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A TARGET-ORIENTED ROBUST OPTIMIZATION APPROACH TO PRODUCTION AND LOGISTICS PLANNING SYSTEMS

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DECLARATION

I hereby declare that the thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Charlle Lee Sy

To Papa, Mama, Diorelle and Justin.

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> Nothing in the world can take the place of persistence. Talent will not, nothing is more common than unsuccessful men with talent. Genius will not; unrewarded genius is almost a proverb. Education will not; the world is full of educated derelicts. Persistence and determination alone are omnipotent. -Calvin Coolidge

Indeed, it has been persistence and determination that has led me to complete this dissertation. My utmost APPRECIATION goes out to each and everyone who has been my source for these:

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SUMMARY

This research presents an integration of a target-oriented approach to decision making. This is depicted in problem applications falling within the contexts of production and logistics decisions in manufacturing and supply systems. The presence of uncertainties in these systems can cause undesirable behavior. Failure to account for these in the design phase can further impair the capability of systems to respond to changes effectively. Problem applications include the analysis of dynamic systems involving dynamic workforce-inventory control and power supply problems. The general objective for these is to develop planning rules to achieve important requirements related to dynamic transient behavior when system parameters are imprecisely known. The application of the target-oriented approach in these settings combines the strengths of recent developments in robust optimization technology and small signal stability analysis of dynamic systems. Numerical case studies of the problem demonstrate significant improvements of the proposed solution in controlling fluctuations and high variability found in both of the systems' state variables.

An offshore gas field development problem has also been considered. This problem seeks to identify the installation and operation plan to achieve a target

profit at the end of the planning horizon. The problem is severely plagued by endogenous uncertainty that is primarily found in the efficacy of well reserves within the field. Similarly, target achievement is integrated into a robust optimization model that maximizes the robustness against uncertainty so that the net present value of the development project could meet or exceed the prescribed performance target. The characteristics of the problem lead to the identification of an equivalent deterministic mixed integer programming model of polynomial size. This enables one to obtain solutions to realistic sized problems. The computational tests show that the proposed model significantly improves target attainment and performs favorably in different problem instances.

The third problem application involves a supply network design problem with performance requirements on CO_2 emissions. The integration of CO_2 emissions in the problem is in response to the increasing emphasis to integrate environmental thinking in supply network design. The decision problem is to select the facilities to install and their respective assignments in order to serve consumer demands and meet cost budget requirements as well as possible under uncertainty. The resulting optimization model using the target-oriented approach is found to be computationally tractable and can be solved efficiently using standard mixed integer programming solvers. Numerical studies using a power system network are performed, which demonstrate that the proposed model performed favorably compared to other optimization models across different performance measures.

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SYMBOLS

Chapter 3

$\boldsymbol{y}$	vector of decision variables
l	index of decision variables
L	total number of decision variables
$\tilde{\boldsymbol{z}}$	vector of uncertain parameters
k	index of uncertain parameters
K	total number of uncertain parameters
$\bar{\boldsymbol{z}}$	vector of nominal values of uncertain parameters
$\boldsymbol{z}$	vector of maximum possible perturbations of uncertain parameters
$\Re$	real number
γ	robustness index
$\boldsymbol{e}$	vector for the coefficient of uncertain parameter relationships
R	matrix for the coefficient of uncertain parameter relationships
W	total number of uncertain parameter relationships
A	matrix for the coefficient of decision variables

B	matrix for the coefficient of uncertain parameters
$\mathcal{Z}_\gamma$	uncertainty set
$\boldsymbol{\tau}$	vector of system targets
i	index for system targets
N	number of system targets
P	matrix for the coefficient of dual variables
Q	matrix for the coefficient of dual variables
$\mathcal{R}$	target-oriented robust optimization model
$\mathcal{C}$	robust counterpart of target-oriented robust optimization model
$\mathcal{Y}$	feasible space of $\boldsymbol{y}$
$\mathcal{Y}^{\mathcal{R}}$	feasible space of $\boldsymbol{y}$ in problem $\mathcal{R}$
$\mathcal{S}^{\mathcal{C}}$	feasible solution space of problem $\mathcal{C}$
$\mathcal{Y}^{\mathcal{C}}$	feasible space of $\boldsymbol{y}$ in problem $\mathcal{C}$
$\tilde{v}$	uncertain level of achievement above the target
ρ	satisficing measure

Chapter 4

x	vector of resource variables
D	state matrix of dynamic system
λ	eigenvalues of state matrix D
μ	real part of eigenvalue of state matrix D

ωi	imaginary part of eigenvalues of state matrix D
$\boldsymbol{w}$	left eigenvectors of state matrix D
$\boldsymbol{v}$	right eigenvector of state matrix D
$S_{\mu}^{\tilde{\boldsymbol{z}}}$	matrix for the sensitivity of the real part of the eigenvalues with respect to uncertain parameters
$S_{\omega}^{\tilde{\boldsymbol{z}}}$	matrix for the sensitivity of the imaginary part of the eigenvalues with respect to uncertain parameters
$S_{\mu}^{\boldsymbol{y}}$	matrix for the sensitivity of the real part of the eigenvalues with respect to decision variables
$S_{\omega}^{\boldsymbol{y}}$	matrix for the sensitivity of the imaginary part of the eigenvalues with respect to decision variables
ϕ	maximum overshoot
t_p	peak time
t_s	settling time
Δ	tolerance parameter for overshoot

Chapter 5

p	production platform $p \in \mathcal{P}$
$\mathcal{P}$	set of production platforms
w	well platform $w \in \mathcal{W}$
$\mathcal{W}$	set of well platforms
a	well-well pipeline connection $a \in \mathcal{A}$

$\mathcal{A}$	set of well-well pipeline connections
b	well-production platform connection $b \in \mathcal{B}$
$\mathcal{B}$	set of well-production platform connections
t	time period $t \in \mathcal{T}$
$\mathcal{T}$	set of time periods in planning horizon
h_w^W	maximum installable capacity for platform $w \in \mathcal{W}$
h_p^P	maximum installable capacity for platform $p \in \mathcal{P}$
h_a^A	maximum installable capacity for pipeline $a \in \mathcal{A}$
h_b^B	maximum installable capacity for pipeline $b \in \mathcal{B}$
f_w^W	fixed cost in installing platform $w \in \mathcal{W}$
f_p^P	fixed cost in installing platform $p \in \mathcal{P}$
f_a^A	fixed cost in installing pipeline $a \in \mathcal{A}$
f_b^B	fixed cost in installing pipeline $b \in \mathcal{B}$
v_w^W	variable cost in installing platform $w \in \mathcal{W}$
v_p^P	variable cost in installing platform $p \in \mathcal{P}$
v_a^A	variable cost in installing platform $a \in \mathcal{A}$
v_b^B	variable cost in installing platform $b \in \mathcal{B}$
$\bar{u}_p$	buffer capacity for production platform $p \in \mathcal{P}$
$\bar{g}_w^W$	operational cost for well platform $w \in \mathcal{W}$
$\bar{g}_p^P$	operational cost for production platform $p \in \mathcal{P}$
$\bar{r}_t$	revenue per unit of gas produced in period $t \in \mathcal{T}$

d_w	initial deliverability of reserve in platform $w \in \mathcal{W}$
s_w	initial size of reserve in platform $w \in \mathcal{W}$
$q^W_{w,t}$	extraction from platform $w \in \mathcal{W}$ in period $t \in \mathcal{T}$
$q^P_{p,t}$	production from platform $p \in \mathcal{P}$ in period $t \in \mathcal{T}$
$q^A_{a,t}$	pipeline flows from $a \in \mathcal{A}$ in period $t \in \mathcal{T}$
$q^B_{b,t}$	pipeline flows from $b \in \mathcal{B}$ in period $t \in \mathcal{T}$
$c^W_{w,t}$	capacity of platform $w \in \mathcal{W}$ in period $t \in \mathcal{T}$
$c^P_{p,t}$	capacity of platform $p \in \mathcal{P}$ in period $t \in \mathcal{T}$
$u_{p,t}$	expansion of platform $p \in \mathcal{P}$ in period $t \in \mathcal{T}$
$x_{w,t}$	1, if platform $w \in \mathcal{W}$ is installed in period $t \in \mathcal{T}$
	0, otherwise
$y^P_{p,t}$	1, if platform $p \in \mathcal{P}$ is installed in period $t \in \mathcal{T}$
	0, otherwise
$y^A_{a,t}$	1, if pipeline $a \in \mathcal{A}$ is installed in period $t \in \mathcal{T}$
	0, otherwise
$y^B_{b,t}$	1, if pipeline $b \in \mathcal{B}$ is installed in period $t \in \mathcal{T}$
	0, otherwise
$b_{p,t}$	1, if buffer capacity is installed on platform p in period t
β	time discount factor
$\tilde{R}$	net present value
$\boldsymbol{\pi}$	vector of decision policies

$\boldsymbol{\mathcal{F}}$	family of admissible policies that captures the non-anticipativity requirements
$\mathcal{V}$	range in which $\boldsymbol{\pi}$ is mapped into

Chapter 6

I	number of nodes
$\mathcal{A}$	set of arc connections
$h_{i,j}$	maximum installable capacities for arc (i, j)
$f_{i,j}$	fixed cost for arc (i, j)
$u_{i,j}$	variable cost for arc (i, j)
d_i	demand from node i
$\bar{d}_i$	nominal value of demand from node i
d_i^n	factor coefficient of demand from node i
e_i	penalty cost for unfulfilled demand in node i
$c_{i,j}$	production cost in arc (i, j)
$\bar{c}_{i,j}$	nominal value of production cost in arc (i, j)
$c_{i,j}^n$	factor coefficient of production cost in arc (i, j)
$w_{i,j}$	amount of emission in arc (i, j)
$\bar{w}_{i,j}$	nominal values of emissions in arc (i, j)
$w_{i,j}^+$	maximum possible perturbations of emissions in arc (i, j)
$p_{i,j}$	installed capacity in arc (i, j)
$x_{i,j}$	flow in arc (i, j)

$y^P_{i,j}$ 1, if arc (i, j) is installed

0, otherwise

ABBREVIATIONS

WIP	Work-in-process inventory
OMIB	One machine infinite bus
WP	Well platforms
PP	Production platforms
TRO	Target oriented robust optimization
SAA	Sample average approximation
SD	System dynamics
EIA	Energy Information Administration
UNFCCC	United Nations Framework Convention on Climate Change
NPV	Net present value
LP	Linear programming

1. INTRODUCTION

1.1 Research Background

It is well-acknowledged in decision theory that real world agents rarely *optimize* their decisions. Doing so implies a need for a perfect model of a system from which the future behavior of variables may be deduced (Sterman, 2000). However, this can never be the case since reality by no means produces the exact conditions as initially planned. Thus, arriving at an optimal decision is virtually impossible even when faced with simple problems. Senge (1990) mentions that the complexities involved in decision making render organizations to fail to perform optimally even if they try to. These complexities lead them to ignore important aspects of a situation and underestimate the consequences of their decisions.

Simon (1959) postulates that the need to achieve real, rather than ideal, representations of systems has imbued an attitude of *satisficing* in decision makers. They are often led to choose the first available actions which ensure that certain desired targets will be achieved. For instance, he illustrates that if business behavior is to be viewed in terms of this approach, it is to be expected that the firm's target would involve the attainment of a certain level of profit or holding a certain share of the market, as oppose to the maximization of profit.

According to Forrester (1969), decisions primarily involve three aspects. The first is the creation of a concept of a desired state of affairs. The second involves the apparent state of actual conditions. And the third is the generation of the kinds of actions that will be taken in accordance with the apparent and the desired conditions. These processes describe the attitude of satisficing and show how intrinsic it is in the decision making process. The evaluation of the actual versus the desired state of a system initiates the creation of actions that would lessen this gap and eventually lead the system to reach its desired state. This behavior consequently accounts for one of the most fundamental dynamics of systems. As targets change and evolve, a system will continuously strive to reach its desired state.

Research efforts have continuously considered the optimization of target achievement probabilities in place of utility functions. Brown and Sim (2010) mention that this is because targets are often more natural for decision makers to specify, whereas traditional approaches based on utility functions depend critically on tolerance parameters which are often difficult for decision makers to intuitively grasp and even harder to appropriately assess. Lanzillotti (1958) concludes in his interviews of senior executives from large companies, that managers are primarily concerned about target returns on investment. Payne et al. (1980a) and Payne et al. (1980b) likewise illustrate that managers consequently tend to disregard investment possibilities that are likely to under perform against their target. Furthermore, in an empirical study by Mao (1970), managers have been found to consider risk as the prospect of not meeting some target rate of return.

Abbas et al. (2009) state that while targets provide the aforementioned advantages in decision making, the use of which can still be aligned with the expected utility-maximizing decisions. This observation has led researchers to prove the equivalence of the use of utility functions and targets in decision making. The earliest work could be attributed to Borch (1968) who uses the concept of ruin probabilities. He shows that the maximization of the expected utility is equivalent to choosing the smallest probability of ruin. Indeed, from the normative perspective, Charnes and Cooper (1963) define a 'satisficing solution' as one that maximizes success probability. They demonstrate how optimization problems could have a set of objectives dependent upon the achievement of target levels. Simon and Kadane (1975) likewise show how traditional optimization algorithms could be translated to evaluate satisficing objectives.

More recently, Castagnoli and LiCalzi (1996) and Bordley and LiCalzi (2000) show that maximizing expected utility is mathematically equivalent to maximizing the probability that the uncertain consequences of a decision are preferable to an uncertain benchmark (i.e., a firm outperforming the uncertain future performance of a major competitor). Meanwhile, Bordley and

Kirkwood (2004) and Tsetlin and Winkler (2007) extend such equivalence for the case of multiple targets.

However, Brown and Sim (2010) argue that maximizing success probabilities alone has the underlying assumption that the modeller is indifferent to the level of losses and gains. This fails to account for the magnitude of losses when extreme scenarios (i.e., tail-end probabilities) occur. In addition, the consideration of such measures also proves to be an intractable problem when coupled with exponential or even an infinite set of alternatives. Brown and Sim (2010) and Brown et al. (2012) address these limitations by proposing a new set of decision criteria known as satisficing and aspiration measures that are inspired by behavior preference for diversification. More importantly, these criteria may be incorporated in optimization problems without degrading their tractability.

This research proposes a target-oriented approach that is inspired by the decision analytic criterion of Brown and Sim (2010) and Brown et al. (2012). This approach is to be applied to production and logistics planning systems, which involve both investment and operational decision problems. These decisions are to be evaluated based on management goals such as the achievement of target investment returns or institutional regulations imposed on the organization.

A challenging issue in production and logistics planning decisions involves the presence of uncertainty. For example, because of the long implementation lead times of design decisions such as infrastructure development, many important system parameters may not be accurately known or projected before implementation of the decision. Hence, planning without explicitly accounting for these uncertainties can yield inferior performance in reality. In addition, data used in the evaluation and analysis of these systems inevitably contain errors and approximations. Results or actions obtained from these would therefore be subjected to uncertainties in the real system as well.

Ben-Tal and Nemirovski (1999) state that even a small degree of uncertainty can make the usual optimal solution completely meaningless. The reality of which creates the need to identify solutions that would be robust to the presence of uncertainty. Bertsimas and Sim (2004) equate this robustness to a solution's immunity to data uncertainty. Such that even if the underlying information deviate from their nominal values, the solution would still be able to achieve the targets of the system across all planning stages.

The motivation of this research is then based on addressing the research challenge of developing a target-oriented optimization approach that leads to the generation of robust solutions. The target-oriented approach preserves computational tractability and also allows for ambiguity in uncertainty information. Moreover, this research aims to show how traditional modeling paradigms can benefit from the use of a target-oriented optimization approach for applications in production and logistics planning problems.

1.2 Production and Logistics Planning Problems

The problem applications involve two-stage and multi-stage decisions found in dynamic system planning and network design planning. The succeeding discussion provides a background on the aforementioned problem applications. It also gives an overview on the type of problems that would be evaluated and the nature of the uncertainties that decision makers need to contend with in these systems.

1.2.1 Dynamic Systems

Dynamic systems are comprised of multiple components and characterized by time-evolutionary processes of change. These types of systems also operate in a constantly changing environment which inevitably contains inherent uncertainties. The presence of uncertainties consequently brings about continuous variations in the outputs and key operating parameters of these systems (Kundur, 2006).

One of the dynamic problems considered in this research involves workforce-inventory control in a manufacturing firm. In the problem, managers seek to meet customer orders through its finished goods inventory. Finished goods are replenished by production, and production releases in turn are determined by the level of manufacturing workforce available. The firm adjusts its workforce through hiring processes by creating job vacancies. It is also desired that the level of workforce and finished goods inventory be as close to specified levels as possible. Consequently, this requires establishing a workforce-inventory feedback control rule that is able to adapt to changing customer orders and requirements dynamically. The other dynamic problem involves a power supply system. The objective of the problem is to control the oscillations in the critical variables of the system such as the voltages and torque to prevent disruption of supply. These oscillations occur when there are disturbances in the system such as mechanical load changes and system faults.

Focus is placed on how the workforce-inventory and power supply systems respond to changes during operation. This refers to the specification of dynamic performance such as stability and transient response requirements for the system under operation. In the workforce-inventory system, these specifications would mean preventing undesirable transient response such as overshoots and large swings in critical variables to occur. Specification of dynamic performance is an important consideration in manufacturing firms and large scale supply systems since they can directly affect a system's capability to respond to changes effectively. Poorly managed dynamic performance can cause instability and hence high variability in the resources such as inventories, work-in-processes and workforce levels. Subsequently, high variability has direct implications in increased operating costs such as the need for higher safety stock coverage, larger hiring and firing overheads and longer quoted lead times. Meanwhile, in power supply systems, instability can lead to cascading outages and shutdowns due to progressive increases in angular separation of generator rotors or progressive decreases in bus voltages.

In view of this, the proposed robust target-oriented approach seeks to evaluate the largest uncertainty space possible such that the required dynamic performance specifications of the dynamic systems are guaranteed to be achieved. It follows that the resulting optimization model will be able to identify the appropriate calibration of controllable system parameters that would meet the dynamic performance specifications as well as possible under uncertainty.

1.2.2 Supply Network Design Planning

Supply network design is a strategic issue of primary importance (Chopra and Meindl 2009, Dekker et al. 2004), involving many expensive decisions such as facility location, inter-connectivity and capacity allocation. Firms are also eventually relegated to making operational decisions involving production quantities, service levels and allocation of supply to demand (Baron et al., 2010).

Gas Field Development Planning

This research looks into a network planning problem that concerns offshore gas field development. Offshore production of oil and gas accounts for approximately 30% of total U.S. production (Humphries et al., 2010). This makes it an extremely important source of supply for petroleum products, in which the Energy Information Administration (EIA) estimates that U.S. consumption has averaged around 20 million barrels per day over the last five years (EIA, 2010). Gas field development projects entail huge investments in excess of $10 billion and span across 10-30 years (Goel and Grossmann, 2004). Consequently, these are among the most important and challenging strategic problems in the petroleum industry.

Gas field development projects involve both investment and operation decisions, including gas field infrastructure installation, gas extraction and production planning. High quality estimation of gas reserves is of central importance in order to ensure judicious decision-making. On the other hand, reservoirs of gas are highly complex entities with significant inherent uncertainties. Even with the support of sophisticated seismic surveys and exploration tests, reserves can remain largely uncertain until after significant capital outlays have been made (Goel et al., 2006).

A challenging characteristic of gas field development problems concerns the presence of the so-called 'endogenous uncertainty' in the decision-making process. Essentially, this means that the resolution of the uncertainties depends on the decisions made. This is prevalent for instance in project activity scheduling where activity durations are uncertain in advance of execution, and the uncertainties that are resolved depend on the choice of activities undertaken. In gas field development planning, knowledge about the subsurface is refined only after a well is drilled and production begins. This means that uncertainty in a particular gas reserve is resolved only if an investment decision is made to produce from the reserve. Unfortunately, this renders modeling using standard stochastic programming formats extremely unwieldy. Furthermore, the problem size suffers from the 'curse of dimensionality', such that the resulting models for realistic problems are too large and very difficult to solve using commercial solvers directly.

The gas field development problem of interest in this research is based on a similar setting as that considered by Goel and Grossmann (2004). In the aforementioned, the authors assume a small set of scenarios with associated probabilities to model the outcomes of the gas reservoir's uncertainties. The uncertainties are defined to arise from the size and initial deliverability of the reservoirs. The point of departure is in the treatment of these uncertainties. Specifically, probabilities are not assigned to the outcome space of the uncertain reserve estimates.

Classical stochastic optimization requires the assumption of repeatability in the problem such that policies can be performed for an indefinite number of time within the planning horizon. However, as described, this is not a valid assumption in the development of offshore fields since once installation decisions have been executed, they would have to remain fixed for the entire duration of the planning and production horizon. The use of the proposed approach addresses these issues as it does not rely on distributional assumptions of the uncertain parameters. Furthermore, the application of the approach reveals unique characteristics on the uncertainties of well reserves. The way these uncertainties relate to the decisions subsequently allows the identification of an equivalent mixed integer programming formulation that is computationally tractable.

Integration of Environmental Consideration

Aside from gas field development planning, the research also looks into the integration of environmental considerations to general supply network design models. Global concerns regarding diminishing natural resources, overflowing waste sites and escalating levels of pollution have motivated various environmental legislations around the world. For example, participants of the United Nations Framework Convention on Climate Change (UNFCCC) have pledged to reduce total greenhouse gases emissions by at least five percent within 2008 to 2012 (Viguier et al., 2003). As a consequence, supply network operators and partners are increasingly benchmarked by their ability to manage the environmental impacts of their products and services. A study from the Confederation of British Industry estimates that environmental legislative actions have already cost companies four billion pounds a year (Willis, 2005). This highlights the important need for effective *green supply chain management* to achieve an acceptable balance of environmental considerations with other supply chain performance requirements.

In the context of green supply chain management, an important consideration is the accounting of greenhouse gas or CO_2 equivalent discharges from supply network activities. Generally, there can be significant levels of uncertainties associated with such accounting due to a host of factors such as incomplete data, lack of consensus on accounting techniques, and natural variability of processes (Schoepp et al. (2005), Ritter and Lev-On (2010)). Finally, there can also be uncertainty due to inaccurate projections on future greenhouse gas emission policies such as penalty charges.

On the topic of target-setting, instead of network cost minimization(profit maximization), the use of the approach in these network design problems enables the the decision makers to specify a cost budget target (profit target) and the aim is to find a solution that would best attain the respective targets. For instance, in the case of the gas field development problem, the net present value (NPV) may be a key performance metric whose target value is often specified as part of the project deliverables. A development plan is then developed in order to achieve this NPV target.

1.3 Research Problem

Organizations are continuously required to make strategic and operational decisions that achieve management goals. Many of these decisions are difficult to reverse, can have long term ramifications, and are often made with a great degree of uncertainty. The use of quantitative models has been acknowledged as a powerful tool to support such business analytics in decision making. However, de facto standard modeling approaches either assume very simplified settings to accurately depict real world scenarios or resort to optimization models that require strong assumptions on the nature of uncertainties. Consequently, the validity of the analysis is compromised and questionable if these assumptions are not well satisfied.

For instance, the paradigm of classical stochastic optimization in which expected profit is maximized tacitly assumes that the problem can be repeated a large number of times so that variations in profits are eliminated. However, the assumption of repeatability may not always be applicable in strategic problems such as those that involve investment decisions in gas field development planning. Hence, the profit variations from the expectation cannot be ignored. These models are also generally highly intractable, which has led to the reliance on approximation techniques to reduce solution complexity. These approximations typically require the assumption of having a discrete sample space or at least some form of discretization of the uncertain parameters. While this may be reasonable for smaller problems, the combinatorial state explosion associated with medium and large sized problems limits the scalability and practical use of such approximations.

Therefore, there is a need to develop a modeling approach for production and logistics planning problems, which not only ensures that management goals are achieved despite the presence of uncertainties but also preserves computational tractability. The proposed robust target-oriented approach does not require specific assumptions on the probability distributions modeling. In addition, the resulting optimization models only require the resolution of a small set of mixed integer linear programming problems, which can be solved efficiently using commercial linear programming solvers.

1.4 Research Objectives

To develop a robust target-oriented optimization approach for production and logistics planning problems that allows the achievement of system targets robustly under any realization within the supports of parameter variation ranges.

The objective centers on the development of an optimization framework that synthesizes recent developments in robust optimization technology and target-oriented decision making under uncertainty. The research likewise considers problem applications involving two-stage and multi-stage problems in production and logistics such as those seen in gas field development planning and production-inventory systems. These problem applications underscore decisions that are structured along the strategic and functional dimensions. The latter dimension can be further distinguished between areas of distribution management, inventory control and production planning. In line with this, the specific research objectives are enumerated as in the following:

1. Demonstrate the effects of uncertainty and the importance of accounting for it in decision making for production and logistics planning systems.

2. Introduce a target oriented robust optimization framework that enables flexibility in specifying target performance objectives.

3. Develop a modeling approach that does not require specific assumptions on the probability distributions of uncertain parameters.

4. Establish the applicability of the robust target-oriented optimization approach as a policy design tool for the achievement of transient performance requirements and stability analysis in dynamic problems.

5. Illustrate that the robust target-oriented optimization approach can be used by decision makers as a tool for fast turnaround planning and decision support through efficient implementation using commercially available solvers.

6. Show that under the target-oriented robust optimization approach, it is sufficient to consider fixed policies to obtain an optimal solution to network design problems.

7. Reduce dynamic planning problems under endogenous uncertainty into a mixed integer programming problem that assumes a single outcome of the uncertain parameters.

1.5 Scope, Limitations and Delimitations

1.5.1 Scope

1. The scope of the research includes the consideration of uncertain parameters with known supports on their nominal values and maximum possible deviations with respect to these nominal values.

2. The scope of the research includes the application of the robust target-oriented approach to production and logistics planning problems.

3. The workforce-inventory planning system to be analyzed is based on the stock management model defined by Saleh et al. (2010). The problem application focuses on the behavior of the system's resource variables, which are namely the finished goods inventory, work-in-process inventory, labor and vacancy. The corresponding targets involve the achievement of stability and transient behavior requirements for these variables.

4. The power supply system is adapted from the one machine infinite bus (OMIB) system defined by Yu and Siggers (1971). The problem application focuses on the stability of the system.

5. The research considers the topology of a gas field development network discussed by Goel and Grossmann (2004). Investments and operations performed within the network incur costs while revenue is obtained from the production of gas. Consequently, the net present value in the devel-

opment of the gas field network represents the target of the development problem.

6. The network design planning problem integrated with environmental considerations is illustrated through a power system generation example obtained from Panida and Singh (2008). The total network cost brought about by investment and operational decisions must remain within a specified cost budget. The emission of greenhouse gases accounts as the environmental component in this network design problem.

1.5.2 Limitations

1. The sources of uncertainties to be considered in the dynamic systems are limited to the system's endogenous parametric uncertainties such as time and oscillation constants for the workforce-inventory and power supply systems, respectively.

2. For the gas field development planning problem, only the endogenous uncertainties found in the initial deliverability and size of gas reservoirs are considered. The efficacy of the gas reservoirs is also characterized by their initial deliverability and size. Furthermore, only a linear reservoir behavior model is considered as a model for the gas reservoirs.

3. The type of greenhouse gases to be considered is limited to carbon dioxide (CO_2). The sources of uncertainties for the problem application

involving network design integrated with environmental considerations are limited to the amount of emissions in facilities and demand from consumers.

1.5.3 Delimitations

1. The research shall not consider the exogenous variables in the dynamic systems. This implies that the exogenous variables are assigned to be zero or constant.

2. The research shall not consider gas price uncertainties for the gas field development planning problem.

3. The research shall not consider capacity expansion for the problem application involving the integration of environmental consideration in network design. In addition, it shall not consider the uncertainties found in area generation, transmission lines, and area loads in the power system network.

1.6 Overview of the Dissertation

The remaining chapters of the dissertation is organized as follows. Chapter 2 presents a review of the relevant literature. A discussion is provided concerning uncertainties in decision making. This is followed by a discussion of the robust optimization framework, which is also accompanied by a comparison between stochastic optimization and robust optimization. This literature review ends with a discussion on target achievement approach to decision making.

Chapter 3 discusses the uncertainty model used in the research. The measure defined to evaluate the ability of solutions to achieve their respective targets are also presented. This is followed by a discussion on the relationship of these performance measures with success probabilities from a decision-theoretic perspective based on the satisficing behavior.

Chapter 4 introduces the main technical components of the proposed robust target-oriented optimization approach for dynamic systems. These consist of the construction of state-space models, analysis through eigenvalue sensitivity and the development of dynamic performance requirements in the form of linear constraints. The workforce-inventory and power supply dynamic system models are then presented and state-space dynamic formulations of the models are provided. The chapter concludes with computational studies on the application of the proposed approach to shape the dynamic response of both systems.

Chapter 5 begins with the introduction of the various components of an offshore gas field development problem, the decisions involved in this problem, and also the linear model of gas reservoirs assumed. This is followed by the presentation of the gas field development problem as a multi-period decision-making process under uncertainty. The results that allow the achievement of a tractable formulation for the problem are likewise shown. The resulting model formulation is then tested in the computational studies that compare the performance of the proposed model with other stochastic optimization models.

Chapter 6 presents the supply network design problem problem integrated with requirements on CO_2 emissions. It is shown that the resulting optimization model is not directly amenable to computation using standard robust optimization methods due to non-linear effects in the uncertain parameters. Consequently, some results based on the problem structure are analyzed and subsequently used to motivate an alternate formulation that can be solved directly using standard mixed integer programming solvers. Computational studies are likewise performed using the proposed model on a power system network design problem. The performance of the model is also compared to a stochastic optimization model based on sample average approximations.

Chapter 7 summarizes the conclusions made from the research. It discusses the main contributions of the research and also possible extensions for future work.

2. REVIEW OF RELATED LITERATURE

2.1 Decision Making Under Uncertainty

The classical paradigm in mathematical programming is to develop a model that assumes that the input data are precisely known and equal to some nominal values (Bertsimas and Sim, 2004). However, real world decision problems are replete with challenges where errors and inaccuracies can seep in. Uncertainties can have corrupting influences on the behavior of systems and failure to account for these in design, model building and analysis can have undesirable and unforeseen consequences. According to Listes (2010), data uncertainty can come from the following:

1. Estimation errors- part of the data is measured or estimated,
2. Prediction errors- part of the data does not exist when problem is solved,
3. Implementation errors- some components of a solution cannot be implemented exactly as computed, which in many models can be mimicked by appropriate data uncertainty.

For instance, estimation errors are prevalent in science and engineering problems while prediction errors can occur since the future demand or prices of commodities are rarely available and are at best, approximations. Finally, even if uncertainties in the model data can be ignored, there can still be implementation errors since solutions cannot be implemented to infinite precision, as assumed in continuous optimization (Bertsimas et al., 2010a).

Ben-Tal and Nemirovski (2000) showed that solutions to optimization problems can exhibit remarkable sensitivity to perturbations in the parameters of the problem. Specifically, they identified that even a 0.1% perturbation in the data coefficients can lead to a constraint violation of up to 125%. Hence, this is indicative of how a computed solution can be potentially rendered as either highly infeasible or suboptimal, or even both. In other words, this can become potentially worthless to a decision maker.

Sensitivity analysis has been widely used to study the influence of uncertainties or data perturbations to optimal solutions. However, while sensitivity analysis offers valuable information with respect to the system under perturbations, it is primarily a post-mortem tool of analysis, rather than a prescriptive and pro-active approach in guiding improvement and decision making under uncertainties. At best, it can quantify locally the stability of a nominal solution with respect to infinitesimal data perturbations, but it does not say exactly how to improve this stability, when necessary (Ben-Tal and Nemirovski, 2000). Furthermore, it is impractical to perform joint sensitivity analysis in models with large number of uncertain parameters (Sim, 2004).

In contrast, approaches that are able to generate solutions that integrate data perturbations in decision problems can be broadly categorized into stochastic optimization and robust optimization approaches. Stochastic optimization has been the de facto standard approach used to handle optimization under uncertainty. It typically requires full specifications of the probability distributions of the uncertain variates. According to Bertsimas et al. (2010a), the probability distribution of the uncertainties can be estimated and incorporated into the model using:

1. Chance constraints (i.e. a constraint which is violated less than p% of the time),
2. Risk measures such as standard deviations, value-at-risk and conditional value-at-risk, or
3. A large number of scenarios emulating the distribution.

Even with these methods, the actual distribution remains to be largely uncertain. Consequently, the need for strong distributional assumptions can be a major drawback of stochastic optimization. For one, this information is rarely available in practice or is difficult to establish. As mentioned, this is especially true in the case of the gas development problem. This is also evident in network problems integrated with environmental consideration, where prior knowledge of new environmental parameters being considered and historical data may be lacking. Furthermore, even if exact probability distributional assumptions can be made, stochastic optimization approaches often yield models that are computationally hard and very expensive to solve. This is for instance seen with chance constraints, which can destroy convexity properties and elevate significantly the complexity of the original problem (Sim, 2004).

In contrast, the key advantages of the robust optimization approach are the following. First, very few assumptions are required to model the uncertain parameters. These typically involve the moments: supports, means or variances. Such information is also easier to elicit and collect from industrial practitioners. Second, if the original deterministic problem is tractable, then the robust optimization approach often yields counterpart problems that are computationally tractable, such as linear programming (LP) or second-order cone programming problems.

In addition, Ben-Tal and Nemirovski (1999) states that although it is not stated explicitly, stochastic programming approaches mainly consider soft constraints. Specifically, the approach allows constraints to be violated, with certain penalties as seen with stochastic programming problems with recourse, scenario optimization, or with certain probability (chance constraints). The authors further add that in the dominating penalty approach, even when the random variables are degenerate (deterministic), the corresponding stochastic programming model does not recover necessarily the original LP constraints, but only a relaxation of these constraints.

Robust optimization, on the other hand, considers hard constraints. As mentioned, the common goal of developing robustness in design and decision-making is to immunize decision models against data uncertainty. That is, the computed solution must satisfy all the constraints regardless of the realization of the data parameters prescribed within an uncertainty set. This is a similar approach used by robust control methods found in Control Theory (further discussion on Control Theory will be provided in Chapter 4), which proposes solutions that are guaranteed to be stable even in the most severe scenarios. This is suitable for systems where deviations from expected scenarios can have serious safety implications. For instance, in civil engineering structures, external forces such as earthquakes or heavy loads acting upon the structure may significantly degrade stability and performance. In many other contexts, such robust solutions however may seem overly conservative, difficult to justify economically and might just not be available even if desired.

2.2 Robust Optimization

In the operations research literature of robust optimization models, Soyster (1973) considered these hard constraints by defining a feasible region via set containment, so as to guarantee the feasibility of the design under all uncertainties arising from a convex set. Consider the following LP problem:

$$\begin{aligned} \min \quad & \boldsymbol{c'x} \\ s.t. \quad & \boldsymbol{Ax} \geq \boldsymbol{b} \\ & \boldsymbol{x} \in \boldsymbol{X} \end{aligned} \tag{2.1}$$

Since one is dealing with hard constraints, it must be guaranteed that every constraint is satisfied for any possible value of $\boldsymbol{A}$ in a given convex uncertainty set $\boldsymbol{K}$ such that,

$$\begin{aligned} \min \quad & \boldsymbol{c'x} \\ s.t. \quad & \boldsymbol{a_i'x} \geq b_i \quad \forall i, \forall\ \boldsymbol{a_i} \in \boldsymbol{K} \\ & \boldsymbol{x} \in \boldsymbol{X} \end{aligned} \tag{2.2}$$

where the uncertainty is assumed to only affect the coefficients $\boldsymbol{a_i}$. This is then equivalent to:

$$\begin{aligned} \min \quad & \boldsymbol{c'x} \\ s.t. \quad & \min_{\boldsymbol{a_i} \in \boldsymbol{K}} \quad \boldsymbol{a_i'x} \geq b_i \quad \forall i \\ & \boldsymbol{x} \in \boldsymbol{X} \end{aligned} \tag{2.3}$$

Bertsimas and Thiele (2006b) state that the key insight that preserves the computational tractability of the robust counterpart approach is that Problem (2.3) can be reformulated as a single convex programming problem for any convex uncertainty set.

Some examples of studies that made use of this approach include, Pishvaee et al. (2011) who consider a closed loop supply chain problem that deals with product recovery and redistribution. The problem has the objective of identifying the flows in the network that minimizes the total cost of the closed loop supply chain under uncertain demands, product returns and transportation costs. Consequently, the robust optimal solution is one that best satisfies all realizations of the constraints arising from a bounded uncertainty set. Similarly, Blanchini et al. (2000) defined the uncertainties for a general dynamic production distribution system as unknown but bounded quantities. The robust framework was applied to this design problem in order to ensure that inventory levels are kept inside prescribed bounds for all possible realizations of customer demands.

While the above approach guarantees feasibility, it may also be considered too pessimistic as the resulting model can generate solutions that give up too much optimality in order to ensure robustness. This has been subsequently addressed in the works of El-Ghaoui and Lebret (1997), El-Ghaoui et al. (1998) and Ben-Tal and Nemirovski (1999) by proposing less conservative models through the use of ellipsoidal uncertainty sets. This specifically involves solving the robust counterparts of the nominal problem in the form of conic quadratic problems. Bertsimas and Sim (2003) state that with properly chosen ellipsoids, such a formulation can be used as a reasonable approximation to more complicated uncertainty sets. However, the authors also point out that a practical drawback of such an approach is that it leads to non-linear, although convex, models, which are more demanding computationally than the earlier linear models by Soyster (1973).

Bertsimas and Sim (2003) alternatively proposed an "uncertainty budget" parameter that controls the degree of robustness to be assumed, so that a designer can choose *a priori* the size of the uncertainty set that he wished to hedge the solution against. Hence, the solution is guaranteed to be feasible if the uncertain parameters change by less than this uncertainty budget. Their proposed approach is found to be at least as flexible as the one proposed by El-Ghaoui and Lebret (1997), El-Ghaoui et al. (1998) and Ben-Tal and Nemirovski (1999). Furthermore, the generated robust counterparts are found to be linear optimization problems, allowing the approach to readily generalize to discrete optimization problems.

The same approach has been used by Bertsimas and Thiele (2006a) to address the optimization of a supply chain when demand is uncertain. The model seeks to determine the optimal ordering policy for a single type of product over a finite discrete time horizon such that the total cost is minimized. The model does not assume any specific distributions and also allows the level of robustness to be adjusted in order to trade off performance and protection against uncertainty.

A practical issue of the above approach however is that the decision-maker might not know how to choose the uncertainty budget parameter in advance. This research instead proposes to search for a solution that maximizes the size of an uncertainty budget, and identify this as the robustness index. This index accounts for the largest degree of uncertainty that can be tolerated by a solution before the solution becomes infeasible. For example, in the workforce-inventory problem, when a solution becomes infeasible, this could imply certain important dynamic performance criteria, such as stability conditions, that can become violated. In this sense, the proposed index is precisely a measure of a system's capability to withstand uncertainties and recover to normal operating conditions in the event of disturbances.

Robust optimization has also been extended to two-stage and more general multi-stage settings. As stated by Ben-Tal et al. (2004), the classical robust optimization approach corresponds to the case when all the variables represent decisions that must be made before the actual realization of the uncertain data becomes known. However, in the modeling of real-world problems, it might be permissible for a subset of the decisions to be made after the realization of all or part of the underlying uncertainties (Goh and Sim, 2010). Consequently, Ben-Tal et al. (2004) partitioned the variables into two sets: adjustable and non-adjustable. The former represents variables whose values can still change depending on the realized uncertainties. For instance, operational decisions and investment decisions can be considered as adjustable and non-adjustable variables, respectively.

Using this partitioning, an approximation was proposed in the form of affinely-adjustable robust counterpart, where the adjustable decisions are constrained to be an affine function of the uncertain data. Ben-Tal et al. (2005) subsequently applied affine decision rules to flexible supplier-retailer adjustable contracts. Chen et al. (2007, 2008) considered piecewise-linear decision rules which are found to improve upon the original linear decision rules. Meanwhile, Goh and Sim (2010) extended the affine decision rules to deflected decision rules and segregated decision rules in order to improve on the solution quality in multi-stage problems.

