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Tesis

Technical efficiency of dairy farms in Uruguay: a stochastic production
frontier analysis

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Abstract

The productivity of Uruguayan dairy farms has been consistently growing for the last 40 years. This process has implied the adoption of new technologies which have had significant effects on the production system. The efficiency with which available technologies are used influence output growth. Hence, assuring and enhancing dairy farms' productivity and efficiency represent an important challenge to improve the competitiveness of the sector and achieve sustained economic growth. The overall objective of this study is to analyze the efficiency performance of dairy farms in Uruguay. Using a cross-sectional database, this study estimates a Cobb-Douglas stochastic production frontier and technical inefficiency model for dairy farms to determine the effect of each input on the production frontier and the principal factors that explain differences in farm efficiency. Results show that the number of milking cows has the highest effect on production, followed by the total consumption of feed, including concentrated feed, hay and silage. Although veterinary, agronomic and accounting assistance matter, the major determinants of efficiency differences are farmers' specialization in dairy farming and the usage of artificial insemination. Overall, farm profiles indicate that those in the high efficiency group achieve a higher level of milk production than those less efficient; and they produce under a more intensive production system than farmers in low efficiency groups.

Key words: stochastic production frontier, Uruguayan dairy farms, technical efficiency, cross-sectional data.

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1. Introduction

Over the past four decades, the Uruguayan dairy sector has exhibited remarkable technological development. This process of technology adoption has implied significant changes in dairy farming's production system. The pastoral extensive model of production based on natural conditions has evolved into intensive farming based on cultivated pastures and a higher supply of better quality feed. Productivity gains are a result of a more intensified farming system which has led to the sustained growth of both total milk production and milk sold to processing industries.

Globalization and the high competitiveness of milk production have driven dairy farmers not only to produce more milk but also to increase their efficiency and productivity to avoid being displaced from international markets. The strong international competition reveals the importance of improving productivity by adopting new technology and making the best use of current practices, as a mechanism to build competitiveness.

The international dairy market is one of the most protected and subsidized worldwide, and it is dominated by a few exporting countries. Although dairy farming in Uruguay has a comparative advantage, meaning a lower cost of production compared to other countries, the whole sector must deal with important challenges to be competitive in the international dairy market (Chaddad, 2009). Therefore, milk production growth obtained by an increase in productivity seems to be the key to remaining competitive in international markets.

Productivity growth can be defined by three components: technical efficiency (TE) change, technological change and scale or size efficiency change (Coelli et al., 2005). The most studied component of productivity is TE because it provides valuable information to policy formulation and farm decisions that are focused on the improvements of farm performance (Bravo-Ureta et al., 2008). When productivity performance studies consider developing countries, the analysis of TE is particularly relevant (Nishimizu and Page, 1982). In these countries, there are more opportunities to improve managerial practice, learn by doing and spread of new technologies, compared with developed economies.

Studying farms efficiency and the potential sources of inefficiency are important factors from a practical and a policy point of view (Solís et al., 2009). The focus of the present study is on the efficiency of Uruguayan dairy farms. The overall objective is to contribute to the understanding of dairy farming efficiency performance. Achieving a higher level of knowledge about the determinants of the farmer's TE allows us to better understand the relationship between the resources used in milk production and the obtained output. In this sense, we explain efficiency differences across farms and determine the potential for dairy farms to increase productivity under current production technology. This study contributes to the dairy farming efficiency and productivity literature available in Uruguay because it uses a Stochastic Frontier Analysis (SFA) methodology for cross-sectional data for the first time.

Frontier production functions have been widely applied in the analysis of TE measurement among farmers in developed and developing countries. Two principal approaches have been developed for efficiency measurement: mathematical programming (nonparametric), commonly known as Data Envelopment Analysis (DEA), and econometrics models (parametric) such as Stochastic Frontier Analysis (SFA). Both methods estimate the production frontier, which represents the best practice for a specific sample of farmers. According to Coelli and Battese (1996) SFA has been the most adopted methodology in measuring farm efficiency performance in studies related to the agricultural sector because of its capacity to deal with stochastic noise.

We implemented a SFA model to estimate the determinants of TE among dairy farms. The data used for empirical estimation is a cross-sectional database that is derived from a survey conducted by the National Institute of Milk (INALE) in 2014. The sample includes 276 dairy farms located in 8 departments of Uruguay. They represent 90% of the total production of milk and are highly specialized with most of their output coming from dairy. The collected data corresponds to the 2013/14 agricultural year.

The structure of the paper is as follows: in section 2 we describe the dairy sector and principal changes in its productive structure. Section 3 presents a literature review. Section 4 defines TE, and presents stochastic production frontier methodology. In section 5 we describe the data and empirical model. Section 6 contains a profile of

Uruguayan dairy farms. Empirical results are presented in section 7. Finally, section 8 presents the conclusions.

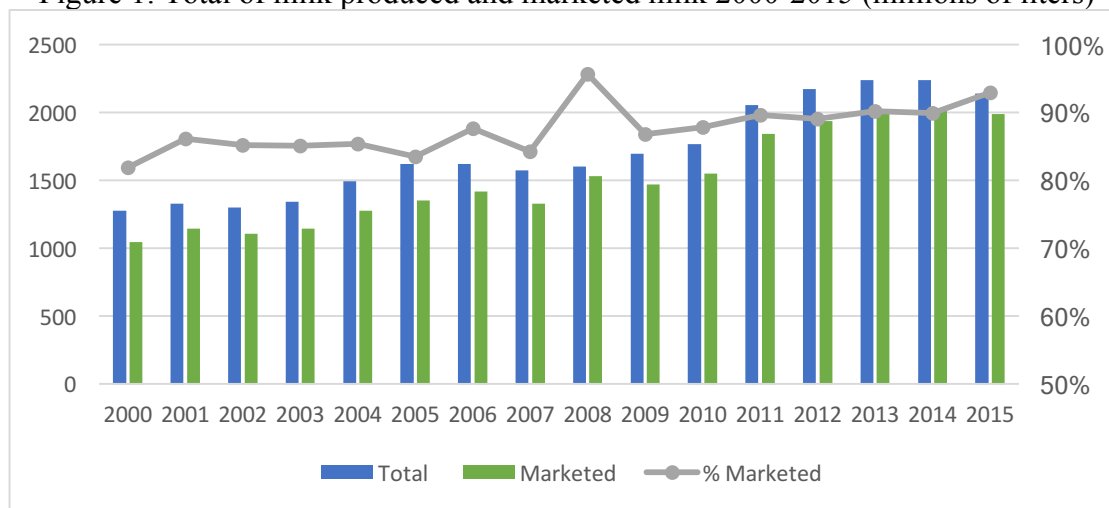
2. Dairy sector background

The dairy sector occupies an important role in Uruguay's economy. Milk production accounted for 9.7% of agricultural gross product in 2015. It is the third most important agricultural product after meat and soybeans which accounted for 30% and 16%, respectively. The total milk production in 2015 was 2,141 million liters, and 93% of it was processed. There are 3,919 dairy farms, with 73.4% of them selling produced milk to the processing industry. The total land used for dairy farming is 771 thousands hectares (DIEA, 2016).

Most of the milk production is oriented to exports (70%). Dairy exports reached 8.1% of the total of Uruguayan goods exported in 2015. Considering only agro-industrial products, dairy accounts for 11%, ranking fourth after exports of crops, meat and forest products (DIEA, 2016). Furthermore, dairy farming is an important employment source. According to the Agricultural Census 2011, the number of permanent workers in dairy farming accounted for 12.8% of the total permanent workers in the agricultural sector. Dairy farming is the second sector in labor hiring after livestock production, which hires 48.4% out of the total permanent workers.

Since the mid-70's the dairy sector has shown continuous growth. Its productive structure has experienced important changes generating a remarkable dynamism in the sector. This dynamism is reflected in the higher volume of produced and marketed milk. In 1976, milk production was 742 million liters, and 47% of it was marketed (347 million liters) (Hernández, 2002). Towards the year 2000, milk production had increased to 72%, reaching 1,278 million liters. Also, marketed milk increased more than three times representing 82% out of the total produced milk (1,047 million liters). As Figure 1 shows, this growing trend of milk production and marketed milk has continued during recent years, with an average annual growth rate of 3.6% and 4.5%, respectively. The amount of marketed milk almost doubled between 2000 and 2015, representing 93% of the produced milk.

Figure 1: Total of milk produced and marketed milk 2000-2015 (millions of liters)



Source: Own elaboration based on DIEA data

The extensive pastoral production system, based on natural grasslands, has evolved into an intensive farming system based on cultivated pastures and the use of feedstuffs. Durán (2004) defines four technological stages or models to explain the technological pathway of the dairy production system. Table 1 shows the most important characteristics of these stages, which have occurred gradually, indicating that the level of adoption of new technologies has been different among farmers. This process of change and transformation has implied a continuous increase in the process of technology adoption that has enabled a more intensive production system.

Table 1: Characteristics of technological models

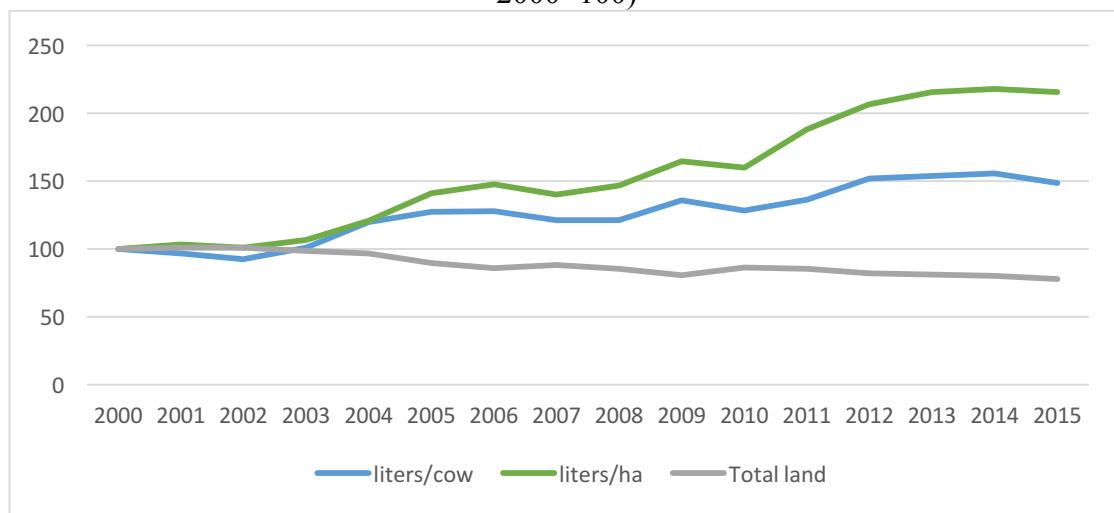
Models	Improved	Organized	Controlled	Advanced
Crop Rotation	No	Yes	Yes	Yes
Pasture (%)	40-50	60	60	60
Dry Matter/ha	Medium	High	Maximum	Maximum
Hay	High	Low	Very low	Low
Silage	Low	Medium	High	Very high
Concentrates kg/cow	500	500	1200	1600
kg/ha	250	250	1200	1712
Stocking rate Milking cow/ha	0.5	0.7	1	1.07
Milk production liters/milking cow	3800	4500	4700	6100
liters/ha	2000	3100	4700	6500

Source: Durán (2004)

The main differences among these models are the usage of cultivated pasture instead of natural rangeland, and a higher supply of concentrated feed and silage. These changes in the diet of cows have led to improvements in dairy cows' performance, and the number of dairy cows per hectare has increased. As Table 1 shows, partial productivity (liters of milk per milking cow and per total land) has increased, and this productivity gain is a result of a more intensified farming system. According to DIEA (1999), the productivity per milking cow was 1,715 lts in 1985. Hence, the annual growth rate between 1985 and 2015 was 3.4%. Moreover, during the period 2000-2015, farmers achieved productivity improvements (Figure 2). However, total used land for dairy farms decreased which shows that land was not a relevant factor in explaining milk production's growth.

Consequently, farmers have achieved better performance by the intensification of the dairy production system. New technologies and more intensive resource usage explain milk production growth. On the other hand, the adoption of new technology implied lower milk production costs which led to an increase in the dairy sector's international competitiveness.

