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An MI-SDP Model for Optimal Location and Sizing of Distributed Generators in DC Grids That Guarantees the Global Optimum

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Abstract: This paper deals with a classical problem in power system analysis regarding the optimal location and sizing of distributed generators (DGs) in direct current (DC) distribution networks using the mathematical optimization. This optimization problem is divided into two sub-problems as follows: the optimal location of DGs is a problem, with those with a binary structure being the first sub-problem; and the optimal sizing of DGs with a nonlinear programming (NLP) structure is the second sub-problem. These problems originate from a general mixed-integer nonlinear programming model (MINLP), which corresponds to an NP-hard optimization problem. It is not possible to provide the global optimum with conventional programming methods. A mixed-integer semidefinite programming (MI-SDP) model is proposed to address this problem, where the binary part is solved via the branch and bound (B&B) methods and the NLP part is solved via convex optimization (i.e., SDP). The main advantage of the proposed MI-SDP model is the possibility of guaranteeing a global optimum solution if each of the nodes in the B&B search is convex, as is ensured by the SDP method. Numerical validations in two test feeders composed of 21 and 69 nodes demonstrate that in all of these problems, the optimal global solution is reached by the MI-SDP approach, compared to the classical metaheuristic and hybrid programming models reported in the literature. All the simulations have been carried out using the MATLAB software with the CVX tool and the Mosek solver.

Keywords: branch and bound method; convex optimization; distributed generation; mixed-integer semidefinite programming; power losses minimization

1. Introduction

Electrical distribution networks are responsible for transporting energy from power systems to end-users in medium- and low-voltage levels [1]. These are typically fed by substations that keep voltage and frequency variables as constant as possible, in the case of alternative current (AC) networks, or voltage magnitude, as in the case of a direct current (DC) distribution network [2]. Due to the use of voltage levels in distribution networks and their lengths (also its radial configurations), these networks have higher

power losses as percentages when compared to transmission networks [1,3]. Multiple methodologies have been reported to improve the performance of electrical distribution networks in terms of power losses and voltage profiles; some of the classical ones are the following:

✓ The optimal location and sizing of capacitor banks [4–6].
✓ The optimal location and sizing of distributed generators [7,8]
✓ The optimal grid reconfiguration [9–11].
✓ The optimal location of voltage regulators and series capacitor banks [12–14].

Note that in the case of DC networks, the most extended approach is the optimal location and sizing of distributed generators. This is the most reported approach in recent scientific literature where DC technologies are gaining more relevance in distribution scenarios [15]. Some of the main characteristics of the rapid advances in DC distribution are the following: (i) lower power losses and better voltage profiles due to the nonexistence of reactances in distribution lines, and (ii) easy controllability as the concept of frequency does not exist, which implies that controls are only focused on constant voltage support. In addition, multiple distributed energy resources, such as batteries and photovoltaic generators, are typically operated under the DC paradigm, which implies that they are easily integrated into these kinds of grids in contrast to AC networks. This is because a lower power electronic interface is needed to connect them in DC networks, which can reduce costs compared to conventional AC networks [15].

Based on these advantages of DC grids over AC grids, in this study, we focus on the problem of the optimal locations and sizes of distributed generators in DC distribution networks. This problem has been largely studied with regard to AC distribution networks [16]. In the case of DC networks, few approaches have been reported in the literature. The authors of [17] proposed a hybrid approach for the optimal location and sizing of dispatchable distributed generators in DC grids. They combined a semidefinite programming model for an optimal power flow analysis with the hyperplane search method, which relaxes the binary variables. A similar approach based on a sequential quadratic optimal power flow was proposed in [18]. Both optimization models reached the optimal global solution for this problem system with 21 and 69 node test feeders. However, the complexity in their implementations lies in the number of hyperplanes that were needed to be evaluated, as these depend on the number of candidate nodes provided by the relaxed optima power flow (OPF) model that can significantly increase the total processing time required for the solution of the problem. In [19], a tutorial was provided on the optimal location and sizing of distributed generators (DGs) in DC grids using the general algebraic modeling system (GAMS), while in [20], the same software was used for the optimal sizing of photovoltaic sources in DC grids while keeping in consideration a day-ahead environmental dispatch. Both approaches present the main advantages of implementing optimization models in GAMS; however, due to the nonlinear, non-convex mixed-integer structure of the studied problem, these approaches do not ensure the optimum global finding. The authors of [21] presented multiple metaheuristic approaches for the optimal location and sizing of DGs in DC grids. For the location, the stage was used as a classical genetic algorithm to define the nodes where all the DGs would be be located, and for the sizing, the stage used constituted the continuous metaheuristics named the continuous genetic algorithm, particle swarm, and black-hole optimizers. Even if these algorithms were easily implementable using sequential programming, the main complication was that multiple tuning parameters and statistical tests were required to measure their efficiency, as there was no guarantee that a global solution would be achieved. The authors of [22] presented a methodology for optimal dimensioning of wind turbines, photovoltaic sources, and batteries in hybrid AC/DC grids by proposing an optimization model that minimizes the life cycle cost of the system while keeping in consideration the life span of renewable energy resources. However, the model employed in the grid modeling is a linear simplification of the grid, which means that it is not possible to ensure that an optimal solution of the linear model is the optimal solution of the exact grid representation.

Based on the review mentioned of the state-of-the-art, we established that specialized literature has not reported an optimization method to address the problem of the optimal location and sizing of distributed generators in DC networks. In this sense, the main contributions of this research are summarized below:

✓ The reformulation of the mixed-integer nonlinear programming (MINLP) model that represents the exact optimization problem in a mixed-integer semidefinite programming model (MI-SDP).
✓ A global and optimal solution found using the combination of the branch and bound and interior point methods to solve the proposed MI-SDP model.

