

# Robust Beamforming for Cognitive and Cooperative Wireless Networks

Ebrahim A. Gharavol

(B.Sc., with Honors and M.Sc., Ferdowsi University of Mashhad, Iran)

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# Summary

In this thesis four different problems in the area of the robust beamforming in cognitive and cooperative wireless networks, namely, robust downlink beamforming in cognitive radio networks, robust joint transceiver optimization in MIMO ad hoc networks, and finally robust relay beamforming for both one-way and two-way relay channels, are studied. In these problems, it is assumed that the channel state information is not perfectly known and its imperfection, is modeled using either a Stochastic Error (SE) model or a Norm Bounded Error (NBE) model. In the case of the SE model of uncertainty, the average performance measure and in the case of the NBE model of uncertainty, the worst case performance measures are optimized. In the former case an algorithm containing second order cone programming problems, and in the latter case, an algorithm containing semidefinite programming problems are proposed to perform the beamforming process. Finally, numerical simulations are provided as well to assess the performance of the proposed algorithms.

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# Acronyms

AF	Amplify and Forward
A.K.A.	Also Known As
BC	Broadcast Channel
BS	Base Station
CDI	Channel Direction Information
CF	Compress and Forward
CR-Net	Cognitive Radio Network
CSC	Conventional Self Cancellation
CSI	Channel State Information
DF	Decode and Forward
ECM	Expectation Conditional Maximization
ExRS	Exact Robust Solution
GIFRC	Gaussian Interference Relay Channel
IP	Interference Power
LBRS	Loosely Bounded Robust Solution
LMI	Linear Matrix Inequality
LP	Linear Program
MAC	Multiple Access Channel
MIMO	Multiple-Input Multiple-Output
MISO	Multiple-Input Single-Output
MMSE	Minimum Mean Square Error
MSE	Mean Square Error
MU	Multiuser
MVDR	Minimum Variance Distortionless Response
NBE	Norm Bounded Error
NP	Nondeterministic Polynomial
OFDMA	Orthogonal Frequency Domain Multiple Access
OWRC	One-Way Relay Channel
P2P	Point to Point
PR-Net	Primary-Radio Network
PU	Primary User
QoS	Quality of Service
RMVB	Robust Minimum Variance Beamforming
Rx	Receive
SBRs	Strictly Bounded Robust Solution
SDP	Semidefinite Programming
SE	Stochastic Error



SIMO	Single-Input Multiple-Output
SINR	Signal to Interference plus Noise Ratio
SNR	Signal to Noise Ratio
SOCP	Second Order Cone Programming
SSC	Strict Self Cancellation
SU	Secondary User
THP	Tomlinson-Harashima Precoding
TWRC	Two-Way Relay Channel
Tx	Transmit
ULA	Uniform Linear Array

# Notations

$a, A, \alpha$	Scalar constants, variables or sets(all normal font letters)
$\mathbf{a}, \boldsymbol{\alpha}$	Vector constants or variables (all bold-faced lowercase letters)
$\mathbf{A}, \boldsymbol{\Delta}$	Matrix constants or variables (all bold-faced uppercase letters)
$\triangleq$	Defined as
$\mathbb{N}$	Set of all natural counting numbers
$\mathbb{R}, \mathbb{C}$	Real and complex number fields, respectively
$\mathbb{R}^n, \mathbb{C}^n$	$n$ -Dimensional real and complex vector spaces, respectively
$\mathbb{R}^{n \times m}, \mathbb{C}^{n \times m}$	$n \times m$ Real and complex matrix fields, respectively
$\mathbf{0}$	Zero matrix
$\mathbf{I}$	Identity matrix
$\mathbf{A}^T$	Transpose of matrix $\mathbf{A}$
$\mathbf{A}^*$	Conjugate transpose of matrix $\mathbf{A}$
$\text{tr}[\mathbf{A}]$	Trace (sum of all diagonal elements) of matrix $\mathbf{A}$
$\text{vec}[\mathbf{A}]$	Vectorized version of matrix $\mathbf{A}$
$\mathbf{A} \succeq \mathbf{0}$	Positive semi-definiteness of matrix $\mathbf{A}$
$\mathbf{A} \otimes \mathbf{B}$	Kronecker product of two matrices $\mathbf{A}$ and $\mathbf{B}$
$\ \mathbf{a}\ , \ \mathbf{a}\ _2$	Euclidean (second) norm of vector $\mathbf{a}$
$\ \mathbf{A}\ _F$	Frobenius norm of matrix $\mathbf{A}$
$x \rightarrow a^+$	$x$ approaches to $a$ from the right side
$A_{-i}$	The other element, in a bi-element set $\mathcal{A} = \{A_1, A_2\}$ $A_{-i} \triangleq \mathcal{A} \setminus \{A_i\}$ , i.e., $A_{-1} = A_2$ and $A_{-2} = A_1$
$(\mathbf{A})_{i,j}$	The $ij$ th element of matrix $\mathbf{A}$
$\text{MAT}[\{\mathbf{A}_i\}_{i=1}^n]$	Stacking of $n$ matrices $\mathbf{A}_1, \dots, \mathbf{A}_n$
$\text{diag}[\{\mathbf{A}_i\}_{i=1}^n]$	Block diagonal matrix with $\mathbf{A}_1$ to $\mathbf{A}_n$ as its diagonal elements
$\mathcal{CN}(\boldsymbol{\mu}, \boldsymbol{\sigma}^2)$	Gaussian random variables with mean $\boldsymbol{\mu}$ and variance $\boldsymbol{\sigma}^2$
$\mathbb{E}_x[f(\cdot)]$	Mathematical expectation of $f(\cdot)$ relative to $\mathbf{x}$
$\nabla_{\mathbf{a}} f(\cdot, \mathbf{a})$	The derivative of function $f$ relative to $\mathbf{a}$
$\text{dom} f$	Domain of function $f$

# Chapter 1

## Introduction

Unlike the single antenna transmission in which the radiation pattern of the antenna is fixed, the multiple-antenna (a.k.a. antenna array or smart antenna) transmission is beneficial to experience higher capacity, higher reliability and space diversity through a process called “beamforming” [1]. Beamforming is the general signal processing technique employed in an antenna array to directionally transmit (Tx) or receive (Rx) data over the communication channel. To do so, beamformer controls the phase and relative amplitude of the stimulating/induced signal of each individual transmit/receive element in the antenna array. To be able to implement the beamforming, at least one party of a signal transmission process should be equipped with an antenna array. Beamforming techniques can be divided into two major categories: conventional beamforming techniques and adaptive beamforming ones. In the former techniques, a set of fixed weights (including phase shifts) is applied to the antenna array to steer the signal towards a known direction or better receive the signal from a priori-known direction, while in the latter techniques, this information (direction of arrival/departure of signals) is combined with the properties of the actual signal received (to be transmitted), typically to boost the desired signals and reject the unwanted signals. It is noteworthy that although in this thesis beamforming is a subject applied to adaptively process the wireless communication physical signals in cognitive radio and cooperative wireless networks, it is also applicable to both radio and sound waves, and is of great importance in other fields like radar, sonar, seismology, radio astronomy, speech, acoustics and biomedicine. In the subsequent sections of this chapter, a

brief review of the cognitive radio and cooperative wireless networks in which the beamforming process is implemented, is given. Since beamforming process conventionally relies on the Channel State Information (CSI), perfect CSI transceiver design and imperfect CSI beamforming, as well as the CSI uncertainty models are studied subsequently.

