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# Note

# A NOTE ON THE EXTENT OF U.S. REGIONAL INCOME CONVERGENCE

# MARK J. HOLMES

Waikato University

Jesús Otero

Universidad del Rosario

## **THEODORE PANAGIOTIDIS**

University of Macedonia

Long-run income convergence is investigated in the U.S. context. We employ a novel pairwise econometric procedure based on a probabilistic definition of convergence. The time-series properties of all the possible regional income pairs are examined by means of unit root and non-cointegration tests, where inference is based on the fraction of rejections. We distinguish between the cases of strong convergence, where the implied cointegrating vector is [1, -1], and weak convergence, where long-run homogeneity is relaxed. To address cross-sectional dependence, we employ a bootstrap methodology to derive the empirical distribution of the fraction of rejections. We find supporting evidence of U.S. states sharing a common stochastic trend consistent with a definition of convergence based on long-run forecasts of state incomes being proportional rather than equal. We find that the strength of convergence between states decreases with distance and initial income disparity. Using Metropolitan Statistical Area data, evidence for convergence is stronger.

Keywords: Panel Data, Cross-section Dependence, Pairwise Approach, Income, Convergence

### 1. INTRODUCTION

In recent years, economists have keenly debated the neoclassical growth model prediction of per capita income convergence. Assuming the same structural characteristics, countries or regions with relatively less capital would be expected to accumulate more capital and grow faster than those with relatively more capital,

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eventually converging to the same steady state. In contrast, models of endogenous growth or the "new" theories of international trade argue that such convergence might fail on account of increasing returns and externalities. Indeed, as pointed out by Krugman (1991) and Krugman and Venables (1995), in the presence of increasing returns, economic activity might be expected to concentrate geographically in a few areas. A wide range of studies that includes early work by Baumol (1986), Barro (1991), Barro and Sala-i-Martin (1991, 1992), Mankiw et al. (1992), Carlino and Mills (1993), Bernard and Durlauf (1995), and Sala-i-Martin (1996) and, more recently, studies such as Papageorgiou and Perez-Sebastian (2004), Inklaar and Timmer (2009), and Deckers and Hanck (2012) have considered convergence across countries, U.S. states, and European regions and industries and provided mixed evidence in favor of convergence. The empirical investigation of the convergence hypothesis has been based on both cross-sectional and time-series approaches. The cross-sectional approach is often encapsulated in the notion of  $\beta$ -convergence, which requires that "poor" regions grow faster than "rich" ones. However, several criticisms have been raised against the conclusions reached in many of these studies on account of Galton's fallacy or "regression towards the mean" (Quah, 1993). In contrast, the time series approach is built on a stochastic definition of convergence in which the per capita disparities are expected to be stationary. Moreover, temporary shocks to key structural variables such as saving rates, population growth, and technological progress are characterized by stationary relative outputs, thereby indicating that economies are stochastically converging. This definition is exemplified by studies such as Bernard and Durlauf (1995), who find little evidence of long-run convergence among OECD countries.

Using per capita income data across many decades, a number of studies have examined stochastic convergence among U.S. states. As pointed out by Choi (2004), among others, because of their almost homogeneous institutional environments and their highly integrated markets for products and factors, the U.S. states satisfy the underlying conditions of the convergence hypothesis in the standard neoclassical growth model. The existing evidence, however, is mixed. For example, Carlino and Mills (1993) provide evidence in favor of stochastic convergence insofar as shocks to relative regional per capita income are temporary, but only after allowing for a structural break in 1946. Evans and Karras (1996) employ a panel unit root test based on Levin et al. (2002). Although this test only allows for fixed effects and common slopes, they reject the null hypothesis of joint nonstationarity of relative per capita incomes. Tsionas (2001) employs vector error correction modeling and finds that multiple common trends are driving the income series, thus concluding against the convergence of real per capita incomes. Choi (2004) applies multiple panel data techniques to state per capita output and finds that output convergence in the United States has proceeded among geographically neighboring states rather than among distant states, notwithstanding the nearly complete integration of product and factor markets. More recently, Mello (2011) examines relative incomes and considers whether low power of unit root tests and high persistence have led researchers to find evidence against convergence. Using a methodology based on fractional integration and interval estimation, support is found for stochastic convergence.

In this paper, we contribute to the debate concerning long-run income convergence among U.S. states. In doing so, we analyze the interaction between nonstationary state income series and draw on the time-series approach, but in a way that also utilizes cross-sectional information. The novelty of our approach is the development of an econometric procedure advocated by Pesaran (2007). Within this framework, a pairwise probabilistic definition of regional convergence forms the basis of our empirical testing strategy. The idea here is that for a sample of N states, unit root tests are conducted on all N(N-1)/2 real per capita income differentials (pairs or gaps). Under the null of nonstationarity or nonconvergence, one would expect the fraction of real per capita income pairs for which the unit root hypothesis is rejected to be close to the size of the underlying unit root tests, denoted as  $\alpha$ . Thus, we can argue that the null of nonstationarity for all state pairs could be rejected if this fraction of rejections exceeded  $\alpha$ . However, the presence of cross-sectional dependence can make inference based on the fraction of rejections difficult, so the bootstrap methodology is employed to derive the empirical distribution of the fraction of rejections.

There already exist a limited number of studies that investigate stochastic convergence using a pairwise approach. Pesaran (2007) considers data for 101 countries and geographical subgroups. Relying on the use of pairwise unit root tests provides little evidence of convergence at a global level, though there is some evidence of club convergence [Quah (1997)]. Deckers and Hanck (2012) do not support the notion of convergence, using a sample of 51 countries. In terms of studies of regional income convergence, Mello (2011) considers the case of the 48 contiguous U.S. states. The pairwise unit root testing procedure indicates that the nonstationary null is rejected in 8.6% of the cases, where  $\alpha$  is set equal to 5%. Le Pen (2011) offers a pairwise study of output convergence between 195 European regions. Although this particular study integrates structural breaks into the analysis, the evidence is not supportive of stochastic convergence. Using these studies as a starting point, we extend the analysis in two important directions. First, none of these regional studies consider an empirical distribution of the fraction of rejections. In our investigation, we construct confidence intervals around the point estimates of the percentage of rejections. Second, and more importantly, the studies mentioned only consider pairwise stationarity with an implied cointegrating coefficient of unity. In sharp contrast to the existing literature, we conduct an analysis that also permits the possibility of pairwise cointegration, that is, relaxing the [1, -1] assumption for the cointegrating vector, as distinct from pairwise stationarity. This extension allows pairwise states to share a common stochastic trend such that state responses to shocks are proportional rather than equal.

The paper is organized as follows. The following section briefly describes the pairwise testing approach to convergence. The third section discusses the data and the results of the empirical analysis. Although pairwise unit root testing is not supportive of long-run convergence among the 48 U.S. states, our pairwise

cointegration approach provides more supportive evidence. Further, we find that the strength of convergence, as measured by the long-run slope coefficient, is negatively related to both distance and initial income disparity. When we consider a more disaggregated data set for 346 Metropolitan Statistical Areas (MSAs), confirmatory evidence of convergence is found (more disaggregation implies stronger convergence). The final section offers some concluding remarks.

