LINER SHIP FLEET PLANNING WITH UNCERTAIN CONTAINER SHIPMENT DEMAND

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2011

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A THESIS SUBMITTED

FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF CIVIL & ENVIRONMENTAL ENGINEERING

NATIONAL UNIVERSITY OF SINGAPORE

2011

ACKNOWLEDGEMENT

Firstly of all, I would like to express my deepest appreciation and sincere gratitude to my supervisor, Associate Professor Meng Qiang for his patient guidance, constructive suggestions and continuous support throughout my Ph.D. Study in National University of Singapore. His strong character and always positive and optimistic attitudes amid difficulties are virtues I have yet to acquire. Whenever the research was in dismay, he always provided me with fresh insights into the issues and encouraged me to solve the problems. Without his help and understanding, this work could not have been completed. Moreover, he often shared his experience in his life with me and exhorted me to be an honest and upright man. Despite his always-tight schedule, he was willing to review my less focused bulky draft, and comment how my research could be improved. I would never forget his guidance, warmth and kindness.

My thanks also extend to members of the thesis committee: Professor Fwa Tien Fang and Associate Professor Chew Ek Peng. Their understanding and willingness to spend so much of their precious time for me in this process is highly appreciated. Their passion and enthusiasm in research has profoundly infected me. I have also benefited from their perspectives, experiences and broad knowledge of diverse fields. Also, I would like to thank Assistant Professor Szeto Wai Yuen in The University of Hong Kong, for his kindness and help.

I would like to specially thank the National University of Singapore for providing the research scholarship for me during my Ph.D study in Singapore. Thanks are also extended to Mr. Foo Chee Kiong, Madam Yap-Chong Wei Leng, Madam Theresa Yu-Ng Chin Hoe and Madam Lim Sau Koon for their assistance. Their kind co-operation has allowed me to complete my research smoothly. Especially, it is very nice to chat with Madam Theresa Yu-Ng Chin Hoe. It is memorable and I would miss her graciousness.

I was lucky enough to meet many friends during my 4 years of stuy at NUS. To name a few, I would like to thank Dr. Shahin Gelareh for his help in programming and Dr. Khoo Hui Ling for her kind encouragement on my study and passionate reception when I went to her wedding ceremony. I am also grateful to my colleagues: Liu Zhiyuan, Weng Jinxian, Wang Xinchang, Qu Xiaobo, Wang Shuaian and Zhang Jian. Particularly, Zhiyuan and Zhang Jian, many a time when I met with depression, they always stand beside me, giving me consolation and support. They are friends in my life.

Last but not least, the most sincere gratitude goes to my parents, my sisters and my brothers-in-law in China. They have always cast never-ending support and love on me. Their unmatched love should deserve the dedication of this humble thesis. Finally, I would like to cite a line from an ancient Chinese poem to express my deepest thanksgiving to my mother for bringing me up: *needlework of the mother, for her child far away*.

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SUMMARY

A liner container shipping company is constantly searching for models and solution procedures for building decision support systems, which help it to create cost-effective plans for operating and upgrading its liner ship fleet and seizing market share in an intensely competitive container shipping market. The plans for operating and upgrading its liner ship fleet aim to make the capacity of the fleet effectively match the current and future demand for container shipment. The container shipment demand is affected by some unpredictable and uncontrollable factors, which indicates that such plans have to be made on the basis of uncertain demand. However, methodologies used by previous researchers are inappropriate here because they make the assumption that container shipment demand is deterministic. Hence, new methodologies are required. This thesis seeks to meet this requirement by proposing new mathematical models and solution algorithms for liner ship fleet planning (LSFP) problems with container shipment demand uncertainty.

LSFP problems with uncertain container shipment demand can be classified according to the length of the planning horizon into short-term and long-term LSFP problems. This thesis first studies short-term LSFP problems and then proceeds to investigate long-term problems with container shipment demand uncertainty.

The short-term LSFP problem with uncertain container shipment demand is, first of all, formulated as a chance-constrained programming (CCP) model. In this model, a confidence parameter is set to represent the probability that the liner container shipping

company cannot satisfy the shippers' demand. However, the CCP model does not allow container transshipment, which is widely used in liner shipping. Therefore, a two-stage stochastic integer programming (2SSIP) model, with the objective of maximizing expected profit, is proposed for the short-term LSFP problem with container transshipment and uncertain container shipment demand. A solution algorithm integrating the sample average approximation method and the dual decomposition and Lagrangian relaxation method is proposed for solving the 2SSIP model. The model only considers the expected value; variance (or risk), which is also an issue of high concern to the decision-maker, is not taken into account. Therefore, next, a robust optimization model (ROM), in which both expected value and variance are considered simultaneously for the short-term LSFP problem, is proposed. By adjusting the penalty parameters of the ROM, decision-makers can determine the optimal liner ship fleet plan, which includes decisions about fleet design and deployment, and which maximizes total profit under different container shipment demand scenarios while at the same time controlling the variance.

The last part of this thesis studies the long-term/multi-period LSFP problem with container transshipment and uncertain demand. The container shipment demand in one period is assumed to be dependent on that in the previous period. A set of scenarios in each single period is used to reflect the uncertainty of container shipment demand, and then the evolution and dependency of this demand across multiple periods is modeled as a scenario tree. The procedure for multi-period LSFP is interpreted as a decision tree and formulated as a multi-period stochastic programming model comprising a sequence of interrelated two-stage stochastic programming models developed for each single period.

Finally, a numerical example is carried out to assess the applicability and performance of the proposed model and solution algorithm.

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GLOSSARY OF NOTATION

Notation in Model Formulation

c_{krt}	Operating cost of a ship k on ship route r per voyage in period t (\$/voyage)
$c_t^{h^{od}}$	Handling cost for a container being carried between O-D port pair $(o,d) \in \mathcal{W}$ incurred on container route $h^{od} \in \mathcal{H}^{od}$ over period t (\$/TEU)
c_{kt}^{IN}	Rate for chartering in a ship of type k over period t (\$/ship)
c_{kt}^{NEW}	Price of a specific new ship k at the beginning of period t (\$)
c_{kt}^{OUT}	Rate for chartering out a ship of type k over period t (\$/ship)
c_{kt}^{SOLD}	Rate for selling out a ship k at the beginning of period t (\$)
e_{kt}	Daily lay-up cost for a specific ship k in period t (\$/day)
$EP_{t,n}^{m}$	Expected profit of strategy n in period t produced by strategy m in period t-1
$EP_{t,n}^{m,s'}$	Expected profit of strategy n in period t produced by strategy m in period t-1 under scenario s'

f_t^{od}	Freight rate for transporting a container between an O-D port pair $(o,d)\in\mathcal{W}$ (\$/TEU) in period *t*
$\mathcal{G}_{t,n}^{\mathrm{KEEP}}$	Set of own ships to be used over period *t* in strategy *n*
$\mathcal{G}_{t,n}^{\mathrm{SOLD}}$	Set of own ships to be sold over period *t* in strategy *n*
$\mathcal{G}_{t,n}^{\mathrm{OUT}}$	Set of own ships to be chartered out over period *t* in strategy *n*
$\mathcal{G}_{t,n}^{\mathrm{IN}}$	Set of ships to be chartered in over period *t* in strategy *n*
$\mathcal{G}_{t,n}^{\mathrm{NEW}}$	Set of new ships at the beginning of period *t* in strategy *n*
$\mathcal{G}_{t,n}$	Set of ships to deliver containers over period *t* in strategy *n*
$\bar{\mathcal{G}}_{T,n^{\circ}}$	Set of ships owned by the liner container shipping company in strategy n° at the end of period *T*
h^{od}	A container route from origin port $o\in\mathcal{P}$ to destination port $d\in\mathcal{P}$
$\mathcal{H}^{od}$	Set of candidate container routes to deliver containers between an O-D port pair $(o,d)\in\mathcal{W}$
$\mathcal{H}$	Set of all predetermined container routes for all O-D ports
$\mathcal{K}$	Set of available ship types $\{1,...,k,...,K\}$
K	Number of ship types in set $\mathcal{K}$
$\mathcal{L}_{t,n}^{n^{\circ},l}$	Binary variable, it equals 1 if a path $l\in\mathbb{P}^{T}(n^{\circ})$ from the dummy root *O* to the leaf node n° passes node *n* of period *t*, otherwise 0
m_r	Number of ports called on route *r*
$\mathcal{M}_r$	Set of port pairs $\{(p_r^i, p_r^j)\|i, j=1,2,\cdots,m_r\}$ on route *r*

N_k^{MAX}	Number of ships of type *k* owned by the liner container shipping company
NCI_k^{MAX}	Number of available ships of type *k* that can be chartered in from the leasing market
N_r	Minimum amount of voyages required on route *r* during the planning horizon in order to maintain a given level of frequency
$\mathcal{N}_t^m$	Set of strategies $\{1,...,N_t^m\}$ proposed for period *t*+1 which are generated from its parent *m* in period *t*
N_t^m	Number of strategies in set $\mathcal{N}_t^m$
$\mathcal{N}_t$	Set of nodes $\{1,...,N_t\}$ in period $t \in \mathcal{T}$
N_t	Number of nodes in set $\mathcal{N}_t$
p_s^t	Probability that container shipment demand scenario *s* occurs in period $t \in \mathcal{T}$
$p_{s\|s'}^t$	Conditional probability that scenario *s* of period *t* happens under the occurrence of scenario s' of period *t*–1
p_r^i	The *i*th port called on ship route r
$\mathcal{P}_r$	Set of ports $\{p_r^1,\ldots,p_r^i,\ldots p_r^{m_r}\}$ called on route *r*
$\mathcal{P}$	Set of ports $\{1,\ldots,p,\ldots P\}$
P	Number of ports in set $\mathcal{P}$
$\mathbb{P}^t(n)$	A path in a subtree from root *O* to a node *n* in period $t \in \mathcal{T}$

$\mathcal{R}$	Set of predetermined ship routes $\{1,...,r,...,R\}$
R	Number of ship routes in set $\mathcal{R}$
$\mathcal{S}_t$	Set of container shipment demand scenarios $\{1,...,s...,S_t\}$ for period $t \in \mathcal{T}$
SV_{T,n°	Salvage value of strategy n° at the end of period T
t_{kr}	Number of voyage days for a ship of type k on route r (days)
$\mathcal{T}$	Set of multiple periods $\{1,...,t,...,T\}$
T	Number of single periods in set $\mathcal{T}$
$\mathbb{T}^t(n)$	A subtree with root n with t-layer
V_k	Capacity of a ship of type k, referring to the number of containers it can carry (TEUs)
$\mathcal{W}$	Set of origin-to-destination (O-D) port pairs $\{(o,d) \mid o \in \mathcal{P}, d \in \mathcal{P}\}$ with container shipment demand
α_r	Confidence parameter on route r
$\rho_{ir}^{h^{od}}$	Binary variable, if a container route $h^{od} \in \mathcal{H}^{od}$ contains leg i of route r, it equals 1, otherwise it equals 0
ξ_t^{od}	Container shipment demand in terms of TEUs between an O-D port pair $(o,d) \in \mathcal{W}$ in a particular single period $t \in \mathcal{T}$
ω_{st}^{od}	Realization of container shipment demand ξ_t^{od} in scenario s

Notation in Solution Algorithms

κ	Step size
m	Index for the number of sample
M	Number of samples
N	Size of each sample
$\hat{\mathbf{v}}_N^m$	Optimal first-stage decisions of the *m-th* **SAA** problem with sample size N
$\hat{v}_N^m$	Optimal objective function value of the *m-th* **SAA** problem with sample size N
$\hat{v}_{N'}^m$	Optimal objective function value of the *m-th* **SAA** problem with sample size N'
v^*	Optimal objective function value of the original 2SSIP model
L_N^M	Lower bound to the optimal objective function value of the original 2SSIP model
$U_{N'}^M$	Upper bound to the optimal objective function value of the original 2SSIP model
$\theta_{M,N,N'}$	Gap between the lower bound and upper bound
$\boldsymbol{\lambda}$	Lagrangian multiplier vector
τ	Threshold for the stopping criteria
$\mathbf{0}$	Square zero matrix in the non-anticipativity constraints
$\mathbf{H}$	Matrix used in the non-anticipativity constraints
$\mathbf{I}$	Square unity matrix in the non-anticipativity constraints

CHAPTER 1 INTRODUCTION

1.1 Preamble

Seaborne trade refers to goods that are transported by ships, and is the main artery of international trade, in a sense, standing at the apex of world economic activity. The increasing globalization and interdependence of various world economies are leading to a tremendous positive growth in the seaborne trade industry. According to the review of maritime transport produced by the United Nations Conference on Trade and Development (UNCTAD) secretariat, international seaborne trade increased from 2.566 billion tons in 1970 to 8.210 billion tons in 2008, showing a 2.95 per cent annual average growth rate during the last four decades, but had fallen to 7.94 billion tons in 2009, due to the depression of the global economy. However, both the global economy and international seaborne trade are expected to recover and grow in 2010, with developing economies, and China in particular, charting the course (Chapter 1 of UNCTAD (2010)). The trend in the growth rate of international seaborne trade for selected years is depicted in Figure 1.1.

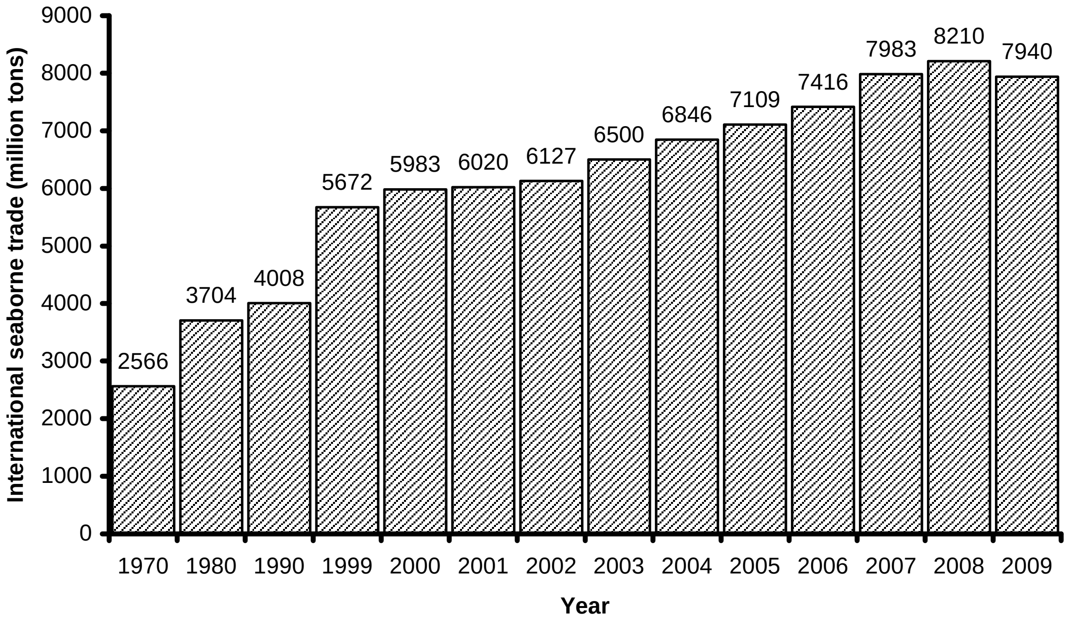

Figure 1. 1 International seaborne trade growths for selected years

In particular, containerized trade is the fastest growing sector in global seaborne transportation, as a result of a combination of various factors, including dedicated purpose-built container vessels, larger vessels capable of achieving increased economies of scale, improved handling facilities in ports, and also the increasing amount of raw materials being carried in containers (here a container refers to the twenty-foot equivalent unit [TEU]).

Maritime transportation can be divided into three different modes of operation: *industrial*, *tramp* and *liner shipping*. In industrial shipping, the container owner or the shipper owns the ships and aims to ship all of his/her containers for as low a cost as possible. In tramp shipping, the carrier or tramp shipping company has to carry containers to specified ports in a specific time frame, according to their contracts with shippers. Additional containers (if any are available in the market) are selected depending on the ship's capacity, so as to bring in as much revenue as possible. In liner shipping, the carrier releases predetermined maritime routes and schedules to the shippers, and then operates according to this. In other words, liner shipping provides a fixed liner service, at regular intervals, between named ports, and offers transport to any goods. In liner shipping, time is very important, since the liner ships have to comply with the schedules even when they are operating at low utilization levels. Thus, one can think of industrial shipping as "owning a car", tramp shipping as "a taxi service" and liner shipping as "a bus service" with definite schedules and a published itinerary. Liner shipping occupies a major posistion within global transportation. With the continuous advancement of ship-building technologies and the increase in global container traffic, the dominance of liner shipping is expected to continue to strengthen (Chapter 4 of UNCTAD (2010)). This thesis focuses on the problems of liner ship fleet planning since liner shipping plays such a central part in the global trading network.

1.2 Research Background

Many researchers have studied the problem of liner ship fleet planning. Their research can be categorized into three groups. The first group focuses on an optimal ship fleet design, including determining the numbers and types of ships needed in a fleet over a particular planning horizon, given a set of liner ship routes and a required regular frequency of liner shipping service for each route. Given a fleet of heterogeneous ships and a set of liner ship routes, the second group focuses on an optimal fleet deployment, which covers the assignment of ships to each route according to the required regular frequency of service for that route, so as to satisfy the container shipping requirements. The third group focuses on a joint optimal ship fleet design and fleet deployment plan, that is, given a set of liner ship routes, decisions on the numbers and types of ships, and ship assignment to routes, are made in order to satisfy the container shipping requirements. The problems tackled by each of these three groups are referred to through this thesis as the liner ship fleet size and mix (LSFSM) problem, the liner ship fleet deployment (LSFD) problem and the liner ship fleet planning (LSFP) problem, respectively.

Tackling these problems has become a key task for both the liner operators and researchers. In the past, liner operators relied mainly on their experience and common sense to choose the best plan from a limited set of alternatives. Sometimes, this task is not difficult when the number of alternatives is small; however, when large fleets are involved, the number of alternatives grows and it is not easy to pick the best among them. Empirically-based selection strategies are too cumbersome. Hence, the focus has switched to analysis-based strategies for all three of these problems. Some mathematical programming models and algorithms have been proposed. Most of the related research is surveyed in four review articles: Ronen (1983, 1993), Perakis (2002) and Christiansen et al. (2004).

Container shipment demand between each port pair is one of the inputs into the liner ship fleet planning problem. The existing research uses forecasted, deterministic demand. However, decisions about fleet design and ship deployment are actually made prior to knowing the exact demand, which is affected by some unpredictable and uncontrollable factors. Container shipment demand can never be forecasted with complete confidence. This implies that the problem of liner ship fleet planning should be investigated under uncertain container shipment demand. This could lead to a new and interesting research area. Hence, there is a need to study and propose stochastic programming models and solution algorithms for the liner ship fleet planning problem, incorporating uncertain container shipment demand.

1.3 Research Scope

This thesis is devoted to studying LSFP problems with container shipment demand uncertainty, as this type of problem joins the LSFSM and LSFD problems together. LSFP problems with container shipment demand uncertainty can be classified according to the length of the planning horizon: short-term or long-term. This thesis firstly studies short-term LSFP problems and then proceeds to investigate long-term LSFP problems with container shipment demand uncertainty.

As the LSFP problem studied in this thesis is a new area of research, the existing linear or integer programming models proposed by previous researchers for deterministic LSFP problems are not applicable here. New programming models are needed to deal with the container shipment demand uncertainty. Stochastic programming has served as a useful tool in decision making problems under uncertain environments. Three different types of stochastic programming model are commonly used: the first is the *expected value model* (Dantzig, 1955); the second is the *chance-constrained model* (Charnes and Cooper, 1959); the third is the *robust optimization model* (Mulvey et al., 1995).

The expected value model aims to maximize the expected earnings or minimize the expected loss within the given constraints; the chance-constrained model aims to achieve the optimum within some probabilistic constraints; finally, the robust optimization model is able to tackle the decision-makers' favored risk aversion or service-level function, and has yielded a series of solutions that are progressively less sensitive to realizations of the data in a scenario set. Since these three models are commonly used tools to deal with problems under uncertain environment, this thesis adopts each of these three models in modeling LSFP problems with container shipment demand uncertainty and proposes solution algorithms for solving these models effectively.

1.4 Research Objectives

The objective of this thesis is to propose models and solution algorithms for more realistic LSFP problems, by taking container shipment demand uncertainty into consideration. More specifically, the following research tasks have been conducted to achieve this objective:

1. developed models and solution algorithms for LSFP problems with container shipment demand uncertainty,
2. evaluated and analyzed the applicability of the proposed models and the performance of the solution algorithms, and
3. proposed some control and management strategies or policies that may assist a liner container shipping company to determine the best liner ship

fleet plan, thereby operating the fleet at as low a cost as possible or earning as much profit as possible.

1.5 Organization of Thesis

The thesis is organized into seven chapters. Chapter 1 introduces the background of the study and provides a general introduction to LSFP problems. In addition, the objectives and scope of the research are highlighted.

Chapter 2 concentrates on a literature review of previous, related studies. It is divided into three parts, which review previous research on ship fleet size and mix problems, ship fleet deployment problems and ship fleet planning problems with deterministic container shipment demand, respectively. Finally, based on the literature review, potential gaps and limitations in the existing literature, which have inspired this research, are highlighted.

Chapter 3 deals with a short-term LSFP problem with container shipment demand uncertainty for a liner container shipping company. The demand uncertainty enables us to propose a chance constraint for each liner ship route, which guarantees that the liner container shipping company can satisfy the shippers' demand, at least with a predetermined probability, on each liner ship route. Assuming that the container shipment demand between port pairs on each liner ship route follows a normal distribution, the proposed short-term LSFP problem is formulated as a chance-constrained programming (CCP) model. In this CCP model, a confidence parameter is set to represent the probability that the liner container shipping company will not be able to satisfy the shippers' demand.

Chapter 4 studies the short-term LSFP problem with container shipment demand uncertainty from the expected value point of view. Besides the consideration of uncertain container shipment demand, the container transshipment issue is also taken into account in this chapeter since container transshipment operation is widely used in practice. The short-term LSFP problem with container transshipment and uncertain demand is formulated as a two-stage stochastic integer programming (2SSIP) model. To effectively solve the proposed model, firstly, the sample average approximation (SAA) method is used to approximate the expected recourse function, and then the dual decomposition and Lagrangian relaxation method is used to solve the model.

The 2SSIP model proposed in Chapter 4 only considers the expected value but not the variance (i.e., the risk), which is also an issue of great concern to decision-makers. Therefore, in Chapter 5 we develop a robust optimization model in which both expected value and variance are considered simultaneously, for the short-term LSFP problem with container shipment demand uncertainty. The robustness and effectiveness of the developed model are demonstrated with numerical results. The trade-off between the solution robustness and the model robustness is also analyzed.

Chapter 6 studies the long-term/multi-period LSFP problem with container transshipment and uncertain container shipment demand. The container shipment demand in a single period is assumed to be dependent on that in the previous period. Using a scenario tree approach to model the evolution of dependent uncertain demand over two successive single periods, and using a decision tree to model the procedure used in liner ship fleet planning, the proposed problem is formulated as a multi-period stochastic programming model comprising a sequence of interrelated two-stage stochastic programming models, developed for each single period. We further show that the multi-period stochastic programming model can be equivalently transformed into a shortest path problem defined on an acyclic network.

Finally, Chapter 7 summarizes the main findings drawn from this research and highlights its contribution to the field. It also provides directions and recommendations for future research.

CHAPTER 2 LITERATURE REVIEW

This chapter presents a critical review of the existing literatures related to the problem of ship fleet planning. Since liner shipping is one of three modes of maritime transportation, the literature review in this chapter is not restricted to liner shipping but also includes related research on industrial and tramp shipping. The chapter is divided into five parts: the first part describes previous studies on fleet size and mix problems, the second part is devoted to fleet deployment problems, the third part reviews the literature on ship fleet planning problems, the fourth part highlights the weaknesses in the existing literature and the need for this current research, and the final part summarizes the contents of this chapter.

2.1 Fleet Size and Mix

Fleet size and mix problems are defined as follows: given a set of routes, the planner must decide on the exact ship types to include in the fleet, their sizes and the number of ships of each size. The analytical models built for fleet size and mix problems can be divided into three classes: linear programming models, integer programming models and dynamic programming models. There are also some simulation models used as decision support systems, in practice. These four types of model are reviewed in the following sections.

2.1.1 Linear Programming Models

Dantzig and Fulkerson (1954) were the pioneers in applying the linear programming approach to the fleet size problem. In this article, they aimed at minimizing the number of tankers required to meet a fixed schedule, and formulated this problem as a linear programming model solved using the simplex algorithm.

Lane et al. (1987) presented a linear programming model for determining a cost-efficient fleet which met the known demand for trade between Australia and the North American West Coast, which incorporates six ports. This problem was dealt with by separating it into three major phases:

Phase Ⅰ: Voyage Option Enumeration

Phase Ⅱ: Vessel Scheduling

Phase Ⅲ: Set Partitioning

Phase Ⅰ is a combinatorial problem which depends on the number of ports on the trade route. In this phase, all feasible itinerary options are enumerated. A feasible itinerary is defined as including at most one ballast or deadheading leg. Phase Ⅱ is the key component of the problem, which is to make cost-minimizing trade-off decisions for vessel scheduling at every origin port. A forward-looking heuristic method is used to decide which cargo will be transported by which route, and the algorithm proceeds to determine the cost-minimal (late-loading cost) schedules for port arrivals and departures. Phase Ⅲ uses the results from Phase Ⅱ to define the most efficient fleet composition, by means of a set partitioning algorithm used to select a subset of the route options which satisfy all shipping demands at the lowest possible cost.

2.2.2 Integer Programming Models

Fagerholt (1999) proposed a three-phase approach for finding the optimal fleet and coherent routes for that fleet. They studied a homogeneous fleet. Each route had to have a weekly service frequency and multiple trips were allowed for each ship. In phase Ⅰ, all feasible single routes are generated for the largest ship available. A single route is defined to be a route that is feasible with respect to the vehicle routing problem (VRP) constraints, that is, originating and terminating at the depot and not visiting it in between. In phase Ⅱ, the single routes generated in phase Ⅰ are combined into multiple routes. Phase Ⅲ involves formulating the problem as a set partitioning problem, as below:

$$\min \sum_{r \in R} \left(C_r^{TC} + C_r^{OP} \right) x_r \tag{2.1}$$

subject to

$$\sum_{r \in R} A_{ir} x_r = 1, \forall i \in N \tag{2.2}$$

$$x_r \in \{0,1\}, \forall r \in R \tag{2.3}$$

$$\sum_{r \in S^k} x_r \le N^k, \forall k \in K \tag{2.4}$$

where *R* is defined as the set of all routes (both single and multiple) generated in phase Ⅰ and phase Ⅱ, indexed by *r*; *N* is defined as the set of nodes or ports to be serviced by the fleet of ships, indexed by *i*; C_r^{TC} is the fixed time-charter cost; C_r^{OP} is the operational cost of route *r* for the lowest-cost ship that has sufficient capacity to perform the given route; x_r is a binary variable which is equal to one if route *r* is chosen in the optimal solution and zero otherwise; S^k denotes the set of routes for ship type *k* and N^k denotes the maximum number of available ships of type *k*. The route generation algorithms of phase

Ⅰ and phase Ⅱ are written and compiled in Borland Pascal 7.0. The set partitioning model is implemented and solved using GAMS/CPLEX 5.0.

Fagerholt and Lindstad (2000) studied a real problem of determining an efficient policy involving the optimal fleet and corresponding weekly schedules, for a supply vessel operation in the Norwegian Sea. The operation involves one onshore service depot located on the northwest coast of Norway and seven offshore installations located in the Norwegian Sea. Six scenarios are developed, in which the opening hours and number of weekly services of the installations are varied, and the best policy is obtained by evaluating the qualitative aspects of the solution for each scenario. The solution algorithm includes two steps for each given scenario. In the first step, a number of feasible candidate schedules are generated for each vessel in the pool. The duration of each schedule is also generated. This consists of the sailing times, the loading/discharging and waiting times at the offshore installations, and the turn-around time at the depot. In the second step, the vessels to be used and their weekly schedules are determined by solving an integer programming model. Finally, a scenario is recommended which incurs the least cost for operating the supply vessels.

Sambracos et al. (2004) considered a problem of dispatching small containers via coastal freight liners. There is only one depot port (Piraeus), from which containers are dispatched to twelve other ports (islands). A homogeneous fleet is used and demand is fulfilled so as to incur minimum costs, including fuel consumption and port costs. This problem was solved along two dimensions. Firstly, strategic planning was analyzed by appropriately introducing an linear programming formulation for the determination of vessel traffic under known supply and demand constraints, where total fuel costs and port dues are minimized. The planning problem is defined on a graph G through a set of ports, for a number, W_{ij}, of containers transported from node i to node j. Supposing n_{ij} is the number of ships traveling from node i to node j, n_{max} is the maximum number of ships in each direction for a link, L_{ij} is the length of link ij (in miles), c_F is the cost of fuel consumption per mile, c_{Pj} is the fee for port i per ship, Q is the capacity of a ship, assumed constant for all ship types, D_i is the demand at port i and S_i is the supply at port i. Then, the problem is formulated as follows:

$$\min \sum_{i,j} [n_{ij} L_{ij} c_F + n_{ij} c_{Pj}] \tag{2.5}$$

$$\sum_j W_{ji} - \sum_k W_{ik} + S_i - D_i = 0 \tag{2.6}$$

$$\left(n_{ij} - 1\right) Q \le W_{ij} \le n_{ij} Q \tag{2.7}$$

$$0 \le n_{ij} \le n_{\max} \tag{2.8}$$

$$n_{ij}, W_{ij} \ge 0 \tag{2.9}$$

Subsequently, the operational dimension of the problem is analyzed by introducing a VRP formulation corresponding to the periodic needs for transportation using smaller containers, and a list-based threshold acceptance (LBTA) algorithm is employed to solve this. LBTA is a stochastic search method that belongs to the class of threshold acceptance-based methods. A typical threshold accepting method iteratively searches the solution space, guided by a deterministic control parameter, in the same units as the cost function, to reveal promising regions for better configurations.

2.1.3 Dynamic Programming Models

Nicholson and Pullen (1971) studied a ship fleet management problem which concerned phasing out a fleet of general cargo ships over a ten-year period with the possibility of premature sales and temporary replacement by charter ships. The objective was to determine a sale and replacement policy which maximized the long-term assets of the company. The method was based on two stages. The first stage determines an order of priority for selling the ships, regardless of the rate at which charter ships are taken on. The second stage uses dynamic programming to determine an optimal level of chartering, given the order of priority for replacement. The first stage essentially reduces the dynamic programming calculation from an N-state variable problem to a one-state variable problem, which is computationally manageable using dynamic programming methods. The order of priority for replacement is calculated by assessing the net contribution to the objective function if each ship, considered individually, was sold in each year and replaced where appropriate by a charter ship. The net contribution of a ship to the final assets consists of the invested earnings of that ship up to the year when it was sold plus the invested net realization from selling it in that year, plus any earnings from a charter ship taken on in lieu of that ship for a limited period. The bigger the net contribution, the higher the ships order of priority. Let $f_t(j)$ be the maximum cash assets accumulated at the end of year *t* if *j* ships are held in year *t* and an optimal policy has been adopted. Let $g_t(i, j)$ be the increase in cash assets in year *t* if *i* ships are held in year *t*-1 and *j* ships are held in year *t*.Then, the dynamic programming recurrence relations between $f_t(j)$ and $f_{t-1}(i)$ can be set up as follows:

$$f_t(j) = \max\left(f_{t-1}(i)(1+r) + g_t(i,j)\right),\ j \le i \le D_{t-1} \tag{2.10}$$

where D_{t-1} is the total number of ships required in year t-1, and $g_t(i, j)$ consists of the earnings in year t from owned and chartered ships, plus the receipts from sales. The dynamic programming recurrence relations are evaluated for $j = M_t$ to D_t and for $t = 1, \ldots, T$ in turn, setting $f_0(i) = 0$. For each evaluation of the recurrence relation, the best value of i, say $q_t(j)$ is recorded. If the largest value of $f_{T+1}(j)$ occurs for $j = x_T$ ships to be held in year T, and in general the number of ships to be held in year t is $x_t = q_{t+1}(x_{t+1})$, then the dynamic programming procedure combined with the order of priority will determine the ships to be held in each year, using the results $x_1, x_2, \ldots, x_T$. Ship numbers N, N-1, …, $x_1 + 1$ are sold in year 1, numbers $x_1, x_1 - 1, x_2 + 1$ in year 2 and so on and $D_t - x_t$ ships are chartered in year t.

2.1.4 Simulation Models

Stott and Douglas (1981) described a Marine Operations Planning and Scheduling System (MOPASS) used for planning and scheduling the ocean transportation of bulk commodities. This model is a collection of integrated models which provide comparisons of voyage costs for different vessels and trades, a financial evaluation and optimization of vessel-to-trade assignment, and the sequencing and scheduling of individual vessels on predefined routes. The main purpose of MOPASS is to evaluate the most profitable opportunities which may arise for a given, controlled fleet of vessels. MOPASS comprises four major subsystems: a linear programming optimization module embedded in one of the subsystems, user-oriented information files, and reports for both management and operating personnel. These various components of MOPASS are accessed, shared, and integrated as needed through a user-oriented executive control program. The model does not deal with the question of overall fleet efficiency but rather with short-run dynamic operations associated with a given fleet and trade opportunities.

Gallagher and Meyrick (1984) developed a cost-based simulation model, designed to analyze the economic characteristics of liner shipping services on a trade route. The model initially defines the components of the shipping system, that is, vessels, ports, trade requirements, and trade routes. Next, cargo assignments are made according to user preference rules and vessel availability. The cargo allocation is then adjusted to obtain feasibility, and finally, the costs of the system are estimated. Like the MOPASS approach, this simulation model is evaluative. Unlike MOPASS, it quantifies system performance with a view to improving the efficiency of the entire shipping system. However, the model does not use a formal optimization model, but rather focuses on evaluating changes to the existing system.

2.2 Fleet Deployment

Fleet deployment problems are described as follows: given a set of ships and a set of routes, the planner must assign the vessels to specific trade routes (i.e., this is a tactical problem). These problems also include the determination of the expected number of lay-up days (if any) for each ship each year. The analytical approaches to fleet deployment problems can be classified into three types: the linear programming approach, the nonlinear programming approach and the integer programming model. Again, there are some simulation models that are used for fleet deployment in practice. These four types of model are introduced in the following sections.

2.2.1 Linear Programming Models

Laderman et al. (1966) developed a linear programming model for ship allocation to satisfy customer commitments. Given a set of ports and vessels, this paper aimed at minimizing the total operating time or maximizing the total unused time such that the fleet carried out the customers' shipment requirements:

$$\max \sum_{k} z_k \tag{2.11}$$

subject to

$$\sum_{i,j} T_{ij}^k X_{ij}^k + \sum_{i,j} t_{ij}^k x_{ij}^k + z_k = T_k \quad (\text{for all } k) \tag{2.12}$$

$$\sum_{k} V_{ij}^k X_{ij}^k = A_{ij} \quad (\text{for all } i, j, \text{ such that } A_{ij} > 0) \tag{2.13}$$

$$\sum_{j} X_{ij}^k = \sum_{j} x_{ij}^k \quad (\text{for all } i, k) \tag{2.14}$$

$$\sum_{i} X_{ij}^k = \sum_{i} x_{ij}^k \quad (\text{for all } j, k) \tag{2.15}$$

$$X_{ij}^k, x_{ij}^k \geq 0 (\text{for all } i, j, k) \tag{2.16}$$

$$z_k \geq 0 \ (\text{for all } k) \tag{2.17}$$

where A_{ij} is the amount to be shipped from origin *i* to destination *j* (tons), V_{ij}^k is the tonnage capacity of vessel *k* when going from origin *i* to destination *j*, T_{ij}^k is the total time required for vessel *k* to load at *i*, go from *i* to *j* and unload at *j*, t_{ij}^k is the time required for an empty vessel *k* to go from *j* to *i*, T_k is the time available for vessel *k* during the shipping season, X_{ij}^{k} is the number of loaded trips to be made by vessel *k* from origin *i* to destination *j*, x_{ij}^{k} is the number of empty trips to be made by vessel *k* from destination *j* to origin *i* and z_k is the amount of "slack" or unused time for vessel *k*. In the paper, the decision variables X_{ij}^{k} and x_{ij}^{k} are relaxed into continuous variables.

Bradley et al. (1977) presented a linear programming formulation for planning the mission and composition of the US merchant marine fleet. The objective was to determine the number of ships of different types, and the voyages which satisfied the annual shipping requirements ("mission") on a defined set of possible routes, at a minimum present value cost. However, a number of simplifying assumptions had to be made in order to facilitate the formulation of this model as a linear programming problem. The restrictions of the linear framework thus limit its accuracy in modeling specific shipping services.

2.2.2 Nonlinear Programming Models

Benford (1981) developed a nonlinear programming model for selecting the most profitable fleet deployment strategy while satisfying customer demands, by means of a trial and error method. The objective of the procedure was to select the mix of available ships and sea speeds that would perform the required service at maximum profitability to the owner. The paper focused on two specific ports with a given quantity of commodities. It assumed that there were more than enough ships to meet the customers' demands, and that there were no appreciable costs or benefits involved in taking excess ships out of service. It first estimated the economic characteristics of each ship when operated at a range of reduced speeds, which involved the annual transport capacity (tons), annual operational cost, unit transportation cost and corresponding speeds. Then, it searched for the minimum operating cost by means of trial and error.

