Income Inequality, Education and Intergenerational Mobility: a General Equilibrium Approach

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Summary

This paper assumes heterogeneous agents with different skill levels and establishes a general equilibrium framework to analyze the relationship between population dynamics, income inequality and intergenerational mobility. Childhood education level plays a key role in determining one's skill level in adulthood. With differentiated educational efficiency, skilled and unskilled families behave differently in fertility choice and decision of child's educational investment level. Analytical results confirm that the population will evolve to be more skilled with higher average income and lower fertility. At the same time, wage premium decreases and social mobility improves.

Skilled-biased technological change (SBTC) has been heavily used to explain the wage premium recently. The model extension incorporating SBTC does bring new insight to our analysis. Numerical simulation shows that STBC completely changes the wage premium pattern and significantly influences the intergenerational mobility.

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1 Introduction

It has been discussed in many economic studies that education plays an important role in shaping the income distribution and the intergenerational mobility, since it relates to both intergenerational efficiency and intergenerational equity. There is a large literature on intergenerational transmission of education and earning (see, e.g., Becker and Tomes (1979), Galor and Tsiddon (1997), Piketty (2000), Mookherjee and Ray (2003), Davies, Zhang and Zeng (2005), Moav (2005),Docquier, Paddison and Pestieau (2007), and Dan and Leigh (2009)). However, it remains a challenging subject once considering such important factors as intergenerational transfer, education investment, intergenerational mobility, fertility and income inequality. Most of the existing discussions assume fixed fertility. By endogenizing fertility, agents from different incomes groups behave differently when deciding the number of children and level of education investment. In addition, existing models on intergenerational mobility and income inequality, however, have mostly developed a partial equilibrium framework where the potential income of an individual agent is exogenous due to fixed interest and fixed wage (e.g., Fan and Zhang, 2011). While this omission is largely innocuous for the purpose of tractability and simplicity, assuming endogenous income gives additional insight into how demographic profile affects individual income thus education investment**,** given a typical relationship between the relative supply of skilled-unskilled labor and wage level in general equilibrium.

In addition, during the past two decades, most of the economies, particularly OECD countries, have experienced a rapid technological progress along with a fundamental change in the pattern of wage premium (the ratio of skilled wage to unskilled wage or the ratio of college graduate's salary to high-school graduate's salary). The wage differential between skilled and unskilled labor has increased significantly despite the increase in the relative supply of skilled labor (college graduates). This somehow contradicts the conventional wisdom of supply-demand theory. Acemoglu (1998) suggests that technological advancement favors skilled individual as it normally better enhances the productivity of skilled labor. The skill-biased technological change (SBTC) has been widely accepted and used to explain the widening wage gap between skilled and unskilled individuals in developed countries from the 1960s. The skill-biased technological change raises the skill premium, rewards skill acquisition, encourages education investment, and thus influences intergenerational mobility at the aggregate level. In recent decades, the skill-biased technological change is the driving force behind the increasing wage premium affecting income distribution and education investment of next generation. However, there is a lack of research on the relationship between skill-biased technological change and intergenerational mobility.

This paper extends the model in Fan and Zhang (2011) and studies intergenerational mobility in a general equilibrium model that incorporates skill-biased technological change and differential fertility with heterogeneous agents differentiated by their skills levels. Different from our approach, the existing literature on intergenerational mobility abstracts from differential fertility and endogenous wage, while the existing models on skilled-biased technological change abstract from the analysis of intergenerational mobility and differential fertility.

Our analysis is based on a unified framework of population dynamics and income dynamics with heterogeneous agents, namely skilled and unskilled, in an overlapping-generation setting. The only channel for intergenerational transfer is through education. During each period, each agent devotes his time to working, rearing, and educating children. Agents receive wage from work and allocate income between own consumption, and physical educational investment of children. The education of each child requires the parental time input and physical educational investment while skilled parents enjoy some advantage in education, such as higher educational efficiency, compared to unskilled parents. The education outcome of a child determines the skill level during his adulthood in a probabilistic way with both upward and downward possibility. In the general equilibrium framework, the education decision and the wage level influence each other in some important ways. On one hand, education outcome determines demographic profile, i.e. the distribution of skilled and unskilled labor, in the subsequent period; the relative supply of skilled/unskilled labor affects the wage of different skill groups. On the other hand, the variation of relative income influences the education decision of each agent and the education outcome of next generation.

To facilitate the analysis, we first discuss the baseline model without skill-biased technological change and then extend our analysis to the case with skill-biased technological change using simulation. In the baseline model, we will demonstrate how two skill groups behave differently when facing a quality and quantity trade-off of children and how the social average fertility rate and education level will be affected. Based on the analytical result from baseline model, we will conduct numerical simulation to reproduce the empirical dynamics of wage inequality and further explore the impact of skill-biased technological change on intergenerational mobility.

The rest of this paper is organized as follows. The next section reviews the related literature. Section three sets up the general equilibrium structure of the model, including final goods production, education production and intergenerational population dynamics. Section four discusses the general equilibrium of the baseline model. In section five, we further extend the analysis to investigate the impact of skill-biased technological change. Section six concludes this thesis and the appendix provides some necessary mathematical proofs.

2 Literature Review

We now provide more details about related literature concerning intergenerational mobility, income inequality, wage differential, endogenous fertility, and skill-biased technological change.

2.1 Intergenerational Mobility, Income Inequality and Fertility

The research on the intergenerational correlation of economic and social status is one of the most important subjects in social sciences. Intergenerational mobility is quite often associated with income inequality and education in economic studies. There is a large literature on intergenerational transmission of education and earning. (see, e.g., Becker and Tomes(1979), Galor and Tsiddon (1997), Mookherjee and Ray (2003), Davies, Zhang and Zeng (2005) and Docquier, Paddison and Pestieau (2007)). However, most of existing discussions assume exogenous fertility. Becker and Lewis (1973) set up an analytical framework to endogenize fertility choice and study the trade-off between the quantity and quality of children. de la Croix and Doepke (2004) further study differential fertility under private and public schooling. Fan and Zhang (2011) endogenize the fertility choice and consider differential fertility in the discussion of intergenerational mobility in an overlapping-generations framework with skilled and unskilled individuals. In their extended model, heterogeneous agents with different skill levels allocate different amount of time to working and educating children and allocate different amount of income to own consumption and education investment of children. Compared with the case of equal fertility, differential fertility makes the allocation of educational resources and education outcome more uneven between the children from skilled and unskilled families and shed some new light on intergenerational mobilty.

There is also a large literature documenting different aspects of the demographic transition and the relationship between income inequality and fertility, such as the Kuznets curve. In a seminal document of empirical regularities of development, Kuznets (1967) observes: “over long periods, fertility has been greater for the poorer and lower social status groups than for the richer and higher social status groups. The negative correlation between birth rates and rates of natural increases, on one hand, and economic status and per capita economic performance, on the other hand, raises problem with respect to the economic advance of the poor and generally less favored groups within any society.” Another significant piece of empirical evidence is that the transition to lower fertility rates is associated with an increase in the investment of child’s education. Caldwell (1980) argues that the beginning of fertility decline is triggered by mass education in the family economy. Birdsall (1983) discusses the inverse correlation between wage and fertility and also reveals the role of education in bringing down fertility rate.

2.2 Wage Differential and Skill-biased Technological Change

In the past decades, most of the OECD countries have experienced a rapid technological advancement along with fundamental changes in the pattern of wage differential. The changes in the labor market can be summarized by the following stylized facts (see, e.g. Katz and Murphy (1992), Juhn, Murphy and Pierce (1993) , Berman, Bound and Machin (1998), Ábrahám (2008)):

1. Wage premium decreases during the 1940s-1960s and then experiences a consistent increase with certain fluctuations from the 1960s onwards. The wage gap has grown significantly since 1980s.