2.3 Target-Achievement Approach in Decision Making

The proposed approach in this research involves the integration of the robust optimization framework and target-achievement decision making. Target-achievement or satisficing differs from utility-based decision making. The set of axioms initially introduced by von Neumann and Morgenstern (1944) showed that cardinal utilities could be assigned to uncertain outcomes. If one behaves according to a manner in line with these axioms, that person would act so as to maximize the expected value (which is the average, weighted by the probabilities of the alternative outcomes of a choice) of his utility (Simon, 1959).

Simon adds that the theory could be tested empirically only on the assumption that the probabilities assigned to the alternatives by the subject are identical with the *objective* probabilities of these events as known to the experimenter.

However, the objective probabilities of these uncertain alternatives are rarely known in real world problems, which leads one to actually view uncertainty as ambiguous. Ambiguity implies that there is very little information that could be used in the decision-making process. In the seminal work of Knight (1921), a distinction has been created between risk and ambiguity (uncertainty). Risk refers to situations in which the decision maker has the ability to assign probabilities to the possible outcomes of events. Runde (1998) adds that this assignment is usually based on the knowledge of *a priori* or empirical information. Meanwhile, ambiguity involves situations that prevent decision makers to assign such probabilities primarily because of their lack of knowledge about these events. As a result, they tend to rely largely on estimates. As mentioned, the use of the robust optimization approach allows one to avoid the use of these subjective probabilities.

Lack of knowledge does not however imply the absence of it. Ellsberg (1961) postulates that even in situations of ambiguity, people tend to behave as though they assign "degrees of belief" to the events impinging on their actions. This is the main reason why the literature has been rife with the usage of subjective probabilities. Ellsberg also shows convincingly by means of paradoxes that ambiguity preference cannot be reconciled with classical expected utility theory (Lam et al., 2012).

In response to these observations, Simon (1959) introduced the concept of bounded rationality to explain the behavior of decision makers. This refers to the limited ability of human agents in formulating and solving complex decision problems in the real world. The theme of bounded rationality in decision behavior plays a very fundamental and intrinsic role in the elucidation of complex systems behavior. Bounded rationality has been able to infer insights in organizational thinking about why, even with the best of intentions, decision policies can result in unintended consequences and organizational failures. Very often, the cause of unintended consequences stem from the bounded rationality of decision makers, such as the failure to use information properly, overlooking systemic influences and underestimating consequences of actions. Bounded rationality can be regarded as a cardinal principle in the modelling of decision behavior. It is important for modelers to account for the bounded rationality of the decision agents in the modelling process in order to achieve real, rather than ideal system representations.

A classical model for *boundedly rational* agents in behavorial economics, due to Simon (1959), is that of a *satisficer* as opposed to a fully rational *optimizer*. Both of which account for two broad approaches to rational behavior in situations where complexity and uncertainty make global rationality impossible. Simon (1972) states that optimization has become the tool for approximation, as it describes the real-world situation radically simplified and reduced in degree of complexity as to make it easier for the decision maker to handle. The author adds that satisficing approaches meanwhile simplify towards another direction, retaining more of the detail of the real-world situation but settling for a satisfactory (instead of the approximate best) decision. That is, when faced with a decision problem, rather than processing and computing the expected utility of every alternative action, a satisficer simply chooses the first available course of action that satisfies his desired targets. Hence, decision makers can satisfice either by finding optimum solutions for a simplified world, or by finding satisfactory solutions for a more realistic world (Simon, 1959).

A satisficing approach to decision making has been integrated to traditional optimization models as seen from the works of Borch (1968), Charnes and Cooper (1963), Castagnoli and LiCalzi (1996), Bordley and LiCalzi (2000) and Bordley and Kirkwood (2004). As discussed in Chapter 1, these works consider the maximization of success probabilities in place of utility functions. However, aside from issues concerning intractability and the inability to account for the magnitude of shortfall, another widely-acknowledged drawback of using success probability is that it does not guarantee diversification preferences. Diversification is a widely accepted axiomatic rule of behavior (Markowitz, 1952) in investment and portfolio choice. Decisions that do not lead to diversified positions are generally viewed as undesirable and inconsistent with risk-aversion because of their tendency to expose investors to large positions (Ng et al., 2012).

These issues are addressed by Chen and Sim (2009), which propose the shortfall aspiration level as a tractable alternative to probability measure. The shortfall aspiration level encompasses the probability of success in achieving the target and an expected level of under-performance or shortfall. As cited by Ng et al. (2012), the authors showed that solving the associated optimization problem, to maximize the shortfall aspiration level, can be reduced to solving a small collection of stochastic optimization problems. The objectives of which are evaluated under the Conditional-Value-at-Risk measure (Rockafellar and Uryasev, 2000), which is known to be computationally tractable. Brown and Sim (2010), Brown et al. (2012) and Lam et al. (2012) further developed these dual relationships between aspiration level measures and risk measures in greater generality. As mentioned, a target-oriented approach inspired by the aforementioned measures is considered in this research. A more in depth discussion of this is provided in the next chapter.

3. ROBUST TARGET-ORIENTED OPTIMIZATION APPROACH

This chapter presents the development of the robust target-oriented optimization approach that will allow planning systems to achieve targets derived under uncertainty. Essentially, the objective is to identify the appropriate settings for the decision variables so that system constraints are feasible for as large a range of uncertain parameters as possible. The decision variables and uncertain parameters are denoted as the vectors $\boldsymbol{y} = (y_1, \cdots, y_L)$ and $\tilde{\boldsymbol{z}} = (\tilde{z}_1, \cdots, \tilde{z}_K)$, respectively. Note that the tilde sign is used to denote uncertainty in the parameters.

3.1 Uncertainty Models

Let $\tilde{\boldsymbol{z}}$ be a vector of uncertain parameters in the system model defined as follows:

$$\tilde{\boldsymbol{z}} = \overline{\boldsymbol{z}} + \boldsymbol{z} \tag{3.1}$$

where $\overline{\boldsymbol{z}}$ denotes the nominal values of the parameters and the perturbations $\boldsymbol{z}$ are such that

$$\left\{\boldsymbol{z} \in \Re^K \mid 0 \leq z_k \leq \bar{z}_k(\gamma),\ \forall\ k = 1, ..., K\right\} \tag{3.2}$$

where the largest perturbations would take on the values $z_k = \bar{z}_k$, for all $k = 1, ..., K$. (3.2) assumes that in the most optimistic case, $\tilde{\boldsymbol{z}}$ would take on the lowest values (i.e., $\tilde{\boldsymbol{z}} = \overline{\boldsymbol{z}}$). For instance, $\tilde{\boldsymbol{z}}$ may be an uncertain cost, to which $\tilde{\boldsymbol{z}} = \overline{\boldsymbol{z}}$ results to the lowest cost achievable.

It can be seen that these perturbations are parameterized by the *robustness index*, $\gamma \in [0, 1]$ where $\bar{z}_k : [0, 1] \to \Re_+,\ k = 1, ..., K$ are nondecreasing functions. Note that the use of $\bar{z}_k(\gamma)$ also allows asymmetry in the definition of the uncertain parameters. Because higher values of γ imply that the uncertainty set includes larger perturbations for the uncertain parameters, a decision maker with a higher robustness index describes a more uncertainty averse attitude that would rather err on the side of caution.

In many practical problems, there are also additional information about the relationships between the uncertain parameters. Such information can help to reduce the uncertain outcome space significantly and hence should be incorporated when available. The below set of linear constraints is used as first-order approximations of such relationships,

$$R\boldsymbol{z} \leq \boldsymbol{e} \tag{3.3}$$

where the $W \times K$ matrix R and the vector $\boldsymbol{e} = (e_1, \cdots, e_W)$ are estimated coefficients of the W relationships shared by the uncertain parameters. Combining the above, for a given γ, the uncertainty set $\mathcal{Z}_\gamma$ is defined as:

$$\mathcal{Z}_\gamma = \left\{ \boldsymbol{z} \in \Re^K \mid 0 \leq z_k \leq \bar{z}_k(\gamma), R\boldsymbol{z} \leq \boldsymbol{e},\ \forall\ k = 1, ..., K \right\} \tag{3.4}$$

The uncertainty set and the robustness index γ described above lead one to think about decisions that can remain feasible in the entire set of outcomes for the perturbations considered, for as large a set of outcomes as possible. Intuitively, this makes sense for decision makers who desire plans that can achieve performance requirements as far as possible under uncertainty.

It is clear that $\mathcal{Z}_{\gamma'} \subseteq \mathcal{Z}_\gamma$ whenever $\gamma \geq \gamma'$. This formulation provides decision makers with a functional means to evaluate the robustness of a solution with regards to uncertainty. In particular, it can be said that a candidate solution has a robustness index γ if the resulting system is able to meet all the requirements for any $\tilde{\boldsymbol{z}} \in \mathcal{Z}_\gamma$.

3.2 Robust Optimization Model

In view of the preceding discussion, a general optimization model is formulated. This model seeks to maximize the robustness index γ through the decision variables $\boldsymbol{y}$, so that targets are always achieved for any realizations of the uncertain parameters arising from the set $\mathcal{Z}_\gamma$ (for the largest possible value of γ). The model is then stated mathematically in the following formulation.

Problem $\mathcal{R}$

$$\max_{\boldsymbol{y}\in\mathcal{Y}} \ \gamma$$

$$A \cdot \boldsymbol{y} + B \cdot \tilde{\boldsymbol{z}} \leq \boldsymbol{\tau} \quad \forall\ \boldsymbol{z} \in \mathcal{Z}_\gamma \tag{3.5}$$

where the feasible space of $\boldsymbol{y}$ is modelled as the polyhedron $\mathcal{Y}$, so that $\boldsymbol{y} \in \mathcal{Y}$ in any feasible solution. System targets are defined by (3.5), where there are N number of targets $\boldsymbol{\tau} \in \Re^N$. Meanwhile, $A \in \Re^{N\times L}$ and $B \in \Re^{N\times K}$ are coefficient matrices of the decision variables and uncertain parameters, respectively.

The targets as specified in (3.5) may imply resource budgets or limits that the system cannot exceed. Targets can also be in the form of performance metrics, which the system must adhere to. Note that the constraints (3.5) are required to hold for all uncertain parameter outcomes $\tilde{\boldsymbol{z}} \in \mathcal{Z}_\gamma$. Since $\mathcal{Z}_{\gamma'} \subseteq \mathcal{Z}_\gamma$ whenever $\gamma' \leq \gamma$, maximizing γ can be interpreted as maximizing the robustness of the system to uncertainty with regards to meeting the specified performance requirements.

Solving the robust problem as it is formulated in (3.5) would require evaluating an infinitely large number of constraints (one constraint for each possible realization of $\tilde{\boldsymbol{z}}$). In effect, this would make the robust formulation considerably more difficult to solve than its nominal counterpart, a linear programming problem (Bertsimas and Thiele, 2006b). However, a more compact formulation could be arrived at due to the convex and polyhedral properties of the uncertainty set $\mathcal{Z}_\gamma$. In Proposition 1, an equivalent formulation of Problem $\mathcal{R}$ is developed to enable application of linear programming methods for solution iteratively.

Proposition 1 *Define $P \in \Re^{N\times K}$ and $Q \in \Re^{N\times W}$ as matrices of variables. Problem $\mathcal{R}$ is then equivalent to the following:*

Problem $\mathcal{C}$

$$\max_{\boldsymbol{y}\in\mathcal{Y}} \quad \gamma$$

s. t.

$$A\cdot\boldsymbol{y} + B\cdot\overline{\boldsymbol{z}} + P\cdot\gamma\overline{\boldsymbol{z}} + Q\cdot\boldsymbol{e} \leq \boldsymbol{\tau} \tag{3.6}$$

$$|B| - P - Q\cdot R \leq 0 \tag{3.7}$$

$$P, Q \geq 0 \tag{3.8}$$

Proof: First, formulation $\mathcal{C}$ can be constructed by considering the constraints in (3.5). In particular, it suffices that:

$$A \cdot \boldsymbol{y} + B \cdot \overline{\boldsymbol{z}} + \max_{\boldsymbol{z} \in \mathcal{Z}_\gamma} \{|B| \cdot \boldsymbol{z}\} \leq \tau \tag{3.9}$$

Since the coefficients of $\tilde{\boldsymbol{z}}$ may be negative, the absolute value is maximized to account for perturbations in both the negative and positive directions. The maximization in the left-hand side can be written explicitly as follows.

$$\max \ |B| \cdot \boldsymbol{z} \qquad \text{s.t. } 0 \leq \boldsymbol{z} \leq \gamma \bar{\boldsymbol{z}}, \ , R\boldsymbol{z} \leq \boldsymbol{e} \tag{3.10}$$

where the constraints imposed on the perturbations $\boldsymbol{z}$ are obtained from the uncertainty set $\mathcal{Z}_\gamma$ and the function $\bar{z}_k(\gamma)$ is defined as $\gamma\bar{\boldsymbol{z}}$. (3.10) is then a linear optimization problem and the dual formulation can be written as:

$$\min P \cdot \gamma \bar{\boldsymbol{z}} + Q \cdot \boldsymbol{e} \tag{3.11}$$

$$\text{s.t. } P + Q \cdot R \geq |B| \ , P, Q \geq 0 \tag{3.12}$$

where $P \in \Re^{N \times K}$ and $Q \in \Re^{N \times W}$ denote the variables in the dual formulation. The formulation $\mathcal{C}$ is then derived by replacing the maximization term in (3.9) with the objective function value in (3.11), thus obtaining (3.6). Finally, (3.7) and (3.8) are obtained by augmenting (3.12) into $\mathcal{R}$. Since both formulations are identical in the objective functions, it suffices to show that for any given γ, the feasible space of $\boldsymbol{y}$ in each of the formulations are the same.

Let $\mathcal{Y}^{\mathcal{R}}$ denote the feasible space of $\boldsymbol{y}$ in Problem $\mathcal{R}$, i.e., $\mathcal{Y}^{\mathcal{R}} = \{y \in \mathcal{Y},\ s.t.\ (3.5)\}$. Let $\mathcal{S}^{\mathcal{C}}$ denote the feasible solution space of Problem $\mathcal{C}$, i.e.,

$$\mathcal{S}^{\mathcal{C}} = \{(\boldsymbol{y}, P, Q)\ s.t.\ (3.6) - (3.8)\}$$

The feasible space (or projection) of $\boldsymbol{y}$ in $\mathcal{C}$, is then defined as

$$\mathcal{Y}^{\mathcal{C}} = \left\{\boldsymbol{y} \in \mathcal{Y} \mid \exists\ (P, Q),\ s.t\ (\boldsymbol{y}, P, Q) \in \mathcal{S}^{C}\right\}$$

The following then completes the equivalence of the two formulations.

1. If $\boldsymbol{y}$ is feasible in $\mathcal{Y}^{\mathcal{R}}$, then it is also feasible in $\mathcal{Y}^{\mathcal{C}}$: Suppose $\boldsymbol{y} \in \mathcal{Y}^{\mathcal{R}}$. Since (3.10) is both feasible and bounded in the objective, its dual (3.11) and (3.12) are also feasible and bounded. Then by the strong duality of linear programming, there must exist P^* and Q^* from the dual space (3.12) so that $P^* \cdot \gamma\bar{\boldsymbol{z}} + Q^* \cdot \boldsymbol{e} = \max_{\boldsymbol{z} \in \mathcal{Z}_\gamma} \{|B| \cdot \boldsymbol{z}\} \leq \boldsymbol{\tau} - A \cdot \boldsymbol{y} - B \cdot \bar{\boldsymbol{z}}$. Hence, $\boldsymbol{y} \in \mathcal{Y}^{\mathcal{C}}$.

2. If $\boldsymbol{y}$ is feasible in $\mathcal{Y}^{\mathcal{C}}$, then it is also feasible in $\mathcal{Y}^{\mathcal{R}}$: Suppose $\boldsymbol{y} \in \mathcal{Y}^{\mathcal{C}}$. By weak duality, $\max_{\boldsymbol{z} \in \mathcal{Z}_\gamma} |B| \cdot \boldsymbol{z} \leq P \cdot \gamma\bar{\boldsymbol{z}} + Q \cdot \boldsymbol{e} \leq \boldsymbol{\tau} - A \cdot \boldsymbol{y} - B \cdot \overline{\boldsymbol{z}}$ for any P and Q from (3.12). Consequently, $\boldsymbol{y} \in \mathcal{Y}^{\mathcal{R}}$. □

For a fixed value of γ', the constraints (3.6)–(3.8) are linear in the decision variables. Model $\mathcal{C}$ can thus be solved for the maximum γ^* by performing a line search in $\gamma \in [0, 1]$. That is, an ascent direction can be identified to improve the value of γ. For a given γ', feasibility in (3.6) – (3.8) can be checked efficiently by solving a Phase 1 linear programming problem. If a feasible solution exists, then $\gamma^* \geq \gamma'$. Otherwise $\gamma^* < \gamma'$. The line search can then proceed by applying a bisection search procedure. The local search procedure is formalized in Chapter 4 of this thesis.

3.3 Relationship of the Robustness Index γ to Success Probability

The use of γ accounts as a measure of the system's ability to achieve performance requirements under uncertainties. In a broader sense, it can also be demonstrated that the proposed measure shares some interesting connections with the probability of success through an axiomatic argument. This section provides further justification for the choice of maximizing the $\gamma - level$ robustness of the system by showing its connection with success probability optimization.

These insights are based on recent developments in the field of risk analytics, in particular, optimization under risk and the concept of satisficing measures (Brown and Sim, 2010). Let $\tilde{a}$ denote the uncertain level of achievement above the target, or *position*, and where inequality relations $\tilde{a} \geq 0$ are applied in the state-wise sense. In this case, the target can be in the form of profits, market share, etc. Brown and Sim (2010) proposed the following axiomatic definition of *satisficing measures* ρ, where $1 \geq \rho \geq 0$, that operate on $\tilde{a}$:

Definition 1 *(from Brown and Sim (2010)): A given function $\rho(\tilde{a})$ is a valid satisficing measure if all the below are true:*

1. Attainment content: If $\tilde{a} \geq 0$, then $\rho(\tilde{a}) = 1$.

2. Non-attainment apathy: If $\tilde{a} < 0$, then $\rho(\tilde{a}) = 0$.

3. Monotonicity: If $\tilde{a} \geq \tilde{a}'$, then $\rho(\tilde{a}) \geq \rho(\tilde{a}')$.

4. Gain continuity: $\lim_{\alpha \downarrow 0} \rho(\tilde{a} + \alpha) = \rho(\tilde{a})$.

Attainment content implies that if the target is always achieved, then the satisficing level ρ attains its maximum (i.e. $\rho = 1$). On the other hand, *Non-attainment apathy* indicates that if the current position never achieves the target (i.e. under any realization), then it is always least preferable (i.e. $\rho = 0$). *Monotonicity* states that if a position $\tilde{a}$ (state-wise) dominates position $\tilde{a}'$, then the former is no less preferable $\rho(\tilde{a}) \geq \rho(\tilde{a}')$. Finally, *Gain continuity* is a technical requirement meaning that if an infinitesimally small increment is made to the current position, the satisficing level would not change. The opposite however is not necessarily true. The reader can readily verify that the success probability $P(\tilde{a} \geq 0)$ is indeed a satisficing measure in accordance to *Definition 1.*

Now for a given $\boldsymbol{y}$, denote $\tilde{a}$ as

$$\tilde{a} = \min_{i=1,\cdots,N} -\boldsymbol{A_i}\boldsymbol{y} - \boldsymbol{B_i}\tilde{\boldsymbol{z}} + \boldsymbol{\tau_i} \tag{3.13}$$

where, $\tilde{a}$ is the extreme-value uncertain variable whose outcome is the worst achievement among the $i = 1, \cdots, N$ design requirements. Next, $\mathcal{Z}_\gamma(\tilde{a})$ for $\tilde{a}$ is defined as follows:

$$\mathcal{Z}_\gamma(\tilde{a}) = \begin{cases} \sup\left\{\gamma \in (0,1) : \min_i \min_{\tilde{\boldsymbol{z}} \in \mathcal{Z}_\gamma} \{-\boldsymbol{A_i}\boldsymbol{y} - \boldsymbol{B_i}\tilde{\boldsymbol{z}} + \boldsymbol{\tau_i}\} \geq 0\right\} & \text{if feasible,} \\ 0 & \text{otherwise.} \end{cases} \tag{3.14}$$

The largest γ achievable by a solution $\boldsymbol{y}$ is given by $\mathcal{Z}_\gamma(\tilde{a})$. The following result shows that the proposed index is in fact coherent with the framework prescribed by Brown and Sim (2010) for satisficing measures.

Proposition 2 *The robustness index $\mathcal{Z}_\gamma(\tilde{a})$ is a satisficing measure in accordance to the decision criteria of attainment content, non-attainment apathy, monotonicity and gain continuity on the uncertain variable $\tilde{a}$.*

Proof: $\mathcal{Z}_\gamma(\tilde{a})$ defined in (3.14), where the random variable

$$\tilde{a} = \min_{i=1,\cdots,N} -\boldsymbol{A_i}\boldsymbol{y} - \boldsymbol{B_i}\tilde{\boldsymbol{z}} + \boldsymbol{\tau_i},$$

satisfies each axiom.

1. *Attainment content.*

$$
\begin{array}{lll}
\textit{Given}: & \tilde{a} \geq 0 & \\
\Rightarrow & \min_{i=1,\cdots N} \{-\boldsymbol{A_i y} - \boldsymbol{B_i \tilde{z}}\} + \boldsymbol{\tau_i} \geq 0 & \forall \boldsymbol{\tilde{z}} \in \mathcal{Z}_\gamma(\gamma = 1) \\
\Rightarrow & \min_{\boldsymbol{\tilde{z}} \in \mathcal{Z}_\gamma} \min_{i=1,\cdots N} \{-\boldsymbol{A_i y} - \boldsymbol{B_i \tilde{z}}\} + \boldsymbol{\tau_i} \geq 0 & \forall \gamma \in [0,1] \\
\Rightarrow & \gamma(\tilde{a}) = 1 &
\end{array}
$$

where the last inequality follows from noting that $\mathcal{Z}'_\gamma \subseteq \mathcal{Z}_\gamma$ whenever $\gamma' \leq \gamma$.

2. *Non-attainment apathy.*

$$
\begin{array}{lll}
\textit{Given}: & \tilde{a} < 0 & \\
\Rightarrow & \min_{i=1,\cdots N} \{-\boldsymbol{A_i y} - \boldsymbol{B_i \tilde{z}}\} + \boldsymbol{\tau_i} < 0 & \forall \boldsymbol{\tilde{z}} \in \mathcal{Z}_\gamma, \forall \gamma \in [0,1] \\
\Rightarrow & \gamma(\tilde{a}) = 0 & \text{(by definition in (3.14)).}
\end{array}
$$

3. *Monotonicity.* Given $\boldsymbol{y}$ and $\boldsymbol{y'}$, and where

$$
\tilde{a}' = \min_{i=1,\cdots N} \{-\boldsymbol{A_i y'} - \boldsymbol{B_i \tilde{z}}\} + \boldsymbol{\tau_i},
$$

we have that:

$$
\tilde{a} \geq \tilde{a}'
$$

$\Rightarrow \forall \tilde{\boldsymbol{z}} \in \mathcal{Z}_\gamma \left(\gamma = 1\right),$

$$\min_{i=1,\cdots N} \{-\boldsymbol{A_i y} - \boldsymbol{B_i \tilde{z}}\} + \boldsymbol{\tau_i} \geq \min_{i=1,\cdots N} \{-\boldsymbol{A_i y'} - \boldsymbol{B_i \tilde{z}}\} + \boldsymbol{\tau_i}$$

$\Rightarrow \forall \tilde{\boldsymbol{z}} \in \mathcal{Z}_\gamma \left(\gamma = \gamma(\tilde{a}')\right),$

$$\min_{i=1,\cdots N} \{-\boldsymbol{A_i y} - \boldsymbol{B_i \tilde{z}}\} + \boldsymbol{\tau_i} \geq \min_{i=1,\cdots N} \{-\boldsymbol{A_i y'} - \boldsymbol{B_i \tilde{z}}\} + \boldsymbol{\tau_i} \geq 0$$

$\Rightarrow \gamma(\tilde{a}) \geq \gamma(\tilde{a}')$

4. *Right continuity.* It is to be shown that the below must hold simultaneously:

$$\lim_{\alpha \downarrow 0} \mathcal{Z}_\gamma(\tilde{a} + \alpha) \leq \mathcal{Z}_\gamma(\tilde{a}) \tag{3.15}$$

$$\lim_{\alpha \downarrow 0} \mathcal{Z}_\gamma(\tilde{a} + \alpha) \geq \mathcal{Z}_\gamma(\tilde{a}) \tag{3.16}$$

(3.15) is shown by contradiction. Suppose $\lim_{\alpha \downarrow 0} \mathcal{Z}_\gamma(\tilde{a} + \alpha) = \mathcal{Z}_\gamma(\tilde{a}) + \epsilon$ for some $\epsilon > 0$. Then we have that:

$$\min_{\tilde{\boldsymbol{z}} \in \mathcal{Z}_\gamma((\tilde{a})+\epsilon)} \min_{i=1,\cdots N} \{-\boldsymbol{A_i y} - \boldsymbol{B_i \tilde{z}}\} + \boldsymbol{\tau_i} + \alpha \geq 0 \quad \forall \alpha > 0 \tag{3.17}$$

Furthermore, by definition of $\mathcal{Z}_\gamma(\tilde{a})$ we must have:

$$\min_{\tilde{\boldsymbol{z}} \in \mathcal{Z}_\gamma((\tilde{a})+\epsilon)} \min_{i=1,\cdots N} \{-\boldsymbol{A_i y} - \boldsymbol{B_i \tilde{z}}\} + \boldsymbol{\tau_i} < 0$$

Clearly, by choosing any $\Delta > 0$ and such that

$$\Delta < - \min_{\tilde{\boldsymbol{z}} \in \mathcal{Z}_\gamma((\tilde{a})+\epsilon)} \min_{i=1,\cdots N} \{-\boldsymbol{A_i y} - \boldsymbol{B_i}\tilde{\boldsymbol{z}}\} - \boldsymbol{\tau_i},$$

we can have

$$\alpha = - \min_{\tilde{\boldsymbol{z}} \in \mathcal{Z}_\gamma((\tilde{a})+\epsilon)} \min_{i=1,\cdots N} \{-\boldsymbol{A_i y} - \boldsymbol{B_i}\tilde{\boldsymbol{z}}\} - \boldsymbol{\tau_i} - \Delta > 0$$

which provides the required contradiction in (3.17).

Finally, (3.16) is straightforward to show by applying monotonicity of γ.

That is, $\mathcal{Z}_\gamma(\tilde{a} + \alpha) \geq \mathcal{Z}_\gamma(\tilde{a})$ for all $\alpha > 0$. □

4. DESIGN AND PLANNING OF DYNAMIC PERFORMANCE UNDER UNCERTAINTY

In this chapter, we focus on the issues of dynamic performance under uncertainty. Yourdon (2004) noted that many systems that are designed to be efficient under normal operating assumptions can be extremely vulnerable and fragile when faced with unanticipated disruptions. Shock events not only impair day-to-day operations but possibly also long term performance (Mitroff and Alpaslan, 2003).

We address these issues through the integration of the robust target-achievement approach and control theory based methods. Control theory is defined as the fully developed branch of engineering that applies differential equations to analyze the time response of physical systems (Ortega and Lin, 2004). Simon (1952) adds that such equations (by means of state space representations) describe, at least approximately, many of the systems with which electrical and mechanical engineering deal with.

These types of systems operate in a constantly changing environment, which inevitably contains inherent uncertainties. In many practical situations, physical systems also require the specification of parameters that may not be accurately known at the time when planning and design decisions need to be made. For instance, analyzing a power supply system involves the identification of a number of parameters that influence its dynamic behavior. However, these parameters can only be roughly specified during the input modeling phase since it is very difficult to measure their exact values. The presence of uncertainty not only impacts the system but also prevents decision makers from coming up with an accurate analysis. This is because the analysis of dynamic performance depends upon the accuracy of the model used (Dong et al., 2005). Control rules that are designed based on a single estimate of these parameters may then perform poorly if the actual parameter values deviate from the assumed values. This therefore underscores the significance of integrating uncertainty in the analysis.

A common approach in practice is to use stochastic simulation to perform the analysis of such models. This unfortunately requires the accurate estimation of the input random variables modeling the uncertain parameters. Furthermore, simulation approaches can be very computationally expensive and impractical since it often requires a large number of repetitions in order to achieve accurate statistical estimates of the performance. Another approach is through robust stability analysis, which involves evaluating whether every element in the system is stable, for a given range of values of the uncertain parameters. However, as shown by Nemirovski (1993) and Vidyasagar and Blondel (2001), this is likewise a NP-hard problem.

Within the field of control theory, robust control (Rollins (1999), Mejia (2003)) accounts for a branch of study which addresses the issue of system stability design under uncertainty. Specifically, the general objective of robust control methods involves the integration of robustness such that a system is able to recover and reorganize from stress or shock events. However, as raised in Chapter 2, robust control can lead to solutions that can be considered as too conservative. This pitfall is avoided in this work by using the robustness index in the optimization models.

It is also well acknowledged in the literature that one of the primary steps to analyze physical systems using these methods is by modeling them through a set of differential equations. System requirements related to dynamic behavior and stability can then be described using a set of linearized constraints based on the differential equations of the systems. Consequently, the use of the robust optimization approach lets us avoid the need for distributional assumptions of the uncertainties, while control theory based methods allow us to build simple approximations of a system's dynamics. Furthermore, the resulting model only requires the resolution of a small set of linear programming problems, each of which can be solved using commercial linear programming solvers. The resulting model could then identify a solution that can achieve all dynamic performance requirements as well as possible under uncertainty.

The next section shows how the techniques and tools developed within the control theory framework have extended beyond traditional engineering systems. These have also been used as a valid alternative to study the dynamic characteristics of production and inventory systems (Ortega and Lin, 2004). The chapter then proceeds by introducing the application of the robust target-achievement approach to dynamic systems and discussing how dynamic performance could be shaped and controlled. This is facilitated by demonstrating its application to a workforce-inventory model. This is then followed by another application of the approach to a power supply system.

4.1 Control Theory in Production - Inventory Control

Control theory based methods traditionally used for engineering systems (such as the power supply system) have consequently been shown to contribute to the study of production and inventory system dynamics. As an illustration, Axslater (1985) models a single-stage system by defining the inventory at time t to be dependent upon the production and demand at time t:

$$\frac{\partial I(t)}{\partial t} - P(t - t_o) - D(t) \tag{4.1}$$

where t_o denotes the production lead time. The time development of the inventory $I(t)$ is then obtained as the difference between the production $P(t - t_o)$ and the demand rate $D(t)$. Ortega and Lin (2004) further note that $P(t)$ is usually chosen to minimize costs when the inventory level deviates from a desired inventory level I_o and when the production rate changes. The production rate $P(t)$ can be obtained by applying a suitable linear operator to the inventory deviation $I(t) - I_o$.

Simon (1952) likewise proposes the application of control theory to study dynamic production and inventory systems. A salient common characteristic in general engineering systems and production inventory systems is the feedback mechanism that refers to the information related to the gap between the actual (output) and reference (input) levels of a system variable. This information is used to alter the gap through implementing certain decision rules and actions. For instance, the production rate is continuously updated based on discrepancies between actual and desired levels of inventory. Vassian (1955) extends the study to enable analysis in discrete time domain, since data on inventories, production rates and orders are often available as series of observations made at different points in time, rather than as continuous functions in time.

Since then, more rigorous approaches have been used. Towill (1982) applies transfer function models to study an inventory control system, and proposes guidelines for tuning control parameters such as inventory adjustments and demand averaging time. Edgehill et al. (1988) study the sensitivity of an inventory system to parameter variations, and provide insights on how stocking policies affect the behavior of the inventory system. Ozveren and Sterman (1989) propose the use of linear controller design to synthesize policies for a problem of business cycles in economics that serve as guidelines to think about actual policies that are generally more complicated. Christensen and Brogan (1971) and Porter and Bradshaw (1974) apply a similar approach to use linear systems models to study specific production-inventory systems. Ortega and Lin (2004) and Sarimveisa et al. (2008) provide more recent and comprehensive reviews of applications of various control methods, such as block diagram algebra, Masons gain formula, Bode plots, Laplace transform, Z transform and optimal control for different production and inventory dynamic control problems.

Alternatively, a methodology termed as *System Dynamics*, originally published by Forrester (1961) as *Industrial Dynamics*, aims to increase the understanding of oscillatory behaviors in production-distribution systems. System Dynamics uses control-theory based models and computer simulation to study the behavior of complex dynamic systems (Sarimveisa et al., 2008). System Dynamics (SD) seeks to understand how and why the dynamics of concern are generated and then eventually searches for policies to further improve the system performance (Vlachos et al., 2007). Saleh et al. (2010) used the SD methodology to model the production-inventory system considered in this research. A more in depth discussion of this methodology is provided in Appendix A.

4.2 A Workforce-Inventory Control Application

4.2.1 Workforce-Inventory System Model

The workforce-inventory system model under study is illustrated in Figure 4.1. This model depicts a simple structure of how inventories in a manufacturing firm can be influenced dynamically by the workforce levels. In the following, the structure of the model is described, in which the production is triggered by the *Prod Start Rate* that releases new work orders into the process.

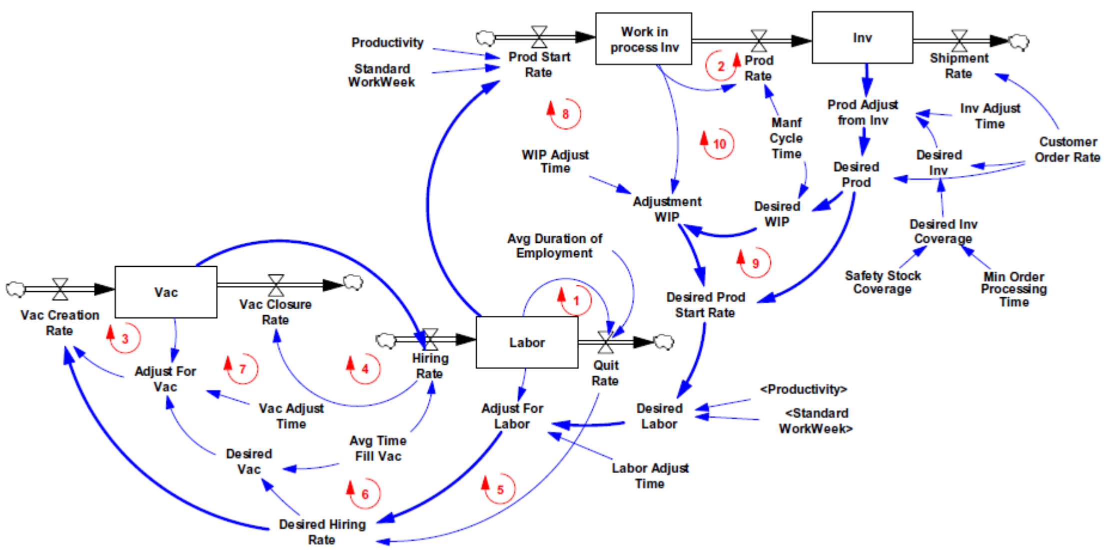

Fig. 4.1. Workforce-inventory planning system from Saleh et al. (2010)

Increasing the *Prod Start Rate* likewise increases the level of work-in-process (WIP) inventory. The WIP inventory is transformed into finished goods inventory (INV) at the production rate *Prod Rate*. Finished goods inventory is depleted by the *Shipment Rate* which is equivalent to the *Customer Order Rate*. The *Prod Start Rate* increases with the level of *Labor*, which models the level of manufacturing workforce resource in the firm. The *Labor* is increased through *Hiring Rate* and decreased by natural attrition through the *Quit Rate*. The *Hiring Rate* of labor in turn, increases when the level of job *Vacancies* increases. The job positions are depleted when hirings take place. In this model, the rate creation of jobs *Vac Creation Rate* is a feedback control policy that is determined based on current statuses of *Vacancies*, *Labor*, *INV* and *WIP*. More detailed formulation of the control rule will be presented in the next section.

The workforce-inventory model is also populated with many time constant parameters such as the *Manufacturing Cycle Time, Adjustment Times for Vacancies, WIP Adjustment Time, Average Duration of Employment* and *Time to Fill Vacancies*. Locally, these time constants determine the speed of response and flow rates of resources. For instance, a large *Manufacturing Cycle Time* implies a lower *Production Rate*, and a longer *Inv Adjust Time* implies a slower action to cover the gap between the *Desired Inv* and the actual finished goods inventory *INV*. Globally, these parameters together determine the dynamic (transient) behavior of the system. Given the values of these model parameters, it is possible to analyze and even design the dynamic behavior of the system accurately according to desired specifications.

However, many of these parameters might not be confidently estimated during input modeling. For instance, a highly variable *manufacturing cycle time* is typical of most realistic manufacturing environments, and similarly for parameters such as *duration of employment* in human resource processes. Hence, a workforce-inventory control rule that is good in a specific setting of the model parameters might not achieve desired performance (even worse, the dynamic system may become unstable) if the actual parameter values differ from the nominal assumed values. Therefore, the objective is to achieve a workforce-inventory control that is able to shape the dynamic response of the system satisfactorily under these parameter uncertainties. In the following, the state-space formulation of the workforce-inventory system described above is first presented. The results from eigenvalue sensitivity analysis are then used to develop the dynamic response requirements as linear constraints in the model parameters.

4.2.2 State Space Model of Workforce-Inventory Dynamic System

The mathematical model of the dynamic system depicted in Figure 4.1 is developed in this section. The following set of notations is defined for the key model parameters. Other notations will be introduced during the discussion as and when appropriate.

Linear and time-invariant state-space representations of dynamic systems are widely accepted as useful means of studying perturbations of the system

INV:	*Inventory*	WIP:	*Work-in-process*
LAB:	*Labor*	VAC:	*Vacancy*
COR:	*Customer Order*	ATF:	*Ave. time to fill vacancy*
PDY:	*Productivity*	DHR:	*Desired hiring rate*
SWW:	*Standard work week*	AFV:	*Adjustment for vacancy*
ADE:	*Ave. duration of employment*	MCT:	*Manufacturing cycle time*
IAT:	*Inventory Adjustment Time*	WAT:	*WIP Adjustment Time*
VAT:	*Vacancy Adjustment Time*	LAT:	*Labour Adjustment Time*

state variable from the nominal values (Pagola et al., 1989). As a result, the workforce-inventory dynamic system model in Figure 4.1 can be written as the following set of ordinary differential equations that describe the net flow rates for each of the critical resources $Inventory$, WIP, *Labor* and $Vacancy$:

$$\dot{INV} = \frac{WIP}{MCT} - COR \tag{4.2}$$

$$\dot{WIP} = LAB \cdot PDY \cdot SWW - \frac{WIP}{MCT} \tag{4.3}$$

$$\dot{LAB} = \frac{VAC}{ATF} - \frac{LAB}{ADE} \tag{4.4}$$

$$\dot{VAC} = AFV + DHR - \frac{VAC}{ATF} \tag{4.5}$$

In the rate of change equation for job vacancies VAC, the inflow rate *Vac Creation Rate* is defined by the sum of the *Desired Hiring Rate* DHR and *Adjustment for Vacancies* AFV, where DHR and AFV are defined as follows:

$$DHR = Adjust\ for\ Labor + Quit\ Rate \tag{4.6}$$

$$AFV = (Desired\ Vac - Vacancies)/VAT \tag{4.7}$$

The desired level of job vacancies is defined as $\textit{Desired Vac} = DHR \cdot ATF$. The above formulation is based on Little's Law equation, which says that at steady state, the achieved level of vacancies should be the product of the desired outflow rate at DHR and the cycle time ATF. In line with this, the creation of new job positions is based on two agenda. The first is to close the discrepancy between the desired and actual levels of job vacancies. The second is to create new jobs to support the planned hiring rate DHR, which in turn is driven by *Adjust for Labor* and for replacements to match the *Quit Rate*. The *Adjust for Labor* variable is formulated as:

$$\textit{Adjust for Labor} = (\textit{Desired Labor} - \textit{Labor})\,/LAT \tag{4.8}$$

$$\textit{Desired Labor} = \textit{Desired Prod Start Rate} \cdot \textit{Productivity} \cdot \textit{Standard WorkWeek} \tag{4.9}$$

That is, the *Adjust for Labor* accounts for the change in hiring rate required due to any gap between the desired labor level and the current labor level. A smaller *Labor Adjustment Time* LAT implies a faster adjustment to close this gap. Meanwhile, the *Desired Labor* is determined by the required production start rate. This is simply a conversion of the *Desired Prod Start Rate* into labor requirements based on productivity and work week considerations.

