Three Essays on Public Policies in R&D Growth Models

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Declaration

I hereby declare that this thesis is my original work and it has been written by me in its entirety.
I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

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24/01/2014

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Contents

Summary

My thesis concerns about how public policies affect the performance of a closed economy. We focus on the growth and welfare effects of these policies. We assume that the economy has perfectly competitive final-good and R&D sectors and monopolized intermediate-good sectors. We consider various policy comparisons in the following three chapters. All the three chapters draw on the same basic model, Romer's 1990 model of technical progress through variety expansion. The main results mainly consist of basic analytical ones and numerical ones from simulations with particular parameter values as well as sensitivity analysis.

Chapter 1 develops an endogenous growth model with innovation and human capital accumulation. In this model, both innovation and human capital accumulation drive economic growth. The growth rate of per capita income depends not only on consumers' preferences and human capital accumulation technologies, but also on firms' production and R&D technologies. Government policies such as subsidies to education and R&D influence the growth rate. We examine the steady-state and transitional effects of education and R&D subsidies on growth and welfare and the relative effectiveness of these subsidies. We find that although both the R&D and education subsidies enhance growth (and the latter generates a higher maximum growth rate than the former), the education subsidies improve welfare while the R&D subsidies do the opposite.

Chapter 2 examines optimal taxation in an R&D growth model with variety expansion. We develop two models. In the basic model, where final good is produced with intermediate good and labor, and intermediate goods are produced with physical capital, we show that, for a given exogenous government expenditure, the optimal tax on physical capital income is always negative while the optimal tax on labor income is positive. The result is driven by the monopoly inefficiency in the intermediate-good sectors. Since the maximum amount of available labor is fixed, the labor income tax distortion is limited, thus it is always optimal to tax labor while subsidizing physical capital accumulation. However, in an extended model with human capital

accumulation, the relationship between the growth rate and physical capital income tax rate depends on the values of the elasticity of marginal utility. In this model, it is optimal to tax physical capital income and subsidize human capital investments as long as the government expenditure is low enough. We find that the optimal policies in the extended model are different from those in the basic model due to the fact that in the extended model, the monopolized intermediate-good sectors have higher capital intensities and the taxation of labor income distorts not just the labor-leisure choice but also the rate of investment in human capital. Our dynamic analysis clearly shows that the physical capital income tax distortion decreases the welfare more than the labor income tax distortion in the basic model, while in the extended model with human capital, the ranking reverses.

Chapter 3 considers both fiscal and monetary policies in an R&D growth model with variety expansion and money-in-production. We investigate how different government policies affect resource allocation, growth and welfare. More specifically, we compare two fiscal policies (a consumption tax and a capital income tax) and one monetary policy (inflation tax) as the instruments of financing the government expenditure. We show that given an exogenous government purchase and in the presence of consumption tax, both the growth-maximizing capital income tax and inflation tax should be negative. We find that the results are driven by the monopoly inefficiency which leads to less than optimal demands for both capital and real money. As a result, the consumption tax will be most favourable. We also consider the case without the consumption tax and show that the capital income tax will be more favourable in terms of improving welfare, and the inflation tax will be more effective in terms of promoting growth.

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Part I

Chapter 1: R&D and Education Subsidies in a Growth Model with Innovation and Human Capital Accumulation

1. Introduction

It is widely believed that human capital accumulation (education) and technological innovation are the two main sources of economic growth. There is a huge literature on the connections between education or innovation on the one hand and economic growth on the other. Many studies focus on endogenous accumulation of human capital through education and therefore emphasize the role of investments in education (e.g., Romer, 1986; Lucas, 1988; Rebelo, 1991). Using this types of models, several authors examine the roles of public education/education subsidies in the process of human capital accumulation and growth (e.g., Glomm and Ravikumar, 1992; Kaganovich and Zilcha, 1999; Zhang and Richard 1998). In particular, Lucas (1988) pointed out clearly that there be a positive education externality, this calls for education subsidies. On the other hand, a large literature takes innovation as the main engine of growth and thus emphasizes the role of investments in innovation activities (e.g., Romer, 1990; Aghion and Howitt, 1992; Grossman and Helpman, 1991). Empirical studies, such as Jones and Williams, (1998, 2000) show a positive R&D externality. The impact of R&D subsidies on growth and welfare is also intensively studied in these models (e.g., Barro and Sala-i-Martin, 2004 (Chapter 6); Davidson and Segerstrom, 1998; Zeng and Zhang, 2007).

However, in the modern economies, both education and innovation simultaneously drive economic growth. They should not be treated as distinct causal factors, since human capital becomes more and more important as an input in innovation activities and new technologies give more economic opportunities for investment in education to take place. As pointed out by Romer (2000) in his discussion about U.S. government policies to encourage R&D spending, "few participants in the political debate surrounding demand-subsidy policies seem to have considered the broad range of alternative programs that could be considered." So the question is not whether public policy should promote growth and welfare but how to do it, especially in the countries with tight government budget. For example, in 2012, individual countries within the OECD experiecned large deficits, such as Ireland (8.1% of GDP) and the United States (8.5%.) Outside the OECD, Brazil and China had deficits of around 2% of GDP. Therefore, theoretically it is very important to integrate innovation and education into a single framework to examine the interactions of the two driving forces and investigate the relative effectiveness of the impact of alternative government policies on growth and welfare.

The objective of this chapter is to develop a dynamic general equilibrium growth model with both innovation and human capital accumulation to study the relative effectiveness of R&D and education subsidies in enhancing economic growth and welfare.[1] We extend the basic model in Romer (1990) by endogenizing human capital accumulation. To consider the subsidies to physical investment in education, we assume that human capital accumulation requires not only time input but also physical inputs such as classrooms and teaching equipments. As in Romer (1990) and Barro and Sala-i-Martin (2004, Chapter 6), the laissez-faire equilibrium is not socially optimal because of the inefficient monopoly pricing of the intermediate goods and the positive externalizes associated with R&D. We then use the extended model to numerically study how the R&D subsidies and education subsidies (to either the physical inputs or time input) affect growth and welfare and compare the relative effectiveness of these subsidies. We consider the impact of the subsidies both in the steady state and

[1] Recently, a few papers study issues similar to that in this chapter. Lloyd-Ellis and Roberts (2002) examines the interaction between skills and technology in driving economic growth; Stadler (2012) integrates human capital accumulation into an R&D growth model to investigate how education subsidies affect growth;

during the transition to the steady state.

We find that both the R&D and education subsidies have positive effects on growth. Moreover, the education subsidies are more effective than the R&D subsidies because the latter can more effectively correct the static inefficiency resulting from the monopoly distortion in the intermediate goods production. We also find that the education subsidies can significantly raise welfare while the R&D subsidies reduce it. The reason for this is that the subsidies and the taxes associated with the subsidies generate two offsetting forces – one raises the growth rate (a gain in dynamic efficiency) and the other further mislocates resources (a loss in static efficiency and that the negative force dominates. More closely related to our analysis are the papers by Zeng (2003) and by Grossmann (2004). In Zeng (2003), he incorporated innovation and human capital accumulation into one endogenous growth model to see the growth effect of innovation subsidies and education subsidies. However, the analysis focuses on the growth effects without a comparison of the effectiveness. Our model is on one hand more general by introducing elastic labor and on the other hand considers not only the growth effects but compares the effectiveness of both the growth and welfare effects of the two subsidies. Grossmann (2004) compared public education expenditure on scientists and engineers and R&D subsidies in an overlapping-generations economy. He claimed that R&D subsidies may be detrimental to both growth and welfare, but education expenditure will not. This chapter while focuses on the analysis in a R&D growth model with variety expansion and also compares R&D subsidies with more general education subsidies instead of only with public education expenditure on scientists and engineers. The rest of this chapter is organized as follows. Section 2 describes the model. Section 3 solves the social planner's problem. We use the solution as the reference point for the decentralized equilibrium. Section 4 characterizes the decentralized equilibrium. Section 5 conducts the steady-state analysis of the growth and welfare effect of the three subsidies. Section 6 performs the dynamic analysis, and the last section concludes.

2. The model

The basic model is due to Romer (1990). We extend the model by incorporating human capital accumulation. As a result, both physical and human capital are endogenously determined in our model. We describe the details of the economic environment in the following sub-sections.

2.1 Technologies

There are five types of production activities in the economy: final good production, intermediate good production, innovations, and physical and human capital accumulation. It is assumed that there exists monopoly power in the intermediate good sectors while all the other sectors are perfectly competitive.

2.1.1 Final good production

A final good producer uses a continuum of intermediate goods and a fixed factor as its inputs subject to the following Cobb-Douglas production function

$$Y_t = AF^{1-\alpha} \int_0^{N_t} {x_{ti}}^{\alpha} di, \quad A > 0, \quad 0 \ < \alpha < 1,$$

where the subscript t refers to time; A is a productivity parameter; α measures the contribution of an intermediate good to the final good production and inversely measures the intermediate monopolist's market power; F is the quantity of the fixed factor; Y_t is final output; x_{ti} is the flow of intermediate good i; N_t is measure of intermediate goods. For simplicity, we normalize the quantity of the fixed factor to unity ($F = 1$). We also omit the time subscript t throughout the chapter whenever no confusion can arise. As a result, the final good production function can be rewritten as

$$Y = A \int_0^{N} {x_i}^{\alpha} di, \quad A > 0, \quad 0 \ < \alpha < 1, \tag{1}$$

Profit maximization in the competitive final good sector gives the demand function for intermediate good i

$$x_i = \left(\frac{\alpha A}{p_i}\right)^{\frac{1}{1-\alpha}}, \quad i \in [0, N],$$

where p_i is the price of intermediate good i in terms of the final good. The final good is used as the numeraire for all prices.

2.1.2 Intermediate goods production

Each intermediate producer i who has a patented technology uses physical and human capital, k_i and m_i, to produce a intermediate good according to

$$x_i = k_i^{\gamma} m_i^{1-\gamma}, \quad 0 < \gamma < 1, \tag{2}$$

where γ measures the contribution of physical capital to the intermediate good production. Given the wage rate w, the interest rate r, and the final good sector's demand for intermediate goods given by equation (1), each intermediate good producer chooses the amounts of physical and human capital to maximize its profit

$$\pi_i = p_i x_i - w m_i - r k_i = \alpha A k_i^{\alpha\gamma} m_i^{\alpha(1-\gamma)} - w m_i - r k_i.$$

The solution to this maximization problem gives the demand functions for k_i and m_i. These in turn give the output x_i and profit π_i of a intermediate good producer

$$m_i = m = \phi(1-\gamma)r, \tag{3}$$

$$k_i = k = \phi w \gamma, \tag{4}$$

$$x_i = x = [(1-\gamma)r]^{1-\gamma}(w\gamma)^{\gamma}\phi, \tag{5}$$

$$\pi_i = \pi = \alpha A[(1-\gamma)r]^{\alpha(1-\gamma)}(w\gamma)^{\alpha\gamma}\phi^{\alpha} - w\phi r, \tag{6}$$

where $\phi = \alpha^{\frac{2}{1-\alpha}} A^{\frac{1}{1-\alpha}} (1-\gamma)^{\frac{\alpha(1-\gamma)}{1-\alpha}} \gamma^{\frac{\alpha\gamma}{1-\alpha}} w^{\frac{\alpha\gamma-1}{1-\alpha}} r^{\frac{\alpha(1-\gamma)-1}{1-\alpha}}$. Here, both x_i and π_i are functions of wage rate w and interest rate r, which then, in turn, are determined by market clearing conditions.

2.2 Innovation

The R&D sector is perfectly competitive and the innovation process is deterministic. An innovator invests η units of final good to discover a technology to produce a new intermediate good. The innovator becomes the sole producer of the intermediate good forever. The value of a new technology equals the present value of the profits from producing the new intermediate good V_t, which is given by

$$V_t = \int_t^{\infty} \pi_s \exp\left(-\int_t^s r_\tau d\tau\right) ds. \tag{7}$$

Assuming free entry in the R&D sector, we have

$$V = (1 - s_\eta)\eta, \tag{8}$$

where s_η is a subsidy to the investment in R&D. From equation (7), we obtain

$$\dot{V} = \pi - rV, \tag{9}$$

where a dot on the top of a variable represents the time change rate of that variable. Combining equations (8) and (9) gives the equilibrium condition in the R&D sector

$$V = \frac{\pi}{r}, \tag{10}$$

which holds true both in and outside the steady state.

2.3 Households

The model economy is populated by a continuum of identical infinitely-lived households with measure one. The representative household is endowed with 1 unit of time which is inelastically allocated among intermediate goods production u, human capital accumulation v and leisure $l \ (= 1 - u - v)$. The household has the following utility function

$$U = \int_0^{\infty} \frac{(Cl^{\epsilon})^{1-\sigma} - 1}{1-\sigma} \exp(-\rho t) dt, \tag{11}$$

where C is per capital consumption; ρ is the constant rate of time preference; and σ is the elasticity of marginal utility; ε is the elasticity of leisure; and l is the amount of time allocated to leisure. The household accumulates human capital H according to

$$\dot{H} = BD^{\beta}(vH)^{1-\beta}, \quad 0 \leq \beta < 1, \tag{12}$$

where B is a productivity parameter and D is physical input in education.

The human capital production technology has been widely used in the literature (e.g. Rebelo, 1991; Stokey and Rebelo, 1995). It is easy to understand that in real world human capital accumulation depends on the physical inputs such as equipments for teaching, lab for experiments and the amount of time devoted to learning. Bowen(1987) and Jones and Zimmer (2001) both suggest that physical investment plays a significant role in the education sector. The representative household has a budget constraint

$$\begin{aligned}(1+\tau_c)C = (1-\tau_k)rK + (1-\tau_h)wuH - (1-s_k)\dot{K} \\ -(1-s_d)D + s_v wvH + P_F + \chi - \zeta,\end{aligned} \tag{13}$$

where K is capital stock; P_F is the price of the fixed factor; χ is the dividends; ζ is the cost of R&D; (τ_c, τ_k and τ_h) are respectively the taxes on consumption, physical capital income and labor income; and (s_k, s_d and s_v) are respectively the subsidies to physical investment, human capital investment and educational time.

2.4 Government budget

Assume that the government's budget is balanced at each point in time, then we have

$$\tau_c C + \tau_k rK + \tau_h wuH = s_k\dot{K} + s_d D + s_v wvH + s_\eta \eta \dot{N}, \tag{14}$$

where the left-hand side is the total tax revenue from consumption ($\tau_c C$), capital income ($\tau_k rK$) and labor income ($\tau_h wuH$) while the right-hand side is the total expenditure on subsidies to investment in physical capital ($s_k\dot{K}$), physical inputs in education ($s_d D$), time spent on education ($s_v wvH$) and investment in R&D ($s_\eta \eta \dot{N}$).

3. Socially optimal solution

In this section, we solve the social planner's problem and use the solution as the reference point to examine the properties of the decentralized equilibrium. Since all the intermediate goods enter the production of final good symmetrically, the quantities of intermediate goods will be the same, i.e., $x_i = x$, for all $i \in [0, N]$. As a result, $k_i = k$ and $m_i = m$ for all $i \in [0, N]$. The resources constraints for physical and human capital, $\int_0^N k_i di = Nk = K$ and $\int_0^N m_i di = Nm = uH$, give the amounts of physical and human capital used in the production of each intermediate good

$$k = \frac{K}{N} \quad \text{and} \quad m = \frac{uH}{N}. \tag{15}$$

Using equation (15), we can rewrite equation (1) as

$$Y = AN^{1-\alpha}K^{\alpha\gamma}(uH)^{\alpha(1-\gamma)}. \tag{16}$$

The social planner then chooses consumption (C), investments in education (D) and R&D (I) and time allocation (u, l) to maximize the representative household's utility equation (11) subject to the human capital accumulation technology equation (12) and the following final output constraint and R&D technology

$$\dot{K} = Y - C - D - I, \tag{17}$$

$$\dot{N} = \frac{I}{\eta}, \tag{18}$$

where Y is given by equation (16). The current-value Hamiltonian function for the social planner's problem

$$\mathcal{L}_{SP} = \frac{(Cl^{\varepsilon})^{1-\sigma} - 1}{1-\sigma} + \mu_1 BD^{\beta}[(1-u-l)H]^{1-\beta}$$

$$+\mu_2[AN^{1-\alpha}K^{\alpha\gamma}(uH)^{\alpha(1-\gamma)} - C - D - I] + \mu_3 I/\eta,$$

where μ_1, μ_2 and μ_3 are respectively the co-state variables associated with equations (12), (17) and (18). The first-order conditions for this optimization problem are equations (12), (17), (18) and the following conditions

$$C^{-\sigma} l^{\varepsilon(1-\sigma)} = \mu_2, \tag{19}$$

$$\beta \mu_1 B D^{\beta-1} [(1-u-l)H]^{1-\beta} = \mu_2, \tag{20}$$

$$\mu_2 = \mu_3/\eta, \tag{21}$$

$$(1-\beta)\mu_1 B D^{\beta} H^{1-\beta}(1-u-l)^{-\beta} = \alpha(1-\gamma)\mu_2 Y/u, \tag{22}$$

$$\varepsilon C^{1-\sigma} l^{\varepsilon(1-\sigma)-1} = (1-\beta)\mu_1 B D^{\beta} H^{1-\beta}(1-u-l)^{-\beta}, \tag{23}$$

$$(1-\beta)\mu_1 B D^{\beta}(1-u-l)^{1-\beta} H^{-\beta} + \alpha(1-\gamma)\mu_2 Y/H = -\dot{\mu}_1 + \rho\mu_1, \tag{24}$$

$$\alpha\gamma\mu_2 Y/K = -\dot{\mu}_2 + \rho\mu_2, \tag{25}$$

$$(1-\alpha)\mu_2 Y/N = -\dot{\mu}_3 + \rho\mu_3, \tag{26}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu_{1t} H_t = 0, \tag{27}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu_{2t} K_t = 0, \tag{28}$$

$$\lim_{t\to\infty} e^{-\rho t}\mu_{3t} N_t = 0. \tag{29}$$

Equation (19) (respectively, (20), (21), (22), (23)) equalizes the social marginal benefit and social marginal cost of consumption (respectively, physical investment in education, investment in R&D, time allocated to production, time allocated to leisure). Equations (24) and (27) (respectively (25) and (28), (26) and (29)) are the socially optimal dynamic conditions for human capital (respectively, physical capital, variety) accumulation.

We now solve the above first-order conditions. From equations (21), (25) and (26), we have: $K/N = \alpha\gamma\eta/(1-\alpha)$ and $\dot{K} = \alpha\gamma\eta\dot{N}/(1-\alpha)$. With $\dot{K} = \alpha\gamma\eta\dot{N}/(1-\alpha)$, equations (17) and (18) give the laws of motion for K and N:

$$\dot{K} = \left[\frac{\alpha\gamma}{1-\alpha(1-\gamma)}\right](Y - C - D), \tag{30}$$

$$\dot{N} = \left[\frac{(1-\alpha}{\eta(1-\alpha(1-\gamma))}\right](Y - C - D). \tag{31}$$

Using equations (19), (22) and (23), we obtain the relationship between consumption (C) and leisure (l):

$$C = \left[\frac{\alpha(1-\gamma)l}{\epsilon u}\right] Y. \tag{32}$$

Combining equations (20) and (22), we have

$$D = \left[\frac{\beta(1-l-u)\alpha(1-\gamma)}{(1-\beta)u}\right] Y. \tag{33}$$

We then solve equations (19), (20), (22), (25), (32) and (33) for the law of motion for u and l:

$$\dot{u} = \left[\frac{u}{1-\alpha(1-\gamma)}\right]\left\{[1-\alpha(1-\gamma)]\left(\frac{Y-C-D}{K+N\eta}\right) - \frac{\alpha\gamma Y}{(1-\beta)K}\right.$$
$$\left. + B\left[u + \alpha(1-\gamma)(1-u-l)\right]\left[\frac{D}{(1-u-l)H}\right]^{\beta}\right\}, \tag{34}$$

$$\dot{l} = \left[\frac{l}{\sigma - \epsilon(1-\sigma)}\right]\left\{\sigma B u\left[\frac{D}{H(1-l-u)}\right]^{\beta} - \alpha\gamma\left(\frac{\sigma}{1-\beta} - 1\right)\frac{Y}{K} - \rho\right\}. \tag{35}$$

The dynamics of the socially planned economy are then described by the system of equations (12), (16) and (30)-(35), along with an initial condition (H_0, K_0, N_0) and the transversally conditions equations (27)-(29). In a steady state, the time allocation (u, l) is constant and all the other variables (consumption C, physical investment in education D, investment in

R&D I, the number of intermediate goods N, physical capital stock K, human capital stock H and final output Y) grow at the same constant rate g. That is, $\dot{u} = \dot{l} = 0$ and $\dot{X}/X = g$, where $X = C, D, I, N, K, H$ and Y. We now derive the steady-state equilibrium conditions that determine the optimal growth rate (g) and leisure (l). From equations (19) and (25), we have

$$g = \frac{1}{\sigma}[\alpha\gamma A N^{1-\alpha} K^{\alpha\gamma-1}(uH)^{\alpha(1-\gamma)} - \rho] \tag{36}$$

Similarly, from equations (22) and (24), we obtain

$$u = (1-l)[1-(1-\beta)\Phi(g)] \tag{37}$$

Combining equations (12), (33), (36) and (37) gives the first equilibrium condition

$$\alpha^{\frac{\beta}{\alpha(1-\gamma)}}(\sigma g + \rho)^q = \Omega(1-l) \tag{38}$$

where $q \equiv 1 + \frac{\beta(\alpha\gamma+1-\alpha)}{\alpha(1-\gamma)} > 1$ and $\Omega \equiv BA^{\frac{\beta}{\alpha(1-\gamma)}}\alpha^{\frac{\beta(1+\alpha)}{\alpha(1-\gamma)}}(1-\alpha)^{\frac{\beta(1-\alpha)}{\alpha(1-\gamma)}}(1-\beta)^{1-\beta}\beta^{\beta}(1-\gamma)^{\beta}\gamma^{\frac{\gamma\beta}{1-\gamma}}\eta^{\frac{\beta(1-\alpha)}{\alpha(\gamma-1)}}$. Next, from equations (32),(33) and (37), we have

$$C = \left\{\frac{\alpha\eta(1-\gamma)(\sigma g+\rho)l}{\varepsilon(1-\alpha)(1-l)[1-(1-\beta)\Phi(g)]}\right\}N \text{ and } D = \left\{\frac{\alpha\beta\eta(1-\gamma)g}{(1-\alpha)[1-(1-\beta)\Phi(g)]}\right\}N,$$

where $\Phi(g) \equiv \frac{g}{g\sigma+\rho}$. Substituting the above expressions into equation (31), we obtain the second equilibrium condition

$$(\alpha\gamma+1-\alpha)\Phi(g) = 1 - \frac{\alpha(1-\gamma)[\frac{l}{\varepsilon(1-l)} + \beta\Phi(g)]}{1-(1-\beta)\Phi(g)}. \tag{39}$$

Equations (38) and (39) determine the socially optimal growth rate and leisure (g^*, l^*).

Solving equation (38) for l and substituting it into equation (39), we obtain the following condition that determines the socially optimal growth rate

$$J(g) \equiv \frac{(g\sigma+\rho)^q}{\alpha^{\frac{\beta}{\alpha(\gamma-1)}}}\left\{\frac{\varepsilon[1-(\alpha\gamma+1-\alpha)\Phi(g)][1-(1-\beta)\Phi(g)]}{\alpha(1-\gamma)} - \beta\varepsilon\Phi(g) + 1\right\} - \Omega = 0. \tag{40}$$

The existence and uniqueness of the solution are given by

Proposition 1: *If (i) $\sigma > 2 - [\beta + \alpha(1-\gamma)(1-\beta)]$ and (ii) $\Omega > \alpha^{\frac{\beta}{\alpha(1-\gamma)}}\rho^q[1+\varepsilon/\alpha(1-\gamma)]$, then there always exists a unique positive growth rate of per capita output.*

Proof. (a) We have $J'(g) > 0$ because

$$J'(g) = \alpha^{\frac{\beta}{\alpha(1-\gamma)}}\frac{\varepsilon(g\sigma+\rho)^{q-2}}{\alpha(1-\gamma)}\left\{\sigma q(g\sigma+\rho)\left\{[1-(\alpha\gamma+1-\alpha)\Phi(g)][1-(1-\beta)\Phi(g)]\right.\right.$$

$$\left.-\beta\Phi(g)\alpha(1-\gamma)+\alpha(1-\gamma)/\varepsilon\right\} - (\alpha\gamma+1-\alpha)\rho-(1-\beta)\rho$$

$$\left.+2(\alpha\gamma+1-\alpha)(1-\beta)\Phi(g)\rho - \beta\rho\alpha(1-\gamma)\right\} > 0$$

if condition (i) holds true. (b) We have $J(0) = \alpha^{\frac{\beta}{\alpha(1-\gamma)}}\rho^q[1+\varepsilon/\alpha(1-\gamma)] - \Omega < 0$ if condition (ii) holds true. (c) Obviously, $J(\infty) = \infty > 0$. By the intermediate value theorem, there must exist a unique positive growth rate $g^* \in (0,\infty)$ such that $J(g) = 0$. *Q.E.D.*

The second condition in this proposition is just equivalent to the condition that the marginal social benefit of investing in R&D (μ_3/η) is greater than its marginal cost(μ_2), i.e., $\mu_3/\mu_2 > \eta$. Consider the economy in a steady with no growth ($g = 0$). When $g = 0$, then $I = D = v = 0$, $l = \frac{\varepsilon}{\alpha(1-\gamma)+\varepsilon}$, $u = \frac{\alpha(1-\gamma)}{\alpha(1-\gamma)+\varepsilon}$, $K = \frac{\alpha\gamma\eta}{1-\alpha}$, $H/N = B^{\frac{1}{\beta}}u^{\frac{1}{\beta}-1}\alpha(1-\alpha)^{-1}\beta(1-\beta)^{\frac{1}{\beta}-1}(1-\gamma)\eta\rho^{1-\frac{1}{\beta}}$[2] and $Y/N = C/N = \frac{\rho\eta}{1-\alpha}$). From equation (26), we have

$$\frac{\mu 3}{\mu 2} = \frac{1}{\rho}\frac{\partial Y}{\partial N} = (1-\alpha)AK^{\alpha\gamma}(uH)^{\alpha(1-\gamma)} = B^{\frac{\alpha(1-\gamma)}{\beta(1-\alpha)}}(A\alpha^\alpha)^{\frac{1}{1-\alpha}}(1-\alpha)$$

$$\times(1-\beta)^{\frac{\alpha(1-\gamma)(1-\beta)}{\beta(1-\alpha)}}[\beta(1-\gamma)]^{\frac{\alpha(1-\gamma)}{1-\alpha}}\gamma^{\frac{\alpha\gamma}{1-\alpha}}\rho^{-\frac{q\alpha(1-\gamma)}{\beta(1-\alpha)}}\left[1+\frac{\varepsilon}{\alpha(1-\gamma)}\right]^{\frac{-\alpha(1-\gamma)}{\beta(1-\alpha)}}. \quad (41)$$

Rewriting the condition $\mu_3/\mu_2 > \eta$ gives $\Omega > \alpha^{\frac{\beta}{\alpha(1-\gamma)}}\rho^q[1+\varepsilon/\alpha(1-\gamma)]$. That is, the marginal social benefit of investing in R&D is greater than its marginal cost. Therefore, it is optimal for the social planner to allocate its sources to the R&D sector. The condition can be guaranteed by various sufficient conditions concerning the values of the technology and preferences such

[2]From equation (12), we obtain $H = B^{\frac{1}{\beta}}u^{\frac{1}{\beta}-1}\alpha(1-\alpha)^{-1}\beta(1-\beta)^{\frac{1}{\beta}-1}(1-\gamma)\eta(\sigma g+\rho)^{1-\frac{1}{\beta}}$, when $g \neq 0$, we take the limit when g approaches to 0 to get the value for H/N

as a sufficiently low subjective discount rate (low ρ), a sufficiently productive human capital accumulation technology (large B), a sufficiently productive parameter for all intermediate goods (large A), a sufficiently low cost of innovation (low η), a sufficiently low elasticity of leisure (low ε), and a sufficiently large elasticity of marginal utility (high σ).

4. Decentralized equilibrium

In this section , we will first solve the representative household's optimization problem. We then use the first-order conditions for this optimization problem and the first-order conditions for (final good, intermediate good and R&D) firms' profit maximization problems to derive a system of equations that describe the dynamics of the decentralized economy. At the end of this section, we compare the decentralized equilibrium with the socially optimal solution to examine the properties of the decentralized equilibrium.

The representative household chooses consumption C, investment in education D, the time allocation u and l to maximize its life-time utility, subject to the human capital accumulation technology and the budget constraint. The current-value Hamiltonian function for this optimization problem is

$$\mathcal{L}_{DE} = \frac{(Cl^{\varepsilon})^{1-\sigma} - 1}{1-\sigma} + \lambda_1 BD^{\beta}[(1-u-l)H]^{1-\beta} + \lambda_2 \frac{1}{1-s_k}[(1-\tau_k)rK + (1-\tau_h)wuH$$

$$-(1+\tau_c)C - (1-s_d)D + s_v wvH + P_F + \chi - \zeta],$$

where λ_{1t} and λ_{2t} are respectively the co-state variables associated with equations (12) and (13). The first-order conditions for this optimization problem are equations (12), (13) and the following conditions

$$C^{-\sigma} l^{\varepsilon(1-\sigma)} = \frac{\lambda_2(1+\tau_c)}{1-s_k}, \tag{42}$$

$$\beta\lambda_1 BD^{\beta-1}[(1-u-l)H]^{1-\beta} = \frac{\lambda_2(1-s_d)}{1-s_k}, \tag{43}$$

$$\varepsilon C^{1-\sigma} l^{\varepsilon(1-\sigma)-1} = (1-\beta)\lambda_1 B D^{\beta} H^{1-\beta}(1-u-l)^{-\beta} + \frac{\lambda_2 s_v w H}{1-s_k}, \tag{44}$$

$$(1-\beta)\lambda_1 B D^{\beta} H^{1-\beta}(1-u-l)^{-\beta} = \frac{\lambda_2[(1-\tau_h)wH - s_v wH]}{1-s_k}, \tag{45}$$

$$(1-\beta)\lambda_1 B D^{\beta}(1-u-l)^{1-\beta} H^{-\beta} + \frac{\lambda_2[(1-\tau_h)wu + s_v wv]}{1-s_k} = -\dot{\lambda}_1 + \rho\lambda_1, \tag{46}$$

$$\frac{\lambda_2 r(1-\tau_k)}{1-s_k} = -\dot{\lambda}_2 + \rho\lambda_2, \tag{47}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{1t} H_t = 0, \tag{48}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda_{2t} K_t = 0. \tag{49}$$

Equation (42) (respectively, (43), (44), (45)) equalizes the private marginal benefit and private marginal cost of consumption (respectively, physical investment in education, time allocated to leisure, time allocated to production). Equations (46) and (48) (respectively (47) and (49)) are the optimal dynamic conditions for human capital (respectively, physical capital) accumulation.