2. The relative supply of skilled labor (college graduates) and college enrollment increases considerably.

3. The real wage of unskilled labor (high-school graduates) decreases despite the increasing relative supply of skilled labor.

A great deal of research has been done on the relationship between technology and inequality. Galor and Tsiddon (1997) and Greenwood and Yorukoglu (1997) argue that the increase in the wage gap between skilled and unskilled labor reveals the increasing demand for skilled labor and higher skilled wage caused by technological progress. Acemoglu (1998) reviews the fact that an exogenous increase in the supply of skilled labor leads to the decline in wage gap between skilled and unskilled labor in the 1970s and suggests technological advancement enhances the relative productivity of skilled worker and skill-biased technological change is the driving force behind the dynamics of wage gap.

3 The Basic Structure of the Model

3.1 Production of Final Goods with Skilled and Unskilled Labor

3.1.1 Production Function

In this economy, the production occurs according to a neoclassical production function with a constant-return-to-scale:

$$Y = AK_t^{\alpha} H_{t,D}^{1-\alpha} \ , \tag{1}$$

where K_t and H_t are the are the quantities of physical and composite labor input used in the production at time t. The technological level A is time-invariant and parameter $\alpha \in (0,1)$ controls relative intensity of capital and composite labor in the production.

The internal structure of the composite labor input is expressed as follows:

$$H_{t,D} = [\beta(D_t L_{s,t})^{\rho} + (1-\beta) L_{u,t}^{\rho}]^{\frac{1}{\rho}} \ , \tag{2}$$

where $L_{s,t}$ and $L_{u,t}$ are the input quantities of skilled and unskilled labor at time t. D_t is the measure of skill-biased technological process. The parameter $\beta \in (0,1)$ controls the intensity with which skilled versus unskilled labor is used in the production. $\rho \in (-\infty, -1]$ determines the degree of substitution between skilled and unskilled labor. Hence, the explicit production function requires three inputs of production, namely capital (K), skilled labor (L_s), and unskilled labor (L_u):

$$Y_t = AK_t^{\alpha} [(D_t L_{s,t})^{\rho} + (1-\beta) L_{u,t}^{\rho}]^{\frac{1-\alpha}{\rho}} \ . \tag{3}$$

Total labor force at time t, denoted by L_t, is the sum of skilled and unskilled labor force, i.e. $L_t = L_{s,t} + L_{u,t}$. The proportion of skilled labor is given by $h_t = \frac{L_{s,t}}{L_t}$, and the proportion of unskilled labor is $1-h_t$.

Production function in per capita term is expressed as follows:

$$y_t = Ak_t^{\alpha}(\beta D_t^{\rho} h_t^{\rho} + (1-\beta)(1-h_t)^{\rho})^{\frac{1-\alpha}{\rho}} \tag{4}$$

where $k_t = \frac{K_t}{L_t}$. The elasticity of technical substitution between skilled and unskilled labor, denoted by σ, is defined as follows:

$$\sigma = \frac{\frac{\partial(L_s / L_u)}{L_s / L_u}}{\frac{\partial(MPL_u / MPL_s)}{MPL_u / MPL_s}} = \frac{1}{1-\rho}. \tag{5}$$

Since $\rho \in (-\infty, 1]$, we have $\sigma \in (0, \infty)$ that affects worker productivity and skill intensity in the production. If $\rho = 0$, i.e. $\sigma \to \infty$, the composite labor input function takes the Cobb-Douglas form as assumed in previous models (e.g. Dahan and Tsiddon (1998)) on wage differential e for the sake of simplicity. However, the estimates of σ from empirical studies range from 1.4 to 5 (see, e.g., Katz and Murphy (1992) and Ciccone and Peri (2005)). Hence, the assumption mentioned above is not realistic and we will preserve the role of ρ in the discussion of wage premium.

3.1.2 Factor Price

Suppose that the production occurs in a small open economy which takes world interest rate, r , as given. The small open economy permits unrestricted borrowing and lending from international capital markets. Production operates in a perfectly competitive market. The producer maximizes his profit and yields the following factor prices.

The return of capital is derived as:

$$r = \alpha A k_t^{\alpha-1}[\beta(D_t h_t)^\rho + (1-\beta)(1-h_t)^\rho]^{\frac{1-\alpha}{\rho}}. \tag{6}$$

The skilled labor wage rate is found to be:

$$w_{s,t} = (1-\alpha)Ak_t^\alpha \beta D_t^\rho [\beta(D_t h_t)^\rho + (1-\beta)(1-h_t)^\rho]^{\frac{1-\alpha}{\rho}-1} h_t^{\rho-1}. \tag{7}$$

The unskilled labor wage rate is:

$$w_{u,t} = (1-\alpha)Ak_t^\alpha (1-\beta)[\beta(D_t h_t)^\rho + (1-\beta)(1-h_t)^\rho]^{\frac{1-\alpha}{\rho}-1} (1-h_t)^{\rho-1}. \tag{8}$$

Given that the production happens in a small open economy which allows free capital flow. The capital level in the economy is determined by the exogenous interest rate r :

$$k_t = (\frac{r}{\alpha A})^{\frac{-1}{1-\alpha}} H_{t,D}.$$

It is now clear that w_s and w_u are determined by the exogenous international interest rate r and by the ratio of skilled workers in the working population, h_t, the economy starts with in period t:

$$w_{s,t} = (1-\alpha)\beta D_t^{\rho} A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} r^{\frac{-\alpha}{1-\alpha}} [\beta(D_t h_t)^{\rho} + (1-\beta)(1-h_t)^{\rho}]^{\frac{1}{\rho}-1} h_t^{\rho-1}, \tag{9}$$

$$w_{u,t} = (1-\alpha)(1-\beta) A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} r^{\frac{-\alpha}{1-\alpha}} [\beta(D_t h_t)^{\rho} + (1-\beta)(1-h_t)^{\rho}]^{\frac{1}{\rho}-1} (1-h_t)^{\rho-1}. \tag{10}$$

For simplicity, let us denote

$$J_{t,D} = \beta(D_t h_t)^{\rho} + (1-\beta)(1-h_t)^{\rho}, \tag{11}$$

$$M = (1-\alpha) A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} r^{\frac{-\alpha}{1-\alpha}}. \tag{12}$$

Hence we can rewrite the wages, w_s, w_u, as

$$w_{s,t} = \beta D_t^{\rho} M \; J_{t,D}^{\frac{1}{\rho}-1} h_t^{\rho-1}, \tag{13}$$

$$w_{u,t} = (1-\beta) M \; J_{t,D}^{\frac{1}{\rho}-1} (1-h_t)^{\rho-1}. \tag{14}$$

The wage premium is defined as the ratio between skilled labor wage and unskilled labor wage, denoted by z_t:

$$\begin{aligned} z_t &= \frac{w_{s,t}}{w_{u,t}} \\ &= \frac{\beta}{1-\beta} D_t^{\rho} \left(\frac{h_t}{1-h_t} \right)^{\rho-1}. \end{aligned} \tag{15}$$

It is important to note that the wage premium in this model is not only affected by factor intensity of labor and the level of skill-biased technological change, but also by the percentage of population accounted for by skilled labor for $\rho \neq 1$. Empirical studies (e.g. Ciccone and Peri (2005)) have indeed found that the rate of technical substitution between skilled and unskilled labor is not infinity, i.e. $\rho \neq 1$. Hence, the wage premium will be lower when the skilled workers become a larger group in the population relative to unskilled workers. However, most of the existing literature (e.g. Dahan and Tsiddon (1998)) on the relationship between wage inequality and intergenerational transition ignores such a feedback effect.

