PROBABILISTIC MODELS FOR RELIABILITY ASSESSMENT OF AGEING EQUIPMENT AND MAINTENANCE OPTIMIZATION

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A THESIS SUBMITTED

FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF
ELECTRICAL AND COMPUTER ENGINEERING

NATIONAL UNIVERSITY OF SINGAPORE

2013

DECLARATION

I hereby declare that this thesis is my original

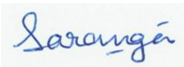
work and it has been written by me in its entirety.

I have duly acknowledged all the sources of

information which have been used in the thesis.

This thesis has also not been submitted for any

degree in any university previously.



Saranga Kumudu Abeygunawardane

24 December 2012

Acknowledgements

I wish to thank everyone who helped me during my doctoral studies.

First, I express my sincere gratitude to my supervisor, Asst. Prof. Panida Jirutitijaroen for giving me an opportunity to pursue my doctoral studies in National University of Singapore. Her constant guidance and sincere advice greatly helped me to overcome difficulties that I encountered in my research. I truly appreciate the efforts that she made to develop my research and communication skills and to revise my papers. I am also thankful to her for giving friendly advice when I faced hard times in my personal life. Her kind and friendly behavior greatly helped to reduce the greatest sorrow that I have ever experienced in my life due to the loss of my beloved father.

Next, I would like to thank Asst. Prof. Huan Xu for his valuable ideas, suggestions and support given towards my research. I am also grateful to my thesis committee members for their time, constructive comments and suggestions.

I would like to acknowledge National University of Singapore and the Department of Electrical and Computer Engineering for providing academic and financial support during my doctoral studies.

I also want to thank Thillainathan Logenthiran, Xiong Peng, Bordin Bordeerath, Shu Zhen, Bai Hong, Bi Yunrui, Sumith Madampath and all my colleagues in the power systems laboratory and the lab officer, Mr. H. C. Seow for the tremendous support given at the lab. I appreciate the valuable friendship of Arunoda Basnayake, Supunmali Ahangama, Chamila Liyanage, Thanuja Kulathunga and Rupika Swarnamala. I think I am fortunate to have such friends and colleagues during my stay in Singapore.

I should not miss to convey my gratitude to all my teachers who strengthened me and supported me, when I was a student at Ferguson High School and University of Peradeniya. I would especially like to mention the names of two teachers whom I adore most; Mrs. Lilani Jayasinghe (FHS) and Prof. Janaka Ekanayake (UOP). I should not forget to thank Ass. Prof.

Saman Abeysekara (NTU), who motivated me to apply for doctoral scholarships in Singapore.

Last but not least, I would like to thank my beloved family members for their love, admiration and encouragement. I wish my father was alive to share the pleasure of completing this PhD thesis. I dedicate this thesis to my late father.

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Abstract

Many electrical devices with considerable life spans are subjected to deterioration throughout their useful lives. Catastrophic failures of such devices in power systems can result in substantial social and economic losses. Maintenance is commonly performed to reduce the occurrence of such catastrophic failures and extend the equipment's lifetime. Probabilistic maintenance models are widely used to quantify the benefits of maintenance in terms of reliability and costs and to determine optimal maintenance policies. This thesis aims to propose analytically solvable probabilistic models to obtain accurate results in power system reliability assessments and maintenance optimization.

The thesis first proposes a new Markov model for scheduled maintenance. This proposed model can accurately assess reliability and costs, while the existing Markov maintenance models provide accurate results only for periodic inspections. The proposed and existing models are applied to assess reliability and costs of circuit breakers. In two other application studies, Markov models are utilized for state prediction of transformers and for analyzing the effects of sub-component characteristics on reliability of a wind energy conversion system. A maintenance optimization problem is formulated to find optimal inspection rates using a grid search algorithm. Optimization results show that practical solutions can be obtained with the careful selection of maintenance models. To obtain adaptive optimal inspection and maintenance policies, a Markov decision process (MDP) model is proposed. This model can explicitly incorporate inspection and maintenance delay times and combine the long term ageing process with frequently observed short term changes in equipment's condition. The applicability of the model is demonstrated using historical condition monitoring and maintenance data of local transformers. System-level maintenance planning is investigated using a system-wide MDP model and through the coordination of MDP models of individual equipment. The proposed models are valuable for reliability evaluation, maintenance-related cost assessments, maintenance decision making and maintenance planning.

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List of Abbreviations

CBM	Condition monitoring based inspection and maintenance
CM	Condition monitoring
DGA	Dissolved gas analysis
FPT	First passage time
IBM	Inspection based maintenance
MDP	Markov decision process
MDT	Mean down time
MTBF	Mean time between failures
MTTF	Mean time to failure
MTTFF	Mean time to first failure
MTTR	Mean time to repair
NRD	Non-redrawing
RD	Redrawing
TDCG	Total dissolved combustible gases
WECS	Wind energy conversion system

List of Symbols

a	Action
A	Transition probability matrix
a_0	Doing nothing
a_1	Inspection/ CM
a_2	Minor maintenance
a_3	Major maintenance
a_4	Replacement
a_5	Repair
$a_t^*(i)$	Optimal action in state i at the decision epoch t
C_i	Last known condition of the equipment
C_I	Costs of performing an activity of inspection
C_M	Costs of performing an activity of minor maintenance
C_{MM}	Costs of performing an activity of major maintenance
C_F	Costs of performing an activity of repair
C_{IC}	Hourly interruption cost
C_{LP}	Hourly loss of profit
C_S	Cost of performing an activity in state S
D	Set of decision epochs
$d(S)$	Mean duration in state S
δ_i	Transition rate from I_i to M_i
F	Failure state
FI	Frequency of interruption
$f(S)$	Frequency of entering or leaving state S
$FI(S)$	Frequency of interruption due to activities in state S
G_i	Grid of values of γ_i

γ_i	Inspection rate at the deterioration stage S_i of the scheduled maintenance model
γ_c	Inspection rate of a transformer in condition c
$\gamma_{\max}$	The maximum inspection rate
i	Present state of the equipment
IC	Annual interruption cost
I_i	Inspection state corresponding to the i^{th} deterioration stage
k	State at the decision epoch t+1
λ_i	Deterioration rate of the i^{th} deterioration stage
λ_c	Deterioration rate of the condition C
λ_1^i	Transition rate from up state to de-rated state of sub-component i, i=1, 2, 4
λ_2^i	Transition rate from de-rated state to down state of sub-component i, i=1, 2, 4
λ^3	Transition rate from up state to down state of sub-component 3
λ^i	Failure rate of sub-component i
λ_{eq}	Equivalent failure rate
λ_{UD}	Transition rate from up state to de-rated state
λ_{DD}	Transition rate from de-rated state to down state
μ_{eq}	Equivalent repair rate
μ^i	Transition rate from down state to up state of sub-component i, i=1, 2, 3, 4
LCC	Annual life cycle cost
LP	Annual loss of profit
M_i	Minor maintenance state corresponding to the i^{th} deterioration stage
MM_i	Major maintenance state corresponding to the i^{th} deterioration stage
$MTTR_i$	Average time required to repair component i
$MTTR_{eq}$	Average down time of the WECS
μ_i	Maintenance rate at M_i
n	Total number of states

n_c	Number of inspections conducted when the condition is c
N	Number of decision epochs
$n_{\mathrm{I},i}$	Number of consecutive times that the inspection is postponed
$n_{\max,i}$	The maximum number of decision intervals that the equipment spends in C_i
Π	Steady state probability vector
$\pi(S)$	Steady state probability that the embedded Markov chain is in state S
P_{U}	Probability of being staying in up state of the intermediate states model
P_{DR}	Probability of being staying in de-rated state of the intermediate states model
P_{D}	Probability of being staying in down state of the intermediate states model
$P_{\mathrm{U*}}$	Probability of being staying in up state of the two-state model
$P_{\mathrm{D*}}$	Probability of being staying in down state of the two-state model
$P(S)$	Steady state probability of state S
P_c	Probability of being found in deterioration condition c
$\mathrm{P}_t(k\|i, a)$	Probability of transiting from state i to any state $k \epsilon \mathrm{S}$, upon choosing action a in state i at the t^{th} decision epoch
$\mathrm{r}_{\mathrm{N}}(i)$	Boundary value of state i
$r_{\mathrm{out}}(S)$	Summation of the transition rates from state S to other neighboring states
$r_t(i,a)$	Immediate reward for choosing action a in state i at the decision epoch t
S	Set of states
S_i	i^{th} deterioration stage
$S_{i,k}$	Sub deterioration state k of the deterioration state i
S_c	Deterioration stage corresponding to the C^{th} condition of a transformer
t	t^{th} decision epoch
t_c	Duration that the transformer spent in condition c
T	Decision horizon
T_o	Total time of observation
τ	Interval at which I & M decision making is performed

τ_I	Time interval between two consecutive inspections
TC	Annual total cost
t_I	Time to perform next inspection
t_M	Time taken to perform maintenance
$t_{I,i}$	Time to perform next inspection when the last known condition is C_i (inspection delay time in C_i)
$t_{max,i}$	The maximum allowable time between two consecutive inspections in C_i
$t_{min,i}$	The minimum time between two consecutive inspections in C_i
$t_{M,i}$	Time spent in C_i (maintenance delay time in C_i)
t_i	The maximum time period spent in condition C_i
t_{S_c}	Average time spent in the condition C
U	Unavailability
$U(S)$	Unavailability caused by the activities in state S
$U_t(i,a)$	Total expected reward received upon choosing action a in state i at time t
$U_N^*(i)$	The maximum total expected reward in state i, at the N^{th} epoch
$U_{t+1}^*(k)$	The maximum total expected reward in state k, at the decision epoch t+1
$U_t^*(i)$	The maximum total expected reward in state i, at the t^{th} epoch
z	Test statistic of hypothesis testing

Chapter 1 : Introduction

Most equipment in electrical transmission and distribution networks has been in use for several decades [1]. Catastrophic failures of such aging equipment can reduce system reliability, while causing substantial economic losses. However, replacing this aging equipment in bulk will be unbearable due to financial constraints. Therefore, electrical utilities adopt different maintenance strategies to minimize the occurrence of catastrophic failures. Too frequent inspection and maintenance would increase the cost of performing inspection and maintenance. On the other hand, lesser inspection and maintenance would result in a lower reliability level. Thus, it is desirable to perform maintenance in an optimal manner. In order to determine optimal maintenance policies, the benefits of maintenance should be quantified in terms of reliability and costs using maintenance models. This chapter reviews the literature on maintenance models after providing some background information related to ageing and maintenance.

1.1 The Background

1.1.1 Ageing of Equipment

In power systems, most electrical equipment is continuously in operation and is subjected to wear out over time. Equipment's physical and electrical strengths gradually deteriorate, until a failure occurs at some point of time causing a termination of equipment's operation. This process is called the deterioration process [2] or the ageing process [3] of equipment. The term "ageing" refers to the deterioration of equipment's physical and electrical strengths as a function of chronological time in operation [4]. There are two main types of equipment failures, namely, random failures and deterioration failures. Random failures which occur at a constant rate are independent of the equipment's deterioration condition. Deterioration failures are the failures that occur due to deterioration of equipment's condition.

The failure rate of equipment is not uniform with the age. In reliability theory, the variation of the failure rate with the equipment's age is given by the bathtub curve which is shown in Figure 1.1 [5]. The bathtub curve is a combination of early failures, wear out failures and random failures of the equipment. Since failures in the early life of the equipment are mostly due to defects in manufacturing and problems in installation, the failure rate decreases in the infant mortality region. In the useful life region, failures occur at random and thus the failure rate is constant. In the wear out region, failure rate increases, as the ageing progresses.

Typically, the design life of equipment spans across the infant mortality and useful life regions. Equipment which is in operation beyond its design lifetime is called aging equipment [3].

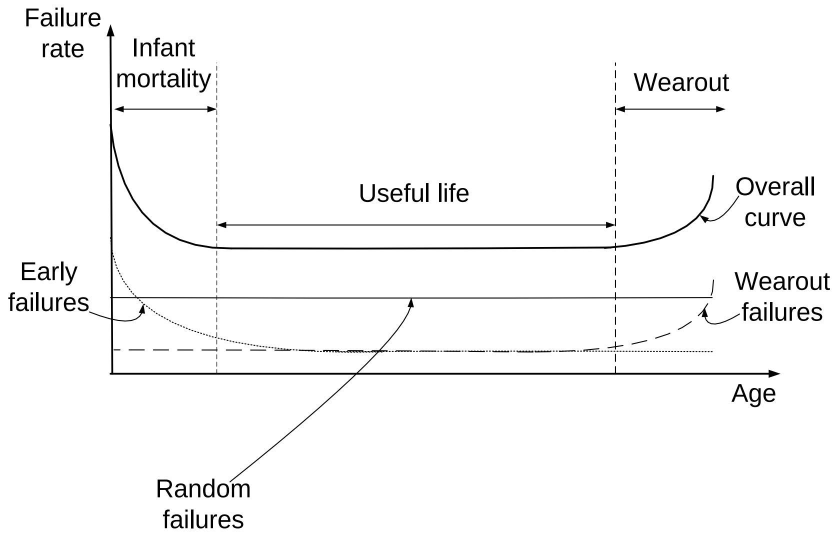

Figure 1.1: Bathtub curve [5]

1.1.2 Maintenance

Many costly electrical devices such as transformers, generators and circuit breakers are usually not replaced at the end of their useful life specified by the manufacturer. Utilities prefer to use them in operation as long as possible. However, in every year, such electrical equipment in power systems gets older and their deterioration mechanisms get accelerated. In order to improve the condition of ageing equipment, maintenance activities are performed. By performing maintenance regularly, the deterioration of the equipment is arrested, reduced or eliminated [2]. It is noteworthy that maintenance is different from the repair activity which is performed on a failed equipment to improve its condition from the failed condition to an operable condition [2].

Utilities adopt different maintenance strategies. According to the classification in [2], an overview of maintenance approaches is shown in Figure 2.1. Basic maintenance approaches described in [2] are maintenance as per manufacturer's specifications, replacement, scheduled maintenance and predictive maintenance. The simplest maintenance approach is to perform maintenance based on the long term experience or according to manufacturers' recommendations given in manuals [2]. Replacement schemes ignore the possible small scale improvements in the equipment's condition which can be performed at a lower cost. Scheduled maintenance is carried out at regular intervals according to a fixed schedule [2]. Predictive maintenance activities are performed when periodic inspections or condition monitoring reveals that it is necessary to perform maintenance [2].

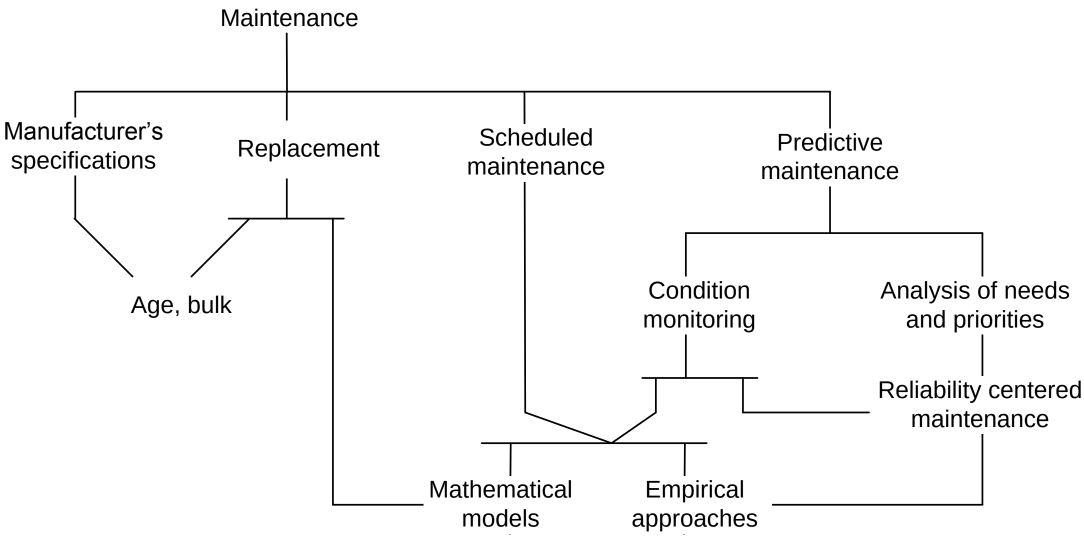

Figure 1.2: Overview of maintenance approaches [2]

Maintenance is beneficial to both electrical utilities and power consumers. Through maintenance strategies, utilities can reduce costly equipment replacements by extending equipment's lifetime. Maintenance also ensures a more reliable power supply. In addition, the social and economic losses experienced by power consumers due to sudden power failures can be minimized through timely inspection and maintenance. However, too frequent inspection and maintenance would unnecessarily increase the cost of inspection and maintenance. It would also increase the number of planned outages, and cause economic losses to consumers [6], especially to industries that consume power in a large scale. During outages, utilities too will experience economic losses due to loss of profit that they generate by selling electricity. Thus, optimal maintenance strategies should be determined considering the trade-off between reliability and costs.

1.2 Literature Review

In order to determine optimal maintenance policies, the effect of inspection and maintenance should be quantified in terms of reliability and costs. Probabilistic maintenance models [7-24] are preferably used for this purpose in preventive maintenance studies as well as in reliability centered maintenance approaches, due to their simplicity and the ability to incorporate uncertainties associated with the deterioration of equipment and the outcomes of inspection and maintenance. Many probabilistic maintenance models are based on state diagrams due to two main advantages. Firstly, state diagrams can combine deterioration, inspection and maintenance processes of a device to form simple and straightforward graphical models which indicate connections between different states of the device. Secondly, state diagrams can be directly converted into mathematical models called Markov models which can be easily solved using standard methods and analytical equations.

Markov maintenance models are firstly used to model scheduled maintenance when inspection rates are periodic [8, 9, 25]. Later, with the change in the maintenance practice to increase the inspection frequency based on the knowledge of the increased deterioration level of the device, non-periodic inspection rates are introduced to state diagrams in maintenance modeling [10-15, 26]. In [7], a non periodic inspection and maintenance model is proposed for the maintenance of high voltage air blast circuit breakers. It is discussed further in [10] and utilized in an asset management planner which can be used to decide the best maintenance option which maximizes reliability with a minimum cost. In [12], a maintenance model is proposed for the inspection and maintenance of oil immersed transformers and it is later used in [13] to analyze the effect of different inspection rates on reliability and all associated costs. Based on the results in [13], it has been suggested to increase the inspection rate with the deterioration for effective maintenance in terms of cost and reliability. A similar probabilistic model has been introduced in [14] for the inspection and maintenance of circuit breakers. This model is utilized in [15] to carry out a sensitivity analysis and this analysis has shown that the probability of failure and the total cost can be reduced by conducting inspections at a higher rate when the device is more deteriorated. Based on the model in [7], a decision varying Markov model is proposed in [27] to occupy different transition probabilities depending on the maintenance decisions made at different time intervals. This model is applied in [27] for optimization of substation maintenance. In [28], the same model is applied for composite power systems to optimize maintenance schedules. However, the above mentioned Markov maintenance models are unable to represent the actual maintenance situation of equipment [29].

Reaching a major milestone, unrealistic properties of the basis of above maintenance models are first discussed in [29]. Some interesting results are provided in [29] by comparing the results of a Markov model with Monte Carlo simulation results. These results prove that existing Markov maintenance models provide accurate results for periodic inspections, but they do not provide accurate results when inspection rates are non-periodic [29]. The author of this paper concludes that any Markov maintenance model based on state diagrams do not provide accurate results.

In view of this, an alternative model is proposed in [29] to obtain accurate results. This model proposed in [29] assumes that the deterioration process and inspection and maintenance process are two independent processes, which are only connected at inspection and maintenance or failures. Due to this assumption, an effort has been made to eliminate direct connections between the two processes. This effort finally led to a complicated state diagram for a probabilistic maintenance model. The main drawback of this graphical model in [29] is the difficulty of finding analytical solutions. To solve this model, Monte Carlo simulation is required. One of the intentions of the work presented in this thesis is to propose scheduled maintenance models based on new state diagrams which can be analytically solved using Markov techniques to obtain accurate results.

In addition, this thesis highlights two main issues which are still not addressed in previously proposed maintenance models. Firstly, time delays in making decisions regarding inspection and maintenance are not included in most previous models [7, 8, 13, 15, 19, 21, 22, 30-41]. Since optimal inspection and maintenance actions may depend on delay times in making decisions regarding inspection and maintenance, these delays should be considered when determining optimal policies. Secondly, time based maintenance models represent equipment's deterioration by the overall condition based on the age [7, 8, 13, 15, 19, 21, 22, 30-35], while condition based maintenance models represent the deterioration of the equipment by some observable measurements [36-43]. However, the deterioration of the equipment's measureable conditions may get accelerated with the ageing. Thus, it is more accurate if models can integrate the deterioration of equipment's measurable conditions with effects of ageing on deterioration. If a model can address the two aforementioned issues, such a model would be able to provide more adaptive inspection and maintenance policies. This thesis intends to propose a Markov decision process model to address the abovementioned two issues.

1.3 Research Objectives

In view of the review in section 1.2, there is a need to propose new maintenance models which address the limitations of maintenance models in the literature. The main objective of this thesis is to propose analytically solvable maintenance models to obtain accurate results in power system reliability assessments and maintenance optimization. The specific objectives within this general objective and the significance of the work are discussed below.

- To propose a new probabilistic model for scheduled maintenance

As reviewed in section 1.2, existing scheduled maintenance models based on state diagrams do not provide accurate results for non periodic inspections, when they are solved using Markov techniques [29]. Although accurate results can be obtained using Monte Carlo simulation, it consumes more time and requires more computational power to run the simulation until convergence. Therefore one of the objectives of this thesis is to develop analytically solvable, more accurate scheduled maintenance models.

- To apply Markov maintenance models to analyse the effect of maintenance on power system equipment

This thesis aims to apply the newly proposed scheduled maintenance model into circuit breakers using real data obtained from the literature. With the use of this circuit breaker maintenance model, several analyses will be performed to study the effect of maintenance on reliability and costs. Considering reliability and cost trade-off, this maintenance model will be further utilized in maintenance optimization. In two other studies, Markov models will be applied for state prediction of transformers and for analyzing the effects of sub-component characteristics on reliability of a wind energy conversion system.

- To propose a Markov decision process model to obtain more adaptive optimal maintenance policies

From the discussion in section 1.2, previous maintenance models do not account for time delays in making decisions regarding inspection and maintenance. In addition, they are unable to integrate the deterioration of equipment's measurable conditions with the effects of ageing on deterioration. In order to address the above mentioned two issues, this thesis aims to propose a maintenance model based on a Markov decision process. This thesis also intends to apply the proposed Markov decision process model to determine optimal inspection and maintenance policies for transformers. The proposed Markov decision process model will be able to provide more adaptive optimal maintenance policies.

This thesis will mainly focus on the two established maintenance strategies in electrical utilities; scheduled maintenance and predictive maintenance [26]. Since random failures cannot be avoided by performing inspection and maintenance activities, such failures will not be considered in the models proposed in this thesis. This thesis only intends to demonstrate the use of maintenance models in finding optimal maintenance policies. Developing efficient algorithms and asset management tools for maintenance scheduling and optimization is beyond the scope of this thesis. It may be required to set several assumptions when the models are developed and those assumptions will be discussed in detail, in coming chapters.

1.4 Thesis Outline and Organization

The organization of this thesis is given below.

Chapter 2: In chapter 2, a new probabilistic maintenance model is proposed for scheduled maintenance, after identifying unrealistic properties of classical maintenance models. The accuracy of the proposed model is proved through a numerical example and a theoretical discussion. In this chapter the first objective of the thesis is met.

Chapter 3: In chapter 3, Markov maintenance models are applied into power systems. First, the scheduled maintenance model proposed in chapter 2 is applied for reliability and cost assessments of circuit breakers using real data. Secondly, this chapter investigates the application of Markov models for state prediction of transformers. Thirdly, with the application of a Markov model developed for a wind energy conversion system, this chapter investigates the effects of subcomponent characteristics on system reliability. The application studies presented in this chapter can be counted towards meeting the second objective of the thesis.

Chapter 4: In chapter 4, circuit breaker maintenance models in chapter 3 are further utilized in maintenance optimization. The optimization problem is formulated by considering the trade-off between six reliability and cost measures. Using a grid search algorithm, optimal inspection and maintenance rates are determined. With the maintenance optimization work presented in this chapter and the application studies presented in chapter 3, the second objective of the thesis is met.

Chapter 5: In chapter 5, a Markov decision process model is proposed to obtain adaptive optimal maintenance policies. The proposed model is applied to transformers using real data. The possibility of extending the MDP model for system level maintenance planning is discussed. In this chapter, the third objective of the thesis is met.

Chapter 6: In chapter 6 conclusions and suggestions for future work are provided.

Chapter 2 : A New Probabilistic Model for Scheduled Maintenance

2.1 Introduction

As stated in chapter 1, previously proposed scheduled maintenance models based on state diagrams provide accurate results only for periodic inspections, and do not provide accurate results for non-periodic inspections [29]. Although a graphical model has been proposed in [29] to obtain accurate results even for non-periodic inspections, it is difficult or impossible to solve this graphical model using analytical equations [29]. This chapter aims to propose a scheduled maintenance model based on a new state diagram, after correctly identifying an impractical property of state diagrams which provide the basis for previously proposed scheduled maintenance models. In addition, this chapter intends to verify the accuracy of the proposed maintenance model through a theoretical discussion and a numerical example. The focus of this chapter is limited to maintenance which assumes that the present condition of the device is improved due to maintenance by one stage. However, in real practice, maintenance is imperfect and may not always improve the present deterioration condition of the device by only one stage. In the forthcoming chapter, the concept behind the scheduled maintenance model proposed in this chapter is applied into imperfect maintenance as well.

This chapter is organized according to the following structure. In section 2.2, a generalized classical state diagram is compared with the practical maintenance situation and an idealistic model property is discussed. In section 2.3, a new state diagram is proposed to eliminate the unwanted idealistic model property discussed in section 2.2. This section also provides a general theoretical discussion to prove the accuracy of the Markov model based on proposed state diagram. In section 2.4, a numerical example is used to validate the results of the Markov maintenance model based on the proposed state diagram. Finally, a short summary is given in section 2.5.

2.2 Classical State Diagrams in Maintenance Modeling

2.2.1 A Generalized Classical State Diagram

Figure 2.1 shows a generalized classical state diagram which provides the basis for previously proposed scheduled maintenance models. As shown in Figure 2.1, the deterioration process of the device is modeled using n deterioration states; S_1, S_2 , … , S_n. If no inspection and maintenance is performed, the deterioration process will be ended up at the failure state F. Model parameters, λ_1, λ_2, … , λ_{n-1} are deterioration rates and λ_n is the transition rate from the last deterioration state to the failure state. If a failure is occurred, the device is replaced to the original state S_1 and μ_F is the repair rate.

In order to minimize catastrophic failures, non-periodic inspection and maintenance activities are carried out. State dependent inspection rates for states S_1, S_2, … and S_n are γ_1, γ_2, … and γ_n, respectively. Inspections at I_1 would reveal that the device is still in good condition and no maintenance is required. Hence, the device is returned to as good as new state S_1. For any i=1 to (n-1), at inspection state I_{i+1}, it is identified that the device is deteriorated to S_{i+1} and maintenance is carried out at M_{i+1}. Since maintenance improves the present condition of the device by one stage [13], due to maintenance activities at M_{i+1} the state of the device is improved to S_i. μ_{i+1} is the maintenance rate and δ_{i+1} is the transition rate from I_{i+1} to M_{i+1}.

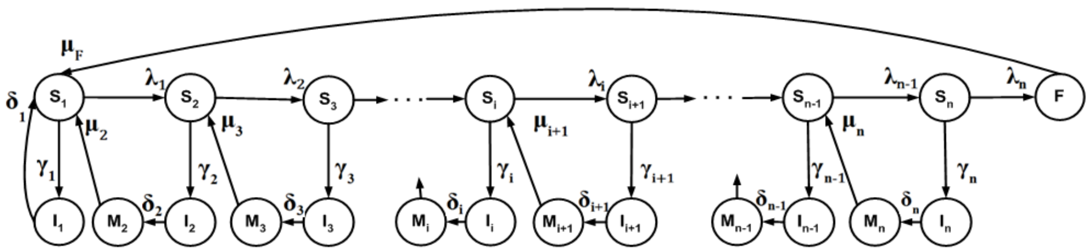

Figure 2.1: A generalized classical state diagram

2.2.2 An Idealistic Modeling Property of Classical State Diagrams

According to the classification of maintenance models, the maintenance models based on classical state diagrams belong to the category of inspection models [44]. The definition of inspection models given in [44] is "*Inspection models usually assume that the state of the system is completely unknown unless an inspection is performed. Every inspection is normally* *assumed to be perfect in the sense that it reveals the true state of the system without error. In the absence of repair or replacement actions, the system evolves as a non-decreasing stochastic process. In general, at every decision epoch there are two decisions that have to be made. One decision is to determine what maintenance action to take, whether the system should be replaced or repaired to a certain state or whether the system should be left as it is. The other decision is to determine when the next inspection epoch is to occur*". The assumptions in this definition reasonably agree with the inspection and maintenance situation in the real world.

Assumptions of classical state diagrams do not agree with the above definition which describes the actual maintenance practice [29]. Classical state diagrams assume that the present state of the device is always known to the operator [29]. However, in practice, the present state of the device is known to the operator only after an inspection or a maintenance activity [29], and this fact is not properly captured in classical state diagrams.

For example, in the classical state diagram in Figure 2.1, whenever there is a transition to deterioration state S_i, inspection rate is set to a fixed inspection rate γ_i. Inspection rate of S_i should be set to γ_i, only if no maintenance is carried out after inspections at I_i or the condition of the device is graded as S_i after maintenance. If the device is deteriorated from S_k to S_i prior to any inspection, the operator does not know that the current condition is S_i. Therefore, inspections are not carried out at a rate of γ_i. Though the device is at S_i, the operator conducts inspections at a rate of γ_k, assuming that the device is still at S_k, where i =2, 3, …, n and k =1, 2, …, (i-1). Therefore, the inspection rate at S_i would vary from γ_1 to γ_i, and such variations in the inspection rate of each deterioration state are not included in classical state diagrams.

2.3 The Proposed Scheduled Maintenance Model

2.3.1 The Proposed State Diagram

The state diagram shown in Figure 2.2 is proposed to better represent the actual maintenance practice. The advantage of this new state diagram is its ability of combining the deterioration process and the inspection and maintenance process using direct connections, while eliminating impractical modeling properties of the classical state diagram in Figure 2.1.

Combining these two processes to obtain a single process is acceptable due to two reasons. Firstly, they are not independent processes, because maintenance activities affect the deterioration status, the deterioration status affects inspection rates, and maintenance activities are done according to the requirements of the present deterioration status of the device. Secondly, operations of the device would be stopped in order to carry out scheduled inspection and maintenance. Therefore, during the deterioration process, maintenance and inspection process is stopped and vice versa. This provides the facility to combine the two processes into a single process in a single state diagram.

In the proposed state diagram in Figure 2.2, the deterioration process of the device is modeled as a combination of (n-1) parallel sub deterioration processes which are ended up at the failure state F. Sub deterioration process i has (n-i+1) deterioration states at which inspections are carried out at a rate of γ_i. Deterioration state S_i in the classical state diagram is represented using i sub states ($S_{i,1}$, $S_{i,2}$, $S_{i,3}$, … , $S_{i,i}$) in the proposed state diagram. When the device is at the sub deterioration state $S_{i,k}$, the inspection rate is γ_k. All other states except for the deterioration states remain the same as in the classical state diagram in Figure 2.1.

In this proposed state diagram in Figure 2.2, sub deterioration states are used to vary the inspection rate of each deterioration state depending on the knowledge about the system. For example, consider the sub deterioration states in the proposed state diagram which are corresponding to S_i in classical state diagram. If the device is at $S_{1,1}$ (that is the device is new or the state of the device is decided as good as new after inspections at I_1 or inspection and maintenance at I_2 and M_2) and deteriorates to the i^{th} deterioration state S_i prior to any other inspection, the current deterioration state is unknown to the operator and hence at $S_{i,1}$ inspections are carried out at a rate of γ_1 assuming that the device is still at the first deterioration state S_1. If the device is at $S_{2,2}$ (that is the state of the device is upgraded to S_2, after inspection and maintenance at I_3 and M_3) and deteriorates to the i^{th} deterioration state S_i prior to any other inspection, the inspection rate at $S_{i,2}$ is γ_2, because the operator conducts inspections assuming that the device is still at the second deterioration state S_2. Likewise, the sub deterioration state k of the deterioration state i ($S_{i,k}$) has the inspection rate γ_k. At last, if the device is at $S_{i,i}$ (that is if the condition of the device is upgraded to S_i, after inspection at I_{i+1} and maintenance at M_{i+1}), the operator knows that the device is in S_i and conducts inspections at a rate of γ_i. Therefore, in this model, the inspection rate of S_i can be varied from γ_1 to γ_i, depending on the knowledge about the device, with the aid of the sub deterioration states.

As discussed in [29], one of the key points which demonstrate beneficial property of the proposed model is the utilization of γ_n. Since there is no transition from I_n, M_n or F to S_n in the classical state diagram in Figure 2.1, γ_n should be neglected, and by doing so, the classical state diagram will be incomplete. On the other hand, in the proposed state diagram, sub states of the last deterioration state S_n has inspection rates varying from γ_1 to γ_{n-1}, and γ_n is not utilized in the proposed state diagram. This illustrates the useful model property of the proposed state diagram.

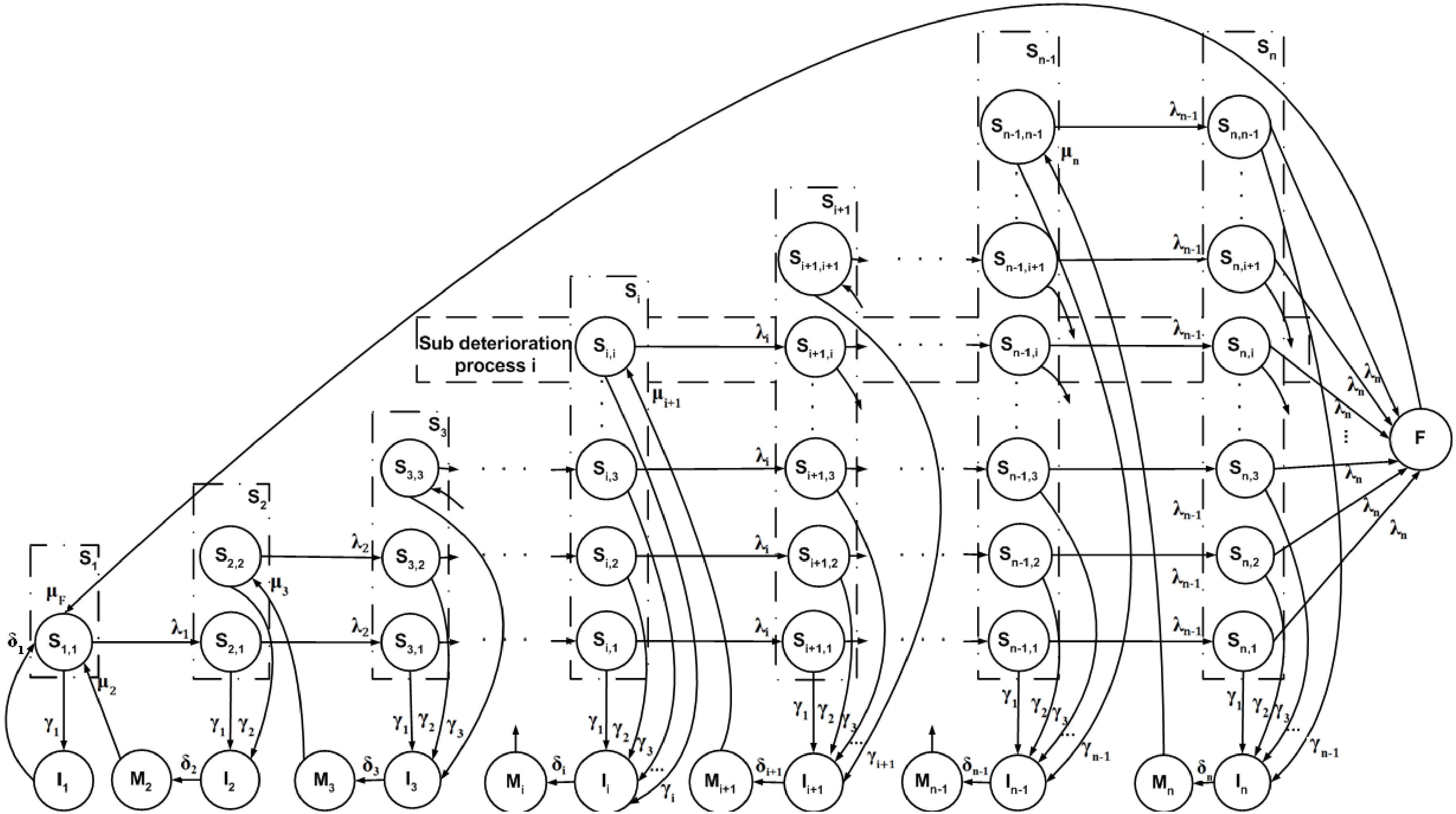

Figure 2.2: The proposed state diagram for probabilistic maintenance models

When comparing the two state diagrams in Figures 2.1 and 2.2, it can be clearly seen that the proposed state diagram has a large number of states compared to the classical state diagram. The number of additional states in the proposed state diagram in Figure 2.2 is found to be $(n-2)\times(n+1)/2$. The increase in number of states can be considered as a disadvantage of the proposed model, especially when the number of deterioration levels is high. However, the proposed state diagram in Figure 2.2 can be reduced to the state diagram shown in Figure 2.3, after computing state probabilities. The only difference between the two state diagrams in Figures 2.1 and 2.3 is their different inspection rates other than the first inspection rate. New inspection rate for the deterioration state i ($\gamma_{i,\text{new}}$) in Figure 2.3 can be calculated using (2.1) which is derived using the frequency balance technique [45]. $P_{i,k}$ is the probability of being in the sub state $S_{i,k}$.

$$\gamma_{i,\text{new}} = \frac{\sum_{k=1}^{i}\left(P_{i,k} \times \gamma_k\right)}{\sum_{k=1}^{i} P_{i,k}} \tag{2.1}$$

This state diagram in Figure 2.3 reduces the complexity of the proposed state diagram in Figure 2.2 and hence, it is advantageous to use this for any other extended analysis beyond the optimization of inspection intervals.

