note:* The solver aborted for some intermediate cases.

Table-1 shows information for several test instances similar to those represented in Figure-1, that is: $\mathcal{K}^n$ over $\mathcal{C}^n$ with $d_{pq} = 1, \forall 1 \leq p < q \leq 4$ and $l_{ij} = 1, \forall (ij) \in P$ over a range of integer $b_1$ values ($|\hat{B}| = 1$). It can be proven that:

**Property 3.1:** *It is always possible to find minimal feasible solutions for these particular topologies and demands when: $b_1 = 2$ and $|V|$ is odd, or when $b_1 = 3$ and*

*$|V|$ is even. In the first situation the complete logical graph is needed, whereas in the second only diagonal links can be disposed of.*

The lowest computation times were found for these extreme cases. On the other hand and for the same set of input data, it can be proven that:

**Property 3.2:** *The cycle configuration for the logical network -the simplest possible- is feasible for every $b_1$ greater or equal to: $|V|^2/4$ when $|V|$ is even, or $(|V|^2-1)/4$ when $|V|$ is odd.*

Very low computation times were found for these cases also.

The time required for finding optimal solutions for non-extreme cases were much higher. CPLEX even aborted for many of them. Aside from a bunch of worthless exceptions, we couldn't find solutions for topologies other than $\mathcal{K}^n$ over $\mathcal{C}^n$. Keeping these physical and logical topologies but trying with simpler matrices of demand (e.g. $d_{1q} = 1, \forall 1 < q \leq |V|$) it was possible to increase the size of the problems to 15 nodes and yet being able to find optimal solutions.

Suffices to say this size bound as well as the simplicity in the topologies and traffic matrices of the previous examples, are incompatible with real network problems. The following property shows this complexity is intrinsic to the problem.

**Proposition 3.1:** *The problem presented in this section is NP-Hard.*

Demonstration lies under reduction of NPP (Number Partitioning Problem) to our particular problem that we will refer as $\mathcal{P}$ within this proof. NPP problem consist in finding two subsets with the same sum for a known multiset of numbers. Formally: given a list of positive integers: $a_1, a_2, \ldots, a_N$, a partition $\mathcal{A} \subseteq \{1, 2, \ldots, N\}$ must be found so that discrepancy:

$$E(\mathcal{A}) = |\sum_{i\in\mathcal{A}} a_i - \sum_{i\notin\mathcal{A}} a_i|,$$

finds its minimum value within the set $\{0, 1\}$. NPP is a very well known NP-Complete problem (see for instance Stephan Mertens (2006)).

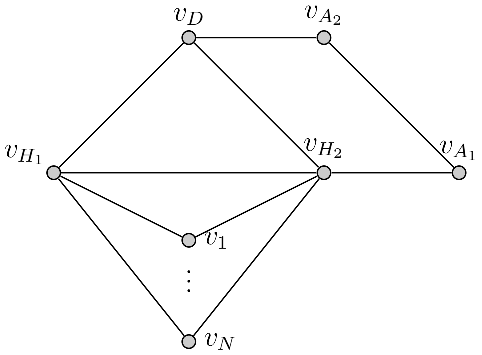
Given such a list of positive integers we create an instance of $\mathcal{P}$ taking: $B = \{\lceil(\sum_{1\leq i\leq N} a_i)/2\rceil\}$, logical and physical graphs with the same topology schematized in Figure-3, $l_{ij} = 1\,\forall(ij) \in P$ and $d_{iD} = a_i$, $\forall 1 \leq i \leq N$.

Since logical and physical topologies are the same and all distances are equal, the logical layer projected over the physical one for any optimal solution must copy the underlying shape. So, if there exists a solution for such an instance of $\mathcal{P}$, this solution must have a feasible routing scenario when transport -and logical-link $(v_{A_1}v_{A_2})$ fails and therefore a way to accommodate traffic requirements over $(v_{H_1}v_D)$ and $(v_{H_2}v_D)$, due to the fact that both links are still in operational state and they are the only way to reach $V_D$.

Because of the limited capacity both links must have been assigned with $(b_1)$, this can only be done when discrepancy is not grater than one, so we indirectly found a solution for the original NPP problem.

