CONTAINER TRANSPORTATION NETWORK MODELING AND OPTIMIZATION

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DECLARATION

I hereby declare that the thesis is my original work and it has been written by me in its entirety. I have duly acknowledged all the sources of information which have been used in the thesis.

This thesis has also not been submitted for any degree in any university previously.

Shuaian Wang

8 Oct 2012

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SUMMARY

Container transportation at sea is fulfilled by liner shipping, which has regularly serviced ship routes with fixed port rotations. The volume of containerized cargo reaches 124 million twenty-foot equivalent units (TEUs) in 2009 as a consequence of the growth of the world's economy and trade. With such a large volume, it can be expected that mathematical optimization tools can bring significant cost reductions for liner shipping companies.

In this dissertation, four tactical-level decision problems for liner shipping companies are addressed: fleet deployment, sailing speed optimization, ship route schedule design, and liner shipping network design. These decision problems focus on different aspects of liner shipping services with different inputs and outputs. However, their common objective is to minimize the total cost, or, to maximize the total profit, for liner shipping companies.

The fleet deployment problem aims to determine the type and number of ships to assign to each ship route, in order to fulfil a given container shipment demand at minimum cost. It is formulated as a mixed-integer linear programming model which captures container transshipment operations. This model adopts origin-based container flow variables, resulting in much fewer decision variables and thereby higher computational efficiency.

The sailing speed optimization problem seeks the optimal sailing speed of ships on each voyage legs to minimize the total operating cost including the bunker cost. The significance of the problem is due to the large proportion of the bunker cost in the total operating cost and the high sensitivity of the bunker consumption with speed. Before optimizing the sailing speed, the bunker consumption - sailing speed relation is calibrated using historical data. Unlike the fleet deployment problem, the sailing speed optimization problem is formulated as a mixed-integer nonlinear programming model. In view of the convexity, non-negativity, and univariate properties of the bunker consumption function, an efficient outer-approximation method is proposed to obtain an ε-optimal solution with a predetermined optimality tolerance level ε.

The ship route schedule design problem determines the arrival time of a ship at each port of call on a ship route and the sailing speed function on each voyage leg by taking into account time uncertainties at sea and at port. A mixed-integer nonlinear stochastic programming model is developed to minimize the ship cost and expected bunker cost while maintaining a required transit time service level. An exact cutting-plane based solution algorithm is proposed to solve the model.

The liner shipping network design problem mainly determines the port rotations of each ship route while considering practical operations and features, including multi-type containers, container transshipment operations, empty container repositioning, origin-to-destination transit time constraint, consistent services with the current network, and joint services with other liner shipping companies. Due to the difficulty of the problem, a successive optimization heuristic is proposed to solve practical-sized problems.

All the above models and algorithms are tested based on realistic data provided by a global liner shipping company. The applicability of the models and the efficacy of the algorithms are demonstrated. Managerial insights from the computational results are obtained.

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GLOSSARY OF NOTATION

Sets

$\mathcal{H}$	Set of container routes
$\mathcal{H}^{od}$	Set of container routes for the O-D port pair $(o,d)\in\mathcal{W}$
$\mathcal{I}_r$	Set of port of call sequences of ship route r, $\mathcal{I}_r=\{1,2,\cdots,N_r\}$
$\mathcal{I}_{rp}$	Set of port of call sequences of ship route r that refer to port $p\in\mathcal{P}$, $\mathcal{I}_{rp}:=\{i\in\mathcal{I}_r \mid p_{ri}=p\}$
$\mathcal{K}$	Set of container types
$\mathcal{P}$	Set of ports
$\mathcal{P}^{+}$	Set of ports with surplus empty containers
$\mathcal{P}^{0}$	Set of ports with balanced empty containers
$\mathcal{P}^{-}$	Set of ports with deficit empty containers
$\mathcal{P}_k^{\mathrm{SUR}}$	Set of ports with surplus empty containers in type k
$\mathcal{P}_k^{\mathrm{BAL}}$	Set of ports with balanced empty containers in type k
$\mathcal{P}_k^{\mathrm{DEF}}$	Set of ports with deficit empty containers in type k
$\mathcal{Q}$	Set of transshipment operations
$\mathcal{R}$	Set of ship routes
$\mathcal{R}_p$	Set of those ship routes containing port p
$\mathcal{R}_v$	Set of ship routes accommodating ship type $v\in\mathcal{V}$
$\hat{\mathcal{R}}$	Set of ship routes that must be used (type 1)

$\bar{\mathcal{R}}$ Set of ship routes that a certain number of them must be used (type 2)

$\tilde{\mathcal{R}}$ Set of ship routes that are optional (type 3)

$\mathcal{W}$ Set of O-D port pairs

$\mathcal{V}$ Set of ship types

$\mathcal{V}_r$ Set of ship types available for the ship route r

$\mathcal{V}_p$ Set of ship types that port $p \in \mathcal{P}$ has enough draft to accommodate

$\mathbb{Z}$ Set of integers

$\mathbb{Z}^+$ Set of positive integers

Parameters

$1-\alpha$ Level of service

α^{bun} Bunker fuel price (USD/ton)

γ_{pk} Weekly number of surplus empty containers of type k at port p; $\gamma_{pk} < 0$ means the port is deficit in empty containers

ε Absolute objective value tolerance (USD/week)

$\bar{\varepsilon}$ Maximum allowable bunker consumption error on every leg (tons/n mile)

$\bar{\varepsilon}_{ri}$ Maximum allowable bunker consumption error on leg i of ship route r (tons)

ρ_{hri} Binary coefficient which is 1 if containers in container route h are transported on leg i of ship route r, and 0 otherwise

δ_h^{rij} Binary coefficient indicating whether containers in container route h are loaded at port p_{ri} and discharged at port p_{rj} on ship route r

δ_h^{rsij} Binary coefficient indicating whether container route h incorporates the transshipment $< r, s, i, j >$

τ_{ri} Realization of the stochastic port time $\tilde{\tau}_{ri}$

$\hat{\tau}_{ri}^{\min}$ Minimum value of τ_{ri}

$\hat{\tau}_{ri}^{\max}$ Maximum value of τ_{ri}

τ_{rv}^{fix} Fixed portion of the time (hours) for the round-trip if service route $r \in \mathcal{R}$ is deployed with ships with type $v \in \mathcal{V}_r$. τ_{rv}^{fix} includes the sailing time and standby time for pilotage in and out at all the ports of call

Λ_r Percentage of ship slots of ship route $r \in \hat{\mathcal{R}}$ controlled by the focal liner shipping company

Cap_v Container capacity (TEUs) of a ship with type $v \in \mathcal{V}$

Cap_r Container capacity (TEUs) of a ship on ship route $r \in \mathcal{R}$

c_h Overall container handling and berth occupancy cost (USD/TEU) associated with delivering one TEU in the container route $h \in \mathcal{H}$

$\bar{c}_p$ Container transshipment cost (USD/TEU) charged by a particular port $p \in \mathcal{P}$, which is the total cost of discharging a container from the incoming ship and loading the container to the outgoing ship

$\hat{c}_p$ Container load cost (USD/TEU) charged by a particular port $p \in \mathcal{P}$

$\tilde{c}_p$ Container discharge cost (USD/TEU) charged by a particular port $p \in \mathcal{P}$

$\bar{c}_{pk}^{\text{EMP}}$ Transshipment cost (USD/TEU) charged by a particular port $p \in \mathcal{P}$ for empty containers in type k

$\hat{c}_{pk}^{\mathrm{EMP}}$ Container load cost (USD/TEU) charged by a particular port $p \in \mathcal{P}$ for empty containers in type k

$\tilde{c}_{pk}^{\mathrm{EMP}}$ Container discharge cost (USD/TEU) charged by a particular port $p \in \mathcal{P}$ for empty containers in type k

c_{od} Penalty (USD/TEU) for not shipping a TEU for the port pair $(o,d) \in \mathcal{W}$

c_r Fixed operating cost (USD/week) for a particular ship route $r \in \mathcal{R}$

c_{pv}^{ber} Berth occupancy charge (USD/hr) at port $p \in \mathcal{P}$ for ships in type $v \in \mathcal{V}$

c_v^{opr} Fixed operating cost (USD/week) for a particular ship in type $v \in \mathcal{V}$

c_r^{ship} Fixed operating cost (USD/week) of one ship deployed on ship route $r \in \mathcal{R}$

c_{rv}^{fix} Fixed voyage cost (USD/week) of ship route $r \in \mathcal{R}$ deployed with ships in type $v \in \mathcal{V}$

c_v^{in} Price (USD/week) for chartering in one ship in type $v \in \mathcal{V}$

c_v^{out} Profit (USD/week) for chartering out one ship in type $v \in \mathcal{V}$

E_k Equivalent volume (TEUs) of a container in type $k \in \mathcal{K}$

g_{od} Freight rate (USD/TEU) charged by other liner shipping companies for shipping one container for port pair $(o,d) \in \mathcal{W}$

L_{ri} Voyage distance (n mile) of the leg i on ship route r

M_{pv} Average productivity (moves/hour) at port $p \in \mathcal{P}$ for ship type v

n_{od} Weekly container shipment demand (TEUs/week) for port pair $(o,d) \in \mathcal{W}$

n_{od}^{k} Weekly number of laden containers in type k to be transported for the O-D pair $(o,d) \in \mathcal{W}$

n_{p}^{EMP} Number of surplus empty containers at port $p \in \mathcal{P}$ and $n_{p}^{\text{EMP}} < 0$ means deficit

N^{SR} Minimum number of ship route type 2 that must be used

N_{r} Number of ports of call on ship route r

N_{v}^{own} Number of owned ships in type $v \in \mathcal{V}$

N_{v}^{in} Maximum number of ships in type $v \in \mathcal{V}$ that the liner shipping company can charter in from the leasing market

t_{r}^{fix} Total standby time (hours) for pilotage in and out at all the ports of call for the round-trip of ship route $r \in \mathcal{R}$ deployed with a fixed type of ships

t_{pv} Average container handling time (hr/TEU) for ship type v at port p, $t_{pv} = 1/M_{pv}$

t_{pv}^{od} Average handling time (hr/TEU) of containers in port pair $(o,d) \in \mathcal{W}$ for ship type v at port p

t_{rh} Additional round-trip time (hr/TEU) posed for ship route r by transporting one TEU according to container route h

$t_{r,i,i+1}^{\min}$ Minimum inter-arrival time between two consecutive ports of call

$t_{r,i,i+1}^{\max}$ Maximum inter-arrival time between two consecutive ports of call which affects bunker consumption

t_{ri}^{ava} Available sailing time on the i^{th} leg of ship route r

$\hat{t}_{ri}^{\text{con}}$ Contingency time at sea for leg i of ship route r

$\hat{t}_p$ Least connection time required at port p

$\hat{T}_h$ Maximum allowable transit time for container route h

$\hat{T}_{od}$ Maximum allowable transit time for port pair $(o,d) \in \mathcal{W}$

U_{ri} Capacity utilization of leg i of ship route $r \in \mathcal{R}$

U_r Capacity utilization of ship route $r \in \mathcal{R}$

$U_r^{\max}$ Hit-haul capacity utilization of ship route $r \in \mathcal{R}$

$V_{ri}^{\min}$ Minimum sailing speed (knot) on leg i of route r

$V_{ri}^{\max}$ Maximum sailing speed (knot) on leg i of route r

W_{od}^{k} Average weight (tons) of a laden container in type k to be transported for the O-D pair $(o,d) \in \mathcal{W}$

W_{od} Average weight (tons/TEU) for all types of containers to be transported for the O-D pair $(o,d) \in \mathcal{W}$

W^k Weight (tons) of an empty container in type k

Wei_v Weight capacity (tons) of a ship with type $v \in \mathcal{V}$

Others

$\tilde{\tau}_{ri}$ Random port time a container ship on ship route r spends at port p_{ri}

$\tilde{t}_{ri}^{\text{ava}}$ Random available sailing time on the i^{th} leg of ship route r

$C_{ri}(t_{ri}^{\text{ava}})$ Optimal bunker consumption function for leg i of ship route r

$\bar{C}_{ri}(t_{r,i,i+1})$ Expected bunker consumption with respect to the inter-arrival time between two consecutive ports of call

$f_{ri}(\tau_{ri})$	Probability distribution function for $\tilde{\tau}_{ri}$
$\hat{g}(v(t))$	Bunker consumption per hour at the speed $v(t)$
$g(v^*(l,t^{\mathrm{ava}}))$	Bunker consumption per nautical mile at the speed $v^*(l,t^{\mathrm{ava}})$
$g_{ri}(v_{ri})$	Bunker consumption (ton/n mile) at the speed v_{ri} on leg i of ship route r
$l_{ri}(t)$	Sailed distance by time t
p_{ri}	The i^{th} port of call on route r
$Q_{ri}(u_{ri})$	Bunker consumption (ton/n mile) at the speed $1/u_{ri}$ on leg i of ship route r
$v(r)$	Type of ship deployed on ship route $r \in \mathcal{R}$
$v^*(t)$	Optimal sailing speed with time t
$v^*(l,t^{\mathrm{ava}})$	Optimal sailing speed with sailed distance l and available sailing time t^{ava}

GLOSSARY OF ABBREVIATIONS

CRM	container route generation model
FD	fleet deployment
FDM	fleet deployment model
FDP	fleet deployment problem
H&S	hub and spoke
IP	pure integer linear programming
LP	linear programming
MIP	mixed-integer linear programming
NDP	network design problem
NDM	network design model
O-D	origin-to-destination
OSM	optimal speed model
SDM	schedule design model
SDP	schedule design problem
SS	sailing speed
SSM	sailing speed model
TEU	twenty-foot equivalent unit
USD	US dollar
VRP	vehicle routing problem

CHAPTER 1. INTRODUCTION

1.1 Classification of Maritime Transportation

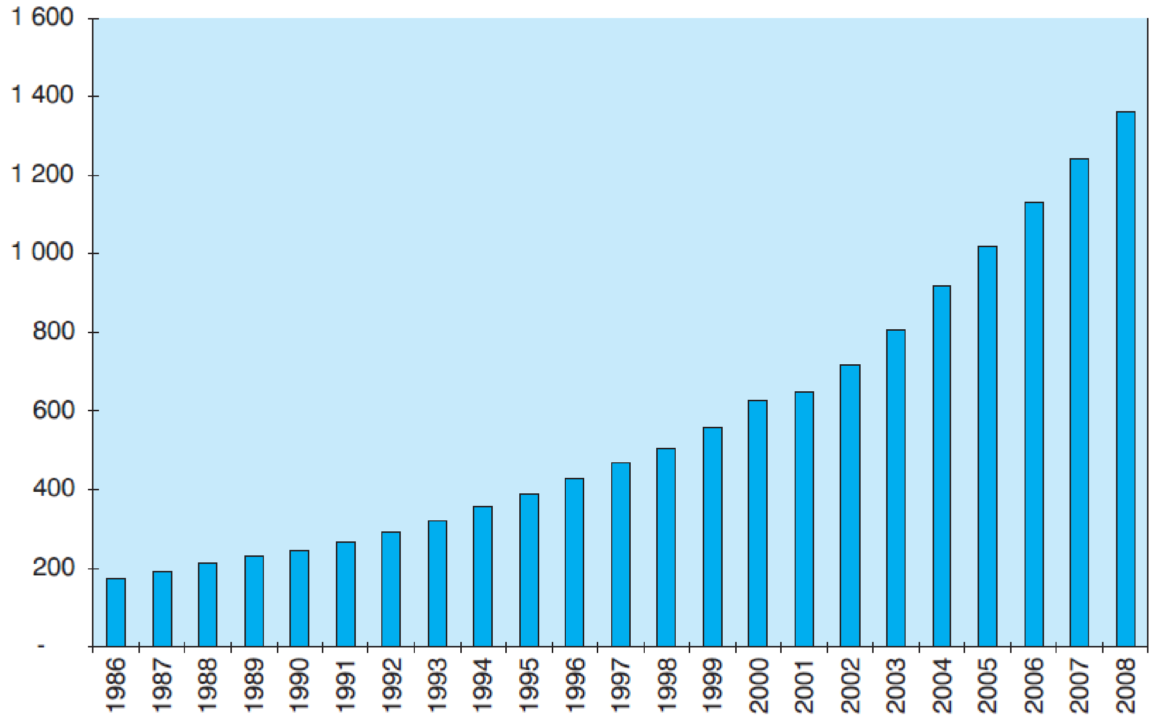
Maritime transportation is the backbone of international trade. UNCTAD (2010) estimates the 2009 international seaborne trade at 7.8 billion tons of goods loaded. There are generally three modes of shipping operations in maritime transportation: industrial, tramp, and liner (Lawrence, 1972). In industrial shipping, the cargo owners control the ships and seek to ship their cargo at minimal cost. Tramp shipping resembles taxis. The ships are sent where cargo is available and usually the cargo is a whole shipload, with one origin and one or two destinations. Liner shipping companies publish their service routes, which have fixed sequence of ports of call, schedule, service frequency and deployed ships, to attract cargo. Liner shipping can hence be likened to bus services. A definition of liner shipping is provided in Stopford (2009): A liner service is a fleet of ships, with a common ownership or management, which provides a fixed service, at regular intervals, between named ports, and offer transport to any goods in the catchment area served by those ports and ready for transit by their sailing dates. A fixed itinerary, inclusion in a regular service, and the obligation to accept cargo from all comers and to sail, whether filled or not, on the date fixed by a published schedule are what distinguish the liner from the tramp. Liner shipping is the focus of this study. Table 1-1 lists the nomenclature used in this study.

Table 1-1 Nomenclature

Nomenclature	Definition/explanation
Fleet size and mix	The types of ships and the number of ships of each type in the ship fleet
Ports of call	A port visited by a ship together with its calling sequence. For example, if a ship visits Hong Kong, Singapore, Colombo, Singapore, and returns to Hong Kong, then the 2nd port of call refers to the call at Singapore after Hong Kong, and the 4th port of call refers to the call at Singapore after Colombo. Both ports of call refer to the same port. However, they are considered to be different ports of call because of the sequence.
Itinerary = loop = port rotation = sequence of ports of call	Port calling sequence in a ship's round-trip journey
Liner service route = liner service = liner ship route = ship route	A port rotation with deployed ships
Liner shipping service network = liner shipping network = liner network	A set of ship routes
Fleet deployment	Assignment of ships to port rotations
Schedule	The arrival and departure time at each port call
Frequency = service frequency	The headway (days) between two adjacent ships on a ship route, or the round-trip time if there is only one ship deployed on the ship route
Transshipment = relay	The shipment of containers to an intermediate destination, and then from there to yet another destination. For example, containers from Australia to Europe may first be transported to Singapore by a small ship. After that, the containers are reloaded to a large ship that sails from Singapore to Europe. The handling operation at Singapore is transshipment.
Bunker	The fuel that the main engine of a ship burns

1.2 Significance and Characteristics of Liner Shipping

Liner shipping mainly involves the transportation of containerized cargo. From the report of UNCTAD (2009), container trade volumes increased to 137 million twenty-foot equivalent units (TEUs) in 2008. The share of containerized trade in the world's total dry cargo increased from 5.1 per cent in 1980 to 25.4 per cent in 2008. The value of world maritime container trade grew from $2 trillion in 2001 to $4 trillion in 2008 accounting for around one in every $14 of global economic output. Figure 1-1 presents the world's containerized cargo trade from 1986 to 2008. The rapid growth in containerized cargo over the last three decades is the result of globalization and increasing world trade volume, dedicated purpose-built large containerships, improved handling facilities in ports, and the rising amount of cargo being containerized. At the same time, the lower cost, fast speed, reliable schedule, and less damage and pilferage of container liner shipping also gives impetus to world trade (UNCTAD, 2009).



Source: UNCTAD (2008)

Figure 1-1 International containerized trade volume from 1986 to 2008 (million tons)

Because of the global recession, container trade recorded its first absolute contraction in 2009 ever since containerization began. The container trade volume in 2009 fell sharply by 9.0 per cent, with the overall volume totaling 124 million TEUs (UNCTAD, 2010). This contraction does not indicate that the liner shipping industry begins to go down at all.

Compared to tramp shipping and other transport modes, liner shipping has a number of unique characteristics:

(a) Liner shipping mainly involves transportation of containerized cargo.

For example, on the trade lanes between the three major market regions (Asia-Europe, Trans-Pacific and Trans-Atlantic), between 75 per cent and 90 per cent of the containerisable cargo is containerised (Fleming, 2003). Containerization not only protects cargo from damage and pilferage, but also facilitates the handling operations at intermodal terminals (including ports). As a consequence, the oceanic transportation and the inland transportation which includes rail, truck, and inland waterway can be seamlessly connected. Thus, cargo can be transported efficiently from its origin to its destination in the intermodal transportation network.

(b) Liner shipping services have fixed schedules.

Liner shipping companies publish their services in advance to attract cargo from the spot market. A liner service route forms a round-trip; hence no origins or destinations can be defined. Usually a string of ships are deployed on a ship route to maintain a fixed schedule (normally weekly services). Ships can pick up and deliver cargo at any port of call, and the ship may never be empty during the voyage. Compared with tramp shipping, each one of the shippers of liner shipping usually has far less than a full shipload of cargo, and liner ships have to maintain the published departure date even when a full payload is not available.

(c) A number of containers are transshipped from their origins to their destinations.

Transshipment enables cargo consolidation for the deployment of large container ships. Transshipment also expands the service scope of liner shipping companies as any port-to-port delivery service is feasible even if there is no ship route connecting these two ports. At the same time, transshipment has the disadvantage of expensive double-handling, increased operational risk, and (possibly) longer transit time than direct shipment. Transshipment is an important feature that must be considered in all levels of planning problems. For example, at Singapore port, one of the world's busiest ports, more than 80 per cent of containers handled are transshipped containers (Petering, 2011). About 28 per cent of the world's port container throughput is transshipped containers (Vernimmen et al., 2007).

1.3 Liner Shipping Network

A liner shipping company provides shipping services over its liner shipping network. The liner shipping service network consists of a number of ports connected by a number of ship routes. The liner shipping company deploys container ships on these ship routes to transport containers from one port to another.

There are several decision issues faced by the liner shipping company regarding the liner shipping network in a medium-term planning horizon (three to six months). (i) Which type of ship and how many ships to deploy on each ship route (fleet deployment)? (ii) How to determine the optimal sailing speed of container ships? (iii) How to design the schedule (on which day to call at which port) for each service route, in order to fulfill a given level of service in terms of port-to-port transit time? (iv) How to design a liner shipping network? These decision issues are very important for liner shipping companies because once the liner shipping network is established, most of the operating costs (ship cost, port charge, bunker cost) are also determined. Also, these problems are challenging because liner shipping companies control a comprehensive ship fleet and operate global shipping services due to merger, consolidation and alliance. Therefore, the traditional planning approach through experience is insufficient to manage the problems. Liner shipping companies are in urgent need of tools that combine operations research and computing techniques in order to optimize their liner shipping services.

Nevertheless, the analysis of liner shipping service network has attracted much fewer research efforts than other transportation areas, such as public transit analysis, airline management, and general vehicle routing problems (Psaraftis, 1999). One reason is that many of the road-, rail-, and air-based services transport passengers, who are more concerned about service quality. Another reason is visibility; the number of trucking companies and airlines is large as compared with shipping operators. Furthermore, the existing reviews (Ronen, 1983, 1993; Christiansen et al., 2004, 2007) on ship routing and scheduling mainly focus on industrial and tramp shipping. The far lower number of studies on liner shipping can be attributed to the even lower visibility of liner shipping operations: liner services are basically provided by a few global companies. Moreover, despite the fact that liner shipping services and container terminal operations are intimately related, container terminal operations have attracted much more attention from researchers. This might be a consequence of the ownership structure; a government is usually the owner of a port, whereas liner shipping companies are privately owned and sometimes the ownership changes. In addition, the liner shipping industry is more conservative than other transportation industries, such as the airline industry, and global liner shipping companies have been reluctant to share their data and concerns with researchers in the past. However, several leading liner shipping companies have recently sought operations research methods to make better decisions because of increased container shipment market competition and bunker prices.

1.4 Objectives

The objective of this thesis is to cover the gap between the needs of liner shipping industry and the scarce relevant literature on liner shipping networks. In detail, the above-mentioned four decision issues – fleet deployment, speed optimization, schedule design, and liner shipping network design - for a liner shipping company will be examined. Both mathematical optimization models and solution algorithms will be proposed. The models and algorithms will be tested on real-case problems.

1.5 Organization of the Thesis

Chapter 1 provides a general introduction to the three shipping modes especially liner shipping, and decision issues associated with the liner shipping network. Furthermore, objectives and organization of the thesis are outlined.

Chapter 2 introduces the liner shipping network in detail. The elements in a liner shipping network are first described and the four decision issues are subsequently elaborated.

Chapter 3 presents a comprehensive literature review on research relevant to fleet deployment, sailing speed optimization, schedule design, and liner shipping network design. Limitations of existing studies are examined and research objectives are highlighted.

Chapter 4 investigates the liner ship fleet deployment (FD) problem with the container transshipment operations. An origin-based nonlinear programming model is developed for the FD problem to capture the practical operations and factors in liner shipping. This model is subsequently transformed into an equivalent mixed-integer linear programming model. Computational studies on instances derived from a realistic Asia-Europe-Oceania shipping network of a global liner shipping company as well as randomly generated large-scale shipping networks demonstrate that the proposed model is able to address fleet deployment problems encountered in practice.

Chapter 5 seeks to optimize the sailing speeds of ships in a liner shipping network. First, it calibrates the bunker consumption - sailing speed relation for container ships using historical operating data from a global liner shipping company. It proceeds to investigate the optimal sailing speed of container ships on each leg of each ship route in a liner shipping network while considering transshipment and container routing. This problem is formulated as a mixed-integer nonlinear programming model. In view of the convexity, non-negativity, and univariate properties of the bunker consumption function, an efficient outer-approximation method is proposed to obtain an ε-optimal solution with a predetermined optimality tolerance level ε. The proposed model and algorithm is applied to a real case study for a global liner shipping company.

Chapter 6 deals with the liner ship route schedule design problem which aims to determine the arrival time of a ship at each port of call on a ship route and the sailing speed function on each voyage leg. The time uncertainties at sea and at port are taken into consideration. It first derives the optimality condition for the sailing speed function with sea contingency and subsequently demonstrates the convexity of the bunker consumption function. A mixed-integer nonlinear stochastic programming model is developed for the proposed liner ship route schedule design problem by minimizing the ship cost and expected bunker cost while maintaining a required transit time service level. In view of the special structure of the model, an exact cutting-plane based solution algorithm is proposed. Numerical experiments on real data provided by a global liner shipping company demonstrate that the proposed algorithm can efficiently solve real-case problems.

Chapter 7 studies a realistic liner shipping network design problem while considering practical operations and features, including multi-type containers, container transshipment operations, empty container repositioning, origin-to-destination transit time constraint, consistent services with the current network, and joint services with other liner shipping companies. It first examines the laden and empty container shipment demand. It proceeds to investigate the routing of laden containers while considering the transit time constraint. Two approaches are proposed. The first one is based on global and regional hubs. It generates container routes efficiently whereas cannot guarantee optimality. The second one is reliant on the construction of a liner shipping operational network and an integer linear programming model. It is an exact algorithm while the computational time is longer. Given a set of candidate ship routes, including ship routes that must be used, ship routes that a minimum number of them must be used, and ship routes that are completely optional, a mixed-integer linear programming model is presented, which gives the ship routes that should be used and the laden and empty container flow in the resulting network. After that, the resulting network is further improved by changing existing ship routes, adding new ship routes, and removing ship routes. Finally, a real case study based on the global shipping network of a liner shipping company, consisting of 166 ports, is reported.

Chapter 8 draws conclusions and recommends future research work.

CHAPTER 2. PRELIMINARIES

In this chapter we first introduce the three major elements in a liner shipping network: ports, ships, and containers. Based on this introduction, we detail the decision problems for a liner shipping company, especially the four tactical-level decision issues associated with liner shipping network.

2.1 Elements in a Liner Shipping Network

A typical liner shipping service network operated by a liner shipping company is shown in Figure 2-1. The network consists of a group of ports denoted by the set $\mathcal{P}$. The liner shipping company operates a number of ship routes denoted by the set $\mathcal{R}$ between these ports. There are a total of 46 ports in Figure 2-1, scattered in Asia, Europe and Oceania. We will frequently refer to this network in subsequent chapters as the Asia-Europe-Oceania shipping network.

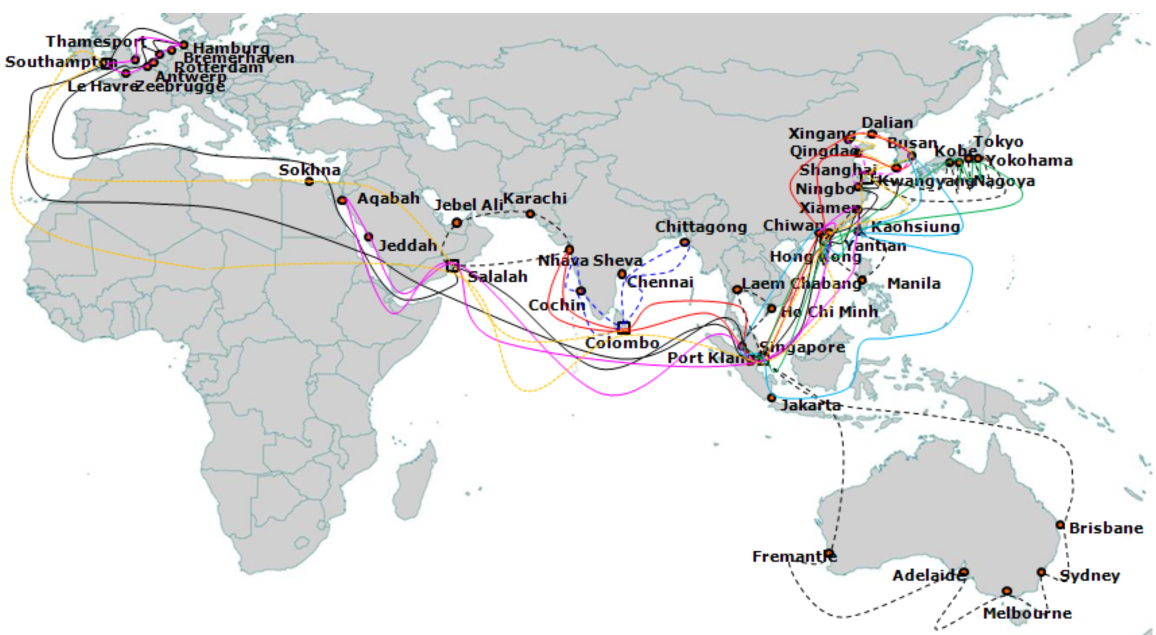


Figure 2-1 An Asia-Europe-Oceania liner shipping service network

2.1.1 Ports and port rotations of ship routes

A container port $p \in \mathcal{P}$ is the interface between inland and maritime transportation. Its main function is container handling and temporary storage. Port productivity usually refers to the number of container moves per hour. Among other factors, port productivity depends on the length of the container ship because a longer ship allows more quay cranes to work simultaneously. Let M_{pv} be the average productivity (moves/hour) at port p for ship type v. Denote by t_{pv} (hours/TEU) the average container handling time of one TEU for ship type v at port p, $t_{pv} = 1/M_{pv}$. A port also has a maximum draft at its berths. A ship whose draft is larger than the port draft cannot be accommodated by the port.

Container handling is very expensive. Container handling cost is an important variable cost for liner shipping companies and the main revenue for container terminal operators. Let $\bar{c}_p$ (USD/TEU) be the container transshipment cost charged by a particular port $p \in \mathcal{P}$. It is usually smaller than the sum of load cost $\hat{c}_p$ (USD/TEU) and discharge cost $\tilde{c}_p$ (USD/TEU) because most port operators encourage container transshipment activities by providing more competitive transshipment prices.

The itineraries of practical ship routes, such as those currently operated by the major liner shipping companies such as OOCL (2010) and Maersk (2010), form loops. A ship route $r \in \mathcal{R}$ can be expressed by its port calling sequence:

$$p_{r1} \to p_{r2} \to \cdots \to p_{rN_r} \to p_{r1} \tag{2.1}$$

where N_r is the number of ports of call on the itinerary and p_{ri} is the i^{th} port of call on route r, $i = 1, 2, \cdots, N_r$. Eq. (2.1) gives a ship route coding scheme and it also describes the loop characteristics of a ship route with a predetermined port calling sequence. Let $\mathcal{I}_r$ be the set of port calling indices of the ship route r, $\mathcal{I}_r = \{1, 2, \cdots, N_r\}$. Since two different port indices shown in Eq. (2.1) may refer to the same port, let $\mathcal{I}_{rp}$ be the set of port indices on ship route r that refer to a particular port $p \in \mathcal{P}$, $\mathcal{I}_{rp} \subset \mathcal{I}_r$. For example, Figure 2-2 illustrates a real ship route. A ship deployed on this ship route visits Busan (BS), Shanghai (SH), Yantian (YT), Hong Kong (HK), Singapore (SG), Yantian (YT) , Hong Kong (HK), and returns to Busan (BS). It can be coded as follows:

$$\begin{aligned} &p_{r1}(\mathrm{BS}) \to p_{r2}(\mathrm{SH}) \to p_{r3}(\mathrm{YT}) \to p_{r4}(\mathrm{HK}) \to p_{r5}(\mathrm{SG}) \to p_{r6}(\mathrm{YT}) \to \\ &p_{r7}(\mathrm{HK}) \to p_{r1}(\mathrm{BS}) \end{aligned} \qquad (2.2)$$

Eq. (2.2) implies that the number of ports of call $N_r = 7$ and Yantian port is served by a ship for two times during a round-trip, namely, $\mathcal{I}_{r,\mathrm{YT}} = \{3, 6\}$.

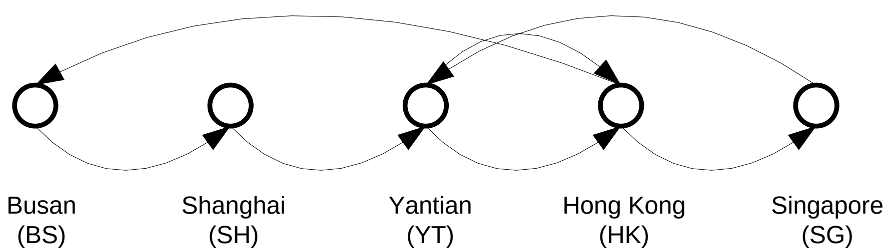


Figure 2-2 The itinerary of a ship route

Defining $p_{r,N_r+1} := p_{r1}$, the voyage between port p_{ri} and port $p_{r,i+1}$ is referred to as *leg* i of ship route $r \in \mathcal{R}$, which can be denoted by the pair of ordered ports $< p_{ri}, p_{r,i+1} >$, $i \in \mathrm{I}_r$. As for the service route shown in Eq. (2.2), it has 7 legs - leg 1: $<p_{r1}$(BS), p_{r2}(SH)>, leg 2: $< p_{r2}$(SH), p_{r3}(YT) >, leg 3: $< p_{r3}$(YT), p_{r4}(HK)>, leg 4: $< p_{r4}$(HK), p_{r5}(SG)>, leg 5: $< p_{r5}$(SG), p_{r6}(YT)>, leg 6: $< p_{r6}$(YT), p_{r7}(HK)> and leg 7: $< p_{r7}$(HK), p_{r1}(BS)>.

2.1.2 Container ships

The liner shipping company deploys container ships on each ship route to transport containers. These ships are categorized into different types denoted by the set $\mathcal{V}$ according to their load capacities, sailing speeds, operating costs and other ship-specific characteristics. The type of ships to deploy on a ship route is mainly dependent on the required shipping capacity. The volume capacity of a ship with type $v \in \mathcal{V}_r$ is represented by Cap_v (TEUs) and the weight capacity is denoted by Wei_v (tons). The liner shipping company tends to use fewer large container ships rather than more small ones. The number of ships required for a ship route depends on the regular service frequency and the round-trip time of the route. Since most major liner shipping companies in practice provide weekly services, we assume that each ship route has to provide a weekly service frequency. Hence, if the round-trip time is 28 days, then 4 container ships are required to maintain a weekly service. We further represent by $\mathcal{V}_p$ the set of types of ships that port $p \in \mathcal{P}$ has enough draft to accommodate.

The round-trip time includes sailing time and port time. The sailing time depends on the sailing speed v_{ri} (knot) and the oceanic distance L_{ri} (n mile) of each leg i of each ship route r. Port time is composed of the standby time for pilotage in and out of port and the time ships spend at berth for container handling. Sometime for the ease of exposition, the sailing time on leg i of ship route r may include the standby time for pilotage out at the $(i-1)^{\text{th}}$ port of call and the standby time for pilotage in at the i^{th} port of call, and the port time is tantamount to the berth time. The berth time is a function of the number of containers handled. Therefore the round-trip time of a ship route depends on the port rotation, type of ships deployed, sailing distance, sailing speed, the pilotage efficiency at port, the number of containers handled and the container handling efficiency.

The following cost components are associated with ships in type $v \in \mathcal{V}$. (i) Fixed operating cost per week for a particular ship, including cost for crew, repair and maintenance, insurance, stores and lubricants, fuel for auxiliary power and administration, denoted by c_v^{opr} (USD/week). Note that c_v^{opr} is independent of voyages and is incurred as long as the ship is in operation. (ii) Fixed port charges when ships visit a port. (iii) Berth occupancy charges at port $p \in \mathcal{P}$, denoted by c_{pv}^{ber} (USD/hr). (iv) Bunker cost, which depends on the bunker price α^{bun} (USD/ton), sailing distance and sailing speed. (v) Canal dues (transit fees) when ships transverse e.g. Suez Canal and Panama Canal.

2.1.3 Containers

The liner shipping company provides liner services to transport containers from one port to another. Since liner services are published in advance, the liner shipping company can only design liner services based on predicted container shipment demand. The liner shipping company forecasts its container shipment demand on the basis of contracted orders, historical data and other affecting factors. Let $\mathcal{W} \subseteq \{(o,d) \mid o \in \mathcal{P}, d \in \mathcal{P}\}$ be the set of origin-destination (O-D) port pairs with container shipment demand. Let n_{od} (TEUs) represent the weekly container shipment demand for port pair $(o,d) \in \mathcal{W}$. n_{od} actually incorporates containers of many types. For example, we can consider a 40-ft container as two TEUs. If more details are to be captured, then the container shipment demand for each type of containers must be provided.

The prevailing container transshipment operations enable the liner shipping company to transport containers originating from one particular port and destined for another one even if these two ports are not on one ship route. In other words, these containers will be transshipped at an intermediate port. A large proportion of containers are transshipped during the trip from origin port to destination and transshipment is an important feature of liner shipping.

The liner shipping company predetermines a set of container routes to deliver containers between an O-D port pair $(o,d)\in\mathcal{W}$, denoted by $\mathcal{H}^{od}$, in accordance with the given set of ship routes $\mathcal{R}$. Define $\mathcal{H}=\bigcup_{(o,d)\in\mathcal{W}}\mathcal{H}^{od}$ to be the set of all container routes for all O-D port pairs. A container route $h\in\mathcal{H}^{od}$ is either a part of one particular ship route or a combination of several ship routes to deliver containers from the original port $o\in\mathcal{P}$ to the destination port $d\in\mathcal{P}$. Container transshipment operations should be involved in a container route with several ship routes. For instance, three container routes h_1, h_2 and h_3 for the liner shipping network in Figure 2-3, can be defined as follows:

$$h_1 = p_{13}(\text{SG})\xrightarrow{\text{Ship Route 1}} p_{11}(\text{HK}) \tag{2.3}$$

$$h_2 = p_{25}(\text{SG})\xrightarrow{\text{Ship Route 2}} p_{21}(\text{HK}) \tag{2.4}$$

$$h_3 = p_{22}(\text{XM})\xrightarrow{\text{Ship Route 2}} p_{24}(\text{CB}) \mapsto p_{31}(\text{CB})\xrightarrow{\text{Ship Route 3}} p_{32}(\text{CN}) \tag{2.5}$$

Container route h_1 is used to directly deliver containers from Singapore to Hong Kong which are loaded at the 3rd port of call of the ship route 1 (Singapore) and discharged at the 1st port of call of the ship route 1 (Hong Kong). Containers along the container route h_2 are delivered by ship route 2. Container route h_3 involves container transshipment operations: Containers are first loaded at the 2nd port of call of the ship route 2 (Xiamen) and delivered to the 4th port of call of the ship route 2 (Colombo). At Colombo, these containers are discharged and reloaded (transshipped) to a ship deployed on the ship route 3, and transported to the destination Chennai.

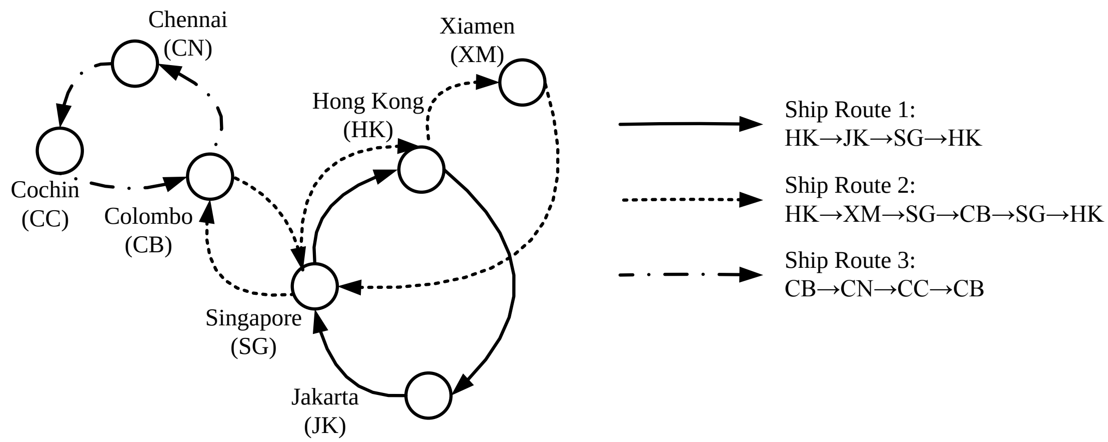


Figure 2-3 An illustrative liner shipping network

2.2 Decision Problems for Liner Shipping Companies

As shown in Figure 2-4, there are three decision-making levels for liner shipping companies: strategic, tactical, and operational (Pesenti, 1995). At the strategic level, a liner shipping company makes long-term decisions such as ship fleet size and mix, targeted service areas (e.g., intra-Asia, Asia-Europe, etc.), and alliance strategies. Tactical-level decisions are made every three to six months in view of the changed container shipment demand. A liner shipping company needs to identify the sequence of ports of call, deploy ships on the itineraries, determine the sailing speed of ships, and design schedules. At the operational level, a liner shipping company determines whether to accept or reject a certain cargo, how to route the accepted cargo, and how to re-route or re-schedule ships to cope with unexpected incidents such as adverse weather and sea conditions or port congestion. There is interplay between decisions made at the three different planning levels. For example, fleet size and mix are necessary inputs for ship fleet deployment, and cargo routing is subject to the shipping services.