Perakis (1985) used Lagrangian multipliers to solve the same problem and obtained a better solution. Based on Benford (1981), the annual capacity of a ship was assumed to be a linear function and the associated operating cost per ton was a quadratic function with respect to speed. This gave the annual capacity of a ship of type *i* operating at speed x_i to be

$$\alpha_i x_i + \beta_i \tag{2.18}$$

and the operating cost per ton was given by

$$\gamma_i x_i^2 + \delta_i x_i + \varepsilon_i \ \gamma_i x_i^2 + \delta_i x_i + \varepsilon_i \tag{2.19}$$

Hence, annual operating costs for each ship can be denoted by:

$$a_i x_i^3 + b_i x_i^2 + c_i x_i + d_i = \left(\alpha_i x_i + \beta_i\right)\left(\gamma_i x_i^2 + \delta_i x_i + \varepsilon_i\right) \tag{2.20}$$

The objective function is:

$$\min \sum_{i=1}^{N} n(i)\left(a_i x_i^3 + b_i x_i^2 + c_i x_i\right) + \sum_{i=1}^{N} n(i)\, d_i \tag{2.21}$$

subject to

$$\sum_{i=1}^{N} n(i)\left(\alpha_i x_i\right) = C_0 - \sum_{i=1}^{N} n(i)\,\beta_i \tag{2.22}$$

The problem can be equivalently stated by using Lagrange multipliers, as follows:

$$\min L = \min \sum_{i=1}^{N} n(i)\left(a_i x_i^3 + b_i x_i^2 + \left(c_i + \lambda \alpha_i\right) x_i\right) + \sum_{i=1}^{N} n(i)\, d_i - \lambda\left(C_0 - \sum_{i=1}^{N} n(i)\,\beta_i\right) \tag{2.23}$$

Solution of (2.23) can be obtained by setting

$$\frac{\partial L}{\partial x_i} = 0 \tag{2.24}$$

$$\frac{\partial L}{\partial \lambda} = 0 \tag{2.25}$$

The paper then supposed that there are N groups of $n(i)$ identical ships, and that C_0 (tons) is the required annual carrying capacity between two given ports. From Eq. (2.25), we have

$$x_i = \frac{\sqrt{b_i^2 - 3a_i\left(c_i + \lambda\alpha_i\right)} - b_i}{3a_i}, i = 1, \ldots, N \tag{2.26}$$

Substituting (2.26) into (2.22), we get:

$$\sum_{i=1}^{N} \frac{n(i)\alpha_i}{3a_i}\left(\sqrt{b_i^2 - 3a_i\left(c_i + \lambda\alpha_i\right)} - b_i\right) = C_0 - \sum_{i=1}^{N} n(i)\beta_i \tag{2.27}$$

Eq. (2.27) can be numerically solved by secant method. The optimal value of λ obtained from (2.27) is then substituted into (2.23) to give us the optimal speeds $x_1, \ldots, x_N$.

Perakis and Papadakis (1987a, b) developed a new nonlinear programming model for the same problem as was considered in Benford (1981) and Perakis (1985). Perakis and Papadakis (1987a) classified the speeds of ships into two classes: ballast speeds and full-load speeds. The objective was to determine each vessel's full-load and ballast speeds such that the total fleet operating cost was minimized and all contracted cargo was transported. Given a fleet of Z ships, each with a given full-load cargo-carrying capacity, and each having known operating cost characteristics as functions of vessel speed, for each individual vessel in the fleet, the total operating costs per ton and total tons carried per year over a specific trade route was expressed as two functions with respect to full-load and ballast speeds, denoted by $F_i\left(X_i, Y_i\right)$ and $G_i\left(X_i, Y_i\right)$. Then, the total operating cost of a vessel per year was given by

$$C_i\left(X_i, Y_i\right) = F_i\left(X_i, Y_i\right) G_i\left(X_i, Y_i\right) \tag{2.28}$$

The optimization problem was to minimize the annual total operating cost of the fleet on the specified route, as follows:

$$\min \sum_{i=1}^{Z} C_i\left(X_i, Y_i\right) \tag{2.29}$$

subject to the following constraints:

$$X_{i\min} \le X_i \le X_{i\max}, i = 1, \ldots, Z \tag{2.30}$$

$$Y_{i\min} \le Y_i \le Y_{i\max}, i = 1, \ldots, Z \tag{2.31}$$

$$\sum_{i=1}^{Z} G_i\left(X_i, Y_i\right) = C_{av} \tag{2.32}$$

where X_i and Y_i respectively denote the full-load and ballast speed of ship i, X_{imax} and X_{imin} denote the upper and lower bounds of X_i respectively (Y_{imax} and Y_{imin} are defined similarly), and C_{av} is the cargo available for transporting by the fleet. The authors employed the Nelder and Mead Simplex Search Technique and the External Penalty Technique to solve their model. However, the number of round trips obtained using the optimal solution is not an integer.

Thus, in order to find the optimal solution with an integer number of round trips, a sequential optimization approach was used by Perakis and Papadakis (1987b). These were taken to be the integral part (or the integral part plus one) of the real numbers of round trips obtained. Also, in this paper, one or more costs were assumed to be random variables with known probability density functions. Those costs were the fuel price, the constant costs (which is the sum of the annual manning, administrative, maintenance, supplies, and equipment costs) for each ship, and the port and route charges for each ship. Analytical expressions for the basic probabilistic quantities, that is, the probability density function, and the mean and variance of the total operating cost were presented in the paper. The objective was to minimize the expected value of the total operating cost:

$$\min \bar{C} = A \cdot \bar{C}_f + B \cdot \bar{C}_{prn} + \sum_{k=1}^{K} \left(D_k + E_k \cdot \bar{C}_{km} \right) + F \tag{2.33}$$

where $\bar{C}_f$, $\bar{C}_{prn}$ and $\bar{C}_{km}$ denote the expected values of the fuel price, the constant costs and the port and route charges for each ship.

In the above articles (Benford, 1981; Perakis, 1985; Perakis and Papadakis, 1987a, b), these authors considered a fleet deployment problem with one origin and one destination, that is, two specific ports. Papadakis and Perakis (1989) extended this to consider a fleet deployment problem with multiple origins and destinations, studying the problem of minimizing the cost of operating a fleet of ships that has to carry a specific amount of cargo from a set of loading ports (origins) to a set of unloading ports (destinations) in a given time period. The paper formulated the operating cost as a nonlinear function with respect to the full-load and ballast speeds of the ships, in the same way as Perakis and Papadakis (1987a, b) did. The objective function is given by:

$$\min \sum_{i,j,k} \left(V_{i,j,k} N_{i,j,k} + U_{i,j,k} M_{i,j,k} \right) + \sum_k L_k Z_k \tag{2.34}$$

subject to

$$\sum_{i,j} \left(\left(\frac{d_{i,j}}{24 X_{i,j,k}} + t_{i,j,k} \right) N_{i,j,k} + \left(\frac{d_{i,j}}{24 Y_{i,j,k}} + t'_{i,j,k} \right) M_{i,j,k} \right) + Z_k = T_k, k = 1, ..., K \tag{2.35}$$

$$\sum_{i,k} W_k N_{i,j,k} = B_j, j = 1, ..., J \tag{2.36}$$

$$\sum_{j,k} W_k N_{i,j,k} = Q_i, i = 1, ..., I \tag{2.37}$$

$$\sum_{i} N_{i,j,k} = \sum_{i} M_{i,j,k}, k = 1,...,K; j = 1,...,J \tag{2.38}$$

$$\sum_{j} N_{i,j,k} = \sum_{j} M_{i,j,k}, k = 1,...,K; i = 1,...,I \tag{2.39}$$

$$X_k^{\min} \le X_{i,j,k} \le X_k^{\max} \tag{2.40}$$

$$Y_k^{\min} \le Y_{i,j,k} \le Y_k^{\max} \tag{2.41}$$

where $M_{i,j,k}$, $N_{i,j,k}$, $X_{i,j,k}$, $Y_{i,j,k}$ and Z_k are the decision variables, $M_{i,j,k}$ denotes the number of ballast trips for vessel *k* from port *j* to port *i*, while $N_{i,j,k}$ denotes the number of full load trips, $X_{i,j,k}$ and $Y_{i,j,k}$ respectively denote the full-load and ballast speeds, Z_k denotes the idle time for vessel *k*, $U_{i,j,k}$ is the total operating cost of vessel *k* traveling from port *i* to port *j* in ballast conditions, while $V_{i,j,k}$ is the equivalent under full-load conditions, L_k is the daily lay-up cost, $d_{i,j}$ is the distance between port *i* and port *j*, $t_{i,j,k}$ is the time required for vessel k to unballast and load at *i* plus any time required to travel from port *i* to port *j*, $t'_{i,j,k}$ is similar to $t_{i,j,k}$ for unloading at port *j* plus a ballast trip from port *j* to port *i*, T_k is the time available for vessel *k* during the shipping season, W_k denotes the cargo capacity of vessel *k*, B_j is the amount of cargo to be delivered to the destination port *j* and Q_i denotes the available amount of cargo at source port *i*. The authors analyzed the properties of their model and found that $Y_{i,j,k}$ could be expressed as a function with respect to $X_{i,j,k}$, and that $X_{i,j,k}$ is the solution to an equation. In other words, the decision variables $X_{i,j,k}$ and $Y_{i,j,k}$ can be eliminated from their model. Finally, they applied a projected, augmented Lagrangian algorithm to find the optimal solution.

2.2.3 Integer Programming Models

Cho and Perakis (1996) considered the liner fleet deployment problem combined with the routing problem. Given a set of ships, the paper aimed to assign each ship to some mix of routes among a finite set of candidate routes so as to minimize the total cost or maximize the total profit. Two programming models were formulated: a linear programming model and a mixed integer programming model. An augmented flow-route incidence matrix was introduced to facilitate the expression of the models. The linear programming model is given by:

$$\max \sum_{r=1}^{R} \sum_{k \in K_r} \pi_{rk} x_{rk} \tag{2.42}$$

subject to

$$\sum_{r=1}^{R} \sum_{k \in K_r} a_{ij,rk} x_{rk} \geq m_{ij}, \forall (i, j) \tag{2.43}$$

$$\sum_{r \in R_k} t_{rk} x_{rk} \leq t_k, k = 1, ..., K \tag{2.44}$$

where x_{rk} is the decision variables (fractions, not integers), π_{rk} denotes the expected profit from a round trip on route *r* by ship *k*, $a_{ij,rk}$ is a component of the augmented flow-route incidence matrix, m_{ij} is the minimum required number of trips from port *i* to port *j*, t_{rk} is the total travel time for ship *k* on route *r* per round trip, t_k is the maximum time ship *k* is available during the planning horizon, R_k is the set of routes *r* to which ship *k* can be assigned, and K_r is a set of available ships that can be assigned to route *r*. The problem can also be represented in matrix form as follows:

$$\max \pi x \tag{2.45}$$

subject to

$$\bar{A}x \geq m \tag{2.46}$$

$$Tx \leq t \tag{2.47}$$

Mourão et al. (2001) presented an application of an integer programming model that could be used to support the decision-making process for assigning ships with hub and spoke constraints, solving the model by means of the MS Excel solver function. In this paper, three levels of ports are identified: The main port is part of the medium-sized transport network that feeds mainline ocean trading and is the principle cargo origin and destination. The hub port represents the consolidation port which links the medium-sized network with the smaller transport network and embeds the terminal ports. Finally, the spoke port is the terminal port, where cargo is delivered to the end consumer. The ships are classified into two types: mainline and feeder ships. The mainline ships move between the main ports and the hub ports, while the feeder ships link the hubs to each set of spoke ports. Two scenarios are proposed: Scenario A consists of scheduling the main and the feeder ships as if a coordinated voyage situation is anticipated, and assumes that a fixed number of voyages are performed each year by each ship, whether main or feeder vessels. Scenario B is constructed exclusively to perform a sensitivity analysis of the solution obtained for Scenario A. Hence, Scenario B sets out to determine the optimum number of voyages each ship should undertake annually, in accordance with each roster. Two integer programming models are formulated for the two scenarios. Finally, MS Excel's solver is employed to solve the two models.

2.2.4 Simulation Models

Some of the simulation models described previously in Section 2.1.2 are also applied to the fleet deployment problem, such as MOPASS (Stott and Douglas, 1981), and the cost-based simulation model (Gallagher and Meyrick, 1984). Since they were described earlier, they are omitted from this section. The following paragraph describes a new simulator, developed for fleet deployment problems.

Xie (1997) proposed a new simulator—the Fleet Planning System (FPS)—which is an optimization-based decision support system for a fleet of heterogeneous vessels, aimed at optimizing their deployment and development planning. FPS takes the characteristics of each type of vessel to be known parameters, such as their size, transportation capacity and the costs incurred on each liner trade route. The number of vessels of each type assigned to each route are the decision variables. The minimum cost of shipping the specified and required amount of cargo is the objective and linear programming techniques are the main method used to optimize the assignment strategy for each vessel and the development planning for the fleet. FPS is coded in the FORTUNE Language and consists of two main programming modules: RDATA and LP. RDATA is designed to read in the initially known data and turn these data into parameters in the linear programming model. LP firstly transforms the linear programming model into a standard linear programming model and then checks the validity of the coefficients of the model and the solvability of the problem. Finally, it optimizes the calculation by means of the simplex algorithm and prints out the results.

2.3 Fleet Planning

Fleet size and mix problems are strategic-level problems while fleet deployment problems are tactical-level problems. Agarwal and Ergun (2008) pointed out that the decisions made at one planning level affected the decision-making at the other. The decisions at the strategic level set the general policies and guidelines for decisions to be made at the tactical level. In the reverse direction, information on the costs and revenues generated by the system, given the set parameters, provides much-needed feedback for decision-making at the higher level. Therefore, fleet size and mix problems and fleet deployment problems are combined by some researchers, who assume that the planner not only decides the fleet size and mix, but also the fleet deployment. These joint problems are referred to as fleet planning problems in this thesis for convenience. Three types of model have been proposed for fleet planning problems: linear programming, integer programming and dynamic programming. They are reviewed in the following sections.

2.3.1 Linear Programming Models

Everett et al. (1972) applied a linear programming approach in order to optimize a fleet of large tankers and bulkers, and proposed the following model to minimize the life-cycle cost of the fleet:

$$\min \sum_s n_s I_s + \sum_s \left(\sum_r C_{sr} x_{sr} + n_s a_s \right) \sum_{t=1}^{T} \left(\frac{1+\beta}{1+\alpha} \right)^t \tag{2.48}$$

subject to

$$\sum_s \sum_r V_{srk} x_{sr} = d_k \tag{2.49}$$

$$\sum_r t_{sr} x_{sr} - 345 n_s \le 0 \qquad (2.50)$$

where n_s and x_{sr} are decision variables denoting the number of ships of type *s* and the number of voyages per annum assigned to ship *s* along route *r*, respectively, I_s is the capital cost for ship type s, C_{sr} is the variable operating cost incurred by ship type s along route *r* while a_s is the annual fixed operating cost of ship type s, α is the annual discount rate and β is the annual inflation rate, *T* is the length of the planning horizon (years), V_{srk} is the maximum amount of commodity *k* which can be carried by ship type *s* along route *r*, d_k is the total annual tonnage of commodity *k* specified in the mission for the pertinent pair of ports, t_{sr} is the time taken to make a round trip by ship type *s* along route *r*, and all ships are assumed to be available 345 days per year. The model was solved by means of the Control Data Corporation's "Ophelie" linear programming system, available on the CDC 6600 model computer. In addition, the necessary inputs to the linear programming model were computed by means of CDC's Matrix Generator Language (MGL) program that allowed for the automatic computation of the basic data required by the model. However, the solutions are fractions rather than integers.

Perakis and Jaramillo (1991) proposed a linear programming model for an optimal fleet size, mix and deployment with detailed cost estimation for liner ships. First of all, they described the costs spent on making a round trip, involving port charges, canal fees, fuel costs, maintenance costs, insurance costs, administrative costs, crew costs, and other miscellaneous costs, and then formulated the cost per voyage as a function of the cruising speed of the container ships. The shipping cost per voyage can be expressed by the following equation:

$$C_{kr}\left(s_{kr}\right)=\bar{\lambda}_{kr}s_{kr}^{2}+\tilde{\lambda}_{kr}/s_{kr}+\hat{\lambda}_{kr} \tag{2.51}$$

where $\bar{\lambda}_{kr}$, $\tilde{\lambda}_{kr}$ and $\hat{\lambda}_{kr}$ are parameters. Eq. (2.51) is a convex function, which implies that the optimal cruising speeds of container ships can be obtained so as to minimize shipping costs. Finally, a linear programming model is developed as follows:

$$\min\sum_{k=1}^{K}\sum_{r=1}^{R}C_{kr}X_{kr}+\sum_{k=1}^{K}e_{k}Y_{k} \tag{2.52}$$

subject to

$$\sum_{r=1}^{R}t_{kr}X_{kr}+Y_{k}=365N_{k}^{\max},\quad \text{for all } k \tag{2.53}$$

$$\sum_{k=1}^{K}X_{kr}\geq\frac{365}{F_{r}},\quad \text{for all } r \tag{2.54}$$

$$Y_{k}\geq\left(365-T_{k}\right)N_{k}^{\max} \tag{2.55}$$

$$X_{kr},Y_{k}\geq 0 \tag{2.56}$$

where X_{kr} and Y_k are decision variables, denote the number of annual voyages and lay-up days of ship *k* on route *r*, respectively, e_k is the total daily lay-up cost for ship *k*, t_{kr} is the voyage time of ship *k* on route *r*, F_r is the frequency of service on route *r*, T_k is the available shipping days for ship *k* per year, $N_k^{\max}$ is the number of ships of type *k* available, and *K* and *R* are the number of ships and routes considered in the paper. The implementation and results were shown in Jaramillo and Perakis (1991), obtained using LINDO solver.

2.3.2 Integer Programming Models

Cho and Perakis (1996) proposed a mixed integer programming model for a long-term ship fleet planning problem. They assumed that a shipping company has to make capital investment decisions over the planning horizon. To meet the expected increasing future cargo demand, the shipping company may consider various options for expanding fleet capacity, such as building or purchasing new ships, or chartering in ships. The objective is to minimize the total cost incurred from operations while meeting the cargo demands over the planning horizon. The total cost included in the objective function is taken to be the sum of the operating cost, the lay-up (or idle) cost, and the (fixed) capital cost incurred over the planning horizon. Let K^0 be the subset of ships that the shipping company considers adding to the existing fleet. The resulting objective function is as follows:

$$\min \sum_{r=1}^{R} \sum_{k \in K_r} c_{rk} x_{rk} + \sum_{k=1}^{K} h_k y_k + \sum_{k \in K^0} f_k z_k \tag{2.57}$$

subject to

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k = t_k, \forall k \notin K^0 \tag{2.58}$$

$$\sum_{r \in R_k} t_{rk} x_{rk} + y_k - t_k z_k = t_k, \forall k \in K^0 \tag{2.59}$$

$$\sum_{r=1}^{R} \sum_{k \in K_r} a_{ij,rk} x_{rk} \geq m_{ij}, \forall (i, j) \tag{2.60}$$

$$x_{rk} \geq 0, y_k \geq 0, z_k \in \{0,1\} \tag{2.61}$$

where x_{rk} is the set of decision variables (fractions, not integers), y_k is another decision variable, denoting the lay-up time of ship k, and the variable z_k is a binary variable denoting whether ship k will be added to the fleet (z_k = 1) or not (z_k = 0). $a_{ij,rk}$ is a component of the augmented flow-route incidence matrix, c_{rk} is the expected operating cost of ship k on route r per round trip, f_k denotes the fixed capital cost involved in adding ship k to the existing fleet, h_k is the lay-up cost of a ship per unit of time, t_{rk} is the total travel time for ship k on route r per round trip, t_k is the maximum time ship k is available during the planning horizon, m_{ij} is the minimum required number of trips from port i to port j, R_k is the set of routes r to which ship k can be assigned, K_r is a set of available ships that can be assigned to route r, K^0 is the subset of ships which the company considers adding to the fleet.

The number of ships allocated to each route in Perakis and Jaramillo (1991) was a real number, and not an integer. A rounding procedure was therefore required to make the number of ships allocated to each route into an integer. The rounding led to some variation in the targeted service frequencies and therefore to sub-optimal results. In order to eliminate any rounding errors in the linear programming formulation, Powell and Perakis (1997) reformulated this problem and proposed an integer programming model which they solved by means of OSL solver. The integer programming optimization model is given by:

$$\min \sum_{k=1}^{K} \sum_{r=1}^{R} C'_{kr} N_{kr} + \sum_{k=1}^{K} Y_k e_k \tag{2.62}$$

Subject to

$$\sum_{r=1}^{R} N_{kr} \leq N_k^{\max} \text{ for each of type } k \tag{2.63}$$

$$\sum_{k=1}^{K} t'_{kr} N_{kr} \geq M_r \text{ for all } r \tag{2.64}$$

$$Y_k = 365N_k^{\max} - T_k \sum_{r=1}^{R} N_{kr} \tag{2.65}$$

$$N_{kr} \geq 0 \tag{2.66}$$

where N_{kr} and Y_k are the decision variables denoting the number of ships of type k operating on route r and the number of lay-up days per year for a ship of type k, respectively, C'_{kr} is the operating cost of a type k ship on route r, e_k is the total lay-up costs per day for a type k ship, M_r is the number of voyages required per year on route r, t'_{kr} is the yearly voyages made by a ship of type k on route r, and T_k is the duration of the shipping season for a ship of type k. The LINDO solver was employed to solve this linear programming model.

Gelareh and Meng (2010) looked at model development for a LSFP problem. First, a mixed integer nonlinear programming model was presented. Then, the proposed nonlinear model was linearized by means of a linearization technique and a mixed integer programming model was obtained that can be solved efficiently using a standard mixed integer programming solver such as CPLEX. The mixed integer programming model determines the optimal route service frequency pattern and takes into account the time window constraints of shipping services.

2.3.3 Dynamic Programming Models

Xie et al. (2000) studied the fleet planning problem for a long-term planning horizon. Due to the strategy involved in fleet planning, a horizon of several years can naturally be deconstructed into a series of consecutive decisions, made at the beginning of each year. The problem is broken down into two optimal subproblems: one is to get the annual optimal fleet deployment plan if fleet and transport demand is fixed, and the other is to get the optimal strategy for fleet development in consecutive years. Therefore, the first subproblem can be formulated as a linear integer programming model which seeks the optimal fleet deployment for a short-term planning horizon (one year) and the second subproblem can be formulated as a dynamic programming model, seeking the best liner fleet size and mix over a long-term planning horizon. In the dynamic programming model, one year is taken as one stage and the quantitative composition of a fleet in terms of ships of various types is taken to be the state of the fleet. The optimization of fleet deployment for the first subproblem can be written as follows:

$$\min Z_{ti} = \begin{cases} \sum_{j=1}^{K} \left(\sum_{h=1}^{G} X_{jhti} R_{jht} + O_{jti} F_{jt} \right) & \text{if } \bar{X}_j \in \bar{\Omega} \\ \infty & \text{if } \bar{X}_t \in \bar{\Psi} \end{cases} \tag{2.67}$$

where $\bar{X}_t$ is the decision vector, denoting the deployment scheme of ships in year *t*, and each element X_{jhti} denotes the number of ships of type *j* distributed on route *h* in the i^{th} state in year *t*, and $\bar{\Omega}$ is a set of $\bar{X}_t$, which meets the following two groups of constraints:

$$\sum_{j=1}^{K} X_{jhti} V_{jht} = W_{ht}, h = 1,...,G \tag{2.68}$$

$$\sum_{h=1}^{G} X_{jhti} + O_{jti} = U_{j,t-1} - WT_{jt} + C_{jti}, j = 1,...,K \tag{2.69}$$

where $\bar{\Psi}$ is the complementary set of $\bar{\Omega}$, C_{jti} is the number of ships of type *j* added to the fleet in the i^{th} state at the beginning of year *t*, F_{jt} is the annual laid-up costs for a ship of type *j* in year *t*, O_{jti} is the number of laid-up ships of type *j* in the i^{th} state in year *t*, R_{jht} is the annual running costs of a ship of type *j* on route *h* in year *t*, $U_{j,t-1}$ is the number of ships of type *j* before the start of year *t*, V_{jht} is the annual transportation capacity of a ship of type *j* on route *h* in year *t*, W_{ht} is the annual transportation demand on route *h* in year *t*, and WT_{jt} is the number of ships of type *j* that are scrapped or out of commission in year *t*. The accumulated sum of the costs of running the fleet in the i^{th} state from year *t* to year *N*, ZP_{ti}, that is, the recursive formulation, is given by:

$$ZP_{ti} = ZP_{t+1,i} + \frac{Z_{ti}}{(1+\alpha)^t} + \frac{1}{(1+\alpha)^t}\sum_{j=1}^{K} C_{jti} S_{jt} - \frac{\beta}{(1+\alpha)^N} L(N-t) \qquad (2.70)$$

where $L(N-t)$ denotes the physical residual value, at the end of the planning horizon, of the new ships that were added into the fleet in year *t*, S_{jt} is the market price for a ship of type *j* in year *t*, α is the discount rate and β is the weight coefficient. The optimal strategy can be obtained by solving the following optimization model:

$$\min_{i=1,\ldots,M} ZP_{0i} \qquad (2.71)$$

where *M* represents the number of various combinations of ships that can be added to the fleet at the beginning of year 0. Finally, a heuristic algorithm is proposed to solve the problem.

2.4 Research Limitations and Gaps

It can be seen from the literature review that there are some limitations and gaps in the existing studies. This section highlights these and shows how they provoke the need for further investigation. The limitations of past studies fall under the following three types.

Firstly and most importantly, all of the previous work reviewed above assume an environment in which the container shipment demand between port pairs is known beforehand and is deterministic. The container shipment demand between port pairs can be forecasted using regressions and time series models. However, they can never be forecasted with complete confidence because they are affected by some unpredictable and uncontrollable factors. Therefore, it is more reasonable to regard the demand as uncertain.

Secondly, the parameters of annual operating cost and transportation capacity of each ship on each route are assumed to be constants in Cho and Perakis (1996) and Xie et al. (2000). Such an assumption is unreasonable because it is inconsistent with reality. In fact, these parameters should be voyage-dependent. For example, a ship sailing twenty voyages on a route in one year will definitely incur greater annual operating costs and have a larger transportation capacity than a ship that sails only ten voyages on this route.

Thirdly, the methodology proposed by Cho and Perakis (1996) for a multi-period ship fleet planning problem is unreasonable. In their methodology, once the decisions about fleet design and fleet deployment are made at the beginning of the planning horizon, these decisions are assumed to be fixed and static over the whole multi-period planning horizon. Such a period-independent model cannot characterize the realistic dynamic decision strategy: the fleet size, mix and ship-to-route allocation should be adjustable period-by-period, due to the fact that container shipment demand is period-dependent.

Based on the limitations observed in previous studies, it is realistic and necessary to take uncertainty of container shipment demand into account in LSFP problems. By considering demand uncertainty, the LSFP problem could become a new and interesting research topic, providing a fresh angle on the classical LSFP problem which is studied under a deterministic environment. The models proposed for classical LSFP problems in previous studies cannot be used directly here. Therefore, the first purpose of this thesis is to propose new models for LSFP problems with container shipment demand uncertainty and then to propose effective solution algorithms to solve the new models.

In addition, this thesis revises the unreasonable assumptions described above, in order to consider a more realistic LSFP problem than has been studied previously in the literature. Moreover, it provides an applicable and feasible way for a liner container shipping company to carry out its liner ship fleet planning in practice.

2.5 Summary

This chapter has presented a critical literature review, focusing on three problems: fleet size and mix problems, fleet deployment problems and fleet planning problems. Through this review, several potential problems and gaps have been identified. Finally, the chapter has described the research purpose of this thesis.

CHAPTER 3 A CHANCE CONSTRAINED PROGRAMMING MODEL FOR SHORT-TERM LSFP

3.1 Introduction

This chapter deals with the short-term LSFP problem encountered by a liner container shipping company. The liner container shipping company (or liner operator) usually operates a fleet of heterogeneous ships on its service routes at regular schedule in order to pick up and deliver containers for shippers. In order to seize market share in an intensely competitive container shipping market, the liner container shipping company is constantly searching for models and solution procedures to build a decision support system that helps to create cost-effective plans to operate its liner ship fleet. In addition, the number of containers transported by a liner container shipping company between two ports often varies season (3 months) by season in practice. For example, container volume from Asia to Europe usually increases dramatically in the fourth quarter of a year due to Christmas Day. To cope with the varying port-to-port container shipment demand, a liner container shipping company has to alter its service routes and redeploy ships according to the estimated container shipment for next season. In other words, its strategic asset management department needs to make a suitable fleet plan for a short-term (3-6 months) planning horizon, which involves considering how to effectively use the ships in its fleet in order to provide efficient shipping services and save on costs. The decisions include the determinations of fleet size (number of ships), mix (ship types) and deployment (ship-to-route assignment). For the sake of presentation, this tactical-level decision is referred to as the short-term liner ship fleet planning (LSFP) problem. The aim is to optimize fleet design and deployment over a short-term planning horizon. The fleet design identifies the types and numbers of ships required, and the fleet deployment covers how the fleet is assigned and operated to transport containers.

Container shipment demand of a port pair on a liner ship route operated by the liner container shipping company is an input of the short-term LSFP problem. The decisions of fleet size, mix and fleet deployment involved in this problem are made prior to knowing the exact market demand. Liner shipping is usually based on a fixed schedule which is generally published up to 6 months into the future. This means the liner ship fleet planning is made depending on the forecasted container shipment demand. The container shipment demand is usually estimated by some shipment demand forecasting methods. Compared with the actual port-to-port container shipment demand, the forecasted shipment demand is inevitably biased because it is usually affected by some unpredictable and uncontrollable factors, such as the shipping contract cancellation by shippers due to manufacturing interruption or transportation delay from plants to ports. Hence, container shipment demand is of high uncertainty in practice. This chapter thus focuses on model development for the short-term LSFP problem by taking into consideration uncertainty of container shipment demand.

It should be pointed out that the existing studies on fleet size and mix problems (such as Dantzig and Fulkerson, 1954; Lane et al., 1987; Fagerholt, 1999; Fagerholt and Lindstad, 2000; Sambracos et al., 2004), fleet deployment problems (such as Laderman et al., 1966; Bradley et al., 1977; Benford, 1981; Perakis, 1985; Perakis and Papadakis, 1987a, b; Papadakis and Perakis, 1989; Mourão et al., 2001); fleet planning problems (such as Everett et al., 1972; Perakis and Jaramillo, 1991; Powell and Perakis, 1997; Gelareh and Meng, 2010) assume that container shipment demand between two ports are deterministic. As discussed previously, container shipment demand has high uncertainty in practice. This uncertainty can be formulated as the stochastic container shipment demand represented by a random variable. Having assumed the stochastic container shipment demand, the above-mentioned three categories of the decision problems should be re-formulated. The existing linear or integer models reviewed above for the deterministic LSFP problems are not applicable for the proposed problem. Therefore, a new stochastic programming model for uncertain container shipment demand is needed to formulate this problem. It should be pointed out that stochastic programming (see Shapiro et al., 2009) has served as a useful modeling tool in decision-making problems under uncertain environment, such as dynamic resource allocation (Cheung and Powell, 1996; Godfrey and Powell, 2001), optimal fleet assignment problems (Norkin et al., 1998a; Sherali and Zhu, 2008), empty container allocation problems (Crainic et al., 1993; Cheung and Chen, 1998), vehicle routing problems (Laporte et al., 1992; Laporte et al., 2002) and supply chain design (Santoso et at., 2005; Schutz et al., 2009).

In this chapter, the container shipment demand between any two ports on each liner ship route is assumed to follow a normal distribution; the probability (chance) that shipping capacity of a liner ship fleet planning scenario cannot meet the demand does exist. In other words, the liner container shipping company fails to make the service for its customers with this probability. To maintain a certain level of service, the company must control this probability (or chance) within a given level called confidence parameter.

The level of service is referred to as chance constraint hereafter. Therefore, a chance constrained programming model is proposed for the short-term LSFP problem with container shipment demand uncertainty. The objective of this chance constrained programming model is to minimize the total operating cost of its fleet subject to a certain level of service.

The remainder of this chapter is organized as follows: Section 3.2 gives notation, assumptions and problem statement. Section 3.3 presents a chance constrained programming model for the short-term LSFP problem with container shipment demand uncertainty. Section 3.4 is a numerical example to assess the proposed model and analyze impact of the confidence parameters and container shipment demand on optimal solutions. Summary is presented in Section 3.5.

3.2 Problem Description, Assumptions and Notations

3.2.1 Code of Port Sequence

Consider a liner container shipping company which provides liner shipping service on a predetermined liner ship route network for shippers within a short-term planning horizon (3-6 months). Let $\mathcal{P}=\{1,\ldots,p,\ldots P\}$ and $\mathcal{R}=\{1,\ldots,r,\ldots,R\}$ denote the set of ports and the set of liner ship routes in the liner ship route network, respectively. The indices p and r represent a particular port and liner ship route, respectively. Additionally, we define $\mathcal{P}_r=\{p_r^1,\ldots,p_r^i,\ldots p_r^{m_r}\}$ as the set of ports called at the liner ship route $r\in\mathcal{R}$, characterized by $\mathcal{P}=\bigcup_{r\in\mathcal{R}}\mathcal{P}_r$, where m_r is the number of ports in the itinerary.

Each liner ship route $r \in \mathcal{R}$ is defined as a sequence of ports called at by ships, which can be expressed by the port calling sequence (or itinerary):

$$p_r^1 \to p_r^2 \to \cdots \to p_r^{m_r} \to p_r^1 \tag{3.1}$$

Eq. (3.1) describes the unique characteristic of a liner ship route: a loop with a given port calling order. Note that the ports on a liner ship route may not all be distinct. For example, Figure 3.1 depicts a liner ship route between the port of Pusan and the port of Singapore. A ship deployed on this liner ship route first calls at Pusan (PS), followed by Shanghai (SH), Yantian (YT), Hong Kong (HK), Singapore (SG), Yantian (YT), and finally back to Pusan (PS). According to the route coding scheme shown in Eq.(3.1), this can be expressed by the port calling sequence:

$$p_r^1(\text{PS}) \to p_r^2(\text{SH}) \to p_r^3(\text{YT}) \to p_r^4(\text{HK}) \to p_r^5(\text{SG}) \to p_r^6(\text{YT}) \to p_r^1(\text{PS}) \tag{3.2}$$

Figure 3.1 also shows that the port calling sequence for the forward direction from Pusan to Singapore is not identical to that for the backward direction from Singapore to Pusan.

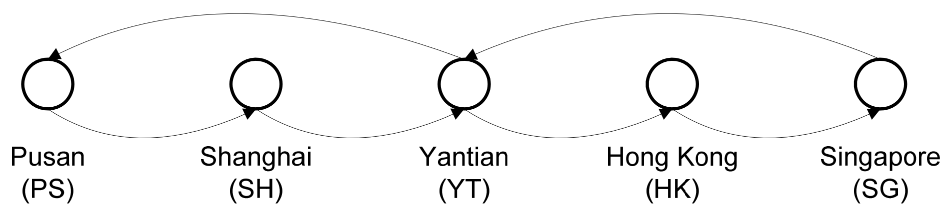

Figure 3. 1 A liner ship route

3.2.2 Container Shipment Flow

To formulate the feature that the first port and last port on a liner ship route are the same, we introduce a generalized mod operator as follows:

$$i \ \underline{\text{mod}} \ m_r = \begin{cases} i \text{ mod } m_r, & i \neq m_r \\ m_r, & i = m_r \end{cases} \tag{3.3}$$

For the sake of presentation, the distance between two consecutive ports p_r^i and $p_r^{(i+1)\underline{\text{mod}} m_r}$ on the liner ship route $r \in \mathcal{R}$ is referred to as *leg* i ($i = 1, 2, \cdots, m_r$), denoted by the pair of ordered ports $< p_r^i, p_r^{(i+1)\underline{\text{mod}} m_r} >$. The liner ship route shown in Eq. (3.2) thus has six legs – 1: $<p_r^1(\text{PS}), p_r^2(\text{SH})>$, 2: $< p_r^2(\text{SH}), p_r^3(\text{YT}) >$, 3: $< p_r^3(\text{YT}), p_r^4(\text{HK})>$, 4: $< p_r^4(\text{HK}), p_r^5(\text{SG}) >$, 5: $< p_r^5(\text{SG}), p_r^6(\text{YT}) >$ and 6: $< p_r^6(\text{YT}), p_r^1(\text{PS}) >$.

The port calling sequence shown in Eq. (3.1) has a limited number of combinations of port pairs which may have container shipment demand on the liner shipping service route $r \in \mathcal{R}$, and these pairs of ports can be expressed by the set

$$\mathcal{M}_r = \left\{ \left(p_r^i, p_r^j \right) \middle| i, j = 1, 2, \cdots, m_r; p_r^i \neq p_r^j \right\} \tag{3.4}$$

An incidence parameter $\rho_l^{\left(p_r^i, p_r^j\right)}$ ($l = 1, 2, \cdots, m_r$) is defined to indicate how the containers are transported from port p_r^i to port p_r^j, namely the itinerary of transporting containers of port pairs (p_r^i, p_r^j). It equals 1 if leg l ($l = 1, 2, \cdots, m_r$) is sailed by ships transporting containers from port p_r^i to port p_r^j and 0 otherwise ($r \in \mathcal{R}$). The incidence parameter thus reflects the relationship between the itinerary for transporting containers from port p_r^i to port p_r^j and the legs l ($l = 1, 2, \cdots, m_r$) in the liner shipping service route. We use the above example to illustrate this. Containers being transported between the port pair $\left(p_r^2, p_r^1\right)$, namely from Shanghai to Pusan, have to be loaded at port p_r^2 (Shanghai) and then carried by ships along legs 2, 3, 4, 5 and 6, before finally being unloaded at port p_r^1 (Pusan). Therefore, $\rho_1^{\left(p_r^2, p_r^1\right)} = 0$ and $\rho_l^{\left(p_r^2, p_r^1\right)} = 1$ $\left(l = 2,3,4,5,6\right)$.