The complementary part of the proof is easier. Given a solution to an instance of NPP, this partition is used to distribute tunnels between $(v_{H_1}v_D)$ and $(v_{H_2}v_D)$. Once in $v_{H_1}$ or $v_{H_2}$ the tunnel is terminated directly in the corresponding node, except for some fault condition in one of these links, in which case a detour through the other $v_{H_x}$ node is always possible. When the fault condition arises in $(v_{H_1}v_D)$ or $(v_{H_2}v_D)$, a detour may be taken through: $(v_DV_{A_2}),(V_{A_2}V_{A_1}),(v_{A_1}v_{H_2})$ or $(v_DV_{A_2}),(V_{A_2}V_{A_1}),(v_{A_1}v_{H_2}),(v_{H_2}v_{H_1})$.

**Figure 3** Graph used for NPP reduction to $\mathcal{P}$.



Since all the transformation are of polynomial complexity it stands that $NPP \preccurlyeq \mathcal{P}$ and $\mathcal{P}$ is NP-Hard. □

The previous result shows that like for most other network design problems, an exhaustive search for the optimal solution of the problem presented in this work is infeasible for real size problems.

## 4 Metaheuristics

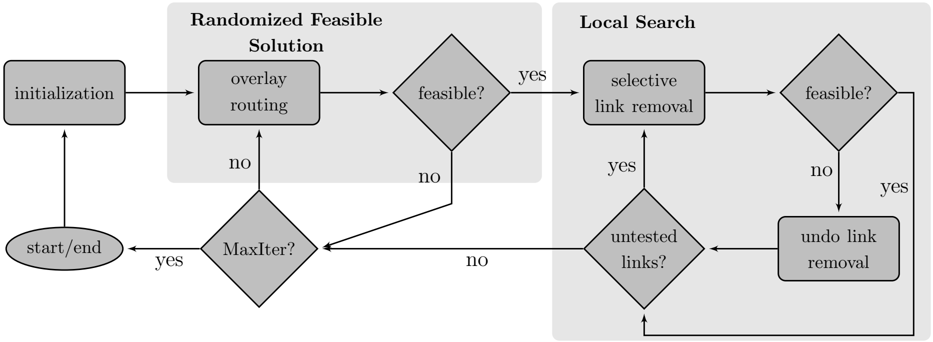
Although hard to solve, this model is very suitable to express many practical applications. Besides the direct self-contained network problems, by adding artificial nodes and links this model allows solving several typical decision problems of an Internet Service Provider (ISP). For instance, since most Internet traffic downloaded by the customers of any ISP usually comes from the outside of the ISP network, it is critical to determine which are the optimal traffic interchange points with other: ISPs, international carriers or content networks. By adding some artificial: nodes, links, and altering the remaining data it is possible to reflect several of the previous application cases -examples will be shown in Section-5-.

The versatility of this model allows rendering complex situations but has a drawback in the higher number of nodes to handle. Nevertheless these artificial nodes are usually very particular and can be treated easier than general nodes. For instance, nodes representing important traffic sources (e.g.: Datacenter) usually have demands against all the remaining nodes, but only share links with a few of them, so they do not affect deeply the core topology of the graph.

We decided to use a metaheuristic algorithm based on GRASP to find good quality solutions for real instances of this problem. A very high level diagram of our algorithm is shown in Figure-4.

As for every GRASP implementation this algorithm has a loop with two phases. The *construction phase* builds a *randomized feasible solution*, from which a local minimum is found during the *local search phase*. This procedure is repeated $MaxIter$ times while the best overall solution is kept as the result. Further information and details in GRASP algorithms can be found in Resende and Ribeiro (2003) or in Resende and Pardalos (2006).

**Figure 4** Block-diagram of the GRASP implementation used.



The *initialization phase* performs computations whose results are invariants among iterations to the other, like the shortest path and distance over the physical layer between each pair of nodes.

The *randomized feasible solution phase* performs a heuristic low cost balanced routing of the logical layer over the physical one. The exact solution for this subproblem is also NP-Complete as it can be seen in Martin Oellrich (2008). The goal is to find a path for every lightpath, such that the number of physical link intersections be minimum. It is also desirable that the total cost be as low as possible but as a second priority.