#### 2. A PAIRWISE APPROACH TO TESTING FOR CONVERGENCE

The unit root and cointegration tests employed in the literature to assess stochastic convergence have typically been applied to regional income benchmarked against national income. However, this approach could be sensitive to the choice of base region or state and susceptible to aggregation bias.<sup>1</sup> For the purposes of our empirical analysis, we employ the Pesaran (2007) pairwise testing procedure to analyze probabilistic convergence across a large number of cross-section units. Let  $y_{it}$  be real per capita income data in U.S. state *i* at time *t*, where i = 1, ..., Nand t = 1, ..., T. Pesaran's pairwise approach is based on examination of the time series properties of all N(N-1)/2 possible real per-capita income gaps between states i and j, denoted  $g_{ijt} = y_{it} - y_{jt}$ , where i = 1, ..., N - 1 and j = i + 1, ..., N. Defining  $\bar{Z}_{NT}$  as the fraction of gaps for which the unit-root hypothesis is rejected, Pesaran (2007) shows that as  $T \to \infty$ , the expected value of  $\bar{Z}_{NT}$  is equal to the nominal size of the unit root test statistic,  $\alpha$ . Thus, evidence of stochastic convergence is found whenever  $\bar{Z}_{NT} > \alpha$ , which is consistent with Definition 2.1 in Bernard and Durlauf (1995). According to this definition, for two states to converge, their incomes must be cointegrated, and the cointegrating vector must be equal to [1, -1]'. This means that in the long run the incomes of the two states contain a common stochastic trend (i.e., they share the same determinants) and respond to permanent shocks with the same weights.

Under the pairwise approach, it is possible that convergence occurs across all states even if cointegration cannot be detected for every pair we choose. An important issue that arises is whether  $\bar{Z}_{NT}$  is statistically different from  $\alpha$ . In a significant departure from Mello (2011), we compute confidence intervals for  $\bar{Z}_{NT}$ . These are based on the derivation of the empirical distribution of the fraction of rejections using the bootstrap methodology, because the derivation of the variance of  $\bar{Z}_{NT}$  is complicated by the fact that individual unit root tests are not independent from each other. Moreover, the pairwise approach outlined so far implicitly assumes that all income pairs are cointegrated, with a known cointegrating vector equal to [1, -1]'. This implies that the output forecasts of any state pair will be equal as  $T \to \infty$ . In practice, this might be regarded as a somewhat strong assumption. Thus, we might expect a case for weaker convergence whenever there is cointegration between two income series but the cointegrated vector is not known. For this reason, we deviate further from Mello (2011) by extending Pesaran's pairwise approach, considering a test for a weaker form of convergence, according to which income pairs are cointegrated with an unknown cointegrating vector. In other words, instead of

testing whether  $g_{ijt} = y_{it} - y_{jt}$  is stationary, we alternatively examine whether  $y_{it}$  and  $y_{jt}$  are cointegrated, possibly with a slope different from unity. This weaker form of convergence is consistent with Definition 2.2 in Bernard and Durlauf (1995), where long-run income forecasts are proportional. This means that the two series contain a common stochastic trend insofar as they share the same determinants in the long run, but in the long run they respond to permanent shocks with different weights. To test for pairwise cointegration between state incomes, we employ the Johansen (1988) maximum likelihood estimator of cointegrated vector autoregressive (VAR) models, which offers the advantage that normalization on a particular state within each bivariate relationship is not an issue, insofar as it does not matter whether one looks at cointegration between  $y_{it}$  and  $y_{jt}$ , or between  $y_{jt}$  and  $y_{it}$ .

At this point it is worth mentioning that attempting to estimate a single vector error correction model (VEC) that incorporates the per capita income series for all the U.S. states would be highly problematic because of the large number of states and lags that would be involved in the modeling exercise, and the number of time observations that are available. The pairwise Johansen approach provides the opportunity to incorporate all bivariate state income relationships that exist. Of course, there already exist panel unit root and noncointegration tests such as Maddala and Wu (1999), Pedroni (2001), Levin et al. (2002), and Im et al. (2003) as potential ways of overcoming the low test power attached to univariate methods. However, following Pesaran et al. (2009), it can be argued that the pairwise methodology provides three key advantages over existing panel techniques in terms of (i) addressing the proportion of cases that are stationary (or cointegrated); (ii) allowing for the presence of cross-sectional dependence across states; and (iii) not being affected by the selection of a base or reference state.

#### 3. EMPIRICAL ANALYSIS

#### 3.1. Data Description

We employ per capita personal income (PCPI) data for 48 U.S. states in dollars.<sup>2</sup> The data, expressed in natural logarithm form, are annual, cover the study period from 1929 to 2009 for a total of 81 observations, and were downloaded from the Federal Reserve Economic Dataset (FRED), assembled by the Federal Reserve Bank of St. Louis. In the data set, each income series is coded as the state abbreviation plus the suffix PCPI; thus, for instance, ARPCPI is per capita personal income in Arkansas, and so on. Because reliable data on state price levels are not available, the PCPI series for each state is then deflated by the overall consumer price index; see, for example, Sala-i-Martin (1996) and Barro and Sala-i-Martin (1999, Ch. 10). Perhaps it is worth noting that in the original pairwise approach advocated by Pesaran (2007), which examines differentials between pairs of series, the results are not affected by the choice of data in nominal or real terms (as long as all series are deflated by the same national deflator, as in this paper). However, when one

considers cointegration between pairs of series, whether the series are nominal or real turns out to be important.

In addition to these state-level data, we also analyze PCPI data obtained from the FRED data set for 346 MSAs over the study period 1969–2009. The reason for analyzing this alternative data source is not only to exploit the information provided by a highly disaggregated data set, but also the fact that the pairwise approach is applicable even when the number of individual series in the panel, N, is large relative to its time dimension, T, as shown by the analytical and Monte Carlo simulation results reported in Pesaran (2007). In particular, the simulation results presented by this author in Tables 1b and 2b illustrate that under the null of nonconvergence, the rejection frequencies vary little across N for a given T, and increase rapidly with T. Overall, the pairwise testing approach performs well for  $T \ge 30$ .

#### 3.2. Discussion of Results

We begin our empirical analysis of convergence by considering a standard approach based on unit root tests applied to income differentials in each state. For this, one could calculate relative income with respect to national real per capita income (United States), as in, e.g., Mello (2011), or with respect to a particular state. For the latter, and in what can be regarded as a somewhat arbitrary choice, we select four benchmark states with respect to which all other incomes are measured, namely California (CA), Florida (FL), Illinois (IL), and New York (NY). As can be seen from the results reported in Table 1, there is considerable variation in the percentage of rejections. In many cases the order of integration of the income differentials depends upon the chosen benchmark. For instance, in the cases of the states of Texas (TX) and Washington (WA), we reject the unit root hypothesis when their incomes are measured relative to the national average (US), Illinois (IL), and New York (NY), but not when they are measured relative to the states of California (CA) and Florida (FL). The results in Table 1 thus illustrate the importance of considering the additional information that emerges when one looks at state incomes in pairs of states, as opposed to the income of a given state relative to U.S. income, or relative to the income of any other (somewhat arbitrarily chosen) state.