In the following equations, the *Desired Prod Start Rate* is determined by two requirements: *Adjustment WIP* and *Desired Prod*. It can be seen that the first requirement *Adjustment WIP* is driven by the discrepancy between the *Desired WIP* and actual *WIP* level. Here, WAT is defined as the *WIP Adjustment Time*, which controls the speed of closing the discrepancies. The *Desired WIP* is determined using Little's Law equation for steady state conditions on the stock of *WIP*, assuming that the outflow rate of *WIP* is at *Desired Prod.*

$$Desired\ Prod\ Start\ Rate = Adjustment\ WIP + Desired\ Prod \tag{4.10}$$

$$Adjustment\ WIP = (Desired\ WIP - WIP)\,/WAT \tag{4.11}$$

$$Desired\ WIP = Desired\ Prod \cdot MCT \tag{4.12}$$

The second requirement *Desired Prod*, is derived from considering the finished goods inventory and customer order fulfillments via the following:

$$Desired\ Prod = Prod\ Adjust\ from\ Inv + Customer\ Order\ Rate \tag{4.13}$$

where

$$Prod\ Adjust\ from\ Inv = (Desired\ Inv - INV)\,/IAT \tag{4.14}$$

Prod Adjust from Inv is the adjustments to the required production rate due to discrepancies between the *Desired Inv* and actual INV level. IAT is the *Inventory Adjustment Time*, which controls the rate which the discrepancies are closed, and the *Desired Inv* level is exogenously determined by safety stock coverage considerations.

4.2.3 Dynamics of Linear Systems under Parameter Variations

The above defines the structure of the control rule used in the model to adjust *Vac Creation Rate*, and hence *Hiring Rate* and *Prod Start Rate* based on current statuses of the *Vacancies*, *Labor*, *WIP* and *Inv*. The control rule is linear in these system states (the four critical resources), and after further manipulation, the system of equations can be written in the state matrix form of:

$$\dot{\mathbf{x}} = D\mathbf{x}$$

where $\mathbf{x}$ refer to the vector of resource variables $Inventory$, $Labor$, $Vacancy$ and WIP, and the state matrix D is given by:

$$\begin{bmatrix} 0 & 0 & 0 & \frac{1}{MCT} \\ 0 & \frac{-1}{ADE} & \frac{1}{ATF} & o \\ -\frac{\frac{ATF(WAT+MCT)}{LATPDYSWWVATWAT}+\frac{WAT+MCT}{LATPDYSWWWAT}}{IAT} & -\frac{1+\frac{ATF}{VAT}}{LAT}+\frac{1+\frac{ATF}{VAT}}{ADE} & \frac{-1}{ATF}-\frac{1}{VAT} & -\frac{\frac{ATF}{LATPDYSWWVAT}+\frac{1}{LATPDYSWW}}{WAT} \\ 0 & PDY \cdot SWW & 0 & \frac{-1}{MCT} \end{bmatrix}$$

In linear dynamic systems, the stability and transient behavior of the system is fully characterized by the eigenvalues of the state matrix D. An eigenvalue is written in the complex number format: μ+iω, where μ and ω refer to the real and imaginary parts of the eigenvalue respectively. The magnitude of

μ dictates the rate of decay of the transients in the system, and ω determines the frequency of oscillations in the system. A system is stable if the the magnitude of the system variables does not grow indefinitely large when inputs or disturbances are not indefinitely large. In order for a system to be stable, it is necessary and sufficient that the real part of all the eigenvalues be negative (i.e. $\mu < 0$).

In classical control engineering, pole placement techniques are applied to design linear controllers to achieve desired system performance, based on choosing the desired eigenvalue positions of the system. On the other hand, if the elements of D contains functions of uncertain variables, the resulting eigenvalues can also become uncertain. Hence, a system originally designed based on a single nominal estimate of D may be unable to achieve desired performance when implemented, or may even become unstable (i.e., when real components of the eigenvalues become positive). The technique discussed by Dong et al. (2005) is applied to analyze the changes in system eigenvalues due to variations in the model parameters. First, the i^{th} left and right eigenvectors $\boldsymbol{w_i}$ and $\boldsymbol{v_i}$ of state matrix D are defined as follows:

$$D\boldsymbol{v_i} = \lambda_i \boldsymbol{v_i}, \tag{4.15}$$

$$\boldsymbol{w_i}^T D = \lambda_i \boldsymbol{w_i}^T. \tag{4.16}$$

Let z be a parameter in the system, of which the state matrix D is a function of. The *eigenvalue sensitivity* $\frac{\partial \lambda_i}{\partial z}$ (Dong et al., 2005) is then the information on the influence of z on the i^{th} eigenvalue defined as:

$$\frac{\partial \lambda_i}{\partial z} = \frac{\boldsymbol{w_i}^T \frac{\partial D}{\partial z} \boldsymbol{v_i}}{\boldsymbol{w_i}^T \cdot \boldsymbol{v_i}} \tag{4.17}$$

Recall from Chapter 3 that the system parameters can be partitioned into a set of controllable parameters (decision variables) $\boldsymbol{y}$ and uncontrollable parameters (uncertain paramters) $\tilde{\boldsymbol{z}}$. Define $S_\mu^{\boldsymbol{y}}$ and $S_\omega^{\boldsymbol{y}}$ as the $N \times L$ (N=number of eigenvalues) matrix such that the il^{th} element of $S_\mu^{\boldsymbol{y}}$ ($S_\omega^{\boldsymbol{y}}$) is the sensitivity of the real (imaginary) components of eigenvalue i with respect to y_l. Similarly, define $S_\mu^{\tilde{\boldsymbol{z}}}$ and $S_\omega^{\tilde{\boldsymbol{z}}}$ as the $N \times K$ (N=number of eigenvalues) matrix such that the ik^{th} element of $S_\mu^{\tilde{\boldsymbol{z}}}$ ($S_\omega^{\tilde{\boldsymbol{z}}}$) is the sensitivity of the real (imaginary) components of eigenvalue i with respect to $\tilde{z}_k$.

In the succeeding discussions, $\boldsymbol{y}$ and $\boldsymbol{z}$ are defined to be changes of the decision and uncertain parameters with respect to their original values, respectively. Hence, for a given operating point where the eigenvalues are denoted as $\bar{\boldsymbol{\mu}}$ and $\bar{\boldsymbol{\omega}}$, and given some small parameter changes $\boldsymbol{y}$ and $\boldsymbol{z}$, the updated eigenvalues (real and imaginary components) are respectively estimated to be

$$\boldsymbol{\mu}(\boldsymbol{y}, \boldsymbol{z}) = \bar{\boldsymbol{\mu}} + S_\mu^{\boldsymbol{y}} \cdot \boldsymbol{y} + S_\mu^{\tilde{\boldsymbol{z}}} \cdot \boldsymbol{z} \tag{4.18}$$

and

$$\boldsymbol{\omega}\left(\boldsymbol{y}, \boldsymbol{z}\right) = \bar{\boldsymbol{\omega}} + S_{\omega}^{\boldsymbol{y}} \cdot \boldsymbol{y} + S_{\omega}^{\tilde{\boldsymbol{z}}} \cdot \boldsymbol{z}. \tag{4.19}$$

4.2.4 Specification of Dynamic Requirements

Stability requirements

In considering dynamic requirements, the key objective is to design a system that has acceptable transient performance. What is 'acceptable' may be rather subjective, but a fundamental requirement is that the system must be stable in the event of disturbances. That is, a system that faces disturbances should be able to return to its normal operating conditions after a finite amount of time. In the workforce-inventory management system, stability is an overriding consideration since it has direct implications on the ability of the management to control and mitigate large changes in the inventory and workforce levels in the event of disruptions and to return to normal operating conditions. These issues are of critical importance to stake holders since they have direct impact on the costs associated with inventory and workforce.

In the presence of uncertainty, it needs to be ensured that the real components μ of the eigenvalues will remain non-negative to ensure that the system remains stable. The system stability requirements can be characterized through the eigenvalue positions while the influence of changes in parameters can be estimated through the eigenvalue sensitivity matrices. For instance, for a given vector of *parameter changes* $\boldsymbol{y}$ and $\boldsymbol{z}$ about the nominal values, the real parts of the eigenvalues ($\boldsymbol{\mu}$) must be less than zero to ensure stability. This requirement is simply stated as:

$$\boldsymbol{\mu}\left(\boldsymbol{y}, \boldsymbol{z}\right) \leq 0 \qquad (4.20)$$

where $\boldsymbol{\mu}\left(\boldsymbol{y}, \boldsymbol{z}\right)$ is as defined in (4.18).

Transient Response Requirements

Aside from stability, there are often other performance requirements that describe how the dynamic system variables respond to changes during operation. For instance, when there is a sudden supply loss in the finished goods inventory, a stable system will adjust production starts, and hence workforce hiring as a response to restore the system to its normal operation, so that the finished goods inventory eventually again achieves its desired level. A fast reaction to the supply loss, while desirable, can cause an unnecessarily large surge in the workforce, large oscillatory swings in the inventory levels due to over-compensation and under-compensation, and a significantly long time before the desired levels are reasonably restored. Too slow reactions on the other hand can be sluggish and unresponsive to required changes in desired inventory levels or customer orders. The design of transient responses is to achieve a good compromise between some of these characteristics qualitatively.

Because it is impractical to predict accurately all types of changes that will occur in the system in advance, the common approach in control engineering is to shape the transient responses based on standard reference input changes, such as a unit step input change to the system. Figure 4.2 illustrates the response of a system output when a unit step input occurs.

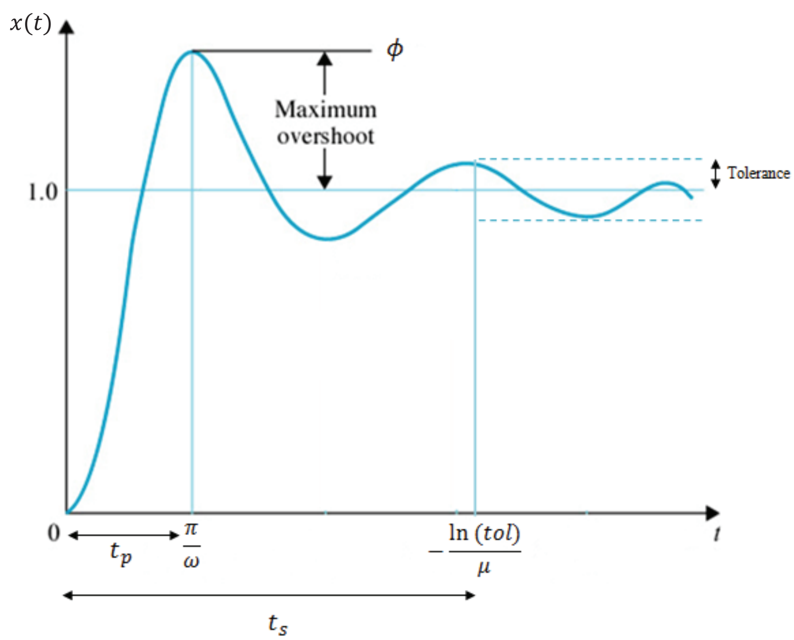

Fig. 4.2. Dynamic responses of a system output with respect to a unit step increase Kuo et al. (2009)

The system output attempts to follow the reference input and achieves the steady-state after a certain time. In the illustration, ϕ denotes the maximum overshoot of the output above the desired level, and the time at which this occurs is termed as the *peak time* t_p. The settling time (t_s) refers to the time taken for the system output to remain bounded within a specified tolerance (tol), which is usually 2% - 5% of the final value of the output.

For second-order systems, simple relationships between the eigenvalues and the dynamic performance measures are well-established. In particular, the peak time $t_p = \frac{\pi}{\omega}$, and the overshoot ratio $\phi = e^{\frac{\mu\pi}{\omega}}$ for a second-order system. A common 'rule-of-thumb' formula for settling time is $t_s = \frac{\Delta}{-\mu}$, where $\Delta = ln(tol)$ and is some pre-specified scalar (e.g., $\Delta = -4$ for a 2% tolerance). For defining the dynamic specifications, the system could then be required to have a *peak time* no greater than t_p, formulated as:

$$\omega\left(\boldsymbol{y}, \boldsymbol{z}\right) \geq \frac{\pi}{t_p} \tag{4.21}$$

The peak time is a surrogate for the speed of response of the system. A short peak time indicates that the system is agile in responding to changes in customer order rates or desired workforce levels. On the other hand, the response frequency ω is related to the bandwidth of the system and should not be too large, otherwise high frequency noise can easily corrupt the system behavior. This can be enforced simply as follows, where $\hat{\omega}$ refers to some upper bound on the frequency allowed in the response:

$$\omega\left(\boldsymbol{y}, \boldsymbol{z}\right) \leq \hat{\omega} \tag{4.22}$$

Similarly, the requirement that settling time should be no greater than t_s is stated by (4.23) while the overshoot requirement is stated by (4.24):

$$\mu(\boldsymbol{y}, \boldsymbol{z}) \leq -\frac{\Delta}{\tau_{t_s}} \tag{4.23}$$

$$\mu(\boldsymbol{y}, \boldsymbol{z}) \leq \frac{\ln(\phi)}{\pi} \cdot \omega(\boldsymbol{y}, \boldsymbol{z}) \tag{4.24}$$

In order to use the above formulations for a general high-order system, a common practice in control engineering design is to position all the eigenvalues other than the dominant eigenvalues to be reasonably far away from the imaginary axis in the complex number plane. This effectively means that all the transients associated with these eigenvalues decay away much faster compared to the dominant eigenvalues and hence can be neglected in the consideration of transient behavior. A simple way to achieve this is to require the magnitude of the real parts of the non-dominant eigenvalues to be at least five times as large as the dominant eigenvalues. This is enforced as a design requirement:

$$|\mu_{i'}(\boldsymbol{y}, \boldsymbol{z})| \geq 5 \cdot |\mu_i(\boldsymbol{y}, \boldsymbol{z})| \quad \forall i' \neq i \tag{4.25}$$

where i is the designated dominant eigenvalue, which can be identified based on the (pair) of eigenvalues nearest to the imaginary axis in the initial system setting.

Summarizing, since the above dynamic performance constraints (4.21), (4.22), (4.23), (4.24) and (4.25) are all linear (or can be cast as linear) in the parameters $\boldsymbol{y}$ and $\boldsymbol{z}$, they can be combined and cast compactly in the form:

$$A \cdot \boldsymbol{y} + B \cdot \boldsymbol{z} \leq \boldsymbol{\tau} \tag{4.26}$$

where $A \in \Re^{N \times L}$, and $B \in \Re^{N \times K}$ are coefficient matrices, $\boldsymbol{\tau} \in \Re^{N}$, and N is the total number of constraints generated from the specified dynamic performance requirements.

4.2.5 Optimization Approach for Workforce-Inventory Dynamics

In this section, the optimization model is developed to obtain a workforce-inventory control that is able to achieve the dynamic performance specifications derived under uncertainty. As mentioned, the goal is to obtain a setting of the controllable parameters $\boldsymbol{y}$ so that the dynamic performance constraints modelled in (4.26) are feasible for as large a range of uncontrollable parameters $\boldsymbol{z}$ as possible. In the workforce-inventory model, the controllable parameters are assumed to be the *Inventory Adjustment Time* IAT and *Labor Adjustment Time* LAT. Since these are time constant parameters associated with the workforce-inventory feedback control rule, different values of IAT and LAT results in different choices of feedback control implemented.

On the other hand, the uncontrollable parameters $\boldsymbol{z}$ are assumed to be the *Average Time to Fill Vacancy* ATF, *Average Duration of Employment* ADE, *Manufacturing Cycle Times* MCT, *WIP Adjustment Time* WAT, and *Vacancy Adjustment Time* VAT. These parameters can be thought of as variables that cannot be easily or economically manipulated in the practical environment. In practice, the discernment of controllable and uncontrollable parameters in a workforce-inventory system may require the inputs of policy-makers and various management. In this work it is assumed that such decisions have already been made.

It is clear that the system requirements as defined in the constraints (4.26), lead to the same problem structure shown for model $\mathcal{R}$. In this regard, the optimization model can be adapted to the workforce-inventory problem. Furthermore, the results of Proposition 1 can likewise be used to obtain an equivalent formulation of Problem $\mathcal{R}$, which can be solved iteratively by linear programming methods.

The search procedure for general design problems involving dynamic constraints is formalized through a local search framework. The framework, as described in Figure 4.3, shows how the maximum robustness level γ^* could be obtained by iteratively solving the proposed linear programming model.

1. Define a step size Δ and identify initial operating points $\bar{\boldsymbol{y}}$, $\bar{\boldsymbol{z}}$ and initialize the *best-so-far* robustness level γ^* to 0.

2. Formulate Problem $\mathcal{R}$ and transform into Problem $\mathcal{C}$.

3. From Problem $\mathcal{C}$, solve for $\boldsymbol{y}$.

4. Check to see if there is a feasible solution and an improvement in γ^*.

5. If there is, then $\gamma^* = \gamma^* + \Delta$ and $\bar{\boldsymbol{y}} = \bar{\boldsymbol{y}} + \boldsymbol{y}$. Continue the search for improvement by going back to Step 2.

6. Otherwise, the optimal design is reached and the search process terminates.

7. The design is then equivalent to $\bar{\boldsymbol{y}} + \boldsymbol{y}$ and γ^*.

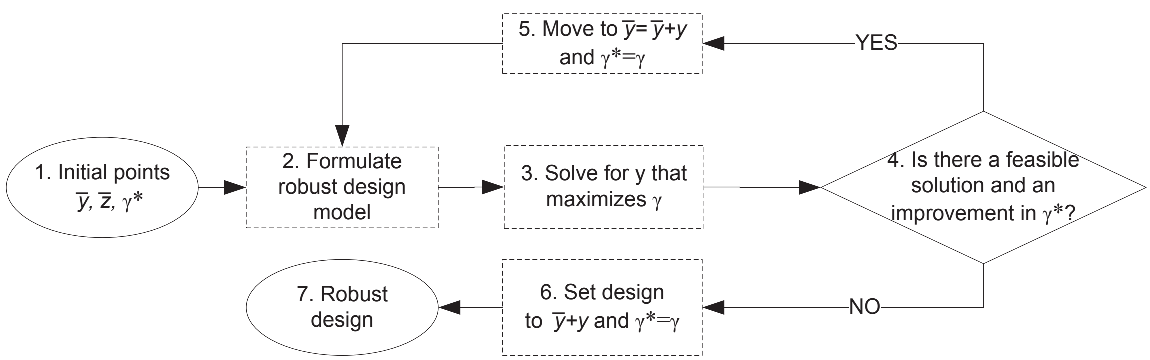

Fig. 4.3. Local search process for solving robust optimization model

In the algorithm, the initial operating points are defined by the nominal values of the control and uncertain parameters. These values can be obtained from the current design of the system. Formulating Problem $\mathcal{R}$ (and $\mathcal{C}$) requires that the local information matrices A and B to be identified and updated in each iteration. These are based on the linearized approximations or sensitivity information around the improved operating point. The step size used in the local search method may account for an additional constraint on the adjustable space of $\boldsymbol{y}$. These considerations can be captured by the feasible set $\mathcal{Y}$ in the formulation $\mathcal{R}$. Such approaches are also similar to design methods such as response surface methods in robust experimental design.

4.2.6 Computational Studies

The computational studies of the workforce-inventory planning system are presented in this section. The first objective of the computational studies is to demonstrate the improvement of dynamic performance of the system under uncertainties using the proposed robust target-oriented optimization approach. Another objective is to compare the performance of the solution to other design alternatives. In line with this, the controllable parameters $\boldsymbol{y}$ are assigned to be the inventory and labor adjustment time parameters IAT and LAT. The uncertain parameters $\boldsymbol{z}$ are: ATF, ADE, WAT, VAT and MCT. The nominal (and initial) values for the parameters are obtained from Saleh et al. (2010). Furthermore, the assumed bounds (changes about the respective nominal values) are shown in Table 4.1 for these parameters.

For the model parameter setting based on Saleh et al. (2010), where $IAT = 12$ and $LAT = 19$, the four eigenvalues are:

Table 4.1
Controllable and uncertain parameters for inventory-workforce system

Controllable	Bounds	Uncertain	Feasible range
IAT	5 to 35 weeks	ATF	-1.80 - 1.80 weeks
LAT	5 to 35 weeks	ADE	-22.60 - 22.60 weeks
		WAT	-1.40 - 1.40 weeks
		VAT	-0.90 - 0.90 weeks
		MCT	-1.80 - 1.80 weeks

$$\lambda_1 = -0.3531 \quad \lambda_2 = -0.0095 + 0.0988\text{ii} \quad \lambda_3 = -0.0095 - 0.0988\text{ii} \quad \lambda_4 = -0.1380$$

The complex conjugate pairs λ_2 and λ_3 are regarded as the dominant eigenvalues because they have the largest real parts amongst the group. Since the real parts of the dominant eigenvalues are non-negative, the system is currently stable. However, because these values are very close to zero, the dampening of such oscillations would be very weak and it can take an extremely long time for the system to achieve stability. Furthermore, when uncertainties are present, the near-zero values of μ_2 and μ_3 can become problematic if an uncertain parameter value alters the eigenvalues to cross the imaginary number axis into the right-hand side of the complex number plane. For instance, it was observed that when the parameters take on the following values: $MCT = 10.5$, $VAT = 7$, $WAT = 8$, $ADE = 100$ and $ATFV = 8$, λ_2 and λ_3 become $0.0002 + 0.0901\text{ii}$ and $0.0002 - 0.0901\text{ii}$, respectively. This indicates the case of dynamic instability.

Dynamic performance specifications

In the following, appropriate dynamic requirements are specified to move the workforce-inventory system away from such undesirable situations to improve its robustness to uncertainties. These requirements are specified by:

- Stability (constraint (4.20)).
- Rise time (constraint (4.21)): t_p is set at 50 weeks.
- Settling time (constraint (4.23)): t_s is set at 190 weeks, with tolerance $\Delta = 4$; that is 2% of the final steady-state values.
- Overshoot limit (constraint (4.24)): ϕ is set at 100% of the nominal value.
- Eigenvalue dominance rule (constraint (4.25)): λ_2 and λ_3 are designated as the dominant eigenvalues.

The formulation of the above dynamic performance constraints for the workforce-inventory system requires the evaluation of the eigenvalue sensitivity matrices. Detailed workings of these can be found in Appendix C.

Results and discussion

The performance of three different solutions of the workforce-inventory control using simulation are compared against each other. The first, denoted as $\boldsymbol{y}^{(\mathrm{I})}$ corresponds to the initial settings based on Saleh et al. (2010), i.e., with $IAT = 12$, $LAT = 19$. The second design, denoted as $\boldsymbol{y}^{(\mathrm{II})}$, attempts to improve on the first by moving $IAT = 18$ and $LAT = 35$, so that the maximum overshoot constraint (4.24) and settling time constraint (4.23) are satisfied exactly with the uncertain parameters at the nominal values, i.e. assume that $\boldsymbol{z} = \mathbf{0}$. It can be verified that the rest of the dynamic requirement constraints are also satisfied. Finally, $\boldsymbol{y}^{(\mathrm{III})}$ denotes the solution obtained using the robust target-oriented optimization approach.

To apply the approach, the optimization model $\mathcal{C}$ in Chapter 3 is formulated and solved iteratively using the search procedure. The computations were performed in MATLAB, with the application of the modeling toolbox ROME version 1.0.8 (Goh and Sim, 2011), designed for robust optimization problems in the MATLAB environment. In addition, the solver engine MOSEK version 6 was called to solve the underlying linear optimization problems. The computations were performed using a 2.50 GHz Intel Core i5-3210M processor, which resulted to an average computer solution time of 10.4 seconds for the algorithm. The optimization procedure was initialized with $\boldsymbol{y}$ set at $\boldsymbol{y}^{(\mathrm{I})}$. The final solution obtained was: $IAT = 20$ weeks and $LAT = 32$ weeks respectively, yielding a robustness index $\gamma = 62.09\%$. Under this design, the workforce-inventory system will be able to achieve the dynamic requirements as long as the uncertain parameters vary by no larger than 62.09% from their nominal values.

In the simulation, 200 realizations of the uncertain parameters were generated by assuming a uniform probability distribution with supports as in Table 4.1. Table 4.2 shows the 95% intervals of the eigenvalues evaluated under the three solution alternatives.

Table 4.2
95% Intervals for simulated eigenvalues

Solution	$\mathrm{Re}(\lambda_2, \lambda_3)$	$\mathrm{Imag}(\lambda_2, \lambda_3)$	λ_1	λ_4
$\boldsymbol{y}^{(\mathrm{I})}$	$[-0.0092, -0.0086]$	$[0.0980, 0.1036]$	$[-0.3634, -0.3572]$	$[-0.1396, -0.1362]$
$\boldsymbol{y}^{(\mathrm{II})}$	$[-0.0135, -0.0134]$	$[0.0650, 0.0679]$	$[-0.3803, -0.3749]$	$[-0.1118, -0.1084]$
$\boldsymbol{y}^{(\mathrm{III})}$	$[-0.0176, -0.0170]$	$[0.0660, 0.0692]$	$[-0.3777, -0.3723]$	$[-0.1071, -0.1032]$

Under $\boldsymbol{y}^{(\mathrm{I})}$, the real parts of λ_2 and λ_3 can become very close to zero (i.e., -0.0086). This implies further perturbations in the parameters may potentially de-stabilize the system. The stability is improved with $\boldsymbol{y}^{(\mathrm{II})}$, since the dominant eigenvalues are more negative than in the previous case. Finally, for $\boldsymbol{y}^{(\mathrm{III})}$, it can be noted that the upper confidence limit for the eigenvalues of λ_2 and λ_3 is -0.0170, which is more negative than the lower confidence limit for $\boldsymbol{y}^{(\mathrm{II})}$. This indicates an even further improvement in the stability of the solution under uncertainty. Qualitatively, this also implies a more effective tracking ability of the workforce-inventory system, since transients such as oscillations decay faster with smaller dominant eigenvalues.

Table 4.3 shows the dynamic performance in the overshoot, peak time and settling time behaviors from the simulation. The confidence intervals for each of the attributes are evaluated with the transient response constraints, using the 200 simulated realizations of the dominant eigenvalues (μ_2 and ω_2). All three solutions had comparable performance in terms of the maximum overshoot ratio ϕ. Meanwhile, the range for the peak time t_p of $\boldsymbol{y}^{(\text{I})}$ is lower than both $\boldsymbol{y}^{(\text{II})}$ and $\boldsymbol{y}^{(\text{III})}$. This just reflects a larger frequency ω in the solution's dominant eigenvalues. More importantly, it can be observed that $\boldsymbol{y}^{(\text{III})}$ had the best performance in the settling time compared to the rest of the two solutions. $\boldsymbol{y}^{(\text{I})}$ produced the worst (largest) settling times. Even though $\boldsymbol{y}^{(\text{II})}$ was designed based on achieving settling time specification under the nominal conditions, there is a significant likelihood that it exceeds the specification of 190 weeks when uncertainties are present. In contrast, only $\boldsymbol{y}^{(\text{III})}$ achieved the settling time specification with high confidence. This highlights the importance of proactively designing the workforce-inventory control parameters to counter the effects of uncertainties.

Table 4.3
Transient performance 95% intervals based on eigenvalue simulation

	Maximum Overshoot ϕ	Peak Time t_p	Settling Time t_s
Specification	100%	50 weeks	190 weeks
$\boldsymbol{y}^{(\text{I})}$	99.96%-99.97%	30.60-31.65	357.91-419.05
$\boldsymbol{y}^{(\text{II})}$	99.96%-99.97%	47.72-49.48	187.06-218.99
$\boldsymbol{y}^{(\text{III})}$	99.95%-99.96%	47.11-49.03	175.04-181.71

The above analysis focuses on the dynamic behavior of the workforce-inventory system under a unit-step reference input. Next, the workforce-inventory system is simulated under more general operational conditions. In particular, the resource variables are initialized at non-steady state conditions. As an example, the work-in-process has a desired level equivalent to 80,000 units but this has been initialized at 60,000 units. This for instance describes a scenario where there is a sudden supply loss in the production work; due to quality reasons or accidents. A stable system then reacts to the disruption by adjusting production and hiring rates and eventually restoring the system states to its steady operating levels. The transients observed in the process of recovery are of interest here.

Figures 4.4 to 4.7 show the behaviors under the control variable solutions of $\boldsymbol{y}^{(\mathrm{I})}$, $\boldsymbol{y}^{(\mathrm{II})}$ and $\boldsymbol{y}^{(\mathrm{III})}$ for the four resource variables: inventory, labor, WIP inventory and vacancy, respectively. Overall, the responses of all resource variables under the three different solutions display decaying oscillations before achieving steady-state levels, which is consistent with the results of the eigenvalue simulations in Table 4.2. To further compare the results, refer to the behavior of the inventory variable in Figure 4.4. The behaviors from $\boldsymbol{y}^{(\mathrm{I})}$ show that the fluctuations in the inventory levels are significantly larger than those from $\boldsymbol{y}^{(\mathrm{II})}$ and $\boldsymbol{y}^{(\mathrm{III})}$. These fluctuations hardly cease by the end of the simulation period, which indicate very weak dampening capability of the system. In contrast, the results for $\boldsymbol{y}^{(\mathrm{II})}$ and $\boldsymbol{y}^{(\mathrm{III})}$ indicate much improved damping, such that most of the transient movements in the stocks are eliminated by the end of the simulation period.

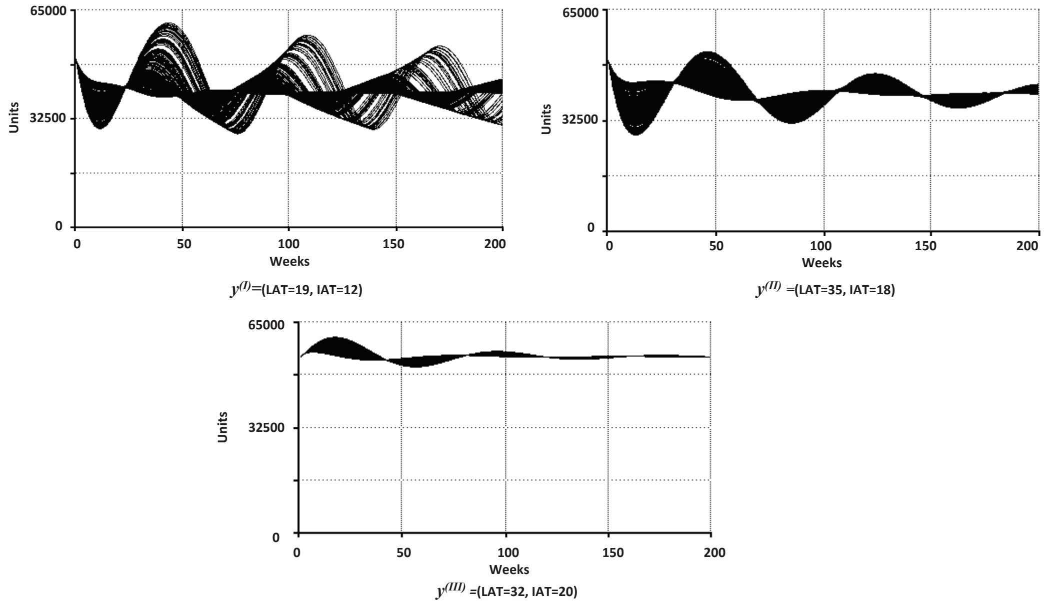

Fig. 4.4. Inventory dynamics under the three control variable solutions.

Furthermore, it can also be seen that the system under the $\boldsymbol{y}^{(\mathrm{II})}$ and $\boldsymbol{y}^{(\mathrm{III})}$ settings are less sensitive to the uncertain parameters. In particular, the trajectory-to-trajectory differences due to variation of the uncertain parameter values are much smaller compared to those from $\boldsymbol{y}^{(\mathrm{I})}$. Under $\boldsymbol{y}^{(\mathrm{I})}$, the inventory levels varied by as much as 14,000 units at the end of the simulation period. Meanwhile, a variation of 2000 and 500 units have been recorded for $\boldsymbol{y}^{(\mathrm{II})}$ and $\boldsymbol{y}^{(\mathrm{III})}$, respectively. From a management and planning point of view, this implies the ability to generate more precise forecasts of the resource statuses in the dynamic business environment. The same observations extend to the other resource variables in Figures 4.5 to 4.7.

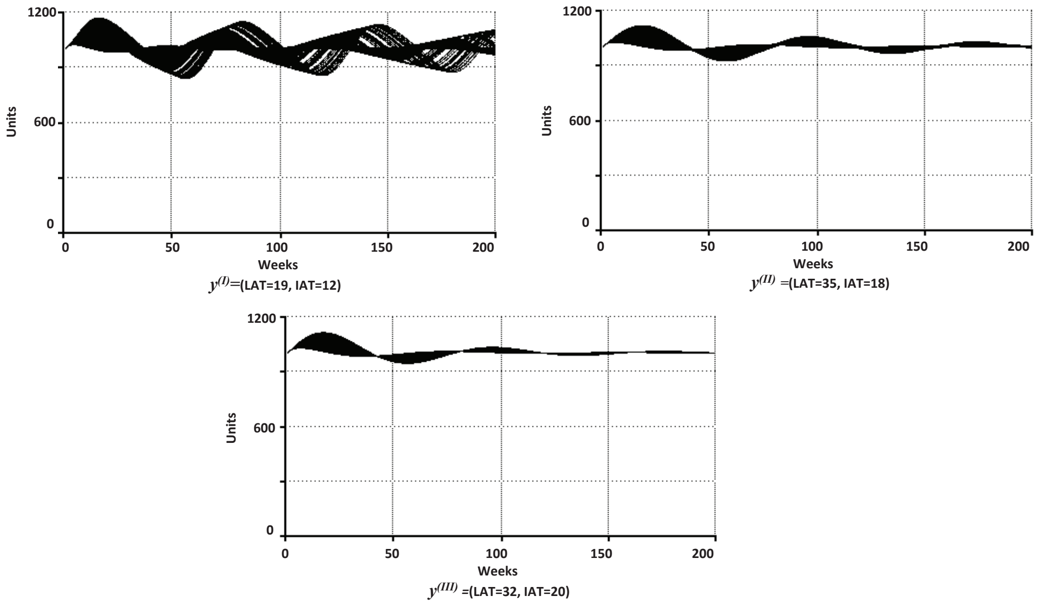

Fig. 4.5. Labor dynamics under the three control variable solutions.

An explanation for such behavior is as follows. In the case of $\boldsymbol{y}^{(\mathrm{I})}$, because of the smaller IAT and LAT values, any discrepancies between the desired and actual levels of the resource variables tend to trigger large compensating actions in the system. For instance, smaller IAT warrants a large *Prod Adjust from Inv* due to discrepancies between the *Desired Inv* and *Inv* levels. A large change in *Prod Adjust from Inv* results in large changes in *Desired Prod Start Rate* and *Desired Labor*, and consequently large adjustments in *Hiring Rate* and *Vacancies Creation Rate*. Due to the time delays in creating job positions (VAT), hiring (*Avg Time to Fill Vac*), and manufacturing (MCT), the finished goods inventory level does not respond immediately to the adjustment

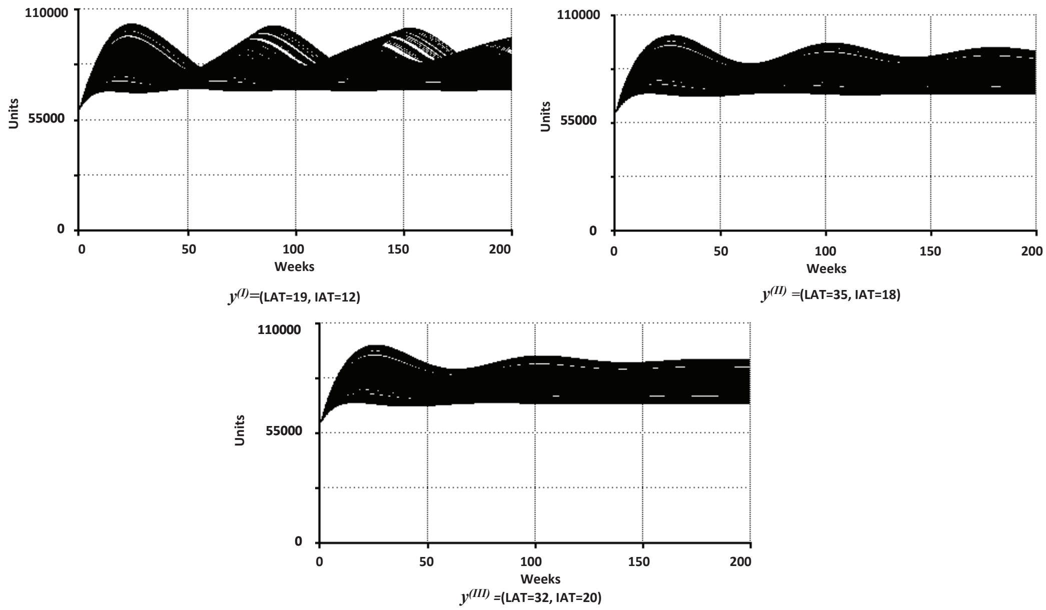

Fig. 4.6. WIP inventory dynamics under the three control variable solutions.

actions, and hence the discrepancy from the *Desired Inv* level is still observed. This causes further large adjustments in the *Desired Prod*.

Eventually, when the finished goods arrive from the manufacturing process, the *Inv* level overshoots the *Desired Inv* significantly, and the compensating action in the opposite direction takes place. These over-compensating and under-compensating actions cause the level of all the four resources to swing back and forth about their steady-state levels. As shown by Figure 4.7, the fluctuations are particularly severe for the *Vacancies* since the control rule for adjusting vacancies implicitly consider all the adjustments required from the other three resources. Obviously, these large fluctuations in organizations are

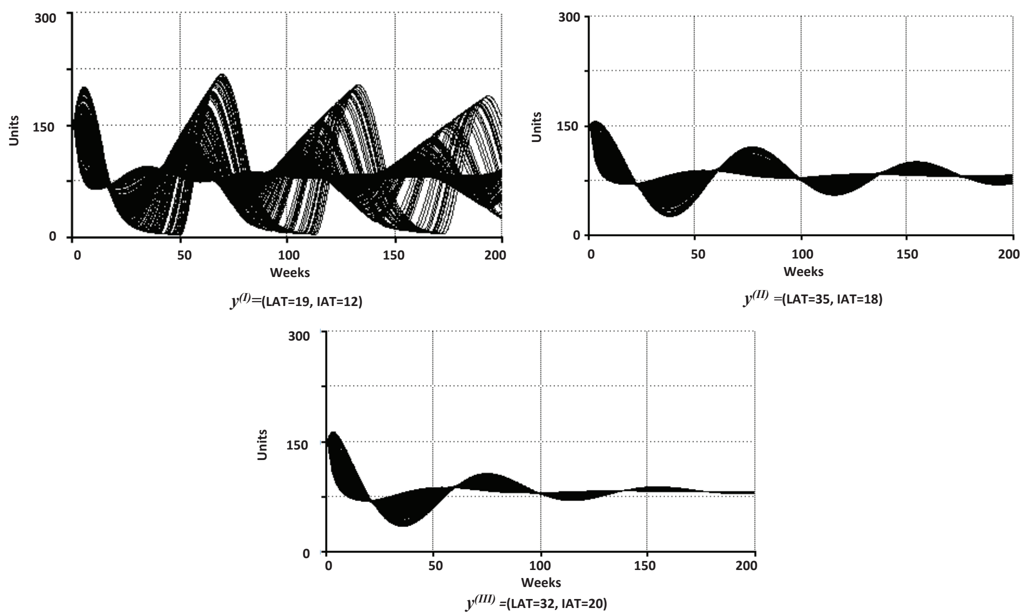

Fig. 4.7. Vacancy dynamics under the three control variable solutions.

undesirable and are viewed as inability of the management to cope and control organizational resources and processes effectively.

