

**CO-EVOLUTIONARY BIDDING AND  
COOPERATION STRATEGIES FOR BUYERS IN  
POWER MARKETS**

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## SUMMARY

Deregulation of electric power industries in recent years has opened many opportunities for electricity buyers. However, the strong influence of network physical constraints may result in economic decisions that adversely affect the interests of the consumers. Compared to the monopolistic economy of yesteryears, electricity buyers may now actually be able to influence the market by cooperating with other buyers in the electrical power network. This research presents different models using agent-based co-evolutionary framework for evolving individual and cooperative strategies of electricity buyers in a power market.

To realize the above objectives, simulations involving evolutionary algorithms and multi-agent systems are used to study a single-node system, where economic agents are modeled by their supply / demand functions, and then a multi-node system, where the technical constraints of the power distribution network are fully taken into account. The results of the single-node model show that it is of great benefit to cooperate but the free rider problem may arise when an individual buyer gains more profit due to the cooperative effort of the others.

The multi-node model is investigated through two situations. First, we focus on deterministic cases where buyers choose their bidding strategies to maximize the profits in different scenarios of playing individually or cooperatively. It is also found that by evolutionary learning, buyers can benefit from cooperation. Next, the uncertain nature of the market is modeled where buyers find optimal cooperation strategies to hedge against the risk of low

payoffs. Our approach is universal since it can be applied to study the behaviors of buyers with any objective for cooperation. We proved a theorem to link the payoff distribution problem in cooperative game theory with the optimal coalition structure generation problem in combinatorial optimization theory. The statistically consistent simulation results show that our approach is able to discover interesting cooperation strategies, and can be easily extended for practical networks with large number of buyers.



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## LIST OF PUBLICATIONS RELATED TO THIS THESIS

Srinivasan, D; Trung, Ly Trong, “Co-Evolutionary Bidding Strategies for Buyers in Electricity Power Markets”, *IEEE Congress on Evolutionary Computation (CEC)*, Pp.2519-2526, 2011.

Trung, Ly Trong; Srinivasan, D, “Bidding and Cooperation Strategies for Buyers in Power Markets”, *Submitted to IEEE Transaction on Evolutionary Computation (TEC)*.

Trung, Ly Trong; Srinivasan, D, “Cooperative Strategies of Buyers in Power Markets – An Evolutionary Game Approach”, *Submitted to Engineering Applications of Artificial Intelligence (EAAI)*.



# **Chapter 1: INTRODUCTION**

In this chapter, we give a brief review on deregulated electricity market. Then the motivation for the work done and structure of the thesis are presented.

## **1.1 Overview of the deregulated power market**

Over the last twenty years, electric power markets have successively experienced a deregulation process related to the opening of gas and electricity industry. Competition, expected to push operators to high efficiency, is presented as the most effective response to the imperfections of the old regulated power industry. Initially implemented by Anglo-Saxon countries, the deregulation of power markets has been gradually taken up by all industrialized countries. By the principle that competition should be introduced whenever possible, this reform has to major implications on the decision of firms initially protected from competition. Moreover, electricity buyer agents also have new opportunities to actively optimize their objectives in a dynamically changing environment.

### **1.1.1 Electricity and natural monopoly**

Electricity is an essential commodity in modern life; the interruption of the electricity supply implies a considerable social cost. Electricity is not storable by

its users; the demand, therefore, must be satisfied in real time. The consumption of electricity is subject to strong randomness which is a function of exogenous factors such as temperature or brightness.

Electricity is transported via high voltage interconnected lines. Transmission and distribution (low voltage) follow nodal rule and mesh rule of Kirchhoff. The lack of storage implies that we must have permanent means of reserve to manage the difference between the predicted quantity and the actually produced and consumed quantity. Transmission is also subject to line loss (part of the electrical power is converted into heat due to Joule effect). If the line temperature exceeds a certain threshold, it will give rise to the rupture of the line. The cost of failure is outrageous as other lines can also collapse in cascade. These features illustrate that the systems must be designed according to the peak demand, with some margin to ensure continuity of supply in case of technical problems.

The electric power industry consists of three major components: central power generation, high voltage transmission and distribution networks. We can therefore recognize the importance of coordination between the various activities related vertically, both in long-term system configuration, and short-term efficient allocation of resources. If we add the economies of scale in production and increasing returns on transportation, electricity markets appear as natural monopolies and vertical integration can significantly reduce transaction costs. This explains why electricity markets have been managed by national or regional monopolies (at least on transportation) in all countries, often vertically integrated, or characterized by close ties between vertically related actors. These companies were often public, particularly because electricity has become a vital product

carrying public service missions. The involvement of the state had also facilitated the mobilization of main material resources that are necessary for the rapid construction of dense and high performance networks.

### **1.1.2 Movement to a new competitive market**

The motivation of movement to competition is driven by a number of criticisms against monopolies in place: inefficiency of production and social debate over surplus sharing. In developing countries, bureaucratic criticism is often used to justify the open to competition and privatization of the electricity industry. Competition, expected to push operators to efficiency, is presented as the most effective response to these imperfections. Thus, allowing consumers to choose their suppliers should guide the latter to better use of resources, reducing waste, improving services or even greater respect for the environment.

The deregulation process has transformed the power market into a competitive environment; firms must therefore change their strategy and organization deeply to adapt. In this free market economy, each participant seeks for the optimal strategy that maximizes its benefit when trading.

The main sectors of power generation, distribution, wholesale and retail have seen an increase in the number of players, who are now able to freely enter and exit the market to seek out economic opportunities. In most countries that have seen the deregulation in power sector, the competitive nature of the new economy has aided the technological push in this area. Coupled with the market forces at work, this has generally led to lower costs and greater market reliability, which has benefited the industry, especially the end users. The result is a market

of stiff competition in which the price and the electricity power traded is decided by the market forces, and where all players are price takers and have to accept the market clearing price (MCP) as dictated by the market. New rules and regulations have been set into place by supervisory bodies to regulate possible technical problems such as system blackouts and transmission security, as well as economic decisions such as curbing possible market power to restrict the ability to set unreasonably high price. Therefore, electricity buyers and sellers have to reconsider their bidding strategies and economic approaches to tackle the changed environment.

### **1.1.3 Deregulated power market models**

The management of the daily operations and ensuring network security are tasked to two independent bodies: the power exchange and the independent system operator. The former determines the market clearing price and market clearing quantity (MCQ) based on the demand and supply bids it receives from the electric power buyers and sellers respectively. The latter monitors and checks the dispatch forecasts to ensure that the security of the system has not been compromised, and advises the power exchange on preventive measures.

Following the restructuring of electricity market, different market models have been proposed to replace the vertically integrated monopoly. There are three basic types of deregulated power market models: PoolCo model, the bilateral contracts model and the hybrid model [1].

A PoolCo is viewed as a centralized marketplace that clears the market for buyers and sellers using a set of rules for trading electricity. Producers submit

their bids for different periods, usually for each hour. Every offer of power quantity is accompanied by a corresponding price representing the minimum level that each producer is willing to accept for each period. The pool centralizes all offers and defines an order of economic efficiency. The last accepted bid that is necessary to cover the level of demand defines the spot price. Sellers compete for selling electricity; if a seller bids too high, it may not be able to sell. On the other hand, buyers compete for buying power, and if their bids are too low, they may not be able to purchase.

In the bilateral contracts model, the supplier and the customer trade directly with each other by signing a contract that defines the kind of service they desire at the price they desire. However, in power market, this model has some drawbacks: Because of its failure to be stored, electricity is extremely price volatile in times of peak demands; hence the market has difficulty in reaching the equilibrium. Moreover, due to the sharing of common transmission network, the transmission losses caused by the action of one participant can affect all others. Because of these negative points, the simulation and analysis of power market often make use of the PoolCo model.

The hybrid model combines features of two previous models. The participants can choose to sign bilateral contracts or to be served by the power pool. Under this mechanism, true customer choice is offered and a variety of services and pricing options to best meet individual customer needs is created.

## **1.2 Motivation of the research**

The deregulation of the electricity power industry has already been

accomplished in many countries and remarkable changes in the management of power systems are introduced. A new environment for the market participants was created since the electricity price is now set by an auction mechanism.

In the global competitive market, electricity buyers are no longer price taker since they are able to influence the market by using different bidding strategies as well as cooperating with other buyers. Therefore it is necessary to develop and investigate individual and cooperative strategies of electricity buyers. That is the inspiration and motivation of this project.

### **1.3 Structure of the thesis**

The thesis is organized in 9 chapters.

Chapter 1 gives an overview on the deregulated power market and the motivation of the research.

In Chapter 2, we give a literature review of different approaches to model power market, with highlights on applying Evolutionary Algorithms in a Multi-Agent framework.

Chapter 3 presents the methodology of the research and gives a brief background on computational tools that will be applied such as Evolutionary / Co-evolutionary Algorithms and Cooperative Game.

In Chapter 4, we propose a single-node model for simulating power market with generators and buyers as two types of participants. The bidding model and market clearing mechanism are also presented.

Chapter 5 presents the simulation results of the proposed single-node model.



Different scenarios of the market are taken into account and economic aspects of the results are investigated.

Chapter 6 develops a multi-node model of the power market where all physical constraints are taken into account. The Optimal Power Flow problem is introduced as a market clearing engine.

Chapter 7 presents the details of the multi-node model implementation, such as the physical power network and market participants' parameters.

Chapter 8 summarizes the simulation results of the multi-node model and discusses the findings with different perspectives.

Chapter 9 concludes this thesis.

## **Chapter 2: REVIEW OF POWER MARKET MODELS**

The electricity market is characterized by complex practical aspects, such as imperfect competition, strategic interaction, asymmetric information, and the possibility of multiple equilibria [2]. Traditional economic modeling techniques face difficulties when taking into account these factors. Therefore, Computational Intelligence is intensively applied to economy, especially economic theories. Recent advances in this field have allowed simulating artificial societies and thus studying economic models by running computer simulations. The concept of “Agent” in computer science is close to that of economic theories [3]. Under a Computational Intelligence framework, the interactions between intelligent agents can be observed and analyzed. With these efficient modeling and simulation tools, researchers are able to investigate economic theories in a complementary framework to the standard analysis.

### **2.1 Background of Agent Based Technology**

From the last decade, information technology grows with an amazing speed. Today, transmission / processing capabilities and networked information resource storage actively interact in the distributed computing paradigm [4] to serve its needs. The current trend in software engineering methodology to build

software system is the object oriented methodology. With the ability to structure data based on inheritance and composition structures, the ability to account for the generic characteristic of behaviors or concepts, the reusability property of objects, object oriented methodology become very attractive for software implementation.

In real world, both the computer system and the problems to be solved are also often physically distributed over a wide area; therefore a large number of experts in different domains is required, coordinating their knowledge and their local view of the problem to reach a global solution. Multi-agent technology can be considered as an extension of the object oriented technology, accounting for the distributed nature of systems and problems.

MAS allows artificially reproducing real life system through autonomous, independent and interacting agent objects. Examples of successful application of MAS to many fields include traffic control simulation, robotics, ecological simulations, videogames...In particular, MAS makes it possible to study individual behaviors and to link them to observations at the macro level, thus allow having a new insight in the field. Indeed, since most collective phenomena result from individual decisions, there is a need to account for phenomena emerging from interaction of individual behaviors.

Agent technology is also commonly used to assist or replace humans in numerous complex tasks. The need for effective and quick decision taking procedures in the increasing global competition involves the support of intelligent systems. Agent-based technologies and international standards developed [5] have taken great steps over the years. The new agent-based approach using object-oriented frameworks [6] and agent-oriented programming

paradigms is far more superior to classical methods in modeling autonomous nature and decision making of market participants.

Multi Agent Systems (MAS) is one of the fastest growing and most interesting fields in agent based technology that models autonomous decision making entities. Recently, encouraging results was produced in a novel approach to deal with multi-player interactive systems [7].

## **2.2 Multi-Agents in economics**

Traditional analytical methods typically have to impose strong and constraining assumptions on the agents of system being studied, so that the models can be tracked mathematically. Therefore, the agent based approach is suitable for simulating and validating the decision making process of various participants in deregulated electricity market. Each agent represents an autonomous participant with independent bidding strategies and responses to market outcomes.

As we saw in the previous section, MAS used in economics is a very particular framework of a fully decentralized economy. The study about this type of economic models comes from the desire of some economists to get out of the standard analytical framework that describes a centralized economy and ignores the interactions between agents. This conventional model functions following the simplifications that do not allow apprehending a number of phenomena, including those rising from the cooperation among agents. The development of MAS follows the development of new economic reflection with game theory as a main tool. Multi-agent simulation is a powerful approach. Indeed, agents are more realistic because they take into account more parameters.

The advantage of using MAS is the ability to show how the collective phenomena arise from the interaction and adaptation of a population of autonomous and heterogeneous agents. These models based on agents are also used as supporting decision tool for firms. These models allow the testing of several market configurations and studying the consequences of individual actions of market participants.

Cooperation and trust between agents, with trust and profit as the determinants of the relationship was investigated using agent-based computational economics in [8]. Similarly, in [9], the agents cooperate with the condition that there is not a reduction in their own benefits.

In [10], it was shown that the joint effort of all rational individuals involved in the economic activities will lead to equilibrium through a sequence of events. The analogy can be applied for a multi agent system, where the concept of rationality can be imbedded into the agents through certain sets of instructions. The agents follow these rules and further develop this rationality by applying penalties or benefits to their actions during their learning process.

It was indicated in [11] that classical economics and computational intelligence are dissimilar because the former is based on mathematical analysis with related simplifications; while the latter is inspired from natural principles and deriving its conclusions by simulating real-world data. Nevertheless, these two approaches are complementary to each other because a convergence in computational intelligence algorithms is equivalent to equilibrium in economics. For that reason, the economic analysis helps to understand the simulation results.

### **2.3 Multi-Agents in power systems**

Particularly, the multi-agent system (MAS) approach is suitable for simulating and validating the participation of various participants in deregulated energy market. Individual entities in the market are represented as agents. Each agent models an autonomous participant with independent bidding strategies and responses to market outcomes. Agents are able to function autonomously and interact actively with their environment. These specific characteristics of agents can be best employed in simulation of autonomous entities as in the situation of the restructured energy market. The administration role of Independent System Operator (ISO) in the restructured energy market can also be considered by an agent entity with decision making policies and market rules to manage efficiently the allocation and dispatch of energy resources on the network. This section gives an overview on the modeling and simulation of energy market and subsequently the application of this thesis using agent based technology.

Multi-agents have been widely applied in power systems. We can find an example of real-world agent representation of power market in [12]. A multi-agent framework was used to realize switching operations of a power system in [13] by considering protective equipment and transmission as agents. A similar multi-agent approach to coordinate secondary voltage control during system contingencies and to create an adaptive over current protection was presented in [14] and [15] respectively.

In [16] was developed an efficient real time power management system using various types of agents to represent the elements of the network. In [17], the competition among intelligent agents was modeled with the goal of obtaining the

quantity of power desired by looking for the optimal electricity energy path.

Chazelas [18] designed a multi-agent electricity market simulator and developed an evolutionary algorithm to solve for unit commitment and dispatch in real-time.

## **2.4 Power market modeling using Evolutionary Algorithms in Agent-based framework**

Intelligent agents possess the capability to learn and evolve from experience; therefore evolutionary algorithms are frequently integrated to model competitive market. In [19], Curzon showed that Genetics Algorithms (GAs) have a high performance in simulating simple standard games. The author also interpreted how GA process discovers the equilibria.

In [20], a refined genetic algorithm was employed to get greatest benefit supplier by finding optimal parameters of linear supply functions. In [21], Richter and Sheblé verified the evolution of bidding strategies of generation companies against the static strategy of a distribution company, without taking into account the transmission constraints. In [22], the optimal selling price for generators was found while taking into account diverse issues such as tariffs, pricing strategy, discount scheme and the elasticity of customer demand.

In [23], Fuji et al. considered a learning multi-agent model to assess different types of generator plants while taking into account real time reserve markets as well as the fluctuation of seasonal and hourly demand. Contreras et al. implemented a simulator for power exchange market in [24] which may be extended to deal with different market clearing mechanisms and incorporate more market rules.

In [25] a Cooperative Co-evolutionary Algorithm was presented, emphasizing on its potential applications to power systems. Cau and Anderson described in [26] another co-evolutionary approach where the agents learn and improve their strategies. Anderson described in [27] another co-evolutionary approach where the agents learn and improve their strategies. They showed that implicit collusion happened even with very limited information available to participants. Chen et al. [28] analyzed supply function equilibrium models of an oligopolistic power market by considering both linear and piece-wise linear supply functions. The results show a robust convergence towards the equilibrium. Adaptive agent based algorithms have also been applied to find equilibria of complex double auction game in a discriminatory pricing electricity market [29].

It was underlined in [30] that a combination of a multi-agent system and an evolutionary algorithm cannot permit the agents to adapt efficiently due to the limitations of the evolutionary algorithm which is set as the external layer. Alternatively, each sub-population or agent should be modeled more similarly to real-world agents who can evolve on their own. The multi-agent system framework should concentrate on providing an environment for the agents to interact. This is the inspiration of the Co-evolutionary Algorithm that will be discussed further.

Although the number of buyers is significantly more than the number of sellers, most of the researches have been concentrating on the supply side. In a competitive market, the agents of both supply side and demand side continuously adapt their strategy according to their objectives. An Agent Based Evolutionary Model can therefore model the double bid auction market. The optimal bidding strategies for generators and large consumers in competitive market was studied in



[31] using the Monte Carlo approach.

Srinivasan et al. [32] focused on minimizing the LMP of buyers using different evolutionary algorithms. In [33], the result was improved by adding a game theoretic decision module. The alliance strategy of buyers was studied in [34] and it was shown that the buyers can lower their costs by evolving their group sizes and memberships.

## **2.5 Cooperative Game and Optimal Coalition**

Game theory provides important concepts and methods when studying the interaction of different agents in competitive markets. In particular, cooperative game theory provides tools to solve the conflicts arising in the interaction, such as in allocating of transmission costs [35]. The solution mechanisms of this approach appreciate fairness, efficiency, and stability in distribution the payoffs among agents. Besides, extensive efforts have been devoted to the area of coalition formation. One direction of research is to partition the agents into coalitions such that the sum of payoffs to all the coalitions is maximized. This is the problem of Optimal Coalition Structure Generation (OCSG).

There are two main classes of available algorithms that have been designed for OCSG problem: exact algorithms use integer programming or dynamic programming, and non-exact algorithms use heuristic or genetic algorithms. In [36], a dynamic programming (DP) that can be directly applied to the OCSG problem with the complexity of  $O(3^n)$  was developed. This complexity is significant less than exhaustive enumeration that runs in  $O(n^n)$  time ( $n$  is the number of agents). Later, the authors in [37] developed an Improved Dynamic

Programming (IDP) algorithm that requires fewer operations and less memory than DP. However, both DP and IDP are not anytime algorithms, meaning they cannot be interrupted at any time to observe the best solution found so far. Given large numbers of agents, this property is a major drawback because agents, usually being limited in time, wouldn't be able to wait until the end of the execution of the algorithm. To overcome this weakness, the first anytime algorithm for coalition structure generation was introduced in [38] by producing solutions within a finite bound from the optimal, and was further improved in [39]. More recently, the OCSG problem was formulated as a mixed integer programming problem and can be solved efficiently in [40].

Non-exact algorithms do not guarantee finding an optimal solution, but they simply offer “good” solutions very quickly, compared to other algorithms. Given larger numbers of agents in this problem, this feature often makes these algorithms more practical. In [41], the authors have proposed an Order Based Genetic Algorithm for optimal coalition structures; the results showed that it surpasses existing deterministic algorithms. Both coalition structure generation and payoff distribution in competitive environments were addressed in [42, 43], where a bound from the optimal can be guaranteed if a kernel-stability is met [43]. More recent research has also modeled dynamic environments, where there are uncertainties; for example the coalition value is not fixed, but it is dependent on context [44].

## **2.6 Chapter conclusions**

This chapter discusses different approaches to model deregulated power. In

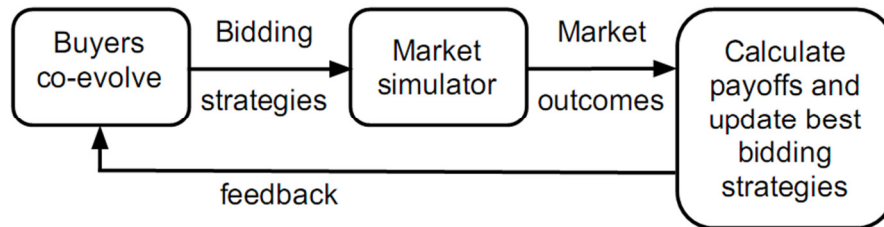
particular, agent-based technology and cooperative game concepts have been highlighted. The overview introduced in this chapter form the grounding for a good and accurate understanding and modeling of the deregulated power market in the later chapters in which two different market simulator frameworks will be developed. Bidding and cooperation strategies of buyers will be implemented and tested on this framework.

