Mathematical argumentation in the classroom

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Mathematical argumentation in the classroom

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Abstract. The article shares some elements of comprehensive type about "mathematical argumentation in the classroom"; whose analysis, was made from two fundamental categories in the development of an oral mathematical argumentation process for the conviction, contradiction and validation of a written mathematical argumentation process. The research addressed two central categories of argumentation as a discursive form, the first one is the epistemic position, and the second one is the discursive position that students unveil at the time of mathematically arguing the solution to a problem situation. The research was developed under the interpretative paradigm through the design of a case study directed by the theory and technique of a focal group, for the collection of information. In the findings, difficulties in the passage were evidenced from the semantic to the theoretical from the epistemic position; regarding the discursive position, the presence of three discursive forms was revealed: description, explanation and argumentation, the latter being the least used by the students.

1. Introduction

Argumentation is a field of study investigated since the Greek era with Aristotle to the present day, and has transcended to different sciences as a field of research, in particular to the science of mathematics.

Studying the argumentation in formal contexts implies to analyze the discursive forms [1], analyzing the argumentative structures that are developed in a process of argumentation [2], an interest that focuses precisely from the treaty of argumentation [3]. This kind of argumentation is developed later by Duval [4], by proposing more precisely a structure for the analysis of an argumentative passage in the resolution of mathematical problems. Figure 1 presents the structure proposed by Duval [5].

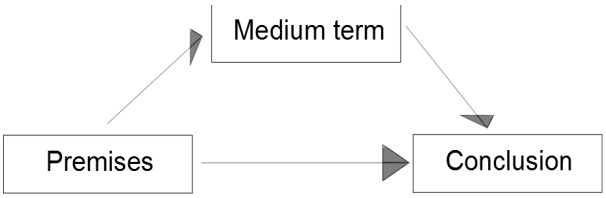

Figure 1. Reasoning passage in argumentation.

Since mathematics has its own theoretical and epistemological corpus, the processes of mathematical argumentation must be developed precisely from the logic of heuristic argumentation and not from the logic of rhetorical argumentation [2,4]. In this way, the analysis of an argumentative

plot written in response to the mathematical that is done from two dimensions: functional and structural [4].

The functional dimension studies the criteria of acceptance or rejection of the arguments produced by the argumentative subjects. The strength and relevance of the arguments [5], the first one is related to the ability to counter-argue and to the epistemic value of the arguments, that is, the appropriate use of logical semiotics and mathematical language [6] and, the second, refers to the semantic epistemic value [7]. For its part, the structural dimension is related to the development of the triad premise, middle terms and conclusion of an argumentative passage [8].

Now, from the perspective of oral argument, the analysis of the production of arguments is called to focus on the epistemic and discursive position of the argumentative subjects. Additionally, it is necessary to identify the conditions of the context where the production of arguments is developed [9], that in this case, it is given by the classroom that belongs to a certain school. In this context, different socio-historical, political and cultural dimensions [10] emerge in each of the subjects, which can affect the construction of the surrounding sense the production of arguments to solve problem situations in mathematical contexts.

2. Materials and methods

The research was based on the interpretative paradigm with a design of case study guided by the theory [11], where it was sought from three stages, to understand the minimum and necessary conditions for the formation and development of mathematical argumentation processes in the classroom, for the solution to a real-world problem situation.

For the first stage, an objective test was applied to 45 students from the "Facultad de Ciencias Administrativas de la Universidad Simón, San José de Cúcuta, Colombia"; whose interest was to analyze the structure of the mathematical argumentation described from the dimensions: functional and structural. In a second moment, a semi-structured interview was developed based on the focus group technique, in which six students of the first phase participated, in order to understand argumentation as a discursive form.

Finally, some conditions were determined, which based on the results obtained, identified as minimum and necessary for the development of argumentative processes in the classroom in the field of mathematics.

The objective test (problem situation), the semi-structured interview and the focus group script were subject to validation by expert judgment. The information was processed by saturation of categories: theoretical coding, open coding, axial coding and selective coding [12], with the support of Atlas Ti software.