Figure 2: Evolution of productivity and total land used for dairy farming (Index 2000=100)

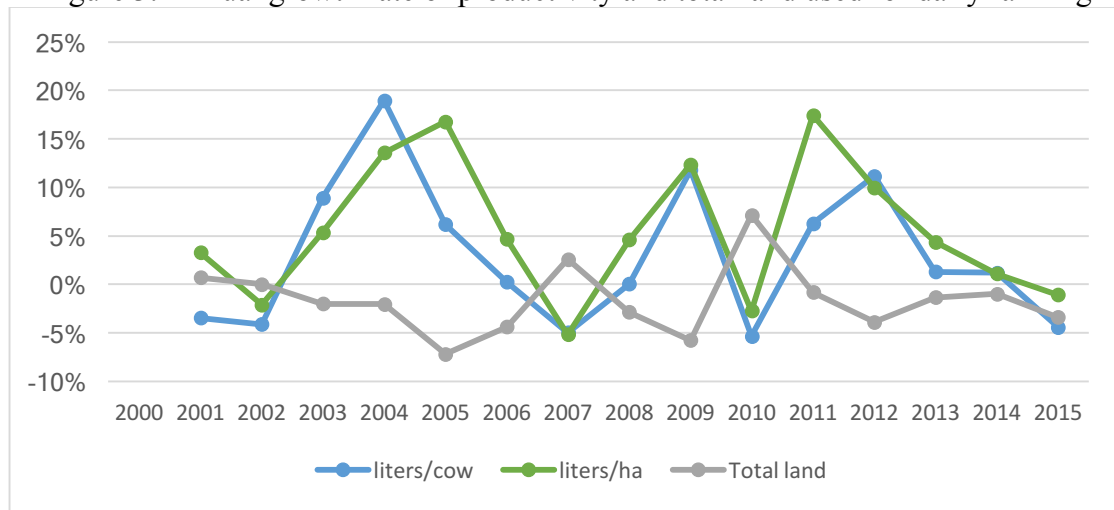


Source: Own elaboration based on DIEA data

As Figure 3 shows, productivity measures exhibit similar behavior with significant oscillations. The average annual rate of productivity, expressed as liters per milking cow and per total land, was 3% and 5%, respectively during 2000-2015. Considering total land, the average annual rate was negative (-2%) which reflects the decrease in total land used for dairy farming. Nevertheless, the annual growth rate for these

variables shows a considerable variability during the considered period (Figure 3). The sharpest falls are explained by climate conditions and price variations, which were particularly significant during the first decade of the 21st century. Because of the intensive usage of feed (concentrates, silage, etc.), dairy farming is highly dependent on climate conditions, and inputs and output prices. Here we can note a limitation of productivity per milking cow and per total land as a measure of productivity. As they are a partial measure of productivity they do not take into account milk production as an interaction of many factors that operates in the production process. However, efficiency measures overcome this weakness present in partial productivity measures.

Figure 3: Annual growth rate of productivity and total land used for dairy farming

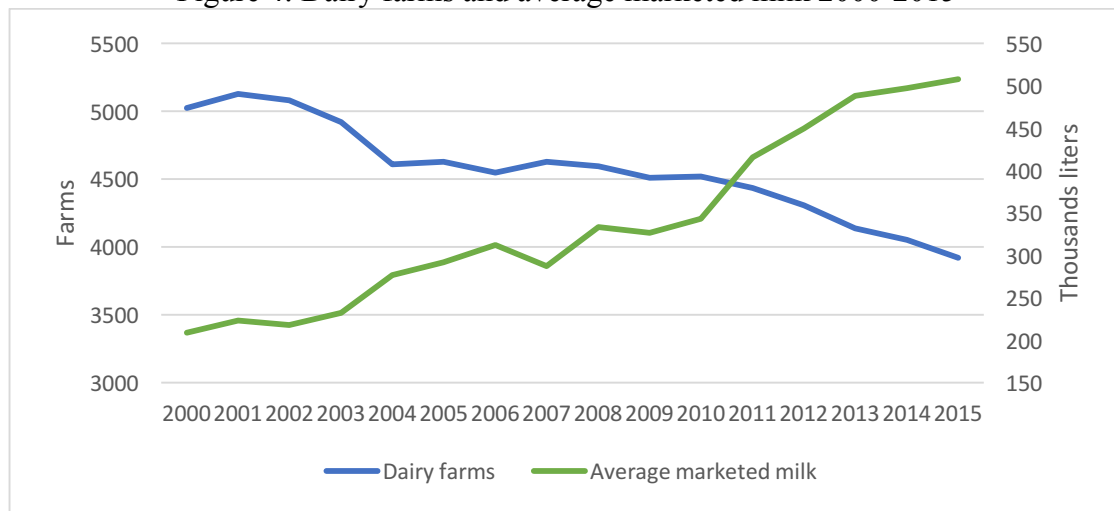


Source: Own elaboration based on DIEA data

Although technological improvements adopted by dairy farms have been key to overcoming the sector's own difficulties and those that have been mainly imposed by the international market requirements, they have had some adverse effects on farmers. For many farmers the intensification of the dairy farm production process involved an increase in the scale of production (Hernández, 2011). Furthermore, the intensive production system requires higher levels of investment, and small or family farms face more restrictions in accessing resources like financial capital or technology. Because of that, the technology adoption process has been asymmetric among farmers and it has led to the exit of family farms (Hernandez, 2002). Mondelli et al. (2013) found that the size of a farm appeared as a restriction to adopting new technologies, particularly the adoption of new production methods, new inputs combinations or significant changes in organizational structure.

According to the Agricultural Census 1960, there were around 9,500 dairy farms while in the latest Census carried out in 2011 the total number of farms was 4,474. This downward trend continued in the following years (Figure 4). In the period considered, the number of farms decreased 22% being around 3919 in 2015. However, the average marketed milk follows a growing trend in the same period showing the sustained growth of milk production.

Figure 4: Dairy farms and average marketed milk 2000-2015



Source: Own elaboration based on DIEA data

The decrease in the number of dairy farms was more significant among smaller farms. During the years 1996 and 2000, the number of small farms (less than 50ha) decreased 29%, while the number of large farms increased 50%. Although some small or family farmers could not face the higher investment requirement that intensive dairy farming imposed, the coexistence of heterogeneous farmers was maintained. Most of the farms were medium (50-500ha) and they represented 64% out of the total dairy farms in 2015, while small and large farms represented 29% and 7%, respectively (Figure 4).

Figure 5: Dairy farms by size group 2000-2015



Source: Own elaboration based on DIEA data

Hernández (2002) shows a concentration of dairy farming which means fewer but larger dairy farms (in terms of milk produced) and more productive. This pattern is also observed through the period 2000-2015 (Table 2). The number of dairy farms decreased while milk production increased at 3.7% average annual rate. This implied an increase in average farm size measured as liters of milk produced (from 707 to 1518 liters per farm per day).

Table 2: Evolution of dairy farming variable 2000-2015

Year	Dairy farms ¹	Mean land (ha)	Herd size ¹	Liters/farm /day	Liters/cow	Liters/ha
2000	5.0	198	720	707	3,195	1,287
2001	5.1	195	760	720	3,084	1,329
2002	5.1	197	763	711	2,957	1,301
2003	4.9	199	734	758	3,221	1,370
2004	4.6	208	708	901	3,831	1,556
2005	4.6	193	724	972	4,068	1,817
2006	4.5	187	728	990	4,078	1,901
2007	4.6	189	743	947	3,875	1,803
2008	4.6	185	744	968	3,877	1,886
2009	4.5	178	710	1045	4,334	2,119
2010	4.5	190	764	1086	4,102	2,061
2011	4.4	192	793	1289	4,359	2,420
2012	4.3	190	755	1403	4,846	2,661
2013	4.1	195	803	1503	4,908	2,777
2014	4.1	197	778	1535	4,967	2,807
2015	3.9	197	783	1518	4,747	2,777

Source: Own elaboration based on DIEA data; (1) In thousands

As we can see in Table 2, neither the average land per farm nor the herd size presented significant variation during the period considered. The average annual growth rate was 0.03% and 0.64%, respectively. Therefore, improvements in productivity (per dairy cow and hectare) explain the important milk production growth, and shows the changes in the dairy farming productive structure described above.

Although, Uruguayan dairy farming has improved its performance, the entire dairy sector faces important challenges in order to remain competitive in the dairy international market. It is one of the most protected and subsidized markets, and it is dominated by a few large scale firms which use modern technology (Chaddad, 2009). In this context, productivity growth seems to be an important mechanism to improve competitiveness. Both technological innovations and the efficiency with which available technologies are used influence output growth (Nishimizu and Page, 1982). Therefore, in developing countries such as Uruguay, where the technology adoption process might face more restrictions, the study of TE can help to generate valuable information about the ability of farmers to obtain the maximum output given a set of inputs and

technology. There exist opportunities to improve dairy farm performance by using resources more efficiently under the current production technology.

3. Literature review

Frontier production functions have been widely applied in the analysis of TE measurement among farmers in developed and developing countries. Battese (1992) presented a survey of empirical applications with estimates of frontier production functions to obtain a measurement of TE. He classified the studies depending on the type of frontier production function estimated: deterministic or stochastic frontiers. The frontier production function methodology seems quite significant to study inefficiency and its determinants.

Bravo-Ureta and Pinheiro (1993) reviewed the frontier production function literature dealing with farm level efficiency in developing countries, and the study shows that considerable effort has been made to measure efficiency using a wide range of frontier models. More recently, Bravo-Ureta et al. (2007) present a meta-regression analysis including farm level TE studies of developing and developed countries. The principal goal of this study was to evaluate the effect of methodological and other study-specific attributes (estimation technique, functional form, sample size) on TE estimates. They found mixed results and conflicting views regarding the merits of different approaches that have been developed to obtain a TE measurement in agriculture.

Another relevant contribution to the existing literature was done by Moreira and Bravo-Ureta (2009). They also examined the impact of study-specific attributes on TE estimates, using a meta-regression analysis focused on dairy efficiency studies. The authors also concluded that the effect of different methodological alternatives on TE estimation was diverse.

Although the dairy sector plays an important role in the Uruguayan economy, TE analysis has not been the focus of studies. There are two studies that have looked at Uruguayan dairy farm efficiency performance: Vaillant (1990) and Grau et al. (1995). In the first study, the author identified the opportunities and limitations of increasing milk production based on improving dairy farmers' productivity. Vaillant (1990) estimated a deterministic production function using a cross sectional sample including

331 Uruguayan dairy farms for 1987. The sample was divided into two farm size groups based on total milk marketed to processing industries. According to the empirical results, larger farms presented higher levels of efficiency compared with smaller farms. It was found that technological practices were heterogeneous among farmers. This was because there existed significant technological changes and the process of incorporation of technologies was different among farms.

Grau et al. (1995) estimated a parametric stochastic production function considering different farm sizes (measured considering hectares of land) using a panel data set which included information about 479 dairy farms that belong to CREA¹ groups. Dairy farms in this group presented higher technology and productivity levels, and higher scale than the rest of the farmers in Uruguay. They found a high level of TE (90.13%) among farms concluding that there was little scope in productivity gains by improving the use of inputs and available technology. Grau et al. (1995) stated that it seems necessary a shift in technological frontier that allows higher levels of production. However, TE is heterogeneous when farmers are individually considered. TE level for small and large farms are 89.31% and 90.94%, respectively, and farm size and TE are not correlated. However, the authors found a positive and significant association between efficiency and milk production, which implies that farms with higher levels of milk production are more efficient. A positive correlation between efficiency and grain feed use was also found.

Bravo-Ureta et al. (2008) applied stochastic production frontier analysis using unbalanced panel data sets for dairy farms from Argentina, Chile and Uruguay. Three SFA models were estimated, one for each country, using a Translog specification. In each case, the same four explanatory variables were used to explain the dependent variable, defined as annual output per farm: average number of cows; labor, measured in equivalent workers; purchased feed and veterinary inputs costs. The frontiers were used to evaluate economies of scale, rate of technological change, and TE. As a result, authors found that TE presented mean values of 87%, 84.9% and 81.1% for Argentina, Chile and Uruguay, respectively. This result means that farmers from the three countries could increase their milk production while maintaining the usage level of inputs. It

¹ Regional Agricultural Experimentation centers.

seems important to note some differences between this study and ours. First, we used updated data to try to capture the increasing technological changes. In addition, the sample used in the present study represents 90% of the total milk production in Uruguay.