## 1.1 Cognitive Radio Networks

The policy of fixed electromagnetic frequency spectrum assignment to different radio communication services has led to the problem of spectrum scarcity in which the reachable spectrum is mostly pre-allocated but is not in frequent use. The Cognitive Radio Network (CR-Net) concept is an intelligent solution to deal with this problem [2]. CR-Nets are mainly categorized as opportunistic and concurrent cognitive networks. In opportunistic CR-Nets (a.k.a. spectrum sensing CR-Nets) the cognitive (secondary) users (SUs) sense the spectrum and then use it whilst that frequency hole is vacant and evacuate the frequency spectrum as soon as they sense that a licensed primary user (PU) is populated. Unlike the opportunistic CR-Net, in the concurrent CR-Net, SUs utilize the same spectrum as do PUs, provided that the amount of interference imposed on the PUs is below a certain threshold. Since each of the PUs and SUs may be equipped with an antenna array, beamforming is also applicable to the CR-Net setup of a communications system. The CR-Net itself may act in different configurations widely known as Broadcast (BC), Multiple-Access (MAC) and Ad Hoc (interfering) networks. In the BC configuration, a SU-Tx (probably a Base Station (BS)) transmits independent data streams towards its intended users while performs the beamforming process to impose little interference on PU-Rxs. In the MAC configuration, a set of independent users transmit independent data streams to a single BS while performing the beamforming process to impose little interference on each other as well as the PU-Rxs. In the Ad Hoc network configuration, there are several independent communicating pairs, who transmit towards their unique destination while protecting at least PU-Rxs.

A CR-Net is actually a series of software defined radio entities which adapt their performance to the physical context and show some kind of “cognitive” behavior. The CR-Net acts based on a three step cognitive cycle.

- Sense Step (spectrum sensing, cognitive pilot channels,...)
- Decide Step (plan, learn, orient, and predict,...)
- Act Step (allocate, code, modulate, transmit,...)

There are three CR-Net paradigms:

1. In an Interweave paradigm, the CR-Net senses the spectrum to find spectrum holes and uses them whilst they are not occupied by a PU. The SU leaves these holes and looks for new ones as soon as it senses a PU starting to use the spectrum. This may disrupt the transmission process of the SU.
2. In the Underlay paradigm, the CR-Net uses the spectrum with this requirement that it will not interfere severely with the actual transmission of the primary radio network. In this paradigm, no cooperation is assumed between the SU's and PU's.
3. In the Overlay paradigm, which is very similar to underlay paradigm, there is cooperation between the primary and secondary networks.

## 1.2 Cooperative Networks

Fading has a destructive effect on the quality of a typical transmission process. Sometimes the destination of a communications system may experience a deep fade resulting in the abruption of the communications process. It is also possible that the destination of the communications system is in a distant place which is not located in the coverage range of the transmitter. To deal with the aforementioned issues, the cooperative (a.k.a. relay) network concepts are introduced [3]. In the relay networks, a separate node sits in between of the source and the destination of the communications, to facilitate the communication between either

ends. Based on the nature of the processing of the relay node, relay networks are divided into three major categories: Compress and forward (CF), Amplify and forward (AF) and Decode and forward (DF). In these categories different levels of processing in the relay node is assumed. In AF relay node, there is no further processing. The relay node receives the noise contaminated and fading degraded version of the transmitted information, simply amplifies it and retransmits it to the destination. The AF relays are the simplest among other types of the relays. On the other hand, the quality of signal and noise are simultaneously affected leading to no improvement in received signal quality. In DF and CF relays, the relay node first decodes the transmitted signal and then independently transmits it to the destination. In these relays the quality of the signal is usually improved in the system. In CF relays, the transmitted signal of the relay is re-encoded to improve the spectral efficiency of the system as well. The relay network itself may operate in different configurations, for example, for a half-duplex communications, a One-Way Relay Channel (OWRC) and for a full-duplex communications, a Two-Way Relay Channel (TWRC) are required, respectively. There is also a more general configuration of the relay networks that cover both OWRC and TWRC, i.e., Gaussian Interference Relay Channel (GIFRC).

### **1.3 Uncertainty models and Imperfect-CSI Transceiver Design**

In the design procedure of a communications system, channel gains play an important role. Conventionally it is assumed that this information (Channel State Information, a.k.a. CSI) is completely known at the transmit and receive sides. Mostly it is assumed that the receive side may obtain this knowledge true pilot transmission process in which a set of certainly known data is transmitted towards the receiver. As the receiver knows both the transmitted and the received data, it can determine (estimate) the channel gain coefficients. Also, it is assumed that there is a perfect feedback channel free of impairments, between the source and the destination of the communication link through which, it is possible to send

back the CSI to the transmitter side. Since the beamformer design process relies on this CSI, it is vital to have CSI perfectly. Unfortunately, due to the erroneous channel gain estimation, limited feedback rate between the source and the destination, and rapidly changing environments, this assumption is not a realistic one, and hence, the CSI is uncertain. Robust beamforming is a methodological solution to treat the uncertainty of the CSI.

Nowadays, uncertain (imperfectly known) CSI is modeled using the following notation. If  $\tilde{\mathbf{H}}$  is to represent the real CSI, uncertain CSI, i.e.  $\mathbf{H}$ , is modeled as

$$\mathbf{H} = \tilde{\mathbf{H}} + \mathbf{\Delta} \quad (1.1)$$

where  $\mathbf{\Delta}$  shows the additive uncertainty of the CSI. To characterize the uncertainty, two models are mainly used: Stochastic Error (SE) model and Norm Bounded Error (NBE) model. In the SE model, the first and the second statistics of the uncertainty are assumed to be known and fixed, and in this case, the average performance measures of the systems are considered in the design process, e.g.,

$$\mathbb{E}_{\mathbf{\Delta}} [\mathbf{\Delta}] = \mathbf{0}, \quad (1.2a)$$

$$\mathbb{E}_{\mathbf{\Delta}} [\text{vec} [\mathbf{\Delta}]^* \text{vec} [\mathbf{\Delta}]] = \delta, \quad (1.2b)$$

where  $\delta$  is a known constant. In the NBE model, however, which is a deterministic model, it is assumed that

$$\|\mathbf{\Delta}\|_F \leq \delta, \quad (1.3)$$

where  $\delta$  is another constant. When the NBE model describes the uncertainty of the system, the performance measures of the system are optimized using a worst-case design concept, in which these performance measures are guaranteed not to fail when the least favorable CSI to the system occurs. In the subsequent chapters these concepts are revealed in more details. It should be clarified that currently most of the papers focusing on the robust design of the networks, use the SE framework to describe the uncertainty and our contribution comes in using the NBE model. We have mentioned the SE framework here to summarize the current

state of the art in this field and to also show that, the methods treating these two different models share a lot of in terms of the basic tools used, and will result in similar trends and behaviour.

