# LINER SHIPPING SPEED AND BUNKERING MANAGEMENT UNDER STOCHASTIC ENVIRONMENT

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**DECLARATION**

I hereby declare that the thesis is my original work and it has been written by me in its entirely. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Sheng Xiaoming 30 July 2013

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SHENG XIAOMING
30 JULY 2013

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Finally, I want to tell my parents and my wife, Charlene, that your **Love** is best thing that ever happened to me.

# Contents

# Summary

Liner shipping operation level decision problem, speed and bunkering management particularly, has been an area that only attracted scarce research attention albeit its significant impact on the profitability of liner companies. The reason is twofold. One is that the shipping industry in general, partly due to its long history, was conservative, fragmented and less willing to adopt changes. It was also largely out of public's sight. "Out of sight, out of mind", researchers' minds as well. The other is that uncertainties involved in the operation level render both the modeling and solving extremely difficult. The mission of this thesis is therefore to fill in this gap and study the operational speed and bunkering management from the liners' perspective.

The first and foremost motivation for this work is the observed fact that in recent years the bunker prices have been increasing and fluctuating dramatically. While the bunker cost takes up more and more percentages of the total operational costs, shipping companies are relentlessly seeking efficient ways to reduce it. One practice that has gradually gained popularity is slow steaming. However, simply slowing down the vessels is not the final answer as they operate under the "stochastic" environment. Bunker prices change everyday and they differ significantly in different ports. Bunker consumption under the same speed for the same distance also depends on weather and sea conditions. Therefore, the first part of this work studies how to dynamically determine the vessel speed and refueling decisions considering the bunker prices and consumption uncertainties. The stochastic nature of the bunker prices is represented by a scenario tree structure. As the model is a large-scale mixed integer programming model, we adopt a modified rolling horizon method to tackle it. Numerical results based on two real liner services with size differences show that our framework provides a lower overall cost and more reliable schedule compared with the stationary model of a related work.

For liner shipping practitioners, it would be highly appealing if there were a simple, yet effective, strategy that guides timely operational decision making on a daily basis. Second part of this work expounds on this issue. We adopt a dynamic $(s, S)$ policy which has been effectively used in inventory management to solve a liner shipping refueling and vessel speed determination problem under both bunker prices and consumption uncertainties. Such a policy allows a more flexible operational bunkering plan; the decision of whether to bunker or not depends on the actual bunker prices as well as the realized ship bunker inventory at every port. In addition, different from the first study where bunker consumption uncertainty is tackled by chance constraints, here we randomly generate a random sample of consumption scenarios and use sample average approximation (SAA) method to handle it. Due to the large size of our stochastic mixed-integer programming model, we propose two variants of the progressive hedging algorithm (PHA) to solve it. Numerical results show that our solving approach is efficient and the $(s, S)$ policy model has the potential to be implemented in the real practice easily and help liners save large amounts of operational costs.

Last part of this work is to coordinate the management of bunker fuel purchasing for all the service routes under the same network. We study the bunker fuel purchasing problem for a whole liner shipping network or even multiple networks under a novel cooperation scheme between liner shipping companies and bunker suppliers. More specifically, bunker suppliers at certain ports offer liner shipping companies some price discounts according to their fleet's weekly or monthly bunker consumption. Under this situation, the bunkering decision of individual shipping routes are no longer independent, and shipping companies need to play the role as the overall decision making center and determine the bunkering plan for all service routes in the shipping network. The resulting model is a very large size mixed integer non-liner programming model which cannot be solved efficiently by the state-of-the-art commercial solvers. However, the problem structure allows us to handle it with a heuristic algorithm based on column generation. In addition, we also devise another two straightforward and effective greedy heuristic algorithms. According to our numerical experiments, our model could help significantly reduce the bunker cost for liner shippers and our heuristic algorithms consistently provide high quality near-optimal solutions.

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# Nomenclature

| | |
|---|---|
| $R$ | total number of price scenarios; |
| $\Pi^r$ | the probability that price scenario $r$ happens; |
| $n$ | number of port of calls; |
| $d_{i,j}$ | distance between port $i$ and port $j$(nautical miles); |
| $t$ | total cycle time (h); |
| $t_i$ | port time(time one ship spends on entering, unloading and loading cargo, idling and exiting) at port $i$ (h); |
| $e_i$ | earliest arrival time at port $i$; |
| $l_i$ | latest arrival time at port $i$; |
| $C_i$ | bunker fuel consumption when the ship is at port $i$; |
| $w$ | bunker fuel capacity for a single ship; |
| $v_{min}$ | minimum ship sailing speed (nautical miles/h); |
| $v_{max}$ | maximum ship sailing speed (nautical miles/h); |
| $P_i^r$ | bunker price for port $i$ under scenario $r$; |
| $f$ | fixed bunkering cost; |
| $\gamma$ | coefficient to control the service level; |
| $h$ | inventory holding cost pmt for bunker; |
| $\eta$ | coefficient of variation for daily bunker consumption rate |
| $V_{i,j}^r$ | ship speed between port $i$ and $j$ under scenario $r$; |
| $S_i^r$ | bunker fuel-up-to level for the ship at port $i$ under scenario $r$; |
| $B_i^r$ | bunkering decision variable. =1 if bunkering at port $i$ under scenario $r$; =0, otherwise; |
| $I_i^r$ | bunker fuel inventory when the ship reaches port $i$ under price scenario $r$; |

| | |
|---|---|
| $\overline{F}^r_{i,j}$ | mean of daily bunker consumption rate for a ship travels from port $i$ to $j$ under price scenario $r$; |
| $\delta^r_{i,i+1}$ | standard deviation of bunker fuel consumption between port $i$ and $i+1$ under price scenario $r$; |
| $D^r_i$ | standard deviation of ship bunker inventory when ship reaches port $i$ under price scenario $r$; |
| $A^r_i$ | ship arrival time at port $i$ under scenario $r$; |
| $K$ | total number of bunker consumption scenarios generated; |
| $\boldsymbol{V}_{i,i+1}$ | The set of discretized feasible sailing speeds (nautical miles/hour) between port $i$ to $i+1$; |
| $G_{i,i+1}$ | set of all possible choices in $\boldsymbol{V}_{i,i+1}$. |
| $\tau_1$ | denotes the penalty for violating the bunker inventory constraint |
| $\tau_2$ | denotes the penalty for violating the time window constraint |
| $X^r_{i,1}$ | amount of time that the ship reaches port $i$ earlier than schedule; $=0$ otherwise; |
| $X^r_{i,2}$ | amount of time that the ship reaches port $i$ later than schedule; $=0$ otherwise; |
| $Y^{k,r}_i$ | indicator variable.$=1$ if bunker inventory when ship reaches port $i$ is less than a certain amount; $=0$ otherwise; |
| $M$ | is a big positive number; |
| $\theta^k_i$ | normal random number with mean 1 and standard deviation $\eta$ during leg $i$ and scenario $k$; |
| $\epsilon$ | is a very small positive number; |
| $Z^{g,r}_{i,i+1}$ | indicator variable.$=1$ if sailing speed $v^g_{i,i+1}(v^g_{i,i+1} \in \boldsymbol{V}_{i,i+1}, g \in G_{i,i+1})$ is chosen under price scenario $r$; $=0$ otherwise; |
| $S^r_i$ | bunker fuel-up-to level for the ship at port $i$ under price scenario $r$; |
| $s^r_i$ | bunker ordering point for the ship at port $i$ under price scenario $r$; |
| $B^{k,r}_i$ | bunkering decision variable. $=1$ if bunkering at port $i$ under bunker consumption scenario $k$, $=0$, otherwise; |
| $I^{k,r}_{i,1}$ | bunker fuel inventory when the ship reaches port $i$ under bunker consumption scenario $k$ and price scenario $r$ ; |
| $I^{k,r}_{i,2}$ | bunker fuel inventory when the ship departs port $i$ under bunker consumption scenario $k$ and price scenario $r$ ; |

| | |
|---|---|
| $\bar{F}^g_{i,i+1}$ | average bunker consumption per nautical mile when the sailing speed is $v^g_{i,i+1}$, and $\bar{F}^g_{i,i+1} = (k_1 \cdot (v^g_{i,i+1})^3 + k_2)/(24 \times v^g_{i,i+1})$; |
| $\theta_{rij}$ | $= 1$ if port $i$ is the $jth$ visiting port on route $r$; $= 0$ otherwise; |
| $n_r$ | the total number of ports visited by route $r$; |
| $f_i$ | fixed bunkering cost at port $i$; |
| $a_{rj}$ | bunker consumption at the $jth$ port on route $r$; |
| $d_{rj}$ | nautical distance at the $jth$ leg on route $r$; |
| $h$ | inventory holding cost per metric ton (pmt); |
| $t_{rj}$ | port time (time one ship spends on entering, unloading and loading cargo, idling and exiting) at the $jth$ leg on route $r$; |
| $e_{rj}$ | earliest arrival time at the $jth$ leg on route $r$; |
| $l_{rj}$ | latest arrival time at the $jth$ leg on route $r$; |
| $Q_r$ | bunker fuel capacity for the ships deployed on route $r$; |
| $\eta_r$ | CV of daily bunker consumption rate for the ships deployed on route $r$; |
| $\beta_r$ | service level coefficient; |
| $x_{rj}$ | binary variable; $= 1$ route $r$ bunkers at its $jth$ port; $= 0$ otherwise; |
| $y_{rj}$ | the bunkering amount of route $r$ at its $jth$ port; |
| $z_{iw}$ | binary variable; $= 1$ if price discount $w \in \Omega_i$ is utilized at port $i \in N$; $= 0$ otherwise; |
| $v_{rj}$ | vessel speed at the $jth$ leg on route $r$; |
| $c_{rj}$ | mean bunker consumption at the $jth$ leg on route $r$; |
| $u_{rj}$ | bunker inventory level when the ship reaches the $jth$ port on route $r$; |
| $T_{rj}$ | ship arrival time at the $jth$ port on route $r$; |
| $sc_{rj}$ | standard deviation of the bunker consumption at the $jth$ leg on route $r$; |
| $su_{rj}$ | standard deviation of the bunker inventory when the ship reaches at the $jth$ port on route $r$; |

# Chapter 1

# Introduction

This thesis contributes to the dynamic speed and bunkering decision support for liner shipping under stochastic environment. In the first two sections of this chapter, we briefly introduce the characteristics of the liner shipping industry and the background of its current business environment. Subsequently, in Section 1.3, we will provide an overview of the research that has been dedicated to this area and highlight the gaps that exist and those of which we are attempting to bridge. The objective and scope of this thesis will be provided in Section 1.4.

## 1.1 Liner shipping industry

For any reader who may not be familiar with the liner shipping, feel free to think of it as the public bus service that almost everybody has at least experienced once or twice. The two of them share an important similarity: regardless of the demand, whether it be cargos or passengers, both have a predetermined and published schedule and a fixed sequence of ports or bus stops to call. This is because the determination of ship or bus routes is usually a mid-term planning problem for companies, which remains unchanged for at least a certain period of time. Other similarities include the network coverage of the service area by a fleet of homogeneous or heterogeneous vessels or buses and the necessity of service reliability. In maritime transport, there are three basic types of shipping operation; besides liner shipping, another two are tramp shipping and industrial shipping. To complete our previous analogy of maritime and public transport, tramp shipping is like taxis which provide more flexible services and industrial shipping is comparable to self-owned vehicles. Amongst these three, there is no doubt that liner shipping has become the most prevalent and important type of maritime shipping service.

Since the 1970$s$, containerization has gained its fast and tremendous popularity in the international maritime transportation due to its high efficiency and low cost of handling. Most of the vessels owned by ship liners are container ships, the size of which is denominated in how many TEUs (TEU stands for Twenty-foot Equivalent Unit and is the volume of a 20-foot-long intermodal container). According to Wikipedia [2013], the title of "the world's largest container ship 2012" was given to Marco Polo, which is operated by the French shipping firm CMA CGM and has a capacity of $16,020$ TEU. However, the crown will soon be given up to a $18,000$ mega container ship which is under building by a Korean dock for Maersk. The general trend is that more and more of those mega containers are going to be built and go into service.

Liner shipping has long been regarded as the world's economy engine either for its direct economical contribution or for its role as the facilitator of international trades. According to Worldshipping [2012], in 2007, liner shipping industry contributed a direct GDP of about US$ 183.3 billion and transported about 60% of the value of total global trade. Major international liners provide extensive coverage of almost every single port over the globe on a timely basis. As of 31st October 2010, there were approximately 400 liner services and 4800 container ships in operation (Marisec [2012]). From an environmental point of view, shipping is considered to be a more carbon-efficient mode of transportation than others, airline or rail industry for instance. Container ships mainly use bunker fuels, which are distillates from the crude oil refinery process, as its energy source. In a report done by Imo [2008], shipping industry was claimed to be accountable for only 2.7% of the global $CO_2$ emissions in year 2007. As a lot of technologies have been invented to improve the engine efficiency of ships and with more and more international regulations on green house gas emissions from shipping operators, the shipping industry will remain a relatively "green" transportation modality for a long time.

## 1.2 Business environment

Despite what we mentioned above, for most of the liner shipping companies, the current market conditions are tough indeed. The trend of increasing bunker prices has threatened the liner shipping companies' accounting bottom line and the oversupply of containers accumulated for the past few years makes the competition especially fierce. Ronen [2010] stated if bunker fuel prices reach around 500 US$/ton, fuel cost constitutes about 75% the total operating cost of a large containership. Therefore, it is not surprising that when the current bunker prices hover around 650 US$/ton, some shipping liners have complained that the cost of bunkers has formed a "lion share" of its operating costs. According to Shipandbunker [2012], Japanese shipping line Nippon Yusen Kabushiki Kaisha (NYK) experienced an average bunker price of 642.01 US$/ton in the first six months of 2011 and in the second half of the year, it jumped to 690.43 US$/ton. The year 2012 had still not been easy for shipping liners. Figure 1.1 shows the bunker prices (380 CST grade) at four major bunkering ports around the world from August to September 2012 (prices data are unavailable on weekends).

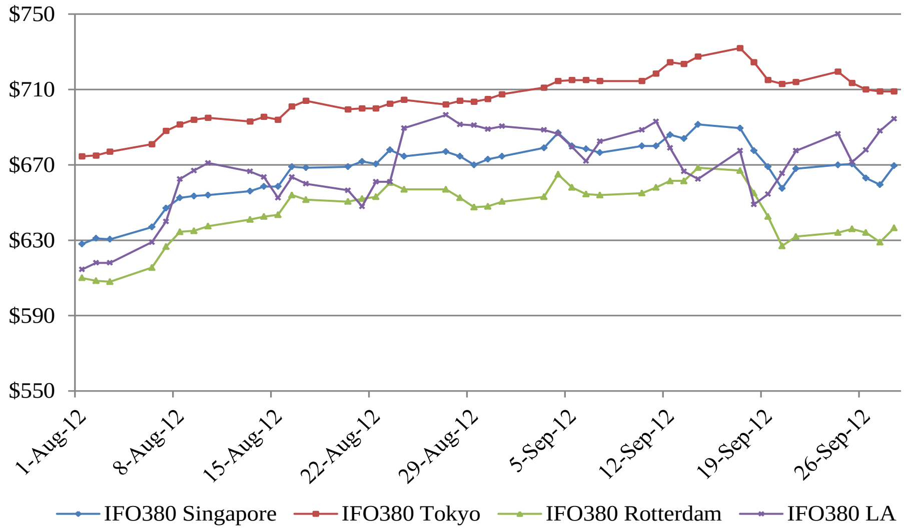


Figure 1.1: IFO380 prices at four major bunkering ports around the world from August to September, 2012 (Data source: http://shipandbunker.com)

To survive, companies need to identify ways to reduce the operating costs. For example, when the oil prices hit $145 a barrel in 2008, Maersk, the world's biggest liner shipping company, spearheaded the strategy of slow steaming. Now over 200 shipping companies have reduced their vessel speeds, especially in those long-haul loops like Asia to Europe and North America. Empirical estimation has shown that when the vessel speed is reduced by 20%, it could reduce the fuel consumption by 50% (Ronen [1982]). Although ship liners have to add one or two more vessels in certain routes to keep a weekly service, which results in an immediate increase of the capital cost as well as the administration and labor costs, the savings from fuel cost has the potential to outweigh those cost increases (Ronen [2011]). Besides, the environmental benefits of less greenhouse gas emission from slow steaming are also significant. Maersk [2010] announced that on average they had successfully reduced the carbon dioxide emission by 14% per vessel during 2008. Another reason is that slow steaming partially mitigates the industry wide over capacity problem, as more ships and containers are deployed in order to keep a weekly service under lower sailing speeds. However, there is a trade-off between sailing speed and service level. Thus, an optimization approach of determining the vessel speeds in the operational level, instead of decisions based on experience, is essential when we are talking about thousands of ships and liner service networks.

Besides increasing rapidly, bunker prices also manifest high volatility. It is a well-observed phenomenon that the crude oil prices fluctuate significantly on a daily basis. As a by-product of the crude oil, bunker prices fluctuate no lesser in the spot market. Figure 1.2 below shows the monthly fluctuation of the bunker prices (380 CST grade) at several major ports and that of the crude oil prices from September 2002 to September 2009. Based on this figure, we can roughly say that there is a high correlation of the bunker prices and the crude oil prices and most of the time, the bunker prices are even more volatile than the crude oil prices. Last but not least, bunker prices at different ports around the world usually have significant differences. For example, on 3 September 2008, bunker fuel prices (380 CST) in Singapore were 677.5 US$/ton. On the same day, bunker prices in Rotterdam were 619 US$/ton and 650 US$/ton in Houston.

The characteristic of the liner shipping is that it usually has a fixed number of port-calls in a cyclical route with a published schedule. While slow steaming would be the general trend when bunker prices are high, high fluctuation and regional differences of the bunker prices complicate the situation because simply

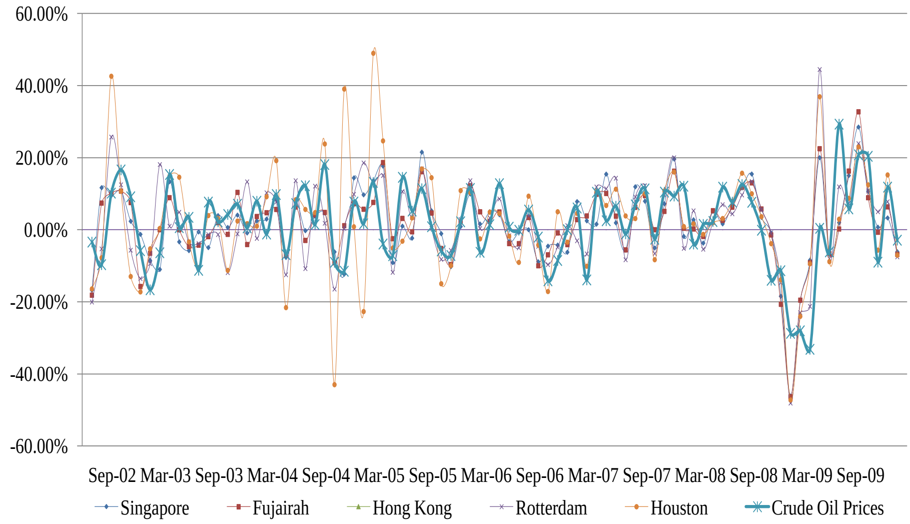


Figure 1.2: Fluctuation of bunker fuel (380 CST) prices at major bunkering ports and world crude oil prices (2002-2009) (Data source: Bloomberg 2009 and www.test.org/doe/)

reducing the vessel speed may miss out the opportunity of reaching the next port when bunker prices there are low. Thus, how to dynamically determine the vessel speed and bunkering decisions with timely updated information has huge cost reduction potential.

## 1.3 Research background

Compared with the uncertainty of bunker prices, the influence of the bunker consumption variation is less significant to the overall planning. However, it cannot be neglected either. Wind force and direction, sea condition, engine efficiency and other factors could change the total bunker consumption even when the ship sails under the same speed and for the same distance. To the best knowledge of authors, there is no published result that considers the bunker consumption uncertainty.

The fluctuation of bunker prices and the uncertainty of bunker consumption can impact the profitability of liner shippers significantly; however most of the previous related works did not tackle the uncertain nature of this problem sufficiently. On the one hand, maritime industry has received relatively less research attention compared to rail or airline industry. On the other hand, when uncertainties come into the picture, the model usually becomes very difficult to solve due to the large size of the problem.

One of the pioneering works in determining the optimal vessel speed was done by Ronen [1982]. It studied the trade-off between the fuel savings from slow steaming and the loss of revenue from the extension of voyage. It approximated the daily bunker consumption as a third power of the ship speed and derived the optimal speed for ships under different operating scenarios, namely, income generating leg, positioning leg and mixed leg. Bunker prices were assumed to be constant, the price difference across different ports was not considered and bunker consumption rate was a deterministic function of vessel speed. These three assumptions were largely retained by most of the subsequent works.

Until very recently, there have been works trying to provide more realistic models by relaxing these assumptions, however only partially (Besbes and Savin [2009], Oh and Karimi [2010] and Yao et al. [2012]). Yao et al. [2012] was considered to be the first work which explicitly took into account the bunker price difference across different ports. It brought up the intuitive idea of optimal bunkering: by bunkering enough fuel under fuel tank capacity limit at those ports with low prices, a large amount of bunkering cost can be reduced. However, as a planning level problem, no uncertainty was tackled in it.

## 1.4 Objective and scope

As indicated in the previous section, there are several research gaps in the area of liner shipping, which can be summarized as follows:

- While the fluctuation or uncertainty of the bunker prices is a matter of reality and hugely impacts the profitability of liner shippers, it has not been sufficiently addressed. The effect of bunker consumption uncertainty cannot be neglected neither. To the best knowledge of authors, there is no existing study which has seriously considered it based on actual data.
- Limited studies on the liner shipping operation level decision support are

available. Examples include how to adjust the vessel speed in the whole voyage, which port to bunker, and how much to bunker based on the available real-time information.

- We found no study devoted to the bunkering management of ships for multiple service routes. Due to the presence of bunker price contract or bunker price discount, a bunkering management plan covering more than one liner service can be beneficial.

The main purpose of this thesis is to apply the stochastic optimization techniques to the liner shipping operation level decision support. The specific goals of this thesis are to:

- Study the uncertainties of bunker prices and bunker consumption and incorporate these two into our mathematical models.
- Propose a model that help to dynamically determine the ship speed and the bunkering decision with all the timely updated information.
- Introduce an effective $(s, S)$ refueling policy which can provide flexible bunkering decision support
- Identify the advantage of considering the bunkering management for one or multiple service networks of a liner company.

The significance of this study lies in the fact that it contributes to the practice of operational decision making in the liner shipping industry as well as the methodology of operations research (OR).

- High and fluctuating bunker prices in recent years have posed an unprecedented challenge for the profitability of the liner industry as a whole. With the consideration of bunker price fluctuation and the regional price difference, our dynamic models can be implemented by liner operators to significantly reduce their operational costs. In addition, the bunkering management plan for liner networks proposed in this thesis can be conveniently adopted by liner practitioners as a handy decision support tool. Besides, the cooperation scheme between liners and bunker suppliers studied in our work could shed light on the future research of bunker price contracts.

- As all the optimization models involved in our work are extremely large in scale, we have devised three state-of-the-art methodologies to tackle them. First, we propose a modified rolling horizon approach. Unlike the standard rolling horizon approach which solves a problem with a shorter horizon than the original problem, our non-standard approach still solves the problem with the whole study horizon. However, we assign a higher level of fidelity for the nearer periods than the later ones. The validity of this non-standard variant is due to our problem nature and the diminishing tail-end effect. Second, by taking advantage of the concepts of "integer convergence" and "variable fixing", we have successfully designed two variants of the progressive hedging algorithm which are capable of dealing with large scale mixed-integer programming problems. Last, a column generation heuristic is implemented in one of our problems where the presence of integer variables prevents the successful application of the classic column generation method. These solving approaches tailored to tackle our large-scale mixed-integer programming problems can be readily applied to other difficult optimization problems.

To simplify the problem and maintain the focus of our work, we have made the following assumptions: first, the service routes and schedules are fixed as the determination of them is more of a tactical level problem. Second, bunker purchasing only happens in the spot market. We do not consider the bunker price contracts as the specific terms and conditions between bunker suppliers and shipping liners vary significantly and are largely kept confidential.

## 1.5 Organization

This thesis consists of 6 chapters. Previous research papers will be reviewed in chapter 2. First, we will go through different levels (namely the strategic level, planning level and operation level) of decision problems in the liner industry. The determination of the level is based on the time horizon of the decision. Despite this difference, the nature of these problems are similar and therefore deserve a discussion of them all even though the focus of this work is on the operation level, real-time bunkering and speed decisions more specifically.

In chapter 3, we study the problem of dynamic bunkering ports selection and ship speed determination for a single vessel in one service route. While previous deterministic works focused more on the planning level of this problem, we aim at providing operational decision support by incorporating two major random factors into our model. Namely, the ship bunker consumption rate and the bunker prices at each port. Based on the real-world data obtained, we establish that the noise of daily bunker consumption follows a normal distribution with zero mean and constant coefficient of variation. For the stochastic nature of the bunker prices, we model it through the scenario tree which is widely used in financial engineering area to depict the randomness of the financial product returns. While solving a whole large dynamic problem is computationally challenging, we propose a solution method that can help to significantly reduce the computer memory requirement and solving time. This method is a combination of scenario tree generation scheme and a non-standard rolling horizon approach. Another advantage about this solving method is that as much new information as possible is used and previous forecasting errors can be easily corrected during the whole study horizon. Numerical examples based on real-world data showing the advantages of our dynamic model over the stationary model will be presented.

In chapter 4, we deal with the problem of devising a dynamic bunkering policy of the $(s, S)$ form which is similar in essence to the one used in the inventory management literature with the consideration of bunker prices and consumption uncertainties. Vessel speed during each leg is still the decision to be made while here we adopt the discrete speed choices for the reasons that we will provide later in the main context. To better incorporate the uncertain bunker consumption into our model, sample average approximation method is used, which randomly generate a random sample of consumption scenarios to feed into the model. As the final model turns out to be a very large scale mixed-integer programming problem, we tackle it with two variations of the progressive hedging algorithm. Numerical results demonstrating the effectiveness of the sample average approximation method and the progressive hedging algorithm will be shown. More importantly, through extensive experiments, we will also showcase the benefits of our dynamic policy model itself.

In chapter 5, we pursue a different approach to help liner shippers reduce the operational cost while facing the high and volatile bunker prices. That is to coordinate the management of bunker fuel purchasing for all the service routes under the same network. Bunker suppliers at certain ports offer liner shipping companies some price discounts according to their fleet's weekly or monthly bunker consumption. Under this situation, the bunkering decision of individual shipping routes are no longer independent, and shipping companies need to play the role as the overall decision making center and determine the bunkering plan for all service routes in the shipping network. With this regard, this chapter dedicates special efforts to the new decision problem, as is referred to as strategic bunkering and speed management in this work, and design an optimal bunker purchasing and speed control plan for a liner shipping company under the cooperation bunkering scheme. As our model is a very large-scale mixed integer non-liner programming model which cannot be solved efficiently by the state-of-the-art commercial solvers, we propose a column generation heuristic (CGH) algorithm and two greedy heuristic algorithms to solve it. Numerical studies based on comprehensive liner networks will be conducted.

The final chapter, Chapter 6, summarizes this thesis and discusses several directions for future research.

# Chapter 2

# Literature review

In this chapter, we will present a survey of literature on liner shipping planning problems in the first section with the emphasis on speed and bunkering determination problem. In the second section, works in other areas that are pertinent to our study will be briefly introduced. The last section will be given to a discussion of several solving approaches that will be used to solve our large scale mixed-integer programming problems. This is the most common difficulty we will encounter in solving a liner shipping speed and bunkering determination problem when either the size of the problem becomes large or when uncertainties come into the picture.

## 2.1 Liner shipping planning problems

Typically, there are three levels of planning in the liner shipping industry, namely the strategic, tactical, and operational planning. As in Christiansen et al. [2007], strategic planning problems are usually long term decision problems (e.g. 2 years) which consist of market and trade selection, ship design, network and transportation system design, fleet size and mix decisions (type, size, and number of vessels), and port/terminal location, size, and design. The tactical problems considers medium term decisions (e.g. 6 months) and covers adjustments to fleet size and mix, fleet deployment (assignment of specific vessels to trade routes), ship routing and scheduling, inventory ship routing, berth scheduling, crane scheduling, container yard management, container stowage planning, ship management, and distribution of empty containers. Operational planning deals with short term decisions (e.g. less than 1 month) which includes cruising speed selection, ship loading, bunkering, and environmental routing. Previous research efforts were largely put on the first two levels of planning problems, while operational planning received relatively less attention.

### 2.1.1 Strategic planning–network design

One important category of problem in the strategic level is the network work design problem. It is the very first decision shipping liners need to make before carrying out the business. Gelareh et al. [2010] addressed a hub-and-spoke network design problem with a newcomer liner service provider and one existing dominating operator. Their mixed integer linear programming model allowed the case of multiple stops along the hub-level network and direct spoke-spoke connections. Due to the large problem size, they devised an accelerated Lagrangian method together with a primal heuristic to derive the bounds which they showed through numerical examples to be quite efficient. Gelareh and Pisinger [2011] presented a model for network design and fleet deployment simultaneously. In order to solve the mixed-integer linear programming, they proposed a primal decomposition approach. One limitation in their work was that the model was designed for a single region and therefore one hub line was considered. Reinhardt and Pisinger [2012] combined the network design with fleet assignment and modeled it as a mixed integer programming problem. Realistic factors like cost of transhipment and a heterogenous fleet were taken into account. Finally, they used a branch and bound algorithm to tackle the problem and computational results on a real scale case were reported. Shintani et al. [2007] combined the study of two problems, liner shipping network design and empty container repositioning (ECR), that had been mostly analyzed separately. They formulated a two-stage model with the first stage to decide the sequence of calling ports and the second stage to evaluate the profit of container management with ECR. Their genetic algorithm-based heuristic was tested on a real container service and compared with the brute force method on the same problem. The genetic algorithm was shown to be effective and also the inclusion of ECR proves to be beneficial. Meng and Wang [2011] studied a liner shipping network design and ECR problem with combined hub-and-spoke and multi-port-calling operations. Dong and Song [2009] considered a joint fleet sizing and ECR problem with the assumption of zero inland transport time. A simulation model was developed and tackled by a combination of genetic algorithms and evolutionary strategy. Later Dong and Song [2012] extended the previous problem to case where inland transport time is stochastic.

