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New anisotropic diffusion operator in images filtering

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Abstract. The anisotropic diffusion filters have become in the fundamental bases to address the medical images noise problem. The main attributes of these filters are: the noise removal effectiveness and the preservation of the information belonging to the edges that delimit the objects of an image. Due to these excellent attributes, through this article, a comparative study is proposed between a new diffusion operator and the Lorentz operator, proposed by the pioneers of anisotropic diffusion. For this, a strategy consisting of two phases is designed. In the first, called operator construction, the composition of functions is used to generate a new diffusion operator that meets with the conditions reported for this kind of the mathematical object. In the second phase, denominated filtering, a synthetic cardiac images database, based on computed tomography, is filtered using the aforementioned operators. According with the value obtained for the peak of the signal-to-noise ratio, the new operator shows similar performance to the Lorentz operator. The implementation of this new operator contributes to the generation of new knowledge in digital image processing context.

1. Introduction

The study of medical images represents a very important issue due to its field health incidence since it allows the incorporation of computational techniques both for obtaining the image and for its processing. The modalities of acquisition of this type of images are based on the principles of: ultrasound, nuclear emission, magnetic resonance and X-ray emission.

The images obtained by ultrasound (US) are essentially a measure of the acoustic response of an impulse to a signal with a particular frequency. Normally, an ultrasonic transducer is capable of producing acoustic waves by converting thermal, electrical and magnetic energy into mechanical energy; the piezoelectric effect being the most efficient process at the time of performing this conversion process, to obtain medical images using US [1].

Magnetic resonance imaging (MRI) allows an exceptional contrast in the images it generates. However, MRI has as its main disadvantages its high cost and the inability to access the patient during the acquisition process. In MRI, the existence of an induced magnetic field, in the patient's body, is used to generate images of specific sections of the human anatomy. When MRI is used to extract information related to cardiac structures, the term cardiovascular magnetic resonance can be used [2].

Nuclear emission imaging is a functional imaging modality and it is a technique used to detect the metabolic activity of cells by injection of radioisotopes. This type of tomography is fundamental in cardiovascular medicine because it is the only modality that allows the early detection of cardiac damage [3].

In this research, cardiac multilayer computed tomography (MSCT) modality is considered. The MSCT uses the principle of X-ray emission and is one of the most used imaging modalities for generating representations about morphology and functioning of the human heart, in two dimensions (2D) and three dimensions (3D) [4]. When the MSCT modality is applied, in medical images, the process of scanning sections of the organ to be studied occurs. This process consists in the passage, through the an specific organ, of a radiation emitted by an X-ray tube that rotates around the patient. The attenuation along the trajectory of such rays is measured by detectors, requiring tomographic reconstruction processes to generate the referred images [5].

On the other hand, the main problems that researchers face, when they intend to extract important information from the MSCT images, are the imperfections that they exhibit classified into three large groups: noise, artifacts and low contrast, which make the information analysis process a real challenge [6]. During the acquisition phase the contamination of the images occurs with a certain type of noise due to the emission of photons from the X-ray source. This noise can be modeled through a Poisson distribution and is generally called Poissonian noise [7], which is manifested by the appearance of random intensities that give a granular appearance to the MSCT images [8].

In MSCT images, the term artifact is theoretically attributed to a systematic discrepancy between the intensities belonging to the images obtained after the tomographic reconstruction and the true values for the attenuation coefficients of the objects present in the real image [9,10]. In a complementary way, the literature reports a huge amount of digital image processing techniques developed with the purpose of addressing the aforementioned imperfections [6]. One of these techniques is the spatial (both linear and non-linear) and frequency filters. Among the non-linear spatial filters is anisotropic diffusion which is characterized by performing, effectively, to address the noise problem exhibited by medical images [11].

Additionally, the anisotropic diffusion filter has a diffusion coefficient that, undoubtedly, represents the main operational focus in the present work. In this sense, it is necessary to establish that, the theoretical foundations of anisotropic diffusion were introduced by Perona and Malik [11]. These authors affirm that the transformation of an image must depend on a function (also known as a diffusion coefficient or operator) that increases diffusion near the edges and stops it in the interior regions. The immediate consequence of this process is the reduction of noise and the preservation of the contours that delimit the objects present in an image. Anisotropic diffusion allows you to take an original image and generate a filtered image that, in theory, is adequately conditioned to ensure the success of subsequent stages, typical of digital image processing.