3.2 Education Production

The education outcome takes a Cobb-Douglas form and requires two inputs, namely a parental time input and a physical educational investment, differentiated between children from skilled versus unskilled parents.

For a skilled worker, the education outcome for each child is given by

$$e_s = \gamma_s v_s^{\delta} d_s^{1-\delta} \; ; \tag{16}$$

for an unskilled worker, the education outcome for each child is

$$e_u = \gamma_u \tilde{v}_u^{\delta} d_u^{1-\delta}, \tag{17}$$

where v_s and $\tilde{v}_u$ are the parental time spent on educating each child, d_s and $\tilde{d}_u$ are the physical capital input to educate each child and $\gamma_s \geq \gamma_u$. $\delta \in (0,1)$ controls the relative factor intensity between educational time and physical educational capital. γ_s and

γ_u represent the productivity factors of education production, and $\gamma_s \geq \gamma_u$ reflects the empirical experience that, given the same amount of educational time and educational spending, the skilled parents have better educational outcome for their children, as documented in the empirical literature, such as Becker (1981) and Ermisch and Francesconi (2002). This is due to the fact that skilled parents have the ability and influence family culture to enhance the learning of their children. By contrast, unskilled parents lack the knowhow to teach children to become skilled.

Assumption 1. *The parental time on child's education has a lower bound $\tilde{v} > 0$. Also, for skilled parents, the marginal contribution of time input to child's education is always positive and diminishing; while for unskilled parents, beyond the necessary level $\tilde{v}$, any additional time input to child's education hardly generates any education output at the margin.*

The assumption reflects the very intuition that the, for unskilled parent, the lack of mental labor, skills and educational background makes the additional parental time on education beyond necessity hardly conducive to the education of children. Bianchi et al. (2004) survey the U.S. household data from 1965 to 2000 and find that college-educated parents spent significantly more time on home education compared to less-than-college-educated parents.

3.3 Individual Preference

This paper assumes an economy with an infinite number of overlapping generations. Each working generation has a mass N_t and every individual lives for two periods, namely childhood and adulthood. In childhood, an individual receives education supported by his parent. In adulthood, childhood education determines the skill level of an individual; he receives salary from his work and decides how many children to bear and how much education investment for each child. There are two types of individuals in the economy, namely skilled and unskilled. Skilled individuals are better educated and receive higher wage because of higher productivity than unskilled individuals.

The preferences of both skilled and unskilled parents are identical as given below:

$$U(n,e,c) = \ln n + \tau \ln e + \eta \ln c\,, \tag{18}$$

where τ and η are positive coefficients indicating the relative tastes for a child's education outcome e and for own consumption c to the taste for the number of children n.

Each individual is endowed with one unit of time and devotes it to working, rearing children, and educating children. We assume the time spending on rearing a child is the same for both skilled and unskilled parents. Parent spends ξ portion of his time rearing a child and spends v portion of his time educating a child. The rest of the time is devoted to working. The labor income, $(1-\xi n - vn)w$, is spent on own consumption c and physical educational investment d per child. Given Assumption 1, an unskilled parent spends only $\tilde{v}$ portion of his time on each child's education.

The budget constraints could be expressed in the following way: A skilled worker has

$$c_{s,t} = (1-\xi n_{s,t} - v_{s,t} n_{s,t}) w_{s,t} - d_{s,t} n_{s,t}\,, \tag{19}$$

and an unskilled worker has

$$c_{u,t} = (1 - \xi n_{u,t} - \tilde{v} n_{u,t}) w_{u,t} - d_{u,t} n_{u,t} \ . \tag{20}$$

Here, the product of two choice variables dn introduces non-convexity in the budget constraints. To ensure a concave maximizing problem, we need:

Assumption 2. $0 < \tau < 1$.

Here, we assume that the preference for the education of children is weaker than the preference of the number of children. Similar assumptions are adopted in Ehrlich and Lui (1991) and Zhang, Zhang and Lee (2003). This assumption helps to ensure the existence of interior solution for the individual optimization problem. It is verifiable that the objective of the utility maximization is concave in all the choice variables.

The lower bound $\tilde{v}$ of the parental educational time is set exogenously and representing the minimum parental time supporting child's education, such as home tutoring. Specifically, we assume that

Assumption 3. $0 < \tilde{v} < \dfrac{\tau\xi\delta}{1-\tau\delta}$.

This assumption helps to ensure that a skilled parent is willing to spend more time educating their children comparing to his unskilled counterpart.

A skilled worker maximizes his utility in (18) subject to (16) and (19). The constrained optimization problem could be simplified to an unconstrained one as follows:

$$\max\{\ln(n_{s,t})+\tau\delta\ln(v_{s,t})+\tau(1-\delta)\ln(d_{s,t})+\eta\ln[(1-\xi n_{s,t}-v_{s,t}n_{s,t})w_{s,t}-d_{s,t}n_{s,t}]$$
$$+\tau\ln\gamma_s\}. \tag{21}$$

Take $n_{s,t}, v_{s,t}, d_{s,t}$ as choice variables and $\gamma_s, w_{s,t}$ as given. First-order-conditions are as follows:

$$n_{s,t}:\frac{1}{n_{s,t}}=\frac{\eta[(\xi+v_{s,t})w_{s,t}+d_{s,t}]}{(1-\xi n_{s,t}-v_{s,t}n_{s,t})w_{s,t}-d_{s,t}n_{s,t}}, \tag{22}$$

$$v_{s,t}:\frac{\tau\delta}{v_{s,t}}=\frac{\eta n_{s,t}w_{s,t}}{(1-\xi n_{s,t}-v_{s,t}n_{s,t})w_{s,t}-d_{s,t}n_{s,t}}, \tag{23}$$

$$d_{s,t}:\frac{\tau(1-\delta)}{d_{s,t}}=\frac{\eta n_{s,t}}{(1-\xi n_{s,t}-v_{s,t}n_{s,t})w_{s,t}-d_{s,t}n_{s,t}}. \tag{24}$$

The first order conditions give the following solution:

$$n_s=\frac{1-\tau}{\xi(1+\eta)},\ \ v_s=\frac{\tau\delta\xi}{1-\tau},\qquad d_{s,t}=\frac{\tau\xi(1-\delta)}{1-\tau}w_{s,t}. \tag{25}$$

Since both n_s and v_s are time-invariant, we drop the time-index hereafter.

An unskilled worker maximizes his utility in (18) subject to(17) and (20). The optimization problem could be simplified to be an unconstrained one as follows:

$$\max\{\ln(n_{u,t})+\tau\delta\ln(\tilde{v})+\tau(1-\delta)\ln(d_{u,t})+\eta\ln[(1-\xi n_{u,t}-\tilde{v}n_{u,t})w_{u,t}-d_{u,t}n_{u,t}]$$
$$+\tau\ln\gamma_u\}. \tag{26}$$

Take $n_{u,t}, d_{u,t}$ as choice variables and $\gamma_u, w_{u,t}$ as given. First-order-conditions are as follows:

$$n_{u,t} : \frac{1}{n_{u,t}} = \frac{\eta[(\xi+\tilde{v})w_{u,t} + d_{u,t}]}{[1-(\xi+\tilde{v})n_{u,t}]w_{u,t} - d_{u,t}n_{u,t}}, \tag{27}$$

$$d_u : \frac{\tau(1-\delta)}{d_{u,t}} = \frac{\eta n_{u,t}}{[1-(\xi+\tilde{v})n_{u,t}]w_{u,t} - d_{u,t}n_{u,t}}. \tag{28}$$

These first order conditions give the following solution:

$$n_u = \frac{1-\tau(1-\delta)}{(\xi+\tilde{v})(1+\eta)}, \; v_u = \tilde{v}, \quad d_{u,t} = \frac{\tau(1-\delta)(\xi+\tilde{v})}{1-\tau(1-\delta)} w_{u,t}. \tag{29}$$

Since both n_u and v_u are time-invariant, we drop the time-index hereafter as well.

