PERSISTENT MIGRATION: WAGES, NETWORKS AND ASSIMILATION

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A THESIS SUBMITTED

FOR THE DEGREE OF

DOCTOR OF PHILOSOPHY IN ECONOMICS

DEPARTMENT OF ECONOMICS

NATIONAL UNIVERSITY OF SINGAPORE

2012

Acknowledgements

Most of my gratitude goes to my supervisor, Aditya Goenka, for being very patient, understanding and for being extremely generous with his time, and whose feedback and advice throughout this entire process have been, and always will be, very valuable to me. The wisdom he has shared with me extends beyond the academic knowledge I have gained from pursuing this degree and, for that, I am extremely grateful. I would also like to thank those who have been a part or are still a part of my thesis committee, or who have been instrumental in the development of this work, either through the Pre-submission Seminar or elsewhere: Zhang Jie, Zeng Jinli, Partha Chatterjee, Parimal Bag, Aamir Hashmi and Joydeep Bhattacharya. Thank you, also, to the three examiners who have provided such great feedback, which has really guided me in the final editing of this document.

To the National University of Singapore, thank you for this opportunity and for providing me with a conducive environment to carry out my work. To the staff of the Economics Department, thank you for being very accommodating and for your support, especially during the times that I was in Manila which, perhaps, required more than was necessary for you to provide.

To De La Salle University - Manila, allow me to express my gratitude for providing me with a solid education. I consider it a blessing to have the chance to work for an institution that has always given me so much. And to the faculty and staff of the DLSU School of Economics, thank you for welcoming me back and allowing me to work with you while completing my dissertation. Thank you, especially, to my former professors, Tereso Tullao, Jr., Roberto Raymundo, Cesar Rufino, Joel Tanchuco, and Bombit Largoza, for your continuous support and inspiration.

Nona Pepito-Antolihao, Neil Sebastian D'Souza and Himani, thank goodness fate has decided for all of us to start our PhDs at the same time. I don't know how I would have gone through all the lectures, problem sets and exam preparations without you! Thank you for the laughter and for keeping me sane.

For helping me get through my final year of thesis writing, thank you to my Century Park karaoke buddies for educating me more on how male economic agents think and for improving both my vocabulary and vocal chords; to Starbucks along Aguirre, B.F. for the "office space" on weekends and the sometimes-free whipped cream; and to my high school *barkada* for never tiring of hearing my updates (or, at least, pretending to) on the progress of my dissertation.

Of course, I cannot forget to thank my parents, who have never questioned my decision to pursue a doctoral degree and who have always let me live my life the way I want to, whether rightly so or not. To Rhia, Juno, Nino, Bom, and Agnes, thank you for always cheering me on and for always pulling me from my thesis work when I needed it - through amazing food, fun basketball games, and a lot of laughter. To Gabby, Lauren, Luis and Santi, thank you for showing me how to see the world through your eyes and for reminding me that life is truly wonderful. And, to God, thank you. Thank You for giving me the strength and faith to continue following my heart. This is all for You.

Contents

Summary

International migration in recent years has been described as being persistent, with the movement of persons from one country to another being shown to be continuous and large in scale. However, migration models have typically shown how migration can lead to the reduction of the wage gap, leading to a convergence in wages and, thus, removing any incentive to migrate and making migration transitory or temporary. Reichlin and Rustichini (1998) employ a model that allows for persistent migration, relying on the size and composition of the workforce in both the migrant-sending and migrant-receiving countries to analyze migration patterns. Thus, by employing the model they introduce, we aim to study persistent migration and the issues underlying the process. In Chapter 2, we adopt their model when there is one type of labor, endogenize the migration choice and introduce a probability of migration, and find a similar result: scale effects is key in the continued existence of a wage gap, leading to persistent migration as the incentive to migrate does not disappear. We also note that the higher the probability of migration, the higher the wages of the migrant-sending country relative to those of the migrant-receiving country. We also assess the possibility of deferred assimilation, i.e. when first-generation immigrants leave the adoption of the foreign birth rate to their children, and we find that deferred assimilation is favorable for the migrant-receiving country as they face higher wages than if assimilation were immediate. In Chapter 3, we continue to adopt the model, but with two types of labor and add an endogenous skill choice. We also consider the possibility of migrating as an unskilled worker. We observe additional results in terms of both the composition of labor (i.e., the ratio of skilled to unskilled labor) as well as the unskilled-to-skilled wage ratios in each country. We find that skilled worker migration leads to higher skilled-to-unskilled workforce ratios for both countries. Unskilled worker migration lowers the skilled-to-unskilled workforce ratio for the migrant-receiving country while skilled worker migration not only improves the

wages of the migrant-sending country relative to those of the migrant-receiving country but it also improves the unskilled-to-skilled wage ratio in each country. In Chapter 4, we then consider the existence of networks and their role in propagating persistent migration based on the premise that network formation by previous migrants leads to a higher probability for others to follow and migrate. This then leads to persistent migration to occur even more while, at steady state, we find that network effects can produce preferable results for either country depending on how the probability of migration is determined by the existing network.

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Chapter 1

Introduction

1.1 Persistent Migration

Migration models have typically predicted the convergence of wages between two countries or regions when a neoclassical growth model, with an assumption of perfect capital mobility, is employed. As the workforce in the migrant-receiving country increases due to the addition of migrant workers, the foreign wage decreases and as this continues to occur, the wage differential becomes nonexistent, and the incentive to migrate simply disappears. Hence, in this scenario, labor migration from one place to another becomes transitory or temporary. With the neoclassical model, and with perfect capital mobility, the prediction is that the direction of the flow of labor is opposite to the direction of the flow of capital, which further lessens the wage differential and any incentive to migrate.

However, certain examples of labor migration across borders show that migration need not be temporary. The stock of migrants from Mexico in the United States, for example, has been rising from the early 1900s until 2009, as shown in Figure A.1, implying that there has been a steady inflow of migrants from Mexico.

To emphasize this point, Figure A.2 shows the steady inflow of migrants from the year 1986, as well as the percentage of migrants from Mexico as a share of total immigrants in the United States. The Mexico-U.S. migration pattern is often referred to as a migration "corridor" as the persistence of this migration flow has led to the existence of around 11.6 million migrants in the United States in year 2010. There are other persistent migration patterns that belong to the top migration corridors in the world involving the United States. One is the inflow of migrants from the Philippines and the other is that from India. Migration from India to the U.S. began in the 1970s and migration from the Philippines began in the 1980s.[1] Both countries have a migrant stock of around 1.7 million people

[1] Source:Migration Policy Institute Data Hub, Washington, D.C., last viewed September 2010, <http:\\www.migrationinformation.org/DataHub/historicaltrends.cfm>

in the United States. Although, worldwide, no other migration corridor has accumulated a migrant stock as large as that of the Mexico-U.S. migration pattern, other migration corridors do exist as a result of persistent migration. Table A.1 shows a list of migration corridors.[2]

While certain examples of migration from the late 1800s to the early 1900s show that migration does not have to be persistent, given that certain migration flows have been shown to have led to a form of convergence in wages[3], the characteristic of migration at present is that labor mobility seems to have a dampened effect on wages, even with capital mobility. The "permanence" of the current migration (Post-World War II) is among the issues that is surrounding debate on migration today (Chiswick and Hatton, 2003). This permanence is more obvious in the migration corridors identified and the Mexico-U.S. flow, in particular, will be analyzed in this study. That permanent migration is a puzzle is due to, therefore, the presence of this characteristic in migration today, which is also not predicted by the neoclassical growth model, which has been traditionally used in studying migration.

Hence, with the onset of persistent migration in recent decades, it then became imperative to address the issue that traditional migration models are not able to show how this can exist. Faini (1996), Reichlin and Rustichini (1998), Larramona and Sanso (2003) address this and, under different assumptions, are able to show how persistent migration can occur where, even if there exists labor mobility, there is no convergence.

Reichlin and Rustichini (1998) adopt the Arrow-Romer endogenous growth model and show that, through the existence of scale effects, a divergence of wages

[2]The migrant stock provided in this table covers the years from 1960 to 2010.

[3]Examples are Scandinavian countries and Italy catching up to Great Britain and the United States (Chiswick and Hatton, 2003).

is possible when there is only one type of labor, even with the assumption of perfect capital mobility. When they make a distinction between skilled and unskilled labor, they show that changes in the wage ratio not only depend on scale effects, but also on the composition of labor in both countries and find that migration may be beneficial to the sending country.[4] Again, in this scenario, they identify an equilibrium path where migration may continue to persist and, in some cases, undergo a reversal, i.e. the flow may change its direction. Their results revolve around the long-run patterns of the flow of migration depending on whether the scale effect or the composition effect is stronger. In their model, however, the proportion of the population that migrates at each time period is taken to be fixed and, thus, ignores the effect that changes in the wage differential may have on the number of migrants.

Faini (1996) specifically studies the literature on convergence, assessing the prediction of the neoclassical model, where there is convergence when there is factor mobility, as well as that of endogenous growth models, where divergence is a possible result. The model presented assumes a two-period overlapping generations model and where production is comprised of two sectors. The first one is that of the tradeable good and the second one is the non-tradeable good which is used as an input to produce the tradeable good, along with labor and capital. Increasing returns to scale is assumed. The study finds that with some level of capital mobility, a more significant level of labor migrations leads to divergence and, thus, more migration, while a lower level of labor migration leads to convergence.

Larramona and Sanso (2003), on the other hand, also employ an endogenous growth model to try to explain persistent migration. They use a production function introduced by Kemnitz (2001), which does not rely on scale effects, and they

[4]The composition of labor is defined as the ratio of skilled labor to unskilled labor.

do not introduce a distinction between skilled and unskilled labor. They use capital-labor ratios and assume that return migration is a possibility. They show that, contrary to Reichlin and Rustichini (1998), migration does not have to be detrimental to the sending country, with one type of labor, and that convergence of growth rates may occur, but not necessarily the convergence of wages, which results in permanent migration. In addition to this, they also assume that individuals get to choose the country where they plan to work and the country where they plan to retire, with a variable indicating preference for their country of birth, which then allows for the migration decision as well as the return migration decision to be endogenous.

One crucial result in the Reichlin-Rustichini model (1998) is the role of scale effects, implying that a relatively larger workforce leads to a relatively higher wage. In the model of homogenous labor, this is immediately apparent in the equilibrium condition. Scale effects are exhibited in studies of economic growth due to Arrow (1962), Lucas (1988), Romer (1986), Simon and Steinman (1984), Backus, et.al. (1992), Jones (1995) and Di Maria (2009). In relation to migration, however, as far as we know, empirical work on the role of scale effects on wages and migration does not exist. While there exist studies on the effect on wages of immigration and emigration, these do not necessarily make specific inferences to the increases in the wage gap to be due to scale effects.

1.1.1 Decisions to Migrate and to Acquire Skill

As Larramona and Sanso (2003) have pointed out, however, not including the decision to migrate based on changing wage differentials is a weakness of the migration literature.

Other studies on migration that allow for the endogeneity of the migration decision are those that have focused on the possibility of a brain drain and that study the impact of migration on the growth of the sending country such as those by Rodriguez (1975), Miyagiwa (1991), Mountford (1997) and Beine, et al. (2001). These studies have typically focused on the sending country and describe equilibrium only according to the country that exports labor. Hence, the foreign wage is assumed to be fixed. This, however, does not reflect the possibility of how foreign wages change due to the inflow of migrants into the receiving countries.

Another note to be made is that most studies on migration do not make a clear distinction between skilled and unskilled workers. This is another weakness in the migration literature as current migration flows are a combination of both highly-skilled workers as well as low-skilled workers.

Hence, these are two weaknesses in the literature which we address as we study the process of persistent migration. Understanding how changing wages may affect the level of potential migrants as well as their level of skill may enhance research on persistent migration.

1.1.2 Probability of Migration

Research on the probability of migration or immigration quotas is scarce. There are several factors that can determine the number of immigrants allowed to enter a country but little has been studied. Factors such as family reunification or lobbying may affect immigration policy (Borjas, 1994; Amegashie, 2004). There has been a study, for example, on how immigration quotas affect voting seeing that one source of friction between immigrants and natives is the possibility of immigrants voting on national issues (Ortega, 2005). Another study looks at quotas and how they affect the skill composition of the workforce (Belletini and Ceroni, 2007).

Our inclusion of a probability of migration is to analyze the effects of "limited" immigration on persistent migration flows. By including a probability of migration, whether interpreted as the ability to cross a border or a policy variable imposed by the migrant-receiving country or even by the migrant-sending country, we hope to analyze the different scenarios resulting from varied probabilities of migration. In our model, the probability of migration is treated as an exogenous variable and is treated as a policy variable, implying that it is the migrant-receiving country's decision of whether a potential migrant is eventually able to migrate or not.

1.2 Remarks

We would like to point out important considerations regarding rural-urban migration, two-way migration, reverse migration and wage differentiation between migrant and native workers.

Although models of international migration may generally be used to explain rural-urban migration, the literature is kept quite separate as rural-urban migration issues involve aspects of urbanization and economic development.[5] However, this is not to imply that the model we employ may not be used to explain certain aspects of rural-urban migration. For example, rural-urban migration may also have a form of permanent migration. However, we adopt the view of persistent migration occurring internationally where the emphasis is on a crossing of borders and, therefore, implies a higher cost of migration and in some cases a longer distance (e.g. Philippines-U.S. and India-U.K.). For our model of heterogenous labor, we also focus on the aspect of migration whereby the movement of labor

[5] An example of work on urbanization is that of Lucas (2004).

may be applicable for only certain types of labor. In rural-urban migration flows, there is no real issue of whether rural-urban migrants need to fulfill certain education requirements and, thus, skill acquisition is not a focus. There is also no true quota imposed on the rural-urban migration flow.

The issues we analyze can also be argued to play a more important role in international migration. In the model we introduce, we assume a probability of migration and this is highlighted more in international migration with the existence of immigration quotas and other immigration policies. We also emphasize the process of applying for migration which involves a cost to the potential migrant. The other issues of network effects and assimilation may also be argued to be issues of a more elevated level when migration involves the crossing of borders. While these are still part of rural-urban migration, they can be recognized as relatively more complex issues when migration occurs internationally.

We also do not consider two-way migration. In certain migration corridors, it may be observed that there is heavy emphasis on only one flow. While there are, of course, instances where there is a process of migration between countries that goes both ways, in most cases, only one way displays a significant volume of migration flows. If we consider the Mexico-U.S., Philippines-U.S., India-U.S. migration flows, migration from the U.S. to these countries is not considered a migration corridor, i.e. volume is not massive and it also does not display any persistence.

Reverse migration is also not tackled in this study. In recent years, there have been more anecdotes of reverse or "returning" migration. There have been cases where migrants have started returning to their home countries after a significant length of time living as a "migrant" in the host country. While this may be true, we are not going to include this process in the current analysis. This, however, may be studied in future research on migration as a strong possibility to return to the home country may have a different impact on the migration decision, as well as on the decision to acquire education.

The last aspect we would like to point out which we do not cover is the differentiation of wages between migrants and the natives. In certain studies on wages, this is of importance in the migration literature. However, the construction of the model only allows us to focus on the home country's wages and the host country's wages so we are currently unable to make any distinction. However, again, this will be considered in future studies.

1.3 Contribution and Objectives

Given the discussion above, we would like to analyze the movement and changes in wages under persistent migration by introducing a model of persistent migration where there is an endogenous migration choice, and an endogenous skill choice in the subsequent chapter. We also assess the role of varying probabilities of migration and by analyzing changes in the ratios of the host and home countries' levels of workforce and the effect on the wage differentials.

Thus, we adopt the dynamic, two-country model introduced by Reichlin and Rustichini (1998) because of three useful points: (1) persistent migration is a possible result of their model, (2) the wage ratio between the sending and receiving country changes as the inflow of migrants affects the workforce levels in both countries, and (3) they make a distinction between skilled and unskilled labor. From this, we aim to achieve three objectives. First, in Chapter 2, from the strand of the migration literature that studies the brain drain issue for the sending country, by endogenizing the decision to migrate based on the wage ratio, which then leads to the number of migrants also changing over time, we assess how the workforce ratio and the wage ratio changes accordingly, whether there is free or limited migration. We also include a short note on the possibility of deferred assimilation by immigrants. Second, in Chapter 3 we also endogenize the decision to acquire skill in the case where heterogenous labor is assumed, and assess the patterns of migration for both unskilled and skilled labor. We also allow for different scenarios resulting from different probabilities of migration given different skill levels, which reflects a characteristic of immigration policies of migrant-receiving countries, in that they are subject to the skill level they expect of their immigrants or of the employment needs of their economies. We then analyze workforce ratios and wage ratios and note their implications. And, third, in Chapter 4, we study the role that network formation has on persistent migration by continuing to study workforce and wage ratios. In all three chapters, we discuss the analytical results whenever possible and include numerical simulations to enhance the studies.

When there is only one type of labor, we find that persistent migration gives the migrant-sending country a relatively lower wage and the migrant-receiving country a higher wage. With the existence of a probability of migration, however, it is better for the migrant-receiving country to limit migration while it is preferable for the migrant-sending country if there were no immigration quotas. We also find that if immigrants choose to defer assimilating into the local culture and leave the assimilation to their children, that those that were not able to migrate will face lower wages while the immigrants themselves and the natives of the host country will receive even higher wages than when assimilation is immediate.

When there are two types of labor, we have results concerning the composition of labor (skilled-to-unskilled workforce ratio) as well as relative wages. We find that if potential migrants are allowed to migrate as skilled workers, this results into a higher skilled-to-unskilled workforce ratio for both the migrant-sending and the migrant-receiving countries. The migrant-receiving country benefits even more when they allow a higher level of skilled worker migration as well as limit unskilled worker migration. In terms of relative wages, skilled worker migration is better for both countries as the unskilled wage is closer to the skilled wage, reducing 'wage inequalities.' The unskilled wage gap and the skilled wage gap between the two countries also decreases with skilled worker migration, which is preferable for the migrant-sending country as they are then receiving wages closer to those of the migrant-receiving country.

When we consider that network formation occurs when there is persistent migration, we find, that by analyzing how the probability of migration is increasing in the proportion of the newborn population that could migrate in the previous period, over time, the level of those that can migrate increases and the divergence of wages is hastened. Hence, when there is one type of labor, the existence of network effects leads the migrant-sending country to face relatively lower wages over time. At steady-state, we find that the results may differ depending on how networks are specified to affect the probability of migration. The slower the occurrence of the network effect, the better for the migrant-sending country as a faster occurring network effect lowers their wages at the steady-state. Hence, for the migrant-receiving country, the faster the occurrence of the network effect, the better, as they face relatively higher wages.

Chapter 2

Persistent Migration and Wages: Homogenous Labor

2.1 Introduction

Due to this existence of persistent migration or 'large-scale migration' (Borjas, 2009), there has been a resurgence of studies done on how migration flows affect labor market outcomes, with the majority of the work being done on the U.S. economy. Immigration in typical migrant-receiving countries has been cause for some concern, as an influx of immigrants represents a labor-supply shock which leads to lower equilibrium wages, and fewer employment opportunities for the native-born.

Studies in this area have typically employed data-mining and regression methodologies, with quite contradictory claims. Recently, Borjas (2009) has attempted to resolve this issue theoretically with the premise that most empirical work have ignored factor demand theory; results show that factors such as the importance of labor in production, the supply elasticity of capital, the elasticity of product demand and the impact of immigration on the economy's consumer base play a role in wage effects.[1]

As solid theoretical work has not been as proliferate, results from empirical studies have resulted into different outcomes. Some studies report to find a negative and significant effect on wages such as Borjas (2003), Mishra (2007) and Peri and Sparber (2007). Other studies claim that the reason why wage effects are negative is due to the assumption that immigrants and natives are perfect substitutes. With this assumption relaxed for some studies, some have found that wage effects are small to insignificant, giving the idea that immigration should not be a cause for concern. Studies by Butcher and Card (1991), Card (2001; 2005), Ottaviano and Peri (2006; 2008), Manacorda, et. al. (2006), Dustmann, et. al.

[1] Studies by Card (2001) and Borjas (2003) also adopt a different approach in determining wage effects, with the former observing that wage effects are small and insignificant and the latter noting that wage effects are significant and negative for workers of different skill-levels to varying degrees.

(2005), Friedburg and Hunt (1995) all claim that immigration does not adversely affect national wages.[2].

A recent study on European countries by Docquier, Ozden and Peri (2010) has found that the wage effects are the opposite of what have been observed so far. For the years 1990 - 2000, they have found that immigration has, in fact, had a positive effect on average wages for native workers.

Hence, in this chapter, we aim to assess the effect of persistent migration on the wage differential and workforce ratio between the migrant-sending and the migrant-receiving countries. We analyze the impact of migration on the steady-state wage ratio and workforce ratio for only one type of labor and include a brief analysis of what may occur when assimilation, or the adoption of the foreign birth rate by migrants, is delayed by one generation[3].

We find that the higher the probability of migrating, with a model of endogenous migration choice, gives a higher workforce ratio and a higher wage ratio. This means that the proportion of the population that decides to migrate is lower as in the steady state, the wage gap is smaller. Hence, while persistent migration leads to the migrant-sending country facing relatively lower wages, and the migrant-receiving country facing relatively higher wages, the higher the probability of migration, the better it is for the migrant-sending country. When we assess the possibility of deferred assimilation, what we find is that whilst assimilation issues have been a difficult challenge for immigrant countries, through this study, failure to adjust birth rates is a burden to the migrant source country as in the long

[2]Manacorda, et. al. (2006) and Dustmann, et. al. (2005) are studies done on Britain, while others are on the U.S. economy

[3]The wage ratio is the ratio between the wage of the migrant-sending country and the wage of the migrant-receiving country. The workforce ratio is the ratio of the workforce in the migrant-sending country to the workforce of the migrant-receiving country. These will be explained further when the model is introduced and solved.

run, those who choose not to migrate end up with relatively lower wages while the migrant-receiving country ends up facing much higher wages than if assimilation was immediate.

Section 2.2 discusses the model of homogenous labor, the results and the numerical simulation results dependent on the probability of migration is less than one. In Section 2.3, we discuss some results on the growth rate of wages and the numerical analysis for how wages change under different probabilities of migration. In Section 2.4, we analyze what occurs when assimilation into the foreign culture is not immediate. And we conclude the findings of this chapter in Section 2.5.

2.2 Model with Homogenous Labor

In this section, we introduce a dynamic, general equilibrium model of migration which is based on the Reichlin and Rustichini (1998) model. It is a two-country model with overlapping generations of individuals with perfect foresight that live for two periods. Both countries face similar technologies and preferences and they differ in initial capital endowments and birth rates. This model is close to Reichlin and Rustichini's model in its basic assumptions and the conditions for equilibrium. The key modification in this model of homogenous labor is that individuals are able to decide on whether they want to migrate at the beginning of the period and face a given probability of successful migration application, or just stay in the country of their birth. We treat the probability of migration as a policy variable, similar to any potential migrant facing the possibility of being granted immigrant status, and is, therefore, exogenous. A migrant-receiving country may have strict immigration policies and the probability of migration can be very low, while some may welcome more immigrants to fulfill job vacancies or as part of family reunification programs for existing immigrants, where the probability may be high. The endogeneity of the migration decision is based on the assumption that individuals are born with different abilities to pay for the cost of migration. Any crossing of borders calls for costs to be incurred. This may be due to the cost of the migration application process, and the costs incurred after the move is made, i.e. adjustment costs, initial accommodation costs and the cost of finding a job. Those that can pay for the cost of migration are then potential migrants and the endogeneity of this variable allows us to consider how changes in the relative wages affects the flow of migrants in the next time period while still be able to explain the existence of persistent migration.

We explain the model below, as adopted from Reichlin and Rustichini (1998), which changes in how the proportion of the population that is able to migrate is determined. This is both through an endogenous migration choice as well as introducing a probability of migration.

2.2.1 Technology

Firms in each country i face a Cobb-Douglas technology

$$Y_t^i = A_t^i (K_t^i)^{\alpha} (N_t^i)^{1-\alpha}, \qquad \alpha \in (0,1), \qquad A^i > 0.$$

A learning-by-doing mechanism is adapted from Arrow (1962) and Romer (1986) and assume that $A_t^i = (\bar{K}^i)_t^{\eta}$ where $\bar{K}_t^i$ is aggregate capital and where $0 < \eta < 1 - \alpha$. As firms are identical, we have $K_t^i = \bar{K}_t^i$ and

$$Y_t^i = (K_t^i)^{\alpha+\eta} (N_t^i)^{1-\alpha} \tag{2.1}$$

Due to perfect competition, the returns to capital are

$$w_t^i = (1-\alpha)(N_t^i)^{\eta}(k_t^i)^{\alpha+\eta} \tag{2.2}$$

$$r_t^i = \alpha(N_t^i)^{\eta}(k_t^i)^{\alpha+\eta-1} - \delta \tag{2.3}$$

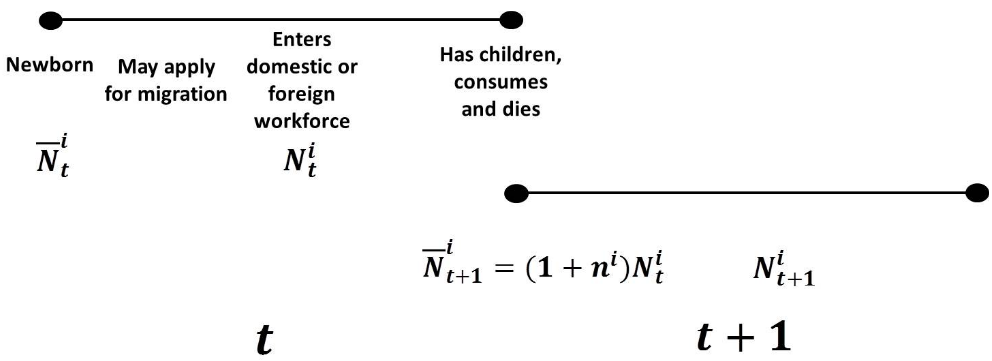
where $k_t^i = K_t^i / N_t^i$ and δ is the depreciation rate. As explained in Reichlin and Rustichini (1998), it is assumed that $\eta < 1 - \alpha$ which “guarantees that the ‘social’ marginal product of physical capital is decreasing in the capital-labor ratio,” k_t^i.

2.2.2 Household Behavior

At the beginning of each time period, there are $\bar{N}_t^i$ of newborn in each country. Each newborn a is born with a cost of applying for migration c^a which is uniformly distributed on space $[0, 1]$. Individuals provide labor when young and consume when old and their utility is linear in second-period consumption. In this study, we adopt the assumption of Reichlin and Rustichini (1998) regarding the specific form of the utility function. This enables us to reduce the migration decision to be based strictly on wages and makes the model more tractable[4]

Below is the OLG structure for this model, where $i = D, F$.

Figure 2.1: OLG Structure with Homogenous Labor



At the beginning of each time period t, each individual then decides on whether he wants to work in their home country or migrate and work in the foreign country. Each individual also faces an exogenous probability p of being accepted as a potential migrant or not. The cost of migration in country i, c^i, then has to be incurred if the individual initiates the migration process and is incurred whether or not the application is successful with a probability p. Hence, they will decide to apply for migration from country i to country j as long as

$$[pw_t^j + (1 - p)w_t^i](1 - c^i) \geq w_t^i$$

[4]This is also true for the case where labor is heterogenous discussed in Chapter 3. Other similar studies have considered different forms such as Larramona and Sanso (2003) who assume that utility is of log-linear form and and is a function of both first and second-period consumption. The migration decision is then based on a preference variable with a Pareto distribution that renders the population heterogenous. Faini (1996) uses a CES utility function and with a parameter representing the preference for location, the choice is also dependent primarily on wages. As discussed in Chapter 1, the results of these related studies have some similarities and differences with the Reichlin-Rustichini (1998) model and the specific form of the utility function does not seem to play a largely significant role. The importance in this particular study is that the choice of migration stems from the wage differential and that there is no convergence is wages even with labor mobility.

where c^i is the constant marginal cost of the migration application process in country i, w_t^j is the wage of the host country and w_t^i is the wage of the home country. The individual that can then migrate can earn wage w_t^j with a given probability p or not be allowed to migrate with a probability $(1-p)$ and earn wage w_t^i.

2.2.3 Flow of Migrants and Evolution of Workforce

There will then be a critical level of cost c^{i*} such that individuals with the cost c^a below this level choose to initiate the migration process. At the beginning of each time period t, a proportion θ_t^i of the newborn population of country i, $\bar{N}_t^i$, equal to the critical level of cost, then decides to apply for migration where

$$\theta_t^i = max\{c_t^{i*}, 0\} \tag{2.4}$$

$$\text{where} \qquad c_t^{i*} = \frac{p(w_t^j - w_t^i)}{pw_t^j + (1-p)w_t^i} \tag{2.5}$$

After the decision to apply for migration is made, an individual is successful with the application process with a probability p so that the workforce for that period in country i consists of those who remain in country i and migrants from country j. Hence, in this model, both θ_t^i and p affect the changes in the workforces over time, N_t^i with θ_t^i being dependent on how the expected wage differential changes over time.

$$N_t^i = (1 - p\theta_t^i)\bar{N}_t^i + p\theta_t^j \bar{N}_t^j \tag{2.6}$$

For each country i, the given population growth rate is n^i so that the newborn at the beginning of the next period is $\bar{N}_t^i = (1+n^i)N_{t-1}^i$, which allows us to express the evolution of country i's workforce as

$$N_{t+1}^{i} = (1 - p\theta_{t+1}^{i})(1 + n^{i})N_{t}^{i} + p\theta_{t+1}^{j}(1 + n^{j})N_{t}^{j} \tag{2.7}$$

This term is composed of those of the newborn of the current period that don't decide to migrate as well as those who decide to migrate but with a probability of $(1 - p)$ are not able to and of those who, with a probability p migrate to that country.

2.2.4 Equilibrium

Similar to Reichlin and Rustichini (1998), we also define equilibrium to be a perfect foresight equilibrium as well as make the same additional assumptions, as stated below.

Definition 1. *We define equilibrium to be a perfect foresight equilibrium for the world economy and is a sequence* $\{K_t^k, N_t^k, w_t^k, r_t^k, \theta_t^k, p_t^{kl}; t = 1, 2...\}_{k,l=i,j}$ *which satisfies the following equations* $\forall k, l = i, j$*:*

$$w_t^k = (1 - \alpha)(N_t^k)^{\eta}(k_t^k)^{\alpha+\eta} \tag{2.8}$$

$$r_t^k = \alpha(N_t^k)^{\eta}(k_t^k)^{(\alpha+\eta-1)} - \delta \tag{2.9}$$

$$N_{t+1}^{k} = (1 - p\theta_{t+1}^{k})(1 + n^{k})N_{t}^{i} + p\theta_{t+1}^{l}(1 + n^{l})N_{t}^{l} \tag{2.10}$$

$$\theta_t^k = \frac{p(w_t^k - w_t^l)}{pw_t^k + (1 - p)w_t^k} \tag{2.11}$$

$$K_{t+1}^k + K_{t+1}^l = w_t^k N_t^k + w_t^l N_t^l \tag{2.12}$$

$$r_t^k = r_t^l \tag{2.13}$$

Eq.(2.12) follows from the assumption that total world savings from the previous period is equal to the world capital stock of the current period, and Eq.(2.13) follows from the assumption of perfect capital mobility. This assumption of perfect capital mobility may be an extreme case; however, this enables us to focus on the changes of the wages and the wage differential. As discussed in Reichlin and Rustichini (1998), in the neoclassical growth model, with perfect capital mobility, capital will flow towards the country with a higher return to capital and the capital-labor ratios for each country will equalize, rendering wages equal as well. Thus, the free mobility of capital reduces the wage gap and removes any incentive to migrate. In this model, we continue to make this assumption as well to show that this assumption in a model of increasing returns does not necessarily lead to the reduction of the wage gap.[5]

2.2.5 Analysis and Results

From the assumption of perfect capital mobility, and Eqs.(2.2) and (2.3), we can express the wage ratio as a function of the ratio between the workforce in country i and country j,

$$\frac{w_t^i}{w_t^j} = \left[\frac{N_t^i}{N_t^j}\right]^{\gamma} \tag{2.14}$$

where $\gamma = \frac{\eta}{1-\alpha-\eta}$. From this expression, it can be noted that the wage ratio is an increasing function in the workforce ratio and that the movement of labor from country i to country j widens the wage gap. Hence, the possibility of persistent migration relies primarily on scale effects. As Reichlin and Rustichini (1998) have pointed out, if there was no externality and $\eta = \gamma = 0$, then this illustrates how using the neoclassical model leads to the automatic equalization of wage rates and causes migration to be temporary.

In the following analysis, we also assume that $w_t^j > w_t^i$ and this restricts

[5]Faini (1996) and Larramona and Sanso (2003) use different technologies and assume that capital mobility is less than fully mobile and achieve similar results, that is, that factor mobility does not lead to convergence.

the flow of migration from country i to country j[6]. This also implies that θ_t^i is positive while θ_t^j is zero. For ease of notation, we then let $j = F$ to denote the migrant-receiving country as the "foreign" country and $i = D$ to denote the migrant-sending country as the "domestic" country.

We also analyze the transition path of the workforce ratio instead of the newborn population ratio. We define the workforce ratio as $\lambda_t = N_t^i/N_t^j$ and the newborn ratio as $\phi_t = \bar{N}_t^i/\bar{N}_t^j$. By analyzing the transition path of the workforce ratio, we are able to assess the relation between changes in the workforce and changes in the wage rate in a more straightforward manner. We show that the workforce ratio and the newborn population ratio are topologically equivalent to carry out the analysis.

Lemma 1. *Let $\phi_{t+1} = f(\phi_t)$ and $\lambda_{t+1} = g(\lambda_t)$, then the discrete dynamical systems of functions f and g are topologically equivalent.*

Proof. Because we assume that one wage rate is larger than the other, then if $\theta^i > 0$, then $\theta^j = 0$ which allows us to derive λ_t from Eq.(2.6) as

$$\lambda_t = \frac{(1 + p\theta_t^i)\bar{N}_t^i}{\bar{N}_t^j + p\theta_t^i\bar{N}_t^i} \tag{2.15}$$

Since $\phi_t = \bar{N}_t^i/\bar{N}_t^j$, we can derive an expression where $\lambda_t = h(\phi_t)$

$$\lambda_t = \frac{(1 + p\theta_t^i)\phi_t}{1 + p\theta_t^i\phi_t} \tag{2.16}$$

Hence, for all time periods t, the mapping $h : \mathbb{R}_+ \to \mathbb{R}_+$ is one-to-one and

[6] In Reichlin and Rustichini (1998) it is assumed that $N_t^j > N_t^i$, which also implies the same condition on wages. Due to the assumption of increasing returns, the total workforce and the capital per worker have a positive effect on wages and we want to have a simple condition that will incite the movement of workers from country j to country i. Hence, we simplify this restriction based on wages and not on capital per worker. The same is true for the model where the workforce is heterogenous, as discussed in Chapter 3. In addition to this, Faini (1996) discusses that the literature on trade theory, a wage differential causes migration to occur.

continuous and h^{-1} is continuous. □

Given $\lambda_t = N_t^D/N_t^F$ and letting $\mu = (1+n^D)/(1+n^F)$ and from Eqs. 2.4 and 2.7, we can then solve for the following system of equations that describe the movement of the workforce ratio over time.

$$\lambda_t = \frac{(1-p\theta_t)\mu\lambda_{t-1}}{1+p\theta_t\mu\lambda_{t-1}} \tag{2.17}$$

where

$$\theta_t = \frac{p(1-\lambda_t^\gamma)}{p+(1-p)\lambda_t^\gamma} \tag{2.18}$$

Eq. (2.18) is the main difference of this model with that of Reichlin and Rustichini (1998) since, in this case, θ_t changes over time and is a function of the expected workforce ratio and, hence, the wage ratio, while this is taken to be fixed in their study.

Case: $p = 1$

In this case, we assume that the probability of migration is equal to one for all time periods. Hence, the system of equations can be simplified to

$$\lambda_t = \frac{\lambda_t^\gamma \mu\lambda_{t-1}}{1+\mu\lambda_{t-1}(1-\lambda_t^\gamma)} \tag{2.19}$$

And at steady state, we can find

$$\mu(\lambda - \lambda^{1+\gamma} - \lambda^\gamma) = -1 \tag{2.20}$$

We apply a first-order Taylor approximation on Eq.(2.20) to the function $\lambda_t = g(\lambda_{t-1})$ in the neighborhood of steady-state λ. And by Implicit Function Theorem, we find $g'(\lambda_{t-1})$ to be

$$g'(\lambda_{t-1}) = -\frac{\mu(\lambda_t - \lambda_t^{1+\gamma} - \lambda_t^{\gamma})}{1 + \mu(\lambda_t - (1+\gamma)\lambda_{t-1}\lambda_t^{\gamma} - \gamma\lambda_{t-1}\lambda_t^{\gamma-1})} \tag{2.21}$$

And at steady state, we have

$$g'(\lambda) = \frac{-\mu(\lambda - \lambda^{1+\gamma} - \lambda^{\gamma})}{1 + \mu(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma})} \tag{2.22}$$

Proposition 1. *For any $\lambda_0 \in (0,1)$, $\gamma > 0$, and μ, if $\mu > 1$, then there exists a perfect foresight equilibrium such that, for all $t \geq 0$, and for two constants c_1 and c_2, where c_1, $c_2 > 0$,*

$$\frac{w_t^F}{w_t^D} \geq c_1 > 1 \qquad \frac{N_t^D}{N_t^F} \leq c_2 < 1 \tag{2.23}$$

Proof. The above proposition follows from Eqs. (2.17) and (2.18). When $\lambda_t \in (0,1)$, in any perfect foresight equilibrium, it must be that $\theta_t \in (0,1)$. Hence, the migration flow will just be from country D to country F.

Since we can find a fixed point λ^* given by Eq.(2.20), and since $(\lambda - \lambda^{1+\gamma} - \lambda^{\gamma}) \in (-1,0)$, λ^* is also less than 1 if $\mu > 1$. This implies that the workforce ratio remains strictly below 1 and the wage ratio remains strictly larger than 1. Thus, the function given by Eq.(2.19) generates a perfect foresight equilibrium where the flow of migrants is from country D to country F. □

Proposition 2. *If $\mu > 1$, then for any $\gamma > 0$, there exists a steady-state solution $\lambda*$.*

Proof. From Eq.(2.20), it is immediate that for any value of $\mu > 1$, the term $(\lambda - \lambda^{1+\gamma} - \lambda^{\gamma})$ has to be equal to a value less than 1. In which case, it must be that steady-state $\lambda^* \in (0,1)$. Since $\theta = (1 - \lambda^{\gamma})$ when $p = 1$, then $\theta \in (0,1)$. □

This implies that at the steady state, there is a proportion θ^* of the domestic population that will migrate from country D to country F because $\lambda^* < 1$. For this to occur, the population growth rate of country D has to be larger than that of country F in order to sustain this movement of labor. This result is similar to that of Reichlin and Rustichini in that the migrant-source country has to have a larger birth rate in order to sustain the movement of migrants to the migrant-receiving country, given that the latter has higher wages. In this case, when $\lambda \in (0,1)$ at steady state, $N^D < N^F$ and, thus, $w^D < w^F$ rendering migration persistent.

We then assess the local stability properties of this nonlinear system below. In the analysis done by Reichlin and Rustichini (1998) in their model of homogenous labor, they find that the steady-state workforce ratio displays divergence. This is a quite straightforward result as the flow of migrants from country D to country F will continuously give the migrant-sending country a relatively lower workforce, and the migrant-receiving country a relatively higher workforce. In the case of this model where we have made the migration choice endogenous, however, and the proportion of migrants, θ can, thus, change over time due to the changes in wages, we find that this local stability properties may differ depending on the values of γ and μ.[7]

Proposition 3. *For $\lambda^* \in (0,1)$, $\mu \in (1,2)$ and any $\gamma > 0$, there are three possible cases:*

(1) there is monotonic divergence away from λ^* when $\mu(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma}) \in (-1,0)$,

[7]This will be explored more in Section 2.2.6

(2) there are explosive oscillations when $\mu(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma}) \in (-2, -1)$*, and*

(3) there are damped oscillations when $\mu(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma}) < -2$*.*

Proof. We first show that $g'(\lambda) \neq 1$ to apply the Hartman-Grobman theorem on local stability properties of nonlinear systems. We use the condition from the steady state, from Eq. (2.20) to simplify Eq.(2.22) to

$$g'(\lambda) = \frac{1}{1 + \mu(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma})} \tag{2.24}$$

To show that $g'(\lambda) \neq 1$, we need to show that $\mu(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma}) \neq 0$. For $g'(\lambda) = 1$, the term $(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma})$ has to be equal to 0 since $\mu \in (1, 2)$. Since $\gamma > 0$, $\lambda \in (0, 1)$, $(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma}) \neq 0$ and $g'(\lambda) \neq 1$.

We then need to show that $g'(\lambda)$ is well-defined. Note that the term $\mu(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma})$ cannot be equal to -1. From Eq.(2.20), for $\mu(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma}) = -1$ to hold, it must be that $\gamma = 0$ and, also, that $\gamma = 1$, which cannot be true. Hence, $\mu(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma}) \neq 1$.

Thus, for any $\mu \in (1, 2)$, $\lambda \in (0, 1)$ and any $\gamma > 0$, $(\lambda - (1+\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma}) < 0$ and conditions for monotonic divergence, explosive oscillations, and damped oscillations are then immediate from Eq.(2.22).

□

Comparative Statics

From the implicit function $G = \mu(\lambda - \lambda^{1+\gamma} - \lambda^{\gamma}) + 1 = 0$, we analyze how steady-state λ changes with respect to the parameters μ, the ratio of birth rates, and γ, the parameter that is a function of the externality, given the relative shares of K and L to output.

By Implicit Function Theorem, we can find the following partial derivatives

$$\frac{\partial \lambda}{\partial \mu} = -\frac{(\lambda - \lambda^{1+\gamma} - \lambda^{\gamma})}{\mu(1-(1+\gamma)\lambda^{\gamma} - \gamma\lambda^{\gamma-1})} \tag{2.25}$$

And for how λ changes with respect to γ, we have

$$\frac{\partial \lambda}{\partial \gamma} = \frac{(\lambda^{1+\gamma} - \lambda^{\gamma}) \ln \lambda)}{(1-(1+\gamma)\lambda^{\gamma} - \gamma\lambda^{\gamma-1})} \tag{2.26}$$

From the possible values of the parameters μ, γ and steady-state λ, we note that $\frac{\partial \lambda}{\partial \mu} < 0$, which implies that at a lower steady state λ, i.e., that at a larger difference between the two countries' workforces, a higher ratio of birth rates is needed to sustain the transfer of people as, in addition to this, with a lower steady-state λ, the proportion of the domestic population that migrates, θ, is higher. This describes the movement of people in the very simple scenario where the pattern of migration is from one country to another. A lower workforce ratio implies a larger wage gap, and hence a larger proportion of migrants, and a birth rate ratio higher than one is needed for this movement to be sustained.