Longer adjustment times can help to desensitize such nervousness and potentially smooth out the fluctuations in the inventories and labor. On the other hand inventory and hiring adjustments that are too slow can also cause the firm to become very sluggish in responding to the volatile customer demands, and this can result in loss of revenue and even reputation in the long run. Improvements to the behavior produced by $\boldsymbol{y}^{(\mathrm{I})}$ are achieved in $\boldsymbol{y}^{(\mathrm{II})}$ and $\boldsymbol{y}^{(\mathrm{III})}$ by appropriately increasing the IAT and LAT. There is a significant reduction in the fluctuations of vacancy creation and closure rates, which lead to lowered changes in the workforce levels of the system. The decreased variability of the WIP and finished goods inventory implies smoother production releases labor force and hence a more stable level of labor.

Tables 4.4 – 4.7 summarize the performance of the three solutions in the settling time, maximum overshoot and peak time behaviors for the four resources. The tabulated results are obtained from the same output values in the Figures 4.4 – 4.7 (note that these are different from the results in Table 4.3, which were obtained using eigenvalue simulations).

Table 4.4
Transient performance for Labor

	Settling Time	Maximum % Overshoot	Peak Time
$\boldsymbol{y}^{(\mathrm{I})}$	241.4- 242.1	7.8-9.0	15.1-16.1
$\boldsymbol{y}^{(\mathrm{II})}$	233.4- 234.2	6.0-6.6	16.9- 17.9
$\boldsymbol{y}^{(\mathrm{III})}$	207.2-212.0	6.2-6.8	16.1-16.9

Table 4.5
Transient performance for Vacancy

	Settling Time	Maximum % Overshoot	Peak Time
$\boldsymbol{y}^{(\mathrm{I})}$	219.9-229.9	103.3-132.1	12.9-20.1
$\boldsymbol{y}^{(\mathrm{II})}$	169.9-182.3	92.3-93.7	1.20-1.28
$\boldsymbol{y}^{(\mathrm{III})}$	142.3-153.7	90.3-91.2	1.4-1.6

For the maximum overshoot, it can be observed that both $\boldsymbol{y}^{(\mathrm{II})}$ and $\boldsymbol{y}^{(\mathrm{III})}$ outperform the results achieved by $\boldsymbol{y}^{(\mathrm{I})}$ significantly. In particular, all the overshoot upper confidence limits of $\boldsymbol{y}^{(\mathrm{II})}$ and $\boldsymbol{y}^{(\mathrm{III})}$ are no greater than the lower confidence limits of the maximum overshoot produced by $\boldsymbol{y}^{(\mathrm{I})}$. Similar

Table 4.6
Transient performance for Inventory

	Settling Time	Maximum % Overshoot	Peak Time
$\boldsymbol{y}^{(\mathrm{I})}$	213.1- 224.9	28.7-33.3	11.5-16.5
$\boldsymbol{y}^{(\mathrm{II})}$	216.8-223.2	23.9-24.1	5.8-10.2
$\boldsymbol{y}^{(\mathrm{III})}$	196-197	23.9-24.1	2.5-5.5

Table 4.7
Transient performance for WIP Inventory

	Settling Time	Maximum % Overshoot	Peak Time
$\boldsymbol{y}^{(\mathrm{I})}$	187.8-202.2	5.5-6.5	31.1-34.9
$\boldsymbol{y}^{(\mathrm{II})}$	141.6-148.4	3.8-4.2	29.1-30.1
$\boldsymbol{y}^{(\mathrm{III})}$	102.1-107.9	3.8-4.2	27.6-28.4

results are also observed for the settling time performance. Finally, the results suggest that the performance of $\boldsymbol{y}^{(\mathrm{II})}$ and $\boldsymbol{y}^{(\mathrm{III})}$ are marginally close for the maximum overshoot behavior, with no more than 2% difference in the results. On the other hand, it can be observed that $\boldsymbol{y}^{(\mathrm{III})}$ achieves significantly shorter settling times than $\boldsymbol{y}^{(\mathrm{II})}$ for all four resources. This is consistent with the results in Table 4.3.

An explanation for the behavior is attributed to longer inventory adjustment time IAT, and shorter labor adjustment time LAT in the case of $\boldsymbol{y}^{(\mathrm{III})}$ compared to $\boldsymbol{y}^{(\mathrm{II})}$. Since in the workforce-inventory control rule, adjustments in finished goods inventory are added towards the adjustments in desired production release rates and consequently desired labor force and hiring rates, a longer IAT tends to decrease the sizes of the adjustment signals that propagate upwards into the labor planning. This serves to reduce the nervousness effects in the system. On the other hand, a shorter LAT increases the labor hiring rate and hence reduce the pipeline delay effects observed downstream at the finished goods inventory. This reduces the tendency for over-ordering to take place at the finished goods inventory. The combined result is an improved damping of the system transients.

4.3 Power Supply System Stability

The one machine infinite bus (OMIB) power supply system in this study is adapted from Yu and Siggers (1971). It involves a 1190 MVA unit connected to an infinite bus through a 575 mile transmission line illustrated in Figure 4.8. Table 4.8 presents the notations used in the model.

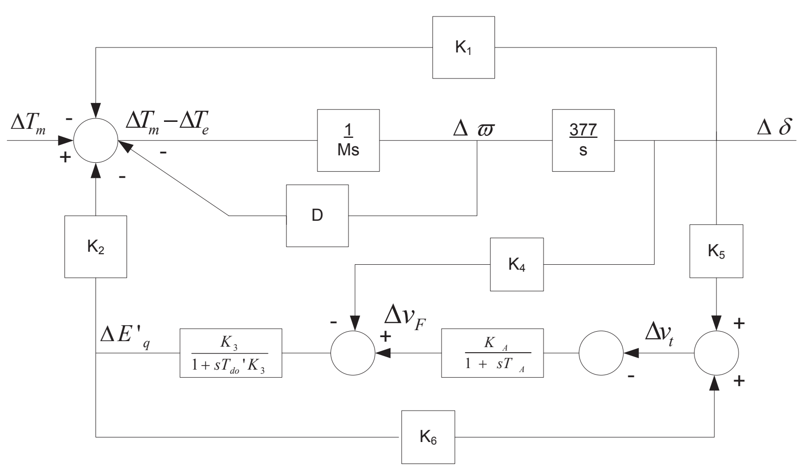

Fig. 4.8. Oscillation model of the OMIB power system (Yu and Siggers (1971))

Table 4.8
Nomenclature used in the OMIB system model

Notation	System Parameter	Notation	State Variables
M	Inertia coefficient	T_M	Mechanical input
Da	Damping coefficient	ω	Angular velocity
s	Laplace operator	v_t	Terminal voltage
T_A	Excited amplifier time constant	v_F	Equivalent excitation voltage
K_A	Excited amplifier gain	T_e	Energy conversion torque
$K_1 \cdots K_6$	Small oscillation constants	δ	Torque angle
$T_{do'}$	D-axis transient open circuit time constant	$E_{q'}$	Q-axis component of voltage behind transient reactance

Oscillations occur when there are disturbances in the system such as mechanical load changes and system faults. In such oscillations, $\Delta\delta$, $\Delta\omega$ and $s\Delta\omega$ may be considered as phasors where $s\Delta\omega$ leads $\Delta\omega$ by 90^o and $\Delta\omega$ leads $\Delta\delta$ again by 90^o (Yu and Siggers, 1971). The disturbances influence the levels of critical variables such as angular velocity, terminal voltage, equivalent excitation voltage and energy torque. These variables are consequently defined as the state variables for the stability analysis in this study.

In practice, the oscillation constants used in the OMIB may drift throughout a range of operating conditions (Yu and Siggers, 1971). Thus, these parameters have associated uncertainties that need to be accounted for in system stability analysis. The state equations $\dot{\mathbf{x}} = \mathbf{D}\mathbf{x}$ are:

$$D = \begin{bmatrix} 0 & 0 & 0 & -1/M \\ 377 \cdot K_5 - \lambda_1\lambda_2 \cdot Da & \frac{-\lambda_2(\lambda_1 \cdot K_2+1)}{K_6} & \lambda_2 K_3 & \lambda_1\lambda_2 \\ 0 & -K_A/T_A & -1/T_A & 0 \\ 377 \cdot K_1 - \lambda_1\lambda_3 \cdot Da & \frac{-\lambda_3(\lambda_1 \cdot K_2+1)}{K_6} & K_2/T_{do} & \lambda_1\lambda_3 - Da/M \end{bmatrix}$$

$$\mathbf{x} = \begin{bmatrix} \Delta\omega & \Delta v_t & \Delta v_F & \Delta T_e \end{bmatrix}$$

As observed, the OMIB system has been directly translated into its state space representation. Hence, the same method used in formulating Problem $\mathcal{C}$ as discussed in the workforce-inventory system problem extends to the OMIB system. In line with this, the original values of the system parameters (from Yu and Siggers (1971)) and the corresponding system eigenvalues are shown in Tables 4.9 and 4.10, respectively. The system exhibits the dynamics of growing oscillations, which indicates instability. In this example, the control variables are the exciter amplifier gain K_A, the exciter amplifier time constant T_A and the dampening coefficient Da. The uncertain variables are the oscillation constants $K_1, \cdots, K_6$. Table 4.11 presents the uncertainty set assumed for these oscillation constants. The respective ranges indicate the values $K_1, \cdots, K_6$ may take on in the OMIB system.

Table 4.9
Initial parameter values for OMIB system

System Parameter	Initial Value	System Parameter	Initial Value
M	9.26	K_1	0.55
Da	0	K_2	1.16
$T_{do'}$	7.76	K_3	0.66
T_A	0.05	K_4	0.67
K_A	130	K_5	-0.09
		K_6	0.82

Table 4.10
Initial system eigenvalues of OMIB system

λ_1	λ_2	λ_3	λ_4
0.1856+5.1846i	0.1856-5.1846i	-10.2833+13.3465i	-10.2833-13.3465i

Table 4.11
Uncertainty set for OMIB system parameters

Uncertain Variables	Uncertainty set	Uncertain Variables	Uncertainty set
K_1	[-0.0272, 0.0272]	K_4	[-0.0332, 0.0332]
K_2	[-0.0574, 0.0574]	K_5	[-0.0045, 0.0045]
K_3	[-0.0327, 0.0327]	K_6	[-0.4059, 0.4059]

Aside from stability targets, there are also prescribed bounds on the control and state variables. In setting the upper and lower bounds of the control variables, Machowski et al. (1997) state that typical values for K_A and T_A are: 20-400 and 0.05-0.20, respectively. A limit of 10 is also assigned for the damping coefficient in line with Yu and Siggers (1971). Note that there are no transient requirements assigned for the power supply OMIB system.

4.3.1 Computational Results

With the defined system requirements, the robust design model $\mathcal{C}$ was formulated and solved iteratively using the search procedure. The model returned a γ of 0.8080, which implies that the system maintains stability even when the uncertain parameters vary up to 80.80% of the size of the uncertainty sets in Table 4.11.

A sampling-based approach is used to test the system performance under the proposed design. 100 realizations of the uncertain parameters were randomly generated by assuming a uniform probability distribution (using MATLAB's random number generator) with supports as in Table 4.11. We observe how the proposed design compares with the performance of the original design of the OMIB system. Figures 4.9 and 4.10 show the behavior of the four state variables under the original settings and the proposed settings, respectively. Table 4.12 shows the 95% confidence interval for the eigenvalues under the proposed design. These were generated with Matlab's edcf routine using 100 simulated samples of the random parameters.

The instability in the behavior of all four state variables is apparent from the results in Figure 4.9. The oscillatory amplitudes grew rapidly, in contrast to the generated behaviors under the proposed settings in Figure 4.10. The proposed settings eliminated the growing oscillations and maintained system stability even in the noisy environment. This thus verifies the improved robustness of the new system design. The observations from the two sets of settings are also consistent with the obtained confidence intervals of Table 4.12. For instance, the real components of λ_1 and λ_2 now range in the stable region of $[-0.4616, -0.4356]$, in contrast to the initial value of 0.1856. Likewise, all other eigenvalues now have confidence bands of real components in the negative region, which indicates the stability of the system.

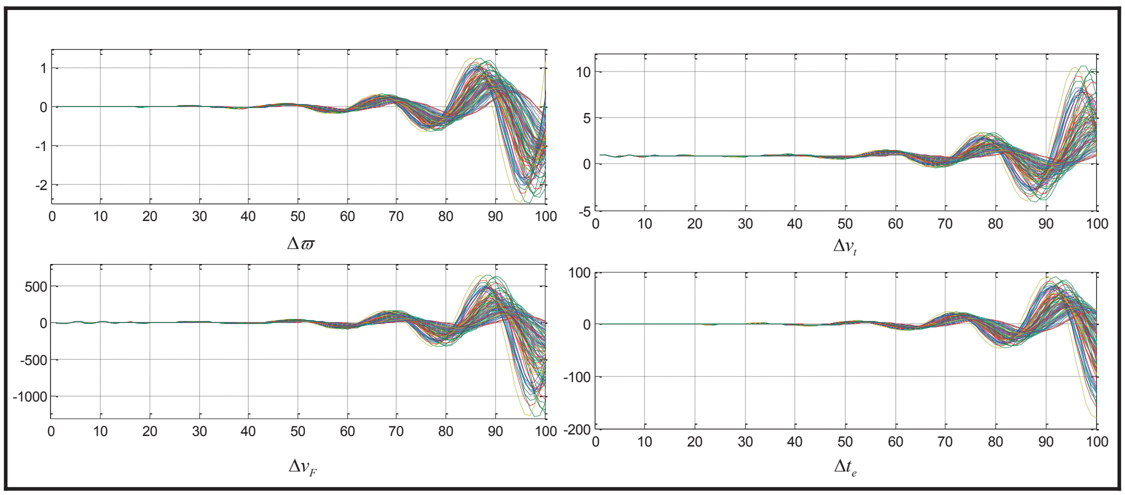

Fig. 4.9. OMIB system behavior under original design

Table 4.12
95% confidence interval for new eigenvalues

	Re(λ_1, λ_2)	Imag(λ_1, λ_2)	Re(λ_3, λ_4)	Imag(λ_3, λ_4)
C. I.	-0.4616 - -0.4356	4.4567 - 4.9369	-4.0062 - -3.8787	1.9166 - 2.3230

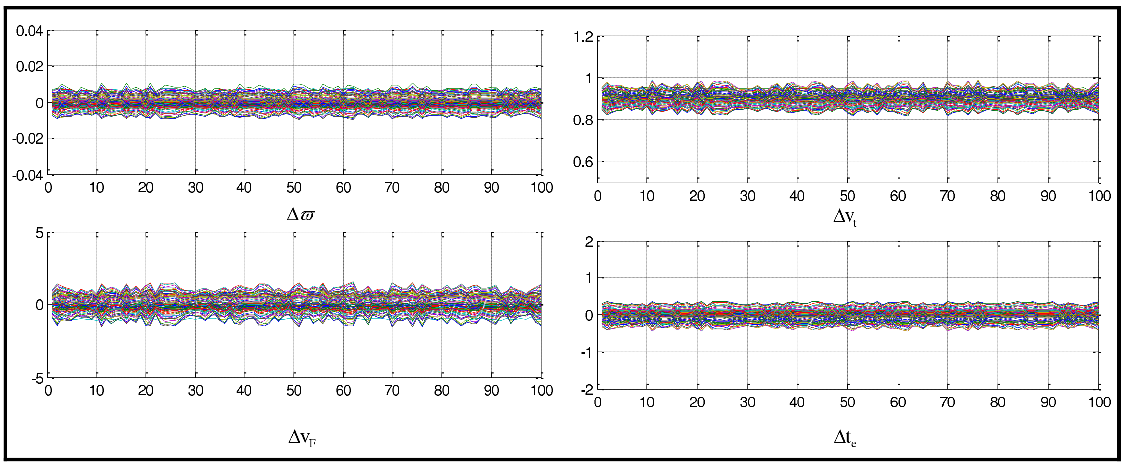

Fig. 4.10. OMIB system behavior under proposed design

5. OFFSHORE GAS FIELD DEVELOPMENT

Gas field development problems contain inherent uncertainties. These uncertainties can be classified according to two categories: exogenous and endogenous. These categories refer to information whose "time of revelation" is independent and dependent of the decisions, respectively (Vayanos et al., 2011). For instance, price and demand for gas are considered to be exogenous uncertainties since they are revealed in due course of time and are not affected by any of the decisions. Meanwhile, endogenous uncertainties are those that get revealed only after a particular decision has been made. This is the case with the gas reservoirs, wherein the actual quality remains largely uncertain until after exploration or capital investments had been undertaken (Goel et al., 2006).

From a modeling perspective, problems involving only exogenous uncertainties are easier to solve than those having endogenous uncertainties. In the case of the former, a solution that is feasible for one scenario remains to be feasible for the other scenarios (Jonsbraten, 1998). In contrast, the presence of endogenous uncertainties creates decision dependencies, which render standard stochastic programming models to be intractable.

Goel and Grossmann (2004) present a review of works that deals with uncertainty in offshore oil and gas exploration problems. Computational difficulties have led most of these works however to focus on exogenous uncertainties or assume very simplified settings. For instance, Jornsten (1992) only considers uncertainty on future demand and assumes that either the investment or operation decisions have already been fixed. Jonsbraten (1998) also restricts the discussion to uncertainty on future oil prices. On the other hand, Haugen (1996) defines a single parameter to represent the uncertainty in the quality of gas reservoirs but narrows down the discussion to scheduling decisions for the gas fields.

To address this gap, Goel and Grossmann (2004) and Goel et al. (2006) consider a gas field development problem where both facility investment and production decisions need to be made in a multi-period setting with gas reservoirs uncertainties. In the aforementioned, the authors assume a small set of scenarios with associated probabilities to model the outcomes of the gas reservoirs' uncertainties. The uncertainties were defined to arise from two parameters involving the size and deliverability of the reserves.

Problems on offshore gas field development are often modelled as stochastic multi-stage mixed-integer programs. However, it is often difficult to obtain exact solutions for these types of problems. This is specially the case for large-scale real-world settings. Tomasgard et al. (2007) analyze a production and distribution network through a two stage stochastic model, representing the uncertainties as scenario trees. The authors had to either decrease the scale of the model or limit the number of scenarios considered in order to come up with an acceptable solution that is not even necessarily optimal. As mentioned, the pervading issue of computational intractability has led the literature on offshore field development to rely on the use of approximation techniques to reduce complexity. Some examples of these techniques include decomposition (Goel and Grossmann, 2004) and branch and bound (Goel et al., 2006) algorithms.

Vayanos et al. (2011) also adapt the problem setting considered by Goel and Grossmann (2004). The authors introduce an approximation scheme based on a set of decision rules, which are commonly used in adjustable robust optimization. Decision rule techniques have often been used to overcome intractability for problems involving exogenous uncertainty. In order to integrate the endogenous uncertainties in gas field development planning, they define the binary and continuous components of the problem to be representable by piecewise constant and linear functions of the uncertain parameters, respectively. The resulting approximate problems were found to be equivalent to mixed-binary linear programs, which can be solved using standard optimization software.

In comparison, through the robust target-oriented approach, this research seeks to solve the gas field development problem under endogenous uncertainties exactly. As mentioned, there is also no explicit use of probability distributions for the uncertain parameters. This is achieved through the use of a fixed policy that is mapped to only a single value of the uncertain initial deliverability and size. Subsequently, the problem is shown to be solved exactly through a deterministic mixed integer linear optimization model.

5.1 The Business of Offshore Gas Field Development

There are three main types of physical infrastructure in an offshore gas field: well platforms, production platforms, and pipeline connections that link these platforms. A schematic of these infrastructure is given in Figure 5.1.

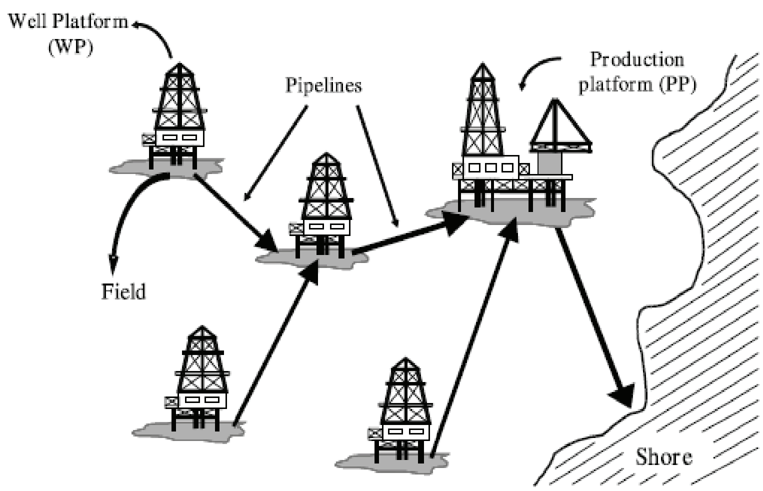

Fig. 5.1. Infrastructure in offshore gas field development (Goel et al., 2006)

An offshore gas field has multiple gas reservoirs or reserves. For each reserve, a dedicated well platform can be installed for gas extraction. The rate of gas extraction is determined by the production capacity of the well platform, as well as the efficacy of the reserve. It is assumed that the capacities of well platforms are fixed on installation, and no expansion can be performed later on.

Gas extracted from well platforms must reach a production platform to undergo further processing and eventually be transported to shore. When production platforms are installed, decision makers determine an initial production capacity, as well as some buffer capacity with relatively low cost, which can later be used for expansion. With this flexibility, decision makers can choose to build smaller production platforms in the early stages of development as a gauge of the gas reserve efficacy, and expand the platforms in the event of *better-than-expected* quantities of gas extracted with some incremental cost.

The transfer of gas from well to production platforms is facilitated through the pipelines connections. However, it is possible that some of the well platforms do not have a direct link to a production platform (see Figure 5.1 for instance) for economic reasons such as the distance from the well to the production platform. Hence, gas obtained from the well platforms may need to be routed to other well platforms before being brought to the production platforms. In practice, each well platform will have one outgoing pipeline that is connected to either another well platform or to a production platform. Since extracted gas needs to reach the production platform, it is clear that at least one well platform must be connected to each of the production platforms in the field.

5.1.1 Gas field development project: installation and operations

An offshore gas field development project consists of many installation and operational decisions to be implemented dynamically across the span of the planning horizon. The following describes the dynamics involved in the development of an offshore field and how these impact the overall profitability of the project.

The system understudy considers a gas field with W wells and P production platforms. Let $\mathcal{W} = \{1, \ldots, W\}$ and $\mathcal{P} = \{1, \ldots, P\}$ represent the sets of indices corresponding to the well and production platforms, respectively. The set of all well-well pipeline connections is denoted by $\mathcal{A} \subseteq \{(w_1, w_2) \;:\; w_1, w_2 \in \mathcal{W}, w_1 \neq w_2\}$. Similarly, the set of all well-production platform pipeline connections is denoted by $\mathcal{B} \subseteq \{(w, p) \;:\; w \in \mathcal{W}, p \in \mathcal{P}\}$. The length of the planning horizon is T and is divided into time periods indexed by $t \in \mathcal{T} = \{1, ..., T\}$.

The major installation decisions of the development project is to choose which infrastructure to build (well platforms, production platforms and pipeline connections), in which period to build them, and their respective capacities. Every well platform, production platform, well-well pipelines and well-production pipelines have specified maximum installable capacities denoted by h_w^W, h_p^P, h_a^A, h_b^B for $w \in \mathcal{W}$, $p \in \mathcal{P}$, $a \in \mathcal{A}$ and $b \in \mathcal{B}$, respectively. In the rest of the development, it is assumed that the capacities of all installed pipeline connections are always chosen at their maximum values h_a^A and h_b^B. The installation of these structures have the following fixed costs: f_w^W, f_p^P, f_a^A, f_b^B, and variable costs: v_w^W, v_p^P, v_a^A, v_b^B. For the platforms, the variable costs are associated with the size and throughput capabilities of extraction and production.

The option on whether a production platform should be equipped with future expansion capability is chosen during the installation phase of the platform. If a platform has expansion capability, then a buffer capacity $\bar{u}_p,\ p \in \mathcal{P}$ for future expansion also needs to be catered for. This incurs an initial capital investment during the installation phase. Furthermore, if decisions are made to actually use the buffer capacity for expansion some time in the future, another set of fixed and variable costs are incurred, where the variable costs are related to the size of the expansion implemented.

The operations of platforms are assumed to be instantaneous upon installation. Operations in the field begin with the extraction of gas in the well platforms, the amount of which will be less than both the capacity of the platforms and the deliverability of the reserves. The latter depends on the initial deliverability d_w and size s_w of the reserve w (to be discussed in more detail in the next section). As mentioned, the extracted gas is transferred in pipelines that can connect to other well platforms and finally reaching the production platforms. The pipelines connecting the installed platforms are uni-directional in flow (i.e., gas flows can only be from either one well platform to another well platform or to a production platform, and not vice versa). The total gas production quantity is then the sum of all flows from the well platforms connected to it.

The revenue r_t, $t \in \mathcal{T}$, is directly derived from the total production of gas in each period. In addition, there are also operational costs g_w^W for $w \in \mathcal{W}$ and g_p^P for $p \in \mathcal{P}$ that arise from the extraction and production activities in the field, respectively.

5.1.2 A model for well reserves

Reserves are estimated quantities of oil, natural gas and related substances to be recovered from known accumulations. These estimations are based on the analysis of drilling, geological, geophysical and engineering data. The methods of reserves estimation fall into three broad categories of volumetric methods, material balance and production decline methods. Material balance methods involve the analysis of pressure behavior as reservoir fluids are withdrawn. Pressure declines as gas is being extracted, and maximum gas extraction is reached when the well declines to its *abandonment pressure* (Hyne, 2001).

This work adopts the well reserve model used by Goel and Grossmann (2004) and Goel et al. (2006), where the efficacy of the reserves is characterized by the two important metrics known as the *initial deliverability* and *size* of the reserve. The initial deliverability is defined as the maximum rate of gas extraction achievable when the reserve becomes developed, while size is the maximum actual amount of gas recoverable. Figure 5.2 illustrates an approximate behavior of the well deliverability as a linear (decreasing) function of the accumulated extraction of gas. Note that the initial deliverability and well size uniquely define the linear reservoir behavior model of Figure 5.2. While more accurate reservoir behavior can be modelled using complex systems of partial differential equations in the literature, simplified algebraic models usually suffice for use in high-level planning problems (Kosmidis et al., 2004). Furthermore, Figure 5.2 is a widely-acknowledged approximation for gas reserves (see Hyne, 2001).

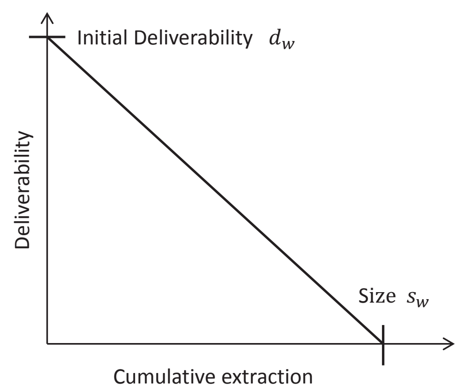

Fig. 5.2. Linear model of gas reserve efficacy

Addressing uncertainties in reserve estimates is an issue of central importance in development projects. Goel and Grossmann (2004) and Goel et al. (2006) use two stochastic parameters, namely initial deliverability and size, to model the uncertainty, and assign to each parameter a discrete probability distribution. However, in reality, the information of subsurface uncertainty is very limited before actual production, and hence it is hard to obtain a good estimation of the probability distribution. As defined in Chapter 3, this work avoids the assignment of probabilities to these estimates. Instead, it only specifies a range for the initial deliverability and size, which is relatively easier to obtain. Secondly, the resulting decision model of Goel and Grossmann (2004) and Goel et al. (2006)'s is highly intractable from an optimization perspective. To illustrate, the uncertainty model leads to a stochastic programming modeling approach, which by design is ill-equipped to handle the endogenous uncertainties in the problem.

The authors formulate the decision problem as a multi-stage stochastic programming model incorporating decision-dependence on the scenario tree of the problem. Assuming a small set of scenarios modeling the outcomes of uncertain parameters, the model is reformulated as a deterministic mixed binary programming problem. The resulting problem size is however extremely formidable due to the presence of the non-anticipativity requirements (Vayanos et al., 2011). While in theory, the problem can be formulated and solved in a stochastic dynamic programming framework, this does not resolve the issues arising from the curse of dimensionality associated with the problem size.

Let the initial deliverabilities and sizes for the W wells in the field be denoted as vectors of uncertain parameters $\tilde{\boldsymbol{d}} = (\tilde{d}_1, ..., \tilde{d}_W)$ and $\tilde{\boldsymbol{s}} = (\tilde{s}_1, ..., \tilde{s}_W)$, respectively. In the scope of this problem, having larger initial deliverabilities and sizes account to a larger extraction yield from the well platforms. Thus, in the most optimistic case, the deliverabilities and sizes of the wells would take on the maximum values, i.e., $\tilde{d}_w = \bar{d}_w$, $\tilde{s}_w = \bar{s}_w$ for all $w \in \mathcal{W}$, which consequently result in the highest profit achievable. However, in practice, such optimistic assumptions may lead to planning solutions that perform badly in practice due to the effects of uncertainties. Similarly, a robust optimization approach in which the uncertain parameters belong to an adjustable uncertainty set $\mathcal{Z}_\gamma$ parameterized by the robustness index $\gamma \in [0, 1]$ is defined as follows:

$$\mathcal{Z}_\gamma = \left\{(\boldsymbol{d}, \boldsymbol{s}) \in \Re^W \times \Re^W \mid \bar{d}_w(\gamma) \leq d_w \leq \bar{d}_w(0), \bar{s}_w(\gamma) \leq s_w \leq \bar{s}_w(0), d_w \leq s_w, \ \forall\, w \in \mathcal{W}\right\}$$

where $\bar{d}_w : [0, 1] \to \Re_+$ and $\bar{s}_w : [0, 1] \to \Re_+$, $w \in \mathcal{W}$ are nonincreasing functions. $\bar{d}_w(\gamma)$ can be interpreted as the deliverability level in which the probability (or subjective probability) of exceeding the level is γ. The same interpretation applies to $\bar{s}_w(\gamma)$. Higher values of γ implies that the uncertainty set $\mathcal{Z}_\gamma$ includes more pessimistic (lower) values of the deliverabilities and sizes of the reserves. Note that the deliverability of a well being less than its size is implied in the definition of $\mathcal{Z}_\gamma$. It is also assumed that $0 < \bar{d}_w(\gamma) \leq \bar{s}_w(\gamma)$ for all $\gamma \in [0, 1]$, $w \in \mathcal{W}$.

5.2 The Dynamics of Offshore Gas Field Development

5.2.1 Gas field development decisions

The decision variables for the gas field development model are discussed in the following. Define binary decision variables $x_{w,t} \in \{0,1\}$ for all $t \in \mathcal{T}$ and $w \in \mathcal{W}$ so that $x_{w,t} = 1$ if installation of well platform on w is performed in period t, and $x_{w,t} = 0$ otherwise. In the similar sense define the installation decisions $y^P_{p,t} \in \{0,1\}$ for production platform $p \in \mathcal{P}$, $y^A_{a,t} \in \{0,1\}$ for well-well pipeline connection $a \in \mathcal{A}$ and $y^B_{b,t} \in \{0,1\}$ for well-production platform pipeline connection $b \in \mathcal{B}$. Define binary decisions $b_{p,t} \in \{0,1\}$ so that $b_{p,t} = 1$ if buffer capacity is invested at platform installation in period t, and $b_{p,t} = 0$ otherwise.

The capacity on installation of a well platform $w \in \mathcal{W}$ and production platform $p \in \mathcal{P}$ in period $t \in \mathcal{T}$ is defined as $c^W_{w,t} \geq 0$ and $c^P_{p,t} \geq 0$ respectively. Denote $u_{p,t} \geq 0$ as the capacity expansion level of production platform $p \in \mathcal{P}$ in period $t \in \mathcal{T}$. Finally, define $q^W_{w,t} \geq 0$, $q^P_{p,t} \geq 0$, $q^A_{a,t}$ and $q^B_{b,t} \geq 0$ as the extraction quantities from platform $w \in \mathcal{W}$, production quantities from platform $p \in \mathcal{P}$, pipeline flows from connection $a \in \mathcal{A}$ and $b \in \mathcal{B}$ in period $t \in \mathcal{T}$, respectively.

It is assumed that none of the structures have been installed when the planning horizon begins, which also implies that all the wells are undeveloped before the project commences. In the following, denote $\mathcal{W}_t$, $\mathcal{P}_t$, $\mathcal{A}_t$ and $\mathcal{B}_t$ as the set of well platforms, production platforms and pipelines installed by the end of period t, respectively. The sequence of decisions in the development project is then as follows:

1. The planning horizon begins at $t = 1$. Initialize the sets $\mathcal{W}_0 = \mathcal{P}_0 = \mathcal{A}_0 = \mathcal{B}_0 = \emptyset$.

2. At the beginning of period t, identify the well platforms $\{w \in \mathcal{W} \mid x_{w,t} = 1\}$, production platforms $\{p \in \mathcal{P} \mid y^P_{p,t} = 1\}$ and pipeline connections $\{a \in \mathcal{A} \mid y^A_{a,t} = 1\}$ and $\{b \in \mathcal{B} \mid y^B_{b,t} = 1\}$ to install.

3. For the well and production platforms installed in the current period (i.e., $\{w \in \mathcal{W} \mid x_{w,t} = 1\}$ and $\{p \in \mathcal{P} \mid y^P_{p,t} = 1\}$), the capacities are set to $c^W_{w,t}$ and $c^P_{p,t}$, respectively. Furthermore, buffer capacity investments $b_{p,t}$ are determined for all production platforms $\{p \in \mathcal{P} \mid y^P_{p,t} = 1\}$.

4. The actual initial deliverability d_w and size s_w of the gas reserve of each newly installed well platform $\{w \in \mathcal{W} \mid x_{w,t} = 1\}$ is observed.

5. Update $\mathcal{W}_t = \mathcal{W}_{t-1} \cup \{w \in \mathcal{W} \mid x_{w,t} = 1\}$, $\mathcal{P}_t = \mathcal{P}_{t-1} \cup \{p \in \mathcal{P} \mid y^P_{p,t} = 1\}$, $\mathcal{A}_t = \mathcal{A}_{t-1} \cup \{a \in \mathcal{A} \mid y^A_{a,t} = 1\}$, and $\mathcal{B}_t = \mathcal{B}_{t-1} \cup \{b \in \mathcal{B} \mid y^B_{b,t} = 1\}$.

6. Before the period ends, determine the expansion quantity $u_{p,t}$ for all platforms with buffer capacity installed, i.e., $p \in \mathcal{P}_t$ with $b_{p,t'} = 1$ for some $t' \leq t$. Additionally, determine production plan which includes well

extraction $q^W_{w,t}$, $w \in \mathcal{W}_t$, platform production $q^P_{p,t}$, $p \in \mathcal{P}_t$ and all pipeline connection flows $q^A_{a,t}$, $a \in \mathcal{A}_t$ and $q^B_{b,t}$, $b \in \mathcal{B}_t$.

5.2.2 Non-anticipative decision dependencies

This section shows the development of the formulation to model non-anticipative requirements in the dynamic decision-making process with endogenous uncertainties. Let the installation policy of the well $w \in \mathcal{W}$ in the first period $t = 1$ be defined by the function $x_{w,1}(\boldsymbol{d}, \boldsymbol{s})$ where $x_{w,1} : \mathcal{Z}_\gamma \to \{0, 1\}$. Note that since none of the wells have been developed at the beginning of the planning horizon, the function should be independent of its inputs $\boldsymbol{d}$ and $\boldsymbol{s}$. Let the set of wells installed by the end of $t = 1$ be given by the function $\mathcal{W}_1 : \mathcal{Z}_\gamma \to 2^{\mathcal{W}}$, where:

$$\mathcal{W}_1(\boldsymbol{d}, \boldsymbol{s}) = \{w \in \mathcal{W} \mid x_{w,1}(\boldsymbol{d}, \boldsymbol{s}) = 1\} \qquad \forall (\boldsymbol{d}, \boldsymbol{s}) \in \mathcal{Z}_\gamma.$$

More generally, given a function $\bar{\mathcal{W}} : \mathcal{Z}_\gamma \to 2^{\mathcal{W}}$ associated with the set of installed wells, we define the following family of measurable functions, which are functions that could be described and whose structure do not change:

$$\mathcal{F}(\bar{\mathcal{W}}) \triangleq \left\{ f : \mathcal{Z}_\gamma \to \Re \;\middle|\; \begin{array}{ll} f(\boldsymbol{d}, \boldsymbol{s}) = & f(\boldsymbol{\delta}, \boldsymbol{\sigma}) \qquad \forall\ (\boldsymbol{d}, \boldsymbol{s}), (\boldsymbol{\delta}, \boldsymbol{\sigma}) \in \mathcal{Z}_\gamma\ : \\ & \qquad (\delta_w, \sigma_w) = (d_w, s_w) \qquad \forall w \in \bar{\mathcal{W}}(\boldsymbol{d}, \boldsymbol{s}) \end{array} \right\}$$

The above is useful for defining the set of admissible policies that are indistinguishable in actions given identical observations of well parameter outcomes. For instance, suppose $\mathcal{W} = \{1, 2, 3, 4\}$ and $\mathcal{W}_1 = \{1, 4\}$. A policy $f \in \mathcal{F}(\bar{\mathcal{W}})$ can then be written as $f((d_1, \bar{d}_2, \bar{d}_3, d_4), (s_1, \bar{s}_2, \bar{s}_3, s_4)) \; \forall \; (\boldsymbol{d}, \boldsymbol{s}) \in \mathcal{Z}_\gamma$, where the values $(\bar{d}_2, \bar{d}_3, \bar{s}_2, \bar{s}_3)$ are arbitrarily chosen, since f is indifferent to these inputs by definition of $\mathcal{F}(\bar{\mathcal{W}})$. Clearly, any function, $x : \mathcal{Z}_\gamma \to \Re$ that is mapped to a constant value also satisfies $x \in \mathcal{F}(\bar{\mathcal{W}})$.

The well installation policy over time is defined as a set of functions, $x_{w,t} \in \mathcal{F}(\mathcal{W}_{t-1})$, $x_{w,t}(\boldsymbol{d}, \boldsymbol{s}) \in \{0, 1\}$, for all $(\boldsymbol{d}, \boldsymbol{s}) \in \mathcal{Z}_\gamma$, $t \in \mathcal{T}$. It then follows that $\mathcal{W}_t : \mathcal{Z}_\gamma \to 2^{\mathcal{W}}$ can be defined recursively as follows:

$$\mathcal{W}_t(\boldsymbol{d}, \boldsymbol{s}) = \mathcal{W}_{t-1}(\boldsymbol{d}, \boldsymbol{s}) \cup \{w \in \mathcal{W} \mid x_{w,t}(\boldsymbol{d}, \boldsymbol{s}) = 1\} \qquad \forall (\boldsymbol{d}, \boldsymbol{s}) \in \mathcal{Z}_\gamma.$$

In summary, the decision policy considered in the model satisfies the following non-anticipativity requirements:

$$\begin{aligned} x_{w,t}, c^W_{w,t} \in \mathcal{F}(\mathcal{W}_{t-1}),\ q^W_{w,t} \in \mathcal{F}(\mathcal{W}_t) &\quad w \in \mathcal{W} \\ y^P_{p,t}, b_{p,t}, c^P_{p,t} \in \mathcal{F}(\mathcal{W}_{t-1}), q^P_{p,t}, u_{p,t} \in \mathcal{F}(\mathcal{W}_t) &\quad p \in \mathcal{P} \\ y^A_{a,t} \in \mathcal{F}(\mathcal{W}_{t-1}), q^A_{a,t} \in \mathcal{F}(\mathcal{W}_t) &\quad a \in \mathcal{A} \\ y^B_{b,t} \in \mathcal{F}(\mathcal{W}_{t-1}), q^B_{b,t} \in \mathcal{F}(\mathcal{W}_t) &\quad b \in \mathcal{B}, \end{aligned}$$

for all $t \in \mathcal{T}$.