Cabrera et al. (2010) and Al-sharafat (2013) are two relevant studies in the sense that they estimated TE using a stochastic production frontier based on a cross-sectional sample of dairy farms. Cabrera et al. (2010) analyzed the effects of practices commonly used by dairy farmers and the effect of intensification on the performance of the farms. A sample of 273 farmers in Wisconsin was used to estimate the stochastic frontier and the technical inefficiency model. The empirical results showed that the average level of TE in the sample was 88%, indicating that farmers could expand milk production using the inputs and technology available. The variable with the highest effect on production was the number of cows followed by the total expenditure on crops, feeding, livestock, and labor. A proportional relationship between the size of the farm and the level of TE was not found, which suggests that improvements in technology and efficiency explain the level of productivity, not the size of the farm.

Al-sharafat (2013) implemented a stochastic production frontier methodology to obtain the level of TE of dairy farms in Jordan. He found a low TE level for most of the dairy farms, and the mean TE in the sample of 100 farms was 39.5%. Herd size, feed intake, labor and veterinary services costs had a positive and significant effect on milk production. However, herd size was the most important explanatory variable. The main determinants of TE which had a positive effect were: the farmer's level of education, the farmer's farming experience, the farmer's contact with extension services and herd size. The author concluded that more education is key to achieving a better efficiency performance among dairy farmers in Jordan.

Finally, Mbagha et al. (2003) estimated the TE level of dairy farms in Québec using a cross-section of 1,143 farms. They measured and assessed the robustness of TE choosing different functional forms for the frontier and different distribution assumptions for the inefficiency term. The stochastic frontier analysis approach was used, and for comparison purposes data envelopment analysis (DEA) was also implemented to estimate TE. High levels of efficiency were obtained for dairy farms in Québec. In addition, results show that the differences in the mean levels of efficiency

were statistically significant across functional forms and inefficiency term's distributions. However, the differences in mean TE magnitudes was not very large.

4. Overview of methodology

The concept of efficiency is at the center of economic theory and efficiency measurement is an important field of applied economics that researchers and policy-makers have focused on. They have been interested in explaining how a given firm can be expected to increase its output by increasing its efficiency without using more inputs (Farrell, 1957). Measuring efficiency is important in order to save resources, which have implications for both policy formulation and firm management (Bravo-Ureta and Rieger, 1991).

The terms productivity and efficiency have been commonly used in the same way but they represent different ideas. The concept of the productivity of a firm refers to the ratio of the output that it produces to the input that it uses. This is a straightforward calculation when the production process involves a single input and output. However, the most common production process implies more than one input and/or output. In this case, methods that aggregate these inputs and outputs into a single index must be used to obtain a productivity measure which is known as total factor productivity.

As the literature states, productivity growth can be defined by three components: technical efficiency (TE) change, technological change and scale or size efficiency change (Coelli et al. 2005). TE change is the relative measure of managerial ability given technology, and scale efficiency change refers to changes in unit costs associated with the growth in the size of the firm. When productivity comparison through time is considered, technology change leads to improvements in productivity that arise from the adoption of new production techniques.

Nishimizu and Page (1982) proposed a methodology that divides total factor productivity change into technological change and TE change. The authors also noted that the distinction between technological change and changes in technical efficiency are very important when we study the productivity performance of developing economies. That is because the productivity gain in these economies due to improvements in technical efficiency seems to be highly relevant. In addition, there are

more opportunities to improve managerial practices, learning by doing and spreading of new technological knowledge, compared with developed economies.

Efficiency is a measure of comparing current performance with the best practice, and the best practice is defined here by the production function. The frontier production function methodology was first introduced by Farrell (1957) in order to measure efficiency. It consists of estimating the frontier production function to obtain the maximum level of output attainable for a firm, and comparing it with the current performance. He presented a relative efficiency measure expressed as the observed deviation from the best performance obtained for a specific group of firms which is given by the production function. As Farrell stated, relative measure means that TE is defined in relation to a given set of firms and considering specific factors, and any modification to these sets will affect the measure.

It is necessary to have an appropriate model of the ideal performance, the production frontier, to compare it with the current performance of a firm. Bogetoft and Otto (2010) define a rational ideal evaluation when all the information is available. This means that the preferences (attain the maximum level of product) and possibilities (given by the production frontier) are specified for the firm. However, in real evaluations the information about preferences and possibilities is not complete which led us to estimate the ideal or optimal relationship between inputs and outputs to know how well the firm is doing.

To obtain an efficiency measurement, Farrell (1957) proposed that the efficiency of a firm can be defined for two components, TE and allocative efficiency (AE). TE refers to the ability of a firm to obtain the maximum output from a given set of inputs, while AE refers to the ability to use the inputs in optimal proportions to produce at least the cost, given the input prices. The product of these two measures provides a measure of total economic efficiency.

Microeconomic theory has mostly focused on AE to the exclusion of other types of efficiencies, whose improvement is an important aspect of the process of growth (Leibenstein, 1966). However, it has become important to know how a given firm can increase its output through appropriate reorganization and increasing its efficiency, without using more resources. Moreover, if the only information available is input and

output quantities, and there is no information about their prices, then it is only possible to measure TE.

Assuming a simple example where there is one input (x) to produce one output (y), the concepts of productivity and TE can be represented as Figure 5 shows. The production function $F(x)$ describes the shape of the current production technology. It is defined in terms of the maximum output attainable by a given set of inputs and technology available to the firms, or the minimum input usage required to produce any given output. It reflects the best technology available due to the fact that it is estimated considering the firms with the best performance. Hence, it represents a standard that can be used to measure the technical efficiency of production (Kumbhakar and Lovell, 2000). Because of this, the frontier production function methodology is accepted and widely applied.

A firm which is at point A is fully technically efficient because it uses (x_0) units of input to produce (y^*). The level of production is on the frontier meaning that it is the maximal of output attainable given the input and technology. However, if the firm operates at point B, below the frontier, it is technically inefficient because it is using the same level of input (x_0) but it produces only (y).

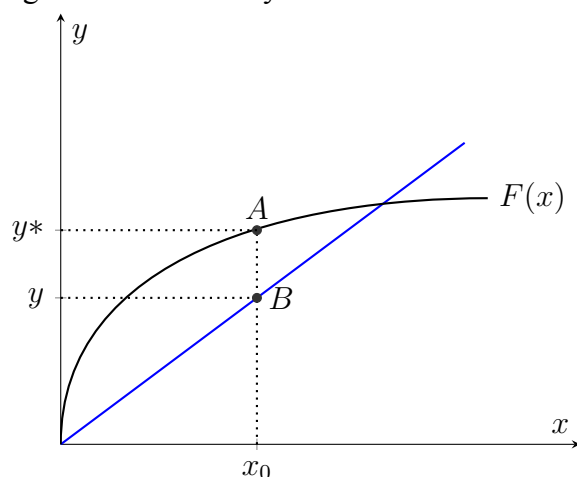
From an output-oriented approach, which assumes a proportional increase of the output quantity while the input quantity is held constant, TE is measured by comparing the observed output with the maximum output that a firm is capable of producing with the technology available and using the same quantity of input.

$$TE = \frac{y}{y^*} \Leftrightarrow y = TE \cdot y^*, 0 \leq TE \leq 1$$

where y is the observed output quantity, and y^* is the maximum output quantity that can be produced with the input quantity x_0 .

On the other hand, productivity of the firm operating at point B is measured by the slope of the ray passing B through the origin: $0x_0/OB$. While TE is a concept relative to the best performance defined by the production function, productivity is an absolute concept. It requires the understanding of growth but it does not require knowing the production technology available.

Figure 5: Productivity and Technical efficiency



Source: Own elaboration based on Battese (1992)

In production theory and under the assumption of rational behavior, we would expect that all the firms operate on the frontier where the production is the maximum attainable. It is implicitly assumed that maximizing behavior is the correct assumption for the agent who makes a decision (Leibenstein, 1977). In this case, firms are making decisions to maximize their profits. Nevertheless, in practice, firms are hardly fully productive efficient. The deviation from the frontier can be explained in terms of TE if we have information about input and output quantities. As Battese (1992) mentioned, the existence of TE of firms engaged in production has been a deliberated topic in economics.

The question that arises is why some firms operate on the frontier while others lie away from it. In others words, why some firms have higher levels of inefficiency than others. Muller (1974) presented several answers for that. First, firms could use different technologies, but since the definition of TE assumes that firms are producing under the same technology, there is no space for this explanation. A second reason states that different levels of inefficiency could be the consequence of random disturbances, factors that are not under the firm's control. This assumption is commonly present when a production function is estimated. Finally, firms that have the same production technology could have different levels of inefficiency because some of them are more successful than others at using it and at combining the inputs to produce.

The ability of a firm to obtain the maximum output from given inputs, or its TE, could be influenced by the information and knowledge that the firm has. A rational firm

makes the best decisions to maximize its profits according to the information available. However, this information is not complete for the firm, and it has a cost that must be incorporated in the production function. Therefore, firms could have different levels of information that impact their ability to obtain the maximum output, and in these cases firms do not operate on the frontier. Leibenstein (1966) concluded that firms could not operate on the frontier, meaning that they are not completely technically efficient. He presented the incomplete knowledge of available techniques, motivation, learning and psychological factors as potential reasons for deviation from the frontier.

Technical efficiency is most frequently associated with the role of management in the production process, and it indicates the gain that can be achieved by simply improving management (Farrell, 1957). Shapiro and Muller (1977) analyzed the roles of information and modernization in the production process on cotton farms in Tanzania, and they found a significant correlation between TE and the stock of information that firms have. Different levels of TE between firms can be explained by differences in the knowledge and information possessed by managers and differences in the quantity and quality of managerial effort supplied to the firm (Page, 1980).

Unlike physical factors such as land, labor, or capital, management is not directly observable. This can complicate any analysis that tries to explain the impact of management on firm performance. Also, this might lead to biased estimates of the parameters of the production function because of omitted variables. One possible solution to avoiding this problem is to define management as a random effect and model it as part of the stochastic element of the production frontier that has a composed error term: a symmetric error term that represents noise, and an asymmetric error term that accounts for TE (Alvarez et al., 2004).

An alternative solution consists of explaining the asymmetric error term and expressing it as a function of certain variables that have an effect on TE. This is common in empirical studies that seek to quantify the influence of management on firm technical performance. The variation in TE is expressed as a function of management ability through the inclusion of socio-economic variables in the analysis. Bravo-Ureta and Pinheiro (1993) reviewed several studies that explained farm level variation in TE, and some of the variables most used for this purpose have been farmer education and experience, access to credit, and farm size. For instance, Battese and Coelli (1993) and

(1995) included variables that reflected the age and the years of formal schooling of the primary decision maker in the farming operation. It is also possible to complement this attempt to quantify the management impact if some non-personal aspects of the decision-making process are considered. For instance, Wilson et al. (2001) estimated and explained the TE of wheat farmers in eastern England by including variables that reflect both personal and decision-making process aspects (motivations, practices, and procedures with respect to business planning). They found that these variables had a significant and positive effect on levels of TE.

According to Coelli et al. (2005), exogenous variables that characterize the environment where the firm operates could have an influence on the ability of a manager to convert inputs into outputs. Hence, it is useful to distinguish between non-stochastic variables that are observable and could be controlled by the firm, and stochastic variables that are not under the firm's control such as the weather. Farrell's approach has a disadvantage that arises when stochastic variables, which affect the environment where the production takes place, are considered (Lau and Yotopoulos, 1971).

Hall and Winston (1959) discussed the ambiguous notion of efficiency, and they stated that technical efficiency could result from environmental variables. For them, it is possible to describe the environment of each firm by the value of one or more variables, and they give an example where climatic variables can be considered as environmental in efficiency problems in which firms cannot choose the place where they operate. Most of the studies assume that environmental conditions are captured by the random error when a stochastic production frontier is estimated. Others studies introduce in the production function some variables in order to capture different environmental conditions. Mukherjee et al. (2013) used climatic indexes in production models to incorporate key climatic variables such as temperature and humidity. They found that those indexes have a significant negative effect on milk production, and their omission in the production function would imply a misspecification error.

As we can see, productivity growth can be explained by technological change, technical efficiency improvements or scale efficiency change. Therefore, in the absence of continuous scientific breakthrough, there might exist opportunities to improve firm performance by a more efficient use of resources under current production technology. Considering that firms have the ability to combine inputs in order to obtain the

maximum output given the technology available, gains in TE can be derived from improvements in decision-making, which are related to variables like knowledge, experience and education (Bravo-Ureta et al., 2007).

4.1. Production frontier estimation

Given that the production frontier cannot be observed directly, several methods have been developed to estimate frontiers to obtain a measure of TE. Among these, the two principal methods that have been useful tools in measurement of the TE of firms are: mathematical programming (nonparametric), commonly known as Data Envelopment Analysis (DEA); and econometric model (parametric) such as Stochastic Frontier Analysis (SFA). Both methods estimate the production frontier which represent the best practice for a specific sample of firms. The estimations methods also depend on the data available, either panel or cross-sectional data. We discuss alternative estimation techniques under the assumption that cross-sectional data is available.

