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Learning the concept of integral through the appropriation of the competence in Riemann sums

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Abstract. It is proposed that the difficulty of engineering students into understanding the concept of the integral, as a way for calculating the area under a curve, can be overcome if students are taught how to translate it into the problem of calculating a Riemann sum. A series of applied problems are proposed to provide a frame that required to calculate the area under a curve to two groups of students. For one of these groups, Geogebra was proposed as a tool that could be used to maintain the focus of students into the concepts, by providing ways to easily calculate and visualize the solutions, while the other group reached to the solutions by analytically making all the calculations. Evidence was found that, to a confidence level of 95%, Riemann sums calculated with Geogebra reduce the score difference in context problems requiring the calculation of integrals, helping students to reach a better understanding on the concept of the integral as the area under the curve of a given function.

1. Introduction

The understanding of calculus and, specifically, solving defined integrals, is a fundamental competence for engineering students. In this aspect, several problems, which involve the relationship between two or more variables, often require students to be proficient on the algebraic manipulation of analytic expressions and on the usage of integration techniques [1, 2]. These integration techniques are usually taught to students by the repetition of algorithmic procedures and, although allowing students the memorization of reliable general rules for integration solving, they do not provide means for the development on their ability for interpretation and modelling of the solutions. Thus, the significant learning on the role of integrals as a tool for solving problems related to engineering and the modelling of their solutions remains incomplete.

Typically, in Mexican universities including engineering careers in their curricula, a complete semester is dedicated for students to learn the important concepts and procedures related to integral calculus. This is the case of the Technological Institute of Colima, part of the National Technological of México (the largest public institution in latin America). Each year students from all the 10 engineering careers offered by the institute take the integral calculus course and, despite the existing additional programs for math tutoring, the approval rate of students remains being too low, reflecting the intrinsic difficulty of the topic but, also, the necessity for improving the current teaching techniques.



A major factor that influences the learning of calculus is the pedagogic focus. In [3] a documentary investigation was made by interviewing students that had took the integral calculus, whose objective was to identify successful approaches used by calculus teachers to teach the concept of integral. From these interviews the following approaches were identified as:

- Discursive approach.
- Learning centred on the resolution of context problems.
- Solving problems by brainstorming previously acquired knowledge.
- Learning how to calculate the integral by associating with the procedure needed to calculate the derivative of a given function.

From these four approaches, an hybrid strategy was designed and applied to students taking the integral calculus course with positive results. Typically, the calculation of the area under a curve relies on the Barrow's rule (*i.e* that the integral of a continuous function, within a closed interval, is equivalent to the numerical difference between the values that the function's primitive takes on the boundaries of that interval). However, the use of this rule requires students to be skilled on the algebraic manipulation of the function for the calculation of its primitive. This represents one of the major difficulties for students, as it has been discussed in [4].

Traditionally, Riemann sums are presented to students taking an integral calculus course, as predecessors methods for calculating the area under a curve, before introducing them to the concept of integral [5-7]. In contrast to integrals, Riemann sums do not require students to apply the Barrow's rule and rely more on their ability for evaluating the function over a partition covering the integration interval [8]. Usually, the major drawback is the time required for students to approximate the value of the area under a curve by using Riemann sums, a difficulty that can be compensated by teaching students how to calculate them by using software.

The usage of communication and information technologies, allowing to easily construct algebraic, graphic and numerical modelling of functions, provides a new way in which mathematics courses can be planned. Geogebra has been used in the past for helping students to develop their mathematical ability [9,10] and multiple studies, specifically, make propositions on how to use this software to teach students on how to calculate the area under a curve, thus helping them to develop their understanding on the concept of the integral. In the work done by [10], Riemann sums were presented as convergent successions of values obtained by evaluating an specific function over a partition of points within a closed interval. A sequence of activities is given to students, that develop them within a time period of 90 minutes, and through the used of Geogebra calculate the convergence values of the succession representing the area under the curve and students are induced to make conjectures over the relation of the exact integral and the convergence value of the succession. Evidence is found that the usage of Geogebra, allows students to concentrate on the concepts, and their relation, instead of consuming most of their time and effort in just trying to calculate a result.

The present investigation was based on a quantitative type of research and a quasi-experimental design [11] and proposes the design and application of an approach consisting on the teaching of the concept of the integral of a function by the calculation of Riemann sums using Geogebra. Multiple context problems, usually covered in physics courses directed to engineering students, are proposed and the solution methodology is decomposed on easy to follow steps in which students can focus on the concepts involving the calculation of the area under a curve.