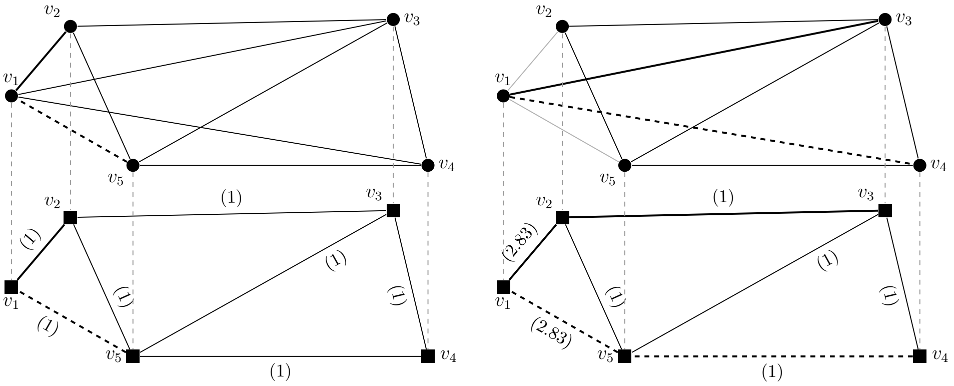
The strategy chosen in this heuristic is the following: nodes are taken randomly (.e.g.: uniformly), and for each node their logical links are also taken randomly but with probabilities in inverse ratio to the minimal possible distance of their lightpaths over the physical layer. Instead of using the real distances of the physical links ($l_{ij}$), from this point on and until the next iteration pseudo-distances: $\bar{l}_{ij}, \forall (ij) \in P$ will be used. Prior to start routing lightpaths, all these pseudo-distances are set to 1. According to these new weights, logical links are routed following the minimal distance without repeating physical links among them.

Usually, after routing some lightpaths the set of not-yet-used physical links empties, and it is necessary to start over a new *control window* by filling again the not-yet-used set. Prior to do this, the pseudo-distances are updated using the following rule: $\bar{l}_{ij} = (1 + n_{ij})^p$ for some fixed penalty $p$, where $n_{ij}$ is the number of lightpaths that are making use of $(ij) \in P$ up to the moment.

For instance, let us guess our networks are like those sketched in Figure-5 and the links drawn are: $(12), (15), (13), (14), (23), (35), \ldots$.

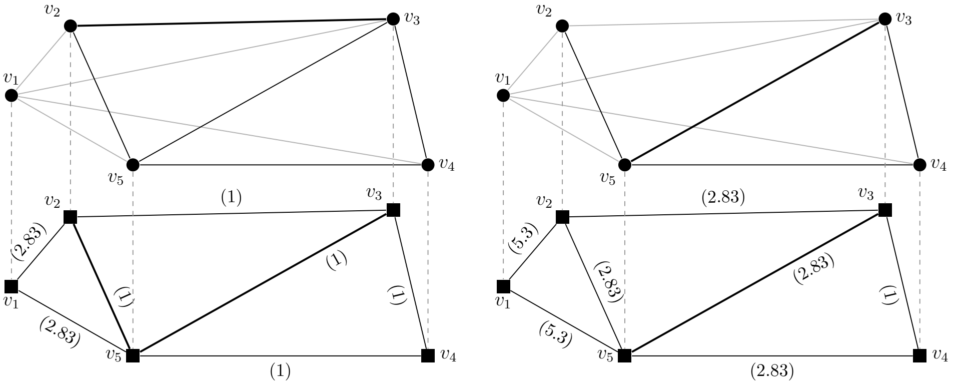
The left half of Figure-5 shows with solid and dashed lines how are routed the lightpaths (12) and (15). At this point we need to update the pseudo-distances and restart the window. If $p = 1.5$ and since $n_{12} = n_{15} = 1$, then $\bar{l}_{12} = \bar{l}_{15} = 2^{1.5} \approx 2.83$ for the next window.

**Figure 5** An example of the balanced routing heuristc.



The next two logical links are (13) and (14). They are routed using the updated values. Their lightpaths are also represented with solid and dashed lines in the right half of Figure-5. The link (23) is the following and it can be routed in two hops. A window restart is necessary to route the lightpath of (35), as it can be seen in Figure-6.

**Figure 6** Lightpaths for logical links (23) and (35).



The elements of the input data in Algorithm-1 are: the logical graph $(V, L)$, the physical graph $(V, P)$, the minimum distance over the physical layer to connect each pair of nodes -computed in the *initialization phase*-, and an auxiliary function *isreal* which indicates whether a node is part of the network or is an artificial node, added to recreate some particular application or constraint. The output is an application between logical links and the subset of physical links used by their lightpaths.