3.4 Population Dynamics

The following notation is used to describe the population dynamics:

p_t : the probability that a child becomes skilled at time $t+1$, given his parent is skilled at time t;

q_t : the probability that a child becomes skilled at time $t+1$, given his parent is unskilled at time t;

n_s : the fertility rate of a skilled parent at time t;

n_u : the fertility rate of an unskilled parent at time t;

As shown in (25) and (29), the fertility rate n_s and n_u is time-invariant. Hence, we drop the time index for n_s and n_u. The probability for a child to become a skilled worker is directly linked to education outcome, e.

The functional form of p, q are assumed as follows:

$$p_t = 1 - \exp(-e_{s,t}), \tag{30}$$

$$q_t = 1 - \exp(-e_{u,t}). \tag{31}$$

Both $p_t \in (0,1)$ and $q_t \in (0,1)$ are strictly increasing and concave functions of education outcome $e_{s,t}$ and $e_{u,t}$ respectively. This reflects the fact that the better education gives a child higher chance to become a skilled worker. Such probabilities lead to intergenerational mobility in this model.

In period t, a population of size N_t with a skilled population ratio h_t will have a demographic profile in period $t+1$ as follows:

Number of skilled worker from skilled family: $p_t n_s h_t N_t$

Number of unskilled worker from skilled family: $(1 - p_t) n_s h_t N_t$

Number of skilled worker from unskilled family: $q_t n_u h_t N_t$

Number of unskilled worker from unskilled family: $(1 - q_t) n_u h_t N_t$

Skilled population ratio in period $t+1$:

$$h_{t+1} = \frac{p_t n_s h_t + q_t n_u (1-h_t)}{n_s h_t + n_u (1-h_t)}, \tag{32}$$

where

$$\begin{aligned} p_t = P(h_t) &= 1-\exp(-e_{s,t}) \\ &= 1-\exp\{-\gamma_s (\frac{\tau\delta\xi}{1-\tau})^{\delta} [\frac{\tau\xi(1-\delta)}{1-\tau} w_{s,t}]^{1-\delta}\} \\ &= 1-\exp\{-\gamma_s (\frac{\tau\delta\xi}{1-\tau})^{\delta} [\frac{\tau\xi(1-\delta)}{1-\tau}]^{1-\delta} [(1-\alpha)\beta D_t^{\rho} A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} r^{\frac{-\alpha}{1-\alpha}} [\beta(D_t h_t)^{\rho} \\ &\quad + (1-\beta)(1-h_t)^{\rho}]^{\frac{1}{\rho}-1} h_t^{\rho-1}]^{1-\delta}\}, \end{aligned} \tag{33}$$

$$\begin{aligned} q_t = Q(h_t) &= 1-\exp(-e_{\mathrm{u,t}}) \\ &= 1-\exp\{-\gamma_u \tilde{v}^{\delta} [\frac{\tau(1-\delta)(\xi+\tilde{v})}{1-\tau(1-\delta)} w_{u,t}]^{1-\delta}\} \\ &= 1-\exp\{-\gamma_u \tilde{v}^{\delta} [\frac{\tau(1-\delta)(\xi+\tilde{v})}{1-\tau(1-\delta)}]^{1-\delta} [(1-\alpha)(1-\beta) A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} r^{\frac{-\alpha}{1-\alpha}} \\ &\quad [\beta(D_t h_t)^{\rho} + (1-\beta)(1-h_t)^{\rho}]^{\frac{1}{\rho}-1} (1-h_t)^{\rho-1}]^{1-\delta}\}. \end{aligned} \tag{34}$$

Define the measure of intergenerational mobility as the relative odds of being skilled for a child from skilled and unskilled family, as used in Iyigun (1999).

$$R_t = \frac{q_t}{p_t} \tag{35}$$

Assumption 4.

$$\beta > \frac{1}{1+e^{(\rho-1)\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}}}}.$$

As we will see later, this assumption helps to ensure the existence of wage premium between skilled and unskilled work, i.e. $w_s > w_u$, for all the feasible value of h_t

A comparison of the solutions to the problems of skilled and unskilled workers yields:

Proposition 1. *A skilled worker has fewer children but invests a greater proportion of his income in education than an unskilled worker.*

$$n_s < n_u \qquad \frac{d_s}{w_s} > \frac{d_u}{w_u}$$

Proof. From (25) and (29), it is obvious that $n_s - n_u < 0$, then $n_s < n_u$.

$$v_s = \frac{\tau\delta\xi}{1-\tau} > \frac{\tau\delta\xi}{1-\tau\delta} > \tilde{v}$$

$$\begin{aligned}\frac{d_s}{w_s} - \frac{d_u}{w_u} &= \frac{\tau\xi(1-\delta)}{1-\tau} - \frac{\tau(1-\delta)(\xi+\tilde{v})}{1-\tau(1-\delta)} \\ &= \frac{\tau(1-\delta)}{(1-\tau)(1-\tau(1-\delta))}[\tau\delta\xi - (1-\tau)\tilde{v}] > 0\end{aligned}$$

The last inequality holds as $\frac{\tau\delta\xi}{1-\tau} > \tilde{v}$ *Q.E.D*

This result has been obtained in Fan and Zhang (2011) with exogenous wage rates. The same result remains in our model with a different production function that links the wage rates as a function and skill levels of workers and the relative proportion of the division into skilled and unskilled groups.

3.5 Baseline Model

In this session, we consider the case where there is no skill-biased technological change. i.e. $D_t = 1$.

For the purpose of simplification, we denote $J_t = \beta h_t^{\rho} + (1-\beta)(1-h_t)^{\rho}$.

Lemma 1. *The feasible range for* h_t *is* $[0, 1-\exp(-\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}})]$.

Proof. Refer to Appendix A.a.

Lemma 2. *Skilled workers always receive higher wage than unskilled workers:*

$$w_s > w_u, \forall h_t \in [0, 1-\exp(-\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}})].$$

Proof. Refer to Appendix A.b.

It is now ready to observe the following result:

Proposition 2: *A child from a skilled family has better education outcome and has better chance to become a skilled worker.*

$$e_s > e_u, \qquad p > q.$$

Proof. From lemma 2 and proposition 1, we know $\frac{d_s}{w_s} > \frac{d_u}{w_u}$ and $w_s > w_u$. It follows immediately that $d_s > d_u$. Together with $v_s > v_u = \tilde{v}$ and $r_s > r_u$. This implies $e_s > e_u$.

Since $f(x) = 1-\exp(-x)$ is an increasing function. We know $p > q$.

Q.E.D.

A skilled parent is willing to invest more resources than an unskilled parent, in the form of parental educational time and physical educational capital, in a child's education. In addition, given the same educational resources, a skilled parent could conduct education more effectively under Assumption 1. Hence, children from skilled families enjoy better education and stand better chance to become skilled when growing up. Such results are also true with exogenous wages in Fan and Zhang (2011).