2. Methodology

The integral of a function, over a well defined interval can be calculated by employing Riemann Sums [12, 13]. Given the closed interval $[a, b]$, a partition of it (P) can be constructed by a

finite set of values obeying $x_0 < x_1 < x_2 \dots < x_{n-1} < x_n$, where $x_0 = 0$ and $x_n = b$. From this partition, n sub-intervals $[x_0, x_1], [x_1, x_2], \dots, [x_{n-1}, x_n]$ are created. Any function defined as Equation (1).

$$F(x) = s_k, x_{k-1} < x < x_k, \quad (1)$$

is called as a step-wise function (meaning that the value of the function remains constant through any open sub-interval of P). The integral of Equation (1) corresponds to the Riemann sum (Equation (2)).

$$\int_a^b F(x) dx = \sum_{k=1}^n s_k (x_k - x_{k-1}) = \sum_{k=1}^n F(c_k) \Delta x_k. \quad (2)$$

Equation (2) was used in this work to provide students with a tool for calculating integrals and a concept that can be used to understand them.

Two groups of second semester students in the career of system engineering of the Technological Institute of Colima were selected to carry on the study and whose results were compared. The control group was assigned to solve the presented problems by the traditional algorithmic procedure for calculating integrals and it was composed by 39 subjects. The test group, composed by 42 students, was taught on the basic usage of Geogebra and was instructed on how to solve each of the presented problems by calculating Riemann sums, as both groups were randomly labeled prior to carrying out the diagnostic test.

Before carrying on the investigation, a test was applied to determine the mathematical ability of students into solving basic algebraic problems. For the design of this diagnostic test, the following abilities were evaluated:

- Basic arithmetic operations and their hierarchy.
- Basic algebraic operations.
- Algebraic manipulation over variables.
- Comprehension on the concept and analysis of functions, specifically, their range, domain and inverse function.

5 practices were designed to be applied for both groups. These practices asked students to solve applied problems, in which the objective was to present them with the need of calculating the defined integral under a curve. Although both groups were given the same problems, it was planned that the control group would solve the problems in the traditional way, using their mathematical skills to calculate the solution, while the test group would solve the problems by employing Geogebra. Each practices asked the students to calculate:

- Practice 1. The geometrical area under the curve and parabolic projectile motion.
- Practice 2. Work employed to move an object and the force exerted by a fluid on one of the walls of its container.
- Practice 3. Work done by a variable force and the pumping withing an spherical container
- Practice 4. Work done by a variable force due to an object that is loosing mass.
- Practice 5. Work done by a pump to fill a conic container and the total force exerted by the fluid on a trapezoidal wall.

Each of the practices were carried on in the same manner. The professor gave the instruction sheets to the students, describing step-by step how to solve each of the context problems, and the students carried on the suggested procedure (the control group doing analytical calculations and the test group by creating their own Geogebra applets).

After all the 5 practices were carried on, a second evaluation test was applied, to determine if there was any evidence supporting that the control group improved their competence on solving problems and their comprehension on the concept of integral. In this test the objective was to measure the comprehension of students on the concept of the defined integral and their ability on the graphic and algebraic analysis of the problems and their solutions.

3. Results

The diagnostic test consisted on 14 questions, of growing difficulty, equally distributed to measure the aforementioned mathematical abilities of students in the control and test group. Result are shown in Table 1.

Table 1. Results from the diagnostic test four both groups of students.

Group	Control	Test
Mean score	2.88	4.56
Median	2.50	4.00
Kurtosis	-0.92	0.03
Skewness	0.43	0.43
Standard error	0.54	0.38
Standard deviation	2.49	3.39

As the average scores were to be compared, the likeness of the data distribution for both groups to the normal distribution was measured. Both data were positively skewed, although within the acceptable range between -0.5 and 0.5 [14]. The kurtosis of the score distribution from the control group was closer to a normal distribution than that of the test group, showing a negative kurtosis and indicating that the tails of the distribution is slightly lighter than that of the normal. The results of the diagnostic test show that the control group had an average score of 2.88 from 14 points. In contrast, the test group obtained an average score of 4.56 out of 14 points. The test group already showed a better average score than the control group but neither of them reached even a 50% of correct answers. The standard deviation of each group was almost the same, showing low-score variability with respect to the mean score on each case.