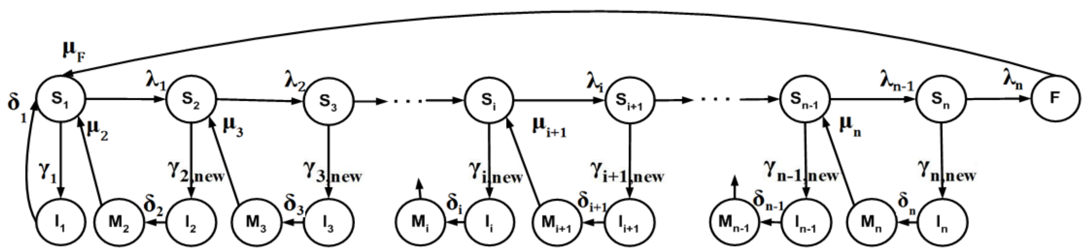

Figure 2.3: The reduced state diagram of the proposed state diagram in Figure 2.2

2.3.2 Mathematical Realization of Maintenance Models

Maintenance models are mathematically solved to compute reliability indices and other performance measures. There are two main methods for mathematical realization of maintenance models based on state diagrams, namely, Markov methods and Monte Carlo simulation techniques.

If the maintenance model is based on a state diagram, it is converted into a Markov model and reliability calculation is easily performed using analytical equations. In a Markov process, next transition only depends on the current state and it is independent from past behavior of the device. If all transition times are exponentially distributed, the Markov model has constant transition rates. On the other hand, if the time spent in a state is random, semi Markov models can be used to solve state diagrams with such non-exponential distributions [19, 45, 46]. Device of stages method is another method which is used to represent non-exponentially distributed states in non-Markovian models with a combination of exponentially distributed states [45, 47, 48].

Monte Carlo simulation is used, if the basis of the maintenance model is not a state diagram, but a complicated graph as suggested in [29]. There are two concepts of conducting Monte Carlo simulation [29]. One concept is redrawing both the next deterioration time and the next inspection time after each state transition due to deterioration, inspection and maintenance and this concept is termed as redrawing (RD) concept [29]. It is found in [29] that the simulation results based on RD concept are as same as the results obtained using Markov models based on classical state diagrams for non-periodic inspections. The other concept is drawing the next deterioration time only after a change in the deterioration state due to deterioration or maintenance and drawing the next inspection time based on the decisions after an inspection or maintenance. This second concept, which better represents the scheduled maintenance practice, is termed as non-redrawing (NRD) concept in [29].

For the following discussion, consider the two Markov processes based on the classical state diagram in Figure 2.1 and the corresponding proposed state diagram in Figure 2.2. As stated in [29], these Markov processes behave according to RD concept and redraw both the time to next deterioration and the time to next inspection after each state transition from S_i to S_{i+1} or I_i to S_i. However, due to the memory less property of Markov process, redrawing does not affect the results, if the state diagram is suitably constructed.

The following discussion shows that the results of the Markov process based on the proposed state diagram are as same as that of NRD concept, although the results of the

Markov process based on the classical state diagram are different from that of NRD concept. Due to the memory less property of Markov processes, the next transition only depends on the current state and it is not affected by past transitions. Therefore, the focus of this discussion is on the time to next inspection and the time to next deterioration. If Monte Carlo simulation is used for mathematical realization, two random numbers are drawn. One random number with rate λ_1 is for the sojourn time in S_1 (time to next deterioration) and the other random number with rate γ_1 is for the time to next inspection. The next transition from S_1 is determined by these two random numbers which are denoted by s_1 and τ_1, respectively.

Consider the first case when $\tau_1<s_1$. Since the time to next inspection is less than the time to next deterioration, the system transits from S_1 to I_1. The inspection at I_1 reveals that the current deterioration state of the system is S_1. Hence, no maintenance is required and the system transits from I_1 to S_1. If we realize any of the two state diagrams according to the previously mentioned RD concept, now at S_1, two numbers are redrawn for the time to next inspection and for the time to next deterioration with the rates γ_1 and λ_1, which are denoted by $\tau_1^*$and s_1^*, respectively. On the other hand, in NRD concept, only the time to next inspection is redrawn with a rate of γ_1, and therefore the time to next inspection is τ_1^*. The time to next deterioration remains unchanged as s_1. Since s_1 and s_1^* are randomly drawn from the same exponential distribution, time to next deterioration also can be considered as the same for both concepts. This shows that the time to next deterioration and the time to next inspection obtained using both classical and proposed state diagrams are not affected by redrawing, and when $\tau_1<s_1$, they are as same as those obtained using NRD concept.

Next, consider the second case where $s_1<\tau_1$. Since next deterioration time is less than the next inspection time, the system transits from S_1 to the next deterioration state. In the classical state diagram this transition is from S_1 to S_2 and therefore, the inspection rate is set to γ_2. However, in the proposed state diagram, the system transits from S_1 ($S_{1,1}$) to $S_{2,1}$ and the inspection rate remains unchanged at γ_1. Now, we realize the two state diagrams according to the RD concept, and compare with the NRD concept. For this case, both concepts redraw the time to next deterioration and the difference is in handling the time to next inspection.

According to the concept of redrawing, the classical state diagram redraws the time to next inspection with a rate of γ_2 and let it to be denoted by τ_2^*. The proposed state diagram redraws the time to next inspection with a rate of γ_1, which is denoted by τ_1^*. On the other hand, the time to next inspection is not redrawn in NRD concept, and it remains unchanged at τ_1. The time to next deterioration can be considered as the same for NRD concept and the proposed state diagram, since τ_1 and τ_1^* are randomly drawn from the same exponential distribution. Since τ_2^* is drawn from a different distribution, the time to next deterioration in the classical state diagram is different from that of NRD concept. It is also clear that this difference does not appear in periodic inspections, because inspection rate does not vary with the deterioration state. This is the reason behind the accurate results provided by classical state diagrams with periodic inspection rates.

Based on the above discussion, it can be concluded that both the Markov process based on the proposed state diagram and Monte Carlo simulation based on NRD concept give accurate results. Whereas, the Markov process based on the classical state diagram gives accurate results only for periodic inspections. The difference in reliability indices between classical model and proposed model is illustrated in a numerical example in the following section.

2.4 A Numerical Example

In this section, a numerical example is used to check the accuracy of the classical model and the proposed model. This example is based on the classical and proposed state diagrams in Figures 2.4 and 2.5. Please note that Figures 2.4 and 2.5 are as same as Figures 2.1 and 2.2, respectively, when the number of deterioration stages (n) of Figures 2.1 and 2.2 is set to three.

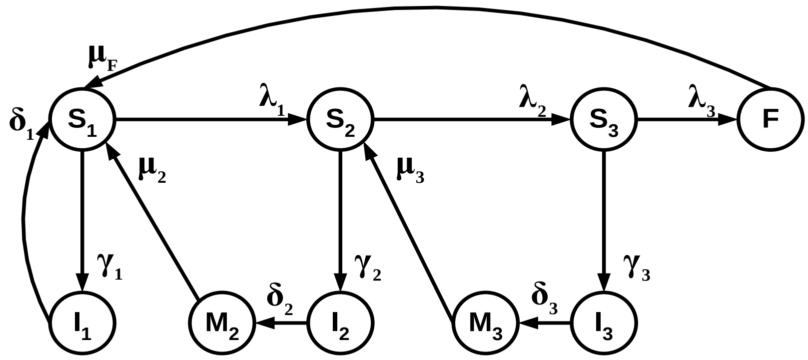

Figure 2.4: Example of a classical state diagram [29]

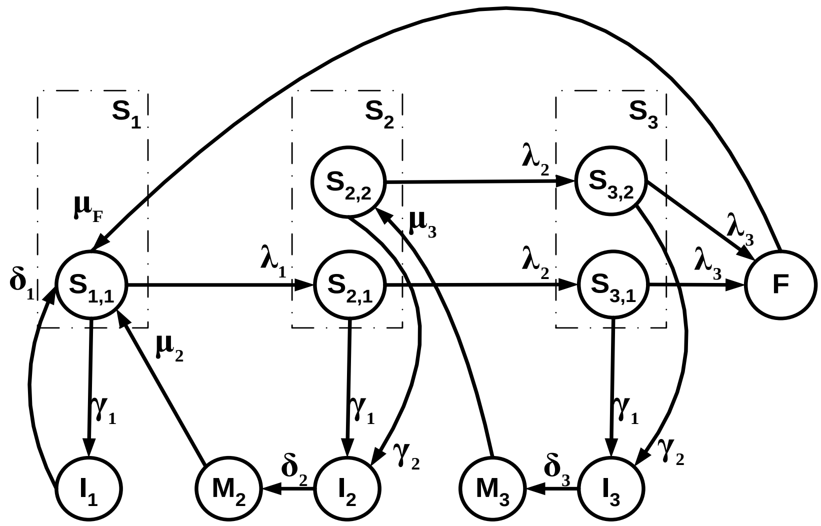

Figure 2.5: The proposed state diagram

Table 2.1: Transition Rates (1/years) [29]

Rate	Value
λ_1	0.33
λ_2	0.29
λ_3	0.5
γ_1	0.5
γ_2	1
γ_3	1
δ_1	360
δ_2	360
δ_3	360
μ_1	12
μ_2	360
μ_3	180

Using the transition rates given in Table 2.1, the models based on the classical state diagram and the proposed state diagram are realized using standard Markov methods to find state probabilities, visit frequencies and mean durations. Some reliability indices such as mean time between failures (MTBF) and mean time to first failure (MTTFF) are also computed. The results for the two models are tabulated in corresponding columns of Tables 2.2, 2.3, 2.4 and 2.5.

The transition rates given in Table 2.1 are as same as the transition rates used in [29]. Therefore the results obtained in [29] by conducting Monte Carlo simulation for a graphical model which represents the real world maintenance situation can be used to verify the accuracy of using Markov methods for the two models based on the classical state diagram and the proposed state diagram. The last columns of Tables 2.2 to 2.5 show the results obtained in [29] by conducting Monte Carlo simulation using NRD concept.

Table 2.2: State Probabilities

State	Classical Model	Proposed Model	Monte Carlo Simulation [29]
S_1	0.7326	0.6197	0.61711
S_2	0.2204	0.2931	0.29641
S_3	0.0426	0.0817	0.08097
I_1	0.0010	0.0009	0.00086
M_2	0.0006	0.0005	0.00046
I_2	0.0006	0.0005	0.00046
M_3	0.0002	0.0002	0.00024
I_3	0.0001	0.0001	0.00012
F	0.0018	0.0034	0.00336

Table 2.3: Visit Frequencies (1/years)

State	Classical Model	Proposed Model	Monte Carlo Simulation [29]
S_1	0.6080	0.5143	0.514
S_2	0.2844	0.2486	0.250
S_3	0.0639	0.0850	0.084
I_1	0.3663	0.3098	0.308
M_2	0.2204	0.1637	0.166
I_2	0.2204	0.1637	0.166
M_3	0.0426	0.0442	0.044
I_3	0.0426	0.0442	0.044
F	0.0213	0.0408	0.041

Table 2.4: Mean Durations (years)

State	Classical Model	Proposed Model	Monte Carlo Simulation [29]
S_1	1.2048	1.2048	1.200
S_2	0.7752	1.1787	1.185
S_3	0.6667	0.9611	0.958
I_1	0.0028	0.0028	0.003
M_2	0.0028	0.0028	0.003
I_2	0.0028	0.0028	0.003
M_3	0.0056	0.0056	0.006
I_3	0.0028	0.0028	0.003
F	0.0833	0.0833	0.083

Table 2.5: Reliability Indices (years)

	Classical Model	Proposed Model	Monte Carlo Simulation [29]
MTBF	46.9283	24.4841	24.7
MTTFF from S_1	46.8450	24.4008	24.5
MTTFF from S_2	43.8105	21.3662	23.5

Table 2.6: Percentage Deviations of Reliability Indices

	Classical Model	Proposed Model
Percentage deviation of MTBF (%)	-89.99	0.87
Percentage deviation of MTTFF from S_1 (%)	-91.20	0.40
Percentage deviation of MTTFF from S_2 (%)	-86.43	9.08

As can be seen from Tables 2.2 to 2.5, the results obtained by solving the maintenance model based on the classical state diagram in Figure 2.4 significantly differ from the results obtained by conducting Monte Carlo simulation. Whereas, the results obtained by the maintenance model based on the proposed state diagram in Figure 2.5 are very much closer to the Monte Carlo simulation results.

Monte Carlo simulation results are considered to be highly accurate, because they are obtained by solving the graphical model in [29] based on NRD concept which is in accordance with the actual maintenance situation. For both models, percentage deviations of reliability indices are calculated with respect to Monte Carlo Simulation results and those percentage deviations are tabulated in Table 2.6. The proposed model has low percentage deviations. Negative percentage differences indicate the overestimation of reliability indices. The reliability indices of the classical model have very high negative percentage deviations due to the misrepresentation of actual maintenance practice. This numerical example verifies the accuracy of the proposed scheduled maintenance model, supporting the theoretical discussion in the previous section.

2.5 Summary

State diagrams are commonly used in maintenance modeling. Classical state diagrams possess an unwanted modeling property that may not be able to represent the maintenance situation well. Inspection rate is typically set based on the knowledge about the deterioration of the device which is only available after inspection and maintenance. Classical state diagrams fail to represent this characteristic, because the inspection rate is changed whenever there is a change in the deterioration state. However, this change in deterioration is unknown to the operator before conducting inspections and in this situation the inspection rate should not be changed. This unwanted model property of classical state diagrams is analyzed in this chapter.

Due to this property, maintenance models based on classical state diagrams provide accurate results only for periodic inspection and do not provide accurate results when inspections are non-periodic. In this chapter, a new state diagram is proposed to eliminate this modeling error from a generalized classical state diagram. The only disadvantage of this proposed state diagram is the increase in number of states. A reduced version for the proposed state diagram is also presented for further extended analyses.

Standard Markov methods and Monte Carlo simulation techniques are used for mathematical realization of probabilistic maintenance models. The two concepts to carry out Monte Carlo simulations are RD and NRD concepts [29]. It is theoretically shown that the results of Markov models based on classical state diagrams are similar to that of unrealistic RD concept, while Markov models based on proposed sate diagrams give similar results as in more realistic NRD concept. In a numerical example, classical and proposed maintenance models are solved using Markov techniques and the results are compared with Monte Carlo simulation results. It is shown that the results of the maintenance model based on the classical state diagram are significantly different from Monte Carlo simulation results, but the results of the maintenance model based on the proposed state diagram are very similar to Monte Carlo simulation results. This numerical example validates that the scheduled maintenance model based on the proposed state diagram represents the real world maintenance situation and can be solved using standard Markov methods to get accurate results.

In the next chapter, we apply this proposed scheduled maintenance model to perform reliability and cost assessments of circuit breakers. The next chapter also applies Markov maintenance models for state prediction of transformers and for analyzing the effect of sub-components' characteristics on the reliability of a wind energy conversion system.

Chapter 3 : Applications of Markov Maintenance Models to Power Systems

3.1 Introduction

This chapter presents three different applications of Markov maintenance models related to power systems. In the first study the scheduled maintenance model proposed in chapter 2 is applied for reliability and cost assessments of circuit breakers. This study utilizes real data [7] which reflects that maintenance is imperfect in actual practice. This application study aims to evaluate the performance of the proposed scheduled maintenance model while relaxing the assumption made in chapter 2 (i.e. maintenance improves the present condition of the equipment by only one stage). The second application study aims to investigate the possibility of utilizing Markov models to predict the states of vulnerable power system equipment. This study is conducted on local transformers. After investigating the effect of loading and operating years on deterioration of transformers using field data, the study utilizes a simple Markov model for state prediction. In the third application of Markov models, the thesis intends to explore the effect of failure and repair characteristics of sub components on the reliability of a system. The Markov model utilized in this study is developed for a wind energy conversion system.

This chapter is organized as follows. In section 3.2, Markov maintenance models discussed in chapter 2 are applied to reliability and cost assessments of circuit breakers using actual data obtained from the literature. In addition, sensitivity analyses are conducted to observe the effect of inspection rate on reliability measures. In section 3.3, a Markov maintenance model is applied to state prediction of different transformer groups which are classified according to the operational age and loading conditions. In section 3.4, a Markov model is applied to conduct a sensitivity analysis to assess which characteristics of the components are critical to the reliability of a wind energy conversion system. Finally, a short summary is given in section 3.5.

3.2 Reliability and Cost Analysis of Circuit Breakers

In section 2.4 of the previous chapter, a numerical example is provided. In this numerical example, it is assumed that maintenance always improves the present condition of the equipment by one stage. However, real maintenance activities are imperfect and may not always improve the present deterioration condition of the device by one stage. This imperfect maintenance may not change the present condition of the device and may in fact degrade its condition. This section aims to show the applicability of the concept behind the maintenance model proposed in chapter 2 into imperfect maintenance of circuit breakers. In addition, the results of the proposed circuit breaker maintenance model are compared with the results of a classical circuit breaker maintenance model.

3.2.1 Reliability and Cost Assessments

Figure 3.1 shows a classical state diagram proposed in [7] for an imperfect maintenance model of circuit breakers. This state diagram is similar to the state diagram in Figure 2.4 except that this diagram models different possible levels of performing maintenance and different possible outcomes of maintenance. As can be seen in this state diagram, inspections are followed by either minor maintenance (M), major maintenance (MM) or no maintenance. There is a possibility to conduct major maintenance after minor maintenance and maintenance may improve or degrade the condition of the device or may not be able to affect the present condition of the device. Based on the concept behind the proposed state diagram in chapter 2, the state diagram shown in Figure 3.2 is proposed for a more realistic imperfect maintenance model.

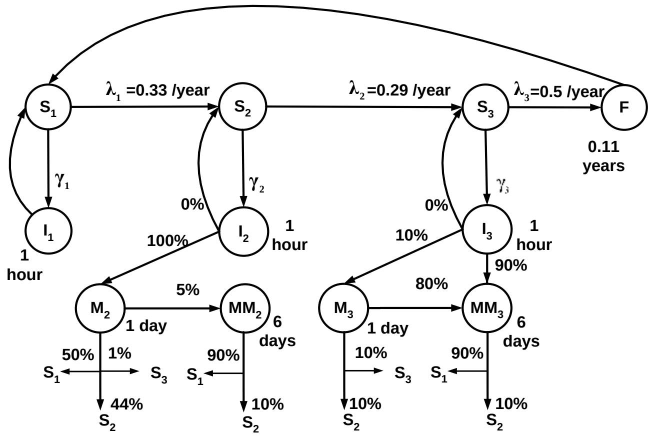

Figure 3.1: Example of a classical state diagram for imperfect maintenance [7]

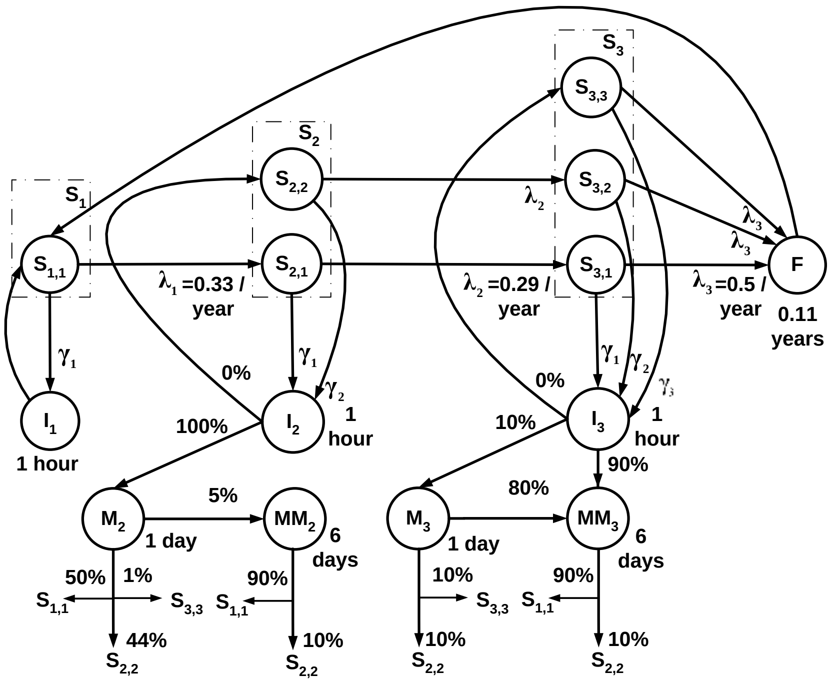

Figure 3.2: The proposed state diagram for imperfect maintenance

The classical and proposed state diagrams in Figures 3.1 and 3.2 are converted into semi Markov models and solved using well-known procedures to compute mean time between failures (MTBF), mean time to first failure (MTTFF), inspection cost, maintenance cost, repair cost and total cost [45, 49]. All data is actual data obtained from [7] for air blast circuit breakers and utilized in this example to solve both classical and proposed models. The transition rates and mean durations are shown in Figures 3.1 and 3.2. Choice and outcome probabilities after inspection and maintenance are also indicated in same figures. In addition to this, the cost details are tabulated in Table 3.1.

Table 3.1: Costs ($) [7]

	Average Cost per Activity ($)
Inspection	200
Minor maintenance	1200
Major maintenance	14400
Repair	144000

The results obtained using the two imperfect maintenance models are shown in Table 3.2. Reliability indices and cost measures of the classical model significantly deviate from those of the proposed model and the percentage deviations are given in Table 3.3. As can be seen in Table 3.3, due to the idealistic model property, the model based on the classical state diagram greatly overestimates reliability and inspection and maintenance costs, while significantly underestimating repair cost and total cost.

Table 3.2: Reliability Indices and Costs for Imperfect Maintenance Models [30]

Reliability/ Cost Measure	Model Based on the Classical State Diagram	Model Based on the Modified State Diagram
MTBF (years)	36.90	23.04
MTTFF from S_1 (years)	36.8	22.9
MTTFF from S_2 (years)	33.8	19.9
MTTFF from S_3 (years)	24.9	12.4
Annual Inspection Cost ($/year)	132	108
Annual Maintenance Cost ($/year)	1288	1101
Annual Repair Cost ($/year)	3903	6251
Annual Total Cost ($/year)	5323	7460

Table 3.3: Percentage Deviations of Reliability Indices and Cost Measures

Reliability Index/ Cost Measure	Percentage Deviation (%)
MTBF (years)	-60.2
MTTFF from S_1 (years)	-60.7
MTTFF from S_2 (years)	-69.8
MTTFF from S_3 (years)	-100.8
Annual Inspection Cost ($/year)	-22.2
Annual Maintenance Cost ($/year)	-17.0
Annual Repair Cost ($/year)	37.6
Annual Total Cost ($/year)	28.6

3.2.2 Effect of Inspection and Maintenance on Reliability

The objective of this section is to investigate the effect of inspection rate on two reliability measures namely, MTBF and availability. This analysis also utilizes the two scheduled maintenance models discussed in section 3.2.1.

3.2.2.1 Sensitivity Analysis of Inspection Rate on MTBF

This section aims to utilize the two maintenance models to observe the sensitivity of different inspection rates on MTBF. In order to check the accuracy of the model, this section especially intends to observe the behavior of MTBF, when γ_1 goes to zero.

Sensitivity Analysis of γ_1 on MTBF

Figures 3.3 and 3.4 are used to discuss different model properties between the two models shown in Figures 3.1 and 3.2. Figure 3.3 shows the variation of MTBF with γ_1, when γ_3 is fixed at 1 and γ_2 is fixed at different values (0.25, 0.5, 1 and 4). It can be seen from this figure that MTBF of the proposed model increases when γ_1 increases and the sensitivity of γ_1 increases with increasing γ_2. On the other hand, the sensitivity of γ_1 on MTBF is negligible for the classical model compared to that of the proposed model.

Figure 3.4 shows the variation of MTBF with γ_1, when γ_2 is fixed at 1 and γ_3 is fixed at different values (0.25, 0.5, 1 and 4). This figure also shows the less sensitivity of γ_1 on MTBF of the classical model. Whereas, γ_1 of the proposed model is more sensitive to MTBF and the effect of γ_3 on this sensitivity is less.

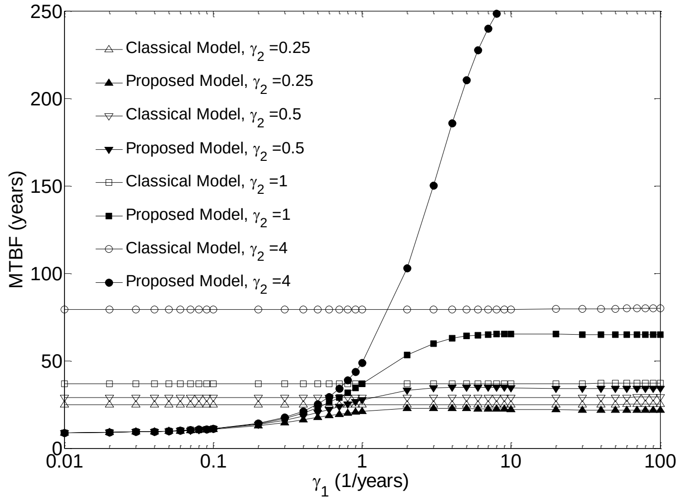

Figure 3.3: The variation of mean time between failures with γ_1, given different values for γ_2

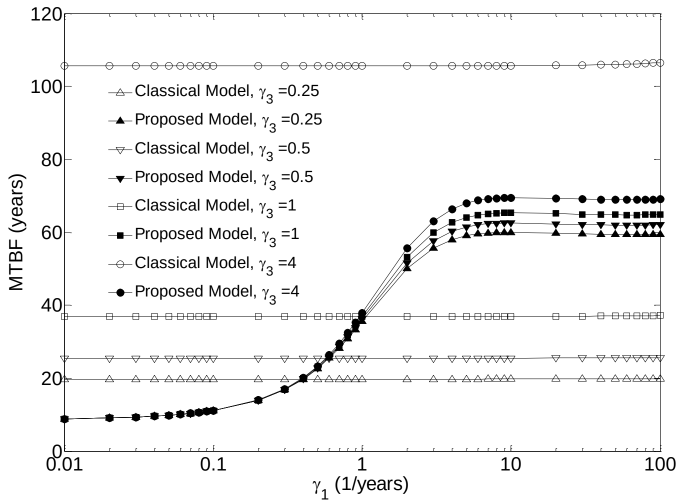

Figure 3.4: The variation of mean time between failures with γ_1, given different values for γ_3

In addition, from Figure 3.3 and 3.4, it can be observed that all curves which are obtained using the proposed model converge to a single value, while the curves which are obtained using the classical model converge to different values. This observation is further discussed in section 3.2.2.3 to verify the accuracy of the proposed model.

Sensitivity Analysis of γ_2 on MTBF

Figure 3.5 shows the variation of MTBF with γ_2, when γ_3 is fixed at 1 and γ_1 is fixed at different values (0.25, 0.5, 1 and 4). MTBF of the classical model increases with γ_2, but the effect of γ_1 on this sensitivity is negligible. In the proposed model, MTBF increases when γ_2 increases, and the sensitivity of γ_2 on MTBF of the proposed model also increases with increasing γ_1.

Figure 3.6 shows the variation of MTBF with γ_2, when γ_1 is fixed at 0.5 and γ_3 is fixed at different values (0.25, 0.5, 1 and 4). According to this figure, MTBF of the classical model rapidly increases when γ_2 increases, and sensitivity of γ_2 on MTBF further increases for higher values of γ_3. Whereas, MTBF of the proposed model increases slightly when γ_2 increases and then remains constant and the effect of γ_3 on this behavior is negligible.

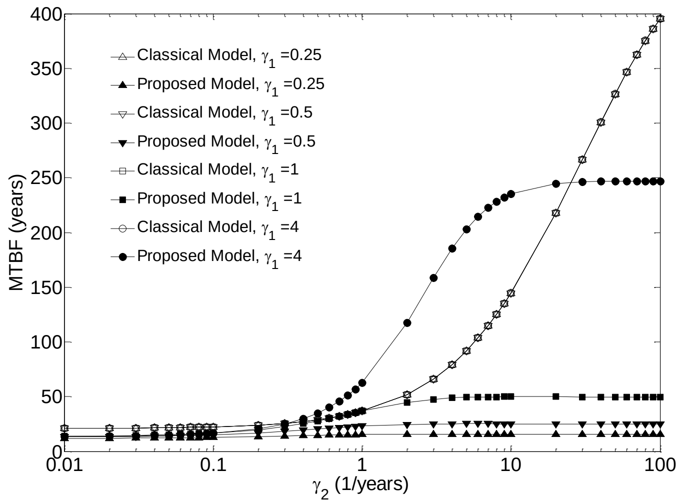

Figure 3.5: The variation of mean time between failures with γ_2, given different values for γ_1

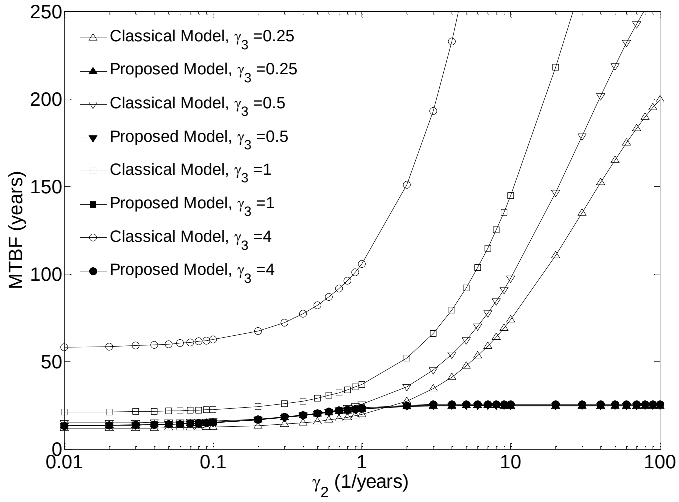

Figure 3.6: The variation of mean time between failures with γ_2, given different values for γ_3

Sensitivity Analysis of γ_3 on MTBF

Figure 3.7 shows the variation of MTBF with γ_3, when γ_2 is fixed at 1 and γ_1 is fixed at different values (0.25, 0.5, 1 and 4). MTBF of the classical model is highly sensitive to γ_3, and this sensitivity is not greatly affected by γ_1. Therefore, this figure confirms that MTBF of the classical model has a negligible effect from γ_1 and a significant effect from γ_3. On the other hand, sensitivity of γ_3 on MTBF of the proposed model is minimal and increases slightly with γ_1. This shows that MTBF of the proposed model is not much affected by γ_3, when γ_1 is low. Figure 3.8 shows the variation of MTBF with γ_3, when γ_1 is fixed at 0.5 and γ_2 is fixed at different values (0.25, 0.5, 1 and 4). This figure shows a significant effect of γ_2 and γ_3 on MTBF of the classical model. It also shows a negligible effect of γ_3 and a minor effect of γ_2 on MTBF of the proposed model. From this figure, we conclude that MTBF of the proposed model cannot be increased significantly by increasing both γ_2 and γ_3, when γ_1 is low.

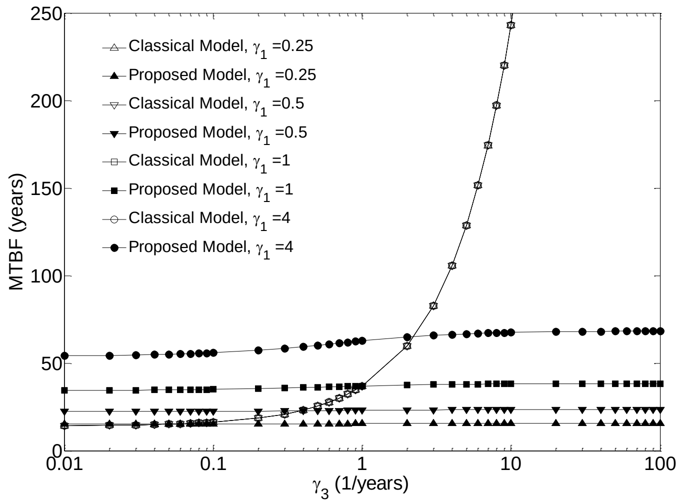

Figure 3.7: The variation of mean time between failures with γ_3, given different values for γ_1

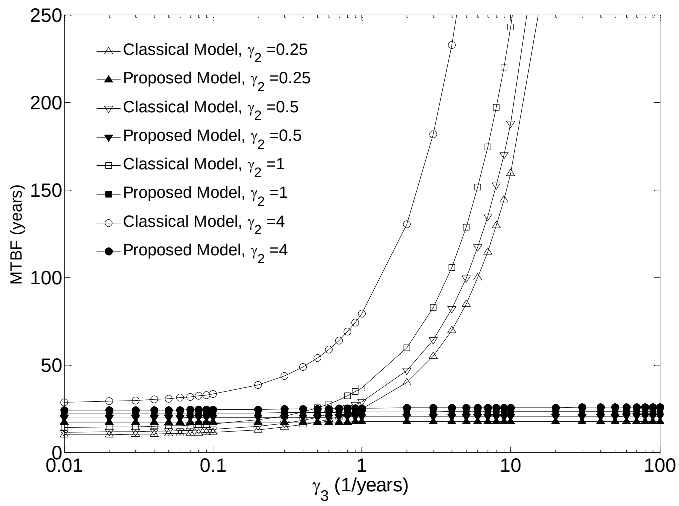

Figure 3.8: The variation of mean time between failures with γ_3, given different values for γ_2

3.2.2.2 Sensitivity Analysis of Inspection Rate on Availability

Availability is another important measure in reliability analysis. If inspection and maintenance is done too frequently, it will ensure a higher reliability but it may disturb the operation of the device too much. Therefore, maximizing availability is one of the criteria in maintenance optimization. This section aims to utilize the two maintenance models to observe the sensitivity of different inspection rates on availability and also to investigate the possibility of selecting inspection rates to maximize availability.

Sensitivity Analysis of γ_1 on Availability

Figure 3.9 shows the variation of availability with γ_1, when γ_3 is fixed at 1 and γ_2 is fixed at different values (0.25, 0.5, 1 and 4). Availability of the proposed model shows a high sensitivity to γ_1 and this sensitivity slightly increases with increasing γ_2. Most importantly, an optimum value can be seen for γ_1 of the proposed model to achieve maximum availability. For the classical model, availability decreases with increasing γ_1 and the sensitivity of γ_1 on availability increases with increasing γ_2. The results of the classical model show a possibility of maximizing availability by minimizing γ_1 as much as possible.