Table 2 reports the percentage of rejections of the augmented Dickey and Fuller (1979) (ADF) tests, based on all 1,128 bivariate income differentials. These tests are conducted at the 5% and 10% significance levels, the number of lags of the test regression is determined using the Akaike information criterion (AIC) with  $p_{\text{max}} = 4$ , and a trend term is included if it is statistically significant at the 5% level.<sup>3</sup> As can be seen, the percentage of rejections exceeds the size of the unit root test statistics, being equal to 33.78% (46.72%) at the 5% (10%) significance level; qualitatively similar results are obtained from employing the more powerful Elliott et al. (1996) (ERS) unit root test (these results are not reported here, though). If we follow Barro (1991) and others in making a distinction between Southern

State	Relative to U.S.	Relative to CA	Relative to FL	Relative to IL	Relative to NY
AL	-2.432	-2.469	-1.636	-1.933	-3.533◆
AR	<i>−</i> 3.364 <sup>◆</sup>	-3.037◆	-1.807	-1.741	-2.040
AZ	<i>−</i> 3.365 <sup>◆</sup>	<i>−</i> 3.288 <sup>◆</sup>	-1.669	-3.225◆	-3.225◆
CA	-1.376	n.a.	-2.287	-2.131	<i>−</i> 2.876 <sup>◆</sup>
CO	-2.101	-2.164	<b>−3.360</b> <sup>◆</sup>	-1.957	-2.471
CT	-2.081	-0.760	-2.662◆	-2.030	<i>−</i> 2.981 <sup>◆</sup>
DE	-1.256	<i>−</i> 2.989 <sup>◆</sup>	-1.924	-1.666	<i>−</i> 2.794 <sup>◆</sup>
FL	-2.193	-2.287	n.a.	<i>−</i> 2.935 <sup>◆</sup>	<i>−</i> 3.494 <sup>◆</sup>
GA	<i>−</i> 3.624 <sup>◆</sup>	<i>−</i> 2.634 <sup>◆</sup>	-2.047	-2.268	<i>−</i> 3.846 <sup>◆</sup>
IA	-2.341	-1.750	<i>−</i> 2.695 <sup>◆</sup>	-2.499	-2.309
ID	<i>−</i> 3.368 <sup>◆</sup>	-3.409◆	-1.202	<i>−</i> 4.984 <sup>◆</sup>	-4.281 <sup>♦</sup>
IL	-2.606	-2.131	<i>−</i> 2.935 <sup>◆</sup>	n.a.	-1.765
IN	-2.473	-2.587	-0.844	<i>−</i> 4.685 <sup>◆</sup>	-2.482
KS	-3.262 <sup>◆</sup>	-1.804	-2.276	<i>−</i> 2.633 <sup>◆</sup>	<i>−</i> 3.687 <sup>◆</sup>
KY	-1.628	-2.255	-1.809	-2.481	-2.310
LA	-1.717	-1.423	-2.424	-1.477	-2.460
MA	-2.468	-0.524	-3.905	-1.653	-1.686
MD	-2.463	-0.801	<i>−</i> 3.434 <sup>◆</sup>	-0.685	-2.155
ME	-2.028	-0.473	-4.471 <sup>◆</sup>	-2.305	-2.713 <sup>◆</sup>
MI	-1.258	-2.504	-0.222	-3.933 <sup>◆</sup>	-3.067 <sup>◆</sup>
MN	-1.942	-1.265	-3.752 <sup>◆</sup>	-1.699	-2.510
MO	-2.621	-1.670	-2.915 <sup>•</sup>	-1.941	-2.869 <sup>◆</sup>
MS	-2.665	-2.570	-1.675	-2.141	-3.742 <sup>◆</sup>
MT	-1.842	-3.525 <b>◆</b>	-1.009	-3.566	-2.557
NC	-3.644 <sup>◆</sup>	-2.479	-1.942	-3.139 <sup>◆</sup>	-4.096 <sup>◆</sup>
ND	-2.057	-1.852	-2.191	-2.071	-2.232
NE	-1.891	-1.435	-2.867 <b></b>	-1.842	-2.060
NH	-1.539	-0.668	-3.024 <sup>◆</sup>	-1.291	-2.292
NJ	-2.328	-0.074	-2.650 <b>◆</b>	-0.782	-2.443
NM	-2.186	-1.970	-1.810	-2.739 <b>◆</b>	-3.802 <sup>◆</sup>
NV	-0.858	-3.079 <sup>◆</sup>	-0.978	-1.965	-3.001 <sup>◆</sup>
NY		-2.876 <sup>◆</sup>		-1.765	n.a.
OH	-0.982	-2.324	-0.822	-5.341 <sup>◆</sup>	-3.056 <sup>◆</sup>
OK	-0.982 -1.570	-2.324 -1.365	-2.274	$-2.759^{\bullet}$	-1.968
OR	-1.515	-2.575	-1.338	-2.739 -3.827◆	-3.370 <sup>◆</sup>
PA	-3.299 <sup>◆</sup>	-2.575 -0.612	-1.558 -3.195◆	-1.225	-2.560
RI	-2.376	-1.663	-3.165◆	-1.223 -1.766	-2.300 -5.185◆
SC	-2.370 -3.597◆	-1.005 -3.151◆	-2.364	-2.178	-3.185 -3.583◆
SD	-2.250	-2.084	-2.098	-2.318	-3.383 -2.727◆
	-2.250 -3.805◆				-2.727* -3.223◆
TN		-2.178	-1.422	-1.690 2.722♠	
TX	-3.265 <sup>◆</sup>	-2.074 2.222◆	-2.013	-2.723 <sup>◆</sup>	-3.929 <sup>◆</sup>
UT	-2.466	-3.332 <sup>◆</sup>	-1.161	-4.255 <sup>◆</sup>	$-3.750^{\diamond}$
VA	-1.497	-3.020 <sup>◆</sup>	-1.316	-2.620 <sup>◆</sup>	-3.484 <sup>◆</sup>
VT	-1.033	-0.086	-4.082 <sup>◆</sup>	-0.702 2.724♠	-2.872 <sup>◆</sup>
WA	-3.068 <sup>◆</sup>	-2.524	-1.845	-3.734 <b>◆</b>	-3.585 <sup>◆</sup>
WI	-2.809 <b>◆</b>	-1.436	-2.632 <b>◆</b>	-2.205	-3.481 <sup>◆</sup>
WV	-2.398	-1.489	-3.527 <b>◆</b>	-1.917	-2.842 <b>◆</b>
WY	-3.204◆	<i>−</i> 2.635 <sup>◆</sup>	-2.205	-1.855	<i>−</i> 2.965 <sup>◆</sup>

TABLE 1. ADF unit root *t*-tests on relative per capita personal income

*Notes*:  $\bullet$  denotes significance at the 10% level; n.a. indicates not applicable. Unit root tests include an intercept term and the number of lags is determined using the AIC with  $p_{max} = 4$ .

