The Temperature/Resistance Curve of NTC Thermistors

NTC thermistors are the most commonly used temperature-sensing elements in modern electronic circuits. Using these sensors in temperature-measuring instruments requires accurately modeling their temperature/resistance curves.

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The temperature-sensing element most often used in electronic circuits is the Negative-Temperature-Coefficient (NTC) thermistor. NTC thermistors are electrically simple and physically rugged, have relatively large temperature coefficients at normal ambient temperatures and are readily available in a range of resistances and packages. Their major drawback is nonlinearity. In order to produce a temperature-measuring instrument of even modest accuracy over a small dynamic range, designers must provide a means of correcting the nonlinearity of the thermistor's temperature/resistance characteristic.

Equation 1 is the simplest formula usually given for an NTC thermistor's resistance, assuming zero power dissipation:

$$R = A_l e^{B_l/T} \tag{1}$$

Parameters A_l and B_l depend on the thermistor's material and dimensions, as well as the operating conditions. T is the absolute temperature. The actual resistance $R_a(T)$ measured for a given thermistor at a given temperature will differ from the resistance calculated using Equation 1 or any other model. Equation 2 defines an error curve $D_r(T)$ that quantifies these errors in a way that allows comparison of results from a number of different thermistor types.

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$$D_r(T) = \frac{100\,[R(T) - R_a(T)]}{R_a(T)} \tag{2}$$

The maximum resistance error E_r, is the greatest absolute value of $D_r(T)$ when T varies in the selected range.

Tables 1 and 2 show empirical resistance versus temperature data published by Steinhart and Hart (SH)[1] and two thermistor manufacturers: Yellow Springs Instruments Co. (YSI) and Western Thermistor Co. (WT). Note that these tables show resistance at the measurement temperature scaled against the resistance measured at 25°C (i.e., $R_a(T)/R_a(25°)$ instead of $R_a(T)$. Table 1 shows data for an extended temperature range while Table 2 covers a much smaller temperature range. The calculations reported here used many more data points than Table 1 shows; data points were separated by 5°C. Table 2 shows all data

TABLE 1. Summary of data used for large temperature range.

	YSI									WT	
Thermistor Identification	44001A	44002A	44003A	44004	44006	44008	44011	44014	44015	2	3
Estimated Uncertainty (%)	0.7	0.3	0.1	0.2	0.1	0.1	0.1	0.1	0.1	0.2	1
Temp (Degrees C)											
−60	40.66	56.43	66.78	140.5	84.59					75.69	49.1
−40	13.74	17.33	19.64	33.65	23.98	29.49	33.56			22.05	16.1
−20	5.389	6.267	6.815	9.711	7.891	9.040	9.898			7.422	6.05
0	2.392	2.592	2.701	3.266	2.949	3.166	3.331	3.627	3.966	2.848	2.57
+20	1.177	1.195	1.206	1.250	1.226	1.243	1.255	1.277	1.299	1.218	1.19
+40	0.631	0.6046	0.5895	0.5329	0.5592	0.5383	0.5219	0.4980	0.4732	0.5727	0.604
+60	0.364	0.3303	0.3119	0.2488	0.2760	0.2533	0.2365	0.2123	0.1891	0.2914	0.328
+80	0.223	0.1930	0.1769	0.1255	0.1458	0.1281	0.1154	0.09783	0.08198	0.1592	0.189
+100	0.143	0.1193	0.1064	0.06785	0.08168	0.06897	0.06005	0.04826	0.03820	0.0923	0.115
+120				0.03894	0.04818	0.03920	0.03307	0.02531	0.01903	0.0565	
+150				0.01861	0.02370	0.01834	0.01481	0.01062	0.007447		

points used in narrow-range calculations.

This report compares the results of six models for NTC thermistor temperature/resistance characteristics with these empirical results. The six models are:

- Equation 1,
- Becker, Green and Pearson (BGP),[2]
- Bosson, Gutmann and Simmons (BGS),[3]
- Steinhart and Hart (SH),
- Cordella[4] and
- Lagrange polynomial expressions of Equation 1.

Temperature/ Resistance-Curve Models

Equation 1 has two parameters, A_1 and B_1, so that two data points, (R_1,T_1) and (R_2,T_2), from the empirical-data tables are enough to calculate these parameters' values. Equations 3a and 3b give the formulas needed to compute the parameters for Equation 1:

$$B_1 = \frac{ln\,(R_1/R_2)}{(1/T_1 - 1/T_2)} \tag{3a}$$

$$A_1 = R_1/e^{B_1/T_1} \tag{3b}$$

The error will be zero at temperatures T_1 and T_2. These are the zero-error points.