Figure 3.10 shows the variation of availability with γ_1, when γ_2 is fixed at 1 and γ_3 is fixed at different values (0.25, 0.5, 1 and 4). For both classical and proposed models, the variation of availability with γ_1 is almost similar to that in Figure 3.9. In addition, this figure shows that sensitivity of γ_1 on availability increases with increasing γ_3 of the classical model. However, the sensitivity of γ_1 on availability is not much affected by γ_3 of the proposed model.

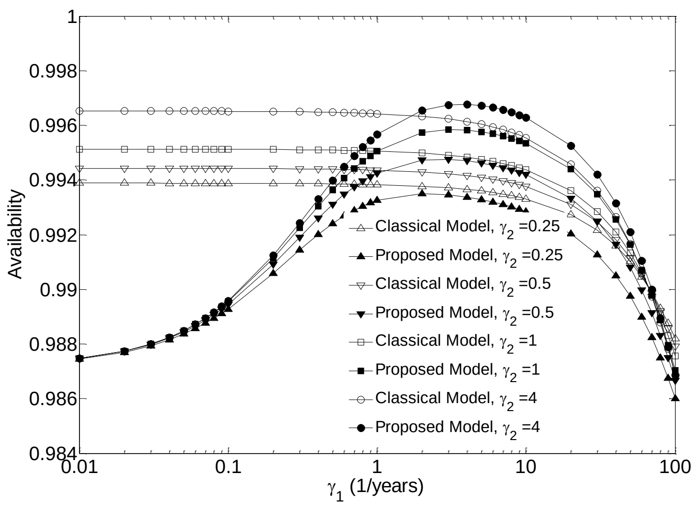

Figure 3.9: The variation of availability with γ_1, given different values for γ_2

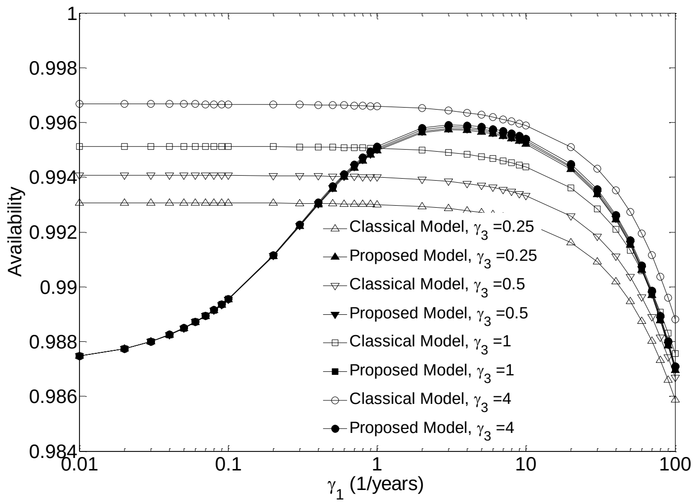

Figure 3.10: The variation of availability with γ_1, given different values for γ_3

Sensitivity Analysis of γ_2 on Availability

Figure 3.11 shows the variation of availability with γ_2, when γ_3 is fixed at 1 and γ_1 is fixed at different values (0.25, 0.5, 1 and 4). Similar behavior can be observed for both classical and proposed models. With increasing γ_2, a rapid increase in availability can be seen, but the rate of increase decreases when γ_2 is further increased. The effect of γ_1 on this behavior is insignificant for the proposed model, but sensitivity of γ_2 on availability increases when γ_1 of the classical model increases.

Figure 3.12 shows the variation of availability with γ_2, when γ_1 is fixed at 0.5 and γ_3 is fixed at different values (0.25, 0.5, 1 and 4). The behavior of availability with γ_2 in this figure is similar to that in Figure 3.11. However, the effect of γ_3 on this behavior is different from the effect of γ_1 observed in Figure 3.11. The sensitivity of γ_2 on availability increases, with the increase in γ_3 of the classical model. γ_3 of the proposed model does not show a considerable effect on this sensitivity.

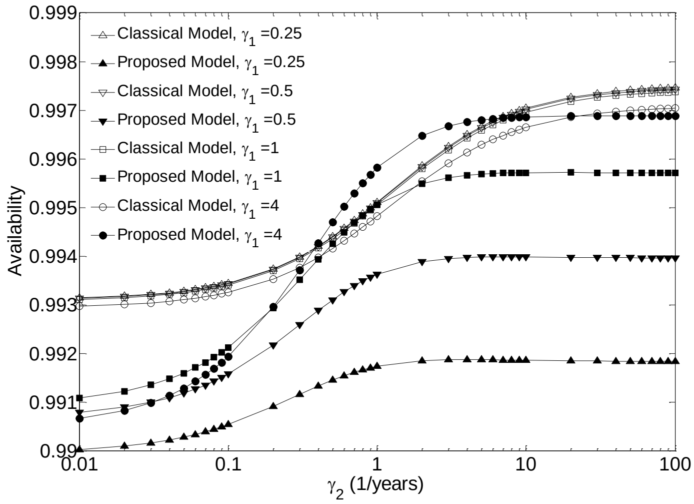

Figure 3.11: The variation of availability with γ_2, given different values for γ_1

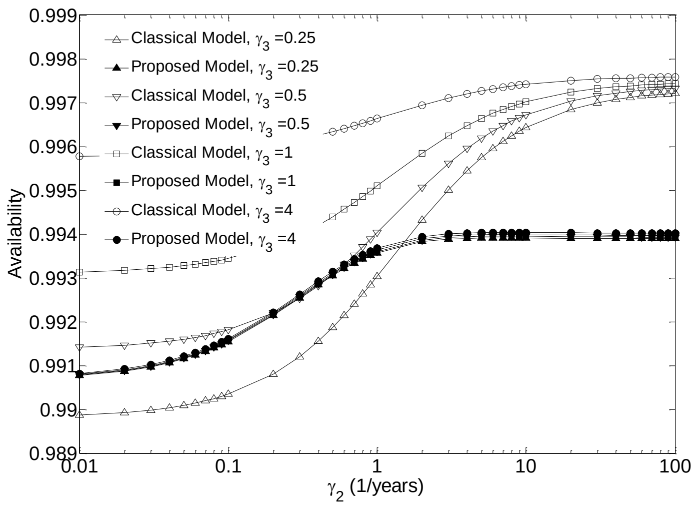

Figure 3.12: The variation of availability with γ_2, given different values for γ_3

Sensitivity Analysis of γ_3 on Availability

Figure 3.13 shows the variation of availability with γ_3 when γ_2 is fixed at 1 and γ_1 is fixed at different values (0.25, 0.5, 1 and 4). Availability of the classical model is highly sensitive to γ_3, and the effect of γ_1 on this sensitivity is insignificant. On the other hand, sensitivity of γ_3 on availability of the proposed model is very less.

Figure 3.14 shows the variation of availability with γ_3, when γ_1 is fixed at 0.5 and γ_2 is fixed at different values (0.25, 0.5, 1 and 4). Availability of the classical model increases significantly with the increase in γ_3, showing a high sensitivity of γ_3 on availability of the classical model. However, this sensitivity decreases when γ_2 increases. This figure also shows an insignificant effect from γ_2 and γ_3 on availability of the proposed model.

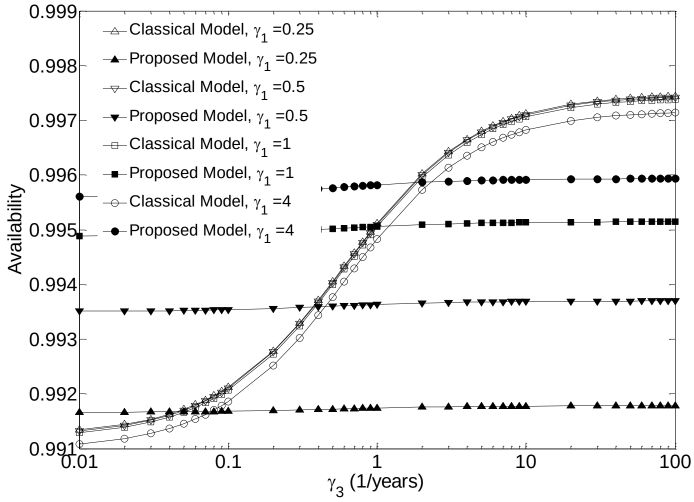

Figure 3.13: The variation of availability with γ_3, given different values for γ_1

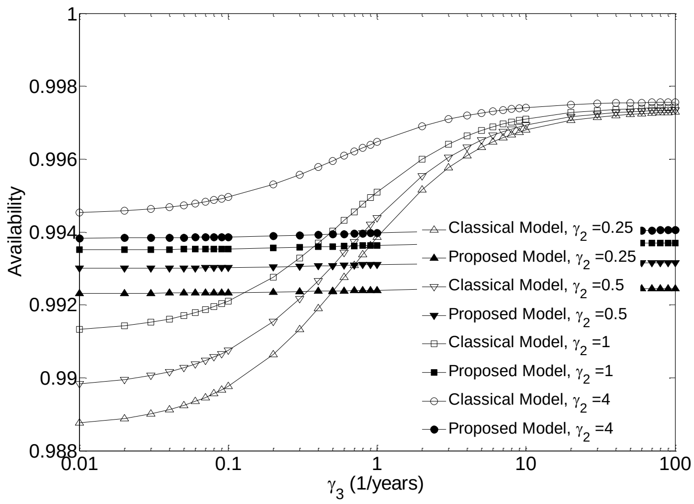

Figure 3.14: The variation of availability with γ_3, given different values for γ_2

3.2.2.3 Discussion

In this section, observations that are found from the sensitivity analyses are discussed. This section also explains different implications of the results considering the discrepancies found between the two models.

The Behavior of MTBF of the Model When γ_1 Goes to Zero

The behavior of MTBF of the model when γ_1 goes to zero is very useful to validate the accuracy of the models [29]. When γ_1 goes to zero, no maintenance will be carried out before the device fails, because the first inspection interval is very large. Therefore, when γ_1 is zero, MTBF is the sum of the expected sojourn times in deterioration states, which is $1/\lambda_1+1/\lambda_2+1/\lambda_3$ equal to 8.5 years for these circuit breakers. MTBF should not be affected by γ_2 or γ_3, if γ_1 is zero, because the system fails even before the first inspection is carried out. If these models are correct, for any value of γ_2 and γ_3, MTBF should converge to 8.5 years, when γ_1 goes to zero. As can be seen in Figure 3.3, the proposed model based on the new state diagram converges to 8.5 years and this behavior is independent from γ_2. In Figure 3.4, all curves which are obtained using the proposed model for different values of γ_3 converge to 8.5 years, further proving the accuracy of the proposed model. This behavior of the proposed model is as same as the realistic behavior according to NRD concept which is discussed in [29] and in chapter 2. However, as can be seen in Figures 3.3 and 3.4, when γ_1 goes to zero, MTBF of the classical model converges to different values depending on γ_2 and γ_3. This confirms the inaccuracy of the maintenance model based on the classical state diagram. This behavior of the two models based on classical and proposed state diagrams proves that the modeling errors only occur at the point of developing state diagrams and they do not occur when realizing state diagrams using Markov techniques. Although Markov processes are similar to RD concept [29], due to their memory less property they can give the same accurate results as NRD concept, if the state diagrams are drawn properly to represent actual maintenance situation.

It should be noted that when all inspection rates (i.e. γ_1, γ_2 and γ_3) are set to zero MTBF is equal to 8.5 years for both classical and proposed models. This is obvious, because setting all inspection rates to zero means that we do not intend to perform any inspection and maintenance and therefore, MTBF should be the sum of the expected sojourn times in deterioration states (which is equal to 8.5 years).

Effects of Inspection Rates on MTBF

MTBF is an average time between the occurrences of two consecutive failures which can be calculated using (3.1), where MTTF is the mean time to failure and MTTR is the mean time to repair. Since inspection and maintenance will increase both MTTF and MTTR of a device, theoretically an increase in any inspection rate will increase MTBF of the device. Results of sensitivity analysis on both models also show this behavior. However, it is impractical to increase inspection rates infinitely and therefore, sensitivity of different inspection rates on MTBF is important to identify better maintenance policies.

$$\text{MTBF}=\text{MTTF}+\text{MTTR} \tag{3.1}$$

Results of sensitivity analysis conducted on the classical model show that MTBF is highly sensitive to γ_2 and γ_3, but not much sensitive to γ_1. In contrast, results of sensitivity analysis conducted on the proposed model show that MTBF is highly sensitive to γ_1 and it cannot be increased considerably by increasing γ_2 and γ_3, while having a low γ_1.

Effects of Inspection Rates on Availability

Availability is the probability that a device is in working condition, which can be calculated using (3.2). This is also a function of MTTF and MTTR, which is influenced by inspection, maintenance and repair. Since operation of a device is stopped during inspection and maintenance activities, higher inspection rates will increase MTTR and an increase in any inspection rate will decrease availability. On the other hand, inspection and maintenance on a deteriorated device will most probably improve condition of the device. Thus, an increase in any inspection rate will also increase availability by increasing MTTF. Due to these increasing behaviors of both MTTF and MTTR with inspection rates, behavior of availability with different inspection rates obtained from sensitivity analysis is difficult to predict by merely observing (3.2).

$$\text{Availability}=\frac{\text{MTTF}}{\text{MTTF}+\text{MTTR}} \tag{3.2}$$

According to results given by the classical maintenance model, availability significantly decreases with increasing γ_1, but significantly increases with increasing γ_2 and γ_3. It can also be seen that the highest availability can be achieved with the lowest value of γ_1. However, results of the proposed model show an optimum value for γ_1 that maximizes availability. It is also shown that availability increases considerably with increasing γ_2, provided that γ_1 is high. Furthermore, γ_3 of the proposed model does not show high sensitivity to availability, when γ_1 or γ_2 is low.

Effect of Model Discrepancies on Results and Implications

As previously mentioned, classical models assume that the condition of a device is known even prior to inspection. Due to this assumption, the classical model allows to set the inspection rate as a device deteriorates, even when the deterioration condition is actually not revealed through inspections. Thus, the results of classical models imply the following.

1) MTBF can be greatly increased by increasing the inspection rate at latter deterioration stages of a device, even though inspection rate is very low at the early stage of deterioration.
2) Availability can be significantly increased by setting the inspection rate with higher values at latter deterioration stages and with a very low value at the early deterioration stage.

When condition of a device is known only after inspection and maintenance, setting the inspection rate to another value depends on the present inspection rate. This assumption is modeled in the proposed state diagram in Figure 3.2. Due to this assumption, inspections must be performed in early deterioration stages at a considerable rate, if it is required to perform more inspections and maintenance in latter deterioration stages. Otherwise, the device would fail without providing any opportunity to conduct inspection and maintenance. This is reflected in the results of the proposed model and the implications of the results are given below.

1) MTBF cannot be increased significantly by increasing the inspection rate at latter deterioration stages, while having a low inspection rate at the early stage of deterioration.
2) Less frequent inspections and maintenance may increase failures and cause a low availability. Too frequent inspection and maintenance could increase the idle time of the

device and hence, availability will be again low. Thus, there is an optimal rate to perform inspections, when a device is at its early deterioration stage. However, when a device is at latter deterioration stages, the risk of failure is higher and therefore, availability can be improved through more frequent inspection and maintenance.

The assumption in classical models which infers that the state of the device is always known is valid only when the inspection rate is very high. Thus, classical models are more suitable to stand for equipment with on-line condition monitoring, where the condition is continuously or more frequently monitored while the equipment is in operation. Since the operation of the device is not disturbed in on-line monitoring, it would be more appropriate to modify classical models by eliminating inspection states. Such application of a modified classical model developed for on-line monitoring of transformers is presented in the next section, section 3.3. The assumption of proposed models is in accordance with the practices in off-line monitoring. Thus, the proposed models could better represent the inspection based scheduled maintenance.

These circuit breaker maintenance models are further utilized in coming chapters with the following terminology. The term "condition monitoring based inspection and maintenance model (CBM model)" is assigned for the model in Figure 3.1, since its assumption is valid only if the deterioration condition of the equipment is always known as in continuous online condition monitoring. The term "inspection based maintenance model (IBM model)" is assigned for the proposed model in Figure 3.2, since its model assumption represents the actual practice of scheduled inspection based maintenance.

3.3 State Prediction of Transformers

Transformers are often the most valuable and indispensible asset in a substation and failures would result in undesirable disturbances to operating systems such as outages and power delivery problems. Therefore transformers are subjected to condition based maintenance. Currently in the industry, condition monitoring is done uniformly and routinely irrespective of the vulnerability of an individual transformer. Insufficient condition monitoring could be done on some transformers, leading to unexpected failures, while more-than-sufficient condition monitoring could be done, in which resources are unnecessarily used.

Condition monitoring activities of transformers can be effectively scheduled by considering the factors which affect the deterioration rate of transformers. Transformers deteriorate with the age and therefore, the failure rates and replacement costs are slowly but definitely steadily increasing with the age [50]. In addition, an increase in loading decreases the strength of the fibrous insulation and the potential of transformer failure is higher. Therefore, there should be a significant effect from the loading pattern of a transformer on its deterioration.

This section investigates the effect of loading and operating years on deterioration conditions of transformers using field data obtained from the local utility. Then, a state prediction tool is proposed using a maintenance model based on a state diagram. Such a tool can be useful for the industry to forecast the condition of transformers and to decide when and how to alter the present maintenance policy.

The approach of this work includes data collection from a large group of transformers in the local utility and data classification according to loading conditions and age. The average of various parameters determining the characteristics of each group of transformers are calculated and statistical hypothesis testing is used to determine if there is a difference in these parameters among the groups. Based on the effects of loading and age on the deterioration of the transformers, a guideline to perform maintenance of an individual transformer is provided. Then, a state prediction model is used to predict the deterioration condition of transformers. Model parameters are obtained from historical data of transformers in different groups. Predicted states obtained by solving the state prediction model are compared with the states in actual data, to verify model predictions.

3.3.1 Deterioration and Condition Monitoring of Transformers

Effects of Loading on Deterioration of Transformers

The winding hottest-spot temperature, which is the most important factor in determining the lifespan and the need for maintenance of a transformer due to loading, increases with the loading of a transformer. The hottest-spot temperature could be obtained from tests done in the laboratory, mathematical models as well as from direct measurement of the top or center of the primary or secondary winding [51].

The two impacts of loading on the transformers are loss of life and dielectric failure. The loss of life of a transformer is related to the deterioration of transformer insulation as a function of time and temperature [52]. The useful life of the paper insulation is 7.42 years at a continuous winding hottest spot temperature of 110°C and up to 50 years with a continuous winding hottest spot temperature of 92°C [53]. Under rated load, the normal loss of life for a transformer is about 0.0369% per day [53]. However, if a transformer is operated within rated capacity, it could be reasonably expected to last in excess of 30 years if routine maintenance and testing is conducted [52].

Dissolved Gas Analysis (DGA)

Deterioration of a transformer can be detected by examining the condition of the transformer oil. The oil in a transformer acts as a dielectric media, an insulator and as a heat transfer agent [54]. There is typically a gradual degradation of the mineral oil, yielding gases that collect in the oil when the transformer is under normal use. However, when an electrical fault arises in the transformer, these gases are generated more rapidly. There are three major types of electrical faults, the least severe being partial discharge and the most severe being arcing. By determining the various gases present and their amounts in the Dissolved Gas Analysis (DGA), one can infer the nature of the fault giving rise to them [54]. DGA is usually performed in accordance with IEEE C57.104 standards or IEC 60599 standards [55].

Local Utility Practices

The utility widely employs DGA to monitor the condition of a transformer. Condition monitoring is conducted at regular intervals which are predetermined for different conditions of the transformer. Based on the DGA of the oil samples, the condition of the transformers will be coded as 1, 2 or 3 according to the total amount of total dissolved combustible gases (TDCG), which is the sum of the concentrations in ppm of hydrogen, methane, ethane, ethylene, acetylene and carbon monoxide in a DGA oil sample [56].

The condition monitoring or maintenance cycle would be shorter for a transformer in condition 3 as compared to a transformer in condition 1 or 2. A transformer may also be replaced if its DGA consistently yields unsatisfactory results.

3.3.2 Classification of Transformers and Hypothesis Testing

Classification of Transformers

In this section, *k*-means clustering is used to classify transformers. *k*-means clustering aims to partition the observations into *k* clusters while minimizing the within-cluster sum of squares. Selected transformers are grouped according to their loading profile and the first year of operation respectively. For example, Figure 3.15 shows three different groups of transformers, formed by *k*-means clustering, each characterized with a unique loading pattern. Then, these different groups of transformers are analyzed using hypothesis testing, in order to identify differences in deterioration among groups.

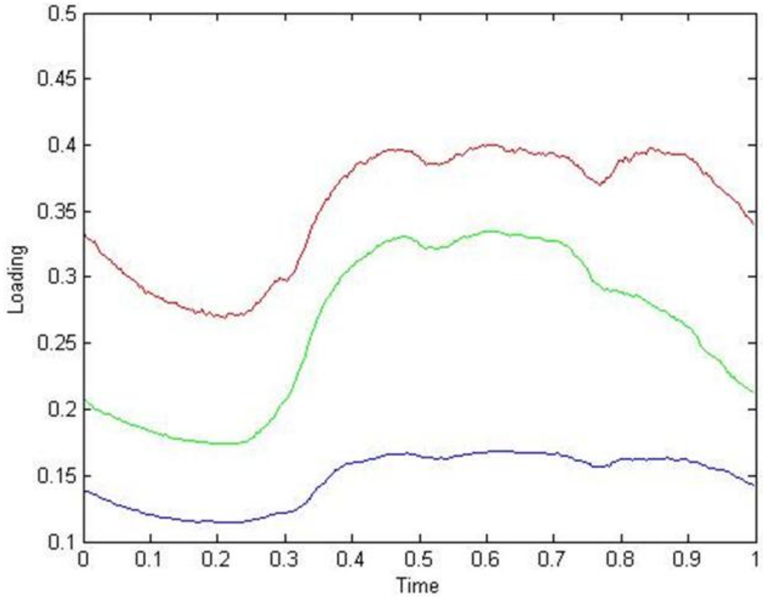

Figure 3.15: Loading profiles of transformers grouped using *k*-means clustering

Transformer Parameters

This section describes some parameters which are considered in hypothesis testing. One of the parameters under consideration is the probability of being found in each condition which is an indicator of the vulnerability of a transformer. Using the historical data of the DGA of a transformer, the probability of being found in each condition can be calculated as given in (3.3). Where P_c is the probability of being found in deterioration condition c, t_c is the duration of the transformer in condition c and T_o is the total time of observation.

$$\mathrm{P}_c = \frac{\mathrm{t}_c}{\mathrm{T}_o} \tag{3.3}$$

The inspection rate of transformers is also considered as it differs with the condition of the transformer. In utility practices, transformers with a greater deterioration are inspected more frequently than less deteriorated transformers. Given the history of DGA, γ_c, the inspection rate of a transformer in condition c, can be determined using (3.4). Where n_c is the number of inspections conducted when the condition is c.

$$\gamma_c = \frac{n_c}{\mathrm{t}_c} \tag{3.4}$$

Besides these parameters, the field measurements of concentration levels of different gases analyzed in the DGA can also indicate the condition and deterioration in transformers. Thus, such measurements of different groups of transformers are also analyzed in hypothesis testing.

Hypothesis Testing

In hypothesis testing, two groups, Group X and Group Y are compared. The difference between the means of a parameter of two groups, μ_x and μ_y is tested with the following one-tailed hypothesis test at level of significance, α = 5%.

H_0: $\mu_x = \mu_y$

H_1: $\mu_x > \mu_y$

Test statistic:

$$z = \frac{(\bar{x} - \bar{y}) - (\mu_x - \mu_y)}{\sqrt{\frac{\sigma_x^2}{n_x} + \frac{\sigma_y^2}{n_y}}} \sim N(0,1)$$

$\bar{x}$ and $\bar{y}$ are the sample means of groups X and Y, respectively. σ_x^2 and σ_y^2 are the population variance of groups X and Y, respectively. n_x and n_y are the number of transformers in groups X and Y, respectively.

Since σ_x^2 and σ_y^2 are unknown, their estimators s_x^2 and s_y^2 are used in the test statistic as given below. As the sample size is large, the central limit theorem allows doing so, because it guarantees that $(\bar{x} - \bar{y})$ has approximately a normal distribution.

Test statistic:

$$z = \frac{(\bar{x} - \bar{y}) - (\mu_x - \mu_y)}{\sqrt{\frac{s_x^2}{n_x} + \frac{s_y^2}{n_y}}} \sim N(0,1)$$

For each group of transformers, the mean, μ, and standard deviation, s, of the relevant parameters are estimated. Then, values are calculated for the test statistic.

If the value of the test statistic does not fall within the critical region ($z < 1.645$) as shown in Figure 3.16, H_0 is not rejected as there is insufficient evidence at 5% level of significance to support the claim that $\mu_x > \mu_y$. If the value of the test statistic falls within the critical region ($z > 1.645$), H_0 is rejected and H_1 is accepted as there is sufficient evidence at 5% level of significance to support the claim that $\mu_x > \mu_y$. Hypothesis testing results which show the differences in deterioration among different transformer groups are provided in section 3.3.3.

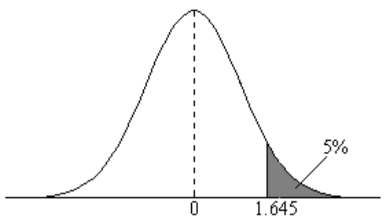

Figure 3.16: Critical region

3.3.3 Results and Analysis of Hypothesis Testing

Effects of Maximum Loading on DGA Gases

Selected results of the hypothesis testing carried out for 2 groups of transformers are shown in Table 3.3.

Group X: Transformers with high maximum loading

Group Y: Transformers with low maximum loading

Table 3.4: Test Statistics for Transformers Grouped by Maximum Loading

Parameter	CH_4	CO	CO_2	C_2H_4
z	0.084	5.129	7.737	4.937

There is sufficient evidence (z > 1.645) to show that the amount of carbon monoxide (CO), carbon dioxide (CO_2) and ethylene (C_2H_4) is larger in the group of transformers with high maximum loading than in the group with low maximum loading at 5% level of significance. Carbon monoxide and carbon dioxide are the only two gases which indicate the deterioration of the paper insulation of the transformer while ethylene, together with other gases, may indicate the deteriorating thermal condition of the transformer oil. As there is no substantial evidence to prove that the amount of methane (CH_4) also increases with increasing maximum loading, it cannot be concluded that thermal fault may arise at a higher probability as maximum loading increases. However, it can be concluded that paper insulation deteriorates with an increased maximum loading, as indicated by the increased amounts in carbon monoxide and carbon dioxide.

This is an important factor for the utility to consider as from the results, the amount of paper insulation degradation essentially increases with maximum loading. This is crucial especially when comparing transformers which have extreme differences in maximum loading as the results indicate that the deterioration of paper insulation is more significant between these groups.

Effects of Age on Deterioration

Selected results of the hypothesis testing carried out for 2 groups of transformers are shown in Table 3.4.

Group X: Transformers with an earlier first year of operation

Group Y: Transformers with a later first year of operation

Table 3.5: Test Statistics for Transformers *K*-Means Clustered by First Year of Operation

Parameter	P_3	λ_2
z	3.133	5.380

The probability of being in condition 3 and the deterioration rate from condition 2 to condition 3 of a transformer with an earlier year of operation are higher than that of a transformer with a later first year of operation (z > 1.645). A transformer which has been in operation for a long time is more prone to the deterioration of its paper insulation, thus giving rise to the deterioration of the transformer.

The utility should factor in the number of years of operation when monitoring transformers, as it is shown that amount of deterioration of a transformer increases with its age.

Effects of Loading on Deterioration

Selected results of the hypothesis testing carried out for 2 groups of transformers are shown in Table 3.5.

Group X: Transformers with high loading

Group Y: Transformers with low loading

Table 3.6: Test Statistics for Transformers *K*-Means Clustered by Loading

Parameter	P_2
z	3.067

The probability of a transformer being in condition 2 is higher for a transformer which is subjected to a high loading than for a transformer with a low or mid load (z > 1.645). There is a greater deterioration in transformers which belong to the former category as a higher load deteriorates the paper insulation at a faster rate due to the larger winding hottest-spot temperature it generates. In addition, higher loading increases the stresses on a transformer which may result in gassing in the solid insulation and oil, leading to the deterioration of a transformer.

Combined Effects of Age and Loading on Deterioration

Selected results of the hypothesis testing carried out for the 2 groups of transformers are shown in Table 3.6.

Group X: Transformers with a high loading and an earlier first year of operation

Group Y: Transformers with a low loading and a later first year of operation

This hypothesis test combines the effects of loading and age on transformer. It can be concluded that transformers which have a high loading and have been in operation for a long time have a higher probability of being in a deteriorated condition 2 or amber condition as compared to transformers which are lowly loaded and are newly-introduced into operation ($z > 1.645$).

Table 3.7: Test Statistics for Transformers *K*-Means Clustered by Loading and Age

Parameter	P_2
z	1.885

3.3.4 State Prediction Model

The state prediction could be useful for utilities to forecast the state or the condition of transformers. When the state of a transformer is predicted to be in a higher deterioration state, the utility can increase the rate of condition monitoring of the transformer.

Figure 3.17 shows a simple state diagram of a Markov model which represents the deterioration and maintenance of transformers. This state diagram provides the basis for state prediction model. In this state diagram, states S_1, S_2 and S_3 represent deterioration conditions corresponding to condition 1, 2 and 3, respectively. F is the failure state. When the device is deteriorated or failed, it is repaired or replaced back to S_1. λ_1 and λ_2 are deterioration rates, and λ_3 is the transition rate from S_3 to F. μ_1 , μ_2 and μ_3 are repair rates. These model parameters are calculated by analyzing historical data of transformers. Using the data of each transformer, λ_1, λ_2 and λ_3 are calculated using (3.5), when c= 1, 2 and 3, respectively. Where t_{S_c} is the average time spent in the deterioration state S_c. μ_1 , μ_2 and μ_3 are the reciprocal of the average repair time which is equal to one week.

$$\lambda_c = \frac{1}{t_{S_c}} \tag{3.5}$$

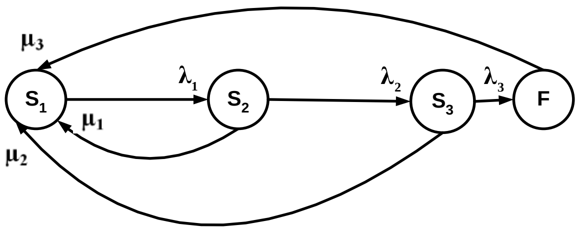

Figure 3.17: The state diagram of the state prediction model

This state diagram can be mathematically represented by a Markov process. Using the transition rate matrix of this Markov process, transition probability matrix is computed. According to these transition probabilities, deterioration states are enumerated in MATLAB 10. The average of 10 enumerations is considered as the predicted state of the transformer. The predicted states are then compared with actual states for verification and results are provided in section 3.3.5.

3.3.5 Results and Analysis of State Prediction

State Prediction for Transformer with Low Loading and an Earlier First Year of Operation

Results obtained for Transformer A with low loading and an earlier first year of operation are shown in Table 3.7.

As can be seen in Table 3.7, the predicted states slightly tally with the actual states. The deterioration of the transformer from condition 1 to 2 and from condition 1 to 3 cannot be simulated very accurately. When a transformer is fairly new, the probability of deterioration from condition 1 to condition 2 and from condition 2 to condition 3 is very low. Due to this reason, it may be difficult to use simulation tools to capture such changes in recently installed transformers.

Table 3.8: Actual and Predicted States of Transformer A

Month	Actual State	Predicted State
1	S_1	S_1
27	S_2	S_1
30	S_1	S_1
33	S_1	S_1
39	S_3	S_2
40	S_1	S_1
50	S_1	S_1
62	S_1	S_1
78	S_1	S_1
103	S_2	S_1
104	S_1	S_1

State Prediction for Transformers with High Loading and a Later First Year of Operation

Results obtained for Transformers B, C and D with high loading and a later first year of operation are shown in Tables 3.8, 3.9 and 3.10. Results show that predicted states almost tally with the actual states. It should be noted that the simulation tool is now able to capture the deterioration of older transformers from condition 1 to 2 and 2 to 3. When the transformers are aged and highly loaded, the actual state data collected from these transformers are dynamic and varied. As such, there is a greater probability of deterioration. The transition probability from state 1 to state 2 and state 2 to state 3 in the model are now slightly higher than case A. The model is now able to produce more accurate predictions for state 2 and state 3.

This proposed tool may be useful for utilities to predict the deterioration condition of transformers. The performance of this simulation tool is better when it is used for state prediction of older transformers. The ability to simulate the state of a transformer is a useful tool for utilities to forecast the amount of deterioration of the transformer and the time to next stage of deterioration. Preventive measures could be taken by the utilities before an eminent failure if the states of the transformer can be predicted. Although results are verified using historical data, it should be noted that the proposed state prediction tool has not been implemented in real time.

Table 3.9: Actual and Predicted States of Transformer B

Month	Actual State	Predicted State
1	S_1	S_1
27	S_1	S_1
28	S_2	S_2
29	S_2	S_2
38	S_2	S_2
42	S_2	S_1
46	S_1	S_1
48	S_1	S_1
50	S_1	S_1
74	S_1	S_1
77	S_1	S_1
87	S_2	S_2
90	S_3	S_3
110	S_1	S_1

Table 3.10: Actual and Predicted States of Transformer C

Month	Actual State	Predicted State
1	S_1	S_1
28	S_2	S_2
29	S_2	S_2
38	S_2	S_2
42	S_2	S_2
46	S_2	S_2
48	S_2	S_2
50	S_1	S_2
74	S_1	S_2
77	S_3	S_2
87	S_3	S_3
90	S_1	S_1
95	S_1	S_1
114	S_1	S_1

Table 3.11: Actual and Predicted States of Transformer D

Month	Actual State	Predicted State
1	S_1	S_1
15	S_2	S_1
30	S_2	S_2
31	S_2	S_2
33	S_3	S_2
34	S_3	S_3
40	S_1	S_1
43	S_2	S_2
46	S_1	S_1
51	S_1	S_1
55	S_2	S_2
60	S_3	S_2
63	S_1	S_1
67	S_1	S_1
75	S_1	S_1

3.4 Effects of Subcomponent Characteristics on Reliability of a Wind Energy Conversion System

Most technical systems are complicated in construction and consist of several sub components. In such complicated systems, a failure of one sub component may lead the system to fail and cause unavailability. Therefore, in order to ensure reliable operation, maintenance activities are conducted to preserve good working conditions of a device. Even though frequent maintenance can improve reliability, at the same time it may cause catastrophic failure if not properly done. It may also not be the optimal or cost effective solution. Therefore, some sub components have condition monitoring to predict failures. Some components do not have condition monitoring and random failures of such components can also cause system failures. Some components fail more frequently and some components require more time to repair. These failure and repair characteristics of several sub components of a system can affect the reliability of the system in different ways. This section aims to apply a Markov model which is developed for a wind energy conversion system (WECS) to study the effect of the characteristics of sub components on the overall system reliability.

3.4.1 A Wind Energy Conversion System

Components of a Wind Energy Conversion System

A typical horizontal-axis geared wind turbine shown in Figure 3.18 has three composite blades (1). These blades which are joined to the rotor hub (2) drive the rotor. The main bearing (4) is positioned to absorb static and dynamic loads and also to support the rotor shaft (5). The rotor drives the gearbox (6), and the generator coupling (8) couples the gearbox to the induction generator (9). The gearbox converts a low speed of the blades to the rated speed of the generator. The safety brake (7) is located between the gearbox and the generator. The gearbox, the generator cooler (10), the control unit (12) and the hydraulic system (13) are also placed in the nacelle. After positioning all these components in the nacelle frame (3), the complete nacelle covered by its cover (16), is mounted on top of the tower (17) with the aid of the yaw bearing (15). The yaw drive (14) aligns the nacelle according to the direction of the wind, sensed by wind sensors (11), in order to harvest maximum energy.