For the next two equations, we note that $\frac{\partial \lambda}{\partial \gamma} > 0$. This implies that a higher value for γ or, since $\gamma = \eta/(1-\alpha-\eta)$, the higher the value of the externality, the steady-state workforce ratio and, consequently, the steady state wage differential are also higher. Hence, the size of the externality plays a role in the basic result of the homogenous model. From Eq.(2.14), when γ has a higher value, for a given workforce ratio, the wage differential is higher. Hence, when η, the externality is higher, the wage ratio is higher, which implies that the wage differential is smaller. Hence, a higher externality results in a smaller proportion of migrants. This can be attributed to the marginal product of labor being higher when the externality is of a larger value.

For a clearer analysis, we discuss the range of values for γ and μ that illustrates this in Section 2.2.6.

Case: $p \in (0, 1)$

We now consider the scenario when the probability of migrating is less than one. In this case, there is a certain proportion of the population that intends to migrate, but only a given probability p will be allowed to migrate, which more closely reflects how immigration policies in most countries are carried out. This is another modification of the Reichlin-Rustichini model as a limit is now imposed on the proportion of the population that can migrate.

In this scenario, at steady state, from Eqs.(2.17) and (2.18), we have

$$\lambda = \frac{(1 - p\theta)\mu\lambda}{1 + p\theta\mu\lambda} \tag{2.27}$$

where

$$\theta = \frac{p(1 - \lambda^{\gamma}}{p + (1 - p)\lambda^{\gamma}} \tag{2.28}$$

We can then re-arrange to arrive at this function

$$G = 1 + p\theta\mu\lambda - (1 - p\theta)\mu = 0 \tag{2.29}$$

where Eq. (2.28) still applies.

From Eq.(2.29), we then find that

$$p\theta(p, \lambda, \gamma)(1 + \lambda) = \frac{(\mu - 1)}{\mu} \tag{2.30}$$

This implies that for a positive p, λ and γ, it must be that $\mu \in (1, 2)$, which is similar to the case where $p = 1$.

However, in this case, we cannot analytically find a particular range for p where λ can be restricted to be between 0 and 1. This will be discussed in Section 2.2.6.

We then find $\frac{\partial \lambda}{\partial p}$ to assess how a lower probability (i.e. $p < 1$) affects this dynamical system. First, we have

$$G_\lambda = p\mu\left(\theta + \frac{\partial \theta}{\partial \lambda}(1+\lambda)\right) \tag{2.31}$$

where

$$\frac{\partial \theta}{\partial \lambda} = -\frac{p\gamma\lambda^{\gamma-1}}{(p + (1-p)\lambda^\gamma)^2} \tag{2.32}$$

Next, we also find

$$G_p = \mu\left(\theta + \frac{\partial \theta}{\partial p}p\right)(1+\lambda) \tag{2.33}$$

where

$$\frac{\partial \theta}{\partial p} = \frac{(1-\lambda^\gamma)\lambda^\gamma}{(p + (1-p)\lambda^\gamma)^2} \tag{2.34}$$

Thus, we find that, by the Implicit Function Theorem, and using Eqs. (2.31) and (2.33),

$$\frac{\partial \lambda}{\partial p} = -\frac{\mu(\theta + \frac{\partial \theta}{\partial p}p)(1+\lambda)}{p\mu(\theta + \frac{\partial \theta}{\partial \lambda}(1+\lambda))} \tag{2.35}$$

We note that, from Eq. (2.34) that $\frac{\partial \theta}{\partial p} > 0$ since $\lambda > 0$, $p > 0$ and, hence, $(1-\lambda^\gamma) > 0$ and $(1-p) > 0$, which implies that the higher the possibility of migration and the higher the possibility of earning the higher foreign wage, then a larger proportion of the domestic population will intend to migrate. Also, from

Eq. (2.32) $\frac{\partial\theta}{\partial\lambda} < 0$, which implies that the smaller the expected workforce ratio (or the larger the difference between the domestic wage and foreign wage), then the higher the proportion of potential migrants.

Thus, from Eq. (2.34), since $\mu(\theta + \frac{\partial\theta}{\partial p}p)(1+\lambda) > 0$, we have two possible scenarios for how the probability of migration affects the workforce ratio λ at the steady-state depending on the sign of $p\mu(\theta + \frac{\partial\theta}{\partial\lambda}(1+\lambda))$ as $\frac{\partial\theta}{\partial\lambda} < 0$: (1) $\frac{\partial\lambda}{\partial p} > 0$ if $\frac{\partial\theta}{\partial\lambda}(1+\lambda) > \theta$ and (2)$\frac{\partial\lambda}{\partial p} < 0$ if $\frac{\partial\theta}{\partial\lambda}(1+\lambda) < \theta$. We see how both these cases occur given certain parameter values in Section 2.2.6. This implies two possible behaviors by which the newborn population deciding on who migrates or not responds to changes in the steady-state workforce ratio. If, as in the first case, θ is more responsive to the expected workforce ratio λ, i.e. the lower the expected workforce ratio (and the larger the wage differential), then the proportion of those who can migrate decreases relatively more. In this case, an increase in the probability of migrating, then leads to a higher workforce ratio, implying that the number of migrants, $p\theta$, will increase. In the second case, if θ is then less responsive to changes in the steady-state workforce ratio, then an increase in the probability of migrating will lead to a lower workforce ratio, implying that the number of migrants, $p\theta$, decreases.

2.2.6 Numerical Analysis

In this section, we discuss examples of the range of values for the parameters γ and μ that display the behavior of λ^*, the steady-state workforce ratio for two cases: when $p = 1$ and when $p \in (0, 1)$.

For this analysis, since $\mu = (1 + n^D)/(1 + n^F)$, with $n^D, n^F \in (0, 1)$, then $\mu \in (1, 2)$. We then limit the values of μ to be $\mu \in (1, 2)$ following Proposition 1 for the case when $p = 1$ as well as for when $p \in (0, 1)$ from the discussion above. The parameter γ can be any number between $(0, \infty)$, and to simplify, we choose $\gamma \in (0, 20)$.[8] With n being the number of elements in each vector of these parameters, we choose $n = 50$.[9]

Case: $p = 1$

Figure B.1 illustrates how steady-state λ changes with respect to μ and γ. This shows what was discussed earlier, with $\frac{\partial \lambda}{\partial \mu} < 0$, and $\frac{\partial \lambda}{\partial \gamma} > 0$, from Eqs. (2.25) and (2.26). Thus, steady-state λ is decreasing in μ, the workforce ratio, and is increasing in γ, with steady-state λ always having a value between $(0, 1)$. This verifies the role of the migrant-sending country having the higher birth rate and, also, the role of the size of the externality in lowering the workforce ratio.

Figure B.2 then illustrates how $g'(\lambda)$ changes with respect to μ and γ. From Proposition 3, it is possible for there to be divergence, explosive oscillations or damped oscillations, depending on which condition is satisfied as stated in the proposition. From the graph, it can be observed that for values of γ very close to 0, for any value of μ, divergence and explosive oscillations are the more possible cases. However, for larger values of γ, the typical scenario is for steady-state λ to display damped oscillations.

This is different from the result found by Reichlin and Rustichini (1998) wherein for a similar condition of $\mu \in (0, 1)$, steady-state λ displayed monotonic divergence or monotonic convergence - in the case where the migration flow may

[8]The values for the parameter $\gamma = \eta/(1 - \alpha - \eta)$ are determined from the the possible range of values of $\alpha \in (0.2, 0.5)$ and the externality $\eta \in (0.1, 1 - \alpha)$.

[9]The numerical simulations are done through MATLAB. The exercise was run numerous times to verify that the movement of the variables is not dependent on the number of elements in the set of possible values for the parameters. Hence, we limit $n = 50$ for a more efficient analysis.

reverse. Hence, in this case, having migration as an endogenous choice leads to different local stability properties of the steady-state workforce ratio.

In later exercises, we specify the values of the parameters to be $\mu = 1.011$, $\alpha = 0.3$, $\eta = 0.2$ and $\gamma = 0.4$. We specify μ based on the average birth rates of Mexico and the United States from the years 1960 to 2009[10]. Using these values, we find the steady-state value of the workforce ratio, λ, to be close to 1, and the stability property is monotonic divergence which reiterates the Reichlin-Rustichini (1998) result. Using Eq. (2.19), Figure B.6 shows the relationship between λ_t and λ_{t-1}, which exhibits monotonic divergence away from the steady-state given the parameters chosen.

Case: $p \in (0, 1)$

We then allow the probability of migrating to be less than one to observe in what range of probabilities permanent migration can still be expected to occur from country D to country F. We use the same parameter specifications as those used when $p = 1$.

Figure B.3 shows us how λ^* changes with respect to μ and γ when $p = 0.80$. The behavior of λ^* with respect to γ and μ when $p < 1$ is similar to the case when $p = 1$.

However, upon checking for the robustness of this exercise for the values chosen for the parameters γ and μ, this similarity in behavior of steady-state λ for the same specified range of values of γ and μ only occurs for higher probabilities, specifically when $p > 0.60$. This implies that we can expect a similar behavior towards permanent migration from country D to country F only if the probability of migration is high enough, at least for the range of values specified for γ and

[10]Source: World Development Indicators (2010)

μ. We can infer that for lower probability values, after some time periods, the incentive to apply for migration is not high enough as there is a lower chance of being accepted as a migrant and, hence, a relatively greater part of the population will choose not to apply for migration and so the one-direction flow cannot be sustained.[11]

The Role of Probabilities in the Workforce Ratio

We also look at the workforce ratio at steady state, λ, and how it depends on the probability of migration, p. For this numerical analysis, similar to the previous exercise, we set $\mu = 1.011$, $\alpha = 0.3$, $\eta = 0.2$, which leads to a $\gamma = \eta/(1-\alpha-\eta) = 0.4$. We use the average birth rates of Mexico and the United States over 60 years and the ratio is 1.011.[12] This ratio of birth rates is also similar to that of Philippines-U.S., and Bangladesh-India, which are also established migration corridors. Although other cases of persistent migration exist where the ratio of birth rates may be less than one, we focus on the case where the migrant-sending country has a higher birth rate than the migrant-receiving country due to the result from Proposition 1, where $\mu > 1$, and the discussion on μ when the probability of migration is less than one. [13]

Figure B.5 shows us that steady-state λ is increasing in p, which implies that for this set of parameters, $\frac{\partial \lambda}{\partial p} > 0$, from the discussion involving Equations (2.31),(2.33) and (2.35). This implies that when the probability of migrating approaches 1, λ also approaches 1, or the workforce level in country D is close to the workforce level in country F. As the probability of migration increases, for

[11] In the following exercises, when we focus on specific values for γ and μ, rather than considering a range of values, we will find that $p > 0$ will work for the model of homogenous labor.

[12] Source: World Development Indicators, 2010

[13] Due to the very small differences in μ for other migration corridors in the numerical exercise, we decide to focus on the Mexico-U.S. migration flow. Also, it is the migration flow studied the most in the migration literature and may be of more interest. In addition, results may be easily compared.

Table 2.1: Robustness Checks for Homogenous Labor Model

Original Parameters	$\alpha = 0.3$, $\eta = 0.25$, $\gamma = 0.625$, $\mu = 1.011$
Check 1a	$\mu = 1.014$ $\alpha = 0.3$, $\eta = 0.25$, $\gamma = 0.625$
Check 1b	$\mu = 1.008$ $\alpha = 0.3$, $\eta = 0.25$, $\gamma = 0.625$
Check 2a	$\alpha = 0.32$ $\eta = 0.2$, $\gamma = 0.4167$, $\mu = 1.011$
Check 2b	$\alpha = 0.28$ $\eta = 0.2$, $\gamma = 0.3846$, $\mu = 1.011$
Check 3a	$\eta = 0.22$ $\alpha = 0.3$, $\gamma = 0.4583$, $\mu = 1.011$
Check 3b	$\eta = 0.18$ $\alpha = 0.3$, $\gamma = 0.3462$, $\mu = 1.011$

the workforce ratio to be at steady-state, there will be fewer individuals applying for migration. Even if the probability of migrating is quite high, relatively fewer move from country D to country F leading λ to be closer to 1.

Robustness Checks

We conduct robustness checks on the results by slightly changing μ, α and η and, thus, γ and compare the steady-state λ for different probabilities. We increase and decrease each of the parameters by 0.2, μ is changed by .003 since we want to keep it above 1, due the the result of Proposition 2, and run the same analysis. The different sets of parameters are listed in Table 2.1 and describe the results below as well as in Figures B.7, B.8 and B.9 in the Appendix.

For robustness checks 1a and 1b, we vary the value of μ by +0.003 and by -0.003. Hence, γ is unchanged. It can be noted that the resulting steady-state λ is higher when μ is lower and is lower when μ is higher for every probability p. Since a lower μ indicates a relatively lower birth rate for country D, this implies that a lower birth rate leads to a lower proportion applying for migrants and, thus, a relatively higher remaining workforce.

For robustness checks 2a and 2b, we vary the value of α by +0.02 and by -0.02. This leads to a different value of γ, even if η is unchanged. The differences in the resulting steady-state λ is smaller than when μ was varied. A higher α leads to a higher γ, and a higher γ leads to a higher steady-state λ, as shown in Figure B.8.

For robustness checks 3a and 3b, we vary the value of η by +0.02 and by -0.02. This also changes the value of γ, even if α remains the same. A higher η leads to a higher γ, which also leads to a higher steady-state λ for every probability p in Figure B.9.

2.3 Movement and Changes in Wages with Homogenous Labor

In this section, we assess how wages change given a model of persistent migration. The exercise is quite simple as the wage ratio is simply dependent on the workforce ratio from Eq.(2.14). We start by first looking at the growth rates of both the workforce of country D and of country F and, as a result of that, the growth rates of the wages w^D and w^F. We then run numerical simulations on the steady-state wage ratio, as an extension of the basic numerical analysis conducted in the previous section.

2.3.1 Analysis and Results

Since $N_{t+1}^D = (1 - p\theta_{t+1})(1 + n^D)N_t^D$, the growth rate of domestic workforce, denoted by g_{N^D}, is calculated below.

$$g_{N^D} = \frac{N_{t+1}^D - N_t^D}{N_t^D} = (1 - p\theta_{t+1})(1 + nD) - 1 \tag{2.36}$$

As for the foreign workforce, since $N_{t+1}^F = (1 + n^F)N_t^F + p\theta_{t+1}N_{t+1}^D$, then the growth rate of the foreign workforce can be calculated as shown below.

$$g_{N^F} = \frac{N_{t+1}^F - N_t^F}{N_t^F} = n^F + p\theta_{t+1}\frac{N_{t+1}^D}{N_t^F} \tag{2.37}$$

In Eqs.(2.36) and (2.37), it can be noted that the growth rate of the workforce in country D is dependent only on its birth rate and the proportion of emigrants, whereas the growth rate of the workforce in country F is not only dependent on its own birth rate but also country D's birth rate due to the term N_{t+1}^D. It is also a function of the proportion of immigrants. This is a result of persistent migration that is one-directional, due to the assumption on wages. We present some basic results below to better understand the analysis on wages.

Lemma 2. *For all time periods t, $g_{N^D} > 0$ and there are three possible cases for g_{N^D}: (1) $g_{N^D} > 0$ if $n^D > \frac{p\theta_t}{1-p\theta_t}$, (2) $g_{N^D} = 0$ if $n^D = \frac{p\theta_t}{1-p\theta_t}$, and (3) $g_{N^D} < 0$ if $n^D < \frac{p\theta_t}{1-p\theta_t}$.*

Proof. We can express Eq.(2.36) as $g_{N^D} = n^D - p\theta_t n^D - p\theta_t$. Hence, we can see if g_{N^D} is greater than, equal to or less than zero by the relation of n^D to $p\theta_t/(1-p\theta_t)$ as $g_{N^D} > 0$ if $n^D(1-p\theta) > p\theta$. □

The above result on g_{N^D} implies that the labor force of country D at time period t may be larger than its labor force in the previous period if the population growth rate is larger than the ratio of emigrants to the remaining workforce. Thus, if the population growth rate is large enough to make up for the emigrants leaving country D, then it will have a larger workforce in the current time period compared to the previous time period.

Lemma 3. *For all time periods t, $g_{N^F} > 0$.*

Proof. From Eq. (2.37), since $n^F > 0$ and $p\theta_t \frac{N_t^D}{N_{t-1}^F} = p\theta - (1-p\theta_t)(1+n^D)\lambda_t > 0$, then $g_{N^F} > 0$. □

This implies that due to the persistent flow of migrants from country D into country F, the workforce of country F constantly grows positively. Therefore, the change in the ratio of the levels of the workforce of country D to country F is dependent on the growth rate of country D's workforce as they may have three different scenarios. Another result below is an observation regarding the ratio of birth rates.

Lemma 4. *If $\mu = (\frac{1}{(1-p\theta_t)^2})(\frac{N_t^F}{N_t^D})$ for some time period t, then $\lambda_t = \lambda_{t-1}$.*

Proof. For $\lambda_t = \frac{N_t^D}{N_t^F} = \frac{N_{t-1}^D}{N_{t-1}^F} = \lambda_{t-1}$, it must be that, based on Lemma 3, $g_{N^D} = g_{N^F>0}$ for time t. This results into the following equation.

$$(1 - p\theta_t)(1 + n^D) - 1 = n^F = p\theta_t(1 - p\theta_t)(1 + n^D)\lambda_t \tag{2.38}$$

From Eq.(2.38), we can simplify this to calculate for $\mu = \frac{1+n^D}{1+n^F}$ and find that $\mu = (\frac{1}{(1-p\theta_t)^2})(\frac{N_t^F}{N_t^D})$. □

The above result simply states that a specific value of μ may lead to a zero change in the workforce ratio and, hence, no change in the wage differential. This implies that if the ratio of birth rates is equal to the proportion of migrants that leave country D to go to country F as well as the newborn of country F, then the newborn of period t simply covers the loss of workforce in period $t-1$ and the addition of newborn in country F.

As for the wage rates, the domestic wage rate is given as $w_t^D = (1-\alpha)(N_t^D)^{-\alpha}(K_t^D)^{\alpha+\eta}$. We then calculate for the domestic wage growth rate as $g_{w^D} = -\alpha g_{N^D} + (\alpha+\eta)g_{K^D}$ where g_{K^D} is the growth rate of capital in country D.

The foreign wage rate is given as $w_t^F = (1-\alpha)(N_t^F)^{-\alpha}(K_t^F)^{\alpha+\eta}$. The growth rate of the wage of country F is then $g_{w^F} = -\alpha g_{N^F} + (\alpha + \eta)g_{F^D}$ where g_{K^F} is the growth rate of capital in country F. Substituting the workforce growth rates of both countries D and F int the wage growth rates, we have the following equations.

$$g_{w^D} = -\alpha[(1 - p\theta_t)(1 + n^D) - 1] + (\alpha + \eta)[g_{K^D}] \tag{2.39}$$

$$g_{w^F} = -\alpha[n^F + p\theta_t(1 - p\theta_t)(1 + n^D)\lambda_{t-1}] + (\alpha + \eta)[g_{F^D}] \tag{2.40}$$

Recall that for this model, we assume perfect capital mobility for all time periods. Hence, $r_t^D = r_t^F \ \forall t$. Since for each country i, we have $r_t^i = \alpha(N_t^i)^\eta(k_t^i)^{(\alpha+\eta-1)} - \delta$, the interest growth rate is $g_{r^D} = (1-\alpha)[g_{N^D}] + (\alpha+\eta-1)[g_{K^D}]$. Because of perfect capital mobility, then for all time periods t, $g_{r^D} = g_{r^F}$, which gives us another result.

It may then be noted that the difference in the changes between the workforces of countries D and F, $(g_{N^F} - g_{N^D})$, is simply a function of the difference in the changes between the countries' capital stocks, $(g_{K^D} - g_{K^F})$ at each time period t. This is due to the assumption on perfect capital mobility. Since $g_{r^D} = g_{r^F}$, we can equate both and re-arrange to find $(g_{N^F} - g_{N^D}) = (\frac{\alpha-\eta-1}{1-\alpha})(g_{K^D} - g_{K^F})$.

From Lemmas 2 and 3 on the growth rate of the workforces, we can then make observations regarding the growth rates of wages.

Proposition 4. *Let $g_N^F > 0 \ \forall t$, then there are three possible cases for g_{w^D}:*
(1)$g_{w^F} > 0$ if $(\frac{\alpha+\eta}{\alpha})[g_{K^F}] > g_{N^F}$,
(2)$g_{w^F} = 0$ if $(\frac{\alpha+\eta}{\alpha})[g_{K^F}] = g_{N^F}$,and
(3)$g_{w^F} < 0$ if $(\frac{\alpha+\eta}{\alpha})[g_{K^F}] < g_{N^F}$.

Proof. From Lemma 3, $g_N^F > 0 \ \forall t$. The size of the growth rate of the foreign wage is is immediate from Eq.(2.39). □

For the growth rate of the wage of country D, there are many more possibilities since the growth rate of the domestic workforce has three possible cases.

Proposition 5. *If $g_N^D > 0$, then (1)$g_{w^D} > 0$ if $(\frac{\alpha+\eta}{\alpha})[g_{K^D}] > g_{N^D}$, (2)$g_{w^D} = 0$ if $(\frac{\alpha+\eta}{\alpha})[g_{K^D}] = g_{N^D}$, (3)$g_{w^D} < 0$ if $(\frac{\alpha+\eta}{\alpha})[g_{K^D}] < g_{N^D}$.*

If $g_N^D = 0$, then (1)$g_{w^D} > 0$ if $(\frac{\alpha+\eta}{\alpha})[g_{K^D}] > 0$, (2)$g_{w^D} = 0$ if $(\frac{\alpha+\eta}{\alpha})[g_{K^D}] = 0$, *(3)*$g_{w^D} < 0$ *if* $(\frac{\alpha+\eta}{\alpha})[g_{K^D}] < 0$.

If $g_N^D < 0$, *then (1)*$g_{w^D} > 0$ *if* $(\frac{\alpha+\eta}{\alpha})[g_{K^D}] > -g_{N^D}$, *(2)*$g_{w^D} = 0$ *if* $(\frac{\alpha+\eta}{\alpha})[g_{K^D}] = -g_{N^D}$, *(3)*$g_{w^D} < 0$ *if* $(\frac{\alpha+\eta}{\alpha})[g_{K^D}] < -g_{N^D}$.

Proof. This is immediate from Eq.(2.40). □

2.3.2 Numerical Analysis

From Eq. 2.14, it can be noted that the steady-state wage ratio is an increasing function of λ, hence, we can calculate the wage ratio for every probability p of migrating given certain values of μ and γ. Figure B.10 in Appendix C shows us the wage ratios given $\mu = 1.011$ and $\gamma = 0.4$, the same parameters used in Section 2.2.6.

It can be noted that the higher the probability of migration (or the higher the immigration quota), the higher the steady-state workforce ratio, λ, and, thus, the higher or the closer to 1 the wage ratio, $\frac{w^D}{w^F}$ at steady-state. At very high levels of allowed immigration, at steady-state, there is then a lower proportion of migrants due to the wage differential being closer to zero. Hence, for the migrant-sending country, a higher probability of migrating allows it to have a wage that is closer to the foreign wage, while the migrant-receiving still receives a relatively higher wage.

2.4 The Possibility of Deferred Assimilation

In this section, we introduce the possibility of immigrants experiencing assimilation into the local culture one generation later as their adoption of the birth rate of the foreign country occurs one period later. In the current model, assimilation is assumed to be immediate. We, thus, consider the idea that assimilation may be deferred by one time period and assess the results.

The ease at which immigrants integrate themselves into the foreign country's cultures, norms and practices may sometimes be a very sensitive but also extremely difficult issue, especially for the migrant-receiving country. The presence of immigrants normally brings about questions of job security, impact on natives' wages and employment opportunities. It is, however, the addition of the possibility of immigrants being unable to assimilate into the culture that can bring immigration issues to the fore and be a cause for debate in political agendas. The difficulties that exist in dealing with differences between immigrant groups and natives, however, are not very well understood from an economic perspective.

Few of the earlier works on immigrant assimilation deal more with how long it takes for newly-arrived immigrants to earn as much as those who had arrived years before and also as compared to the native workers (Chiswick, 1978; Borjas, 1985). These studies are, therefore, more concerned with assimilation in the labor market. In terms of other means of immigrant assimilation and the economic impact of the different processes of assimilation taken by immigrant groups, there is really an absence of thorough studies done.

In this chapter, we focus on the birth rate or the fertility of immigrants. Research in this area has focused primarily on demography and less on the economic impact; however, there are some interesting insights which might be useful to explore in the study of persistent migration. There exist studies which have looked at how fertility and migration are related, basically finding that immigrants that have settled in areas with lower fertility rates tend to adopt those fertility rates even if they have emigrated from areas that have higher fertility rates (Ahlburg and Jensen, 2004). While studies in the demographic literature have attributed this change in behavior to being distinct from spouses, or to a basic disruption of their normal lives, some have also associated this change in fertility rates to changes in the economic environment, i.e. prices and wages. Immigrants have been found to simply adapt due to the receipt of higher wages and a higher family income leading to a choice of changing their lifestyles to mirror the lifestyles of the natives that they interact with (Rosenzweig and Schultz, 1985).[14]

Therefore, in this section, we introduce this idea of deferred assimilation into the baseline model. In the earlier model, we assume that assimilation, i.e. adapting to the migrant-receiving country's birth rate, is immediate, i.e. it occurs for the first-generation migrants. First-generation migrants immediately have the same birth rate as the natives as soon as they are able to migrate. However, we would like to explore the effect of having the immigrants delay their assimilation into the native culture by maintaining their birth rate for one generation. Hence, the immigrants maintain the birth rate of their home country but their children, the second-generation migrants adopt the birth rate of the destination country and, hence, assimilation is deferred by one time period.

[14]This study has looked at rural-urban migration; however, the basic premise is the same in that migration caused by an expectation to receive higher wages affects fertility rates to be lower or there is a need to adapt to the fertility rates of the area they are migrating to.

2.4.1 Model of Homogenous Labor with Deferred Assimilation

We reflect this deferred assimilation through the birth rate, with the migrants still maintaining their birth rate n^D until they die and with their children adopting the foreign birth rate n^F as soon as they are born. Hence, adoption of the foreign population growth is delayed by one generation as opposed to immediate assimilation that was assumed earlier and occurs at time period $(t + 1)$ for immigrants of time period t.

In the model presented, the foreign workforce was composed of the native workers and the migrants and was expressed as $N_t^F = \bar{N}_t^F + \theta_t \bar{N}_t^D$. The newborn population is then simply the birth rate multiplied by the previous period's workforce, $\bar{N}_t^F = (1 + n^F) N_{t-1}^F$ implying that the entire workforce of the previous period, including both the natives and the migrants, had the same birth rate.

Under this model, we now assume that the adoption of the foreign birth rate is delayed by one generation. Hence, the newborn is expressed as

$$\bar{N}_t^F = (1 + n^F)\bar{N}_{t-1}^F + (1 + n^D)\theta_{t-1}\bar{N}_{t-1}^D. \tag{2.41}$$

The equation is made up of two terms explaining that the newborn at time period t includes the children of the natives, $\bar{N}_{t-1}^F$ born with the foreign birth rate, and the children of the migrants, $\theta_{t-1}\bar{N}_{t-1}^D$ born with the domestic birth rate.

2.4.2 Workforce Evolution with Assimilation

The primary equation affected by this assumption is the workforce ratio, λ_t. With deferred assimilation, the workforce ratio is now expressed as

$$\lambda_t = \frac{(1-\theta_t)(1+n^D)N^D_{t-1}}{(1+n^F)\bar{N}^F_{t-1} + (1+n^D)\theta_{t-1}\bar{N}^D_{t-1} + \theta_t(1+n^D)N^D_t} \tag{2.42}$$

This can then be simplified to

$$\lambda_t = \frac{(1-\theta_t)\mu\lambda_{t-1}}{n_t + \mu m_t + \theta_{t+1}\mu\lambda_t} \tag{2.43}$$

where $n_t = \frac{\bar{N}^F_{t-1}}{N^F_{t-1}}$ is the proportion of natives in the workforce of the previous period and $m_t = \frac{\theta_{t-1}\bar{N}^D_{t-1}}{N^F_{t-1}}$ is the proportion of migrants in the workforce of the previous period. And, also, where $\theta_t = (1-\lambda_t^\gamma)$. Here, we simplify the analysis by assuming that $p = 1$.

2.4.3 Analysis and Results

We then analyze this model of deferred assimilation under steady state. At steady state, the workforce ratio is

$$\lambda = \frac{(1-\theta)\mu\lambda}{\omega + \theta\mu\lambda} \tag{2.44}$$

where $\omega = n + \mu m$ and $\theta = 1 - \lambda^\gamma$. The analysis here is somewhat similar to the analysis of the case when $p = 1$ under homogenous labor. We can first observe the main parameter that needs to change for this homogenous model with deferred assimilation to hold, as discussed below.

Proposition 6. *Let $\lambda \in (0,1)$, $\gamma > 0$ and $\mu_a \in (1,2)$ where μ_a is the ratio of birth rates under a model of deferred assimilation, then $\mu_a > \mu$.*

Proof. Given the steady state Eq. (2.44), we may derive the following equation

$$\mu_a(\lambda^\gamma + \lambda^{1+\gamma} - \lambda^\gamma) = -\omega \tag{2.45}$$

Since $\lambda \in (0,1)$ and $\gamma > 0$, it must be that $(\lambda^\gamma + \lambda^{1+\gamma} - \lambda^\gamma) \in (-1,0)$. We note that since n is the proportion of natives in the workforce of country F and

m is the proportion of migrants in the workforce of country F, then it must be that $n + m = 1$. Since $\mu \in (1, 2)$, then $\omega = n + \mu m > 1$.

For $(\lambda^{\gamma} + \lambda^{1+\gamma} - \lambda^{\gamma}) \in (-1, 0)$, it must be that $\mu_a > \omega$, and since $\omega > 1$, we have $\mu_a > \mu$. □

At this point, it must be noted that Proposition 3 applies in this model as well since we can find the following equation.

$$g'(\lambda) = -\frac{\mu(\lambda - \lambda^{1+\gamma} - \lambda^{\gamma})}{\omega + \mu(\lambda - (1-\gamma)\lambda^{1+\gamma} - \gamma\lambda^{\gamma})} \tag{2.46}$$

This is similar to Eq.(2.24) and implies that the system of equations have the same local stability properties, except that for the same λ and γ, we note that μ_a is higher but still within the range $(0, 1)$.

We can also observe what happens to the steady-state workforce ratio in the case of deferred assimilation.

Proposition 7. *Let $\gamma_a = \gamma$, $\mu_a = \mu$, then for $\omega > 1$, $\lambda_a < \lambda$.*

Proof. From Equation (2.24) and from Proposition 3, we observed that $\mu_a > \mu$ if steady-state λ is to hold from the baseline model.

Hence, for ω very close to 1, then for the same γ and a larger μ, then it must be that $(\lambda - \lambda^{1+\gamma} - \lambda)_a < (\lambda - \lambda^{1+\gamma} + \lambda)$.

Since we have $(\lambda - \lambda^{1+\gamma} - \lambda) \in (-1, 0)$ for $\lambda \in (0, 1)$ and $\gamma > 0$, then the term $(\lambda - \lambda^{1+\gamma} - \lambda)$ must be closer to zero, in which case, it has to be that for the same γ, $\lambda_a < \lambda$.

□

This implies that when there is deferred assimilation, we can expect a lower steady-state λ, implying that in the long run, deferred assimilation by the immigrants leads to a lower workforce ratio, i.e. country D has a much lower workforce than country F implying that the migrant-receiving country ends up with an even larger workforce relative to that of the migrant-sending country.

2.4.4 Wages

The above analysis implies that when the migrants from country D choose to delay assimilating into country F's culture, or adopt the foreign birth rate one generation later, then at steady state, the domestic wage, w^D is much lower than the foreign wage w^F. Hence, those who choose to stay in their home country's workforce end up with much relatively lower wages while both the immigrants and natives end up with relatively higher wages.

For the migrant-receiving country, the effect is positive since its wages will be relatively higher. This is a different result from what may be expected with recent migration issues regarding assimilation. Assimilation issues are generally perceived to be negative for the migrant-receiving country; however, what this exercise essentially implies is that if country D's wages are to hold, then it is both the natives of country D and the migrants themselves that must adjust, i.e. they should have a higher birth rate than in the case when assimilation occurs immediately. Without any adjustment in their practices or if the birth rate of country D holds, then this implies that those who stay in country D will have to bear the consequences of the deferred assimilation, while the natives of country F, as well as the migrants and the migrants' offspring enjoy relatively higher wages. Thus, contrary to what migrant-receiving countries may feel, any difficulties associated with assimilating into the local culture allows them to face relatively higher wages at the steady state.

This simple result highlights that the assimilation of migrants into the migrant-receiving country is a burden held by country D and, more importantly, by its natives, and is not at all a burden held by country F.

2.4.5 Implications of Deferred Assimilation

This simple exercise of allowing for deferred assimilation in the model of homogenous labor shows that, first, the gap in the size of the labor force between the two countries is larger with deferred assimilation and, second, the wage gap is also larger. What this implies is that the burden of the deferred assimilation falls on the migrant-sending country and the proportion of the population that does not choose to migrate. As for the migrant-receiving country, the result is favorable in that their wages become relatively higher with deferred assimilation. This result is different from what issues of assimilation may be expected to show, but this highlights that the true burden of assimilation falls on the shoulders of the immigrants and those that were not able to migrate and stayed in their home country.

2.5 Conclusions

Under the model of homogenous labor, the main conclusion is that for any two countries, if one country has higher wages than the other, then there will be a proportion of the population in the migrant-sending country that will migrate to the country with higher wages. Migration will display persistence as long as the birth rate is higher in the migration-sending country. The role of scale effects in the model with only type of labor is quite strong. The movement of people from one country to another leads to the increase in the workforce and in the wage gap. This is similar to the Reichlin-Rustichini (1998) model and, hence, rendering persistent migration with one type of labor unfavorable for the migrant-sending country as their wages will be relatively lower in the long run. The additional analytical result in this study is the different dynamics of the workforce ratio and, hence, the wage ratio, due to the endogeneity of the migration choice.

Another important result we have in addition to the results of Reichlin and Rustichini (1998) is regarding the probability of migration. Given the wage differential between the two countries, a higher level of a probability of migration is better for the migrant-sending country. At steady-state, a higher probability of migration implies that a lower proportion of newborn will consider migrating as given a higher probability, the resulting workforce ratio is smaller, leading the wage gap to be lower. This implies that the wage level in the migrant-sending country will be relatively higher. For the migrant-receiving country, however, allowing free migration will result in lower wages as compared to the case where they limit migration. For a migration flow that is primarily of one type of labor, it is then more favorable, in terms of wages, for the migrant-receiving country to limit migration.

The analysis on wages due to endogenous choices in the model of Reichlin and

Rustichini (1998) then adds another dimension to the overall analysis. When we consider the probabilities of migration, if persistent migration is to occur then, at steady-state, in terms of relative wages, it is more favorable for the migrant-receiving country to limit migration while it is more favorable for the migrant-sending country to prefer unlimited migration. While this is quite an obvious result, that this is based on a dynamic model of general equilibrium and where the migration choice is endogenous makes it a more interesting result.

When we then relax the assumption of immediate assimilation, we find that the migrant-receiving country is better off when assimilation is deferred as they face higher wages. When the adoption of the local birth rate occurs for second-generation immigrants, then those that were not able to migrate face lower wages and, thus, deferred assimilation becomes more a burden for the migrant-sending country than for the migrant-receiving country.

However, all of the analysis are of course not without their limitations since we are currently only looking at one type of labor. In the following chapter, we then introduce two types of labor in the model as well as additional choice of skill acquisition for an extended analysis on workforce ratios and wage ratios.

Chapter 3

Persistent Migration and Wages: Heterogenous Labor

3.1 Introduction

While the previous chapter showed how persistent migration may be beneficial to one country and not so beneficial to the other, current migration flows are characterized by different skill levels in immigrants. Countries that are predominantly a source of migrants are concerned with issues such as the loss of skilled workers while those that are primarily a destination for migrants are concerned with how the existence of low-skilled workers affects labor market outcomes. In this chapter, we address this aspect of migration by identifying two types of labor: skilled and unskilled.

It has been reported that 35 percent of all migrants from developing countries found in developed countries have a university degree, 30 percent graduated from high school, and around 35 percent left school before finish high school.(International Organization for Migration, 2008) It then becomes important for migration of different types of labor to be analyzed as the migration of skilled labor may have different characteristics to the migration of unskilled labor, and that allowing both to occur may lead to more interesting results for persistent migration.

Most migrant-receiving countries today have several programmes for the intake of workers of different skill levels and they also differ in their approach. Countries such as Australia, Canada and the U.K. base their immigration decisions on certain migrant characteristics including skills and qualifications, where as some countries such as the U.S. and Canada base their decisions on available job offers. This implies that successful migration can only occur with a given probability depending on the skill level or qualifications of the migrants. Potential migrants who are looking to work in the medical field may have a higher chance of migrating to Canada or Australia, while those who are looking to work in construction have been welcomed more in the Middle East. This is something which we hope to address in this paper.

Differences in skill level obtained by the immigrant community is one of the characteristics of international migration. Thus, another dimension of persistent migration that we would like to include in this study is the flow of skilled and less-skilled workers into destination countries. A study by the Organisation for Economic Co-operation and Development (OECD) in 2008 reports that in the OECD area, the immigrant population holds a high share of both tertiary-educated individuals and those that have low educational attainment of all foreign-born individuals, as compared to the native born. Figure C.1 shows the different levels of education attained by the immigrants from the main countries of origin for the U.S.[1]

This illustrates that defining skill levels across migrants is interesting to analyze and to include in the model. From the perspective of the migrant-sending country, the emigration of skilled workers is an issue which in the development literature has been assessed to see if it is detrimental or not to the economy, normally referred to as "brain drain.". Brain drain, according to Kwok and Leland (1982), "is an expression of British origin commonly used to describe one of the most sensitive areas in the transfer of technology. It refers to skilled professionals who leave their native lands in order to seek more promising opportunities elswhere." In the development literature, the focus has been on skilled workers or highly-educated individuals leaving developing countries for developed countries (Lien and Wang, 2005; Beine, et.al. 2008). Bhagwati and Hamada (1974) assess that brain drain may have negative effects welfare while Haque and Kim (1995) show that it may decrease the growth of income per capita of the migrant-sending country in the long run. Some studies also show how the emigration of

[1]The report by the OECD (2008) contains similar data on all OECD countries.

skilled workers can be beneficial for the migrant-sending country such as those of Miyagiwa (1991), Mountford (1997), Vidal (1998), Beine, et. al.(2001) and Fan and Stark (2007) either through growth or some measure of the level of educated workers left behind in the migrant-sending country. Although Beine, et.al. (2001) argue this may be more of a "theoretical curiosity" given that the education decision is dependent upon the migration possibility. In this study, while we neither assess the effect of migration on growth nor welfare, we are able to asses the level of skilled workforce in the migrant-sending country relative to the level of unskilled workers since we are able to study the ratios of the different workforces in the both countries.

Another issue we consider is the existence of "overqualified" immigrants in migrant-receiving countries. This issue may still be studied further in the migration literature as accounts of immigrants adopting employment of a lower skill qualification do exist. Anecdotes of domestic helpers having acquired degrees in their home countries, for example, exist in Canada and Singapore (Momsen, 2007). Gonzalez (1997) reported that in the year 1997, 17 percent of domestic helpers from the Philippines that were located in Singapore held a university degree and around 45 percent spent at least some years in university even if they were not able to complete their degree. Similar occurrences can be found in the Mexico-U.S. migration flow where "trained teachers and nurses" accept employment as maids due to the higher wages (Momsen, 2007). According to the Organisation for Economic Co-operation and Development, around the year 2000, 25 percent, 17 percent and 19 percent of the foreign-born populations in Canada, U.S., and Australia were holding a job for which they were overqualified.[2] Hence, this issue cannot be ignored and we include this scenario in our analysis by allowing migrants who acquired education in their home countries to accept less-skilled jobs

[2] In the report, it is explained that they distinguish jobs as low, intermediate or high, in terms of the level of qualification and "overqualified" is defined as having a job that is one level lower to the potential level.

in the host country.

In this chapter, where we discuss the model of heterogenous labor and the effect of persistent migration on wages, three additional assumptions are made. First, all newborn at each time period t are born unskilled for both countries. This is different from Reichlin and Rustichini (1998) as they assume that there is an intergenerational transfer of skill. Hence, we add another decision to be made at the beginning of the time period, and that is before an individual decides to apply for migration, they first decide whether they are going to acquire skill or not. This is the key addition to the model. The next two additional assumptions are made to simplify the model. One is that only individuals that are skilled can migrate. This implies that even if unskilled labor wish to migrate to gain an unskilled job in the migrant-receiving country, they would need to acquire skill first. Hence, at the beginning of each time period, all newborn for both countries decide on whether they would like to acquire skill or not and if they have opted to become skilled workers, they then make a decision on whether to apply for migration or not. And, the other is that all those who apply for migration face the possibility of migrating as an unskilled worker, even if they get a higher wage.

We find that, in terms of the composition of labor, i.e. the skilled to unskilled labor ratio, both the migrant-sending and the migrant-receiving countries benefit from a higher probability of migration, either as a skilled migrant or as an unskilled migrant. This holds true even if migrants were allowed to migrate only as skilled workers. For the migrant-receiving country, an expected result is that their workforce composition is better the more immigrants acquire jobs as skilled workers.

In terms of wages, we find that the unskilled to skilled wage gap is lower in the migrant-sending country the higher the probability of migrating, whether as a skilled worker or an unskilled worker. For the migrant-receiving country, however, a higher probability of skilled migration while limiting unskilled worker migration makes their unskilled to skilled wage ratio improve. It is also interesting to note that if the migrant-receiving country were to accept more migrants as unskilled workers, then both countries experience wider wage gaps between their unskilled and skilled workers, than if they accepted more skilled workers. And, a higher probability of skilled worker migration is very beneficial to the migrant-sending country as both their skilled and unskilled wages are closer to those of the migrant-receiving country.