## **Chapter 3: PROPOSED METHODOLOGY FOR MODELING POWER MARKETS**

In the global competitive market, electricity buyers are no longer price takers since they are able to influence the market by using different bidding strategies as well as cooperating with other buyers. Therefore it is necessary to develop and investigate individual and cooperative strategies of electricity buyers. However, as mentioned above, most of the research efforts have been targeted at power generation and transmission; whereas research in demand side has not been sufficiently forthcoming. Moreover, to the best of our knowledge, OCSG problem has not been studied for electricity market, although many applications of this problem arise from e-commerce; for example, coalitions allow buyer to benefit the price discounts by purchasing in bulk [45].

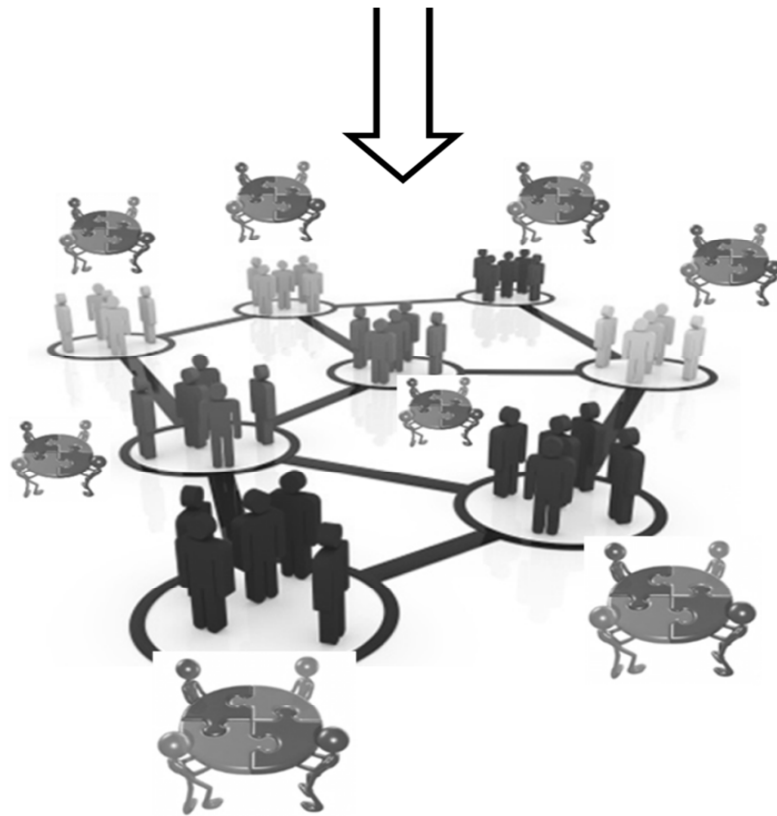
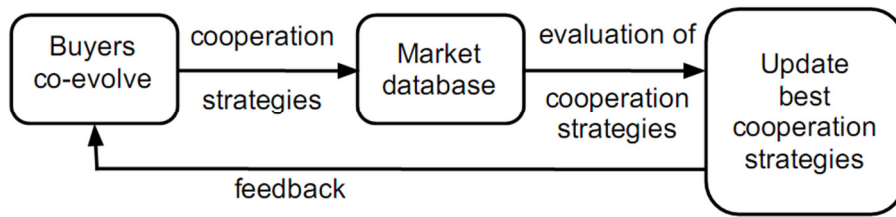
In that perspective, we seek to understand the cooperative behavior of electricity buyers using evolutionary approach in a cooperative game framework. In this study, a theorem was proved and served as a link between the payoff distribution problem in cooperative game theory and the OCSG problem, thus forming a theoretically fundamental background for the proposed methodology. Moreover, while existing literature over-simplifies the market model by introducing only a few participants (typically less than 6), our studies can handle much larger number of buyers, taking fully into account the physical and technical constraints of the power network.

This research seeks to understand the cooperative behavior of electricity buyers through two situations: deterministic situation and stochastic situation. In the deterministic situation as presented in Figure 3.1, buyers co-evolve and find out the optimal bidding strategies to maximize their payoffs. The solution to the problem corresponds to a particular market state, which is the outcome from the market simulation. A market state includes information about the bidding strategies of players, the generated and dispatched electric power, the nodal prices, as well as the payoffs of players.



**Figure 3.1: Co-evolutionary approach for deterministic situation**

In the stochastic situation as presented in Figure 3.2, a market database consisting of different market states has been generated. Using the information from this database, buyers co-evolve and find out the optimal cooperation strategy to hedge against the risk of low payoffs. The quality of a coalition is measured through a characteristic function that depends on the nature and purpose of cooperation. After different coalitions are formed, members in each coalition can use a fair scheme to share the payoffs among themselves. A theorem will be proved to clarify the rational link between these two stages.



**Figure 3.2: Cooperative Game approach for stochastic situation**

In perspective of modeling the market using agent based approach and cooperative game, we use the terms “agent” and “player” interchangeably in the contexts without potential confusion. Similarly, the term “payoff” is used alternatively with “profit”. Moreover, these terms correspond to buyers since we always focus on demand side.

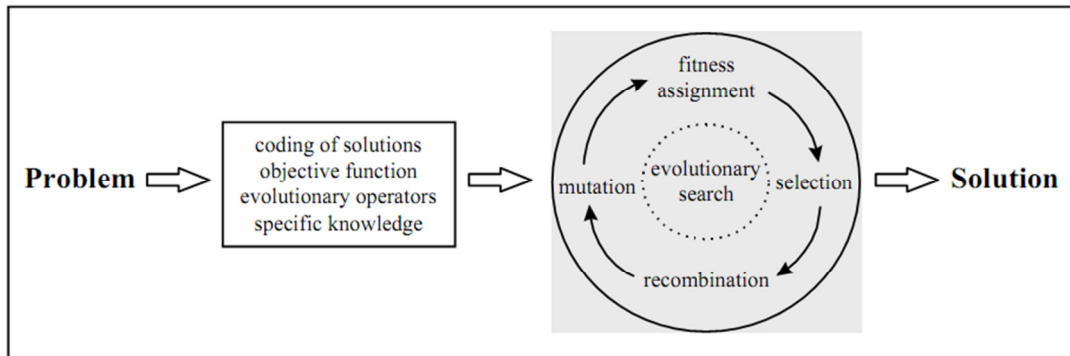
### **3.1 Co-evolutionary approach for deterministic situation**

Standard evolutionary algorithms are highly simplified models inspired from the famous Darwinian theory of natural selection. They are applied directly on a well-defined objective function: all individuals are evaluated using the same objective function. In a more complicated manner, co-evolution between individuals of different species in their environment can give various feedback mechanisms to computing complex objective functions. The purpose of co-evolution in computer science is to produce a dynamic similar to that of the arms race. Informally, the arms race best performance is achieved by each species while incrementing the performance of other species. The idea behind this concept is that a system may evolve better through reciprocal performance. In a co-evolutionary system, the evolution of different species must be considered simultaneously, because the evolutionary adaptation of a species can force the adaptation of others.

#### **3.1.1 Principles of Evolutionary Algorithms**

The idea of Evolutionary Algorithms is simply to build a random population of potential solutions to the problem. The “individuals” are then evaluated to encourage the reproduction of the fittest individuals, i.e. those who are closest to the optimal solution. The mechanisms of selection, recombination of most adapted individuals and mutation permit to gradually approach the desired solution. Evolutionary Algorithms have common core mechanism: it consists of making a population evolving by random transformation of some of its elements and application of the natural selection principle [46]. The principle of problem

solution using Evolutionary Algorithms is summarized in Figure 3.3.



**Figure 3.3: Problem solving using Evolutionary Algorithms**

The representation space that we actually study (where the evolution operators operate, also called the genotypes space) is often different from space in which the fitness is calculated (phenotypes space). To move from phenotypes space to genotypes space, an additional modeling or coding step is necessary. The representation or coding of an individual has to include fundamental characteristics of the problem. It must also be easily to be manipulated by recombination and mutation operators, allow easy transformation on the search space and generate feasible solutions. Coding can be binary or real valued. In general, the  $N$  individual population  $P(0) = \{X_1, \dots, X_N\}$  is initialized through uniform drawing from the search space  $E$  while ensuring that all individuals meet the constraints.

The Darwinian part of Evolutionary Algorithm consists of two steps: the reproduction step where parents are selected to recombine and the replacement step which replaces the worse individuals by better ones. The selection is an essential operator whose principle is to allow the best individuals of a population to reproduce. The adjustment of this mechanism is critical in the performance



of the Evolutionary Algorithm. If the individuals of a population are too similar, the following next generations may become more and more homogeneous. In this case, the evolution of a population may be summarized in the evolution of a single dominant individual, thus less exploration the search space. To perform an efficient search, we have to maintain a balance between the exploitation of good solutions found so far and the exploration of unknown areas of the search space. Excessive exploitation can lead to stagnation in a local optimum (premature convergence) while as an excessive exploration could lead to an almost random search (no convergence).

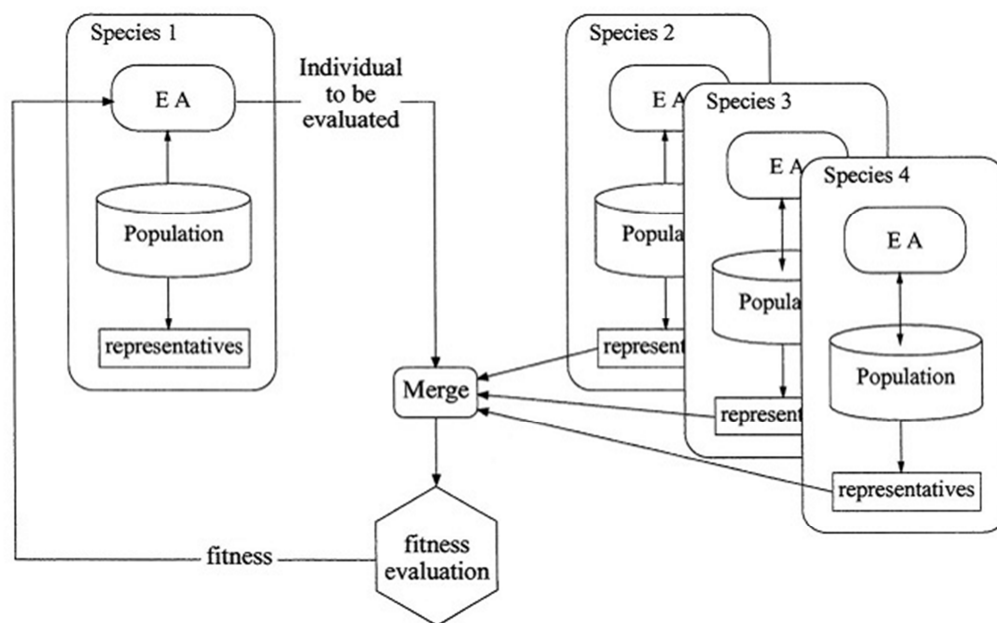
In Evolutionary Algorithms, the exploration is realized by variation operators, which aim to generate new individuals from those previously selected. We distinguish between recombination and mutation. The principle of recombination is analogous to biological reproduction: The children inherit the qualities from their parents. Recombination is usually called crossover for binary representation. Mutation has the general idea of introducing variability in the population. This operator modifies one or more genes of the selected individual with a certain probability  $p_m$  ( $0 \leq p_m \leq 1$ ). Mutation ensures ergodicity property (the capacity to cover the whole search space) for the Evolutionary Algorithms and the reintroduction of lost diversity.

### **3.1.2 Towards Co-evolution**

In ecology a living individual is not only influenced by its own environment but also by other individuals in the environment as well as other processes as changes in climate or geographical structure. The notion of mutual dependence or inter-

specific relationship between different species is named co-evolution. In a co-evolutionary system, the evolution of a species must be considered simultaneously, because the evolutionary adaptation of a species can force the adaptation of others.

Co-evolutionary algorithms are based on the principle of subjective function, where the fitness of an individual becomes estimation for other individuals interacting with it [47]. In co-evolutionary algorithms, individuals are evaluated based on their interactions with others. The nature of these interactions depends on the problem to be solved. In many problems, the individuals or populations compete with one another. This is called competitive co-evolution, which is widely applied in game playing strategies. On the other hand, an individual is rewarded when it contributes well in cooperation with other individuals in cooperative co-evolution.



**Figure 3.4: Framework of Co-evolutionary Algorithms**

The mechanism in which a participant determines its collaborators or competitors is among the most important factors for a successful application of co-evolutionary algorithms. The most obvious (and computationally expensive) method to evaluate an individual is to let it interact with all potential collaborators or competitors, this is sometimes called pair-wise or complete interaction. Alternatively, collaborators / competitors can be selected by a variety of ways: uniformly random methods or methods based on fitness [48].

The framework of Co-evolutionary Algorithm is represented in Figure 3.4. In this framework, every buyer is represented by a species, which is also an intelligent learning agent. The species interact with one another in the ecosystem, which in this case is the electric power market being simulated. They learn from the interaction and evolve. The fitness of an individual of a species is calculated when it interacts with other representatives from other species. The fitness function depends on different simulation scenarios. It is important to make a clear distinction between the stochastic nature of the proposed co-evolutionary approach and the deterministic nature of the situation being studied. As mention earlier, the co-evolutionary process leads to a particular market state, which is referred by being “deterministic”. This approach ultimately results in an equilibrium strategy vector that represents an ideal solution. However, in practice uncertainty is always present. For example, when a player varies its strategy even by a small amount, there could be large impact on the payoffs of all players. This fact is due to the physical constraint of the system and the incompleteness of information. Therefore, a practical study requires risk to be taken into account. That is also the motivation of the second approach in this paper.

## 3.2 Evolutionary Cooperative Game approach for stochastic situation

### 3.2.1 Cooperative game concepts

In this section, we introduce several concepts of cooperative game theory that will be later used. A cooperative game is a game where players can communicate freely with each other and enforce cooperative behavior by forming coalitions (e.g. in form of contract). Hence competition appears at level of coalitions of players, rather than between individual players.

Let  $N = \{1, 2, \dots, n\}$  be a finite set of  $N$  players. A coalition  $S$  is a subset of  $N$ , in which the player members of  $S$  cooperate together. An empty coalition is a null set; a singleton coalition has only one member whereas the grand coalition is the set  $N$  of all players. The collection of coalitions can be formed by  $N$  players is denoted by  $2^N$ , which is actually the power set of  $N$ . A game  $(N, v)$  on  $N$  is defined by a characteristic function  $v: 2^N \rightarrow \mathbb{R}$ , where  $v(S)$  represents the collective payoff that coalition  $S$  can assure by cooperation among its member, and is independent of the strategies of other coalitions. If the domain of the characteristic function  $v$  is restricted on a specific non-empty set  $2^S$  instead of  $2^N$ , by abusing the notation  $v$ , we have a subgame  $(S, v)$  defined on  $S$ . We note that the grand coalition of the subgame  $(S, v)$  is the set  $S$ .

The game  $(N, v)$  is called superadditive if its characteristic function satisfies the following property for all  $S$  and  $T$  subsets of  $N$ :

$$v(S \cup T) \geq v(S) + v(T) \tag{3.1}$$

Superadditivity tells that a union coalition of player is at least as efficient as the

ensemble of disjoint separate coalitions. We note that in a superadditive game, the grand coalition will form since it is the most efficient. On the other hand, the game is subadditive if

$$v(S \cup T) \leq v(S) + v(T). \quad (3.2)$$

In this case, singleton coalitions will form, where all player act individually.

Classically, it is often assumed that the characteristic function is superadditive in a cooperative game. However, in this study, we will consider both cases of superadditive and non-superadditive characteristic functions.

In a Transferable Utility Game [49], the goal of cooperation is to maximize the total gain of the grand coalition and then distribute this amount among the members. A challenging problem in a cooperative superadditive game is the distribution of gains from cooperation. A payoff that satisfies individual and global rationality conditions is called an imputation – a distribution that benefits each player who cooperates in a game. Moreover, an imputation that satisfies group rationality is said to lie in the core of the game – a collection of stable imputations that no coalition can improve upon.

In an alternative approach to the core theory, Shapley proposed a distribution of gains from the grand coalition of  $n$  players that calculates the payoff a player could reasonably expect before the game begins. Being the unique solution concept of a cooperative game which holds the axioms of symmetry, efficiency, additivity, and dummy player, the Shapley value is considered to be “fair” in that sense [50]. The Shapley value  $\phi_i(v)$  of the game  $(N, v)$  for player  $i$  is calculated as the average of its marginal contribution to all possible coalitions:

$$\varphi_i(v) = \sum_{S, i \in S} \frac{(|S|-1)!(n-|S|)!}{n!} (v(S) - v(S \setminus \{i\})) \quad (3.3)$$

### 3.2.2 Optimal Coalition Structure Generation problem

As we have seen previously, the grand coalition will be formed in case of a superadditive characteristic function. In reality, the characteristic function can be non-superadditive, giving rise to the problem of finding optimal coalition structure where different coalitions can be formed.

Let consider the game  $(N, v)$  of  $n$  players and characteristic function  $v$  as defined above. A partition of  $N$  into disjoint and exhaustive coalitions is called a coalition structure. For example, a coalition structure  $CS$  of 5 players can be  $CS = \{(1,2), 3, (4,5)\}$  where  $(1,2)$ ,  $(3)$  and  $(4,5)$  are three coalitions. The value of a coalition structure is defined in term of its social welfare

$$V(CS) = \sum_{S_k \in CS} v(S_k). \quad (3.4)$$

Where  $v(S_k)$  is the value of coalition  $S_k$ , calculated using the characteristic function of the game. The OCSG problem seeks to find a coalition structure  $CS^*$  that maximizes its social welfare

$$CS^* = \arg \max V(CS). \quad (3.5)$$

It is natural to ask whether the optimal coalition structure, found from a single objective optimization problem, is a reasonable group formation that can be accepted by all players.

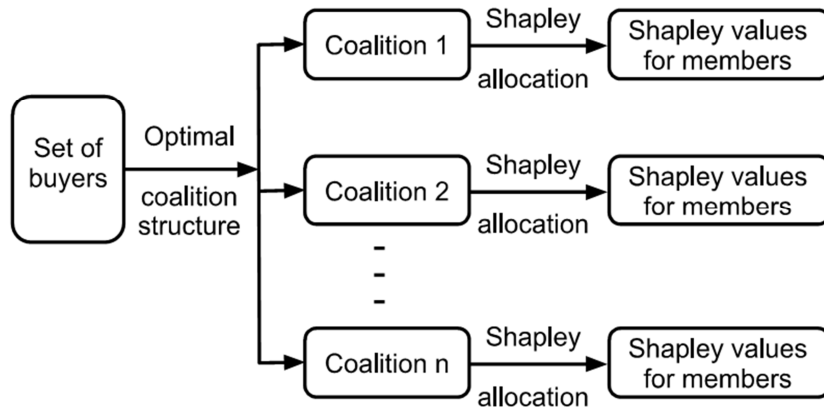
***Theorem:*** *Each coalition in the optimal coalition structure defines a cooperative superadditive subgame.*

***Proof:*** Let the optimal coalition structure of a game  $(N, v)$  be  $CS^* = \{S_1, S_2, \dots, S_k\}$ . We need to prove that each  $S_i$  defines a cooperative superadditive subgame  $(S_i, v)$ . We will prove by contradiction.

Assume that there exists a coalition  $S_i$  such that the subgame  $(S_i, v)$  is non-superadditive. That means we can partition  $S_i$  into  $S_{i1}$  and  $S_{i2}$  such that  $v(S_{i1}) + v(S_{i2}) > v(S_i)$ . Let's consider a new coalition structure  $CS'$  by replacing  $S_i$  with  $S_{i1}$  and  $S_{i2}$ . It is obvious that  $V(CS') > V(CS^*)$ , which means  $CS^*$  is not the optimal coalition structure. Therefore, the theorem is proved by contradiction.

This theorem is theoretically fundamental to our methodology. In this study about optimal cooperation strategies, we need to solve two problems simultaneously: The first problem is how players partition into coalitions, and the second problem is how the gains are fairly distributed among group members. To solve the second problem using Shapley value, each coalition must be superadditive, and this condition is satisfied by solving the first problem – optimal coalition structure generation. Figure 3.5 depicts our approach: Given a cooperative game  $(N, v)$ , we first find the optimal coalition structure. Then Shapley allocation is applied for each coalition to provide its players with reasonable payoff shares. In particular, the characteristic function  $v$  is context defined since it depends on what the buyers seek for when cooperating. That

feature makes our approach universal.



**Figure 3.5: Shapley allocation for Optimal Coalition Structure**

Our approach is very general in the sense that it can be applied for any characteristic function, whether superadditive or otherwise.

### 3.3 Value at Risk and group characteristic function

In financial industry, the most popular risk measure is Value at Risk ( $VaR$ ), which is essentially a quantile on a loss distribution. In particular, Value at Risk ( $VaR$ ) estimates how much a portfolio could lose due to market uncertainty over a time horizon and within a given confidence interval. In this study, the  $VaR$  is defined as the expected minimum profit for a given confidence level  $(1-\alpha)$ :

$$\Pr(\pi \geq VaR) = 1 - \alpha .$$

Under this perspective, the  $VaR$  can be recognized as downside risk measure. The  $VaR$  is more efficient than a symmetric risk measure such as the variance because the later also includes the case where the profit values are better



than the expected profit.