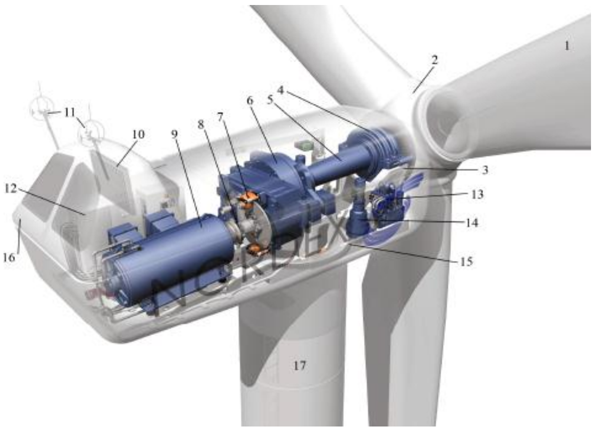

Figure 3.18: Nacelle of a typical geared wind turbine [57]

1. Rotor blade	7. Safety brake	13. Hydraulic system
2. Rotor hub	8. Generator coupling	14. Yaw drive
3. Nacelle frame	9. Induction generator	15. Yaw bearing
4. Main bearing	10. Generator and gearbox cooler	16. Nacelle cover
5. Rotor shaft	11. Wind sensors	17. Tower
6. Gearbox	12. Nacelle control	

Reliability Block Diagram of a Wind Energy Conversion System

When all of the above mentioned sub-components are taken into consideration, a reliability block diagram of the complete WECS [58] can be rather complex. The reliability block diagram of a typical WECS, shown in Figure 3.19, has six major sub-systems that significantly affect the system reliability. The block diagram is connected in series even though all sub-components of a WECS are not physically connected in series. This is due to the fact that failure of each sub-component causes the WECS to fail.

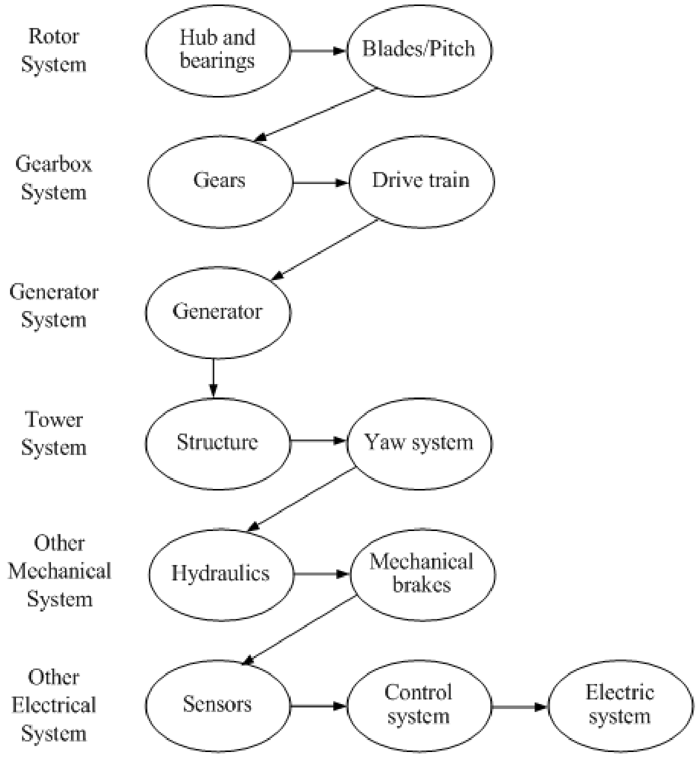

Figure 3.19: Reliability block diagram of a typical wind energy conversion system [58]

Failure Statistics

The critical components of WECS are identified by observing the failure statistics of individual sub-components [57, 59-62]. Figures 3.20 and 3.21 show percentage of failures and percentage of downtime per component respectively in Swedish wind power plants during 2000 to 2004 [60]. From Figure 3.20, it can be seen that most failures are related to the electric system followed by sensors and blades/pitch.

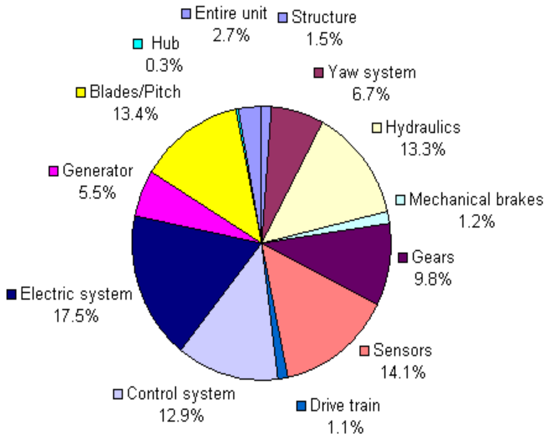

Figure 3.20: Distribution of number of failures

Figure 3.21 shows that the gearbox has the highest downtime. This is closely followed by the control system and the electric system.

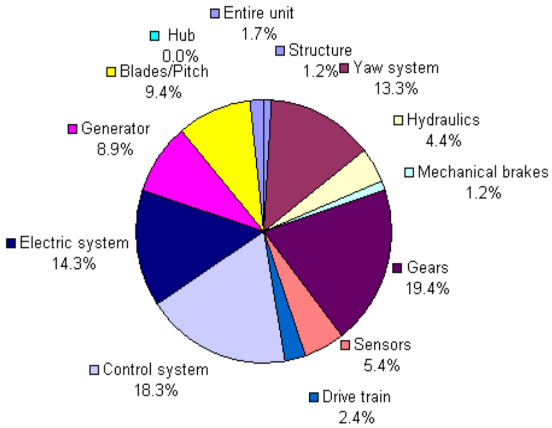

Figure 3.21: Percentage of downtime per component

Condition Monitoring of Wind Energy Conversion System

Most modern WECS are incorporated with condition monitoring (CM) systems. Blades, generator and gearbox and drive train are the three main sub-components with CM techniques in most wind turbines [63]. In addition, CM of yaw system and mechanical brake is newly proposed considering their high downtime and failure frequency [64].

3.4.2 A Markov Model for a Wind Energy Conversion System

A WECS comprises of several devices that make it challenging to incorporate all sub-components in the reliability model. Therefore, selecting and modeling sub-components are major issues in modeling a WECS.

In reliability models, only a few sub-components (generator, gearbox, electronics and blades) have been selected to represent the entire WECS [61, 62, 65]. These selections are based on the previous attempts for identifying crucial components of a WECS using historical data. In [59], electrical system, rotor and converter are found as the most unreliable sub-assemblies due to their high failure frequencies. A recently conducted survey on failures of WECS in Sweden stated that the gearbox is the most crucial component due to its high down time per failure [60]. Therefore, in addition to the failure frequency, downtime is also considered as a selection criterion. This is reasonable, because some components with a very high downtime and a low failure rate may disturb the system operation, more than those with a very short downtime but a high failure frequency. In order to analytically investigate effects of sensitive components to the WECS reliability, a proper model for a WECS with CM is needed.

A probabilistic model was developed for a WECS with CM in [61, 62] considering published data, reliability data and opinions of industry experts. This model accounts for failures related to generator, gearbox, blades and electronics of a WECS. Sub-component selection criteria are downtime distributions, failure rates and the presence of CM [61, 62]. Instead of modeling a few selected sub-components, in this section a new model for a WECS is proposed to incorporate CM effects and failure data of all sub-components. To reduce the complexity of the model, sub-components with similar characteristics are grouped together.

The Proposed Markov Model for a Wind Energy Conversion System

The complexity and number of states of the model depend on the number of sub-components and the selected reliability models to represent each sub-component. In this proposed model, all sub components are grouped into four categories. Gearbox, generator and blades/pitch are considered as separate groups as they are having CM, and all other components without CM are considered as one group.

- Gearbox: This group includes gears and drive train.
- Generator
- Electronics and others: This group represents all other components that are not included in other three groups.
- Blades/pitch

There are two basic models available to model sub components. They are the two-state model and the intermediate states model. The two-state model is used to model all sub components which do not have CM. As can be seen in Figure 3.24, this model has only two states; the up state and the down state. Due to simplicity, this model is widely used in reliability studies [61, 62, 66]. Intermediate states model which is shown in Figure 3.23 introduces a better representation for devices with CM. This model is ideal for a single component having up, down and intermediate state(s). This is widely used to model the deterioration process of a component. In this work, this model is used to represent a component under continuous condition monitoring. Transition between states depends on failure characteristics of the sub-component and the maintenance activities.

In the proposed Markov model, sub component 1, 2 and 4 are represented using a three-state model in order to include the intermediate state at which the CM system detects faults. Sub-component 3 is modeled using the two-state model as it has only up and down states to consider. Then the WECS is modeled by combining the states of sub components.

State Space Diagram

The state space diagram of the proposed WECS model is shown in Figure 3.22.

Theoretically, the developed Markov model for a WECS has 54 ($3^3 \times 2$) states. As in most reliability models, using following assumptions, the number of system states are reduced to 28. Failure states are drawn with grey color in Figure 3.22.

- Simultaneous failures or degradations of two components are negligible.

- System must transit to a failure states via a de-rated state.
- All failure states are considered as absorbing states.

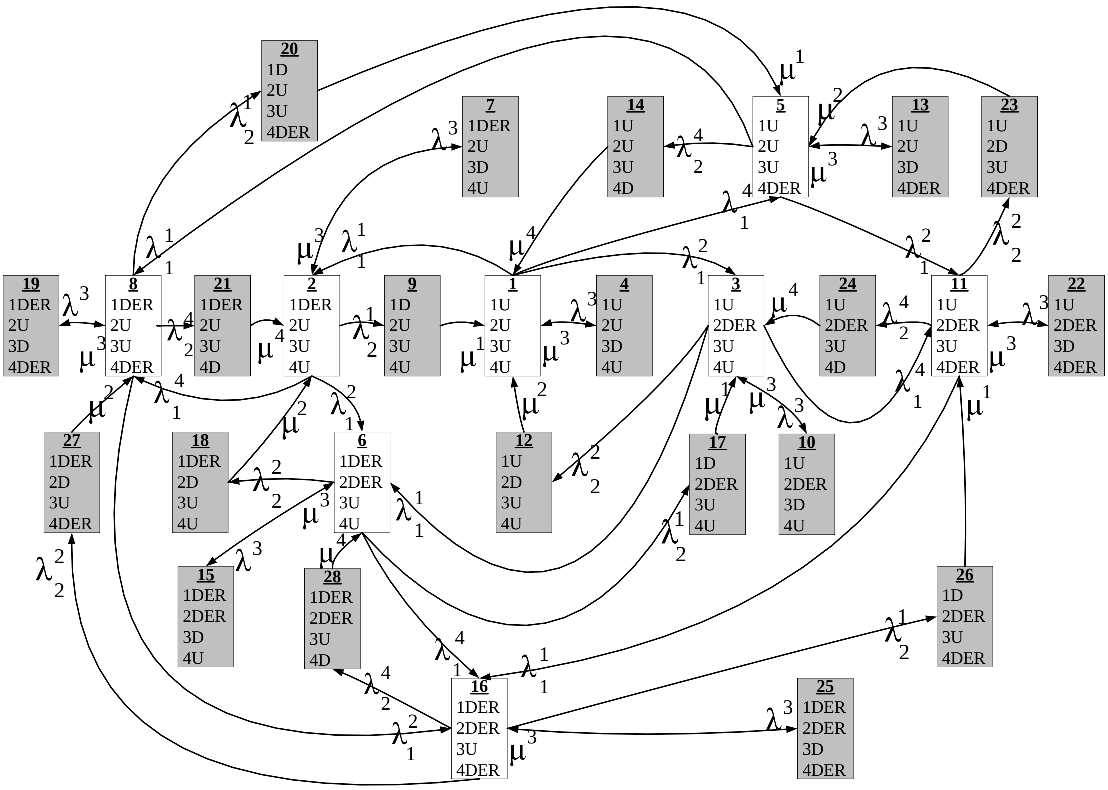

Figure 3.22: State space diagram of the proposed wind energy conversion system

Transition Rate Matrix

The deterioration and failure characteristics of each sub component are incorporated into the developed model using the transition rate matrix, the parameters of which can be calculated from historical data collected over a long period of time. For this study, transition rate parameters are computed using average number of failures and average downtimes available in the literature for the test system. This calculation procedure is explained in detail under section 3.4.3. Following transition rates are employed in the developed model.

- λ_1^i is the transition rate from up state to de-rated state of sub-component *i*, *i*=1,2,4
- λ_2^i is the transition rate from de-rated state to down state of sub-component *i*, *i*=1,2,4
- λ^3 is the transition rate from up state to down state of sub-component 3
- μ^i is the transition rate from down state to up state of sub-component *i*, *i*=1,2,3,4

3.4.3 A Test System

The above Markov model is incorporated into a test system with failure rates and repair rates obtained from [60]. The data is based on a recent survey conducted for more than 600 Swedish wind turbines over a period of five years. Equivalent failure rates and repair rates are calculated for each group of sub components of the selected test system.

Calculation of Equivalent Rates

For a series system with frequency of failure approximation, equivalent failure rate (λ_{eq}) and equivalent repair rate (μ_{eq}) can be calculated using following equations.

$$\lambda_{eq} = \sum \lambda^i \tag{3.6}$$

$$MTTR_{eq} = \frac{\sum(\lambda^i \times MTTR_i)}{\sum \lambda^i} \tag{3.7}$$

$$\mu_{eq} = \frac{1}{MTTR_{eq}} \tag{3.8}$$

Where λ^i is the failure rate of sub-component *i*, $MTTR_i$ is the average time required to repair component *i* and $MTTR_{eq}$ is the average down time of the WECS.

Table 3.12: Equivalent Failure Rates and Repair Rates of Sub-groups

Component	Equivalent Failure Rate, λ_{eq} (per year)	Equivalent Repair Rate, μ_{eq} (per day)
Gearbox (1)	0.000134	0.0925
Generator (2)	0.000058	0.1139
Electronics and other (3)	0.000740	0.2183
Blades/pitch (4)	0.000142	0.2620

Since components of each group are in a series, equivalent failure and repair rates in Table 3.11 are calculated using individual failure rates and mean down times (MDTs).

Determining the Transition Rates for Intermediate State Model

Intermediate states model for gearbox, generator and blades/pitch is shown in Figure 3.23. For gearbox, generator and blades/pitch that have an intermediate state, it is needed to calculate the transition rate from up state to de-rated state (λ_{UD}) and the transition rate from de-rated state to down state (λ_{DD}). The procedure of calculating λ_{UD} and λ_{DD} is briefly described in this section.

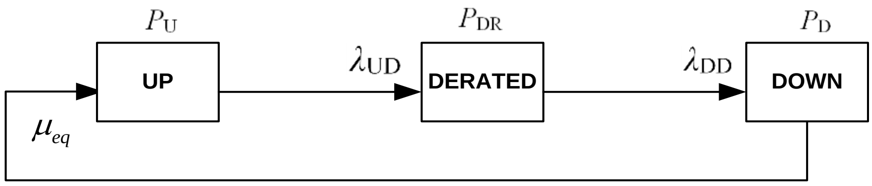

Figure 3.23: Intermediate states model

The following equations are obtained by applying the frequency balancing technique to the model shown in Figure 3.23.

$$P_{\mathrm{U}} \times \lambda_{\mathrm{UD}} = P_{\mathrm{D}} \times \mu_{eq} \tag{3.9}$$

$$P_{\mathrm{U}} \times \lambda_{\mathrm{UD}} = P_{\mathrm{DR}} \times \lambda_{\mathrm{DD}} \tag{3.10}$$

$$P_{\mathrm{DR}} \times \lambda_{\mathrm{DD}} = P_{\mathrm{D}} \times \mu_{eq} \tag{3.11}$$

$$P_{\mathrm{U}} + P_{\mathrm{DR}} + P_{\mathrm{D}} = 1 \tag{3.12}$$

Where P_{U}, P_{DR} and P_{D} are probabilities of being staying in up, de-rated and down states of the intermediate states model, respectively.

From (3.9), (3.10), (3.11) and (3.12),

$$P_{\mathrm{D}} \times \mu_{eq} \times \left(\frac{1}{\lambda_{\mathrm{UD}}} + \frac{1}{\lambda_{\mathrm{DD}}} + 1\right) = 1 \tag{3.13}$$

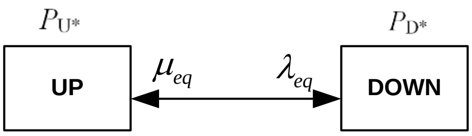

Figure 3.24: Two-state model

Similarly, using frequency balancing technique for the equivalent two-state model in Figure 3.24, (3.14) and (3.15) are obtained.

$$P_{\mathrm{U}*} \times \lambda_{eq} = P_{\mathrm{D}*} \times \mu_{eq} \tag{3.14}$$

$$P_{U*} + P_{D*} = 1 \tag{3.15}$$

Where P_{U*} and P_{D*} are probabilities of being staying in up and down states of the two-state model, respectively.

From (3.14) and (3.15),

$$P_{D*} \times \mu_{eq} \times \left(\frac{1}{\lambda_{eq}} + 1\right) = 1 \tag{3.16}$$

Since P_D=P_{D*},

$$\frac{1}{\lambda_{eq}} = \frac{1}{\lambda_{UD}} + \frac{1}{\lambda_{DD}} \tag{3.17}$$

To determine λ_{UD} and λ_{DD} using (3.17), one of them must be known. Since historical data is unavailable for λ_{UD} and λ_{DD}, considering fast transitions from de-rated state to down state, λ_{DD} is assumed to be equal to the repair rate; μ_{eq}. With this approximation, λ_{UD} can be calculated using (3.17) and the values for λ_{UD} and λ_{DD} are tabulated in Table 3.12.

Table 3.13: Transition rates from up state to de-rated state and from de-rated state to down state

Component	Transition rate from up state to de-rated state, λ_{UD} (per day $\times 10^{-4}$)	Transition rate from de-rated state to down state, λ_{DD} (per day)
Gearbox	1.3444	0.0925
Generator	0.57563	0.1139
Blades/pitch	1.4254	0.2620

3.4.4 A Sensitivity Analysis of Sub Component Characteristics on the System Reliability

Maintaining high availability with minimum maintenance cost is important to minimize operation cost of WECS. CM of sub-components can predict the failures in advance and hence maintenance scheduling can be effectively conducted. To conduct CM in a cost effective manner, identifying the more sensitive sub-components to the availability of a WECS is required.

The system availability, given by (3.18), shows that a high availability can be achieved either by a high mean time to failure (MTTF) or a low mean time to repair (MTTR) or a combination of both.

$$Availability = \frac{MTTF}{MTTF + MTTR} \tag{3.18}$$

Failure statistics show that 20 percent of downtime of a wind turbine is due to gearbox failures and hence gearbox is identified as the most critical component to the availability of a WECS [60]. Since the availability depends on both MTTF and MTTR, high downtime of a component may or may not contribute to low system availability. It is therefore important to analytically identify critical components through a sensitivity analysis. The test system in section 3.4.3 is used for this analysis. It is also expected that the sensitivity analysis provide insightful information on the effects of failure rates and repair rates of sub-assemblies to the overall WECS reliability.

Three sensitivity analyses are carried out to observe the effects of failure rates and repair rates of each sub component on the reliability measures of the WECS namely MTTF, MTTR, and system availability. By observing the sensitivity of the sub-components to MTTF, MTTR and the availability, the components that mostly affect the system availability can be identified. In this analysis, failure rate and repair rate of each group of sub-components are varied from 0 to 0.001 and 0.05 to 0.3 respectively. These ranges are selected by adding a margin to the ranges of actual failure and repair rates.

Sensitivity Analysis of Failure/Repair Rates on Mean Time to Failure

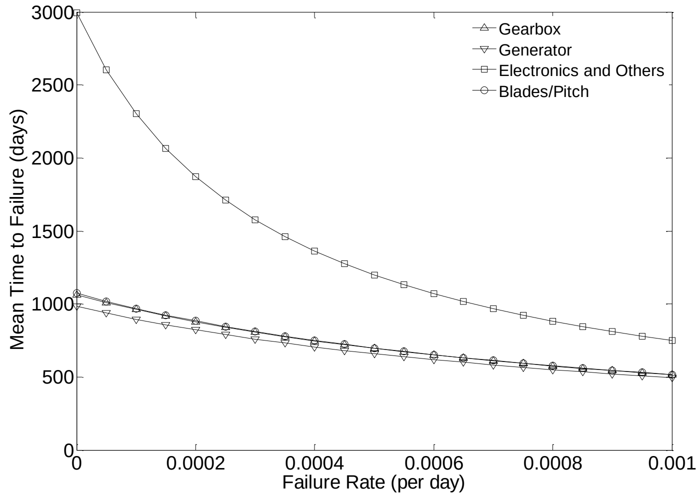

Figure 3.25: The variation of mean time to failure with failure rates

As can be seen from Figure 3.25, MTTF decreases when failure rate increases. Gearbox, generator and blades/pitch with comparatively low failure rates are not highly sensitive to MTTF of a WECS. On the other hand, electronics and other sub components with high failure rates are very sensitive to MTTF. By reducing the failure rate of this category, MTTF can be decreased. Theoretically, this trend can be well explained using (3.20).

For a WECS, MTTF can be computed using (3.19).

$$MTTF = \frac{1}{\lambda_{eq}} \tag{3.19}$$

From (3.6) and (3.19), for a series system with frequency of failure approximation,

$$MTTF = \frac{1}{\sum \lambda^{i}} \tag{3.20}$$

According to (3.20), MTTF is inversely proportional to the failure rates and hence, as can be seen from Figure 3.25, MTTF decreases with increasing failure rates.

In the reliability model of a WECS, all sub components of a WECS are connected in series. For this series system MTTF is a function of failure rates of all components and not a function of repair rates (3.20). Therefore, MTTF remains constant with varying repair rates.

Sensitivity Analysis of Failure/Repair Rates on Mean Time to Repair

As can be seen from Figure 3.26, there are two major trends in the variation of MTTR with failure rates. For sub components with high average down times such as gearbox and generator, MTTR increases, when failure rate increases. It is due to the fact that the system MDT increases as components with high average down times fail frequently than the others. On the other hand, for sub-components with low average down times, MTTR decreases, when failure rate increases. This implies that the quick repairable failures are more desirable than the failures that take a long time to repair.

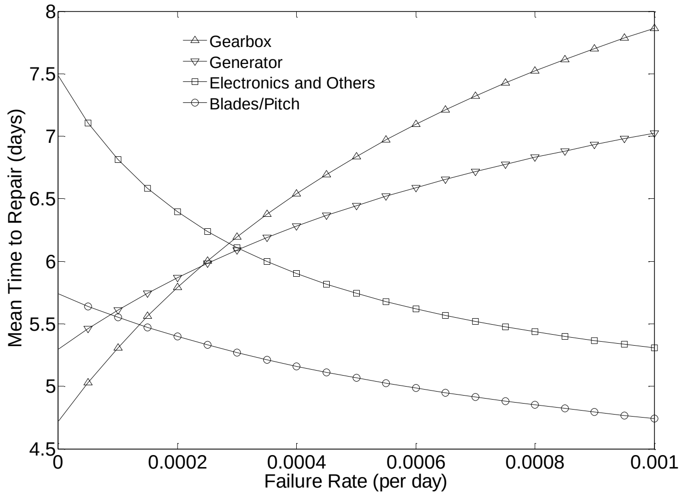

Figure 3.26: The variation of mean time to repair with failure rates

The above trends can be theoretically explained using following mathematical expressions.

For a component i,

$$MTTR_i = \frac{1}{\mu^i} \tag{3.21}$$

From (3.7) and (3.21),

$$MTTR_{eq} = \frac{\sum \left(\lambda^i \times \frac{1}{\mu^i}\right)}{\sum \lambda^i} \tag{3.22}$$

As given in (3.22), MTTR of a WECS is a function of repair rates and failure rates of its sub components. To discuss the trends in Figure 3.26, the first derivative of $MTTR_{eq}$ with respect to λ^i can be computed as given in (3.23).

$$\frac{d(MTTR_{eq})}{d\lambda^i} = \frac{\frac{\sum_{\forall k\neq i}(\lambda^k)}{\mu^i} - \sum_{\forall k\neq i}\left(\frac{\lambda^k}{\mu^k}\right)}{(\lambda^i + \sum_{\forall k\neq i}\lambda^k)^2} \tag{3.23}$$

The numerator that decides the sign of the derivative can be rearranged as follows.

$$\sum_{\forall k\neq i}\left(\lambda^k \times \left(\frac{1}{\mu^i} - \frac{1}{\mu^k}\right)\right)$$

For a component having a comparatively small repair rate, this term is positive and leads to an increasing trend, as for the gearbox and generator in Figure 3.26. On the other hand, for a component with a high repair rate, this term is negative and leads to a decreasing trend.

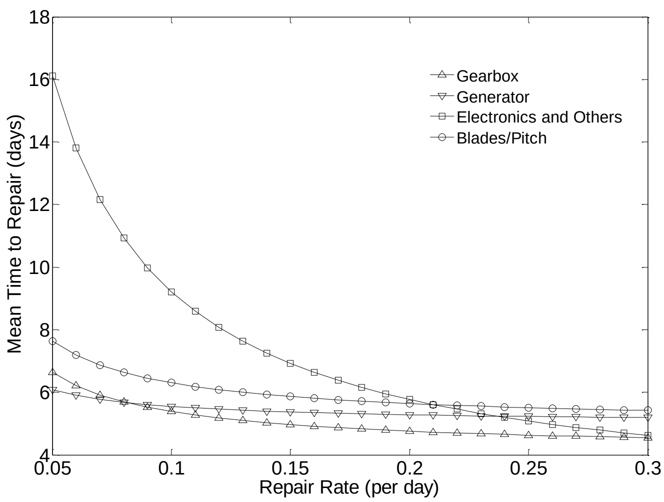

Figure 3.27: The variation of mean time to repair with repair rates

MTTR of the system is inversely proportional to the repair rates of individual sub components (3.22). Figure 3.27 shows that MTTR decreases when repair rate increases. By increasing the repair rate of the components that fail frequently, MTTR of the system can be dramatically reduced. Therefore by increasing the repair rate of the components with high failure rates such as electronics and blades, MTTR can be decreased.

Sensitivity Analysis of Failure/Repair Rates on Availability

Figure 3.28 shows that the failure rates of generator and gearbox are more sensitive to the availability than the failure rates of others. The gearbox has the most sensitive failure rate to the availability of the WECS. Although failure rate of electronics is comparatively less sensitive to the availability, its small failure rate is also important to guarantee a significantly high availability of the system.

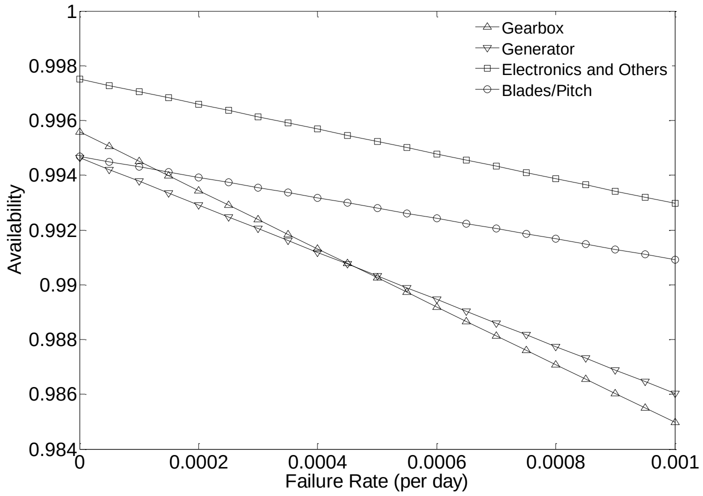

Figure 3.28: The variation of availability with failure rates

Figure 3.29 shows that the repair rate of electronics is highly sensitive to the availability of a WECS, than the repair rates of other components. However this high sensitivity decreases when repair rate increases.

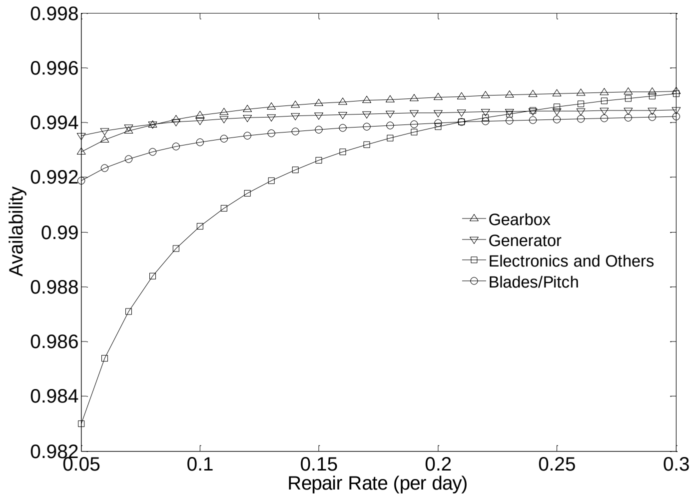

Figure 3.29: The variation of availability with repair rates

Discussion

The availability of a WECS depends on both the MTTF and MTTR. This sensitivity analysis examines the effect of failure rates and repair rates of three major components with CM techniques to the overall reliability of a WECS. Combined effect of all other components without CM techniques is also examined. Although some components are lumped together in the model, there exist significant differences in failure and repair rates among sub-groups (Table 3.11). For example, generator and gearbox have low failure rates and high repair rates, while electronics and other components considered as group 3 is having a high failure rate and a low repair rate. These different characteristics of sub-groups are useful to generalize the results for any sub component with known failure and repair rates, to predict the sensitivity to the availability of a WECS.

Results show that MTTF decreases rapidly with the failure rates of the components that typically have high failure rates. Therefore, a decrease in failure rates of the components with high failure frequency can significantly increase MTTF of the system. MTTR can be reduced significantly by reducing failure rates of sub-components with high MDTs or by increasing repair rate of sub-components with high failure frequency.

The results of this analysis emphasize the importance of minimizing sudden failures of the components with high MDTs such as gearbox and generator using CM techniques. Most unreliable components such as electronics are also critical and investigating new techniques to minimize failures of these components or to replace them with highly reliable components is also favorable for a higher availability of WECSs. Another possibility is to further increase the repair rates of the components that fail frequently.

Since this study is conducted by only using failure and repair rates in the literature [60], in section 3.4.3, when λ_{UD} and λ_{DD} are determined using (3.17), an assumption is made that λ_{DD} is equal to the repair rate; μ_{eq}. This assumption is highly speculative and therefore, numerical results should be treated with caution. However, if condition monitoring and failure data is available, we can find the average duration that a sub component takes to transit from up state to derated state and from derated state to down state. Thus, λ_{UD} and λ_{DD} can be determined using more detailed data in the future.

3.5 Summary

This chapter presents three possible applications of Markov maintenance models in power systems. First, two scheduled maintenance models are applied into reliability and cost assessments of circuit breakers using actual data obtained from [7]. It is shown that the results of the classical model are significantly different from that of the proposed model. Sensitivity analyses are conducted to provide a better insight for effect of inspection rates on MTBF and availability. The behavior of MTBF of proposed and classical models when γ_1 goes to zero, further validates the accuracy of the proposed state diagram. From the results of sensitivity analysis, it is concluded that the proposed model is more applicable to devices with off-line monitoring where the classical model is more applicable to those with on-line monitoring.

Secondly, a Markov model is applied for state prediction of transformers. First transformers are classified according to the operational age and loading conditions. The effect of ageing and loading on transformer deterioration is investigated. Then states are predicted for different groups of transformers and predicted states are compared with actual states. Results show that the state of transformers can be predicted rather accurately, especially for old transformers which have a higher loading. This is crucial as highly-loaded transformers are more vulnerable to deterioration and it would be useful if the state of such transformers can be predicted so that preventive action can be taken.

Thirdly, a Markov model is utilized for a preliminary study to identify sensitive components of a WECS by conducting several sensitivity analyses. Results show that the components with high mean down times and high failure frequencies are more critical to the availability of a WECS than the others.

In the next chapter, the circuit breaker maintenance models utilized in the section 3.2 of this chapter are further utilized to perform maintenance optimization.

Chapter 4 : Reliability and Cost Trade-off in Maintenance Strategies Using Probabilistic Models

4.1 Introduction

This chapter aims to formulate a maintenance optimization problem considering the trade-off between reliability and cost, and obtain optimal maintenance policies using the two scheduled maintenance models described in section 3.2.

The effect of inspection and maintenance on reliability and costs is expressed by means of various performance measures. These measures include cost of performing inspection, maintenance and repair [13, 15, 16, 19-22, 24, 27, 67-70], unavailability (or availability) [19-22], frequency of failure [68], first passage time (FPT) [13, 18, 20], cost of interruption (cost of consequences of power interruptions) [68] and cost of lost energy (revenue loss due to energy not served) [20, 68].

Typically, optimizing the performance measures becomes the objective of maintenance optimization. Single-objective optimization formulations usually aim to minimize the cost [24, 70, 71]. The objective of the optimization problem presented in [20] is to maximize substation availability. The work in [21, 22] has two objectives, i.e. to maximize the availability and to minimize the cost. The optimization model in [19] attempts to maximize FPT, while minimizing life cycle cost and unavailability. In the formulation in [19], the objective function has been defined by assigning different weighting factors for FPT, unavailability and life cycle cost. However, the optimal policies become subjective, when these factors are assigned for different measures under consideration. Although there are several performance measures to be considered in the objective of maintenance optimization, relationships may exist among some of these performance measures. One of the goals in this chapter is to identify such relationships which may lead to a simple yet more accurate problem formulation.

Maintenance optimization is generally performed either through sensitivity analyses or by using optimization algorithms. In [13, 15], sensitivity analyses have been performed to analyze the behavior of reliability and cost measures by varying inspection rates. Optimization tasks in [19] and [21, 22] are based on a simulated annealing algorithm and Markov decision processes, respectively. In [16], the best maintenance scenario has been chosen with the aid of a computer tool based on a probabilistic model. The optimum maintenance rates that maximize substation availability are obtained in [20] by using particle swarm optimization. In [24], a genetic algorithm is utilized to determine optimal maintenance policies. In this chapter, a grid search algorithm is used to find optimal policies.

There are two main contributions of the work presented in this chapter. First, this chapter analyzes commonly used reliability and cost measures and proposes a maintenance optimization formulation in a simple manner to describe the trade-off between reliability and cost. Secondly, this chapter presents a comparative study between two probabilistic models which has different model assumptions. This study investigates to which extent the selection of the underlying maintenance model affects the results of a maintenance optimization problem.

This chapter is organized as follows. Section 4.2 provides the background information. Section 4.3 describes the optimization formulation and the procedure of finding optimal inspection rates. In section 4.4, results of several case studies are presented. In section 4.5, results and findings are discussed. Finally, a short summary is given in section 4.6.

4.2 Maintenance Models, Performance Measures and Decision Variables

In this section, the maintenance models utilized in maintenance optimization are first presented. Then, some reliability and cost measures are described. These measures are considered in this study to evaluate the effect of inspection and maintenance on both the utilities and power consumers. Finally, the decision variables that affect reliability and cost measures are discussed.

4.2.1 Maintenance Models

In this chapter, we utilize the same scheduled maintenance models described in section 3.1. These models are again shown in Figures 4.1 and 4.2 of this chapter. In this chapter, the following terminology is used for the two maintenance models, based on the discussions in section 3.1. The term “inspection based maintenance model (IBM model)” is assigned for the model in Figure 4.1, since its model assumption represents the actual practice of scheduled inspection based maintenance. The term “condition monitoring based inspection and maintenance model (CBM model)” is assigned for the model in Figure 4.2, since its assumption is valid only if the deterioration condition of the equipment is always known as in continuous online condition monitoring.