The BGP model includes a potential factor, T^n:

$$R = A_2 T^n e^{B_2/T} \tag{4}$$

Equations 5a, 5b and 5c are formulas for the three parameters A_2, B_2 and n in the BGP model. These three formulas derive from applying the model to three zero-error points:

$$n = \frac{\alpha_1 ln(R_2/R_3) - \beta_1 ln(R_1/R_3)}{\alpha_1 ln(T_2/T_3) - \beta_1 ln(T_1/T_3)} \tag{5a}$$

$$B_2 = \frac{ln[T_1/T_3)^{-n}\, R_1/R_3]}{1/T_1 - 1/T_3} \tag{5b}$$

$$A_2 = \frac{R_1}{(T_1{}^n)\, e^{B_2/T_1}} \tag{5c}$$

where

$$\alpha_1 = 1/T_1 - 1/T_3 \text{ and}$$
$$\beta_1 = 1/T_2 - 1/T_3 \tag{6}$$

Equation 7 gives the empirical model due to BGS:

$$R = A_3 e^{B_3/(T+\theta)} \tag{7}$$

Evaluating the three parameters A_3, B_3 and θ requires plugging

TABLE 2. Summary of data used for very small temperature range.

	SH			
Thermistor Identification	T3		T4	
Estimated Uncertainty %	0.03		0.02	
Temp°C	R/R (25)		Temp°C	R/R (25)
-0.0046	2.8447		-0.0070	2.6809
2.1856	2.5794		2.4845	2.4140
3.7905	2.4028		6.9282	2.0089
3.9970	2.3811		9.0130	1.8463
4.2675	2.3530		11.0004	1.7052
4.4715	2.3320		16.9169	1.3536
5.0174	2.2770		23.5792	1.0536
6.8358	2.1043		30.2011	.82912
7.0747	2.0827		34.9111	.70315

empirical results at three zero-error points into the model yielding Equations 8a, 8b and 8c:

$$\theta = \frac{-(T_2\beta_3 - T_1\alpha_3)}{\beta_3 - \alpha_3} \tag{8a}$$

$$B_3 = \frac{ln(R_1/R_3)}{1/(T_1 + \theta) - 1/(T_3 + \theta)} \tag{8b}$$

$$A_3 = \frac{R_1}{e^{B_3/(T_1+\theta)}} \tag{8c}$$

Thermistor-based instruments normally infer temperature from measurements of the thermistor's resistance.

where

$$\alpha_3 = (T_3 - T_2)ln(R_1/R_3) \text{ and}$$
$$\beta_3 = (T_3 - T_1)ln(R_2/R_3) \tag{9}$$

The SH model is another empirical three-parameter formula shown in Equation 10:

$$R = e^{\{(1/3)[(-F_1+F_2)^{1/3} - (F_1+F_2)^{1/3}]\}} \tag{10}$$

where

$$F_1 = \frac{(27/2)(A_4 - 1/T)}{C_4} \tag{11a}$$

$$F_2 = (3/2)\sqrt{3} \times \sqrt{27[(A_4 - 1/T)/C_4]^2 + 4(B_4/C_4)^3} \tag{11b}$$

Equations 12a, 12b and 12c give the parameters A_4, B_4 and C_4 from empirical results at three zero-error points:

$$C_4 = \frac{L_3(1/T_1 - 1/T_3) - L_4(1/T_2 - 1/T_3)}{L_3L_1 - L_4L_2} \tag{12a}$$

$$B_4 = \frac{(1/T_1 - 1/T_3) - C_4L_1}{L_4} \tag{12b}$$

$$A_4 = 1/T_1 - B_4 lnR_1 - C_4(ln\,R_1)^3 \tag{12c}$$

where

$$L_1 = (ln\,R_1)^3 - (ln\,R_3)^3 \text{ and}$$
$$L_2 = (ln\,R_2)^3 - (ln\,R_3)^3 \tag{13a}$$

$$L_3 = ln(R_2/R_3) \text{ and}$$
$$L_4 = ln(R_1/R_3) \tag{13b}$$

Steinhart and Hart originally proposed their equation in the form $T{=}T(R)$.

Recently, Cordella proposed the heuristic model in Equation 14:

$$R = R_3(T/T_3)^{[A_5 e^{(B_5 T/T_3)}]} \tag{14}$$

where the two parameters, A_5 and B_5 depend on two zero-error points, with R_3, T_3 being the third zero-error point:

$$B_5 = \left(\frac{T_3}{T_1 - T_2}\right) 1n \left[\frac{ln(R_1/R_3)\; ln(T_2/T_3)}{ln(R_2/R_3)\; ln(T_1/T_3)}\right] \tag{15a}$$

$$A_5 = \frac{ln(R_1/R_3)}{e^{[B_5(T_1/T_3)]} ln(T_1/T_3)} \tag{15b}$$

Other approximations may be obtained by using the Lagrange polynomial expressions from Equation 1. Developing the actual function $lnR = f(1/T)$ as a second degree polynomial produces Equation 16:

$$R = e^{[A_6(1/T)^2 + B_6(1/T) + C_6]} \tag{16}$$

Evaluating the three coefficients,

TABLE 3. Extended temperature range comparation of E_r values.