Note that the scope of this research focuses on the optimal sizing and location of DGs in DC distribution networks in medium- and low-voltage levels. We considered that DGs are fully dispatchable, i.e., they are constant power sources. In the case of renewable energy resources such as photovoltaic or wind turbines, we assumed that these would be connected in parallel with battery packages that controlled power output at a constant value at the node where the DG is located.

The remainder of this study is organized as follows: Section 2 presents the classical MINLP model for the problem of the optimal location and sizing of distributed generators in DC grids, which considers the minimization of total grid losses as its objective function. Section 3 presents the MI-SDP programming reformulation of the original MINLP model by focusing on power flow equations and the product of voltage variables and their restructuring via semidefinite programming. Section 4 presents the main aspects of the solution methodology via the branch and bound method. Section 5 offers the main characteristics of the 21- and 69-node test feeders. In Section 6, we report the numerical validation of our proposed MI-SDP approach and compare it with nonlinear and metaheuristic optimization methods. Finally, Section 7 presents the main conclusions derived from this research and some recommendations for future studies.

2. Minlp Formulation

The problem of optimal location and sizing of distributed generation in DC power grids is represented as a mixed-integer nonlinear programming model due to the following facts: (i) The problem of the optimal location of DGs is a binary optimization problem, as there exists a variable y_i that takes the value of 1 if a node i is located at one DG, or 0 if this node does not have assigned any DG. (ii) The problem of sizing DGs is indeed an optimal power flow problem due to the presence of power balance constraints that generate hyperbolic relations between voltages and powers at each node. The optimization model that represents this problem is presented below in full.

2.1. Objective Function

The objective function in this problem, as presented in [20], is typically a technical index related to the minimization of power losses in all the distribution conductors, as presented in Equation (1).

$$\min p_{\text{loss}} = \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{N}} G_{ij} v_i v_j, \tag{1}$$

where p_{loss} represents the objective function value, G_{ij} represents the component of the conductance matrix that relates nodes i and j, and v_i and v_j are the voltage values at nodes i and j, respectively. Note that $\mathcal{N}$ is the set that contains all the nodes of the network.

Remark 1. *The objective function (1) is convex, as it is the sum of square variables regarding the voltage profiles between pairs of nodes. The convexity of this function is ensured by the fact that the conductance matrix G is a positive semidefinite function [17].*

2.2. Set of Constraints

The set of constraints on the optimal location and sizing of power sources in DC distribution networks are as follows: power balance at each node, voltage regulation bounds, and device and element capacities. Each one of these constraints is described below:

$$p_i^s + p_i^{dg} - p_i^d = v_i \sum_{j \in \mathcal{N}} G_{ij} v_i v_j, \ \forall i \in \mathcal{N} \tag{2}$$

$$p_{km} = g_{km} v_k (v_k - v_m), \ \forall km \in \mathcal{L} \tag{3}$$

$$p_{km}^{\min} \leq p_{km} \leq p_{km}^{\max}, \ \forall km \in \mathcal{L} \tag{4}$$

$$v^{\min} \leq v_i \leq v^{\max}, \ \forall i \in \mathcal{N} \tag{5}$$

$$\sum_{i \in \mathcal{N}} y_i \leq N_{gd}^{\max}, \tag{6}$$

$$0 \leq p_i^{dg} \leq y_i p_i^{\max}, \ \forall i \in \mathcal{N} \tag{7}$$

$$0 \leq p_i^s \leq p_i^{s,\max}, \ \forall i \in \mathcal{N} \tag{8}$$

$$y_i \in \{0,1\}, \ \forall i \in \mathcal{N} \tag{9}$$

where p_i^s represents the power injected by the slack node connected at node i, p_i^{dg} represents the amount of power generated by the distributed source connected at node i, and p_i^d represents the total power consumption at the same node. p_{km} represents the total power flow on the line that connects nodes k and m, which has a conductive parameter g_{km}. $p_{km}^{\max}$ and $p_{km}^{\min}$ represent the maximum and minimum power flow bounds allowed in the line that connects nodes k and m, respectively. $v^{\min}$ and $v^{\max}$ represent the minimum and maximum voltage bounds allowed for the voltage variables in all the nodes. y_i stands for the binary variable that defines whether a DG is located ($y_i = 1$) or not ($y_i = 0$) at node i. $N_{gd}^{\max}$ represents the total number of DGs available for installation in the DC network. $p_i^{\max}$ and $p_i^{s,\max}$ represent the maximum capabilities of power injection of the DG and the slack source connected at node i, respectively.

The complete mathematical formulation presented from (1) to (9) is completely described below: Equation (1) defines the objective function value that corresponds to a technical measure related to the minimization of grid power losses. Equation (2) defines the power balance equilibrium at each node of the network. In Equation (3), the power flow at each line of the network is calculated as a function of the voltage difference between the sending and receiving nodes of the line and its conductance. Equations (4) and (5) are box-type constraints that bound the power flows of all the lines and voltage profiles in all the nodes. The inequality constraint (6) determines the availability of distributed generators inside the DC grid. Equation (7) determines the power generation capabilities of a DG connected at node i; the inequality constraint (8) defines the lower and upper bounds of the power injection in a slack source, and Equation (9) presents the binary nature of the decision variable regarding the location (or not) of a DG in the DC network.

Remark 2. *The optimal location and sizing of distributed generators in DC networks is indeed a nonlinear, non-convex optimization problem due to the following:* (i) *the presence of binary variables associated with the location problem, and* (ii) *the power balance and line power flow constraints that contain products between voltages, i.e., nonlinear quadratic equality constraints.*

Due to the binary and nonlinear constraints of the model (1)–(9), this model corresponds with an MINLP formulation, with the main characteristics presented in Figure 1.
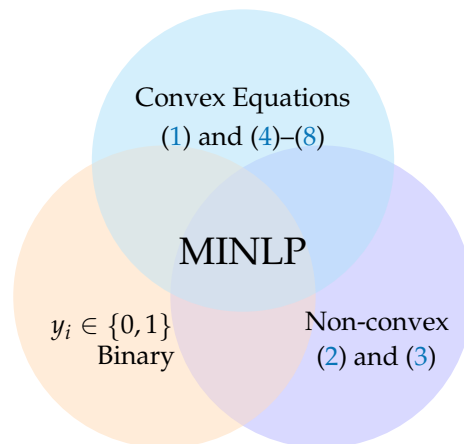
Figure 1. Characterization of the mathematical model associated with the optimal location and sizing of DGsin DC grids.