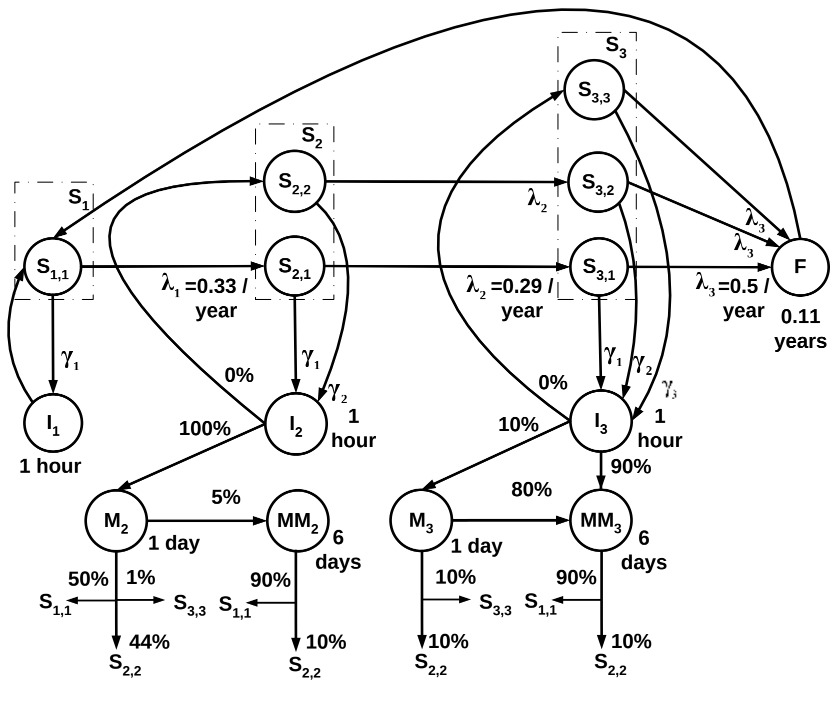

Figure 4.1: The state diagram of the condition monitoring based inspection and maintenance model

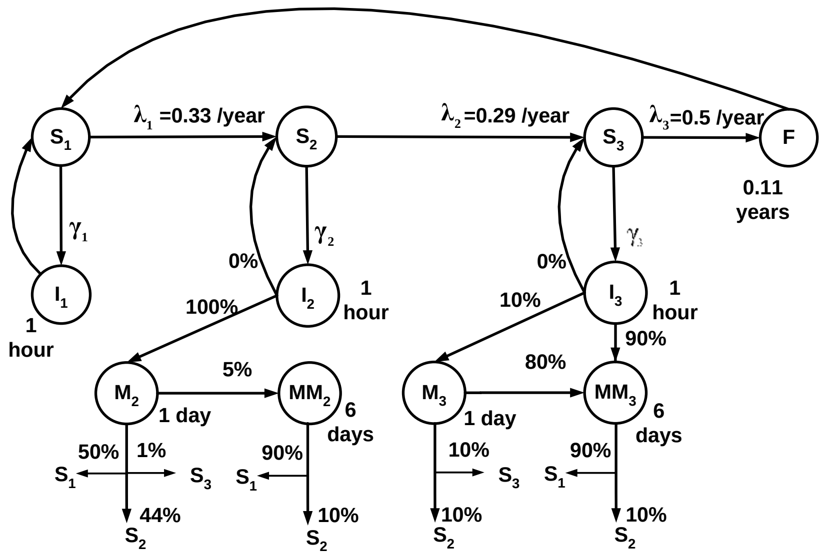

Figure 4.2: The state diagram of the inspection based maintenance model [7]

4.2.2 Performance Measures

In order to quantify the effect of maintenance on reliability and costs, several performance measures can be computed with the use of probabilistic maintenance models. For the analyses in this chapter, we choose the following measures.

1) FPT: FPT is the average time taken to reach the failure state for the first time, and this can be calculated using Markov equations [45].
2) Life cycle cost: This includes inspection, maintenance and repair costs of the equipment [19]. Annual life cycle cost (*LCC*) can be calculated using (4.1), where C_I, C_M, C_{MM} and C_F are costs of performing an activity of inspection, minor maintenance, major maintenance and repair, respectively. $P(S)$ and $d(S)$ are the steady state probability of state S and the mean duration in state S, respectively.

$$LCC = \mathrm{C_I} \times \sum_{i=1}^{3} \frac{P(\mathrm{I}_i)}{\mathrm{d}(\mathrm{I}_i)} + \mathrm{C_M} \times \sum_{j=2}^{3} \frac{P(\mathrm{M}_j)}{\mathrm{d}(\mathrm{M}_j)} + \mathrm{C_{MM}} \times \sum_{k=2}^{3} \frac{P(\mathrm{MM}_k)}{\mathrm{d}(\mathrm{MM}_k)} + \mathrm{C_F} \times \frac{P(\mathrm{F})}{\mathrm{d}(\mathrm{F})} \tag{4.1}$$

3) Unavailability (U): Unavailability is the probability that the equipment is not in operation. As shown in (4.2), unavailability can be calculated by simply adding the state probabilities of inspection, maintenance and failure states.

$$U = \sum_{i=1}^{3} P(\mathrm{I}_i) + \sum_{j=2}^{3} P(\mathrm{M}_j) + \sum_{k=2}^{3} P(\mathrm{MM}_k) + P(\mathrm{F}) \tag{4.2}$$

4) Frequency of interruption (*FI*): *FI* is the average number of interruptions occurred within a year. As in (4.3), this can be calculated by adding the frequencies of inspection and the failure frequency. *f*(*S*) denotes the frequency of entering or leaving state *S*.

$$FI = \sum_{i=1}^{3} f(\mathrm{I}_i) + f(\mathrm{F}) \tag{4.3}$$

5) Interruption cost: Interruption cost is the economic loss experienced by power consumers due to interruptions caused by inspection, maintenance and failures. When interruption cost is evaluated, several factors such as the load curtailed, the type of power consumers involved and the duration of the outage are taken into account [72, 73]. Depending on factors such as the structure of the power system, the location of the equipment in the power system and redundancy, the load curtailed can be zero, even though the operation of some equipment is interrupted. For such equipment, interruption cost will be zero. However, in many power systems, equipment operates closer to its limits and therefore, interrupting the operation of equipment will likely to cause load curtailment.

   Annual interruption cost (*IC*) can be calculated using (4.4), where $\mathrm{C_{IC}}$ is the hourly interruption cost experienced by consumers. It should be noted that this hourly interruption cost, $\mathrm{C_{IC}}$ is a function of the amount of load curtailed and the type of affected consumers. In this thesis we consider a device which contributes to supply 1 MW to average consumers. If the operation of such equipment is interrupted, the hourly interruption cost during the peak is \$ 29000 / hour [6].

$$IC = \mathrm{C_{IC}} \times \left\{ \begin{array}{l} \sum_{i=1}^{3} [f(\mathrm{I}_i) \times \mathrm{d}(\mathrm{I}_i)] + \sum_{j=2}^{3} [f(\mathrm{M}_j) \times \mathrm{d}(\mathrm{M}_j)] \\ + \sum_{k=2}^{3} [f(\mathrm{MM}_k) \times \mathrm{d}(\mathrm{MM}_k)] + f(\mathrm{F}) \times \mathrm{d}(\mathrm{F}) \end{array} \right\} \tag{4.4}$$

6) Loss of profit: During interruptions, utilities fail to generate revenue by selling electricity and as a result, they lose some profit. Annual loss of profit (*LP*) due to interruptions can

be calculated using (4.5). C_{LP} is the average profit which is lost by not operating the equipment for 1 hour and it is a function of the amount of load affected by not operating the equipment for 1 hour. For equipment which contributes to supply 1 MW to consumers, the average loss of profit is $70 / hour [68].

$$LP = C_{LP} \times \left\{ \begin{array}{l} \sum_{i=1}^{3} [f(\mathrm{I}_i) \times \mathrm{d}(\mathrm{I}_i)] + \sum_{j=2}^{3} [f(\mathrm{M}_j) \times \mathrm{d}(\mathrm{M}_j)] \\ + \sum_{k=2}^{3} [f(\mathrm{MM}_k) \times \mathrm{d}(\mathrm{MM}_k)] + f(\mathrm{F}) \times \mathrm{d}(\mathrm{F}) \end{array} \right\} \tag{4.5}$$

4.2.3 Decision Variables

Since every inspection is associated with a cost and off line inspections temporarily stop the operation of the device, inspection rate directly affects reliability and cost measures. Thus, inspection rate is considered as a variable parameter in many maintenance optimization tasks [13, 15, 19-22]. Similarly, we consider the inspection rate (i.e. γ_1, γ_2 and γ_3) as our decision variables.

Performance measures are related to steady state probability of state *S*, *P*(*S*), and steady state frequency of entering or leaving state *S*, *f*(*S*). The process is semi-Markov and *P*(*S*) can be found using (4.6), where $\pi(S)$ is the steady state probability that the embedded Markov chain is in state *S* [45].

$$P(S) = \frac{\mathrm{d}(S) \times \pi(S)}{\sum_{\forall S} \mathrm{d}(S) \times \pi(S)} \tag{4.6}$$

$\pi(S)$, which is an element of the steady state probability vector Π can be found using (4.7) and (4.8).

$$\Pi A = \Pi \tag{4.7}$$

$$\sum_{\forall S} \pi(S) = 1 \tag{4.8}$$

where *A* is the transition probability matrix and can be constructed based on the state transition diagrams shown in Figures 4.1 and 4.2.

Elements of A contain γ_1, γ_2 and γ_3 and therefore, $P(S)$ can have nonlinear relationships with γ_1, γ_2 and γ_3. Frequency of being in state S, i.e. $f(S)$ is computed using (4.9) [45], where, $r_{\text{out}}(S)$ is the summation of the transition rates from state S to other neighboring states. As can be seen in (4.9), $f(S)$ is related to $P(S)$ and transition rates, thus $f(S)$ also has a nonlinear relationship with γ_1, γ_2 and γ_3.

$$f(S) = P(S) \times r_{\text{out}}(S) \tag{4.9}$$

It is clear that the functions of performance measures are nonlinear with respect to decision variables γ_1, γ_2 and γ_3. Due to such non-linear relationships, it is difficult to mathematically describe the behavior of performance measures with γ_1, γ_2 and γ_3 by simply observing the equations. We perform sensitivity analysis in the following section to observe such behavior of some performance measures with respect to γ_1, γ_2 and γ_3.

4.3 Selection of Optimal Inspection Rates

In this section, we formulate the maintenance optimization problem, after analyzing the relationships among performance measures and results of two sensitivity analyses. Then, we describe the grid search algorithm which is used to find the optimal policies.

4.3.1 Relationships among Different Performance Measures

Two relationships can be observed in the performance measures mentioned in section 4.2.2. First, from (4.1) and (4.2), a relationship is observed between life cycle cost and unavailability. In order to show this relationship more clearly, (4.10) is obtained by rewriting (4.1) using the notations related to unavailability. In (4.10), $U(S)$ is the unavailability caused by the activities in state S. C_S is the cost of performing an activity in state S and $\mathrm{K}_1(S) = \mathrm{C}_S/\mathrm{d}(S)$. Since C_S and $\mathrm{d}(S)$ are constants for state S, $\mathrm{K}_1(S)$ is also a constant for state S. Thus, according to (4.10), life cycle cost can be expressed as a function of unavailability caused by different inspection, maintenance and repair activities. $\mathrm{K}_1(S)$ in the LCC function can be considered as the weightage assigned for the unavailability caused by the activities in state S.

$$LCC = \left\{ \begin{array}{c} \sum_{i=1}^{3} \mathrm{K}_1(\mathrm{I}_i) \times U(\mathrm{I}_i) + \sum_{j=2}^{3} \mathrm{K}_1(\mathrm{M}_j) \times U(\mathrm{M}_j) \\ + \sum_{k=2}^{3} \mathrm{K}_1(\mathrm{MM}_k) \times U(\mathrm{MM}_k) + \mathrm{K}_1(\mathrm{F}) \times U(\mathrm{F}) \end{array} \right\} \tag{4.10}$$

Secondly, from (4.3) and (4.4), another relationship can be observed between the frequency of interruption and the interruption cost. We rewrite (4.4) to obtain (4.11). $FI(S)$ is the frequency of interruption due to activities in state S and $\mathrm{K}_2(S) = \mathrm{C}_{\mathrm{IC}} \times \mathrm{d}(S)$ for any inspection or failure state S. Since $\mathrm{K}_2(S)$ is a constant for state S, it can be considered as a weight assigned for the frequency of interruption due to activities in state S.

$$IC = \mathrm{C}_{\mathrm{IC}} \times \left\{ \begin{array}{c} \sum_{j=2}^{3} [f(\mathrm{M}_j) \times \mathrm{d}(\mathrm{M}_j)] \\ + \sum_{k=2}^{3} [f(\mathrm{MM}_k) \times \mathrm{d}(\mathrm{MM}_k)] \end{array} \right\} + \sum_{i=1}^{3} [\mathrm{K}_2(\mathrm{I}_i) \times FI(\mathrm{I}_i)] + \mathrm{K}_2(\mathrm{F}) \times FI(\mathrm{F}) \tag{4.11}$$

Above relationships show that unavailability and frequency of interruption are implicitly included in the objective of maintenance optimization, if the objective function includes life cycle cost and interruption cost, respectively. Thus, we omit unavailability and frequency of interruption from the objective function and consider the corresponding cost components. If it is required to guarantee a minimum reliability level, it is possible to impose constraints on these reliability measures.

Further, as given in (4.12), we add all cost components, i.e. *LCC*, *IC* and *LP* to compute annual total cost (*TC*). The addition of all cost components does not require any weighting factors because all terms are expressed as cost functions. Weighting factors can be introduced, if the significances of cost terms are different.

$$TC = LCC + IC + \ LP \tag{4.12}$$

Now, the performance measures that should be considered in the objective function of the maintenance optimization problem are FPT and total cost.

4.3.2 Sensitivity Analyses of Inspection Rate on First Passage Time and Total Cost

Through sensitivity analyses, this section investigates the possibilities of further simplifying the maintenance optimization problem. In these analyses, the model parameters γ_1, γ_2 and γ_3 are varied to find the behaviors of FPT and total cost. When one parameter is varied from 0.01 per year to 45 per year, the other two parameters are kept constant at their original values.

4.3.2.1 Sensitivity Analysis of Inspection Rate on First Passage Time

Figures 4.3, 4.4 and 4.5 show the variation of FPT with γ_1, γ_2 and γ_3 of the two maintenance models. According to the results of the IBM model, FPT can be greatly increased by increasing γ_1 and the effect of γ_2 and γ_3 on FPT is less, when γ_1 is kept at its original low value. On the other hand, the results of the CBM model show that the effect of γ_1 on FPT is negligible, but FPT can be greatly increased by increasing γ_2 and γ_3. Although γ_1, γ_2 and γ_3 of the two models show more or less effects on FPT, in overall both models show that FPT increases with increasing γ_1, γ_2 and γ_3.

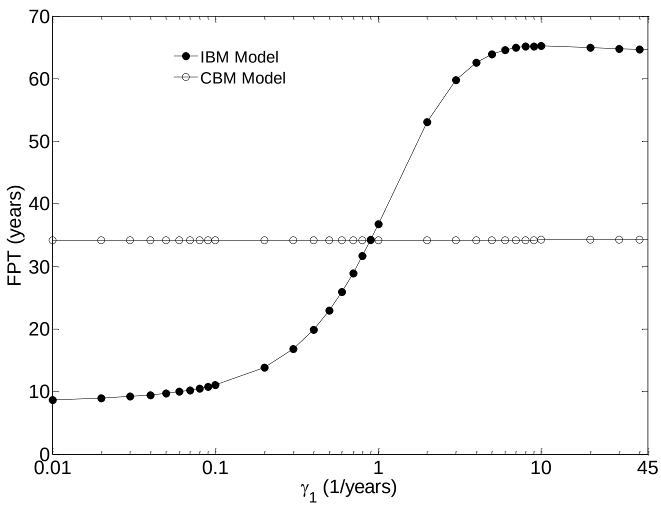

Figure 4.3: The variation of first passage time with γ_1

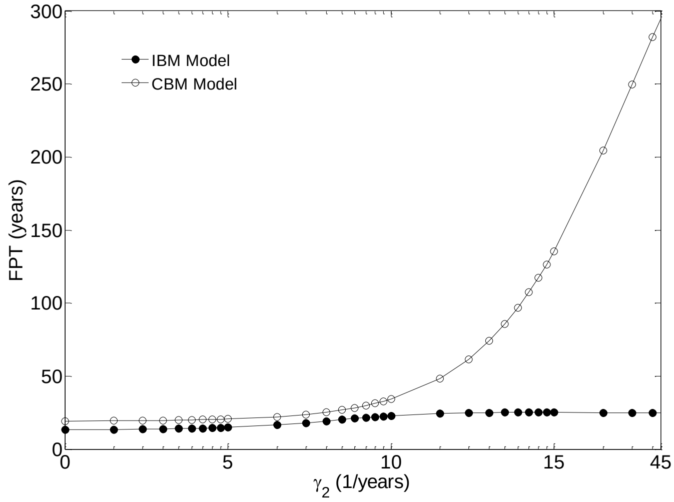

Figure 4.4: The variation of first passage time with γ_2

However, it is practically impossible to infinitely increase the inspection rate. Thus, maintaining the FPT above a required level by imposing a constraint would be more meaningful than trying to maximize the FPT by including it in the objective function.

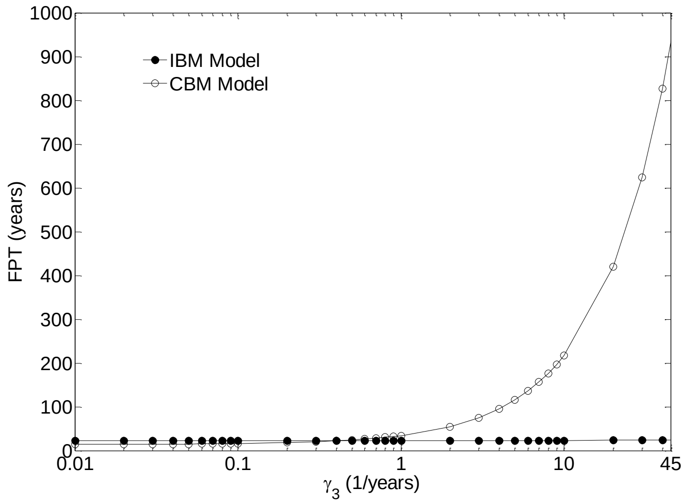

Figure 4.5: The variation of first passage time with γ_3

4.3.2.2 Sensitivity Analysis of Inspection Rate on Total Cost

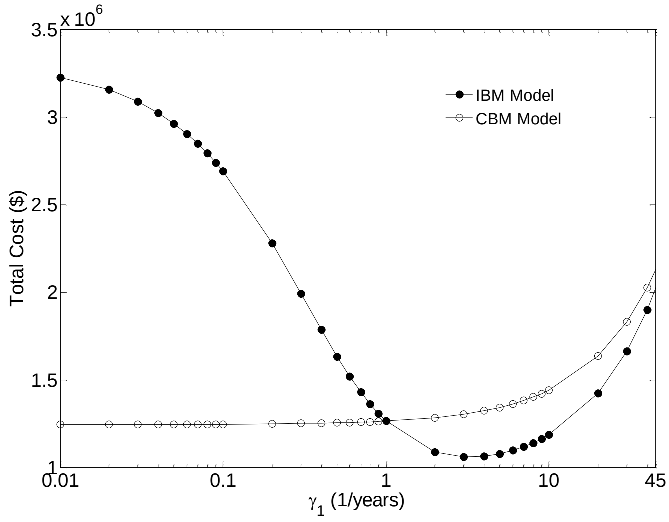

Figure 4.6: The variation of total cost with γ_1

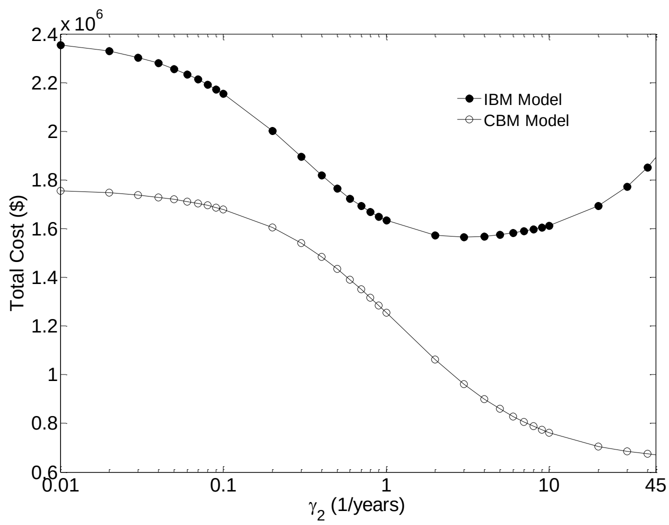

Figure 4.7: The variation of total cost with γ_2

Figures 4.6, 4.7 and 4.8 show the variation of total cost with γ_1, γ_2 and γ_3 of the two maintenance models. As can be seen in these figures, results of the two models are contradictory. Results of the IBM model show the possibility of selecting optimal values for γ_1, γ_2 and γ_3, which minimize the total cost. In contrast, results of the CBM model suggests that total cost can be minimized by minimizing γ_1 and maximizing γ_2 and γ_3.

Since the results of the IBM model show the existance of optimal values for γ_1, γ_2 and γ_3 which minimize the total cost, total cost is considered in the objective of optimization.

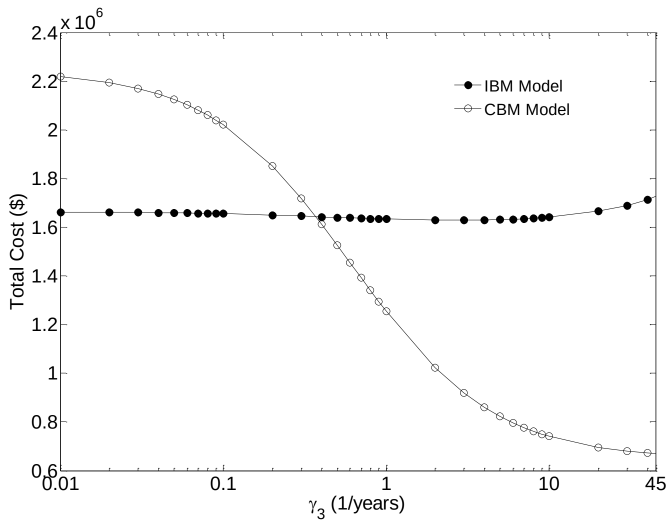

Figure 4.8: The variation of total cost with γ_3

4.3.3 Problem Definition

Objective Function: Based on the analyses in sections 4.3.1 and 4.3.2, the objective of the maintenance optimization problem is simplified to minimizing the total cost.

$$\text{Minimize}\{TC\}$$

Decision Variables: γ_1, γ_2 and γ_3.

Constraints: As discussed in sections 4.3.1 and 4.3.2, constraints can be imposed on reliability measures. In this formulation, a constraint is imposed on FPT.

4.3.4 A Grid Search Algorithm

As discussed in section 4.2.3, total cost function is a highly nonlinear function of γ_1, γ_2 and γ_3. Thus, it would be difficult to find the optimal values of γ_1, γ_2 and γ_3 that minimize total cost through the use of mathematical programming tools. Some alternative methods would be heuristic algorithms such as simulated annealing [21, 22], partical swam optimization [20], or genetic algorithms [24], to name a few. As the focus of the chapter is to analyze the performance of two maintenance models (IBM and CBM models), we apply a simple grid search algorithm to find the optimal inspection rates.

The following grid search algorithm is implemented in MATLAB to solve the optimization problem defined in section 4.3.3. Through a discrete grid search, this algorithm provides optimal values for the inspection rate. The steps of the algorithm are given below.

1) A discrete set of values is defined for the inspection rate considering practical constraints.
2) Then, the defined discrete set of values for the inspection rate is assigned for grids of values of γ_1, γ_2 and γ_3. Let G_1=the grid of values of γ_1, G_2=the grid of values of γ_2 and G_3=the grid of values of γ_3.
3) Cartesian product of grids of values of γ_1, γ_2 and γ_3 are computed as shown below.

$$\Omega = G_1 \times G_2 \times G_3$$

   Each element set $k \in \Omega$ consists of the values of γ_1, γ_2 and γ_3 corresponding to one possible maintenance policy.
4) Probabilistic maintenance model is analytically solved using Markov equations for each $k \in \Omega$ to compute total cost.
5) By comparing total cost of each $k \in \Omega$, k corresponding to the minimum total cost (k_{opt}) is obtained. k_{opt} represents the optimal maintenance policy and consists of the optimal values of γ_1, γ_2 and γ_3. The two reliability measures, FPT and unavailability corresponding to the optimal γ_1, γ_2 and γ_3 are also computed.

It should be noted that the efficiency of a grid search algorithm decreases with the size of Ω. A device usually has three or less deterioration stages and therefore, for a single equipment the size of Ω is small. Hence, this grid search algorithm performs favorably for finding optimal inspection rates for a single device. However, this algorithm may not perform well in system level maintenance optimization. In the system level, there exist system constraints on the budget and reliability and hence, it is not accurate to consider each equipment seperately and decompose the problem into sub problems. It is required to find optimal inspection rates for every equipment in the system considering system constraints. This will increase the numer of decision variables in the problem and the size of Ω and decrease the performance of the algorithm.

4.4 Case Studies

In this section, case studies are conducted on both maintenance models, using the grid search algorithm presented in section 4.3.4. In these case studies, the parameters γ_1, γ_2 and γ_3

are varied from 0 to the maximum inspection rate (γ_{max}) with a step size of 0.2 per year. Thus, the set of values assigned for the inspection rate is {0, 0.2, 0.4, 0.6, …, γ_{max}}. For each of the following three constraints on FPT, eight case studies are conducted by setting γ_{max} and C_{IC} according to Table 4.1.

1. FPT $\geq$ 30 years
2. FPT $\geq$ 50 years
3. FPT $\geq$ 100 years

Table 4.1: Constraints on γ_{max} and Hourly Interruption Costs

Case Study	γ_{max} (per year)	C_{IC} ($ per hour)
1	1	29000
2	3	29000
3	4	29000
4	6	29000
5	10	29000
6	12	29000
7	12	200
8	12	0

As shown in Table 4.1, from case study 1 to 6, γ_{max} is increased. Hence, the results of these case studies together show how the optimal policy varies depending on the maximum possible inspection rate, γ_{max}.

According to Table 4.1, from case studies 6-8, different values are set for C_{IC} i.e. for the hourly interruption cost experienced by consumers. Since different power consumers experience different hourly interruption costs [6], this hourly interruption cost depends on the type of consumers that the equipment serves. Thus, case studies 6-8 show how the optimal policy varies depending on the type of consumers that the equipment serves.

In each case study, the optimal policy (i.e. the optimal values of γ_1, γ_2 and γ_3) and total cost, FPT and unavailability corresponding to the optimal policy are determined. These results obtained using the IBM model and the CBM model are tabulated in Tables 4.2 to 4.5.

4.4.1 Results of Case Studies with the Constraint FPT ≥ 30 Years

Tables 4.2 and 4.3 show the results obtained using the IBM model and the CBM model, with the constraint, FPT ≥ 30 years. The followings are revealed from the results of case studies 1-6:

1) As can be seen in the results of the IBM model, the optimal values of γ_1, γ_2 and γ_3 (in the selected range i.e. from 0 to 12 per year) are 4, 8.6 and 12 per year, respectively. These optimal values of the inspection rate confirms that it is beneficial to increase the inspection rate with the deterioration as in the common practice of inspection based maintenance.
2) The optimal policy suggested by the IBM model is to conduct inspections at the optimal values of γ_1, γ_2 and γ_3. However, if the maximum inspection rate is less than the optimal values, the results suggest conducting inspections at the maximum inspection rate.
3) According to the results of the CBM model, the optimal value of γ_1 is zero and the optimal values of γ_2 and γ_3 are the maximum possible inspection rate.
4) The optimal policy suggested by the CBM model is to conduct no inspections at the early stage and to conduct inspections at the maximum rate at latter deterioration stages.
5) When γ_{max} is increased, optimal policies of both models show an increase in benefits i.e. an increase in FPT and a reduction in total cost and unavailability (due to the increase in one or more out of optimal γ_1, γ_2 and γ_3). However, the CBM model shows an impossible increase in benefits, especially in FPT. In contrast, the IBM model shows a comparatively less yet reasonable increase in benefits.

The results of case studies 6 to 8 suggest the followings:

1) When the hourly interruption cost is low, the IBM model provides a slightly low value for the optimal γ_1 and a higher value for the optimal γ_2. However, the optimal γ_3 remains the same at the maximum possible inspection rate. As the hourly interruption cost decreases, a great reduction can be observed in total cost corresponding to the optimal policy of the IBM model. This reduction could be mainly due to the reduction in interruption cost which is a component of total cost. Since the optimal values of γ_1 and γ_2 change with the

changes in the hourly interruption cost, a change can be observed in FPT, however the change in unavailability is insignificant.

2) Optimal values of γ_1, γ_2 and γ_3 of the CBM model do not change with the changes in hourly interruption cost. Thus, no changes are observed in FPT and unavailability, but total cost is high, when the hourly interruption cost is high.

Table 4.2: Results Obtained Using the Inspection Based Maintenance Model for FPT ≥ 30 Years

Case Study	Optimal γ_1 (per year)	Optimal γ_2 (per year)	Optimal γ_3 (per year)	FPT (years)	Unavailability	Total Cost
1	1	1	1	37	0.0049	1264
2	3	3	3	151	0.0033	833
3	4	4	4	233	0.0031	804
4	4	6	6	297	0.0031	786
5	4	8.4	10	347	0.0030	780
6	4	8.6	12	353	0.0030	780
7	3.8	10	12	339	0.0030	9.80
8	3.8	12	12	348	0.0030	4.44

Table 4.3: Results Obtained Using the Condition Monitoring Based Inspection and Maintenance Model for FPT ≥ 30 Years

Case Study	Optimal γ_1 (per year)	Optimal γ_2 (per year)	Optimal γ_3 (per year)	FPT (years)	Unavailability	Total Cost
1	0	1	1	34	0.0049	1244
2	0	3	3	138	0.0030	760
3	0	4	4	212	0.0027	702
4	0	6	6	400	0.0025	650
5	0	10	10	904	0.0024	616
6	0	12	12	1208	0.0024	609
7	0	12	12	1208	0.0024	7.16
8	0	12	12	1208	0.0024	2.99

4.4.2 Results of Case Studies with the Constraint FPT ≥ 50 Years or FPT ≥ 100 Years

Tables 4.4 and 4.5 show the results obtained using the IBM model and the CBM model, when the constraint on FPT is FPT ≥ 50 years or FPT ≥ 100 years.

These results of case studies 2 to 8 are as same as the results of case studies 2 to 8 in section 4.4.1, which are for the constraint FPT ≥ 30 years. In case study 1, when γ_{max} is 1 per year, solution is infeasible when the constraint on FPT is FPT ≥ 50 years or FPT ≥ 100 years. This is beacause when γ_{max} is 1 per year, for any combination of γ_1, γ_2 and γ_3, FPT that can be achieved is less than 30 years.

Table 4.4: Results Obtained Using the Inspection Based Maintenance Model for FPT ≥ 50 Years or for FPT ≥ 100 Years

Case Study	Optimal γ_1 (per year)	Optimal γ_2 (per year)	Optimal γ_3 (per year)	FPT (years)	Unavailability	Total Cost (k$)
1	Infeasible					
2	3	3	3	151	0.0033	833
3	4	4	4	233	0.0031	804
4	4	6	6	297	0.0031	786
5	4	8.4	10	347	0.0030	780
6	4	8.6	12	353	0.0030	780
7	3.8	10	12	339	0.0030	9.80
8	3.8	12	12	348	0.0030	4.44

Table 4.5: Results Obtained Using the Condition Monitoring Based Inspection and Maintenance Model for FPT ≥ 50 Years or for FPT ≥ 100 Years

Case Study	Optimal γ_1 (per year)	Optimal γ_2 (per year)	Optimal γ_3 (per year)	FPT (years)	Unavailability	Total Cost (k$)
1	Infeasible					
2	0	3	3	138	0.0030	760
3	0	4	4	212	0.0027	702
4	0	6	6	400	0.0025	650
5	0	10	10	904	0.0024	616
6	0	12	12	1208	0.0024	609
7	0	12	12	1208	0.0024	7.16
8	0	12	12	1208	0.0024	2.99

4.5 Discussion

Section 4.3 investigate the reliability and cost measures that should be essentially included in the objective of maintenance optimization. In section 4.3.1, it is shown that the two reliability measures, unavailability and the frequency of interruption are implicitly included within the calculations of life cycle cost and cost of interruption, respectively. Thus, we conclude that it is not necessary to consider unavailability and frequency of failure in the objective function, if life cycle cost and cost of interruption are considered in the objective of maintenance optimization. In section 4.3.2, it is shown that the reliability measure FPT can be maximized by maximizing the inspection rate. However, it is impossible to infinitely increase the inspection rate. Thus, it is more suitable to impose a constraint on the minimum FPT, rather than trying to maximize FPT. From the results of studies in sections 4.3.1 and 4.3.2 it can be concluded that it is more appropriate to consider cost measures in the objective function, while imposing constraints on the minimum required reliability level.

Based on the results of sensitivity analyses in section 4.3.2 and the results of case studies in section 4.4, the CBM model shows that the benefits of inspection and maintenance can be increased by minimizing γ_1, and maximizing γ_2 and γ_3. In contrast, the IBM model suggests optimal values for γ_1, γ_2 and γ_3 that maximize the benefits of inspection and maintenance. The optimal inspection rates suggested by the IBM model appear more practical, when results are analyzed considering the behaviors of different cost components. If the inspection rate is very low, due to high repair cost and interruption cost associated with the increased number of failures, the total cost should increase. If the inspection rate is very high, the total cost should again increase due to high inspection and maintenance cost as well as high interruption cost associated with the increased number of inspection and maintenance activities. Therefore, in between the minimum and maximum possible values of the inspection rate (i.e. in between zero and highest possible γ_{max}), optimal values that minimize the total cost should exist for γ_1, γ_2 and γ_3. Such optimal values suggested by the IBM model seem more reasonable.

Results of the last three case studies in section 4 also show that the policies suggested by the IBM model are more reasonable than the policy suggested by the CBM model. As shown in the results, the IBM model provides different optimal maintenance policies, when the hourly interruption cost experienced by the consumers is different. On the other hand, the CBM model suggests adopting the same optimal maintenace policy irrespective of the hourly interruption cost experienced by the consumers. However, when power consumers experience different hourly interruption costs [6], power interruptions should affect them differently. Therefore, maintenance policy of equipment should depend on the type of consumers that it serves. For example, households experience a lower or nil hourly interruption cost. Thus, the annual interruption cost that they experience would be very less compared to the replacement cost of the equipment. Hence, it should be cost effective to perform frequent inspection and maintenance in order to avoid costly replacements. If the equipment serves to an industrial zone, the hourly interruption cost is higher and frequent power interuptions are undesirable. Thus, only an adequate amount of inspection and maintenance should be performed, in order to tolerate with the interruptions due to failures.

The reasons for contraditions in the results provided by the two models can be explained considering different underlying model assumptions of the two models. The assumption of the CBM model (i.e. the deterioration condition of the equipment is always known) is true for some maintenance strategies such as continuous online condition monitoring. Thus, the optimal policies suggested by the CBM model would be suitable for maintenance strategies which continuously reveal the condition of the equipment. However, in inspection based maintenance, the condition of the equipment is known only after inspection and maintenance. Thus, inspecting the equipment is vital to get the knowledge about the equipment's deterioration condition that requires for the decision making regarding maintenance and the next inspection. As discussed in section 3.1, if γ_1 is zero, the equipment would fail even before any inspection and maintenance is performed. Unlike the CBM model, the IBM model assumes that the condition is known only after inspection and maintenance. Thus, its results show the need of performing inspections even at the early stage of deterioration, in order to get the knowledge about the deterioration status of the equipment.

4.6 Summary

This chapter applies probabilistic models developed for scheduled maintenance of ageing equipment to find optimal maintenance policies. First, the study investigates how reliability and cost can be traded-off in maintenance optimization tasks. Initially, six reliability and cost measures are considered in the objective of the optimization problem. Analytical equations show that it is redundant to consider the two reliability measures, unavailability and the frequency of interruption in the objective function, when life cycle cost and cost of interruption are considered. After eliminating unavailability and the frequency of interruption and combining all cost components together, the measures considered in the objective function are reduced to FPT and total cost. Sensitivity analyses show that it is more appropriate to impose a constraint on FPT. Therefore, the objective of the maintenance optimization problem is set to minimizing total cost.

A discrete grid search algorithm is used to find optimal inspection rates that minimize total cost. Several case studies are conducted using two probabilistic models, the IBM model and the CBM model. It is shown that the IBM model is capable of providing optimal inspection rates for each deterioration stage. The results of the IBM model are in accordance with the common practice of scheduled maintenance. In addition, the IBM model suggests different optimal inspection rates for equipment that serves different type of consumers. Results of the CBM model seem less applicable for scheduled maintenance. The reasons for differences in results of the two models are explained. It is concluded that the IBM model is well applicable for the selection of optimal scheduled maintenance policies. The optimal policies provided by the CBM model would be applicable for maintenance strategies which continuously monitor the condition of equipment.

In the next chapter, a maintenance optimization model is proposed based on a Markov decision process. The model proposed in the next chapter can provide more adaptive optimal inspection and maintenance policies than the optimal policies obtained in this chapter by using scheduled maintenance models.