Bravo-Ureta et al. (2007) studied the effect that different specification models (estimation technique, functional form, panel or cross-sectional data, etc.) might have on TE estimation. They concluded that the extent to which TE estimates are sensitive to model specification is still inconclusive and it is under debate.

Non-parametric models provide a mathematical programming method of estimating the best practice production frontier which is used to measure the relative efficiency of different firms. This approach to efficiency measurement obtains TE estimators as optimal solutions to mathematical programming problems. This deterministic non-parametric frontier model was first presented by Farrell (1957).

Charnes et al. (1978) proposed a DEA model that had an input orientation and assumed constant returns to scale. Authors such as Fare et al. (1983) and Banker et al. (1984), considered alternative sets of assumptions giving, as a result, models with variable returns to scale.

The main advantage of DEA is that it does not require specification of the functional form of the production function. It can be implemented without knowing the algebraic form of the relationship between outputs and inputs (Coelli et al., 2005). However, it is

a deterministic model that means it attributes all of the deviations of the observed output from the frontier to inefficiencies. Hence, it is not possible to make inferences about estimated DEA results as it is a non-parametric model. In addition, all deterministic frontier models are sensitive to extreme observations.

According to Battese (1992), the estimation of the production frontier using econometrics models provides a useful representation of the best practice technology. The implementation of econometrics models to efficiency measurement requires an explicit assumption regarding the specific form of the underlying production function (linear, quadratic, Cobb-Douglas, Translog and generalized Leontief). A specific functional form is assumed for the relationship between inputs and an output, and the unknown technological parameters of the production function need to be estimated using econometric techniques. The parametric frontier can be either deterministic or stochastic frontier.

4.1.1. Deterministic production frontier model

The deterministic frontier model is defined as the following:

$$y_i = f(x_i, \beta) \exp\{-u_i\}, \quad i = 1, 2, \dots, n$$

where y_i is the level of output of the firm i , x_i is a vector of inputs used by the firm i , $f(x_i, \beta)$ is the production frontier and β is a vector of technology parameters to be estimated which describes the shape of the production frontier. The term u_i is a non-negative random variable associated with firm specific factors which contribute to the firm i not achieving the maximum level of output given by the production frontier.

The u_i represents the technical inefficiency (TI) of the firm, and the entire shortfall of observed output y_i from the maximum feasible output $f(x_i, \beta)$ is attributed to technical inefficiency. This model ignores the possibility that output can be affected by random shocks that are not under the control of the firm.

The most common output oriented measurement of TE can be defined as the ratio of the observed output (y_i) and the maximum feasible output (y_i^*) given the levels of inputs

used by the farmer (Coelli et al., 2005). Using the deterministic frontier model the TE for the firm i is:

$$TE_i = \frac{y_i}{y_i^*} = \frac{f(x_i, \beta) \exp\{-u_i\}}{f(x_i, \beta)} = \exp\{-u_i\}$$

$$TI_i = 1 - TE_i$$

Technical efficiency takes values between zero and one which means that y_i is bounded above by the deterministic quantity $f(x_i, \beta)$, ensuring that the observed output lies below the frontier. When $TE_i = 1$, the firm is fully efficient, and the observed output reaches its maximum value; $TE_i < 1$ provides a measure of the deviation of the observed output from the maximum level attainable. As $0 \leq TE_i \leq 1$, the following inequality is met:

$$y_i \leq f(x_i, \beta), \quad i = 1, 2, \dots, n$$

The objective is to obtain estimates of the parameter vector β and u_i , in order to obtain estimates of TE_i for each firm.

Aigner and Chu (1968) estimated a deterministic frontier model using a Cobb-Douglas production function. They proposed mathematical programming to estimate the model. It was shown that the parameters of the models could be estimated using linear or quadratic programming models. The goal is to calculate a parameter vector β for which the sum of the proportionate deviations of the observed output u_i (or u_i^2) of each firm beneath maximum feasible output is minimized subject to the constraints of $u_i \geq 0$.

An important weakness of the mathematical programming techniques is that the parameters are calculated rather than estimated which complicates statistical inference regarding the calculated parameter value (Kumbhakar and Lovell, 2000).

Winsten (1957), cited by Kumbhakar and Lovell (2000), suggested the corrected ordinary least square (COLS) method as an alternative to estimating the deterministic production frontier. It involves two steps. In the first one, the ordinary least square (OLS) is used to obtain consistent and unbiased estimates of the slope parameters β . However, the estimate of the intercept parameter is consistent but biased. In the second

step, the biased intercept parameter is corrected to ensure that the observed output lies below the estimated frontier. But the COLS frontier does not necessarily bound the data from above as closely as possible, since it is required to be parallel to the OLS regression.

Modified ordinary least square (MOLS) is a variation on COLS which was presented by Afriat (1972) and Richmond (1974). It differs from COLS in that disturbances (u_i) are assumed to follow an explicit one-side distribution, such as exponential or half normal. The MOLS procedure also consists of two steps. After estimation by OLS, the biased intercept is modified by the mean of the assumed one-side distribution. Although MOLS is easy to implement, there is no guarantee that the modification of OLS shifts the estimated intercept up by enough to ensure that all firms are bounded from above by the estimated production frontier. In addition, the MOLS frontier is parallel to the OLS regression as a result of the fact that only the OLS intercept is modified.

To conclude, the three techniques described are easy to implement, but a serious deficiency of the deterministic frontier model is that it attributes all deviations from the frontier to inefficiency (Kumbhakar and Lovell, 2000). All variations in output that are not explained by a variation in inputs is associated with technical inefficiency. There is no space for the effect of random shocks which could also have an impact (positively or negatively) on the output. Bravo-Ureta and Pinheiro (1993) found that stochastic models yield higher average TE than their deterministic versions, and for the authors this is a consequence of stochastic models being more reliable than deterministic models since the former consider statistical noise.

4.1.2. Stochastic production frontier model

A more recent approach to measuring efficiency is the stochastic production frontier model that simultaneously accounts for statistical noise and technical inefficiency. It was independently developed by Aigner et al. (1977) and Meeusen and Broeck (1977), and it resolves the most serious deficiency in the deterministic frontier approach: all deviations from the frontier are a consequence of inefficiency. Using cross-sectional data and a generalized production function the model can be represented as follows:

$$y_i = f(x_i, \beta) \exp\{\varepsilon_i\}$$

$$\varepsilon_i = v_i - u_i$$

Here y is the scalar output of the firm i ($i = 1, 2, \dots, n$), x_i is a vector of inputs, β is a vector of unknown parameters, and ε is the “composed error” term. The error term is farm specific and is composed of two independent components. The first element v_i is a symmetric error component that captures random shocks and statistical noise, which are outside farmer’s control, such as weather, natural disasters, and measurement error. This term is assumed to be an independent and identically distributed normal random variable with zero mean and constant variance ($v_i \sim N(0, \sigma_v^2)$). In addition, v_i is distributed independently of u_i implying that the error term ε_i is asymmetric, since $u_i \geq 0$.

The one-side, non-negative error term $u_i \geq 0$ captures TI relative to the stochastic frontier. If a farmer is technically efficient ($u_i = 0$), he or she operates on its stochastic frontier, $f(x_{ij}, \beta) \exp\{v_i\}$. If a farmer is technically inefficient ($u_i > 0$), he or she operates beneath its stochastic frontier. The stochastic frontier defines the farmer’s maximum feasible output given inputs and available technology in the presence of random shocks. The principal idea of SFA is that the distance from the observed output to the frontier output is due partly to inefficient production and partly to the random shocks experienced by the farmer. It is possible for a farmer to operate above the deterministic production frontier when the noise effect is positive and larger than the inefficiency effect.

Much of SFA is directed towards the prediction of the TE. In this case, following the output oriented measurement and given a stochastic production frontier, technical efficiency can be defined as:

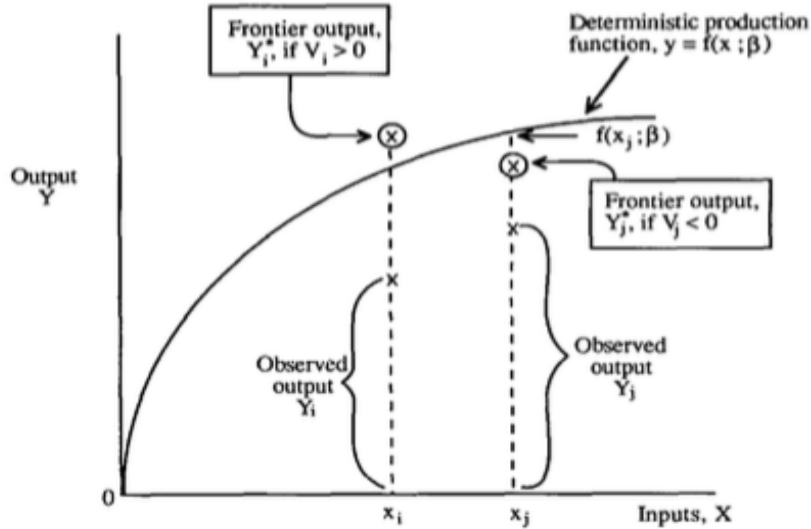
$$TE_i = \frac{y_i}{y_i^*} = \frac{f(x_{ij}, \beta) \exp\{v_i - u_i\}}{f(x_{ij}, \beta) \exp\{v_i\}} = \exp\{-u_i\}$$

$$TI_i = 1 - TE_i$$

where y_i is the observed output, and y_i^* is the maximum output that can be produced given the inputs and technology available. The inequality $0 \leq TE_i \leq 1$ is also met due to $y_i \leq y_i^*$.

The SFA allows farmers to operate above the deterministic production frontier if they suffer positive random shocks that are larger in magnitude than the inefficiency effect.

Figure 6: Stochastic frontier production function



Source: Battese (1992)

Although the expression of TE of a firm calculated with the deterministic and stochastic frontier models is the same, it is important to note that the values are different comparing both models. As it is shown in figure 6, the TE of firm j is greater considering the stochastic frontier model than for the deterministic model. Therefore, firm j is judged technically more efficient relative to the unfavorable conditions associated with its productive activity than if its production is judged relative to the maximum associated with the value of the deterministic function (Battese, 1992).

To estimate the parameters of the SFA model, that is, to determine the unknown parameters $(\beta, \sigma_v^2, \sigma_u^2)$, the following assumptions are assumed (Coelli et al., 2005):

- (1) v_i is distributed independently of u_i , and both errors are uncorrelated with the explanatory variables x_i
- (2) $E(v_i) = 0$
- (3) $E(v_i^2) = \sigma_v^2$ (homoskedastic)
- (4) $E(v_i v_j) = 0$ for all $i \neq j$ (uncorrelated)
- (5) $E(u_i^2) = \text{constant}$ (homoskedastic)
- (6) $E(u_i u_j) = 0$ for all $i \neq j$ (uncorrelated)

The OLS method can be applied under these assumptions to obtain consistent estimates of the slope parameters β , but not of the intercept, which is biased downwards, because $E(\varepsilon_i) = -E(u_i) \leq 0$. Also, OLS cannot be used to estimate firm-specific TE. However, using OLS estimations, a simple test to check the presence of technical inefficiency in the data can be constructed. If $u_i = 0$, then $\varepsilon_i = v_i$, the error term is symmetric and there is no evidence of technical inefficiency. On the other hand, if $u_i > 0$, then $\varepsilon_i = v_i - u_i$ is negatively skewed, and the data supports the presence of technical inefficiency suggesting that is a correct of estimate a SPF. Although this test is easy to obtain, its weakness is that it is based on asymptotic theory and many samples are relatively small (Kumbhakar and Lovell, 2000). As in the case of deterministic frontier, the COLS estimator is one solution to correct the bias in the intercept term. However, there is a better solution that applies some distributional assumptions concerning the two error terms and estimates the model using the method of maximum likelihood (ML). Estimators obtained from ML method have been preferred to other estimators such as COLS, because they present many desirable large sample properties (Coelli et al., 2005).