The confidence level depends on the extent of the player's risk-aversion. Normally, a 95% confidence level is adopted by a player with moderate risk-aversion. Under normal distribution assumption, the variance – covariance approach calculates the *VaR* of the payoff  $\pi$  by

$$VaR(\pi) = E(\pi) - z_{1-\alpha} \sigma(\pi) \quad (3.6)$$

$E(\pi)$  is the expected value of the pay off,  $\sigma(\pi)$  is the standard deviation of the payoff,  $z_{1-\alpha}$  depends on the confidence level. For 95% confidence level,  $z_{1-\alpha}$  is equal to 1.65.  $E(\pi)$  and  $\sigma(\pi)$  are calculated from our random simulation database.

As stated in Section II, our approach is very general in the sense that it can be applied for any characteristic function. In this part, we propose an explicit form of the characteristic function for the game:

$$v(S) = VaR(S) \cdot e^{-a(|S|-1)} = [E(\pi_S) - z_{1-\alpha} \sigma(\pi_S)] \cdot e^{-a(|S|-1)} \\ \pi_S = \sum_{i \in S} \pi_i \quad (3.7)$$

$S$  is a coalition of buyer, which is a subset of  $N$ ,  $\pi_S$  is the total payoff of coalition  $S$  and  $|S|$  is the number of members in  $S$ . The first factor of the characteristic function is the *VaR* of the total payoff of this coalition  $S$ , while the second measures the effect of group size through parameter  $a$ . Larger value of  $a$  means higher transaction cost among the group and thus having negative effect the characteristic function of the coalition. When parameter  $a$  is equal to zero, the transaction cost is zero and thus the grouping environment is ideal. This setting is

reasonable since in a larger group more transaction and communication cost is incurred; thus there exists a certain negative effect of the group size on the group efficiency.

### **3.4 Chapter conclusions**

This chapter presents a general methodology to simulate power markets and study the behaviors of economic participant. Concepts of evolutionary / co-evolutionary algorithms and Cooperative Game theory have been highlighted. The following chapter will introduce the first model in this research, where the interactions in only one bus are considered.

## **Chapter 4: SINGLE-NODE POWER MARKET MODEL**

In this section, we build a single-node power market model with uniform non-discriminatory pricing, which means buyers and generators on only one bus are studied. Since the power market is supposed to be single-nodal, we do not take into account the congestion of transmission lines. Therefore, the local marginal prices are equal to the market clearing price. Moreover, since we focus on studying the behavior of the electricity buyers, the bidding strategies of the generators are assumed to be fixed.

### **4.1 The single-node power market model**

The PoolCo model is chosen among the three models described in Chapter 2. The reasons of this choice are as follows:

- PoolCo allows a greater number of autonomous agents than Bilateral Contracts model.
- PoolCo model is more complex and dynamic than Bilateral Contracts model
- PoolCo model can validate the proposed co-evolutionary methodology more efficiently than the Hybrid model, which is too complicated within the framework of this research.

The operation of the electricity spot market takes place every hour from days to days. This is modeled as a repeated game in which the players compete against one another to maximize its own profit or cooperate to maximize the total profit of the group. A group here may include several buyers or all buyers.

At the start of each round, the participants submit their bidding curves, and the Independent System Operator clears the market by intersecting the aggregated demand curve of buyers and the aggregated supply curve of generators. Each generator is paid at the market clearing price for the quantity of power they have supplied, and each buyer has to pay at the market clearing price for the quantity of power they have received.

## 4.2 Generator and buyer models

We approximate the total production cost of a generator as a quadratic function:

$$C_G(Q) = b_0 + b_1Q + b_2Q^2 \quad (b_j > 0 \forall j) \quad (4.1)$$

$Q$  is the quantity the generator sells in this round, and  $b_j$  are the cost coefficient of this generator. Each generator has its minimum and maximum power output. The data of 4 generators used in this work is given in the table below:

**Table 4.1: Data of generators**

Generators	$b_0$	$b_1$	$b_2$	$Q_{Gmin}$ (MWh)	$Q_{Gmax}$ (MWh)
1, 2	3000	32	0.0065	200	3000
3, 4	2000	30	0.0060	200	3000
4, 6	1500	35	0.0077	200	3000

A buyer is characterized by the revenue function

$$R = a_1Q - a_2Q^2 \quad (a_j > 0 \forall j) \quad (4.2)$$

The revenue function of a buyer stands for its performance. Intuitively, the revenue function tells us how much profit a buyer can make using the quantity of power  $Q$  it has purchased.

*The efficiency level* of each buyer is determined by the coefficients  $a_1$  and  $a_2$ . A buyer is efficient if he has large value of  $a_1$  and small value of  $a_2$ .

The coefficients used in this work are:

**Table 4.2: Data of buyers**

Buyers	$a_1$	$a_2$
1, 2	61	0.002
3, 4, 5, 6, 7, 8	60	0.002
9, 10, 11, 12, 13, 14	59	0.002
15, 16, 17, 18, 19, 20	58	0.002

The reason of dividing 20 buyers into 4 groups of efficiency level is to facilitate the observation of their strategic behavior. We expect that the buyers with same level of efficiency will behave similarly throughout the simulation.

If the MCP of the current round is  $\lambda$  and the quantity of power the buyer received is  $Q$ , the buyer will earn a profit of

$$\pi(Q, \lambda) = a_1Q + a_2Q^2 - \lambda Q \quad (4.3)$$

This profit depends on both the market clearing price  $\lambda$  and the quantity  $Q$  that the buyer receives from the auction (the market equilibrium is the intersection point of the aggregated demand function and the aggregated supply function). The buyers will play a bidding game to find out the strategy that maximizes their profit.

### 4.3 The bidding model and market calculation

The bidding curve of a participant in the market is a piece-wise linear function with  $K$  segments. For simplification,  $K$  prices are defined in advance, and are the same for all participants. The participants only bid  $K$  quantities corresponding to  $K$  fixed prices to form a decreasing demand curve or an increasing supply curve.

In this work, the predefined prices are 45, 50, 55, 60, 65, 70 (\$/MW). A seller bids increasing supply curve and a buyer bids decreasing demand curve. The illustration of bidding curves of sellers and buyers are shown below.

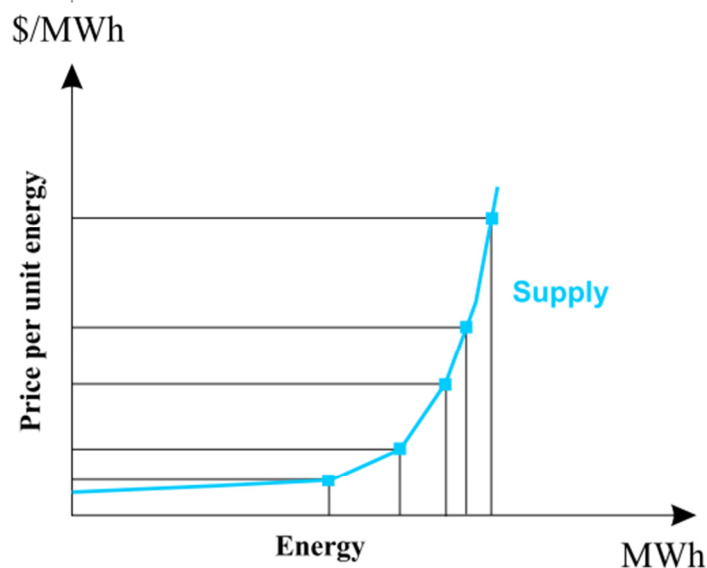
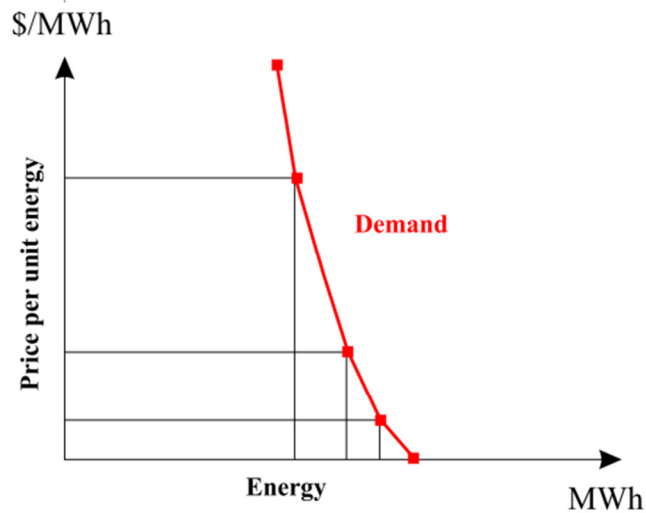


Figure 4.1: Bidding curve of sellers



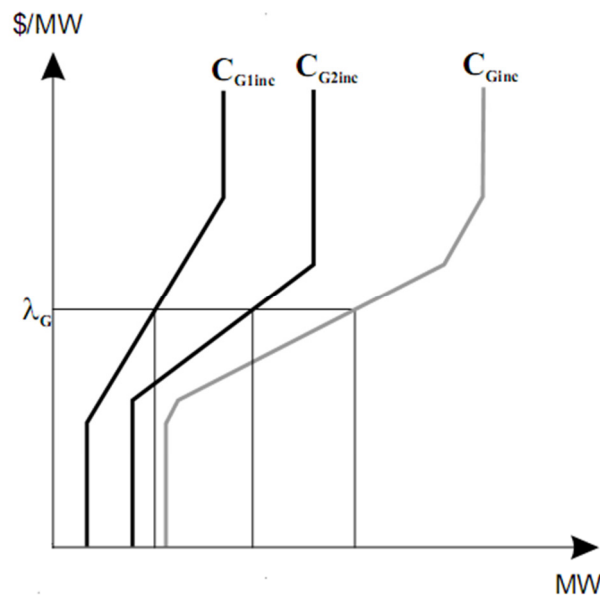
**Figure 4.2: Bidding curve of buyers**

As discussed previously, the sellers keep their bidding strategy unchanged. We suppose that they follow marginal bidding procedure, i.e. at a given price  $P$ , the sellers bid a power quantity:  $Q = b_1 + 2b_2P$ . If the corresponding quantity  $Q$  exceeds the generator capacity, it will bid  $Q_{Gmax}$ . The buyers are allowed to bid any quantity between 0 and 700 MW, which is their maximum capacity.

So far, we have only taken into account the bidding curves of just one seller and of just one buyer. Usually there are many sellers and buyers with different supply and demand functions who participate in the market. To compute the market equilibrium, we have to aggregate these curves into one aggregated supply function and one aggregated demand function. The aggregated curves will be used to calculate the Market Clearing Price (MCP) and the total traded power volume. First we consider the combining of supply functions followed by the combining of demand functions.

The purpose of combining the supply functions is to find out how much

energy the generators are willing to sell at most to a certain per-unit price. Therefore, at each price the quantities bid by all generators have to be added. Due to the capacity limit of the generators and the piece-wise linear form of the bidding curves, a compact formula cannot express the supply functions. Consequently, it is not easy to do the aggregation symbolically. An efficient solution is to discretize the prices and aggregate all quantities at each discrete price value. Figure 4.3 shows an example where  $C_{G1inc}$  and  $C_{G2inc}$  are aggregated to get  $C_{Ginc}$  at the price  $\lambda_G$ .



**Figure 4.3: Aggregation of demand curves**

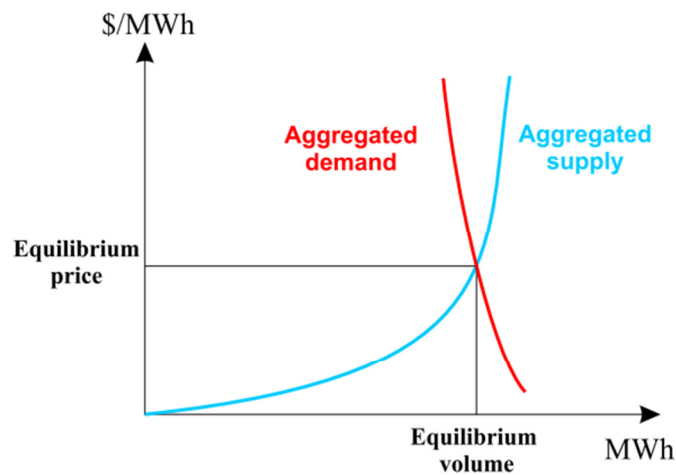
The only difference between demand curves and supply curves is that a demand curve has negative slope. The aggregation procedure for demand curves is exactly the same as for supply curves.

Once we have obtained the aggregated supply and demand curves, we can apply the method of computing the MCP and the total traded volume as in the spot



market model. The aggregated incremental supply and demand curves are presented in the same graph. The per-unit price at the intersection of the two curves is the MCP. The power-value at this point corresponds to the total produced and purchased power. The intersection determines the MCP and the total traded volume because it is where the quantities of sellers and consumers match.

We also have to take note that sometimes the aggregated curves do not intersect. This is the case when the maximum power the buyers want to purchase is smaller than the minimum power the generators produce. Another case where there is no intersection is when the minimum power volume the buyers want to purchase is bigger than the maximum volume the generators are able to produce. If one of these two cases happens, there is no solution for this market round.



**Figure 4.4: Calculation of Market Clearing Price**

#### **4.4 The co-evolution model**

In this simulation model, every buyer is represented by a species, which is also a continuous learning agent. The species interact with one another in the ecosystem,

which is the competitive power market. They “learn” from the interaction and evolve. The fitness of an individual of a species is calculated when it interacts with other representatives from other species. The fitness function depends on different simulation scenarios. If a buyer  $i$  tries to maximize his own profit, his fitness function is simply given by (3):

$$Fitness_i = \pi(Q, \lambda) = (a_1Q - a_2Q^2) - \lambda Q \quad (4.4)$$

On the other hand, if buyer  $i$  cooperates in a group  $G$  with  $L$  members,  $G = \{j_1, j_2, \dots, j_L\}$ , his fitness function is the total profit of all buyers in this group:

$$Fitness_i = \sum \pi_j(Q_j, \lambda) \quad \text{with } j \in G \quad (4.5)$$

Each species is a population consisting of a number of chromosomes. The length of the chromosomes is the number of pairs power quantify – price in one bid; each chromosome encodes one bidding strategy of that buyer species.

We build a simple market clearing block. The input to the market clearing block will be each bidding strategy of the buyer we are considering, combined with the representative strategies from the rest, together with the fixed bidding strategies of the sellers. The output from the market clearing block is the market clearing price (MCP) and power received by each buyer corresponding to the above situation. Base on this information, we can calculate the benefit of every buyer, which will serve in calculating the fitness of the corresponding strategy chromosome. Here, in order to facilitate good convergence of the co-evolutionary algorithm, we choose heuristically the best chromosome of each species to be the representative. The pseudo code of the algorithm is as follow:

***Initialization:***

*t = 1*

*For each buyer*

*Randomly initialize a sub population of strategies for round t = 1*

*Choose a representative strategy for this round*

*End*

***Main loop:***

*While not stop do*

*t = t + 1*

*For each buyer i*

*Evaluate the fitness of each strategy j*

*(Based on the representatives from round t - 1)*

*Choose the representative strategy for this round t*

*Evolution of buyer i: selection, crossover, mutation*

*End for*

*All representative strategies are combined to get the market output of this round t*

*End while*

**Figure 4.5: Pseudo code of the proposed Co-evolutionary Algorithm**

The key of co-evolutionary algorithms is the choice of the representatives. At generation  $t$ , a buyer has to forecast the strategies that other buyers will use in this generation. In competitive co-evolution, each buyer only knows the fitness of his own strategies. Therefore, each buyer assumes that the rest will use their most updated strategies, which are the strategies from previous round  $t-1$ . In

cooperative co-evolution, each buyer in the group evolves one after another, and the strategies to be used in this round are gradually made available within the group. After going through evolution, a buyer will inform other buyers in the group his best fitness strategy of this round.

## **Chapter 5: SIMULATION OF SINGLE-NODE POWER MARKET MODEL**

This chapter simulates the proposed single-node market model. In our implementation, one buyer is associated with a population of 20 chromosomes. Intermediate recombination is used to generate new individuals from the selected parents. We also use elitism by replacing the worst strategy in each generation with the best strategy found so far. The mutation rate is set as 0.1. Moreover, we allow each buyer to realize a total of 2 evolutionary generations against the representative strategy of other players, before the evolution of the next buyer takes place. This is called sub-evolution, and it aims to accelerate the convergence of the algorithm.

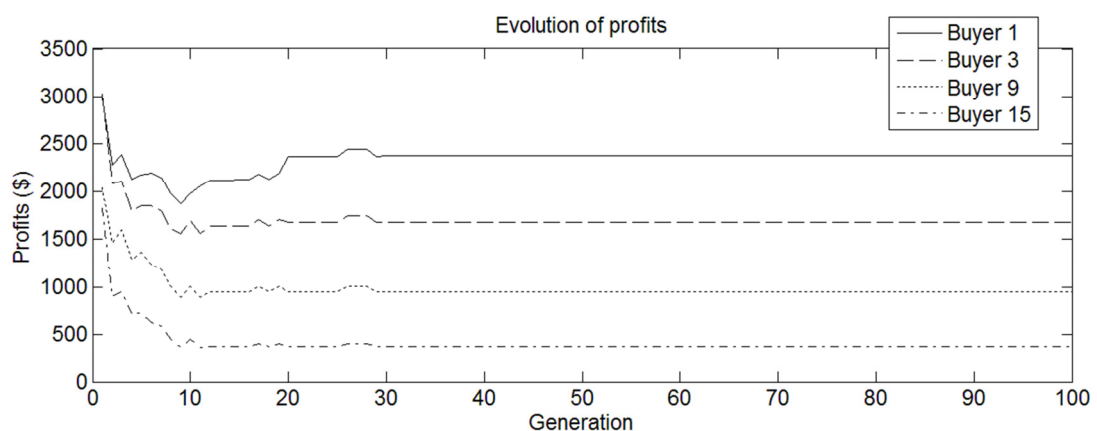
### **5.1 Competition scenario**

In this scenario, all buyers play individually to maximize their own profits. The fitness of each buyer is calculated using (4). Since all bids are submitted individually, each buyer has no information about the bids of other participants in this round, only the bids from previous rounds are known. Thus, each buyer forecasts that others will use their previous round strategies. This is actually his choice of representatives. We have run a simulation of 500 generations with 6 generators and 20 buyers and the results are described in Figures 5.1, 5.2 and 5.3. We observed that buyers with similar level of efficiency behave similarly;

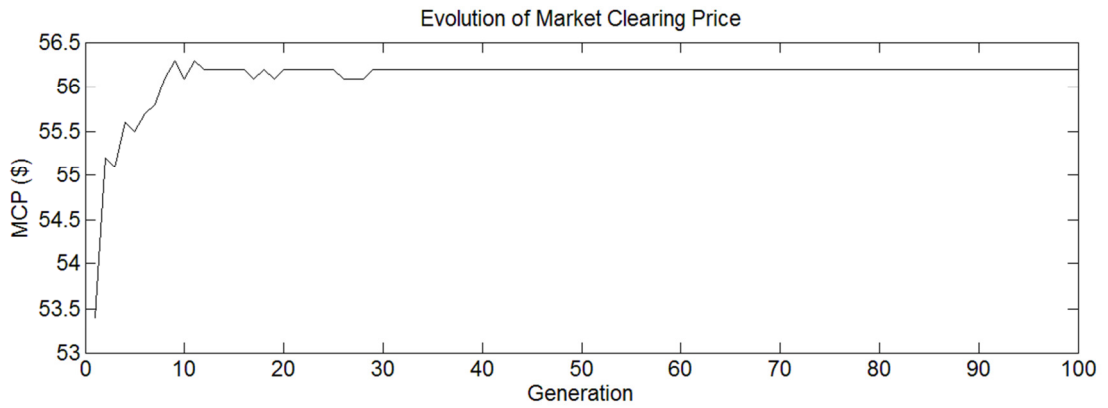
therefore we choose to report the evolution of buyers 1, 3, 9 and 15.

We observe that the profits of all buyers decrease compare to what they gain in the first randomly initialized generation. All buyers try to adjust their bids to get maximum profits in response to their opponents' strategies. Therefore there is a competition between them that leads to an equilibrium situation. As expected, all buyers with same efficiency level will behave similarly, and thus get quite similar profits. Buyers 1 and 2 who are most efficient get highest profits. Next are buyers 3 to 8, then following by buyers 9 to 14 and buyers 15 to 20 get least profits because they are least efficient.