We now derive the equilibrium conditions. According to equation (5), we know that all the intermediate good producers will produce the same quantity, so we have $x_i = x$. Using the capital and labor market clearing conditions, i.e., $\int_0^N k_i di = Nk = K$ and $\int_0^N m_i di = Nm = uH$, we have $k = K/N$, $m = uH/N$ and $x = K^{\gamma}(uH)^{1-\gamma}/N$. Since each intermediate good enters the production of final good symmetrically, we can rewrite equation (1) as equation (16) as in the social planner's problem. From equations (42) and (44), we obtain the relationship between consumption (C) and leisure (l):

$$C = \left[\frac{(1-\tau_h)lw}{\epsilon(1+\tau_c)}\right] H. \tag{50}$$

Combining equations (43) and (45), we have

$$D = \left[\frac{\beta(1-\tau_h)(1-l-u)w}{1-\beta}\right] H. \tag{51}$$

From equations (8) and (10), we have $K/N = \alpha\gamma(1-s_\eta)\eta/(1-\alpha)$ [3]and thus $\dot{K} = \alpha\gamma(1-s_\eta)\eta\dot{N}/(1-\alpha)$. With $\dot{K} = \alpha\gamma(1-s_\eta)\eta\dot{N}/(1-\alpha)$, the final goods market clearing condition $\dot{K} = Y - C - D - \eta\dot{N}$, gives the laws of motion for K and N:

$$\dot{N} = \left[\frac{(1-\alpha)}{\eta(1-\alpha(1-\gamma(1-s_\eta)))}\right](Y-C-D), \tag{52}$$

$$\dot{K} = \left[\frac{\alpha\gamma(1-s_\eta)}{1-\alpha(1-\gamma(1-s_\eta))}\right](Y-C-D). \tag{53}$$

From equations (52) and (53), we can easily see that an increase in the R&D subsidy (s_η) speed up the expansion of varieties while the same change slows down the accumulation of physical capital. Equations (3) and (4), along with $k = K/N$ and $m = uH/N$, gives $r = \alpha^2\gamma Y/K$ and $w = \alpha^2(1-\gamma)Y/(uH)$. Substituting the expressions for r and w into equations (50) and (51) yields

$$C = \left[\frac{\alpha^2(1-\tau_h)(1-\gamma)}{\epsilon\mu(1+\tau_c)l}\right]Y, \tag{54}$$

$$D = \left[\frac{\beta(1-\tau_h-s_v)\alpha^2(1-\gamma)(1-l-u)}{(1-\beta)\mu(1-s_d)}\right]Y. \tag{55}$$

We can see from equations (54) and (55) that an increase in the tax on consumption (τ_c) or labor income (τ_h) will reduce the share of consumption in final output. An increase in the tax on labor income (τ_h) or the subsidy to educational time (s_v) will decrease the share of physical investment in final output while a rise in the subsidy to physical investment in education will increase this share. We then solve equations (42), (43), (45), (47), (54) and (55) for the laws of motion for u and l:

$$\dot{u} = \left[\frac{u}{1-\alpha(1-\gamma)}\right]\left\{[1-\alpha(1-\gamma)]\left(\frac{Y-C-D}{K+N\eta}\right) - \frac{(1-\tau_k)\alpha^2\gamma Y}{(1-\beta)(1-s_k)K}\right.$$

[3]From equations (8) and (10), we have $\frac{\pi}{r} = (1-s_\eta)\eta$. Also from the intermediate goods production sector, we know the profit is proportional to the final output, i.e., $\frac{\pi}{y} = \alpha(1-\alpha)$. Lastly, capital income $rk = \alpha^2\gamma y$ could also be derived. The three equations give us a constant value of $k = K/N = \alpha\gamma(1-s_\eta)\eta/(1-\alpha)$. The main reason is that in the model we have a constant innovation cost, i.e., to increase N by 1, the cost in terms of final good is constat at η. It is similar as the AK model, where the growth rate of capital is constant.

$$+B\left[\frac{u(1-\tau_h)+s_v v}{1-\tau_h-s_v}+\alpha(1-\gamma)(1-u-l)\right]\left[\frac{D}{(1-u-l)H}\right]^{\beta}\Bigg\}, \tag{56}$$

$$\dot{l}=\left[\frac{l}{\sigma-\epsilon(1-\sigma)}\right]\Bigg\{\sigma B\frac{u(1-\tau_h)+s_v v}{1-\tau_h-s_v}\left[\frac{D}{H(1-l-u)}\right]^{\beta}$$
$$-(1-\tau_k)\alpha^2\gamma\left(\frac{\sigma}{1-\beta}-1\right)\frac{Y}{(1-s_k)K}-\rho\Bigg\}. \tag{57}$$

The dynamics of the decentralized economy are then characterized by the system of equations (12), (16), (52), (53), (54), (55), (56) and (57), along with an initial condition (H_0, K_0, N_0) and the transversality conditions equations (48) and (49). In a steady state, the time allocation (u, l) is constant and all the other variables (consumption C, physical investment in education D, the number of intermediate goods N, physical capital stock K, human capital stock H and final output Y) grow at the same constant rate g. That is, $\dot{u}=\dot{l}=0$ and $\dot{X}/X=g$, where $X=C, D, N, K, H$ and Y. We now derive the steady-state equilibrium conditions that determine the decentralized economy growth rate (g) and leisure (l). From equations (42) and(43), we have

$$g=\frac{1}{\sigma}\left[\frac{r(1-\tau_k)}{1-s_k}-\rho\right], \tag{58}$$

which gives

$$r=\frac{(g\sigma+\rho)(1-s_k)}{(1-\tau_k)}.$$

Since $r=\alpha^2\gamma Y/K$ and $K/N=\alpha\gamma(1-s_\eta)\eta/(1-\alpha)$, we obtain

$$\frac{Y}{N}=\frac{(g\sigma+\rho)(1-s_k)\eta}{\alpha(1-\alpha)(1-\tau_k)}. \tag{59}$$

The conditions equations (45) and (46) yield the steady-state allocation of time (v, u):

$$v=\frac{(1-\beta)g}{g\sigma+\rho}\frac{1-\tau_h}{1-\tau_h-s_v}(1-l), \tag{60}$$

$$u=\frac{(g\sigma+\rho)(1-\tau_h-s_v)-(1-\beta)g(1-\tau_h)}{(g\sigma+\rho)(1-\tau_h-s_v)}(1-l). \tag{61}$$

Using equations (59), (60) and (61), we can rewrite equations (54) and (55) as

$$C = \left\{ \frac{l(1-\tau_h)(1-\tau_h-s_v)\alpha(1-\gamma)(g\sigma+\rho)(1-s_k)(1-s_\eta)\eta}{\varepsilon(1+\tau_c)[(1-\tau_h-s_v)-(1-\beta)\Phi(g)(1-\tau_h)](1-l)(1-\alpha)(1-\tau_k)} \right\} N, \tag{62}$$

$$D = \left\{ \frac{\Phi(g)\beta(1-\tau_h)(1-\tau_h-s_v)\alpha(1-\gamma)(g\sigma+\rho)(1-s_k)(1-s_\eta)\eta}{(1-s_d)[(1-\tau_h-s_v)-(1-\beta)\Phi(g)(1-\tau_h)](1-\alpha)(1-\tau_k)} \right\} N. \tag{63}$$

Combining equations (62), (63) and the final goods market clearing condition $\dot{K} = Y - C - D - \eta\dot{N}$, we obtain the first equilibrium condition

$$\frac{\alpha[\alpha\gamma(1-s_\eta)+1-\alpha](1-\tau_k)}{(1-s_\eta)(1-s_k)}\Phi(g) = 1 - \frac{\alpha^2(1-\gamma)(1-\tau_h-s_v)\left[\frac{l(1-\tau_h)}{\varepsilon(1-l)(1+\tau_c)} + \frac{\beta(1-\tau_h)}{(1-s_d)}\Phi(g)\right]}{(1-\tau_h-s_v)-(1-\beta)\Phi(g)(1-\tau_h)}. \tag{64}$$

As in the social planner' problem, we combine equations (12), (55), (61) and (58) to obtain the other equilibrium condition

$$(g\sigma+\rho)^q = \Omega\Psi(1-l), \tag{65}$$

where $\Psi \equiv (1-s_k)^{-\frac{\beta(1-\alpha(1-\gamma))}{\alpha(1-\gamma)}}(1-s_\eta)^{-\frac{\beta(1-\alpha)}{\alpha(1-\gamma)}}(1-s_d)^{-\beta}(1-\tau_h)(1-\tau_k)^{\frac{\beta(1-\alpha+\alpha\gamma)}{\alpha(1-\gamma)}}(1-\tau_h-s_v)^{\beta-1}$. By now we have reduced the equilibrium system of equations to the two conditions (equations (64) and (65)) that determine the growth rate g and time devoted to leisure time l. This steady-state equilibrium has the following features: First, there is no scale effect in terms of the size of population, as in those recent non-scale endogenous growth models (Jones, 1995; Kortum, 1997; Segerstrom, 1998; Young, 1998; Howitt, 1999; Zeng and Zhang, 2002). This is because the scale effect is nullified by human capital accumulation. Second, the long run growth rate depends on preferences, human capital accumulation and R&D activities as long as $0 < \beta < 1$. Third, similar to some R&D models without scale effects (e.g., Aghion and Howitt, 1998; Howitt, 1999), government policies such as taxes on physical, labor incomes and consumption and subsidies to R&D , investments in physical and human capital and time allocated to human accumulation have permanent effects on growth.

Next, we find the conditions under which there exists a unique steady-state equilibrium. For simplicity, we consider a laissez faire equilibrium. Without government intervention, the equilibrium conditions equations (64) and (65) are simplified to[4]

$$(g\sigma+\rho)^q = \Omega(1-l), \tag{66}$$

and

$$(\alpha\gamma+1-\alpha)\alpha\Phi(g) = 1-\frac{\alpha^2(1-\gamma)[\frac{l}{\varepsilon(1-l)}+\beta\Phi(g)]}{1-(1-\beta)\Phi(g)}. \tag{67}$$

Solving equation (66) for l and substituting it into equation (67), we can further simplify the equilibrium conditions to the following condition that determines the long-run decentralized equilibrium growth rate g

$$E(g) \equiv (g\sigma+\rho)^q\left\{\frac{\varepsilon[1-\alpha(\alpha\gamma+1-\alpha)\Phi(g)][1-(1-\beta)\Phi(g)]}{\alpha^2(1-\gamma)}-\beta\varepsilon\Phi(g)+1\right\}-\Omega=0. \tag{68}$$

The existence and uniqueness of the steady-state equilibrium are given by

Proposition 2: *If (i)* $\sigma > 1-[\beta-\alpha+\alpha^2(1-\gamma)(1-\beta)]$ *and (ii)* $\Omega > \rho^q[1+\varepsilon/\alpha^2(1-\gamma)]$ *, then there always exists a unique positive growth rate of per capita output.*

Proof. (a) We have $E'(g) > 0$ because

$$E'(g) = \frac{\varepsilon(g\sigma+\rho)^{q-2}}{\alpha^2(1-\gamma)}\{\sigma q(g\sigma+\rho)\{[1-\alpha(\alpha\gamma+1-\alpha)\Phi(g)][1-(1-\beta)\Phi(g)]$$

$$-\beta\Phi(g)\alpha^2(1-\gamma)+\alpha^2(1-\gamma)/\varepsilon\Big\}-\alpha(\alpha\gamma+1-\alpha)\rho-(1-\beta)\rho$$

$$+2\alpha(\alpha\gamma+1-\alpha)(1-\beta)\Phi(g)\rho-\beta\rho\alpha^2(1-\gamma)\Big\} > 0$$

[4]When human capital accumulation does not require physical inputs ($\beta=0$), the equilibrium conditions equations (66) and (67) becomes $g\sigma+\rho = B(1-l)$ and $(\alpha\gamma+1-\alpha)\alpha\Phi(g) = 1-\alpha^2(1-\gamma)\frac{l}{\varepsilon(1-l)}/(1-\Phi(g))$.

if condition (i) holds true. (b) We have $E(0) = \rho^q[1 + \varepsilon/\alpha^2(1-\gamma)] - \Omega < 0$ if condition (ii) holds true. (c) Obviously, $E(\infty) = \infty > 0$. By the intermediate value theorem, there must exist a unique positive growth rate $\tilde{g} \in (0, \infty)$ such that $E(g) = 0$. *Q.E.D.*

The second condition in this proposition is equivalent to the condition that $V > \eta$. As in the social planner's problem, consider the economy in a steady state with no growth ($g = 0$). When $g = 0$, we have $D = I = v = 0$, $l = \frac{\varepsilon}{\alpha^2(1-\gamma)+\varepsilon}$, $u = \frac{\alpha^2(1-\gamma)}{\alpha^2(1-\gamma)+\varepsilon}$, $r = \rho$, $w = \beta\{\frac{\rho[1+\varepsilon/\alpha^2(1-\gamma)]}{B(1-\beta)^{1-\beta}}\}^{\frac{1}{\beta}}$. These results give

$$\pi = B^{\frac{\alpha(1-\gamma)}{\beta(1-\alpha)}}(A\alpha^{1+\alpha})^{\frac{1}{1-\alpha}}(1-\alpha)(1-\beta)^{\frac{\alpha(1-\gamma)(1-\beta)}{\beta(1-\alpha)}}(\beta(1-\gamma))^{\frac{\alpha(1-\gamma)}{1-\alpha}}\gamma^{\frac{\alpha\gamma}{1-\alpha}}\rho^{1-\frac{q\alpha(1-\gamma)}{\beta(1-\alpha)}}$$

$$\times(1 + \frac{\varepsilon}{\alpha^2(1-\gamma)})^{\frac{-\alpha(1-\gamma)}{\beta(1-\alpha)}},$$

and

$$V = B^{\frac{\alpha(1-\gamma)}{\beta(1-\alpha)}}(A\alpha^{1+\alpha})^{\frac{1}{1-\alpha}}(1-\alpha)(1-\beta)^{\frac{\alpha(1-\gamma)(1-\beta)}{\beta(1-\alpha)}}(\beta(1-\gamma))^{\frac{\alpha(1-\gamma)}{1-\alpha}}\gamma^{\frac{\alpha\gamma}{1-\alpha}}\rho^{-\frac{q\alpha(1-\gamma)}{\beta(1-\alpha)}}$$

$$\times(1 + \frac{\varepsilon}{\alpha^2(1-\gamma)})^{\frac{-\alpha(1-\gamma)}{\beta(1-\alpha)}}.$$

Then the condition $V > \eta$ leads to $\Omega > \rho^q[1+\varepsilon/\alpha^2(1-\gamma)]$. Since the marginal private benefit of investing in R&D is greater than its marginal cost, it is optimal for the profit-maximizing R&D firms to invest in R&D until $V = \eta$.

This condition can be guaranteed by various sufficient conditions concerning the values of the technology and preferences such as a sufficiently low subjective discount rate (low ρ), a sufficiently productive human capital accumulation technology (large B), a sufficiently productive parameter for all intermediate goods (large A), a sufficiently low cost of innovation (low η), and a sufficiently low elasticity of leisure (low ε).

Comparing the decentralized steady-state equilibrium with the socially optimal steady-state solution, we can see that the decentralized economy has a lower growth rate and a higher level of leisure. More formally, we have

Proposition 3: *If (i)* $\sigma > 2 - [\beta + \alpha(1-\gamma)(1-\beta)]$ *and (ii)* $\Omega > \rho^q[1+\varepsilon/\alpha^2(1-\gamma)]$*, then the decentralized economy has a lower growth rate and a higher level of leisure compared with the socially optimal solution. i.e.,* $\tilde{g} < g^*$ *and* $\tilde{l} > l^*$*.*

Proof. (a) By definitions, we have $J(g^*) = E(\tilde{g}) = 0$. We also have $J(g) < E(g)\ \forall\ g$. As a result, $\tilde{g} < g^*$. (b) Since $\frac{\tilde{l}}{1-\tilde{l}} = \frac{\varepsilon}{\alpha^2(1-\gamma)}[1-\alpha(\alpha\gamma+1-\alpha)\Phi(\tilde{g})][1-(1-\beta)\Phi(\tilde{g})] - \beta\varepsilon\Phi(\tilde{g}) + 1 > \frac{l^*}{1-l^*} = \frac{\varepsilon}{\alpha(1-\gamma)}[1-(\alpha\gamma+1-\alpha)\Phi(g^*)][1-(1-\beta)\Phi(g^*)] - \beta\varepsilon\Phi(g^*) + 1$, we have $\tilde{l} > l^*$. *Q.E.D.*

As in the R&D model with variety expansion in Barro and Sala-i-Martin (2004, Chapter 6), the decentralized equilibrium is sub-optimal in our model because of the existence of monopoly power in the intermediate good sector and the knowledge spillover from R&D. On the one hand, monopoly pricing in the intermediate good sector leads to a lower than optimal level of demand for labor (human capital) and thus a greater than optimal level of leisure (*static inefficiency*). On the other hand, the positive externality generated by knowledge spillover from R&D results in less than optimal investment in R&D and thus too low a growth rate (*dynamic inefficiency*).

5. Steady state results

From Proposition 3, we know that the steady-state laissez-faire equilibrium is not optimal, in particular, the level of leisure is too high and the growth rate is too low. In this section, we consider how the government can intervene to move the steady-state equilibrium towards the first best solution. We consider two commonly-used policies: R&D subsidies and education subsidies (to either physical inputs or time input). These subsidies are financed by a uniform tax rate ($\tau_k = \tau_h = \tau$) on physical capital and labor income.

With $\tau_k = \tau_h = \tau$ (and $\tau_c = 0$ and $s_k = 0$), we can rewrite the government budget constraint as

$$\frac{\tau}{1-\tau} - \frac{(1-\gamma)\left[\frac{\beta s_d(1-\tau-s_v)\Phi(g)}{1-s_d} + (1-\beta)s_v(1-\tau)\Phi(g)\right]}{1-\tau-s_v-(1-\beta)(1-\tau)\Phi(g)} = \frac{(1-\alpha)s_\eta\Phi(g)}{\alpha(1-s_\eta)}. \tag{69}$$

In addition, we can express the household's welfare (in terms of l ,g and subsidies) as

$$U = \frac{l^{(\epsilon+1)(1-\sigma)}}{(1-\sigma)(\rho - g + \sigma g)} \left[\frac{\alpha(1-\gamma)(\sigma g + \rho)\eta(1-s_\eta)}{\varepsilon(1-\alpha)(1-(1-\beta)\Phi(g))(1-l)} \right]^{1-\sigma} - \frac{1}{\rho(1-\sigma)}, \tag{70}$$

which is the integral of the consumer's utility function along the steady state path.

Given the equilibrium conditions, equations (64) and (65), the government budget constraint equation (69) and the welfare function equation (70), we are now in a position to examine the long-run effects of the two types of subsidies on growth and welfare. Since the equilibrium equations are highly non-linear, analytical results are difficult to obtain. We do numerical simulations instead.

To do the simulations, we first need to specify the values of the model's parameters in a benchmark economy. We have preference parameters $(\varepsilon, \sigma, \rho)$ and technology parameters $(\alpha, \beta, \gamma, A, B, \eta)$. First, using the parameter values employed by the growth calibration exercises in Lucas (1990), King and Rebelo (1990), and Stokey and Rebelo (1995), we set $(\sigma, \rho, \alpha, \gamma\ ) = (1.5, 0.05, 0.8, 0.35)$ (implying a labor's share of 0.7). In the absence of more precise information about the human capital technology, we set β equal to 0.5 which is close to the value used in in King and Rebelo (1990).

Following Prescott(1986), we choose ε to be 0.2, implying that the representative household spends about 30 per cent of its available time working. The last three parameters $(A, B, \eta) = (0.5, 0.5, 1)$ are chosen for the steady-state equilibrium to generate a growth rate close to that for the US economy (3%).

Given the values of the model's parameters we have chosen, we can now simulate the model. Table 1.1 reports the growth rate and leisure in the laissez-faire equilibrium and the socially optimal solution. As expected, the equilibrium growth rate (2.60%) is substantially lower than the optimal growth rate (3.9%) while the equilibrium level of leisure (0.25) is significantly higher than the optimal level (0.18).

Table 1.1: Steady state results of decentralized economy and social optimal

Parameters: $\alpha = 0.8$,$\beta = 0.5$,$\gamma = 0.35$,$\sigma = 1.5$, $\eta = 1$, $\rho = 0.05$, $\epsilon = 0.2$,$A = B = 0.5$		
	Growth rate	Leisure
Decentralized economy	0.0260	0.2511
Socially optimal problem	0.03959	0.1830

Tables 1.2, 1.3 and 1.4 respectively show the growth and welfare effects of the subsidies to R&D, physical investment in education and educational time. [5] It is clear that starting with a equilibrium with a lower growth rate and a higher level of leisure, an increase in the R&D subsidy (respectively, the subsidy to physical investment in education, subsidy to educational time) encourages R&D investment (respectively, physical investment in education, time investment in education) and thus stimulate economic growth. However, on the other hand, an accompanying increase in the labor income tax discourages physical and time investments in education (and thus raising leisure further above its socially optimal level); Similarly, an increase in the physical capital income tax decreases discourages investment in physical capital. As a result, the income taxes tend to reduce the growth rate. When the subsidy rate is low (for the three subsidies), the tax distortion is weak, leading to a net increase in the growth rate. When the subsidy rate is high enough, the tax distortion becomes stronger and eventually dominates the positive growth effect, leading to a lower growth rate. In other words, there exists a positive rate of the subsidy at which growth is maximized.

[5] In the simulation, we first express taxation rate in terms of subsidy rate according to equation (69), except for time subsidy s_v (table 1.4), in which we express subsidy rate in terms of taxation rate, since the expression of taxation rate in terms of subsidy rate is more complicated.

Table 1.2: Growth and welfare effects of R&D subsidies.

s_η	Growth rate	Leisure	Consumption per intermediate good	Welfare
0	0.0260	0.2511	0.4543	-14.0829
0.04	0.0262	0.2510	0.4377	-15.0077
0.08	0.0264	0.2509	0.4211	-15.9916
0.12	0.0266	0.2509	0.4044	-17.0413
0.16	0.0268	0.2509	0.3877	-18.1647
0.2	0.0270	0.2509	0.3709	-19.3712
0.24	0.0272	0.2510	0.3539	-20.6717
0.28	0.0275	0.2511	0.3369	-22.0795
0.32	0.0277	0.2513	0.3198	-23.6107
0.36	0.0279	0.2516	0.3027	-25.2847
0.40	0.0280	0.2520	0.2854	-27.1255
0.44	0.0282	0.2525	0.2679	-29.1636
0.48	0.0283	0.2531	0.2504	-31.4376
0.52	0.0285	0.2539	0.2328	-33.9973
0.56	0.02860	0.2550	0.2150	-36.9089
0.60	0.02865	0.2563	0.1970	-40.2620
0.64	0.02864	0.2581	0.1789	-44.1812
0.68	0.0285	0.2604	0.1606	-48.8459
0.72	0.0283	0.2634	0.1421	-54.5256
0.76	0.0279	0.2675	0.1233	-61.6465
0.80	0.0271	0.2733	0.1043	-70.9288
0.84	0.0260	0.2816	0.0850	-83.7064
0.88	0.0240	0.2946	0.0651	-102.7971
0.92	0.0207	0.3167	0.0448	-135.5485
0.96	0.0147	0.3621	0.0237	-210.9592

Table 1.3: Growth and welfare effects of education subsidies.

s_d	Growth rate	Leisure	Consumption per intermediate good	Welfare
0	0.0260	0.2511	0.4543	-14.0829
0.04	0.0267	0.2503	0.4589	-13.5335
0.08	0.0274	0.2496	0.4637	-12.9710
0.12	0.0281	0.2490	0.4688	-12.3948
0.16	0.0289	0.2484	0.4743	-11.8040
0.2	0.0296	0.2478	0.4800	-11.1978
0.24	0.0304	0.2474	0.4862	-10.5753
0.28	0.0313	0.2470	0.4926	-9.9356
0.32	0.0321	0.2468	0.4996	-9.2774
0.36	0.0329	0.24676	0.5072	-8.5996
0.40	0.0338	0.24672	0.5153	-7.9008
0.44	0.0347	0.2470	0.5241	-7.1792
0.48	0.0356	0.2475	0.5338	-6.4332
0.52	0.0365	0.2483	0.5445	-5.6605
0.56	0.0374	0.2495	0.5563	-4.8585
0.60	0.0382	0.2513	0.5696	-4.0244
0.64	0.0390	0.2537	0.5848	-3.1541
0.68	0.0398	0.2570	0.6022	-2.2428
0.72	0.0404	0.2615	0.6229	-1.2840
0.76	0.0408	0.2679	0.6478	-0.2686
0.80	0.0409	0.2770	0.6789	0.8172
0.84	0.0404	0.2904	0.7197	1.9960
0.88	0.0390	0.3114	0.7772	3.3089
0.92	0.0357	0.3480	0.8683	4.8471
0.96	0.0283	0.4259	1.0525	6.8998

Table 1.4: Growth and welfare effects of time subsidies.

τ	s_v	Growth rate	Leisure	Consumption per intermediate good	Welfare
0	0	0.0260	0.2511	0.4543	-14.0829
0.04	0.2308	0.0307	0.2405	0.5013	-9.8327
0.08	0.3407	0.0334	0.2338	0.5413	-7.0936
0.12	0.4015	0.0352	0.2288	0.5789	-5.0572
0.16	0.4359	0.0362	0.2248	0.6158	-3.4378
0.2	0.4539	0.0367	0.2216	0.6528	-2.0956
0.24	0.4610	0.0368	0.2190	0.6907	-0.9501

We compare the magnitudes of the growth effects of the subsidies in Table 1.5. In general, if only one subsidy is varied at a time, setting the other ones 0, comparing the growth rates at the growth-maximizing subsidy rate, we find that education subsidies (s_d, s_v) are more effective than R&D subsidies (s_η) in promoting growth. Although the R&D subsidies directly lower the cost of R&D investment (which improves dynamic efficiency), their tax distortions increase the share of time allocated to leisure further above the social optimal level, and decreases the share of time allocated to production and education further more below the social optimal level (which reduces static efficiency). While the tax distortions of financing education subsidies are similar to those in the case with R&D subsidies, the education subsidies promote growth by encouraging physical or time investment in human capital accumulation and at the same time decreasing the share of time allocated to leisure (which improves both static and dynamic efficiency). It turns out that education subsidies are more effective than R&D subsidies in stimulating economic growth. Comparing the two education subsidies (s_d, s_v) in the benchmark case, we also find that subsidizing physical inputs education (s_d) is more effective than subsidizing educational time (s_v).

Table 1.5: Growth-maximizing subsidies.

Parameters: $\alpha = 0.8$,$\beta = 0.5$,$\gamma = 0.35$,$\sigma = 1.5$, $\eta = 1$, $\rho = 0.05$, $\epsilon = 0.2$,$A = B = 0.5$						
	s_η		s_d		s_v	
	Subsidy rate	Growth rate	Subsidy rate	Growth rate	Subsidy rate	Growth rate
Benchmark results						
	0.6	0.0287	0.8	0.0409	0.4611	0.0368
Sensitivity analysis						
$\alpha = 0.5$	0.68	0.0110	0.8	0.0127	0.5833	0.0130
$\alpha = 0.9$	0.6	0.0379	0.8	0.0567	0.4333	0.0486
$\beta = 0.2$	0.32	0.0761	0.8	0.0917	0.2605	0.1036
$\beta = 0.8$	0.68	0.2131	0.76	0.3063	0.6324	0.0206
$\gamma = 0.2$	0.6	0.0309	0.8	0.0459	0.4284	0.0402
$\gamma = 0.8$	0.52	0.0490	0.68	0.0517	0.4993	0.0520
$\sigma = 2$	0.72	0.0218	0.84	0.0341	0.5120	0.0302
$\sigma = 2.5$	0.76	0.0178	0.88	0.0298	0.5494	0.0257
$\rho = 0.03$	0.4	0.0425	0.68	0.0501	0.3807	0.0473
$\rho = 0.08$	0.84	0.0117	0.88	0.0298	0.5587	0.0246
$\epsilon = 0.1$	0.6	0.0346	0.8	0.0474	0.4398	0.0418
$\epsilon = 0.3$	0.64	0.024	0.8	0.0356	0.4739	0.0326

Changes in the model's parameters do not reverse the ranking of education subsidies (s_d, s_v) and R&D subsidies (s_η), but changes in the parameters may reverse the ranking of the two education subsidies (s_d, s_v). For example, when β decreases to 0.2, the subsidy to educational time (s_v) is more effective than the subsidy to physical investment (s_d). This is mainly because, β measures the contribution of physical investment to education, while $(1 - \beta)$ measures the contribution of time investment to education. We find that the relative effectiveness of the two education subsidies (s_d, s_v) varies with the values of several parameters. Thus, the governments should rank and choose the two different education subsidies according to the relative contribution of physical input and time input in their countries. Bowen (1987) estimated that D account for 22% of the total explicit cost of acquiring higher education. And it is clear, as technology advances, physical investment is playing an increasingly important role in education. Thus, the government should pay more attention to the physical input subsidies. Result 1 summarizes the results concerning the growth effects of subsidies.

Result 1: *The education subsidies are always more effective than the R&D subsidies in stimulating growth. However, the relative effectiveness of the two education subsidies depends on the values of the model's parameters.*

We also consider the combinations of R&D subsidy and educational subsidies, e.g., a combination of R&D subsidies and physical input subsides, the growth-maximizing combination of the R&D subsidies and physical input subsidies are $(0.6, 0.72)$, generating a higher maximizing growth rate at 0.4124 then using each of them alone. The welfare effects of the subsidies are reported in Table 1.6. Similar to the ranking of the subsidies in terms of their growth effects, the education subsidies (s_d, s_v) always produce a higher level of welfare than the R&D subsidies (s_η). The reasons for this result are similar to those for the growth effects.

Table 1.6: Welfare-maximizing subsidies.

Parameters: $\alpha = 0.8$,$\beta = 0.5$,$\gamma = 0.35$,$\sigma = 1.5$, $\eta = 1$, $\rho = 0.05$, $\epsilon = 0.2$,$A = B = 0.5$						
	s_η		s_d		s_v	
	Subsidy rate	Welfare	Subsidy rate	Welfare	Subsidy rate	Welfare
Benchmark results						
	0	-14.0829	0.96	6.8998	0.4611	-0.9501
Sensitivity analysis						
$\alpha = 0.5$	0	-46.9112	0.92	-19.0582	0.5833	-26.9166
$\alpha = 0.9$	0	3.1861	0.96	17.0181	0.4345	12.8125
$\beta = 0.2$	0	9.4796	0.96	20.0002	0.2605	20.6332
$\beta = 0.8$	0	-21.999	0.96	2.9759	0.6830	-14.738
$\gamma = 0.2$	0	-12.7015	0.96	11.7034	0.4284	1.6116
$\gamma = 0.8$	0	-0.51914	0.84	3.4480	0.5541	4.1692
$\sigma = 2$	0	-19.8912	0.96	5.5489	0.5120	-1.6657
$\sigma = 2.5$	0	-26.8837	0.96	4.6905	0.5494	-3.2385
$\rho = 0.03$	0	-5.4135	0.96	19.2187	0.3974	10.6434
$\rho = 0.08$	0	-13.5782	0.96	2.4528	0.5587	-1.9771
$\epsilon = 0.1$	0	-7.9930	0.96	11.3988	0.4488	4.0493
$\epsilon = 0.3$	0	-19.4932	0.96	3.2356	0.4739	-4.4447

Similarly, the relative effectiveness of the two education subsidies (s_d, s_v varies with the values of the model's parameters. For instance, in the benchmark case, subsidizing physical inputs education (s_d) is more effective than subsidizing educational time (s_v). However, when β decreases to 0.2, the opposite is true. One surprising result in Table 1.6 is the negative relationship between the R&D subsidy and the level of welfare, which is contrary to conventional wisdom. [6].As shown in Table 1.2, although an increase in R&D subsidy raises the growth rate (the *positive growth* effect on welfare), the higher subsidy (along with its tax distortions) further misallocates capital and labor (the *negative level* effect on welfare). In addition, it decreases the level of consumption much more than education subsidies. It turns out that the negative level effect on welfare dominates the positive growth effect on welfare (in particular, the loss from the decrease in consumption per intermediate good and leisure dominates the gain from the increase in the growth rate), as a result, welfare falls as the subsidy rises. We summarize the results as follows:

Result 2: *The education subsidies are always more effective than the R&D subsidies in improving welfare. However, the relative effectiveness of the two education subsidies varies with the values of the model's parameters.*

We also consider the combinations of R&D subsidy and educational subsidies, e.g., a combination of R&D subsidies and physical input subsides as Figure 1.1. We show the combination will not generate a higher welfare than the educational subsidies alone, since the R&D subsidies have a negative effect on welfare.

[6]Under some values of parameters, the increase in R&D subsidies could improve welfare when the subsidies rate is low enough, e.g., when $\alpha = 0.3$, $\beta = 0.5$, $\gamma = 0.35$, $\eta = 1$, $\rho = 0.05$, $A = 0.5$, $B = 0.325$, $\sigma = 1$, $\epsilon = 0$. However the ranking remains the same.