## 1.4 Related Works

Robust beamforming has a long story which goes back to 1970s with pioneering works of Frost and Abramovich [4, 5]. In the first days of the research on robust beamforming, the scholars adopted ad hoc methods to impose the robustness to the beamforming process. For example, in [4], the author used additional points or derivative constraints to secure a priori-desired main beam area which requires too many degrees of freedom for the beamforming problem and reduces the applicability of this method. In [5], in which the author aimed to design a Minimum Variance Distortionless Response (MVDR), a penalty term was added to the objective function resulting to a diagonal loading of the sample covariance matrix. This method, however, does not promote a rigorous way of choosing the diagonal loading scale factor in which its optimal value is scenario dependent. In this line, a systematic overview of limited feedback in wireless communication systems is provided in [6].

The modern treatment of the robust beamforming started with the seminal works of Bengtson and Ottersten [7, 8]. In these works, the authors used the worst-case design approach to guarantee the performance of the beamformer even when the least favorite channel realizations are occurring. The authors recast the original formulation to be in a semidefinite optimization form and then relaxed it to be convex. It is noteworthy that this way of treatment had a great impact for upcoming research in this area, as the semidefinite relaxation is an important tool, and is adopted in many beamforming research papers afterwards. A similar approach was employed in [9] where the authors showed that there is a tight relation between the worst-case design procedure and the diagonal loading, in which the optimal value of the diagonal loading factor is computed based on the known levels of the uncertainty in the steering vector. Robust Capon beamforming was



the focus of [10, 11, 12]. Capon beamformer [13] is known to be benefited from a better resolution and a much better interference rejection capability relative to the standard data-independent beamformers, provided that the Direction of Interest (DOI) steering vector is correctly and perfectly known. However, steering vector impairment due to uncertainty might degrade the performance of the Capon beamformer to become even worse than the standard ones. To settle this issue a robust counterpart was proposed in [10] and its relation to diagonal loading was clarified in [11]. A doubly constrained Capon beamformer was also proposed in [12] to cover both constant norm constraints as well as spherical uncertainty set constraints. The Robust Minimum Variance Beamforming (RMVB) problem was studied in [14]. In this paper an ellipsoid based uncertainty set was employed to characterize the uncertainty. Lagrange multiplier method was used to solve the beamforming problem and it was also shown that when the uncertainty set is a singleton, the performance of RMVB is similar to the Capon beamformer. A framework for designing Multiple-Input Multiple-Output (MIMO) Point to Point (P2P) systems with nonlinearities, i.e., decision feedback equalization and Tomlinson-Harashima Precoding (THP) schemes was studied in [15]. A similar approach, but for linear precoding and decoding was described in [16]-[24]. In the former paper, a minimum error rate and an average Bit Error Rate (BER) were selected as the objective of the design procedure.

Unlike the single-user (P2P) communications reviewed above, Multiuser (MU) communications has attracted many researchers. Multiuser communications research works can be divided into two categories: Multiuser Multiple-Input Single-Output (MU-MISO), when only one party in the transmission process is equipped with a single antenna and the other party has multiple antennas; and MU-MIMO, where both transmit and receive sides come with multiple antennas. In [25] the robust design of a MU-MISO MAC was dealt, while in [26]-[41] the linear and nonlinear precoding schemes for downlink (BC) was treated. In [26, 27] the authors exploited the nonlinear precoding schemes while in [28]-[41] the linear precoding configurations were used. In this line, [42] also focused on the SINR balancing

problem.

MU-MIMO MAC was the focus of [43]-[49]. In the former research ([43]) a sum Mean Square Error (MSE) objective function was used to design the MAC while in the latter one a probabilistically constrained MVDR approach was employed. In [50, 51] a robust THP-based nonlinear treatment for downlink beamforming was presented. The DFE treatment of a MIMO-MMSE based system was also studied in [52]. While the information theoretic aspects of the capacity of MIMO-BC with partial side information was studied in [53], the robust beamforming of the MU-MIMO BC was studied extensively in [54]-[67]. In these works both SE model and NBE model were considered to model the channel uncertainty. In case of SE model, the mathematical expectation of the objective function and the constraints were optimized while in the presence of the NBE model, the worst case design procedure was employed. It is noteworthy that although the original problem formulation based on Signal to Interference plus Noise Ratio (SINR) or MSE is a Second Order Cone Programming (SOCP) problem, the robust version is a Semidefinite Programming (SDP) problem. Since the robust version is usually in the Linear Matrix Inequality (LMI) form, it is possible to resort to efficient interior point methods to solve these problems numerically. Just recently in [68] the authors investigated that the robust counterpart of a SOCP is again another SOCP with more constraints and variables, which makes this finding a milestone in the robust optimization framework. Although the complexity of the original SOCP is increased in this new framework, it is expected that the complexity of a sparse SOCP, though a larger problem, is smaller than the SDP counterpart. Unfortunately, although this framework is mathematically appealing, no research in this area exploited this new framework. It is necessary to resort to this new framework and benchmark it against the conventional S-Procedure SDP-based methods. In this line, it is noteworthy that the cognitive radio setup, for both MIMO and MISO configurations in downlink, is extensively studied by the authors of [69]-[76].

In [77]-[82] the MIMO ad hoc networks were studied. In [81] a game theo-

retic approach was used to study the cognitive radio configuration of an ad hoc network while in [82] a maximum sum-rate capacity objective was used to design the cognitive radio configuration of an ad hoc network. Although both conventional and cognitive radio configuration were studied, these works lack to cover the robust design of the linear and nonlinear precoding/decoding schemes. It is also necessary to study the cooperative transceiver optimization in MIMO ad hoc networks.

