DECISION SUPPORT FOR OPTIMAL WELL PLACEMENT, INFRASTRUCTURE INSTALLATION AND PRODUCTION PLANNING IN OIL FIELDS

MOHAMMAD SADEGH TAVALLALI

(BE in Chemical Engineering, Shiraz University, Shiraz, Iran)

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DECLARATION

I hereby declare that the thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Tavallali

Mohammad Sadegh Tavallali

5 March 2014

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[1] Tavallali, Ee, Birgersson, and Karimi IA (2012) Optimization of a Direct Methanol Fuel Cell with 2D Approximate Analytical Solutions and Pattern Search Algorithm, 14th Asia Pacific Confederation of Chemical Engineering Congress (APCChE)– Feb 2012 Singapore

[2] Etemadnia, Goetz, Canning, and Tavallali (2014) Optimal Wholesale Facilities Location within the Fruit and Vegetables Supply Chain With Bimodal Transportation Options: An LP-MIP Heuristic Approach, 93rd Annual Meeting of the Transportation Research Board, Washington, USA

[3] Afshang, Tavallali, Chong, Han Joo Chong, Teh Kah Chan, C.B Chan and Karimi (2014) On The Performance Study of Time Frame Duration for Cognitive Radio in OFDMA based Network, The International Wireless Communications and Mobile Computing Conference, Nicosia, Cyprus, Under review

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CONTENTS

SUMMARY

Oil and gas will continue to be the major sources of energy for the near future, and drilling of wells in new, marginal, and mature fields is a growing activity. In fact, optimal well placement is critical to the success of exploration and production of oil and gas. That should be efficiently linked with other field development planning activities, especially with optimal design of surface network facilities. These facilities provide the ultimate connection of the new well to the transfer pipeline or jetties. However, the physical complexities of multiphase flow in the reservoirs and well strings, numerous combinations of well locations and allocations to surface gathering manifolds and processing centers, as well as complicated interaction between the elements of this network, make this integrated reservoir study a very complex problem. Model-based optimization methods offer powerful tools for tackling some of these challenges. Therefore, this PhD study focuses on the application of advanced mathematical programming technique in addressing well placement and a number of related issues to well-drillings within integrated field development scope.

This study started with a detailed expository survey of available literature on well placement. It described and evaluated the different approaches for formulating and solving this problem, and identified the main challenges and important research gaps in the current techniques. To our knowledge, this was the first and most extensive survey of systematic methodologies for optimal well placement. Using the findings of this survey and through industrial consultation, three main problems related to well-drillings in an oil field was chosen to be addressed via mathematical programming framework, namely (1) placement problem (the optimal sites for well drillings and

infrastructure installations), (2) allocation problem (the optimal connection between wells, manifolds and surface centers) and finally (3) ordering / timing problem (optimal planning of the drillings and installations).

The first work was targeted at the placement problem for well-drillings in single rectangular oil reservoirs and included both modeling and algorithm design. Most of the available literature on optimal well placement has employed numerical simulators in a black box manner linked to an external search engine. In the first study, we formulated the contents of that box inside a mixed integer nonlinear programming model (MINLP) for optimal well placement. We provided a unified model that integrated the subsurface and wells in an upstream production project. It linked the production plan with the aforementioned elements, and economics and market. This resulted in a complex spatiotemporal mixed integer nonlinear model, for whose solution we modified and augmented an existing outer approximation algorithm. The model solution provided the optimal number of new producers, their locations, and optimal production plan over a given planning horizon. To our knowledge, this was the first contribution in optimal well placement studies that used mathematical programming in a real dynamic sense by honoring the constituent partial differential equations.

The multi-reservoir oil field development planning which includes both placement and allocation problem was studied in the next chapter. This study involved decisions regarding well-drillings, infrastructure placement, and allocation of different elements of the surface network infrastructure. It extended and strengthened the previous work by extensively including the surface elements of an oil field and also by deploying a more detailed economic analysis. Moreover, it went beyond single rectangular reservoirs and studied sub-surface dynamics in irregular shaped multi-reservoirs oil

fields. The resulting dynamic and non-convex MINLP model provided the number and location for drilling new wells and/or installing new infrastructure (gathering manifolds and surface separation /processing centers), optimal well-to-manifold/center and manifold-to-center allocation, as well as throughputs and pressure settings at various points of the production/injection network. Furthermore, few limitations of the solution algorithm (that was developed for the first work) were effectively addressed.

In the final work, the placement, allocation and timing/ordering problems were addressed together. The MINLP model developed in the second study was extended in order to include the time domain into the design variables (binaries). To solve this model, a solution algorithms was suggested. First, the placement and allocation problems (i.e to locate the new wells and other infrastructure) were solved, and then the timing/ordering problem for the fixed solution from the first part was addressed.

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NOMENCLATURE

Chapter 2

Sets and subscripts

f	fluid (oil, water) index
i	well location index
n	iteration index
t	time period index $(1,2,3,\dots,T)$
M	middle point (of a well)

Parameters

a, b	exponent terms- Corey's correlation
BHP	bottom hole pressure
B_f	Formation volume factor of f
C_{WD}	well drilling cost
d	uniform well-to-well spacing
$\bar{d}$	uni-directional, normalized average distance from a reservoir boundary
D_{ij}	distance between cells i and j,
D_{min}	minimum allowable well-to-well distance
dS	search direction
g	gravity acceleration
G	planning time horizon
H	Heel (of a well)

I, J	number of cell blocks (x, y)
$\boldsymbol{K}$	absolute permeability
k_r	relative permeability
kr_0^o, kr_0^w	end point relative permeability
$l_{w,i}$	trunk length (of a well w at location i)
$l_{z,i}$	projected depth (of well)
L	well length
N	Number of potential wells
P_c	capillary pressure
S_{or} , S_{wr}	residual oil and water saturations
T	Toe (of a well)
THP	tubing head pressure
x_i, y_i, z_i	location of site i in the Cartesian coordinates
Z	depth
$\mathit{\Delta P}$	frictional pressure drop
Δp	pressure drop due to acceleration
δ	Dirac delta function
σ	a parameter for 2D bivariate Gaussian function
κ	reference distance
μ	viscosity
$\boldsymbol{r}$	model parameters of model (O)
θ	projection angle
τ	junction location
ρ	density
χ	step size

Ψ	connection transmissibility factor for wells

Variables

e	integer
E	pattern selection operator
P	pressure
q	total production/injection
q_f	oil/water production
S	water saturation
u	transformation operator
y	binary variable
$\boldsymbol{v}$	continuous variables of model (O)
$\boldsymbol{w}$	integer variables of model (O)
ψ,γ	binaries

Subscripts and superscripts

o	oil phase
U	upper bound
w	water phase

Chapter 3

Sets and subscripts

IX	interior cells with $i < I$
IY	interior cells with $j < J$
IW	cells with old injector
f	fluid (oil, water) index
n	cells, location index
nL	pipe segment index
t	time period index $(1,2,3, \ldots, T)$
$x-, y-$	Backward upstream weighting index
$x+, y+$	Forward upstream weighting index

Parameters

A	accumulation function
A_x, A_y	directional cross-sectional area
a, b	exponent terms- Corey's correlation
BHP	bottom hole pressure
B_f	Formation volume factor of f
b_f	Inverse of B_f
c_v	water viscosibility
c_w	water compressibility
g	gravity acceleration
h^t	time
H	production horizon
I, J	number of cell blocks (x, y)

$\boldsymbol{K}$	absolute permeability
k_r	relative permeability
kr_0^o, kr_0^w	end point relative permeability
ℓ	pipe /well length
NL	number of pipe segments
P_r	reference pressure
S_{or} , S_{wr}	residual oil and water saturations
T	number of time periods
T^x, T^y	directional transmissibility
THP	tubing head pressure
V	volume of cell
α	regression parameters
$\mu_{Bo,1}, \mu_{Bo,2}$	regression coefficients for $(\mu_o \times B_o)$
$\mu_{o,1}, \mu_{o,2}$	regression coefficients for oil viscosity
ρ	density
Ψ	connection transmissibility factor for wells

Variables

$d_{f,1}, d_{f,2}$	terms in accumulation
M_f	mobility of phase f
P	pressure
q	total production/injection
q_f	oil/water production
S	water saturation

Subscripts and superscripts

o	oil phase

w	water phase

Chapter 4

Sets and subscripts

AW	Active wells
IX	interior cells with $i < I$
IY	interior cells with $j < J$
IW	cells with old injector
PW	cells with old producer
f	fluid (oil, water) index
n	cells, location index
r	iteration index
t	time period index $(1,2,3, \dots, T)$
τ	time interval index $(1, 2, 3, \dots, \sum_t \mathcal{T}_t)$
$x-, y-$	Backward upstream weighting index
$x+, y+$	Forward upstream weighting index

Parameters

A	accumulation function
A_x, A_y	directional cross-sectional area
a, b	exponent terms- Corey's correlation
BHP	bottom hole pressure
B_f	Formation volume factor of f
b_f	Inverse of B_f
c_d	drilling cost for one well

c_{iw}	water injection cost per bbl
c_o	oil selling profit per oil bbl
c_{pw}	water production cost per bbl
c_v	water viscosibility
c_w	water compressibility
D	demand
DC	total drilling cost
g	gravity acceleration
h^t	time
H	production horizon
I, J	number of cell blocks (x, y)
IE^t	injection capacity expansion
IIC	initial injection capacity
IPC	initial production capacity
$\boldsymbol{K}$	absolute permeability
k_r	relative permeability
kr_0^o, kr_0^w	end point relative permeability
L	well length
P_c	capillary pressure
P_r	reference pressure
PE^t	production capacity expansion
S_{or}, S_{wr}	residual oil and water saturations
T	number of time periods
$\mathcal{T}_t$	number of time intervals in period t
T^x, T^y	directional transmissibility

THP	tubing head pressure
TIC	total injection capacity
TLPC	total liquid production capacity
TWP	total water production
V	volume of cell
WC	water-cut ratio limit
α	regression parameters
δ	a very small number
β	economic parameter in NPV definition
$u_r, \boldsymbol{w}_r, \boldsymbol{\omega}_r$	penalty parameters
$\mu_{Bo,1}, \mu_{Bo,2}$	regression coefficients for ($\mu_o \times B_o$)
μ_r^0, $\boldsymbol{\lambda}_r$, $\boldsymbol{\mu}_r$	Lagrange multiplier
$\mu_{o,1}, \mu_{o,2}$	regression coefficients for oil viscosity
ρ	density
γ	regression coefficient for well equations
Ψ	connection transmissibility factor for wells

Variables

$d_{f,1}, d_{f,2}$	terms in accumulation
F	net convective flow from each cell
M_f	mobility of phase f
P	pressure
q	total production/injection
q_f	oil/water production
R	intermediate free variable for IPR equation
S	water saturation

TLP^t	Total liquid production in period t
TOP^t	Total oil production in period t
TWI^t	Total water injection in period t
y	binary variable
ϕ	an unrestricted continuous variable
θ, η, σ	slack variables for MILP master subproblem

Subscripts and superscripts

L	lower bound
o	oil phase
R	rock
U	upper bound
w	water phase

Chapter 5

Sets and subscripts

$\mathbf{C}_{nr}$	cell n of reservoir r is or can be connected to center c
$\mathbf{C}_m$	manifold m is or can be connected to center c
$\mathbf{IP}_r$	interior cell in reservoir r
$\mathbf{IX}_r$	interior cells with $i < I$
$\mathbf{IY}_r$	interior cells with $j < J$
$\mathbf{IW}_r$	existing injector well
$\mathbf{M}_{nr}$	cell n of reservoir r is or can be connected to manifold m
$\mathbf{NW}_r$	potential producer candidate

$\mathbf{PW}_r$	existing producer well
PM	producer manifold
PC	producer center
$c = 1, 2, \ldots, C$	processing centers with C_e existing processing centers ($1 \leq c \leq C_e$)
f	fluid (oil, water) index
$m = 1, 2, \ldots, M$	manifold with M_e existing manifolds ($1 \leq m \leq M_e$)
n	cells, location index
k	iteration index
$r = 1, 2, \ldots, R$	reservoir
t	time period index ($1,2,3, \ldots, T$)
τ	time interval index ($1, 2, 3, \ldots, \sum_t \mathcal{T}_t$)
$x-, y-$	Backward upstream weighting index
$x+, y+$	Forward upstream weighting index

<u>Parameters</u>

A	accumulation function
AD	annual discount factor.
a, b	exponent terms- Corey's correlation
B_f	formation volume factor of f
$\overline{B_o}$	average oil formation volume factor
BM_{nr}, BM_m, BM_c	big-M parameters
C_w	water compressibility
$CP_{c,avg}$	average or nominal pressure at the surface center
$P_{r,avg}^t$	average reservoir pressure
d^t	annual rate of depreciation for interval t
D^t	projected oil demand

E^k	percentile value for $\lvert F_{f,nr}^{t,k} \rvert$ at iteration k.
g	gravity acceleration
h^t	time
H	production horizon
I_r, J_r	number of cell blocks
$\boldsymbol{K}$	absolute permeability
k_r	relative permeability
kr_0^o, kr_0^w	end point relative permeability
L_{nr}	vertical depth of a well at cell (n, r),
L_{nrc}	length of well-to-center pipeline
L_{mc}	length of manifold-to-center pipeline
L_{nrm}	length of well-to-manifold pipeline
CMW_c^U	maximum center-to- well/manifold connections
MW_m^U	maximum manifold-to-well connections
MC_m^U	maximum manifold-to-center connections
WMC_{nr}^U	maximum well-to-manifold/center connections
P_r	reference pressure
PF_{nr}	fracture pressure
Q_m^*	maximum liquid flow capacity for manifold m
Q_c^{t*}	maximum liquid flow capacity for center c
Rs	solubility ratio
S_{or} , S_{wr}	residual oil and water saturations
T	number of time periods
$\mathcal{T}_t$	number of time intervals in period t
T^x, T^y	directional transmissibility

V	volume of cell
WC	water-cut ratio limit
$\Delta x_{ir}, \Delta y_{jr}$	cells lengths and widths
α_{nr}	cost of drilling per unit vertical depth for a well at cell (n, r),
α_c	cost of installing center c
α_m	cost for installing manifold m
α_{nrc}	unit-length cost for well-to-center piping
α_{nrm}	unit-length cost of well-to-manifold piping
α_{mc}	unit-length cost of manifold-to-center piping
α_o	unit cost of oil/gas production
α_{pw}	unit cost of water production and disposal
α_{iw}	unit cost of water injection.
β_o	oil price ($ per unit volume)
β	regression parameters
γ^t	tax rate during interval t
ϵ	a very small number
$\mu_{Bo,1}, \mu_{Bo,2}$	regression coefficients for $(\mu_o \times B_o)$
$\mu_{o,1}, \mu_{o,2}$	regression coefficients for oil viscosity
ρ	density
Ψ	connection transmissibility factor for wells

<u>Variables</u>

$d_{f,1}, d_{f,2}$	terms in accumulation
BHP_{nr}^t	bottom hole pressure
$CAPEX$	capital expenditure
CF^t	cash flow

CP_c^t	pressure at center c
$F_{f,nr}^t$	net convective flow from each cell
M_f	mobility of phase f
MP_m^t	pressure at manifold m
P_{nr}^t	pressure at cell n
q_n^t	well liquid flow rate
$q_{f,n}^t$	well phase flow rate
qwm_{nrm}^t	total flows through the connection between well n and manifold m
$qwm_{o,nrm}^t$	oil flows through the connection between well n and manifold m
qwc_{nrc}^t	total flows through the connection between well n and center c,
$qwc_{o,nrc}^t$	oil flows through the connection between well n and center c,
qmc_{mc}^t	total flow through the connection between manifold m and center c
$qmc_{o,mc}^t$	oil flow through the connection between manifold m and center c
QM_m^t	total fluid flows through manifold m
$QM_{o,m}^t$	oil fluid flows through manifold m
QC_c^t	total fluid flows through processing center c
$QC_{o,c}^t$	total fluid flows through processing center c
QC^t	field liquid production rates
QC_o^t	field oil production rates
QC_w^t	field water injection rate
$\mathbf{R}_n^{r,t}$	intermediate free variable for IPR equation
$S_n^{r,t}$	water saturation
SR^t	total revenue
TI^t	taxable income

THP_{nr}^{t} tubing head pressure

y, u, z, yz, yu, zu binary variable

ΔP_{nr}^{t} total pressure drop function through well tubing

ΔP_{nrm}^{t} total pressure function between well (n, r) and manifold m

ΔP_{nrc}^{t} total pressure function between well (n, r) and center c

ΔP_{mc}^{t} total pressure function between manifold m and center c

θ slack variables for MILP master subproblem

Subscripts and superscripts

L lower bound (function / value)

o oil phase

OF approximation for open-flow condition

R rock

U upper bound

w water phase

Chapter 6[4]

Variables

y, u, z, yz, yu, zu binary variable for placement and connection allocation

$yc, uc, zc, yzc, yuc, zuc$ continuous variable for planning

$yb, ub, zb, yzb, yub, zub$ binary variables for planning

[4] Chapter 5 and 6 share the same notation with few exceptions indicated here.

CHAPTER 1 INTRODUCTION

Oil and gas are not only the most widely used energy sources, but they are the precursors to most petrochemical products [1]. Although the world crude oil production in 2008 was only 72.03 mb/d, some projections [2] indicate that the oil demand would rise from 85.6 mb/d in 2008 to 105.6 mb/d in 2030. According to the predictions of OPEC, the recent global reserves-to-production (R/P) ratio (including both existing and anticipated reserves) is nearly 84 years. It means that crude oil and natural gas liquids (NGLs) can be sustainably produced only for another 84 years at the current production level [3]. Clearly, the age of easy oil production is coming to an end [4], and it is becoming more and more critical to optimize the recovery of the remaining oil resources.

The first step in exploiting a given hydrocarbon field is obviously well drilling. It is important not only for new fields, but also for mature and marginal fields. As hydrocarbon prices climb higher, the recovery from both marginal and mature fields [5] is becoming more important. Drilling is an important key to that. As Figure 1-1 shows, the increasing oil price over the last decade has generally motivated oil and gas exploration and production companies to increase their drilling activities worldwide [6]. More drilling rigs are employed to extend the wells to deeper targets. As onshore reservoirs are depleting, offshore drilling is increasing significantly to exploit new and potentially huge fields such as in Brazil's Pre-salt offshore Santos Basin [7]. Thus, in spite of the possible environmental risks especially in offshore ventures, well drilling is still an activity that is critical to the energy needs of the world.

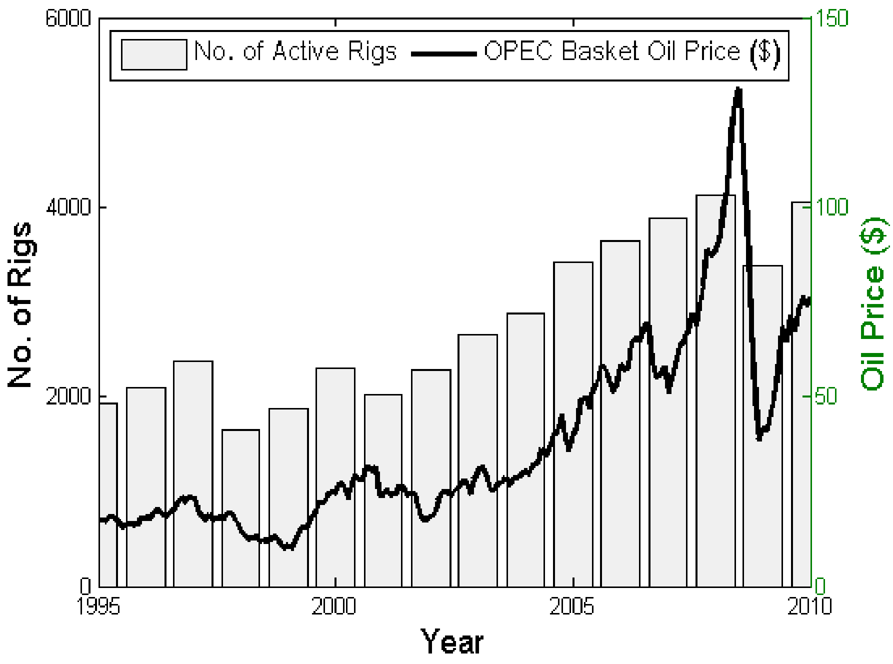

Figure 1-1: Profiles of OPEC oil basket price (monthly) and active drilling rigs worldwide.

Production drilling activities are inevitably preceded and proceeded by series of significantly expensive and technically complicated activities. Amongst all these activities, well placement appears to be a critical step with permanent impact, because a well is our only connection to the subsurface energy treasure. Once drilled, this connection is irreversible. Due to this unique role in the return of the investment in the exploitation projects, well placement should be efficiently planned. To this end, this PhD dissertation mainly focuses on optimal well placement and issues related to that. Specially, this study targets (a) assessing available techniques and approaches for optimal well placement in order to understand their strengths and weaknesses as a foundation for a detailed mathematical study, (b) attempting to propose a shift of paradigm in addressing optimal well placement, (c) preparing a decision support tool for well and infrastructure placement and installation studies and finally (d) highlighting possible future research frontiers based on the current study and the industrial need in this area.

The aim of the current chapter is to provide a general overview of different elements involved in oil and gas production in order to pave the path for modeling this process in an optimization framework. In the next sections of this chapter a brief introduction to oil reservoirs and fields are provided, and then the wells in general and well placement in particular are discussed. After that the problem of well placement is defined. Next, flow dynamics for this problem is described to further clarify the challenges which are then discussed. Finally, the research objectives and the structure of this dissertation are presented.

1.1 Oil reservoirs and fields

Figure 1-2 represents an oil reservoir under injection. Oil, gas and water are trapped in this porous formation where a group of production and injection wells work together to produce the hydrocarbon content of the reservoir.

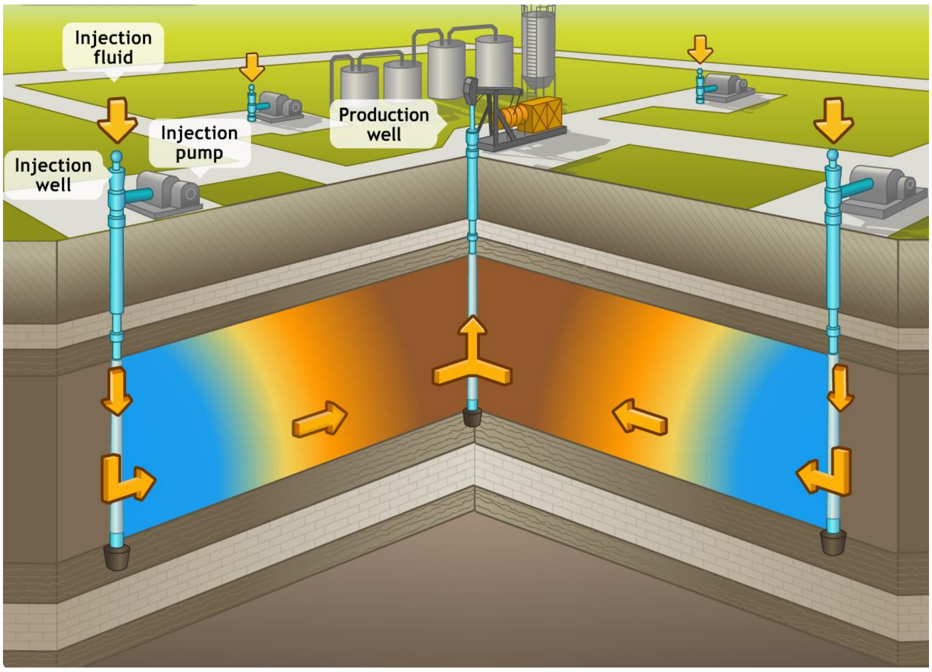

Figure 1-2 : An oil reservoir under injection drive mechanism[5].

[5] From http://snf-oil.com/ with permission.

As Figure 1-3 shows, a collection of reservoirs form a field and the field production is directed to a shared surface processing network. The surface network includes wellheads, manifolds (headers) that collect/mix the oil/gas flows from the wellheads, processing centers that receive the commingled flows from the manifolds, a maze of valves and flow-lines that interconnect the wellheads, manifolds, and processing centers, and finally long pipelines that supply the fluids to the market after pressurization. The multiphase flow relations that govern the fluid flow in different parts of this system are concisely discusses in Chapter 3.

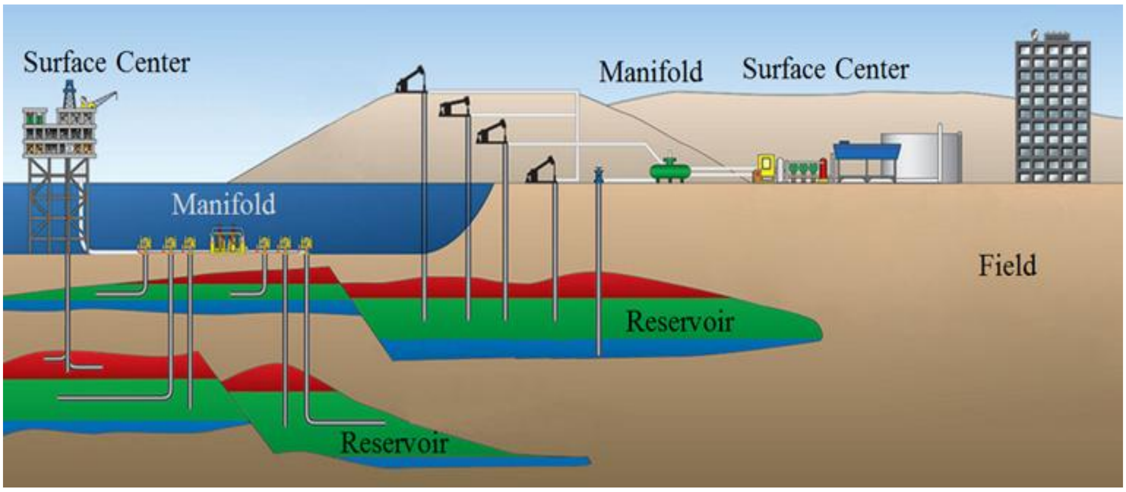

Figure 1-3 : A multi-reservoir field with surface network [8].

In brief, there are three main elements involved in upstream production: (a) sub-surface element (reservoir/ field), (b) wells and (c) surface network. Wells link the subsurface elements with the surface network to begin and continue the production. Before beginning the production, a potential oil/gas reservoir undergoes several steps to evaluate those three elements [9] (Figure 1-4). The subsurface evaluation is very extensive. A lengthy pre-development (exploration and appraisal) phase is required, before the well drilling begins in a reservoir. This phase involves seismic studies, wild-cat drilling, core sampling, well-testing, and well-logging to gather petro-physical and geological data to enable basic understanding of the static and dynamic characteristics of a reservoir. This understanding and resulting numerical models play a critical role in well-drilling decisions and the eventual long-term exploitation of the reservoir.

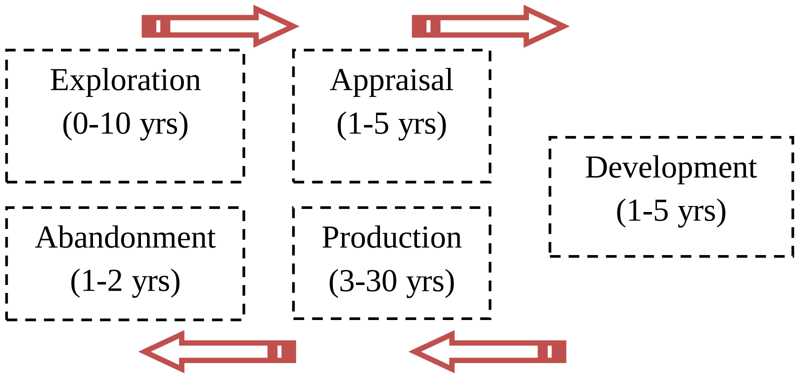

Figure 1-4 : Typical scenario of reservoir life.

1.2 **Wells**

Wells are drilled using different types of drilling rigs. They are drilled either to produce gas/oil, or to inject a driving fluid such as water and CO_2. The fluid injection helps sweep the zones that are not drained from oil/gas. A well can be of two types: conventional or unconventional. The former are vertical or slightly deviated (up to 60^o), whereas the latter are horizontal, highly deviated, or multi-lateral [10] (Refer to reference [11] for a complete discussion on nonconventional wells). Although horizontal wells are more expensive than vertical wells, they are more promising for production [12], because they can intersect natural fractures, produce from different layers of thin and/or tight reservoirs and in some cases hinder water/gas coning [see 13] . They can improve net present value (NPV) and/or total revenue by accelerating production and/or reducing the number of required wells. However, the industry does not always prefer horizontal wells. For instance, vertical wells are preferred to horizontal wells in a high-permeability reservoir with an active aquifer. This is because the latter can cause early water breakthrough and high water-cut as water coning is very likely to happen, which is propagated by high contact area of horizontal wells in these types of the reservoirs.

Well location determines the recoverable volume over time [14]. The geology and heterogeneity of the reservoir, physical properties of the rocks and fluids, well type, driving mechanism, surface facilities, economics, production timeframe [15-17], etc. are the key factors that dictate the type and location of a well. The actual cost of drilling a well depends on many factors including its type and location. These [18] are the lengths of main bore, lateral drilling, and completion, number of junction milling, bit costs, rotating time, round-trip time, footage per bit, angle of inclination, rig type and location (offshore / onshore), etc. [19, 20]. Offshore wells are more expensive and they need offshore production facilities or a subsea well tied back to either an onshore or offshore facility [21]. Van Den Heever et al. [22] and Bitterncourt [23] have discussed the various economic parameters in the assessment of a hydrocarbon field development.

1.3 Well placement

In practice, the industry uses a variety of data, tools, and heuristics to select well locations. They include the various numerical models and understanding developed in the pre-development phase. For fields with ongoing production, past production history is a key information. Much of the experience in well drilling has been formalized into several heuristics such as placing injector-wells from flank to crest in water/gas-drive reservoirs with dome shaped structures, keeping the well trajectory as far as possible from gas and water. This is usually done by drilling in high-permeability zones for the fields with uniform pressure and saturation at early production time and at a later stage using field saturation as a major guide to avoid high water-cut or gas-oil ratio [24]. The engineering team applies these heuristics to define a variety of development scenarios and examines them by extensive simulations. Such an ad-hoc procedure is inherently myopic and carries shortcomings. First, it is not possible to identify and examine all potential scenarios manually. Second, while heuristics are useful and important, they are by nature specific, fallible, and carry no guarantees. Indeed, studies have challenged some of them, for example uniform well spacing and uniform patterns such as five, seven or nine-spot and staggered line drive drilling are very well known heuristics [25]. However, recently several studies [26, 27] have shown that non-uniform well spacing is superior. This supports the need for the optimization process in non-uniform well spacing, which is more complex than the uniform one. Therefore, there has been much recent interest in a systematic approach to evaluate and optimize different feasible drilling scenarios [28-30].

Systematic optimization methods have the best potential for a holistic and rigorous approach. In contrast to heuristics, most optimization approaches keep the entire set of scenarios in view, while searching for the best. However, they do not enumerate all scenarios exhaustively [31]. During their search, they change all variables simultaneously and not one at a time or in some pre-determined manner. This strategy usually leads to a better solution compared to that from a myopic, piece-meal, and manual search based on only heuristics [32]. This has motivated the application of optimization at different stages of the petroleum production industry such as field development, production planning, etc. [33-38].

1.4 **Well Placement Problem Statement**

An oil or gas field consists of one or more reservoirs. The description of a reservoir/field and its production plan requires a variety of information:

- Geological data such as the dimensions, porosity, and permeability of the formation from seismic studies or history matching.

- PVT data such as the fluid volume formation factor and fluid properties (viscosity, density, and compressibility) from core samples or previous production history.

- Existing wells (if any), their functionality (producer vs. injector), type (conventional vs. unconventional) and locations.

- Current state (i.e. pressure and saturation profiles) of the reservoir and the next projects production horizon.

- Economic data such as drilling costs, injection costs, discount rate, and oil revenue forecasts, etc.

- The future plan for enhanced oil recovery (EOR) process which could be suggested even from the early age production of the oil reservoir using any EOR screening software[39]. This would help to come up with the best scenario needed for better oil sweep efficiency during the proposed EOR agent injection.

Given the above information regarding a reservoir exploitation project, the primary concern for well placement is to determine the number, types, and spatial locations of well-drillings that will optimize some performance measures. For each well, its inclination (horizontal or vertical), trajectory, perforation length, and throughput (production or injection) rate must also be determined. It is insisted that the latter should be specifically considered. In addition, the oil company must decide the number and locations of well/production platforms and allocations of wells to different platforms. These decisions must consider multiple factors and policies such as acceptable bottom hole pressure (BHP) or tubing head pressure (THP), well spacing, and distance to surface facilities.

In a strict sense, well placement is a dynamic rather than a static problem as defined above. Every drilling must occur at some time and that ***time*** is also an optimization decision. In the early life of a reservoir, most or all wells would naturally be production wells. However, as the time progresses, one may decide to either convert some of these to be injector wells [40], or drill one or more injectors to inject some driving fluid to help drain the reservoir. Thus, these functionality switches/initialization, their spatial coordinates, and their timings are also key optimization decisions.

Each chapter of this dissertation will provide a very clear and specific problem definition that is addressed from the above domain.

1.5 Research objectives

The goal of this PhD study is to develop an advanced decision support methodology for integrated reservoir management. In the core of that, this research targets optimal well placement, and a variety of other field development decisions related to the well placement. That includes decisions related to well-drillings and infrastructure (manifold and processing center) installation, their allocation, and planning the order and time of drilling/installing each. Integrated reservoir management requires understanding, evaluating and incorporating different elements of both sub-surface and surface sections in a unified model. Such a unified surface-subsurface approach is very crucial for well placement activities. However, the available literatures have either focused on subsurface (usually from petroleum engineering discipline) or on surface problems (usually from chemical engineering and operation research disciplines). They are extensively analyzed in the next chapter. The current study tries to bridge this gap between the subsurface and surface studies to an applicable extent from computational point of view. Therefore, the specific objectives of this study are (a) to comprehend the strengths and limitations of the available strategies in order to address some of their weaknesses, utilize their positive features, and possibly to extend them (b) to build a model based on accurate governing equations of multiphase flow in porous media to guarantee the accuracy of the final solution, and verify that by comparing with industrial standard reservoir simulators, (c) to prepare a holistic model that examines the flow from the reservoir, through the wells and into the manifolds and the separation centers, (d) complete the former by efficiently modeling the well and infrastructure placement, allocation and timing to build a specialized integrated field management model (e) propose and extend an advanced mathematical solution strategy for this specialized model, and finally to (f) highlight possible future research frontiers based on the current study and the industrial need for this problem.

1.6 Outline of Thesis

This thesis consists of seven chapters. After a brief introduction in Chapter 1, the second chapter presents a detailed literature review on existing techniques and approaches to model and solve well placement problem. This chapter also identifies the main challenges and important research gaps in the current techniques. To our knowledge, Chapter 2 is the first and currently the most extensive survey of systematic methodologies for optimal well placement.

Chapter 3 lays the modeling foundations for the next chapters. This chapter discusses the modeling approach used for representing the multiphase flow inside the porous media of the reservoir. Moreover, it describes the steps taken to prepare the multiphase flow pressure drop equations for the wells, flow lines and pipes.

Using the multiphase flow model developed in the previous chapter, Chapter 4 provides a modeling and algorithm design study. Most of the available literature on optimal well placement has employed numerical simulators in a black box manner linked to an external search engine. However, in Chapter 4, the content of that box is formulated inside a mixed integer nonlinear programming model for optimal well placement in one rectangular reservoir. It provides a unified model that integrates the subsurface, and (to some extend) wells and surface levels of an upstream production project. It links the production plan with the aforementioned elements, and economics and market. This results in a complex spatiotemporal mixed integer nonlinear model, for whose solution, the second important part of this chapter is prepared. An existing outer approximation algorithm is modified and augmented. The model solution provides the optimal number of new producers, their locations, and optimal production plan over a given planning horizon. To the best of our knowledge, this is the first contribution that uses mathematical programming in a real dynamic sense by honoring the constituent partial differential equations.

Chapter 5 extends the previous single-rectangular-reservoir study in Chapter 4, and addresses well placement / surface network design and production/injection planning in a field with multiple irregular-shaped reservoirs supplying to a shared surface production-network facility. This chapter focuses more on the surface infrastructure by maintaining strong link to the subsurface environment. As a result, in addition to well placement, Chapter 5 addresses infrastructure placement and allocation problem. Here the dynamic, economic, and operational inter-dependencies of the entire field and its reservoirs are considered through a novel deterministic model. It holistically includes the entire field financial considerations and market demand, dynamic and structural constraints in a surface network of well-manifold-separators, and provides drilling /network design decisions on a long term horizon and detailed production/injection plan on several short horizons (integrated into the main long horizon). Finally, this chapter suggests an adaptive procedure to increase the accuracy of a relaxation used in Chapter 4.

Apart from well and infrastructure placement and allocation problems in the field design, a vital task in upstream development activities is to optimally determine the order of drilling and infrastructure installation in the field. Therefore, Chapter 6 modifies the MINLP model developed in Chapter 5 to tackle this problem. In this chapter a solution strategy is suggested and tested. It employs two successive MINLPs to solve placement/allocation and then ordering problems.

Finally, the conclusions of this study are presented in Chapter 7 where potential future research areas based on that are summarized through a list of recommendations.

CHAPTER 2 TECHNIQUES AND APPROACHES – LITERATURE SURVEY [6]

2.1 Introduction

Well placement problem can be studied from different perspectives. The reservoir and drilling engineers address related issues such as well trajectory optimization, locating the right drilling point, integrating surface and subsurface facilities, etc. As an example, Ayodele [41] briefly addressed well trajectory optimization. Recently, Nasrabadi et al. [42] have published a brief literature survey on well placement with focus on gas/gas condensate reservoirs. However, a comprehensive review or analysis focusing on systematic optimization does not exist in the literature to our knowledge. This chapter aims to fill this gap. It concentrates on the subsurface and deterministic well placement problem.

In the remaining of this chapter, we start with a concise description of the fluid dynamics in the oil reservoir and wells, and that is followed by presenting challenges towards well placement studies. Then we use that understanding to present / compare / contrast the three approaches commonly used in the literature to address well placement problem. Next, we briefly discuss uncertainty handling and available computational tools. Lastly, we identify current gaps in the literature and highlight the open opportunities for future research.

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[6] Tavallali., Karimi, Teo, Ayatollahi and Baxendale (2013). Optimal Well Placement - An Expository Survey of Techniques and Approaches. Optimization and Analytics in the Oil and Gas Industry, In-print.

2.2 Flow dynamics

An oil reservoir is a complex multi-phase (water, oil, gas, and solid) system whose dynamic behavior can be described by coupled spatiotemporal differential equations. For a 2-phase (oil-water) reservoir, these are [43]:

$$\frac{\partial}{\partial t}\left[\varepsilon \frac{S_f}{B_f}\right] + q_f - \nabla\left[\frac{kr_f}{\mu_f B_f}\boldsymbol{K}\left(\nabla P_f - \rho_f \frac{g}{g_c}\nabla z\right)\right] = 0 \quad (1)$$

where, $\boldsymbol{K}$ is the absolute permeability tensor, ε is the porosity, μ_f is the viscosity of phase f ($f = o$ for oil and $f = w$ for water), ρ_f is the density, B_f is the formation volume factor, kr_f is the relative permeability, S_f is the saturation, P_f is the pressure, and q_f is the flow from (+ve for out, $-$ve for in) the reservoir. Interested reader is suggested to refer to reference [43] for the case of three phase (water/oil/gas) flow. The relative permeability is a function of saturation, and for mathematical analysis and modeling, relative permeability is usually defined by Corey's equation [25] for each phase:

$$kr_o = kr_o^0\left(\frac{S_o - S_{or}}{1 - S_{wr} - S_{or}}\right)^a \quad (2)$$

$$kr_w = kr_w^0\left(\frac{S_w - S_{wr}}{1 - S_{wr} - S_{or}}\right)^b \quad (3)$$

where, S_{or} and S_{wr} are the residual oil and water saturations respectively, kr_0^o and kr_0^w are the end-point relative permeabilities for oil and water respectively, a and b are the exponents in Corey's correlation.

The saturation and pressures of water and oil phases are related to each other through following equations:

$$S_o + S_w = 1 \quad (4)$$

$$P_c = P_o - P_w = f(S_o, S_w) \quad (5)$$

where P_c is the capillary pressure. In the remaining of this section the phase index f is dropped for brevity.

At least three pressure types are critical in determining the flow rate q in Eqn. (1). These are (1) the reservoir pressure (P), (2) pressure at the well bottom called the bottom hole pressure (BHP) and (3) pressure before the choke valve near the surface called the tubing head pressure (THP). Then the flow rate q at the well bore can be defined using the following well production equation (IPR or Inflow Performance Relation):

$$q = \Psi(kr/\mu B)(P - BHP) \tag{6}$$

where, Ψ is the connection transmissibility factor for wells (see Eqn. 76.5 of [44]) and the BHP is determined by the Vertical Flow Performance (VFP) curve:

$$BHP = THP + \rho gL + \Delta P + \Delta p \tag{7}$$

where, ΔP is the frictional pressure drop through the well tubing and Δp is the pressure drop due to acceleration, L is the well depth, and ρ is the density of the well output. In practice, the choke valve controls well production and fixes THP, which is measured continually in the field. For a given well string and completion type [45], total flow, water to oil ratio, (gas to oil ratio), and inlet/outlet pressure determine the pressure drops in Eqn. (7).

The initial saturation and pressure distribution provides the initial condition; additionally, a combination of no flow boundary conditions, water influx rate from aquifer and bottom hole pressure forms the boundary conditions of the above set of equations. While Eqs. (1)-(7) constitute a rigorous reservoir and well model, they cannot be solved analytically. Sophisticated commercial simulators such as ECLIPSE [44], CMG [46], etc. exist for solving these accurately.

2.3 Challenges

Amongst many challenges in addressing well placement problem, few are mentioned here:

- The large number of decision variables such as the potential well positions, their types, their allocation to different gathering points, and drilling schedules make the optimal well placement a highly combinatorial optimization problem with considerable computational challenges. The solution of Eqs. (1)-(7) normally requires discretization in both spatial and temporal domains [43]. Depending on the geological formation, up to 10^6 cells or more grids may be required with at least two unknowns (say *P* and *S* in a 2-phase black oil model) associated with each grid. The simulation over the production life of a reservoir may take hours to days of computation time [47]. Optimizing well placements may require hundreds of reservoir simulations under varying conditions, and hence can be computationally prohibitive.

- Apart from these considerable number of design variables and combinations, a production project is characterized through a numerous number of system parameter including geological, operational, petro-physical, geometric and economic data. Working with all these data in an integrated manner and ensuring that the design parameters do not conflict each other is not a trivial task. Probably, that explains why the modern reservoir simulators such as ECLIPSE [44], are equipped with pre-processing, simulation and post-processing tool boxes to reduce this workload.

- Fluid properties such as relative permeability and fluid mobility are nonlinear functions of saturation and pressure. Sharp changes in field permeability often result in stiff ODEs, which increase the simulation time even further. Consequently, the resulting optimization problem is not only highly nonlinear; it possesses complex trade-offs, and substantial non-convexity. As an example, consider the use of water injection. On the one hand, water acts as the driving fluid and helps to maintain pressure in the reservoir. On the other hand, after it breaks through, it starts to play a negative role by increasing the surface processing load downstream and acting as an energy sink.

- Most numerical simulators discretize the continuous differential Eqs. (1)-(7). Embedding this discretized model inside an optimization algorithm makes it challenging to use the well-established continuous optimization solvers for well placement, as the well locations are no longer continuous.

- Many known and unknown uncertainties naturally exist in a real-world system such as a complex multiphase reservoir[48]. For example, the huge dimensions limit our knowledge, as we cannot sample the entire reservoir to determine various geological properties. This leads to substantial amount of computational efforts, since most approaches address uncertainties through simulations under many alternative scenarios.

2.4 Optimization Modeling and Algorithm

Let vector $\boldsymbol{v}$ represent the continuous variables (e.g. BHP, THP, pressure and saturation), $\boldsymbol{w}$ represent the integer variables (e.g. location indices and number of wells), and $\boldsymbol{r}$ the model parameters (e.g. geological realization, production time horizon, and techno-economic data). The majority of variables in v are technically or physically bounded. For example, the maximum curvature of a well is limited by the current technology; the BHP of an injector must not exceed the formation's fracturing pressure; and the BHP/THP of a producer must be sufficient to lift the produced fluids to the surface.

All production scenarios should satisfy the various physical, hydrodynamic, operational, and market constraints. These constraints, listed below, can be represented as nonlinear equalities $h(\boldsymbol{v}, \boldsymbol{w}, \boldsymbol{r})$ and inequalities$g(\boldsymbol{v}, \boldsymbol{w}, \boldsymbol{r})$:

- Appropriate constitutive and conservation relations for the reservoir, well strings and pipes, such as inflow performance relations, vertical flow performance relations, pressure drop for multiphase flow, and heat and mass balance equations.
- Preventing intersection of wells, specifying drilling path and following the perforation policies. Well perforation in inactive grids of the reservoir is not permitted; however, non-perforated well segments can cross these inactive grids. Finally, perforated segments must logically remain in the lease boundaries of the reservoir.
- Maximum/minimum injection/production rates of the field or wells.
- Limits on techno-economically acceptable water-oil ratio (WOR), gas-oil ratio (GOR) and water-cut.
- Minimum well-to-well and well-to-platform spacing
- Maximum or minimum number of wells to be drilled and platforms to be installed
- Production demand, and field processing capacity

Due to the nonlinear and PDE nature of continuity equations, these relations are one of the most important constraints amongst others in a well placement model. Indeed, the major role of different commercial and in-house software packages is to solve these coupled PDE equations, while, internally adhering to the constraints and bounds, and reacting to their violation based on pre-defined polices. They rigorously simulate the production scenario for a given specific well configuration. We will later discuss available reservoir simulator packages and their role in optimization algorithms.

The optimal well placement problem attempts to optimize a techno-economical measure such as total drilling costs, production profit, recovery factory, sweep efficiency or remaining oil in place, simultaneous maximum pressure and temperature support in geothermal water re-injection operations[49], cumulative production and estimated ultimate recovery [50], while satisfying the mentioned operational and techno-economic constraints. This techno-economical measure is represented by $f(\boldsymbol{v},\boldsymbol{w},\boldsymbol{r})$. From literature, the most widely used measure has been NPV. However, for companies with restricted production based on their quotas (such as national oil companies in the OPEC countries), ultimate recovery may be more relevant [51].

Given the above definitions, the optimal well placement optimization problem can be stated as Model (O) below:

Model (O)

$$min \quad f(\boldsymbol{v},\boldsymbol{w},\boldsymbol{r})$$

Subject to

$$\begin{bmatrix} h(\boldsymbol{v},\boldsymbol{w},\boldsymbol{r}) = 0 \\ g(\boldsymbol{v},\boldsymbol{w},\boldsymbol{r}) \leq 0 \\ v \in V = [v: v \in \Re^n, \boldsymbol{Ax} \leq \boldsymbol{a}] \subseteq \Re^n \\ w \in W = [w: w \in (0,1,\ldots)^m] \subseteq \mathbb{Z}^{m+} \end{bmatrix} \tag{8}$$

Although the well locations should be continuous in principle, discretization of Eqn. (1) makes them discrete variables. Depending on how the potential well locations and their structures are represented, model (O) can be either a mixed integer or a continuous (i.e. W is an empty set) optimization problem. The optimization approaches invariably exploit the unique features of O. Three approaches are commonly used, namely (a) evolutionary and direct search, (b) mathematical programming, and (c) gradient-based methods. For each, we introduce how the well location is modeled and the well placement problem is formulated and solved. We use the following notation for the remainder of the paper.

- N = Number of potential wells
- $I = [i \; or \; j = 1, 2, \ldots, N \mid site \; i \; is \; a \; potential \; well \; location]$
- (x_i, y_i, z_i) = Location of site i in the Cartesian coordinates.

2.4.1 Evolutionary and Direct Search Methods

As Figure 2-1 depicts, the simulation-optimization method [52] drives the search procedure by using a search engine on top of a simulator. The search engine assesses the previous solutions to propose a new solution, which the simulator then evaluates. These two elements are usually augmented with different local search methods and proxy models. Typically, these algorithms are ideally meant for unconstrained optimization problems and cannot deal with external constraints without using some penalty terms in the objective function. The first challenge in using these methods is the appropriate representation of the solution vectors that may involve discrete decisions such as well locations of single or multiple patterned wells. The second challenge is to fine-tune the algorithms to obtain fast solutions and reliable performance.

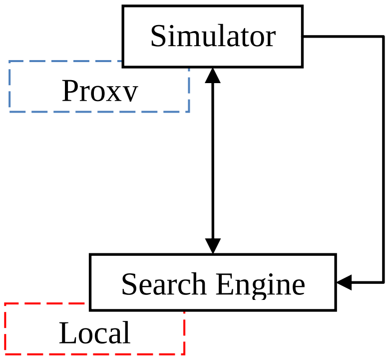


Figure 2-1: The simulation-optimization framework.

2.4.1.1 **Parameterization of Decision Variables**

To illustrate how integer and continuous variables are used to model well location and configuration, first consider a single deviated, multi-lateral well. Any conventional and vertical well can be geometrically defined as a special case of a nonconventional and deviated well. Figure 2-2 depicts the general trajectory of a linear multilateral well. Such a well usually consists of one main trunk and multiple laterals. We first model the main trunk, and then extend it to include the laterals. Finally, we discuss group well-modeling.

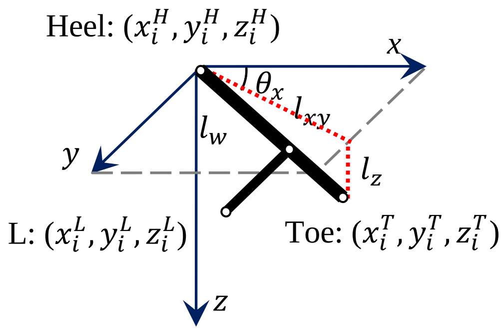


Figure 2-2: Geometrical representation of wells.

The figure is modified from Yeten et al. [53].