The algorithm detailed in Algorithm-1 is the one depicted in Figure-5 and Figure-6. It is worth mentioning that instructions from (2) to (17) are repeated twice; along the first iteration only *real nodes* are taken, while in the second iteration all the remaining artificial nodes are processed too. The reason is that "the earlier a lightpath is routed the deeper its influence over the routes of the remaining lightpaths". By taking earlier the *real nodes* we are intending to shape the overlay network prioritizing its proper structure.

**Overlay routing (logical over physical). Algorithm 1**

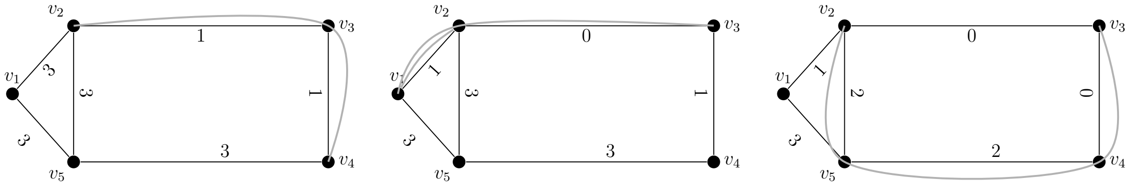
```
Input: $(V, L), (V, P)$, isreal : $V \to \{0, 1\}$, $d : V \times V \to \mathbb{R}_0^+$.
Output: $\Psi : L \to 2^P$.
1: Set $\Psi(e) = \emptyset, \forall e \in L$ and $pd : P \to \mathbb{R}_0^+, pd(e) = 1 \, \forall e \in P$.
2: while $\exists v \in V \,/\, (\text{isreal}(v)$ AND not-processed$(v))$ do
3:   Select randomly $v \in V \,/\, (\text{isreal}(v) = 1$ AND not-processed$(v))$;
4:   Set $prob(vw) = \frac{1}{d(v,w)}, \forall (vw) \in L \,/\, \Psi(vw) = \emptyset$;
5:   Normalize prob such that: $\sum_{e \in L} prob(e) = 1$;
6:   Create new control window;
7:   while $\exists w \in V \,/\, ((vw) \in L$ AND $\Psi(vw) = \emptyset)$ do
8:     Draw such $w \in V$ randomly weighted by $prob(vw)$;
9:     Find shortest-lp, a pd-distance shortest lightpath for $(vw)$ avoiding
       repeating physical links within this window;
10:    if (shortest-lp=$\emptyset$) AND (there are not unprocessed $(vw)$) then
11:      Update $\text{pd}(v, w) = \text{pd}(w, v) = (1 + \sum_{e \in L} |\Psi(e) \cap \{(vw)\}|)^p$;
12:      Create new control window;
13:    else
14:      $\Psi(v_e v_f) = \Psi(v_f v_e) = shortest\text{-}lp$;
15:    end if
16:  end while
17: end while
18: Repeat from (2) to (17), but using $NOT(\text{isreal}(v))$ instead of isreal$(v)$;
19: return $\Psi : L \to 2^P$.
```

The outcome of the *randomized feasible solution phase* is a candidate configuration for the route of each lightpath over the physical network. We did not make use of capacity and traffic information yet; and before going any further we must state that -as in the exact examples- in our practical applications we limited the capacities set to only one capacity ($|\hat{B}| = 1$). The main reason was that the telecommunications company we developed this application for, wanted the maximum possible bitrate for all the interfaces of its core network; but to be honest using more than one capacity increases the complexity significantly and this was an important motive too.

The next issue is determining whether the configuration found is feasible or not. The answer to this question is far from being easy, since this sub-problem is NP-Complete and it is in fact the base of our proof to Theorem-3.1. We have based on a heuristic to answer this question, which is in addition inspired in a very well known heuristic for NPP (choosing up sides in a ball game). The heuristic is the following: demands are taken in decreasing order of volume ($d_{pq}$) and each tunnel is routed over the logical layer following the minimal number of hops, but using only links with remaining capacity to allocate the new tunnel demand.