3. Results

The first stage, sought to perform an analysis of the functional and structural dimension of the argumentative plot developed by students around the solution of the problem posed. Overall, it was found that 7% of the fraction of students tested, developed a heuristic argument with true logic value (premises-medium term-conclusion), another 7% showed a heuristic argument with true logic value but incomplete, since did not develop the findings of the argument frame (premises-medium term), 77% of the fraction of students developed heuristic argumentation with false logical value and 9% of the remaining students showed argument rhetorical type.

The arguments rejected because of the lack of dimensions of force and relevance in the context of mathematics, in other words, the students solved the situation from rhetorical elements and not from the elements of the epistemological and theoretical status of mathematics.

In the analysis of the argumentative structure, there are recurrent errors of significance (Figure 2), conceptual errors (Figure 3) and rhetorical conclusions (Figure 4).

In Figure 5, it can be observed a concrete example of an argumentative passage in the field of mathematics, developed by a student.

Each element of the structural dimension, premises, middle term and conclusion, reveals the construction of arguments with characteristics of the functional dimension, that is, arguments validated from the dimensions of force and relevance.

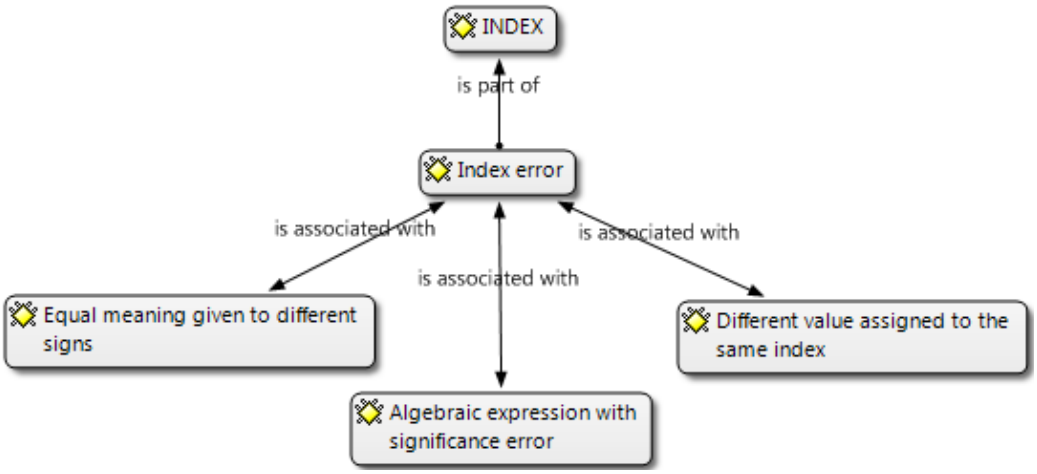

Figure 2. Significance errors.

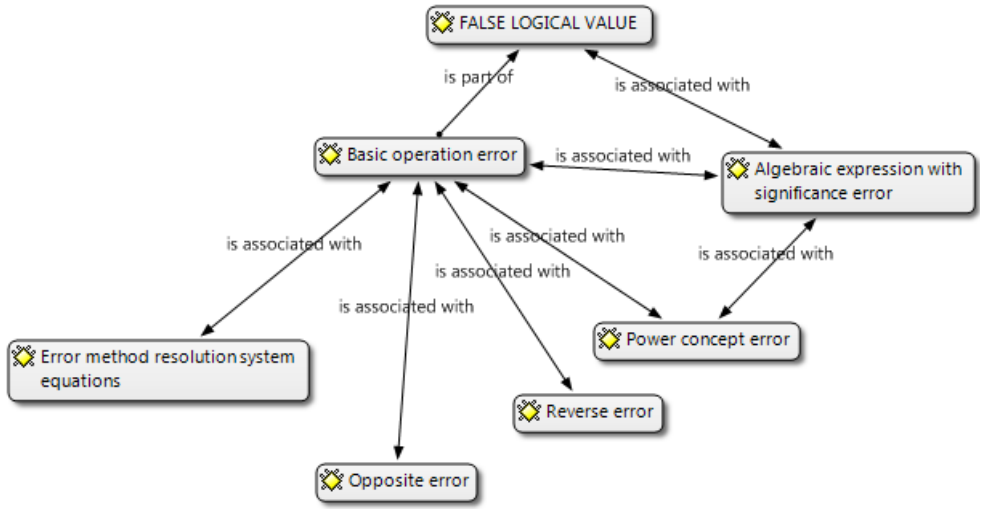

Figure 3. Conceptual errors.