### 2.1.2 Strategic planning–port selection

Port selection is closely related to network design as it decides which ports should be included in the service networks and which ports should not. Slack [1985] explored the factors that influenced ocean liners' choice of ports. Their study showed that price and service were the top two priorities contributing to a port's attractiveness while port infrastructures played a relatively less important role. Murphy et al. [1992] looked at the port competitiveness problem from a multi-participants perspective, larger or smaller liners, international water carriers and ports and international freight forwarders for instance. They showed through both univariate and multivariate analyses that, in terms of port competitiveness factors, different participants had varied evaluations. Guy and Urli [2006] studied the port selection problem based on a multi-criteria analysis (quality of infrastructures, cost and service). Their analysis were then applied on the port competitiveness comparison between New York and Montreal and suggested liners should bypass Montreal and call New York unless extensive hinterland coverage would be established in Montreal. Wiegmans et al. [2008] revisited this port and terminal selection problem with the focus on identifying key factors for deep-sea container shippers, some of which were largely neglected by previous research, for example, feeder connectivity and environmental issues. Another interesting finding, practically reasonable, was that the decision making was actually different across different type of carriers, trades and ports.

### 2.1.3 Tactical planning–fleet deployment problem

Decision of fleet deployment is carried out after the service networks are determined. Lane et al. [1987] provided a dynamic model to decide a cost efficient way of deploying fleets on pre-defined trade routes with known demand for each shipping service. Before this, there had been a scarcity of research work on general applicable analytical models for liner shipping for the reasons mentioned in their paper. One limitation when it came to the solution approach of its model was that since only a subset of the feasible voyage options were chosen, there was no guarantee that an global optimal solution could be reached. Perakis and Jaramillo [1991] and Jaramillo and Perakis [1991] used linear programming to solve a fleet deployment problem (FDP) which was to assign an existing available fleet of vessels to a given set of service routes. Details for estimating the operating costs of liner vessels on different routes were also given. Because of the non-linearity of the vessel speed, it was determined independently before solving the LP model. A real world case study was conducted in their papers. Later Powell and Perkins [1997] extended those two works by formulating the FDP as an IP model. The objective was to minimize the total operating and layup costs for a fleet of liner ships on several different routes. Cho and Perakis [1996] suggested two optimization models (one profit maximization model and one cost minimization model) for the optimal fleet size and design of liner shipping. Before solving the problem as a Linear Programming model, a set of candidate routes were generated a priori for different ships. Fagerholt [1999] studied the problem of optimal determination of fleet size and mix as well as routing policy for the liner ships. A three phase solution method was proposed and efficiently implemented on a network of 19 ships and 40 ports. One of the limitations was that the feasible routes have a route time no longer than one week. In addition, both aforementioned works did not consider transhipment. Xinlian et al. [2000] presented an algorithm which used a combination of the linear programming and dynamic programming techniques to improve the solution to linear model of a long-term FDP for liner shippers. Gelareh and Meng [2010] modeled a short-term FDP as a mixed integer nonlinear programming problem. The optimal vessel speeds for different type of vessels and routes were obtained indirectly by their realistic optimal travel times. The model was then linearized and solved by CPLEX.

### 2.1.4 Tactical planning–ship scheduling

Ship scheduling is another important tactical level decision. A good schedule immediately translates into an attractive service. It also interacts with fleet deployment as the ship scheduling determines how many ships are needed for each route. In addition, the choice of ship sailing speeds is also a direct result of published schedules. Boffey et al. [1979] developed both an interactive computer program and an optimization model based on heuristics for the scheduling of container ships on the North Atlantic. Even though the interactive program was not very sophisticated as it asked for a lot of direct human control, it was a quite realistic representation of the real world ship scheduling problem. Rana and Vickson [1988] studied a problem of optimal routing of chartered container ships. More specifically, to study whether a container should be chartered or not. Benders' partitioning method and a dynamic programming scheme were used together to achieve an effective solving of the Non-linear integer programming model. Rana and Vickson [1991] presented a model for routing multiple ships. The decisions to make were the optimal sequence of port calls, the frequency of service and the amount of cargo transported between any pair of ports for each ship. To handle the mix-integer programming, they used Lagrangian Relaxation to decompose the problem as well as the network. Ronen [1993] did a very thorough summarization of works on the ship scheduling and related problems for the past decade before that review paper. In lieu of the set partitioning method which had been largely adopted in solving ship scheduling problems, Brønmo et al. [2007] proposed a multi-start local search heuristic. This heuristic combined an initial random generation of large amount of solutions with either a quick or extended local search that was designed to improve the best incumbent solution so far. Numerical studied presented in that paper demonstrated the consistent good performance for real-world-size ship scheduling problems. Agarwal and Ergun [2008] modeled the simultaneous ship-scheduling and cargo-routing as a mixed-integer program. Realistic constraints like weekly frequency and transhipment were incorporated. An optimal decision simultaneously decides the optimal choice of service routes, cargo to deliver, and the delivering paths. As for the solving approaches, they used three different algorithms: a greedy heuristic, column generation and Benders decomposition, all of which, as claimed, could reach within 1% to 1.5% of optimality. A recent review on ship scheduling and routing was done by Christiansen et al. [2012], who claimed that the volume of research on this area almost doubles every decade. Also the scope of the research had been widened and more specialized problems had been addressed.

### 2.1.5 Operational planning–operational scheduling and environment routing

In the previous section, we discussed about ship scheduling as a tactical planning problem. However, under certainty circumstance (significant demand uncertainty of cargo delivery for example), it can be operational too. In addition, even though many of the ship schedules are determined on the tactical level, they rarely follow them on the actual day of sailing because of the change of weather conditions, port congestion and mechanical problems. Ronen [1986] studied the ship scheduling problem of a fleet of non-homogeneous ships (different sizes and cost functions) to deliver a set of cargo to several destinations. Three solution algorithms were proposed and their results are compared with industry practice, which showed that substantial cost reduction could be achieved by operational scheduling.

Due to the unpredictable nature of the sea environment, vessels may need to deviate from their pre-scheduled routes on the actual day of sailing. Environment routing helps shippers avoid the negative influence of unpredictable environment as much as possible or even take advantage of it.

Papadakis and Perakis [1990] studied a vessel minimal time routing problem between an origin and destination pair with stationary or time dependent seas. The decisions to make were the route as well as the power setting of the vessel based on the knowledge of wave which was a function of location. Variational calculus and optimal control theory were used respectively for stationary and time dependent case. Two major extensions of the previous work were made in Perakis and Papadakis [1989]. Firstly, time-dependent routing, which allows the sea condition at any point to change over time, was considered. Secondly, instead of a single origin and destination pair, they considered multiple legs with known leg length. Their solving approach was based on a combination of first variation considerations with global boundary conditions. Lo et al. [1991] studied the value of knowing the ocean current information in strategic routing by calculating the fuel consumption difference with or without using this information. They claimed that, for US only, more than $10 million of fuel cost could be saved by taking advantage of the ocean currents. With the development of more sophisticated technologies which can predict the ocean current information more accurately, the cost reduction potential can be ever higher.

### 2.1.6 Operational planning–liner ship bunkering and speed determination

Even though the ship speed determination problem, as we claimed, is a relatively new research area, there were some early works touching on this topic which dated back as early as 1970$s$, Avi-Itzhak [1974], Jun [1975] and Ryder and Chappell [1979] for instance. However, we think that the first work that gave this topic a serious discussion was done by Ronen [1982]. There were some very interesting points discussed that laid the foundation for future research. Firstly, there is a trade-off between bunker fuel saving by reducing the ship speed and the loss of revenue associated with slow steaming. Secondly, by estimation, a third power relationship exists between the bunker consumption and ship speed. This means if the ship speed is reduced by 20%, around 50% of bunker consumption can be saved. Lastly, voyages of ships mostly consists of incoming generating leg, position leg and mixed leg. Optimal speeds for ships under those three different operating scenarios were derived. Perakis and Papadakis [1987a] studied the cost minimization problem that a fixed amount of cargo needs to be delivered within a specific period of time between one loading port and one unloading port by a fleet of ships under fixed contract prices. Total fleet operating costs were minimized by choosing the optimal full load and ballast vessel speeds. It modeled the all-purpose fuel (fuel that includes propulsion fuel and all that used during ship operation) rate as a quadratic function of the power of a vessel, which in return was expressed in a power function of the ship speed. In a subsequent study, Perakis and Papadakis [1987b] extended the problem with multiple loading and unloading ports. Benford [1981] studied a problem of transporting a given quantity of commodity between two specific ports and proposed a simple algorithm to maximize the profit by selecting the ships and their respective sea speeds.

Notteboom and Vernimmen [2009] studied how liner shipping, facing high bunker fuel prices, had adapted their liner service schedules. In this study, the authors provided real-world data about the relationship between the fuel consumption per day and the ship speed for different size of container ships. From the data shown, we can see that the fuel consumption rate against speed for different sizes of ships is actually different although the authors did not look into the details of this issue. Ronen [2011] investigated into the trade-off between slow steaming and adding additional vessels in a container route. The objective was to minimize the annual operating cost of the route by deciding the optimal vessel speed and number of ships to deploy. Yao et al. [2012] studied the bunker fuel purchasing and vessel speed determination problem for an individual shipping route at the planning level. For a particular shipping route, the bunkering planning translated into selecting the bunkering ports along its shipping route as well as determining the bunkering amount at these ports, so that the total bunker purchasing cost was minimized. Their study facilitates the bunkering decision for individual service route with the underlying assumption that each route operates independently. On the relationship between the bunker consumption rate and the ship speed, they separated its analysis by the different sizes of ships. In addition, instead of assuming a single third power relationship, they added a constant coefficient in the regression model, which they proved to be non-trivial by numerical experiments. Wang and Meng [2012] presented a sailing speed optimization problem for a liner ship network with the consideration of transhipment and container routing. In all of those above studies, bunker prices were either assumed to be constant or not explicitly considered.

Oh and Karimi [2010] presented a mixed-integer liner programming model that optimized the operation of a multiparcel tanker under uncertain bunker prices. However, only a small number of independent price scenarios were generated before solving the model. Therefore, it was a stationary model in essence. For two types of vessel, “liners” and “trampers”, Besbes and Savin [2009] constructed a dynamic profit maximization problem and derived the optimal refueling policies. In the liner scenario, the problem reduced to a refueling cost minimization problem subject to random bunker fuel prices and limited vessel fuel capacity. However, vessel sailing speed was given in the problem formulation and bunker consumption uncertainty was not considered. They modeled the bunker prices as a sum of three parts: spot crude oil prices with a global price adjustment factor, local supply correction factor for the bunker prices and geographical adjustments due to some other factors. The spot crude oil prices were forecasted using a AR(1) mean-reverting process and the local supply correction factor is described as a two states Markovian process. Sheng et al. [2013] studied an operational level bunkering and vessel speed determination problem considering the bunker prices and consumption uncertainty. Bunker price uncertainty was modeled by a scenario tree structure and the standard deviation of bunker consumption during each leg was shown through statistical analysis based on actual data to be a constant percentage of mean consumption. Numerical studies in the work demonstrated the benefit of considering uncertainties.

## 2.2 Related works

We found that the research in the inventory management area is very relevant to our work. As the ship sails along, it consumes bunker fuel which is similar to the product demand for a manufacturing plant. We mentioned that the bunker fuel consumption is stochastic in nature, so is product demand. Bunker fuel inventory is similar to product inventory. Most of the time, a ship can only refuel when it reaches a port, therefore it is a "periodic review system". Other concepts like prices, inventory holding costs, capacity etc. apply to both situations.

### 2.2.1 Capacitated lot-sizing problem

Either in the real practice or in the research literature of production planning, capacitated lot sizing is a very important class of problems. In general, the target of production planning is to satisfy all the product demand with minimum cost for a certain period of time. If there are resource constraints, as in most of the real situations, it becomes a capacitated lot sizing problems (CLSP). The main challenge in CLSP is to solve it to optimality when the problem size is big.

Florian et al. [1980] analyzed the computational complexity for a class of production planning problems with know demand. Unless some assumptions regarding to the cost function are made, CLSP is NP-hard. As in Bitran and Yanasse [1982], who studied the algorithm complexity for both single-item and multi-item CLSP with a particular cost structure. It assumed the continuous components of the production as well as a liner holding cost. This rendered the objective function to be concave and the resulting CLSP polynomial solvable. When it came to the multi-item case, CLSP became NP-hard even with similar conditions like in the single-item case. Or if setup times were incorporated, Maes et al. [1991] stated that it was even NP-hard to find a feasible solution. Karimi et al. [2003] did a thorough review of literature on CLSP models and solution approaches before 2003. It classified all the approaches into three main categories, namely exact methods, specialized heurisitcs and mathematical programming-based heuristics. Due to the complexity of CLSP, mathematical programming-based heuristics are the most promising area of exploration, which in turn includes heuristics based on relaxation, branch and bound, set partitioning and column generation, and some other approaches.

Understandably, stochastic capacitated lot sizing problem (SCLSP) is even more challenging to solve as introducing uncertainties further increases the problem size. Haugen et al. [2001] used progressive hedging algorithm (PHA) as a meta-heuristic to solve the SCLSP. Even though we feel more numerical experiments could have been done in that work to demonstrate the effectiveness of the proposed solution method, the idea of implementing PHA in SCLSP was very interesting. Tarim and Kingsman [2004] studied the multi-period single-item CLSP with stochastic demands under the "static–dynamic uncertainty" strategy. A two stage solution heuristic was proposed with the first stage determining the replenishment periods and the second stage making adjustments of orders according to the realized demands. The accuracy of the heuristic was tested under the case of zero unit purchase/production cost. Buschkühl et al. [2010] presented a comprehensive review of CLSP models as well as solution approaches for the past four decades.

### 2.2.2 $(s, S)$ replenishment policy

Another stream of research in the inventory management which caught our attention is replenishment policy problems. The $(s, S)$ policy, because of its simplicity, has been widely adopted as a good approximation of the optimal policy under general conditions. (Arrow et al. [1951] and Veinott and Wagner [1965]). Here $s$ is the re-order point. When the inventory level is less than or equal to $s$, an order is triggered, which increases the inventory level up to $S$. Some other researchers (Karlin [1960], Scarf [1993], Zheng and Federgruen [1991] etc.) have established that, under mild assumptions, this $(s, S)$ replenishment policy is indeed optimal.

Many solution methods to find the optimal policy parameters have been proposed. Ehrhardt [1979] presented a power approximation for computing the $(s, S)$ policy. Under the assumptions of single item, periodic review with a set-up cost, linear holding and shortage costs, fixed replenishment lead time, and backlogging of unfilled demand, this approximation was able to achieve good results with robustness. Zheng and Federgruen [1991] devised an efficient algorithm to compute the $(s, S)$ policy for a single-item inventory system by taking advantage of some newly found properties of the cost function $c(s, S)$. A key condition for this algorithm to work is a quasi-convex cost rate. In addition, the deterministic lead time assumption ensured that orders arrived in the same sequence as they were placed. Bashyam and Fu [1998] relaxed this assumption by considering a periodic review $(s, S)$ problem with random lead time. A simulation optimization algorithm based on the feasible directions approach was proposed to solve the problem. Numerical case studies demonstrating the effectiveness of this algorithm were provided.

All previous works do not explicitly consider the product price changes in their models. However, price fluctuations are very common in reality; Prices of source materials and end products keep changing. One setting of optimal $(s, S)$ under current prices may not be good for a different price. In Kalymon [1971], a single-item multi-period inventory problem was studied where the future purchase prices for the item was modeled by a markovian stochastic process and convex holding and shortage costs and a set-up cost for ordering were assumed. It showed that a policy of the form $(s_i(p), S_i(p))$ was optimal based on the aforementioned assumptions in the finite horizon case. Here, $p$ is the realized item prices at current time. The idea of a $(s, S)$ policy for our bunkering decision came from this work.

## 2.3 Solution methods

### 2.3.1 Scenario reduction algorithms

In our models which will be discussed later, we use a scenario tree to model the uncertainty of bunker prices. One of the very first problems associated with a scenario tree formulation is the huge scenario size when either the number of nodes at each stage or the number of stages becomes big. Therefore, a very natural idea is to reduce the tree size while ensuring that the final optimal solutions and optimal objective value do not change too much. Dupačová et al. [2003] studied the problem of optimal scenario reduction for a convex stochastic programming problem with a discrete initial probability distribution. The purpose was to find a subset with pre-specified cardinality and a probability measure of the initial scenarios. This subset was also closest to the initial distribution in terms of some canonical probability metric. They argued that Fortet-Mourier type probability metrics could well serve the purpose based on the stability analysis. They also provided two algorithms, a forward reduction algorithm and a backward reduction algorithm, with numerical experiments. In Heitsch and Römisch [2003], two improved versions of the forward reduction algorithm and the backward reduction algorithm were introduced.

Based on the stability analysis results for multistage stochastic programs in Heitsch et al. [2006] (the distance of optimal values of original and approximate models can be bounded by the filtration distance of the underlying stochastic processes), Heitsch and Römisch [2009] extended the previous work to the case of multistage stochastic programs. An efficient algorithm was provided.

One thing we need to mention here is that we have not directly implemented the scenario reduction algorithms in our problems, instead we use one of them as a benchmark to show the efficiency of our own solving method. The comparison will be provided in the Appendix A.

### 2.3.2 Sample average approximation approach

Another uncertainty in our problem we need to handle is the bunker consumption. As we will show in our analysis based on actual data in Chapter 3, the bunker consumption rate has a mean which is a function of the ship speed and a noise which follows a zero mean normal distribution with standard deviation a constant percentage of the mean consumption rate. However, solving the model which incorporates a continuously distributed stochastic variable will be extremely hard. One promising direction which has been intensively studied by Operations Research (OR) researchers is the sampling method, which can be further divided into internal sampling and external sampling. While internal sampling adds newly generated samples to the already generated old group of samples, external sampling works by generating independent samples at each iteration. We will use the external sampling in our work. Instead of directly inputting the continuous distribution into the model and obtaining the expected objective function, a certain number of scenarios based on the initial distribution is sampled and the sample average estimate of the objective function is derived, in the hope that this approximation provides acceptable results with significantly reduced solving difficulty by avoiding the multiple integrations (A formal presentation of this concept can be found in Rubinstein and Shapiro [1990]).

The aforementioned method is usually referred to as sample average approximation (SAA). From both theoretical and practical perspective, convergence property of this method is of great interest. Shapiro [1996] studied the convergence rate of the SAA for stochastic linear problem and stated that in smooth cases (under independent and identically distributed (iid) sampling) with some mild regularity conditions, the SAA estimators is asymptotically equivalent to stochastic approximation estimators implemented with optimal step sizes. Driven by the need to apply SAA to a wider range of stochastic optimization problems, convergence analysis of SAA with non-iid sampling were also extensively studied (Balaji and Xu [2008], Shapiro and Xu [2007] and Xu and Meng [2007]). For example, Xu and Meng [2007] discussed the convergence of Karush-Kuhn-Tucker points as well as the exponential convergence of global minimizers of SAA problems for a class of stochastic programs with non-smooth equality constraints.

### 2.3.3 Progressive hedging algorithm

Decomposition algorithms have been widely applied in solving multi-stage stochastic programming problems. Basically, they belong to two major categories, namely the primal decomposition which breaks the original problem down in terms of stages and the dual decomposition which works on subproblems of individual scenarios. PHA falls into the latter category.

It is based on the idea that after relaxing a relatively small number of side constraints, many complex stochastic programming problems can be viewed as a group of easy-to-solve subproblems which can then be solved individually. In the next step, penalties of the violations of these side constraints are put in the objective function of each individual problem. Coefficients of those penalty terms are updated step by step. Algorithm stops until a predetermined accuracy or solving time limit is reached.

Rockafellar and Wets [1991] was one of the first several works that discussed PHA. It brought up the idea of scenario analysis where the uncertainty about future is modeled by a number of deterministic subproblems. By studying the similarities and trends of the optimal solutions of individual sub-problems, there is hope that a "well-hedged" overall solution which performs well under all scenarios can be found. It also proved that in the case of convex problem, PHA converges to the global optimal and if it converges in the non-convex case, it achieves a local optimum. Unfortunately, our problems are mixed-integer (a large number of binary variables indeed) in nature and therefore hinder a straightforward application of PHA.

In the case of multi-stage stochastic problems with $0-1$ variables, Løkketangen and Woodruff [1996] suggested a combination of PHA with tabu search as a solution technique. Tabu search was used to find the solutions for the induced quadratic $0-1$ mixed-integer sub-problems. They also introduced a very interesting idea called integer convergence, which means all the integer variables have converged before the full convergence of other variables. Haugen et al. [2001] applied the PHA as a meta-heuristic to their stochastic lot-sizing problem. What brought our attention to this work was their application of the integer convergence concept in their PHA. The main cornerstone of our PHA framework is also based on this integer convergence. When it comes to the actual implementation, Watson and Woodruff [2011] pointed out that there are several directions (computing effective penalty values, accelerating convergence, termination criteria and detecting cyclic behavior) that have the potential to significant improve the performance of PHA.

### 2.3.4 Column generation

Column generation is an efficient solving approach when dealing with problems with a huge number of variables. It is based on the observation in the Simplex method that, in the optimal solution, most of the variables will be non-basic and assume a value of zero. Therefore, a very natural idea is to only consider a subset of the total variables when solving the problem and bring in new basic variables when necessary. Column generation achieves this end by first separating the original problem into a master problem and a set of sub-problems. In the master problem, only a subset of the original variables are included and those sub-problems are used to identify new variables based on the dual information from the master problem. Algorithm works in an iterative way until no new variable is eligible for entering into the master problem.

Column generation method was initially proposed by [Gilmore and Gomory, 1961]. Due to its ability to deal with problems with a huge number of variables ([Lübbecke and Desrosiers, 2005]), column generation method has been widely used in vehicle routing and other logistics planning problems. [Ahn et al., 2012] devised heuristics based on column generation for a generalized location routing problem with profits. [Mufalli et al., 2012] studied an aerial vehicle routing problem in military reconnaissance missions using heuristics augmented by column generation approach. Numerical experiments showed that column generation could improve the heuristic solutions while it only marginally increased the solution time. [Jin et al., 2013] investigated a train network design optimization problem using a column generation based hierarchical solving approach. [Parragh and Schmid, 2012] applied column generation and large neighborhood search to study a Dial-a-ride problem. Column generation is also applied in areas like scheduling and network optimization: for instance, aircrew scheduling ([Vance et al., 1997]), wireless network routing scheduling ([El-Najjar et al., 2009]) and telecommunication network optimization ([Santos et al., 2010]).

# Chapter 3

# Dynamic determination of vessel speed and selection of bunkering ports for liner shipping under stochastic environment

We have shown in Chapter 1 that in recent years the bunker prices have been increasing and fluctuating dramatically. While the bunker cost takes up more and more percentages of the total operational cost, shipping companies are relentlessly seeking for ways to reduce it. One practice that has gradually obtained popularity is slow steaming. However, the characteristic of the liner shipping is that it usually has a fixed number of port-calls in a cyclical route with a published schedule. While slow steaming would be the general trend when bunker prices are high, high fluctuation and regional differences of the bunker prices complicate the situation because simply reducing the vessel speed may miss out the opportunity of reaching the next port when bunker prices there are low. In addition, bunker consumption under the same speed for the same distance also varies depends on weather and sea conditions.

From the literature review in Chapter 2, we see that bunker prices and consumption uncertainties in this problem have not been sufficiently tackled. Thus this study is devoted to the dynamic determination of the vessel speed from the current port to the next one and the bunkering amount in the current port considering the uncertain bunker prices and bunker consumption. This is an operational problem when, on the planning level, ships' deployment, scheduling and routing have been decided. Also due to the service nature of the liner shipping, interaction between ships of the same service route and interaction between different service routes, if no transhipment is considered, is very low. Therefore, we only need to consider a single vessel in one service route because of the operational independence of ships and routes.

## 3.1 Problem description

In this study, we consider the operational level decision making for a single liner ship in one cyclical route (start from one port, travel through all other ports at least once and go back to the original port) with fixed number of port calls and time windows. Time window states the ship arrival and departure times at each port. Two uncertain factors considered in our work are the bunker fuel prices and the bunker fuel consumption rate. A more detailed discussion on how to capture the randomness of these two parts would be given later in this section.

Two key decisions to be made are where and how much to bunker. In the real practice, prior to the arrival of the next port, ship owners would ask the bunker suppliers for quotations, based on which, bunkering decision is made. Once determined, the quotations will rarely change until the ship reaches the port. Therefore, we can conveniently assume that bunkering only happen when a ship reaches one port. Bunkering decision depends on the bunker prices at each port which are usually different across those ports due to local supply–demand factors. The evolution of the bunker prices at each port can be modeled as a discrete-time Markovian process which describes all the possible states and transition matrix between those states. Without loss of generality, we assume that port calls are on a weekly basis and hence we only need to describe the bunker prices evolution on a weekly basis. While this is a drawback of our work, rolling horizon approach can help to mitigate this problem. This is so because we can always update the bunker price scenario tree based on timely real world situations.

Aside from bunkering, another important decision is the ship speed between each leg, which has been commonly assumed to be constant during each leg. How to reach each one port within the scheduled arriving time and save bunker consumption as much as possible through slow steaming is a question faced by most of the practitioners. Traditionally, ships are designed to sail at high speed. Speed that utilizes lower than 40% of engine load is considered to be damaging to engines according to the recommendation of ship manufacturers. However, a recent experiment done by Maersk [2010], on its own fleet of 110 ships, showed that it is possible for vessels to slow down if necessary.

In our problem, the objective is to minimize the total operational cost in one service loop. The costs considered here are the bunker cost and inventory holding cost. Bunker cost mainly consists of two parts, fixed bunkering cost incurred each time a bunkering takes place and variable cost that depends on the bunkering amount and bunker prices at the time being. Inventory holding cost can be interpreted as a combination of the capital committed in the bunker purchase which could otherwise generate profits through some investment activities and a loss of revenue due to less capacity to carry revenue-generating cargo. As a simplification, we assume that the inventory carrying cost per metric ton (pmt) is constant. Because our study horizon is one service loop which is finite, inevitably, there would be bunker fuel left in the ship fuel tank at the end of voyage. For this amount of bunker fuel left, we deduct it from the total cost based on the bunker prices at the time being.

### 3.1.1 Model for bunker prices

To model the evolution of the bunker prices, we use the percentage changes in each leg of the voyage, but the difficulty is that the percentage change can take any continuous value within a reasonable range. Incorporating a random variable with continuous distribution into an optimization model would make solving the model extremely hard, if not impossible.

Therefore, we discretize the bunker price percentage changes and assume they follow a Markovian process, which means current bunker prices only depend on previous period price percentage changes. At first, we determine an interval within which the bunker price percentage changes between two subsequent periods can take place and then we divide this interval into several smaller sub-intervals. Transition matrix depicting the transition among those sub-intervals is constructed. In the end, one discrete point value is chosen to represent each sub-interval. We can either choose the mean of the sub-interval or generate it by random sampling.

For example, if we denote $P_t^i$ and $\theta_t^i$ as the bunker prices and bunker price percentage changes at port $i$ and time period $t$, and $P_0^i$ as the baseline bunker prices at port $i$ and time 0, then bunker prices at each port and time period $t$ are based on baseline bunker prices as well as all the percentage changes during previous periods. For example, $P_1^i = P_0^i \times \theta_1^i$ and $P_k^i = P_{k-1}^i \times \theta_k^i$. As mentioned, we approximate the port-by-port bunker price change evolution by the weekly bunker price change evolution.

### 3.1.2 Model for bunker consumption rate

In Yao et al. [2012], they assumed that the daily bunker consumption rate could be expressed as $F = k_1 \cdot V^3 + k_2$, within which, $F$ is the daily bunker consumption (tons/day), $k_1$ and $k_2$ are two constants (can be different for different vessel sizes), and $V$ is the ship speed (knots/hour). Due to the reasons we mentioned earlier, a noise term is added to depict the uncertainty of bunker consumption. This means

$$F = k_1 \cdot V^3 + k_2 + \varepsilon(V)$$

Based on the data we obtained from a real liner company, we found that the noise $\varepsilon$ is a function of the ship speed and the noise term follows a normal distribution with zero mean and constant coefficient of variation under different ship speeds. Table 3.1 below shows the results of our analysis.

We have grouped the original data according to the different speed intervals. Notice we have different intervals for different sizes of ships. This is simply because larger ships usually sail under a greater speed. However, this would not be an issue here because for ships of each size category, Table 3.1 shows that the coefficient of variation is approximately constant. This means that the standard deviation of the bunker consumption within a specific period of time is

Table 3.1: Analysis of daily bunker consumption rate

| Ship size (TEU) | $K_1$ | $K_2$ | CV for different speed intervals | | | |
|---|---|---|---|---|---|---|
| 0 – 1000 | 0.004476 | 6.17 | speed interval | 12.5 ∼ 13.5 | 13.5 ∼ 14.5 | 14.5 ∼ 15.5 |
| | | | CV | 0.34 | 0.39 | 0.3 |
| 1000 – 2000 | 0.004595 | 16.42 | speed interval | 12.5 ∼ 13.5 | 13.5 ∼ 14.5 | 14.5 ∼ 15.5 |
| | | | CV | 0.21 | 0.24 | 0.21 |
| 2000 – 3000 | 0.004501 | 29.28 | speed interval | 12.5 ∼ 13.5 | 13.5 ∼ 14.5 | 14.5 ∼ 15.5 |
| | | | CV | 0.13 | 0.15 | 0.10 |
| 3000 – 4000 | 0.006754 | 37.23 | speed interval | 17.5 ∼ 18.5 | 18.5 ∼ 19.5 | 19.5 ∼ 20.5 |
| | | | CV | 0.09 | 0.09 | 0.075 |
| 4000 – 5000 | 0.006732 | 55.84 | speed interval | 18.5 ∼ 19.5 | 19.5 ∼ 20.5 | 20.5 ∼ 21.5 |
| | | | CV | 0.068 | 0.08 | 0.08 |
| 5000 – 6000 | 0.007297 | 71.4 | speed interval | 17.5 ∼ 18.5 | 18.5 ∼ 19.5 | 19.5 ∼ 20.5 |
| | | | CV | 0.07 | 0.07 | 0.08 |

proportional to the mean consumption. Considering that wind and sea current are two of the main factors for the bunker consumption uncertainty and the fact that their influence increases with ship speed would not surprise us with such a conclusion. Also, for different sizes of ships, we state that the coefficient of variation actually decreases with ship size. The intuitive explanation is that bigger ships are usually equipped with more powerful engines, and thus wind and sea current impose relatively less influence on them.

In our dynamic model, we will use chance constraints to control the probability of a ship running out of bunker before reaching the next port to be less than one fixed percentage value (e.g. 0.99%).