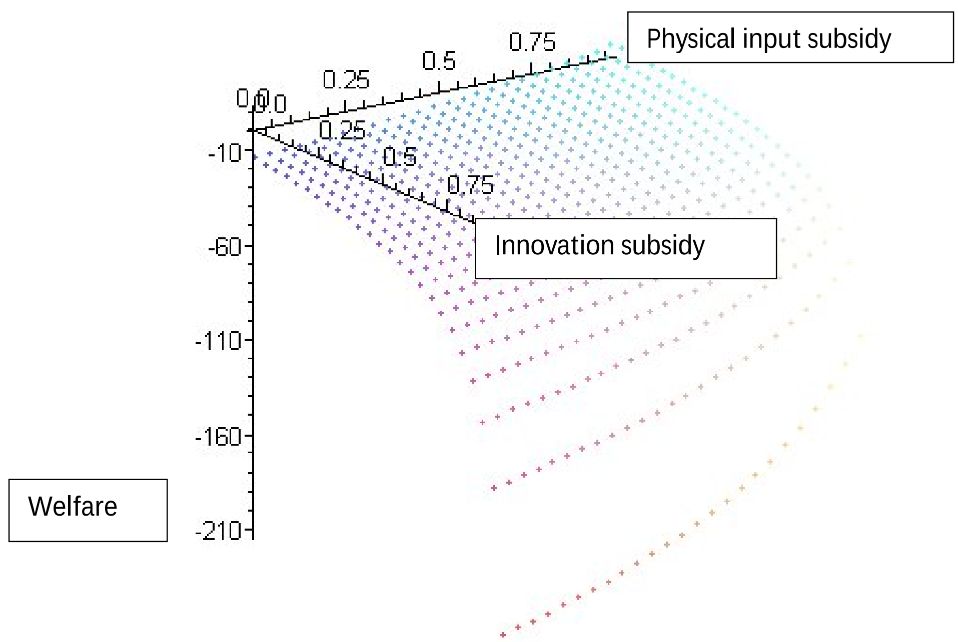


Figure 1.1: Combination of R&D subsidies and physical input subsidies

6. Dynamic results

In the previous section, we examine the growth and welfare effects of the subsidies by comparing the before- and after-subsidy steady states. In this section, we take into account the transitional adjustments of the economy after a subsidy and a tax are implemented. We use exactly the same set of parameter values as in the steady-state analysis and focus on the welfare effects of the subsidies. To simulate the dynamics of the model, we rescale the variables to convert the system of equilibrium equations into a stationary system of equations.

Let $x = X/N$, where $X = N, H, K, C, D$ and Y, we convert the original equilibrium system of equations ((12), (16), (52), (53), (54), (55), (56) and (57)) into the following stationary equilibrium system:

$$\dot{h} = Bd^{\beta}(vh)^{1-\beta} - \frac{(y-c-d)}{k+\eta},$$

$$\dot{u} = \left[\frac{u}{1-\alpha(1-\gamma)}\right]\left\{[1-\alpha(1-\gamma)]\left(\frac{y-c-d}{k+\eta}\right) - \frac{(1-\tau_k)\alpha^2\gamma y}{(1-\beta)(1-s_k)k} + B\left[\frac{u(1-\tau_h)+s_v v}{1-\tau_h-s_v} + \alpha(1-\gamma)(1-u-l)\right]\left[\frac{d}{(1-u-l)h}\right]^{\beta}\right\},$$

$$\dot{l} = \left[\frac{l}{\sigma-\epsilon(1-\sigma)}\right]\times \left\{\sigma B\frac{u(1-\tau_h)+s_v v}{1-\tau_h-s_v}\left[\frac{d}{h(1-l-u)}\right]^{\beta} - (1-\tau_k)\alpha^2\gamma\left(\frac{\sigma}{1-\beta}-1\right)\frac{y}{(1-s_k)k} - \rho\right\},$$

$$y = A(uh)^{\alpha(1-\gamma)}k^{\alpha\gamma},$$

$$c = \alpha^2 l(1-\tau_h)(1-\gamma)y/(u\epsilon(1+\tau_c)),$$

$$d = \frac{\beta(1-\tau_h-s_v)(1-l-u)\alpha^2(1-\gamma)y}{u(1-\beta)(1-s_d)}.$$

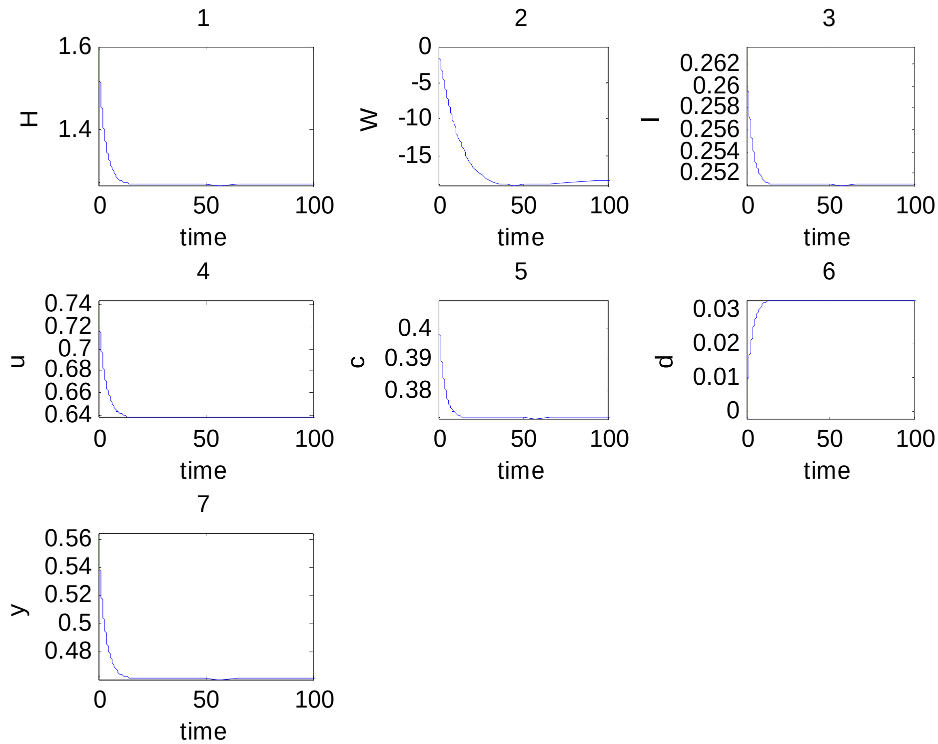
We then adopt the relaxation algorithm developed in Trimborn et al. (2005) to numerically solve the above system and compute the household's welfare. Tables 1.7 and 1.8 report the steady-state values of the above re-scaled variables in a economy with and without subsidies. Table 1.8 also gives the level of welfare in an equilibrium with each of the three subsidies. We use one subsidy at a time, setting the rates at levels the lead to the same amount of government expenditure in each case. We choose the government expenditure as 0.0054, and calculate the corresponding subsidy and tax rates according to equations (14) and (69). Figures 1.2-1.4 show the transitional dynamics of the key variables associated with each of the three subsidies while Figures 1.5-1.7 show the relationship between the subsidies and the welfare level. Figure 1.8 compares the levels of welfare resulting from the three expenditure-equivalent subsidies. We also perform various sensitivity analyses, the results are not reported here but they are available upon request.

Table 1.7: Values of re-scaled variables in laissez-faire equilibrium.

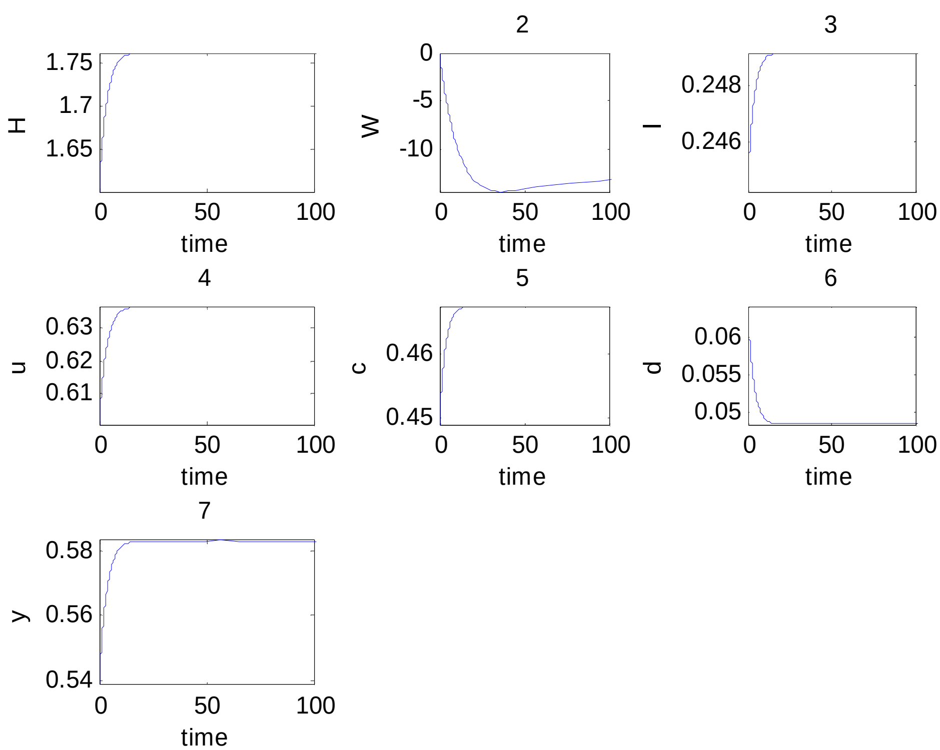
$h = 1.6013$	$u = 0.6395$	$l = 0.2511$
$c = 0.4543$	$d = 0.0396$	$y = 0.5562$

Table 1.8: Welfare effect in terms of equalized government budget:0.0054.

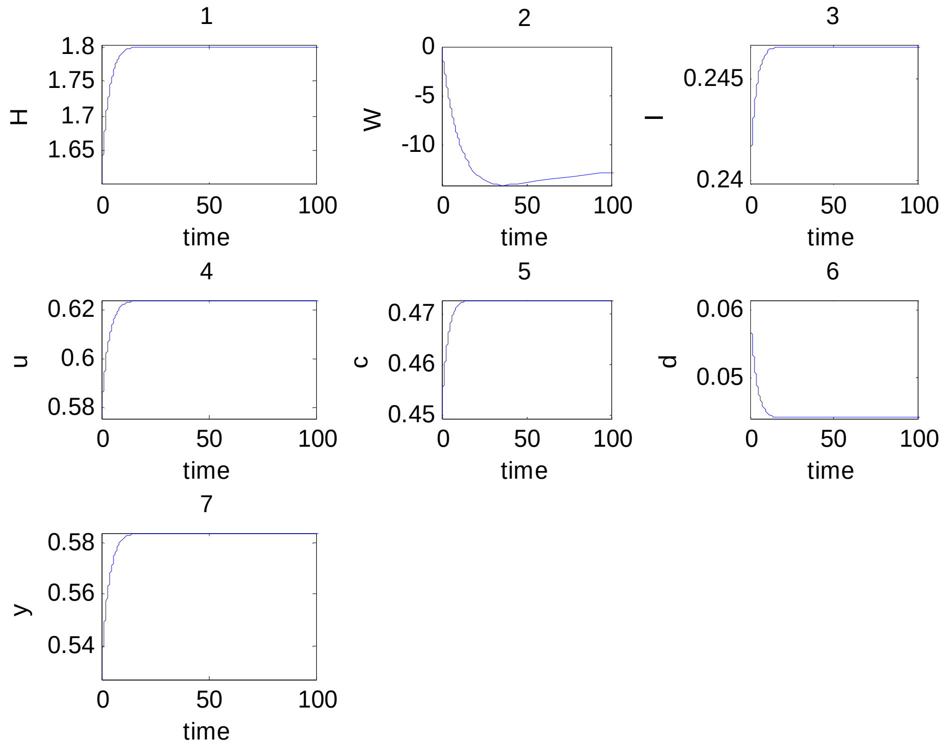
	Values of re-scaled variables		
	$s_\eta = 0.2$	$s_d = 0.1119$	$s_v = 0.108$
h	1.2640	1.7619	1.8013
u	0.6372	0.6366	0.6245
l	0.2509	0.2491	0.2465
c	0.3708	0.4677	0.4727
d	0.033	0.482	0.044
y	0.4612	0.5832	0.5827
τ	0.0183	0.0144	0.0145
W	-18.264	-12.945	-12.79



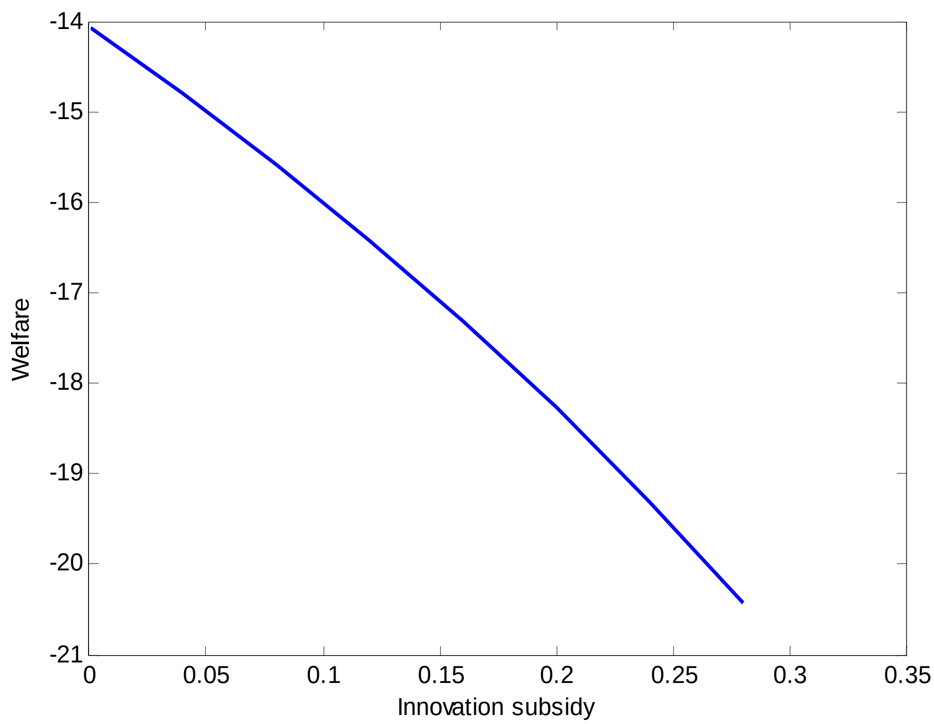
1.2 Transition dynamics of re-scale variables when R&D subsidies=0.2



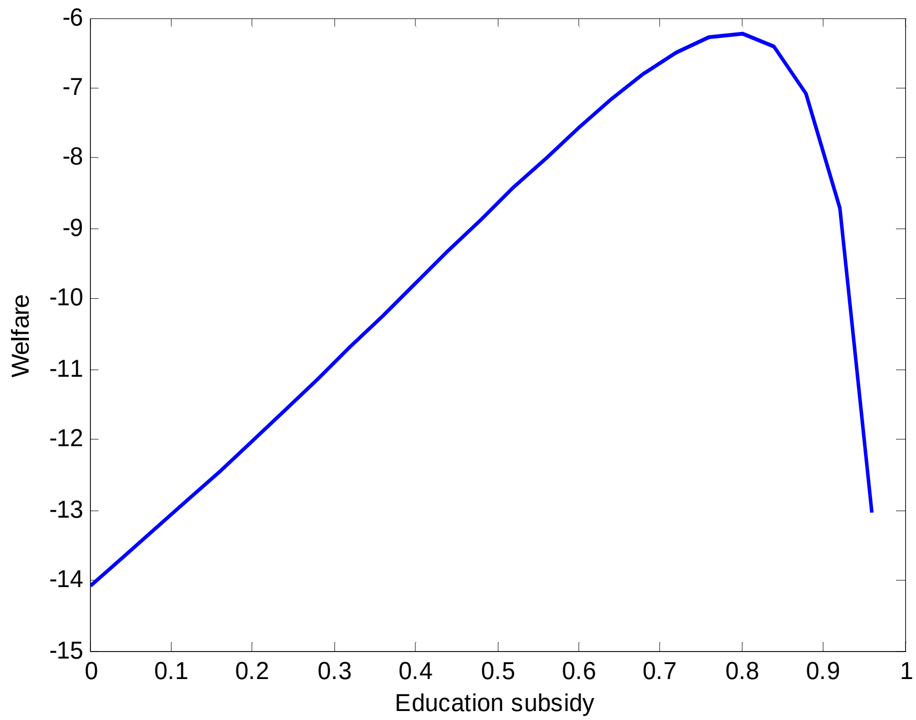
1.3 Transition dynamics of re-scale variables when education subsidies=0.1119



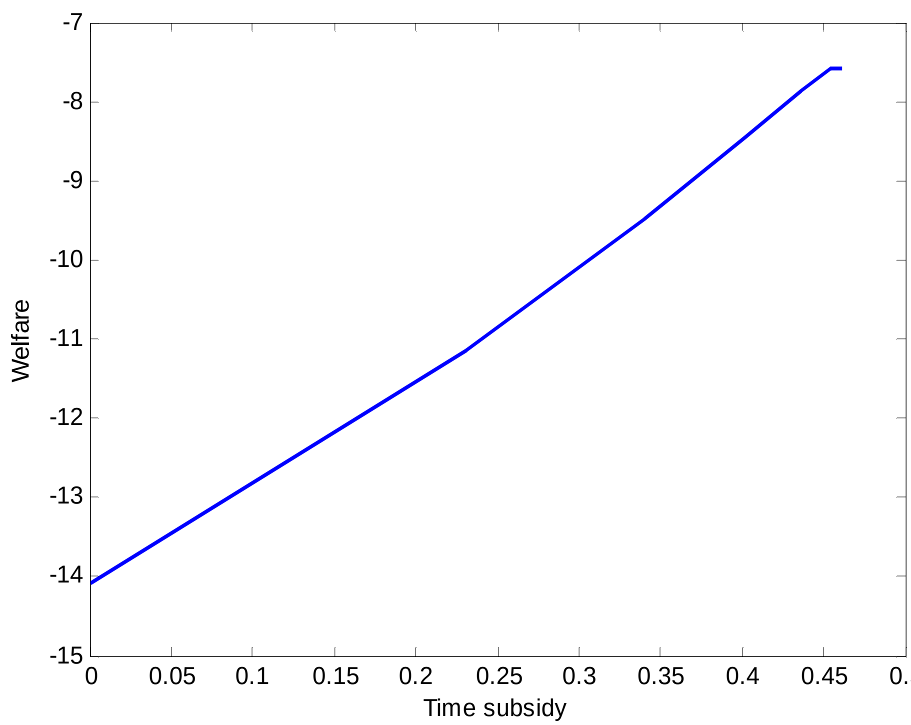
1.4 Transition dynamics of re-scale variables when time subsidies=0.108



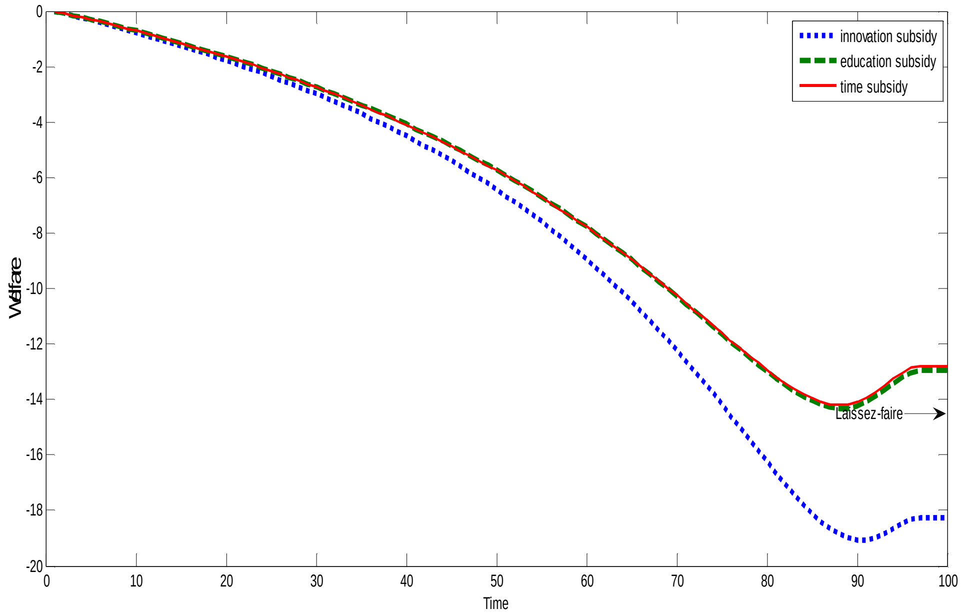
1.5 Relationship of innovation subsidy and welfare from a dynamic analysis



1.6 Relationship of education subsidy and welfare from a dynamic analysis



1.7 Relationship of time subsidy and welfare from a dynamic analysis



1.8 Comparison of the effect on welfare

There are several interesting aspects regarding the differences between the steady-state and dynamic behaviors of the economy. For R&D subsidies(Figure 1.2), starting with $h = 1.601$ (the steady-state value in an equilibrium without subsidies), the fractions of time used in production (u) and leisure (l) jump to higher levels. That is, more time is used in production and leisure and thus less time is spent on education. As a result, the values of c and d drop to lower levels and the value of y jumps to a higher level (due to the rise in the amount of labor used in production). The value of h falls, leading to a decrease in the values of y and c. Then further adjustments in the economy lead to decreases in u and l and increase in both time and physical investments in education. We could observe from Table 1.8, that R&D subsidies is detrimental to human capital accumulation (h decreases) since it alloctes more resouces from education to innovation (d decreases). The negative market size effect (y decreases) may be detrimental to welfare.

For education subsidies (Figures 1.3 and 1.4), starting with $h = 1.601$, the two control variables (u, l) drop to lower levels (i.e., less time is used in production and leisure and thus more time is spent on education). At the same time, physical investment (d) in education rises; The values of c and y fall to lower levels; The value of h rises, leading to to increases in the values of y and c. Further adjustments in the economy result in increases in u and l decreases in both time and physical investment in education.

Similar to the result in the steady-state analysis, we find that

Result 3: *The education subsidies are always more effective than the R&D subsidies comparing the welfare at the welfare-maximizing subsidy rate. However, the relative effectiveness of the two education subsidies depends on the values of the model's parameters.*

7. Conclusion

How to promote R&D activity and education to enhance productivity growth is at the center of public policy debates. This chapter contributes to the debate by comparing the growth and welfare effects of two alternative measures to foster R&D-based growth. We develop a dynamic general equilibrium growth model with both innovation and capital accumulation to investigate the relative effectiveness of various government subsidies. The model is an extension of the Romer (1990) model by incorporating human capital accumulation. In particular, we assume that in addition to the time input, human capital accumulation also requires physical inputs in order to consider the subsidy to physical investment in education. In our model, the growth rate of output depends on household's preferences, human capital production technology and R&D activities and government policies affect the growth rate. Even if the human capital accumulation do not requires physical inputs (when $\beta = 0$), the growth also depends on both human capital production technology and R&D activities. This is different from the results in Zeng (2003), in which as long as the human capital is the only input in human capital accumulation, then the long-run growth rate does not depend on R&D technology. The difference is mainly because of the introduction of elastic labor supply in our model, i.e, the R&D subsidies not only affect the rate of return of the investment in R&D but also time allocation. As in Romer (1990) and Barro and Sala-i-Martin (2004, Chapter 6), the laissez-faire equilibrium growth rate is less than optimal because of the existence of the monopoly pricing of the intermediate goods and the positive externality associated with innovation.

We consider both R&D and education subsidies and examine their relative effectiveness in stimulating economic growth and improving welfare. We find that both the R&D and education subsidies can raise the long-run growth rate and that the education subsidies are always more effective than the R&D subsidies. However, the relative effectiveness of the two education subsidies depends on the values of the model's parameters.

We also find from both steady-state and dynamic analysis that subsidizing education improves welfare while subsidizing innovation does the opposite and that the welfare effects of the two education subsides varies with the values of the model's parameters. The result that the R&D subsidies are welfare reducing is surprising at first glance. However, as discussed in Section 4, the R&D subsidies have two offsetting effects on welfare (a negative level effect and a positive growth effect). It turns out that the negative effect always more than offsets the positive effect. As a result, the R&D subsidies always reduce welfare. Our results are consistent with those in Volker grossmann (2004), in which they claimed, due to the rivalry of education expenditure, effective aggregate supply of skilled labor may not increase in response to higher R&D subsidies. While in our model, the R&D subsidies directly decreases the physical input into human capital accumulation, which is weaken the human capital accumulation, generating a negative market size effect.

Although both the R&D and education subsidies are commonly used in real world and both can stimulate economic growth, this chapter concludes that subsidizing education is more promising than subsidizing R&D in enhancing welfare. Sapir et al. (2004) recommended a substantial increase in government and EU spending for postgraduate education. Our analysis has given a theoretical foundation to this policy prescription by showing that such a policy will unambiguously boost efficiency and growth. Although in the real world, almost a third of R&D expenditure in the OECD is financed by the government sector through grants, funding or tax incentives, (OECD, 1999), as pointed out by Goolsbee (1998), when the government increases R&D spending subsidies or direct provision, a significant fraction of the increased spending goes directly into higher wages, an increase in the price rather than the quantity of inventive activity in the long run. It is fair to say that R&D policy needs to be re-evaluated.

The findings in this chapter suggest that governments should seriously consider their policy-mix of different subsidies to promote growth and welfare especially in the case of tight government budgets. Although the finding is based on a closed economy, they may also hold in a small open economy. The empirical study on Taiwan (a small open economy) in Lin (2003) indirectly supports our findings. Lin (2003) examines the impact of education and technical progress on economic growth in Taiwan over the period from 1965 to 2000 and finds that education has a positive and significant effect on growth, but the role of technical progress does not appear to be extraordinarily important. In a small economy, there will be cross-country higher education externalities, which means not only there should be education subsidies but also cooperate education project between countries. At the meantime, when the bulk of technological advances occur as a result of R&D in other countries, there will be even higher R&D externalities, so the results regarding that the R&D subsidies may decrease welfare may change.

With the numerical example and sensitivity analysis, all the results are general in the economy environment. However, if the same issue are examined in a total different model, such as a growth model with overlapping generations, some results may still hold, but some may change. The results in Chapter 2&3 also subject to this.

Part II

Chapter 2: Optimal Taxation in an R&D Growth Model with Variety Expansion

1. Introduction

It is well known that in Chamley and Judd's neoclassical growth model with capital accumulation and infinitely-lived agents, the optimal tax system to finance an exogenous government expenditure involves a zero tax rate on capital in the long run, with the required tax revenue being collected by a distortional tax on labor income. Many studies based on the Chamley-Judd model reconfirm this result. Judd (1999) claimed not only the tax on physical capital but also the tax on human capital should be zero if human capital is not a final good. Chamley (1986) also showed that optimal tax rate is zero in the long run in a general equilibrium model.

However, Jones et al. (1993) studied the connection between government policies and growth. They compared the capital and labor income tax in an AK model, and found that a positive tax on capital income becomes optimal under special conditions. In addition, Jones et al. (1997) extended the model to an economy with both physical and human capital, and found that the zero optimal taxation on both capital incomes is due to three strict conditions: constant return to scale, a sufficiently rich tax code, and no possibilities for relative prices to affect wealth. They found a positive optimal tax on capital with inelastic labor supply or restrictions on what taxes could be levied. Therefore, there are still rationales for also taxing capital, in an economy with credit-constrained agents (e.g., Chamley(2001)), or with private information (e.g., Golosov et al. (2006), Kocherlakota (2010)), or with two sources of inequality (e.g., Piketty and Saez (2012)). Turnovsky (1996) suggested the trade off between consumption and income taxes should depend on the level of government expenditure relative to its social optimum in an economy with public goods. In other aspect, Devereux & Love (1994) compared the growth and welfare effects of capital income and wage taxes and claimed that firstly for equal percentage changes, the wage tax has a larger effect on growth than the capital tax, but for revenue-equivalent changes, they have similar effects on growth. Secondly, the capital income tax will generate a higher welfare cost than wage tax.

More recently, some attention has been paid to the effects of taxation in innovation-based growth model. Atkeson and Burstein (2012) compared R&D tax credits, federal expenditures on R&D, and corporate profit tax in a semi-endogenous growth model with horizontal innovation, focusing on the impact of these policies on innovation and welfare. More closely related to our analysis is the paper by Aghion et al. (2013), in which optimal taxation was discussed in a Schumpeterian growth model. Their results showed that a zero tax on capital becomes suboptimal for a given required trend of public expenditure due to a market size effect and claimed that Chamley-Judd result is not robust to introducing endogenous technical progress.

This study, parallel to Aghion et al. (2013), aims at examining the robustness of the results in Chamley-Judd by introducing endogenous technical progress. We imply an R&D growth model with variety expansion. More specifically, in Section 2, we firstly introduce a basic model without human capital accumulation, where final good is produced with intermediate good and labor, and intermediate good is produced with only physical capital. We show that the optimal tax on physical capital is always negative and there is an negative relationship between the tax on physical capital and the steady-state growth rate given a fixed government expenditure share. The first main difference between our study and Aghion et al. (2013) is that economic growth in our model is driven by horizontal innovation through variety expansion instead of vertical innovation through quality improvement. It is shown in Aghion and Howitt (1992) that there can be too much or too little R&D investment in a vertical innovation model. However, R&D is always under-invested in variety expansion model due to the monopoly inefficiency and knowledge spillover. Thus, with the non-accumulable labor income tax distortion, it is always optimal to tax labor while subsidizing physical capital accumulation.

The second difference is that we also introduce an extended model with human capital accumulation in Section 4 to make the model more general, we show that, in this extended model, it is optimal to tax physical capital to subsidize human capital accumulation as long as the government expenditure share is low enough. We claim that the difference in optimal taxation in the two models mainly comes from the fact that human capital can be accumulated and plays a more important role (relative to physical capital) in the monopolized production sector in the extended model. Lastly, we also apply dynamic analysis to find the respective tax distortion, and it clearly shows that the physical capital income tax distortion decreases the welfare more than labor income tax distortion in the model without human capital, while in an economy with human capital accumulation, the ranking reverses.

The remaining part of the study is organized as follows. Section 2 describes the basic model and characterizes the dynamic system of the equilibrium. Section 3 analyzes optimal taxation in steady-state. Section 4 extends the analysis to a model with human capital. Section 5 concludes.

2. Basic Model

The basic model is due to Romer (1990). We will firstly use a model without human capital and then extend the model by incorporating human capital accumulation in section 4. We describe the details of the economic environment in the following sub-sections.

2.1 Technologies

There are four types of production activities in the economy: final good production, intermediate good production, innovation, and physical capital accumulation. It is assumed that there exists monopoly power in the intermediate good sector while all the other sectors are perfectly competitive.

2.1.1 Final good production

A final good producer uses a continuum of intermediates goods and labor as its inputs subject to the following Cobb-Douglas production function

$$Y_t = AL_t^{1-\alpha} \int_0^{N_t} {z_{it}}^{\alpha} di, \quad A > 0, \quad 0 < \alpha < 1, \tag{71}$$

where the subscript t refers to time; A is a productivity parameter; α measures the contribution of an intermediate good to the final good production and inversely measures the intermediate monopolist's market power; Y_t is the final output; z_{it} is the flow of intermediate good i; N_t is the number of intermediate goods; L_t is the labor used in final good production. For simplicity, we omit the time subscript t throughout the chapter whenever no confusion can arise. The profit function for the final good producer is

$$AL^{1-\alpha} \int_0^N {z_i}^{\alpha} di - \int_0^N p_i z_i di - wL,$$

where we normalize the price of final good to 1 and p_i is the price of intermediate good i in terms of the final good. Profit maximization in the competitive final good sector gives the demand functions for intermediate good i and labor,

$$p_i = \alpha A \left(\frac{L}{z_i}\right)^{1-\alpha}, \quad i \in [0, N], \tag{72}$$

$$w = \frac{(1-\alpha)Y}{L}. \tag{73}$$

That is, in the competitive final good sector, the price of an input should equal to its marginal product.

2.1.2 Intermediate good production

Each monopoly intermediate producer i who has a patented technology for one variety uses physical capital k_i to produce an intermediate good according to

$$z_i = k_i. \tag{74}$$

Given the interest rate r, and the final good sector's demand for intermediate good i given by equation (2), intermediate good i producer chooses the amount of capital to maximize its profit

$$\pi_i = p_i z_i - r k_i.$$

The solution to this maximization problem gives the producer's demand function for capital k_i as well as the output z_i.

$$z_i = k_i = L \left(\frac{\alpha^2 A}{r} \right)^{\frac{1}{1-\alpha}}, \quad i \in [0, N] \tag{75}$$

by which we obtain

$$\pi_i \equiv \pi = \alpha(1-\alpha) A^{\frac{1}{1-\alpha}} \left(\frac{\alpha^2}{r} \right)^{\frac{\alpha}{1-\alpha}} L,$$

which will not depend on i. Substituting equation (75) into equation (1), we rewrite Y as

$$Y = A^{\frac{1}{1-\alpha}} \left(\frac{\alpha^2}{r} \right)^{\frac{\alpha}{1-\alpha}} NL.$$

As a result, the profit π could be written as

$$\pi = \frac{\alpha(1-\alpha)Y}{N}. \tag{76}$$

Denoting $\Pi = N\pi$, we have $\Pi/Y = \alpha(1-\alpha)$, that is, the aggregate profit is proportional to the aggregate output, which accords with the result in Aghion et al. (2013). In addition, the model is symmetric so that each new intermediate good raises N by one unit. Also, each unit of capital here is one infinitely durable unit of the final good, and r is the rental rate for each such unit, making it the interest rate that expresses the relative prices of final goods today and in the future.