Although there have been attempts to consider curved trajectories [54] for the trunk, the majority of contributions has assumed linear orientation. Such a linear trunk can be represented by the relative position of its heel and toe, which are the highest [$H{:}\,(x_i^H, y_i^H, z_i^H)$] and the lowest [$T{:}\,(x_i^T, y_i^T, z_i^T)$] points on the main trunk, respectively. Logically, these two points can be represented in two ways: (a) using the point-based notation i.e. (x_i, y_i, z_i) or (b) using the location of one point and (semi) cylindrical information of the trunk $(l_{w,i}, l_{z,i}, \theta_i)$. Here, $l_{w,i}$ is the trunk length, $l_{z,i}$ is its projected depth, and θ_i is its projected angle in the xy plane with either x or y axis. To choose between the two approaches, compare the below constraints on the maximum well length (l_w^{max}):

$$\sqrt{(x_i^H - x_i^T)^2 + (y_i^H - y_i^T)^2 + (z_i^H - z_i^T)^2} \leq l_w^{max} \qquad i = 1,2,..N \qquad (9)$$

$$l_{w,i} \leq l_w^{max} \qquad i = 1,2,..N \qquad (10)$$

Eqn. (9) uses the point-based format and it is a nonlinear inequality. However, Eqn. (10) uses the trunk length and it is a straightforward linear constraint (or a bound). The same holds for the angular limits on the deviated wells. Furthermore, while the three variables of (x_i^T, y_i^T, z_i^T) are discrete, those in $(l_{w,i}, l_{z,i}, \theta_i)$ are continuous. Therefore, specifying the well position using the heel location and trunk information has more advantages.

Heel and toes can be represented via different combinations of angles and projections on xy, xz and yz planes, such as using both horizontal and vertical angles and one length variable. However, due to the usually small ratio of horizontal to vertical grids in simulators, the deviation angle in the vertical plane (θ_z) can become very small. Hence, it is better to use only one horizontal angle (θ_x or θ_y) combined with two length properties. Clearly, the trunk part is modeled with six variables. As an example, Yeten et al. [53] used [$(x_i^H, y_i^H, z_i^H), (l_{xy,i}, l_{z,i}, \theta_{x,i})$] and Farshi [54] used $[(x_i^M, y_i^M, z_i^M), (l_{w,i}, l_{z,i}, \theta_{x,i})]$ where superscript M refers to the middle point on the trunk.

Having defined the main trunk position in space, we can now cover the laterals. The laterals are connected to the trunk at the junction points. The location of each junction (τ_i) between toe and heel can be shown by a variable m ($0 \leq m \leq 1$) such that:

$$\tau_i = H_i + m_i(T_i - H_i) \qquad i = 1,2,..N \qquad (11)$$

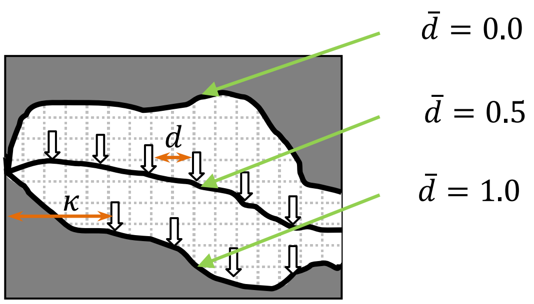
Therefore, each lateral can be represented by four variables: $[m_i, (l_{w,i}, l_{z,i}, \theta_{x,i})]$.

Vector parameterization can be used to extend the above definition to consider the well type (injector or producer) and state of each well (close/open or drilled/undrilled) using a binary variable. As an example, if at most two new wells are to be drilled, the solution vector can be defined as $[(H_1, T_1, m_1, L_1, \psi_1, \gamma_1)(H_2, T_2, m_2, \mathrm{L}_2, \psi_2, \gamma_2)]$. Here, ψ_i and γ_i are binaries ($\psi_i\ = 0$, if an injector, else a producer and $\gamma_i = 1$, if the well is open, else closed). Therefore, the constraint on the maximum number of wells (and hence the optimal number of wells) can be implicitly addressed by defining a fixed size solution vector along with a set of binaries. Moreover, since numerical simulators use grid-grid connections, the above linear representation makes the deviated wells into a staircase like structure. Correct well indexing can reduce this approximation error [55].

The above approach models wells individually. Now, consider a pattern well configuration, such as linear drive or seven spot, where more than one well is used in a predetermined configuration. Using the above formulation for each well in a placement problem can lead to an intractable model. Therefore, an alternative formulation is required, which models the configuration rather than single wells. Ozdogan et al. [56] investigated the linear drive pattern by using three variables for each line of wells. As shows, these variables are (1) uni-directional, normalized average distance from a reservoir boundary ($\bar{d}$), (2) uniform well-to-well spacing (d) and (3) reference distance showing the distance of the first well from a specific boundary (κ). They used separated zones to investigate producers versus injectors.

Figure 2-3 : Variables for modelling a linear pattern of wells.

The figure is adapted from Ozdogan et al. [56] .



Recently, Onwunalu and Durlofsky [57] used a combination of integer and continuous variables to model each closed well-pattern by a polygon. As Figure 2-4 depicts, there is a well at the center point (x, y). The well spacing variables (a, b) relate other wells to this center point.

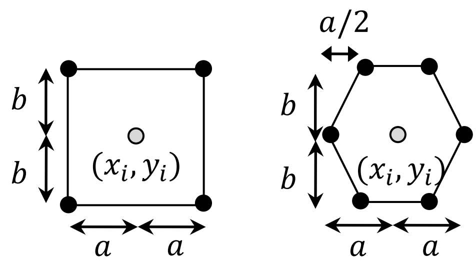


Figure 2-4 : Closed well patterns represented by a general polygon.

The figure is from the work of Onwunalu and Durlofsky [57].

An integer variable E selects the pattern (e.g. seven-spot, nine-spots, etc.). To capture any alteration to the standard form of these polygons, they define a set of operators u ∈ [rotational, scaling, shearing, and switching]. At most two continuous mapping variables $(arg1_u, arg2_u)$ determine the degree of rotation, factor of scaling, shearing and switching of the pattern around a reference well R_u^{ref} in the x and y directions. That defines the transformation operator as $O_u = [R_{u,}^{ref}, arg1_u, arg2_u]$. The reference well may change after each transformation. Finally, the order of U potential pattern transformations are selected by integer variables e_u, $(u = 1,2,\dots,U)$. Therefore, a model can be represented by:

$$\left[\underbrace{(E,[x,y,a,b])}_{\text{pattern parameters}}\ \underbrace{(e_1,e_2,\dots,e_Y)}_{\text{operator sequences}}\ \underbrace{(O_1,O_2,\dots,O_Y)}_{\text{pattern operators}}\right] \tag{12}$$

By using integer variables, the model of Onwunalu and Durlofsky [57] accommodates a wide range of patterns in contrast to that of Ozdogan et al. [56] discussed earlier. However, none of these models considers the presence of previous wells in the field.

All above approaches represent the well information via vector parameterization. Having defined the various approaches for modeling the well location and structure, we now discuss the algorithms that use them.

2.4.1.2 **Solution Algorithms**

Evolutionary methods usually adopt the same general idea: generate many samples randomly from different zones of a feasible region, evaluate them and use the best samples to generate the next batch of sampling points. In every iteration of the search procedure, there are three major steps: (1) solution generation, (2) feasibility check of solutions generated, and (3) objective value evaluation using dynamic reservoir simulation of feasible solutions. Figure 2-5 illustrates these three steps. In step two, the geometrical constraints such as minimum well-to-well distance are typically verified before the reservoir simulator in step three is used to check if operational constraints are satisfied and evaluate the quality of the solution.

Such an approach is a black box search - a search without exploiting the physics and characteristics of the problem; the simulator acts as a black box and provides the objective value when given a solution vector. Hence, a key strength of this approach is that it is compatible with different types of simulators. It also falls under the family of gradient free methods. This is an essential characteristic of this approach due to the discretized nature of the model. (See References [58, 59] for a review on gradient free methods in reservoir engineering.). Consequently, the range of input variables (different well locations, types and functionality, etc.) can be specified without consideration for linearity, nonlinearity or convexity via solution vector parameterization.

Early attempts using evolutionary methods for well placement [23, 27, 60] showed promising results, but also revealed deficiencies. Mainly, large computational effort and generation of infeasible well configurations are the common problems. After the early work of Beckner and Song [27], many latter contributions have tried to address these shortcomings. To do so, all three stages of evolutionary methods have been targeted for modifications. We describe the proposed modifications in each category here:

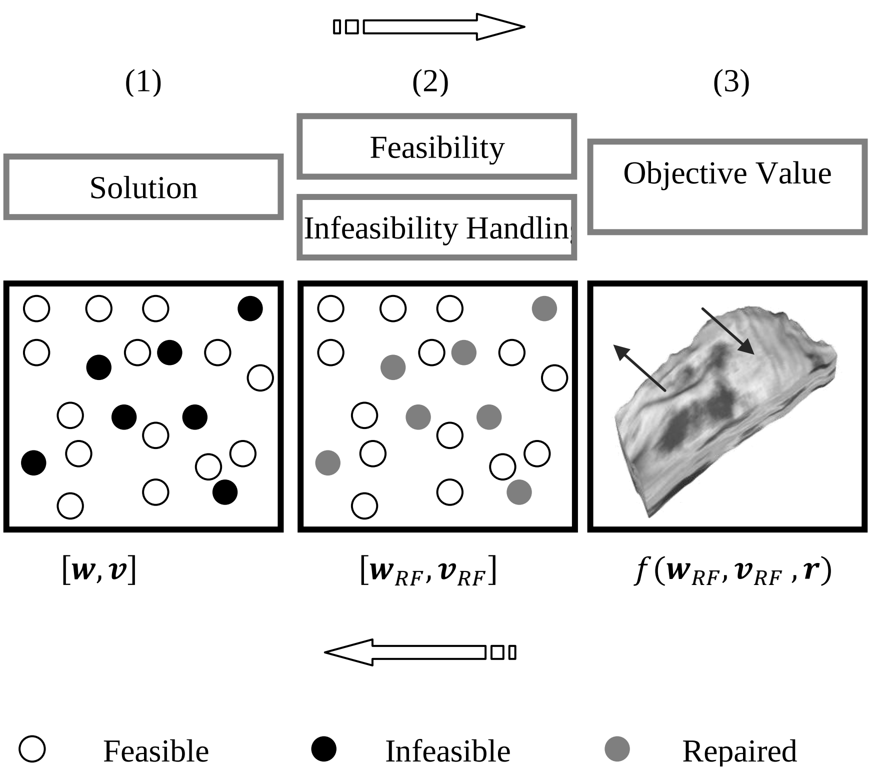


Figure 2-5: The flow diagram of simulation-optimization approach.

Subscript RF refers to screened and repaired feasible solutions.

2.4.1.2.1 Stage 1: Generating Solutions

The objective of solution generation is to generate promising solution vectors in the unexplored search space, while avoiding infeasible vectors. From our observations, researchers have developed four different strategies.

The first strategy adds extra information to the information extracted from the main random search. Bittencourt [23] proposed a hybrid genetic algorithm (HGA) approach that generates new solution vectors using GA and partly the polytope search. Polytope search is a simplex based search strategy and generates candidates far away from the worst well site evaluated in previous iterations. The idea of HGA is used in most subsequent contributions. Bangerth et al. [61] used a Simultaneous Perturbation Stochastic Approximation (SPSA) approach to estimate the gradient information. Under SPSA, the directional gradient along a randomly chosen vector is estimated by means of the simulator. In that sense, it can be called a direct search technique that adds more information to the random search.

The second strategy lies in refining the search region. Evolutionary methods sample several solutions which can span a large part of the objective surface, but when no rigorous mathematical termination criterion is used, the final solution can often be improved by a neighbouring solution. To avoid this problem, Güyagüler et al.[62], Yeten et al. [53] and Ciaurri et al. [59] added a local search to the evolutionary method. The local search is equivalent to perturbing a portion of the wells in their neighbourhood. Güyagüler et al.[62] called it local mutation, while Yeten et al. [53] and Ciaurri et al. [59] described it as a heuristic adaptation of the Hooke-Jeeves pattern search. All of them reported improvement in the final solution. Similarly, dynamic search parameter tuning, as in Afshari et al.[63], can perform global search initially and be converted to a local search in later iterations.

Evolutionary methods often generate solution vectors from previous generations using a multivariate normal distribution. The third strategy, the Covariance Matrix Adaptation- Evolutionary Strategy, attempts to improve the search performance by modifying the covariance matrix of this mutation distribution. It is proposed by Ding[64] and later used by Bouzarkouna et al. [65, 66].

Yeten et al [53] reported that for a specific case study, almost 30% of solution vectors generated in majority of generations were infeasible. As the number of wells increases or the well configuration becomes more complicated, this figure can increase drastically. To get around this problem, there have been increasing interests to shift from discrete binary genetic algorithm (bGA), the dominant search algorithm during the last decade which suffers from the problem of infeasible solution generation, to another strategy of continuous evolutionary methods in recent years. In this strategy, the well locations are expressed in continuous space, and a simple function maps these values into integer space for introducing to reservoir simulator. Farshi [54] and Abukhamsin [51] studied continuous GA (cGA). Bukhamsin et al.[67] compared the performance of cGA and bGA and reported that while both have comparable performance, cGA solutions cluster around the average. Moreover, they reported stepwise behavior for bGA in contrast to gradual behavior of cGA. Hence, cGA can be more stable as a result. In another study, Onwunalu and Durlofsky [68] use particle swarm optimization (PSO), another evolutionary optimization algorithm that works in continuous space. They tried to minimize the number of infeasible solutions by forcing the velocity parameter of infeasible solutions to zero. This is to prevent more infeasible solutions in the next iteration. In their computational studies, PSO outperformed GA on average.

Although useful, such strategies at the solution generation stage cannot completely prevent the infeasibility. Therefore an infeasibility screening step is required to prevent such infeasible solutions.

2.4.1.2.2 Stage 2: Screening for Infeasibility

In the well location problem, the most troublesome constraints for the evolutionary methods are:

- Minimum well-to-well and subsequently avoiding intersection of wells
- Well-to-surface facility distance
- Preventing completion in inactive cells and placing producers in aquifer zone [60]
- Placing all wells inside the reservoir (feasible region)

The solutions generated in Stage 1 may be infeasible, i.e. may violate the above constraints. Stage 2 detects such solutions, but discarding them may not be the best decision. Instead, researchers have tried to recover feasible solutions from them.

Penalizing the infeasibility is commonly used in evolutionary methods [53, 63, 64], however, one should do that cautiously. To discuss that, consider a well that is completely located in the inactive grids. Though not a feasible producer, it can be a promising injector, especially if it is near the boundary of a sealed reservoir. Moreover, the vicinity of this infeasible location can be a promising drilling site, which can be found by a local search. However, penalizing this infeasible solution too high can cause the algorithm to ignore this region [62]. Therefore, Güyagüler et al. [62] suggest assigning 90% of the objective value of closest feasible location to these infeasible sites.

Numerous ways can be used to find a neighboring feasible solution from an infeasible one. Where variables are supposed to be integral, such as the definition of well locations in the simulator, values can be rounded up or down to the closest integer to recover feasibility. For another example, suppose wells are to be located inside the reservoir, wells currently outside can be projected onto the nearest reservoir boundaries [68, 69]. Emerick et al. [70] used a rather different approach. They allow the infeasible solution to interact with a reference feasible population through crossover to produce feasible individuals. The resulted solution can update the reference population if it is promising.

2.4.1.2.3 Stage 3: Evaluating the Objective

The most basic deterministic well placement problem with simultaneous well drilling already requires extensive simulations. By including the considerations for drilling sequence [27, 71] and uncertainties [72-74], and the computational load increases significantly. Therefore, most studies concentrate on reducing this computational cost by: (a) search space reduction, (b) surrogate modeling, and (c) changing the computational platforms.

(a) Search space can be reduced by screening and model/algorithm modification. Screening omits non-promising well locations: Beckner and Song [27] used a predetermined list of potential locations and a variation of the "travelling salesman" problem to handle the order of drilling problem. Santellani et al. [75] used maximum water saturation, minimum completion layers and well distancing as criteria to screen the well locations before simulation. Johnson and Rogers [76] restricted the number of new wells in Pompano offshore field (in Gulf of Mexico) to 25 sites and searched for their optimal locations. In each instance, prescreening was essential to limit the computation power required. However, with more powerful computing hardware and algorithms, prescreening requirements may be relaxed. For example, by studying the same Pompano field in Johnson and Rogers [76] but bypassing the prescreening stage, Güyagüler et al. [62] found better solutions (at least for mono-well placement problem), proving that pre-selection may cause a better optimum to be omitted from the search.

Additionally, model can be approximated by a problem with fewer number of variables, as in Onwunalu and Durlofsky[57, 59] and Ozdogan et al. [56]. A trivial example is modeling the order of drilling n wells using vector of $(n-1)$ binaries[71] , so in a 3-well problem, drilling order of (1,2,3) and (1,3,2) can be represented by (1,1) and (1,2) with one fewer variable.

(b) Surrogate modeling aims to replace the expensive evaluation of the objective function $f(\boldsymbol{v}, \boldsymbol{w}, \boldsymbol{r})$ with an inexpensive approximation, thereby saving computational effort. This approach has also been called proxy models, meta-models, meta-heuristics models etc. in the literature. Such a surrogate model can be constructed using systematic design of experiments where maximum information about a response surface are extracted with minimal simulation effort by choosing evaluation points carefully [77, 78]. Surrogate models can be used to rapidly evaluate and pre-rank alternative scenarios, so that the fewer actual simulator calls are required [5]. However, it is important that the surrogate model is accurate enough: regular update during optimization progress is often required. Common proxy modeling approaches used in well placement problems include: Kriging [62, 72, 74, 79-81], neural network[5, 53, 62, 76, 82], neuro-fuzzy [83], clustering[84, 85] and quality map and regression [5, 86-93]. Recently, there is a growing interest in using reduced order modeling [94-98] in reservoir control. However, to the best of our knowledge, this concept has not been used directly well placement yet. For a detailed accuracy assessment of these proxies, refer to references [62, 99].

(c) The last approach tries to evaluate $f(\boldsymbol{v}, \boldsymbol{w}, \boldsymbol{r})$ more quickly by parallelization [58, 100] using multi-processor simulation, grid and distributed computing [101]. Significant gains have been reported [61, 70, 71]. Afshari et al.[63] also suggested the use of streamline simulators, rather than finite difference simulators. Of course, the limitation of streamline simulators from the viewpoint of reservoir engineering should be considered beforehand.

2.4.2 Mathematical Programming

The optimal well placement problem can be modeled as a mathematical programming problem, and may be solved using the wealth of theory, techniques and algorithms developed for the same. In fact, mathematical programming has been already used for several real (especially combinatorial) problems such as refinery blending, planning and scheduling [102-106], pharmaceutical enterprises [107, 108], bio desulfurization [109], heat and work exchange networks [110-115], and offshore infrastructure planning and scheduling [116].

Most research has been on the surface level of field development, rather than the subsurface well placement that requires experience and expertise in dynamic, multi-phase, subsurface reservoir modeling. Thus, problems such as economic analysis and determination of number, type and size of production and gathering platforms as well as location-allocation of these platforms and wells have received more attention [22, 117-123]. These studies are further reviewed in Chapter 5 and this chapter focuses on the subsurface problem subject to the surface constraints.

2.4.2.1 Models with Discrete Variables

Binary variables are a natural choice to model options for well location, segment, and perforation. For instance, we can define the following binary variable (w_i) to model well placement at a grid i.

$$w_i = \begin{cases} 1 & \text{if a well is drilled at location } i \\ 0 & \text{otherwise} \end{cases}$$

Then, one can use this binary variable to regulate production from that grid [124] or enforce a minimum well-to-well distance[125] as follows:

$$q_i \le w_i . q_i^{max} \tag{13}$$

$$D_{ij} + D_{min}\left(2 - w_i - w_j\right) \ge D_{min} \qquad i \ne j \tag{14}$$

where, q_i^{max} is the maximum possible production at cell i, D_{ij} is distance between cells i and j, and D_{min} is the minimum allowable well-to-well distance. q_i^{max} can be estimated from absolute open flow rate, available surface processing capacity, and projected demand.

Several works have addressed the well placement problem, but the focus of most has been surface [126] rather than subsurface details. In general, the modeling of subsurface flows determines the linear or nonlinear nature of O. If f, h and g are linear (nonlinear), then O becomes an MILP (MINLP). The first reported models use different forms of linear relations and static data to reflect the dynamics of the subsurface flows, thus they are MILPs. In their pioneering work, Rosenwald and Green [124] reported an MILP by defining something called influence function. This function describes how the reservoir pressure at cell i and time n is affected by unit production at site j, and it is computed by running extensive reservoir simulations. Using this function, they related the reservoir pressures at various sites as a linear function [127] of various production rates up to and including time n. They called this the superposition method, which essentially approximated the real nonlinear reservoir dynamics in terms of linear functions. Using the same method, Haugland et al. [128] employed a linear and simplified well production equation to determine well placement and scheduling, platform capacity, and production plan concurrently. Later, Iyer et al. [116] used a piecewise linear approximation of reservoir pressure and GOR versus cumulative oil production to describe the reservoir response to production. However, they made several excessive simplifying assumptions such as homogenous fluid mixture at the same pressure in each reservoir, constant productivity index throughout the planning horizon, well productions being independent, and linear pressure drops versus flow rates in pipes. In spite of these, their contribution is important because their MILP model includes many complex operational considerations including well selection in different reservoirs and different fields, drilling schedule, platform installation and sizing, production planning, and even the availability of drilling rig.

In contrast to the above work that considers the reservoir as dynamic, some researchers have assumed it as static and defined various metrics to approximate the properties of wells at sites. Dogru [129] formulated the offshore well platform and drilling location-allocation problem by defining a productivity index for each potential well site and using oil-in-place data to indicate production quality. Vasantharajan and Cullick [125] defined a static metric, which describes the connected hydrocarbon pore volume at a site inversely weighted by tortuosity. Ierapetritou et al. [130] used some available data on the quality of various sites, which considers the geological information and static state of the reservoir at each site. Their model assumes vertical wells, but allows multiple geo-objects with multiple layers in the reservoir, and a well to have a perforation in each layer. They used one binary variable to model the existence of a well at a site, and others to locate perforations in various layers at each site. They allowed the quality to vary with layer, and imposed a minimum well-to-well distance for well sites or even perforations. These are important considerations in practice, and hence their MILP model is of significant interest. Cullick et al. [131] further extended the work of Ierapetritou et al. [130] to deviated wells using a sequential heuristic approach. Their model considers well spacing, bending angle, and total well length in a 3D domain. In contrast to Ierapetritou et al. [130] who used two binary variables, Cullick et al. [131] used one binary variable for each site, and one continuous variable for each layer at each site.

Among the surface-directed works, the work of Van Den Heever and Grossmann [132] is notable. They extended the MILP model of Iyer et al. [116] by fitting an exponential function to describe reservoir pressure versus cumulative oil flow rate, and quadratic functions to describe the cumulative gas productions and GOR versus cumulative oil flow rates. Their MINLP model employs generalized disjunctive programming.

2.4.2.2 **Algorithms**

The above binary-based modeling approaches lead to MILP and MINLP models, where branch and bound techniques are primarily used along with continuous LP/NLP solvers. They can guarantee successive improvement in the objective function. However, because of the details captured, the resulting MIP models tend to be sizeable. To reduce the computation time, researchers have tried several strategies.

One is to pre-process reservoir data to remove inferior well candidates based on attributes such as net pay, permeability, productivity index, oil-in-place map before executing the optimization algorithm [125, 130]. We have discussed similar approach and its deficiencies in section 2.4.1.2.1.

The second is to use heuristic or decomposition procedures, which may give suboptimal solutions. For instance, Iyer et al. [116] proposed a sequential decomposition algorithm to obtain an upper bound for their maximization model. They aggregated wells in each reservoir and time steps, and employed a piecewise linear approximation for the reservoir. They computed the lower bound using disaggregation and relaxing the declining profile constraint. Their MILP model requires a huge number of binary variables due to their piecewise linearization, which Van Den Heever and Grossmann [132] avoided by directly using nonlinear correlations. Since the resulting MINLP is non-convex, they employed convex envelopes instead of linearizations in the master problem of their outer approximation algorithm. Finally to obtain a tighter bound, they proposed a dynamic programming strategy to derive an effective aggregation scheme at every iteration.

The work of Ierapetritou et al. [130] is an example of how a MILP model can be decomposed to reduce the search space. To find the best well configurations, Ierapetritou et al. [130] used iterative ranking of candidates, cut-off criterion, and feasibility tests in both the decomposed and the full decision space. In a similar study, Cullick et al. [131] proposed a two-stage MILP algorithm to reduce the number of candidates and refine the search space for deviated wells. Their algorithm first locates vertical wells by using definitions of quality and geo-objects. In the second stage, it uses the solution from the first stage to check for 3D completion and design deviated trajectory using ideas from graph theory.

2.4.3 Gradient-based Methods

Recent developments in adjoint gradient calculation method [133] bring about more efficient gradient evaluation and optimization. Consequently, continuous well location models have been developed to exploit the techniques and to overcome the shortcomings of mixed integer modeling.

2.4.3.1 **Models with Continuous Variables**

These models use one or more continuous variables to model well locations. They use the idea of pseudo-well completely or to some extent. A pseudo-well is a well with negligible (or no) flow rate and its presence does not affect the reservoir's performance. Both models are compatible with reservoir simulators; a simple function can be defined to map these decision variables to the discrete location indices before introducing the proposed solution to the simulator.

2.4.3.1.1 State-based Well Models

The key decision variables in these models are the flow rate and/or the pressure difference at the well [134]. Therefore, we refer to this approach as state-based models. Flow rate is used more frequently. In this case, all feasible locations are represented by pseudo wells, which together produce/inject a fixed total field flow rate; a zero flow rate for a pseudo well location means that this candidate is not selected.

A shortcoming of this method is its efficiency in addressing the optimal number of wells. Therefore, Wang et al. [135] and Zhang et al.[136] added the following fractional approximation to the objective function to minimize the well number and simultaneously avoid using binary variables:

$$f_D(q_{,i}) = -\sum_i \left(\frac{q_{\omega,i}}{q_{\omega,i}+\epsilon}\right).C_{WD} \tag{15}$$

where C_{WD} is the well drilling cost, $q_{\omega,i}$ is well flow rate at location i and ϵ is a small enough number. When the flow rate is zero (non-zero), the drilling cost is eliminated (considered). A similar approach in water treatment context is taken by Cunha [137]. However, as pointed out by Murray and Edgar [127] and Forouzanfar et al.[138], this approach can lead to numerical difficulties because the derivative (gradient) of the objective function is insensitive to variations in flow rate. To address this problem, Forouzanfar et al. [138] proposed the following approach:

The derivative-based algorithms solve the well placement problem iteratively. Let us show each iteration by n. Therefore $q_{\omega,i}^{n}$ is the flow rate of phase ω at location i and at iteration n. The value of $q_{\omega,i}^{n-1}$ is already known at iteration n, but not that of $q_{\omega,i}^{n}$. Therefore, to have a sensitive function to variation in the flow rate, Forouzanfar et al. defined f_D as:

$$f_D\left(q_{\omega,i}^{n}\right) = -\sum_i \left(\frac{q_{\omega,i}^{n}}{q_{\omega,i}^{n-1}}\right)^{\frac{1}{4}} . C_{WD} \qquad i \in q_{\omega,i}^{n-1} \neq 0 \qquad (16)$$

Only, the differentiation of the above term is used in the algorithm. In other cases, $f_D(q_{\omega,i}^{n})$ is set to $(-C_{WD})$ if $q_{\omega,i}^{n-1} \neq 0$, otherwise it is zero. At every iteration, the flow rate is assumed to be constant for the entire time horizon G. However, this assumption can limit the scope of this approach.

2.4.3.1.2 Ring-based Well Models

Dirac delta function $\delta(x, y, z)$ is generally used to represent the well term after discretizing Eqn.(1):

$$q(x, y, z) = \delta(x_w, y_w, z_w) \times q_0 \qquad (17)$$

where

$$\delta(x, y, z) = \begin{cases} 1 & (x, y, z) = (x_w, y_w, z_w) \\ 0 & \text{otherwise} \end{cases}; \qquad (18)$$

In the ring-based models, the non-differentiable Dirac delta function, is approximated using a continuous function to recover differentiability. A chain of pseudo wells surrounding each real well is usually employed for this approximation. Therefore, we refer to this approach as ring-based well modeling. Under this, the derivative of the objective function with respect to the spatial variables is computable if the ring is directly defined as a spatial function. Moreover, it is notable that the ring-based models offer a more realistic solution. Although the state-based models share the assumption that the well flow rates stay constant over the entire time horizon G (which may not be true), the chain-based models do not require such an assumption.

In an early work, Virnovsky and Kleppe [17] used an unknown function $\eta_i(z)$ as a control variable to approximate the production of a horizontal well (i) over an active zone of $0 \leq z \leq \mathrm{Z}$. The horizontal locations of these wells were predetermined and their vertical locations were to be calculated. The vertical production spectrum of each well was represented using piece-wise constant function $\eta_i(z)$. Each piece of this piece-wise constant function served similar to a pseudo well. The vertical location of these pseudo wells determined the objective value. They finally suggested the heuristic approach of setting the vertical position of the main well at the center of mass for these production densities (i.e. pseudo wells):

$$\mathrm{z_i} = \frac{\int_0^{\mathrm{Z}} \eta_\mathrm{i}(\mathrm{z}).\mathrm{z}\, \mathrm{dz}}{\int_0^{\mathrm{Z}} \eta_\mathrm{i}(\mathrm{z})\, \mathrm{dz}} \tag{19}$$

Such an approach is equivalent to representing a well with several pseudo wells.

More recently, Zandvliet et al. [139] improved upon the idea. By approximating the Dirac delta function with a ring of pseudo wells around the main well. Ayda-Zade and Bagirov [140] and later Sarma and Chen [141] further extend the idea by placing a series of rings and using a 2D bivariate Gaussian function:

$$\lim_{\sigma\to 0} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{1}{2\sigma^2}[(x-x_w)^2 + (y-y_w)^2]\right) = \delta^2(x-x_w, y-y_w); \tag{20}$$

where σ is a parameter that determines the accuracy of the approximation. This approach approximates the well term with a flow distribution with the main well located at the peak point (i.e. at x_w, y_w). Clearly, equation (20) is differentiable with respect to location, and gradient-based optimization algorithms can be easily used.

2.4.3.2 **Algorithms**

Gradient estimation by finite differences is usually costly, because $2N + 1$ reservoir simulations are required for a 2D N-well location problem [139]. Direct gradient evaluation is also limited to cases with shortcut and proxy models [142, 143] due to the complexity of the dynamic equations. Therefore, gradient-based approaches were seldom investigated until the development of adjoint gradient evaluation techniques [133].

The adjoint technique has its roots in optimal control theory [144], which transforms a constrained problem into an unconstrained problem. Adjoint technique efficiently computes the gradient in two steps:

(1) A forward reservoir simulation is performed to evaluate the objective value and the state, output, and control variables for a given set of decision variables and input parameters. This typically involves decomposing the discretized simulation model into smaller problems along the temporal domain and solving them sequentially.

(2) A backward simulation is done to compute the Lagrangian multipliers for the complete problem, which are used to obtain the gradient.

The survey paper by Jansen [133] provides detailed discussions on adjoint gradient calculations and its theory.

To exploit the computational efficiency of adjoint gradient calculation, most recent researches in this category have utilized the optimal control framework, the essence of which is to find the best control policy that would guide a dynamic system from its initial to its final state. This matches the requirements for typical well placement problems, where the dynamic system is associated with a list of dynamic and static control variables. For example, Ayda-Zade and Bagirov [140] formulated in-fill drilling as a parametric problem for optimal control of distributed systems with concentrated sources, where the parameters and control actions are well location and well flow rates respectively. Ebadat et al.[143] also formulated the well placement problem as a tracking control problem and included seasonal changes in the market demand.

Wang et al. [135] developed a state-based water injector model by assigning injectors to each well-free grid (i) and covering all N_{inj} potential points. The total field injection rate in their method is constant (q_t):

$$\sum_{i=1}^{N_{inj}} q_{inj,i}^{n} = q_t \tag{20}$$

The injection rates $q_{inj,i}^{n}$ are constant throughout the production horizon G. They are modified via steepest ascent algorithm in each iteration (n). Inefficient wells are rejected by: (a) incorporating drilling cost into the objective function as in Eqn. (16) and (b) employing the following line search:

$$q_{inj,i}^{n+1} = q_{inj,i}^{n} + \chi_{max}^{n}.\, d_i^n; \tag{21}$$

where $dS_i^n = df/d_{inj,i}^n$ is the search direction, f is the objective function to be maximized and $\chi_{max}^{\mathrm{n}} = min(\chi_{max,i}^{n})$ is the step size. $\chi_{max,i}^{n}$ is defined as:

$$\chi_{max,i}^{n} = \begin{cases} -q_{inj,i}^{n}/dS_i^n & if\ d_i^n < 0 \\ (q_t - q_{inj,i}^{n})/dS_i^n & if\ d_i^n > 0 \end{cases} \tag{22}$$

However, this algorithm is inefficient because step (b) results in rejection of only one injector at every iteration.

Zhang et al. [136] try to overcome this difficulty by iterating between the line search and an extra gradient projection step. The projection step makes sure that after the line search shuts an injector, the active bounds and the projected search direction (dS^n) are updated so that all linear constraints (total injection) and bounds on well flow rates are satisfied at every iteration. The maximum number of wells to be screened per iteration specifies the number of inner iterations between these two steps. Together, these two steps reduce the number of injectors before proceeding to the next major iteration. Finally, a forward simulation checks the quality of the solution. This approach can explore the optimal number of wells at a convergence rate faster than the method in Wang et al. [135]. More recently, Forouzanfar et al. [138] tried to strengthen this approach by providing heuristics to approximate the value of q_t and by imposing explicit bounds (with respect to the simulator) on the bottom hole pressure.

Zandvliet et al. [139] used the ring-based approach. The pseudo wells produce/inject at a very small fraction of the main well in the center of the ring. The gradient information based on these flow rates determines the direction for shifting the central wells. The number of wells can be reduced if two potential wells merge into a same cell. The exit criterion is based on oscillation between two well configurations. Clearly, these are heuristics and cannot guarantee an improving direction, therefore the optimality. Although Zandvliet et al. [139] considered only vertical wells, Vlemmix et al. [145] attempted to locate the trajectory of a deviated well with the same idea. They assigned side-tracks to the main trajectory, acting with small flow rates. Similar to the previous work, the gradient-based on these flow rates helps shifting each segment of the well and forming a new trajectory. The updated trajectory is subject to the constraint on the dog leg severity (i.e. degrees of inclination, and/or azimuth per 100 *ft* or 30 *m* of well length [146]). Therefore, they heuristically modify the trajectory, in case it is necessary.

Although both above studies [139, 145] have used ring-based models, their objective functions are not directly dependent on explicit and continuous well location terms. In contrast, the method of Sarma and Chen [141] represent the well locations by continuous variables instead of integer variables, hence the objective function directly depends on well locations. The geometrical index of each pseudo-well on the ring is a fraction of the index of the central well. The scaling factor is computed based on the well-to-pseudo well distance using the bivariate Gaussian function. Since, the pseudo-wells are distributed everywhere, there is no limit on the search directions and the step size. In contrast, the search direction and the step size in the work of Zandvliet et al. [139] are limited to the eight surrounding directions and to one grid per iteration, respectively.

Gradient-based methods are famous for monotonically improving the objective function and offering local solutions. However, it is notable that these may not be the same here. A part of algorithms that employ the ring-based models [139, 145, 147], cannot guarantee monotonic improvement. Moreover, the final solution strongly depends on the initial guess and is either local solution or a good solution. This method is faster than the evolutionary and direct search approach. However, it demands more data from the reservoir simulator. If the simulator does not provide the adjoint information, this method requires sophisticated preparations and programming.

2.5 Tools

Perhaps reservoir simulator software packages are the most common tool-boxes of the majority of above well placement techniques. These simulators have provided the required data for the optimization algorithms, either directly or through proxies. They also take care of different operational constraints such as maximum water injection flow-rates. Furthermore, they are the main tool of validating the optimization results. Some of the most famous packages include CMG[46], ECLIPSE[44], VIP, KAPPA[148], BOSS [149], 3DSL [89], Chevron Texaco's CHEARS [55]. We briefly comment on the software packages that are more accessible to academia.

Three main industrial and numerical software packages are CMG[46] , ECLIPSE[44] and VIP [150]. In the academic sector, Stanford University has also developed GPRS (Stanford's General Purpose Research Simulator) that is both black oil and compositional simulators. Here we discuss the first two simulators (CMG and ECLIPSE) which have been actively used in academic researches worldwide. Both simulators have three phase black oil packages (IMEX/CMG and E100/ECLIPSE), full compositional reservoir simulators (GEM/CMG and E300/ECLIPSE) and optimization modules (CMOST/CMG and PlanOpt/ECLIPSE). Therefore, both can cover a diverse range of upstream operations and analysis. CMG uses CMOST to perform the sensitivity analysis, history matching, uncertainty assessment and optimization [46]. It uses particle swarm optimization, random search and brute force search. On the other hand, optimization tool-box of ECLIPSE uses adjoint gradient and line search method (steepest descent and conjugate gradient). The adjoint gradient option is available for E300. The reservoir model linked with this optimizer should be fully implicit. The "PlanOpt" toolbox of ECLIPSE finds the optimal vertical well location using predefined screening criteria and simulation [44].

Both CMG and ECLIPSE can be connected to other geological, downstream, uncertainty assessment and economical software packages for detailed and integrated field development studies. PETREL, Roxar RMS, JOA JewelSuite, Earth vision and MEPO are some of the geological software packages. PipeSim, GAP and FORGAS, are downstream software. EnABLE by ROXAR is statistical uncertainty analysis and assisted history matching tool that can be used for examining multiple development scenarios. COUGAR is another reservoir's uncertainty analysis software, which produces response surfaces by using the experiment design techniques [151]. Finally, PEEP is an economic package. Connecting these packages enable us to do an integrated study of the field [152].

On the optimization side, very few contributions have used general-purpose optimization packages to solve the well placement problem as a mathematical and technical problem. GAMS [153] is the most important such software. It solves optimization problems by mathematical programming algorithms. To solve the MILP and MINLP models, a user need access to MILP and NLP solvers (and MINLP). CPLEX package, available in GAMS, is already used to address the MILP well placement problem [154, 155]. The current state-of-art MILP solver is GUROBI, which has also shown signs of good performance in this problem[156]. Similarly, we have successfully experienced solving very large size NLP problems with IPOPT[157] in an acceptable time. The solution of GAMS to nonlinear programming problems (NLP) is more accurate with less iterations compared to other packages that use finite difference methods. Using these powerful tools, a user can implement any variation of different decomposition methods and heuristics in GAMS. The interested reader is advised to refer [155, 158] for a general discussion on available algorithms and computer codes for MINLP. Since GAMS can interface with MATLAB through gdxmrw tool box [157], it can also interface with previously mentioned reservoir simulators, and geological and economical software packages to exchange data.

2.6 **Uncertainty**

Changes in oil basket price of OPEC from 12.28 $ in 1998 to 94.45 $ in 2008 and even higher afterwards clearly shows the uncertainty in the market and risk of economic investment in oil development projects [159]. Additionally, the inherently limited, expensive, and difficult nature of reservoir sampling and analyses introduce significant uncertainty in data. In fact, an exact description is nearly impossible at the beginning of the field life and may be achieved only after the reservoir has been fully depleted. Even then, significant uncertainties [26] may exist. While the uncertainty can be reduced by drilling appraisal wells [160] at the beginning and using the production data of drilled wells later for history matching, the fact remains that it is difficult to fully understand and accurately model / describe a reservoir. Thus, a typical well placement problem has many uncertain parameters, especially in the geological realizations. However, because the nominal problem without uncertainties is already difficult to handle, very few results have been reported to handle such uncertainties using mathematical programming [161, 162]. To the best of our knowledge, there is also no reported work using gradient related methods. Nevertheless, recent advances in general robust optimization techniques [163, 164] mean that if the nominal problem without uncertainties can be handled, then the uncertainties can be handled with some more effort.

Comparatively, there has been more work on handling uncertainties using the evolutionary and direct searches. This is usually done by simulating production of a well configuration on all (or most of) probable geological realizations to evaluate the expected value of the corresponding objective function. A risk aversion parameter can also be used to reflect the perspective of the decision maker [53]. Güyagüler and Horne [72] took such an approach using the utility framework. However, such approaches are computationally cumbersome [165] and are intractable as the number of required simulations becomes too large. Consequently, only small size reservoirs can be handled.

Different researchers have tried to balance between reliable uncertainty handling and reducing computational load. Ozdogan and Horne [74], included time-dependent uncertainties (or pseudo-history), and Morales et al.[73] applied probabilities of success.

More recently Wang et al. [166] considered a retrospective optimization framework. That starts with one possible realization, and adds more scenarios in the optimization process. Therefore, all possible realizations are considered as the procedure converges to the optimum. Cluster-based sampling can also be employed after characterizing all possible realizations based on their static and (updated) dynamic attributes. This approach can reduce the number of required simulations.

Because the subject of uncertainties is not within the main scope of this paper, we have included only a short introduction. A brief discussion can be found in reference[42].

2.7 Discussion and Research Opportunities

Table 2-1 provides a concise summary of the present survey. It compares the three different methods discussed earlier. For simplicity, we refer to these methods as (1) parameterized well models solved by evolutionary (and direct search) methods, (2) MILP and MINLP models solved by mathematical programming, and finally (3) continuous models solved by gradient-based search techniques.

As Table 2-1 shows, the first set of methods is not affected by non-convexity and nonlinearity, provided that enough computational resources are available. Moreover, considering the simplicity of vector parameterization and the availability of reservoir simulators and general global optimization toolboxes, their implementation is straightforward. Therefore, they are very versatile, and they have attracted most publications on well placement. While their key disadvantage is their huge computational cost, they can at least give some solution from the effort for large problems. The other important key disadvantage is their inability to implicitly honor the geometrical and logical constraints such as feasible well configuration, which can add to computational cost.

Table 2-1: Characteristics of algorithms used to address well placement problem.

Algorithm	Evolutionary and Direct Search	Mathematical Programming	Gradient-based Search
Well representation	Mixed vector parameterization	Mixed integer	Continuous variables
Examples of algorithms	GA, PSO, SA[a], HS[b]	Branch and bound	Steepest descent
Parallel Computing	Compatible	Possible	Useful for multi-start strategy
Number of Simulations	High	Low	Low
Successive improvement in objective	No guarantee	Yes	In principle, yes
Gradient information	Not required	Required	Required
Stopping criteria	Heuristic	Theoretical and/or Heuristic	Theoretical and/or Heuristic
Suitability for large problems	Good	Fair	Good
Uncertainty	Work exists	No work for subsurface issues	No work so far
Handling of geometrical constraints	Weak: Externally	Strong: Internally	Weak: Externally
Intermediate infeasible solutions	High	Least/Impossible	Low
Simplicity of implementation	Easy	Difficult	Average

[a] Simulated Annealing [27]
[b] Harmony Search [63]

The second set of methods is the first approach for well placement that appeared in the literature [124]. However, they are not currently common in field applications. From the viewpoint of a petroleum engineering team, their main deficiency is the approximations used by the early contributions in modeling the nonlinear subsurface flows. They are problem-specific and they can be complex. Cullick et al. [167] argue this to be the main reason why application of the second method is limited in practice. However, with the progress of computational tools and advanced algorithms, ability to handle geometrical and logical constraints, and advanced and efficient search algorithms, mathematical programming is attracting more attention and application in many fields. Thus, they hold much promise for well placement as well.

The third set of methods represents the most recent approach. The idea of state-based modeling is similar to the role of Eqn. (13) in MILP and MINLP models, where unpromising pseudo wells are rejected by pushing their flow rate to zero. However, the assumption of constant flow rate of each well for the entire planning horizon G in the state-based models limits their applicability. On the other hand, using the Dirac delta function in ring-based well models seems promising. However, none of these ideas are adequately tested for complicated multilateral and deviated wells. The only known contribution is the initial study by Vlemmix et al. [145]. In contrast to well models in the first two methods, both state and ring-based models are differentiable. Therefore, the gradient-based algorithms that solve them are much faster than the first two sets of methods. The use of reservoir simulator allows them to take care of the nonlinearity and complex trade-off with no loss of fidelity. However, they are local optimizers, and the quality of the final solution strongly depends on the initialization step. Finally, they require special explicit handling of logical and geometrical constraints.

In the view of these comments, we identify the below opportunities for further computational research on well placement:

1) It is vital to evaluate the existing academic studies by assessing the technical constraints in more details and removing simplifying assumptions. For instance, consider perforation location on multilateral wells. The available literature simply assumes that the laterals are completely perforated, which may not be realistic. Additionally, any vertical or horizontal well can be perforated at different segments to connect small trapped zones of hydrocarbon. Ierapetritou et al. [130] investigated vertical wells penetrating different geo-objects. In case of a

sandwiched pay zone between gas and water zones [168], the majority of studies has focused only on linear trajectory and horizontal location of the main trunk. However, wells in reality may be drilled in a snake shape. Therefore, the design and configuration of complex wells in non-uniform formations remains an open research area. That will give a strong boost to the field application of academic research.

2) Perhaps, the biggest practical challenge is to address the uncertainty in model parameters and data. At this time, few contributions except those using evolutionary methods have included uncertainty in the context of subsurface issues. While considering uncertainty with evolutionary methods is computationally expensive; their suitability for parallel computing offers much promise.

3) Deriving better and simpler analytical approximations (e.g. reduced order models [94-98]) for the hydrodynamic equations governing the reservoir behavior in unsteady state mode is a key area of research [169]. This is already being done in other multi-physics fields such as fuel cell modeling [170, 171]. It can certainly make the gradient-based and mathematical programming methods much more powerful.

4) Consideration of critical and practical aspects such as uncertainty and risk assessment, design and configuration of complex wells, determination of drilling sequence [5, 27, 56, 116], allocation of well-to-surface facilities, and economic analyses is certainly essential and needs much attention. However, that will undoubtedly complicate and enlarge optimization models [172] , so the development of novel algorithms and efficient solution approaches for well placement will be must.

5) To better serve the needs of the industry, it is necessary to develop a user friendly and integrated analyzer [70] that would provide a variety of solver options available in the literature. No comparative study exists on the available toolboxes for well placement in commercial software packages such as PlanOpt in ECLIPSE, and their comparison with academic methods. Such a study will be very useful.

6) The hydrocarbon fields are getting older and oil price is rising. Consequently, EOR methods are becoming more important (Refer [173] for a review on mature fields and to [174, 175] for EOR processes). There is a need to consider the specific demands and impact of EOR processes. To this end, the models and methods need to go beyond just water injection scenarios to fluid injections. Furthermore, one needs to develop methods for determining EOR injection sites, selecting the EOR agent types and scheduling their injection [168, 176-179].

7) HSE (Health, Safety, and the Environment) has always been a real concern for drilling activities. Incidents such as Piper Alpha disaster in 1988 [180] and oil spill in Gulf of Mexico in 2010 [181] have highlighted their importance even further. It is important to define more holistic merits of comparison for different well drilling scenarios to accommodate all concerns regarding long-term environmental and techno-economic issues such as drilling safety, drilling rig abandonment, and well commissioning; especially for offshore fields and infrastructures.

8) The methodologies for addressing well placement have much wider applications to other fields such as underground water well placement [182, 183], windmill placement, and fin placement in heating/cooling problems.

2.8 Selected methodology and research focus

Well placement problem involves numerous logical, structural, operational and economic constraints. From above discussion, it is clear that evolutionary methods and gradient based searches are unconstrained optimization approaches and handling discrete constraints remains computationally costly for them. They produce many infeasible configurations, which deteriorates their performance [59], and require external intervention to recover from infeasibility [70]. However, mathematical programming is a *constrained* optimization approach and therefore can efficiently handle such constraints. Moreover, this approach is flexible and versatile; it can embed the reservoir physics inside the optimization model to benefit from its mathematical structure. This allows one to include the production/injection profiles along with the location decisions in the model and improve computational speed. The improvements of computational software and hardware have significantly extended flexibilities of mathematical programming studies. The changes in the trend of such studies from LPs/MILPs to MINLPs can confirm that. Thus, mathematical programming approach is chosen as the main methodology of this research. For achieving the best performance, some of the limitations, shortcomings and gaps of current studies based on this technique should be addressed. In particular, the following topics are addressed in this dissertation:

1) The strategy should go beyond the black box approaches, and employ a rigorous subsurface fluid flow model. So that the final solution is accurate and comparable with the solution from industrial standard reservoir simulators. That is addressed in Chapter 3 and tested in Chapter 4.
2) This rigorous subsurface model should be embodied in a novel holistic model that integrates both subsurface and surface elements to address well and infrastructure placement, installation and allocation as well as production planning. That is presented in Chapter 4 and Chapter 5 .

3) The above integrated model should be extended to optimally determine, the time and order of drillings and installations. That is tackled in Chapter 5.
4) An efficient and flexible algorithm should be tailored to effectively solve each of these integrated models. Chapters 4,5 and 6 discuss that.

2.9 Summary

Wells demand significant investment in any hydrocarbon production plan and may constitute up to 60% or more of the capital expenditure (CAPEX). This high financial burden has motivated a range of operational research studies on optimal well placement. In parallel, the recent advances in high performance computers have given the engineers more sophisticated analysis tools. Despite all these advances and interests, there is a lack of detailed review of existing important contributions on the specific problem of well placement. By providing a comprehensive literature survey, this article has made a significant contribution.

Nonlinear PDEs, problem size, and model uncertainty are the three main challenges in the well placement problem. In this chapter, our focus was largely on the first two challenges. We did not address uncertainty and risk assessment in detail. Three approaches have been commonly used to solve optimal well placement problems, of which evolutionary and direct search are the most popular and simplest. These features have allowed them to address all three challenges reasonably well, although with possibly high computational effort. They are the best at handling nonlinearity, non-convexity, and large problems. They have been the focus of most publications in this field. The gradient-based search approaches are new and novel with very limited work. They need to be further investigated, tested, and improved. With the progress in computing hardware and general-purpose optimization tools, mathematical programming methods offer the most promise due to their versatility and theoretical foundations. Although they were the first to be used on this problem, their application has lagged behind in practice and much work is needed. They offer versatility in formulating complex problems with a variety of real physical constraints, but cannot solve truly large problems. Their application has also been limited due to their complexity, and the representation by the early works of multiphase flow in the reservoir needs much to be desired. Clearly, finding an analytical solution for or an accurate approximation to the main conservation equations can significantly reduce the computational cost in all three methods.

The future work on this problem will be directed on offshore drilling, infill drilling for both EOR and routine production, complex well design, and uncertainty handling. The current industrial situation points to a shift from on-shore to offshore well placement. Although the exploration sector has been very successful during the past few years, the chance of exploring new fields is decreasing. Hence, the focus is shifting to complex well designs and in-fill drilling for both production and EOR purposes considering surface facility and processing constraints.

Our survey and discussion are clearly not exhaustive; they are probably the first to focus on systematic optimization techniques covering most aspects of this problem. Lots of algorithmic, computational, modeling and data gathering challenges remain unaddressed as described in our discussion section. Considering the major challenge of energy facing the world, these require attention from both the research and industrial community. Extensive interaction between the two will be essential for attaining fruitful results and tools.

From the above study, mathematical programming was chosen as the main framework for the current study. The research focus and gaps were also discussed.