**TABLE 2.** Fraction of rejections assuming state income pairs are cointegrated with known cointegrating vector [1, -1]'

α	All states	Non-Southern states	Southern states
5%	33.78	35.29	38.18
10%	46.72	47.15	58.18

*Notes*: Sample period 1929–2009. In line with Barro and Sala-i-Martin (1992), the following 11 states are classified as Southern: Alabama, Arkansas, Florida, Georgia, Kentucky, Louisiana, Mississippi, North Carolina, South Carolina, Tennessee, and Virginia. The remaining states are non-Southern. The ADF unit-root test regressions include a linear trend if it is statistically significant at the 5% level. The number of lags of the dependent variable is determined using the AIC with  $p_{max} = 4$ . Critical values for the ADF test are based on response surfaces estimated by Cheung and Lai (1995).

and non-Southern states, the corresponding point fraction of rejections is slightly higher for Southern states (38.18%) than the 35.29% obtained for non-Southern states.<sup>4</sup> The results just described, however, focus only on the point estimate of the proportion of the pairwise tests that reject the null hypothesis of no convergence. It is important to consider the precision of these estimates, because potential cross-section dependence between the test outcomes is likely to increase the uncertainty considerably. We therefore move the analysis forward by employing the factor-augmented sieve bootstrap approach outlined in Appendix 1. In doing so, the cross-sectional dependence is interpreted in terms of a factor model. As explained, the parameters of an underlying factor model are estimated directly, and we subsequently use these estimates to bootstrap the pairwise rejection rates, treating this factor model as an approximation to the true data generation process (the bootstrap results are based on 5,000 replications).

To implement the bootstrap, we start off by considering the time series properties of the cross-sectional mean of all income series in real terms, denoted as  $\bar{y}_t$  in the previous section, as an estimate of the common factor.<sup>5</sup> The results indicate that the ADF and ERS tests (including constant and trend) provide mixed evidence regarding the order of integration of  $\bar{y}_t$ ; that is, whereas ADF(1) = -3.075 suggests that the null hypothesis of a unit root is not rejected at the 10% significance level, ERS(1) = -3.263 provides evidence in favor of stationarity. Thus, for the purposes of the implementation of the bootstrap, we consider two cases, one in which a unit root is imposed on  $\bar{y}_t$ , and another one in which a unit root is not imposed.

Table 3 reports the respective distributions of the bootstrapped fraction of rejections for the income gaps in real terms. Focusing on the case where a unit root is imposed on the common factor, the results of the ADF test reveal that the mean of the bootstrap distribution is 16.03% for  $\alpha = 10\%$ , a value that is much lower than the corresponding point estimate of 46.72% reported in Table 2. The lower bound of the 90% bootstrap confidence interval is 7.27%, which includes 10%. It should be recalled that for convergence one would expect a fraction of rejections

States	α	Mean	Median	SD	2.5%	5%	10%	90%	95%	97.5%
Imposing a unit root on common factor										
All	5%	8.79	7.80	4.57	<u>2.75</u>	3.28	3.98	14.98	17.73	20.57
	10%	16.03	14.89	6.63	6.38	7.27	8.59	25.09	28.82	31.74
Non-Southern	5%	9.09	8.26	4.33	<u>3.00</u>	3.45	4.35	15.02	17.27	<u>19.52</u>
	10%	16.16	15.32	6.37	6.46	<u>7.36</u>	8.71	24.78	<u>27.78</u>	30.78
Southern	5%	10.63	9.09	6.43	1.82	1.82	3.64	18.18	21.82	<u>25.46</u>
	10%	19.08	18.18	8.85	5.46	7.27	9.09	30.91	<u>34.55</u>	40.00
		Not in	nposing a	unit ro	oot on	commoi	n factor			
All	5%	11.46	10.73	4.78	<u>4.43</u>	5.14	6.12	17.73	20.39	23.05
	10%	20.10	19.33	6.41	9.66	<u>10.81</u>	12.50	28.81	<u>31.65</u>	34.93
Non-Southern	5%	13.17	12.16	6.20	<u>4.35</u>	5.11	6.29	21.47	24.93	28.38
	10%	21.83	21.02	7.99	9.16	10.21	12.16	32.58	<u>36.49</u>	40.24
Southern	5%	13.08	12.73	7.08	<u>1.82</u>	3.64	5.46	21.82	25.46	<u>29.09</u>
	10%	22.82	21.82	9.40	7.27	<u>9.09</u>	10.91	34.55	<u>40.00</u>	43.64

**TABLE 3.** Distribution of the bootstrapped fraction of rejections assuming state income pairs are cointegrated with known cointegrating vector [1, -1]'

*Notes*: Sample period 1929–2009. Pairwise ADF unit root tests. The bounds of the confidence intervals are given by the underlined figures. The number of bootstrap replications used to derive the empirical distribution of the fraction of rejections is 5,000.

larger than 10%, which is the significance level at which the tests are conducted. If a unit root is not imposed on the common factor, the lower bound of the bootstrap distribution is in the boundary of  $\alpha = 10\%$  for the ADF test (i.e., 10.81%). Similar results are observed when  $\alpha = 5\%$  is used as the significance level, or when instead of the ADF unit root test the ERS test is employed in the analysis. It is therefore clear that cross-section dependence introduces a large degree of uncertainty into the point estimate of the proportion of rejections. As can be seen from Table 3, qualitatively similar results are also obtained when Southern and non-Southern states are considered.

Our findings so far do not provide strong support for long-run convergence between U.S. state incomes in real terms. The possibility we have considered so far is one of strong convergence where the implied cointegrating vector is restricted to [1, -1]. In the spirit of Definition 2.2 in Bernard and Durlauf (1995), it can be argued that a weaker form of convergence is more relevant, whereby state income pairs are cointegrated, but with an unknown cointegrating vector not necessarily equal to [1, -1].<sup>6</sup> To explore this possibility, we employ and develop the Johansen (1988) maximum likelihood estimator of cointegrated VAR models within the Pesaran pairwise setting. The starting point of the analysis is to estimate for each possible state income pair a VAR model with an unrestricted constant term (because the variables exhibit a positive drift). The optimal lag length of the VAR models is determined using the AIC with  $p_{max} = 4$ . Then we use the trace and the maximum eigenvalue tests to determine the number of cointegrating vectors, which is denoted r. The former involves testing the null hypothesis that

Но	На	α	Fraction of rejections
	Jo	hansen trac	ce test
r = 0	$r \ge 1$	5%	62.68
r = 0	$r \ge 1$	10%	71.72
	Johansen i	maximum e	eigenvalue test
r = 0	r = 1	5%	53.55
r = 0	r = 1	10%	65.60

**TABLE 4.** Fraction of rejections assuming state income pairs are cointegrated with unknown cointegrating vector

*Notes*: Sample period 1929–2009. The Johansen cointegration test results are based on the estimation of bivariate VAR models with a constant term that enters unrestrictedly. The number of lags of the VAR models is determined using the AIC with  $p_{\text{max}} = 4$ . *r* denotes the number of cointegrating vectors. Critical values are based on response surfaces estimated by MacK-innon et al. (1999).