The problem of non-regenerative MIMO relay design with/without a direct link is introduced in [83, 84] where the authors optimize the capacity or the SNR between the source and the destination. Both of the optimum canonical coordinates of the relay matrix and the upper and the lower bounds of the optimal system capacity are discussed respectively. The problem of joint MIMO beamforming and power adaptation at source and relay with Quality of Service (QoS) constraints is discussed in [85, 86]. In these papers, a broadcasting relay scheme with a MIMO architecture in a source-to-relay link is studied. Both receive and transmit beamforming is accomplished in the relay station. As a result, optimum beamforming weights and power adaptation are calculated. The linear relay beamformer for a MIMO relay broadcast channel with limited feedback with zero forcing and minimum mean square error measures is described in [87]. There it is concluded that only Channel Direction Information (CDI) feedback is sufficient to determine the beamforming vector in the proposed scheme which highly reduces the amount of the required feedback. A similar treatment is also covered in [88] for the Single-Input Multiple-Output (SIMO)/MISO case with only one set of CSI imperfectly known. In this paper both systems are designed iteratively. The iterative design of the linear relay precoder and the destination equalizer is considered in [89]. The joint power and time allocation for multi-hop relay networks are addressed in [90, 91] based on the SOCP and SDP problems, respectively [92, 93]. In [94] the problem of collaborative uplink transmit beamforming with robustness against channel estimation errors for a DF relay is addressed. The CSI has a stochastic error model and the proposed algorithms can be applied to both line of

sight propagation and fading channels.

Optimal beamforming for a MIMO TWRC with analogue network coding is addressed in [96] where only the relay station is equipped with multiple antennas. The optimum relay beamforming structure as well as an efficient algorithm to compute the optimal beamforming matrix are proposed in [96]. The effects of transmit CSI at a DF-based MIMO TWRC with two different re-encoding processes, namely superposition coding and bitwise XOR operation is studied in [97]. The upper bound of the achievable sum-rate capacity of an AF-based MIMO TWRC and the relay beamforming matrix design are dealt in [98]. It is assumed that the relay station has perfect CSI in this study. Optimal distributed beamforming for TWRC, for three different relaying schemes is the center of [99]. To solve this problem they exploit two different approaches, a Signal to Noise Ratio (SNR) balancing approach, as well as a total power minimization approach. In each approach they provide different properties of the beamforming weights. Multiuser two-way AF relay processing and power control methods for the beamforming systems are studied in [100]. The relay is optimized based on both zero-forcing and Minimum Mean-Square-Error (MMSE) criteria under relay power constraints, and various transmit and receive beamforming methods, for example, eigen beamforming, antenna selection, random beamforming, and modified equal gain beamforming are examined. In [101]-[103], the optimum resource allocation for a two-way relay-assisted Orthogonal Frequency Domain Multiple Access (OFDMA) is explained. A new transmission protocol, named hierarchical OFDMA, is proposed to support two-way communications between the base station (BS) and each mobile user with or without an assisting relay station (RS). An iterative receiver for a MIMO TWRC is presented in [104]. A MMSE-based iterative soft interference cancellation (SIC) unit and an Expectation Conditional Maximization (ECM)-based estimation algorithm is used to build the receiver. The training-based channel estimation under the AF relay scheme is studied in [105] and a two-phase training protocol for channel estimation is proposed. The uncertainty of the CSI is modeled using both stochastic and deterministic models.

## 1.5 Motivation and Objectives

Research gaps for the current study of robust beamforming for cognitive and cooperative wireless networks are summarized as follows:

- In current studies mostly one model of uncertainty is central to the undergone research. Either the stochastic or the deterministic error model is targeted by the researchers. As mentioned before, there are two different types of uncertainties that require distinct treatments. Based on the nature and the amount of information given for CSI, either one of these models is selected.
- In the study of the robust beamformer for MISO BC, researchers mostly use SINR as an objective and to maximize SINR, they resort to approximate solutions while an exact solution is preferred.
- Although the ad hoc networks may be the prominent configuration of wireless networks in future, there is no specific study of the robust transceiver optimization in these networks.
- Relaying is an important concept to extend the coverage area of a telecommunication system. But a unified study of the robust beamforming in both one-way and two-way relay channel is of great importance.

The main purpose of this study was to propose algorithms to robustly design the transceivers in different configurations of cognitive and cooperative wireless networks, specifically:

- Except for the MISO BC cognitive beamforming design, we use both SE and NBE models to model the uncertainty of the CSI and subsequently, and we propose two different algorithms to optimize the transceivers.
- In the MISO BC cognitive beamforming, we propose an exact solution that maximizes the SINR.

- We propose both linear and nonlinear transceiver optimization for MIMO ad hoc networks.
- We propose algorithms to robustly design the beamformer of the one-way and two-way relay channels.

It is understood that the current treatments of the uncertainty are conservative, but we are aware that sometimes this conservative methods are the only way that may be used to guarantee the performance of the system.

## 1.6 Thesis Structure

This thesis is composed of seven chapters. In current chapter, which is Chapter 1, a general overview about cognitive and cooperative wireless networks, uncertainty and robust design, as well as related works and list of publications are summarized.

In Chapter 2, a general and introductory presentation of mathematical preliminaries is presented. The content of this chapter reviews the basic materials from linear algebra and convex optimization. This chapter is included to have this thesis self contained, but it is not intended to be comprehensive. For more details of the reviewed content, classical texts are cited as well.

In Chapter 3, the problem of cognitive robust beamforming in a multi-user MISO broadcast channel is presented. It is assumed that the uncertainty of CSI is modeled using a ball-shaped uncertainty set. The original problem formulation is a NP-hard problem, and therefore a SDP-relaxed version is solved instead. In this chapter, unlike the other studies that aim to approximate the maximization of SINR, an exact solution is provided. In Chapter 4, a multi-user MIMO ad hoc (interfering) network is studied. Both the robust linear and nonlinear joint optimization of precoder and equalizers are presented. In this chapter and subsequent chapters the uncertainty is modeled using both SE and NBE models. It is shown that using SE model, the MU-MIMO ad hoc network transceiver design is a SOCP while using NBE model, this problem is a SDP.

The problems of robust beamforming in a half-duplex MIMO one-way relay

channel and a full-duplex MIMO two-way relay channel is described in Chapters 5 and 6. In these problems as well, it is shown that for the SE model of uncertainty the robust beamformer design is a SOCP and for the NBE model of uncertainty, the same problem would be a SDP. Finally Chapter 7 concludes this thesis and proposes few ways to extend these studies.

## 1.7 List of Publications

In this section the list of publications are summarized. The content of Chapter 3 covers the following two papers:

1. **Ebrahim. A. Gharavol**, Liang Ying Chang, and Koen Mouthaan, “Robust downlink beamforming in multiuser MISO cognitive radio networks,” *Proc. IEEE Int. Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC09)*, pp. 808 - 812, Sep. 2009.
2. **Ebrahim. A. Gharavol**, Liang Ying Chang, and Koen Mouthaan, “Robust downlink beamforming in multiuser MISO cognitive radio networks with imperfect channel-state information,” *IEEE Trans. Vehicular Technology*, vol. 59, no. 6, pp. 2852 - 2860, Jul. 2010.