A key question is how a higher proportion of skilled population influences the skilled and unskilled wages.

Proposition 3. *An increase in the proportion of skilled population causes a decrease in unskilled wage and an increase in unskilled wage:*

$$\frac{dw_s}{dh_t} < 0, \qquad \frac{dw_u}{dh_t} > 0.$$

Proof. Denote $M = (1-\alpha)A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}r^{\frac{-\alpha}{1-\alpha}}$ and $J_t = \beta h_t^{\rho} + (1-\beta)(1-h_t)^{\rho}$. Hence we can rewrite w_s, w_u as follows:

$$\begin{aligned} w_{s,t} &= (1-\alpha)\beta A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}r^{\frac{-\alpha}{1-\alpha}}J_t^{\frac{1}{\rho}-1}h_t^{\rho-1} \\ &= \beta M\ J_t^{\frac{1}{\rho}-1}h_t^{\rho-1} \end{aligned}, \tag{36}$$

$$\begin{aligned} w_{u,t} &= (1-\alpha)(1-\beta)A^{\frac{1}{1-\alpha}}\alpha^{\frac{\alpha}{1-\alpha}}r^{\frac{-\alpha}{1-\alpha}}J_t^{\frac{1}{\rho}-1}(1-h_t)^{\rho-1} \\ &= (1-\beta)M\ J_t^{\frac{1}{\rho}-1}(1-h_t)^{\rho-1} \end{aligned}, \tag{37}$$

$$\begin{aligned}\frac{dw_s}{dh_t} &= (\frac{1}{\rho}-1)\beta M\ J_t^{\frac{1}{\rho}-2} h_t^{\rho-1}\frac{dJ_t}{dh_t} + (\rho-1)\beta M\ J_t^{\frac{1}{\rho}-1} h_t^{\rho-2} \\ &= (\rho-1)\beta M\ J_t^{\frac{1}{\rho}-2} h_t^{\rho-2}[-\beta h_t^{\rho} + (1-\beta)(1-h_t)^{\rho-1}h_t + J_t] \ , \\ &= (\rho-1)(1-\beta)\beta M\ J_t^{\frac{1}{\rho}-2} h_t^{\rho-2}(1-h_t)^{\rho-1}\end{aligned} \tag{38}$$

$$\begin{aligned}\frac{dw_{u,t}}{dh_t} &= (\frac{1}{\rho}-1)(1-\beta)M\ J_t^{\frac{1}{\rho}-2}(1-h_t)^{\rho-1}\frac{dJ_t}{dh_t} - (\rho-1)(1-\beta)M\ J_t^{\frac{1}{\rho}-1}(1-h_t)^{\rho-2} \\ &= (1-\rho)(1-\beta)M\ J_t^{\frac{1}{\rho}-2}(1-h_t)^{\rho-2}[\beta h_t^{\rho-1}(1-h_t) - (1-\beta)(1-h_t)^{\rho} + J_t] \\ &= (1-\rho)(1-\beta)\beta M\ J_t^{\frac{1}{\rho}-2} h_t^{\rho-1}(1-h_t)^{\rho-2}.\end{aligned} \tag{39}$$

Hence, $\frac{dw_s}{dh_t} < 0, \frac{dw_u}{dh_t} > 0$. *Q.E.D.*

This implies that an increase in the skilled population will result in a decrease in skilled wage and an increase in unskilled wage, which reflects the fundamental rule of supply and demand and the law of diminishing marginal product. This result is new compared to models with exogenous wages such as Fan and Zhang (2011).

Let us now look at the long run equilibrium.

Proposition 4. *There exists a stable equilibrium steady state* h^*.

Proof. Given the intergenerational transition:

$$h_{t+1} = H(h_t) = \frac{p_t n_s h_t + q_t n_u (1-h_t)}{n_s h_t + n_u (1-h_t)},$$

takin differentiation gives

$$\frac{dh_{t+1}}{dh_t} = \frac{n_s h_t \frac{dp_t}{dh_t} + n_u(1-h_t)\frac{dq_t}{dh_t}}{n_s h_t + n_u(1-h_t)} + \frac{(p_t - q_t) n_s n_u}{[n_s h_t + n_u(1-h_t)]^2}. \qquad (40)$$

Denote $N_t = (1-\rho)(1-\beta)\beta M \ J_t^{\frac{1}{\rho}-2} h_t^{\rho-1}(1-h_t)^{\rho-1}$. We can rewrite

$$\frac{dw_s}{dh_t} = -\frac{N_t}{h_t} \qquad \frac{dw_u}{dh_t} = \frac{N_t}{1-h_t}.$$

Hence, we have

$$\frac{dp_t}{dh_t} = -\exp(-e_{s,t})\frac{e_{s,t}}{w_{s,t}}\frac{N_t}{h_t}, \qquad (41)$$

$$\frac{dq_t}{dh_t} = \exp(-e_{u,t})\frac{e_{u,t}}{w_{u,t}}\frac{N_t}{1-h_t}. \qquad (42)$$

We can also rewrite the term $n_s h_t \frac{dp}{dh_t}$ and $n_u(1-h_t)\frac{dq}{dh_t}$ as

$$\begin{aligned} n_s h_t \frac{dp_t}{dh_t} &= n_s h_t \exp(-e_{s,t})\frac{e_{s,t}}{w_{s,t}}\frac{dw_{s,t}}{dh_t} \\ &= -n_s \exp(-e_{s,t})\frac{e_{s,t}}{w_{s,t}} N_t < 0 \end{aligned},$$

$$\begin{aligned} n_u(1-h_t)\frac{dq_t}{\partial h_t} &= n_u(1-h_t)\exp(-e_{u,t})\frac{e_{u,t}}{w_{u,t}}\frac{dw_{u,t}}{dh_t} \\ &= n_u \exp(-e_{u,t})\frac{e_{u,t}}{w_{u,t}} N_t > 0 \end{aligned}.$$

Consider $\text{sign}(n_s h_t \frac{dp_t}{dh_t} + n_u(1-h_t)\frac{dq_t}{dh_t})$:

$$\left|\frac{n_u(1-h_t)\dfrac{\partial q}{\partial h_t}}{n_s h_t \dfrac{\partial p}{\partial h_t}}\right| = \frac{n_u w_s \dfrac{e_u}{\exp(e_u)}}{n_s w_u \dfrac{e_s}{\exp(e_s)}}.$$

Since $n_u > n_s$, $w_{s,t} > w_{u,t}$ and $e_{u,t} < e_{s,t} \Rightarrow \frac{e_{u,t}}{\exp(e_{u,t})} > \frac{e_{s,t}}{\exp(e_{s,t})}$, and since

$\frac{x}{\exp(x)}$ is a decreasing function with respect to x, we have

$$\frac{n_s h_t \dfrac{dp_t}{dh_t} + n_u(1-h_t)\dfrac{dq_t}{dh_t}}{n_s h_t + n_u(1-h_t)} > 0.$$

As shown in proposition 1, $p_t > q_t \Rightarrow \frac{(p_t - q_t) n_s n_u}{[n_s h_t + n_u(1-h_t)]^2} > 0$. We have

$$\frac{dh_{t+1}}{dh_t} = \frac{n_s h_t \dfrac{dp_t}{dh_t} + n_u(1-h_t)\dfrac{dq_t}{dh_t}}{n_s h_t + n_u(1-h_t)} + \frac{(p_t - q_t) n_s n_u}{[n_s h_t + n_u(1-h_t)]^2} > 0 \ .$$

Hence h_{t+1} is a strictly increasing function of h_t .