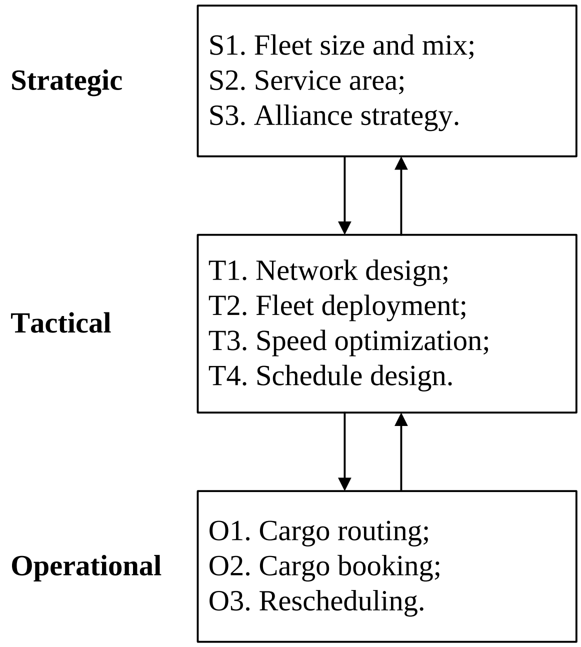


Figure 2-4 Three levels of decision-making for liner shipping companies

Strategic planning problems are mostly business issues, and they cannot be easily addressed simply through operations research methods because of the high levels of uncertainty in the future shipping market. Still, there are a few studies on the fleet size and mix problem (Xinlian et al., 2000; Meng and Wang, 2011) and alliance strategies (Song and Panayides, 2002; Agarwal and Ergun, 2010). Some studies describing tactical problems also consider ship fleet composition by having a variable fleet; see, for example, Cho and Perakis (1996), Fagerholt (1999), and Alvarez (2009). For a general review of fleet composition, see Hoff et al. (2010). Tactical decisions are very important for liner shipping companies because once the liner shipping services are established, most of the operating costs (ship costs, port charges, bunker costs) are also determined. By contrast, the operational-level plans do not impact total cost as much as tactical decisions. Moreover, operational planning problems are less organized and hence less amenable to mathematical representations. As a result, there is not much literature on operational-level planning problems.

This study focuses on tactical-level decisions faced by liner shipping companies. Since most global liner shipping companies provide a weekly service frequency, the service frequency is not a decision problem in most studies; for exceptions, see for example, Bendall and Stent (2001) and Alvarez (2009). Of course, to provide better services, especially for connection with mainline services, some feeder services are twice or thrice weekly. Four major decision issues at the tactical planning level are listed in Table 2-1. It should be mentioned that the sailing speed determination problem may also happen at the operational level by setting appropriate speeds considering weather and current conditions. Port rotation is a major input of fleet deployment, sailing speed determination, and schedule design, and a major output of network design.

Table 2-1 Tactical-level decision issues

Problem	Major input	Major output
Fleet deployment	• Port rotation	• Type and number of ships deployed on each ship route
Sailing speed determination	• Port rotation • Type of ships to deploy on each route	• Number of ships deployed on each route • Sailing speed on each leg
Schedule design	• Port rotation • Type of ships to deploy on each route • Maximum port-to-port transit time	• Arrival day at each port of call on each route
Network design	• Container shipment demand • Ship fleet • Inland transportation system	• Liner shipping service routes with deployed ships

2.2.1 Fleet deployment

A liner shipping company deploys container ships on each ship route to transport containers. This decision problem is termed as the fleet deployment (FD) problem. The liner shipping company makes fleet deployment decisions every three to six months in order to cope with the seasonal variation of the container shipment demand. Since the daily operating cost of a containership is usually as high as tens of thousands of dollars, fleet deployment decisions determine a large proportion of the total cost of a liner shipping company. Consequently, to maximize its profit, a liner shipping company needs to assess the trade-off between the cost and capacity of ships.

Not all the ships can be deployed on a specific ship route $r \in \mathcal{R}$ because of those physical constraints imposed on the ship route such as limited port draft, and we denote by $\mathcal{V}_r \subseteq \mathcal{V}$ the set of ship types available for ship route r. It is reasonable as well as practical to assume that the string of ships deployed on a specific ship route is of the same ship type. For one reason, it is difficult for ships with different sailing speeds to keep a constant and stable service frequency. For another reason, the operational homogeneity could be decreased if ships of different capacities are deployed.

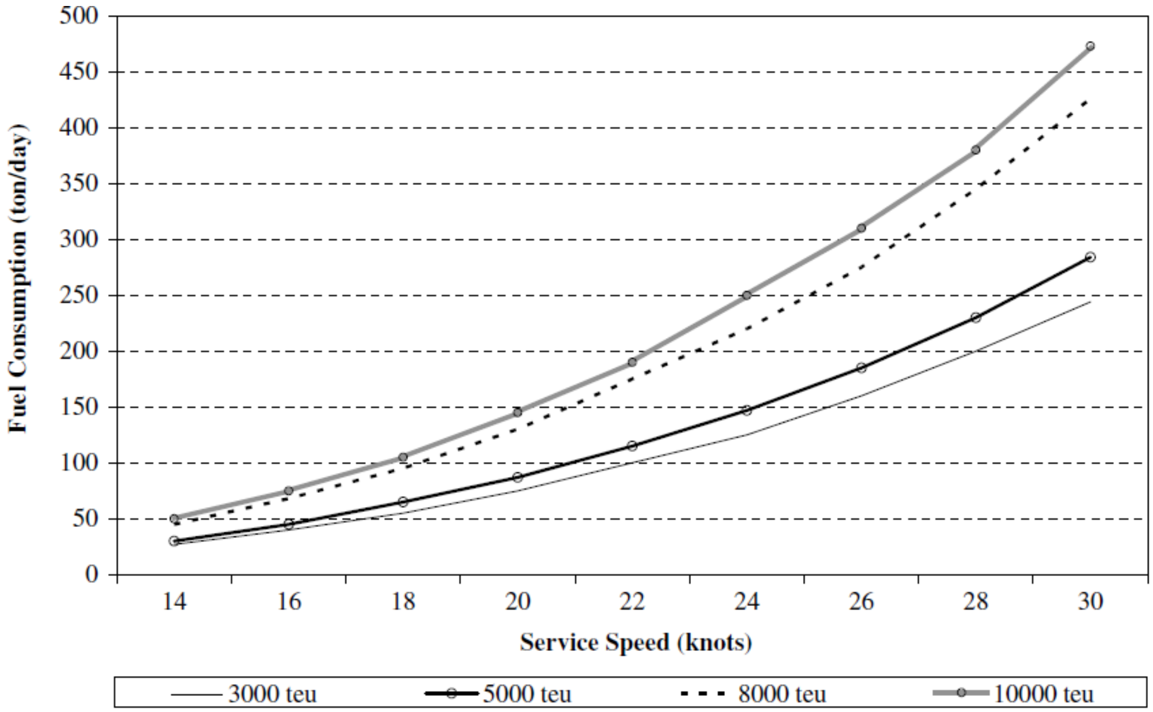
The number of ships required for a ship route depends on the regular service frequency and the round-trip time of the route. The term "service frequency" is often used to denote the time in terms of days between sailings from a port of call on a given route. Most literature imposes a weekly service frequency because global liner shipping companies normally operate weekly services. We believe that the dominance of weekly services cannot be changed within a short time for three reasons. First, a weekly service is a trade-off between the shipper's needs for frequent services in order to reduce the inventories, and carrier's needs for cargo accumulation so as to utilize large container ships for economies of scale. Second, a weekly service is easy for consigners and consignees to arrange their plans for production and/or sales. Third, since most liner shipping companies operate weekly services, container terminal operators also allocate the berth time on a weekly basis. If a ship route has a 10-day service frequency, for example, then ships on the route arrive at a certain port at different time in each week and the container terminal operator would have difficulty in berth allocation. Some feeder services do not operate on a regular basis since the container shipment demand is very low and has a large fluctuation. These feeder services do not follow the weekly convention and their scheduling problems mostly occur at the operational level. We require weekly services for all liner routes in this study. The round-trip time depends on the round-trip distance, the sailing speed, and the port time. If the round-trip time is 28 days, then 4 container ships should be deployed to maintain the weekly service.

The type of ships to deploy on a liner service route is mainly dependent on the required shipping capacity. Nevertheless, this problem is not straightforward since containers can be transshipped at any port of call during the trip from their origin port to their destination. In sum, transshipment brings a great challenge for making the optimal fleet deployment decisions. At the same time, transshipment offers opportunities for maximizing fleet utilization and increased load factor for large ships.

2.2.2 Sailing speed optimization

The sailing speed (SS) of container ships has a significant impact on the total operating cost. On one hand, an increase in sailing speed with just a few knots already results in a dramatic increase of bunker consumption (Notteboom and Vernimmen, 2009), see Figure 2-5.

On the other hand, the bunker cost accounts for a large proportion of the total operating cost, e.g., 20% to 60% according to Ronen (1993).



Source: Notteboom and Vernimmen (2009)

Figure 2-5 Daily fuel consumption for four types of container ships at different speeds

Liner shipping companies generally sail ships at the normal service speed. Bunker consumption can be reduced with a slow sailing speed, whereas more container ships may be needed to provide the same shipping capacity. As a result, liner shipping companies choose a slow speed in order to save the bunker cost under two circumstances: (i) when the bunker price is extremely high, or (ii) when there is a large excess shipping capacity. For example, both the Grand Alliance and CMA CGM each decided to add a ninth ship to one of their respective Asia-Europe routes during the summer of 2006 to cope with the high bunker price. The resulting fuel cost savings generated by each of the other eight vessels more than compensated for the cost of hiring and operating the ninth vessel (Vernimmen et al., 2007).

Another example is that in 2009, to deal with the decreased container shipment demand and the large container ship fleet, liner shipping companies took measures including slow or super slow steaming (at half speed of around 13 knots) in an attempt to curb shipping capacity and thus boost the freight rate (UNCTAD, 2010). The major obstacle to design an optimal sailing speed is the nonlinear relationship between service speed and bunker consumption, which poses difficulties for designing a proper solution algorithm.

2.2.3 Schedule design

Transit time from the origin port to the destination port is an important service factor in liner shipping because shippers demand fast services to reduce their inventories. Offering short transit time is a competitive factor, in particular when the goods involved are time sensitive (Notteboom, 2006). However, the benefit of shorter transit time to a liner shipping company is difficult to quantify since the customers to serve are from various industries.

The schedule of a liner service route is the arrival date at each port of call on the route. The schedules of services in the liner shipping network determine the transit time from the origin port to the destination port. When containers are delivered from the origin port to the destination port without transshipment, the port-to-port transit time is basically dependent on the intermediate ports of call and the sailing speed. When containers are transshipped, their O-D transit time not only depends on the time onboard ships (port-to-port sailing time), but also the connection time at the transshipment port(s). The connection time is the wait time depending on the schedules of the two connecting ship routes. For instance, the connection time is one day if the incoming ship arrives on Monday and the outgoing ship arrives on Tuesday, and the connection time is six days in the converse case due to the weekly service frequency.

Therefore, the schedule design problem comes when the liner shipping company aims to improve its services by providing short transit time, either because other companies are providing shorter transit time than it does, or because it seeks to outperform its competitors. The company will define a maximum allowable transit time for each port pair and thereby design the schedules of the service routes to satisfy the transit time constraint. Apparently the sailing speed optimization is part of the schedule design issue.

The sailing speeds of ships may be adjusted in practice as a hedge against a number of uncertain factors that liner shipping services are subject to, in order to maintain the integrity of ship route schedules. The uncertainties can be classified into two categories: uncertainty at sea (adverse weather conditions such as rain, snow, winds, low visibility, tornado, hurricane, and thunderstorm and sea conditions including currents and tides) and uncertainty at port (lack of navigation experience of the ship master; insufficient berth planning system; fluctuation of quay crane handling efficiency; and variation of the number of containers handled in each week). To cope with the uncertainty at sea, liner shipping companies build some buffer time (sea contingency) for each voyage leg. A ship sails at a higher speed at the beginning of the voyage on a leg to ensure that it has enough time to hedge against possible adverse weather and sea conditions it may encounter. The ship can slow down when approaching the destination port because a shorter residual voyage distance means less possibility of adverse weather and sea conditions. The uncertainty at port affects the port time and thereby the available sailing time for the subsequent voyage leg. As a consequence, the sailing speed function for the subsequent voyage leg has to be adjusted accordingly. Thus, sea contingency time and uncertain port time must be taken into consideration in the schedule design. To design the schedules with consideration of the uncertain environment under which ships are operating is no easy task, whereas this topic is worth research efforts.

2.2.4 Liner shipping network design

In the aforementioned three issues of fleet deployment, sailing speed optimization, and schedule design, the port rotations of the ship routes are already known. Liner shipping companies may need to design new ship routes or altering existing ship routes in order to optimize its shipping operations. As a result of the regular shipping services, a large proportion of the total operating cost is determined at the shipping network design stage. Therefore, it is important for a liner shipping company to design a cost-effective intermodal liner shipping service network.

A liner shipping company has to redesign its service network whenever there are significant changes in container shipment demand. It first examines the current network and then determines whether to introduce a new service, or change portcall sequence of an existing service, or remove a portcall, or add a portcall, or deploy a larger/smaller type of ship, or make other alterations. A liner shipping company cannot change its whole network overnight because ships are located all over the world and the liner shipping company tries to provide consistent services.

A liner shipping company must deliver containers to the destination port within a certain time because longer transit time of containers means higher inventory cost and depreciation. Shippers are generally not concerned about the intermediate transfer points as long as containers can be delivered to the destination on the designated date. Thus, the shipping network has to be designed such that the requirement of origin-to-destination transit times of containers is satisfied.

When designing the liner services, a liner shipping company must bear in mind that there are many types of containers, such as dry TEU (twenty-foot equivalent unit), dry FEU (forty-foot equivalent unit), reefer TEU, and reefer FEU. Different types of containers are different in volume and port handling cost. Besides laden containers, a liner shipping company has to reposition its empty containers due to the imbalance in international trade. Take the trans-Pacific trade lane for example: container flow from Asia to North America is estimated at 15.4 million TEUs in 2007, while in the opposite westbound direction, the flow is only 4.9 million TEUs (UNCTAD, 2008). This imbalance leads to the empty container accumulation in import-dominant areas (North America) and shortage in export-dominant ones (Asia). Hence, empty containers have to be repositioned from the former to the latter. Empty containers are different from laden containers in that (i) they are not so time-sensitive, and (ii) they do not have fixed destinations. The flexibility in the choice of destinations for empty containers facilitates the repositioning operations while posing challenges for shipping service network design.

It is clear that the design of liner shipping service network may nest fleet deployment, sailing speed optimization, and schedule design as special cases. Hence, the network design problem is a even more challenging task.

2.3 Summary

A liner shipping network consists of ports, ships, and containers. Ships are deployed on fixed port rotations at a certain speed following published schedules to transport containers from their origin port to their destination port. Under different circumstances, a liner shipping company needs to make different decisions associated with its liner shipping network. These decisions determine, to a large extent, the services (shipping capacity, transit time, and schedule integrity) and the operating costs (fixed ship operating cost, bunker, fixed port calling fees, berth occupancy charges, canal dues, and container handling charges). Consequently, an in-depth analysis, modeling and optimization of these issues has practical significances for a liner shipping company.

CHAPTER 3. LITERATURE REVIEW

This chapter first reviews the studies on fleet deployment, sailing speed optimization, schedule design, and liner shipping network design. These problems are interrelated and some studies address more than one topic and consequently are discussed in more than one subsection. The limitations of the existing studies are outlined. Finally, the research objectives of this thesis are presented.

3.1 Literature search methods and summary

We use a computerized literature search approach to find all the relevant studies. First the databases of Scopus, the Sciences Citation Index and Google Scholar were searched with the following key words: “liner”, “container”, “shipping”, “(maritime or sea or waterway) and (transportation or transport)”, and “(ship or vessel) and (routing or schedule or scheduling)”. We also looked at the personal websites of researchers active within maritime transportation and reviewed our own research. We also retrieved studies by tracking the references cited in papers we had already found. We identified 41 papers using operations research methods to examine liner shipping management and operations. Table 3-2 lists these papers and the country/region of the affiliation of the corresponding authors, the problems solved and main concerns, and the model and solution approach.

Researchers from Singapore, Denmark, and Norway are the most active in investigating liner shipping management and operations using operations research methods (Figure 3-1), most likely because the maritime industry plays such an important role in the national economy of these three countries. Although researchers from the United States have a slightly larger number of publications, there are also many more universities and research institutes in the United States. It is interesting to note that all of the 41 publications originate from a coastal country/region. In addition, there are a number of descriptive and qualitative studies on liner shipping that we did not include in our review because they are not the focus of this study. Considerable research efforts have been devoted solely to the repositioning of empty containers, but they also are not included in our review. Although the number of studies on mathematical modeling of liner shipping operations is small, this research topic has begun to draw more academic attention in recent years. In particular, the number of publications has increased considerably since 2006 (Figure 3-2), most likely because many leading liner shipping companies have recently started to collaborate with the research community to develop better decision support systems and the increasing volume of containerized trade resulting from an increasingly globalized economy.

Table 3-1 Summary of literature on liner shipping routing and scheduling

Paper and country/region		Problem and major considerations	Approach
Álvarez (2009)	NO	Shipping network design; transshipment; heterogeneous fleet; container routing; speed determination	Column generation
Álvarez (2012)	NO	Dwell time at transshipment ports	Analytical
Agarwal and Ergun (2008)	US	Shipping network design; transshipment; heterogeneous fleet; container routing	Space time network; Benders decomposition
Agarwal and Ergun (2010)	US	Alliance strategy	Game theory
Bell et al. (2011)	UK	Container routing; random ship arrivals	LP
Bendall and Stent (2001)	AU	Flexible demand	MIP
Brouer et al. (2011)	DK	Container routing; empty containers	LP
Cho and Perakis (1996)	US	Fleet deployment; container routing	MIP
Corbett et al. (2010)	US	Speed optimization; green shipping	Analytical
Du et al. (2011)	CN	Speed optimization	Second-order cone programming
Fagerholt (1999)	NO	Feeder network design; homogeneous fleet	MIP
Fagerholt (2004)	NO	Feeder network design; heterogeneous fleet	MIP
Fagerholt et al. (2009)	NO	Fleet deployment	Multi-start local search heuristic
Fagerholt and Lindstad (2000)	NO	Feeder network design	IP
Gelareh and Meng (2010)	SG	Fleet deployment; speed optimization; transit time	MIP

Gelareh et al. (2010)	DK	Liner hub-and-spoke network design	Lagrangian relaxation
Gelareh and Pisinger (2011)	DK	Liner hub-and-spoke network design; main ship route design	Benders decomposition
Golias et al. (2010)	US	Speed optimization	GA
Halvorsen-Weare and Fagerholt (2011)	NO	Robust feeder network design	IP
Jaramillo and Perakis (1991)	US	Fleet deployment	LP
Jepsen et al. (2011)	DK	Liner shipping network design, aggregated demands, green logistics	Column generation
Karlaftis et al. (2009)	GR	Feeder network design; heterogeneous fleet; pickup and delivery; time window	Hybrid GA
Løfstedt (2010)	DK	Liner shipping network design	Construction heuristic
Løfstedt et al. (2010)	DK	Liner shipping network design	MIP
Meng and Wang (2010)	SG	Fleet deployment; uncertain container shipment demand; level of service	MIP
Meng and Wang (2011)	SG	Fleet planning; fleet deployment	Dynamic programming; MIP
Mourão et al. (2001)	PT	Fleet deployment; hub-and-spoke network; transshipment; weekly service frequency; inventory cost	MIP
Perakis and Jaramillo (1991)	US	Fleet deployment	LP
Powell and Perakis (1997)	US	Fleet deployment	IP
Rana and Vickson (1988)	CA	Single liner route design; fixed port calling sequence; single ship	Lagrangian relaxation, decomposition
Rana and Vickson (1991)	CA	Multiple liner route design; fixed port calling sequence; heterogeneous fleet; container routing	Lagrangian relaxation, decomposition
Reinhardt and Pisinger (2012)	DK	Container routing; transshipment	B&C
Ronen (2011)	US	Sailing speed	Analytical
Sambracos et al. (2004)	GR	Feeder network design; homogeneous fleet	Meta-heuristic
Shintani et al. (2007)	JP	Single liner route design; empty containers	GA
Song and Panayides (2002)	UK	Alliance strategy	Game theory
Ting and Tzeng (2003)	TW	Ship scheduling	Dynamic programming
Wu et al. (2009)	CN	Time reliability	Simulation
Xinlian et al. (2000)	CN	Fleet planning	Dynamic programming
Yan et al. (2009)	TW	Schedule design; container routing; transshipment	Space time network; Lagrangian relaxation
Yao et al. (2012)	SG	Speed optimization	MIP

Note: Countries SG: Singapore; DK: Denmark; NO: Norway; CN: Mainland China; GR: Greece; PT: Portugal; JP: Japan; TW: Taiwan; AU: Australia; CA: Canada. Methods MIP: mixed-integer linear programming; B&B: branch-and-bound; B&C: branch-and-cut; LP: linear programming; IP: integer linear programming; GA: genetic algorithm;

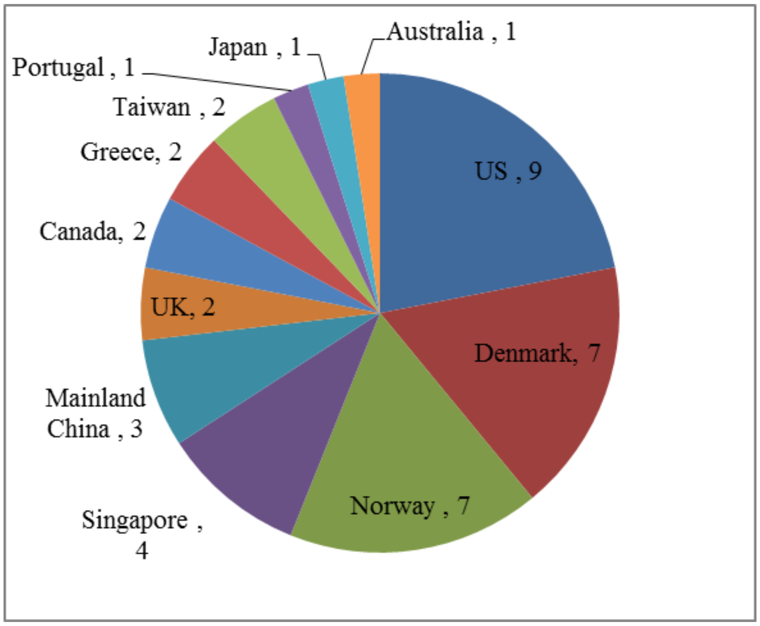


Figure 3-1 Classification of publications on mathematical modeling of liner shipping by country/region

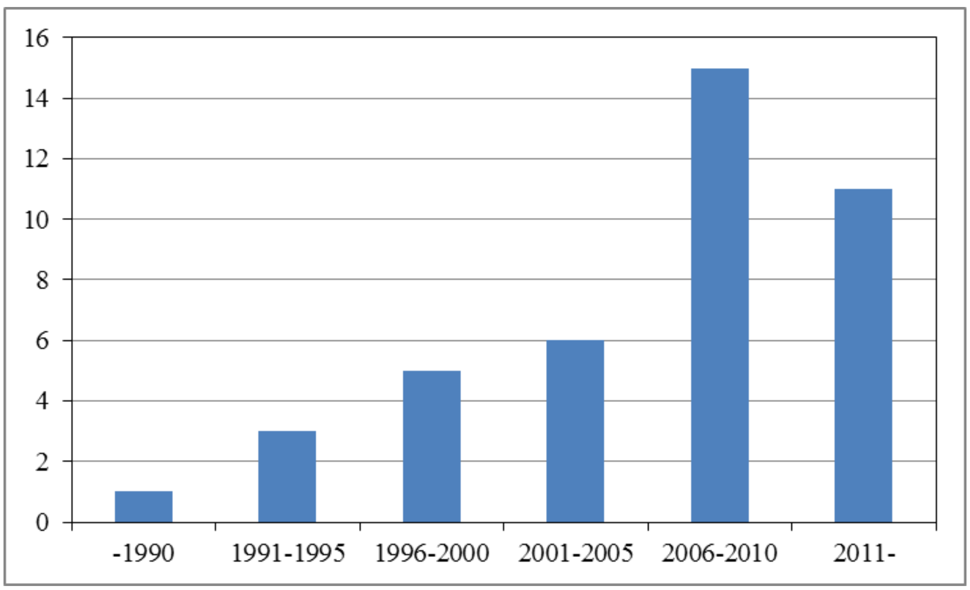


Figure 3-2 Number of publications on mathematical modeling of liner shipping by year

3.2 Fleet Deployment

Fleet deployment (FD) seeks to assign ships to liner service routes to maximize profit or minimize cost. Port rotation is one of the prime inputs for liner ship fleet deployment. A number of pure or mixed-integer linear programming models for the FD problem have been developed to account for various restrictions arising in liner shipping operations, as shown in Table 3-2.

Table 3-2 Literature on liner ship fleet deployment

Paper	Problem and major considerations			Method
	Flexible container routing	Allow transshipment at some ports	Allow transshipment at any port	
Perakis and Jaramillo (1991) and Jaramillo and Perakis (1991)				LP
Powell and Perakis (1997)				IP
Cho and Perakis (1996)	√			MIP
Gelareh and Meng (2010)				MIP
Meng and Wang (2010)				MIP
Mourão et al. (2001)		√		MIP
Fagerholt et al. (2009)				Multi-start local search heuristic

In a pioneering modeling work on FD, Perakis and Jaramillo (1991) and Jaramillo and Perakis (1991) built a linear programming model incorporating ship capacity constraints, minimum service frequency requirements, and ship chartering issues. The objective of this linear programming model is to minimize the total fleet's operating costs, including fuel consumption costs, daily operating costs, port charges, and canal fees. It implicitly and unrealistically assumes that the number of ships allocated to a service route is a continuous rather than an integer decision variable. To remedy this unrealistic assumption, Powell and Perakis (1997) presented an integer linear programming model. These three studies all assume a service route based port-to-port shipment demand pattern; the number of containers between a pair of ports on each service route is known a priori. To relax this assumption, Cho and Perakis (1996) formulated a mixed-integer linear programming model for the FD problem, in which the container shipment demand between two specific ports can be served by any service route passing through both ports. Because the sailing speed of ships has direct implications on bunker consumption, Gelareh and Meng (2010) developed a mixed-integer linear programming model for the FD problem in which the sailing speed of a ship is a decision variable. Unlike the aforementioned models with deterministic container shipment demand, Meng and Wang (2010) developed a chance constrained programming model for the FD problem with uncertain container shipment demand. They assumed that a certain level of service for each route has to be maintained. The level of service is defined as the probability that all container shipment demand on the service route can be fulfilled. The chance constraints can be transformed into equivalent deterministic linear constraints.

In most of the above models, containers must be delivered from their origin port to their destination port by direct services, and transshipment is not allowed. In the literature that takes into account container transshipment operations, Mourão et al. (2001) proposed a simple model for a specific FD problem defined on a small hub-and-spoke network consisting of two routes—a feed route and a main route—and one pair of ports by assuming that all containers must be transshipped at the hub port in the feeder route.

In addition, Fagerholt et al. (2009) developed an FD model that requires ships to fulfill a given number of voyages without explicitly considering the container flow. They further integrated the model into a decision support system. Compared with the network design problem, FD is easier since the number of ship types is limited and not all ship types are compatible with each port rotation due to commercial and physical restrictions. Even though the fleet deployment topic has been extensively investigated, modeling and solving real size problems, especially richer versions of the problem, remain a challenge.

3.3 Sailing Speed Optimization

The sailing speed of ships has a significant impact on the total operating cost because an increase of just a couple of knots results in a dramatic increase in bunker consumption (Notteboom and Vernimmen, 2009), and bunker costs account for a large proportion of the total operating cost, for example, 20–60 percent according to Ronen (1993). Higher speed means shorter transit time and fewer ships required to maintain weekly services, but it also mean higher bunker consumption. Sailing speed is an important decision for all levels of decision making. At the strategic level, there is a tradeoff between fleet composition and the speed of the ships; fewer ships means that each ship must sail faster. When there is flexibility in the duration of the trades that are serviced, speed decisions become important in the fleet deployment phase. At the operational level, weather and currents greatly influence speed. We, however, review the optimal sailing speed problem at the tactical level, which is challenging because of the nonlinear relation between sailing speed and bunker consumption. Most studies have assumed that ships sail at a predetermined speed. Some researchers (Perakis and Jaramillo, 1991; Corbett et al., 2010; Meng and Wang, 2010; Ronen, 2011) have derived the optimal sailing speed by assuming that the sailing speed is constant during the voyage and considering one ship route. To optimize the sailing speed in a more general setting, different approaches have been used. The first approach bypasses the nonlinearity by assuming that bunker consumption varies linearly with sailing speed (Lang and Veenstra, 2010). This approach is a good approximation only when the possible speed range is very narrow. The second approach is to use heuristic methods, such as the genetic algorithm used in Golias et al. (2010), which cannot guarantee optimality. In the third approach presented by Gelareh and Meng (2010) and Yao et al. (2012), the sailing speed is discretized into many small intervals and additional binary decision variables are introduced to indicate the adopted sailing speed interval. Nevertheless, the addition of these binary decision variables significantly increases the computational burden. Du et al. (2011) proposed a fourth approach by exploiting the property of the power relation between sailing speed and bunker consumption. They transformed the constraints with power functions to second-order cone programming (SOCP) constraints and took advantage of state-of-art solvers to solve the SOCP problem. This exact algorithm is efficient when the power of speed in the sailing speed–bunker consumption function takes specific values, such as 3.5, 4.0, or 4.5. When the power takes other values, for example, 3.31, each power function constraint has to be represented by a substantial number of SOCP constraints and the problem can no longer be solved efficiently.

Sailing speed optimization is related to a number of other decision problems, for example, network design (e.g., Alvarez, 2009), fleet deployment (e.g., Gelareh and Meng, 2010), and schedule design, which is described in the next subsection. Due to increased environmental awareness, CO_2 emissions, which are related to bunker consumption, should also be formulated in optimization models. The determination of optimal speed with consideration of CO_2 emissions is an interesting research topic.

3.4 Schedule Design

Transit time (on a port-to-port basis or on a door-to-door basis) is an important service factor in liner shipping because shippers demand fast service to reduce their inventories. Offering short transit time is a competitive factor, particularly when the goods involved are time sensitive (Notteboom, 2006). When containers are delivered from the load port to the discharge port without transshipment, the port-to-port transit time is basically only dependent on the intermediate port calls and the sailing speed. Karlaftis et al. (2009), Gelareh and Meng (2010) considered the transit time constraint for direct deliveries without transshipment. Bell et al. (2011) formulated a container routing model to minimize the total transit time of containers by assuming that ships arrive at ports randomly. In the aforementioned model by Mourão et al. (2001), the feeder route has two possible schedules: Tuesday and Thursday departures from the hub. The two schedules are examined on the basis of inventory cost. Ting and Tzeng (2003) proposed a dynamic programming model for scheduling decisions under time window restrictions. Yan et al. (2009) investigated the schedules at the operational level in a space-time network. Alvarez (2012) formulated the connection time at a transshipment port for regular shipping services.

Schedule design for liner service routes may not have attracted much attention because liner schedules are subject to a number of uncertain factors. For example, port congestion, weather conditions or mechanical problems at sea, delay when transiting canals, unexpected wait time at bunkering sites, and cascading effects from previous ports of call. It would be worthwhile for future research efforts to address these uncertainties.

To summarize, schedule design for liner service routes has not attracted much attention. There are two explanations for this. First, the maximum wait time at transshipment port is one week with weekly serviced liner routes. Second, a carefully designed schedule may be hampered by possible delay at port and at sea (e.g., port congestion, adverse weather). Despite these two reasons, an optimized schedule is still meaningful because the inventory cost of containerized cargo for one week cannot be neglected (0.5 per cent the value of a container per day according to Bakshi and Gans, 2010) and liner shipping companies may try to keep the schedule integrity by steaming fast in case of delay. All the prior literature with consideration of schedules assumes that ships sail at a constant speed on each voyage leg and that the port time is deterministic (may depend on the number of containers handled). The designed schedules under such assumption may easily be undermined by the uncertain and uncontrollable external factors. To design the schedules with consideration of the uncertain environment under which ships are operating is no easy task, whereas this topic is worth research efforts.

3.5 Liner Shipping Service Network Design

The aim of liner shipping network design is to determine which ports to visit and in what order they should be visited. The network design problem is associated with the other tactical-level problems and thus cannot be investigated in isolation. Most existing literature is devoted to itinerary design and ship deployment by assuming a fixed sailing speed and weekly service frequency, and does not consider schedules.

The liner shipping network design problem is NP-hard (Agarwal and Ergun, 2008), and we cannot expect to find a polynomial-time algorithm that obtains the optimal solution for a general liner shipping network design problem. Research on liner shipping network design can be classified into four categories, and illustrative networks for these four categories are shown in Figure 3-3. The first category examines the feeder shipping network design problem, which consists of a hub port and many feeder ports, as shown in Figure 3-3 (a). Containers either originate from or are destined for the hub port and transshipment is excluded within the feeder network. Fagerholt (1999) contributed a pioneering study in this category. He proposed a set-partitioning model by enumerating all possible shipping service routes and combining these single shipping service routes into multiple shipping service routes if possible. This model relies on the assumption that all ships have the same sailing speed. Fagerholt (2004) extended the set-partitioning model to address a heterogeneous ship fleet with a given cost structure, capacity, and in particular, sailing speed, for each type of ship. Results on 40 ports and 20 ships are reported. Fagerholt and Lindstad (2000) formulated an integer programming model to optimize the service of offshore installations from an onshore depot. Unlike Fagerholt (1999, 2004) where each feeder port is serviced once each week, this model requires that each offshore installation be serviced at least a minimum number of times. Halvorsen-Weare and Fagerholt (2011) extended the model of Fagerholt and Lindstad (2000) by taking into consideration the impact of weather at sea. Note that the above four models on liner shipping are not dedicated to the transportation of containerized cargo, but the models and algorithms are applicable to container liner shipping. Sambracos et al. (2004) carried out a case study on the feeder ship route design to dispatch small containers in the Aegean Sea, from one depot port (Piraeus) to 12 other ports (islands). They assumed a homogeneous fleet to meet container shipment demand with minimum operating costs, including fuel consumption and port charges. They used a list-based threshold acceptance meta-heuristic method. Results show that at least a 5.1 percent cost savings may be realized over current shipping practices. This problem was later generalized by Karlaftis et al. (2009) to account for container pickup and delivery operations as well as time deadlines. They formulated this extended problem as a vehicle routing problem with pickup and delivery and time windows and used a hybrid genetic algorithm to solve the problem.

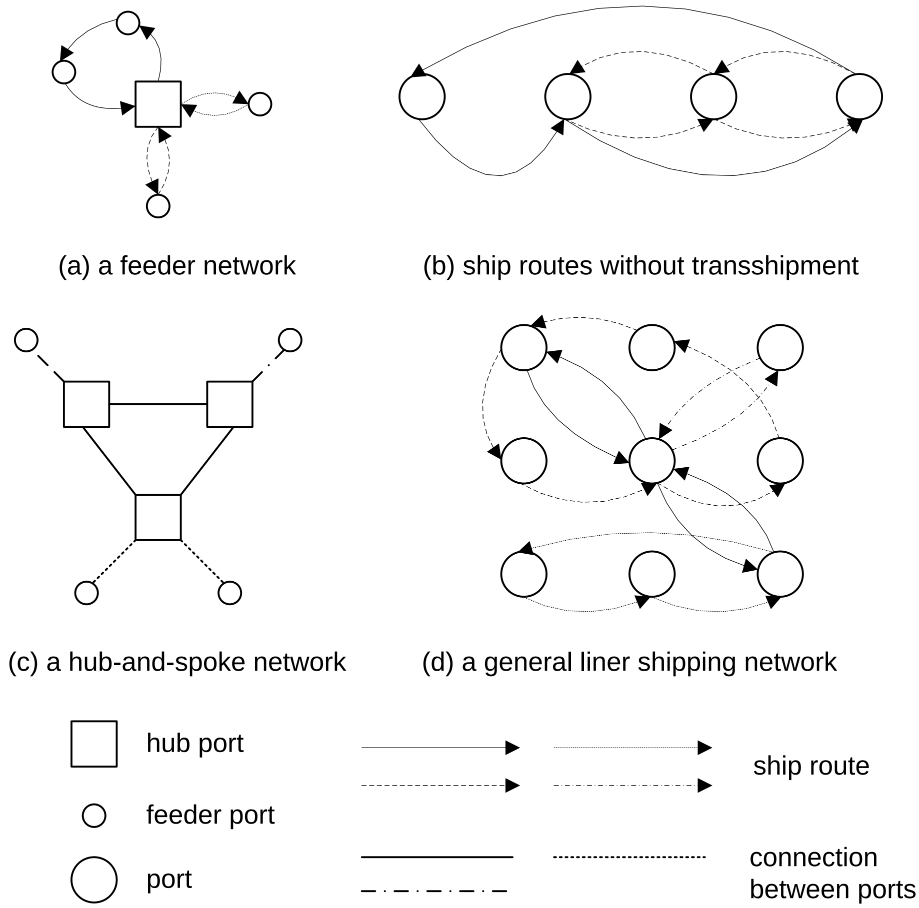


Figure 3-3 Liner shipping service network design categories

The second category aims to design one or a few liner service routes without container transshipment operations, as shown in Figure 3-3 (b). In this category of research, Rana and Vickson (1988) contributed a seminal work by building a mixed-integer linear programming model for a single ship route design problem. Rana and Vickson (1991) later extended this model to design multiple ship routes. They employed Lagrangian relaxation to solve the mixed-integer linear programming model. Both models assume that the port calling sequence by a ship is predetermined. Shintani et al. (2007) relaxed the assumption of port calling precedence relations and also considered empty container repositioning to design a single ship route. They employed a genetic algorithm to solve the problem.

The third group of studies seeks to design a hub-and-spoke liner shipping network similar to airline and telecommunication systems, as shown in Figure 3-3 (c). Gelareh et al. (2010) developed a mixed-integer programming formulation for the liner shipping hub-and-spoke network design problem. Based on this work, Gelareh and Pisinger (2011) further designed a main liner ship route connecting the hubs.

The fourth line of research investigates the general liner shipping network design problem, which usually involves more ports in the network and allows for container transshipment operations, as shown in Figure 3-3 (d). Agarwal and Ergun (2008) proposed a multi-commodity-based space-time network model for the liner shipping service network design problem with cargo routing. This model covers a heterogeneous fleet, a weekly service frequency, multiple ship routes, and cargo transshipment operations, but transshipment cost is not considered in the network design stage. Results on 20 ports and 100 ships are reported. Alvarez (2009) formulated the transshipment cost in network design and introduced the notion of "run", defined as a combination of vessel type, speed, and ports of call, to facilitate model formulation. He also applied a column generation-based heuristic to design the service network with 120 ports and 5 types of ships. Løfstedt (2010) and Løfstedt et al. (2010) developed constructive heuristic methods to design liner shipping service networks. Reinhardt and Pisinger (2012) presented a model that allows a port to be visited twice in a route, stating that such butterfly routes are common in real-world situations. Their model also incorporates transshipment costs and route dependent capacities. An exact branch-and-cut algorithm is developed to solve instances with up to 15 ports.

Obtaining data on liner shipping is not a trivial matter, and the various authors have used different sources or artificial data for their test cases. To make comparisons of future solution methods for the liner shipping network design problem easier, Løfstedt et al. (2010) proposed a general model and a suite of benchmark instances based on real-world data. The model is similar to Alvarez's (2009) model and is used to clearly define the problem and to compare various solution methods.

Because of their inherent complexity, most liner shipping network design problems cannot be solved to optimality. Also, there is no report on the comparison between the designed network and the existing one using real cases for general network design problems. One possible reason is the high sensitivity of commercial data for liner shipping companies. Still, some companies, such as COSCO, OOCL, and Maersk Line, have started to work with academia to improve their shipping network designs.

3.6 Limitations of the Existing Literature

Despite the above mentioned advancements in the research on liner shipping network, there are still a number of practically significant issues that have seldom been touched. In this section we examine these issues and suggest some future research directions.

(a) The container transshipment operation is not fully addressed.

Transshipment is a unique feature of liner shipping. As a comparison, in tramp shipping the cargo volume between each port pair is very large, and hence cargo consolidation at hubs is unnecessary. Transshipment arises as a result of the improved handling operations at ports and the requirement for deploying large containerships. In spite of the importance of transshipment in liner shipping, not enough efforts have been devoted to modelling the transshipment operations. The relay cost, container handling time, and wait time at transshipment port have attracted the least attention. Apart from the special problem characteristics (e.g., feeder network) in some studies, the most important reason that transshipment is not extensively investigated is the modelling difficulty. Transshipment makes the liner shipping problems much more difficult than similar transportation problems, e.g., vehicle routing problems (VRPs).

(b) Optimization of sailing speed is not well investigated.

Notwithstanding the importance of the sailing speed to the operating cost, existing models are either too simplistic or inefficient to address realistic speed optimization problems. The nonlinear speed-bunker consumption relation is the major challenge for modelling and algorithm design. It should be mentioned that nearly all problems with liner ship routing and scheduling have integer decision variables. Hence it may be difficult to apply the traditional nonlinear programming method (e.g., Frank-Wolfe method) directly to address the sailing speed optimization problem. In order to attack this problem, one might use linear formulations to approximate the nonlinear speed-bunker consumption relation and take advantage of the progress of state-of-art MIP solvers.

(c) Very few studies have considered liner service schedules with O-D transit time constraints.

There are studies on the port-to-port transit time without transshipment. However, the port-to-port time with transshipment is not well established. Both the port-to-port transit time and the connection time at transshipment ports are determined by the schedules. Hence the transit time issue is in fact the design of liner service schedules. To investigate the service schedule and thereby the transit time issue, the sailing speed problem cannot be circumvented. Moreover, to build a robust liner service schedule, uncertainty at sea and uncertainty at port must be accounted for. This further complicates the schedule design problem.

(d) Many practical issues are missing in the existing liner shipping network design models.