Note that from the mathematical structure depicted in Figure 1, the main complication of the model is given by the power balance and power flow constraints (2) and (3), as these are nonlinear equality constraints; however, as these only contain products among variables, it is possible to make them into a convex formulation via semidefinite programming. This implies that the MINLP model can be reformulated as a mixed-integer semidefinite programming (MI-SDP) model, with the main advantage being that it is efficiently solvable with the branch and bound method, in conjunction with interior points, thereby ensuring the optimum global finding due to the convex structure of the SDP formulation. In the next section, the proposed MI-SDP approach for locating and sizing DGs in DC networks is widely explained.

3. Mi-Sdp Reformulation

The MI-SDP reformulation of the problem of the optimal location and sizing of DGs in DC distribution networks is based on the possibility of reformulating the nonlinear non-convex power balance and power flow constraints into convex restrictions via semidefinite matrices [17]. This is possible since the nonlinearities regarding continuous variables, i.e., voltages, powers, and currents, are products, which makes them easily treatable with semidefinite programming, as described in [23].

Remark 3. *The main property of an MI-SDP model is the possibility of guaranteeing the global optimum if the binary part is addressed with a modified version of the B&B method where each explored node can be represented with a convex optimization model [24,25].*

3.1. Sdp Model for Power Flow Equations

To obtain a convex equivalent of the constraints (2) and (3), let us introduce the matrix of variables X as follows:

$$X = VV^T, \tag{10}$$

where

$$
V = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}, \qquad
X = \begin{bmatrix}
v_1^2 & v_1 v_2 & \cdots & v_1 v_n \\
v_2 v_1 & v_2^2 & \cdots & v_2 v_n \\
\vdots & \vdots & \ddots & \vdots \\
v_n v_1 & v_n v_2 & \cdots & v_n^2
\end{bmatrix}.
$$

Note that each column of the matrix X contains all the products between the voltage variables of each node. In addition, this matrix exhibits asymmetric behavior; i.e., $X = X^T$.

Now if we remember that the conductance matrix G is a positive semidefinite matrix (also symmetric) with the structure presented below:

$$
G = \begin{bmatrix}
G_{11} & G_{12} & \cdots & G_{1n} \\
G_{21} & G_{22} & \cdots & G_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
G_{n1} & G_{n2} & \cdots & G_{nn}
\end{bmatrix},
$$

and that with the vectors of the power in slack nodes, power generation in distributed sources and power demands can be rewritten as the following:

$$
P^s = \begin{bmatrix} p_1^s \\ p_2^s \\ \vdots \\ p_n^s \end{bmatrix}, \qquad
P^{gd} = \begin{bmatrix} p_1^{gd} \\ p_2^{gd} \\ \vdots \\ p_n^{gd} \end{bmatrix}, \qquad
P^d = \begin{bmatrix} p_1^d \\ p_2^d \\ \vdots \\ p_n^d \end{bmatrix};
$$

then, the power balance constraint (2) can be expressed using a vector structure as follows:

$$
P^s + P^{gd} - P^d = \mathbf{diag}\,(GX), \tag{11}
$$

where $\mathbf{diag}\,(\cdot)$ is an operation that takes the diagonal of a matrix with dimensions $n \times n$ and translates it into a vector with dimensions $n \times 1$. Note that the power flow constraint defined in Equation (11) is now an affine constraint with the form $Ax = b$, which has allowed the rewriting of the optimal power flow problem for optimal dimensioning of DGs in DC grids as a convex constraint.

In the case of power flow in lines, this expression is easily written by observing in the definition of the matrix X that $X_{km} = v_k v_m$ and $X_{kk} = v_k^2$, which implies that (3) takes the following form:

$$
p_{km} = g_{km}\,(X_{kk} - X_{km}),\ \forall km \in \mathcal{L} \tag{12}
$$

Remark 4. *It is important to highlight that in the case of semidefinite programming, for the X matrix, a nonlinear constraint associated with the rank of this matrix appears. This implies that for recovering exactly all the voltage variables V, it is mandatory that **rank** (X) = 1. However, these conditions are a non-convex constraint that we can relax as recommended in [26] to obtain a complete convex reformulation of the optimal power flow model associated with the problem of the optimal dimensioning of DGs in DC networks.*

3.2. Objective Function Representation

Even if the objective function (1) is a convex function due to the properties of the conductance matrix, we need to rewrite it as a function of the new variables X_{ij}. For doing so, let us to take the right-hand part

of the power balance constraint (11) by using the property name trace, which is applicable to matrices and has the function of adding all the components of the diagonal to produce a scalar function [26]. Based on this function, Equation (1) takes the following form:

$$\min p_{\text{loss}} = \textbf{trace}\,(GX), \tag{13}$$

3.3. Voltage Bounds and Solution Recovering

In the case of the voltage regulation bounds reported by Equation (5), it is necessary to obtain an equivalent box-type constraint as a function of the new variables X. For doing so, let us to represent it as follows:

$$\left(v^{\min}\right)^2 \le v_i^2 \le (v^{\max})^2 \,.\, \forall i \in \mathcal{N} \tag{14}$$

Now, if we define a matrix filled by ones with dimensions $n \times n$ as $1_{n \times n}$, then the Equation (14) takes the following structure:

$$1_{n \times n}\left(v^{\min}\right)^2 \le X \le 1_{n \times n}\,(v^{\max})^2\,. \tag{15}$$

Note that to recover the voltage variables from the SDP approximation of the power flow equations, we can easily take the square root of each component of the diagonal of the matrix X, i.e., $v_i = \sqrt{X_{ii}}$. This is possible, as in the matrix X, there is only one eigenvalue that is different from zero, while the other ones are quite near zero, as demonstrated in [26], which implies that X can be adequately reconstructed by using this eigenvalue and its corresponding eigenvectors.