there are r = 0 cointegrating vectors against the alternative that  $r \ge 1$ . The latter involves testing the null hypothesis that there are r = 0 cointegrating vectors against the alternative that r = 1. In both cases, if the null hypothesis is rejected, then this would provide support for the view that the two real income series share the same stochastic trend.<sup>7</sup>

The results in Table 4 indicate that the proportion of rejections of  $H_o: r = 0$ (against  $H_a: r \ge 1$ ) is 71.72% (i.e., 809 out of 1,128 possible real income gaps) setting  $\alpha = 10\%$ , whereas the corresponding number of rejections for  $\alpha = 5\%$  is 62.68%. Once again, these initial point estimates of the percentage of rejections of noncointegration fail to account for the presence of potential cross-section dependence, so we implement the bootstrap procedure. These results thus offer support for the presence of a weaker form of convergence; see Table 5. Indeed, let us again consider the results obtained for the trace test when a unit root is imposed on  $\bar{y}_t$ . Looking at  $\alpha = 10\%$ , the mean proportion of rejections is 37.31%, and the 90% bootstrap confidence interval around this mean estimate ranges from 12.94% to 69.59%. Therefore, this 90% confidence interval does not cover values below 10%. Qualitatively similar results are obtained using the maximum eigenvalue test, or setting  $\alpha = 5\%$ , or when a unit root is imposed on the common factor  $\bar{y}_t$ (irrespective of the significance level).

These results are more favorable toward the presence of cointegration between bivariate state pairs. It can be argued that strong or weak convergence is reflected in the long-run slope coefficient that depicts each long-run relationship. Conditioning on the cases for which the trace test provides evidence in favor of cointegration, that is, 809 when  $\alpha = 10\%$  and 707 when  $\alpha = 5\%$ , the null hypothesis that the cointegrating vector can be set equal to [1, -1] cannot be rejected in less than half of the possible cases—more precisely  ${}^{303}/_{809} = 0.37$  when  $\alpha = 10\%$  and  ${}^{310}/_{707} = 0.44$  when  $\alpha = 5\%$ .

Johansen tests											
Но	На	α	Mean	Median	SD	2.5%	5%	10%	90%	95%	97.5%
Imposing a unit root on common factor											
Trace:											
r = 0	$r \ge 1$	5%	25.87	22.52	14.80	<u>6.20</u>	7.71	9.93	47.08	55.33	<u>62.41</u>
r = 0	$r \ge 1$	10%	37.31	34.62	17.31	10.90	<u>12.94</u>	16.58	62.59	<u>69.59</u>	74.91
Max. E	Eigenval	ue:									
r = 0	r = 1	5%	21.54	17.64	13.66	5.41	6.56	8.07	40.78	50.36	56.83
r = 0	r = 1	10%	31.65	28.10	16.22	9.66	<u>11.44</u>	13.74	55.50	<u>64.27</u>	69.42
			Not i	mposing a	a unit ro	ot on c	ommon	factor			
Trace:											
r = 0	$r \ge 1$	5%	27.80	25.53	12.77	9.22	11.26	13.56	45.48	52.75	<u>58.95</u>
r = 0	$r \ge 1$	10%	36.10	34.31	13.57	14.71	<u>17.29</u>	20.21	55.14	<u>61.53</u>	67.11
Max. E	Eigenval	ue:									
r = 0	r = 1	5%	20.90	18.44	11.22	<u>6.56</u>	7.80	9.49	35.90	43.71	<u>50.44</u>
r = 0	r = 1	10%	27.92	25.44	12.22	11.26	<u>12.86</u>	14.89	44.60	<u>52.58</u>	59.22

**TABLE 5.** Distribution of the bootstrapped fraction of rejections assuming state

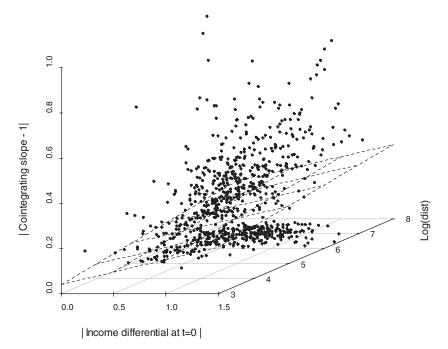
 income pairs are cointegrated with unknown cointegrating vector

*Notes*: Sample period 1929–2009. *r* denotes the number of cointegrating vectors. The bounds of the confidence intervals are given by the underlined figures. The number of bootstrap replications used to derive the empirical distribution of the fraction of rejections is 5,000.

Given the presence of weak as opposed to strong convergence across state pairs and heterogeneity in the estimated long-run slopes, it is of interest to consider what factors might drive the estimated values of the slopes themselves and whether it is possible to define a basis for convergence clusters. Choosing  $\beta_2^{(ij)}$  as the cointegrating slope, we measure the strength of convergence between real per capita personal incomes in states *i* and *j* as the absolute value of the difference between  $\beta_2^{(ij)}$  and one, that is,  $|\beta_2^{(ij)} - 1|$ , and consider the roles played by two potential drivers. The first is the absolute value of the difference between (the logs of) initial per capita income in states *i* and *j*, denoted by  $|\log y_{i0} - \log y_{j0}|$ . The second driver is (the log of) distance between states *i* and *j*, which we denote log  $D_{ij}$ . For this, we employ the Euclidean distance between the population centers of any two states, based on the geographic coordinates (latitude and longitude) obtained from the Census Bureau for the year 2000.<sup>8</sup>

Estimation by OLS for the 809 cases where the trace test rejects the null hypothesis of no cointegration at the 10% significance level yields the following regression result:

$$\begin{vmatrix} \beta_2^{(ij)} - 1 \end{vmatrix} = -\underbrace{0.032}_{(0.076)} + \underbrace{0.110}_{(0.028)} \left| \log y_{i0} - \log y_{j0} \right| + \underbrace{0.024}_{(0.011)} \log D_{ij}, \\ \hat{\sigma} = 0.251, \end{aligned}$$
(1)



**FIGURE 1.** Three-dimensional scatter plot and regression plane of convergence determinants. To improve the visualization of the scatterplot, the observations for which the value of the *z*-axis is greater than one are excluded. Nonetheless, the regression plane is fitted using all 809 observations for which the Johansen trace test rejects the null hypothesis of no cointegration at the 10% significance level.

where White's heteroskedasticity-consistent standard errors are reported in parentheses.<sup>9</sup> The estimated positive coefficient on  $|\log y_{i0} - \log y_{j0}|$  supports the view that the likelihood of strong convergence, or  $|\beta_2^{(ij)} - 1| = 0$ , is enhanced if two pairwise states are characterized by similar initial per capita incomes. The estimated coefficient on (the log of) distance is positive and statistically significant, supporting the view that convergence between any two states is stronger, the closer they are in terms of distance. Thus, although our findings are supportive of cointegrating relationships across state pairs, it is on this basis that convergence clubs or groupings may arise.<sup>10</sup> To illustrate graphically the convergence results, Figure 1 presents the regression plane estimated in the preceding, along with the corresponding three-dimensional scatterplot of the data, in which to improve the visualization of the results the z-axis is restricted to the [0, 1] interval. As can be seen from the figure, the shorter the distance between any two states, and the smaller the absolute difference between their initial incomes, the closer the cointegrating slope will be to unity; that is, the more likely is convergence Definition 2.1 in Bernard and Durlauf (1995) to hold.