CHAPTER 3 MODELS FOR FLUID FLOW [7]

3.1 Introduction

Statistical methods, decline curve analysis, material balance, multiplication method of partial efficiencies, stream tube model and numerical modeling are the main methods for simulating, forecasting and analyzing the oil and gas recovery processes. Refer to [184] and [25] for discussion on each method. As can be seen in Table 3-1 [184] numerical modeling offers the most versatile option. In contrast to others, numerical models are capable of studying the effects of the following factors on the process recovery: well distance, heterogeneity, production rate, conning, gravitational flow, cross flow, and most importantly enhanced oil/gas recovery technologies. These factors can have substantial effects on the final decisions for well placement and reservoir development. Commercial reservoir simulators are the main tools that utilize these functionalities. The researchers and the engineering teams usually use them in a black box manner, and the contents of this box remain intact, hidden and untouched. However, opening this black box and embedding its governing spatiotemporal equations inside an optimization model enables the researchers to provide quicker and better guidance to the optimization engine. The powerful and versatile technique of mathematical programming offers significant potential and promise for this. The first step is preparing a rigorous model for this purpose; hence we discretize the governing spatio-temporal relations introduced in the last chapter, i.e. eqs. (1)-(7) of Chapter 2.

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[7] Tavallali, and Karimi (2011). Optimal well placement using dynamic mathematical programming. Presented in 73rd European Association of Geoscientists and Engineers Conference and Exhibition 2011 - Incorporating SPE EUROPEC 2011, Austria - Vienna.

Table 3-1 : Methods of Determination of Recovery Process [184]

Parameter	Methods of Calculation					
	Statistical Methods	Decline Curve Analysis	Material Balance	Multiplication method of Partial Efficiencies	Stream Tube Model	Numerical Modelling
Recovery factor	+	-	+	+	+	+
Resources	+	+	+	+	+	+
Production over time	-	+	+	+	+	+
Displacement mechanism	-	-	+	-	-	+?
Effect of distance of the wells	-	+?	-		-	+
Effect of heterogeneity	-	-	-	+?	+?	+
Effect of production rate	-	-	+	-	-	+
Conning	-	-	-	-	-	+
Effect of gravitational flow (cross flow)	-	-	+?	+?	-	+
EOR and EGR technologies	-	-	+?	+?	+?	+

Explanation: (+) Can be determined
(-) Cannot be determined
(+?) Can be determined approximately
Refer to [184] for discussion on each method.

This chapter is organized as follows. First, our approach for building a spatio-temporal discrete model (based on the governing PDE equations of multiphase fluid flow inside the reservoir) is presented. Then, pressure drop due to fluid flow (specifically multiphase flow) inside the wells, flow lines and pipes are discussed and the data generation scheme for building empirical models for these pressure drops are explained.

3.2 Model for reservoir dynamics

In this study, we address two dimensional, under-saturated reservoirs with oil and water (and negligible capillary pressure). Many practical problems can be modeled using 2D fluid flow equations. Some practical examples include: thin reservoirs of large areal extent, reservoirs with narrow or approximately uniform pay zone, cross sectional problems, well testing models for single wells. In such cases the 2D modeling offers very good approximations for 3D models, provided that the variation of thickness is not large [43]. Moreover, 2D models can provide the foundations for studying 3D problems.

As it was discussed in the previous chapter, a set of coupled spatiotemporal differential equations [43] represent the dynamic behavior of two phase fluid flow inside the reservoir. Equation (1) of Chapter 2 shows this dynamic behavior and different reservoir simulators such as ECLIPSE [44], CMG [46], etc. solve that accurately. As discussed earlier, previous studies based on evolutionary and gradient-based techniques have employed these simulators as black box models. In contrary, in our approach based on mathematical programming, we capture the detailed dynamics of the reservoir via analytical algebraic equations in our optimization formulation.

To the above end, we can convert Eq. (1) of Chapter 2 into a set of algebraic equations using backward finite difference approximation in a Cartesian system. Here we assume that (a) the reservoir is a horizontal 2D plane and therefore there is no flow in z-direction, (b) capillary pressure is negligible (c) both water and oil phases are compressible.

Consider eqs (4) and (5) of Chapter 2. Negligible capillary pressure results in same oil and water pressure i.e. $P_o = P_w$; additionally the water saturation is $S_w = 1 - S_o$. We can replace these two equations in Eq. (1) to (3) of chapter 2 and their variables change to oil pressure and water saturation. In the remaining of this thesis we refer to them as P and S respectively.

Considering the above descriptions we discretize Eq. (1) to (3) of Chapter 2 along both spatial and temporal dimensions for a single rectangular reservoir; we discretize the x-dimension into I elements of lengths Δx_i, $i = 1, 2, \dots, I$; y-dimension into J elements of lengths Δy_i, $j = 1, 2, \dots, J$; and the planning horizon H into T intervals of arbitrary lengths Δh^t, $t = 1, 2, \dots, T$. All elements and intervals have arbitrary lengths. This decomposes the entire reservoir into ($N = I \times J$) grids. Instead of dealing with two independent indices (i and j), it is customary to order the grids into a single array of N grids ($n = 1, 2, \dots, N$) with volumes V_n, where $n = i + (j-1) \times I$ for $i = 1, 2, \dots, I$; $j = 1, 2, \dots, J$). For a fully interior (i.e. not on any boundary of the 2D reservoir) grid n, grid $(n-1)$ proceeds and $(n+1)$ follows grid n in the x-direction; while $(n-I)$ proceeds and $(n+I)$ follows grid n in the y-direction.

If P_n^t and S_n^t denote the pressure and water saturation respectively in grid n during interval t, then the following equations [43] represent the implicit discretized version of eq. 1 to 3 of Chapter 2 using backward finite difference approximation:

$$\left(\frac{V_n}{dh^t}\right) \cdot \left\{d_{f,1,n}^t \left[P_n^t - P_n^{t-1}\right] + d_{f,2,n}^t . \left[S_n^t - S_n^{t-1}\right]\right\} + q_{f,n}^t +$$

$$\begin{pmatrix} \left\{T_{n-1}^x . M_{f,x-}^t . \left[P_n^t - P_{n-1}^t\right]\right\}_{(n-1)\in \mathbf{IX}} + \left\{T_n^x . M_{f,x+}^t . \left[P_n^t - P_{n+1}^t\right]\right\}_{n\in \mathbf{IX}} \\ + \left\{T_{n-I}^y . M_{f,y-}^t . \left[P_n^t - P_{n-I}^t\right]\right\}_{(n-I)\in \boldsymbol{IY}} + \left\{T_n^y . M_{f,y+}^t . \left[P_n^t - P_{n+I}^t\right]\right\}_{n\in \boldsymbol{IY}} \end{pmatrix} = 0; \quad (1)$$

$$M_{o,n}^t = kr_0^o \left(\frac{1 - S_n^t - S_{or}}{1 - S_{wr} - S_{or}}\right)^a / (\mu_{Bo,1} P_n^t + \mu_{Bo,2}) \quad (2)$$

$$M_{w,n}^t = kr_0^w \left(\frac{S_n^t - S_{wr}}{1 - S_{wr} - S_{or}}\right)^b / B_{wr}^0 \mu_w \quad (3)$$

$q_{f,n}^t$ is the well flow rate, $d_{f,1,n}^t$ and $d_{f,2,n}^t$ are the variable accumulation multipliers (see sections 3.2.1), $M_{f,n}^t$ is the phase mobility, $M_{f,x+}^t$, $M_{f,x-}^t$, $M_{f,y+}^t$ and $M_{f,y-}^t$ are upstream weighted mobilities, which are taken as those of the neighbouring cells with higher pressures, (T_n^x , T_n^y) are the transmissibility in the x-direction and y-direction respectively (see section 3.2.2); they are defined in the following sections. In the above equation, **IX** and **IY** are two subsets of the temporal domain, and are defined as:

$\mathbf{IX} = \{n \mid n \text{ is an interior grid with } i < I\}$; $\mathbf{IY} = \{n \mid n \text{ is an interior grid with } j < J\}$

A bracketed term that does not satisfy its validity condition (e.g. $n \in \mathbf{IX}$) is omitted from Eq. (1). This equation uses the simultaneous solution method [43]. The initial conditions are provided through the initial pressure and saturation maps; no-flow boundary conditions are used for cells at the boundary and the borehole flow or pressure settings determine the boundary condition for the well hosting cell.

The first two terms in Eq. (1) are accumulation terms and the last four terms are convective flows along the four faces of grid n. The following two sections describe various equations and relations that define these terms:

3.2.1 The variable accumulation multipliers

Accumulation multipliers are defined [43] as:

$$d_{o,1,n}^t = (1 - S_n^{t-1})(\varepsilon_n^{t-1} \cdot {b'}_{o,n}^{t-1} + {\varepsilon'}_n^{t-1} \cdot b_{o,n}^t) \quad (4)$$

$$d_{o,2,n}^t = -\varepsilon_n^t \cdot b_{o,n}^t \quad (5)$$

$$d_{w,1,n}^t = S_n^{t-1}\left(\varepsilon_n^{t-1} \cdot {b'}_{w,n}^{t-1} + {\varepsilon'}_n^{t-1} \cdot b_{w,n}^t\right) \quad (6)$$

$$d_{w,2,n}^t = \varepsilon_n^t \cdot b_{w,n}^t - \varepsilon_n^{t-1} \cdot S_n^{t-1} \cdot b_{w,n}^{t-1} \cdot P_c' \quad (7)$$

where, ε is the porosity, $'$ refers to differentiation, P_c is the capillary pressure, b_w and b_o are the inverse of the formation volume factor of phase water and oil phases (i.e. $b_w = 1/B_w$ and $b_o = 1/B_o$). The pressure dependencies of B_w and ε are modeled using Eq. (3.157) of [185] and Eq. (2.41) of [43] respectively:

$$B_{w,n}^t = \frac{B_{wr}}{1 + Y_n^t + 0.5{Y_n^t}^2} \quad (8)$$

$$\varepsilon_n^t = \varepsilon_0(1 + c_R[P_n^t - P_r^R]) \tag{9}$$

$$Y_n^t = (c_w - c_v)(P_n^t - P_r^w) \tag{10}$$

where, c_v, c_w, and B_{wr} respectively are the viscosibility, compressibility, and formation volume factor (B_w) at reference pressure P_r^W for water, c_R is the rock compressibility, and ε_0 is the porosity at the reservoir reference pressure P_r^R. Using (a) quadratic regression for $\varepsilon_n^t \cdot b_{o,n}^t$ term (based on P_n^t) and subsequent differentiation with respect to pressure in order to approximate the second parenthesis in Eq. 4, (b) linear regression for 5, (c) substituting Eqs. 8 and 9 into 6 and 7, and (d) ignoring the negligible terms [43] and P_c', we get:

$$d_{o,1,n}^t = (1 - S_n^{t-1})(\alpha_{o,11} \cdot P_n^t + \alpha_{o,12}) \tag{11}$$

$$d_{o,2,n}^t = -\left(\alpha_{o,21} \cdot {P_n^t}^2 + \alpha_{o,22} P_n^t + \alpha_{o,23}\right) \tag{12}$$

$$d_{w,1,n}^t = S_n^{t-1}(c_R + c_w)\varepsilon_0 / B_{wr}^0 \tag{13}$$

$$d_{w,2,n}^t = P_n^t(c_R + c_w)\varepsilon_0 / B_{wr}^0 + (1 - c_R P_r^R - c_w P_r^W)\varepsilon_0 / B_{wr}^0 \tag{14}$$

where, $\alpha_{o,11}$, $\alpha_{o,12}$, $\alpha_{o,21}$, and $\alpha_{o,22}$ are regression parameters. The above four terms are directly embedded in Eqn. (1).

3.2.2 The convective flow terms

We use the following definitions of transmissibility. They are similar to the definition of technical manual of ECLIPSE [44], which are slightly different from those used by Aziz and Settari [43].

$$T_n^x = 2cA_n^x / \left(\frac{\Delta x_n}{K_{x,n}} + \frac{\Delta x_{n+1}}{K_{x,n+1}}\right) \qquad n \in \mathbf{IX} \tag{15}$$

$$T_n^y = 2cA_n^y / \left(\frac{\Delta y_n}{K_{y,n}} + \frac{\Delta y_{n+I}}{K_{y,n+I}}\right) \qquad n \in \mathbf{IY} \tag{16}$$

where $K_{x,n}$ and $K_{y,n}$ are the permeabilities in the x and y-directions respectively, c is a coefficient, S_{or} and S_{wr} are the residual oil and water saturations respectively, kr_0^o and kr_0^w are the end-point relative permeabilities for oil and water respectively, a and b are the exponents in Corey's correlation, μ_w is the water viscosity, B_{wr}^0 is the formation volume factor for water at reference pressure P_r, and finally $(\mu_{Bo,1}, \mu_{Bo,2})$ are regression parameters for the product $(\mu_o \times B_o)$ of the viscosity and formation volume factor of oil.

In order to well-represent the direction of convective flow in equation 1 of Chapter 2, we assume and use the upstream-weighting of the mobility (Aziz and Settari, 1979). To model the convective flow between two adjacent cells, this approach chooses the mobility term of the higher pressure cell. Therefore using the pressure map at time $(t-1)$ and defined these upstream weighted mobilies using following eqs. (17)-(20):

$$M_{f,x+}^t = \begin{cases} M_{f,n}^t & P_n^{t-1} \geq P_{n+1}^{t-1} \\ M_{f,n+1}^t & P_n^{t-1} < P_{n+1}^{t-1} \end{cases} \qquad n \in \mathbf{IX} \tag{17}$$

$$M_{f,x-}^t = \begin{cases} M_{f,n}^t & P_n^{t-1} \geq P_{n-1}^{t-1} \\ M_{f,n-1}^t & P_n^{t-1} < P_{n-1}^{t-1} \end{cases} \qquad (n-1) \in \mathbf{IX} \tag{18}$$

$$M_{f,y+}^t = \begin{cases} M_{f,n}^t & P_n^{t-1} \geq P_{n+I}^{t-1} \\ M_{f,n+I}^t & P_n^{t-1} < P_{n+I}^{t-1} \end{cases} \qquad n \in \mathbf{IY} \tag{19}$$

$$M_{f,y-}^t = \begin{cases} M_{f,n}^t & P_n^{t-1} \geq P_{n-I}^{t-1} \\ M_{f,n-I}^t & P_n^{t-1} < P_{n-I}^{t-1} \end{cases} \qquad (n-\mathrm{I}) \in \mathbf{IY} \tag{20}$$

This necessitates that we solve the above equations sequentially one-period at a time. Thus, while solving for period t, the weights will be set based on the solution for period $t-1$. Eqs. (2) and (3) show the pressure and saturation dependencies of the multiphase flow. These dependencies are captured by Corey's correlation for saturation [25], and regression based correlations for pressure (of oil phase).

In the next chapter, we will describe an adaptive stability check for the above discretization approach and verify the accuracy of the discretization by comparing its solution with ECLIPSE [44] as an industrial standard reservoir simulator.

3.3 Models for flow in pipes

The relationship between flow rate into the well and flowing pressure are shown through the Inflow Performance Relationship (IPR):

$$q_{f,n}^{t} = \Psi_n \cdot M_{f,n}^{t}(P_n^t - BHP_n^t) \tag{21}$$

Here, Ψ_n is the connection transmissibility factor for wells (see eq. 76.5 of [44]), and BHP_n^t is the bottom hole pressure of the well drilled at cell n. For an injector well at grid n, the procedure is similar except for one key difference arising due to the direction of flow. Moreover, in the absence of cross flow at an injector, it is a standard practice to combine water mobility with oil's relative permeability and viscosity [44]. This is for example to prevent injection blockage, when $S_n^t = S_{wr}$ [186]. Therefore, it is customary to add a term $M_{cw,n}^t$ for an injector and we introduce that as:

$$q_{w,n}^{t} = \Psi_n \cdot (M_{w,n}^{t} + M_{cw,n}^{t}) \cdot (P_n^t - BHP_n^t) \tag{22}$$

$$M_{cw,n}^{t} = (kr_0^o \left(\frac{1-S_n^t-S_{or}}{1-S_{wr}-S_{or}}\right)^a / [\mu_{o,1}P_n^t + \mu_{o,2}]) \times \frac{1}{B_{wr}^o} \tag{23}$$

where, $\mu_{o,1}$ and $\mu_{o,2}$ are regression parameters for oil viscosity.

The pressure-flow relation of the flow from the bottom hole to the top side of tubing are modeled via following Vertical Flow Performance (VFP) relationship:

$$BHP_n^t = THP_n^t + \rho_n^t g L_n + \Delta P_n^t + \Delta p_n^t + dp_n^t \tag{24}$$

THP_n^t is the pressure of the same well string before the choke valve and near the surface, ΔP_n^t is the frictional pressure drop through the well tubing and Δp_n^t is the pressure drop due to acceleration, dp_n^t is the pressure drop through the valves, L_n is the well depth, and ρ_n^t is the density of the fluid. Preparing the VFP eqn for single phase flow (e.g. water injector) is straight forward; however, that is very complicated for multiphase flow happening at the producer wells. That requires special attention.

Pressure drop in a multiphase flow is a complex function of liquid flow rate (or oil flow rate), water-oil ratio (or water flow rate), gas-oil ratio (or gas flow rate) and absolute pressure levels. Several models exist for capturing these dependencies. Homogenous flow models ignore flow pattern and are the simplest. Homogenous models that consider phase slip are called drift flux models [187-189]. Empirical methods such as those of Beggs and Brills [190] and Mukherjee and Brill [191], and the mechanistic model of Petalas and Aziz model [192], predict flow patterns and consider phase slip.Accurate predictions for pressure drops involve highly complex and nonlinear expressions. Moreover, in majority of cases, these models are discontinuous. Brill [193] provides an excellent monograph on this topic. In this study, we represented above pressure drop functions via empirical and regression models, which are regressed from the data generated by the VFPi package of ECLIPSE [194].

The following section describes the data generation proceadure using VFPi.

3.3.1 Pressure drop data

VFPi software [194] was used to generate the pressure drop data for numerous pipe lengths and fluid flow regimes. This software works as a characterization and pre-processing tool for ECLIPSE [44] and VIP [150] reservoir simulators and is equipped with several multiphase flow correlations, and mechanistic models. Following tasks are performed to ensure the accuracy of the generated data before doing the regression:

1) Consider the upward flow with inlet pressure of BHP and outlet pressure of THP. Figure 3-1 is generated with VFPi software and shows the BHP variations with flow rate for two fixed THPs. In order to increase the upward flow rate under a fixed THP, normally BHP should increase. However, as Figure 3-1 depicts, in some low flow rates, an unstable zone is created where the reverse happens. As can be seen, the BHP vs. liquid flow rate forms a J-shaped function in which the sections with negative (positive) derivative indicates the instability (stability) and the production should be in the stable zone. Usually more

number of data points should be employed to model the unstable region. VFPi provides a convenient tool for identifying this zone before generating the regression data.

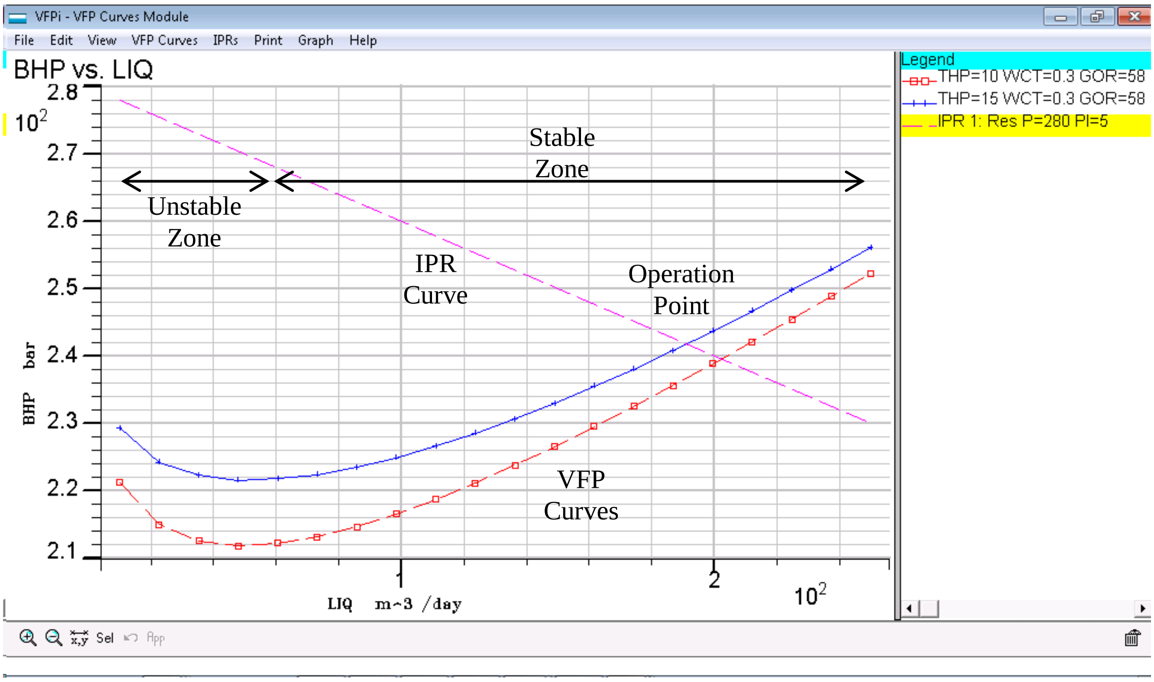


Figure 3-1: Well performance analysis for identifying the unstable zone and the operating conditions of a producer well.

2) Additionally, the operating point (q_n^t, BHP_n^t) of a well is where the VFP curve of Eqn. (7) intersects with the IPR curve of Eqn. (21). That is also shown in Figure 3-1. If these two curves do not intersect for a nominal condition, their corresponding well might not be a good candidate and that can be used as a pre-screening criterion.
3) The choked flow conditions and negative values in the pressure traverses should be identified and screened after the pressure drop calculations. VFPi tags these conditions by assigning $\pm$1E10 values to the final BHP/THP pressure report.
4) Pressure drop calculations should be specific to each pipe according to its length and properties; hence numerous number of VFP calculations for a well placement study should be done. To reduce the simulation load, a representative range of pipe lengths $(L_{min} : L_{max})$ is divided into NL elements of lengths ℓ_{nL} , $nL = 1, 2, \ldots, NL$, such that

$L_{min} \leq \ell_1 < \cdots < \ell_{nL} < \cdots < \ell_{NL} = L_{max}$. Then NL number of VFP calculations are performed with pipe lengths of ℓ_{nL}. The final regression function for a well length of L_n is done using the data related to nL element.

5) VFPi package works under two modes of interactive and batch modes. The former requires the interactive user input, whereas the latter employs the command script files (CMD) to perform a series of tasks without any interruption. CMDs can be loaded with instruction for performing numerous VFP calculations in a generic form. That is specially an important task for the studies in Chapter 5 and 6. CMD files included the following tasks and information for each individual wells: (a) description of pipe geometry (b) instructions and inputs for performing VFP calculation, and finally (c) instructions for labelling the generated VFP curve and saving it into a VFP text file. After playback of this CMD file, individualized VFP tables for each string is automatically generated. A MATLAB code is prepared to read the text files and gather the required data for regression. The final empirical models are represented in each chapter.

For all above calculation we used Aziz and Petalas [192] mechanistic model and loose emulsion to generate the data. The regressions are done using nlinfit function of MATLAB.

3.4 Summary

In this Chapter we prepared a spatiotemporal, discrete and dynamic model to rigorously consider the reservoir dynamics in the MINLP models that will be developed in the next three chapters. Furthermore, the fluid flow (and the pressure traverse) from the reservoir into the well, and up to the tubing head was discussed. The procedure for building an accurate regressed vertical flow performance relationship was also described.

CHAPTER 4 OPTIMAL PRODUCER WELL PLACEMENT AND PRODUCTION PLANNING IN AN OIL RESERVOIR [8]

4.1 Introduction

The continuous depletion of oil reserves and rise in global oil demand have created a challenge for the oil exploration and production (E&P) industry. In 2011, the global oil production and demand were 88.4 mb/d and 88.2 mb/d respectively [195]. OPEC estimates the demand to be 109.7 mb/d in 2035 [195]. To meet this demand, the oil companies are expanding [6] their drilling activities (see Figure 4-1). However, drilling oil wells is highly expensive and uncertain, and involves potential environmental hazards and economic risks. For instance, a vertical onshore (offshore) well can cost $MM\$2-5$ ($MM\$8.3$) on an average and a horizontal one can cost $MM\$2.6-6.5$ ($MM\$10.2$). Even after such expense, there is no guarantee that a well will be productive. In 2010, 56 of 227 exploration wells and 5 of 726 development wells of Shell Company [196] turned out to be dry holes. BP's recent drilling blowout and resulting oil spill in the Gulf of Mexico keeps attracting news even now and BP has so far spent [197] more than $B\$8$ in compensation. With such high financial and environmental stakes and significant uncertainty, there exist clear incentive and much recent interest to increase the overall economic efficiency and success rate of the hydrocarbon

[8] Tavallali, Karimi, Teo, Baxendale and Ayatollahi (2013). "Optimal Producer Well Placement and Production Planning in an Oil Reservoir." Computers & Chemical Engineering **55**: 109-125.

recovery processes by using systematic optimization approaches to obtain the best drilling and production scenarios.

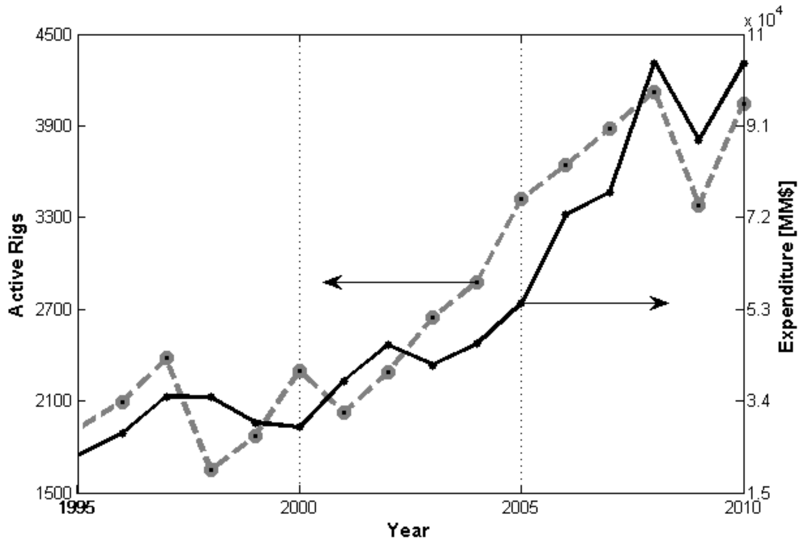


Figure 4-1: Total upstream (exploration and development) expenditure of oil majors and number of active drilling rigs worldwide.

Typically, the industrial approach for selecting of drilling sites involves two stages, which employs a variety of data bases, computational tools, and engineering heuristics. In the first stage, the engineering team defines a variety of development scenarios. In the second, it evaluates those scenarios via extensive simulations and develops various field production/injection profiles. Although intuitive and useful, such a sequential procedure is inherently empirical, ad-hoc, and myopic, and has shortcomings. Much scope and benefits exist for the application of advanced optimization methods. A systematic model-based approach that simultaneously considers the drilling decisions along with the production/injection profiles over the planning horizon can yield significant returns in terms of economics, success, and recovery.

An integrated strategy for optimal well placement would encompass at least five elements in one single optimization model: (a) subsurface physics, (b) well geometry and dynamics, (c) surface facilities, (d) production/injection profiles, and (e) market and economics. Formulating and solving such an optimization model is a tremendous challenge. First, the myriad of decisions such as potential well locations, types, functionalities (producer/injector) [53], trajectories and inclinations [41], drilling schedules [27], and flow distributions [35, 58, 179] make this a highly combinatorial optimization problem . Second, the physics of multi-phase flow in the reservoir is highly nonlinear and spatiotemporal, which makes the optimization problem large, complex, and nonconvex. Guaranteeing the best solution becomes a huge challenge. Last, the inevitable discretization of the governing continuity equations renders the problem non-differentiable in the spatial domain and limits the application of derivative-based optimization algorithms.

As it was extensively discusses in Chapter 2, three main approaches for optimal well placement are studied in the existing literature: (a) mathematical programming (b) evolutionary and direct search [53, 63, 66, 68, 81, 198], (c) gradient-based search [139, 141, 145, 199]. Biegler and Grossmann [158], and Grossmann and Biegler [200] present an excellent overview of these methods, while our detailed literature survey [201] and Nasrabadi et al. [42] specifically discuss their applications to well placement. While mathematical programming has been the first reported approach [124], the other two approaches (call them search methods) have received much more attention. The two methods usually search for better well locations [62, 135], and then use commercial reservoir simulators in a black box manner to evaluate the performance of these locations. Thus, in a sense, they parallel the conventional industrial approach. The simulator acts as a mere function evaluator that numerically solves the system of governing equations for a given set of heuristic control policies. The optimizer then uses the black box to determine a feasible production/injection plan. To obtain a near optimal solution, the optimizer must evaluate many such plans and simulate many scenarios. This can easily become computationally expensive for the evolutionary and direct search methods due to the dynamic nature of the reservoir and time-dependent decisions regarding entire production/injection profiles. While the gradient-based methods [133] used in the work of Forouzanfar et al. [138], Li and Jafarpour [202], Wang et al. [135], and Zhang et al. [136], have the potential to fare better, they all assume constant well/field production rates over the planning horizon. This makes it difficult for them to handle dynamic events (e.g. water breakthrough) and infeasible pre-fixed production profiles.

While the aforementioned searches are basically unconstrained optimization methods, mathematical programming is a constrained optimization technique. It can embed the reservoir physics inside its model and benefit from its structure. In another word, this allows one to include the production/injection profiles along with the location decisions in the model and potentially improve computational speed. This combination is a promising technique; instead of using heuristics to set the flow rates (as is done in the numerical simulators), an optimization model (equipped with flow equations) can use optimization principles to plan the production, and hence can achieve a better solution. In fact, mathematical programming has been successfully used in a variety of industries and applications such as energy systems [203, 204], petroleum refining and blending [104-106], pharmaceutical enterprises[205, 206], chemical process design and integration [110-112, 114, 115], chemical logistics [207, 208], and others. It offers several advantages over other methods.

Most previous mathematical programming work on well placement and production planning has focused on the surface issues and related problems such as numbers, types, capacities, locations, and allocations of wells and platforms [22, 117-123, 126]. Those that have included the subsurface issues have usually empirically approximated the reservoir response to various production scenarios. These approximations being largely linear have resulted in mixed integer linear programs (MILPs). Rosenwald and Green [124] developed an MILP model by using influence function and superposition to approximate the flow dynamics. While the former is an approximation derived from several reservoir simulations, the latter relates pressure drop at each well to production rate (see [127]). Using the same approach, Haugland et al. [128] studied well placement and scheduling, platform capacity, and production planning. Later, Iyer [116] used piecewise linear approximations of reservoir pressure and gas-oil-ratio (GOR) versus cumulative oil production to describe the subsurface dynamics. They also included the well and surface elements in their MILP model. Although comprehensive, their model uses several simplifying assumptions such as linear pressure drop vs. flow relation for pipes, constant productivity index for each well throughout the planning horizon, non-interacting and independent wells, uniform fluid pressure and composition throughout the reservoir. These assumptions can affect production estimates significantly. On the other hand, their work also addresses several important issues: well selection in reservoirs belonging to multiple fields, well drilling and platform installation scheduling considering the drilling rig availability, and finally platform sizing and production planning. Among the surface-directed studies, Van Den Heever and Grossmann [132] extended the MILP model of Iyer et al. [116] by fitting an exponential function to describe reservoir pressure vs. cumulative oil flow, and quadratic functions to describe cumulative gas production and GOR vs. cumulative oil flow. In contrast to these works that have used dynamic approximations, several others have used static approximations. Dogru [129] employed productivity index and oil-in-place data to formulate the offshore well platform and drilling location-allocation problem. Vasantharajan and Cullick [125] used connected hydrocarbon pore volume in a specified drainage area, inversely weighted by tortuosity, to define another static metric. Ierapetritou et al. [130] used a similar approach. Their MILP model allows multiple geo-objects and layers with perforated wells spanning multiple layers. Later, Cullick et al. [131] extended this approach and included deviated wells based on a sequential heuristic.

Most of the above optimization models are large; hence several solution approaches have also been used or developed in the literature. These include heuristic or decomposition procedures [116, 130-132] and pre-processing steps (e.g. reservoir data [125, 130]).

In spite of its potential and versatility, the application of mathematical programming in practice has been limited due to several reasons. One is the complexity in their model formulations and executions. Another is the lack of their awareness in the industry. Furthermore, their various approximations of the nonlinear multiphase flow dynamics have been largely problem-specific and far less accurate than rigorous numerical simulations. Significant advances in computing hardware and the solvers and tools for mathematical programming enable us to go beyond *approximating* the subsurface multi-phase flow. This is one of the main objectives of this work. In addition, we relate the subsurface flow to the flow inside the well tubing and consider the surface and economic constraints to obtain a very detailed and comprehensive model for the upstream drilling and production activities. Their foundations are discussed in the previous chapters and they are applied and extended here.

In this chapter, we consider the deterministic problem of optimally locating the drilling sites for new/infill producers and deciding the optimal production and injection plans for all active wells. We first state and define the scope of our well placement and production planning problem. Then, we describe our modeling approach and devise a solution algorithm, as the commercial solvers fail to solve the formulated problem. We then present two case studies to demonstrate the effectiveness of our proposed approach, and conclude with a concise discussion.

4.2 **Problem Statement**

Consider a typical oil reservoir with existing producer and injector wells as shown in Figure 4-2. It is desired to increase its production by infill-drilling some new producer wells. The problem can be stated as follows:

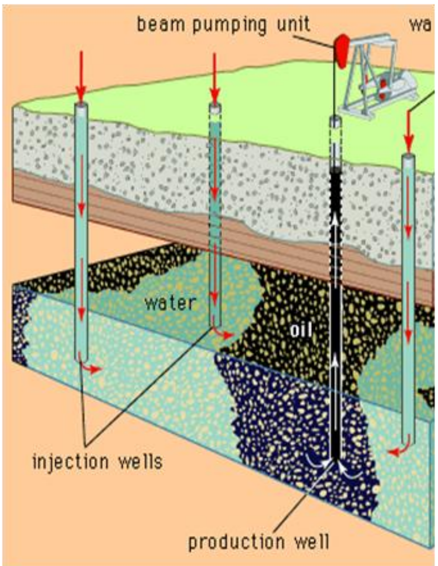


Figure 4-2: An oil reservoir with producer and injector wells.

Given:

1. Geological information such as dimensions, porosity, and permeability about the formation from seismic studies or history matching.
2. Pressure-Volume-Temperature (PVT) related information such as formation volume factor and fluid properties (viscosity, density, compressibility) from core samples or previous production data.
3. Existing wells (if any), their types (producer vs. injector), and locations.
4. Current pressure and saturation profiles in the reservoir at time zero.
5. Minimum allowable well-to-well distance.
6. Operational data such as water-cut limits, maximum injection pressure, minimum production pressure, and capacity expansion plans for surface facilities.
7. Production horizon of H years.
8. Relevant economic data such as demand curve, drilling budget and costs, injection costs, discount rate, oil revenue forecasts, etc.

Obtain:

1. Number and locations of new producer wells.
2. Production and injection profiles for all wells.

Aiming to maximize the net present value (NPV) of oil production over the planning horizon.

Assuming:

1. Reservoir is a horizontal 2D plane and all wells are/will be vertical wells.
2. Capillary pressure in the reservoir is negligible.
3. Main driving mechanism is water-drive injection.
4. Water phase is incompressible, but not the oil phase.

Following policies:

1. Respect minimum well-to-well distance.
2. Open all new wells to production simultaneously.
3. Shut in a well that hits its water-cut limit.

4.3 Formulation

In the previous chapter we developed a spatiotemporal discrete model. Here, we use the same discretization approach and model. Then I cells has arbitrary lengths Δx_i $(i = 1, 2, \dots, I)$ in the x-direction and J cells of arbitrary lengths Δy_j $(j = 1, 2, \dots, J)$ in the y-direction and each cell in the reservoir is named by a single index, $n = i + (j - 1) \times I$. Following sets are also defined:

$\mathbf{IX} = \{n \mid n \text{ is an interior grid with } i < I\}$;

$\mathbf{IY} = \{n \mid n \text{ is an interior grid with } j < J\}$ **;**

here, **IX** eliminates the reservoir's border cells in the x-direction, and **IY** eliminates them in the y-direction. Also we discretize the planning horizon H into T time periods of arbitrary lengths Δh^t $(t = 1, 2, \dots, T)$.

4.3.1 Well placement decision

We first model the well placement decisions. To decide the locations of additional producers, we give the opportunity of being a well to all grids in the reservoir. Therefore, we define:

$$y_n = \begin{cases} 1 & \text{if a well exists at grid } n \\ 0 & \text{otherwise} \end{cases}$$

$$\mathbf{IW} = \{n \mid \text{injector well exists at grid } n\}$$

$$\mathbf{PW} = \{n \mid \text{producer well exists at grid } n\}$$

We set $y_{n \in \mathbf{IW} \cup \mathbf{PW}} = 1$ for all wells that already exist.

In practice, minimum well-to-well distance is a key consideration. Locating multiple wells in a small area can potentially cause a depleted pressure sink which would limit a well's production. It would also encourage the water to flow to that area due to the lower pressure. This would prevent the full sweeping of the reservoir and reduce overall oil recovery. Typically, the industry practice is to have at most one well per certain area. This serves as a heuristic but practical guide to spread out wells. It helps distribute the pressure field and avoid water breaking into the wells. In terms of grid elements, the industry prefers to have a distance of at least one, and preferably three, grid elements between adjacent wells. In this work, we assume a minimum well-to-well distance of one grid element. To ensure it, we first set $y_n = 0$, if a grid n is adjacent to any existing producer or injector well. Then, for all others grids ($n \notin \mathbf{IW} \cup \mathbf{PW}$, $y_n \neq 0$), we use the following constraints.

$$y_n + y_{n+1} \leq 1 \qquad n \in \boldsymbol{IX} \qquad (1)$$

$$y_n + y_{n+I} \leq 1 \qquad n \in \boldsymbol{IY} \qquad (2)$$

$$y_n + y_{n+I-1} \leq 1 \qquad (n + I - 1) \in \boldsymbol{IX} \qquad (3)$$

$$y_n + y_{n+I+1} \leq 1 \qquad (n + I) \in \boldsymbol{IX} \qquad (4)$$

The budget (DC^U) available for drilling obviously limits the number of wells. If cd_n is the cost of drilling a new well at location n, then the total drilling cost ($DC \leq DC^U$) is given by,

$$DC \geq \sum_{n \notin \boldsymbol{IW} \cup \boldsymbol{PW}} cd_n y_n \tag{5}$$

4.3.2 Reservoir Dynamics

The mass balances for the two fluids are defined using the discrete spatiotemporal model developed in the previous chapter:

$$(V_n/dh^t)\{d_{o,1,n}^t\,[P_n^t - P_n^{t-1}] + d_{o,2,n}^t\,[S_n^t - S_n^{t-1}]\} + q_{o,n\notin\mathbf{IW}}^t +$$
$$\begin{pmatrix} \{M_{o,x-}^t . T_{n-1}^x . [P_n^t - P_{n-1}^t]\}_{(n-1)\in IX} + \{M_{o,x+}^t . T_n^x . [P_n^t - P_{n+1}^t]\}_{n\in IX} \\ +\{M_{o,y-}^t . T_{n-I}^y . [P_n^t - P_{n-I}^t]\}_{(n-I)\in IY} + \{M_{o,y+}^t . T_n^y . [P_n^t - P_{n+I}^t]\}_{n\in IY} \end{pmatrix} = 0 \tag{6}$$

$$(V_n/dh^t)\{d_{w,1,n}^t\,[P_n^t - P_n^{t-1}] + d_{w,2,n}^t . [S_n^t - S_n^{t-1}]\} + \left(q_n^t - q_{o,n\notin\mathbf{IW}}^t\right) +$$
$$\begin{pmatrix} \{M_{w,x-}^t . T_{n-1}^x . [P_n^t - P_{n-1}^t]\}_{(n-1)\in IX} + \{M_{w,x+}^t . T_n^x . [P_n^t - P_{n+1}^t]\}_{n\in IX} \\ +\{M_{w,y-}^t . T_{n-I}^y . [P_n^t - P_{n-I}^t]\}_{(n-I)\in IY} + \{M_{w,y+}^t . T_n^y . [P_n^t - P_{n+I}^t]\}_{n\in IY} \end{pmatrix} = 0 \tag{7}$$

Here, P_{nr}^t and S_{nr}^t are pressure and saturation at cell n and the end of time period t, $d_{f,1,nr}^t$ and $d_{f2,nr}^t$ are the variable accumulation multipliers, T_{nr}^x and T_{nr}^y are transmissibilities in x and y-directions, $M_{f,(x+)r}^t$, $M_{f,(x-)r}^t$, $M_{f,(y+)r}^t$ and $M_{f,(y-)r}^t$ are upstream weighted mobilities, which are taken as those of the neighbouring cells with higher pressures, $q_{f,n}^t$ is the phase flow from (+ve for out, $-$ ve for in) the reservoir. The variable accumulation multipliers, transmissibilities, and upstream weight mobility terms are as in eqs. 2-14, 17-20 of Chapter 3. $M_{w,nr}^t$ and $M_{o,nr}^t$ are the water and oil mobilities defined as:

$$M_{w,n}^t = kr_0^{\mathrm{w}} \left(\frac{S_n^t - S_{wr}}{1 - S_{wr} - S_{or}}\right)^b / B_{wr}^0 \mu_w \tag{8}$$

$$M_{o,n}^t = kr_0^{\mathrm{o}} \left(\frac{1 - S_n^t - S_{or}}{1 - S_{wr} - S_{or}}\right)^a / (\mu_{Bo,1} P_n^t + \mu_{Bo,2}) \tag{9}$$

here, kr_0^o and kr_0^w are the end-point relative permeabilities of oil and water, S_{or} and S_{wr} are the residual oil and water saturations respectively, a and b are the exponents in Corey's correlation, $(\mu_{Bo,1}, \mu_{Bo,2})$ are regression parameters for the product $(\mu_o \times B_o)$ of the viscosity and formation volume factor of oil, B_{wr}^{0} is the formation volume factor for water at reference pressure P_r, finally μ_w is the water viscosity.

4.3.3 Flow Balances

To relate the binary well placement variables to the dynamic state of the reservoir, we define q_n^t as the total (oil plus water) flow and $q_{o,n}^t$ as the oil flow from a well at grid n. Clearly, the oil flow from a well cannot exceed the total flow, so:

$$q_{o,n}^t \leq q_n^t \qquad n \notin \mathbf{IW} \qquad (10)$$

Note that we are not using water flow as a separate variable, as the difference between the total and oil flows gives us the water flow.

Now, if a grid does not host a well, then both the oil and total flows must be zero.

$$q_n^t \leq y_n \cdot (IPC + PE^t) \qquad n \notin \mathbf{IW} \qquad (11)$$

$$q_{o,n}^t \leq y_n D^t \qquad n \notin \mathbf{IW} \qquad (12)$$

where, IPC is the initial processing capacity of the reservoir at time zero, PE^t is the planned incremental expansion in the capacity of surface processing facilities during period t, and D^t is the total oil demand during period t.

The total liquid and oil productions (TLP^t, TOP^t) and water injection (TWI^t) are:

$$TLP^t = \sum_{n \notin \mathbf{IW}} q_n^t \qquad t > 1 \qquad (13)$$

$$TOP^t = \sum_{n \notin \mathbf{IW}} q_{o,n}^t \qquad t > 1 \qquad (14)$$

$$TWI^t = \sum_{n \in \mathbf{IW}} q_n^t \qquad t > 1 \qquad (15)$$

Clearly, TLP^t, TOP^t and TWI^t cannot exceed their processing/injection capacities. Thus, we have $0 \leq TLP^t \leq IPC + PE^t$, $0 \leq TOP^t \leq \min[D^t, IPC + PE^t]$, and $-(IIC + IE^t) \leq TWI^t \leq 0$), where IPC (IIC) is the initial production (injection) capacity, and PE^t (IE^t) is the planned incremental capacity expansion for production (injection) during period t.

4.3.4 Well flow terms

For a given tubing, the frictional pressure drop across the tubing will depend on factors such as length and diameter of the tubing, total flow rate from the well, fluid density, and fluid viscosity. We used VFPi package [45] to generate pressure drop data for ranges of liquid flow rates and water cuts but fixed THP and GOR (Gas Oil Ratio) as required by VFPi. GOR is constant due to the assumption of undersaturated reservoir. We used the loose-emulsion option in VFPi and included all three forms (hydrostatic, frictional, and acceleration) of pressure drop. Then, we fitted the following empirical correlation for the BHP:

$$BHP_n^t = THP_n^L + \rho_n^t gL + \Delta P_n^t + \Delta p_n^t = \tag{16}$$

$$\gamma_1(q_n^t + \gamma_2)^{\gamma_3} + \gamma_4\left(q_{o,n}^t + \gamma_5\right)^{\gamma_6} + \gamma_7\, q_{o,n}^t/(q_n^t + \gamma_8) + \gamma_9 q_n^t + \gamma_{10} q_{o,n}^t + \gamma_{11}$$

where, $\gamma_1, \gamma_2, \gamma_3, \gamma_4, \gamma_5, \gamma_6, \gamma_7, \gamma_8, \gamma_9, \gamma_{10}$ and γ_{11} are regression parameters for a specific well string and reservoir fluid properties. Eq. 16 captures the relative importance of q_n^t and $q_{o,n}^t$ and their interaction on the pressure drop across the tubing. This equation is an empirical equation that captures the string fluid dynamics well and ensures that zero flow condition does not create infeasibilities. We developed it mainly by trial and error, and better expressions could also be possible. Using IPR and VFP equations (defined in Chapter 3) and Eq. (16), we can bound the output oil flow from the well as follows.

$$q_n^t \leq \psi_n(M_{o,n}^t + M_{w,n}^t) \cdot \left(P_n^t - \left[\gamma_1(q_n^t + \gamma_2)^{\gamma_3} + \gamma_4\left(q_{o,n}^t + \gamma_5\right)^{\gamma_6} + \gamma_7\, q_{o,n}^t/(q_n^t + \gamma_8) + \gamma_9 q_n^t + \gamma_{10} q_{o,n}^t + \gamma_{11}\right]\right) \quad n \notin \mathbf{IW} \tag{17}$$

Eq. (17) will prevent an unrealistic full drainage of the reservoir system down to the atmospheric pressure. In case of low reservoir pressure, the bracketed term on the right hand side can become negative. That can cause infeasibility, since $q_n^t \geq 0$. Therefore, we define an intermediate free variable ($\mathbf{R}_n^t$) and use the below constraints $\left(M_{o,n}^t + M_{w,n}^t \neq 0\right)$:

$\mathbf{R}_n^t/\left(M_{o,n}^t + M_{w,n}^t\right) \le \psi_n \cdot \left(P_n^t - \left[\gamma_1(q_n^t+\gamma_2)^{\gamma_3} + \gamma_4\left(q_{o,n}^t+\gamma_5\right)^{\gamma_6} + \gamma_7\, q_{o,n}^t/(q_n^t+\gamma_8) + \gamma_9 q_n^t + \gamma_{10} q_{o,n}^t + \gamma_{11}\right]\right) \qquad n \notin \mathbf{IW} \qquad (18)$

$$q_n^t \le 0.5\left(\mathbf{R}_n^t + \sqrt{{\mathbf{R}_n^t}^2 + \delta^2}\right) \qquad n \notin \mathbf{IW} \qquad (19)$$

Here δ is a small ($\cong 10^{-4}$) scalar. Eq. 19 is a smooth approximation for $\max(\mathbf{R}_n^t, 0)$ with a maximum error of δ at $\mathbf{R}_n^t = 0$. The approximation error reduces to zero as $\mathbf{R}_n^t$ goes to infinity [209]. When the reservoir pressure is low, $\mathbf{R}_n^t$ becomes negative, and $q_n^t \cong 0$. Otherwise, $q_n^t \cong \mathbf{R}_n^t$. Thus, Eqs. 18 and 19 effectively provide a tight upper-bound on q_n^t for active wells. The optimizer will force these two inequalities into equalities for active wells.

To ensure the correct proportion of oil and water flows from the well, we use IPR equation (Eq. 21 Chapter 3) to derive the below constraint:

$$q_{o,n}^t = q_n^t \cdot M_{o,n}^t/\left(M_{w,n}^t + M_{o,n}^t\right) \qquad n \notin \mathbf{IW} \qquad (20)$$

The above can be written as two separate inequalities:

$$q_{o,n}^t \le q_n^t \cdot M_{o,n}^t/\left(M_{w,n}^t + M_{o,n}^t\right) \qquad n \notin \mathbf{IW} \qquad (21)$$

$$q_{o,n}^t \ge \mathbf{R}_n^t \cdot M_{o,n}^t/\left(M_{w,n}^t + M_{o,n}^t\right) \qquad n \notin \mathbf{IW} \qquad (22)$$

Eqs. 21 and 22 provide a tighter relation between $\boldsymbol{R}_n^t$ and q_n^t. From Eq. 19, $q_n^t \le \max(\mathbf{R}_n^T, 0)$ and from Eqs. 21 and 22, $\mathbf{R}_\mathrm{n}^\mathrm{t} \le q_o^t \le q_n^t$. We will further comment on these equations later.

For injectors, we define THP_n^U as the maximum possible tubing head pressure for a water injector at grid n. Similar to the production wells, we fit another regression model for $BHP_n^t(q_n^t)$ at THP_n^U based on the data from the VFPi software package. Moreover, we use an equivalent of Eq. (22) of Chapter 3 $\left(M_{cw,n}^t = M_{o,n}^t \overline{B_o}/B_{wr}^o\right)$ to formulate the following equation that governs the flow in the injectors:

$$q_n^t \ge \psi_n\left(M_{w,n}^t + M_{o,n}^t \overline{B_o}/B_{wr}^o\right).\left(P_n^t - \left[\gamma_1' {q_n^t}^2 + \gamma_2' q_n^t + \gamma_3'\right]\right) \qquad n \in \mathbf{IW} \qquad (23)$$

where, γ_1', γ_2' and γ_3' are regression parameters for a specific well string and water properties and $\overline{B_o}$ is the average oil formation volume factor. We could also consider $q_n^t \geq \psi_n M_{w,n}^U . \left(P_n^t - \left[\gamma_1' {q_n^t}^2 + \gamma_2' q_n^t + \gamma_3'\right]\right)$, where $M_{w,n}^U$ is an upper bound on $M_{w,n}^t$.