Note that in the above, the non-anticipativity conditions for the installation decisions $x_{w,t},\ c^W_{w,t},\ y^P_{p,t},\ c^P_{p,t},\ b_{p,t},\ y^A_{a,t}$ and $y^B_{b,t}$ are defined over $\mathcal{W}_{t-1}$, i.e. the set of wells developed by the end of period $t-1$. On the other hand, the decisions $q^W_{w,t},\ q^P_{p,t},\ u_{p,t},\ q^A_{a,t},\ q^B_{b,t}$ are performed after observing the deliverabilities and sizes of all developed wells $w \in \mathcal{W}_t$. Some of these decision policies are mapped to binary values, while others are assigned to nonnegative values as follows:

$$\tilde{x}_{w,t}, \tilde{b}_p, \tilde{y}^P_{p,t}, \tilde{y}^A_{a,t}, \tilde{y}^B_{b,t} \in \{0,1\} \quad w \in \mathcal{W}, p \in \mathcal{P}, a \in \mathcal{A}, b \in \mathcal{B}$$

$$\tilde{c}^W_{w,t}, \tilde{q}^W_{w,t}, \tilde{c}^P_{p,t}, \tilde{u}_{p,t}, \tilde{q}^A_{a,t}, \tilde{q}^B_{b,t} \in \Re_+ \quad w \in \mathcal{W}, p \in \mathcal{P}, a \in \mathcal{A}, b \in \mathcal{B},$$

where the tilde accent ˜ is used as a shorthand, say $\tilde{x} \in \mathcal{X} \subseteq \Re$ to mean $x(\boldsymbol{d}, \boldsymbol{s}) \in \mathcal{X}$ for all $(\boldsymbol{d}, \boldsymbol{s}) \in \mathcal{Z}_\gamma$ for some set $\mathcal{X}$. In particular, constraints involving decision functions such as $x(\boldsymbol{d}, \boldsymbol{s}) \geq t$ and $y(\boldsymbol{d}, \boldsymbol{s}) = t$ for all $(\boldsymbol{d}, \boldsymbol{s}) \in \mathcal{Z}_\gamma$ are represented using $\tilde{x} \geq t$ and $\tilde{y} = t$ respectively.

5.2.3 Gas production requirements

Any feasible development plan must respect physical and logical requirements such as capacity and flow balance constraints. It is emphasized that feasibility is only required to be satisfied if the uncertainty occurs within $\mathcal{Z}_\gamma$, and that depends on the level of robustness that a decision maker would like to achieve.

The following constraints refer to the installation of structures in the field.

$$\tilde{c}^W_{w,t} \leq h^W_w \tilde{x}_{w,t} \qquad \forall\, w \in \mathcal{W},\ t \in \mathcal{T} \tag{5.1}$$

$$\tilde{c}^P_{p,t} \leq h^P_p \tilde{y}^P_{p,t} \qquad \forall\, p \in \mathcal{P},\ t \in \mathcal{T} \tag{5.2}$$

$$\tilde{y}^A_{w,w',t} \leq \tilde{x}_{w',t} \qquad \forall\, (w,w') \in \mathcal{A}, w' \in \mathcal{W}, t \in \mathcal{T} \tag{5.3}$$

$$\tilde{y}^B_{w,p,t} \leq \tilde{y}^P_{p,t} \qquad \forall\, (w,p) \in \mathcal{B}, p \in \mathcal{P}, t \in \mathcal{T} \tag{5.4}$$

$$\tilde{x}_{w,t} = \sum_{\{w'|(w,w') \in \mathcal{A}\}} \tilde{y}^A_{w,w',t} + \sum_{\{p|(w,p) \in \mathcal{B}\}} \tilde{y}^B_{w,p,t} \qquad \forall\, w \in \mathcal{W},\ t \in \mathcal{T} \tag{5.5}$$

$$\sum_{t \in \mathcal{T}} \tilde{x}_{w,t} \leq 1 \qquad \forall\, w \in \mathcal{W} \tag{5.6}$$

$$\sum_{t \in \mathcal{T}} \tilde{y}^P_{p,t} \leq 1 \qquad \forall\, p \in \mathcal{P} \tag{5.7}$$

$$\sum_{t \in \mathcal{T}} \tilde{y}^A_{a,t} \leq 1 \qquad \forall\, a \in \mathcal{A} \tag{5.8}$$

$$\sum_{t \in \mathcal{T}} \tilde{y}^B_{b,t} \leq 1 \qquad \forall\, b \in \mathcal{B} \tag{5.9}$$

In (5.1) and (5.2), the capacities on installation for the well $w \in \mathcal{W}$ and production platforms $p \in \mathcal{P}$ are limited by upper bounds h^W_w and h^P_p, respectively. (5.3) and (5.4) require that a well can only be connected to another platform that is already installed. (5.5) states that there can only be one outgoing pipeline connection from a well platform, which can either be to another well platform or to a production platform. (5.6)-(5.9) ensure that well, production platforms and pipeline connections are installed at most once during the planning horizon.

$$\tilde{u}_{p,t} \leq (h^P_p + \bar{u}_p)\tilde{b}_{p,t} \qquad \forall\, p \in \mathcal{P},\ t \in \mathcal{T} \tag{5.10}$$

$$\sum_{t \in \mathcal{T}} (\tilde{c}^P_{p,t} + \tilde{u}_{p,t}) \leq h^P_p + \bar{u}_p \qquad \forall\, p \in \mathcal{P} \tag{5.11}$$

$$\tilde{b}_{p,t} \leq \tilde{y}^P_{p,t} \qquad \forall\, p \in \mathcal{P},\ t \in \mathcal{T} \tag{5.12}$$

The capacities of the production platforms can be increased through expansion as shown in (5.10) and (5.11). Expansion quantity, as well as the total capacity of a production platform, must be within the sum of the maximum initial capacity h^P_p and the maximum buffer capacity $\bar{u}_p$ for $p \in \mathcal{P}$. (5.12) states that the decision to expand a production platform should be made upon its installation since the platform needs to be fitted with dedicated expansion capability.

Next, the gas extraction levels, production levels and pipeline flow levels are constrained by the capacities of the relevant structures. These are depicted in (5.13)-(5.16) below:

$$\tilde{q}^W_{w,t} \leq \sum_{t'=1}^{t} \tilde{c}^W_{w,t'} \qquad \forall\, w \in \mathcal{W},\ t \in \mathcal{T} \tag{5.13}$$

$$\tilde{q}^P_{p,t} \leq \sum_{t'=1}^{t} \left(\tilde{c}^P_{p,t'} + \tilde{u}_{p,t'}\right) \qquad \forall\, p \in \mathcal{P},\ t \in \mathcal{T} \tag{5.14}$$

$$\tilde{q}^A_{w,w',t} \leq h^A_{w,w'} \sum_{t'=1}^{t} \tilde{y}^A_{w,w',t'} \qquad \forall\, (w,w') \in \mathcal{A},\ t \in \mathcal{T} \tag{5.15}$$

$$\tilde{q}^B_{w,p,t} \leq h^B_{w,p} \sum_{t'=1}^{t} \tilde{y}^B_{w,p,t'} \qquad \forall\, (w,p) \in \mathcal{B},\ t \in \mathcal{T} \tag{5.16}$$

Mass balance equations for a feasible production plan are as follows:

$$\tilde{q}^W_{w,t} + \sum_{\{w'|(w',w)\in\mathcal{A}\}} \tilde{q}^A_{w',w,t} = \sum_{\{w'|(w',w)\in\mathcal{A}\}} \tilde{q}^A_{w,w',t} + \sum_{\{p|(w,p)\in\mathcal{B}\}} \tilde{q}^B_{w,p,t} \qquad \forall\, w \in \mathcal{W},\; t \in \mathcal{T} \tag{5.17}$$

$$\tilde{q}^P_{p,t} = \sum_{\{w|(w,p)\in\mathcal{B}\}} \tilde{q}^B_{w,p,t} \qquad \forall\, p \in \mathcal{P},\; t \in \mathcal{T} \tag{5.18}$$

$$q^W_{w,t}(\boldsymbol{d},\boldsymbol{s}) \le d_w \left(1 - \frac{1}{s_w}\sum_{t'=1}^{t-1} q^W_{w,t'}(\boldsymbol{d},\boldsymbol{s})\right) \qquad \forall\, (\boldsymbol{d},\boldsymbol{s}) \in \mathcal{Z}_\gamma,\; w \in \mathcal{W},\; t \in \mathcal{T} \tag{5.19}$$

Constraints (5.17) and (5.18) are the mass balance of gas flows for well platforms and production platforms respectively. Finally, (5.19) requires that for each well platform, the gas extraction rate cannot be greater than the deliverability of a reserve in the well platforms.

Note that in order to ensure feasibility in the well extraction constraints, the accumulated extraction at any time period $t \in \mathcal{T}$ should never exceed the size of a well platform. Observe that at $t = T$,

$$q^W_{w,T}(\boldsymbol{d},\boldsymbol{s}) \le d_w \left(1 - \frac{1}{s_w}\sum_{t'=1}^{T-1} q^W_{w,t'}(\boldsymbol{d},\boldsymbol{s})\right) \qquad \forall\, (\boldsymbol{d},\boldsymbol{s}) \in \mathcal{Z}_\gamma.$$

Since $s_w \ge d_w > 0$, this implies that

$$\sum_{t\in\mathcal{T}} q^W_{w,t}(\boldsymbol{d},\boldsymbol{s}) \le \sum_{t=1}^{T-1} q^W_{w,t}(\boldsymbol{d},\boldsymbol{s}) + \frac{s_w}{d_w} q^W_{w,T}(\boldsymbol{d},\boldsymbol{s}) \le s_w \qquad \forall (\boldsymbol{d},\boldsymbol{s}) \in \mathcal{Z}_\gamma.$$

Hence, the constraints in (5.19) imply that the total quantity extracted from each well is bounded above by its size.

5.2.4 Project net present value

The installation decisions are capital investments that result in cash outflows in the offshore field problem defined as follows:

$$\begin{aligned}
\tilde{C}_t^O = & \underbrace{\sum_{w \in \mathcal{W}} \left(f_w^W \tilde{x}_{w,t} + v_w^W \tilde{c}_{w,t}^W \right)}_{\text{Installation cost for well platforms}} + \underbrace{\sum_{(w,w') \in \mathcal{A}} f_{w,w'}^A \tilde{y}_{w,w',t}^A}_{\text{Installation cost for well-well pipelines}} \\
& + \underbrace{\sum_{p \in \mathcal{P}} \left(f_p^P \tilde{y}_{p,t}^P + f_p^B \tilde{b}_{p,t} + v_p^P \left(\tilde{c}_{p,t}^P + \tilde{u}_{p,t} + \bar{u}_p \tilde{b}_p \right) \right)}_{\text{Installation cost for production platforms}} \\
& + \underbrace{\sum_{(w,p) \in \mathcal{B}} f_{w,p}^B \tilde{y}_{w,p,t}^B}_{\text{Installation cost for well-production pipelines}} \qquad \forall\, t \in \mathcal{T} \qquad (5.20)
\end{aligned}$$

Note that each infrastructure installation incurs both fixed and variable costs. Furthermore, the installation cost for the production platform has additional fixed and variable components related to expansion capability.

The annual cash inflow obtained from total gas production is defined by (5.21). For all $t \in \mathcal{T}$, this is made up of the revenue from the production of gas (at price r_t per unit) and the operating cost from the extraction (at g_w^W

per unit) and production (at g_p^P per unit) of gas in the platforms $w \in \mathcal{W}$ and $p \in \mathcal{P}$, respectively. Hence,

$$\tilde{C}_t^I = r_t \sum_{p \in \mathcal{P}} \tilde{q}_{p,t}^P - \sum_{w \in \mathcal{W}} g_w^W \tilde{q}_{w,t}^W - \sum_{p \in \mathcal{P}} g_p^P \tilde{q}_{p,t}^P \quad \forall\, t \in \mathcal{T} \tag{5.21}$$

Finally, the *net present value* (NPV) of the gas field development is determined as follows:

$$\tilde{R} = \sum_{t \in \mathcal{T}} \beta^{-t} \left(\tilde{C}_t^I - \tilde{C}_t^O \right)$$

where β^{-t} is a time-discount factor for $t \in \mathcal{T}$.

5.3 Target oriented robust optimization

In the development planning problem, the NPV is a key performance metric whose target value is often specified as part of the project deliverables. A development plan is then developed to achieve the NPV target. However, in the presence of uncertainty, the actual NPV is also subject to uncertainty. The proposed target oriented robust optimization approach searches for a planning solution that can achieve the target NPV over as large an uncertainty space as possible. The notations are first simplified to present the target oriented robust optimization model. In the following exposition, the uncertain parameters $(\boldsymbol{d}, \boldsymbol{s})$ are denoted by $\boldsymbol{z}$ and the decision policy $\boldsymbol{\pi} : \mathcal{Z}_\gamma \rightarrow \Re^D$ is denoted as a vector of functions of appropriate dimensions. $\boldsymbol{\mathcal{F}}$ is used to denote the family of admissible policies that captures the non-anticipativity requirements of $\boldsymbol{\pi}$ while $\mathcal{V}$ to denote the range in which $\boldsymbol{\pi}$ is mapped into. Note that each dimension of $\mathcal{V}$ is restricted to either binary values or to nonnegative reals. Finally, the linear constraints (5.1)-(5.19) over the decision policy are presented by

$$\boldsymbol{A}(\boldsymbol{z})\boldsymbol{\pi}(\boldsymbol{z}) \leq \boldsymbol{b}(\boldsymbol{z}) \qquad \forall \boldsymbol{z} \in \mathcal{Z}_{\gamma},$$

where $\boldsymbol{A}(\boldsymbol{z})$ and $\boldsymbol{b}(\boldsymbol{z})$ are respectively the matrix and vector that are influenced by $\boldsymbol{z}$. The model can be extended to consider uncertain coefficients in evaluating the net present value such as to encompass uncertainty in gas prices, production costs among others. Therefore, in general, $\boldsymbol{r}(\boldsymbol{z})'\boldsymbol{\pi}(\boldsymbol{z})$ is used to represent the net present value of the gas field development planning problem in which the uncertainty associated with the coefficients $\boldsymbol{r}(\boldsymbol{z})$ extends beyond the status of the explored wells.

The target oriented robust optimization problem (TRO) is as follows:

$$
\begin{aligned}
\gamma^* = \quad & \max \quad \gamma \\
& \text{s.t.} \quad \boldsymbol{r}(\boldsymbol{z})'\boldsymbol{\pi}(\boldsymbol{z}) \geq \tau \qquad \forall \boldsymbol{z} \in \mathcal{Z}_\gamma \\
& \qquad \boldsymbol{A}(\boldsymbol{z})\boldsymbol{\pi}(\boldsymbol{z}) \leq \boldsymbol{b}(\boldsymbol{z}) \quad \forall \boldsymbol{z} \in \mathcal{Z}_\gamma \\
& \qquad \boldsymbol{\pi}(\boldsymbol{z}) \in \mathcal{V} \qquad \forall \boldsymbol{z} \in \mathcal{Z}_\gamma \\
& \qquad \boldsymbol{\pi} \in \boldsymbol{\mathcal{F}} \\
& \qquad \gamma \in [0,1].
\end{aligned} \tag{5.22}
$$

The TRO is related to the linear adjustable robust optimization framework introduced in Ben-Tal et al. (2004), with the additional complications of having non affine disturbance in $\boldsymbol{A}(\boldsymbol{z})$ and $\boldsymbol{r}(\boldsymbol{z})$, decision policy being in a non convex set $\mathcal{V}$ and endogenously dependent non-anticipative restrictions in $\boldsymbol{\mathcal{F}}$. Therefore, linear decision rules (a.k.a adjustable robust counterpart), which have been shown to be effective in robust and stochastic optimization problems (see Ben-Tal et al. 2005; Bertsimas et al. 2010b; Chen et al. 2008; Goh and Sim 2011), cannot be used to approximate the decision polices.

5.3.1 Fixed policy and its optimality

One way to address the computation of the target oriented robust optimization problem is to impose restriction to *fixed policy* $\boldsymbol{\pi}$, which is a function that is mapped to a single value and hence, not affected by the information $\boldsymbol{z} \in \mathcal{Z}_\gamma$.

Definition 2 *A fixed decision policy* $\boldsymbol{\pi} : \mathcal{Z}_\gamma \to \mathcal{V}$ *is one that satisfies*

$$\boldsymbol{\pi}(\boldsymbol{z}) = \boldsymbol{v} \qquad \forall \boldsymbol{z} \in \mathcal{Z}_\gamma,$$

for some $\boldsymbol{v} \in \mathcal{V}$.

It is easy to examine that any fixed policy, $\boldsymbol{\pi}$, will automatically satisfy the non-anticipativity requirement, i.e., $\boldsymbol{\pi} \in \boldsymbol{\mathcal{F}}$. With a fixed policy, TRO model can be simplified as follows:

$$\begin{array}{rll} \gamma^\dagger = & \max & \gamma \\ & \text{s.t.} & \boldsymbol{r}(\boldsymbol{z})'\boldsymbol{v} \geq \tau \qquad \forall \boldsymbol{z} \in \mathcal{Z}_\gamma \\ & & \boldsymbol{A}(\boldsymbol{z})\boldsymbol{v} \leq \boldsymbol{b}(\boldsymbol{z}) \quad \forall \boldsymbol{z} \in \mathcal{Z}_\gamma \\ & & \boldsymbol{v} \in \mathcal{V} \\ & & \gamma \in [0, 1]. \end{array} \tag{5.23}$$

Clearly, since a fixed decision policy is not necessarily optimal, one can say that $\gamma^\dagger \leq \gamma^*$. Nevertheless, a fixed policy has immense computation advantage to address this seemingly intractable problem. Moreover, it is possible to show an important condition in which a fixed decision policy can also be optimal in the TRO problem.

Definition 3 *The TRO model (5.22) has a* solution-independent worst case scenario, $\hat{\boldsymbol{z}}_\gamma \in \mathcal{Z}_\gamma$ *if for all* $\boldsymbol{v} \in \mathcal{V}$ *satisfying*

$$\boldsymbol{r}(\hat{\boldsymbol{z}}_\gamma)'\boldsymbol{v} \geq \tau$$

$$\boldsymbol{A}(\hat{\boldsymbol{z}}_\gamma)\boldsymbol{v} \leq \boldsymbol{b}(\hat{\boldsymbol{z}}_\gamma)$$

then $\boldsymbol{v}$ *is also feasible in*

$$\boldsymbol{r}(\boldsymbol{z})'\boldsymbol{v} \geq \tau \qquad \forall \boldsymbol{z} \in \mathcal{Z}_\gamma$$

$$\boldsymbol{A}(\boldsymbol{z})\boldsymbol{v} \leq \boldsymbol{b}(\boldsymbol{z}) \qquad \forall \boldsymbol{z} \in \mathcal{Z}_\gamma.$$

Intuitively, this implies that if one is able to identify a solution in an optimization problem using the worst case scenario, $\hat{\boldsymbol{z}}_\gamma \in \mathcal{Z}_\gamma$, then this solution will also be feasible for any realization of the uncertain parameters $\boldsymbol{z} \in \mathcal{Z}_\gamma$.

Theorem 5.3.1 *Suppose the TRO model (5.22) has a solution-independent worst case scenario, given by* $\hat{\boldsymbol{z}}_\gamma \in \mathcal{Z}_\gamma$*, then there exist a fixed policy that is* *optimal. Moreover, the fixed policy* $\pi(\boldsymbol{z}) = \boldsymbol{v}$ *for all* $\boldsymbol{z} \in \mathcal{Z}_\gamma$ *can be obtained by solving the following deterministic optimization problem:*

$$
\begin{aligned}
\gamma^{\ddagger} = \quad \max \quad & \gamma \\
\text{s.t.} \quad & \boldsymbol{r}(\hat{\boldsymbol{z}}_\gamma)'\boldsymbol{v} \geq \tau \\
& \boldsymbol{A}(\hat{\boldsymbol{z}}_\gamma)\boldsymbol{v} \leq \boldsymbol{b}(\hat{\boldsymbol{z}}_\gamma) \\
& \boldsymbol{v} \in \mathcal{V}, \\
& \gamma \in [0,1].
\end{aligned}
\tag{5.24}
$$

Hence, $\gamma^{\dagger} = \gamma^* = \gamma^{\ddagger}$.

Proof : Clearly, $\gamma^{\dagger} \leq \gamma^*$. However, it can be observed that

$$
\begin{aligned}
\gamma^{\dagger} &= \max\{\gamma \in [0,1] : \boldsymbol{r}(\boldsymbol{z})'\boldsymbol{v} \geq \tau,\ \boldsymbol{A}(\boldsymbol{z})\boldsymbol{v} \leq \boldsymbol{b}(\boldsymbol{z}),\ \forall \boldsymbol{z} \in \mathcal{Z}_\gamma,\ \boldsymbol{v} \in \mathcal{V}\} \\
&\geq \max\{\gamma \in [0,1] : \boldsymbol{r}(\hat{\boldsymbol{z}}_\gamma)'\boldsymbol{v} \geq \tau, \boldsymbol{A}(\hat{\boldsymbol{z}}_\gamma)\boldsymbol{v} \leq \boldsymbol{b}(\hat{\boldsymbol{z}}_\gamma),\ \boldsymbol{v} \in \mathcal{V}\} \\
&= \gamma^{\ddagger} \\
&= \max\{\gamma \in [0,1] : \boldsymbol{r}(\hat{\boldsymbol{z}}_\gamma)'\boldsymbol{\pi}(\hat{\boldsymbol{z}}_\gamma) \geq \tau, \boldsymbol{A}(\hat{\boldsymbol{z}}_\gamma)\boldsymbol{\pi}(\hat{\boldsymbol{z}}_\gamma) \leq \boldsymbol{b}(\hat{\boldsymbol{z}}_\gamma),\ \boldsymbol{\pi}(\hat{\boldsymbol{z}}_\gamma) \in \mathcal{V}, \boldsymbol{\pi} : \mathcal{Z}_\gamma \to \Re^D\} \\
&\geq \max\{\gamma \in [0,1] : \boldsymbol{r}(\hat{\boldsymbol{z}}_\gamma)'\boldsymbol{\pi}(\hat{\boldsymbol{z}}_\gamma) \geq \tau, \boldsymbol{A}(\hat{\boldsymbol{z}}_\gamma)\boldsymbol{\pi}(\hat{\boldsymbol{z}}_\gamma) \leq \boldsymbol{b}(\hat{\boldsymbol{z}}_\gamma),\ \boldsymbol{\pi}(\hat{\boldsymbol{z}}_\gamma) \in \mathcal{V}, \boldsymbol{\pi} \in \mathcal{F}\} \\
&\geq \max\{\gamma \in [0,1] : \boldsymbol{r}(\boldsymbol{z})'\boldsymbol{\pi}(\boldsymbol{z}) \geq \tau, \boldsymbol{A}(\boldsymbol{z})\boldsymbol{\pi}(\boldsymbol{z}) \leq \boldsymbol{b}(\boldsymbol{z}),\ \boldsymbol{\pi}(\boldsymbol{z}) \in \mathcal{V}, \forall \boldsymbol{z} \in \mathcal{Z}_\gamma, \boldsymbol{\pi} \in \mathcal{F}\} \\
&= \gamma^*,
\end{aligned}
$$

where the first inequality is the result of $\hat{\boldsymbol{z}}_\gamma$ being the solution-independent worst case scenario and the last inequality is due to the fact that $\hat{\boldsymbol{z}}_\gamma \in \mathcal{Z}_\gamma$. ■

The TRO model (5.24) can be solved by decomposing it in a sequence of mixed integer optimization problems

$$\begin{aligned} \rho(\gamma) = \quad \max \quad & \boldsymbol{r}(\hat{\boldsymbol{z}}_\gamma)'\boldsymbol{v} \\ \text{s.t.} \quad & \boldsymbol{A}(\hat{\boldsymbol{z}}_\gamma)\boldsymbol{v} \leq \boldsymbol{b}(\hat{\boldsymbol{z}}_\gamma) \\ & \boldsymbol{v} \in \mathcal{V}, \end{aligned} \tag{5.25}$$

and performing binary search on $\gamma \in [0, 1]$ so that $\rho(\gamma^*) = \tau$.

Theorem 5.3.2 *In regard to the offshore gas field development problem, the solution-independent worst case deliverabilites and sizes are*

$$\bar{d}_w(\gamma), \bar{s}_w(\gamma)$$

for all $w \in \mathcal{W}$*. Therefore, the corresponding problem to (5.25) is as follows:*

$$
\begin{aligned}
\rho(\gamma) = \quad \max \quad & \textstyle\sum_{t\in\mathcal{T}} \beta^{-t}\left(C_t^I - C_t^O\right) \\
\text{s.t.} \quad & C_t^O = \sum_{w\in\mathcal{W}} \left(f_w^W x_{w,t} + v_w^W c_{w,t}^W\right) + \\
& \quad \textstyle\sum_{(w,w')\in\mathcal{A}} f_{w,w'}^A y_{w,w',t}^A + \sum_{(w,p)\in\mathcal{B}} f_{w,p}^B y_{w,p,t}^B + \\
& \quad \textstyle\sum_{p\in\mathcal{P}} \left(f_p^P\left(y_{p,t}^P + b_p\right) + v_p^P\left(c_{p,t}^P + u_{p,t} + \bar{u}_p b_p\right)\right) && \forall t\in\mathcal{T} \\
& C_t^I = r_t \textstyle\sum_{p\in\mathcal{P}} q_{p,t}^P - \sum_{w\in\mathcal{W}} g_w^W q_{w,t}^W - \sum_{p\in\mathcal{P}} g_p^P q_{p,t}^P && \forall\ t\in\mathcal{T} \\
& c_{w,t}^W \le h_w^W x_{w,t} && \forall\ w\in\mathcal{W},\ t\in\mathcal{T} \\
& c_{p,t}^P \le h_p^P y_{p,t}^P && \forall\ p\in\mathcal{P},\ t\in\mathcal{T} \\
& y_{w,w',t}^A \le x_{w',t} && \forall\ (w,w')\in\mathcal{A},\ w'\in\mathcal{W},\ t\in\mathcal{T} \\
& y_{w,p,t}^B \le y_{p,t}^P && \forall\ (w,p)\in\mathcal{B},\ p\in\mathcal{P},\ t\in\mathcal{T} \\
& x_{w,t} = \textstyle\sum_{\{w'|(w,w')\in\mathcal{A}\}} y_{w,w',t}^A + \sum_{\{p|(w,p)\in\mathcal{B}\}} y_{w,p,t}^B && \forall\ w\in\mathcal{W},\ t\in\mathcal{T} \\
& \textstyle\sum_{t\in\mathcal{T}} x_{w,t} \le 1 && \forall\ w\in\mathcal{W} \\
& \textstyle\sum_{t\in\mathcal{T}} y_{p,t}^P \le 1 && \forall\ p\in\mathcal{P} \\
& \textstyle\sum_{t\in\mathcal{T}} y_{a,t}^A \le 1 && \forall\ a\in\mathcal{A} \\
& \textstyle\sum_{t\in\mathcal{T}} y_{b,t}^B \le 1 && \forall\ b\in\mathcal{B} \\
& u_{p,t} \le (h_p^P + \bar{u}_p) b_p && \forall\ p\in\mathcal{P},\ t\in\mathcal{T} \\
& \textstyle\sum_{t\in\mathcal{T}} (c_{p,t}^P + u_{p,t}) \le h_p^P + \bar{u}_p && \forall\ p\in\mathcal{P} \\
& b_{p,t} \le y_{p,t}^P && \forall\ p\in\mathcal{P},\ t\in\mathcal{T} \\
& q_{w,t}^W \le \textstyle\sum_{t'=1}^t c_{w,t'}^W && \forall\ w\in\mathcal{W},\ t\in\mathcal{T} \\
& q_{p,t}^P \le \textstyle\sum_{t'=1}^t \left(c_{p,t'}^P + u_{p,t'}\right) && \forall\ p\in\mathcal{P},\ t\in\mathcal{T} \\
& q_{w,w',t}^A \le h_{w,w'}^A \textstyle\sum_{t'=1}^t y_{w,w',t'}^A && \forall\ (w,w')\in\mathcal{A},\ t\in\mathcal{T} \\
& q_{w,p,t}^B \le h_{w,p}^B \textstyle\sum_{t'=1}^t y_{w,p,t'}^B && \forall\ (w,p)\in\mathcal{B},\ t\in\mathcal{T} \\
& q_{w,t}^W + \textstyle\sum_{\{w'|(w',w)\in\mathcal{A}\}} q_{w',w,t}^A = \\
& \textstyle\sum_{\{w'|(w,w')\in\mathcal{A}\}} q_{w,w',t}^A + \sum_{\{p|(w,p)\in\mathcal{B}\}} q_{w,p,t}^B && \forall\ w\in\mathcal{W},\ t\in\mathcal{T} \\
& \ldots
\end{aligned}
$$

$$
\begin{aligned}
&q^P_{p,t} = \textstyle\sum_{\{w|(w,p)\in\mathcal{B}\}} q^B_{w,p,t} && \forall\ p \in \mathcal{P},\ t \in \mathcal{T} \\
&q^W_{w,t} \le \bar{d}_w(\gamma)\left(1 - \tfrac{1}{\bar{s}_w(\gamma)}\textstyle\sum_{t'=1}^{t-1} q^W_{w,t'}\right) && \forall\ w \in \mathcal{W},\ t \in \mathcal{T} \\
&x_{w,t}, b_{p,t}, y^P_{p,t}, y^A_{a,t}, y^B_{b,t} \in \{0,1\} && \forall w \in \mathcal{W}, p \in \mathcal{P}, a \in \mathcal{A}, b \in \mathcal{B} \\
&c^W_{w,t}, q^W_{w,t}, c^P_{p,t}, u_{p,t}, q^A_{a,t}, q^B_{b,t} \in \Re_+ && \forall w \in \mathcal{W}, p \in \mathcal{P}, a \in \mathcal{A}, b \in \mathcal{B} \\
&C^I_t, C^O_t \in \Re && \forall t \in \mathcal{T}.
\end{aligned}
$$

Proof : To show that $\bar{d}_w(\gamma), \bar{s}_w(\gamma)$ correspond to the solution-independent worst case scenario, it only needs to be shown that any solution feasible in these values is also always feasible in the well reserve constraints (5.19). Suppose $q^W_{w,t} \ge 0$, satisfy

$$q^W_{w,t} \le \bar{d}_w(\gamma)\left(1 - \frac{1}{\bar{s}_w(\gamma)}\sum_{t'=1}^{t-1} q^W_{w,t'}\right)$$

for all $w \in \mathcal{W}$, $t \in \mathcal{T}$, then, since $0 < \bar{d}_w(\gamma) \le d_w$ and $0 < \bar{s}_w(\gamma) \le s_w$ for all $(\boldsymbol{d}, \boldsymbol{s}) \in \mathcal{Z}_\gamma$ it is straightforward to see that the following

$$q^W_{w,t} \le d_w\left(1 - \frac{1}{s_w}\sum_{t'=1}^{t-1} q^W_{w,t'}\right) \qquad \forall (\boldsymbol{d}, \boldsymbol{s}) \in \mathcal{Z}_\gamma$$

are also satisfied. The resulting formulation in the statement of the theorem then follows. ■

Remark : The fact that the TRO problem can be solved exactly as a deterministic optimization is a critical insight for addressing this difficult multistage optimization problem under uncertainty. The result holds because of two important conditions. First, the target oriented robust optimization framework requires the objective function to be evaluated at its worst value. Other decision criteria such as expected utility and risk measure would not necessarily work. Second, the existence of the solution independent worst case scenario depends on the structure of the problem and also the geometry of the uncertainty set, $\mathcal{Z}_\gamma$. Uncertainty sets such ellipsoids may render the worst case scenario to be dependent on the solution.

5.3.2 Target setting and folding horizon implementation

In target setting, the decision maker has to bear the risk of shortfalls when setting a high target. On the other hand, if the target is set too low, one may be criticized for being over conservative. Hence, it is essential to know the reasonable range in which an appropriate target might be set. Observe that $\rho(0)$ corresponds to the highest possible profit attainable under the most optimistic conditions and $\rho(1)$ corresponds to the lowest profit attainable in the worst case. Therefore, it is reasonable to set the target τ within $[\rho(1), \rho(0)]$. The *aggressiveness index* of a target, τ is defined as follows:

$$\alpha \triangleq \frac{\tau - \rho(1)}{\rho(0) - \rho(1)}.$$

Hence, a target with an aggressiveness index of one corresponds to extreme risk seeking or optimistic behavior, while an aggressiveness index of zero corresponds to extreme risk aversion or pessimistic behavior. In practice, the *aggressiveness index* can be incorporated in decision making to calibrate expectation for target setting. In implementing the TRO in practice, we propose the folding horizon approach, where at any stage, only the here-and-now decisions are implemented. Proceeding to the next stage, new information concerning the status of the installed wells becomes available and it is likely that the NPV target may be reassessed accordingly. This strategy is similar to the model predictive control approach, which is immensely practical and widely used in control engineering practice.

5.4 Computational Experiments

To evaluate the performance of the proposed TRO model, it should ideally be compared to the traditional optimum policy in which expected return is optimized. However, one is unable to do so due to the dire computational intractability of obtaining the optimal risk neutral policy. A common way to simplify the computation is by grossly reducing the number of scenarios such as having small number of wells whose reserves are uncertain and assuming that reserve distributions are limited to three-point estimates (low, best, high estimates) instead of being continuously distributed. See for instance, Goel and Grossmann (2004) and Goel et al. (2006). These approaches are not scalable enough to tackle the actual problem of interest.

For the purpose of obtaining a meaningful computational study, evaluation is limited to a single period and solutions are benchmarked against stochastic programming solutions obtained via sample average approximations (SAA). A triangular distribution is used to generate the deliverabilities and sizes for the reserves. The goal is to show that the solutions obtained via the proposed TRO model perform reasonably well despite the fact that probability distributions are not being used explicitly in the model. Moreover, quite apart from the stochastic programming approach, the model allows decision makers to obtain meaningful solutions as they vary the desired target accordingly.

In the following computational set up, the number of wells (WP) and production platforms (PP) that could be installed have been set to 15 and 5, respectively. Similarly, the first stage decisions refer to installation activities while the second stage decisions are the flows between platforms and expansion capacities. The model parameter values used in the study are presented in Table 5.1. The initial deliverabilities and sizes of the reserves (in Billion Standard Cubic Feet) refer to the optimistic estimates, i.e. $\bar{d}_w(0)$ and $\bar{s}_w(0)$, for all $w \in \mathcal{W}$).

In the specification of the probability distributions, the supports for the deliverabilities and sizes are defined as $d_w \in [\bar{d}_w(1), \bar{d}_w(0)]$ and $s_w \in [\bar{s}_w(1), \bar{s}_w(0)]$, respectively for all $w \in \mathcal{W}$. $\bar{d}_w(0)$ and $\bar{s}_w(0)$ are the values stated in Table 5.1, while $\bar{d}(1)$ and $\bar{s}(1)$ are set at $0.60 * \bar{d}_w(0)$ and $0.60 * \bar{s}_w(0)$ for all $w \in \mathcal{W}$. Furthermore, the most likely values of these parameters are assumed to be at the midpoint of the support intervals.

Table 5.1
Model parameters for well platforms (WP) and production platforms (PP)

	WP_1	WP_2	WP_3	WP_4	WP_5	WP_6	WP_7	WP_8
Initial Deliverability (BSCF)	130	150	100	100	130	130	150	120
Size (BSCF)	400	350	350	200	290	300	330	400
Maximum initial capacity	130	140	100	120	100	100	130	140
Fix. cost- WP to WP pipelines	110	110	110	110	110	110	110	110
Fix. cost- PP to WP pipelines	234	219	214	199	229	229	234	219
Fix. cost- installation/expansion	1500	1500	1500	1500	1500	1500	1500	1500
Var. cost- installation/expansion	140	140	140	140	140	140	140	140
	WP_9	WP_{10}	WP_{11}	WP_{12}	WP_{13}	WP_{14}	WP_{15}	$PP_1 - PP_5$
Initial Deliverability (BSCF)	150	130	150	180	200	130	200	-
Size (BSCF)	400	180	250	390	410	470	390	-
Maximum initial capacity	100	120	100	110	130	140	100	200
Allowable expansion	-	-	-	-	-	-	-	150
Fix. cost- WP to WP pipelines	110	110	110	110	110	110	110	-
Fix. cost- PP to WP pipelines	214	199	229	229	234	219	214	-
Fix. cost- installation/expansion	1500	1500	1500	1500	1500	1500	1500	1000
Var. cost-installation/expansion	140	140	140	140	140	140	140	140

For the TRO model, NPV target is set with respect to the aggressiveness index α, from zero to one at increments of 0.1. In each case, a binary search on γ is performed to identify γ^*. Consequently, γ^* reflects the degree of conservativeness used in deriving a development plan since setting a higher target would require considering a lower robustness index (i.e., higher estimates of the uncertain deliverabilities and sizes).

The TRO model shows an average solution time of 24.8 seconds. Out-of-sample testing has been performed using 5000 realizations of the deliverabilities and sizes simulated using the triangular distributions described above. This leads to the largest sampling error of 0.02 with respect to the coefficient of variation of the results. Tables 5.2 and 5.3 present the performance of the TRO model in terms of mean NPV, standard deviation of the NPV and target achievement probability. The values for the mean and standard deviations have been rounded off to the nearest hundred.

The computational studies strongly suggest that the probability of target attainment is significantly improved by TRO. It can be observed that the expected NPV increases until the target level reaches an α of 0.7 and then declines when α (and τ) increases further. Furthermore, it is interesting to note that the expected profit at the highest level of $\alpha = 1.0$ is worse than the expected profit obtained from using the lowest possible target level. Moreover, the standard deviation of the realized NPV can be observed to be increasing with higher targets. These observations are consistent with the fact that higher targets can only be achieved at the expense of greater risks.

Table 5.2
Performance of TRO model solution based on triangular distributions

Target	$\tau_0 = 29,336$	$\tau_1 = 29,929$	$\tau_2 = 30,522$	$\tau_3 = 31,115$	$\tau_4 = 31,708$
α	0	0.1	0.2	0.3	0.4
γ^*	1.0	0.9619	0.9248	0.8857	0.8486
Mean NPV	29,336	29,930	30,495	31,057	31,554
Std. Dev. NPV	0	63	233	523	900
$P(NP \geq \tau_0)$	1.00	0.9980	0.9930	0.9840	0.9730
$P(NP \geq \tau_1)$	0	0.9975	0.9920	0.9810	0.9695
$P(NP \geq \tau_2)$	0	0	0.9900	0.9770	0.9685
$P(NP \geq \tau_3)$	0	0	0	0.9755	0.9630
$P(NP \geq \tau_4)$	0	0	0	0	0.9590
$P(NP \geq \tau_5)$	0	0	0	0	0
$P(NP \geq \tau_6)$	0	0	0	0	0
$P(NP \geq \tau_7)$	0	0	0	0	0
$P(NP \geq \tau_8)$	0	0	0	0	0
$P(NP \geq \tau_9)$	0	0	0	0	0
$P(NP \geq \tau_{10})$	0	0	0	0	0

What follows is the comparison between the performance of the TRO model solution with those obtained from SAA optimization models. Two sample sizes (i.e., 50 and 100) have been used in the optimization of the expected

Table 5.3
Performance of TRO model based on triangular distributions (continued)

Target	$\tau_5 = 32,302$	$\tau_6 = 32,895$	$\tau_7 = 33,488$	$\tau_8 = 34,081$	$\tau_9 = 34,674$	$\tau_{10} = 35,267$
α	0.5	0.6	0.7	0.8	0.9	1.0
γ^*	0.7979	0.7373	0.6768	0.6162	0.5557	0.1924
Mean NPV	31,995	32,324	32,501	32,470	32,179	20,129
Std. Dev. NPV	1,372	1,965	2,683	3,502	4,391	11,860
P($NP \geq \tau_0$)	0.9610	0.9405	0.9035	0.8640	0.8075	0.2485
P($NP \geq \tau_1$)	0.9570	0.9290	0.8885	0.8500	0.7910	0.2220
P($NP \geq \tau_2$)	0.9530	0.9210	0.8805	0.8305	0.7715	0.2055
P($NP \geq \tau_3$)	0.9435	0.9095	0.8690	0.8145	0.7480	0.1830
P($NP \geq \tau_4$)	0.9310	0.8955	0.8565	0.7995	0.7265	0.1615
P($NP \geq \tau_5$)	0.9260	0.8835	0.8380	0.7810	0.7065	0.1455
P($NP \geq \tau_6$)	0	0.8755	0.8210	0.7575	0.6820	0.1285
P($NP \geq \tau_7$)	0	0	0.8075	0.7355	0.6565	0.1110
P($NP \geq \tau_8$)	0	0	0	0.7145	0.6360	0.0950
P($NP \geq \tau_9$)	0	0	0	0	0.6115	0.0775
P($NP \geq \tau_{10}$)	0	0	0	0	0	0.0680

NPV from the SAA model. Under each sample size, three sets of samples are used to solve the SAA model, generating three different planning solutions. The performance of these solutions are tabulated in Table 5.4. The average computer solution times for the SAA model are 84.6 and 320.2 seconds for sample sizes of 50 and 100, respectively.

