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Mathematical modeling with digital technological tools for interpretation of contextual situations

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Abstract. This article has the goal of proposing physical contextual situations modeling as a way to interpret mathematical representations that are produced by digital technological tools. Thus, there is an experimental situation-problem about a physical phenomenon that is modeled through video analysis and dynamic geometry software; the methodological model Cuvima conducts the experimental activity. Pre-testing and post-testing measuring instruments were designed to obtain the information and previous conceptions of ten graduate students in Mathematical Education, which showed a conceptual change. Similarly, results prove that digital technology, from a didactical sequence, supports and strengthens experimental work simplifying modeling processes of a physical phenomenon, promoting the use of mathematical representations to solve a situation-problem.

1. Introduction

Mathematics and Physics are two knowledge areas that have developed interactively [1-5]. Nonetheless, there is a dichotomy in its teaching, polarized like this: in physics it is usual to find math as an operative and memoristic tool for formula calculation, and sometimes as a reasoning tool to interpret the physical world; on the other hand, physics is considered as an example of application of mathematical concepts previously defined abstractly in math teaching. This situation does not favor the conceptual comprehension of any of both sciences [2,6,7].

The above-mentioned is reflected in the expression's students use to interpret a physical phenomenon, called previous ideas [8] or common reasoning [9], which are inadequate explanations; in some cases, incomplete, and in others, not according to the accepted concepts from a scientific viewpoint. So, this raises a question: how to promote a significate mathematical learning to interpret physical situations?

To this end, the present article has as its goal to present a physical situation that may be modeled through digital devices following the Cuvima methodological model [10]. We refer to significate learning from the perspective of Ausubel, Hanesian & Novak [11], as acquired knowledge to be applied later to a contextual situation, which, in this case, it refers to the physical phenomenon of a ball rolling over an inverted half-cycloid and a mathematical modeling perspective through digital technology will be used.

2. Theoretical framework

Mathematical modeling substantially, may be described as the process of translating a real-world situation to a mathematical model that represents it, considering that this representative model allows predicting and interpret the ideal situation in the real world [10,12-14].

Mathematical modeling is vital because it integrates mathematics with other disciplines such as physics, biology, and engineering, making sense of them [12]. Nevertheless, conducting a modeling process is not a simple task. There are diverse ways to do it, as the varied cycles of mathematical modeling show [4,10,15-17]. One of the modeling cycles to be used to guide the mathematical modeling activity in this article is the Cuvima methodological model [10], which incorporates the use of digital devices (smartphones, tablets, and computers) as significate mediating tools in the experimentation of a physical phenomenon and obtaining modeling data. The model has four frames and will be adapted, as shown in Figure 1.

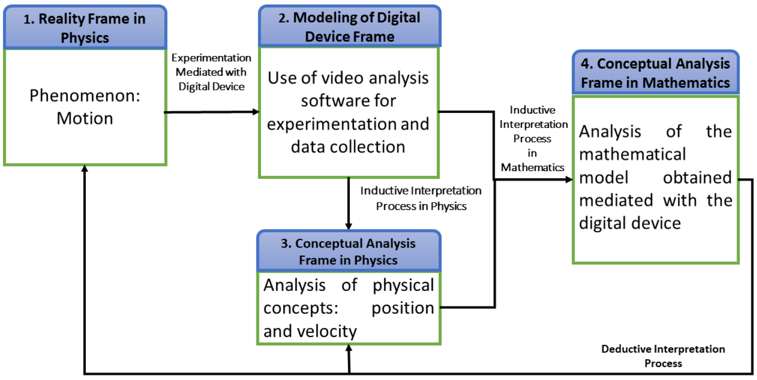

Figure 1. Modeling of the motion through Cuvima.

In the Figure 1, the four frames of the Cuvima model are described:

- Reality frame in physics: parts from the situation-problem about the phenomenon of motion of a ball over the inverted cycloid.
- Modeling of digital device frame: in this frame, the student must conduct the experiment based on digital resources to obtain data. The digital device provides data through some representation that may be an image, tabular registry, or plane graphic.
- Conceptual analysis frame in physics: it is connected to the second frame through an inductive interpretation physical process, which consists in conducting an analysis on implicit physical concepts in representations of the experiment provided by the digital device (can be done through group discussion between the teacher and the students).
- Conceptual analysis frame in mathematics: it is conducted through an inductive interpretation of the mathematics process, that is to say, in obtaining a math model that represents the phenomenon. In this case, digital technology will serve as a mediator in obtaining this model through an algebraic expression or a geometric or graphic representation. The mathematical model obtained through this process that represents the physical phenomenon in reality must be validated through inductive interpretation, which means using the model to describe reality and concepts through Physics.

