

CHAOTIC SPACE-TIME

ESPACIO-TIEMPO CAÓTICO

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RECIBIDO EL 23 DE ENERO DE 2014 Y APROBADO EL 25 DE JUNIO DE 2014

RESUMEN ABSTRACT

En este artículo se demuestra cómo la consideración de una mecánica caótica suministra una redefinición del espacio-tiempo en la teoría de la relatividad especial. En particular, el tiempo caótico significa que no hay una posibilidad de definir el ordenamiento temporal lo que implica una ruptura de la causalidad. Las nuevas transformaciones caóticas entre las coordenadas espaciotemporales 'indeterminadas' no son más lineales y homogéneas. Los principios de inercia y el impulso de la conservación de la energía ya no son bien definidos y en todo caso no son más invariantes.

In this paper we have shown how the consideration of a chaotic mechanics supplies a redefinition of special-relativistic space-time. In particular chaotic time means no possibility of defining temporal ordering and implies a breakdown of causality. The new chaotic transformations among 'undetermined' space-time coordinates are no more linear and homogeneous. The principles of inertia and of energy-impulse conservation are no longer well defined and in any case no more invariant.

PALABRAS CLAVE KEY WORDS

Caos, dinámica no lineal, relatividad, espacio-tiempo.

Chaos, non-linear dynamics, relativity, space-time.

Introduction

Since special relativity has appeared (Cf. Poincaré, "L'état actuel", "Sur la dynamique", "On the dynamics", Cf. Einstein, "Zur Elektrodynamik")¹ the hierarchy between geometry and dynamics in physical theories has been questioned and turned up down.² Following Klein's "Erlanger Programm" (Cf. Klim), as well known, geometry can be defined by its invariance transformation group. Thus, one can define geometry as, a non — a priori given object, but as a physical, operational structure given by the dynamical invariance transformation group. This is also in a more evident way at the ground of general relativity theory.

In recent times, Poincaré's (Cf. Poincaré, *Les méthodes*) and Born's³ analysis on the problem of the actual predictability and determinism of classical mechanics have been independently rediscovered and developed, pointing out the very general emergence of chaos as an invariant feature of classical and quantum mechanics. (Cf. Lorentz)⁴ Indeed, already very simple and common mechanical systems give rise to chaos.⁵ Hence dynamical chaos has very fastly and relevantly modified our mechanical representation of the world, and in some way it has found a geometrical counterpart in the idea of fractals.⁶ However, this kind of relation between dynamical chaos and fractal geometry rests only on external grounds and has never affected our idea of time. Only Arecchi (Cf. Arecchi, Basti, Boccaletti and Perrone) has recently attempted a new definition of time starting from bifurcations of nonlinear systems and Prigogine has recognized as a consequence of chaos the breakdown of time reversal symmetry (Cf. Prigogine, *La nascita, Le leggi*).

In this paper we would like to show how the consideration of a general chaotic mechanics supplies a redefinition of special relativistic

¹ See also: Eddington (*The Mathematical*). For a complete historical and theoretical analysis of special relativity see: Tyapkin, "'Expression of", Giannetto, "Henri Poincaré and".

² Cf. Barut, *Geometry*, and references therein. For the physical and epistemological relevance of this step in the construction of the physical theory see: Finkelstein, "Matter".

³ Born's work is almost unknown; see in particular: Born and Hooton ("Statistical dynamics"). Cf. Giannetto, "Max Born and".

⁴ For a review see, for example: Bai-lin, Eckmann and Ruelle. Cf. Ruelle (*Elements*), Rasband, Schroeder. See also: Earman. Indeed, it has been challenged that chaos could be an invariant feature Zak. However, when we use a general-covariant formulation of classical mechanics or special relativity, see for example: Havas, we yield an inescapable invariant appearance of chaos.

⁵ See Born's papers quoted in note 3, and also: Cf. Moore, Cf. da Costa and Doria.

⁶ For some trials to use fractals in a priori characterization of space-time, see: Svozil, Svozil and Zeilinger, *The Dimension*, Zeilinger and Svozil ("Measuring the Dimension").

chrono-geometry (space-time) as a whole. In such a framework a new chaotic space-time is defined, of which Lorentz space-time are found as a limiting case corresponding to a non-chaotic (relativistic) mechanical regime.

As well known, Lorentz transformations can be obtained by the only requisite of the preservation of temporal ordering of events, which is relativistic causality (Cf. Whitehead, Cf. Zeeman, Cf. Agodi and Cassarino) for a critical and historical review of such argument (see also: Segal).

Chaotic time means no possibility of a local or global, absolute or relativistic, temporal ordering of events, that is a breakdown of causality; hence chaotic time implies a breakdown of Lorentz transformations. The transformations defining the new chrono-geometry become event-dependent (already for inertial reference frames, in relation to dynamical chaos). We have always to consider transformations from 'undetermined' space-time coordinates $(\chi_\mu \pm \Delta\chi_\mu)$ to other 'undetermined' space-time coordinates $(\chi'_\mu \pm \Delta\chi'_\mu)$ (Cf. Il-Tong).