In practice, the industry uses water cut limit (WC_n) as a measure for continued well production to avoid excessive water production. While it is possible to use such a criterion in this model as well, given the proper quantitative incentive for oil production and disincentive for water production, the optimizer will have the innate ability to decide when to shut in a well. This is true for both existing and new producers. Thus, although not necessary in principle, we do use water cut ($WC_n^t = q_{w,n}^t / q_n^t$) to monitor wells dynamically in our work as follows.

$$q_{o,n}^t \geq (1 - WC_n) q_n^t \qquad n \notin \mathbf{IW} \qquad (24)$$

Any well that violates this constraint at any time will be shut in ($q_n^t = 0$).

4.3.5 Well Placement Objective

We maximize NPV (Net Present Value) as the well placement objective. It comprises four parts: drilling costs, oil selling profit, water production costs, and water injection costs.

$$\text{Max NPV} = \sum_{t>1} \left[c_o TOP^t - c_{pw} TLP^t + c_{iw} TWI^t\right] \frac{\Delta h^t}{(1+\beta)^{\frac{h^t}{365}}} - DC \qquad (25)$$

where, c_o is the [oil revenue - oil production cost] ($ per unit flow), c_{pw} is water production and disposal cost ($ per unit flow), c_{iw} is the water injection cost ($ per unit flow), and β is the annual discount factor.

Combination of Eq. 11 with any of Eqs. 10, 21 and 24 can take the role of Eq. 12 to set $q_{o,n}^t = 0$ for rejected grids ($y_n = 0$). Therefore, in order to reduce the number of equations while fulfilling the requirements of our model, we drop Eq. 12 from our model.

This completes our model (**G**). It involves Eqs.(1)-(11), (13)-(15), (18)-(19), (21)-(25) with variable bounds: $0 \leq DC \leq DC^U$, $0 \leq M^t_{f,n} < M^U_{f,n}$, $P^L \leq P^t_n \leq P^U$, $S_{wr} \leq S^t_n \leq S_{or}$, $0 \leq TLP^t \leq (IPC + PE^t)$, $0 \leq TOP^t \leq min(D^t, IPC + PE^t)$, $-(IIC + IE^t) \leq TWI^t \leq 0$, $0 \leq q^t_{o,n\notin\mathbf{IW}}$, $0 \leq q^t_{n\notin\mathbf{IW}}$, and $q^t_{n\in\mathbf{IW}} \leq 0$. It is a nonconvex, dynamic, mixed integer nonlinear programming (MINLP) model. Its nonlinear programming (NLP) part optimizes the dynamic production and injection profiles. During our formulation of **G**, we preferred inequalities over equalities. For instance, we converted several equalities into inequalities (including Eqs. 18-19 and 21-22 for active wells) and relied on the objective function to force them to be equalities. By solving **G**, we obtain the optimal number and locations of wells, and the optimum production and injection profiles. From these, we can compute the water flow rate $(q^t_n - q^t_{o,n})$ and also BHP^t_n and THP^t_n at each well n.

G integrates the elements and effects of wells, subsurface and surface facilities, production economics, and market demand. Such an integrated approach is more general and complex, but completely different from using several specialized software packages in piecemeal manner. The piecemeal approach is straightforward, but it is limited in scope and bounded by the requirements of the packages.

4.4 Solution Strategy

G is a huge model with spatiotemporal decisions. For a sample reservoir with $I = 20, J = 50$, $n = 1000, t = 150$, and $|\mathbf{IW}| = 5$, **G** has nearly 1.65×10^6 equations (mostly nonlinear and partly endogenous) with 995 binary and 1048501 continuous variables. This size along with the endogenous equations and highly complex nonlinear equations make it virtually impossible to solve **G** using existing solvers such as BARON and DICOPT. In some practical cases, the number of grids (n) can even reach 10^6 [95], which would mean a much larger **G**. Thus, although current optimization solvers have a lot to offer, we need a specialized solution algorithm. Since the NLP subproblem in **G** is complex and nonconvex, we decided to modify the outer-approximation algorithm with equality relaxation and augmented penalty (OA/ER/AP algorithm) proposed by Grossmann and coworkers [210-213]. It is suited for MINLPs with complex nonconvex NLP subproblems, as it does not rely on the convexity assumption (which matches our problem). It has been used in several applications [214] and it exists in DICOPT [215]. The interested reader may refer [213] and [155] for its detailed mathematical description. However, to keep this paper self-sufficient, we give next a brief description of the algorithm in its original form, before we proceed to modify to suit our present problem.

4.4.1 OA/ER/AP Algorithm

The algorithm [213] addresses the following MINLP, where *f*, $\boldsymbol{g}$ and $\boldsymbol{h}$ are continuously differentiable functions.

MINLP:

$$\min \quad \boldsymbol{cy} + f(\boldsymbol{x}, \boldsymbol{m})$$

$$\text{Subject to} \quad \begin{cases} \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{m}) = 0 \\ \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{m}) \leq 0 \\ \boldsymbol{Cx} + \boldsymbol{By} \leq \boldsymbol{d} \\ x \in \boldsymbol{X} = [x : x \in \Re^n, \boldsymbol{A}_1\boldsymbol{x} \leq \boldsymbol{a}_1] \subseteq \Re^n \\ y \in \boldsymbol{Y} = [y : y \in (0,1)^T], \boldsymbol{A}_2\boldsymbol{y} \leq \boldsymbol{a}_2 \end{cases} \tag{26}$$

$\boldsymbol{x}$ is the vector of continuous variables (e.g. saturations, pressures, and flows in the present problem), $\boldsymbol{y}$ is the vector of binary variables (e.g. existence of well), and $\boldsymbol{A}, \boldsymbol{B}, \boldsymbol{C}, \boldsymbol{a}, \boldsymbol{c}, \boldsymbol{d},$ and $\boldsymbol{m}$ are the vectors or matrices of fixed parameters (e.g. geological realization, production time horizon, economic coefficients, and so on). In the present problem, $\boldsymbol{cy}$ represents drilling costs, $f(\boldsymbol{x}, \boldsymbol{m})$ represents gross profit from oil production, $\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{m})$ represents the constitutive and conservation equations, $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{m})$ represents the various physical and operational constraints such as water-cut limits, total field production/injection, and $\boldsymbol{Cx} + \boldsymbol{By} \leq \boldsymbol{d}$ represents other linear constraints. For the sake of simplicity, we drop $\boldsymbol{m}$ from f, $\boldsymbol{h}$, and $\boldsymbol{g}$.

The algorithm (Figure 4-3) decomposes the above MINLP into (1) an NLP primal subproblem and (2) an MILP master subproblem. At each iteration r, it first solves the NLP primal problem, and then the MILP master problem. The NLP subproblem at iteration $(r+1)$ is obtained by fixing the binary variables in the MINLP at their values $(\boldsymbol{y}_r)$ in the solution of the MILP subproblem at iteration r as follows [213] :

$$\begin{aligned} &min && \boldsymbol{c}\boldsymbol{y}_r + f(\boldsymbol{x}) \\ &\text{Subject to} && \left[\begin{array}{c} \boldsymbol{h}(\boldsymbol{x}) = 0 \\ \boldsymbol{g}(\boldsymbol{x}) \leq 0 \\ \boldsymbol{Cx} + \boldsymbol{B}\boldsymbol{y}_r \leq \boldsymbol{d} \\ x \in \boldsymbol{X} = [x : x \in \Re^n, \boldsymbol{A_1 x} \leq \boldsymbol{a}_1] \subseteq \Re^n \end{array}\right. \end{aligned} \tag{27}$$

The master MILP at iteration r is then obtained by linearizing the nonlinear constraints in the MINLP at the optimal solution of the primal NLP in that iteration as follows [213].

$$\begin{aligned} &min && \boldsymbol{c}^{\boldsymbol{T}}\boldsymbol{y} + \phi + \textstyle\sum_{r'=r^L}^{r} u_{r'}\sigma_{r'} + \sum_{r'=r^L}^{r} \boldsymbol{v}_{r'}\boldsymbol{\theta}_{r'} + \sum_{r'=r^L}^{r} \boldsymbol{w}_{r'}\boldsymbol{\eta}_{r'} \\ &\text{Subject to} && \left[\begin{array}{l} \phi + \sigma_{r'} \geq f(\boldsymbol{x}_{r'}) + \nabla f(\boldsymbol{x}_{r'}) \cdot (\boldsymbol{x} - \boldsymbol{x}_{r'}) \\ \boldsymbol{\theta}_{r'} \geq signum(\boldsymbol{\lambda}_{r'}).[\boldsymbol{h}(\boldsymbol{x}_{r'}) + \nabla \boldsymbol{h}(\boldsymbol{x}_{r'}) \cdot (\boldsymbol{x} - \boldsymbol{x}_{r'})] \\ \boldsymbol{\eta}_{r'} \geq \boldsymbol{g}(\boldsymbol{x}_{r'}) + \nabla \boldsymbol{g}(\boldsymbol{x}_{r'}) \cdot (\boldsymbol{x} - \boldsymbol{x}_{r'}) \\ \qquad \boldsymbol{Cx} + \boldsymbol{By} \leq \boldsymbol{d} \\ \qquad x \in \boldsymbol{X} = [x : x \in \Re^n, \boldsymbol{A}_1\boldsymbol{x} \leq \boldsymbol{a}_1] \subseteq \Re^n \\ \qquad y \in \boldsymbol{Y} = [y : y \in (0,1)^T], \boldsymbol{A}_2\boldsymbol{y} \leq \boldsymbol{a}_2 \\ \sum_{n \in \boldsymbol{E}_{r'}} y_n - \sum_{n \notin \boldsymbol{E}_{r'}} y_n \leq |\boldsymbol{E}_{r'}| - 1, \ \ \boldsymbol{E}_{r'} = [n, y_{n,r'} = 1]; \\ \qquad \sigma_{r'}, \boldsymbol{\eta}_{r'}, \boldsymbol{\theta}_{r'} \geq 0, \end{array}\right] r^L \leq r' \leq r \end{aligned} \tag{28}$$

where, ϕ is an unrestricted continuous variable defined by $\phi - f(\boldsymbol{x}) \geq 0$; σ_r, $\boldsymbol{\theta}_r$, and $\boldsymbol{\eta}_r$ are slack variables; and u_r, $\boldsymbol{v}_r$, and $\boldsymbol{w}_r$ are penalty parameters; and $r^L \leq r-1$. Viswanathan and Grossmann [213] suggested $u_r > |\mu_r^0|$, $\boldsymbol{v}_r > |\boldsymbol{\lambda}_r|$, and $\boldsymbol{w}_r > |\boldsymbol{\mu}_r|$, where μ_r^0, $\boldsymbol{\lambda}_r$, and $\boldsymbol{\mu}_r$ are the Lagrange multipliers [216] for $\phi - f(\boldsymbol{x}) \geq 0$, $\boldsymbol{h}(\boldsymbol{x}) = 0$, and $\boldsymbol{g}(\boldsymbol{x}) \leq 0$ respectively.

When solving the MILP subproblem, it adds an integer cut to eliminate the integer solution from the master MILP of the previous iteration. Furthermore, it adds the linearizations of the primal NLP from iteration r^L to r to improve the outer-approximation.

The algorithm repeats until the objective value increases or the master MILP is infeasible [155, 213]. In the present problem, the master MILP will identify promising well positions, while the primal NLP will evaluate the qualities of proposed locations by optimizing the production/injection profiles.

In this work, we made several significant modifications to the above general algorithm to suit the present problem.

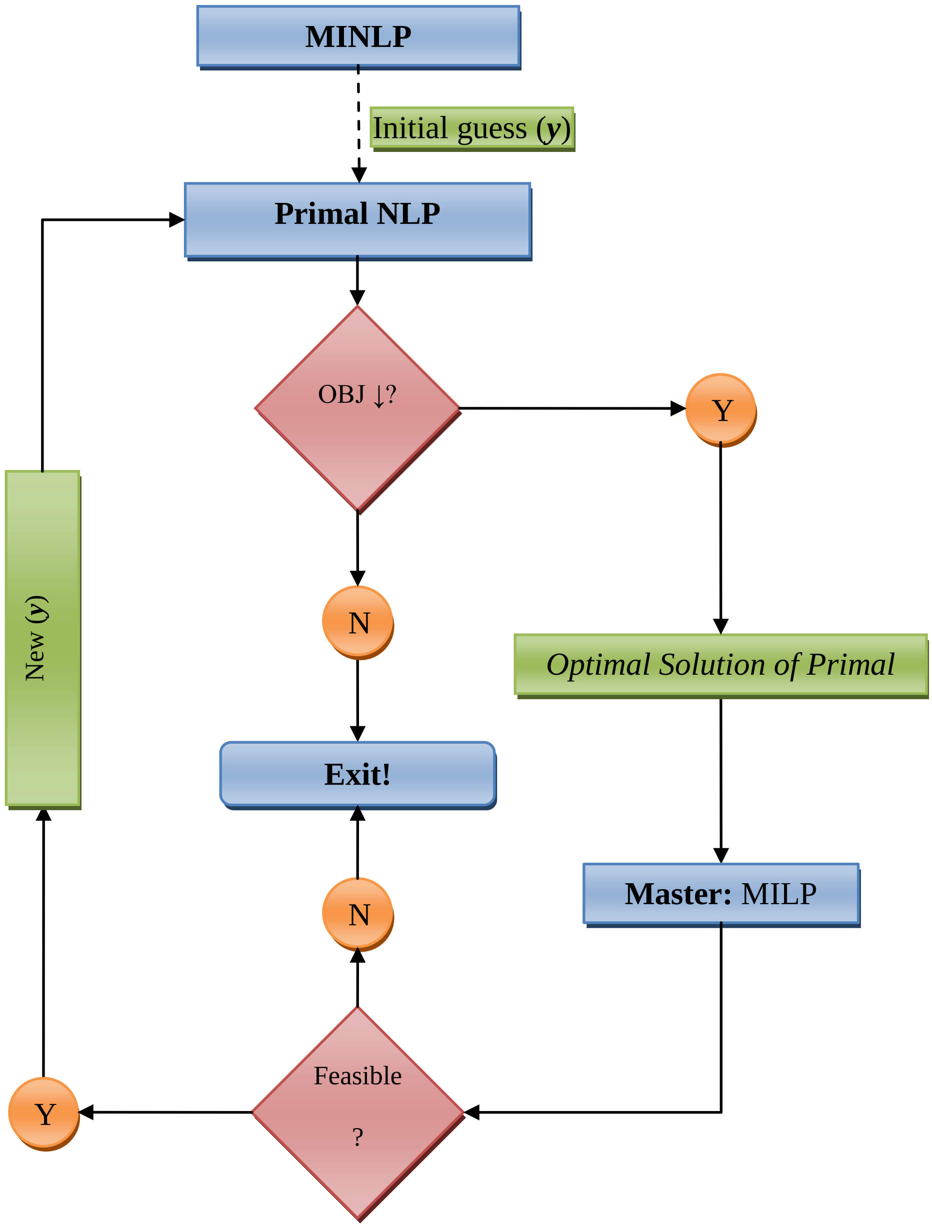


Figure 4-3 : The original OA/ER/AP algorithm.

4.4.2 Modifications to the OA/ER/AP Algorithm

Let us call **GM** (**GP**) as the master (primal) subproblem of **G**. Our modifications involve (a) solution strategy for **GP**, (b) reformulation of **GM**, and (c) termination criteria for the algorithm.

(a) Solution strategy for **GP**: **GP** in its current form involves the flow related terms ($q_n^t, q_{o,n}^t$ and $\mathbf{R}_n^t$) even for well-free grids as in Eq. 25. It also considers all periods together. These along with the upstream weighting of mobility make the formulation endogenous. **GP** in its present form is large and virtually impossible to solve using existing solvers. Therefore, we employ two strategies to simplify **GP**.

We know that q_n^t and $q_{o,n}^t$ are zero for well free grids and shut-in wells, and their $\mathbf{R}_n^t$ value is not important for **GP**. Therefore, we define a set $\mathbf{AW}(t)$ ($t > 1$) = $\{n \mid \text{grid } n \text{ had an active producer well during } t-1\}$, and use it specifically for **GP**, and not **GM**. We define an active well as the one producing or on stand-by, but not shut-in. With that, we set $q_{o,n\notin\mathbf{AW}(\mathrm{t})}^t = 0$ and $q_{n\notin(\mathbf{AW}(\mathrm{t})\cup\mathbf{IW})}^t = 0$ and remove them from **GP**. This restricts the domains of Eqs. 10-11, 13-15, 18-19, 21-22 and 24 to $n \in \mathbf{AW}(t)$, and reduces the size of **GP** considerably by eliminating several constraints and variables (including $\mathbf{R}_{n\notin(\mathbf{AW}(\mathrm{t})\cup\mathbf{IW})}^t$). After solving reduced **GP**, we later compute $\mathbf{R}_{n\notin(\mathbf{AW}(\mathrm{t})\cup\mathbf{IW})}^t$ by assuming that Eq. 18 is an equality constraint for use in **GM**.

The above simplification of **GP** is not sufficient, so we decompose **GP** further into smaller problems ($\mathbf{GP}_\tau$) along the temporal domain by discretizing each period t into $\mathcal{T}_t$ intervals ($\tau = 1, 2, \dots, \sum_t \mathcal{T}_t$). Then, instead of solving **GP** for the entire planning horizon, we solve $\mathbf{GP}_\tau$ for each interval separately in the sequence $\tau = 1, 2, \dots, \sum_t \mathcal{T}_t$. Then, we use only the optimal solution of $\mathbf{GP}_\tau$ at the end of each period t, which is $\mathbf{GP}_\tau$ with $\tau = \sum_{t' \le t} \mathcal{T}_{t'}$, for linearization in **GM**. For solving each $\mathbf{GP}_\tau$, we use the solution of $\mathbf{GP}_{(\tau-1)}$ as the initial solution and also to provide required initial conditions. This allows us to make the formulation exogenous, and update **AW** sequentially. For example, for formulating and solving $\mathbf{GP}_2$, we use the optimal solution of $\mathbf{GP}_1$ to (1) determine **AW**(2), (2) estimate the flow directions from Eqs.(17)-(20) of Chapter 2., (3) fix (P_n^1, S_n^1), and (4) initialize the remaining variables. Figure 4-4 depicts this stage by an orange dashed rectangle. We call this sequential solution (SS).

(b) MILP Reformulation: Here, our main goal is to exploit the special characteristics of the mass balance equations to develop appropriate linearizations for use in **GM.**

As well locations change from iteration to iteration, the fluid flow directions will also change. The new directions may be different from the previous ones, and this can create problems with the use of upstream weighting for mobility. This may invalidate mass balance linearizations and increase the values of the slack variables corresponding to the mass balance equations in **GM**. Moreover, the mass balances are dense, as the convective flow terms involve many variables. Therefore, to simplify the linear expansions of mass balances, we first define a new variable ($F_{f,n}^t$) to represent the net convective flow from a central grid n to its adjacent grids. The convective flow is a directional flow term, i.e. the flow from grid a to b is negative of that from grid b to a. Therefore, summing the net convective flows over grids must give us zero [44]:

$$\sum_n F_{f,n}^t = 0 \tag{29}$$

Then, we replace the four individual convective flow terms (which depend on P_n^t, P_{n-1}^t, P_{n+1}^t, P_{n-I}^t, P_{n+I}^t, $M_{f,x-}^t$, $M_{f,x+}^t$, $M_{f,y-}^t$, and $M_{f,y+}^t$) with one **net** convective flow term, namely $F_{f,n}^t$, in the linear expansions of Eqs. 35 and 36. With this, the mass balance reduces to:

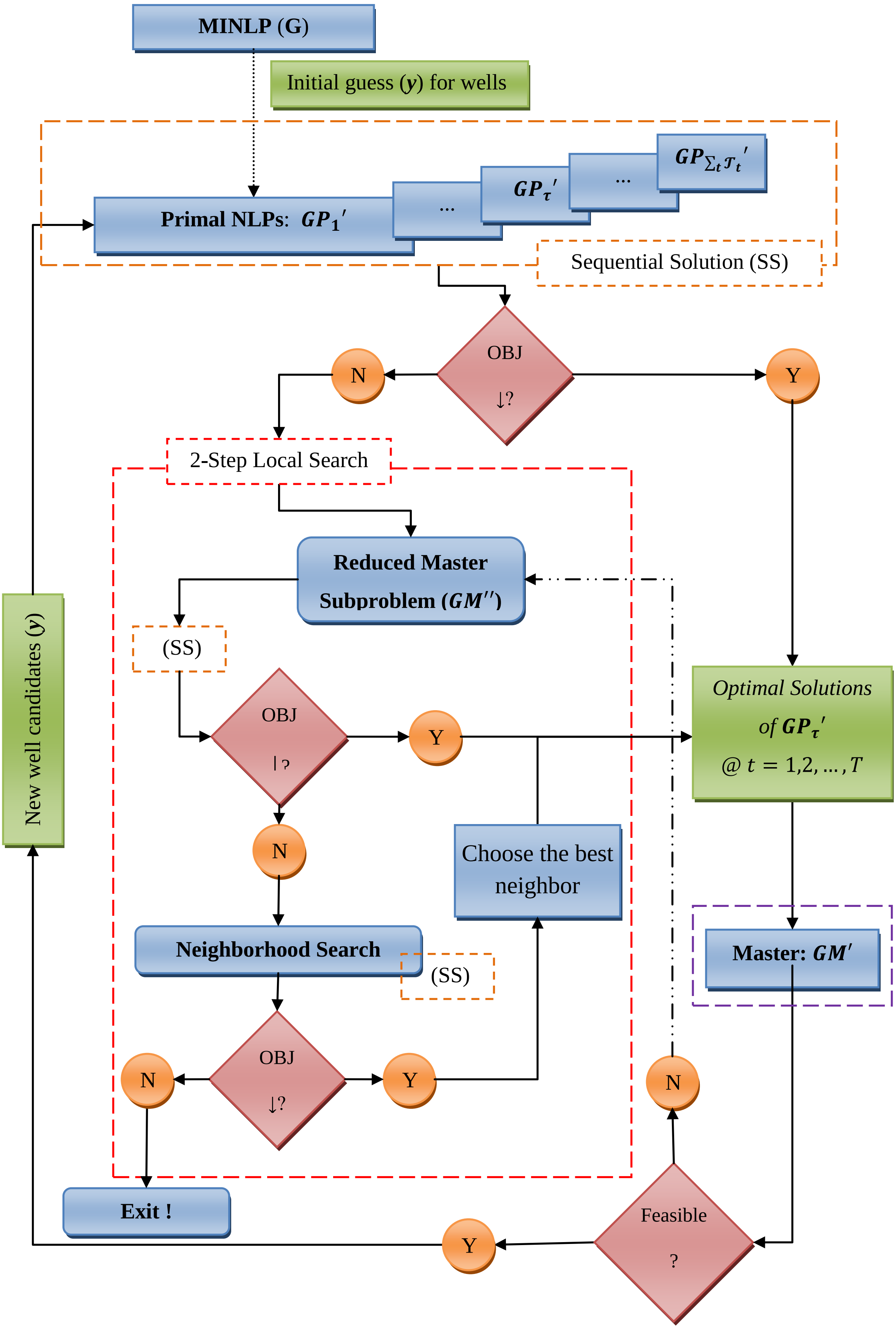


Figure 4-4 : Our modified OA/ER/AP algorithm.

$$\theta_{f,n,r'}^{t} \geq signum\left(\lambda_{f,n,r'}^{t}\right) \cdot \left(\begin{array}{c} \frac{\partial \mathrm{A}_{f,n,r'}^{t}}{\partial P_{n}^{t}}\left[P_{n}^{t}-P_{n,r'}^{t}\right]+\frac{\partial \mathrm{A}_{f,n,r'}^{t}}{\partial S_{n}^{t}}\left[S_{n}^{t}-S_{n,r'}^{t}\right]+ \\ \frac{\partial \mathrm{A}_{f,n,r'}^{t}}{\partial P_{n}^{t-1}}\left[P_{n}^{t-1}-P_{n,r'}^{t-1}\right]+\frac{\partial \mathrm{A}_{f,n,r'}^{t}}{\partial S_{n}^{t-1}}\left[S_{n}^{t-1}-S_{n,r'}^{t-1}\right]+ \\ \left[q_{o,n}^{t}-q_{o,n,r'}^{t}\right]_{n\notin \mathbf{IW}}+\left[q_{n}^{t}-q_{n,r'}^{t}\right]_{f=w}+\left[F_{f,n}^{t}-F_{f,n,r'}^{t}\right] \end{array} \right)$$

$$r^{L} \leq r' \leq r \qquad (30)$$

where, A is the accumulation function defined in Appendix A.

We then limit $F_{f,n}^{t}$ by $F_{f,n}^{L} \leq F_{f,n}^{t} \leq F_{f,n}^{U}$:

$$F_{f,n}^{U} = Min\left(Max_{r^{L}\leq r'\leq r}\left(F_{f,n,r'}^{t}\right), M_{f,n}^{U}.\left[T_{n\in\mathbf{IX}}^{x}+T_{n-1\in\mathbf{IX}}^{x}+T_{n\in\mathbf{IY}}^{y}+T_{n-I\in\mathbf{IY}}^{y}\right].dP_{cf}^{U}\right) \qquad (31)$$

$$F_{f,n}^{L} = Max\left(Min_{r^{L}\leq r'\leq r}\left(F_{f,n,,r'}^{t}\right), -M_{f,n}^{U}.\left[T_{n\in\mathbf{IX}}^{x}+T_{n-1\in\mathbf{IX}}^{x}+T_{n\in\mathbf{IY}}^{y}+T_{n-I\in\mathbf{IY}}^{y}\right].dP_{cf}^{U}\right) \qquad (32)$$

$$dP_{cf}^{U} = Max\left(Max_{r^{L}\leq r'\leq r}\left(dP_{x,r'}, dP_{y,,r'}\right), dP_{U}\right) \qquad (33)$$

Here $dP_{x,r}$ and $dP_{y,r}$ are the absolute pressure gradients along the x and y directions at iteration r, dP_{U} is a user specified pressure gradient.

Note that we have not removed the convective flow terms entirely, rather we replaced them by defining a net convective flow in the MILP approximation step and imposed an overall mass balance constraint to maintain some accuracy in our approximation. This approach definitely has its pros and cons. On the one hand, the convective flow terms are clearly important, but on the other hand, we need to solve **GM** in an acceptable time. While it is indeed an approximation at the MILP stage that determines the well locations, it does not impact the actual flows because they are done at the NLP stage. The only thing that it can do is compromise the quality of well placements. However, by making sure that we improve NPV at each iteration, hopefully we can reduce the impact of this approximation. That calls for the next point to work on the termination criterion.

We call the resulting master subproblem as **GM'**. Figure 4-4 depicts this stage by a purple dashed rectangle.

(c) Termination: We observed that the termination criteria of increased objective or infeasible $\mathbf{GM}'$ caused premature termination and inferior solutions for our problem. Therefore, to continue algorithmic progress, we perform a 2-step *local* search, whenever the objective increases or $\mathbf{GM}'$ is infeasible. For this search, we rank the new well locations in the solution from the previous iteration in the decreasing order of their total production amounts over the horizon. We then fix all new wells except the least productive new well. This gives us a *reduced master subproblem* $\mathbf{GM}''$.

As step 1 of our 2-step local search, we solve $\mathbf{GM}''$ to see if any new wells beyond the ones fixed will improve the current solution. If $\mathbf{GM}''$ gives an integer solution, then we get the corresponding production profiles and compute the NPV by solving $\mathbf{GP}_{\boldsymbol{\tau}}$ for that solution. If the NPV increases, then we use it as the incumbent for the next iteration in the OA/ER/AP algorithm.

If $\mathbf{GM}''$ is unsuccessful in giving a better solution, then we execute step 2 of our local search. We perform a partial enumerative search in the neighborhood of the least productive new well to get a better location. For this, we examine the eight grids adjacent to the least productive well as possible locations, and solve $\mathbf{GP}_{\boldsymbol{\tau}}$ for each to get eight new solutions and their NPVs. Note that the locations of the other better wells are already fixed as in $\mathbf{GM}''$. However, unlike step 1, this second step of our search cannot add new wells. If this step 2 finds a better solution, then we use it as the incumbent for the next iteration of the OA/ER/AP algorithm.

If the 2-step local search fails to find a better solution, then we terminate the algorithm. Note that during this local search, we do not revisit solutions that are already examined. As we show later, the local search helps the algorithm continue its progress. The red dashed rectangle in Figure 4-4 represents this 2-step local search.

The original OA/ER/AP algorithm or our algorithm along with its above modifications cannot guarantee a globally best solution for this complex MINLP. However, the same is true for any other evolutionary or gradient-based algorithm [69]. According to Yeten [55], even in the case of evolutionary techniques (widely known as global optimization methods), the global solution can theoretically be achieved only by using either infinite generations or large population sizes, both of which are intractable.

4.4.3 Remarks

Recall that we deliberately replaced the equality of Eq. 20 by two inequalities of Eqs. 21 and 22. While this does increase the numbers of constraints and variables, it has certain crucial advantages. To better appreciate them, consider the linear expansion of Eq. 20 needed for the MILP approximation. Since the derivative of Eq. 20 with respect to $M_{f,n}^{t}$ is zero at $q_n^t = 0$, its linear expansion loses the mobility variables at non-producing grids, which is the majority. However, Eq. 22 allows such derivatives to remain nonzero for non-producing grids with $\mathbf{R}_n^t < 0$ and thus partially retains the effect of the mobility terms.

The other issue is the stability of $\mathbf{GP}_\tau$ solutions. Although not very often, we do face situations when we cannot solve a $\mathbf{GP}_\tau$. In such cases, we first try another NLP solver. If that does not help, then we divide the interval into a few (6 and then 10, if necessary) smaller subintervals, and solve these subintervals one at a time. Once we complete that interval by solving all the subintervals, we revert to our normal procedure for the next interval. Furthermore, since saturation is a particularly sensitive variable, we monitor its changes. If $\max_n(S_n^\tau - S_n^{\tau-1})$ exceeds some acceptable pre-fixed ΔS_{min} at any interval, then we reduce the length of that interval and solve $\mathbf{GP}_\tau$ again.

In spite of its success, our algorithm does have some shortcomings. First, our decomposition of the primal NLP into sequential sub-NLPs along the time domain is a clear limitation, as it modifies the objective function. Instead of maximizing the sum of interval

NPVs over time, it takes the sum of the maximum NPVs at each interval. While this can obviously lead to suboptimal solutions [217]; we can justify it due to the complexity of our model. Second, it is possible and desirable to guarantee smooth production curve [116] by limiting the changes in the variable values at successive iterations. Although we have not done this fully in our study, limiting saturation changes does help smoothen results in general. Last, our master MILP subproblem still poses a challenge for larger problems.

We now use two synthetic reservoirs of different sizes to demonstrate the accuracy and performance of our optimization model and algorithm.

4.5 Case Studies

Each case study involves a synthetic 2D reservoir, for which the oil production for a horizon of 2180 days is to be planned. We assume that each reservoir has some existing active injectors and producers that began operation 380 days ago, when the pressure and saturation maps in the reservoir were uniform. We assume that the horizon consists of the first period of 20 days (with intervals of 5 days) and the remaining periods of 360 days (with intervals of 10 days) for both cases studies. Table 4-1 shows the various petro-economical, geometrical, operational and economical parameters used for both reservoirs. We assume the fluid properties to be identical for the reservoirs.

For comparison, we define a base case for each reservoir, where no new wells are drilled, but the existing wells continue the production. We eliminate the base case solution by using a simple integer cut (Eq. 28). Example 1 is mainly to illustrate the progress and performance of the algorithm at each iteration. It also explains the optimal production plan and compares $\mathbf{GM}'$ with $\mathbf{GM}$. Example 2 studies the optimal production plans and reservoir dynamics for a larger problem and compares its accuracy with ECLIPSE as a commercial reservoir simulator.

We used GAMS 23.7.3 to implement our model and algorithm. The computations were done on a Dell Precision T7500 with two Intel® Xeon® X5690 CPUs (3.47 GHz and 3.46 GHz) and 192 GB of RAM. For the master subproblems, we mainly used CPLEX 12.2.0.2 and if CPLEX could not solve the MILP we would switch to GUROBI 4.5.1. For the primal subproblems, we used CONOPTD 0.1 except in (rare) cases of failure, where we used iPOPT and SNOPT. Please see Appendix B for the list of solver settings and Appendix C for implementation issues. Since our algorithm cannot guarantee a globally best solution, we solved each case study with ten initial guesses on wells and locations. Here, we report only the run that gave the best solution for each case study.

4.5.1 Example 1

A $1882m \times 3184m \times 5.56m$ isotropic pay zone initially has four producers and nine injectors. It is discretized into 20×40 ($I = 20$ and $J = 40$) grids. Figure 4-5a shows the well locations and the reservoir's current oil-in-place (OIP) map, which is after 380 days of production. P1-P4 denote the four existing producers and I1-I9 denote the four existing injectors. Figure 4-6 shows the statistical variations in porosity and permeability. MM\$40 are available for drilling and each new well-drilling costs MM\$5. Thus, at most eight new wells can be drilled. The base case scenario with no new wells has an NPV of MM\$588.7.

Our algorithm yields one new producer well with an NPV of MM\$631.6 and drilling expense of MM\$5. Figure 4-5b depicts the final OIP and the new well location where PN1 denotes the new producer.

Figure 4-7 and Figure 4-8 show the progress of our algorithm. Figure 4-7 shows sequentially the NPVs for solutions (S0-S38) generated during our algorithm. The number in each bar gives the number of new wells and Figure 4-8 shows well locations at iterations 0, 3, and 8. Each vertical dashed line in Figure 4-7 marks the end of a major iteration of the algorithm. S0 corresponds to the base case with an NPV of MM\$ 588.7 and zero new wells.

To initialize our algorithm, we fix eight new wells (Figure 4-8a) at (6,21), (6,37), (11,8), (11, 32), (14,4), (15,21), (16,37), and (17,13) and solve $\mathbf{GP}_\tau$ to obtain S1 in Figure 4-7. The linearizations of S0 form $\mathbf{GM}'$ for the first iteration, whose solution identifies one new well in S1. Since S1 has a slightly higher NPV than S0, we use this as the incumbent solution for the first major iteration of our algorithm. The linearizations of S1 and S2 form $\mathbf{GM}'$ for the second iteration, whose solution identifies S3 with another one new well and slightly higher NPV to shape the current incumbent solution. After the third iteration, the NPV increases again (S4) (Figure 4-8b), however, the forth iteration initially gives us S5 in Figure 4-7 with a worse NPV. The original OA/ER/AP algorithm would have terminated here. But, our algorithm retracts back to S4 (Figure 4-8b) and initiates the 2-step local search. It removes the least productive (in this example, the only) new well at (2, 31) and fixes the remaining four (in this example, the old) producer wells. Then, the solution of the reduced master subproblem $\mathbf{GM}''$ identifies another new well at (11, 31) that yields S6 in Figure 4-7 with better NPV than that of S4. In the next iteration, both $\mathbf{GM}'$ and $\mathbf{GM}''$ fail (S7 and S8 respectively) and hence, our algorithm now executes a neighborhood search around (11, 31), probing the unvisited grids ((10, 32) and (11, 32) were previously visited). It solves $\mathbf{GP}_\tau$ with the six wells located at (10, 30), or (10, 31), or (11, 30), or (12, 30), or (12, 31), or (12, 32), S9-S14 in Figure 4-7 represent these solutions. Of these, S12 has the best NPV, which is even better than that of S6. This ends iteration 5, and allows our algorithm to proceed further to iteration 6 by using S12 as the incumbent solution. In Figure 4-7, each filled circle shows the best solution after each iteration, the unfilled bars denote the solutions during the local searches and each vertical dashed line marks the end of a major iteration of the algorithm. The similar switches between $\mathbf{GM}'$ and the two step local search are repeated in the next iterations until the termination criterion is met at S37 and the algorithm suggests S30 (Figure 4-8c) as the best solution.

As discussed above, the addition of our local search to the OA/ER/AP algorithm was very useful. Such an approach has been used with other optimization methods. For instance, Ciaurri et al. [59], Güyagüler et al. [62], and Yeten et al. [53] all added a local search to their evolutionary search engines. Their local search involved perturbing wells in the neighborhood of a solution. Güyagüler et al. [62] called it local mutation, while Yeten et al. [53] and Ciaurri et al. [59] described it as a heuristic adaptation of the Hook-Jeeves pattern search. All of them reported improvement in the final solution. However, in contrast to these three previous works, our local search has a unique feature that it allows the addition/deletion of wells through $\mathbf{GM}''$ (as we partially saw in this example (S6)). For instance, it may remove one well and instead add more number of wells in other places.

Let us now examine the final solution S30. The algorithm has located the new producer well in the zone with both high initial OIP and good geological connection to the injectors. Moreover, the optimizer has reacted efficiently to the changes during the production horizon. Figure 4-9 clearly illustrates that. After the water breakthrough at about 900 d, the total oil production drops slightly for the first time. However the optimizer increases the liquid production and simultaneously pumps more water into the reservoir; that can finally fulfills the oil demand after another 230 days. In the second event of oil production drop, the reservoir condition changes and it may not be feasible to fulfill the required production demand with the same previous approach. Hence, the optimizer reacts differently by fully supporting the injection and simultaneously *reducing* the total production, probably to provide sufficient pressure support. Indications of success can be seen slightly before the end of production horizon. We will comment on individual well throughputs and changes in the reservoir state variables in the next example.

We also compared the effect of our MILP reformulation (i.e. using $\mathbf{GM}'$ instead of **GM**) on the performance of our algorithm. With **GM**, the algorithm took almost 27 h of clock time for one major iteration when we had to stop. The final NPV was $MM\$617.8$ .However, with **GM′**, we needed only 21 h and 9 iterations to solve the problem with a final NPV of $MM\$631.6$. This shows the utility of our MILP reformulation.

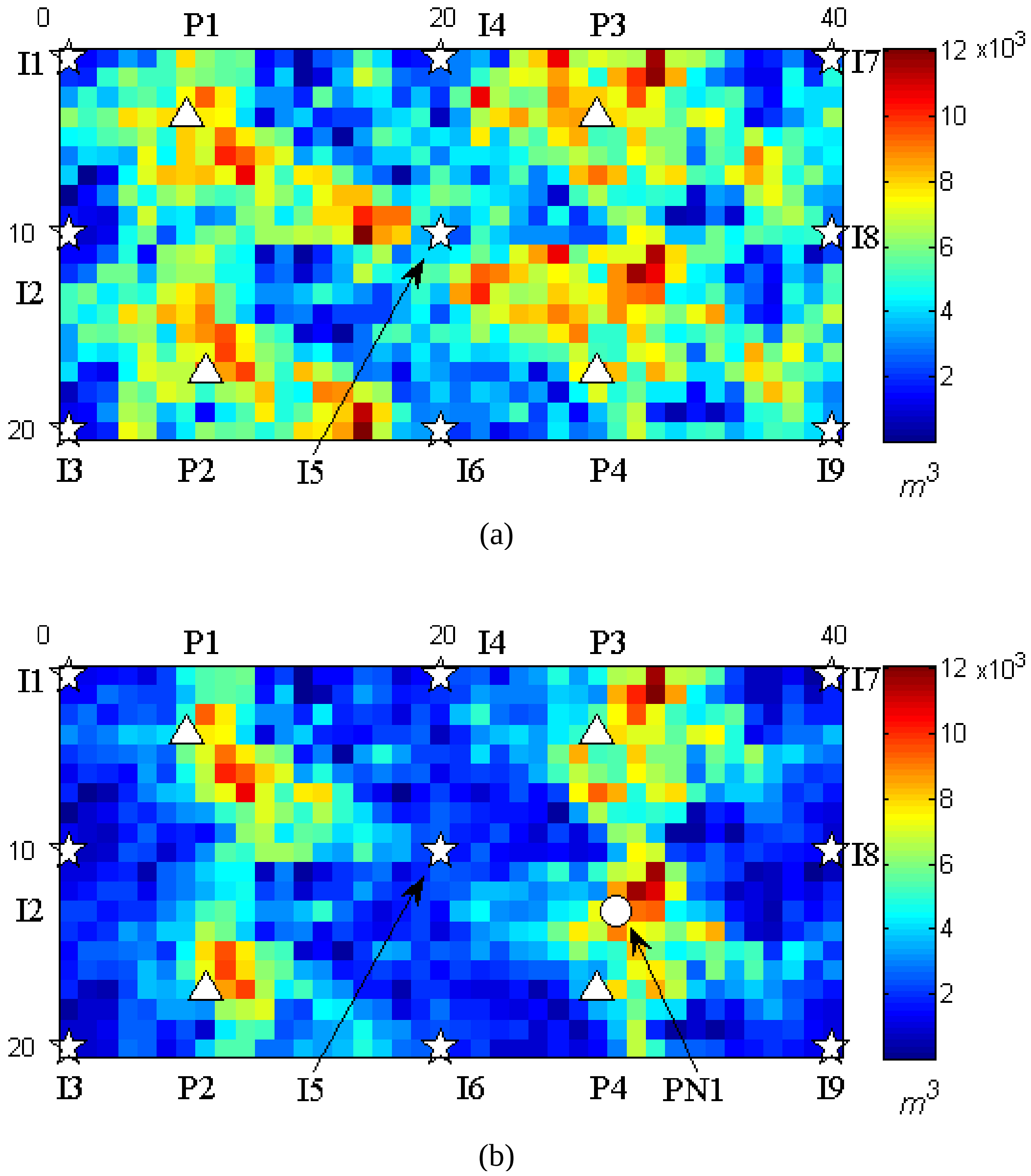
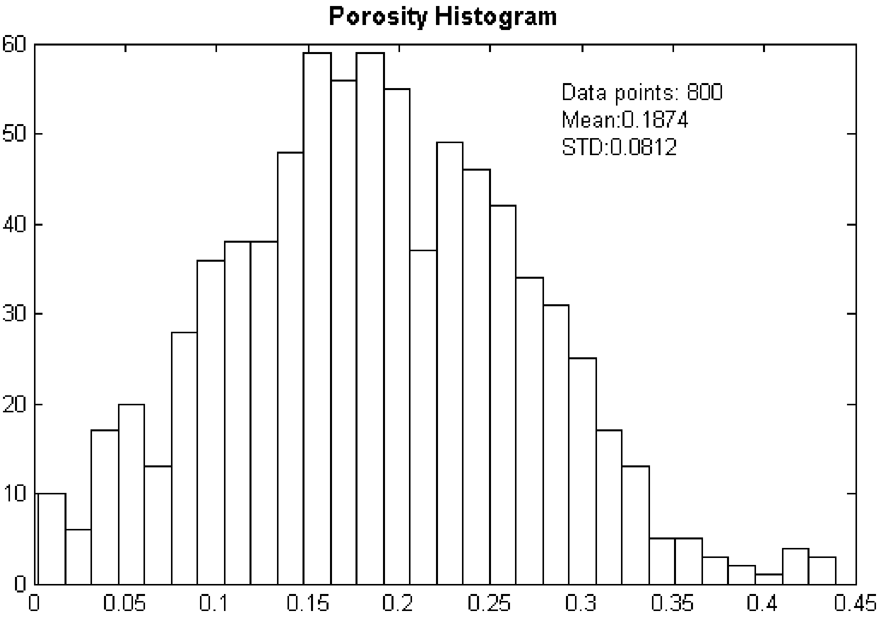
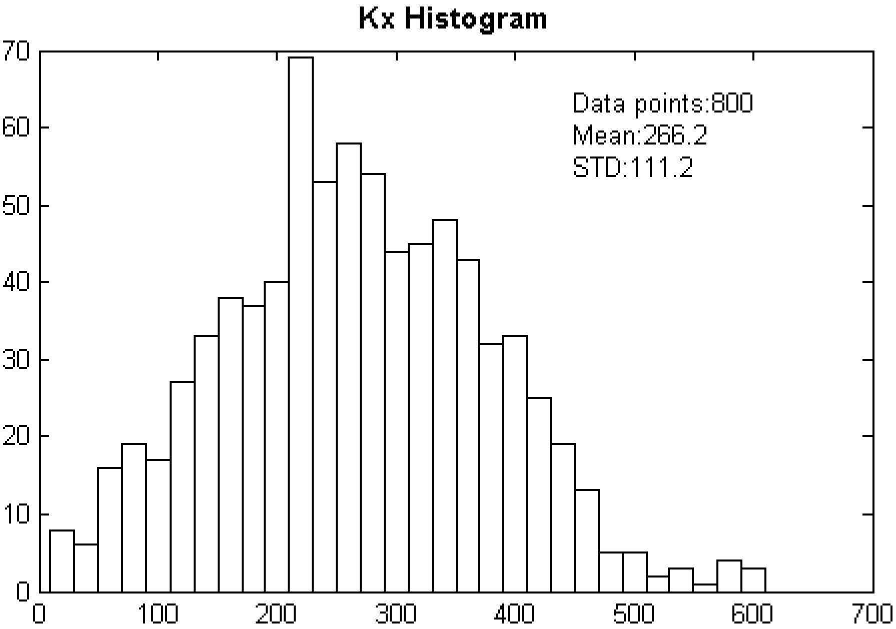


Figure 4-5: Oil-in-place (OIP) (m3) for Example 2 at (a) t = 0 d, (b) t = 2180 d.

Pentagrams, triangles and circles represent the old injectors, old producers, and new producers respectively.



(a)



(b)

Figure 4-6: Variations in (a) reference porosity and (b) permeability for Example 1.

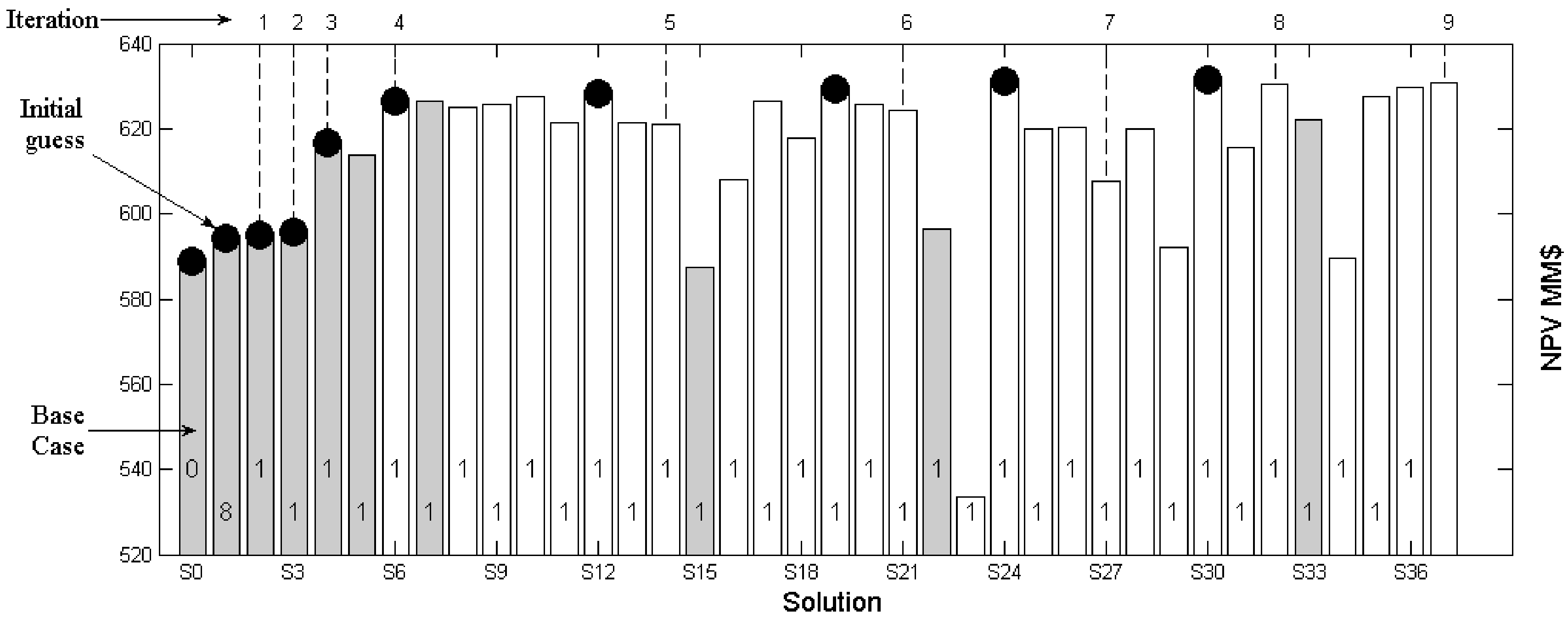


Figure 4-7: The progress of NPV and number of new wells for Example 1 during our algorithm.

Each filled circle shows the best solution after each iteration, the unfilled bars denote the solutions during the local searches and each vertical dashed line marks the end of a major iteration of the algorithm.

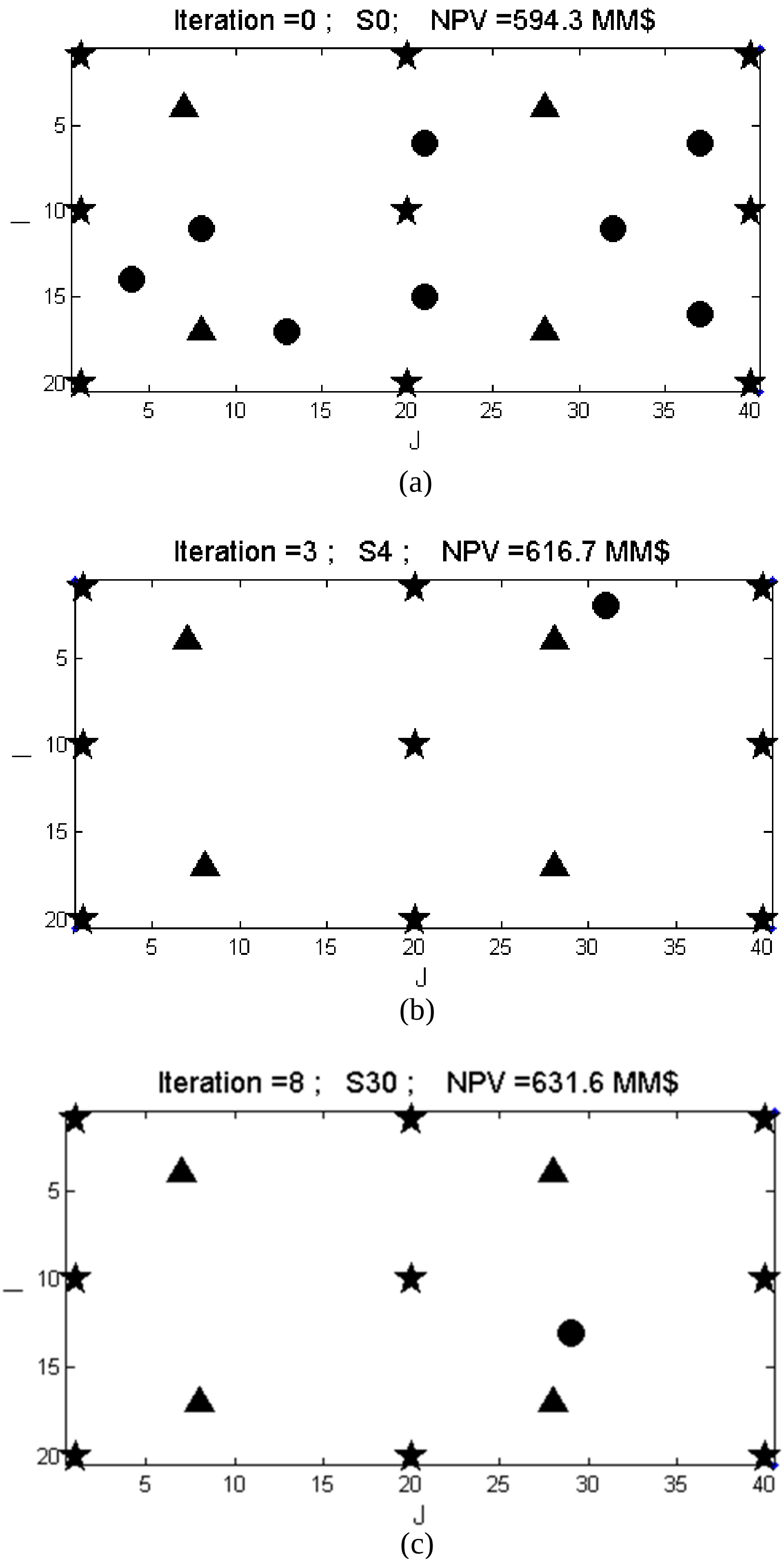


Figure 4-8: The NPV and the relative locations of new producer wells (a) initially, (b) after iteration 3, and (c) after final iteration for Example 1.
Pentagrams, triangles and circles represent the old injectors, old producers, and new producers respectively.