Chapter 5 : Adaptive Maintenance Policies Using a Markov Decision Process

5.1 Introduction

Asset management is essential for reliable and economic operation of power systems. With deregulation, asset management procedures became more complicated [74]. In such environments, an asset owner can perform preventive maintenance only after the independent system operator schedules a planned outage upon the request of the asset owner. In some situations, the operator may delay certain requested outages, in order to fulfill the overall aim of serving the power consumers [74]. Such situations require equipment owners to adjust their asset management plans accordingly. This highlights the need for adaptive asset management policies which can deal with maintenance delays.

Adaptive asset management policies would also be more economical than fixed policies. When the equipment is new and in good condition, too frequent inspection or condition monitoring (CM) would not reveal any additional information about the equipment's condition, and thus, unnecessarily increases the operational cost. On the other hand, when the equipment is aged or its condition is more deteriorated, delaying inspection and maintenance may cause huge economic losses through unexpected failures. Hence, it would be more economical to perform inspection and maintenance, considering the equipment's age, condition and delay times in making decisions regarding inspection and maintenance.

In this chapter, a new maintenance optimization model is proposed based on a Markov decision process (MDP) for inspection and maintenance of ageing equipment. The proposed decision making model additionally considers time delays in making decisions regarding inspection and maintenance. Moreover, this model represents the deterioration of equipment using a quantifiable condition, while allowing the parameters of the deterioration process to vary with the operational age. Due to the above features of the model, it can provide more adaptive inspection and maintenance policies which allow the asset owners to choose the optimal action, based on the knowledge about the equipment's *condition*, the operational *age* and *time delays* in making decisions regarding inspection and maintenance.

The structure of this chapter is as follows. In section 5.2, the background theories and information are provided. In section 5.3, the formulation of the maintenance optimization model is presented. In section 5.4, the solution procedure is explained. In section 5.5, a case study demonstrates a model application. Section 5.6 investigates the accuracy of using individual MDP models for coordinating maintenance in power systems. Finally, a short summary is given in section 5.7.

5.2 Background

This section first reviews some applications of MDPs to power system decision making problems. Next, the framework of a finite horizon discrete time MDP is reviewed with reference to [75]. Then, the decision making process regarding inspection and maintenance of equipment is discussed. Lastly, this section describes how inspection and maintenance decision making process is modeled in the framework of a finite horizon discrete time MDP.

5.2.1 Markov Decision Processes in Power Systems

MDPs have been widely applied to electricity markets in order to perform dynamic decision making while considering uncertainties in energy demand and energy prices. In [76], an MDP has been proposed for generation expansion planning. MDPs proposed in [77, 78] provide optimal electricity supply bidding decisions over a planning horizon considering uncertainties associated with price and load. In [79], a competitive MDP is proposed to assess market powers associated with the electricity bidding prices. The reinforcement learning algorithm presented in [80] for dynamic load allocation in automatic generation control systems is also formulated as an MDP. In [81], another reinforcement learning approach is proposed based on an MDP to find dynamic optimal generation command dispatch for automatic generation control.

In addition to the above applications of MDPs, some MDP models have been proposed for power system maintenance decision making problems [21, 22, 33, 42, 43]. A partially observed MDP is proposed in [43] to find static optimal strategies for season dependent maintenance of wind turbines. By using time varying model parameters which depend on weather conditions, this model is further improved in [42] to obtain season dependent dynamic maintenance strategies. This partially observed MDP suits well for wind turbine maintenance, where the information is mainly gathered using unreliable remote sensors. However, this model may be unnecessarily complex for maintenance of some other components equipped with more reliable online CM facilities.

In [21], optimal preventive maintenance policies for power equipment are found in two stages. First, a Markov model is solved to find the optimal maintenance rate which maximizes the availability. Then, at this availability, cost effective maintenance actions are found by solving an MDP using policy iteration method. The same method is applied in [22] to find optimal maintenance policies using a semi Markov model and a semi MDP. The semi MDP proposed in [33] is also solved using the policy iteration method to find optimal condition based maintenance (CBM) policies for transformers.

As mentioned in section 1.2, some of the above mentioned maintenance decision making models are unable to incorporate time delays in making decisions regarding inspection and maintenance [21, 22, 33]. The models in [42, 43] represent the deterioration of the equipment by some observable measurements and are unable to incorporate the effect of ageing on the deterioration of equipment's observable condition. The new MDP model which will be presented in section 5.3 addresses the above limitations of previous MDP models which are proposed for power system maintenance decision making.

5.2.2 The Framework of a Markov Decision Process

An MDP is a sequential decision making model which considers uncertainties in outcomes of current and future decision making opportunities. At each decision making time, the system/equipment occupies a state. Based on this state, a decision is made on choosing an action from the set of actions associated with this state. Upon choosing an action, a reward is received and a state transition occurs from the present state to a new state which is determined by a transition probability distribution. Since the process holds the Markov property, both transition probabilities and rewards only depend on the present state and the action chosen in the present state. As the process evolves, the decision maker receives a sequence of rewards. When choosing actions, the decision maker intends to maximize the total expected reward received over the total decision making period. If the total decision making period of an MDP is finite and the decisions are made in discrete time, the MDP is called a finite horizon discrete time MDP.

In standard practice, decisions regarding inspection and maintenance of equipment are made in discrete time. In addition, no equipment can be used over an infinitely long period and therefore, decisions regarding inspection and maintenance of equipment are made over a finite time horizon. Due to these reasons, we represent the decision making process of equipment's inspection and maintenance using a finite horizon discrete time MDP.

The five basic components of a finite horizon discrete time MDP are as follows.

1) Decision epochs: Decision epochs are the point of times at which decisions are made. In a discrete time MDP, the total decision making period (decision horizon) is divided into intervals which are called decision intervals, and at the beginning of each decision interval, a decision epoch occurs. The set of decision epochs is given by $D = \{1, 2, 3, \dots, N\}$. In a finite horizon MDP, N is finite and according to the convention, decisions are not made at the N^{th} decision epoch. Decision horizon, decision intervals and decision epochs of a discrete time finite horizon MDP are shown in Figure 5.1.

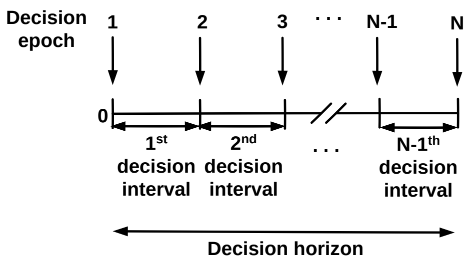

Figure 5.1: Decision horizon, decision intervals and decision epochs

2) States: Different statuses of a system/equipment are modeled using a finite number of states.

3) Actions: Each state is connected with a finite number of actions.

4) Transition probabilities: As a result of choosing any action a connected with state i at the t^{th} decision epoch, a state transition occurs. The new state at the decision epoch t+1 is determined by the probabilities of transiting from state i to possible states in the state space S. The probability of transiting from state i to any state $k \in$ S, upon choosing action a in state i at the t^{th} decision epoch is denoted by $\mathrm{P}_t(k|i, a)$. It should be noted that $\sum_{k=1}^{n} \mathrm{P}_t(k|i, a) = 1$.

5) Rewards: At each decision epoch t <N, the decision maker receives a reward, as a result of choosing an action. The reward received upon choosing action a in state i at the t^{th} decision epoch is denoted by $\mathrm{r}_t(i, a)$. The reward received at the N^{th} decision epoch is assigned based on the state that the equipment is being found at the N^{th} decision epoch. This is called the boundary value, and the boundary value of state i is denoted by $\mathrm{r}_\mathrm{N}(i)$.

An MDP can be symbolically represented using a state transition diagram. A simple state transition diagram is given in Figure 5.2 to provide a better understanding about some of the aforementioned basic components. This model in Figure 5.2 has two states namely S_1 and S_2, and three actions namely a_1, a_2 and a_3. The state S_1 is connected with actions a_1 and a_3, while the state S_2 is connected with actions a_2 and a_3. Rewards and transition probabilities associated with actions and states at any decision epoch t <N are also given in Figure 5.2.

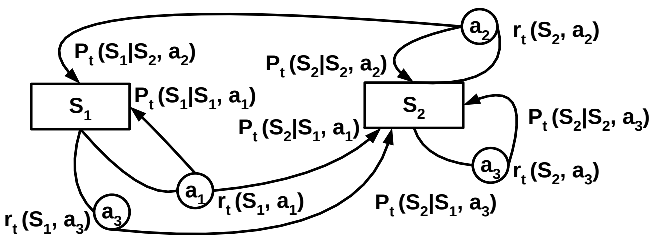

Figure 5.2: The simple state transition diagram of a Markov decision process model

5.2.3 Inspection and Maintenance Decision Making in Actual Practice

In practice, the condition of equipment is assessed through online or offline inspections. These inspections are usually performed at scheduled intervals as specified in the standards or by the manufacturer. Based on the results of inspections, the condition of equipment can be interpreted using one of the finite number of deterioration stages i.e. C_1, C_2, …, C_j, where C_1

and C_j represent the best and the worst conditions, respectively. In order to improve the present condition C_i of the equipment, maintenance is performed. If no maintenance is performed, the condition gradually deteriorates from C_i to C_j. However, maintenance may also degrade the present condition C_i or may not change C_i. When the equipment is at any deterioration stage, there is a certain probability of failure, which usually increases with the deterioration.

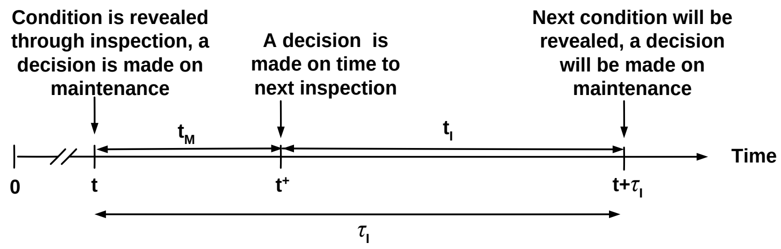

Figure 5.3: Decision making process regarding inspection and maintenance

Two consecutive decisions are made repeatedly throughout the equipment's operational life; one on maintenance and the other on time to next inspection. As shown in Figure 5.3, when the condition is revealed through inspection at time t, the first decision is made regarding the required maintenance action. Followed by this decision, the second decision is made at time t^+ regarding the time to perform next inspection i.e. t_I. In Figure 5.3, t_M denotes the time taken to perform maintenance, and τ_I is the time interval between two consecutive inspections. Since $t_M << t_I$, whether maintenance is performed or not, the time gap between t and t^+ is considered small and thus, $\tau_I \approx t_I$.

Generally, asset owners tend to decrease the time to next inspection with the deterioration of the equipment's condition. Therefore, the value of t_I depends on the equipment's condition C_i. We denote this time interval corresponding to the last known condition C_i by $t_{I,i}$. In order to achieve more cost effective inspection and maintenance policies, it is possible to vary $t_{I,i}$ within a range $t_{min,i} \leq t_{I,i} \leq t_{max,i}$, depending on other considerations such as the equipment's age and time delays in making decisions regarding inspection and maintenance. $t_{min,i}$ and $t_{max,i}$ refer to the minimum and the maximum allowable time between two consecutive inspections in stage C_i. These parameters can be determined based on inspection histories and experts' opinion.

5.2.4 Modeling the Process of Decision Making

In the MDP framework, it is required that the decisions are made at constant intervals. When modeling the decision making process of inspection and maintenance, a common time slot (τ) is first determined to perform decision making on inspection and maintenance. It would be more accurate to choose a small duration for τ and let the model make decisions regarding inspection and maintenance in each τ. However, in practice, when the equipment is in good condition, inspections are performed at a lower frequency. A large data set is required to accurately calculate deterioration probabilities for each interval τ, especially when the equipment's deterioration condition is good. As shown in (5.1), τ is set to the greatest common divisor of all $t_{min,i}$ values suggested by industry experts.

$$\tau = \gcd(t_{min,1}, t_{min,2}, \cdots, t_{min,j}) \tag{5.1}$$

Within the interval τ, two decisions are made. The first decision is regarding maintenance activities, whereas the second decision is regarding the next inspection. Decision making regarding inspection and maintenance within each interval τ is modeled similar to the actual situation shown in Figure 5.3. That is, if a decision is made regarding maintenance at time t and if this decision is implemented within a time interval t_M, the decision regarding the next inspection is made at time t^+ (or $t+t_M$). Then, the time to next decision making on maintenance is τ-t_M. It is worth to note that MDP theory does not require t_M (the time from the maintenance decision to the inspection decision) and τ-t_M (the time from the inspection decision to the maintenance decision) to be equal. However, it is required to keep these time intervals consistent for different inspection and maintenance trajectories. In the proposed MDP model, t_M is the same for each inspection and maintenance trajectory. Since τ is kept constant for each inspection and maintenance trajectory, τ-t_M is also the same for each trajectory. Moreover, in each and every possible trajectory, the decision maker makes the maintenance decisions on odd decision stages only and the decisions of next inspection on even decision stages only. Thus, the time intervals t_M and (τ-t_M) are kept consistent for different inspection and maintenance trajectories.

In the MDP framework, the second decision is not regarding time to next inspection, but regarding whether to perform the next inspection at the end of the current interval τ or to wait till the next interval τ to make a decision on the next inspection. However, time to next inspection ($t_{\mathrm{I},i}$) is related to the number of the next consecutive intervals of τ in which the inspection is postponed ($n_{\mathrm{I},i}$). For example, assume that inspection was performed in the previous interval τ. If it is decided to perform inspection again at the end of the current interval τ (i.e. if inspection is not postponed to the next interval τ), $n_{\mathrm{I},i}$ is zero and $t_{\mathrm{I},i}$ is τ. If the next inspection is postponed by $n_{\mathrm{I},i}$ consecutive times, $t_{\mathrm{I},i}$ is given using (5.2).

$$t_{\mathrm{I},i} = \tau \times (n_{\mathrm{I},i} + 1) \tag{5.2}$$

Although the decision making interval regarding inspection and maintenance is considered to be τ, inspection and maintenance can be practically performed only once in each $\mathrm{t}_{\mathrm{min},i}$. Thus, the MDP model should allow choosing inspection and maintenance actions only once in each $\mathrm{t}_{\mathrm{min},i}$. When $\mathrm{t}_{\mathrm{min},i} > \tau$, this property is incorporated into the model by eliminating inspection and maintenance actions connected with some states of the equipment. This will be further explained in section 5.3.2, using the state transition diagram of the proposed MDP model.

5.3 Problem Formulation

This section describes the concept of the proposed MDP model for inspection and maintenance decision making of ageing equipment. In subsections 5.3.1, 5.3.2 and 5.3.3, components of the proposed MDP model are described using the state diagram of the proposed model in Figure 5.4. This description is based on a simplified model developed for equipment having two deterioration stages, C_1 and C_2 with $t_{min,1}$, $t_{max,1}$ $t_{min,2}$, and $t_{max,2}$ of 3τ, 6τ, τ, and 2τ, respectively. However, the proposed model can be applied to equipment with any number of deterioration stages. Subsection 5.3.4 discusses how the proposed MDP model combines the effect of ageing with the deterioration process of the equipment's measurable condition.

5.3.1 Decision epochs

These are the points of time at which decisions are made regarding equipment's inspection or maintenance. The number of decision epochs (N) of the proposed MDP is given by (5.3), where T is the decision horizon. Since inspection and maintenance decisions are made throughout the equipment's total operational period, the expected operational life of the equipment is set for T. T/τ in this equation gives the number of inspection and maintenance decision making intervals in the decision horizon T. It should be noted that two decision epochs occur within each inspection and maintenance decision making interval τ. If the decision epoch is odd, decisions are made only regarding inspection. Otherwise, decisions are made only regarding maintenance.

$$N=\left(\frac{T}{\tau}\right)\times 2+1 \tag{5.3}$$

5.3.2 States and Actions

Different statuses of the equipment at time points of inspection and maintenance decision making are modeled using different states. In order to decouple decision making on inspection and decision making on maintenance, two types of equipment states are defined, namely main states and intermediate states. This decoupling is essential, to model the practical scenario, where a decision is made on inspection prior to decision making on maintenance and the decision regarding maintenance is made depending on the outcomes of the inspection. At main states, decisions are made only regarding maintenance and at intermediate states, decisions are made only regarding the next inspection. (For example, with respect to Figure 5.3, the possible states of the equipment at t and $t+\tau_I$ are called main states and the possible states at t^+ are called intermediate states.) These main and intermediate states of the proposed model are shown in Figure 5.4 using solid and dashed rectangles, respectively. The set of actions includes doing nothing (a_0), inspection (a_1), minor maintenance (a_2), major maintenance (a_3), replacement (a_4) and repair (a_5).

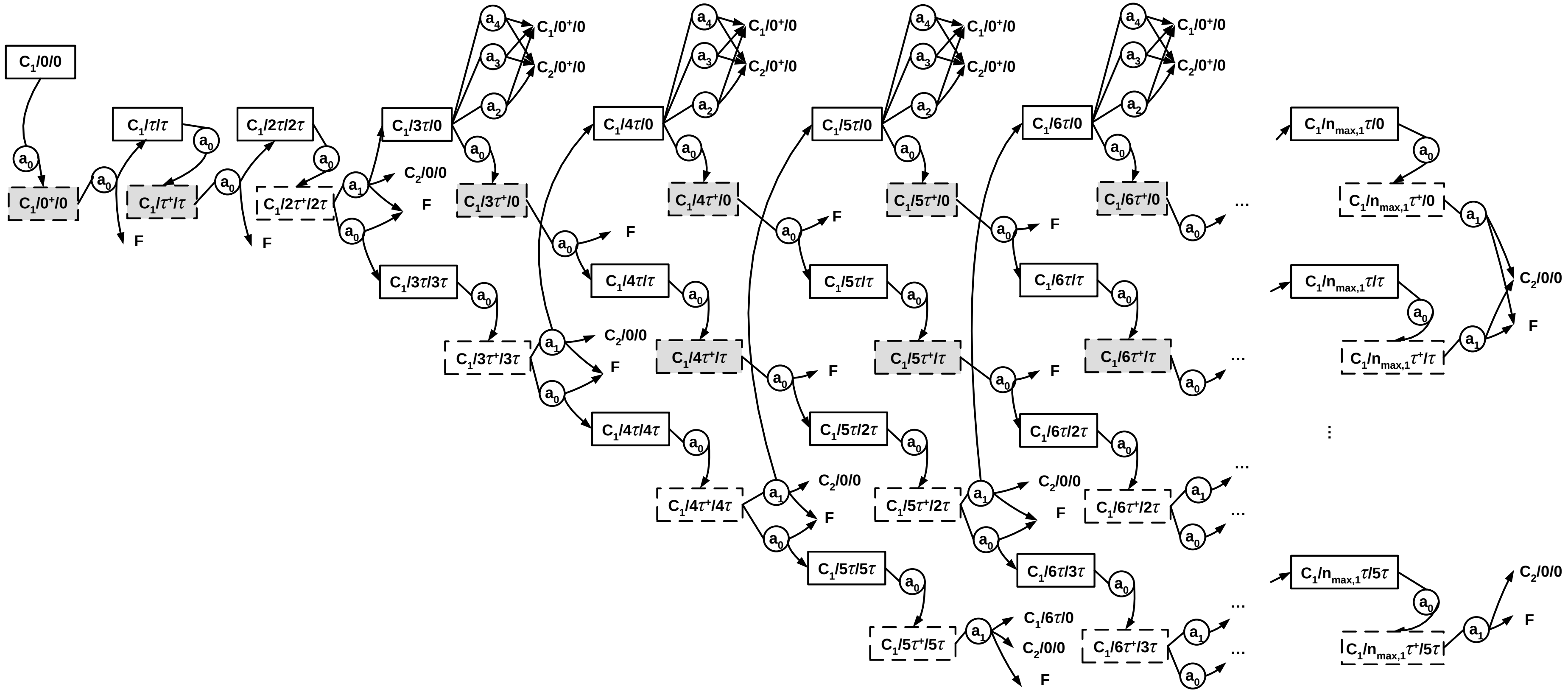

Figure 5.4 (a): The state transition diagram of the proposed Markov decision process model for maintenance decision making

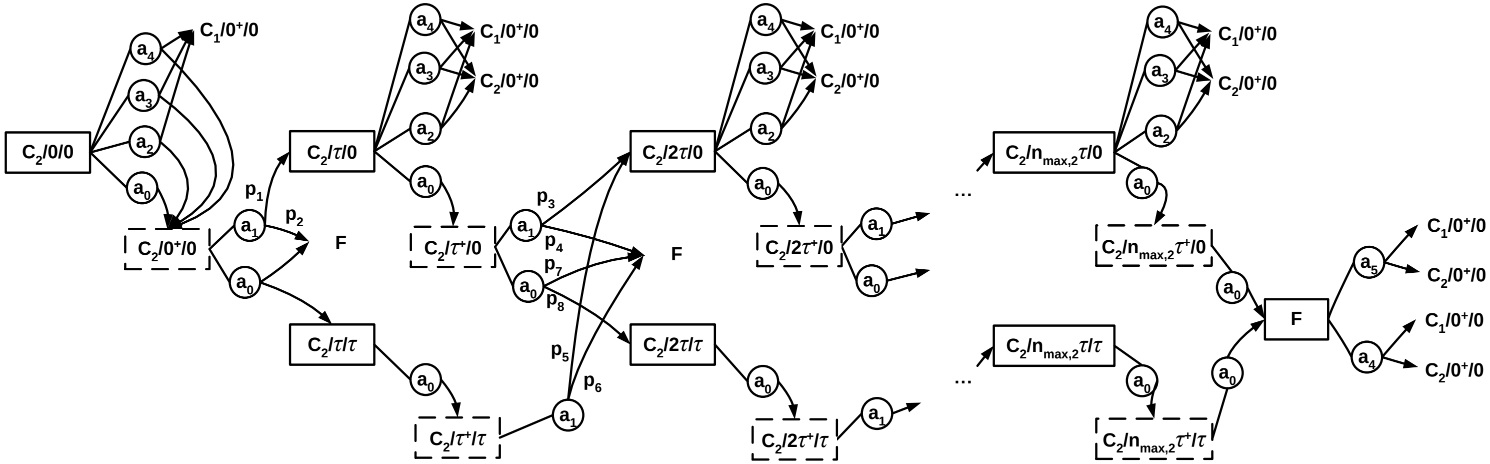

Figure 5.4 (b): The state transition diagram of the proposed Markov decision process model for maintenance decision making

In the MDP model, we describe a deterioration state by $C_i/t_{M,i}/t_{I,i}$. C_i denotes the deterioration stage of the equipment where i = 1, 2. $t_{M,i}$ is the time spent in C_i, which can also be called as the maintenance delay time in stage C_i. $t_{I,i}$ is the time from the most recent inspection i.e. the inspection delay time in stage C_i. According to this state convention, the status of newly installed equipment is represented by the state $C_1/0/0$. The failure state is denoted by F. This failure state F only stands for the deterioration failures of the equipment. Since random failures cannot be avoided by performing inspection and maintenance activities, such failures are not considered in the model.

States are connected with associated actions, as shown in the state transition diagram in Figure 5.4. This diagram also shows how possible state transitions occur upon choosing each action at each state. Since the decisions made at intermediate states are about postponing the next inspection, intermediate states are connected only with actions a_0 and a_1. If the equipment does not fail (i.e. if the next state is not the state F), the two actions a_0 and a_1 lead to different main states. If action a_0 is chosen at an intermediate state, these next possible main states are connected only with action a_0. If action a_1 is chosen at an intermediate state, next possible main states are connected with actions a_0, a_2, a_3 and a_4. However, the following exceptions can be noted in Figure 5.4.

- When equipment is newly installed, it is not required to perform maintenance, replacement or repair. Therefore, the main state $C_1/0/0$ is only connected with action a_0.
- Once the equipment fails, it must be replaced or repaired and hence the main state F is connected with a_4 and a_5.
- The minimum possible time between two consecutive inspections in stage C_1, $t_{min,1}$ is 3τ. Due to this reason, when the condition is C_1, the model permits to choose inspection only if the inspection delay time $t_{I,1} \geq 2\tau$. Otherwise, inspection is not allowed and therefore, action a_1 is not connected to the grey colored intermediate states in Figure 5.4.
- The maximum time that the inspection can be delayed at C_1 and C_2 (i.e. $t_{max,1}$ and $t_{max,2}$) are 6τ and 2τ, respectively. Therefore, decisions must be made to perform inspection,

when $t_{I,1}$ is 5τ or $t_{I,2}$ is τ, at an intermediate state. As Figure 5.4 shows, such intermediate states are connected only with a_1.

- There is a maximum time period that the equipment spends in each condition, before it deteriorates further or fails. This maximum time period spent in condition C_i (t_i) can be determined from inspection and failure history. With the use of t_i, the maximum number of decision intervals that the equipment spends in C_i ($n_{max,i}$) can be determined as shown in (5.4).

$$n_{max,i} = \frac{t_i}{\tau} - 1 \qquad (5.4)$$

The MDP model assumes that inspections must be performed, when the time spent in stage C_i is $n_{max,i}\tau$. Thus, if $t_{M,i}$ of an intermediate state is $n_{max,i}\tau$, decisions are made to perform inspection. As can be seen in Figure 5.4, such intermediate states are connected only with a_1. However, if C_i is the last deterioration stage, at the end of the interval $n_{max,i}\tau$, the equipment fails whether inspection is performed or not. Therefore, as shown in Figure 5.4, corresponding intermediate states are connected only with action a_0.

5.3.3 Transition Probabilities and Rewards

Some notations given in Figure 5.4 (i.e. transition probabilities p_1-p_8) are used to illustrate the calculation procedure of transition probabilities for the proposed MDP model. Transition probabilities corresponding to action a_1 are the deterioration/failure probabilities of the equipment. Deterioration/failure probabilities associated with the zero inspection delay time (e.g. p_1-p_4 in Figure 5.4) can be directly calculated using inspection and failure history. For example, let us denote the number of transformers found to be in condition C_2 for a period of τ by n_1, the number of transformers found to be in condition C_2 for a period of 2τ by n_2. From this data, $p_3=n_2/n_1$ and $p_4=(n_1- n_2)/n_1$.

When the inspection delay time is greater than zero, deterioration/failure probabilities can be calculated with the use of deterioration/failure probabilities corresponding to zero inspection delay time. For example, consider the calculation of p_5 and p_6, which are the probabilities of being found in C_2 for a period of 2τ and being found in the failure state F, if the inspection is delayed by an interval τ. Let the events,

$$\mathrm{A} = \{\text{Equipment being found in } \mathrm{C}_2 \text{ for a period } \tau\}$$

$$\mathrm{B} = \{\text{Equipment being in } \mathrm{C}_2 \text{ for a period } 2\tau\}$$

$$\mathrm{B|A} = \left\{\begin{matrix}\text{Equipment being found in } \mathrm{C}_2 \text{ for a period } 2\tau | \\ \text{Equipment being found in } \mathrm{C}_2 \text{ for a period } \tau\end{matrix}\right\}$$

Using the conditional probability rule,

$$\mathrm{P(B|A)=P(A \cap B)/P(A)=P(B)/P(A}$$

$$\therefore \mathrm{P(B)=P(A) \times P(B|A)}$$

By substituting values,

$$p_5=p_1 \times p_3 \tag{5.5}$$

Similarly, p_6 can also be computed as follows.

$$p_6=p_1 \times p_4+p_2 \tag{5.6}$$

Since failures can occur whether inspection is performed or not, failure probability is the same for both actions a_0 and a_1. When the condition is C_i, if the equipment does not fail upon choosing action a_0, the model assumes that the condition would remain same as C_i. Thus, probabilities corresponding to a_0 can be simply calculated using the failure probabilities calculated for a_1. For example, $p_7 = p_4$ and p_8=1-p_7.

Transition probabilities corresponding to maintenance and repair actions (i.e. a_2, a_3 and a_5) can be calculated using maintenance/repair records and the above method of calculation which is used to find deterioration probabilities. When equipment is replaced (i.e. action a_4 is chosen), the probability of transiting to state $C_1/0^+/0$ is considered to be 1.

For each action a reward is allocated equal to the negative of the cost of performing that particular action. Boundary value of each state is set to zero, assuming that the value of the equipment at the end of the expected operational life is zero.

5.3.4 Incorporating the Effects of Aging

Deterioration of equipment's condition generally gets accelerated with the ageing. In addition, when equipment is aged, it would be less possible to improve the condition through maintenance. These effects of ageing on deterioration of equipment's measurable condition should be reflected in inspection and maintenance history. Thus, deterioration probabilities and the probabilities of improving the condition after maintenance/repair would be different for different age levels of the equipment. Since the transition probabilities (i.e. $P_t(k|i,\ a)$) of an MDP model can vary with time t, the proposed MDP model can easily incorporate the effects of ageing.

Incorporating the effects of ageing can be done as follows, given that data is available over the equipment's expected life. First, the expected life is divided into an appropriate number of age levels (j), as shown in Figure 5.5. Then, inspection and maintenance data collected over the expected life of the equipment is categorized into different groups corresponding to these age levels. Next, the classified data is used to compute deterioration probabilities and maintenance/repair outcome probabilities for each age level. These probabilities calculated for a particular age level are set for transition probabilities of the decision epochs which belong to that particular age level. For example, with reference to Figure 5.5, the probabilities calculated using the data corresponding to the 2nd age level are set for all $P_t(k|i,\ a)$, where $t \in \{x, x+1, x+2, \dots, y\}$.

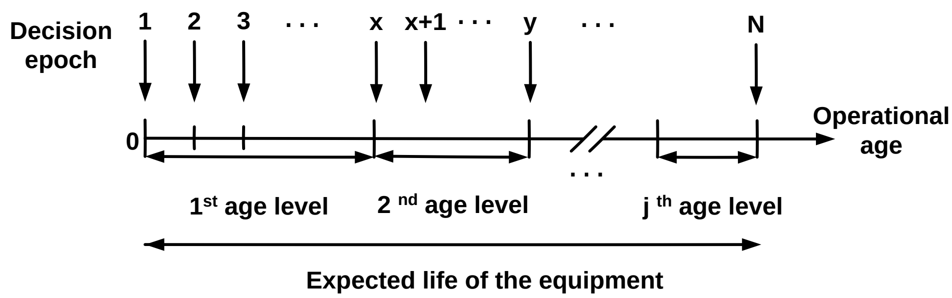

Figure 5.5: Decision epochs at different age levels of the equipment

5.4 Solution Procedure

The combinations of states and actions of the proposed MDP model result in a large number of possible maintenance policies. In order to solve this model efficiently, backward induction (i.e. dynamic programming) is used [75]. This technique can provide the optimal policies without analyzing every possible policy. In backward induction, the final stage of decision making at t=N is first attended and the decisions on optimal actions are made by moving one step backward at each decision epoch in the desired time horizon. Intuitively, backward induction works because an action of an intermediate state s is optimal only if it is optimal for a reduced MDP starting from s [75].

When solving an MDP, the objective is to decide the optimal set of actions which maximizes the total expected reward. In backward induction method, when t=N, for any state i, the maximum total expected reward $\mathrm{U}_{\mathrm{N}}^{*}(i)$ is set to the boundary value of state i. Then, when t<N, the maximum total expected reward for state i at time t, (i.e. $\mathrm{U}_t^*(i)$) is found as follows.

- Using (5.7), $\mathrm{U}_t(i,a)$ i.e. the total expected reward received upon choosing action a in state i at time t is calculated. In (5.7), $\mathrm{r}_t(i,a)$ is the immediate reward received upon choosing action a, which is basically the reward assigned for action a in state i at time t. The term $\sum_{k=1}^{\mathrm{n}} \mathrm{P}_t(k|i,\ a) \times \mathrm{U}_{t+1}^{*}(k)$ is the expected terminal reward, where n is the total number of states, $\mathrm{P}_t(k|i,\ a)$ is the probability of transiting to state k, if action a is chosen in state i at the epoch t and $\mathrm{U}_{t+1}^{*}(k)$ is the maximum total expected reward in state k, at the epoch t+1.

$$\mathrm{U}_t(i,a)=\mathrm{r}_t(i,a)+\sum_{k=1}^{\mathrm{n}}\left\{\mathrm{P}_t(k|i,\ a)\times\mathrm{U}_{t+1}^{*}(k)\right\} \tag{5.7}$$

- Once the total expected reward is found for every possible action in state i, the maximum total expected reward in state i, at the t^{th} epoch is found using the criterion in (5.8).

$$\mathrm{U}_t^*(i) = \max\{\mathrm{U}_t(i,a)\} \tag{5.8}$$

The optimal action in state i at the decision epoch t can be obtained using (5.9).

$$a_t^*(i) = \arg\max\{\mathrm{U}_t(i,a)\} \tag{5.9}$$

Likewise, at each decision epoch t, an optimal action which maximizes the expected total reward can be found for all relevant states. As mentioned before in section 5.3.1, at odd decision epochs optimal actions are found for all intermediate states where decisions are made regarding inspection. At even epochs, optimal actions are found for all main states, where decisions are made regarding maintenance. The set of optimal actions of all relevant states at the decision epoch t is called the solution of MDP at time t.

To find optimal maintenance policies by solving our proposed MDP model, the following backward induction algorithm is implemented in MATLAB 10.

Step 1: Set t=N and $\mathrm{U}_{\mathrm{N}}^{*}(i) = \mathrm{r}_{\mathrm{N}}(i)$ for all $i \in (1,n)$

Step 2: Set $t = t-1$

Step 3: Set i =1

Step 4: Continue only if, t is odd and i is an intermediate state, or t is even and i is a main state. Otherwise, it is not required to find $a_t^*(i)$ and thus, skip steps 5 and 6 and set $\mathrm{U}_t^*(i)=\mathrm{U}_{t+1}^*(i)$.

Step 5: Compute $\mathrm{U}_t(i,a)$ for each action a available in state i using (5.7).

Step 6: Find the optimal action for state i, using (5.8) and (5.9).

Step 7: If i=n, stop. Otherwise, set $i=i+1$ and go to step 4.

Step 8: If t=1, stop. Otherwise repeat from step 2.

5.5 Case Study

This section presents a case study in which the proposed MDP model is applied to CBM of oil insulated distribution transformers.

5.5.1 Condition Based Maintenance of Oil Insulated Transformers

Utilities basically assess the condition of oil insulated transformers through dissolved gas analysis (DGA) [82, 83]. In DGA, insulation oil is sampled at scheduled intervals, while the transformer is in operation, and the amounts of dissolved gases are measured and analyzed. Then, the condition is determined using the total amount of dissolved combustible gases (TDCG) according to the criterion specified in the IEEE standards [56]. Next, based on the revealed condition, maintenance decisions are made considering recommendations in the standards [56].

5.5.2 The Markov Decision Process Model of Transformers

The data required for the MDP model of transformers include DGA results, maintenance, repair and replacement records of transformers and costs of performing CM, maintenance, repair and replacement actions. DGA results are only available over past 7 years, as DGA is recently introduced for distribution transformers in the local utility. However, in order to demonstrate the model applicability, the case study is conducted with these DGA results and maintenance records which belong to the transformers' age range of 20 to 30 years. CM, maintenance and replacement costs are assumed based on [7]. Using this data and considering current CM and maintenance practices and experts' opinion, model parameters are determined.