3. Methodology

The following problem-situation was presented to 10 graduate-level students in mathematics education: by letting role two balls simultaneously, one on a straight trajectory, and the other on a curve (inverted cycloid) (Figure 2), and the following questions were asked: - which of them will arrive from point A to point B first? Which will arrive with a higher speed to point B? How would you describe the components of the ball velocity over the curved trajectory? Do they increase, decrease, alternate, or remain constant? Also, students were asked to use mathematical reasoning to interpret the situation. Such a situation of motion of a body over an inclined plane was studied by Galileo [17], and the motion over an inverted cycloid was resolved elegantly by Isaac Newton [18,19].

Figure 2. Experimentation of the problem situation.

Questions about the situation-problem were presented to the student before the experimentation (allowing them to imagine what may happen). After, the experimentation was carried out, but it was not modeled and analyzed with digital technology. Finally the situation was modeled and analyzed with the support of digital technology resources, students conducted the experimentation together, using the following devices: digital camera (smartphone), tripod, trajectory structure, balls, video beam, board, a computer per student, a video analysis software Tracker Physics, and a dynamic geometry software Geogebra.

The study is qualitative, and the information on the explanations provided by the students was collected through print guides, photos, and videos. The researcher had a journal for each session. Three one-hour sessions were conducted, referring to: (1) Situation approach, exploration of previous ideas, and experimentation, (2) analysis of results through an analysis video software, (3) a compilation of explanations posterior to the experimentation with digital technology and the interpretation of the experimental results to show the conceptual change.

4. Results

4.1. Results before the experimentation

Here the previous ideas were collected. Four of the ten students considered that the ball rolling over the inverted cycloid would reach the ground first and with higher speed because the inclination would result in greater acceleration. Four of the ten students considered the ball rolling over the inclined plane would reach first and with higher speed because it has a shorter trajectory. Finally, two of the ten students considered that both balls would reach at the same time and with the same speed because velocities, although they change, they compensate at a point because they are homogeneous. None of the explanations were mathematical.

4.2. Results posterior to the experimentation without digital technology

Once the experimentation without digital technology was conducted, students changed their responses. When observing that the ball that reaches point B is the one rolling over the inverted cycloid, they considered that it reached it with greater speed. Between the arguments they mentioned is that the curved surface gave a higher velocity to the ball. The fact they say this shows that students manifest their interpretations based on the form of trajectories, and in no case, they used mathematical reasoning for their explanations. About the horizontal speed in the inverted cycloid, 3 of 10 students said that it is constant, 6 of 10 said it always increases, and 1 of 10 says that it initially increases and that it decreases afterward.

4.3. Results after digital technology modeling
Students conducted mathematical modeling with the guidance of the Cuvima model (Figure 1). The experiment was recorded, and the video was uploaded to the video analysis software Tracker Physics to capture the movement position data of each ball in time (see Figure 3).

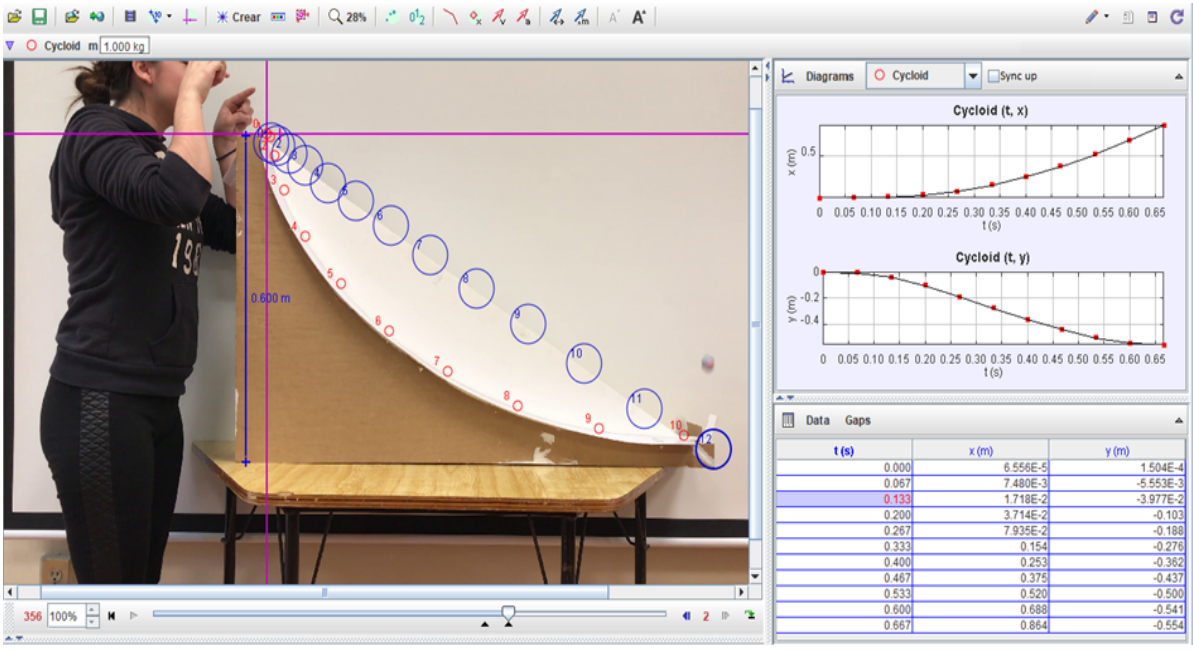

Figure 3. Modeling conducted with Tracker Physics and graphical representations of the movement of the ball over the inverted cycloid.