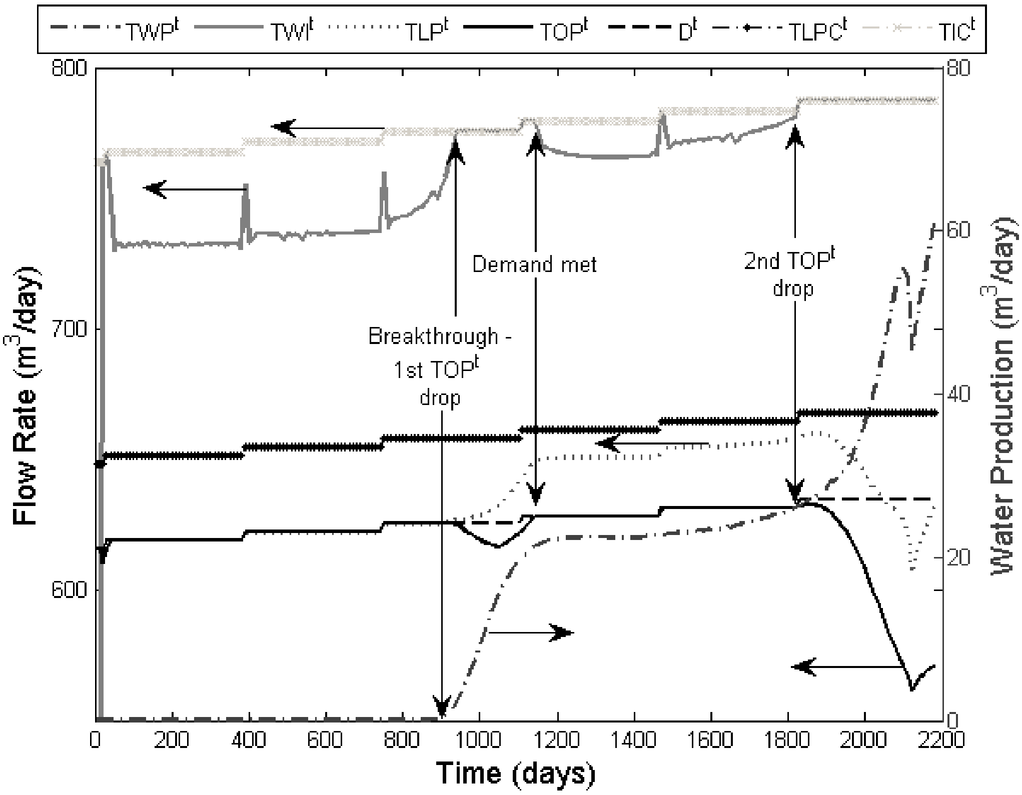


Figure 4-9: Total flow profile in the reservoir of Example 1.

Total water production ($\boldsymbol{TWP^t}$), total water injection ($\boldsymbol{TWI^t}$), total liquid (oil+water) production ($\boldsymbol{TLP^t}$), total oil production ($\boldsymbol{TOP^t}$), oil demand ($\boldsymbol{D^t}$), total liquid production capacity ($\boldsymbol{TLPC^t}$), total water injection capacity ($\boldsymbol{TWIC^t}$).

4.5.2 Example 2

A $2823m \times 6502.4m \times 5.56m$ anisotropic pay zone has been in production with three producers (P1-P3) and five injectors (I1-I5) for the past 380 days. It is discretized to 30×44 ($I = 30$ and $J = 44$) grids and Figure 4-10a shows its current OIP map. Figure 4-11 shows the statistical variations of its porosity and permeability. These geological data are obtained from upscaling SPE10 [218] into dimensions of $30 \times 44 \times 17$. We used the data of the 12th layer that has higher average permeability compared to Example 1. $MM\$55$ are available for new drilling activities. The drilling cost is $MM\$5$ per well, so the team can drill up to 11 new producers. The base case with no new wells has an NPV of $MM\$580.7$.

We used different number of wells (up to 11) and locations as the initial guesses. Amongst all, the test with three initial well locations at (13, 40), (18, 30) and (20,14) achieved the highest NPV of MM\$701.7, almost 22% higher than the base case. Our algorithm suggested three new wells in the vicinity of the initial well locations and needed nearly 12 h of CPU time. Figure 4-10b shows these new producer wells (P1-P3) on the final OIP map. In the remaining part we first check the accuracy of our GP_τ solution for production/injection of these wells and then discuss the production plan profiles to better understand the interactions between the producers and injectors in the reservoir based on GP_τ solution.

In order to assess the accuracy of our discretized model and GP_τ solutions, we can compare the production/injection profiles obtained from our optimizer with those from ECLIPSE. However, ECLIPSE is a simulation package and not an optimization package. Hence, it needs appropriate controls derived from optimization. The GP_τ solutions give us the desired well injection rates, actual liquid (oil plus water) production rates, and pressure and oil/water mobilities in grids. We substitute these in

IPR equation (Eqs. (21) of Chapter 3) to compute BHP_n^t. Now, we use these BHP_n^t as control bounds inside ECLIPSE to restrict the bottom hole pressures in active injector/producer wells. With these, ECLIPSE gives the liquid/oil production and water injection profiles for the entire planning horizon, which we can then compare with our model predictions. Figure 4-12 shows that the profiles obtained from our model (the dashed lines) and ECLIPSE (the solid lines) match very well. The oil in place values are expected to match automatically based on these profiles. The THP values at the producer/injector are also within their specified ranges of $THP_{n \in IW} \leq 50$ bar and $THP_{n \notin IW} \geq 10$ bar.

Now, consider the optimal (individual) well flow rates from Figure 4-12. The production and injection rates are affected between 790 d and 1580 d (zone 1) and 1580 d to 1970 d (zone 2) marked by the dash lines. In the first time zone, water breakthrough occurs at P3 (Figure 4-12b), which triggers a series of changes in other wells. First, I2 and I4, the two nearest injectors to P3, reduce their water injections to slow down excessive water production at P3. However, this weakens the pressure support in that part of the reservoir. To compensate for the loss of oil production at P3, I1 (the farthest injector from P3) increases water injection to expand the waterfront and thus push oil to increase production at P1 and PN1. Meanwhile, the waterfronts from I4 and I3 slowly approach PN3 and PN2 respectively (Figure 4-12-b). This transition in the first time zone is well captured in Figure 4-13, showing the changes in pressure and oil saturation between 790 d and 1580 d. The pressure field changes due to the injectors (and specifically I1 in the northwest) are clearly visible in Figure 4-13a and Figure 4-13b. The saturation map shows that the waterfront breaks through via P3 at 790 d, and it is very close to PN2 and PN3 (Figure 4-13b) at 1580 d.

In the second time zone, PN2 undergoes a water breakthrough at 1850 d, PN3 does the same at 1970 d (Figure 4-12-b). In both instances the optimizer reduces their liquid production to control the water production (Figure 4-12-a). These two along with the earlier breakthrough at P3 reduce the total oil production considerably (Figure 4-12-c). Additionally, I4 goes through a series of on-off injections in its attempts to strike a balance between supplying the driving fluid and avoiding high water cuts at PN3 and P3. However, P1, P2, and PN1 sustain their productions, as they are not too far from or near the injectors. Despite these responses, the reservoir is unable to compensate for the considerable loss in oil production. This situation can be addressed by sequencing the drilling and well-opening operations properly along with the optimal placement of injectors and producers. However, this is beyond the scope of this work.

In brief, the optimizer increased water injections at some places to ensure pressure support and increase oil production, while it decreased injections at other places to avoid excessive water production. In the event of water breakthrough, it also decreased production to contain water in the reservoir.

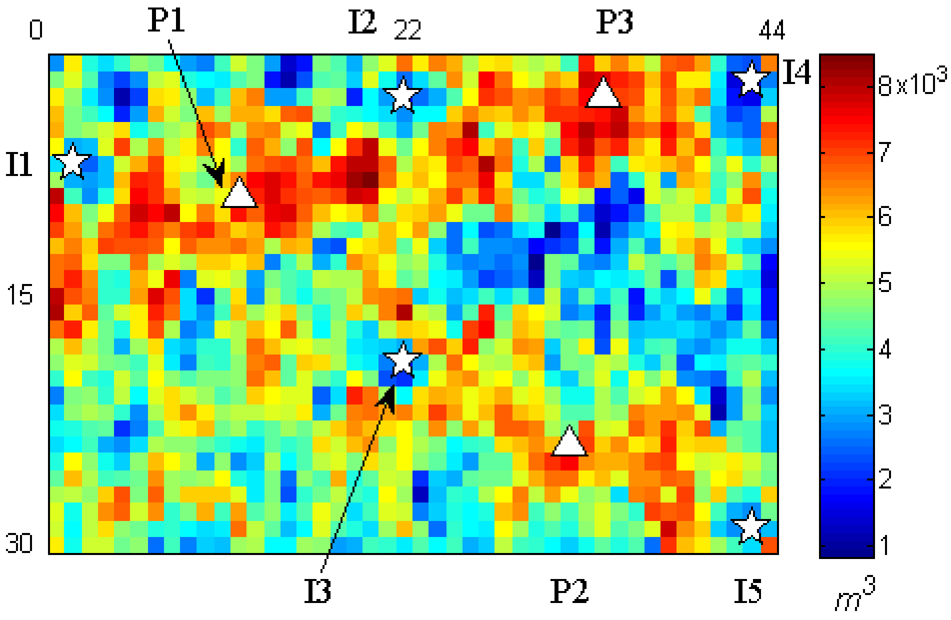
Table 4-1: Parameters for Examples 1 and 2.

Parameter	Value	Parameter	Value
μ_w at 245 bar	0.38 cP	$\alpha_{o,11}$	0.37 x10^{-6}
B_{wr}^0 at 245 bar	1.0 rm^3/sm^3	$\alpha_{o,12}$	0.23 x10^{-3}
c_R at 245 bar	1.0 x10^{-5} bar^{-1}	$\alpha_{o,21}$	0.18 x10^{-6}
S_{or} and S_{wr}	0.2	$\alpha_{o,22}$	0.23 x10^{-3}
kr_0^o and kr_0^w	0.875 , 0.3	$\alpha_{o,23}$	0.802
a and b	2.5 , 2.9	$\gamma_1{}^*$	2.7x10^{-3}
$\overline{B_o}$	1.1749	γ_2	0.5
WC_n	96%	γ_3	1.6805
Well Length	2600 m	γ_4	-7.093
Wellbore diameter	4.5 cm	γ_5	0.5
$\Delta x, \Delta y, \Delta z$	94.1, 79.6, 5.56 m	γ_6	0.3610
ΔS_{min}	0.3	γ_7	-62.181
dP_U	250 bar	γ_8	0.0791
β	0.1	γ_9	-0.0226
c_o	100 $/STB	γ_{10}	0.1716
c_{pw}	1 $/STB	γ_{11}	277.55
c_{iw}	1.5 $/STB	$\gamma_1'{}^{**}$	-2.63x10^{-4}
q_n^L	30 m^3/d	γ_2'	9.2 x10^{-3}
		γ_3'	313.3

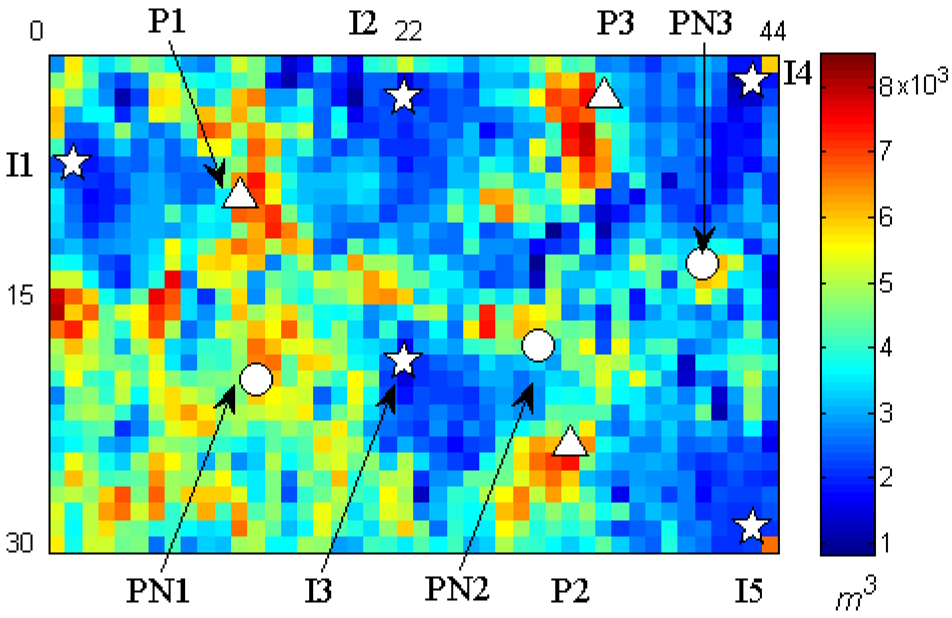
The regressions data are for

* THP_n^L =10 bar; 1≤ $|q_n^t| \leq 450\ m^3/d$

** THP_n^U=50 bar; 1≤ $|q_n^t| \leq 450\ m^3/d$



(a)



(b)

Figure 4-10: Oil-in-place (OIP) (m3) for Example 2 at (a) t = 0 d, (b) t = 2180 d.

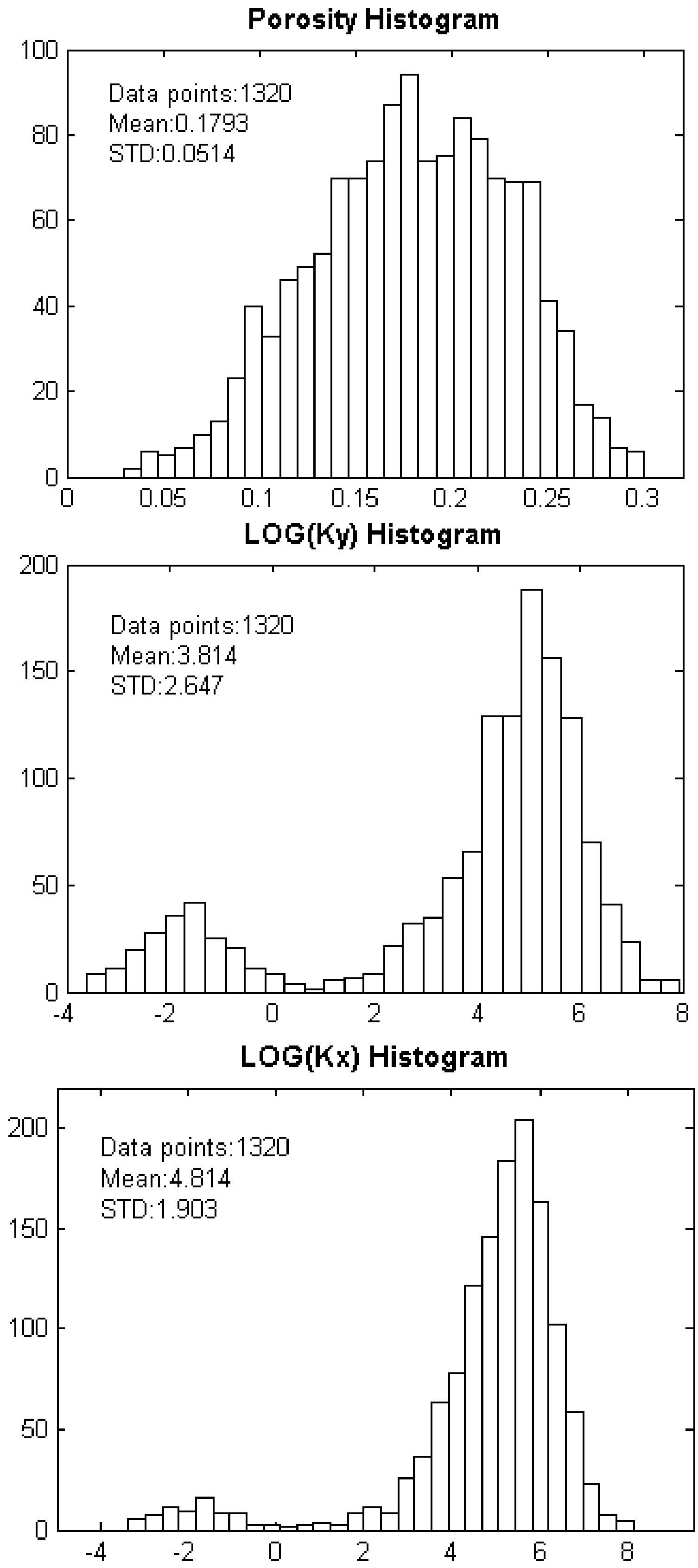
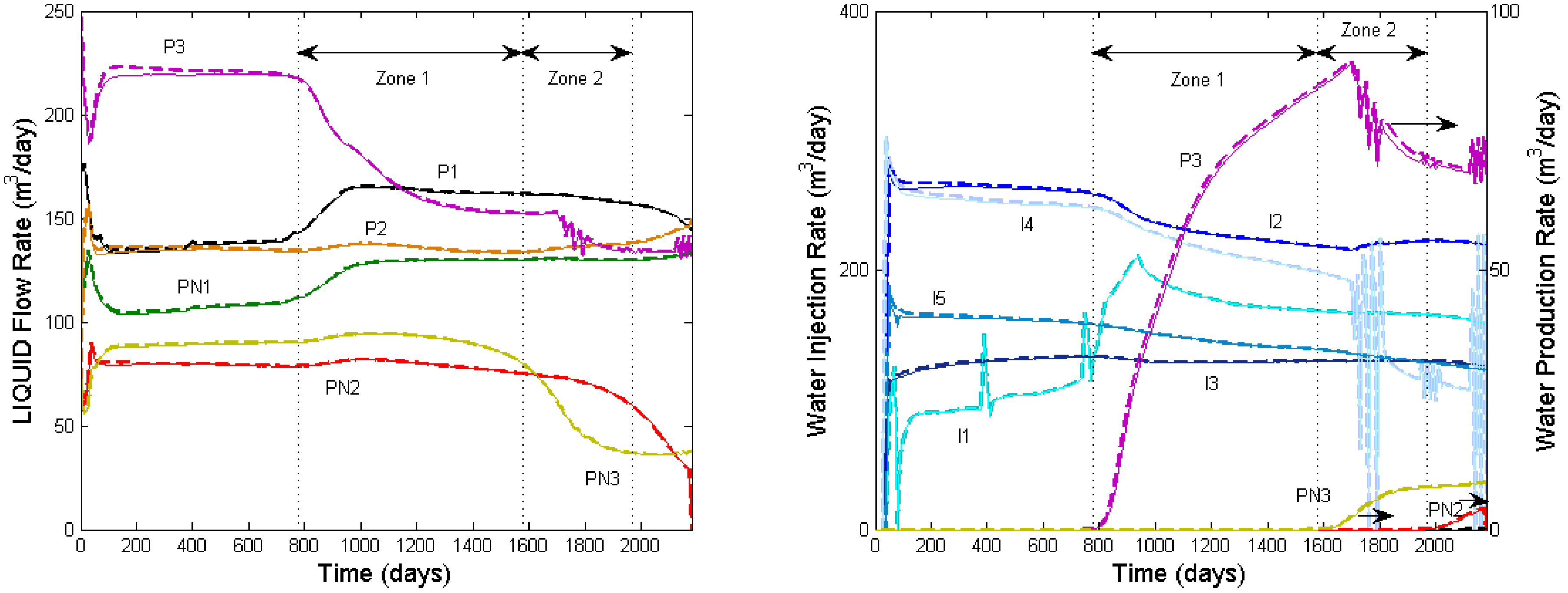


Figure 4-11: Statistical distributions of reference porosity and log-permeability (log(mD)) for Example 2.



LIQUID Flow Rate (m³/day)
Time (days)
P3
P1
P2
PN1
PN2
PN3
Zone 1
Zone 2
Water Injection Rate (m³/day)
Water Production Rate (m³/day)
I1
I2
I3
I4
I5

(a) (b)

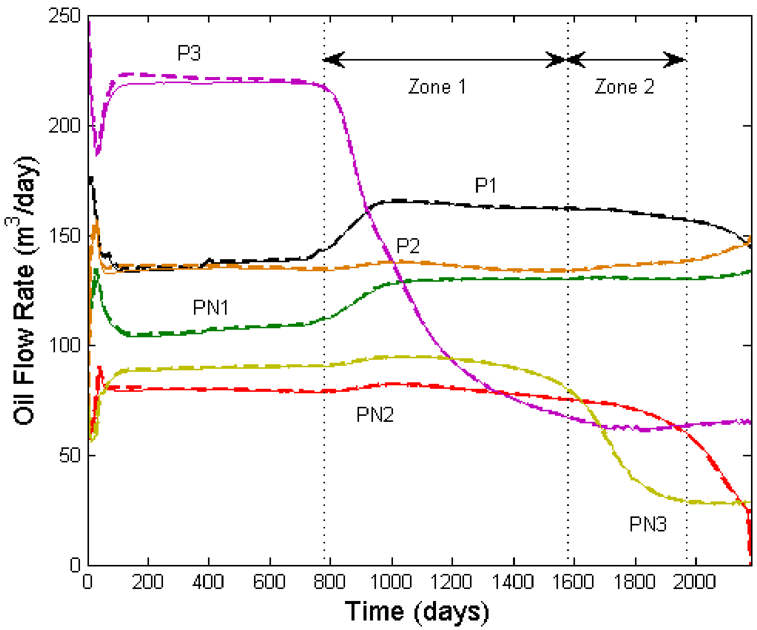
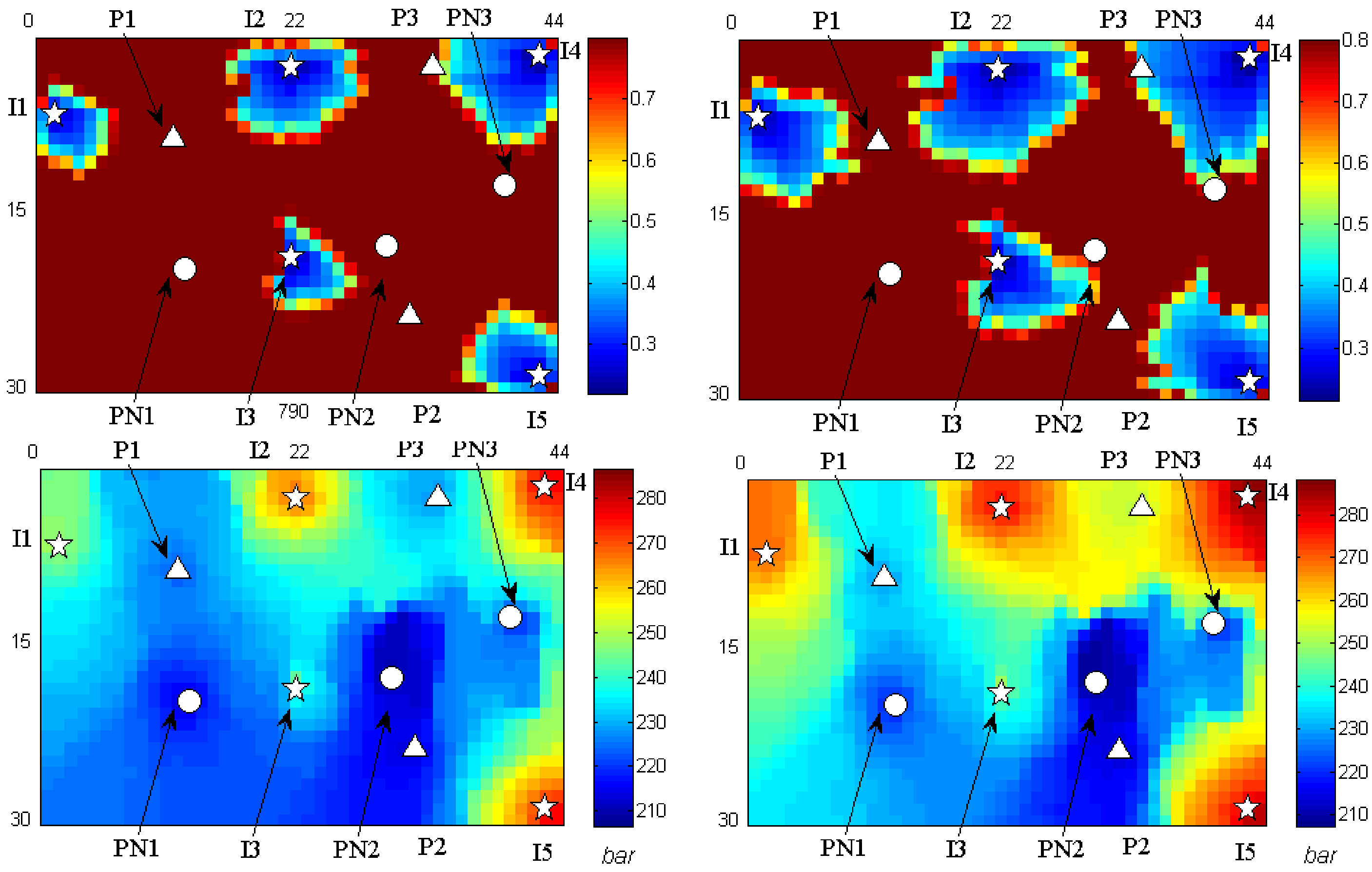


Figure 4-12: Production and injection profiles – Accuracy comparison of $\mathbf{GP}_\tau$ with ECLIPSE as a commercial simulator for Example 2.

(a) total (oil plus water) flows, (b) water flows (injection and production) and (c) the oil production. The solutions from $\mathbf{GP}_\tau$ and ECLIPSE are shown with dashed and solid lines respectively.



(a) 790 d

(b) 1850 d

Figure 4-13: Pressure and saturation field of Example 2 at (a) 790 d and (b) 1580 d.

The top and bottom rows show the oil saturation (also waterfront) and pressure fields respectively. The data are captured from our **GP$_{\tau}$** solution.

4.6 Summary

In this chapter, we presented an integrated and practically useful model for oil well placement and production planning in a petroleum reservoir, and proposed an effective solution algorithm for the same. The major contribution of our work is that we considered subsurface flow dynamics much more rigorously than any other previous study. Most studies thus far on mathematical programming neglected or grossly approximated this dynamics. In fact, none of them, to our knowledge, considered optimal production plan using such a detailed spatiotemporal model. Thus, this is the first contribution to integrate most of the critical elements of upstream production and spatiotemporal subsurface dynamics in a multiperiod mathematical programming approach. Furthermore, in contrast to most previous work, our approach does not require pre-fixing wells and locations or production/injection rate patterns.

We also successfully tailored and modified the OA/ER/AP algorithm to improve its success in solving this large and complex problem and improving its performance. Our modifications of the primal NLP and master MILP subproblems along with a 2-step local search before termination were critical in ensuring progress and good solutions for the two illustrative examples.

While much further work is needed to address the size and complexity of this important problem, we have taken the first step in rigorously applying the powerful and versatile technique of mathematical programming and addressing some of the challenges associated with the industry-scale well placement problem.

4.7 Appendices

Appendix A: Solver settings in GAMS

We used an infeasibility tolerance of 10^{-4} in GAMS for both the NLP and MILP solvers. Other settings for specific solvers are as follows:

A. GUROBI : feasibilitytol=0.0001; optimalitytol=0.0001; solvefixed=0; threads=2

B. CONOPT: RVHESS=1000000; LFSTAL=200;

C. CPLEX: advind=0; relaxfixedinfeas=1; solvefinal=0; startalg=2; threads=2

D. IPOPT: mumps_mem_percent=20000;

Appendix B: Some implementation details for our algorithm

In order to avoid the unstable region of the vertical flow performance relation while solving $\mathbf{GP}_\tau$, we shut producing wells with $q_n^t \le q_n^L$, where q_n^L is a low flow rate. This is only done for wells with nonzero cumulative production to avoid killing a well (on stand-by) which has not yet started the production. Similarly, we shut highly water flooded producers with $|WC_n^t - WC_n| \le 10^{-3}$. In both cases we remove the shut well from $\mathbf{AW}$(n) set.

Instead of using $signum(\boldsymbol{\lambda}_{r'})$ in $\boldsymbol{\theta}_{r'} \ge signum(\boldsymbol{\lambda}_{r'}).[\boldsymbol{h}(\boldsymbol{x}_{r'}) + \nabla\boldsymbol{h}(\boldsymbol{x}_{r'}) \cdot (\boldsymbol{x} - \boldsymbol{x}_{r'})]$ of Eq. 28, we define $\Lambda_{r'}$ as

$$\Lambda_{r'} = \begin{cases} 1 & \lambda_{r'} \ge 0 \\ -1 & \lambda_{r'} < 0 \end{cases}$$

and we set $\boldsymbol{\theta}_{r'} \ge \Lambda_{r'}.[\boldsymbol{h}(\boldsymbol{x}_{r'}) + \nabla\boldsymbol{h}(\boldsymbol{x}_{r'}) \cdot (\boldsymbol{x} - \boldsymbol{x}_{r'})]$.

For the MILP master subproblem $\mathbf{GM'}$ (Eq. 38), we used $r^L = r - 1$. In other words, we used the linearizations from the last and current iterations only.

To solve $\mathbf{GM'}$, we need the terms at each period t in the mass balance equation 30. For this, we need information such as pressure and saturation and their derivatives from period $(t-1)$. Because, we solve the NLP primal subproblem in terms of intervals rather than periods, it is not clear what values we should use for these terms. In this work, we used a very rough approximation as follows. Let $\tau = \sum_{t' \le t} \mathcal{T}_{t'}$ denote the last interval in period t. Then, for the pressure, saturation, and their derivatives at $(t-1)$, we use from their values at $(\tau - 1)$. Thus, we write Eq. 30 as:

$$\theta_{f,n,r'}^{t} \ge$$

$$signum\left(\lambda_{f,n,r'}^{t}\right) \cdot \left(\begin{array}{c} \frac{\partial \mathrm{A}_{f,n,r'}^{t}}{\partial P_n^t}\left[P_n^t - P_{n,r'}^t\right] + \frac{\partial \mathrm{A}_{f,n,r'}^{t}}{\partial S_n^t}\left[S_n^t - S_{n,r'}^t\right] + \\ \frac{\partial \mathrm{A}_{f,n,r}^{t}}{\partial P_n^{\tau-1}}\left[P_n^{t-1} - P_{n,r'}^{\tau-1}\right] + \frac{\partial \mathrm{A}_{f,n,r'}^{t}}{\partial S_n^{\tau-1}}\left[S_n^{t-1} - S_{n,r'}^{\tau-1}\right] + \\ \left[q_{o,n}^t - q_{o,n,r'}^t\right]_{n \notin \mathbf{IW}} + \left[q_n^t - q_{n,r'}^t\right]_{f=w} + \left[F_{f,n}^t - F_{f,n,r'}^t\right] \end{array} \right)$$

$$r^L \le r' \le r \qquad \text{(B-1)}$$

CHAPTER 5 WELL AND INFRASTRUCTURE PLACEMENT, ALLOCATION AND PRODUCTION PLANNING IN MULTI-RESERVOIR OIL FIELDS WITH SURFACE FACILITY NETWORKS [9,10]

5.1 Introduction

The world population has increased from almost 5.4 billion in 1990 to about 7.3 billion in 2012. The global energy demand has thus been constantly increasing [219]. With the advent of new drilling technologies such as horizontal drilling and hydraulic fracturing and discoveries of shale gas [220], fossil fuels such as oil and gas will continue to be the major energy resources for the world. Thus, it is critical to exploit these limited resources (both existing and new) in a wise, efficient, and cost-effective manner.

A large oil and gas field has three main elements: (1) the porous subsurface formation, (2) the well strings that bring the oil from the subsurface to the surface, and (3) the surface network that gathers and processes oil from the various wells. Cost-effective production of oil requires optimal drilling and operating of wells and optimal integration of the three field elements. Well-drilling is expensive and poses a considerable financial risk. Up to 60% or more of the

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[9] Tavallali, Karimi, Baxendale, Halim and Teo (2013). Well and Infrastructure Placement, Allocation and Production Planning in Multi-Reservoir Oil Fields With Surface Facility Networks. Journal of Industrial & Engineering Chemistry Research – Under review

[10] Tavallali, A Halim, Karimi and Teo (2013). Producer Well Placement for Integrated Multi-Reservoir Oil Fields. 2013 AICHE Annual Meeting. San Francisco, CA - USA.

capital expenditure (CAPEX) on an exploitation project might be due to well drilling and associated activities alone. Yet, once a well is drilled and operated successfully, it can provide huge revenue, and return the initial investment. Therefore, optimal well placement is a critical step that must consider the integration of the subsurface and surface elements.

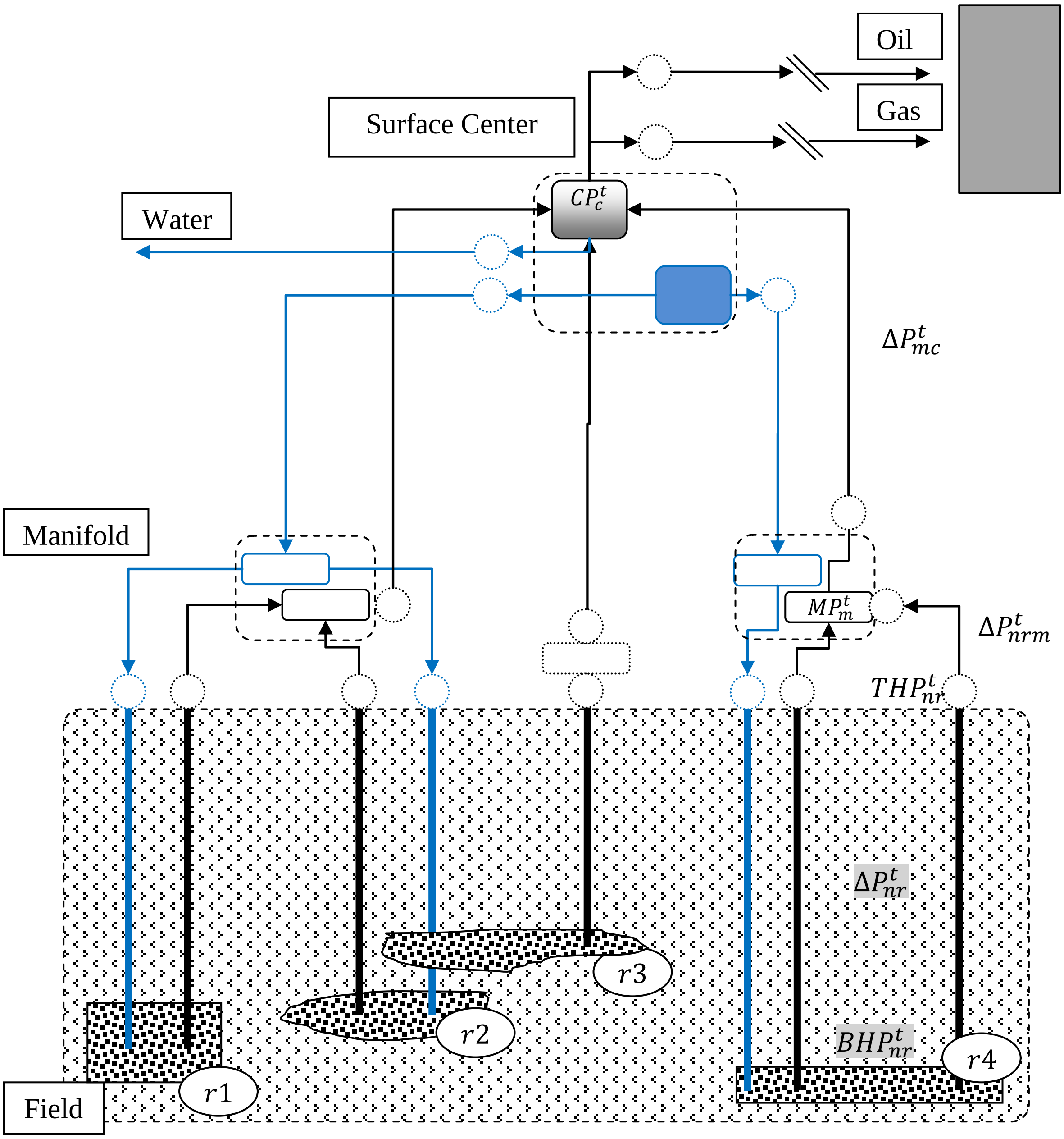

Figure 5-1: A schematic diagram of sub-surface and surface components of a hydrocarbon field.

The thick black and blue lines represent the production and injection wells respectively, r1, r2, r3 and r4 are four reservoirs (with possibly different production mechanism) in this field.

5.1.1 Oil fields & production challenges

Figure 1-1 shows a schematic for a multi-reservoir field with surface facility network. The surface network includes wellheads, manifolds (headers) that collect/mix the oil/gas flows from the wellheads, processing centers that receive the commingled flows from the manifolds, a maze of valves and flow-lines that interconnect the wellheads, manifolds, and processing centers, and finally long pipelines that supply the fluids to the market after pressurization.

In addition to the local subsurface pressure inside the reservoir, the production rate from a well depends on the various pressure levels in the surface network. In this regard, the bottom hole pressure (BHP) at the bore hole in the well and the tubing head pressure (THP) at the well head are critical [221]. Since the wells may share manifolds, and the manifolds may share processing centers, the production variations at wells connected to the same flow path can alter the productions at other wells. Thus, when new producer wells are drilled to increase/sustain the production rates, and connected to the surface network, the conditions of the surface network may change drastically. These changes may even necessitate changes in the design and operation of the entire surface network. Therefore, it is crucial to consider both subsurface and surface conditions and their dynamic variations for optimal well drilling and placement.

Well placement in a single reservoir [221] is already a complex and difficult problem. In practice, fields often have multiple reservoirs that share common surface infrastructure and production facilities. These facilities, as described earlier, strongly interlink the operations of these reservoirs. Since each reservoir can no longer be studied separately, the well placement and production planning becomes even more challenging for various reasons. Firstly, each reservoir may have different geological characteristics and production mechanism. While one reservoir might be producing via a secondary or tertiary mechanism, another might be using the primary mechanism. Treating these separate reservoirs as one aggregated reservoir with several inactive portions is very inefficient [44]. Secondly, the surface settings would depend on the conditions of the connected wells from multiple reservoirs. Clearly, addressing well placement jointly with the design and planning of surface facilities at the ***field level*** - rather than the individual reservoir levels - is of paramount importance, and poses special challenges. Lastly, the complex multiphase flow regimes in the subsurface, wells, and surface network add even more complexities, and make integrated reservoir management even more challenging.

5.1.2 Literature review

Upstream production in oil and gas industry involves long-term planning over several years. Long-term horizon problems include the capital investment assessments for infrastructure design and well placement activities on a time span, whereas shorter horizon problems include the operation planning problems with operational cost/revenue such as well production planning, gas lift studies, flow routing from wells to manifolds and they are subject to surface network facilities on a daily / weekly basis. In an oil field, the reservoir engineers handle the subsurface issues, and production engineers handle the surface tasks. Well drilling decisions are amongst the long term horizon problems, and production planning subject to surface facility network is very close to (and even can be absolutely within) the domains of short horizon planning. However, an ideal design must integrally consider both horizons [222]. In the following we briefly discuss some of the studies on long term planning (for well placement, and production planning subject to surface network) and the short term planning (daily production planning, surface routing, etc.).

Well placement is an important instance of long term planning problems. In the literature, there are numerous studies addressing well placement with only focusing on the reservoir itself without almost any connections to the dynamics of the surface network facilities. Evolutionary and direct search techniques [53, 63, 66, 68, 81, 198, 223, 224] have attracted the majority of such studies followed by few gradient based optimization approaches [139, 141, 145, 199]. In fact, most of these studies even do not consider optimal production planning when subsurface media is segregated from surface network. The production planning is left to the reservoir simulator which uses a set of useful heuristics for simulating the production life subject to limited production/injection rates or bounded BHP/THP. However, this trend is changing and more and more studies pay attention to the well placement jointly with production planning [225, 226], yet we have not observed any solid studies in this category that considers the dynamic effects of surface network. It is possible to extend these works from reservoir to field level; however the surface network should be included. In fact ECLIPSE reservoir simulator offers reservoir coupling and network option which uses master-slave heuristic. In a rather different approach, few other contributions using mathematical programming have employed reservoir's static data as a well screening tool [125, 129, 130], and naturally cannot reflect the dynamics of the problem.

In contrast to the above category, there are studies that have addressed well placement and production planning (and some) *subject to the surface network facility* [116] on long term horizon. They are mainly within mathematical programming framework and are multi-period mixed integer (non) linear programming models (MILP / MINLP). In general they address upstream infrastructure planning with decisions and plans regarding well drillings, capacity determination, platform installations, well-to-platform allocation and production profile. These studies have not considered fluid injection (which is a usual field activity) and moreover they have grossly approximated the subsurface flow which may not be applicable for well placement studies [184]. In such cases, there is no meaningful distinction between single reservoir and multi-reservoir (field) subsurface models. Some of these empirical approximations include influence function and superposition [124, 128], piecewise linear [116] and nonlinear approximations [132] of reservoir pressure and gas-oil-ratio (GOR) versus cumulative oil production. Additionally, in different cases the multiphase flow in the surface flow-lines is coarsely approximated by linear pressure drop vs. flow [116, 120]. Despite these simplifications the above contributions are important as they address a broad range of surface design applications.

The aforementioned two categories cover the long term planning. When short-term scheduling on a scale of days or weeks is studied, there is usually no capital cost involved, reservoir dynamics is ignored and the studies focus on surface operation, production planning and flow-lines routing. Apart from other methods [47, 217, 227], mathematical programming has contributed several studies to this field. Usually, these models integrate the multiphase flow from wellbore into well tubing, thorough well string, choke valves, manifolds and flow lines, and finally into separation unit. If flow routing is addressed the model becomes MILP/MINLP. Since the reservoir dynamics is not included, two key elements in comparing these contributions are (a) the way they tackled the multiphase flow in the flow-lines (b) the algorithm used for optimization. Kosmidis et al. [228] developed an MINLP model for the production planning and flow routing in fields subject to gas lift on a daily basis, and solved the model by an outer approximation algorithm. They applied piecewise linear approximation to approximate the well and flow line momentum balances and calculated the pressure drop of choke valves in a post-processing step. In contrast, Barragán-Hernández [229] studied the integrated surface production planning for a fixed topology on a daily basis. They compiled a detailed set of equations to model the flow in the well and surface network. Later, Gunnerud and Foss [230] developed an MILP model for the production planning and surface routing of an offshore multiphase field for a short period. They used piecewise linearization to approximate the pressure drops and applied two decomposition methods (Lagrange relaxation and Dantzig-Wolfe) to solve the model. In another study, Gunnerud et al. [231] modeled the surface routing and production planning in a multiphase field for a short period using an

MILP frame. The also used piecewise linearization to approximate the pressure drop equations and also largely approximated the pressure effect at the inlet. Finally, they decomposed the main MILP into smaller problems and applied so-called column generation in branch & price framework to solve those sub-problems. Similarly, Codas et al. [232] studied the short term planning of the flow routing and production in an oil and gas field. They approximated the (multi-phase flow) pressure drop in pipes by (multidimensional) piecewise linearization technique. They considered both multiphase and one phase (gas) flow pressure drops. For generating the initial pressure drop data, they assumed a nominal pressure at the pipeline outlet based on prevailing conditions and later assessed that by an error analysis. They allowed at most one outlet from each manifold and stressed the need for considering multiple outlets (that would complicate the formulation). To overcome some of the complexities of their MILP problem, they applied branching priories, reduced the size of piecewise-linear approximations and excluded the infeasible flows by valid inequalities. This work provides a good industrial overview of the surface studies. Yet, few of their assumptions are questionable. Mainly, they artificially impose positive drawdown pressure at wells. However, it is obvious that some of the wells lose their production capability when the well cannot provide enough support to push the flow upward.

In brief, optimal well placement, facility design and production planning subject to surface network facilities are complex mixed horizon planning and design problems. Most of the available studies either have not used rigorous multiphase flow reservoir models, or have not included the surface facility network effects. Moreover, in most cases, this problem is either not addressed at the field wide level, or it is addressed by significant compromise on governing equations. The industry has already identified the importance of such integrated studies and software packages such as AVOCET by Schlumberger [233] are working in that direction. The complexity and curse of dimensionality is a big obstacle to that. In the previous chapter, we addressed joint well placement and production planning in a *single* reservoir with a rectangular shape, and focused on rigorous reservoir model and (multiphase) well flow up to the well head. Although we did not consider the surface network dynamics, this problem was already a challenging dynamic optimization problem requiring a spatiotemporal and dynamic nonconvex MINLP model. The complexity of the model motivated us to modify an outer approximation algorithm to solve our model. That work provides a good platform for our current study.

The aim of this work is to extend our previous single-rectangular-reservoir study [221] to address well placement / surface network design and production/injection planning in a field with multiple irregular-shaped reservoirs supplying to a shared surface production-network facility. We simultaneously address all the dynamic, economic, and operational inter-dependencies of the entire field and its reservoirs. Our deterministic model holistically includes the entire field financial considerations and market demand, dynamic and structural constraints in a surface network of well-manifold-separators, and provides drilling /network design decisions on a long term horizon and detailed production/injection plan on several short horizons (integrated into the main long horizon). The remaining of this article is organized as follows to achieve these goals. First, we define and describe the scope of our problem. After that, we discuss our modeling approach and concisely describe some important remark on the solution strategy. Then, to assess our approach we present a case study and conclude with a discussion.

5.2 Problem Statement

It is desired to initiate or increase oil production from a multi-reservoir hydrocarbon field (Figure 3-1) by drilling new/infill vertical producer wells and installing the required surface network (manifolds, processing centers, valves, flow-lines, etc.). If the field is already producing, then it may have some existing wells, manifolds, and processing centers. In this case, the surface network may need changes to accommodate new wells and production. The well placement problem is then as follows:

Given:

- Geological information for each reservoir such as location, dimensions from seismic studies or history matching, porosity, and permeability.
- Pressure and saturation profiles in each reservoir at time zero.
- Pressure-Volume-Temperature (PVT) related information such as viscosity, density, compressibility, surface tension, and formation volume factor (from core samples or previous production data) for each reservoir.
- Existing wells (if any), their locations, types (producer vs. injector), diameters, lengths, tubing roughness, etc.
- Potential/existing manifolds, their locations, capacities, maximum numbers of connections, and existing/potential well-to-manifold and manifold-to-separation center allocations.
- Operational data such as the required inlet pressures at the separation centers, separation-center capacity for each phase, water-cut limits, and incremental capacity expansion plans for surface facilities.
- Field production horizon of H years.
- Relevant economic data such as drilling budget and costs, manifold installation cost, costs per well-to-manifold and manifold-to-center connections, injection costs, oil/gas revenue forecasts, discount rate, demand curve, etc.

Obtain:

- Number and locations of new producer wells (and hence the reservoirs to be exploited) and their production profiles
- Number and location of manifolds and processing centers

- Potential well-to-manifold, well-to-surface-center, and manifold-to-surface-center allocations
- Throughput profiles for all producer/injector wells, flow-lines, manifolds, and processing centers
- Dynamic pressure profiles along the network at processing centers, manifolds, wellheads, well bore holes (and corresponding valve settings)
- Dynamic pressure and saturation profiles for each reservoir

Aiming to maximize the net present value (NPV) of oil/gas production over the planning horizon.

Assuming:

- The reservoirs are horizontal and planar. They may overlap, but they are disconnected. Field surface elevation may vary from point to point.
- Wells are vertical, and can pass through multiple reservoirs, but can be perforated to access only one reservoir.
- A wellhead may be connected to one or more manifolds/centers.
- The manifold capacities remain constant over time.
- Each well (existing or potential) is pre-allocated to some manifolds/centers (existing or potential) based on some criteria such as distance, from which the best allocations will be selected. This can be relaxed but will increase computation time. [116]
- Capillary pressure in the reservoir is negligible and the reservoir is under-saturated (i.e. its pressure exceeds bubble point pressure).
- All phases (oil, water, and gas) are compressible. The driving mechanisms in the field can be water-drive injection and/or primary expansion. Each reservoir might have different pressure and saturation distribution; however all have the same fluid.
- Existing well-to-manifold and manifold-to-center connections continue to exist.

Following:

- All new wells, manifolds and surface centers begin operations simultaneously at time zero.
- A well that hits its water-cut limit is shut in.
- Each well must be beyond some minimum distance from all other wells.

Allowing:

- Reservoirs may have arbitrary and irregular shapes.
- Processing center capacities may vary with time.
- Existing manifolds and centers can make/receive new connections.
- Processing centers can receive fluids from wells directly or manifolds.
- Central water processing units supply water to injector wells directly or via shared manifolds.