When a ship sails along leg *l* of route *r*, it carries containers including those new containers loaded at port p_r^l and also those loaded at previous ports that have remained on the ship, which is referred to as container shipment flow on leg *l*. Continuing with the above example, when a ship sails on leg 6, it carries containers corresponding to eight port pairs: $\left(p_r^2, p_r^1\right)$, $\left(p_r^3, p_r^1\right)$, $\left(p_r^4, p_r^1\right)$, $\left(p_r^5, p_r^1\right)$, $\left(p_r^3, p_r^2\right)$, $\left(p_r^4, p_r^2\right)$, $\left(p_r^5, p_r^2\right)$, and $\left(p_r^5, p_r^4\right)$, of which the containers being transported between the port pair $\left(p_r^3, p_r^1\right)$ and between the pair $\left(p_r^3, p_r^2\right)$ were newly loaded at port p_r^3, and the containers for the other six port pairs were loaded at previous ports.

3.2.3 Liner Ship Fleet Planning

Let $\mathcal{K} = \{1,...,k,...,K\}$ be the set of ship types available to the liner container shipping company, where the index *k* denotes a particular type of ships. The container capacity in terms of twenty-foot-equivalent unit (TEU) of a particular ship type *k* is denoted by V_k. The liner container shipping company has to determine the number of ships of type $k \in \mathcal{K}$ in its ship fleet and deploy them on each liner ship route $r \in \mathcal{R}$ to pick up and deliver containers for shippers at a regular schedule on each route.

In the short-term LSFP problem, the liner container shipping company not only uses its own ships to deliver containers, but also charters ships from other liner shipping companies. Generally, there are three types of chartering ships: bareboat charter, voyage charter and time charter. Bareboat charter is the simplest way in which the charterer manages the ship and pays all costs except the capital repayment, tax and depreciation. In other words, the owner does not bear any cost except collecting the rent from the charterer. In order to simplify the problem, bareboat charter is the one only adopted in this thesis. The chartering rate of a ship of type k in the planning horizon is denoted by c_k^{IN} (\$/ship). Besides paying the chartering rate to the ship owner, the ship charterer takes other charges of operating the chartered ship, such as routine maintenance cost and insurance and etc. Therefore, it is rational to assume that

$$c_k^{\mathrm{OUT}} < c_k^{\mathrm{IN}} \tag{3.5}$$

where c_k^{OUT} denotes the rate of a ship of type k chartered out (\$/ship).

Let N_k^{MAX} and NCI_k^{MAX} denote the number of available ships of type k owned and chartered by the liner container shipping company, respectively. Given these candidate ships, the liner container shipping company chooses some ships to form a liner ship fleet, namely, a ship fleet design plan comprising mix and size of the ship fleet; and then assigns the ships in the fleet to those ship routes. The objective is to make an efficient joint ship fleet design and ship fleet deployment plan in order to maximize the expected value of the total profits subject to some constraints. It is noted that regular shipping service is required to be maintained on each ship route because liner shipping is characterized by providing regular shipping service in contrast to tramp and industrial shipping (Christiansen et al., 2004). In practice, most liner shipping companies generally provides a weekly shipping service on a ship route.

3.2.4 Container Shipment Demand Uncertainty

In practice, the container shipment demand over the short-term planning horizon T (3-6 months) is estimated by using some demand forecast methods based on historical data. However, the estimated container shipment demand is biased and it thus causes uncertainty of the container shipment demand used for a short-term planning decision. Uncertainty of container shipment demand comes also from transactions between shippers and the liner container shipping company briefed as follows. A shipper firstly needs to book space from the shipping company according to ship schedule and itinerary launched by the company, to deliver its containers through a shipping agent by filling in a shipping application (S/A). If the S/A is accepted, the shipper will receive a shipping order (S/O) from the shipping company to load its containers on a ship operated by the shipping company. Then, the carrier (i.e. the liner container shipping company) will offer a mates receipt (M/R) to the shipper to show its containers are loaded on the ship. The shipper bears the M/R to exchange bill of lading (B/L) and posts it to the consignee. The shipping agent at the discharge port informs the consignee to retrieve the containers when they arrive. After the payment of all fees, the consignee uses B/L to exchange the delivery order (D/O) and takes delivery of goods. However, the shipper is allowed to cancel the transaction or contract signed with the shipping company in advance. The cancellation as an uncontrollable factor brings uncertainty of the estimated container shipment demand.

The distribution-based approach is a typical method to characterize the parameter uncertainty issue. It is usually used to describe the issue with exact concept or essence but whether it happens depends on some random factors, such as in the trial of flipping coin, the concept "face of coin" is exact but the occurrence is related to some unpredictably or uncontrollably uncertain factors. The container shipment demand uncertainty considered in this chapter has an exact concept but is related to some random factors. Therefore, the distribution-based approach is reasonable to be employed to formulate uncertainty of container shipment demand. Following the distribution-based uncertainty characterization approach, the container shipment demand is assumed normally distributed with given mean and standard deviation. The rationale of assuming the normal distribution is that the deviation of the forecasted demand and the real demand is often approximately normally distributed and especially the normal distribution has been established to be one of the suitable probability distribution to describe the demand uncertainty by Brown (1959). Without loss of generality, these normal random container shipment demands are assumed independent.

3.2.5 Problem Statement

As aforementioned, the container shipment demand between any two ports on each liner ship route is assumed following a normal distribution. This assumption may lead to another problem: since the demands are uncertain, one can hardly find any decision which would definitely exclude later constraint violation caused by unexpected random effects, in other words, once the decisions in LSFP problem are determined, the fleet of ships may be unable to fully meet the pickups and deliveries requirement for its customers, even though the expected demands along the route do not exceed the fleet capacity. Once such case happens, it implies losing money for this liner container shipping company. Since it is hardly unavoidable, the liner container shipping company hopes that it happens at a low possibility as possible.

In order to reduce the possibility of the occurrence that the liner container shipping company cannot satisfy the customers' demand, such constraint violation can often be balanced afterwards by some compensating decisions which are considered as a penalization for constraint violation. However, the compensation cannot be modeled by cost in this chapter because the container shipment demand is not realized. In such circumstances, we would rather insist on decisions guaranteeing feasibility 'as much as possible'. This loose term refers once more to the fact that constraint violation can almost never be avoided because of unexpected extreme events. On the other hand, when knowing or approximating the distribution of the random parameter, it makes sense to call decisions feasible whenever they are feasible with high probability, i.e., only a low percentage of realizations of the random parameter leads to constraint violation under this fixed decision. Therefore, we formulate the constraint that the liner container shipping company should satisfy the customers' demand as a probabilistic form in this chapter, which is called chance constraint. The probability of the constraint violation is called a confidence parameter in this chance constraint. It indicates that if the liner container shipping company makes a decision which satisfies the chance constraint, the event that the customers' demand cannot be met will occur at most with this probability. For those unmet cargoes, we regarded they are lost.

Therefore, the short-term LSFP problem with container shipment demand uncertainty aims to determine the best decision variables to minimize the total operating cost while maintaining the chance constraints. It is formulated as a chance constrained programming model.

3.3 Model Development

Before the development of mathematical programming model for the short-term LSFP problem with container shipment demand uncertainty, we firstly introduce the decision variables shown as follows:

n_{kr}^{OWN} number of owned ships of type k ($k \in \mathcal{K}$) assigned on route r ($r \in \mathcal{R}$)

n_{kr}^{IN} number of chartered in ships of type k ($k \in \mathcal{K}$) assigned on route r ($r \in \mathcal{R}$)

x_{kr} number of voyages of ships of type k ($k \in \mathcal{K}$) on route r ($r \in \mathcal{R}$)

3.3.1 Chance Constraints

Let $\xi^{\left(p_r^i, p_r^j\right)}$ be the random variable representing the container shipment demand of a port pair $\left(p_r^i, p_r^j\right) \in \mathcal{M}_r$, the container shipment flow on leg l $\left(l = 1, ..., m_r\right)$ of route $r \in \mathcal{R}$, denoted by η_l^r, is given by:

$$\eta_l^r = \sum_{\left(p_r^i, p_r^j\right) \in \mathcal{M}_r} \rho_l^{\left(p_r^i, p_r^j\right)} \xi^{\left(p_r^i, p_r^j\right)}, l = 1, \cdots, m_r; \forall r \in \mathcal{R} \tag{3.6}$$

Since the container shipment demand of any port pair, $\xi^{\left(p_r^i, p_r^j\right)}$ $\left(\left(p_r^i, p_r^j\right) \in \mathcal{M}_r, r \in \mathcal{R}\right)$, is assumed following an inter-independent normal distribution with a mean value denoted by $\mu^{\left(p_r^i,p_r^j\right)}$ and a variance denoted by $\sigma^{\left(p_r^i,p_r^j\right)^2}$, namely $\xi^{\left(p_r^i,p_r^j\right)} \sim N\left(\mu^{\left(p_r^i,p_r^j\right)},\sigma^{\left(p_r^i,p_r^j\right)^2}\right)$, according to the probability theory, η_l^r also follows a normal distribution, then we have:

$$\eta_l^r \sim N\left(\sum_{\left(p_r^i,p_r^j\right)\in\mathcal{M}_r} \rho_l^{\left(p_r^i,p_r^j\right)}\mu^{\left(p_r^i,p_r^j\right)}, \sum_{\left(p_r^i,p_r^j\right)\in\mathcal{M}_r} \rho_l^{\left(p_r^i,p_r^j\right)}\sigma^{\left(p_r^i,p_r^j\right)^2}\right), l=1,\cdots,m_r;\forall r\in\mathcal{R} \quad (3.7)$$

Let α_r denote the confidence parameter on route *r*, therefore, the constraints that the transportation capacity of containerships operated on this route is not less than each container shipment flow on leg *l* with a least probability of $1-\alpha_r$ can be formulated as the following chance constraints:

$$\Pr\left(\sum_{k\in\mathcal{K}} x_{kr}V_k \geq \eta_l^r\right)\geq 1-\alpha_r, l=1,\cdots,m_r;\forall r\in\mathcal{R} \quad (3.8)$$

3.3.2 Chance Constrained Programming Model

The proposed short-term LSFP problem with uncertain container shipment demand can be formulated as the chance constrained programming (CCP) model:

[CCP]

$$\min\ C=\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}}\left(c_{kr}x_{kr}+n_{kr}^{\text{IN}}c_k^{\text{IN}}\right) \quad (3.9)$$

Subject to:

$$\Pr\left(\sum_{k\in\mathcal{K}} x_{kr}V_k \geq \eta_l^r\right)\geq 1-\alpha_r, l=1,\cdots,m_r;\forall r\in\mathcal{R} \quad (3.10)$$

$$x_{kr}\leq\left(n_{kr}^{\text{OWN}}+n_{kr}^{\text{IN}}\right)\times\left\lfloor\frac{T}{t_{kr}}\right\rfloor,\forall r\in\mathcal{R},k\in\mathcal{K} \quad (3.11)$$

$$\sum_{k\in\mathcal{K}} x_{kr}\geq N_r,\forall r\in\mathcal{R} \quad (3.12)$$

$$\sum_{r\in\mathcal{R}} n_{kr}^{\mathrm{OWN}} \le N_k^{\mathrm{MAX}}, \forall k \in \mathcal{K} \tag{3.13}$$

$$\sum_{r\in\mathcal{R}} n_{kr}^{\mathrm{IN}} \le NCI_k^{\mathrm{MAX}}, \forall k \in \mathcal{K} \tag{3.14}$$

$$n_{kr}^{\mathrm{OWN}}, n_{kr}^{\mathrm{IN}}, x_{kr} \in \mathbb{Z}^{+} \bigcup \{0\}, \forall k \in \mathcal{K}, r \in \mathcal{R} \tag{3.15}$$

where c_{kr} denotes the operating cost of ships of type *k* on route *r* per voyage ($/voyage). It includes the fuel cost, daily running cost, port charge and canal fee (if any). The chartering rate of a ship of type *k* in the planning horizon is denoted by c_k^{IN} ($/ship). t_{kr} is the voyage time of a ship of type *k* on a route *r* (days), *T* is the length of the short-term planning horizon (3-6 months), N_r is the minimal number of voyages required on route *r* during the planning horizon in order to maintain a given liner shipping service frequency, V_k denotes the capacity of a ship of type *k* referring to the number of containers it can be loaded.

Eq. (3.9) is the objective function of the CCP model. The first term in the bracket presents the shipping cost and the second term is the cost of chartering in containerships of type *k* in the short-term planning horizon. Constraints (3.10) are the chance constraints which show that the liner container shipping company can satisfy the customers' demand at least with a probability of $1-\alpha_r$. Constraint (3.11) compute the maximal number of voyage that ships of type *k* can complete on route *r*, where $\lfloor a \rfloor$ denotes the maximum integer not greater than *a*. The constraints (3.12) guarantee the number of voyages required on ship route *r* in order to maintain the given liner shipping frequency. For example, if a weekly shipping service is required on ship route *r* during a planning horizon of six months, then $N_r = 26$. Constraints (3.13) and (3.14) ensure the number containerships of own and chartered in should not exceed its corresponding maximum available containerships, respectively. Constraint (3.15) requires that all variables are nonnegative integers.

It is not difficult to find that constraints (3.10) can be respectively rewritten as follows:

$$\sum_{k\in\mathcal{K}} x_{kr}V_k \geq \Phi^{-1}\left(1-\alpha_r\right)\sqrt{\sum_{\left(p_r^i,p_r^j\right)\in\mathcal{M}_r} \rho_l^{\left(p_r^i,p_r^j\right)}\sigma^{\left(p_r^i,p_r^j\right)^2}} + \sum_{\left(p_r^i,p_r^j\right)\in\mathcal{M}_r} \rho_l^{\left(p_r^i,p_r^j\right)}\mu^{\left(p_r^i,p_r^j\right)}, l=1,\cdots,m_r; \forall r\in\mathcal{R} \tag{3.16}$$

where $\Phi^{-1}\left(1-\alpha_r\right)$ is the inverse cumulative probability of $1-\alpha_r$. Eqs. (3.16) imply that constraints (3.10) have the equivalent linear function expressions. Objective function shown by Eq. (3.9) and the other constraints (3.11) to (3.14) are all linear functions with respect to the decision variables. Therefore, the CCP model is an integer linear programming model. As the CCP model is an integer liner programming model, it can be thus solved by any optimization solver such as CPLEX. CPLEX actually employs the branch-and-cut algorithm for solving an integer liner programming problem.

The proposed CCP model involves two blocks of costs: shipping costs and chartering in cost. The shipping costs include fuel cost, daily running cost, port charge and canal fee. The rationale behind port charge is that port authorities levy various fees against ships and/or containers for the use of the facilities and services provided by them; and the main canal dues payable are for transiting the Suez and Panama canals. As for chartering in ships, it is commonly adopted by liner shipping companies in practice. For example, APM-Maersk, the largest maritime container shipping operator in the world, operates totally 524 ships, in which it owns 184 ships and charters 340 ships in 2007. Therefore, chartering in cost is also involved.

3.4 Numerical Example

In this section, we use a numerical example to assess the CCP model and then take this numerical example as a benchmark pattern, we investigate impact of container shipment demand and the confidence parameter on the optimal decisions made in the proposed short-term LSFP problem.

3.4.1 Example Design

In the numerical example, we assume that a liner container shipping company intends to make a 6-month fleet plan. In order to make the example close to a realistic case, we design a liner ship route network consisting of 8 routes operated by a liner container shipping company, OOCL in Hong Kong (see Figure 3.2).

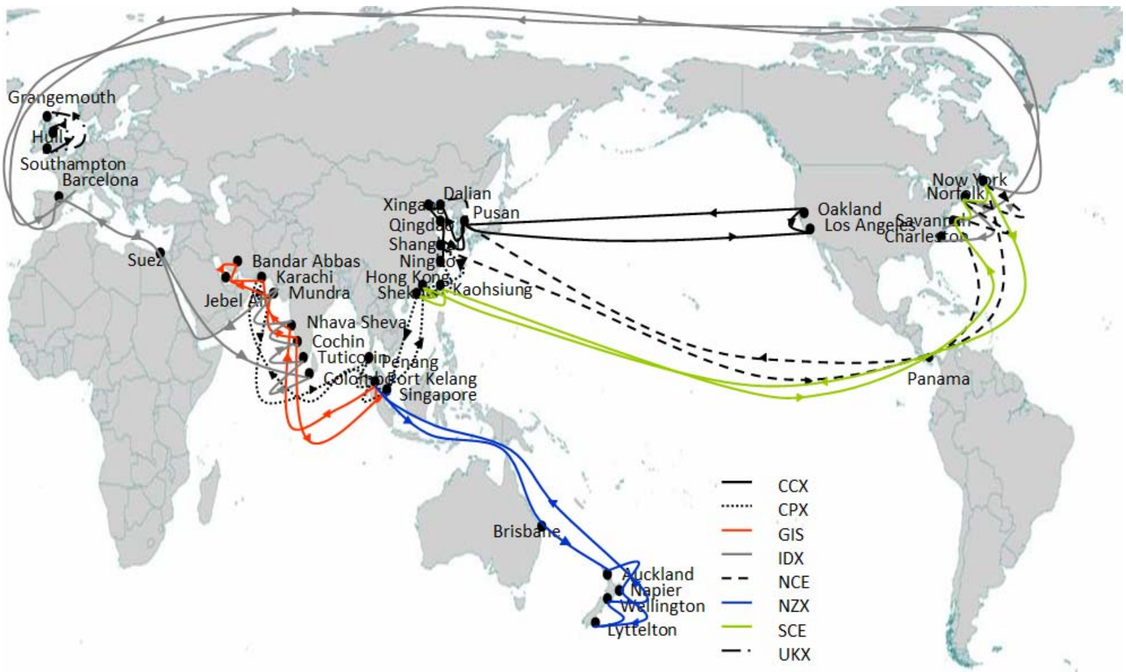

Figure 3. 2 Liner shipping network for the numerical example

The liner shipping topology involves a total of 36 calling ports and 390 O-D pairs. The ports called on each liner ship route and their digital number codes are shown in Table 3.1. Table 3.2 gives distance of each leg in each liner ship route. The numbers, sizes, market prices, daily operating cost and design speed of each ship type are listed in Table 3.3. It is noted that the daily operating cost of each ship type is estimated by using the following regression equation (Shintani et al., 2007) since the exact data is unavailable:

$$\text{daily operating cost} = 6.54 \times \text{ship size } (\text{TEU}) + 1422.5 \tag{3.17}$$

Table 3. 1 Port calling sequence and number code for each route

Routes	Port calling sequence and number code
CCX	Los Angeles/Oakland/Pusan/Dalian/Xingang/Qingdao/Ningbo/Shanghai /Pusan/Los Angles (1-2-3-4-5-6-7-8-9-1)
CPX	Shanghai/Ningbo/Shekou/Singapore/Karachi/Mundra/Penang/PortKelang /Singapore/Hong Kong/Shanghai (1-2-3-4-5-6-7-8-9-10-1)
GIS	Singapore/Port Kelang/Nhava Sheva/Karachi/Jebel Ali/Bandar Abbas /Jebel Ali/ Mundra/Cochin/Singapore (1-2-3-4-5-6-7-8-9-1)
IDX	Colombo/Tuticorin/Cochin/Nhava Sheva/Mundra/Suez/Barcelona/NewYork /Norfolk/Charleston/Barcelona/Suez/Colombo (1-2-3-4-5-6-7-8-9-10-11-12-1)
NCE	New York/Norfolk/Savannah/Panama/Pusan/Dalian/Xingang/Qingdao /Ningbo/Shanghai/Panama/New York (1-2-3-4-5-6-7-8-9-10-11-1)
NZX	Singapore/Port Kelang/Brisbane/Auckland/Napier/Lyttelton/Wellington/ Brisbane/Singapore (1-2-3-4-5-6-7-8-1)
SCE	New York/Norfolk/Savannah/Panama/Kaohsiung/Shekou/Hong Kong /Panama/New York (1-2-3-4-5-6-7-8-1)
UKX	Southampton/Hull/Grangemouth/Southampton (1-2-3-1)

Table 3. 2 Distances of each leg in each liner ship route

Routes	Distance (nautical miles)
CCX	298-4985-543-158-356-339-60-492-5230
CPX	60-845-1460-2887-261-2510-172-210-1460-845
GIS	210-3097-261-711- 151-151-962-953-1853
IDX	140-161-158-186-2809-1673-3721-287-429-4124-1673-3394
NCE	287-505-982-13 831-543-158-356-339-60-13 565-1359
NZX	210-4050-1358-377-336-174-1448-3840
SCE	287-505-982-12949-342-33-12 788-1359
UKX	324-256-528

Table 3. 3 Example data

Item	Ship types				
	1	2	3	4	5
Ship size (TEUs)	2808	3218	4500	5714	8063
Daily cost (10^3 $)	19.8	22.5	30.9	38.8	54.2
Design speed (knots)	21.0	22.0	24.2	24.6	25.2
Chartering in rate (million $)	2	2.6	3.5	4.7	6.0
N_k^{MAX}	2	2	9	2	12
NCI_k^{MAX}	5	5	3	5	5

Although this example is hypothetical, it is close to a "realistic" case. This is because some data of the numerical example are extracted from a real liner shipping company-OOCL; for example, calling ports in a liner ship route, types of ships and their sizes and sailing speeds and so on. However, some data is still unavailable, including miscellaneous shipping costs and container shipment demand between two ports on a liner ship route since they are business confidential. These data are thus determined in a reasonable manner. As for the data of container shipment demand, though OOCL provides the annual business report, the port-to-port container shipment demand on a liner ship route is not elaborated. These data are hypothetical in this example. Since the data of miscellaneous shipping costs and port-to-port container shipment demand are too many (more than one thousand), they are not listed for reasons of space.

3.4.2 CCP Model Assessment

Table 3.4 shows the confidence parameter predetermined set on each route. With liner shipping services at level $1-\alpha$ (α is given in Table 3.4), the optimal solution of fleet size, mix and deployment for this example is obtained by CPLEX Ver. 11 and shown in Table 3.5. It can be seen from Table 3.4 that the confidence parameters set on Route NCE and Route SCE are small, which indicates that high level of service has to be maintained on these two routes. Therefore, most ships are allocated to these two routes in order to maintain the high level of service.

Table 3. 4 Confidence parameters on each liner ship route

	Route							
	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
α	0.10	0.15	0.05	0.15	0.05	0.10	0.05	0.15

Table 3. 5 Results to benchmark pattern

	Route / Ship Type	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
Ship allocations	1					1	2		1
	2			2					
	3	2	2				1	6	
	4			1	2			2	
	5				4	8			
Number of voyages	1					8	17		26
	2			16					
	3	26	26				10	21	
	4			10	14			8	
	5				22	24			

3.4.3 Sensitivity Analysis

To study impact of container shipment demand, ten sets of container shipment demands are tested with the same confidence parameters in Table 3.4. These 10 sets of container shipment demands are generated by setting 60%, 70%,…,150% of the benchmark demand pattern. The trend of corresponding optimal objective function value with each of these 10 sets is shown in Figure 3.3. This figure indicates that with the increase of container shipment demand, more cost is taken to maintain the same level of service. The ratios of costs corresponding to other sets with the costs of benchmark demand pattern increase, from 70% to 130%, shown in Figure 3.4. We take three sets of different confidence parameters as shown in Table 3.6 to analyze their impacts on the optimal fleet planning solution.

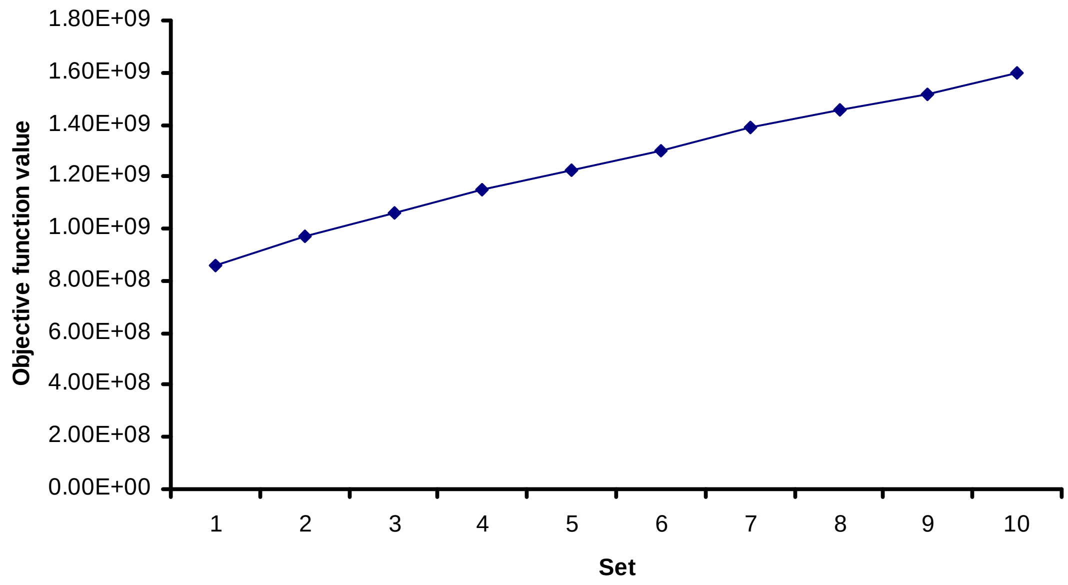

Figure 3. 3 Objective function value for different container shipment demand sets

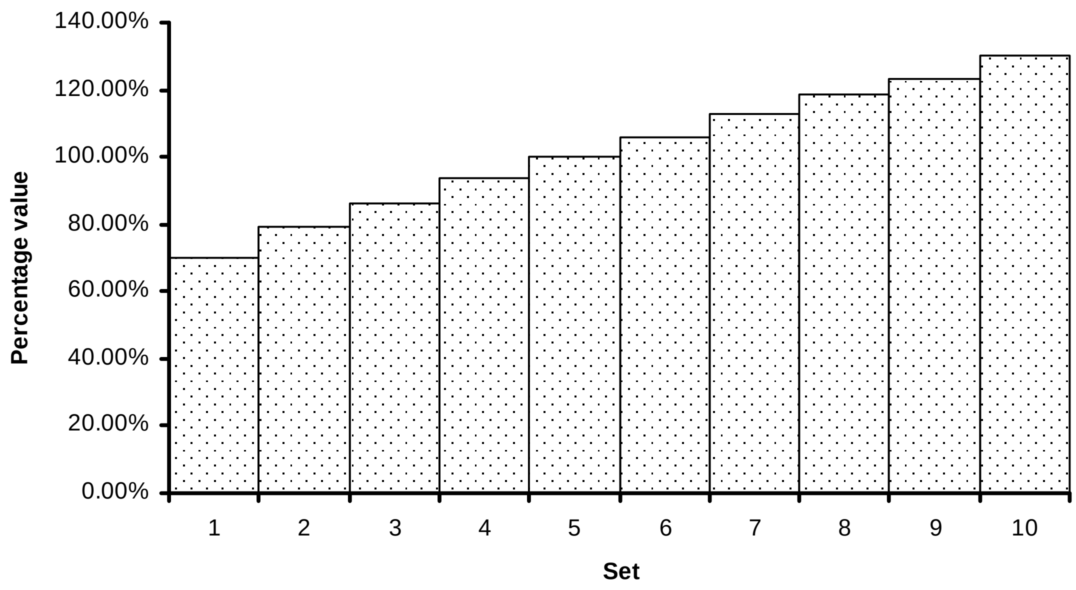

Figure 3. 4 Ratio of optimal objective function value for different sets with benchmark pattern

Table 3. 6 Three sets of confidence parameters

Route / α	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
Set 1	0.20	0.15	0.20	0.10	0.15	0.20	0.15	0.15
Set 2	0.15	0.10	0.15	0.05	0.10	0.15	0.10	0.10
Set 3	0.10	0.05	0.10	0.05	0.05	0.10	0.05	0.05

Table 3. 7 Results with respect to confidence parameters in set 1

	Route / Ship Type	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
Ship allocations	1					1	2		1
	2			2					
	3	2	2				1	6	
	4			1	2			2	
	5				4	8			
Number of voyages	1					8	17		26
	2			20					
	3	26	26				10	21	
	4			8	14			8	
	5				22	25			
Cost (million $)		949.5924							

Since the level of service equals to $1-\alpha$, it implies that a lower level of service corresponds to a larger value of α. Hence, Set 1 in Table 3.6 indicates a low level of service, Set 2 shows a medium level of service and Set 3 suggests a high level of service. The optimal solutions corresponding to these three sets of confidence parameters are listed in Tables 3.7 – 3.9, respectively. These three tables imply that the confidence parameter has significant impact on the optimal fleet size and deployment. It can be also found that that more ships are needed and more cost are taken in order to maintain a higher level of service.

Table 3. 8 Results with respect to confidence parameters in set 2

	Route / Ship Type	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
Ship allocations	1						2		1
	2			2					
	3	2	3			1	1	6	
	4			1	2			2	
	5				4	8			
Number of voyages	1						17		26
	2			20					
	3	26	26			5	10	21	
	4			7	14			8	
	5				22	24			
Cost (million $)		1162.2142							

Table 3. 9 Results with respect to confidence parameters in set 3

	Route / Ship Type	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
Ship allocations	1						2		1
	2			2					
	3	3	3			1	1	6	
	4			1	2			2	
	5				4	8			
Number of voyage	1						17		26
	2			24					
	3	27	27			3	10	24	
	4			3	14			6	
	5				22	29			
Cost (million $)		1340.1157							

3.5 Summary

This chapter takes the initiative to investigate the container shipment demand uncertainty issue arising from practice for the short-term liner fleet planning problems. Assuming that container shipment demand of a port pair on each liner ship route follows a normal distribution, the probability (chance) that shipping capacity of a liner ship fleet planning scenario cannot meet the demand does exist. In other words, the liner container shipping company failed to make the service for its customers with this probability. The level of service is proposed to represent the probability of satisfying the customers' requirement, and it can be formulated as a chance constraint. To maintain a certain level of service, the company must control this probability (or chance) within a given level called confidence parameter. We therefore develop a chance constrained programming model for the short-term LSFP problem with container shipment demand uncertainty. The proposed model can be solved by many optimization solvers such as CPLEX because it is an integer linear programming model. A numerical example has been carried out for the model assessment and impact analysis of the confidence parameters and cargo shipment demand.

It is noted that in this Chapter, the container shipping company aims to maintain a certain level of service on each leg, namely Eq. (3.8). This equation makes us simplify the CCP model and obtain an analytical form, namely Eq. (3.16). If the container shipping company aims to maintain a certain leve lof service on each route, Eq.(3.8) should be rewritten as follows:

$$\Pr\left(\sum_{k\in\mathcal{K}} x_{kr}V_k \geq \max_{l=1,\cdots,m_r}\left(\eta_l^r\right)\right) \geq 1-\alpha_r, \forall r\in\mathcal{R} \tag{3.18}$$

In this case, it is difficult to obtain the analytical form of Eq. (3.18). To deal with this issue, we can use Mento Carlo simulation to approximate. The procedures are described as follows:

Step 1: Generate a sample for each O-D port pair, namely $\xi^{\left(p_r^i, p_r^j\right)}$.

Step 2: Calculate the realization of $\eta_l^r \ \left(l=1,...,m_r;\forall r\in\mathcal{R}\right)$ according to Eq.(3.6).

Step 3: Find the maximal value of the realizations of all $\eta_l^r \left(l=1,...,m_r;\forall r\in\mathcal{R}\right)$, namely to find $\max_{l=1,\cdots,m_r}\left(\eta_l^r\right)$ for each $r\in\mathcal{R}$.

Step 4: Repeat the three steps addressed above for a number of times, say 1000 times.

Step 5: Sort the 1000 values of $\max_{l=1,\cdots,m_r}\left(\eta_l^r\right)$ by acesending order for each $r\in\mathcal{R}$.

Step 6: Note the value of $\max_{l=1,\cdots,m_r}\left(\eta_l^r\right)$ corresponding to an order of $1000\times\left(1-\alpha_r\right)$ for each $r\in\mathcal{R}$, denoted by $\eta_{\max}^r$.

Therefore, we can rewrite Eq. (3.18) as the following closed form:

$$\sum_{k\in\mathcal{K}} x_{kr}V_k \ge \eta_{\max}^r, \forall r\in\mathcal{R} \tag{3.19}$$

CHAPTER 4 A TWO-STAGE STOCHASTIC INTEGER PROGRAMMING MODEL FOR SHORT-TERM LSFP

4.1 Introduction

Chapter 3 deals with the uncertain container shipment demand by assuming that the container shipment demand of each port pair is a random variable following a normal distribution with a given mean value and variance. Further, a confidence parameter on a liner ship route is set to represent the probability that a liner container shipping company fails to meet the container shipment demand on this liner ship route. Thus the short-term LSFP problem with uncertain container shipment demand is then formulated as a CCP model. However, transshipment of containers, which is an intrinsic characteristic of liner shipping services and widely used in liner shipping, is not taken into account in Chapter 3.

Transshipping containers at a hub port is a typical liner shipping operations nowadays because it enables to deploy large ships calling at hub ports to benefit the economies of scale in ship size (Cullinane and Khanna, 1999). As reported by Vernimmen et al. (2007), about one third of the laden container throughput in the world is made up of transshipped containers. Mourão et al. (2001) made the first attempt on the liner ship fleet deployment problem with container transshipment and deterministic container shipment demand. They investigated a hypothetical hub-and-spoke (H&S) network with one pair of ports and two ship routes - one feeder route and one main route. All containers had to be transshipped at the hub port in the feeder route. This model is too simple to reflect the realistic ship fleet deployment, however. Therefore, this chapter studies the short-term LSFP problem with container transshipment and uncertain container shipment demand.

Container transshipment operations mean that there can be multiple container routes between an origin and a destination, and some of these container routes involve more than one ship route. The short-term LSFP problem should therefore choose the best container routes and assign the right number of containers to each of these container routes. In fact though, the short-term LSFP problem taking into account container transshipment and demand uncertainty is a new research issue with practical importance; most of the existing relevant literature (Ronen, 1983 and 1993; Perakis, 2002 and Christiansen et al.2004) assumes deterministic container shipment demand. This chapter thus focuses on model formulation and algorithm development for this new research issue.

In this chapter, we investigate the short-term LSFP problem with container transshipment and uncertain container shipment demand. To characterize the uncertainty, we first assume that the number of containers transported from an origin port to a destination port is a random variable. With these random container shipment demands, the proposed LSFP problem can be formulated as a two-stage stochastic integer programming model with the objective of maximizing the expected value of the total profit. To solve this, a solution algorithm integrating the sample average approximation method and a dual decomposition and Lagrangian relaxation approach will be developed.

The remainder of this chapter is organized as follows: Section 4.2 firstly introduces a novel concept of container route to formulate the issue of container transshipment and then develops a two-stage stochastic integer programming (2SSIP) model for the short-term LSFP problem with container transshipment and uncertain container shipment demand. Section 4.3 presents the solution algorithm which integrates the sample average approximation method and dual decomposition and Lagrangian relaxation approach. Section 4.4 uses a numerical example to evaluate the model and solution algorithm proposed in this study and analyzes the numerical example to verify the necessity and rationality of our 2SSIP model. Finally, Section 4.5 summarizes the work of this chapter.

4.2 Model Development

Before the development of a two-stage stochastic programming model which aims to maximize the expected profit for the short-term LSFP problem with container transshipment and uncertain container shipment demand, we firstly introduce the concept of container route to deal with the container transshipment issue.

4.2.1 Container Routes with Container Transshipment Operations

Let $\mathcal{W} = \{(o,d) | o \in \mathcal{P}, d \in \mathcal{P}\}$ be the set of origin-to-destination (O-D) port pairs with container shipment demand, and ξ^{od} be the number of containers in terms of TEUs (acronyms of twenty-foot equivalent unit) to be transported between an O-D port pair $(o,d) \in \mathcal{W}$ in the short-term planning horizon (6 months). As aforementioned in Chapter 3, the liner container shipping company provides regular shipping service on a predetermined liner ship route network, in other words, the route of ships is fixed (illustrated in Figure 3.1). However, the route of containers may be different from the route of ships because there are usually many candidate routes for transporting containers from their origin to destination due to transshipment. Given the set of ship routes $\mathcal{R}$, the liner container shipping company can predetermine a set of candidate container routes to deliver containers between an O-D port pair $(o,d)\in\mathcal{W}$, denoted by $\mathcal{H}^{od}$. A container route $h^{od}\in\mathcal{H}^{od}$ is either a part of one particular ship route or a combination of several ship routes to deliver containers from original port $o\in\mathcal{P}$ to destination port $d\in\mathcal{P}$. Container transshipment operations are involved in any container route made up of several ship routes. For example, there are two possible container routes from Jakarta (JK) to Shanghai (SH) in Figure 4.1:

$$h_1^{\text{JK-SH}} = p_1^1(\text{JK}) \xrightarrow{\text{Ship Route 1}} p_1^2(\text{SG}) \mapsto p_3^2(\text{SG}) \xrightarrow{\text{Ship Route 3}} p_3^3(\text{SH}) \tag{4.1}$$

$$h_2^{\text{JK-SH}} = p_2^1(\text{JK}) \xrightarrow{\text{Ship Route 2}} p_2^2(\text{SH}) \tag{4.2}$$

The first container route $h_1^{\text{JK.SH}}$, made up of two ship routes, involves container transshipment operations: containers are loaded at the first port call of ship route 1 (Jakarta) and delivered to the second port of call of ship route 3 (Singapore). At Singapore port, these containers are discharged and reloaded (transshipped) to a ship deployed on ship route 3, and transported to the destination port, Shanghai. However, the second container route $h_2^{\text{JK.SH}}$ provides direct delivery service via ship route 2 without container transshipment.