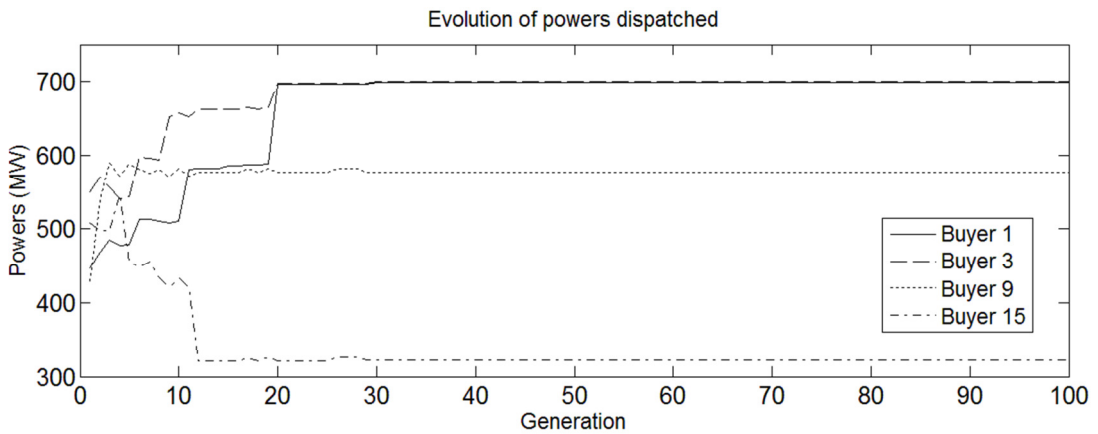
The reason of the reduction in profit is the increasing of the market clearing price as we can see on Figure 5.2. Because all buyers want to gain more profit, they bid more quantities at the same price as before. This also means a right shift of the aggregated demand curve, which results in a higher equilibrium price. We observe that efficient buyers 1 to 8 could manage to get the maximum power of 700 MW while less efficient buyers cannot get their maximum capacities. The low efficient level of these buyers has limited their quantity bidding: higher market clearing price will just cause them a loss.



**Figure 5.1: Evolution of profits (Competition scenario)**



**Figure 5.2: Evolution of MCP (Competition scenario)**



**Figure 5.3: Evolution powers dispatched (Competition scenario)**

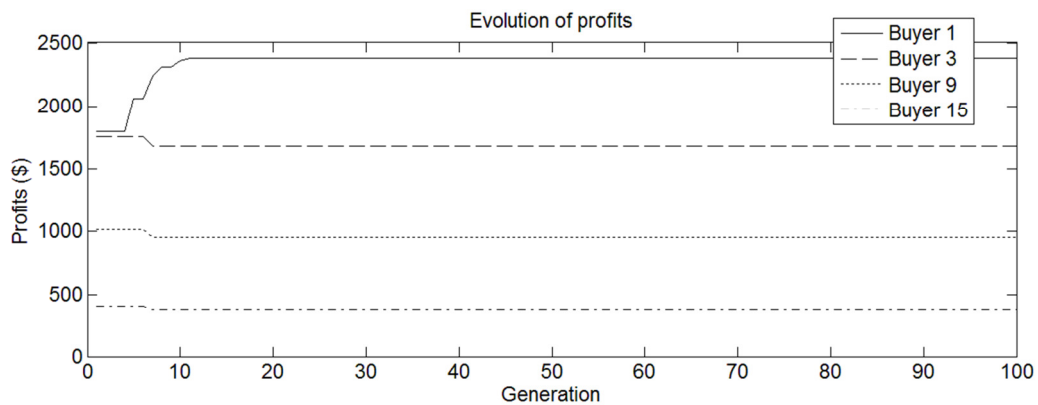
## 5.2 Verification of Nash equilibrium

An interesting question is whether the equilibrium is a Nash equilibrium. Nash equilibrium is the situation where every buyer has no incentive to unilaterally change his current strategy, which is the strategy that maximizes his payoff whatever the strategies played by the others.

We can give an answer to this question by using co-evolutionary approach, in which we let one buyer evolve while the strategies of others are

fixed. If the evolving buyer cannot get a better situation than his equilibrium profit, the stable situation is Nash equilibrium. A typical case when buyer 1 evolves and other buyers use their stable strategies is shown in Figure 5.4.

It is found that the whole system gets back to its stable situation in less than 15 generations. The result shows that the evolving buyer cannot get more than what he got in the equilibrium (2380.00\$). The tests for other buyers give similar result. Thus we have strong evidence to believe that the equilibrium is Nash equilibrium.



**Figure 5.4: Evolution of buyer 1's profit (Nash equilibrium)**

### 5.3 Cooperation scenario

In this scenario, all buyers cooperate with the goal of maximizing the total profit. Therefore, the fitness of a strategy chromosome which is calculated using (5) is the total profit of all buyers when the buyer in consideration uses that strategy. This is equivalent to solving a multi-objective optimization problem, where each objective is to maximize the profit of one buyer. The choice of maximizing the total profit of all buyers is equivalent to using an aggregate objective function,



which is in the form of a non-weighted linear sum.

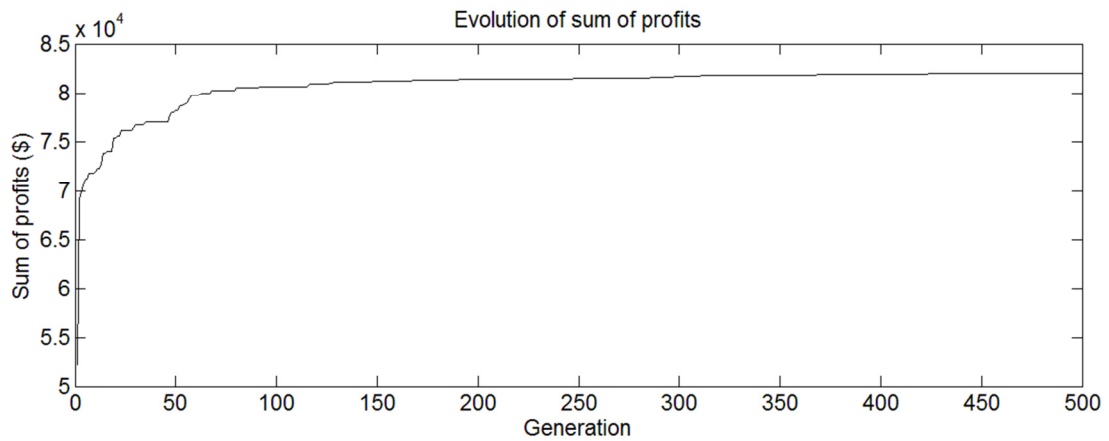
Since all buyers cooperate, the information about the bid to be submitted this round is made available step by step. With this mechanism, a worse total profit due to the ill-cooperation of the buyers can be avoided. In our implementation this is modeled as following: The first buyer evolves according to the bids from previous round (that means he assumes that the representatives of other buyers are their bids from previous round), then he informs his best strategy – the strategy he will submit this round to other buyers. The second buyer evolves according the bid from previous round, plus the “sure to happen” strategy of buyer 1 who has just informed him. The information gradually becomes certain and the last buyer can evolve with complete knowledge of the strategies of other buyers in this round. This approach is somehow similar to elitism: the buyers who evolve later keep track of the best strategies so far found by those evolved before him.

The results of the simulation are shown in Figures 5.5 and 5.6. We note that it takes longer time to reach equilibrium in this case. To facilitate the comparison, we report in Table 5.1 the profits and powers dispatched at the generation 500 in the cooperation scenario, together with the percentage change compared to the equilibrium situation in the competition scenario. As expected, the total profit keeps increasing. The total profit in this case is 81996.85\$, which increases 262.02% compared to the previous competition case. It is clear that the cooperation helps to increase the total profit.

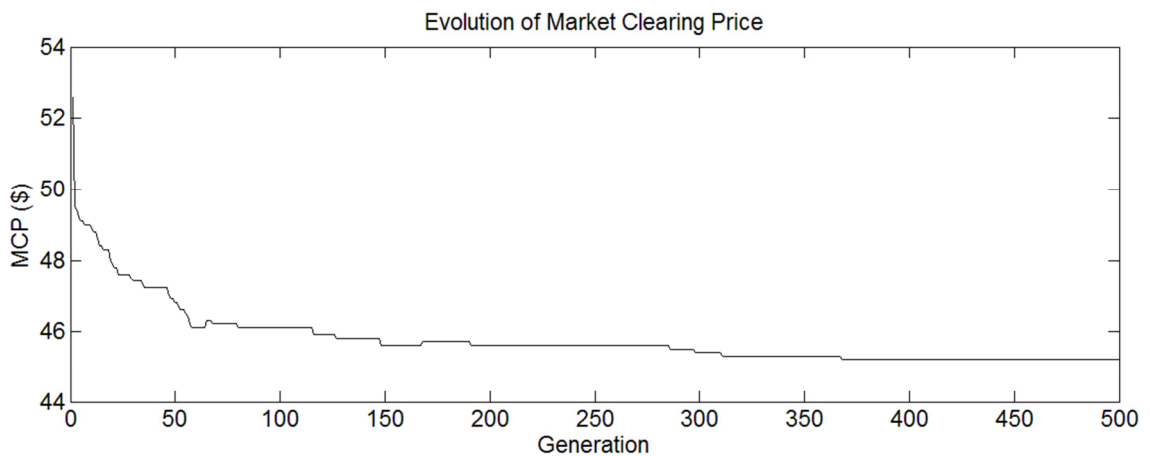
But it is also interesting to look at individual profits. We see that while the profits of almost every buyer increase, the profit of buyer 19 decreases by 79.7%. We note that this buyer is of lowest level of efficiency. A possible explanation is that the worst buyers will “sacrifice” by limit their quantity bids to help increase

the total profit of the group, which is now the common goal. It is clear that the result reflects real life fact. If we consider all buyers as a population, with the total profit as the fitness, the disappearance of least efficient buyers reflects the core principle of evolution: only the best will survive.

Moreover, the profits of the buyers with same level of efficiency may vary. That is because of the goal is no more maximizing individual profits, but the total profit of all buyers.



**Figure 5.5: Evolution of total profit (Cooperation scenario)**



**Figure 5.6: Evolution of MCP (Cooperation scenario)**

The reason for an increase in profits is the decrease of MCP. All buyers have cooperated to pull down the MCP by decreasing their quantity bids. In other words, they have tried to make the MCP lower by shifting the aggregated demand function to the left. That is why the power dispatched decreases.

**Table 5.1: Equilibrium profits and powers dispatched (Cooperation scenario)**

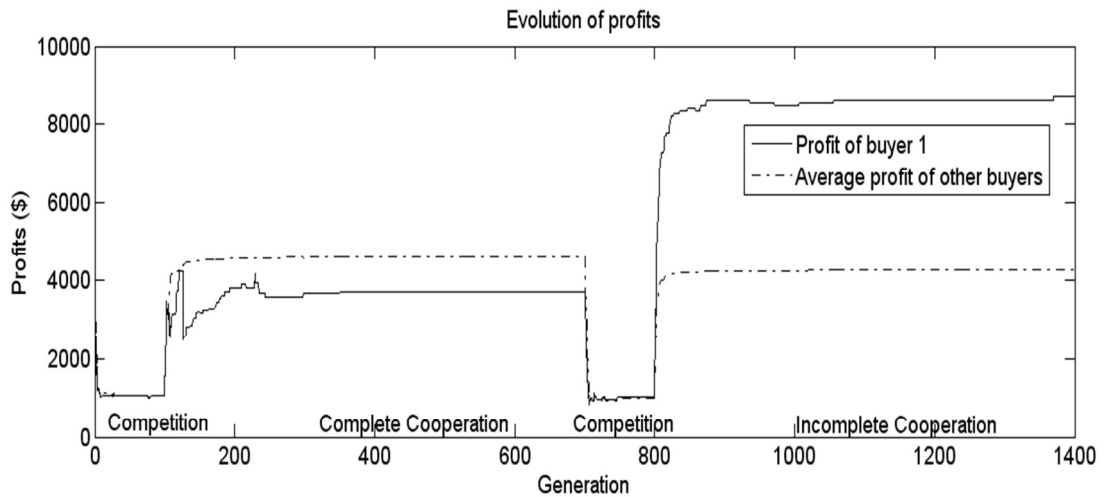
Buyers	Profits (\$)	% Change in profits	Powers Dispatch-ed (MW)	% Change in Powers Dispatched
1	9386.70	294.40%	647.10	-7.56%
2	8658.04	263.78%	592.40	-15.37%
3	5800.18	245.25%	415.20	-40.69%
4	4819.61	186.88%	341.40	-51.23%
5	4876.00	198.37%	345.60	-47.46%
6	7405.01	354.56%	539.70	-17.40%
7	7021.42	319.46%	509.50	-26.59%
8	6480.59	286.63%	467.40	-32.86%
9	4198.68	341.84%	319.00	-44.82%
10	3037.28	222.82%	227.60	-59.36%
11	4054.39	345.51%	307.50	-40.06%
12	3408.12	272.54%	256.50	-50.63%
13	3248.13	238.26%	244.00	-59.37%
14	3634.86	279.36%	274.30	-53.94%
15	592.14	58.95%	46.60	-85.55%
16	1549.06	308.23%	123.40	-63.38%
17	1507.20	293.49%	120.00	-65.24%
18	1217.82	224.6%	96.60	-70.52%
19	75.45	-79.7%	5.90	-98.18%
20	1026.17	157.1%	81.20	-79.46%
<b>Total</b>	<b>81996.85</b>	<b>262.02%</b>	<b>5960.90</b>	<b>-45.41%</b>

## 5.4 The free rider problem

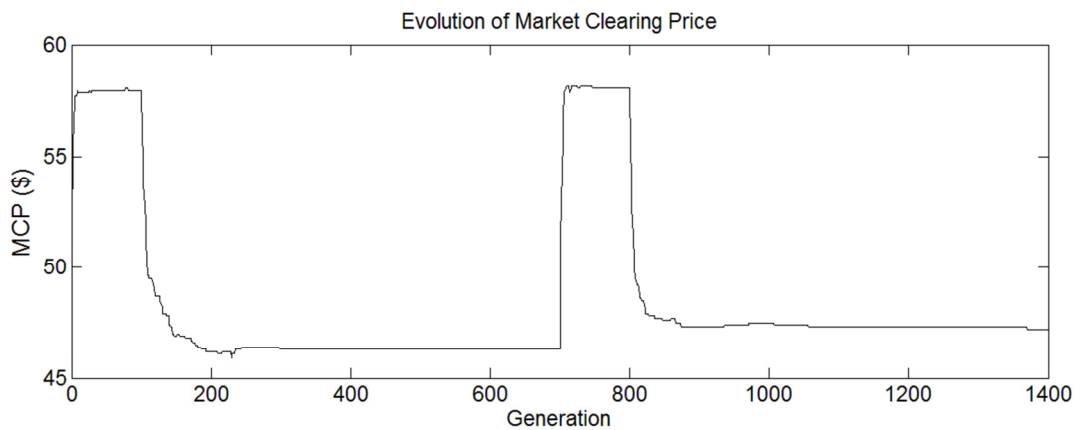
So far we have observed the scenarios where all buyers compete against one other or cooperate together. In this section, we simulate the case of incomplete cooperation where buyer 1 plays individually while others cooperate. Since we have observed the effect of different buyer's efficiency levels, in this experiment we choose to simulate 20 similar buyers in term of efficiency level to facilitate the observation of results. The 6 generators are kept unchanged and 20 buyers are copies of buyer 1 in Table 4.2. In order to compare different scenarios, we run a simulation with 1400 generations as followed: Competition from generation 1 to 100, complete cooperation from generation 101 to 700, competition from generation 701 to 800 and incomplete cooperation from generation 801 to 1400. The reason to insert a scenario of competition in the beginning and between complete and incomplete scenarios is to give a same starting point for all buyers. Since all buyers have similar revenue functions, we report the results of buyer 1 – the buyer playing alone in incomplete cooperation scenario and the average result of other buyers.

We observe in Figure 5.7 that when all buyers cooperate, they get better profits compared to competition. However, when buyer 1 plays individually against the cooperation of others, he gets even more profit. As we have known from previous experiments, when buyers cooperate they try to pull down the MCP by limit their quantity bids. On the other hand, buyer 1 who is now playing individually doesn't need to limit his quantity bids, but he still enjoys the low MCP thanks to the cooperative effort of other buyers. That is why buyer 1 gets very high profit in this case. He is called a free rider.

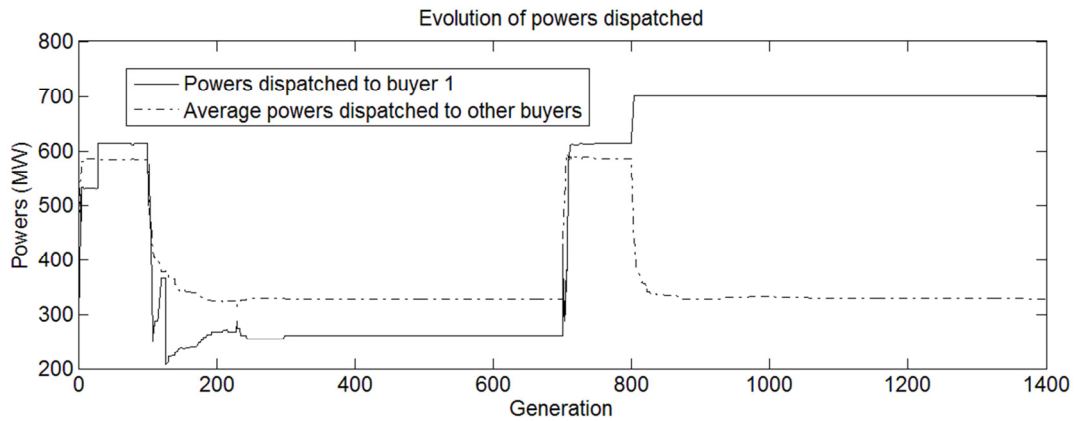
We see in Figure 5.8 that the MCP in incomplete cooperation scenario is lightly higher than in complete cooperation case, that's because of the non-cooperation of buyer 1. Therefore the average profit of other buyers is slightly less in this case compared to complete cooperation.



**Figure 5.7: Evolution of profit (different scenarios)**



**Figure 5.8: Evolution of MCP (different scenarios)**



**Figure 5.9: Evolution of powers dispatched (different scenarios)**

The simulation results can lead to a situation similar to the prisoner dilemma. A buyer, desiring to become a free rider to get very high profit, will play individually with the hope that others will cooperate. Since every buyer has incentive to do so, the market will ultimately become completely competitive, and thus all buyers get low profit.

## 5.5 Cooperation schemes for small buyers

In previous experiments, every buyer can bid any quantity from 0 to 700 MW which is their capacity limits. In this section, buyers 3 to 20 are chosen to be small buyers, they can bid maximum 200 MW; buyers 1 and 2 are kept unchanged because they are large buyers. All coefficients of buyers' revenue functions are as in Table 4.2. We propose 3 algorithms to study different cooperation schemes of small buyers:

- *Algorithm 1*: This is the algorithm we have been using so far in our simulations.

- *Algorithm 2*: This is a modification of algorithm 1. In this algorithm, a group is coded by a chromosome. A chromosome thus represents the strategies of all buyers in the group.
- *Algorithm 3*: This is another modification of algorithm 1. The small buyers will cooperate to form a large buyer representing their group. That means instead of bidding individually, the group will bid the total power of every buyer in the group, then the received power will be shared to the members proportionally to their maximum capacity.

The simulation results of 3 algorithms after 500 generations where the small buyers 3-20 cooperate and large buyers 1, 2 play individually are reported and compared with the competition scenario in Table 5.2. We observe that cooperation has helped most small buyers get higher profits in all three algorithms. Algorithm 1 gives best total profit of small buyers, followed by algorithms 2 and 3. We recall that in algorithm 2, a chromosome represents a group of small buyers. Therefore, the co-evolution happens actually among large buyers 1, 2 and the group of small buyer. In algorithm 3, the co-evolution is again among large buyers 1, 2 and the newly formed large buyer representing the group of 18 small buyers.

**Table 5.2: Profits of small buyers in different cooperation schemes (\$)**

Buyers	Competition	Algorithm 1	Algorithm 2	Algorithm 3
1	1334.16	6510.00	6230.00	7140.00
2	1315.94	6493.40	6230.00	7140.00
3	471.16	1860.00	1667.45	760.97
4	480.00	1860.00	844.92	760.97
5	480.00	1693.77	1586.02	760.97
6	480.00	1721.57	1329.11	760.97
7	480.00	1738.59	1316.04	760.97
8	480.00	1860.00	1635.78	760.97
9	280.00	1041.48	304.36	688.18
10	280.00	460.57	721.27	688.18
11	280.00	0.00	871.50	688.18
12	280.00	24.34	85.28	688.18
13	280.00	199.90	397.03	688.18
14	260.71	7.83	198.05	688.18
15	64.69	53.03	199.96	615.39
16	59.15	108.94	6.57	615.39
17	53.92	0.00	759.58	615.39
18	61.53	65.31	430.09	615.39
19	53.27	241.32	206.43	615.39
20	63.91	115.81	122.80	615.39
Total profits Of small buyers (3 to 20)	4888.34	13052.47	12682.23	12387.18

Although algorithm 3 gives less total profit of small buyers than algorithm 1 and 2, it ensures a good sharing of the electricity power received for less efficient small buyers. In algorithms 1 and 2, inefficient buyers could get very low



profits (buyers 11 and 17 get zero profit in algorithm 1 and buyer 16 get 6.57 \$ of profit in algorithm 2). Therefore, small buyers with low level of efficiency (buyers 9 to 20) would highly appreciate the scheme of cooperation as in algorithm 3 where they get high profits thanks to the efficiency of their group mates. On the other hand, efficient small buyers (buyers 3 to 8) would appreciate the cooperation schemes as in algorithm 1 and 2, where they are ensured high profits thanks to their efficiency.