Hence, in order to use ML method and obtain more efficient estimations, explicit assumptions about the distribution of the inefficiency error term u_i need to be imposed. Meeusen and Broeck (1977) assumed an exponential distribution of u_i , Battese and Corra (1977) assigned a half normal distribution, and Aigner et al. (1977) considered both distributions of u_i . More flexible distributional assumptions of u_i were considered by Greene (1980), who assumed gamma distribution, and Stevenson (1980) who assumed gamma and truncated normal distribution. A more detailed analysis of inefficiency error term distributional forms can be found in (Kumbhakar and Lovell, 2000).

The choice of inefficiency error term distribution is sometimes a matter of computational convenience (Coelli et al., 2005). For instance, in this study, FRONTIER 4.1 package² is used to estimate the parameters under the assumption that the u_i follows

² Tim Coelli and Arne Henningsen (2013). Frontier: Stochastic Frontier Analysis. R package version 1.0. <http://CRAN.R-Project.org/package=frontier>.

a half-normal or truncated-normal distributions. However, sample mean efficiencies are sensitive to the distribution of the inefficiency error term u_i (Kumbhakar and Lovell, 2000).

Ahmad and Bravo-Ureta (1996) compared alternative model specification and their effect on TE measure for dairy farms. A stochastic frontier model with a half-normal inefficiency term was rejected when tested against a stochastic frontier model which assumed a truncated-normal distribution. They concluded that the half-normal distribution, which implied a mean equal to zero, was too restrictive for the data analyzed. Rivas and Bravo-ureta (2001) stated that the effect of different inefficiency error term distributional assumptions on TE estimations was still inconclusive. They found that the studies that assumed a distributional form different to half-normal obtained a lower average TE level compared to those studies which did not assume any distributional form. On the other hand, studies that assumed half-normal distribution gave an average TE that was not statistically different from those that did not specify any distributional form.

Often it is assumed that the noise term v_i follows a normal distribution while the inefficiency term u_i follows a positive half-normal ($\mu = 0$) or a positive truncated normal ($\mu \neq 0$) distribution. These assumptions imply a left-skewed distribution of the composed error term $\varepsilon_i = v_i - u_i$ which means that it is more frequent that a firm has a large negative residual than a large positive one (Henningsen, 2014):

$$v_i \sim N(0, \sigma_v^2)$$

$$u_i \sim N^+(\mu, \sigma_u^2)$$

Following Kumbhakar and Lovell (2000), we focus on the model which assumes a truncated normal distribution for u_i . Then, the density functions of v_i and u_i are given by:

$$f(v) = \frac{1}{\sqrt{2\pi}\sigma_v} \exp\left\{-\frac{v^2}{2\sigma_v^2}\right\}$$

$$f(u) = \frac{1}{\sqrt{2\pi}\sigma_u \Phi(-\mu/\sigma_u)} \exp\left\{-\frac{(u-\mu)^2}{2\sigma_u^2}\right\}$$

As the truncated normal distribution is a generalization of half-normal distribution, when $\mu = 0$, $f(u)$ collapses to the half-normal density function. As we can observe, the truncated normal distribution must estimate one additional parameter (μ) which is not estimated in the half-normal distribution. This parameter is the mode of the normal distribution and $\Phi(\cdot)$ is the standard normal cumulative distribution function.

Given that v_i is distributed independently of u_i , the joint density function is defined as the product of their individual density functions:

$$f(u, v) = \frac{1}{2\pi\sigma_u\sigma_v\Phi(-\mu/\sigma_u)} \exp\left\{-\frac{(u-\mu)^2}{2\sigma_u^2} - \frac{v^2}{2\sigma_v^2}\right\}$$

and since $\varepsilon = v - u$, the joint density function for u and ε is:

$$f(u, \varepsilon) = \frac{1}{2\pi\sigma_u\sigma_v\Phi(-\mu/\sigma_u)} \exp\left\{-\frac{(u-\mu)^2}{2\sigma_u^2} - \frac{(\varepsilon+u)^2}{2\sigma_v^2}\right\}$$

By defining $f(u, \varepsilon)$ it is possible to obtain the marginal density of ε as follows:

$$f(\varepsilon) = \int_0^\infty f(u, \varepsilon) du = \frac{1}{\sigma} \phi\left(\frac{\varepsilon+\mu}{\sigma}\right) \Phi\left(\frac{\mu}{\sigma\lambda} - \frac{\varepsilon\lambda}{\sigma}\right) \left[\Phi\left(-\frac{\mu}{\sigma_u}\right)\right]^{-1}$$

where $\sigma = (\sigma_u^2 + \sigma_v^2)^{1/2}$, $\lambda = \sigma_u/\sigma_v$, and $\Phi(\cdot)$ and $\phi(\cdot)$ are the normal cumulative distribution and density functions³. This parametrization was presented by Aigner et al. (1977) where λ gives information about the relative contribution of u and v to the composed error term (ε). If $\lambda \rightarrow 0$ either $\sigma_v^2 \rightarrow \infty$ or $\sigma_u^2 \rightarrow 0$ implying that there are no technical inefficiency effects and all deviation from the frontier are explained for noise Coelli et al. (2005).

Once $f(\varepsilon)$ is defined, the log-likelihood function for a sample of I firms can be obtained, and it can be maximized with respect to the parameters to obtain the ML estimates of the parameters:

³ The mean and variance of $f(\varepsilon)$ can be seen in Kumbhakar et al. (2000).

$$\ln L = b - l \ln \sigma - l \ln \Phi \left[-\frac{\mu}{\sigma_u} \right] + \sum_i \ln \Phi \left[\frac{\mu}{\sigma \lambda} - \frac{\varepsilon_i \lambda}{\sigma} \right] - \frac{1}{2} \sum_i \left(\frac{\varepsilon_i + \mu}{\sigma} \right)^2$$

where b is the constant $\sigma_u = \lambda \sigma / \sqrt{1 + \lambda^2}$. The log-likelihood is maximized with respect to the parameters to obtain ML estimators.

Battese and Corra (1977) provided a similar parametrization and they found that the log-likelihood function could be expressed in terms of the variance parameters: $\sigma^2 = \sigma_v^2 + \sigma_u^2$ and $\gamma = \sigma_u^2 / \sigma^2$. The variance ratio γ reflects which part of the total deviation of the optimal product, given by the frontier, is attributed to technical inefficiency effects. Hence, the parameter γ , which takes values between zero and one, represents the importance of the inefficiency term. It is irrelevant if γ is equal to zero. In this case the results should be equal to the Ordinary Least Square (OLS) results that imply $\gamma = 0$. On the other hand, if γ is one, the noise term is irrelevant and all deviations from the production frontier are explained by technical inefficiency.

Besides the mean TE, it is desirable to be able to estimate the technical inefficiency u_i for each individual firm. To do this, it is necessary to separate the composed error term (ε_i), easily estimated for each observation, into its two components to get information about u_i . However, the prediction of the technical efficiencies of individual firms associated with the stochastic frontier production function was impossible until the study of Jondrow et al. (1982) who proposed a method for separating the composed error term. The principal idea presented was that the conditional distribution of the non-negative random variable u_i , given the random variable $\varepsilon_i = v_i - u_i$, was observable. Jondrow et al. (1982) proposed that either the mean or the mode of the conditional distribution ($u_i | \varepsilon_i$) could be used to estimate TE of each firm. The mean is more commonly used than the mode, though the mode is a more attractive interpretation as ML estimator (Kumbhakar et al., 2000).

The conditional distribution $f(u/\varepsilon)$ is given by:

$$f(u/\varepsilon) = \frac{f(u, \varepsilon)}{f(\varepsilon)} = \frac{1}{\sqrt{2\pi}\sigma_* [1 - \Phi(-\tilde{\mu}/\sigma_*)]} \exp \left\{ -\frac{(u - \tilde{\mu})^2}{2\sigma_*^2} \right\}$$

where $f(u/\varepsilon) \sim N^+(\tilde{\mu}_i, \sigma_*^2)$, $\tilde{\mu}_i = (-\sigma_u^2 \varepsilon_i + \mu \sigma_v^2) / \sigma^2$ and $\sigma_*^2 = \sigma_u^2 \sigma_v^2 / \sigma^2$. Thus, the mean and the mode of $f(u/\varepsilon)$ are:

$$E(u_i/\varepsilon_i) = \sigma_* \left[\frac{\tilde{\mu}_i}{\sigma_*} + \frac{\phi(\tilde{\mu}_i/\sigma_*)}{1 - \Phi(-\tilde{\mu}_i/\sigma_*)} \right]$$

$$M(u_i/\varepsilon_i) = \begin{cases} \tilde{\mu}_i, & \tilde{\mu}_i \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

After we have the estimations for u_i , where \hat{u}_i is either $E(u_i/\varepsilon_i)$ or $M(u_i/\varepsilon_i)$, the TE measure for each firm is equal to:

$$\widehat{TE}_i = \exp(-\hat{u}_i)$$

Kumbhakar et al. (1991) extended the stochastic production frontier model in which determinants of TI are explicitly introduced in the model. They assumed that TI is composed of a deterministic component, that it is a function of some firm specific characteristics, and a random component. The mean of TI is no longer invariant across observations. It is considered a function of exogenous variables specific of each firm. Thus, TI and the composed error term can be expressed as:

$$u_i = \delta' z_i + w_i$$

$$\varepsilon_i = v_i - (\delta' z_i + w_i)$$

where z_i is a vector of explanatory variables that may influence firm efficiency performance, δ is the associated vector of parameters to be estimated and w_i is a random variable whose distribution is $N^+(0, \sigma_w^2)$. The requirement that $u_i \geq 0$ implies that $w_i \geq -\delta' z_i$. Consequently, the inefficiency effects in the frontier model have positive truncated normal distributions that vary with z_i , $u_i \sim N(\delta' z_i, \sigma_u^2)$. Simultaneous estimation of parameters in the stochastic production frontier and in the technical inefficiency model $(\beta, \delta, \sigma_v^2, \sigma_u^2)$, can be obtained using ML method under the assumptions that v_i and u_i are distributed independently of each other and of the regressors⁴.

⁴ All these calculations are done using the FRONTIER package.

The log-likelihood function is a generalization of that of the truncated-normal model presented above. It is important to note that constant mode μ is replaced with variable mode $\mu_i = \delta' z_i$.

$$\ln L = b - \frac{1}{2} \ln(\sigma_v^2 + \sigma_u^2) - \sum_i \ln \Phi \left(\frac{\delta' z_i}{\sigma_u} \right) + \sum_i \ln \Phi \left(\frac{\mu_i^*}{\sigma^*} \right) - \frac{1}{2} \sum_i \frac{(\varepsilon_i + \delta' z_i)^2}{\sigma_v^2 + \sigma_u^2}$$

where,

$$\mu_i^* = \frac{\sigma_v^2 \delta' z_i - \sigma_u^2 \varepsilon_i}{\sigma_v^2 + \sigma_u^2}$$

$$\sigma^{*2} = \frac{\sigma_v^2 \sigma_u^2}{\sigma_v^2 + \sigma_u^2}$$

Finally, TE of production for each firm is defined as follows, and it also can be estimated using the method proposed by Jondrow.

$$TE_i = \exp(-u_i) = \exp(-\delta' z_i - w_i)$$

$$\widehat{TE}_i = \exp(-\hat{u}_i)$$

where \hat{u}_i can be the mean or the mode $f(u/\varepsilon)$:

$$E(u_i/\varepsilon_i) = \mu_i^* + \sigma^* \frac{\phi(\mu_i^*/\sigma^*)}{\Phi(\mu_i^*/\sigma^*)}$$

$$M(u_i/\varepsilon_i) = \begin{cases} \mu_i^*, & \mu_i^* \geq 0 \\ 0, & \text{otherwise} \end{cases}$$

5. Data and empirical model

The data used in this study is a cross-sectional sample that was derived from a survey conducted by INALE in 2014. The bureau of agricultural statistics in Uruguay (DIEA⁵) designed the sample, using the Agricultural Census carried out in 2011 as a sampling

⁵ Dirección de Estadísticas Agropecuarias – Ministerio de Ganadería, Agricultura y Pesca del Uruguay.

frame. The sample has five strata which were defined considering the farm size, including 276 dairy farms located in 8 departments⁶ of Uruguay. They represent 90% of the total production of milk and they are highly specialized in dairy production. Considering that the sample is representative of the dairy farm population, the empirical results obtained from it can be expanded to the total population using the corresponding weights. We use the analytic weights provided by STATA. These weights are inversely proportional to the variance of an observation. The collected data corresponds to the 2013/14 agricultural year. Table 3 depicts a summary of the data with the different variables, dependent and explanatory, which are included in our stochastic production frontier model.