5.3 Formulation

Let R $(r = 1, 2, \dots, R)$ denote the number of reservoirs, C $(c = 1, 2, \dots, C)$ denote the number of processing centers (existing/potential), and M $(m = 1, 2, \dots, M)$ denote the number of manifolds (existing/potential). Of these, we assume that the first M_e manifolds $(1 \le m \le M_e)$ and first C_e processing centers $(1 \le c \le C_e)$ already exist. Following the discretization approach that was presented in Chapter 3, we discretize the spatial coordinates. To this end, we use the field's geological map to define the tightest possible rectangle around each r. Then, we discretize each rectangle r by defining I_r cells of arbitrary lengths Δx_{ir} $(i = 1, 2, \dots, I_r)$ in the x-direction and J_r cells of arbitrary lengths Δy_{jr} $(j = 1, 2, \dots, J_r)$ in the y-direction. Then, we name each cell in a reservoir by a single index, $n = i + (j - 1) \times I_r$, and define three sets for each reservoir as follows:

$\mathbf{IP}_r = \{n \mid \text{cell } n \text{ belongs to reservoir } r\}$

$\mathbf{IX}_r = \{n \mid n \in \mathbf{IP}_r, (n + 1) \in \mathbf{IP}_r, \text{and } i < I_r\}$

$\mathbf{IY}_r = \{n \mid n \in \mathbf{IP}_r, (n + I_r) \in \mathbf{IP}_r, \text{and } j < J_r\}$

$\mathbf{IP}_r$ excludes from the subscribing rectangle the cells that do not belong to reservoir r, $\mathbf{IX}_r$ excludes the reservoir's border cells in the x-direction, and $\mathbf{IY}_r$ excludes them in the y-direction.

A typical water-drive reservoir will have two types of wells. The ones producing fluids from the reservoirs are called producer wells, and the others that inject water into the reservoirs are called injector wells.

$\mathbf{PW}_r = \{n \mid n \in \mathbf{IP}_r \text{ is an existing producer well}\}$

$\mathbf{IW}_r = \{n \mid n \in \mathbf{IP}_r \text{ is an existing injector well}\}$

$\mathbf{NW}_r = \{n \mid n \in \mathbf{IP}_r \text{ is a potential location for a new producer well}\}$

$= \mathbf{IP}_r - \mathbf{PW}_r - \mathbf{IW}_r - \{n \in \mathbf{IP}_r \mid \text{a new producer well is infeasible at cell } n\}$

Similarly, it will have two types of manifolds and processing centers. The ones associated with processing oil/gas are called production manifolds or oil production centers, and the ones associated with treating and injecting water are called injector manifolds and water treatment centers. To differentiate among these different types of manifolds and centers, we define:

$\mathbf{PM} = \{m \mid m \text{ is an existing/potential producer manifold}\}$

$\mathbf{PC} = \{c \mid c \text{ is an existing/potential oil processing center}\}$

Thus, if $m \notin \mathbf{PM}$, then m is an injector manifold, and if $c \notin \mathbf{PC}$, then c is a water treatment center.

Lastly, to manage well-manifold, well-center, and manifold-well connections, we define:

$\mathbf{M}_{nr} = \{m \mid \text{cell n of reservoir r is or can be connected to manifold } m\}$

$1 \le r \le R, n \in \mathbf{IP}_r$

$\mathbf{C}_{nr} = \{c \mid \text{cell n of reservoir r is or can be connected to center } c\}$

$1 \le r \le R, n \in \mathbf{IP}_r$

$\mathbf{C}_m = \{c \mid \text{manifold } m \text{ is or can be connected to center } c\}$

$1 \leq r \leq R, n \in \mathbf{IP}_r$

Unless stated otherwise, all variables and constraints involving index t (defined later), r, m, and c are to be written for all their valid values, i.e. $1 \leq t \leq T$, $1 \leq r < R$, $1 \leq m < M$, $1 \leq c < C$.

5.3.1 Drilling and infrastructure design decisions

We define the following binary variables.

$$u_c = \begin{cases} 1 & \text{if center } c \text{ should exist} \\ 0 & \text{otherwise} \end{cases}$$

$$z_m = \begin{cases} 1 & \text{if manifold } m \text{ should exist} \\ 0 & \text{otherwise} \end{cases}$$

$$y_{nr} = \begin{cases} 1 & \text{if a well should exist at cell } n \text{ of reservoir } r \\ 0 & \text{otherwise} \end{cases} \qquad n \in \mathbf{IP}_r$$

$$yz_{nrm} = \begin{cases} 1 & \text{if a well at cell } n \text{ of reservoir } r \text{ should be connected to manifold } m \\ 0 & \text{otherwise} \end{cases}$$

$$n \in \mathbf{IP}_r, m \in \mathbf{M}_{nr}$$

$$yu_{nrc} = \begin{cases} 1 & \text{if a well at cell } n \text{ of reservoir } r \text{ should be connected to center } c \\ 0 & \text{otherwise} \end{cases}$$

$$n \in \mathbf{IP}_r, c \in \mathbf{C}_{nr}$$

$$zu_{mc} = \begin{cases} 1 & \text{if manifold } m \text{ should be connected to center } c \\ 0 & \text{otherwise} \end{cases} \qquad c \in \mathbf{C}_m$$

If any facility or connection from the above already exists, then we set its binary to 1. Thus, $u_c = 1$ and installation cost zero for $1 \leq c \leq C_e$, $z_m = 1$ and installation cost zero for $1 \leq m \leq M_e$, $y_{nr} = 1$ and drilling cost zero for $n \in \mathbf{PW}_r \cup \mathbf{IW}_r$, and $y_{nr} = 0$ and drilling cost infinity for $n \in \mathbf{NW}_r - \mathbf{PW}_r - \mathbf{IW}_r$. Similarly, if a well (n, r) is already connected to manifold m, then $yz_{nrm} = 1$ and installation cost zero.

For preventing wells (injectors or producers) from being adjacent, we set following constraints:

$$y_{nr} + y_{(n+1)r} \leq 1 \qquad n \in \mathbf{IX}_r \qquad (1)$$

$$y_{nr} + y_{(n+I)r} \leq 1 \qquad n \in \mathbf{IY}_r \qquad (2)$$

$$y_{rn} + y_{(n+I_r-1)r} \leq 1 \qquad (n + I_r - 1) \in \mathbf{IX}_r \qquad (3)$$

$$y_{nr} + y_{(n+I_r+1)r} \leq 1 \qquad (n + I_r) \in \mathbf{IX}_r \qquad (4)$$

They are suitable for clique cuts in CPLEX. So we add those constraints too. Based on above constraints, we exclude the cells adjacent to the existing injector and producer wells from $\mathbf{NW}_r$, and fix their y_{nr} to zero as discussed earlier.

Suppose that a well $n \in \mathbf{IP}_r$ can have at most WMC_{nr}^U manifold/center connections, a manifold m can have at most MW_m^U well connections and at most MC_m^U center connections, and a center c can have at most CMW_c^U well/manifold connections. Similarly, if a well, manifold or center should exist, then it must have at least one connection. Then, we have,

$$WMC_{nr}^U y_{nr} \geq \sum_{m \in \mathbf{M}_{nr}} yz_{nrm} + \sum_{c \in \mathbf{C}_{nr}} yu_{nrc} \geq y_{nr} \qquad 1 \leq r \leq R, n \in \mathbf{IP}_r \qquad (5)$$

$$MW_m^U z_m \geq \sum_{r=1}^{R} \sum_{n \in \mathbf{IP}_r \ni m \in \mathbf{M}_{nr}} yz_{nrm} \geq z_m \qquad 1 \leq m \leq M \qquad (6)$$

$$MC_m^U z_m \geq \sum_{c \in \mathbf{C}_m} zu_{mc} \geq z_m \qquad 1 \leq m \leq M \qquad (7)$$

$$CMW_c^U u_c \geq \sum_{m \ni c \in \mathbf{C}_m} zu_{mc} + \sum_{r=1}^{R} \sum_{n \in \mathbf{IP}_r \ni c \in \mathbf{C}_{nr}} yu_{nrc} \geq u_c \qquad 1 \leq c \leq C \qquad (8)$$

5.3.2 Reservoir Dynamics

Here we use the same generic partial differential equation that was introduced in Chapter 2 to model the reservoir dynamics, namely:

$$\frac{\partial}{\partial t}\left[\varepsilon \frac{S_f}{B_f}\right] + q_f - \nabla\left[\frac{kr_f}{\mu_f B_f}\boldsymbol{K}(\nabla P_f - \rho_f \frac{g}{g_c} \nabla z)\right] = 0 \qquad (9)$$

where, ε is the porosity, B_f is the formation volume factor of phase f ($f = o$ for oil and $f = w$ for water), S_f is the saturation, P_f is the pressure, and q_f is the flow from (+ve for out, −ve for in) the reservoir, kr_f is the relative permeability, μ_f is the viscosity, ρ_f is the density and $\boldsymbol{K}$ is the absolute permeability tensor. It is important to note that all flow rates in eq. 1 are at the processing center (versus *in situ*) conditions. The formation volume factors (B_o and B_w)

map back the center flows to flows at the reservoir conditions. Discretizing the planning horizon H into T intervals of arbitrary lengths dh^t $(t = 1, 2, \ldots, T)$ and employing backward finite difference approximation for the derivatives, we obtain,

$$(V_{nr}/dh^t)\{d^t_{o,1,nr}\,[P^t_{nr} - P^{t-1}_{nr}] + d^t_{o,2,nr}\,[S^t_{nr} - S^{t-1}_{nr}]\} + q^t_{o,n\notin \mathbf{IW_r}} + \tag{10}$$

$$\begin{pmatrix} \{M^t_{o,(x-)r} T^x_{(n-1)r}[P^t_{nr} - P^t_{(n-1)r}]\}_{(n-1)\in \mathbf{IX}_r} + \{M^t_{o,(x+)r} T^x_{nr}[P^t_{nr} - P^t_{(n+1)r}]\}_{n\in \mathbf{IX}_r} + \\ \{M^t_{o,(y-)r} T^y_{(n-I_r)r}[P^t_{nr} - P^t_{(n-I_r)r}]\}_{(n-I)\in \mathbf{IY}_r} + \{M^t_{o,(y+)r} T^y_{nr}[P^t_{nr} - P^t_{(n+I_r)r}]\}_{n\in \mathbf{IY}_r} \end{pmatrix} = 0;$$

$$(V_{nr}/dh^t)\{d^t_{w,1,nr}\,[P^t_{nr} - P^{t-1}_{nr}] + d^t_{w,2,nr}\,[S^t_{nr} - S^{t-1}_{nr}]\} + \left(q^t_{n,r} - q^t_{o,n\notin \mathbf{IW}_r}\right) + \tag{11}$$

$$\begin{pmatrix} \{M^t_{w,(x-)r} T^x_{(n-1)r}[P^t_{nr} - P^t_{(n-1)r}]\}_{(n-1)\in \mathbf{IX}_r} + \{M^t_{w,(x+)r} T^x_{nr}[P^t_{nr} - P^t_{(n+1)r}]\}_{n\in \mathbf{IX}_r} + \\ \{M^t_{w,(y-)r} T^y_{(n-I_r)r}[P^t_{nr} - P^t_{(n-I_r)r}]\}_{(n-I)\in \mathbf{IY}_r} + \{M^t_{w,(y+)r} T^y_{nr}[P^t_{nr} - P^t_{(n+I_r)r}]\}_{n\in \mathbf{IY}_r} \end{pmatrix} = 0;$$

where, $n \in \mathbf{IP}_r$, S^t_{nr} and P^t_{nr} are saturation and pressure at the end of interval t, S^0_{nr} and P^0_{nr} are the initial saturation and pressure at time zero, $d^t_{f,1,nr}$ and $d^t_{f2,nr}$ are the variable accumulation multipliers, T^x_{nr} and T^y_{nr} are transmissibilities in x and y-directions, $M^t_{f,(x+)r}$, $M^t_{f,(x-)r}$, $M^t_{f,(y+)r}$ and $M^t_{f,(y-)r}$ are upstream weighted mobilities, which are taken as those of the neighboring cells with higher pressures. Outcoming flow is positive, and ingoing flow is negative. The transmissibilities, upstream weight mobility terms, and variable accumulation multipliers are introduced in Chapter 3. $M^t_{o,nr}$ and $M^t_{w,nr}$ are the oil and water mobilities defined as:

$$M^t_{o,nr} = kr^o_0 \left(\frac{1 - S^t_{nr} - S_{r,or}}{1 - S_{r,wr} - S_{r,or}}\right)^a / (\mu_{r,Bo,1} P^t_{nr} + \mu_{r,Bo,2}) \tag{12}$$

$$M^t_{w,nr} = kr^w_0 \left(\frac{S^t_{nr} - S_{r,wr}}{1 - S_{r,wr} - S_{r,or}}\right)^b \times [1 + C_w(P^t_{nr} - P_r)]/B_{r,wr}\mu_{w,r} \tag{13}$$

where, kr^o_0 and kr^w_0 are the end-point relative permeabilities of oil and water, S_{or} and S_{wr} are the residual oil and water saturations respectively, a and b are the exponents in Corey's correlation, $(\mu_{Bo,1}, \mu_{Bo,2})$ are regression parameters for the product $(\mu_o \times B_o)$ of the viscosity and formation volume factor of oil, C_w is the water compressibility factor, B^0_{wr} is the formation volume factor for water at reference pressure P_r, finally μ_w is the water viscosity.

5.3.3 Flow Balances

Non-existent wells, manifolds, and centers cannot send/receive any flows at any time. If QM^t_m and QC^t_c are the total fluid flows through manifold m and processing center c during interval t, then we have,

$$\delta_{nr} q^t_{nr} \le q^{t,U}_{nr} y_{nr} \qquad n \in \mathbf{IP}_r \tag{14}$$

$$QM^t_m \le QM^{t,U}_m z_m \tag{15}$$

$$QC^t_c \le QC^{t,U}_c u_c \tag{16}$$

where, $\delta_{nr} = -1$ for $n \in \mathbf{IW}_r$ and 1 otherwise, and $q^{t,U}_{nr}$, $QM^{t,U}_m$, and $QC^{t,U}_c$ are reasonable upper bounds. Similar constraints apply for the various connections.

$$qwm^t_{nrm} \le qwm^{t,U}_{nrm} yz_{nrm} \qquad n \in \mathbf{IP}_r, m \in \mathbf{M}_{nr} \tag{17}$$

$$qwm^t_{o,nrm} \le qwm^t_{nrm} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r, m \in \mathbf{M}_{nr} \tag{18}$$

$$qwc^t_{nrc} \le qwc^{t,U}_{nrc} yu_{nrc} \qquad n \in \mathbf{IP}_r, c \in \mathbf{C}_{nr} \tag{19}$$

$$qwc^t_{o,nrc} \le qwc^t_{nrc} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r, c \in \mathbf{C}_{nr} \tag{20}$$

$$qmc^t_{mc} \le QM^{t,U}_m zu_{mc} \qquad c \in \mathbf{C}_m \tag{21}$$

$$qmc^t_{o,mc} \le qmc^t_{mc} \qquad c \in \mathbf{C}_m \tag{22}$$

where, qwm^t_{nrm} and $qwm^t_{o,nrm}$ are the total and oil flows through the connection between well n and manifold m, qwc^t_{nrc} and $qwc^t_{o,nrc}$ are the total and oil flows through the connection between well n and center c, and qmc^t_{mc} and $qmc^t_{o,mc}$ are the total and oil flow through the connection between manifold m and center c. Note that we do not write eqs. (18), (20), and (22) for injector wells, as their oil flows are zero.

We demand that a new well, manifold, or center be installed, only if it handles/produces some minimum amount ($q^L_{o,nr}$, QM^L_m, and QC^L_c) of flow over the horizon:

$$\sum_t q_{o,nr}^t \Delta h^t \geq q_{o,nr}^L y_{nr} \qquad n \in \mathbf{NW}_r \qquad (23)$$

$$\sum_t QM_m^t \Delta h^t \geq QM_m^L z_m \qquad M_e + 1 \leq m \leq M \qquad (24)$$

$$\sum_t QC_c^t \Delta h^t \geq QC_c^L u_c \qquad C_e + 1 \leq c \leq C \qquad (25)$$

where, $q_{o,nr}^t$ is the oil flow through well (n, r).

As per our earlier assumption, a well should be shut in, if its water cut exceeds certain minimum. We used this constraint explicitly in the pervious chapter. In this chapter, we handle it implicitly in our solution algorithm, as discussed later. Therefore, we do not write the water cut constraints for the production wells. However, another analogous constraint is of relevance to this work. This relates to the maximum water cut that a separation center can handle. Again, we handle it implicitly in our algorithm, and do not write it explicitly in our formulation.

The flow balances at wellheads, manifold inlets, manifold outlets, and processing center inlets give us:

$$\delta_{nr} q_{nr}^t = \sum_{m \in \mathbf{M}_{nr}} qwm_{nrm}^t + \sum_{c \in \mathbf{C}_{nr}} qwc_{nrc}^t \qquad n \in \mathbf{IP}_r \qquad (26)$$

$$q_{o,nr}^t = \sum_{m \in \mathbf{M}_{nr}} qwm_{o,nrm}^t + \sum_{c \in \mathbf{C}_{nr}} qwc_{o,nrc}^t \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r \qquad (27)$$

$$QM_m^t = \sum_{r=1}^R \sum_{n \in \mathbf{IP}_r \ni m \in \mathbf{M}_{nr}} qwm_{nrm}^t = \sum_{c \in \mathbf{C}_m} qmc_{mc}^t \qquad (28)$$

$$QM_{o,m}^t = \sum_{r=1}^R \sum_{n \in (\mathbf{IP}_r - \mathbf{IW}_r) \ni m \in \mathbf{M}_{nr}} qwm_{o,nrm}^t = \sum_{c \in \mathbf{C}_m} qmc_{o,mc}^t \qquad m \in \mathbf{PM} \qquad (29)$$

$$QC_c^t = \sum_{m \ni c \in \mathbf{C}_m} qmc_{mc}^t + \sum_{r=1}^R \sum_{n \in \mathbf{IP}_r \ni c \in \mathbf{C}_{nr}} qwc_{nrc}^t \qquad (30)$$

$$QC_{o,c}^t = \sum_{m \ni c \in \mathbf{C}_m} qmc_{o,mc}^t + \sum_{r=1}^R \sum_{n \in (\mathbf{IP}_r - \mathbf{IW}_r) \ni c \in \mathbf{C}_{nr}} qwc_{o,nrc}^t \qquad c \in \mathbf{PC} \qquad (31)$$

where, $QM_{o,m}^t$ and $QC_{o,c}^t$ are the oil flows through manifold m and center c.

The field production/injection rates are:

$$QC^t = \sum_{c \in \mathbf{PC}} QC_c^t \qquad (32)$$

$$QC_o^t = \sum_{c \in \mathbf{PC}} QC_{o,c}^t \qquad (33)$$

$$QC_w^t = \sum_{c \notin \mathbf{PC}} QC_c^t \qquad (34)$$

where, QC^t and QC_o^t are field liquid and oil production rates, and QC_w^t is the field water injection rate.

Finally, careful bounding of various flows is essential. Let Q_m^* and Q_c^{t*}be the maximum liquid flow that manifold m and center c are designed to handle. Q_m^* is time-invariant but not Q_c^{t*}, so the manifolds would need to be overdesigned to accommodate potential surface capacity expansions later in the horizon. If $q_{nr}^{t,OF}$ denotes the maximum flow (or an approximation to open flow) possible at each well, and D^t denotes the projected oil demand in interval t, then the bounds on various variables are:

$$q_{nr}^{t,U} = \min\left(q_{nr}^{t,OF}, \min(\textstyle\sum_{m\in\mathbf{M}_{nr}} Q_m^*, \sum_{c\in\mathbf{C}_m \ni m\in\mathbf{M}_{nr}} Q_c^{t*}) + \sum_{c\in\mathbf{C}_{nr}} Q_c^{t*}\right) \quad n\in\mathbf{IP}_r \tag{35}$$

$$q_{o,nr}^t \le \min\left[D^t, q_{nr}^{t,U}\right] \quad n\in\mathbf{IP}_r - \mathbf{IW}_r \tag{36}$$

$$qwm_{nrm}^{t,U} = \min\left[q_{nr}^{t,OF}, Q_m^*, \textstyle\sum_{c\in\mathbf{C}_m} QC_c^{t,*}\right] \quad n\in\mathbf{IP}_r, m\in\mathbf{M}_{nr} \tag{37}$$

$$qwm_{o,nrm}^t \le \min\left[D^t, qwm_{nrm}^{t,U}\right] \quad n\in\mathbf{IP}_r, m\in\mathbf{M}_{nr} \tag{38}$$

$$qwc_{nrc}^{t,U} = \min\left[q_{nr}^{t,OF}, QC_c^{t,*}\right] \quad n\in\mathbf{IP}_r, c\in\boldsymbol{C}_{nr} \tag{39}$$

$$qwc_{o,nrc}^t \le \min\left[D^t, qwc_{nrc}^{t,U}\right] \quad n\in\mathbf{IP}_r, c\in\boldsymbol{C}_{nr} \tag{40}$$

$$QM_m^{t,U} = \min\left[Q_m^*, \textstyle\sum_{c\in\mathbf{C}_m} QC_c^{t,*}, \sum_{r=1}^R \sum_{n\in\mathbf{IP}_r \ni m\in\mathbf{M}_{nr}} qwm_{nr}^{t,U}\right] \tag{41}$$

$$QM_{o,m}^t \le \min\left[D^t, QM_m^{t,U}\right] \tag{42}$$

$$qmc_{mc}^{t,U} = \min[QM_m^{t,U}, Q_c^{t*}] \quad c\in\mathbf{C}_m \tag{43}$$

$$qmc_{o,mc}^t \le \min\left[D^t, qmc_{mc}^{t,U}\right] \quad c\in\mathbf{C}_m \tag{44}$$

$$QC_c^{t,U} = \min\left[Q_c^{t*}, \textstyle\sum_{m\in\boldsymbol{C}_m} qmc_{mc}^{t,U} + \sum_{r=1}^R \sum_{n\in\mathbf{IP}_r \ni c\in\mathbf{C}_{nr}} q_{nr}^{t,U}\right] \quad c\in\mathbf{PC} \tag{45}$$

$$QC_{o,c}^t \le \min\left[D^t, QC_c^{t,U}\right] \quad c\in\mathbf{PC} \tag{46}$$

$$QC^t \le \textstyle\sum_{c\in\mathbf{PC}} QC_c^{t,U} \tag{47}$$

$$QC_o^t \le \min(D^t, \textstyle\sum_{c\in\mathbf{PC}} QC_c^{t,U}) \tag{48}$$

$$QC_w^t \le \textstyle\sum_{c\notin\mathbf{PC}} QC_c^{t,U} \tag{49}$$

Here, $q_{nr}^{t,OF}$ is the maximum possible flow (open flow condition) during t for a well at cell (n,r), and is defined later.

5.3.4 Pressure Balances

Let BHP_{nr}^{t} be the pressure at the well-bore of cell (n,r), THP_{nr}^{t} be the pressure at its wellhead, MP_{m}^{t} be the pressure at manifold m, and CP_{c}^{t} be the pressure at center c. Figure 3-1 shows the different pressures and pressure drops. CP_{c}^{t} enables the flow downstream of center c. While it may be taken as fixed [227], we allow it to be time-dependent. If ΔP_{nr}^{t} ($n \in \mathbf{IP}_r$) denotes the total pressure drop through well tubing, ΔP_{nrm}^{t} ($n \in \mathbf{IP}_r$, $m \in \mathbf{M}_{nr}$) denotes that between well (n,r) and manifold m, ΔP_{nrc}^{t} ($n \in \mathbf{IP}_r$, $c \in \mathbf{C}_{nr}$) denotes that between well (n,r) and center c, and ΔP_{mc}^{t} ($c \in \mathbf{C}_m$) denotes that between manifold m and center c, then we have,

$$BHP_{nr}^{t} + (1 - y_{nr})BM_{nr} \geq THP_{nr}^{t} + \Delta P_{nr}^{t} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r \qquad (50)$$

$$THP_{nr}^{t} \geq BHP_{nr}^{t} + \Delta P_{nr}^{t} \qquad n \in \mathbf{IW}_r \qquad (51)$$

$$THP_{nr}^{t} + (1 - yz_{nrm})BM_{m} \geq MP_{m}^{t} + \Delta P_{nrm}^{t} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r,\ m \in \mathbf{M}_{nr} \qquad (52)$$

$$MP_{m}^{t} \geq THP_{nr}^{t} + \Delta P_{nrm}^{t} \qquad n \in \mathbf{IW}_r,\ m \in \mathbf{M}_{nr} \qquad (53)$$

$$THP_{nr}^{t} + (1 - yu_{nru})BM_{c} \geq CP_{c}^{t} + \Delta P_{nrc}^{t} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r,\ c \in \mathbf{C}_{nr} \qquad (54)$$

$$CP_{c}^{t} \geq THP_{nr}^{t} + \Delta P_{nrc}^{t} \qquad n \in \mathbf{IW}_r,\ c \in \mathbf{C}_{nr} \qquad (55)$$

$$MP_{m}^{t} + (1 - zu_{mc})BM_{c} \geq CP_{c}^{t} + \Delta P_{mc}^{t} \qquad m \in \mathbf{PM},\ c \in \mathbf{C}_m \qquad (56)$$

$$CP_{c}^{t} \geq MP_{m}^{t} + \Delta P_{mc}^{t} \qquad m \notin \mathbf{PM},\ c \in \mathbf{C}_m \qquad (57)$$

where, BM_{nr}, BM_{m}, are BM_{c} big-M parameters.

Pressure drop in a multiphase flow is a complex function of liquid flow rate (or oil flow rate), water-oil ratio (or water flow rate), gas-oil ratio (or gas flow rate) and absolute pressure levels. Several models exist for capturing these dependencies. Homogenous flow models ignore flow pattern and are the simplest. Homogenous models that consider phase slip are called drift flux models [187-189]. Empirical methods such as those of Beggs and Brills [190] and Mukherjee and Brill [191], and the mechanistic model of Petalas and Aziz model [192], predict flow patterns and consider phase slip. Accurate predictions for pressure drops involve highly complex and nonlinear expressions. Brill [193] provides an excellent monograph on this topic. In this work, we represent ΔP_{nr}^{t}, ΔP_{nrm}^{t}, ΔP_{nrc}^{t}, and ΔP_{mc}^{t} via empirical correlations given in Appendix A, which are regressed from the data generated by the VFPi package of Eclipse [194].

In addition, we impose reasonable lower bounds on the various pressures to avoid reverse flows. These lower bounds (BHP_{nr}^{L}, THP_{nrm}^{L}, THP_{nrc}^{L}, THP_{nr}^{L}, MP_{mc}^{L}, MP_{nrm}^{L}, CP_{mc}^{L}, and CP_{nrc}^{L}) are not pre-assigned constants, but analytical functions (see Appendix B) of fluid properties and flow variables, which are regressed from the data generated by the VFPi package of Eclipse [194].

$$BHP_{nr}^{t} \geq BHP_{nr}^{L} \qquad n \in \mathbf{IP}_r \tag{58}$$

$$THP_{nr}^{t} \geq THP_{nrm}^{L} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r,\ m \in \mathbf{M}_{nr} \tag{59}$$

$$THP_{nr}^{t} \geq THP_{nrc}^{L} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r,\ c \in \mathbf{C}_{nr} \tag{60}$$

$$THP_{nr}^{t} \geq THP_{nr}^{L} \qquad n \in \mathbf{IW}_r \tag{61}$$

$$MP_{m}^{t} \geq MP_{mc}^{L} \qquad m \in \mathbf{PM},\ c \in \mathbf{C}_m \tag{62}$$

$$MP_{m}^{t} \geq MP_{nrm}^{L} \qquad n \in \mathbf{IW}_r,\ m \in \mathbf{M}_{nr} \tag{63}$$

$$CP_{c}^{t} \geq CP_{mc}^{L} \qquad m \notin \mathbf{PM},\ c \in \mathbf{C}_m \tag{64}$$

$$CP_{c}^{t} \geq CP_{nrc}^{L} \qquad n \in \mathbf{IW}_r,\ c \in \mathbf{C}_{nr} \tag{65}$$

Since BHP_{nr}^{t} at an injector must not exceed fracture pressure (PF_{nr}), we use $BHP_{nr}^{t} \leq PF_{nr}$ for $n \in \mathbf{IW}_r$.

5.3.5 Well Flow Rates

The well flow rate depends on the differential between the internal reservoir and bottom hole pressures as follows.

$$q_{nr}^{t} = \psi_{nr}\left(M_{o,nr}^{t} + M_{w,nr}^{t}\right)\left(P_{nr}^{t} - BHP_{nr}^{t}\right) \qquad n \in \mathbf{IP}_{r} \qquad (66)$$

where, $\Psi_{r,n}$ is the connection transmissibility factor for wells, which includes geological information (see eq. 76.5 of [44]). For injectors, the usual practice [221] in reservoir simulators is to use $M_{o,nr}^{t} = M_{o,nr}^{t}\overline{B_o}/B_{wr}^{o}$, where $\overline{B_o}$ is the average oil formation volume factor. That is also used for all equations related to injector wells in this section.

As pointed out in the previous chapter, Eqn. (66) has a serious flaw when cross flow is not allowed, because Eqn. (66) is not valid for $\delta_{nr}(P_{nr}^{t} - BHP_{nr}^{t}) < 0$. The correct way to write is,

$$\delta_{nr}q_{nr}^{t} = \max[\delta_{nr}\psi_{nr}\left(M_{o,nr}^{t} + M_{w,nr}^{t}\right)\left(P_{nr}^{t} - BHP_{nr}^{t}\right), 0] \qquad n \in \mathbf{IP}_{r} \qquad (67)$$

We used a smooth approximation [209] for the above max function to eliminate the flaw. In this work, we propose a more efficient formalism for this approximation. We define $\mathbf{R}_{nr}^{t}$ as an unrestricted (in sign) variable:

$$\mathbf{R}_{nr}^{t} = \delta_{nr}\psi_{nr}\left(P_{nr}^{t} - BHP_{nr}^{t}\right) \qquad n \in \mathbf{IP}_{r} \qquad (68)$$

Then, we replace eq. 66 by,

$$\delta_{nr}q_{nr}^{t} \geq \mathbf{R}_{nr}^{t}\left(M_{o,nr}^{t} + M_{w,nr}^{t}\right) \qquad n \in \mathbf{IP}_{r} \qquad (69)$$

$$\delta_{nr}q_{nr}^{t} \leq 0.5\left(M_{o,nr}^{t} + M_{w,nr}^{t}\right)\left(\mathbf{R}_{nr}^{t} + \sqrt{{\mathbf{R}_{nr}^{t}}^{2} + \epsilon^{2}}\right) \qquad n \in \mathbf{IP}_{r} \qquad (70)$$

where ϵ is a small ($\cong 10^{-4}$) scalar. Thus, $\delta_{nr}q_{nr}^{t} \approx \mathbf{R}_{nr}^{t}\left(M_{o,nr}^{t} + M_{w,nr}^{t}\right)$ for $\mathbf{R}_{nr}^{t} \gg \epsilon$, and $q_{nr}^{t} \leq 0.25\epsilon\left(M_{o,nr}^{t} + M_{w,nr}^{t}\right)$ for $\mathbf{R}_{nr}^{t} \leq 0$.

Then, the oil flow from a producer well, given by $q_{o,nr}^{t} = q_{nr}^{t} \cdot M_{o,nr}^{t}/\left(M_{w,nr}^{t} + M_{o,nr}^{t}\right)$, can be computed as,

$$q_{o,nr}^{t} \leq 0.5M_{o,nr}^{t}\left(\mathbf{R}_{nr}^{t} + \sqrt{{\mathbf{R}_{nr}^{t}}^{2} + \epsilon^{2}}\right) \qquad n \notin \mathbf{IW}_{r} \qquad (71)$$

$$q_{o,nr}^{t}\left(M_{w,nr}^{t} + M_{o,nr}^{t}\right) \geq q_{nr}^{t} \cdot M_{o,nr}^{t} \qquad n \notin \mathbf{IW}_{r} \qquad (72)$$

Since $q_{nr}^{t} \approx \mathbf{R}_{nr}^{t}\left(M_{o,nr}^{t} + M_{w,nr}^{t}\right)$ for $\mathbf{R}_{nr}^{t} \gg \epsilon$, Eqn. (72) can be changed to

$$\mathbf{R}_{nr}^{t} \cdot M_{o,nr}^{t} \leq q_{o,nr}^{t} \qquad n \notin \mathbf{IW}_{r} \tag{73}$$

In this way, both q_{nr}^{t} and $q_{o,nr}^{t}$ are properly bounded and the inequalities are guaranteed to be tight constraints.

5.3.6 Objective function

The main objective is to maximize NPV (Net present value). To obtain NPV, we first compute the total capital expenditure, which includes the costs of drilling wells and installing manifolds, centers, and their flow lines.

$$CAPEX = \sum_{c} u_c \alpha_c + \sum_{m} z_m \alpha_m + \sum_{r=1}^{R} \sum_{n \in \boldsymbol{IP}_r} y_{nr} L_{nr} \alpha_{nr} + \sum_{m} \sum_{c \in C_m} zu_{mc}\, \alpha_{mc}$$

$$+ \sum_{r=1}^{R} \sum_{n \in \boldsymbol{IP}_r} \left[\sum_{c \in C_{nr}} yu_{nrc} L_{nrc} \alpha_{nrc} + \sum_{m \in \boldsymbol{M}_{nr}} yz_{nrm} L_{nrm} \alpha_{nrm} \right] \tag{74}$$

where, α_{nr} is the cost of drilling per unit vertical depth for a well at cell (n, r), α_c shows the cost of installing center c, α_m is the cost for installing manifold m, α_{nrc} ($ per unit length) is the unit-length cost for well-to-center piping, α_{nrm} is the unit-length cost of well-to-manifold piping, α_{mc} is the unit-length cost of manifold-to-center piping, L_{nr} is the vertical depth of a well at cell (n, r), L_{nrc} is the length of well-to-center pipeline, L_{mc} is the length of manifold-to-center pipeline, and L_{nrm} is length of well-to-manifold pipeline. We assume that all CAPEX occurs at time zero, or before the start of interval 1. However, since the budget for fixed costs may be limited, we impose $CAPEX \leq CAPEX^{U}$. Furthermore, we ignore the depreciation for existing infrastructure.

The total revenue is from the sale of oil and gas:

$$SR^{t} = \beta_o QC_o^{t} \Delta h^{t} \tag{75}$$

where, β_o is the oil price ($ per unit volume).

The operating expense includes the costs of water production/injection and oil/gas production/processing.

$$EX^{t} = \left(\alpha_o QC_o^{t} + \alpha_{pw} [QC^{t} - QC_o^{t}] + \alpha_{iw} QC_w^{t} \right) \Delta h^{t} \tag{76}$$

where, α_o is the unit cost of oil/gas production, α_{pw} is the unit cost of water production and disposal, and α_{iw} is unit cost of water injection. Assuming d^t as the annual rate of depreciation for interval t, we get the taxable income and taxes for interval t as,

$$TI^t \geq SR^t - EX^t - d^t CAPEX \tag{77}$$

Then, we obtain the actual cash flow (CF^t) during interval t as:

$$CF^t = SR^t - EX^t - \gamma^t TI^t \tag{78}$$

where, γ^t is the tax rate during interval t. Then, our objective is:

$$\max NPV = \sum_t \left[CF^t / (1 + AD)^{h^t/365} \right] - CAPEX \tag{79}$$

where, AD is the annual discount factor.

This completes our model **E** which includes Eqs (1)-(8), (10)-(65), (68)-(25). Model **E** is a nonconvex, spatiotemporal MINLP.

5.4 Solution Strategy

We modify slightly the OA-ER-AP algorithm developed in our previous chapter. It decomposes MINLP **E** into a master MILP for design and a primal NLP for operational decisions. The master MILP at iteration k provides the binary variables, which are fixed in the primal NLP to obtain continuous variables. The NLP involves multiple periods, and each period has multiple intervals. We solve a series of NLPs sequentially, one for each interval. The NLP solutions at the ends of periods provide the linearization points to form the next master problem. This finishes iteration k. The combination of binary variables that underlie the solutions at iteration k is prohibited from subsequent iterations by a suitable integer-cut.

The sequential approach described above for solving the primal NLP poses a problem in that it is not possible to ensure Eqs. (23) to (25). This is because they require the knowledge of the full throughput profile, which is only possible at the master problem. Hence, we relax eqs. (23) to (25) in the primal problem, but use them fully in the master problem. Note that the lack of access to the entire production plan, while making decisions at an intermediate interval, is also a limitation for the numerical reservoir simulators. But, the difference is that the reservoir simulators solve a series of different *sets of nonlinear equations,* while our approach solves a sequence of *constrained NLP problems*. As a result, while our method gives a series of local optimum solutions; the simulators can at best provide a sequence of good feasible solutions to the NLP problem.

When the master problem fails to improve the best available solution, we initiate a 2-stage local search around the location of the least productive new producer (LPNW) in that solution. The first stage fixes all the active binaries except $(y_{nr}, yz_{nrm}, yu_{nrc})$ for LPNW and (z_m, zu_{mc}) or u_c of manifold/center which is only connected LPNW. It then solves a reduced master problem to locate new wells and/or reject the LPNW. If the first stage of local search fails to improve the objective function, the second stage is activated which removes LPNW and probes its neighborhood for a better location of another well. In order to avoid infeasible configurations, we only consider locations that can be connected to the same manifold/center that LPNW was connected to. Other policies are also possible for the neighborhood probing.

The decisions on drilling, manifold, and center installations are clearly interconnected. However, they follow a natural hierarchy of center, manifold, and then well. We use this priority for branching in the MILP solution. The order of priorities is u_c^k, z_m^k, zu_{mrc}^k, y_{nr}^k, and (yz_{nrm}^k, yu_{nrc}^k). Moreover, we define the integer cut only for the triplet of $(yz_{nrm}, yu_{nrc}, zu_{mc})$, which ensures that all six sets of binary variables are screened properly. It is because, the triplet represents the connections and a connection is only established, if the corresponding infrastructure is installed. In another word, screening the connections is subsumes the screening of units as well.

5.4.1 Adaptive Mass Balance Approximation in MILP Master

In our previous algorithm, we had tested two approaches for linearizing the convective flow terms in the master MILP. In the first, we linearized all individual flow terms at each cell separately. In the second, we defined a net convective flow term ($F_{f,nr}^{t}$) for each phase f as follows.

$$F_{f,n}^{t} = \left\{M_{f,(x-)r}^{t} T_{(n-1)r}^{x} \left[P_{nr}^{t} - P_{(n-1)r}^{t}\right]\right\}_{(n-1)\in \mathbf{IX}_r} + \left\{M_{f,(x+)r}^{t} T_{nr}^{x} \left[P_{nr}^{t} - P_{(n+1)r}^{t}\right]\right\}_{n\in \mathbf{IX}_r} + \left\{M_{f,(y-)r}^{t} T_{(n-I_r)r}^{y} \left[P_{nr}^{t} - P_{(n-I_r)r}^{t}\right]\right\}_{(n-I)\in \mathbf{IY}_r} + \left\{M_{f,(y+)r}^{t} T_{nr}^{y} \left[P_{nr}^{t} - P_{(n+I_r)r}^{t}\right]\right\}_{n\in \mathbf{IY}_r} \quad (80)$$

Then, we replaced all individual flow terms by this net flow, and defined the mass balance by:

$$\theta_{f,nr}^{t,k} \geq signum\left(\lambda_{f,nr}^{t,k}\right) \cdot \{\frac{\partial A_{f,nr}^{t,k}}{\partial P_{nr}^{t}}\left[P_{nr}^{t} - P_{nr}^{t,k}\right] + \frac{\partial A_{f,nr}^{t,k}}{\partial S_{nr}^{t}}\left[S_{nr}^{t} - S_{nr}^{t,k}\right] + \frac{\partial A_{f,nr}^{t,k}}{\partial P_{nr}^{t-1}}\left[P_{n}^{t-1} - P_{n,r'}^{t-1}\right] + \frac{\partial A_{f,nr}^{t,k}}{\partial S_{n}^{t-1}}\left[S_{n}^{t-1} - S_{nr}^{t-1,k}\right] + \left[q_{o,n}^{t} - q_{o,nr}^{t,k}\right]_{n\notin IW} + \left[q_{n}^{t} - q_{nr}^{t}\right]_{f=w} + \left[F_{f,n}^{t} - F_{f,nr}^{t}\right]\}; \quad (81)$$

$$\sum_{r=1}^{R} \sum_{n\in \mathbf{IP}_r} F_{f,n}^{t} = 0 \quad (82)$$

While the use of net flow term saved computational effort considerably compared to keeping the individual flow terms, it involved a loss of invaluable flow information in the subsurface cell-to-cell network. In this work, we devise an adaptive approach that combines the benefits of both methods by using a mix of both.

We begin by using the second approach. As iterations proceed, we begin mixing the two approaches. To see this, consider the NLP solution at iteration k. We first sort the absolute net convective flow terms ($\left|F_{f,nr}^{t,k}\right|$) at $y_{nr}^{k} = 0$ in the decreasing order at each period for each phase. Then, we identify a flow value ($PE_{f,r}^{t,k}$) that corresponds to $\mathcal{F} = \min[100 - 5(k-1), 5]$ percentile of sorted $\left|F_{f,nr}^{t,k}\right|$. Conceptually, the locations with $\left|F_{f,nr}^{t,k}\right| \geq PE_{f,r}^{t,k}$ show higher convection activity compared to the cells with $\left|F_{f,nr}^{t,k}\right| < PE_{f,r}^{t,k}$. We use this distinction to screen the cells by defining $\mathrm{ACF}_{f,nr}^{t,k} = [(n,r) \in \mathbf{IP}_r \mid \left(\left|F_{f,nr}^{t,k}\right| \geq PE_{f,r}^{t,k}\right) \, or \; y_{nr}^{k} = 1]$. For these cells, we use both net and individual convective flow terms in the master problem; eqs (81) and (82) employ the net flows, and the following uses the individual convective flow terms for $\mathrm{ACF}_{f,nr}^{t,k}$ at iteration k:

$$\theta'^{t,k}_{f,nr} \geq -\left[F_{f,n}^{t} - F_{f,nr}^{t}\right] + \{\frac{\partial A_{f,nr}^{t,k}}{\partial P_{(n-1)r}^{t}}\left[P_{(n-1)r}^{t} - P_{(n-1)r}^{t,k}\right]_{(n-1)\in IX_r} +$$

$$\left.\begin{matrix}\frac{\partial A_{f,nr}^{t,k}}{\partial P_{(n+1)r}^{t}}\left[P_{(n+1)r}^{t} - P_{(n+1)r}^{t,k}\right]_{n\in IX_r} + \frac{\partial A_{f,nr}^{t,k}}{\partial P_{(n-I_r)r}^{t}}\left[P_{(n-I_r)r}^{t} - P_{(n-I_r)r}^{t,k}\right]_{(n-I_r)\in IY_r} + \\ \frac{\partial A_{f,nr}^{t,k}}{\partial P_{(n+I_r)r}^{t}}\left[P_{(n+I_r)r}^{t} - P_{(n+I_r)_r}^{t,k}\right]_{n\in IY_r} + \frac{\partial A_{f,nr}^{t,k}}{\partial M_{f,(x-)r}^{t}}\left[M_{f,(x-)r}^{t} - M_{f,(x-)r}^{t,k}\right]_{(n-1)\in IX_r} + \\ \frac{\partial A_{f,nr}^{t,k}}{\partial M_{f,(x+)r}^{t}}\left[M_{f,(x+)r}^{t} - M_{f,(x+)r}^{t,k}\right]_{n\in IX_r} + \frac{\partial A_{f,nr}^{t,k}}{\partial M_{f,(y-)r}^{t}}\left[M_{f,(y-)r}^{t} - M_{f,(y-)r}^{t,k}\right]_{(n-1)\in IY_r} + \\ \frac{\partial A_{f,nr}^{t,k}}{\partial M_{f,(y+)r}^{t}}\left[M_{f,(y+)r}^{t} - M_{f,(y+)r}^{t}\right]_{n\in IY_r}\end{matrix}\right\}$$

$$(n,r) \in \mathrm{ACF}_{f,nr}^{t,k} \qquad (83)$$

As the iterations progress, $PE_{f,r}^{t,k}$ decreases, and more cells employ detailed linearization. Thus, the MILP linearizations are more relaxed and flexible in the earlier iterations; and they become tighter and more accurate in the later iterations. We call this new strategy adaptive linear approximation of the mass balance.

Irrespective of the linearization approach, proper scaling of the master problem is important. This is because numerous slack variables exist in this spatio-temporal model, and their cumulative values (with their multipliers) can sum up to very large values, which may cause numerical difficulties, if not managed properly for each individual problem.

5.5 Case Study

We consider a synthetic 2D field with two reservoirs ($r = 1$ on the left and $r = 2$ on the right in Figure 5-2a) at different depths. The geological data are extracted and adapted from an up-scaled version of SPE10 [218] benchmark example (layer 12th). The two reservoirs are discretized using 21×44 and 16×30 cells. The field has been producing for 750 days already. Figure 5-2a the initial oil in place at time zero for both reservoirs and the locations of existing wells and infrastructure. The field has five injector wells (I1-I5), four oil producer wells (P1-P4), two production manifolds (PM1-PM2), two injection manifolds (IM1-IM2), one production center, and one injection center. The two potential manifold locations (NM1-NM2) are at (11, 12) and (7, 29) in $r = 1$. The new wells can be directly connected to the production center, or they can be connected to any of the four manifolds (PM1, PM2, NM1, or NM2). Table 2-1 presents some of the geometrical and financial parameters for this example.

Table 5-1: Parameters for field development case study.

Parameter	Value	Parameter	Value
α_{nr}	2.2E3 $\$/m$	α_{mc}	8E2 \$
α_c	5E5 \$	L_{n1}	2600 m
α_m	5E4 \$	L_{n2}	2100 m
$\alpha_{nrc}, \alpha_{nrm}$	6E2 \$	$\Delta x, \Delta y, \Delta z$	94.1, 79.6, 5.56 m

It is desired to plan the next 1460 days of production by locating new production wells and/or manifolds, and determining the best well-to-manifold/center and manifold-to-center connections. We define a base case scenario as the one in which no new drillings or installations occur and the field continues its production using the available infrastructure and wells. We optimize its production plan by solving a primal problem. Using this example, we (1) test our optimal well placement methodology, (2) compare the two linear approximations used in the master problem, and (3) analyze the dynamic features of various solutions.

We implemented our methodology in GAMS 23.8.2 platform and used CONOPT 3.15D and GUROBI as the main NLP and MILP solvers respectively. We employed SNOPT/iPOPT and CPLEX 12.4.0.0 in the rare cases of solver failure. Appendix C lists the solver settings. All computations were performed on a Dell Precision T5500 with two Intel®

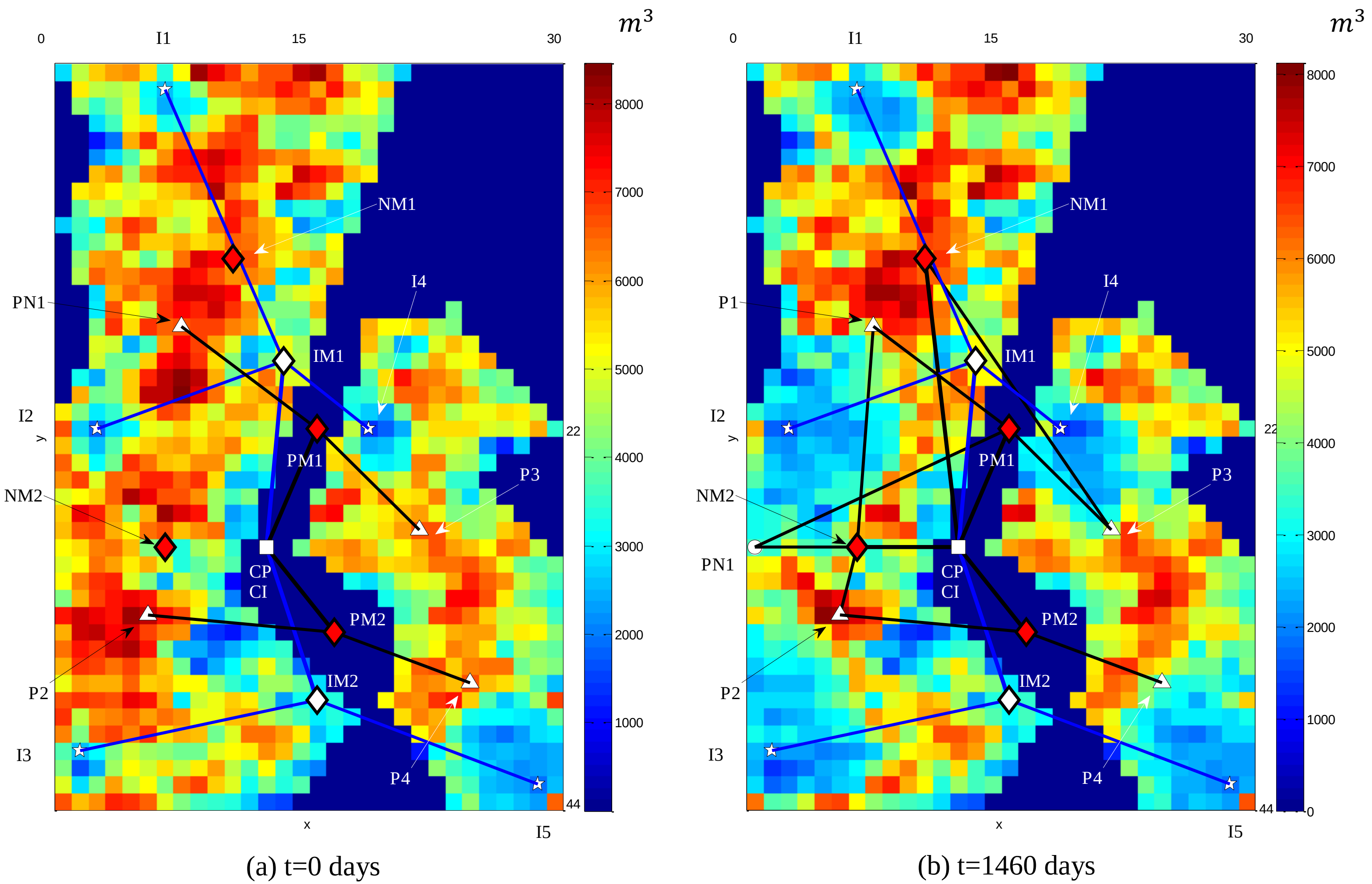

Figure 5-2: Surface network and oil-in-place (OIP) (m3) for at (a) t = 0 d, (b) t = 1460 d.

Pentagrams, triangles and circles represent the old injectors, old producers, and new producers respectively.
Also the red and white diamonds show the production and injection manifolds, and the white rectangle is the surface center.

Xeon® X5650 CPUs (2.67 GHz and 2.66 GHz) and 48 GB of RAM. While we have already verified and validated with ECLIPSE [221] our subsurface model implementation in the previous study, this work is even further extension.

First, we tried to compare the two approaches for linearization, namely the one involving the net convective flow term alone, and the other involving a mix of individual and net terms. We call the former normal, and the latter adaptive. Then, to compare these two approaches, we consider five arbitrary initial location/allocation scenarios. Table 5-2 shows the final NPV values from and solution times for the two approaches for each scenario. There seems no clear choice in terms of NPV, as each approach outperforms the other in two of the five scenarios. However, on an average, the adaptive method has higher NPVs and lower solution times. Clearly, more scenario tests are needed to draw more reliable conclusion. Based on this limited testing, it seems that the adaptive approach has the potential to be better. For the remainder of this section, we restrict ourselves to scenario A (the highest NPV) and use the adaptive approach.