## 3.2 Modeling

As discussed, we model the evolution of the bunker prices by a Markovian process.

In the financial engineering area, researchers use scenario tree models to formulate their problems in which the returns of financial products possess stochastic nature. Mulvey et al. [1997] reviewed the application of multi-stage stochastic optimization on asset/liability management. When it came to the tradeoff between realism and computational tractability, they listed several essential characteristics that a mathematical model for investment problem should possess to render useful application. One possible way that they claimed to be effective in covering all of those characteristics was a scenario tree model. Considering the similar nature of those financial products with bunker prices, here we use the scenario tree to model the randomness of bunker prices.

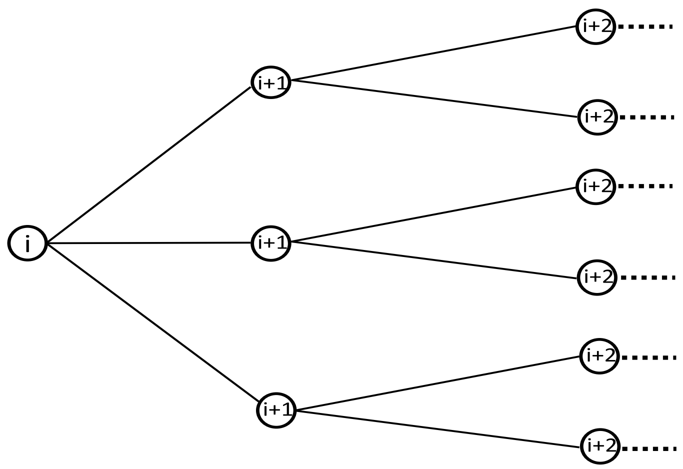


Figure 3.1: A simple example of scenario tree

Bunker price uncertainty in the future times is modeled by a discrete stochastic process $\xi$ that is defined on a probability space of $(\Omega, \mathcal{F}, \mathcal{P})$ with

$$\xi = \{\xi_{\mathbf{t}} := \theta^{\mathbf{i}}_{\mathbf{t}}\}_{\mathbf{t}\in\mathbf{T}}.$$

$\theta^i_t$ denotes the bunker prices percentage change at time period $t$ and port $i$. To make our multi-stage stochastic optimization problems computationally tractable, following assumptions on the property of $(\Omega, \mathcal{F}, \mathcal{P})$ are made: first, $\Omega$ is finite and $\Omega = \{w_r\}_{r\in\mathcal{R}}$ with $\mathcal{R} = \{\mathbf{1}, ..., \mathcal{R}\}$; $\mathcal{F}$ is the power set of $\Omega$; $\mathcal{P}(\{w\}) = p_r$ with $r \in \mathcal{R}$. Second, $\{\mathcal{F}_t\}_{t\in\mathcal{T}}$ is the filtration induced by $\xi$ with $\mathcal{F}_t \subseteq \mathcal{F}$ as the $\sigma$-algebra generated by $\xi^t$. At the beginning of every service loop, the most recent bunker price changes are known. This means $\xi_1$ is deterministic and $\mathcal{F}_1 = \{\emptyset, \Omega\}$. For the future bunker price changes, we only know the discrete probability distribution. Bunkering and speed decisions at any stage do not depend on future realization of bunker price changes, but on the probability specification $(\Omega, \mathcal{F}, \mathcal{P})$. This is a non-anticipative constraint commonly used in many multi-stage stochastic optimization problems. When it comes to the end of the studying horizon, all the random information is realized and $\mathcal{F}_t = \mathcal{F}$. A series of realizations $(\xi_1^r, ..., \xi_T^r)$ over the entire study horizon consist of a scenario. All the scenarios are combined into a scenario tree representation. Figure 4.1 above shows an example of a scenario tree.

### 3.2.1 Assumptions

Now, we summarize all the assumptions made in our paper:

1. Bunkering and ship speed decisions are made when ship reaches one port.
2. We consider one ship in one service route with time windows. Port time (time one ship spends on entering, unloading and loading cargo, idling and exiting) at each port is deterministic and known.
3. Relationship between the ship speed and the bunker consumption is established in Section 3.1.2.
4. Bunker prices at different ports are not necessarily the same. In addition, bunker price changes follow a discrete-time Markovian process.
5. Bunkering cost includes the fixed cost which is constant over time by assumption and the variable cost. Bunker inventory cost pmt is assumed to be constant and independent of bunker prices. Bunker left at the end of one service loop is refunded.

### 3.2.2 Notations

Following notations are used to express our dynamic stochastic problem:

| | |
|---|---|
| $R$ | total number of price scenarios; |
| $\Pi^r$ | the probability that price scenario $r$ happens; |
| $n$ | number of port of calls; |
| $d_{i,j}$ | distance between port $i$ and port $j$(nautical miles); |
| $t$ | total cycle time (h); |
| $t_i$ | port time(time one ship spends on entering, unloading and loading cargo, idling and exiting) at port $i$ (h); |

| | |
|---|---|
| $e_i$ | earliest arrival time at port $i$; |
| $l_i$ | latest arrival time at port $i$; |
| $C_i$ | bunker fuel consumption when the ship is at port $i$; |
| $w$ | bunker fuel capacity for a single ship; |
| $v_{min}$ | minimum ship sailing speed (nautical miles/h); |
| $v_{max}$ | maximum ship sailing speed (nautical miles/h); |
| $P_i^r$ | bunker price for port $i$ under scenario $r$; |
| $f$ | fixed bunkering cost; |
| $\gamma$ | coefficient to control the service level; |
| $h$ | inventory holding cost pmt for bunker; |
| $\eta$ | coefficient of variation for daily bunker consumption rate |

The following decision variables are defined:

| | |
|---|---|
| $V_{i,j}^r$ | ship speed between port $i$ and $j$ under scenario $r$; |
| $S_i^r$ | bunker fuel-up-to level for the ship at port $i$ under scenario $r$; |
| $B_i^r$ | bunkering decision variable. =1 if bunkering at port $i$ under scenario $r$; =0, otherwise; |

Dependent variables:

| | |
|---|---|
| $I_i^r$ | bunker fuel inventory when the ship reaches port $i$ under price scenario $r$; |
| $\overline{F}_{i,j}^r$ | mean of daily bunker consumption rate for a ship travels from port $i$ to $j$ under price scenario $r$; |
| $\delta_{i,i+1}^r$ | standard deviation of bunker fuel consumption between port $i$ and $i+1$ under price scenario $r$; |
| $D_i^r$ | standard deviation of ship bunker inventory when ship reaches port $i$ under price scenario $r$; |
| $A_i^r$ | ship arrival time at port $i$ under scenario $r$; |

### 3.2.3 Mathematical model

The major difference between our model and the one in Yao et al. [2012] is that ours can provide dynamic decision making. We included two uncertainties, which render our model more realistic, but make the solving extremely difficult. We will discuss the solving issues in the next section. There are some other minor modeling differences. For example, as our focus is on the operational level, we study the optimization problem for a finite horizon, while the model in Yao et al. [2012] is an infinite horizon problem. Compared with their model, we also add one variable which is the fixed bunkering cost and delete the maximum bunkering times constraint in their model. We believe that in this way our model is more general and better conforms to the reality.

We present a mathematical model to describe our problem.

$$\min \sum_{r=1}^{R} \Pi^r \cdot \big( \sum_{i=1}^{n} [(S_i^r - I_i^r) P_i^r + f \cdot B_i^r + (S_i^r - C_i^r) \cdot h] - I_{n+1}^r \cdot P_{n+1}^r \big)$$

$$I_1^r = 0, \qquad D_1^r = 0 \qquad \forall r \in R \tag{3.1}$$

$$I_i^r = S_{i-1}^r - C_{i-1}^r - \overline{F}_{i-1,i}^r \cdot d_{i-1,i}/24 \cdot V_{i-1,i}^r \qquad \forall r \in R, i \in 2, 3, ..., n+1 \tag{3.2}$$

$$S_i^r - I_i^r \leq B_i^r \cdot w \qquad \forall r \in R, i \in 1, 2, ..., n \tag{3.3}$$

$$S_i^r \leq w \qquad \forall r \in R, i \in 1, 2, ..., n \tag{3.4}$$

$$\delta_{i-1,i}^r + (1 - B_{i-1}^r) \cdot D_{i-1}^r = D_i^r \qquad \forall r \in R, i \in 2, 3, ..., n+1 \tag{3.5}$$

$$\overline{F}_{i,i+1}^r = k_1 (V_{i,i+1}^r)^3 + k_2 \qquad \forall r \in R, i \in 1, 2, ..., n \tag{3.6}$$

$$\delta_{i-1,i}^r = \eta \times \overline{F}_{i-1,i}^r \cdot d_{i-1,i}/24 \cdot V_{i-1,i}^r \qquad \forall r \in R, i \in 2, 3, ..., n+1 \tag{3.7}$$

$$I_i^r \geq \gamma \times D_i^r \qquad \forall r \in R, i \in 2, 3, ..., n+1 \tag{3.8}$$

$$v_{min} \leq V_{i,i+1}^r \leq v_{max} \qquad \forall r \in R, i \in 1, 2, ..., n \tag{3.9}$$

$$A_i^r + t_i + d_{i,i+1}/V_{i,i+1}^r = A_{i+1}^r \qquad \forall r \in R, i \in 1, 2, ..., n \tag{3.10}$$

$$e_i \leq A_i^r \leq l_i \qquad \forall r \in R, i \in 1, 2, ..., n \tag{3.11}$$

$$A_{n+1}^r = t \qquad \forall r \in R \tag{3.12}$$

$$B_i^r = 0 \text{ or } 1 \qquad \forall r \in R, i \in 1, 2, ..., n \tag{3.13}$$

$$V_{i,i+1}^r = V_{i,i+1}^{r'} \qquad \forall (r, r') \in R\, identical\ past\ to\ i+1, i \in 1, 2, ..., n \tag{3.14}$$

$$S_i^r = S_i^{r'} \qquad \forall (r, r') \in R\, identical\ past\ to\ i \in 1, 2, ..., n \tag{3.15}$$

$$B_i^r = B_i^{r'} \qquad \forall (r, r') \in R\, identical\ past\ to\ i \in 1, 2, ..., n \tag{3.16}$$

$$F_{n,n+1} = F_{n,1}, d_{n,n+1} = d_{n,1}, V_{n,n+1}^r = V_{n,1}^r \qquad \forall r \in R \tag{3.17}$$

The objective function is to minimize the expected total cost, which includes the fixed and variable bunkering cost and inventory holding cost. Bunker inventory left at the end of one service loop or beginning of a new loop is dealt as if we can sell it in the market at the prices of that time being. Constraint (3.1) sets the initial ship bunker inventory and standard deviation of it at zero. Constraint (3.2) is a flow conservation constraint. Constraints (3.3) and (3.4) ensure that the maximum bunkering amount and bunker-up-to level is less than the fuel tank capacity. Constraint (3.5) states that if the ship bunkered at the previous port, then standard deviation of the ship bunker inventory at current port is equal to the standard deviation of bunker consumption from previous port to the current port. Otherwise the standard deviation of ship bunker inventory at previous port should also be added. This is because, as discussed, standard deviation of bunker consumption is proportional to the total bunker consumption. Constraints (3.6) and (3.7) express the mean daily consumption rate at a certain speed and stand deviation of bunker consumption between ports $i$ and $i+1$. Constraint (3.8) is the deterministic equivalent for chance constraint $P\{I_i^r \geq D_i^r\} \geq \gamma^*$, which ensures that the probability of bunker inventory being greater than a certain amount is greater than a pre-specified value. Constraint (3.9) is simply to limit the ship speed within a reasonable range, while constraints (3.10) to (3.12) are about time window constraints and put additional restraints on the allowable sailing speed choice. Constraint (3.13) is a binary constraint. Constraints (3.14) to (3.16) are non-anticipative constraints which ensure that scenarios that share the same history up to port $i$ should take the same action.

## 3.3 Solution method

There are two potential challenges in solving our problem. The first one is the non-linearity constraints related to the ship speed. We deal with this by following the method used in Yao et al. [2012], which applied a piece wise linear function to approximate the non-linear terms.

Another challenge is that when a scenario tree procedure is used to model the stochastic parameters in a multi-stage stochastic problem, solving the problem is usually difficult because of the large problem size. For example, in a case where there are 15 ports and for each period (ship reaches a new port) there are four price scenarios, the total number of scenarios in a scenario tree construction would be $4^{16}$ (because the ship needs to sail back to the first port after visiting all other ports).

Mulvey et al. [1997] reviewed several different solution algorithms for multi-period stochastic problem with discrete-time decisions. Their focus was on medium size of problems: problems with 1,000–3,000 scenarios and nonlinear objective functions. Direct solvers like OBI, MINOS, GRG, CPLEX, LOQO, etc., and decomposition algorithms like L-shaped proposal, progressive hedging algorithm and diagonal quadratic approximation were mentioned. Another possible way is to look at how to trim down the tree size. Growe-Kuska et al. [2003] proposed scenario reduction algorithms which select a subset of the initial scenarios and assign new probability to the remaining ones. Also the tree construction algorithms help to reduce the number of nodes through modifying the tree structure and bundling similar scenarios. Other interesting works in scenario reduction are Dupačová et al. [2003], Heitsch and Römisch [2003] and Heitsch and Römisch [2009].

In this work, however, because the problem size could be extremely large when the number of ports involved becomes large, all those direct solvers are not able to solve the problem. Also considering our problem nature, instead of trying decomposition algorithms or scenario reduction algorithms, we propose to use a slightly different method of generating scenario tree and combine it with a modified rolling horizon approach to solve a liner shipping operational level problem. The rationale behind this combination is first, bunker price forecasting which covers a long period of time, if not impossible, suffers greatly in terms of forecasting quality. Instead of making one-time forecast only at the very beginning for the whole horizon, constantly updating the forecast and resolving the optimization problem are beneficial; second, this successfully circumvents the trouble of solving a large-scale stochastic optimization problem.

### 3.3.1 A modified rolling horizon solving scheme

The essence of the standard rolling horizon planning scheme is that a problem with the study horizon shorter than the original one (to reduce the problem size) is solved and the first period decision is implemented. With newly available information, the problem is updated and resolved. Still the decision is only acted on for the imminent period. This process goes on and on until the end of the study horizon. For example, Baker [1977] implemented the standard rolling horizon approach in a production planning problem and numerical results in his work showed that rolling horizon approach produced results that were well within 10% of optimality and if the model construction was well tailored for specific circumstance, the optimality gap could be further reduced within 1%. In addition, he mentioned two key reasons ("uncertain information about the future" and "limited information about the future") that legitimized the use of finite-horizon model for the purpose of decision making in infinite-horizon system.

In our case, we will use a non-standard rolling horizon approach. Unlike the standard one which solves a problem with a shorter horizon than the original problem, our non-standard approach still solves the problem with the whole study horizon. However, we assign a higher level of fidelity for the nearer periods than the later ones by modifying the way we generate the scenario tree. For the first few number of periods (could be 1, 2 or any number of periods depending on the problem), all the price change alternatives are generated as shown in Figure 3.1, while a relatively small number of realizations (also problem specific) are randomly generated for all the remaining periods till the end. Therefore, an example of our modified version of scenario tree would look like Figure 3.2, in which scenarios for periods after $i + 2$ are randomly generated for each parent node. The validity of this non-standard variant is due to our problem nature and the diminishing tail-end effect. We will further show the suitability of using this non-standard solving horizon approach through our first numerical example.

The modified rolling horizon solving procedure tailored for our problem is given below:

1. When the ship reaches the port $i (i = 1, 2..., n)$, generate the price scenario tree which looks ahead $n_1^i$ periods and randomly generate $n_2^i$ scenarios for all the remaining periods and each parent node. The choices of $n_1^i$ and $n_2^i$ are problem specific.

2. Solve the dynamic optimization problem and get the optimal bunkering and speed decisions for the ship at port $i$.

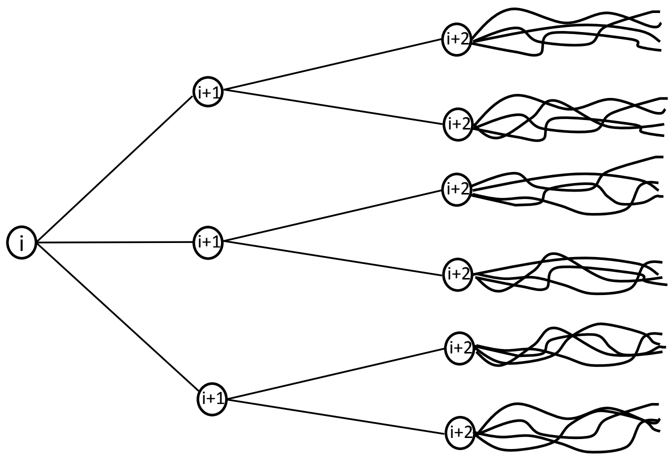


Figure 3.2: Modified scenario tree with randomly generated siblings

3. When the ship reaches the port $i+1$, generate the price scenario tree again based on newly available information.

4. Repeat steps 2 and 3 until the ship reaches the destination port.

## 3.4 Case study

Here, we implement our model in two real-world service routes, Malaysia Service (MAS) and Asia-Europe Express (AEX), offered by a real liner. The MAS route consists of three port-of-calls; therefore, direct solving of the whole dynamic problem is possible and we will use this example to illustrate the effectiveness of our modified rolling horizon solving approach by testing its optimality gap. AEX route has 15 port-of-calls. We use the modified rolling horizon approach to solve it and compare the results provided by the stationary model in Yao et al. [2012].

However, we have modified their model to make a fair comparison. The main modification is about the ending bunker inventory. In their stationary model, because it is an infinite horizon problem, bunker inventory at the end of one service loop is the starting inventory of the next loop. However, in our comparison, we only consider one service loop; thus the ending bunker inventory in the stationary model will be refunded as in our dynamic model. Also, we have removed the maximum bunkering times constrain in the stationary model and redesigned the bunker cost to include fixed bunkering cost instead. Modified version of the stationary model will be presented in the Appendix.

We run all our numerical studies with CPLEX-11.2 on a 3 GHz Dual Core PC with 4 GB of RAM. Stationary model in Yao et al. [2012] can be solved by CPLEX in seconds.

### 3.4.1 Parameter setting for bunker price changes

Our model has no problem accommodating the case where every port has a different parameter setting for their bunker price scenario trees; in our numerical study here, for ease of illustration, we assume that the bunker price percentage changes for all the ports at each period will be the same .

One of the most commonly used methods in generating scenarios for continuous distribution function is the Discretization technique. For a general introduction and application of this method, please refer to Kotsiantis and Kanellopoulos [2006] and Dougherty et al. [1995]. For example, based on the bunker prices in Singapore from August 7, 2002 to September 3, 2009, we discretize the weekly bunker price changes into four intervals with equal probability and Table 3.2 below lists the mean values of each interval. Since we model the evolution of bunker prices as an one-stage Markovian process, we also derive the conditional transition matrix among those intervals in Table 3.3. However, problems associated with the Discretization method in deriving bunker price percentage change scenarios based on historical data are that periods of highly volatile prices would be evened out by mild ones and it assumes that history will repeat. Our numerical experiments show that under this setting of bunker price percentage changes, dynamic model only has marginal benefits than the stationary model. Considering our work is more relevant in times when bunker prices are highly fluctuating (September 2008, for example, IFO380 averaged slightly over $600 pmt in Singapore, however, it dropped to average $410 in October), we construct three cases of weekly bunker price percentage changes as shown in Tables 3.4–3.9, which are more volatile.

Table 3.2: Weekly bunker price change alternatives: historical case

| Scenario | value | Probability | value |
|---|---|---|---|
| $C_1$ | -5.7% | $P(C_1)$ | 0.25 |
| $C_2$ | -0.75% | $P(C_2)$ | 0.25 |
| $C_3$ | 1.5% | $P(C_3)$ | 0.25 |
| $C_4$ | 6.5% | $P(C_4)$ | 0.25 |

Table 3.3: Transition matrix of the weekly bunker price changes: historical case

| Scenario | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $C_1$ | 29% | 22% | 20% | 29% |
| $C_2$ | 24% | 28% | 23% | 25% |
| $C_3$ | 20% | 24% | 32% | 24% |
| $C_4$ | 27% | 23% | 25% | 25% |

Table 3.4: Weekly bunker price change alternatives: Case 1

| Scenario | value | Probability | value |
|---|---|---|---|
| $C_1$ | -10% | $P(C_1)$ | 0.25 |
| $C_2$ | -5.0% | $P(C_2)$ | 0.25 |
| $C_3$ | 5.0% | $P(C_3)$ | 0.25 |
| $C_4$ | 10% | $P(C_4)$ | 0.25 |

Table 3.5: Transition matrix of the weekly bunker price changes: Case 1

| Scenario | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $C_1$ | 40 % | 30 % | 20 % | 10 % |
| $C_2$ | 30 % | 40 % | 20 % | 10 % |
| $C_3$ | 10 % | 20 % | 40 % | 30 % |
| $C_4$ | 10 % | 20 % | 30 % | 40 % |

Table 3.6: Weekly bunker price change alternatives: Case 2

| Scenario | value | Probability | value |
|---|---|---|---|
| $C_1$ | -15.0 % | $P(C_1)$ | 0.25 |
| $C_2$ | -7.50 % | $P(C_2)$ | 0.25 |
| $C_3$ | 7.50 % | $P(C_3)$ | 0.25 |
| $C_4$ | 15.0 % | $P(C_4)$ | 0.25 |

Table 3.7: Transition matrix of the weekly bunker price changes: Case 2

| Scenario | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $C_1$ | 40 % | 30 % | 20 % | 10 % |
| $C_2$ | 30 % | 40 % | 20 % | 10 % |
| $C_3$ | 10 % | 20 % | 40 % | 30 % |
| $C_4$ | 10 % | 20 % | 30 % | 40 % |

Table 3.8: Weekly bunker price change alternatives: Case 3

| Scenario | value | Probability | value |
|---|---|---|---|
| $C_1$ | -20.0 % | $P(C_1)$ | 0.25 |
| $C_2$ | -10.0 % | $P(C_2)$ | 0.25 |
| $C_3$ | 10.0 % | $P(C_3)$ | 0.25 |
| $C_4$ | 20.0 % | $P(C_4)$ | 0.25 |

Table 3.9: Transition matrix of the weekly bunker price changes: Case 3

| Scenario | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $C_1$ | 40 % | 30 % | 20 % | 10 % |
| $C_2$ | 30 % | 40 % | 20 % | 10 % |
| $C_3$ | 10 % | 20 % | 40 % | 30 % |
| $C_4$ | 10 % | 20 % | 30 % | 40 % |

### 3.4.2 MAS service route

#### 3.4.2.1 Parameter Setting

Parameters for the MAS route is provided in Table 3.10:

Table 3.10: Parameters for MAS service

| Parameter | Value |
|---|---|
| Number of port of calls | 3 |
| Service frequency | Weekly |
| Ship size | 3,000TEU |
| Total cycle time | 168 h |
| Ship speed interval | 8–15 knots |
| Mean bunker consumption rate | $F = 0.006743V^3 + 37.23$ |
| Fixed bunkering cost pmt | $1,000$ |
| Inventory holding cost pmt | 50 |
| Coefficient of variation of bunker consumption rate | 0.09 |

#### 3.4.2.2 Numerical results

With 3 ports, there are altogether 256 scenarios, so we can solve the whole dynamic problem with CPLEX. One scenario means a series of price percentage change realizations from the start till the end of the route. For example, if bunker prices increase $\theta_i\%$ ($i = 1, 2, 3$) when the ship reaches port $i$ ($\theta_i$ can be less than 0 which means it is actually a decrease of prices), and in the end when the ship sails back to port 1, bunker prices increase another $\theta_0\%$. Hence, we denote this scenario as $[\theta_1\%, \theta_2\%, \theta_3\%, \theta_0\%]$.

We obtain the speed and refueling decisions given by the stationary model, direct solving of the dynamic model and dynamic model solved by the modified rolling horizon approach, respectively, under all three cases of bunker price percentage changes. Comparison of the results from direct solving of the dynamic model and dynamic model solved by the modified rolling horizon approach is to test the effectiveness of the modified rolling horizon approach. For the modified rolling horizon approach, we look ahead one period for which we generate all four possible alternatives and for the remaining three periods (it is not two because, as mentioned, the ship needs to sail back to the first port), three bunker price change realizations are generated for each parent node. All those three bunker price change realizations belong to the same parent node should share the same decision. For the modified rolling horizon approach, it is based on which specific scenario happens to solve the problem. All 256 scenarios will be solved by our modified rolling horizon approach.

Under Case 1 of the bunker price changes setting, the optimal expected average cost of those 256 scenarios solved by the stationary model is $\$123,637$, the optimal expected average cost solved by the direct solving of the dynamic model is $\$117,194$ and the optimal expected average cost solved by the modified rolling horizon approach is $\$118,779$. The failure rate (probability that a ship will run out of fuel before finishing the voyage) in these three models is controlled at the same level by setting the service level coefficients. We see that the optimality gap between the rolling horizon approach and the direct dynamic solving is only about 1.3%. In terms of performance, direct solving of dynamic model is better than the dynamic model solved by the modified rolling horizon approach, which is better than the stationary approach. The cost saving of using the modified rolling horizon approach compared with the stationary model is 3.9%.

Under Case 2 of the bunker price changes setting, the optimal expected average cost of those 256 scenarios solved by the stationary model is $\$122,739$, the optimal expected average cost solved by the direct solving of the dynamic model is $\$113,422$ and the optimal expected average cost solved by the modified rolling horizon approach is $\$116,637$. The failure rate in these three models is controlled at the same level by setting the service level coefficients. The optimality gap between the modified rolling horizon approach and the direct dynamic solving is only about 2.8%. The cost saving of using the modified rolling horizon approach compared with the stationary model is 5.0%

Under Case 3 of the bunker price changes setting, the optimal expected average cost of those 256 scenarios solved by the stationary model is $\$118,878$, the optimal expected average cost solved by the direct solving of the dynamic model is $\$95,580$ and the optimal expected average cost solved by the modified rolling horizon approach is $\$100,502$. The failure rate in these three models is controlled at the same level by setting the service level coefficients. The optimality gap between the modified rolling horizon approach and the direct dynamic solving is only about 4.9%. The cost saving of using the modified rolling horizon approach compared with the stationary model is 15.5%

Table 3.11 and 3.12 below summarize the results so far for three different solving methods under three different cases of bunker price percentage changes. $R$ denotes the modified rolling horizon solving approach, $D$ denotes the direct dynamic solving approach and $S$ denotes the solving of stationary model.

Table 3.11: Comparison between the modified rolling horizon approach and direct dynamic solving approach

| Methods | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| $D$ | $\$117,194$ | $\$113,422$ | $\$95,580$ |
| $R$ | $\$118,779$ | $\$116,637$ | $\$100,502$ |
| $\frac{R-D}{R}(\%)$ | 1.3 | 2.8 | 4.9 |

Table 3.12: Comparison between the modified rolling horizon approach and the solving of stationary model

| Methods | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| $R$ | $\$118,779$ | $\$116,637$ | $\$100,502$ |
| $S$ | $\$123,637$ | $\$122,739$ | $\$118,878$ |
| $\frac{S-R}{S}(\%)$ | 3.9 | 5.0 | 15.5 |

The above results show that the modified rolling horizon approach performs quite well compared with the direct solving of the dynamic model, though the optimality gap tends to be bigger when bunker prices become more volatile. Also, as price fluctuations increase, the cost saving of using the dynamic model, either solved directly or by the modified rolling horizon approach, increases as well.

Next we look into details of the optimal speed and refueling decisions given by the modified rolling horizon approach and the direct solving of the dynamic model. We take Case 3 setting of the bunker price percentage changes for example. Table 3.13 below lists numerical results from both solving approaches under some illustrative scenarios. For some scenarios, our experiments show that the dynamic approach and the modified rolling horizon approach give the same or similar optimal solutions, scenarios 1–3 in Table 3.13, for example. We also find that, for both approaches, if there is a bunker prices increase when the ship reaches the port 2, it will bunker more. The bigger the increase, the more it bunkers. We can see this from the comparison between scenario 1 with scenario 2 for example. There is a 10% prices increase in stage 2 at scenario 2 and −20%

Table 3.13: Comparison of the direct solving of dynamic model and the modified rolling horizon approach

| Scenarios | Speed (knot/h): Port 1–Port 2–Port 3 | Bunkering Amount (ton): Port 1–Port 2–Port 3 | Optimal Cost: |
|---|---|---|---|
| $1, [-10\%, -20\%, -10\%, 10\%]$ | R:8.75–7.17–7.83<br>D:8.75–7.17–7.83 | R:48.89–39.36–87.99<br>D:48.89–39.36–87.99 | R:$74,800$<br>D:$74,800$ |
| $2, [-20\%, 10\%, -10\%, -20\%]$ | R:8.75–6.09–8.59<br>D:8.75–6.88–8.00 | R:48.89–46.27–80.81<br>D:48.89–40.98–86.24 | R:$76,854$<br>D:$76,139$ |
| $3, [-20\%, 10\%, 10\%, 20\%]$ | R:8.75–7.17–7.83<br>D:8.75–6.88–8.00 | R:48.89–46.27–1992.2<br>D:48.89–40.98–1993.0 | R:$46,729$<br>D:$46,762$ |
| $4, [-10\%, 20\%, -20\%, 10\%]$ | R:8.75–5.78–8.90<br>D:8.75–6.69–8.13 | R:48.89–48.80–78.38<br>D:48.89–134.38–0 | R:$91,564$<br>D:$103,841$ |
| $5, [10\%, 20\%, -20\%, 10\%]$ | R:8.75–5.78–8.91<br>D:8.75–6.69–8.13 | R:48.89–1992.9–0<br>D:48.89–134.35–0 | R:$705,837$<br>D:$124,408$ |
| $6, [10\%, 20\%, 20\%, -10\%]$ | R:8.75–6.69–8.13<br>D:8.75–6.69–8.13 | R:48.89–1992.9–47.8<br>D:48.89–134.35–1990.58 | R:$294,238$<br>D:$474,717$ |

decrease in scenario 1. Bunkering amount of the modified rolling horizon approach at port 2 in scenario 2 is 46.27 and it is 39.39 in scenario 1. Bunkering amount of the direct solving approach at port 2 in scenario 2 is 40.98 and it is 39.36 in scenario 1. Comparison between scenario 3 with scenario 4 shows the same conclusion. This is because there are altogether only three ports in one service loop. Port 2 is relatively more important in the overall planning for the whole loop. When it spots a increase of bunker prices, it tends to bunker more at port 2.