3.4. Complete Mi-Sdp Model and Characterization

Based on the SDP formulation presented in previous subsections, the MINLP model is transformed into an MI-SDP model with the structure presented below:
Objective function:

$$\min p_{\text{loss}} = \textbf{trace}\,(GX), \tag{16}$$

Set of constraints:

$$P^s + P^{gd} - P^d = \textbf{diag}\,(GX), \tag{17}$$

$$p_{km} = g_{km}\,(X_{kk} - X_{km}),\ \forall km \in \mathcal{L} \tag{18}$$

$$p_{km}^{\min} \le p_{km} \le p_{km}^{\max},\ \forall km \in \mathcal{L} \tag{19}$$

$$1_{n \times n}\left(v^{\min}\right)^2 \le X \le 1_{n \times n}\,(v^{\max})^2, \tag{20}$$

$$1_{n \times 1}^T Y \le N_{gd}^{\max}, \tag{21}$$

$$0_{n \times 1} \le P^{gd} \le Y \otimes P^{gd,\max}, \tag{22}$$

$$0_{n \times 1} \le P^s \le P^{s,\max}, \tag{23}$$

$$Y \in \{0,1\}, \tag{24}$$

where $\otimes$ is the Hadamard product between vectors [27]. Note that the classification of the mathematical model (16)–(24) is presented in Figure 2.
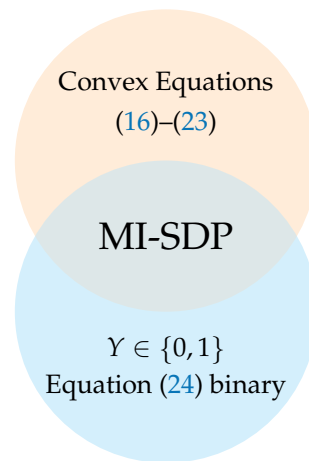
Figure 2. Equivalent MI-SDP model for the problem of the optimal location and sizing of distributed generators in DC distribution networks.

4. Solution Methodology

To address the optimal solution of the problem of the optimal location and dimensioning of distributed generators in direct current networks, the classical B&B method is combined with the SDP relaxation of the power flow problem as depicted in Figure 2. Note that an MI-SDP problem has the following general structure:

$$
\begin{aligned}
\sup\ & b^T y, \\
\text{s.t.}\ & C - \sum_{i=1}^{m} A_i y_i, \\
& l_i \leq y_i \leq u_i,\ \forall i \in [m] \\
& y_i \in \mathbb{Z},\ \forall i \in I
\end{aligned}
\tag{25}
$$

where $C \in S_n$, $b \in \mathbb{R}^m$, and $A_i \in S_n$, $l_i \in \mathbb{R} \cup \{-\infty\}$, $u_i \in \mathbb{R} \cup \{\infty\}$ for all $i \in [m]$. The set of indices of integer variables is given by $I \subseteq [m]$. Note that S_n is the space where symmetric semidefinite matrices are defined.

MI-SDPs with the mathematical form (25) can be solved with a straightforward branch and bound algorithm. Branch and bound was first proposed for integer linear programs by Land and Doig [28]. In 1965, Dakin [29] realized that the problems do not have to be linear, but that the same technique can also be applied to general problems with integrality restrictions, as long as the sub-problems obtained by relaxing the integrality conditions and possibly adding variable bounds can be solved optimally [24]. Figure 3 presents a small example of the evolution of an MI-SDP approach through a solution space with two variables.
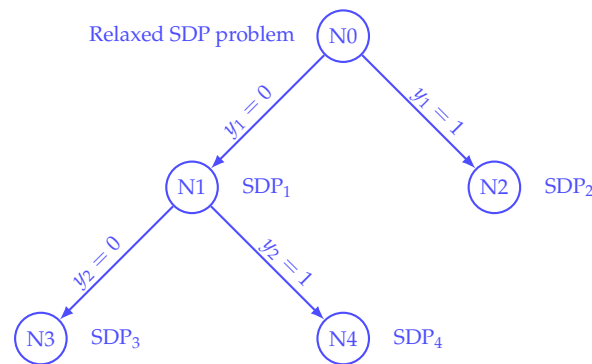
Figure 3. Schematic representation of the B&B method for addressing MI-SDP problems.

Complete details about the implementation of the SDP-based branch and bound algorithm can be found in [25].

5. Test Systems

Two radial DC distribution test feeders composed of 21 and 69 nodes were used to validate our proposed MI-SDP. They have been employed in specialized literature to validate methodologies regarding the optimal location and sizing of distributed generation. The complete information of each one of these test feeders is presented below.

5.1. 21-Node Test Feeder

This DC distribution network is composed of 21 nodes and 20 branches (i.e., radial configuration), where the slack source is connected at node 1, and it supports a constant voltage output of 1.0 pu [17]. The configuration of this test feeder, and the information regarding its loads and branches, are reported in Figure 4 and Table 1, respectively.
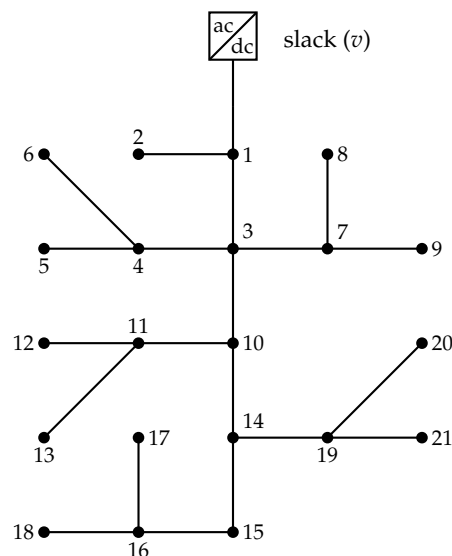

Figure 4. Schematic connection among nodes for the 21-node test feeder.