TABLE 6. Fraction of rejections
assuming MSA income pairs are
cointegrated with known cointe-
grating vector $[1, -1]'$

α	Fraction of rejections
5%	15.48
10%	25.38

*Notes*: The ADF unit-root test regressions include a linear trend if it is statistically significant at the 5% level. The number of lags of the dependent variable is determined using the AIC with  $p_{max} = 4$ . Critical values for the ADF test are based on response surfaces estimated by Cheung and Lai (1995).

In the final part of our investigation, we examine convergence using MSA-level data that provide 346 annual income series over the study period 1969–2009. As with the case of state-level data, we are using the maximum study period dictated by data availability. Once again, we begin our analysis by applying the ADF unit root test to MSA income differentials calculated with respect to national real per capita income. Results, not reported here, indicate that the null hypothesis of nonstationarity can be rejected only in 26 (49) instances when a 5% (10%) significance level is used. Table 6 reports the percentage of rejections of the ADF unit root tests based on all 59,585 bivariate MSA-level income differentials.<sup>11</sup> The percentage of rejections exceeds the size of the unit root test statistics, being equal to 15.48% (25.38%) at the 5% (10%) significance level. Table 7 reports the respective distributions of the bootstrapped fraction of rejections. These results are supportive of strong convergence when MSA-level data are analyzed. Focusing on the case where a unit root is imposed on the common factor, for example, the lower bound of the 90% bootstrap confidence interval is 16.26%. In contrast to

α	Mean	Median	SD	2.5%	5%	10%	90%	95%	97.5%		
Imposing a unit root on common factor											
5%	12.65	12.46	2.29	8.65	9.19	9.88	15.64	16.72	17.84		
10%	21.16	20.99	3.12	15.39	16.26	17.32	25.21	26.59	27.93		
	Not imposing a unit root on common factor										
5%	12.68	12.49	2.25	8.80	9.30	9.98	15.62	16.68	17.74		
10%	21.20	21.01	3.07	15.71	16.44	17.42	25.24	26.57	27.78		

**TABLE 7.** Distribution of the bootstrapped fraction of rejections assuming MSA income pairs are cointegrated with known cointegrating vector [1, -1]'

*Notes*: Pairwise ADF unit root tests. The bounds of the confidence intervals are given by the underlined figures. The number of bootstrap replications used to derive the empirical distribution of the fraction of rejections is 5,000.

Но	На	α	Fraction of rejections			
	Jo	hansen trace te	est			
r = 0	$r \ge 1$	5%	15.32			
r = 0	$r \ge 1$	10%	24.64			
	Johansen	maximum eige	nvalue test			
r = 0	r = 1	5%	12.57			
r = 0	r = 1	10%	20.39			

**TABLE 8.** Fraction of rejections assuming MSA income pairs are cointegrated with unknown cointegrating vector

*Notes*: The Johansen cointegration test results are based on the estimation of bivariate VAR models with a constant term that enters unrestrictedly. The number of lags of the VAR models is determined using the AIC with  $p_{\text{max}} = 4$ . *r* denotes the number of cointegrating vectors. Critical values are based on response surfaces estimated by MacKinnon et al. (1999).

the earlier results based on state-level data, this is greater than 10%. This finding is qualitatively unchanged when  $\alpha = 5\%$  is used as the significance level, or when a unit root is not imposed on the common factor. Further results based on the application of the pairwise Johansen test are reported in Tables 8 and 9. Focusing on the trace test, the fraction of rejections of H<sub>o</sub> : r = 0 (against H<sub>a</sub> :  $r \ge 1$ ) is 24.64% when  $\alpha = 10\%$ , whereas the corresponding fraction of rejections for  $\alpha = 5\%$  is 15.32%. If a unit root is imposed on  $\bar{y}_t$ , when looking at  $\alpha = 10\%$ the mean proportion of rejections is 27.40%, and the 90% bootstrap confidence interval around this mean estimate ranges from 17.98% to 42.66%. This 90% confidence interval does not cover values below 10%, and so offers further support for income convergence. Qualitatively similar results are obtained by using the maximum eigenvalue test, by setting  $\alpha = 5\%$ , or when a unit root is imposed on the common factor  $\bar{y}_t$  (irrespective of the significance level).

#### 3.3. Comparison with Existing Findings

The earlier pairwise findings reported by Le Pen (2011) and Mello (2011) focus only on strong convergence and are based on point estimates, making no allowance for confidence intervals. Our investigation, based on confidence intervals, suggests that in fact, a weaker form of convergence at the state level is likely to hold in terms of state incomes sharing a common stochastic trend. Although Le Pen (2011) provides evidence against convergence in the case of a European sample, it is most likely that the point pairwise estimate for U.S. states reported by Mello (2011) lies within a confidence interval based on the size of the individual unit root tests. The results that we report here are supportive of convergence across U.S. states, but we find that relative state incomes might respond proportionately to shocks such as those based on saving, demography, and technology. Therefore, the long-run forecasts of relative state incomes are proportional rather than equal. This is a form of convergence that is not considered by Le Pen (2011) or by Mello (2011).

Johansen tests											
Но	На	α	Mean	Median	SD	2.5%	5%	10%	90%	95%	97.5%
Imposing a unit root on common factor											
Trace:											
r = 0	$r \ge 1$	5%	19.57	18.23	6.32	11.37	12.33	13.33	27.38	31.34	36.22
r = 0	$r \ge 1$	10%	27.40	25.74	7.95	16.73	<u>17.98</u>	19.36	37.45	<u>42.66</u>	48.17
Max. H	Eigenval	ue:									
r = 0	r = 1	5%	17.72	16.79	4.65	11.45	12.15	13.01	23.12	26.31	29.34
r = 0	r = 1	10%	25.15	24.04	6.01	16.96	<u>17.91</u>	18.95	32.40	<u>36.21</u>	40.26
			Not in	nposing a	unit r	oot on o	commo	n factor	•		
Trace:											
r = 0	$r \ge 1$	5%	20.14	19.68	3.85	13.97	14.71	15.61	25.23	27.24	28.91
r=0	$r \ge 1$	10%	28.28	27.78	4.83	20.25	<u>21.29</u>	22.55	34.58	<u>37.14</u>	38.99
Max. H	Eigenval	ue:									
r = 0	r = 1	5%	18.46	18.09	3.36	13.06	13.70	14.51	22.89	24.66	26.16
r = 0	r = 1	10%	25.96	25.59	4.23	19.00	<u>19.88</u>	20.97	31.56	<u>33.57</u>	35.39

**TABLE 9.** Distribution of the bootstrapped fraction of rejections assuming MSA income pairs are cointegrated with unknown cointegrating vector

*Notes*: *r* denotes the number of cointegrating vectors. The bounds of the confidence intervals are given by the underlined figures. The number of bootstrap replications used to derive the empirical distribution of the fraction of rejections is 5,000.

earlier study by Tsionas (2001) investigates common trends, but concludes that multiple common trends, rather than a single common trend, drive real incomes across states. On this basis, Tsionas (2001) is not able to conclude that convergence is present. Our finding of weak convergence is robust to a pairwise grouping that permits us to distinguish between Southern and non-Southern states, and we do not find evidence that supports the presence of convergence clubs along these particular lines.