One more finding is that when scenarios $[10\%, 20\%, x\%, x\%]$, $[20\%, 10\%, x\%, x\%]$ or $[20\%, 20\%, x\%, x\%]$ (x denotes either −20,−20,10 or 20) happen, the modified rolling horizon approach will bunker up to the maximum capacity at port 2 while the direct solving approach never does this. This means the modified rolling horizon approach is tend to be myopic compared with the direct solving approach because if in later stages, bunker prices actually decrease, then the modified rolling horizon approach results in much higher cost than the direct solving, as shown in scenario 5. However, if in later stages, bunker prices actually increase, as shown in scenario 6, the modified rolling horizon approach will outperform the direct solving approach (when all scenarios are considered, and on the expected average sense, the direct dynamic solving will still be better). In this sense, we can also say that direct solving approach is conservative compared with the modified rolling horizon approach.

Overall, we can say that the modified rolling horizon approach provides a quite good solving scheme for our dynamic programming problem. With this example in mind, we could have the confidence to implement our rolling horizon approach in a larger problem where direct solving of the dynamic model is practically impossible due to the computer memory restraint or extremely long solving time. Another example we are going to show belongs to this category.

### 3.4.3 AEX service route

AEX service route consists of 15 ports which means there are altogether $4^{16}$ scenarios and the parameter setting is given below. It is the same with that in Yao et al. [2012] for the purpose of fair comparison. In this example, we are going to solve the problem using the modified rolling horizon approach and then compare the results with the stationary model. Besides the three cases of bunker price percentage changes just given, we want to see another special case 0 of bunker prices uncertainty as represented by Tables 3.14 and 3.15. We set all four bunker price percentage changes to be 0. The purpose is to test the benefit of introducing bunker consumption uncertainty by controlling the bunker prices to be constant. In addition, we will study the effect of ship size difference on the overall operational decisions.

Table 3.14: Weekly bunker price change alternatives: Case 0

| Scenario | value | Probability | value |
|---|---|---|---|
| $C_1$ | 0 % | $P(C_1)$ | 0.25 |
| $C_2$ | 0 % | $P(C_2)$ | 0.25 |
| $C_3$ | 0 % | $P(C_3)$ | 0.25 |
| $C_4$ | 0 % | $P(C_4)$ | 0.25 |

Table 3.15: Transition matrix of the weekly bunker price changes: Case 0

| Scenario | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $C_1$ | 40 % | 30 % | 20 % | 10 % |
| $C_2$ | 30 % | 40 % | 20 % | 10 % |
| $C_3$ | 10 % | 20 % | 40 % | 30 % |
| $C_4$ | 10 % | 20 % | 30 % | 40 % |

#### 3.4.3.1 Parameter setting

Parameters for the AEX route is provided below in Table 3.16:

Table 3.16: Parameters for AEX service

| Parameter | Value |
|---|---|
| Number of port of calls | 15 |
| Service frequency | Weekly |
| Ship size | 6,000TEU |
| Total cycle time | 1512 h |
| Ship speed interval | 14–24 knots |
| Mean bunker consumption rate | $F = 0.007297V^3 + 71.4$ |
| Fixed bunkering cost pmt | $1,000$ |
| Inventory holding cost pmt | 50 |
| Coefficient of variation of bunker consumption rate | 0.07 |

#### 3.4.3.2 Comparison between the dynamic model solved by the modified rolling horizon approach and the stationary model

Failure rate in both models is controlled to be 0.01. In the modified rolling horizon method of this example, we look ahead 3 periods which we fully generate all the alternatives for them and for the remaining 13 periods, 8 price realizations are generated. In our comparison, 40 price scenarios have been generated.

Under Case 1 of the bunker price changes setting, average cost for the dynamic model solved by the modified rolling horizon approach is $\$3.66 \times 10^6$ and average cost for the stationary model is $\$3.84 \times 10^6$ which is about 4.9% of cost saving. Under Case 2 of the bunker price changes setting, average cost for the dynamic model solved by the modified rolling horizon approach is $\$3.79 \times 10^6$ and average cost for the stationary model is $\$4.07 \times 10^6$ which is about 7.4% of cost saving. Under Case 3 of the bunker price changes setting, average cost for the dynamic model solved by the modified rolling horizon approach is $\$3.82 \times 10^6$ and average cost for the stationary model is $\$4.30 \times 10^6$ which is about 12.6% of cost saving.

From Case 1 to Case 3, as the bunker prices become more volatile, the cost saving of the dynamic model solved by modified rolling horizon approach compared with the stationary model increases from 4.9% to 12.6%. Considering the huge amount of operational costs for a liner shipping company, this means a significant total cost reduction. Also this result does not surprise us because dynamic model should become more superior to stationary model when prices are more fluctuating.

Under Case 0 of the bunker price changes setting, the cost saving is 4.6%. This is the cost saving by introducing bunker consumption uncertainty solely. Therefore, we can see, under Case 1 of bunker price changes setting, the benefit of introducing stochastic bunker prices is only about 0.3%. However, under Cases 2 and 3, this increases to $(7.4-4.6) = 2.8\%$ and $(12.6-4.6) = 8.0\%$. This finding conforms with our intuition that the more volatile the bunker prices, the more benefits of considering the stochastic bunker prices.

Table 3.17 below summarizes the results so far for AEX service example.

Table 3.17: Comparison between the modified rolling horizon approach and the solving of stationary model

| Methods | Case 0 | Case 1 | Case 2 | Case 3 |
|---|---|---|---|---|
| $R$ | $\$3.51 \times 10^6$ | $\$3.66 \times 10^6$ | $\$3.79 \times 10^6$ | $\$3.82 \times 10^6$ |
| $S$ | $\$3.68 \times 10^6$ | $\$3.84 \times 10^6$ | $\$4.07 \times 10^6$ | $\$4.30 \times 10^6$ |
| $\frac{R-S}{S}(\%)$ | 4.6 | 4.9 | 7.4 | 12.6 |

Bunker inventory holding cost per ton in our problem is assumed to be constant and independent of the bunker prices. Thus we want to see how sensitive the result is to this parameter. In this AEX route example, bunker prices of all the ports at the initial stage are set around \$460 pmt with minimum \$456 and maximum \$471. Our previous results are based on bunker inventory holding cost being \$50 pmt. In our subsequent analysis, we want to see what will happen if we vary this parameter.

Take Case 1 of the bunker price changes setting for example, our numerical results show that when bunker inventory cost is \$100 pmt, dynamic model solved by the modified rolling horizon approach has 8.56% of cost saving to the stationary model, compared with 4.9% if bunker inventory cost is \$50 pmt. When inventory cost is \$150 pmt, this cost saving increases to 13.4%. Or if we set inventory cost to be \$25 pmt, the cost saving is 3.48%. This means generally when bunker inventory cost pmt increases, the dynamic model becomes even more superior to the stationary approach.

In addition, we can expect that the benefit of introducing bunker consumption uncertainty (Case 0 of the bunker price changes setting) increases with bunker inventory holding cost. If we set the bunker inventory holding cost to be \$25 pmt, the cost saving is 2.56%. If it is \$100 pmt, the cost saving is 8.53%, and if it is \$150 pmt, the cost saving increases to 11.2%.

Therefore, the benefits of introducing these two uncertainties increase with the volatility of the bunker prices and the bunker inventory holding cost.

#### 3.4.3.3 Effect of the ship size difference on the overall operational planning

In the end, we want to discuss the effect of using a different size of ship. For instance, what if the 3000-TEU ship is used here in this AEX route. All other parameters for the AEX service route remain the same, except for these related to the ship size. Based on the bunker prices scenarios generated in previous analysis under Case 1 setting of the percentage changes, and under \$50 pmt of the bunker inventory holding cost, the average cost for the dynamic model solved by the modified rolling horizon approach is $\$2.65 \times 10^6$ and the average cost for the stationary model is $\$2.82 \times 10^6$ which is about 6.0% of cost saving (4.9% for a 5000-TEU ship). The average costs are lower compared with the case when a 5000-TEU ship is used, because we can see from the bunker consumption rates in Table 3.10 and 3.16 that smaller ships burn less bunker sailing under the same speed and distance. Also the dynamic model performs even better than the stationary model when one smaller ship is used. This is largely due to the fact that smaller ships have a higher coefficient of variation of the bunker consumption rate. However, we notice that the average cost (dynamic model solved by the modified rolling horizon approach) per TEU for the 3000-TEU ship is $8.84 \times 10^2$ and that for the 5000-TEU ship is $7.32 \times 10^2$. This means, cost per TEU wise, bigger ships are more efficient.

Next, we want to see the effect of ship size difference on the bunkering and refueling decisions. In the stationary model, we find that the bunkering ports selection and bunkering amount will be different. However, speed choice is the same because there are no bunker prices and consumption uncertainties. In the dynamic model solved by the modified rolling horizon approach, both bunkering and speed decisions can differ when different sizes of ships are deployed. For example, under scenario [5 %,10 %,10 %,-5 %,-10 %,10 %,5 %,5 %,5 %,10 %,10 %,-5 %,-5 %,-5 %,-5 %,-10 %] (randomly selected one), the modified rolling horizon approach suggests to bunker at port 12 when a 5000-TEU ship is used, and not to bunker there if it is a 3000-TEU ship. Bunkering amount at every port are significantly different too. As for the ship speed, Figure 3.3 shows that during some legs, different sailing speeds are suggested for these two sizes of ships, although the difference is not very significant. However, under some scenarios, the difference can be larger as shown in Figure 3.4 under scenario [-5 %,5 %,-5 %,-5 %,5 %,-5 %,10 %,5 %,10 %,-5 %,5 %,-5 %,-5 %,-5 %,5 %,5 %].

To conclude, we think ship size differences impose significant effect on the overall operational planning for the liner shipping companies.

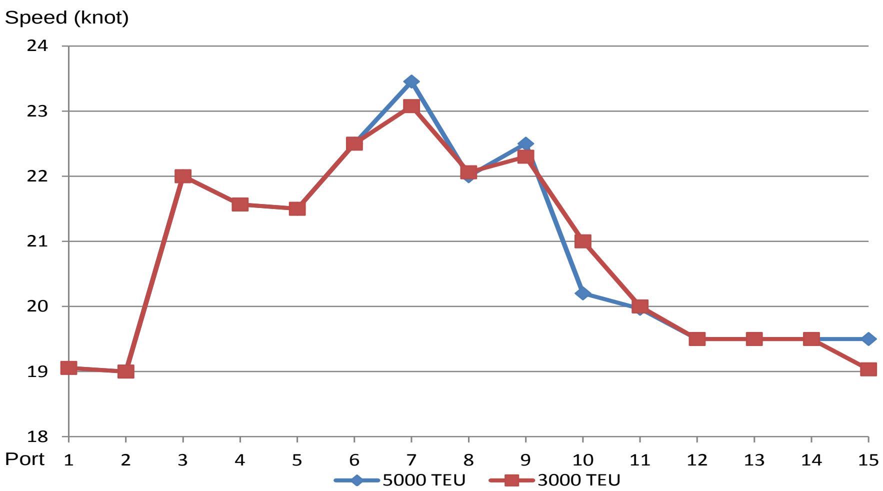


Figure 3.3: Optimal speed decisions given by the modified rolling horizon approach 1

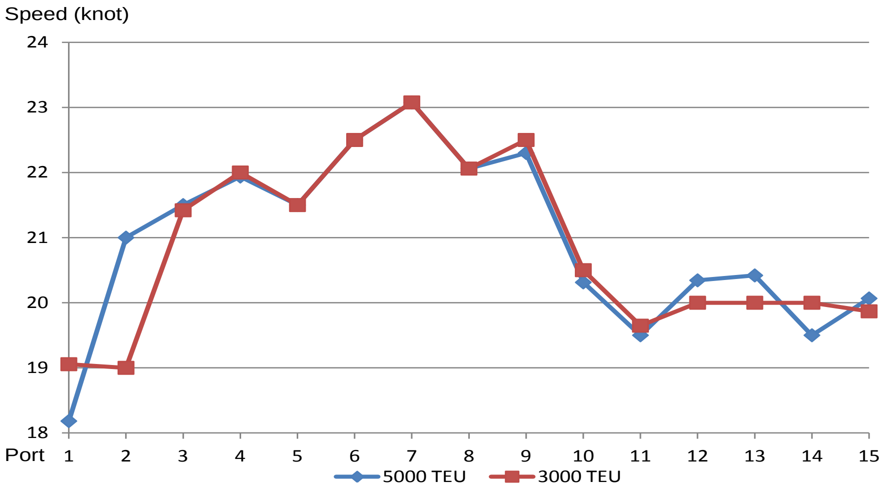


Figure 3.4: Optimal speed decisions given by the modified rolling horizon approach 2

## 3.5 Summary

This chapter studies the problem of dynamic bunkering port selection and ship speed determination for a single vessel in one service route. While previous deterministic works focus more on the planning level of this problem, we aim at providing operational decision support by incorporating two major random factors into our model. Namely, the ship bunker consumption rate and the bunker prices at each port. Based on the bunker consumption model in Yao et al. [2012], we further established that the noise of daily bunker consumption follows a normal distribution with zero mean and constant coefficient of variation. For the stochastic nature of the bunker prices, we have modeled it through the scenario tree which is widely used in financial engineering area to depict the randomness of the financial product returns. While solving a whole large dynamic problem is computational challenging, we proposed a solving method that could help to significantly reduce the computer memory requirement and solving time. This method is a combination of scenario tree generation scheme and a non-standard rolling horizon approach. Another advantage about this solving method is that as much new information as possible is used and previous forecasting errors could be easily corrected during the whole study horizon. Our numerical examples based on real-world data have shown that the dynamic model improves significantly in terms of overall cost and service level (or failure rate) compared with the stationary model. With the reasonable solving time, we think our model could be implemented by liner shipping companies to give operational level decision support in order to lower the overall operation cost and provide more reliable service.

Some possible future research directions are first, we have noticed that though the number of scenarios would be very huge with just a few number of ports, many of them share the same optimal decision. This is a phenomenon determined by our problem nature. In our problem, time window determines the ship speed range during each leg, which determines the bunker consumption. In the end, how much bunker consumed determines how much needs to be bunkered. Also, a change of optimal decision in our problem usually means a change of bunkering ports. However, a change of bunkering ports would not happen unless bunker prices at one port become significantly attractive considering the bunker inventory holding cost. Therefore, the optimal decision is not very sensitive to the bunker price changes and we could look for ways to group those scenarios which give the same, or close, optimal solution. Second, in our current work, no structured policy is followed. The bunkering decision and speed selection could always change along with external factors. As a future research, we want to propose a $(s, S)$ policy like that in the inventory management problem. When the ship bunker inventory drops below $s$, we bunker fuel up to the level of $S$. Careful readers will find our problem has a lot of similarities with inventory management problem. Bunker inventory is equivalent to product inventory, and bunker consumption is equivalent to product demand, running out of fuel before finishing a voyage leg is equivalent to an inventory being out of stock. Also, for the bunker fuel consumption, instead of using chance constraints, we could also use the sample average approximation method to model the uncertainty of bunker consumption. Last but not least, soft time windows associated with penalty cost and inventory holding cost depending on bunker price could be introduced into our model to render it more realistic.

# Chapter 4

# $(s, S)$ policy model for liner shipping refueling and sailing speed optimization problem

Based on the current intense business environment of high volatile bunker prices, fierce market competition, large fleet of ships in operation to name a few, it would be highly appealing for liner shipping practitioners if there were a simple, yet effective, strategy that guides timely operational decision making on a daily basis. This work expounds on this issue. We adopt the $(s, S)$ policy which has been effectively used in inventory management to solve a liner shipping refueling and vessel speed determination problem under both bunker prices and consumption uncertainties.

We found that our problem nature is very similar with that of the inventory management. Bunker fuel in our case is the "product"; Bunker consumption within each leg is the "product demand" during each period; And inventory holding costs are similar in both cases. In the inventory management literature, many researchers ([Scarf, 1993], [Karlin, 1960], [Zheng and Federgruen, 1991] etc.) have established that, under mild assumptions, a simple $(s, S)$ replenishment policy is optimal. Here $s$ is the re-order point. When the inventory level is less than or equal to $s$, an order is triggered, which increases the inventory level up to $S$. In [Kalymon, 1971], a single-item multi-period inventory problem was studied where the future purchase prices for the item was modeled by a markovian stochastic process and convex holding and shortage costs and a set-up cost for ordering were assumed. It showed that a policy of the form $(s_i(p), S_i(p))$ was optimal based on the aforementioned assumptions in the finite horizon case. Here, $p$ is the realized item prices at the current time. Thus, very naturally, we want to implement a similar $(s_i(p), S_i(p))$ policy in our bunkering decision to provide a contingent bunkering plan, the execution of which depends on the actual bunker prices and consumption realizations. What is different is that in our problem, $p$ means the historical realized bunker prices up till the current time. At each port, there is a bunker re-order point $s_i(p)$ associated with it. If bunker inventory is below this critical point, a bunkering decision takes place and bunker inventory is increased to the bunker up to level $S_i(p)$. This allows a more flexible operational bunkering plan; the decision of whether to bunker or not depends on the actual bunker price realizations as well as the ship bunker inventory at every port.

However, what further complicates our problem is that our policy parameters are a combination of discrete and continuous variables, which make solving approaches based on dynamic programming practically impossible. Moreover, we need to make the vessel speed decision for each leg. This poses yet another additional challenge in our problem. Hence, special effort has been dedicated to devise an effective solving scheme which will be discussed later.

## 4.1 Problem description

Liner service network and schedule are determined in advance before the actual customer demand is realized, which is different from that of tanker service. Starting from one port, ships travel through all the ports in the network (sometimes one port may be visited more than once) and return back to the starting port. Due to this nature, the study of liner service is more focused on cost minimization rather than profit maximization. Also, fleet mix and deployment problem is a tactical level decision, thus we assume the number of ships deployed in each service is known and we can conveniently study one ship for our operation level problem.

Denote $\boldsymbol{L}$ as the set of all port calls. $L_1$ is the first port-of-call and $L_i$ is the

$ith$ port-of-call. $L_i$ and $L_j$ $(i \neq j)$ may represent the same port. Assume that in the published schedule, there is a time window associated with each port-of-call. It is preferable that a ship could reach within the time window, otherwise, a lump sum penalty cost or a cost proportional to the duration of violation is incurred which represents the cost of the surcharge by port operators, compensations to customers, loss of goodwill and so on. When the ship reaches one port, based on the current bunker inventory and all the available information for the bunker prices, a refueling decision is made. In our model, we adopt a $(s_i(p), S_i(p))$ refueling policy which tells the ship to bunker up to $S_i(p)$ at port $i$ if the bunker inventory is less than $s_i(p)$ and not bunker if otherwise. As mentioned, $p$ means the historical realized bunker prices up till the current time.

Aside from the refueling decision, we need to decide the ship's speed for the next leg simultaneously. As mentioned, slow sailing speed saves fuel and thus reduces cost. However, violating the time window constraints incurs a cost. Therefore, there is a trade off between low sailing speed and high schedule reliability which we need to balance. Ship speed also determines the bunker consumption within each leg. In [Sheng et al., 2013], it is empirically shown that the daily bunker consumption rate under a certain speed has a mean $\bar{F} = k_1 * V^3 + k_2$, where $k_1$ and $k_2$ are two constants and $V$ denotes the sailing speed, and noise which follows a zero mean normal distribution with standard deviation a constant percentage of $\bar{F}$. Due to this consumption uncertainty, a good refueling policy should avoid these scenarios that a ship runs out of bunker in between two ports as much as possible. Note that the bunkering policy and the speed decision only depend on the bunker prices but not the bunker consumption. In our model, "backlogging" of fuel is allowed, however, a high penalty cost (for example, pay for a fast emergency refueling) will be charged or for the amount of shortage, fuel will be bunkered at a premium price.

As the bunker consumption rate is a cubic function of the ship speed, either a non-linear programming problem or a problem with a large amount of integer variables, if we use a piece wise linear approximation, needs to be solved. One nice way to circumvent this problem, which also conforms to real practice, is to discretize the ship speed into a small set of choices. For instance, [Oh and Karimi, 2010] and [Gelareh and Meng, 2010] used this discretized sailing speed in their problems respectively. We denote $\boldsymbol{V}_{i,i+1}$ as the set of discretized feasible sailing speeds (nautical miles/h) between port $i$ to $i+1$. There are $G_{i,i+1}$ possible choices in set $\boldsymbol{V}_{i,i+1}$.

As for modeling uncertain bunker prices, we learnt that the scenario tree structure has been widely applied in the financing industry to model the uncertain returns of various financial products. Taking into account the similarity between the nature of bunker fuel and those financial products, we also use a scenario tree structure as shown in Figure 4.1 to model the evolution of bunker prices. However, a serious problem associated with this approach is the huge problem size. If a large amount of integer variables are associated with each decision stage as in our case, then it becomes even more computationally challenging to solve the problem. Here we propose a method which takes advantage of an effective combination of progressive hedging algorithm ([Rockafellar and Wets, 1991], [Løkketangen and Woodruff, 1996] and [Watson and Woodruff, 2011]) and rolling horizon approach. Implementation details will be discussed in Section 4.3.

In the end, the objective is to minimize the total cost in a single service loop, which consists of fixed and variable bunkering cost, bunker inventory holding cost, penalties for violating the time window and minimum bunker inventory constraints. Fixed bunkering cost is assumed to be constant across all ports and variable bunkering cost is dependent on the bunker prices and bunkering amount. Bunker inventory holding cost per metric ton (pmt) is also assumed to be constant across ports and depends on the bunker inventory when a ship leaves one port. For the fuel left at the end of service loop, we assume that we can sell it at the bunker prices of that time period and deduct the revenue from the total cost.

## 4.2 Modeling

### 4.2.1 Sample average approximation

As discussed, the bunker consumption between each leg has a mean $\bar{F} = k_1 * V^3 + k_2$ and noise which follows a zero mean normal distribution with standard deviation a constant percentage of $\bar{F}$. However, solving the model which incorporates a continuously distributed stochastic variable will be extremely hard if not

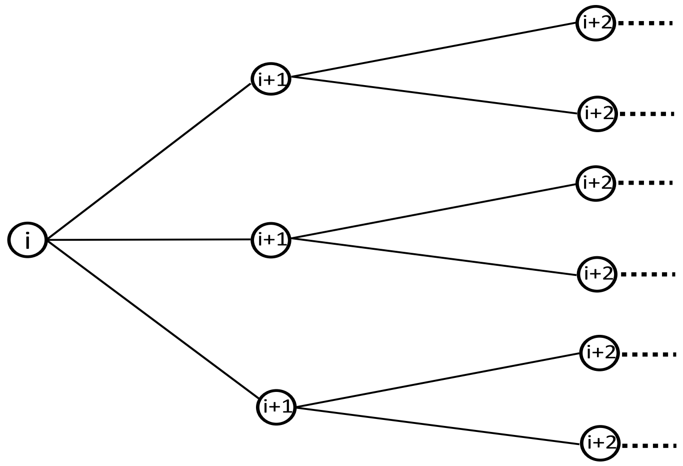


Figure 4.1: A simple example of scenario tree

impossible. Therefore, following the basic idea of sample average approximation (SAA) method, we randomly generate a random sample of consumption scenarios to feed into our model and run several replications with different samples to reach a candidate solution. As the fluctuation of the bunker prices and uncertainty of the bunker consumptions are independent, common consumption scenarios are generated for different price scenarios. One thing we need to mention is that because the mean bunker consumption is determined by the vessel speed, generating consumption scenarios thus means generating different scenarios of noise of the mean consumption. Our numerical studies show that the optimality gap of SAA method in our problem is rather small. For a general introduction of the SAA method, please refer to [Kleywegt et al., 2002].

### 4.2.2 Scenario tree for bunker prices uncertainty

When we use the scenario tree to model the uncertainty of bunker prices, constraints which enforce non-anticipativity or implementability must be added to conform to the reality that decisions are made before the realizations of future prices, of which we only know their probability distributions and, if two scenarios are indistinguishable up to time $T$, their corresponding decisions before $T$ should be the same. Section 4.2.4 summarizes and defines all the notations that are going to be used. Section 4.2.5 presents our mathematical model.

### 4.2.3 Assumptions

Here, we summarize the two important assumptions made in our paper:

1. The bunkering policy and the speed decision depend only on the bunker prices but not the consumption.

2. Common consumption noise scenarios are generated for different price scenarios.

### 4.2.4 Notations

Following notations are used to express our dynamic stochastic problem:

| | |
|---|---|
| $R$ | total number of bunker price scenarios; |
| $\Pi^r$ | the probability that price scenario $r$ happens; |
| $K$ | total number of bunker consumption scenarios generated; |
| $P_i^r$ | bunker prices for port $i$ under price scenario $r$; |
| $n$ | total number of port of calls; |
| $d_{i,i+1}$ | distance between port $i$ and $i+1$ (nautical miles); |
| $t_i$ | total port time at port $i$ (hours); |
| $e_i$ | earliest arrival time (EAT) at port $i$; |
| $l_i$ | latest arrival time (LAT) at port $i$; |
| $C_i$ | bunker fuel consumption when the ship is at port $i$; |
| $W$ | bunker fuel capacity for a single ship; |
| $\boldsymbol{V}_{i,i+1}$ | The set of discretized feasible sailing speeds (nautical miles/hour) between port $i$ to $i+1$; |
| $G_{i,i+1}$ | set of all possible choices in $\boldsymbol{V}_{i,i+1}$. |
| $\tau_1$ | denotes the penalty for violating the bunker inventory constraint |
| $\tau_2$ | denotes the penalty for violating the time window constraint |
| $X_{i,1}^r$ | amount of time that the ship reaches port $i$ earlier than schedule; $= 0$ otherwise; |
| $X_{i,2}^r$ | amount of time that the ship reaches port $i$ later than schedule; $= 0$ otherwise; |
| $Y_i^{k,r}$ | indicator variable.$= 1$ if bunker inventory when ship reaches port $i$ is less than a certain amount; $= 0$ otherwise; |

| | |
|---|---|
| $f$ | fixed bunkering cost; |
| $h$ | inventory holding cost for bunker; |
| $\eta$ | coefficient of variation for daily bunker consumption rate |
| $M$ | is a big positive number; |
| $\theta_i^k$ | normal random number with mean 1 and standard deviation $\eta$ during leg $i$ and scenario $k$; |
| $\epsilon$ | is a very small positive number; |

The following decision variables are defined:

| | |
|---|---|
| $Z_{i,i+1}^{g,r}$ | indicator variable.= 1 if sailing speed $v_{i,i+1}^g (v_{i,i+1}^g \in \boldsymbol{V}_{i,i+1}, g \in G_{i,i+1})$ is chosen under price scenario $r$; = 0 otherwise; |
| $S_i^r$ | bunker fuel-up-to level for the ship at port $i$ under price scenario $r$; |
| $s_i^r$ | bunker ordering point for the ship at port $i$ under price scenario $r$; |

Dependent variables:

| | |
|---|---|
| $B_i^{k,r}$ | bunkering decision variable. = 1 if bunkering at port $i$ under bunker consumption scenario $k$, = 0, otherwise; |
| $I_{i,1}^{k,r}$ | bunker fuel inventory when the ship reaches port $i$ under bunker consumption scenario $k$ and price scenario $r$ ; |
| $I_{i,2}^{k,r}$ | bunker fuel inventory when the ship departs port $i$ under bunker consumption scenario $k$ and price scenario $r$ ; |
| $\bar{F}_{i,i+1}^g$ | average bunker consumption per nautical mile when the sailing speed is $v_{i,i+1}^g$, and $\bar{F}_{i,i+1}^g = (k_1 \cdot (v_{i,i+1}^g)^3 + k_2)/(24 \times v_{i,i+1}^g)$; |
| $A_i^r$ | ship arrival time at port $i$ under price scenario $r$; |

### 4.2.5 Model

To reiterate, one of the outputs of this model, bunkering policy parameters $(s_i^r, S_i^r)$, allows a more flexible operational bunkering plan; the decision of whether to bunker or not depends on the bunker price realization as well as the actual bunker inventory at every port. To put it simply: for each price scenario realization, our work provides a contingent bunkering plan, the execution of which also depends on the actual consumption.