Table 1. Parametric information of the 21-node test system.

Node i	Node j	R_{ij} [pu]	P_j [pu]	From	To	R_{ij} [pu]	P_j [pu]
1	2	0.0053	0.70	11	12	0.0079	0.68
1	3	0.0054	0.00	11	13	0.0078	0.10
3	4	0.0054	0.36	10	14	0.0083	0.00
4	5	0.0063	0.04	14	15	0.0065	0.22
4	6	0.0051	0.36	15	16	0.0064	0.23
3	7	0.0037	0.00	16	17	0.0074	0.43
7	8	0.0079	0.32	16	18	0.0081	0.34
7	9	0.0072	0.80	14	19	0.0078	0.09
3	10	0.0053	0.00	19	20	0.0084	0.21
10	11	0.0038	0.45	19	21	0.0082	0.21

It is worth mentioning that all of the values in Table 1 were calculated considering 1 kV and 100 kW as the voltage and power bases, respectively.

5.2. 69-Node Test Feeder

The 69-node test feeder is a classical AC distribution network employed for power loss minimization studies in the literature [7]. However, in this study, we used its DC adaptation, as reported in [17]. Figure 5 presents the 69-node test feeder configuration, and Table 2 reports the branch parameters and load information. Furthermore, for simulation purposes, 12.66 kV and 100 kVA were used as the voltage and power bases for this test system.
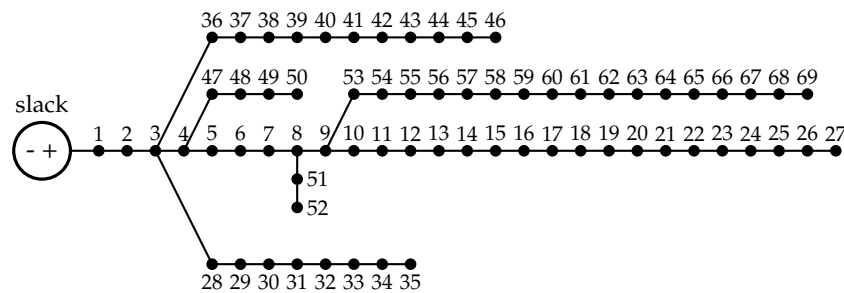

Figure 5. Schematic connection among nodes for the 69-node test feeder.

Table 2. Parametric information of the 69-node test system.

Node i	Node j	R_{ij} [Ω]	P_j [kW]	From	To	R_{ij} [Ω]	P_j [kW]
1	2	0.0005	0	3	36	0.0044	26
2	3	0.0005	0	36	37	0.0640	26
3	4	0.0015	0	37	38	0.1053	0
4	5	0.0215	0	38	39	0.0304	24
5	6	0.3660	2.6	39	40	0.0018	24
6	7	0.3810	40.4	40	41	0.7283	102
7	8	0.0922	75	41	42	0.3100	0
8	9	0.0493	30	42	43	0.0410	6
9	10	0.8190	28	43	44	0.0092	0
10	11	0.1872	145	44	45	0.1089	39.22
11	12	0.7114	145	45	46	0.0009	39.22
12	13	1.0300	8	4	47	0.0034	0
13	14	1.0440	8	47	48	0.0851	79
14	15	1.0580	0	48	49	0.2898	384.7
15	16	0.1966	45	49	50	0.0822	384.7
16	17	0.3744	60	8	51	0.0928	40.5
17	18	0.0047	60	51	52	0.3319	3.6
18	19	0.3276	0	9	53	0.1740	4.35
19	20	0.2106	1	53	54	0.2030	26.4
20	21	0.3416	114	54	55	0.2842	24
21	22	0.0140	5	55	56	0.2813	0
22	23	0.1591	0	56	57	1.5900	0
23	24	0.3463	28	57	58	0.7837	0
24	25	0.7488	0	58	59	0.3042	100
25	26	0.3089	14	59	60	0.3861	0
26	27	0.1732	14	60	61	0.5075	1244
3	28	0.0044	26	61	62	0.0974	32
28	29	0.0640	26	62	63	0.1450	0
29	30	0.3978	0	63	64	0.7105	227
30	31	0.0702	0	64	65	1.0410	59
31	32	0.3510	0	65	66	0.2012	18
32	33	0.8390	10	66	67	0.0047	18
33	34	1.7080	14	67	68	0.7394	28
34	35	1.4740	4	68	69	0.0047	28

6. Numerical Validation

The proposed MI-SDP model was implemented in the MATLAB software version 2019*b* using the CVX and the branch and bound method on a desktop computer with an INTEL(R) Core(TM) *i7*-7700 2.8-GHz processor and 16.0 GB of RAM running on a 64-bit version of Microsoft Windows 10 Home. To validate the effectiveness and robustness of our mixed-integer convex reformulation of the optimal location and dimensioning of DGs in DC networks, we compared this with heuristic and metaheuristic approaches reported in recent literature. The comparative approaches are combinations of the discrete genetic algorithm (GA) with continuous methods, such as GA black hole optimizer (GA-BHO), GA continuous genetic algorithm (GA-CGA), and GA particle swarm optimization (GA-PSO) [18]. In addition, we compared our results with hyperplanes combined with semidefinite and sequential quadratic programming methods, i.e., HSDP and HSQP, respectively [17].

For both test feeders, we followed the simulating conditions reported in [18], where

✓ The maximum power penetration in the DC test feeder was 60% of the total power consumption in the 21-node test feeder and 40% in the case of the 69-node test system.