We introduce the model in Section 3.2, following a similar process of analysis as to Chapter 2, including the role that the probabilities play in migration in Section 3.2.7. We then analyze the results for wage ratios in Section 3.3 and discuss our conclusions in Section 3.4.

3.2 Model with Heterogenous Labor

The model description and analysis that follows is only for the case where only those who have acquired skilled can migrate. It may be argued that all migrants have some form of education, regardless of the type of job they acquire upon migrating and they have more chances of being allowed migrant status. We also make an assumption on wages to restrict the flow of migration from one country to another, which will be discussed in Section 3.2.2. To simplify notation, we follow from the previous case of homogenous labor and label the sending country as $'D'$, the domestic country and the receiving country as $'F'$, the foreign country.

3.2.1 Technology with Two Types of Labor

Firms in each country face a similar technology to that of the case of homogenous labor, but in this case, we let N be unskilled labor, and L be skilled labor for both countries $j = D, F$.

$$Y_t^j = A(K_t^j)^{\alpha}(L_t^j)^{\beta}(N_t^j)^{1-\alpha-\beta} \tag{3.1}$$

$$\tag{3.2}$$

where $A = \bar{K}^{\eta}$. Again, due to perfect competition, the returns to capital and wages for skilled and unskilled workers are

$$r_t^j = \alpha(K_t^j)^{\alpha+\eta-1}(\pi_t^j)^{\beta}(N_t^j)^{1-\alpha} - \delta \tag{3.3}$$

$$w_t^j = \beta(K_t^j)^{\alpha+\eta}(\pi_t^j)^{\beta-1}(N_t^j)^{-\alpha} \tag{3.4}$$

$$v_t^j = (1-\alpha-\beta)(K_t^j)^{\alpha+\eta}(\pi_t^j)^{\beta}(N_t^j)^{-\alpha} \tag{3.5}$$

where $\pi_t^j = L_t^j/N_t^j$, w_t^j is the skilled labor wage and v_t^j is the unskilled labor wage.

3.2.2 Assumption on Wages

There are two patterns of migration which can be assessed. First, we consider the flow of migration from country D to country F where immigrants become skilled workers - Pattern 1. Second, we consider the possibility where migrants from country D can become either a skilled worker or an unskilled worker in country F - Pattern 2.

Pattern 1

For this pattern of migration, we make the following assumption on wages. For both countries D and F, it must be that $w_t^j > v_t^j$ where $i, j = D, F$, would then hold as otherwise, there would be those who have acquired skill and would still want to get hired for the unskilled job, as noted in Reichlin and Rustichini (1998). Thus, all newborn individuals are unskilled and make a decision on whether to acquire skill or not and this applies to both countries.

Our assumption on skilled wages and unskilled wages for both countries in Pattern 1 is as follows: $w_t^F > w_t^D > v_t^F > v_t^D$.

This automatically restricts the flow of migration from country D to country F and that only those with skill from country D can apply for migration (i.e., unskilled workers from country D face a probability of migration of zero.) This rests on the assumption that only those who have acquired skill can apply for migration.

In this pattern, there are two possibilities for the educated individuals in country D, apply for migration or stay and work at home.

Pattern 2

As in Pattern 1, we first assume that $w_t^j > v_t^j$. The difference is in v^F and w^D. Our assumption on skilled wages and unskilled wages in this pattern is as follows: $w_t^F > v_t^F > w_t^D > v_t^D$.

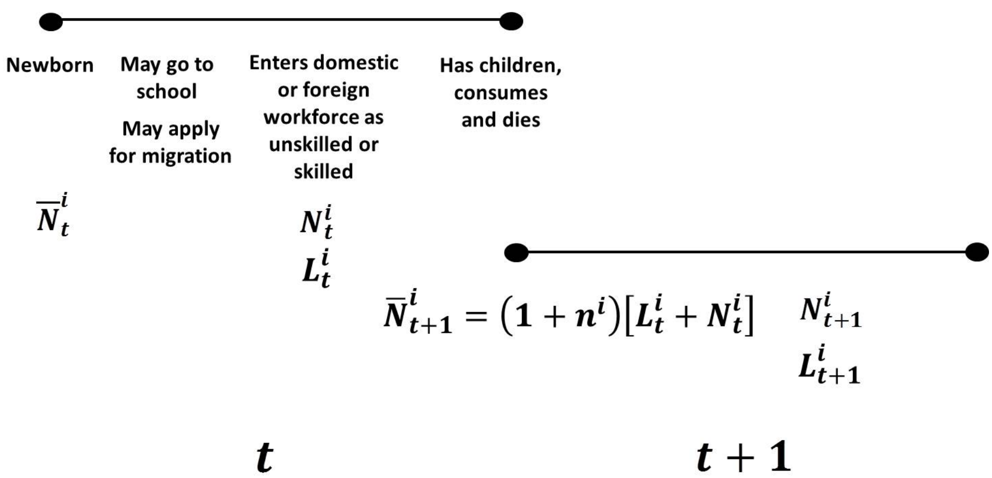
Hence, for this pattern, those who apply for migration from country D now face three possibilities. They may be accepted as a skilled migrant in country F, an unskilled migrant in country F or neither, and in which case the applicant will work as a skilled worker in country D.

In the following two sections, we then discuss the household behavior for both countries D and F where we have Pattern 1: $w_t^F > w_t^D > v_t^F > v_t^D$, and Pattern 2: $w_t^F > v_t^F > w_t^D > v_t^D \ \forall\ t$.

3.2.3 Household Behavior

The OLG structure for the model with heterogenous labor is found below.

Figure 3.1: OLG Structure with Heterogenous Labor



Pattern 1

Household Behavior in Country D

All newborn individuals a are endowed with c^{Da} (the cost of migration) and u^{Da} (the cost of skill acquisition) and each is uniformly distributed on space[0,1]. The costs c and u are independent of each other. Each individual is then faced with a decision of whether to acquire skill or not, and if skill is acquired, then whether to apply for migration or not.

We focus on the case where $[pw_t^F+(1-p)w_t^D](1-c^D)(1-u^D) \geq w_t^D(1-u^D) \geq v_t^D$ where p is the exogenous probability of becoming a skilled migrant.

There are then three available options for the unskilled newborn in country D:

1. Stay unskilled when $w_t^D(1-u^D) < v_t^D$
2. Acquire skill but not apply for migration when $w_t^D(1-u^D) \geq v_t^D$ and $[pw_t^F + (1-p)w_t^D](1-c^D)(1-u^D) < w_t^D(1-u^D)$
3. Acquire skill and apply for migration when $w_t^D(1-u^D) \geq v_t^D$ and $[pw_t^F + (1-p)w_t^D](1-c^D)(1-u^D) \geq w_t^D(1-u^D)$

Therefore, the domestic newborn population changes by σ_t^D which is dependent on the critical level of cost of skill acquisition u_t^{D*} and by θ_t which is dependent on the critical level of cost of applying for migration c_t^{D*}, as in the case of homogenous labor, which gives us the following expressions.

$$\sigma_t^D = max\{u_t^{D*}, 0\} \tag{3.6}$$

$$\theta_t = max\{c_t^{D*}, 0\} \tag{3.7}$$

(3.8)

$$
\begin{aligned}
u_t^{D*} &= 1 - v_t^D / w_t^D && (3.9)\\
c_t^{D*} &= \frac{p(w_t^F - w_t^D)}{p w_t^F + (1-p) w_t^D} && (3.10)
\end{aligned}
$$

The variable σ^D is then the proportion of the newborn population that chooses to acquire skill. Since it is assumed that $v^D < w^D$, it must be that $\sigma^D \in (0, 1)$. As in the model of homogenous labor, θ_t describes that proportion of the newborn that fulfill the skill requirement and can migrate.

The workforce of country D at time period t is then made up of skilled labor and unskilled labor. Skilled labor is composed of those who have acquired skill and decided not to migrate as well as those who have acquired skill, have applied for migration but were not accepted. And unskilled labor is simply the proportion of the population that have chosen to remain unskilled, as expressed below.

$$
\begin{aligned}
L_t^D &= [\sigma_t^D (1 - p\theta_t)] \bar{N}_t^D && (3.11)\\
N_t^D &= (1 - \sigma_t^D) \bar{N}_t^D && (3.12)
\end{aligned}
$$

Household Behavior in Country F

Similar to country D, all newborn individuals a are endowed with c^{Fa} and u^{Fa} and each is uniformly distributed on space [0,1]. For country F, however, due to our assumption on wages, there are only two available options for the newborn unskilled individuals as no one will want to migrate, which are:

1. Stay unskilled when $w_t^F(1 - u^F) < v_t^F$

2. Acquire skill when $w_t^F(1-u^F) \geq v_t^F$

A proportion of the foreign newborn population that acquires skill is then

$$\sigma_t^F = max\{u_t^{F*}, 0\} \tag{3.13}$$

where

$$u_t^{F*} = 1 - v_t^F / w_t^F \tag{3.14}$$

The workforce of country F at time period t is then composed of skilled labor and unskilled labor, including those who have migrated from country D. Skilled labor for country F is composed of those who have acquired skill and those who were allowed to migrate from country D. Unskilled labor is then composed of those who did not acquire skill, as expressed in the equations below.

$$L_t^F = \sigma_t^F \bar{N}_t^F + p\theta_t \sigma_t^D \bar{N}_t^D \tag{3.15}$$

$$N_t^F = (1-\sigma_t^F)\bar{N}_t^F \tag{3.16}$$

Pattern 2

Household Behavior in Country *D*

The process in this pattern is exactly the same as that of Pattern 1, but with the inclusion of a probability of migrating as unskilled, q.

We focus on the case where $[pw_t^F + qv_t^F + (1-p-q)w_t^D](1-c^D)(1-u^D) \geq w_t^D(1-u^D) \geq v_t^D$ where p is the exogenous probability of becoming a skilled migrant and q is the exogenous probability of becoming an unskilled migrant.

As above, there are three available options for the unskilled newborn in country D:

1. Stay unskilled when $w_t^D(1-u^D) < v_t^D$

2. Acquire skill but not apply for migration when $w_t^D(1-u^D) \geq v_t^D$ and $[pw_t^F + qv_t^F + (1-p-q)w_t^D](1-c^D)(1-u^D) < w_t^D(1-u^D)$

3. Acquire skill and apply for migration when $w_t^D(1-u^D) \geq v_t^D$ and $[pw_t^F + qv_t^F + (1-p-q)w_t^D](1-c^D)(1-u^D) \geq w_t^D(1-u^D)$

The newborn population of country D changes by σ_t^D and by θ_t as in Pattern 1, which gives us the following expressions.

$$\sigma_t^D = max\{u_t^{D*}, 0\} \tag{3.17}$$

$$\theta_t = max\{c_t^{D*}, 0\} \tag{3.18}$$

$$\tag{3.19}$$

where

$$u_t^{D*} = 1 - v_t^D / w_t^D \tag{3.20}$$

$$c_t^{D*} = \frac{p(w_t^F - w_t^D) + q(v_t^F - w_t^F)}{[pw_t^F + qv_t^F + (1-p-q)w_t^D]} \tag{3.21}$$

In this case, σ^D is similar to the previous case, but θ_t is now dependent not only on the level of skilled wages but also on the foreign unskilled wage since those that acquire skill can also acquire an unskilled job upon migrating.

The workforce of country D at time period t is then made up of skilled labor and unskilled labor, as expressed in Eqs. (3.15) and (3.16). Note that the skilled labor of country D is different from Pattern 1 as there is the possibility of leaving the domestic country as a skilled worker and migrating to work as an unskilled worker in the foreign country.

$$L_t^D = [\sigma_t^D(1 - p\theta_t - q\theta_t)]\bar{N}_t^D \tag{3.22}$$

$$N_t^D = (1 - \sigma_t^D)\bar{N}_t^D \tag{3.23}$$

Household Behavior in Country F

For country F, there are only two available options for the newborn unskilled individuals as no one will want to migrate, which are:

1. Stay unskilled when $w_t^F(1 - u^F) < v_t^F$

2. Acquire skill when $w_t^F(1 - u^F) \geq v_t^F$

The foreign newborn population then changes by

$$\sigma_t^F = max\{u_t^{F*}, 0\} \tag{3.24}$$

where

$$u_t^{F*} = 1 - v_t^F / w_t^F \tag{3.25}$$

Skilled and unskilled labor for country F are expressed below, noting that the addition of the probability of migrating as an unskilled worker changes the equation for country F's unskilled labor.

$$L_t^F = \sigma_t^F \bar{N}_t^F + p_t\theta_t\sigma_t^D\bar{N}_t^D \tag{3.26}$$

$$N_t^F = (1 - \sigma_t^F)\bar{N}_t^F + q\theta_t\sigma_t^D\bar{N}_t^D \tag{3.27}$$

3.2.4 Equilibrium

Definition 2. *A perfect foresight equilibrium for the world economy is a sequence* $\{K_t^j, N_t^j, L_t^j, w_t^j, v_t^j, r_t^j, \theta_t, \sigma_t^j, p_t; j = D, F; t = 1, 2...\}$ *which satisfies the following* *equations:*

$$
\begin{aligned}
r_t^j &= \alpha(N_t^j)^\eta(k_t^j)^{\alpha+\eta-1} - \delta && (3.28)\\
w_t^j &= (1-\alpha)(N_t^j)^\eta(k_t^j)^{\alpha+\eta} && (3.29)\\
v_t^j &= (1-\alpha-\beta)K^{\alpha+\eta}\pi^\beta N^{-\alpha} && (3.30)\\
\theta_t &= p_t(w_t^F - w_t^D)/[p_t w_t^F + (1-p_t)w_t^D] && (3.31)\\
\sigma_t^j &= 1 - v_t^j/w_t^j && (3.32)\\
\sum_{j=1}^{2} K_{t+1}^j &= \sum_{j=1}^{2}(w_t^j L_t^j + v_t^j N_t^j) && (3.33)\\
r_t^D &= r_t^F && (3.34)
\end{aligned}
$$

We assume that total world savings in the current period is equal to total capital stock in the next period, and also assume perfect capital mobility, as in the model of homogenous labor.

3.2.5 Analysis and Results

From the given equations above, we can then solve for the relative wage structure and the capital flows which follows from Reichlin-Rustichini (1998). For each time period t, we have these equations

$$
\begin{aligned}
v_t^j/w_t^j &= \rho\pi_t^j && (3.35)\\
w_t^D/w_t^F &= \left(\frac{N_t^D}{N_t^F}\right)^\gamma \left(\frac{\pi_t^D}{\pi_t^F}\right)^{\zeta-1} && (3.36)\\
v_t^D/w_t^F &= \rho\left(\frac{N_t^D}{N_t^F}\right)^\gamma \left(\frac{\pi_t^D}{\pi_t^F}\right)^{\zeta} \pi_t^F && (3.37)\\
v_t^D/v_t^F &= \left(\frac{N_t^D}{N_t^F}\right)^\gamma \left(\frac{\pi_t^D}{\pi_t^F}\right)^{\zeta} && (3.38)
\end{aligned}
$$

where $\zeta = \beta/1-\alpha-\eta$, $\gamma = \eta/1-\alpha-\eta$ and $\rho = \beta/1-\alpha-\beta$.

Similar to the case of homogenous labor, we analyze the behavior of the ratio of unskilled workforce $\lambda_t = N_t^D/N_t^F$. In this model, however, we include an analysis of two additional variables: the workforce composition of country D, $\pi^D = L_t^D/N_t^D$, and the workforce composition of country F, $\pi_t^F = L_t^F/N_t^F$. In order to do this, we show that the mentioned ratios are topologically equivalent to the ratio of newborn populations. This allows us to analyze the workforce ratios at steady state as their properties would be the same as that of the ratio of newborn populations.

Lemma 5. *Let* $\phi_{t+1} = f_1(\phi_t)$, $\bar{\pi}_{t+1}^D = f_2(\bar{\pi}_t^D)$, $\bar{\pi}_{t+1}^F = f_3(\bar{\pi}_t^F)$, $\lambda_{t+1} = g_1(\lambda_t)$, $\pi_{t+1}^D = g_2(\pi_t^D)$, $\pi_{t+1}^F = g_3(\pi_t^F)$, *where* $\phi_t = \bar{N}_t^D/\bar{N}_t^F$ *then the discrete dynamical systems of functions* f_i *and* g_i, $i = 1, 2, 3$, *are topologically equivalent.*

Proof. From Eqs. (3.8),(3.9),(3.11),(3.12),(3.15),(3.16),(3.18),(3.19), it can be noted that each workforce variable is a function of the newborn variable. Hence, any ratio of these workforce variables is a function of a ratio of the newborn population variables. And we can solve for $\lambda_t = h_1(\phi_t)$, $\pi^D = h_2(\bar{\pi}^D)$ and $\pi^F = h_3(\bar{\pi}^F)$ and each mapping $h_i : \mathbb{R}_+ \to \mathbb{R}_+$, $i = 1, 2, 3$, is one-to-one, and continuous, and h_i^1 is continuous, $i = 1, 2, 3$, for all time periods t. □

We then have the following equations that describe the evolution of the ratio of unskilled workforces, the workforce compositions of country D and country F for both patterns. It can be noted that when $q = 0$, then the equations are similar for both patterns.

Pattern 1

Below is the system of equations that describe the steady state when wages have this relation: $w^F > w^D > v^F > v^D$.

$$\lambda_t = \frac{(1-\sigma_t^D)\mu(\pi_{t-1}^D\lambda_{t-1}+\lambda_{t-1})}{(1-\sigma_t^F)(\pi_{t-1}^F+1)} \tag{3.39}$$

$$\pi_t^D = \frac{(1-p\theta_t)\sigma_t^D}{(1-\sigma_t^D)} \tag{3.40}$$

$$\pi_t^F = \frac{\sigma_t^F(\pi_{t-1}^F+1)+p\theta_t\sigma_t^D\mu(\pi_{t-1}^D\lambda_{t-1}+\lambda_{t-1})}{(1-\sigma_t^F)(\pi_{t-1}^F+1)} \tag{3.41}$$

where

$$\sigma_t^D = 1-\rho\pi_t^D \tag{3.42}$$

$$\sigma_t^F = 1-\rho\pi_t^F \tag{3.43}$$

$$\theta_t = \frac{p(1-\lambda_t^\gamma(\frac{\pi_t^D}{\pi_t^F})^{\zeta-1})}{p+(1-p)\lambda_t^\gamma(\frac{\pi_t^D}{\pi_t^F})^{\zeta-1}} \tag{3.44}$$

$$\tag{3.45}$$

Pattern 2

And here we have the system of equations that describe the steady state when wages have this relation: $w^F > v^F > w^D > v^D$

$$\lambda_t = \frac{(1-\sigma_t^D)\mu(\pi_{t-1}^D\lambda_{t-1}+\lambda_{t-1})}{(1-\sigma_t^F)(\pi_{t-1}^F+1)+q\theta_t\sigma_t^D(\pi_{t-1}^D\lambda_{t-1}+\lambda_{t-1})} \tag{3.46}$$

$$\pi_t^D = \frac{(1-p\theta_t-q\theta_t)\sigma_t^D}{(1-\sigma_t^D)} \tag{3.47}$$

$$\pi_t^F = \frac{\sigma_t^F(\pi_{t-1}^F+1)+p\theta_t\sigma_t^D\mu(\pi_{t-1}^D\lambda_{t-1}+\lambda_{t-1})}{(1-\sigma_t^F)(\pi_{t-1}^F+1)+q\theta_t\sigma_t^D\mu(\pi_{t-1}^D\lambda_{t-1}+\lambda_{t-1})} \tag{3.48}$$

where

$$\sigma_t^D = 1 - \rho\pi_t^D \tag{3.49}$$

$$\sigma_t^F = 1 - \rho\pi_t^F \tag{3.50}$$

$$\theta_t = \frac{p(1 - \lambda_t^\gamma (\frac{\pi_t^D}{\pi_t^F})^{\zeta-1}) + q(\rho\pi_t^F - \lambda_t^\gamma (\frac{\pi_t^D}{\pi_t^F})^{\zeta-1})}{p + q\rho\pi_t^F + (1-p-q)\lambda_t^\gamma (\frac{\pi_t^D}{\pi_t^F})^{\zeta-1}} \tag{3.51}$$

(3.52)

Case: $p \in (0,1), q = 0$

In this section, we explore results that may occur in the case where the migrant-receiving country decides to receive only skilled migrants and sets $q = 0$, and $p \in (0,1)$. For a such a case, at steady-state, we arrive at the following set of equations.

$$\lambda = \frac{\mu(1-\sigma^D)(\pi^D\lambda + \lambda)}{(1-\sigma^F)(\pi^F + 1)} \tag{3.53}$$

$$\pi^D = \frac{(1-p\theta)\sigma^D}{1-\sigma^D} \tag{3.54}$$

$$\pi^F = \frac{\sigma^F(\pi^F + 1) + p\theta\sigma^D\mu(\pi^D\lambda + \lambda)}{(1-\sigma^F)(\pi^F+1)} \tag{3.55}$$

(3.56)

where

$$\sigma^D = 1 - \rho\pi^D \tag{3.57}$$

$$\sigma^F = 1 - \rho\pi^F \tag{3.58}$$

$$\theta = \frac{p(1-\lambda^\sigma)(\frac{\pi^D}{\pi^F})^{\zeta-1}}{p + (1-p)\lambda^\gamma(\frac{\pi^D}{\pi^F})^{\zeta-1}} \tag{3.59}$$

We may note that from re-arranging Eq. (3.53), we have this equation.

$$\lambda = \mu(\frac{1-\sigma^D}{1-\sigma^F})(\frac{N^D+L^D}{N^F+L^F}) \tag{3.60}$$

Eq. (3.60) confirms that at steady-state, when the migrant-receiving country only prefers to receive skilled migrants with a probability p, that the ratio of unskilled workers is essentially equal to the ratio of birth rates and the ratio of the proportion of the total workforce that decides to stay unskilled. Thus, in the scenario, the steady-state ratio of unskilled workers is unaffected by the proportion of those that decide to migrate and the probability of migration.

3.2.6 Numerical Analysis at Steady State

In this section, we analyze the steady state properties of this system of equations at steady state numerically. Based on the equilibrium conditions, the ratio of the composition of labor between the two countries affects the skilled wage ratio with an exponent of $(\zeta - 1)$. This implies that if ζ were less than one, then an improvement, for example, of the composition of labor in the migrant-sending country, country D, relative to that of the migrant-receiving country, country F, may have a negative effect on the relative skilled wage ratio[3], which could go against our assumption on relationship between the wages. Hence, we restrict ζ to be greater than 1.

Since $\zeta = \frac{\beta}{1-\alpha-\eta}$, this implies that β is greater than $(1-\alpha-\eta)$ or that η is larger than the relative share of unskilled labor N to output, or that the combined share of physical capital K and skilled labor L is larger than that of unskilled labor.[4]

We choose to use the following parameters: $\alpha = 0.35$; $\beta = 0.45$ and $\eta = 0.25$,

[3] This is also discussed in Reichlin and Rustichini (1998).

[4] Reichlin and Rustichini (1998) assess the possibility of ζ not being greater than one, and explore the pattern of the flow of migration reversing. For this paper, we don't consider this possibility due to the differences in our model wherein we endogenize both the education and migration choices.

which give us $\gamma = 0.625$; $\zeta = 1.125$ and $\rho = 0.444$. We choose these parameters to be as close to typically used values for these parameters while still allowing for the condition on ζ to hold.

The other important parameter choice is that of μ, the ratio of birth rates. For this model, we also need $\mu > 1$ for the migration to be sustained from country D to country F.[5]

By choosing $\mu > 1$, we are essentially exploring the migration pattern that is apparent in the migration flows of migration corridors such as Mexico-U.S., Philippines-U.S., Bangladesh-India, India-US. While other migration corridors exhibit a μ that is less than 1, we do not consider that possibility here due to the equations in our model. We then use the average population growth rates of Mexico and the United States from 1960 to 2009.[6] This gives us a $\mu = 1.011$, which is similar to the case with homogenous labor.[7]

We discuss the results below of λ, the ratio of unskilled labor, π^i, the ratio of skilled-to-unskilled labor for each country $i = D, F$, as a function of both p and q, the probabilities of migrating as skilled labor and unskilled labor, respectively, noting that $p + q \leq 1$ since there are only three possibilities: that they are accepted as a skilled migrant with a probability p, accepted as an unskilled migrant with a probability q or are not accepted as a migrant. Given a certain set of parameters (i.e., γ, ζ, and ρ), we can then see the steady-state results for the different workforce ratios dependent on the probabilities of migrating.

[5] We have conducted numerous exercises on the value of μ and find that the equations are satisfied at steady-state only when $\mu > 1$, though this cannot be verified analytically. This condition is similar to the case of homogenous labor where the birth rate of the migrant-sending country has to be greater than the birth rate of the migrant-receiving country.

[6] Source: World Bank Development Indicators, 2010

[7] We also ran exercises using the ratio of birth rates between the Philippines and the United States, as well as Bangladesh and India, but the results don't differ significantly so we opt to discuss the results from the Mexico-U.S. example.

Pattern 1

We note that when the probability of migrating as an unskilled worker is zero, or $q = 0$, and migrants are only allowed to migrate as skilled workers, the model only works numerically for cases where $p > 14\%$, indicating that extremely low levels of probabilities of migration cannot sustain a persistent migration where our assumption on wages holds.

Steady-state λ

From Figure C.2, we may note that steady-stateλ is increasing in p. This implies that for any given probability of migrating as an unskilled worker, the foreign country's unskilled workforce will be relatively smaller as they receive more skilled workers as migrants. This then leads to an increase in steady-state λ.

In this case, as discussed previously, steady-state λ is not affected by the proportion that decides to migrate but only by the proportion that decides to acquire education.

Steady-state π^D

We can observe from Figure C.3 that as the probability of migrating as a skilled worker increases, π^D then increases. This implies that there are now relatively more of the unskilled newborn that choose to go to school, and relatively fewer of those that actually get to migrate and, thus, the domestic country is left with relatively more skilled workers than unskilled workers as the probability of migrating as a skilled worker increases..

Steady-state π^F

Figure C.4 illustrates that steady-state π^F is increasing in p. This implies that as the probability of migrating as a skilled worker increases, then there will be relatively more skilled workers in country F, and thus have a higher skilled to unskilled workforce ratio.

Pattern 2

Pattern 2 works only for certain probabilities of migration; mostly when p is close to zero and $q > 0$. Thus, the model works only if the probability of migrating as a skilled worker is almost zero and the probability of migrating as an unskilled worker is high enough. We base this on the assumption on the relationship among the wages. For our assumption on wages to hold, we only focus on these probabilities given our chosen parameters.

The sets of probabilities we can assess are the following: 1) $p = 0$, $q \in (0, 1]$, 2) $p = 0.02$, $q \in (0, 0.72)$, 3) $p = 0.04$, $q \in (0, 0.34)$, 4) $p = 0.06$, $q \in (0, 0.20)$, 5) $p = 0.08$, $q \in (0, 0.12)$, and 6) $p = 0.10$, $q \in (0, 0.06)$

In the graphs found in the appendix, we show how the steady-state variables respond to changes in the probability of migrating as unskilled worker given certain probabilities of p, selected from the values above.

Steady-state λ

Compared to Pattern 1, λ is very low. However, the unskilled workforce ratio between country D and F is still increasing in the probability of migrating as an unskilled worker, as shown in Figure C.5. This implies that while there is some incentive to acquire skill and then invest in education due to probability of migrating as an unskilled worker, there is a much lower proportion that do, leaving country D with relatively more unskilled workers than country F, even if they are receiving significantly more unskilled migrants.

Steady-state π^D

From Figure C.6, π^D is increasing in q which implies that there are still relatively more of newborn population that opt to stay unskilled as probabilities of migration are quite low.

Steady-state π^F

As for π^F, it is decreasing in q as in Figure C.7 as there are relatively more unskilled migrants and they are thus increasing their unskilled workforce.

3.2.7 The Role of the Probability of Migration

To focus the discussion, we can assess the size and composition ratios given Patterns 1 and 2. With Pattern 1, we have $q = 0$ and $p \in (0, 1]$, specifically for $p > 0.14$, and with Pattern 2, we have p close to zero and $q > 0$, but with the upper limit dependent on the given p. We can compare different scenarios for the workforce ratios under these scenarios.

Table 3.1: The Role of Probabilities under Heterogenous Labor

Pattern 1: $q = 0$, $p > 0$					
	$p = 0.20$	$p = 0.40$	$p = 0.60$	$p = 0.80$	$p = 1$
λ	0.62431	0.89760	0.95481	0.97515	0.98461
π^D	1.07643	1.07712	1.07724	1.07728	1.07730
π^F	1.08421	1.08490	1.08502	1.08506	1.08508
Pattern 2: $p = 0$, $q > 0$					
	$q = 0.20$	$q = 0.40$	$q = 0.60$	$q = 0.80$	$q = 1$
λ	0.16277	0.26214	0.28724	0.29655	0.30093
π^D	1.07451	1.07503	1.07515	1.07519	1.07521
π^F	1.08003	1.07945	1.07932	1.07927	1.07925

In the table above, we have the results on the steady-state variables of workforce ratios given different probabilities of migration. The more interesting variables are those of π^D and π^F as these reflect the composition of the labor force in both countries. The study of λ add to this analysis since it is indicative of the size of the unskilled workforce in both countries.

In understanding the composition of labor in both countries, we adopt the use of the term of Reichlin and Rustichini (1998) that the higher the ratio of skilled-to-unskilled labor for either country, the composition improves or is "better." This is true in the sense that the economy's workforce is made up of relatively more skilled workers than unskilled workers. From the values above, a few observations may be made. First, both π^D and π^F are higher in every case of $p > 0$, $q = 0$, than $q > 0$, $p = 0$. This indicates that for both the migrant-receiving country as well as the migrant sending country, the composition of the workforce (i.e., the ratio of skilled to unskilled workers) is higher when the probability of migrating as a skilled worker is higher than if the probability of migrating as an unskilled worker is higher. This shows that when we allow for endogenous skill acquisition, that the higher the possibility of migrating as a skilled worker encourages education and that the concept of "brain drain" does not occur in this model[8]. Second, it can be noted that for the case where $p > 0$, π^D and π^F become higher as p approaches 1. This again indicates the inclusion of endogenous skill choice as both countries end up with a higher composition of labor the higher the chance to migrate. For the migrant-receiving country, this implies that they end up with a higher labor composition, when there is no limit imposed on skilled worker immigration. Third, when the migrant-receiving country is accepting only unskilled workers, then π^F is better as q approaches zero. This is quite intuitive as the more unskilled workers country F accepts, the lower the composition of its workforce becomes. In terms

[8]For this analysis, we do not make inferences to growth or welfare, rather we are just stating that "brain drain," as defined in 3.1, does not necessarily occur in the sense that there is no depletion of skilled workers, given that the migrant-sending country faces a higher skilled-to-unskilled workforce ratio the higher the probability of migrating as a skilled worker. This reiterates the statement of Fan and Stark (2007) that most theoretical analyses that find that the emigration of highly skilled workers may result in the sending country having a higher level of educated workers is due to the endogeneity of the education choice being dependent on the possibility of migration.

of comparing the size of the unskilled workforce, another glaring result is that λ is much lower when $q > 0$ than when $p > 0$. Thus, the size of the unskilled labor force is relatively larger for country F when it is accepting more unskilled workers than skilled workers.

What this exercise shows us is that the modifications made on the Reichlin-Rustichini model do allow us to gain more insightful results. The endogeneity of skill acquisition as well as the inclusion of probabilities of migration allow us to see how the effects of the size and the composition of labor are affected in persistent migration given different scenarios. The basic result that the composition of labor in both countries is greater than 1, i.e., there is more skilled labor than unskilled labor, for all cases shows us that when migration is characterized by endogenous skill acquisition, that both countries can end up with an improved composition of labor.

3.2.8 Robustness Checks

The results in graphical form for the robustness checks for Pattern 1 are in the Appendix, labeled Figures C.8-C.19 while the results for the robustness checks for Pattern 2 are in the Appendix, labeled Figures C.20-C.31.

In Table 3.2.8, we show the variations in the parameters applied. All the variables seem to behave well even with slight variations in some parameters. The only slight differences occur when either p or q are very close to zero, an indication that the model may be sensitive when there is virtually no possibility of migration which is not covered by this model of persistent migration.

Table 3.2: Robustness Checks for Heterogenous Labor Model

Original Parameters	$\alpha = 0.35$, $\beta = 0.45$, $\eta = 0.25$, $\gamma = 0.625$, $\zeta = 1.125$, $\rho = 0.444$, $\mu = 1.011$
Check 1a	$\alpha = 0.36$ $\beta = 0.45$, $\eta = 0.25$, $\gamma = 0.641$, $\zeta = 1.154$,$\rho = 0.422$, $\mu = 1.011$
Check 1b	$\alpha = 0.34$ $\beta = 0.45$, $\eta = 0.25$, $\gamma = 0.610$, $\zeta = 1.098$,$\rho = 0.467$, $\mu = 1.011$
Check 2a	$\beta = 0.46$ $\alpha = 0.35$, $\eta = 0.25$, $\gamma = 0.625$, $\zeta = 1.150$,$\rho = 0.413$, $\mu = 1.011$
Check 2b	$\beta = 0.44$ $\alpha = 0.35$, $\eta = 0.25$, $\gamma = 0.625$, $\zeta = 1.100$,$\rho = 0.477$, $\mu = 1.011$
Check 3a	$\eta = 0.26$ $\alpha = 0.35$, $\eta = 0.45$, $\gamma = 0.667$, $\zeta = 1.154$,$\rho = 0.444$, $\mu = 1.011$
Check 3b	$\eta = 0.24$ $\alpha = 0.35$, $\beta = 0.4b5$, $\gamma = 0.5855$, $\zeta = 1.098$,$\rho = 0.444$, $\mu = 1.011$
Check 4a	$\mu = 1.014$ $\alpha = 0.35$,$\beta = 0.45$, $\eta = 0.25$, $\gamma = 0.641$, $\zeta = 1.154$,$\rho = 0.422$
Check 4b	$\mu = 1.008$ $\alpha = 0.35$, $\beta = 0.45$, $\eta = 0.25$, $\gamma = 0.610$, $\zeta = 1.098$,$\rho = 0.467$

3.3 Movement of Wages with Heterogenous Labor

Numerous studies on migration have looked at labor market outcomes, specifically on wages as this is of utmost concern for both the migrant-sending country as well as the migrant-receiving country, especially for the latter. This aspect of migration literature aims to study the effect of immigration on the distribution of wages, i.e. the differences in wages for skilled and unskilled workers, whether native- or foreign-born. Most studies that have looked at how immigration affects the wages of the less-skilled natives have found that the effect is negative (Johnson (1980), Altonji and Card (1991), De New and Zimmerman (1994), Orrenius and Zavodny (2007), Ottaviano and Peri (2006)). As for the effect of an influx of immigrants on skilled natives, it is less clear. Johnson (1980) and De New and Zimmerman (1994) find that skilled natives' wages increase due to immigration, while Borjas (2005) and Ottaviano and Peri (2006) find that wages are lower. Orrenius and

Zavodny (2007) find no significant effects on skilled natives' wages.

One aspect of migration that is often overlooked in research is the effect of migration on the sending country's labor market outcomes. As most of the studies done on the sending country have involved the issue of brain drain, relatively fewer studies exist that look at how emigration affects the sending country's wages and employment opportunities. Mishra (2007) claims to be the only study to be done on Mexico and finds that emigration leads to skilled wages to increase.

When we allow for heterogeneity in labor, and endogenize skill acquisition, we assess how wages differ given different probabilities of migration for unskilled and skilled labor. Our contributions lie in the aspects of current migration research that are lacking: the effect of emigration on the source country's wages as well as the effect of migration on both skilled and unskilled wages for both the migrant-sending and migrant-receiving countries.

We find the growth rates of the workforces and, consequently, the growth rates of wages. In this model, there are two types of labor: skilled and unskilled, and there are two types of wages: skilled and unskilled. We first calculate the growth rates for the labor of country D.

3.3.1 Analysis and Results

Since the unskilled workforce of country D at time period t is given by $N_t^D = (1-\sigma_t^D)(1+n^D)[L_{t-1}^D + N_{t-1}^D]$, its growth rate is given by

$$g_{N^D} = (1-\theta_t^D)(1+n^D)(1+\pi_{t-1}^D) - 1. \tag{3.61}$$

Hence, the growth of the remaining unskilled workforce is determined by those who decide not to invest in education and the natural birth rate.

The skilled workforce of country D is given by $L_t^D = \theta_t^D(1 - p\theta_t - q\theta_t)(1 + n^D)[L_{t-1}^D + N_{t-1}^D]$ and we can calculate its growth rate as shown below.

$$g_{L^D} = \sigma_t^D(1 - p\theta_t - q\theta_t)(1 + n^D)(1 + \frac{1}{\pi_{t-1}^D}) - 1 \tag{3.62}$$

This implies that the growth rate of skilled labor in country D is dependent on the birth rate, the proportion of the newborn that invests in education and the proportion of those who apply for migration but do not get accepted as a skilled nor as an unskilled migrant.

For country F, unskilled labor at time period t is given by $N_t^F = (1 - \sigma_t^F)(1 + n^F)[L^F t - 1 + N_{t-1}^F] + q\theta_t\sigma_t^D(1 + n^D)[N_{t-1}^D + L_{t-1}^D]$. Since the workforce in country F is also affected by the proportion of migrants coming from country D, the equations are slightly more complex. The growth rate of unskilled labor in country F is then calculated as shown below.

$$g_{N^F} = (1 - \sigma_t^F)(1 + n^D)(1 + \pi_{t-1}^F) + q\theta\sigma_t^D(1 + n^D)(\lambda_{t-1}(1 + \pi_{t-1}^D)) - 1 \tag{3.63}$$

This equation indicates that country F's unskilled workforce growth is dependent on two important terms. The leftmost term would be its growth rate without migration and it would only, therefore, be affected by its own birth rate and the proportion of the population that do not go to school. Since a migration process occurs, then its growth rate is also affected by what occurs in country D and is dependent on country D's birth rate, the proportion of newborn that decide to acquire skill and the proportion of those that get accepted as unskilled migrants in country F. As for country F's skilled labor, at time period t, it is given by $L_t^D = \sigma_t^F(1 + n^D)[L_{t-1}^F + N_{t-1}^F] + p\theta\sigma_t^D(1 + n^D)[L_{t-1}^D + N_{t-1}^D]$. This allows us to find its growth rate as

$$g_{L^F} = \sigma_t^F(1+n^D)(1+\frac{1}{\pi_{t-1}^F}) + p\theta\sigma_t^D(1+n^D)(\frac{\lambda_{t-1}(1+\pi_{t-1}^D)}{\pi_{t-1}^F}) - 1 \tag{3.64}$$

Similarly, this equation shows us how the proportion of migrants accepted as skilled workers also affects the growth rate of country F and if were not accepting any migrants, then its skilled labor's growth rate would only be a function of the proportion of its own newborn that decide to acquire education.

Since the unskilled wage of country D is $v_t^D = (1-\alpha-\beta)(K_t^D)^{\alpha+\eta}(L_t^D)^{\beta}(N_t^D)^{-\alpha-\beta}$ we can then find the following equation to express its growth rate in time period t.

$$g_{v^D} = (\alpha+\eta)[g_K) + \beta[g_{L^D}] - (\alpha+\beta)[g_{N^D}] \tag{3.65}$$

where

$$g_{L^D} = [\sigma_t^D(1-p\theta_t-q\theta_t)(1+n^D)(1+\frac{1}{\pi_{t-1}^D}) - 1] \tag{3.66}$$

$$g_{N^D} = [(1-\theta_t^D)(1+n^D)(1+\pi_{t-1}^D) - 1] \tag{3.67}$$

For country D's skilled wage, we have $w_t^D = \beta(K_t^D)^{\alpha+\eta}(L_t^D)^{\beta-1}(N_t^D)^{1-\alpha-\beta}$, which gives us the following growth rate.

$$g_{w^D} = (\alpha+\eta)[g_K) + (\beta-1)[g_{L^D}] - (1-\alpha+\beta)[g_{N^D}] \tag{3.68}$$

where

$$g_{L^D} = [\sigma_t^D(1 - p\theta_t - q\theta_t)(1 + n^D)(1 + \frac{1}{\pi_{t-1}^D}) - 1] \quad (3.69)$$

$$g_{N^D} = [(1 - \theta_t^D)(1 + n^D)(1 + \pi_{t-1}^D) - 1] \quad (3.70)$$

For country F, the unskilled wage is $v_t^F = (1-\alpha-\beta)(K_t^F)^{\alpha+\eta}(L_t^F)^{\beta}(N_t^F)^{-\alpha-\beta}$ and its growth rate can be calculated as shown below.

$$g_{v^F} = (\alpha+\eta)[g_K) + \beta[g_{L^F}] - (\alpha+\beta)[g_{N^F}] \quad (3.71)$$

where

$$g_{L^F} = \sigma_t^F(1+n^D)(1+\frac{1}{\pi_{t-1}^F}) + p\theta\sigma_t^D(1+n^D)(\frac{\lambda_{t-1}(1+\pi_{t-1}^D)}{\pi_{t-1}^F}) - 1 \quad (3.72)$$

$$g_{N^F} = (1-\sigma_t^F)(1+n^D)(1+\pi_{t-1}^F)$$
$$+q\theta\sigma_t^D(1+n^D)(\lambda_{t-}(1+\pi_{t-1}^D)) - 1 \quad (3.73)$$

And the calculation for country F's skilled wage, $w_t^F = \beta(K_t^F)^{\alpha+\eta}(L_t^F)^{\beta-1}(N_t^F)^{1-\alpha-\beta}$ is as follows.

$$g_{w^D} = (\alpha+\eta)[g_K) + (\beta-1)[g_{L^D}] - (1-\alpha+\beta)[g_{N^D}] \quad (3.74)$$

where

$$g_{L^F} = \sigma_t^F(1+n^D)(1+\frac{1}{\pi_{t-1}^F}) + p\theta\sigma_t^D(1+n^D)(\frac{\lambda_{t-1}(1+\pi_{t-1}^D)}{\pi_{t-1}^F}) - 1 \quad (3.75)$$

$$g_{N^F} = (1-\sigma_t^F)(1+n^D)(1+\pi_{t-1}^F) +$$
$$q\theta\sigma_t^D(1+n^D)(\lambda_{t-}(1+\pi_{t-1}^D)) - 1 \quad (3.76)$$

3.3.2 Numerical Analysis

For Pattern 1, we refer to the C.32-C.35 in the Appendix and for Pattern 2, we refer to Figures C.36-C.39. In both patterns, we again highlight the role that probabilities of migration play in this model of persistent migration.