2.1.3 Innovation

The R&D sector is perfectly competitive and the innovation process is deterministic. An innovator invests η units of final good to discover a technology to produce a new intermediate good. The innovator then becomes the sole producer of the intermediate good forever. The value of a new technology equals the present value of the profits from producing the new intermediate good V_t, which is given by

$$V_t = \int_t^\infty \pi_s \exp\left(-\int_t^s r_\tau d\tau\right) ds. \tag{77}$$

Assuming free entry in the R&D sector, we have

$$V = \eta. \tag{78}$$

From equation (77), we obtain

$$\dot{V} = \pi - rV, \tag{79}$$

where a dot on the top of a variable represents the time change rate of that variable. Combining equations (78) and (79) gives the equilibrium condition in the R&D sector

$$V = \frac{\pi}{r} = \eta, \tag{80}$$

which holds true both in and outside the steady state.

2.2 Households

The model economy is populated by a continuum of identical infinitely-lived households with measure one. The representative household is endowed with 1 unit of time which is inelastically allocated between final good production u, and leisure $l \quad (= 1 - u)$. The representative household seeks to maximize utility defined as

$$U = \int_0^\infty \frac{(Cl^\epsilon)^{1-\sigma} - 1}{1-\sigma} \exp(-\rho t) dt, \tag{81}$$

where C is consumption; ρ is the constant rate of time preference; σ is the elasticity of marginal utility; ε is the elasticity of the flow of utility with respect to leisure; l is the amount of time allocated into leisure. The representative household has a budget constraint

$$C = (1-\tau_k)rK + (1-\tau_l)wu - \dot{K} + D, \tag{82}$$

where two taxes are introduced to illustrate the problem of optimal taxation: one on physical capital income at rate τ_k and the other on labor income at rate τ_l and D [7]is the total dividend from intermediate producers.

The representative household chooses consumption C, and the time allocation to maximize its life-time utility equation (81), subject to the budget constraint equation (82). The current-value Hamiltonian function for this optimization problem is

$$\mathcal{L} = \frac{[C(1-u)^{\varepsilon}]^{1-\sigma} - 1}{1-\sigma} + \lambda\left[(1-\tau_k)rK + (1-\tau_l)wu + D - C\right],$$

where λ is the costate variable associated with equation (82). The first-order conditions for this optimization problem are equation (82) and the following equations,

$$C: \qquad C^{-\sigma}(1-u)^{\epsilon(1-\sigma)} = \lambda, \tag{83}$$

$$u: \qquad \epsilon C^{1-\sigma}(1-u)^{\epsilon(1-\sigma)-1} = \lambda(1-\tau_l)w, \tag{84}$$

$$K: \qquad \lambda r(1-\tau_k) = -\dot{\lambda} + \rho\lambda, \tag{85}$$

$$TVC: \qquad \lim_{t\to\infty} e^{-\rho t}\lambda_t K_t dt = 0. \tag{86}$$

Equation (83) (respectively, (84)) equalizes the marginal benefit and marginal cost of consumption (respectively, time allocated into production). Equation (85) is the optimal dynamic condition for physical capital accumulation and equation (86) is the transversalty condition.

[7]Although in real world tax laws, capital income and dividends are taxed at the same rate, the monopoly profits in the intermediate goods sector are treated differently from the income from renting out capital goods. If the dividends were taxed at the same rate as capital income, this increases the capital income tax distortion and will may change the ranking of the two taxes.

2.3 Government

We assume that the government's budget is balanced at each point in time. At the meantime, same as Aghion et al. (2013), we assume the government has an exogenous government expenditure, whose share of final output is fixed at x, i.e. $G = xY$. Both taxes on capital and labor income are collected to finance the exogenous government expenditure. Thus, we have [8]

$$\tau_k r K + \tau_l w u = G, \tag{87}$$

where the left-hand side is the sum of tax revenue collected from capital and labor income and the right-hand side is the government expenditure which is proportional to final output.

2.4 Decentralized equilibrium

We now derive the equilibrium condition. According to equation (75), all the intermediate good producers will produce the same quantity, so we have $z_i = z = K/N$, since from the capital good clearing condition, we have $\int_0^N k_i di = \int_0^N z_i di = K$. From the labor market clearing conditions, i.e., $u = L$. We could rewrite Y as

$$Y = AK^{\alpha}(uN)^{1-\alpha}. \tag{88}$$

Also the final good (Y) market clearing condition will be satisfied,

$$Y = C + \dot{K} + \eta \dot{N} + G, \tag{89}$$

i,e, final output will be distributed among consumption, investment, innovation as well as government expenditure. We now begin to derive the equilibrium conditions for each of these components. We first derive the expression for consumption. From equations (83) and (84), we have the relationship between C and u, which is

$$C = (1-u)(1-\tau_l)w/\epsilon. \tag{90}$$

[8] In our analysis, either tax rates can be negative. Thus, it is possible to use only one tax to finance both the government expenditure and a subsidy.

Substituting equation (73) into equation (90), we have

$$C = (1-u)(1-\tau_l)(1-\alpha)Y/(u\epsilon). \tag{91}$$

This expression implies that consumption in equilibrium is always proportional to final output. At the meantime, its share to final output depends negatively on the amount of time devoted into production, the tax on labor income as well as the elasticity of the flow of utility with respect to leisure. We then combine equations (76) and (80) to obtain the expression for physical capital,

$$\frac{K}{N} \equiv k = \frac{\alpha\eta}{1-\alpha}, \tag{92}$$

which is constant [9], thus, we have

$$\frac{\dot{K}}{K} = \frac{\dot{N}}{N}. \tag{93}$$

Some observations are worth noting based on the fact that capital to productivity in terms of N (k) is constant. Firstly, since we could rewrite $\frac{Y}{N} \equiv y = Au^{1-\alpha}k^{\alpha}$, y will depend positively on labor supply (u) as k is constant. Policies which could increase time allocated into production will always have a positive market size effect. [10] Also from the subsection describing intermediate goods production, we know that the aggregate profit (Π) is proportional to the aggregate final output (Y), thus the profit to productivity (π) is also proportional to the final output to productivity (y). In addition, $\frac{\pi}{r} = \eta$ holds true both in and outside the steady state. Thus, because of the positive relationship between the final output to productivity (y) and the labor supply (u), we also have a positive relationship between the profit to productivity (π) and the labor supply (u) and between the interest rate (r) and the labor supply (u) as well. Combining the final good market clearing condition equation (89) and

[9] Please see details in footnote 3 in Chapter 1

[10] The definition of market size effect is the same as in Aghion et al. (2013)

equation (93), we have the transition dynamics for the measure of intermediate goods N and physical capital K,

$$\dot{N} = \frac{[(1-x)Y - C]\, N}{K + N\eta}, \tag{94}$$

$$\dot{K} = \frac{[(1-x)Y - C]\, K}{K + N\eta}. \tag{95}$$

In addition, from equation (75), we have

$$r = \alpha^2 A N^{1-\alpha} K^{\alpha-1} (u)^{1-\alpha},$$

which implies that the interest rate will be lower than the marginal product of capital, highlighting the important implication in variety-expansion models. Using equations (83) and (85), we have

$$-\sigma\frac{\dot{C}}{C} - \varepsilon(1-\sigma)\frac{\dot{u}}{1-u} = \frac{\dot{\lambda}}{\lambda} = \rho - r(1-\tau_k),$$

by which we obtain

$$\dot{u} = \frac{(1-u)u\{\rho + \sigma\frac{[(1-x)Y-C]}{K+N\eta} - \alpha^2 A N^{1-\alpha} K^{\alpha-1}(u)^{1-\alpha}(1-\tau_k)\}}{\alpha\sigma(1-u) + [\epsilon(\sigma-1)+\sigma]\, u}. \tag{96}$$

The dynamics of the decentralized economy are then characterized by the system of equations (88), (91), (94), (95) and (96), along with an initial condition (K_0, N_0) and the transversally condition equation (86). We construct a stationary system by re-scaling the variables to variable to productivity. Letting $s = S/N$, where $S = N, K, C$ and Y, (We always have $n = 1$, and $k = \frac{\alpha\eta}{1-\alpha}$ which are constant), we convert the original equilibrium system of equations (88), (91), (94), (95) and (96) into the following stationary equilibrium system:

$$\dot{u} = \frac{(1-u)u\{\rho + \sigma\frac{[(1-x)y-c]}{k+\eta} - \alpha^2 A k^{\alpha-1} u^{1-\alpha}(1-\tau_k)\}}{\alpha\sigma(1-u) + [\epsilon(\sigma-1)+\sigma]\, u}, \tag{97}$$

$$y = A u^{1-\alpha} k^{\alpha}, \tag{98}$$

$$c = (1-u)(1-\tau_l)(1-\alpha) A u^{-\alpha} k^{\alpha}/\epsilon. \tag{99}$$

Since the expressions for y and c are just static equations and we already know the values of n and k, the key variable in the above system is labor supply (u).

3. Balanced growth equilibrium

In the balanced growth equilibrium, the time allocation (u, l) remains the same and all the other variables (consumption C, the number of intermediate good N, physical capital stock K, and final output Y) grow at the same constant rate g, that is, $\dot{u} = 0$ and $\dot{S}/S = g$, where $S = C, N, K$ and Y. We now derive the steady-state equilibrium conditions that determine the decentralized economy growth rate (g) and time allocation. Setting $\dot{u} = 0$, we obtain

$$\rho + \sigma \frac{[(1-x)y - c]}{k+\eta} - \alpha^2 A k^{\alpha-1}(1-\tau_k)u^{1-\alpha} = 0. \tag{100}$$

Since $\dot{N}/N = \frac{(1-x)y-c}{k+\eta} = g$, from equation (100), we have

$$\sigma g + \rho = A\alpha^2 \left(\frac{\alpha\eta}{1-\alpha}\right)^{\alpha-1}(1-\tau_k)u^{1-\alpha}.$$

From the above equation we derive

$$u = \frac{\alpha\eta}{(1-\alpha)}\left[\frac{\sigma g + \rho}{\alpha^2 A(1-\tau_k)}\right]^{\frac{1}{1-\alpha}}. \tag{101}$$

Dividing both sides of the final good market clearing condition equation (89) by Y, we could use the expression of u to compute each component's share of final output. Denoting $\zeta_c = \frac{C}{Y}$, from equation (91), we have $\zeta_c = \left\{1 - \frac{\alpha\eta}{(1-\alpha)}\left[\frac{\sigma g+\rho}{\alpha^2 A(1-\tau_k)}\right]^{\frac{1}{1-\alpha}}\right\}(1-\tau_l)(1-\alpha)^2\left[\frac{\alpha^2 A(1-\tau_k)}{\sigma g+\rho}\right]^{\frac{1}{1-\alpha}}/\alpha\epsilon\eta$.

From equations (76) and (78), we have

$$y = \frac{\eta(\sigma g + \rho)}{\alpha(1-\alpha)(1-\tau_k)},$$

from which, we get $\frac{\dot{K}}{Y} = \frac{\dot{K}}{K}\frac{K}{Y} = \frac{kg}{y} = \frac{\alpha^2(1-\tau_k)g}{\sigma g+\rho}$, and $\frac{\eta\dot{N}}{Y} = \frac{\eta\dot{N}}{N}\frac{N}{Y} = \frac{\eta g}{y} = \frac{\alpha(1-\alpha)(1-\tau_k)g}{\sigma g+\rho}$.

In addition, the government expenditure is a fixed share of final output, i.e., $\frac{G}{Y} = x$.

In summary we could re-write the final output clearing condition as

$$\frac{\alpha(1-\tau_k)g}{\sigma g + \rho} = 1 - x - \zeta_c. \tag{102}$$

Equation (102) determines the steady-state growth rate in the decentralized equilibrium.

3.1 Existence of the balanced growth equilibrium

For simplicity, we consider a laissez faire equilibrium to show the existence of the decentralized equilibrium, then if τ_k and τ_l are sufficiently small, by the continuity of the equations, the equilibrium should exist as well. When there is no government intervention, we could re-write equation (101) as $u = \frac{\alpha\eta}{(1-\alpha)}\left[\frac{\sigma g+\rho}{\alpha^2 A}\right]^{\frac{1}{1-\alpha}}$, thus there is a one-to-one relationship between the labor supply u and the growth rate g. As a result, it is enough to use the equation involving u to prove the existence, uniqueness and stability of the equilibrium. Looking back into the stationary equilibrium system equations (97) to (99), setting $\dot{u} = 0$, when $x = \tau_k = \tau_l = 0$ and substituting equations (98), (99) and the values of n and k into equation (97), we have

$$\rho + A\alpha\left(\frac{(1-\alpha)u}{\alpha\eta}\right)^{1-\alpha}\left\{\sigma\left[1-\frac{(1-u)(1-\alpha)}{u\epsilon}\right]-\alpha\right\} = 0. \tag{103}$$

We denote the value of u that solves the above equation as u^*. We now derive the existence condition of the balanced growth equilibrium and then examine the stability of the system. Firstly, since consumption could not exceed total output, we have $1 - \frac{(1-u)(1-\alpha)}{u\epsilon} > 0$, i.e., $u > \frac{1-\alpha}{1-\alpha+\epsilon}$, and since $u + l = 1$, we should have $u < 1$. In summary, we have $\frac{1-\alpha}{1-\alpha+\epsilon} < u < 1$.

Proposition 1: There always exists a positive unique value of u^* such that $\frac{1-\alpha}{1-\alpha+\epsilon} < u^* < 1$, as long as (i) $\sigma > 1$ and (ii) $\rho < A\alpha^2\left[\frac{(1-\alpha)^2}{\alpha\eta(1-\alpha+\epsilon)}\right]^{1-\alpha}$.

Proof. We will prove the proposition by intermediate value theorem. Firstly, it is obvious that the LHS of equation (103) is increasing in u. In addition, when $u = 1$, we have $LHS = \rho + A\alpha(\frac{1-\alpha}{\alpha\eta})^{1-\alpha}(\sigma-\alpha) > 0$, as long as $\sigma > 1$.[11] Lastly, when $u = \frac{1-\alpha}{1-\alpha+\epsilon}$, we have $LHS = \rho - A\alpha^2\left[\frac{(1-\alpha)^2}{\alpha\eta(1-\alpha+\epsilon)}\right]^{1-\alpha} < 0$, as long as condition (ii) holds. *Q.E.D.*

[11]The value of σ is just a sufficient condition for the existence of an unique positive value of u^*. To be more precise, as long as $\sigma > \alpha - \frac{\rho}{\alpha A(\frac{1-\alpha}{\alpha\eta})^{1-\alpha}}$, the statement is true. In addition, in most empirical studies, the value of the elasticity of marginal utility (σ) is found to be greater 1.

These conditions can be guaranteed by various sufficient conditions concerning the values of the preferences such as a sufficiently low subjective discount rate (low value of ρ), and a sufficiently high elasticity of marginal utility (high value of σ).

Lastly, under the conditions of the existence of the balanced growth equilibrium: $\sigma > 1$, since the LHS of equation (103) is increasing in u, we know from equation (97) that when $u > u^*$, $\dot{u} > 0$, and when $u < u^*$, $\dot{u} < 0$, thus the dynamic system is globally unstable. So the economy must jump to the steady state. Thus in the basic model, we will focus only on balanced growth analysis. And we will perform dynamic analysis in an extended model in section 4.

3.2 Government's problem

The government will choose τ_k and τ_l to maximize the representative consumer's life time utility equation (81), such that the optimal choice for representative consumer: equation (102) and the government budget constraint equation (87) are satisfied. We plug the values of r, k, w and u to reduce equation (87) to,

$$x = \alpha^2 \tau_k + \tau_l (1 - \alpha), \tag{104}$$

from which we could express τ_l in terms of τ_k and x,

$$\tau_l = \frac{x - \alpha^2 \tau_k}{1 - \alpha}. \tag{105}$$

Equation (105) implies that for any given government expenditure share x, 1% increase in physical capital income tax will decrease the labor income tax by $\frac{\alpha^2}{1-\alpha}$%. The government then needs to find an optimal combination of the two taxes in terms of improving growth/welfare. In addition, since both rates should be less than 1, we have $\frac{x+\alpha-1}{\alpha^2} < \tau_k < 1$. Meanwhile for the representative consumer, the labor supply u should be less than 1, thus we have $\frac{\sigma g+\rho}{\alpha^2 A(1-\tau_k)}^{\frac{1}{1-\alpha}} < \frac{1-\alpha}{\alpha\eta}$. Plugging equation (105) back into equation (102), we could rewrite equation (102) as

$$g = \frac{(1 - x - \zeta_c)(\sigma g + \rho)}{\alpha(1 - \tau_k)}, \tag{106}$$

where

$$\zeta_c = \frac{(1-u)(1-\alpha - x + \alpha^2 \tau_k)}{u\epsilon}.$$

Thus, the steady-state growth rate will be determined by the values of x, τ_k and other parameters. We graph both the LHS and the RHS of equation (106) in Figure 2.1 for the proof in Proposition 2, noticing that both the LHS and the RHS are increasing functions with g.

Proposition 2: *Given any given x, we have $\frac{\partial g}{\partial \tau_k} < 0$, as long as $\sigma > 1$.*

The steady-state growth rate will be determined by the intersection of the LHS and the RHS cruves. Since, the LHS curve will not be affected by the value of τ_k, we only need to show that there should be a left-ward moving of the RHS curve given a higher rate of physical capital income tax τ_k.

Proof. 1. Given any τ_1 and τ_2, $\tau_1 < \tau_2$. From equation (101), when $u = 1$, we have $\bar{g}_2 = \frac{1}{\sigma}\left[\alpha^2 A(1-\tau_2)\left(\frac{1-\alpha}{\alpha\eta}\right)^{1-\alpha} - \rho\right] < \bar{g}_1 = \frac{1}{\sigma}\left[\alpha^2 A(1-\tau_1)\left(\frac{1-\alpha}{\alpha\eta}\right)^{1-\alpha} - \rho\right]$. Meanwhile at both $(\tau_1, \bar{g}_1)$ and $(\tau_2, \bar{g}_2)$, the RHS of equation (106) equals to the same value $(1-x)\alpha A(\frac{1-\alpha}{\alpha\eta})^{1-\alpha}$. 2. From $\underline{u}_1 = \frac{1-\alpha-x+\alpha^2\tau_1}{1-\alpha-x+\alpha^2\tau_1+(1-x)\epsilon}$ and $\underline{u}_2 = \frac{1-\alpha-x+\alpha^2\tau_2}{1-\alpha-x+\alpha^2\tau_2+(1-x)\epsilon}$ (the minimum value of labor supply to ensure that consumption does not exceed the total final output net of government expenditure), we have $\underline{g}_1 = \frac{1}{\sigma}\left\{\alpha^2 A(1-\tau_1)\left[\frac{(1-\alpha)\underline{u}_1}{\alpha\eta}\right]^{1-\alpha} - \rho\right\}$ and $\underline{g}_2 = \frac{1}{\sigma}\left\{\alpha^2 A(1-\tau_2)\left[\frac{(1-\alpha)\underline{u}_2}{\alpha\eta}\right]^{1-\alpha} - \rho\right\}$. It is easy to show that $\underline{g}_2 < \underline{g}_1$, and at $(\tau_1, \underline{g}_1)$ and $(\tau_2, \underline{g}_2)$, the RHS of equation (106) equals to 0.

As shown in the Figure 2.1, that we have $g_2^* < g_1^*$. *Q.E.D.*

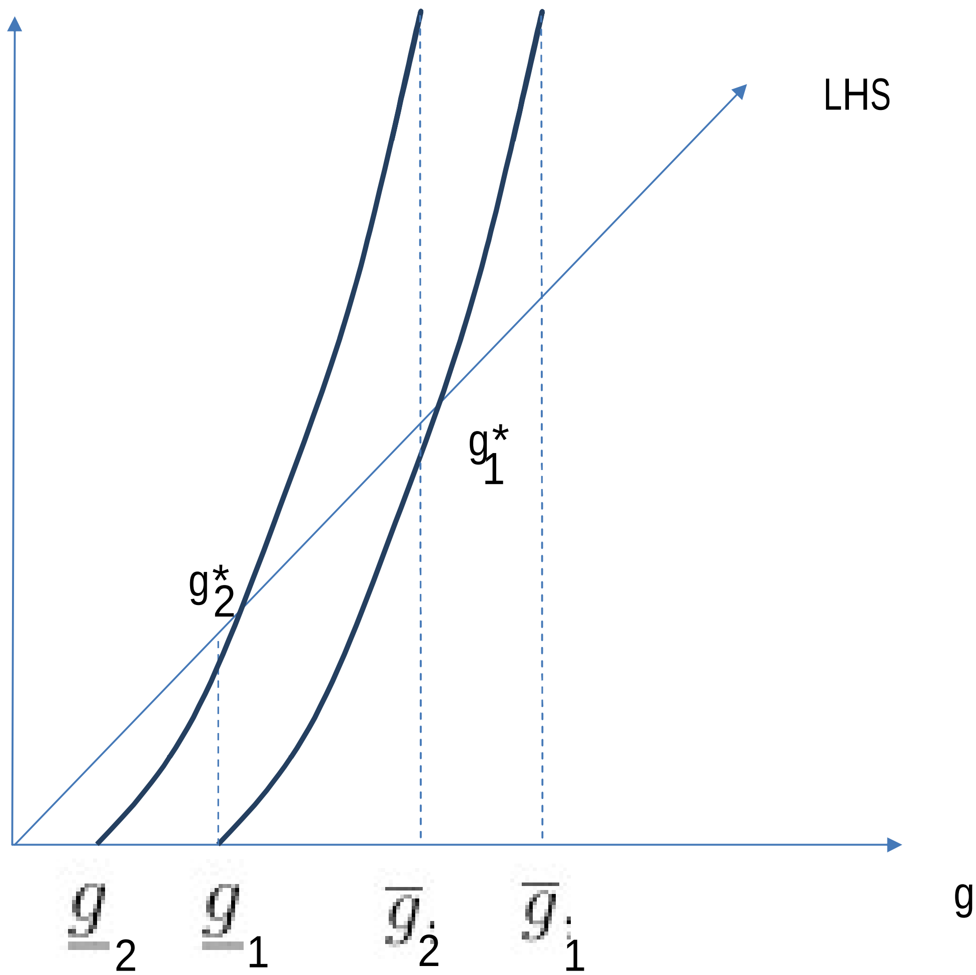


Figure 2.1 Steady state growth rate in the basic model

Since $y = \frac{(\sigma g+\rho)\eta}{\alpha(1-\alpha)(1-\tau_k)}$, we could write the growth rate as $g = \tilde{b}_1 y(1-\tau_k) + \tilde{b}_2$, where $\tilde{b}_1$ and $\tilde{b}_2$ are constants. An increase in τ_k has two growth effects: (i) market size effect (y), and (ii) innovation incentive effect $(1-\tau_k)$. Arising from the main characteristics of the variety-expansion model: the monopoly power in the intermediate good sector and technology spillover, the decentralized economy has both a lower level of production as well as a lower growth rate in the steady state. Firstly, an increase in τ_k will reduce incentive for physical capital investment which will in turn decrease the steady-state growth rate, which is a negative innovation incentive effect ($(1-\tau_k)$ decreases). However, on the other hand, the corresponding decrease in τ_l will increase people's incentive to devote time into production, which will increase the growth rate, due to the positive market size effect (y increases as u increases). The positive effect through labor is limited, because the maximum amount of time used in production is limited, while the negative effect through physical capital could accumulate, the negative effect will always more than offset the positive effect, resulting in a negative relationship between the growth rate and the tax on physical capital income.
Our results accord with those widely accepted studies which claimed that there should be a negative relationship between taxation and growth (Barro (1990, 1991), Gordon and Lee (2006)). However, they are different from Aghion et al. (2013) in which the relationship between the growth rate and the tax on physical capital income is not always negative. The differences arise from the fact that intermediate goods are monopolized. Aghion et al. (2013) examined the optimal taxation problem in a Schumpeterian growth model, in which R&D could be over-invested or under-invested by monopoly power, while in our model with variety expansion, the R&D will always be under-invested, thus, it will be optimal to subsidize the physical capital which is the only input in intermediate goods sector.

3.3 Numerical results

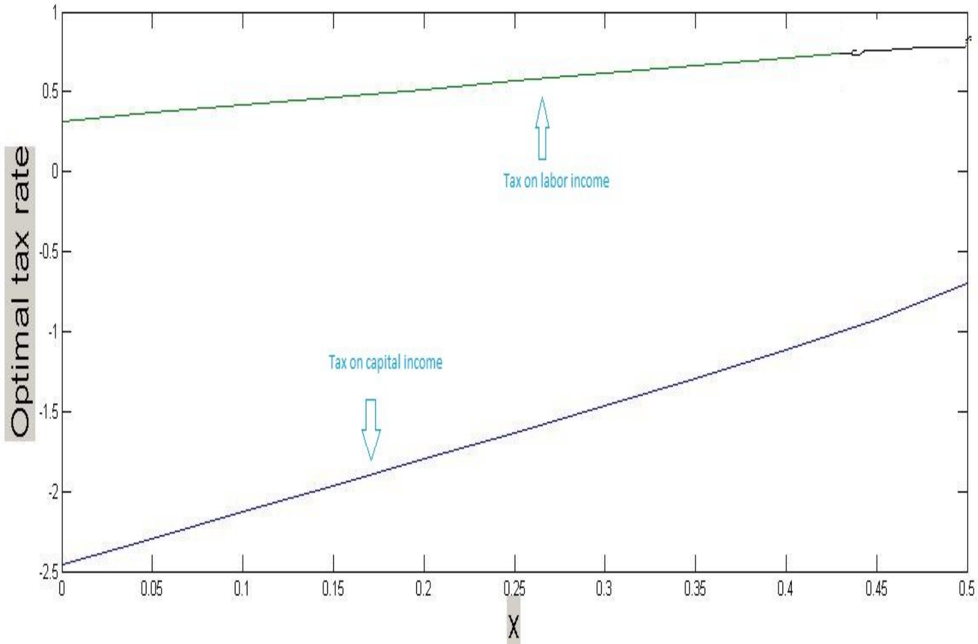
Next we perform numerical simulations for the model. The parameter values we require are preference parameters (ϵ, σ, ρ), and technology parameters (α, A, η). We first pin down a bench mark economy by choosing the values of the parameters, and then examine growth and welfare effects of the taxes in the decentralized economy.

First we choose the values of the time preference ρ to be 0.05 and contribution of an intermediate good $\alpha = 0.3$ (implying a conventional labor's share of 0.7) based on the growth calibration exercises in Lucas (1990), King and Rebelo (1990), and Stokey and Rebelo (1995). We follow Zeng and Zhang (2007) to set the elasticity of marginal utility σ to be 1.5.

Following Prescott (1986), we choose, the elasticity of labor supply ε, to be 0.5, leading to the representative agent spends about 60 per cent of his available time working. The last two parameters, final good productivity A and innovation cost η ($A = 1$, $\eta = 1.3$, $g = 0.0304$) are chose to generate a growth rate of about 3.0% (the average growth rate in the United States for the last 30 years.) All values of the parameters meet the requirements in Proposition 1.

Table 2.1 and Figure 2.2 report the optimal tax rates (τ_k and τ_l) for various government expenditure shares.[12]

[12]We consider the range of x between 0% to 50%, since based on the observation, total United States government expenditure's share on GDP raised from 8% in 1929 to around 37% in 2011. We believe the range we choose for the value of x reflects the reality.



2.2 Optimal tax on physical capital benchmark

Table 2.1: Optimal taxes for various government expenditure shares

x	Optimal tax on capital	Optimal tax on labor
0	-2.4600	0.3163
0.05	-2.2949	0.3665
0.1	-2.1298	0.4167
0.15	-1.9645	0.4669
0.2	-1.7990	0.5170
0.25	-1.6329	0.5671
0.3	-1.4654	0.6170
0.35	-1.2950	0.6665
0.4	-1.1181	0.7152
0.45	-0.9263	0.7619
0.5	-0.6994	0.8042

Observations are summarized in Result 1.

Result 1: *The optimal tax on capital is always negative. In addition, the optimal tax on capital will increase with the share of government expenditure.*

In a variety expansion growth model, the optimal tax on capital is always negative, which is financed by a positive tax on labor income (which also finances the government expenditure). The basic trend of Figure 2.2 is the same as the Figure 1 in Aghion et al. (2013). However when the government expenditure's share on the final output is large enough, they claim a positive optimal capital tax rate which is in contrast with the negative optimal capital tax rate in our model. The reason is that in variety-expansion models, innovation activities are always under-invested. But in the quality-ladder models, the innovation activities could be under-invested or over-invested.

At the meantime, as the government expenditure share x increases, the optimal tax on capital gradually increases (the subsidy on capital gradually decreases). Since the tax on labor must grow with the increase in the size of government expenditure, so does the tax distortion. Thus, we need to reduce the subsidy to capital to offset this negative impact. In the above analysis, the distortion of labor income tax is limited and will not accumulate, however, if there is human capital accumulation (instead of labor) which will accumulate, the results concerning the optimal tax on capital may change. The optimal tax problem in an extended model is the main objective of the next section.

4. Extension: Model with human capital

This section extends the model to an economy with human capital accumulation. There are several differences between the two models, as discussed in details in the following subsections.

4.1 Technologies

4.1.1 Final good production

A final good producer only uses a continuum of intermediate goods and a fixed factor as its inputs subject to the following Cobb-Douglas production function, we eliminates labor as a factor in final goods production.

$$Y = AF^{1-\alpha}\int_0^N {z_i}^{\alpha} di, \quad A > 0, \quad 0 \ < \alpha < 1,$$

where F is the quantity of the fixed factor. For simplicity, we normalize the quantity of the fixed factor to unity ($F = 1$). As a result, the final good production function could be rewritten as

$$Y = A\int_0^N {z_i}^{\alpha} di, \quad A > 0, \quad 0 \ < \alpha < 1.$$

4.1.2 Intermediate good production

In addition, each intermediate monopolist i uses both physical (k_i) and human capital (m_i) to produce an intermediate good according to

$$z_i = k_i^{\gamma} m_i^{1-\gamma}, \quad 0 < \gamma < 1, \tag{107}$$

where γ measures the contribution of physical capital to the intermediate good production. Each intermediate good producer chooses the amounts of physical and human capital to maximize its profit, given the wage rate (w) and interest rate (r).

$$\pi_i = p_i z_i - w m_i - r k_i.$$

The solution to the maximization problem gives the demand functions for k_i and m_i. Similarly we find that all the intermediate good producers will produce the same quantity, so we have $z_i = z$ and $\pi_i = \pi$. Re-writing $Y = AN^{1-\alpha}K^{\alpha\gamma}(uH)^{\alpha(1-\gamma)}$, we obtain the following equations,

$$m = \frac{\alpha^2(1-\gamma)Y}{wN}, \tag{108}$$

$$k = \frac{\alpha^2\gamma Y}{rN}, \tag{109}$$

$$z = \left(\frac{Y}{AN}\right)^{\frac{1}{\alpha}}, \tag{110}$$

$$\pi = \frac{\alpha(1-\alpha)Y}{N}. \tag{111}$$

Thus, in the extended model, the aggregate profit $N\pi$ is still proportional to the aggregate final output Y, which is exactly the same as in the basic model.

4.1.3 Innovation

The behavior of an R&D firm is the same as in the model without human capital. As a result, we have $V = \frac{\pi}{r} = \eta$ holds true both in and outside the steady state.

4.2 Households

The representative household is endowed with 1 unit of time which is inelastically allocated among intermediate good production u, human capital accumulation v and leisure l $(= 1 - u - v)$. The household has the same utility function, as in equation (81).

The household accumulates human capital H according to

$$\dot{H} = BvH, \tag{112}$$

where B is a productivity parameter and v is time input in education.

The representative household has the following budget constraint

$$C = (1 - \tau_k)rK + (1 - \tau_h)wuH - \dot{K} + D + P_F, \tag{113}$$

where τ_k and τ_h are now taxes on physical capital income and human capital income. And P_F is the fixed factor (F) income, which equals to $(1 - \alpha)Y$.