The data was exported to a Geogebra spreadsheet, allowing a much more dynamic analysis of the results to obtain a mathematical model of graphic representations over a cartesian plane (Figure 4). As there were no responses based on mathematical analysis, students used the representations (cartesian plane, value table) to, through a socialization process, conclude that horizontal speed always increases, while vertical speed increases in a determinate time frame and then decreases until reaching cero (repose).

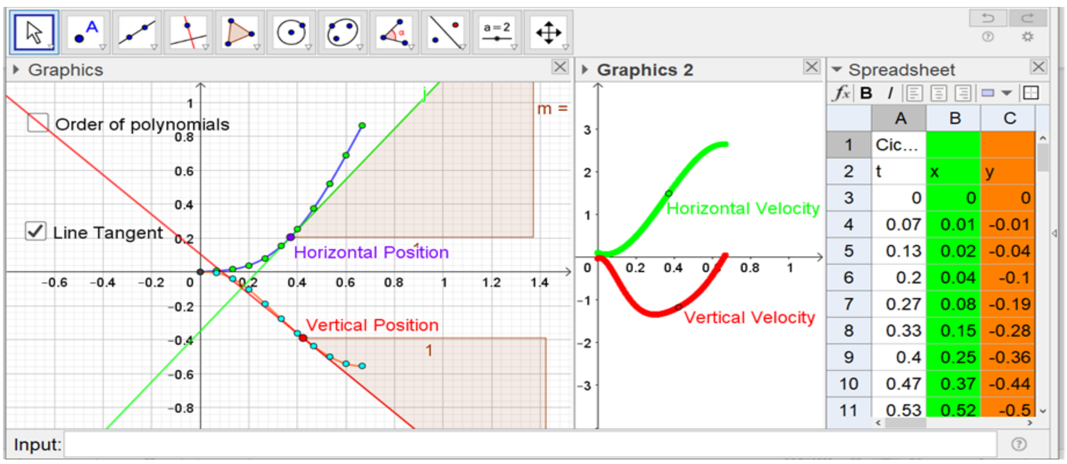

Figure 4. Mathematical modeling with Geogebra.

Another way to interpret and solve the situation-problem is following Tall [20] idea of exploring mathematical through sensitivity, using the sense of tact. Using this model, the tangent line of the curve was represented horizontally and vertically versus time (x vs. t and y vs. t) with one of their hands (see Figure 5). When they slide the extended hand from left to right over the graphs of position against time, the students detected when the inclination of the tangent line to the curve (x vs. t and y vs. t) was changing and determined there was an inflection point, which allowed them to understand if speed increased or decreased in a period of time, considering that the tangent slope represents speed.

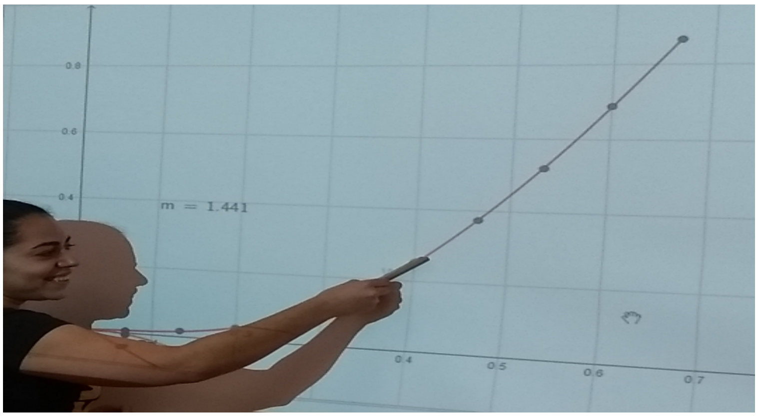

Figure 5. Sensibilization of the tangent slope to interpret speed.

Upon obtaining the graphical representations of the velocity of each ball, the students concluded that they arrived at approximately the same speed, which is consistent with the theorem of the conservation of mechanical energy.

Regarding the concepts of physics, there was a discussion about the brachistochrone property of the inversed cycloid from speed representations obtained using both software programs.