The content of Chapter 4 contain the following four papers:

1. **Ebrahim A. Gharavol**, Liang Ying Chang, and Koen Mouthaan, “Robust linear transceiver design in MIMO ad hoc cognitive radio networks,” *Proc. IEEE Vehicular Technology Con. (VTC10-Spring)*, pp. 1 - 5, 16-19 May, 2010.
2. **Ebrahim. A. Gharavol**, Liang Ying Chang, and Koen Mouthaan, “Robust cooperative nonlinear transceiver design in multi-party MIMO cognitive radio networks with stochastic channel uncertainty,” *Proc. IEEE Vehicular Technology Conf. (VTC10-Fall)*, pp. 1 - 6, Sep. 2010.
3. **Ebrahim. A. Gharavol**, Liang Ying Chang, and Koen Mouthaan, “Collaborative nonlinear transceiver optimization in multi-tier MIMO cognitive

radio networks with deterministically imperfect CSI,” *Accepted to be published in Proc. IEEE Global Communication Conf. (GLOBECOM10)*, 6-10 Dec., 2010.

4. **Ebrahim. A. Gharavol**, Liang Ying Chang, and Koen Mouthaan, “Robust linear transceiver design in MIMO ad hoc cognitive radio networks with imperfect channel state information,” *Accepted to be published in IEEE Trans. Wireless Communications*, Nov., 2010.

The content of Chapter 5 is drawn from the following two papers:

1. **Ebrahim. A. Gharavol**, Liang Ying Chang, and Koen Mouthaan, “Robust linear beamforming for MIMO relay with imperfect channel state,” *Proc. IEEE Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC10)*, 26-30 Sep. 2010.



# Chapter 2

## Mathematical Preliminaries

In this chapter few identities, theorems and lemmas that are frequently used in the subsequent chapters are summarized. This chapter is included to make this thesis self-contained. For more information on the details and proofs of the theorems and lemmas, the reader should consult the [106, 107, 108, 109] for linear algebra and [110, 111, 112] for robust and convex optimization.

### 2.1 Linear Algebra

**Lemma 2.1.** *For any vector  $\mathbf{x}$  and matrix  $\mathbf{A}$ , we have the following identity:*

$$\mathbf{x}^* \mathbf{A} \mathbf{x} = \text{tr} [\mathbf{x}^* \mathbf{A} \mathbf{x}] \quad (2.1)$$

$$= \text{tr} [\mathbf{A} \mathbf{x} \mathbf{x}^*]. \quad (2.2)$$

*Proof.* Please see [106]. □

The following identities are also frequently used in case of encountering with matrix norms:

**Lemma 2.2.** *For any matrix  $\mathbf{A}$  we have*

$$\|\mathbf{A}\|_F^2 = \text{tr} [\mathbf{A}^* \mathbf{A}] \quad (2.3a)$$

$$\|\mathbf{A}\|_F^2 = \|\text{vec} [\mathbf{A}]\|_2^2. \quad (2.3b)$$

*Proof.* Please see [109]. □

The following identity, helps a lot when using the vectorized version of a matrix:

**Lemma 2.3.** *For any three compatible matrices*

$$\text{vec}[\mathbf{ABC}] = (\mathbf{C}^T \otimes \mathbf{A})\text{vec}[\mathbf{B}]. \quad (2.4)$$

*Proof.* Please see [108].  $\square$

An important special case of the above identity occurs when either of  $\mathbf{A}$  or  $\mathbf{C}$  is the identity matrix  $\mathbf{I}$ . It is also possible to aggregate the sum-of-norms expressions in terms of the norm of a single vector.

**Lemma 2.4.** *For all  $\alpha, \beta, \theta, \phi \in \mathbb{R}$  and  $\mathbf{a} \in \mathbb{C}^\alpha, \mathbf{b} \in \mathbb{C}^\beta$ ,*

$$\theta \|\mathbf{a}\|_2^2 + \phi \|\mathbf{b}\|_2^2 = \left\| \begin{bmatrix} \sqrt{\theta} \mathbf{a} \\ \sqrt{\phi} \mathbf{b} \end{bmatrix} \right\|_2^2. \quad (2.5)$$

*Proof.* Please see [110].  $\square$

**Lemma 2.5.** *Assume  $\mathbf{x}$  is a random variable with the statistics of  $\mathbb{E}_x[\mathbf{x}] = \boldsymbol{\mu}$  and  $\text{Var}\{\mathbf{x}\} = \boldsymbol{\Sigma}$  and also assume that  $\mathbf{A}$  is a symmetric matrix, then*

$$\mathbb{E}_x[\mathbf{x}^* \mathbf{A} \mathbf{x}] = \text{tr}[\mathbf{A} \boldsymbol{\Sigma}] + \boldsymbol{\mu}^* \mathbf{A} \boldsymbol{\mu}. \quad (2.6)$$

*Proof.* Please refer to [113].  $\square$

Schur Complement Lemma which is one of the most important lemmas that is used in this thesis, is stated in the following lemma.

**Lemma 2.6. [Schur Complement Lemma]** *Let  $\mathbf{Q}$  and  $\mathbf{R}$  be symmetric matrices. Then the following two expressions are equivalent.*

$$\begin{cases} \mathbf{R} \succeq 0, \\ \mathbf{Q} - \mathbf{S}^* \mathbf{R}^{-1} \mathbf{S} \succeq 0, \end{cases} \quad (2.7a)$$

$$\begin{bmatrix} \mathbf{Q} & \mathbf{S} \\ \mathbf{S}^* & \mathbf{R} \end{bmatrix} \succeq 0. \quad (2.7b)$$

*Proof.* Please refer to [113].  $\square$

Especially when  $\mathbf{Q}$  is a scalar,  $\mathbf{S}$  is a vector and  $\mathbf{R}$  is equal to the identity matrix  $\mathbf{I}$ , (2.7a) is to express a second-norm constraint in terms of an LMI like (2.7b).

**Lemma 2.7. [S-Procedure]** Let  $\mathbf{F}_0, \dots, \mathbf{F}_N$  be quadratic functions of the variable  $\boldsymbol{\zeta} \in \mathbb{R}^n$ :  $\mathbf{F}_i(\boldsymbol{\zeta}) \triangleq \boldsymbol{\zeta}^T \mathbf{T}_i \boldsymbol{\zeta} + 2\mathbf{u}_i^T \boldsymbol{\zeta} + v_i, i = 0, \dots, N$  where  $\mathbf{T}_i = \mathbf{T}_i^T$ . The following condition  $\mathbf{F}_0(\boldsymbol{\zeta}) \geq \mathbf{F}_i(\boldsymbol{\zeta}), \forall i = 1, \dots, N$  holds if and only if there exists  $N$  positive reals  $\tau_1 \geq 0, \dots, \tau_N \geq 0$  such that  $\mathbf{F}_0(\boldsymbol{\zeta}) - \sum_{i=1}^N \tau_i \mathbf{F}_i(\boldsymbol{\zeta}) \succeq 0$ . This last expression is simple a LMI:

$$\begin{bmatrix} \mathbf{T}_0 & \mathbf{u}_0 \\ \mathbf{u}_0^T & v_0 \end{bmatrix} - \sum_{i=1}^N \tau_i \begin{bmatrix} \mathbf{T}_i & \mathbf{u}_i \\ \mathbf{u}_i^T & v_i \end{bmatrix} \succeq 0.$$