Table 5.1 (a): Deterioration/Failure Probabilities

C_i	$t_{M,i}$ / (years)	Probability of transition to condition C_1, C_2, C_3 or F											
		0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
		C_1	C_2	C_3	F	C_1	C_2	C_3	F	C_1	C_2	C_3	F
C_1	0	1	0	0	0	1	0	0	0	1	0	0	0
	0.33	1	0	0	0	1	0	0	0	1	0	0	0
	0.67	1	0	0	0	1	0	0	0	1	0	0	0
	1	1	0	0	0	1	0	0	0	0.94	0.06	0	0
	1.33	1	0	0	0	1	0	0	0	0.94	0.06	0	0
	1.67	1	0	0	0	1	0	0	0	0.94	0.06	0	0
	2	1	0	0	0	0.94	0.06	0	0	0.90	0.10	0	0
	2.33	1	0	0	0	0.94	0.06	0	0	0.89	0.11	0	0
	2.67	1	0	0	0	0.94	0.06	0	0	0.88	0.12	0	0
	3	1	0	0	0	0.90	0.10	0	0	0.67	0.33	0	0
	3.33	1	0	0	0	0.89	0.11	0	0	0.50	0.50	0	0
	3.67	1	0	0	0	0.88	0.12	0	0	0	1	0	0
	4	0.94	0.06	0	0	0.67	0.33	0	0	-	-	-	-
	4.33	0.94	0.06	0	0	0.50	0.50	0	0	-	-	-	-
	4.67	0.94	0.06	0	0	0	1	0	0	-	-	-	-
	5	0.90	0.10	0	0	-	-	-	-	-	-	-	-
	5.33	0.89	0.11	0	0	-	-	-	-	-	-	-	-

Table 5.1 (b): Deterioration/Failure Probabilities

C_i	$t_{M,i}$ / (years)	Probability of transition to condition C_1, C_2, C_3 or F											
		0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
		C_1	C_2	C_3	F	C_1	C_2	C_3	F	C_1	C_2	C_3	F
C_1	5.67	0.88	0.12	0	0	-	-	-	-	-	-	-	-
	6	0.67	0.33	0	0	-	-	-	-	-	-	-	-
	6.33	0.50	0.50	0	0	-	-	-	-	-	-	-	-
	6.67	0	1	0	0	-	-	-	-	-	-	-	-
C_2	0	0	1	0	0	0	1	0	0	0	1	0	0
	0.33	0	1	0	0	0	1	0	0	0	0.89	0.11	0
	0.67	0	1	0	0	0	1	0	0	0	0.75	0.25	0
	1	0	1	0	0	0	0.89	0.11	0	0	0.67	0.33	0
	1.33	0	1	0	0	0	0.75	0.25	0	0	0	1	0
	1.67	0	1	0	0	0	0.67	0.33	0	-	-	-	-
	2	0	0.89	0.11	0	0	0	1	0	-	-	-	-
	2.33	0	0.75	0.25	0	-	-	-	-	-	-	-	-
	2.67	0	0.67	0.33	0	-	-	-	-	-	-	-	-
	3	0	0	1	0	-	-	-	-	-	-	-	-
C_3	0	0	0	1	0	0	0	1	0	0	0	1	0
	0.33	0	0	1	0	0	0	1	0	0	0	0.8	0.2
	0.67	0	0	1	0	0	0	0.8	0.2	0	0	0.6	0.4
	1	0	0	0.8	0.2	0	0	0.6	0.4	0	0	0	1
	1.33	0	0	0.6	0.4	0	0	0	1	-	-	-	-
	1.67	0	0	0	1	-	-	-	-	-	-	-	-

Table 5.2: Transition Probabilities upon Choosing Maintenance Actions at C_3

Action	$t_{M,i}$ / (years)	Probability of transition from C_3 to other conditions											
		0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
		C_1	C_2	C_3	F	C_1	C_2	C_3	F	C_1	C_2	C_3	F
a_2	0	0	1	0	0	0	1	0	0	0	0.7	0.3	0
	0.33	0	1	0	0	0	0.7	0.3	0	0	0.4	0.6	0
	0.67	0	0.7	0.3	0	0	0.4	0.6	0	0	0.2	0.8	0
	1	0	0.4	0.6	0	0	0.2	0.8	0	0	0	0.5	0.5
	1.33	0	0.2	0.8	0	0	0	0.5	0.5	-	-	-	-
	1.67	0	0	0.5	0.5	-	-	-	-	-	-	-	-
a_3	0	1	0	0	0	1	0	0	0	0.9	0.1	0	0
	0.33	1	0	0	0	0.9	0.1	0	0	0.8	0.2	0	0
	0.67	0.9	0.1	0	0	0.8	0.2	0	0	0.6	0.4	0	0
	1	0.8	0.2	0	0	0.6	0.4	0	0	0.5	0.5	0	0
	1.33	0.6	0.4	0	0	0.5	0.5	0	0	-	-	-	-
	1.67	0.5	0.5	0	0	-	-	-	-	-	-	-	-

Based on TDCG, IEEE standards specify four deterioration conditions of transformers [56]. This model considers them as three conditions i.e. by separately considering the first two conditions and by combining the third and fourth conditions. According to the degree of deterioration, we denote the three conditions by C_1, C_2 and C_3. The minimum and maximum CM intervals, $t_{min,1}$, $t_{max,1}$, $t_{min,2}$, $t_{max,2}$, $t_{min,3}$, and $t_{max,3}$ in years are 1, 3, 0.33, 1.33, 0.33 and 1, respectively. Using (5.1), inspection and maintenance decision making interval is chosen as 0.33 years. Data shows that the maximum time spent in C_1, C_2 and C_3 are 5, 2.33 and 1.67 years, and therefore, $n_{max,1}$, $n_{max,2}$ and $n_{max,3}$ are 14, 6 and 4, respectively. According to these model parameters, the state diagram was developed and given in Appendix A. This state diagram consists of 369 states. The set of actions that are performed on local transformers includes doing nothing (a_0), CM (a_1), minor maintenance (a_2), major maintenance (a_3) and replacement (a_4), respectively.

Assuming that the total expected life of a transformer is 40 years, T is set to 40 years. Then, from (5.3), N = 241. In this case study, three sets of transition probabilities are utilized for three age levels, i.e. 0 to 20 years, 20 to 30 years and 30 to 40 years. Deterioration/failure probabilities for these three age levels are given in Table 5.1. For the age range of 20 to 30 years, these deterioration probabilities are computed using the available DGA results. As there are no occurrences of failures, failure probabilities are interpolated. Probabilities corresponding to the age range of 20 to 30 years are appropriately amended to obtain other deterioration/failure probabilities in Table 5.1 which are corresponding to the other two age ranges. When the inspection delay time is greater than zero, the probability of deterioration is calculated according to the procedure mentioned in section 5.3.3. These probabilities are given as Appendix B. Using these deterioration probabilities in Appendix B, transition probabilities corresponding to actions a_1 and a_0 are calculated. Maintenance history shows that, actions a_2, a_3 and a_4 are not performed, when the condition is C_1. By performing a_2 or a_3, the condition is improved from C_2 to C_1. Upon choosing a_2 or a_3 in C_3, transitions occur according to the probabilities given in Table 5.2. If action a_4 is chosen at any state, the condition is improved to C_1. Rewards assigned for actions a_0, a_1, a_2, a_3 and a_4 are 0, -200, -1200, -14400 and -144000, respectively [7]. Boundary values are set to zero.

The MDP model with these parameters is solved using the backward induction algorithm to find optimal actions.

5.5.3 Results and Discussion

Although it is possible to select optimal actions directly from the solution of the MDP, for easy reference, the solution is converted into look up tables which are given in Tables 5.3 to 5.6.

Based on the current condition, the time spent in this condition, the CM delay time, and the operational age, the optimal decision regarding CM can be chosen from Tables 5.3 to 5.5. According to the model, these decisions are to be implemented at the end of the next 4 months. In Tables 5.3 to 5.5, CM actions which are pre-specified during the modeling of the state diagram are mentioned in bold. Apart from these pre-specified CM actions, the results in Tables 5.3 to 5.5 suggest performing some additional CM. The overall implications of the results in Tables 5.3 to 5.5 are explained below.

1) With the ageing of a transformer, the probability of deterioration and failure increases. Therefore, it is not cost effective to delay CM too much, if the equipment is old.
2) Similarly, when the equipment is more deteriorated, the equipment is at a higher risk of failure. Thus, it would be cost effective to perform CM with a less delay.
3) With the increase in time that the equipment spends in a condition, the probability of deterioration increases. If CM is delayed too much, the transformer may further deteriorate unknown to the maintenance staff and require more maintenance to improve the condition or it may fail unexpectedly. Thus, it would be more cost effective to perform CM without any delay, if the time spent in a condition (or maintenance delay time) is high.

Table 5.3 (a): Optimal Actions to Perform Condition Monitoring at C_1

$t_{M,i}$ / (years)	$t_{C,i}$ / (years)	Optimal action		
		0≤ age <20 years	20≤ age <30 years	30≤ age <40 years
0	0	a_0	a_0	a_0
0.33	0.33	a_0	a_0	a_0
0.67	0.67	a_0	a_0	a_0
1	0, 1	a_0	a_0	a_0
1.33	0, 0.33, 1.33	a_0	a_0	a_0
1.67	0, 0.33, 0.67, 1.67	a_0	a_0	a_0
2	0, 0.33, 0.67, 1, 2	a_0	a_0	a_0
2.33	0, 0.33, 0.67, 1	a_0	a_0	a_0
	1.33, 2.33	a_0	a_0	a_1
2.67	0, 0.33, 0.67, 1	a_0	a_0	a_0
	1.33, 1.67	a_0	a_0	a_1
	2.67	a_1	a_1	a_1
3	0, 0.33, 0.67, 1	a_0	a_0	a_0
	1.33, 1.67, 2	a_0	a_0	a_1
3.33	0, 0.33, 0.67, 1	a_0	a_0	a_0
	1.33, 1.67, 2, 2.33	a_0	a_0	a_1

Table 5.3 (b): Optimal Actions to Perform Condition Monitoring at C_1

$t_{M,i}$ / (years)	$t_{C,i}$ / (years)	Optimal action		
		0≤ age <20 years	20≤ age <30 years	30≤ age <40 years
3.67	0, 0.33, 0.67, 1, 1.33, 1.67, 2, 2.33	a_0	a_0	a_1
	2.67	a_1	a_1	a_1
4	0, 0.33, 0.67, 1, 1.33, 1.67, 2, 2.33	a_0	a_0	-
	2.67	a_1	a_1	-
4.33	0, 0.33, 0.67, 1, 1.33, 1.67, 2	a_0	a_0	-
	2.33	a_0	a_1	-
	2.67	a_1	a_1	-
4.67	0, 0.33, 0.67, 1, 1.33, 1.67, 2, 2.33	a_0	a_1	-
	2.67	a_1	a_1	-
5, 5.33, 5.67, 6, 6.33	0, 0.33, 0.67, 1, 1.33, 1.67, 2, 2.33	a_0	-	-
	2.67	a_1	-	-
6.67	0, 0.33, 0.67, 1, 1.33, 1.67, 2, 2.33, 2.67	a_1	-	-

Based on the condition, the maintenance delay time, and the operational age of a transformer, the optimal decision regarding maintenance can be chosen from Table 5.6. These maintenance decisions are for immediate implementation. Implications of the results in Table 5.6 are given below.

1) With the ageing of the equipment, the failure probability would increase and therefore, the time that the maintenance can be delayed decreases.
2) It is not cost effective to delay maintenance, when the equipment is more deteriorated and at a higher risk.
3) Cost effective maintenance actions would change with the maintenance delay time. For example, as the time spent in C_3 increases, the probability of improving the condition by

performing minor maintenance decreases and thus, it will be more cost effective to perform major maintenance.

Table 5.4 (a): Optimal Actions to Perform Condition Monitoring at C_2

$t_{M,i}$ / (years)	$t_{C,i}$ / (years)	Optimal action		
		0≤ age <20	20≤ age <30	30≤ age <40
0	0	a_0	a_0	a_0
0.33	0	a_0	a_0	a_0
	0.33	a_0	a_0	a_0
0.67	0	a_0	a_0	a_0
	0.33	a_0	a_0	a_1
	0.67	a_0	a_0	a_1
1	0	a_0	a_0	a_1
	0.33	a_0	a_0	a_1
	0.67	a_0	a_0	a_1
	1	a_1	a_1	a_1
1.33	0	a_0	a_0	a_1
	0.33	a_0	a_0	a_1
	0.67	a_0	a_0	a_1
	1	a_1	a_1	a_1
1.67	0	a_0	a_0	-
	0.33	a_0	a_0	-
	0.67	a_0	a_1	-
	1	a_1	a_1	-
2	0	a_0	a_1	-
	0.33	a_0	a_1	-
	0.67	a_0	a_1	-
	1	a_1	a_1	-
2.33	0	a_0	-	-
	0.33	a_0	-	-
	0.67	a_0	-	-
	1	a_1	-	-

Table 5.4 (b): Optimal Actions to Perform Condition Monitoring at C_2

$t_{M,i}$ / (years)	$t_{C,i}$ / (years)	Optimal action		
		0≤ age <20	20≤ age <30	30≤ age <40
2.67	0	a_0	-	-
	0.33	a_0	-	-
	0.67	a_0	-	-
	1	a_1	-	-
3	0	a_1	-	-
	0.33	a_1	-	-
	0.67	a_1	-	-
	1	a_1	-	-

Table 5.5: Optimal Actions to Perform Condition Monitoring at C_3

$t_{M,i}$ / (years)	$t_{C,i}$ / (years)	Optimal action		
		0≤ age <20	20≤ age <30	30≤ age <40
0	0	a_0	a_0	a_1
0.33	0	a_0	a_1	a_1
	0.33	a_0	a_1	a_1
0.67	0	a_1	a_1	a_1
	0.33	a_1	a_1	a_1
	0.67	a_1	a_1	a_1
1	0	a_1	a_1	-
	0.33	a_1	a_1	-
	0.67	a_1	a_1	-
1.33	0	a_1	-	-
	0.33	a_1	-	-
	0.67	a_1	-	-

Table 5.6: Optimal Actions to Perform Maintenance

Condition	$t_{M,i}$ / (years)	Optimal action		
		0≤ age <20	20≤ age <30	30≤ age <40
C_2	0	a_0	a_0	a_0
	0.33	a_0	a_0	a_0
	0.67	a_0	a_0	a_2*
	1	a_0	a_0	a_2
	1.33	a_0	a_0	a_2
	1.67	a_0	a_2	-
	2	a_0	a_2	-
	2.33	a_0	-	-
	2.67	a_0	-	-
	3	a_0	-	-
C_3	0	a_0	a_2	a_2
	0.33	a_0	a_2*	a_2
	0.67	a_2*	a_2	a_2
	1	a_2	a_2	a_3
	1.33	a_2	a_3	-
	1.67	a_3	-	-

Since CM must be performed before maintenance, some of the suggested maintenance actions are not implementable. Such actions in Table 5.6 are denoted using an additional "*". In order to guarantee that CM is performed before each maintenance activity, the equipment operators should first refer Tables 5.3 to 5.5 for the optimal CM action, and only if Tables 5.3 to 5.5 suggest performing CM, they should refer Table 5.6 for the optimal maintenance action.

5.6 Using Markov Decision Process Models in System-level Maintenance Planning

Through a numerical example, this section investigates the possibility of utilizing the individual MDP models in system-level maintenance planning.

In this numerical example, a simple system with two equipment i.e. equipment A and B is considered. Three-state MDP models of equipment A and B are shown in Figures 5.6 and 5.7 respectively. As can be seen in these figures, equipment A and B have two actions, i.e. a_0

and a_1. Upon choosing an action, state transitions occur as shown in Figures 5.6 and 5.7. Transition probabilities and rewards are different for equipment A and equipment B. These transition probabilities and rewards are also mentioned in the figures. The system model consists of 3^2 system states and 2^2 action combinations. The system states are ($S_{A,1}$, $S_{B,1}$), ($S_{A,1}$, $S_{B,2}$), ($S_{A,1}$, $S_{B,3}$), ($S_{A,2}$, $S_{B,1}$), ($S_{A,2}$, $S_{B,2}$), ($S_{A,2}$, $S_{B,3}$), ($S_{A,3}$, $S_{B,1}$), ($S_{A,3}$, $S_{B,2}$) and ($S_{A,3}$, $S_{B,3}$). Action combinations include (a_0, a_0), (a_0, a_1), (a_1, a_0) and (a_1, a_1). Transition probabilities and rewards for the system model are calculated using the values of those parameters of the individual MDP models.

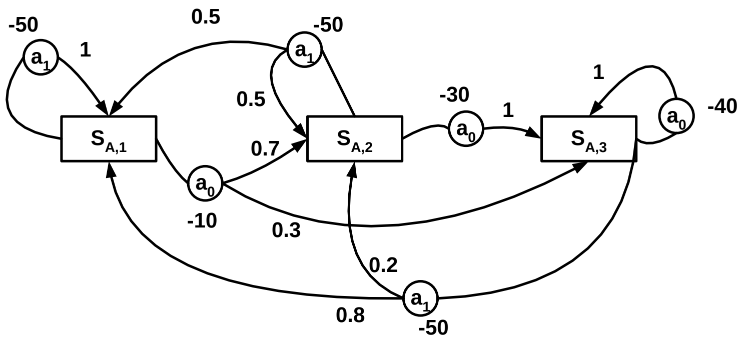

Figure 5.6: The Markov decision process model of equipment A

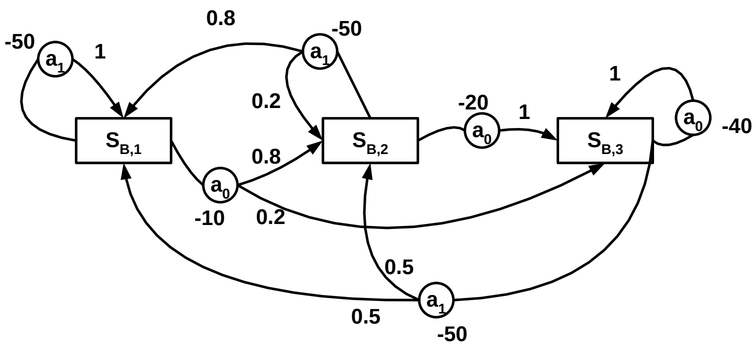

Figure 5.7: The Markov decision process model of equipment B

According to the solution procedure given in section 5.4, optimal actions are obtained by separately solving the MDP models of equipment A and B and by solving the system model. Decision horizon and boundary values are set to 30 years and zero, respectively. Three case studies are conducted, under the budget constraints in Table 5.7. Results of case studies are tabulated in Tables 5.8 to 5.10.

Table 5.7: Budget Constraints

Case study	Budget Constraints
1	No budget constraints
2	Budget ≤ 95
3	Budget ≤ 85

Table 5.8: Optimal Actions for Case Study 1

Equipment states		Optimal action		
Equipment A	Equipment B	System model	Model of equipment A	Model of equipment B
$S_{A,1}$	$S_{B,1}$	(a_0, a_0)	a_0	a_0
$S_{A,1}$	$S_{B,2}$	(a_0, a_0)	a_0	a_0
$S_{A,1}$	$S_{B,3}$	(a_0, a_1)	a_0	a_1
$S_{A,2}$	$S_{B,1}$	(a_1, a_0)	a_1	a_0
$S_{A,2}$	$S_{B,2}$	(a_1, a_0)	a_1	a_0
$S_{A,2}$	$S_{B,3}$	(a_1, a_1)	a_1	a_1
$S_{A,3}$	$S_{B,1}$	(a_1, a_0)	a_1	a_0
$S_{A,3}$	$S_{B,2}$	(a_1, a_0)	a_1	a_0
$S_{A,3}$	$S_{B,3}$	(a_1, a_1)	a_1	a_1

Table 5.9: Optimal Actions for Case Study 2

Equipment states		Optimal action		
Equipment A	Equipment B	System model	Model of equipment A	Model of equipment B
$S_{A,1}$	$S_{B,1}$	(a_0, a_0)	a_0	a_0
$S_{A,1}$	$S_{B,2}$	(a_0, a_0)	a_0	a_0
$S_{A,1}$	$S_{B,3}$	(a_0, a_1)	a_0	a_1
$S_{A,2}$	$S_{B,1}$	(a_1, a_0)	a_1	a_0
$S_{A,2}$	$S_{B,2}$	(a_1, a_0)	a_1	a_0
$S_{A,2}$	$S_{B,3}$	(a_0, a_1)	a_0	a_1
$S_{A,3}$	$S_{B,1}$	(a_1, a_0)	a_1	a_0
$S_{A,3}$	$S_{B,2}$	(a_1, a_0)	a_1	a_0
$S_{A,3}$	$S_{B,3}$	(a_0, a_1)	a_0	a_1

Table 5.10: Optimal Actions for Case Study 3

Equipment states		Optimal action		
Equipment A	Equipment B	System model	Model of Equipment A	Model of Equipment B
$S_{A,1}$	$S_{B,1}$	**(a_1, a_0)**	**a_0**	a_0
$S_{A,1}$	$S_{B,2}$	**(a_0, a_1)**	a_0	**a_0**
$S_{A,1}$	$S_{B,3}$	(a_0, a_1)	a_0	a_1
$S_{A,2}$	$S_{B,1}$	(a_1, a_0)	a_1	a_0
$S_{A,2}$	$S_{B,2}$	(a_1, a_0)	a_1	a_0
$S_{A,2}$	$S_{B,3}$	(a_0, a_1)	a_0	a_1
$S_{A,3}$	$S_{B,1}$	(a_1, a_0)	a_1	a_0
$S_{A,3}$	$S_{B,2}$	(a_1, a_0)	a_1	a_0
$S_{A,3}$	$S_{B,3}$	(a_0, a_0)	a_0	a_0

Tables 5.8 to 5.10 show that the optimal actions vary with the budget constraints. When the budget limit is low, the optimal action given by the system model can be different from the optimal action suggested by the individual MDP models. This can be observed in case study 3 and the different optimal actions given by the individual models and the system model are shown in Table 5.10 in bold.

This numerical example shows that the individual MDP models may not be capable of representing the system, when there are system constraints. Thus, it is concluded that the individual MDP models of transformers cannot be considered in system level maintenance planning.

However, the MDP model proposed for transformers in section 5.5 consists of 370 states and 5 actions. If a system has n_{tf} number of transformers, the MDP model of the system of transformers consists of $370^{n_{tf}}$ number of states and $5^{n_{tf}}$ number of action combinations. Such a system may not be computationally tractable. To solve the curse of dimensionality, methods should be applied such as approximate dynamic programming [84].

5.7 Summary

With the power system deregulation, asset owners would prefer to adopt more adaptive and cost effective maintenance policies. In this chapter, a maintenance optimization model based on an MDP is proposed to find such maintenance policies for ageing electrical equipment. Deterioration states of this proposed MDP model are more detailed. Thus, this model is capable of incorporating the effect of delay times in making decisions regarding inspection and maintenance on equipment's deterioration and failure. In addition, this model integrates the deterioration of equipment's measurable conditions with effects of ageing on deterioration. The proposed model is solved using backward induction to obtain adaptive optimal policies with a less computational effort.

Using the solution of the proposed model, asset owners can perform inspection more cost effectively considering the knowledge about the current deterioration condition, the time that the equipment spent in that condition, the inspection delay time and the equipment's age. The solution also helps to perform maintenance more cost effectively considering the age, last known condition and the maintenance delay time. These adaptive policies are more useful, when maintenance has to be delayed in order to satisfy system requirements.

In a case study, we use CM and maintenance histories of transformers to demonstrate the model applicability. It is shown that the optimal CM actions vary with the equipment's condition, the time spent in the condition, CM delay time and the age of the equipment. The optimal maintenance actions vary with the equipment's condition, maintenance delay time and the equipment's age. A numerical example showed that it is not accurate to perform system-level maintenance planning by coordinating the optimization results provided by the MDP models of individual equipment.

Chapter 6 : Conclusions and Future Work

In this final chapter, major findings of this research are summarized, and some areas for future research are suggested.

6.1 Conclusions

Many electrical utilities adopt maintenance strategies to improve the system reliability, while extending the equipment's life time. In order to maximize benefits of maintenance, maintenance models are utilized. In this thesis, new probabilistic maintenance models are proposed for reliability assessment and maintenance optimization of power system equipment. The proposed models are applied to circuit breakers and transformers using real data. In addition, this thesis discusses the application of Markov maintenance models for state prediction and for analyzing the effects of sub component characteristics on system reliability.

In chapter 2, a probabilistic scheduled maintenance model is proposed based on a new state diagram. The advantage of this model is its capability of accurately assessing reliability and costs using analytical equations. In a numerical example, the results of this new model are compared with the results of existing Markov maintenance models and Monte Carlo simulation. This comparison verifies the accuracy of the proposed scheduled maintenance model. A theoretical discussion is also provided to prove the accuracy of the proposed model. The work presented in this chapter shows that maintenance models based on state diagrams are capable of modeling the scheduled maintenance practice accurately.

In chapter 3, three applications of Markov maintenance models are presented. First, the scheduled maintenance model proposed in chapter 2 is applied to reliability and cost assessment of circuit breakers using real data obtained from the literature. For comparison purposes, an existing maintenance model is also utilized in this application study. It is shown that the results of the two models are significantly different due to their different model assumptions. In addition, sensitivity analyses are conducted on both models to investigate the effect of inspection rate on reliability. Based on the results, the study concludes that the proposed model is well applicable for scheduled maintenance of equipment where inspections are performed off-line, whereas the existing models are more applicable to equipment with continuous on-line condition monitoring. In the second application study, a simple Markov model is utilized for state prediction of transformers. The results show that the state of transformers can be predicted rather accurately, especially for old transformers which are operated under high loading conditions. In the third study, a Markov model is applied to a wind energy conversion system to observe the effect of failure and repair rates of different sub components on system reliability. The results show that the components with high failure rates and high mean down times are equally important for the reliability of a system.

In chapter 4, a maintenance optimization problem is formulated to obtain optimal inspection and maintenance rates using scheduled maintenance models. The significance of this formulation is its simplified objective function which is set after conducting pre-analyses on six reliability and cost measures. The considered reliability and cost measures are FPT, unavailability, frequency of interruption, cost of inspection, maintenance and repair, interruption cost and loss of profit due to interruptions. Through analytical equations, it is shown that unavailability and frequency of interruption are implicitly included when the objective function considers the cost of inspection, maintenance and repair and the cost of interruption, respectively. A sensitivity analysis shows that it is more appropriate to incorporate FPT as a constraint. The objective of the optimization problem is then simplified to minimizing the total cost which is the sum of all considered cost measures. A grid search algorithm is employed to find optimal inspection rates using newly proposed and existing maintenance models of circuit breakers. Results show that practical solutions can be obtained for the optimal inspection rates with the selection of appropriate probabilistic maintenance models.

In chapter 5, a maintenance optimization model is proposed based on an MDP. The significance of this model is that it is capable of integrating the deterioration of equipment's measurable conditions with effects of ageing on deterioration. In addition, this model is capable of incorporating the effect of delay times in making decisions regarding inspection and maintenance on equipment's deterioration and failure. Due to the aforementioned added features, this model can provide more adaptive and cost effective maintenance policies. This proposed model is particularly applied to transformers using real historical condition monitoring and maintenance data obtained from the local utility. The adaptive optimal maintenance policies provided by this model may aid planning engineers to conduct condition monitoring and maintenance activities effectively and efficiently. With small modifications, this model can be generalized to represent any other aging equipment.

In conclusion, chapter 2 and chapter 5 of this thesis have proposed new probabilistic models for reliability assessment and maintenance optimization. The proposed scheduled maintenance model in chapter 2 can assess reliability and costs more accurately than the existing Markov maintenance models in the literature. The MDP model proposed in chapter 5 addresses several issues which are not addressed in existing maintenance models. Hence, this model is capable of providing more adaptive optimal maintenance policies. The application studies in chapter 3, chapter 4 and chapter 5 have shown the applicability and usefulness of the proposed models for maintenance scheduling and maintenance related decision making in power systems. The models proposed in this thesis can be implemented in future to perform maintenance of power system equipment in an optimal manner.

6.2 Future Research Work

The work presented in this thesis may provide a basis for further research work in model development and applications, maintenance optimization and system-level maintenance planning.

6.2.1 Model Development and Applications

- In this thesis, the proposed scheduled maintenance model is only applied to circuit breakers and the proposed MDP model is only applied to transformers. These models are very generic and in future, similar models can be developed with the application to other ageing equipment as well.
- Since condition monitoring techniques are fairly new, the case study in chapter 5 is conducted with data obtained from transformers over a limited operational period of 7 years. Once data is available over an extended period of time, the transition probabilities

of different age ranges can be updated in future to obtain more accurate results.

- If required, the MDP model proposed in chapter 5 can easily incorporate the present value of money, when the model is solved using the backward induction method.

6.2.2 Maintenance Optimization

- The grid search algorithm used in chapter 4 may not perform efficiently for systems with a large number of deterioration stages. In future, more efficient algorithms can be applied in extended studies.
- As discussed in [85], outcome probabilities may vary with the maintenance effort, but such model parameter adjustments are not considered in the scope of the optimization task presented in chapter 4. Such studies greatly need the inputs from experienced maintenance engineers. In future, with their help the optimization work in chapter 4 can be further improved to incorporate more decision variables such as money allocated for each maintenance action and the duration of performing each maintenance action [85].

6.2.3 System-level Maintenance Planning

- The proposed scheduled maintenance model in chapter 4 consists of additional states. However, the thesis has discussed the possibility of reducing the model complexity, when the model is used in system level studies. In future, the reduced version of the proposed scheduled maintenance model can be utilized to prioritize/coordinate scheduled maintenance activities in power systems.
- The MDP model proposed in chapter 5 of this thesis can be efficiently solved to find optimal adaptive maintenance policies for single equipment. The thesis shows that a system-wide MDP model should be employed to obtain optimal adaptive maintenance policies for a system with more equipment. A system-wide MDP model may have a large number of system states and action combinations. In order to tackle the curse of dimensionality associated with a system-wide MDP model, techniques such as approximate dynamic programming can be investigated [84]. If the curse of dimensionality can be tackled with the use of some technique, a system-wide MDP model

can be used to conduct system-level maintenance planning.

- In further work, the probabilistic maintenance models proposed in this thesis can be used to develop an asset management tool for optimizing and prioritizing maintenance activities in power systems. Such a tool may be very useful for electrical utilities to make effective decisions more efficiently on managing costly electrical assets.

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List of Publications

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[2] S. K. Abeygunawardane, P. Jirutitijaroen and H. Xu, "Adaptive maintenance policies for ageing devices using a Markov decision process", accepted for publication in *IEEE Trans. Power Syst*.

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Appendix A : The Proposed Markov Decision Process Model for Transformers