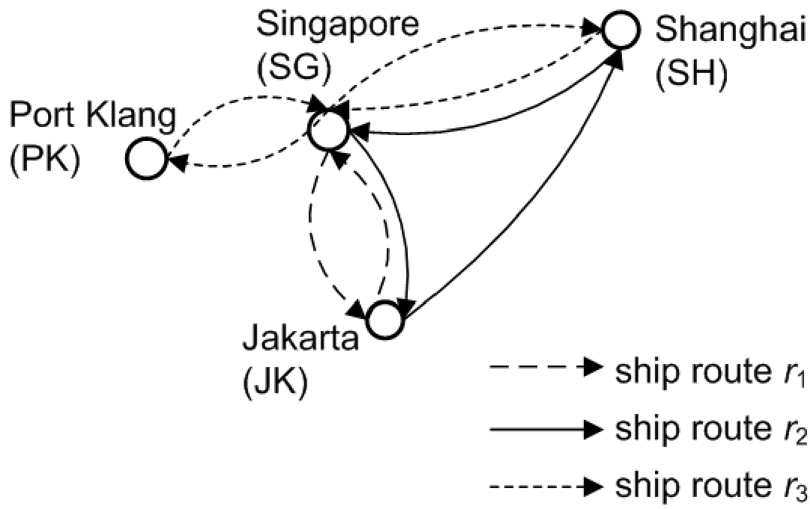

Figure 4. 1 Three liner ship route

A container route contains all the information on how containers will be transported, such as origin, destination, ports called along the route and transshipment port(s). The introduction of the concept of container route facilitates the model formulation as the complex container delivery process is simplified and represented by a finite number of container routes. Some container routes for the liner shipping network in Figure 4.1 are provided in Table 4.1. An O-D port pair may have several container routes, and the volume of containers to be transported between this O-D port pair could be spitted among these container routes. Let $\mathcal{H}$ be set of all these predetermined container routes for all the O-D port pairs, namely,

$$\mathcal{H}= \bigcup_{(o,d)\in\mathcal{W}} \mathcal{H}^{od} \tag{4.3}$$

Table 4. 1 Container route plans for different O-Ds

O-D	Container route plans
JK-SH	$h_1^{\mathrm{JK.SH}}: p_1^1(\mathrm{JK}) \to p_1^2(\mathrm{SG}) / p_3^2(\mathrm{SG}) \to p_3^3(\mathrm{SH})$ $h_2^{\mathrm{JK.SH}}: p_2^1(\mathrm{JK}) \to p_2^2(\mathrm{SH})$
SH-PK	$h_1^{\mathrm{SH.PK}}: p_3^3(\mathrm{SH}) \to p_3^4(\mathrm{SG}) \to p_3^1(\mathrm{PK})$
SH-SG	$h_1^{\mathrm{SH.SG}}: p_2^2(\mathrm{SH}) \to p_2^3(\mathrm{SG})$ $h_2^{\mathrm{SH.SG}}: p_3^3(\mathrm{SH}) \to p_3^4(\mathrm{SG})$

4.2.2 Two-Stage Stochastic Integer Programming Model

Before the development of an optimization model which aims to maximize the expected value of profit for the short-term LSFP problem with container transshipment and uncertain demand, the following decision variables are introduced as follows:

n_{kr}^{OWN} number of owned ships of type k ($k \in \mathcal{K}$) assigned on route r ($r \in \mathcal{R}$)

n_{kr}^{IN} number of chartered in ships of type k ($k \in \mathcal{K}$) assigned on route r ($r \in \mathcal{R}$)

n_{k}^{OUT} number of chartered out ships of type k ($k \in \mathcal{K}$)

x_{kr} number of voyages of ships of type k ($k \in \mathcal{K}$) on route r ($r \in \mathcal{R}$)

$z^{h^{od}}$ number of containers between O-D port pair $(o,d) \in \mathcal{W}$ carried by ships deployed on the container route $h^{od} \in \mathcal{H}^{od}$.

The revenue earned by the liner container shipping company comes from two resources: one is the rent of chartering out ships to other liner operators; and the other is freight rate of shipping containers for shippers. Let c_k^{OUT} denote the rate received for chartering out a ship of type k in the planning horizon ($/ship), then the revenue gained from chartering ships out is given by:

$$\sum_{k\in\mathcal{K}} c_k^{\mathrm{OUT}} n_k^{\mathrm{OUT}} \tag{4.4}$$

As for the revenue of shipping containers, it is uncertain due to the uncertainty of container shipment demand. Let ξ be a random vector defined over a probability space (Ω, F, P) where Ω is the set of elementary outcomes ω, F is the event space and P is the probability measure. The container shipment demand of an O-D port pair $(o,d)\in\mathcal{W}$ denoted by ξ^{od} is a random variable. Given ω^{od}, which is a realization of the random parameter ξ^{od}, then $z^{h^{od}}$ is obviously a function with respect to ω^{od}. Let f^{od} denote the freight rate of delivering a container with an O-D port pair $(o,d)\in\mathcal{W}$ ($/TEU), the revenue of shipping containers for all O-D port pairs along all liner ship routes is given by:

$$\sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} f^{od} z^{h^{od}}\left(\omega^{od}\right) \tag{4.5}$$

The total costs incurred by the liner container shipping company consist of three components: container handling cost, ship operating cost and ship chartering in cost. The container handling cost incurred on a container route includes the loading cost at the origin port, container discharging cost at the destination port and transshipment costs at any transshipment ports. Different container routes between an O-D port pair may result in different container handling cost. For example, the first container route shown in Eq. (4.1) and the second container route shown in Eq. (4.2) both involve the container loading cost at JK and container discharging cost at SH, but the first container route is associated with an additional transshipment cost at SG. Let $c^{h^{od}}$ (\$/TEU) denote the container handling cost per TEU incurred on the container route $h^{od} \in \mathcal{H}^{od}$ and the total container handling cost can be calculate as:

$$\sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} c^{h^{od}} z^{h^{od}} \left(\omega^{od}\right) \tag{4.6}$$

Let c_{kr} denote the operating cost of ships of type k on ship route r per voyage, including fuel consumption costs, administration costs, fixed daily operating costs, port charges and canal fees (if any). The total ship operating cost plus the rent paid for chartering in ships is given by:

$$\sum_{r\in\mathcal{R}} \sum_{k\in\mathcal{K}} c_{kr} x_{kr} + \sum_{r\in\mathcal{R}} \sum_{k\in\mathcal{K}} c_k^{\text{IN}} n_{kr}^{\text{IN}} \tag{4.7}$$

It should be noted that the decision regarding n_{kr}^{OWN}, n_{kr}^{IN}, n_k^{OUT} and x_{kr} are made prior to a realization of the random container shipment demand $\xi^{od}\left((o,d)\in\mathcal{W}\right)$, which is denoted by ω^{od}. In reality, the number of containers between an O-D port pair $(o,d)\in\mathcal{W}$ assigned to a particular container route, denoted by $z^{h^{od}}$, can be determined only after the realization of container shipment demand. In other words, n_{kr}^{OWN}, n_{kr}^{IN}, n_k^{OUT} and x_{kr} are made before $z^{h^{od}}$. We can thus break down the decisions into two stages. In a two-stage stochastic optimization model, the set of decisions are divided into two groups: the first-stage decision variables are those that have to be decided before the actual realization of the uncertain parameters and often referred to as *here-and-now* decisions. Subsequently, when the random events have presented themselves, further design or operational policy improvements can be made by selecting the values of the second-stage decision variables, which are often referred to as *wait-and-see* decisions. Therefore, in our LSFP problem, the set of decisions is broken down into two groups. n_{kr}^{OWN}, n_{kr}^{IN}, n_{k}^{OUT} and x_{kr} are first-stage decision variables because they are determined before knowing the actual container shipment demand of each O-D port pairs; once they are determined, the number of containers picked up and delivered by ships are then further determined, namely $z^{h^{od}}$ are second-stage decision variables. Since the objective of the two-stage stochastic integer programming (2SSIP) model is to choose the first-stage variables in a way that the sum of the profit of the first stage and the expected profit of the second stage is maximized, the optimization model of the short-term LSFP is given by:

$$\max Z\left(\tilde{\mathbf{v}}\right) = \sum_{k\in\mathcal{K}} c_k^{\text{OUT}} n_k^{\text{OUT}} - \sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}}\left(c_{kr}x_{kr} + c_k^{\text{IN}} n_k^{\text{IN}}\right) + \mathbb{E}\left[\!\left[\hat{Q}_{\xi}\left(\tilde{\mathbf{v}},\boldsymbol{\xi}(\omega)\right)\right]\!\right] \tag{4.8}$$

subject to:

$$\sum_{r\in\mathcal{R}} n_{kr}^{\text{OWN}} \le N_k^{\text{MAX}}, \forall k \in \mathcal{K} \tag{4.9}$$

$$\sum_{r\in\mathcal{R}} n_{kr}^{\text{IN}} \le NCI_k^{\text{MAX}}, \forall k \in \mathcal{K} \tag{4.10}$$

$$n_k^{\text{OUT}} = N_k^{\text{MAX}} - \sum_{r\in\mathcal{R}} n_{kr}^{\text{OWN}}, \forall k \in \mathcal{K} \tag{4.11}$$

$$x_{kr} \le \left(n_{kr}^{\text{OWN}} + n_{kr}^{\text{IN}}\right) \times \left\lfloor \frac{T}{t_{kr}} \right\rfloor, \forall r \in \mathcal{R}, k \in \mathcal{K} \tag{4.12}$$

$$\sum_{k\in\mathcal{K}} x_{kr} \ge N_r, \forall r \in \mathcal{R} \tag{4.13}$$

$$n_{kr}^{\text{OWN}}, n_{kr}^{\text{IN}}, n_k^{\text{OUT}}, x_{kr} \in \mathbb{Z}^+ \cup \{0\}, \forall r \in \mathcal{R}, k \in \mathcal{K} \tag{4.14}$$

where vector $\tilde{\mathbf{v}} = \left(\cdots n_{kr}^{\text{OWN}} \cdots n_{kr}^{\text{IN}} \cdots n_k^{\text{OUT}} \cdots x_{kr} \cdots\right)$ contains all of the first-stage decision variables for succinctness, t_{kr} is the voyage time of a ship of type *k* on a particular ship route *r* (in days), N_r is the minimal number of voyages required on route *r* during the planning horizon in order to maintain a given liner shipping service frequency. $\mathbb{E}\left[\!\left[\hat{Q}_{\xi}\left(\tilde{\mathbf{v}},\boldsymbol{\xi}(\omega)\right)\right]\!\right]$ is the expected recourse function in which $\hat{Q}_{\xi}\left(\tilde{\mathbf{v}},\boldsymbol{\xi}(\omega)\right)$ is the optimal objective function value for the following second-stage optimization problem with a given vector $\tilde{\mathbf{v}}$ and a given realization $\boldsymbol{\xi}(\omega)$ of the random container shipment demand vector $\boldsymbol{\xi}=\left(\xi^{od}:(o,d)\in\mathcal{W}\right)$:

$$\hat{Q}_{\xi}\left(\tilde{\mathbf{v}},\boldsymbol{\xi}(\omega)\right)=\max\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\left(f^{od}-c^{h^{od}}\right)z^{h^{od}}\left(\omega^{od}\right) \tag{4.15}$$

subject to

$$\sum_{k\in\mathcal{K}}x_{kr}V_k\geq\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\rho_{ir}^{h^{od}}z^{h^{od}}\left(\omega^{od}\right),i=1,\cdots,m_r,\forall r\in\mathcal{R} \tag{4.16}$$

$$\sum_{h^{(o,d)}\in\mathcal{H}^{(o,d)}}z^{h^{od}}\left(\omega^{od}\right)\leq\xi^{od}\left(\omega^{od}\right),\forall(o,d)\in\mathcal{W} \tag{4.17}$$

$$z^{h^{od}}\geq 0,\forall(o,d)\in\mathcal{W},\forall h^{od}\in\mathcal{H}^{od} \tag{4.18}$$

where V_k is size of a particular ship *k* (TEUs) and $\rho_{ir}^{h^{od}}$ is a binary variable, which equals 1 if a container route $h^{od}\in\mathcal{H}^{od}$ contains leg *i* of ship route *r*, or 0 otherwise.

Eq. (4.8) is the objective function of the two-stage stochastic integer programming model, which is equivalent to maximizing the expected value of the total profit. Constraints (4.9) and constraints (4.10) ensure the number of owned and chartered in ships should not exceed the maximum available ships. The number of chartering out ships is given by Eqs. (4.11). Constraints (4.12) compute the maximal number of voyage that ships of type *k* can complete on route *r*, where $\lfloor a \rfloor$ denotes the maximum integer not greater than a. N_r represents the minimal amount of voyages required on route *r* during the planning horizon in order to maintain a given level of frequency. For example, if a weekly frequency is required on each route, then $N_r = 26$ for a 6-month planning horizon. Therefore, Constraints (4.13) guarantee the number of voyages required on route *r* in order to maintain a given level of liner shipping frequency. The constraints (4.14) require that the decision variables n_{kr}^{OWN}, n_{kr}^{IN}, n_k^{OUT} and x_{kr} are nonnegative integers.

The left-hand side of Constraints (4.16) represents the total transportation capacity of ships deployed on the liner ship route $r \in \mathcal{R}$. The right-hand side computes the total number of containers carried by ships sailing on leg *i* of route $r \in \mathcal{R}$, including the containers loaded at previously calling ports but still remained on ships and the containers loaded or transshipped at port p_r^i. Therefore, Constraints (4.16) ensure that the container flow on each leg carried on the ships cannot exceed the ship capacity deployed on the ship route. Constraints (4.17) imply the containers carried on the ships cannot exceed the realization of the demand. Constraints (4.18) defines the range for decision variables of $z^{h^{od}}$.

Substituting n_k^{OUT} in the objective function expressed by Eq. (4.8) with the right-hand sides of Eqs. (4.11) yields the following two-stage stochastic integer programming (2SSP) model with fewer first-stage decision variables denoted by the vector $\mathbf{v} = \left(\cdots n_{kr}^{\text{OWN}} \cdots n_{kr}^{\text{IN}} \cdots x_{kr} \cdots\right)$.

[2SSIP]

$$\min Z\left(\mathbf{v}\right) = \mathbf{c}^T \mathbf{v} + \mathbb{E}\left[\!\left[Q\left(\mathbf{v}, \xi\left(\omega\right)\right) \right]\!\right] - \sum_{k \in \mathcal{K}} c_k^{\text{OUT}} N_k^{\text{MAX}} \tag{4.19}$$

subject to constraints (4.9)-(4.10), (4.12)-(4.14). where vector $\mathbf{c} = \left(\cdots c_k^{\text{OUT}} \cdots c_k^{\text{IN}} \cdots c_{kr} \cdots\right)$ groups the cost coefficients in the first-stage problem and $Q_\xi\left(\mathbf{v}, \xi\left(\omega\right)\right)$ is the optimal objective function value for the following second-stage optimization problem with a given vector $\mathbf{v}$ and a given realization $\xi(\omega)$ of the random container shipment demand vector $\xi = \left(\xi^{od} : (o,d) \in \mathcal{W}\right)$:

$$Q_{\xi}\left(\mathbf{v}, \xi(\omega)\right) = \min \sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} \left(c^{h^{od}} - f^{od}\right) z^{h^{od}}\left(\omega^{od}\right) \quad (4.20)$$

subject to constraints (4.16)-(4.18).

4.3 Solution Algorithm

As presented by Ahmed (2004), there are some potential sources of difficulty in solving a 2SSIP model. Firstly, for given first-stage decisions, an evaluation of the 2SSIP problem (4.19) involves a very huge number of programming problems (4.9)-(4.18), one for each scenario of the realization of the uncertain parameters in this problem. Additionally, the expected recourse function $\mathbb{E}\left[\!\left[Q_{\xi}\left(\mathbf{v}, \xi(\omega)\right) \right]\!\right]$ is only implicitly defined and depends on the current decisions and usually involves optimization problems embedded in expectation, making the problem very difficult to solve. If the uncertain parameters have a continuous distribution, the evaluation of $\mathbb{E}\left[\!\left[Q_{\xi}\left(\mathbf{v}, \xi(\omega)\right) \right]\!\right]$ involves integrating the value function of an integer program and is in general impossible; if the uncertain parameters have a discrete distribution, it involves solving a huge number of similar integer programs. Therefore, the conventional integer programming tools are quite cumbersome and computational intractable due to the inherent problem complexity and its large number of variables and constraints.

The implicit definition of expected recourse function $\mathbb{E}\left[\!\left[Q_{\xi}\left(\mathbf{v},\xi(\omega)\right)\right]\!\right]$ is the essential factor making the proposed 2SSIP model difficult to solve, so an effective way is needed to handle it. Some methods can be used to deal with the expected recourse function, such as Stochastic Linearization Method (SLM) (Ermoliev, 1988); Successive Linear Approximation Procedure (SLAP) method (Frantzeskakis and Powell, 1990); network recourse decomposition (NRD) methods (Powell and Cheung, 1994); Successive Convex Approximation Method (SCAM, Cheung and Powell, 1996); Stochastic Hybrid Approximation Procedures (SHAPE, Cheung and Powell, 2000); and Sample Average Approximation (SAA) method (Kleywget et al., 2001). However, SLM, SLAP and SHAPE cannot obtain an integral solution; SLAP requires the expected recourse function is convex; SCAM and SHAPE use a convex, piecewise linear and separable function to replace the expected recourse function; and NRD is applicable in a problem with a tree structure. Therefore, those methods, SLM, SLAP, NRD, SCAM and SHAPE are improper to be employed as a solution approach for our problem.

The proposed 2SSIP model (4.19) has three characteristics: (i) the expected value function $\mathbb{E}\left[\!\left[Q_{\xi}\left(\mathbf{v},\xi(\omega)\right)\right]\!\right]$ does not have a closed form and its values cannot be calculated easily, (ii) the optimal objective function expressed by Eq. (4.20) of the second-stage optimization problem can be calculated easily for a given first-stage decision and a realization of the random container shipment demand, by means any efficient algorithm for solving the linear programming problems, (iii) the number of feasible first-stage decisions is very large so that the enumeration approaches are not feasible. These three characteristics enable us to employ the sample average approximation (SAA) method proposed by Kleywegt et al. (2001) for solving the 2SSIP model.

The main procedure involved in using the SAA method to solve the 2SSIP model is as follows: first, a sample $\boldsymbol{\xi}_1,\cdots,\boldsymbol{\xi}_N$ of N realizations of the random container shipment demand vector $\boldsymbol{\xi}(\omega)$ is generated and the expected value function $\mathbb{E}\left[\!\left[Q_{\xi}\left(\mathbf{v},\boldsymbol{\xi}(\omega)\right)\right]\!\right]$ is approximated by the sample average function $N^{-1}\sum_{l=1}^{N}Q_{\xi}\left(\mathbf{v},\boldsymbol{\xi}_l\left(\boldsymbol{\omega}\right)\right)$. The 2SSIP model expressed by Eqs. (4.19)-(4.20) can be thus approximated by the SAA problem:

[SAA]

$$\min Z\left(\mathbf{v}\right)=\mathbf{c}^T\mathbf{v}+\frac{1}{N}\sum_{l=1}^{N}Q_{\xi}\left(\mathbf{v},\boldsymbol{\xi}_l\left(\boldsymbol{\omega}\right)\right)-\sum_{k\in\mathcal{K}}c_k^{\mathrm{OUT}}N_k^{\mathrm{MAX}} \tag{4.21}$$

subject to the constraints (4.9)-(4.10) and (4.12)-(4.14), where $Q_{\xi}\left(\mathbf{v},\boldsymbol{\xi}_l\left(\omega\right)\right)$ $\left(l=1,2,\cdots,N\right)$ is the optimal objective function value for the following second-stage optimization problem with a given vector $\mathbf{v}$ and a given realization $\boldsymbol{\xi}_l\left(\omega\right)=\left(\xi_l^{od}\left(\omega^{od}\right):\left(o,d\right)\in\mathcal{W}\right)$.

$$Q_{\xi}\left(\mathbf{v},\boldsymbol{\xi}_l\left(\omega\right)\right)=\min\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\left(c^{h^{od}}-f^{od}\right)z_l^{h^{od}}\left(\omega_l^{od}\right) \tag{4.22}$$

subject to

$$\sum_{k\in\mathcal{K}}x_{kr}V_k\geq\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\rho_{ir}^{h^{od}}z_l^{h^{od}}\left(\omega_l^{od}\right),\forall i=1,\cdots,m_r,r\in\mathcal{R} \tag{4.23}$$

$$\sum_{h^{(o,d)}\in\mathcal{H}^{(o,d)}}z_l^{h^{od}}\left(\omega_l^{od}\right)\leq\xi_l^{od}\left(\omega_l^{od}\right),\forall\left(o,d\right)\in\mathcal{W} \tag{4.24}$$

$$z_l^{h^{od}}\geq 0,\forall\left(o,d\right)\in\mathcal{W},\forall h^{od}\in\mathcal{H}^{od} \tag{4.25}$$

4.3.1 Dual Decomposition and Lagrangian Relaxation

It can be seen that the SAA problem expressed by Eqs. (4.21)-(4.25) involves N linear programming problems shown by Eqs. (4.22)-(4.25). Each of these linear programming problems corresponds to one realization (or scenario) of the random container shipment demand and need to be solved to obtain the expected value associated with a given first-stage decisions. One possible way to solve the SAA problem is as follows. We first enumerate all feasible first-stage solutions, and then calculate the value of the objective function shown in Eq. (4.21) with respect to each feasible first-stage decision after solving the corresponding N linear programming problems. Finally, we choose a feasible first-stage decision with the minimum objective function value. This method might be workable for very small scale problems but not for real-life problems.

The dual decomposition and Lagrangian relaxation approach, proposed by Carøe and Schultz (1999), can be used for solving the SAA problem effectively because it can decompose the SAA problem into N sub-problems based on those the container shipment demand realization. To make the decomposition, the first-stage decision variables are duplicated with respect to each container shipment demand realization, denoted by $\mathbf{v}_l, \left(l = 1, 2, \cdots, N\right)$. The SAA problem can be rewritten as follows:

$$Z_N = \min \frac{1}{N} \sum\nolimits_{l=1}^{N} \mathbf{c}^T \mathbf{v}_l + \frac{1}{N} \sum\nolimits_{l=1}^{N} \left[\sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} \left(c^{h^{od}} - f^{od}\right) z_l^{h^{od}} \right] - \sum_{k\in\mathcal{K}} c_k^{\text{OUT}} N_k^{\text{MAX}} \quad (4.26)$$

subject to the constraints (4.9)-(4.10) and (4.12)-(4.14), duplicated with respect to each container shipment demand realization, the constraints (4.23)-(4.25) for each container shipment demand realization, and the non-anticipativity constraints:

$$\mathbf{v}_1 = \cdots = \mathbf{v}_N \quad (4.27)$$

The above non-anticipativity constraints imply that the first-stage decision should not depend on the container shipment demand realizations that prevails in the second-stage optimization problem; and they can be alternatively expressed as

$$\mathbf{v}_l = \mathbf{v}_{l+1}, l = 1, 2, \cdots, N-1 \tag{4.28}$$

Eqs. (4.28) can be written below using the matrix notation:

$$\sum_{l=1}^{N} \mathbf{H}_l \mathbf{v}_l = \mathbf{0} \tag{4.29}$$

where $\mathbf{H}_l$ is a suitable matrix with $(N-1)\times(3KR)$ rows and $3KR$ columns for l = 1,…,N ($3KR$ is the total number of first-stage decision variables, n_{kr}^{OWN}, n_{kr}^{IN} and x_{kr}), defined as follows:

$$\begin{aligned}&\mathbf{H}_1 = (\mathbf{I}, \mathbf{0}, \cdots, \mathbf{0})^T, \mathbf{H}_2 = (-\mathbf{I}, \mathbf{I}, \mathbf{0}, \cdots \mathbf{0})^T, \mathbf{H}_3 = (\mathbf{0}, -\mathbf{I}, \mathbf{I}, \cdots \mathbf{0})^T, \cdots,\\ &\mathbf{H}_{N-1} = (\mathbf{0}, \cdots, -\mathbf{I}, \mathbf{I})^T, \mathbf{H}_N = (\mathbf{0}, \cdots \mathbf{0}, -\mathbf{I})^T\end{aligned} \tag{4.30}$$

where **I** and **0** are the square unity matrix and the zero matrix of size $3KR$, respectively.

Let $\boldsymbol{\lambda}$ be a $(N-1)\times(3KR)$-dimensional vector of Lagrangian multiplier associated with the non-anticipativity constraints. The corresponding Lagrangian relaxation of the SAA problem can be formulated below:

[LR]

$$LR(\boldsymbol{\lambda}) = \min \sum_{l=1}^{N} \left[\frac{1}{N} \mathbf{c}^T \mathbf{v}_l + \frac{1}{N} \sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} \left(c^{h^{od}} - f^{od} \right) z_l^{h^{od}} + \boldsymbol{\lambda}^T \mathbf{H}_l \mathbf{v}_l - \frac{1}{N} \sum_{k\in\mathcal{K}} c_k^{\text{OUT}} N_k^{\text{MAX}} \right] \tag{4.31}$$

subject to the constraints (4.9)-(4.10) and (4.12)-(4.14), duplicated with respect to each container shipment demand realization, the constraints (4.23)-(4.25) for each container shipment demand realization and the non-anticipativity constraints. This LR model can be further decomposed into N separate mixed-integer linear programming problems corresponding to the N container shipment demand realizations, namely:

$$LR(\boldsymbol{\lambda}) = \sum_{l=1}^{N} LR_l(\boldsymbol{\lambda}) \tag{4.32}$$

where

$$LR(\boldsymbol{\lambda}) = \min \frac{1}{N}\mathbf{c}^T\mathbf{v}_l + \frac{1}{N}\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\left(c^{h^{od}} - f^{od}\right)z_l^{h^{od}} + \boldsymbol{\lambda}^T\mathbf{H}_l\mathbf{v}_l - \frac{1}{N}\sum_{k\in\mathcal{K}} c_k^{\mathrm{OUT}} N_k^{\mathrm{MAX}} \tag{4.33}$$

subject to the constraints (4.9)-(4.10), (4.12)-(4.14) and (4.23)-(4.25) associated with the l^{th} container shipment demand realization.

Each sub-problem shown in Eq. (4.33) can be solved using an efficient optimization solver such as CPLEX. It is straightforward to demonstrate that $LR(\boldsymbol{\lambda})$, the optimal objective function value of the LR model with respect to a given Lagrangian multiplier $\boldsymbol{\lambda}$, is a lower bound on the optimal function value of the SAA problem (4.21). The best or tightest lower bound can be found by solving the Lagrangian dual model:

[LD]

$$LD = \max_{\boldsymbol{\lambda}} LR(\boldsymbol{\lambda}) \tag{4.34}$$

This Lagrangian dual model is a concave maximization problem with the non-differentiable objective function $LR(\lambda)$. Eqs. (4.31)-(4.32) further show that $\sum_{l=1}^{N}\mathbf{H}_l\mathbf{s}_l^*$ is a subgradient of the convex and non-differentiable function $LR(\lambda)$ where $\mathbf{s}_l^*$ is the optimal solution to the l^{th} sub-problem shown in Eq.(4.33), namely:

$$\sum_{l=1}^{N}\mathbf{H}_l\mathbf{s}_l^* \in \partial LR(\lambda) \tag{4.35}$$

With this subgradient, the LR model can be solved by the following subgradient method:

Step 0: Take an initial Lagrangian multiplier vector $\boldsymbol{\lambda}^{(1)}$ and a predetermined step size sequence $\left\{\kappa^{(1)},\kappa^{(2)},\cdots\right\}$. Let the number of iterations $h=1$.

Step 1: Calculate the subgradient $\sum_{l=1}^{N}\mathbf{H}_l\mathbf{v}_l^{*(h)}$ by solving the subproblem shown in Eq. (4.33) with respect to the Lagrangian multiplier vector $\boldsymbol{\lambda}^{(h)}$.

Step 2: Update the Lagrangian multiplier vector according to the formula:

$$\boldsymbol{\lambda}^{(h+1)}=\boldsymbol{\lambda}^{(h)}+\kappa^{h}\sum_{l=1}^{N}\mathbf{H}_l\mathbf{v}_l^{*(h)} \tag{4.36}$$

Step 3: If the following criterion is fulfilled, the algorithm is terminated. Otherwise, let $h=h+1$ and go to Step 1.

$$\left|\left(LR\left(\boldsymbol{\lambda}^{h+1}\right)-LR\left(\boldsymbol{\lambda}^{h}\right)\right)\Big/LR\left(\boldsymbol{\lambda}^{h}\right)\right|\leq\tau \tag{4.37}$$

where τ is a given tolerance value.

The global convergence of this subgradient method has already been proved, provided that the step size satisfying the square is summable but the step size conditions are not summable (see Shore, 1985):

$$\kappa^{(h)}\geq 0\left(h=1,2,\cdots,\infty\right),\ \sum_{h=1}^{\infty}\kappa^{(h)}=\infty \text{ and } \sum_{h=1}^{\infty}\left[\kappa^{(h)}\right]^2<\infty \tag{4.38}$$

This study adopts the typical step size sequence $\left\{\kappa^{(h)}=1/h,h=1,2,\cdots,\infty\right\}$ that fulfills the above condition.

4.3.2 Sample Average Approximation

The SAA method proposed by Kleywegt et al. (2001) is a Monte Carlo simulation-based approach to the 2SSIP model. The quality of the solution from the SAA method, depending on the number of samples and the sample size, can be assessed by statistical analysis techniques. The optimal solution of (4.21), $\hat{\mathbf{v}}_N$, and the optimal objective function value, $\hat{v}_N$, can converge to an optimal solution of the original problem (4.19)-(4.20) as the sample size increases (Kleywegt et al., 2001). We can choose N considering the trade-off between the qualities of the solution obtained for the **SAA** problem and the computational effort needed to solve it in practice.

Let

m index for the number of sample

M number of samples

N size of each sample (i.e., the number of the realizations)

$\hat{\mathbf{v}}_N^m$ optimal first-stage decisions of the *m-th* **SAA** problem with sample size N

$\hat{v}_N^m$ optimal objective function value of the *m-th* **SAA** problem with sample size N

$\hat{v}_{N'}^m$ optimal objective function value of the *m-th* **SAA** problem with sample size N'

v^* optimal objective function value of the original problem

L_N^M lower bound to the optimal objective function value of the original problem

$U_{N'}^M$ upper bound to the optimal objective function value of the original problem

$\theta_{M,N,N'}$ gap between the lower bound and upper bound

The SAA method, incorporating with the dual decomposition and Lagrangian relaxation approach and the relevant statistical moment estimation, is presented as follows:

Step 0: Generate M samples of the random container shipment demand, where each sample has size N, namely, $\left\{\left(\boldsymbol{\xi}_1^m,\cdots,\boldsymbol{\xi}_l^m,\cdots,\boldsymbol{\xi}_N^m\right)\middle| m=1,2,\cdots,M\right\}$.

Step 1: Solve the SAA problem corresponding to each container shipment demand sample $\left(\boldsymbol{\xi}_1^m,\cdots,\boldsymbol{\xi}_l^m,\cdots,\boldsymbol{\xi}_N^m\right)$, $m=1,2,\cdots,M$, by using the above-mentioned dual decomposition and Lagrangian relaxation approach. Let $\hat{v}_N^1,\cdots,\hat{v}_N^M$ and $\hat{\mathbf{v}}_N^1,\cdots,\hat{\mathbf{v}}_N^M$ be the optimal first-stage solutions and objective function value of the SAA problem (4.21), respectively.

Step 2: Calculate a point estimation of a lower bound on the optimal function value of the 2SSIP model (4.19) using

$$L_N^M=\frac{1}{M}\sum_{m=1}^{M}\hat{v}_N^m \tag{4.39}$$

Step 3: For each optimal solution $\hat{\mathbf{v}}_N^m$ obtained in Step 1 ($m=1,2,\cdots,M$), independently generating another container shipment demand sample $\left(\hat{\boldsymbol{\xi}}_1^m,\hat{\boldsymbol{\xi}}_2^m,\cdots,\hat{\boldsymbol{\xi}}_{\hat{N}}^m\right)$ where the sample size here is $\hat{N}$ ($\hat{N}$ is much larger than N), and calculate

$$\hat{v}_{N'}^m\left(\hat{\mathbf{v}}_N^m\right)=\mathbf{c}^T\hat{\mathbf{v}}_N^m+\frac{1}{\hat{N}}\sum_{l=1}^{N'}Q'_{\xi}\left(\hat{\mathbf{v}}_N^m,\hat{\boldsymbol{\xi}}_l\left(\boldsymbol{\omega}\right)\right)-\sum_{k\in\mathcal{K}}c_k^{\text{OUT}}N_k^{\text{MAX}} \tag{4.40}$$

$\hat{v}_{\hat{N}}^m\left(\hat{\mathbf{v}}_N^m\right)$ is an unbiased estimation of an upper bound for the optimal function value of the 2SSIP model because $\hat{\mathbf{v}}_N^m$ is one of its feasible solutions. The best upper bound on v^* is given by:

$$U_{N'}^{M} = \min_{m \in \{1,\cdots,M\}} \left\{ \hat{v}_{N'}^{m} \left(\hat{\mathbf{v}}_{N}^{m} \right) \right\} \tag{4.41}$$

Step 4: Calculate an estimate of the gap between L_N^M and $U_{N'}^M$ as follows:

$$\theta_{M,N,N'} = U_{N'}^{M} - L_{N}^{M} \tag{4.42}$$

It has been proved by Norkin et al. (1998b) and Mak et al. (1999) that the expected value of $\hat{v}_N$ is less than or equal to the optimal value v^* of the original problem (4.19), namely $\mathbb{E}[\![\hat{v}_N]\!] \le v^*$. However, it is impossible to get the exact value of $\mathbb{E}[\![\hat{v}_N]\!]$, which indicates that $\mathbb{E}[\![\hat{v}_N]\!]$ has to be approximated by its sample mean since the sample mean is an unbiased estimator of $\mathbb{E}[\![\hat{v}_N]\!]$. In order to get the sample mean, *M* samples are generated (i.e. Step 0) and then the sample mean, denoted by L_N^M, is calculated (i.e. Step 1) using Eq. (4.39). Since the calculated sample average L_N^M in Eq. (4.39) is an unbiased estimator of $\mathbb{E}[\![\hat{v}_N]\!]$, L_N^M is less than or equal to the optimal value v^* of the original problem. Thus, L_N^M is a lower bound for v^*.

4.4 Numerical Example

In this section we implement the model for the data in a numerical example in order to validate the applicability of the proposed methodology. We firstly describe the characteristics of the numerical example and then do a sensitivity analysis of the SAA parameters, and finally explore the effect of variance value of uncertain container shipment demand on the objective function value of the stochastic programming solution and comment on the quality of the stochastic programming solutions in comparison to those obtained using a deterministic approach.

4.4.1 Experiment Design

We use an example to assess the 2SSIP model and the solution algorithm developed for solving the short-term LSFP problem with container transshipment and uncertain container shipment demand. We assume that a liner container shipping company intends to make a 6-month fleet plan. The liner shipping network depicted in Figure 3.2 is used here as a numerical example. This liner shipping network consists of 8 routes involving a total of 36 calling ports, serves 390 O-D pairs and generates 443 container routes. (see Figure 3.2). The ports called on each liner ship route and their digital number codes are shown in Table 3.1. The distance of each leg in each liner ship route is given in Table 3.2. The parameters of the numbers, sizes, chartering in rent, daily operating cost and design speed of each ship type are set the same value with that in the numerical of Chapter 3, listed in Table 3.3. The parameters of chartering out ships of each type are shown in Table 4.1. The daily operating cost of each ship type is computed by Eq. (3.17).

Table 4. 2 Parameters of chartering out rate

Item	Ship types				
	1	2	3	4	5
Chartering out rate (million $)	1.82	2.34	3.21	4.32	5.12

The random container shipment demand of each O-D pair $d \in \mathcal{D}$ is assumed to follow a normal distribution, that is $\xi_d \sim N\left(\mu_d, \sigma_d\right)$ with probability distribution function F_d (the demands are assumed to be independent) in this numerical example. The rationale of assuming normal distribution is that the deviation of the forecasted demand and the real demand is often approximately normally distributed and especially the normal distribution has been established to be suitable to describe the demand uncertainty by Brown (1959). The ratio μ_d / σ_d is assumed the same for all O-D pairs $d \in \mathcal{D}$ in order to simplify the data generation process. Since σ_d reflects the uncertainty level of container shipment demand, it can be set as different levels in order to investigate the effect of level of container shipment demand uncertainty on profit. The investigation is shown in Section 4.4.3. It is noted that the distribution tails are truncated in order to guarantee all generated values of demand are positive. In this way the values we generated are all from a range where the distribution has higher density and none values are from two-side tail regions.

We set the stop tolerance $\tau = 10^{-6}$ in the subgradient method, and the number of samples $M = 20$ and $\hat{N} = 1000$ in the SAA method. The solution algorithm is programmed using the programming language Lua (v5.1) with a mixed integer linear programming solver. All computations are carried out on a desktop personal computer with Intel (R) Core (TM) 2 CPU 1.86 GHz and 2.0 GB of RAM under Microsoft Windows 7.

4.4.2 Sensitivity Analysis of the Sample Size N in the SAA Method

Within the sample average approximation method, the sizes of the deterministic equivalents of the **SAA** problems corresponding to the different values of N are presented in Table 4.3. Table 4.4 gives the lower bound, upper bound, gap and 95% confidence interval of the gap, for each sample size $N \in \{20, 30, 40, 50, 60\}$, obtained using the proposed solution method. According to Table 4.4, the confidence interval of the optimality gap becomes narrower as the sample size increases. We thus take the sample size $N = 60$ in the subsequent analysis, in view of the acceptable confidence interval that results from this sample size.