For instance, Figure-7 shows an example logical topology whose link capacities are 3. Let the demands be: $d_{24} = 2,\ d_{12} = 1,\ d_{13} = 1$ and $d_{23} = 1$. The path followed by every tunnel is sketched in Figure-7 using grey curves, so it is the remaining capacity in every link after routing each tunnel -two tunnels in the central image-.

**Figure 7** Routes for the tunnels (24), (12), (13), and (23) over a Logical Layer.



This constraint based routing algorithm is straightforward and it is based on Dijkstra's algorithm. Nevertheless an efficient implementation is quite complicated because of the following fact: to be sure a solution is feasible this algorithm must be repeated for each single failure scenario. In order to improve the efficiency: routes cache, optimized data structures and several others low-level programming techniques were used. This *isFeasible* function is used in both: *construction* and *local search* phases. The performance of this function is critic since it is used several times within the same iteration in the *local search*, as it is represented in Figure-4.

Up to this point and before entering the *local search phase*, we have a feasible configuration for the routes of every lightpath; but we are still using all of the initial logical links and this input network is very likely to be over-sized.

Moreover, in the *construction phase* we attempted to distribute the routes of the lightpaths uniformly over the physical layer, but it is still possible that many logical links fail simultaneously because of a single physical link failure. Therefore, it is very likely that many of these "redundant links" may be disposed of, if they are not really adding useful capacity.

It is worth mentioning that from this point on and until the next iteration, lightpaths costs are revealed because we have their lengths -from the configuration for their routes- and there is only one possible capacity.

Through the *local search phase* we intend to remove the most expensive and unnecessary logical links for the current configuration. The process is the following: logical links are taken in decreasing order of cost for their lightpaths, each one is removed and the feasibility of the solution is tested again. If the solution remains feasible the current logical link is permanently removed, otherwise it is reinserted and the sequence follows for the remaining logical links. Once this processes is finished the result is a minimal solution.

After $MaxIter$ iterations the best solution found is chosen to be the output of the algorithm. Since the construction procedure we have used in this work privileges the nodes drawn earlier to shape the routes of the lightpaths, we presume that adding path-relinkg to this algorithm could significantly improve the quality of the result, if the initial lightpaths routes of the elite solutions are prioritized to explore new solutions. We are planning to check this assumption in a future work. For further information in path-relinking enhancement to GRASP, please refer to: Resende and Ribeiro (2003) and F. Glover (2006).

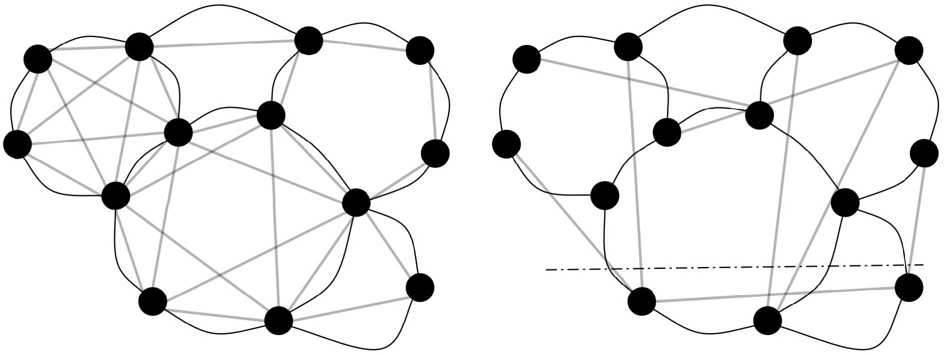
## 5 Application case context and results

The physical layer in our example was a planar graph. Due to the fact that most optical fiber networks are deployed over the Earth's surface we presume this is the usual situation. Since the physical graph is planar and because of Reinhard Diestel -P.3.1.1- (2010) it can be decomposed in *faces*: independent cycles but for their borders, with no interior vertices.

The quality of the solutions found with the metaheuristic presented in this work relies mainly in the *construction phase*, and this relies in turns in: the density of the logical layer and the value of the penalty parameter $p$ presented in Algorithm-1. Best combination of both parameters happened when $p = 1.5$ and $|L| \leq 4|P|$. Results were much better when most (65%-75%) of the logical links could be deployed within particular faces of the physical link.