Several metrics have been reported in the literature for the evaluation of the performance of filtering techniques. In the present work, it is planned to consider a metric related to noise removal for which, theoretically, synthetic images or numerical phantoms are required [6]. A numerical phantom, usually, consists of images and it can be considered as synthetic images of cardiac MSCT, in which the structure and the characteristics of a given real model, as for example real images of cardiac MSCT, are re-created. The conditions of an anisotropic diffusion operator (f) are presented as follows and Figure 1 [6,12].

(i) f(x) must be infinitely differentiable, where x represents the domain values of the anisotropic diffusion function.
(ii) f must be monotonous decreasing.
(iii) The flow (ϕ) is given by: ϕ = x* f (x). Additionally, ϕ must have a maximum in the set of positive real numbers.
(iv) The flow first derivative (ϕ') must generate ranges of positive real values, for domain values less than or equal to the maximum of f; while ϕ' must produce ranges of negative values for domain values greater than this maximum.

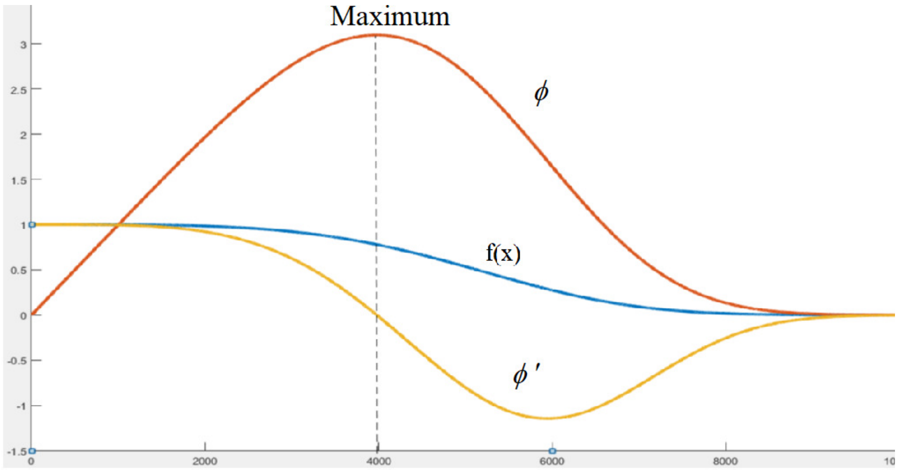

Figure 1. Anisotropic diffusion operator typical properties.

The pioneers of anisotropic diffusion were Perona and Malik who proposed the Lorentz [f1(x)] and the Leclerc [f2(x)] functions as diffusion operators [11]. The mathematical models of these functions are given by Equation (1) and Equation (2), respectively.

$$f1(x) = exp(-\frac{x}{k})^2 \tag{1}$$

$$f2(x) = \frac{1}{1 + (\frac{x}{k})^2} \tag{2}$$

In Equation (1) and Equation (2), k represents an arbitrary parameter, belonging to the integers set, linked to the image contrast. The k parameter is also known as the image gradient threshold. This parameter, usually, is estimated developing a tuning process in order to obtain its optimum value for a specific application.

Finally, the main purpose of this paper is to propose a comparative study between a new diffusion operator and the Lorentz operator. The importance of this type of work is that it allows enriching the filtering techniques available to address the imperfections described above, which drastically affect the quality of the information contained in the images. In this way, it could contribute to the generation of new knowledge through the proposal, in this case, of a new diffusion operator whose performance will be evaluated in the development of this research.

2. Materials and methods

2.1. Dataset

Poisson noise that is present in the real images of cardiac computed tomography, is re-created in synthetic databases represented by phantoms. To construct these synthetic databases, all images of a real cardiac MSCT database are selected, in which the left ventricle (LV) is present and an analysis of its most outstanding characteristics is made. The information obtained from the aforementioned analysis is used to construct a total of 2 numerical phantoms or synthetic databases.