Table 4. 3 Size of the deterministic equivalent of the SAA problem

N	Constraints	Variables
1	510	510
10	5,100	5,100
20	10,200	10,200
30	15,300	15,300
40	20,400	20,400
50	25,500	25,500
60	30,600	30,600

Table 4. 4 Statistical lower bound, upper bound, estimated gap and confidence interval with *M* = 20 and *N'* = 1000

N	Lower bound ($\times10^6$)	Upper bound ($\times10^6$)	Estimated gap ($\times10^6$)	95% Confidence interval ($\times10^6$)		
	Average	Average	Average	Min	Max	Interval
20	8875.531	8877.881	2.350	-1.291	5.991	7.282
30	8876.902	8878.574	1.672	-1.763	5.108	6.871
40	8875.375	8877.636	2.261	-0.680	5.202	5.882
50	8875.220	8877.193	1.973	-0.899	4.845	5.744
60	8874.668	8877.005	2.337	-0.010	4.685	4.695

4.4.3 Results Discussions

We now investigate the effect of container shipment demand uncertainty, by comparing the average profits obtained from the proposed 2SSIP model to those obtained from the expected value problem (EVP), that is the profits obtained where the uncertain container shipment demands are replaced by their mean values from the 2SSIP model. After solving the EVP, the optimal first-stage solutions, that is the fleet design and deployment decisions, are obtained. Given these optimal first-stage solutions obtained from the EVP, we compute the EEV (see Birge and Louveaux, 1997), that is the expected value of the EVP solution, by computing the expected value of the EVP first-stage solution across a large number of different scenarios of container shipment demand.

In order to investigate the effect of container shipment demand uncertainty, three different levels (low, medium, and high) of standard deviations of the uncertain container shipment demands are considered. They are set as 5%, 10% and 15% of the expected mean value respectively for each level. In Figure 4.2, the average profits obtained using the 2SSIP model, corresponding to these three levels, are compared with those obtained using the EVP. It is clearly observed that the estimated average profits in the 2SSIP model corresponding to all three levels of variance are smaller than those in the EVP. This is reasonable because, in the EVP, the container shipment demands are deterministic rather than random, their values given by the mean values used in the 2SSIP model; thus the EVP could be regarded as a problem with deterministic container shipment demand. Therefore, the EVP with deterministic container shipment demand would be expected to have a higher yield than the 2SSIP model with uncertain container shipment demand. This indicates that the precision of the estimate of container shipment demand is significant for a liner container shipping company. Moreover, it is found that the expected profits decrease with an increase in the variance of container shipment demands. This further verifies the significance of container shipment demand information for the liner container shipping company.

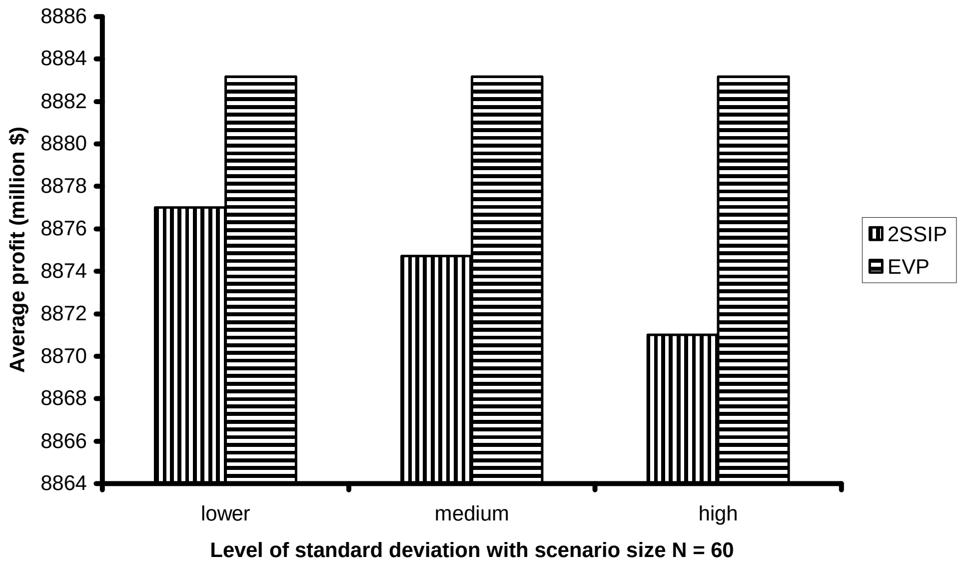

Figure 4. 2 Average profits of 2SSIP model and EVP with *N* = 60

After solving the EVP, the optimal first-stage solutions, that is the decisions about the numbers and types of ships in the fleet and the ship-to-route allocation, are obtained; then, the EEV can be computed by implementing the EVP first-stage solution for a large number of different scenarios of container shipment demand. The EEVs and the expected profits from the 2SSIP model associated with low, medium and high standard deviations of container shipment demand are depicted in Figure 4.3. It is clear that the estimated average profits of the 2SSIP model corresponding to the three levels of variance are all higher than the EEVs, which shows that the 2SSIP model would be expected to have a higher yield than the EVP and indicates that the 2SSIP model is superior to the EVP. Also, we find that the ratios between the objectives for 2SSIP and EEVs increase with increasing variance (as expected). However, we have to acknowledge that the average profit obtained from the 2SSIP model is weak because we can only set *proper* but not precise values of the SAA parameters, *M*, *N* and *N'*. Additionally, although Shore (1985) proved that, theoretically, $LR\left(\boldsymbol{\lambda}^{h}\right) \rightarrow LD$ in the dual decomposition method, it is quite difficult to reach the convergence point in practice. We can only set a tolerance τ in order to find a relative better solution with an acceptable level of precision.

The liner ship fleet plans suggested by the 2SSIP model and the EVP with a low variance of container shipment demand are shown in Tables 4.5 and Table 4.6, respectively. Both fleets contain a total of 34 ships. However, the two plans are different. Under the 2SSIP model, the liner container shipping company makes up its fleet using four types of ships, charters out three ships and charters in ten ships; under EVP, its fleet contains five types of ship, eight ships are chartered out and one ship is chartered in.

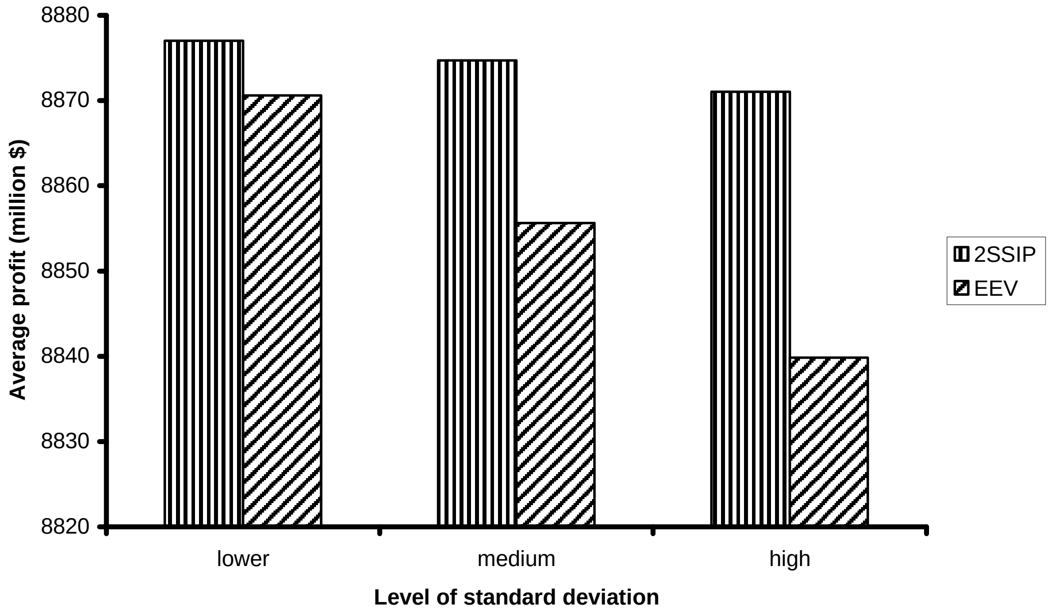

Figure 4. 3 Average profits of 2SSIP model and EEV for different variances

Table 4. 5 Liner ship fleet plan produced by the 2SSIP model with low variance

	Ship Type \ Route	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
Ship allocations	1			2			2	3	
	2								1
	3	3	3	1			1	3	
	4								
	5				5	8		2	
Number of voyages	1			15			17	8	
	2								26
	3	28	28	12			10	12	
	4								
	5				32	31		8	

Table 4. 6 Liner ship fleet plan produced by the EVP

	Route / Ship Type	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
Ship allocations	1			2			2	3	
	2							1	1
	3	3	3	1			1	2	
	4				1				
	5				4	8		2	
Number of voyages	1			15			16	11	
	2							4	26
	3	27	27	11			10	7	
	4				6				
	5				28	31		8	

4.5 Summary

This chapter continues to study a realistic planning problem with container transshipment and demand uncertainty faced by a liner container shipping company from view point of maximizing expected profit. The problem was formulated as a 2SSIP model. Actually, it is possible to adapt the mathematical formulation of the problem to any planning problem that consists of two stages of decision variables. The greatest difficulty in solving the 2SSIP model is determining how to deal with the expected recourse function, which is only implicitly defined and depends on the first-stage decisions and usually involves optimization problems embedded in expectation. To effectively solve the proposed model, firstly, the sample average approximation method is used to approximate the expected recourse function, and then the dual decomposition and

Lagrangian relaxation method is used to solve the model. The proposed model and solution methods are tested using a numerical example. The gaps between the lower and upper bounds are small, which indicates that the solution methods are effective. It is also found that the variability of the uncertain parameters has a significant effect on the solutions. As the variability increases, the profit obtained by a liner container shipping company decreases.

CHAPTER 5 A ROBUST OPTIMIZATION MODEL FOR SHORT-TERM LSFP

5.1 Introduction

Chapter 4 studies a liner ship fleet planning problem with container transshipment under uncertain container shipment demand and formulates it as an optimization model which aims to maximize the expected profit. However, this type of expected value model only aims to minimize or maximize the expected value of a key variable, such as cost or profit. As for the variance (namely the risk), which is another issue of great concern to decision-makers, is not taken into account. Therefore, this chapter develops a robust optimization model in which both expected value and variance are considered simultaneously. By adjusting penalty parameters of the robust optimization model, decision-makers can determine an optimal liner ship fleet planning, including decisions about fleet design and fleet deployment, in order to maximize total profit under different container shipment demand scenarios while at the same time control the variance. The simplicity of the implementation and operation of the model should enable the decision-makers to manage liner ship fleet in terms of fleet design and deployment without having to learn complex operations and programming procedures. It is realized that this model should yield solutions that are less sensitive to the uncertain data of container shipment demand. The robustness and effectiveness of the developed model are demonstrated with numerical results. The trade-off between solution robustness and model robustness is also analyzed.

The remainder of this chapter is organized as follows. After this introductory section, Section 5.2 firstly introduces the general modeling framework of robust optimization, and then describes the concept of container shipment demand scenarios, finally proposes a robust optimization model for the short-term LSFP problem with container transshipment and uncertain container shipment demand. Section 5.3 demonstrates the applicability of the proposed robust optimization model by applying it on a numerical example, and also analyzes the trade-off between solution robustness and model robustness. Finally, summary is given in Section 5.4.

5.2 Model Development

5.2.1 General Modeling Framework of Robust Optimization

Before the development of robust optimization model for the proposed short-term liner ship fleet planning problem with container shipment demand uncertainty, we briefly introduce the framework of a robust optimization model.

Robust optimization, developed by Mulvey et al. (1995), is able to tackle decision-makers' level of risk aversion, and has yielded a series of solutions progressively less sensitive to realizations of data in a scenario set. It has been applied in some real-life problems, such as capacity expansion planning problems (Malcolm and Zenios, 1994; Laguna, 1998), logistics problems (Leung et al., 2002) and production planning problems (Leung et al., 2007a, 2007b). The optimal solution provided by a robust optimization model is called *solution robust* if it remains "close" to optimal for all scenarios of the input data. The solution is called *model robust* if it remains "almost" feasible for all data scenarios. Robust optimization includes two distinct constraints: a structural constraint and a control constraint. Structural constraints are formulated following the concept of linear programming and its input data are free of any noise, while control constraints are taken as an auxiliary constraint influenced by noisy data. Moreover, robust optimization includes two sets of variables: *design variables* and *control variables*. Design variables cannot be adjusted once a specific realization of the data has been observed and their optimal values is not conditioned on the realization of the uncertain parameters; while control variables are subject to adjustment once uncertain parameters are observed and their optimal value depend both on the realization of uncertain parameters and on the optimal value of the design variables.

Let $\mathbf{x} \in \Re^{n_1}$ be a vector of the design variables and $\mathbf{y} \in \Re^{n_2}$ be a vector of control variables. Then a general linear programming model has the following structure:

$$\min \mathbf{c}^T \mathbf{x} + \mathbf{d}^T \mathbf{y} \tag{5.1}$$

subject to

$$\mathbf{Ax} = \mathbf{b}, \tag{5.2}$$

$$\mathbf{Bx} + \mathbf{Cy} = \mathbf{e}, \tag{5.3}$$

$$\mathbf{x}, \mathbf{y} \geq \mathbf{0}. \tag{5.4}$$

Eq. (5.2) denotes the structural constraints whose coefficients are fixed and free of noise, while Eq. (5.3) denotes the control constraints whose coefficients are subject to noise. Constraints (5.4) ensure non-negative vectors.

To define the robust optimization problem, a set of scenarios $\mathcal{S}=\{1,2,3,...,S\}$ is introduced and the index *s* represents a specific scenario. With each scenario $s\in\mathcal{S}$ we associate the set $\{\mathbf{d}_s,\mathbf{B}_s,\mathbf{C}_s,\mathbf{e}_s\}$ of realizations for the coefficients of the control constraints, and the probability of the scenario p_s which is characterized by $\sum_{s=1}^{S} p_s = 1$. Moreover, we introduce a set $\{\mathbf{y}_1,\cdots,\mathbf{y}_S\}$ of control variables for each scenario $s\in\mathcal{S}$ and a set $\{\boldsymbol{\varepsilon}_1,\cdots,\boldsymbol{\varepsilon}_S\}$ of error vectors that measure the infeasibility allowed in the control constraints under scenario *s*. Then the general form of the robust optimization model has the following structure:

$$\min \sigma\left(\mathbf{x},\mathbf{y}_1,...,\mathbf{y}_S\right)+\varpi\rho\left(\boldsymbol{\varepsilon}_1,...,\boldsymbol{\varepsilon}_S\right) \tag{5.5}$$

subject to

$$\mathbf{Ax}=\mathbf{b}, \tag{5.6}$$

$$\mathbf{B}_s\mathbf{x}+\mathbf{C}_s\mathbf{y}_s+\boldsymbol{\varepsilon}_s=\mathbf{e}_s, \forall s\in\mathcal{S} \tag{5.7}$$

$$\mathbf{x},\mathbf{y}_s\geq\mathbf{0}, \forall s\in\mathcal{S} \tag{5.8}$$

It is noted that since the robust optimization model considers multiple scenarios, the objective function in (5.1), $\zeta=\mathbf{c}^T\mathbf{x}+\mathbf{d}^T\mathbf{y}$ becomes a random variable, taking the value $\zeta_s=\mathbf{c}^T\mathbf{x}+\mathbf{d}^T\mathbf{y}_s$, with probability p_s. The first term of the objective function (5.5), $\sigma(\cdot)$, is called aggregate function, used to measure optimality robustness, whereas the second term $\rho(\cdot)$ is called feasibility penalty function and is used to penalize violations of the control constraints under some of the scenarios and to measure the model robustness. The goal programming weight ϖ is used to derive a spectrum of answers that tradeoff solution for model robustness. Using the weight ϖ, the tradeoff between solution robustness measured from the aggregate function $\sigma(\cdot)$ and model robustness measured from the penalty function $\rho(\cdot)$ can be modeled under the multi-criteria decision-making process. For instance, if $\varpi = 0$, the objective is to minimize the term $\sigma(\cdot)$ and the solution may be infeasible; while if ϖ is assigned to be sufficiently large, the term $\rho(\cdot)$ dominates the objective and results in a higher cost. An appropriate form of term the aggregate function $\sigma(\cdot)$ and the penalty function $\rho(\cdot)$ is proposed by Mulvey et al. (1995):

$$\sigma(\mathbf{x}, \mathbf{y}_1, \ldots, \mathbf{y}_S) = \sum_{s \in \mathcal{S}} p_s \zeta_s + \lambda \sum_{s \in \mathcal{S}} p_s \left(\zeta_s - \sum_{s' \in \mathcal{S}} p_{s'} \zeta_{s'} \right)^2 \tag{5.9}$$

$$\rho(\boldsymbol{\varepsilon}_1, \ldots, \boldsymbol{\varepsilon}_S) = \sum_{s \in \mathcal{S}} p_s \max\{0, \boldsymbol{\varepsilon}_s\} \tag{5.10}$$

The aggregation function $\sigma(\cdot)$ in Eq. (5.9) is the mean value plus a constant λ times the variance $\sum_{s \in \mathcal{S}} p_s \left(\zeta_s - \sum_{s' \in \mathcal{S}} p_{s'} \zeta_{s'} \right)^2$, in which the quadratic form is hard to tackle. Yu and Li (2000) proposed another replacement for the aggregation function $\sigma(\cdot)$, which takes the following form:

$$\sigma(\mathbf{x}, \mathbf{y}_1, \ldots, \mathbf{y}_S) = \sum_{s \in \mathcal{S}} p_s \zeta_s + \lambda \sum_{s \in \mathcal{S}} p_s \left| \zeta_s - \sum_{s' \in \mathcal{S}} p_{s'} \zeta_{s'} \right| \tag{5.11}$$

Yu and Li (2000) developed an efficient method to deal with the absolute deviations in (5.11), with a framework described as follows:

$$\min \sum_{s \in \mathcal{S}} p_s \zeta_s + \lambda \sum_{s \in \mathcal{S}} p_s \left[\left(\zeta_s - \sum_{s' \in \mathcal{S}} p_{s'} \zeta_{s'} \right) + 2\vartheta_s \right] \tag{5.12}$$

subject to

$$\zeta_s - \sum_{s' \in \mathcal{S}} p_{s'} \zeta_{s'} + \vartheta_s \geq 0, \forall s \in \mathcal{S} \tag{5.13}$$

$$\vartheta_s \geq 0, \forall s \in \mathcal{S} \tag{5.14}$$

It is verified that the solution from models (5.12) to (5.14) is identical to that from objective (5.11). Therefore, the general framework of a robust optimization model is formed as follows:

$$\min \sum_{s \in \mathcal{S}} p_s \zeta_s + \lambda \sum_{s \in \mathcal{S}} p_s \left[\left(\zeta_s - \sum_{s' \in \mathcal{S}} p_{s'} \zeta_{s'} \right) + 2\vartheta_s \right] + \varpi \sum_{s \in \mathcal{S}} p_s \boldsymbol{\varepsilon}_s \tag{5.15}$$

subject to

$$\mathbf{Ax} = \mathbf{b}, \tag{5.16}$$

$$\mathbf{B}_s \mathbf{x} + \mathbf{C}_s \mathbf{y}_s + \boldsymbol{\varepsilon}_s = \mathbf{e}_s, \forall s \in \mathcal{S} \tag{5.17}$$

$$\zeta_s - \sum_{s' \in \mathcal{S}} p_{s'} \zeta_{s'} + \vartheta_s \geq 0, \forall s \in \mathcal{S} \tag{5.18}$$

$$\mathbf{x}, \mathbf{y}_s, \boldsymbol{\varepsilon}_s, \vartheta_s \geq \mathbf{0}, \forall s \in \mathcal{S} \tag{5.19}$$

5.2.2 Scenarios of Uncertain Container Shipment Demand

The uncertainty of container shipment demand is included in the model by specifying a set of discrete demand scenarios. In each scenario, we specify values for container shipment demand for each port pair over the planning horizon. Associated with each scenario is a weight; these weights are often thought of as the probabilities that each scenario will occur. In other words, the container shipment demand of a given O-D port pair over the planning horizon is assumed to be a discrete random variable $\xi^{od} \left((o,d) \in \mathcal{W} \right)$, taking a limited number of possible values with known probabilities. Let $s \in \mathcal{S} = \{1, 2, 3, ..., S\}$ be the set of container shipment demand scenarios. The realization of the random parameter ξ^{od} in scenario $s \in \mathcal{S}$ is denoted by ω_s^{od} and the probability that scenario s happens is represented by p_s and is characterized by $\sum_{s=1}^{S} p_s = 1$.

To illustrate the concept of container shipment demand scenarios, consider a simple network shown in Figure 5.1. For simplicity, consider three O-D pairs: Pusan (PS) $\rightarrow$ Shanghai (SH), Shanghai (SH) $\rightarrow$ Yantian (YT), Yantian (YT) $\rightarrow$ Hong Kong (HK). A liner shipping operator has to acquire a fleet of ships before knowing the demand, and then provides liner shipping service to shippers in order to fulfill as much demand as possible (given their available fleet). To make this illustrate as clear as possible, suppose that there are only five discrete scenarios for the unknown demands, as shown in Table 5.1, and the corresponding probability that these scenarios each occurs is shown in Table 5.2.

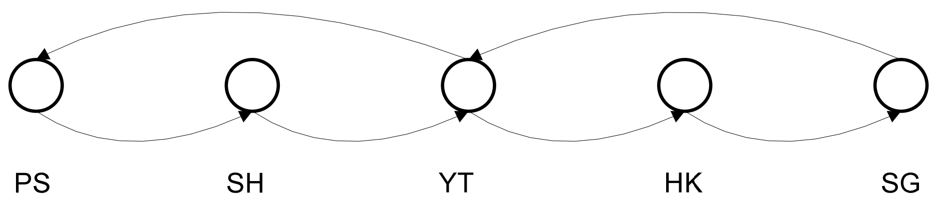

Figure 5. 1 A liner ship route

Table 5. 1 Demand scenarios for illustrative example

O-D	Container shipment demand				
	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
PS $\rightarrow$ SH	1000	2000	3000	4000	5000
SH $\rightarrow$ YT	800	1000	1500	2000	2500
YT $\rightarrow$ HK	1000	1500	2000	2500	3000

Table 5. 2 Probability of scenarios for the illustrative example

	Scenario 1	Scenario 2	Scenario 3	Scenario 4	Scenario 5
Probability	0.15	0.25	0.25	0.25	0.1

5.2.3 Robust Optimization Model

Before the development of robust optimization model for the short-term LSFP problem with container transshipment and uncertain container shipment demand, we firstly introduce the following decision variables used in the formulation of this chapter.

n_{kr}^{OWN} number of owned ships of type k ($k \in \mathcal{K}$) assigned on route r ($r \in \mathcal{R}$)

n_{kr}^{IN} number of chartered in ships of type k ($k \in \mathcal{K}$) assigned on route r ($r \in \mathcal{R}$)

x_{kr} number of voyages of ships of type k ($k \in \mathcal{K}$) on route r ($r \in \mathcal{R}$)

$z_s^{h^{od}}$ number of containers carried by ships deployed on the container route $h^{od} \in \mathcal{H}^{od}$ between O-D port pair $(o,d) \in \mathcal{W}$ under container shipment demand scenario s

Based on the above introduction of robust optimization in Section 5.2.1, we can easily distinguish the design and control variables for the proposed LSFP problem studied in this chapter. n_{kr}^{OWN}, n_{kr}^{IN} and x_{kr} are design variables; $z_s^{h^{od}}$ are control variables.

The revenue earned by the liner container shipping company comes from two sources: rent from chartering out ships to other liner operators and revenue from shipping containers for shippers. Let c_k^{OUT} denoted the of chartering out a ship of type k over the planning horizon (\$/ship), then the revenue from chartering out ships is given by:

$$\sum_{k\in\mathcal{K}} c_k^{\text{OUT}} \left(N_k^{\text{MAX}} - \sum_{r\in\mathcal{R}} n_{kr}^{\text{OWN}} \right) \tag{5.20}$$

Let f^{od} denote freight rate for transporting a container between an O-D port pair $(o,d)\in\mathcal{W}$ (\$/TEU), given scenario s, and realization of container shipment demand, ω_s^{od}, for an O-D port pair $(o,d)\in\mathcal{W}$, the revenue from shipping containers along all possible routes is given by:

$$\sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} f^{od} z_s^{h^{od}} \left(\omega_s^{od} \right) \tag{5.21}$$

The total costs consist of the container handling cost, the operating costs of the ships and the investment made by chartering in ships. The container handling cost incurred on a given container route includes the loading cost at the origin, the discharging cost at the destination and the transshipment costs (if any). Let $c^{h^{od}}$ denote the handling cost of a container being carried between O-D port pair $(o,d)\in\mathcal{W}$ incurred on container route $h^{od}\in\mathcal{H}^{od}$ (\$/TEU), the handling costs for delivering all containers under the container shipment demand scenario s are given by:

$$\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}} c^{h^{od}} z_s^{h^{od}}\left(\omega_s^{od}\right) \tag{5.22}$$

Let c_{kr} denote the operating cost of a ship of type k on route r per voyage ($/voyage), and c_k^{IN} denote the rate of chartering in a ship of type k over the planning horizon ($/ship), then the operating costs of the ships plus the rent paid for chartering in ships is given by:

$$\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}} c_{kr}x_{kr} + \sum_{k\in\mathcal{K}} c_k^{\mathrm{IN}} n_{kr}^{\mathrm{IN}} \tag{5.23}$$

Therefore, the maximized profit obtained by the liner container shipping company under container shipment demand scenario s is given by:

$$\max \sum_{k\in\mathcal{K}} c_k^{\mathrm{OUT}}\left(N_k^{\mathrm{MAX}} - \sum_{r\in\mathcal{R}} n_{kr}^{\mathrm{OWN}}\right) + \sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\left(f^{od} - c^{h^{od}}\right)\times z_s^{h^{od}}\left(\omega_s^{od}\right) - \sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}}\left(c_{kr}x_{kr} + c_k^{\mathrm{IN}} n_{kr}^{\mathrm{IN}}\right) \tag{5.24}$$

$\sum_{k\in\mathcal{K}} c_k^{\mathrm{OUT}} N_k^{\mathrm{MAX}}$ can be removed from Eq. (5.24) since it is a constant value, therefore Eq. (5.24) can be rewritten as follows:

$$\max -\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}}\left(c_k^{\mathrm{OUT}} n_{kr}^{\mathrm{OWN}} + c_{kr}x_{kr} + c_k^{\mathrm{IN}} n_{kr}^{\mathrm{IN}}\right) + \sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\left(f^{od} - c^{h^{od}}\right)\times z_s^{h^{od}}\left(\omega_s^{od}\right) \tag{5.25}$$

Mathematically, Eq. (5.25) is equivalent to the equation below:

$$\min \sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}}\left(c_k^{\mathrm{OUT}} n_{kr}^{\mathrm{OWN}} + c_{kr}x_{kr} + c_k^{\mathrm{IN}} n_{kr}^{\mathrm{IN}}\right) - \sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\left(f^{od} - c^{h^{od}}\right)\times z_s^{h^{od}}\left(\omega_s^{od}\right) \tag{5.26}$$

Therefore, the notation ζ_s in the robust optimization model described in Section 5.2.1 is given below:

$$\zeta_s = \sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}}\left(c_k^{\mathrm{OUT}} n_{kr}^{\mathrm{OWN}} + c_{kr}x_{kr} + c_k^{\mathrm{IN}} n_{kr}^{\mathrm{IN}}\right) - \sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\left(f^{od} - c^{h^{od}}\right)\times z_s^{h^{od}}\left(\omega_s^{od}\right) \tag{5.27}$$

The aggregation function $\sigma(\cdot)$ for the proposed LSFP problem is thus expressed as follows:

$$
\begin{aligned}
&\sigma(\cdot) \\
&= \sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}}\left(c_k^{\mathrm{OUT}} n_{kr}^{\mathrm{OWN}} + c_{kr}x_{kr} + c_k^{\mathrm{IN}} n_{kr}^{\mathrm{IN}}\right) + \sum_{s\in\mathcal{S}}\sum_{(o,d)\in\mathcal{W}}\sum_{h^{(o,d)}\in\mathcal{H}^{(o,d)}} p_s \times \left(c^{h^{od}} - f^{od}\right)\times z_s^{h^{od}}\left(\omega_s^{od}\right) \qquad (5.28)\\
&\quad + \lambda\sum_{s\in\mathcal{S}} p_s \left[\left(\begin{array}{l}\displaystyle\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\left(c^{h^{od}} - f^{od}\right)\times z_s^{h^{od}}\left(\omega_s^{od}\right)\\ \displaystyle -\sum_{s'\in\mathcal{S}}\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}} p_{s'}\times\left(c^{h^{od}} - f^{od}\right)\times z_{s'}^{h^{od}}\left(\omega_{s'}^{od}\right)\end{array}\right) + 2\vartheta_s\right]
\end{aligned}
$$

Therefore, the robust optimization model for the proposed LSFP problem is given by:

[ROM]

$$
\begin{aligned}
\min &\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{K}}\left(c_k^{\mathrm{OUT}} n_{kr}^{\mathrm{OWN}} + c_{kr}x_{kr} + c_k^{\mathrm{IN}} n_{kr}^{\mathrm{IN}}\right) + \sum_{s\in\mathcal{S}}\sum_{(o,d)\in\mathcal{W}}\sum_{h^{(o,d)}\in\mathcal{H}^{(o,d)}} p_s \times \left(c^{h^{od}} - f^{od}\right)\times z_s^{h^{od}}\left(\omega_s^{od}\right)\\
&+ \lambda\sum_{s\in\mathcal{S}} p_s \left[\begin{array}{l}\displaystyle\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\left(c^{h^{od}} - f^{od}\right)\times z_s^{h^{od}}\left(\omega_s^{od}\right)\\ \displaystyle -\sum_{s'\in\mathcal{S}}\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}} p_{s'}\times\left(c^{h^{od}} - f^{od}\right)\times z_{s'}^{h^{od}}\left(\omega_{s'}^{od}\right) + 2\vartheta_s\end{array}\right] + \varpi\sum_{s\in\mathcal{S}}\sum_{r\in\mathcal{R}}\sum_{i=1}^{m_r} p_s\varepsilon_s^{ir}
\end{aligned}
$$

(5.29)

subject to:

$$\sum_{r\in\mathcal{R}} n_{kr}^{\mathrm{OWN}} \le N_k^{\mathrm{MAX}}, \forall k\in\mathcal{K} \qquad (5.30)$$

$$\sum_{r\in\mathcal{R}} n_{kr}^{\mathrm{IN}} \le NCI_k^{\mathrm{MAX}}, \forall k\in\mathcal{K} \qquad (5.31)$$

$$x_{kr} \le \left(n_{kr}^{\mathrm{OWN}} + n_{kr}^{\mathrm{IN}}\right)\times\left\lfloor\frac{T}{t_{kr}}\right\rfloor, \forall r\in\mathcal{R}, k\in\mathcal{K} \qquad (5.32)$$

$$\sum_{k\in\mathcal{K}} x_{kr} \ge N_r, \forall r\in\mathcal{R} \qquad (5.33)$$

$$\sum_{k\in\mathcal{K}} x_{kr}V_k + \varepsilon_s^{ir} \geq \sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} \rho_{ir}^{h^{od}} z_s^{h^{od}}\left(\omega_s^{od}\right), i=1,\cdots,m_r, \forall r\in\mathcal{R}, \forall s\in\mathcal{S} \tag{5.34}$$

$$\begin{bmatrix} \sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} \left(c^{h^{od}} - f^{od}\right)\times z_s^{h^{od}}\left(\omega_s^{od}\right) \\ -\sum_{s'\in\mathcal{S}} \sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} p_{s'}\times\left(c^{h^{od}} - f^{od}\right)\times z_{s'}^{h^{od}}\left(\omega_{s'}^{od}\right) + \vartheta_s \end{bmatrix} \geq 0, \forall s\in\mathcal{S} \tag{5.35}$$

$$\sum_{g\in\mathcal{G}_d} z_{dg}^s\left(\omega_d^s\right) \leq \xi_d\left(\omega_d^s\right), \forall d\in\mathcal{D}, \forall s\in\mathcal{S} \tag{5.36}$$

$$n_{kr}^{\text{OWN}}, n_{kr}^{\text{IN}}, x_{kr} \in \mathbb{Z}^+ \cup \{0\}, \forall k\in\mathcal{K}, \forall r\in\mathcal{R} \tag{5.37}$$

$$\varepsilon_s^{ir} \geq 0, \vartheta_s \geq 0, i=1,\cdots,m_r; \forall r\in\mathcal{R}, s\in\mathcal{S} \tag{5.38}$$

Eq. (5.29) is the objective function of the robust optimization model. The sets of constraints (5.30) and (5.31) ensure the numbers ship owned and chartered in do not exceed the number available. Constraints (5.32) compute the maximal number of voyage that ships of type *k* can complete on route *r*, where $\lfloor a \rfloor$ denotes the maximum integer not greater than *a*. Constraint (5.33) specifies the number of voyages required on route *r* in order to maintain a given level of liner shipping frequency. The constraints (5.30) to (5.33) are structural constraints, while Constraints (5.34) are control constraints in the robust optimization model. The first term of the left-hand side of Constraints (5.34) represents the transportation capacity of fleet on route *r*, and the second term represents the errors in the robust optimization model, the right-hand side of Constraints (5.34) represents the container flow of each leg *i* on route *r*. Constraints (5.34) are used to measure the feasibility of the solution of x_{kr} and $z_s^{h^{od}}\left(\omega_s^{od}\right)$. If the transportation capacity of ships on route *r* (the solution of x_{kr}) is greater than the container assignment flow of each leg on this route *r* (the solution of $z_s^{h^{od}}$ given a demand scenario ω_s^{od}) which indicates the solution of x_{kr} and $z_s^{h^{od}}\left(\omega_s^{od}\right)$ is feasible, then the deviation $\varepsilon_s^{ir}=0$ under minimization; otherwise $\varepsilon_s^{ir}=\sum_{(o,d)\in\mathcal{W}}\sum_{h^{od}\in\mathcal{H}^{od}}\rho_{ir}^{h^{od}}z_s^{h^{od}}\left(\omega_s^{od}\right)-\sum_{k\in\mathcal{K}}x_{kr}V_k$ which indicates under fulfillment of transportation capacity constraints. Thus, an infeasible solution is obtained. Constraints (5.35) are the application of Constraints (5.18) in the robust optimization model for the proposed LSFP problem. Constraints (5.36) imply the containers carried on the ships of each scenario cannot exceed the realization of the demand. Constraints (5.37) and Constraints (5.38) require that n_{kr}^{OWN} , n_{kr}^{IN} and x_{kr} are nonnegative integer decision variables and define the range of ε_s^{ir} and ϑ_s is nonnegative, respectively. It is noted that the proposed ROM is an integer linear programming model; therefore, it can be solved by any optimization solvers such as CPLEX.

5.3 Numerical Example

We use the same liner shipping network depicted in Figure 3.2 as the numerical example. Assume that a liner container shipping company intends to make a 6-month fleet plan. This liner shipping network consists of 8 routes involving a total of 36 calling ports, 390 O-D pairs and 443 container route plans (see Figure 3.2). The ports called on each liner ship route and their digital number codes are shown in Table 3.1. The distance of each leg in each liner ship route is given in Table 3.2. The parameters of the numbers, sizes, chartering in rent, daily operating cost and design speed of each ship type are set the same value with that in the numerical of Chapter 3, listed in Table 3.3. The parameters of chartering out ships of each type are shown in Table 4.1. The daily operating cost of each ship type is computed by Eq. (3.17).

5.3.1 Sensitivity Analysis of λ

The variance in the objective function (5.15) measures the level of risk encountered by a decision-maker. Therefore, λ can be regarded as representing the risk attitude of the decision-maker. In the case when $\lambda = 0$, the decision-maker can be thought of as risk neutral because the variance is not involved in the decision. In the case when $\lambda > 0$, the decision-maker is risk averse. When the value of λ increases, this indicates that the decision-maker will pay more attention to the variance $\sum_{s \in \mathcal{S}} p_s \left[\left(\zeta_s - \sum_{s' \in \mathcal{S}} p_{s'} \zeta_{s'} \right) + 2\vartheta_s \right]$ when aiming to achieve their minimization objective, because the variance $\sum_{s \in \mathcal{S}} p_s \left[\left(\zeta_s - \sum_{s' \in \mathcal{S}} p_{s'} \zeta_{s'} \right) + 2\vartheta_s \right]$ will dominate the expected cost $\sum_{s \in \mathcal{S}} p_s \zeta_s$. Therefore, it is expected that the value of variance will decrease when the value of λ increases. This expectation has been mathematically proven, see Proposition 1 in Appendix A.

In the numerical example, we vary the parameter λ from 0 to 2 with increments of 0.1 and show the trend in the variance as λ changes in Figure 5.2. The parameter λ reflects the risk appetite level of the liner container shipping company; in other words, their desire or reluctance to confront risk. When $\lambda > 0$, and the company is risk averse, this implies that it will implement a safe strategy to manage and operate its fleet. When $\lambda = 0$, and the company is risk neutral, it will be relatively confident in confronting risk. However, the expected profit in the case of risk neutrality is not guaranteed to be higher than that in the case of risk aversion. As can be seen from Figure 5.2, the expected profit in the case where $\lambda=0$ is less than that in the case where $\lambda=0.5$.