Note that $h_t = 0 \Rightarrow h_{t+1} = q$, and that $h_t = 1 \Rightarrow h_{t+1} = p$. Thus, h_{t+1} is a continuous and increasing function bounded between $[q, p]$ and $p < 1$. It is obvious 45^0 line intersects h_{t+1} from below. So h_{t+1} has a smaller slope than 1 at intersection, i.e. $\frac{dh_{t+1}}{dh_t} < 1$. Hence h_t^* is locally stable. *Q.E.D.*

The term $\dfrac{n_s h_t \dfrac{dp_t}{dh_t} + n_u(1-h_t)\dfrac{dq_t}{dh_t}}{n_s h_t + n_u(1-h_t)}$ describes the income effect of a higher starting level of the ratio of skilled to unskilled population on population dynamics. Also, $\dfrac{dw_s}{dh_t} < 0$ and $\dfrac{dw_u}{dh_t} > 0$ show that the increase in the skilled population ratio h_t results in a decrease in the wage of skilled worker and an increase in the wage of unskilled worker. With the opposite changes in skilled and unskilled wages, the child of a skilled worker will have a lower chance to become skilled but the child of unskilled workers will be more likely to become skilled, i.e. $\dfrac{dp_t}{dh_t} < 0$ and $\dfrac{dq_t}{dh_t} > 0$.

However, by taking into account of the existing population size of skilled and unskilled workers, $n_s h_t$ and $n_u(1-h_t)$, the increase in future skilled population from current unskilled families dominates the decrease in future skilled population from current skilled families. Overall, the income effect results in an increase in the ratio of skilled to total working population. The term $\dfrac{(p-q)n_s n_u}{[n_s h_t + n_u(1-h_t)]^2}$ shows a positive impact of h_t on the ratio of future skilled to total owrking population. With a rise in the proportion of skilled labor, the next generation will become more skill-intensive since the offspring of skilled families will be more likely to be skilful than that of unskilled families, i.e. $p_t > q_t$. Both of the effects above show that a higher percentage of skilled population in the current generation will increase the percentage of skilled population in the next generations.

The actual income for a skilled and an unskilled worker are $I_{s,t} = (1-\xi n_s - v_s n_s)w_{s,t}$ and $I_{u,t} = (1-\xi n_u - \tilde{v} n_u)w_{u,t}$ respectively. The average income in period t is:

$$\begin{aligned} E(I_t) &= h_t I_{s,t} + (1-h_t) I_{u,t} \\ &= h_t(1-\xi n_s - v_s n_s) w_{s,t} + (1-h_t)(1-\xi n_u - \tilde{v} n_u) w_{u,t} \\ &= [\frac{\eta + \tau(1-\delta)}{1+\beta}][h_t w_{s,t} + (1-h_t) w_{u,t}] \end{aligned} . \tag{43}$$

The implications of a higher ratio of skilled to working population for average income and average fertility of the economy are:

Proposition 5: A *higher percentage of skilled to working population leads to higher average income but lower fertility.*

Proof. Differentiating (43) with respect to h_t leads to

$$\begin{aligned} \frac{dE(I_t)}{dh_t} &= [\frac{\eta + \tau(1-\delta)}{1+\eta}]M \ \frac{d[\beta h_t^{\rho} + (1-\beta)(1-h_t)^{\rho}]^{\frac{1}{\rho}}}{dh_t} \\ &= [\frac{\eta + \tau(1-\delta)}{1+\eta}]M \ [\beta h_t^{\rho} + (1-\beta)(1-h_t)^{\rho}]^{\frac{1}{\rho}-1}[\beta h_t^{\rho-1} - (1-\beta)(1-h_t)^{\rho-1}] \end{aligned} . \tag{44}$$

As shown in the proof of lemma 2, $\beta h_t^{\rho-1} - (1-\beta)(1-h_t)^{\rho-1} > 0$. Hence $\frac{dE(I_t)}{dh_t}$ >0.

Average fertility could be expressed as $h_t n_s + (1-h_t) n_u$. Since $n_s < n_u$, we have $\frac{d[h_t n_s + (1-h_t) n_u]}{dh_t} = n_s - n_u < 0$. *Q.E.D.*

Even though an increase in the skilled population will result in a decrease of skilled wage and an increase of unskilled wage, the increase of unskilled wage generates a dominating effect and leads to the increase of average wage.

A a measure of income inequality in this model, the variance of income is

$$\begin{aligned} Var(I_t) &= h_t[I_{s,t} - E(I_t)]^2 + (1-h_t)[I_{u,t} - E(I_t)]^2 \\ &= [\frac{\eta + \tau(1-\delta)}{1+\eta}]h_t(1-h_t)(w_s - w_u)^2 \end{aligned} . \tag{45}$$

We can see that the variance of income is determined by two factors: the product of the skilled and unskilled proportions, $h_t(1-h_t)$, and the wage gap, $w_s - w_u$.

Expressing w_s, w_u as a function of h_t, we rewrite (45) as:

$$\begin{aligned} Var(I_t) = [\frac{\eta + \tau(1-\delta)}{1+\eta}]h_t(1-h_t)[\beta h_t^{\rho-1} - (1-\beta)(1-h_t)^{\rho-1}]^2 \\ [\beta h_t^{\rho} - (1-\beta)(1-h_t)^{\rho}]^{2(\frac{1}{\rho}-1)} M^2. \end{aligned} \tag{46}$$

Based on (46), Figure 1 shows the evolution of income inequality resembling the Kuznets curve with an inverted U pattern. Many existing literature (Galor and Zeira (1993)) derives the Kuznets curve under the framework of exogenous wage. Our model resembles the Kuznets curve when wage is endogenous.

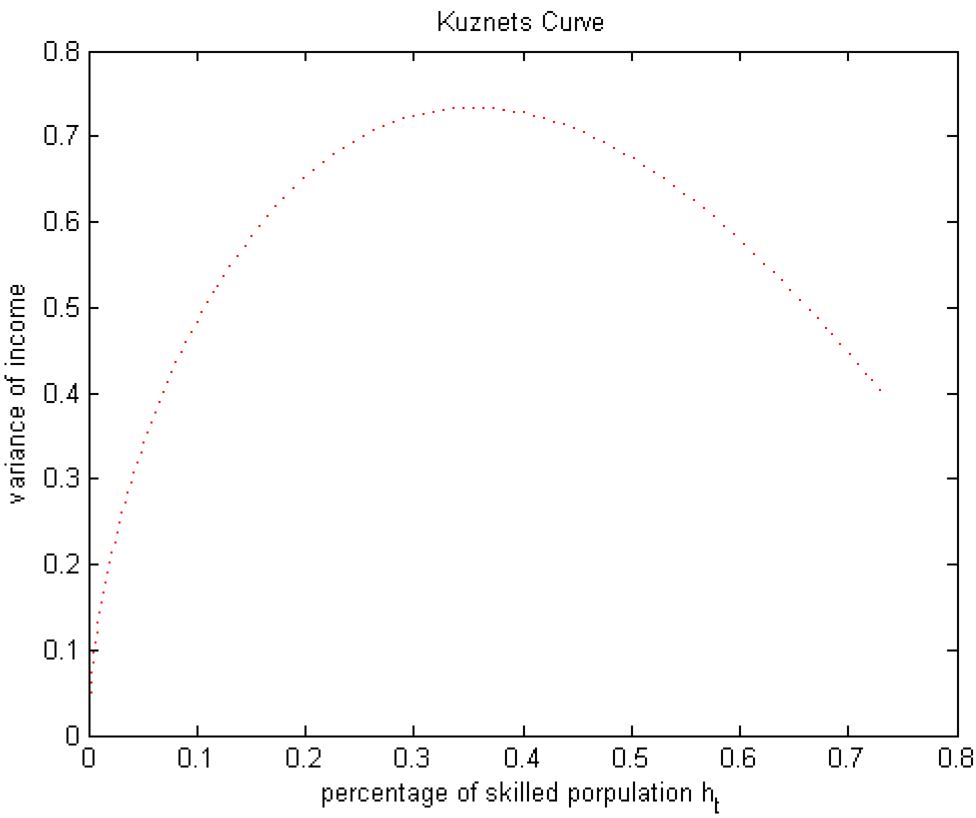

Figure 1. Kuznets Curve

Let us consider how a higher starting ratio of skilled to working population affects intergenerational mobility.