Liner shipping has many practical and important operations and features, including multi-type containers, container transshipment operations, empty container repositioning, origin-to-destination transit time constraint, consistent services with the current network, and joint services with other liner shipping companies. Existing studies usually assume that there are only one type of containers – TEU, or transform all types of containers to TEU. For example, a dry 40’ container is equivalent to two TEUs. While this is acceptable in terms of the ship slot used, it is no longer true in terms of the handling cost because the handling cost of a dry 40’ container should be less than two times the handling cost of a dry 20’ container. Transshipment operations and empty container repositioning are not accounted for in all the research. O-D transit time constraint, consistent services with the current network, and joint services with other liner shipping companies are hardly investigated by the literature.

3.7 Commercial Software

Liner shipping companies must have some optimization software for their shipping services. For instance, one can log on the website of a liner shipping company, and search for available services from one port to another within a specified time range. The liner shipping company will provide all practical services, including combination of services with transshipment. However, such software is confidential. We have investigated a global liner shipping company, and found that it has dedicated tools for optimizing the flow of containers based on manually designed container routes. However, the design of container routes and port rotations cannot be automated. The deployment of ships, speed, and schedule are designed from experience. We are also aware of a liner shipping management tool: Round Trip Simulation System (TRIPS), developed by Institute of Shipping Economics and Logistics, Germany. Given a simple configuration of shipping service networks, TRIPS could calculate the resulting costs, which works an a decision support tool for liner shipping companies. In sum, liner shipping companies are in need of optimization-based decision support tools.

3.8 Research Objectives

The objective of this research is to examine the tactical-level decision problems associated with liner shipping networks in order to cover the gap between industrial requirement and academic research. In detail, the following four issues will be investigated:

(a) Fleet deployment with container transshipment operations;

(b) Ship speed optimization in liner shipping networks;

(c) Ship route schedule design with sea contingency time and port time uncertainty;

(d) Large-scale liner shipping network design;

We will apply operations research methods to analyze these issues. Each issue is first formulated as a mathematical optimization model. Then efficient solution algorithms based on optimization theory, liner shipping characteristics, and state-of-art computer technology, are designed. The models and algorithms are tested for real-case problems and managerial insights for the liner shipping company are provided.

CHAPTER 4. FLEET DEPLOYMENT WITH TRANSSHIPMENT

This chapter first proposes a realistic ship fleet deployment problem (FDP) arising in the liner shipping industry while considering practical operations and features, including container transshipment operations, transshipment cost, and container flow dependent port time. Second, an interesting origin-based mixed-integer linear programming model for the proposed FDP is developed. This formulation allows container transshipment operations at any port, any number of times, without explicitly defining the container transshipment variables. Experiments on the Asia-Europe-Oceania shipping network of a global liner shipping company show that more than one third (17 to 22 ports) of the total 46 ports have transshipment throughputs. Computational studies based on randomly generated large-scale shipping networks demonstrate that the proposed model can be solved efficiently by CPLEX.

4.1 Problem Statement

Given a set of ports $\mathcal{P}$, a set of weekly-serviced ship routes $\mathcal{R}$, and the container shipment demand n_{od} (TEUs), $(o,d)\in\mathcal{W}$, the liner shipping company aims to determine the type of ship and number of ships to deploy on each ship route, in order to fulfill the container shipment demand while minimizing the total cost.

4.1.1 Ship Fleet

The liner shipping company deploys both owned and chartered-in ships on its ship routes within the medium-term planning horizon. These ships are categorized into different types denoted by the set $\mathcal{V}$ according to their load capacities, sailing speeds, operating costs and other ship-specific characteristics. For a particular ship type $v\in\mathcal{V}$, let N_v^{own} be the number of owned ships by the liner shipping company, N_v^{in} be the maximum number of ships that the liner shipping company can charter in from the leasing market, and c_v^{in} (USD/week) be the price for chartering in one ship for one week. It is assumed that all the unused ships will be chartered out, and let c_v^{out} (USD/week) be the weekly profit of chartering out one ship in type v. Because of the overhead of making a ship-chartering deal, it is obvious and practical that:

$$c_v^{\text{out}} < c_v^{\text{in}}, v \in \mathcal{V} \tag{4.1}$$

4.1.2 Ship Assignment

The liner shipping company deploys ships on its ship routes within the medium-term planning horizon to fulfill the container shipment demand at minimum cost. Not all the ships can be deployed on a specific ship route $r \in \mathcal{R}$ because of those physical constraints imposed on the ship route such as limited port draft, and we denote by $\mathcal{V}_r \subseteq \mathcal{V}$ the set of ship types available for ship route r. It is reasonable as well as practical to assume that the string of ships deployed on a specific ship route is of the same ship type. For one reason, it is difficult for ships with different sailing speeds to keep a constant and stable service frequency. For another reason, the operational homogeneity could be decreased if ships of different capacities are deployed.

Assuming that the sailing speed of a ship on a leg of a particular service route $r \in \mathcal{R}$ is already specified, the sailing time of a ship with type $v \in \mathcal{V}_r$ deployed on the service route, denoted by τ_{rv}^{fix} (hours), is also determined. For the sake of presentation, τ_{rv}^{fix} also includes the standby time for pilotage in and out at all the ports of call. Port time here only includes the time ships spend at berth for container handling, and it is thus a function of the number of containers handled.

The operating cost of a ship route r deployed with m_r ships of type v can be classified into three portions: cost associated with ships, which can be calculated by the term $m_r c_v^{\text{opr}}$; cost related to the voyage (including bunker cost, canal dues, and fixed charges of calling at ports), denoted by c_{rv}^{fix} (USD/week), which depends only on the ship type v (note that since the port rotation of ship route r and sailing speed of ships of type v are known, c_{rv}^{fix} is also determined); cost charged for berth occupancy, which is based on the ship type and time spent at berth. Besides the operating cost of the ship routes, the total fleet's operating cost also includes the container handling cost.

The FDP faced by the liner shipping company can be described as follows: Given a set of liner ship routes with fixed port rotations, fixed sailing speed of ships between any two ports, and known weekly container shipment demand, determine the type and number of ships to deploy on each ship route, in order to minimize the total fleet's operating cost while allowing container transshipment operations and fulfilling the container shipment demand.

4.2 Origin-Based Mathematical Formulation

4.2.1 Origin-based decision variables

There are three types of interrelated decision variables for the proposed FD problem. The first type relate to the ships that are chartered in and chartered out, the second concerns ship fleet deployment, and the third is container flow with container transshipment operations. The first two types of decision variables are straightforward and specified as follows:

n_v^{in}: Number of chartered in ships of type $v \in \mathcal{V}$;

n_v^{out}: Number of chartered out ships of type $v \in \mathcal{V}$;

m_r: Number of ships deployed on route $r \in \mathcal{R}$;

x_{rv}: A binary variable which takes value 1 if route $r \in \mathcal{R}$ is deployed with ships of type v, $v \in \mathcal{V}_r$, and 0 otherwise;

Containers loaded and/or discharged at a particular port can be classified into three categories. The first category consists of those containers originating from this port, the second comprises those containers originating from other ports in the shipping network and destined for this particular port, and the third category is composed of those containers being transshipped at this port. To describe these three categories of containers, we differentiate them according to their origin ports, by defining the following three sets of decision variables, which we refer to as origin-based container flow variables:

$\hat{z}_{ri}^{o}$: Number of containers (TEUs/week) originating from port $o \in \mathcal{P}$ and loaded at the i^{th} port of call on route $r \in \mathcal{R}$;

$\tilde{z}_{ri}^{o}$: Number of containers (TEUs/week) originating from port $o \in \mathcal{P}$ and discharged at the i^{th} port of call on route $r \in \mathcal{R}$;

f_{ri}^{o}: Number of containers (TEUs/week) originating from port $o \in \mathcal{P}$ and stowed onboard ships sailing on the i^{th} leg of route $r \in \mathcal{R}$.

As there are two container handling operations—discharging and loading—for one container being transshipped at a particular port $p \in \mathcal{P}$, the number of transshipped containers at this port equals the difference between the total number of containers handled at the port and the number of containers originating from or destined for this port, divided by 2. Let $\mathcal{R}_p \subseteq \mathcal{R}$ be the set of ship routes containing port p. The number of containers transshipped at a particular port $p \in \mathcal{P}$ can thus be calculated by

$$\frac{\sum_{r\in\mathcal{R}_p}\sum_{i\in\mathcal{I}_{rp}}\sum_{o\in\mathcal{P}}(\hat{z}_{ri}^o+\tilde{z}_{ri}^o)-\sum_{d\in\mathcal{P}}n_{pd}-\sum_{o\in\mathcal{P}}n_{op}}{2} \tag{4.2}$$

Eq. (4.2) implies that the number of transshipped containers can be implicitly represented by the origin-based container flow variables, without explicitly introducing variables indicating the number of containers transshipped from one ship route to another.

We could alternatively represent the aforementioned container flows according to their origin and destination ports, that is, using O-D container flow variables. However, the number of O-D container flow variables is much greater than the number of origin-based container flow variables. For instance, the Asia-Europe-Oceania liner shipping service network that will be used in the subsequent case study consists of 46 ports (origin ports), as shown in Figure 2-1, and a total of 652 O-D port pairs between these 46 ports have container shipment demand. Assuming that there are 10 ship routes and each ship route has 10 legs, in contrast to a total of 3×10×10×652=195,600 O-D based container flow variables, the number of origin-based container flow variables is 3×10×10×46=13,800. Therefore, the origin-based container flow representation, without variables explicitly representing transshipment decisions, is more suitable for our model building. For the sake of presentation, let vector $\mathbf{x}$ denote all the decision variables, namely:

$$\mathbf{x}=(n_v^{\text{in}},n_v^{\text{out}},m_r,x_{rv},\hat{z}_{ri}^o,\tilde{z}_{ri}^o,f_{ri}^o \mid r\in\mathcal{R},v\in\mathcal{V},i\in\mathcal{I}_r,o\in\mathcal{P}) \tag{4.3}$$

4.2.2 Mathematical models

For a given decision vector $\mathbf{x}$, the total cost (USD/week) for the liner shipping company, denoted by $\text{TC}(\mathbf{x})$, can be expressed as:

$$\min \mathrm{TC}(\mathbf{x}) = \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}_r}(m_r c_v^{\mathrm{opr}} + c_{rv}^{\mathrm{fix}})x_{rv} + \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r}\sum_{o\in\mathcal{P}}\sum_{v\in\mathcal{V}_r} c_{p_{ri}v}^{\mathrm{ber}} t_{p_{ri}v} x_{rv}(\hat{z}_{ri}^o + \tilde{z}_{ri}^o)$$
$$+\frac{1}{2}\sum_{p\in\mathcal{P}}\bar{c}_p[\sum_{r\in\mathcal{R}_p}\sum_{i\in\mathcal{I}_{rp}}\sum_{o\in\mathcal{P}}(\hat{z}_{ri}^o + \tilde{z}_{ri}^o) - \sum_{d\in\mathcal{P}} n_{pd} - \sum_{o\in\mathcal{P}} n_{op}] + \sum_{(o,d)\in\mathcal{W}}(\hat{c}_o + \tilde{c}_d)n_{od} \quad (4.4)$$
$$+\sum_{v\in\mathcal{V}} c_v^{\mathrm{in}} n_v^{\mathrm{in}} - \sum_{v\in\mathcal{V}} c_v^{\mathrm{out}} n_v^{\mathrm{out}}$$

Eq. (4.4) has six terms: the first is the total cost associated with operating ships and voyages, the second term is the total berth occupancy charge, the third is the total container transshipment cost, the fourth is the total loading and discharge cost, the fifth term is the cost of chartering in ships, and the sixth is profit from chartering out ships.

Define $f_{r0}^o := f_{rN_r}^o$. The container flows involved in the given vector of decision variables $\mathbf{x}$ should fulfill the fundamental flow conservation equations:

$$f_{r,i-1}^o + \hat{z}_{ri}^o = f_{ri}^o + \tilde{z}_{ri}^o, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r, \forall o\in\mathcal{P} \quad (4.5)$$

$$\sum_{r\in\mathcal{R}_d}\sum_{i\in\mathcal{I}_{rd}}(\tilde{z}_{ri}^o - \hat{z}_{ri}^o) = n_{od}, \forall (o,d)\in\mathcal{W} \quad (4.6)$$

Eq. (4.5) guarantees that the number of incoming containers is equal to the number of outgoing containers at each port of call on each ship route. Eq. (4.6) enforces that the container shipment demand is fulfilled.

Let $\mathcal{R}_v \subseteq \mathcal{R}$ be the set of ship routes accommodating ship type $v\in\mathcal{V}$, that is, $\mathcal{R}_v := \{r \mid r\in\mathcal{R}, v\in\mathcal{V}_r\}$. The proposed FDP can be formulated as the following mixed-integer nonlinear programming model, named fleet deployment model 1 (FDM1):

[FDM1]

$$\min_{\mathbf{x}} \mathrm{TC}(\mathbf{x}) \quad (4.7)$$

subject to:

$$\sum_{v\in\mathcal{V}_r} x_{rv} = 1, \forall r \in \mathcal{R} \tag{4.8}$$

$$168m_r \geq \sum_{v\in\mathcal{V}_r} x_{rv}[\tau_{rv}^{\text{fix}} + \sum_{i\in\mathcal{I}_r}\sum_{o\in\mathcal{P}} t_{p_{ri}v}(\hat{z}_{ri}^o + \tilde{z}_{ri}^o)], \forall r \in \mathcal{R} \tag{4.9}$$

$$\sum_{o\in\mathcal{P}} f_{ri}^o - \sum_{v\in\mathcal{V}_r} \text{Cap}_v x_{rv} \leq 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{4.10}$$

$$f_{r,i-1}^o + \hat{z}_{ri}^o = f_{ri}^o + \tilde{z}_{ri}^o, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{4.11}$$

$$\sum_{r\in\mathcal{R}_d}\sum_{i\in\mathcal{I}_{rd}} (\tilde{z}_{ri}^o - \hat{z}_{ri}^o) = n_{od}, \forall (o,d) \in \mathcal{W} \tag{4.12}$$

$$f_{ri}^o = 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, o = p_{r,i+1} \tag{4.13}$$

$$\tilde{z}_{ri}^o = 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, o = p_{ri} \tag{4.14}$$

$$\sum_{r\in\mathcal{R}_v} m_r x_{rv} - n_v^{\text{in}} + n_v^{\text{out}} = N_v^{\text{own}}, \forall v \in \mathcal{V} \tag{4.15}$$

$$n_v^{\text{in}} \leq N_v^{\text{in}}, \forall v \in \mathcal{V} \tag{4.16}$$

$$x_{rv} \in \{0,1\}, \forall r \in \mathcal{R}, \forall v \in \mathcal{V}_r \tag{4.17}$$

$$m_r \in \mathbb{Z}^+, \forall r \in \mathcal{R} \tag{4.18}$$

$$\hat{z}_{ri}^o \geq 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{4.19}$$

$$\tilde{z}_{ri}^o \geq 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{4.20}$$

$$f_{ri}^o \geq 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{4.21}$$

$$n_v^{\text{in}}, n_v^{\text{out}} \in \mathbb{Z}^+ \bigcup \{0\}, \forall v \in \mathcal{V} \tag{4.22}$$

Constraints (4.8) impose the condition that exactly one type of ship is deployed on each ship route. Constraints (4.9) ensure that the number of ships deployed on a ship route is large enough to maintain a weekly service frequency, where 168 is the number of hours in a week. Constraints (4.10) are the capacity constraints on each leg of each ship route. Constraints

(4.11) and (4.12) enforce flow conservation at each port of call on each ship route. Constraints (4.13) require that containers originating from a given port o do not return to this port. Constraints (4.14) require that containers originating from a given port o are not discharged at this port. Constraints (4.15) are the ship number conservation equations. Constraints (4.16) are the upper bound constraints on the number of chartered in ships. Finally, constraints (4.17)-(4.22) define non-negativity and/or integer attributes of the decision variables.

FDM1 is a mixed-integer optimization model with nonlinear terms $m_r x_{rv}$ and $x_{rv}(\hat{z}_{ri}^o + \tilde{z}_{ri}^o)$. We can transform FDM1 to an equivalent and solvable mixed-integer linear programming model, by introducing vector $\hat{\mathbf{x}}$:

$$\hat{\mathbf{x}} = (m_{rv}, z_{riv} \,|\, r \in \mathcal{R}, v \in \mathcal{V}_r, i \in \mathcal{I}_r) \tag{4.23}$$

where m_{rv} denotes the number of ships of type v deployed on ship route r and z_{riv} denotes the number of containers handled (loaded and discharged, including containers originating from all ports) for a ship of type v at the i^{th} port of call on ship route r. Both m_{rv} and z_{riv} are zero when $x_{rv} = 0$. The FDM1 can be transformed into the following mixed-integer linear programming model, called FDM2, by means of the big-M modeling method:

[FDM2]

$$\begin{aligned} \min_{\mathbf{x},\hat{\mathbf{x}}} \mathrm{TC}(\mathbf{x}) = & \sum_{r\in\mathcal{R}} \sum_{v\in\mathcal{V}_r} (m_{rv} c_v^{\text{opr}} + c_{rv}^{\text{fix}} x_{rv}) + \sum_{r\in\mathcal{R}} \sum_{i\in\mathcal{I}_r} \sum_{v\in\mathcal{V}_r} c_{p_{ri}v}^{\text{ber}} t_{p_{ri}v} z_{riv} \\ & + \frac{1}{2} \sum_{p\in\mathcal{P}} \bar{c}_p [\sum_{r\in\mathcal{R}_p} \sum_{i\in\mathcal{I}_{rp}} \sum_{o\in\mathcal{P}} (\hat{z}_{ri}^o + \tilde{z}_{ri}^o) - \sum_{d\in\mathcal{P}} n_{pd} - \sum_{o\in\mathcal{P}} n_{op}] + \sum_{(o,d)\in\mathcal{W}} (\hat{c}_o + \tilde{c}_d) n_{od} \\ & + \sum_{v\in\mathcal{V}} c_v^{\text{in}} n_v^{\text{in}} - \sum_{v\in\mathcal{V}} c_v^{\text{out}} n_v^{\text{out}} \end{aligned} \tag{4.24}$$

subject to:

$$\sum_{v \in \mathcal{V}_r} x_{rv} = 1, \forall r \in \mathcal{R} \tag{4.25}$$

$$m_{rv} \le M_1 x_{rv}, \forall r \in \mathcal{R}, \forall v \in \mathcal{V}_r \tag{4.26}$$

$$168 m_{rv} + M_2 (1 - x_{rv}) \ge \tau_{rv}^{\text{fix}} + \sum_{i \in \mathcal{I}_r} t_{p_{ri} v} z_{riv}, \forall r \in \mathcal{R}, \forall v \in \mathcal{V}_r \tag{4.27}$$

$$\sum_{o \in \mathcal{P}} f_{ri}^o - \sum_{v \in \mathcal{V}_r} \text{Cap}_v x_{rv} \le 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{4.28}$$

$$f_{r,i-1}^o + \hat{z}_{ri}^o = f_{ri}^o + \tilde{z}_{ri}^o, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{4.29}$$

$$\sum_{r \in \mathcal{R}_d} \sum_{i \in \mathcal{I}_{rd}} (\tilde{z}_{ri}^o - \hat{z}_{ri}^o) = n_{od}, \forall (o,d) \in \mathcal{W} \tag{4.30}$$

$$z_{riv} \le M_3 x_{rv}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall v \in \mathcal{V}_r \tag{4.31}$$

$$z_{riv} + M_4 (1 - x_{rv}) \ge \sum_{o \in \mathcal{P}} (\hat{z}_{ri}^o + \tilde{z}_{ri}^o), \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall v \in \mathcal{V}_r \tag{4.32}$$

$$\sum_{r \in \mathcal{R}_v} m_{rv} - n_v^{\text{in}} + n_v^{\text{out}} = N_v^{\text{own}}, \forall v \in \mathcal{V} \tag{4.33}$$

$$f_{ri}^o = 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, o = p_{r,i+1} \tag{4.34}$$

$$\tilde{z}_{ri}^o = 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, o = p_{ri} \tag{4.35}$$

$$n_v^{\text{in}} \le N_v^{\text{in}}, \forall v \in \mathcal{V} \tag{4.36}$$

$$x_{rv} \in \{0,1\}, \forall r \in \mathcal{R}, \forall v \in \mathcal{V}_r \tag{4.37}$$

$$m_{rv} \in \mathbb{Z}^+ \cup \{0\}, \forall r \in \mathcal{R}, \forall v \in \mathcal{V}_r \tag{4.38}$$

$$\hat{z}_{ri}^o \ge 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{4.39}$$

$$\tilde{z}_{ri}^o \ge 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{4.40}$$

$$f_{ri}^o \ge 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall o \in \mathcal{P} \tag{4.41}$$

$$z_{riv} \ge 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall v \in \mathcal{V}_r \tag{4.42}$$

$$n_v^{\text{in}} \geq 0, n_v^{\text{out}} \geq 0, \forall v \in \mathcal{V} \tag{4.43}$$

$\text{TC}(\mathbf{x}, \hat{\mathbf{x}})$ in the objective function (4.24) is an equivalent formulation for $\text{TC}(\mathbf{x})$. Constraints (4.26)-(4.27) are the weekly service frequency constraints. Eqs. (4.31)-(4.32) define z_{riv} because constraints (4.32) must be binding in the optimal solution if $x_{rv} = 1$. M_1 to M_4 can be determined from practical considerations. A round trip seldom exceeds 15 weeks in practice; hence, we can set $M_1 = 15$ and $M_2 = 15 \times 168 \text{ hours / week} = 2520$. Both M_3 and M_4 could be set to $2 \times \max\{\text{Cap}_v, \forall v \in \mathcal{V}\}$, accounting for the extreme case that a full shipload of containers is discharged, and another full shipload is loaded, at a given port. The integrality constraint (4.22) is relaxed in Eq. (4.43) as a consequence of the following proposition.

Proposition 4-1: Let $(n_v^{in})^*$ and $(n_v^{out})^*$ denote the optimal solution for n_v^{in} and n_v^{out} in FDM2, respectively. Both $(n_v^{in})^*$ and $(n_v^{out})^*$ take integer values.

Proof: We prove the proposition by contradiction. Let $\lfloor a \rfloor$ be the maximum integer not greater than a. Suppose that $(n_v^{in})^*$ is not an integer for a particular ship type v, i.e., $\lfloor (n_v^{in})^* \rfloor < (n_v^{in})^*$. As a result of constraints (4.33) and the integrality property of m_{rv} and N_v^{own}, $\lfloor (n_v^{out})^* \rfloor < (n_v^{out})^*$. The converse also holds.

Due to the condition (4.1), the optimal value of FDP2 can be decreased if we use $\lfloor (n_v^{in})^* \rfloor$ and $\lfloor (n_v^{out})^* \rfloor$ in place of $(n_v^{in})^*$ and $(n_v^{out})^*$, respectively, while satisfying all the constraints. Therefore $(n_v^{in})^*$ and $(n_v^{out})^*$ are not the optimal solution. □

We note that for pratcial-sized instances, the proposed mixed-integer linear programming model FDM2 can be solved efficiently using off-the-shelf optimization solvers such as CPLEX.

4.3 Practical Considerations

In this section, we introduce how to handle other practical considerations in the model FDM2. In fact, FDM2 can easily accommodate these additional constraints.

First, the container flow conservation equations (4.11) and (4.12) cannot ensure that the origin-based container flow on a leg of a particular ship route, f_{ri}^{o}, reflects the actual container flow. For example, assuming that the ship route in Figure 2-2 has only one TEU shipped from Busan to Shanghai, the actual container flow is shown in Figure 4-1 (a). However, given a positive integer number δ, Figure 4-1 (b) is also a possible optimal flow, as long as the ship capacity constraint is not violated. Although the case in Figure 4-1 (b) has no effect on fleet deployment decisions, the leg container flow f_{ri}^{o} (or rather, ship utilization, which is defined as the ratio between the leg flow and the ship capacity) is an important criterion when a liner shipping company is evaluating a ship fleet deployment decision. In order to obtain the actual container flow, we first solve FDM2 to obtain the optimal decisions, denoted by $n_v^{*\mathrm{in}}$, $n_v^{*\mathrm{out}}$, m_{rv}^{*}, x_{rv}^{*}, $\hat{z}_{ri}^{*o}$, $\tilde{z}_{ri}^{*o}$, and z_{riv}^{*}, and subsequently solve the following linear programming model to obtain the optimal actual origin-based leg container flows:

$$\min_{f_{ri}^{o}} \sum_{r\in\mathcal{R}} \sum_{i\in I_r} \sum_{o\in\mathcal{P}} f_{ri}^{o} \tag{4.44}$$

subject to

$$f_{r,i-1}^{o} + \hat{z}_{ri}^{*o} = f_{ri}^{o} + \tilde{z}_{ri}^{*o}, \forall r\in\mathcal{R}, \forall i\in I_r, \forall o\in\mathcal{P} \tag{4.45}$$

$$f_{ri}^{o} \geq 0, \forall r\in\mathcal{R}, \forall i\in I_r, \forall o\in\mathcal{P} \tag{4.46}$$

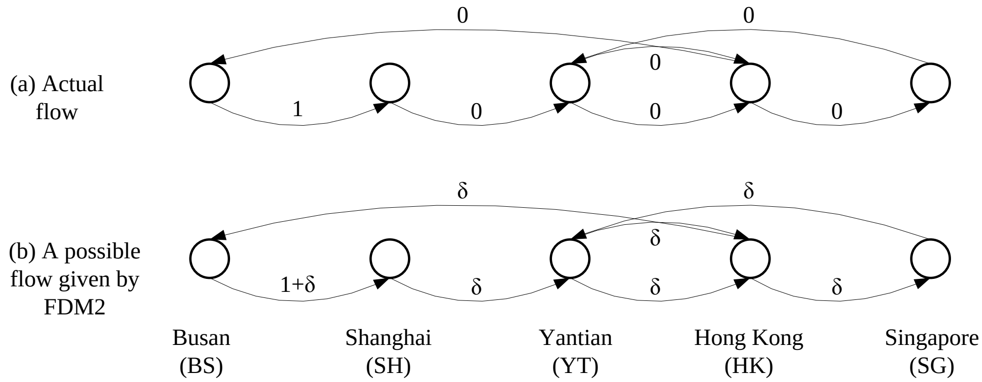


Figure 4-1 Actual container flow and a possible container flow by FDM2

Second, the liner shipping company may not transport all containers using its own services. Instead, it can buy ship slots from its shipping alliances. To incorporate slot-buying operations in FDM2, we define y_{od} (TEUs/week) as the number of containers shipped for port pair $(o,d) \in \mathcal{W}$ along the ship routes operated by the liner shipping company. Thus, $n_{od} - y_{od}$ is the number of containers outsourced to other liner shipping companies. We further define g_{od} (USD/TEU) as the freight rate charged by other liner shipping companies for shipping one container for port pair $(o,d) \in \mathcal{W}$. FDM2 can thus be reformulated as:

[FDM3]

$$\min_{\mathbf{x},\hat{\mathbf{x}},y_{od}} \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}_r}(m_{rv}c_v^{\text{opr}} + c_{rv}^{\text{fix}}x_{rv}) + \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r}\sum_{v\in\mathcal{V}_r} c_{p_{ri}v}^{\text{ber}} t_{p_{ri}v} z_{riv}$$
$$+\frac{1}{2}\sum_{p\in\mathcal{P}}\overline{c}_p[\sum_{r\in\mathcal{R}_p}\sum_{i\in\mathcal{I}_{rp}}\sum_{o\in\mathcal{P}}(\hat{z}_{ri}^o + \tilde{z}_{ri}^o) - \sum_{d\in\mathcal{P}} y_{pd} - \sum_{o\in\mathcal{P}} y_{op}] + \sum_{(o,d)\in\mathcal{W}}(\hat{c}_o + \tilde{c}_d)y_{od} \quad (4.47)$$
$$+\sum_{v\in\mathcal{V}} c_v^{\text{in}} n_v^{\text{in}} - \sum_{v\in\mathcal{V}} c_v^{\text{out}} n_v^{\text{out}} + \sum_{(o,d)\in\mathcal{W}} g_{od}(n_{od} - y_{od})$$

subject to:

$$\sum_{r\in\mathcal{R}_d}\sum_{i\in\mathcal{I}_{rd}}(\tilde{z}_{ri}^o - \hat{z}_{ri}^o) = y_{od}, \forall (o,d)\in\mathcal{W} \quad (4.48)$$

$$0 \le y_{od} \le n_{od}, \forall (o,d)\in\mathcal{W} \quad (4.49)$$

and relevant constraints. The objective function (4.47) is similar to the objective function defined by Eq. (4.24). The last term in Eq. (4.47) is the freight rate charged by other liner shipping companies. Constraints (4.48)-(4.49) denote the fact that some containers can be delivered by other liner shipping companies.

The liner shipping company may be reluctant to predict the freight rate g_{od} for FDM3 because it might be difficult to obtain a good prediction. Alternatively, the company may require that at least a fraction, $1-\alpha$, of its total demand must be fulfilled by its own services, with $1-\alpha$ taking a value of 95 per cent, for example. FDM2 is now reformulated as:

[FDM4]

$$\begin{aligned}\min_{\mathbf{x},\hat{\mathbf{x}},y_{od}} & \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}_r}(m_{rv}c_v^{\text{opr}}+c_{rv}^{\text{fix}}x_{rv})+\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r}\sum_{v\in\mathcal{V}_r}c_{p_{ri}v}^{\text{ber}}t_{p_{ri}v}z_{riv}\\ &+\frac{1}{2}\sum_{p\in\mathcal{P}}\overline{c}_p[\sum_{r\in\mathcal{R}_p}\sum_{i\in\mathcal{I}_{rp}}\sum_{o\in\mathcal{P}}(\hat{z}_{ri}^o+\tilde{z}_{ri}^o)-\sum_{d\in\mathcal{P}}y_{pd}-\sum_{o\in\mathcal{P}}y_{op}]+\sum_{(o,d)\in\mathcal{W}}(\hat{c}_o+\tilde{c}_d)y_{od}\\ &+\sum_{v\in\mathcal{V}}c_v^{\text{in}}n_v^{\text{in}}-\sum_{v\in\mathcal{V}}c_v^{\text{out}}n_v^{\text{out}}\end{aligned} \tag{4.50}$$

subject to:

$$\sum_{r\in\mathcal{R}_d}\sum_{i\in\mathcal{I}_{rd}}(\tilde{z}_{ri}^o-\hat{z}_{ri}^o)=y_{od},\forall(o,d)\in\mathcal{W} \tag{4.51}$$

$$\sum_{(o,d)\in\mathcal{W}}y_{od}\geq(1-\alpha)\sum_{(o,d)\in\mathcal{W}}d_{od} \tag{4.52}$$

$$0\leq y_{od}\leq n_{od},\forall(o,d)\in\mathcal{W} \tag{4.53}$$

and relevant constraints.

Third, the liner shipping company may have committed to some port operators that at least a certain number of container moves (loading or discharging operations) would be conducted at their port, in order to obtain a preferential container handling price. For example, if, at a particular port $p \in \mathcal{P}$, at least β_p container moves per week are required, then we can add the following constraints to FDM2:

$$\sum_{r\in\mathcal{R}_p}\sum_{i\in\mathcal{I}_{rp}}\sum_{o\in\mathcal{P}}(\hat{z}_{ri}^{o}+\tilde{z}_{ri}^{o}) \geq \beta_p, \forall p \in \mathcal{P} \tag{4.54}$$

Conversely, suppose that the loading, discharge, and transshipment prices at port $p \in \mathcal{P}$ are $\hat{c}'_p$, $\tilde{c}'_p$, and $\overline{c}'_p$, respectively, if the liner shipping company does not commit to a particular volume of container moves; if the company has committed to a volume of at least β_p moves per week, and the volume is achieved, then the company enjoys preferable prices of $\hat{c}_p$, $\tilde{c}_p$, and $\overline{c}_p$, respectively; however, if the committed volume is not achieved, the corresponding penalty prices are $\hat{c}''_p$, $\tilde{c}''_p$, and $\overline{c}''_p$, respectively. Hence, $c''_p > c'_p > c_p$, $\tilde{c}''_p > \tilde{c}'_p > \tilde{c}_p$, and $\overline{c}''_p > \overline{c}'_p > \overline{c}_p$. We can thus use binary decision variables, w_p, to indicate whether the liner shipping company should commit to a minimum container move volume at port p ($w_p = 1$) or not ($w_p = 0$). We further represent, by u_p and u'_p, the number of transshipment operations at port p at prices $\overline{c}_p$ and $\overline{c}'_p$, respectively. u_p and u'_p cannot both be non-negative at port p. Let M_5 and M_6 be two large numbers. Then, FDM2 is rewritten as:

[FDM5]

$$\begin{aligned}\min_{\mathbf{x},\hat{\mathbf{x}},u_p,u'_p,w_p} & \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}_r}(m_{rv}c_v^{\text{opr}}+c_{rv}^{\text{fix}}x_{rv})+\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r}\sum_{v\in\mathcal{V}_r}c_{p_{ri}v}^{\text{ber}}t_{p_{ri}v}z_{riv} \\ & +\sum_{p\in\mathcal{P}}\overline{c}_p u_p+\sum_{p\in\mathcal{P}}\overline{c}'_p u'_p+\sum_{o\in\mathcal{P}}\sum_{d\in\mathcal{P}}\Big[w_p(\hat{c}_o+\tilde{c}_d)+(1-w_p)(\hat{c}'_o+\tilde{c}'_d)\Big]n_{od} \\ & +\sum_{v\in\mathcal{V}}c_v^{\text{in}}n_v^{\text{in}}-\sum_{v\in\mathcal{V}}c_v^{\text{out}}n_v^{\text{out}}\end{aligned} \tag{4.55}$$

subject to:

$$\sum_{r\in\mathcal{R}_p}\sum_{i\in I_{rp}}\sum_{o\in\mathcal{P}}(\hat{z}_{ri}^o+\tilde{z}_{ri}^o)+M_5(1-w_p)\geq\beta_p,\forall p\in\mathcal{P} \tag{4.56}$$

$$u_p+u'_p=\frac{1}{2}\left\{\sum_{r\in\mathcal{R}_p}\sum_{i\in I_{rp}}\sum_{o\in\mathcal{P}}(\hat{z}_{ri}^o+\tilde{z}_{ri}^o)-\sum_{d\in\mathcal{P}}n_{pd}-\sum_{o\in\mathcal{P}}n_{op}\right\},\forall p\in\mathcal{P} \tag{4.57}$$

$$u_p\leq M_6w_p,\forall p\in\mathcal{P} \tag{4.58}$$

$$u'_p\leq M_6(1-w_p),\forall p\in\mathcal{P} \tag{4.59}$$

$$u_p\geq 0,u'_p\geq 0,w_p\in\{0,1\},\forall p\in\mathcal{P} \tag{4.60}$$

and relevant constraints.

Fourth, if the liner shipping company needs to determine whether a particular ship route $r\in\mathcal{R}$ should be operated, it can simply add a dummy ship type v^{dummy} with zero capacity and zero cost to the set of candidate ship types $\mathcal{V}_r$. If, under the optimal solution, the dummy ship type is chosen, then ship route r should not be operated.

Fifth, the empty container repositioning issue can also be integrated into FDM2 by using flow variables similar to $\hat{z}_{ri}^o$, $\tilde{z}_{ri}^o$, and f_{ri}^o. The main difference is that empty containers do not have fixed destinations and all empty containers can be considered as a single special type of container. Suppose that the number of surplus empty containers at port $p\in\mathcal{P}$ is n_p^{EMP} (TEUs/week). $n_p^{\text{EMP}}<0$ means that port p has a deficit of empty containers and $n_p^{\text{EMP}}=0$ means that port p has a balance between incoming and outgoing empty containers. Let $\mathcal{P}^+:=\{p\in\mathcal{P}:n_p^{\text{EMP}}>0\}$, $\mathcal{P}^0:=\{p\in\mathcal{P}:n_p^{\text{EMP}}=0\}$, and $\mathcal{P}^-:=\{p\in\mathcal{P}:n_p^{\text{EMP}}<0\}$. We need the following decision variables:

$\hat{z}_{ri}^{\text{EMP}}$: Number of empty containers (TEUs/week) loaded at the i^{th} port of call on ship route $r\in\mathcal{R}$;

$\tilde{z}_{ri}^{\text{EMP}}$: Number of empty containers (TEUs/week) discharged at the i^{th} port of call on ship route $r \in \mathcal{R}$;

f_{ri}^{EMP} : Number of empty containers (TEUs/week) stowed on board ships sailing on the i^{th} leg of ship route $r \in \mathcal{R}$.

With the above decision variables, FDM2 can be reformulated as:

[FDM6]

$$\begin{aligned}
\min_{\mathbf{x},\hat{\mathbf{x}},\hat{z}_{ri}^{\text{EMP}},\tilde{z}_{ri}^{\text{EMP}},f_{ri}^{\text{EMP}}} & \sum_{r\in\mathcal{R}}\sum_{v\in\mathcal{V}_r}(m_{rv}c_v^{\text{opr}}+c_{rv}^{\text{fix}}x_{rv})+\sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r}\sum_{v\in\mathcal{V}_r}c_{p_{ri}v}^{\text{ber}}t_{p_{ri}v}z_{riv} \\
&+\frac{1}{2}\sum_{p\in\mathcal{P}}\bar{c}_p[\sum_{r\in\mathcal{R}_p}\sum_{i\in\mathcal{I}_{rp}}\sum_{o\in\mathcal{P}}(\hat{z}_{ri}^o+\tilde{z}_{ri}^o)-\sum_{d\in\mathcal{P}}n_{pd}-\sum_{o\in\mathcal{P}}n_{op}]+\sum_{(o,d)\in\mathcal{W}}(\hat{c}_o+\tilde{c}_d)n_{od} \\
&+\frac{1}{2}\sum_{p\in\mathcal{P}}\bar{c}_p[\sum_{r\in\mathcal{R}_p}\sum_{i\in\mathcal{I}_{rp}}(\hat{z}_{ri}^{\text{EMP}}+\tilde{z}_{ri}^{\text{EMP}})-|n_p^{\text{EMP}}|]+\sum_{p\in\mathcal{P}^+}\hat{c}_p n_p^{\text{EMP}}-\sum_{p\in\mathcal{P}^-}(-\tilde{c}_p n_p^{\text{EMP}}) \\
&+\sum_{v\in\mathcal{V}}c_v^{\text{in}}n_v^{\text{in}}-\sum_{v\in\mathcal{V}}c_v^{\text{out}}n_v^{\text{out}}
\end{aligned} \tag{4.61}$$

subject to:

$$(\sum_{o\in\mathcal{P}}f_{ri}^o+f_{ri}^{\text{EMP}})-\sum_{v\in\mathcal{V}_r}\text{Cap}_v x_{rv}\le 0, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{4.62}$$

$$f_{r,i-1}^{\text{EMP}}+\hat{z}_{ri}^{\text{EMP}}=f_{ri}^{\text{EMP}}+\tilde{z}_{ri}^{\text{EMP}}, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{4.63}$$

$$\sum_{r\in\mathcal{R}_p}\sum_{i\in\mathcal{I}_{rp}}(\hat{z}_{ri}^{\text{EMP}}-\tilde{z}_{ri}^{\text{EMP}})=n_p^{\text{EMP}}, \forall p\in\mathcal{P} \tag{4.64}$$

$$z_{riv}+M_4(1-x_{rv})\ge\sum_{o\in\mathcal{P}}(\hat{z}_{ri}^o+\tilde{z}_{ri}^o)+(\hat{z}_{ri}^{\text{EMP}}+\tilde{z}_{ri}^{\text{EMP}}), \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r, \forall v\in\mathcal{V}_r \tag{4.65}$$

$$\hat{z}_{ri}^{\text{EMP}}\ge 0, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{4.66}$$

$$\tilde{z}_{ri}^{\text{EMP}}\ge 0, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{4.67}$$

$$f_{ri}^{\text{EMP}}\ge 0, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{4.68}$$

and relevant constraints.

4.4 Computational Study

4.4.1 Real-case problems

To assess the performance of the proposed mixed-integer linear programming model and to gain insight into the fleet deployment problem, we first take advantage of eight test cases with different container shipment demands, provided by a global shipping company. The test cases apply to the Asia-Europe-Oceania shipping network of the global liner shipping company, which has a total of 46 ports, as shown in Figure 2-1. All 46 ports are assumed to have the same characteristics, except for geographical location and container shipment demand. The loading cost $\hat{c}_p = 150$ USD/TEU, discharge cost $\tilde{c}_p = 150$ USD/TEU, and transshipment cost $\overline{c}_p = 200$ USD/TEU. There are four types of ships, as shown in Table 4-1. A total of 12 ship routes are operated over these 46 ports, as shown in Table 4-2. Each ship route has two or three candidate ship types. The candidate ship types for the 12 ship routes and the fixed operating costs c_{rv}^{fix} and fixed round-trip voyage times τ_{rv}^{fix} are shown in Table 4-3. A blank in Table 4-3 means that the corresponding ship type is not a candidate for the relevant ship route. c_{rv}^{fix} and τ_{rv}^{fix} are computed using the data from Table 4-1 and Table 4-2. c_{rv}^{fix} is calculated by summing the fixed cost of calling at a port and the bunker cost. τ_{rv}^{fix} is derived by summing the fixed time spent calling at each port on the ship route and the sailing time on each voyage leg. There are 652 O-D port pairs with container shipment demand. The mixed-integer linear programming model FDM2 is solved using CPLEX-12.1, running on a 3 GHz Dual Core PC with 4 GB of RAM.