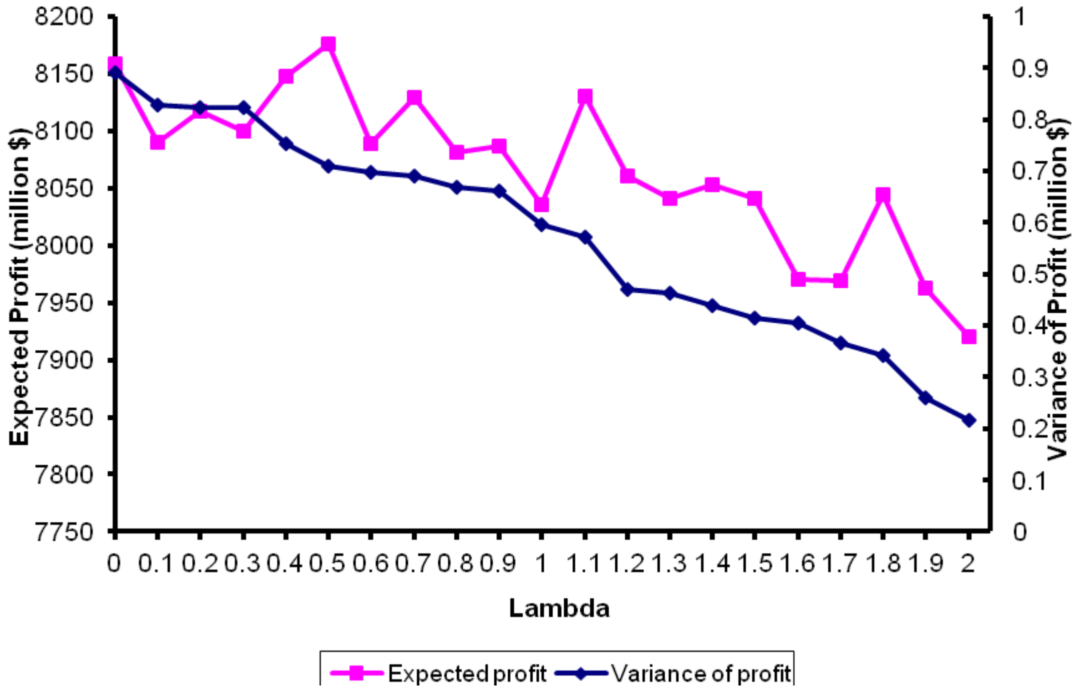

Figure 5. 2 Trend in variance and expected profit with lambda

5.3.2 Sensitivity Analysis of ϖ

In the general form of ROM (5.5), the feasibility penalty function of $\rho(\cdot)$ is used to penalize violations of the control constraints that occur under certain scenarios and to measure model robustness. Therefore, for the ROM (5.29) proposed for the short-term LSFP problem in this study, ε_s^{ir} in (5.29) can be regarded as the underfulfillment of the transportation capacity and the weight ϖ can be regarded as the unit penalty cost for underfulfillment ($/TEU). The role of weight ϖ in the objective function (5.29) is to find a tradeoff between solution robustness (close to an optimal solution) and model robustness (close to a feasible solution). In the case when $\varpi = 0$, ε_s^{ir} in the set of constraints (5.34) is equal to $\sum_{(o,d)\in\mathcal{W}} \sum_{h^{do}\in\mathcal{H}^{od}} \rho_{ir}^{h^{od}} z_s^{h^{od}} \left(\omega_s^{od}\right)$ due to minimization, which indicates that the underfulfillment (i.e., ε_s^{ir}) is at its highest. The suggested plan obtained in this case cannot be adopted. As the weight ϖ increases, which means the penalty cost increases, the underfulfillment (i.e., ε_s^{ir}) is expected to decrease. This expectation has been mathematically proven in Proposition 2 of Appendix A.

As the weight ϖ increases, the expected total profit which represents the solution robustness, and the expected underfulfillment, which represents model robustness, both decrease. The intuition behind these results is that, when the weight ϖ increases, ε_s^{ir} in the objective function (5.29) of ROM generally decreases due to the minimization of the objective, which results in a decrease in expected underfulfillment; simultaneously, it also results in a decrease of $z_s^{h^{od}}$ to meet the constraints (5.34), which implies that the expected total profit decreases. The results show that, for larger values of ϖ, the solution obtained approaches 'almost' feasible for any realization of scenario. The expected underfulfillment will eventually drop to zero with an increase in the value of ϖ. The trade-off between feasibility and expected profits for this numerical example is plotted in Figure 5.3.

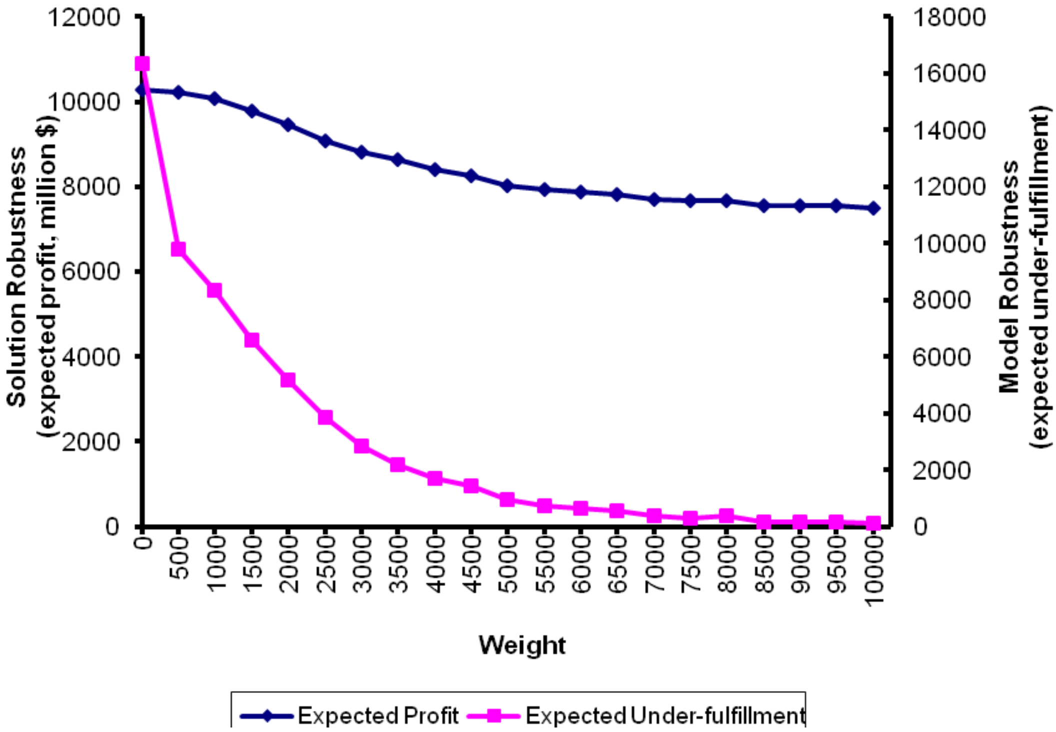

Figure 5. 3 Trade-off between expected profit and expected under-fulfillment

5.3.3 Comparison between ROM and EVM

Here, the parameters λ and ϖ are set to $\lambda = 1$ and $\varpi = 5000$. The optimal liner ship fleet plan is shown in Table 5.3. The results generally show that bigger ships are mostly assigned to longer routes while smaller ships are mostly assigned to shorter routes; also, the results generally indicate that more of the ships are assigned to longer routes than to the shorter ones. For example, a total of seven ships are assigned to the longest route NCE, including three ships with a size of 8063 TEUs while only one ship, with a size of 2808 TEUs, is assigned to the shortest route, UKX. These results are reasonable. A possible explanation is that the container flow on each leg of a long route (such as route NCE) generally consists of more containers from different port pairs than would be the case on a short route (such as route UKX). Therefore, bigger and more ships have to be assigned to the long route to meet the high level of container flow. Additionally, ships take more time to complete a round voyage on a long route than on a short route, which indicates that more ships are deployed on the long route in order to provide the same frequency level as on a short route.

Table 5.4 shows the liner ship fleet plan obtained from the EVM. Comparing it to the plan shown in Table 5.3, it is found that the fleets suggested by the EVM and ROM both contain a total of 33 ships. However, the two plans are different. The liner container shipping company is expected to have a yield of 7373 million dollars if it employs the plan suggested by the EVM, but 8036 million dollars if it uses the ROM's plan; meanwhile, the company will face a higher risk under the plan obtained from the EVM because the variance of the profit is higher than that of the ROM plan. The results show that the liner ship fleet plan suggested by the ROM is superior to that produced by the EVM.

Table 5. 3 Liner ship fleet plan produced by the ROM

	Route / Ship Type	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
Ship allocations	1							3	1
	2			2				2	
	3	2				3		2	
	4			2	1	1			
	5	1	3		5	3	2		
Number of voyages	1							12	29
	2			26				8	
	3	28				12		8	
	4			26	7	4			
	5	13	39		35	12	26		
Expected profit (million $)		8036.098							
Variance (million $)		1.788							

Table 5. 4 Liner ship fleet plan produced by the EVM

	Route / Ship Type	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
Ship allocations	1							3	1
	2			2				2	
	3					2	4	2	
	4				1	2		2	
	5	2	4	1	4	3			
Number of voyages	1							12	29
	2			26				8	
	3					8	52	4	
	4				7	8		4	
	5	26	52	13	28	12			
Expected profit (million $)		7273.046							
Variance (million $)		2.14							

5.4 Summary

In this chapter, a liner ship fleet planning problem with container shipment demand uncertainty is considered. Firstly, an expected value model (EVM) is developed aiming to maximize the expected profit. However, the variance, which means the risk, is not involved in EVM. Therefore, we continue to develop a robust optimization model which involves the expected profit and variance simultaneously. By analyzing penalty parameters for the tradeoff between optimality and infeasibility, the planners can obtain a feasible liner ship fleet plan which is less sensitive to the change in the noisy and uncertain data of container shipment demand within an acceptable level of under-fulfillment.The model may provide a credible and effective methodology for real world liner ship fleet planning problem in an uncertain environment.

CHAPTER 6 A MULTI-PERIOD STOCHASTIC PROGRAMMING MODEL FOR LONG-TERM LSFP

6.1 Introduction

This chapter studies a realistic long-term (multi-period) LSFP problem with container transshipment for a liner container shipping company. Traditional multi-period liner ship fleet planning begins with a forecasted container shipment demand of each single period based on forecasting techniques such as regression and time series models. However, the forecasted container shipment demand, which is a key input in multi-period LSFP problem, can never be forecasted with complete confidence. Moreover, the historical data fully show that the current container shipment demand has effect on the future demand, which indicates the container shipment demands of different periods are dependent. Therefore, it is realistic and necessary to take the uncertainty and dependency of container shipment demand into account in multi-period LSFP problem. Here, the container shipment demand between two successive single periods is assumed to be dependent. During a multi-period planning horizon, the container shipment demand in each single period is possibly different, which implies that the liner ship fleet plans vary with periods and depend on container shipment demand. Therefore, to cope with the period-dependent container shipment demand pattern, the liner container shipping company has to adjust its liner ship fleet plan of determining fleet size, mix and deployment period-by-period.

Under the consideration of container transshipment and the uncertainty and dependency of container shipment demand, the multi-period LSFP problem could be a new and interesting research topic, expanding the classical multi-period LSFP problem, which is studied under a deterministic environment and without container transshipment, into a fresh and worthwhile research area. This chapter thus focuses on model development and the design of solution method for the multi-period LSFP problem with container transshipment as well as uncertain and dependent container shipment demand.

Multi-period/long-term ship fleet planning problems have been studied for several decades. However, the research on the topic all makes the assumption of deterministic demand. Nicholson and Pullen (1971) were the pioneers in the field, developing a dynamic programming model for a ship fleet management problem that aimed to find the best sale and replacement policy, with the objective of maximizing the multi-period company assets. They proposed a two-stage decision strategy: the first stage determines a priority order for selling a ship, based on its assessment of the net contribution to the objective function if it is sold in each year, regardless of the rate at which charter ships are taken on; the second stage uses the dynamic programming approach to find the optimal level of chartering for a given priority replacement order. Cho and Perakis (1996) developed an integer linear programming model for a multi-period liner ship fleet planning problem looking to determine the optimal fleet size, mix and ship-to-route allocation. In their model, as long as those decisions are made at the beginning of the planning horizon, they remain static over the whole horizon. Such a period-independent model cannot characterize a realistic dynamic decision strategy: the fleet size, mix and ship-to-route allocation should be adjustable period-by-period, since the container shipment demand is period-dependent. In other words, it is more rational and practical to assume that the fleet size, mix and ship-to-route allocation are period-dependent (dynamic) decisions rather than static ones. Xie et al. (2000) thus reformulated the multi-period liner shipping problem proposed by Cho and Perakis (1996) by applying a dynamic programming approach. They first divided the multi-period planning horizon into a number of single periods (each single period being one year). For each period, they used integer linear programming to determine the fleet size, mix and ship-to-route assignment incurring minimal cost. However, they assumed that the annual operating cost and transportation capacity of each ship on each route were constant. This assumption is unrealistic because the costs are voyage-dependent. For example, a ship sailing 20 voyages on a given route over a given year would certainly incur greater annual operating costs and have a greater transportation capacity than a ship that sails ten voyages on the same route. Recently, Meng and Wang (2011b) proposed a realistic multi-period LSFP problem for a liner container shipping company and formulated this problem as a scenario-based dynamic programming model. However, as well as the deterministic container shipment demand assumption, these studies (i.e. Nicholson and Pullen, 1971; Cho and Perakis, 1996; Xie et al., 2000; Meng and Wang, 2011b) do not take container transshipment operations into account.

Compared with the few relevant papers on the MPLSFP problem with uncertain container shipment demand, much research has been devoted to other problems under the assumption of uncertain multi-period demand, such as capacity expansion problems (Berman et al., 1994; Wagner and Berman, 1995; Laguna, 1998; Ahmed et al., 2003; Ahmed and Sahinidis, 2003; Singh et al., 2009), airline fleet composition and allocation problem (Listes and Dekker, 2005), a multi-site production planning problem (Leung et al., 2007a and b), portfolio management problems (Celikyurt and Özekici, 2007; Gülpinar and Rustem, 2007; Osorio et al., 2008), and others. Their objectives are to minimize or maximize the expected value of a key variable, such as cost or profit, over a multi-period planning horizon, which is defined as the sum of the cost or profit in each single period. However, the methodologies applied or proposed in those studies did not involve the dependency of the uncertain multi-period demand. Shapiro and Philpott (2007) did in fact previously mention the dependency of uncertain demand in a multi-stage stochastic programming problem. Unfortunately, no application or study involving dependency has been reported so far.

Therefore, the model formulation for the multi-period LSFP problem integrating uncertainty and dependency is a challenge and a goal of this chapter. The objective is to seek an optimal multi-period liner ship fleet plan (i.e. a joint ship fleet development and deployment plan for the multi-period planning horizon) that will be implemented before the container shipment demands are known, such that the expected profit reaped across the whole multi-period planning horizon, by a liner container shipping company implementing this plan, is maximized.

To formulate uncertainty of container shipment demand during a particular period within the multi-period planning horizon, it is assumed to be a discrete random variable taking a limited number of possible values with a known occurrence probability. It has to be pointed out that the container shipment demand in a given period is dependent on demand in previous periods. Therefore, the probability is actually a conditional probability so as to reflect this dependency.

In order to capture a characteristic of the realistic dynamic planning strategy, the multi-period planning horizon is divided into a number of single periods and a stochastic programming model is developed for each one, with the aim of determining fleet deployment for that period. Using a scenario tree approach to model the evolution of dependent uncertain demand of two successive single periods, and using a decision tree to interpret the procedure of liner ship fleet planning, the proposed problem in this study is formulated as a multi-period stochastic programming model, comprising a sequence of interrelated stochastic programming models developed for each single period.We further show that the multi-period stochastic programming model can be equivalently transformed into a shortest path problem defined on an acyclic network. A path on the acyclic network corresponds to a multi-period liner ship fleet planning. Finally, a numerical example is carried out to assess applicability and performance of the proposed model and solution algorithm.

This remainder of this chapter is organized as follows: Section 6.2 gives assumptions and problem statement and discusses the dependency of container shipment demand in details. Section 6.3 elaborates the procedure for determining a multi-period liner ship fleet plan as a decision tree and develops a multi-period stochastic programming model for it. Section 6.4 designs the solution algorithm used to solve the multi-period stochastic programming model. Section 6.5 provides a numerical example, to illustrate the model and solution method. Summary is presented in Section 6.6.

6.2 Problem Statement

6.2.1 Uncertainty and Dependency of Container Shipment Demand

Assume that the multi-period planning horizon consists of T single periods, denoted by $\mathcal{T} = \{1,...,t,...,T\}$. The length of one single period can be determined according to the changes in container shipment demand forecasted within the multi-period planning horizon; for example, one period could be one year. Let ξ_t^{od} be the number of containers in terms of TEUs (acronyms of twenty-foot equivalent unit) to be transported between an O-D port pair $(o,d) \in \mathcal{W}$ in a particular single period $t \in \mathcal{T}$. The uncertainty of container shipment demand in the multi-period LSFP problem is included in the model by specifying a set of discrete demand scenarios. Let $\mathcal{S}_t = \{1,...,s...,S_t\}$ be the set of container shipment demand scenarios for period $t \in \mathcal{T}$. In each scenario $s \in \mathcal{S}_t$, values are specified for the container shipment demand between each port pair in period $t \in \mathcal{T}$ are specified. Associated with each scenario $s \in \mathcal{S}_t$ is a weight; these weights are often thought of as the probabilities that each scenario will occur, and are denoted by p_s^t and characterized by $\sum_{s=1}^{S_t} p_s^t = 1$. In other words, the container shipment demand between each port pair during a particular period, namely $\xi_t^{od}\left((o,d) \in \mathcal{W}, t \in \mathcal{T}\right)$, is assumed to be a discrete random variable, which takes a limited number of possible values with known occurrence probabilities.

Moreover, the historical data fully show that the current container shipment demand has an effect on the future demand, which indicates that the container shipment demand in one period is dependent on that in previous periods. Since the effect on demand in some faraway future is quite weak, we simply assume that the container shipment demand is only dependent on that of the previous period. Therefore, the scenario $s \in \mathcal{S}_t$ is dependent on the scenario $s' \in \mathcal{S}_{t-1}$. Let $p_{s|s'}^t$ be the conditional probability that scenario s occurs in period t given that scenario s' occurs in period t–1, then p_s^t is given by $\sum_{s'=1}^{S_{t-1}} p_{s'}^{t-1} p_{s|s'}^t$.

Since scenario $s \in \mathcal{S}_t$ occurs in period t with conditional probability $p_{s|s'}^t$, given that scenario $s' \in \mathcal{S}_{t-1}$ occurs in period t-1, all scenarios for the whole T-period planning horizon can be depicted as a scenario tree with T layers, where each layer corresponds to a single period. The following example is provided for clarity.

An example

Let us consider the liner shipping service route shown in Figure 6.1 in order to illustrate the scenarios of container shipment demand. For simplicity, consider two periods (say two years) and three O-D pairs: Pusan (PS) → Shanghai (SH), Shanghai (SH) → Yantian (YT), Yantian (YT) → Hong Kong (HK). Suppose that there are three discrete scenarios of demand in each year: L (low), M (medium) and H (high), as shown in Table 6.1, and these six scenarios for the two years are illustrated by a two-layer scenario tree, shown in Figure 6.2. The value on each branch in the two-layer scenario tree is the probability or conditional probability of each scenario's occurrence. Accordingly, the probabilities of each of the three scenarios in year 2 are computed as follows:

$$\begin{aligned}
p_H^2 &= p_H^1 \times p_{H|H}^2 + p_M^1 \times p_{H|M}^2 + p_L^1 \times p_{H|L}^2 = 0.7\times 0.6 + 0.2\times 0.5 + 0.1\times 0.1 = 0.53 \\
p_M^2 &= p_H^1 \times p_{M|H}^2 + p_M^1 \times p_{M|M}^2 + p_L^1 \times p_{M|L}^2 = 0.7\times 0.3 + 0.2\times 0.3 + 0.1\times 0.2 = 0.29 \quad (6.1) \\
p_L^2 &= p_H^1 \times p_{L|H}^2 + p_M^1 \times p_{L|M}^2 + p_L^1 \times p_{L|L}^2 = 0.7\times 0.1 + 0.2\times 0.2 + 0.1\times 0.7 = 0.18
\end{aligned}$$

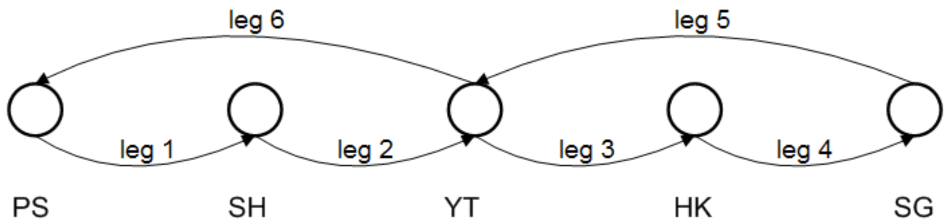

Figure 6. 1 A liner shipping service route

Table 6. 1 Container shipment demand scenarios for illustrative example

	Year 1			Year 2		
O-D pairs	Low	Medium	High	Low	Medium	High
PS $\rightarrow$ SH	1000	2000	3000	1500	2500	3500
SH $\rightarrow$ YT	800	1000	1500	1200	2000	2500
YT $\rightarrow$ HK	1000	1500	2000	1500	2000	2500

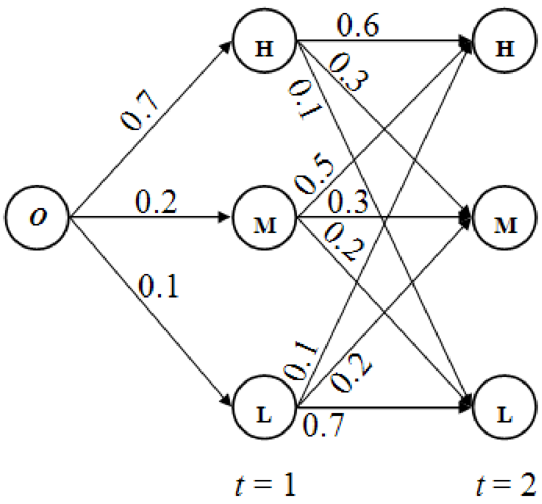

Figure 6. 2 Probability of scenarios for the illustrative example

6.2.2 Fleet Size and Mix Strategies

The liner container shipping company can use its own ships to pick up and deliver containers for shippers, and may also charter ships from other liner container shipping companies or purchase new ships to meet its container shipment demand. The company may also charter out some of its own ships, depending on their capacity in terms of TEUs. *A fleet size and mix strategy* associated with a particular period within the *T*-period planning horizon is defined as a plan comprising the number of ships to be chartered, the number of the company's own ships to be chartered out, the number of its own ships to be used during the period and the number of new ships to be purchased. The order time for new ships is ignored since this is generally known in practice.

At the beginning of the period $t \in \mathcal{T}$, experts from the strategic development department of the liner container shipping company would propose several possible fleet size and mix strategies for the period, based on their experiences, and/or the available budget of the company for the period. It is thus assumed that there are a number of suggested fleet size and mix scenarios at the beginning of each period $t \in \mathcal{T}$. There is an inherent and implicit relation between these strategies from one period to the next. For example, assuming that the liner container shipping company currently owns three ships named by A, B and C, the experts might propose two possible fleet size and mix strategies at the beginning of period t. Strategy 1 might be to use the existing three ships, while strategy 2 might be to purchase a new ship D to use as well. These two strategies would lead to two different states of the ship fleet at the beginning of the next period $t+1$: in the first state, there are three ships in the fleet, while in the second state there are four. Each of these two states becomes a possible initial state of the fleet at the beginning of period $t+1$. At the beginning of period $t+1$, the experts will propose a group of possible fleet size and mix strategies with respect to each of these two ship fleet states. This strategy decision process will be repeated until the end of the last period T, that is, the beginning of period $T+1$. The entire decision process of fleet size and mix strategies thus actually forms a decision tree containing T layers.

6.2.3 The Multi-Period Liner Ship Fleet Planning

The multi-period LSFP problem with container transshipment and uncertain container shipment demand aims to maximize the total expected profit reaped over the whole T-period planning horizon by making an optimal joint fleet development and deployment plan. A joint fleet development and deployment plan consists of (1) a fleet size and mix strategy proposed by the experts at the beginning of each period (i.e., a fleet development plan), and (2) a fleet deployment plan, including the allocation of the ships in the fleet to liner ship routes, the number of voyages by each ship on each liner shipping route $r \in \mathcal{R}$ required to maintain a given liner shipping service frequency on the route, and the number of lay-up days allocated to each ship for maintenance. The objective of the deployment plan is expected profit maximization under various scenarios of container shipment demand, for each of the given fleet size and mix strategies.

The rationale behind the adoption of this a period-by-period planning is that the liner container shipping company can flexibly adjust its ship fleet size and mix according to the varying container shipment demand in each period. Moreover, the ships are assets with finite lives, which implies that the ship fleet needs to be renewed as old ships are removed from the fleet when they reach a given age limit and new ships are added in their place. The adoption of period-by-period planning thus also satisfies the physical requirement of the renewal of the fleet over time. We assume the liner container shipping company makes its planning decisions at the beginning of each single period and this process is repeated until all periods in the multi-period planning horizon have been covered. Therefore, the multi-period fleet plan consists of a number of single-period fleet plans. At the end of the planning horizon, without loss of generality, we assume that all ships owned by the liner container shipping company are disposed of for their salvage values.

6.3 Model Development

6.3.1 Decision Tree of Fleet Development Plan

The procedure of determining a fleet development plan for a T-period planning horizon can be interpreted as a decision tree with T layers, where each layer represents a period and each node in layer t of the tree represents a fleet size and mix strategy proposed at the beginning of period t. Dummy node O is introduced as the root of the decision tree, to represent the current ship fleet state, that is, the decision tree grows from the root O. Each node in period t ($t = 1, 2, \cdots, T-1$) can be regarded as a parent and will generate some offspring in period t+1, that is, the fleet size and mix strategies for the next period. Each parent and its offspring are connected by an arc. It is noted that different parents may produce the same offspring. Each node of the decision tree, except the root, has a parent (which may not be unique). A parent n at period t and its offspring from period t = 1,…,T-1 to the end of the whole T-period planning horizon form a sub-tree, denoted by $\mathbb{T}^t(n)$. Each parent n, namely a non-terminal node in period t = 1,…,T-1, is the root of the sub-tree $\mathbb{T}^t(n)$. Thus $\mathbb{T}^0$ denotes the entire tree over the whole T-period planning horizon. The set of paths from root O to a node n in period t , is denoted by $\mathbb{P}^t(n)$, and each path $l \in \mathbb{P}^t(n)$, represents a development plan of fleet sizes and mixes for t periods. If n is a terminal node (i.e. a leaf), then path l corresponds to a development plan for all T periods.

Figure 6.3 schematically illustrates the decision tree. In Figure 6.3, let $\mathcal{N}_t = \{1,...,N_t\}$ be the set of nodes in period $t \in \mathcal{T}$, where N_t is the number of nodes in this set, and let $\mathcal{N}_t^m = \{1,...,N_t^m\}$ be the set of strategies proposed for period t+1 which are generated from a particular strategy m proposed for period t where N_t^m represents the number of strategies of the set $\mathcal{N}_t^m$. If each offspring node has a unique parent, we then have:

$$\mathcal{N}_{t+1} = \bigcup_{m \in \mathcal{N}_t} \mathcal{N}_t^m, t = 1,...,T-1 \tag{6.2}$$

and

$$N_{t+1} = \sum_{m=1}^{N_t} N_t^m, t = 1,...,T-1 \tag{6.3}$$

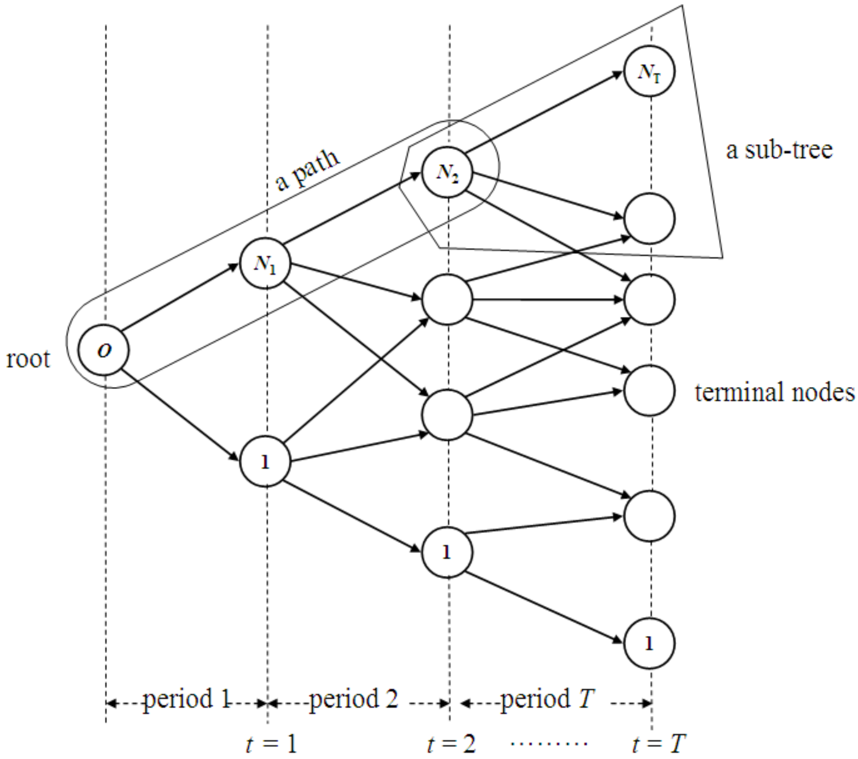

Figure 6. 3 Decision tree for fleet development plan

The following notation is introduced for the sake of presentation:

$\mathcal{G}_{t,n}^{\text{KEEP}}$: set of company's own ships to be used at the beginning of period t in strategy n

$\mathcal{G}_{t,n}^{\text{SOLD}}$: set of company's own ships to be sold at the beginning of period t in strategy n

$\mathcal{G}_{t,n}^{\text{OUT}}$: set of own ships to be chartered out at the beginning of period t in strategy n

$\mathcal{G}_{t,n}^{\text{IN}}$: set of ships to be chartered in at the beginning of period t in strategy n

$\mathcal{G}_{t,n}^{\text{NEW}}$: set of new ships bought at the beginning of period t in strategy n

$\mathcal{G}_{t,n}$: set of ships that are used to deliver containers as at the beginning of period t in strategy n

For a node (strategy) n in period t, the ships that can be used to deliver containers include the company's own ships, which are kept in service, new ships purchased at the beginning of period t ($\mathcal{G}_{t,n}^{\mathrm{NEW}} = \varnothing$ if no available new ships) and ships chartered in from other liner container shipping companies. The set of ships used in strategy n to deliver containers is given by:

$$\mathcal{G}_{t,n} = \mathcal{G}_{t,n}^{\mathrm{KEEP}} \cup \mathcal{G}_{t,n}^{\mathrm{NEW}} \cup \mathcal{G}_{t,n}^{\mathrm{IN}}, \forall t \in \mathcal{T} \tag{6.4}$$

The relationship between a parent m in period t and its offspring n in period t+1 (t = 1,…,T-1) is given by:

$$\mathcal{G}_{t,m}^{\mathrm{KEEP}} \cup \mathcal{G}_{t,m}^{\mathrm{OUT}} \cup \mathcal{G}_{t,m}^{\mathrm{NEW}} = \mathcal{G}_{t+1,n}^{\mathrm{KEEP}} \cup \mathcal{G}_{t+1,n}^{\mathrm{OUT}} \cup \mathcal{G}_{t+1,n}^{\mathrm{SOLD}}, m = 1,..., N_t, n = 1,..., N_t^m, t = 1,..., T-1 \tag{6.5}$$

6.3.2 2SSP Models for Fleet Deployment Plans

In Section 6.3.1, each node n in period $t \in \mathcal{T}$ represents a fleet size and mix strategy proposed by the liner container shipping company's experts, based on their experience and the available budget (the budget is used for investment in the chartering in or purchase of new ships). However, the decisions of how to properly deploy the ships in the fleet, as given by the fleet size and mix strategy n in period $t \in \mathcal{T}$, in order to maximize the profit gained from shipping containers over period t, have not yet been determined. Four types of decision variables are now defined as follows:

δ_{nt}^{kr}: binary variables equal to 1 if ship k is assigned to route r in strategy n of period t and 0 otherwise

x_{nt}^{kr}: number of voyages sailed by ship k on route r in strategy n of period t

y_{nt}^{k}: number of lay-up days of ship k in strategy n of period t

$z_{snt}^{h^{od}}$: number of containers carried by ships deployed on the container route $h^{od} \in \mathcal{H}^{od}$ between O-D port pair $(o,d) \in \mathcal{W}$ under container shipment demand scenario s in strategy n of period t

Given the set of ships under strategy n of period t, namely $\mathcal{G}_{t,n}$, the values of ξ_t^{od} for a port pair $(o,d) \in \mathcal{W}$ under scenario $s \in \mathcal{S}_t$ in period $t \in \mathcal{T}$, denoted by ω_{st}^{od}, and the freight rate of transporting a container from its origin port o to its destination port d in period t ($/TEU), denoted by f_t^{od}, the revenue gained from shipping containers along all possible routes in period t under container shipment demand scenario s is given by:

$$\sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} f_t^{od} z_{snt}^{h^{od}} \left(\omega_{st}^{od}\right) \tag{6.6}$$

Other revenue gained in strategy n over period t includes earnings from chartering out the company's ships and the salvage value gained from selling its ships. This is given by the following:

$$\sum_{k\in\mathcal{G}_{t,n}^{\mathrm{OUT}}} c_{kt}^{\mathrm{OUT}} + \sum_{k\in\mathcal{G}_{t,n}^{\mathrm{SOLD}}} c_{kt}^{\mathrm{SOLD}} \tag{6.7}$$

where c_{kt}^{OUT} is the amount received for chartering out a particular ship k at the beginning of period t ($) and c_{kt}^{SOLD} is the amount received for selling out a ship k at the beginning of period t ($).

The total costs incurred in strategy n of period t usually consist of the following components: container handling costs, the voyage costs of ships in the fleet to transport containers, the lay-up costs of ships for maintenance, the costs of chartering in ships from other liner container shipping companies and capital investment of purchasing new ships. The container handling cost incurred in a container route, includes container loading cost at origin port, container discharging cost at destination port and container transshipment cost at transshipment ports (if any). Let $c_t^{h^{od}}$ ($/TEU) denote the container handling cost per TEU incurred in the container route $h^{od} \in \mathcal{H}^{od}$ over period t and then the total container handling cost can be calculate by

$$\sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} c_t^{h^{od}} z_{snt}^{h^{od}} \left(\omega_{st}^{od}\right) \tag{6.8}$$

The voyage costs of the ships in the fleet that are used to transport containers, plus lay-up costs of those ships undergoing maintenance, plus the costs of chartering in ships from other liner container shipping companies and the capital investment of purchasing new ships is given by:

$$\sum_{r\in\mathcal{R}} \sum_{k\in\mathcal{G}_{t,n}} c_{krt} x_{nt}^{kr} + \sum_{k\in\mathcal{G}_{t,n}} e_{kt} y_{nt}^{k} + \sum_{k\in\mathcal{G}_{t,n}^{\mathrm{IN}}} c_{kt}^{\mathrm{IN}} + \sum_{k\in\mathcal{G}_{t,n}^{\mathrm{NEW}}} c_{kt}^{\mathrm{NEW}} \tag{6.9}$$

where c_{krt} is the voyage cost of operating a specific ship k on route r in period t ($/voyage), e_{kt} is the daily lay-up cost for a specific ship k in period t ($/day), c_{kt}^{IN} is the cost of chartering in a specific ship k at the beginning of period t ($), c_{kt}^{NEW} is the price of the new ship k at the beginning of period t ($).

As mentioned earlier, the fleet deployment plan of a specific fleet size and mix strategy n in period t is dependent on the container shipment demand of the previous period $t-1$. Therefore, given a fleet size and mix strategy n in period t which is produced by a parent m of in period t-1, the optimal fleet deployment plan under this given strategy n is dependent on the container shipment demand scenario s' over the previous period $t-1$, which can be formulated as a 2SSP model with the objective of maximizing the expected profit accross all container shipment demand scenarios s in period t, denoted by $EP_{t,n}^{m,s'}$.