From this point of view, we have no longer a unique spacetime coordinatization even for a particular reference frame and no space-time invariants as the metric interval.

Chaotic time

In the last few years, chaotic *phenomenology* has been extensively revealed in various disciplines and, in particular, in several domains of the physics.⁷ More specifically, chaos has been observed and studied ranging from General Relativity to Quantum Relativistic systems (Cf. Zardecki, Cf. Chernikov, Tél, Vattay and Zaslavsky, Cf. Balakirev, Cf. Barrow, "Chaotic behaviour", "General relativistic", Cf. Di Prisco, Herrera, Carot, Cf. Dilts). As it is well known an important feature of chaotic mechanical problems is the sensitive dependence on initial condition of the dynamical evolution: two different trajectories starting very close rapidly diverge. The above property causes an exponential growth of initial errors.

⁷ For a recent review see, for example: Anderson, Arrow, Pines.

In this paper we focus our attention on Special Relativistic systems that exhibit non-linear chaotic behaviours. Here, we give only the general features. First of all, we would like to point out the argument to justify our new analysis: as long as we have to recover classical mechanics as a limiting case of special relativistic mechanics, chaos must emerge also within special relativity. Its dynamical equations are formally equivalent to the classical ones. In this case, the total error bar, associated with one of spatial coordinates of x_μ for a moving particle, evolves, as a function of the proper time τ , as follows:

$$\varepsilon(\tau) = \varepsilon_0 2^{\lambda\tau} (\lambda > 0) \quad (1)$$

where λ is the standard Lyapunov exponent (Cf. Rasband).

Equation (1) gives the τ -evolution of the initial error bar ε_0 as analytically determined by the mechanical laws.

After using eq. (1), one can easily calculate the uncertainty $\Delta[u]$ associated with i -th chaotic component of u_μ , directly from kinematic definition of the four-velocity:

$$u_\mu \equiv \frac{dx_\mu}{d\tau} \quad (2)$$

$$\Delta[u_i] = K 2^{\lambda\tau}. \quad (3)$$

In the classical physical-mathematical framework of continuum space-time the more preservative and consistent position is one to assume ε_0 as infinitesimal. Consequently it seems correct to consider K as a finite constant.⁸

The first three components of the four-velocity are related to the three-vector $\vec{v} \equiv d\vec{x}/dt$ by the well-known relation:

$$u_i = \gamma v_i \quad (4)$$

In equation (4):

$$\gamma \equiv \frac{1}{(1 - v^2/c^2)^{1/2}} \quad (5)$$

⁸ For the probabilistic approach to classical mechanics, see Born's papers quoted in Ref. 5.

We have to note that, in order to relate a generic coordinate time to proper time, the quantity V , which we must consider in eq. (5), is nothing else than v . Thus, the error on particle velocity is an error also on the velocity associated to the motion of proper reference frame. Here, we stress that when we study accelerated motions Special Relativity can be formulated only for instantaneous inertial reference frames. A simple algebraic calculation of error propagation permits to write explicitly the total uncertainties:

$$\Delta[u^2] \Delta[v^2/c^2] \Delta[(1-v^2/c^2)]^{1/2}$$

and $\Delta[\gamma]$ induced by $\Delta[u_i]$ on u_i^2 ,

$$v^2/c^2, (1-v^2/c^2)^{1/2}$$

and γ respectively:

$$\Delta[u_k^2] = \begin{cases} 2u_k \Delta[u_k] & |u_k| > \frac{\Delta[u_k]}{2} \\ |u_k| \Delta[u_k] + \left(\frac{\Delta[u_k]}{2}\right)^2 + u_k^2, & |u_k| < \frac{\Delta[u_k]}{2} \end{cases}, \quad (6a)$$

$$\Delta[v^2/c^2] = \frac{2(2u^2 + c^2) \Delta[u^2]}{(u^2 + c^2)^2 - \Delta^2[u^2]}, \quad (6b)$$

$$\Delta[(1-v^2/c^2)^{1/2}] = \frac{2\Delta[v^2/c^2]}{[1-v^2/c^2 + \Delta(v^2/c^2)]^{1/2} + [1-v^2/c^2 - \Delta(v^2/c^2)]^{1/2}}, \quad (6c)$$

$$\Delta[\gamma] = 2 \frac{\Delta[(1-v^2/c^2)^{1/2}]}{(1-v^2/c^2)^{1/2} - \Delta[(1-v^2/c^2)^{1/2}]}, \quad (6d)$$