Proposition 6. *Intergenerational mobility increases with* h_t, $\dfrac{dR_t}{dh_t} > 0$.

Proof. We know $\dfrac{dp_t}{dh_t} < 0$ from (41) and $\dfrac{dq_t}{dh_t} > 0$ from (42).

Hence, $\dfrac{dR_t}{dh_t} = \dfrac{p_t \dfrac{dq_t}{dh_t} - q_t \dfrac{dp_t}{dh_t}}{p_t^2} > 0$. *Q.E.D.*

This result is intuitive. As discussed earlier, an increase in the skilled population percentage will cause a decrease of skilled wage and an increase of unskilled wage, which is an important channel for intergenerational mobility yet ignored in existing studies with fixed wages. Children from unskilled families will receive more physical education investment and enjoy better education outcome, while children in skilled families will receive less physical education investment and worse education outcome. Hence, starting from a higher ratio of skilled to working population, children from unskilled families have better chance to become skilled and children from unskilled families have less chance to become skilled when growing up. Even though it is still true that children from skilled families still enjoy better education compared to those from unskilled families, the educational inequality is declining. Hence, a higher skilled population percentage results in greater intergenerational mobility.

4 The Model with Skill-biased Technological Change

In this section, we will present a model that can reproduce some empirical dynamics of income inequity and education by incorporating skill-biased technological change.

We set up the simulation framework similar to that in Ábrahám (2008). The skill-biased technological progress is an exogenous process measured by D_t, and it is assumed to follow such a process:

$$D_{t+1} = \Omega D_t, \tag{47}$$

where $D_t > 1$ and $\Omega > 1$, this shows that skill-biased technical change enhances the return of skilled workers and grows in an accumulative pattern.

The following table displays the parameters used in the simulation

Final Goods production						Education Production			Individual Preference				
α	β	ρ	D_0	r	Ω	γ_s	γ_u	δ	τ	η	δ	ξ	$\tilde{v}$
0.6	0.55	0.3	1	0.1	1.5	8	2.5	0.5	8	2.5	0.5	0.05	0.05

Table 1. Parameter for simulation under skill-biased technological change

Figure 2 shows the evolution of skilled wage premium

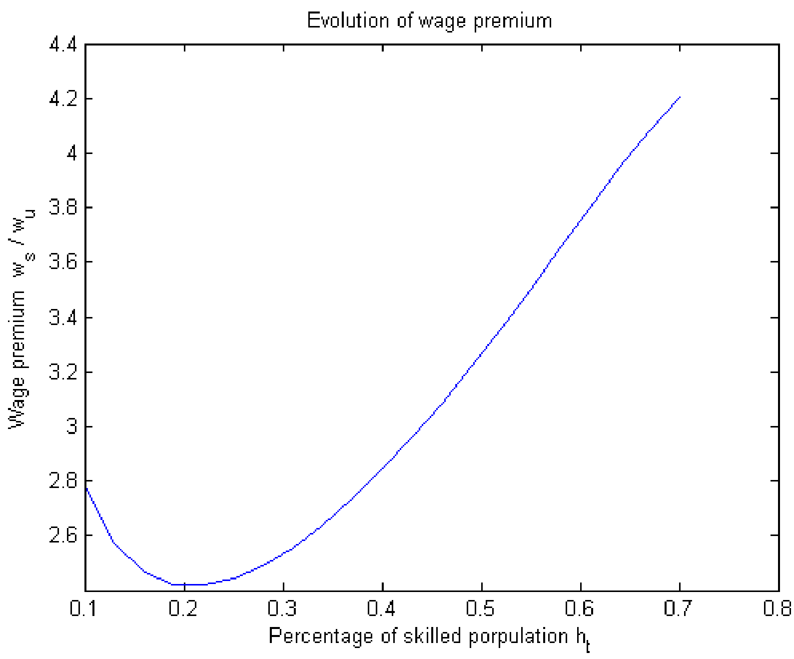

Figure 2. Evolution of wage premium under skill-biased technological change

The wage premium is governed by $z_t = \frac{\beta}{1-\beta} D_t^\rho \left(\frac{h_t}{1-h_t} \right)^{\rho-1}$ where there are two factors affecting wage premium. First, the increase in the supply of skilled labor depresses skilled wage, represented by the term $\left(\frac{h_t}{1-h_t} \right)^{\rho-1}$. Second, skill-biased technological change directly enhances the return of skilled labor, represented by the term D_t^ρ. In the early stage, the supply-demand relationship plays a major role in wage determination and leads to a decrease in wage premium. When skill-biased technology accumulates to a certain level, the relative wage of skilled worker starts to increase in a steady manner. The result of this simulation also reassembles the wage pattern for most of the OECD countries from 1950s to 2000s (see, e.g. Katz and

Murphy (1992), Juhn, Murphy and Pierce (1993) , Berman, Bound and Machin (1998)).

In addition, technological change also has a direct impact on the marginal productivity of unskilled labor. Since skilled and unskilled labor are complementary inputs in this simulation, as $0 < \rho < 1$, the increase in the supply of skilled labor will increase the marginal productivity of unskilled labor. Moreover, the decrease in the unskilled labor proportion, $1-h_t$, will enhance the marginal productivity of unskilled worker because it becomes more scarce.