## **5.6 Summary of result analysis**

With the simulations of the single-node power market model, we have examined some important issues in the bidding strategies of buyers in electricity market. In the first scenario where all buyers play individually, a competition among them takes place and pull up the MCP due to their large quantity bids. The result is that the market comes to equilibrium where everybody gets low profit compared to other scenarios. Moreover, we have shown that the equilibrium is actually a Nash equilibrium by evolving the strategy of one buyer and letting other buyers play their equilibrium strategies. In the second scenario where all buyers cooperate, we can see a significant drop in MCP thanks to the reduction in quantity bids of all buyers. Therefore, most of buyers get better profits. The first lesson taken from these two simulations is that trying to get more quantity is not always a good choice because this can make the MCP become very high. A better strategy is to cooperate by limiting the quantity bids and thus get lower MCP, which can lead to very high profits. The second lesson is that inefficient buyers might not want to cooperate with efficient buyers to maximize the total profit, because cooperation

in a group can also mean to sacrifice by giving the priority of bidding large quantities to efficient buyers in order to maximize the total profit of the group.

In the third scenario where there is one buyer plays individually against the cooperation of others, we find that the free rider problem arises. The free rider is the buyer who does not cooperate. Without cooperation, the free rider doesn't have to limit his quantity bids, but still enjoys the low MCP thanks to the cooperation of others. It's true that cooperation helps each buyer to get more profit, but it is actually the free rider who benefits the most. We also point out that this result might affect the decision to cooperate or not of the buyers through a mechanism similar to the prisoner dilemma. In fact, since the free rider benefits the most, all buyers hope to be a free rider. As a consequence, none of them will cooperate, and the market will be completely competitive, which is the least profit situation for most of the buyers.

In the last section, three different cooperation schemes for small buyers have been proposed. It is found that efficient buyers would appreciate the cooperation scheme where they can draw more profits thanks to the "sacrifice" in the bid quantities of inefficient small buyers. Another cooperation scheme for small buyers is to form a large buyer by bidding their total quantities demanded, and then share the quantities received. This scheme of cooperation is highly appreciated by inefficient small buyers because they are equally shared the power quantities and enjoy good MCP thanks to the high performance of efficient small buyers. However, the formation of a new large buyer from small buyers may not be easily feasible.

## **Chapter 6: MULTI-NODE POWER MARKET MODEL**

We build a multi-node model of a power market, which means buyers and generators are located on different buses. Therefore it is necessary to take into account the technical constraints and congestion limits of the transmission network. The spot prices depend on buses - the locations of generators / buyers in the network, and are called local marginal price (LMP) or nodal price. Moreover, since we focus on studying the behavior of electricity buyers, the bidding strategies of generators are assumed to be fixed.

### **6.1 The multi-node power market model**

In this paper, the PoolCo model is chosen because of the same reasons as in the single-node model. We simulate the power market using spot pricing theory [51]. In each bidding round, generators and buyers submit bid curves to the pool operator which runs an optimization routine to determine the power dispatch results, which are generation, load dispatchs and spot prices. Generators are then paid a price according to their bids and consumers must pay a price according to their bids.

The operation of the power spot market is modeled as a game in which the actual players are electricity buyers, because generators use fixed strategies. Players can choose to compete against one another or cooperate to accomplish

their goal in an optimal way.

## 6.2 Generator and buyer models

As usually seen in power system studies, the total production cost of a generator  $j$  is approximated as a quadratic function:

$$C_j(s_j) = b_{j0} + b_{j1}s_j + b_{j2}s_j^2 \quad \forall j \in G \quad (6.1)$$

$G$  is the set of generators,  $s_j$  is the electric power that generator  $j$  supplies in this round, and  $b_{j0}$ ,  $b_{j1}$ ,  $b_{j2}$  are the cost coefficients of this generator  $j$ . The cost coefficients are positive and each generator has its minimum and maximum power output.

A buyer  $i$  is characterized by the revenue function, which is symmetric to the cost function of a generator.

$$R_i(d_i) = a_{i1}d_i - a_{i2}d_i^2 \quad \forall i \in L \quad (6.2)$$

$L$  is the set of buyers,  $d_i$  is the electric power that buyer  $i$  is dispatched in this round, and  $a_{i1}$ ,  $a_{i2}$  are the revenue coefficients of this buyer  $i$ . The cost coefficients are positive and each buyer also has its minimum and maximum power demand as will be discussed further in this section. In a particular round, if the LMP of buyer  $i$  is  $\lambda_i$  and the electricity power received is  $d_i$ , this buyer will earn a profit of

$$\pi_i(d_i, \lambda_i) = (a_{i1}d_i - a_{i2}d_i^2) - \lambda_i d_i \quad (6.3)$$

Since the profit depends on both the LMP and the power received, each buyer has

to choose an optimal bidding strategy to maximize their profits.

The revenue function of a buyer stands for its intrinsic performance. Intuitively, the revenue function tells us how much profit a buyer can make using the electricity power it has purchased. The efficiency level of each buyer is determined by the coefficients  $a_{i1}$  and  $a_{i2}$ . A buyer is efficient if it has large value of  $a_{i1}$  and small value of  $a_{i2}$ . However, in this study we assume that buyers are homogenous, which means they all have the same revenue function. The convenience of this is, besides simplicity, a better interpretation of the interaction among buyers in different bus on the network.

In reality, the electricity power  $d_i$  a buyer can buy is bounded and consists of a fixed amount  $Q_{\min i}$  and a variable amount  $q_{dispi}$  which is the *dispatchable electricity power*:

$$d_i = Q_{\min i} + q_{dispi} \leq Q_{\max i} \quad (6.4)$$

The variable  $Q_{\min i}$  is the minimum electricity power that the buyer needs to maintain a certain level of production or to satisfy certain consumption demand,  $Q_{\max i}$  is the maximum electricity power it can buy. By this mechanism, buyer  $i$  is assured to receive at least  $Q_{\min i}$  MW, and the extra dispatchable electric power  $q_{dispi}$  depends on the dispatch results, which in its turn depend partially on the bids of the players. The revenue of a buyer  $i$  is therefore

$$\begin{aligned} R_i(d_i) &= a_{i1}(Q_{\min i} + q_{dispi}) - a_{i2}(Q_{\min i} + q_{dispi})^2 \\ &= (a_{i1}Q_{\min i} - a_{i2}Q_{\min i}^2) + (a_{i1} - 2a_{i2}Q_{\min i})q_{dispi} - a_{i2}q_{dispi}^2 \end{aligned} \quad (6.5)$$

We note that the term  $(a_{i1}Q_{\min i} - a_{i2}Q_{\min i}^2)$  is constant and the revenue actually depends on the dispatchable electricity power  $q_{dispi}$ .

### 6.3 The bidding model and market calculation

In our model, each generator is allowed to bid a supply function and each buyer is allowed to bid a demand function to the system operator. A supply function  $P_j(s_j)$  represents the price at which a generator  $i$  is ready to sell if the power it has produced is  $s_j$ . Similar interpretation is applied for buyers.

Base on the bidding information as well as the network configuration, the operator solves an optimal power flow (OPF) problem to determine the generation, load dispatch and LMPs while satisfying physical and operational constraints. The objective of OPF problem is to maximize the social welfare  $W$ , which is equal to the total buyer benefits minus the total generator costs.

$$\begin{aligned} \max_{\mathbf{x}, \mathbf{s}, \mathbf{d}} \quad & W(\mathbf{s}, \mathbf{d}) = \sum_{i \in L} R_i(d_i) - \sum_{j \in G} C_j(s_j) \\ \text{s.t.} \quad & \mathbf{h}(\mathbf{x}, \mathbf{s}, \mathbf{d}) = \mathbf{0} \\ & \mathbf{g}(\mathbf{x}, \mathbf{s}, \mathbf{d}) \leq \mathbf{0} \end{aligned} \tag{6.6}$$

$\mathbf{x}$  is the state vector consisting of system voltages and angles,  $\mathbf{s}$  is the vector of generated power,  $\mathbf{d}$  is the vector of dispatched power,  $\mathbf{h}(\mathbf{x}, \mathbf{s}, \mathbf{d})$  are equality constraints such as the power flows equations,  $\mathbf{g}(\mathbf{x}, \mathbf{s}, \mathbf{d})$  are inequality constraints such as line flow limits. Details on OPF problem could be found in textbooks about electrical power system.

It is well known from microeconomics theory that in a market with perfect competition, the social welfare is maximized when the players bid their marginal cost / revenue function. However, in an electric power market, the physical constraints of the network gives certain market power to some players and thus discourage them from bidding marginally.

In this study, the bids of generators are assumed to be fixed. More specifically, the generators always bid their true marginal cost functions as supply functions:

$$P_j(s_j) = \frac{\partial C_j(s_j)}{\partial s_j} = b_{j1} + 2b_{j2}s_j \quad (6.7)$$

Buyers bid strategically rather than bid their true marginal revenue function. A strategy or a bid of a certain buyer is defined by a coefficient  $k_i$  that is multiplied to the true marginal revenue function to get demand function:

$$P_i(d_i) = k_i \frac{\partial R_j(d_i)}{\partial d_i} = k_i(a_{i1} - 2a_{i2}d_i) \quad (6.8)$$

We note that a rational buyer would bid  $k_i \leq 1$ . This method of bidding also means to multiply the revenue function used in OPF formulation by  $k_i$ . In fact, from the view of the pool operator, submitted bids are considered to reflect the true marginal curves of the participants. Therefore, the revenue function of buyer  $i$  as being viewed by the pool operator is:

$$\begin{aligned} R_i^{bid}(d_i) &= k_i R(d_i) = k_i(a_{i1}Q_{\min i} - a_{i2}Q_{\min i}^2) \\ &\quad + k_i(a_{i1} - 2a_{i2}Q_{\min i})q_{dispi} - k_i a_{i2}q_{dispi}^2 \end{aligned} \quad (6.9)$$

For a specific strategy  $k_i$ , the constant  $k_i(a_{i1}Q_{\min i} - a_{i2}Q_{\min i}^2)$  can be excluded from the OPF problem. The objective function of the maximization problem is therefore:

$$W(\mathbf{s}, \mathbf{d}_{disp}) = \sum_{i \in L} k_i(a_{i1} - 2a_{i2}Q_{\min i})q_{dispi} - k_i a_{i2}q_{dispi}^2 - \sum_{j \in G} C_j(s_j) \quad (6.10)$$

Note that the objective function depends on the supply and dispatchable power. New constraints must be included to take into account the fixed power required

$Q_{\min i}$  .

A black box simulator is built to integrate bidding strategies and solve OPF problem. The input to OPF solver consists of a network configuration, the cost function coefficients of generators, the maximum powers of generators, the revenue function coefficients of buyers, the must serve powers  $Q_{\min i}$  and maximum capacities  $Q_{\max i}$  of buyers. The output from the simulator consists of the LMPs  $\lambda_i$  and dispatchable powers  $q_{\text{dis}i}$  for each buyer.



# Chapter 7: IMPLEMENTATION OF MULTI-NODE POWER MARKET MODEL

## 7.1 Test network

Using the multi-node power market model as proposed in previous chapter, we implement an IEEE 14-bus network with 7 generators representing electricity sellers and 18 loads representing electricity buyers. This test network is chosen because it integrates physical and technical constraints as in practice and consists of a large enough number of seller / buyer agents to validate the proposed approach. Besides, our approach will also be tested on a IEEE 30 bus system. The 14 bus network is shown in Figure 7.1, where loads are represented by arrows and generators are represented by circular objects. Power is delivered into and drawn from the network busbars. Busbars are interconnected via transmission lines which have an upper limit to the amount of power they can transmit.

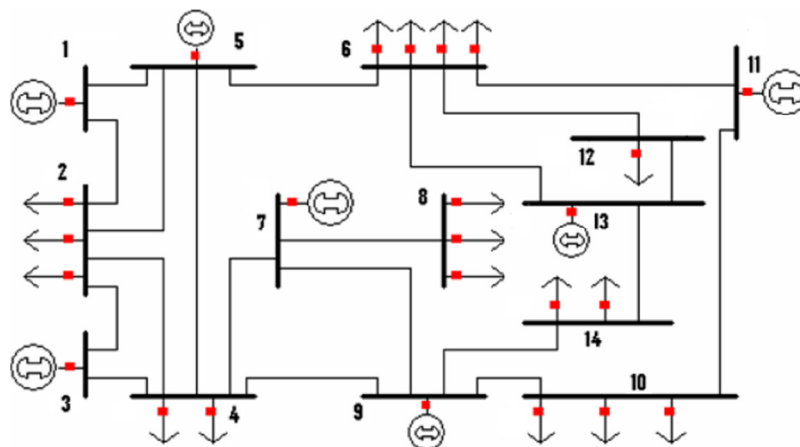


Figure 7.1: IEEE 14 bus test system

To observe the influence of the physical network on players' performance, we suppose that all buyers are homogenous, which means they have the same revenue function. The common revenue function is characterized by  $a_{i1} = a_1 = 80$ ,

for all buyers. Other parameters of buyers are summarized in Table 7.1. The last column, which will be discussed later, is the profits of buyers when they all bid marginally ( $k_i = 1.0$  for all buyers).

**Table 7.1: Data of Buyers**

Buyer	Bus	Must serve Power (MW)	Max Power (MW)	Dispatchable Power (MW)	Marginal bid profit (\$)
1	2	45.0	67.5	22.5	2186.72
2	2	49.0	73.5	24.5	2336.99
3	2	55.0	82.5	27.5	2548.9
4	4	48.0	72.0	24.0	2177.76
5	4	50.0	75.0	25.0	2245.99
6	6	44.0	66.0	22.0	773.48
7	6	55.0	82.5	27.5	830.72
8	6	47.0	70.5	23.5	794.49
9	6	50.0	75.0	25.0	811.45
10	8	42.0	63.0	21.0	266.58
11	8	55.0	82.5	27.5	214.95
12	8	46.0	69.0	23.0	273.57
13	10	50.0	75.0	25.0	1004.35
14	10	54.0	81.0	27.0	1036.1
15	10	55.0	82.5	27.5	1042.91
16	12	50.0	75.0	25.0	453.72
17	14	48.0	72.0	24.0	350.63
18	14	55.0	82.5	27.5	366.22
				<b>Total</b>	<b>19715.53</b>

## 7.2 Market database

Firstly, a database of the power market was constructed. This will provide the reference case for studying the deterministic situation and permit the calculation of characteristic function in the stochastic situation. The database can be viewed as historical data of a market in which agents possess no intelligence. We have performed 100 thousands random simulations using the proposed model with strategies  $k_i$  of buyers uniformly distributed between 0.3 and 1. The rationale for that choice of as follow: We ran 60, 70, 80, 85, 90, 95 and 100 thousands random simulations sequentially. It was observed that the relevant statistics (average values and standard deviations of powers dispatched, LMPs and profits) converge from the number of 85 thousands samples. Therefore it is convincible that 100 thousands random simulations could statistically represent the system with high confidence level.

The average dispatchable power, the average LMP as well as the average and standard deviation of profits of buyers are reported in Table 7.2. The fluctuation index, which is equal to the standard deviation value divided by the average value, is also calculated for ease of comparison. This index reflects the relative profit fluctuation, measured in percentage of the average value. A low value of fluctuation index indicates a stable payoff, while a high value of fluctuation index indicates a highly sensitive payoff.

**Table 7.2: Results from 100 000 Random Simulations**

Buyer	Average Dispatchable power (MW)	Average LMP (\$/MW)	Average payoff (\$)	Payoff Sdv (\$)	Fluctuation index
1	18.08	38.84	2187.95	254.67	0.12
2	19.94	38.84	2350.45	260.41	0.11
3	22.69	38.84	2579.89	267.20	0.10
4	18.46	40.00	2204.33	274.21	0.12
5	19.39	40.00	2280.75	277.15	0.12
6	14.77	45.22	1683.94	271.91	0.16
7	19.23	45.22	2008.07	299.81	0.15
8	15.96	45.22	1776.64	280.82	0.16
9	17.17	45.22	1866.12	288.78	0.15
10	7.10	51.57	1151.19	362.62	0.31
11	11.15	51.57	1406.51	413.55	0.29
12	8.31	51.57	1241.03	387.07	0.31
13	16.57	44.95	1867.01	262.74	0.14
14	18.12	44.95	1980.73	265.88	0.13
15	18.52	44.95	2008.50	267.01	0.13
16	8.19	55.62	1028.89	501.05	0.49
17	4.16	59.80	768.00	396.94	0.52
18	4.44	59.80	830.60	455.01	0.55
		<b>Total</b>	<b>31220.60</b>		

As can be observed from Table 7.2, buyers who are in the same bus in the network have similar fluctuation indices. This observation suggests the high influence of the physical constraints to the payoff of buyers. In fact, due to the particular location on the network, a buyer may have some market power - which is the ability to alter the electricity price with its strategy. On the other hand, a buyer in a location with high fluctuation index highly depends on the strategies of

others. Moreover, we can see a correlation between fluctuation indices and LMPs, which also vary largely among bus. On buses with low LMPs, buyers can make more profit by buying larger amount of electric power; therefore the average dispatchable powers are higher in these buses.

In brief, a bus is considered stable if the fluctuation indices and the LMP of buyers on this bus are low, leading to high dispatchable powers. Bus 2 (buyers 1, 2, 3) is a typically stable bus. Contrarily, a bus with high fluctuation indices is considered unstable, such as bus 14 (buyers 17, 18). This simple statistical analysis has provided a very good overview on the performance of buyers in the network. In reality, buyers may have this kind of knowledge by learning through experience, and the large number of bidding simulations in this study actually models a long period the players participate in the market. Moreover, comparison between the last column of Table I and the 4<sup>th</sup> column of Table 7.2 proposes that even random bidding can return in better profits than marginal bidding. This observation confirms the incomplete competitive nature of the market.

### 7.3 Chromosome structures

Co-evolutionary algorithm was used in the deterministic situation. The fitness function depends on different simulation scenarios. If buyer  $i$  cooperates in a group  $S$  having  $l$  members,  $S = \{i_1, i_2, \dots, i_l\}$ , its fitness function is the total profit of all buyers in this group:

$$Fitness(i) = \sum_{k \in S} \pi_k(d_k, \lambda_k) \quad (7.1)$$

Each species is a population consisting of a number of chromosomes. Each

chromosome is a real number in the interval  $[0.3, 1]$  that represents the coefficient  $k_i$  in the bid demand function and encodes one bidding strategy of that buyer species.

In the stochastic situation, each chromosome encodes a coalition structure. More specifically, the length of a chromosome is the number of buyers  $n$  in the market. The value of the  $i^{\text{th}}$  allele of the chromosome, which can be any number between 1 and  $n$ , represents the coalition that buyer  $i$  is joining. It is noted that two different chromosomes can actually represent one coalition structure. Let's take an example where there are 5 buyers: the chromosomes  $chrom_1 = [11232]$  and  $chrom_2 = [22141]$  both represent the coalition structure  $CS = \{(1, 2), (4), (3, 5)\}$  with 3 coalitions in total. While the search space is significantly inflated with this representation, such many-to-one mappings simplify the problem and guards against disruptive crossover. The fitness of a chromosome  $c$  is the value of the coalition structure it encodes:

$$Fitness(c) = V(CS) = \sum_{S_k \in CS} v(S_k) \quad (7.2)$$

$CS$  is the coalition structure coded by chromosome  $c$  and the sets  $S_k$  are the coalitions in  $CS$ .

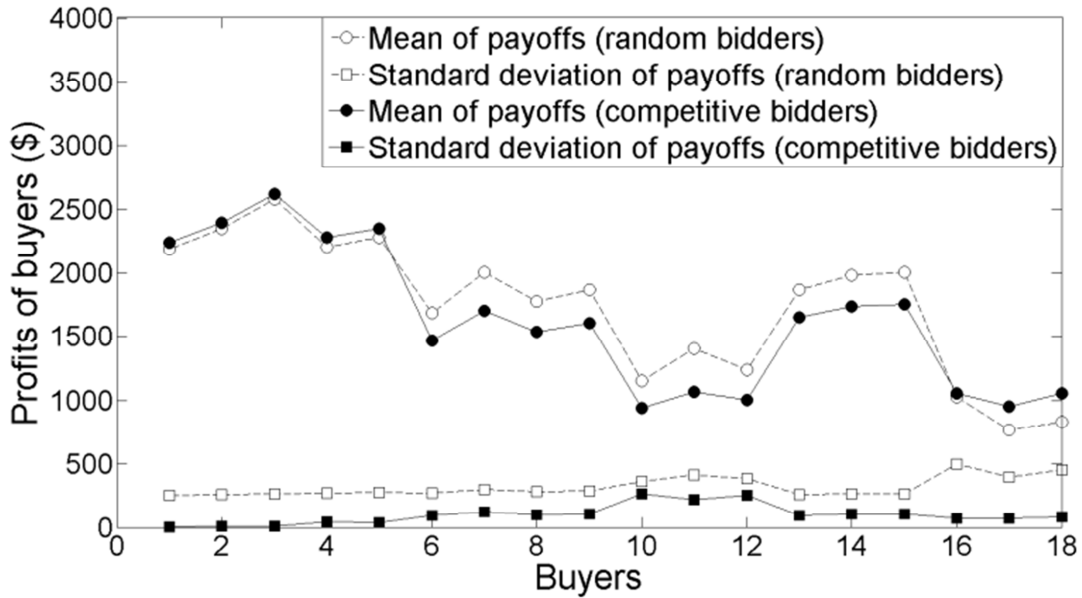
## **Chapter 8: SIMULATION OF MULTI-NODE POWER MARKET MODEL**

To validate the proposed approach, the system was simulated in different scenarios. The deterministic situation was studied firstly through individual bidding. Then players were allowed to cooperate by different schemes, where cooperation occurs in the whole player set or in smaller groups, and the condition to cooperate is or is not imposed. In the stochastic situation, the cooperation strategies were studied with different characteristic functions representing different group properties.