With the exception of SAA_2^{50}, the performance measures do not significantly deviate from each other and are comparable to the results obtained under $\alpha \in [0.8, 0.9]$. Hence, the TRO model is generally on par with the results obtained from the SAA model. While the expected NPV from $\alpha \in [0, 0.5]$ may be lower for the TRO model, their respective standard deviations are significantly lower than the SAA's. More importantly, for the targets that are achieved under this range of α, the achievement probabilities are also consistently higher.

Table 5.4
Out-of-sample performance evaluation of SAA model based on triangular distributions

	SAA_1^{50}	SAA_2^{50}	SAA_3^{50}	SAA_1^{100}	SAA_2^{100}	SAA_3^{100}
Mean NPV	32,395	30,124	32,331	32,333	32,377	32,396
Std. Dev. NPV	3,640	4,408	3,997	3,768	3,537	3,443
$P(NP \geq \tau_0)$	0.8560	0.7240	0.8330	0.8440	0.8555	0.8605
$P(NP \geq \tau_1)$	0.8365	0.7045	0.8165	0.8235	0.8425	0.8495
$P(NP \geq \tau_2)$	0.8200	0.6815	0.8010	0.8100	0.8220	0.8300
$P(NP \geq \tau_3)$	0.8055	0.6565	0.7805	0.7905	0.8080	0.8135
$P(NP \geq \tau_4)$	0.7885	0.6100	0.7590	0.7725	0.7880	0.7950
$P(NP \geq \tau_5)$	0.7655	0.5585	0.7355	0.7540	0.7710	0.7760
$P(NP \geq \tau_6)$	0.7440	0	0.7165	0.7355	0.7520	0.7580
$P(NP \geq \tau_7)$	0.7070	0	0.6960	0.7185	0.7330	0.7385
$P(NP \geq \tau_8)$	0.6575	0	0.6695	0.6700	0	0
$P(NP \geq \tau_9)$	0	0	0	0	0	0
$P(NP \geq \tau_{10})$	0	0	0	0	0	0

There can be cases when the decision maker does not have a specific NPV target in mind. As shown in this section, it may be essential to determine a range of targets that would be appropriate for the problem. It is to be noted that even if this will result to solving the model repeatedly (for each target level), the solution time from doing so would still be less then solving an appropriately-sized SAA model.

Figure 5.3 plots the expected profit versus standard deviation performance of both the TRO and SAA solutions. The plot indicates that the TRO solutions (with the exception of $\alpha = 1$) are nondominated (best possible combinations of risk and expected profit) and describes an efficient frontier of solutions. On the other hand a SAA solution using 50 samples is clearly dominated. An interesting implication of the efficient frontier description is that it can be generated by simply scanning through the levels of α in the TRO model. Knowledge of the efficient frontier is valuable to decision-makers looking for a suitable trade-off between risk and the expected NPV.

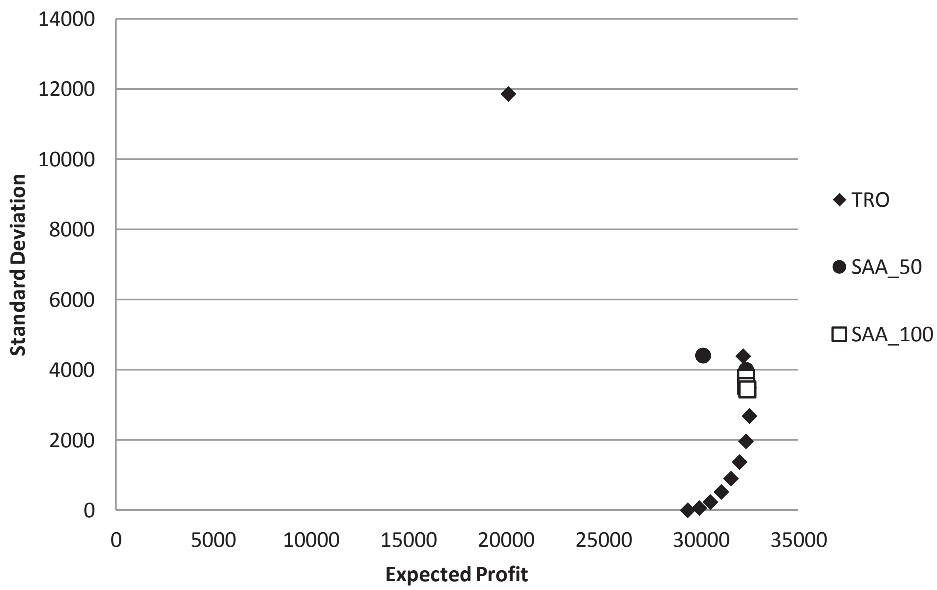

Fig. 5.3. Solution performance comparison plot based on triangular distribution.

5.4.1 Multi-stage Development Planning Case Study

In this section, computational studies are performed for a multi-stage gas field development problem using the TRO model. In the problem, there is a choice of six well platforms and one production platform (using data of WP_1-WP_6, and PP_1 in Table 5.1) to be developed over a planning horizon of ten years (stages). A discount rate β of 12% is assumed and the constraint that no more than four well platforms can be installed on the first year of the project is imposed.

The folding horizon approach is adopted for the multi-stage planning, so that at each stage, only the here-and-now decisions are implemented. As before, NPV target levels are set based on varying the aggressiveness index $\alpha \in [0, 1]$ in steps of 0.1. At each of the subsequent stages, the targets are then allowed to change accordingly so that the aggressiveness index remains fixed. To evaluate the performance of the TRO solution, 200 realizations of the deliverabilities and sizes are generated using a uniform distribution. The average solution time for one replication of the rolling horizon implementation is 37 seconds. Table 5.5 displays the evaluated performance at each target level. The values for the mean and standard deviations have been rounded off to the nearest hundred.

Table 5.5
Mean and Standard deviation for targets

α	0	0.1	0.2	0.3	0.4	0.5
Mean NPV	308,900	309,350	309,320	310,790	308,890	308,670
Std. dev	40,664	40,699	40,639	41,489	45,841	46,027
α	0.6	0.7	0.8	0.9	1.0	
Mean NPV	308,160	307,530	307,140	306,170	306,040	
Std. dev	46,645	47,764	48,158	48,846	48,719	

The above results are generally qualitatively consistent with the observations from the computational study in the previous subsection. There is a general trend that as α increases, the standard deviation of NPV increases and the expected NPV increases up to a certain level before declining again. It can be noted that in order to achieve the most aggressive target (for $\alpha = 1$), the resulting development plan has to suffer in its robustness to uncertainties. The results in Table 5.5 show that this position leads to the worst performance of expected NPV and standard deviation.

For a more detailed analysis, observe the well platform installation solutions under the extreme cases of $\alpha = 0$ and $\alpha = 1$. These are shown in Tables 5.6 and 5.7, respectively.

Table 5.6
Well platform installation schedule for target $\alpha = 0$.

Well \ Year	1	2	3	4	5	6	7	8	9	10	Capacity
W1	1.00	-	-	-	-	-	-	-	-	-	58.91
W2	1.00	-	-	-	-	-	-	-	-	-	39.40
W3	1.00	-	-	-	-	-	-	-	-	-	57.26
W4	-	0.45	0.26	0.25	0.04	-	-	-	-	-	29.08
W5	-	0.82	0.18	-	-	-	-	-	-	-	35.71
W6	1.00	-	-	-	-	-	-	-	-	-	44.47
P1	1.00	-	-	-	-	-	-	-	-	-	200.00

Table 5.7
Well platform installation schedule for target $\alpha = 1.0$.

Well \ Year	1	2	3	4	5	6	7	8	9	10	Capacity
W1	1.00	-	-	-	-	-	-	-	-	-	65.82
W2	-	1.00	-	-	-	-	-	-	-	-	80.77
W3	1.00	-	-	-	-	-	-	-	-	-	53.85
W4	1.00	-	-	-	-	-	-	-	-	-	33.33
W5	1.00	-	-	-	-	-	-	-	-	-	47.00
W6	-	1.00	-	-	-	-	-	-	-	-	69.47
P1	1.00	-	-	-	-	-	-	-	-	-	200.00 (+ 134.93 in year 2)

The values tabulated under columns of the first to tenth year are the probabilities of developing a particular well in a particular year, evaluated over the 200 replications. First, under the least aggressiveness level $\alpha = 0$, well installations are observed to take place up to the fifth year. Under the most aggressiveness level $\alpha = 1$, all wells are installed by the end of the second year. A likely explanation for this is that in the aggressive target setting, the projected well parameter values (and hence their associated profitabilities) are more optimistic (larger). On the other hand, in the conservative target setting, the well parameter estimates (in particular for W_4 and W_5) are too low and unattractive in returns (compared to the other, larger wells) for development in the first year. These are only developed in the later years to leverage on the investment in production capacity when the deliverabilities of the other wells have diminished to comparable levels.

It can also be observed that with $\alpha = 0$, the first set of explored wells tend to be the largest amongst the group (in the size parameter). For $\alpha = 1$, the installation of some of the larger wells are delayed until the second year. In this case, the higher estimated values of the well deliverabilities lead to a higher projected consumption of production capacity. This limited the development of some of the large wells in the first year. In the second year, when the realized deliverabilities and sizes turn out to be less than expected, the remaining wells are developed, and production capacity is also expanded.

At lower aggressiveness levels, the corresponding projections for the well parameters are also considerably lower. Specifically, having $\alpha = 0$ would refer to generating a plan under the worst possible values for the deliverabilities and sizes. This results in lower capacity levels assigned to the well platforms. Furthermore, under these conservative projections, there is also little incentive of expansion for the production platform. Thus, when the observed well deliverabilities and sizes are larger than expected, there would still be sufficient reserves to produce up to capacity in the succeeding year without exploring all of the remaining wells immediately.

When the highest aggressiveness index is employed, i.e. $\alpha = 1$, the projections for the deliverabilities and sizes are at their most optimistic (highest) levels. This leads to the belief that the expansion of the production platform would be most profitable. Therefore, when the actual deliverabilities and sizes are found to be smaller than expected, the production and well platforms become under-utilized. In effect, larger-than-necessary capital expenditures are made in this plan.

Finally, Table 5.8 shows the well installation schedule when the target is set with aggressiveness $\alpha = 0.3$ (which from Table 5.5 yielded highest expected NPV with comparably low standard deviation). It can be observed that the solution in this case generally resembles that with aggressiveness $\alpha = 0$, the main difference being in the higher assigned capacities of the well platforms. Also, well platform installations take place only up to the third year, instead of the fourth and production platform will be expanded in year 2. More specifically, the probability that well $W4$ is developed in the second year is significantly higher than the case with $\alpha = 0$. As before, this is due to the relatively higher projections used for the deliverability of $W4$, which justifies its development by the second year.

Table 5.8
Well platform installation schedule for fixed target $\alpha = 0.3$.

Well \ Year	1	2	3	4	5	6	7	8	9	10	Capacity
W1	1.00	-	-	-	-	-	-	-	-	-	53.62
W2	1.00	-	-	-	-	-	-	-	-	-	39.91
W3	1.00	-	-	-	-	-	-	-	-	-	56.18
W4	-	0.83	0.17	-	-	-	-	-	-	-	32.03
W5	-	1.00	-	-	-	-	-	-	-	-	44.06
W6	1.00	-	-	-	-	-	-	-	-	-	50.29
P1	1.00	-	-	-	-	-	-	-	-	-	200.00(+ 49.64 in year 2)

6. SUPPLY NETWORK DESIGN

Stochastic and robust optimization approaches have been extensively applied to supply network design problems. Most stochastic models seek to optimize the expected costs or profits of the problem of concern. Balachandran and Jain (1976), LeBlanc (1977), Franca and Luna (1982) considered joint capacitated facility location, production, and distribution problems facing continuous random demands. The objective is to minimize the expected cost of location, production, transportation, and underage and overage. Similarly, Gregg et al. (1988) minimize a weighted sum of the production cost (a one-time cost for establishing capacity), the transportation cost, and expected overage and underage costs. The authors applied the model to a case study involving the Queens borough public library system in New York City. Daskin et al. (2002) developed a joint location-inventory model that minimizes the expected cost of facilities location, transportation, and holding inventory under stochastic demands.

Snyder (2006) broadly classifies the robust optimization literature as minimax cost and minimax regret approaches. A minimax cost solution is one that minimizes the maximum cost across all scenarios. The other robustness measure is the *regret* of a solution, that is the difference between the cost of a solution in a given scenario and the cost of the optimal solution for that scenario. Models that seek to minimize the maximum (absolute or relative) regret across all scenarios are called minimax (absolute or relative) regret models. Averbakh and Berman (1997) uses a minimax-regret formulation of the weighted p-center problem on a network where the uncertain weights are represented by interval. They showed that the problem can be solved through $n+1$ deterministic p-center problems. Subsequently, they have also developed a polynomial-time algorithm for the general problem (Averbakh and Berman, 2000).

Unfortunately, with the exception of very specially structured problems (e.g. Chen and Lin (1998), Averbakh and Berman (1997)), minimax regret models are extremely difficult to solve, even if the deterministic problem was easy. On more general networks, heuristic approaches are used to solve the problems. Serra et al. (1996) solve the maximum capture problem (to locate a given number of facilities in order to capture the maximum market share, given that the firm's competitors have already located their facilities) under scenario-based demand uncertainty. Killmer et al. (2001) considered a stochastic network design problem, with the objective of minimizing the expected cost plus penalties for regret, demand shortfalls, and capacity surpluses. The expected cost and regret penalty are the solution robustness terms (encouraging solutions to be close to optimal), while the demand and capacity variation penalties are model robustness terms (encouraging solutions to be close to feasible). The non-linear programming model is applied to a small case study involving the location of hazardous waste treatment facilities in Albany, NY and is solved using MINOS.

6.1 Network Design and the Environment

A substantial body of literature exists on the integration of environmental considerations into network design problems. Beamon and Fernandes (2004) consider a firm that manufactures new products and remanufactures used products. Products at the end of their useful life are recovered, processed and eventually reintroduced to the market. These recovery activities require additional decisions such as identifying which warehouses and collection centers to open and the amount of material to transport between each pair of sites. They model the problem as a mixed integer linear program that identified the types of facilities to be installed and the respective capacities to be allocated.

Jayaraman et al. (2003) consider a recovery network for products that have either been returned, recalled or disposed of. These products are assumed to originate from customer collection points or retailers, which are then brought to collection facilities and eventually to the recovery facilities for processing. Similar to Beamon and Fernandes (2004), a mixed integer programming model was developed to minimize the transfer costs from customer collection to recovery and the fixed charge in opening these facilities. Sahyouni et al. (2007)

extend the problem by considering bidirectional distribution facilities in the network alongside dedicated facilities for forward and reverse flows. That is, these facilities handle the flows for both new (forward) and used (reverse) products.

Wang et al. (2011) likewise consider a network design problem. But rather than dealing with it from a product recovery perspective (i.e., creating a network to support recovery activities such as collection, remanufacture and disposal), decisions have instead been made for the "forward" or downstream flows in the supply chain. Their work includes decisions on environmental investments to facility location and capacity allocation. Furthermore, it also determines an initial investment on environmental protection equipment and techniques for the flow of materials and products between facilities.

There are relatively fewer published works in network planning with consideration of greenhouse gas emissions. Diabat and Simchi-Levi (2009) consider a design problem of a green supply chain network consisting of facility location and distribution decisions in the presence of a carbon cap requirement. Carbon emissions can come from facilities and distribution activities. A mixed integer programming model is developed to maximize the supply chain profits while meeting the carbon emission constraint. Wang et al. (2011) considers a similar problem, integrating the environmental component by designing a network that curtailed the adverse environmental impacts brought about by CO_2 emissions. However, they model the network problem as a bi-objective optimization model that seeks to minimize costs and CO_2 emissions in the supply chain. They obtain a Pareto Frontier by using an aggregate objective function on these two objectives.

In the area of reverse logistics, Listes and Dekker (2005) consider the presence of uncertainty for a network problem based on a case of sand collection, recycling, and reuse from demolition sites in the Netherlands. The authors propose a three-stage stochastic mixed integer programming model for choosing facility locations and product flow decisions. Lieckens and Vandaele (2007) consider the stochasticity of lead times, quantity and quality of products flowing back to the supply chain. They combine a mixed integer programming and a queuing model to identify which facilities to open that minimize the investment, operation and penalty costs of the network subject to the uncertainties. Finally, Pishvaee et al. (2011) use the minimax cost approach in designing a supply chain that accommodates the recovery and re-introduction of products to and from the consumers. They consider uncertainties in product demand, returns and transportation costs.

6.2 Supply Network Design

In this chapter, a mixed integer programming model is developed for the design of general supply networks under uncertainty, including the consideration of curbing harmful emissions such as greenhouse gases related to production and distribution activities. Examples of relevant supply networks include electricity generation and distribution, oil and gas, and semiconductor supply chains, which are among the largest contributors of harmful emissions in the industry. In the models, uncertainties are assumed to arise as a consequence of inaccurate projections of demand (consumption) levels and emission-related cost parameters. The supply network design problem of concern is framed as a two-stage planning problem. The first stage involves the determination of production facilities installation, the related production capacities, and also supporting infrastructure that connect supply and demand nodes. These decisions are made before the uncertainties are completely resolved. The second stage decisions involve formulating a feasible distribution plan for supply to meet demands when the uncertainties are observed. The robust optimization approach is adopted to mitigate the effects of parametric uncertainties in network design planning. The need for relatively few assumptions to model the uncertain parameters, as compared to stochastic optimization approaches, is viewed as particularly relevant for green supply chain management. This is because it may be difficult and subjective to assign probabilities confidently to uncertain outcomes associated factors such as new emission control technologies, the effects of emerging contaminants, and also future environmental policy outlooks.

Baron et al. (2010) considered a supply network design problem under uncertainty using the robust optimization approach. As in most of the literature in supply network design under uncertainties, the authors consider uncertainties arising from consumer demands. All other planning data are assumed to be precisely known. This assumption is relaxed in this work to allow also uncertainties in model data coefficients motivated by emission calculations. Another key difference of this work from Baron et al. (2010) is that, instead of fixing the uncertainty set and optimizing the worst-case cost, the goal of the proposed robust design model is to achieve a solution that maximizes the size of the uncertainty set with respect to cost budget requirements. Finally, the model proposed in Baron et al. (2010) assumes that all capacity and distribution decisions are made robustly before the uncertainty unfolds. This work instead allows distribution decisions to be adjustable in the uncertain parameters using affine decision rules models. That is, the distribution of the supply to the consumers is assumed to be performed after learning the consumer demands and emission penalty rates. This can exploit the flexibility of the supply network to improve planning.

6.3 A Basic Model for Supply Network Design

This section first describes the basic supply network design problem with the assumption that all planning data are accurately known *a priori* (i.e., assume no uncertainties). Define the set of nodes $\mathcal{I} := \{1, \cdots, I, s, t\}$, where nodes $i = 1, \cdots, I$ can represent either production or demand points, or both, and where s and t are respectively defined as 'dummy' source and sink nodes. An example of such a set up is found in multi-area power system networks (depicted in Figure 6.1 from Panida and Singh (2008)), where nodes are used to model a given region or area that can facilitate both power generation (production) and load (demand) activities (the source and sink nodes are not included in Figure 6.1 to simplify the presentation).

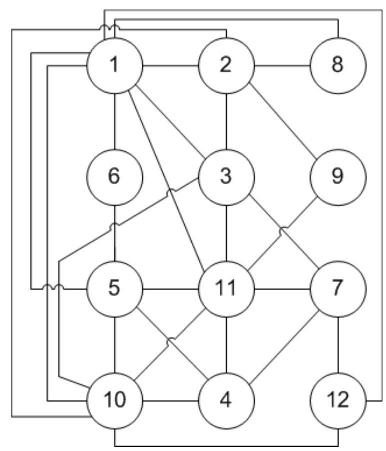

Fig. 6.1. 12-area test system

The transfer of supply between any two nodes is enabled through an installed *arc* connecting the pair of nodes. Generally, nodes in the network can behave as transit nodes, that is, unconsumed supply that arrives at a node can be directed to another node. Flow through the connected nodes may also be bi-directional. For example, bi-directional flows are common in closed loop supply chains where distribution centers partake in both forward and reverse logistics. In energy supply networks, each area may both transmit or receive energy from other areas depending on load situations. Define the arc (i, j) as the connection between nodes i and j, where $i, j \in \mathcal{I}$, and

$\mathcal{A} \subseteq \{(i,j) \ : \ i,j \in \mathcal{I}, i \neq j\}$ as the set of feasible arc connections. Note that we do not distinguish between (i,j) and (j,i) so that if $(i,j) \in \mathcal{A}$ then $(j,i) \notin \mathcal{A}$. The capacity of an arc (i,j) $\forall (i,j) \in \mathcal{A}$, i, $j \neq s$ models the transfer capacity of supply between production or demand nodes i and j, where $i,j = 1,\cdots,I$, $i \neq j$. On the other hand, the capacity of an arc (s,i) $\forall (s,i) \in \mathcal{A}$ models the production or supply capacity of the node i.

The supply network design problem addresses the following issues: (a) the configuration (interconnectivity of the nodes) of the supply network, (b) the capacity of the interconnections, and (c) distribution of supply via the interconnections to satisfy demands. Denote $h_{i,j}$ as the maximum installable capacity of arc $(i,j) \in \mathcal{A}$. The installation of these structures has fixed cost $f_{i,j}$ and capacity cost rate $u_{i,j}$. Define also d_i as the demand in node i, and e_i as the penalty cost per unit of unfulfilled demand. Demand shortages are allowed since the presence of uncertainties is of interest in the subsequent sections. Operational cost rates $c_{i,j}$ are associated with the costs of production and distribution activities on (i,j). In particular, these include CO_2 (equivalent) emission tax penalties for activities on (i,j). For instance, the emissions from the shipment of supplies from node i to j should be appropriately accounted for in the computation of $c_{i,j}$. For arcs (s,j), the cost $c_{s,i}$ models the production cost rate at node i, including emission costs of the production. For instance, the CO_2 emissions from a coal-fired power generation plant stationed in node i should be appropriately accounted for in the computation of $c_{s,i}$.

Define binary decision variables $y_{i,j} \in \{0,1\}$ so that $y_{i,j} = 1$ if the arc $(i,j) \in \mathcal{A}$ is installed, and $y_{i,j} = 0$ otherwise. Define also the capacity of the arc $p_{i,j} \geq 0$, which is determined upon its installation. Finally, let $x_{i,j} \geq 0$ be the flow of supply on the arc $(i,j) \in \mathcal{A}$. Note that for the arc (s,i) originating from the source node s, the flow $x_{s,i}$ models the supply level produced or generated at node i. For the arc (i,t) connecting the sink node t, the flow $x_{i,t}$ represents the level of supply consumed at node i. The supply network design network requirements are then stated as follows:

$$p_{i,j} \leq h_{i,j} y_{i,j} \qquad \forall \ \{(i,j) \in \mathcal{A} |\ j \neq t\} \tag{6.1}$$

$$x_{s,i} \leq p_{s,i} \qquad \forall \ s,i \in \mathcal{I} \tag{6.2}$$

$$|x_{i,j} - x_{j,i}| \leq p_{i,j} \qquad \forall \ \{(i,j) \in \mathcal{A} |\ i,j \neq s,t\} \tag{6.3}$$

$$\sum_{\{j|(j,i)\in\mathcal{A}\}} x_{j,i} - \sum_{\{j|(i,j)\in\mathcal{A}\}} x_{i,j} = 0 \qquad \forall \ \{i \in \mathcal{I} |\ i \neq s,t\} \tag{6.4}$$

$$x_{i,t} \leq d_i \qquad \forall \ i,t \in \mathcal{I} \tag{6.5}$$

$$\sum_{i\in\mathcal{I}} e_i d_i + \sum_{(i,j)\in\mathcal{A}} (f_{i,j} y_{i,j} + u_{i,j} p_{i,j} + c_{i,j} x_{i,j}) \leq \tau \tag{6.6}$$

$$y_{i,j} \in \{0,1\}, \quad p_{i,j}, x_{i,j} \in \Re_+ \qquad \forall \ (i,j) \in \mathcal{A} \tag{6.7}$$

Constraint (6.1) limits the installed capacity $p_{i,j}$ on (i,j) to the maximum of $h_{i,j}$, and (6.2) and (6.3) restrict flows on (i,j) to $p_{i,j}$. (6.4) are mass-balance equations on the nodes, and (6.5) are requirements of flows to the sink node t modeling resource consumptions at each node i, so that $d_i - x_{i,t} \geq 0$ are the unfilled demands or shortages. The cost budget constraint (6.6) requires that the total cost of the designed supply network should be no greater than τ, where τ is a specified budget level. The total costs include fixed costs and variable costs of capacity installation, operational costs and demand shortage costs. Note that the operational cost rate associated with $x_{i,t}$ is defined as $c_{i,t} = -e_i$, so that the left hand side in (6.6) correctly accounts for the demand shortage cost, $e_i(d_i - x_{i,t})$. That is:

$$\begin{aligned} & \sum_{i\in\mathcal{I}} e_i d_i + \sum_{(i,j)\in\mathcal{A}} (f_{i,j}y_{i,j} + u_{i,j}p_{i,j} + c_{i,j}x_{i,j}) \\ = \;& \sum_{i\in\mathcal{I}} e_i (d_i - x_{i,t}) + \sum_{(i,j)\in\mathcal{A}\setminus(i,t)} (f_{i,j}y_{i,j} + u_{i,j}p_{i,j} + c_{i,j}x_{i,j}) \end{aligned}$$

Finally, (6.7) specifies the domains of the decision variables in the model. In the deterministic problem, any supply network design that satisfies (6.1)–(6.7) is an acceptable design.

6.4 Supply Network Design Under Uncertainty: A Robust Design Approach

6.4.1 Modelling the Uncertainties

In the supply network design problem of interest in this work, the demand d_i, $i \in \mathcal{I}$ and the operational cost rates $c_{i,j}$, $(i,j) \in \mathcal{A}$ are assumed to be uncertain, or imprecisely known before implementing the design solution. The uncertainty in the operational costs c_{ij} in this work is motivated primarily by the uncertain outlook of the severity and degree of emission taxation imposed on the production and distribution activities. The computation of accountable emissions can be in itself rather complex, coupled with issues such as insufficient or inappropriate data, or lack of consensus on accounting techniques.

Let $\tilde{\boldsymbol{v}} \in \Re^K = [\tilde{v}_1, \cdots, \tilde{v}_K]$ denote a K-vector of uncertain factors or *primitive uncertainties*. It is assumed that $\tilde{v}_k$, $\forall\ k = 1, \cdots, K$ can take realizations contained in a discrete sample space $\tilde{v}_k \in \left\{v_k^1, \cdots, v_k^M\right\}$ where $-1 \leq v_k^m \leq 1,\ \forall\ m = 1, \cdots, M,\ k = 1, ..., K$. Define the following outcome space parameterized by the scalar $\gamma \in [0, 1]$:

$$\mathcal{Z}_\gamma^k = \left\{\tilde{v}_k \in \left\{v_k^1, \cdots, \tilde{v}_k^M\right\} \mid\ |\tilde{v}_k| \leq \gamma\right\} \quad \forall\ k = 1, ..., K \tag{6.8}$$

For a given γ, define an uncertainty set $\mathcal{Z}_\gamma$ containing a subset of $\tilde{\boldsymbol{v}}$ using the concatenation $\mathcal{Z}_\gamma = \mathcal{Z}_\gamma^1 \times \cdots \times \mathcal{Z}_\gamma^K$.

The factor model approach is adopted for the demand and cost uncertainties, which is a widely-used modeling assumption in robust optimization literature; see for instance, Ben-Tal et al. (2004) Chen et al. (2008); See and Sim (2010). A factor model of uncertainty formulates each uncertain parameter as an affine function of a set of uncertain factors. The uncertain parameters are then defined as affine functions of $\tilde{\boldsymbol{v}}$ using a set of factor coefficients. Similarly, it is assumed that the factor coefficients have been estimated (i.e., using past data studies, or experiments) *a priori*. The factor model can thus be regarded as a first order approximation of the uncertain parameter. Specifically, denote the demand and cost factor coefficients as d_i^k and $c_{i,j}^k$, respectively. The parameters then take on the following affine functions:

$$d_i(\tilde{\boldsymbol{v}}) = \overline{d}_i + \sum_{k=1}^{K} d_i^k \tilde{v}_k \qquad \forall\ i \in \mathcal{I},\ \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma \tag{6.9}$$

$$c_{i,j}(\tilde{\boldsymbol{v}}) = \overline{c}_{i,j} + \sum_{k=1}^{K} c_{i,j}^k \tilde{v}_k \quad \forall\ (i,j) \in \mathcal{A},\ \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma \tag{6.10}$$

where $\overline{d}_i$ and $\overline{c}_{i,j}$ are the nominal values of the consumer demand and production cost.

6.4.2 A Robust Design Model for Supply Network Planning

In the presence of uncertainties, the supply network planning decisions are made in two stages. The "here and now" (non-adjustable) decisions are long term planning decisions and need to be made before the uncertainties are resolved. These include the network configuration $\boldsymbol{y}$ and capacity installation $\boldsymbol{p}$. Upon observation of demands $\boldsymbol{d}$ and operational cost rates $\boldsymbol{c}$, the "wait and see" (adjustable) decisions, i.e. the distribution $\boldsymbol{x}$, are then implemented.

For a given value of γ, define the following *robust mixed integer optimization problem* over the uncertainty set $\mathcal{Z}_\gamma$:

$$r(\gamma) = \min z \tag{6.11}$$

s.t.

$$z \geq \sum_{i \in \mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j) \in \mathcal{A}} (f_{i,j} y_{i,j} + u_{i,j} p_{i,j} + c_{i,j}(\tilde{\boldsymbol{v}}) x_{i,j}(\tilde{\boldsymbol{v}})) - \tau \qquad \forall\ \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma \tag{6.12}$$

$$p_{i,j} \leq h_{i,j} y_{i,j} \qquad \forall\ (i,j) \in \mathcal{A}, j \neq t \tag{6.13}$$

$$x_{s,i}(\tilde{\boldsymbol{v}}) \leq p_{s,i} \qquad \forall\ i \in \mathcal{I},\ \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma \tag{6.14}$$

$$|x_{i,j}(\tilde{\boldsymbol{v}}) - x_{j,i}(\tilde{\boldsymbol{v}})| \leq p_{i,j} \qquad \forall (i,j) \in \mathcal{A}, i,j \neq s,t \quad \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma \tag{6.15}$$

$$\sum_{\{j | (j,i) \in \mathcal{A}\}} x_{j,i}(\tilde{\boldsymbol{v}}) - \sum_{\{j | (i,j) \in \mathcal{A}\}} x_{i,j}(\tilde{\boldsymbol{v}}) = 0 \qquad \forall\ i \in \mathcal{I},\ i \neq s,t \quad \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma \tag{6.16}$$

$$x_{i,t}(\tilde{\boldsymbol{v}}) \leq d_i(\tilde{\boldsymbol{v}}) \qquad \forall\ i \in \mathcal{I},\ \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma \tag{6.17}$$

$$y_{i,j} \in \{0,1\},\quad p_{i,j}, x_{i,j}(\tilde{\boldsymbol{v}}) \in \Re_+ \qquad \forall\ (i,j) \in \mathcal{A} \quad \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma \tag{6.18}$$

The objective function $r(\gamma)$ in (6.11) minimizes the worst-case budget shortfall formulated in the right-hand side of (6.13) achievable over $\mathcal{Z}_\gamma$, and each of the constraints (6.13)–(6.18) are required to hold for all uncertain parameter outcomes over $\mathcal{Z}_\gamma$. Note that positive values of $r(\gamma)$ indicates that the total cost exceeds the stipulated budget τ and is hence unacceptable.

The *robust design problem* then seeks to achieve non-positive budget shortfalls over $\mathcal{Z}_\gamma$ for as large γ as possible:

$$\gamma^* = \max_{\gamma \in [0,1]} \gamma \ s.t.\ r(\gamma) \leq 0 \tag{6.19}$$

where γ^* is the highest achievable robustness level across all feasible design solutions. Hence, the solution returned by the robust optimization problem (6.11)–(6.18) under γ^* is the required robust design solution. A simple bisection search algorithm (Algorithm 1) can be applied to solve (6.19). In each pass of the algorithm, the robust optimization model (6.11)–(6.18) is solved for a fixed γ.

Set tolerance level α;

Initialize the maximum possible γ to $\gamma^+ = 1$ and lowest possible γ to $\gamma^- = 0$;

while $(\gamma^+ - \gamma^-) > \alpha$ **do**

  $\gamma^* = \frac{\gamma^+ + \gamma^-}{2}$;

  Solve problem (6.11)–(6.18) given γ^* ;

  if $r(\gamma^*) \leq 0$ **then**

    $\gamma^- = \gamma^*$;

  else

    $\gamma^+ = \gamma^*$;

  end

end

Algorithm 1: Bisection search procedure on γ

6.4.3 Model Variation: Greenhouse Gas Emissions Budget

In some situations, practitioners may prefer to articulate design requirements such as a cap on CO_2 emissions, rather than monetizing the effects of emissions in a total cost model. The problem is then to achieve as high a robustness as possible in achieving both a cost budget τ and a CO_2 emission budget target τ'. Defining $w_{ij}(\tilde{\boldsymbol{v}})$ as the uncertain per unit CO_2 emissions associated with supply activity x_{ij}, the following two constraints describe the cost budget shortfall and emission budgets respectively.

$$\sum_{i\in\mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j)\in\mathcal{A}} (f_{i,j}y_{i,j} + u_{i,j}p_{i,j} + c_{i,j}(\tilde{\boldsymbol{v}})x_{i,j}(\tilde{\boldsymbol{v}})) \leq \tau,\ \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma \tag{6.20}$$

$$\sum_{(i,j)\in\mathcal{A}} \tilde{w}_{i,j}x_{i,j}(\tilde{\boldsymbol{v}}) \leq \tau', \quad \forall\ \tilde{w}_{i,j} \in W_{i,j},\ (i,j) \in \mathcal{A},\ \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma \tag{6.21}$$

As in the previous sections, $\tilde{w}_{i,j}$ can be modelled using the affine random factors uncertainty model where $W_{i,j}$ represents its uncertainty set for $(i,j) \in \mathcal{A}$. The results in the rest of this chapter can also be applied to this model variation without loss of generality. The connection of the robustness index γ to success probability given the presence of adjustable affine decisions is discussed in Appendix E.

6.5 Tractable Approximations for the Robust Design Model

6.5.1 Convex Uncertainty Sets

Solving the robust design problem (6.19) involves solving the following robust optimization problem (6.11)–(6.18) for a fixed robustness level γ repeatedly (see Algorithm 1). Since the constraints in $r(\gamma)$ are required to hold for all uncertain outcomes $\tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma$, the number of constraints in (6.11)–(6.18) is of the order M^K (M realizations, K uncertain parameters), which is clearly intractable for even moderate size problems. For clarity of exposition, in the following the discussion is focused on the cost budget constraint (6.12) (since these are the most complicated in the model), re-written in the following form for convenience.

$$\sum_{i\in\mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j)\in\mathcal{A}} (f_{i,j} y_{i,j} + u_{i,j} p_{i,j} + c_{i,j}(\tilde{\boldsymbol{v}}) x_{i,j}(\tilde{\boldsymbol{v}})) - z - \tau \leq 0 \quad \forall \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma$$

The above set of constraints can be replaced with the following single inequality, or *robust counterpart*:

$$\sum_{(i,j)\in\mathcal{A}} (f_{i,j} y_{i,j} + u_{i,j} p_{i,j}) - z + \sup_{\tilde{\boldsymbol{v}}\in\mathcal{Z}_\gamma} \left(\sum_{i\in\mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j)\in\mathcal{A}} c_{i,j}(\tilde{\boldsymbol{v}}) x_{i,j}(\tilde{\boldsymbol{v}}) - \tau \right) \leq 0 \tag{6.22}$$

Applying the affine factors uncertainty model for $c_{ij}(\tilde{\boldsymbol{v}}) = c^0_{i,j} + c^1_{i,j}\tilde{v}_1 + \cdots + c^K_{i,j}\tilde{v}_K$, the supremum operator term in (6.22) can be rewritten as:

$$\begin{aligned}
&\sup_{\tilde{\boldsymbol{v}}\in\mathcal{Z}_\gamma} \left\{ \sum_{i\in\mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j)\in\mathcal{A}} \left(c^0_{i,j} + c^1_{i,j}\tilde{v}_1 + \cdots + c^K_{i,j}\tilde{v}_K\right) x_{i,j}(\tilde{\boldsymbol{v}}) - \tau \right\} \\
&= \sup_{\tilde{\boldsymbol{v}}\in\mathcal{Z}_\gamma} \left\{ \sum_{i\in\mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j)\in\mathcal{A}} \left(c^0_{i,j} x_{i,j}(\tilde{\boldsymbol{v}}) + \sum_{k=1}^{K} c^k_{i,j}\tilde{v}_k \cdot x_{i,j}(\tilde{\boldsymbol{v}}) \right) - \tau \right\}
\end{aligned}$$

In the following, the uncertainty associated with the adjustable decisions $\boldsymbol{x}$ (i.e. assume that they are non-adjustable) is temporarily ignored and its dependence on $\tilde{\boldsymbol{v}}$ is suppressed. Furthermore, since the terms

$\sum_{(i,j)\in\mathcal{A}} (f_{i,j}y_{i,j} + u_{i,j}p_{i,j}) - z$ in (6.22) do not involve uncertainty, the discussion can be focused on the following constraint without loss of generality:

$$\sup_{\tilde{\boldsymbol{v}}\in\mathcal{Z}_\gamma} \left\{ \sum_{i\in\mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j)\in\mathcal{A}} \left(c_{i,j}^0 x_{i,j} + \sum_{k=1}^{K} c_{i,j}^k \tilde{v}_k \cdot x_{i,j} \right) - \tau \right\} \leq 0 \qquad (6.23)$$

For a given $\boldsymbol{x}$, the left-hand side of the above inequality involves the maximization of an objective function linear in $\boldsymbol{v}$ over a discrete set $\mathcal{Z}_\gamma$. Generally, this is a non-convex optimization problem that can be extremely difficult to solve for $\mathcal{Z}_\gamma$ of high cardinality. The key idea of achieving tractability in robust optimization is to construct uncertainty sets that result in safe and tractable robust counterparts. Consider the following constraint with the hypercube $\mathcal{H}_\gamma = \{\tilde{v}_k \in [-\gamma, \gamma], \forall\, k, \cdots, K\}$, :

$$\sup_{\tilde{\boldsymbol{v}}\in\mathcal{H}_\gamma} \left\{ \sum_{i\in\mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j)\in\mathcal{A}} \left(c_{i,j}^0 x_{i,j} + \sum_{k=1}^{K} c_{i,j}^k \tilde{v}_k \cdot x_{i,j} \right) - \tau \right\} \leq 0 \qquad (6.24)$$

Since $\mathcal{Z}_\gamma \subseteq \mathcal{H}_\gamma$, clearly (6.24) implies (6.23), and is hence a safe approximation of (6.23). Tractability is achieved by applying the established results in the robust linear optimization(see for instance Ben-Tal and Nemirovski (1999)). In particular, for any given $\boldsymbol{x}$, the left-hand side of (6.24) involves the maximization of the objective affine in $\boldsymbol{v}$ over the convex set $\mathcal{H}_\gamma$. Furthermore, since $\mathcal{H}_\gamma$ is a hypercube, it can be described using a small number of linear inequalities. This admits a reformulation of the robust counterpart in linear programming format of polynomial size (Bertsimas and Sim, 2004) similar to what has been shown from Proposition 1 in Chapter 3.