✓ There were three distributed generators available for installation on the DC test feeder, where in the case of the 21-node test system their maximum individual allowed size was 1.5 pu, and in the case of the 69-node test feeder, this bound was 12 pu.

6.1. 21-Node Test Feeder

For this test system, the initial power losses without distributed generation were about 0.2760 pu. When the MI-SDP method was applied to the 21-node test feeder, we found that the distributed generators must be located at nodes 9, 12, and 16 with power injections of 0.8350, 1.0258, and 1.4632 pu, respectively; these produce final power losses of about 0.0306 pu. This implies a total reduction of 88.91% in the power losses with respect to the best case. For comparative purposes, we used the general algebraic modeling system (GAMS) with different MINLP solvers to verify the efficiency of our MI-SDP approach. The numerical results are summarized in Table 3.

Table 3. Comparison of the MI-SDP with the MINLP model solved in GAMS for the 21-node test feeder.

Solver	Nodes	Sizes [pu]	p_{loss} [pu]
BARON	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306
BONMINH	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306
DICOPT	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306
KNITRO	$\{9, 12, 17\}$	$\{0.9297, 1.1491, 1.2452\}$	0.0356
LINDO	$\{9, 12, 17\}$	$\{0.9297, 1.1491, 1.2452\}$	0.0356
SBB	$\{9, 12, 16\}$	$\{0.8441, 1.0254, 1.4544\}$	0.0306

From Table 3, we can observe that due to the strong non-convexities of the power flow problem being added to the binary nature of the problem of installations of DGs in DC current, all the solvers available in GAMS have the ability to find the global optimal solution of the studied problem. In addition, we can observe that in the case of the local solutions, i.e., KNITRO and LINDO solvers, they got stuck in a sub-optimal solution wherein node 17 was selected in the final report instead of node 16. This is important, as in the case of large-scale distribution networks, the local solutions can be far from optimal. On the other hand, to verify this in the 21-node test feeder, we exhaustively evaluated all the possible solution points, i.e., 1140 options in the MATLAB software using nested-loops, where we found that the solution reported by the proposed MI-SDP is indeed the global optimum.

Regarding the comparison with metaheuristics in Figure 6, the best results reported for the 21-node test feeder in the literature are presented in comparison with our proposed MI-SDP model.
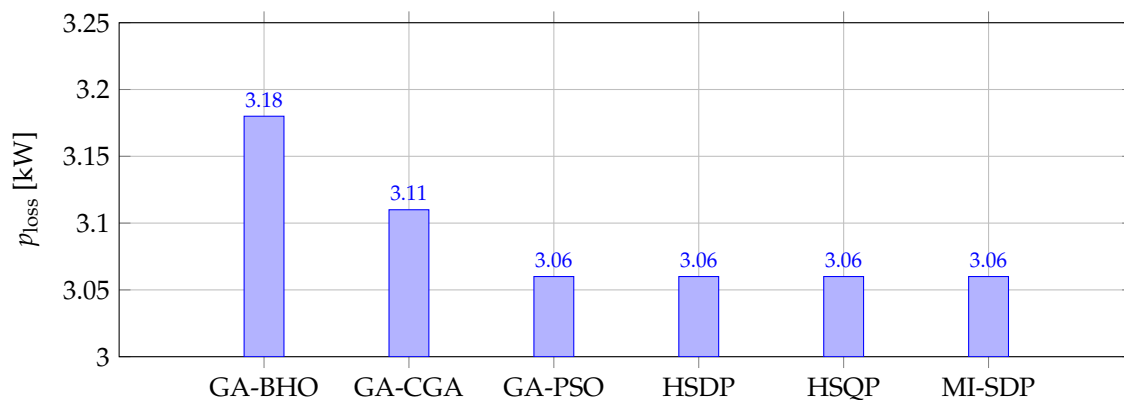

Figure 6. Final power losses when heuristics and metaheuristics are compared to the MI-SDP approach.

Note that results in Figure 6 show that some metaheuristics can find the global optimal solution of the problem. However, as mentioned in [18], at each simulation, the optimal solution has little changes, which implies that these present standard deviations different from zero. At the same time, the standard deviations of the HSDP, the HSQP, and the proposed approach are zero. This is because they work with convex solution spaces regarding the optimal power flow, which does not happen in the metaheuristic cases.

To present in a graphical manner the proposed MI-SDP approach that has the ability to find the global optimal solution in the 21-node test feeder, we plotted the information obtained after the exhaustive evaluation of the solution space, i.e., 1140 possible locations for the constant power sources, in Figure 7.
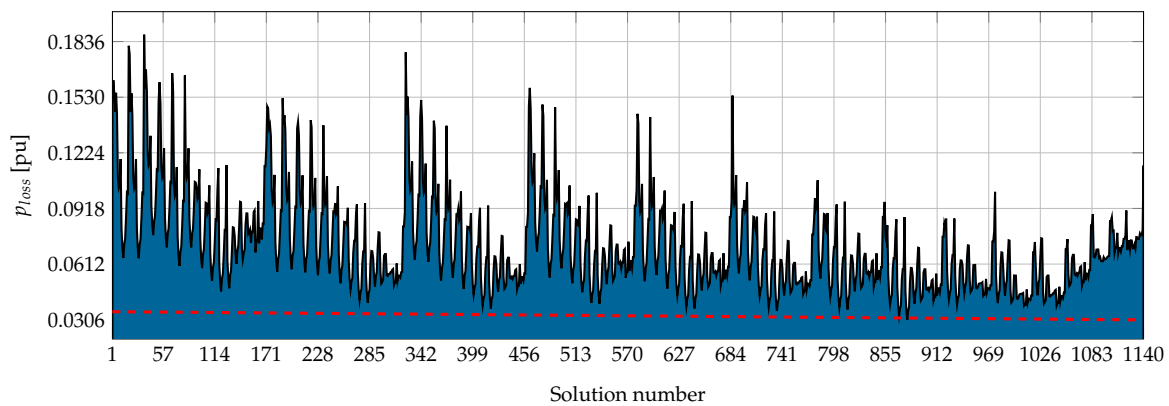

Figure 7. Solutions reached by the exhaustive evaluation in the 21-node test feeder.