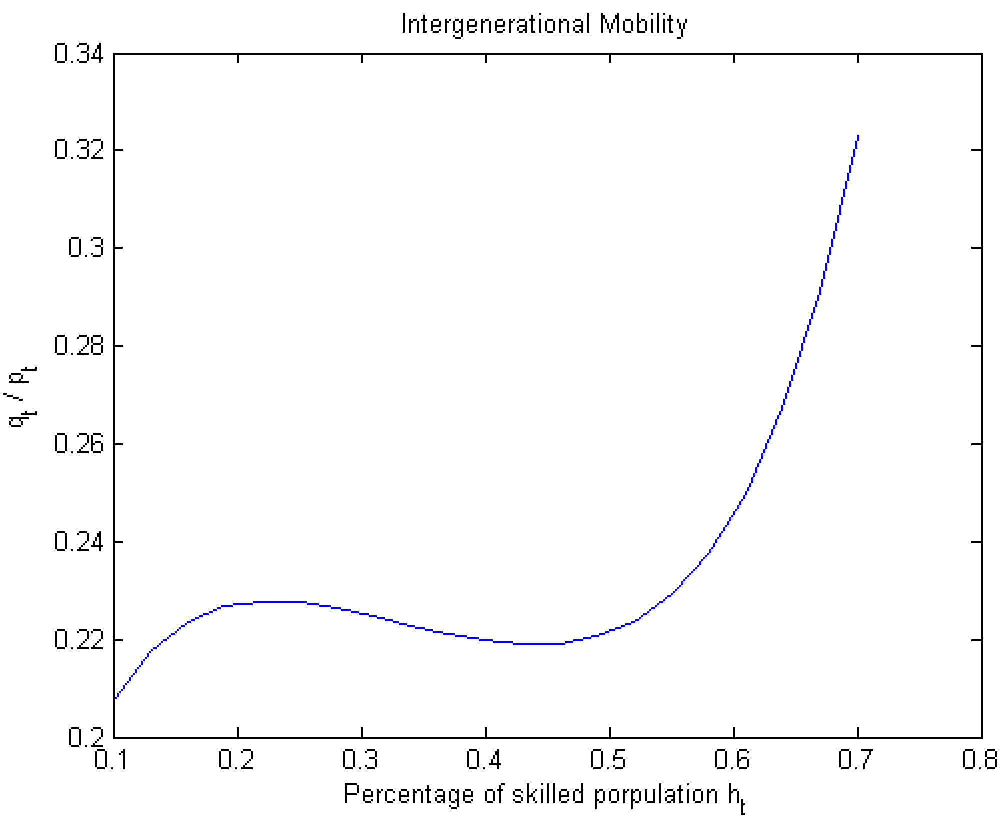

Figure 3. Intergenerational mobility under skill-biased technological change

Figure 3 shows an interesting pattern of the response of intergenerational mobility to a rising ratio of skilled population. The first increase in the social mobility is explained by the higher relative wage of unskilled worker so that an unskilled family has more resources to spend in the education. The additional unit of physical educational investment gives considerable return to an unskilled family when the education investment level is low, hence social mobility improves. Thereafter, the social mobility is slightly worse off when the wage premium begin to increase: the turning point coincides with the turning point of wage premium. This is due to the significant increase of skilled wage when the accumulation of skill-biased technological change is strong enough. Skilled family starts to increase the spending in the physical education investment. This reduces downward mobility of children from skilled families and thus slightly depresses the social mobility.

However, through the Cobb-Douglas form of education function, the constant increase in the physical capital education investment suffers from diminishing return. The persistent increase of skilled wage does not translate into a significant decline of downward mobility for children in skilled families. On the other hand, the increase in the unskilled wage and the corresponding increase of physical education investment in unskilled families improve their child's upward mobility, as the physical education investment level is still low for the unskilled family. Hence social mobility improves quite significantly in that regime.

5 Conclusion

Intergenerational mobility and income inequality attract great attention from both economists and politicians. Income gap and intergenerational mobility are interrelated and affect the social equity of current and next generation. The widening income gap across skilled and unskilled individuals could be driven by skill-biased technological change, while education is a key determinant of intergenerational mobility. To better explain the intergenerational mobility, one should develop a comprehensive understanding of education, intergenerational transfer, endogenous fertility, population dynamics, and wage premium. Hence, this paper develops a general equilibrium model that incorporates skill-biased technological change and differential fertility with heterogeneous agents differentiated by their skills levels to analyze fertility, education investment, income inequality and intergenerational mobility.

Under some reasonable assumptions, the baseline model shows that, when facing the trade-off between quantity and quality of child, skilled worker has fewer children but invests a greater proportion of their income and time in education than an unskilled worker. Since the skilled parents have some advantage in educating a child, they are more willing to invest in educating next generation, both in the form of educational time and physical capital. Hence, a child from a skilled family has better education outcome and has a better chance to become a skilled worker in his adulthood. When the economy develops towards a higher proportion of skilled population, average fertility decreases, average income increases, and the evolution of income inequality reassembles the Kuznets curve. Intergenerational mobility improves when the skilled group becomes larger, as the relative increase of unskilled wage gives the poor child better education resources.

We further extend the baseline model to incorporate skill-biased technological change and conduct numerical simulation to reproduce the empirical dynamics of wage inequality and further explore the impact of skill-biased technological change on intergenerational mobility. Different from the previous case, intergenerational mobility generally improves but with some setbacks along the way. This setbacks arises from the endogenous response of wage premium the a rising ratio of skilled to working population.

The results in this paper are based on the assumptions which simplify the model. One may try to investigate the relationship between education and technological progress and endogenize skill-biased technological change. This will give additional insight into education and income inequality.

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Appendices

A.a The feasible range for h_t is $[0, 1-\exp(-\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}})]$.

Proof.

$$n_u - n_s = \frac{1-\tau(1-\delta)}{(\xi+\tilde{v})(1+\eta)} - \frac{1-\tau}{\xi(1+\eta)}$$
$$= \frac{1}{\xi(\xi+\tilde{v})(1+\eta)}[\tau\delta\xi - (1-\tau)\tilde{v}]$$

Since we assume $\tilde{v} < \frac{\tau\xi\delta}{1-\tau\delta}$, this leads to $n_u > n_s$

$$h_{t+1} = \frac{p_t n_s h_t + q_t n_u (1-h_t)}{n_s h_t + n_u (1-h_t)} < p_t$$

$$h_t < \max_{\forall t}(p_t)$$

Given $r_t \geq 1$ and $h_t \in [0,1]$

$$p_t(h_t) = 1-\exp\{-\gamma_s (\frac{\tau\delta\xi}{1-\tau})^{\delta} [\frac{\tau\xi(1-\delta)}{1-\tau}]^{1-\delta} [(1-\alpha)\beta A^{\frac{1}{1-\alpha}} \alpha^{\frac{\alpha}{1-\alpha}} r^{\frac{-\alpha}{1-\alpha}} (\beta h_t^{\rho} + (1-\beta)(1-h_t)^{\rho})^{\frac{1}{\rho}-1} h_t^{\rho-1}]^{1-\delta}\}$$
$$< 1-\exp[-\gamma_s \frac{\tau\xi}{1-\tau} \delta^{\delta} (1-\delta)^{1-\delta} (1-\alpha)^{1-\delta} \beta^{1-\delta} A^{\frac{1-\delta}{1-\alpha}} \alpha^{\frac{\alpha(1-\delta)}{1-\alpha}}]$$
$$< 1-\exp(-\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}})$$

The feasible range of h_t is $[0, 1-\exp(-\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}})]$. *Q.E.D.*

A.b Skilled workers always receive higher wage than unskilled workers:

$$w_s > w_u, \forall h_t \in [0, 1-\exp(-\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}})]$$

Proof. Given the assumption that $\beta > \dfrac{1}{1+e^{(\rho-1)\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}}}}$, we can derive the following inequality:

$$\begin{aligned}
\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}} &< \log[(\frac{1}{\beta}-1)^{\frac{1}{\rho-1}}], \\
\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}} &< \log[(\frac{1}{\beta}-1)^{\frac{1}{\rho-1}}+1], \\
1-\exp(-\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}}) &< 1-\frac{1}{(\frac{1}{\beta}-1)^{\frac{1}{\rho-1}}+1}.
\end{aligned}$$

This implies $h_t < 1-\dfrac{1}{(\frac{1}{\beta}-1)^{\frac{1}{\rho-1}}+1}$ for all the feasible value of h_t.

The wage premium $z(h_t) = \dfrac{w_{s,t}}{w_{u,t}} = \dfrac{\beta}{1-\beta}\left(\dfrac{h_t}{1-h_t}\right)^{\rho-1}$. Given $\rho < 1$, $z(h_t)$ is an decreasing function of h_t. Observe that

$$\min z(h_t) = z(1-\frac{1}{(\frac{1}{\beta}-1)^{\frac{1}{\rho-1}}+1}) > 1 \ .$$

Hence $w_s > w_u, \forall h_t \in [0, 1-\exp(-\gamma_s \frac{\tau\xi}{1-\tau} A^{\frac{1-\delta}{1-\alpha}})]$. *Q.E.D.*