Table 3: Descriptive statistics for variables used in the frontier (n=276)

Variable	Description	Mean	SD	Min	Max
y	Milk (L) ¹	1,676	1,672	26	9,579
x_1	Cow (n)	308	298	7	2,250
x_2	Labor (n)	8	6	1	30
x_3	Feed (kg) ¹	898	997	4	6,633
x_4	Pasture (ha)	226	237	5.6	1,456

(1) In thousands

As we implement an econometric model, a specific functional form for the production frontier is required. Giannakas et al. (2003) analyzed the effect of functional form specification on the estimation of TE. They concluded that both estimates of production structure and TE measurements were sensitive to the choice of functional form specification. In this sense, the choice of an appropriate functional form affects the identification of the factors that determine individual performance. Similarly, Battese and Broca (1997) compared the Translog and Cobb-Douglas functional forms using panel data from wheat farms in Pakistan. The authors also highlighted the possible differences in TE measurement that may arise when different functional forms or inefficiency effects model are specified. Bravo-Ureta et al. (2007) concluded that the effect of functional form on TE was inconclusive. A likelihood ratio test⁷ was used to

⁶ Canelones, Colonia, Flores, Florida, Paysandú, Río Negro, San José and Soriano.

⁷ The likelihood-ratio test statistic is calculated as:

$\lambda = -2[\log(\text{likelihood}(H_0)) - \log(\text{likelihood}(H_1))]$, and it has a χ^2 -distribution with parameter equal to the number of parameters assumed to be zero under the null hypothesis.

help confirm which functional form fits the data significantly better. The null hypothesis is that all Translog coefficients are zero. Results show that it is not possible to reject the null hypothesis at a 5% significance level meaning that the Cobb-Douglas is preferred instead of the Translog. Thus, the empirical model in this study is based on the estimation of a Cobb-Douglas stochastic production function in which dependent and explanatory variables are expressed in natural logarithmic form:

$$\ln y_i = \beta_0 + \beta_1 \ln x_{1i} + \beta_2 \ln x_{2i} + \beta_3 \ln x_{3i} + \beta_4 \ln x_{4i} + v_i - u_i$$

where the subscript i ($i=1, 2, \dots, n$) refers to the i th sample farm. The dependent variable (y_i) represents the total liters of milk produced during the year for each farmer i . Following the literature and the data available we include four explanatory variables: x_1 denotes the total number of milking cows, x_2 is the total number of employees including family and hired labor, x_3 is defined as the total consumption of feed including concentrated feed, hay and silage (kg), and x_4 is the pasture variable measured as the total area under cultivated forage (ha).

Management plays a key role in any farm performance, even more so if it affects the level of technical efficiency within which the farm operates. We include the following explanatory variables to define the inefficiency model, and to capture some farm specific management characteristics. The maximum level of education achieved by the primary decision-maker is measured as a categorical variable⁸, where z_1 and z_2 are dummies equal 1 if the maximum level is secondary school or university, respectively. The category that is not included is primary school which is used for comparison. A higher level of education could have some positive effect on the farmer's ability to combine inputs to obtain the maximum output. Also, a well-educated farmer could perform better and use more modern production practices than a less educated one.

As more than half of farmers do other productive activities as their second source of income, it is important to measure how specialized farmers are. To do this, variable z_3 is defined as the ratio of the total land⁹ that is used exclusively for milk production to

⁸ Years of schooling were not available in the data.

⁹ Including land owned plus land leased

the total land available for any other production. Land used for milk production includes land devoted to milking cows, and heifers. We compare the performance of those farmers who use most of their land for milk production (z_3 close to one) and the performance of those who use part of their land to carry out other productive activities. Farmers who are specialized in milk production tend to concentrate all of their resources and effort on this activity which may allow them to increase their knowledge and experience.

Finally, we include three dummy variables that reflect other management strategies among the farmers. z_4 equals 1 if the farmer used artificial insemination to improve herd genetics. Although artificial insemination could be defined as an explanatory variable of milk production, we include it in the inefficiency model for the following reasons. Firstly, the database does not have data about artificial insemination's costs in order to define a quantitative variable. Secondly, artificial insemination requires some degree of precision to be implemented. Also, the farmer needs to have some specific knowledge about this technique to be able to apply it successfully. In these sense, artificial insemination might be thought of as a proxy for farmer's management abilities.

Finally, farmers who receive professional assistance could improve their efficiency because they can make better decisions about the productive process and its organization. Therefore, two variables are defined: z_5 equals 1 if the farmer paid for veterinary or agronomic assistance, and z_6 equals 1 if the farmer paid for accounting assistance.

$$u_i = \delta_0 + \delta_1 z_{1i} + \delta_2 z_{2i} + \delta_3 z_{3i} + \delta_4 z_{4i} + \delta_5 z_{5i} + \delta_6 z_{6i} + w_i$$

6. Profile of dairy farms

Given that we are working with a sample of dairy farms which represents 90% of the total production of milk in Uruguay, the results obtained for the sample are expandable to the entire sector. Hence, we can describe the most relevant characteristics of Uruguayan dairy farms. It is important to note that we only consider dairy farms that sell the milk produced to the processing industry.

Most of the farms (69.7%) have total land for production that varies between 50 and 500 ha, while only 11.9% have 500 ha or more. In Table 4, the total number of farms is divided into three groups according to the total land available for production -small, medium and large. Farms with more than 500ha produced on average 2,414,385lt annually which is almost four times greater than the average milk production for the whole sector. However, if we consider milk production per hectare of land that is exclusively used for milk production, land productivity is higher on small farms with 3,923lt/ha, while in larger farms it is 3,727lt/ha. As Table 4 shows, small farmers use 92.3% of their total land to produce milk. However, this percentage decrease for medium and large farmers who use 85.7% and 69.8%, respectively, of the total land that they have available for milk production.

Table 4: Average values of variables by farm size group

Farm group (Ha)	Farms (n)	Milk (L) ¹	Total land (Ha)	Land-milk prod (Ha)	Cows (n)	Liters/cow ¹	Liters/ha ¹	Cow/Ha
<50	509	126	34.9	32.2	30	4.2	3.9	0.93
50-500	1,929	507	169.2	145.0	107	4.5	3.5	0.74
>=500	328	2,414	928.6	647.7	446	5.2	3.7	0.69
Total	2,766	663	234.7	184.0	133	4.5	3.6	0.72

(1) In thousands

Regarding a milking cow's productivity, large farms achieve a level of 5,239lt/cow in the agricultural year, which is superior compared with medium and small farms (4,528lt/cow and 4,181lt/cow respectively).

The mean of milking cows per farm is 133. Farmers who have less than 100 milking cows account for 59.6% of the total number of farms, while those who have between 100 and 500 milking cows reached 36.1%, and only 4.2% of the farmers have more than 500.

All of the farms have milk production as the principal productive activity. Nevertheless, 51.2% of them have a secondary productive activity, and the two activities more frequently practiced together with milk production are livestock and cereal crops. In the case of larger farms, 69.8% have another productive activity as a second source of income. Farms with less than 50 ha are more specialized than larger ones and only

38.9% of them have a secondary activity. Half of those who have between 50 and 500 ha have another activity as a source of income.

It is interesting to analyze some of the socio-economic characteristics of the farmers in order to have a more complete description. We found that 62.6% of the farmers are more than 50 years old, while only 3.3% are less than 30 years old. Hence, Uruguay has an aging dairy farmer population. Besides this fact, only 40.6% of the farmers have a relative who is working on the farm and that will probably continue with the business. Young farmers (less than 30 years old) represent 16.5% of the total small farmers, while this percentage is 2% among large farmers.

Table 5: Age of primary farmer decision-maker

	Farm group (ha)			Total
	<50	50-500	>=500	
<30	84	-	6	90
(%)	16.5	-	2.0	3.3
30-50	113	731	86	930
(%)	22.2	38.3	27.3	34.1
>50	312	1,179	223	1,714
(%)	61.3	61.7	70.7	62.6
Total	509	1,910	315	2,734 ¹
(%)	100	100	100	100

(1) Missing values

Considering the maximum education level of the primary farmer decision-maker, primary school was the maximum level achieved for 40.6% of them, while 31.7% have secondary school as maximum level of education, and 13.2% have technical studies. A total of 13.5% achieved a higher level of education at university. If we consider the maximum level of education among the three farm size groups defined above, we can conclude that farmers in the larger farm size group have a higher level of education than those farmers in the small-size group. More than a half of the smallest farmers have primary school as a maximum level of education, while this percentage is 11.9% for largest farmers. The percentage of farmers that achieved an education level equal to or higher than secondary in each group is 44.4%, 58,8% and 88.1% respectively.

Table 6: Maximum education level of the primary farmer decision-maker

	Farm group (ha)			Total
	<50	50-500	>=500	
Primary	283	787	38	1,108
(%)	55.6	41.2	11.9	40.6
Secondary	113	673	85	871
(%)	22.3	35.3	26.6	31.7
Technical	85	206	68	359
(%)	16.6	10.7	21.3	13.2
University	0	244	128	372
(%)	-	12.8	40.2	13.5
Other	28	1	0	29
(%)	5.5	-	-	1.0
Total	509	1,911	319	2,739 ¹
(%)	100	100	100	100

(1) Missing values

Most of the farmers (66.6%) work full time on their dairy farms. This percentage is higher in smaller farms (72.2%) which reflect that farmers are more dedicated when farms are small. In the case of medium and large farms, the working time allocated to milk production is lower (67% and 56% respectively).

Table 7: Time dedicated in dairy farms by the primary farmer decision-maker

	Farm group (ha)			Total
	<50	50-500	>=500	
Full time	368	1,280	183	1,831
(%)	72.2	67.0	56.0	66.6
Other	141	633	144	918
(%)	27.8	33.0	44.0	33.4
Total	509	1,913	327	2,749 ¹
(%)	100	100	100	100

(1) Missing values

Regarding to the amount of family and hired labor on each farm, we found that 40.8% of the farms use only family labor, and 59.2% use family or paid labor. 83.4% of the smallest farms use only family labor while this percentage is null for the largest farms.

Table8: Farmers that only use family labor

	Farm group (ha)			Total
	<50	50-500	>=500	
Family	425	705	0	1,130
(%)	83.4	36.5	-	40.8
Hired	84	1,224	328	1,636
(%)	16.6	63.5	100.0	59.2
Total	509	1,929	328	2,766
(%)	100	100	100	100

(1) Family and hired labor

Considering only hired labor, 72.2% of the farms have two or less employees, while 27.8% have three or more. For smallest farms, the maximum number of hired labor is two. Also, most of the medium farms (74.4%) have two or less employees. On the other hand, 95.5% of the largest farms have three or more employees.

Table 9: Hired labor by farm size group

	Farm group (ha)			Total
	<50	50-500	>=500	
0-2	509	1,474	15	1,998
(%)	100	74.4	4.5	72.2
3 or more	0	455	313	768
(%)	-	23.6	95.5	27.8
Total	509	1,929	328	2,766
(%)	100	100	100	100

In addition, 81.6% of farmers live on their own farms. As is expected, this percentage is higher for smaller farms (89%) than for larger (64%). These results show that most of the Uruguayan farms are family farms.

7. Results and discussion

We first defined an Error Components Frontier model (ECF) where the stochastic frontier production function was estimated predicting the technical inefficiency effects under the assumption that this inefficiency was identically distributed. This means that technical inefficiency did not depend on farm specific variables. The mean of u_i was invariant across observations. Hence, we assumed that u_i followed a positive half-normal distribution. In the second stage, we defined an Efficiency Effects Frontier model (EEF) that allowed the mean of u_i to be a function of farm specific variables that

were supposed to explain differences in technical inefficiency among farms. This implies that we specified a regression model for the technical inefficiency effects on the stochastic frontier. In this case, u_i followed a positive truncated normal distribution. The estimated parameters of ML regression for both frontier production functions are shown in Table 10.