Table 3.3: The Role of Probabilities in Relative Wages with Heterogenous Labor

Pattern 1: $q = 0$, $p > 0$					
	$p = 0.20$	$p = 0.40$	$p = 0.60$	$p = 0.80$	$p = 1$
$\frac{v^D}{w^D}$	0.47837	0.47867	0.47872	0.47874	0.47875
$\frac{v^F}{w^F}$	0.48182	0.48213	0.48218	0.48220	0.48221
$\frac{v^D}{v^F}$	0.73894	0.92717	0.96367	0.97646	0.98236
$\frac{w^D}{w^F}$	0.74428	0.93387	0.97064	0.98351	0.98946
Pattern 2: $p = 0$, $q > 0$					
	$q = 0.20$	$q = 0.40$	$q = 0.60$	$q = 0.80$	$q = 1$
$\frac{v^D}{w^D}$	0.47751	0.47774	0.47780	0.47782	0.47782
$\frac{v^F}{w^F}$	0.47996	0.47971	0.47965	0.47963	0.47962
$\frac{v^D}{v^F}$	0.31969	0.43111	0.45658	0.46581	0.47013
$\frac{w^D}{w^F}$	0.32133	0.43288	0.45835	0.46758	0.47189

For Pattern 1, what we find is that all the relevant wage ratios, v^D/w^D, v^F/w^F, v^D/v^F, and w^D/w^F are all increasing in the probability of migrating as a skilled worker. This can be noted in Table 3.3.2 as well as in the Figures in the Appendix. For Pattern 2, we find that v^D/w^D, v^D/v^F, and w^D/w^F all increase with the probability of migrating as an unskilled worker. However, v^F/w^F, the unskilled wage to skilled wage ratio decreases as q increases. This implies that the wage gap, or wage inequality, between unskilled workers and skilled workers increases as the migrant-receiving country opts to receive more unskilled labor.

From Table 3.3.2, it can be observed that in either case where $p > 0$ or $q > 0$, the unskilled to skilled wage ratio for both countries D and F is just below 0.5, implying that the unskilled wage is just half of the skilled wage. Although in

Section 3.2.7, we saw that there is an improvement in the composition of labor for both patterns of migration, when wages are observed, we find that the wage gap within each country widens with migration. However, it may be noted that this inequality decreases the higher the probability of migration, whether as an unskilled or as a skilled worker.

It is also worth noting that when we are comparing domestic to foreign wages, the case of skilled worker migration (i.e. Pattern 1), results in more favorable scenarios for the wage ratios when comparing the wages of country D to those of county F. The skilled wage and unskilled wage ratios between country D and country F are closer to 1. This implies that a high probability for skilled worker migration leads to a decrease in the gaps of both the skilled wage and unskilled wage between both countries.

In addition to this, one result for Pattern 2 shows that the wage differences between country D and country F are much wider. In fact, when q is low (in this particular example, when $q = 20\%$), the wage gap is larger for both skilled and unskilled wages.

3.4 Conclusions

From this study of analyzing a model of persistent migration, we find that endogenizing the migration choice in both the homogenous and heterogenous labor models, that endogenizing the skill acquisition choice in the heterogenous labor model, and that introducing a probability of migration that may serve as a quota, changes the characteristics of persistent migration and our results differ from those originally found by Reichlin and Rustichini (1998).

In this model of heterogenous labor, we are able to analyze the composition of labor from a different perspective. Since there exists this aspect of endogenous skill acquisition and, again, the probability of migration, we gain more insights regarding the differences between skilled labor and unskilled labor in each country. In this case, again we find that probabilities of migration have an important role. First, we can identify two patterns for persistent migration. One is when skilled labor from the migrant-sending country migrate only as skilled migrants in the migrant-receiving country. Second is when the skilled labor from the migrant-sending country migrate as unskilled workers with a higher probability or as skilled workers with a lower probability.

From the numerical analysis conducted, an important result is that both countries develop higher compositions of labor the higher the probability of migration, whether as skilled or unskilled. A higher composition of labor means a higher skilled workforce relative to the unskilled workforce. This implies that, because only skilled labor are allowed to migrate, the increase in the probabilities of migration results in a larger incentive to invest in education. For the migrant-sending country, this implies that applying stricter rules on the education of its migrants or increasing the standards for potential migrants may be important for emigration policy. For the migrant-receiving country, if such a policy is in place in the country where their immigrants are from, then increasing the probability of migrating, as a skilled or an unskilled worker, also results in a higher labor composition. Another result is that if the migrant-receiving country does intend to receive either skilled or unskilled migrants, it achieves a higher skilled-to-unskilled workforce ratio when it chooses to receive only skilled migrants than unskilled migrants.

For the model of heterogenous labor, given our chosen parameters, we find that for both countries, that the unskilled wage is approximately half the skilled wage, and are closer to each other as the probability of migrating as an unskilled worker or as a skilled worker increases. This is true except for the unskilled to skilled wage ratio of the migrant-receiving country, and in this sense, skilled migration as a pattern becomes more favorable. However, if the migrant-receiving country would like to maintain relatively higher wages than the migrant-sending country, then unskilled migration is more favorable as the wage ratios are smaller, or the wage gaps are larger.

For the migrant-sending country, skilled migration allows both the skilled wage and the unskilled wage to approach those of the migrant-receiving country. Although, due to persistent migration, the wages will not equalize, the wage gap is smaller with the pattern of migration where migrants gain skilled employment.

Chapter 4

The Effect of Networks

4.1 Introduction

One of the main observations of modern migrations flows is the formation of migrant networks in migrant-receiving countries. Clustering has been noted as a characteristic of migration where a specific location may attract a growing group of immigrants and, in addition, these immigrants also emigrate from similar areas of their home countries. Mexican migrants, for example, have been found to group themselves in the states of California, Texas, Florida and Chicago. While this is indicative of the formation of networks in the destination country, another interesting aspect of research in this area of migrant networks points to the observation that migrants in the same area also come from the same area. For example, 58 percent of migrants coming from Guanajuato in Mexico find themselves located in California. (Bauer, et. al. 2002) For the most part, existing networks, whether family networks or community networks, may primarily provide potential migrants with information on migrating, information on available jobs and, in some cases, a source of credit. (Dolfin and Genicot, 2006)

Hence, we also explore the effect that existing migrant networks may have on persistent migration. Work done by Carrington, et. al. (1996) has proposed that the presence of networks has a decreasing effect on the cost of migrating, while Munshi (2003) showed that a Mexican migrant to the US is more likely to be employed and to get a non-agricultural job, compared to another without an already existing network.

These networks may also allow immigrants to partake in entrepreneurial activities, have access to financial capital and have more work opportunities through referrals. Munshi (2003), in his study on Mexican immigrants in the U.S., for instance, found that networks determine both the magnitude of migration flows and the employment of future migrants. Frijters, et.al. (2005) have found that networks account for job success among immigrant groups in the United Kingdom.

Though difficult to measure, and primarily an issue for the migrant-receiving country, migrant networks have recently received more attention in research as they have been seen to allow immigrants to integrate themselves with relative ease, which is seen as a decrease in the cost of migration. Whether formal or informal, migrant organizations provide services in order for immigrants to be able to do things such as learn the local language, take training courses, and familiarize themselves with the local community.

Most of the other research done in this area has also been empirical in their approach. Goel and Lang (2009), for example, analyzes immigration in Canada using a theoretical model that shows that the larger the difference between the strength of the network and the strength of formal sources in finding employment, the smaller the wage differential between the formal and informal networks themselves. The study also shows that networks contribute to the assimilation of immigrants in the sense that network strength induces a faster arrival rate of jobs. A study looking at 27 OECD countries by Pedersen, et.al. (2004) also highlighted network effects in European countries as they have not been focused on as much as North America. One of their key results is that network effects, which is measured by the coefficient of the stock of migrants of the same nationality, has a large and positive effect on the inflow of migrant workers in OECD countries from 1990 to 2000. Along with this, other factors such as language, colonial history and business ties also have an impact on migrant flows. Bauer, et. al. (2002) also focused on the Mexico-U.S. migration corridor and try to differentiate network effects from herd effects, implying that it is herd behavior and not network effects that have a stronger positive influence on the migration decision. The study concludes that this difference is not very clear but both do have significant effects on the location of the immigrant.[1]

Munshi (2003) has a discussion on how networks emerge in the process of labor migration. He states that positive unemployment is a necessary condition for the existence of networks. However, other causes have to justify their continued existence such as the savings on the cost of migration on the part of the individual, and also incomplete information on the ability of potential employees on the part of the firm. The latter prompts the firm to enlist the help of its existing workers which could then make his network valuable. He also discusses that it is the "older" migrants that have a higher propensity to help new immigrants and, as such, contribute more to the network. By looking at data from the Mexico-U.S. migration corridor, he is able to verify that if the network is larger, an individual is more likely to find a job and to hold a preferred non-agricultural job and find that it is the presence of the more established members of the network who contribute the most and that it is the disadvantaged members (i.e. women or elderly) that benefit the most. Dolfin and Genicot (2006) add to this by stating three ways in which a network has positive migration effects: (1) the provision of information, (2) the ease of assimilation, and (3) potential source of credit. Their study, which distinguishes between family and community networks, verifies that networks do have strong positive effects on the migration decision.

Mackenzie and Rapaport (2007) look at the effect of these formed networks on income inequality in the home country, through the premise that the formed networks lower the cost of migration and, hence, might improve inequality in the source economy. They find that at very high levels of migration due to the presence of large networks, the benefits of migration spread to the those in the lower income brackets in the source economy as the cost of migration is lower while the reverse is also true, i.e. smaller networks benefit those with higher incomes in the

[1] See Epstein (2002) for more discussion on herd behavior, along with informational cascades.

home economy.

Carrington, et. al. (1996) use the presence of networks in determining migration flows in a dynamic setting which attempt to explain the Great Migration from the South to the North in the U.S. The presence of networks essentially decreases the cost of migration and this model better explains increasing migration flows than models that assume fixed or decreasing costs of migration.[2]

The main objective of this chapter is then to assess how migrant networks affect persistent migration. Network effects is an area that has also garnered research attention over the years. It banks on the concepts brought forth by research on the network effects on the labor-market in a local economy such as those from Montgomery (1991; 1992). And also uses theories and ideas put forth by social networks and ethnicity such as that of Borjas (1995). The main idea from labor economics is that work referrals through friends or relatives contributes to the resulting workforce. It is understood that referrals through networks is not only less costly but can also result into a higher probability of generating a job offer. Hence, individuals who are well-connected tend to fare better from this informal process than those who are not and, also, firms who hire through this process could earn higher profits.

In Section 3.2, we introduce network effects into the model of homogenous labor, as presented in Chapter 2 and show how network effects may occur through a higher probability of migration. In Section 3.3, we show our analysis and results and conclude in Section 3.4.

[2]See Sjaastad (1962). They also test this idea empirically on the incomes of African-American migrants in the North and South and find that the income differential decreases due to the presence of networks.

4.2 Network Effects

In this section, we introduce the effect that existing migrant networks have on the patterns of migration. Some migrant-receiving countries have implemented programs of family reunification for migrant workers, and this has made the migration process easier for some applicants. The migration process also becomes less expensive as the existence of networks enables individuals to settle in their foreign environments more efficiently, including finding accommodation as well as acquiring employment. In the job search literature, it has been shown that networks do increase the probability of finding a job due to the increase in job referrals (Montgomery, 1991; 1992).

We can, thus, identify one way in which networks may affect migration, where an existing migrant network may increase the probability of migration. The increase of the probability of migration due to the presence of network effects may be interpreted as the ease at which a potential migrant is able to cross the border, find a job in the host country, and also settle in the new environment. It may also be interpreted as the ease of applying for migration with the migration applicant facing a better chance of being granted an immigrant visa to the country of their choice. This may represent programs such as family reunification where people who have families in the destination country find it easier to migrate or are given a higher chance to migrate by the host country itself. In the United States, for example, family reunification was found the the largest avenue for potential migrants to gain residence.[3] Hence, family ties may be a source for increasing the probability of migration.

[3]Source: Migration Information Source, 2003, http://www.migrationinformation.org/usfocus/display.cfm?id=122#1, Retrieved 2011.

4.2.1 Increasing Probability Due to Networks

The assumption in this case is that an increase in the proportion of migrants in the previous period makes it easier for an individual applying for migration to be accepted by the migrant-receiving country so we assume that the probability of migration is increasing in the previous period's proportion of migrants. Hence, the probability of migration becomes endogenous and is determined by the initial probability of migration and a term which is increasing in the proportion of those who apply for migration in the previous period.

$$p_t = p(\theta_{t-1}) \tag{4.1}$$

where $p : (0,1) \to (0,1)$ is twice continuously differentiable and where $p'(\cdot) > 0$.

Hence, the proportion of migrants at time period t is

$$\theta_t = \frac{p(\theta_{t-1})(1-\lambda_t^\gamma)}{p(\theta_{t-1}) + (1-p(\theta_{t-1}))\lambda_t^\gamma} \tag{4.2}$$

4.2.2 Analysis and Results

Lemma 6. *Let $p_t \in (0,1)$ and if $p_t = p(\theta_{t-1})$ where $p : (0,1) \to (0,1)$ is twice continuously differentiable and where $p'(\cdot) > 0$, we have $(\theta_t)_n > \theta$.*

Proof. Let $(\theta_t)_n$ be the proportion of those that intend to migrate when there are network effects. When $p_t \in (0,1)$, $\theta_t = \frac{p_t(1-\lambda_t^\gamma)}{p_t+(1-p_t)\lambda_t^\gamma}$. We can find the following partial derivative.

$$\frac{\partial \theta_t}{\partial p_t} = \frac{(1-\lambda_t^\gamma)\lambda_t^\gamma}{(p_t+(1-p_t)\lambda_t^\gamma)^2} \tag{4.3}$$

Since $\lambda \in (0,1)$ and $\gamma > 0$, we have $\lambda_t^\gamma \in (0,1)$ and $\frac{\partial \theta_t}{\partial p_t} > 0$.

Therefore, with network effects, $(\theta_t)_n > \theta_t$. □

Hence, when the network effect takes place, the result is a higher proportion of migrants θ_t when there is an increase in the previous period's θ, or $(\theta_t)_n > \theta_t$ where the subscript n stands for network effects. A higher proportion that applies for or intends to migrate in the previous period leads to a higher probability of migration in the current period, leading to a higher proportion of potential migrants in the current period, showing how existing networks formed by immigrant from the previous period can encourage more migration.

Proposition 8. *If there is an increase in θ_{t-1}, then λ_t will decrease.*

Proof. When $p \in (0,1)$, $\theta_t = \frac{p_t(1-\lambda_t^\gamma)}{p_t+(1-p_t)\lambda_t^\gamma}$.

We can calculate, via Implicit Function Theorem,

$$\frac{\partial \lambda_t}{\partial \theta_t} = -\frac{p_t + (1-p_t)\lambda_t^\gamma}{\theta_t(1-p_t)\gamma\lambda_t^{\gamma-1} + p_t\gamma\lambda_t^{\gamma-1}} \tag{4.4}$$

Hence, $\frac{\partial \lambda_t}{\partial \theta_t} < 0$ and a higher θ_t due to network effects leads to a lower λ_t or a lower workforce ratio between countries D and F. □

This implies that when network effects take place, and a higher probability of migration occurs in the next period, then there will be a higher proportion intending to migrate, which leads to a lower workforce ratio in the next period, i.e. the domestic workforce will be smaller while the foreign workforce will be larger. From this, it should then be noted that the wage differential is larger with network effects. The migrant-receiving country thus enjoys relatively higher wages over time when network effects take place while the migrant-sending country experiences relatively lower wages. This may then be interesting to note for migrant-receiving countries. The increase of immigrants due to the formation of networks may be of an issue in certain societies; however, in this simple case, the host country ends up facing relatively higher wages over time and network effects for their country tends to be positive.

We also assess the new system of equations under steady state.

Lemma 7. *Let* $p = p(\theta)$ *where* $p'(\theta) > 0$*, then if* $\theta p'(\theta) + p(theta) + (1-p)\lambda^\gamma > p'(\theta)[\theta\lambda^\gamma + (1-\lambda^\gamma)]$*, then* $\frac{\partial\theta}{\partial p} > 0$*.*

Proof. At steady state, Eq.(4.2) becomes

$$\theta = \frac{p(\theta)(1-\lambda^\gamma)}{p(\theta) + (1-p(\theta))\lambda^\gamma} \tag{4.5}$$

where $p(\theta)$ is the function of the probability of migration where $p'(\theta) > 0$.

By Implicit Function Theorem, we can find $\frac{\partial\theta}{\partial p}$ as follows.

$$\frac{\partial\theta}{\partial p} = -\frac{(\theta-1)(1-\lambda^\gamma)}{\theta p'(\theta) + p(\theta) + (1-p(\theta))\lambda^\gamma - p'(\theta)(\theta\lambda^\gamma + (1-\lambda^\gamma))} \tag{4.6}$$

Since $(\theta-1) < 0$, the term $\frac{\partial\theta}{\partial p} < 0$ or > 0 depending on $\theta p'(\theta) + p(\theta) + (1-p)\lambda^\gamma$ and $p'(\theta)(\theta\lambda^\gamma + (1-\lambda^\gamma))$. Since, by assumption, $p'(\theta) > 0$, then if $\theta p'(\theta) + p(\theta) + (1-p)\lambda^\gamma > p'(\theta)(\theta\lambda^\gamma + (1-\lambda^\gamma))$, then $\frac{\partial\theta}{\partial p} > 0$.

□

Therefore, analytically, at steady state, the effect of an increasing probability function on the steady-state proportion of the newborn population that applies for migration is ambiguous. It is dependent on the steady-state probability as well as the steady-state proportion of potential migrants and the steady-state workforce ratio. Given this, we may observe a result for the steady-state workforce ratio.

Proposition 9. *Let* $\mu(2\theta-1) > 1$*, then if* $p = p(\theta)$ *where* $p'(\theta) > 0$ *and* $\frac{\partial\theta}{\partial p} > 0$*, then* $\frac{\partial\lambda}{\partial p} > 0$*.*

Proof. At steady state, we have $\lambda = \frac{(1-p(\theta)\theta)\mu - 1}{p(\theta)\theta\mu}$. We can find $\frac{\partial\lambda}{\partial p} > 0$ to be

$$\frac{\partial \lambda}{\partial p} = \frac{\theta\mu(\frac{\partial \theta}{\partial p})(\mu(2\theta - 1) - 1)}{(p\theta\mu)^2} \tag{4.7}$$

Thus, if, by assumption, $\mu(2\theta - 1) > 1)$, if $\frac{\partial \theta}{\partial p} > 0$, then we have $\frac{\partial \lambda}{\partial p} > 0$. □

The above result explains that there are two possible scenarios for the workforce ratio, and this also implies that there are two possible scenarios for the wage ratio, depending on what happens to the steady-state proportion of migrants if the probability of migration is endogenous and is increasing in the proportion of potential migrants. Hence, if network effects take place, and if the result is a higher steady-state proportion of migration applicants, then the steady-state workforce ratio is higher, and the wage ratio is higher, which implies that the wage in the migrant-sending country is closer to that of the migrant-receiving country. However, it is still possible that at steady-state, the workforce ratio and the wage ratio are lower than when network effects do not take place, and the migrant-sending country is left with a much lower wage compared to that of the migrant-receiving country. Therefore, at steady-state, the existence of network effects may bode well either for the home country or the host country, depending on the effect of the increasing probability of migration on the steady-state proportion of the newborn population that applies for migration.

4.2.3 Numerical Analysis

In this section, we provide a numerical simulation by specifying the functional form of the probability of migration as an example. We assess the change in the workforce ratio over time when network effects take place, and we also assess the effect on the steady-state workforce ratio and the steady-state wage ratio with network effects.

In the Appendix, Figure D.1 shows us how the workforce ratio changes over several time periods given different probabilities. We use the same parameters used in Chapter 2 where $\mu = 1.011$ and $\gamma = 0.4$. We set the initial workforce ratio at $\lambda_0 = 0.90$. And, from the analysis in Chapter 2, we note that given these parameters, there is divergence from the steady-state, and the workforce ratio decreases over time, as the workforce of the migrant-sending country, Country D decreases and the workforce of the migrant-sending country, Country F increases as persistent migration occurs. We assess three cases: (1) where the initial probability of migration is set at $p = 0.40$, (2) where the probability of migration is increasing in the proportion of potential migrants from the last period at an increasing rate, $p_t = 0.40 + \theta_{t-1}^2$, and (3) another case with a different exponent, where $p_t = 0.40 + \theta_{t-1}^{0.80}$., where the additive term is increasing in the previous period's proportion of potential migrants at a decreasing rate.

From the figure, it can be noted that the decrease in the workforce ratio over time is faster in both cases where the probability of migration is increasing in the proportion of migrants in the last period, with the decrease occurring faster when the exponent is 0.80 than when it is 2. This basically implies that with the probability of migration increasing in the previous period's proportion of potential migrants, more migration occurs and migration becomes more persistent. This also implies that the wage ratio decreases faster over time as network effects take place. Thus, with more generations participating in persistent migration, over time, the migrant-sending country faces a relatively lower wage than the migrant-receiving country. The migrant-receiving country, over time, experiences relatively higher wages due to network effects.

At steady-state, we analyze the effect of network formation on the workforce ratio. Below is Table 4.1 where we find the steady-state values of λ and θ given an initial probability of either $p = 0.50$ or $p = 0.80$ and with p increasing in steady-state θ with an exponent taking on different values. From the numerical examples below, it can be noted that if the exponent were less than one, that the steady-state values for the workforce ratio, λ, are higher. If the exponent were more than one, then λ is lower.

Table 4.1: Steady-State Variables with Network Effects

Probability of Migration p	Workforce Ratio λ	Proportion of Potential Migrants θ
$p = 0.50$	0.9456	.0112
$p = 0.50 + \theta^{0.2}$	0.9816	0.064
$p = 0.50 + \theta^{0.8}$	0.9499	0.0106
$p = 0.50 + \theta^{1.4}$	0.9448	0.0111
$p = 0.50 + \theta^{2}$	0.9444	0.0112
$p = 0.80$	0.9787	.0069
$p = 0.80 + \theta^{0.2}$	0.9895	0.048
$p = 0.80 + \theta^{0.8}$	0.9795	0.0067
$p = 0.80 + \theta^{1.4}$	0.9786	0.0069
$p = 0.80 + \theta^{2}$	0.9785	0.0069

The numerical example above shows that if the probability of migration was increasing in the previous period's proportion of migrants at an increasing rate, that the resulting steady-state workforce ratio is lower. This implies that the network effects occurring at an increasing rate leads to country D having a relatively lower workforce, and also relatively lower wages than country F. If network effects occurred at a decreasing rate, then country D will end up having a higher workforce, and also face higher wages compared to the previous case. We can then note from this simple exercise that network effects taking place at a slower rate leads to a more favorable outcome for the migrant-sending country as it faces relatively higher wages compared to the other case. However, for the migrant-receiving country, at steady-state, they face a more favorable outcome under network effects taking place at a faster rate as they result in having a relatively higher workforce, and relatively higher wages at steady state.

Given this scenario of persistent migration where network effects take place, it is then of more interest to explore how network effects take place and the rate at which it may increase potential migrants. As is the case in many migration corridors, network formation is a part of the migration process and assessing the specifics of how network effects occur may bring in more comprehensive results regarding its long-run impact on the host and home countries.

4.3 Conclusions

A model of persistent migration enables network effects to take place and we can therefore analyze the effects on the workforce ratios as well as the wage ratios between the two countries involved. With the assumption that a larger migrant network leads to positive externalities of a potential migrant via an increase in the probability of migration, the effects on migration flow are the same, albeit the analysis is different and is also dependent on the specific functional form of how network effects take place and determine the probability of migration.

First, we observe a higher proportion of the population that does intend to migrate in the subsequent time periods. This is a very basic result and relies heavily on the functional form of the probability of migration. To our knowledge, however, that this is shown for a model of persistent migration when the choice of migration is endogenous is perhaps new to the literature.

Second, the impact on the workforce ratio in the next period between the two countries is lower, implying that the source country has a relatively lower workforce than the destination country. In this case, under the assumption of homogenous labor, when comparing the size of the workforce, the size of the migrant-sending country's workforce is, thus, relatively smaller than if there were no network effects. This implies that persistent migration leads to the migrant-sending country having a relatively smaller workforce, simply due to the effect of the network encouraging a larger outflow of emigrants, and over time, also experiences relatively lower wages. The divergence is thus hastened with network effects.

At steady state, however, the results may vary. The effect on the steady-state workforce and wage ratios may vary depending on how the probability of migration is affected by the network effects. It may either be more favorable for the migrant-sending country, i.e. the workforce and the wage differentials are smaller, or it may be favorable for the migrant-receiving country, i.e. they have a much higher workforce and face higher wages.

Numerically, we show how this may occur. When the network effects occur at a slower rate, it is favorable for the home country and when the network effects occur at a faster rate, it is more favorable for the host country. Hence, for host countries, a strong network formation process of their immigrants is better in that they have relatively higher wages at steady state. Whereas for the home country, a slower process of network formation becomes more favorable. While network effects enhance the chances of potential migrants of working in the country of their choice for a few generations, at steady state, they are better off with a slow occurrence of network effects.

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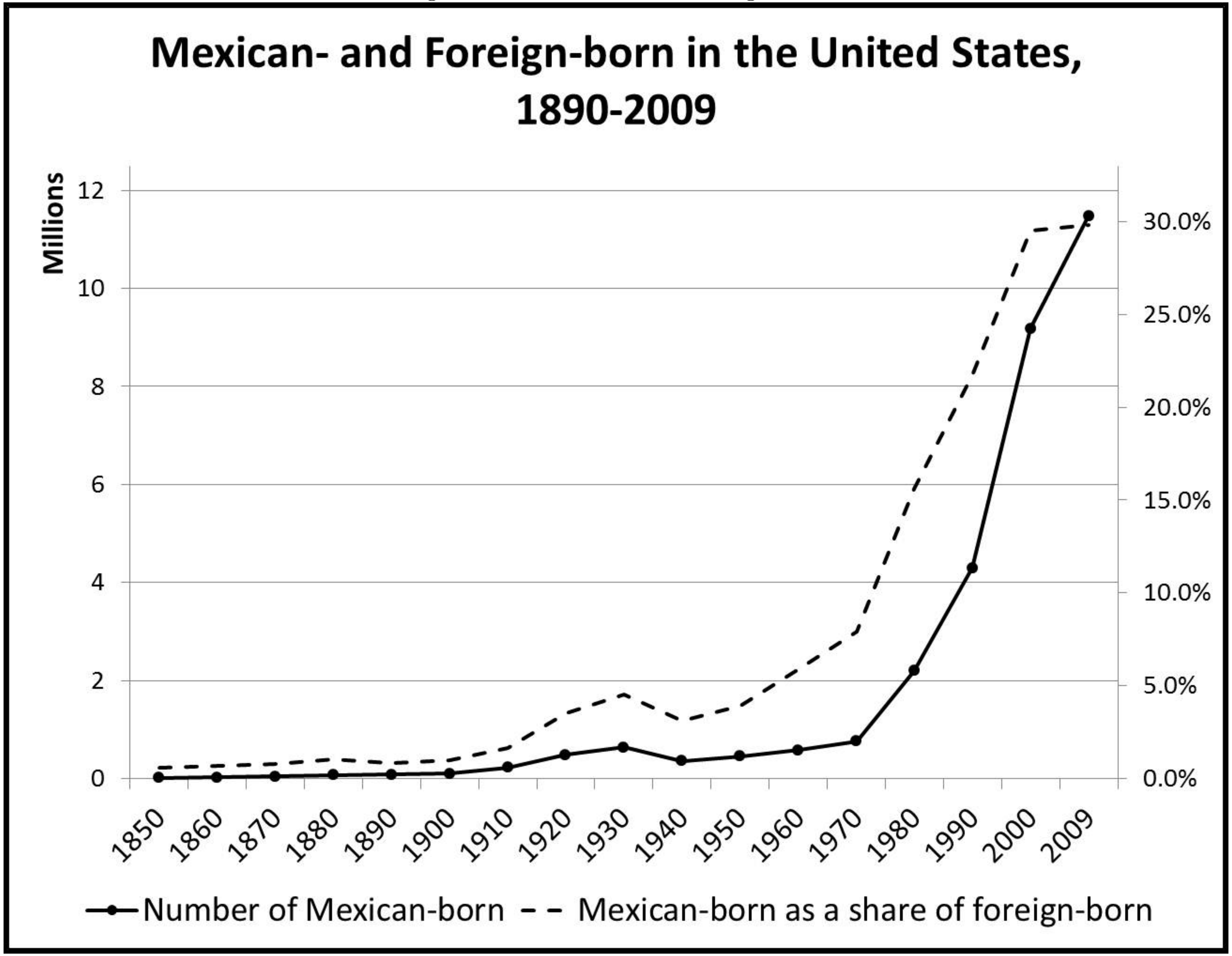
Appendices

Appendix A

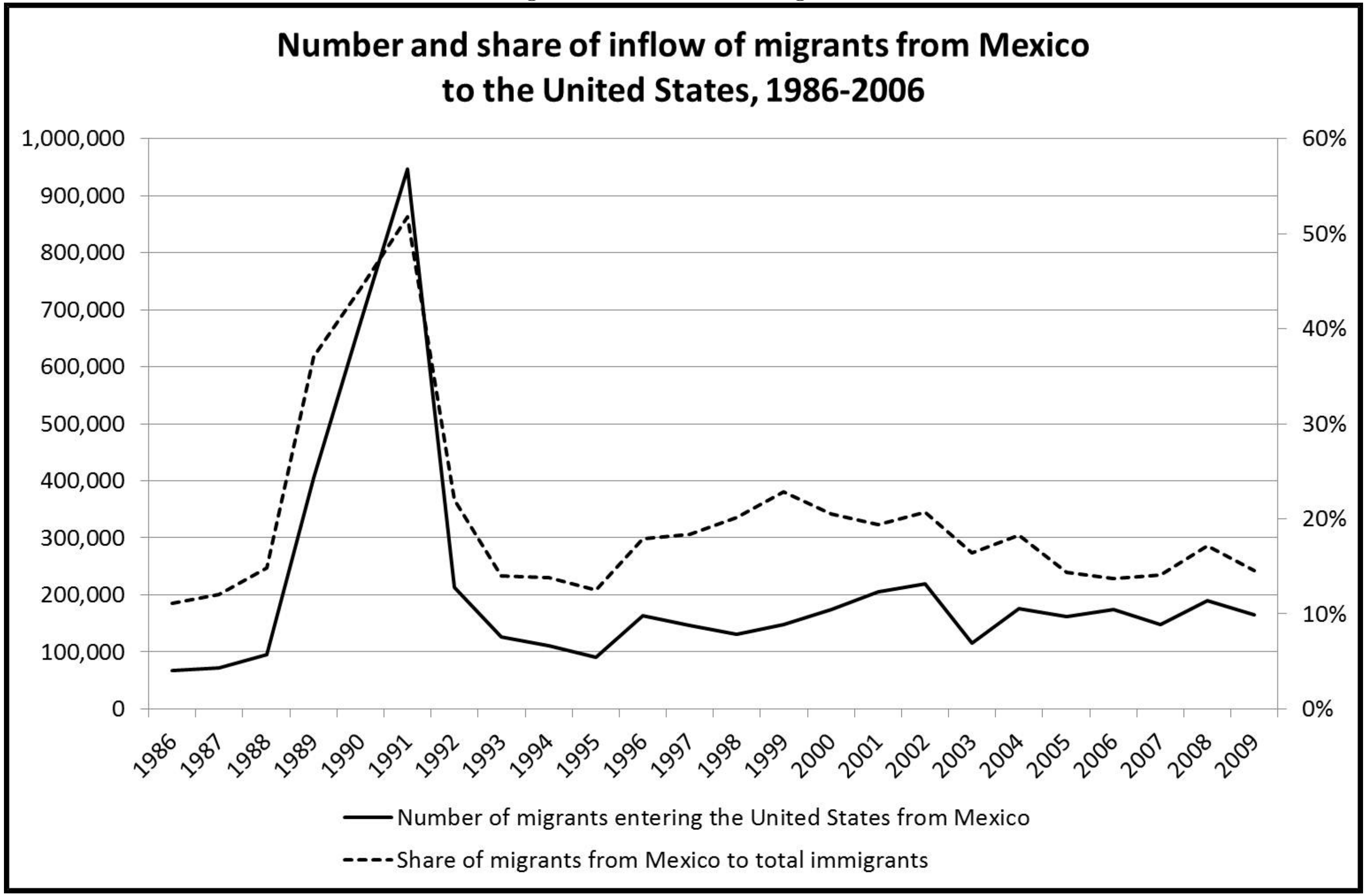
Introduction: Figures and Tables

Figure A.1: Mexican- and Foreign-born



Source: Migration Policy Institute Data Hub, Washington D.C., last viewed April 2011, http:\\www.migrationinformation.org/DataHub/charts/fb-mexicans.cfm

Figure A.2: Inflow of Migrants



Source:Migration Policy Institute Data Hub, Washington, D.C., last viewed September 2010, http:\\www.migrationinformation.org/DataHub/historicaltrends.cfm

Table A.1: Top 15 Migration Corridors

Migration Pattern	**Number of migrants, millions**
Mexico-United States	11.6
Bangladesh-India	3.3
Turkey-Germany	2.7
China-Hong Kong SAR, China	2.2
India-United Arab Emirates	2.2
China-United States	1.7
Philippines-United States	1.7
Afghanistan-Iran, Islamic Rep.	1.7
India-United States	1.7
Puerto Rico-United States	1.7
West Bank and Gaza-Syrian Arab Republic	1.5
India-Saudi Arabia	1.5
Indonesia-Malaysia	1.4
Burkina Faso-Côte d'Ivoire	1.3
United Kingdom-Australia	1.2

[a] Source: World Bank, Migration and Remittances Factbook 2011

[a]This table excludes any migration involving the former Soviet Union.

Appendix B

Chapter 2: Figures

Figure B.1: Steady-State Workforce Ratio, λ

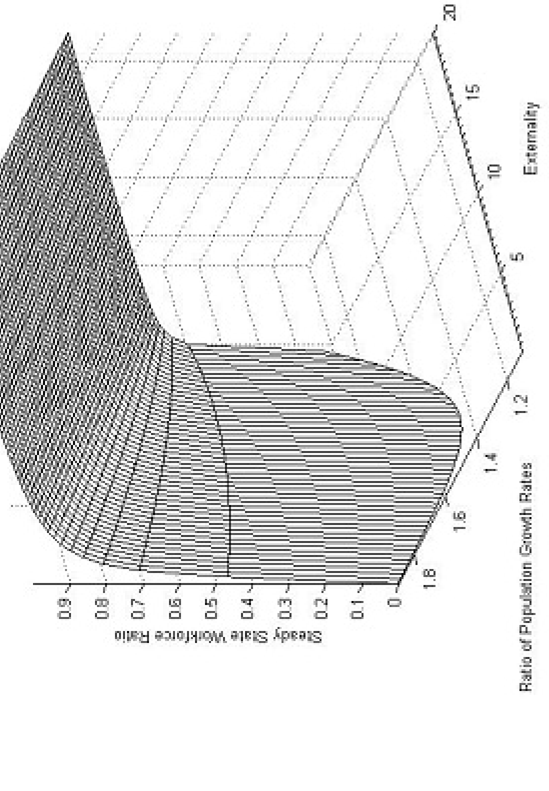


Figure B.2: $g'(\lambda)$

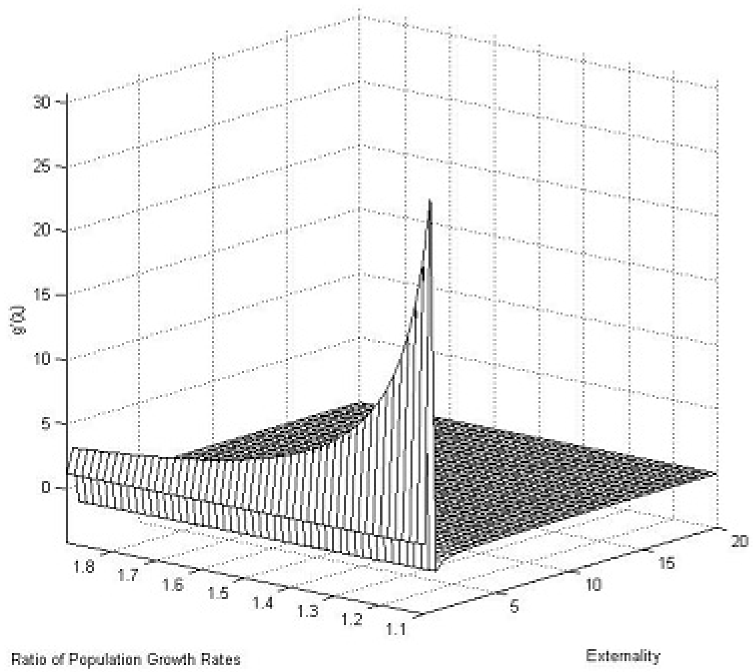


Figure B.3: Steady-State Workforce Ratio, λ when $p = 0.80$

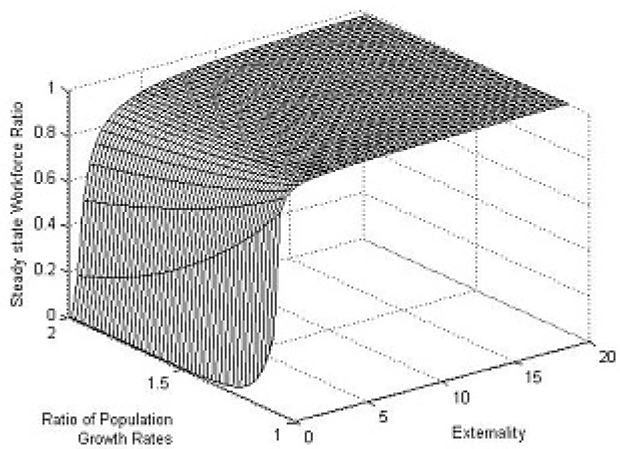
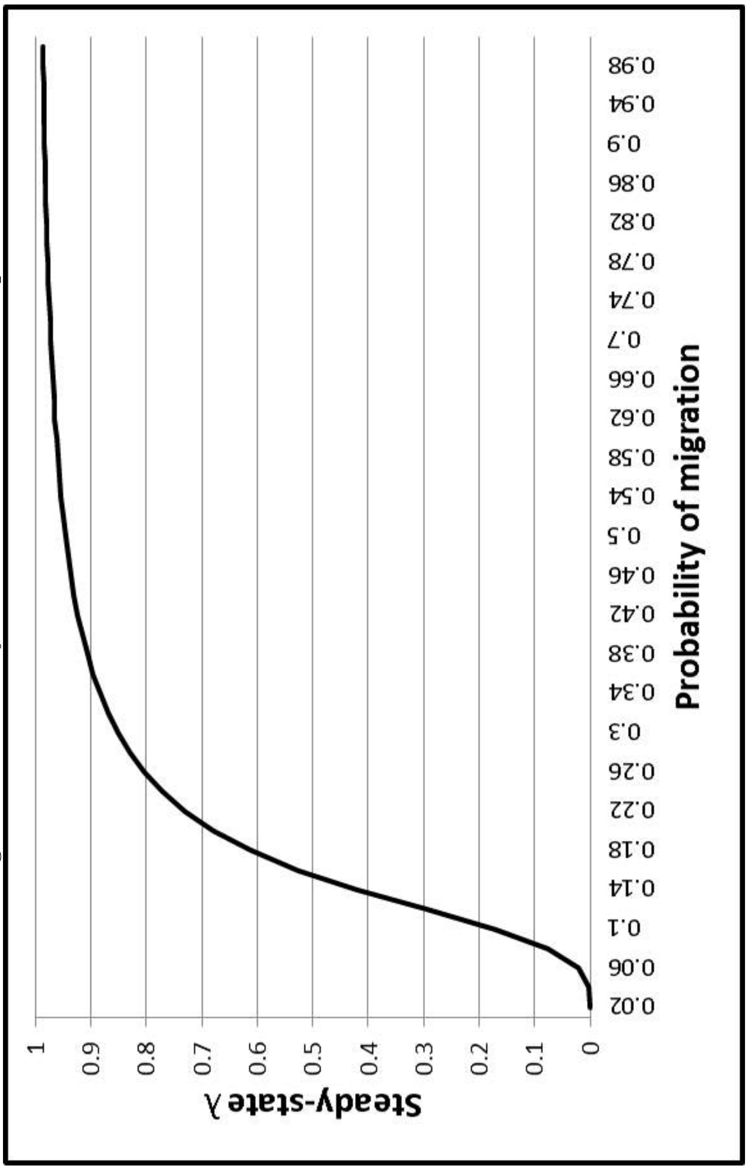