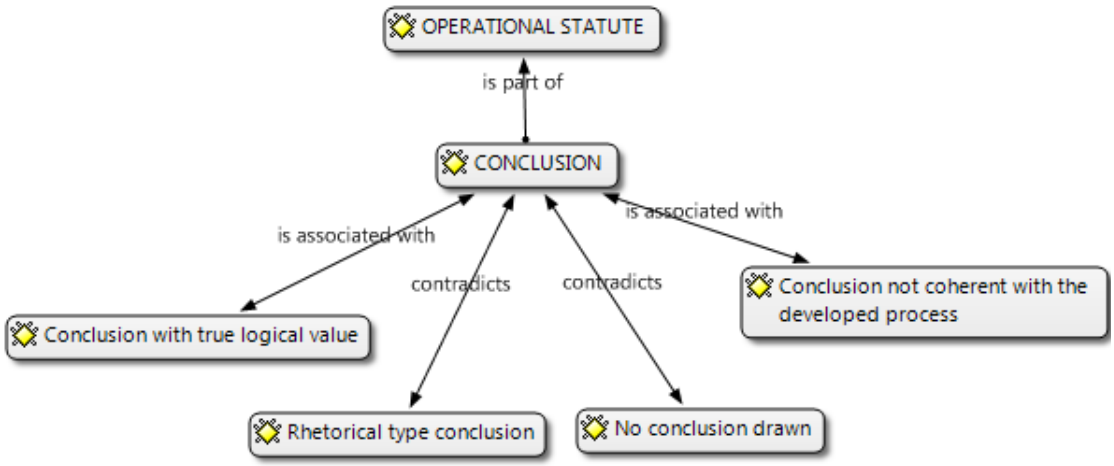

Figure 4. Rhetorical argumentation.