6.5.2 Affine Decision Rules and Adjustable Budget Partitioning

The robust supply design problem involves decisions $\boldsymbol{x}$ that are adjustable in the observed outcomes of the uncertain factors $\tilde{\boldsymbol{v}}$. Generally, robust counterparts of uncertain linear programming problems with adjustable decision variables are computationally intractable (Ben-Tal et al., 2004). In the robust optimization literature, a resolution is to restrict the policy space of all adjustable decisions to the set of policies affine in the uncertain data (i.e. affine decision rules). In particular, for the adjustable decisions $\boldsymbol{x}(\tilde{\boldsymbol{v}})$:

$$x_{i,j}(\tilde{\boldsymbol{v}}) = x_{i,j}^0 + \sum_{k=1}^{K} x_{i,j}^k \tilde{v}_k \quad \forall\ (i,j) \in \mathcal{A},\ \tilde{\boldsymbol{v}} \in \mathcal{H}_\gamma \tag{6.25}$$

where $x_{i,j}^0, \cdots, x_{i,j}^k$ are the affine rule parameters approximating the adjustable decision $x_{i,j}(\tilde{\boldsymbol{v}})$ to be optimized. For robust linear programming problems with deterministic left-hand side coefficients, affine decision rules results in tractable robust counterpart models that can be formulated in linear programming format of polynomial size (Ben-Tal et al., 2004). Affine decision rules are widely acknowledged to be very useful constructs that permits scalability of robust optimization models (Chen et al., 2008).

We now consider the robust formulation of the constraints (6.24). Unfortunately, (6.24) involves uncertain data coefficients (i.e., the operational costs $c_{i,j}(\tilde{\boldsymbol{v}})$), and hence, the resulting robust counterpart formulation remains intractable even with the application of affine decision rules for $\boldsymbol{x}(\tilde{\boldsymbol{v}})$. More specifically, the resolution of the supremum operation in (6.24) is generally a non-convex optimization due to the product terms of $\boldsymbol{v}$ in $c_{i,j}(\tilde{\boldsymbol{v}})x_{i,j}(\tilde{\boldsymbol{v}})$.

To circumvent the issue of tractability, in the following proposition a simple but safe approximation of (6.23) is proposed. The chief advantage of the approximation is that it retains the linear structure of the model, so that mixed-integer programming solvers can still be directly applied for solution purpose. The idea of the proposed approximation is based on creating artificial but adjustable partitions of the cost budget τ under uncertainty so that each resulting 'partitioned' constraint is more amenable to computation.

Proposition 3 *Consider the following set of constraints on $\boldsymbol{x}(\tilde{\boldsymbol{v}})$:*

$$\sup_{\tilde{\boldsymbol{v}} \in \mathcal{H}_\gamma} \left\{ \sum_{(i,j)\in\mathcal{A}} c_{i,j}^k \tilde{v}_k \cdot x_{i,j}(\tilde{\boldsymbol{v}}) - \tau_k \right\} \leq 0 \quad \forall\ k = 1, ..., K \tag{6.26}$$

$$\sup_{\tilde{\boldsymbol{v}} \in \mathcal{H}_\gamma} \left\{ \sum_{i\in\mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j)\in\mathcal{A}} c_{i,j}^0 x_{i,j}(\tilde{\boldsymbol{v}}) - \tau_0 \right\} \leq 0 \tag{6.27}$$

$$\sum_{k'=0}^{K} \tau_{k'} \leq \tau \qquad \forall\ \tilde{\boldsymbol{v}} \in \mathcal{H}_\gamma \tag{6.28}$$

where $\tau_0, \cdots, \tau_K$ are defined as real valued decisions adjustable in $\tilde{\boldsymbol{v}}$. (6.26) - (6.28) is then a safe approximation to the budget shortfall constraint (6.24).

Proof : In order for (6.26) - (6.28) to be a safe approximation of (6.24), it suffices to show that any $\boldsymbol{x} \in \{\boldsymbol{x} \mid \exists (\boldsymbol{x}, \boldsymbol{\tau}) \ \textit{feasible in} \ (6.26) - (6.28)\}$ is also feasible in (6.24). The following then holds true:

$$
\begin{aligned}
&\sup_{\tilde{\boldsymbol{v}} \in \mathcal{H}_\gamma} \left\{ \sum_{i \in \mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j) \in \mathcal{A}} c^0_{i,j} x_{i,j}(\tilde{\boldsymbol{v}}) + \sum_{k'=1}^{K} \sum_{(i,j) \in \mathcal{A}} c^{k'}_{i,j} \tilde{v}_{k'} x_{i,j}(\tilde{\boldsymbol{v}}) - \tau \right\} \\
&\quad \le \sup_{\tilde{\boldsymbol{v}} \in \mathcal{H}_\gamma} \left\{ \sum_{i \in \mathcal{I}} e_i d_i(\tilde{\boldsymbol{v}}) + \sum_{(i,j) \in \mathcal{A}} c^0_{i,j} x_{i,j}(\tilde{\boldsymbol{v}}) - \tau_0 \right\} + \sum_{k=1}^{K} \left[\sup_{\tilde{\boldsymbol{v}} \in \mathcal{H}_\gamma} \left\{ \sum_{(i,j) \in \mathcal{A}} c^k_{i,j} \tilde{v}_k \cdot x_{i,j}(\tilde{\boldsymbol{v}}) - \tau_k \right\} \right] \\
&\quad \le 0
\end{aligned}
$$

The first inequality follows from applying (6.28), and the basic argument that for any set of real valued functions, the sum of the supremum of each function is no less than the supremum over the sum of the functions. The second inequality follows from the assumption that $\boldsymbol{x}$ is feasible in (6.26) - (6.27).

□

The approximation in Proposition 2 is not yet directly amenable to tractable solution since (6.26) involves the product term $c^k_{i,j} \tilde{v}_k \cdot x_{i,j}(\tilde{\boldsymbol{v}})$. However, this can be resolved by enumeration of $\tilde{v}_k$ on its discrete outcome space. Define $\tilde{\boldsymbol{v}}^-_k = [\tilde{v}_1, \cdots, \tilde{v}_{k-1}, \tilde{v}_{k+1}, \cdots, \tilde{v}_K] \in \Re^{K-1}$, that is, all the components of

$\tilde{\boldsymbol{v}}$ less $\tilde{v}_k$. In a similar sense define also $\mathcal{H}_\gamma^{k-} = \mathcal{H}_\gamma^1 \times \cdots \times \mathcal{H}_\gamma^{k-1} \times \mathcal{H}_\gamma^{k+1} \times \cdots \times \mathcal{H}_\gamma^K$. Constraint (6.26) is then equivalent to the following:

$$\sup_{\tilde{\boldsymbol{v}}_k^- \in \mathcal{H}_\gamma^{k-}} \left\{ \sum_{(i,j)\in\mathcal{A}} c_{i,j}^k \tilde{v}_k \cdot x_{i,j}(\tilde{\boldsymbol{v}}) - \tau_k \right\} \leq 0 \quad \forall\ \tilde{v}_k \in \left\{ v_k^1, \cdots, v_k^M \right\} \tag{6.29}$$

Note that in each of the above constraints (one for each $\tilde{v}_k \in \left\{ v_k^1, \cdots, v_k^M \right\}$), the supremum operation is performed over $\tilde{\boldsymbol{v}}_k^- \in \mathcal{H}_\gamma^{k-}$. Also, in this scheme, the number of constraints required is of the order $K \times M$, and hence the resulting robust counterpart model is polynomial in size. Finally, since the linear constraints (6.27)–(6.29) are deterministic in the constraint coefficients, affine decision rules can be applied to $\boldsymbol{x}(\tilde{\boldsymbol{v}})$ and $\boldsymbol{\tau}$ to achieve a tractable approximation for the adjustable robust optimization problem.

6.6 Computational Experiments

This section presents the numerical studies of the proposed robust design model for supply network design. We consider a power supply and distribution planning problem involving the installation of a set of generation facilities and their interconnections (i.e. via transmission lines). The cost of the network includes installation, capacity, demand shortage and environmental (CO_2 equivalent emission) costs. The objective of the design problem is to meet a given cost budget as well as possible under data uncertainty.

6.6.1 Power Generation Case Problem

The case study involves a hypothetical twelve-area (node) power supply-demand system (refer to Figure 6.1), consisting of twelve power generation units and 27 connections between areas. The parameters used for the annual generation and load activities in each area are presented in Table 6.1. Note that there are different types of generation plants in the system, including coal (CL), hydroelectric (HY), geothermal (GT) and natural gas (NG) plants.

Table 6.1
Generation and load parameters

Area	Load (MW)	Generation (MW)	Type of Generation	Fixed Cost ($M)
1	-	8,220	CL	3,200
2	6,750	8,220	CL	3,200
3	5,850	8,220	CL	3,200
4	10,650	4,795	NG	550
5	-	4,795	GT	250
6	1,500	822	HY	1,650
7	-	822	HY	1,650
8	2,000	5,000	GT	250
9	1,300	822	HY	1,650
10	2,550	4,795	GT	250
11	3,000	4,795	NG	550
12	-	4,795	NG	550

In this example, the load requirements and the emission rates are assumed to be uncertain data, modelled using the affine factor model approach with K=12 primitive uncertainties $\tilde{v}_k \in [-1, 1], \ \forall \ k = 1, \cdots, K$. An increment of 0.05 is used for the interval $[-1, 1]$, which leads to the discrete sample space $\tilde{\boldsymbol{v}} \in \{-1, -0.95, \cdots, 0.95, 1\}$, where the number of samples is equivalent to M=41.

The load factor coefficients d_i^k, $k = 1, \cdots, 12$ are randomly generated and assumed to be at most one half of the nominal values in Table 6.1. The nominal values and range of the uncertain emission rates are stated in Table 6.2 (from Lenzen (2008)). Table 6.3 shows the cost rate parameters used in the numerical study. The capacity cost is assumed to be \$0.01 per MW, where each connection has a maximum installable capacity of 5,000 MW. The activity cost parameter $\tilde{c}_{i,j}$ includes both operational cost (\$0.01 per MW) and emission cost (assumed to be \$20.00 per ton CO_2 (Trembath et al., 2012)). A shortage penalty of \$2.00 per MW is charged on every unit load that is not met by the system.

Table 6.2
Emission Parameters from Generation Plants (from Lenzen (2008))

Area	Range of emissions (g $CO_2 \setminus$ MWh)	Nominal emissions (g $CO_2 \setminus$ MWh)
Coal (1,2,3)	0.8430-1.0460	0.8430
Natural Gas (4,11,12)	0.4910-0.6550	0.4910
Geothermal (5,8,10)	0.0700-0.1100	0.0700
Hydroelectric (6,7,9)	0.0100-0.0440	0.0100

Table 6.3
Cost Parameters

Parameter	Value
Capacity cost	\$0.01 per MW
Operational cost	\$0.01 per MW
Connection installation cost	\$20 per connection
Emission cost	\$20.00 per ton CO_2 equivalent
Shortage penalty cost	\$2 per MW unfulfilled load

6.6.2 Performance Study of Robust Design Model

The performance of the proposed robust design model for the energy supply network problem is first studied through computational examples. All the models were developed using ROME version 1.0.8 beta (Goh and Sim, 2011). CPLEX was used in conjunction with ROME to solve the mixed integer programming models.

Table 6.4 presents the (worst-case) minimum cost $r(\gamma)$ obtained by fixing γ at various levels and solving the corresponding robust optimization problem (6.11)–(6.18). For the purpose of minimizing worst-case cost, the τ parameter in (6.12)–(6.18) is set to zero. The average computer solution time of the model is around 19.5 seconds. Generally, the optimal cost $r(\gamma)$ is non-decreasing in γ, which is not surprising, since higher γ levels correspond to more averse projections of the uncertain emission rates and loads. Also, under increased robustness, solutions favoring higher levels of capacity investments and stockpiling may be more preferable (this will be verified later). Note that the case $\gamma = 0$ corresponds to solving the nominal data problem without consideration of uncertainties. From Table 6.4, the optimal cost $r(\gamma) = 35,443$. An interpretation of this is that if one had specified the budget target $\tau = \$35,443$, the best achievable robustness $\gamma^*(\tau = \$35,443) = 0$. Since $\gamma = 0$ is least preferable from a robustness point of view, this implies that the budget might be too optimistic in the presence of uncertainties. A useful interpretation of

Table 6.4 is then the following. Increasing the budget levels (as in Table 6.4) enables developing solutions that improve the design robustness. For instance, increasing the total cost budget to $\$40,148$ increases robustness level γ to 0.6. More specifically, if τ is set to $\$40,148$ in the robust design problem (6.19), then the best achievable robust design solution permits hedging against all uncertainties arising from $\mathcal{Z}(\gamma = 0.6)$. Hence, Table 6.4 provides the *efficient trade-off* between the cost budget and the robustness level that is acceptable to the decision-maker. Clearly, the case when $\gamma = 1$ (with $r(\gamma) = \$48,177$ in Table 6.4) implies that the achieved solution is 'as good as it gets' from the robustness point of view. This implies that, for the purpose of robustness, one would never justify budgets above $\$48,177$.

Table 6.4
Equivalent minimum network costs with respect to the robustness of the supply design network

γ	0	0.40	0.60	0.80	1.00
$r(\gamma)$ ($'100)	$35,443$	$38,553$	$40,148$	$44,612$	$48,177$

The performance of the robust design solutions for the various robustness levels in Table 6.4 is evaluated using out-of-sample testing with 3,000 randomly-generated realizations of the uncertain data. Figure 6.2 plots the frequency diagrams of the total costs. Table 6.5 summarizes some important out-of-sample performance measures of the realized total cost (sample average, sample standard deviation, probabilities of budget target $\tau = r(\gamma)$, and expected shortfall with respect to $r(\gamma)$).

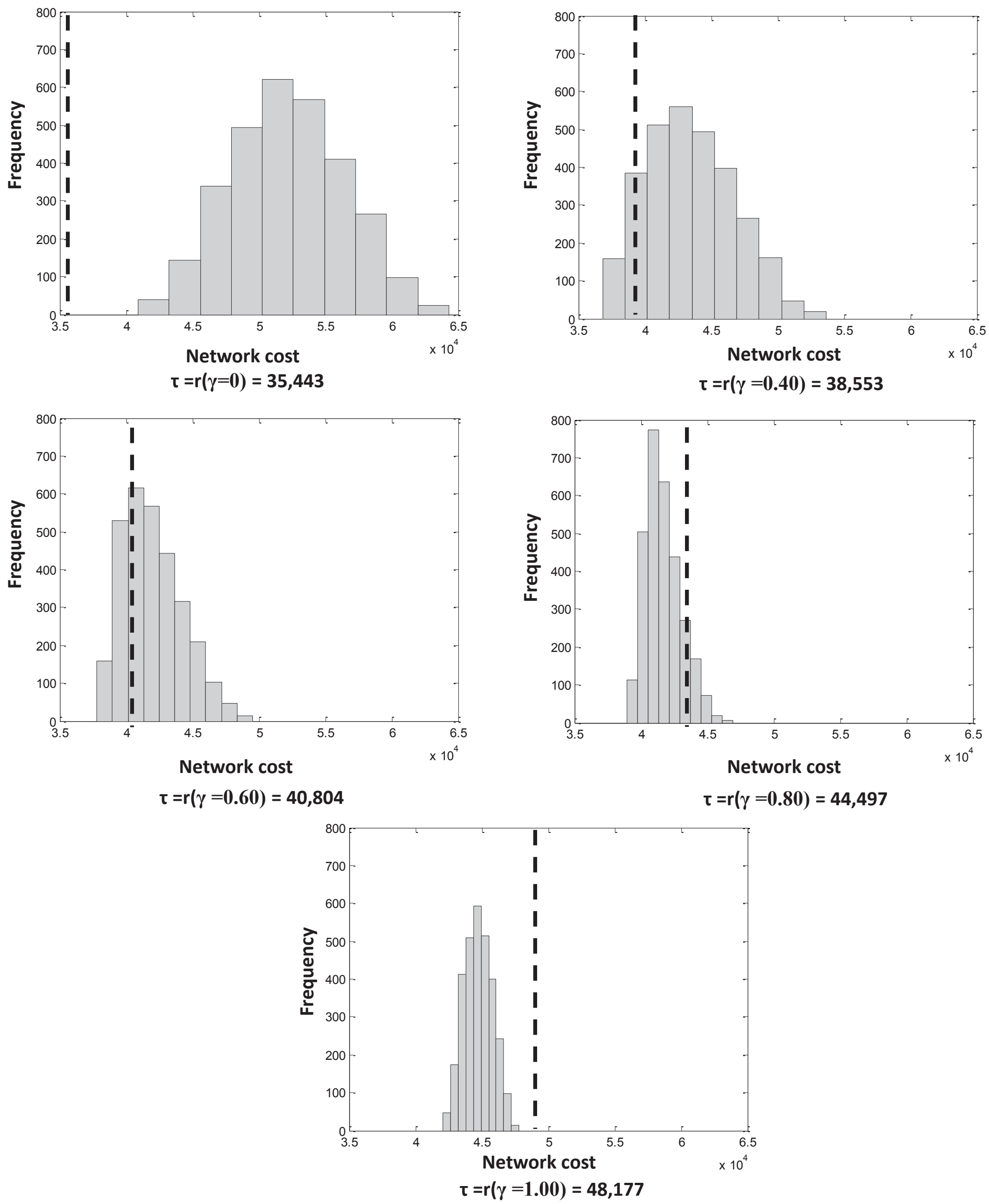

Fig. 6.2. Histogram of total network costs for the robust design model. The targets $\boldsymbol{\tau}$ refer to the minimum network costs $r(\gamma)$ (represented by dashed lines) and are superimposed on the respective histograms.

Table 6.5
Performance evaluation of robust design models based on uniform distributions

Robustness γ	0	0.40	0.60	0.80	1.00
Budget target τ ($'100)	35,443	38,553	40,148	44,612	48,177
Mean ($'100)	52,151	43,469	42,016	41,656	44,742
SD ($'100)	4,260	3,303	2,220	1,348	1,056
P($TC \leq r(\gamma = 0)$)	0	0	0	0	0
P($TC \leq r(\gamma = 0.40)$)	0	0.0557	0.0183	0	0
P($TC \leq r(\gamma = 0.60)$)	0	0.2367	0.3403	0.2957	0
P($TC \leq r(\gamma = 0.80)$)	0.0267	0.5970	0.8270	0.9453	0.3147
P($TC \leq r(\gamma = 1.00)$)	0.1887	0.9040	0.9947	1.0000	1.0000
EL($TC - r(\gamma = 0)$)	16,708	8,025	6,572	6,213	9,299
EL($TC - r(\gamma = 0.40)$)	13,598	4,946	3,468	3,103	6,189
EL($TC - r(\gamma = 0.60)$)	11,347	3,007	1,578	1,022	3,938
EL($TC - r(\gamma = 0.80)$)	8,017	1,057	256	35	776
EL($TC - r(\gamma = 1.00)$)	4,369	140	3	0	0

Qualitatively, the histogram plots in Figure 6.2 indicate that as robustness level γ increases, so does the probability of achieving the respective budget targets. Furthermore, the variability of the cost realizations decreases significantly with increasing robustness level. These trends are also observed in the summary statistics in Table 6.5. In addition, as the robustness level γ increases from 0 to 0.8, the sample average costs in Table 6.5 first decrease, and then increase when $\gamma = 1.0$ (an explanation for this will be discussed later). The probability of achieving a given budget target generally increases with solutions that are more robust (higher γ), with the exception of that corresponding to $\gamma = 1$. Note that in Table 6.5, the sample probability of achieving $\tau = r(\gamma)$ is zero; verifying again that the minimum cost from the nominal data model is too optimistic to be used as a budget in the presence of uncertainties.

The observed trends for the success probabilities are similar for the expected shortfalls so that generally, solutions with higher robustness also suffered lower budget target shortfalls. In summary these observations strongly suggest that the robustness level γ gives reasonably good gauge of solution performance that is consistent with the descriptive statistics performance measures.

A reason for the sample average costs observed in Table 6.5 can be explained using Figure 6.3 which plots, for each robustness level, the sample average cost components of capacity investments, load shortage penalties and emissions penalties. The plots show that capital investments increase while shortage penalties decrease as γ increases. This verifies the intuition that design solutions that offer higher capacity buffer are more favorable with increasing uncertainty aversion that is associated with the increasing robustness levels. Indeed, the simulation results in Figure 6.3 show that at $\gamma = 0$ and $\gamma = 0.4$, a (relatively) small increase in capacity investment costs is rewarded by a significantly higher level of savings in shortage penalties. The effectiveness of the trade-off however diminishes with increasing uncertainty aversion. In particular, when $\gamma = 1.0$, huge levels of capital investments are made. The large uncertain load projections however, are possibly too conservative from practical perspective, and consequently yields no significant marginal benefit from reducing load shortages. Finally, from Figure 6.3, emission penalty costs increase (very slightly) with increasing γ. It is important to note that the total realized loads served increases as γ increases, and hence the associated emission levels and operational costs are also expected to increase on average. However, the observed increase in the operational cost is very small, possibly due to the ability of the robust design solution to mitigate the emission penalties by diverting increased loads to cleaner power generators.

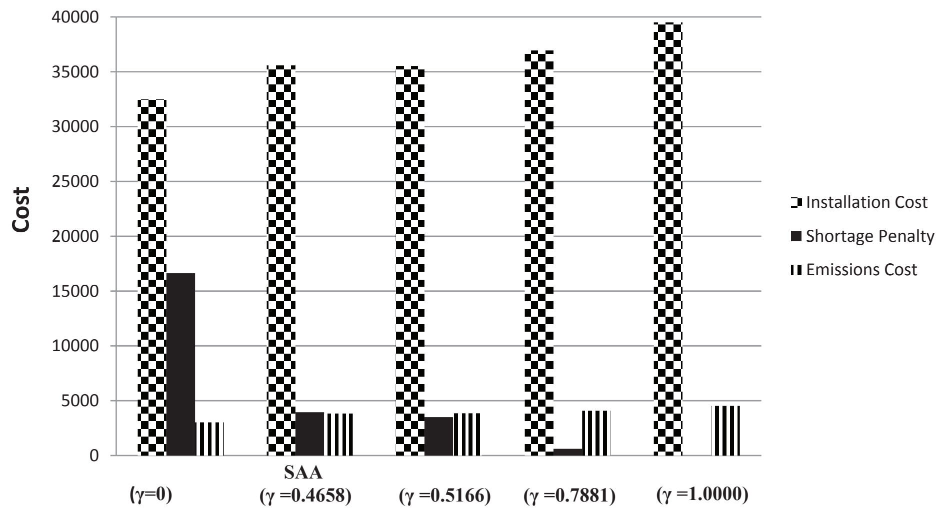

Fig. 6.3. Cost contributions of installation, emission and shortage penalty

6.6.3 Comparison of Robust Design with Sample Average Design

This section compares the performance of the robust design solution model with a design solution obtained using the *sample average approximation* (SAA) method. To build the SAA model, a discretized uniform distribution is assigned to the outcome space of $\tilde{v}_k$, for each $k = 1, \cdots, K$. Random samples of $\tilde{\boldsymbol{v}}$ were then generated as inputs for the SAA optimization model. Since the SAA solutions depend on the samples used, three sets of samples, for each of three different sample sizes (250, 500 and 1,000) were generated to obtain nine different design solutions. The average computer solution times for the SAA model are 76.8, 403.9 and 1,830.1 seconds for sample sizes of 250, 500 and 1,000, respectively. To facilitate a meaningful comparison with the robust design model, the largest objective function value from the nine instances of the SAA problem ($\$40,148$) was used as the budget target (i.e. set $\tau = \$40,148$) in all the subsequent computations. The robust design model (6.19) is then solved at this level of budget target to yield the robust design solution.

Performance Comparison

Out-of-sample performance of the two design solutions are evaluated using 3,000 randomly generated realizations of the uncertain parameters. The robustness and out-of-sample results are summarized in Table 6.6.

Table 6.6
Performance evaluation of models based on uniform distributions

	SAA Design	Robust Design
Robustness γ	0.45	0.55
Mean ($'100)	43,375	42,519
SD ($'100)	2,830	2,647
P($TC \leq \$40,148$)	0.1353	0.1633
EL($TC - \$40,148$)	3,325	2,880

Note that in Table 6.6, the robustness level of the SAA solution ($\gamma = 0.45$) is evaluated by fixing the budget level $\tau = \$40,148$ and the design variables (with the SAA solution) in the constraints (6.12)–(6.18), and then maximizing γ in (6.19). For the same budget level, the optimal robust design yields $\gamma = 0.55$. This indicates that there exists designs that strictly improves the robustness level over the SAA, the best of which allows protection against 55% of the uncertain data. Furthermore, the results in Table 6.6 show that for the out-of-sample mean, variance, success probability and expected losses, the robust design solution also outperforms the SAA design solution marginally. Note that the fact that the success probability is relatively low, is not indicative of the capability of the robust design model. Rather, what it means is that using the SAA optimal cost objective as a budget target itself, may be quite optimistic (since SAA is concerned only with expected performance). In summary, the results in Table 6.6 indicates that the improved robustness $\gamma = 0.55$ comes at no cost of the other performance measures, despite the fact that probability assumptions were not used explicitly in the robust design model.

Table 6.7 shows the cost breakdown of the sample average costs in the components of capacity installations, shortage and emission penalties for both solutions. The results are entirely consistent with those in the previous section: the robust design favors higher capacity investments in order to hedge against larger uncertain loads, and this is rewarded by substantial cost savings in the shortage penalty. The increase in emission costs, due to the larger volume of load served, on the other hand is comparatively much smaller.

Table 6.7
Cost contributions of capacity, shortage and emissions penalty

	SAA Design	Robust Design
Installation Cost ($'100)	35,130	35,516
Shortage Penalty ($'100)	4,514	3,100
Emissions Cost ($'100)	3,731	3,903
Total($'100)	43, 375	42, 519

Solution Comparison

We next study the supply network design solutions obtained by the SAA and robust design models. Table 6.8 shows the respective capacities assigned for the power generator at each node.

Table 6.8
Capacity of generation plants installed. Values in parenthesis are percentages with respect to maximum capacity of plant.

Plants	SAA Design	Robust Design
$1 - CL$	-	-
$2 - CL$	7,882 (95.89)	6,907(84.03)
$3 - CL$	7,993 (97.24)	7,361 (89.55)
$4 - NG$	4,795 (100.00)	4,795 (100.00)
$5 - GT$	4,795 (100.00)	4,795 (100.00)
$6 - HY$	822 (100.00)	822 (100.00)
$7 - HY$	-	-
$8 - GT$	4,795 (100.00)	4,395 (91.68)
$9 - HY$	822 (100.00)	822 (100.00)
$10 - GT$	4,795 (100.00)	4,795 (100.00)
$11 - NG$	4,726 (98.56)	3,774 (78.73)
$12 - NG$	-	3,810(79.48)

The total installed generation capacities for the robust design is 42,276 MW, slightly higher than that in the SAA design (41,425 MW). This is expected, and consistent with the insights from Table 6.7, since the robust design anticipates higher loads. In both designs, although coal plants are used to generate the largest amount of power supply (MW per plant), cleaner sources such as geothermal and natural gas have almost close to maximum capacity installed, compared to the coal plants. This indicates a strategy of prioritizing the use of cleaner technologies (whenever physically and economically feasible) to mitigate the emission penalties. Outstanding loads are then fulfilled with coal generated power. The fact that the hydro-electric generator in Area 7 is not installed in both solutions is due to the lack of economy of scale, since the hydro-electric plant has very small capacity and high fixed costs compared to the other plants.

Figure 6.4 plots the total capacity installed (as a percentage of maximum capacity) for each type of generation technology. It is interesting to note that for the robust design solution, there is a significant increase in natural gas generation capacity (and a corresponding decrease in coal-fired generation capacity) over the SAA design. This is chiefly due to the installation of the natural gas plant in Area 12 in the robust design solution (see Table 6.8). An explanation for this is that under the more averse projections of the loads and emission rates, the robust design model attempts to mitigate the increase in emission penalties by diverting the additional required generation from the coal-fired plants to natural gas plants. A more detailed discussion of the emission mitigation strategy based on the supply network configurations shown in Figure 6.5 is as follows.

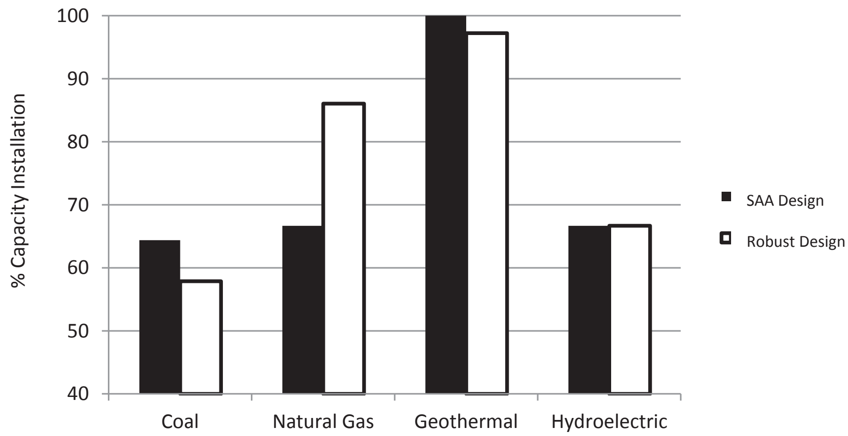

Fig. 6.4. Capacity installation (percentage of maximum capacity) of generation technologies

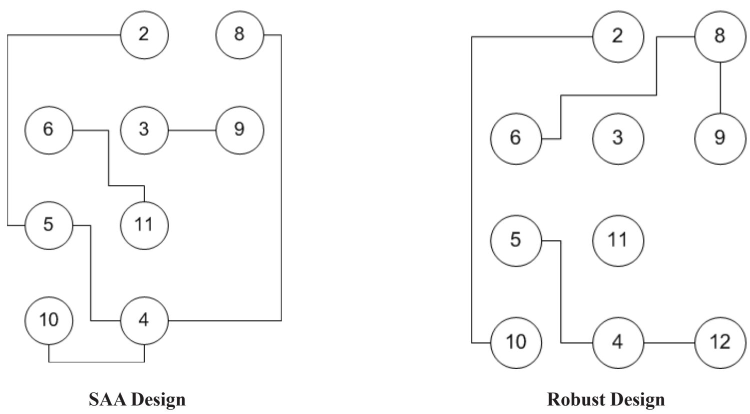

Fig. 6.5. Supply network configuration of SAA design and Robust design

We first discuss how a reduction in the coal use is achieved in the robust design solution for Area 2 (which contains a coal-fired plant). In the robust design solution, the geothermal plant in Area 10 is installed at maximum capacity (4795 MW) and is dedicated to serve the load in Area 2. Outstanding loads are supported by the coal-fired plant in Area 2 itself. On the other hand, in the SAA solution, Area 2 is supported by only a part of the generation capacity from Area 5 (also a geothermal plant with maximum capacity 4795). This is because Area 5 is also connected to Area 4 to jointly support the load requirement in Area 4 (together with the generators in Areas 8 and 10). Consequently, the coal-fired generator in Area 2 itself is installed at higher capacity. In contrast, for the robust design solution, the additional natural gas plant in Area 12 is dedicated to serve the load requirements in Area 4. This then frees up the capacity of the geothermal plant in Area 10, which is 100% dedicated to serve the load requirements in Area 2, hence effectively reducing coal use in Area 2.

Similarly, the capacity of the coal plant in Area 3 in the robust design is also lower compared to that in the SAA design. In the SAA design, Area 3 is connected to Area 9, in order to help supplement the supply for Area 9 (note that the hydro-electric generator in Area 9 is not installed; and Area 9 is not connected to anywhere else). In contrast, in the robust design, the coal plant in Area 3 is essentially 'off-the-grid' (i.e. it is not connected to any other areas), and is hence only required to generate enough supply for its own load requirements. Furthermore, the load from Area 9 is now supported by the cleaner geothermal plant in Area 8. This is again possible due to the addition of the natural gas plant in Area 12, which frees the geothermal capacity in Area 8 (from contributing to Area 4).

In summary, the robust design solution not only recommends the appropriate additional capacity to install, in order to improve hedging against load uncertainty (and hence reducing unserved loads), it effectively exploits the flexibility of the network connections to help divert power generation using coal to using cleaner sources, hence effectively mitigating the potential increase in CO_2 emissions.

7. CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

There has been a continuous stream of research works that make use of the satisficing approach within the context of optimization. This can be attributed to how inherent the approach is in the decision making process. This research proposed its application to production and logistics planning problems to contribute to the growing literature in this field of study.

This work initially applies the TRO model to dynamic workforce-inventory system and power supply system. The computational studies demonstrate that the application of the proposed optimization model is able to improve the behavior of both systems significantly under uncertainty. The results of the TRO model lead to the creation of stability and achievement of transient requirements in the inventory-workforce system. The oscillations in the power supply system are also seen to be controlled despite the fluctuations in the uncertain parameters. The respective models are able to generate policy parameter settings, which lead to the reduction in the frequency of oscillations and at the same time meet the required projections for the state variables throughout the simulation run. All in all, this applications show the effectiveness of the satisficing approach in the design of policies for dynamic systems. The approach has been able to identify appropriate settings that allowed the system to achieve its targets throughout the planning period.

Meanwhile, offshore gas field development is an important strategic problem in the oil and gas industry since it entails huge amounts of capital investment. The research highlights the importance of uncertainty in development planning arising from the efficacies of the unexplored gas reserves. The complexity in gas field development is further complicated by the issue of endogenous uncertainty in a dynamic decision-making process. To address these challenges, the target oriented robust optimization framework has been applied to the problem understudy. This framework allows a decision maker to flexibly specify target performance objectives such as net present value. The optimization model is solved to maximize the robustness of the planning solution to reservoir uncertainty in achieving the NPV target. Unlike in stochastic optimization approaches, the target oriented robust optimization model does not require specific assumptions on the probability distributions modeling the uncertain subsurface parameters

Finally, the third problem application involves a supply network design problem integrated with CO_2 considerations under uncertainty. A two-stage robust design mixed-integer programming model is proposed to maximize the protection of the supply design against uncertainties in demands and emission rate parameters. The design problem is to first determine the production facilities to install in the network, their capacity levels, and the network connectivity. When the uncertainties are revealed, a supply distribution plan is then formulated to minimize the costs of emissions and demand shortfalls. The problem is translated into a TRO model having the objective of achieving a total cost budget target as well as possible under uncertainty.

7.2 Summary of Contributions

7.2.1 Chapter 3-Robust Target Oriented Optimization Approach

This research develops a target-oriented robust optimization model that maximizes the degree of uncertainty for a given set of feasible solutions. The degree of uncertainty is defined as the robustness index, which is found to be coherent with an axiomatic definition of satisficing measures. Furthermore, the index is also shown to share fundamental similarities with the use of probability measures from the aforementioned axiomatic definition.

The robustness index is integrated into a decision-analytic framework that could be used in conjunction with the modeling paradigms of robust optimization and control theory based methods such as SD. A key advantage of the underlying mathematical programming model is its ease in computation, and that it does not require specific probability distributions to be assumed for the uncertain variables. Furthermore, it does not require the user to specify subjective weights as in typical utility maximization approaches. Rather, the user specifies the set of design goals to be satisfied. The computed solution is then one that seeks to achieve these goals as well as possible under uncertainty.

7.2.2 Chapter 4-Dynamic Systems

SD and stability analysis represent good platforms for the application of the TRO approach since these take on a perspective that a system could only be understood by considering a certain planning period rather than by mere instances of this period. Thus, decisions and policies would inevitably have to be made within the context of such operating conditions.

Currently, formal methods of SD analysis do not extend readily to design problems in the presence of parametric uncertainties. The TRO approach addresses this gap by incorporating eigenvalue analysis with a mathematical programming approach. It has been developed as a means to calibrate model parameters subject to the achievement of targets in the presence of uncertainty. It enables the achievement of dynamic performance specifications under uncertainties, so that stability and transient behaviors such as overshoots and oscillations in the system can be appropriately shaped.

The approach involves the approximation of nonlinear systems into their linear form. It utilizes information obtained from current eigenvalues and behavior of state variables to set the stability and performance measure requirements. Through the use of these approximations and the formulation of a TRO model, the approach has been able to simultaneously evaluate the effects of policy settings to both the behavior modes and state variable projections. The integration of the satisficing measure to the TRO model identifies the set of policy parameters that not only achieves the stability and performance measures requirements, but also maximizes the magnitude of uncertainty in the system. Decision makers can thus ensure that system targets would always be met even when uncertain parameters deviate from their nominal values.

7.2.3 Chapter 5-Offshore Gas Field Development Planning

A key technical contribution in this research is the demonstration that under the target oriented robust optimization approach, it is sufficient to consider fixed policies to obtain an optimal solution to the problem. Also, the optimal fixed policy can be evaluated by solving a single solution-independent worse case instance of the problem, which can be easily identified in the development planning problem. This is important since the resolution of a difficult dynamic planning problem under endogenous uncertainty is now reduced to solving a mixed integer programming problem that assumes a single outcome of the uncertain parameters. This was not possible for stochastic programming approaches in previously published works. Hence, the TRO model is highly attractive for practitioners who require fast turnaround planning and decision-support that can be efficiently implemented using commercially available solvers.

The computational studies demonstrate that the performance of the TRO model is very competitive if not better compared with SAA models. This is encouraging since the TRO model uses much less information (in particular the probability distributions) than the SAA models. It has also been shown that an efficient frontier (in the expected NPV and standard deviation) of solutions can be described systematically by solving the TRO model over a range of aggressiveness in target setting. Finally, the computational study of a multi-stage development planning problem indicates that while the model may initially plan for the worst-case outcomes, the folding horizon implementation allows the re-optimization to leverage on newly available information in the case of better-than-expected outcomes of the gas well parameters.

7.2.4 Chapter 6-Supply Network Design

This problem application shows how the TRO model could be used alongside the development of convex uncertainty sets and affine decision rules. To achieve computational tractability, a safe approximation is proposed based on the partitioning of the cost budget target. Consequently, solving the TRO model involves only solving a small collection of mixed integer programming models. Using a case example based on a power generation and distribution network system, the computational studies show that the proposed robust design model is highly competitive with an SAA model in various performance measures despite the fact that probability distributions were not explicitly used in the robust design model. Furthermore, investigating the solutions of the robust design model demonstrates its effectiveness in mitigating emissions level. This strongly suggests that the proposed robust design approach has an important role to play in green supply chain design and optimization under uncertainty.

7.3 Direction for Further Study

The satisficing measure presented in this paper has been developed based on a set of axioms. This therefore presents an opportunity to look into other properties of satisficing measures found in the literature. Among these properties include quasi-convexity, risk duality and measures that reward diversification. In connection, a future direction of this research may also involve the development of additional types of satisficing measures. This may primarily be done through the extension of the general framework defined by Brown and Sim (2010) and Brown et al. (2012), which will consider the magnitude of shortfall, computational tractability and joint probability of attributes meeting the targets.

This research also lays detailed methodological foundations and concepts that are valuable in developing comprehensive decision-support technology for resilient large scale systems in the future work. In this regard, one important consideration for future work will be the implementation and integration of the proposed approach with a SD modeling software platform. With the increasing availability of free optimization software in the Internet, a promising approach is an open source initiative to develop the software components of the proposed design framework. These can include linear dynamic system evaluation modules, local search algorithmic modules, and interface modules to various SD simulation programs. An open source approach can enable a rapid and reliable software development process, and can also encourage technical and methodological advances through networking of SD modelers and optimization experts interested in these areas.

In the case of the offshore gas field development problem, a future direction can be in the consideration of both exogenous and endogenous types of uncertainties. It is important to account for both types since regardless of the nature of uncertainties, each one would inevitably impact decisions and the economic profitability of a project. As an example, the price structure of gas depends on the type of contract agreed upon by the producer and customer. Long-term contracts generally assign fixed prices on the amount of gas to be supplied to the customers whereas short term contracts allow for more price volatility and demand uncertainty. The profiles of reserves are also at best obtained from estimates at the planning stage. An objective for a future work is then to give a comprehensive analysis of a development project that captures properties found in decision parameters such as price, demand and well reserves. One can look into how these parameters affect different areas of gas production and distribution networks.

Finally, the proposed robust design model can also be easily extended to incorporate multiple performance goals. For instance, multiple CO_2 budgets can be incorporated. This may arise due to decentralized budget allocation, or can represent different annual emission requirements. The CO_2 budgets themselves may even be uncertain, i.e. representing uncertain outlook of environmental legislations. Furthermore, the capital investment budget requirements may also contain uncertainties. Future work includes extension of the model to a multi-stage planning environment, where facilities and connecting infrastructures can be installed based on realized information at each stage.