*Proof.* Please see [114]. □

**Lemma 2.8. [“Nemirovski” Lemma]** Given matrices  $\mathbf{P}, \mathbf{Q}, \mathbf{A}$  with  $\mathbf{A} = \mathbf{A}^*$ , the semi-infinite LMI of the form of

$$\mathbf{A} \succeq \mathbf{P}^* \mathbf{X} \mathbf{Q} + \mathbf{Q}^* \mathbf{X}^* \mathbf{P}, \quad \forall \mathbf{X} : \|\mathbf{X}\| \leq \rho,$$

holds if and only if  $\exists \epsilon \geq 0$  such that

$$\begin{bmatrix} \mathbf{A} - \epsilon \mathbf{Q}^* \mathbf{Q} & -\rho \mathbf{P}^* \\ -\rho \mathbf{P} & \epsilon \mathbf{I} \end{bmatrix} \succeq 0.$$

*Proof.* Please See [115]. □

For real valued matrices, the aforementioned lemma is called Petersen’s Lemma on matrix uncertainty [116] and is generalized in [117]. This lemma is handy when there is only one uncertain matrix. In case of having multiple uncertainty sources, it is possible to extend this lemma as follows:

**Theorem 2.1. [Generalized “Nemirovski” Lemma]** Given matrices  $\{\mathbf{P}_i, \mathbf{Q}_i\}_{i=1}^N$  with  $\mathbf{A} = \mathbf{A}^*$ , the semi-infinite LMI of the form of

$$\mathbf{A} \succeq \sum_{i=1}^N (\mathbf{P}_i^* \mathbf{X}_i \mathbf{Q}_i + \mathbf{Q}_i^* \mathbf{X}_i^* \mathbf{P}_i), \quad \forall \mathbf{X}_i : \|\mathbf{X}_i\| \leq \varkappa_i, \quad i = 1, \dots, N; \quad (2.8)$$

holds if and only if  $\exists \epsilon_1, \dots, \epsilon_N \geq 0$  such that

$$\begin{bmatrix} \mathbf{A} - \sum_{i=1}^N \epsilon_i \mathbf{Q}_i^* \mathbf{Q}_i & \text{mat} \left[ \{-\varkappa_i \mathbf{P}_i\}_{i=1}^N \right]^* \\ \text{mat} \left[ \{-\varkappa_i \mathbf{P}_i\}_{i=1}^N \right] & \text{diag} \left[ \{\epsilon_i \mathbf{I}\}_{i=1}^N \right] \end{bmatrix} \succeq 0. \quad (2.9)$$

*Proof.* It is known that

$$\mathbf{A} \succeq \sum_{i=1}^N (\mathbf{P}_i^* \mathbf{X}_i \mathbf{Q}_i + \mathbf{Q}_i^* \mathbf{X}_i^* \mathbf{P}_i), \quad \forall \mathbf{X}_i : \|\mathbf{X}_i\| \leq \varkappa_i, \quad i = 1, \dots, N; \quad (2.10)$$

holds if and only if for every  $\mathbf{x}$

$$\mathbf{x}^* \mathbf{A} \mathbf{x} \geq \max_{\{\mathbf{c}_i\}_{i=1}^N} \sum_{i=1}^N (\mathbf{x}^* \mathbf{P}_i^* \mathbf{X}_i \mathbf{Q}_i \mathbf{x} + \mathbf{x}^* \mathbf{Q}_i^* \mathbf{X}_i^* \mathbf{P}_i \mathbf{x}) \quad (2.11)$$

$$= 2 \sum_{i=1}^N \kappa_i \|\mathbf{P}_i \mathbf{x}\| \|\mathbf{Q}_i \mathbf{x}\| \quad (2.12)$$

where

$$\mathfrak{C}_i = \{\forall \mathbf{X}_i : \|\mathbf{X}_i\| \leq \kappa_i\}, \quad i = 1, \dots, N. \quad (2.13)$$

Using the Cauchy-Schwarz inequality the above equation can be expressed as

$$\mathbf{x}^* \mathbf{A} \mathbf{x} - 2 \sum_{i=1}^N \kappa_i \Re\{\mathbf{y}_i^* \mathbf{P}_i \mathbf{x}\} \geq 0, \quad \forall \mathbf{x}, \mathbf{y}_i : \|\mathbf{y}_i\| \leq \|\mathbf{Q}_i \mathbf{x}\|, \quad i = 1, \dots, N. \quad (2.14)$$

Since  $\|\mathbf{y}_i\| \leq \|\mathbf{Q}_i \mathbf{x}\|$  is equivalent to  $\mathbf{x} \mathbf{Q}_i^* \mathbf{Q}_i \mathbf{x} - \mathbf{y}_i^* \mathbf{y}_i \geq 0$ , it is possible to express it in terms of a quadratic expression. By choosing  $\mathbf{z} = [\mathbf{x}^T, \mathbf{y}_1^T, \dots, \mathbf{y}_N^T]^T$ , it is possible to write the above quadratic expression as the  $\mathbf{z}^* \mathbf{M}_i \mathbf{z} \geq 0$  where  $\mathbf{M}_i$  is a block partitioned matrix, i.e.,  $\mathbf{M}_i \triangleq [\mathbf{M}^{[i]}]_{j,k} (N+1) \times (N+1)$  where

$$(\mathbf{M}^{[i]})_{j,k} = \begin{cases} \mathbf{Q}_i^* \mathbf{Q}_i & j = k = 1, \\ -\mathbf{I} & j = k = i, \\ \mathbf{0} & \text{otherwise.} \end{cases} \quad (2.15)$$

Based on this notation it is possible to write the result of the implication (2.14) as another quadratic form, i.e.,

$$\mathbf{x}^* \mathbf{A} \mathbf{x} - 2 \sum_{i=1}^N \kappa_i \Re\{\mathbf{y}_i^* \mathbf{P}_i \mathbf{x}\} \geq 0 \quad \equiv \quad \mathbf{z}^* \mathbf{M}_0 \mathbf{z} \geq 0 \quad (2.16)$$

where  $\mathbf{M}_0$  is a block partitioned matrix as well,

$$\mathbf{M}^{[0]} = \begin{bmatrix} \mathbf{A} & \text{MAT} [\{-\kappa_i \mathbf{P}_i\}_{i=1}^N]^* \\ \text{MAT} [\{-\kappa_i \mathbf{P}_i\}_{i=1}^N] & \mathbf{0} \end{bmatrix} \quad (2.17)$$