Note that information plotted in Figure 7 confirms that the global optimal solution in the case of the 21-node test feeder is about 0.0306 pu of power loss, which is evidenced as no one solution is lower than the dashed line (see upper and lower peaks in the shaded figure). It is important to mention that the exhaustive evaluation took about 1150 s, whereas the GAMS and the proposed MI-SDP approach did not take more than 10 s, which confirms the efficiency of the optimization models in contrast with heuristic approaches.

6.2. 69-Node Test Feeder

In the 69-node test feeder case, the base case without distributed generation produces total power losses of about 1.5385 pu. When the MI-SDP model is implemented, the optimal nodes to locate GDs are 21, 61, and 64 with power generations of 1.4140, 10.2627, and 3.8803 pu, respectively. These power injections produce final power losses of 0.1573 pu, which implies that these generators allow the reduction of total grid power losses by about 89.78%, which corresponds with the results reported in [17,18] using the HSDP and HSQP approaches, respectively. It is worth mentioning that the solution determined by the proposed MI-SDP approach is indeed the optimum global, as it was verified by implementing an exhaustive approach that evaluates all the possibles node combinations for locating DGs, i.e., 50,116 options for this test feeder.

Table 4 reports the numerical results obtained by different solvers in the GAMS package.

Table 4. Comparison of the MI-SDP with the MINLP model solved in GAMS for the 69-node test feeder.

Solver	Nodes	Sizes [pu]	p_{loss} [pu]
DICOPT	$\{24, 61, 66\}$	$\{1.4969, 11.6024, 2.4577\}$	0.1577
CONOPT	$\{21, 61, 66\}$	$\{1.4969, 11.6024, 2.4577\}$	0.1577
MINOS	$\{24, 61, 64\}$	$\{1.4552, 10.2855, 3.8163\}$	0.1574
SNOPT	$\{26, 61, 68\}$	$\{1.3981, 12.0000, 2.1589\}$	0.1599
SBB	$\{22, 61, 64\}$	$\{1.4961, 10.2446, 3.8163\}$	0.1573
BONMINH	$\{21, 61, 64\}$	$\{1.4973, 10.2434, 3.8163\}$	0.1573

From Table 4, we can observe that only two solvers in the GAMS determined the global optimum regarding final power losses (see BONMINH and SBB solvers). However, the BONMINH solver identified the same nodes of the proposed MI-SDP approach, with some variations in the total power injections, while the SBB changed node 21 for the node 22, with similar results regarding power generation when compared with the BONMINH and the MI-SDP approaches. These variations may be attributed to the higher connectivity between nodes 21 and 22, as observed by the small resistance value of 0.014 pu (hence, higher connection) between nodes as shown in Table 4. This means that there are multiples power injections that can achieve the same objective function value, as shown in [17].

Remark 5. *Despite some GAMS's solvers reaching the global optimum of the problem, this cannot always be guaranteed. Furthermore, as the solution space grows, these solvers may fall to local optima, whereas the proposed technique will continue to reach the global optimum.*

Regarding the comparison with metaheuristics, Figure 8 reports the final power losses between the MI-SDP and the comparative approaches.
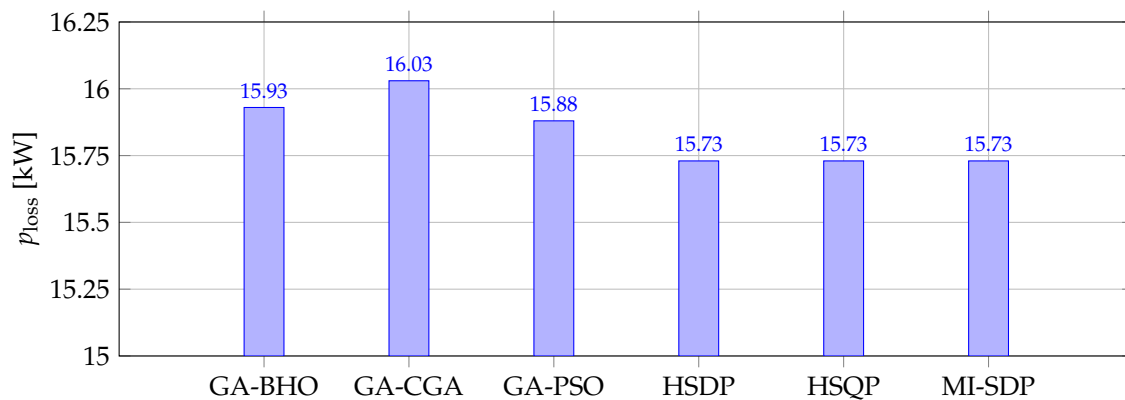

Figure 8. Final power losses when heuristics and metaheuristics are compared to the MI-SDP approach in the 69-node test feeder.

Based on the results reported in Figure 8, we can confirm that when the solution space increases for the metaheuristic approaches, it is difficult to achieve the global optimum, as it gets stuck in a local optimum. This occurs due to not having a sense-solve optimal power flow problem with continuous metaheuristics, as in the literature, this problem can be solved precisely with convex formulations.

Remark 6. *Note that HSDP and HSQP methods also achieve the global optimum of the problem. However, when presenting these results, it cannot be ensured their solutions are the global optimum, whereas the proposed method*

can ensure it. This indicates that the HSDP and HSQP methods may take out optimal nodes depending on the size of the test systems.

To demonstrate that the proposed MI-SDP approach can find the global optimum in the 69-node test feeder, at the same time, the GAMS solvers were stuck in locally optimal solutions; we present a simulation case where four distributed generators were available for installation considering 40% of thte distributed generation penetration, as reported in Table 5.