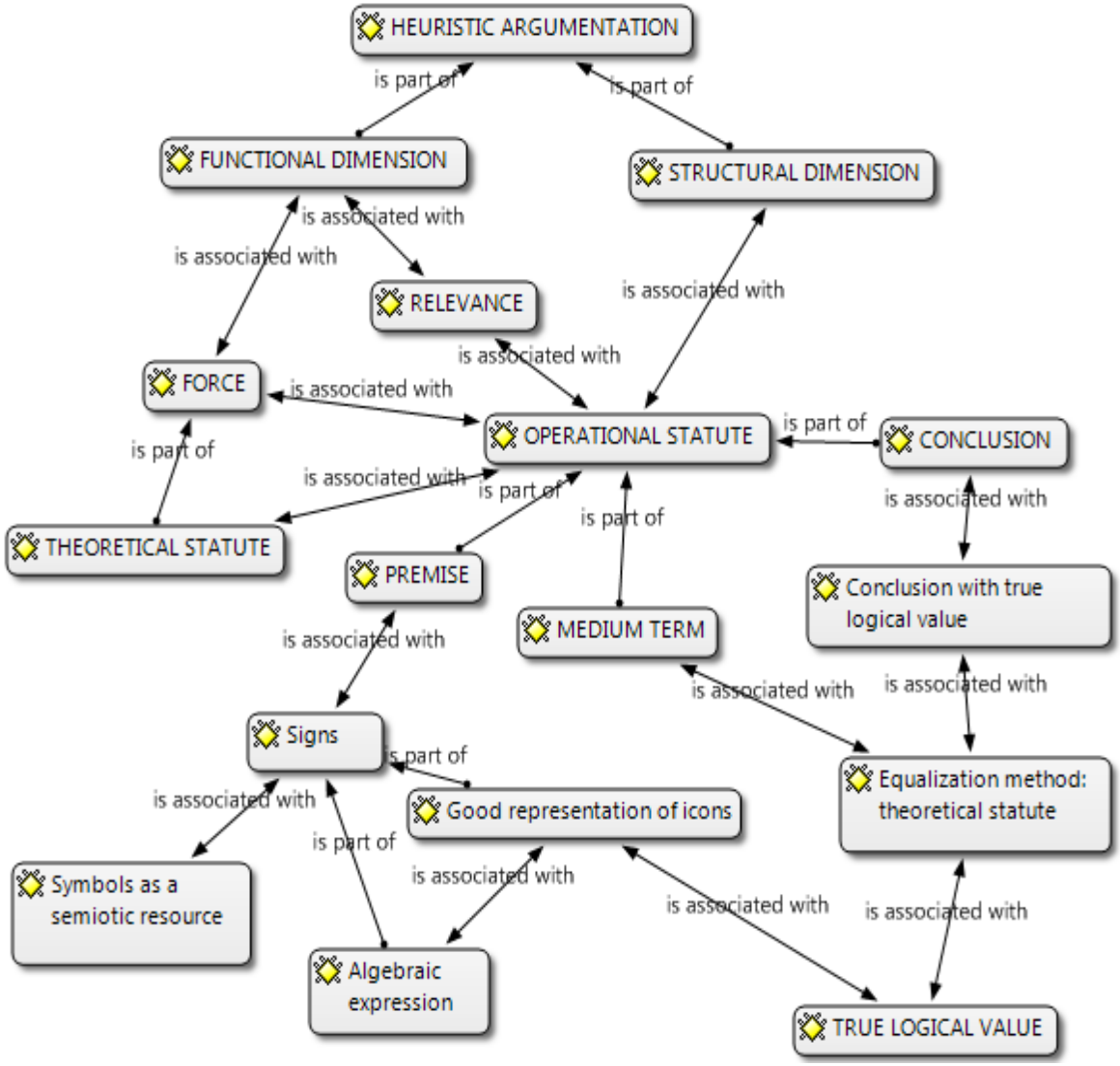

Figure 5. Argumentative passage in mathematics.

Regarding the analysis of the focus group, whose objective was to understand argumentation as a discursive form, environments were found to be difficult to understand the meaning of the statement (process of interpretation), difficulties related to the processes of meaning of the use of signs (logical semiotics - language mathematical) and, a mechanistic view of mathematics (see Figure 6).

Understanding argumentation as a discursive and non-structural dimension, allowed us to identify three discursive forms developed by participating students: description, explanation and argumentation, with the explanatory level being the one with the most recurrence and the argumentative level the least evident. Likewise, different forms of consensus were unveiled as a social agreement in the construction of an oral argumentative plot; these were the acceptance of its own solution and other solutions, the rejection of its own solution and acceptance to other solutions, and the rejection of other solutions and acceptance of its own solution.

Based on the above, certain essential conditions are proposed to promote the processes of formation and development of argumentative passages in the classroom, in contexts of teaching and learning processes of mathematics.

The first condition is to look at mathematics beyond a simple reduction to the instrumental and mechanical, that is, to develop skills, virtues and skills for the competent formation of mathematical

knowledge and know-how, involving mathematical thoughts. As well as, the different logical-formal and structural processes that evidence the formation of quantitative reasoning and the dimensions that it involves: interpretation, representation, modeling, and argumentation.