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APPENDIX

A. SYSTEM DYNAMICS METHODOLOGY

An important purpose of the SD method is to facilitate the inquiry of complex socio-economic dynamics. Forrester (1961) initially defined it as,

> "...the study of the information feedback characteristics of industrial activity to show how organizational structure, amplification (in policies), and time delays (in decision and actions) interact to influence the success of the enterprise. It treats the interactions between the flows of information, money, orders, materials, personnel, and capital equipment in a company, an industry, or a national economy."

The focus on capturing system interactions, feedback loops and delay processes has made the use of SD invaluable in the elucidation of complex systems. An outcome of the work by Forrester (1961) is a simulation game that is commonly used in supply chain classrooms known as the *Beer game*. This originated from the "Forrester Model" shown in Figure A.1, which is described by six interacting flow systems, namely the flows of information, materials, orders, money, manpower, and capital equipment (Angerhofer and Angelides, 2000).

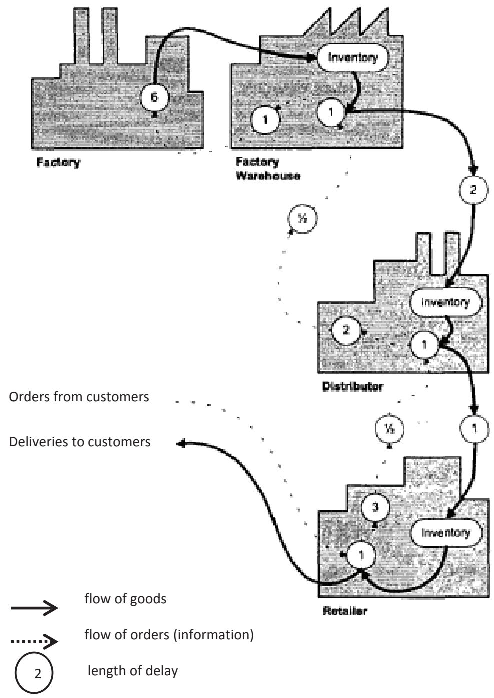

Fig. A.1. Forrester Model of a production-distribution system (Forrester, 1961)

Through his use of SD, he was able to point out issues such as demand amplification, inventory swings, and effect of advertising policies on production variations, decentralized control, or management process. Specifically, he was able to demonstrate the unintended consequences termed as the *bullwhip* phenomenon in supply chains, due to order placement decisions made with limited information.

However, SD has also been often criticized for not having sufficient rigor and analytical support (see for instance, Ansof and Slevin (1968) and Towill (1982)). Model analysis is usually approached in an ad-hoc manner, and very few systematic guidelines are provided to improve performance. Forrester (1980) has likewise stressed the importance of appropriate policy design methods in shaping and controlling system behavior in SD applications. As a consequence, there has been considerable interest and progress in the development of formal approaches to analyze the influence of system structure on dynamic behavior.

A.1 Formal Approaches in System Dynamics

Distinct from 'flight simulator' approaches of combining intuition with repeated simulations to understand system behavior, formal approaches are based on mathematical analysis of the system structure using tools from various relevant fields. Formal approaches are useful in supporting the analysis of large systems that need to be represented using large scale dynamic models. These include the application of modal control theory to SD, for instance by pole assignment (Forrester, 1982; Keloharju, 1987; Mohapatra and Sharma, 1985), or by sensitivity analysis (Diallo and Rahn, 1987). An essential step enabling the modal control approach of analysis is a linearization of the original (generally non-linear) dynamic system model. Diallo and Rahn (1990) describe the procedure to obtain linear dynamic model approximations amenable to analysis.

A related class of formal approaches in SD based on linear system theory is the use of eigenvalue elasticity analysis (EEA) pioneered by Forrester (1982). EEA focuses on studying the contribution of each feedback loop to each mode of behavior in the SD model. A number of researches have since attempted to improve on the EEA approach. Kampmann (1996), Kampmann and Oliva (2006) and Saleh et al. (2010) show that it is possible to analyze only a subset of the feedback loops in order to scale down the number of loops to be analyzed. Goncalves (2009) studies how eigenvectors, together with eigenvalues, can be used to explain overall behavior trajectories of the state variables.

This research proposes an extension of the formal approach in SD based on the methods of modal control and eigenvalue analysis. In the above cited works on applying modal control in SD (Mohapatra and Sharma, 1985; Ozveren and Sterman, 1989), the objective is to synthesize state feedback controllers that can achieve desired dynamic system performance. In EEA, the primary objective is to evaluate the influence of the feedback loops in an SD model (Forrester, 1982; Kampmann, 1996; Kampmann and Oliva, 2006) on the state trajectories. This involves analyzing the behavior modes with respect to the parameters in the model through the use of eigenvalues in sensitivity calculations.

While the proposed approach in this research also makes use of eigenvalues, the objectives are very distinct from these key works. In particular, the aim is to achieve the desired behavior modes of the system (through the eigenvalues) under uncertainty. In a broad sense, it can be viewed as an extension of sensitivity analysis to address design issues under parametric uncertainties. None of the abovementioned works consider explicitly the issues of system behavior and stability in the presence of external disturbances or uncertainties.

Dynamic stability of a system refers to the ability of a system to achieve and maintain equilibrium under external disturbances. Dynamic system stability under uncertainties is also often the goal in control theory (Sontag, 1998). In SD applications, this can have important economic implications. For instance, in the context of Forrester's Industrial Dynamics (1961), achieving stability can be associated with the reduction of inventory swings in the presence of information delays within a production system.

A.2 Parametric Uncertainties in System Dynamics Models

Any practitioner of SD will acknowledge that the SD modeling process is replete with challenges where errors and inaccuracies can seep in, among which include numerical parameter estimation and graphical function solicitation. Uncertainties can have corrupting influences on the behavior of dynamic systems and failure to account for these in design and analysis can have undesirable and unforeseen consequences. The model analysis and system design are consequently subject to inaccuracy and need to be treated with a degree of suspicion (Coyle, 1977).

Consider the following simple example of the hare and lynx model as adapted from Jensen (2008), where the hare catch rate in Figure A.2 is an uncertain parameter in the model. In the stock flow model, hare deaths are defined to be a function of the catch rate, the number of hares and the number of lynx. The effects of varying the *hare catch rate* parameter are presented in Figure A.3. The trajectories represented by dashed lines show the behaviors of the hare and lynx when the value of the hare catch rate is 0.04, while the trajectories represented by solid lines show the behaviors when the same parameter is set at 0.08.

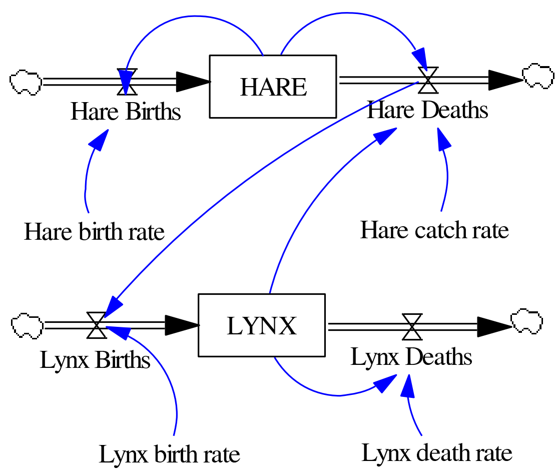

Fig. A.2. Stock flow diagram of the system

When the hare catch rate is 0.04, the hare and lynx populations both exhibit oscillatory behaviors throughout the time horizon considered. However, when the hare catch rate is 0.08, an 'overshoot and collapse' behavior mode is observed for both the hare and lynx populations. The change in the parameter setting not only affects the level of the populations but more importantly changes the behavior modes in the system. These differences indicate that for some systems, even a slight change in the settings can lead to drastically different behavior patterns. Thus, the realization of policies may entirely be

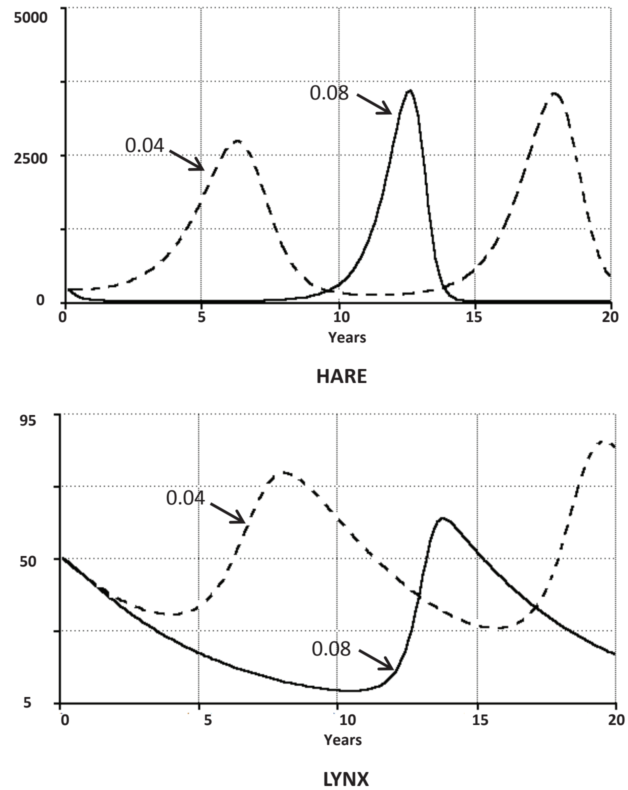

Fig. A.3. Dynamics of the hare (top) and lynx (bottom) populations.

different from what has been designed upon the introduction of parametric uncertainties.

SD has often been compared to other quantitative approaches and criticized for its lack of rigor in the evaluation of model validity, specifically with regards to the effects of accuracy in parameter estimation (Meadows, 1980). Some proponents of SD have defended against these issues by arguing that SD is concerned only with the general behavior patterns of systems rather than short-term precision forecasts. This position may be reasonable in certain situations. For instance, in the seminal urban dynamics model (Forrester, 1969), urban decay (i.e., unemployment rate) was observed to persist over large range of parameter values (i.e., building construction rate) as long as the policy structure remains (i.e., attracting more businesses).

Consequently, Forrester's view is that the model parameter values are of secondary importance in the presence of interlocking feedback loop structures. Similarly, in discussing about the calibration of table functions, Sterman (2000) points out that systems in which strong balancing feedback mechanisms are present tend to self-correct its behavior when parameter values fluctuate. While it may be justifiably argued that in such cases (such as the urban dynamics model), the influence of parameter values is not overly important, the point of view taken in this research is that one cannot always depend on the nature of the system presented to sufficiently dampen the effects of parameter uncertainties in the analysis (the hare and lynx model is one such counter-example).

B. WORKFORCE-INVENTORY STOCK MANAGEMENT STRUCTURE

As mentioned, the workforce-inventory model in Saleh et al. (2010) was based from the work of Sterman (2000). This section provides a supplementary discussion on how the stock and flow structure of the model had been formed.

The *stock management structure* accounts for the foundation of the workforce-inventory model. This structure is described in Figure B.1 and demonstrates how a firm maintains an inventory of finished goods and fills orders as they arrive.

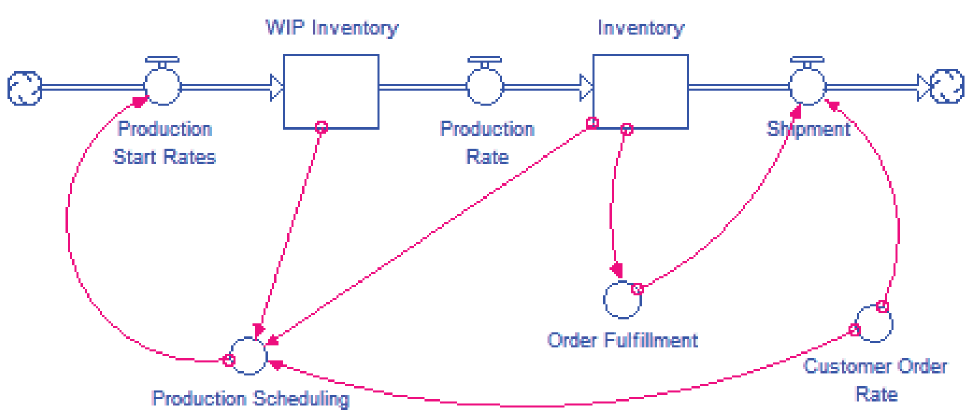

Fig. B.1. Stock Management Structure Sterman (2000)

Aside from the finished goods inventory, the firm also maintains an inventory of WIP goods, which are increased by production starts and subsequently decreased by production. Orders that the firm fail to fulfill are assumed to be lost sales. As mentioned by Sterman (2000), the key production control and inventory management decisions made by the firm include order fulfillment (determining the ability to fill customer orders based on the adequacy of inventory) and production scheduling (determining the rate of production starts based on the demand forecast and inventory position of the firm, including the WIP inventory). Both of these decisions involve an important feedback control rule which adjusts the production starts to move the levels of inventory and WIP toward their desired levels as depicted in Figure B.2.

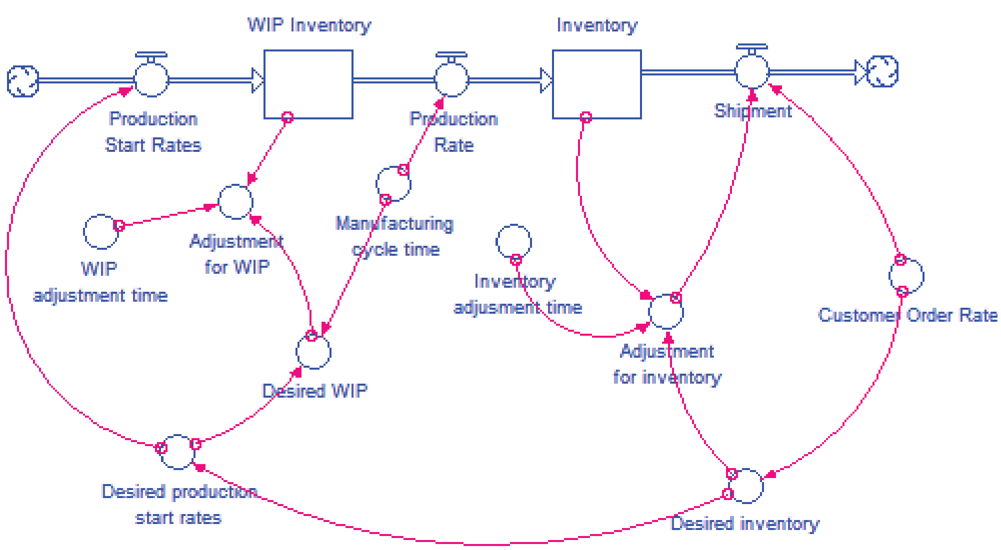

Fig. B.2. Stock Management Structure Sterman (2000)

The above figure expounds on how order fulfillment and production scheduling decisions are carried out in the system. The system could only fulfill customer orders if inventory is larger than the desired shipments. Consequently, if inventory falls below the desired levels, this will trigger the production scheduling to increase production starts. The adjustment variables are then proportional to the difference of the desired and the actual inventory levels, delayed by a certain adjustment time. The delay is incorporated to account for the time it will take production schedulers or management to react to the discrepancies between actual on-hand inventory and the desired quantities.

In the workforce-inventory system, workforce or labor is defined to be the limiting resource for the rate of production starts. Similarly, Figure B.3 shows that this is also modelled to follow a stock management structure, wherein the system seeks to maintain a workforce level that could support the desired production in order to fulfill customer orders.

However, the hiring of workers does not occur instantaneously on the event that the firm needs additional workers. As discussed by Sterman (2000), hiring takes time since positions must be authorized and vacancies must be created. Furthermore, job openings must be posted and advertised, followed by interviews, background checks, training, and other delays. As shown in Figure B.4, these considerations are represented by the level of vacancies in the system. The amount of vacancies is increased by the vacancy creation rate and decreased by the vacancy closure rate. The latter is determined by the hiring

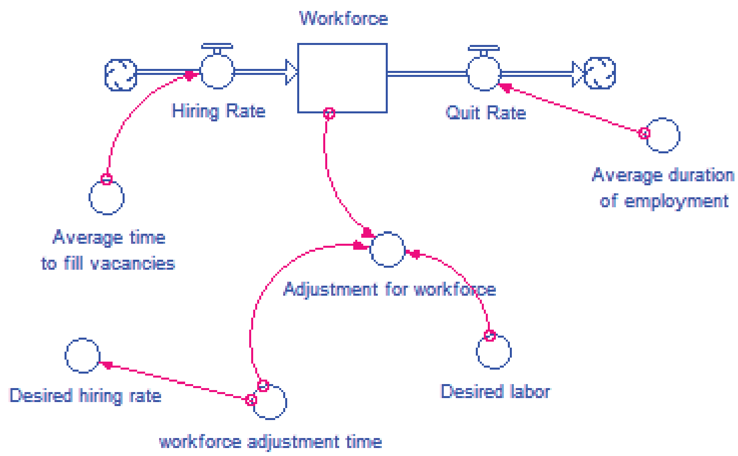

Fig. B.3. Stock Management Structure Sterman (2000)

rate. Hence, the stock of vacancies is the supply line of orders for workers that have been placed but not yet filled. The Time to Fill Vacancies represents the average delay between creating and filling a vacancy (Sterman, 2000).

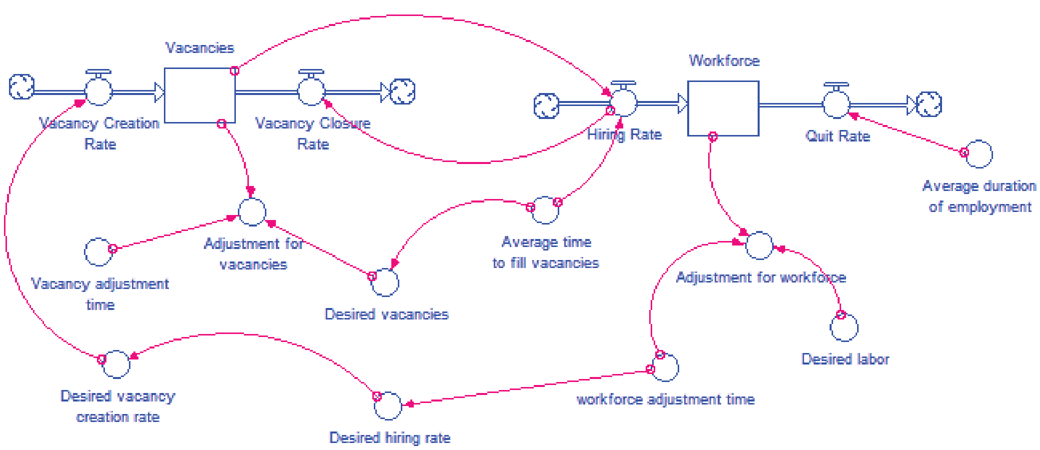

Fig. B.4. Stock Management Structure Sterman (2000)

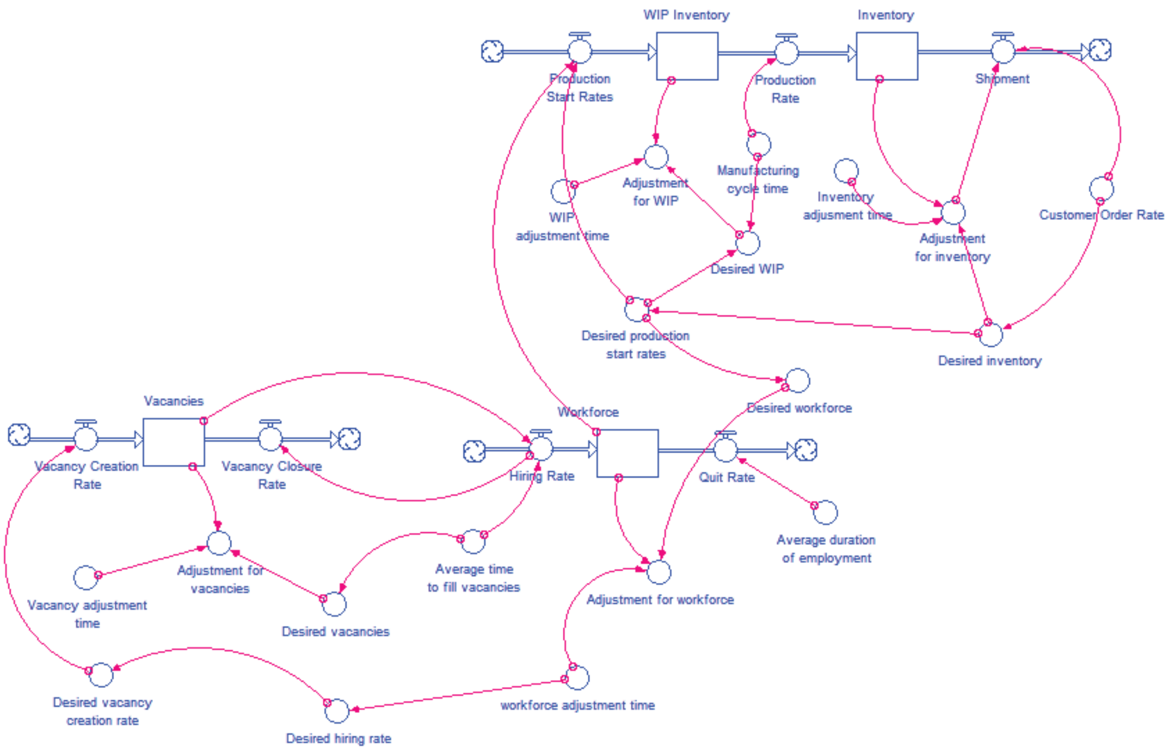

Fig. B.5. Stock Management Structure Sterman (2000)

Finally, the workforce and inventory components of the system are integrated through the identified connections in Figure B.5. Specifically, the desired workforce level is based on the desired production level. In turn, the production rates is determined by the current workforce level.

C. DERIVATION OF PERFORMANCE CONSTRAINTS FOR WORKFORCE-INVENTORY SYSTEM

In order to translate the targets to the dynamic response constraints, calculate for the right (V) and left (W) eigenvectors using the nominal values of the current system, which are respectively found as in the following:

$$V = \begin{bmatrix} 0.3331 & 0.7832 & 0.7832 & -0.6713 \\ 0.0215 & -0.0068 + 0.0066i & -0.0068 - 0.0066i & -0.0010 \\ -0.0589 & -0.0052 - 0.0053i & -0.0052 + 0.0053i & 0.0010 \\ -0.9408 & -0.0593 + 0.6188i & -0.0593 - 0.6188i & 0.7412 \end{bmatrix}$$

$$W = \begin{bmatrix} 0.0034 + 0.0091i & 0.0034 - 0.0091i & -0.0104 & -0.0086 \\ 0.9495 & 0.9495 & -0.8844 & -0.1727 \\ 0.3026 - 0.0818i & 0.3026 + 0.0818i & -0.4665 & -0.9849 \\ 0.0039 + 0.0083i & 0.0039 - 0.0083i & 0.0054 & -0.0067 \end{bmatrix}$$

Subsequently, it is also necessary to obtain the matrices corresponding to the partial derivatives of state matrix D with respect to the uncertain and control variables. For instance, in the case of the uncertain parameter ATF, this is equivalent to:

$$\frac{\partial D}{\partial ATF} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{-1}{ATFV^2} & 0 \\ \frac{-(MCT+WAT)}{(10*IAT*LAT*VAT*WAT)} & \frac{1}{(ADE*VAT)} - \frac{1}{(LAT*VAT)} & \frac{1}{ATFV^2} & \frac{-1}{(10*LAT*VAT*WAT)} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Thus, the change in the eigenvalues with respect to ATF is,

$$\frac{W^T \cdot \frac{\partial D}{\partial ATF} \cdot V}{W^T V} \cdot \Delta ATF$$

The same procedure is carried out for the remaining variables and targets in the system, where the partial derivatives of matrix D with respect to each variable are as follows:

$$\frac{\partial D}{\partial MCT} = \begin{bmatrix} 0 & 0 & 0 & \frac{-1}{MCT^2} \\ 0 & 0 & 0 & 0 \\ -\frac{\frac{1}{10*LAT*WAT} + \frac{ATF}{10*LAT*VAT*WAT}}{IAT} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{MCT^2} \end{bmatrix}$$

$$\frac{\partial D}{\partial WAT} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\frac{\frac{1}{10*LAT*WAT} - \frac{\frac{MCT}{10} + \frac{WAT}{10}}{LAT*WAT^2} + \frac{ATF}{10*LAT*VAT*WAT} - \frac{ATF*(MCT+WAT)}{10*LAT*VAT*WAT^2}}{IAT} & 0 & 0 & \frac{\frac{1}{10*LAT} + \frac{ATF}{10*LAT*VAT}}{WAT^2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial D}{\partial ADE} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{ADE^2} & 0 & 0 \\ 0 & -\frac{\frac{ATF}{VAT}+1}{ADE^2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial D}{\partial VAT} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{ATF*(MCT+WAT)}{10*IAT*LAT*VAT^2*WAT} & \frac{ATF}{LAT*VAT^2} - \frac{ATF}{ADE*VAT^2} & \frac{1}{VAT^2} & \frac{ATF}{10*LAT*VAT^2*WAT} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial D}{\partial LAT} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\frac{\frac{MCT}{10}+\frac{WAT}{10}}{LAT^2*WAT}+\frac{ATF*(MCT+WAT)}{10*LAT^2*VAT*WAT}}{IAT} & \frac{\frac{ATF}{VAT}+1}{LAT^2} & 0 & \frac{\frac{1}{10*LAT^2}+\frac{ATF}{10*LAT^2*VAT}}{WAT} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\frac{\partial D}{\partial IAT} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{\frac{\frac{MCT}{10}+\frac{WAT}{10}}{LAT*WAT}+\frac{ATF*(MCT+WAT)}{10*LAT*VAT*WAT}}{IAT^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Using the notations $z_1, ..., z_5$ and y_1 and y_2 for the uncertain and design variables, respectively, it therefore follows that $S_\mu^{\boldsymbol{y}} + S_\omega^{\boldsymbol{y}}$ and $S_\mu^{\boldsymbol{z}} + S_\omega^{\boldsymbol{z}}$ can be obtained by the summation of the eigenvalue sensitivity matrices,

$$S_\mu^{\boldsymbol{y}} + S_\omega^{\boldsymbol{y}} = \sum_{m=1}^{2} \frac{W^T \cdot \frac{\partial D}{\partial y_m} \cdot V}{W^T V}$$
$$S_\mu^{\boldsymbol{z}} + S_\omega^{\boldsymbol{z}} = \sum_{l=1}^{5} \frac{W^T \cdot \frac{\partial D}{\partial z_l} \cdot V}{W^T V}$$

Thus, once the sensitivity matrices have been classified according to the real and imaginary parts, the updated eigenvalues are then equivalent to:

$$\boldsymbol{\mu}(\boldsymbol{y}, \boldsymbol{z}) = \bar{\boldsymbol{\mu}} + S_\mu^{\boldsymbol{y}} \cdot \boldsymbol{y} + S_\mu^{\boldsymbol{z}} \cdot \boldsymbol{z}$$

and

$$\boldsymbol{\omega}(\boldsymbol{y}, \boldsymbol{z}) = \bar{\boldsymbol{\omega}} + S_\omega^{\boldsymbol{y}} \cdot \boldsymbol{y} + S_\omega^{\boldsymbol{z}} \cdot \boldsymbol{z}$$

where the stability constraints can be formulated for $\boldsymbol{u}$ as $\boldsymbol{\mu}(\boldsymbol{y}, \boldsymbol{z}) \leq 0$. A similar procedure is applied to the remaining dynamic response constraints.

D. ALTERNATIVE UNCERTAINTY SET DEFINITION

This section shows an alternative definition for the uncertainty set, one that allows $\boldsymbol{z}$ to be negative. To model the uncertain parameters $\boldsymbol{z}$, we write

$$\tilde{\boldsymbol{z}} = \bar{\boldsymbol{z}} + \boldsymbol{z} \tag{D.1}$$

where $\bar{\boldsymbol{z}}$ denotes the nominal values of the parameters and the perturbations $\boldsymbol{z}$ are such that

$$\boldsymbol{z} \in [-\gamma\overline{\boldsymbol{z}}, \gamma\overline{\boldsymbol{z}}]$$

with the scalar parameter $\gamma \in [0,1]$ and $\overline{\boldsymbol{z}} \geq 0$. Hence, $\gamma\overline{\boldsymbol{z}}$ or $-\gamma\overline{\boldsymbol{z}}$ represents the maximum possible perturbations in the set. Similarly, the relationships between the uncertain parameters could be defined as follows

$$R\boldsymbol{z} \leq \boldsymbol{e} \tag{D.2}$$

where the WxK matrix R and the vector $\boldsymbol{e} = [e_1, \cdots, e_W]$ are estimated coefficients of the W relationships. Combining the above, for a given γ, we define the set of possible outcomes $\mathcal{Z}_\gamma$ as:

$$\mathcal{Z}_\gamma = \left\{\boldsymbol{z} \in \Re^K : \boldsymbol{z} \in [-\gamma\overline{\boldsymbol{z}}, \gamma\overline{\boldsymbol{z}}], R\boldsymbol{z} \leq \boldsymbol{e}\right\}$$

From the above discussion, the design problem is then to maximize the γ-*level* robustness of the system to meet the specified system requirements through the calibration of the control parameters. That is, we seek a design such that the system requirements are always satisfied under perturbations of the uncertain parameters arising from the set $\mathcal{Z}_\gamma$ with the largest achievable value of γ. The robust design problem is stated in the following formulation.

Problem $\mathcal{R}$

$$\max_{\boldsymbol{y}\in\mathcal{Y}} \ \gamma$$

$$A \cdot \boldsymbol{y} + B \cdot \boldsymbol{z} \leq \boldsymbol{\tau} \quad \forall \boldsymbol{z} \in \mathcal{Z}_\gamma \tag{D.3}$$

(D.3) must hold for all $\boldsymbol{z} \in \mathcal{Z}_\gamma$. In order to make the formulation amendable for solution using linear programming techniques, we apply the following Proposition to develop an equivalent formulation of Problem $\mathcal{R}$ termed as the robust counterpart problem $\mathcal{C}$.

Proposition 1a: *Define P, $Q \in \Re^{N\times K}$ and $E \in \Re^{N\times W}$ as matrices of variables. Problem $\mathcal{R}$ is then equivalent to the following:*

Problem $\mathcal{C}$

$$\max_{\boldsymbol{y}\in\mathcal{Y}} \gamma$$

s.t.

$$A \cdot \boldsymbol{y} + (P + Q) \cdot \gamma\overline{\boldsymbol{z}} + E \cdot \boldsymbol{e} \leq \boldsymbol{\tau} \tag{D.4}$$

$$Q - P - E \cdot R \leq -B \tag{D.5}$$

$$E, P, Q \geq 0 \tag{D.6}$$

Proof: First, it suffices that:

$$A \cdot \boldsymbol{y} + \max_{\boldsymbol{z} \in \mathcal{Z}_\gamma} \{B \cdot \boldsymbol{z}\} \leq \boldsymbol{\tau} \tag{D.7}$$

The maximization in the left-hand side can be written explicitly as follows.

$$\max \ B \cdot \boldsymbol{z} \text{ s.t. } -\gamma \overline{\boldsymbol{z}} \leq \boldsymbol{z} \leq \gamma \overline{\boldsymbol{z}} \ , \ R\boldsymbol{z} \leq \boldsymbol{e} \tag{D.8}$$

The above is a linear optimization problem and the dual formulation can be written as:

$$\min \gamma \overline{\boldsymbol{z}} \cdot (P + Q) + \boldsymbol{e}E \tag{D.9}$$

$$\text{s.t. } P - Q + R \cdot E \geq B \ , \ P, Q, E \geq 0 \tag{D.10}$$

where $P, Q \in \Re^{N \times K}$ and $E \in \Re^{N \times W}$ denote the variables in the dual formulation. The formulation $\mathcal{C}$ is then derived by replacing the maximization term in (D.7) with the objective function value in (D.9), thus obtaining (3.6). Finally, (D.5) and (D.6) are obtained by augmenting (D.10) into $\mathcal{R}$.

E. CONNECTIONS OF DESIGN ROBUSTNESS WITH SATISFICING BEHAVIOR

In the following, define the *position level* $a(\tilde{\boldsymbol{v}})$ as the uncertain level above a given target (i.e., a profit target), so that positive realizations of $a(\tilde{\boldsymbol{v}})$ indicate that the target is achieved, and negative realizations indicate a target shortfall. For convenience, inequality relations $a(\tilde{\boldsymbol{v}}) \geq 0$ are applied in the state-wise sense. We then have the following axiomatic definition of *satisficing measures* ρ, where $1 \geq \rho \geq 0$, that operate on $a(\tilde{\boldsymbol{v}})$:

Definition 4 *(from Brown and Sim (2010)): A given function $\rho\left(a(\tilde{\boldsymbol{v}})\right)$ is a valid satisficing measure if all the below are true:*

1. Attainment content: If $a(\tilde{\boldsymbol{v}}) \geq 0$, then $\rho(a(\tilde{\boldsymbol{v}})) = 1$.

2. Non-attainment apathy: If $a(\tilde{\boldsymbol{v}}) < 0$, then $\rho(a(\tilde{\boldsymbol{v}})) = 0$.

3. Monotonicity: If $a(\tilde{\boldsymbol{v}}) \geq a'(\tilde{\boldsymbol{v}})$, then $\rho(a(\tilde{\boldsymbol{v}})) \geq \rho(a'(\tilde{\boldsymbol{v}}))$.

4. Gain continuity: $\lim_{\alpha \downarrow 0} \rho(a(\tilde{\boldsymbol{v}}) + \alpha) = \rho(a(\tilde{\boldsymbol{v}}))$.

For clarity of notation, in the the following we use the generic compact form of the constraints of the robust optimization model (6.11)–(6.18) involving adjustable decisions:

$$f_k(\tilde{\boldsymbol{v}}) \geq 0 \quad \forall\ k = 1, \cdots, K,\ \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma$$

where f_k is a given function of $\tilde{\boldsymbol{v}}$ (and hence uncertain in general), and the above requires f_k to be non-negative for all $\tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma$. Defining the position level $a(\tilde{\boldsymbol{v}}) = \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}})$, the above can be re-written as:

$$a(\tilde{\boldsymbol{v}}) \geq 0 \quad \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma$$

The robustness level associated with $a(\tilde{\boldsymbol{v}})$, for the given set of functions f_k, $\forall k = 1, \cdots K$, is then:

$$\gamma\left(a(\tilde{\boldsymbol{v}})\right) = \begin{cases} \sup\{\gamma \in [0,1] : \min_{\tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma} a(\tilde{\boldsymbol{v}}) \geq 0\} & \text{if feasible,} \\ 0 & \text{otherwise.} \end{cases} \tag{E.1}$$

The robustness level γ thus describes the largest uncertainty set $\mathcal{Z}_\gamma$ that can be tolerated, given the set of functions f_k, $\forall k = 1, \cdots K$. We then have the following result:

Proposition 4 $\gamma(a(\tilde{\boldsymbol{v}}))$ *is a satisficing measure as defined in Definition 1 on the extreme-value uncertain variable* $a(\tilde{\boldsymbol{v}})$*.*

Proof : We show that $\gamma(a(\tilde{\boldsymbol{v}}))$ defined in (E.1), where the uncertain variable

$$a(\tilde{\boldsymbol{v}}) = \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}}) \quad \forall\ \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma$$

satisfies each axiom in Definition 1.

1. *Attainment content.*

$$
\begin{aligned}
\textit{Given}: \quad & a(\tilde{\boldsymbol{v}}) \geq 0 \\
\Rightarrow \quad & \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}}) \geq 0 \qquad \forall\, \tilde{\boldsymbol{v}} \in \mathcal{Z}(\gamma = 1) \\
\Rightarrow \quad & \min_{\tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma} \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}}) \geq 0 \quad \forall\, \gamma \in [0,1] \\
\Rightarrow \quad & \gamma(a(\tilde{\boldsymbol{v}})) = 1
\end{aligned}
$$

where the last inequality follows from noting that $\mathcal{Z}'_\gamma \subseteq \mathcal{Z}_\gamma$ whenever $\gamma' \leq \gamma$.

2. *Non-attainment apathy.*

$$
\begin{aligned}
\textit{Given}: \quad & a(\tilde{\boldsymbol{v}}) < 0 \\
\Rightarrow \quad & \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}}) < 0 \quad \forall\, \tilde{\boldsymbol{v}} \in \mathcal{Z}_\gamma,\ \forall\, \gamma \in [0,1] \\
\Rightarrow \quad & \gamma(a(\tilde{\boldsymbol{v}})) = 0 \qquad \text{(by definition in (E.1))}.
\end{aligned}
$$

3. *Monotonicity.* Given $f_k(\tilde{\boldsymbol{v}})$ and $f'_k(\tilde{\boldsymbol{v}})$, and where $a'(\tilde{\boldsymbol{v}}) = \min_{k=1,\cdots,K} f'_k(\tilde{\boldsymbol{v}})$, we have that:

$$
\begin{aligned}
& a(\tilde{\boldsymbol{v}}) \geq a'(\tilde{\boldsymbol{v}}) \\
\Rightarrow \quad & \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}}) \geq \min_{k=1,\cdots,K} f'_k(\tilde{\boldsymbol{v}}) \qquad \forall\, \tilde{\boldsymbol{v}} \in \mathcal{Z}\,(\gamma = 1) \\
\Rightarrow \quad & \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}}) \geq \min_{k=1,\cdots,K} f'_k(\tilde{\boldsymbol{v}}) \geq 0 \quad \forall\, \tilde{\boldsymbol{v}} \in \mathcal{Z}\left(\gamma = \gamma(a'(\tilde{\boldsymbol{v}}))\right) \\
\Rightarrow \quad & \gamma(a(\tilde{\boldsymbol{v}})) \geq \gamma(a'(\tilde{\boldsymbol{v}}))
\end{aligned}
$$

4. *Gain continuity.* We proceed by showing that the below must hold simultaneously:

$$\lim_{\alpha \downarrow 0} \gamma(a(\tilde{\boldsymbol{v}}) + \alpha) \leq \gamma(a(\tilde{\boldsymbol{v}})) \tag{E.2}$$

$$\lim_{\alpha \downarrow 0} \gamma(a(\tilde{\boldsymbol{v}}) + \alpha) \geq \gamma(a(\tilde{\boldsymbol{v}})) \tag{E.3}$$

We show (E.2) by contradiction. Suppose $\lim_{\alpha \downarrow 0} \gamma(a(\tilde{\boldsymbol{v}}) + \alpha) = \gamma(a(\tilde{\boldsymbol{v}})) + \epsilon$ for some $\epsilon > 0$. Then we have that:

$$\min_{\tilde{\boldsymbol{v}} \in \mathcal{Z}(\gamma(a(\tilde{\boldsymbol{v}}))+\epsilon)} \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}}) + \alpha \geq 0 \quad \forall\ \alpha > 0 \tag{E.4}$$

Furthermore, by definition of $\gamma(a(\tilde{\boldsymbol{v}}))$ we must have:

$$\min_{\tilde{\boldsymbol{v}} \in \mathcal{Z}(\gamma(a(\tilde{\boldsymbol{v}}))+\epsilon)} \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}}) < 0$$

Clearly, by choosing any $\Delta > 0$ and such that $\Delta < -\min_{\tilde{\boldsymbol{v}} \in \mathcal{Z}(\gamma(a(\tilde{\boldsymbol{v}}))+\epsilon)} \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}})$, it follows that

$$\alpha = -\min_{\tilde{\boldsymbol{v}} \in \mathcal{Z}(\gamma(a(\tilde{\boldsymbol{v}}))+\epsilon)} \min_{k=1,\cdots,K} f_k(\tilde{\boldsymbol{v}}) - \Delta > 0$$

which provides the required contradiction in (E.4).

Finally, (E.3) is straightforward to show by applying monotonicity of γ. That is, we must have that $\gamma(a(\tilde{\boldsymbol{v}}) + \alpha) \geq \gamma(a(\tilde{\boldsymbol{v}}))$ for all $\alpha > 0$. □