Mathematical model for our problem with a consumption scenario sample size $K$:

$$\min \sum_{k=1}^{K} \frac{1}{K} \sum_{r=1}^{R} \Pi^r \big( \sum_{i=1}^{n} [(I_{i,2}^{k,r} - I_{i,1}^{k,r} + C_i) P_i^r + B_i^{k,r} f + I_{i,2}^{k,r}\, h + \tau_1\, Y_i^{k,r} + \tau_2 (X_{i,1}^r$$
$$+ X_{i,2}^r)] - I_{n+1,1}^{k,r}\, P_{n+1}^r \big)$$

$$I_{1,1}^{k,r} = 0 \qquad \forall k \in K, \forall r \in R \tag{4.1}$$

$$I_{i,2}^{k,r} \leq S_i^r - C_i + (1 - B_i^{k,r}) \cdot M \qquad \forall k \in K, \forall r \in R, i \in 1,2,..n \tag{4.2}$$

$$I_{i,2}^{k,r} \geq S_i^r - C_i - (1 - B_i^{k,r}) \cdot M \qquad \forall k \in K, \forall r \in R, i \in 1,2,..n \tag{4.3}$$

$$I_{i,2}^{k,r} \leq I_{i,1}^{k,r} - C_i + B_i^{k,r} \cdot M \qquad \forall k \in K,, \forall r \in R, i \in 1,2,..n \tag{4.4}$$

$$I_{i,2}^{k,r} \geq I_{i,1}^{k,r} - C_i - B_i^{k,r} \cdot M \qquad \forall k \in K, \forall r \in R, i \in 1,2,..n \tag{4.5}$$

$$I_{i+1,1}^{k,r} = I_{i,2}^{k,r} - \sum_{g=1}^{G_{i,i+1}} \bar{F}_{i,i+1}^{g} \cdot Z_{i,i+1}^{g,r} \cdot d_{i,i+1} \times \varepsilon_i^k \quad \forall k \in K, \forall r \in R, i \in 1,2,..n \tag{4.6}$$

$$\sum_{g=1}^{G_{i,i+1}} Z_{i,i+1}^{g,r} = 1 \qquad \forall r \in R, i \in 1,2,,..n \tag{4.7}$$

$$0 < W \cdot B_i^{k,r} + (I_{i,1}^{k,r} - s_i^r) \leq W \qquad \forall k \in K, \forall r \in R, i \in 2,3,..n+1 \tag{4.8}$$

$$s_i^r \leq S_i^r \qquad \forall r \in R, i \in 1,2,,..n \tag{4.9}$$

$$S_i^r \leq w \qquad \forall r \in R, i \in 1,2,,..n \tag{4.10}$$

$$I_{i,1}^{k,r} + M \cdot Y_i^{k,r} \geq 0 \qquad \forall k \in K, \forall r \in R, i \in 2,3,..n+1 \tag{4.11}$$

$$A_i^r + t_i + \sum_{g=1}^{G_{i,i+1}} Z_{i,i+1}^{g,r} \cdot d_{i,i+1}/v_{i,i+1}^{g} = A_{i+1}^r \qquad \forall r \in R, i \in 1,2,,..n \tag{4.12}$$

$$e_i \leq A_i^r + X_{i,1}^r \qquad \forall r \in R, i \in 1,2,,..n+1 \tag{4.13}$$

$$A_i^r - X_{i,2}^r \leq l_i \qquad \forall r \in R, i \in 1,2,,..n+1 \tag{4.14}$$

$$Z_{i,i+1}^{g,r} = Z_{i,i+1}^{g,r'} \qquad \forall (r,r') \in R,\ indistinguishable\, up\, to\, i, i \in 1,2,,..n \tag{4.15}$$

$$S_i^r = S_i^{r'} \qquad \forall (r,r') \in R,\ indistinguishable\, up\, to\, i, i \in 1,2,,..n \tag{4.16}$$

$$s_i^r = s_i^{r'} \qquad \forall (r,r') \in R,\ indistinguishable\, up\, to\, i, i \in 1,2,,..n \tag{4.17}$$

$$B_i^{k,r} = 0 \text{ or } 1 \qquad \forall r \in R, i \in 1,2,,..n \tag{4.18}$$

$$Y_i^{k,r} = 0 \text{ or } 1 \qquad \forall k \in K, r \in R, i \in 1,2,,..n+1 \tag{4.19}$$

$$Z_{i,i+1}^{g,r} = 0 \text{ or } 1 \qquad \forall g \in G_{i,i+1}, r \in R, i \in 1,2,,..n \tag{4.20}$$

$$X_{i,1}^r \geq 0, X_{i,2}^r \geq 0 \qquad \forall r \in R, \forall i \in 1,2,,..n+1 \tag{4.21}$$

$$I_{i,2}^{k,r} \geq 0 \qquad \forall r \in R, \forall k \in K, \forall i \in 1, 2, , ..n \tag{4.22}$$

$$S_i^r \geq 0 \forall r \in R, \forall i \in 1, 2, , ..n \tag{4.23}$$

$$G_{n,n+1} = G_{n,1}, d_{n,n+1} = d_{n,1}, Z_{n,n+1}^{g,r} = Z_{n,1}^{g,r} \tag{4.24}$$

The objective function is to minimize the expected total cost, which includes fixed and variable bunkering costs, bunker inventory holding cost and penalty costs for violating the bunker inventory and time window constraints. Bunker left at the end of one service loop or the beginning of a new loop is deducted as though it could be sold in the spot market. Constraint (4.1) sets initial ship bunker inventory at zero under different bunker consumption and price scenarios, however this can be problem specific. Constraints (4.2) to (4.5) state that if ship bunkers fuel at port $i$, then bunker inventory when it departs the port is bunker up to level minus the bunker consumption at port $i$, otherwise it is equal to the bunker inventory when the ship reaches port $i$ minus the bunker consumption in port $i$. Constraint (4.6) is flow conservation constraint. Constraint (4.7) ensures that only one sailing speed is chosen. Constraint (4.8) ensures that a bunkering decision is made when and only when the bunker inventory is less than or equal to the re-order point. Constraint (4.9) sets the bunker-up-to level higher than the re-order point. Constraint (4.10) puts a upper limit on bunker-up-to level which is the maximum fuel capacity. Constraint (4.11) is related to the objective function. When the bunker inventory drops to zero before finishing one voyage leg, a penalty cost is paid. Constraints (4.12) to (4.14) are about soft time windows. If a ship reaches port $i$ earlier than EAT or later than LAT, then a penalty is incurred. Constraints (4.15) to (4.17) are non-anticipative constraints. In the end, constraints (4.18) to (4.23) are binary and non-negative constraints.

## 4.3 Solution method and numerical examples

In terms of solving the model, the biggest challenge was posed by the large number of integer variables. Speed choice decision is associated with every price scenario and every stage; bunkering decision is further associated with every consumption scenario. This causes the number of integer variables to grow out of control very easily. In the following sections 4.3.1 and 4.3.2, we will present our solution methodologies.

### 4.3.1 Progressive hedging algorithm

Decomposition algorithms are commonly used when the size of the problem on hand is too large to be solved directly. Basically, decomposition algorithms for multi-stage stochastic programming belong to two major categories, namely the primal decomposition which breaks the original problem down in terms of stages and the dual decomposition which works on subproblems of individual scenarios. PHA falls into the latter category. In the case of convex problem, PHA converges to the global optimal and if it converges in the non-convex case, it achieves a local optimum. As our problem is a mixed-integer programming problem, it is non-convex and therefore we cannot ensure that a global optimal solution can be reached.

The idea of PHA is that by relaxing constraints which force implementability, the problem can be solved much more easily according to individual scenarios. In our problem, we will relax the implementability constraints (4.15) to (4.17) which are related to bunker price scenarios. The remaining problem is decomposable according to different price scenarios. For example, for price scenario $r$, $\forall r \in R$, the objective function for the sub-problem is:

$$\min \sum_{k=1}^{K} \frac{1}{K} \Big( \sum_{i=1}^{n} [(I_{i,2}^{k,r} - I_{i,1}^{k,r} + C_i) P_i^r + B_i^{k,r} f + I_{i,2}^{k,r}\, h + \tau_1\, Y_i^{k,r} + \tau_2 (X_{i,1}^r + X_{i,2}^r)] - I_{n+1,1}^{k,r}\, P_{n+1}^r \Big)$$

Subject to those constraints in (4.1)-(4.14) and (4.18)-(4.24) that are related to price scenario $r$.

The above sub-problem can be solved easily with all the bunker consumption scenarios. For example, we can obtain optimal reorder points $s_i^r$ and order-up-to levels $S_i^r$, $\forall i \in n$, for individual scenario sub-problem $r$, $\forall r \in R$. However, optimal solutions for individual scenarios are not very meaningful since we cannot know exactly which scenario will happen in the future, but only its probability distribution. Therefore we need a way to construct an implementable solution from those individual scenario solutions. One straightforward method is to take the weighted average of them. Suppose $\bar{s}_i^{\mathcal{A}_{r,i}}$, $\forall i \in n$, ($\mathcal{A}_{r,i}$ is the set of scenarios who share the same history with scenario $r$ till port $i$), is an implementable solution obtained by this way. The issue here is that $\bar{s}_i^{\mathcal{A}_{r,i}}$, $\forall i \in n$, may not be feasible for the original problem or they are still far from the real optimal solutions, or both.

Therefore, in the next iteration of PHA, it will solve these scenario subproblems again with additional augmented Lagrangian penalty terms in the objective functions. For the reorder point $s_i^r$, $\forall i \in n$ and $\forall r \in R$, these two penalty terms are: $w_i^r \times (s_i^r - \bar{s}_i^{\mathcal{A}_{r,i}})$ and $\frac{\rho_i^r}{2} \times (s_i^r - \bar{s}_i^{\mathcal{A}_{r,i}})^2$. They penalize the violation of individual re-order point $s_i^r$ to the implementable solution $\bar{s}_i^{\mathcal{A}_{r,i}}$. $w_i^r$ and $\rho_i^r$ are two penalty coefficients which get updated in each iteration. Penalty terms for other decision variables are added in the same fashion. Such a PHA iteration terminates until a good enough solution is obtained or the total running time is reached. For a more detailed discussion on PHA, interested readers could refer to the seminal work done by [Rockafellar and Wets, 1991].

However, the implementation of the aforementioned classical PHA for our problem is not very straightforward and our preliminary numerical runs based on it did not produce satisfactory results. This is due to our problem nature which is related to constraint (4.8): $0 < W \cdot B_i^{k,r} + (I_{i,1}^{k,r} - s_i^r) \leq W$. Since there are altogether $K$ consumption scenarios for each price scenario and at each period, there are $K$ discrete $I_{i,1}^{k,r}$ values ($\forall k \in K$). Assume that we sort them from the smallest to the largest. When $s_i^r$ takes the value in between two bunker inventory levels, $I_{i,1}^{k_1,r}$ and $I_{i,1}^{k_2,r}$ ($k_1, k_2 \in K$ and suppose $I_{i,1}^{k_1,r} < I_{i,1}^{k_2,r}$) for example, the objective value of our model does not change when $s_i^r$ takes value in the range $[I_{i,1}^{k_1,r}, I_{i,1}^{k_2,r})$. When $s_i^r$ further increases to the value of $I_{i,1}^{k_2,r}$, there will be a sudden change of the objective function value because $B_i^{k_2,r}$ changes from 0 to 1. Figure 4.2 is a more detailed illustration of the aforementioned phenomenon, in which $opts_i^r$ is the optimal re-order point for price scenario $r$ at port $i$ and $obj$ is the objective value of the scenario subproblem corresponding to price scenario $r$.

This discontinuity does not pose any problem—the minimum is achieved in the range $[I_{i,1}^{k_*}, I_{i,1}^{k_j})$–until the second iteration of PHA and penalty terms we just described come into the picture. With the additional penalty terms in the objective function, one possible situation looks like Figure 4.3. Clearly, we can

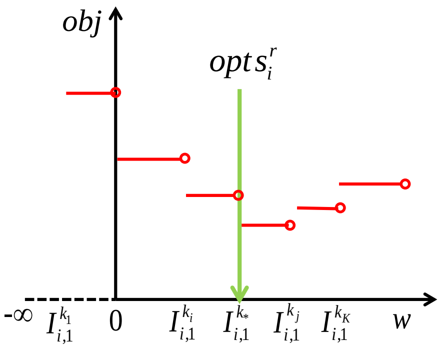


Figure 4.2: Discontinuous nature of our problem

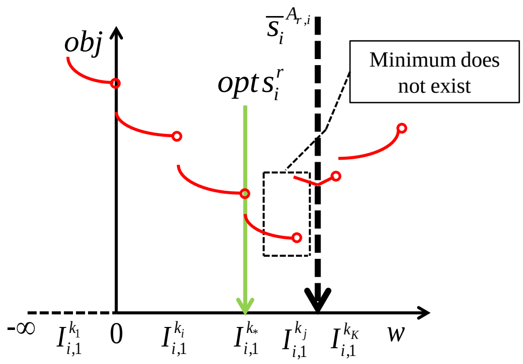


Figure 4.3: Objective function with the penalty term

see that the minimum does not exist in this case. Therefore, we remark that if the aforementioned progressive hedging algorithm is adopted in our problem, minimum does not exist at certain iteration of the algorithm. This renders the inability of PHA to solve our problem satisfactorily.

Recent advancement in PHA was made by taking advantage of the concept of "integer convergence" ([Løkketangen and Woodruff, 1996] and [Haugen et al., 2001]). They claimed that integer variables usually take less time to converge than other continuous variables and once these integer variables converge, the original problem can be solved in the deterministic extensive form with known integer variables. However, the aforementioned problem of the nonexistence of minimum still exists and even though we can partially circumvent it by adding one more constraint: $|s_i^r - I_{i,1}^{k,r}| \geq \epsilon, \forall r \in R, \forall k \in K$, where $\epsilon$ is a very small number (this is to ensure that $s_i^r$ does not take any one of the values of bunker inventory level), the choice of a good $\epsilon$ poses another problem. Moreover, our preliminary experiments showed that the performance of the PHA with integer convergence was still not very satisfactory. Algorithm terminated prematurely with very large optimality gap compared to the direct solving by CPLEX (for smaller model sizes, CPLEX can solve the problem to optimality). This can be explained as so. First, under best situation, PHA only guarantees a local optimal solution in the non-convex case, and second, when so many different decision variables, continuous and binary, come into the original objective function as well as the penalty terms, the solution quality is very vulnerable to the inappropriate choice of any one of the following sets of parameters during each iteration: implementable solutions for all the scenario sub-problems and two types of penalty coefficients in the PHA procedure.

While the previous analysis helps us understand the reasons why the typical PHA or PHA with integer convergence does not perform well, it also leads us to the direction of improvement. Our approach is to only penalize the violation of the binary dependent decision variable $B$, governing whether to bunker at each port under different price and consumption scenarios, from an implementable solution in each PHA iteration. In this way, we overcome the problem of the nonexistence of minimum. Also, we can still use the "integer converge" concept; when all the $Bs$ converge across all scenarios and stages, we fix them and go back to solve the original whole problem. In our numerical testings, we have recognized that $B$ poses the greatest computational challenge. Even if we know the values of all the $s_i^r$ and $S_i^r$ ($\forall r \in R, i \in 1, 2, ..., n$), when the number of consumption scenario increases, CPLEX still cannot solve our problem. However, when all the values of the $Bs$ are given, the problem can be solved in a matter of seconds.

For a start, we would like to slightly modify our original problem by adding one term, $\epsilon \times s_i^r$, in the objective function. Therefore, the new objective function is:

$$\min \sum_{k=1}^{K} \frac{1}{K} \sum_{r=1}^{R} \Pi^r \Big( \sum_{i=1}^{n} [(I_{i,2}^{k,r} - I_{i,1}^{k,r} + C_i) P_i^r + B_i^{k,r} f + I_{i,2}^{k,r}\, h + \tau_1\, Y_i^{k,r} + \tau_2 (X_{i,1}^r + X_{i,2}^r) + \epsilon \times s_i^r] - I_{n+1,1}^{k,r}\, P_{n+1}^r \Big)$$

The rationale of doing so is that by approximating the original problem, we have the following proposition for the new modified problem. In addition, we will state in the proof that the approximation error is limited by $\epsilon \times \sum_{r=1}^{R} \sum_{i=1}^{n} s_i^r$.

**Proposition 4.3.1** *In the new problem, reorder point $s_i^r$ will only take one of these discrete values: $-\infty$ and bunker inventory levels $I_{i,1}^{k,r}$, $\forall k \in K$.*

**Proof 4.3.2** *This proposition is rather intuitive. Only three different bunkering situations will happen:* (1)*, no bunkering is needed for every consumption scenario.* (2)*, bunkering is needed for certain consumption scenarios.* (3)*, bunkering is needed for every consumption scenario.*

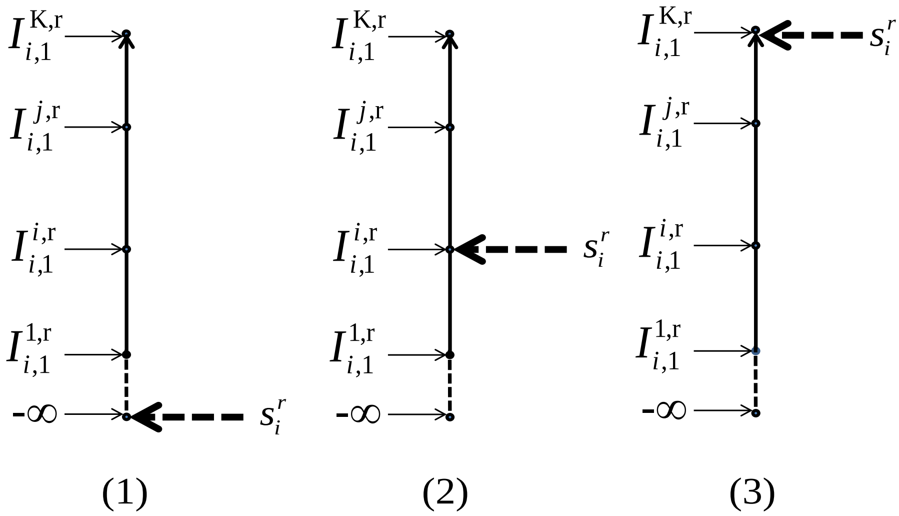


Figure 4.4: Figure illustration of Proposition 4.3.1 (a)

*If, for the ease of illustration, we re-order all the bunker inventory levels from the smallest to the largest and re-index them from 1 to $K$, together with $-\infty$. we can arrange them all in the Y-axis as shown in Figure 4.4. In the original model, as we have analyzed, the objective value would not change when the re-order point varies in the range $[I_{i,1}^{1,r}, I_{i,1}^{k,r})$ for example. However, after we add the term $\epsilon \times s_i^r$ in the objective function, the re-order point will be forced to choose $I_{i,1}^{1,r}$ as this is a minimization problem and the difference of the objective values for both models is $\epsilon \times \sum_{r=1}^{R} \sum_{i=1}^{n} s_i^r$. When we choose $\epsilon$ to be sufficiently small, the difference is negligible.*

*(1), (2) and (3) in Figure 4.4 correspond to the three different bunkering situations we just mentioned. This completes our proof.*

Proposition 4.3.1 helps significantly reduce the searching space of our optimization problem and allows us to determine $s_i^r$ based on the values of $B_{i,1}^{k,r}$, $\forall k \in K$, as shown in Figure 4.5. This was not true before because $s_i^r$ could vary within a certain range.

After relaxing constraints (4.15) to (4.17) in our model, it can be separated into sub-problems. Denote $P_r^{(0)}$, $\forall r \in R$, as the individual sub-problem for price scenario $r$ at iteration 0. By solving these sub-problems, we have $B_{i,1}^{k,r}$ for each price scenario $r$ and consumption scenario $k$. We then check how many of those

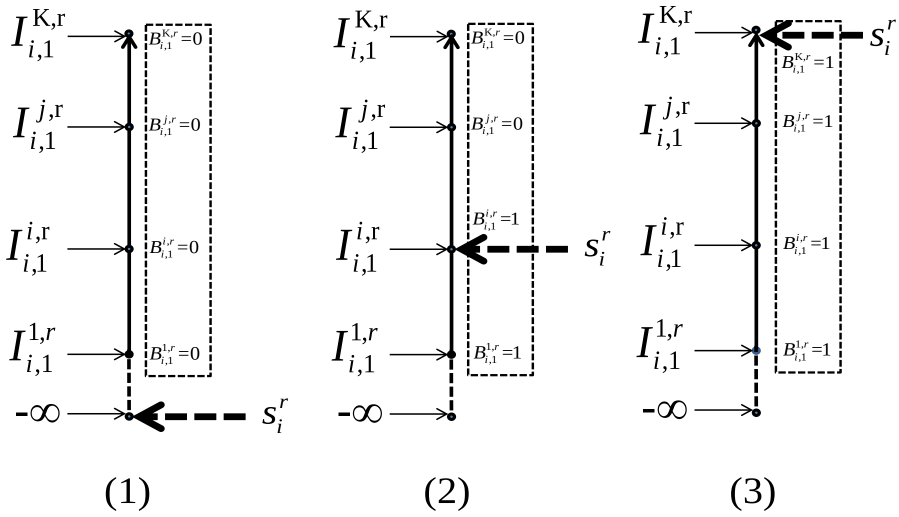


Figure 4.5: Figure illustration of Proposition 4.3.1 (b)

$Bs$ have converged and for those which have not converged, calculate the weighted average of these individual solutions based on the probability of each price scenario and obtain an implementable solution for the next iteration. We use $\Pi(\mathcal{A})$ to denote the sum of $\Pi(r)$ over all $r$ for scenarios emanating from node $\mathcal{A}$. Therefore, $\bar{B}_{i,1}^{k,r} = \sum_{r \in \mathcal{A}} \Pi(r) B_{i,1}^{k,r} / \Pi(\mathcal{A})$. Unconverged $Bs$ produce a fractional weighted average $\bar{B}_{i,1}^{k,r}$, so we need to apply a consistent way of rounding it to an integer value. This is the first iteration of our PHA.

At the second iteration, we will solve sub-problems $P_r^{(1)}$, $\forall r \in R$, which is different from $P_r^{(0)}$ by adding two additional penalty terms in the objective function. They are:

$$\sum_{k=1}^{K} \frac{1}{K} \sum_{i=1}^{n} w_{B,i}^{k,r} B_{i,1}^{k,r},$$

and

$$\sum_{k=1}^{K} \frac{1}{K} \sum_{i=1}^{n} \rho_{B,i}^{k,r} (B_{i,1}^{k,r} - \bar{B}_{i,1}^{k,r})^2,$$

where $\bar{B}_{i,1}^{k,r}$ is from the first iteration. For typesetting convenience, we use $\Omega_r$ to denote the constraints (4.1) to (4.14) and (4.18) to (4.23) for scenario $r$. Similarly, $P^{(1)}$ breaks down into individual sub-problems $P_r^{(1)}$. Each of them is a quadratic mixed-integer programming problem and by solving all of them, we obtain an updated implementable solution. Repeating the aforementioned steps for $P^{(q)}$ until all of the $Bs$ converge. Following the most widely used way to update all the $w$ and $\rho$, $w_{B,i}^{(q+1),k,r} \leftarrow w_{B,i}^{(q),k,r} + \rho_{B,i}^{(q),k,r}(B_{i,1}^{(q),k,r} - \bar{B}_{i,1}^{(q),k,r})$ and $\rho_{B,i}^{(q+1),k,r} \leftarrow \alpha\rho_{B,i}^{(q),k,r}$ where $\alpha > 1$ is a chosen constant.

A formal description of the PHA-WLB is given below.

**PHA-WLB procedure:**

```
input: an instance, all the related parameters
output: the best feasible solution Best

q ← 0;
    for ∀r ∈ R, i ∈ n
        set w_{B,i}^{(0),k,r} = 0 and solve all the sub-problems P_r^{(0)};
    end for
    Calculate the reference points B̄_{i,1}^{(0),k,r};
repeat progressive hedging iteration
q ← q + 1;
    for ∀r ∈ R, i ∈ n
        Update w_{B,i}^{(q),k,r}, ρ_{B,i}^{(q),k,r} accordingly and Solve sub-problems P_r^{(q+1)};
    end for
    Calculate the reference points B̄_i^{(q+1),k,r};
until all the B_{i,1}^{k,s} converge
Solve the original problem with all the known Bs and return Best;
```

We have one more variant of the above algorithm. That is, for those $Bs$ which have already converged in one iteration, we fix them in the next iteration of the PHA. While in the previous version, there is no such fixing scheme as in the typical progressive hedging algorithm. We name the first version of our algorithm as PHA-Without Locking $B$ (PHA-WLB) and the second version as PHA-Locking $B$ (PHA-LB). A figure illustration of the difference between these two is shown in Figure 4.6.

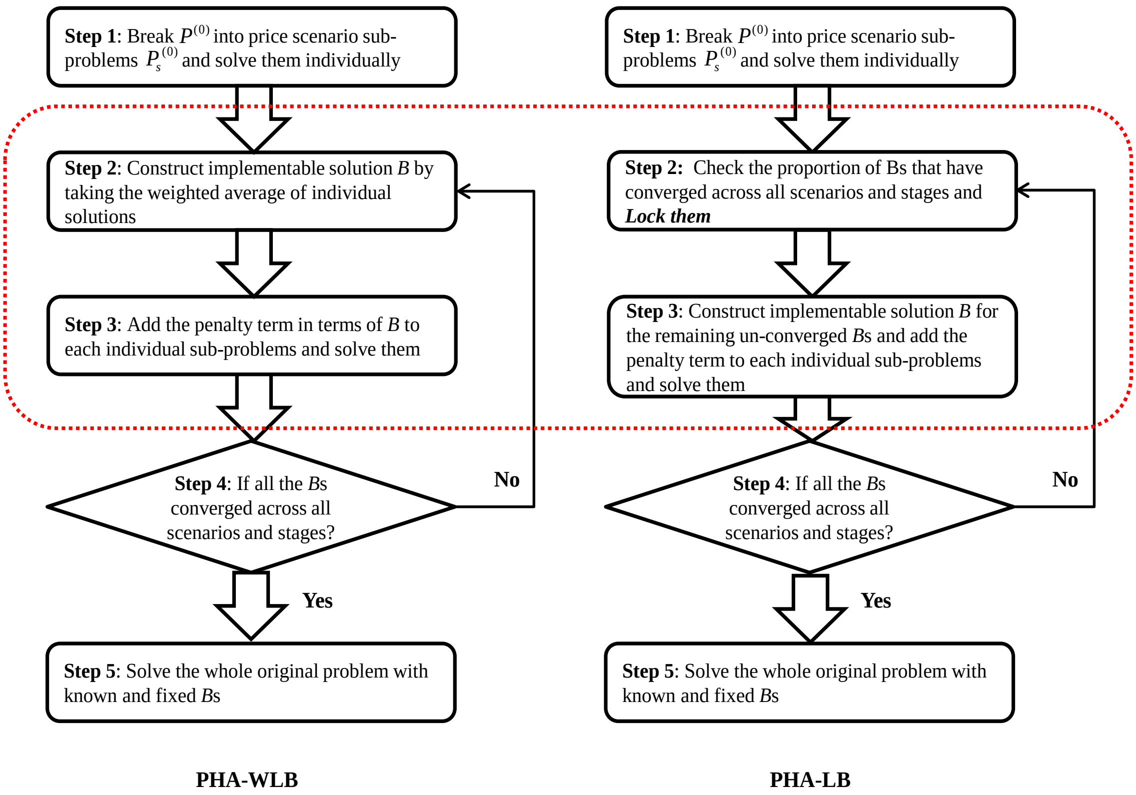


Figure 4.6: Flowchart representations of PHA-WLB and PHA-LB

### 4.3.2 Rolling horizon solving approach

When the problem size increases exponentially with the number of ports involved, even PHA cannot solve the problem within reasonable time. This is because, for one, PHA needs to solve price scenario sub-problems individually at each iteration and, for another, the size of individual sub-problem also keeps increasing. The modified rolling horizon approach we devised in our last work [Sheng et al., 2013] can well handle this kind of situation. We also made a detailed comparison of our modified rolling horizon approach with the scenario reduction algorithms mentioned in [Dupačová et al., 2003], [Heitsch and Römisch, 2003] and [Heitsch and Römisch, 2009]. Interested readers are suggested to refer to the Appendix B.

The essence of the standard rolling horizon planning scheme is: A problem with the study horizon shorter than the original one (to reduce the problem size) is solved and the first period decision is implemented. With newly available information, the problem is updated and resolved. Again, the decision is taken on the current period only. This process goes on and on until the end of the study horizon. Figure 4.7 below illustrates the solving procedure of the standard rolling horizon solving approach.

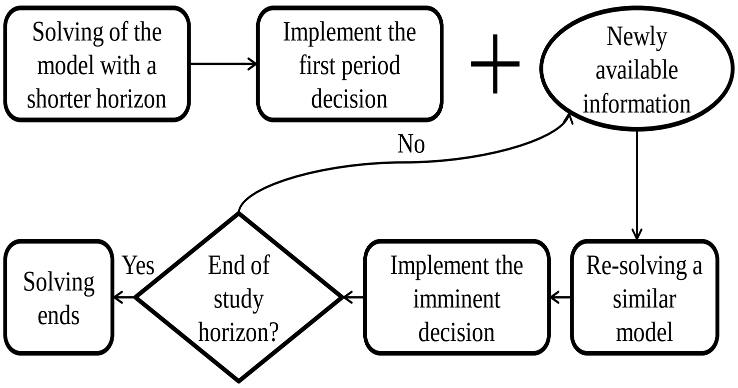


Figure 4.7: Diagram for the rolling horizon solving approach

Unlike the standard one which solves a problem with a shorter horizon than the original problem, our non-standard approach still solves the problem with the whole study horizon. However, we assign a higher level of fidelity for the nearer periods than the later ones by modifying the way we generate the scenario tree. For the first few number of periods (could be 1,2 or any number of periods depending on the problem), all the bunker price change alternatives are generated as shown in Figure 4.1, while a relatively small number of price realizations (also problem specific) are randomly generated for all the remaining periods till the end. Therefore, an example of our modified version of scenario tree would look like Figure 4.8, in which price scenarios for periods after $i + 2$ are randomly generated for each parent node. The validity of this non-standard variant is due to our problem nature and the diminishing tail-end effect.

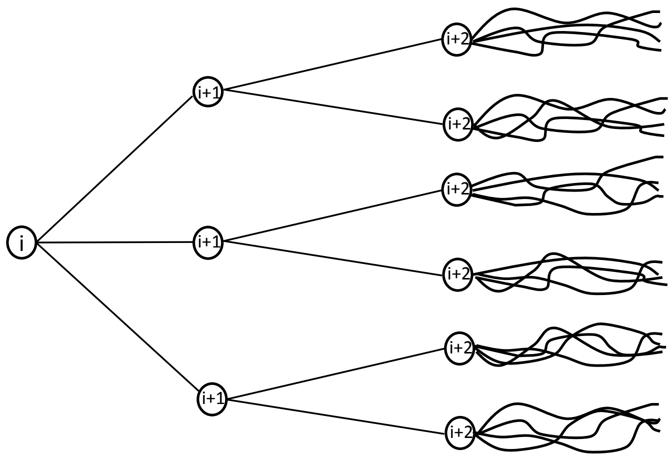


Figure 4.8: Modified scenario tree with randomly generated siblings

## 4.4 Numerical examples

### 4.4.1 Parameter setting

In this study, we will apply our model based on two actual liner services, namely the Malaysia Service (MAS) and the Asia-Europe Express (AEX), offered by a liner shipping company headquartered in Singapore. The MAS route consists of 3 port-of-calls, while the AEX route has 15 port-of-calls. Some key parameters of these two services are provided in table 4.1 and 4.2. For the MAS service with a relatively small number of consumption scenarios, CPLEX can solve it to optimality within reasonable time. We also use our PHA-WLB and PHA-LB to solve it so that the efficiency of these two variants of PHA can be tested.

We model the bunker prices evolution by a one-stage Markovian process with the percentage change in each period given by the scenario tree, of which the parameter setting is shown in Table 4.3 and 4.4. Without loss of generality, we assume that port calls are on a weekly basis and hence we only need to describe the bunker prices evolution on a weekly basis. One price scenario means a series of realizations of price percentage change in each port from the start to the end of voyage. Bunker prices at any port and time is calculated by the baseline prices as well as all the percentage changes during previous periods. For example, if we denote $\delta_t^i$ as the bunker prices percentage change at port $i$ and time period

Table 4.1: Parameters for MAS service

| Parameter | Value |
|---|---|
| Number of port of calls | 3 |
| Service frequency | weekly |
| Ship size | 3,000TEU |
| Ship speed choices | $6, 7, 8, 9, 10$ knots |
| Mean bunker consumption rate | $F = 0.006743V^3 + 37.23$ |
| Coefficient of variation of bunker consumption rate | 0.09 |
| Fixed bunkering cost pmt | 1000 |
| Inventory holding cost pmt | 50 |
| Hourly penalty for violating time windows | $1,000$ |
| Penalty for bunker inventory below a minimum level | $100,000$ |

Table 4.2: Parameters for AEX service

| Parameter | Value |
|---|---|
| Number of port of calls | 15 |
| Service frequency | weekly |
| Ship size | $6,000$TEU |
| Ship speed choices | $18, 19, 20, 21, 22$ knots |
| Mean bunker consumption rate | $F = 0.007297V^3 + 71.4$ |
| Coefficient of variation of bunker consumption rate | 0.07 |
| Fixed bunkering cost | $1,000$ |
| Inventory holding cost pmt | 50 |
| Hourly penalty for violating time windows | $1,000$ |
| Penalty for bunker inventory below a minimum level | $1,000,000$ |

$t$. There are baseline bunker prices $P_i^0$ at each port $i$ at time 0. Then, for all $i$, $P_i^1 = P_i^0 \times \delta_i^1$ and $P_i^j = P_i^{j-1} \times \delta_i^j$.