The process to create the input data-set starts by adding all the artificial nodes and links of both layers necessary to model the aimed reality. After doing so, all the faces of the resulting physical graph are identified and logical links are added in order to fulfill with: Property-3.1, Property-3.2, and other theoretical properties or particular solutions like those shown in Section-3. To apply these properties demand considerations must be included. Moreover, it is convenient to consider also some noticeable critic physical failure scenarios. Because of the extension of this article some details were omitted.

**Figure 8** Criteria followed to construct input data for the logical layer.



The left sub-figure of Figure-8 sketches an example of this sort, were "black thin arcs" represent physical links while "grey thick lines" represent the logical ones. Those nodes with degree greater or equal to three allow routing traffic between faces, and even detouring it against a fault condition.

The quality of the solutions found with this kind of input data-set is quite satisfactory, so it is the behavior of the algorithm. Nevertheless, the overall quality may be improved by adding selected logical links "between faces". The second-half of Figure-8 shows some links of this sort.

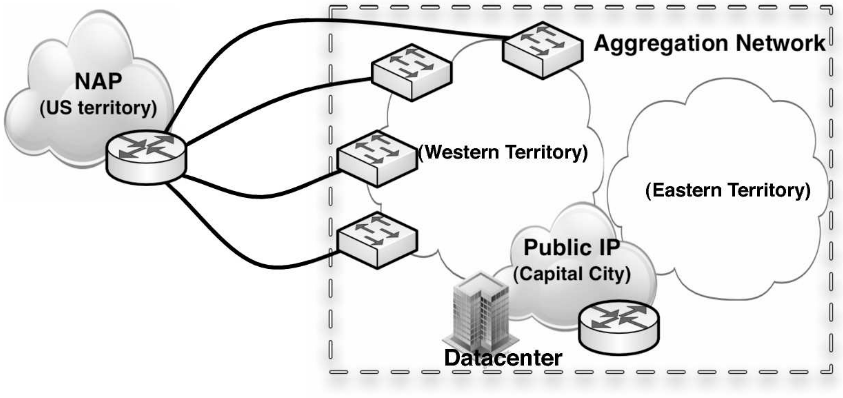
We will focus now in the context of the telecommunication company we applied this metaheuristic to, and prior to doing so we are giving some basic elements of the overall Internet architecture.

Internet is actually a network that could be disaggregated into several separate smaller networks also known as *Autonomous Systems* (AS). Typically every AS is a portion of the global Internet owned/governed by a particular Internet Service Provider (ISP).

Internet users access content residing in servers of: companies, universities, government sites or even from other residential customers (e.g. P2P applications). A portion of this content is located within the own network of the ISP this customer lease the service to -into some of the *Points Of Presence* (POP) of the ISP-, but most content is scattered over the Internet.

Since traffic interchange is necessary among different ISPs, the Internet architecture needs special POPs known as *Network Access Points* (NAPs). Within these NAPs: Carriers, ISPs and important content providers (e.g.: Google, Akamai) connect to each other in order to interchange traffic.

**Figure 9** Remarkable aspects of the particular network architecture.



This company had two different IP/MPLS networks referred to as: *aggregation network* and *public Internet network*. The *aggregation network* is geographically dispersed all over the country and it is responsible of gathering and delivering the traffic of the customers to the *public Internet network*.

The *public Internet network* is where the AS of this ISP is implemented; centralizes the international connections with other ISPs as well as those to Datacenters of local content providers. The *public Internet network* is geographically concentrated and only has POPs in the Capital City and in an important NAP of the US territory (see grey clouds in Figure-9).

In terms of the model covered in this article we may stand that the physical network has all of its nodes but one -the NAP- within the national boundaries.

Although end-to-end connected, the physical network comprises four subregions with a higher degree of connectivity: Capital City, Eastern and Western Territories and the NAP portion itself. Every path from east to west must go through Capital City and every path from Capital City to the NAP has to make its way through the Western Territory. There are four independent paths for international connections -leased to Carriers- between the NAP and the national boundaries.

The *aggregation* and *public Internet* networks are both logical. The *public Internet network* only has presence in a few POPs of the Capital City and in the NAP; and although the *aggregation network* has full-national presence it does not span the NAP.

Another remarkable aspect of this architecture is that whereas the *aggregation network* is deployed directly over the physical layer, there is an extra SDH layer between the *public Internet network* and the physical one.