Based on these notations, it is possible to reformulate (2.14) as follows:

$$\mathbf{z}^* \mathbf{M}_i \mathbf{z} \geq 0, \quad i = 1, \dots, N \quad \Rightarrow \quad \mathbf{z}^* \mathbf{M}_0 \mathbf{z} \geq 0 \quad (2.18)$$

Using the general form of S-procedure for quadratic functions and non-strict inequalities which is summarized in the following lemma, (2.18) holds if there exists  $\epsilon_1, \dots, \epsilon_N \geq 0$  such that

$$\begin{bmatrix} \mathbf{A} - \sum_{i=1}^N \epsilon_i \mathbf{Q}_i^* \mathbf{Q}_i & \text{mat} [\{-\kappa_i \mathbf{P}_i\}_{i=1}^N]^* \\ \text{mat} [\{-\kappa_i \mathbf{P}_i\}_{i=1}^N] & \text{diag} [\{\epsilon_i \mathbf{I}\}_{i=1}^N] \end{bmatrix} \succeq \mathbf{0} \quad (2.19)$$

which completes the proof.  $\square$

## 2.2 Convex and Robust Optimization

The fundamental concepts of convex sets and convex functions are central to the convex, biconvex and robust optimization theories. A set  $C$  is convex if the line segment between any two points in  $C$  lies in  $C$ , i.e., for any  $\mathbf{x}_1, \mathbf{x}_2 \in C$  and any  $\theta \in \mathbb{R}$  with  $0 \leq \theta \leq 1$  we have

$$\theta \mathbf{x}_1 + (1 - \theta) \mathbf{x}_2 \in C. \quad (2.20)$$

A set  $B \subseteq X \times Y$  is said to be biconvex, if both  $x$ - and  $y$ -sections of it, which are defined below, are themselves convex sets. The  $x$ - and  $y$ -sections of  $B$  which are denoted using  $B_x$  and  $B_y$ , respectively, are defined as follows:

$$B_x = \{y \in Y : (x, y) \in B\}, \quad (2.21a)$$

$$B_y = \{x \in X : (x, y) \in B\}. \quad (2.21b)$$

A function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is convex if  $\text{dom} f$  is a convex set and if for all  $\mathbf{x}, \mathbf{y} \in \text{dom} f$ , and  $\theta \in \mathbb{R}$  with  $0 \leq \theta \leq 1$ , we have

$$f(\theta \mathbf{x} + (1 - \theta) \mathbf{y}) \leq \theta f(\mathbf{x}) + (1 - \theta) f(\mathbf{y}). \quad (2.22)$$

A concave function  $f : \mathbb{R}^n \rightarrow \mathbb{R}$  is a function such that  $-f$  is convex.

A function  $f : B \rightarrow \mathbb{R}$  over a biconvex set  $B$  is called biconvex if

$$f_x(\cdot) \triangleq f(\mathbf{x}, \cdot) : B_x \rightarrow \mathbb{R} \quad (2.23)$$

is a convex function over  $B_x$  for every fixed  $\mathbf{x} \in X$  and

$$f_y(\cdot) \triangleq f(\cdot, \mathbf{y}) : B_y \rightarrow \mathbb{R} \quad (2.24)$$

is a convex function over  $B_y$  for every fixed  $\mathbf{y} \in Y$ .

### 2.2.1 Convex Optimization

The generic standard form of an optimization problem is as follows:

$$\underset{\mathbf{x}}{\text{minimize}} \quad f_0(\mathbf{x}) \quad (2.25a)$$

$$\text{subject to} \quad f_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m; \quad (2.25b)$$

$$h_i(\mathbf{x}) = 0, \quad i = 1, \dots, p. \quad (2.25c)$$

This notation is to describe a problem which tries to find the minimum value of the objective function  $f_0(\mathbf{x})$  subject to  $m$  and  $p$  inequality and equality constraint, respectively. Domain of this problem is the set of all points for which the objective and constraint functions are defined:

$$\mathcal{D} = \bigcap_{i=0}^m \text{dom} f_i(\mathbf{x}) \cap \bigcap_{i=1}^p \text{dom} h_i(\mathbf{x}) \quad (2.26)$$

A point in this domain is feasible if it satisfies all the inequality and equality constraints. It is possible to reorder this problem to have a linear objective function. Sometimes this linear-objective problem is called the standard form of an optimization problem:

$$\underset{\mathbf{x}, t}{\text{minimize}} \quad t \quad (2.27a)$$

$$\text{subject to} \quad f_0(\mathbf{x}) - t \leq 0; \quad (2.27b)$$

$$f_i(\mathbf{x}) \leq 0, \quad i = 1, \dots, m; \quad (2.27c)$$

$$h_i(\mathbf{x}) = 0, \quad i = 1, \dots, p. \quad (2.27d)$$

This version is called the epigraph form of an optimization problem and is mostly used in this thesis.

A convex optimization problem, which is the most general subclass of this problem that can be solved efficiently using interior point methods [122], is an optimization problem in which  $f_0, \dots, f_m$  are all convex functions and  $h_1, \dots, h_p$  should be affine ones. Since systematically may absorb the equality constraints in inequality ones, from now on, we will not mention them in the problem formulations. There are especial cases of convex optimization problems that are mostly used in science and technology, namely Linear Programming (LP) problems, Second Order Cone Programming (SOCP) problems and Semidefinite Programming (SDP) problems. In this thesis we mostly focus on SOCP and SDPs, as the objective and constraints of beamforming design problems are stated using these problems.

A SOCP is a problem of the form of

$$\underset{\mathbf{x}}{\text{minimize}} \quad \mathbf{c}_0^T \mathbf{x} \quad (2.28a)$$

$$\text{subject to} \quad \|\mathbf{A}_i \mathbf{x} + \mathbf{b}_i\|_2 \leq \mathbf{c}_i^T \mathbf{x} + d_i, \quad i = 1, \dots, m. \quad (2.28b)$$

The constraints of this problem are called to be in SOC form. Sometimes the constraints of our problems are not in SOC form. The most frequent form of our constraints are SOC-squared constrained, i.e.,  $\|\mathbf{A}_i \mathbf{x} + \mathbf{b}_i\|_2^2 \leq \mathbf{c}_i^T \mathbf{x} + d_i$ . Using the following trick it is possible to convert them back to the standard form.

$$\|\mathbf{A}_i \mathbf{x} + \mathbf{b}_i\|_2^2 \leq t \quad (2.29a)$$

$$= \left(\frac{t+1}{2}\right)^2 - \left(\frac{t-1}{2}\right)^2 \quad (2.29b)$$

resulting in

$$\left\| \begin{bmatrix} \mathbf{A}_i \mathbf{x} + \mathbf{b}_i \\ (t-1)/2 \end{bmatrix} \right\|_2 \leq \frac{t+1}{2}, \quad (2.29c)$$

which is in its standard form. In the remainder of this thesis, it is shown that the objective and constraint functions of our interest are all in SOC form. It is noteworthy that these SOCP problems are usually in semi-infinite forms, i.e., our problems of interest have infinite constraints. It is also shown that the robust treatment of our problems would lead to SDPs. In the remainder of this chapter, first an standard SDP and then biconvex and robust optimization problems are introduced. A SDP is a convex optimization problem as follows:

$$\underset{\mathbf{x}}{\text{minimize}} \quad \mathbf{c}_0^T \mathbf{x} \quad (2.30a)$$

$$\text{subject to} \quad \sum_{i=1}^n x_i \mathbf{F}_i + \mathbf{G} \succeq 0 \quad (2.30b)$$

where  $\mathbf{G}$  and  $\mathbf{F}_i$  are symmetric matrices.