Table 4-1 Characteristics of the liner ship fleet

	Ship type			
	1	2	3	4
Capacity (TEUs)	1500	3000	5000	10000
Fixed operating cost c_v^{opr} (USD/week)	51923	76923	115384	173076
Speed (knots)	16.2	18.9	22.5	26
Bunker cost (USD/n mile)	59	72	78	100
Fixed cost for calling at a port (USD)	3873	5477	7071	10000
Berth occupancy charge c_{pv}^{ber} (USD/hour)	500	1000	1666	3333
Fixed time when calling at a port (hours)	4	4	4	4
Container handling time (hours/TEU)	1/40	1/85	1/95	1/120
Number of owned ships N_v^{own}	20	20	20	20
Profit gained from chartering out a ship c_v^{out} (USD/week)	52500	77000	98000	140000
Maximum number of ships that can be chartered in N_v^{in}	30	30	30	30
Cost of chartering in a ship c_v^{in} (USD/week)	66500	94500	122500	175000

Table 4-2 Ports of call and voyage distance (n miles) for each leg of the 12 ship routes

No.	Ports of call (voyage distance)
1	Yokohama(15)→Tokyo(177)→Nagoya(201)→Kobe(734)→Shanghai(745)→Hong Kong(1568) → Yokohama
2	Ho Chi Minh(589)→Laem Chabang(755)→Singapore(187)→Port Klang(830) →Ho Chi Minh
3	Brisbane(419)→Sydney(512)→Melbourne(470)→Adelaide(1325)→Fremantle(1733)→ Jakarta(483)→Singapore(3649)→Brisbane
4	Manila(527)→Kaohsiung(164)→Xiamen(260)→Hong Kong(15)→Yantian(19)→Chiwan(17) →Hong Kong(620)→Manila
5	Dalian(187)→Xingang(379)→Qingdao(303)→Shanghai(93)→Ningbo(93)→ Shanghai(383)→ Kwangyang(72)→Busan(487)→Dalian
6	Chittagong(872)→Chennai(573)→Colombo(306)→Cochin(723)→Nhava Sheva(723)→ Cochin(306)→Colombo(573)→Chennai(872)→Chittagong
7	Sokhna(265)→Aqabah(554)→Jeddah(1268)→Salalah(885)→Karachi(688)→Jebel Ali(862) →Salalah(1878)→Sokhna
8	Southampton(165)→Thamesport(386)→Hamburg(82)→Bremerhaven(196)→Rotterdam(42)→ Antwerp(51)→Zeebrugge(168)→Le Havre(103)→Southampton
9	Port Klang(187)→Singapore(483)→Jakarta(1917)→Kaohsiung(904)→Busan(904) →Kaohsiung(342)→Hong Kong(17)→Chiwan(1597)→Port Klang
10	Southampton(3162)→Sokhna(1878)→Salalah(1643)→Colombo(1560)→Singapore(1415) →Hong Kong(260)→Xiamen(486)→Shanghai(448)→Busan(487)→Dalian(187)→ Xingang(379)→Qingdao(303)→Shanghai(745)→Hong Kong(1415)→Singapore(1560)→ Colombo(1643)→Salalah(5029)→Southampton
11	Brisbane(419)→Sydney(512)→Melbourne(470)→Adelaide(1325)→Fremantle(3148)→ Colombo(1643)→Salalah(5244)→Rotterdam(5244)→Salalah(1643)→Colombo(5191)→ Brisbane
12	Yantian(9956)→Hamburg(3621)→Sokhna(620)→Jeddah(4156)→Port Klang(187)→ Singapore(1309)→Manila(629)→Yantian

Table 4-3 Candidate ship types, fixed costs (USD/week) and fixed voyage times (hours) of the 12 ship routes

No.	Ship type 1		Ship type 2		Ship type 3		Ship type 4	
	c_{rv}^{fix}	τ_{rv}^{fix}	c_{rv}^{fix}	τ_{rv}^{fix}	c_{rv}^{fix}	τ_{rv}^{fix}	c_{rv}^{fix}	τ_{rv}^{fix}
1	226198	236	280542	206			404000	161
2	154791	161	191900	140			276100	110
3	533980	558	656891	482			929100	371
4			155123	113	176013	100	232200	92
5	148807	155	187600	137			279700	111
6	322916	337	400072	293			574800	229
7	404711	423	499139	366			710000	284
8			129712	95	149622	85	199300	79
9			501088	368	551946	314	715100	286
10					1883007	1072	2430000	972
11	1504231	1573	1843178	1354			2583900	1033
12	1235313	1292	1512755	1111			2117800	847

Table 4-4 shows the total container shipment demand for the 652 O-D port pairs (TEUs), the number of ports with transshipment containers, the optimal total number of container transshipment operations, the optimal total cost (million USD/week), and CPU time (minutes) for the eight test cases. First, we observe that FDM2 can be solved efficiently for all test cases: the maximum CPU time required is nine minutes. Second, the total number of transshipment containers is significant compared with the total container shipment demand. This highlights the importance of taking into consideration container transshipment operations in the FDP. It is noteworthy that, among the total of 46 ports, 17 to 22 ports have transshipment container throughput in the optimal solutions. This high proportion of transshipment ports is not captured in the literature on the hub-and-spoke (H&S) network of liner shipping services, whereas the proposed model fully captures this feature, since it allows container transshipment at any port. For example, in practice a number of global liner shipping companies do not have direct ship routes from Australia to Europe. However, they have ship routes that connect Australia to Asia and ship routes that connect Asia to Europe. Therefore containers from Australia to Europe can be transshipped at any of the common Asian ports of call, such as Singapore, Hong Kong, and Kaohsiung. If the transshipment cost at e.g. Singapore were very high, global liner shipping companies would choose to transship containers from Australia to Europe at Hong Kong or Kaohsiung. The load, discharge, and transshipment container throughput at the 22 transshipment ports in test case 1 is plotted in Figure 4-2. Major transshipment ports, such as Singapore, Salalah, Colombo and Hong Kong, can easily be identified.

Table 4-4 Computational results for the real-case fleet deployment problems

Case No.	Total container shipment demand	No. of ports with transshipment	Total no. of transshipment containers	Total cost (million USD/week)	CPU Time (mins)
1	23353	22	21846	20.73	9
2	24308	18	22080	21.37	7
3	25176	22	23155	21.91	4
4	26233	17	23815	22.71	5
5	27321	19	24811	23.42	5
6	28610	20	26412	25.00	4
7	30189	18	27417	26.55	8
8	32559	18	30507	28.37	3

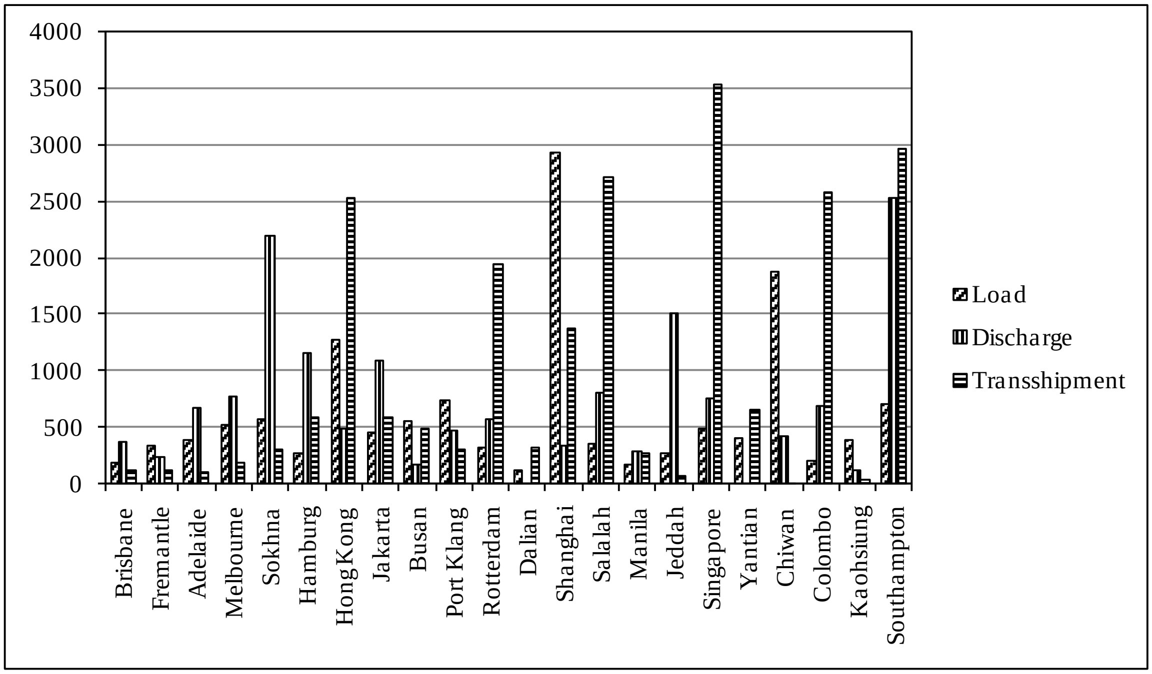


Figure 4-2 Load, discharge and transshipment container throughput (TEUs) at the 22 transshipment ports in test case 1

Ship utilization can provide important insights into liner shipping services. For instance, the ship utilization on each leg of ship route 9 in test case 1 is shown in Figure 4-3. It can be observed that ship utilization between Kaohsiung and Busan is very low. Given that Busan is far away to the north of the other ports on the ship route, the liner shipping company may consider removing Busan from this ship route. Figure 4-4 shows the ship utilization on each leg of ship route 10 in test case 1. Similarly, the ship utilization is very low when the ship sails from Shanghai to Busan, Dalian, Xingang, Qingdao, and back to Shanghai. Therefore, these four ports—Busan, Dalian, Xingang, and Qingdao—might also be removed from the ship route and a feeder service to Shanghai could possibly be introduced instead. It should be mentioned that ship utilization on the westbound route (from Shanghai to Southampton) is high, whereas ship utilization on the eastbound route (from Southampton to Shanghai) is much lower. This phenomenon is due to the trade imbalance between the east and the west. Unlike the above two cases of low utilization between Kaohsiung and Busan, and between the two calls at Shanghai, removing some ports of call on the eastbound route would not significantly shorten the round-trip distance. In fact, it would be difficult to improve the low ship utilization due to trade imbalance.

Figure 4-3 Ship utilization on each leg of ship route 9 in test case 1

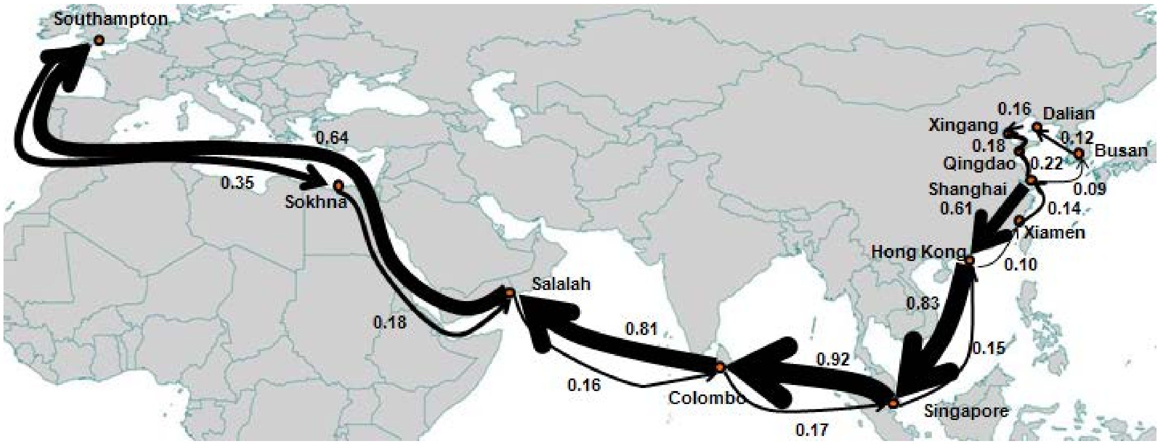


Figure 4-4 Ship utilization on each leg of ship route 10 in test case 1

4.4.2 Randomly generated test instances

To further test the computational efficiency of the proposed model, we make use of ten randomly created cases, each of which has a network of 300 ports and 20,000 O-D port pairs. The scale of these problems is thus larger than the shipping network of the world's largest liner shipping company Maersk, which has a total of 234 unique ports and 14,000 O-D port pairs (Løfstedt, 2010). In the test cases, the ports are uniformly distributed on a 20,000×15,000 (unit: *n mile*) rectangular area. The container shipment demand for each O-D port pair is generated according to the following distribution:

$$\Pr(n_{od} \geq 1) = 20000 / \left(300 \times 300\right) \tag{4.69}$$

$$\Pr(1 \leq n_{od} \leq 30 \mid n_{od} \geq 1) = 0.95 \tag{4.70}$$

$$\Pr(31 \leq n_{od} \leq 100 \mid n_{od} \geq 1) = 0.045 \tag{4.71}$$

$$\Pr(101 \leq n_{od} \leq 1000 \mid n_{od} \geq 1) = 0.005 \tag{4.72}$$

The container volume defined by Eqs. (4.70), (4.71) or (4.72) obeys the discrete uniform distribution, which means that n_{od} takes only integer values. For the generation of ship routes, we first generate the maximum number of ports of call on a certain ship route, according to the discrete uniform distribution $\text{U}(2,15)$. The i^{th} ship route starts from the last port of call on the $(i-1)^{\text{th}}$ route (the 1$^{\text{st}}$ ship route starts from Port 1). The j^{th} port of call on the i^{th} ship route is chosen randomly from the 10 nearest ports to the $(j-1)^{\text{th}}$ port of call. The i^{th} ship route finishes either when j equals the randomly generated maximum number of ports of call or when the round-trip distance has exceeded 30,000 *n miles*. Using this procedure, we first generate 20 ship routes. After that, the ports not included in these ship routes are randomly connected through feeder services to one of the ports on these ship routes. We consider a total of nine ship types, of which one dummy mega-ship type has a capacity of 1,000,000 TEUs. This dummy mega-ship type is a candidate ship type for all ship routes, ensuring that FDM2 has a feasible solution. Each of the generated ship routes has two or three candidate ship types. With these settings, we solve ten randomly generated large-scale networks with CPLEX, using the same computer as was used in the eight test cases described above. The average CPU time required over the ten cases was 67 minutes and the maximum time was 236 minutes. Hence, the large-scale problems encountered in practice can be solved with the proposed model.

4.5 Conclusions

A realistic FDP with container transshipment operations has been investigated. Based on the novel concept of origin-based container flow variables, a mixed-integer linear programming model for the FDP is formulated, which allows container transshipment operations at any port for any number of times.

The computational experiments on the Asia-Europe-Oceania shipping network of a global liner shipping company are conducted. The result that 17 to 22 ports out of the total of 46 ports have transshipment throughputs in the optimal solutions cannot be captured by the existing literature on H&S liner shipping networks. We showed how ship utilization in the optimal solution can be used to redesign liner services. Further tests on randomly generated large-scale networks show that the proposed model is able to address large-scale problems encountered in practice.

CHAPTER 5. SAILING SPEED OPTIMIZATION

This chapter first calibrates the bunker consumption - sailing speed relation for container ships using historical operating data from a global liner shipping company. It proceeds to investigate the optimal sailing speed of container ships on each leg of each ship route in a liner shipping network while considering transshipment and container routing. This problem is formulated as a mixed-integer nonlinear programming model. In view of the convexity, non-negativity, and univariate properties of the bunker consumption function, an efficient outer-approximation method is proposed to obtain an ε-optimal solution with a predetermined optimality tolerance level ε. The proposed model and algorithm is applied to a real case study for a global liner shipping company.

5.1 Calibration of Bunker Consumption - Sailing Speed Function

Before describing the sailing speed optimization problem, we first calibrate the relation between bunker consumption and sailing speed. According to the literature review, we assume that the daily bunker consumption Q (tons/day) and sailing speed v (knot) has the power relation:

$$Q = a \times v^b \tag{5.1}$$

where a and b are coefficients to be calibrated from real data. Function (5.1) is more general than the third power relation assumed in most existing studies. We can transform Eq. (5.1) into an equivalent form:

$$\ln Q = \ln a + b \ln v \tag{5.2}$$

Therefore, we can consider $\ln v$ as the independent variable and $\ln Q$ as the response variable, and use the conventional linear regression method to calibrate parameters $\ln a$ and b.

5.1.1 Data description

We take advantage of 5 groups of data provided by a global liner shipping company as shown in Table 5-1. This dataset is representative because it covers three types of ships – 3000-TEU (acronym for twenty-foot equivalent unit), 5000-TEU, and 8000-TEU ships – on five voyage legs. There are 20 historical data for each voyage leg on the average sailing speed and daily bunker consumption. We hence calibrate the parameter a and b for each group of data using the linear regression method.

Table 5-1 Historical data on sailing speed and bunker consumption

Ship type and voyage leg	Average speed (knot)	Bunker (ton/day)	Average speed (knot)	Bunker (ton)
3000-TEU Singapore-Jakarta (SG-JK)	16.0	42	17.3	53
	17.0	47	16.9	47
	16.3	43	15.7	39
	15.0	37	17.1	50
	17.6	56	18.0	60
	16.4	44	16.2	43
	15.9	40	17.5	55
	15.5	38	15.8	39
	16.6	45	15.4	38
	17.7	58	16.5	45
3000-TEU Singapore-Kaohsiung (SG-KS)	20.0	82	18.7	69
	20.5	87	20.7	93
	18.5	67	19.0	73
	19.7	77	20.6	90
	19.6	76	19.8	78
	18.8	72	20.4	85
	20.9	98	19.4	75
	20.8	96	21.5	108
	18.2	65	21.1	101
	21.2	103	20.3	84
5000-TEU Hong Kong-Singapore (HK-SG)	17.0	51	17.0	51
	18.5	70	16.8	48
	17.9	64	16.5	46
	17.4	56	17.0	52
	16.9	49	18.9	74
	16.4	46	17.1	53
	17.1	52	18.5	71
	16.1	45	17.6	59
	19.8	80	18.4	69
	19.0	75	18.2	67
8000-TEU Yantian-Los Angeles (YT-LA)	19.0	109	19.6	121
	19.5	118	18.4	98
	18.8	103	18.5	100
	21.0	147	21.4	155
	19.7	123	22.0	178
	20.4	135	20.2	132
	18.9	106	19.7	124
	17.9	89	19.4	117
	19.3	115	18.9	105
	17.0	81	23.0	200
8000-TEU Tokyo-Xiamen (TK-XM)	17.5	86	18.4	97
	17.8	87	17.4	85
	16.8	76	18.0	92
	20.1	128	17.0	82
	16.1	70	18.6	102
	15.8	68	17.2	83
	21.0	146	16.9	79
	20.5	135	18.0	93
	16.5	75	17.8	88
	17.2	84	17.0	81

5.1.2 Calibration results with statistical analysis

Figure 5-1 shows the calibrated bunker consumption - sailing speed function using Excel, the historical operating data (the diamonds in the figure) and the calibrated function (the curve). Table 5-2 reports the calibration and relevant statistical test results. First, we observe that the coefficient of determination R^2 is at least 0.96 and the adjusted R^2 is at least 0.95. Also, at the 5% significance level, the hypothesis that the residual errors are normally distributed is not rejected for any of the five data sets by using the Anderson-Darling Test. Therefore, using the power function (5.1) to approximate the bunker consumption function is appropriate.

Second, at the 5% significance level, the hypothesis that b=1 is rejected for all of the five data sets as shown in Table 5-2. We can hence conclude that the daily bunker consumption is a not a linear function of the sailing speed. Table 5-2 also shows that at the 5% significance level, the hypothesis that b=3 is rejected for two of the five data sets. Among the five data sets, the largest coefficient b is 3.3 and the smallest is 2.7. As a consequence, we reach the conclusion that the third power relationship is indeed a good approximation. Hence, we argue that the third power relation can be used if not enough historical data are available. Once enough historical data are ready for obtaining b through regression, we should use a more accurate bunker consumption function.

Third, the regression analysis also shows that bunker consumption is dependent on voyage legs. For example, the bunker consumption - sailing speed functions for 3000-TEU ships on the leg Singapore-Jakarta and the leg Singapore-Kaohsiung are different. This can be explained by the fact that different legs have different weather conditions and sea conditions such as currents. As a consequence, the optimal sailing speeds on different legs may be different.

Table 5-2 Statistical analysis of the linear regression model

		3000-TEU SG-JK	3000-TEU SG-KS	5000-TEU HK-SG	8000-TEU YT-LA	8000-TEU TK-XM
	$a^{\wedge}$	0.014	0.010	0.004	0.011	0.037
	b	2.892	3.002	3.314	3.118	2.709
	R^2	0.964	0.960	0.977	0.993	0.990
	Adjusted R^2	0.962	0.958	0.976	0.993	0.990
p-values	H_0:b=1*	0.000	0.000	0.000	0.000	0.000
	H_0: b=3#	0.425	0.990	0.018	0.066	0.000
	Normality of residual error&	0.790	0.095	0.112	0.150	0.638

^Computed from $\ln a$; * t-Test; #t-Test; &Anderson-Darling Test

5.2 Problem Description

Given a set of ports $\mathcal{P}$, a set of weekly-serviced ship routes $\mathcal{R}$, each of which already has a given type of ship to deploy, and the container shipment demand n_{od} (TEUs), $(o,d) \in \mathcal{W}$, the liner shipping company aims to determine the number of ship to deploy on each ship route and the sailing speed on each voyage leg, in order to fulfill the container shipment demand while minimizing the total cost.

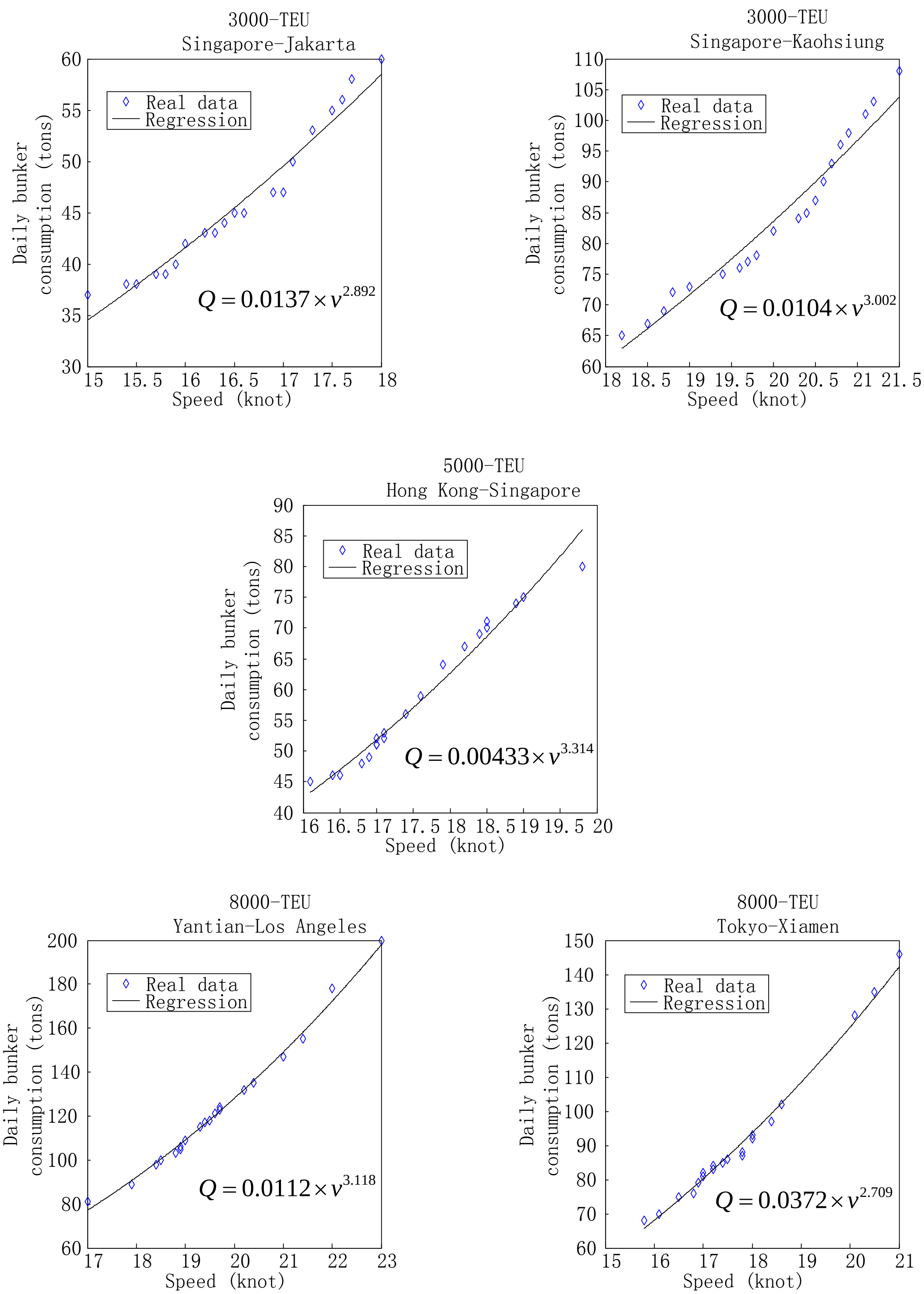


Figure 5-1 Bunker consumption – sailing speed relation

5.2.1 Weekly service, sailing speed, and bunker consumption

It is assumed that each ship route $r \in \mathcal{R}$ is deployed with a given type of ship. The capacity of a ship deployed on the ship route r is denoted by Cap_r (TEUs) and let c_r^{ship} (USD/week) be the fixed operating cost of a ship on the ship route r. The liner shipping company maintains weekly service frequency on the ship routes. That is, if the round-trip time of a ship route is 42 days, then six ships are deployed to ensure that each port of call is visited one time every week. The round-trip time consists of sea time and port time. The sailing speed on leg i of the ship route r, $r \in \mathcal{R}$, $i \in \mathcal{I}_r$, is denoted by v_{ri} (knot). The sailing speed v_{ri} should be within the economic sailing interval $[V_{ri}^{\min}, V_{ri}^{\max}]$. Let L_{ri} be the oceanic distance (n mile) of leg i of the ship route r, and the sailing time on leg i is L_{ri}/v_{ri} (hr). The port time consists of standby time for pilotage in and out and container handling time. The total standby time for pilotage in and out of ports in a round-trip is denoted by t_r^{fix} (hr). Since the weekly service frequency has to be maintained, assuming that a total of m_r ships are deployed on the ship route r, we have

$$\sum_{i \in \mathcal{I}_r} \frac{L_{ri}}{v_{ri}} + t_r^{\text{fix}} + \text{total container handling time at all portcalls} \le 168 m_r, \forall r \in \mathcal{R} \tag{5.3}$$

As the bunker consumption function is dependent on voyage legs, we denote by $g_{ri}(v_{ri})$ (tons/n mile) the bunker consumption per nautical mile at the speed v_{ri} on leg i of ship route r. We further use α^{bun} (USD/ton) to represent the bunker fuel price, the total operating cost of the ship route r (USD/week) can thus be calculated by

$$\sum_{i \in \mathcal{I}_r} \alpha^{\text{bun}} L_{ri} g_{ri}(v_{ri}) + c_r^{\text{ship}} m_r \tag{5.4}$$

5.2.2 Properties of container routes

Each container route contains full information on how containers are transported from the origin port to the destination port. The container routing problem needs to determine how many containers to be transported on each container route h, denoted by y_h (TEUs). All container shipment demand has to be fulfilled, namely

$$\sum_{h \in \mathcal{H}^{od}} y_h = n_{od}, \forall (o,d) \in \mathcal{W} \tag{5.5}$$

Let t_{rh} (hr/TEU) be the additional round-trip time posed for the ship route r by transporting one TEU according to container route h. For instance, each TEU routed on the container route h_3 shown by Eq. (2.5) will be loaded at Xiamen and discharged at Colombo on the ship route 2, thereby increasing the round-trip time of the ship route 2 by the sum of handling time for one TEU at the two ports: Xiamen and Colombo. Therefore, the total container handling time (hr) at all ports of call on the ship route r can be calculated by

$$\sum_{h \in \mathcal{H}} t_{rh} y_h, \forall r \in \mathcal{R} \tag{5.6}$$

Let c_h (USD/TEU) be the handling cost associated with transporting one TEU on a container route h. For instance, c_{h_3} for the container route h_3 in Eq. (2.5) is the sum of loading cost at Xiamen, transshipment cost at Colombo, and discharge cost at Chennai. Both t_{rh} and c_h are known parameters. The total container handling cost can be calculated by

$$\sum_{h \in \mathcal{H}} c_h y_h \tag{5.7}$$

We further let binary coefficient ρ_{hri} be 1 if containers on the container route h are transported on leg i of ship route r, and 0 otherwise. For example, the container route h_3 consists of the 2nd and the 3rd legs of the ship route 2 and the 1st leg of the ship route 3. We hence have $\rho_{h_3 22}=1$, $\rho_{h_3 23}=1$, and $\rho_{h_3 31}=1$. To ensure that ship capacity constraint is respected on all the legs of the ship route r, we should have

$$\sum_{h\in\mathcal{H}} \rho_{hri} y_h \le \text{Cap}_r, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{5.8}$$

5.2.3 Mixed-integer nonlinear programming model

The sailing speed optimization problem has the following decision variables:

m_r: Number of ships to deploy on the ship route $r\in\mathcal{R}$;

v_{ri}: Sailing speed on leg i of a ship route $r\in\mathcal{R}$;

y_h: Number of containers (TEUs) routed on container route $h\in\mathcal{H}$;

The objective aims to minimize the total operating cost. The port charges and canal dues are fixed; hence we only consider the fixed ship operating cost, bunker cost and container handling cost.

The sailing speed optimization problem can be formulated as a mixed-integer nonlinear programming model: optimal speed model 1 (OSM1):

[OSM1]
$$\min_{m_r, v_{ri}, y_h} \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r} \alpha^{\text{bun}} L_{ri} g_{ri}(v_{ri}) + \sum_{r\in\mathcal{R}} c_r^{\text{ship}} m_r + \sum_{h\in\mathcal{H}} c_h y_h \tag{5.9}$$

subject to
$$\sum_{i\in\mathcal{I}_r} \frac{L_{ri}}{v_{ri}} + \sum_{h\in\mathcal{H}} t_{rh} y_h + t_r^{\text{fix}} \le 168 m_r, \forall r\in\mathcal{R} \tag{5.10}$$

$$\sum_{h\in\mathcal{H}} \rho_{hri} y_h \le \text{Cap}_r, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{5.11}$$

$$\sum_{h\in\mathcal{H}^{od}} y_h = n_{od}, \forall (o,d)\in\mathcal{W} \tag{5.12}$$

$$V_{ri}^{\min} \le v_{ri} \le V_{ri}^{\max}, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{5.13}$$

$$m_r \in \mathbb{Z}^+, \forall r\in\mathcal{R} \tag{5.14}$$

$$y_h \geq 0, \forall h \in \mathcal{H} \tag{5.15}$$

The objective function (5.9) minimizes the total operating cost. The first term is the bunker consumption cost, the second term is the vessel operating cost, and the third term is the container handling charges. Constraint (5.10) enforces the weekly service requirement. Constraint (5.11) imposes the capacity constraint. Eq. (5.12) require that all the container shipment demands are satisfied. Constraint (5.13) defines the lower and upper bounds of sailing speed. Constraints (5.14) and (5.15) define m_r as positive integer variable, and y_h as nonnegative continuous variable, respectively.

5.3 Solution Methodology

OSM1 is a mixed-integer nonlinear programming model with nonlinear terms shown in Eqs. (5.9) and (5.10). In order to take advantage of state-of-art mixed-integer linear programming solvers, we intend to linearize the OSM1. The nonlinearity of Eqs. (5.10) can be overcome by using the reciprocal of sailing speed as a decision variable. We will prove that the nonlinear objective function (5.9) is convex and therefore an efficient outer-approximation method can be employed by using sum of many piecewise-linear functions to approximate the convex function shown in Eq. (5.9). The approximation error can be controlled within a predetermined tolerance level with a suitable outer-approximation scheme. Due to the convexity of the objective function, the piecewise-linear approximating functions are also convex. Therefore the model with the sum of the piecewise-linear approximating functions as the objective function can be transformed to a mixed-integer linear programming model. Unlike the discretization approach (e.g., Gelareh and Meng, 2010), no additional integer variables are needed. Hence, the mixed-integer linear programming model can be efficiently solved by state-of-art mixed-integer linear programming solvers such as CPLEX.

5.3.1 Convexification

We define new decision variables

$$u_{ri} = 1/v_{ri}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{5.16}$$

All the constraints expressed by Eqs. (5.10) become linear constraints with respect to the alternative decision variable u_{ri}:

$$\sum_{i \in \mathcal{I}_r} L_{ri} u_{ri} + \sum_{h \in \mathcal{H}} t_{rh} y_h + t_r^{\text{fix}} \le 168 m_r, \forall r \in \mathcal{R} \tag{5.17}$$

The constraints (5.13) can be rewritten by

$$1/V_{ri}^{\max} = U_{ri}^{\min} \le u_{ri} \le U_{ri}^{\max} = 1/V_{ri}^{\min}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{5.18}$$

The bunker consumption function $g_{ri}(v_{ri})$ can be alternatively expressed as a function of the reciprocal of the sailing speed. We define:

$$Q_{ri}(u_{ri}) = g_{ri}(1/u_{ri}), \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{5.19}$$

Hence, the model OSM1 is equivalent to the following model by rewriting objective function shown in Eq. (5.9) as a function of the alternative decision variables.

[OSM2]

$$\min_{m_r, u_{ri}, y_h} \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} \alpha^{\text{bun}} L_{ri} Q_{ri}(u_{ri}) + \sum_{r \in \mathcal{R}} c_r^{\text{ship}} m_r + \sum_{h \in \mathcal{H}} c_h y_h \tag{5.20}$$

Note that OSM2 is a mixed-integer nonlinear programming model with linear constraints and nonlinear objective function (5.20).

We exploit the special structure of OSM2 and prove that the objective function (5.20) is convex. To this end, we can prove the convexity of the function $Q_{ri}(u_{ri})$ as a consequence of the convexity, non-negativity, and univariate property of $g_{ri}(v_{ri})$. Suppose that daily bunker consumption (tons) on leg i of ship route r is $a_{ri} \times (v_{ri})^{b_{ri}}$. Noting that $Q_{ri}(u_{ri})$ represents the bunker consumption (tons) per nautical mile, we have

$$Q_{ri}(u_{ri}) = g_{ri}(1/u_{ri}) = \frac{a_{ri} \times (1/u_{ri})^{b_{ri}}}{24 \times (1/u_{ri})} = a_{ri}(u_{ri})^{1-b_{ri}} / 24, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{5.21}$$

According to the regression analysis, the coefficient b_{ri} is between 2.7 and 3.3 and $a_{ri} > 0$. Therefore $Q_{ri}(u_{ri})$ is convex in u_{ri} on the interval $[U_{ri}^{\min}, U_{ri}^{\max}]$. Even if considering that b_{ri} may be out of the range $[2.7, 3.3]$, $Q_{ri}(u_{ri})$ is convex in u_{ri} as long as $b_{ri} > 1$. Therefore it is reasonable to conclude that the objective function (5.20) is convex.

5.3.2 Outer-approximation method

In view of the convexity of the function $Q_{ri}(u_{ri})$, we use a piecewise-linear function to approximate it. To control the approximation error, we can define an absolute objective value tolerance ε (USD/week), namely, the solution obtained by approximation should not be worse than the optimal one by more than ε in the objective value. In our algorithm, we allocate the total tolerance ε among the voyage legs in proportion to the voyage distance. Define $\bar{\varepsilon}$ (tons/n mile) as:

$$\bar{\varepsilon} = \frac{\varepsilon}{\alpha^{\text{bun}}} \times \frac{1}{\sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} L_{ri}} \tag{5.22}$$

If the approximation error for $Q_{ri}(u_{ri})$ is not greater than $\bar{\varepsilon}$, then the overall objective value error is not greater than ε.

Now, we develop an algorithm that generates a piecewise-linear function with as few pieces as possible for approximating $Q_{ri}(u_{ri})$ while controlling the approximation error within $\bar{\varepsilon}$ for any $u_{ri} \in [U_{ri}^{\min}, U_{ri}^{\max}]$. Let $Q'_{ri}(u_{ri})$ denote the derivative of $Q_{ri}(u_{ri})$ at u_{ri}, namely

$$Q'_{ri}(u_{ri}) = a_{ri}(1 - b_{ri})(u_{ri})^{-b_{ri}} / 24 \tag{5.23}$$

We describe the algorithm below by using Figure 5-2 to schematically illustrate it.

Algorithm 5-1: Generation of a piecewise-linear approximation function

Step 0: Let Ψ be a set of lines, $\Psi = \varnothing$. Set $k = 0$, $u_{ri}^1 = U_{ri}^{\min}$, $Q_{ri}^1 = Q_{ri}(u_{ri}^1) - \overline{\varepsilon}$.

Step 1: Set $k = k+1$. If the following inequality holds:

$$\frac{Q_{ri}(U_{ri}^{\max}) - Q_{ri}^k}{U_{ri}^{\max} - u_{ri}^k} \geq Q_{ri}'(U_{ri}^{\max}) \tag{5.24}$$

namely, point $(u_{ri}^k, Q_{ri}(u_{ri}^k))$ is on or below the tangent line of $Q_{ri}(u_{ri})$ at $U_{ri}^{\max}$, see Figure 5-2 (a), add to Ψ line k as follows

$$Q_{ri} - Q_{ri}(U_{ri}^{\max}) = Q_{ri}'(U_{ri}^{\max})(u_{ri} - U_{ri}^{\max}) \tag{5.25}$$

and go to Step 3. Else, add to Ψ line k that passes the point (u_{ri}^k, Q_{ri}^k) and supports the epigraph of $Q_{ri}(u_{ri})$. Line k can be obtained in the following manner. Suppose that line k supports the epigraph of $Q_{ri}(u_{ri})$ at the point $(\hat{u}_{ri}^k, \hat{Q}_{ri}^k)$. According to Eq.(5.21), we have

$$\hat{Q}_{ri}^k = a_{ri}(\hat{u}_{ri}^k)^{1-b_{ri}} / 24 \tag{5.26}$$

By definition,

$$\frac{\hat{Q}_{ri}^k - Q_{ri}^k}{\hat{u}_{ri}^k - u_{ri}^k} = Q_{ri}'(\hat{u}_{ri}^k) = a_{ri}(1-b_{ri})(\hat{u}_{ri}^k)^{-b_{ri}} / 24 \tag{5.27}$$

Combining Eqs. (5.26) and (5.27), we can numerically estimate $\hat{u}_{ri}^k$ by the bisection search method. Hence, line k is defined to be

$$Q_{ri} - Q_{ri}^k = \frac{\hat{Q}_{ri}^k - Q_{ri}^k}{\hat{u}_{ri}^k - u_{ri}^k}(u_{ri} - u_{ri}^k) \tag{5.28}$$

Go to Step 2.

Step 2: For line k, when u_{ri} takes the value $U_{ri}^{\max}$, Q_{ri} will take the following value:

$$Q_{ri}^{k} + \frac{\hat{Q}_{ri}^{k} - Q_{ri}^{k}}{\hat{u}_{ri}^{k} - u_{ri}^{k}} (U_{ri}^{\max} - u_{ri}^{k}) \tag{5.29}$$

If Eq. (5.29) is not less than $Q_{ri}(U_{ri}^{\max}) - \overline{\varepsilon}$, namely, the gap between line k and function $Q_{ri}(u_{ri})$ is not greater than $\overline{\varepsilon}$ even when u_{ri} takes the value $U_{ri}^{\max}$, then we can conclude that the gap between line k and function $Q_{ri}(u_{ri})$ is not greater than $\overline{\varepsilon}$ when u_{ri} takes any value between u_{ri}^{k} and $U_{ri}^{\max}$, see Figure 5-2 (b), go to Step 3. Else, there exists exactly one point $(u_{ri}^{k+1}, Q_{ri}^{k+1})$ on line k such that $u_{ri}^{k} < u_{ri}^{k+1} < U_{ri}^{\max}$ and $Q_{ri}^{k+1} = Q_{ri}(u_{ri}^{k+1}) - \overline{\varepsilon}$, see Figure 5-2 (c). Similar to $\hat{u}_{ri}^{k}$, u_{ri}^{k+1} can also be numerically estimated by bisection search method. Go to Step 1.

Step 3: Let K_{ri} be the current value of k, namely, the number of lines in Ψ, and use the generic form

$$Q_{ri} = \text{slope}_{rik} \times u_{ri} + \text{Q-intercept}_{rik} \tag{5.30}$$

to represent a line k in Ψ that is defined by Eq. (5.25) or Eq. (5.29), $k = 1, 2 \cdots K_{ri}$. The piecewise-linear approximation function represented by $\overline{Q}_{ri}(u_{ri})$ can be written as

$$\overline{Q}_{ri}(u_{ri}) = \max\left\{\text{slope}_{rik} \times u_{ri} + \text{Q-intercept}_{rik}, k = 1, 2 \cdots K_{ri}\right\} \tag{5.31}$$

$\overline{Q}_{ri}(u_{ri})$ is schematically shown by the thickest solid line in Figure 5-2 (d).

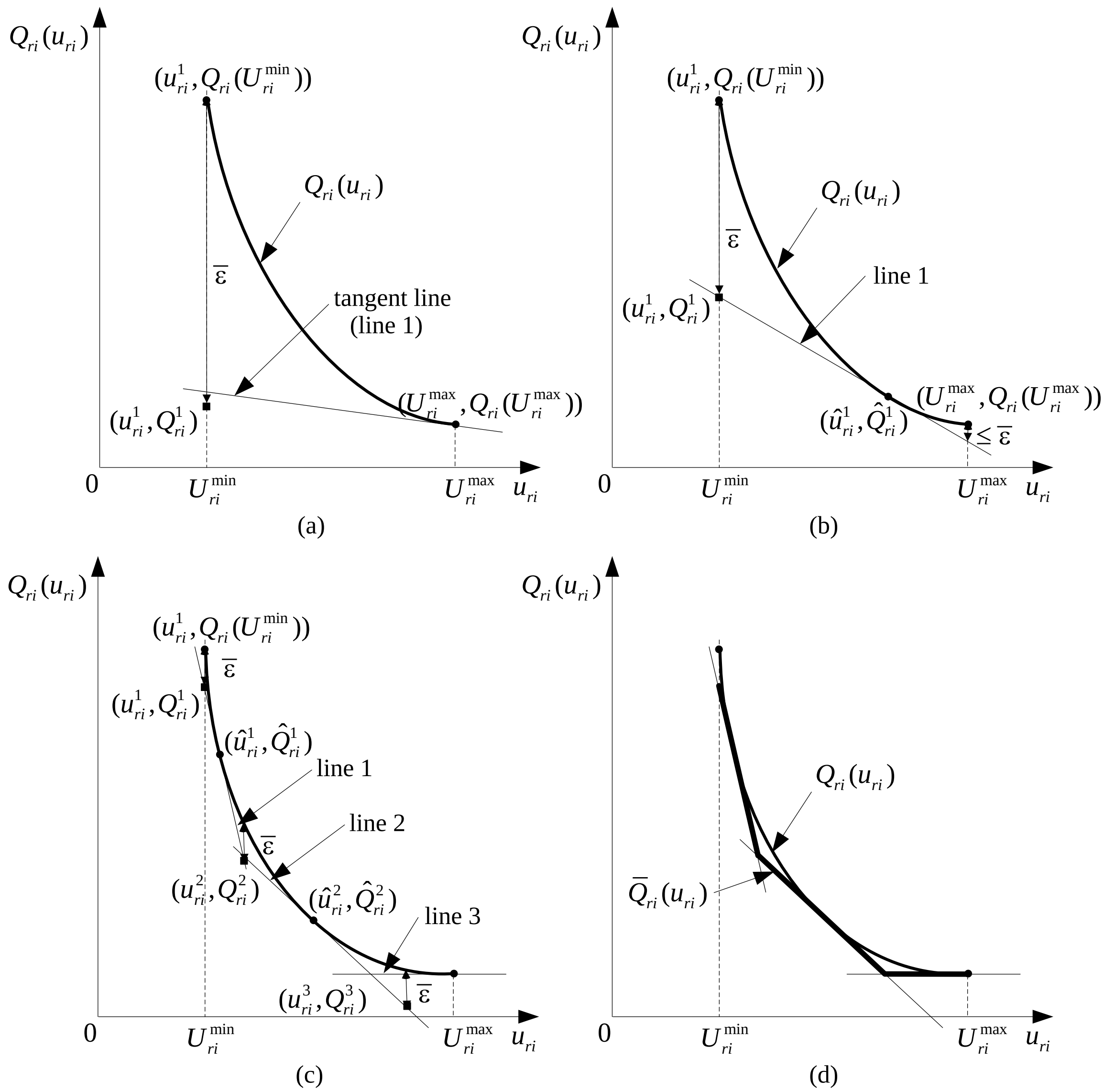


Figure 5-2 Generation of a piecewise-linear approximation function

We can replace $Q_{ri}(u_{ri})$ in the objective function (5.20) of OSM2 by $\bar{Q}_{ri}(u_{ri})$ and obtain an approximation model represented by OSM3:

[OSM3] $$\min_{m_r, u_{ri}, y_h} \sum_{r\in\mathcal{R}} \sum_{i\in I_r} \alpha^{\text{bun}} L_{ri} \bar{Q}_{ri}(u_{ri}) + \sum_{r\in\mathcal{R}} c_r^{\text{ship}} m_r + \sum_{h\in\mathcal{H}} c_h y_h \tag{5.32}$$

Algorithm 1 and the convexity of function $Q_{ri}(u_{ri})$ implies that

$$Q_{ri}(u_{ri}) - \overline{\varepsilon} \leq \overline{Q}_{ri}(u_{ri}) \leq Q_{ri}(u_{ri}), \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall u_{ri} \in [U_{ri}^{\min}, U_{ri}^{\max}] \tag{5.33}$$

Therefore, by using the piecewise-linear function $\overline{Q}_{ri}(u_{ri})$ as a surrogate for $Q_{ri}(u_{ri})$, the total objective value error of OSM3 with respect to the original model OSM1 can be controlled within the tolerance level ε.