Table 5-2: Final solutions using the normal and adaptive approximations (MM$).

Scenario	Approximation Type in Master Formulation			
	Normal		Adaptive	
	NPV (MM$)	Time (min)	NPV (MM$)	Time (min)
A	345.65	392	347.97	320
B	341.42	364	328.53	139
C	342.15	154	328.68	146
D	289.20	155	322.12	165
E	342.80	104	342.80	101
Average	332.24	233.8	334.02	174.2

For scenario A, the initial location/allocations are as follows. NM2 is selected with connections to wells at (2, 36) and (3, 28) in $r = 1$. The well at (11, 18) in $r = 2$

is connected to PM2. The base case, namely maintaining the field as is without any new wells/manifolds, has NPV = $MM\$271$. The best solution (Figure 5-2b) from our algorithm has NPV = $MM\$347.97$, which is almost 28% higher. This required 320 min, 5 major iterations, and 20 primal problems. In Figure 5-2b, pentagrams represent old injectors, triangles represent old producers, circles represent new producers, red diamonds represent production manifolds, white diamonds represent injection manifolds, and the white rectangle represents the production cum injection center. The best solution suggests drilling a new well (PN1) at the lower Eastern zone (1, 29) of reservoir 1, installing both new manifolds (NM1-NM2), and making five new well-to-manifold connections of PN1-PM1, P3-NM1, P1-NM2, P2-NM2 and PN1-NM2. It provides a wealth of interesting information and observations on the operations of field and surface network. We now discuss a few of these.

Consider the field production curves first. Figure 5-3 compares the curves for the base case with those of the best solution. OPR denotes the oil production rate, WPR represents the water flow rate, subscript B refers to the base case solution, and subscript S refers to the best solution. The base case solution has higher water production, lower oil production, and unfulfilled oil demand for the majority of the planning horizon. In contrast, the decisions from our methodology reduce and control water production, increase oil production, and meet the demand for more than 1200 days of the 1460-day horizon.

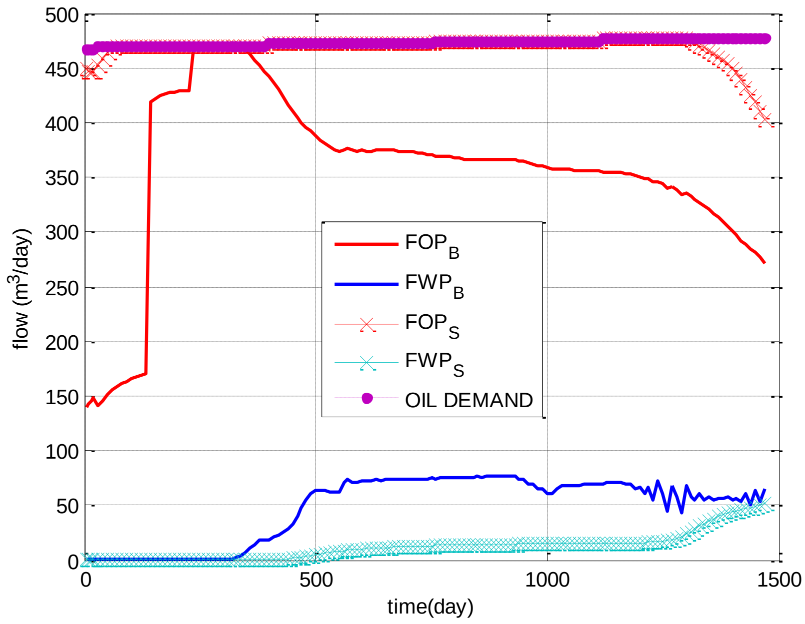

Figure 5-3 Oil demand curve, and the total oil and water production profiles of the field.

FOP and FWP are the total oil and water production rates respectively. The subscript B refers to the base case scenario and subscript S refers to the optimal scenario obtained from our methodology.

Now, consider the production rates of individual wells. Figure 5-4 shows water productions from active wells. Three water breakthroughs happen, the first at P3 (~ 400 days), the second at PN1 (~ 1200 days), and the third at P1 (~ 1400 days). Our optimizer reacted beautifully to these events. We describe these first at the manifold level, and then at the well levels. As Figure 5-5 depicts, at the first water breakthrough at P3, the optimizer regulated the flows through the manifolds based on their connections with this water-flooded well. It reduced the total flows through PM1 and NM1 manifolds, and simultaneously initiated/increased flows through PM2/NM2. Note that the former two manifolds are connected to P3, but not the latter ones. Thus, water production is reduced.

For further discussion, consider Figure 5-6 and Figure 5-7 .They show the flows to/from various wells and manifolds, and the wellhead THPs. In these figures, the colour codes refer to the wells, and the line patterns to the manifolds. For instance, the solid blue line denotes the flow from P1 to PM1, the solid red line denotes the flow from P3 to PM1, and the dashed red line denotes the flow from P3 to NM1. P4 is inactive for the entire production horizon. As can be seen from Figure 5-2a, P4 is located near the waterfront supported by I5 even at the beginning of the project, so its production would lead to very early water breakthrough. At the first breakthrough, when the optimizer reduces the flows through P3-PM1 and P3-NM1 (Figure 5-6a), it temporarily attempts to compensate production losses by increasing the flows through P1-PM1, P1-NM2, PN1-PM1, and PN1-NM2 (Figure 5-6a and Figure 5-6b). However, this turns out to be insufficient. Therefore, it opens P2 along with P2-PM2 and P2-NM2 connections to support the production (Figure 5-6c), which works very well. Once this occurs at t = 460 days, and P2 is working strongly, the optimizer cuts the flows through PN1-PM1 for about 40 days as seen in Figure 5-6b. As seen from Figure 5-7, all wells experience significant variations in their THPs due to these events. Furthermore, as P3 production declines, the production load is shifted largely to reservoir 1.

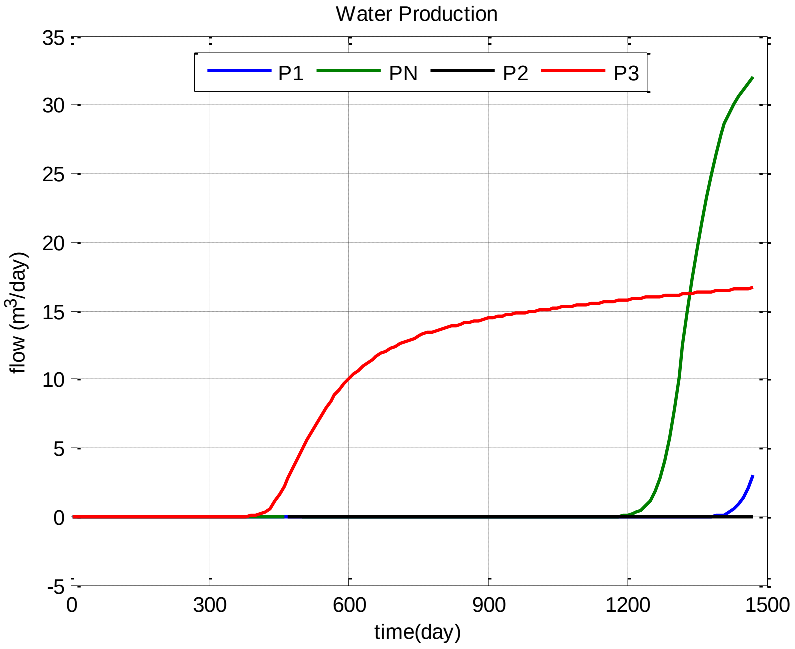

Figure 5-4: Water production profiles for the best solution.

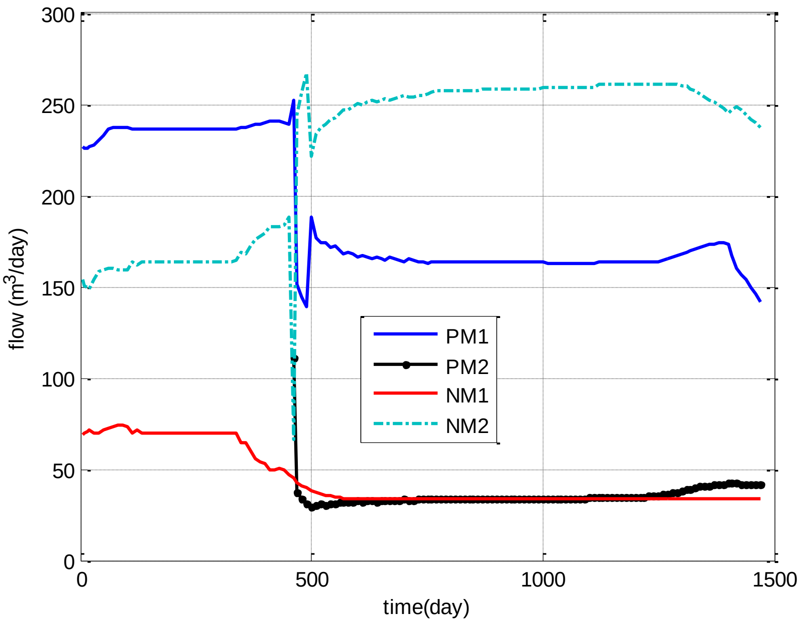

Figure 5-5: Total flow rate at the gathering manifolds.

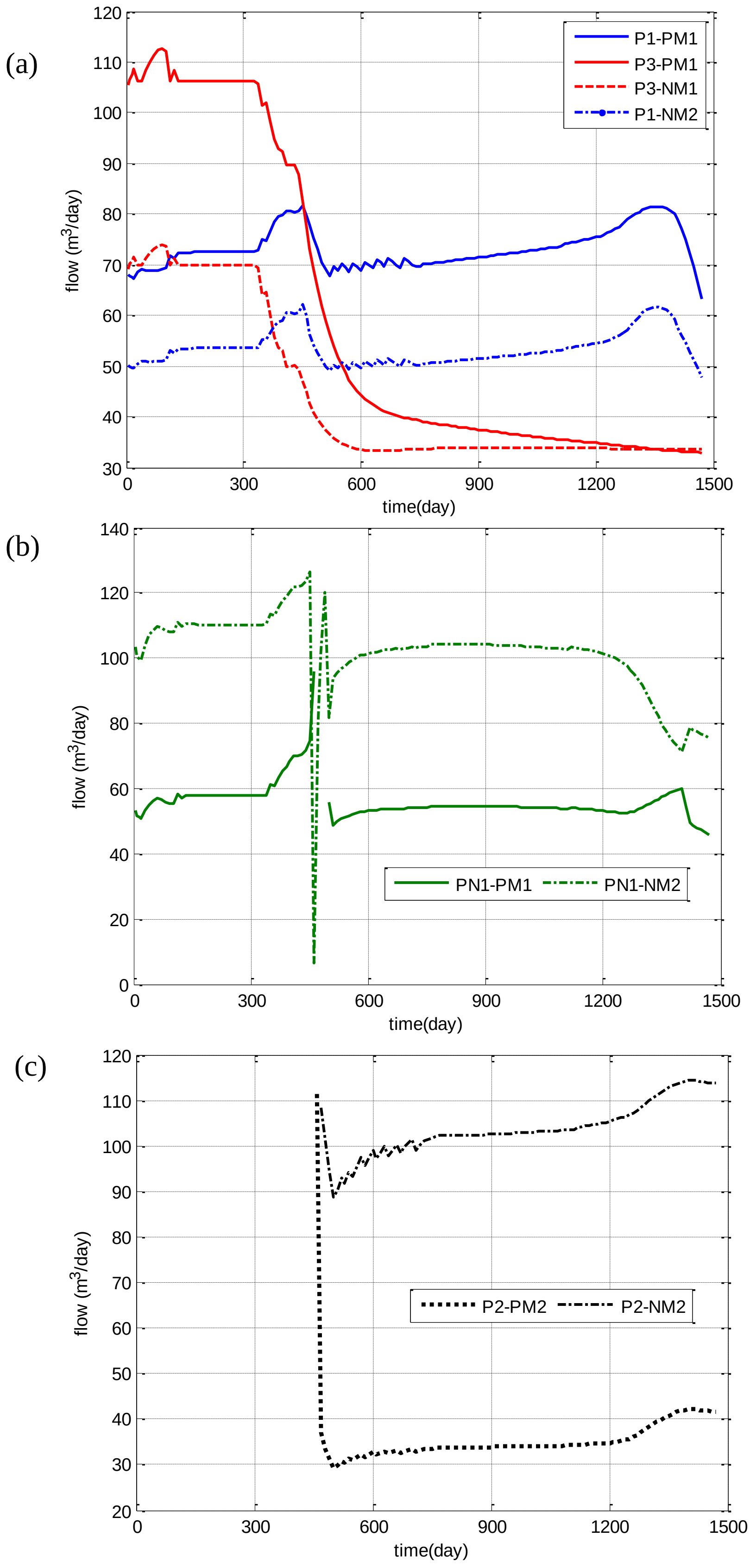

Figure 5-6: Total flow rates between wellheads and gathering manifolds.

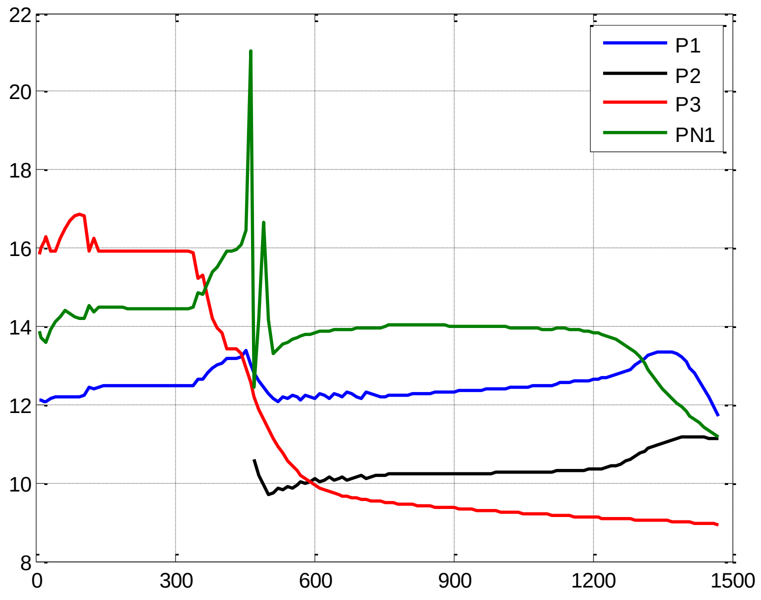

Figure 5-7: THP profiles at wellheads in the best solution.

All active producer wells are connected to multiple manifolds. One reason might be the relatively lower capital cost for surface piping compared to drilling costs. Another reason can be the system dynamics. For instance, P2 supplies both PM2 and NM2, probably due to its rich oil content (Figure 5-2a) and the considerable pressure support from I3. This pressure support is especially important, when the waterfront is close to P2. It pushes and ultimately sweeps the remaining oil in place in the zone between I3 and P2, but does not cause water breakthrough. Yet as discussed previously, we should also examine the various surface interactions causing back pressures, or pressure and flow redistributions to explain and interpret such connections.

Although not presented in Figure 5-2b, the master stage of the algorithm suggested connecting P3 directly to the production center, however the primal solution did not allocate any flow rates to this connection. This is because most demand was met by the connections in Figure 5-2b and without using the P3-CP connection.

Therefore, we removed P3-CP from the solution. Clearly, the primal stage here modifies the suggested solution from the master stage. On the other hand, it may also suggest that P3-CP could be influential, if for example we had not assumed the simultaneous opening of all connections.

5.6 Summary

In this chapter, we extended the study in Chapter 3 and 4. We mainly developed a holistic and integrated model for location-allocation problem of well-drilling and infrastructure installation in a multi-reservoir oil field with a shared surface processing network. Here we combined the sub-surface, well and surface elements of a production project and generalized the sub-surface model to consider irregular-shaped reservoirs. That led to a detailed non-convex, dynamic, multi-period MINLP model.

This work aids decision-making for (a) number and locations of new producer wells (hence the eligible reservoirs for new drilling), new manifolds and processing centers (b) the well-to-manifold/center and manifold-to-center connections (c) production/injection planning for each well, (c) pressure settings at various valves, manifolds, and separation centers over time, and (d) the spatiotemporal profiles of pressure and saturation (hence the oil in place and water front maps) in each reservoir. By allowing irregular shaped reservoirs, this work expands the realism and application of our work [234].

Finally, we tackled a number of limitations of the modified OA algorithm (developed in Chapter 4.) to improve its performance.

5.7 Appendix

Pressure drop equations

$$\Delta P_{nr}^{t} = \beta_{1nr}{q_{nr}^{t}}^{2} + \beta_{2nr}q_{nr}^{t} + \beta_{3nr}q_{o,nr}^{t} + \beta_{4nr}q_{o,nr}^{t}/(q_{nr}^{t}+1) + (\beta_{5nr}q_{o,nr}^{t} + \beta_{6nr})/(BHP_{nr}^{t}+100) + \beta_{7nr}q_{o,nr}^{t}{BHP_{nr}^{t}}^{2} + \beta_{8nr} \qquad nr \in \mathbf{IP}_r - \mathbf{IW}_r \qquad \text{(A-1)}$$

$$\Delta P_{nr}^{t} = \beta_{1nr}{q_{nr}^{t}}^{2} + \beta_{2nr}q_{nr}^{t} + \beta_{3nr}q_{nr}^{t}BHP_{nr}^{t} + \beta_{4nr} \qquad nr \in \mathbf{IW}_r \qquad \text{(A-2)}$$

$$\Delta P_{nrm}^{t} = \beta_{1nrm}{qwm_{nrm}^{t}}^{2} + (\beta_{2nrm}qwm_{nrm}^{t} + \beta_{3nrm}qwm_{o,nrm}^{t})/(BHP_{nr}^{t}+0.01) + \beta_{4nrm}L_{nrm} + \alpha_{5nrm} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r, m \in \mathbf{M}_{nr} \qquad \text{(A-3)}$$

$$\Delta P_{nrm}^{t} = \beta_{1nrm}{qwm_{nrm}^{t}}^{2} + \beta_{2nrm}qwm_{nrm}^{t} + \beta_{3nrm}qwm_{nrm}^{t}L_{nrm}^{t} + \beta_{4nrm} \qquad n \in \mathbf{IW}_r, m \in \mathbf{M}_{nr} \qquad \text{(A-4)}$$

$$\Delta P_{nrc}^{t} = \beta_{1nrc}{qwc_{nrc}^{t}}^{2} + (\beta_{2nrc}qwc_{nrc}^{t} + \beta_{3nrc}qwc_{o,nrc}^{t})/(BHP_{nr}^{t}+0.01) + \beta_{4nrc}L_{nrc} + \beta_{5nrc} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r, c \in \mathbf{C}_{nr} \qquad \text{(A-5)}$$

$$\Delta P_{nrc}^{t} = \beta_{1nrc}{qwc_{nrc}^{t}}^{2} + \beta_{2nc}qwc_{nrc}^{t} + \beta_{3nrc}qwc_{nrc}^{t}L_{nrc}^{t} + \beta_{4nrc} \qquad n \in \mathbf{IW}_r, c \in \mathbf{C}_{nr} \qquad \text{(A-6)}$$

$$\Delta P_{mc}^{t} = \beta_{1mc}{qmc_{mc}^{t}}^{2} + (\beta_{2mc}qmc_{mc}^{t} + \beta_{3mc}qmc_{o,mc}^{t})/(BHP_{nr}^{t}+0.01) + \beta_{4mc}L_{mc} + \beta_{5mc} \qquad m \in \mathbf{PM}, c \in \mathbf{C}_m \qquad \text{(A-7)}$$

$$\Delta P_{mc}^{t} = \beta_{1mc}{qmc_{mc}^{t}}^{2} + \beta_{2mc}qmc_{mc}^{t} + \beta_{3mc}qmc_{mc}^{t}L_{mc}^{t} + \beta_{4mc} \qquad m \notin \mathbf{PM}, c \in \mathbf{C}_m \qquad \text{(A-8)}$$

Minimum flowing pressures

$$BHP_{nr}^{L} = \beta_{1nr}q_{o,nr}^{t}/(q_{nr}^{t} + q_{o,nr}^{t} + 1) + \beta_{2nr}q_{nr}^{t}q_{o,nr}^{t} + \beta_{3nr}q_{nr}^{t} + \beta_{4nr}{q_{o,nr}^{t}}^{2} + \beta_{5nr}q_{o,nr}^{t} + \beta_{5nr} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r \qquad \text{(A-9)}$$

$$BHP_{nr}^{L} = \beta_{1nr}{q_{nr}^{t}}^{2} + \beta_{2nr} \qquad nr \in \mathbf{IW}_r \qquad \text{(A-10)}$$

$$THP_{nrm}^{L} = \beta_{1nrm}{qwm_{o,nrm}^{t}}^{2} + \beta_{2nrm}qwm_{o,nrm}^{t}\sqrt{L_{nrm}} + \beta_{3nrm}qwm_{nrm}^{t} + \beta_{4nrm} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r, m \in \mathbf{M}_{nr} \qquad \text{(A-11)}$$

$$THP_{nrc}^{L} = \beta_{1nrc}{qwm_{o,nrc}^{t}}^{2} + \beta_{2nrc}qwm_{o,nrc}^{t}\sqrt{L_{nrc}} + \beta_{3nrc}qwm_{nrc}^{t} + \beta_{4nrc}$$

$$n \in \mathbf{IP}_r - \mathbf{IW}_r,\ c \in \mathbf{C}_{nr} \qquad \text{(A-12)}$$

$$MP^L_{mc} = \beta_{1mc} {qmc^t_{o,mc}}^2 + \beta_{2mc} qmc^t_{o,mc} \sqrt{L_{mc}} + \beta_{3mc} qmc^t_{mc} + \beta_{4mc}$$

$$m \in \mathbf{PM},\ c \in \mathbf{C}_c \qquad \text{(A-13)}$$

$$MP^L_{nrm} = \beta_{1nrc} {qwc^t_{nrc}}^2 + \beta_{2nrc} L_{nrc} + \beta_{3nrc} \qquad n \in \mathbf{IW}_r,\ c \in \mathbf{C}_{nr} \qquad \text{(A-14)}$$

$$CP^L_{mc} = \beta_{1mc} {qmc^t_{mc}}^2 + \beta_{2mc} L_{mc} + \beta_{3mc} \qquad m \notin \mathbf{PM},\ c \in \mathbf{C}_c \qquad \text{(A-15)}$$

$$CP^L_{nrc} = \beta_{1nrc} {qwm^t_{o,nrc}}^2 + \beta_{2nrc} L_{nrc} + \beta_{3nrc} \qquad n \in \mathbf{IW}_r,\ c \in \mathbf{C}_{nr} \qquad \text{(A-16)}$$

Where $\boldsymbol{\beta}$ is the matrix regression parameters. If the pressure drop is not significant it can be ignored. A sensitivity analysis can help for this purpose (specially for determining the critical length, beyond which the pressure drop should be considered [235]).

CHAPTER 6 PLACEMENT AND PLANNING THE DRILLINGS AND INFRASTRUCTURE INSTALLATIONS IN OIL FIELDS

6.1 Introduction

Well drilling is an important step towards fulfilling the demand of energy market, and it is a critical activity in oil and gas exploitation projects due to its considerable costs and risks. Well drilling determines the amount of accessible hydrocarbon deposit in the subsurface formation, and hence to a great extent it dictates the profitability of the exploitation project. Therefore, drilling has continued to be a major task in oil and gas fields. Some statistics suggests that the average active drilling rigs in 2002 was about 1829 rigs and it has almost doubled to 3518 rigs in 2012 [236]. According to International Association of Drilling Contractors (IADC) nearly 572.334 million man-hours work was spent in drilling, and that only reflects almost 74% of (and not the entire) worldwide oil and gas well drilling rig-fleets [237]. Such a huge and expensive workload should be efficiently planned priory by determining the best drilling and installation strategy. Such strategy involves many important techno-economic decisions, including (and not limited to) instructions for locating the best drilling and installation sites, their numbers and times of installations, the best production network structure and processing capacities. These decisions should consider numerous subsurface and surface factors and conditions as well as the market and economy constraints.

6.1.1 Background

Figure 5-1 shows a general multi-reservoir oil field. Each field consists of number of reservoirs with sub-surface fluids (oil, gas and water) deposited in these porous media. The production from the subsurface formation is sent to the shared surface production network via different producer wells. This surface production network is consisted of different manifolds which receive the production and transfer that to the surface treatment units. Multiphase flow happens in considerable portion of the entire system. The fluid is pushed by the reservoir pressure support and traverses the bottom hole pressure (in the well bore), the tubing head pressure (at the wellhead), the manifold pressure and surface center pressure; different valves regulate the flow along this path. Similarly, there is an injection network sending back the water as the driving fluid. The field cannot continue the operation with the same production level forever, let alone meeting an increasing oil demand. Therefore, new strategies should be implemented and new wells should be drilled and connected to the surface network at the right time.

Well drillings are subject to drilling-rig availability. Moving a rig to a new location involves several steps such as preparation for moving, transition to a new location, and re-installation. These steps are followed by other activities to prepare a new well, including installation of the blowout preventer (BOP), drilling into the reservoir, casing and cementing, and perforation. These tasks are time consuming and considerably expensive. Therefore, these well preparation activities are preceded by careful studies of subsurface and surface conditions, and are proceeded by connecting the new wells to the surface production network. Once a new well is opened for production not only the surface conditions, but also the subsurface dynamics get affected. Since this system is spatio-temporally dynamic, any drilling activity is subject to *time and location* decisions for drilling. If the well is drilled too early, besides the economic drawbacks in terms of time value of the money, it might reduce the efficiency of water flooding by scattering the waterfront and creating unnecessary pressure sinks. On the other hand, if it is drilled too late, the producer well might not receive enough pressure support. It is essential to notice that there are complicated interactions between all these elements (wells, manifolds, surface centers and the rest of the network) and wells are not the only players and the rest of surface network also should be modified accordingly at the right time. Decisions for installing new manifolds or processing centers and determining the processing capacities are dependent on the well placement and drilling planning, if done correctly, the production decline can be postponed. Therefore, it is vital to critically study the order of drilling and infrastructure installations.

6.1.2 Literature Review

The central part of the aforementioned field development problem remains to be the well placement studies, and that is completed by considering the surface infrastructure planning. The majority of well placement researches (implicitly) assume that all the wells are opened for production at the beginning of production horizon [53, 62, 63, 74, 139]. The previous chapters [201, 221] provide detailed literature survey on this category. Therefore, here we only review the well placement studies that also address the order of drilling. Haugland et al. [128] studied the well placement, drilling and production planning, and platform capacity design by using a linear simplified form of well production equation and superposition method. They reported high computational cost for the problem of determining order of drilling, and suggested utilizing the model structure and reformulation, rather than using commercial solvers directly. In another study, Beckner and Song [27] used a predetermined list of potential locations and a variation of the "travelling salesman" problem to address the order of drilling problem. Later, Iyer et al. [116] performed a land mark study. They developed an MILP model to address the well selection, the well drilling and platform installation timing considering the drilling rig availability, platform sizing and production planning. They used piecewise linear approximation of reservoir pressure and GOR versus cumulative oil production to consider the reservoir response to the production. However, these approximations do not depend on the number of wells and their locations. Iyer et al. [116] did not consider any injection operation and employed a set of simplifying assumptions, such as constant productivity index for each well throughout planning horizon, independent production of wells from each other, and linear pressure drop based on the flow rate in pipes. Most of these assumptions can cause either overestimation or underestimation of the production performance. They applied a sequential decomposition algorithm to suggest an upper bound for the proposed maximization model via aggregation of the wells in each reservoir and time steps, as well as relaxation of the piecewise linear expansion. In the reverse procedure they calculated the lower bound by disaggregation and relaxing the proposed declining profile constraint and finally they reduced the sharp changes in the suggested profile. Later, their MILP model is extended in several studies. Van Den Heever and Grossmann [132] extended that by fitting an exponential function to describe reservoir pressure versus cumulative oil flow rate, and other quadratic functions to describe the cumulative gas productions and GOR versus cumulative oil flow rates. Their MINLP model employs generalized disjunctive programming. Additionally, Aseeri et al. [238] extended the deterministic model of Iyer et al. [116] into a stochastic model which caused even more serious issues with the dimensionality. Therefore, they used a sampling average algorithm to overcome this problem. Oil price and productivity index were considered as stochastic parameters. Cavalho and Pinto [123] studied the platform location-allocation problem as well as drilling and connection timing and well flow rate assignment using a multi-period MILP model. They used the algorithm suggested by Iyer et al. [116] to solve this MILP problem. The well-platform assignments are addressed in the master problem and the timing for fixed assignment is determined in the sub-problems. They assume linear decline in pressure with the oil removal to define the maximum flow rate from each well (open flow rate productivity index × pressure).

In another study, Barnes et al. [126] suggested an MILP followed by an MINLP to address design and operation of oil and gas. The MILP registers the design decisions (location and capacity of platforms as well as drilling centers), and the MINLP determines the well operation variables. Similar to Iyer et al. [116] they assumed independent production of wells in a reservoir based on their productivity index. Therefore, the nonlinear interactions between the wells are not considered and the subsurface dynamic is grossly approximated. In the next study, Gupta and Grossmann [239] developed a nonconvex MINLP model for a deterministic multi-field problem and later reformulated that into a MILP model. Their 3-phase model addresses the decisions for production planning, well drilling planning, floating production storage and offloading (FPSO)-field connection, surface unit installation and expansion planning. Similar to the previous works of Grossmann and co-workers, they employ regression models to capture the field's dynamic. These regression models include maximum oil flow rate, WOR and GOR versus fractional oil recovery. Although this is an extensive, detailed, and important work, it also has limiting assumptions such as identical well performance in each reservoir.

Apart from optimization studies and on the simulation path, ECLIPSE reservoir simulator [44] provides an option of "drilling queue" which gets activated when the production target is not met. This queue is a list of pre-defined wells which can be deployed either sequentially or based on a priority. Güyagüler [71] in his PhD thesis suggests solving an ordering problem for this queue whenever the queue is called. He puts this problem into the travelling sales person framework and solves it with genetic algorithm after some modifications. He also reports high computational cost, yet the connection to the surface network and required changes in the surface is not addressed.

A similar problem to well placement and drilling planning is the rig scheduling, that is allocating rigs to different wells for a specific service, and determining the service sequence and movement route for rigs. Specifically work-over rig scheduling (WRS) problem falls under this category and it is to ensure that the right rig is moved to a right well at a right time, through a right path [240]. Ribeiro et al. [241] provides a concise review on different algorithms used to solve WRS. Additionally, they solved a model of WRS by enhancing simulated annealing with neighbourhood local search based on re-ordering wells, re-allocating and swapping well moves. They also provided comparison with CPLEX, dynamic assembly heuristic, greedy randomized adaptive search procedure, scatter search, bubble swap and genetic algorithm. Their method beats CPLEX, however for other methods, a general conclusion in terms of solution time and objective value may not be clear. Bassi et al. [240] studied the drilling rig scheduling to find good (not necessarily optimum) set of solutions and consider the mobilization/demobilization, travelling and uncertain service time in their model. They used constructive heuristics and greedy randomized adaptive search procedure in simulation optimization framework to solve the problem and as a result they penalized the constraint violation in their objective function. Finally, Duhamel et al. [242] focused on mathematical formulation of scheduling the work-over rigs which consists of allocating rigs to wells and rig movement routing. They modified and proposed three mixed integer linear models: scheduled-based formulation, open vehicle routing approach and an extended model. For the last one they employed a heuristics and column generation method.

Finally, in the last two chapters ([221] and [243]), we addressed joint well placement and production planning in a single reservoir with a rectangular shape, and focused on rigorous reservoir model and (multiphase) well flow up to the well head [221], and then extended that to multi-reservoir oil fields with irregular shapes and completed the connection to surface network by addressing the surface infrastructure installation and allocation problem. We formulated these dynamic optimization problems within a spatiotemporal and dynamic nonconvex MINLP model and modified an outer approximation algorithm to solve that. In both of these studies, we assumed that all the wells are opened to production at time zero.

Despite of numerous findings from above studies, there are still several important gaps and from the above literature survey, they are briefly as follows:

1) In majority, the *subsurface dynamics* and sometimes the multiphase flow to the surface are either ignored or grossly approximated; hence in such studies the prediction of water/oil front is not accurate.

2) In cases where the subsurface dynamic is accurate (using simulation-optimization) the order of drilling is not addressed, or the interactions between the subsurface and surface network are not considered.

3) Some of the studies use a predetermined list of potential locations/connections and then try to determine the order of drillings/installations. However the processes of preparing this list are either based on limited heuristics [27, 44] (if any), or based on very general approximations [126]. Both of which can cause significant departure from an optimal (or very good) solutions.

In light of above points, in this study we extend our previous studies to address well and infrastructure placement, allocation, and timing as well as planning the capacity of the surface centers which equip this study with significant novelty. We consider the drilling rig availability, yet we do not address the rig routing and scheduling problem in details. With the elements provided here, those problems can also be studied within the same framework after required modifications. In the remaining of this chapter, we initially define the problem and then discuss our model. Then we introduce our solution strategy and after that we evaluate its performance with a case study. Finally, we conclude with a concise discussion.

6.2 Problem Definition

We would like to boost the oil and gas production in a multi-reservoir oil field (Figure 5-1), by (some or all of the) following tasks: (1) drilling new vertical producer wells, (2) installing new manifolds/processing centers, with appropriate new connections, (3) regulating all throughputs during the planning horizon, (4) incremental capacity expansion of the surface centers. For brevity, we refer to the vertical wells, manifolds, and processing centers as “elements” and each element can be attached to several flow-lines and valves. We consider “drilling” as a special form of “installation”; hence, in the remaining of this article, the term “element installation” can refer to well-drillings or infrastructure installations. If the field is already producing, we need to modify the surface network to respond to the new installations and production plan. That requires determining the optimal element-to-element location-allocation, as well as time and the order of these installations and modifications. Therefore the problem can be defined as follows:

Let us consider a field with water injection and/or primary expansion driving mechanisms in each reservoir. Its reservoir might have different saturation and pressure distribution with compressible fluid phases (oil, water, and gas). Any wellhead in this field can be connected to one or more manifolds/centers. Moreover, processing centers can receive fluids from wells directly or via manifolds; similarly central water processing units supply water to injector wells directly or via shared manifolds.

The status of this oil field, the oil market requirements, and the future financial projections be deterministically explained through various groups of data: (1) geological, petro-physical and dynamic data (including permeability, porosity, field structure, as well as compressibility factors, viscosity, and also initial saturation and pressure map), (2) structural data (such as surface connections and each well’s diameter, length and roughness), (3) the operational data and their limits (minimum / maximum manifold or bottom hole pressure, inlet pressures at the separation centers, maximum water-cut, the processing capacities of the manifolds and centers, oil demand and production horizon of H years), and (4) the economic data (such as fluid production/injection and various drilling and installation expenses). These data are usually provided through multi-disciplinary studies; let these data be given.

We would like to **maximize** the net present value (NPV) of the oil/gas exploitation project over the planning horizon by determining:

1. Number, locations, time and order of installing new elements (with required flow-lines and valves) and their throughputs.
2. New surface connection allocations (well-to-manifold, well-to-surface-center, and manifold-to-surface-center), time and order of connecting them.
3. The capacity and incremental capacity expansion of each center and the field.
4. Dynamic pressure profiles along the network at processing centers, manifolds, wellheads, well bore holes, and corresponding valve settings
5. Dynamic pressure and saturation profiles (waterfront location) for each reservoir

The above decisions determine the reservoirs to be exploited and each reservoir’s share in the total production. For that, we use the following assumptions:

1. The required time for installation activities for each element is not longer than each time period. Moreover, the manifold capacities remain constant over time; however, processing center capacities may vary with time as more separators can be installed.
2. Field surface elevation may vary from point to point, however the reservoirs are horizontal and planar, and may have arbitrary and irregular shapes. They may overlap, but they are disconnected. Wells are vertical, and can pass through multiple reservoirs, but can be perforated to access only one reservoir.
3. Capillary pressure in the reservoir is negligible and the reservoir is under-saturated (i.e. its pressure exceeds bubble point pressure). All reservoirs have the same fluid.

4. Each well (existing or potential) is pre-allocated to some manifolds/centers (existing or potential) based on some criteria such as distance, from which the best allocations will be selected. This can be relaxed but will increase computation time [116]. Existing manifolds and centers can make/receive new connections.

Our solution follows three policies: (1) a well that hits its water-cut limit is shut in and is not opened again. (2) Each well must be located beyond some minimum distance from all other wells. (3) Unlike our previous study we allow rejecting or delaying the production through existing well-to-manifold and manifold-to-center connections.

6.3 Modeling

We extend and modify the study in the previous chapter [243] to expand its applicability for addressing the capacity design and ordering / timing problem. Here, we provide the complete set of equations with very concise descriptions of the common features to keep the paper self-sufficient and focus on the new aspects of our study. We define our model **ET** as follows:

Let us use R ($r = 1, 2, \dots, R$) to present the number of reservoirs, C ($c = 1, 2, \dots, C$) to refer to the number of processing centers (existing/potential), and M ($m = 1, 2, \dots, M$) to show the number of manifolds (existing/potential). Of these, we assume that the first C_e processing centers ($1 \leq c \leq C_e$) and first M_e manifolds ($1 \leq m \leq M_e$) already exist. We discretize the spatial coordinates following the previous chapters to model each reservoir r bounded in the tightest possible rectangle. Each rectangle is discretized by defining I_r cells of arbitrary lengths Δx_{ir} ($i = 1, 2, \dots, I_r$) in the x-direction and J_r cells of arbitrary lengths Δy_{jr} ($j = 1, 2, \dots, J_r$) in the y-direction. Then, each cell in a reservoir can be indexed by an integer as $n = i + (j - 1) \times I_r$. These sets help defining other subsets as follows:

Interior points: $\mathbf{IP}_r = \{n \mid \text{cell } n \text{ belongs to reservoir } r\}$

Geometry: $\mathbf{IX}_r = \{n \mid n \in \mathbf{IP}_r, (n + 1) \in \mathbf{IP}_r, \text{and } i < I_r\}$

$$\mathbf{IY}_r = \{n \mid n \in \mathbf{IP}_r, (n + I_r) \in \mathbf{IP}_r, \text{and } j < J_r\}$$

Potential allocations: $\mathrm{M_{nr}} = \{\mathrm{m} \mid \text{cell n of reservoir r is/ can be connected to manifold m}\}$

$$1 \leq r \leq R, n \in \mathbf{IP}_r$$

$$\mathbf{C}_{nr} = \{c \mid \text{cell } n \text{ of reservoir } r \text{ is/ can be connected to center } c\}$$

$$1 \leq r \leq R, n \in \mathbf{IP}_r$$

$$\mathbf{C}_m = \{c \mid \text{manifold } m \text{ is/ can be connected to center } c\}$$

$$1 \leq r \leq R, n \in \mathbf{IP}_r$$

Existing and candidate wells:

$$\mathbf{PW}_r = \{n \mid n \in \mathbf{IP}_r \text{ is an existing producer well}\}$$

$$\mathbf{IW}_r = \{n \mid n \in \mathbf{IP}_r \text{ is an existing injector well}\}$$

$$\mathbf{NW}_r = \{n \mid n \in \mathbf{IP}_r \text{ is a potential location for a new producer well}\}$$

$$= \mathbf{IP}_r - \mathbf{PW}_r - \mathbf{IW}_r - \{n \in \mathbf{IP}_r \mid \text{a new producer well is infeasible at cell } n\}$$

Finally, discretizing the planning horizon H into T periods of arbitrary lengths Δh^t $(t = 1, 2, \ldots, T)$ determines the planning time periods. Unless stated otherwise, all variables and constraints involving index t, r, m, and c are to be written for all their valid values, i.e. $0 < t \leq T$, $1 \leq r < R$, $1 \leq m < M$, $1 \leq c < C$.

Based on the discretization approach used in previous chapters, variables are defined at the end of each time period; e.g. S_{nr}^t and P_{nr}^t are saturation and pressure at the end of interval t. Then the initial point of the first time period is indexed by 0 , so S_{nr}^0 and P_{nr}^0 are the initial saturation and pressure at time zero.

Logical constraints: Each installation is an important point in time and happens only once; after which the status of the installed element changes to *"ready for exploitation"*. Hence we define following two sets of design variables:

The first set is used to monitor the installation period. Since the production through a new element is only possible after its installation, new elements can be installed /connected at any time period except the last one ($0 \le t < T$). These binary variables show the drilling and installation (one-time) events:

$$u_c^t = \begin{cases} 1 & \text{if center } c \text{ is installed at the end of period } t \\ 0 & \text{otherwise} \end{cases}$$

$$z_m^t = \begin{cases} 1 & \text{if manifold } m \text{ is installed at the end of period } t \\ 0 & \text{otherwise} \end{cases}$$

$$zu_{mc}^t = \begin{cases} 1 & \text{if manifold } m \text{ is connected to center } c \text{ at the end of period } t \\ 0 & \text{otherwise} \end{cases} \qquad c \in \mathbf{C}_m$$

$$y_{nr}^t = \begin{cases} 1 & \text{if a well is drilled at cell } n \text{ of reservoir } r \text{ at the end of period } t \\ 0 & \text{otherwise} \end{cases} \qquad n \in \mathbf{IP}_r$$

$$yz_{nrm}^t = \begin{cases} 1 & \text{if a well at cell } n \text{ of reservoir } r \text{ is connected to manifold } m \text{ at the end of period } t \\ 0 & \text{otherwise} \end{cases} \qquad n \in \mathbf{IP}_r, m \in \mathbf{M}_{nr}$$

$$yu_{nrc}^t = \begin{cases} 1 & \text{if a well at cell } n \text{ of reservoir } r \text{ should be connected to center } c \text{ at the end of period } t \\ 0 & \text{otherwise} \end{cases} \qquad n \in \mathbf{IP}_r, c \in \mathbf{C}_{nr}$$

The second set is utilized to flag the continuous availability for production. Since there is no production at $t = 0$ they are defined over $0 < t \le T$:

$$us_c^t = \begin{cases} 1 & \text{if center } c \text{ can be used during period } t \\ 0 & \text{otherwise} \end{cases}$$

$$zs_m^t = \begin{cases} 1 & \text{if manifold } m \text{ can be used during period } t \\ 0 & \text{otherwise} \end{cases}$$

$$zus_{mc}^t = \begin{cases} 1 & \text{if the connection between manifold } m \text{ and center } c \text{ can be used during period } t \\ 0 & \text{otherwise} \end{cases} \qquad c \in \mathbf{C}_m$$

$$ys_{nr}^t = \begin{cases} 1 & \text{if the well at cell } n \text{ of reservoir } r \text{ can be productive during } t \\ 0 & \text{otherwise} \end{cases} \qquad n \in \mathbf{IP}_r$$

$$yzs_{nrm}^t = \begin{cases} 1 & \text{if the connection between well at cell } n \text{ of reservoir } r \text{ and manifold } m \text{ can be used during period } t \\ 0 & \text{otherwise} \end{cases} \qquad n \in \mathbf{IP}_r, m \in \mathbf{M}_{nr}$$

$$yus_{nrc}^{t} = \begin{cases} 1 & \text{if the connection betweeb well at cell } n \text{ of reservoir } r \\ & \text{and center } c \text{ can be used during period } t \\ 0 & \text{otherwise} \end{cases} \qquad n \in \mathbf{IP}_r, c \in \mathbf{C}_{nr}$$

The above two sets are tightly related to each other. That is ($0 \le t' < T$, $0 < t \le T$):

$$us_c^t = \sum_{t'<t} u_c^{t'} \tag{1}$$

$$zs_m^t = \sum_{t'<t} z_m^{t'} \tag{2}$$

$$zus_{mc}^t = \sum_{t'<t} zu_{mc}^{t'} \qquad c \in \mathbf{C}_m \tag{3}$$

$$ys_{nr}^t = \sum_{t'<t} y_{nr}^{t'} \qquad n \in \mathbf{IP}_r \tag{4}$$

$$yzs_{nrm}^t = \sum_{t'<t} yz_{nrm}^{t'} \qquad n \in \mathbf{IP}_r\ , m \in \mathbf{M}_{nr} \tag{5}$$

$$yus_{nrc}^t = \sum_{t'<t} yu_{nrc}^{t'} \qquad n \in \mathbf{IP}_r\ , c \in \mathbf{C}_{nr} \tag{6}$$

Each installation happens only once and after that the connections are not removed; both of which are satisfied through Eqs. (1) to (6). Since these equations are written in a cumulative manner, they guarantee that no off-on-off behavior will happen for selected solutions, additionally as can be seen later this also to increase the sparsity of the model.

Figure 6-1 represents the idea of defining the above two sets. As an example, consider a well candidate which is selected to be drilled by the end of time period t'. With the above definitions $ys_{nr}^t = 1$ for $t > t'$ and $ys_{nr}^t = 0$ for $t \le t'$, also $y_{nr}^t = 1$ for $t = t'$ and $y_{nr}^t = 0$ for $t \ne t'$.

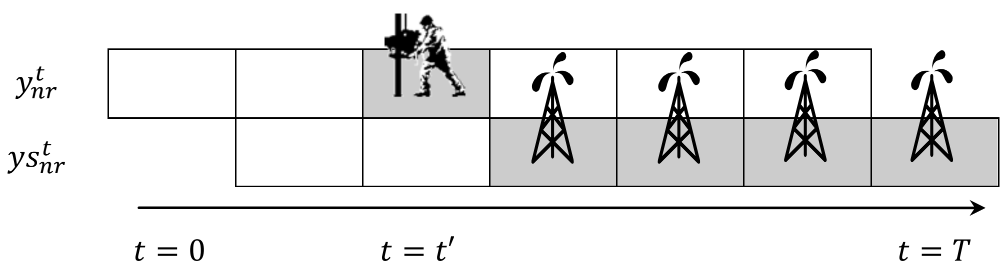

Figure 6-1: Schematic representation of the binary and continuous design variables for drilling scheduling. The shaded and white slots represent 1 and 0 values respectively.

The second set can also be represented as bounded continuous variables. We tested both and preferred the binary definitions. The first set of binary variables for installations is mainly defined as one-time-activating variables for modelling the installation cost (which is paid only once), whereas the other set is defined to model the operational costs/revenues effectively. Therefore, if any element of the infrastructure already exists, its installation binary at $t = 0$ and its production binary variables at $0 < t \leq T$ are fixed at 1 and its installation cost is rendered as zero.

The above definitions enable us to establish proper constraints for maximum possible drillings at each time step (ADR^t is the number of available drilling rigs) (Eqn. (7)), minimum number of producer wells to continue the production (at least one) (Eqn. (8)), minimum/maximum number of connections as well as simultaneous drilling / installation and connection (Eqs. (9-12)):

$$\sum_{r=1}^{R} \sum_{n\in\mathbf{IP}_r, n\notin\mathbf{IW}_r} y_{nr}^{t} \leq ADR^t \qquad t \leq T \qquad (7)$$

$$\sum_{r=1}^{R} \sum_{n\in\mathbf{IP}_r, n\notin\mathbf{IW}_r} ys_{nr}^{t=T} \geq 1 \qquad (8)$$

$$WMC_{nr}^{U} ys_{nr}^{t} \geq \sum_{m\in\mathbf{M}_{nr}} yzs_{nrm}^{t} + \sum_{c\in\mathbf{C}_{nr}} yus_{nrc}^{t} \geq ys_{nr}^{t} \qquad n \in \mathbf{IP}_r,\ 0 < t \qquad (9)$$

$$MW_{m}^{U} zs_{m}^{t} \geq \sum_{r=1}^{R} \sum_{n\in\mathbf{IP}_r \ni m\in\mathbf{M}_{nr}} yzs_{nrm}^{t} \geq zs_{m}^{t} \qquad 0 < t \qquad (10)$$

$$MC_{m}^{U} zs_{m}^{t} \geq \sum_{c\in\mathbf{C}_{m}} zus_{mc}^{t} \geq zs_{m}^{t} \qquad 0 < t \qquad (11)$$

$$CMW_{c}^{U} u_{c}^{t} \geq \sum_{m\ni c\in\mathbf{C}_{m}} zus_{mc}^{t} + \sum_{r=1}^{R} \sum_{n\in\mathbf{IP}_r \ni c\in\mathbf{C}_{nrc}} yus_{nrc}^{t} \geq us_{c}^{t} \qquad 0 < t \qquad (12)$$

Furthermore, following previous chapters, two wells are prevented to be drilled in two adjacent cells ($t = T$):

$$ys_{nr}^{t} + ys_{(n+1)r}^{t} \leq 1 \qquad n \in \boldsymbol{IX}_r \qquad (13)$$

$$ys_{nr}^{t} + ys_{(n+I_r)r}^{t} \leq 1 \qquad n \in \boldsymbol{IY}_r \qquad (14)$$

$$ys_{nr}^{t} + ys_{(n+I_r-1)r}^{t} \leq 1 \qquad (n + I_r - 1) \in \boldsymbol{IX}_r \qquad (15)$$

$$ys_{nr}^{t} + ys_{(n+I_r+1)r}^{t} \leq 1 \qquad (n + I_r) \in \boldsymbol{IX}_r \qquad (16)$$

Notice that the clique cuts can be added due to the structure of above set of equations.