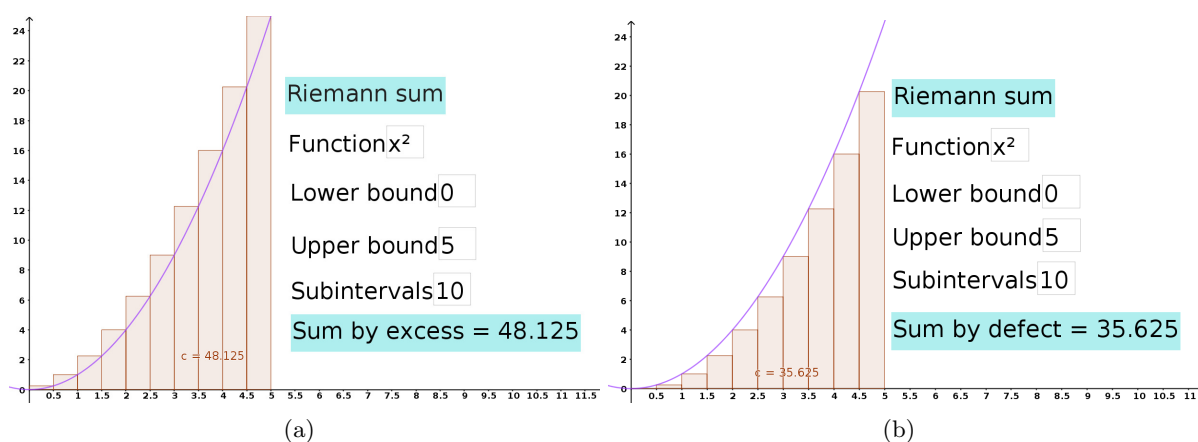


Figure 1. Activities developed for the first part of practice 1 by students of the test group. First, the geometrical area under a curve is approximated as the Riemann sum by excess of the area of 10 rectangles (a). Then, the result is then compared with the Riemann sum by defect on the same interval (b).

Figures 1 and Figure 2, show the activities done by students on the test group when developing the first practice (while students on the control group made the calculations by hand). Figure 1, is obtained after students are asked to approximate the area under a curve as the Riemann sum by excess Figure 1(a) and by defect Figure 1(b) and then to compare to the exact result given by the teacher (in both exercises the integration interval is subdivided in 10 parts). After this, the second activity (Figure 2) involved the calculation of the distance (measured in feet) travelled by a projectile thrown vertically into the air by using the given formula for its velocity as a function of time (measured in seconds and represented by the variable x). First, the exact distance is approximated by calculating the Riemann sum by excess and defect using 5 subintervals Figure 2(a). Then, the exact value of the travelled distance is inferred by using a growing number of subintervals (Figure 2(b) shows the results of using 50 subintervals).

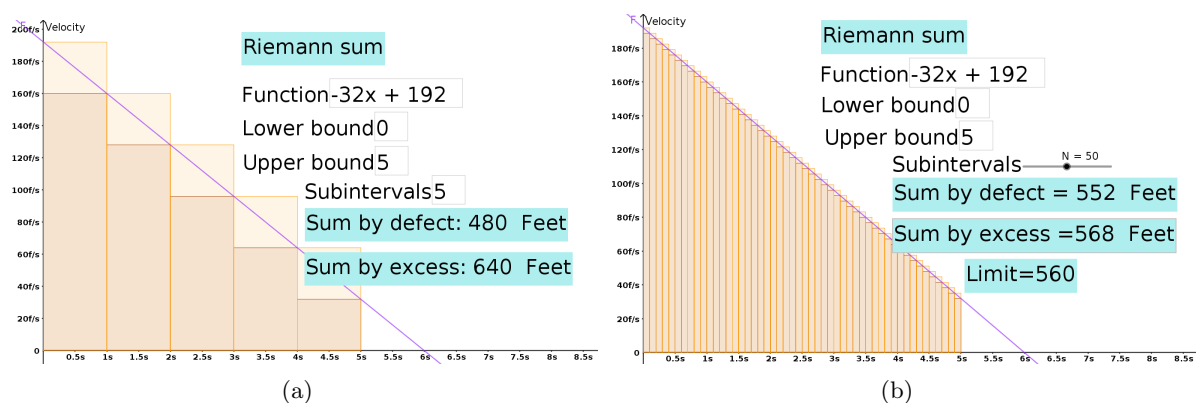


Figure 2. Activities developed for the second part of practice 1 in the control group. First the distance travelled by a projectile is calculated as the average value of the Riemann sums by excess and defect (a). Next the exact integral is approximated as the limit in which the number of subintervals tends to infinity (b).

The scores of both groups after taking the second test are shown in Table 2. First, the data was analyzed to determine if the hypothesis “The mean score of the test group will be significantly higher, supporting that the claim that the concept of integral can be better understood if students solve problems using Riemann’s sums”. The average score of the control and test group after taking the second test was 62.5 and 55.2, indicating a slight difference between both. The standard deviation of their scores was 30.613 and 23.7, showing that there was a higher degree of dispersion in the scores among the students of the control group. Taking these results, the Z-score test was performed and allowed to conclude that, to a significance level of 5%, the average score of students on the test groups was not significantly higher than that of the control group (given that $Z_{test} = 1.23 < Z_{critical} = 1.67$). Thus, no evidence was found whatsoever than the usage of Riemann sums and Geogebra helped students to reach a better understanding on the concept of the integral.