The convexity of function $Q_{ri}(u_{ri})$ further implies the convexity of $\overline{Q}_{ri}(u_{ri})$. Therefore OSM3 can be transformed into an equivalent mixed-integer linear programming model OSM4 by introducing new decision variables Q_{ri}:

[OSM4]
$$\min_{m_r, u_{ri}, y_h, Q_{ri}} \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} \alpha^{\text{bun}} L_{ri} Q_{ri} + \sum_{r \in \mathcal{R}} c_r^{\text{ship}} m_r + \sum_{h \in \mathcal{H}} c_h y_h \tag{5.34}$$

subject to

$$Q_{ri} \geq \text{slope}_{rik} \times u_{ri} + \text{Q-intercept}_{rik}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall k = 1, 2 \cdots K_{ri} \tag{5.35}$$

OSM4 can be efficiently solved by state-of-art mixed-integer linear programming solvers such as CPLEX. Let the optimal objective value to the original mixed-integer nonlinear programming model OSM1 be Opt. The optimal objective value to the approximation model OSM4, denoted by LB, is a lower bound for Opt. Let the optimal solution to OSM4 be m_r^*, u_{ri}^*, y_h^*, Q_{ri}^*. It is evident that $m_r = m_r^*$, $v_{ri} = 1/u_{ri}^*$, $y_h = y_h^*$ is a feasible solution to OSM1, hence an upper bound for Opt can be determined by

$$UB = \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} \alpha^{\text{bun}} L_{ri} g_{ri}(1/u_{ri}^*) + \sum_{r \in \mathcal{R}} c_r^{\text{ship}} m_r^* + \sum_{h \in \mathcal{H}} c_h y_h^* \tag{5.36}$$

According to the piecewise-linear approximation scheme, it follows that

$$LB \leq Opt \leq UB \leq LB + \varepsilon \tag{5.37}$$

5.4 Computational Study

5.4.1 Description of parameter settings

To evaluate the applicability of the proposed model and the efficiency of the algorithm, we use a real-case example provided by a global liner shipping company. This example has 46 ports in an Asia-Europe-Oceania shipping network as shown in Figure 2-1. There are a total of 652 O-D port pairs with container shipment demand and the overall demand is 22,054 TEUs/week. There are 3 types of ship and 11 ship routes, as shown in Table 5-3 and Table 5-4, respectively. The 11 ship routes have 87 legs altogether. A total of 814 container routes for the 652 O-D port pairs in the shipping network are provided by the global liner shipping company. The load cost, discharge cost, and transshipment cost is assumed to be the same for all the 46 ports at 60 USD/TEU, 60 USD/TEU, and 100 USD/TEU, respectively. The container handling efficiency is assumed to be the same for all ports and is only related to ship type, as shown in Table 5-3. The pilotage in and out of any port by any ship is 4 hours. The coefficients a and b for daily bunker consumption function (5.2) on each voyage leg is calibrated using the data provided by the global liner shipping company. The mixed-integer nonlinear programming model OSM1 has a total of 11 integer decision variables m_r and 87+814=901 continuous decision variables v_{ri} and y_h. The total number of constraints is 11+87+652+2×87=924. The mixed-integer linear programming model OSM4 has an additional 87 continuous decision variables Q_{ri} and a number of approximation constraints (5.35). OSM4 is solved by CPLEX-12.1 running on a 3 GHz Dual Core PC with 4 GB of RAM.

We set the absolute objective value tolerance ε at 0.01(10^6 USD/week) and solve the model OSM4 with different bunker price settings. Given the bunker price α^{bun}, we first calculate $\overline{\varepsilon}$ according to Eq. (5.22). After that, we obtain the approximation constraints (5.35) using the proposed algorithm for each of the 87 voyage legs in the shipping network. With these constraints, we can formulate model OSM4, which is subsequently solved by CPLEX. The objective value of OSM4, LB, is a lower bound of the original problem, and an upper bound, UB, can also be obtained by Eq. (5.36).

Table 5-3 Ship fleet

Ship Type (TEUs)	3000	5000	10000
Min speed (knot)	15	20	21
Max speed (knot)	23	26	30
Container move per hour	85	95	120
Time for pilotage in and out of a port (hr)	4	4	4
Weekly operating cost (1000 USD)	76.9	115.4	173.1

Table 5-4 Ship route

No.	Ship Type	Ports of call
1	5000-TEU	Singapore → Brisbane → Sydney → Melbourne → Adelaide → Fremantle
2	5000-TEU	Xiamen → Chiwan → Hong Kong → Singapore → Port Klang → Salalah → Jeddah → Aqabah → Salalah → Singapore
3	3000-TEU	Yokohama → Tokyo → Nagoya → Kobe → Shanghai
4	3000-TEU	Ho Chi Minh → Laem Chabang → Singapore → Port Klang
5	3000-TEU	Brisbane → Sydney → Melbourne → Adelaide → Fremantle → Jakarta → Singapore
6	3000-TEU	Manila → Kaohsiung → Xiamen → Hong Kong → Yantian → Chiwan → Hong Kong
7	3000-TEU	Dalian → Xingang → Qingdao → Shanghai → Ningbo → Shanghai → Kwangyang → Busan
8	3000-TEU	Chittagong → Chennai → Colombo → Cochin → Nhava Sheva → Cochin → Colombo → Chennai
9	5000-TEU	Sokhna → Aqabah → Jeddah → Salalah → Karachi → Jebel Ali → Salalah
10	10000-TEU	Southampton → Thamesport → Hamburg → Bremerhaven → Rotterdam → Antwerp → Zeebrugge → Le Havre
11	10000-TEU	Southampton → Sokhna → Salalah → Colombo → Singapore → Hong Kong → Xiamen → Shanghai → Busan → Dalian → Xingang → Qingdao → Shanghai → Hong Kong → Singapore → Colombo → Salalah

5.4.2 Result analysis

Table 5-5 shows the computational results regarding the number of approximation constraints (5.35), the CPU time used to solve the mixed-integer linear programming model OSM4, the LB (10^6 USD/week), UB, and their relative difference. We observe that the proposed algorithm generates a piecewise-linear approximation function with a moderate number of pieces. Considering that there are 87 voyage legs, less than 10 pieces are needed to approximate $Q_{ri}(u_{ri})$. As a consequence, OSM4 can be efficiently solved (less than 0.2 second). By setting ε at 0.01, we can see that $UB - LB$ is indeed not greater than 0.01, which is consistent with (5.36). The setting of ε is actually very tight: the relative difference of LB and UB is less than 0.1% for all bunker prices.

Table 5-5 Computational results

Bunker price (USD/ton)	Number of Constraints (5.35)	CPU time (second)	LB	UB	$(UB - LB)/LB$
300	398	0.125	10.491	10.497	0.06%
400	453	0.047	11.157	11.162	0.05%
500	497	0.047	11.822	11.828	0.05%
600	546	0.047	12.488	12.493	0.04%
700	585	0.062	13.136	13.140	0.03%
800	620	0.063	13.772	13.776	0.03%
900	655	0.047	14.398	14.401	0.02%
1000	692	0.063	15.023	15.027	0.02%

Table 5-6 reports the bunker cost (10^6 USD/week), ship cost, and the number of each type of ship deployed with the bunker price varying from 300 to 1000 USD/ton. It is evident that when the bunker price is high, more ships are deployed at the optimal solution in order to lower down the sailing speed and control bunker consumption. Our model provides the optimal trade-off between bunker cost and ship cost.

Table 5-6 Ship number and ship cost at different bunker prices

Bunker price (USD/ton)	Cost (10^6 USD/week)		Ship number			
	Bunker	Ship	1500-TEU	3000-TEU	5000-TEU	Sum
300	1.992	3.577	9	10	10	29
400	2.659	3.577	9	10	10	29
500	3.324	3.577	9	10	10	29
600	3.990	3.577	9	10	10	29
700	4.445	3.769	10	11	10	31
800	5.082	3.769	10	11	10	31
900	5.630	3.846	11	11	10	32
1000	6.256	3.846	11	11	10	32

We further examine the optimal sailing speed structure. Table 5-7 shows the bunker consumption coefficients *a* and *b* and the optimal sailing speed on each leg of ship route 1 at the bunker price 800 USD/ton. We observe that at the same sailing speed, the daily bunker consumptions on legs 1 and 2 are higher than legs 3, 5, 6 since both coefficients *a* and *b* on legs 1 and 2 are larger than legs 3, 5, 6, and the optimal sailing speeds on legs 1 and 2 are lower than legs 3, 5, 6. However, one should note that higher daily bunker consumption does not necessarily mean lower sailing speed. Suppose that a ship route has two legs with bunker consumption function shown in Figure 5-3. Evidently, leg 1 has higher bunker consumption than leg 2 at the same speed. However, the same speed increment would result in a much more dramatic increase in bunker consumption on leg 2 than leg 1. Hence, ships in general should sail at higher speed on leg 1 and lower speed on leg 2. In fact, the optimal sailing speed is mainly related to the sensitivity (derivative) of bunker consumption to speed, rather than the absolute bunker consumption value. In Table 5-7, since both coefficients *a* and *b* on legs 1 and 2 are larger than legs 3, 5, 6, the bunker consumption on legs 1 and 2 are more sensitive to speed than legs 3, 5, 6.

Table 5-7 Optimal sailing speed on each leg of ship route 1 at the bunker price 800 USD/ton

Leg	1 Singapore→	2 Brisbane→	3 Sydney→	4 Melbourne→	5 Adelaide→	6 Fremantle→
a ($\times 10^{-2}$)	1.296	1.326	1.293	1.215	1.260	1.254
b	2.958	2.970	2.436	2.928	2.442	2.538
Speed (knot)	20.0	20.0	26.0	20.0	26.0	23.3

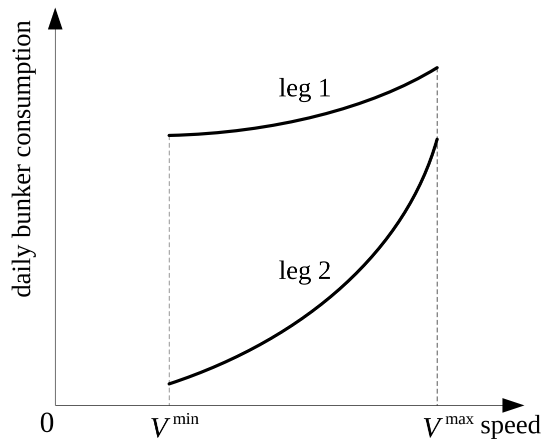


Figure 5-3 A counter-example

5.5 Conclusions

The bunker consumption - sailing speed relation for container ships is calibrated using historical operating data from a global liner shipping company. Results show that the extensively used third power relationship is indeed a good approximation. Therefore, the third power relation can be used if not enough historical data are available. Once enough historical data are available for the calibration purpose, a more accurate bunker consumption function should be used. The bunker consumption - sailing speed relation is also dependent on voyage legs. Therefore, this chapter investigated the optimal sailing speed of container ships on each leg of each ship route in a liner shipping network while considering transshipment and container routing. It is formulated as a mixed-integer nonlinear programming model. In view of the convexity, non-negativity, and univariate properties of the bunker consumption - sailing speed function, an efficient outer-approximation method is proposed to obtain an ε-optimal solution with a predetermined optimality tolerance level ε. The proposed model and algorithm is applied to a real case study for a global liner shipping company.

CHAPTER 6. SHIP ROUTE SCHEDULE DESIGN

This chapter deals with a tactical-level liner ship route schedule design problem (SDP) which aims to determine the arrival time of a ship at each port of call on a ship route and the sailing speed function on each voyage leg by taking into account time uncertainties at sea and at port. It first derives the optimality condition for the sailing speed function with sea contingency and subsequently demonstrates the convexity of the bunker consumption function. A mixed-integer nonlinear stochastic programming model is developed for the proposed liner ship route schedule design problem by minimizing the ship cost and expected bunker cost while maintaining a required transit time service level. In view of the special structure of the model, an exact cutting-plane based solution algorithm is proposed. Numerical experiments on real data provided by a global liner shipping company demonstrate that the proposed algorithm can efficiently solve real-case problems.

6.1 Problem description

The liner ship route schedule design problem aims to determine the arrival time at each port of call on each ship route to minimize the ship cost and expected bunker cost, while considering sea contingency and uncertain port time and fulfilling the port-to-port transit time constraints.

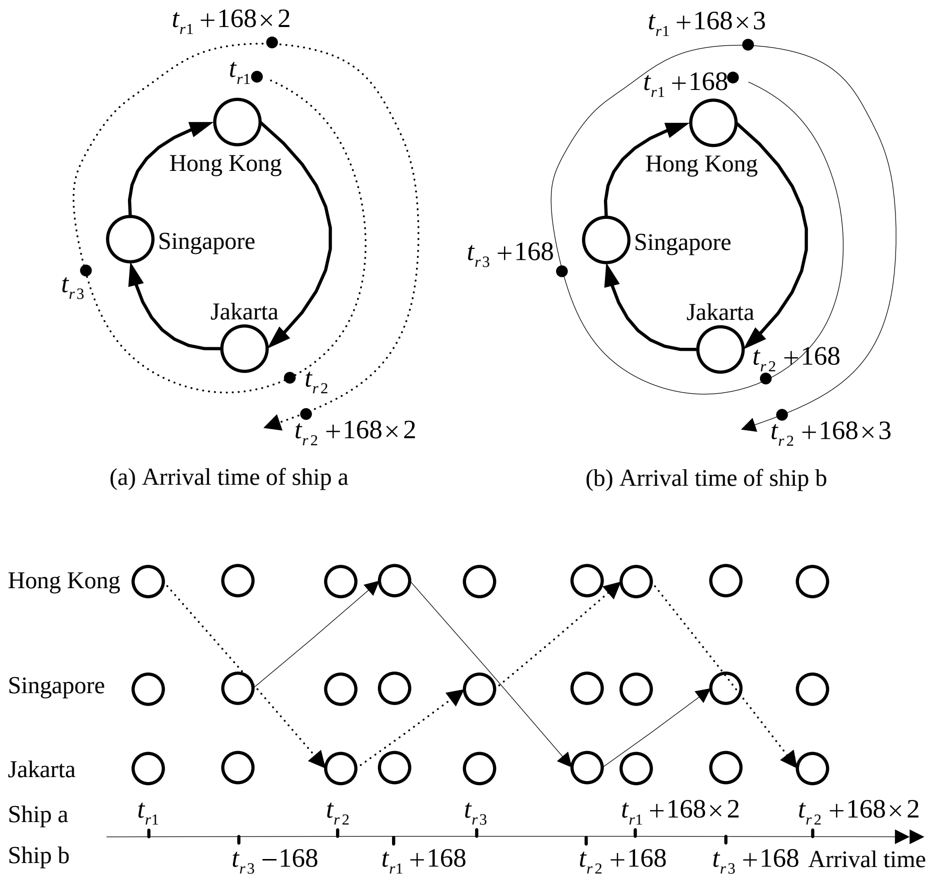
6.1.1 Weekly service-based schedule

For the sake of presentation, time 0 (hours) in this study is defined as the time 00:00 on a particular Sunday. Hence time 12:00 of the next Monday can be represented by the time 204 (hours), namely, $7 \text{ days} \times 24 \text{ hours/day} + 24 \text{ hours} + 12 \text{ hours} = 204$ (hours). Each ship route has a weekly service frequency, which means that ships on the same ship route arrive at each port of call at the same time (e.g., 9:00 a.m. on Tuesday) of each week.

Let $\mathbf{t}_r = (t_{r1}, t_{r2}, \cdots, t_{rN_r})$ (hours) denote the arrival time at the 1st, 2nd … N_r^{th} port of call of ship route r in a round trip by a ship deployed on the ship route. The round trip time is the time interval between the arrival at the 1st port of call and the next arrival at the 1st port of call. Hence, the round trip time is $t_{rN_r} - t_{r1}$ plus the inter-arrival time between the last port of call and the next arrival at the 1st port of call. To maintain a weekly service, if the round trip time is 56 days, then eight ships must be deployed on the ship route. Let m_r be the number of homogeneous ships deployed on the ship route r to maintain the weekly service. Thus, a ship completes a round-trip journey in $168m_r$ hours ($168m_r = 7$ days $\times\, 24$ hours/day $\times\, m_r$). We choose any one of these m_r ships, and the weekly service based schedule for the ship route $r \in \mathcal{R}$ can be represented by a row vector $\left(m_r, \mathbf{t}_r\right)$. Given this schedule, inter-arrival time between the i^{th} port of call and the j^{th} port of call for the same ship on ship route r, denoted by t_{rij} (hours), can be calculated by

$$t_{rij} = \begin{cases} t_{rj} - t_{ri}, & i < j \\ t_{rj} + 168 \times m_r - t_{ri}, & i > j \end{cases}, \forall r \in \mathcal{R}, \forall i, j \in \mathcal{I}_r, i \neq j \tag{6.1}$$

Figure 6-1 intuitively depicts one ship route r served by two container ships *a* and *b* to maintain the weekly service and shows the arrival time at each port of call, namely, $\left(t_{r1}, t_{r2}, t_{r3}\right)$. We define Hong Kong as the 1st port of call. Figure 6-1 (a) and (b) separately show the arrival time at each port by ship *a* and ship *b*, and Figure 6-1 (c) gives a space time network that combines the arrival times of both ships. According to Figure 6-1 (a), it can be easily seen that $t_{r2} - t_{r1}$ is the inter-arrival time between the 1st port of call and the 2nd port of call by ship *a* on the ship route r. For simplicity, hereafter when we mention inter-arrival time we mean the inter-arrival time of the same ship. In reality, the inter-arrival time between two consecutive ports of call consists of the time spent at the former port of call, plus the sailing time on the voyage leg. Ship *a* visits the 1st port of call at time t_{r1}, and the next time it visits the 1st port of call is time $t_{r1} + 168 \times 2$ because a round-trip journey time is two weeks. Consequently, the inter-arrival time between the N_rth port of call and the 1st port of call $t_{r,N_r,1} = t_{r1} + 168 \times 2 - t_{rN_r}$. According to Figure 6-1 (c) it is easy to see that ship *b* visits the 1st port of call at time $t_{r1} + 168$. As a consequence, the 1st port of call (as well as other ports of call) is visited exactly once a week.



(c) Space time network representation of arrival times for both ships

Figure 6-1 Ship route schedule

6.1.2 Random port time and sea contingency time

The total time spent by a ship deployed on a particular ship route $r \in \mathcal{R}$ at the i^{th} port of call is referred to as the *port time* at this port of call, denoted by $\tilde{\tau}_{ri}$. The port time consists of time for pilotage in and out of the port and container handling time. The container handling time, which depends on the number of containers loaded and discharged, the number of quay cranes allocated to the ship, and efficiency of quay crane operators, amongst others, is a major constituent of the port time. Because the ship route schedule design is a tactical-level planning decision, the number of containers to be handled cannot be predicted accurately. Furthermore, the number of quay cranes allocated to serve a ship relies on the container terminal operator, and the efficiency of quay crane operators fluctuates. Therefore, the port time $\tilde{\tau}_{ri}$ cannot be predicted accurately at the stage of ship route schedule design. To address this issue, we formulate $\tilde{\tau}_{ri}$ as a random variable with a predetermined probability distribution function $f_{ri}(\tau_{ri})$ defined on an interval $[\hat{\tau}_{ri}^{\min}, \hat{\tau}_{ri}^{\max}]$, where τ_{ri} represents a realization of the random variable $\tilde{\tau}_{ri}$, and $\hat{\tau}_{ri}^{\min}$ and $\hat{\tau}_{ri}^{\max}$ are two given non-negative parameters. This probability distribution function can be estimated from the historical port time when a similar ship visited the port. Defining $t_{r,N_r,N_r+1} = t_{r,N_r,1}$, the available voyage time for a container ship sailing on the i^{th} leg of the ship route r can be calculated by

$$\tilde{t}_{ri}^{\text{ava}} = t_{r,i,i+1} - \tilde{\tau}_{ri}, \forall r \in \mathcal{R}, \forall i = 1, 2, \cdots, N_r \tag{6.2}$$

Eq. (6.2) implies that the available voyage time $\tilde{t}_{ri}^{\text{ava}}$ is also a random variable.

The liner shipping company has to maintain some contingency time at sea for each leg i of each ship route r, denoted by $\hat{t}_{ri}^{\text{con}}$ (hours), as a hedge against uncertainty, like adverse weather, to ensure the schedule integrity. At the beginning of the voyage, namely, when a ship has just departed from port p_{ri}, the sailing speed has to be at least $L_{ri}/(t_{ri}^{\text{ava}} - \hat{t}_{ri}^{\text{con}})$, where L_{ri} (n mile) is the voyage distance of the i^{th} leg of ship route r, and t_{ri}^{ava} is a realization of the random variable $\tilde{t}_{ri}^{\text{ava}}$. In other words, the ship will arrive at the next port of call at least $\hat{t}_{ri}^{\text{con}}$ hours before the scheduled arrival time if the sailing speed does not change. In this context, even if the ship is delayed during the voyage, it can still arrive at the next port of call at the scheduled time as long as the delay does not exceed $\hat{t}_{ri}^{\text{con}}$ hours. After having sailed for a certain distance, the sea contingency time is reduced because a shorter residual voyage distance means less uncertainty. It is assumed that the sea contingency time required during the voyage of the leg is proportional to the residual voyage distance. Given the sailed distance $l_{ri}(t)$ by time t , $0 < l_{ri}(t) < L_{ri}$, the required sea contingency time for the remaining voyage can thus be computed by

$$\hat{t}_{ri}^{\text{con}} \times \frac{L_{ri} - l_{ri}(t)}{L_{ri}} \tag{6.3}$$

According to Eq. (6.2), the corresponding realization of the random available voyage time $\tilde{t}_{ri}^{\text{ava}}$ on the i^{th} leg can be calculated by

$$t_{ri}^{\text{ava}} = t_{r,i,i+1} - \tau_{ri}, \forall r \in \mathcal{R}, \forall i = 1, 2, \cdots, N_r \tag{6.4}$$

Since the remaining voyage distance is $L_{ri} - l_{ri}(t)$ and the residual available voyage time is $t_{ri}^{\text{ava}} - t$, the instantaneous sailing speed of a ship at the time t, denoted by $v_{ri}(t)$, can be lowered down to no less than

$$\left(L_{ri} - l_{ri}(t)\right) / \left(t_{ri}^{\text{ava}} - t - \hat{t}_{ri}^{\text{con}} \times (L_{ri} - l_{ri}(t)) / L_{ri}\right) \tag{6.5}$$

The sailing speed has direct implications on the bunker consumption. Denote the bunker consumption per hour (tons/hr) at the speed $v_{ri}(t)$, $0 \le t \le t_{ri}^{\text{ava}}$, as a function $\hat{g}_{ri}(v_{ri}(t))$. For the ease of exposition, this chapter assumes that the hourly bunker consumption $\hat{g}_{ri}(v_{ri}(t))$ is proportional to the 3rd power of sailing speed, namely:

$$\hat{g}_{ri}(v_{ri}(t)) = a_r[v_{ri}(t)]^3, v_{ri}(t) \in [V_{ri}^{\min}, V_{ri}^{\max}] \tag{6.6}$$

where a_r is a known coefficient related to the type of ship deployed on ship route r, $V_{ri}^{\min}$ and $V_{ri}^{\max}$ are the minimum and maximum economic sailing speeds, respectively.

6.1.3 Transit time of container routes

The transit time for a container route $h \in \mathcal{H}^{od}$ for port pair $(o,d) \in \mathcal{W}$, denoted by t_h (hours), is defined as the difference between the arrival time at the origin port $o \in \mathcal{P}$ of the ship that will carry the containers and the arrival time at the destination port $d \in \mathcal{P}$ of the ship (possibly not the same ship as at the origin port) that carries the containers to the destination port. The transit time consists of the time that containers are stowed onboard, and it also incorporates the connection time at transshipment ports. The onboard time is dependent on the schedule of the particular ship route and the connection time is reliant on the schedules of the two connecting ship routes.

Container transshipment operations at a particular port can occur only when this port is visited by ships at least twice a week. Given the set of ship routes $\mathcal{R}$, all the possible container transshipment operations can be represented by the following set:

$$\mathcal{Q} = \left\{ < r,s,i,j > \middle| \forall r,s \in \mathcal{R}, \forall i \in \mathcal{I}_r, \forall j \in \mathcal{I}_s, p_{ri} = p_{sj} \right\} \tag{6.7}$$

A quadruplet $< r,s,i,j > \in \mathcal{Q}$ represents a container transshipment operation from one ship on the ship route r to another ship on the ship route s at their common calling port $p_{ri} = p_{sj}$. The set $\mathcal{Q}$ can be easily identified. For example, the set of transshipment quadruplets for the network shown in Figure 2-3 is:

$$\mathcal{Q} = \left\{ \begin{array}{l} <2,3,4,1>, <3,2,1,4>, \\ <2,1,1,1>, <1,2,1,1>, \\ <2,2,3,5>, <2,2,5,3>, <2,1,3,3>, <1,2,3,3>, <2,1,5,3>, <1,2,3,5> \end{array} \right\} \tag{6.8}$$

where $<2,3,4,1>$ and $<3,2,1,4>$ correspond to transshipments at Colombo, $<2,1,1,1>$ and $<1,2,1,1>$ are transshipments at Hong Kong, and the other six quadruplets represent transshipments at Singapore.

Denote by $\hat{t}_p$ (hours) the least connection time required at a particular port p, namely, if an incoming ship arrives at port p earlier than an outgoing ship by at least $\hat{t}_p$ hours, then containers can be transshipped from the former ship to the latter; otherwise containers have to wait until the next week when a ship on the outgoing ship route arrives. The connection time for a transshipment $<r,s,i,j>\in\mathcal{Q}$, denoted by t_{rsij}^{conn} (hours), can be expressed by

$$t_{rsij}^{\text{conn}} = \min_k \{t_{sj} - t_{ri} + 168k : k \in \mathbb{Z} \text{ and } t_{sj} - t_{ri} + 168k \geq \hat{t}_{p_{ri}}\}, \forall <r,s,i,j> \in \mathcal{Q} \tag{6.9}$$

where $\mathbb{Z}$ is the set of integers.

The topological relation between a container route $h\in\mathcal{H}^{od}$ and a ship route $r\in\mathcal{R}$ can be represented by binary indicators $\delta_h^{rij}\in\{0,1\}$ and $\delta_h^{rsij}\in\{0,1\}$. $\delta_h^{rij}=1$ means that the $|j-i|$ consecutive legs, namely, the i^{th}, $(i+1)^{\text{th}}\ldots(j-1)^{\text{th}}$ legs of the ship route r are contained in container route h, if $i<j$; or the $|j+N_r-i|$ consecutive legs, namely, the i^{th}, $(i+1)^{\text{th}}\ldots {N_r}^{\text{th}}$, 1^{st}, $2^{\text{nd}}\ldots(j-1)^{\text{th}}$ legs of the ship route r are contained in container route h, if $i>j$. $\delta_h^{rsij}=1$ if container route h incorporates the transshipment $<r,s,i,j>\in\mathcal{Q}$, and 0 otherwise.

The transit time of container route $h\in\mathcal{H}^{od}$ can be calculated by

$$t_h = \sum_{r\in\mathcal{R}}\sum_{i\in I_r}\sum_{j\in I_r}\delta_h^{rij}t_{rij} + \sum_{<r,s,i,j>\in\mathcal{Q}}\delta_h^{rsij}t_{rsij}^{\text{conn}}, \forall(o,d)\in\mathcal{W}, \forall h\in\mathcal{H}^{od} \tag{6.10}$$

For example, the transit time for the container route h_3 show in Eq. (2.5) is

$$t_{h_3} = t_{224} + t_{312} + t_{2341}^{\text{conn}} \tag{6.11}$$

Note that Eq. (6.10) is applicable to all container routes no matter whether transshipment is involved. The transit time t_h should not be too long because otherwise shippers may turn to other liner container shipping companies. The liner container shipping company thus must provide a certain level of service in terms of the maximum allowable transit time for each container route h, denoted by $\hat{T}_h$ (hours), for shippers, namely,

$$t_h \le \hat{T}_h, \forall (o,d) \in \mathcal{W}, \forall h \in \mathcal{H}^{od} \tag{6.12}$$

The schedule design problem with sea contingency time and uncertain port time can be summarized as follows. Determine the weekly service based schedule $\left(m_r, \mathbf{t}_r\right)$ for each ship route $r \in \mathcal{R}$ and the sailing speed function $v_{ri}(t)$ on each voyage leg of each ship route $r \in \mathcal{R}$ for every realization of the random port time $\tilde{\tau}_{ri}$, so as to minimize the expected total cost, including the ship cost and bunker cost, while respecting sea contingency on each voyage leg and satisfying the transit time constraint for all the container routes.

6.2 Optimal Sailing Speed Function

We first investigate the optimal sailing speed and minimum bunker consumption on a specific leg i of a particular ship route $r \in \mathcal{R}$ with a given available sailing time. Given a liner ship route schedule $(m_r, \mathbf{t}_r)$ and a realization τ_{ri} of the uncertain port time $\tilde{\tau}_{ri}$, the corresponding available sailing time t_{ri}^{ava} on the i^{th} leg of the ship route r can be calculated by Eq. (6.4).

To simplify the notation, we drop the subscript r and i in the following discussion. We use L, t^{ava}, $\hat{t}^{\text{con}}$ to denote the voyage distance, available sailing time, and sea contingency time, respectively. We denote by $v(t)$, $\hat{g}(v(t))$, $C(t^{\text{ava}})$ the sailing speed at time t, hourly bunker consumption at speed $v(t)$, and minimum bunker consumption on the voyage leg given the available sailing time t^{ava}.

6.2.1 Bunker consumption minimization model

The optimal sailing speed problem (SSP) with the available sailing time t^{ava} aims to find a function $v(t)$ that minimizes the bunker consumption on the voyage leg subject to the sea contingency time constraint, and it can be formulated as a minimization model named sailing speed model (SSM).

[SSM]
$$C(t^{\text{ava}}) := \min_{v(t)} \int_0^{t^{\text{ava}}} \hat{g}(v(t))dt \tag{6.13}$$

$$v(t) \times \left[t^{\text{ava}} - t - \hat{t}^{\text{con}} \times \frac{L - \int_0^t v(\tau)d\tau}{L} \right] \geq L - \int_0^t v(\tau)d\tau, \forall 0 \leq t \leq t^{\text{ava}} \tag{6.14}$$

$$L - \int_0^t v(\tau)d\tau \geq 0, \forall 0 \leq t \leq t^{\text{ava}} \tag{6.15}$$

$$\int_0^{t^{\text{ava}}} v(\tau)d\tau = L \tag{6.16}$$

$$v(t) \in \{0\} \bigcup [V^{\min}, V^{\max}], \forall 0 \leq t \leq t^{\text{ava}} \tag{6.17}$$

The objective function seeks for the minimum bunker consumption $C(t^{\text{ava}})$ on the voyage leg. Eq. (6.14) ensures that the sea contingency is maintained, in which $\int_0^t v(\tau)d\tau$ is the sailed distance by time t. Eqs. (6.15)-(6.16) guarantee that the ship arrives at the next port of call by time t^{ava}. Eq. (6.17) enforces that the ship either sails within the economic speed range, or stands still at sea. To guarantee the feasibility of the [SSP] model, it follows that:

$$t^{\text{ava}} \geq t^{\text{ava}}_{\min} \tag{6.18}$$

where

$$t^{\text{ava}}_{\min} = \hat{t}^{\text{con}} + \frac{L}{V^{\max}} \tag{6.19}$$

SSM is a variant of the continuous optimal control problems like rocket launching (see, Stengel, 1994; Weinstock, 2008), and it can be approximated by a discrete optimization model. We divide the time interval $[0, t^{\text{ava}}]$ into Υ uniform sub-intervals and assume that the ship sails at a constant speed denoted by $\bar{v}(\xi)$ in each time sub-interval $\xi = 1, 2, \cdots, \Upsilon$. SSM can be approximated by the nonlinear programming model:

[SSM (Υ)]
$$\min_{\bar{v}(\xi)} \frac{t^{\text{ava}}}{\Upsilon} \sum_{\xi=1}^{\Upsilon} \hat{g}(\bar{v}(\xi)) \tag{6.20}$$

$$\begin{aligned} &\bar{v}(\xi) \times \left\{ t^{\text{ava}} - \frac{\xi - 1}{\Upsilon} t^{\text{ava}} - \hat{t}^{\text{con}} \times \frac{L - \sum_{\xi'=1}^{\xi-1} [\bar{v}(\xi') \times (t^{\text{ava}} / \Upsilon)]}{L} \right\} \\ &\geq L - \sum_{\xi'=1}^{\xi-1} [\bar{v}(\xi') \times (t^{\text{ava}} / \Upsilon)], \xi = 1, 2, \cdots, \Upsilon \end{aligned} \tag{6.21}$$

$$L - \sum_{\xi'=1}^{\xi-1} [\bar{v}(\xi') \times (t^{\text{ava}} / \Upsilon)] \geq 0, \xi = 2, 3, \cdots, \Upsilon \tag{6.22}$$

$$\sum_{\xi=1}^{\Upsilon} [\bar{v}(\xi) \times (t^{\text{ava}} / \Upsilon)] = L \tag{6.23}$$

$$\bar{v}(\xi) \in \{0\} \bigcup [V^{\min}, V^{\max}], \xi = 1, 2, \cdots, \Upsilon \tag{6.24}$$

6.2.2 Optimality condition

For the nonlinear programming model (6.20)-(6.24), we define a feasible speed function as follows:

$$\bar{v}^*(1) := \max\{L / (t^{\text{ava}} - \hat{t}^{\text{con}}), V^{\min}\} \tag{6.25}$$

$$\bar{v}^*(\xi) := \begin{cases} \beta(\xi), \beta(\xi) \in \{0\} \bigcup [V^{\min}, V^{\max}] \\ V^{\min}, 0 < \beta(\xi) < V^{\min} \end{cases}, \xi = 2, 3 \cdots \Upsilon \tag{6.26}$$

where

$$\beta(\xi) := \frac{L-\sum_{\xi'=1}^{\xi-1}[\overline{v}^*(\xi')\times(t^{\text{ava}}/\Upsilon)]}{t^{\text{ava}}-[(\xi-1)\times(t^{\text{ava}}/\Upsilon)]-\hat{t}^{\text{con}}\times\dfrac{L-\sum_{\xi'=1}^{\xi-1}[\overline{v}^*(\xi')\times(t^{\text{ava}}/\Upsilon)]}{L}} \tag{6.27}$$

This feasible speed function $\overline{v}^*(\xi)$ can be alternatively interpreted as follows. At the beginning of any time sub-interval ξ, $\xi=1,2\cdots\Upsilon$, the residual distance to sail $L(\xi):=L-\sum_{\xi'=1}^{\xi-1}[\overline{v}^*(\xi')\times(t^{\text{ava}}/\Upsilon)]$, the residual available time $t^{\text{ava}}(\xi):=t^{\text{ava}}-(\xi-1)\times(t^{\text{ava}}/\Upsilon)$, the required sea contingency time for the remaining voyage $\hat{t}^{\text{con}}(\xi):=\hat{t}^{\text{con}}(L-L(\xi))/L$, and the sailing speed is exactly enough to maintain the sea contingency time $\hat{t}^{\text{con}}(\xi)$, that is, $\overline{v}^*(\xi)=L(\xi)/[t^{\text{ava}}(\xi)-\hat{t}^{\text{con}}(\xi)]$. If $\overline{v}^*(\xi)\notin\{0\}\cup[V^{\min},V^{\max}]$, $\overline{v}^*(\xi)$ should be adjusted to $V^{\min}$ due to constraint (6.24). Note that $\overline{v}^*(\xi)\le V^{\max}$ due to Eqs. (6.18)-(6.19) and (6.25)-(6.26).

Figure 6-2 intuitively plots the feasible sailing speed function $\overline{v}^*(\xi)$ with three possible scenarios of the available sailing time t^{ava}. In the first scenario, t^{ava} is very small and $\overline{v}^*(\xi)>V^{\min}$ throughout the time interval $[0,t^{\text{ava}}]$. In the second scenario, $\overline{v}^*(0)>V^{\min}$, and after some time $\overline{v}^*(\xi)$ is set at $V^{\min}$. As a result, the ship arrives at the destination earlier than t^{ava}. In the third scenario, t^{ava} is very large and $L/(t^{\text{ava}}-\hat{t}^{\text{con}})$ is already smaller than $V^{\min}$. As a result, $\overline{v}^*(\xi)$ is set at $V^{\min}$ until the ship arrives at the destination (at a time earlier than t^{ava}).

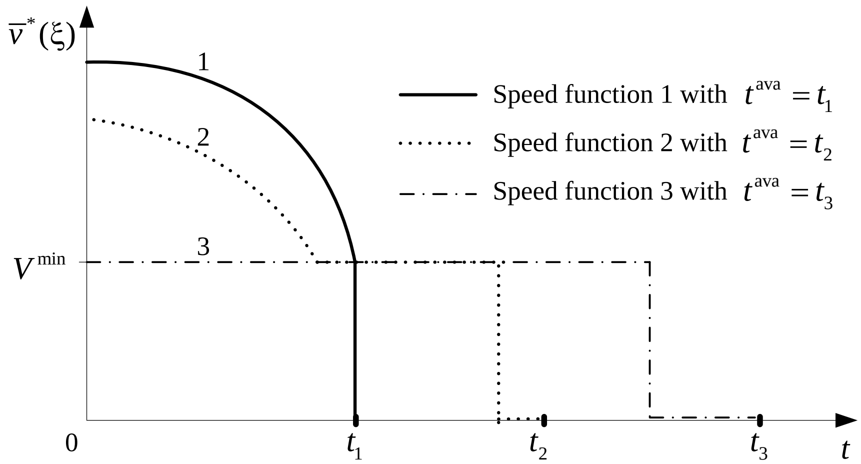


Figure 6-2 The sailing speed function with available sailing time

To simplify the exposition, unless explicitly stated, in the remainder we assume that $V^{min} = 0$. The speed function $\bar{v}^*(\xi)$ has the following property:

Lemma 6-1: $\bar{v}^*(\xi_1) \ge \bar{v}^*(\xi_2)$ for $1 \le \xi_1 < \xi_2 \le \Upsilon$.

Proof: This property can easily be demonstrated in an inductive manner by proving that (i) $\bar{v}^*(\xi_1 = 1) \ge \bar{v}^*(\xi_2 = 2)$ and (ii) $\bar{v}^*(\xi_1 + 1) \ge \bar{v}^*(\xi_1 + 2)$ if $\bar{v}^*(\xi) \ge \bar{v}^*(\xi + 1)$ for any $\xi = 1, 2 \cdots \xi_1$.□

By using the Lemma 6-1 and the third power relationship between the bunker consumption and sailing speed, we have the following important proposition:

Proposition 6-1: Function $\bar{v}^*(\xi)$ is the optimal solution to the SSM (Υ) model.

Proof: We assume that the optimal solution to the [SSP(Υ)] is $\tilde{v}(\xi)$ rather than $\bar{v}^*(\xi)$. For example, Figure 6-3 shows $\bar{v}^*(\xi)$ and $\tilde{v}(\xi)$ that are discretized into $\Upsilon = 5$ sub-intervals. In both Figure 6-3 (a) and (b), $\bar{v}^*(\xi)$ and $\tilde{v}(\xi)$ overlap when $\xi = 1$ and $\xi = 2$. $\tilde{v}(\xi)$ shown in Figure 6-3 (a) is infeasible to the model [SSP(Υ)]. This is because by definition, at the beginning of the time sub-interval $\xi = 3$, namely, at time $(\xi - 1) \times (t^{\text{ava}} / \Upsilon) = 2t^{\text{ava}} / 5$, equality holds in constraint (6.21) for $\overline{v}^*(\xi)$. Since $\tilde{v}(\xi) = \overline{v}^*(\xi)$ for $\xi = 1$ and $\xi = 2$, and $\tilde{v}(\xi) < \overline{v}^*(\xi)$ for $\xi = 3$, it is evident that constraint (6.21) does not hold at this time point for $\tilde{v}(\xi)$. That is, $\tilde{v}(\xi)$ is infeasible to the model [SSP(Υ)]. By contrast, $\tilde{v}(\xi)$ shown in Figure 6-3 (b) is feasible. In other words, any feasible $\tilde{v}(\xi)$ that is different from $\overline{v}^*(\xi)$ must satisfy: there exist a $\xi_1 \in \{1, 2 \cdots \Upsilon - 1\}$ and a $\xi_2 \in \{\xi_1 + 1, \xi_1 + 2 \cdots \Upsilon\}$ such that

$$\tilde{v}(\xi) = \overline{v}^*(\xi), \forall \xi \leq \xi_1 - 1 \tag{6.28}$$

$$\tilde{v}(\xi_1) > \overline{v}^*(\xi_1) \tag{6.29}$$

$$\tilde{v}(\xi) \geq \overline{v}^*(\xi), \forall \xi_1 + 1 \leq \xi \leq \xi_2 - 1 \tag{6.30}$$

$$\tilde{v}(\xi_2) < \overline{v}^*(\xi_2) \tag{6.31}$$

In the example of Figure 6-3 (b), $\xi_1 = 3$ and $\xi_2 = 4$. Eqs. (6.28)-(6.29) ensure that in the first time sub-interval ξ_1 where $\tilde{v}(\xi_1) \neq \overline{v}^*(\xi_1)$, we must have $\tilde{v}(\xi_1) > \overline{v}^*(\xi_1)$. If $\xi_1 = 1$, there is no Eq. (6.28) since no time sub-interval ξ satisfies $\xi \leq \xi_1 - 1$. Eqs. (6.28)-(6.29) follow due to the definition of $\overline{v}^*(\xi)$ in Eq. (6.26) and the constraint (6.21). Eqs. (6.30)-(6.31) guarantee that there exists a time sub-interval $\xi_2 > \xi_1$ such that $\tilde{v}(\xi_2) < \overline{v}^*(\xi_2)$. Eqs. (6.30)-(6.31) hold as a result of Eq. (6.23). The time sub-intervals after ξ_2, namely, after the first time sub-interval where $\tilde{v}(\xi) < \overline{v}^*(\xi)$, does not affect the proof.