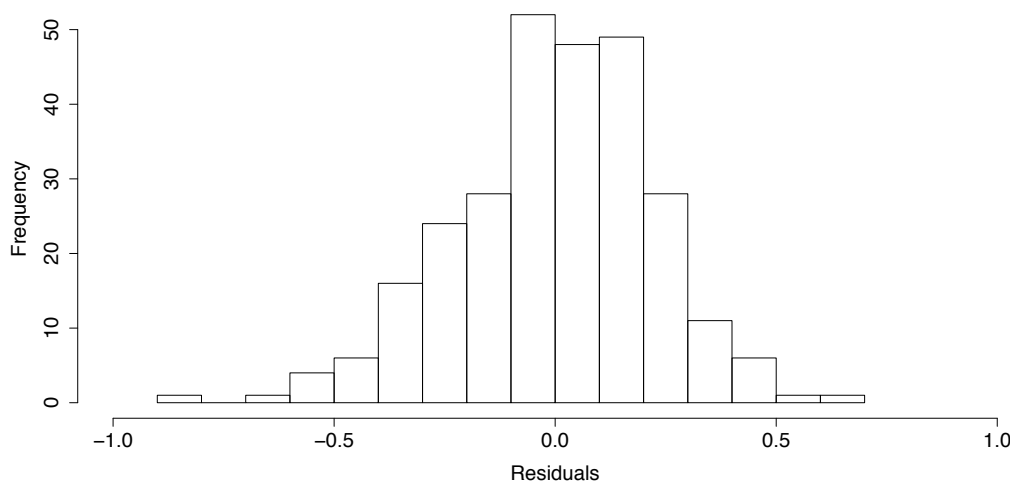
We estimated the ECF model and used a likelihood ratio test to check if adding the inefficiency term significantly improved the fit of the model. This test compared the stochastic frontier model with the corresponding OLS model where γ is equal to zero. The null hypothesis that OLS better fits the data was widely rejected. We obtained an estimated γ equal to 0.839, which confirmed that both statistical noise and inefficiency are important for explaining deviations from the production function. Therefore, the stochastic frontier model was more suitable.

Then, we estimated the EEF model where we extended the frontier production function including an inefficiency model to explain differences in technical inefficiency among farms. The explanatory variables which were included to explain the inefficiency were statistically significant except for the dummies variables that reflected the maximum achieved level of education. The estimated average TE in the EEF model was higher than in the ECF model (0.810 and 0.799 respectively). This implies that the explanatory variables have a negative effect on TI which means a positive effect on TE. Because of all these results, it is possible to conclude that the second model is more suitable to explain TE among dairy farms.

Consequently, the analysis developed from this point will be focused on the empirical results attaining from using the EEF model. The estimate for parameter γ is equal to 0.789, which lets us conclude that both statistical noise and inefficiency are important for explaining deviations from the production function. However, inefficiency is more important than noise, which means that part of the difference between observed and maximum frontier output can be explained by the difference in a farmer's level of TE by adopting different management practices. Besides, it is possible to test the relevance of inefficiency component using a likelihood ratio test. It compares the stochastic frontier model with the corresponding OLS model, and the null hypothesis that TI effects are absent ($\gamma = \delta_0 = \delta_1 = \delta_2 = \delta_3 = \delta_4 = \delta_5 = \delta_6 = \delta_7 = 0$) and OLS better fits the data is strongly rejected by the data. Moreover, as Figure 6 shows the residuals

of the Cobb-Douglas frontier from OLS estimation are left-skewed indicating that it is likely that not all dairy farmers are fully technical efficient.

Figure 6: Residuals of Cobb-Douglas production function



As Table 10 shows, all production function coefficients are non-negative meaning that the function satisfies the monotonicity property¹⁰. The sum over the coefficients of all inputs is very close to one, indicating that technology might present constant returns to scale (CRS). To confirm this result, we used a likelihood ratio test. The null hypothesis that the production frontier present constant returns to scale ($\beta_1 + \beta_2 + \beta_3 + \beta_4 = 1$) was not rejected at a 5% significance level. CRS implied that farm size and level of production were proportional. There was no evidence for economies scale. Therefore, improvements in technology and efficiency could be more significant in explaining productivity change than farm size. However, Bravo-Ureta et al. (2008) found increasing returns to scale in their study on technological change and TE for dairy farms in Uruguay. As Giannakas et al. (2003) show, the magnitude of production elasticities is model specific.

In the Cobb-Douglas function, the output elasticities of the inputs are equal to the corresponding coefficient if all inputs are measured in logarithmic form. As we can see in Table 10, all output elasticities are positive and statistically significant. These results

¹⁰ The monotonicity property of a production function says that additional units of an input will not decrease output.

reveal that the variables milking cows, labor, feed and pasture positively influence milk production. This implies that a 1% increase in any of the independent variables, i.e. the herd size, the number of workers, the feed consumption and the area under cultivated forage, results in an estimated increase in milk production of 0.570%, 0.070%, 0.270% and 0.074% respectively. Of all input variables, the number of milking cows have the highest effect on milk production level with elasticity equal to 0.570. This result is consistent with other studies, including Kumbhakar et al. (1991), Heshmati and Kumbhakar (1994), Cabrera et al. (2010) and Bravo-Ureta et al. (2008). Regarding the labor variable, it is important to note that it is significant at a 10% significant level, and Bravo-Ureta et al. (2008) found that this variable is not significant in explaining milk production.

Table 10: Stochastic production frontier estimates

Variable	ECF		EEF	
	Coefficient	SD	Coefficient	SD
Frontier				
Constant	6.313***	0.257	6.987***	0.250
Cow	0.620***	0.050	0.570***	0.047
Labor	0.023	0.042	0.070.	0.039
Feed	0.312***	0.030	0.270***	0.027
Pasture	0.065.	0.034	0.074*	0.031
Inefficiency Model				
Constant			1.093***	0.174
Secondary			-0.020	0.066
University			-0.082	0.103
Specialization			-0.542**	0.186
Insemination			-0.341***	0.089
Vet or agronomic assistance			-0.225**	0.073
Accounting assistance			-0.220**	0.079
σ^2	0.107***	0.016	0.076***	0.017
γ	0.839***	0.070	0.789***	0.081
Log-likelihood				
	28.28		61.64	
Mean TE				
	0.799		0.810	

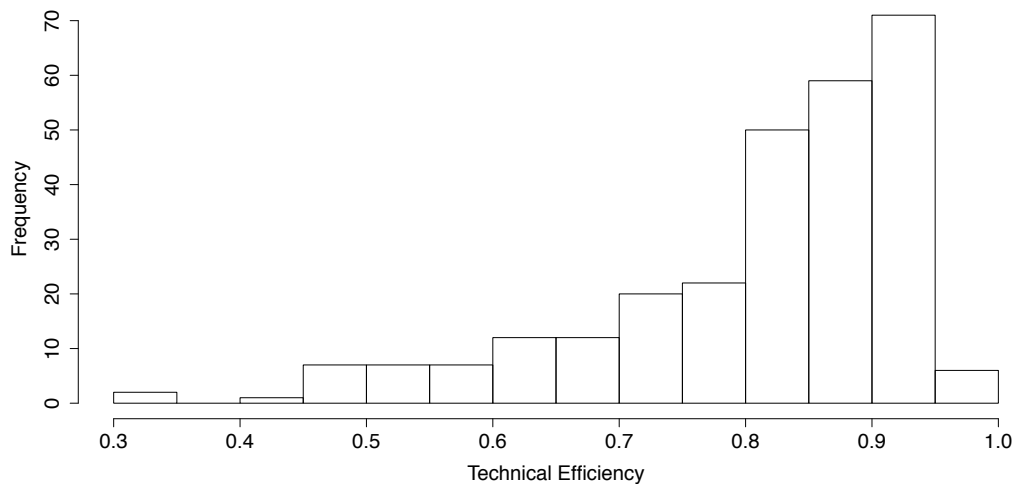
(.), *, **, ***, estimated coefficients significant at the 10%, 5%, 1%, 0.1% respectively.

In terms of the technical inefficiency model, a negative sign on a coefficient indicates that an increase in the value of that variable results in a fall in inefficiency and a positive value an increase in inefficiency. The empirical results show that all the explanatory variables that were included, except for the dummies that reflect educational level, have a significant negative impact on technical inefficiency. The non-significance of educational level might arise from measurement problems since schooling years were not available in the data. Holding everything else constant, those

farmers who are more specialized in milk production, and those who use artificial insemination, veterinary, agronomic or accounting assistance, achieved better performance that is associated with a lower technical inefficiency level, compared to farmers who have other productive activity, or do not use artificial insemination or receive professional assistance. However, the major determinants of efficiency differences are the level of specialization in milk production (-0.542) and the use of artificial insemination (-0.341).

The mean TE level is 0.810 indicating that on average the sample farmers reached 81% of their technical abilities and the remaining percentage were not realized (Table 10). Despite the fact that Bravo-Ureta et al. (2008) used unbalanced panel data to estimate stochastic production frontiers, they found that the mean TE of dairy farms in Uruguay was 0.811 and its maximum level was 0.971. Also, the average level of TE that we obtained is comparable to the average TE that Bravo-Ureta et al. (2007) presented in their meta-regression analysis of TE in agriculture. For the TE studies that consider countries from Latin America and implement a stochastic frontier analyses, the authors found an average TE level equal to 0.78.

Figure 7: Technical efficiency estimates of Cobb-Douglas production frontier



As the sample represents the total dairy farm population, it is possible to analyze the empirical results for the entire sector. Hence, when we expand the sample's results to the population we obtain that the mean TE of dairy farms in Uruguay is 0.7415. This suggests that farmers are not fully technically efficient. Farmers could increase milk

production using the current level of inputs and production technology available. They can improve their productivity and efficiency if they implement more efficient farm practices.

Table 11: Descriptive statistics for estimated TE

Min	Q ₁	Median	Mean	Q ₃	Max
0.3310	0.6249	0.7651	0.7415	0.8722	0.9636

Analyzing some descriptive statistics for the estimated TE of the whole population, we observe that half of the farmers have a TE level equal to or lower than 0.7651, which is higher than the mean. On the other hand, the maximum TE level is 0.9636, meaning that there are not any farmers that are completely efficient.

As efficiency has a direct effect on the output quantity, it is expected that the efficiency estimates are highly correlated with the output (Table 12). A positive and significant correlation (0.64) was found between TE and milk production, meaning that the higher the milk production the more efficient the farmer is. However, the association between TE and the total land that is used for dairy farming is significant but weaker (0.35). These results were also presented by Grau et al. (1995) and Vaillant (1990). Furthermore, no clear association between TE and farm size was found by Bravo-Ureta and Rieger (1991). The correlation is also weak (0.41) if we consider the relationship between farm size, measured as herd size, and efficiency. Moreira et al. (2012) found that farm size was not associated with productivity growth in dairy production in Chile. Although TE is also positively associated with labor, feed consumption and pasture, correlations are relatively weak. A Pearson product-moment correlation coefficient was used to measure the linear correlation between efficiency and the variables. The null hypothesis that the correlation was equal to zero was widely rejected meaning that all correlations were statistically significant.

Table 12: Efficiency and explanatory variables correlation

	Milk	Cow	Labor	Feed	Pasture
Efficiency	0.64	0.41	0.30	0.48	0.38

The distribution of TE scores for dairy farms is presented in Table 13. As the table indicates, 35.5% of the farms present a level of TE below 0.7, while almost 50% of

them achieve a level of TE between 0.7 and 0.89. Only 16.3% are in the higher group where the mean TE is 0.92.

Table 13: Distribution of the farm level measures of technical efficiency (TE)

TE	Farms (n)	Mean TE	Farms in TE groups (%)
<0.5	265	0.44	9.6
0.5-0.59	282	0.56	10.2
0.6-0.69	407	0.63	14.7
0.7-0.79	582	0.74	21.0
0.8-0.89	780	0.85	28.2
>0.9	450	0.92	16.3
Total	2,766	0.74	100

Using the farm level efficiency measures from the frontier estimates, we can obtain a profile of dairy farms by efficiency ranking, which are divided into five groups as Table 14 shows. The Bonferroni test was used to analyze differences in average values of each variable between groups.

Table 14: Average value of milk production and explanatory variables by efficiency groups¹

TE farm group	Farms (n)	Milk (l) ²	Cows (n)	Labor (n)	Feed (kg) ²	Pasture (ha)	Land (ha) ³
0.88-1	601	1,261 c	199 c	5.6 b	575 b	138 c	258 c
0.81-0.87	544	876 bc	170 bc	5.1 b	454 b	117 bc	227 bc
0.72-0.80	527	625 ab	137 abc	4.8 ab	357 ab	100 abc	208 abc
0.60-0.71	547	321 a	91 ab	3.2 a	165 a	71 ab	129 ab
0-0.59	547	174 a	62 a	2.9 a	98 a	41 a	91 a

(1) Values sharing the same letter between groups are not significantly different at a 5% significance level. (2) In Thousands; (3) Land used exclusively for milk production.

Milk production is on average statistically and significantly different between low and high efficiency groups. The most efficient farmers achieve a higher level of milk production than those less efficient. This result confirms the positive correlation between efficiency and milk production.