Table 5. Comparison of the MI-SDP with the MINLP model solved in GAMS for the 69-node test feeder with four distributed generators.

Solver	Nodes	Sizes [pu]	p_{loss} [pu]	Proc. Times [s]
DICOPT	$\{21, 61, 64, 69\}$	$\{1.4985, 10.2452, 2.7200, 1.0931\}$	0.1557	12.4270
CONOPT	$\{21, 61, 64, 69\}$	$\{1.4985, 10.2452, 2.7200, 1.0931\}$	0.1557	3.0360
MINOS	$\{21, 27, 61, 66\}$	$\{1.1471, 0.3497, 11.6025, 2.4577\}$	0.1575	4.8520
SBB	$\{21, 22, 61, 64\}$	$\{0.8870, 0.6103, 10.2434, 3.8163\}$	0.1573	5.4250
BONMINH	$\{21, 61, 64, 65\}$	$\{1.4985, 10.2452, 2.3027, 1.5105\}$	0.1557	4.5550
MI-SDP	$\{21, 61, 64, 67\}$	$\{1.4986, 10.2452, 2.4010, 1.4122\}$	0.1556	14.4688

From Table 5, we can observe that all tested solvers in GAMS were stuck in locally optimal solutions, while the proposed MI-SDP approach dealt with the global optimal one by selecting nodes 21, 61, 64, and 67. It is worth mentioning that none of the GAMS solvers identified node 67 in the optimal solution, as GAMS solvers identified node 69 as the best solution, reported with an objective function value of 0.1557 pu (see CONOPT and DICOPT). These solutions are suboptimal, as the global optimum found by the MI-SDP approach was 0.1556 pu.

Regarding the processing times required by the different tested approaches, it was observed that all of them took less than 15 s, which can be considered very efficient for problems related to the optimal location and sizing of distributed generators (planning problems). These times are irrelevant, as the main interest is to guarantee the global optimum. In addition, the procedure of installing these power sources can take weeks or months. This means that the times in the order of seconds in the planning stage are negligible for any practical purpose.

6.3. Isolated Operation Scenario

One of the most important applications of distributed generation is the possibility of providing electrical service to remote (i.e., rural areas), where conventional power systems are non-existent [30]. To demonstrate the ability of the proposed MI-SDP model to be applicable to isolated DC distribution networks, we considered the following: (i) the slack source was removed from the 69-node test feeder, (ii) the operative voltage bounds were assigned between 0.90 and 1.00 pu, (iii) three distributed generators could be located in the DC network, and *iv)* the capability of each distributed generator was 20 pu.

Once that simulation scenario was evaluated, the proposed MI-SDP approach found that the best locations of these distributed generators were nodes 19, 50, and 61, with power injections of about 5.1695, 15.0132, and 18.7636 pu, respectively. The final power loss in this isolated operation scenario was about 0.0538 pu, while the low-voltage profile was about 0.9952 pu at node 11. These results confirm the possibility of extending the proposed MI-SDP model to rural applications, especially for providing service to non-interconnected zones.

7. Conclusions and Future Works

This paper presents a new mathematical formulation for the problem of optimal location and sizing of distributed generators in DC networks with interconnected or isolated operative conditions. This new formulation transforms the classical MINLP model into the MI-SDP to find the global optimum by using the classical branch and bound method, as each node explored is convex, based on the semidefinite programming reformulation of the power flow model. Numerical results demonstrated that the MI-SDP approach reached the best solution reported in the literature for both test feeders (global optimum), which was not possible with some MINLP solvers available in GAMS and the classical metaheuristic approaches. Based on the results reported in this research, it was possible to affirm that the proposed MI-SDP approach solved the problem of the optimal location and sizing of DGs in DC networks, as this approach guarantees the determination of the global optimum in a formal mathematical manner.

As for future works, it would be beneficial to work on the following topics: (i) the extension of the MI-SDP approaches to the problem of the optimal location of DGs and capacitor banks in AC distribution networks; (ii) the reformulation of the problem of the optimal operation of battery energy storage systems in DC networks using convex models instead of combinatorial approaches, and (iii) exploration of the convexification of the problem of the optimal reconfiguration of DC networks.

Abbreviations

The following abbreviations are used in this manuscript:

Acronyms

AC	Alternating Current
B&B	Branch and Bound
DC	Direct Current
DG	Distributed Generator
NLP	Nonlinear Programming
MINLP	Mixed-Integer NLP
CVX	Matlab Software for Disciplined Convex Programming
GAMS	General Algebraic Modeling System
OPF	Optimal Power Flow
SDP	Semidefinite Programming

Sets and subscripts

$\mathcal{L}$	Set of lines
$\mathcal{N}$	Set of nodes
d	Demand
gd	Distributed source
s	Slack node
i or j	Node

Parameters

G	Conductance matrix
G_{ij}	Component of the conductance matrix that relates nodes i and j
g_{km}	Conductance between nodes k and m
p^d	Power demanded by loads
p^s	Power generated by generators
p^{gd}	Power generated by distributed source

Variables

v_i	Voltage profile at node i
V	Voltage profile vector
X	Matrix of cross-product of voltage i with voltage j
W_{ij}	cross-product of voltage i with voltage j

Control variables

p^s	Power generated by conventional generator
p^{gd}	Power generated by renewable energies
p_{km}	Power flow between nodes k and m
y_i	Binary variable for location of DG
P^s	Power generated vector by conventional generator
P^{gd}	Power generated vector by renewable energies

Limits

N_{gd}^{max}	Maximum number of DG
v^{min}, v^{max}	Minimum and maximum voltage profile
V^{min}, V^{max}	Minimum and maximum voltage profile squared
$p^{s,min}, p^{s,max}$	Minimum and maximum by conventional generator
$p^{gd,min}, p^{gd,max}$	Minimum and maximum by generated source
$p_{km}^{min}, p_{km}^{max}$	Minimum and maximum power flow between nodes k and m

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