We now show that the speed function $\tilde{v}(\xi)$ is not optimal. To simplify the notation, we define $v_1 = \tilde{v}(\xi_2 = 4)$, $v_2 = \overline{v}^*(\xi_2 = 4)$, $v_3 = \overline{v}^*(\xi_1 = 3)$, $v_4 = \tilde{v}(\xi_1 = 3)$. According to Lemma 1, we have

$$v_4 > v_3 \geq v_2 > v_1 \tag{6.32}$$

Recall that the bunker consumption function $\hat{g}(v) = av^3$. The total bunker consumption of $\overline{v}(\xi)$ at time sub-intervals ξ_1 and ξ_2 is $[a(v_4)^3 + a(v_1)^3] \times (t^{\text{ava}} / \Upsilon)$. Assuming that $v_4 - v_3 \geq v_2 - v_1$, we define a new speed function $\overline{v}'(\xi)$:

$$\overline{v}'(\xi) = \begin{cases} v_4 - (v_2 - v_1), \xi = \xi_1 \\ v_2, \xi = \xi_2 \\ \tilde{v}(\xi), \text{ otherwise} \end{cases} \tag{6.33}$$

It is evident that $\overline{v}'(\xi)$ is feasible to the model [SSP(Υ)]. The total bunker consumption of $\overline{v}'(\xi)$ at time sub-intervals ξ_1 and ξ_2 is $[a(v_4 - (v_2 - v_1))^3 + a(v_2)^3] \times (t^{\text{ava}} / \Upsilon)$, which is smaller than the bunker consumption of $\tilde{v}(\xi)$. At other time sub-intervals, the bunker consumption of $\overline{v}'(\xi)$ is the same as $\tilde{v}(\xi)$. Therefore $\tilde{v}(\xi)$ is not the optimal solution to the model [SSP(Υ)]. The case when $v_4 - v_3 < v_2 - v_1$ can be proved analogously. □

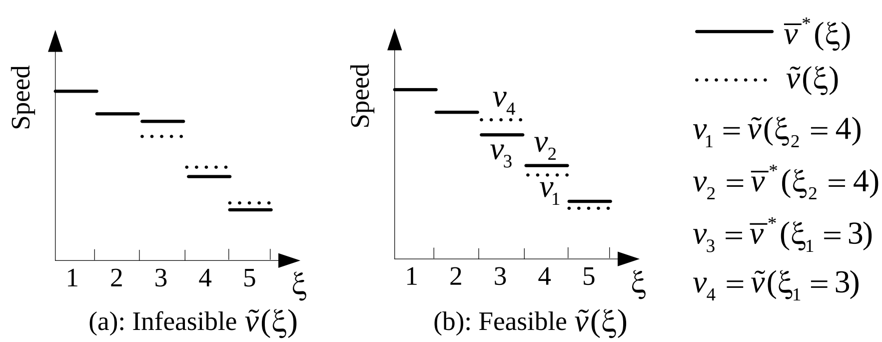


Figure 6-3 Proof of the optimal sailing speed function

Represent by $v^*(t)$ the optimal solution to SSM. When the number of sub-intervals Υ approaches infinity, $\bar{v}^*(\xi)$ converges to $v^*(t)$. Referring to Eq. (6.26) when Υ approaches infinity, the optimality condition of the optimal speed function $v^*(t)$ is:

$$v^*(t)\times\left[t^{\text{ava}}-t-\hat{t}^{\text{con}}\times\frac{L-\int_0^t v^*(\tau)d\tau}{L}\right]=L-\int_0^t v^*(\tau)d\tau,\forall 0\le t\le t^{\text{ava}} \tag{6.34}$$

The difference between the optimality condition (6.34) and constraint (6.26) is that "=" replaces "≥". By Lemma 6-1, we have:

Lemma 6-2: $v^*(t)$ decreases with t.

6.2.3 Optimal sailing speed function

Based on the optimality condition (6.34), we can obtain a deeper insight into the optimal sailing speed function $v^*(t)$. To this end, the distance travelled by a ship at the time-varying speed $v^*(t)$ by a given time t can be calculated by

$$l^*(t):=\int_0^t v^*(\tau)d\tau \tag{6.35}$$

It is straightforward to check that the function $l^*(t)$ satisfies the following differential equation and boundary condition according to Eq. (6.34):

$$\frac{dl^*(t)}{dt}=\frac{L-l^*(t)}{b_1\times l^*(t)-t+b_2},\forall 0\le t\le t^{\text{ava}} \tag{6.36}$$

where parameters $b_1=\hat{t}^{\text{con}}/L$, $b_2=t^{\text{ava}}-\hat{t}^{\text{con}}$ and

$$l^*(0)=0 \tag{6.37}$$

Eqs. (6.36)-(6.37) imply that $l^*(t)$ has the expression as follows:

$$l^*(t)=L-e^{-b_0+LambertW\left(\frac{t-b_2-Lb_1}{b_1}\times e^{b_0}\right)} \tag{6.38}$$

where b_0 is a constant determined by the boundary condition shown in Eq. (6.37) and $LambertW(\bullet)$ is the Lambert W function (Corless et al., 1996). Note that the Lambert W function $LambertW(z)$ satisfies the condition:

$$z = LambertW(z) \times e^{LambertW(z)} \tag{6.39}$$

However, it cannot be expressed in terms of elementary functions. Hence the optimal sailing speed function $v^*(t)$ does not have a closed-form expression because $v^*(t) = dl^*(t)/dt$. Fortunately, we can use $\bar{v}^*(\xi)$ shown in Eqs. (6.25)-(6.26) to approximate $v^*(t)$ in numerical computation.

6.2.4 Properties of the optimal sailing speed function

We analyze some useful properties of the optimal sailing speed function. Rather than considering the optimal sailing speed as a function of time t, we alternatively regard it as a function of the sailed distance l, $0 \le l \le L$. It is apparent that the optimal sailing speed at a particular time t (or, at a particular sailed distance l) also relies on t^{ava}. In this sub-section we treat t^{ava} as a variable, and therefore with a little abuse of notation, we represent the optimal sailing speed when the ship has sailed a distance of l, as $v^*(l, t^{\text{ava}})$. According to the optimality condition (6.34), it follows that

$$\begin{aligned} v^*(l,t^{\text{ava}}) &= \frac{L-l}{t^{\text{ava}} - \int_0^l \frac{1}{v^*(x,t^{\text{ava}})}dx - \hat{t}^{\text{con}} \bullet \frac{L-l}{L}} \\ &= \frac{L-l}{(t^{\text{ava}} - \hat{t}^{\text{con}}) + l\hat{t}^{\text{con}}/L - \int_0^l \frac{1}{v^*(x,t^{\text{ava}})}dx} \end{aligned} \tag{6.40}$$

We further have:

Lemma 6-3: $\partial v^*(l,t^{\text{ava}})/\partial t^{\text{ava}} \le 0$.

Proof: To simplify the notation, we use v in place of $v^*(l,t^{\text{ava}})$. Taking the first-order partial derivative with respect to t^{ava} for both sides of Eq. (6.40), we have

$$\frac{\partial v}{\partial t^{\text{ava}}} = -\frac{L-l}{\left((t^{\text{ava}}-\hat{t}^{\text{con}})+l\times\hat{t}^{\text{con}}/L-\int_0^l \frac{1}{v}dx\right)^2}\left\{1+\frac{1}{v^3}\times\frac{\partial v}{\partial t^{\text{ava}}}\right\} \tag{6.41}$$

Thus, if $\partial v/\partial t^{\text{ava}} > 0$, then the left-hand side of Eq. (6.41) is positive, while the right-hand side of Eq. (6.41) would be negative. Therefore $\partial v/\partial t^{\text{ava}} \le 0$.□

Proposition 6-2: $\partial^2 v^*(l,t^{\text{ava}})/\partial(t^{\text{ava}})^2 \ge 0$.

Proof: To simplify the notation, we use v in place of $v^*(l,t^{\text{ava}})$. Combining Eqs. (6.40) and (6.41) yields that

$$\frac{\partial v}{\partial t^{\text{ava}}} = -\frac{v^2}{L-l}\left\{1+\frac{1}{v^3}\times\frac{\partial v}{\partial t^{\text{ava}}}\right\} \tag{6.42}$$

Hence,

$$v\frac{\partial v}{\partial t^{\text{ava}}} = -\frac{1}{L-l}\left\{v^3+\frac{\partial v}{\partial t^{\text{ava}}}\right\} \tag{6.43}$$

Computing the partial derivative with respect to t^{ava} for both sides of Eq. (6.43), we obtain:

$$\left(\frac{\partial v}{\partial t^{\text{ava}}}\right)^2 + v\frac{\partial^2 v}{\partial(t^{\text{ava}})^2} = -\frac{1}{L-l}\left\{3v^2\frac{\partial v}{\partial t^{\text{ava}}}+\frac{\partial^2 v}{\partial(t^{\text{ava}})^2}\right\} \tag{6.44}$$

Thus,

$$\begin{aligned}&\frac{\partial^2 v}{\partial(t^{\text{ava}})^2}\\&=\frac{-3v^2(\partial v/\partial t^{\text{ava}})/(L-l)-(\partial v/\partial t^{\text{ava}})^2}{v+1/(L-l)}\\&=\frac{-(\partial v/\partial t^{\text{ava}})/(L-l)}{v+1/(L-l)}\left[3v^2+(L-l)\frac{\partial v}{\partial t^{\text{ava}}}\right]\end{aligned}\tag{6.45}$$

According to Eq. (6.43), we have

$$(L-l)\frac{\partial v}{\partial t^{\text{ava}}}=-\left\{v^2+\frac{1}{v}\bullet\frac{\partial v}{\partial t^{\text{ava}}}\right\}\tag{6.46}$$

Combining Eqs. (6.45) and (6.46), we obtain:

$$\begin{aligned}&\frac{\partial^2 v}{\partial(t^{\text{ava}})^2}\\&=\frac{-(\partial v/\partial t^{\text{ava}})/(L-l)}{v+1/(L-l)}\left[3v^2-\left(v^2+\frac{1}{v}\bullet\frac{\partial v}{\partial t^{\text{ava}}}\right)\right]\\&=\frac{-(\partial v/\partial t^{\text{ava}})/(L-l)}{v+1/(L-l)}\left[2v^2-\frac{1}{v}\bullet\frac{\partial v}{\partial t^{\text{ava}}}\right]\end{aligned}\tag{6.47}$$

Since $\partial v/\partial t^{\text{ava}}\le 0$, it follows that $\partial^2 v/\partial(t^{\text{ava}})^2\ge 0$. □

Since $v^*(t)$ is the optimal sailing speed function, the minimum bunker consumption $C(t^{\text{ava}})$ shown in Eq. (6.13) can be represented by

$$C(t^{\text{ava}})=\int_0^{t^{\text{ava}}}\hat{g}(v^*(t))dt\tag{6.48}$$

Alternatively, $C(t^{\text{ava}})$ can be expressed in terms of $v^*(l,t^{\text{ava}})$. Let $g(v^*(l,t^{\text{ava}}))$ denote the bunker consumption per nautical mile (tons/n mile) at the speed of $v^*(l,t^{\text{ava}})$. According to Eq. (6.6), it follows

$$g(v^*(l,t^{\text{ava}}))=\frac{\hat{g}(v^*(l,t^{\text{ava}}))}{v^*(l,t^{\text{ava}})}=a[v^*(l,t^{\text{ava}})]^2\tag{6.49}$$

Using bunker consumption per nautical mile $g(v^*(l,t^{\text{ava}}))$ in place of bunker consumption per hour $\hat{g}(v^*(t))$, the minimum bunker consumption function $C(t^{\text{ava}})$ can be rewritten as:

$$C(t^{\text{ava}}) = \int_0^L g(v^*(l,t^{\text{ava}}))dl \tag{6.50}$$

Lemma 6-3 and Eq. (6.50) imply the following lemma:

Lemma 6-4: The minimum bunker consumption $C(t^{\text{ava}})$ decreases with the available sailing time t^{ava}.

We further have the proposition:

Proposition 6-3: The minimum bunker consumption $C(t^{\text{ava}})$ is convex on the available sailing time t^{ava}.

Proof: Let $t_\lambda^{\text{ava}} = \lambda t_1^{\text{ava}} + (1-\lambda)t_2^{\text{ava}}$, $0 \le \lambda \le 1$. According to Eq. (6.50),

$$\begin{aligned}
& C(t_\lambda^{\text{ava}}) \\
& = \int_0^L g(v^*(l,t_\lambda^{\text{ava}}))dl \\
& = \int_0^L g(v^*(l,\lambda t_1^{\text{ava}} + (1-\lambda)t_2^{\text{ava}}))dl \\
& \le \int_0^L g(\lambda v^*(l,t_1^{\text{ava}}) + (1-\lambda)v^*(l,t_2^{\text{ava}}))dl \\
& \le \int_0^L \left[\lambda g(v^*(l,t_1^{\text{ava}})) + (1-\lambda)g(v^*(l,t_2^{\text{ava}}))\right]dl \\
& = \lambda\int_0^L g(v^*(l,t_1^{\text{ava}}))dl + (1-\lambda)\int_0^L g(v^*(l,t_2^{\text{ava}}))dl \\
& = \lambda C(t_1^{\text{ava}}) + (1-\lambda)C(t_2^{\text{ava}})
\end{aligned} \tag{6.51}$$

The first inequality holds due to the convexity of $v^*(l,t^{\text{ava}})$ by Proposition 6-2 and the monotonically increasing property of $g(v^*(l,t^{\text{ava}}))$ when $v^*(l,t^{\text{ava}}) \ge 0$ by its definition, and the second inequality holds because of the convexity of $g(v^*(l,t^{\text{ava}}))$. Thus, $C(t^{\text{ava}})$ is convex on t^{ava}.□

6.3 Mixed-Integer Nonlinear Stochastic Optimization Model

Suppose that we have identified the optimal bunker consumption function $C_{ri}(t_{ri}^{\text{ava}})$ in SSM for each leg i of each ship route r. The schedule design problem aims to determine the arrival time $\mathbf{t}_r$ and the number of ships m_r for each ship route, in order to minimize the sum of ship cost and expected total bunker cost while satisfying the transit time constraints. Let c_r^{ship} be the fixed operating cost (USD/week) of one ship deployed on ship route $r \in \mathcal{R}$, which includes the cost components such as crew cost, insurance, depreciation, which are incurred as long as a ship is in operation. c_r^{ship} relies solely on the type of ship that is deployed on ship route r and hence may be different for ships on different ship routes. Let α^{bun} (USD/ton) be the bunker price. Represent by $\mathbb{Z}^+$ the set of positive integers. The schedule design problem can be formulated as the following schedule design model (SDM):

[SDM1]
$$\min_{m_r,\mathbf{t}_r} \sum_{r\in\mathcal{R}} c_r^{\text{ship}} m_r + \alpha^{\text{bun}} \mathbb{E}\left[\sum_{r\in\mathcal{R}} \sum_{i\in\mathcal{I}_r} C_{ri}(\tilde{t}_{ri}^{\text{ava}}(m_r, \mathbf{t}_r, \tilde{\tau}_{ri}))\right] \tag{6.52}$$

subject to

$$t_{rsij}^{\text{conn}} = t_{sj} - t_{ri} + 168 z_{rsij}^{+} - 168 z_{rsij}^{-}, \forall < r,s,i,j > \in \mathcal{Q} \tag{6.53}$$

$$t_{rsij}^{\text{conn}} \geq \hat{t}_{p_{ri}}, \forall < r,s,i,j > \in \mathcal{Q} \tag{6.54}$$

$$z_{rsij}^{+}, z_{rsij}^{-} \in \mathbb{Z}^+ \bigcup \{0\}, \forall < r,s,i,j > \in \mathcal{Q} \tag{6.55}$$

$$t_{rij} = \begin{cases} t_{rj} - t_{ri}, i < j \\ t_{rj} + 168 \times m_r - t_{ri}, i > j \end{cases}, \forall r \in \mathcal{R}, \forall i,j \in \mathcal{I}_r, i \neq j \tag{6.56}$$

$$t_h = \sum_{r\in\mathcal{R}} \sum_{i\in\mathcal{I}_r} \sum_{j\in\mathcal{I}_r} \delta_h^{rij} t_{rij} + \sum_{<r,s,i,j>\in\mathcal{Q}} \delta_h^{rsij} t_{rsij}^{\text{conn}}, \forall (o,d) \in \mathcal{W}, \forall h \in \mathcal{H}^{od} \tag{6.57}$$

$$t_h \leq \hat{T}_h, \forall (o,d) \in \mathcal{W}, \forall h \in \mathcal{H}^{od} \tag{6.58}$$

$$t_{r,i,i+1} - \hat{\tau}_{ri}^{\max} \geq \hat{t}_{ri}^{\text{con}} + L_{ri} / V_{ri}^{\max}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{6.59}$$

$$t_{11} = 0 \tag{6.60}$$

$$0 \le t_{r1} \le 168, \forall r \in \mathcal{R} \setminus \{1\} \tag{6.61}$$

$$t_{ri} \ge 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{6.62}$$

$$m_r \in \mathbb{Z}^+, \forall r \in \mathcal{R} \tag{6.63}$$

The objective function (6.52) minimizes the total ship cost and expected bunker cost. Note that the optimal bunker consumption $C_{ri}(\tilde{t}_{ri}^{\text{ava}})$ is a random variable since it is dependent on $\tilde{t}_{ri}^{\text{ava}}$, which is reliant on the random port time $\tilde{\tau}_{ri}$. Constraints (6.53)-(6.55) define the connection time at transshipment ports. Eqs. (6.56)-(6.58) impose the transit time constraints. Eq. (6.59) guarantees that ships can maintain the schedule integrity in the worst case, namely, when port time takes the largest value $\hat{\tau}_{ri}^{\max}$. Constraints (6.60) and (6.61) are introduced to eliminate the symmetric solutions of the schedules due to the weekly services. Constraint (6.60) requires that the ship on the 1st ship route arrives at the 1st port of call at time 0. Constraints (6.61) enforce that the arrival time at the 1st port of call on all ship routes other than the 1st one is in the 1st week. Constraints (6.62) impose that t_{ri} are non-negative variables and constraints (6.63) require that m_r are positive integers.

The objective function (6.52) can be rewritten as follows:

$$\min_{m_r, \mathbf{t}_r} \sum_{r \in \mathcal{R}} c_r^{\text{ship}} m_r + \alpha^{\text{bun}} \sum_{r \in \mathcal{R}} \sum_{i \in \mathcal{I}_r} \bar{C}_{ri}(t_{r,i,i+1}) \tag{6.64}$$

where the function $\bar{C}_{ri}(t_{r,i,i+1})$ is the expected value of bunker consumption with respect to $t_{r,i,i+1}$, namely,

$$\begin{aligned}
&\bar{C}_{ri}(t_{r,i,i+1}) \\
&= \mathbb{E}\left[C_{ri}(\tilde{t}_{ri}^{\text{ava}}(m_r, \mathbf{t}_r, \tilde{\tau}_{ri})) \right] \\
&= \int_{\tau_{ri}=\hat{\tau}_{ri}^{\min}}^{\tau_{ri}=\hat{\tau}_{ri}^{\max}} C_{ri}(t_{ri}^{\text{ava}}(m_r, \mathbf{t}_r, \tau_{ri})) f_{ri}(\tau_{ri}) d\tau_{ri} \\
&= \int_{\tau_{ri}=\hat{\tau}_{ri}^{\min}}^{\tau_{ri}=\hat{\tau}_{ri}^{\max}} C_{ri}(t_{r,i,i+1} - \tau_{ri}) f_{ri}(\tau_{ri}) d\tau_{ri}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r
\end{aligned} \tag{6.65}$$

Proposition 6-4: $\bar{C}_{ri}(t_{r,i,i+1})$ is convex with respect to $t_{r,i,i+1}$.

Proof: Let $t_{r,i,i+1}^{\lambda} = \lambda t_{r,i,i+1}^{1} + (1-\lambda) t_{r,i,i+1}^{2}$, $0 \le \lambda \le 1$. We have

$$\begin{aligned}
&\bar{C}(t_{r,i,i+1}^{\lambda}) \\
&= \int_{\tau_{ri}=\hat{\tau}_{ri}^{\min}}^{\tau_{ri}=\hat{\tau}_{ri}^{\max}} C_{ri}(t_{r,i,i+1}^{\lambda} - \tau_{ri}) f_{ri}(\tau_{ri}) d\tau_{ri} \\
&= \int_{\tau_{ri}=\hat{\tau}_{ri}^{\min}}^{\tau_{ri}=\hat{\tau}_{ri}^{\max}} C_{ri}[\lambda t_{r,i,i+1}^{1} + (1-\lambda) t_{r,i,i+1}^{2} - \tau_{ri}] f_{ri}(\tau_{ri}) d\tau_{ri} \\
&= \int_{\tau_{ri}=\hat{\tau}_{ri}^{\min}}^{\tau_{ri}=\hat{\tau}_{ri}^{\max}} C_{ri}[\lambda (t_{r,i,i+1}^{1} - \tau_{ri}) + (1-\lambda)(t_{r,i,i+1}^{2} - \tau_{ri})] f_{ri}(\tau_{ri}) d\tau_{ri} \\
&\le \int_{\tau_{ri}=\hat{\tau}_{ri}^{\min}}^{\tau_{ri}=\hat{\tau}_{ri}^{\max}} [\lambda C_{ri}(t_{r,i,i+1}^{1} - \tau_{ri}) + (1-\lambda) C_{ri}(t_{r,i,i+1}^{2} - \tau_{ri})] f_{ri}(\tau_{ri}) d\tau_{ri} \\
&= \lambda \int_{\tau_{ri}=\hat{\tau}_{ri}^{\min}}^{\tau_{ri}=\hat{\tau}_{ri}^{\max}} C_{ri}(t_{r,i,i+1}^{1} - \tau_{ri}) f_{ri}(\tau_{ri}) d\tau_{ri} + (1-\lambda) \int_{\tau_{ri}=\hat{\tau}_{ri}^{\min}}^{\tau_{ri}=\hat{\tau}_{ri}^{\max}} C_{ri}(t_{r,i,i+1}^{2} - \tau_{ri}) f_{ri}(\tau_{ri}) d\tau_{ri} \\
&= \lambda \bar{C}(t_{r,i,i+1}^{1}) + (1-\lambda) \bar{C}(t_{r,i,i+1}^{2})
\end{aligned} \tag{6.66}$$

The inequality holds due to the convexity of $C_{ri}(t_{ri}^{\text{ava}})$ by Proposition 6-3. Therefore Proposition 6-4 holds. □

Proposition 6-4 also implies the convexity of the nonlinear stochastic programming model SDM1.

The constraints (6.58) in SDM1 may be over conservative in practice. The liner shipping company may also define a certain level of service $1-\alpha$, $0 \le \alpha < 1$, where at least $(1-\alpha)\sum_{(o,d)\in\mathcal{W}} |\mathcal{H}^{od}|$ container routes have to satisfy the transit time constraints. This can be achieved by replacing constraints (6.58) with the following relaxed constraints:

$$t_h \le \hat{T}_h + Mx_h, \forall (o,d) \in \mathcal{W}, \forall h \in \mathcal{H}^{od} \tag{6.67}$$

$$\sum_{(o,d)\in \mathcal{W}, h\in \mathcal{H}^{od}} x_h \le \left\lfloor \alpha \times \sum_{(o,d)\in \mathcal{W}} |\mathcal{H}^{od}| \right\rfloor \tag{6.68}$$

$$x_h \in \{0,1\}, \forall (o,d) \in \mathcal{W}, \forall h \in \mathcal{H}^{od} \tag{6.69}$$

where M is a large number and $\lfloor a \rfloor$ represents the largest integer not greater than a.

6.4 Solution Algorithm

The difficulty in solving the nonlinear convex stochastic programming model SDM1 lies in that the function $\bar{C}_{ri}(t_{r,i,i+1})$ does not have analytical expression. Nevertheless, in view of its convexity, we first propose a procedure to numerically estimate $\bar{C}_{ri}(t_{r,i,i+1})$ and subsequently develop a tailored cutting-plane based algorithm for solving the nonlinear convex stochastic model SDM1.

6.4.1 A numerical procedure for calculating bunker consumption

We evaluate $\bar{C}_{ri}(t_{r,i,i+1})$ at many discrete points of $t_{r,i,i+1}$ and thereby derive a piecewise linear function as an approximation, as shown in Figure 6-4. The points of $t_{r,i,i+1}$ used in the piecewise linear function are denoted by $t^1_{r,i,i+1} = t^{\min}_{r,i,i+1}$, $t^2_{r,i,i+1} \ldots t^{\kappa}_{r,i,i+1}$, $t^{\kappa+1}_{r,i,i+1} \ldots t^{K_{ri}}_{r,i,i+1} = t^{\max}_{r,i,i+1}$, where K_{ri} is a very large number representing the number of points used in the piecewise linear function for leg i of ship route r. Note that for clarity only a few points are shown in Figure 6-4. The minimum time $t^{\min}_{r,i,i+1}$ can be determined by Eq. (6.19) and the maximum port time $\hat{\tau}^{\max}_{ri}$:

$$t^{\min}_{r,i,i+1} = \hat{t}^{\text{con}}_{ri} + \frac{L_{ri}}{V^{\max}_{ri}} + \hat{\tau}^{\max}_{ri}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{6.70}$$

Similarly, $t_{r,i,i+1}^{\max}$ is achieved when the ship can always sail at its lowest speed:

$$t_{r,i,i+1}^{\max} = \hat{t}_{ri}^{\text{con}} + \frac{L_{ri}}{V_{ri}^{\min}} + \hat{\tau}_{ri}^{\max}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{6.71}$$

It is easy to show that $\bar{C}_{ri}(t_{r,i,i+1}) = \bar{C}_{ri}(t_{r,i,i+1}^{\max})$ if $t_{r,i,i+1} \geq t_{r,i,i+1}^{\max}$. Thus $\bar{C}_{ri}(t_{r,i,i+1})$ is not evaluated at any time $t_{r,i,i+1} > t_{r,i,i+1}^{\max}$ in Figure 6-4.

Given $t_{r,i,i+1}$, obtaining $\bar{C}_{ri}(t_{r,i,i+1})$ involves a two-dimensional integration: the outer integration is on the random port time $\tilde{\tau}_{ri}$ as in Eq. (6.65) and the inner integration is to find the value $C_{ri}(t_{ri}^{\text{ava}})$ with given t_{ri}^{ava}. According to Eqs. (6.6) and (6.20), $C_{ri}(t_{ri}^{\text{ava}})$ can be approximated by

$$C_{ri}(t_{ri}^{\text{ava}}) = \sum_{\xi=1}^{\Upsilon} a_r (v_{ri}^*(\xi))^3 \bullet t_{ri}^{\text{ava}} / \Upsilon, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{6.72}$$

The outer integration can be approximated by discretizing the port time range $[\hat{\tau}_{ri}^{\min}, \hat{\tau}_{ri}^{\max}]$ analogous to the inner integration. Such a two-dimensional integration can be efficiently calculated at a high precision. Therefore, we assume that there is no gap between the obtained piecewise linear function and the function $\bar{C}_{ri}(t_{r,i,i+1})$.

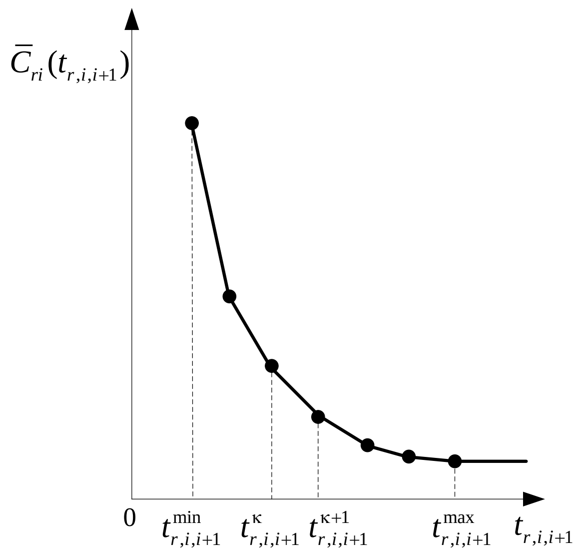


Figure 6-4 Relation between the inter-arrival time and expected bunker consumption

6.4.2 A cutting-plane based solution algorithm

The brutal-force approach via dynamic programming is unworkable for the liner ship route schedule design problem. In fact, although the bunker consumption on a leg depends solely on the designed inter-arrival time between the two end ports of call, $t_{r,i,i+1}$, the number of states explodes exponentially with the increase in voyage legs as a result of the linking constraints (6.57)-(6.58). Therefore, new solution approaches have to be proposed.

The piecewise linear function shown in Figure 6-4 is convex due to the convexity of $\bar{C}_{ri}(t_{r,i,i+1})$. Therefore in theory SDM1 can be transformed into a mixed-integer linear optimization model. Nevertheless, we must use quite a large number of points to approximate $\bar{C}_{ri}(t_{r,i,i+1})$ in order to achieve a high accuracy. Under this circumstance, SDM1 will have too many constraints defining the line segments for the piecewise linear function. These constraints would drastically increase the computational burden and computer memory requirement. To circumvent this difficulty, we observe that at most two such constraints are binding on each voyage leg at the optimality. Therefore we propose a cutting-plane based algorithm to solve SDM1. This cutting-plane based algorithm dynamically generates potentially binding cuts. As a result, only a very small subset of the line segments in the piecewise linear function is used. This algorithm may terminate finitely since $\bar{C}_{ri}(t_{r,i,i+1})$ is piecewise linear. In practical applications, the liner shipping company can define an absolute objective value tolerance ε (USD/week). When the difference in objective value between the obtained solution and the optimal one is below ε, the algorithm terminates. We can allocate the total tolerance ε among the voyage legs in proportion to the voyage distance. Define $\bar{\varepsilon}_{ri}$ (tons) as:

$$\bar{\varepsilon}_{ri} = \frac{\varepsilon}{\alpha^{\text{bun}}} \bullet \frac{L_{ri}}{\sum_{s\in\mathcal{R}}\sum_{j\in\mathcal{I}_s} L_{sj}}, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{6.73}$$

$\bar{\varepsilon}_{ri}$ is the maximum allowable bunker consumption error on leg i of ship route r in the cutting-plane based algorithm. Note that if we set $\varepsilon = 0$, the algorithm finds the optimal solution. The cutting-plane based algorithm is as follows.

Algorithm 6-1: Cutting-plane based algorithm:

Step 1: Reformulate SDM1 by introducing new decision variables $\bar{C}_{ri}$ as:

[SDM2]
$$\min_{m_r, \mathbf{t}_r, \bar{C}_{ri}} \sum_{r\in\mathcal{R}} c_r^{\text{ship}} m_r + \alpha^{\text{bun}} \sum_{r\in\mathcal{R}}\sum_{i\in\mathcal{I}_r} \bar{C}_{ri} \tag{6.74}$$

$$\bar{C}_{ri} \geq 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r \tag{6.75}$$

with the same constraints as SDM1.

Step 2: Solve the mixed-integer linear programming model SDM2 to find the optimal solution denoted by m_r^*, $\mathbf{t}_r^*$, $\bar{C}_{ri}^*$. Recall that $\bar{C}_{ri}(t_{r,i,i+1})$ is the piecewise linear function shown in Figure 6-4. We check the error in bunker consumption for each leg

$i \in \mathcal{I}_r$ on each ship route $r \in \mathcal{R}$. If $\bar{C}_{ri}(t^*_{r,i,i+1}) - \bar{C}^*_{ri} > \bar{\varepsilon}_{ri}$, we add a new cut to the model SDM2 in such a manner: If $t^*_{r,i,i+1} \geq t^{\max}_{r,i,i+1}$, add the cut:

$$\bar{C}_{ri} \geq \bar{C}_{ri}(t^{\max}_{r,i,i+1}) \tag{6.76}$$

Otherwise, there exists $1 \leq \kappa \leq K_{ri} - 1$ such that $t^{\kappa}_{r,i,i+1} \leq t^*_{r,i,i+1} < t^{\kappa+1}_{r,i,i+1}$. Add the following cut to SDM2:

$$\bar{C}_{ri} \geq \bar{C}_{ri}(t^{\kappa}_{r,i,i+1}) + \frac{\bar{C}_{ri}(t^{\kappa+1}_{r,i,i+1}) - \bar{C}_{ri}(t^{\kappa}_{r,i,i+1})}{t^{\kappa+1}_{r,i,i+1} - t^{\kappa}_{r,i,i+1}} \times (t_{r,i,i+1} - t^{\kappa}_{r,i,i+1}) \tag{6.77}$$

If there are new cuts added, repeat Step 2. Otherwise, the gap between the obtained solution and the optimal one is less than ε, stop.

6.5 Case Study

We use an Asia-Europe-Oceania shipping network provided by a global liner shipping company to assess the models and solution algorithms proposed in this study. The network has a total of 46 ports in Asia, Europe, and Oceania, as shown in Figure 2-1. These 46 ports are served by 11 ship routes with three types of ships, as shown in Table 6-1 and Table 6-2, respectively. There are a total of 100 container routes in the shipping network.

A basic setting of the case study is as follows: the bunker price is 300 USD/ton; the contingency time for each leg is proportional to the voyage distance at 10 hours per 1000 n miles; the random port time is uniformly distributed and the maximum (minimum) port time is 10% larger (smaller) than the average port time shown in Table 6-2; the tolerance ε is taken as 0.1% of the estimated total cost. The total cost is estimated as the sum of the estimated ship cost and estimated bunker cost. The estimated ship cost is the average of the maximum ship cost (when all ships sail at the lowest speed) and the minimum ship cost (when all ships sail at the highest speed). The estimated bunker cost is the average of the maximum bunker cost (when all ships sail at the highest speed) and the minimum bunker cost (when all ships sail at the lowest speed).

The mixed-integer linear programming model [SDM2] is solved by CPLEX-12.1 running on a 3.2 GHz Dual Core PC with 4 GB of RAM. The relative mixed-integer programming (MIP) gap tolerance is set at 0.1%, and therefore the obtained solution is at most 0.2% worse than the optimal one. In order to further improve the computational performance of the cutting-plane based algorithm, we first set ε at 1% of the estimated total cost and set the relative MIP gap tolerance in CPLEX at 1% and solve [SDM2] (fast cut-generation stage). When all cuts have been generated, we reset ε at 0.1% of the estimated total cost and the relative MIP gap tolerance in CPLEX at 0.1% and solve [SDM2] together with the generated cuts (refining stage). The advantage of this setting is that a number of high quality cuts will be generated without considerable computational efforts in the fast cut-generation stage.

With the above basic parameter settings, the fast cut-generation stage generates 258 cuts after 8 iterations, and the refining stage generates another 145 cuts after 8 iterations. The total CPU time for the cutting-plane based algorithm is 1.6 seconds. Therefore this algorithm is very efficient to find high quality solutions. Next, we analyze the solutions in different parameter settings to gain managerial insights for liner shipping companies.

Table 6-1 Ship fleet profile

Ship Type	1	2	3
TEU Capacity	3000	5000	10000
Min Speed (knot)	15	20	21
Max Speed (knot)	23	26	30
Bunker Consumption Coefficient a (1e-4)	3.95	4.64	5.33
Operating Cost (million USD/(year•ship))	2.0	3.0	4.5

Table 6-2 Ship route information

No.	Ship Type	Ports of call (average port time, hour)
1	5000-TEU	Singapore(12) → Brisbane(11) → Sydney(12) → Melbourne(12) → Adelaide(12) → Fremantle(11) → Singapore
2	5000-TEU	Xiamen(12) → Chiwan(14) → Hong Kong(13) → Singapore(12) → Port Klang(12) → Salalah(12) → Jeddah(13) → Aqabah(12) → Salalah(12) → Singapore(12) → Xiamen
3	3000-TEU	Yokohama(8) → Tokyo(8) → Nagoya(8) → Kobe(8) → Shanghai(14) → Yokohama
4	3000-TEU	Ho Chi Minh(8) → Laem Chabang(9) → Singapore(10) → Port Klang(10) → Ho Chi Minh
5	3000-TEU	Brisbane(9) → Sydney(10) → Melbourne(10) → Adelaide(10) → Fremantle(9) → Jakarta(10) → Singapore(10) → Brisbane
6	3000-TEU	Manila(8) → Kaohsiung(8) → Xiamen(10) → Hong Kong(11) → Yantian(8) → Chiwan(12) → Hong Kong(11) → Manila
7	3000-TEU	Dalian(8) → Xingang(8) → Qingdao(8) → Shanghai(14) → Ningbo(10) → Shanghai(14) → Kwangyang(8) → Busan(9) → Dalian
8	3000-TEU	Chittagong(8) → Chennai(9) → Colombo(9) → Cochin(8) → Nhava Sheva(9) → Cochin(8) → Colombo(9) → Chennai(9) → Chittagong
9	5000-TEU	Sokhna(15) → Aqabah(12) → Jeddah(13) → Salalah(12) → Karachi(10) → Jebel Ali(11) → Salalah(12) → Sokhna
10	10000-TEU	Southampton(21) → Thamesport(15) → Hamburg(17) → Bremerhaven(17) → Rotterdam(16) → Antwerp(19) → Zeebrugge(15) → Le Havre(15) → Southampton
11	10000-TEU	Southampton(21) → Sokhna(20) → Salalah(17) → Colombo(16) → Singapore(17) → Hong Kong(18) → Xiamen(17) → Shanghai(21) → Busan(16) → Dalian(15) → Xingang(15) → Qingdao(15) → Shanghai(21) → Hong Kong(18) → Singapore(17) → Colombo(16) → Salalah(17) → Southampton

6.5.1 Impact analysis of the maximum allowable transit time

First, we make a sensitivity analysis of the solution with the maximum allowable transit time. We vary the maximum allowable transit time for all the container routes with the incremental of 2 days. Figure 6-5 shows the total cost and the number of deployed ships, where “0” on the horizontal axis corresponds to the maximum allowable transit time in the basic setting, 100% and 95% represent the level of service, namely, 100% means the maximum allowable transit time has to be satisfied for all the container routes, and 95% implies that at most (1-95%)×100=5 container routes may violate this constraint. It can be seen that with the shortening of the maximum allowable transit time from 0 to -10 on the horizontal axis in Figure 6-5, the number of deployed ships drops because the ships have to sail at a higher speed. At the same time, the total cost increases due to the dramatic increase in bunker cost. Given the same maximum allowable transit time, the 95% level of service may require fewer ships and incur lower cost than the 100% level of service. When the maximum allowable transit time is very tight (corresponding to -10 on the horizontal axis in Figure 6-5), the 100% level of service can no longer be fulfilled, while with the 95% level of service, the total cost only increases moderately. When the maximum allowable transit time is very slack (e.g., -4, -2, and 0 on the horizontal axis in Figure 6-5), a slight change in the maximum allowable transit time and the level of service has no impact on the number of ships deployed or the total cost.

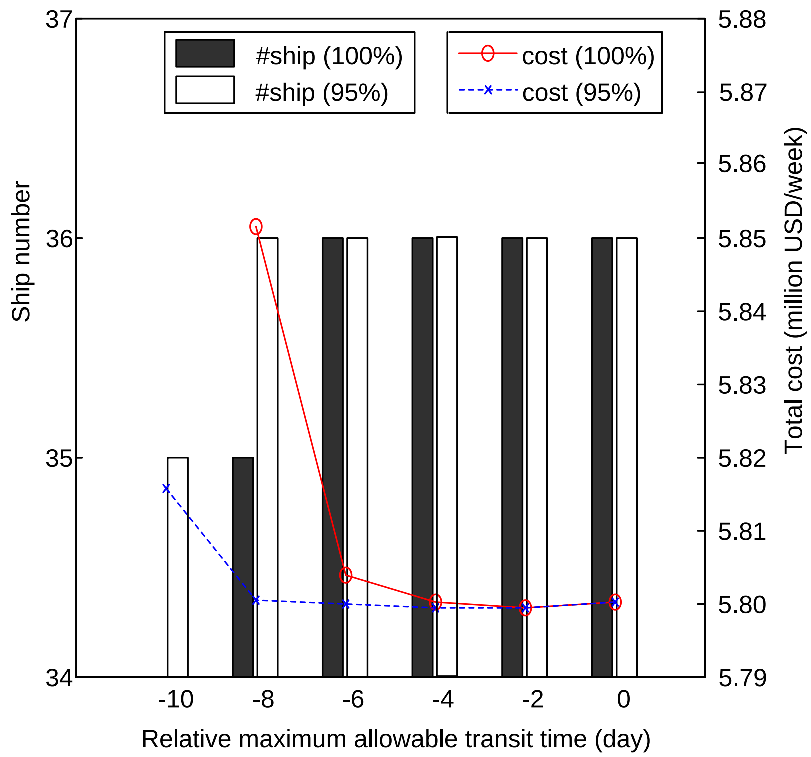


Figure 6-5 Sensitivity of ship number and total cost with transit time

6.5.2 Change in bunker price and sea contingency

The bunker price and sea contingency also have implications on the schedule design. Fig. 6 illustrates the total bunker consumption and the number of deployed ships at the bunker price of 300 USD/ton and 600 USD/ton for different sea contingencies. It should be pointed out that the total cost increases with sea contingency and bunker price and is not plotted in Figure 6-6. According to Figure 6-6, it can be observed that the liner shipping company will choose to sail at lower speeds to control the bunker consumption when the bunker price is high. This observation is consistent with industrial practices. For example, both the Grand Alliance and CMA CGM each decided to add a ninth ship to one of their respective Asia-Europe routes during the summer of 2006 to cope with the high bunker price. The resulting fuel cost savings generated by each of the other eight ships more than compensated for the cost of hiring and operating the ninth ships (Vernimmen et al., 2007). Another example is that in 2009, to deal with the decreased container shipment demand and the large ship fleet, liner shipping companies took measures including slow or super slow steaming in an attempt to curb shipping capacity and thus boost the freight rate (UNCTAD, 2010).