Herd sizes is statistically different comparing high and low efficiency groups, indicating that larger farms, in terms of milking cows, achieve a higher efficiency level than smaller ones. Nevertheless, the difference is not significant considering medium efficiency groups. When we measure farm size in terms of the land available for milk production, we observe that the most efficient farmers are larger than the least efficient.

However, the differences in average values of milking cows and land among efficiency groups are not very large in magnitude among groups with a higher efficiency level. These results confirm the weak correlation presented above between TE and farm size (in terms of milking cows or land).

Finally, labor, feed and pasture are also statistically different when we compare high and low efficiency groups (Table 14). These results indicate that farms in the high efficiency group are larger in terms of the used labor, feed consumption and area under cultivated forage than those in the lower efficiency group. Similar results are presented in Kompas and Che (2006) which compared the average value of farm characteristics by efficiency groups.

When we divide farms into groups according to their size (in terms of milk production, milking cows and land used for dairy farming), we observe that the average TE among milk production groups differ more than in the other cases (Table 15). For instance, average TE is statistically different between the largest and smallest farms in terms of milk produced. Also, this value is different comparing medium farms and the smallest ones.

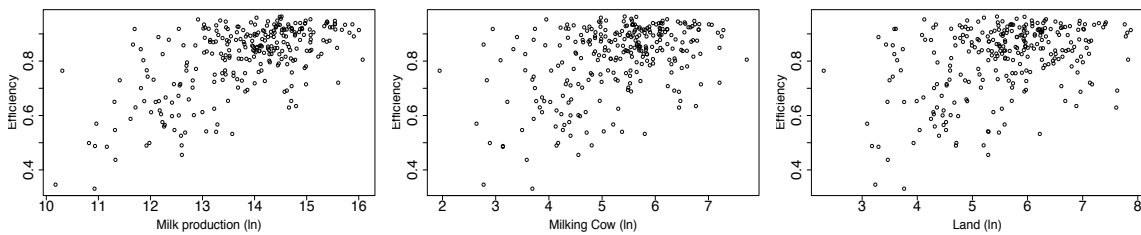
On the other hand, TE is statistically different between the largest and the smallest farms when farm size is measured in terms of milking cows or land, but the difference among medium farms is not very clear. Average TE is significantly different comparing the two largest farmers' groups and the three smallest groups in terms of milking cows. However, TE is not significantly different among these smallest farms. In the land groups case, average TE is clearly different between the largest and smallest farms but it is not significant among medium groups. According to these results, larger farms in terms of milk production are more efficient. Also, larger farms in terms of herd size and land used for milk production are more efficient. However, in this case farm size does not clearly involve higher levels of TE. Figure 8 shows that the correlation between TE and each farm size variable is positive, but as we mention above, the strongest correlation is between TE and milk production (0.64).

Table 15: Average value of TE by farm size group¹

Milk group	Farms (n)	TE	Cow group	Farms (n)	TE	Land group	Farms (n)	TE
≥856	549	0.86 b	≥175	545	0.83 b	≥175	551	0.83 c
479-855	547	0.82 b	101-174	532	0.80 b	101-174	542	0.77 bc
280-478	541	0.73 a	71-100	574	0.72 a	71-100	524	0.72 ab
152-279	600	0.68 a	42-70	525	0.71 a	42-70	588	0.70 ab
≤ 151	529	0.60 c	≤ 41	590	0.66 a	≤ 41	561	0.67 a

(1) In terms of total milk produced (thousands of liters), milking cows and land used for dairy farming.

Figure 8: Correlation among efficiency and milk production, milking cows and land



As can be seen in Table 16, there is no doubt about the association between milking cow productivity and efficiency. It is statistically different across all of the TE farm groups indicating that the most efficient farmers combine resources in a better way than those least efficient to achieve a higher level of production per milking cow. The level of productivity per milking cow in the high efficiency group is more than twice than that of low efficiency group. On the other hand, milk production per hectare of land that is used exclusively for milk production is also significantly different if we compare the least and the most efficient farms. Furthermore, the number of milking cows per hectare in the high efficiency group is higher and significantly different than in the low efficiency group.

The proportion of pasture to total land used for milk production is not statistically and significantly different among efficiency groups. Grau et al. (1995) found a weak and not statistically significant correlation between TE and the percentage of pasture for CREA farmers.

Table 16: Average value of farm characteristics by efficiency groups

TE farm group	Liters/cow ¹	Liters/Ha ¹	Feed/l	Concen/l	Forage/l	Labor/Ha	Cow/Ha	Ratio of pasture
0.88-1	6.3 a	5.5 a	0.40 a	0.21 ab	0.19 a	0.031 a	0.86 b	0.53 a
0.81-0.87	5.3 b	4.4 b	0.46 ab	0.21 ab	0.25 ab	0.031 a	0.83 ab	0.52 a
0.72-0.80	4.7 c	3.2 c	0.46 ab	0.21 ab	0.25 ab	0.038 ab	0.70 a	0.47 a
0.60-0.71	3.5 d	2.7 c	0.43 a	0.18 a	0.24 ab	0.030 a	0.77 ab	0.52 a
0-0.59	2.7 e	2.0 d	0.53 b	0.25 b	0.27 b	0.04 b	0.73 a	0.46 a

(1) In thousands

Comparing the average amount of feed used to produce a liter of milk, we can observe that it is statistically different between the most efficient farmers and the least efficient. The most efficient farmers use less concentrated feed and forage to produce a liter of milk than those less efficient. However, when we divide feed consumption into concentrated feed and forage, we find that only the consumption of forage per liter of milk is statistically different between farmers in the high efficiency group and those in the low efficiency group.

Considering the differences in magnitude of the average value of feed per liter, they are not very large among the efficiency groups. Therefore, the outstanding difference in productivity per milking cow between efficiency groups might be not a direct consequence of the usage of feed. It seems that the consumption of concentrated feed and forage should be complemented with a higher consumption of pasture or better herd genetics to obtain productivity improvements. As we have already pointed out, the percentage of pasture in the extensive production model was low, though it has grown through the intensification of dairy farming. Durán (2004) noted that in the advanced model the higher usage of concentrated feed and forage gave better yields because of better herd genetics.

On the other hand, the consumption of feed per milking cow is higher for the most efficient farms and the average value is significantly different between TE farm groups (Table 17). Kompas & Che (2006) also found that concentrated feed per cow was largest for the high efficiency group. The higher supply of concentrated feed allows for an increase in the number of cows per hectare of land. This is another feature of intensive dairy farming.

Table 17: Average value of feed per milking cow (kg/cow)

TE farm group	Feed/cow	Concen/cow	forage/cow
0.88-1	2.6 a	1.4 c	1.3 a
0.81-0.87	2.5 a	1.1 bc	1.4 a
0.72-0.80	2.2 a	1.0 b	1.2 ac
0.60-0.71	1.6 b	0.7 a	0.9 bc
0-0.59	1.5 b	0.7 a	0.8 b

From this analysis among efficiency groups arises the fact that the production system of the most efficient farms tend to be that of intensive dairy farming. Variables, like productivity measured as liters of milk per milking cow and per land, number of dairy cows per hectare of land, the amount of feed in the diet, are different measurements of dairy farming intensity.

Considering the most and the least efficient farms, the empirical results show that the former achieve productivity levels that are more than twice the productivity level of less efficient farms. Furthermore, the number of dairy cows per hectare of land is 17.8% higher on the most efficient farms than for the less efficient. Finally, farmers in the highest efficiency group use larger quantities of concentrated feed and forage per dairy cow. As we have already described, the higher supply of concentrated feed and forage was a fundamental change that occurred during the technological advance of the dairy production system in Uruguay.

Alvarez et al. (2008) explained the possibility that intensive production system implies fewer technical challenges than the extensive one. For instance, farmers in the low efficiency group closer to the extensive system use more pasture to feed the cows. This involves several activities on the farm like planting, fertilizing, harvesting, etc. On the other hand, the most efficient farmers whose production systems are more intensive avoid these activities on their farms because they base feeding mainly on purchased concentrates. Consequently, the level of TE might be negatively affected by these additional production activities which require more effort and resources.

As we use an output oriented TE measure, it is possible to calculate how much milk production can be expanded by given the input quantities and current technology. As defined in previous section:

$$TE_i = \frac{y_i}{y_i^*} \rightarrow TI_i = 1 - TE_i$$

where y_i and y_i^* are the observed and the maximum level of milk produced for each farm, respectively. Given that there are not completely efficient farmers ($TE_i = 1$), we take the maximum level of TE to estimate by how much each farmer might increase milk production if they were as efficient as the most efficient farmer.

Table 18: Total level of milk produced (millions of liters)

y (observed)	1,836
y^* (with $TE_i = 0.9636$)	2,091

Total milk production could increase by 14% (225 millions of liters) if all farmers achieve the maximum level of efficiency (0.9636). By dividing farms into milk production groups, we can observe that the smallest farms could increase average milk production by 34% using the current input quantities and technologies available. This increase in production can be achieved if farms improve their managerial ability.

Table 19: Average observed and maximum output by milk production group

Milk group	Farms (n)	y	y^*	Increase (%)
≥ 856	549	2.024	2.221	10
479-855	547	644	737	14
280-478	541	364	443	22
152-279	600	211	273	29
≤ 151	529	91	122	34

8. Conclusion

This study estimated a stochastic production frontier and an associated technical inefficiency model to determine the effect of inputs on dairy production, and farm specific characteristics that explain the differences in efficiency among dairy farms in Uruguay.

The empirical results showed that the Cobb-Douglas functional form was superior to the Translog form, and that dairy production exhibits constant returns to scale. This

suggests that productivity gains will depend more strongly on improvements in technology and efficiency and not necessarily on farm size.

All input variables were statistically significant with a positive effect on milk production. The highest effect on production was the number of milking cows followed by feed, pasture and labor. The average level of TE in the whole sector was 74%, which suggests that dairy farmers in Uruguay can expand milk production by 26% using the current level of inputs and production technology available. They can improve their productivity and efficiency implementing more efficient farm practices.

On the other hand, total milk production could increase by 14% if all farmers were as efficient as the most efficient farmer. The smallest farms, in terms of milk production, have more opportunities to increase their output (34%) using the current input quantities and technologies available but improving managerial ability.

The principal determinants of TE differences were the level of specialization in milk production (-0.542) and the artificial insemination (-0.341). These results show that farmers who focus on dairy farming or use artificial insemination, can achieve higher levels of efficiency than those who have less experience or are not using artificial insemination. Also, veterinary, agronomic and accounting assistance have a significant negative impact on technical inefficiency.

A positive and significant correlation (0.64) exist between TE and milk production, meaning that the higher the milk production the more efficient the farmer is. The correlation is weaker between TE and farm size (in terms of milking cows or land used for milk production). Even though more efficient farms are also larger in terms of herd size and land used for milk production, the association with TE levels is less significant (0.41 and 0.35, respectively). Also, farms in the high efficiency group are larger in terms of labor, feed consumption and area under cultivated forage than those in the lower efficiency group.

Empirical results show that farmers in the high efficiency group follow a more intensive production system than farmers in the low efficiency group. The most efficient farms achieve productivity levels which are more than twice of those of the least efficient farms. Furthermore, the number of dairy cows per hectare of land is 17.8% higher for

the most efficient farms than for the less efficient. Finally, farmers in the highest efficiency group use larger quantities of concentrated feed and forage per dairy cow.

The striking difference in productivity per milking cow between efficiency groups might not be a direct consequence of the usage of feed. Results show that the average amount of feed used to produce a liter of milk is not very different among the efficiency groups. Also, when we divide feed consumption into concentrated feed and forage, we find that only the consumption of forage per liter of milk is statistically different between farmers in the high efficiency group and those in the low efficiency group. Therefore, it seems that the consumption of concentrated feed and forage should be complemented with a higher consumption of pasture or better herd genetics to obtain productivity improvements.

As this study shows, there exists an intensification process in dairy farming. The smallest farms are the ones facing more difficulties in obtaining better performance. However, they could increase milk production if they improve efficiency. This means that the ability of a farmer to obtain the maximum output with the current quantities of inputs and technology available, can be improved. Therefore, productivity gains for the smallest farms due to improvements of TE seems to be more relevant than for the largest farms who already present higher TE levels.

From a policymaking point of view, it seems important to make policies focused on improving the ability of farms to use new techniques and combine inputs. Policies which attempt to promote the adoption of new technologies should be accompanied by policies oriented to improving managerial practice, learning by doing and spreading new technological knowledge. The information that farms use to make their decisions is different among farms, and that impacts TE.

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