5. Conclusions

Mathematical model as a process that integrates math with physics or other disciplines promote significate learning, in the sense that mathematical objects are used to represent, interpret, and solve a contextual situation, in this case, movement. Likewise, the Cuvima model served as a methodological frame for the professor to guide the modeling process, and organizing the used resources, that is to say, to propose the adequate time to model the object with digital technology, and, later, discuss and interpret the obtained results where the inclusion of digital technologies established a didactical bridge for experimentation in the classroom and obtaining representations. Generally, the mathematical model of a physical situation is algebraic; nevertheless, here, it is considered that, in the learning process, physical or mathematical concepts in a physical phenomenon may manifest in diverse representations.

Technology as a mediating tool in the experimentation of mathematical modeling is a support element to analyze previous ideas as it provides complex models of representations and promote an adequate interpretation of the contextual situation to approach a conceptual change. However, the interpretation of mathematical and physical concepts must reach a didactical sequence that is implicit in the activity.

References

[1] Gingras Y 2001 What did mathematics do to physics? *History of Science* **39(4)** 383
[2] Karam R 2015 Introduction of the Thematic issue on the interplay of physics and mathematics *Science & Education* **24(5)** 487
[3] Kjeldsen T H, Lützen J 2015 Interactions between mathematics and physics: The History of the concept of function teaching with and about nature of mathematics *Science & Education* **24(5)** 543
[4] Kline M 1981 *Mathematics and the Physical World* (New York: Dover)

[5] Poincaré H 1907 *The Value of Science* (New York: The Science Press)
[6] Tuminaro J, Redish EF 2007 Elements of a cognitive model of physics problem solving: Epistemic games *Physical Review Special Topics-Physics Education Research* **3(2)** 020101-1
[7] Uhden O Karam R, Pietrocola M and Pospiech G 2012 Modelling mathematical reasoning in physics education *Science & Education* **20(4)** 485
[8] Hierrezuelo J, Montero A 2006 *La Ciencia de los Alumnos. Su Utilización en la Didáctica de la Física y la Química* (México: Fontamara)
[9] Viennot L 1979 *Le Raisonnement Spontané en Dynamique Élemetaire* (París: Hermann)
[10] Cuevas C A Villamizar F Y, Martínez A 2017 Aplicaciones de la tecnología digital para actividades didácticas que promuevan una mejor comprensión del tono como cualidad del sonido para cursos tradicionales de física en el nivel básico *Enseñanza de las Ciencias* **35(3)** 129
[11] Ausubel D, Novak J, Hanesian H 2012 *Psicología educativa: Un punto de vista cognoscitivo* (México: Trillas)
[12] Cirillo M, Pelesko J A, Felton-Koestler M D, Rubel L 2016 Perspectives on modeling in school mathematics *Annual Perspectives in Mathematics Education 2016: Mathematical Modeling and Modeling Mathematics* (Electronic materials vol 1) ed Hirsch C and McDuffie A (Sweden: National Council of Teachers of Mathematics) chapter 1 p 3
[13] Espinosa-Castro J F 2018 Software: Aplicaciones de ecuaciones diferenciales de primer orden *Investigación y Praxis en la Enseñanza de las Matemáticas* ed Salazar Torres J P, Contreras Santander Y L, Espinosa-Castro J F (Barranquilla: Ediciones Universidad Simón Bolívar)
[14] Villamizar F Y 2018 *Modelo metodológico para promover conceptos físicos y matemáticos: hacia la orquestación de actividades didácticas con tecnologías digitales* (México: Centro de Investigación y de Estudios Avanzados del Instituo Politécnico Nacional)
[15] Kaiser G, Sriraman B 2006 A Global Survey of international perspectives on modeling in mathematics education *ZDM Mathematics Education* **38(3)** 302
[16] Rodríguez R, Quiroz S 2016 El papel de la tecnología en el proceso de modelación matemática para la enseñanza de las ecuaciones diferenciales *Revista Latinoamericana de Investigación en Matemática Educativa* **19(1)** 99
[17] Drake S 1975 The role of music in Galileo´s experiments *Scientific American* **232(6)** 98
[18] Boyer C 1986 *Historia de la Matemática* (Madrid: Alianza Universidad Textos)
[19] Goldstein H 1986 *Mecánica Clásica* (Barcelona: Reverté)
[20] Cuevas Vallejo C A, Pluvinage F, Dorier J-L, Hitt F, Tall D, Madrid H, Dufour S, Flores A, Moreno L, Martinez M, Cantoral R, Soto J, Parraguez M 2013 Una Aproximación Sensible al Cálculo *La Enseñanza del Calculo Diferencial e Integral: Compendio de Investigaciones y Reflexiones para Profesores, Formadores e Investigadores en Matemética Educativa* (Ciudad de México: Pearson) capitulo 6 p 127