Another interesting finding is that the increase in sea contingency leads to either the increase in the number of deployed ships or the increase in sailing speed. As a consequence, when the sea contingency increases and more ships are deployed, the sailing speed may decrease. Therefore we cannot reach a straightforward conclusion regarding the relation between sea contingency and sailing speed. This highlights the inherent difficulty of the problem.

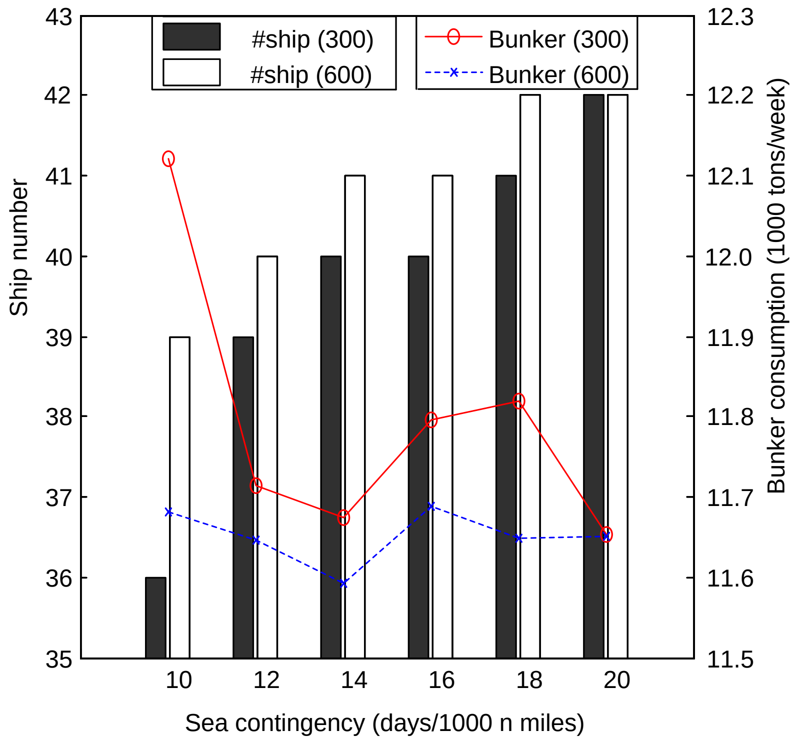


Figure 6-6 Sensitivity of bunker consumption and ship number with bunker price and sea contingency

6.5.3 Different port time profiles

We investigate the effect of average port time and the range of port time $\tau_{ri}^{\max} - \tau_{ri}^{\min}$ on the optimal solution. Figure 6-7 shows the bunker consumption and the number of deployed ships with the increase of average port time (the range of port time keeps constant), where "0%" on the horizontal axis corresponds to the port time range in the basic setting. Figure 6-8 presents the bunker consumption and the number of deployed ships with the increase in the range of port time (The average port time keeps constant), where the range 10% denotes that the maximum and minimum port times are 10% larger and smaller than the average, respectively. The total cost increases with the average port time and the range of port time, and is not plotted.

Compared with larger sea contingency as shown in Figure 6-6, longer average port time and a wider port time range have a similar effect on the optimal ship number and sailing speed. That is, either the number of ships increases, or ships sail at higher speeds. The increase in the average port time, port time range, and sea contingency leads to a non-decreasing optimal number of ships deployed, but does not necessarily result in a higher sailing speed.

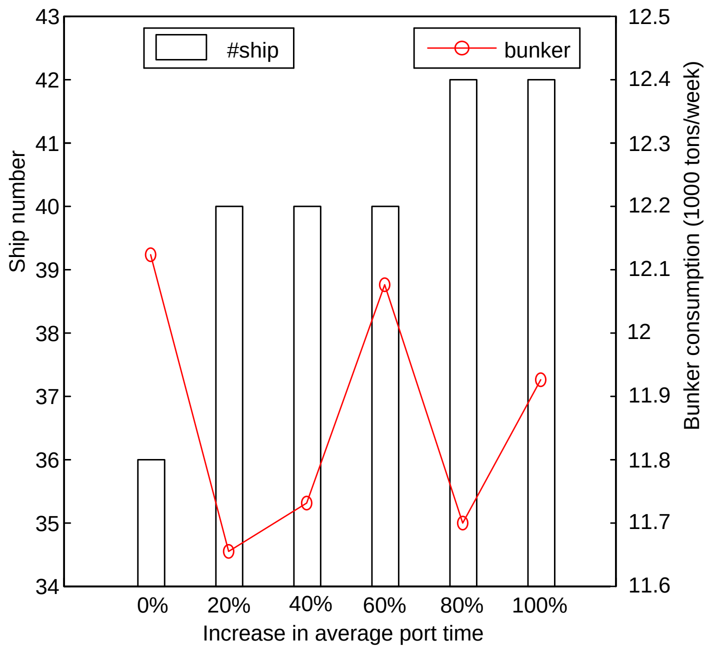


Figure 6-7 Sensitivity of bunker consumption and ship number with average port time

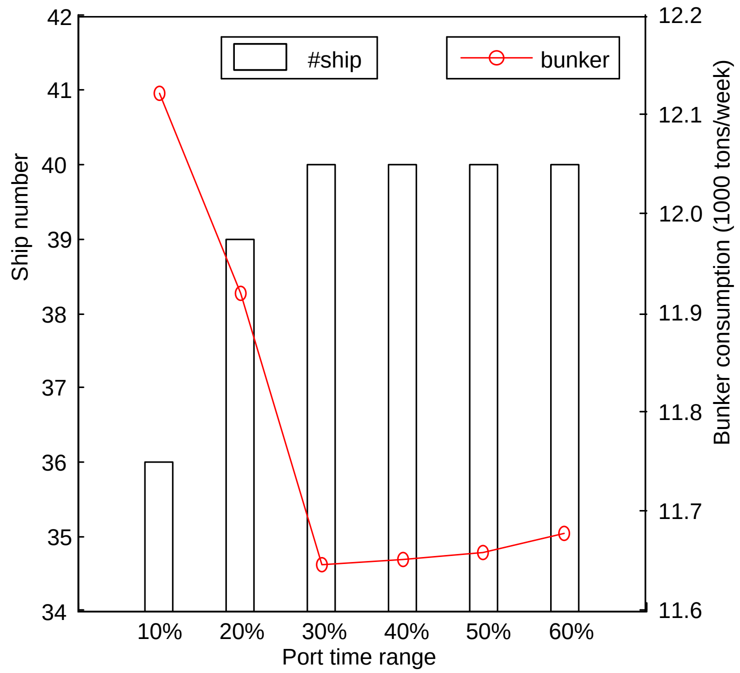


Figure 6-8 Sensitivity of bunker consumption and ship number with range of port time

6.6 Conclusions

This chapter addresses a practical liner ship route schedule design problem accounting for uncertainties at sea and at port. The optimality condition for the sailing speed with sea contingency is derived by analyzing the characteristics of the bunker consumption – sailing speed relation. Based on the optimal sailing speed function, the convexity of bunker consumption on the available sailing time for each voyage leg is proved. A mixed-integer nonlinear stochastic programming model is developed to minimize the ship cost and expected bunker cost while satisfying transit time constraints. An exact and efficient cutting-plane based solution algorithm is proposed. Experiments on data provided by a global liner shipping company yield the following managerial insights. First, when the maximum allowable transit time is tight, allowing a small percentage of container routes to violate the transit time requirement not only reduces the total cost but also provides feasible schedules when an extremely tight transit time is required. Second, liner shipping companies should lower down sailing speed of ships to control bunker consumption in case of high bunker price. Third, with the increase in sea contingency, average port time, and port time range, the optimal number of ships deployed will not decrease, however, the optimal sailing speed may increase or decrease.

CHAPTER 7. LINER SHIPPING NETWORK DESIGN

This chapter studies a realistic liner shipping network design problem while considering practical operations and features, including multi-type containers, container transshipment operations, empty container repositioning, origin-to-destination transit time constraint, consistent services with the current network, and joint services with other liner shipping companies. It first examines the laden and empty container shipment demand. It proceeds to investigate the routing of laden containers while considering the transit time constraint. Two approaches are proposed. The first one is based on global and regional hubs. It generates container routes efficiently whereas cannot guarantee optimality. The second one is reliant on the construction of a liner shipping operational network and an integer linear programming model. It is an exact algorithm while the computational time is longer. Given a set of candidate ship routes, including ship routes that must be used, ship routes a minimum number of which must be used, and ship routes that are completely optional, a mixed-integer linear programming model is presented, which gives the ship routes that should be used and the laden and empty container flow in the resulting network. After that, the resulting network is further improved by changing existing ship routes, adding new ship routes, and removing ship routes. Finally, a real case study based on the global shipping network of a liner shipping company, consisting of 166 ports, is reported.

7.1 Problem Description

The liner shipping network design problem (NDP) aims to construct the ship routes (port rotations, type and number of ships to deploy) that constitute the network, which fulfills the container shipment demand at minimum cost.

7.1.1 Container shipment demand

There are many types of containers to transport for each O-D pair, such as dry 20-ft, dry 40-ft, reefer 20-ft, and reefer 40-ft. Different types of containers are different in volume and port handling cost. Let $\mathcal{K}$ represent the set of container types. Denote by E_k the twenty-foot equivalent volume (TEUs) of a container in type $k \in \mathcal{K}$. For example, a dry 40-ft is 2 TEUs. Denote by n_{od}^k the weekly number of laden containers in type k to be transported for the O-D pair $(o,d) \in \mathcal{W}$. Let W_{od}^k be the average weight (tons) of a laden container in type k to be transported for the O-D pair $(o,d) \in \mathcal{W}$.

Due to trade imbalance, some locations have surplus empty containers and other locations are deficit in empty containers. Define $\gamma_{pk} := \sum_{o \in \mathcal{P}} n_{op}^k - \sum_{d \in \mathcal{P}} n_{pd}^k$. Let $\mathcal{P}_k^{\text{SUR}}$ be the set of ports with surplus empty containers in type k, that is, $\mathcal{P}_k^{\text{SUR}} := \{p \in \mathcal{P} \mid \gamma_{pk} > 0\}$. Similarly, let $\mathcal{P}_k^{\text{BAL}}$ and $\mathcal{P}_k^{\text{DEF}}$ be the set of ports with balanced and deficit empty containers in type k, respectively. We further represent by W^k the weight (tons) of an empty container in type k.

7.1.2 Container shipping network

The liner shipping company does not design a shipping network from scratch. In fact, it usually designs the network based on its current shipping network. Therefore, we use the current network as an input of the network design problem. We assume that the company has an initial network which consists of three types of ship routes: ship routes that must be used (type 1) denoted by $\hat{\mathcal{R}}$; ship routes that at least N^{SR} of them must be used (type 2) denoted by $\bar{\mathcal{R}}$; and ship routes that are optional (type 3) denoted by $\tilde{\mathcal{R}}$. $\mathcal{R} = \hat{\mathcal{R}} \cup \bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$.

There are two constituents of ship routes in type 1: ship routes that are jointly operated with other shipping companies and ship routes that are proved to be very successful. Each ship route of type 1 has a specified type of ship deployed on it. It should be mentioned that if a ship route is operated with other companies, it is possible that e.g. six 9k-TEU ships are deployed, while the focal company controls only two of them. Therefore, the focal company can utilize 3k-TEU ship slots on each ship. To capture this feature, we use Λ_r to represent the percentage of ship slots of ship route $r \in \hat{\mathcal{R}}$ controlled by the focal company. The number of ships deployed as well as the arrival and departure time at each port of call are also known for ship routes of type 1.

Ship routes of type 2 are important and the liner shipping company does not intend to change all of them. Hence, it requires that at least N^{SR} of them must be used, $N^{\mathrm{SR}} \le |\bar{\mathcal{R}}|$. Ship routes of type 3 are less important. Each ship route of type 2 and type 3 also has a specified type of ships to deploy. However, the type of ship can be changed. The number of ships deployed on a ship route of type 2 and type 3 is a decision variable. When we say that a ship route of type 2 is used, we mean that the pre-specified ship type is not changed.

7.2 Laden Container Routing

7.2.1 Assumptions and constraints

As we have mentioned, a port pair $(o,d) \in \mathcal{W}$ may have more than one type of container to ship. In reality, different types of containers are shipped together. Therefore, we assume that whenever containers of the same O-D are shipped, different types of containers must be shipped proportional to the container shipment demand n_{od}^k. This assumption significantly simplifies the modeling difficulties because we can treat the containers of the same O-D as a single type of containers, which is elaborated below.

We can use TEU to compute the container shipment demand of port pair $(o,d) \in \mathcal{W}$. Thus, in terms of TEU, the container shipment demand n_{od} can be calculated by

$$n_{od} = \sum_{k \in \mathcal{K}} E_k n_{od}^k, \forall (o,d) \in \mathcal{W} \quad (7.1)$$

The average weight (tons) per TEU, denoted by W_{od}, is computed by

$$W_{od} = \sum_{k \in \mathcal{K}} W_{od}^k n_{od}^k / n_{od}, \forall (o,d) \in \mathcal{W} \quad (7.2)$$

Other parameters can be computed in a similar manner. For example, suppose that the productivity of port $p \in \mathcal{P}$ is M_{pv} moves/hour for ships of type $v \in \mathcal{V}$. Then the average container handling time (hour) per TEU, denoted by t_{pv}^{od}, can be calculated by

$$t_{pv}^{od} = \sum_{k \in \mathcal{K}} n_{od}^k / (M_{pv} n_{od}), \forall (o,d) \in \mathcal{W}, \forall p \in \mathcal{P}, \forall v \in \mathcal{V}_p, \quad (7.3)$$

The liner container shipping company thus must provide a certain level of service in terms of the maximum allowable transit time for each port pair $(o,d) \in \mathcal{W}$, denoted by $\hat{T}_{od}$ (hours), for shippers. This is one of the major constraints in the generation of container routes.

To capture the transit time of containers, the arrival and departure time at each port of call on each ship route in $\mathcal{R}$ must be known. As mentioned above, the arrival and departure time of ship routes in type 1 are already given. However, the time components and the number of ships deployed on ship routes in type 2 and type 3 are not known. Before routing containers, we have to set the schedules for these ship routes. This is implemented as follows. We first estimate the time spent at each port of call based on historical data, and the sea time based on the voyage distance and the design speed of ships. After that, we obtain the round-trip time, thereby calculating the number of ships required. If this number is not an integer, we round it up or down by adjusting the speed of ships. After that, we can obtain the sea time on each leg and the port time at each port of call. It should be mentioned that the transit time of containers includes the connection time (or dwell time) at transshipment ports. This connection time is dependent on the schedules of the two ship routes. However, we do not capture so many details in the model and simply assume that the connection time at a port is a constant value $\hat{t}_p$ hours. We make this simplifying assumption because otherwise we have to consider the available berth time window at each port of call, which dramatically increases the difficulties of the model.

In the generation of container routes, maritime cabotage has to be taken into account. Maritime cabotage is imposed in most of the countries in the world. It requires that a global liner shipping company that is not based in the country does not allow transporting laden container within the country. For example, Maersk Line is not allowed to relay containers from Qingdao at Shanghai, because this operation involves the delivery of containers from Qingdao to Shanghai and it would reduce the profit of local shipping companies such as COSCO and CSCL.

7.2.2 A hub-based container route generation approach

To facilitate the generation of container routes, we take advantage of the current operating rules of the liner shipping company. It has a set of global hubs and regional hubs for transshipment. For example, Figure 7-1 depicts the 166 ports of a global liner shipping company. Figure 7-2 shows its two global hubs: Singapore and Hong Kong, and 14 regional hubs which are Balboa, Manzanillo, Miami, Hamburg, Rotterdam, Port Said, Djibouti, Salalah, Jebel Ali, Colombo, Singapore, Hong Kong, Kaohsiung, and Pusan (note that a global hub is also a regional hub). We assume that global hubs can be used to relay containers of any O-D pair, while regional hubs can only relay containers associated with its feeder ports. Because of the voluminous data, we do not show the associated feeder ports of each regional hub in Figure 7-2.

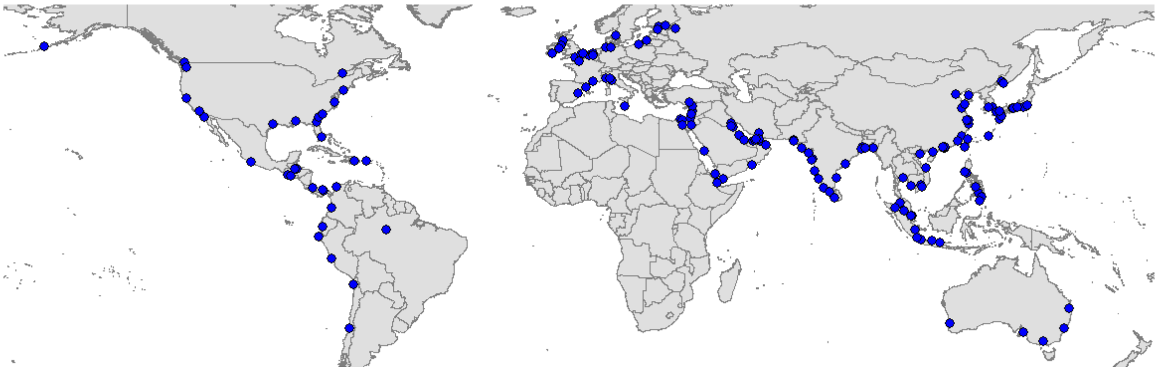


Figure 7-1 A global shipping network of 166 ports

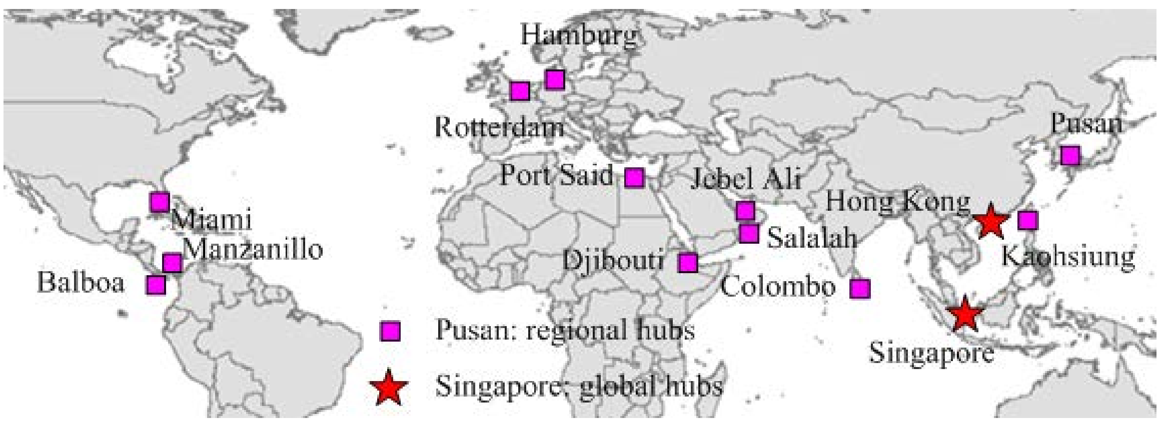


Figure 7-2 Global and regional hubs

Take the O-D pair Hakata-Kotka shown in Figure 7-3 as an example. Hakata is a feeder port assigned to Pusan, and Kotka is a feeder port assigned to both Hamburg and Rotterdam. This corresponds to the most complicated case where containers are from one feeder to another feeder and these two feeders are not assigned to the same regional hub. The possible routing choices are shown in Table 7-1. Containers can be shipped without relay (ID1), or relayed once at a regional hub or a global hub (ID2-ID6), or relayed twice (ID7-ID14), or relayed three times at two regional hubs and a global hub (ID15-ID18).

It should be noted that if a routing alternative violates the maritime cabotage restriction, it should be removed. Table 7-1 does not provide container routes because it does not specify which leg of which ship route to use. To generate container routes, we enumerate all possible combinations of ship routes for each routing choice in Table 7-1 and check the transit time constraint. We do not consider all the container routes for each O-D at the network design stage. Rather, we only consider the 30 container routes with the lowest operating cost.

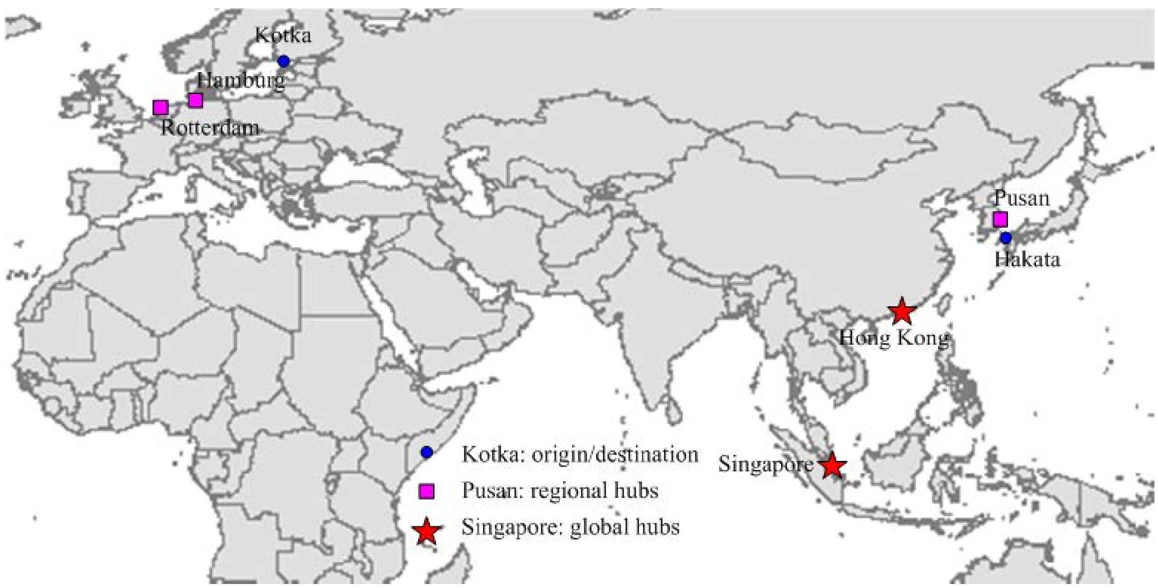


Figure 7-3 Relevant ports for the O-D pair Hakata-Kotka

Table 7-1 Routing choices for containers from Hakata to Kotka

ID	Possible routing choices
1	Hakata-Kotka
2	Hakata-Pusan-Kotka
3	Hakata-Hamburg-Kotka
4	Hakata-Rotterdam-Kotka
5	Hakata-Hong Kong-Kotka
6	Hakata-Singapore-Kotka
7	Hakata-Pusan-Hamburg-Kotka
8	Hakata-Pusan-Rotterdam-Kotka
9	Hakata-Pusan-Hong Kong-Kotka
10	Hakata-Pusan-Singapore-Kotka
11	Hakata-Hong Kong-Hamburg-Kotka
12	Hakata-Hong Kong-Rotterdam-Kotka
13	Hakata-Singapore-Hamburg-Kotka
14	Hakata-Singapore-Rotterdam-Kotka
15	Hakata-Pusan-Hong Kong-Hamburg-Kotka
16	Hakata-Pusan-Hong Kong-Rotterdam-Kotka
17	Hakata-Pusan-Singapore-Hamburg-Kotka
18	Hakata-Pusan-Singapore-Rotterdam-Kotka

7.2.3 An exact container route generation approach

The hub-based container route generation approach is very efficient in practice. Nevertheless, it may not generate all the best container routes because it does not allow laden containers to be relayed at ports other than hubs. We further develop a mathematical model for generating container routes while considering operational constraints.

To facilitate the generation of container routes, we reformulate a liner shipping network as an operational network. For example, Figure 7-4 is the corresponding operational network for the O-D pair Xiamen-Singapore of the network in Figure 2-3. The operational network

(N, A) for the O-D pair of $(o,d) \in \mathcal{P} \times \mathcal{P}$, where N represents the node set and A is the arc set, is constructed as follows. First, add a virtual source node and a virtual sink node to N. Each node in N except the virtual source and sink nodes corresponds to a port of call on a liner ship route. Hence, every node in N except the source and sink nodes can be represented by (r,i), i.e., the *i*th port of call on ship route $r \in \mathcal{R}$. The arc set $A := A^v \cup A^t \cup A^{\text{source}} \cup A^{\text{sink}}$, which represent the voyage arcs, transshipment arcs, source arcs, and sink arcs, respectively. A voyage arc $a \in A^v$ can be represented by its tail node (r,i). In other words, voyage arc (r,i) is the voyage from node (r,i) to node $(r,i+1)$. Note that voyage arc (r, N_r) is the voyage from node (r, N_r) to node $(r,1)$ because each ship route forms a loop. To simplify the notation, in the sequel we define node $(r,i+1)$, $i = N_r$, to be node $(r,1)$. A relay arc $a \in A^t$ can be represented by $((r,i),(s,j))$ where $p_{ri} = p_{sj}$. In other words, containers are transshipped at port $p_{ri} = p_{sj}$ from a ship on ship route r to a ship on ship route s. A source arc is an arc from the source node to a node (r,i) referring to the origin port o, namely, $p_{ri} = o$. A sink arc is defined similarly. To summarize, we can use $a \in A$ to refer to an arc of any type. We can also use $(r,i) \in A^v$ to refer to the voyage arc from node (r,i) to node $(r,i+1)$, and use $((r,i),(s,j)) \in A^t$ to refer to the transshipment arc from the *i*th port of call of ship route r to the *j*th port of call of ship route s.

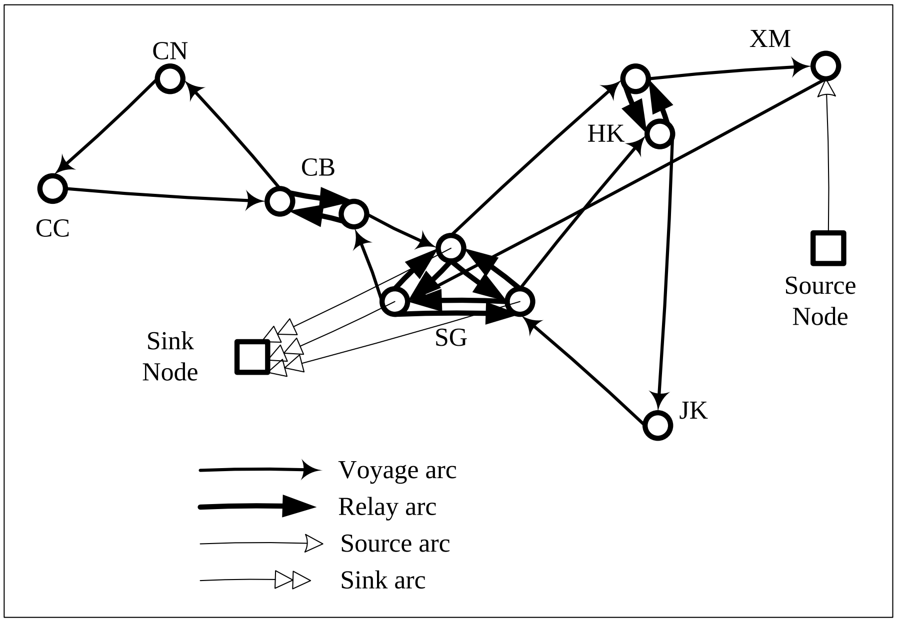


Figure 7-4 A liner shipping operational network for the O-D of Xiamen-Singapore

Each arc $a \in A$ is associated with a time, denoted by t_a, which is determined by the liner service schedules. Suppose that each ship route provides a weekly service frequency and consider a particular ship on each ship route. Suppose that the ship on ship route $r \in \mathcal{R}$ visits the first port of call at time t_{r1} (the time 0 can be arbitrarily defined, e.g., 00:00:00 01/01/2011), and then visits the second port of call at time t_{r2}, etc. In this context, the time of a voyage arc $a = (r,i) \in A^v$ is the time from the arrival at the *i*th port of call to the arrival at the (*i*+1)th port of call, i.e., $t_a = t_{r,i+1} - t_{ri}$. The time of a relay arc $a = ((r,i),(s,j)) \in A^t$ is $\hat{t}_{p_{ri}}$ for simplicity.

Similar to t_a, $a \in A$, each arc $a \in A$ is also associated with a cost, denoted by c_a. The cost of a source arc is the loading cost of containers at their origin port, and the cost of a sink arc is the discharge cost of containers at their destination port. The cost of a voyage arc can be set at 0 because the marginal cost of transporting one more container is insignificant compared with the handling cost. The cost of a transshipment arc is mainly the transshipment cost of containers at the port associated with the transshipment arc.

In the operational network, a container route is a path from the source node to the sink node. Theoretically, there may be an infinite number of paths for each O-D pair. However, practically the number of paths is quite limited because of operational constraints and business considerations. Before presenting the mathematical model, we present the notation:

$a^{\text{source}}(r,i)$: The source arc from the virtual source node to node (r,i);

$a^{\text{sink}}(r,i)$: The sink arc from node (r,i) to the virtual sink node;

A_p^t: Set of transshipment arcs at port $p \in \mathcal{P}$, $A_p^t := \{((r,i),(s,j)) \in A^t : p_{ri} = p_{sj} = p\}$;

A_p^{v+}: Set of voyage arcs entering port $p \in \mathcal{P}$, $A_p^{v+} := \{(r,i) \in A^v : p_{r,i+1} = p\}$;

A_p^{v-}: Set of voyage arcs leaving port $p \in \mathcal{P}$, $A_p^{v-} := \{(r,i) \in A^v : p_{ri} = p\}$;

Θ: The set of countries where the set of ports $\mathcal{P}$ belong to

$\hat{\Theta}$: The set of countries where maritime cabotage is imposed for the liner shipping company

$\Theta \setminus \hat{\Theta}$: The set of countries within each of which the liner shipping company can freely provide maritime transport services

θ_p: The country that port $p \in \mathcal{P}$ belongs to

P_θ: The set of ports located in country θ, $\theta \in \Theta$

The decision variable $x_a \in \{0,1\}, a \in A$. $x_a = 1$ if and only if arc a is contained in the container route. Hence, the values of all x_a fully represent a container route. The container route generation model (CRM) is:

[CRM]
$$\min_{x_a} \sum_{a \in A} c_a x_a \tag{7.4}$$

subject to:

$$\sum_{a \in A^{\text{source}}} x_a = 1 \tag{7.5}$$

$$\sum_{a \in A^{\text{sink}}} x_a = 1 \tag{7.6}$$

$$x_{(r,i-1)} + \sum_{((s,j),(r,i)) \in A^t} x_{((s,j),(r,i))} = x_{(r,i)} + \sum_{((r,i),(s,j)) \in A^t} x_{((r,i),(s,j))}, \quad \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, p_{ri} \in \mathcal{P} \setminus \{o,d\} \tag{7.7}$$

$$x_{a^{\text{source}}(r,i)} + x_{(r,i-1)} + \sum_{((s,j),(r,i)) \in A^t} x_{((s,j),(r,i))} = x_{(r,i)} + \sum_{((r,i),(s,j)) \in A^t} x_{((r,i),(s,j))}, \quad \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, p_{ri} = o \tag{7.8}$$

$$x_{(r,i-1)} + \sum_{((s,j),(r,i)) \in A^t} x_{((s,j),(r,i))} = x_{(r,i)} + \sum_{((r,i),(s,j)) \in A^t} x_{((r,i),(s,j))} + x_{a^{\text{sink}}(r,i)}, \quad \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, p_{ri} = d \tag{7.9}$$

$$x_{(r,i-1)} = 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, p_{ri} = o \tag{7.10}$$

$$x_{(r,i)} = 0, \forall r \in \mathcal{R}, \forall i \in \mathcal{I}_r, p_{ri} = d \tag{7.11}$$

$$x_{((r,i),(s,j))} = 0, \forall ((r,i),(s,j)) \in A_o^t \bigcup A_d^t \tag{7.12}$$

$$\sum_{((r,i),(s,j)) \in A_p^t} x_{((r,i),(s,j))} \le 1, \forall p \in \mathcal{P} \setminus \{o,d\} \tag{7.13}$$

$$x_{((r,i),(s,j))} = 0, \forall \theta \in \hat{\Theta} \bigcap \{\theta_o, \theta_d\}, \forall p \in P_\theta, \forall ((r,i),(s,j)) \in A_p^t \tag{7.14}$$

$$\sum_{p \in P_\theta} \sum_{((r,i),(s,j)) \in A_p^t} x_{((r,i),(s,j))} \le 1, \forall \theta \in \hat{\Theta} \setminus \{\theta_o, \theta_d\} \tag{7.15}$$

$$x_{(r,i-1)} \geq x_{((r,i),(s,j))}, \forall ((r,i),(s,j)) \in A^t \quad (7.16)$$

$$x_{(s,j)} \geq x_{((r,i),(s,j))}, \forall ((r,i),(s,j)) \in A^t \quad (7.17)$$

$$x_a \leq 1 - x_{((r,i),(s,j))}, \forall p \in \mathcal{P}, \forall ((r,i),(s,j)) \in A_p^t, \forall a \in (A_p^{v+} \cup A_p^{v-}) \setminus \{(r,i-1),(s,j)\} \quad (7.18)$$

$$\sum_{a \in A^v \cup A^t} t_a x_a \leq \hat{T}_{od} \quad (7.19)$$

$$x_a \in \{0,1\}, a \in A \quad (7.20)$$

The objective function (7.4) minimizes the total cost for transporting one container. Constraints (7.5)-(7.9) impose flow conservation. Eqs. (7.10)-(7.11) require that containers never visit their origin port from other ports and containers never visit other ports from their destination port (we define arc $(r,i-1)$, $i=1$, to be arc (r,N_r)). Eqs. (7.12)-(7.13) require that containers should never be transshipped at their origin or destination port and containers can be transshipped at a port other than their origin or destination at most once. Eqs. (7.14)-(7.15) impose the maritime cabotage restriction. Eq. (7.14) requires that if the liner shipping company is subject to the maritime cabotage restriction in the country of the origin (destination) port, the liner shipping company cannot transship the containers at ports in the same country as the origin (destination) port. Eq. (7.15) enforces that for other countries that impose the maritime cabotage, the liner shipping company can transship the containers at most once at all ports within a country. Eqs. (7.16)-(7.18) require that if a transshipment arc $((r,i),(s,j)) \in A^t$ is visited, then the voyage arcs $(r,i-1) \in A^v$, $(s,j) \in A^v$ must also be visited, and none of the other voyage arcs entering or leaving the port associated with the transshipment arc is visited. Eq. (7.19) imposes the transit time constraint and Eq. (7.20) defines x_a to be a binary decision variable.

After the container route with the lowest cost while satisfying the transit time limit is generated, we can generate the container route with the 2nd lowest cost as follows. Let the binary vector $(x_a^*)_{a\in A} \in \{0,1\}^{|A|}$ represent the container route with the lowest cost. Add to the above model the following constraint to exclude this container route to be generated again:

$$\sum_{a\in A^1}(1-x_a)+\sum_{a\in A^0}x_a \geq 1 \tag{7.21}$$

where

$$A^1 := \{a \in A : x_a^* = 1\} \tag{7.22}$$

$$A^0 := \{a \in A : x_a^* = 0\} \tag{7.23}$$

The other container routes can be generated one by one in a similar manner by excluding all the previously generated container routes until the model is infeasible.

The above model is an integer (binary) linear programming model and can be efficiently solved by state-of-art integer linear programming solvers. Moreover, the container routes for different O-D pairs can be simultaneously generated with different computing units.

7.3 Network Design with Candidate Ship Routes

Given an initial network with ship routes classified into three groups $\hat{\mathcal{R}}$, $\bar{\mathcal{R}}$, and $\tilde{\mathcal{R}}$, we aim to determine which ship routes in $\bar{\mathcal{R}}\cup\tilde{\mathcal{R}}$ should be chosen in order to minimize the total cost of transporting both laden and empty containers. It should be noted that since the predicted container shipment demand cannot match the true demand exactly, we allow some laden and empty containers not shipped while incurring some penalty cost.

7.3.1 Decision variables

The NDP has the following decision variables:

x_r: Binary decision variable which equals 1 if and only if ship route $r \in \mathcal{R}$ is used;

n_v^{in}: Number of ships in type $v \in \mathcal{V}$ that are chartered in;

y_h: Number of laden containers (TEUs) shipped on container route $h \in \mathcal{H}$;

y_{od}: Number of laden containers (TEUs) unfulfilled for the port pair $(o,d) \in \mathcal{W}$;

f_{ri}^k: Number of empty containers in type k flowing on leg i of ship route $r \in \mathcal{R}$;

$\hat{z}_{ri}^k$: Number of empty containers in type k loaded at the port of call i of ship route $r \in \mathcal{R}$;

$\tilde{z}_{ri}^k$: Number of empty containers in type k discharged at the port of call i of ship route $r \in \mathcal{R}$;

$\hat{z}_p^k$: Number of loading operations for empty containers in type k at port $p \in \mathcal{P}$;

$\tilde{z}_p^k$: Number of discharge operations for empty containers in type k at port $p \in \mathcal{P}$;

$\overline{z}_p^k$: Number of transshipments for empty containers in type k at port $p \in \mathcal{P}$;

z_p^k: Number of unshipped empty containers in type k at port $p \in \mathcal{P}$;

y_{od}: Number of laden containers (TEUs) unfulfilled for the port pair $(o,d) \in \mathcal{W}$;

7.3.2 Mixed-integer linear programming model

Let $\hat{c}_{pk}^{\text{EMP}}$, $\tilde{c}_{pk}^{\text{EMP}}$, and $\overline{c}_{pk}^{\text{EMP}}$ be the load, discharge, and relay cost of empty containers in type k at port $p \in \mathcal{P}$, respectively. Let c_{od} (USD/TEU) be the penalty for not shipping a TEU for the port pair $(o,d) \in \mathcal{W}$. We define $\Lambda_r = 1$ for ship routes $r \in \overline{\mathcal{R}} \cup \tilde{\mathcal{R}}$. The weight (tons) of an empty container in type k is denoted by W_k. The type of ship deployed on ship route $r \in \mathcal{R}$ is $v(r)$, and the number of ships is $m(r)$. $\mathcal{R}_v := \{r \in \mathcal{R} \mid v(r) = v\}$. The network design model (NDM) is:

[NDM]

$$\min \sum_{r\in\mathcal{R}} c_r x_r + \sum_{h\in\mathcal{H}} c_h y_h + \sum_{k\in\mathcal{K}} \sum_{p\in\mathcal{P}} \left(\hat{c}_{pk}^{\mathrm{EMP}} (\hat{z}_p^k - \overline{z}_p^k) + \tilde{c}_{pk}^{\mathrm{EMP}} (\tilde{z}_p^k - \overline{z}_p^k) + \overline{c}_{pk}^{\mathrm{EMP}} \overline{z}_p^k \right)$$
$$+ \sum_{v\in\mathcal{V}} c_v^{\mathrm{in}} n_v^{\mathrm{in}} + \sum_{(o,d)\in\mathcal{W}} c_{od} y_{od} + \sum_{k\in\mathcal{K}} \sum_{p\in\mathcal{P}} c_{pk}^{\mathrm{EMP}} z_p^k \tag{7.24}$$

$$y_{od} + \sum_{h\in\mathcal{H}^{od}} y_h = n_{od}, \forall (o,d)\in\mathcal{W} \tag{7.25}$$

$$\hat{z}_p^k = \sum_{r\in\mathcal{R}_p} \sum_{i\in\mathcal{I}_r, p_{ri}=p} \hat{z}_{ri}^k, \forall k\in\mathcal{K}, \forall p\in\mathcal{P} \tag{7.26}$$

$$\tilde{z}_p^k = \sum_{r\in\mathcal{R}_p} \sum_{i\in\mathcal{I}_r, p_{ri}=p} \tilde{z}_{ri}^k, \forall k\in\mathcal{K}, \forall p\in\mathcal{P} \tag{7.27}$$

$$\hat{z}_p^k - \tilde{z}_p^k = 0, \forall k\in\mathcal{K}, \forall p\in\mathcal{P}_k^{\mathrm{BAL}} \tag{7.28}$$

$$\hat{z}_p^k - \tilde{z}_p^k + z_p^k = \gamma_{pk}, \forall k\in\mathcal{K}, \forall p\in\mathcal{P}_k^{\mathrm{SUR}} \tag{7.29}$$

$$\hat{z}_p^k - \tilde{z}_p^k - z_p^k = \gamma_{pk}, \forall k\in\mathcal{K}, \forall p\in\mathcal{P}_k^{\mathrm{DEF}} \tag{7.30}$$

$$\hat{z}_{ri}^k + f_{ri}^k = \tilde{z}_{ri}^k + f_{r,i-1}^k, \forall k\in\mathcal{K}, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{7.31}$$

$$\overline{z}_p^k = \min\{\hat{z}_p^k, \tilde{z}_p^k\}, \forall k\in\mathcal{K}, \forall p\in\mathcal{P} \tag{7.32}$$

$$\sum_{h\in\mathcal{H}} \rho_{hri} y_h + \sum_{k\in\mathcal{K}} E_k f_{ri}^k \le \Lambda_r \mathrm{Cap}_{v(r)} x_r, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{7.33}$$

$$\sum_{(o,d)\in\mathcal{W}} \sum_{h\in\mathcal{H}_{od}} \rho_{hri} W_{od} y_h + \sum_{k\in\mathcal{K}} W_k f_{ri}^k \le \Lambda_r \mathrm{Wei}_{v(r)} x_r, \forall r\in\mathcal{R}, \forall i\in\mathcal{I}_r \tag{7.34}$$

$$\sum_{r\in\mathcal{R}_v} m(r) x_r \le N_v^{\mathrm{own}} + n_v^{\mathrm{in}}, \forall v\in\mathcal{V} \tag{7.35}$$

$$\sum_{r\in\overline{\mathcal{R}}} x_r \ge N^{\mathrm{SR}} \tag{7.36}$$

$$x_r = 1, \forall r\in\hat{\mathcal{R}} \tag{7.37}$$

$$x_r \in \{0,1\}, \forall r\in\overline{\mathcal{R}} \cup \tilde{\mathcal{R}} \tag{7.38}$$

$$n_v^{\mathrm{in}} \ge 0, \forall v\in\mathcal{V} \tag{7.39}$$

$$y_h \geq 0, \forall h \in \mathcal{H} \tag{7.40}$$

$$y_{od} \geq 0, \forall (o,d) \in \mathcal{W} \tag{7.41}$$

$$\hat{z}_{ri}^k \geq 0, \tilde{z}_{ri}^k \geq 0, f_{ri}^k \geq 0, \forall k \in \mathcal{K}, \forall r \in \mathcal{R} \tag{7.42}$$

$$\hat{z}_p^k \geq 0, \tilde{z}_p^k \geq 0, \overline{z}_p^k \geq 0, z_p^k \geq 0, \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \tag{7.43}$$

The objective function (7.24) minimizes the total cost. The first component is the cost associated with ship routes, the second term is the laden container routing cost, the third term is empty container handling cost, the fourth term is the cost for using more ships than owned, and the last two terms are penalty cost for not fulfilling the laden and empty container shipment demand. Eq. (7.25) is the laden container flow conservation equation. Eqs. (7.26)-(7.27) define the load and discharge volumes of empty containers, respectively. Eqs. (7.28)-(7.31) are empty container flow conservation equations. Eq. (7.32) defines the transshipped empty containers. Eqs. (7.33)-(7.34) impose the ship volume and capacity constraints, respectively. Eq. (7.35) requires that the number of ships used cannot exceed the number of ships owned. Eq. (7.36) requires that at least N^{SR} ship routes of type 2 must be chosen. Eq. (7.37) requires that all ship routes of type 1 must be used. Eqs. (7.38)-(7.43) define the decision variables.