The representative household chooses consumption C, the time allocation u and l to maximize its life-time utility, subject to the human capital accumulation technology and the budget constraint. The current-value Hamiltonian function for this optimization problem is

$$\mathcal{L} = \frac{(Cl^{\varepsilon})^{1-\sigma} - 1}{1 - \sigma} + \lambda_1 B[(1 - u - l)H]$$

$$+\lambda_2[(1 - \tau_k)rK + (1 - \tau_h)wuH - C + D + P_F], \tag{114}$$

where λ_{1t} and λ_{2t} are respectively the costate variables associated with equations (112) and (113). The first-order conditions for this optimization problem are equations (112), (113) and the following conditions

$$C: \quad C^{-\sigma} l^{\epsilon(1-\sigma)} = \lambda_2, \tag{115}$$

$$l: \quad \epsilon C^{1-\sigma} l^{\epsilon(1-\sigma)-1} = \lambda_1 BH, \tag{116}$$

$$u: \quad \lambda_1 BH = \lambda_2[(1 - \tau_h)wH], \tag{117}$$

$$H: \quad \lambda_1 B(1 - u - l) + \lambda_2(1 - \tau_h)wu = -\dot{\lambda}_1 + \rho\lambda_1, \tag{118}$$

$$K: \quad \lambda_2 r(1 - \tau_k) = -\dot{\lambda}_2 + \rho\lambda_2, \tag{119}$$

$$TVC_H: \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{1t}H_t = 0, \tag{120}$$

$$TVC_K: \quad \lim_{t\to\infty} e^{-\rho t}\lambda_{2t}K_t = 0, \tag{121}$$

where λ_{1t},λ_{2t} are the costate variables. Equation (115) (respectively (116),(117)) equalizes the marginal benefit and marginal cost of consumption (respectively time allocated to leisure, time allocated to production). Equations (119) and (121) (respectively (118), (120)) are the optimal dynamic conditions for physical capital (respectively, human capital) accumulation.

4.3 Decentralized equilibrium

We now derive the equilibrium conditions. Using the physical and human capital market clearing conditions, i.e., $\int_0^N k_i di = Nk = K$ and $\int_0^N m_i di = Nm = uH$, we have $k = K/N$, $m = uH/N$ and $z = K^{\gamma}(uH)^{1-\gamma}/N$. Thus Y could be expressed as $Y = AN^{1-\alpha}K^{\alpha\gamma}(uH)^{\alpha(1-\gamma)}$. As usual, there is final good clearing condition,

$$Y = C + \dot{K} + \eta\dot{N} + G, \tag{122}$$

which is the same as in the basic model. From equations (115) and (116), we can easily get the relationship between C and l,

$$C = l(1-\tau_h)wH/\epsilon. \tag{123}$$

Similar to the model without human capital, we combine $\frac{\pi}{r} = \eta$ and equation (111) to obtain $k = \frac{\alpha\gamma\eta}{1-\alpha}$, which is constant. [13]Thus, we have $\frac{\dot{K}}{K} = \frac{\dot{N}}{N}$. Using this condition, along with the final good market clearing condition, we have

$$\dot{N} = \frac{[(1-x)Y - C]\, N}{K + N\eta}, \tag{124}$$

$$\dot{K} = \frac{[(1-x)Y - C]\, K}{K + N\eta}. \tag{125}$$

From physical and human capital markets clearing conditions, we obtain

$$r = \alpha^2\gamma AN^{1-\alpha}(uH)^{\alpha(1-\gamma)}K^{\alpha\gamma-1},$$

[13]Refer to the details in footnote 3

and

$$w = \alpha^2(1-\gamma)AN^{1-\alpha}(uH)^{(\alpha(1-\gamma)-1)}K^{\alpha\gamma}. \tag{126}$$

Thus, not only the interest rate is less than the marginal product of physical capital, but also the wage rate is less than the marginal product of human capital. Thus, there is less than optimal demand for both physical capital and human capital. In addition, we observe that $rK = \alpha^2\gamma Y$, and $wuH = \alpha^2(1-\gamma)Y$, i.e. because of the existence of monopoly, the sum of three factors income $rK + wuH + P_F = (1-\alpha+\alpha^2)Y$ does not equal to Y. The remaining $\alpha(1-\alpha)Y$ is the monopoly profit. Plugging the value of w into equation (123), we have

$$C = \alpha^2 l(1-\tau_h)(1-\gamma)Y/(u\epsilon). \tag{127}$$

Obviously, consumption is again proportional to output. The ratio of consumption to output depends negatively on the amount of time allocated into production, and the tax rate on human capital income and positively on the amount of time allocated into leisure.

Using equations (115) and (116), we have

$$\begin{aligned}\dot{u} = \left[\frac{u}{1-\alpha(1-\gamma)}\right]\Bigg\{&[1-\alpha(1-\gamma)]\left(\frac{[(1-x)Y-C]}{K+N\eta}\right) - \frac{(1-\tau_k)\alpha^2\gamma Y}{K} \\ &+B\left[u+\alpha(1-\gamma)(1-u-l)\right]\Bigg\}, \end{aligned} \tag{128}$$

$$\dot{l} = \frac{l}{\sigma-\epsilon(1-\sigma)}\left[\sigma B u - \rho - \alpha^2\gamma Y/K(\sigma-1)(1-\tau_k)\right]. \tag{129}$$

The dynamics of the decentralized economy are then characterized by the system of equations (112), (122), (124), (125), (127),(128) and (129), along with an initial condition (H_0, K_0, N_0) and the transversally conditions equations (120) and (121).

4.3.1 Steady state analysis

We will first focus on the steady-state balanced growth equilibrium, where the number of intermediate goods N, human capital H and physical capital K, consumption C, and final output Y will grow at the same rate g. While time allocation (l and u) and factor prices (r and w) will remain constant. We could reduce the system to two equations describing the balanced growth path. Since $\frac{\dot{H}}{H} = g$, from equation (112), we have $g = Bv = B(1-u-l)$. Since $rK = \alpha^2\gamma Y$ and $\frac{\dot{N}}{N} = \frac{[(1-x)Y-C]}{K+N\eta} = g$, setting $\dot{l} = \dot{u} = 0$, we have

$$\sigma g + \rho = B(1-l), \tag{130}$$

and

$$r = \frac{\sigma g + \rho}{1-\tau_k}. \tag{131}$$

Equation (130) is the first equation from the condition: in steady state all sectors will have the same rate of return. Next re-writing $rK = \alpha^2\gamma Y$ and recalling $k = \frac{\alpha\gamma\eta}{1-\alpha}$, we have

$$y = \frac{(\sigma g + \rho)\eta}{\alpha(1-\alpha)(1-\tau_k)}. \tag{132}$$

This expression is the same as in the basic model. Equation (132) implies that a capital income tax has two effects on the growth rate as well, i.e., a market effect and an innovation incentive effect. Next from equations (123) and (126) (setting $\zeta_c = \frac{C}{Y}$), we have

$$\zeta_c = \frac{\alpha^2(1-\gamma)(1-\tau_h)l}{\varepsilon u}. \tag{133}$$

Thus, dividing both sides of the final good market clearing condition by Y, we have

$$\alpha(\alpha\gamma + 1 - \alpha)(1-\tau_k)\frac{g}{\sigma g + \rho} = 1 - x - \zeta_c. \tag{134}$$

Equation (134) is the second equation from the final good clearing condition.

Equations (130) and (134) will determine growth rate g and leisure l in steady state in decentralized economy. Combining equations (130) and (134), we obtain

$$\alpha(\alpha\gamma + 1 - \alpha)(1-\tau_k)\frac{g}{\sigma g + \rho} = 1 - x - \zeta_c, \tag{135}$$

where

$$\zeta_c = \frac{\alpha^2(1-\gamma)(1-\tau_h)[B-(\sigma g + \rho)]}{\varepsilon[(\sigma - 1)g + \rho]}.$$

Equation (135) determines the steady-state growth rate g.

4.4 Existence of the equilibrium

The existence and uniqueness of the steady-state equilibrium are given by

Proposition 3: *If (i)* $B > \rho[1 + \frac{\varepsilon(1-x)}{\alpha^2(1-\gamma)(1-\tau_h)}]$ *and (ii)* $\sigma > 1$ *then there always exists a unique positive growth rate.*

Proof. We will use Figure 2.3 for the curves of both LHS and RHS of equation (135) to show the existence of an positive growth rate. Firstly, both LHS and RHS are positive monotonic functions with g. When $g = \underline{g}$, we have $\zeta_c = 1 - x$, thus $RHS = 0 < LHS$, where $\underline{g} = \frac{\alpha^2(1-\gamma)(B-\rho)(1-\tau_h)-(1-x)\epsilon\rho}{(1-x)\epsilon(\sigma-1)+\alpha^2(1-\gamma)(1-\tau_h)\sigma}$. When $g = \overline{g} = \frac{B-\rho}{\sigma}$, we have $\zeta_c = 0$. Moreover, we have $\underline{g} > 0$ as long as the $B > \rho[1+\frac{\varepsilon(1-x)}{\alpha^2(1-\gamma)(1-\tau_h)}]$ holds, and $RHS = 1 > LHS = \alpha(\alpha\gamma+1-\alpha)(1-\tau_k)\frac{B-\rho}{B\sigma}$ as long as condition (ii) holds. *Q.E.D.*

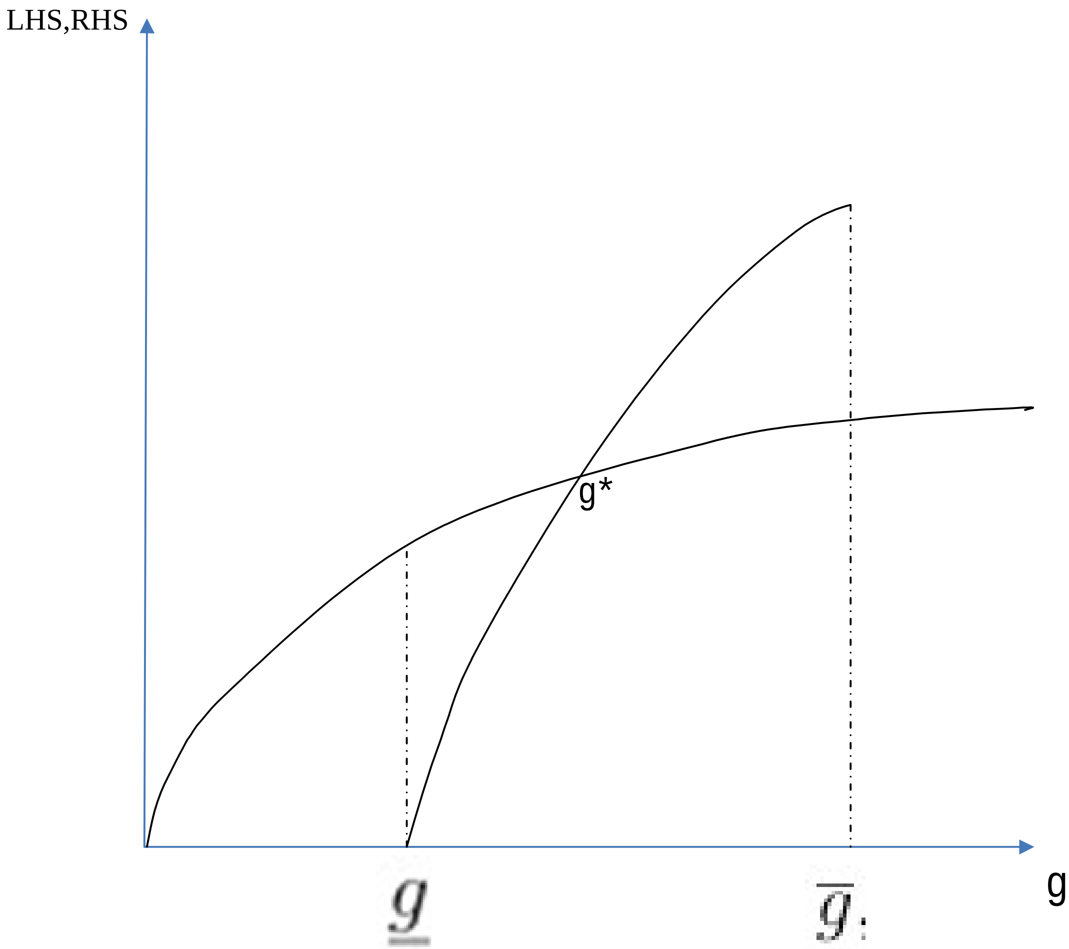
Condition (i) in this proposition is equivalent to the condition that $V > \eta$. Consider the economy in a steady state with no growth ($g = 0$). When $g = 0$, we have $v = 0$, $l = \frac{(1-x)\varepsilon}{\alpha^2(1-\gamma)(1-\tau_h)+(1-x)\varepsilon}$, $u = \frac{\alpha^2(1-\gamma)(1-\tau_h)}{\alpha^2(1-\gamma)(1-\tau_h)+(1-x)\varepsilon}$, $r = \frac{\rho}{1-\tau_k}$, $w = \frac{\rho(1-\tau_k)[1+\frac{\varepsilon(1-x)}{\alpha^2(1-\gamma)(1-\tau_h)}]}{B}$. These results give

$$\pi = \frac{B\eta}{(1-\tau_k)[1+\frac{\varepsilon(1-x)}{\alpha^2(1-\gamma)(1-\tau_h)}]},$$

and

$$V = \frac{B\eta}{\rho(1+\frac{\varepsilon(1-x)}{\alpha^2(1-\gamma)(1-\tau_h)})}.$$

Then the condition $V > \eta$ leads to $B > \rho[1+\frac{\varepsilon(1-x)}{\alpha^2(1-\gamma)(1-\tau_h)}]$. Since the marginal private benefit of investing in R&D is greater than its marginal cost, it is optimal for the profit-maximizing R&D firms to invest in R&D until $V = \eta$.



2.3 Steady state growth rate in the extended model

This condition can be guaranteed by various sufficient conditions concerning the values of the technology and preferences such as a sufficiently low subjective discount rate (low value of ρ), a sufficiently productive human capital accumulation technology (high value of B), a sufficiently high elasticity of marginal utility and a sufficiently low elasticity of the flow of utility with respect to leisure (low value of ε).[14](high value of σ)

4.5 Government's problem

The government will now choose τ_k and τ_h to maximize the life-time utility for the representative consumer equation (81), such that the optimal conditions equations (130) and (134) and the following government's balanced budget condition are satisfied,

$$\tau_k rK + \tau_h wuH = G = xY,$$

where the left-hand side is the sum of tax revenue from physical capital and human capital income, and the right-hand side is the government expenditure. Since $rK = \alpha^2\gamma Y$, and $wuH = \alpha^2(1-\gamma)Y$, the above equation could be reduced to

$$x = \alpha^2\tau_k\gamma + \alpha^2\tau_h(1-\gamma). \tag{136}$$

In order to make consumption (C) positive, we should have $\frac{x-\alpha^2(1-\gamma)}{\alpha^2\gamma} < \tau_k$. Also, we should have $\tau_k < 1$. To compare the results in the extended model with those in the basic model, we consider the cases with $x < \alpha^2(1-\gamma)$. [15]

4.6 Effects of taxation

To examine the impact of taxation on the economy, we first use the government's budget constraint equation (136)) to express τ_h as a function of x and τ_k and then rewrite the

[14]This elasticity is referred to as the coefficient of relative risk aversion.

[15]The basic model, the optimal capital income tax is negative. When $x > \alpha^2(1-\gamma)$, τ_k has to be positive since $\tau_h \leq 1$. Such values of parameters will give us an ambiguous comparison of the sign of optimal physical capital tax. Thus, we rule out such combinations of parameters.

equilibrium condition equation (135) as,

$$\alpha(\alpha\gamma+1-\alpha)(1-\tau_k)\frac{g}{\sigma g+\rho}=1-x-\frac{(\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k)[B-(\sigma g+\rho)]}{\varepsilon[(\sigma-1)g+\rho]}. \quad (137)$$

We need to firstly find the valid range of τ_k. In order to satisfy the condition for existence of a unique positive growth rate as in Proposition 3, we need $B>\rho[1+\frac{(1-x)\varepsilon}{\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k}]$ or equivalently $\frac{x-\alpha^2(1-\gamma)+\frac{(1-x)\epsilon\rho}{B-\rho}}{\alpha^2\gamma}<\tau_k$. In addition as the condition for a positive value of consumption, we need $\frac{x-\alpha^2(1-\gamma)}{\alpha^2\gamma}<\tau_k<1$. In summary we should have $\underline{\tau}_k=\frac{x-\alpha^2(1-\gamma)+\frac{(1-x)\epsilon\rho}{B-\rho}}{\alpha^2\gamma}<\tau_k<1$.

4.6.1 Effect on growth

We start to examine the gross effect of taxation on growth rate. To see the effect of taxation on growth, we re-write equation (137) as a function of only g and τ_k

$$G(g,\tau_k)=\frac{1-x-\frac{[\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k][B-(\sigma g+\rho)]}{\varepsilon[(\sigma-1)g+\rho]}}{1-\tau_k}-\alpha(\alpha\gamma+1-\alpha)\frac{g}{\sigma g+\rho}=0. \quad (138)$$

From equation (138), we use explicit differentiation to see the relationship of τ_k and g by getting the first order partial differentiation of equation (138) with respect to the both variables. Firstly we have,

$$\frac{\partial G}{\partial\tau_k}=\frac{1-x-\frac{(\alpha^2-x)[B-(\sigma g+\rho)]}{\varepsilon[(\sigma-1)g+\rho]}}{(1-\tau_k)^2},$$

and combining equation (137), we re-write it as

$$\frac{\partial G}{\partial\tau_k}=\frac{\alpha\left[(\alpha^2-x)(\alpha\gamma+1-\alpha)\frac{g}{\sigma g+\rho}-\alpha\gamma(1-x)\right]}{(1-\tau_k)[\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k]}. \quad (139)$$

With respect to g, from equation (138), we derive

$$\frac{\partial G}{\partial g}=\frac{[\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k]\,[(\sigma-1)B+\rho]}{\epsilon(1-\tau_k)\,[(\sigma-1)g+\rho]^2}-\frac{\alpha(\alpha\gamma+1-\alpha)\rho}{(\sigma g+\rho)^2}. \quad (140)$$

We will now examine the growth effects using the two equations, reported in Proposition 4-6 as follows.

Proposition 4: Within the valid range of τ_k, there does not exist an interior growth-maximizing capital income tax rate.

Proof. Since $\frac{dg}{d\tau_k} = -\frac{\frac{\partial G}{\partial \tau_k}}{\frac{\partial G}{\partial g}}$, if there is an interior solution, we will have $\frac{\partial G}{\partial \tau_k} = 0$. From equations (137) and (139), we could not find a value of τ_k such that $\frac{\partial G}{\partial \tau_k} = 0$ (When $\frac{\partial G}{\partial \tau_k} = 0$, $\tau_k = \frac{x-\alpha^2(1-\gamma)}{\alpha^2\gamma} < \underline{\tau}_k$). *Q.E.D.*

Denote $\underline{\sigma} \equiv \frac{\left[(B-\rho)(\alpha^2-x)-(1-x)\epsilon\rho\right](1-\alpha)(\alpha^2-x-\alpha\gamma-\alpha^2\gamma)}{\alpha\gamma B(1-x)(\alpha^2-x)} + \frac{B-\rho}{B}$, we have the following propositions:

Proposition 5: *If $\sigma > \underline{\sigma}$, then an increase in the tax rate on physical capital will increase the growth rate; when the value of τ_k increases and approaches* 1*, the growth rate will increase and converge to $g^* = \frac{(B-\rho)(\alpha^2-x)-(1-x)\epsilon\rho}{\sigma(\alpha^2-x)+(1-x)\epsilon(\sigma-1)}$.*

Proof. Since we have already shown that there is no interior solution, it is sufficient to show that $\frac{dg}{d\tau_k} > 0$ at $\tau_k = \underline{\tau}_k$. Plugging $\underline{\tau}_k = \frac{x-\alpha^2(1-\gamma)+\frac{(1-x)\epsilon\rho}{B-\rho}}{\alpha^2\gamma}$ into equation (137), we have $\frac{g}{\sigma g+\rho} = 1 - \frac{\alpha\gamma(1-x)[B(\sigma-1)+\rho]}{[(B-\rho)(\alpha^2-x)-(1-x)\epsilon\rho](\alpha\gamma+1-\alpha)} \equiv \underline{\Phi}$. If $\tau_k = \underline{\tau}_k$, we have (a) $\frac{\partial G}{\partial g} = \frac{\alpha\rho(\alpha\gamma+1-\alpha)(1-\underline{\Phi})\underline{\Phi}}{[(\sigma-1)g+\rho]^2} > 0$; (b) $\frac{\partial G}{\partial \tau_k} < 0$, if $\sigma > \underline{\sigma}$ holds. Thus $\frac{dg}{d\tau_k} > 0$. *Q.E.D.*

Proposition 6: *If $\sigma < \underline{\sigma}$, then an increase in the tax rate on physical capital will decrease the growth rate, thus maximum growth rate g^* happens at $\underline{\tau}_k$, and $g^* = \frac{\underline{\Phi}\rho}{1-\sigma\underline{\Phi}}$.*

Proof. The proof is parallel to that of Proposition 6. *Q.E.D.*

4.6.2 Effect on human capital

Next we will decompose the gross effect to examine the effect of taxation on human capital supply ("the market size effect"). In the extended model, the human capital stock is $m = uH/N$. This effect is worth noting since we could always rewrite Y as $Y = AN(\frac{K}{N})^{\alpha\gamma}(\frac{uH}{N})^{\alpha(1-\gamma)}$ and $y = \frac{Y}{N} = Ak^{\alpha\gamma}m^{\alpha(1-\gamma)}$. Since k has been shown to be constant, output to productivity y depends positively on human capital supply m which is the market size effect. In addition, since $V = \frac{\pi}{r} = \eta$ holds true as well both in and outside the steady state and the profit to productivity (π) is proportional to the final output to productivity (y) by equation (111), we have a positive relationship not only between the human capital supply m and y but also between m and π and m and r.

Proposition 7: *If $\sigma > 1$, then an increase in the tax rate on physical capital will increase human capital supply, the final output to productivity as well as the gross interest rate.*

Proof. We have shown that there is a positive relationship between m and τ_k if and only if there is a positive relationship between r and τ_k. So we only need to prove that $r = \frac{\sigma g+\rho}{1-\tau_k}$ will increase with τ_k, from which we have $g = \frac{r(1-\tau_k)-\rho}{\sigma}$. We could re-write equation (137) as

$$1 - x + \frac{\alpha(\alpha\gamma + 1 - \alpha)\rho}{\sigma r} = \frac{\sigma(\alpha^2(1-\gamma) - x + \alpha^2\gamma\tau_k)[B - r(1-\tau_k)]}{\varepsilon[(\sigma-1)r(1-\tau_k) + \rho]}$$
$$+\frac{\alpha(\alpha\gamma + 1 - \alpha)(1-\tau_k)}{\sigma}.$$

Both the LHS and the RHS will decrease with r and the LHS does not depend on τ_k, so we only need to prove that $\frac{\partial RHS}{\partial \tau_k} > 0$, where

$$\frac{\partial RHS}{\partial \tau_k} = \frac{\alpha^2\sigma\gamma[B - r(1-\tau_k)]}{\epsilon[(\sigma-1)r(1-\tau_k) + \rho]} + \frac{\sigma(\alpha^2(1-\gamma) - x + \alpha^2\gamma\tau_k)r[\rho + (\sigma-1)B]}{\epsilon[(\sigma-1)r(1-\tau_k) + \rho]^2}$$
$$-\frac{\alpha(\alpha\gamma + 1 - \alpha)}{\sigma}$$

If Proposition 5 holds true, i.e. $\frac{dg}{d\tau_k} > 0$, then it is obvious that $r = \frac{\sigma g+\rho}{1-\tau_k}$ will increase with τ_k. If Proposition 6 applies, i.e., $\frac{dg}{d\tau_k} < 0$, then we have $\frac{\partial RHS}{\partial \tau_k} > 0$, because $\frac{\alpha^2\sigma\gamma[B-r(1-\tau_k)]}{\epsilon[(\sigma-1)r(1-\tau_k)+\rho]}$, which decreases with g will be greater than $\frac{\alpha(\alpha\gamma+1-\alpha)}{\sigma}$ at $g^* = \frac{\Phi\rho}{1-\sigma\underline{\Phi}}$, which is the maximum value of g. *Q.E.D.*

Intuitively, the increase in the tax rate on physical capital will decrease the tax rate on human capital given any required government budget. Thus, the consumer has incentive to supply more human capital into production. Thus, the tax on physical capital always has a positive market size effect on the growth rate. In summary the net sign of the effect of physical capital income taxation on growth depends on the value of the elasticity of marginal utility (σ). [16] Thus, for a high (low) enough value of σ, the tax on physical capital income will increase (decrease) growth. This is because for $\sigma > 1$ in CRRA utility function, with a higher value of σ, the tax will provoke a stronger positive effect on employment and on the gross interest rate, which is a stronger positive market size effect on growth.

The tax on physical capital has two opposite effects on the growth: the market size effect which has been shown to be positive, that is, physical capital income tax will increase the human capital supply and thus increase the total market size to promote growth and the negative innovation incentive effect, leading to a lower growth rate. Their relative magnitude determines the sign of the growth effect. Propositions 6 and 7 imply that the sign mainly depends on several important parameters: the elasticity of marginal utility (σ), capital share (γ) and labor supply elasticity (ϵ). Firstly, the elasticity of marginal utility (σ) will enforce the positive market size effect, as a result of which, physical capital taxation is more effective than human capital taxation in promoting growth with a high value of σ. Also, from equation (136), we have already shown that, a 1% increase in physical capital tax will decrease the human capital tax by $\frac{\gamma}{1-\gamma}$%, because $x = \alpha^2\gamma\tau_k + \alpha^2(1-\gamma)\tau_h$. As a result, with a higher γ the same amount of increase in physical capital income tax leads to a higher amount of decrease in the human capital income tax (a stronger market size effect). Lastly with a higher value of labor supply elasticity (ϵ) human capital taxation becomes more distortional. In summary, with a high value of elasticity of marginal utility (σ), a high capital share (γ) and a high value of labor supply elasticity (ϵ), the total effect of physical capital taxation on growth is positive. As a result, increasing in the physical capital tax rate stimulate growth.

[16] Our results are consistent with the proposition 6 in Long and Pelloni (2011) which is noticed by us until the completion stage of this chapter.

4.6.3 Effect on welfare

First we could express the welfare in the steady state as

$$W = \frac{(cl^{\epsilon})^{1-\sigma}}{(1-\sigma)(\rho - g + g\sigma)} - \frac{1}{\rho(1-\sigma)}.$$

The welfare function is in terms of (τ_k, g, x), because both c and l are functions of (τ_k, g. x).

$$c = \frac{(1-x)(\sigma g + \rho)\eta}{\alpha(1-\alpha)(1-\tau_k)} - \frac{g\eta(\alpha\gamma + 1 - \alpha)}{1-\alpha},$$

$$l = 1 - \frac{\sigma g + \rho}{B}.$$

Differentiating W with respect to τ_k, we obtain the total effect of physical capital income tax on welfare

$$\frac{dW}{d\tau_k} = \frac{\partial W}{\partial g}\frac{dg}{d\tau_k} + \frac{\partial W}{\partial c}\frac{dc}{d\tau_k} + \frac{\partial W}{\partial l}\frac{dl}{d\tau_k}$$

where the first term is the growth effect, and the last two terms are the level effect. For simplicity in calculation we could rewrite

$$\frac{dW}{d\tau_k} = \left(\frac{\partial W}{\partial c}\frac{\partial c}{\partial g} + \frac{\partial W}{\partial l}\frac{\partial l}{\partial g} + \frac{\partial W}{\partial g}\right)\frac{dg}{d\tau_k} + \frac{\partial W}{\partial c}\frac{\partial c}{\partial \tau_k}.$$

Firstly, it is easy to get $\frac{\partial W}{\partial c} = \frac{c^{-\sigma} l^{\epsilon(1-\sigma)}}{\rho - g + g\sigma} > 0$, and $\frac{\partial c}{\partial \tau_k} = \frac{(1-x)(\sigma g + \rho)\eta}{\alpha(1-\alpha)(1-\tau_k)^2} > 0$. In addition, we derive

$$\frac{\partial c}{\partial g} = \frac{(1-x)\sigma\eta}{\alpha(1-\alpha)(1-\tau_k)} - \frac{\eta(\alpha\gamma + 1 - \alpha)}{1-\alpha}, \tag{141}$$

$$\frac{\partial W}{\partial l} = \frac{\epsilon c^{1-\sigma} l^{\epsilon(1-\sigma)-1}}{\rho - g + g\sigma},$$

$$\frac{\partial l}{\partial g} = -\frac{\sigma}{B}, \tag{142}$$

and

$$\frac{\partial W}{\partial g} = \frac{c^{1-\sigma} l^{\epsilon(1-\sigma)}}{(\rho - g + g\sigma)^2}. \tag{143}$$

Thus we could simplify $\frac{dw}{d\tau_k}$ to,

$$\frac{dw}{d\tau_k} = \frac{c^{-\sigma} l^{\epsilon(1-\sigma)}}{(\rho - g + \sigma g)} \left\{ \left(\frac{\partial c}{\partial g} + \frac{\epsilon c}{l} \frac{\partial l}{\partial g} + \frac{c}{\rho - g + g\sigma} \right) \frac{dg}{d\tau_k} + \frac{\partial c}{\partial \tau_k} \right\}.$$

Denoting $\bar{W} = \left(\frac{\partial c}{\partial g} + \frac{\epsilon c}{l} \frac{\partial l}{\partial g} + \frac{c}{\rho - g + g\sigma} \right) \frac{dg}{d\tau_k} + \frac{\partial c}{\partial \tau_k}$, we have sign $(\frac{dW}{d\tau_k})$= sign $(\bar{W})$.

Proposition 8: *If $\sigma > 1 + \frac{1-\alpha}{\alpha\gamma}$, then an increase in the tax rate on physical capital will increase the welfare, i.e., $\frac{dW}{d\tau_k} > 0$.*

Proof. (a) According to Proposition 4, $r = \frac{\sigma g + \rho}{1 - \tau_k}$ will increase with τ_k. Thus, we have $\frac{dr}{d\tau_k} = \frac{\partial r}{\partial g} \frac{dg}{d\tau_k} + \frac{\partial r}{\partial \tau_k} > 0$. As a result of that, if $\frac{dg}{d\tau_k} < 0$, $|\frac{dg}{d\tau_k}| < \frac{\sigma g + \rho}{\sigma(1 - \tau_k)}$.

(b) From equations (132), (133), (141), (142) and (143), we have

$$\frac{\partial c}{\partial g} + \frac{\epsilon c}{l} \frac{\partial l}{\partial g} + \frac{c}{\rho - g + g\sigma} = \frac{\eta \Omega}{\alpha(1-\alpha)(1-\tau_k)},$$

where

$$\Omega = (1-x)\sigma - \frac{\sigma g + \rho}{\sigma g + \rho - g} \left[(\alpha^2(1-\gamma) - x + \alpha^2 \gamma \tau_k)\sigma - 1 + x + \alpha(\alpha\gamma + 1 - \alpha)(1 - \tau_k) \right].$$

Since $\sigma > 1 + \frac{1-\alpha}{\alpha\gamma}$, Ω will decrease with τ_k. In addition, $\frac{\sigma g + \rho}{\sigma g + \rho - g} < \frac{\sigma}{\sigma - 1}$, as a result of which, we have $\Omega > 0 \Leftrightarrow \frac{\partial c}{\partial g} + \frac{\epsilon c}{l} \frac{\partial l}{\partial g} + \frac{c}{\rho - g + g\sigma} > 0$. At the meantime, when $\tau_k = \underline{\tau}_k$, we have $\Omega \leq (1-x)\sigma$, that is to say, we have $\frac{\partial c}{\partial g} + \frac{\epsilon c}{l} \frac{\partial l}{\partial g} + \frac{c}{\rho - g + g\sigma} < \frac{(1-x)\sigma\eta}{\alpha(1-\alpha)(1-\tau_k)}$.

(c) Thus, if $\frac{dg}{d\tau_k} > 0$, we have $\frac{dw}{d\tau_k} > 0$, i.e., the net effect on welfare is positive. However, if $\frac{dg}{d\tau_k} < 0$, $\left(\frac{\partial c}{\partial g} + \frac{\epsilon c}{l} \frac{\partial l}{\partial g} + \frac{c}{\rho - g + g\sigma} \right) \frac{dg}{d\tau_k}$ will be less than $\frac{(1-x)\sigma\eta}{\alpha(1-\alpha)(1-\tau_k)} \frac{\sigma g + \rho}{\sigma(1-\tau_k)} = \frac{(1-x)(\sigma g + \rho)\eta}{\alpha(1-\alpha)(1-\tau_k)^2} = \frac{\partial c}{\partial \tau_k}$. Thus, there is still a positive net effect on welfare. *Q.E.D.*

The condition $\sigma > 1 + \frac{1-\alpha}{\alpha\gamma}$ in Proposition 8 applies to economies with either a high enough value of elasticity of marginal utility σ, or a high enough value of α. Thus, the physical capital tax in more developed countries with high value of α, i.e., less contribution of fixed inputs in production, such as land, will be more likely to have a positive net effect on welfare.