It is noted that the decision about δ_{nt}^{kr}, x_{nt}^{kr} and y_{nt}^{k} are made prior to a realization of the random container shipment demand. In reality, the number of containers transported between an O-D port pair $(o,d)\in\mathcal{W}$ assigned to a particular container route can be determined only after the realization of the random container shipment demand. We can thus break down the set of all decision variables into two stages. The first-stage decision variables are δ_{nt}^{kr}, x_{nt}^{kr} and y_{nt}^{k}, and the second-stage variables are $z_{snt}^{h^{od}}$. Therefore, the 2SSP model is as follows:

[2SSP]

$$\begin{aligned} EP_{t,n}^{m,s'} = \max \sum_{k\in\mathcal{G}_{t,n}^{\text{OUT}}} c_{kt}^{\text{OUT}} + \sum_{k\in\mathcal{G}_{t,n}^{\text{SOLD}}} c_{kt}^{\text{SOLD}} - \sum_{r\in\mathcal{R}} \sum_{k\in\mathcal{G}_{t,n}} c_{krt} x_{nt}^{kr} - \sum_{k\in\mathcal{G}_{t,n}} e_{kt} y_{nt}^{k} - \sum_{k\in\mathcal{G}_{t,n}^{\text{IN}}} c_{kt}^{\text{IN}} \\ - \sum_{k\in\mathcal{G}_{t,n}^{\text{NEW}}} c_{kt}^{\text{NEW}} + \sum_{s\in\mathcal{S}_t} p_{s|s'}^{t} Q_{\xi}^{ts}\left(\mathbf{v},\xi(\boldsymbol{\omega})\right) \end{aligned} \tag{6.10}$$

subject to

$$\delta_{nt}^{kr} \le x_{nt}^{kr} \le M^{kr}\delta_{nt}^{kr}, \forall r\in\mathcal{R}, \forall k\in\mathcal{G}_{t,n} \tag{6.11}$$

$$\sum_{k\in\mathcal{K}} x_{kr} \ge N_r, \forall r\in\mathcal{R} \tag{6.12}$$

$$\Delta t - T_k^t \le y_{nt}^{k}, \forall k\in\mathcal{G}_{t,n} \tag{6.13}$$

$$x_{nt}^{kr} t_t^{kr} + y_{nt}^{k} = \Delta t, \forall r\in\mathcal{R}, \forall k\in\mathcal{G}_{t,n} \tag{6.14}$$

$$\sum_{r\in\mathcal{R}} \delta_{nt}^{kr} = 1, \forall k\in\mathcal{G}_{t,n} \tag{6.15}$$

$$x_{nt}^{kr} \in \mathbb{Z}^{+} \bigcup \{0\}, \forall k\in\mathcal{K}, \forall r\in\mathcal{R} \tag{6.16}$$

$$y_{nt}^{k} \ge 0, \forall k\in\mathcal{K} \tag{6.17}$$

$$\delta_{nt}^{kr} = \{0,1\}, \forall k \in \mathcal{G}_{t,n}, \forall r \in \mathcal{R} \tag{6.18}$$

where, for succinctness, $\mathbf{v} = \left(\cdots \delta_{nt}^{kr} \cdots x_{nt}^{kr} \cdots y_{nt}^{k} \cdots\right)$ contains all first-stage decision variables, M^{kr} represents the maximum number of voyages ship k can complete on route r during period t, N_r is the number of voyages required on route r during period t in order to maintain a given level of service frequency, Δt is the duration of period t (days), T_k^t represents the shipping season for ship k in period t (days), referring to the number of days on which it is safe and appropriate for the ship to sail on the sea, t^{kr} is the voyage time of ship k on route r (days/voyage), and $\mathbb{Z}^+$ is the set of positive integers. $Q_\xi^{ts}\left(\mathbf{v}, \xi(\boldsymbol{\omega})\right)$ is a function used for the following second-stage optimization problem, which depends on the first-stage decision variables and the realization of container shipment demand, $\boldsymbol{\omega}$, under scenario s, its value is obtained by solving the following optimization problem:

$$Q_\xi^{ts}\left(\mathbf{v}, \xi(\boldsymbol{\omega})\right) = \max \sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} \left(f_t^{od} - c_t^{h^{od}}\right) z_{snt}^{h^{od}}\left(\omega_{st}^{od}\right) \tag{6.19}$$

subject to

$$\sum_{k\in\mathcal{G}_{t,n}} x_{nt}^{kr} V_k \geq \sum_{(o,d)\in\mathcal{W}} \sum_{h^{od}\in\mathcal{H}^{od}} \rho_{ir}^{h^{od}} z_{snt}^{h^{od}}\left(\omega_{st}^{od}\right), \forall i = 1, \cdots, m_r, \forall r \in \mathcal{R}, \forall s \in \mathcal{S}_t \tag{6.20}$$

$$\sum_{h^{(o,d)}\in\mathcal{H}^{(o,d)}} z_{snt}^{h^{od}}\left(\omega_{st}^{od}\right) \leq \xi^{od}\left(\omega_{st}^{od}\right), \forall (o,d) \in \mathcal{W}, \forall s \in \mathcal{S}_t \tag{6.21}$$

$$z_{snt}^{h^{od}} \geq 0, \forall (o,d) \in \mathcal{W}, \forall h^{od} \in \mathcal{H}^{od}, \forall s \in \mathcal{S}_t \tag{6.22}$$

where V_k is the capacity of a particular ship k (TEUs), $\rho_{ir}^{h^{od}}$ is a binary coefficient which equals 1 if a container route $h^{od} \in \mathcal{H}^{od}$ contains leg i of route r and otherwise equals 0.

Eq. (6.10) is the objective function of the 2SSP model. The set of constraints (6.11) applies the big-M method to ensure that if δ_{tn}^{kr} equals to 0 then x_{tn}^{kr} equals to 0; else if δ_{tn}^{kr} equals to 1 then x_{tn}^{kr} would be a positive integer. The value of M^{kr} can be given by $M^{kr} = \lfloor \Delta t / t^{kr} \rfloor$, where $\lfloor a \rfloor$ denotes the maximum integer not greater than a. The set of constraints (6.12) gives the number of voyages required on route r in order to maintain a given level of liner shipping frequency. For example, if a weekly shipping service is required on each liner ship route during a planning horizon of six month, then $N_r = 26$. The set of constraints (6.13) provide the minimum lay-up days of ship k on route r. Eq. (6.14) indicates that the total voyage time of ship k on route r sailing on sea and the lay-up time should not exceed one single period. Eq. (6.15) ensures that each ship only serves on one route. Constraints (6.16)-(6.18) defines the range of decision variables, x_{nt}^{kr}, y_{nt}^{k}, and δ_{nt}^{kr}, respectively.

Eq. (6.19) is the objective function of the second-stage optimization problem. The left-hand side of the constraints (6.20) is the total transportation capacity of ships deployed deployed on the liner ship route $r \in \mathcal{R}$. The right-hand side of the constraints (6.20) is the total number of containers carried by ships sailing on leg l of the liner ship route $r \in \mathcal{R}$, including the containers loaded at previously calling ports which have remained on the ships plus any containers loaded or transshipped at port p_r^i. Therefore, the constraints (6.20) guarantee the total number of containers transported on each leg of a liner shipping service route does not exceed the ship capacity deployed on the route. The constraints (6.21) imply that the total number of containers assigned to all the ship routes between an O-D port pair does not exceed the corresponding container shipment demand. Eq. (6.22) requires the decision variables z_{snt}^{hod} should be non-negative.

After $EP_{t,n}^{m,s'}$ is obtained by solving the 2SSP model above, we can then calculate the expected profit under strategy *n* in period *t* given strategy *m* was applied in period *t*-1, which is denoted by $EP_{t,n}^{m}$ $\left(t=1,...,T; n=1,...,N_t\right)$, and given by:

$$EP_{t,n}^{m} = \sum_{s' \in \mathcal{S}_{t-1}} p_{s'}^{t-1} \times EP_{t,n}^{m,s'} \tag{6.23}$$

6.3.3 Multi-Period Stochastic Programming Model

At the end of period *T*, the set of ships owned by the liner container shipping company under strategy n°, denoted by $\bar{\mathcal{G}}_{T,n^{\circ}}$, includes ships that were kept, ships that were chartered out and ships that were bought at the beginning of period T:

$$\bar{\mathcal{G}}_{T,n^{\circ}} = \mathcal{G}_{T,n^{\circ}}^{\text{KEEP}} \cup \mathcal{G}_{T,n^{\circ}}^{\text{OUT}} \cup \mathcal{G}_{T,n^{\circ}}^{\text{NEW}}, n^{\circ} = 1,...,N_T \tag{6.24}$$

Without loss of generality, we assume that all ships owned by the liner container shipping company are disposed of at the end of period *T* for their salvage values, which is denoted by $SV_{T,n^{\circ}}$. The objective of the MPLSFP problem is to find the best policy that maximizes the sum of the expected profits across the whole *T*-period planning horizon plus the salvage value. Here a policy refers to a path from the dummy root *O* to the leaf node $n^{\circ} \in \mathcal{N}_T = \left\{1,...,N_T\right\}$ in the decision tree. Therefore, the best policy refers to the path from the dummy root *O* to a leaf node n° in the decision tree, with the maximal sum of expected profits plus salvage values. The length of a path is, as usual, the sum of the length of the arcs that it contains.

Let $\mathcal{L}_{t,n}^{n^\circ,l}$ be 1 if a path $l \in \mathbb{P}^T\left(n^\circ\right)$ from the dummy root O to the leaf node n° passes node n of period t, and 0 otherwise (n° = 1,…,N_T). The best path, with the maximal sum of expected profits across all period plus salvage value, that is, the optimal plan for the MPLSFP problem, is given by:

$$Z = \max_{\substack{l\in\mathbb{P}^T\left(n^\circ\right) \\ n^\circ\in\mathcal{N}_T}} \sum_{n,m\in\mathbb{T}^0} \sum_{t=1}^{T} \frac{EP_{t,n}^m \mathcal{L}_{t,n}^{n^\circ,l}}{\left(1+r\right)^t} + \frac{SV_{T,n^\circ}}{\left(1+r\right)^T} \tag{6.25}$$

where r is the discount rate of each period during the multi-period planning horizon.

Remark

The Eq. (6.25) can be rewritten as a dynamic recurse function. Let TEP_t^m denote the maximal present value of the profit from strategy m over the periods from period t ($t \geqslant 1$) to period T, the Bellman equations can be established as follows:

$$TEP_t^m = \max_{n\in\mathcal{N}_t^m} \left\{ \frac{EP_{t+1,n}^m}{\left(1+r\right)^t} + TEP_{t+1}^n \right\} \tag{6.26}$$

At the end of period T, all the ships owned by the liner container shipping company are assumed to be disposed of in order to get the salvage value. Therefore, the boundary condition is given by:

$$TEP_T^{n^\circ} = \frac{SV_{T,n^\circ}}{\left(1+r\right)^T}, n^\circ \in \mathcal{N}_T \tag{6.27}$$

Therefore, the maximal present value of the profit from the beginning of period 1 to the end of period T, namely from the root O to a leaf node in the decision tree, denoted by TEP_T^O, is given by:

$$TEP_T^O = \max_{n\in\mathcal{N}_1} EP_{1,n}^O + TEP_1^n \tag{6.28}$$

6.4 Solution Algorithm

As shown in Figure 6.3, the expected profit on each arc contributes to the total profits along a given path from the dummy root *O* to a leaf node n°. In order to find the path with the greatest total profits across all periods, the attribute of each arc, $EP_{t,n}^{m}$, and the salvage value $SV_{T,n^{\circ}}$ have to be obtained. Once each $EP_{t,n}^{m}$ is obtained, the path from the dummy root *O* to a leaf node n° with the maximal total profit can be found. Therefore, the key aspect of the solution method is to obtain $EP_{t,n}^{m}$, that is to solve the 2SSP model. The following firstly proposes a solution method to deal with the 2SSP model in order to get $EP_{t,n}^{m}$, and then describes an algorithm for finding the best path for the proposed MPLSFP problem in this chapter.

6.4.1 Dual Decomposition and Lagrangian Relaxation Method for Solving 2SSP Models

It is noted that each 2SSP model under strategy *n* for period *t* involves a number of scenarios of the uncertain container shipment demand. Even when the first-stage decisions are given and fixed, S_t (t = 1,…, T) optimization models (6.19) have to be solved in order to obtain the expected value associated with this given set of fixed first-stage decisions.

In order to effectively solve a 2SSP model under strategy *n* for period *t* (n = 1,…,N_t; t = 1,…,T), the dual decomposition and Lagrangian relaxation method proposed by Carøe and Schultz (1999) is used because it can decompose the 2SSP model into S_t

sub-problems based on the scenarios of container shipment demand. In order to do that, the first-stage variables are copied for each scenario. Such duplication might result in a new problem: the first-stage decision variables $\mathbf{v}_s$ for each scenario s ($s = 1,\ldots,S_t$) could be different. However, the first-stage decision variable vector $\mathbf{v}_s$ ($s = 1,\ldots,S_t$) in the 2SSP model should be independent of uncertain container shipment demand because they are made prior to knowing the exact market demand. Therefore, the non-anticipativity constraints $\mathbf{v}_1 = \mathbf{v}_2 = \cdots = \mathbf{v}_{S_t}\ (t = 1,\cdots,T)$ are added, to guarantee that the first-stage decisions in period t do not depend on the scenarios. The non-anticipativity constraints are implemented through the equation $\sum_{s\in\mathcal{S}_t}\mathbf{H}^s\mathbf{v}^s = \mathbf{0}$ ($t = 1,\ldots, T$) where $\mathbf{H}^s$ is a suitable matrix with $(S_t-1)\times(2K_{tn}R{+}K_{tn})$ rows and $2K_{tn}R{+}K_{tn}$ columns (K_{tn} is the cardinality of set $\mathcal{G}_{t,n}$, namely the number of ships; $2K_{tn}R{+}K_{tn}$ is the number of first-stage decision variables x_{nt}^{kr}, y_{nt}^{k} and δ_{nt}^{kr}) for $s = 1,\ldots,S_t$ defined as follows:

$$\begin{aligned}&\mathbf{H}^1 = (\mathbf{I},\mathbf{0},\cdots,\mathbf{0})',\mathbf{H}^2 = (-\mathbf{I},\mathbf{I},\mathbf{0},\cdots\mathbf{0})',\mathbf{H}^3 = (\mathbf{0},-\mathbf{I},\mathbf{I},\cdots\mathbf{0})',\cdots,\\&\mathbf{H}^{S_t-1} = (\mathbf{0},\cdots,-\mathbf{I},\mathbf{I})',\mathbf{H}^{S_t} = (\mathbf{0},\cdots\mathbf{0},-\mathbf{I})'\end{aligned} \tag{6.29}$$

where $\mathbf{I}$ and $\mathbf{0}$ are the square unity matrix and zero matrix of size $2K_{tn}R{+}K_{tn}$, respectively. Let $\boldsymbol{\lambda}$ be a $(S_t-1)\times(2K_{tn}R{+}K_{tn})$-dimensional vector of Lagrangian multiplier associated with the non-anticipativity constraints. The resulting Lagrangian relaxation is as follows:

[LR$_{\mathbf{t,n}}$]

$$LR_{t,n}(\boldsymbol{\lambda}) = \max\sum_{s\in\mathcal{S}_t} p_{s|s'}^{t}\left(\begin{aligned}&\sum_{k\in\mathcal{G}_{t,n}^{\mathrm{OUT}}} c_{kt}^{\mathrm{OUT}} + \sum_{k\in\mathcal{G}_{t,n}^{\mathrm{SOLD}}} c_{kt}^{\mathrm{SOLD}} - \sum_{k\in\mathcal{G}_{t,n}^{\mathrm{IN}}} c_{kt}^{\mathrm{IN}} - \sum_{k\in\mathcal{G}_{t,n}^{\mathrm{NEW}}} c_{kt}^{\mathrm{NEW}}\\&-\sum_{r\in\mathcal{R}}\sum_{k\in\mathcal{G}_{t,n}} c_{krt}x_{nt}^{krs} - \sum_{k\in\mathcal{G}_{t,n}} e_{kt}y_{nt}^{ks} + Q_{\xi}^{ts}\left(\mathbf{v}^s,\xi(\boldsymbol{\omega})\right)\end{aligned}\right) + \boldsymbol{\lambda}'\mathbf{H}^s\mathbf{v}^s \tag{6.30}$$

subject to constraints (6.11)-(6.18), (6.20)-(6.22) for each scenario of container shipment demand. This Lagrangian relaxation model **LR_{t,n}** can be further decomposed into S_t separate mixed-integer linear programming problems according to the S_t container shipment demand scenarios, namely:

$$LR_{t,n}\left(\boldsymbol{\lambda}\right)=\sum_{s\in\mathcal{S}_t} LR_{t,n}^{s}\left(\boldsymbol{\lambda}\right) \tag{6.31}$$

where

$$LR_{t,n}^{s}\left(\boldsymbol{\lambda}\right)=\max\, p_{s|s'}^{t}\left(\begin{array}{l}\sum\limits_{k\in\mathcal{G}_{t,n}^{\mathrm{OUT}}} c_{kt}^{\mathrm{OUT}}+\sum\limits_{k\in\mathcal{G}_{t,n}^{\mathrm{SOLD}}} c_{kt}^{\mathrm{SOLD}}-\sum\limits_{k\in\mathcal{G}_{t,n}^{\mathrm{IN}}} c_{kt}^{\mathrm{IN}}-\sum\limits_{k\in\mathcal{G}_{t,n}^{\mathrm{NEW}}} c_{kt}^{\mathrm{NEW}}\\ -\sum\limits_{r\in\mathcal{R}}\sum\limits_{k\in\mathcal{G}_{t,n}} c_{krt}x_{nt}^{krs}-\sum\limits_{k\in\mathcal{G}_{t,n}} e_{kt}y_{nt}^{ks}+Q_{\xi}^{ts}\left(\mathbf{v}^{s},\xi\left(\boldsymbol{\omega}\right)\right)\end{array}\right)+\boldsymbol{\lambda}'\mathbf{H}^{s}\mathbf{v}^{s} \tag{6.32}$$

subject to constraints (6.11)-(6.18), (6.20)-(6.22) associated with the s^{th} scenario of container shipment demand.

Each subproblem shown in Eq. (6.32) can be solved efficiently using an optimization solver of linear integer programs such as CPLEX. It is straightforward to demonstrate that $LR_{t,n}\left(\boldsymbol{\lambda}\right)$, the objective function value of the **LR_{t,n}** model with respect to a given Lagrangian multiplier $\boldsymbol{\lambda}$, is an upper bound on the optimal value of Eq.(6.10). The best or tightest upper bound is found by solving the Lagrangian dual:

[LD_{t,n}]

$$LD_{t,n}=\min_{\boldsymbol{\lambda}} LR_{t,n}\left(\boldsymbol{\lambda}\right) \tag{6.33}$$

which is solved by the subgradient method, a brazen adaptation of the gradient method in which gradients are replaced by subgradients. Carøe and Schultz (1999) shown that $\sum_{s\in\mathcal{S}_t}\mathbf{H}^{s}\mathbf{v}^{s^*}$ is the subgradient of (6.30) where $\mathbf{v}^{s^*}$ is the optimal solution of the s^{th}

subproblem (6.32). With this subgradient, the **$LR_{t,n}$** model can be solved using the following subgradient method:

Step 0: Give an initial Lagrangian multiplier vector $\boldsymbol{\lambda}^{(1)}$. Let the number of iterations $h = 1$.

Step 1: Calculate the subgradient $\sum_{s \in \mathcal{S}_t} \mathbf{H}^s \mathbf{v}^{s^*(h)}$ by solving the subproblem shown in Eq. (6.32) with respect to the Lagrangian multiplier vector $\boldsymbol{\lambda}^{(h)}$.

Step 2: Update the Lagrangian multiplier vector according to the formula:

$$\boldsymbol{\lambda}^{h+1} = \boldsymbol{\lambda}^h + \kappa^h \sum_{s \in \mathcal{S}_t} \mathbf{H}^s \mathbf{v}^{s^*(h)} \tag{6.34}$$

where κ^h is a positive scalar step size and is given by

$$\kappa^h = 1/h \tag{6.35}$$

Step 3: If the following criterion is fulfilled, the algorithm is terminated. Otherwise, let $h = h + 1$ and go to Step 1.

$$\left| \left(LR_{t,n}\left(\boldsymbol{\lambda}^{h+1}\right) - LR_{t,n}\left(\boldsymbol{\lambda}^{h}\right) \right) \big/ LR_{t,n}\left(\boldsymbol{\lambda}^{h}\right) \right| \le \tau \tag{6.36}$$

6.4.2 Shortest Path Algorithm for the Multi-period LSFP Problem

Once the attribute of each arc has been obtained using the solution method described in Section 6.4.1, the next step is to find the longest path from the dummy root *O* to a leaf node, with the maximal profit (summed across all arcs contained in this path) plus salvage value. Each leaf node, n^{o}, is connected to a dummy destination node, *D* (shown in Figure 6.4), by a dummy arc, and the value on each dummy arc is set equal to the salvage value of this leaf node, SV_{T,n°.

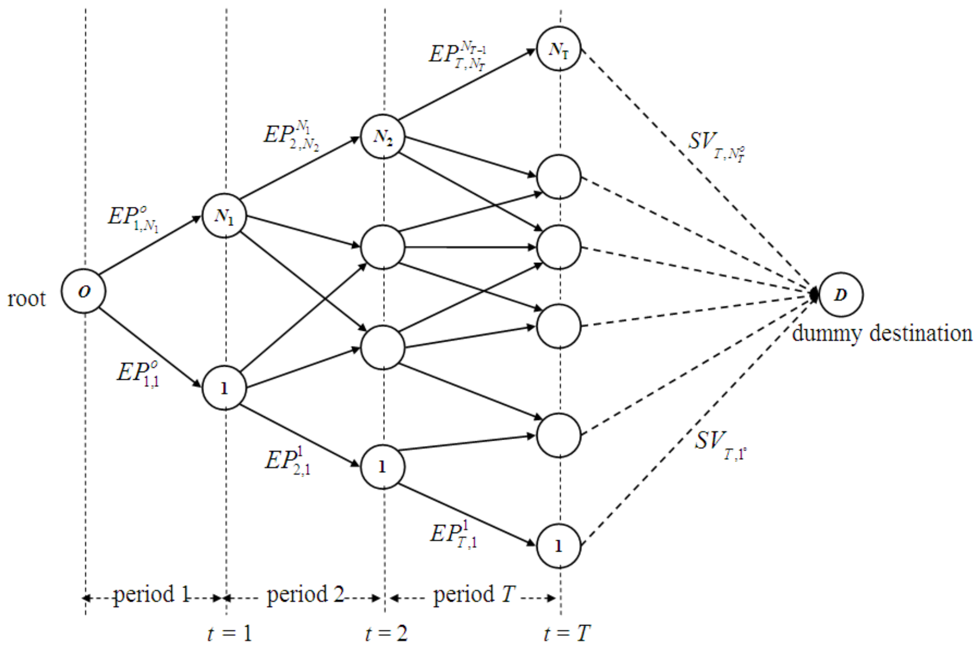

Figure 6.4 An acyclic network representation

Then, finding the longest path from the dummy root O to a leaf node is equivalent to finding the longest path from O to D in the acyclic network shown in Figure 6.4. Any shortest path algorithms applicable to an acyclic network can be applied to find the longest path, such as Dijkstra's algorithm (Denardo and Fox, 1978; Ahuja et al., 1996). It is noted that, in order to use shortest path algorithms, a negative sign is added to the attribute of each branch, that is we consider $-EP_{t,n}^{m}$. Then, the shortest path, found using shortest path algorithms, is actually the longest path that we are seeking.

6.5 Computational Results

6.5.1 A Numerical Example Design

In order to illustrate the applicability of the proposed approach to the MPLSFP problem with container transshipment and demand uncertainty, we provide a numerical example. The liner shipping topology and 36 calling ports depicted in Figure 3.2 are taken as a numerical example here. In the example, we assume that a liner container shipping company intends to make a 10-year liner ship fleet plan for providing a weekly liner shipping service. The relevant ship data are presented in Table 6.2.

Table 6. 2 Ship fleet at the beginning of research horizon

Item	Ship types				
	1	2	3	4	5
Ship size (TEUs)	2808	3218	4500	5714	8063
Design speed (knots)	21.0	22.0	24.2	24.6	25.2
Daily operating cost (10^3 $)	19.8	22.5	30.9	38.8	54.2
Daily lay-up cost (10^3 $)	2.8	3.2	4.5	6	8
Annual chartering out rate (million $)	3.64	4.68	6.42	8.64	10.24
Annual chartering in rate (million $)	4	5.2	7.0	9.4	12.0
Selling price (million $)	85	105	175	225	345
Purchasing price (million $)	135	155	215	275	385

6.5.2 Generation of Demand Scenarios and Fleet Size and Mix Strategies

We assume there are three scenarios of container shipment demand: high, medium and low for each single period (i.e. one year) shown in Figure 6.5. Additionally, we assume three feasible strategies are proposed by the liner container shipping company's experts at the beginning of each year (see Table 6.3). A strategy involves five options: keep, charter out, sell, charter in and buy ships. We use five capital letters: *K*, *O*, *S*, *I* and *B* to represent those five options, respectively. Additionally, the superscript and the subscript of the capital letters in a strategy represent the ship type and the number of ships of this type, respectively. For example, the strategy $K_2^1K_2^2K_9^3I_1^3K_2^4K_{12}^5$ in year 1 indicates that a total of 28 ship are contained in the ship fleet, of which two ships of type 1, two ships of type 2, nine ships of type 3, two ships of type 4 and twelve ships of type 5 are kept in the ship fleet and one ship of type 3 is chartered in.

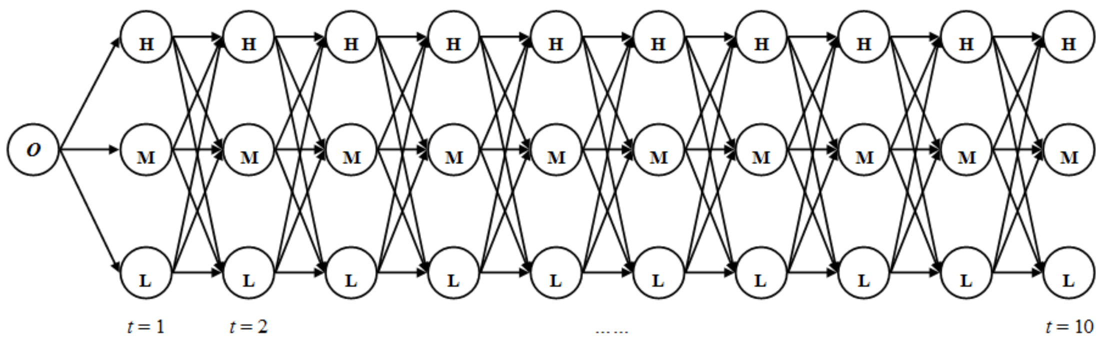

Figure 6.5 Scenario tree for the numerical example

6.5.3 Profit Comparison

The results of the numerical example are illustrated as an acyclic network representation. It is found that the longest path from O to D is $O \to 1 \to 3 \to 3 \to 1 \to 1 \to 2 \to 2 \to 2 \to 2 \to 3 \to D$ with total profits of 95.2586 billion dollars.

As mentioned, the most significant contribution of this study is to take the dependency of uncertain container shipment demand between periods into account in the multi-period LSFP problem. In order to evaluate whether it is worthwhile considering container shipment demand dependency and to investigate the effect of the dependency on profit, we then compute the total profit over the whole multi-period planning horizon for the same numerical example, with the assumption that the container shipment demand in each period is independent of that in other periods, and compare the results with those produced above. For the sake of presentation, in the remainder of this paper, the case with dependency of container shipment demand is called case Ⅰ (i.e. the problem studied in this paper) while the case with independent container shipment demand is called case Ⅱ.

In case Ⅱ, $EP_{t,n}^{m}$ $\left(t=1,\ldots,T;n=1,\ldots,N_t\right)$ is given by:

$$EP_{t,n}^{m} = \max \sum_{k\in\mathcal{G}_{t,n}^{\mathrm{OUT}}} c_{kt}^{\mathrm{OUT}} + \sum_{k\in\mathcal{G}_{t,n}^{\mathrm{SOLD}}} c_{kt}^{\mathrm{SOLD}} - \sum_{r\in\mathcal{R}} \sum_{k\in\mathcal{G}_{t,n}} c_{krt} x_{nt}^{kr} - \sum_{k\in\mathcal{G}_{t,n}} e_{kt} y_{nt}^{k} - \sum_{k\in\mathcal{G}_{t,n}^{\mathrm{IN}}} c_{kt}^{\mathrm{IN}} - \sum_{k\in\mathcal{G}_{t,n}^{\mathrm{NEW}}} c_{kt}^{\mathrm{NEW}} + \sum_{s\in\mathcal{S}_t} p_s^t Q_{\xi}^{ts}\left(\mathbf{v},\boldsymbol{\xi}\left(\boldsymbol{\omega}\right)\right) \tag{6.37}$$

subject to constraints (6.11) to (6.22).

Table 6. 3 Strategies proposed for each year

Year	Scenario 1	Scenario 2	Scenario 3
1	$K_2^1K_2^2K_9^3K_2^4K_{12}^5$	$K_2^1K_2^2K_9^3I_1^3K_2^4K_{12}^5$	$K_1^1O_1^1K_2^2K_9^3I_4^3K_2^4K_{12}^5$
2	$K_2^1K_2^2K_9^3I_5^3K_2^4K_{12}^5$	$K_2^1K_2^2K_9^3I_2^3K_2^4I_2^4K_{12}^5$	$K_2^1K_2^2K_9^3I_3^3K_2^4I_2^4K_{12}^5$
3	$S_1^1K_1^1K_2^2K_9^3I_4^3K_2^4I_2^4K_{12}^5$	$S_2^1K_2^2K_9^3B_5^3K_2^4K_{12}^5$	$S_1^1K_1^1K_2^2K_9^3K_2^4I_6^4K_{12}^5$
4	$K_1^1K_2^2K_9^3I_4^3K_2^4I_5^4K_{12}^5$	$K_2^2K_{14}^3K_2^4K_{12}^5$	$K_1^1K_2^2K_9^3B_5^3K_2^4I_3^4K_{12}^5$
5	$S_1^1K_2^2K_9^3B_5^3K_2^4I_3^4K_{12}^5$	$K_2^2K_{14}^3K_2^4I_5^4K_{12}^5$	$K_1^1K_2^2K_{14}^3K_2^4I_5^4K_{12}^5$
6	$K_2^2K_{14}^3K_2^4I_8^4K_{12}^5$	$S_1^2K_1^2K_{14}^3K_2^4B_4^4K_{12}^5I_2^5$	$S_1^1K_2^2K_{14}^3K_2^4B_4^4K_{12}^5$
7	$S_2^2K_{14}^3K_2^4B_4^4K_{12}^5I_3^5$	$K_1^2K_{14}^3K_6^4K_{12}^5I_5^5$	$K_2^2K_{14}^3K_6^4K_{12}^5I_5^5$
8	$K_{14}^3K_6^4K_{12}^5I_6^5$	$S_1^2K_{14}^3K_6^4K_{12}^5B_6^5$	$S_1^2K_1^2K_{14}^3K_6^4K_{12}^5B_6^5$
9	$K_{14}^3K_6^4I_4^4K_{12}^5B_3^5$	$K_{14}^3K_6^4K_{18}^5$	$S_1^2K_{14}^3K_6^4K_{18}^5$
10	$K_{14}^3K_6^4K_{15}^5I_5^5$	$K_{14}^3K_6^4I_3^4K_{18}^5$	$K_{14}^3K_6^4I_4^4K_{18}^5$

We found that the longest path from *O* to *D* for the numerical example was the same as the path in case Ⅰ, but the total profit was 95.0217 billion dollars. The results show that the total profits in case Ⅱ are lower than those in case Ⅰ, which indicates that the dependency of container shipment demand has a significant effect on profits and verifies that the importance of considering dependency between the container shipment demand in different periods. Actually, we have also theoretically proven that the profit in case Ⅱ will be less than or equal to that in case Ⅰ (see Appendix B).

6.5.4 Comparison of Fleet Deployment Plans

Section 6.5.3 evaluates whether it is worthwhile considering of container shipment demand dependency, by investigating the effect of this dependency on profit.

Similarly, this section investigates the effect of the dependency on the resulting fleet deployment plans. The 2SSP model (6.10) indicates that the fleet deployment plan under a given fleet strategy n in period t is dependent on the container shipment demand scenario s' of the previous period t-1. Since there are S_{t-1} container shipment demand scenarios in period t-1, it is possible that there are S_{t-1} different fleet deployment plans for a strategy n in year t (t = 2,…,T), where each fleet deployment plan corresponds to a container shipment demand scenario s' from the previous period t-1 and is obtained by solving the 2SSP model (6.10). This shows that, in case Ⅰ, the fleet deployment decisions for period t take the container shipment demand from the previous period into account, and therefore, the fleet deployment plans are demand-dependent. In case Ⅱ, the container shipment demand between periods is assumed to be independent, that is the container shipment demand in period t-1 is not taken into consideration in the fleet deployment plan developed for period t, which indicates that the fleet deployment plans are demand-independent. The optimization model (6.37) shows that, in case Ⅱ, a strategy n in year t (t = 2,…,T) has only one fleet deployment plan, which is obtained by solving the optimization model. Obviously, the demand-dependent fleet deployment plans in case Ⅰ are more reasonable and flexible because the consideration of container shipment demand dependency in this case means that the liner container shipping company can adopt a proper fleet deployment plan based on the container shipment demand that came about in the previous period; in case Ⅱ, meanwhile, the same fleet deployment plan must be adopted regardless of the scenario of container shipment demand that materialized in the previous year.

In the numerical example, each fleet strategy has three fleet deployment plans corresponding to three scenarios of demand: high, medium and low. For example, for the strategy $K_2^1K_2^2K_9^3I_5^3K_2^4K_{12}^5$ of year 2 in case Ⅰ, three fleet deployment plans are shown in Table 6.4. The fleet deployment plan for the same strategy in case Ⅱ is shown in Table 6.5. It is found that those fleet deployment plans are different; the reason for this is that the probabilities involved in the optimization models are different.

Table 6. 4 Ship-to-route allocation of strategy $K_2^1K_2^2K_9^3I_5^3K_2^4K_{12}^5$ in case I for year 2

Demand scenario	Route / Ship Type	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
High	1							2	
	2							2	
	3	3	3			5	3		
	4			1	1				
	5			1	3	4		3	1
Medium	1							2	
	2							2	
	3	3	3			5	3		
	4					2			
	5			2	4	2		3	1
Low	1							2	
	2			2					
	3	3	3				4	4	
	4					1		1	
	5				4	6		1	1

Table 6. 5 Ship-to-route allocation of strategy $K_2^1K_2^2K_9^3I_5^3K_2^4K_{12}^5$ in case II for year 2

Route / Ship Type	CCX	CPX	GIS	IDX	NCE	NZX	SCE	UKX
1							2	
2							2	
3	3	3		2	2	3	1	
4					2			
5				3	4		2	1

6.6 Summary

This chapter considers a multi-period LSFP problem with container transshipment and uncertain container shipment demand. The uncertain container shipment demand in each period is assumed to be dependent on that of the previous period. A set of scenarios in each single period is used to reflect the uncertainty of container shipment demand, and then the evolution and dependency of container shipment demand across multiple periods is modeled as a scenario tree. A decision tree is used to interpret the procedure of fleet development over the multi-period planning horizon. Then, the proposed multi-period LSFP problem is formulated as a multi-period stochastic programming model comprising a sequence of interrelated 2SSP models. In order to solve this model, the dual decomposition and Lagrangian relaxation method is employed to solve the 2SSP models; and then the solution to the multi-period LSFP problem is found by using a shortest path algorithm. We illustrate the applicability and performance of our proposed model and solution method on a numerical example. We also investigated the effect of considering container shipment demand dependency. The results show that the profit obtained when considering dependency is higher and the ship fleet plans are more flexible than when dependency is not considered.

It is worth highlighing that the most significant contribution of this study is that it takes the first step towards a more realistic multi-period LSFP problem than has been studied in previous literature and provides an applicable and feasible method for handling such a problem in practice. It has to be pointed out that the feasible fleet size and mix strategies in each single period are assumed to be proposed by experts at the liner container shipping company, rather than being regarded as decision variables. The rationale behind such an assumption is that it effectively reduces the searching space from the viewpoint of operation research and makes the multi-period LSFP problem solvable in practice; otherwise, the multi-period LSFP problem would be highly intractable. However, the quality of the solution, i.e. the longest path provided by this study, is *relatively* better than the others, but possibly not a *global* optimum. We also need to reduce the runtime further because the convergent rate of the harmonic series, i.e. the step size sequence adopted in the solution algorithm, is inefficient. It might be worhwhile investigating whether a more sophisticated heuristic for finding feasible solutions would produce even better results.

Currently, only the expected profit is studied in this paper, no attempt is mde to control the variance (that is the risk that results from the uncertain environment). This will be the subject of our future research work.

CHAPTER 7 CONCLUSIONS

This chapter summarizes the research work presented in this thesis and highlights its outcomes and contributions. Additionally, future work of potential interest is proposed.

7.1 Outcomes and Contributions

This thesis addressed the need to investigate LSFP problems with container shipment demand uncertainty. A review of the current literature showed that there are many limitations and gaps in the LSFP problems studied so far. For example, there are no systematic methodologies proposed in the existing literature to deal with the uncertainty issue arising in LSFP problems. Besides this, the multi-period LSFP problem is not properly addressed in the existing studies. This thesis worked on eliminating these limitations and gaps, by proposing new methodologies. Also, solution algorithms were proposed in order to efficiently solve the new optimization models. Numerical examples were implemented to illustrate the efficiency and applicability of the proposed models and solution algorithms.

Chapter 3 made an initial investigation of the short-term LSFP problem with container shipment demand uncertainty. To deal with the uncertainty, we assumed that the container shipment demand between each port pair on a liner ship route follows a normal distribution with a given mean and variance. This assumption may lead to a problem: since the demands are uncertain, once the decisions in the short-term LSFP problem have been determined, the fleet of ships may be unable to meet the pickup and delivery requirements of its customers, even though the expected demand along the routes does not exceed the fleet capacity. Since this is not completely avoidable, the liner container shipping company simply hopes that it will have a very low possibility of occurring. In order to reduce the possibility that the liner container shipping company cannot satisfy the customers' demand, we decided to insist that the decisions guarantee feasibility 'as much as possible'. Therefore, we formulated the constraint that the liner container shipping company satisfies customer demand in a probabilistic form in this chapter, which is called a chance constraint. The level of service was proposed, to represent the probability of satisfying the customers' requirements, and this was formulated as a chance constraint. The short-term LSFP problem with uncertain container shipment demand was then formulated as a CCP model. The model is actually an integer linear programming model, which can be solved efficiently using any optimization solver, for example CPLEX. A numerical example was carried out to assess the model and analyze the impact of the confidence parameters and cargo shipment demand. The results implied that the level of service has a significant impact on the optimal fleet size and deployment. It was also found that more ships are needed and more costs must be incurred in order to maintain a higher level of service.