Figure B.4: Steady-State λ as a Function of p



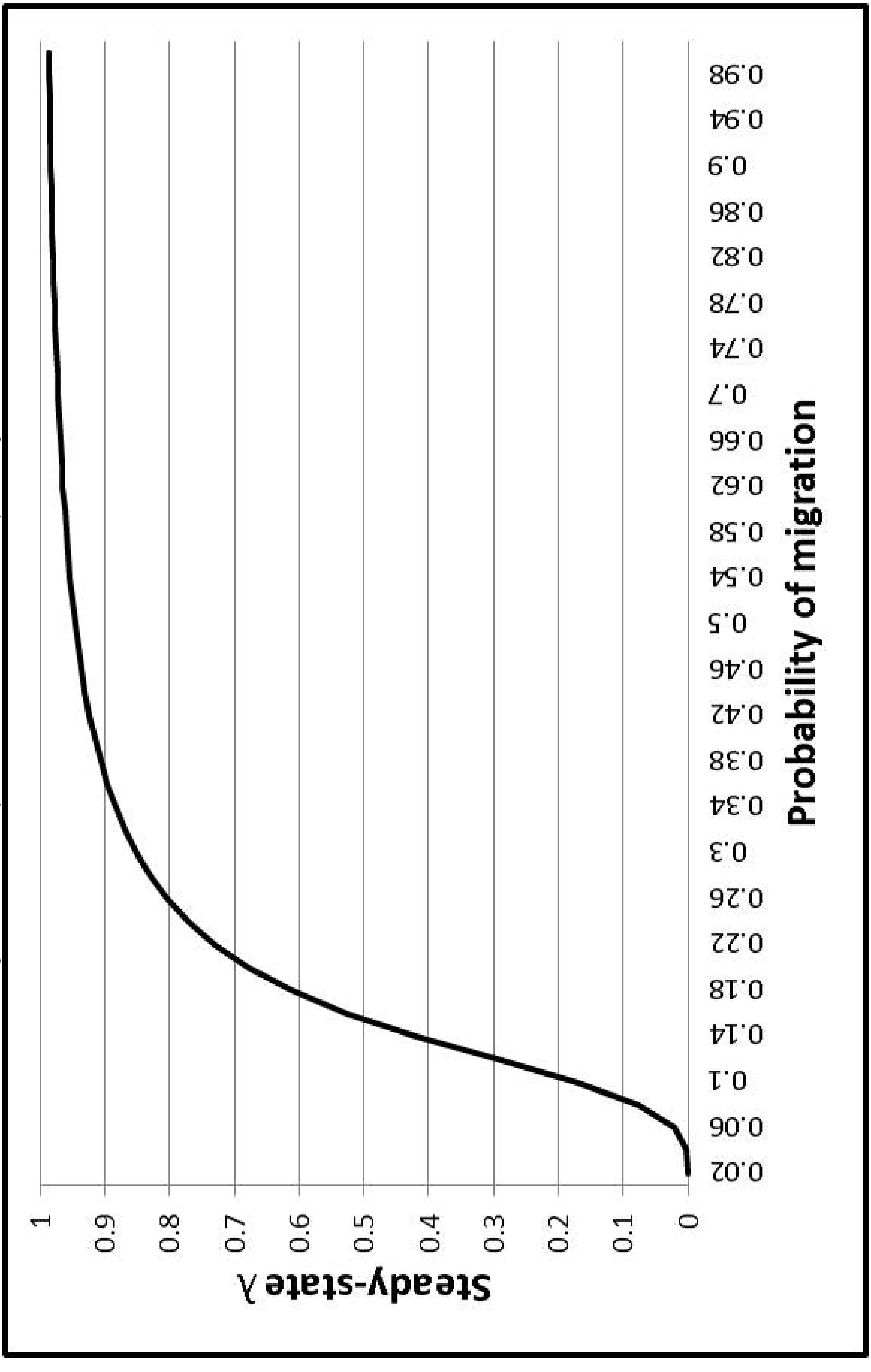
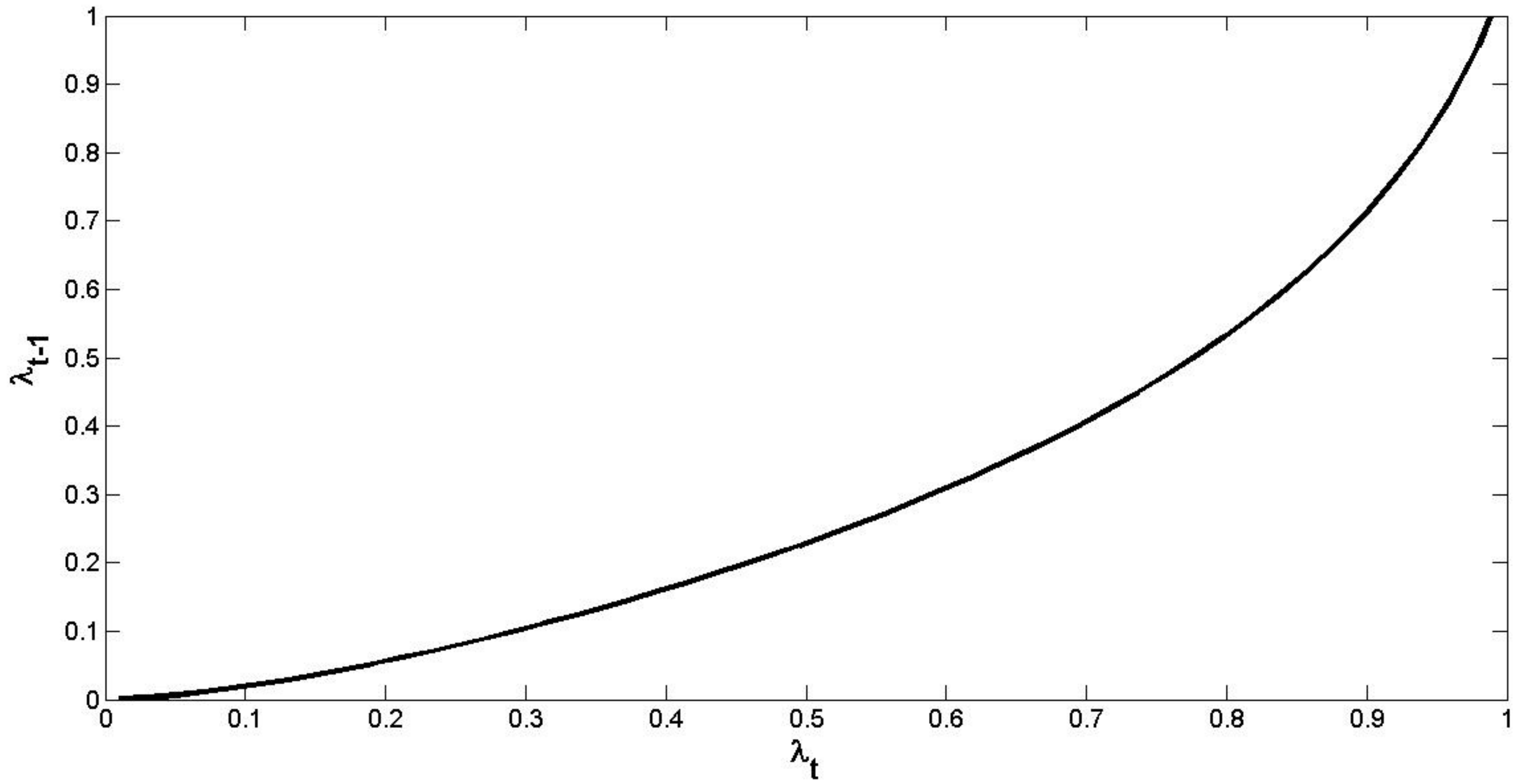


Figure B.5: Steady-State λ and the Probability of Migration

Figure B.6: Evolution of λ for a Given Set of Parameters



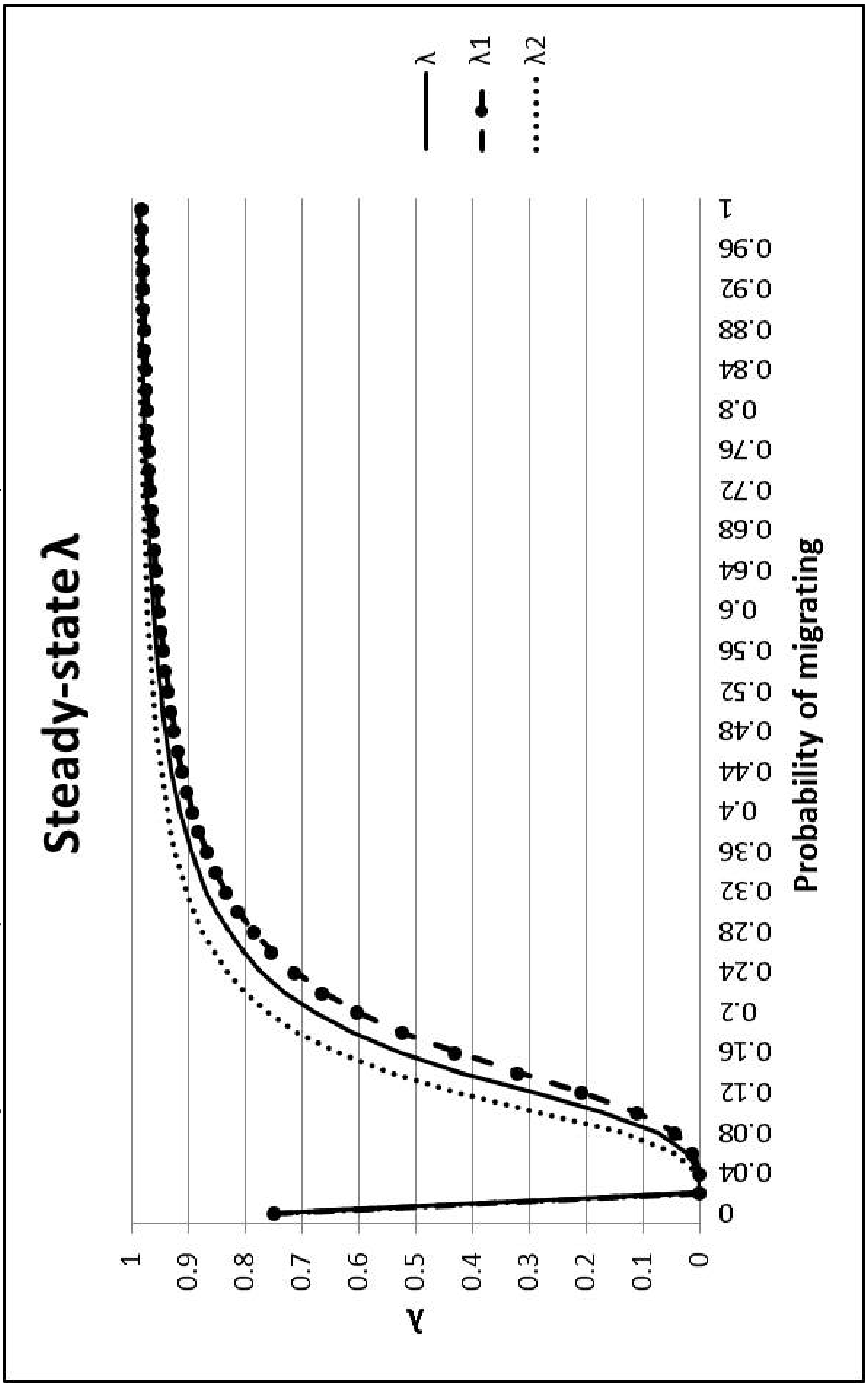


Figure B.7: Steady-State λ under Different Values of μ, Checks 1a and 1b

Figure B.8: Steady-State λ under Different Values of α, Checks 2a and 2b

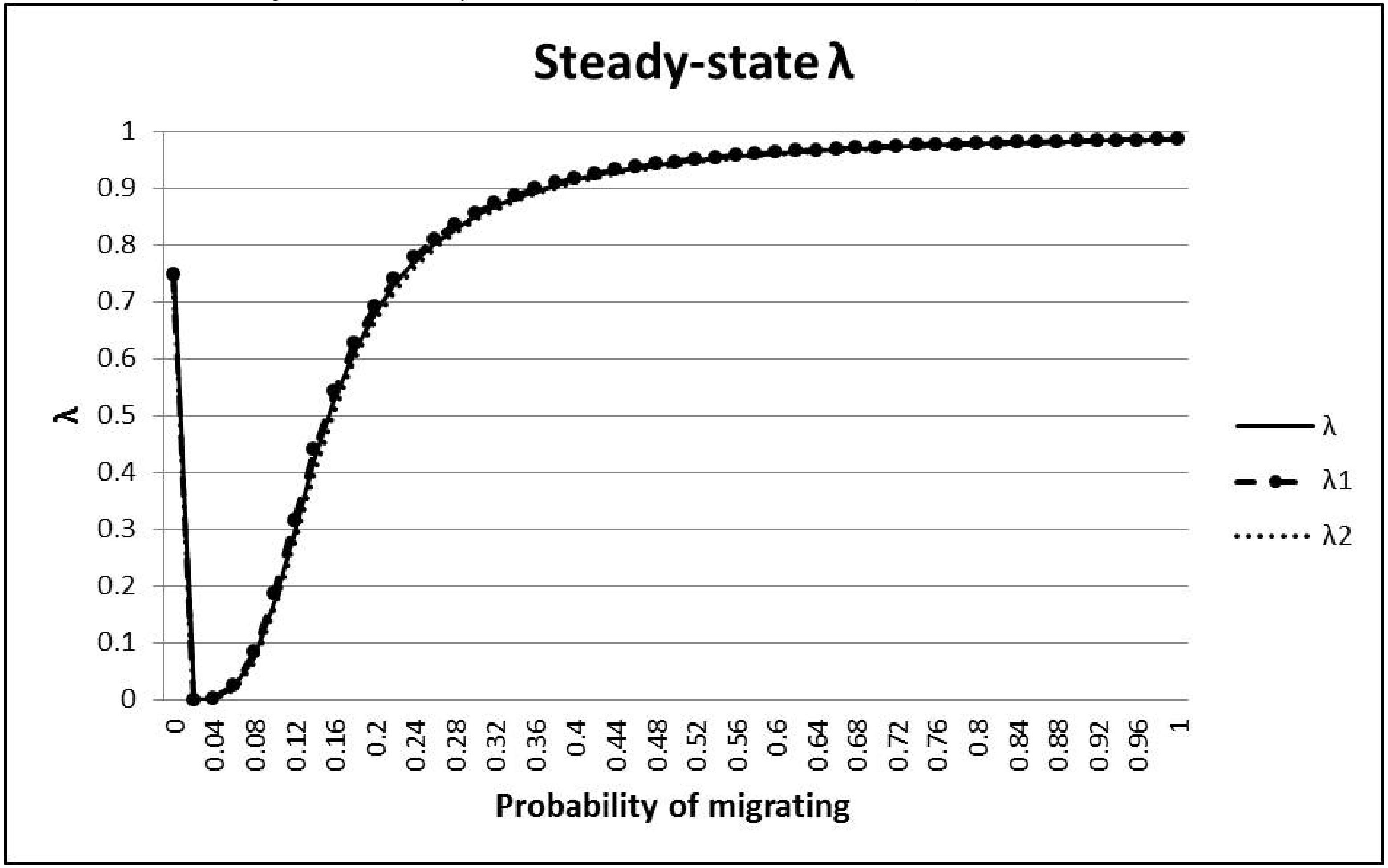


Figure B.9: Steady-State λ under Different Values of η, Checks 3a and 3b

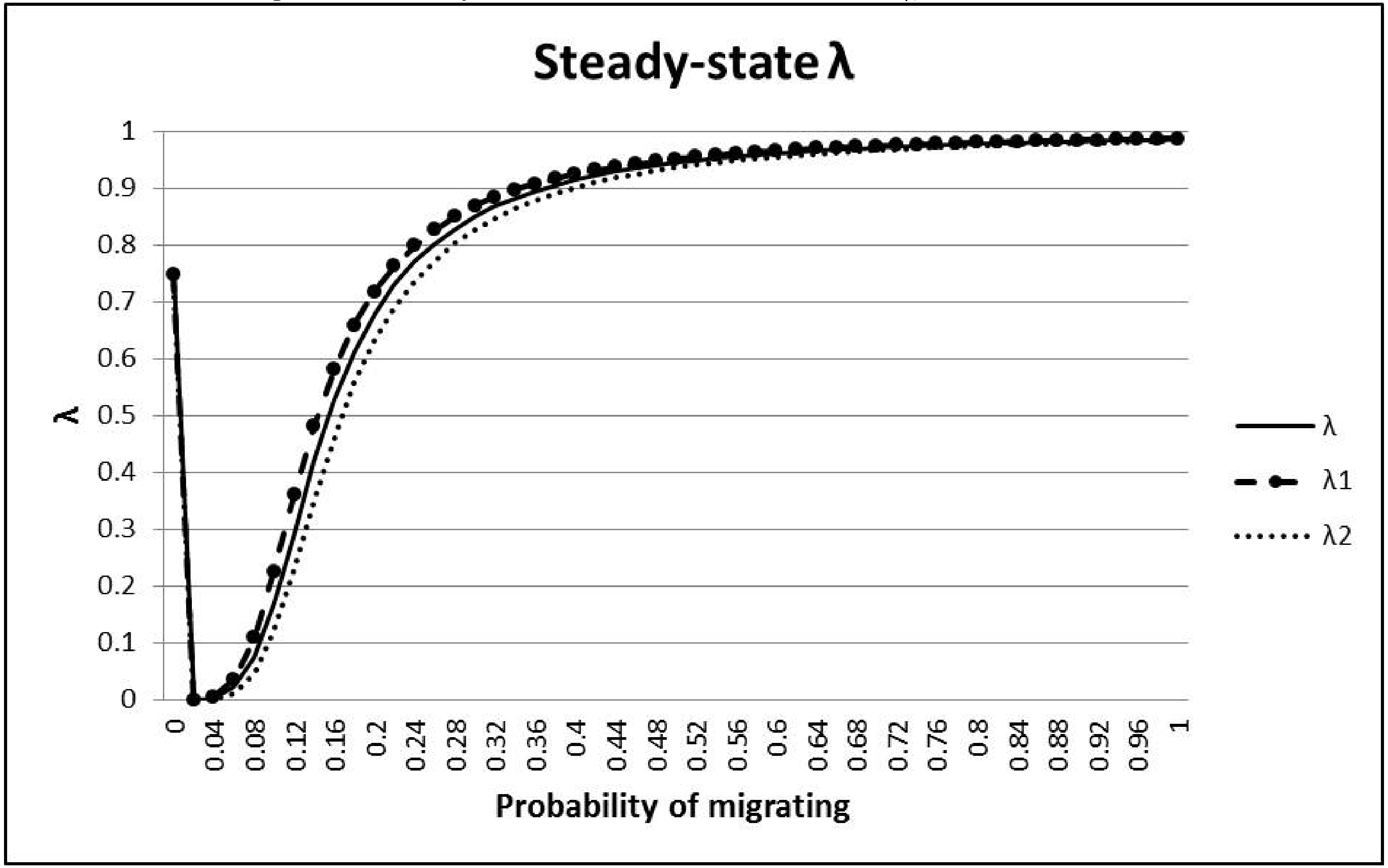
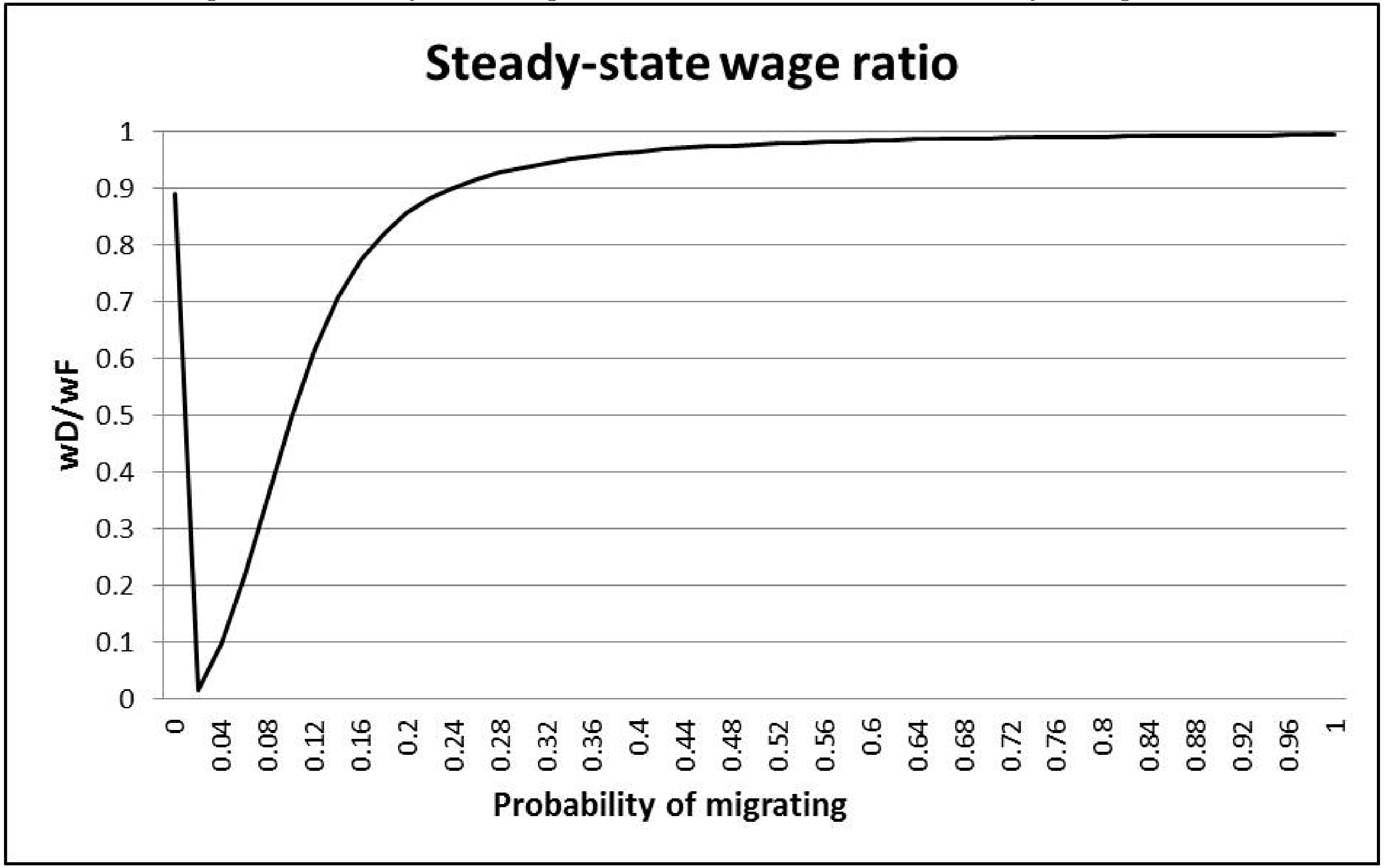


Figure B.10: Steady-State Wage Ratio as a Function of the Probability of Migration