However, when it comes to more general cases, the sign of $\frac{dW}{d\tau_k}$ becomes more complicated, since the equilibrium equations are highly non-linear. We do numerical simulations instead in the next section.

In summary there are two interesting new features in the model with human capital. The first one is that we have human capital taxation, which affects not only labor-leisure choice but also affect the rate of investment in human capital, generating a higher level and growth effects. The second is that the monopolized sector (intermediate goods sector) uses both kinds of capital, instead of only one. Because of this, the government should subsidize both types of capital due to the intermediate producer's lower than optimal demand. The two new features determine the different results between the basic and extended model. In summary, the government should prefer capital income tax more with a higher value of elasticity of marginal utility σ. Gandelman and Hernndez-Murillo (2013) estimates the coefficient of relative risk aversion (equivalent to the elasticity of marginal utility σ) for 80 countries using data on self-reports of personal well-being from the Gallup World Poll. The estimates range from 0.03 to 3.02. The median and simple averages of the country estimates are both equal to 0.97. The average coefficient among developing countries is 1.01, while the average coefficient among developed countries is 0.88. As a possible implication for our results, we find some countries with relative high σ has also a high rate of capital income tax, e.g., U.S. (σ=1.37, the average capital income tax between year 2009-2012 is 52.125%), and U.K. (σ=0.96, the average capital income tax between year 2009-2012 is 34.925%). While, some countries with relative low σ has also a low rate of capital income tax, e.g., Paraguay (σ=0.5, the average capital income tax between year 2009-2012 is 11.825%) and Ukraine (σ=0.38, the average capital income tax between year 2009-2012 is 11.3%). However, since the capital income tax will also be affected by the required government expenditure, as well as how many taxes could be levied, empirical results are far from conclusive. Further empirical evidence regarding the relationship of σ and τ_k could be done.

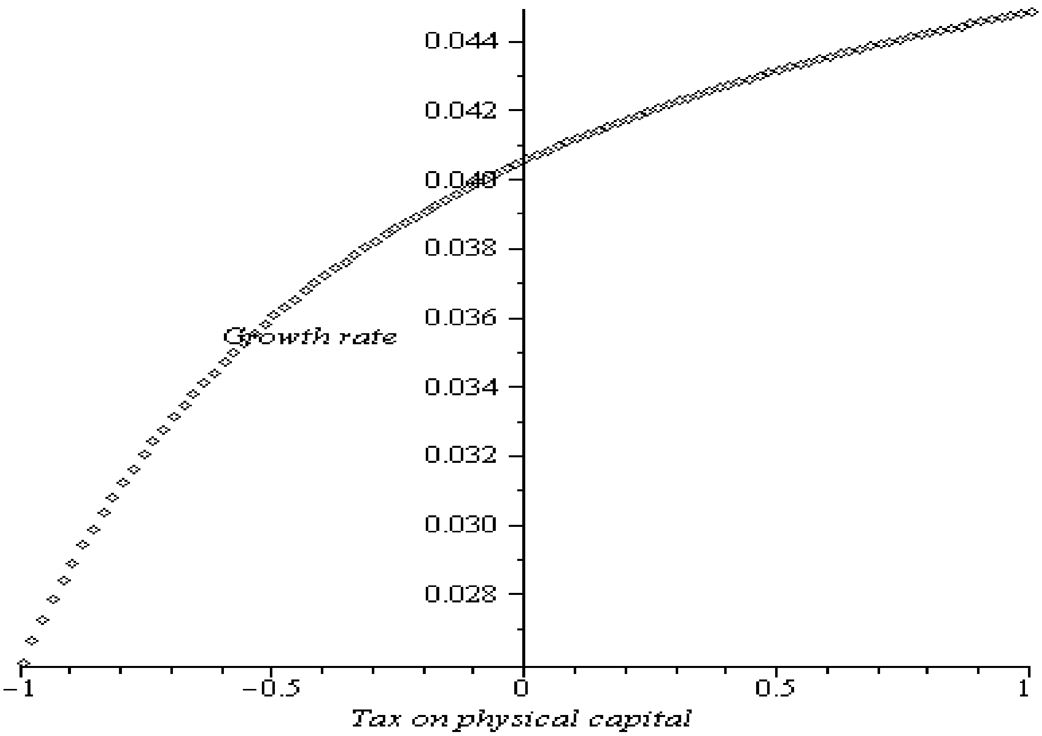
4.7 Numerical results

We now perform numerical simulations for the model. The parameter values we require are preference parameters (ϵ, σ, ρ) and technology parameters $(\alpha, \beta, \gamma, A, B, \eta)$. We first choose the values of the model's parameters to pin down a benchmark economy, and then investigate the growth and welfare effects of the taxation.

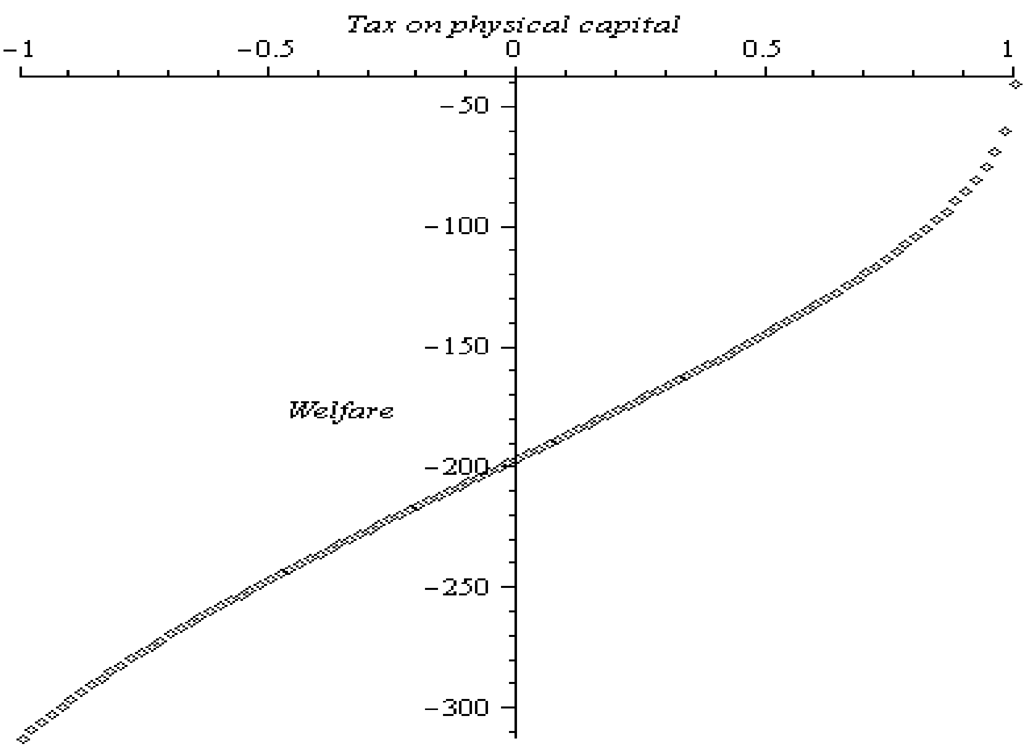
The values of σ and ρ remain the same. We choose $\alpha = 0.8$, $\gamma = 0.35$ (implying a labor's share of 0.7) and ϵ to be 0.3, leading to that the representative agent spends about 30 per cent of his available time working. The last three parameters A, B and η $(A = 1, B = 0.14, \eta = 1)$ are chose to generate the average United States growth rate. The benchmark values (Case 1) satisfy the condition in proposition 5. While to check on the sensitivity of the results, we also decrease $\gamma = 0.05$. (Parameter values in Case 2 satisfy the condition in Proposition 6.)

It is very important to restrict the tax rates to ensure a positive growth rate. Too high a distortionary tax may induce a negative growth rate. As claimed in Bond et al. (1996), if factor taxes are too distortional, there may be instability or indeterminacy. Thus, we will strictly follow the lower bound of the capital income tax.

Figure 2.4 shows the relationship between the growth rate and the physical capital income tax in Case 1, while Figure 2.5 illustrates this relationship in Case 2. Numerical results confirm proposition 5 and 6.



2.4 Growth rate and tax on physical capital, case 1



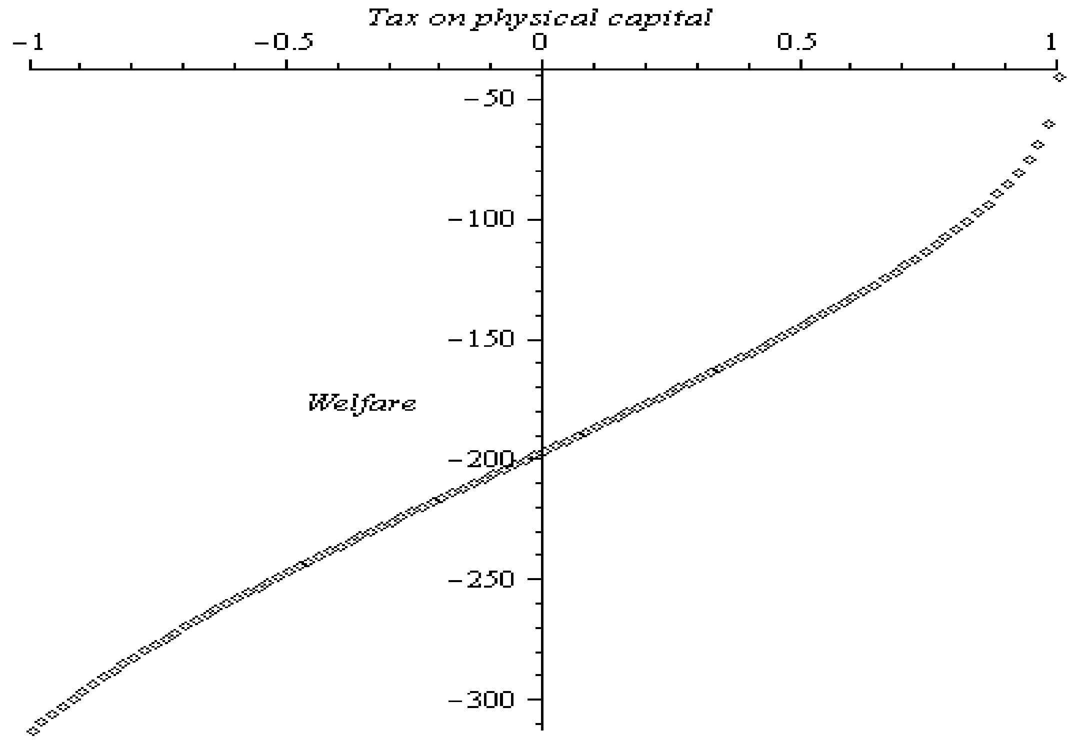
2.5 Growth rate and tax on physical capital, case 2

These results re-confirm the result in King and Rebelo (1990) that taxation can substantially affect long run growth in a two sector endogenous growth model. They also highlight the differences between the basic model and extension model. In the basic model, we show that the decrease in physical capital taxation will always increase the growth rate. While, in a model with human capital, we have both types of capital in the monopoly sector, so both types of capital should be subsidized to correct this inefficiency. Subsidizing physical (human) capital will accelerate physical (human) capital accumulation, promote aggregate activity and stimulate innovation activities, and in turn promote growth. However, since at least one of them should be positive to finance the exogenous government expenditure, now it is a trade-off between the two capital taxes. Thus, the relationship between growth and the physical capital income tax could be positive or negative depending on the values of parameters.

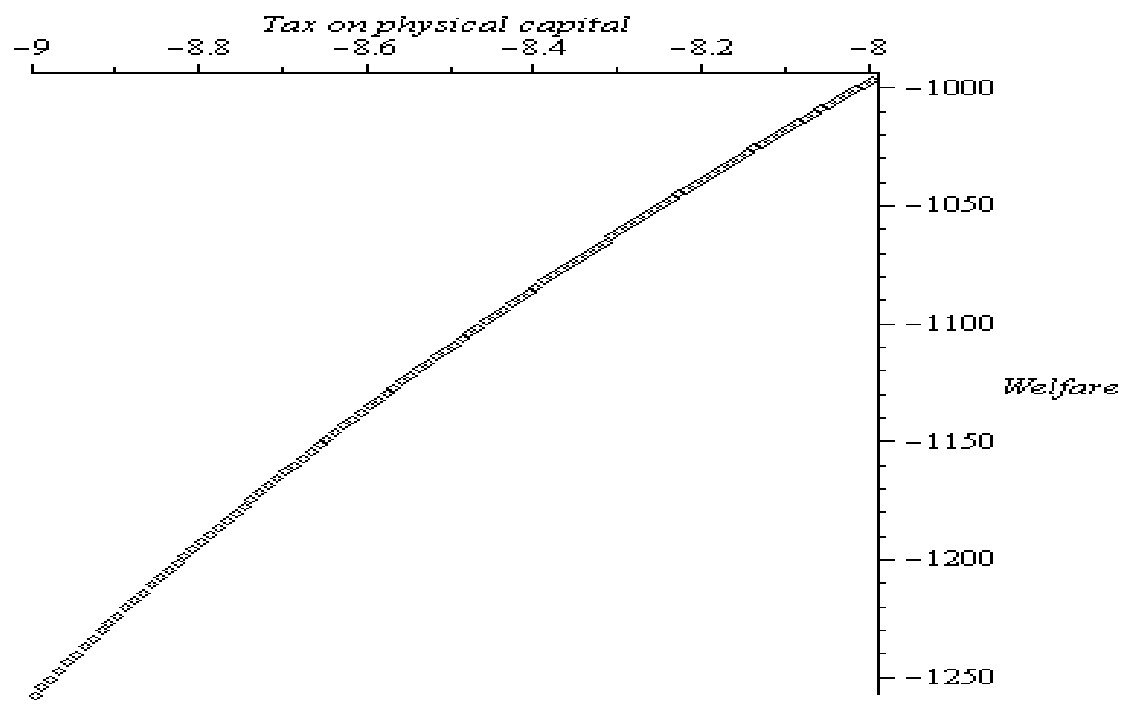
Figures 2.6 and 2.7 show the relationship between the physical capital income tax and the level of welfare correspondingly.

Result 2: *The optimal tax on physical capital is always positive in both cases. The corresponding optimal tax on human capital is negative when the share of government expenditure is low, in other words it is always optimal to tax physical capital to finance both human capital subsidy and the government expenditure. While, we have to tax on both kinds of capital when the share of government expenditure is high enough.*

We find that in Case 2 when the tax on physical capital always decreases the growth rate, it could also increase welfare. This finding goes against the traditionally accepted belief that the dynamic effect will always dominate static effect, i.e., when the steady-state growth rate is less than optimal, policies that will further decrease it could not be a Pareto improvement, even though it will bring static gain. However, the numerical results in our study show the possibility that static gains may dominate growth losses.



2.6 Welfare and tax on physical capital, case 1



2.7 Welfare and tax on physical capital, case 2

4.8 Dynamic analysis

In the previous section, we examine the impact of the taxes in the steady-state. In this section, we take into account the transitional adjustments of the economy after taxes are implemented. We use exactly the same set of benchmark parameter values as in the steady-state analysis. To simulate the dynamics of the model, we re-scale the variables to convert the system of equilibrium equations into a stationary system of equations. We follow Trimborn et al. (2005) to perform the dynamic analysis.

Letting $s = X/N$, where $X = N, H, K, C$ and Y, we convert the original equilibrium system of equations ((112), (122), (124), (125), (127), (128) and (129)) into the following stationary equilibrium system:

$$\dot{h} = Bvh - \frac{[(1-x)y - c]h}{k+\eta}, \tag{144}$$

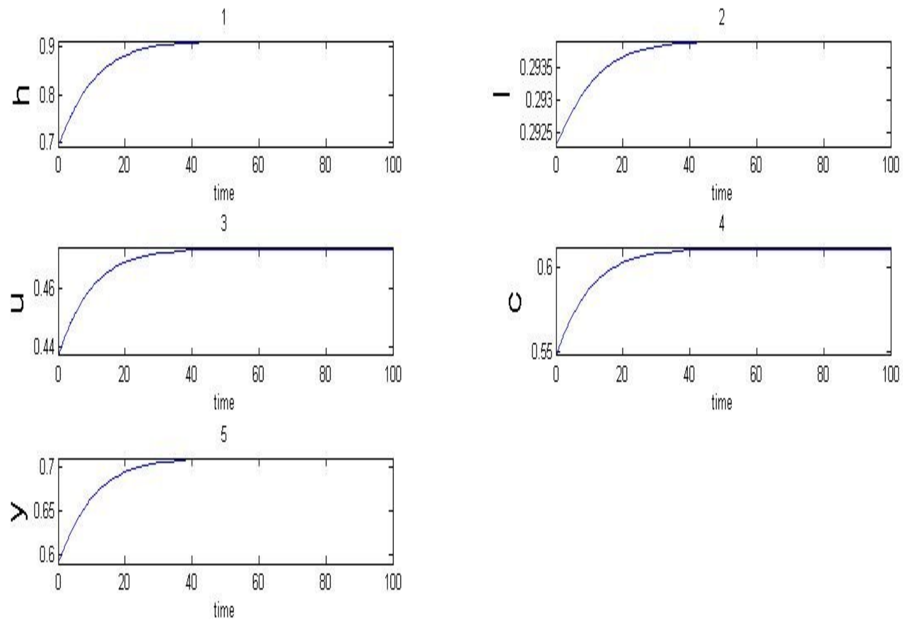
$$\dot{u} = \left[\frac{u}{1-\alpha(1-\gamma)}\right]\left\{[1-\alpha(1-\gamma)]\left(\frac{[(1-x)y-c]}{k+\eta}\right) - \frac{(1-\tau_k)\alpha^2\gamma y}{k}\right.$$
$$\left. + B\left[u + \alpha(1-\gamma)(1-u-l)\right]\right\}, \tag{145}$$

$$\dot{l} = \frac{l}{\sigma - \epsilon(1-\sigma)}[\sigma B u - \rho - \alpha^2\gamma y/k(\sigma-1)(1-\tau_k)]. \tag{146}$$

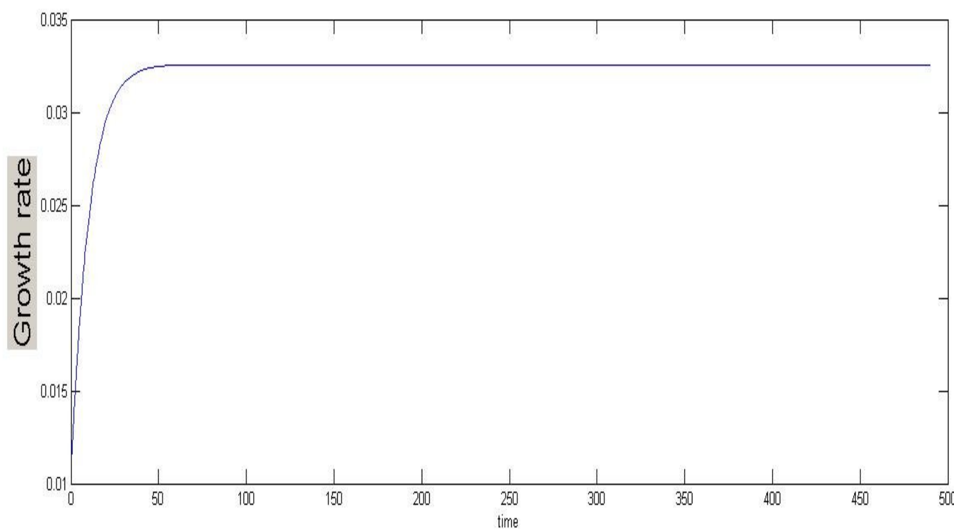
$$y = A(uh)^{\alpha(1-\gamma)}k^{\alpha\gamma}, \tag{147}$$

$$c = \alpha^2 l(1-\tau_h)(1-\gamma)y/(u\epsilon). \tag{148}$$

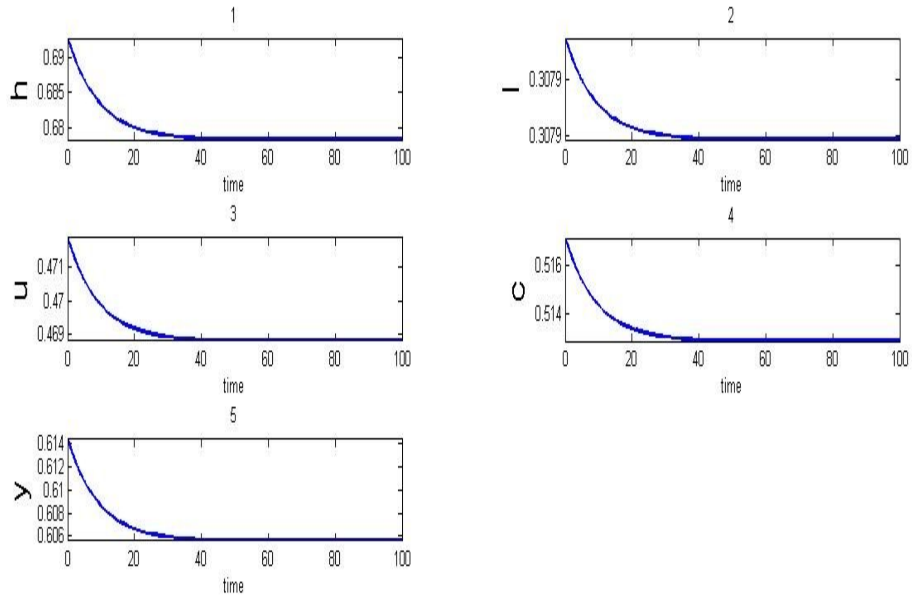
Since equations (147) and (148) are just static equations, in the system there will be one state variable h and two control variables l and u. We consider one tax at a time, for a given government expenditure share x. Table 2.2 displays the tax rates necessary to generate the same government expenditure share, and the corresponding steady state values of variables. Figures 2.8 and 2.9 display the transitional dynamics for τ_k. Figures 2.10 and 2.11 display the transition dynamics for τ_h. Figure 2.12 compares the transition dynamic of welfare under two taxes.



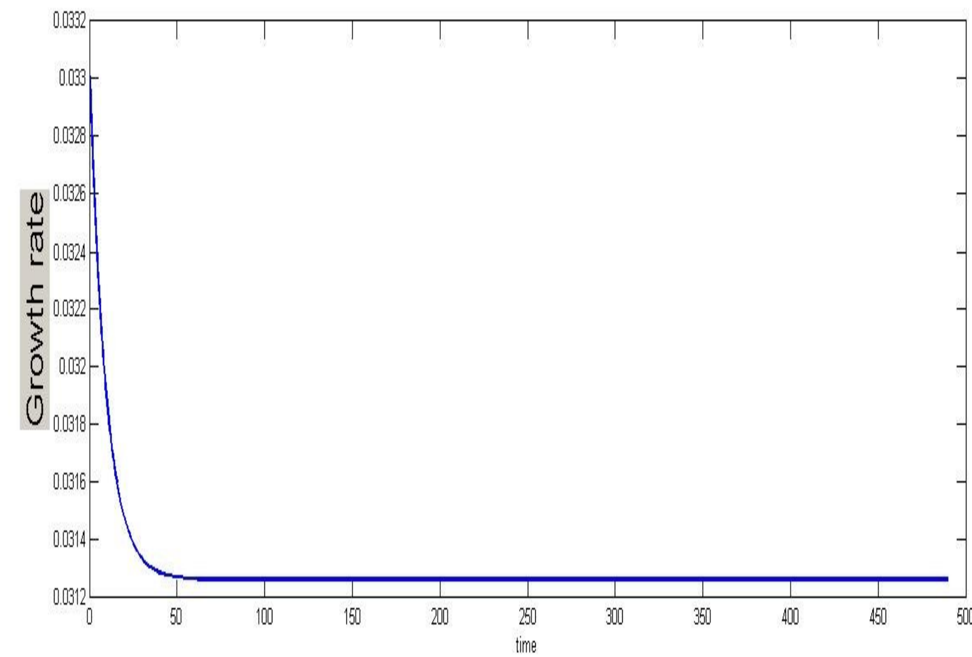
2.8Dynamic Transition: τ_k=0.13



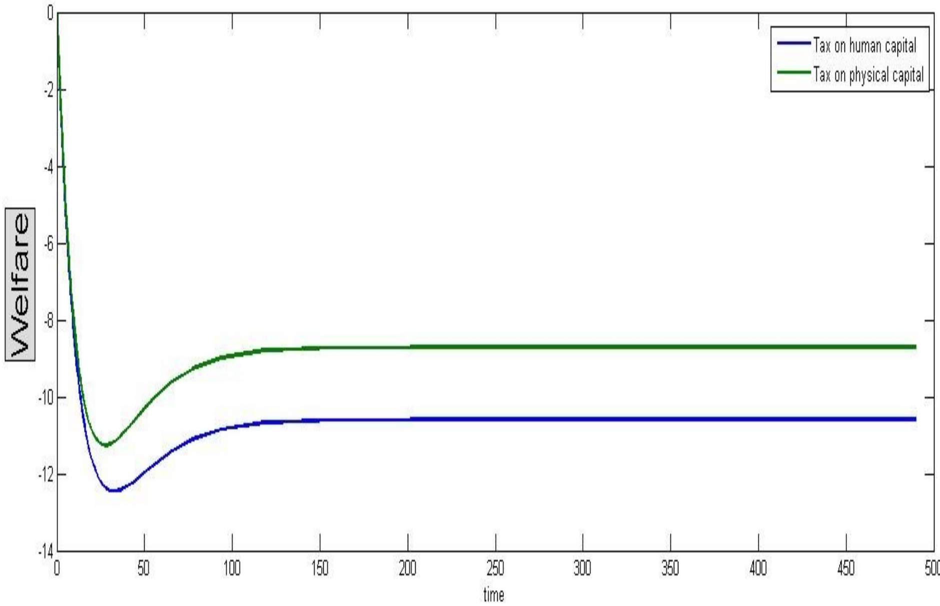
2.9 Growth rate dynamic transition: τ_k =0.13



2.10 Dynamic Transition τ_h =0.07



2.11 Growth rate dynamic transition: τ_h =0.07



2.12 Welfare dynamic transition

Table 2.2: Steady state values: government expenditure share=0.0291

	laissez faire	$\tau_k = 0.13$	$\tau_h = 0.07$
human captial to productivity	0.6927	0.9124	0.6783
physical captial to productivity	1.4000	1.4000	1.4000
consumption to productivity	0.5372	0.6113	0.5129
final ouput to productivity	0.6145	0.7101	0.6056
time into leisure	0.2977	0.2939	0.3079
time into production	0.4722	0.4735	0.4688
time into education	0.2301	0.2326	0.2233
growth rate	0.0322	0.0326	0.0313
Welfare (static)	-9.5040	-6.3724	-10.7727
Welare (dynamic)	-9.5040	-8.7138	-10.5938

Result 3: *A human capital income tax carries a higher welfare cost than a physical capital income tax.*

From Figures 2.8 and 2.10, we could understand the transition dynamics for both taxes. We apply the same value of state variable (h) as that in the laissez-faire case ($h = 0.6927$) as the starting point. When we only have the physical capital income tax, the steady-state analysis over-estimate the welfare level. While when we only have the human capital income tax, the steady-state analysis under-estimate the welfare level.

Next we assume the existence of both taxes, we repeat this dynamic method, and find the following result.

Result 4: *Both the dynamic and steady-state analysis show the same result: a tax on physical capital income dominates a tax on human capital income in terms of improving welfare.*

We could see clearly from Figures 2.12 that with the human capital accumulation, the distortions from the human capital income tax are more serious than those from physical capital income tax. A tax on human capital income will decrease the growth rate dynamically by a larger percentage than a tax on physical capital income, since it directly increases leisure, and decreases the time devoted into production as well as human capital accumulation which will further decrease both consumption and growth rate. Our results accord with the result in Devereux & Love (1994) that for equal percentage changes, the wage tax has a larger effect on growth than the capital tax. However, our result concerning welfare costs is different from that in Devereux & Love (1994). We find that a physical capital income tax will induce a lower welfare cost than a human capital income tax.

5. Conclusion

In this chapter, we examine the optimal taxation in an R&D growth model with variety expansion. We first introduce a model without human capital accumulation, where final good is produced with intermediate good and labor, and intermediate good is produced with physical capital. As in Aghion et al. (2013), we show that the optimal capital income tax will increase with the size of the government expenditure and Chamley-Judd result is not robust to introducing endogenous technical progress. However, different from their paper, we show that the optimal tax on physical capital is always negative and there is a negative relationship between the physical capital income tax and the steady-state growth rate given a fixed government expenditure share. The different assumption of technology progress is responsible for the difference. Since R&D activities are always under-invested in a variety expansion model due to the monopoly inefficiency and knowledge spillover, while it could be under or over-invested in a quality-ladder R&D model. With non-accumulable labor income tax distortions, it is always optimal to tax labor while subsidizing physical capital accumulation.

We then introduce a model with human capital accumulation. The results show that now it is optimal to tax physical capital to subsidize human capital accumulation as long as the government expenditure share is low enough. This conclusion is different from Devereux and Love (1994), which claimed that the capital income tax is the least efficient method of generating revenue in a two sector model of endogenous growth. We use dynamic analysis to find the reason of the change in the welfare rankings, and it clearly shows that the physical capital income tax distortion is smaller than the labor income tax distortion in the model without human capital, while in an economy with the human capital accumulation, the ranking reverses. Since the human income tax distorts not just the labor-leisure choice but also the rate of investment in human capital in the extended model.

If the same issue is examined in a small open economy with capital mobility, countries may have incentives to decrease the capital income tax because of the international tax competition, then the ranking of the two taxes may change.

Part III

Chapter 3: Fiscal Policy versus Monetary Policy in an R&D Growth Model with Money in Production

1. Introduction

Should the government implements taxes or inflation to collect revenue? So far, this debate has been controversial. On the one hand, some studies supported a positive inflation taxation supported by maintaining a positive rate of money growth and claimed that inflation taxation will be favorable to income taxation as the instrument to finance public expenditure in terms of promoting welfare (Phelps (1973), Braun (1994) and Palivos and Yip (1995)). Cooley and Hansen (1991) particularly found the welfare costs of financing a given government expenditure are dramatically lower for economies that substitute consumption tax or inflation tax for the tax on capital income tax. On the other hand, Ho et al. (2007) showed that switching from consumption taxation to inflation taxation has a negative welfare effect in an economy without a production externality.

This chapter compares how different government interventions affect resource allocation, growth and welfare in an R&D growth model with variety expansion and money-in-production. We assume that the economy has perfectly competitive final good production and innovation and monopolized intermediate good production with both physical capital and the real money balances as its inputs. There are large empirical evidence showing that real money balances have a significant role as an explicit input in the production process (e.g., Sinai and Stokes, 1972; John, 1980; Simos, 1981; Hasan and Mahmud, 1993; Alexander, 1994; Lotti and Marcucci, 2007; Apergis, 2010). More specifically, Sinai and Stokes (1972) estimated an aggregate Cobb-Douglas production function in which the factors are capital, labor and real balances and claimed an elasticity of gross private domestic product with respect to real money balances of 0.17. As claimed in Fischer (1974), firms hold money to facilitate production, on the grounds that money enables them to economize the use of other inputs, and spares the cost of running short of cash. This setting of the model gives the monetary authority an ability to influence the equilibrium allocation of resources across sectors through the nominal interest rate or the money growth rate. Some recent literature, such as Morellec et al (2013) and Lyandres and Palazzo (2012) also show firms have incentive to hold cash. Our main objective is to compare the effectiveness of two fiscal policies (consumption tax and capital income tax) and one monetary policy (inflation tax) as the instruments of the government. In the meantime, the values of the consumption tax rate, capital tax rate and nominal money growth rate could be negative (i.e., subsidies), as long as there exists at least one instrument to finance both the government expenditure and any subsidies. We first show that given an exogenous government expenditure and in the presence of consumption tax, the growth-maximizing capital tax rate should be negative. Meanwhile, the growth-maximizing monetary policy is deflation, i.e., the growth rate of the supply of nominal money should be negative. We claim the results mainly arise from the monopoly inefficiency which makes the demands for both capital and real money less than optimal. Thus, we conclude that consumption taxes are favorable to the other two instruments. However, in the case without the consumption tax we re-consider the problem and show that the capital tax will be favourable in terms of improving welfare, and the inflation tax will be better in terms of promoting growth.