### **8.1 Deterministic situation**

#### **8.1.1 Individual bidding**

In this scenario, all buyers bid individually to maximize their own profits. The fitness of each buyer is calculated using (18), where each group  $C$  contains only one member. Since all bids are submitted individually, each buyer has no information about the bids of other participants in this round; therefore they are considered as competitive bidders. Each buyer chooses the previous round strategies of their rivals as representatives to evaluate the fitness function in the co-evolution process.



**Figure 8.1: Comparison of random bidding and competitive bidding**

The co-evolutionary progress doesn't lead to a stable state. It is observed that the payoffs of buyers fluctuate around certain average values. In this scenario where all players bid individually, there is strong competition among them for highest possible payoffs. Similar to real life deregulated markets, no players have enough market power to dominate the market; thus the strategic bidding progress is just like a fight with no winner. That's why the market is not settled to a perfect equilibrium, but rather some kind of "dynamic equilibrium" around certain average values. The mean and standard deviation of payoffs in the competitive scenario is compared with the reference case in Figure 8.1. Unlike the reference case when all players bid randomly (in other word, they don't have intelligence), in this competition, every player learns and evolves to choose the best strategy. Therefore the fluctuation of payoffs is smaller than the one in the random bidding case, as can be seen on Figure 8.1. It is also noted that some buyers can manage for better profits compared to random bidding, while the rest get lower profits.



With intelligence, certain players can make use of physical advantage, which depends on the location on the network, to improve the payoffs (buyers 1, 2, 3, 4, 5, 16, 17, 18) while others, even with intelligence, perform worse because they don't have that much advantage. Referring to Table 7.2, it is also interesting to note that players who perform better all have either lowest or highest fluctuation indices.

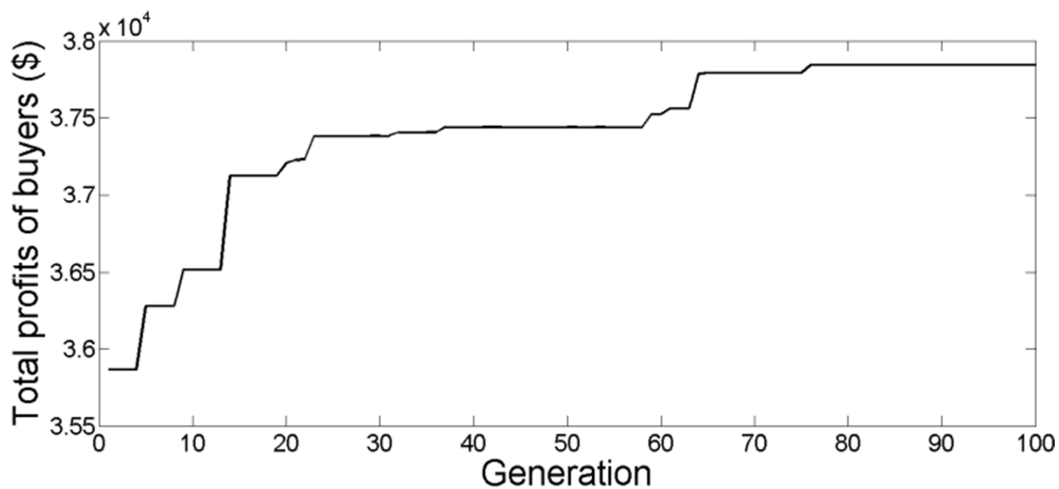
### **8.1.2 Total cooperation**

In this scenario, all buyers cooperate with the goal of maximizing the total profit. The total cooperation is modeled by two key points: Cooperation in goal and cooperation in information. The common goal is reflected the fitness of a strategy chromosome, which is the total profit of all buyers when the buyer in consideration uses that strategy. Therefore the sum in (18) is taken over the whole set of buyers. This is similar to solving a multi-objective optimization problem, where each objective is to maximize the profit of one buyer. The choice of maximizing the total profit of all buyers is equivalent to using an aggregate objective function, which is in the form of a non-weighted linear sum of payoffs. Secondly, the cooperation in information is modeled as follows: the strategies of buyers are informed to the whole group and the optimal strategy vector is chosen under the agreement of all members.

The best total profits over generations of one simulation are shown in Figure 8.2. It takes about 80 generations to reach equilibrium in this case. As expected, the total profit keeps increasing. The total profit in this case is 37846.61 \$, which increases by 20% compared to the reference case. It is clear that

cooperation helps to increase the total profit.

But it is also interesting to look at individual profits reported in Table 8.1. We see that while the profits of almost every buyer increase, the profits of buyers 7, 9, 13, 15 decrease. This fact suggests an improved cooperation scheme that can assure acceptable payoff for every player.



**Figure 8.2: Evolution of total profit under total cooperation**

### **8.1.3 Total cooperation with Pareto improvement**

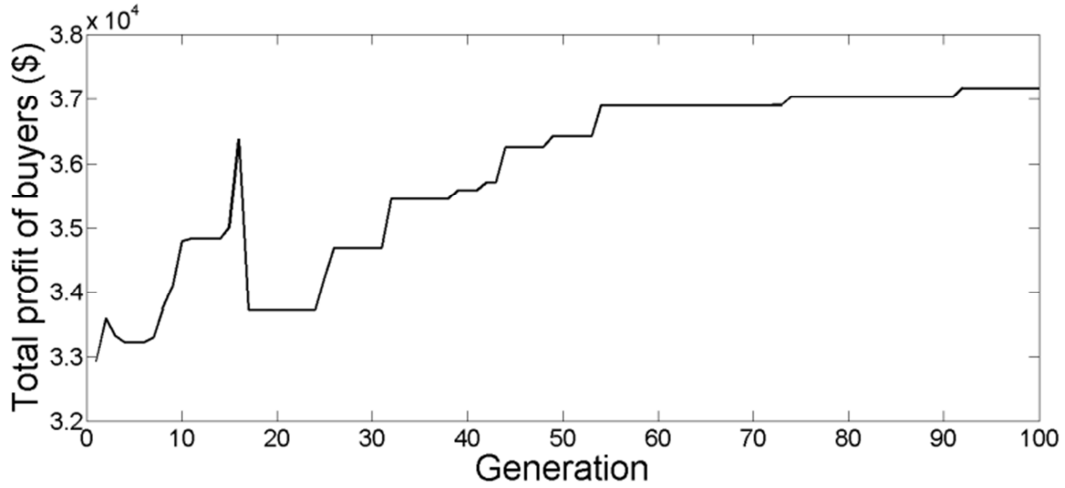
Pareto efficiency is a concept in economics with many applications in engineering. Given an initial allocation of payoff among a set of players, a *Pareto improvement* is defined as a change in the allocation that makes at least one individual better off and no worse for any other. An allocation where no further Pareto improvements can be made is called Pareto efficiency. In a multi criteria decision making problem, there are usually many different Pareto efficient allocations and they form the Pareto frontier. However, seeking for that frontier is

not our objective. In this part about total cooperation, we only seek for a Pareto improvement that is good as possible, compared to the reference case.

The common objective function, which is the total payoff of all buyers, remains the same as previous simulation. The selection process in the co-evolution is modified as followed to integrate Pareto constraint:

- Between two strategy vectors that both lead to Pareto improvements, the one with higher fitness will be chosen.
- A strategy vector that leads to a Pareto improvement will always be preferred to a strategy vector that does not, regardless of its fitness value.

A typical simulation result is shown in Figure 8.3. In the first 25 generations, there are rises and drops in total profits. Each drop in total profit marks a discovery of a new strategy vector that leads to Pareto improvement, and thus being the replacement for previous strategy vectors (that do not satisfy Pareto condition). From Table 8.1, we observe that the payoffs in the end of the simulation are Pareto-improved compared to the reference case. More specifically, buyers 7, 9, 13, 15 who had worse payoff than the reference case in previous cooperation now achieve better results. Of course, the tradeoff is slightly lower profits for other buyers compared to the previous cooperation without Pareto constraint. Nevertheless, all players still have considerably better payoffs than in the reference case. Pareto constraint in some sense has implied the redistribution of payoffs, which makes this cooperation scheme acceptable and equitable for all players.



**Figure 8.3: Evolution of total profit under total cooperation with Pareto improvement**

#### 8.1.4 Group cooperation

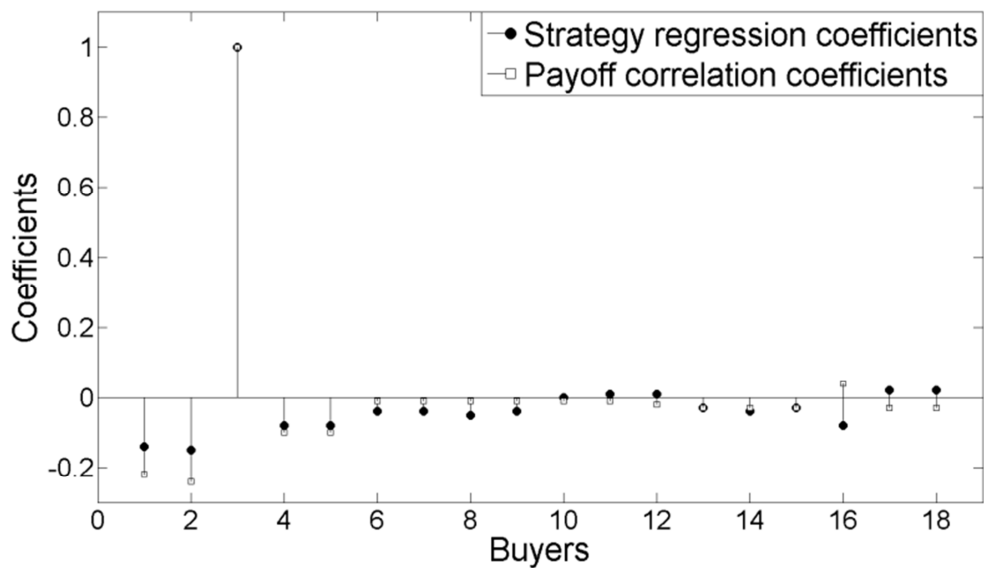
As we have seen in previous simulations, cooperation with the participation of all players leads to better outcome compared to the reference case. However, in reality, the number of players could be very large and such total cooperation would be impracticable. In fact, cooperation naturally happens among smaller groups in which members have close relationship and share common goal.

To verify this assumption, statistical analysis on the market database has been carried out. Firstly, for each buyer, we implemented a linear regression of the payoff on the strategies of all buyers in the market. Mathematically, with  $n$  buyers, we want to find coefficients  $c_j$  in the regression equation:

$$\pi_i = \sum_{j=1}^n c_j k_j + c_0 \quad (8.1)$$

The regression coefficients measure the sensitivity of a particular player payoff versus the strategies of others. The larger the absolute value of  $c_j$  is, the higher the

influence of player  $j$  on player  $i$  is. Secondly, for each player, we calculated the correlations between its payoff and others' payoffs. It is observed that the statistical analysis results are similar for all buyers; therefore only the results for buyer 3 are reported. The regression coefficients  $c_j$  have been normalized for easy comparison.



**Figure 8.4: Statistical analysis of buyer 3's correlation with others**

From the analysis results, the profit of a buyer is most affected by the strategies of other buyers on the same bus. Specifically, Figure 8.4 shows that the payoff of buyer 3 is highly affected by buyers 1 and 2 – those on the same bus. Moreover, the profits of buyers on the same bus are highly correlated. In fact, these buyers have common LMP and share a number of network technical parameters. This proximity suggests that they should form a group. Since buyers are located on 7 buses, we assume that 7 groups will be formed: group 1 consists of buyers 1, 2, 3; group 2 consists of buyers 4, 5 and so on.

Similarly to the total cooperation scheme, the fitness function of a buyer is now the total profit of buyers within the bus, and the bidding information is shared among group mates. As in first simulation of total cooperation without Pareto constraint, while the total profit of a group increases, the profit of some members could be worse off. This fact is shown on Table 8.1. The profit of buyer 6 is less than in reference case.

**Table 8.1: Results of different cooperation schemes**

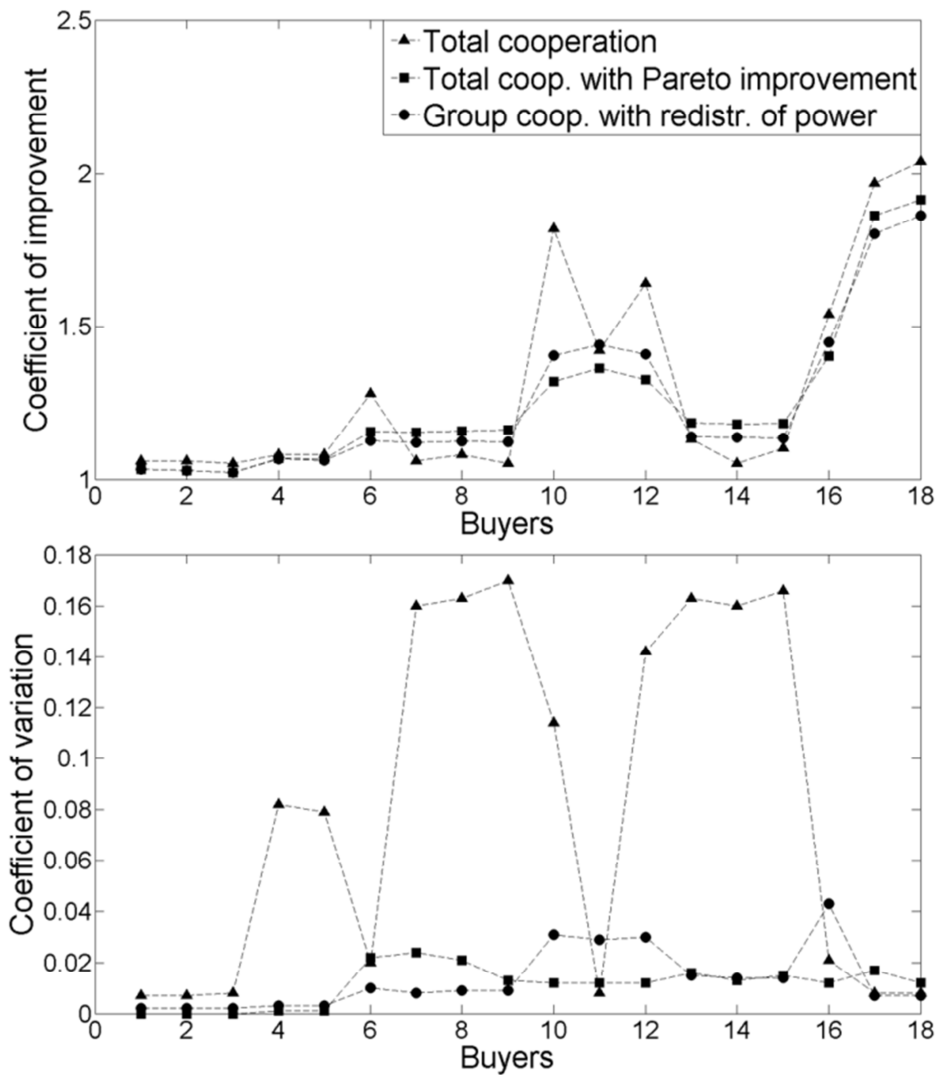
Buyer	Average profit (\$)	Total coop. (\$)	Total coop. with Pareto impr. (\$)	Group coop. (\$)	Group coop. with redistr. of power (\$)
1	2187.95	2338.47	2257.16	2259.05	2259.05
2	2350.45	2502.24	2413.69	2415.76	2415.76
3	2579.89	2734.38	2635.00	2637.32	2637.32
4	2204.33	2451.76	2355.53	2345.41	2345.41
5	2280.75	2531.42	2431.18	2420.63	2420.63
6	<b>1683.94</b>	2176.76	1947.09	<b>1445.03</b>	<b>1901.18</b>
7	<b>2008.07</b>	<b>1874.46</b>	<b>2297.73</b>	2391.81	2259.96
8	1776.64	2293.45	2048.12	2128.51	2003.65
9	<b>1866.12</b>	<b>1729.06</b>	<b>2145.10</b>	2230.63	2102.65
10	1151.19	1591.61	1546.07	2034.17	1626.71
11	1406.51	2792.24	1953.12	1819.87	2036.17
12	1241.03	1724.79	1674.92	1563.47	1757.43
13	<b>1867.01</b>	<b>1758.98</b>	<b>2272.84</b>	2137.32	2137.32
14	1980.73	2598.45	2302.21	2259.7	2259.7
15	<b>2008.50</b>	<b>1907.38</b>	<b>2438.24</b>	2289.17	2289.17
16	1028.89	1600.34	1417.23	1484.12	1484.12
17	768.00	1528.22	1435.27	1391.68	1391.68
18	830.60	1712.59	1606.08	1556.13	1556.13
Total	<b>31220.60</b>	<b>37846.61</b>	<b>37176.55</b>	<b>36809.78</b>	<b>36884.04</b>

An improved version of group cooperation is achieved when we integrate a method for redistribution of dispatchable power. More specifically, after the OPF solver decides the power to be dispatched, buyers on same bus will receive their minimum power required  $Q_{\min i}$  and share the total dispatchable power proportionally to their maximum capacity  $Q_{\max i}$ . This redistribution of dispatchable power makes use of technical advantages of power transmission in same bus. It is also necessary to emphasize that the redistribution is consistent because we assume homogeneous buyers. In fact, the common revenue function of buyers evaluates one MW of electricity the same way no matter it belongs to which buyer.

The last column of Table 8.1 shows the outcome of group cooperation with redistribution of power. As expected, buyer 6 has improved the profit to 1901.18\$. We also note that the total profit in this case is slightly higher than in previous case without power redistribution (36884.04\$ versus 36809.78\$). The explanation is that power redistribution has allocated the resource in a better way.

### **8.1.5 Comparison of different schemes of cooperation**

To evaluate the performance and stability of the proposed cooperation schemes, each of them was simulated 100 times; then the mean and standard deviation values of outcome payoffs were calculated. Each mean value is divided by the corresponding value in reference case to get the coefficient of improvement, and each standard deviation value is normalized by the corresponding mean value to get the coefficient of variation. These indices are plotted in Figure 8.5.



**Figure 8.5: Evaluation of different cooperation schemes**

The coefficient of improvement reflects how much a player is better off compared to the reference case. From the plot we observe that buyers in unstable buses such as buyer 10, 11, 12 (bus 8), buyer 16 (bus 12), buyers 17, 18 (bus 14) benefit the most by cooperation schemes. Other buyers with stable payoffs don't have much improvement. The coefficients of improvement under total cooperation vary a lot from buyers to buyers, while total cooperation with Pareto constraint and group cooperation have restricted that variation. In other words, the two later schemes are more equitable for all players.