Table 4.3: Weekly Price Change Alternatives

| Scenario | value | Probability | value |
|---|---|---|---|
| $C_1$ | -10% | $P(C_1)$ | 0.25 |
| $C_2$ | -5.0% | $P(C_2)$ | 0.25 |
| $C_3$ | 5.0% | $P(C_3)$ | 0.25 |
| $C_4$ | 10% | $P(C_4)$ | 0.25 |

Table 4.4: Transition Probability between Alternatives

| Scenario | $C_1$ | $C_2$ | $C_3$ | $C_4$ |
|---|---|---|---|---|
| $C_1$ | 40% | 30% | 20% | 10% |
| $C_2$ | 30% | 40% | 20% | 10% |
| $C_3$ | 10% | 20% | 40% | 30% |
| $C_4$ | 10% | 20% | 30% | 40% |

Care should be given when generating the scenarios for bunker consumption because the noise of it follows a zero mean normal distribution with standard deviation a constant percentage of the mean consumption. Therefore, it is possible that numbers with very large absolute values will be generated. Based on the data we obtained, bunker consumption deviating from more than 20% of the mean consumption almost never happens. This prompts us to use a truncated normal distribution when generating consumption scenarios. In our defense, if the weather and/or sea conditions go so bad that significantly more than 20% of the mean consumption would be used, the ship should not sail on that day at all due to safety reasons. In the following numerical experiments, noise of the bunker consumption follows a normal distribution truncated at ±20%

Our problem is solved by CPLEX-12.4 running on a 3 GHz Dual Core PC with 4 GB of RAM.

## 4.4.2 Numerical results for MAS service

### 4.4.2.1 Performance of PHA-WLB and PHA-LB

When it comes to the evaluation of the performance of a heuristic algorithm, the rate of convergence and the solution quality are the two most important criteria. In our numerical experiments, both variants of PHA converge very fast. They are able to achieve full convergence within 20 iterations for almost all of our trials. PHA-LB converges slightly faster than PHA-WLB due to the locking mechanism. For example, Figure 4.9 is taken from one of our numerical runs and shows the proportion of convergence along with the number of algorithm iterations for both versions.

With regards to the solving time, when the number of consumption scenarios increases, our algorithms save a lot of time compared to the direct solving by CPLEX. Figure 4.10 is a comparison of the solving time among the CPLEX direct solving, PHA-WLB and PHA-LB. When there is only a small number of scenarios, direct solving uses less time because both variants of PHA need to solve a large number of price scenario sub-problems during each iteration. However, as more and more consumption scenarios are involved, the time savings by applying PHA increases exponentially. Compared to PHA-WLB, PHA-LB solves significantly faster.

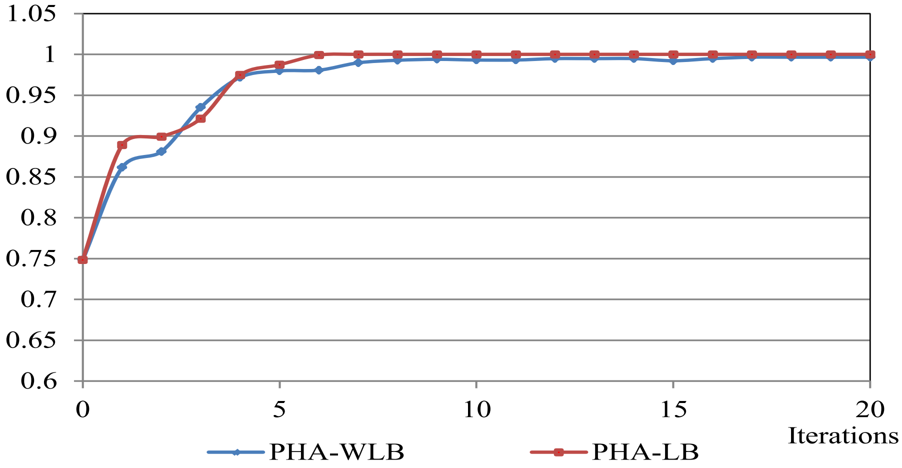


Figure 4.9: Convergence for both PHA-WLB and PHA-LB

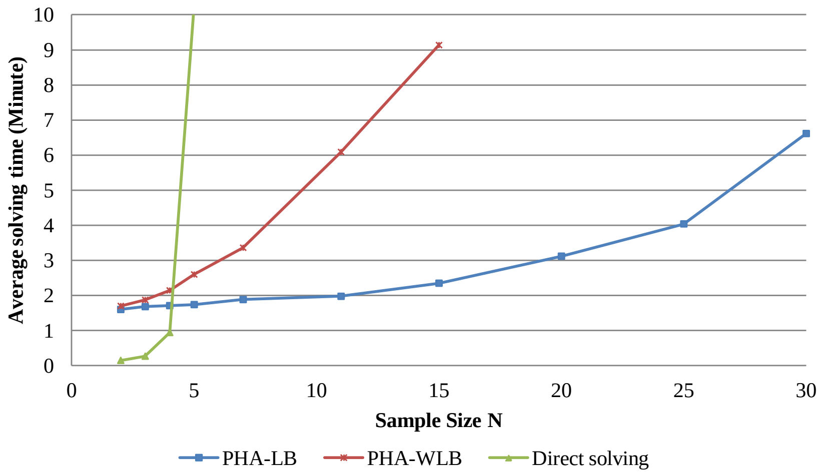


Figure 4.10: Solving time comparison between direct solving, PHA-WLB and PHA-LB

In terms of the the solution quality, results from PHA-WLB and PHA-LB are compared with the optimal values from direct solving by CPLEX with different numbers of consumption scenarios as shown in Table 4.5. For each fixed number of scenarios, we randomly generate new samples and re-run the test for 30 times and obtain the average. We can see that both algorithms are able to obtain results that are within 2% of the optimal values obtained by CLPEX and there is no significant difference between them in this respect.

Table 4.5: Optimality gap of PHA-WLB and PHA-LB

| Number of consumption scenarios | 3 | 5 | 7 | 9 | 11 | 16 |
|---|---|---|---|---|---|---|
| Optimal value from direct solving | 118, 387 | 117, 543 | 117, 665 | 117, 675 | 118, 788 | *N.A.** |
| PHA-WLB optimal value | 120, 457 | 119, 736 | 119, 807 | 119, 805 | 119, 982 | 120.110 |
| PHA-WLB optimality gap | 1.75% | 1.87% | 1.82% | 1.81% | 1.86% | *N.A.* |
| PHA-LB optimal value | 120, 457 | 119, 750 | 119, 822 | 119, 810 | 119, 997 | 120.130 |
| PHA-LB optimality gap | 1.75% | 1.88% | 1.83% | 1.81% | 1.88% | *N.A.* |

* Unable to solve within one hour

All in all, the above analysis regarding our two versions of progressive hedging algorithm shows that they work fairly well in our problem. They are able to efficiently solve instances of our problem with large sizes. This gives us the confidence to implement them in our later case studies where even larger size problems come into the picture.

#### 4.4.2.2 Sensitivity of the SAA method to $K$

In our model, $K$ denotes the total number of bunker consumption scenarios generated. We name our original model as $p_{TRUE}$ when $K \to \infty$, and $p_{SAA}$ if $K$ is another pre-specified relatively small number. Based on the SAA method of deriving lower and upper bounds of our optimization model discussed in [Kleywegt et al., 2002], we obtain the estimated optimality gap from using the SAA method, which is given below in Table 4.6 (number of consumption scenarios generated for each price scenario to evaluate the solution: 100, 000. Those scenarios are generated based on a truncated normal distribution with truncations at ±20%; replication number: 50). We can only evaluate up to $K = 11$ as true optimal solution cannot be obtained with larger $K$ by CPLEX.

Table 4.6: Optimality gap of SAA

| Number of consumption scenarios | 3 | 7 | 11 |
|---|---|---|---|
| Optimality gap | 24.04% | 9.2% | 3.9% |

Based on Table 4.6, we can see that even when $K = 11$, which is rather small, the optimality gap of 3.9% is quite tolerable already. When our two variants of PHA are used, we can solve instances of problem with much larger $K$, say $K = 30$. In this case, the optimality gap of using SAA will be further reduced.

Even though the use of our PHA will slightly impair the solution quality a little bit (Table 4.5), authors believe that such a solution scheme, SAA plus PHA, is suitable for our specific problem on hand. Therefore, what we have achieved so far is a solving scheme that can handle small to medium size of our problem which is a $(s, S)$ dynamic policy model that provides operation level bunkering and vessel speed determination decision support. What we are going to do next is to showcase the value of this dynamic policy model itself.

#### 4.4.2.3 Performance of our dynamic $(s, S)$ refueling policy model

Our refueling policy model is dynamic in the sense that policy parameters and ship speed depend on bunker price realizations. One natural question is that what is the advantage by taking the uncertainty of bunker prices into account. Put it in another way, if it is a stationary model that the policy parameters $(s, S)$ and ship speed only depend on port location, how much additional cost will incur. The comparison is done in two ways: first, we generate 50 random sets of consumption scenarios; each set consists of $K$ consumption scenarios. We then compare the expected costs of the dynamic and stationary policy models for each consumption scenario set. Second, from the solutions of those 50 replications, we choose the candidate solution which provides the lowest evaluation cost under $100,000$ randomly generated consumption scenarios. Candidate solutions from the dynamic and the stationary policy model are to be compared.

Different values of $K$ will be tested too. When $K$ becomes too large ($K \geq 11$) and CPLEX can not solve the dynamic policy model directly, we will use our PHA to solve it. Stationary policy model is much easier to be solved, therefore, in the comparisons below, we use all its true optimal solutions.

The comparison between the dynamic and the stationary policy models in terms of individual replication for $k = 7$ is shown in Figures 4.11. Comparison results for other values of $k$ are summarized in Table 4.7.

The comparison between the dynamic and the stationary policy models in terms of SAA solutions is shown in Table 4.8.

The above analysis has clearly demonstrated the advantage by considering the bunker prices uncertainty and adopting a dynamic policy. The cost reduction

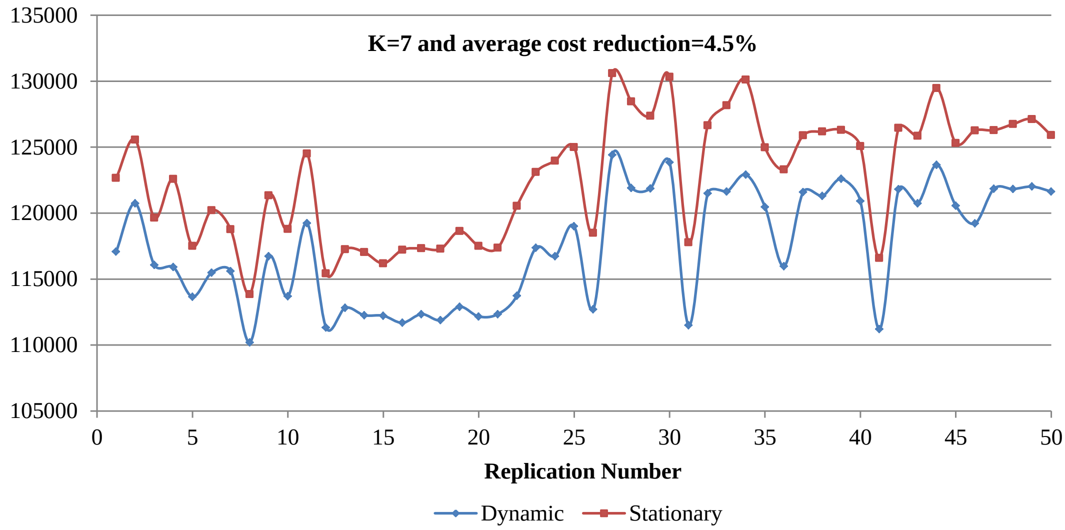


Figure 4.11: Comparison between dynamic and stationary policy models (K=7)

Table 4.7: Comparison between dynamic and stationary policy models in terms of individual replication

| Number of consumption scenarios | 7 | 11 | 15* | 25* |
|---|---|---|---|---|
| Average cost reduction | 4.5% | 4.6% | 3.3%* | 3.7%* |

* PHA is used

Table 4.8: Cost reduction of the dynamic policy model over the stationary policy model

| Number of consumption scenarios | 7 | 11 | 15* | 25* |
|---|---|---|---|---|
| Average cost reduction | 3.5% | 3.9% | 4.3%* | 4.3%* |

* PHA is used

of the dynamic policy model over the stationary one is approximately 4%, which can be rather attractive for liner companies. During times when the bunker prices are more volatile, this cost saving is even higher. In the next section, we will implement this dynamic policy model on a much larger service network and showcase its superiority.

### 4.4.3 Numerical results for AEX service

Due to the large size of this service route, our modified rolling horizon approach will be used. We look ahead 2 ports and each of them has four possible price change alternatives as shown in Table 4.3. For the remaining 13 ports, 8 price realizations are generated. Therefore the total number of price scenarios is $S = 4^2 \times 8$. For each price scenario, 25 common consumption scenarios are generated. To obtain a SAA solution at each stage, 50 independent replications are run and each replication is evaluated under $100,000$ randomly generated consumption scenarios. When the ship reaches one port, the bunkering and speed decisions are made based on the actual price and consumption realization. In this sense, while our model is an offline optimization tool which precomputes the policy for every possible scenario, the implementation of the rolling horizon approach turns it into an online one as well. This helps to circumvent the limitation of an offline optimization model whose size increases exponentially with the study horizon.

In this example, our dynamic policy model will be compared with a case where the bunker reorder point is determined by an ad hoc way as in the inventory management: reorder point is set to be the sum of the demand during lead time plus a safety inventory. In the context of our problem, the bunker reorder point at one port is set to cover the total mean consumption until the ship reaches the next port plus the safety inventory. This means that instead of determining the reorder point dynamically based on the bunker consumption scenarios as we did before, reorder point can be calculated as: $s_i = \sum_{g=1}^{G_{i,i+1}} \bar{F}^g_{i,i+1} \cdot Z^{g,r}_{i,i+1} \cdot d_{i,i+1}$+safety inventory. This safety inventory can be conveniently set as a fixed percentage of the mean consumption. We use $\rho$ to denote this fixed percentage. Therefore, $s_i = \sum_{g=1}^{G_{i,i+1}} \bar{F}^g_{i,i+1} \cdot Z^{g,r}_{i,i+1} \cdot d_{i,i+1} \times (1 + \rho)$. We name this new variation of our model as model-$\rho$. Even though it is much easier to be solved than the original dynamic policy model, it is still out of the capability of CPLEX. Hence we will use the same modified rolling horizon approach to solve it.

To evaluate both our dynamic policy model and model-$\rho$, we randomly generate 30 price scenarios and 30 consumption scenarios. This results in a total number of 900 different combinations. The average costs of both models under those 900 scenario combinations are obtained.

Different values of $\rho$ has be chosen as this is a key parameter which determines the trade-off between bunker inventory holding cost and penalty cost for violating the positive bunker inventory constraint. In the Figure 4.12 below, these small diamonds on the black line depict the average costs with varying $\rho$ values from 0.10, 0.12, 0.14 to 0.30. We notice that there is a fast decrease of the average cost from $\rho = 0.10$ to $\rho = 0.20$. This is because that the real consumption scenarios are generated based on a truncated normal distribution with truncations at $\pm 20\%$, when $\rho \leq 0.2$, the chance of "running out of bunker" decreases rapidly with increasing $\rho$. As $\rho$ increases from 0.20 to 0.30 , there will be no penalty cost for "running out of bunker". However, the inventory holding cost keeps increasing. This explains the general almost linearly increasing trend after $\gamma = 0.20$. This trend of increasing cost is rather mild because we set the penalty cost for violating the bunker inventory constrain much larger than the bunker inventory cost pmt.

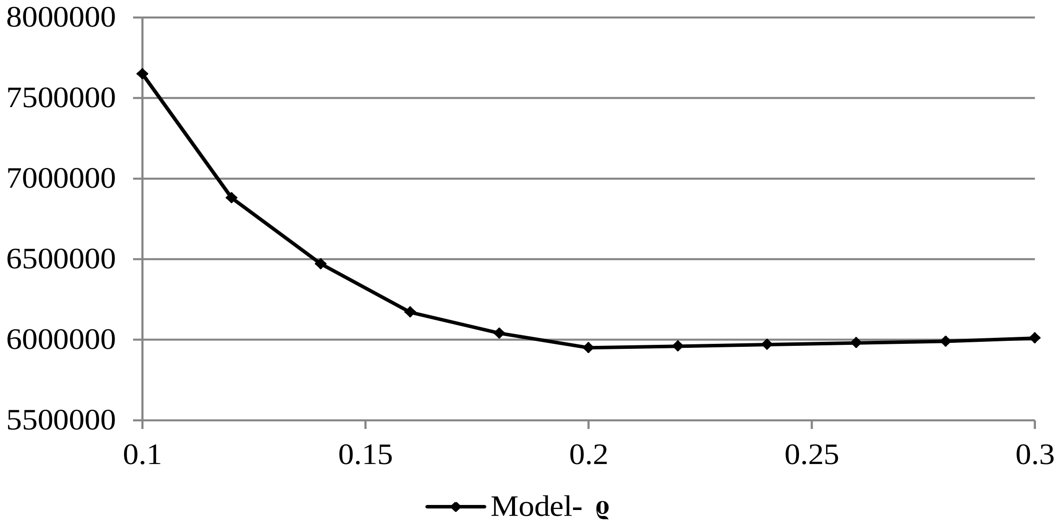


Figure 4.12: Average cost of the model-$\rho$ ($\rho = 1.0, 1.2, ..., 3.0$)

In terms of bunkering decision, we note that the optimal solution of model-$\rho$ is always a feasible solution for our dynamic policy model. This is reflected by our numerical experiments: the average cost for our dynamic policy model under those 900 scenario combinations is $5.68 \times 10^6$ and the average cost for model-$\rho$ (when $\rho = 0.20$) is $5.95 \times 10^6$. This is approximately 4.8% of cost reduction. Since model-$\rho$ under $\rho = 0.20$ does not incur penalty cost for violating the bunker inventory constraint, this cost reduction is mainly from wiser bunkering decisions by determining the bunker reorder point in a more strategic way. In addition, this will also result in a reduction of the overall bunker inventory holding cost.

For the model-$\rho$, when $\rho$ is poorly chosen, say 0.16, the average cost increases another 3.7%, while our dynamic policy model does not suffer from this problem. This is another advantage of our dynamic policy model and, for this, it is more practically useful for liner companies to implement it in the real business.

## 4.5 Summary

In this work, we study a $(s, S)$ optimal refueling policy and speed determination problem for liner shipping faced with tough market conditions. Two major uncertainties we tackled here are bunker prices and bunker consumption. The first and foremost contribution of this current work lies in the introduction of the refueling policy. As mentioned, bunkering policy allows a more flexible operational bunkering plan; the decision of whether to bunker or not depends on the bunker price realization as well as the actual bunker consumption during the previous leg.

In terms of modeling and solving the model, the biggest challenge was posed by the large number of integer variables. Similar to [Sheng et al., 2013], the uncertain bunker prices are formulated as a one-stage Markovian process using a scenario tree structure. For the bunker consumption uncertainty, instead of dealing with it with chance constraints, we used the random generation of scenarios and the SAA to circumvent the difficulty of incorporating a continuous distribution and calculating the expectation. However, speed choice decision is associated with every price scenario and every stage; bunkering decision is further associated with every consumption scenario. This causes the number of integer variables to grow out of control. Hence the second major contribution of our work is the introduction of two variants of the progressive hedging algorithm to solve the aforementioned large-scale mixed-integer problem. The efficiency of these two algorithms have been clearly demonstrated through our numerical studies.

Finally, with extensive numerical experiments, we have shown that the proposed $(s, S)$ refueling policy model is a practical, useful and applicable model with high cost saving potential.

Some possible future research directions are: first, instead of modeling the possible bunker price changes by discrete percentage values, we can use an interval to represent one scenario in the scenario tree. For example, the interval $[-10\%, -5\%)$ can be one scenario and $[-5\%, 0\%)$ can be another. The benefit of this new scheme is of course a more accurate representation of the uncertainty of bunker prices. However, the potential drawback would be the even huger problem size. Therefore, our second possible future research direction is to revisit the scenario reduction algorithm discussed by [Dupačová et al., 2003], [Heitsch and Römisch, 2003] and [Heitsch and Römisch, 2009]. Originally, when we use discrete values to represent one scenario, the problem associated with the scenario reduction algorithm is that once one scenario is deleted in the tree, optimal solution to the reduced tree does not tell us how to act when that deleted scenario happens in the reality. With the interval representation of scenarios, we, however, can solve this problem by simply combining scenarios. For instance, scenario $[-10\%, -5\%)$ and $[-5\%, 0\%)$ can be combined into $[-10\%, 0\%)$ during a certain stage. When either $-7.8\%$ or $-1.8\%$ happens, the same action is taken at that stage according to the optimal solution of the reduced tree. Hence, it can also be called a scenario combination algorithm.

Last but not least, bunkering and speed management problem for liner shipping can be extended from a single service route to a whole or multiple networks.

# Chapter 5

# Strategic bunkering and speed management in liner shipping networks

Remember that bunkering decisions for each service are not actually independent under common market conditions, we have identified another direction for bunkering cost reduction. That is to coordinate the management of bunker fuel purchasing for all the service routes under the same network. Figure 5.1 showcases the rationale behind this idea. There are two shipping routes and four ports. The market bunker prices of the four ports are the same, denoted as $p_0$. Assume that the port SG offers a bunker discount of 10% for ordering quantity large than 1000 ton. The optimal bunker amounts for the two shipping routes with and without the bunker discount are shown in the figure. It can be easily verified that the total bunker purchasing cost without discount is $2000p_0$ while the cost with discount reduces to $1900p_0$. More importantly, with the presence of this discount, bunkering decisions are no longer the same.

This is, to the best knowledge of authors, a problem that has yet to be studied in the liner shipping area so far. Refueling management problem in the airline industry has also been restricted to a single aircraft or a single route mostly. Stroup and Wollmer [1992] looked into the minimum cost refueling problem for an airline flight schedule under prices, station and supplier constraints. Even though the

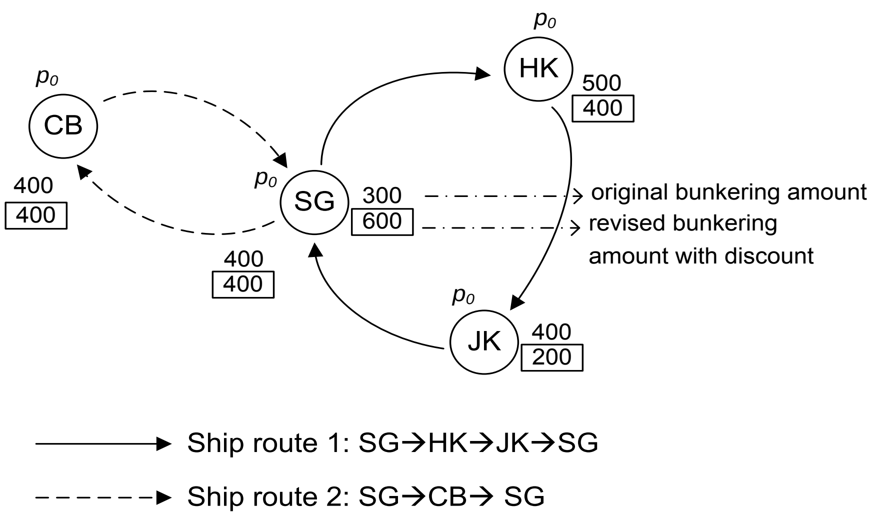


Figure 5.1: An illustrative example with two shipping routes

case of multiple aircrafts was discussed, because of the assumption of operational independence, each aircraft's refueling policy was optimized individually. Zouein et al. [2002] analyzed a optimal aircraft refueling problem based on a multiple period capacitated inventory model. The objective was to minimize the fuel cost along a predetermined route for a single plane.

In the railroad industry, only very recently, we found some works that consider the optimal refueling problem for a whole network. Nourbakhsh and Ouyang [2010] studied an optimal refueling problem for locomotive fleets in railroad networks which was formulated as a mix-integer linear programming model. In order to use the facility of a fixed station and refuel under a regular price, a flat contract fee must be paid. Otherwise railroad companies have to resort to emergency refueling which is more expensive. Kumar and Bierlaire [2011] and Nag and Murty [2012] revisited the locomotive refueling problem with slight modeling assumption difference from Nourbakhsh and Ouyang [2010].

One significant difference in the shipping industry is that shipping companies can always purchase fuel from local suppliers based on the spot market prices without committing to any contracts. Even though there are also several different types of bunker price contracts exist in the market[1], the specific terms

[1]For example, according to the website of one leading international bunker supplier (BP

and conditions of these contracts are confidential and may vary between different participants, and hence hinder a general discussion of the efficiency of those contracts. Therefore, in this paper, we study the bunker fuel purchasing problem for a whole liner shipping network under a novel cooperation scheme between liner shipping companies and bunker suppliers without considering the option of bunker price contracts. More specifically, bunker suppliers at certain ports offer liner shipping companies some price discounts according to their fleet's weekly or monthly bunker consumption. We believe that this kind of cooperation can be beneficial to both parties: shipping companies may jointly organize the bunkering plans of all fleets instead of individual vessels so as to take advantage of the price discounts and thus lower the overall bunkering cost, while bunker suppliers could attract more customers and therefore increase the revenue by offering this sort of "loyalty benefits". Under this situation, the bunkering decision of individual shipping routes are no longer independent, and shipping companies need to play the role as the overall decision making center and determine the bunkering plan for all service routes in the shipping network. With this regard, this study dedicates special efforts to the new decision problem, as is referred to as *strategic bunkering and speed management* in this paper, and design an optimal bunker purchasing and speed control plan for a liner shipping company under the cooperation bunkering scheme. As our model is a very large scale mixed integer non-liner programming model which cannot be solved efficiently by the state-of-the-art commercial solvers, we propose a column generation heuristic (CGH) to solve a linearized version of our model. The CGH reformulates the MIP model as a master problem and a set of sub-problems; each sub-problem considers bunkering and speed decisions for a single route.

## 5.1 Problem description

A liner network consists of several service routes, each of which in turn includes a predetermined order of port-calls. Denote the shipping network as a graph

[2013]), it offers following forms of bunker price contracts: Basic Physical Fuel Contract, Fixed Price Physical, Capped Price Physical, Participation, Swap, Cap, Collar, Reseller Hedge and The Fuel Contract with Min and Max Price.

$G(N, A)$ where $N$ represents the set of ports and $A$ is the set of service legs connecting two ports. The network contains a number of service routes, denoted as set $R$. Each route is operated by a fleet of vessels. Vessels for the same route are usually the same type for operational and administrative convenience. However, authors note that our model will not rely on this restriction.

Time windows are associated with every port-call. They state the ship arrival and departure times at each port. In this current work, we assume that the realized schedule cannot violate the time windows. This is equivalent to put a constraint on one of our decision variables, the ship speed $v$. As for the bunker consumption within each leg, in Sheng et al. [2013], it is empirically shown that the daily bunker consumption rate under a certain speed has a mean $\bar{F} = k_1 * v^3 + k_2$, where $k_1$ and $k_2$ are two constants, and noise which follows a zero mean normal distribution with standard deviation a constant percentage of $\bar{F}$. A chance constraint formulation will be used to control the probability of one ship running out of fuel during each leg to be less than one pre-defined value.

The key decision to make here is the bunkering decision for all the ships in the network: where and how much to bunker. This is largely determined by the bunker prices. We assume that each port $i \in N$ is associated with basic bunker prices $p_i^0$ and a set of incremental quantity discount offers $\Omega_i$; Each quantity discount offer $w \in \Omega_i$ is characterized by two parameters $\{p_{iw}, q_{iw}\}$. For example, if the weekly total bunkering amount $q_i$ exceeds $q_{iw}$, the bunker prices will be $p_{iw}$. Note that we treat the basic bunker price option as a special case $\{p_{i0}, q_{i0}\}$ and include it in the discount offer set $\Omega_i$. Obviously, we have $p_{i0} = p_i^0$ and $q_{i0} = 0$. For any two different discount offers $w_1$ and $w_2$, it is reasonable to have $p_{iw_1} < p_{iw_2}$ if $q_{iw_1} > q_{iw_2}$.

In the end, the objective is to minimize the total costs for all the ships in one or multiple networks. The costs considered here are the bunker cost and inventory holding cost. Bunker cost mainly consists of two parts, fixed bunkering cost incurred each time a bunkering takes place and variable cost that depends on the bunkering amount and bunker purchase prices. As a simplification, we assume that the inventory carrying cost per metric ton (pmt) is constant. Ship bunker inventory at the beginning of one service route is equal to the ending inventory of the same route.

## 5.2 Model formulation

### 5.2.1 Assumptions

Now, we state all the other assumptions made in our paper:

1. Port time (time one ship spends on entering, unloading and loading cargo, idling and exiting) and bunker consumption at each port is deterministic and known.