More accurate information and details are protected by a Non-Disclosure Agreement (NDA) signed between the telecommunications company and our research institute. The costs and traffic information shown in the rest of this article are only referential.

Several planning concerns arose from the situation exposed:

- Is it convenient the current architecture? or It would be better to merge both IP/MPLS networks?
- Are profitable the IT infrastructure investments necessary to increase the percentage of local content?
- Which would be the optimal network to fulfill every demand requirements scenario?

We helped to answer these questions by identifying representative scenarios and creating their associated data-set to feed the metaheuristic. Although the overall performance of the algorithm described in Section-4 was very good -under the two hours of execution time in every scenario-, we have been very careful with the topology of the input logical graph to avoid an undesirable behavior.

We tried several scenarios based on the following considerations: traffic volume, network architecture and the percentage of locally terminated traffic. We selected eight remarkable scenarios to detail in Table-2.

According on traffic forecasts it is expected that some years from now the total volume of traffic be placed somewhere between 56 and 100 (reference values). If some IT investments and agreements were made it is expected that the percentage of locally terminated traffic (national traffic) could be greater. These new potential sources of traffic would be placed in the Capital City; specifically in the same POPs where the *public Internet network* is present.

**Table 2** Referential results for representative scenarios

| *scenario index* | *aggregated traffic demand* | *% local content* | *merged networks* | *number of nodes* | *required lightpaths* | *total cost* |
|---|---|---|---|---|---|---|
| 1 | 100 | Low | False | 56 | 81 | 10,000,000 |
| 2 | 100 | Low | True | 68 | 118 | 7,662,651 |
| 3 | 100 | High | False | 56 | 81 | 7,578,234 |
| 4 | 100 | High | True | 68 | 133 | 5,713,563 |
| 5 | 57 | Low | False | 56 | 75 | 6,319,470 |
| 6 | 57 | Low | True | 63 | 94 | 4,872,987 |
| 7 | 57 | High | False | 56 | 75 | 5,108,587 |
| 8 | 57 | High | True | 63 | 105 | 4,064,597 |

Those scenarios where *merged networks* is set to *False* inherit the current network architecture. Since the *public Internet network* only has a few nodes and its protection relies on the 1+1 protection mechanism of SDH, its optimal value can be estimated easily. The only portion where we needed computer assistance is that of the *aggregation network*. The columns *number of nodes* and *required lightpaths* refers exclusively to the values for this last network. On the other hand and in order to compare solutions fairly, the column *total cost* represents the combined cost of both networks -when they are not joined-.

It is worth observing that scenarios: 1 and 3, as well as 5 and 7 require the same number of lightpaths. Moreover, their solutions use exactly the same lightpaths. This result should be expected because in both pairs of scenarios share the same traffic and non-merged network architecture; since Datacenters -the only difference- are connected to the *public Internet network*, the *aggregation network* is unaware of the percentage of local content. The only changes are in the *total cost* because of the saving of international capacity, but the IT infrastructure has an important cost itself -not considerer in problem- to take in count.

Less intuitive are those savings arising exclusively from the merging of both networks like: 1 and 2, 3 and 4, an so-on. The reason is the following: "the routing search-space of the IP/MPLS technology is much greater than that of the SDH equivalent, so it is much more efficient".

Let us guess for a while that traffic does not need to be fitted in tunnels and instead can behave as a fluid. Since the length of international connections is measured in thousands of kilometers, this links are the most expensive of the physical network. As it was showed if Figure-9 there are four independent connections to the NAP, hence if we needed to guarantee 60Gbps of international traffic we could reserve 20Gbps in every one of these links, because a single failure could only affect one of them. Therefore the efficiency in the usage of international connections could raise to 75% if the efficiency of IP/MPLS would be available.

The protection mechanism of SDH (1+1) cannot exploit this degree of connectivity. To protect 60Gbps of traffic using SDH active/stand-by independent paths, we always need other 60Gbps of reserved capacity, so the efficiency of SDH it is limited to 50%. The improved efficiency of IP/MPLS to exploit the extra connectivity degree between local and international traffic explains by itself most of the savings.

We presume that the application this work dealt with is not an exception, and the potential savings might replicate from one ISP to the other.

The previous example and its later analysis justifies the convenience behind the update to existing models this work introduced.

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