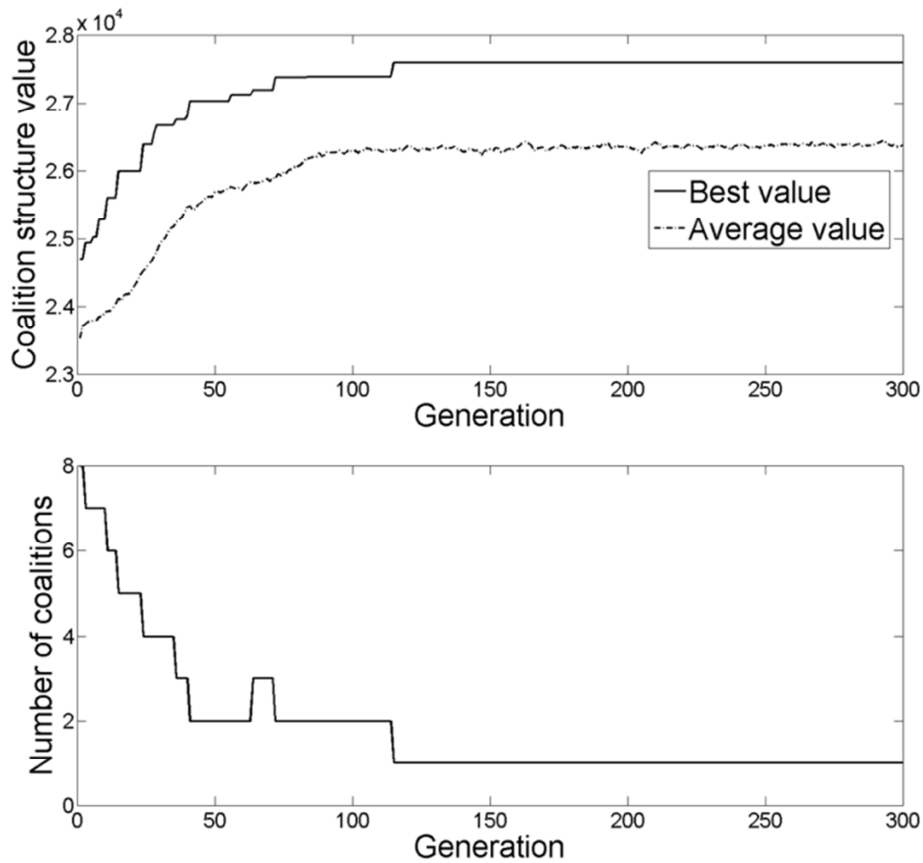


The coefficient of variation measures the stability of the algorithm. It is clear that the coefficients of variation under total cooperation scheme without Pareto constraint are high for most players, which means this scheme is unstable for most of them. Two other cooperation schemes assure good stability with variations in payoff of less than 5% for all players.

## **8.2 Stochastic situation**

### **8.2.1 Test on IEEE 14 bus system**

We implement the evolutionary algorithm for finding optimal coalition structure on IEEE 14 bus network. Firstly, the parameter  $a$  in the characteristic function (17) is set to zero, which means there are no transaction cost for grouping and no restriction of the group size. Figure 8.6 depicts the evolution process.

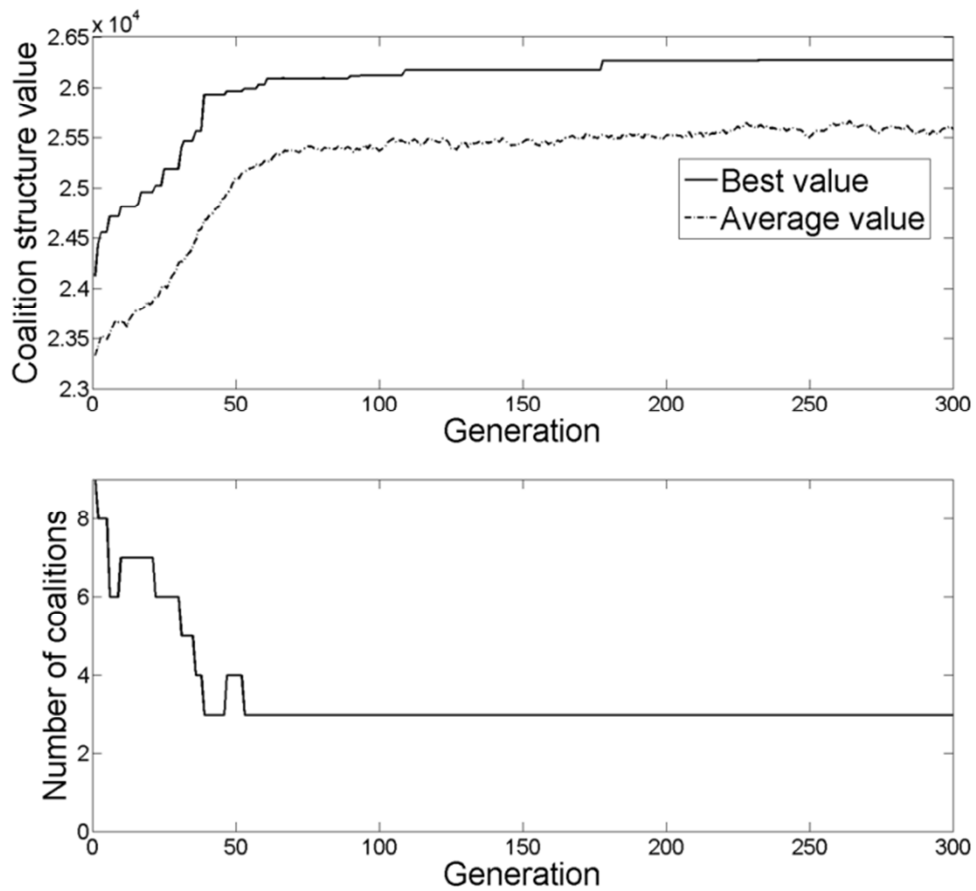


**Figure 8.6: Evolution of coalition structure (case without group size effect)**

The best coalition structure value of 27605.30\$ was achieved after about 120 generations. The number of coalition decreases over generations, and ultimately there is only one coalition – the grand coalition. This is an expected result. In fact, when the parameter  $a$  is set to zero, the characteristic function is superadditive and the optimal coalition structure is the grand coalition, as noted in part B - Section II.

We run a second simulation where parameter  $a$  is set to 0.05. In this case, there exist transaction costs when forming a group and the group size causes certain negative effect on the coalition value. This simulation tests the capability of the proposed algorithm when dealing with non-superadditive characteristic

function. The results are plotted in Figure 8.7.



**Figure 8.7: Evolution of coalition structure (case with group size effect)**

The evolutionary algorithm reaches the optimal coalition structure value of 26269.78\$ after nearly 200 generations. Due to the negative effect of group size, the optimal coalition structure is lower compared to previous case. Moreover, since the characteristic function is no longer superadditive, the grand coalition was not formed, but 3 different coalitions. The members of 3 coalitions and their location on the network are reported in Table 8.2.

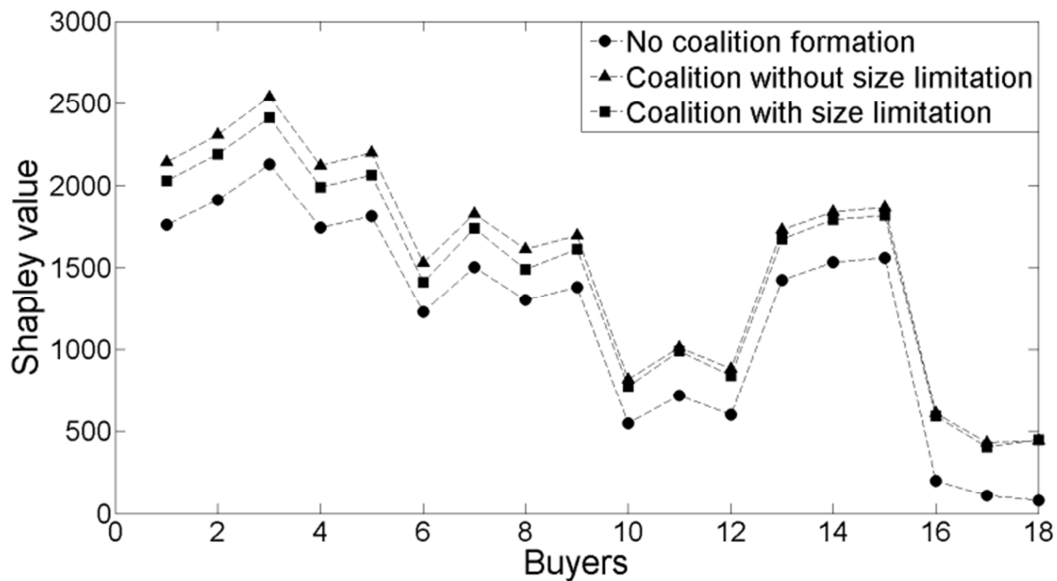
**Table 8.2: Distribution of optimal coalition structure**

Buyer members (bus)							
Coalition 1	6 (6)	8 (6)	<b>13</b> <b>(10)</b>	<b>14</b> <b>(10)</b>	<b>15</b> <b>(10)</b>		
Fluctuation indices	0.16	0.16	<b>0.14</b>	<b>0.13</b>	<b>0.13</b>		
Coalition 2	<b>1</b> <b>(2)</b>	<b>2</b> <b>(2)</b>	<b>3</b> <b>(2)</b>	10 (8)	12 (8)	16 (12)	18 (14)
Fluctuation indices	<b>0.12</b>	<b>0.11</b>	<b>0.10</b>	0.31	0.31	0.49	0.55
Coalition 3	<b>4</b> <b>(4)</b>	<b>5</b> <b>(4)</b>	7 (6)	9 (6)	11 (8)	17 (14)	
Fluctuation indices	<b>0.12</b>	<b>0.12</b>	0.15	0.15	0.29	0.52	

Since the characteristic function is based on a measure of risk in payoff, buyers on the same bus with low fluctuation indices prefer to form coalition. In fact, buyers 1, 2, 3 (bus 2) who have lowest fluctuation indices all join one coalition (coalition 2); the same strategy is observed with buyers 4, 5 (bus 4) and 13, 14, 15 (bus 10). With group size effect, grouping decision has to take into account the number of members to join the coalition; therefore those with stable payoffs are the first to form coalitions. Other buyers with high fluctuation indices have to distribute themselves in existing coalitions so that they can also hedge against the risk making use of the payoff stability of their group mates.

After partitioning the buyers into coalitions, we apply Shapley distribution within each coalition. The Shapley values of buyers in three different situations are plotted in Figure 8.8. When there is no coalition formation among buyer, the Shapley value of a buyer is simply the Value at Risk of its payoff. Two other

situations of coalition with or without group size limit were discussed above.



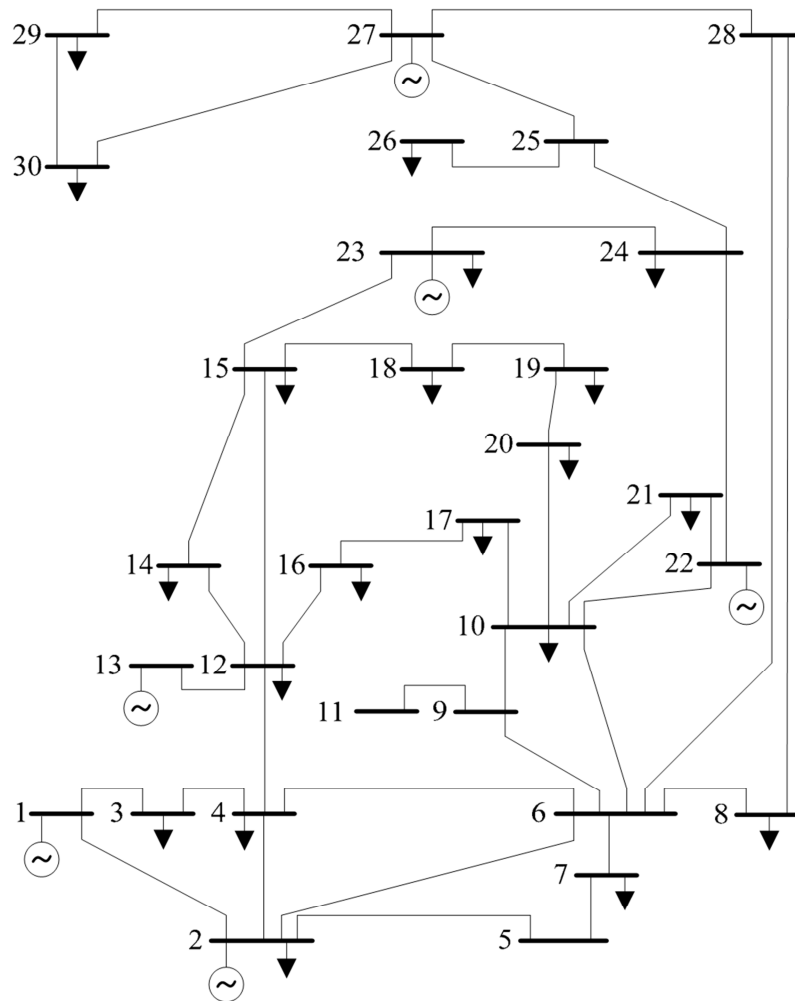
**Figure 8.8: Shapley values for different coalition structures**

From the plot, we see that coalition helps to increase the Shapley value compared to the case of playing individually. That means buyers can better hedge against risk by forming coalition. The ideal grand coalition results in highest Shapley values, while Shapley values decrease because of restriction in group size.

### 8.2.2 Test on IEEE 30 bus system

To check the efficiency of the proposed method, simulation was performed on IEEE 30 bus system. In this implementation, there are 6 generators and 90 buyers in total. The buyers are equally distributed on 30 buses so that on each bus there are three buyers. The single line diagram for IEEE 30 bus system is shown in

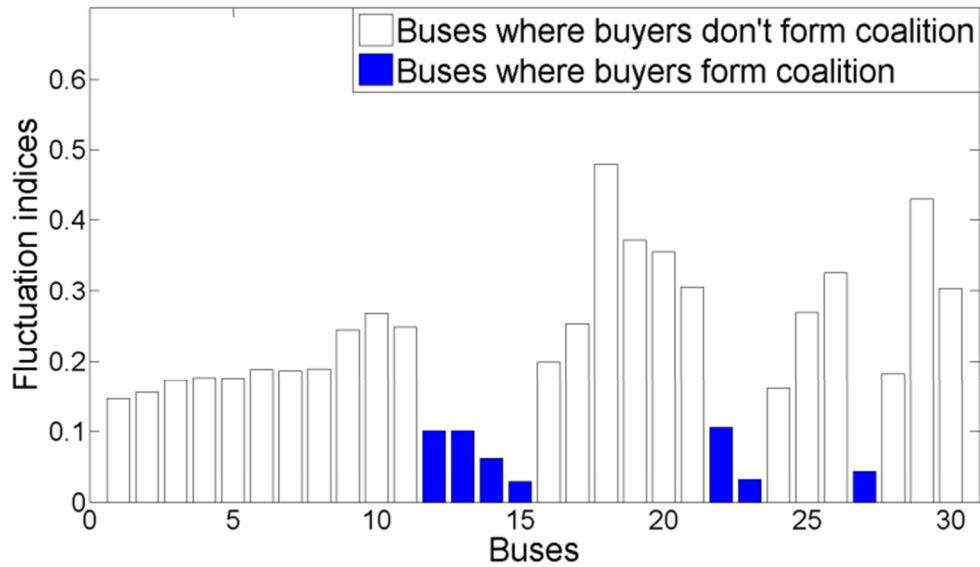
Figure 8.9.



**Figure 8.9: IEEE 30 bus test system**

A market database was also built using random bids from buyer agents. As in the previous test on IEEE 14 bus system, it is observed that the fluctuation indices highly effect the formation of coalitions. Since all buyers in a same bus have very close fluctuation indices, we define the fluctuation index of a bus as the average of the fluctuation indices of the buyers on that bus. High attention is reserved for the buses where buyers cooperate to form coalition; therefore we make a distinction between those buses and other buses where buyers have to distribute themselves

in different existing coalitions. The fluctuation indices of 30 buses of the system, coupled with the proposed distinction are reported in Figure 8.10.



**Figure 8.10: Fluctiation indices of different buses in the test system**

It is clear that on buses with low fluctuation indices, buyers tend to cooperate to form coalition by themselves because they already have the advantage of stability in payoffs. On the other hand, buyers on buses with high fluctuation indices need to cooperate with buyers in other more stable buses to hedge against this fluctuation. Therefore a highly unstable bus doesn't become a coalition, while a stable bus likely does.

### 8.3 Summary of result analysis

The proposed power market model has been simulated through deterministic and stochastic situation. The results show various aspects on the bidding and

cooperation strategies of electricity buyers.

When buyers bid individually, their payoffs fluctuate around certain average values and the co-evolution progress doesn't lead to a stable state due to the strong competition among them for highest possible payoffs. The market is driven to some "dynamic equilibrium".

On the other hand, total cooperation happens when all buyers cooperate to maximize the total profit. As expected, cooperation helps to increase the total profit; however, some buyers may not fairly enjoy the advantage of cooperation since their payoffs decrease. This fact suggests an improved scheme of cooperation which allows redistribution of payoffs and itself acceptable and equitable for all players: total cooperation with Pareto improvement.

Since the profits of buyers on the same bus are highly correlated (these buyers have common LMP and share a number of network technical parameters), it is suggested that they should form a group. This cooperation helps to increase the total profit of a group, but similarly to the first simulation of total cooperation without Pareto constraint, the profit of some members could be worse off. Therefore, certain method for redistribution of dispatchable power could be used.

In the stochastic situation where buyers cooperate to hedge against the risk of unstable payoffs, coalition helps to increase the Shapley value compared to the case of playing individually. It is noted that when the characteristic function is superadditive, the optimal coalition structure is the grand coalition. The ideal grand coalition results in highest Shapley values, while Shapley values decrease because of restriction in group size. The implicit factor that drives the cooperation progress is the fluctuation indices: Buyers on the same bus with low fluctuation indices prefer to form coalition, while other buyers with high fluctuation indices



have to distribute themselves in existing coalitions so that they can also hedge against the risk by making use of the payoff stability of their group mates.

## **Chapter 9: CONCLUSION**

In deregulated power markets, consumers are given more choices through flexible bidding and cooperation strategies, but they have to consider the transmission network and its physical limitations. Therefore, active demand side participation in the market is both a reasonable requirement and an economic necessity for greater efficiency. In this research, bidding and cooperation strategies of buyers in a deregulated electricity market have been studied through designing different simulations by incorporating co-evolutionary algorithms in an agent based framework.

### **9.1 Contributions**

In order to fully develop the electricity market, this research proposes and evaluates a single-node model and a multi-node model. It was found from the simulations of the single-node model that a competitive market can lead to equilibrium, which has been shown to be a Nash equilibrium. We also found that cooperation helps to increase the profit of most buyers. The strategy used in cooperation is to limit the quantity bids and thus get lower MCP. However, inefficient buyers have a risk of being limited in bidding large quantities and get low profits. The free rider problem when one buyer plays individually against the cooperation of others was also investigated. The free rider doesn't have to limit his quantity bids, but still enjoys the low MCP thanks to the cooperation of others

and thus benefits the most. This result can lead to a situation similar to the prisoner dilemma, where all buyers hope to be a free rider and play individually.

The multi-node model was then developed by taking into account the physical limitations of the system. Moreover, besides the payoff, another estimation of buyers' performance was proposed to capture the risks while trading in a very volatile environment like electricity market. In this multi-node model, all physical and technical constraints of the network were taken into account by using an IEEE 14 bus test system and an Optimal Power Flow solver. The first finding is that players should not bid marginally but strategically, since the Local Marginal Price depends heavily on the physical location.

In the deterministic situation, we also found that cooperation helps to increase the profit of most buyers, while individual bidding introduces a competitive environment and prevents the market from getting to an equilibrium state. Moreover, total cooperation with Pareto constraint can assure an improvement in profits for all buyers and make it somehow more equitable and globally acceptable. In reality, total cooperation is difficult to achieved, therefore group cooperation was investigated, where buyers formed groups and bid strategically to maximize the profit of the group. Similarly to the total cooperation case, it has been shown that a payoff redistribution in group cooperation is necessary. The performance and stability of different cooperation schemes have also been analyzed and verified statistically.

Stochastic situation was modeled by building a database that represents the historical data of buyers in the market. The theoretical base of the approach was studied in the framework of an optimal coalition structure problem. In this study, we assume that buyers cooperate to hedge against the risk in low payoffs. Both

mathematical and simulation results show that when there are no limitation of coalition size, the grand coalition is optimal. It is then shown that when there are limitations of coalition size, such as the transaction cost in practice, different coalitions will be formed. The partitioning way of buyers in coalitions was also discovered: Buyers in the same bus with stable payoffs tend to form coalition by themselves, while buyers with highly fluctuating payoffs have to join existing coalitions to make use of the stability of others. The efficiency and applicability of the proposed evolutionary algorithm were verified by an additional test on a much larger system of 30 buses and 90 buyers.

To summarize, we list down the main contributions of this research:

- The proposed agent based co-evolutionary framework has been demonstrated to be especially suitable for modeling market participants. In fact, the restructured electricity market with its large number of participants is spread over wide geographical areas, and the interactions and coordination of independent participants have been effectively simulated using this approach.
- The simulation results have successfully illustrated common observations in bidding and cooperation strategies of electricity buyers. The practicability of the proposed methodology is also verified by successfully dealing with large number of buyers.
- We proved an important theorem that serves as a link between the problems of payoff distribution in cooperative game theory and optimal coalition generation in combinatorial optimization theory. The main

advantage of the approach is that this methodology is general and any characteristic function can be applied.

- This study would be helpful for electric power buyers in finding attractive cooperative strategies, while assuring certain payoff stability in a volatile trading environment. For power market operators and policy makers, our findings give a deeper and more dynamic view into the deregulated electricity market.

## **9.2 Suggestions for future work**

In this study, cooperative buyers are assumed to have unconditional trust among them, which does not fully reflect the real world situation. Therefore, conditional trust should be modeled, where cooperative buyers may have the possibility to turn their backs on their groups in order to gain greater benefits. As such, modeling conditional and fuzzy trust is one possible area for further research and development.