2.2. Research phases

2.2.1. Design phase. For the purpose of pre-processing the synthetic images of cardiac MSCT containing the LV, were implemented in Matlab environment filtering strategies. These strategies are based on the comparative study of several operators in the context of the anisotropic diffusion filter.

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2.2.2. Experimental phase. Experimental tests were performed to establish the optimal parameters linked to each considered operators. For this, the tuning parameters of the anisotropic diffusion filters (number of iterations (t) and enhancement threshold (k)) were, iteratively, optimized considering unit step sizes and the following ranges: $0 < t < 100$ and $0 < k < 10000$.

2.2.3. Validation phase. Once the images were filtered, the comparative performance, between the operator proposed in [11] and the proposed by the present investigation, was evaluated. For this, the metric called peak signal to noise ratio (PSNR) will be considered [13]. This metric is measured in decibels (db) and it allows us to establish the quality of an image after being subjected to filtering processes. It is very important to note here, that the filtering must be done from three-dimensional domain. Due the anisotropic diffusion requires discretization the continuous model, given by Equation (3), uses an approach based on finite differences [14].

$$\frac{dI(x,y,t)}{dt} = \nabla c(x,y) \nabla I(x,y,t), \tag{3}$$

where $I(x,y,0)$ represents the original image, t is equal to time, and c is the conductance or diffusion operator which is a function of (x,y).

In this sense, the 3D filtering method is carried out considering cubic neighborhoods (observation window) that run through the image that is to be conditioned, generating a filtered version, with the same spatial dimensions. Initially, the observation window is fixed in the first column and the first axial layer. In each iteration the cubic neighborhood is displaced considering the rows. Upon reaching the last row, the observation window is located in the next column, staying in the same axial layer. Then the displacement in the direction of the rows is performed again and so on until reaching the last column.

Finally, it is fixed on the next axial layer and begins the same displacement by rows then by columns until all the axial layers of the three-dimensional image are swept.

3. Results

Both the original image and the image contaminated with Poisson noise were considered in order to calculate the PSNR reference value (PSNRRef), which was of 37.92 db. In addition, the database described in section 2.1 was processed by applying the anisotropic diffusion filter, considering the Lorentz operator, showed by Equation (1), and the new proposed operator given by Equation (4), which was obtained by applying functions composition principles considering, for this, the Leclerc and Lorentz operators.

$$f3(x) = \frac{1}{1 + \frac{exp(x)}{k}} \tag{4}$$

In order to obtain the best performance for each operator, a tuning process of the parameters (t and k), that control the performance of both operators, was developed. In this sense, the size of the cubic observation window, for both operators, was arbitrarily set at (3×3×3), that is, each time the filtering process is performed, 27 elements of the image contaminated with Poisson noise are simultaneously considered.