2. As a planning level problem, bunker price fluctuation is not considered.

### 5.2.2 A mixed integer non-linear program

We further denote the following notations:

$\theta_{rij}$ $= 1$ if port $i$ is the $jth$ visiting port on route $r$; $= 0$ otherwise;
$n_r$ the total number of ports visited by route $r$;
$f_i$ fixed bunkering cost at port $i$;
$a_{rj}$ bunker consumption at the $jth$ port on route $r$;
$d_{rj}$ nautical distance at the $jth$ leg on route $r$;
$h$ inventory holding cost per metric ton (pmt);
$t_{rj}$ port time (time one ship spends on entering, unloading and loading cargo, idling and exiting) at the $jth$ leg on route $r$;
$e_{rj}$ earliest arrival time at the $jth$ leg on route $r$;
$l_{rj}$ latest arrival time at the $jth$ leg on route $r$;
$Q_r$ bunker fuel capacity for the ships deployed on route $r$;
$\eta_r$ CV of daily bunker consumption rate for the ships deployed on route $r$;
$\beta_r$ service level coefficient;

the following decision variables are defined:

$x_{rj}$ binary variable; $= 1$ route $r$ bunkers at its $jth$ port; $= 0$ otherwise;
$y_{rj}$ the bunkering amount of route $r$ at its $jth$ port;
$z_{iw}$ binary variable; $= 1$ if price discount $w \in \Omega_i$ is utilized at port $i \in N$; $= 0$ otherwise;
$v_{rj}$ vessel speed at the $jth$ leg on route $r$;

following are dependent variables:

$c_{rj}$ mean bunker consumption at the $jth$ leg on route $r$;
$u_{rj}$ bunker inventory level when the ship reaches the $jth$ port on route $r$;
$T_{rj}$ ship arrival time at the $jth$ port on route $r$;
$sc_{rj}$ standard deviation of the bunker consumption at the $jth$ leg on route $r$;
$su_{rj}$ standard deviation of the bunker inventory when the ship reaches at the $jth$ port on route $r$;

Mathematical model to describe our problem:

$$\sum_{r\in R}\sum_{i\in N}\sum_{j=1}^{n_r}\theta_{rij}f_i x_{rj} + \sum_{i\in N}\sum_{\omega\in\Omega_i}\sum_{r\in R}\sum_{j=1}^{n_r}p_{iw}\theta_{rij}z_{iw}y_{rj} + \sum_{r\in R}\sum_{j=1}^{n_r}h(u_{rj}+y_{rj}-a_{rj})$$

$$y_{rj} \leq Q_r x_{rj} \quad \forall r\in R, \forall j=1,...,n_j \tag{5.1}$$

$$u_{rj}+y_{rj} \leq Q_r \quad \forall r\in R, \forall j=1,...,n_j \tag{5.2}$$

$$u_{rj} \geq \beta_r^{'} su_{rj} \quad \forall r\in R, \forall j=1,...,n_j+1 \tag{5.3}$$

$$u_{rj}+y_{rj}-a_{rj}-c_{rj} = u_{r(j+1)} \quad \forall r\in R, \forall j=1,...,n_j \tag{5.4}$$

$$u_{rn_j} = u_{r1} \quad \forall r\in R \tag{5.5}$$

$$\sum_{w\in\Omega_i} z_{iw} = 1 \quad \forall i\in N \tag{5.6}$$

$$\left(\sum_{r\in R}\sum_{j=1}^{n_r}\theta_{rij}y_{rj} - q_{iw}\right) z_{iw} \geq 0 \quad \forall i\in N, \forall j=1,...,n_j+1, \forall w\in\Omega_i \tag{5.7}$$

$$T_{rj}+t_{rj}+\frac{d_{rj}}{v_{rj}} = T_{r(j+1)} \quad \forall r\in R, \forall j=1,...,n_j \tag{5.8}$$

$$e_{rj} \leq T_{rj} \leq l_{rj} \quad \forall r\in R, \forall j=1,...,n_j+1 \tag{5.9}$$

$$v_r^{min} \leq v_{rj} \leq v_r^{max} \quad \forall r\in R, \forall j=1,...,n_j \tag{5.10}$$

$$c_{rj} = \left(k_{1r}v_{rj}^3 + k_{2r}\right)\frac{d_{rj}}{24v_{rj}} \quad \forall r\in R, \forall j=1,\cdots,n_j \tag{5.11}$$

$$sc_{rj} = \eta_r c_{rj} \quad \forall r\in R, \forall j=1,\cdots,n_j \tag{5.12}$$

$$su_{rj}(1-x_{rj}) + sc_{rj} = su_{r(j+1)} \quad \forall r\in R, \forall j=1,\cdots,n_j \tag{5.13}$$

$$x_{rj} \in \{0,1\} \quad \forall r\in R, \forall j=1,\cdots,n_j \tag{5.14}$$

$$y_{rj} \geq 0 \quad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.15}$$

$$z_{iw} \in \{0, 1\} \quad \forall i \in N, \forall w \in \Omega_i \tag{5.16}$$

$$v_{rj} \geq 0 \quad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.17}$$

$$c_{rj} \geq 0 \quad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.18}$$

$$u_{rj} \geq 0 \quad \forall r \in R, \forall j = 1, \cdots, n_j + 1 \tag{5.19}$$

$$T_{rj} \geq 0 \quad \forall r \in R, \forall j = 1, \cdots, n_j + 1 \tag{5.20}$$

$$sc_{rj} \geq 0 \quad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.21}$$

$$su_{rj} \geq 0 \quad \forall r \in R, \forall j = 1, \cdots, n_j + 1 \tag{5.22}$$

The first and the second terms in the objective function are fixed and variable bunkering costs. $\theta_{rij}$ in them is a known parameter which matches port index $i$ to the location index $j$ in each route $r$. It is equal to $= 1$ if port $i$ is the $jth$ visiting port on route $r$; $= 0$ otherwise. The third part of the objective function is the bunker inventory holding cost. Constraint 5.1 states that only those ports with bunker decisions are allowed to purchase bunker. Constraint 5.2 is the bunker capacity restriction. Constraint 5.3 is the deterministic equivalent for chance constraint $P\{u_{rj} \geq 0\} \geq \beta_r$, which ensures that the probability of bunker inventory being greater than a certain amount is greater than a pre-specified value. Constraints 5.4 and 5.5 are flow conservation constraints. Constraint 5.6 means that exactly one price is utilized at each port. Constraint 5.7 expresses the minimum bunker purchase requirement to enjoy price discounts. Constraints 5.8 to 5.9 are time window constraints, while constraint 5.10 is simply to limit the ship speed within a reasonable range. Constraints 5.11 and 5.12 express the mean daily consumption rate at a certain speed ($k_{1r}$ and $k_{2r}$ are two constants) and the stand deviation of bunker consumption during each leg as a constant percentage of the mean consumption. Constraint 5.13 states that if the ship bunkered at the previous port, then standard deviation of the ship bunker inventory at current port is equal to the standard deviation of bunker consumption from previous port to the current port. Otherwise the standard deviation of ship bunker inventory at previous port should also be added. This is because, as discussed, standard deviation of bunker consumption is proportional to the total bunker consumption.

Constraints 5.14 to 5.22 simply clarify the domain of decision variables.

### 5.2.3 Model linearization

Note that quadratic terms are involved in the objective function, constraints (5.7) and (5.13). Additional auxiliary decision variables are introduced to linearize the model:

- $\varphi_{iw}$: $\geq 0$, equals $\sum_{r \in R} \sum_{j=1}^{n_r} \theta_{rij} y_{rj} z_{iw}, \forall i \in N, \forall w \in \Omega_i$
- $\phi_{rj}$: $\geq 0$, equals $su_{rj} x_{rj}, \forall r \in R, \forall j = 1, \cdots, n_j$

The related additional constraints are defined as follows:

$$\varphi_{iw} \leq \sum_{r \in R} \sum_{j=1}^{n_r} \theta_{rij} y_{rj} \qquad \forall i \in N, \forall w \in \Omega_i \tag{5.23}$$

$$\varphi_{iw} \leq M z_{iw}, \forall i \in N \qquad \forall w \in \Omega_i \tag{5.24}$$

$$\varphi_{iw} \geq \sum_{r \in R} \sum_{j=1}^{n_r} \theta_{rij} y_{rj} + M(z_{iw} - 1) \qquad \forall i \in N, \forall w \in \Omega_i \tag{5.25}$$

$$\varphi_{iw} \geq 0 \qquad \forall i \in N \qquad \forall w \in \Omega \tag{5.26}$$

$$\phi_{rj} \leq su_{rj} \qquad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.27}$$

$$\phi_{rj} \leq M x_{rj} \qquad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.28}$$

$$\phi_{rj} \geq su_{rj} + M\left(x_{rj} - 1\right) \qquad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.29}$$

$$\phi_{rj} \geq 0 \qquad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.30}$$

where $M$ is a sufficiently large constant. Constraints (5.23) to (5.26) mean that if $z_{iw} = 0$, then $\varphi_{iw} = 0$, or if $z_{iw} = 1$, then $\varphi_{iw} = \sum_{r \in R} \sum_{j=1}^{n_r} \theta_{rij} y_{rj}$. Similarly, constraints (5.27) to (5.30) mean that if $x_{rj} = 0$, then $\phi_{rj} = 0$, or if $x_{rj} = 1$, then $\phi_{rj} = su_{rj}$. With the additionally defined decision variables, the objective function, constraints (5.7) and (5.13) can be expressed as follows:

$$\sum_{i \in N} \sum_{\omega \in \Omega_i} p_{iw} \varphi_{iw} \tag{5.31}$$

$$\varphi_{iw} - q_{iw} z_{iw} \geq 0 \qquad \forall i \in N, \forall w \in \Omega_i \tag{5.32}$$

$$su_{rj} - \phi_{rj} + sc_{rj} = su_{r(j+1)} \qquad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.33}$$

The non-linear problem also comes from the terms related with speed decision variable $v_{rj}$ in constraints (5.8) and (5.11). We use the piecewise linear approximation technique to replace the non-linear terms. Firstly, we get $\bar{s}$ fixed points $v_{rs}$ by discretizing the feasible interval $[v_r^{min}, v_r^{max}]$ into $\bar{s}-1$ segments. Then, ......

$$\frac{1}{v_{rj}} = \sum_{s=1}^{\bar{s}} \frac{1}{v_{rs}} \lambda_{rjs} \qquad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.34}$$

$$v_{rj}^2 = \sum_{s=1}^{\bar{s}} v_{rs}^2 \lambda_{rjs} \qquad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.35}$$

$$\lambda_{rj1} \leq \pi_{rj1} \qquad \forall r \in r, \forall j = 1, \cdots, n_j \tag{5.36}$$

$$\lambda_{rjs} \leq \pi_{rj(s-1)} + \pi_{rjs} \qquad \forall r \in r, \forall j = 1, \cdots, n_j, \forall s = 2, \cdots, \bar{s}-1 \tag{5.37}$$

$$\lambda_{rj\bar{s}} \leq \pi_{rj(\bar{s}-1)} \qquad \forall r \in r, \forall j = 1, \cdots, n_j \tag{5.38}$$

$$\sum_{s=1}^{\bar{s}} \lambda_{rjs} = 1 \qquad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.39}$$

$$\sum_{s=1}^{\bar{s}-1} \pi_{rjs} = 1 \qquad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.40}$$

$$\lambda_{rjs} \geq 0 \qquad \forall r \in R, \forall j = 1, \cdots, n_j, \forall s = 1, \cdots, \bar{s} \tag{5.41}$$

$$\pi_{rjs} \in \{0, 1\} \qquad \forall r \in R, \forall j = 1, \cdots, n_j, \forall s = 1, \cdots, \bar{s}-1 \tag{5.42}$$

Then, Constraints (5.8), (5.10) and (5.11) can be updated as:

$$T_{rj} + t_{rj} + d_{rj} \sum_{s=1}^{\bar{s}} \frac{1}{v_{rs}} \lambda_{rjs} = T_{r(j+1)} \qquad \forall r \in R, \forall j = 1, ..., n_j \tag{5.43}$$

$$\frac{1}{v_r^{max}} \leq \sum_{s=1}^{\bar{s}} \frac{1}{v_{rs}} \lambda_{rjs} \leq \frac{1}{v_r^{min}} \qquad \forall r \in R, \forall j = 1, ..., n_j \tag{5.44}$$

$$c_{rj} = \left( k_{1r} \sum_{s=1}^{\bar{s}} v_{rs}^2 \lambda_{rjs} + k_{2r} \sum_{s=1}^{\bar{s}} \frac{1}{v_{rs}} \lambda_{rjs} \right) \frac{d_{rj}}{24} \qquad \forall r \in R, \forall j = 1, \cdots, n_j \tag{5.45}$$

Therefore, the strategic bunkering and speed management problem can be formulated as a mixed integer linear program as follows:

$$\textbf{[P]} \quad \min \quad \sum_{r \in R} \sum_{i \in N} \sum_{j=1}^{n_r} \theta_{rij} f_i x_{rj} + \sum_{i \in N} \sum_{\omega \in \Omega_i} p_{iw} \varphi_{iw} + \sum_{r \in R} \sum_{j=1}^{n_r} h(u_{rj} + y_{rj} - a_{rj}) \tag{5.46}$$

s.t. (5.1) – (5.6), (5.9), (5.12), (5.14) – (5.16), (5.18) – (5.30), (5.32), (5.33) and (5.36) – (5.45)

## 5.3 Solution methods

Our preliminary numerical experiments show that when the total number of ports increases or under certain settings of bunker price discounts, commercial optimization softwares like CPLEX cannot solve our problem (large optimality gap after long hours of solving). Therefore, this section is devoted to the development of efficient solution methods that can handle instances of our problem with large size. More specifically, we will devise a column generation heuristic (CGH) and two greedy heuristic algorithms. Results from these three heuristic algorithms can be compared with each other. We will present our two greedy algorithms first, followed by a detailed discussion of the CGH.

### 5.3.1 Greedy algorithms

The basic idea of greedy algorithm-1 is to start with the optimal solution when no price discount option is available. We then identify those ports where bunker price discounts can already be enjoyed by the liner (the total bunker purchase amount at that port in the previous optimal solution has exceeded the threshold level). Next step is to find those ports where the total bunkering amount is "very close" to the requirement of being entitled to a discount and observe the cost reduction if such a discount be enjoyed with the minimum purchase being met. A formal description of greedy algorithm-1 is given below:

---

**input**: an instance, heuristic parameters
**output**: the best feasible solution *Best*

*Best* ← optimal solution without price discount option;
identify those ports where $q_i \geq q_{iw}, \forall w \in \Omega_i$ ;
fix the values of $z_{iw} = 1$ for those ports and update *Best*;
**repeat** greedy heuristic search procedure
  **for** all the discount options
    **for** those ports where $q_i$ is within $(q_{i(W-1)},\, q_{iW})$
      find the port where $(q_{iW} - q_i)$ is the smallest;
      evaluate the optimal cost if the $q_i$ is forced to be $q_{iW}$;
      **If** there is a reduction of total cost
      fix $z_{iW} = 1$, update *best* and delete port $i$ from the future search;
      **else**
      delete port $i$ from the future search;
    **end for**
    $W = W - 1$;
  **end for**
**until** the stopping condition is met
Return *Best*;

---

$W$ corresponds to the option with the highest discount

Our problem is computationally challenging when the number of price discount options at each port (cardinality of $\Omega_i$, $\forall i \in N$, in constraint (5.7) of our original model) is large. We assume that discount options $w_{i_1}$, $w_{i_2}$,...,$w_{i_k} \in \Omega_i$ with $w_{i_1} < w_{i_2} <, ..., < w_{i_k}$ are available at port $i$ in the original model. The idea of greedy algorithm-2 is to start by solving the problem with only the first few price discounts available at each port. For example, begin with only $w_{i_1}$ and $w_{i_2}$, $\forall i \in N$, available at each port. The next step is to increase, one port at a time, the number of bunker price discounts by one (until the original number of price discounts), solve our problem with the current available discounts at each port and choose the setting which provides the lowest optimal cost. Subsequently start with this setting and continue the search by increasing again, one port at a time, the number of bunker price discounts by one (until the original number of price discounts) and choose the next setting which provides the lowest optimal cost. It works in a way that is similar to a tree structure. Figure 5.2 illustrates this idea:

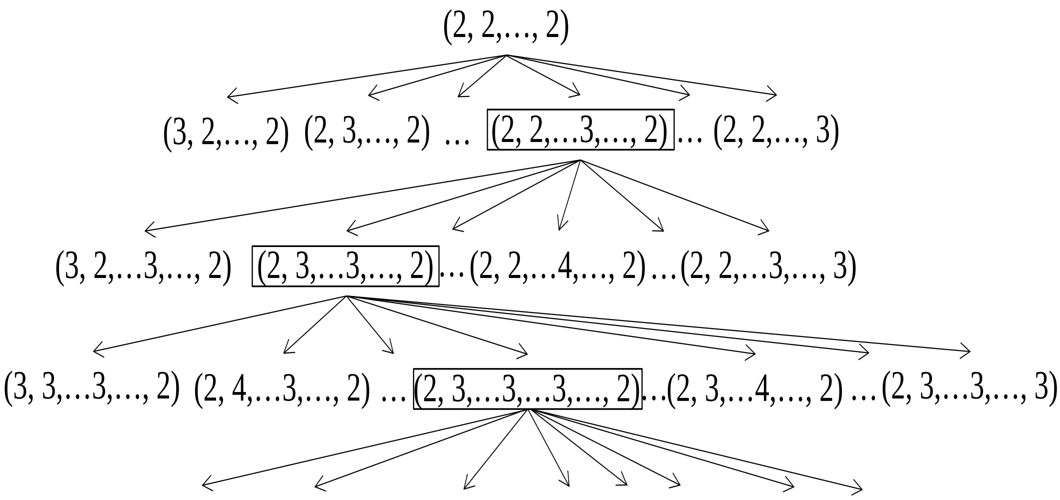


Figure 5.2: Tree structure of the greedy algorithm-2

$(2, 2, ..., 2)$ means that there are only the first two discount options available at each port and $(3, 2, ..., 2)$ means that there are the first three discount options available at port 1 and all other ports have the first two discount options. The setting which is enclosed in a square is the one which provides the current best solution. This tree structure search goes on until it reaches the bottom and no more different settings can be explored or a user-specified time period has passed. Under either situation, we will be able to obtain a best-so-far solution. A formal description of greedy algorithm-2 is given below:

1: **input**: an instance, heuristic parameters;
2: **output**: the best feasible solution *Best*;

3: *Best* ← optimal solution with $w_{i_1}$ and $w_{i_2}$ available at each port;
4: **do**
5: **for** each port
6: **if** the number of discount options at this port is less than the original number
7: increase the number of discount options by one;
8: **end if**
9: solve the problem with available discount options at each port;
10: **end for**
11: find the setting related to the lowest optimal cost in step 9, which serves as the starting point for the next search;
12: **while** the termination conditions are not met
13: *Best* ← the best available solution obtained so far;
14: Return *Best*;

### 5.3.2 Column generation heuristic

We noticed that without the price discount options at each port, our strategic bunkering and speed management can be decomposed into each route individually. This means that without constraint (5.7), our original problem can be decomposed into individual service routes, which is a very useful structure that we can take advantage of. The typical column generation method is not very suitable here due to the presence of integer variables. However, its general framework of dividing the original problem into a restricted master problem and some sub-problems is still relevant. Therefore, we devise a column generation heuristic to solve our strategic bunkering and speed management problem.

We first introduce the restricted master problem, based on which, we discuss how to extract useful information from the solving of it and use the information in route sub-problems to generate new columns. Each column corresponds to a bunkering plan of a shipping route with aggregated information of bunkering and speed decisions of that route.

#### 5.3.2.1 Restricted master problem for column generation

The decision in the master problem is to choose which bunkering plan is used for each route. Mathematically, we define decision variable $\delta_{rg}$ be 1 if bunkering plan $g \in G_r$ is adopted by route $r \in R$; and 0 otherwise, where $G_r$ is a set of feasible bunkering plans of route $r$. Another decision variable in the master problem is $z_{iw}$ which governs the utilization of price discount option at each port. The corresponding parameters for each bunkering plan $g \in G_r$ are further defined as follows:

- $c^1_{rg}$: total fixed bunkering cost along route $r$ if plan $g$ is adopted;
- $c^2_{rg}$: inventory holding cost of route $r$ if plan $g$ is adopted;
- $q_{rgj}$: bunkering amount at the $j^{th}$ port along route $r$ if plan $g$ is adopted;
- $Q_i$: total bunkering amount at port $i$;

Then, the restricted master problem could be formulated as follows:

$$\textbf{[RMP]} \quad \min \quad \sum_{r\in R}\sum_{g\in G_r}\left(c^1_{rg}+c^2_{rg}\right)\delta_{rg}+\sum_{i\in N}\sum_{w\in\Omega_i}p_{iw}z_{iw}Q_i \tag{5.47}$$

$$\sum_{g\in G_r}\delta_{rg}=1 \quad \forall r\in R \tag{5.48}$$

$$\sum_{r\in R}\sum_{g\in G_r}\sum_{j=1}^{n_r}\theta_{rij}q_{rgj}\delta_{rg}=Q_i \quad \forall i\in N,\forall r\in R,\forall g\in G_r,\forall j\in n_r \tag{5.49}$$

$$\sum_{w\in\Omega_i}z_{iw}=1 \qquad \forall i\in N \tag{5.50}$$

$$\left(Q_i-q_{iw}\right)z_{iw}\geq 0 \quad \forall i\in N,\forall w\in\Omega_i \tag{5.51}$$

$$\delta_{rg}\in\{0,1\} \quad \forall r\in R,\forall g\in G_r \tag{5.52}$$

The first part in the objective function is the sum of the fixed bunkering cost and bunker inventory holding cost for all the routes and the second part is the variable bunkering cost based on bunker price discounts. Constraint (5.48) ensures that one and only one column (bunkering plan) is chosen for each route.

Constraint (5.49) expresses the relationship between $q_{rgj}$ and $Q_i$. $q_{rgj}$ is a given parameter obtained from route sub-problems and $\theta_{rij}$, as mentioned, is a $0-1$ mapping parameter between route and port indexes. Constraints (5.50) and (5.51) determine exactly which bunker price discount can be enjoyed at a certain port. Constraint (5.51) would be linearized in the same way as we did to constraint (5.7).

#### 5.3.2.2 Sub-problems for column generation

The purpose of route sub-problems is to provide promising new columns to the master problem. New columns can be generated by simply changing the bunker prices at some or all of the ports. To make sure that those new columns are promising indeed, we need to make use of the information from the master problem. In the sub-problems we define $k_i$ as the bunker prices at port $i$. It is a given parameter and here we assign the bunker prices at each port in the **[RMP]** (after obtaining the discount information) to $k_i$, $\forall i \in N$. The rationale is that if a certain price discount is enjoyed at one port, by adjusting the bunker prices accordingly in the sub-problems, there is a higher chance that a more promising column can be obtained. This approach can be too rigid sometimes. Therefore, we can slightly and randomly perturb $k_i$ to generate more columns. For example, we can uniformly generate one value in the range of $[(1-b\%)k_i,\ (1+b\%)k_i\ ]$, where $b$ is a given constant small number.

The route sub-problems are to find, for each shipping routes, new bunkering plans. The decision variables of the pricing sub-problems include binary bunkering decision variable $x_{rj}$, bunkering amount $y_{rj}$ and ship speed $v_{rj}$. All other parameters and variables defined in our original model remain unchanged unless otherwise stated. Therefore, the parameters of the restricted master problem could be expressed as:

$$c_r^1 = \sum_{i \in N} \sum_{j=1}^{n_r} \theta_{rij} f_i x_{rj} \quad \forall r \in R \tag{5.53}$$

$$c_r^2 = \sum_{j=1}^{n_r} h(u_{rj} + y_{rj} - a_{rj}) \quad \forall r \in R \tag{5.54}$$

$$q_{rj} = y_{rj} \quad \forall r \in R, \forall j = 1, \cdots, n_r \tag{5.55}$$

The total cost of a bunkering plan for shipping route $r$ is:

$$\tilde{c}_r = c_r^1 + c_r^2 + \sum_{i \in N} \sum_{j=1}^{n_r} k_i \theta_{rij} y_{rj} \tag{5.56}$$

For route $r \in R$, the route sub-problem could be formulated as:

$$\min \qquad \tilde{c}_r \tag{5.57}$$

$$y_{rj} \leq Q_r x_{rj} \qquad \forall j = 1, ..., n_j \tag{5.58}$$

$$u_{rj} + y_{rj} \leq Q_r \qquad \forall j = 1, ..., n_j \tag{5.59}$$

$$u_{rj} \geq \beta_r^{'} s u_{rj} \qquad \forall j = 1, ..., n_j + 1 \tag{5.60}$$

$$u_{rj} + y_{rj} - a_{rj} - c_{rj} = u_{r(j+1)} \qquad \forall j = 1, ..., n_j \tag{5.61}$$

$$u_{rn_j} = u_{r1} \tag{5.62}$$

$$T_{rj} + t_{rj} + \frac{d_{rj}}{v_{rj}} = T_{r(j+1)} \qquad \forall j = 1, ..., n_j \tag{5.63}$$

$$e_{rj} \leq T_{rj} \leq l_{rj} \qquad \forall j = 1, ..., n_j + 1 \tag{5.64}$$

$$v_r^{min} \leq v_{rj} \leq v_r^{max} \qquad \forall j = 1, ..., n_j \tag{5.65}$$

$$c_{rj} = \left(k_{1r} v_{rj}^3 + k_{2r}\right) \frac{d_{rj}}{24 v_{rj}} \qquad \forall j = 1, \cdots, n_j \tag{5.66}$$

$$sc_{rj} = \eta_r c_{rj} \qquad \forall j = 1, \cdots, n_j \tag{5.67}$$

$$su_{rj}(1 - x_{rj}) + sc_{rj} = su_{r(j+1)} \qquad \forall j = 1, \cdots, n_j \tag{5.68}$$

$$x_{rj} \in \{0, 1\} \qquad \forall j = 1, \cdots, n_j \tag{5.69}$$

$$y_{rj} \geq 0 \qquad \forall j = 1, \cdots, n_j \tag{5.70}$$

$$z_{iw} \in \{0, 1\} \qquad \forall i \in N, \forall w \in \Omega_i \tag{5.71}$$

$$v_{rj} \geq 0 \qquad \forall j = 1, \cdots, n_j \tag{5.72}$$

$$c_{rj} \geq 0 \qquad \forall j = 1, \cdots, n_j \tag{5.73}$$

$$u_{rj} \geq 0 \qquad \forall j = 1, \cdots, n_j + 1 \tag{5.74}$$

$$T_{rj} \geq 0 \qquad \forall j = 1, \cdots, n_j + 1 \tag{5.75}$$

$$sc_{rj} \geq 0 \qquad \forall j = 1, \cdots, n_j \tag{5.76}$$

$$su_{rj} \geq 0 \qquad \forall j = 1, \cdots, n_j + 1 \tag{5.77}$$

The linearization of the sub-problems is similar to that of the original formulation [**P**] and is not presented here for the sake of brevity.

#### 5.3.2.3 Column generation procedure

The column generation heuristic procedure for the strategic bunkering and speed management problem is summarized as following: initial columns for a certain route are obtained by randomly generating some sets of bunker prices $k_i$ for each port on the route and solving this route sub-problem. The aforementioned two greedy algorithms can also be used to generate initial columns. When all the initial columns for every route are obtained, solve the restricted master problem with these columns and obtain the optimal solution. As a by-product, we also get the bunker prices at every port after discount and assign those prices to $k_i$, $\forall i \in N$ for the next iteration of route sub-problems. A solution from each sub-problem will serve as a new column to be input into the master problem (as mentioned, to get more columns, randomly and slightly perturb some values of $k_i$ in the objective function (5.56) and solve the route sub-problem). Repeat the aforementioned procedures until there is no further improvement of the overall cost after several consecutive repetitions. A formal description of the CGH is given below:

---

**input**: an instance, heuristic parameters;
**output**: the best feasible solution $Best$;

  obtain the initial columns for each route;
  **repeat** heuristic algorithm
    solve the master problem and update values of $k_i$;
    solve route sub-problems with given values of $k_i$;
    solutions from sub-problems become new columns in the master problem;
    **if** more columns need to be generated

9: randomly and slightly perturb some values of $k_i$ in the objective function (5.56);
10: **until** the stopping condition is met
11: *Best* ← optimal solution from solving the master problem;
12: Return *Best*;

## 5.4 Case studies

We have identified 3 important real liner service networks (a total coverage of 55 service routes and 117 ports) provided by an ocean liner headquartered in Singapore. These 3 liner networks are Intra-Asia, Asia-North America and Asia-Europe which include almost all of the important ports around the world. In our first case study, we apply our model on one of the smallest network within these 3, the size of which allows the model to be solved directly by CPLEX most of the time. However, under certain parameter settings, CPLEX cannot solve the problem with a satisfying result within reasonable time. Therefore, CHG and the two greedy heuristic algorithms we just described will be implemented and tested their efficiency on the smaller example. In the second case study, to fully demonstrate the advantage of our model by jointly considering the bunkering decisions for all the service routes, our model will be applied on all 3 service networks. Under this case, CPLEX can no longer solve the problem while the heuristic algorithms can obtain good quality near-optimal solutions efficiently.

All parameters related to service routes and schedules are readily available on the company website. Port-to-port distance was not given, so we collected the data from some external internet sources (Ports [2012], Searates [2012], Portworld [2012] and Sea-distances [2012]). Those collected distances may bear a certain degree of error and not represent the real distances; however this by no means influences the final comparison between our model and the model which considers every single route individually, so long as both models use the same inputs. Due to the space constraint, we are not going to present all of those parameters.

As for the bunker prices at each port, we used their monthly average prices in November, 2012. For the available discount options, which are also the key parameters in this work, we will resort to reasonable assumptions and a sensitivity analysis. For simplicity but without the loss of generality, we first assume that all ports have the same available bunker price discount options. The case of each port having different options can be easily adopted. Secondly, we assume that this discount option is on a weekly basis which means that when the weekly total purchase at a certain port reaches a predetermined amount, the discount can be enjoyed. Finally, Table 5.4 below provides a base case of the price discount options, upon which we will conduct further sensitivity analysis.

Table 5.4: Bunker price discount options I

| Amount | $\geq 0$ | $\geq 3,000$ | $\geq 5,000$ |
|---|---|---|---|
| Discount | 0% | 3% | 5% |

Two different sizes of vessels are assumed to be deployed in our case studies as shown in Table 5.5. They differ in terms of capacity, allowable sailing speed range, mean bunker consumption rate and its coefficient of variation (CV). As a real practice, vessels with bigger capacity usually perform deep sea shipping or intercontinental shipping while smaller vessels operate in short sea shipping. There is a combination of both types of liner business in our considered service networks. In the end, for the other remaining required inputs for our model, we assign convenient values to them as they are incidental to the final outputs and comparison.

Table 5.5: Parameters related to two types of vessel

| Parameter | Type I | Type II |
|---|---|---|
| Ship size | 3000TEU | 6000TEU |
| Ship speed interval | 5–25 knots | 5–35 knots |
| Mean bunker consumption rate | $F = 0.006743V^3 + 37.23$ | $F = 0.007297V^3 + 71.4$ |
| CV of bunker consumption rate | 0.09 | 0.07 |

We ran all our numerical studies with CPLEX-12.5 on a 3GHz Dual Core PC with 4 GB of RAM.