Hence, in the chaotic hypothesis, we can exactly extrapolate that, running τ , the error bar on γ diverges as $\Delta[u^2]^{1/2}$:

$$\Delta[\gamma]_{\tau \rightarrow \infty} = (\Delta[u^2])^{1/2}. \quad (7)$$

From equations (3) and (6a) we thus obtained:

$$\Delta[\gamma] \sim A 2^{\frac{1}{2}\tau} \quad (8)$$

Finally, standard equations of Special Relativity permit to resolve t as a function of τ :

$$t = \int_{\tau_0}^{\tau} \gamma d\tau. \quad (9)$$

Equation (9) provides a direct route in order to calculate error bar $\Delta[t]$:

$$\Delta[t] = \int_{\tau_0}^{\tau} \{(\gamma + \Delta[\gamma]) - (\gamma - \Delta[\gamma])\} d\tau \quad (10)$$

From this formula, we can see that there is no reference frame for which the error on time is zero: also in the case $v=0$ (proper time) the error is not zero, that is also proper time as evolution variable is chaotic. Combining equations (8) and (9) it is immediate to demonstrate that the uncertainty on the time $t(\tau)$ results:

$$\Delta[t] \sim B 2^{\frac{1}{2}\tau}. \quad (11)$$

On the basis of the above concluding result we can assert that: even an infinitesimal initial error, which affects one of the spatial coordinates, induces a finite error on the relative time t . Moreover, in the chaotic hypothesis, the proper-time evolution of $\Delta[t]$, as analytically governed by mechanical laws, diverges very rapidly with, at least, an exponential growth.

Chaos and Lorentz transformations

Let us now consider how temporal ordering and causality are violated. Temporal ordering of events can be defined as:

$$x_\mu < y_\mu \text{ iff } x_0 < y_0 \rightarrow \Delta s^2 = c^2 \Delta t^2 - \Delta |\vec{x}|^2 > 0, \quad (12)$$

here $x_0 \equiv ct_x$ and $y_0 \equiv ct_y$.

Thus, if we have a $x_0 \pm \Delta^{\pm}[x_0]$, with $\Delta^+[x_0] + \Delta^-[x_0] = c\Delta[t_x]$ non-negligible total error bar, we in general can write:

$$x_0 \pm \Delta^{\pm}[x_0] \prec y_0 \pm \Delta^{\pm}[y_0] \rightarrow \Delta s^2(x_{\mu}, y_{\mu}) \neq 0 \quad (13)$$

If we now perform new Lorentz transformations

$$t' \pm \Delta^{\pm}[t'] = (t \pm \Delta^{\pm}[t] - (x \pm \Delta^{\pm}[x])(v \pm \Delta^{\pm}[v])/c^2) \gamma \pm \Delta^{\pm}[\gamma] \quad (14a)$$

$$x' \pm \Delta^{\pm}[x'] = (x \pm \Delta^{\pm}[x] - (v \pm \Delta^{\pm}[v])(t \pm \Delta^{\pm}[t])/c^2) \gamma \pm \Delta^{\pm}[\gamma] \quad (14b)$$

we have also that $x_0 < y_0$ does not imply $x'_0 < y'_0$. So temporal ordering cannot even be defined and in any case it is not an invariant feature of the world of events. Thus, of course, we also find as obvious consequences Prigogine's result of the breakdown of time reversal: irreversibility. The principles of inertia and of energy-impulse conservation are no longer well defined in correspondence with the error $\Delta[u]$ and they are however no more invariant. In fact these new transformations must be used to define a chaotic space-time: they are no more linear and homogeneous and they change hypothetical-inertial in non-inertial reference frames. As it could be derived directly from the uncertainty on $\Delta[u]$, it is no more possible to distinguish between inertial and non-inertial reference frames.

Conclusions and prospects

In this paper we have shown that it is enough only an infinitesimal error on the initial condition (due, for example, to an experimental measure performed with an ideal infinitesimal precision), in order that the analytical chaotic dynamics of the system is affected by finite and rapidly increasing error bars $\Delta[u]$ and $\Delta[t]$.

In the previous section we have already discussed as the implications of the above results break down the usual mechanical special-relativistic theory, involving a new chaotic space-time.

These preliminary considerations anticipate a systematic analysis about the way the presence of initial error imposes a revision of the physical-mathematical framework of relativistic and classical mechanics and paves the way for a new finite or, equivalently, probabilistic perspective.

Therefore, we could or should at least introduce, as it was done in quantum (relativistic) physics and how it was suggested by Born even for classical mechanics, probability distributions for the space-time and the other related physical variables, which can no longer be considered as actual physical variables, by changing to an intrinsic event representation where the event-‘fields’ themselves are the physical variables.

It can be shown as this is the case also for Galilei transformations and classical mechanics, because we have to consider a chaotic neo-newtonian space-time.

The general features of this new theoretical framework imply the need to link operational definitions of physical quantities with error theories, to which the chaotic phenomenology has given big importance.

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Como citar:

Giannetto, Enrico, Giunta, Gaetano and Marino Domenico. "Chaotic space-time". *Discusiones Filosóficas*. ene.-jun. 2014: 87-97.