Appendix C

Chapter 3: Figures

Figure C.1: Educational Attainment

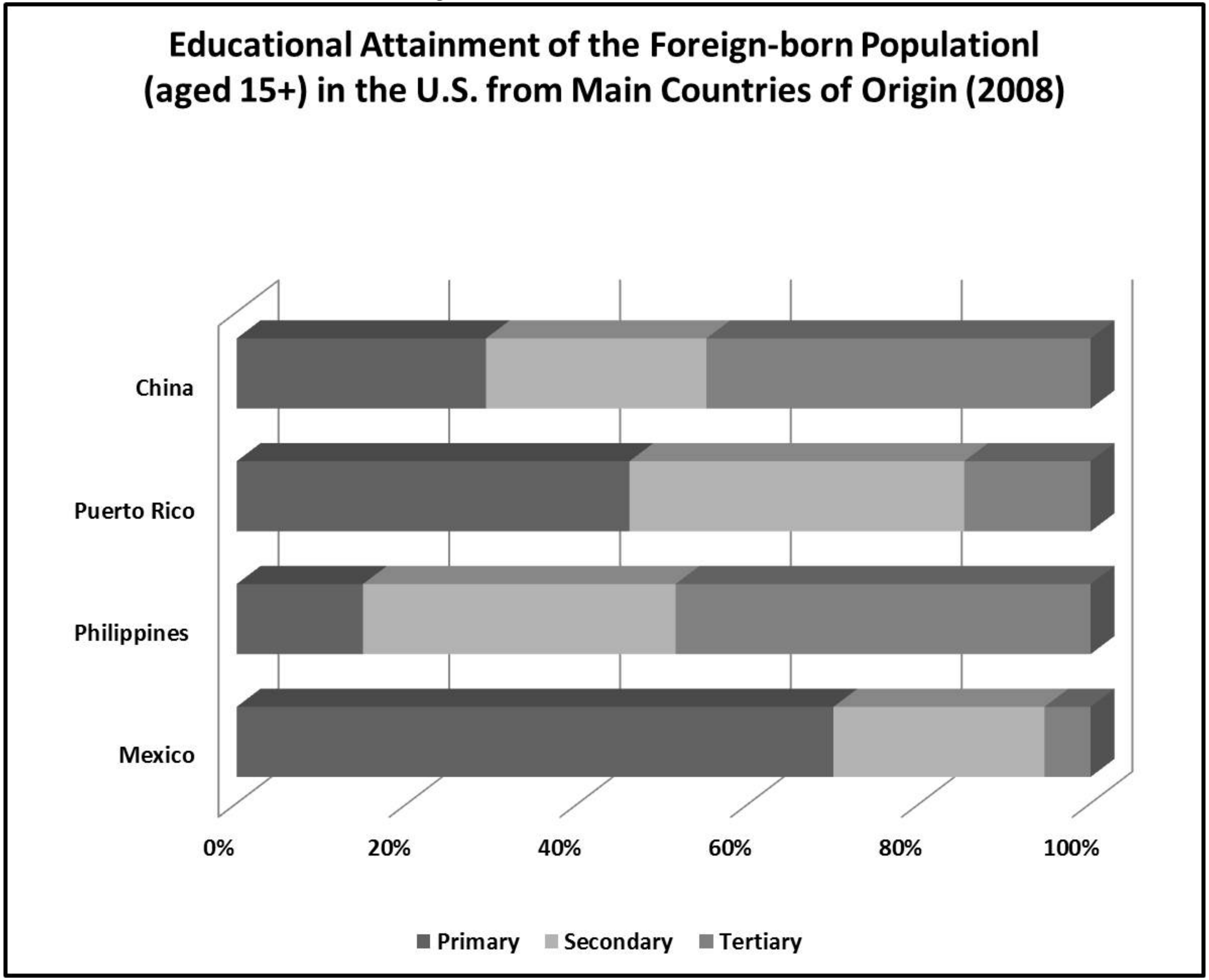


Figure C.2: Steady-State λ for Pattern 1

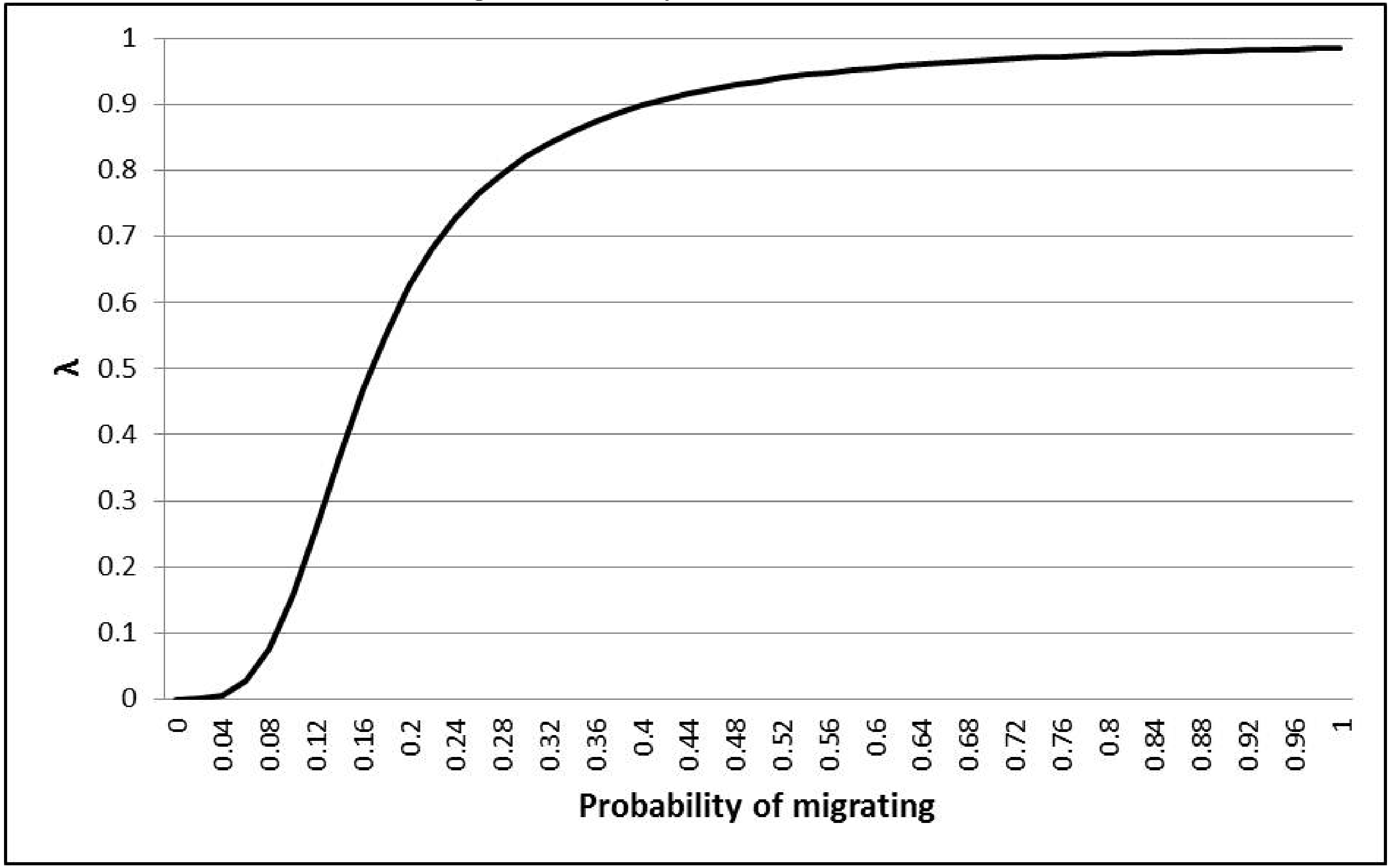


Figure C.3: Steady-State π^D for Pattern 1

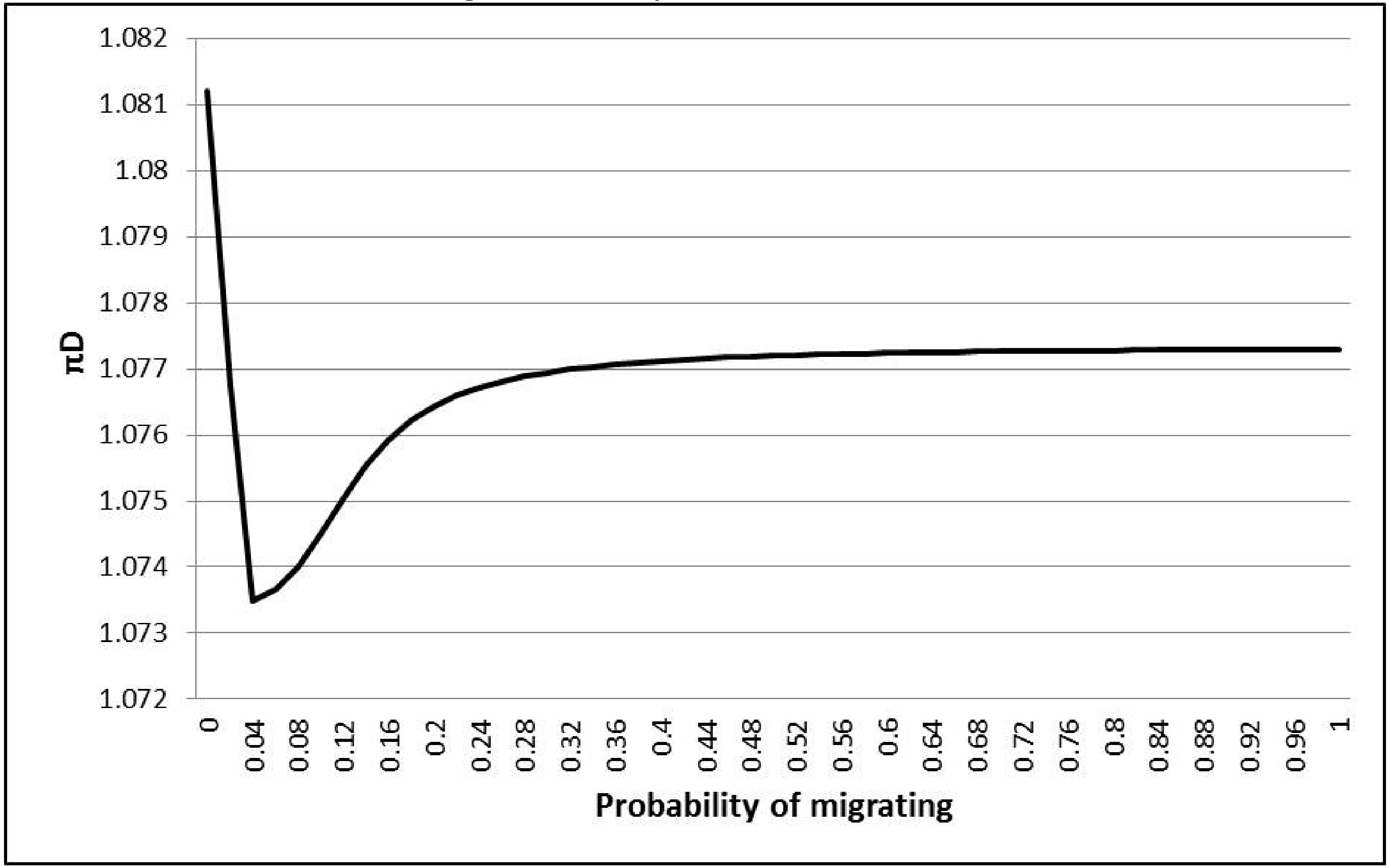


Figure C.4: Steady-State π^F for Pattern 1

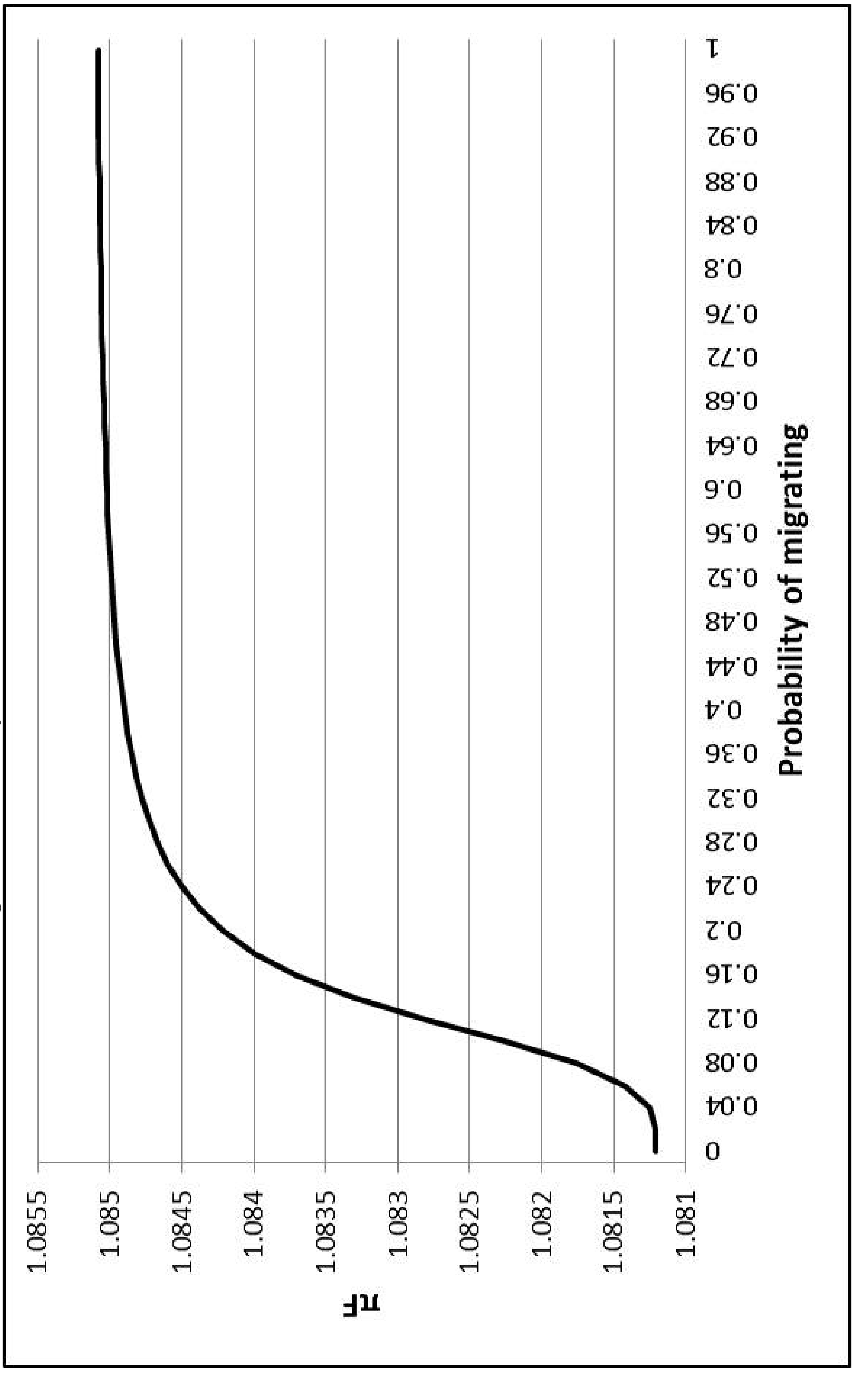


Figure C.5: Steady-State λ for Pattern 2

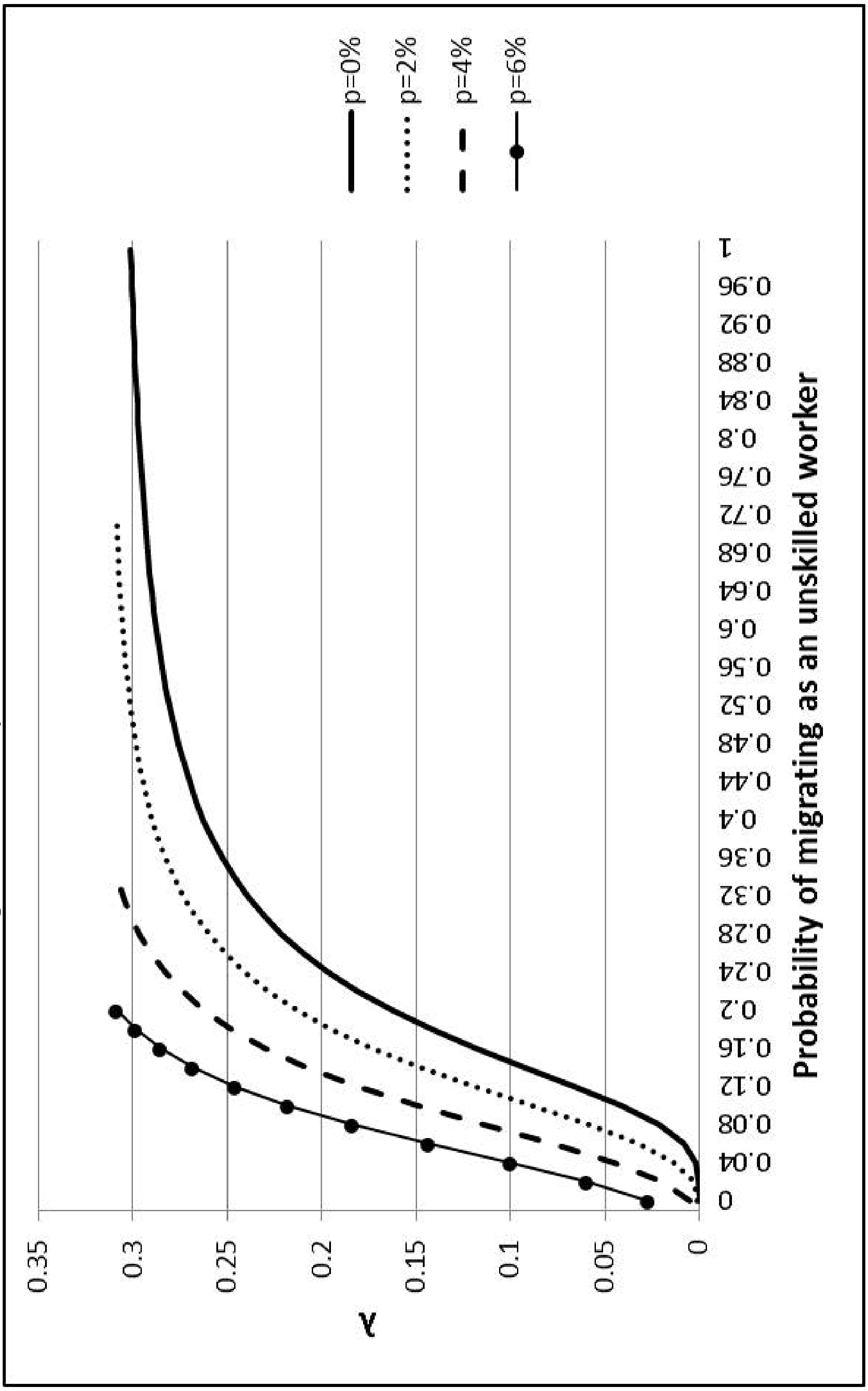


Figure C.6: Steady-State π^D for Pattern 2

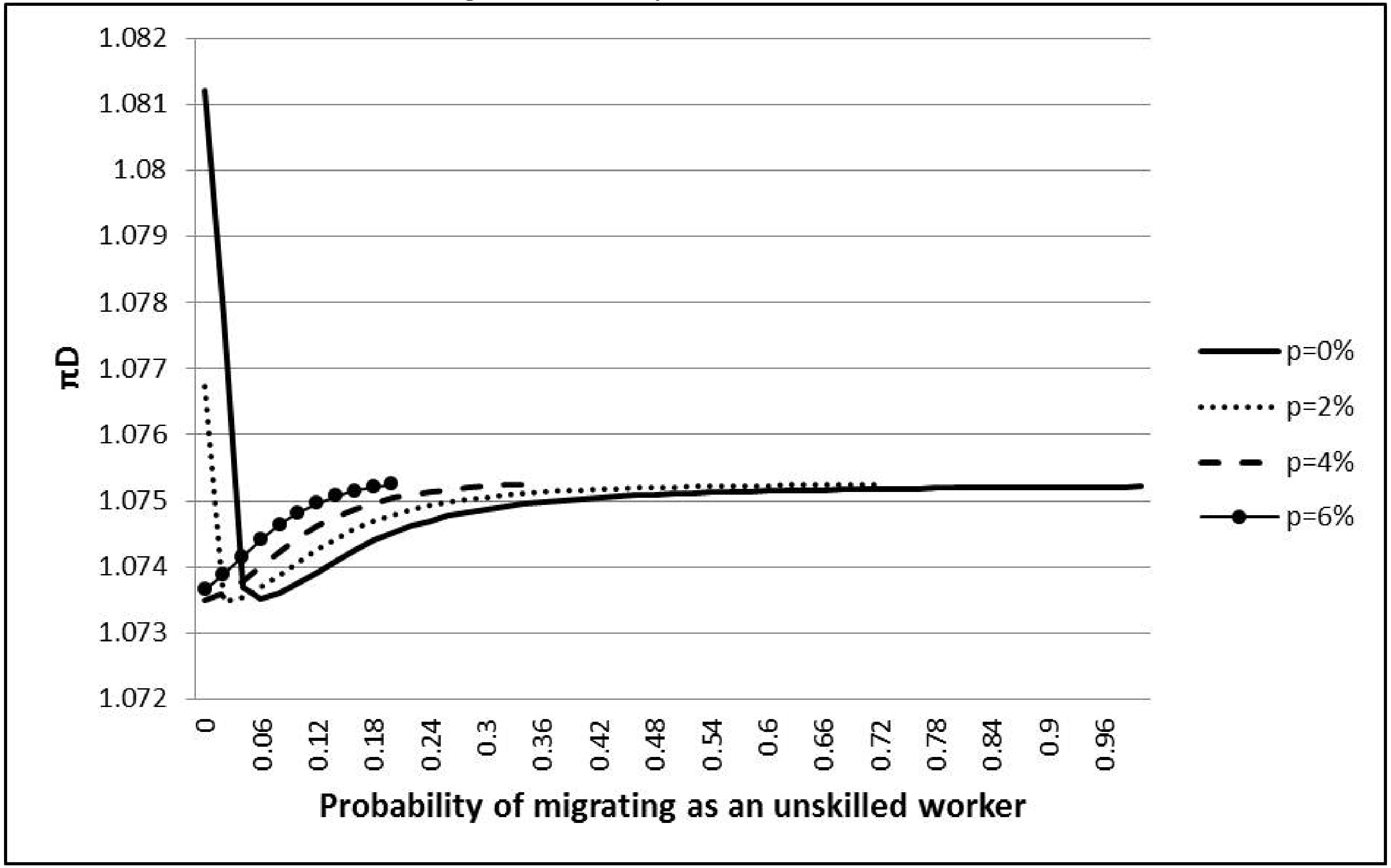


Figure C.7: Steady-State π^F for Pattern 2

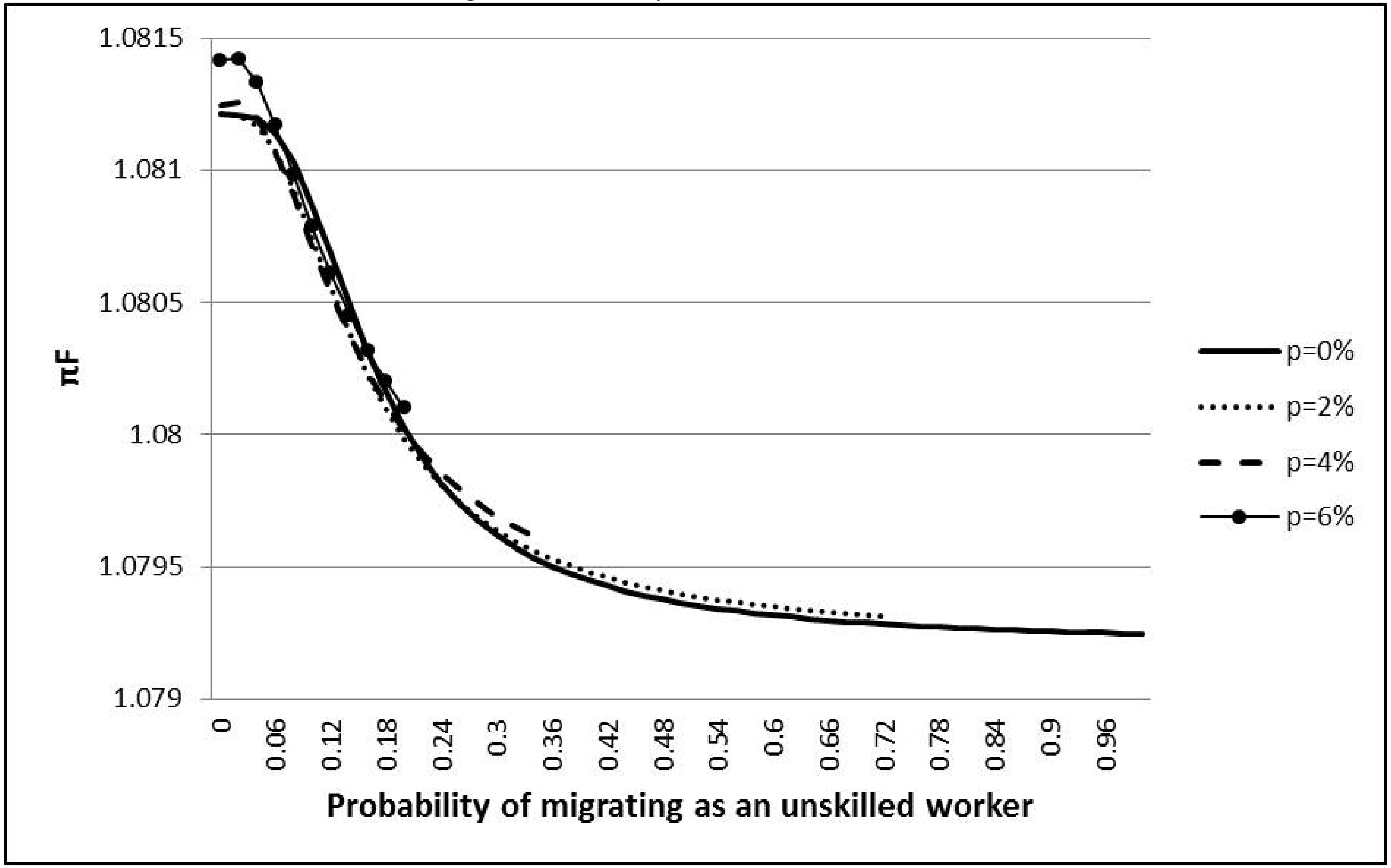


Figure C.8: Steady-State λ under Different Values of α, Checks 1a and 1b

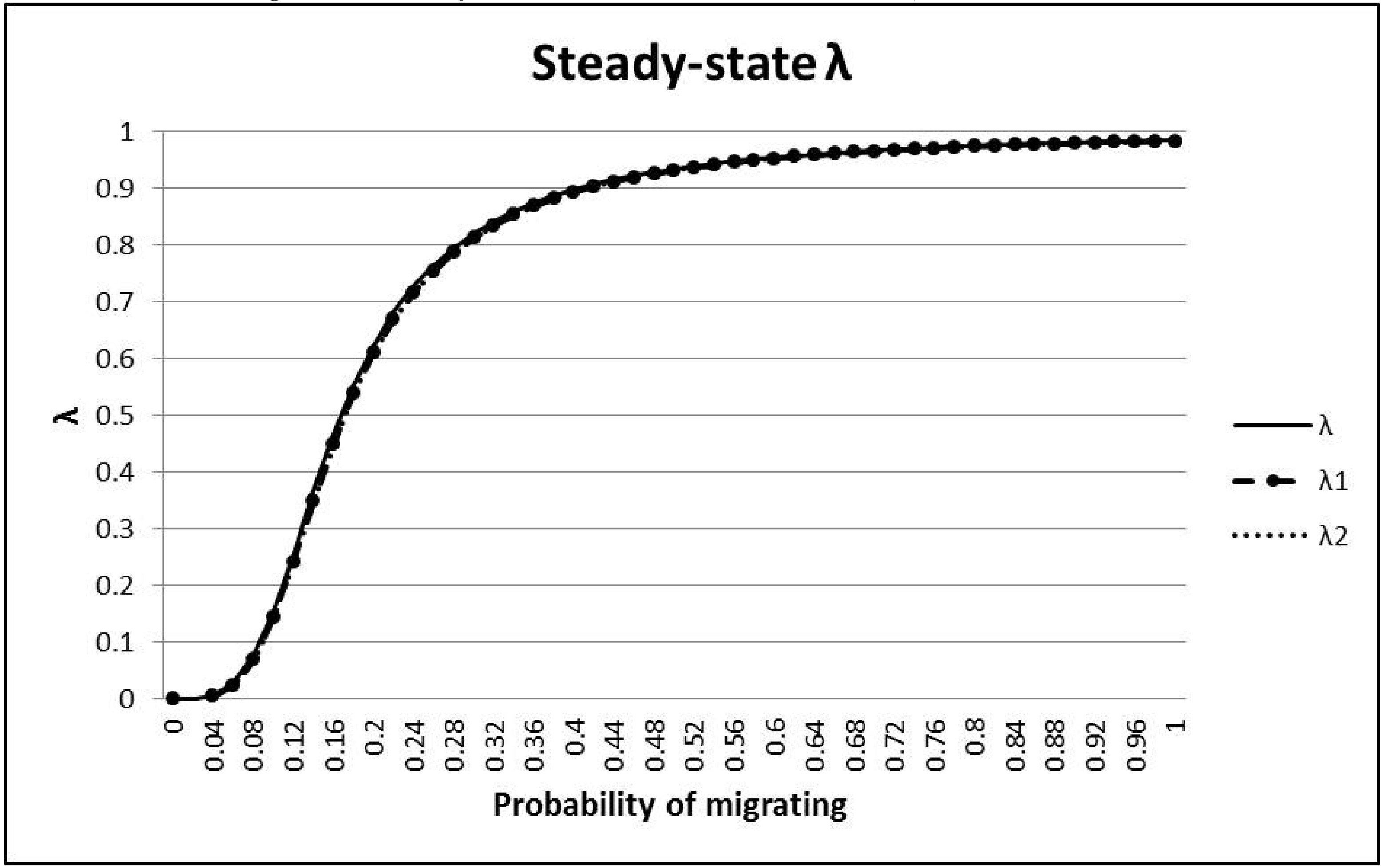


Figure C.9: Steady-State π^D under Different Values of α, Checks 1a and 1b

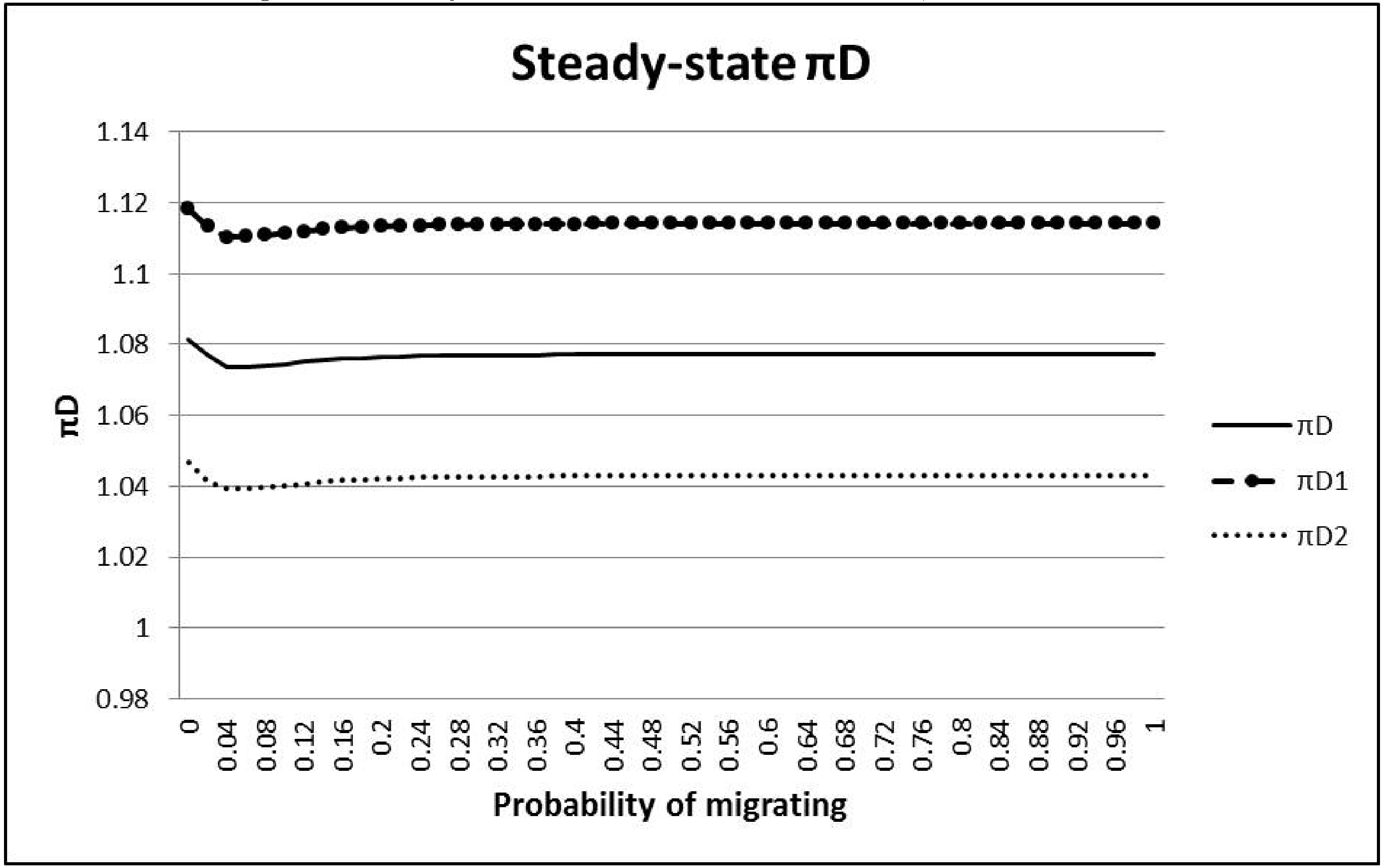


Figure C.10: Steady-State π^F under Different Values of α, Checks 1a and 1b

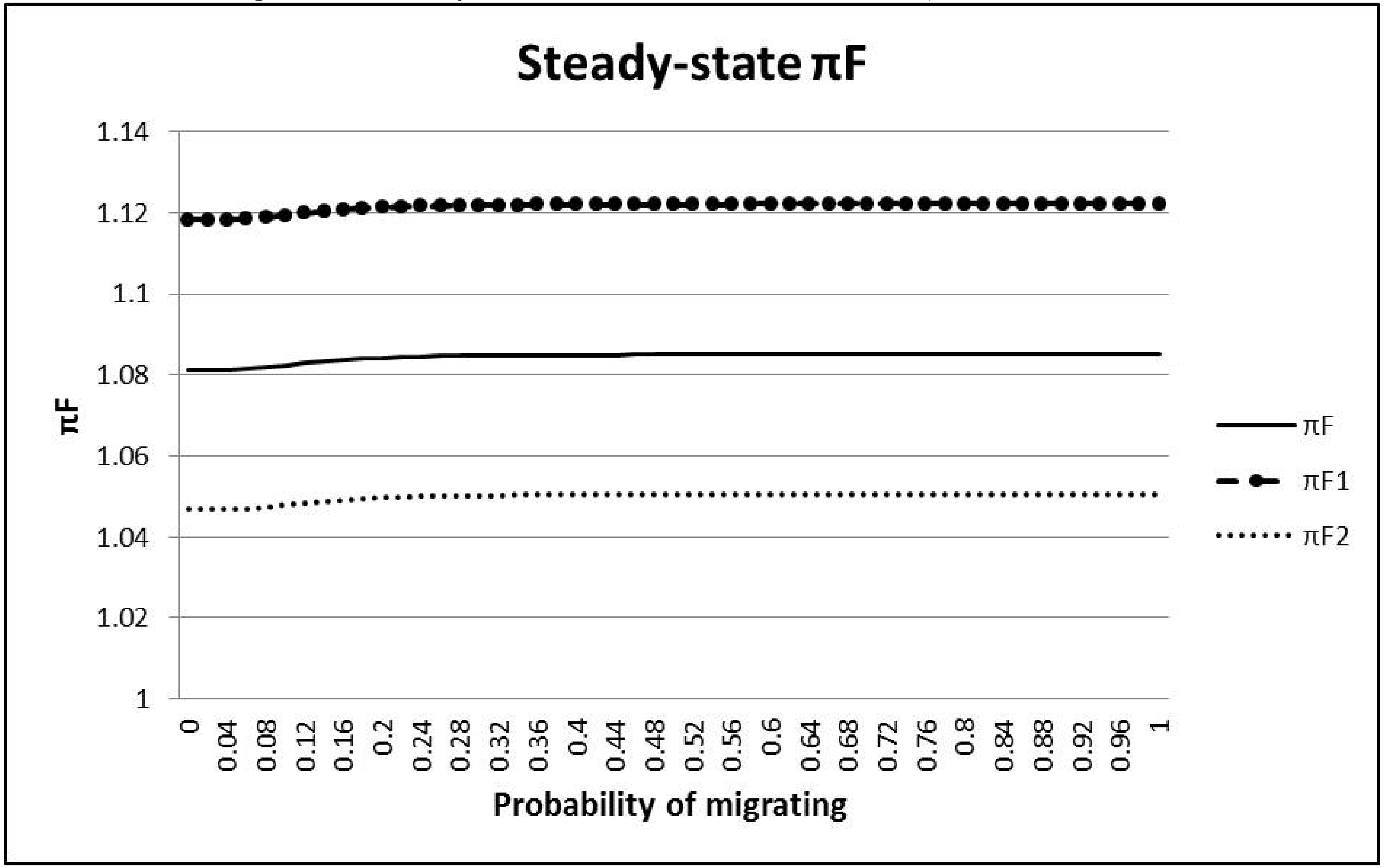


Figure C.11: Steady-State λ under Different Values of β, Checks 2a and 2b

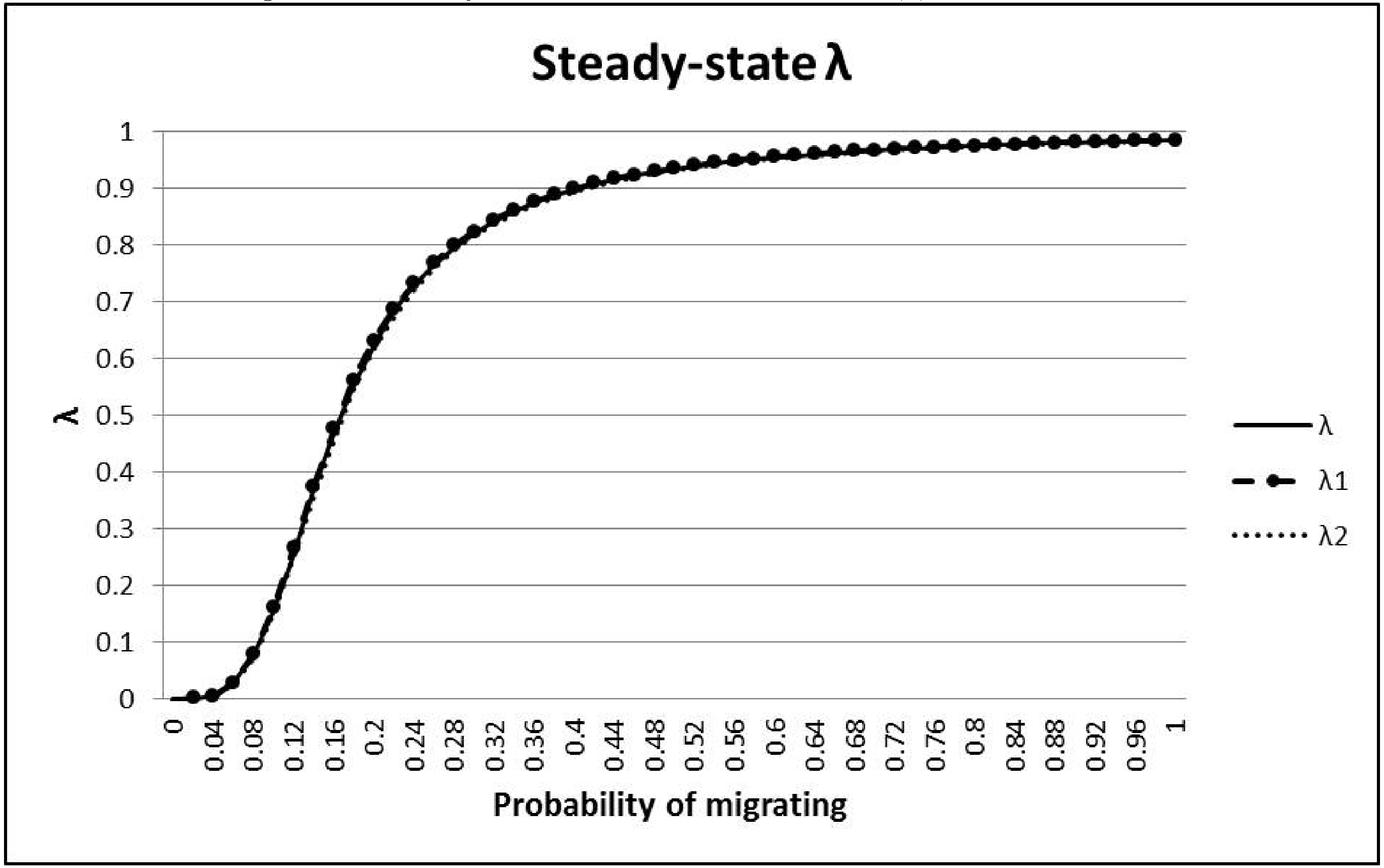


Figure C.12: Steady-State π^D under Different Values of β, Checks 2a and 2b

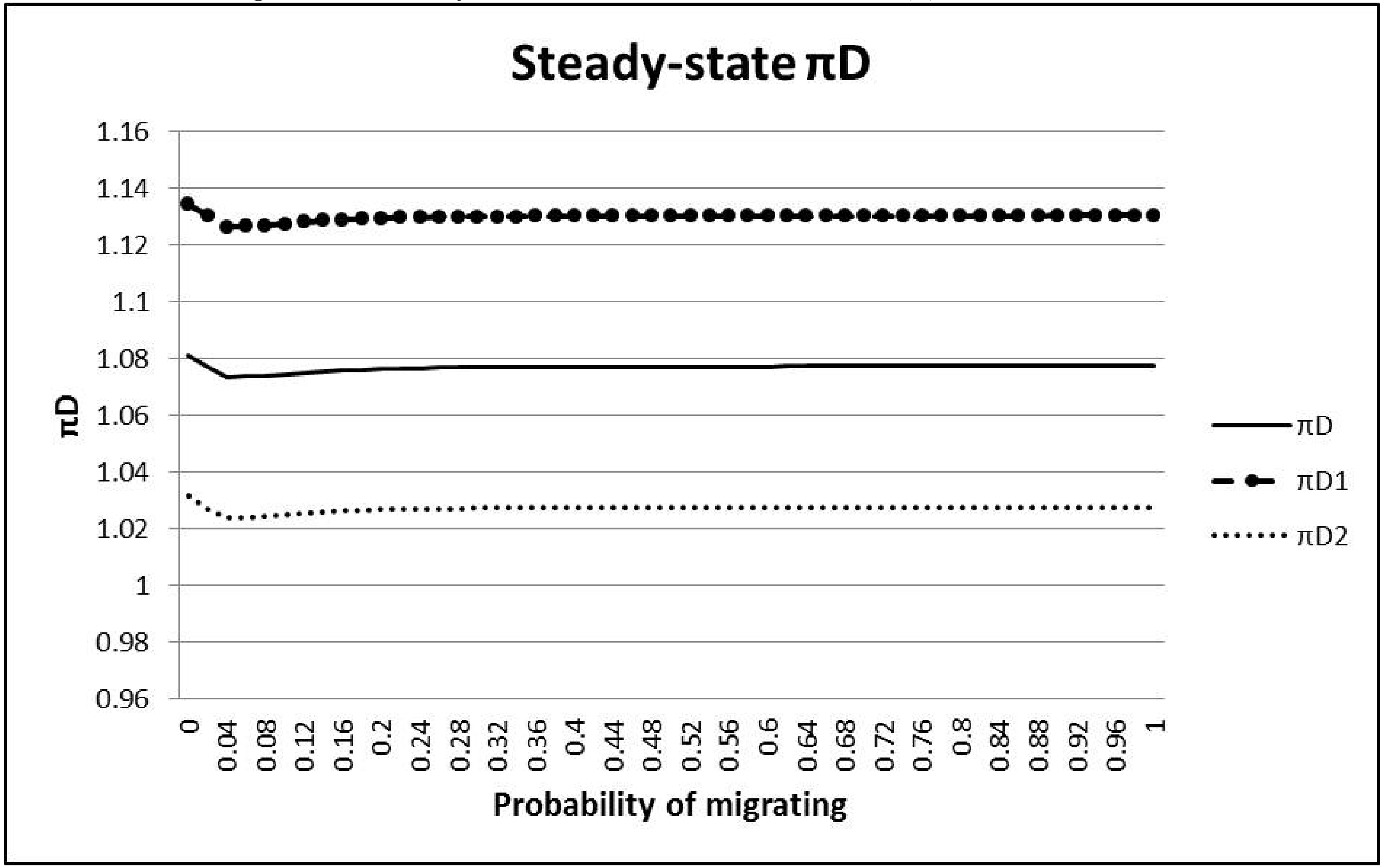


Figure C.13: Steady-State π^F under Different Values of β, Checks 2a and 2b

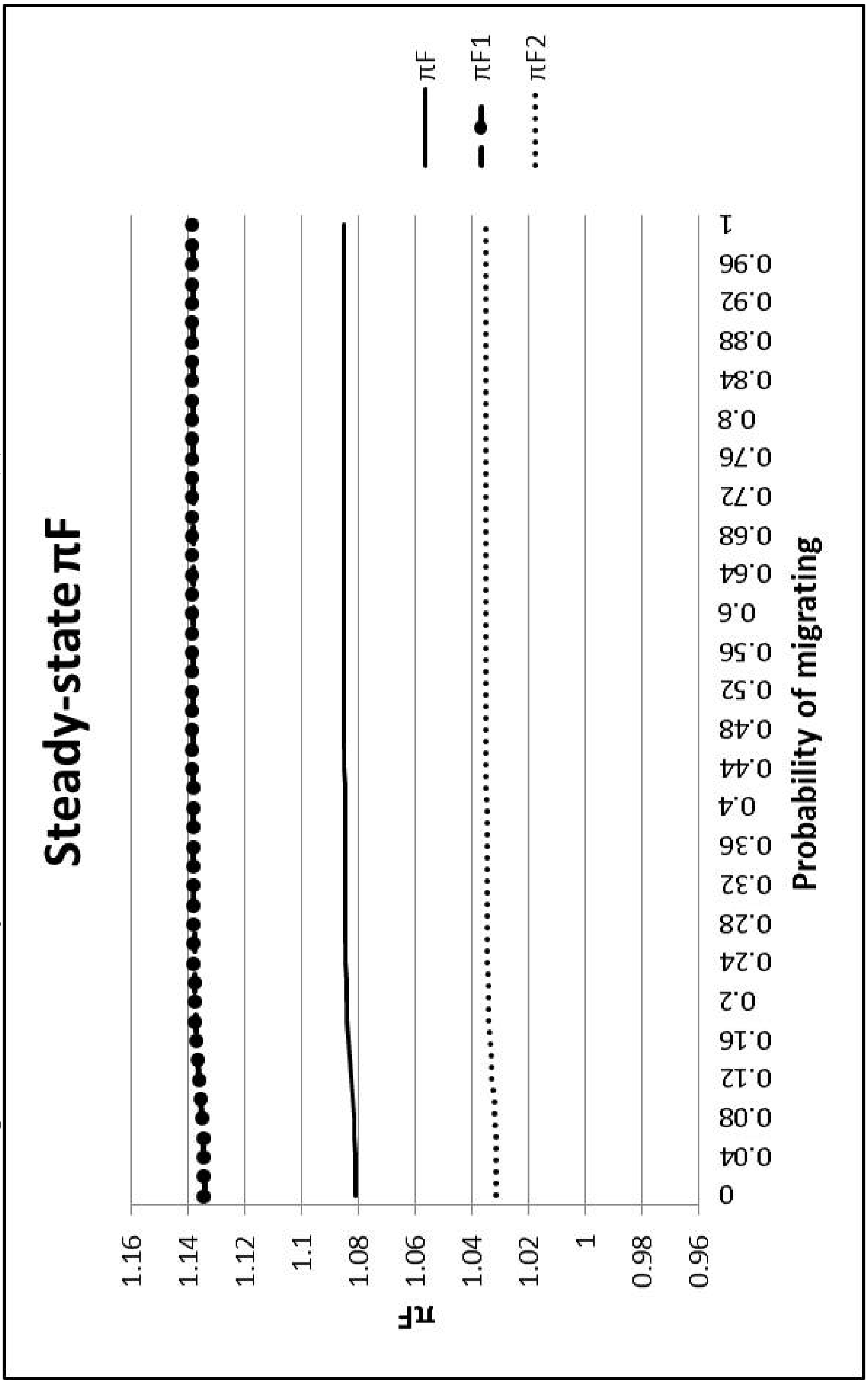


Figure C.14: Steady-State λ under Different Values of η, Checks 3a and 3b

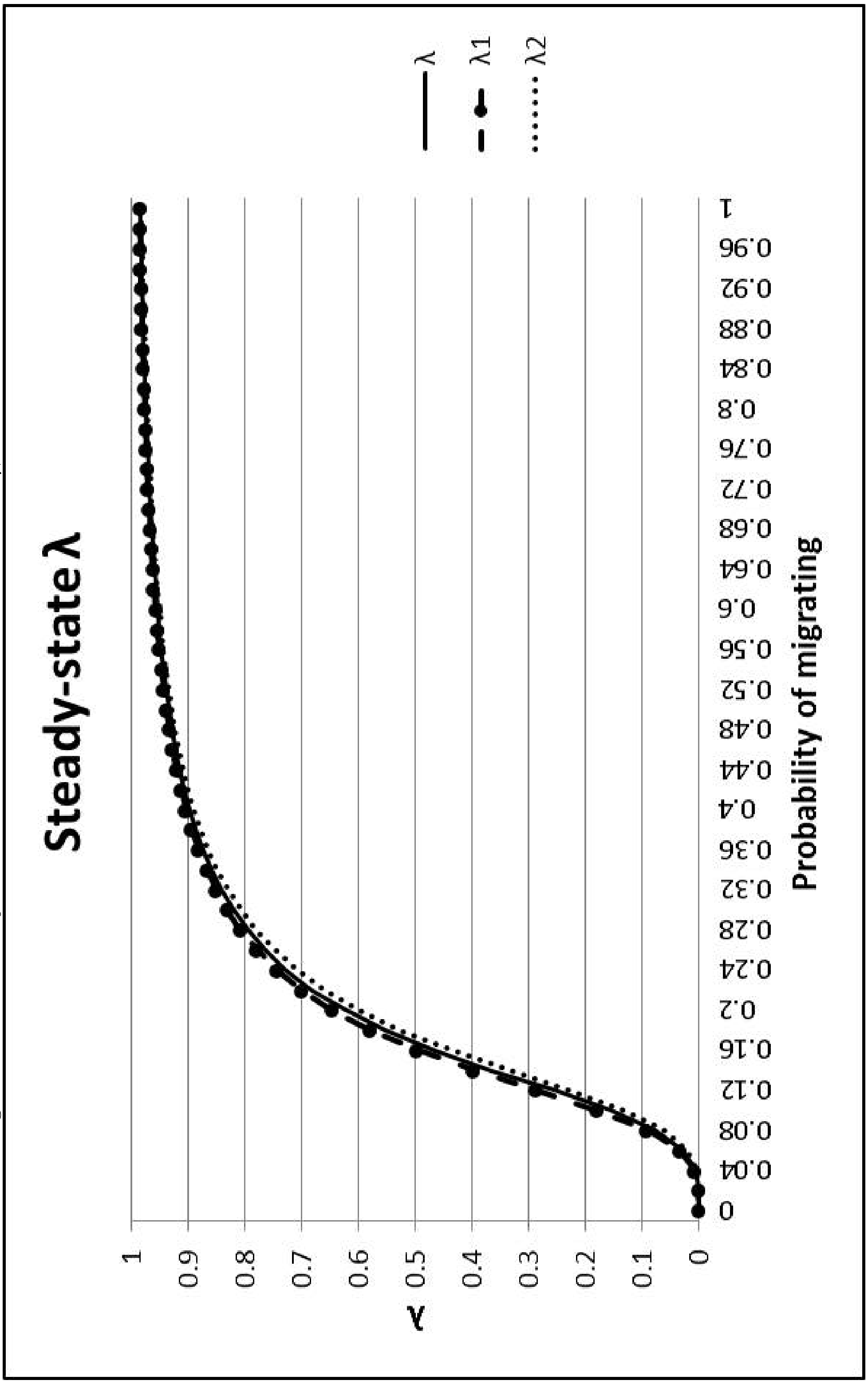


Figure C.15: Steady-State π^D under Different Values of η, Checks 3a and 3b

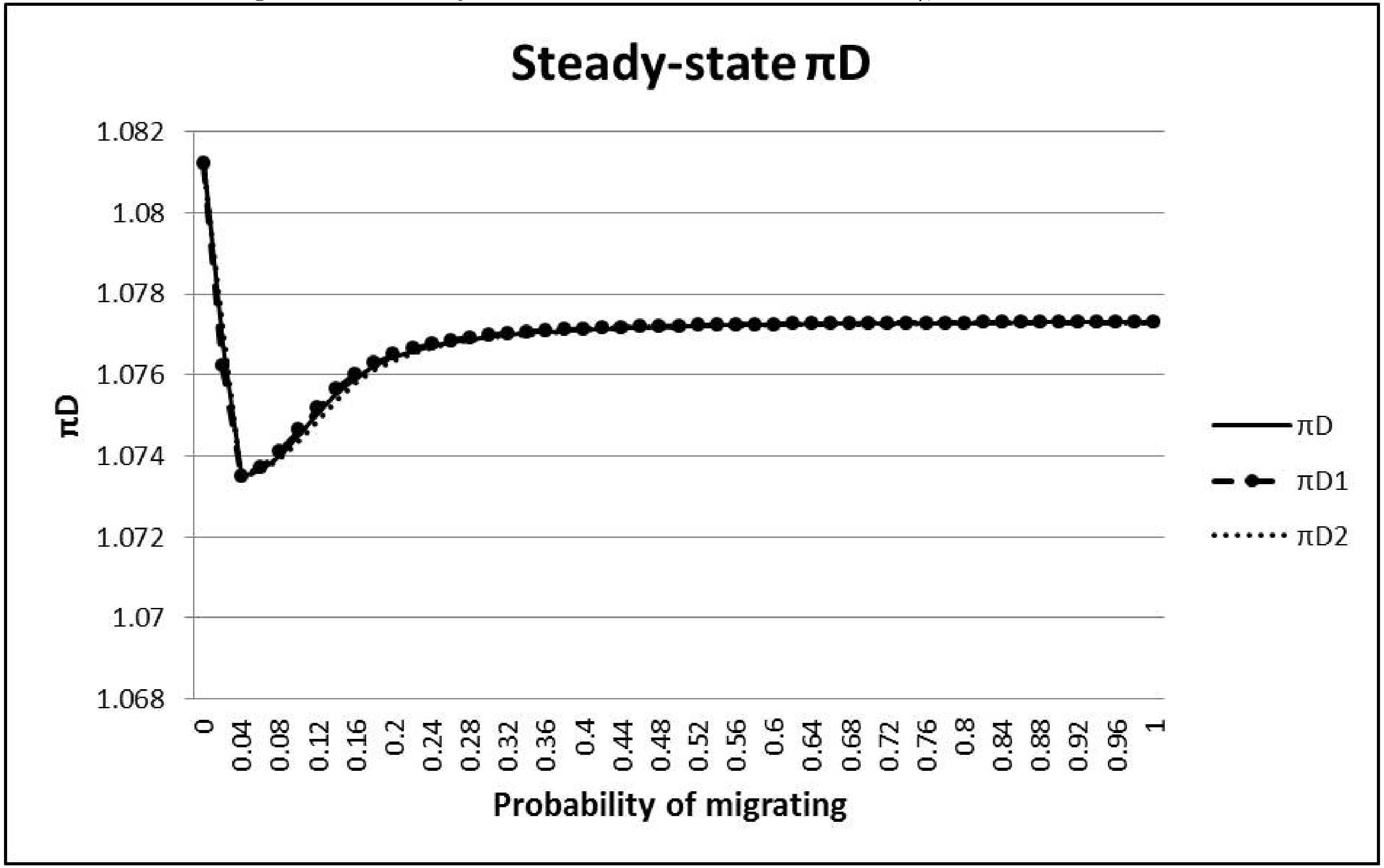


Figure C.16: Steady-State π^F under Different Values of η, Checks 3a and 3b

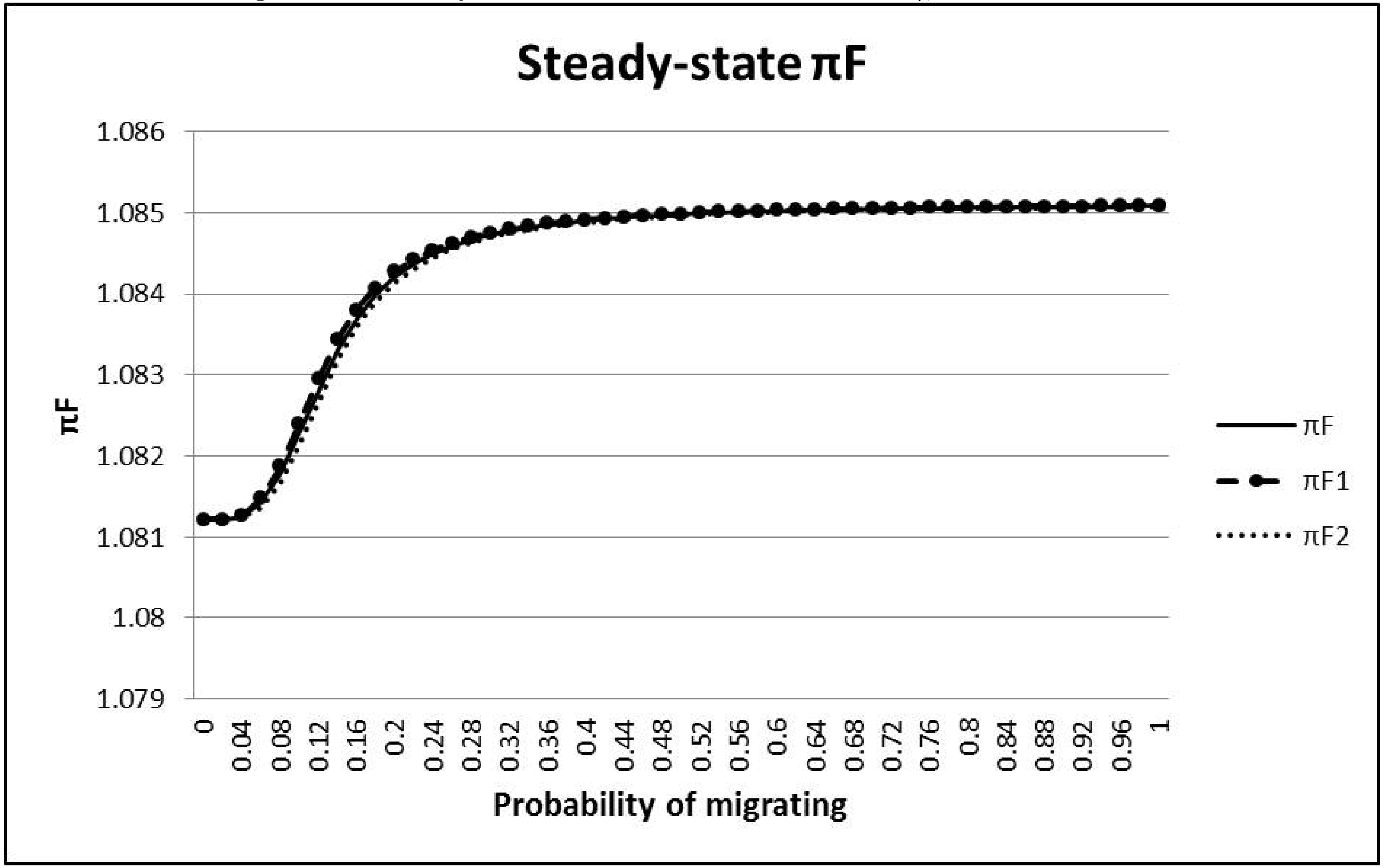
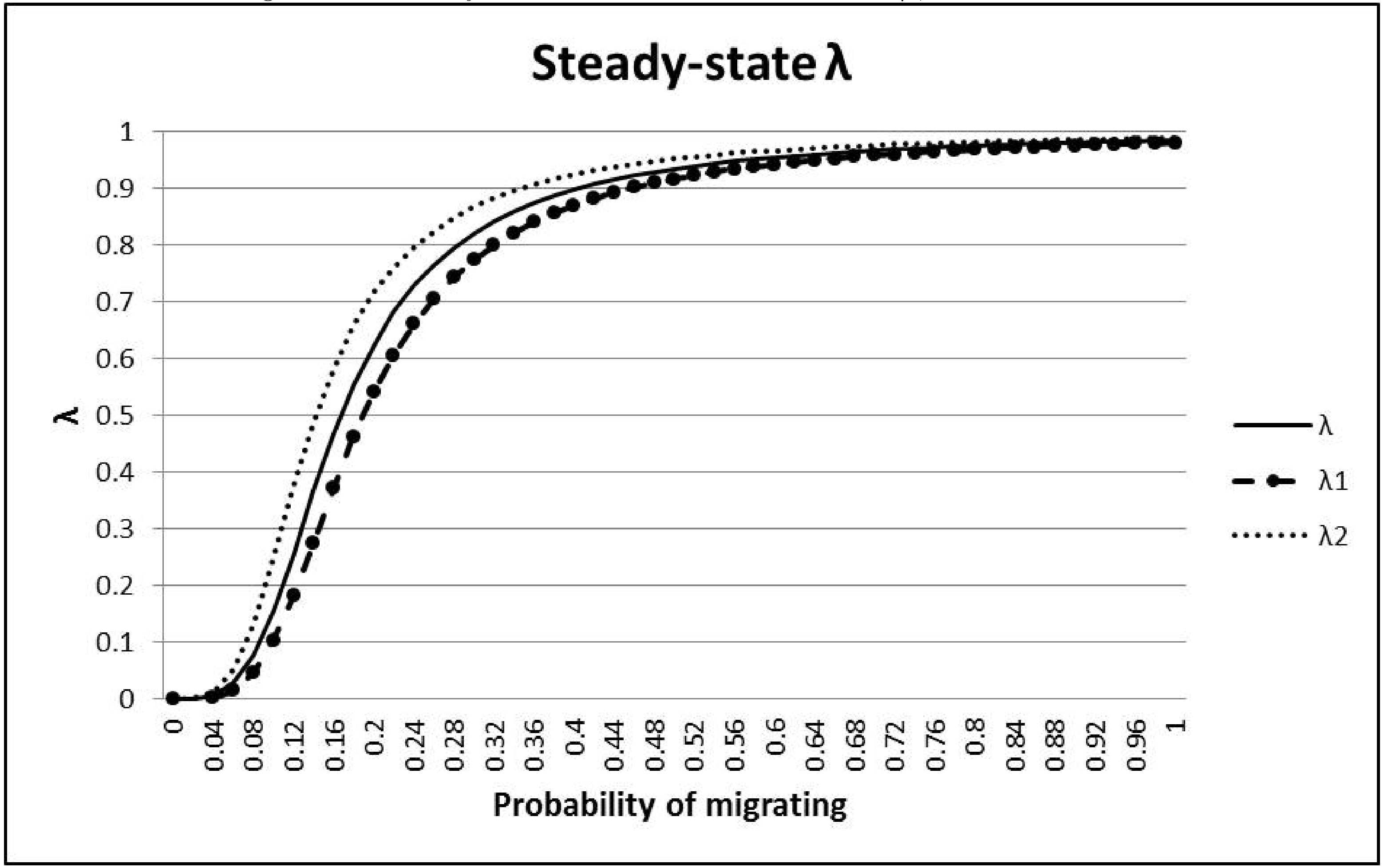