Another suggestion for future research is to consider different characteristic functions in the Cooperative Game model. As we have seen, a characteristic function models the rationale and motivation for cooperation, which can vary through different market situations. Therefore, considering different characteristics function may help to discover implicit reasons and mechanism of cooperation.

Thirdly, we have introduced the problem of Optimal Coalition Structure Generation as a tool for studying cooperation between buyers. In future research, more effort should be spent to develop an efficient algorithm to solve the above

problem. Optimal Coalition Structure Generation is an important and difficult combinatorial optimization problem that has close relation with multi-agent systems. Therefore, we strongly believe that an agent-based evolutionary framework could be a potential candidate.

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## **APPENDIX**

### **A. From Evolutionary Algorithms to Co-Evolutionary Algorithms**

This section helps us understand and distinguish the artificial evolution and artificial co-evolution. Two main models of co-evolution are cooperative and competitive co-evolution although many variations can be used. Many researchers have used these co-evolutionary optimization models in which the evaluation of fitness of an individual is subjective i.e. it depends on the relations with other individuals. It is reported that such models give higher values of fitness and require a lower computational cost than the classical evolutionary models.

#### **A.1 Evolutionary algorithms**

Many optimization problems broad areas of research have no direct “analytical solutions”. The idea of Evolutionary Algorithms is simply to build a random population of potential solutions to the problem. The “individuals” are then evaluated to encourage the reproduction of the fittest individuals, i.e. those who are closest to the optimal solution. The mechanisms of selection, recombination of most adapted individuals and mutation permit to gradually approach the desired solution.

Evolutionary Algorithms have common core mechanism: it consists of making a population evolving by random transformation of some of its elements and application of the natural selection principle. Several techniques have been elaborated. The main ones are as follows:

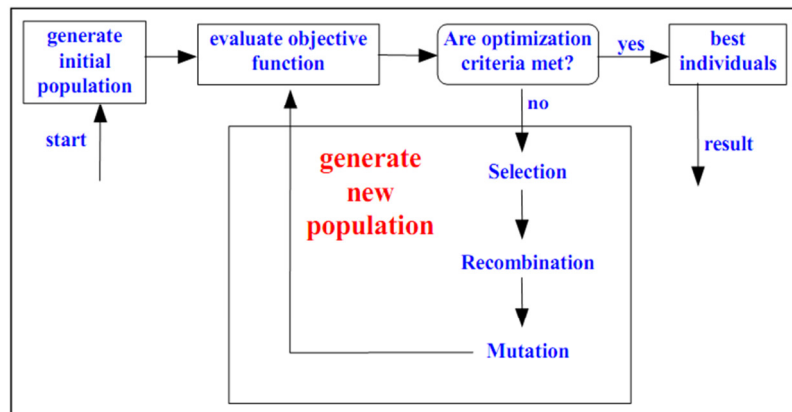
- Genetic Algorithms: they are probably the best known algorithms in evolutionary computation. They were developed in the 60s to study the complex adaption process of natural species.
- Evolution Strategies were developed to solve numerical optimization problems in the space of real parameters
- Programming Evolutionary first appeared in the finite state automata space for the prediction of time-series.
- Genetic Programming is to evolve structures of trees representing programs.

Although the applications of evolutionary algorithms are varied and they have given good results in different areas, the mathematical study of these algorithms is still very limited due to their theoretical complexity. It was until the 90s that complete and rigorous proofs of convergence in probability are established. Nevertheless, these theoretical results are difficult to use in practice.

To optimize a given objective function  $F$  (also known as performance or fitness) defined over a search space  $E$ , a population of individuals (points of  $E$ ) is subjected to a series of generations (the initial population is randomly chosen in  $E$ ). A generation begins with the selection of the most adapted individuals (relative to  $F$ ) for reproduction. These individuals generate offspring using stochastic operators called crossover for binary operators, and mutations for unary operators (applying to a single individual). Finally, some of the descendants replace some of the parents to complete the process of generation. The selection and replacement paradigms which represent the Darwinian rule of survival of the fittest may be stochastic or determinist. As in natural evolution, it is hoped to observe the gradual emergence of more and more adapted individuals: the best

individuals in the final population should be close to solutions of the optimization problem.

The representation space that we actually study (where the evolution operators operate, also called the genotypes space) is often different from space in which the fitness is calculated (phenotypes space). To move from phenotypes space to genotypes space, an additional modelling or coding step is necessary.



**Figure A.1: Operation of Evolutionary Algorithms**

The stopping criterion can take several forms: for example, the maximum number of evaluations or satisfaction in the objective value of the best individual. If the individuals of a population are too similar, the following next generations may become more and more homogeneous. In this case, the evolution of a population may be summarized in the evolution of a single dominant individual, thus less exploration the search space. To perform an efficient search, we have to maintain a balance between the exploitation of good solutions found so far and the exploration of unknown areas of the search space. Excessive exploitation can lead to stagnation in a local optimum (premature convergence) while as an excessive exploration could lead to an almost random search (no convergence).



## A.2 Representation of an individual

The representation or coding of an individual has to include fundamental characteristics of the problem. It must also be easily to be manipulated by recombination and mutation operators, allow easy transformation on the search space and generate feasible solutions. A good coding is as follow:

- Facilitate the development and application of variation operators (recombination, mutation) to adequately cover the space of individuals;
- Be simple in its construction and consistent with the addressed problem
- Provide simple and effective transition to the search space (and vice versa).

### A.2.1 Binary representation

By analogy with the natural genetics, evolutionary algorithms use bits by tradition to represent the chromosomes. Indeed, biological genes are encoded by nucleotide sequences built from four varieties: adenine (A), guanine (G), cytosine (C) and thymine (T). Biological genes allow the synthesis of amino acid sequences, i.e. proteins in charge of the phenotype of an individual. For an optimization problem on  $n$  integer variables  $X_i$ , we can represent each of these variables by a binary string of  $k_i$  bits and we obtain the chromosome of size  $\sum_{i=1..n} k_i$

The first results on convergence were established on such sequences of bits, and showed that the coding of chromosomes with genes whose alphabet has low cardinal was theoretically more efficient. Binary encoding also gives Evolutionary Algorithms good robustness because it is independent from the domain of the problem and standard operators can be used systematically. However, this type of coding has some drawbacks:

- Two elements close in search space does not necessarily decode two neighbouring individuals in terms of Hamming distance (the number of different bits). We can avoid this problem by using Gray coding [31] which maintains a Hamming distance of “1” between two consecutive individual voters.
- Additionally, for problems requiring high precision, the binary encoding can quickly become inadequate.

### **A.2.2 Real valued presentation**

The principle of this representation is to directly encode the variables of the problem in the individual without using the binary coding means. Thus, the individuals are no longer strings of bits but real vectors. One major advantage of this representation is to keep variables of the problem in the coding itself, thus allowing it to take better account of the structure of the problem. This direct representation using real parameters requires defining new specific operators.

### **A.3 Initialization of the population**

In general, the  $N$  individual population  $P(0) = \{X_1, \dots, X_N\}$  is initialized through uniform drawing from the search space  $E$  while ensuring that all individuals meet the constraints. Moreover, if we have priori information about a region where the optimal solution is likely located, it is obvious to manually add good solutions into the initial population, while ensuring a sufficient diversity of population. The initial population can also be the result of a previous evolution.

### **A.4 Artificial Darwinism and evolution engine**

The Darwinian part of Evolutionary Algorithm consists of two steps: the reproduction step where parents are selected to recombine and the replacement

step which replaces the worse individuals by better ones.

The selection is an essential operator whose principle is to allow the best individuals of a population to reproduce. The adjustment of this mechanism is critical in the performance of the Evolutionary Algorithm: an excess of selection leads to a loss of diversity and results in unreachable areas in the search space, and an insufficient selection can lead random walk, thus no convergence. We can find in literature a large number of selection strategies that are more or less adapted to the problem they address. We present here the most popular selection procedures [31].

#### **A.4.1 Proportional selection**

There are two popular proportional selection methods: Roulette wheel selection and stochastic universal sampling.

The Roulette wheel selection represents each individual of the population  $P(t) = \{X_1, \dots, X_N\}$  on a contiguous segments of a line such that the individual's segment is proportional to their fitness. A random number is generated and the individual whose segment spans the random number is selected. We repeat the process until the desired number of individuals is obtained. This method is similar to a roulette wheel with each slice size proportional to the fitness. The expected number of copies of an element  $X_i$  of the current population is given by:

$$n_i = \frac{N \cdot F(X_i)}{\sum_{j=1..N} F(X_j)}$$

This method of selection favours the best individuals, but the bad ones also have chance of being selected. However, the cost of execution is high and the minimum spread (minimum range of possible values for the number of offspring of an

individual) is not guaranteed. Moreover, the loss of diversity is possible because the copies produced only from the best individuals can represent the whole the next population.

Stochastic universal sampling bases on roulette wheel selection, except that a deterministic aspect is added. Here we place equally spaced pointers over the line; the number of pointers is the number of individuals to be selected. Let  $M$  be this number, then the distance between the pointers are  $1/M$  and the position of the first pointer is given by a randomly generated number in the range  $[0, 1/M]$ . The interest of this selection method is that it reduces the spread.

#### **A.4.2 Tournament selection**

The tournament selection also uses comparisons between individuals, and does not even require sorting the fitness of the population. The results depend on the size  $T$  of the tournament. To select an individual, we draw  $T$  individual uniformly in the population, and we select the best of them. Over a generation ago, the number of individuals to be selected is the number of tournaments. This method is characterized by a selection pressure that is in general stronger than the proportional method (for a less adapted individual to be selected, it is necessary that its tournament opponents are even worse). Moreover, this method is the cheapest in terms of execution cost and the selection pressure is easily configurable by the value of  $T$ . However, it does not guarantee minimum spread.

#### **A.4.3 Reinsertion**

Various reinsertion strategies can be used, the principle is to replace the old population by new, after applying recombination and mutation. In standard genetic algorithms, the children simply replace the parent

population. Nevertheless, there exist alternative strategies:

- Replacing a percentage of the parents by the best children;
- The systematic replacement of the worst individual;
- The random replacement (while maintaining a coherent research strategy).

The goal is to increase the speed of convergence of simple Genetic Algorithm, but reinsertion can still produce a premature convergence towards local optima.

Another technique is deterministic replacement, widely used in evolution strategies (ES). The purely deterministic characteristic plays a key role in evolution as it guides the search towards areas with better individuals. Two distinct versions were introduced:

- The scheme  $(\mu, \lambda)$ -ES:  $\mu$  denotes the number of parents in the population that generates  $\lambda > \mu$  new individuals (by recombination and mutation). The reinsertion takes place by selecting the  $\mu$  best individuals among  $\lambda$  children and replacing  $\mu$  parents with  $\mu$  chosen children.
- The scheme  $(\mu + \lambda)$ -ES: This scheme looks at the best  $\mu$  individuals from the union of  $\mu$  parents and  $\lambda$  children.

The  $(\mu + \lambda)$  scheme is called elitism and it guarantees a monotonic improvement of the fitness of best individual through generations, but it fits poorly with a possible change of environment. On the other hand, with the  $(\mu, \lambda)$  scheme we may lose the best individual, but the algorithm is more flexible in dynamic optimization where the environment changes. It should be noted that elitism is often used. This mechanism keeps the best individuals (often only the best one) of the population in generation  $t$  for the next population in generation  $t + 1$  if there are no better children.

## A.5 Variation operators

Variation operators aim to generate new individuals from those previously selected. We distinguish between recombination and mutation.

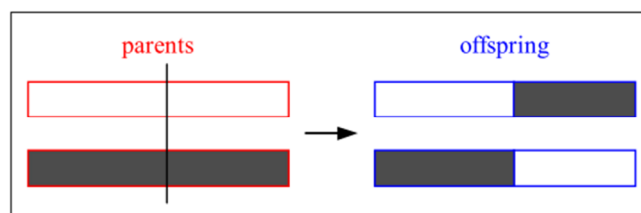
### A.5.1 Recombination

The principle is analogous to biological reproduction: The children inherit the qualities from their parents. Recombination is usually called crossover for binary representation. The standard form of the recombination operator is  $c: E \times E \rightarrow E \times E$  ( $E$  is the search space), that recombines two parents  $P_1, P_2$  with a certain probability  $p_c$  ( $0 \leq p_c \leq 1$ ). Other forms of recombination are available such as when one child is produced by more than two parents. Among different types of recombination, there are:

- **Binary crossover:**

This is an operator on  $E \times E \rightarrow E \times E$ , with  $E = \{0, 1\}$ . It corresponds to an exchange of genes (bits) between the two parents. There are three most popular variants: single point crossover, multi-point crossover and uniform crossover.

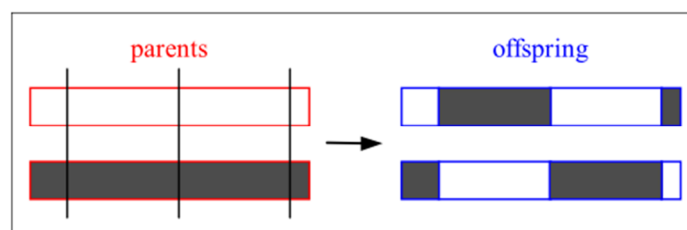
Single point crossover is the simplest and the most classical recombination technique of Genetic Algorithms. It randomly selects a breakpoint in each of the two parents  $P_1$  and  $P_2$ , and builds two offspring by exchanging their genes on both sides of this point.



**Figure A.2: Single-point crossover**

Choosing a single cross point biases the effect of crossover: if the chosen point is close to one end of the chromosome, the children will be almost identical to the parents. On the other hand, if the chosen point in the middle, they will be very different from their parents.

Multi-point crossover avoids the problem above by considering the chromosome as circular rather than linear, and by choosing  $k$  breakpoints. Figure A.3 shows an example of multi-point crossover with  $k = 3$ .



**Figure A.3: Multi-point crossover**

Uniform crossover uses a randomly generated binary mask with the same size as the chromosomes to indicate which parent will provide the gene at each locus. Other crossover operators exist, they can either make modifications to those presented above, or be specific to a class of problems, but nevertheless they obey to a common principle: the exchange of information between individuals.

#### - **Real valued recombination**

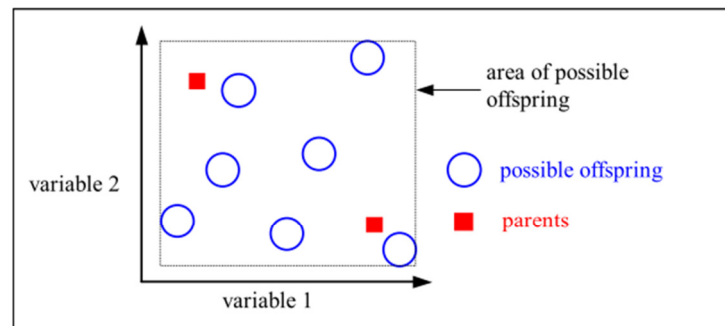
The standard real valued recombination is very close to the crossover described for binary coding in the previous section. It differs from binary crossover only by the nature of the genes altered. Bits are no longer exchanged, but the actual values. The real valued representation allows us to develop a new mating type which is called arithmetic recombination, mainly based on linear combination of two individuals that are now real vectors. It consists of choosing two genes  $P_1(i)$

and  $P_2(i)$  in each parent at the same position  $i$ , and define the corresponding genes  $E_1(i)$  and  $E_2(i)$  of the children using linear combination:

$$E_1(i) = \alpha P_1(i) + (1 - \alpha) P_2(i) \quad (3.2)$$

$$E_2(i) = (1 - \alpha) P_1(i) + \alpha P_2(i) \quad (3.3)$$

Where  $\alpha$  is a uniform random variable belonging to the interval  $[0, 1]$ . It is also possible to generate individuals outside the segment joining the two parents by choosing the parameter  $\alpha$  in  $[-d, 1 + d]$ , with caution to stay within the bounds of the problem domain.



**Figure A.4: Possible area of the offspring after intermediate recombination**

## A.5.2 Mutation

The general idea of mutation is to introduce variability in the population. This operator modifies one or more genes of the selected individual with a certain probability  $p_m$  ( $0 \leq p_m \leq 1$ ). Mutation ensures ergodicity property (the capacity to cover the whole search space) for the Evolutionary Algorithms and the reintroduction of lost diversity.

### - Binary Mutation

Binary mutation is a random modification of the gene values that happens with a



fixed probability  $p_m$  by individual. The most frequently used binary mutations are:

- Single-bit mutation chooses randomly a position in the chromosome and changes the value of the corresponding bit.

- $c/l$  – mutation changes the bit value of each position independently with probability  $c/l$ , where  $l$  is the length of the chromosome and  $c > 0$ .

- **Real valued mutation**

The principle of real valued mutation is generally to add a random Gaussian perturbation to the various components of the individual  $X$ :

$$X_i := X_i + s.N(0, 1) \quad (3.4)$$

where  $s$  is the standard deviation of the mutation and  $N(0, 1)$  is a random normal standard variable.

The difficulty of this approach is the adjustment of the standard deviation  $s$ . Indeed, if the standard deviation is too small, the movement in the search space is insufficient, thus the algorithm can be stuck near to a local optimum and cannot visit new areas. On the other hand, if the standard deviation is high, the algorithm can reach to the region containing the optimum, but the convergence quality will not be good. Thus at the beginning of the evolution, the standard deviation  $s$  should be high enough to quickly explore the search space, and ultimately become a lower for better exploration of solutions.

## **A.6 Properties of Evolutionary Algorithms:**

At each step of the Evolutionary Algorithm, we must make a trade-off between exploring the search space to avoid getting stuck in local optima and exploiting the best individuals obtained in order to achieve better solutions. Exploration in

Evolutionary Algorithms is done with mutation and exploitation is done with the selection and recombination. We can therefore adjust exploration and exploitation through various algorithm parameters.

The term genetic diversity indicates the variety of genotypes in the population. It is a key feature of Evolutionary Algorithms. Genetic diversity becomes zero when all individuals are identical, and when diversity is very low, there is very little chance that it increases again. If the loss of diversity occurs too early, the convergence takes place to a local optimum.

The advantage of Evolutionary Algorithms is to that they can be applicable to wide classes of problems: multi nodal, convex or non-convex problems.... Moreover, they are able to work on any space research: continuous, discrete, or mixed-space... However, the success and search execution time depend heavily on the representation (genotype space) and variation operators (recombination, mutation) selected. Also, the choice of the fitness function is a crucial point since the algorithm requires a large number of evaluations of the objective function.

The computation time required to obtain significant results on real problems leads to the use of other techniques such as parallelization: Distribution of calculation on a set of synchronous or asynchronous processors, using island and distributed population models.

## **A.7 Towards Co-evolutionary Algorithms**

As mentioned before, in ecology a living individual is not only influenced by its own environment but also by other individuals in the environment as well as other processes as changes in climate or geographical structure. The notion of mutual dependence or inter-specific relationship between different species is named co-

evolution.

### **A.7.1 Definition of Co-evolution**

In classical evolutionary algorithm, each individual evolves independently, which is not the case in real ecosystems. In an ecosystem, the fitness of an individual is defined according to its interactions with other individuals. Co-evolution arises because of interactions between different species. In a co-evolutionary system, the evolution of a species must be considered simultaneously, because the evolutionary adaptation of a species can force the adaptation of others. In other word, the actions of each species affect all other species in the same physical environment.

Co-evolution has many advantages that can renew the evolutionary performance of system. It is based on the principle that when a population becomes superior to the other, the later has to amplify the selection pressure and evolve more quickly to survive. The class of Co-evolutionary Algorithms is an extension of classical Evolutionary Algorithms to solve problems that are potentially complex, with too large search space or problems without an objective function such as strategy games. Co-evolutionary Algorithms are based on the principle of subjective function, where the fitness of an individual becomes estimation for other individuals interacting with it [32].

In co-evolutionary algorithms, individuals are evaluated based on their interactions with others. The nature of these interactions depends on the problem to be solved. In many problems, the individuals or populations compete with one another. This is called competitive co-evolution, which is widely applied in game playing strategies. On the other hand, an individual is rewarded when contribute

well in cooperation with other individuals in cooperative co-evolution.

### **A.7.2 Interaction and selection of collaborators**

The mechanism in which a participant determines its collaborators or competitors is among the most important factors for a successful application of algorithms co-evolutionary. The most obvious (and computationally expensive) method to evaluate an individual is to let it interact with all potential collaborators or competitors, this sometimes called pair-wise or complete interaction. Alternatively, collaborators / competitor can be selected by a variety of ways: uniformly random methods or methods based on fitness.

## **A.8 Properties of Co-evolutionary Algorithms**

The scope of Co-evolutionary Algorithms is extremely broad. It can approach problems with large search space, or having no intrinsic objective function or with complex structure. To obtain better results, it is therefore reasonable to divide a large search into sub-spaces. It is also more efficient to divide a complex structure into simple structures that co-evolve.

Co-evolutionary Algorithms are more difficult to control compared to classical Evolutionary Algorithms. The reasons often stem from the complicated internal dynamics of co-evolutionary systems. Sometimes, this can lead to a system behaving in an incomprehensible manner, and whose progress is difficult to diagnose.