The NDM can be transformed to a mixed-integer linear programming model due to the proposition below:

Proposition 7-1: Replacing Eq. (7.32) by

$$\overline{z}_p^k \leq \hat{z}_p^k, \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \tag{7.44}$$

$$\overline{z}_p^k \leq \tilde{z}_p^k, \forall k \in \mathcal{K}, \forall p \in \mathcal{P} \tag{7.45}$$

At the optimal solution, Eq. (7.32) holds automatically.

Proof: If $\overline{z}_p^k > \min\{\hat{z}_p^k, \tilde{z}_p^k\}$, we can decrease the objective function (7.24) by reducing the value of $\overline{z}_p^k$ while not violating any constraint. Hence, the proposition holds.□

7.3.3 A heuristic solution approach

NDM cannot be solved directly by off-the-shelf solver if the cardinality of $\bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$ is large because it has hundreds of thousands of continuous decision variables. To obtain a high quality solution efficiently, we propose a heuristic approach.

First, denoting by y_h^* and f_{ri}^{*k} the resulting flow of laden and empty containers, respectively, in a given network, we define the capacity utilization of leg i of ship route $r \in \mathcal{R}$, denoted by U_{ri}, as follows:

$$U_{ri} := \max\left\{ \frac{\sum_{h\in\mathcal{H}} \rho_{hri} y_h^* + \sum_{k\in\mathcal{K}} E_k f_{ri}^{*k}}{\Lambda_r \mathrm{Cap}_{v(r)}}, \frac{\sum_{(o,d)\in\mathcal{W}} \sum_{h\in\mathcal{H}_{od}} \rho_{hri} W_{od} y_h^* + \sum_{k\in\mathcal{K}} W_k f_{ri}^{*k}}{\Lambda_r \mathrm{Wei}_{v(r)}} \right\}, \forall r \in \mathcal{R}, \forall i \in I_r \tag{7.46}$$

We further define the capacity utilization of ship route $r \in \mathcal{R}$, denoted by U_r, as follows:

$$U_r := \frac{\sum_{i\in I_r} U_{ri} L_i}{\sum_{i\in I_r} L_i}, \forall r \in \mathcal{R} \tag{7.47}$$

and the hit-haul capacity utilization of ship route $r \in \mathcal{R}$, denoted by $U_r^{\max}$, as follows:

$$U_r^{\max} := \max_{i\in I_r}\{U_{ri}\}, \forall r \in \mathcal{R} \tag{7.48}$$

A heuristic algorithm that obtains a high quality solution is as follows:

Algorithm 7-1: Optimizing the initial network

Step 0: Specify the value N^{OPT} (e.g. 20). Solve NDM by requiring $x_r = 1$ for any $r \in \mathcal{R}$. Obtain the container flow y_h^* and f_{ri}^{*k}.

Step 1: Compute the capacity utilization of ship route $r \in \bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$. Set $x_r \in \{0,1\}$ for the N^{OPT} ship routes with the lowest capacity utilization.

Step 2: Solve NDM by requiring $x_r = 1$ for any $r \in \mathcal{R}$ except the N^{OPT} ship routes identified. Obtain the container flow y_h^* and f_{ri}^{*k}. If all the N^{OPT} ship routes are chosen, stop. Otherwise remove the ship routes that are not chosen, go to Step 1.

Proposition 7-2: Algorithm 7-1 terminates in a finite number of iterations.

Proof: At least one ship route in $\mathcal{R}$ is removed each time the algorithm repeats Step 2. Since the number of ship routes in $\mathcal{R}$ is limited, Proposition 7-2 holds. □

In the above algorithm, the number N^{OPT} is used to balance between the solution quality and computational time. For instance, if $N^{\mathrm{OPT}} = |\mathcal{R}|$, we obtain the optimal solution. If N^{OPT} is small, NDM can be solved efficiently in each iteration. Nevertheless, some potentially good ship routes may be removed at early stages of the algorithm.

7.4 Successive Optimization Heuristic

The approach mentioned above chooses a set of high quality ship routes to operate from a set of candidate ship routes. However, no new ship routes are generated. As a result, it may not be sufficient for designing the liner shipping network. Therefore, we propose a successive optimization heuristic algorithm that improves the network. The flowchart of the algorithm is shown in Figure 7-5. After the initial network is optimized, there are three stages to improve the resulting network, which are elaborated below.

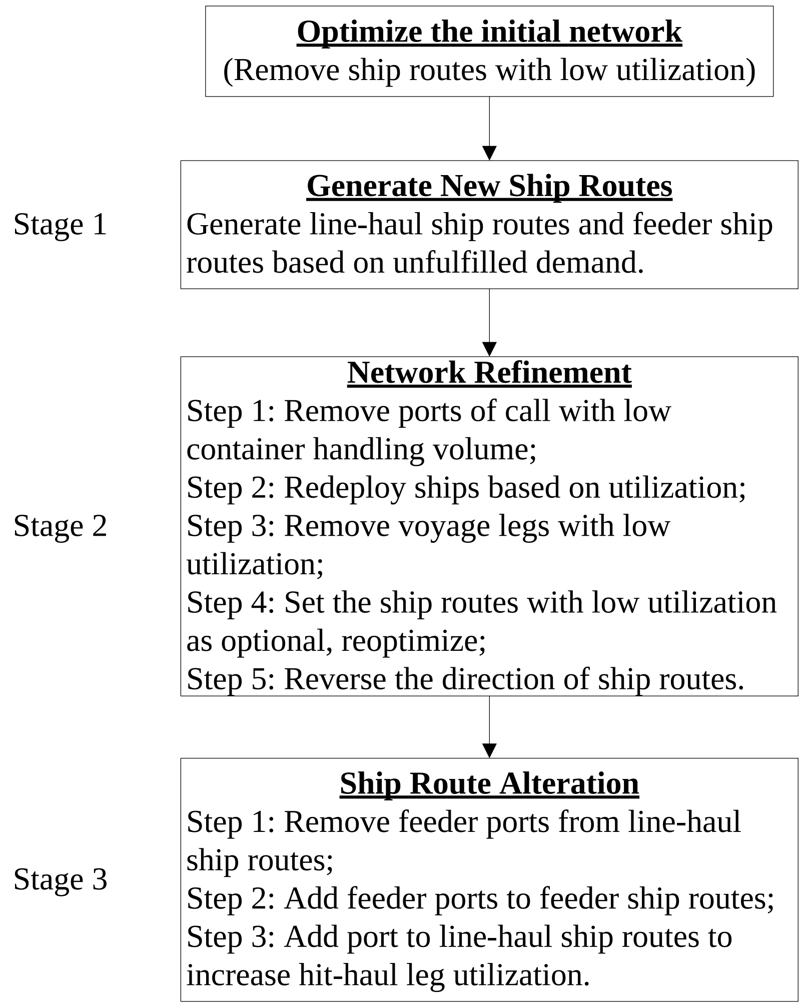


Figure 7-5 Flowchart of the successive optimization heuristic algorithm

7.4.1 Generate new ship routes

If the projected container shipment demand is much larger than the current demand, or if the demand covers shipping markets that are not serviced by the current network, or if there is no initial network, the resulting network after optimizing the initial network may have a large number of unshipped containers. In such a setting, we need to design ship routes from scratch.

We first design the line-haul ship routes, which are classified according to trade lanes as e.g. intra-Asia, Asia to American West Coast, trans-Atlantic, Asia to American East Coast, etc. For each trade lane of line-haul ship routes, we first examine whether the volume of unshipped containers exceeds a threshold value. If it is not true, we do not need to design line-haul ship routes of this trade lane. Otherwise, we need to design a line-haul ship route.

To design a line-haul ship route, first, we sequence the ports in the regions covered by the trade lane according to their unshipped laden containers to and from other ports in the regions. It should be mentioned that if a port is a regional hub, we add to it 30% of its feeders' unshipped containers. We then choose a maximum of e.g. 10 ports with the largest volume of unshipped containers such that the volume of unshipped containers at each of these ports must be above a certain threshold. The port calling sequence is determined such that the round-trip journey distance is minimized. We design three line-haul ship routes with the same port rotations and different types of ships. The type of ship is determined based on 1/3, 1/2, and 100% of the total unshipped demand of all the ports on the ship route.

Feeder ship routes must be designed accordingly. After having design the line-haul ship routes, we connect the feeder ports to the regional hubs on the line-haul ship route if necessary.

All these newly designed line-haul and feeder ship routes are set as optional and we solve NDM. This process is repeated until there are no new line-haul ship routes designed. After this stage, the volume of unshipped containers is usually less than 5% of the overall demand, which can be acceptable by the liner shipping company.

7.4.2 Network refinement

The network refinement stage aims to improve the designed network without adding ports of call.

In step 1, if the number of laden and empty containers handled at some ports of call is very small, for example, less than 10 containers, then these ports of call can be considered as removed. The number of empty containers handled can be obtained directly from the values of $\hat{z}_{ri}^{k}$ and $\tilde{z}_{ri}^{k}$. The number of laden containers handled can be derived by examining the value of y_h and the property of the container route $h \in \mathcal{H}$. It should be highlighted that we do not remove a port of call directly. Instead, we let the algorithm determine whether a port of call should be removed. The algorithm is elaborated below, and the similar idea is applied to all the other steps of the incremental network alteration algorithm.

Algorithm 7-2: Removing ports of call

Step 0: Specify the value N^{OPT} (e.g. 20).

Step 1: Let $\tilde{\mathcal{R}}^{\text{new}} := \mathcal{R}^{\text{old}} := \emptyset$. Obtain the port of call with the smallest number of containers handled for each ship route $r \in \bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$. Sequence these ports of call from the smallest number of containers handled to the largest. For each of the first $\lfloor N^{\text{OPT}} / 2 \rfloor$ ship routes, denoted by r, set $\mathcal{R}^{\text{old}} := \mathcal{R}^{\text{old}} \cup \{r\}$ and add a new ship route r' with the same type of ship and port rotation as r except that the port of call with the smallest number of containers handled is removed, and set $\tilde{\mathcal{R}}^{\text{new}} := \tilde{\mathcal{R}}^{\text{new}} \cup \{r'\}$ and $\tilde{\mathcal{R}} := \tilde{\mathcal{R}} \cup \{r'\}$.

Step 3: Set $x_r = 1$ for any $r \in \mathcal{R} \setminus (\tilde{\mathcal{R}}^{\text{new}} \cup \mathcal{R}^{\text{old}})$ and $x_r \in \{0,1\}$ for any $r \in \tilde{\mathcal{R}}^{\text{new}} \cup \mathcal{R}^{\text{old}}$. Solve NDM. If all ship routes in $\mathcal{R}^{\text{old}}$ are chosen and no ship routes in $r \in \tilde{\mathcal{R}}^{\text{new}}$ is chosen, stop. Otherwise, repeat Step 1.

In step 2 of the network refinement stage, if the capacity utilization U_r of ship route $r \in \bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$ is very large or small, we consider replacing the ships with a string of larger or smaller ships, respectively. Still, we let the algorithm determine whether the ship size should be changed.

An example of step 3 of the network refinement stage is shown in Figure 7-6. If the leg capacity utilizations U_{ri} of the legs Yantian to Pusan, Pusan to Shanghai and Shanghai to Yantian are all very low, we consider removing all these legs altogether.

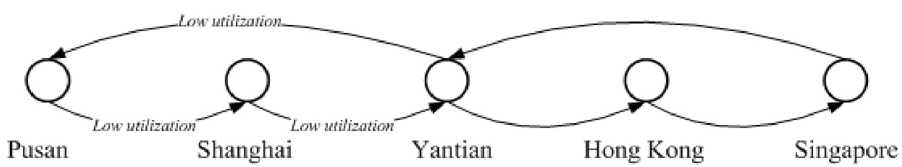


Figure 7-6 Remove voyage legs

In step 4 of the network refinement stage, we choose the N^{OPT} ship routes from $\bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$ with the lowest capacity utilization, set them as optional, and re-optimize.

In step 5 of the network refinement stage, we choose the $\lfloor N^{\text{OPT}}/2 \rfloor$ ship routes from $\bar{\mathcal{R}} \cup \tilde{\mathcal{R}}$ with the lowest capacity utilization, set them as optional, set the new ship routes by reversing their port calling sequences as optional, too, and re-optimize.

7.4.3 Ship route alteration

The ship route alteration stage aims to improve the designed network by adding or removing ports of call in an intelligent manner.

In step 1, if the capacity utilization of a feeder ship route, which is defined as a ship route consisting of a regional hub and its feeders, is low (e.g., less than 50%), we then consider removing the feeder ports included in the feeder ship route from existing line-haul ship routes.

In step 2, if the capacity utilization of a feeder ship route is low, we also consider adding a new feeder port, which is assigned to the regional hub in the feeder ship route, to the feeder ship route based on the unshipped demand.

In step 3, if the hit-haul capacity utilization of a line-haul ship route is low, we then consider adding a new port to ship route based on the unshipped demand.

7.5 Case Study

7.5.1 Data description

We apply the network design models and solution algorithms to the global liner shipping network shown in Figure 7-1. The network has a total of 166 ports between which 6085 port pairs has container shipment demand. The general demand information regarding the ports are shown in Table 7-2 to Table 7-8, Figure 7-7, and Figure 7-8. There are four types of containers: dry 20-ft, dry 40-ft, reefer 20-ft, and reefer 40-ft. In terms of volume, there are totally 253383 laden TEUs and 106364.8 empty TEUs to ship per week. Three types of ships are used: 1500-TEU, 3000-TEU, and 5000-TEU ships. All other parameters and inputs are provided or estimated by the global liner shipping company. The initial network is based on the current network that the global liner shipping company is operating. The mixed-integer linear programming model NDM is solved by CPLEX-12.1 running on a 3.2 GHz Dual Core PC with 4 GB of RAM.

Table 7-2 Ports in North Asia

#	Port Name	Country Name	Total Import (TEUs)	Total Export (TEUs)
1	SHEKOU	CHINA	0	88
2	TOKYO	JAPAN	3024	1919
3	FUZHOU	CHINA	64	1543
4	DAVAO	PHILIPPINES	1175	2303
5	KOBE	JAPAN	3279	3313
6	HONG KONG	HONG KONG	5143	5027
7	SHIMIZU	JAPAN	122	178
8	ZHANJIANG	CHINA	0	262
9	NAGOYA	JAPAN	943	696
10	OITA	JAPAN	59	217
11	SUBIC BAY	PHILIPPINES	442	334
12	VOSTOCHNY	RUSSIAN FEDERATION	941	303
13	VLADIVOSTOK	RUSSIAN FEDERATION	646	12
14	MANILA	PHILIPPINES	2585	1638
15	KWANGYANG	KOREA	2220	3652
16	NAHA	JAPAN	1377	701
17	YOKOHAMA	JAPAN	3521	4115
18	OSAKA	JAPAN	43	0
19	QINGDAO	CHINA	2913	7035
20	UBE	JAPAN	5	165
21	NAN SHA	CHINA	218	1461
22	NHAVA SHEVA	PHILIPPINES	54	494
23	SHIBUSHI	JAPAN	246	328
24	XIAMEN	CHINA	713	7564
25	PUSAN	KOREA	5367	5278
26	MINDANAO	PHILIPPINES	850	1050
27	CEBU	PHILIPPINES	795	152
28	YANGSHAN	CHINA	1612	4876
29	NINGBO	CHINA	1217	10022
30	LIANYUNGANG	CHINA	346	1996
31	TAIPEI PORT	TAIWAN	818	48
32	KAOHSIUNG	TAIWAN	4711	7512
33	SHANGHAI	CHINA	5082	6214
34	HAKATA	JAPAN	599	454
35	XINGANG	CHINA	2588	2712
36	YANTIAN	CHINA	926	6225
37	DA CHAN BAY	CHINA	0	35
38	DADIANGAS	PHILIPPINES	166	387
39	DALIAN	CHINA	1172	1972
40	HOSOSHIMA	JAPAN	81	207
41	CHIWAN	CHINA	1370	9735

Table 7-3 Ports in South Asia

#	Port Name	Country Name	Total Import (TEUs)	Total Export (TEUs)
1	ADELAIDE	AUSTRALIA	362	601
2	SYDNEY	AUSTRALIA	1338	708
3	HALDIA	INDIA	63	108
4	PANJANG	INDONESIA	438	465
5	LAEM CHABANG	THAILAND	9388	10805
6	PORT QASIM	PAKISTAN	81	174
7	JAWAHARLAL NEHRU	INDIA	7496	7654
8	MELBOURNE	AUSTRALIA	1423	855
9	COCHIN	INDIA	282	0
10	KAMPONG SAOM	CAMBODIA	546	614
11	VUNG TAU	VIETNAM	613	1500
12	BELAWAN	INDONESIA	332	408
13	CHITTAGONG	BANGLADESH	2535	1738
14	SINGAPORE	SINGAPORE	7983	6895
15	SURABAYA	INDONESIA	2409	2116
16	SEMERANG	INDONESIA	1040	1077
17	KARACHI	PAKISTAN	7783	3125
18	FREMANTLE	AUSTRALIA	954	682
19	COLOMBO	SRI LANKA	2170	1392
20	PALEMBANG	INDONESIA	194	227
21	MONGLA	BANGLADESH	59	324
22	JAKARTA	INDONESIA	7404	5516
23	BRISBANE	AUSTRALIA	670	246
24	MUNDRA	INDIA	1146	1858
25	TUTICORIN	INDIA	876	0
26	PENANG	MALAYSIA	566	289
27	DA NANG	VIETNAM	22	0
28	HO CHI MINH	VIETNAM	4558	3713
29	MANGALORE	INDIA	6	0
30	MADRAS	INDIA	2048	2085
31	PIPAVAV	INDIA	3	0
32	HAIPHONG	VIETNAM	833	1400
33	PORT KLANG	MALAYSIA	4038	3441
34	VISHAKHAPATNAM (VISAG)	INDIA	138	224
35	PASIR GUDANG	MALAYSIA	591	0
36	MORMUGAO	INDIA	14	0
37	CALCUTTA	INDIA	396	444

Table 7-4 Ports in West Asia

#	Port Name	Country Name	Total Import (TEUs)	Total Export (TEUs)
1	AJMAN	UNITED ARAB EMIRATES	877	0
2	SALALAH	OMAN	349	110
3	HODEIDAH	YEMEN	1220	138
4	SOKHNA	EGYPT	5100	972
5	JEBEL ALI	UNITED ARAB EMIRATES	8018	5492
6	SHUAIBA	KUWAIT	403	525
7	MINA KHALID	UNITED ARAB EMIRATES	2693	34
8	PORT SULTAN QABOOS	OMAN	4	0
9	JEDDAH	SAUDI ARABIA	4539	1817
10	DOHA	QATAR	1356	201
11	BANDAR ABBAS	IRAN	119	0
12	BAHRAIN	BAHRAIN	942	960
13	DAMMAN	SAUDI ARABIA	4844	2265
14	DJIBOUTI	DJIBOUTI	838	112
15	ADEN	YEMEN	856	115
16	FUJAIRAH	UNITED ARAB EMIRATES	243	83
17	AQABAH	JORDAN	1626	307
18	ABU DHABI	UNITED ARAB EMIRATES	1948	1551
19	SHUWAIKH	KUWAIT	2951	615
20	UMM QASAR	IRAQ	1492	40
21	SOHAR	OMAN	615	1489

Table 7-5 Ports in West Europe

#	Port Name	Country Name	Total Import (TEUs)	Total Export (TEUs)
1	KOTKA	FINLAND	460	420
2	GOTHENBERG	SWEDEN	185	70
3	DUBLIN	IRELAND	651	758
4	LEHAVRE	FRANCE	1218	996
5	THAMESPORT	UNITED KINGDOM	276	651
6	KLAIPEDA	LITHUANIA	1	0
7	CORK	IRELAND	111	66
8	TALLIN	ESTONIA	1008	432
9	FELIXSTOWE	UNITED KINGDOM	766	721
10	HAMBURG	GERMANY	4897	4276
11	FOS SUR MER	FRANCE	159	139
12	ST.PETERSBURG	RUSSIAN FEDERATION	1633	1409
13	SOUTHAMPTON	UNITED KINGDOM	4720	1973
14	ANTWERP	BELGIUM	2237	1792
15	ZEEBRUGGE	BELGIUM	65	24
16	ROTTERDAM	NETHERLANDS	6857	5773
17	HELSINKI	FINLAND	174	271
18	GREENOCK	UNITED KINGDOM	4	153
19	BELFAST	IRELAND	63	60
20	GDYNIA	POLAND	977	1335
21	BREMERHAVEN	GERMANY	887	1560

Table 7-6 Ports in the Mediterranean Sea

#	Port Name	Country Name	Total Import (TEUs)	Total Export (TEUs)
1	GENOA	ITALY	788	744
2	LA SPEZIA	ITALY	0	5
3	LATAKIA	SYRIAN ARAB REPUBLIC	38	0
4	MARSAXLOKK MALTA	MALTA	49	13
5	PORT SAID	EGYPT	187	289
6	BEIRUT	LEBANON	19	0
7	VALENCIA	SPAIN	216	144
8	SUEZ CANAL CONTAINER TERMINAL	EGYPT	98	518
9	MERSIN	TURKEY	862	605
10	BARCELONA	SPAIN	188	554
11	DAMIETTA	EGYPT	27	115
12	HAIFA	ISRAEL	194	192
13	LIVORNO	ITALY	1	0

Table 7-7 Ports on West Coast of America

#	Port Name	Country Name	Total Import (TEUs)	Total Export (TEUs)
1	LAZARO CARDENAS	MEXICO	647	462
2	PUERTO QUETZAL	GUATEMALA	478	373
3	BALBOA	PANAMA	244	175
4	OAKLAND	UNITED STATES	3904	4926
5	ACAJUTLA	EL SALVADOR	874	904
6	SAN PEDRO	UNITED STATES	12789	13480
7	DUTCH HARBOR	UNITED STATES	3	347
8	SEATTLE	UNITED STATES	6256	5312
9	VANCOUVER	CANADA	2094	2241
10	SAN ANTONIO	CHILE	237	482
11	ENSENADA	MEXICO	189	242
12	BUENAVENTURA	COLOMBIA	206	360
13	GUAYAQUIL	ECUADOR	218	843
14	PAITA	PERU	0	451
15	IQUIQUE	CHILE	15	8
16	CALLAO	PERU	188	549

Table 7-8 Ports on East Coast of America

#	Port Name	Country Name	Total Import (TEUs)	Total Export (TEUs)
1	JACKSONVILLE	UNITED STATES	226	143
2	MONTREAL	CANADA	543	589
3	PUERTO BARRIOS	GUATEMALA	358	272
4	PUERTO LIMON	COSTA RICA	442	607
5	PORTSMOUTH	UNITED STATES	2003	2076
6	HOUSTON	UNITED STATES	800	1256
7	MOBILE	UNITED STATES	95	240
8	PUERTO CORTES	COSTA RICA	368	456
9	NEW YORK / NEW JERSEY	UNITED STATES	8189	2810
10	CHARLESTON	UNITED STATES	2076	1704
11	MANZANILLO	PANAMA	4089	1641
12	MIAMI	UNITED STATES	1378	1104
13	SAVANNAH	UNITED STATES	3064	2388
14	SAN JUAN	PUERTO RICO	674	372
15	CARTAGENA	COLOMBIA	595	526
16	MANAUS	BRAZIL	490	35
17	RIO HAINA	DOMINICAN REPUBLIC	371	218

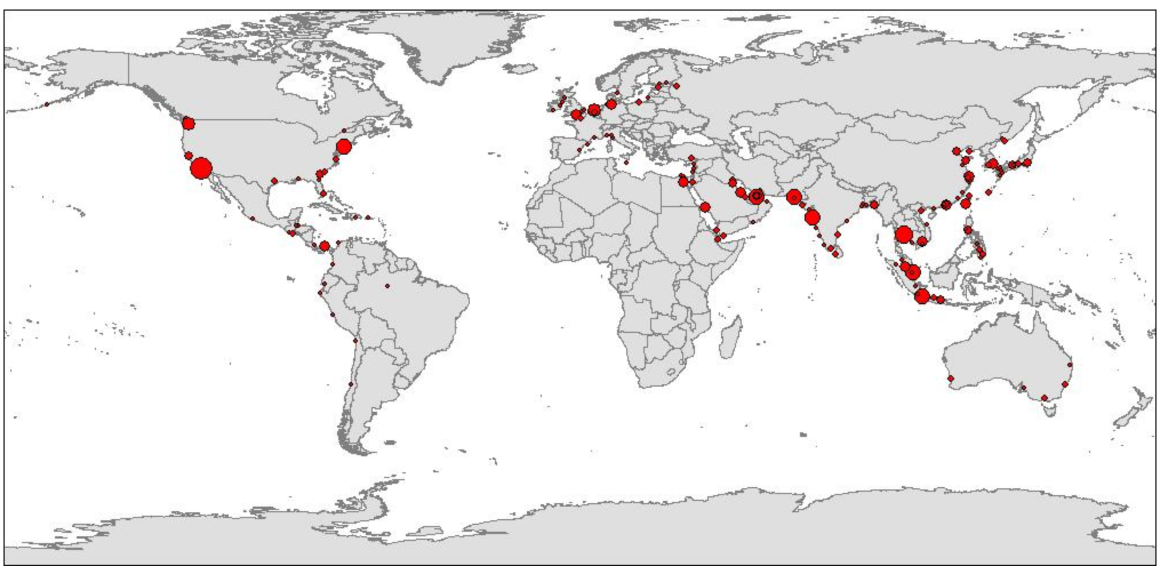


Figure 7-7 The import volume of each port

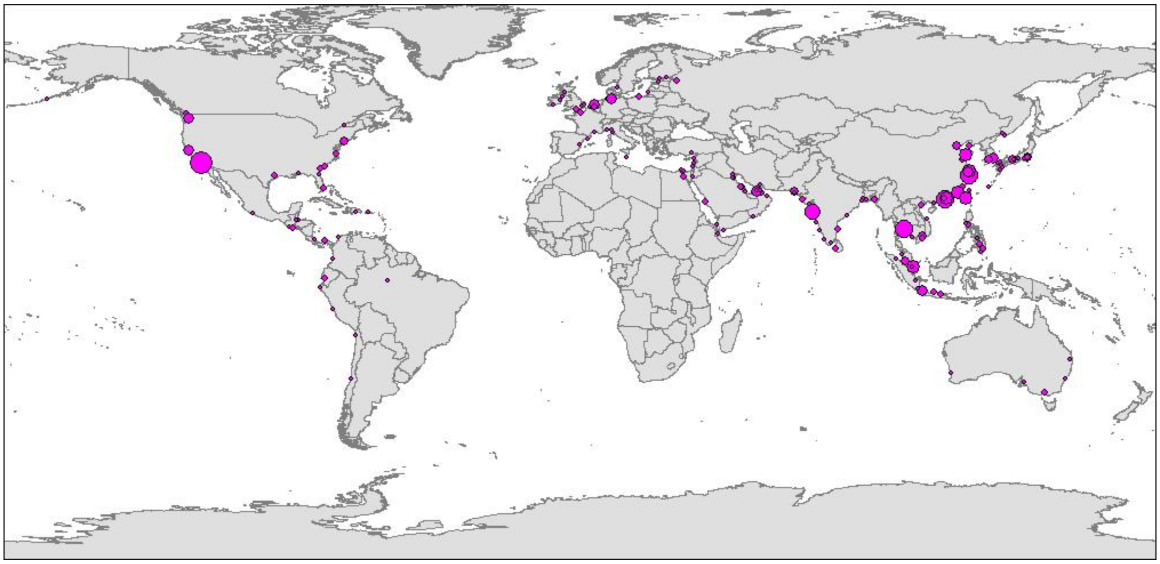


Figure 7-8 The export volume of each port

7.5.2 Computational results

The algorithm finishes after 7 minutes. A total of 81 ship routes are designed in the final network, as shown in Table 7-9. 304 ships are deployed in the network, with a total of ship board capacity of 1.157 million TEUs. An Asia-Europe ship route is shown in Figure 7-9, and its detailed information is presented in Table 7-10.

Since many of the input data are estimated, it is difficult to evaluate the quality of the designed network. Liner shipping companies tend to use two indicators. The first one is the ratio of ship board capacity over the demand, and the second one is the hit-haul utilization of ship routes. The proposed algorithm designs a network that shipped 248692 TEUs per week (4691 TEUs are unshipped) with the ship board capacity of 1.157 million TEUs. This result outperforms the status quo of the liner shipping company. Most of the hit-haul utilization of the designed ship routes exceeds 90%, with the exception of a few feeder ship routes. The hit-haul utilization of the feeder ship routes is low because the smallest ship size used is 1500 TEUs. In practice, smaller ships can be deployed. Moreover, the liner shipping company can buy feeder shipping services if the demand is not large enough. Considering these limitations of the input data, the quality of the designed liner shipping network is high.

Table 7-9 Ship routes and ships deployed

Region-to-region	Number of ship routes	Number of ships		
		1500-TEU	3000-TEU	5000-TEU
Intra-Asia	50	49	22	46
Asia-Europe	6	8	0	43
Trans-Pacific	10	3	7	82
Trans-Atlantic	2	0	10	0
Intra-Europe	5	4	3	2
Intra-America	7	14	3	0
Asia-US-Europe	1	0	0	8
Total	81	78	45	181

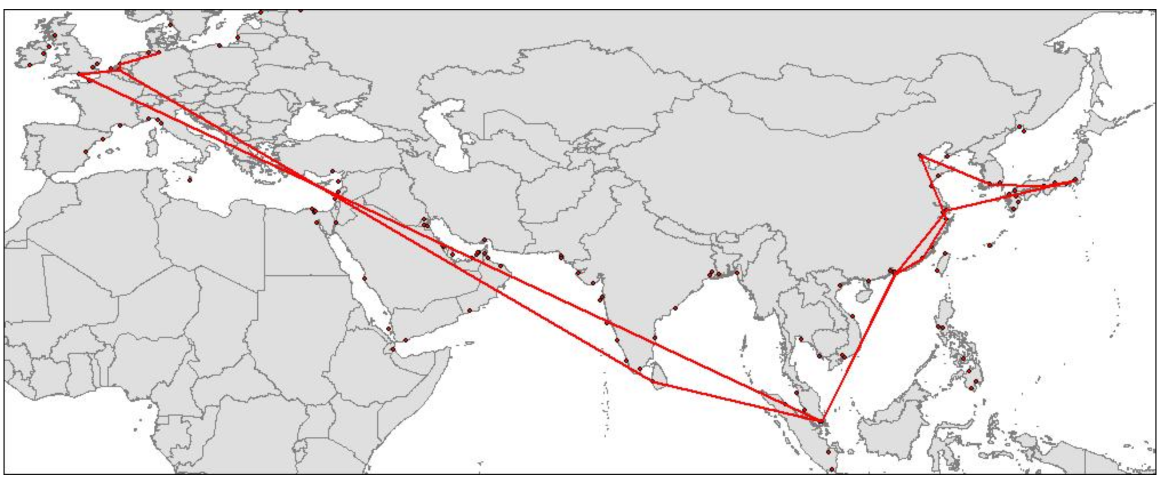


Figure 7-9 A designed Asia-Europe ship route

Table 7-10 Details of a designed Asia-Europe ship route

Ship route ID: [54]; Ship type: 5000-TEU; Ship number: 10				
Shipped laden containers (TEUs) for different O-D pairs		Intra-Asia	Asia-Europe	Trans-Pacific
		5724	9601	224
		Trans-Atlantic	Intra-Europe	Intra-America
		152	37	0
Leg capacity utilization (%)				
1- KOBE	2- KWANGYANG	3- XINGANG	4- NINGBO	5- XIAMEN
27	28	33	79	100
6- HONG KONG	7- SINGAPORE	8-SOUTHAMPTO	9- ANTWERP	10- ROTTERDAM
100	100	47	24	16
11- HAMBURG	12- ROTTERDAM	13- ANTWERP	14- COLOMBO	15- SINGAPORE
35	62	100	100	100
16- YANTIAN	17- HONG KONG	18- SHANGHAI	19- YOKOHAMA	
78	70	74	42	

7.6 Conclusions

This chapter addresses a realistic liner shipping network design problem while considering practical operations and features, including multi-type containers, container transshipment operations, empty container repositioning, origin-to-destination transit time constraint, consistent services with the current network, and joint services with other liner shipping companies. A practical and efficient method to handle the routing of laden containers is proposed to account for different types of containers. The container routes are generated based on both a heuristic method and an exact approach.

A network design algorithm is presented. It first optimizes the initial network by removing the ship routes with low capacity utilization. After that, new ship routes are designed to ensure that the container shipment demand is fulfilled. The resulting network is further improved by two more stages. In one stage, the network is refined by removing ports of call with low handling volume, redeploying ships based on ship route capacity utilization, removing voyage legs with low leg capacity utilization, removing ship routes with low capacity utilization, and reversing the port rotations of ship routes with low capacity utilization. In the next stage, both feeder ship routes and line-haul ship routes are further altered by adding or removing ports of call.

A real case study based on the global shipping network of a liner shipping company, consisting of 166 ports, is reported. The algorithm efficiently designs a liner shipping network. Two indicators, the ratio of ship board capacity over the demand and the hit-haul capacity utilization of ship routes, demonstrate that the design network is of high quality.

CHAPTER 8. CONCLUSIONS AND RECOMMENDATIONS

8.1 Overview and Contributions of the Work

The research work carried out so far concerns with four issues: an origin-based fleet deployment model which fully captures the transshipment operations in liner shipping, a cutting-plane based ε-optimal algorithm for optimizing the sailing speed of container ships, a ship route schedule design problem with sea contingency and port time uncertainty, and the design of a practical liner shipping network.

In the origin-based fleet deployment model, we use the flow volume on each leg, load volume and discharge volume at each port for containers originating from the same port as the decision variables. The origin-based formulation is much more compact than O-D based formulation. By comparing the container shipment demand and the volume of containers loaded and discharged at the ports, the number of containers transshipped at each port of call is derived. Thereby the transshipment cost can be accounted for in the model. In addition, the load volume and discharge volume at each port of call can be converted to container handling time. Hence, the transshipment cost and container handling time are included in the model. Since there are decision variables representing the load and discharge of containers at all ports of call, containers are allowed to be transshipped at any port of call in the model. Extensive test experiments show that the proposed origin-based model can be efficiently solved by CPLEX for real-case problems.

In the speed optimization problem, we first examine the bunker consumption - sailing speed relation using historical operating data from a global liner shipping company. This work closes the gap in literature by providing empirical data to test how good the third power approximation is. Results show that the extensively used third power relationship is indeed a good approximation. Therefore, the third power relation can be used if not enough historical data are available. Once enough historical data are available for the calibration purpose, a more accurate bunker consumption function should be used. We subsequently investigate the optimal sailing speed problem of container ships on each leg of ship routes in a liner shipping network. Practical issues which are seldom examined by existing studies on the speed optimization, including unique bunker consumption function for each leg, transshipment, container routing, and container handling time, are taken into account in the model. The sailing speed optimization problem for container ships in a liner shipping network with container routing is a practical research issue arising in the liner shipping industry. This problem is formulated as a mixed-integer nonlinear programming model. In view of the convexity, non-negativity, and univariate properties of the bunker consumption - sailing speed function, a novel outer-approximation algorithm is proposed to obtain an ε-optimal solution. Similar to the discretization method or the SOCP approach, the proposed algorithm is exact in that it obtains an ε-optimal solution. At the same time, the outer-approximation algorithm is very efficient compared to the discretization method and not subject to the restriction of the SOCP approach. Practical test instances demonstrate that the algorithm is efficient enough to solve problems encountered in practice to optimality with guaranteed precision.

In the ship route schedule design problem with sea contingency and port time uncertainty, an effective solution algorithm is proposed in order to minimize the ship cost and bunker cost, while fulfilling the port-to-port transit time constraints. This problem is a new research issue with practical significances. The contributions to the literature can be summarized into three-fold. First, to the best of our knowledge, it makes the first attempt to examine the optimal sailing speed function in view of sea contingency to minimize bunker consumption. The optimality condition for the sailing speed and the optimal sailing speed function with time are derived. Second, it contributes to the line of literature on optimization of sailing speed to control bunker consumption by providing an efficient and exact cutting-plane based solution algorithm. This algorithm is capable of incorporating practical considerations, such as different bunker consumption – sailing speed relations on different legs, port-to-port transit time requirement, and sailing speed optimization at the network level. Third, it addresses the practical schedule design problem arising in liner shipping industry while considering port-to-port transit time with transshipment and sea contingency and uncertain port time. In other words, the transit time issue in liner shipping is addressed to its generality. The port-to-port transit time with transshipment issue is solved with a mixed-integer programming model; sea contingency is investigated in the optimality condition of sailing speed; and the uncertain port time is addressed by proving the convexity of the expected bunker cost on each voyage leg in the inter-arrival time between the two consecutive ports of call of the leg. The novel holistic solution algorithm exploits the special structure of the decision problem and integrates several techniques in a nice manner. The proposed model and algorithm are very efficient to solve practical-scale problems and thus provide a useful planning tool for liner shipping companies.

The liner shipping network design problem addressed in this study is different from existing literature in the following major aspects. First, multi-type containers are considered. In the literature all containers are transformed to TEUs, which may not be accurate because the volume of a dry 40-ft container is two TEUs, while its handling cost is less than the handling cost of two dry 20-ft containers. A practical and efficient method to handle the routing of laden containers is proposed to account for different types of containers. Second, the origin-to-destination transit time of containers and the maritime cabotage restriction are included. To this end, both a heuristic method and an exact approach are proposed to generate container routes that satisfy these two constraints. Third, the network is designed in a manner that accounts for the initial network. This is a very practical consideration because a liner shipping cannot change its network overnight. As a result, a network that shares some degree of common features is desirable for liner shipping companies. The proposed models and algorithms are applied to the global shipping network of a liner shipping company, consisting of 166 ports. The ratio of ship board capacity over the demand and the hit-haul capacity utilization of ship routes demonstrate the efficacy of the proposed algorithms.

8.2 Recommendations for Future studies

8.2.1 Intermodal container transportation

The origin and destination of containerized cargo are usually inland locations. Sometimes shippers arrange the inland transportation from the origin to the export port and from the import port to the destination. Under this circumstance, liner shipping companies can take ports as the origin and destination of containers. Otherwise, liner shipping companies not only provide maritime transportation services, but also take charge of inland transportation to fulfill the supply chain management requirement of the shippers.

The literature on routing and scheduling in liner shipping focuses exclusively on the ocean side. Although there are a few studies on the inland transportation of containers (e.g., Bontekoning et al., 2004; Macharis and Bontekoning, 2004), hardly any research is directed at the optimization of both inland and maritime transportation systems. The interaction between the inland and maritime transportation lies in the choice of load (export) port and discharge (import) port as well as the origin–destination transit time consideration. It is evident that a holistic optimization of the intermodal transportation network has implications for a liner shipping company.

8.2.2 Joint planning between liner shipping companies and port operators

In general, liner shipping companies and port operators make decisions independently. Nevertheless, a holistic optimization approach may improve the operating efficiency of both parties. For example, when a port is congested, ships may slow their speed to save bunker because they will have to wait for a berth even if they arrive early at the port. There have been some research in this area, for example, Golias et al. (2010) and Du et al. (2011). Another example is that liner shipping companies and port operators would agree on a berth time window for ships over a planning horizon. However, in practice, ships frequently miss the allocated time slots because liner shipping companies build too little buffer time in the schedule. This would adversely affect both berth and yard planning for port operators. A major challenge here is how to design mechanisms to coordinate different parties involved in both the decision making and the execution of the decisions.

8.2.3 Shipping network reliability and vulnerabilities

Maintaining a high quality of service is a great concern for liner shipping companies and maintaining reliable schedules has proven to be a challenge. In the container shipping segment, only 56 percent of all shipments arrived on time in the second quarter of 2011. (Drewry Maritime Research, 2011). The effect of unreliable deliveries from liner shipping companies on the supply chain as a whole is analyzed in Vernimmen et al. (2007). They proposed that if schedule reliability is increased, customers' inventory costs may be reduced by 20 percent. A model for calculating time reliability in a container liner shipping network is presented in Wu et al. (2009). Understanding and correctly formulating models that take reliability and vulnerability into account is an important topic for future research.

8.2.4 Green shipping

Designing shipping networks that are not only efficient, but also minimize environmental impact is becoming more important. Emissions from commercial shipping are currently the subject of intense scrutiny (Psaraftis, 2005). Among the top fuel-consuming categories of ships and hence air polluters are container vessels and the main reason is their high service speed. There are a few research papers in this area, such as Psaraftis and Kontovas (2010), Corbett et al. (2010), Kontovas and Psaraftis (2011) and Jepsen et al. (2011). In general, not many papers to date address the challenges of incorporating environmental issues into the models. In practice, increased environmental awareness will affect all levels of decision making, from deciding the fleet composition and which ports to call to selecting the sailing route between two ports in the presence of a storm or favorable currents. It will therefore be important to consider these challenges in future models.

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ACCOMPLISHMENTS DURING PHD STUDY

Awards and Honours

1. Awardee: President’s Graduate Fellowship of NUS

The President's Graduate Fellowship (PGF) is awarded to candidates who show exceptional promises or accomplishments in research.

Journal Publications

1. Wang, S., Meng, Q., 2012. Liner ship route schedule design with sea contingency time and port time uncertainty. Transportation Research Part B, Vol. 46, No. 5, pp. 615-633.
2. Wang, S., Meng, Q., Bell, M.G.H., 2012. Liner ship route capacity utilization estimation with a bounded polyhedral container shipment demand pattern. Transportation Research Part B, in press.
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