Next, the data was analyzed to determine if the hypothesis “the proportion of successful students of the test group will be significantly higher, supporting that the claim that the concept of integral can be better understood if students solve problems using Riemann sums”. The number of students of the test group that approved the second exam was 23, compared to the 16 students in the control group (Table 2). These results are equivalent to a 56% and 38% of success. By applying the Z-score test for proportions, requiring that the result has a significance of 5%, one gets the value $Z_{test} = 1.87$, while the critical value for rejecting the null hypothesis is $Z_{critical} = 1.64$. This results suggest that, at least to a confidence level of 95%, the method of Riemann sums developed with Geogebra helped students to reach a better understanding on the concept of the integral and to be more proficient in solving problems related to their calculation.

Table 2. Results from the second test. The data shown on the left was used to test the statistical significance of the difference in mean scores for both groups. The data shown on the right was used to test the statistical significance for the difference in proportional success rate.

Group	Control	Test	Group	Control	Test
Number of students	39.00	42.00	Number of students	39.00	42.00
Mean score	62.56	55.21	Proportions of success	23.00	26.00
Standard deviation	30.13	23.79	Proportions of fails	16.00	16.00
Z-score	1.23		Z-score	1.89	
P-value	0.22		P-value	0.06	

4. Conclusions and future work

This work presented the result of an effort for making the concept of the integral better understood by engineering students, by teaching them how that operation can be approximated as a Riemann sum. Geogebra proved to be a valuable tool in helping students to maintain their focus on the concepts needed to solve the problems, instead of the involved calculations. However, the evidence gathered by this study was enough only to indicate that the success rate of students in test group was higher than in the control group and that the variability of scores appears to be lower between students in the control group, suggesting that the typical skill differences among students could have been diminished when solving integrals by Riemann sums and Geogebra.

Although not shown in the Table 1 and Table 2 presented here, the perception of students in both groups was also considered. Students that used Geogebra and Riemann sums showed more confidence as each of the practices were completed, were able to deduce the general algorithm employed to solve all the 5 practices and also showed more eagerness into further learning new ways to confront the mathematical difficulties related to the calculation of integrals, while the students within the control group manifested insecurity about the way in which they could solve each of the context problems and were not able to deduce a general solution procedure. This suggest that the use of Geogebra could boost the confidence of students into solving mathematical problems by providing a way to tackle the usual difficulties related to the algebraic manipulation and solving of equations and will be further investigated.

It is intended that this study will be replicated in a near future and expanded to provide students with a way to understand multiple integrals, line integrals and other important topics in the subject of vector calculus.

References

- [1] Thomas G B 2006 *Cálculo: Una variable* (México: Pearson Educación)
- [2] Stewart J 2008 *Cálculo de una variable* (México: International Thomson Editors)
- [3] Villanueva M, Cabrera G and Bernabé M 2018 *Revista Iberoamericana de Producción Académica y Gestión Educativa* **5** 10
- [4] Fuster J and Gómez F 1997 *Divulgaciones Matemáticas* **5** 61
- [5] Larson R and Edwards B 2014 *Matemáticas II (Cálculo Integral)* (USA: CENGAGE Learning)
- [6] González L, González E and De León G 2016 *Revista Iberoamericana de Producción Académica y Gestión Educativa* **2** 3
- [7] Ugarte F and Martínez M 2017 *Acta Latinoamericana de Matemática Educativa* **2** 459
- [8] Salas S, Hille E and Etgen G 2002 *Calculus: Una y varias variables* (México: Reverté Ediciones)
- [9] Álvarez R, Fernández F, Martínez F and Zamora A 2015 *Análisis y métodos numéricos con Geogebra, prácticas de Matemáticas 2* (España: Universidad de Alicante)
- [10] Espinoza Vásquez I G 2015 Construcción de los conceptos partición y sumas de Riemann con Geogebra *Educación Matemática en las Américas 2015* vol 4 (*XIV Conferencia Interamericana de Educación Matemática*) ed Patrick (Rick) Scott y Ángel Ruiz (México: Comité Interamericano de Educación Matemática) p 165

- [11] Hernández S, Fernández C y Baptista L 2014 *Metodología de la Investigación* (México: Mc Graw Hill Education)
- [12] Apostol T 2001 *Cálculo* (México: Reverté Ediciones)
- [13] Ramos J 2018 *Cálculo Integral* (México: Alfaomega Grupo Editor)
- [14] Field A 2009 *Discovering statistics using SPSS* (London: SAGE)