We do find that strong convergence is present when examining data at the MSA level. In convergence studies, the concept of aggregation bias suggests that it is more likely that convergence will be found at the state level. It should, however, be remembered that the MSA-level study period used in this study starts much later, in 1969 rather than in 1929. On this basis, our study suggests that strong convergence could be a phenomenon associated with the more recent decades. After all, the support for stochastic convergence offered by Carlino and Mills (1993) relies on allowing for a structural break in 1946. Last, we find that distance is an important consideration in reflecting on the strength of convergence. Indeed, we support the findings of Choi (2004) insofar as there is convergence with respect to geographically neighboring states. Although we find that convergence is present with respect to the states separated by larger distances.

#### 4. CONCLUDING REMARKS

This paper employs time series annual information for U.S. states to assess one of the key predictions of the neoclassical growth model, namely that of real per capita income convergence. Our empirical modeling exercise uses a pairwise probabilistic approach to examine stochastic convergence. This approach is based on the fraction of rejections of nonstationarity or noncointegration across all bivariate state per capita income pairs. Our results confirm convergence over a long time period as well as convergence with highly disaggregated data. Although we reject strong convergence at the state level insofar as testing the nonstationarity of pairwise state income differentials, these tests are characterized by implied cointegrating vectors of the form [1, -1] under the alternative hypothesis. Further results based on the development and application of a pairwise Johansen cointegration test offer more empirical support. In this respect, there is a weaker form of convergence characterized by cointegration between state incomes where the elements of the cointegrating vector are unrestricted. However, we find that the likelihood of convergence between any two states is strongest for those states that have similar initial per capita incomes and are closest in terms of distance. Additional analysis at a more disaggregated level using metropolitan statistical area data from a more recent study period provides stronger evidence of long-run convergence characterized by stationary income differentials.

#### NOTES

1. For example, real per capita income in states i and j might be found nonstationary when they are measured against a third numeraire state k, but stationary when they are measured against one another. This would be the case when there was a highly persistent factor that was common to states i and j, but that was not shared by state k. The pairwise methodology considers all possible bivariate relationships and does not involve what can be a problematic choice of a single reference state across the sample.

2. Alaska and Hawaii are excluded from our analysis on the grounds that these states are not geographically contiguous with any other state in the United States, so the mechanisms that may underpin long-run constancy of income ratios across states within the United States may not operate in these cases.

3. As indicated by Le Pen (2011), authors such as Carlino and Mills (1993), among others, argue that including a deterministic component in the output gap is not always incompatible with convergence. This situation could appear when regions or countries have not yet reached their long-run equilibrium. The deterministic trend would thus reflect a catch-up process. At the end of the catch-up phase, the deterministic trend becomes insignificant.

4. We classify states as Southern and non-Southern following Barro and Sala-i-Martin (1992); see the notes in Table 1.

5. An application of the Bai and Ng (2002) test confirmed the presence of a single common factor driving U.S. state incomes.

6. See also the early work by Quintos (1995) and others in the context of the relationship between government revenue and expenditure and the sustainability of the budget deficit,

7. It is worth mentioning that a test of the null hypothesis that  $r \le 1$  against the alternative that r = 2 is not undertaken, as it opens up the possibility of obtaining counterintuitive results. Indeed, this second test rejection of the null would indicate the presence of two cointegrating vectors or, in other words, that each real income series was stationary in levels.

8. We are most grateful to Gary Wagner, who kindly provided these data, which were used in Garrett et al. (2007).

9. It should also be noted that when the null of homogeneity is not rejected at the 10% significance level (i.e., in 303 instances), the corresponding value of the dependent variable is set equal to zero.

10. To assess whether convergence is stronger when two states are both Southern, as defined in Barro and Sala-i-Martin (1992), inclusion in the regression model of a dummy variable that takes the value of one if this condition is satisfied, and zero otherwise, was also tried. However, the estimated coefficient on this additional regressor was not found to be statistically different from zero.

11. The significant increase in the number of income differentials that accompanies an analysis at MSA-level poses no additional difficulties for the pairwise convergence approach, other than the requirement for substantially more computing time.

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# APPENDIX A: BOOTSTRAPPING THE $\bar{Z}_{NT}$ STATISTIC

To bootstrap the  $\bar{Z}_{NT}$  statistic, we follow Pesaran et al. (PSYH) (2009). The model setup considered by these authors is based on the following set of equations:

$$\mathbf{y}_{it} = \boldsymbol{\alpha}'_i \mathbf{d}_t + \boldsymbol{\gamma}'_i \mathbf{f}_t + \varepsilon_{it}, \qquad (\mathbf{A.1})$$

$$\Delta \varepsilon_{it} = \eta_i + \lambda_i \varepsilon_{i,t-1} + \sum_{l=1}^{p_i} \psi_{il} \Delta \varepsilon_{i,t-l} + \upsilon_{it}, \qquad (A.2)$$

$$\Delta f_{st} = \boldsymbol{\mu}'_{s} \mathbf{d}_{t} + \phi f_{s,t-1} + \sum_{l=1}^{p_{s}} \xi_{sl} \Delta f_{s,t-l} + e_{st}, \qquad (\mathbf{A.3})$$

where s = 1, 2, ..., m is the number of common factors,  $\mathbf{d}_t = (1, t)'$  is a vector of deterministic components that includes intercept and trend,  $\mathbf{f}_t$  is an  $m \times 1$  vector of unobserved

common factors, with elements denoted  $f_{st}$ , and  $\varepsilon_{it}$  denotes the corresponding idiosyncratic elements. The unobserved common factors  $f_{st}$  and/or the idiosyncratic elements  $\varepsilon_{it}$  may be I(0) or I(1).

In line with PSYH, we use the cross-sectional average of  $y_{it}$ , denoted as  $\bar{y}_t = N^{-1} \sum_{i=1}^{N} y_{it}$ , as an estimate of the common factor that may induce cross-section dependence across state incomes. To account for cross-section dependence, real per capita income in each state is regressed on  $\bar{y}_i$ :

$$y_{it} = \hat{\alpha}_i + \hat{\delta}_i t + \hat{\gamma}_i \bar{y}_t + \hat{\varepsilon}_{it}.$$
(A.4)

In Appendix B we report the results of estimating the factor equations for the state-level data used in the paper. It should be noted that in these factor equations the trend term is included if it turns out to be statistically significant at the 5% level.