Chapter 4 studied the short-term LSFP problem with container transshipment and uncertain container shipment demand from another point of view: the goal of this chapter was to maximize the expected profit for the liner container shipping company. The container shipment demand for each port pair on a liner ship route was assumed to be a random variable. This problem was reformulated as a 2SSIP model. In a stochastic model, some decisions have to be made before the uncertain terms are observed, and these are termed as first-stage decisions. Furthermore, after the uncertain terms become known, recourse actions can take place, which are called second-stage decisions. Since the decisions about fleet design and deployment are made before the realization of container shipment demand, they should be first-stage decision variables, while the number of containers shipped between a port pair on a liner ship route should be second-stage decision variables. The objective of this 2SSIP model was to choose the first-stage decision variables, such as the numbers, types and deployment of ships, in such a way that the sum of the first-stage profit and the expected value of the second-stage profit from shipping containers was maximized. To effectively solve the proposed model, firstly, the SAA method was used to approximate the expected recourse function, and then the dual decomposition and Lagrangian relaxation method was used to solve the model. Finally, the performance of the proposed model and solution algorithms was tested using a numerical example. The results indicate that the solution methods are effective. It was also found that the variability of the uncertain parameters has a significant effect on the solution. As the variability increases, the profit obtained by a liner container shipping company decreases.

Chapter 5 extended the work of Chapter 4 by taking the expected value and variance into account simultaneously. In Chapter 4, the 2SSIP model only considers the expected value, but ignores the variance (namely the risk), which is also an issue of great concern to decision-makers. Therefore, Chapter 5 developed a robust optimization model in which both expected value and variance are considered simultaneously, for the short-term LSFP problem with container shipment demand uncertainty. Robust optimization is able to tackle the decision-makers' favored risk aversion or service-level function. By adjusting the penalty parameters of the robust optimization model, it was shown how decision-makers could determine an optimal liner ship fleet plan, including fleet design and deployment, which maximizes total profit under different container shipment demand scenarios while at the same time controlling the variance. The robustness and effectiveness of the developed model was demonstrated using numerical results. These results generally show that larger ships are mostly assigned to longer routes while smaller ships are mostly assigned to shorter routes, and that more ships are assigned to the longer routes while fewer ships are assigned to the shorter routes.

Chapter 6 studied the long-term/multi-period LSFP problem with container shipment demand uncertainty. This chapter proposed a more realistic problem for a liner container shipping company by taking the uncertainty and the dependency of container shipment demand into account. Using a scenario tree approach to model the evolution of dependent uncertain demand across two successive single periods, and using a decision tree to model the procedure of LSFP, the proposed problem was formulated as a multi-period stochastic programming model, comprising a sequence of interrelated two-stage stochastic programming models developed for each single period. Each two-stage model was solved by the dual decomposition and Lagrangian relaxation method. Each path from the root to a leaf on the decision tree corresponds to a multi-period liner ship fleet plan. A numerical example was carried out to assess the applicability and performance of the proposed model and solution algorithm. We compared the profit in case I, in which dependency and uncertainty of container shipment demand were both included, to the profit in case II, where only uncertainty was included. The results showed that the profit in case I was higher than in case II, indicating that case I was superior.

In short, the contributions of the thesis are as follows:

1. It proposes more realistic LSFP problems than have been studied previously in the literature.
2. It provides a fresh and worthwhile research area for classical LSFP problems by taking the uncertainty and dependence of container shipment demand into account in such problems.
3. It improves the existing mathematical programming models proposed for classical LSFP problems with deterministic container shipment demand.
4. It proposes algorithms and systematic methodologies for formulating LSFP problems with uncertain container shipment demand.
5. It provides an applicable and feasible way for a liner container shipping company to produce its liner ship fleet plans in practice.

7.2 Recommendations for Future Work

This thesis provides many potential future research topics. Firstly, in this thesis all containers are assumed to be of the same size, that is they are all standard twenty-foot equivalent units (TEUs). However, in practice there are multiple types of containers with different sizes and weights, such as eight-foot equivalent units (EEUs), forty-foot equivalent units (FEUs), refrigerated containers, high cube containers, flat rack containers, platform containers, and others. The combination of these multiple types of container makes the operation of loading and unloading them much more complicated, and their inclusion in the LSFP problem would make it more realistic. This would be an interesting and worthwhile research area.

Secondly, although the solution algorithm that integrates the sample average approximation approach with a dual decomposition technique, as proposed in Chapter 4, can produce high-quality results, its convergence speed in approaching the optimum is slow and thus the computational time required is unsatisfactory. How one could increase the convergence speed and reduce the computational time would be a further interesting and challenging issue to explore. Distributed computing would be a useful tool for efficiently reducing the computational time (MirHassani et al., 2000).

Thirdly, Chapters 4, 5 and 6 assume that the liner ship route network is predetermined and fixed. Such an assumption is reasonable in Chapters 4 and 5 because the planning horizon of the LSFP problems studied in these chapters is short-term. During a short-term horizon, a liner container shipping company would be unlikely (and probably unable) to change its liner ship route network. For the long-term LSFP problem, however, as studied in Chapter 6, the liner ship route network may not be fixed, as the company may change it. This assumption was made here in order to simplify the problem. In future, however, we could extend this research work by integrating the dynamic routing problem with the long-term LSFP problem.

Forthly, most liner trade is unbalanced because of the different economic needs in different regions. The number of inbound loaded containers can be quite different from the number of outbound loaded containers at any given port. Liner container shipping companies often need to reposition their empty containers or lease containers from vendors to meet customer demand. It would be interesting and worthwhile to look into repositioning empty containers and to discuss where and when companies should lease containers from vendors so as to meet the demand at different ports.

Finally, since ships are assets with finite lives, the liner container shipping company often has to consider when and which ships should be replaced. Therefore, building a control model to capture ship utilization and replacement decisions would make the problem more realistic. The control model should jointly involve investment timing and trading strategies.

APPENDIX A

Proposition 1: The variance $Var = \sum_{s\in\mathcal{S}} p_s \left[\left(\zeta_s - \sum_{s'\in\mathcal{S}} p_{s'}\zeta_{s'} \right) + 2\vartheta_s \right]$ in the optimization model (5.15) decreases when the value of λ increases.

Proof: Assume that $\lambda^1 < \lambda^2$, and that $\mathbf{x}^1, \mathbf{y}_s^1, \boldsymbol{\varepsilon}_s^1, \vartheta_s^1 \left(\forall s \in \mathcal{S}\right)$ and $\mathbf{x}^2, \mathbf{y}_s^2, \boldsymbol{\varepsilon}_s^2, \vartheta_s^2 \left(\forall s \in \mathcal{S}\right)$ are the optimal solutions for $\mathbf{x}, \mathbf{y}_s, \boldsymbol{\varepsilon}_s, \vartheta_s \left(\forall s \in \mathcal{S}\right)$ in the optimization models (5.15) associated with λ^1 and λ^2, respectively. The objective functions of the optimization models (5.15) with λ^1 and λ^2, denoted by $Z\left(\mathbf{x}^1, \mathbf{y}_s^1, \boldsymbol{\varepsilon}_s^1, \vartheta_s^1\right)\Big|_{\lambda=\lambda^1}$ and $Z\left(\mathbf{x}^2, \mathbf{y}_s^2, \boldsymbol{\varepsilon}_s^2, \vartheta_s^2\right)\Big|_{\lambda=\lambda^2}$ respectively, are as follows:

$$Z\left(\mathbf{x}^1, \mathbf{y}_s^1, \boldsymbol{\varepsilon}_s^1, \vartheta_s^1\right)\Big|_{\lambda=\lambda^1} = \sum_{s\in\mathcal{S}} p_s\zeta_s^1 + \lambda^1 \sum_{s\in\mathcal{S}} p_s \left[\left(\zeta_s^1 - \sum_{s'\in\mathcal{S}} p_{s'}\zeta_{s'}^1 \right) + 2\vartheta_s^1 \right] + \varpi \sum_{s\in\mathcal{S}} p_s \boldsymbol{\varepsilon}_s^1 \quad \text{(A-1)}$$

$$Z\left(\mathbf{x}^2, \mathbf{y}_s^2, \boldsymbol{\varepsilon}_s^2, \vartheta_s^2\right)\Big|_{\lambda=\lambda^2} = \sum_{s\in\mathcal{S}} p_s\zeta_s^2 + \lambda^2 \sum_{s\in\mathcal{S}} p_s \left[\left(\zeta_s^2 - \sum_{s'\in\mathcal{S}} p_{s'}\zeta_{s'}^2 \right) + 2\vartheta_s^2 \right] + \varpi \sum_{s\in\mathcal{S}} p_s \boldsymbol{\varepsilon}_s^2 \quad \text{(A-2)}$$

Also $Z\left(\mathbf{x}^1, \mathbf{y}_s^1, \boldsymbol{\varepsilon}_s^1, \vartheta_s^1\right)\Big|_{\lambda=\lambda^2}$ and $Z\left(\mathbf{x}^2, \mathbf{y}_s^2, \boldsymbol{\varepsilon}_s^2, \vartheta_s^2\right)\Big|_{\lambda=\lambda^1}$ are given by:

$$Z\left(\mathbf{x}^1, \mathbf{y}_s^1, \boldsymbol{\varepsilon}_s^1, \vartheta_s^1\right)\Big|_{\lambda=\lambda^2} = \sum_{s\in\mathcal{S}} p_s\zeta_s^1 + \lambda^2 \sum_{s\in\mathcal{S}} p_s \left[\left(\zeta_s^1 - \sum_{s'\in\mathcal{S}} p_{s'}\zeta_{s'}^1 \right) + 2\vartheta_s^1 \right] + \varpi \sum_{s\in\mathcal{S}} p_s \boldsymbol{\varepsilon}_s^1 \quad \text{(A-3)}$$

$$Z\left(\mathbf{x}^2, \mathbf{y}_s^2, \boldsymbol{\varepsilon}_s^2, \vartheta_s^2\right)\Big|_{\lambda=\lambda^1} = \sum_{s\in\mathcal{S}} p_s\zeta_s^2 + \lambda^1 \sum_{s\in\mathcal{S}} p_s \left[\left(\zeta_s^2 - \sum_{s'\in\mathcal{S}} p_{s'}\zeta_{s'}^2 \right) + 2\vartheta_s^2 \right] + \varpi \sum_{s\in\mathcal{S}} p_s \boldsymbol{\varepsilon}_s^2 \quad \text{(A-4)}$$

Then we have:

$$Z\left(\mathbf{x}^1, \mathbf{y}_s^1, \boldsymbol{\varepsilon}_s^1, \vartheta_s^1\right)\Big|_{\lambda=\lambda^1} \le Z\left(\mathbf{x}^2, \mathbf{y}_s^2, \boldsymbol{\varepsilon}_s^2, \vartheta_s^2\right)\Big|_{\lambda=\lambda^1} \quad \text{(A-5)}$$

$$Z\left(\mathbf{x}^2, \mathbf{y}_s^2, \boldsymbol{\varepsilon}_s^2, \vartheta_s^2\right)\Big|_{\lambda=\lambda^2} \le Z\left(\mathbf{x}^1, \mathbf{y}_s^1, \boldsymbol{\varepsilon}_s^1, \vartheta_s^1\right)\Big|_{\lambda=\lambda^2} \quad \text{(A-6)}$$

Summing both sides of Eqs. (A-5) and (A-6), the following Eq. (A-7) can be obtain:

$$Z\left(\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\right)\Big|_{\lambda=\lambda^1}+Z\left(\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\right)\Big|_{\lambda=\lambda^2} \le Z\left(\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\right)\Big|_{\lambda=\lambda^1}+Z\left(\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\right)\Big|_{\lambda=\lambda^2} \tag{A-7}$$

Substituting Eqs. (A-1) to (A-4) into Eq. (A-7), and then we get:

$$\left(\lambda^1-\lambda^2\right)\left\{\sum_{s\in\mathcal{S}}p_s\left[\left(\zeta_s^1-\sum_{s'\in\mathcal{S}}p_{s'}\zeta_{s'}^1\right)+2\vartheta_s^1\right]-\sum_{s\in\mathcal{S}}p_s\left[\left(\zeta_s^2-\sum_{s'\in\mathcal{S}}p_{s'}\zeta_{s'}^2\right)+2\vartheta_s^2\right]\right\}\le 0 \tag{A-8}$$

From the assumption that $\lambda^1<\lambda^2$, we can easily derive that

$$\sum_{s\in\mathcal{S}}p_s\left[\left(\zeta_s^1-\sum_{s'\in\mathcal{S}}p_{s'}\zeta_{s'}^1\right)+2\vartheta_s^1\right]\ge\sum_{s\in\mathcal{S}}p_s\left[\left(\zeta_s^2-\sum_{s'\in\mathcal{S}}p_{s'}\zeta_{s'}^2\right)+2\vartheta_s^2\right] \tag{A-9}$$

Thus, $Var\big|_{\lambda=\lambda^1}\ge Var\big|_{\lambda=\lambda^2}$. □

Similarly, we can derive the following proposition:

Proposition 2: The underfulfillment $\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s$ in the optimization model (5.15) decreases when the value of the weight ϖ increases.

Proof: The proof is similar to that used for Proposition 1. Assume that $\varpi^1<\varpi^2$, and that $\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\left(\forall s\in\mathcal{S}\right)$ and $\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\left(\forall s\in\mathcal{S}\right)$ are the optimal solutions for $\mathbf{x},\mathbf{y}_s,\boldsymbol{\varepsilon}_s,\vartheta_s\left(\forall s\in\mathcal{S}\right)$ in the optimization models (5.15) associated with ϖ^1 and ϖ^2, respectively. The objective functions of the two optimization models, denoted by $Z\left(\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\right)\Big|_{\varpi=\varpi^1}$ and $Z\left(\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\right)\Big|_{\varpi=\varpi^2}$, respectively, are as folllows:

$$Z\left(\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\right)\Big|_{\varpi=\varpi^1}=\sum_{s\in\mathcal{S}}p_s\zeta_s^1+\lambda\sum_{s\in\mathcal{S}}p_s\left[\left(\zeta_s^1-\sum_{s'\in\mathcal{S}}p_{s'}\zeta_{s'}^1\right)+2\vartheta_s^1\right]+\varpi^1\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^1 \tag{A-10}$$

$$Z\left(\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\right)\Big|_{\varpi=\varpi^2}=\sum_{s\in\mathcal{S}}p_s\zeta_s^2+\lambda\sum_{s\in\mathcal{S}}p_s\left[\left(\zeta_s^2-\sum_{s'\in\mathcal{S}}p_{s'}\zeta_{s'}^2\right)+2\vartheta_s^2\right]+\varpi^2\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^2 \quad \text{(A-11)}$$

Also $Z\left(\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\right)\Big|_{\varpi=\varpi^2}$ and $Z\left(\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\right)\Big|_{\varpi=\varpi^1}$ are given by:

$$Z\left(\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\right)\Big|_{\varpi=\varpi^2}=\sum_{s\in\mathcal{S}}p_s\zeta_s^1+\lambda\sum_{s\in\mathcal{S}}p_s\left[\left(\zeta_s^1-\sum_{s'\in\mathcal{S}}p_{s'}\zeta_{s'}^1\right)+2\vartheta_s^1\right]+\varpi^2\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^1 \quad \text{(A-12)}$$

$$Z\left(\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\right)\Big|_{\varpi=\varpi^1}=\sum_{s\in\mathcal{S}}p_s\zeta_s^2+\lambda\sum_{s\in\mathcal{S}}p_s\left[\left(\zeta_s^2-\sum_{s'\in\mathcal{S}}p_{s'}\zeta_{s'}^2\right)+2\vartheta_s^2\right]+\varpi^1\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^2 \quad \text{(A-13)}$$

Then we have:

$$Z\left(\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\right)\Big|_{\varpi=\varpi^1}\le Z\left(\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\right)\Big|_{\varpi=\varpi^1} \quad \text{(A-14)}$$

$$Z\left(\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\right)\Big|_{\varpi=\varpi^2}\le Z\left(\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\right)\Big|_{\varpi=\varpi^2} \quad \text{(A-15)}$$

Summing both sides of Eqs. (A-14) and (A-15), Eq. (A-16) can be obtain:

$$\begin{gathered}Z\left(\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\right)\Big|_{\varpi=\varpi^1}+Z\left(\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\right)\Big|_{\varpi=\varpi^2}\\ \le \\ Z\left(\mathbf{x}^2,\mathbf{y}_s^2,\boldsymbol{\varepsilon}_s^2,\vartheta_s^2\right)\Big|_{\varpi=\varpi^1}+Z\left(\mathbf{x}^1,\mathbf{y}_s^1,\boldsymbol{\varepsilon}_s^1,\vartheta_s^1\right)\Big|_{\varpi=\varpi^2}\end{gathered} \quad \text{(A-16)}$$

Substituting Eqs. (A-10) to (A-13) into Eq. (A-16), and then we get:

$$\varpi^1\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^1+\varpi^2\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^2\le\varpi^2\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^1+\varpi^1\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^2 \quad \text{(A-17)}$$

which can be rewritten as follows:

$$\left(\varpi^1-\varpi^2\right)\left(\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^1-\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^2\right)\le 0 \quad \text{(A-18)}$$

From the assumption that $\varpi^1<\varpi^2$, we can easily derive that

$$\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^1\ge\sum_{s\in\mathcal{S}}p_s\boldsymbol{\varepsilon}_s^2 \quad \text{(A-19)}$$

which proves the proposition. □

APPENDIX B

Proposition 1: The profit using Eq. (6.23). in case Ⅰ is larger or equal to that using Eq. **Error! Reference source not found.** in case Ⅱ.

Proof: In case Ⅰ, $EP_{t,n}^{m}$ is given by:

$$EP_{t,n}^{m} = \sum_{s' \in \mathcal{S}_{t-1}} p_{s'}^{t-1} \times EP_{t,n}^{m,s'} \tag{B-1}$$

In Eq. (6.10), the terms $\sum_{k \in \mathcal{G}_{t,n}^{\text{OUT}}} c_{kt}^{\text{OUT}}$, $\sum_{k \in \mathcal{G}_{t,n}^{\text{SOLD}}} c_{kt}^{\text{SOLD}}$, $\sum_{k \in \mathcal{G}_{t,n}^{\text{IN}}} c_{kt}^{\text{IN}}$ and $\sum_{k \in \mathcal{G}_{t,n}^{\text{NEW}}} c_{kt}^{\text{BUY}}$ can be removed since they are fixed when the sets of $\mathcal{G}_{t,n}^{\text{OUT}}, \mathcal{G}_{t,n}^{\text{SOLD}}, \mathcal{G}_{t,n}^{\text{IN}}$ and $\mathcal{G}_{t,n}^{\text{NEW}}$ are given. Then Eq. (B-1) could be rewritten as follows after substituting Eq. (6.10) to replace $EP_{t,n}^{m,s'}$:

$$\begin{aligned}
EP_{t,n}^{m} &= \sum_{s' \in \mathcal{S}_{t-1}} p_{s'}^{t-1} \times \max\left(\sum_{s \in \mathcal{S}_t} p_{s|s'}^{t} Q_{\xi}^{ts}\left(\mathbf{v}, \xi(\boldsymbol{\omega})\right) - \sum_{r \in \mathcal{R}} \sum_{k \in \mathcal{G}_{t,n}} \left(c_{kt}^{r} x_{nt}^{kr} + e_{kt} y_{nt}^{k} \right) \right) \\
&= \sum_{s' \in \mathcal{S}_{t-1}} \max\ p_{s'}^{t-1} \times \left(\sum_{s \in \mathcal{S}_t} p_{s|s'}^{t} Q_{\xi}^{ts}\left(\mathbf{v}, \xi(\boldsymbol{\omega})\right) - \sum_{r \in \mathcal{R}} \sum_{k \in \mathcal{G}_{t,n}} \left(c_{kt}^{r} x_{nt}^{kr} + e_{kt} y_{nt}^{k} \right) \right) \\
&\geq \max \sum_{s' \in \mathcal{S}_{t-1}} p_{s'}^{t-1} \times \sum_{s \in \mathcal{S}_t} p_{s|s'}^{t} Q_{\xi}^{ts}\left(\mathbf{v}, \xi(\boldsymbol{\omega})\right) - \sum_{r \in \mathcal{R}} \sum_{k \in \mathcal{G}_{t,n}} \left(c_{kt}^{r} x_{nt}^{kr} + e_{kt} y_{nt}^{k} \right) \\
&= \max \sum_{s \in \mathcal{S}_t} \sum_{s' \in \mathcal{S}_{t-1}} p_{s'}^{t-1} \times p_{s|s'}^{t} Q_{\xi}^{ts}\left(\mathbf{v}, \xi(\boldsymbol{\omega})\right) - \sum_{r \in \mathcal{R}} \sum_{k \in \mathcal{G}_{t,n}} \left(c_{kt}^{r} x_{nt}^{kr} + e_{kt} y_{nt}^{k} \right) \\
&= \max \sum_{s \in \mathcal{S}_t} p_{s}^{t} \times Q_{\xi}^{ts}\left(\mathbf{v}, \xi(\boldsymbol{\omega})\right) - \sum_{r \in \mathcal{R}} \sum_{k \in \mathcal{G}_{t,n}} \left(c_{kt}^{r} x_{nt}^{kr} + e_{kt} y_{nt}^{k} \right)
\end{aligned} \tag{B-2}$$

In case Ⅱ, $EP_{t,n}^{m}$ is given by Eq. **Error! Reference source not found.**. Similarly, the terms $\mathcal{G}_{t,n}^{\text{OUT}}, \mathcal{G}_{t,n}^{\text{SOLD}}, \mathcal{G}_{t,n}^{\text{IN}}$ and $\mathcal{G}_{t,n}^{\text{NEW}}$ are removed and then $EP_{t,n}^{m}$ is given by:

$$EP_{t,n}^{m} = \max \sum_{s \in \mathcal{S}_t} p_{s}^{t} Q_{\xi}^{ts}\left(\mathbf{v}, \xi(\boldsymbol{\omega})\right) - \sum_{r \in \mathcal{R}} \sum_{k \in \mathcal{G}_{t,n}} \left(c_{kt}^{r} x_{nt}^{kr} + e_{kt} y_{nt}^{k} \right) \tag{B-3}$$

Therefore, $EP_{t,n}^{m}$ in case Ⅰ $\geq$ $EP_{t,n}^{m}$ in case Ⅱ.

REFERENCES

Agarwal, R. and Ergun O., 2008. Ship schedule and network design for cargo routing in liner shipping. *Transportation Science*, 42 (2), 175-196.

Ahmed, S., 2004. Stochastic *Integer Programming: An Algorithmic Perspective*. Available from: http://www2.isye.gatech.edu/~sahmed/siptutorial.pdf

Ahmed, S., King, A.J. and Parija, G., 2003. A multi-stage stochastic integer programming approach for capacity expansion under uncertainty, *Journal of Global Optimization*, 26, 3-24.

Ahmed, S. and Sahinidis, N.V., 2003. An approximation scheme for stochastic integer programs arising in capacity expansion, *Operations Research*, 51 (3), 461-471.

Ahuja, R.A., Magnanti, T.L., Orlin, J.B., 1996. *Network flows: theory, algorhtims and applications*. Prentice Hall, New Jersey.

Benford, H., 1981. A simple approach to fleet deployment. *Maritime Policy and Management*, 8 (4), 223-228.

Berman, O., Ganz, Z. and Wagner, J.M., 1994. A stochastic optimization model for planning expansion in a service industry under uncertain demand, *Naval research logistics*, 41, 545-564.

Birge, J.R. and Louveaux, F.V., 1997. *Introduction to Stochastic Programming*. Springer-Verlag, New York.

Bradley, S.P., Hax, A. C. and Magnanti, T. L., 1977. *Applied Mathematical Programming*. Addison-Wesley, Mass., Chap. 7.

Brown, R.G., 1959. *Statistical forcasting for inventory control*. New York: McGrav-Hill.

Carøe, C.C. and Schultz, R., 1999. Dual decomposition in stochastic integer programming. *Operations Research Letter* 24, 37-45.

Celikyurt, U. and Özekici, S., 2007. Multiperiod portfolio optimization models in stochastic markets using the mean-variance approach, *European Journal of Operational Research*, 179, 186-202.

Charnes, A. and Cooper, W. W., 1959. Chance-Constrained Programming. *Management Science*, 6 (1), 73-79.

Cheung, R.K. and Chen, C.Y., 1998. A two-stage stochastic network model and solution methods for the dynamic empty container allocation problem. *Transportation Science*, 32 (2), 142-162.

Cheung, R. K. and Powell, W. B., 1996. An algorithm for multistage dynamic networks with random arc capacities, with an application to dynamic fleet management. *Operations Research*, 44 (6), 951-963.

Cheung, R. K. and Powell, W. B., 2000. SHAPE-A stochastic hybrid approximation procedure for two-stage stochastic programs. *Operations Research*, 48 (1), 73-79.

Cho, S.C. and Perakis, A.N., 1996. Optimal liner fleet routing strategies. *Maritime Policy and Management*, 23 (3), 249-259.

Christiansen, M., Fagerholt, K. and Ronen, D., 2004. Ship routing and scheduling: status and perspectives. *Transportation Science*, 38 (1), 1-18.

Crainic, T.G., Gendreau, M. and Dejax, P., 1993. Dynamic and stochastic models for the allocation of empty containers, *Operations Research*, 41 (1), 102-125.

Cullinane, K. and Khanna, M., 1999. Economics of scale in large container ships. *Journal of Transport Economics and Policy*, 33 (2), 185-208.

Dantzig, G.B.,1955. Linear programming under uncertainty. *Management Science*, 1, 197-206.

Dantzig, G.B. and Fulkerson, D.R., 1954. Minimizing the number of tankers to meet a fixed schedule. *Naval Research Logistics Quarterly*, 1, 217-222.

Denardo, E.V. and Fox, B.L., 1978. Shortest –route methods: 1. reaching, pruning, and buckets. *Operations Research*, 27 (1), 161-186.

Ermoliev, Y., 1988. *Stochastic quasigradient methods*. In: Ermoliev, Yu., Wets, R. J-B. (Eds.), Numerical Techniques for Stochastic Programming, Springer-Verlag, Berlin, 141-185.

Everett, J. L., Hax, A.C., Lewinson, V.A. and Nudds, D., 1972. Optimization of a Fleet of Large Tankers and Bulkers: A linear Programming Approach. *Marine Technology*, October, 430-438.

Fagerholt, K, 1999. Optimal fleet design in a ship routing problem. *International Transactions in Operational Research*, 6 (5), 453-464.

Fagerholt, K. and Lindstad, H., 2000. Optimal policies for maintaining a supply service in the Norwegian Sea. *Omega*, 28 (3), 269-275.

Frantzeskakis, L. F. and Powell, W. B., 1990. A successive linear approximation procedure for stochastic, dynamic vehicle allocation problems. *Transportation Sciences*, 24 (1), 40-57.

Gallagher, F. D. and Meyrick, S. J., 1984. ASEAN-Australia liner shipping: a cost-based simulation analysis. ASEAN-Australia Joint Research Project (Kuala Lumpur and Canberra), ASEA-Australia Economic Papers No. 12.

Gelareh, S. and Meng, Q., 2010. A novel modeling approach for the fleet deployment problem within a short-term planning horizon. *Transportation Research Part E*, 46 (1), 76-89.

Godfrey, G.A. and Powell, W.B., 2001. An adaptive, distribution-free algorithm for the newsvendor problem with censored demands, with applications to inventory and distribution, *Management Science*, 47 (8), 1101-1112.

Gülpinar, N. and Rustem, B., 2007. Worst-case robust decisions for multi-period mean-variance portfolio optimization, *European Journal of Operational Research*, 183, 981-1000.

Jaramillo, D. I. and Perakis, A.N., 1991. Fleet deployment optimization for liner shipping Part 2. Implementation and results. *Maritime Policy and Management*, 18 (3), 235-262.

Kleywegt, A. J., Shapiro, A. and Homem-De-Mello, T., 2001. The sample average approximation method for stochastic discrete optimization. *SIAM Journal of Optimization*, 12, 479-502.

Laderman, J., Gleiberman, L. and Egan, J.F., 1966. Vessel allocation by linear programming. *Naval Research Logistics Quarterly*, 315-320.

Laguna, M., 1998. Applying robust optimization to capacity expansion of one location in telecommunications with demand uncertainty. *Management Science*, 44, 11, 101-110.

Lane, D.E., Heaver. T.D. and Uyeno. D., 1987. Planning and scheduling for efficiency in liner shipping. *Maritime Policy and Management*, 12 (3), 109-125.

Laporte, G., Louveaux, F.V. and Mercure, H., 1992. The vehicle routing problem with stochastic travel times, *Transportation Science*, 26 (3), 161-170.

Laporte, G., Louveaux, F.V. and Hamme, L.V., 2002. An integer L-shpaed algorithm for the capacitated vehicle routing problem with stochastic demands, *Operations Research*, 50 (3), 415-423.

Leung, S. C. H, Lai, K. K., Ng, W-L, and Wu, Y., 2007a. A robust optimization model for production planning of perishable products. *Journal of Operational Research Society*, 58, 4, 413-422.

Leung, S. C. H, Tsang, S. O. S., Ng, W-L, and Wu, Y., 2007b. A robust optimization model for multi-site production planning problem in an uncertain environment. *European Journal of Operational Research*, 181, 1, 224-238.

Leung, S.C.H., Wu, Y. and Lai, K.K., 2002. A robust optimization model for a cross-border logistics problem with fleet composition in an uncertain environment. *Mathematical and Computer Modelling*, 36 (11-13), 1221-1234.

Listes, O. and Dekker, R., 2005. A scenario aggregation-based approach for determining a robust airline fleet composition for dynamic capacity allocation, *Transportation Science*, 39 (3), 367-382.

Mak, W.K., Morton, D.P. and Wood, R.K. 1999. Monte Carlo bounding techniques for determining solution quality in stochastic programs. *Operations Research Letters*, 24 (1-2), 47-56.

Malcolm, S. A. and Zenios, S.A., 1994. Robust optimization for power systems capacity expansion under uncertainty. *Journal of Operational Research Society*, 45, 9, 1040-1049.

MirHassani, S.A., Lucas, C., Mitra, G., Messina, E. and Poojari, C.A., 2000. Computational solution of capacity planning models under uncertainty. *Parallel Computing*, 26, 511-538.

Mourão, M.C., Pato, M.V. and Paixão, A.C., 2001. Ship assignment with hub and spoke constraints. *Maritime Policy and Management*, 29 (2), 135-150.

Mulvey, J. M., Vanderbei, R. J. and Zenios, S. A., 1995. Robust optimization of large-scale systems. *Operations Research*, 43, 2, 264-281.

Nicholson, T.A.J. and Pullen, R.D., 1971. Dynamic Programming Applied to Ship Fleet Management. *Operational Research Quarterly*, 22 (3), 211-220.

Norkin, V. I., Ermoliev, Y.M. and Ruszczyński, A., 1998a. On optimal allocation of indivisibles under uncertainty, *Operations Research*, 46 (3), 381-395.

Norkin, V.I., Pflug, G.C. and Ruszczyński, A., 1998b. A branch and bound method for stochastic global optimization. *Mathematical Programming*, 83, 425-450.

Osorio, M.A., Gülpinar, N. and Rustem, B., 2008. A mixed integer programming model for multistage mean-variance post-tax optimization, *European Journal of Operational Research*, 185, 451-480.

Papadakis, N.A. and Perakis, A.N., 1989. A Nonlinear Approach to the Multiorigin, Multidestination Fleet Deployment Problem. *Naval Research Logistics*, 36, 515-528.

Perakis, A.N., 1985. A second look at fleet deployment. *Maritime Policy and Management*, 12 (3), 209-214.

Perakis, A.N., 2002. *Fleet operation optimization and fleet deployment.* Costas Th. Grammernos, ed. The Handbook of Maritime Economics and Business. Lloyds of London Publications, London, U.K. ,580-597.

Perakis, A.N. and Jarammillo, D. I., 1991. Fleet deployment optimization for liner shipping Part 1. Background, problem formulation and solution approaches. *Maritime Policy and Management*, 18 (3), 183-200.

Perakis, A.N. and Papadakis, N., 1987a. Fleet deployment optimization models Part 1. *Maritime Policy and Management*, 14 (2), 127-144.

Perakis, A.N. and Papadakis, N., 1987b. Fleet deployment optimization models Part 2. *Maritime Policy and Management*, 14 (2), 145-155.

Powell, B.J. and Perakis, A.N., 1997. Fleet deployment optimization for liner shipping: an integer programming model. *Maritime Policy and Management*, 24 (2), 183-192.

Powell, W. B. and Cheung, R. K., 1994. A network recourse decomposition method for dynamic networks with random arc capacities. *Networks*, 24, 369-384.

Ronen, D., 1983. Cargo ships routing and scheduling: Survey of models and problems. *European Journal of Operational Research*, 12 (2), 119-126.

Ronen, D., 1993. Ship scheduling: The last decade. *European Journal of Operational Research*, 71 (3), 325-333.

Sambracos, E., Paravantis, J.A., Tarantilis, C.D. and Kiranoudis, C.T., 2004. Dispatching of small containers via coastal freight liners: The case of the Agenan Sea. *European Journal of Operational Research*, 152 (2), 365-381.

Santoso, T., Ahmed, S., Goetschalckx M. and Shapiro A., 2005. A stochastic programming approach for supply chain network design under uncertainty, *European Journal of Operation Research*, 167, 96-115.

Schutz, P, Tomasgard, A., Ahmed, S., 2009. Supply chain design under uncertainty using sample average approximation, *European Journal of Operation Research*, 199, 409-419.

Shapiro, A., Dentcheva, D. and Ruszczyński, A., 2009. *Lectures on Stochastic Programming: Modeling and Theory*. Philadelphia: Society of Industrial and Applied

Mathematics and Mathematical Programming Society.

Shapiro, A. and Philpott, A., 2007. *A tutorial on stochastic programming*. http://www2.isye.gatech.edu/people/faculty/Alex_Shapiro/TutorialSP.pdf.

Sherali, H.D. and Zhu, X.M., 2008. Two-stage fleet assignment model considering stochastic passenger demands. *Operations Research*, 56 (2), 383-399.

Shore, N. Z. 1985. *Minimization Methods for Non-differentiable Functions*. Springer-Verlag New York, Inc.

Shintani, K., Imai, A., Nishimura, E. and Papadimitriou, S., 2007. The container shipping network design problem with empty container repositioning. *Transportation Research Part E*, 43 (1), 39-59.

Singh, K.J., Philpott, A.B. and Wood, P.K., 2009. Dantzig-Wolfe decomposition for solving multistage stochastic capacity-planning problems, *Operations Research*, 57 (5), 1271-1286.

Stott, K.L. and Douglas, B.W., 1981. A model-based decision support system for planning and scheduling ocean-borne transportation. *Interfaces*, 11(4), 1-10.

UNCTAD, 2010. *Review of Maritime Transprotation 2010*. Paper presented at the United Nations Conference on Trade and Development. New York and Geneva, USA. http://www.unctad.org/en/docs/rmt2010_en.pdf.

Vernimmen, B., Dullaert, W. and Engelen, S., 2007. Schedule unreliability in liner shipping: origins and consequences for the hinterland supply chain. *Maritime Economics and Logisitics*, 9, 193-213.

Wagner, J.M. and Berman, O., 1995. Models for planning capacity expansion of convenience stores under uncertain demand and the value of information, *Annals of*

Operations Research, 59, 19-44.

Xie, X.L., 1997. *Fleet Management and Deployment*, Renmin Jiao Tong Press, Beijing. Textbook. ISBN-7-114-03490-3 (in Chinese).

Xie, X.L., Wang, T.F. and Chen, D.S., 2000. A dynamic model and algorithm for fleet planning. *Maritime Policy and Management*, 27 (1), 53-63.

Yu, C. S. and Li, H. L., 2000. A robust optimization model for stochastic logistic problem. *International Journal of Production Economics*, 64, 1, 385-397.

PUBLICATIONS

- **Journal Papers**

1. Meng, Q., **Wang, T.,** 2011, A scenario-based dynamic programming model for multi-period liner ship fleet planning, *Transportation Research Part E-Logistics and Transportation Review*, 47 (4), 401-413.
2. Wang, S., **Wang, T.** and Meng, Q., 2011, A note on liner ship fleet deployment, *Flexible Services and Manufacturing Journal*, 1-9 (in press). DOI: 10.1007/s10696-011-9089-0.
3. Meng, Q. and **Wang, T.,** 2010, A chance constrained programming model for short-term liner ship fleet planning problems, *Maritime Policy & Management*, 27 (4), 329-346.
4. Meng, Q., **Wang, T.** and Wang, S., 2010, Liner ship fleet planning with uncertain container shipment demand, *Transportation Science*, (under review).
5. Meng, Q., **Wang, T.** and Wang, S., 2010, A multi-period liner ship fleet planning problem with dependent uncertain container shipment demand, *Transportation Research Part E-Logistics and Transportation Review*, (under review).
6. **Wang, T.** and Meng, Q., 2010, A robust optimization model for liner ship fleet planning under uncertain demand, *Flexible Services and Manufacturing Journal*, (under review).

- **Book Chapters**

1. Meng, Q., **Wang, T.** and Gelareh, S., 2010, A linearized approach for the liner containership fleet planning with demand uncertainty, *Recent Advances in Maritime Logistics and Supply Chain Systems*, (to appear).

- **International Conferences**

1. Reviewer, the 14th International IEEE Annual Conference on Intelligent Transportation Systems, to be held on October 5-7, 2011 at The George Washington, University, Washington, DC, USA.
2. Reviewer, the 13th International IEEE Annual Conference on Intelligent Transportation Systems, Madeira Island, Portugal, September, 19-22, 2010.
3. Speaker, "A long-term liner ship fleet planning problem with container shipment demand uncertainty", the 7th Triennial Symposium on Transportation Analysis (TRISTAN VII), Tromsø, Norway, June, 20-25, 2010.
4. Speaker, "A dynamic programming approach for long-term containership fleet planning", the 89th Annual Meeting of Transportation Research Board, U.S.A., December, 2009.
5. Speaker, "A linearized approach for the liner containership fleet planning with demand uncertainty", the International Symposium on Maritime Logistics & Supply Chain Systems (MLOG2009), Singapore, 23-24， April, 18-10, 2009.
6. Speaker, "Optimal fleet planning with cargo demand uncertainty for liner shipping", the 18th Triennial Conference of the International Federation of Operation Research Societies (INFORS), Sandton, South Africa, 13-18, July, 2008.