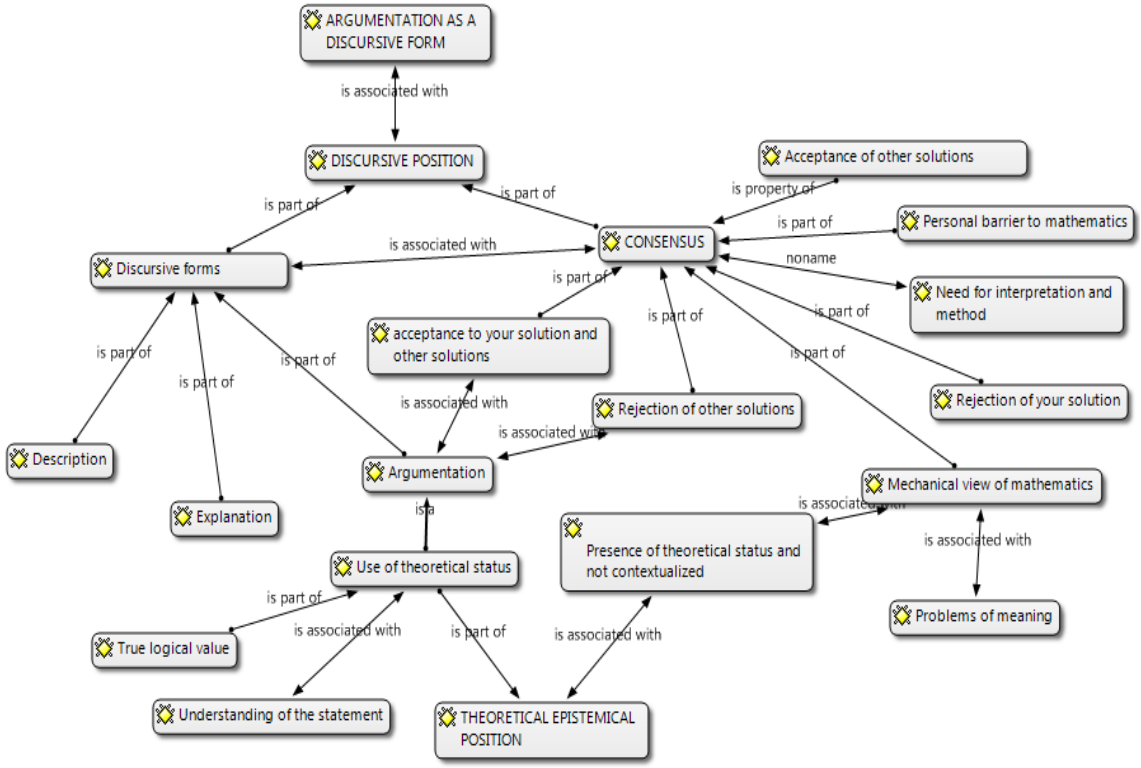

Figure 6. Argumentation as a discursive form in the field of mathematics.

The second condition is related to the need to create a teacher qualification plan around the field of heuristic argumentation, since it presents a completely theoretical and investigative corpus that relates central categories in the teaching of mathematics. As they are: specific didactics, the elements of logical semiotics and mathematical language, the construction of sense and meaning processes and the production of argumentative passages, the latter, evaluated from the dimensions of functionality and the structuring of the argument.

The third condition is related to the need to implement the problem situation, or any didactic tendency that allows analyzing real problems in contexts of mathematics, allowing the students to develop an epistemic, semantic and consensus-building dimension now of produce arguments that make up an entire argumentative process.

4. Conclusions

The investigation allowed revealing two types of argumentation, the heuristic argumentation in the field of mathematics and the rhetorical type of argumentation. Regarding heuristic argumentation, problems were observed in the construction of meaning of mathematical statements, which are generating problems around the processes of meaning (use of mathematical language), which affect the production of arguments in the development of a plot argumentative. These are, not strong arguments, nor pertinent from the logic of the functional dimension, as well as incomplete argumentative passages, since, although they present coherence in the premises and medium terms of the argumentative process, a large part of the students do not develop conclusions of the statement (structural dimension).

Likewise, three discursive forms were found in the oral argumentation process, the description, the explanation and the argumentation, the latter being the one that was least presented in the students who participated in the focus group.

Finally, three essential conditions for the formation and development of argumentative processes within the mathematics classroom were determined. They were: (i) The approach of mathematics beyond a simple mechanical and instrumental reduction. (ii) The training of teachers in the field of mathematical argumentation, which involves the semiotic, discursive, epistemic and theoretical dimensions. (iii) The use of didactic tendencies based on the application of problem situations that allow the formation of students' logical and mathematical reasoning.

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