Figure C.17: Steady-State λ under Different Values of μ, Checks 4a and 4b



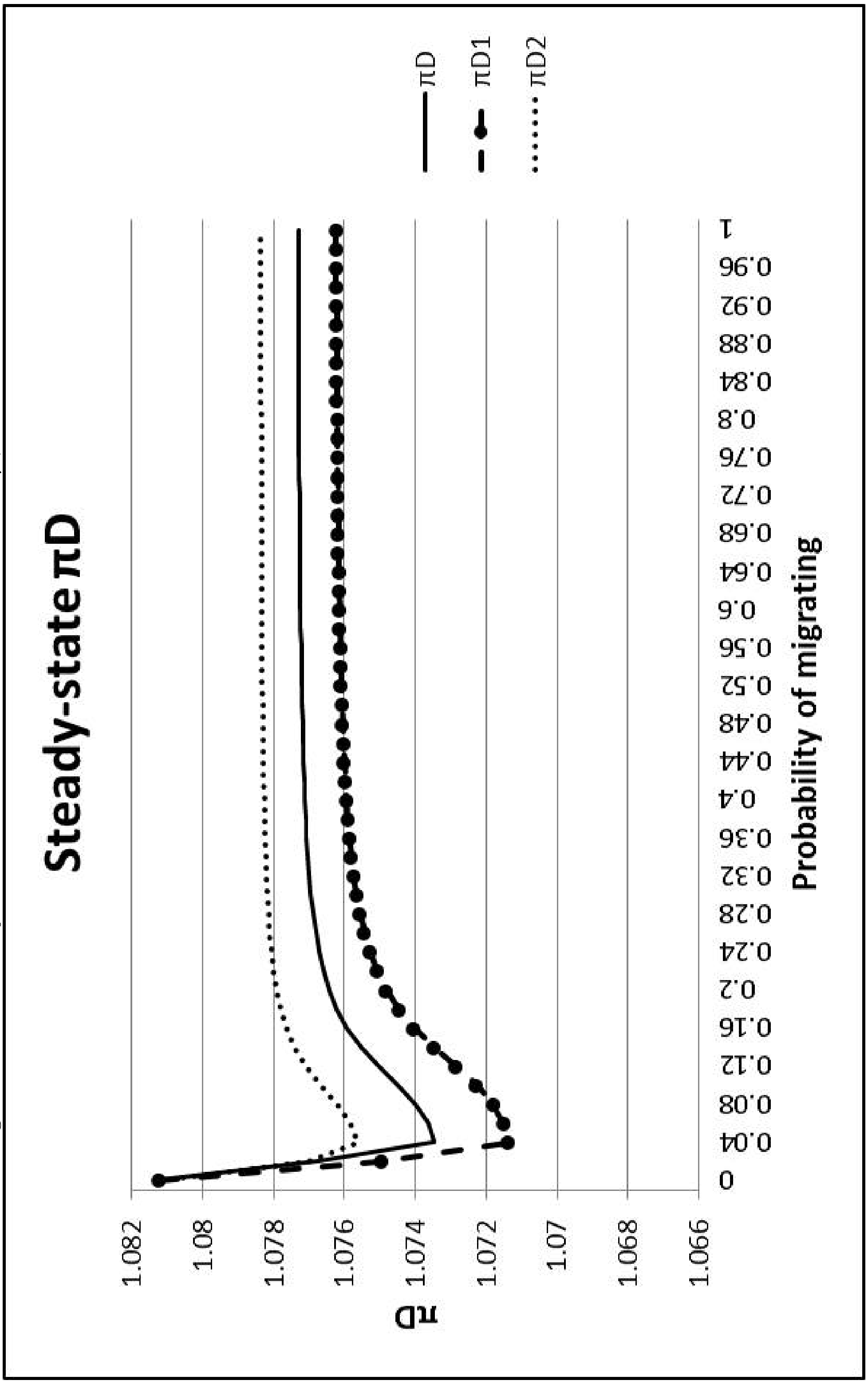


Figure C.18: Steady-State π^D under Different Values of μ, Checks 4a and 4b

Figure C.19: Steady-State π^F under Different Values of μ, Checks 4a and 4b

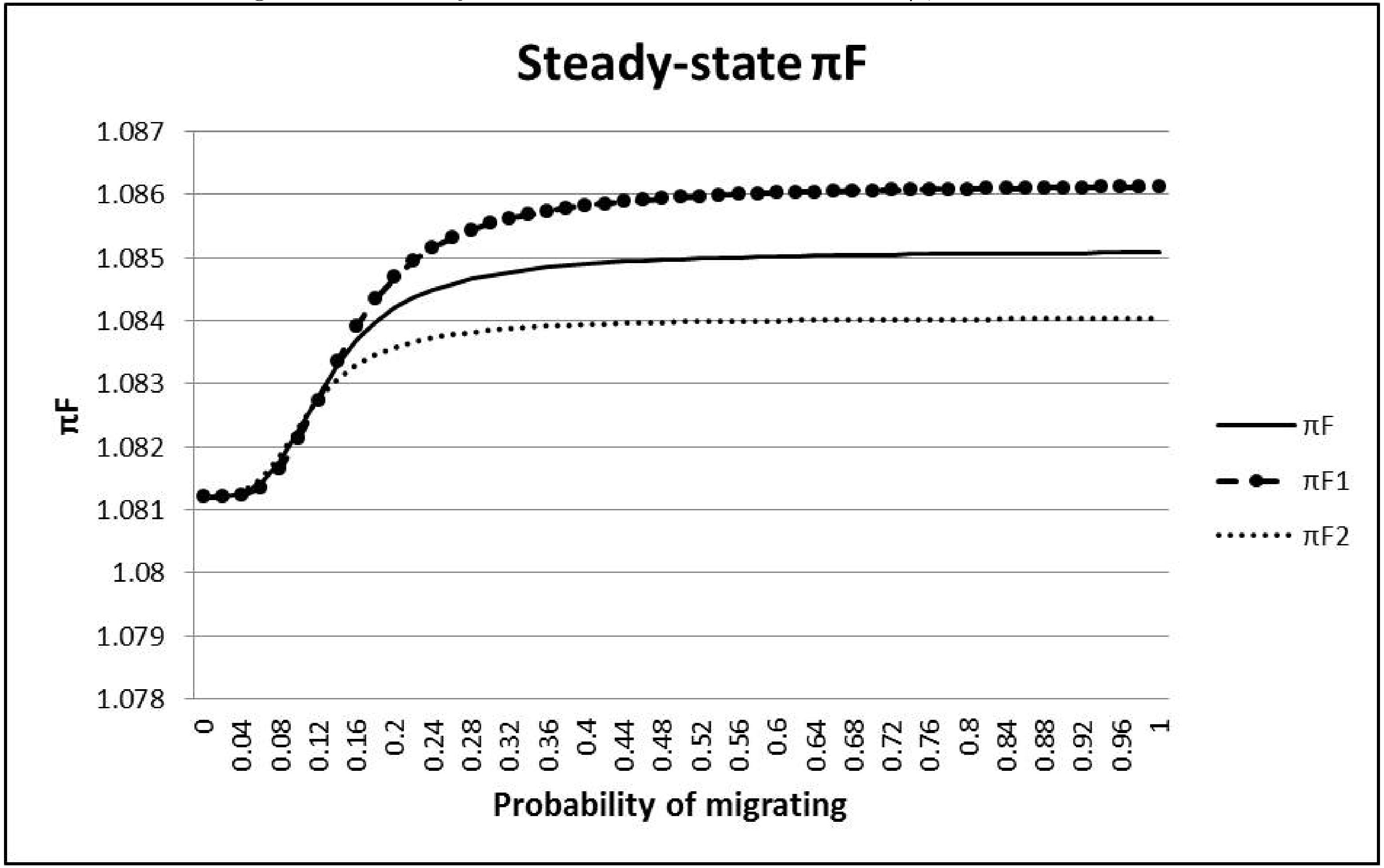


Figure C.20: Steady-State λ under Different Values of α, Checks 1a and 1b

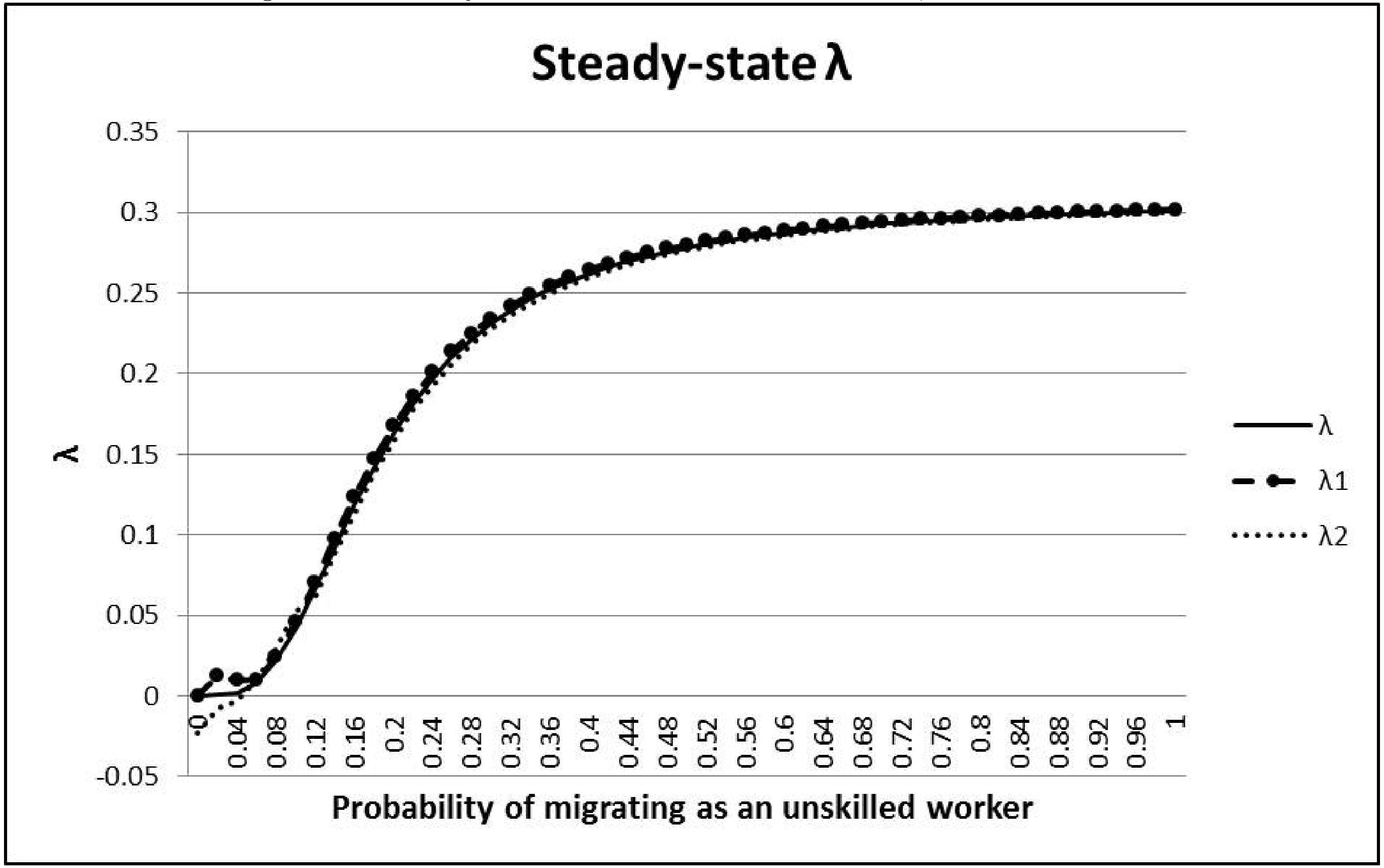


Figure C.21: Steady-State π^D under Different Values of α, Checks 1a and 1b

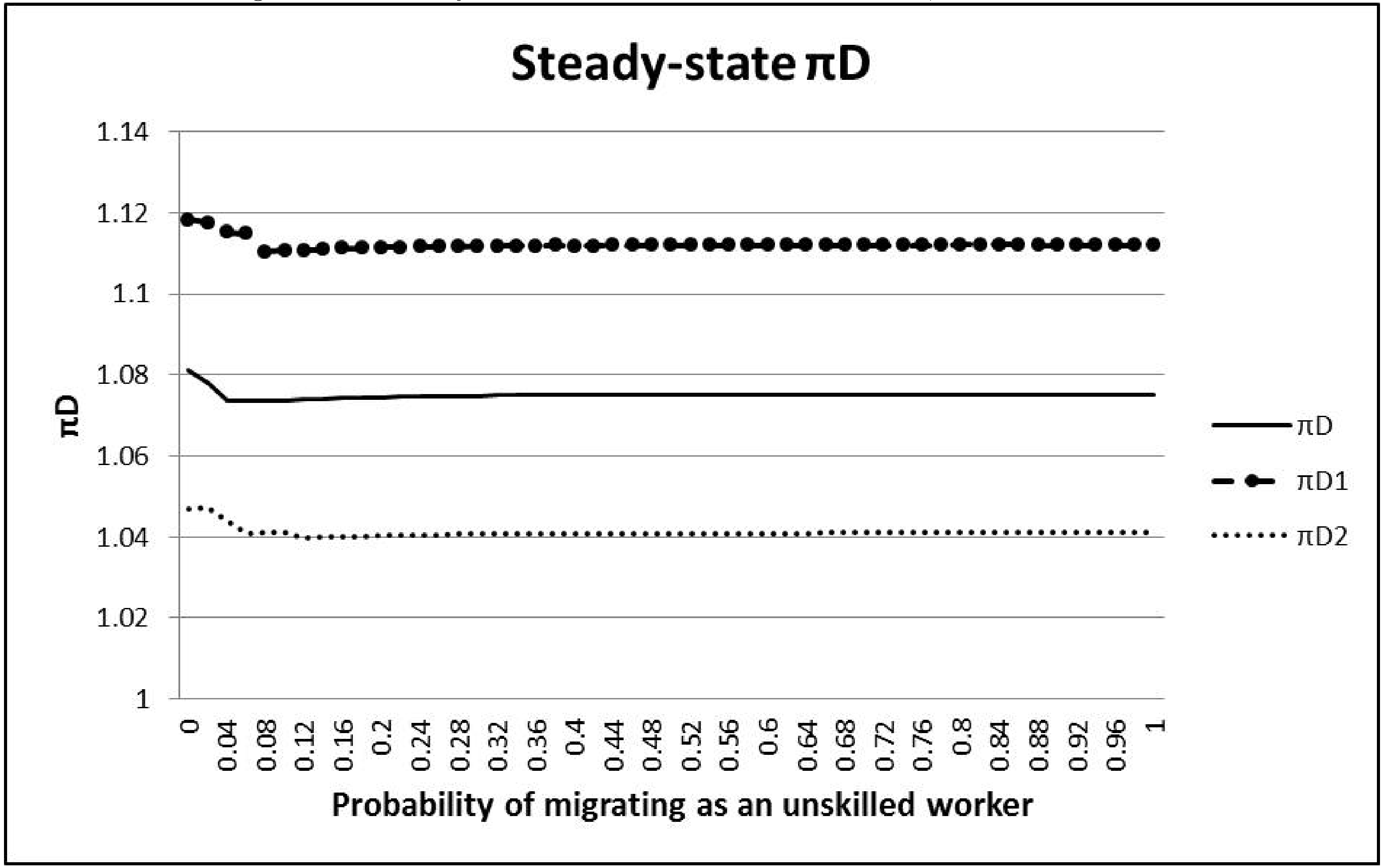


Figure C.22: Steady-State π^F under Different Values of α, Checks 1a and 1b

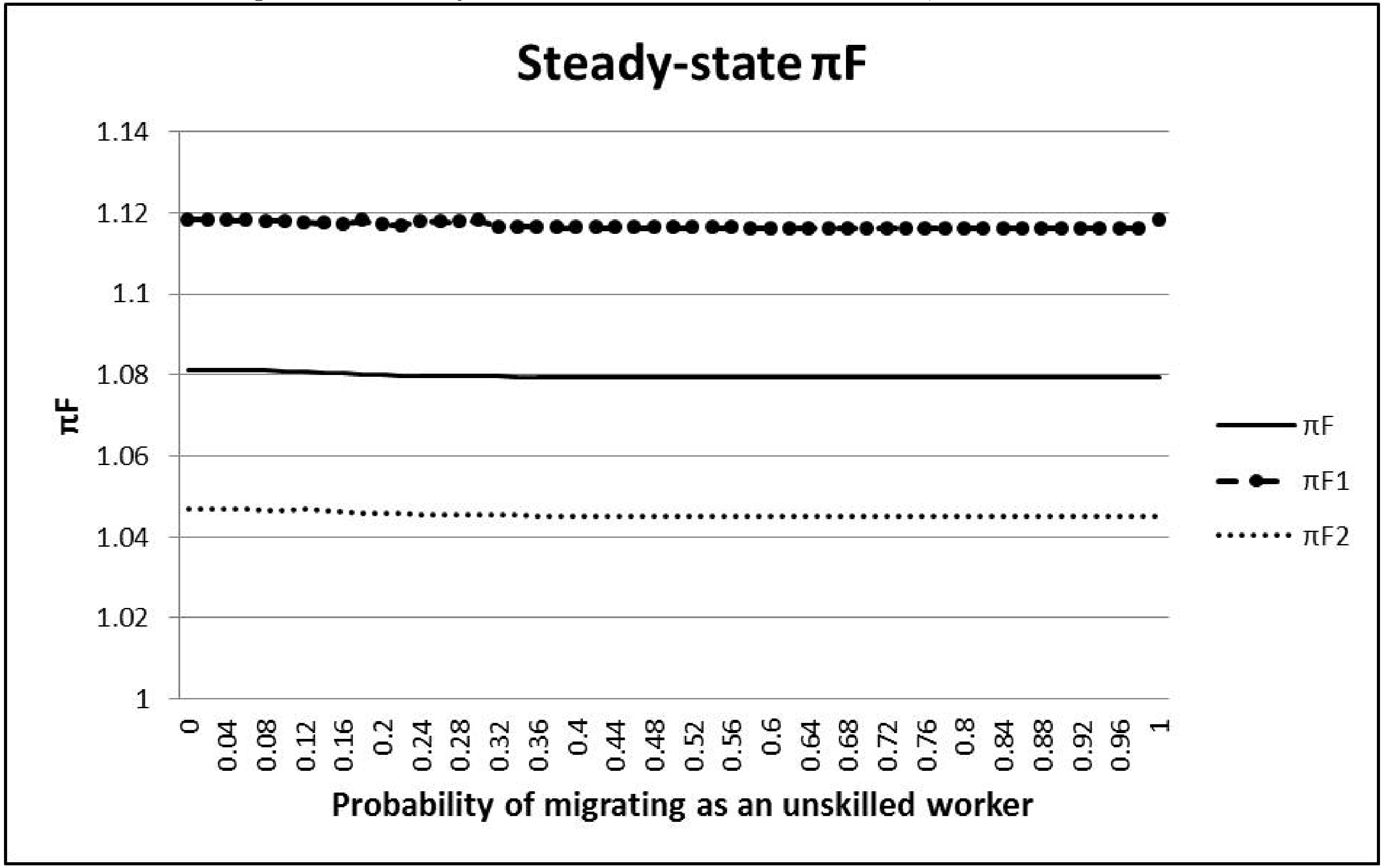


Figure C.23: Steady-State λ under Different Values of β, Checks 2a and 2b

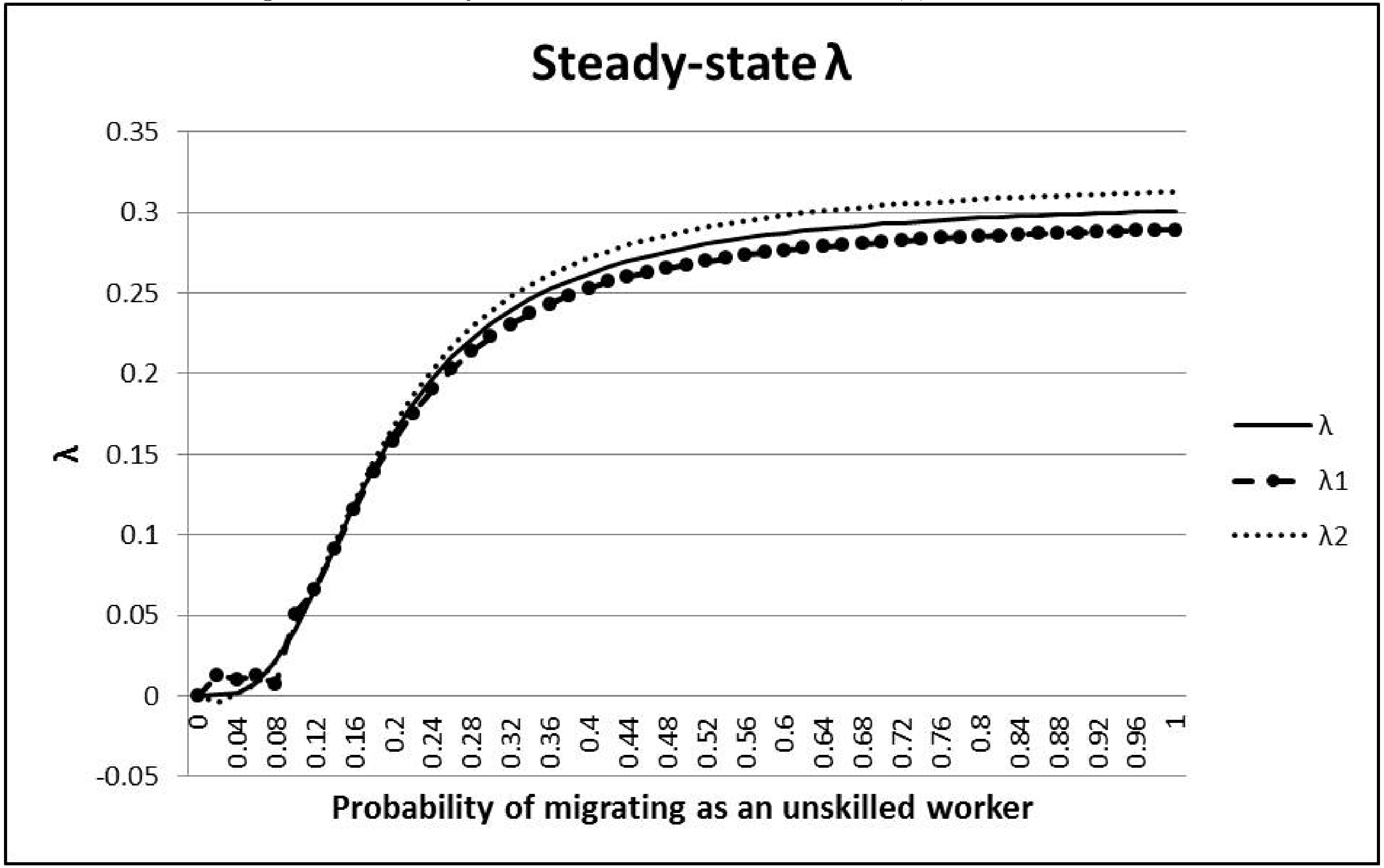


Figure C.24: Steady-State π^D under Different Values of β, Checks 2a and 2b

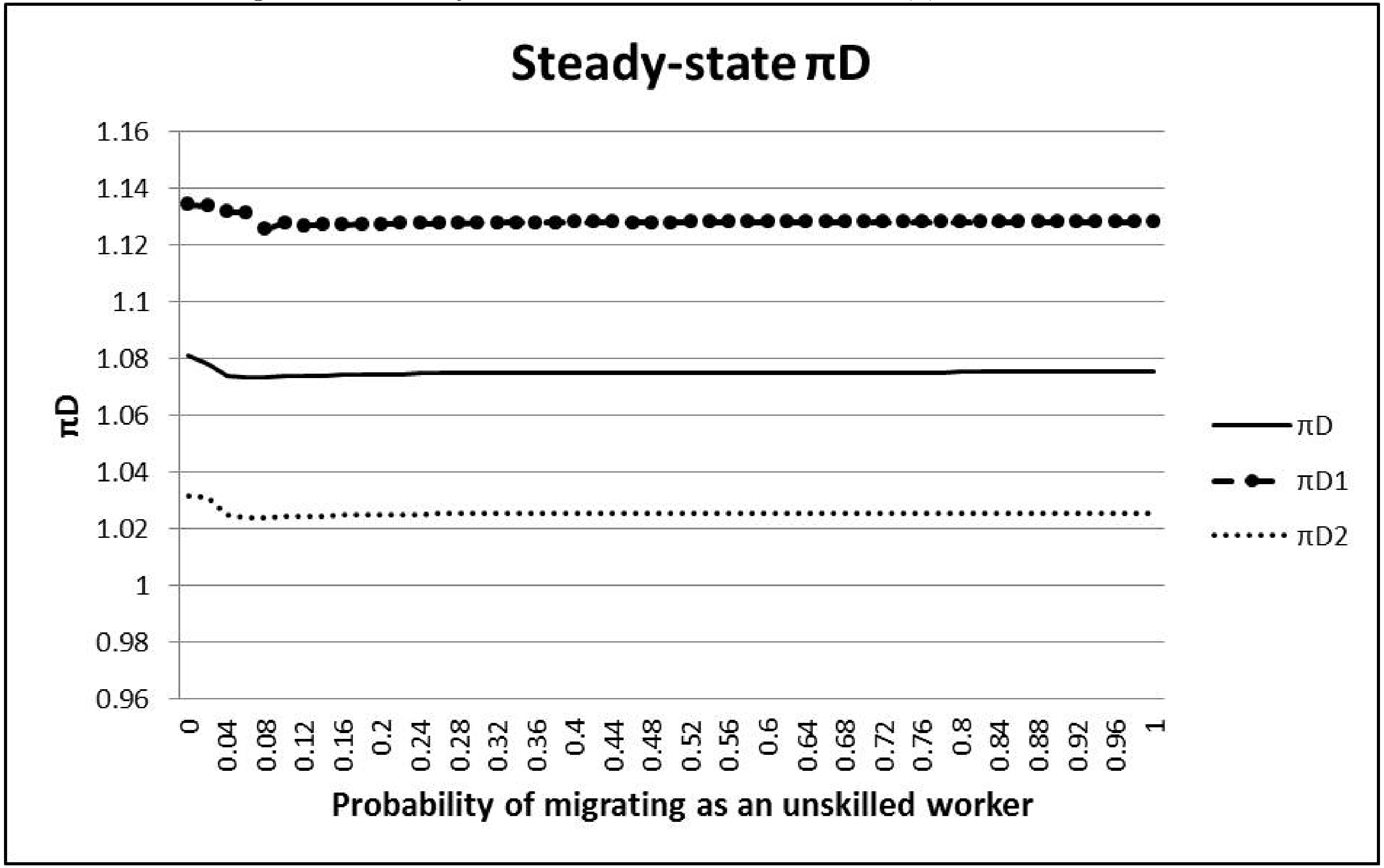


Figure C.25: Steady-State π^F under Different Values of β, Checks 2a and 2b

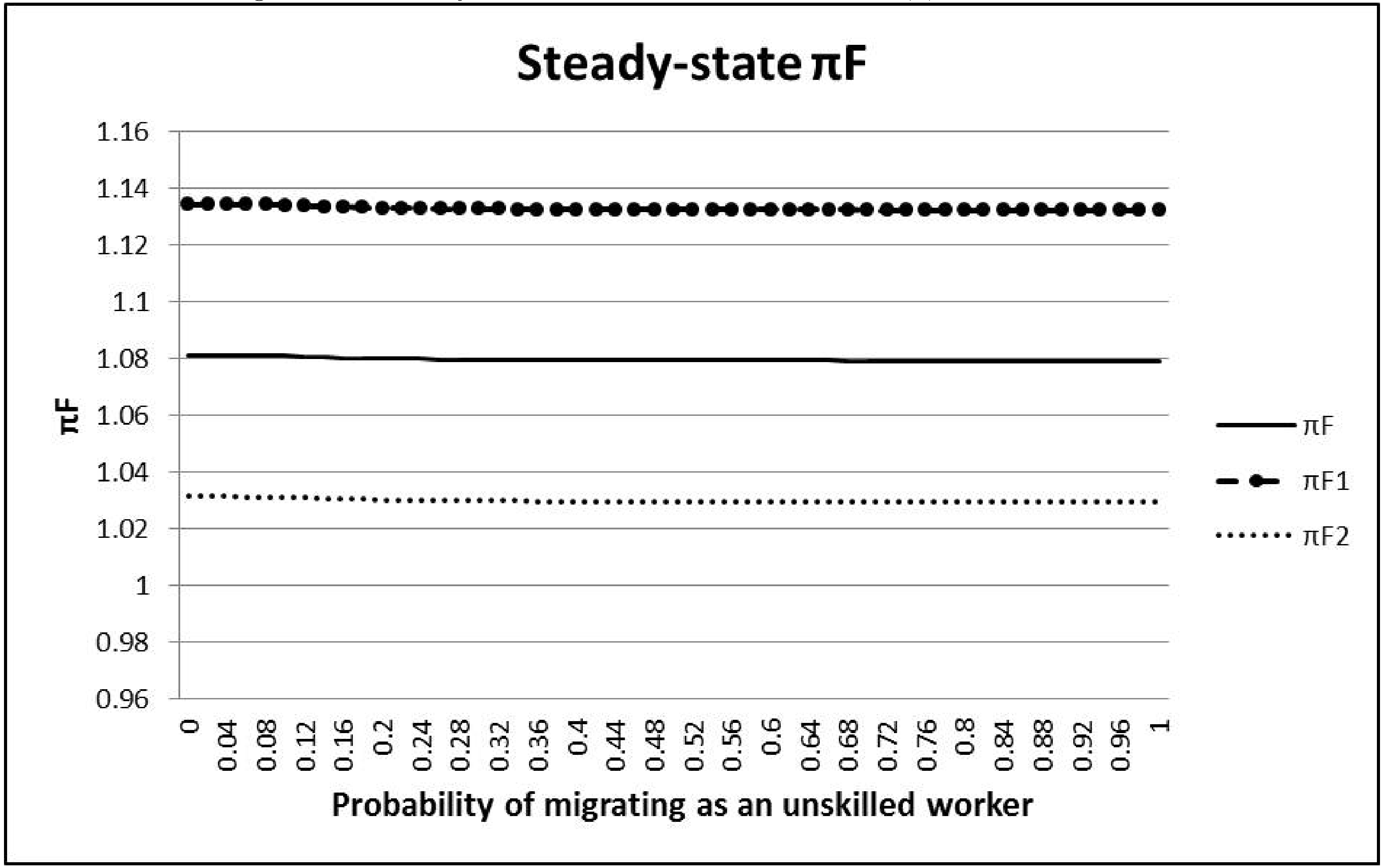


Figure C.26: Steady-State λ under Different Values of η, Checks 3a and 3b

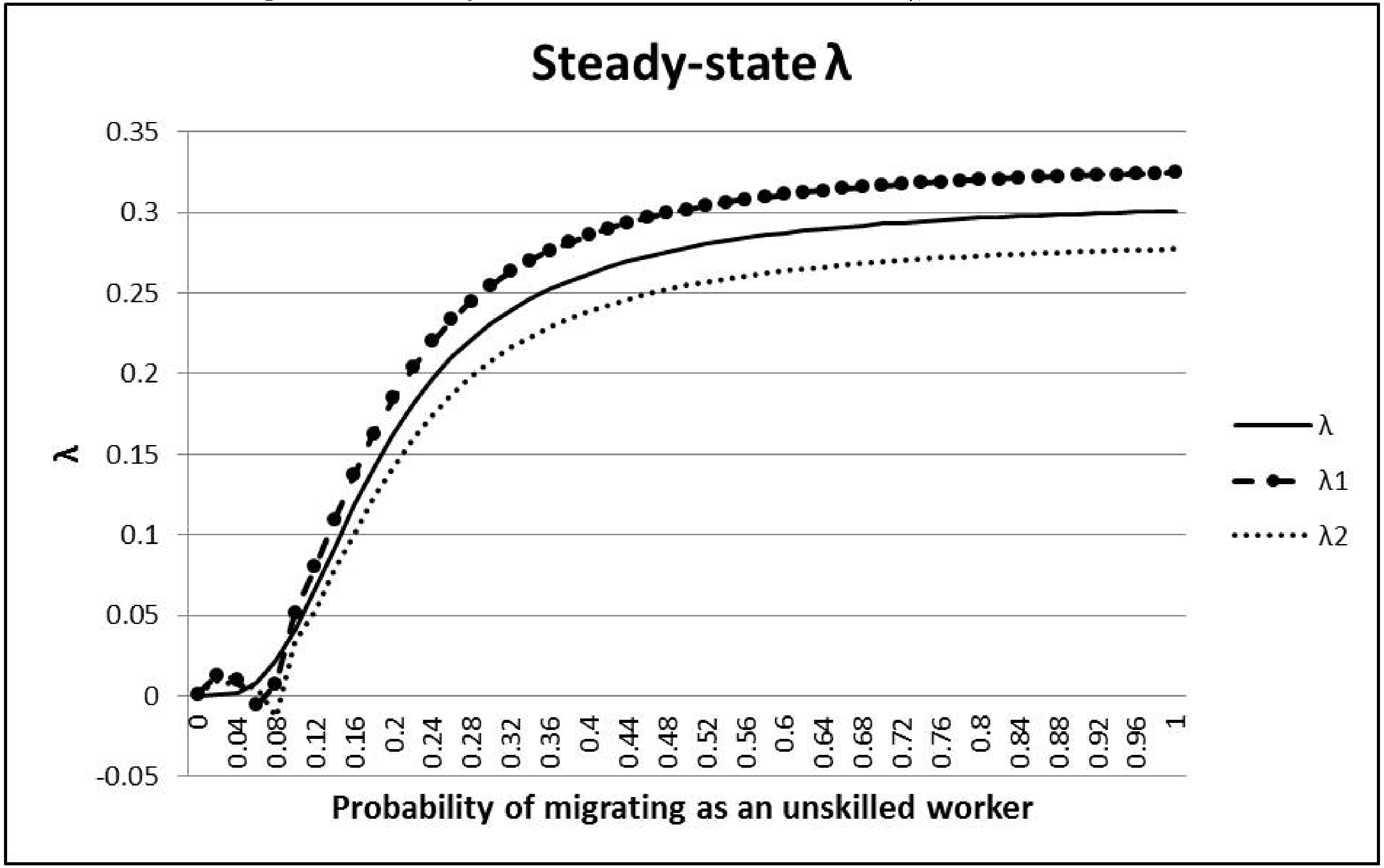


Figure C.27: Steady-State π^D under Different Values of η, Checks 3a and 3b

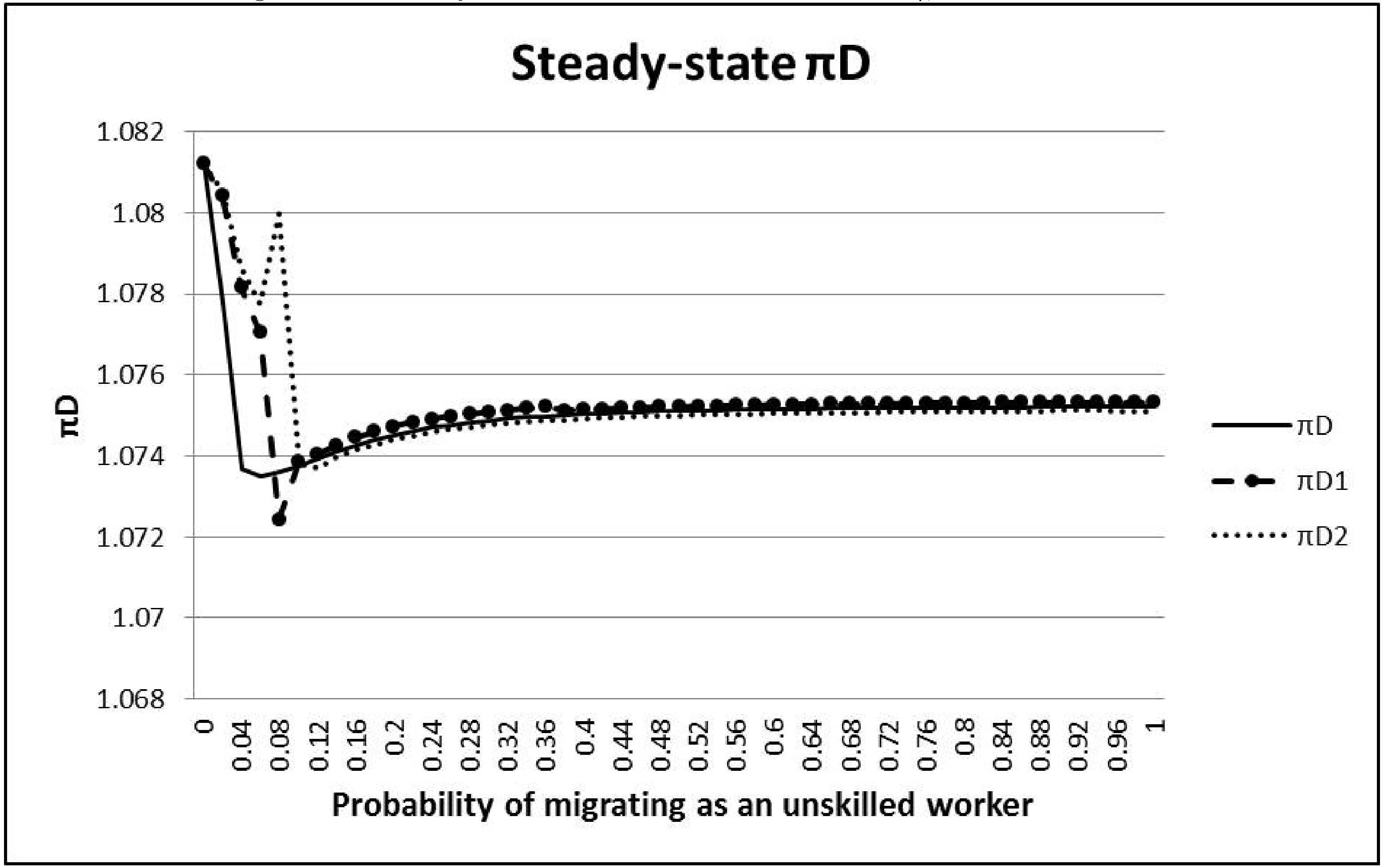