The next step is to examine the time series properties of  $\bar{y}_t$ , which may be I(0) or I(1). This involves estimating the following ADF(p) regression for  $\bar{y}_t$ :

$$\Delta \bar{y}_{t} = \hat{\mu} + \hat{\phi} \bar{y}_{t-1} + \sum_{l=1}^{p} \hat{b}_{l} \Delta \bar{y}_{t-l} + \hat{e}_{t}, \qquad (A.5)$$

which may also include a trend term if it is statistically significant, and where *p* may be determined, e.g., using the AIC. To illustrate the implementation of the bootstrap, let us consider, for instance, the case in which  $\bar{y}_i$  has a unit root with a drift and no deterministic trend. Imposing a unit root on (A.5), i.e., letting  $\hat{\phi} = 0$ , and allowing for a drift implies the following restricted version of (A.5):

$$\Delta \bar{y}_t = \hat{\mu} + \sum_{l=1}^p \hat{c}_l \Delta \bar{y}_{t-l} + \hat{u}_t.$$
(A.6)

Thus, when a unit root and a drift term are imposed on the factor  $\bar{y}_t$ , the bootstrap samples of  $\bar{y}_t$ , denoted  $\bar{y}_t^{(b)}$ , can be computed using the following generating mechanism:

$$\bar{\mathbf{y}}_{t}^{(b)} = \hat{\mu} + \bar{\mathbf{y}}_{t-1}^{(b)} + \sum_{l=1}^{p} \hat{c}_{l} \Delta \bar{\mathbf{y}}_{t-l}^{(b)} + \hat{u}_{t}^{(b)},$$
(A.7)

where bootstrap residuals  $\hat{u}_t^{(b)}$  are generated by randomly drawing with replacement from the set of estimated and centered residuals  $\hat{u}_t$  in (A.6), and where the first (p + 1) values of  $\bar{y}_t$  are used to initialize the process  $\bar{y}_t^{(b)}$ .

In turn, the bootstrap samples of  $y_{it}$ , denoted as  $y_{it}^{(b)}$ , are generated as

$$y_{it}^{(b)} = \hat{\alpha}_i + \hat{\delta}_i t + \hat{\gamma}_i \bar{y}_{it}^{(b)} + \hat{\varepsilon}_{it}^{(b)}, \qquad (A.8)$$

where  $\hat{\alpha}_i$ ,  $\hat{\delta}_i$ , and  $\hat{\gamma}_i$  are the OLS estimates of  $\alpha_i$ ,  $\delta_i$ , and  $\gamma_i$  in (5), respectively, and

$$\varepsilon_{it}^{(b)} = \hat{\eta}_i + (1 + \hat{\lambda}_i)\varepsilon_{i,t-1}^{(b)} + \sum_{l=1}^{p_i} \hat{\psi}_{il} \Delta \varepsilon_{i,t-1}^{(b)} + \upsilon_{it}^{(b)},$$
(A.9)

where bootstrap residuals  $v_{it}^{(b)}$  are generated by randomly drawing with replacement from the set of estimated residuals  $v_{it}$  in (A.2), and the first (p + 1) values of  $\hat{\varepsilon}_{it}$  are used to initialize the process  $\varepsilon_{it}^{(b)}$ . The AIC is used to select the optimal lag order  $p_i$ .

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After  $y_{it}^{(b)}$  has been obtained, it is possible to compute all possible bootstrap income gaps between states *i* and *j*, that is,  $g_{ijt}^{(b)} = y_{it}^{(b)} - y_{jt}^{(b)}$ , so that one can then calculate the fraction of these income gaps for which the unit root hypothesis can be rejected using either the ADF(*p*) or the ERS(*p*) test. The procedure already described is repeated b = 1, ..., Btimes to derive the empirical distribution of the bootstrapped fraction of rejections.

State	Intercept	S.E.	Trend	S.E.	$\bar{y}_t$	S.E.	$\bar{R}^2$
AL	-1.840	0.100	-0.002	0.001	1.369	0.029	0.998
AR	-2.005	0.121	-0.003	0.001	1.396	0.035	0.997
AZ	-0.095	0.083	-0.003	0.001	1.039	0.024	0.997
CA	1.317	0.028			0.766	0.006	0.995
CO	0.480	0.085	0.003	0.001	0.884	0.024	0.997
CT	2.472	0.174	0.011	0.001	0.423	0.050	0.985
DE	2.028	0.136	0.003	0.001	0.574	0.039	0.986
FL	-0.221	0.034			1.051	0.008	0.996
GA	-1.276	0.038			1.248	0.009	0.996
IA	-0.903	0.147	-0.006	0.001	1.263	0.042	0.992
ID	-0.731	0.141	-0.006	0.001	1.204	0.041	0.992
IL	0.965	0.025			0.833	0.006	0.996
IN	-0.546	0.083	-0.007	0.001	1.188	0.024	0.997
KS	-1.389	0.107	-0.008	0.001	1.386	0.031	0.996
KY	-0.969	0.037			1.165	0.008	0.996
LA	-0.681	0.037			1.110	0.008	0.996
MA	2.875	0.156	0.015	0.001	0.264	0.045	0.988
MD	1.686	0.125	0.009	0.001	0.588	0.036	0.993
ME	1.334	0.144	0.007	0.001	0.618	0.042	0.991
MI	0.003	0.110	-0.006	0.001	1.087	0.032	0.994
MN	0.514	0.079	0.005	0.001	0.855	0.023	0.998
MO	0.259	0.025			0.945	0.006	0.997
MS	-2.297	0.123	-0.002	0.001	1.435	0.035	0.997
MT	-0.361	0.166	-0.007	0.001	1.141	0.048	0.986
NC	-1.293	0.038			1.247	0.009	0.996
ND	-3.389	0.293	-0.017	0.002	1.886	0.084	0.979
NE	-0.898	0.148	-0.005	0.001	1.250	0.043	0.992

# APPENDIX B: FACTOR ESTIMATE EQUATIONS

NH	1.924	0.129	0.013	0.001	0.469	0.037	0.993
NJ	1.958	0.131	0.008	0.001	0.548	0.038	0.992
NM	-1.357	0.153	-0.006	0.001	1.319	0.044	0.992
NV	1.029	0.135	-0.003	0.001	0.854	0.039	0.988
NY	2.549	0.156	0.009	0.001	0.413	0.045	0.984
OH	0.488	0.065	-0.003	0.001	0.936	0.019	0.998
OK	-1.459	0.154	-0.006	0.001	1.351	0.044	0.993
OR	-0.041	0.106	-0.005	0.001	1.076	0.031	0.994
PA	1.278	0.079	0.004	0.001	0.697	0.023	0.997
RI	2.670	0.152	0.011	0.001	0.326	0.044	0.986
SC	-1.546	0.043			1.281	0.010	0.996
SD	-2.618	0.242	-0.012	0.002	1.665	0.070	0.985
TN	-1.147	0.026			1.213	0.006	0.998
ΤX	-0.855	0.106	-0.003	0.001	1.211	0.031	0.996
UT	-0.414	0.102	-0.005	0.001	1.121	0.030	0.995
VA	0.080	0.100	0.006	0.001	0.922	0.029	0.997
VT	1.190	0.130	0.008	0.001	0.645	0.038	0.993
WA	0.233	0.117	-0.003	0.001	1.009	0.034	0.994
WI	0.316	0.019			0.941	0.004	0.998
WV	0.067	0.103	0.002	0.001	0.920	0.030	0.996
WY	0.422	0.063			0.929	0.014	0.982