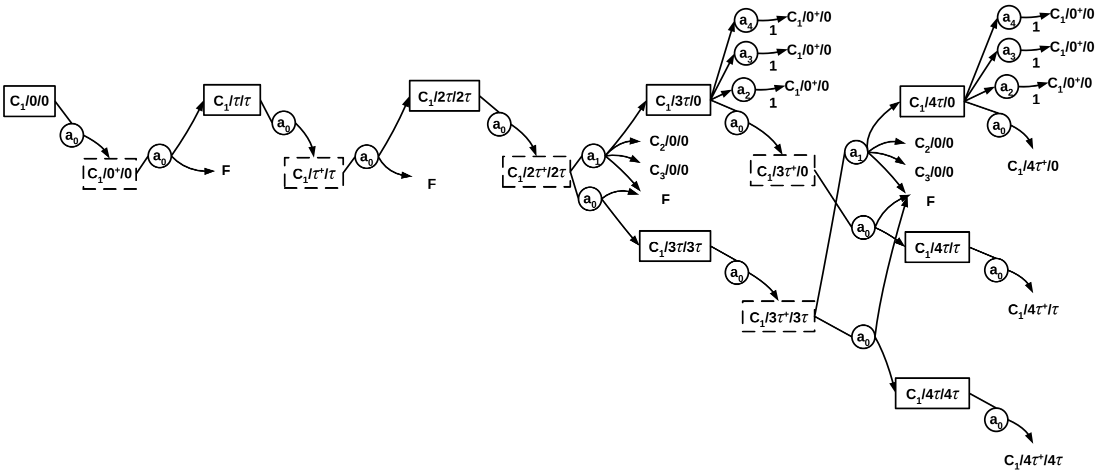

Figure A.1 (a): The proposed Markov decision process model for transformers

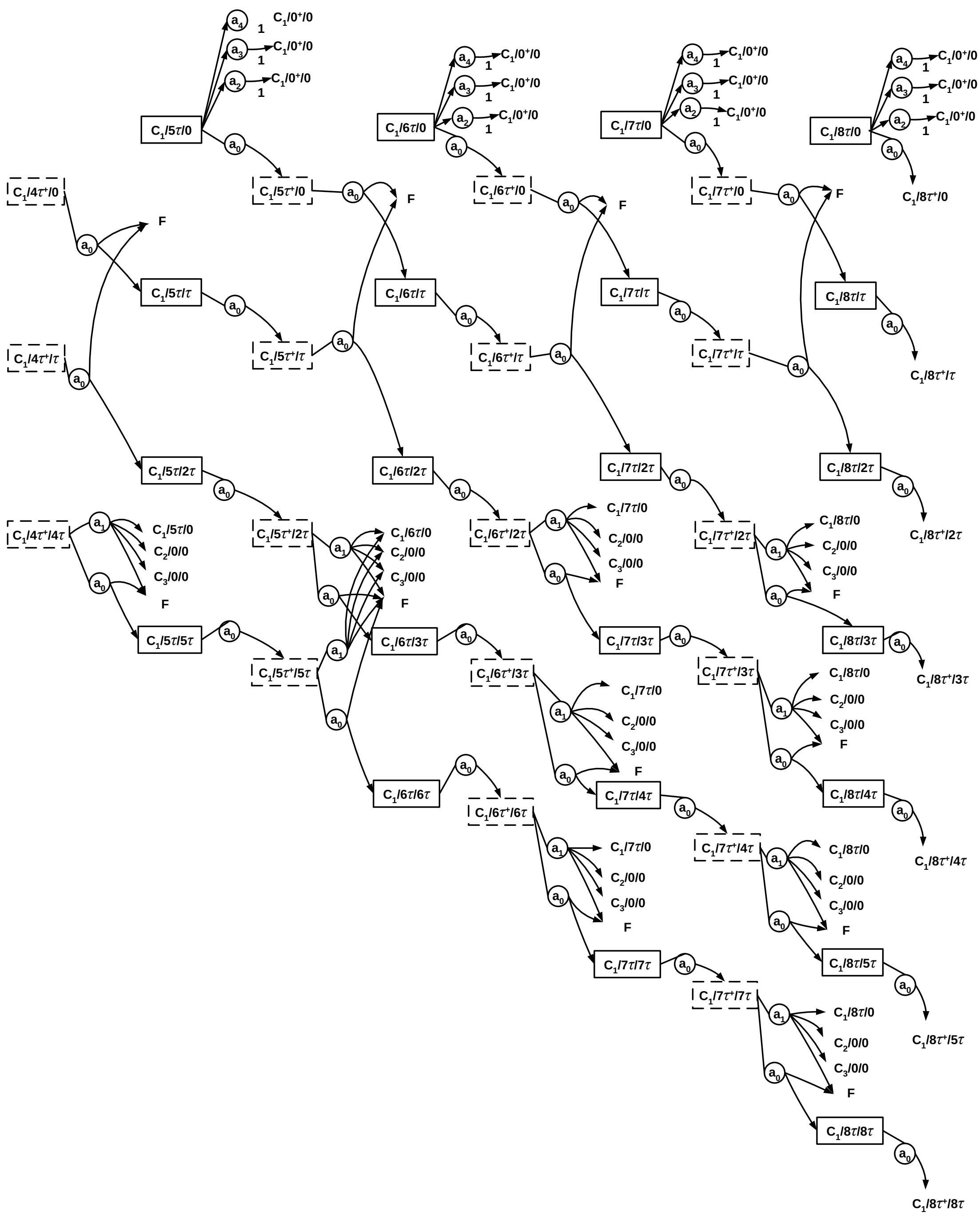

Figure A.1 (b): The proposed Markov decision process model for transformers

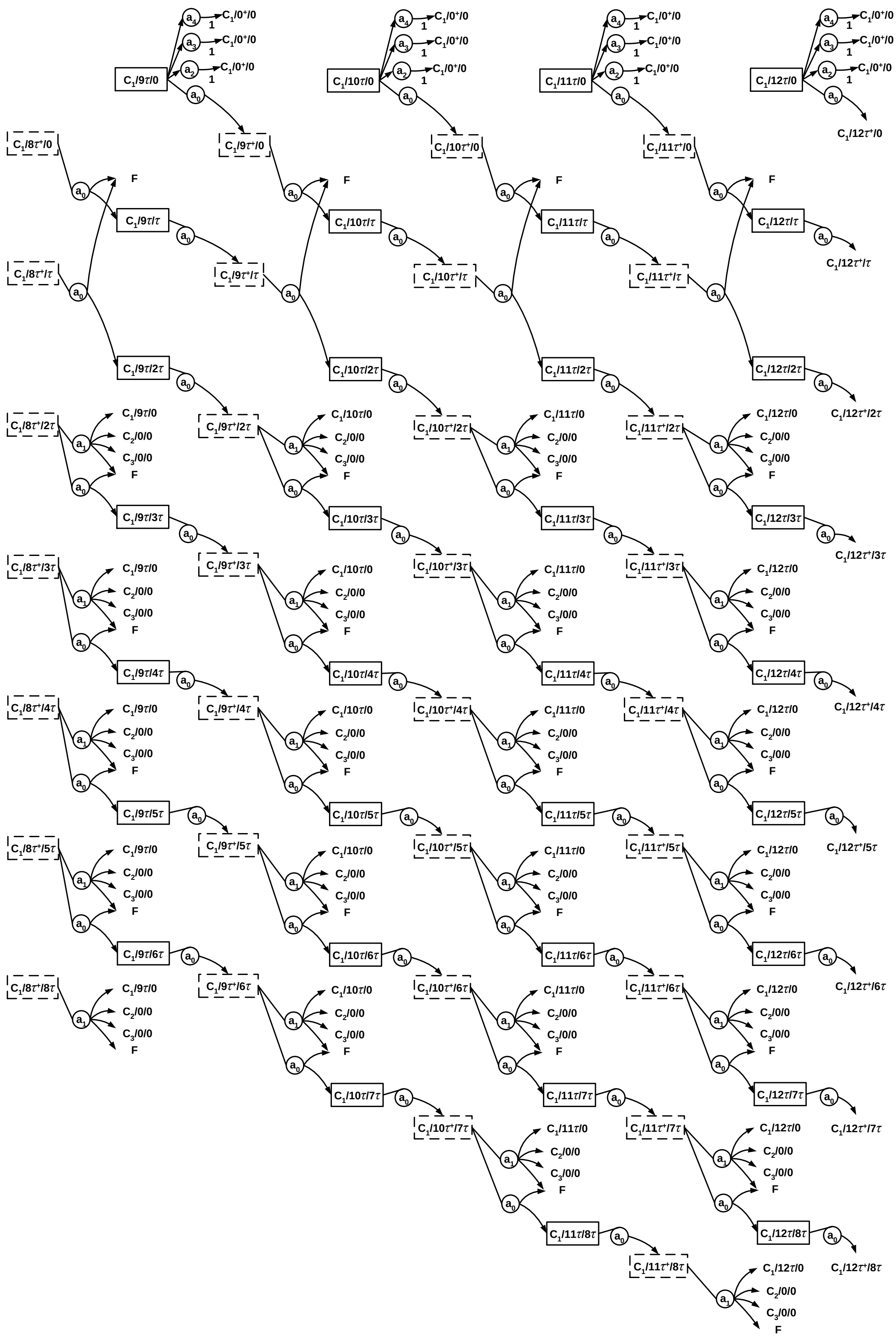

Figure A.1 (c): The proposed Markov decision process model for transformers

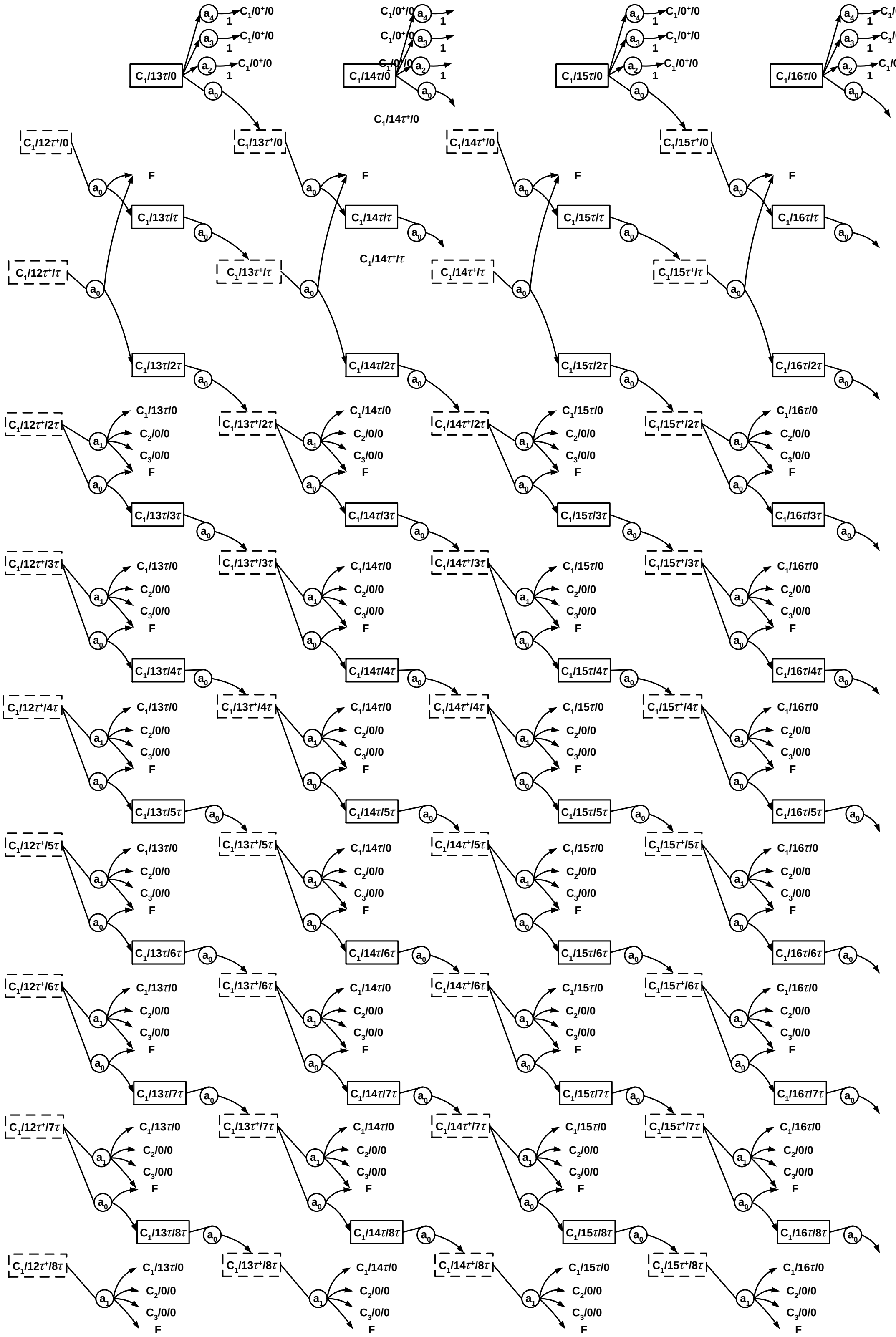

Figure A.1 (d): The proposed Markov decision process model for transformers

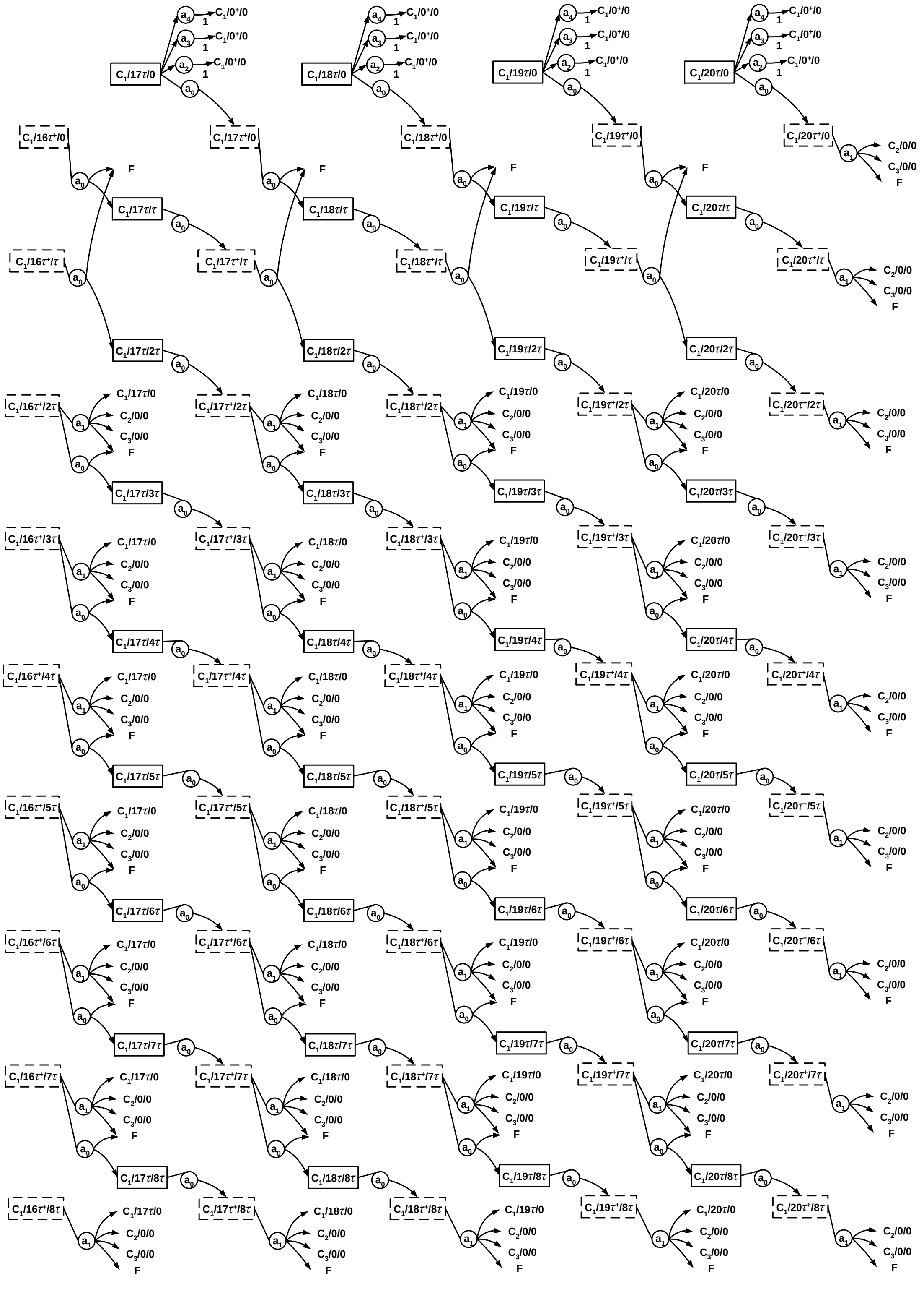

Figure A.1 (e): The proposed Markov decision process model for transformers

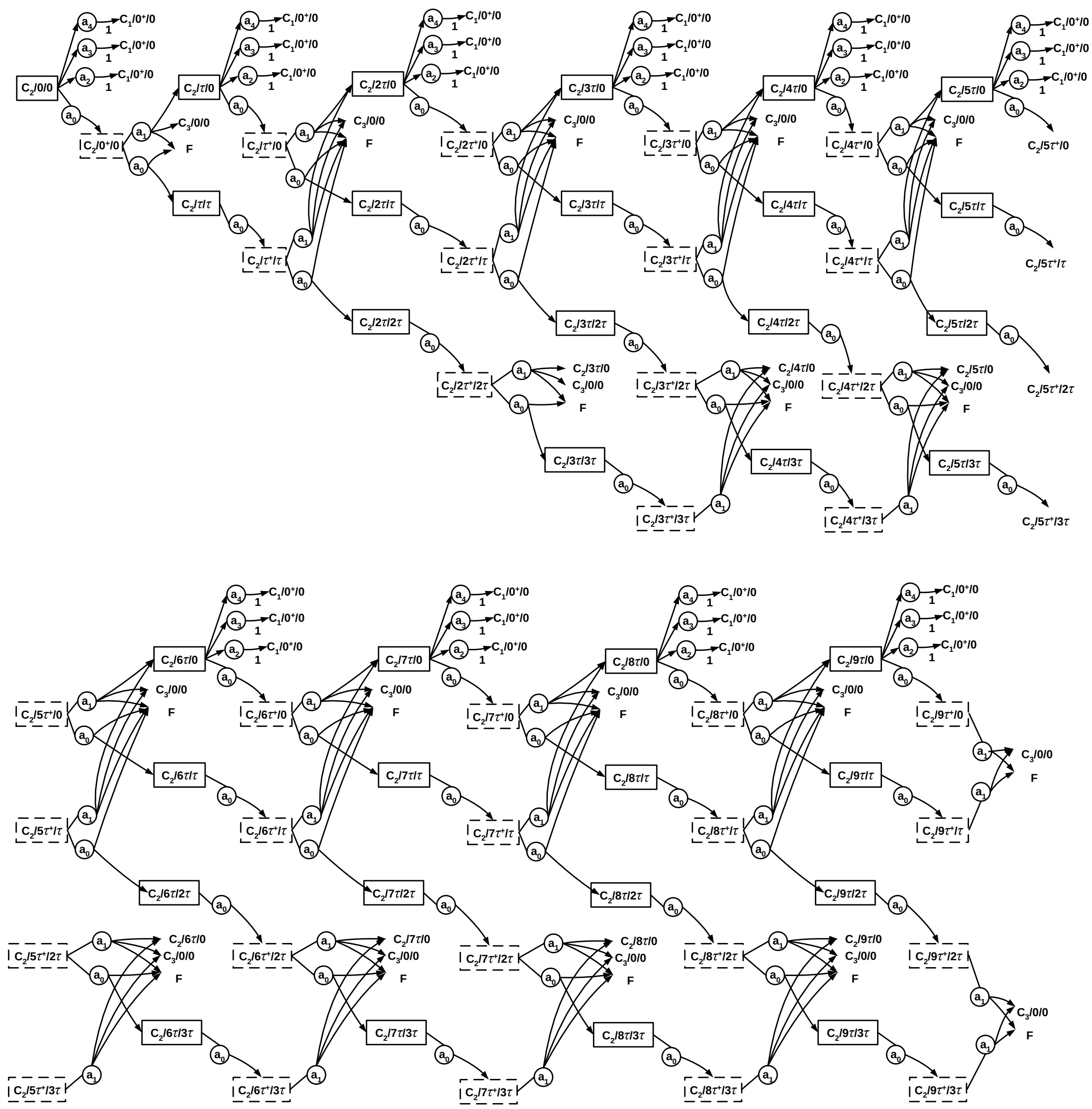

Figure A.1 (f): The proposed Markov decision process model for transformers

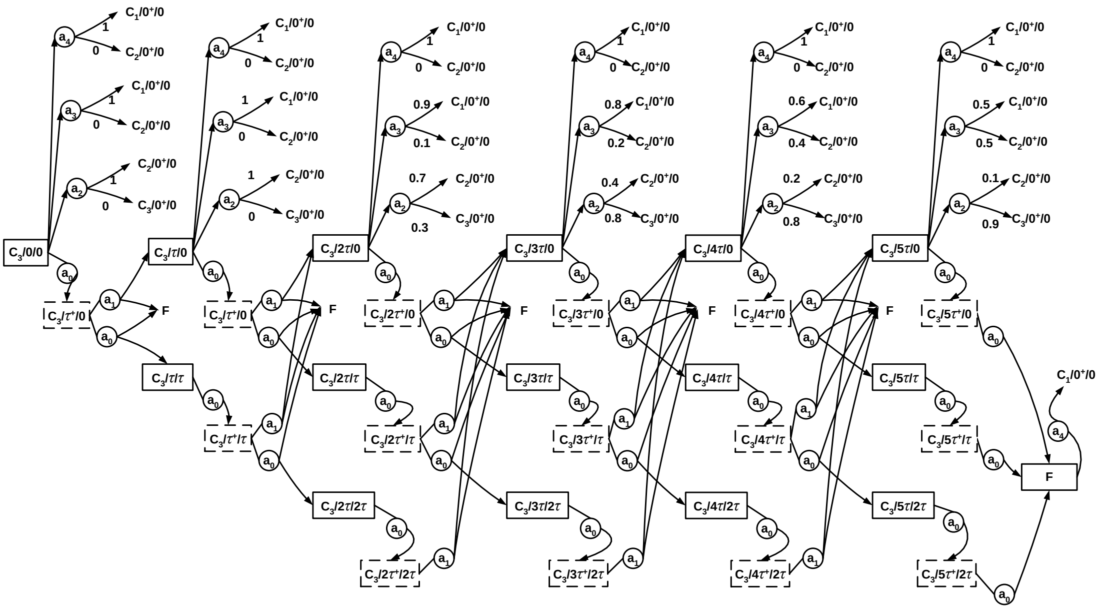

Figure A.1 (g): The proposed Markov decision process model for transformers

Appendix B : Deterioration Probabilities for the Markov Decision Process Model of Transformers

Table B.1 (a): Deterioration Probabilities for the Markov Decision Process Model of Transformers

From			To											
State	Time spent in the condition/ (years)	Time from last CM	0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
			S_1	S_2	S_3	F	S_1	S_2	S_3	F	S_1	S_2	S_3	F
S_1	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
	0.33	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
	0.67	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
		0.67	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00
	1.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00
		0.67	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00
		1.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00
	1.33	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00
		0.67	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00
		1.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00
		1.33	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00
	1.67	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00
		0.67	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.83	0.16	0.01	0.00
		1.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.83	0.16	0.01	0.00
		1.33	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.83	0.16	0.01	0.00
		1.67	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.83	0.16	0.01	0.00

Table B.1 (b): Deterioration Probabilities for the Markov Decision Process Model of Transformers

From			To											
State	Time spent in the condition/ (years)	Time from last CM	0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
			S_1	S_2	S_3	F	S_1	S_2	S_3	F	S_1	S_2	S_3	F
S_1	2.00	0.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00	0.90	0.10	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00	0.85	0.15	0.00	0.00
		0.67	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00	0.79	0.20	0.01	0.00
		1.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00	0.75	0.23	0.02	0.00
		1.33	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00	0.75	0.23	0.02	0.00
		1.67	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00	0.75	0.23	0.02	0.00
		2.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00	0.75	0.23	0.02	0.00
	2.33	0.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00	0.89	0.11	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00	0.80	0.20	0.00	0.00
		0.67	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00	0.75	0.24	0.01	0.00
		1.00	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00	0.71	0.26	0.03	0.00
		1.33	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00	0.66	0.28	0.06	0.00
		1.67	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00	0.66	0.28	0.06	0.00
		2.00	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00	0.66	0.28	0.06	0.00
		2.33	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00	0.66	0.28	0.06	0.00
	2.67	0.00	1.00	0.00	0.00	0.00	0.94	0.06	0.00	0.00	0.88	0.12	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00	0.78	0.22	0.00	0.00
		0.67	1.00	0.00	0.00	0.00	0.83	0.17	0.00	0.00	0.71	0.28	0.01	0.00
		1.00	1.00	0.00	0.00	0.00	0.83	0.17	0.00	0.00	0.66	0.31	0.03	0.00
		1.33	1.00	0.00	0.00	0.00	0.83	0.17	0.00	0.00	0.62	0.32	0.06	0.00
		1.67	1.00	0.00	0.00	0.00	0.83	0.17	0.00	0.00	0.58	0.30	0.11	0.01
		2.00	1.00	0.00	0.00	0.00	0.83	0.17	0.00	0.00	0.58	0.30	0.11	0.01
		2.33	1.00	0.00	0.00	0.00	0.83	0.17	0.00	0.00	0.58	0.30	0.11	0.01
		2.67	1.00	0.00	0.00	0.00	0.83	0.17	0.00	0.00	0.58	0.30	0.11	0.01

Table B.1 (c): Deterioration Probabilities for the Markov Decision Process Model of Transformers

From			To											
State	Time spent in the condition/ (years)	Time from last CM	0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
			S_1	S_2	S_3	F	S_1	S_2	S_3	F	S_1	S_2	S_3	F
S_1	3.00	0.00	1.00	0.00	0.00	0.00	0.90	0.10	0.00	0.00	0.67	0.33	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	0.85	0.15	0.00	0.00	0.59	0.41	0.00	0.00
		0.67	1.00	0.00	0.00	0.00	0.80	0.20	0.00	0.00	0.53	0.46	0.01	0.00
		1.00	1.00	0.00	0.00	0.00	0.75	0.25	0.00	0.00	0.47	0.48	0.05	0.00
		1.33	1.00	0.00	0.00	0.00	0.75	0.25	0.00	0.00	0.45	0.48	0.07	0.00
		1.67	1.00	0.00	0.00	0.00	0.75	0.25	0.00	0.00	0.42	0.45	0.12	0.01
		2.00	1.00	0.00	0.00	0.00	0.75	0.25	0.00	0.00	0.39	0.43	0.16	0.02
		2.33	1.00	0.00	0.00	0.00	0.75	0.25	0.00	0.00	0.39	0.43	0.16	0.02
		2.67	1.00	0.00	0.00	0.00	0.75	0.25	0.00	0.00	0.39	0.43	0.16	0.02
	3.33	0.00	1.00	0.00	0.00	0.00	0.89	0.11	0.00	0.00	0.50	0.50	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	0.75	0.25	0.00	0.00	0.34	0.66	0.00	0.00
		0.67	1.00	0.00	0.00	0.00	0.71	0.29	0.00	0.00	0.30	0.69	0.01	0.00
		1.00	1.00	0.00	0.00	0.00	0.66	0.33	0.01	0.00	0.26	0.69	0.05	0.00
		1.33	1.00	0.00	0.00	0.00	0.66	0.33	0.01	0.00	0.24	0.66	0.10	0.00
		1.67	1.00	0.00	0.00	0.00	0.66	0.33	0.01	0.00	0.22	0.63	0.14	0.01
		2.00	1.00	0.00	0.00	0.00	0.66	0.33	0.01	0.00	0.21	0.59	0.18	0.02
		2.33	1.00	0.00	0.00	0.00	0.66	0.33	0.01	0.00	0.20	0.55	0.20	0.05
		2.67	1.00	0.00	0.00	0.00	0.66	0.33	0.01	0.00	0.20	0.55	0.20	0.05
	3.67	0.00	1.00	0.00	0.00	0.00	0.88	0.12	0.00	0.00	0.00	1.00	0.00	0.00
		0.33	1.00	0.00	0.00	0.00	0.78	0.22	0.00	0.00	0.00	1.00	0.00	0.00
		0.67	1.00	0.00	0.00	0.00	0.70	0.30	0.00	0.00	0.00	0.96	0.04	0.00
		1.00	1.00	0.00	0.00	0.00	0.66	0.34	0.00	0.00	0.00	0.93	0.07	0.00
		1.33	1.00	0.00	0.00	0.00	0.62	0.37	0.01	0.00	0.00	0.88	0.12	0.00
		1.67	1.00	0.00	0.00	0.00	0.58	0.39	0.03	0.00	0.00	0.79	0.20	0.01
		2.00	1.00	0.00	0.00	0.00	0.58	0.39	0.03	0.00	0.00	0.74	0.23	0.03
		2.33	1.00	0.00	0.00	0.00	0.58	0.39	0.03	0.00	0.00	0.70	0.24	0.06
		2.67	1.00	0.00	0.00	0.00	0.58	0.39	0.03	0.00	0.00	0.66	0.24	0.10

Table B.1 (d): Deterioration Probabilities for the Markov Decision Process Model of Transformers

			To											
State	Time spent in the condition/ (years)	Time from last CM	0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
			S_1	S_2	S_3	F	S_1	S_2	S_3	F	S_1	S_2	S_3	F
S_1	4.00	0.00	0.94	0.06	0.00	0.00	0.67	0.33	0.00	0.00	-	-	-	-
		0.33	0.94	0.06	0.00	0.00	0.59	0.41	0.00	0.00	-	-	-	-
		0.67	0.94	0.06	0.00	0.00	0.52	0.48	0.00	0.00	-	-	-	-
		1.00	0.94	0.06	0.00	0.00	0.47	0.53	0.00	0.00	-	-	-	-
		1.33	0.94	0.06	0.00	0.00	0.44	0.55	0.01	0.00	-	-	-	-
		1.67	0.94	0.06	0.00	0.00	0.41	0.56	0.03	0.00	-	-	-	-
		2.00	0.94	0.06	0.00	0.00	0.39	0.55	0.06	0.00	-	-	-	-
		2.33	0.94	0.06	0.00	0.00	0.39	0.55	0.06	0.00	-	-	-	-
		2.67	0.94	0.06	0.00	0.00	0.39	0.55	0.06	0.00	-	-	-	-
	4.33	0.00	0.94	0.06	0.00	0.00	0.50	0.50	0.00	0.00	-	-	-	-
		0.33	0.88	0.12	0.00	0.00	0.34	0.66	0.00	0.00	-	-	-	-
		0.67	0.88	0.12	0.00	0.00	0.30	0.70	0.00	0.00	-	-	-	-
		1.00	0.88	0.12	0.00	0.00	0.26	0.74	0.00	0.00	-	-	-	-
		1.33	0.88	0.12	0.00	0.00	0.24	0.75	0.01	0.00	-	-	-	-
		1.67	0.88	0.12	0.00	0.00	0.22	0.75	0.03	0.00	-	-	-	-
		2.00	0.88	0.12	0.00	0.00	0.21	0.73	0.06	0.00	-	-	-	-
		2.33	0.88	0.12	0.00	0.00	0.20	0.68	0.12	0.00	-	-	-	-
		2.67	0.88	0.12	0.00	0.00	0.20	0.68	0.12	0.00	-	-	-	-
	4.67	0.00	0.94	0.06	0.00	0.00	0.00	1.00	0.00	0.00	-	-	-	-
		0.33	0.88	0.12	0.00	0.00	0.00	1.00	0.00	0.00	-	-	-	-
		0.67	0.83	0.17	0.00	0.00	0.00	1.00	0.00	0.00	-	-	-	-
		1.00	0.83	0.17	0.00	0.00	0.00	1.00	0.00	0.00	-	-	-	-
		1.33	0.83	0.17	0.00	0.00	0.00	0.99	0.01	0.00	-	-	-	-
		1.67	0.83	0.17	0.00	0.00	0.00	0.96	0.04	0.00	-	-	-	-
		2.00	0.83	0.17	0.00	0.00	0.00	0.92	0.08	0.00	-	-	-	-
		2.33	0.83	0.17	0.00	0.00	0.00	0.87	0.13	0.00	-	-	-	-
		2.67	0.83	0.17	0.00	0.00	0.00	0.82	0.17	0.01	-	-	-	-

Table B.1 (e): Deterioration Probabilities for the Markov Decision Process Model of Transformers

From			To											
State	Time spent in the condition/ (years)	Time from last CM	0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
			S_1	S_2	S_3	F	S_1	S_2	S_3	F	S_1	S_2	S_3	F
S_1	5.00	0.00	0.90	0.10	0.00	0.00	-	-	-	-	-	-	-	-
		0.33	0.85	0.15	0.00	0.00	-	-	-	-	-	-	-	-
		0.67	0.80	0.20	0.00	0.00	-	-	-	-	-	-	-	-
		1.00	0.75	0.25	0.00	0.00	-	-	-	-	-	-	-	-
		1.33	0.75	0.25	0.00	0.00	-	-	-	-	-	-	-	-
		1.67	0.75	0.25	0.00	0.00	-	-	-	-	-	-	-	-
		2.00	0.75	0.25	0.00	0.00	-	-	-	-	-	-	-	-
		2.33	0.75	0.25	0.00	0.00	-	-	-	-	-	-	-	-
		2.67	0.75	0.25	0.00	0.00	-	-	-	-	-	-	-	-
	5.33	0.00	0.89	0.11	0.00	0.00	-	-	-	-	-	-	-	-
		0.33	0.80	0.20	0.00	0.00	-	-	-	-	-	-	-	-
		0.67	0.75	0.25	0.00	0.00	-	-	-	-	-	-	-	-
		1.00	0.71	0.29	0.00	0.00	-	-	-	-	-	-	-	-
		1.33	0.67	0.33	0.00	0.00	-	-	-	-	-	-	-	-
		1.67	0.67	0.33	0.00	0.00	-	-	-	-	-	-	-	-
		2.00	0.67	0.33	0.00	0.00	-	-	-	-	-	-	-	-
		2.33	0.67	0.33	0.00	0.00	-	-	-	-	-	-	-	-
		2.67	0.67	0.33	0.00	0.00	-	-	-	-	-	-	-	-
	5.67	0.00	0.88	0.12	0.00	0.00	-	-	-	-	-	-	-	-
		0.33	0.78	0.22	0.00	0.00	-	-	-	-	-	-	-	-
		0.67	0.70	0.30	0.00	0.00	-	-	-	-	-	-	-	-
		1.00	0.66	0.34	0.00	0.00	-	-	-	-	-	-	-	-
		1.33	0.62	0.38	0.00	0.00	-	-	-	-	-	-	-	-
		1.67	0.59	0.41	0.00	0.00	-	-	-	-	-	-	-	-
		2.00	0.59	0.41	0.00	0.00	-	-	-	-	-	-	-	-
		2.33	0.59	0.41	0.00	0.00	-	-	-	-	-	-	-	-
		2.67	0.59	0.41	0.00	0.00	-	-	-	-	-	-	-	-

Table B.1 (f): Deterioration Probabilities for the Markov Decision Process Model of Transformers

From			To											
			0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
State	Time spent in the condition/ (years)	Time from last CM	S_1	S_2	S_3	F	S_1	S_2	S_3	F	S_1	S_2	S_3	F
S_1	6.00	0.00	0.67	0.33	0.00	0.00	-	-	-	-	-	-	-	-
		0.33	0.59	0.41	0.00	0.00	-	-	-	-	-	-	-	-
		0.67	0.52	0.48	0.00	0.00	-	-	-	-	-	-	-	-
		1.00	0.47	0.53	0.00	0.00	-	-	-	-	-	-	-	-
		1.33	0.44	0.56	0.00	0.00	-	-	-	-	-	-	-	-
		1.67	0.42	0.58	0.00	0.00	-	-	-	-	-	-	-	-
		2.00	0.39	0.61	0.00	0.00	-	-	-	-	-	-	-	-
		2.33	0.39	0.61	0.00	0.00	-	-	-	-	-	-	-	-
		2.67	0.39	0.61	0.00	0.00	-	-	-	-	-	-	-	-
	6.33	0.00	0.50	0.50	0.00	0.00	-	-	-	-	-	-	-	-
		0.33	0.34	0.66	0.00	0.00	-	-	-	-	-	-	-	-
		0.67	0.30	0.70	0.00	0.00	-	-	-	-	-	-	-	-
		1.00	0.26	0.74	0.00	0.00	-	-	-	-	-	-	-	-
		1.33	0.24	0.76	0.00	0.00	-	-	-	-	-	-	-	-
		1.67	0.22	0.78	0.00	0.00	-	-	-	-	-	-	-	-
		2.00	0.21	0.79	0.00	0.00	-	-	-	-	-	-	-	-
		2.33	0.19	0.80	0.01	0.00	-	-	-	-	-	-	-	-
		2.67	0.19	0.80	0.01	0.00	-	-	-	-	-	-	-	-
	6.67	0.00	0.00	1.00	0.00	0.00	-	-	-	-	-	-	-	-
		0.33	0.00	1.00	0.00	0.00	-	-	-	-	-	-	-	-
		0.67	0.00	1.00	0.00	0.00	-	-	-	-	-	-	-	-
		1.00	0.00	1.00	0.00	0.00	-	-	-	-	-	-	-	-
		1.33	0.00	1.00	0.00	0.00	-	-	-	-	-	-	-	-
		1.67	0.00	1.00	0.00	0.00	-	-	-	-	-	-	-	-
		2.00	0.00	1.00	0.00	0.00	-	-	-	-	-	-	-	-
		2.33	0.00	0.99	0.01	0.00	-	-	-	-	-	-	-	-
		2.67	0.00	0.97	0.03	0.00	-	-	-	-	-	-	-	-

Table B.1: (g): Deterioration Probabilities for the Markov Decision Process Model of Transformers

From			To											
State	Time spent in the condition/ (years)	Time from last CM	0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
			S_1	S_2	S_3	F	S_1	S_2	S_3	F	S_1	S_2	S_3	F
S_2	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00
	0.33	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.89	0.11	0.00
		0.33	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.89	0.11	0.00
	0.67	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.75	0.25	0.00
		0.33	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.67	0.33	0.00
		0.67	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.67	0.33	0.00
	1.00	0.00	0.00	1.00	0.00	0.00	0.00	0.89	0.11	0.00	0.00	0.67	0.33	0.00
		0.33	0.00	1.00	0.00	0.00	0.00	0.89	0.11	0.00	0.00	0.50	0.50	0.00
		0.67	0.00	1.00	0.00	0.00	0.00	0.89	0.11	0.00	0.00	0.45	0.53	0.02
		1.00	0.00	1.00	0.00	0.00	0.00	0.89	0.11	0.00	0.00	0.45	0.53	0.02
	1.33	0.00	0.00	1.00	0.00	0.00	0.00	0.75	0.25	0.00	0.00	0.00	1.00	0.00
		0.33	0.00	1.00	0.00	0.00	0.00	0.67	0.33	0.00	0.00	0.00	1.00	0.00
		0.67	0.00	1.00	0.00	0.00	0.00	0.67	0.33	0.00	0.00	0.00	0.95	0.05
		1.00	0.00	1.00	0.00	0.00	0.00	0.67	0.33	0.00	0.00	0.00	0.90	0.10
	1.67	0.00	0.00	1.00	0.00	0.00	0.00	0.67	0.33	0.00	-	-	-	-
		0.33	0.00	1.00	0.00	0.00	0.00	0.50	0.50	0.00	-	-	-	-
		0.67	0.00	1.00	0.00	0.00	0.00	0.45	0.55	0.00	-	-	-	-
		1.00	0.00	1.00	0.00	0.00	0.00	0.45	0.55	0.00	-	-	-	-
	2.00	0.00	0.00	0.89	0.11	0.00	0.00	0.00	1.00	0.00	-	-	-	-
		0.33	0.00	0.89	0.11	0.00	0.00	0.00	1.00	0.00	-	-	-	-
		0.67	0.00	0.89	0.11	0.00	0.00	0.00	1.00	0.00	-	-	-	-
		1.00	0.00	0.89	0.11	0.00	0.00	0.00	0.98	0.02	-	-	-	-
	2.33	0.00	0.00	0.75	0.25	0.00	-	-	-	-	-	-	-	-
		0.33	0.00	0.67	0.33	0.00	-	-	-	-	-	-	-	-
		0.67	0.00	0.67	0.33	0.00	-	-	-	-	-	-	-	-
		1.00	0.00	0.67	0.33	0.00	-	-	-	-	-	-	-	-

Table B.1 (h): Deterioration Probabilities for the Markov Decision Process Model of Transformers

From			To											
State	Time spent in the condition/ (years)	Time from last CM	0≤ age <20 years				20≤ age <30 years				30≤ age <40 years			
			S_1	S_2	S_3	F	S_1	S_2	S_3	F	S_1	S_2	S_3	F
S_2	2.67	0.00	0.00	0.67	0.33	0.00	-	-	-	-	-	-	-	-
		0.33	0.00	0.50	0.50	0.00	-	-	-	-	-	-	-	-
		0.67	0.00	0.45	0.55	0.00	-	-	-	-	-	-	-	-
		1.00	0.00	0.45	0.55	0.00	-	-	-	-	-	-	-	-
	3.00	0.00	0.00	0.00	1.00	0.00	-	-	-	-	-	-	-	-
		0.33	0.00	0.00	1.00	0.00	-	-	-	-	-	-	-	-
		0.67	0.00	0.00	1.00	0.00	-	-	-	-	-	-	-	-
		1.00	0.00	0.00	1.00	0.00	-	-	-	-	-	-	-	-
S_3	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00
	0.33	0.00	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.80	0.20
		0.33	0.00	0.00	1.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.80	0.20
	0.67	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.80	0.20	0.00	0.00	0.60	0.40
		0.33	0.00	0.00	1.00	0.00	0.00	0.00	0.80	0.20	0.00	0.00	0.48	0.52
		0.67	0.00	0.00	1.00	0.00	0.00	0.00	0.80	0.20	0.00	0.00	0.48	0.52
	1.00	0.00	0.00	0.00	0.80	0.20	0.00	0.00	0.60	0.40	0.00	0.00	0.00	1.00
		0.33	0.00	0.00	0.80	0.20	0.00	0.00	0.48	0.52	0.00	0.00	0.00	1.00
		0.67	0.00	0.00	0.80	0.20	0.00	0.00	0.48	0.52	0.00	0.00	0.00	1.00
	1.33	0.00	0.00	0.00	0.60	0.40	0.00	0.00	0.00	1.00	-	-	-	-
		0.33	0.00	0.00	0.48	0.52	0.00	0.00	0.00	1.00	-	-	-	-
		0.67	0.00	0.00	0.48	0.52	0.00	0.00	0.00	1.00	-	-	-	-
	1.67	0.00	0.00	0.00	0.00	1.00	-	-	-	-	-	-	-	-
		0.33	0.00	0.00	0.00	1.00	-	-	-	-	-	-	-	-
		0.67	0.00	0.00	0.00	1.00	-	-	-	-	--	-	-	-