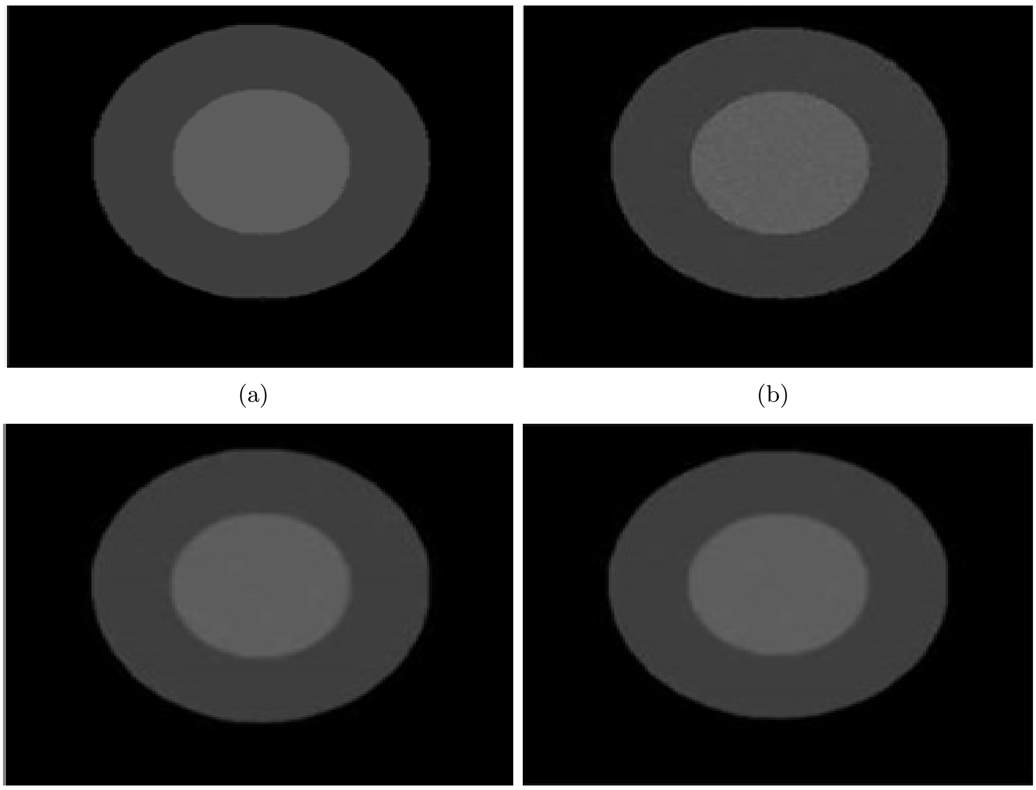
The quantitative results derived from the filtering process, based on the PSNR metric, optimal parameters (OP) and computational time (Ct), are presented by Table 1. This PSNRRef value is vital for analyzing the operators performance, with respect to the noise level presents in the considered images. In this paper, the filter performance can be analyzed using the next criterion: A PSNR value higher than PSNRRef is a good filter, in other case, the filter is considered non-adequate. Normally, if the filter generates PSNR values one decibels above of PSNRRef we can considered that this filter development an acceptable filtering process.

Table 1. PSNR, OP and Ct for each operator.

Operator	PSNR (db)	t	k	Ct (min)
$f1$(x)	39.49 ± 1.12	7	200	21.33
$f3$(x)	39.16 ± 2.59	10	200	22.25

According to the quantitative information, presented in Table 1, it can be affirmed that both the operator proposed by Malik–Perona and the one proposed, in the present paper, are capable of generating comparable results to each other. These statements are supported by the PSNR values obtained by both operators, which differ only in their decimal part. These PSNR values outperform the PSNRRef in approximately two decibels which, according with the criterion considered in this paper, is a clear indicator of the excellent performance of both operators. But the Perona-Malik operator is a little better that the operator proposed, in this article, due its PSNR value is almost 0.5 decibels higher than the PSNR of our diffusion operator.

Now, considering the computational cost, even the computation times also exhibit similar values noting, only, a slight efficiency advantage for the operator proposed by the pioneers of anisotropic diffusion. In addition, Figure 2 shows the results from a qualitative point of view. Note in Figure 2, that almost no differences are perceived between the filtered image considering the operators presented, which is reaffirmed with the quantitative results shown in Table 1.



(a) (b) (c) (d)

Figure 2. Axial views showing the images: (a) original, (b) contaminated with Poisson noise, (c) filtered with operator proposed by Perona-Malik, (d) processed with the diffusion operator proposed in this paper.

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In Figure 2, also we can observe the excellent edges preservation and adequate smoothing of the information contained within the filtered images contours. This fact let us say that both operators exhibiting similar smoothing performance. These quantitative and qualitative results indicate us that both operators could be used interchangeably to diminish the effect of the poisson noise displayed by the images considered.

4. Conclusions

Through the present work a diffusion operator, not reported in the literature, has been proposed. In this sense, the creation of this new operator contributes to the generation of new knowledge in digital image processing context, specifically, in the images filtering techniques area. From the point of view of computation time and PSNR metric both operators generated a comparable values, that is, we are in the presence of a new diffusion operator that behaves with an efficiency and effectiveness similar to the operator proposed by the pioneers of anisotropic diffusion filtering. As an extension of this work, a larger number of metrics and databases (both synthetic and real images) could be considered to establish the robustness of the new operator proposed in this paper. For example, our operator could be applied in real images filtering context in order to exploit its useless to address the artifacts problem and to elevate the MSCT real images quality, generating a better conditioning and doing more effective the image information analysis process.

Finally, the design of hybrid techniques using our operator and smart operators can generate a excellent results to address the low contrast problem presents in medical images, particularly, in cardiac MSCT images.

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