Figure C.28: Steady-State π^F under Different Values of η, Checks 3a and 3b

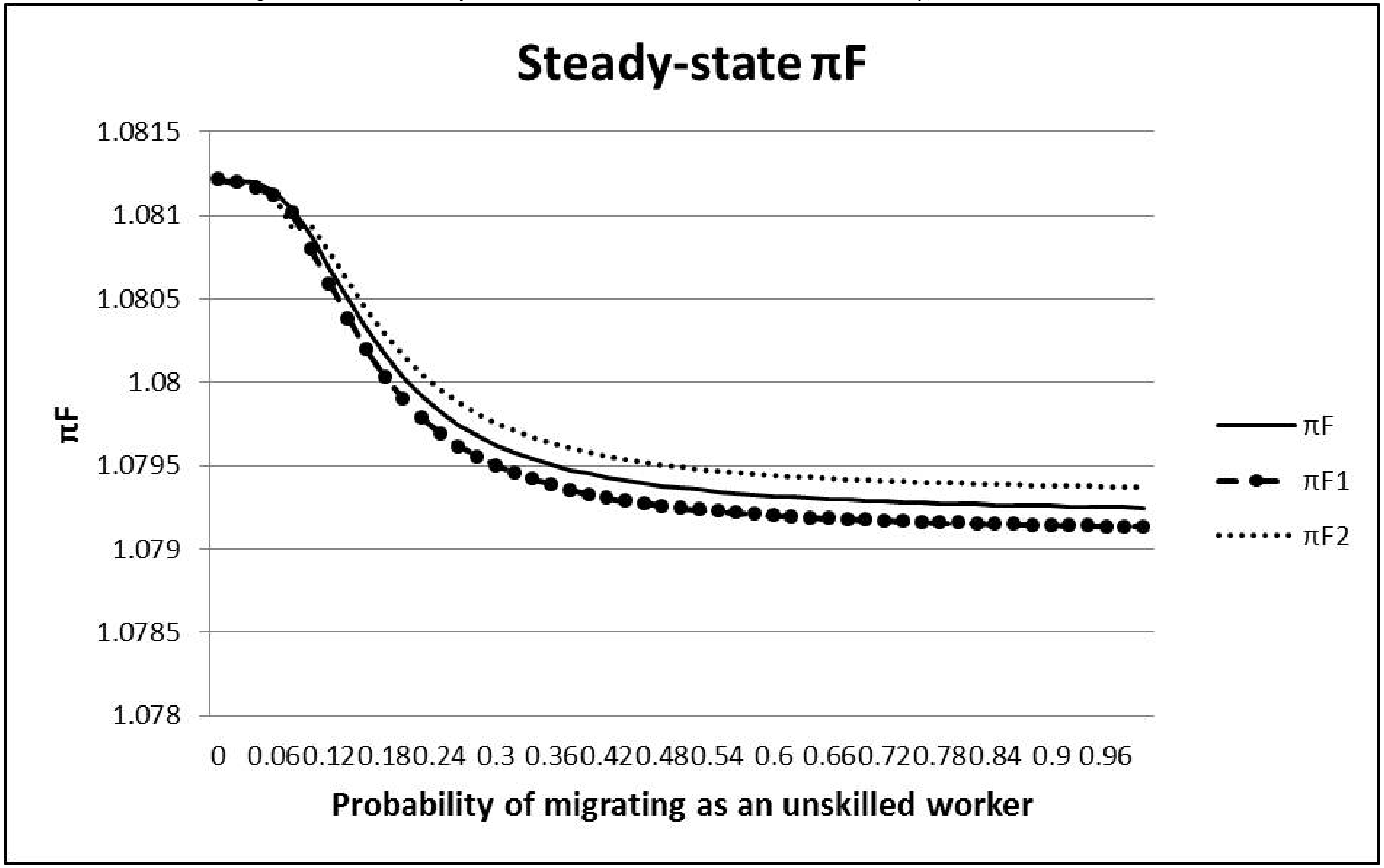


Figure C.29: Steady-State λ under Different Values of μ, Checks 4a and 4b

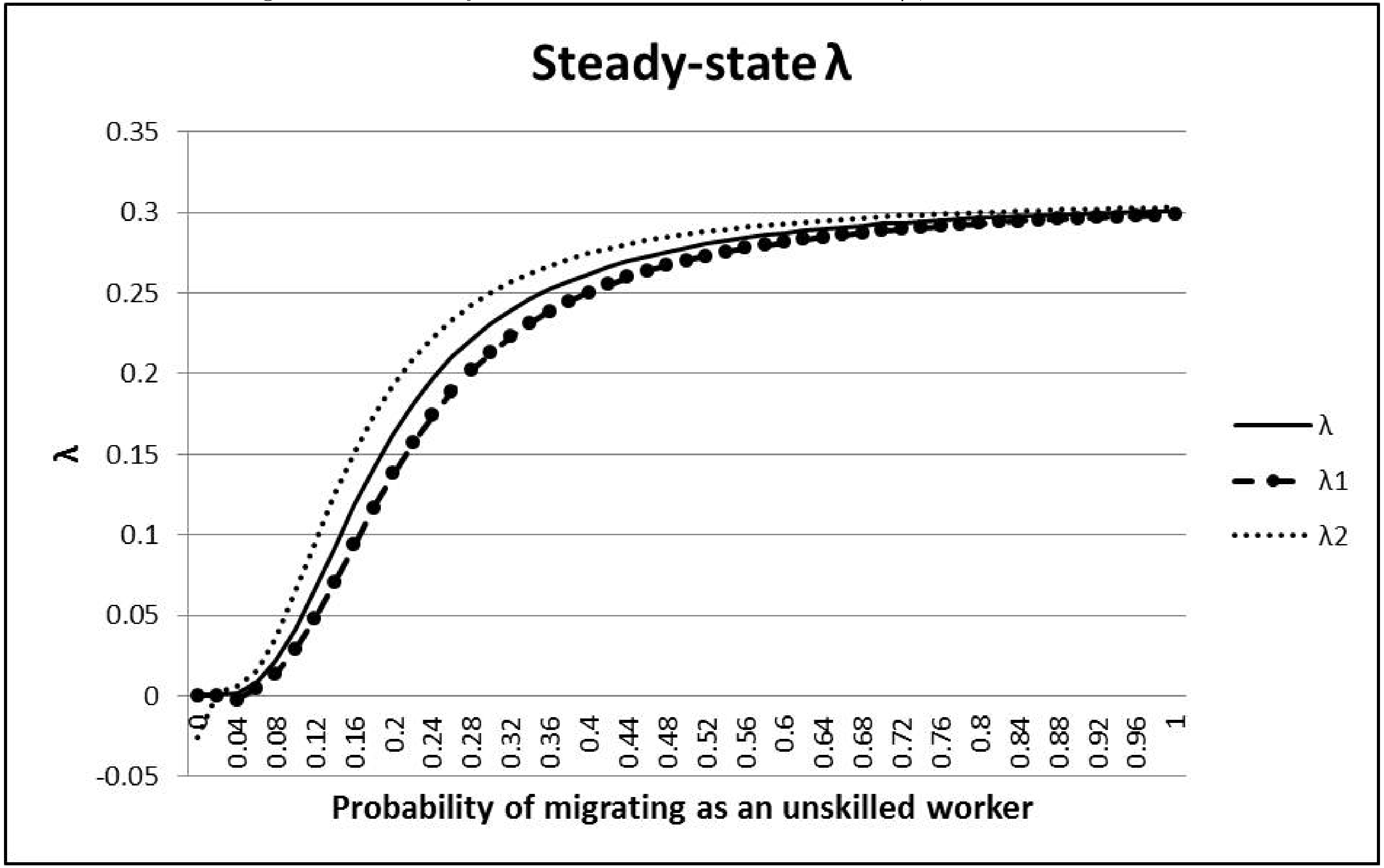


Figure C.30: Steady-State π^D under Different Values of μ, Checks 4a and 4b

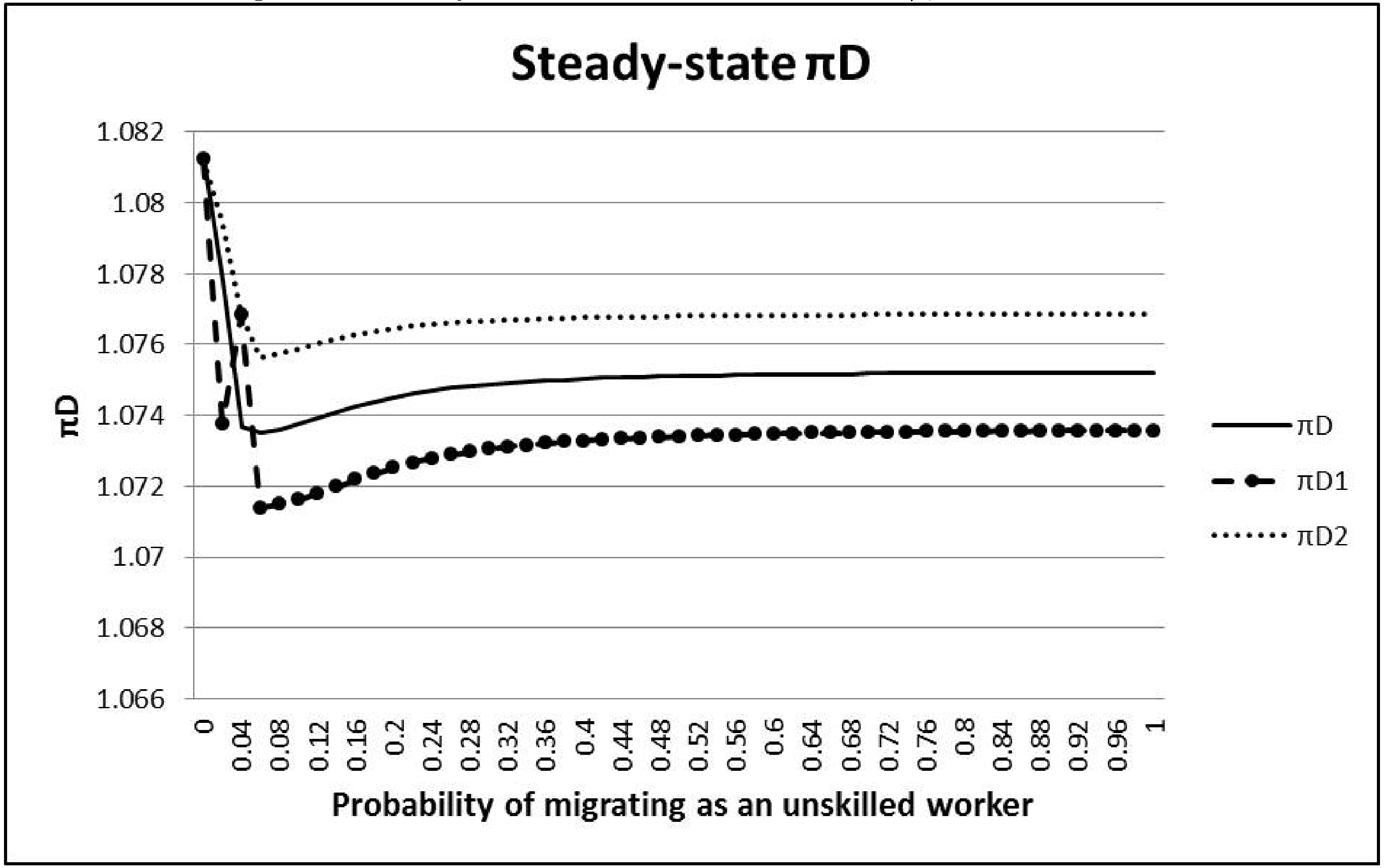


Figure C.31: Steady-State π^F under Different Values of μ, Checks 4a and 4b

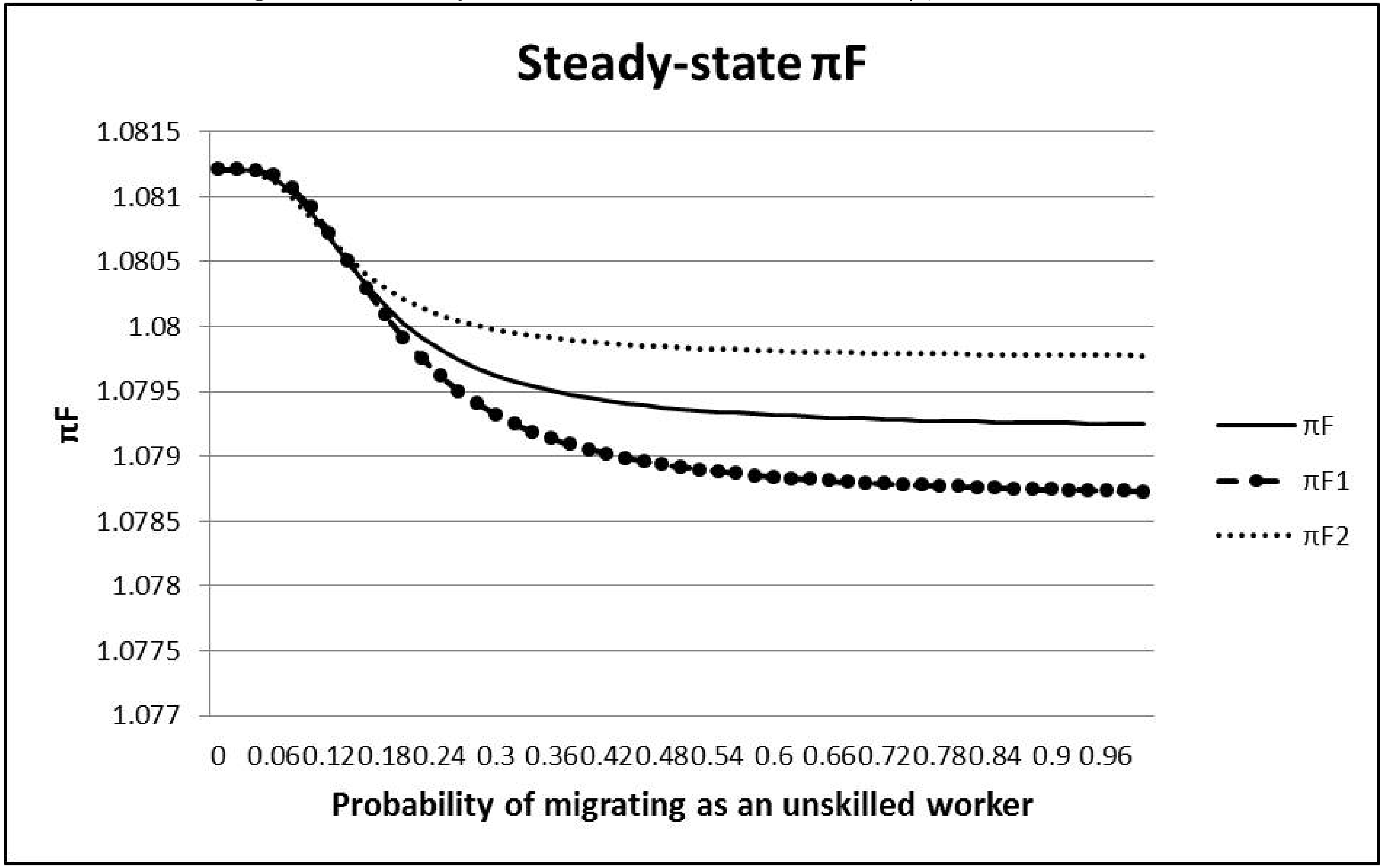


Figure C.32: Unskilled to Skilled Wage Ratio in Country D

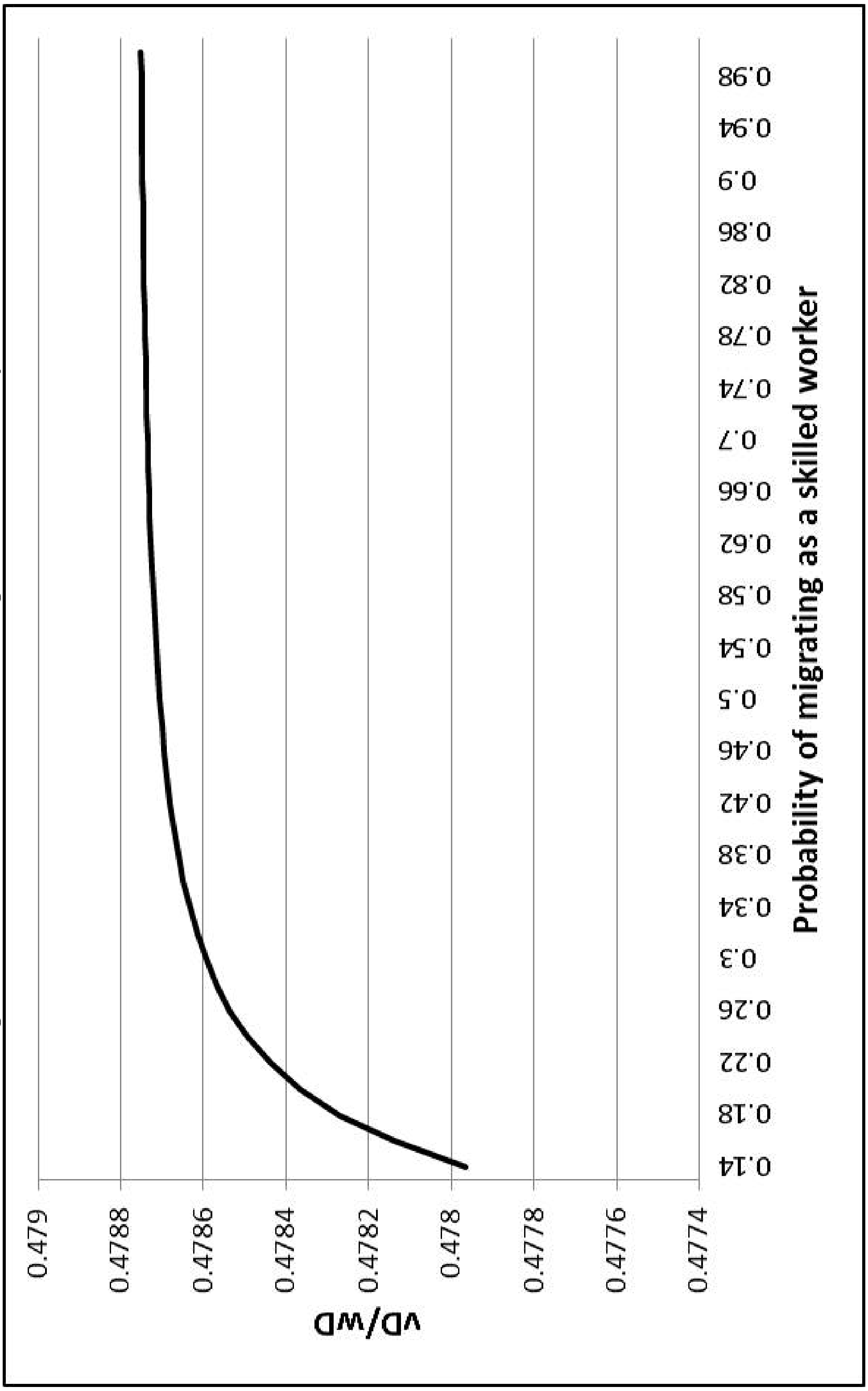


Figure C.33: Unskilled to Skilled Wage Ratio in Country F

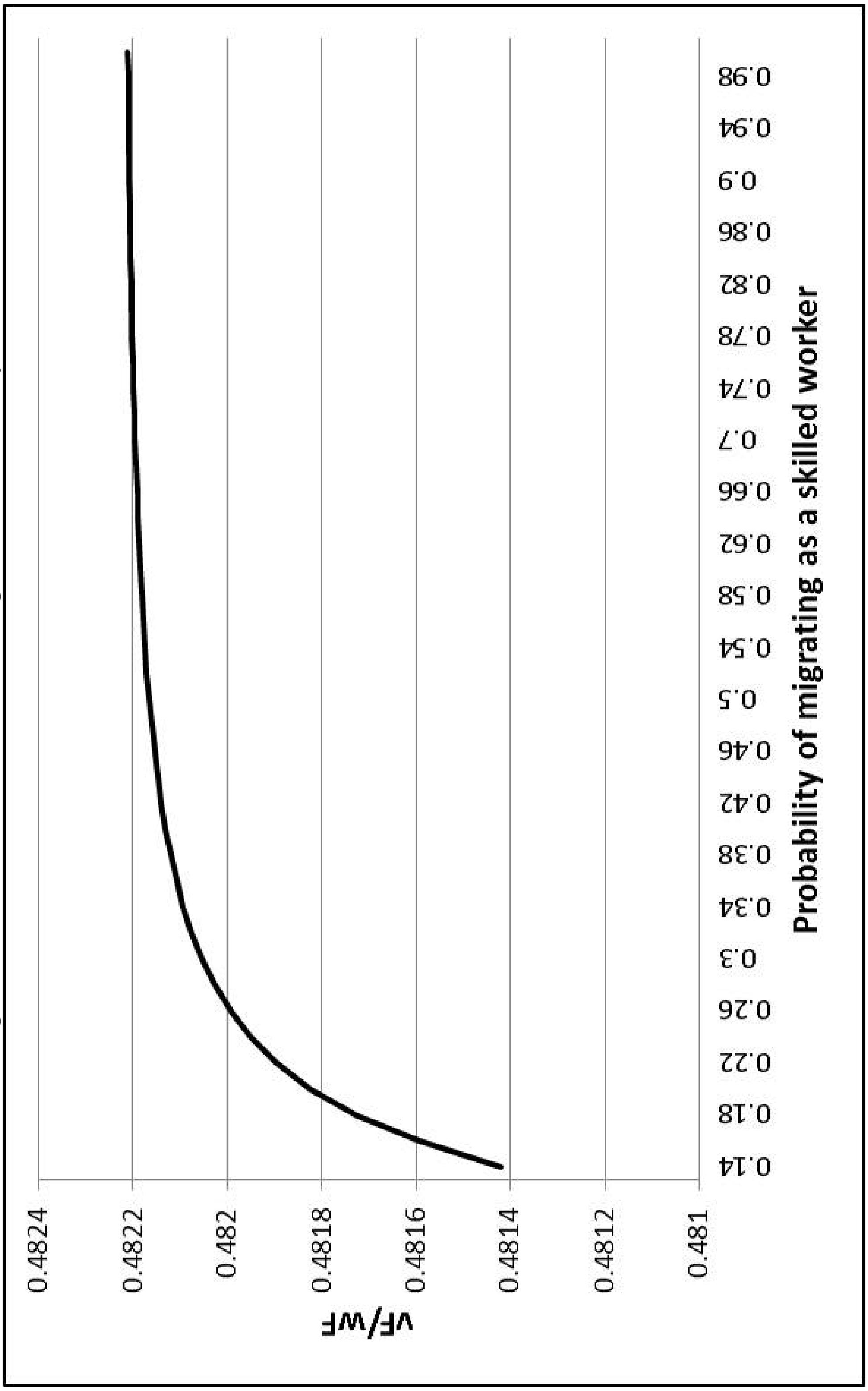


Figure C.34: Unskilled Wage Ratio

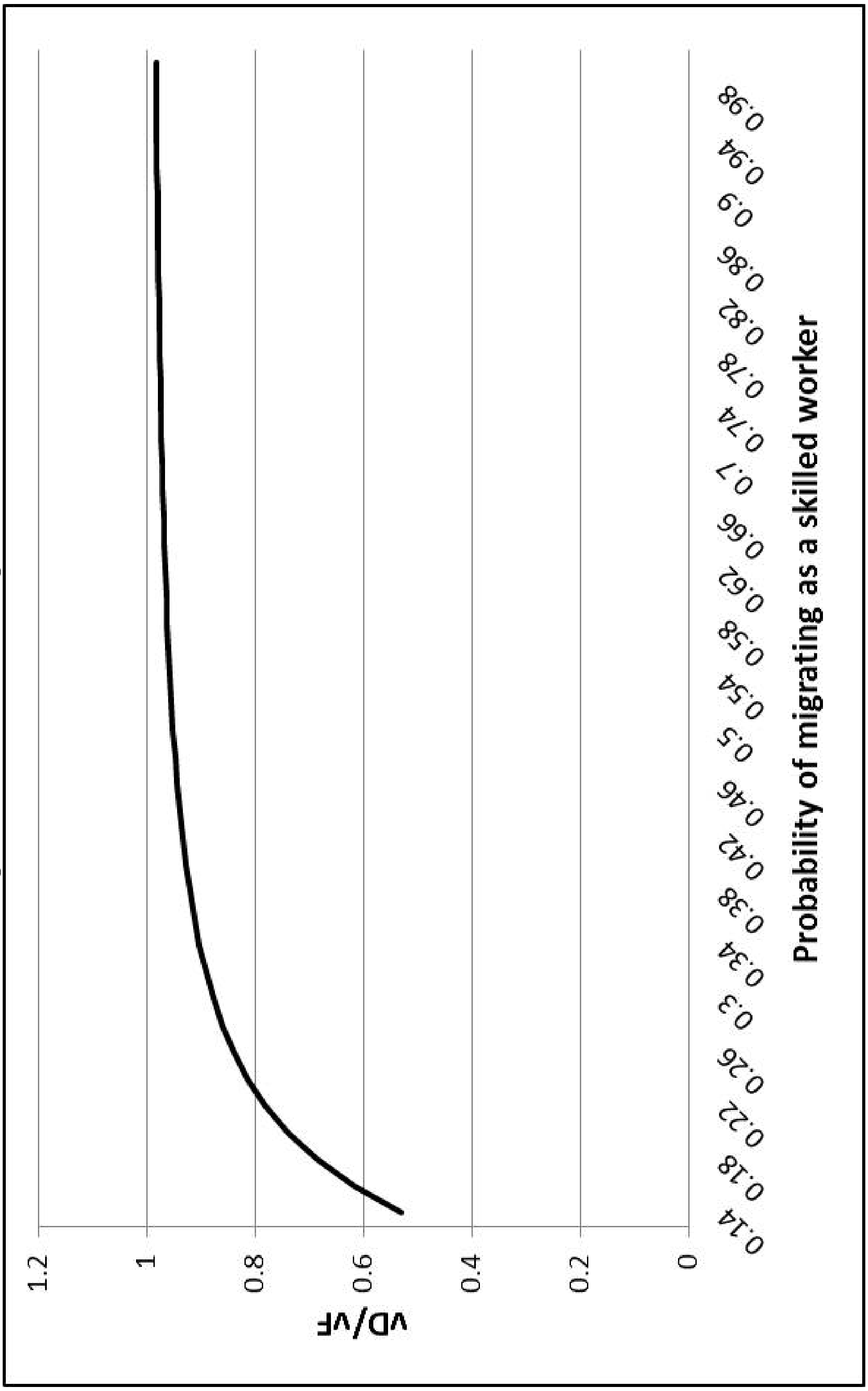


Figure C.35: Skilled Wage Ratio

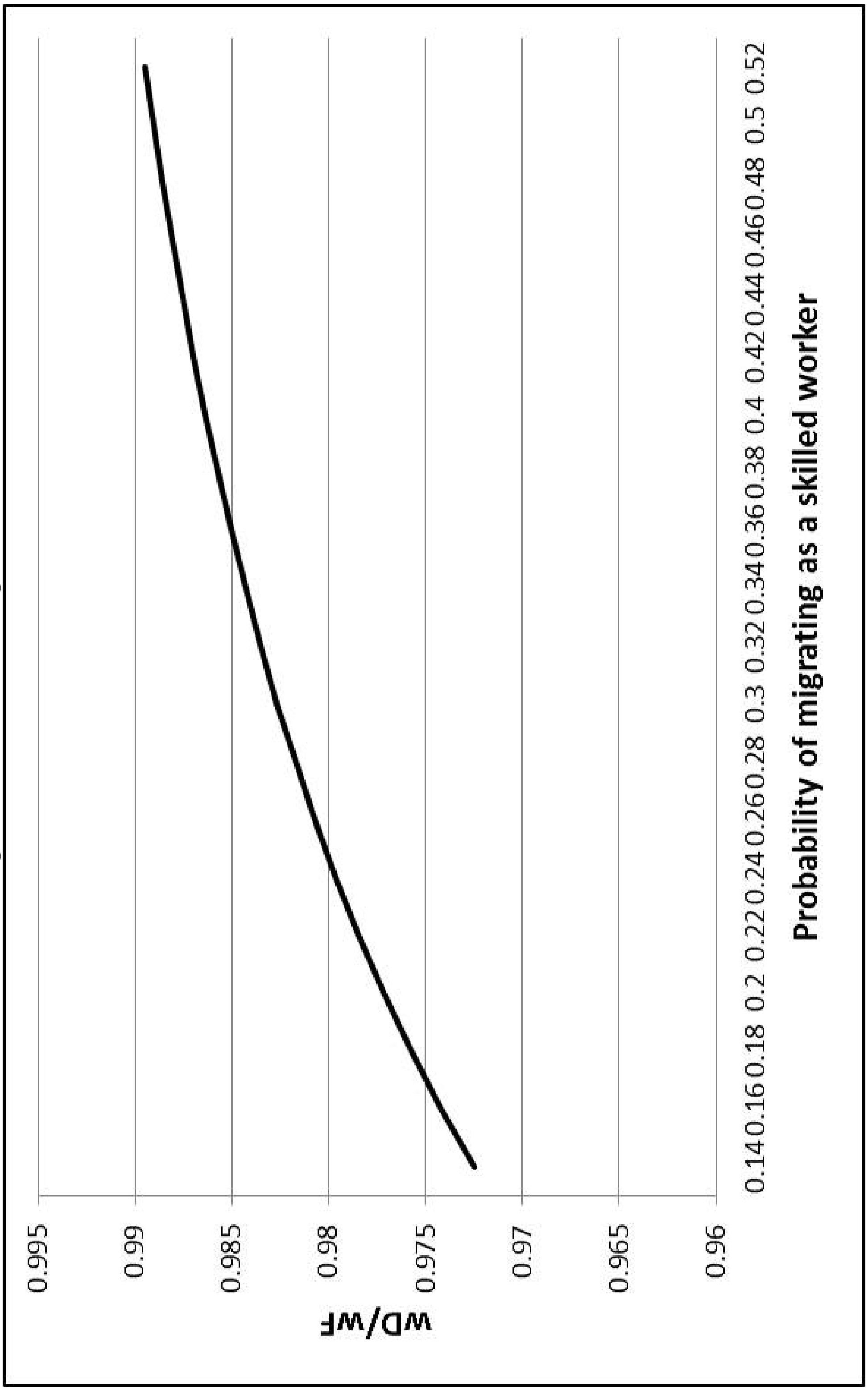


Figure C.36: Unskilled to Skilled Wage Ratio in Country D

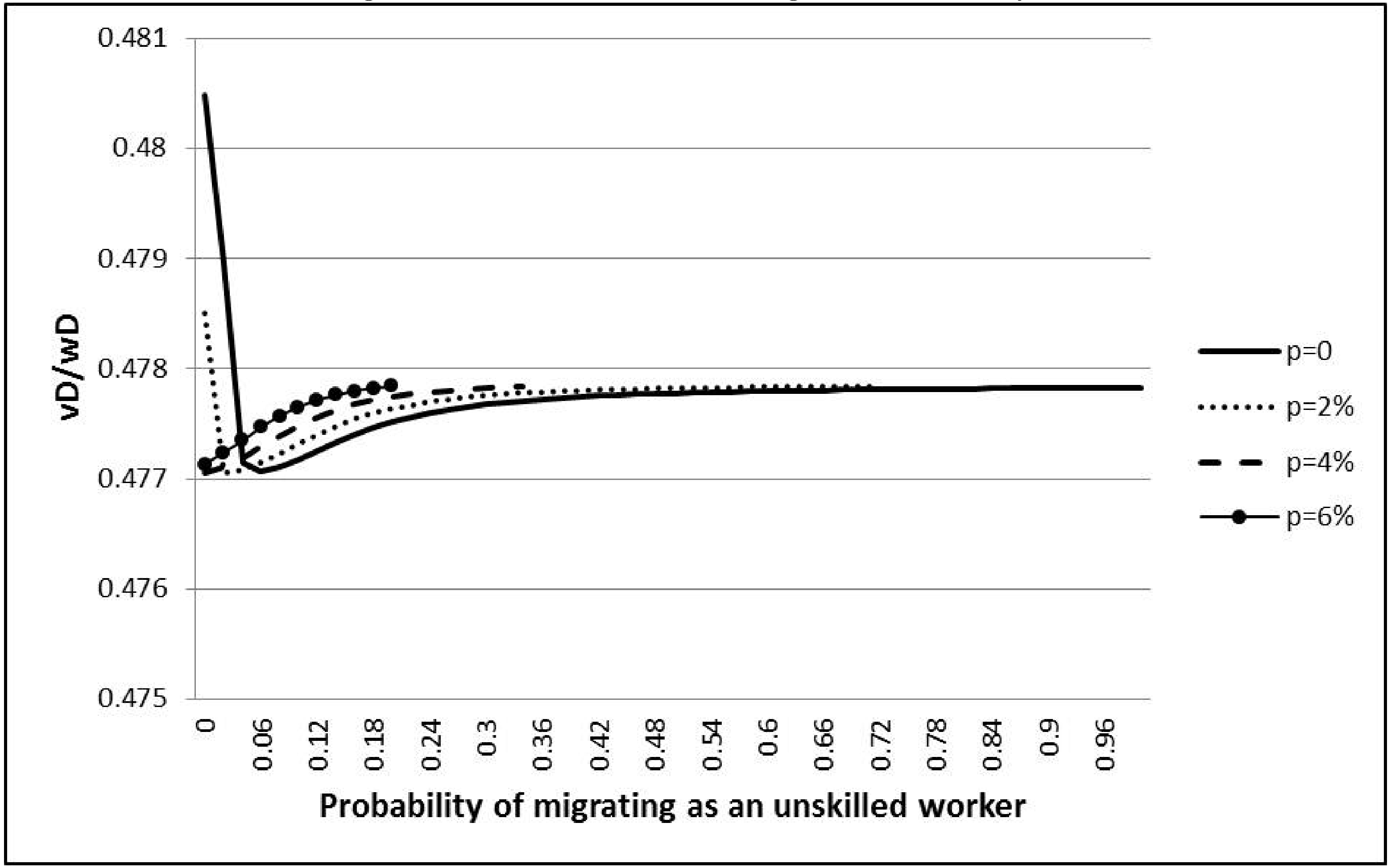
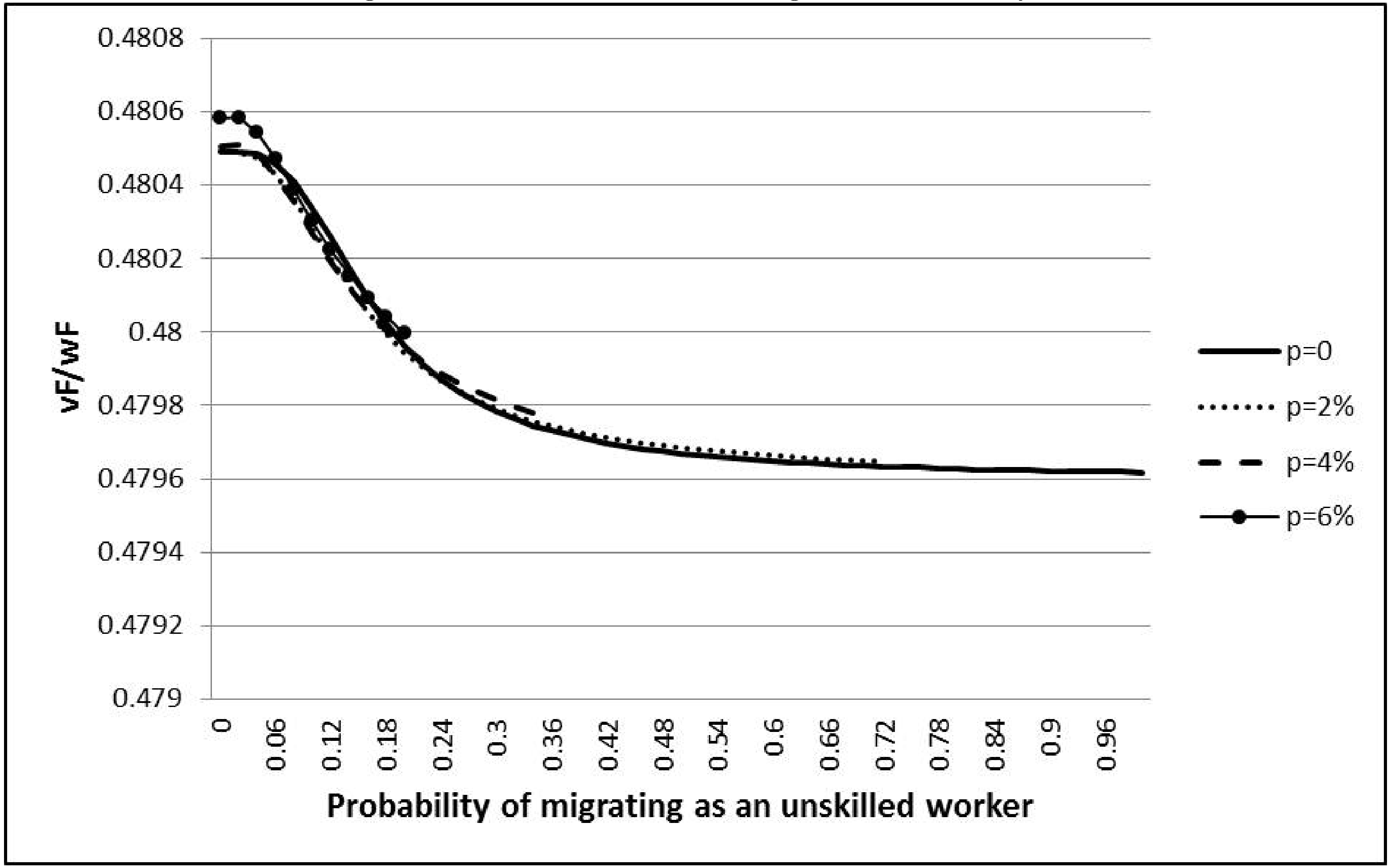


Figure C.37: Unskilled to Skilled Wage Ratio in Country F



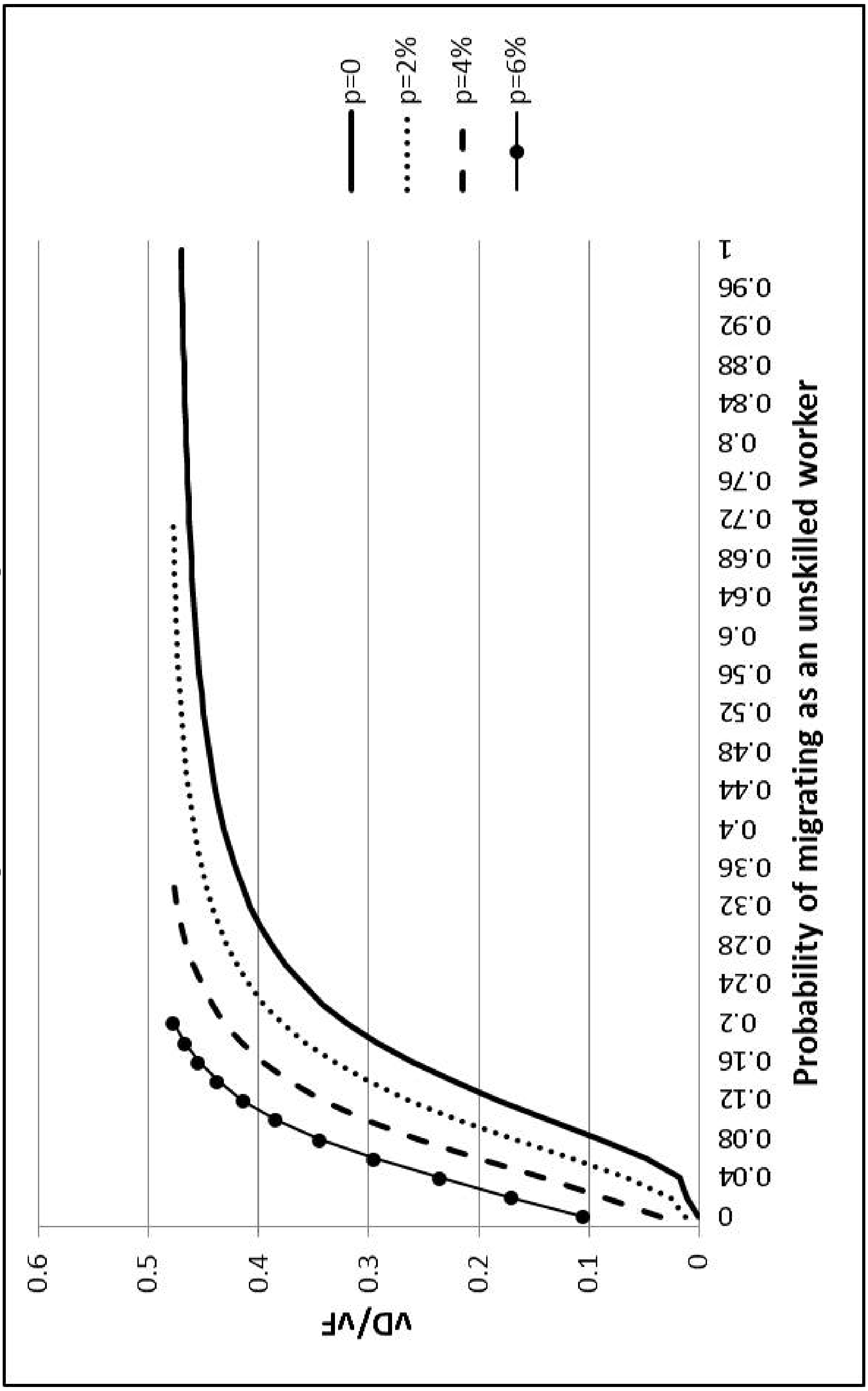
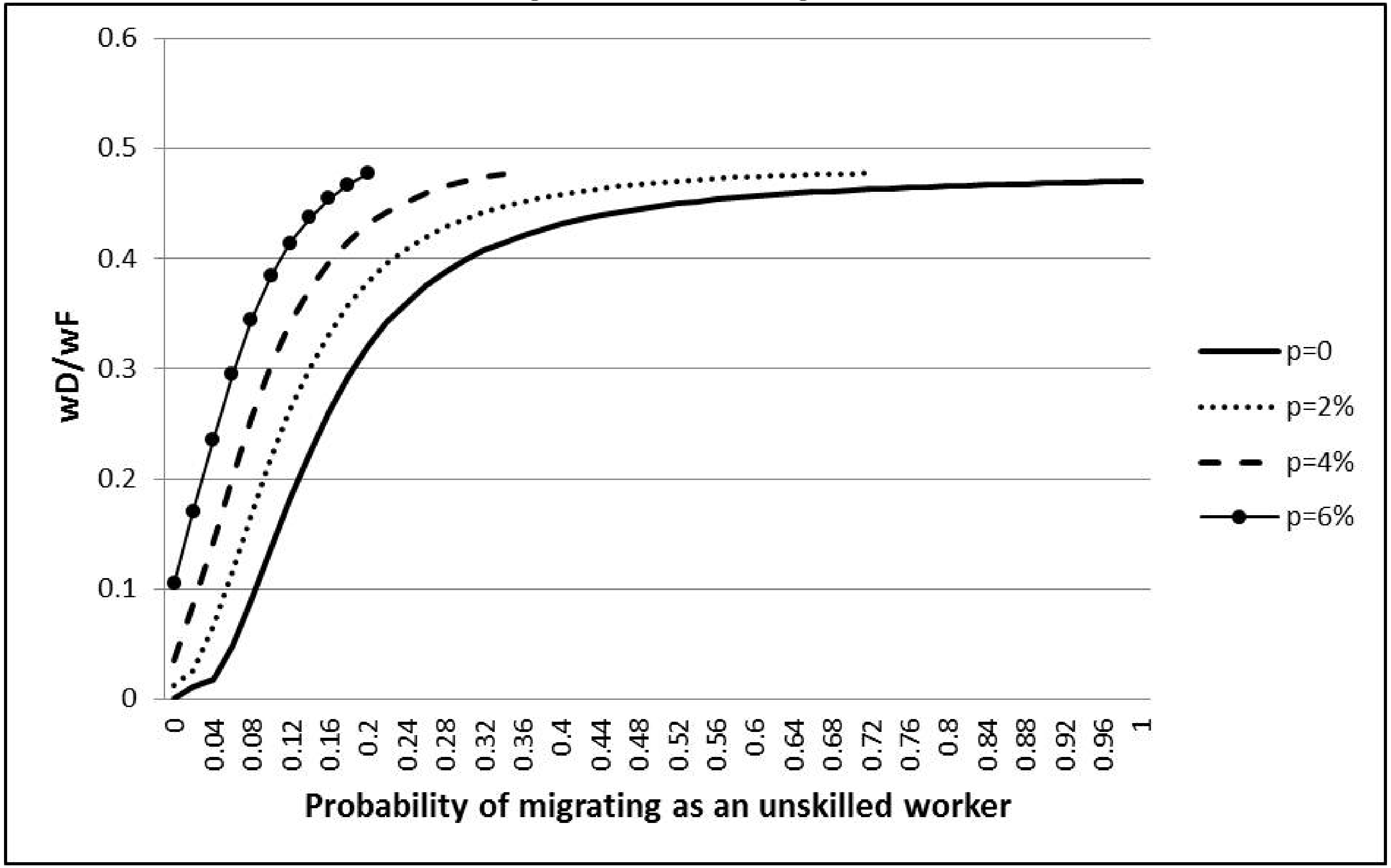


Figure C.38: Unskilled Wage Ratio

Figure C.39: Skilled Wage Ratio



Appendix D

Chapter 4: Figure

Figure D.1: Steady-state Workforce Ratio Over Time with Different Probabilities