Our analysis also relates to the other two main strands of the literature. First is the literature on the effects of monetary policy on economic growth and social welfare. Basically, there are three ways to model the money demand: from a cash-in-advance constraint, a transaction technology with money as an intermediate input and real money balances in the utility function. Several studies claimed that inflation taxation reduces welfare (e.g., Cole and Stockman (1992), Schreft (1992), Grillman (1993), Gomme (1993), Dotsey and Ireland (1996), Aiyagari, et al. (1998), Wu and Zhang (1998), (2000), Lucas (2000)). More recently, the effects of monetary policy on economic growth and social welfare have been examined in R&D models. Chu and Cozzi (2012) assumed cash-in-advance constraints on consumption, R&D investment, and manufacturing in a Schumpeterian model, and showed that increases in the nominal interest rate would decrease economic growth. Meanwhile, the increase in the nominal interest rate may have an overall negative effect on R&D. Chu and Lai (2013) analyzed the effects of inflation on R&D and innovation-driven growth, incorporating money demand into a quality-ladder model with elastic labor supply. They found that if the elasticity of substitution between consumption and the real money balances is less (greater) than unity, then R&D and the growth rate of output would be decreasing (increasing) in the growth rate of money supply. Except for that we use a variety expansion model instead of a quality-ladder model, the main difference between our study and Chu and Lai (2013) is that we assume money in production, more specifically in a monopolized production sector. As a result of this, real money demand will always be less than optimal value. By which, we claim deflation would be optimal in our model both in terms of promoting growth and improving welfare in the presence of consumption taxes. Also in Chu and Lai (2013), they considered the welfare effects of inflation with the inflation tax as the only government instrument. To be more general, this chapter evaluates the welfare effects of inflation when it serves as a source of tax revenue along with other distortionary tax instruments.

A second related literature is that on optimal fiscal taxation in terms of promoting growth and improving welfare. It is well known that in Chamley and Judd's neoclassical growth model with capital accumulation and infinitely-lived agents, the optimal tax system to finance an exogenous government expenditure involves a zero tax rate on capital in the long run. Many studies based on the Chamley-Judd model reconfirmed this result (Judd (1999), Chamley (1986)). However, Jones et al. (1993) studied the connection between government polices and growth. They compared the capital and labor income tax as in an AK model, and found that a positive tax on capital income becomes optimal under special conditions. In addition, Jones et al. (1997) extend the model for an economy with both capital and human capital accumulation and showed that the zero optimal taxation on both capital income is due to infinite supply elasticity and should be based on three strict conditions, i.e., constant return to scale, sufficiently rich tax code, and no possibilities for relative prices to affect wealth. They found that the optimal tax on capital is positive with inelastic labor supply. Therefore, there is still a rationale for taxing capital (e.g., Chamley (2001), Golosov et al. (2006), Kocherlakota (2010), Piketty and Saez (2012), Turnovsky (1996)). Devereux & Love (1994) compared the growth and welfare effect of taxes on capital income and wages and claimed that firstly for equal percentage changes, the wage tax has a larger effect on growth than the capital tax, but for revenue-equivalent changes, they have similar effects on growth. Secondly, the capital income tax will generate more welfare cost than the wage tax. More closely related to our analysis is the paper by Aghion et al. (2013), in which optimal taxation was discussed in a Schumpeterian growth model. They showed that a zero tax on capital becomes suboptimal for a given required trend of public expenditure due to a market size effect and claimed that the Chamley-Judd result is not robust to introducing endogenous technical progress. The main difference between our study and Aghion et al. (2013) is that we employ an R&D model with horizontal innovation through variety expansion instead of vertical innovation through quality improvement. It is shown in Aghion and Howitt (1992), that both over-investment and under-investment in R&D can accur in a vertical innovation model. However, R&D is always under-invested in a variety expansion model due to the monopoly inefficiency and knowledge spillover. That is the reason that we obtain a negative relationship between the capital tax rate and the growth rate in our analysis.

The rest of this chapter is organized as follows. Section 2 describes the model. Section 3 characterizes the decentralized equilibrium. Section 4 considers the government problem of choosing optimal taxation in terms of promoting growth and improving welfare. Section 5 characterizes a special case without consumption taxes. The last section concludes.

2. The model

We assume that the model economy is a closed, money-in-production economy populated by a continuum of identical infinitely-lived households with measure one.

2.1 Technologies

There are four types of production activities in the economy: final good production, intermediate good production, innovations, and capital accumulation. It is assumed that the intermediate good sector has monopoly power while the rest are perfectly competitive.

2.1.1 Final good production

A final good is produced by a large number of identical competitive firms. The producers use z_{it} units of intermediate good i and $1 - l_t$ units of labor to produce Y units of the final good according to

$$Y_t = A(1 - l_t)^{1-\alpha} \int_0^{N_t} z_{it}{}^{\alpha} di, 0 \ < \alpha < 1, \tag{149}$$

where Y_t is the final output; z_{it} is the flow of the intermediate good i; A is the fix productivity parameter for all intermediate goods; α inversely measures the intermediate monopolist's market power; N_t is the measure of intermediate goods. We assume the consumer is endowed with 1 unit of time which will be allocated into leisure (l_t) and final good production ($1 - l_t$).

Assuming $\tilde{P}_t$ is the market price level,[17] we take the price of final good: $p_{yt} \equiv \frac{1}{\tilde{P}_t}$. We omit the time subscript t whenever no confusion can arise.

Given the price q_i of intermediate good i, in terms of the final good and the wage rate of the labor w, the competitive final good producers choose the demand functions for intermediate goods as well as labor to maximize the profit, thus, we have

$$q_i = \alpha A(1-l)^{1-\alpha} z_i^{\alpha-1}, \quad i \in [0, N], \tag{150}$$

$$w = \frac{(1-\alpha)Y}{1-l}, \tag{151}$$

i,e., the price of an input should equal to its marginal product in the competitive sector.

2.1.2 Intermediate good production

Monopoly intermediate producer i to whom permanent monopoly rights over the production and sale of their invented intermediated goods are given uses both capital k_i and real money balances m_i to produce an intermediate good, subject to[18]

$$z_i = k_i^{\gamma} m_i^{1-\gamma}.$$

Given the interest rate for capital r and the interest rate for real money balances i , as well as the final sector's demand for intermediate good i equation in (150), the intermediate good i producer chooses a monopolistic quantities of capital and real money balances to maximize its profit, Γ_i, where

$$\Gamma_i = q_i z_i - r_i k_i - i m_i.$$

[17]The numeraire in the model continues to be one unit of the final good, and $\tilde{P}_t$ is the price of a unit of the final good in terms of money.

[18]Shaw et al. (2005) illustrated how we can derive the Cobb-Douglas form of a money in production from more general specification. It is also consistent with the viewpoint proposed by Dornbusch and Frenkel (1973) that involves transaction costs.

The solution to this maximization problem gives the producer's demand functions for capital k_i, real money balances m_i and output z_i,

$$m_i = \phi(1-\gamma)r, \tag{152}$$

$$k_i = \phi i \gamma, \tag{153}$$

$$z_i = [(1-\gamma)r]^{1-\gamma}(w\gamma)^{\gamma}\phi,$$

where $\phi = \alpha^{2/(1-\alpha)}A^{1/(1-\alpha)}(1-\gamma)^{\alpha(1-\gamma)/(1-\alpha)}\gamma^{\alpha\gamma/(1-\alpha)}i^{(\alpha\gamma-1)/(1-\alpha)}r^{(\alpha-\alpha\gamma-1)/(1-\alpha)}(1-l)$.

Since the value of z_i is independent of i, we denote $z \equiv [(1-\gamma)r]^{1-\gamma}(w\gamma)^{\gamma}\phi$.

Thus we rewrite Y as

$$Y = AN(1-l)^{1-\alpha}z^{\alpha}, \tag{154}$$

where output growth is driven by expanding the variety of intermediate goods N, and capital K. As a result, the profit Γ_i could be written as

$$\Gamma_i = \Gamma = \alpha(1-\alpha)\frac{Y}{N}, \tag{155}$$

i.e., aggregate monopoly profit $N\Gamma$ will be proportional to aggregate final output Y.

2.1.3 Innovation

We adopt two assumptions in this section. First the R&D process is deterministic, i.e., by investing η units of the final good, the intermediate good producer will create a new type of intermediate good and become the sole producer forever, i.e., costing η will increase N by one unit. Thus, the value of a new technology equals the discounted present value of the gross profit from producing the new intermediate good V_t, i.e.

$$V_t = \int_t^{\infty} \Gamma_s \exp\left(-\int_t^s r_\tau d\tau\right) ds. \tag{156}$$

The second assumption is there is free entry in the R&D sector, thus we have

$$V = \eta. \tag{157}$$

From equation (156), we obtain

$$\dot{V} = \Gamma - rV, \tag{158}$$

where a dot on the top of a variable represents the time change rate of that variable. Combining equations (157) and (158) gives the equilibrium condition in the R&D sector,

$$V = \frac{\Gamma}{r}, \tag{159}$$

which holds true both in and outside the steady state.

2.2 Households

The model economy is populated by a continuum of identical infinitely-lived households with measure one. The representative household's preferences are defined over an infinite horizon

$$U = \int_0^\infty \frac{(Cl^\epsilon)^{1-\sigma} - 1}{1-\sigma} \exp(-\rho t) dt, \tag{160}$$

where C is consumption; ρ is the constant rate of time preference; σ is the elasticity of marginal utility; ϵ measures the importance of leisure relative to consumption.

The representative household income from capital and real money balances rental, work and dividend, is allocated between consumption and saving after tax.

$$(1+\tau_c)C = (1-\tau_k)rK + iM + w(1-l) - \dot{K} - \dot{M} - \pi M + D, \tag{161}$$

where two fiscal taxes are introduced: one on consumption at rate τ_c and the other on capital income at rate τ_k. M is the real money balances, i.e., $M = \frac{\tilde{M}}{\tilde{P}}$, where $\tilde{M}$ is the nominal money supply. π is the inflation rate, i.e., πM is the amount of inflation tax. D is the total dividend from intermediate producers.

Let $B = K + M$, we could rewrite equation (161) as,

$$\dot{B} = (1-\tau_k)rK + iM + w(1-l) - (1+\tau_c)C - \pi M + D.$$

The representative household then chooses consumption C, the time allocation l, real money balances M and capital K to maximize its life-time utility equation (160), subject to the budget constraint equation (161). The current value Hamiltonian function for the decentralized problem is as follows

$$\mathcal{L} = \frac{(Cl^{\epsilon})^{1-\sigma} - 1}{1-\sigma} + \lambda[(1-\tau_k)rK + iM + w(1-l) - (1+\tau_c)C - \pi M + D]$$
$$+v(B - K - M),$$

where, λ is the co-state variable and v is a multiplier. The first-order conditions for the representative household's optimization problem are

$$C: \quad C^{-\sigma} l^{\epsilon(1-\sigma)} = \lambda(1+\tau_c), \tag{162}$$

$$l: \quad \epsilon C^{1-\sigma} l^{\epsilon(1-\sigma)-1} = \lambda w, \tag{163}$$

$$K: \quad \lambda(1-\tau_k)r = v, \tag{164}$$

$$M: \quad \lambda(i - \pi) = v, \tag{165}$$

$$B: \quad v = -\dot{\lambda} + \rho\lambda, \tag{166}$$

$$TVC: \quad \lim_{t\to\infty} e^{-\rho t}\lambda_t B_t = 0. \tag{167}$$

Equation (162) (respectively, (163), (164), (165)) equalizes the marginal benefit and marginal cost of consumption (respectively, time allocated to leisure, capital and real money balances). Equation (166) is the optimal dynamic condition for capital and real money balances accumulation.

From equations (164) and (165), we could derive the conventional no-arbitrage condition, i.e.,

$$i = (1 - \tau_k)r + \pi.$$

2.3 Government's problem

The government uses a consumption tax, a capital income tax and an inflation tax to finance the government expenditure G_t. We assume that the government expenditure is a fixed fraction x of final output, i.e., $G_t \equiv xY_t$, with $0 < x < 1$. The task of the government is to choose the welfare-maximizing rates of different types of taxes to finance the required government expenditure.[19] Assuming that the government budget is balanced at each point in time, we have

$$\tau_k rK + \tau_c C + \frac{\dot{\tilde{M}}}{\tilde{P}} = G = xY,$$

where left-hand side is the sum of tax revenue from capital income, consumption and seignorage revenue, and right-hand side is the total government expenditure which is proportional to the final output. We consider the money growth rate as the policy instrument which is directly controlled by the monetary authority. Given an exogenously chosen $\frac{\dot{\tilde{M}}}{\tilde{M}} = \theta$, i.e., $\frac{\dot{\tilde{M}}}{\tilde{P}} = \frac{\dot{\tilde{M}}}{\tilde{M}}\frac{\tilde{M}}{\tilde{P}} = \theta M$, we could rewrite and above equation as

$$\tau_k rK + \tau_c C + \theta M = G = xY. \tag{168}$$

3. Equilibrium

3.1 Stability of the equilibrium

First ,we will determine the stability of the equilibrium. Thus we derive the optimal dynamic equations for the system. From the intermediate goods section, we already shown that all

[19] In our analysis, all the three tax rates could be negative (i.e., subsidies).

the intermediate good producers will use the same quantities of capital k_i and real money balances m_i to produce the same level of intermediate goods z_i. With the capital and real money balances market clearing conditions, i.e., $\int_0^N k_i di = Nk = K$ and $\int_0^N m_i di = Nm = M$, we have $z = \frac{K^\gamma M^{1-\gamma}}{N}$. Thus, we could express the final output Y as

$$Y = A(1-l)^{1-\alpha} N^{1-\alpha} K^{\alpha\gamma} M^{\alpha(1-\gamma)}. \tag{169}$$

In addition, the final good clearing condition should be satisfied, i.e.,

$$C + \dot{K} + \dot{N}\eta = (1-x)Y, \tag{170}$$

i.e., the final output net of government expenditure (fixed fraction of the final output: xY) will be allocated into consumption, innovation and investment. We now begin to derive the equilibrium conditions for each of these components. We first derive the expression for capital stock. From equations (153), (155) and (159), we have

$$k = \frac{\alpha\gamma\eta}{1-\alpha}, \tag{171}$$

which is constant.[20] Thus, we have $\frac{\dot{N}}{N} = \frac{\dot{K}}{K}$. Combining this condition and the final good clearing condition equation (170), we have

$$\dot{N} = \frac{[(1-x)Y - C]N}{K + N\eta}, \tag{172}$$

$$\dot{K} = \frac{[(1-x)Y - C]K}{K + N\eta}. \tag{173}$$

Since we could denote output to productivity in terms of N as $\frac{Y}{N} \equiv y = \frac{Y}{N} = A(1-l)^{1-\alpha} k^{\alpha\gamma} m^{\alpha(1-\gamma)}$, and k is constant, y will depends positively on labor supply $(1-l)$ and real money balances to productivity(m). Also by equations (153), (155) and (159), we have

$$r = A\alpha^2\gamma(1-l)^{1-\alpha} k^{\alpha\gamma-1} m^{\alpha(1-\gamma)}. \tag{174}$$

[20] Refer to the details in footnote 3

This expression implies that the interest rate will be lower than the marginal product of capital in the economy, which results in a less than optimal value of the demand for capital. We then use equations (151) and (162) to capture the relationship between consumption and leisure,

$$C = \frac{(1-\alpha)lY}{\epsilon(1+\tau_c)(1-l)}, \tag{175}$$

i.e., consumption will be proportional to final output. Its share depends positively on the amount of time devoted into leisure, and negatively on the consumption tax rate. Using equations (166),(164) and (162), we have

$$-\sigma\frac{\dot{C}}{C} + \epsilon(1-\sigma)\frac{\dot{l}}{l} = \rho - (1-\tau_k)r. \tag{176}$$

Plugging equation (163) into equation (151), we have

$$(1-\sigma)\frac{\dot{C}}{C} + [\epsilon(1-\sigma)-1]\frac{\dot{l}}{l} = \rho - (1-\tau_k)r + \alpha(1-\gamma)\frac{\dot{M}}{M} + [1-\alpha(1-\gamma)]\frac{\dot{N}}{N} + \alpha\frac{\dot{l}}{1-l}, \tag{177}$$

where

$$\frac{\dot{M}}{M} = \frac{\dot{\tilde{M}}}{\tilde{M}} - \frac{\dot{\tilde{P}}}{\tilde{P}} = \theta - \pi, \tag{178}$$

i.e., the growth rate of real money balances equals to the exogenously given money growth rate minus the inflation rate. Now we only need the expression for differential equation of real money balances $\dot{M}$ to finish the solving for optimal dynamic system. From equation (152), we derive

$$M[(1-\tau_k)r + \pi] = \alpha^2(1-\gamma)Y,$$

thus, we have

$$\dot{M} = (1-\tau_k)rM + \theta M - \alpha^2(1-\gamma)Y. \tag{179}$$

Lastly plugging equation (179) into equation (177) and combining equation (176), we could solve for $\frac{\dot{l}}{l}$, where

$$\dot{l} = \frac{(1-l)l}{\alpha\sigma\gamma l + [\sigma - \epsilon(1-\sigma)](1-l)}$$

$$\left\{(1-\tau_k)r[1-\alpha\sigma(1-\gamma)] - \rho - \sigma\frac{\dot{N}(1-\alpha+\alpha\gamma)}{N} - \alpha\sigma(1-\gamma)\theta + \frac{\sigma\alpha^3(1-\gamma)^2 Y}{M}\right\}. \quad (180)$$

In the system of differential equations, three of them are for state variables: equations (172), (173) and (179), one is for control variable: equation (180) and the rest are static equations: equations (169) and (175). We next construct a stationary system by re-scaling variables to productivity. The transformed variables are $n = 1, m = \frac{M}{N}, k = \frac{K}{N} = \frac{\alpha\gamma\eta}{1-\alpha}, c = \frac{C}{N}, y = \frac{Y}{N}$, and l for the decentralized economy. In summary we have

$$\dot{n} = 0,$$

$$\dot{k} = 0,$$

$$\dot{m} = (1-\tau_k)rm + \theta m - \alpha^2(1-\gamma)y - g_N m,$$

$$\dot{l} = \frac{(1-l)l}{\alpha\sigma\gamma l + [\sigma - \epsilon(1-\sigma)](1-l)}$$

$$\times\left\{(1-\tau_k)r[1-\alpha\sigma(1-\gamma)] - \rho - \sigma g_N(1-\alpha+\alpha\gamma) - \alpha\sigma(1-\gamma)\theta + \frac{\sigma\alpha^3(1-\gamma)^2 y}{m}\right\},$$

$$y = A(1-l)^{1-\alpha}k^{\alpha\gamma}m^{\alpha(1-\gamma)},$$

$$c = \frac{(1-\alpha)ly}{\epsilon(1+\tau_c)(1-l)},$$

where $g_N = \frac{[(1-x)y-c]}{k+\eta}$. Then plugging the two static equations into the system noting that $k = \frac{\alpha\gamma\eta}{1-\alpha}$ is constant, we could reduce the above system into two equations with only one state variable and one control variable as follows to examine the stability of the equilibrium,

$$\dot{m} = \theta m + A\alpha\gamma(k)^{\alpha\gamma-1}(1-l)^{-\alpha}m^{\alpha(1-\gamma)}$$

$$\times \left\{ (1-\tau_k)\alpha(1-l)m - \frac{\alpha^2(1-\gamma)(1-l)\eta}{1-\alpha} - \frac{[(1-x)(1-l) - \frac{(1-\alpha)l}{\epsilon(1+\tau_c)}]m}{\alpha\gamma + 1 - \alpha} \right\},$$

$$\dot{l} = \frac{(1-l)l}{\alpha\sigma\gamma l + [\sigma - \epsilon(1-\sigma)](1-l)}$$

$$\times \left\{ \frac{\left\{ (1-\tau_k)\alpha^2\gamma(1-l) - \frac{\sigma\alpha\gamma[(1-x)(1-l) - \frac{(1-\alpha)l}{\epsilon(1+\tau_c)}]}{\alpha\gamma+1-\alpha} \right\}}{A^{-1}(1-l)^{\alpha} m^{\alpha(\gamma-1)}(k)^{1-\alpha\gamma}} - \rho - \alpha\sigma(1-\gamma)\frac{\dot{m}}{m} \right\}.$$

Setting $\dot{m} = \dot{l} = 0$, we have

$$\theta + \frac{A\alpha\gamma(1-l)^{-\alpha}}{(k)^{1-\alpha\gamma} m^{\alpha(\gamma-1)}} \left\{ (1-\tau_k)\alpha(1-l) - \frac{\alpha^2(1-\gamma)(1-l)\eta}{(1-\alpha)m} - \frac{[(1-x)(1-l) - \frac{(1-\alpha)l}{\epsilon(1+\tau_c)}]}{\alpha\gamma + 1 - \alpha} \right\} = 0$$

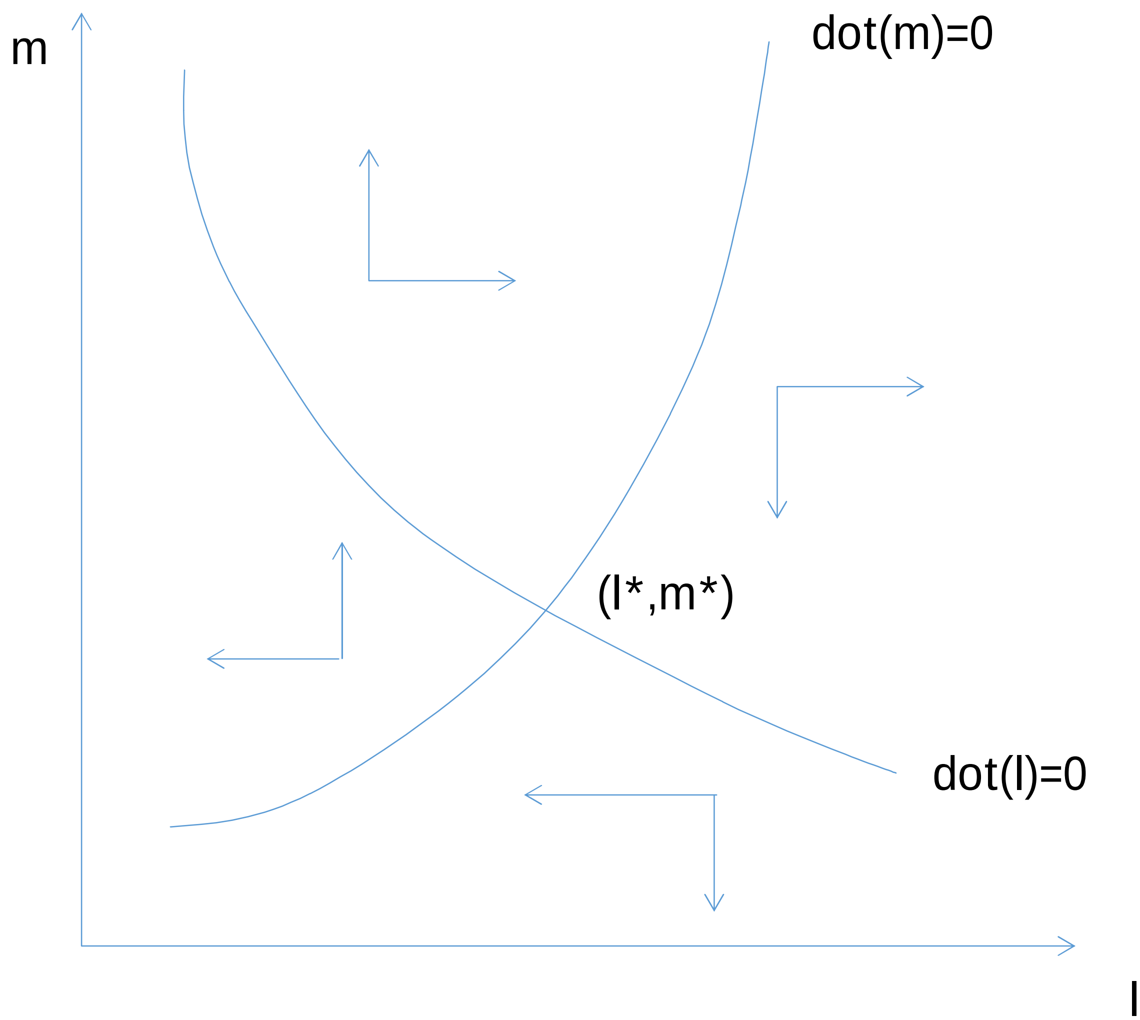
$$\frac{\left\{ (1-\tau_k)\alpha^2\gamma(1-l) - \frac{\sigma\alpha\gamma[(1-x)(1-l) - \frac{(1-\alpha)l}{\epsilon(1+\tau_c)}]}{\alpha\gamma+1-\alpha} \right\}}{A^{-1}(1-l)^{\alpha} m^{\alpha(\gamma-1)}(k)^{1-\alpha\gamma}} - \rho = 0. \tag{181}$$

The phase diagram is plotted in Figure 3.1. Given the global instability of this dynamic system, the economy must jump to the steady state given by m^* and l^*. Therefore, given a constant money growth rate θ, the economy is on a unique BGP along which consumption, capital and the real money balances grow at the same rate. The dynamic system has the same global instability as in Chu and Lai (2013).

3.2 Balanced growth equilibrium

A steady-state balanced growth equilibrium will be considered in this section, in which stationarity is imposed on the allocation of time l as well as the interest rates. While, final output (Y), the number of intermediate goods (N), capital (K), consumption (C) and real money balances (M) will grow at the same rate g. Since $\frac{\dot{C}}{C} = g$ and $\frac{\dot{l}}{l} = 0$, from equation (176) we have,

$$r = \frac{\sigma g + \rho}{1-\tau_k}. \tag{182}$$



3.1 Phase Diagram

Combining equations (155),(159) and (182), we have

$$y = \frac{(\sigma g + \rho)\eta}{\alpha(1-\alpha)(1-\tau_k)}. \tag{183}$$

From the above equation, we could express $g = \tilde{d}_1 y(1-\tau_k) + \tilde{d}_2$, where $\tilde{d}_1$ and $\tilde{d}_2$ are just constants. In summary, there are two main effects on growth of different government polices, among which the effect on y is the market size effect, and the effect on $(1-\tau_k)$ is the innovation incentive effect. The sign and magnitude of the two effects determine the net effect of different polices on growth.

We next derived the balances growth condition of money. From equation (178) we have already shown that the inflation rate will be endogenously determined according to

$$\pi = \theta - g,$$

i.e., the inflation rate (π) will be positively correlated with the exogenous money growth rate (θ) and negatively correlated with the long run growth rate (g). In addition, we obtain

$$i = (1-\tau_k)r + \pi = \sigma g + \rho + \theta - g.$$

Combining the values of π and i, we have

$$m = \frac{\alpha^2(1-\gamma)y}{\sigma g + \rho + \theta - g}. \tag{184}$$

Combining equation (184) and equation (152), we obtain

$$l = 1 - \xi y i^{\frac{\alpha - \alpha\gamma}{1-\alpha}} r^{\frac{\alpha\gamma}{1-\alpha}}, \tag{185}$$

where

$$\xi = \alpha^{-2\alpha/(1-\alpha)} A^{-1/(1-\alpha)} (1-\gamma)^{(\alpha\gamma-\alpha)/(1-\alpha)} \gamma^{-\alpha\gamma/(1-\alpha)}.$$

The last step is to determine the value of consumption per intermediate good.

From equation (175), denoting $c = \frac{C}{N}$, we have

$$c = \frac{(1-\alpha)ly}{\epsilon(1+\tau_c)(1-l)}. \tag{186}$$

Further more, plugging equation (185) into (186), we obtain

$$c = \frac{(1-\alpha)(1-\xi y i^{\frac{\alpha-\alpha\gamma}{1-\alpha}} r^{\frac{\alpha\gamma}{1-\alpha}})y}{\epsilon(1+\tau_c)\xi y i^{\frac{\alpha-\alpha\gamma}{1-\alpha}} r^{\frac{\alpha\gamma}{1-\alpha}}}.$$

As a result, consumption's share of final output: ζ_c could be denoted as

$$\zeta_c = \frac{(1-\alpha)(1-\xi y i^{\frac{\alpha-\alpha\gamma}{1-\alpha}} r^{\frac{\alpha\gamma}{1-\alpha}})}{\epsilon(1+\tau_c)\xi y i^{\frac{\alpha-\alpha\gamma}{1-\alpha}} r^{\frac{\alpha\gamma}{1-\alpha}}}. \tag{187}$$

Dividing both sides of the overall resource constraint $C + \dot{K} + \dot{N}\eta = (1-x)Y$ by N, we have

$$\frac{g\eta(1-\alpha+\alpha\gamma)}{1-\alpha} = (1-x-\zeta_c)y. \tag{188}$$

Plugging equation (183) into equation (188), we have

$$\frac{\alpha(1-\tau_k)(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} = 1-x-\zeta_c. \tag{189}$$

Equation (189) will determine the growth rate in the long run which depends on the three different taxes rate and the values of all parameters.

3.2.1 Existence and uniqueness of the equilibrium

Next we will determine the conditions for the existence and uniqueness of the steady state equilibrium from the laissez faire case such that $\tau_k = \theta = \tau_c = 0$. Since by the continuity of the equations, when all the tax rates are sufficiently small, the equilibrium should exist as well. Re-write equation (189) as $E(g) = \frac{\alpha(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} + \frac{\alpha(1-\alpha)^2}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}} = 1 + \frac{1-\alpha}{\epsilon}$, we derive

$$E'(g) = \frac{\alpha(1-\alpha+\alpha\gamma)\rho}{(\sigma g+\rho)^2} - \frac{\alpha(1-\alpha)^2}{\epsilon\xi\eta}(\sigma g+\rho)^{\frac{-\alpha\gamma}{1-\alpha}-2}(\sigma g+\rho+\theta-g)^{\frac{\alpha\gamma-1}{1-\alpha}}$$

$$\times \left[\frac{(\alpha\gamma + 1 - \alpha)\sigma}{1-\alpha}(\sigma g + \rho + \theta - g) + \frac{(\alpha - \alpha\gamma)}{1-\alpha}(\sigma - 1)(\sigma g + \rho) \right].$$

There is also one important inequality which will be used frequently for the proofs below

$$\frac{\xi\eta(\sigma g + \rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g + \rho + \theta - g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}{\alpha(1-\alpha)} < 1, \tag{190}$$

representing the amount of time allocated into leisure should be positive but not exceeding total endowment time 1,

$$0 < l = 1 - \xi y i^{\frac{\alpha-\alpha\gamma}{1-\alpha}} r^{\frac{\alpha\gamma}{1-\alpha}} < 1. \tag{191}$$

Proposition 1: *If (i) $\alpha\epsilon < 1$, (ii) $\sigma > 1$ and (iii) $\rho < \left(\frac{\alpha(1-\alpha)}{\xi\eta(1+\frac{\epsilon}{1-\alpha})}\right)^{1-\alpha}$, then there always exists a unique positive long run growth rate.*

Proof. We will use the intermediate value theorem to prove that $\exists\ g^* \in (0, \infty)$ such that $E(g) = 1 + \frac{1-\alpha}{\epsilon}$.

(a) The first step is that to show $E(g)$ is a monotonic function, more specifically we show $E'(g) < 0$, i.e., $\frac{(1-\alpha)}{\epsilon\xi\eta}(\sigma g+\rho)^{\frac{-\alpha\gamma}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{\alpha\gamma-1}{1-\alpha}}\ [(\sigma-1)\sigma g + \rho(\sigma+\alpha-\alpha\gamma)]\ > (1-\alpha+\alpha\gamma)\rho$. From equation (190), we have, $\frac{(1-\alpha)}{\epsilon\xi\eta}(\sigma g+\rho)^{\frac{-\alpha\gamma}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{\alpha\gamma-1}{1-\alpha}} > \frac{\sigma g+\rho}{\alpha\epsilon(\sigma g+\rho-g)}$, thus, the sufficient condition for the above inequality is $\frac{\sigma g+\rho}{\alpha\epsilon(\sigma g+\rho-g)}[(\sigma-1)\sigma g + \rho(\sigma+\alpha-\alpha\gamma)] > (1-\alpha+\alpha\gamma)\rho$. Since now the LHS will increase with g, and the RHS is constant we only need to prove $\frac{1}{\alpha\epsilon}(\sigma+\alpha-\alpha\gamma) > (1-\alpha+\alpha\gamma)$, which could be guaranteed by conditions (i) and (ii).

(b) When g takes the minimum value 0, $E(0) = \frac{\alpha(1-\alpha)^2}{\epsilon\xi\eta\rho^{\frac{1}{1-\alpha}}} > 1 + \frac{1-\alpha}{\epsilon}$, if condition *(iii)* holds true.