### 5.4.1 Asia-Europe service

There are 6 service routes and altogether 32 port-of-calls in this service network as shown in Table 5.6 and Figure 5.3. We have highlighted those ports which have been shared by more than 3 routes for the reason that they are more likely to be chosen as strategic bunkering locations than other ports.

Table 5.6: Asia-Europe service

| | |
|---|---|
| Services: | Asia Europe Loop 1, 4, 5, 6, 7 and EUM |
| Ports: | **Rotterdam**, **Hamburg**, **Southampton**, Le Havre, **Singapore**, **Hong Kong**, Kobe, Nagoya, Tokyo, Shimiza, Cai Mep, Jeddah, **Yantian**, **Ningbo**, Shanghai,Thamesport, Gwangyang, Busan, **Shekou**, Antwerp, Jebel Ali, Kaohsiung, Xiamen, Colombo, Salalah, Qing dao, Genoa, Fos, Barcelona, Valencia, Damietta, Yangshan |

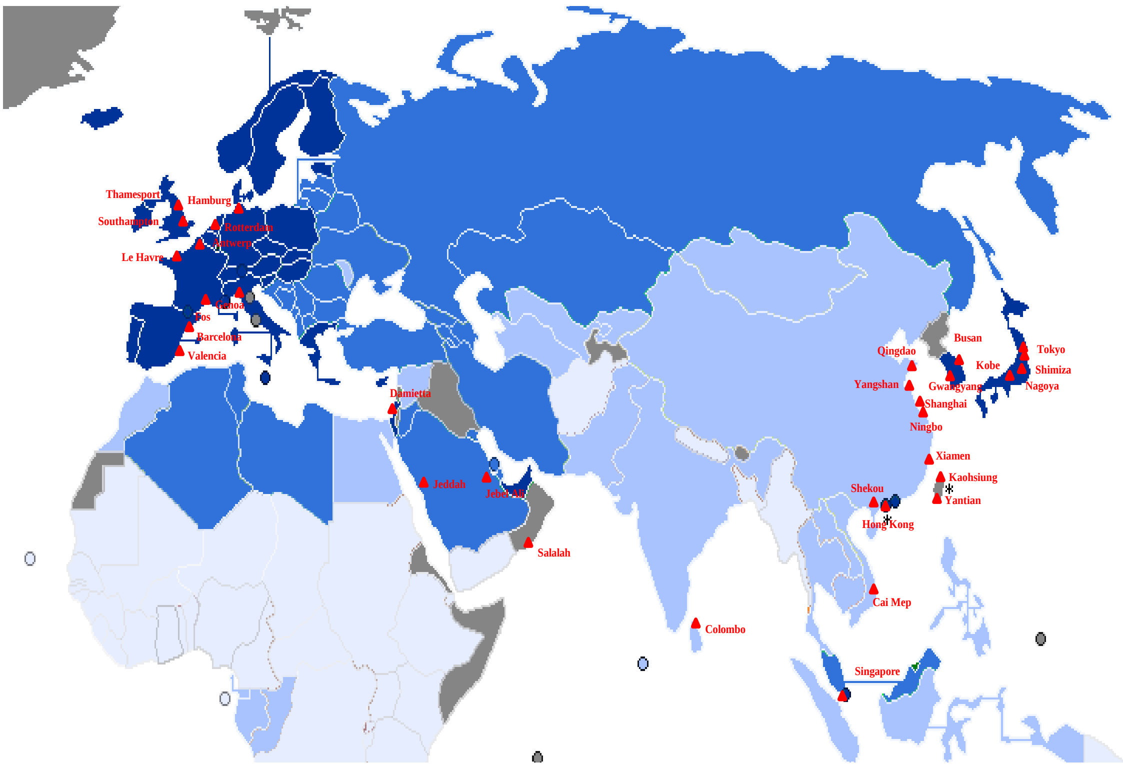


Figure 5.3: Port distribution of the Asia-Europe service

Our numerical study starts from the base case where no bunker discount option is available. CPLEX can solve the problem to optimality within seconds and the optimal cost is $\$2.36 \times 10^7$. Next we consider the case that the bunkering decisions are made independently for each individual service route as in Yao et al. [2012] and the discount options as shown in Table 5.4 are available, CPLEX can also quickly solve the problem to optimality and the optimal cost is $\$2.34 \times 10^7$. When our model is implemented, the optimal cost is $\$2.32 \times 10^7$. This translates into 0.9% of cost saving when we consider the bunkering decision for the whole network, instead of individual routes.

Those ports in our model where discount options are actually utilized are: Rotterdam ($\geq 3,000$), Salalah ($\geq 3,000$), Jeddah ($\geq 3,000$) and Singapore ($\geq 5,000$). Table 5.7 shows the total bunkering amount at these four ports without the discount options, with each route optimized individually and in our model.

Table 5.7: Bunkering amount comparison between different scenarios

| Ports: | Rotterdam | Salalah | Jeddah | Singapore |
|---|---|---|---|---|
| Without discount | 3,848 | 2,653 | 4,159 | 9,759 |
| Individual route | 3,821 | 2,652 | 4,151 | 9,612 |
| Our model | 3,709 | 3,000 | 4,150 | 9,598 |

Under current parameter settings, 3 out of the 4 ports actually "automatically" enjoy the price discounts as their original weekly total bunkering amount is greater than 3,000 tons or 5,000 tons. At port Salalah, the bunkering amount increases from 2,653 tons weekly, when there is no bunker price discount, to 3,000 tons in our model to enjoy the 3% price discount. As a result, bunkering amounts at the other three ports (also holds true to most of the other 28 ports) slightly decrease. There are three reasons for this. Firstly, these 3 ports already enjoy the price discounts. Secondly, for port Rotterdam and Jeddah, only significant increase of the bunkering amount can entitle them to enjoy a higher discount. Lastly, total bunker consumption for all the service routes is the same with or without the price discounts. Another observation is that the optimal solution in the case when each route is optimized individually is closer to the optimal solution in our model, compared to the case when no discount option is available. This would be helpful when we implement our column generation heuristic.

Let us see what is the impact if we raise the bar to enjoy price discounts, say a minimum 5,000 tons instead of 3,000 tons for 3% discount and a minimum

10,000 tons instead of 5,000 tons for 5% discount (Table 5.8).

Table 5.8: Bunker price discount options II

| Amount | $\geq 0$ | $\geq 5,000$ | $\geq 10,000$ |
|---|---|---|---|
| Discount | 0% | 3% | 5% |

With the presence of discount options, the optimal cost when each route is considered individually is $\$2.35 \times 10^7$ and the optimal cost of our model slightly increases to $\$2.33 \times 10^7$. The cost savings marginally decrease from 0.9%. The increase of optimal cost in our model is rather intuitive since it is more difficult to entitle for discounts. And under this situation, only Singapore utilizes the discount option. Total bunkering amount at Singapore increases from 9,759 to 10,000. In the case when each route is optimized individually, no bunker price discount is enjoyed.

Understandably, if bunker suppliers at every port agree to offer higher discounts at lower minimum purchases, the benefit of overall bunkering management for a whole network will be more significant and bunker discount options at more ports will be strategically chosen. However, CPLEX fails to solve the problem to optimality (more than 20% of the optimality gap after 2 hours of solving) due to insufficient computer memory under discount option setting as shown in Table 5.9. This is because CPLEX uses a branch-and-bound technique to solve MIPs and as more bunker discount options have the potential to be used, the search space becomes much larger. Understandably, if the network size increases and more ports get involved, CPLEX will still fail to solve our problem. Therefore, we will have to resort to our heuristic algorithms.

Table 5.9: Bunker price discount options III

| Amount | $\geq 0$ | $\geq 2,000$ | $\geq 4,000$ |
|---|---|---|---|
| Discount | 0% | 5% | 10% |

We tested our heuristic algorithms on the previous numerical experiment that CPLEX failed to solve. The best available costs for all three of our heuristic algorithms are $\$2.24 \times 10^7$. Compared to the case where no price discount option is available, the cost saving is 6.8% and compared to the case that each route is optimized individually with discount options, the cost saving is 1.3%. Regarding the solving time and solution quality, our heuristic algorithms outperform CPLEX: Table 5.10 compares the solving time and estimated optimality gap between three heuristic algorithms and CPLEX. Estimated optimality gaps of our heuristic algorithms are obtained by deriving a lower bound for the optimal cost in our model. When $(2000, 5\%)$ and $(4000, 10\%)$ are replaced with a single discount option $(2000, 10\%)$, the optimal objective value is $\$2.20 \times 10^7$, which serves as a lower bound and is 1.8% lesser than the cost for our heuristic algorithms.

Table 5.10: Solving time comparison between heuristic algorithms and CPLEX

| Method | Greedy algorithm-1 | Greedy algorithm-2 | CGH | CPLEX |
|---|---|---|---|---|
| Estimated optimality gap | 1.8% | 1.8% | 1.8% | $\geq 20\%$ |
| Solving time (mins) | $\leq 1$ | $\approx 20$ | $\approx 10$ | $\geq 120$ |

Another different angle to demonstrate the efficiency of our heuristic algorithms is to compare them with CPLEX on a problem whose size allows CPLEX to solve it to optimality.

Under both discount option settings I and II, greedy algorithm-2 obtained the exact true optimal solutions as the direct solving by CPLEX. CGH obtained solutions with negligible difference (optimality gap is less than 0.05%) from the true optimal solutions. Regarding the solving time, CGH is much faster than greedy algorithm-2 as similar to the case in Table 5.10. While under the discount option setting I, greedy algorithm-1 obtained a high quality near-optimal solution: (1) the optimality gap is 0.13%; (2) exactly the same bunker discount options at all ports are used; (3) only bunkering amounts at some individual ports are slightly different. Under the discount option setting II, the optimality gap is 0.8%, which is still within 1%.

From the previous experiments, we gain the confidence that our proposed heuristic algorithms suit the specific problem on hand and when the network size becomes even larger, they can be implemented to obtain satisfactory results.

### 5.4.2 Intra-Asia service

Intra-Asia is a much larger service network which consists of 55 routes and 117 ports worldwide. Figure 5.4 shows the degree of port sharing among those 55 routes. For example, we can see that there are 24 ports shared by more than 4 routes, 8 ports shared by more than 7 routes and 1 port shared by more than 32 routes. The solving of our problem by CPLEX directly with such a large size is practically impossible, and hence must also rely on our heuristic algorithms. For the numerical experiments of this larger network, we will use the discount option setting II which is the most restrictive amongst these above three. The rationale is that since more ports are shared by many routes, it is more appropriate to use a setting with a higher purchase requirement from the numerical experiment's point of view, even though the opposite is always more appealing to liner companies in the real world. In addition, we will also test one more discount setting with four options as shown in Table 5.11.

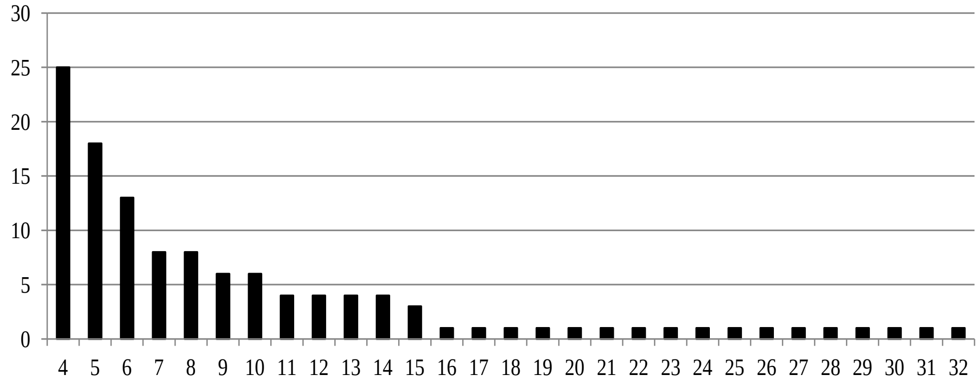


Figure 5.4: Degree of port sharing among 55 routes

Table 5.11: Bunker price discount options IV

| Amount | $\geq 0$ | $\geq 3,000$ | $\geq 5,000$ | $\geq 12,000$ |
|---|---|---|---|---|
| Discount | 0% | 3% | 4% | 5% |

Under discount setting II, we start from the base case where no bunker discount option is available. CPLEX can solve the problem to optimality within seconds and the optimal cost is $\$9.42 \times 10^7$. If each route is considered independently with the available discount options, the total optimal cost is $\$9.38 \times 10^7$. In our model, greedy algorithm-1 provides a solution with a cost of $\$9.28 \times 10^7$. This is 1.1% of cost saving by planning the bunkering decisions for the whole network instead of individual routes. 10 out of 117 ports are where price discount options are actually utilized. Greedy algorithm-2 and CGH also provide solutions with the same cost of $\$9.28 \times 10^7$. Compared to greedy algorithm-1, there is one more port where price discount option is actually utilized in greedy algorithm-2 and CGH. Regarding the solving time, it takes approximately 2 mins for greedy algorithm-1, 90 mins for greedy algorithm-2 and 20 mins for CGH to solve the problem. The results are summarized in Table 5.12 and 5.13.

Table 5.12: Cost comparison under different scenarios

| Scenarios: | Without discount | Individual route | Our model |
|---|---|---|---|
| Cost | $\$9.42 \times 10^7$ | $\$9.38 \times 10^7$ | $\$9.28 \times 10^7$ |

Table 5.13: Comparison between heuristic algorithms

| Method | Greedy algorithm-1 | Greedy algorithm-2 | CGH |
|---|---|---|---|
| Cost | $\$9.28 \times 10^7$ | $\$9.28 \times 10^7$ | $\$9.28 \times 10^7$ |
| Solving time (mins) | $\approx 2$ | $\approx 90$ | $\approx 20$ |

A lower bound of the optimal cost is $\$9.23\times10^7$, which is obtained by replacing $(5000, 3\%)$ and $(10000, 5\%)$ with a single discount option $(5000, 5\%)$. Therefore, our heuristic algorithms provide a result that is 0.5% higher than the lower bound. We have shown that our heuristic algorithms perform very well and by planning the bunkering decision for a whole network, instead of single routes, there is a significant cost reduction potential (1.1% or $\$1.0 \times 10^6$ in absolute value under the current parameter setting).

Under discount setting IV, CGH demonstrates its superiority over the other two greedy algorithms. Due to the even larger problem size under this setting, greedy algorithm-2's solving time is significantly increased. After 2 hours of solving, we terminated the algorithm with a solution that provides a cost 1.2%

higher than that of CGH ($\$9.30 \times 10^7$ and $\$9.19 \times 10^7$ respectively). Comparing greedy algorithm-1 to CGH, the cost of the solution from greedy algorithm-1 is 0.5% higher than that of CGH ($\$9.24 \times 10^7$ and $\$9.19 \times 10^7$ respectively). This is because greedy algorithm-1 determines the highest price discount option first and then works on lower discount options, this myopic nature causes some ports to use the discount options of $(12000, 5\%)$ and $(5000, 4\%)$ which are not used in the solution of CGH. Although the cost difference of 0.5% is not very significant because our model is quite robust and numerically stable to its parameters, the advantage of CGH regarding solution quality has been clearly demonstrated. The results are summarized in Table 5.14. Optimal cost when each route is considered individually with bunker discount options is $\$9.30 \times 10^7$. This shows that 1.2% of cost reduction has been achieved under the discount setting IV.

Table 5.14: Comparison between heuristic algorithms

| Method | Greedy algorithm-1 | Greedy algorithm-2 | CGH |
|---|---|---|---|
| Cost | $\$9.24 \times 10^7$ | $\$9.30 \times 10^7$ | $\$9.19 \times 10^7$ |
| Solving time (mins) | $\approx 5$ | 120 | $\approx 35$ |

All in all, authors recommend the use of greedy algorithm-1 and CGH when liner companies want to implement our strategic bunkering management model in the real practice. In light of the fact that greedy algorithm-1 solves faster than CGH and CGH can provide solutions of higher quality, they can work together with greedy algorithm-1 providing initial columns for CGH to obtain high quality near-optimal solutions in a timely fashion.

## 5.5 Summary

In this work, we study the bunker fuel purchasing problem for a whole liner shipping network under a novel cooperation scheme between liner shipping companies and bunker suppliers. More specifically, bunker suppliers at certain ports offer liner shipping companies some price discounts according to their fleet's weekly or monthly bunker consumption. Under this situation, the bunkering decision of individual shipping routes are no longer independent, and shipping companies play the role as the overall decision making center and determine the bunkering plan for all service routes in the shipping network. As shown in our numerical studies, the bunker cost reduction by this strategic bunkering management is significant. Considering the millions of dollars of bunker cost involved for even a medium size liner company, our model has the potential to be implemented in practice and help save a large amount of operational cost. Besides, our model can decide the optimal sailing speeds for each leg in each service route and help further reduce the cost while maintaining the required service level and reliability. As our model is a very large size mixed integer non-liner programming model which cannot be solved efficiently by the state-of-the-art commercial solvers, we propose one column generation heuristic and two greedy heuristic algorithms which take advantage of our problem structure to solve it. Through extensive numerical experiments, we have shown that CGH is the most versatile of the three heuristic algorithms. The advantage of greedy algorithm-1 is its fast solving time. Therefore, in the real practice, we think both can work together with greedy algorithm-1 providing initial columns for CGH to obtain high quality near-optimal solutions in a timely fashion.

Some possible future research directions are: Firstly, the incorporation of bunker price contracts can be considered. For example, one type of bunker price contract works in this way: bunker supplier agrees to offer a fixed amount of fuel with a pre-determined price regardless of the future market conditions. If the liner wants to buy more than the fixed amount, it has to pay the prevailing spot market prices. And if the liner cannot fulfill the amount in the contract, it has to pay a damage charge. Authors note that our current model can accommodate this type of contract with only marginal modification of notations and formulations. However, the solving would be even more difficult as such a contract can be regarded as a more restricted form of cooperation scheme from the liners' perspective. Therefore, the second future research direction is to modify our column generation heuristic or devise some new efficient solution algorithms for this type of network strategic bunkering management problem. Last but not least, the inclusion of bunker suppliers as well in a game-theory framework can be a very interesting arena for future exploration.

# Chapter 6

# Conclusions and future research

This thesis contributes to the liner shipping operation level speed and bunkering management decision support. In this chapter, we summarize the main results of our work. Limitations and possible future research will also be discussed.

## 6.1 Summary of results

Chapter 3 studied the problem of dynamic bunkering port selection and ship speed determination for a single vessel in one service route. While previous deterministic works focused more on the planning level of this problem, we aimed at providing operational decision support by incorporating two major random factors into our model. Namely, the ship bunker consumption rate and the bunker prices at each port. Based on the bunker consumption model in Yao et al. [2012], we further established that the noise of daily bunker consumption follows a normal distribution with zero mean and constant coefficient of variation. For the stochastic nature of the bunker prices, we modeled it through the scenario tree. While solving a whole large dynamic problem was computational challenging, we proposed a solving method that could help to significantly reduce the computer memory requirement and solving time. This method is a combination of scenario tree generation scheme and a non-standard rolling horizon approach. Another advantage about this solving method is that as much new information as possible is used and previous forecasting errors could be easily corrected during the whole study horizon. Our numerical examples based on real-world data have shown that the dynamic model improves significantly in terms of overall cost and service level (or failure rate) compared with the stationary model. With the reasonable solving time, we think our model could be implemented by liner shipping companies to give operational level decision support in order to lower the overall operation cost and provide more reliable service.

In chapter 4, we incorporated a dynamic $(s, S)$ bunkering policy into the previous problem. Bunkering policy allows a more flexible operational bunkering plan; the decision of whether to bunker or not depends on the bunker price realization as well as the actual bunker consumption during the previous leg. In terms of modeling and solving the model, the biggest challenge was posed by the large number of integer variables. The uncertain bunker prices were formulated as a one-stage Markovian process using a scenario tree structure. For the bunker consumption uncertainty, instead of dealing with it with chance constraints, we used the random generation of scenarios and the SAA to circumvent the difficulty of incorporating a continuous distribution and calculating the expectation. However, speed choice decision was associated with every price scenario and every stage; bunkering decision was further associated with every consumption scenario. This caused the number of integer variables to grow out of control. Hence the second major contribution of our work is the introduction of two variants of the progressive hedging algorithm to solve the aforementioned large-scale mixed-integer problem. The efficiency of these two algorithms have been clearly demonstrated through our numerical studies. We have also shown that the proposed $(s, S)$ refueling policy model is a practical, useful and applicable model with high cost saving potential.

Chapter 5 analyzed the bunker fuel purchasing problem for a whole liner shipping network under a novel cooperation scheme between liner shipping companies and bunker suppliers. More specifically, bunker suppliers at certain ports offer liner shipping companies some price discounts according to their fleet's weekly or monthly bunker consumption. Under this situation, the bunkering decision of individual shipping routes are no longer independent, and shipping companies play the role as the overall decision making center and determine the bunkering plan for all service routes in the shipping network. As shown in our numerical studies, the bunkering cost reduction by this strategic bunkering management is significant. Considering the millions of dollars of bunker cost involved for even a medium size liner company, our model has the potential to be implemented in the real world and help save a large amount of operational cost. Besides, our model can decide the optimal sailing speeds for each leg in each service route and help further reduce the cost while maintaining the required service level and reliability. As our model is a very large size mixed integer non-liner programming model which cannot be solved efficiently by the state-of-the-art commercial solvers, we proposed one column generation heuristic and two greedy heuristic algorithms which take advantage of our problem structure to successfully solve it.

## 6.2 Future research

Authors concede that several limitations are existing in our current work. It can be improved and extended in several directions:

Firstly, instead of modeling the possible bunker price changes by discrete percentage values as in chapter 3 and 4, we can use an interval to represent one scenario in the scenario tree. For example, the interval $[-10\%, -5\%)$ can be one scenario and $[-5\%, 0\%)$ can be another. The benefit of this new scheme is of course a more accurate representation of the uncertainty of bunker prices. However, the potential drawback would be the even huger problem size.

Therefore, our second possible future research direction is to revisit the scenario reduction algorithm discussed by Dupačová et al. [2003], Heitsch and Römisch [2003] and Heitsch and Römisch [2009]. Originally, when we use discrete values to represent one scenario, the problem associated with the scenario reduction algorithm is that once one scenario is deleted in the tree, optimal solution to the reduced tree does not tell us how to act when that deleted scenario happens in the reality. However, with the interval representation of scenarios, we can solve this problem by simply combining scenarios. For instance, scenario $[-10\%, -5\%)$ and $[-5\%, 0\%)$ can be combined into $[-10\%, 0\%)$ during a certain stage. When either $-7.8\%$ or $-1.8\%$ happens, the same action is taken at that stage according to the optimal solution of the reduced tree. Hence, it can also be called a scenario combination algorithm.

Thirdly, the incorporation of bunker price contracts in the model of chapter 5 can be considered. For example, one type of bunker price contract works in this way: bunker supplier agrees to offer a fixed amount of fuel with a pre-determined price regardless of the future market conditions. If the liner wants to buy more than the fixed amount, it has to pay the prevailing spot market prices. And if the liner cannot fulfill the amount in the contract, it has to pay a damage charge. Authors note that our current model can accommodate this type of contract with only marginal modification of notations and formulations. However, the solving would be even more difficult as such a contract can be regarded as a more restricted form of cooperation scheme from the liners' perspective. Therefore, another future research direction is to modify our column generation heuristic or devise some new efficient solution algorithms for this type of network strategic bunkering management problem.

Last but not least, the inclusion of bunker suppliers as well in a game-theory framework can be a very interesting arena for future exploration.

# Appendix A

Following notations are used to express model.1:

| | |
|---|---|
| $n$ | number of port of calls; |
| $d_{i,j}$ | distance between port i and port j(nautical miles); |
| $t$ | total cycle time(hours); |
| $t_i$ | port time(time one ship spends on entering, unloading and loading cargo, idling and exiting) at port $i$(hours); |
| $e_i$ | earliest arrival time at port $i$; |
| $l_i$ | latest arrival time at port $i$; |
| $C_i$ | bunker fuel consumption when the ship is at port i; |
| $w$ | bunker fuel capacity for a single ship; |
| $v_{min}$ | minimum ship sailing speed (nautical miles/hour); |
| $v_{max}$ | maximum ship sailing speed (nautical miles/hour); |
| $k_1, k_2$ | bunker fuel consumption coefficients; |
| $P_i$ | bunker price for port $i$; |
| $f$ | fixed bunkering cost; |
| $h$ | inventory holding cost pmt for bunker; |
| $\gamma$ | coefficient to control the service level; |

Decision Variables:

| | |
|---|---|
| $V_{i,j}$ | ship speed between port $i$ and $j$; |
| $S_i$ | bunker fuel-up-to level for the ship at port $i$; |
| $B_i$ | bunkering decision variable. $= 1$ if bunkering at port $i$; $= 0$, otherwise; |
| $I_i$ | bunker fuel inventory when the ship reaches port $i$; |

Dependent Variables:

| | |
|---|---|
| $F_{i,j}$ | daily bunker consumption rate for a ship travels from port $i$ to $j$; |
| $A_i$ | ship arrival time at port $i$; |

$$\min \sum_{i=1}^{n} [(S_i - I_i)P_i + f \cdot B_i + (S_i - C_i) \cdot h] - P_1 \cdot I_{n+1}$$

$$I_1 = 0 \tag{1}$$

$$I_i = R_{i-1} - a_{i-1} - F_{i-1,i} \cdot d_{i-1,i}/24 \cdot V_{i-1,i} \qquad i \in 2, 3, ..., n+1 \tag{2}$$

$$R_i - I_i \leq B_i \cdot w \qquad i \in 1, 2, ..., n \tag{3}$$

$$R_i \leq w \qquad i \in 1, 2, ..., n \tag{4}$$

$$I_i \geq \gamma \cdot w \qquad i \in 1, 2, ..., n \tag{5}$$

$$F_{i,i+1} = k_1 (V_{i,i+1})^3 + k_2 \qquad i \in 1, 2, ..., n \tag{6}$$

$$v_{min} \leq V_{i,i+1} \leq v_{max} \qquad i \in 1, 2, ..., n \tag{7}$$

$$A_i + t_i + d_{i,i+1}/V_{i,i+1} = A_{i+1} \qquad i \in 1, 2, ..., n \tag{8}$$

$$e_i \leq A_i \leq l_i \qquad i \in 1, 2, ..., n \tag{9}$$

$$A_{n+1} = t \tag{10}$$

$$B_i = 0 \text{ or } 1 \qquad i \in 1, 2, ..., n \tag{11}$$

$$F_{n,n+1} = F_{n,1}, d_{n,n+1} = d_{n,1}, V_{n,n+1} = V_{n,1} \tag{12}$$

The objective function is to minimize the expected total cost, which includes the fixed and variable bunkering cost and inventory holding cost. Bunker left at the end of the service loop is refunded. Constraint 1 sets the initial inventory to be 0. Constraint 2 is a flow conservation constraint. Constraints 3 and 4 ensure that the maximum bunkering amount and bunker-up-to level are less than the bunker fuel capacity. Constraint 5 controls the minimum bunker inventory to be a fixed percentage of the total bunker capacity. Constraint 6 expresses the daily consumption rate at a certain speed between port $i$ and $i+1$. Constraint 7 is simply to limit the ship speed within a reasonable range, while constraint 8 to 10 are about time window constraints. Constraint 11 is a binary constraint.

# Appendix B

We applied the fast forward selection algorithm in Heitsch and Römisch [2003] to reduce the bunker price scenario tree size in our first case study, MAX service route, and compared the result with that of our modified rolling horizon approach. As mentioned, the size of the MAX service route allows us to solve the whole dynamic model by CPLEX directly, so we can easily derive the optimality gap of the scenario reduction method and our modified rolling horizon approach, respectively.

Table 1 shows the optimality gap of the scenario reduction method under different parameter settings. There are altogether 256 price scenarios initially and we still look at 3 different cases of bunker price fluctuation. "Number of scenarios" means the total number of scenarios retained after reduction and these percentage numbers in the table denote the optimality gap between the scenario reduction method and the direct solving of the dynamic model.

Table 1: Optimality gap of the scenario reduction method

| Number of scenarios | 10 | 30 | 60 | 90 | 100 | 200 |
|---|---|---|---|---|---|---|
| Case 1 | 32.31% | 29.29% | 17.70% | 10.19% | 2.56% | $< 0.1\%$ |
| Case 2 | 93.17% | 32.91% | 26.55% | 18.40% | 13.15% | $< 0.1\%$ |
| Case 3 | 91.41% | 53.81% | 29.97% | 17.52% | 13.20% | $< 0.1\%$ |

Table 2 shows the optimality gap of our modified rolling horizon approach under three different cases of bunker price percentage change. Table 3 is a comparison of the solving time between these two methods under case 1.

Comparing the results in Tables 1, 2 and 3, we can see that the modified

Table 2: Optimality of the modified rolling horizon approach

| Methods | Case 1 | Case 2 | Case 3 |
|---|---|---|---|
| $D$ | $\$117,194$ | $\$113,422$ | $\$95,580$ |
| $R$ | $\$118,779$ | $\$116,637$ | $\$100,502$ |
| $\frac{R-D}{R}(\%)$ | 1.3 | 2.8 | 4.9 |

Table 3: Solving time comparison of these two methods

| Methods/Case 1 | 60 | 100 | 200 |
|---|---|---|---|
| Reduction | $35s$ | $65s$ | $139s$ |
| Rolling horizon | $5.6s$ | $5.6s$ | $5.6s$ |

rolling horizon approach is a good approach to be used for our problem. Under all three cases, the modified rolling horizon approach is better than the scenario reduction method when the scenario reduction method retains less than 100 scenarios. Moreover, the solving time for the modified rolling horizon approach remains unchanged while the solving time for the scenario reduction method increases considerably with the number of scenarios. And under all these 3 cases, the optimality gap of our modified rolling horizon approach is under 5%, which is encouraging.

When it comes to our second case study where there are a total of $4^{16}$ price scenarios, the implementation of the scenario reduction method becomes even harder. As CPLEX can only solve the problem with less than 500 price scenarios, this means only 5 out of $4^{14}$ scenarios is retained. Not only will the scenario reduction algorithm take a long time to reduce the scenarios, but the optimality gap of the reduced tree might be big based on our study of the small size problem.

In summary, we feel that the scenario reduction technique might not work well in our problem. However, having said so, we still feel that there is a potential in this method to be applied to this type of the problem, but this will need an in-depth research work. As for our proposed method, it can be viewed as a special type of scenario reduction technique in the sense that all the branches in the near future are enumerated, but the branches far away from the decision point are not enumerated fully and so we have a reduction in scenarios. Moreover, by forcing it to solve at every decision point, we are able to obtain decisions using the most updated information.

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