Approx. Equation	(1)	(2)	(3)	(4)	(5)	(6)
Material						
44001A	3.4	0.7	0.5	0.3	1.1	0.5
44002A	4	0.8	0.4	0.2	1.0	0.4
44003A	5	0.9	0.5	0.2	0.9	0.4
44004	10	1.8	0.8	0.2	1.7	0.6
44006	13	1.8	0.6	0.1	1.1	0.1
44008	10	1.1	0.3	0.1	0.8	0.0
44011	12	1.4	0.5	0.2	0.6	0.1
44014	7	0.7	0.3	0.1	0.2	0.1
44015	8	0.8	0.4	0.3	0.1	0.2
2	10	1.8	1.0	0.6	0.5	0.9
3	8	1	0	1	1	0
T3	0.028	0.002	0.002	0.002	0.002	0.002
S4	0.30	0.02	0.02	0.02	0.02	0.02

A_6, B_6 and C_6 requires three zero-error points:

$$A_6 = \alpha_6 + \beta_6 + \gamma_6 \tag{17a}$$

$$B_6 = -(1/T_2 + 1/T_3)\alpha_6 - (1/T_1 + 1/T_3)\beta_6 - (1/T_1 + 1/T_2)\gamma_6 \tag{17b}$$

$$C_6 = \frac{\alpha_6}{T_2T_3} + \frac{\beta_6}{T_1T_3} + \frac{\gamma_6}{T_1T_2} \tag{17c}$$

where

$$\alpha_6 = \frac{\ln R_1}{(1/T_1 - 1/T_2)(1/T_1 - 1/T_3)} \tag{18a}$$

$$\beta_6 = \frac{\ln R_2}{(1/T_2 - 1/T_1)(1/T_2 - 1/T_3)} \tag{18b}$$

$$\gamma_6 = \frac{\ln R_3}{(1/T_3 - 1/T_1)(1/T_3 - 1/T_2)} \tag{18c}$$

Evaluating the Models

Tables 3 and 4 show the maximum resistance error E_r, expressed as percentages. Table 3 uses all the temperature values for all the materials shown in Tables 1 and 2. Table 4 covers a more limited temperature range extending between −10°C and 100°C. Calculations for materials that have a limited intrinsic temperature range covered only those limited ranges.

These results show that the exponential approximation (Equation 1) makes errors that are approximately ten times greater than errors made by the other models. Over the small temperature spread that Table 4 covers, all three parameter approximations yield similar errors. In the very small ranges covered by T_3 and S_4 materials, the errors produced by all three parameter models are equal to within the estimated uncertainty. Only over large temperature ranges do significant differences emerge. The SH and Lagrange polynomial models prove best for most of the thermistor materials analyzed, with errors being on the order of the estimated uncertainty. The differences between resistance errors in calculations using the various three-parameter approximations are not great.

Thermistor-based instruments normally infer temperature from measurements of the thermistor's resistance, implying the need to view resistance as the independent variable and evaluate the *temperature*-measurement errors

TABLE 4. Limited temperature range comparation of E_r values.

Approx. Equation	(1)	(2)	(3)	(4)	(5)	(6)
Material						
44001A	0.8	0.3	0.4	0.4	0.5	0.3
44002A	0.9	0.2	0.2	0.2	0.2	0.2
44003A	1.3	0.3	0.2	0.1	0.2	0.2
44004	2.4	0.2	0.2	0.1	0.3	0.2
44006	3.3	0.2	0.1	0.1	0.2	0.1
44008	3.6	0.2	0.1	0.0	0.2	0.0
44011	4.2	0.3	0.1	0.1	0.2	0.1
44014	4.0	0.2	0.1	0.0	0.1	0.0
44015	4.8	0.3	0.1	0.1	0.0	0.0
2	2.2	0.3	0.2	0.1	0.2	0.2
3	2	0	0	0	0	0

induced by relying on the different models to convert resistance measurements to temperature readings. Equation 19 defines the error curve as a function of resistance, rather than of temperature:

$$D_t(R) = \frac{100[T(R) - T_a(R)]}{T_a(R)} \qquad (19)$$

Where $T(R)$ is the inverse function of $R(T)$, and $T_a(R)$ is the inverse table of the $R_a(T)$ table (i.e., Tables 1 and 2). The maximum temperature errors that arise from using the six approximations presented here are between 10 and 20 times smaller than the corresponding resistance errors in Tables 3 and 4.

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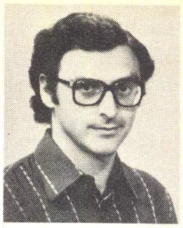

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