(c) When g takes the maximum value, such that $\frac{\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}{\alpha(1-\alpha)} = 1$, the LHS of $E(g)$ equals to $\frac{\alpha(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} + \frac{1-\alpha}{\epsilon}$ which is definitely less than $1 + \frac{1-\alpha}{\epsilon}$. *Q.E.D.*

Conditions (i) to (iii) in Proposition 1 could be guaranteed by various sufficient conditions concerning the values of the technology and preferences such as a sufficiently low subjective discount rate (low ρ), a sufficiently productive parameter for all intermediate goods (large A), a sufficiently low cost of innovation (low η), a sufficiently low elasticity of leisure (low ϵ), and a sufficiently high elasticity of marginal utility (high σ).

4. Government's problem

The government will choose a capital tax rate τ_k, a consumption tax rate τ_c and a money growth rate θ to maximize the representative consumer's life time utility equation (160), such that the representative consumer's choice equation (189) and government budget constraint equation (168) are satisfied. Since there are three kinds of taxes in the system to finance an exogenous given government expenditure, as long as two of them: the money growth rate (θ) and capital tax rate (τ_k) are chosen, the other one, consumption tax rate (τ_c) will be automatically determined by the government budget constraint. As a result, we will first eliminate τ_c from the system. From equation (186), we could rewrite ζ_c as

$$\zeta_c = \frac{(1-\alpha)l}{\epsilon(1+\tau_c)(1-l)}. \tag{192}$$

Meanwhile from the government budget constraint equation (168), along with the values of r, k, m and y, we have

$$\zeta_c = \frac{1}{\tau_c}\left[x - \alpha^2\gamma\tau_k - \frac{\alpha^2(1-\gamma)\theta}{\sigma g + \rho + \theta - g}\right].$$

Plugging the above equation back into equation (187) , we have

$$\tau_c = \frac{\epsilon(1-l)[x - \alpha^2\gamma\tau_k - \frac{\alpha^2(1-\gamma)\theta}{\sigma g+\rho+\theta-g}]}{(1-\alpha)l - \epsilon(1-l)[x - \alpha^2\gamma\tau_k - \frac{\alpha^2(1-\gamma)\theta}{\sigma g+\rho+\theta-g}]}. \tag{193}$$

Lastly, plugging the above equation back into (192), we obtain

$$\zeta_c = \frac{(1-\alpha)l - \epsilon(1-l)[x - \alpha^2\gamma\tau_k - \frac{\alpha^2(1-\gamma)\theta}{\sigma g+\rho+\theta-g}]}{\epsilon(1-l)}. \tag{194}$$

As a result, we could re-write equation (189) as

$$\frac{\alpha(1-\tau_k)(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} = 1 - x - \zeta_c, \tag{195}$$

where $1 - x - \zeta_c = 1 - \alpha^2\gamma\tau_k + \frac{1-\alpha}{\epsilon} - \frac{(1-\alpha)}{\epsilon(1-l)} - \frac{\alpha^2(1-\gamma)\theta}{\sigma g+\rho+\theta-g}$.

Since equation (195) is independent of x, we show that the growth rate will be determined only by capital tax rate τ_k, and the money growth rate θ as well as the values of all the model's parameters. τ_c will only be endogenously determined by the value of any given x. We observe that the value of government expenditure share x will not affect maximum growth rate, which is different from Aghion et al. (2013), in which a higher government expenditure share always leads to a lower maximum long run growth rate.

4.1 Effects on growth

In this section we will see the effects of capital tax (τ_k) and the money growth rate (θ) on growth. To examine these effects, we rewrite equation (189) as $E(g) = \frac{\alpha(1-\tau_k)(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} + \alpha^2\gamma\tau_k + \frac{\alpha(1-\alpha)^2(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}} + \frac{\alpha^2(1-\gamma)\theta}{\sigma g+\rho+\theta-g} = 1 + \frac{1-\alpha}{\epsilon}$, based on which we have Proposition 2 below.

Proposition 2: An increase in the capital tax rate (τ_k) or the money growth rate (θ) will decrease the long run growth rate, as long as the conditions in Proposition 1 hold.

Proof. Based on the results in Proposition 1 we know that when $\tau_k \neq 0$ and $\theta \neq 0$ but sufficiently small, we have $E'(g) < 0$. In order to show $\frac{dg}{d\tau_k} < 0$ and $\frac{dg}{d\theta} < 0$, we only need to prove that $\frac{\partial E}{\partial \tau_k} < 0$ and $\frac{\partial E}{\partial \theta} < 0$. Noticing that when $\tau_k \neq 0$, $\theta \neq 0$, the important inequality condition equation (190) becomes to

$$\frac{\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}{\alpha(1-\alpha)(1-\tau_k)^{\frac{1-\alpha+\alpha\gamma}{1-\alpha}}} < 1. \tag{196}$$

(a). To prove $\frac{\partial E}{\partial \tau_k} < 0$, we need to show

$$\frac{\partial E}{\partial \tau_k} = -\frac{\alpha(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} + \alpha^2\gamma - \frac{\alpha(\alpha\gamma+1-\alpha)(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}} < 0. \tag{197}$$

From equation (196), we have $\frac{\alpha(\alpha\gamma+1-\alpha)(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}} > \frac{\alpha\gamma+1-\alpha}{\epsilon(1-\tau_k)}$, thus, the sufficient condition for inequality (197) is $\alpha^2\gamma < \frac{\alpha(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} + \frac{\alpha\gamma+1-\alpha}{\epsilon(1-\tau_k)}$. This inequality holds true if $\alpha\epsilon(1-\tau_k) < 1$, which could be guaranteed by condition (i) in Proposition 1.

(b). To prove $\frac{\partial E}{\partial\theta} < 0$, we need to show

$$\frac{\partial E}{\partial\theta} = \frac{\alpha^2(1-\gamma)(\sigma g+\rho-g)}{(\sigma g+\rho+\theta-g)^2} - \frac{\alpha^2(1-\gamma)(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{1-\alpha\gamma}{1-\alpha}}} < 0. \tag{198}$$

From equation (196), we have $\frac{\alpha^2(1-\gamma)(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{1-\alpha\gamma}{1-\alpha}}} > \frac{\alpha(1-\gamma)}{\epsilon(\sigma g+\rho+\theta-g)}$, thus the sufficient condition for equation (198) is $\frac{\alpha^2(1-\gamma)(\sigma g+\rho-g)}{(\sigma g+\rho+\theta-g)^2} - \frac{\alpha(1-\gamma)}{\epsilon(\sigma g+\rho+\theta-g)} < 0 \Leftrightarrow \frac{\sigma g+\rho-g}{\sigma g+\rho+\theta-g} < \frac{1}{\alpha\epsilon}$. Since the LHS is less than 1 while the RHS is greater 1 by condition (i) in Proposition 1, the inequality holds. *Q.E.D.*

Proposition 2 clearly shows that subsidizing the capital and reducing money growth always promote growth. It is mainly because of the inefficiency arising from the monopolized intermediate good sector, where the demands of both capital and real money balances will be less than optimal. Thus, an increase in the capital subsidy rate or an decrease in the money growth rate could correct this inefficiency and increase the long run growth rate.

In other words, a consumption tax is the most effective in terms of promoting growth. The growth-maximizing consumption tax rate is always positive while both the capital income tax rate and the money growth rate are negative.

Our results accord with the ideas in the literature on taxation that a consumption tax is the most favorable one (Devereux and Love (1994), Chamely (1981), Lucas (1990) and Cooley and Hansen (1991)).

4.2 Effects on Welfare

To examine the effects of the polices on welfare, we first express the life-time utility in the long run as

$$W = \frac{(cl^{\epsilon})^{1-\sigma}}{(1-\sigma)(\rho - g + g\sigma)} - \frac{1}{\rho(1-\sigma)}.$$

The level of welfare W will be in a function of (τ_k, θ, g, x), if we denote both consumption to productivity c as well as leisure l as functions of (τ_k, θ, g, x), where

$$c = \frac{C}{N} = \frac{(1-x)(\sigma g + \rho)\eta}{\alpha(1-\alpha)(1-\tau_k)} - \frac{g\eta(\alpha\gamma + 1 - \alpha)}{1-\alpha},$$

$$l = 1 - \frac{\xi\eta(\sigma g + \rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g + \rho + \theta - g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}{\alpha(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}}.$$

The growth rate g will be implicitly determined by the values of the model's parameters and policies (τ_k, θ). The form of W in terms of (τ_k, θ, g , x) is as follows,

$$W = \frac{\left[\frac{(1-x)(\sigma g+\rho)\eta}{\alpha(1-\alpha)(1-\tau_k)} - \frac{g\eta(\alpha\gamma+1-\alpha)}{1-\alpha}\right]^{1-\sigma}\left[1 - \frac{\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}{\alpha(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}}\right]^{\epsilon(1-\sigma)}}{(1-\sigma)(\rho-g+g\sigma)}$$
$$-\frac{1}{\rho(1-\sigma)}. \qquad (199)$$

Thus, given the government expenditure share x, the government will choose (τ_k, θ) to maximize equation (199) subject to equation (195). Differentiating W with respect to τ_k and θ, we have the following two first-order conditions:

$$\frac{\partial W}{\partial \tau_k} = \frac{c^{-\sigma}l^{\epsilon(1-\sigma)-1}}{\rho - g + g\sigma}\left[\left(lu_3 + \epsilon cu_4 + \frac{cl}{\rho - g + g\sigma}\right)u_1 + lu_5 + \epsilon cu_6\right] = 0, \qquad (200)$$

$$\frac{\partial W}{\partial \theta} = \frac{c^{-\sigma}l^{\epsilon(1-\sigma)-1}}{\rho - g + g\sigma}\left[\left(lu_3 + \epsilon cu_4 + \frac{cl}{\rho - g + g\sigma}\right)u_2 + \epsilon cu_7\right] = 0, \qquad (201)$$

where

$$u_1 = \frac{dg}{d\tau_k} = \frac{\frac{\alpha(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} - \alpha^2\gamma + \frac{\alpha(\alpha\gamma+1-\alpha)(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}}{E'(g)} < 0,$$

$$u_2 = \frac{dg}{d\theta} = \frac{\frac{\alpha^2(1-\gamma)(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{1-\alpha\gamma}{1-\alpha}}} - \frac{\alpha^2(1-\gamma)(\sigma g+\rho-g)}{(\sigma g+\rho+\theta-g)^2}}{E'(g)} < 0,$$

$$u_3 = \frac{\partial c}{\partial g} = \frac{(1-x)\sigma\eta}{\alpha(1-\alpha)(1-\tau_k)} - \frac{\eta(\alpha\gamma+1-\alpha)}{1-\alpha},$$

$$u_4 = \frac{\partial l}{\partial g} = -\frac{\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{2\alpha-\alpha\gamma-1}{1-\alpha}}}{\alpha(1-\alpha)^2(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}}$$

$$\times\{(\alpha\gamma+1-\alpha)\sigma\theta + \sigma(\sigma-1)g + [\sigma-\alpha(1-\gamma)]\rho\} < 0,$$

$$u_5 = \frac{\partial c}{\partial \tau_k} = \frac{(1-x)(\sigma g+\rho)\eta}{\alpha(1-\alpha)(1-\tau_k)^2} > 0,$$

$$u_6 = \frac{\partial l}{\partial \tau_k} = -\frac{\xi\eta(\alpha\gamma+1-\alpha)(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}{\alpha(1-\alpha)^2(1-\tau_k)^{\frac{\alpha\gamma}{1-\alpha}+2}} < 0,$$

$$u_7 = \frac{\partial l}{\partial \theta} = -\frac{\xi\eta(\alpha-\alpha\gamma)(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}(\sigma g+\rho+\theta-g)^{\frac{2\alpha-\alpha\gamma-1}{1-\alpha}}}{\alpha(1-\alpha)^2(1-\tau_k)^{\frac{\alpha\gamma}{1-\alpha}+1}} < 0.$$

The welfare-maximizing combination of $(\tau_k,\ \theta)$ is the subset of the solutions $(\tau_k,\ \theta,\ g)$, such that equations (200), (201) and (195) are satisfied. Denoting $\underline{x} = 1 - \frac{\alpha\gamma+1-\alpha}{\frac{\sigma}{\alpha} - \frac{(\sigma-\alpha+\alpha\gamma)(1-\alpha)}{\xi\rho^{\frac{1}{1-\alpha}}\eta}}$, we have Proposition 3 below.

Proposition 3: The optimal combination of $(\tau_k,\ \theta)$ requires that $\theta^* < 0$ as long as $x < \underline{x}$.

Proof. To simplify the proof, we will look at the problem starting with the point when $\tau_k = 0$, $\theta = 0$, and any $x \in [0, 1)$. As we have shown Proposition 1, the equilibrium exists at a small neighborhood of this point. Our objective is to prove that at $(\tau_k = 0,\ \theta = 0)$, $\frac{\partial W}{\partial \theta} < 0 \Leftrightarrow lu_3 + \epsilon c u_4 > 0$, i.e.,

$$l[\tfrac{(1-x)\sigma\eta}{\alpha(1-\alpha)} - \tfrac{\eta(\alpha\gamma+1-\alpha)}{1-\alpha}] > \epsilon c \tfrac{\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma}{1-\alpha}}(\sigma g+\rho-g)^{\frac{2\alpha-\alpha\gamma-1}{1-\alpha}}}{\alpha(1-\alpha)^2}[\sigma(\sigma-1)g + (\sigma-\alpha(1-\gamma))\rho].$$

Given equation (190), the sufficient condition for the above inequality is

$$\frac{l}{\epsilon c}\left[\frac{(1-x)\sigma\eta}{\alpha(1-\alpha)} - \frac{\eta(\alpha\gamma+1-\alpha)}{1-\alpha}\right] > \frac{\sigma(\sigma-1)g + (\sigma-\alpha(1-\gamma))\rho}{(1-\alpha)(\sigma g+\rho)(\sigma g+\rho-g)}.$$

Because of equation (186), the above inequality is equivalent to

$$\frac{(1+\tau_c)(1-l)}{y}\left[\frac{(1-x)\sigma\eta}{\alpha(1-\alpha)}-\frac{\eta(\alpha\gamma+1-\alpha)}{1-\alpha}\right]>\frac{(\alpha\gamma+1-\alpha)\sigma}{(\sigma g+\rho)}+\frac{(\alpha-\alpha\gamma)(\sigma-1)}{(\sigma g+\rho-g)}\equiv RHS.$$

From equation (191), we can see that the sufficient condition for the above inequality is

$$(1+\tau_c)\xi(\sigma g+\rho)^{\frac{\alpha\gamma}{1-\alpha}}(\sigma g+\rho-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}[\frac{(1-x)\sigma\eta}{\alpha(1-\alpha)}-\frac{\eta(\alpha\gamma+1-\alpha)}{1-\alpha}]>RHS,$$

which is equivalent to

$$\frac{(1-\alpha)l}{(1-\alpha)l-\epsilon(1-l)x}\xi(\sigma g+\rho)^{\frac{\alpha\gamma}{1-\alpha}}(\sigma g+\rho-g)^{\frac{\alpha-\alpha\gamma}{1-\alpha}}[\frac{(1-x)\sigma\eta}{\alpha(1-\alpha)}-\frac{\eta(\alpha\gamma+1-\alpha)}{1-\alpha}]>RHS.$$

Since the LHS will increase with g, while the RHS will decrease with g, if the inequality holds true at $g=0$, it will hold for any value of g.

When $g=0$, from equation (189), we have

$$l=\frac{\epsilon}{1-\alpha+\epsilon}. \tag{202}$$

Thus, it is sufficient to show that $\frac{\sigma}{\alpha}-\frac{\alpha\gamma+1-\alpha}{1-x}>\frac{(\sigma-\alpha+\alpha\gamma)(1-\alpha)}{\xi\rho^{\frac{1}{1-\alpha}}\eta}\Leftrightarrow x<\underline{x}$. *Q.E.D.*

Thus as long as the government expenditure share is low enough, it is welfare-enhancing to maintain a negative growth rate of nominal money balances. Our results support the studies in the literature which are in favor of using deflation to improve welfare and promote growth (e.g., Cole and Stockman (1992), Schreft (1992), Grillman (1993), Gomme (1993), Dotsey and Ireland (1996), Aiyagari, et al. (1998), Wu and Zhang (1998), (2000), Lucas (2000)).

4.3 Numerical results

Since the first-order conditions for the welfare maximization are highly non-linear, analytical results are difficult to obtain. We do numerical simulations instead. We choose the benchmark value as follows.

Firstly we choose the value of time preference $\rho=0.05$ and the contribution of an intermediate good $\alpha=0.8$ based on the growth calibration exercises in Lucas (1990), King and Rebelo (1990), and Stokey and Rebelo (1995). We choose the elasticity of marginal utility $\sigma = 1.5$, as in Zeng and Zhang (2007). In the absence of more precise information about the real money balances intensity in intermediate good sector, we choose a medial value of $\gamma = 0.5$ to compare the effectiveness of taxation on the two inputs in intermediate good sector.

Following Prescott (1986) we choose, ϵ, to be 0.1, making that the representative agent allocates about 30 per cent of his available time into leisure. And it seems reasonable in a life-cycle view that people spend about 30 per cent of their time on leisure, and about 70 percent of their time on working.

The last two parameters, A and η ($A = 0.33$, $\eta = 1$), are chose to generate a growth rate of about 3.0% in the decentralized economy (the average growth rate in the United States for the last 30 years.)

Table 3.1 reports the results under benchmark values without government policies. ($x = \tau_k = \tau_c = \theta = 0$)

Table 3.1: Results under benchmark value for laissez-faire equilibrium

Parameters: $\alpha = 0.8$, $\gamma = 0.5$,$\epsilon = 0.1$, $\sigma = 1.5$, $\eta = 1$, $\rho = 0.05$, $A = 0.33$	
Welfare=-3.609	
Growth rate: 0.034	Real money per intermediate good: 3.02
Leisure: 0.295	Consumption share of final output: 0.838

Then we assign different values to τ_k and θ, from negative to positive, ensuring a positive growth rate to see their effects on welfare. We have the following result.

Result 1: *For low enough value of x, the optimal policy combination is $\theta^* < 0$, $\tau_k^* > 0$ and $\tau_c^* > 0$. While for high enough values of x, we have $\theta^* > 0$, $\tau_k^* > 0$ and $\tau_c^* > 0$.*

Table 3.2 reports the results.[21] Several observations are worth highlighting. Firstly, the values of both θ^* and τ_k^* will increase with the value of x, which is straightforward. Secondly, the optimal τ_c is always the maximum value 0.2 under the restriction, i.e., we should tax consumption as much as possible, or consumption taxation should be ranked as the best in terms of improving welfare. The results are consistent with those in Cooley and Hansen (1991), which particularly claimed the welfare costs of financing a given government expenditure are dramatically lower for economies that substitute consumption tax for the tax on capital income tax. Thirdly, with a low enough value of x, we have $\theta^* < 0$ and $\tau_k^* > 0$ as in the table. In such case, the two fiscal taxes dominate the inflation tax in terms of improving welfare.

Table 3.2: Optimal welfare-maximizing policies for benchmark values.

	Optimal τ_k	Optimal θ	Corresponding τ_c
$x = 0$	0.16	-0.0290	0.2
$x = 0.1$	0.38	-0.0228	0.2
$x = 0.35$	0.54	0.0089	0.2

Our ranking is different from Devereux and Love (1994), in which they claim that a capital income tax carries the highest welfare cost. Until now, we have shown that consumption taxation will dominate the other two taxes in terms of promoting growth and improving welfare in some cases as well. Then, the next step, we would like to examine, if there is no consumption taxation, how will the government tradeoff between the other two taxes, i.e., a capital income tax and an inflation tax.

[21] We only consider x ranging form 0% to 35%. We believe it is enough for calibration, since U.S. government's spending to output ranging from 8% to 35% from 1929 to 2003. In addition, we also restrict the value of τ_c to be no more than 20%, based on reality.

5. A special case with no consumption tax

When there are only two policy instruments (τ_k and θ), we could reduce (168) to

$$\frac{\alpha^2(1-\gamma)\theta}{\sigma g+\rho+\theta-g} = x-\alpha^2\gamma\tau_k. \tag{203}$$

Plugging equation (203) into equation (189) to eliminate θ ($\tau_c = 0$), we could get the relationship between g and τ_k,

$$\frac{\alpha(1-\tau_k)(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} + \frac{(1-\alpha)}{\epsilon(1-l)} = 1-x+\frac{1-\alpha}{\epsilon},$$

where

$$l = 1-\frac{\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}\left[\frac{\alpha^2(1-\gamma)(\sigma g+\rho-g)}{\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k}\right]^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}{\alpha(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}}.$$

Thus we have

$$\frac{\alpha(1-\tau_k)(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} + \frac{\alpha(1-\alpha)^2(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}\left[\frac{\alpha^2(1-\gamma)(\sigma g+\rho-g)}{\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k}\right]^{\frac{\alpha-\alpha\gamma}{1-\alpha}}} = 1-x+\frac{1-\alpha}{\epsilon}, \tag{204}$$

Equation (204) will determine the growth rate based on which we have the following Proposition 4.

Proposition 4: The growth-maximizing τ_k is negative, when $x < \min[\frac{1-\alpha}{\epsilon}, \frac{\alpha^2(1-\gamma)(1-\alpha)}{1-\alpha+\alpha\gamma}]$ and $\sigma > 1$.

Proof. Re-write equation (204) as

$$F(g) = \frac{\alpha(1-\tau_k)(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} + \frac{\alpha(1-\alpha)^2(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}\left[\frac{\alpha^2(1-\gamma)(\sigma g+\rho-g)}{\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k}\right]^{\frac{\alpha-\alpha\gamma}{1-\alpha}}} = 1-x+\frac{1-\alpha}{\epsilon}.$$

We have

$$\frac{\partial F}{\partial g} = \frac{\alpha(1-\tau_k)(1-\alpha+\alpha\gamma)\rho}{(\sigma g+\rho)^2} + \frac{\alpha(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}[\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k]^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+2-2\alpha}{1-\alpha}}[\alpha^2(1-\gamma)(\sigma g+\rho-g)]^{\frac{1-\alpha\gamma}{1-\alpha}}}\left[\sigma(\sigma-1)g+\rho(\sigma+\alpha-\alpha\gamma)\right],$$

$$\frac{\partial F}{\partial \tau_k} = -\frac{\alpha(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} + \frac{\alpha(1-\alpha)(1-\tau_k)^{\frac{\alpha\gamma}{1-\alpha}}[\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k]^{\frac{2\alpha-\alpha\gamma-1}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}[\alpha^2(1-\gamma)(\sigma g+\rho-g)]^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}\left[\alpha^2\gamma(1-\tau_k)+(1-\alpha+\alpha\gamma)(x-\alpha^2)\right].$$

We want to find the optimal τ_k such that $\frac{dg}{d\tau_k} = 0 \Leftrightarrow$ at τ_k^*, we have $\frac{\partial g}{\partial \tau_k} = \frac{-\frac{\partial F}{\partial \tau_k}}{\frac{\partial F}{\partial g}} = 0$, i.e.,

$$\frac{\alpha(1-\alpha+\alpha\gamma)g}{(\sigma g+\rho)} = \frac{\alpha(1-\alpha)(1-\tau_k^*)^{\frac{\alpha\gamma}{1-\alpha}}[\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k^*]^{\frac{2\alpha-\alpha\gamma-1}{1-\alpha}}}{\epsilon\xi\eta(\sigma g+\rho)^{\frac{\alpha\gamma+1-\alpha}{1-\alpha}}[\alpha^2(1-\gamma)(\sigma g+\rho-g)]^{\frac{\alpha-\alpha\gamma}{1-\alpha}}}[\alpha^2\gamma(1-\tau_k^*)+(1-\alpha+\alpha\gamma)(x-\alpha^2)].$$

Plugging the above equation into equation (204),we have

$$\frac{\alpha(1-\alpha+\alpha\gamma)g(1-\tau_k^*)}{(\sigma g+\rho)}\left\{1+\frac{(1-\alpha)[\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k^*]}{\alpha^2\gamma(1-\tau_k^*)+(1-\alpha+\alpha\gamma)(x-\alpha^2)}\right\} = 1-x+\frac{1-\alpha}{\epsilon}. \quad (205)$$

When $x < \min[\frac{1-\alpha}{\epsilon}, \frac{\alpha^2(1-\gamma)(1-\alpha)}{1-\alpha+\alpha\gamma}]$ and $\sigma > 1$, if $\tau_k^* \geq 0$, the RHS is greater 1, however, the LHS is less than 1, since $(1-\alpha)[\alpha^2(1-\gamma)-x+\alpha^2\gamma\tau_k^*] > 0$, $\alpha^2\gamma(1-\tau_k^*)+(1-\alpha+\alpha\gamma)(x-\alpha^2) < 0$, and $\max(\frac{g}{\sigma g+\rho}) = \frac{1}{\sigma} < 1$. Contradiction! Thus, we have $\tau_k^* < 0$, as long as $x < \min[\frac{1-\alpha}{\epsilon}, \frac{\alpha^2(1-\gamma)(1-\alpha)}{1-\alpha+\alpha\gamma}]$ and $\sigma > 1$. *Q.E.D.* As along as the government expenditure share is low enough, it is optimal to subsidize the capital.

5.1 Numerical results

Next, we calibrate the special case under the benchmark values of the model parameters in section 4.3. And the following results report the growth and welfare effect of the two policy instruments.

Result 2: *When capital income tax is low, an increase in it will reduce leisure and ratio of consumption to output and promote growth. When capital income tax is high, an increase in it will then increase leisure and ratio of consumption to output and decrease the growth rate. The growth-maximizing capital income tax is negative when x is low, and positive when x is high and will increase with x.*

Result 3: *When capital income tax is low, an increase in it will increase welfare. When capital income tax is high, an increase in it will then decrease welfare. The welfare-maximizing capital income tax is positive and will increase with x.*

Table 3.3 reports the results in detail. In addition, since both the growth and welfare maximizing nominal interest rates are strictly positive, the Friedman rule is suboptimal. We also do sensitivity analysis with respect to the value of input intensity γ (1-γ). Table 3.4

shows that the magnitude of the optimal capital tax rate depends largely on the value of the intensity γ of capital in the monopoly sector. On one hand, an increase in γ will increase the effectiveness of the capital income tax in collecting tax revenue , i.e., a higher γ will induce a higher reduction in the money growth rate given the same amount of increase in the capital income tax, which will generate a higher positive market size effect. On the other hand, since γ measures the capital intensity, and $(1-\gamma)$ measures the money intensity, i.e., a higher γ means money will contribute less in the monopoly production, i.e., the reduction in the money growth rate will have a lower market size effect. Under the bench mark a higher value of γ just affects the magnitude of the two policies, decreasing both the optimal capital income tax and the money growth rate, however, will not affect the ranking of the two polices in terms of improving welfare. [22]

Table 3.3: Results for special case.

	Growth-Maximizing				Welfare-Maximizing			
	τ_k	θ	π	i	τ_k	θ	π	i
$x = 0$	-0.21	0.01806	-0.0179	0.0860	0.49	-0.01899	-0.0345	0.0388
$x = 0.05$	-0.11	0.02346	-0.0059	0.0881	0.54	-0.0152	-0.0254	0.0398
$x = 0.1$	-0.02	0.03057	0.0078	0.0920	0.59	-0.01138	-0.0161	0.0410
$x = 0.15$	0.07	0.03854	0.0223	0.0967	0.64	-0.0072	-0.0067	0.0425

[22]Under other values of the parameters, the value of γ may change the ranking of the two policies, e.g., when $\alpha = 0.8$, $\gamma = 0.5$, $\eta = 1$, $\rho = 0.05$, $A = 0.24$, $\sigma = 1$, $\epsilon = 0$ and $x = 0$, we prefer capital income tax to inflation tax if $\gamma \geq 0.3$, and the ranking reverses if $\gamma \leq 0.3$.

Table 3.4: Results for special case: sensitivity analysis on γ, when $x = 0$.

	Optimal capital income tax	Optimal money growth rate
$\gamma = 0.2$	0.91	-0.008987
$\gamma = 0.4$	0.59	-0.01606
$\gamma = 0.5$ (Benchmark value)	0.49	-0.01899
$\gamma = 0.6$	0.42	-0.02266
$\gamma = 0.7$	0.34	-0.02688
$\gamma = 0.8$	0.26	-0.032660
$\gamma = 0.9$	0.16	-0.04123
$\gamma = 0.99$	0.03	-0.06044

In summary, we could rank the three taxes as follows: consumption tax (τ_c), inflation tax, capital income tax (τ_k) in terms of promoting growth and the ranking is: consumption tax (τ_c), capital income tax (τ_k), inflation tax in terms of improving welfare. Thus, a capital income tax carries the highest growth cost, but an inflation tax carries the highest welfare cost. Our ranking is different from those which claim that an inflation taxation is better than a income taxation as the instrument to finance public expenditure in terms of promoting welfare (Phelps (1973), Braun (1994) and Palivos and Yip (1995)). We next compare growth and welfare costs under money financing and capital income tax financing given the same size of government expenditure share. The costs refer to the difference in the growth rate and welfare compared to the values in a Laissez-faire equlibrium ($g = 0.034$, $W = -3.6090$). Table 3.5 reports the results.

Table 3.5: Growth and welfare costs of financing equalized government expenditure share.

	Capital tax	Inflation tax	Capital tax	Inflation tax
	Growth cost	Growth cost	Welfare cost	Welfare cost
$x = 0.1$	-0.01667	-0.01127	-4.697	-10.9942
$x = 0.15$	-0.0259	-0.0181	-7.367	-19.1677

Accord with Palivos and Yip (1995), for any given government expenditure size, the decrease in the growth rate is smaller under money financing than under income tax financing, i.e., the inflation tax will be more favorable in terms of promoting growth. However, in their paper, this ranking remains the same for welfare, while in ours, the welfare cost is smaller under income tax financing. These results also are conflict with the ideas in Cooley and Hansen (1991) that substituting inflation tax for the tax on capital income tax will generate a lower welfare cost. The reason behind this is that inflation tax will induce a larger decrease in leisure and consumption compare with capital income tax leading to a much higher static loss. Milton Friedman (1969) stated that, different productive activities may differ in cash-intensity, just as they differ in labor - or land - intensity. Dennis and Smith (1978) presented a detailed evaluation of the role of real cash balances as a productive factor for 11 two-digit (SIC code) industries over the period 1952-73 and claimed real cash balances do play an important role in the production technology for the industries evaluated. They estimated the interest elasticity of demand for real cash balances varies over the industries studied, ranging from -0.219 to -0.409. In Sinai and Stokes (1972) with the US data in the sample period 1929-67, they estimated an aggregate Cobb-Douglas production function in which the factors are capital, labor and real balances and claimed an elasticity of gross private domestic product with respect to real money balances of 0.17 and an elasticity of gross private domestic product with respect to capital of 0.585. i.e., the equivalent γ ($\frac{\gamma}{1-\gamma} = \frac{0.585}{0.17}$) is 0.775.

As we have stated in the intermediate good production sector, following Shaw et al. (2005), the monopoly production function is derived by an AK technology with a financial intermediation system. Capital's share in intermediate goods production γ could measure how developed is the financial system, i.e., with a high γ, more resources would be allocated from financial intermediaries to production sector. Our results are consistent with the data of average money growth rate collected by the world bank between year 2009-2013, i.e., most of the developed countries (associates with high γ) have a relative low average money growth rate, such as -0.825% in Austria, -0.15% in Germany, and 0.075% in United Kingdom, while, most of the developing countries (associates with low γ) have a relative high average money growth rate, such as 23.075% in Vietnam, 19.75% in China, and 25.05% in Cambodia.

6. Conclusion

This chapter compares the effects of a consumption tax, a capital income tax and an inflation tax on resource allocation, growth and welfare in an R&D growth model with variety expansion and money-in-production. As mentioned in the possible extensions for future research in Chu and Lai (2013), our parallel R&D growth model evaluating the welfare effects of inflation when it serves as a source of tax revenue along with other distortionary tax instruments. Similarly, we show that given an exogenous government expenditure and in the presence of consumption tax, both the money growth rate and the tax on capital income has a pure negative growth effect, which are consistent with the results developed in their quality ladder innovation model. We also show the optimal rate of both consumption and capital income tax should be positive and it is optimal to have deflation if the government expenditure level is low enough. The results mainly arise from the monopoly inefficiency which leads to less than optimal demands for both capital and real money balances.

As a result, we then re-consider the problem in the case without the consumption tax, and show that the capital tax will be more favourable in terms of improving welfare, and the inflation tax will be more effective in stimulating growth. We claim that inputs intensity in the inefficient sector affects the magnitudes of growth and welfare effects. Our ranking towards improving welfare are different from those in Palivos and Yip (1995) and in Cooley and Hansen (1991), since inflation tax will induce a larger decrease in leisure and consumption compare with capital income tax leading to a much higher static loss.

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