Reservoir Dynamics

The spatio-temporal discretization of the governing PDE of the flow inside the reservoir provides the below dynamic set of equations:

$$(V_{nr}/dh^{t})\{d^{t}_{o,1,nr}\,[P^{t}_{nr}-P^{t-1}_{nr}]+d^{t}_{o,2,nr}\,[S^{t}_{nr}-S^{t-1}_{nr}]\}+q^{t}_{o,n\notin \mathbf{IW_r}}+ \quad (17)$$

$$\begin{pmatrix}\{M^{t}_{o,(x-)r}T^{x}_{(n-1)r}[P^{t}_{nr}-P^{t}_{(n-1)r}]\}_{(n-1)\in \mathbf{IX}_r}+\{M^{t}_{o,(x+)r}T^{x}_{nr}[P^{t}_{nr}-P^{t}_{(n+1)r}]\}_{n\in \mathbf{IX}_r}+\\ \{M^{t}_{o,(y-)r}T^{y}_{(n-I_r)r}[P^{t}_{nr}-P^{t}_{(n-I_r)r}]\}_{(n-I)\in \mathbf{IY}_r}+\{M^{t}_{o,(y+)r}T^{y}_{nr}[P^{t}_{nr}-P^{t}_{(n+I_r)r}]\}_{n\in \mathbf{IY}_r}\end{pmatrix}=0;$$

$$(V_{nr}/dh^{t})\{d^{t}_{w,1,nr}\,[P^{t}_{nr}-P^{t-1}_{nr}]+d^{t}_{w,2,nr}\,[S^{t}_{nr}-S^{t-1}_{nr}]\}+\left(q^{t}_{n,r}-q^{t}_{o,n\notin \mathbf{IW}_r}\right)+ \quad (18)$$

$$\begin{pmatrix}\{M^{t}_{w,(x-)r}T^{x}_{(n-1)r}[P^{t}_{nr}-P^{t}_{(n-1)r}]\}_{(n-1)\in \mathbf{IX}_r}+\{M^{t}_{w,(x+)r}T^{x}_{nr}[P^{t}_{nr}-P^{t}_{(n+1)r}]\}_{n\in \mathbf{IX}_r}+\\ \{M^{t}_{w,(y-)r}T^{y}_{(n-I_r)r}[P^{t}_{nr}-P^{t}_{(n-I_r)r}]\}_{(n-I)\in \mathbf{IY}_r}+\{M^{t}_{w,(y+)r}T^{y}_{nr}[P^{t}_{nr}-P^{t}_{(n+I_r)r}]\}_{n\in \mathbf{IY}_r}\end{pmatrix}=0;$$

And phase mobilites are defined as:

$$M^{t}_{o,nr}=kr^{o}_{0}\left(\frac{1-S^{t}_{nr}-S_{r,or}}{1-S_{r,wr}-S_{r,or}}\right)^{a}/(\mu_{r,Bo,1}P^{t}_{nr}+\mu_{r,Bo,2}) \quad (19)$$

$$M^{t}_{w,nr}=kr^{w}_{0}\left(\frac{S^{t}_{nr}-S_{r,wr}}{1-S_{r,wr}-S_{r,or}}\right)^{b}\times[1+C_{w}(P^{t}_{nr}-P_{r})]/B_{r,wr}\mu_{w,r} \quad (20)$$

Flow Balances and Capacity Planning

<u>Installation and flow</u>: A new element can only be productive after it is selected and after that its throughput is bounded (Eqs. (21)-(26)),. Furthermore, oil flow rate cannot exceed total liquid flow rates (Eqs. (27)-(29)) .

$$\delta_{nr}q^{t}_{nr}\leq q^{t,U}_{nr}ys^{t}_{nr} \qquad n\in \mathbf{IP}_r \quad (21)$$

$$QM^{t}_{m}\leq QM^{t,U}_{m}zs^{t}_{m} \quad (22)$$

$$QC^{t}_{c}\leq QC^{t,U}_{c}us^{t}_{c} \quad (23)$$

$$qmc^{t}_{mc}\leq QM^{t,U}_{m}zus^{t}_{mc} \qquad c\in \mathbf{C}_m \quad (24)$$

$$qwm^{t}_{nrm}\leq qwm^{t,U}_{nrm}yzs^{t}_{nrm} \qquad n\in \mathbf{IP}_r, m\in \mathbf{M}_{nr} \quad (25)$$

$$qwc^{t}_{nrc}\leq qwc^{t,U}_{nrc}yus^{t}_{nrc} \qquad n\in \mathbf{IP}_r, c\in \mathbf{C}_{nr} \quad (26)$$

$$qmc^{t}_{o,mc}\leq qmc^{t}_{mc} \qquad c\in \mathbf{C}_m \quad (27)$$

$$qwm_{o,nrm}^{t} \leq qwm_{nrm}^{t} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r, m \in \mathbf{M}_{nr} \qquad (28)$$

$$qwc_{o,nrc}^{t} \leq qwc_{nrc}^{t} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r, c \in \mathbf{C}_{nr} \qquad (29)$$

Oil flow rate of different elements of the injection line is set to zero.

Minimum flow: The total production from a selected element should be economically acceptable:

$$\sum_t q_{o,nr}^{t} \Delta h^t \geq q_{o,nr}^{L} ys_{nr}^{t'=T} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r \qquad (30)$$

$$\sum_t QM_m^t \Delta h^t \geq QM_m^L zs_m^{t'=T} \qquad (31)$$

$$\sum_t QC_c^t \Delta h^t \geq QC_c^L us_c^{t'=T} \qquad (32)$$

Flow Balances: The flow balances should be satisfied at every point in the field.

$$\delta_{nr} q_{nr}^{t} = \sum_{m \in \mathbf{M}_{nr}} qwm_{nrm}^{t} + \sum_{c \in \mathbf{C}_{nr}} qwc_{nrc}^{t} \qquad n \in \mathbf{IP}_r \qquad (33)$$

$$q_{o,nr}^{t} = \sum_{m \in \mathbf{M}_{nr}} qwm_{o,nrm}^{t} + \sum_{c \in \mathbf{C}_{nr}} qwc_{o,nrc}^{t} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r \qquad (34)$$

$$QM_m^t = \sum_{r=1}^{R} \sum_{n \in \mathbf{IP}_r \ni m \in \mathbf{M}_{nr}} qwm_{nrm}^{t} = \sum_{c \in \mathbf{C}_m} qmc_{mc}^{t} \qquad (35)$$

$$QM_{o,m}^t = \sum_{r=1}^{R} \sum_{n \in (\mathbf{IP}_r - \mathbf{IW}_r) \ni m \in \mathbf{M}_{nr}} qwm_{o,nrm}^{t} = \sum_{c \in \mathbf{C}_m} qmc_{o,mc}^{t} \qquad m \in \mathbf{PM} \qquad (36)$$

$$QC_c^t = \sum_{m \ni c \in \mathbf{C}_m} qmc_{mc}^{t} + \sum_{r=1}^{R} \sum_{n \in \mathbf{IP}_r \ni c \in \mathbf{C}_{nr}} qwc_{nrc}^{t} \qquad (37)$$

$$QC_{o,c}^t = \sum_{m \ni c \in \mathbf{C}_m} qmc_{o,mc}^{t} + \sum_{r=1}^{R} \sum_{n \in (\mathbf{IP}_r - \mathbf{IW}_r) \ni c \in \mathbf{C}_{nr}} qwc_{o,nrc}^{t} \qquad c \in \mathbf{PC} \qquad (38)$$

$$QC^t = \sum_{c \in \mathbf{PC}} QC_c^t \qquad (39)$$

$$QC_o^t = \sum_{c \in \mathbf{PC}} QC_{o,c}^t \qquad (40)$$

$$QC_w^t = \sum_{c \notin \mathbf{PC}} QC_c^t \qquad (41)$$

Center capacity

This capacity (Q_c^{t*}) can be expanded incrementally (dQ_c^{t*}). The dQ_c^{t*} capacity expansion happens at t and is only available to the surface facility at $t+1$. Furthermore the incremental capacity expansion is limited and depends on the available capacity at each period; and these expansions increase the field capacity. Obviously the new capacities control the maximum field fluid flow :

$$Q_c^{(t+1)*} = dQ_c^{t*} + Q_c^{t*} \qquad 0 \le t < T \qquad (42)$$

$$dQ_c^{t*} = fr.Q_c^{t*} \qquad (t < T) \qquad (43)$$

$$QC^{t*} = \sum_{c\in\mathbf{PC}} Q_c^{t*} \qquad (44)$$

$$QC_w^{t*} = \sum_{c\notin\mathbf{PC}} Q_c^{t*} \qquad (45)$$

$$QC_c^t \le Q_c^{t*} \qquad (46)$$

$$QC^t \le QC^{t*} \qquad (47)$$

$$QC_w^t \le QC_w^{t*} \qquad (48)$$

Q_c^{0*} is the initial center capacity and $\boldsymbol{fr}$ is the maximum possible expansion fraction from each available capacity.

Bounds on flow variables: All above flow variables should be properly bounded with available information:

$$q_{nr}^{t,U} = \min[q_{nr}^{t,OF}, \sum_{m\in\mathbf{M}_{nr}} Q_m^* + \sum_{c\in\mathbf{C}_{nr}} Q_c^{t*,U}, \sum_{m\in\mathbf{M}_{nr}\ni c\in\mathbf{C}_m + c\in\mathbf{C}_{nr}} Q_c^{t*,U}, QC^{t*,U}|_{nr\notin IW_r} + QC_w^{t*,U}|_{nr\in IW_r}] \qquad nr \in \mathbf{IP}_r \qquad (49)$$

$$q_{o,nr}^t \le \min\left[D^t, q_{nr}^{t,U}\right] \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r \qquad (50)$$

$$qwm_{nrm}^{t,U} = \min\left[q_{nr}^{t,U}, Q_m^*, \sum_{c\in\mathbf{C}_m} Q_c^{t*,U}\right] \qquad n \in \mathbf{IP}_r, m \in \mathbf{M}_{nr} \qquad (51)$$

$$qwm_{o,nrm}^t \le \min\left[D^t, qwm_{nrm}^{t,U}\right] \qquad n \in \mathbf{IP}_r, m \in \mathbf{M}_{nr} \qquad (52)$$

$$qwc_{nrc}^{t,U} = \min\left[q_{nr}^{t,U}, Q_c^{t*,U}\right] \qquad n \in \mathbf{IP}_r, c \in \boldsymbol{C}_{nr} \qquad (53)$$

$$qwc_{o,nrc}^t \le \min\left[D^t, qwc_{nrc}^{t,U}\right] \qquad n \in \mathbf{IP}_r, c \in \boldsymbol{C}_{nr} \qquad (54)$$

$$QM_m^{t,U} = \min\left[Q_m^*, \sum_{c\in\mathbf{C}_m} Q_c^{t*,U}, \sum_{r=1}^{R} \sum_{n\in\mathbf{IP}_r\ni m\in\mathbf{M}_{nr}} qwm_{nr}^{t,U}\right] \qquad (55)$$

$$QM_{o,m}^t \le \min\left[D^t, QM_m^{t,U}\right] \qquad (56)$$

$$qmc_{mc}^{t,U} = \min[QM_m^{t,U}, Q_c^{t*,U}] \qquad c \in \mathbf{C}_m \qquad (57)$$

$$qmc_{o,mc}^t \le \min\left[D^t, qmc_{mc}^{t,U}\right] \qquad c \in \mathbf{C}_m \qquad (58)$$

$$QC_c^{t,U} = \min\left[QC^{t*,U}|_{c\in\mathbf{PC}} + QC_w^{t*,U}|_{c\notin\mathbf{PC}}, Q_c^{t*,U}, \sum_{m\in C_m} qmc_{mc}^{t,U} + \sum_{m\in \boldsymbol{M}_{nr}\ni c\in \boldsymbol{C}_m} q_{nr}^{t,U}\right] \quad c \in \mathbf{PC} \quad (59)$$

$$QC_{o,c}^t \le \min\left[D^t, QC_c^{t,U}\right] \quad c \in \mathbf{PC} \quad (60)$$

$$QC^{t,U} = \min(QC^{t*,U}, \textstyle\sum_{c\in\mathbf{PC}} QC_c^{t,U}) \quad (61)$$

$$QC_o^t \le \min(D^t, QC^{t,U}) \quad (62)$$

$$QC_w^t \le \min(QC_w^{t*,U}, \textstyle\sum_{c\notin\mathbf{PC}} QC_c^{t,U}) \quad (63)$$

$$q_{nr}^{t,OF} = \psi_{nr} M_{o,nr}^U \left(P_{r,avg}^t - \mathcal{P}_{c,avg}\right) \quad nr \notin \mathbf{IW}_r \quad (64)$$

where, $\delta_{nr} = -1$ for $n \in \mathbf{IW}_r$ and 1 otherwise.

Voidage Displacement

The injected water should replace the produced liquid to maintain the reservoir pressure.

$$\overline{B_o} QC_o^t \le \overline{B_w} QC_w^t \quad (65)$$

Pressure traverses

Pressure Balances: These equations monitor the automatic pressure variation across the production network, from well bore, to the well head, manifolds and finally the centers.

$$BHP_{nr}^t + (1 - ys_{nr}^t)BM_{nr} \ge THP_{nr}^t + \Delta P_{nr}^t \quad n \in \mathbf{IP}_r - \mathbf{IW}_r \quad (66)$$

$$THP_{nr}^t \ge BHP_{nr}^t + \Delta P_{nr}^t \quad n \in \mathbf{IW}_r \quad (67)$$

$$THP_{nr}^t + (1 - yzs_{nrm}^t)BM_m \ge MP_m^t + \Delta P_{nrm}^t \quad n \in \mathbf{IP}_r - \mathbf{IW}_r, m \in \mathbf{M}_{nr} \quad (68)$$

$$MP_m^t \ge THP_{nr}^t + \Delta P_{nrm}^t \quad n \in \mathbf{IW}_r, m \in \mathbf{M}_{nr} \quad (69)$$

$$THP_{nr}^t + (1 - yus_{nru}^t)BM_c \ge CP_c^t + \Delta P_{nrc}^t \quad n \in \mathbf{IP}_r - \mathbf{IW}_r, c \in \mathbf{C}_{nr} \quad (70)$$

$$CP_c^t \ge THP_{nr}^t + \Delta P_{nrc}^t \quad n \in \mathbf{IW}_r, c \in \mathbf{C}_{nr} \quad (71)$$

$$MP_m^t + (1 - zus_{mc}^t)BM_c \ge CP_c^t + \Delta P_{mc}^t \quad m \in \mathbf{PM}, c \in \mathbf{C}_m \quad (72)$$

$$CP_c^t \ge MP_m^t + \Delta P_{mc}^t \quad m \notin \mathbf{PM}, c \in \mathbf{C}_m \quad (73)$$

Minimum Pressures:

$$BHP_{nr}^t \ge BHP_{nr}^L \quad n \in \mathbf{IP}_r \quad (74)$$

$$THP_{nr}^t \ge THP_{nrm}^L \quad n \in \mathbf{IP}_r - \mathbf{IW}_r, m \in \mathbf{M}_{nr} \quad (75)$$

$$THP_{nr}^{t} \geq THP_{nrc}^{L} \qquad n \in \mathbf{IP}_r - \mathbf{IW}_r,\ c \in \mathbf{C}_{nr} \qquad (76)$$

$$THP_{nr}^{t} \geq THP_{nr}^{L} \qquad n \in \mathbf{IW}_r \qquad (77)$$

$$MP_{m}^{t} \geq MP_{mc}^{L} \qquad m \in \mathbf{PM},\ c \in \mathbf{C}_m \qquad (78)$$

$$MP_{m}^{t} \geq MP_{nrm}^{L} \qquad n \in \mathbf{IW}_r,\ m \in \mathbf{M}_{nr} \qquad (79)$$

$$CP_{c}^{t} \geq CP_{mc}^{L} \qquad m \notin \mathbf{PM},\ c \in \mathbf{C}_m \qquad (80)$$

$$CP_{c}^{t} \geq CP_{nrc}^{L} \qquad n \in \mathbf{IW}_r,\ c \in \mathbf{C}_{nr} \qquad (81)$$

Well Flow Rates

These flexible constraints ensure feasible response to the low (high) reservoir pressure for supporting (receiving) the flows at producer (injector) wells and preventing the flow as required.

$$\mathbf{R}_{nr}^{t} = \delta_{nr}\psi_{nr}(P_{nr}^{t} - BHP_{nr}^{t}) \qquad n \in \mathbf{IP}_r \qquad (82)$$

$$\delta_{nr}q_{nr}^{t} \geq \mathbf{R}_{nr}^{t}\left(M_{o,nr}^{t} + M_{w,nr}^{t}\right) \qquad n \in \mathbf{IP}_r \qquad (83)$$

$$\delta_{nr}q_{nr}^{t} \leq 0.5\left(M_{o,nr}^{t} + M_{w,nr}^{t}\right)\left(\mathbf{R}_{nr}^{t} + \sqrt{{\mathbf{R}_{nr}^{t}}^{2} + \epsilon^{2}}\right) \qquad n \in \mathbf{IP}_r \qquad (84)$$

$$q_{o,nr}^{t} \leq 0.5M_{o,nr}^{t}\left(\mathbf{R}_{nr}^{t} + \sqrt{{\mathbf{R}_{nr}^{t}}^{2} + \epsilon^{2}}\right) \qquad n \notin \mathbf{IW}_r \qquad (85)$$

$$\mathbf{R}_{nr}^{t} \cdot M_{o,nr}^{t} \leq q_{o,nr}^{t} \qquad n \notin \mathbf{IW}_r \qquad (86)$$

where ϵ is a small ($\cong 10^{-4}$) scalar. Moreover, for injectors we use $M_{o,nr}^{t} = M_{o,nr}^{t}\overline{B_o}/B_{wr}^{o}$, where $\overline{B_o}$ is the average oil formation volume factor and that is discussed in [221].

Objective function:

<u>Capital expenditures:</u>

$$CAPEX^{t} = \sum_{c} u_{c}^{t}\alpha_{c} + \sum_{m} z_{m}^{t}\alpha_{m} + \sum_{r=1}^{R}\sum_{n\in \boldsymbol{IP}_r} y_{nr}^{t}L_{nr}\alpha_{nr} + \sum_{m}\sum_{c\in C_m} zu_{mc}^{t}\,\alpha_{mc} + \sum_{r=1}^{R}\sum_{n\in \boldsymbol{IP}_r}\left[\sum_{c\in C_{nr}} yu_{nrc}^{t}L_{nrc}\alpha_{nrc} + \sum_{m\in \boldsymbol{M}_{nr}} yz_{nrm}^{t}L_{nrm}\alpha_{nrm}\right] + IPE^{t}\alpha_{pe} + IIE^{t}\alpha_{ie}$$

$$(0 \leq t < T) \qquad (87)$$

In the last time period no installation/drilling activities are permitted, hence we set $CAPEX^{t=T} = 0$.

Total capital expenditures: $\sum_{0\leq t<T} \boldsymbol{CAPEX^t} \leq \boldsymbol{T\_CAPEX^U}$ (88)

Where $\boldsymbol{T\_CAPEX^U}$ is the maximum available budget.

Total revenue: $SR^t = \beta_o QC_o^t \Delta h^t$ (89)

Operating expense: $EX^t = \left(\alpha_o QC_o^t + \alpha_{pw} QC_f^t + \alpha_{iw} QC_w^t\right)\Delta h^t$ (90)

Taxable income: $TI^t \geq SR^t - EX^t - d^t \sum_{t\geq 0} CAPEX^t$ (91)

Actual cash flow: $CF^t = SR^t - EX^t - \gamma^t TI^t - CAPEX^{t<T}$ $(0 \leq t < T)$ (92)

with $SR^0 = EX^0 = TI^0 = 0$.

Objective function based on Net Present Value:

$$\max NPV = \sum_{t\geq 0} CF^t/(1+\tau)^{(h^t+ih)/365} \quad (93)$$

Since we have allowed initial drilling/installation and expansion, we add ih days to the time exponent of NPV definition to take into account the fact that the investments and costs starts few months before the actual production, during which we do not consider any production.

That completes definition of our model **ET**. This model rigorously includes the subsurface dynamics and the multiphase flow to the surface, the interactions between the subsurface and surface network and the economic parameters. Additionally it is to address the order of installations. Hence, **ET** is addressing the first two gaps in the literature which was highlighted earlier.

6.4 Solution Strategy

6.4.1 Overview

Model **ET** is a nonconvex, dynamic, multi-period and spatio-temporally discrete MINLP model, and hence that is a complex model which is computationally expensive to solve.

Consider an example problem with $R = 2$, $|\mathbf{IP}_r| = 1000$, $M = 5$, $N = 2$ and $T = 100$. Assuming that each cell can be potentially connected to all manifolds/centers, there are more than 2.8×10^6 and 4.6×10^6 binary and continuous variables respectively. In addition to this dimensionality, the significant possible combinations, the nonlinearity of the system and the presence of endogenous equations make this problem a huge combinatorial and complicated problem. It is almost impossible (at least now) to use the standard MINLP solvers for such models. Hence, designing a specific algorithm for solving model **ET** is currently unavoidable.

In light of above difficulties, we propose decomposing **ET** into (a) a production network design problem and (b) a network design and installation planning problem. The former supplies the latter with a list of potential element locations, element-to-element allocations and incremental production expansion plan. Since this potential list is prepared via solving an optimal design and location-allocation problem in a systematic manner, it is not limited to heuristics and very general approximations. Consequently, this approach can overcome the third shortcoming of the previous literatures to some good extent.

Let model **E** be the production network design problem formed by fixing (u_c^t, z_m^t, zu_{mc}^t, y_{nr}^t, yz_{nrm}^t, yu_{nrc}^t) binaries to zero at ($t > 0$), removing Eqs. (1) to (6) and (8), replacing all (us_c^t, zs_m^t, zus_{mc}^t, ys_{nr}^t, yzs_{nrm}^t, yus_{nrc}^t) binaries with the respective binaries (u_c^0, z_m^0, zu_{mc}^0, y_{nr}^0, yz_{nrm}^0, yu_{nrc}^0), and finally reducing the time domain of Eqs. (8) to (16) to $t = 0$. Therefore the only binary variables in model **E** are (u_c^0, z_m^0, zu_{mc}^0, y_{nr}^0, yz_{nrm}^0, yu_{nrc}^0) and its solution helps forming the network design and installation planning problem. Let us refer to the second model as model **RET**.

Model **RET** is similar to model **ET** with the more restricted search domain. The active binary solutions of model **E** reduce the search space by limiting the sets of potential manifolds/ centers, $\mathbf{C}_m$, $\mathbf{PW}_r$, $\mathbf{NW}_r$, $\mathbf{M}_{nr}$, and $\mathbf{C}_{nr}$ to subsets of their original definitions. The elements of these sets corresponding to inactive binaries from solution to model **E** – i.e. (u_c^0, z_m^0, zu_{mc}^0,

y^0_{nr}, yz^0_{nrm}, yu^0_{nrc}) equal to zero – are removed from the solution space. Therefore, now the problem changes to a network design *and* installation planning problem.

Both models **E** and **RET** are also nonconvex, dynamic, multi-period and spatio-temporally discrete MINLP. We have previously modified an outer approximation algorithm to solve such MINLPs [221, 243]. Here we modify this algorithm separately for models **E** and **RET**.

6.4.2 Solving Model E

Model **E** is similar to our recent model [243] for element location-allocation and production planning in multi-reservoir oil fields with surface facility networks. The algorithm that we previously tailored is based on the outer approximation and equality relaxation and augmented penalty (OA/ER/AP) algorithm (of Grossman and co-workers [213]). OA/ER/AP decomposes an MINLP into a primal (NLP) stage and a master (MILP) stage, and solves the MINLP through successive iterations. Refer to [213] for the detailed formulation of the master problem.

We equipped OA/ER/AP with sequential solution of sub-NLPs, and a two-stage local search. Let the primal and master problems in the k-th major iteration be $\mathbf{EP}^k$ and $\mathbf{EM}^k$ respectively. We solve $\mathbf{EP}^k$ using a sequential solution approach; it discretizes each period t into $\mathcal{T}_t$ intervals ($\tau = 1, 2, \ldots, \sum_t \mathcal{T}_t$) and solves several $\mathbf{EP}^k_\tau$ by marching into the time domain. After each time interval $\max_n(S^\tau_n - S^{\tau-1}_n)$ is checked to ensure solution stability. The initial solutions and required initial conditions for $\mathbf{EP}^k_\tau$ are based on the solution of $\mathbf{EP}^k_{\tau-1}$. In each major iteration, the primal stage provides the linearization points (at the end of each period t) for the master problem ($\mathbf{EM}'^k$), and the master problem in turn prepares the binary solutions for the primal problem. Integer-cuts ensure that the same solution is not revisited. If the algorithm cannot improve the solution or the master problem becomes infeasible, it is directed to a two-stage search step, where (1) a reduced master MILP ($\mathbf{EM}''^k$) problem is formed by fixing the binary solution for all active wells except the least productive well with lowest initial oil in place (IOIP); it is solved to prepare a new primal problem. If this cannot improve the solution (2) a local neighborhood search is performed by probing the adjacent locations to the location of the least productive well while maintaining the other well locations. The capacity expansion plan in the latter case is borrowed from the current incumbent solution. The two-stage search is shown in a green box in Figure 6-2. Finally, the algorithm is terminated if these two stages are unsuccessful. It is notable that the local search increases the number of possible primal problems in each iteration; therefore instead of k-index we used κ-index to refer to the number of primal problems and we have $\mathbf{EP}^{\kappa}$. Interested reader is referred to [221] for more information; Figure 3 of this reference represents our previous algorithm schematically.

Our studies ([243] and [221]) assured us that the two-stage local search has significant role in improving the solutions. Additionally, Huang and Karimi [244] also applied the similar methodology and reported the same. Hence, here we investigated the chance of increasing the role of the local search. To do so, we focused on the capacity expansion planning feature of model **E** which is its most important difference with our previous model [243]. We fix all binary variables of model $\mathbf{EM'}^{k}$, and turn this MILP model into a LP model and call that model $\mathbf{EL'}^{k}$. That provides us with another local search stage which updates the incremental capacity expansion plan. $\mathbf{EL'}^{k}$ is used to update the initial guesses/values of the capacity plan (a) after the first primal problem and (b) after the successful neighborhood probing search. It is obvious that $\mathbf{EL'}^{k}$ is not required to be solved for the primal problems that follow $\mathbf{EM'}^{k}$ or $\mathbf{EM''}^{k}$ models and end with improved the NPVs. They are shown in red boxes in Figure 6-4. That changes our previous two-stage local search into a three-stage local search.

Finally, after termination, the above algorithm prepares the binary design variables and the incremental capacity expansion plans to be transferred to model **RET**.

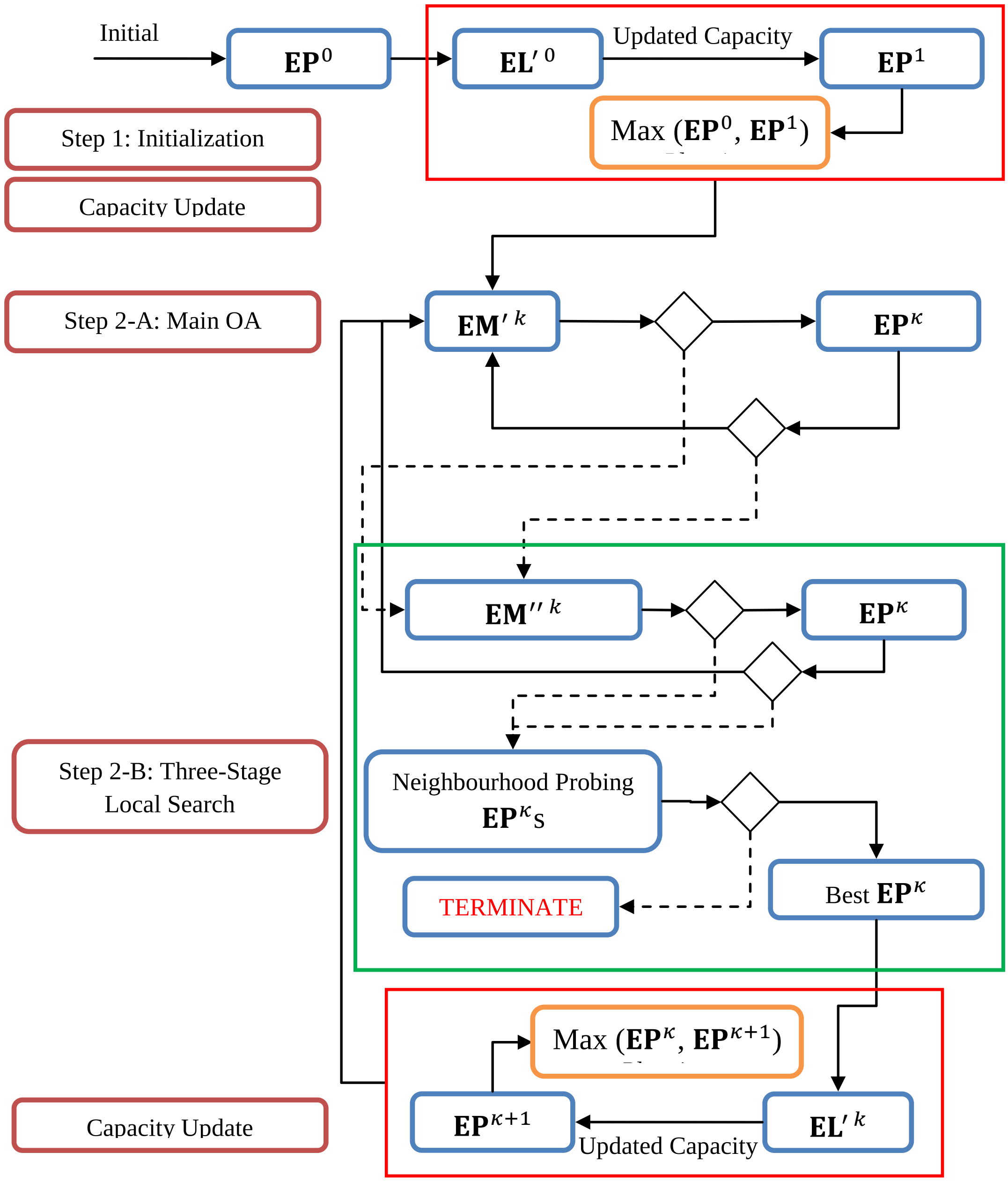

Figure 6-2: The structure of the three-stage MINLP algorithm.

The diamond boxes refer to logical check boxes, dashed and solid lines in order represent failed and passed criteria checks. The criteria are feasible MILP solution and improved NPV for master and primal problems respectively.

6.4.3 Solving Model RET

Let the primal and master problems for **RET** at *k*-th iteration be $\mathbf{RETP}^k$ and $\mathbf{RETM'}^k$ respectively. Since the drilling rig availability constraint (Eqn. 8) was relaxed in model **E,** its solution can be infeasible for model **RET**, hence for $\mathbf{RETP}^k$ and $\mathbf{RETM'}^k$. Therefore, we identify the most productive new producer well with the greatest IOIP value and use this well with its connections to initialize the model **RET**. The next steps are similar to the algorithm described for solving model **E**. However there are two main differences:

1) Adaptive time discritization: We usually use large time steps in model $\mathbf{EM'}^k$ to strike a balance between accuracy and solvability. However, potentially model **RET** (and hence $\mathbf{RETM'}^k$) has more restricted search domain and therefore we can reduce the step size of each time period for stressing the time effect on installation planning. Therefore, we increase the number of periods from T to $T\Theta$ with $t = 1, 2, \ldots, T\Theta$ and accordingly we decrease the number of intervals in each period to $\mathcal{T}/\Theta$. However we ensure maintaining the same interval step sizes for $\mathbf{EP}^k$ and $\mathbf{RETP}^k$ to keep the time impact on the primal NPV formulation intact. Figure 6-3 represents and compares the time discritization for the master and primal problems of **E** and **RET**; the points with an arrow show the linearization points passed from the primal to the master formulations. This procedure can be potentially iterated however we just used the first round.
2) Neighborhood probing: Here, we employ the similar idea of the neighborhood probing. Instead of location perturbation of the worst new producer well with the lowest IOIP, we probe the adjacent time steps of its drilling time. Its well-to-manifold and/or well-to-center connections (and consequently the appropriate manifolds and center, if required) are perturbed accordingly.

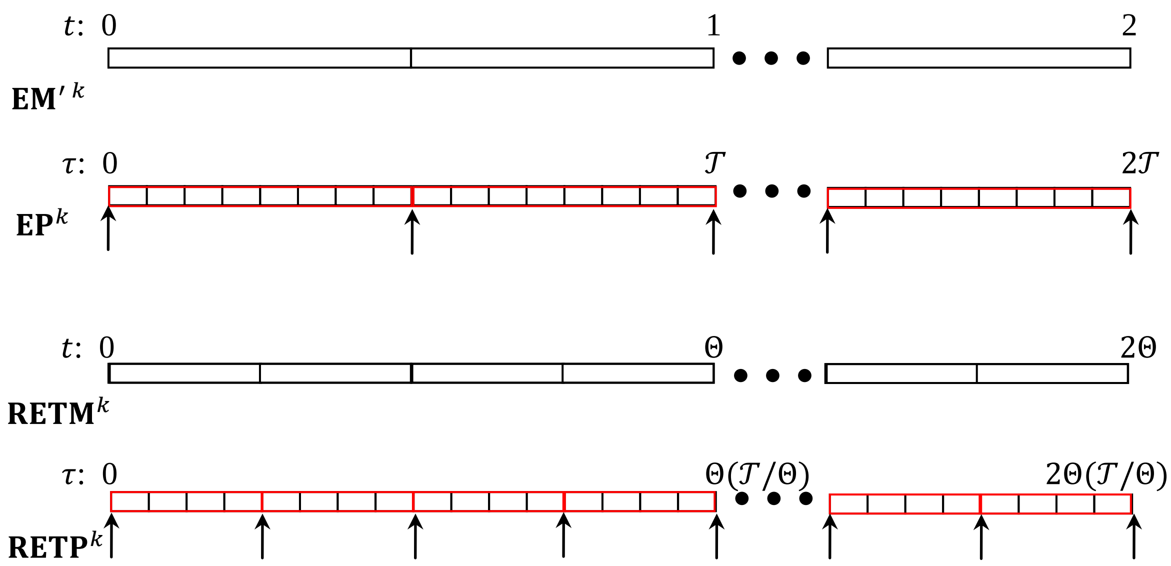

Figure 6-3: Time discritization structure for master and primal formulation of model **E** and **ERT**.

The final solution of **RET** determines the time, order and location of installing new elements, decides the various element-to-element connections and plans the incremental surface center capacity expansion.

6.5 **Case Study**

In order to test the above two-stage MINLP methodology, we use the field development problem introduced in the previous chapter and modify that based on model **ET.** Then we can test our solution strategy for drilling and installation planning. Therefore, Figure 5-2a shows the field under study with the initial surface facility network. There are minor changes to the parameters used, though.

Let each time period be 100 days. We again provide five different wells and infrastructure configurations as the initial guess to the model and solve each of them. The best solution progresses to the highest NPV of $MM\$\ 240.4$ which is 73% higher than its base case and suggests drilling 6 new wells (PN1-PN6), establishing 8 new connections and installing two new manifolds (NM1-NM2). It took 90 minutes with 3 major iterations and 6 primal problems to reach to the final solution. The timing part of the solution had a prime role in improving the final NPV. Figure 6-4 depicts the location of these new wells and manifold. The next figure, Figure 6-5, represents the order of opening the wells, installing the manifolds and connecting the connections.

It is informative to inspect the response of wells and field flow rates to the new installations. Figure 6-6 represents the field oil production (FOP), water injection (FIW) and field liquid production curves and the black arrows depicts the time of new drillings. Furthermore, Figure 6-7 depicts the flow rates enetring each manifold As can be seen from these figures, once each new well is opened for production the water injection rate decreased and oil production rate increased. That is the major trend during the production horizon, except for the last two time periods where the reservoir is in general not able to return to a high production level.

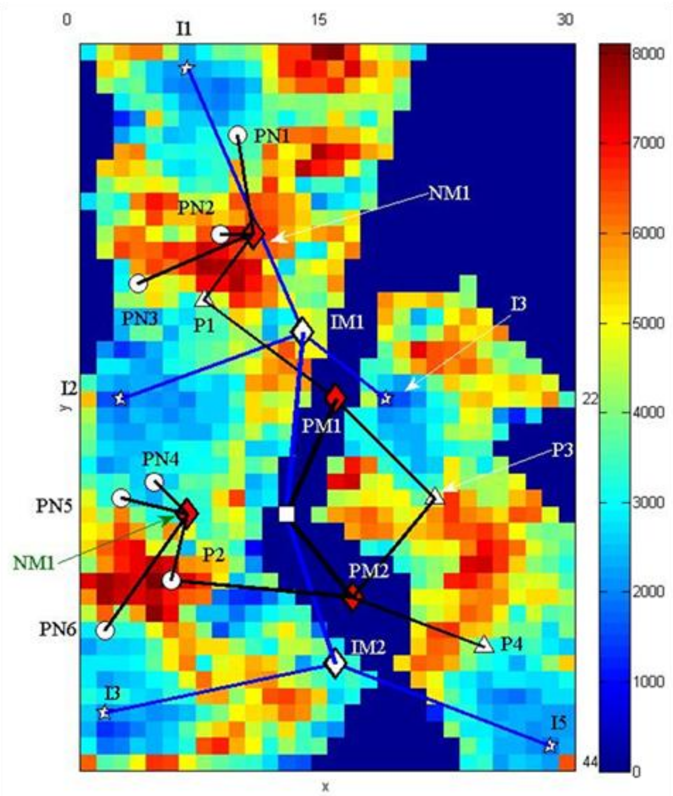

Figure 6-4 The final positions of the wells and surface manifolds on the final oil in place map (m^3).

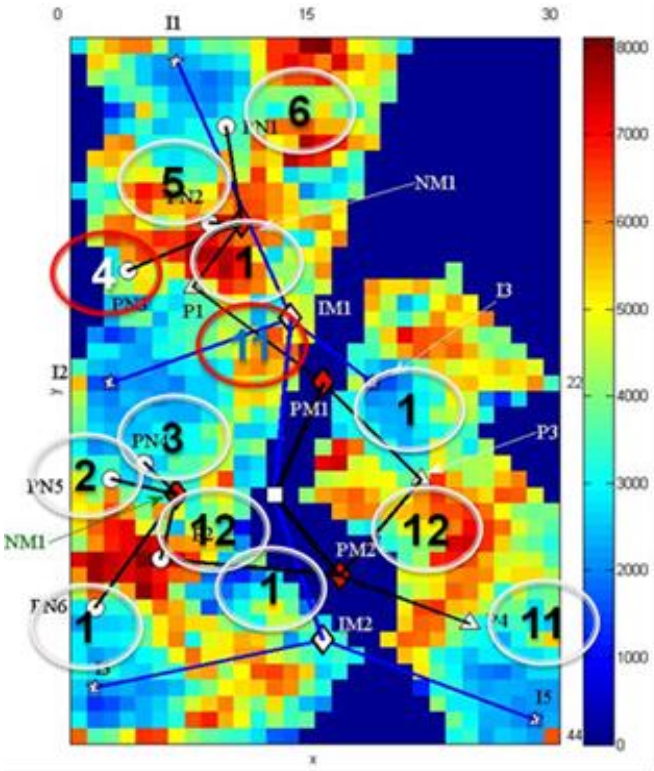

Figure 6-5 The order of opening the new wells

The numbers in each ellipsoid refer to the time period of installation. Each period is 100 days. The background map is the final oil in place.

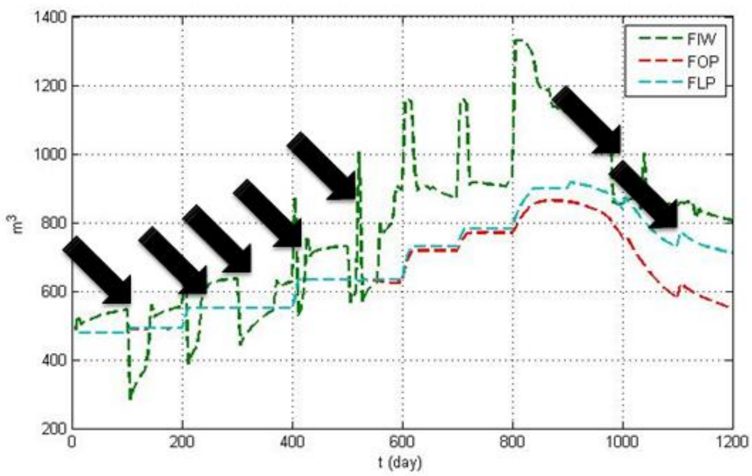

Figure 6-6 Field flow rates

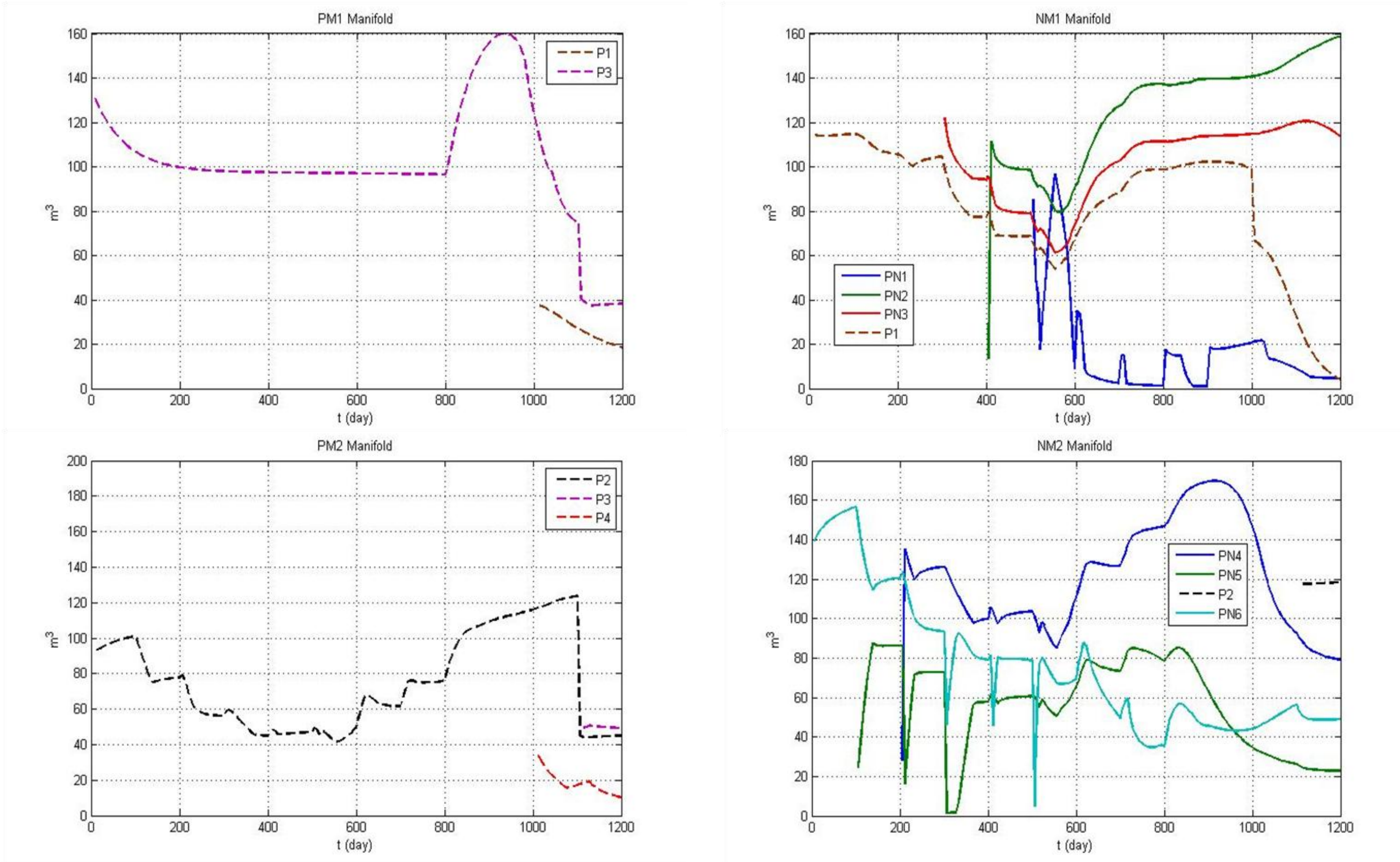

Figure 6-7: Well-manifold flow rates.

6.6 Summary

To our knowledge, work-over rig scheduling (WRS) has received more attention in the open literature comparing to well placement and drilling planning. It can be due to the fact that usually there are so many wells operating in a field, and in some point of time they need the work-over operation. Yet, there are usually very few work-over rigs available and that motivates optimally scheduling and routing these work-over rigs. Therefore there is a strong momentum from industry for WRS study. From computational perspective, in typical WRS problems, the target wells are limited to the existing wells in the field. However in mixed well placement and drilling scheduling problem, the search space for well placement is substantially larger than the first case. That provides the idea behind this chapter. We basically solved the location / allocation / timing problem in two stages. In the first stage we obtained the optimal location / allocations through the model we developed in the previous work. In the second stage we solved the drilling and installation timing problem. Although sub-optimal, that could provide improvement in the financial objective value.

CHAPTER 7 CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

This PhD study addressed optimal oil field development planning and mainly focused on well placement decisions. In particular, three central problems were addressed: (1) placement problem (the optimal sites for well drillings and infrastructure installations), (2) allocation problem (the optimal connection between wells, manifolds and surface centers) and finally (3) ordering / timing problem (optimal planning of the drillings and installations). Throughput planning based on the rigorous subsurface model was the mutual part embedded in all those. These problems were formulated as mixed integer nonlinear programming (MINLP) based models, and they were non-convex, dynamic and discrete. The major contributions of this thesis are as follows:

The extensive literature survey in Chapter 2 indicated that the model-based optimal well placement studies lacked (a) rigorous sub-surface fluid flow model, and (b) a holistic approach that would integrate subsurface, wells and surface dynamics and their features. Moreover, it suggested that the potential power of mathematical programming method had not been fully utilized to address well placement problems. This survey showed that the important limitation of previous mathematical programming studies was the gross approximations of subsurface multiphase flow dynamics, whereas these studies are practically rich in addressing surface issues. These findings motivated us to employ mathematical programming technique in the current study.

After this thorough literature survey, we developed a rigorous, spatiotemporal discrete model to represent the subsurface dynamics in Chapter 3 and verified that in Chapter 4 by comparing its performance with an industry standard reservoir simulator. We embodied this subsurface model into an MINLP frame, and integrated the subsurface and well dynamics to address well placement and production planning problems in a single rectangular reservoir. The key contribution of this modeling part is that we considered subsurface flow dynamics much more rigorously than any other previous study. This is, to our knowledge, the first contribution to integrate most of the critical elements of the upstream production and spatiotemporal subsurface dynamics in a multi-period mathematical programming approach to address well placement. Furthermore, to solve this holistic model, we modified and extended an outer approximation algorithm (of Grossmann and coworkers [211-214, 245]) and empowered that by (a) a sequential solution strategy for the primal problem, (b) master problem reformulation, and (c) a two stages local search. Our model clearly considered the nonlinear interaction between all the wells, and the sequential solution strategy could optimally plan the various throughputs of all these wells at each time period. Although the sequential solution strategy appeared locally efficient, it lacked a global behavior. Our tests showed that the master reformulation considerably reduced the solution time, and the two stages local search had significant impacts on the solution. In contrast to most previous work, our approach did not require pre-fixing wells and locations or production/injection rate patterns.

In the second study in Chapter 5, we further generalized the approach developed in Chapter 4. Instead of a single rectangular reservoir, we modeled multiple irregular-shaped reservoirs of an oil field connected to a shared surface network facility. Then we used that to address both placement and allocation problems in Chapter 5. Additionally, further modifications to the algorithm developed in the previous chapter improved its performance.

The third study in Chapter 6 tackled the timing and ordering problem for well-drillings and infrastructure placement. Two solution algorithms were suggested and their efficiency was tested. It was shown that using an optimal plan for drilling new wells and installing new infrastructures can delay the production decline; and hence that can significantly affect the profitability of the project.

While much further work is needed to address the size and complexity of this important problem, we have taken the first step in rigorously applying the powerful and versatile technique of mathematical programming and addressing some of the challenges associated with the industry-scale well placement problem.

7.2 Recommendations

Chapter 2 provided an insight into possible future researches in the optimal well placement field. In addition to those, the following studies are recommended to extend the current PhD research:

Production planning study: The current study can certainly be improved by studying the production planning in more details. Currently the sequential solution approach shows a myopic behavior. At each time step, the optimizer can only see the reservoir response within the same time step, and that can potentially affect the future response of the reservoir. Some of the researchers [47] have used gradient based technique (such as adjoint based gradient), and evolutionary methods [217] to optimally plan the production. We had some initial attempts [246] to use adjoint information for this purpose, however as different researchers has identified [133], there are complexities in using adjoint based gradient calculation with models with nonlinear inequalities involving control and state variables. Any improvement on production planning can certainly help to improve the overall solution.

Surface network study: Compressors are used after separators to pressurize the processed gas [120]. The compressor performance can be integrated with the current model. Moreover, it was assumed that all reservoirs have similar fluids with same properties [116]. This assumption should be attacked in order to better generalize the current approach. However, that complicates the mixings at the manifolds and separators.

Drilling planning: Another possible way of solving the drilling planning problem that was presented in Chapter 6, might be to start again with the MINLP formulation for placement problem in the kth iteration of the outer approximation algorithm ($\mathbf{ET}_1^k$). However, instead of solving a detailed MINLP problem ($\mathbf{ET}_2^k$) for the planning problem, we may use the flow rate profile from solution of $\mathbf{ET}_{1\mathrm{m}}^k$ in the k-th iteration to determine the time and order of drilling/installing the new wells/ infrastructure. Therefore, in addition to the binary solution, we should extract the throughput profiles from $\mathbf{ET}_{\boldsymbol{1m}}^k$. However, we may not interpret a zero flow rate at the later stages as a stopping flag. After this $ub_c^{t,k+1}, zb_m^{t,k+1}, yb_{nr}^{t,k+1}, yzb_{nrm}^{t,k+1}, yub_{nrc}^{t,k+1}$ and $ub_c^{t,k+1}, zb_m^{t,k+1}, yb_{nr}^{t,k+1}, yzb_{nrm}^{t,k+1}$, and $yub_{nrc}^{t,k+1}$ can be updated and then introduced to $\mathbf{ET}_{\boldsymbol{1p}}^{k+1}$. We did not test this thoroughly; therefore we just present that as a recommendation here.

In addition to the above, planning the drilling rig movement [116] and its path selection could potentially be mixed with our model.

Well type selection: A natural extension of this study is to include injection well placement into the model. If the potential locations of injector and producer wells are exclusively separated, it is easier to update the model. However, if a potential site can be producer, injector or a normal cell, more preparations are required. We have done some initial study on the former [247] however that requires a more detailed analysis.

Three dimensional model and nonconventional wells: Since in this study, the general form of equations showing multiphase flow in porous media is used, the present model is extendable to three dimensions. However, special tactics should be employed to tackle to dimensionality problem.

Point wise representation of wells was used in this study. As discussed in Chapter 2, Yeten [55] has translated the well type into the number of junction points on the main wellbore. An integer variable showing the junction is zero for mono-bore wells and nonzero for multilateral wells. Each mono-bore well can be represented by pointing to its heel and toe positions. In the case of multi-lateral wells, the conjunction and toe points of laterals should be added to the specifications of the main trunk. A similar formulation might be imbedded in our model; however, the definition of productivity index should be updated to account for the nonconventional wells and the nonconventional well will be represented in a staircase.

Uncertainty modeling: Uncertainty assessment is the important element that closes the loop of optimal well placement and stochastic approaches are the vehicle for that. Stochastic models are built on the foundation of deterministic models. This study has provided such platform.

Other applications: The novel methodology developed here can be potentially applied to similar problems from different application domains, particularly dynamic problems which include some types of location/allocation decisions. As an example, the work in Chapter 4 can be modified to potentially study wind mill placement. Additionally, the same methodology developed in Chapter 5 might be applicable to Coal Bed Methane (CBM) developments to tackle related placement and allocations issues. In these developments thousands of wells are drilled and they are connected to very complex network facilities. They are too complex to use the conventional approaches in oil and gas developments. Both above examples can be formulated in the frame work of the sixth chapter to study the timing of installations.

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