## 2.2.2 Biconvex Optimization

Any problem of the form

$$\underset{\mathbf{x}, \mathbf{y}}{\text{minimize}} \quad f(\mathbf{x}, \mathbf{y}) \quad (2.31a)$$

$$\text{subject to} \quad (\mathbf{x}, \mathbf{y}) \in B \quad (2.31b)$$

where  $B$  is a biconvex set and  $f(\mathbf{x}, \mathbf{y})$  is a biconvex function is called a biconvex optimization problem. Since this problem is not a convex problem, originally it is hard to be solved. In the following, an algorithm which is well-studied in the mathematics texts [118], is summarized. This algorithm is called Alternate Convex Search (ACS). It should be noted that this algorithm cannot guarantee the global optimality of the solutions. Only sub-optimal solutions are provided generally. In the following algorithm, the decision variables are divided into two disjoint groups and in each iteration, one group is assumed to be fixed. The resulting sub-problem is a convex problem and can be solved to update the selected set of decision variables. In the next iteration, the role of the variables is changed, and since the problem is bilinear, the resulting sub-problem is also a convex problem. This new sub-problem can be solved efficiently to update the value of the rest of the decision variables. In the case of a multi-linear problem, this process can be done for any number of disjoint variable sets. In this text, however, we only consider the bilinear case.

---

**Algorithm 2.1.** [Alternate Convex Search (ACS) Algorithm]

*Assume that a biconvex optimization problem like (2.31) is given.*

**Step 1:** *Choose an arbitrary starting point  $\mathbf{z}_0 = (\mathbf{x}_0, \mathbf{y}_0)$  and set  $i = 0$ .*

**Step 2:** *Solve for fixed  $\mathbf{y}_i$  the convex optimization problem*

$$\underset{\mathbf{x}}{\text{minimize}} \quad f(\mathbf{x}, \mathbf{y}_i) \tag{2.32a}$$

$$\text{subject to} \quad \mathbf{x} \in B_{\mathbf{y}_i}. \tag{2.32b}$$

*If there exists an optimal solution  $\mathbf{x}^{opt} \in B_{\mathbf{y}_i}$  to this problem, set  $\mathbf{x}_{i+1} = \mathbf{x}^{opt}$ , otherwise STOP.*

**Step 3:** *Solve for fixed  $\mathbf{x}_{i+1}$  the convex optimization problem*

$$\underset{\mathbf{y}}{\text{minimize}} \quad f(\mathbf{x}_{i+1}, \mathbf{y}) \tag{2.33a}$$

$$\text{subject to} \quad \mathbf{y} \in B_{\mathbf{x}_{i+1}}. \tag{2.33b}$$



If there exists an optimal solution  $\mathbf{y}^{opt} \in B_{x_{i+1}}$  to this problem, set  $\mathbf{y}_{i+1} = \mathbf{y}^{opt}$ , otherwise STOP.

**Step 4:** Set  $\mathbf{z}_{i+1} = (\mathbf{x}_{i+1}, \mathbf{y}_{i+1})$ . If an appropriate stopping criterion is satisfied, then STOP, otherwise increase  $i$  by 1 and go to Step 2.

---

It should be noted that the order of Step 2 and Step 3, in the aforementioned algorithm can be permuted. It is also important that it is possible to define different stopping criterion, e.g., the absolute value of the difference of  $\mathbf{z}_i$  and  $\mathbf{z}_{i-1}$ , the absolute value of the difference in their function values, or the relative change in the  $\mathbf{z}$  variable relative to the last iteration. The convergence of the sequence of  $\{f(\mathbf{z}_i)\}_{i \in \mathbb{N}}$  is proven in [118].

### 2.2.3 Robust Optimization

In practice, usually the data based on which the optimization problems are built, is uncertain. As stated before, there are different models to describe this uncertainty. An uncertain optimization problem is as follows:

$$\underset{\mathbf{x}}{\text{minimize}} \quad f_0(\mathbf{x}, \mathbf{D}_0) \quad (2.34a)$$

$$\text{subject to} \quad f_i(\mathbf{x}, \mathbf{D}_i) \leq 0, \quad i = 1, \dots, m. \quad (2.34b)$$

where  $\mathbf{D}_0, \dots, \mathbf{D}_m$  represent uncertain data. It is usually assumed that  $\mathbf{D}_i \in \mathcal{D}_i$ , where  $\mathcal{D}_i$  describes the uncertainty set, and the objective and constraints must be satisfied for any occurrences of the data in the uncertainty set  $\mathcal{D}_i$ . Since there are infinite number of realizations for this data, the uncertain optimization problems are called semi-infinite problems. As it is known, these problems are hard to solve, and therefore, three different approaches are considered to relax the semi-infinite problems in this area. In each approach, a known mechanism is exploited to reduce the number of constraints. These approaches are namely:

1. **Stochastic Approach:** Especially when the data uncertainty is described using the SE model, the optimization is to optimize the average (mathematical expectation) of the objective and constraints (performance measures).

In this case, the robust counterpart of the uncertain problem would be:

$$\underset{\mathbf{x}}{\text{minimize}} \quad \mathbb{E}_{D_0} [f_0(\mathbf{x}, D_0)] \quad (2.35a)$$

$$\text{subject to} \quad \mathbb{E}_{D_i} [f_i(\mathbf{x}, D_i)] \leq 0, \quad i = 1, \dots, m. \quad (2.35b)$$

**2. Sampling Approach:** In this approach, the original problem is replaced with a regular problem which contains a large but finite sample of realization of the uncertain data. This method is an approximate way to replace the uncertainty with special cases.

$$\underset{\mathbf{x}}{\text{minimize}} \quad t \quad (2.36a)$$

$$\text{subject to} \quad f_0(\mathbf{x}, D_0^{[k]}) \leq t, \quad k \in \mathcal{K} \quad (2.36b)$$

$$f_i(\mathbf{x}, D_i^{[k]}) \leq 0, \quad i = 1, \dots, m. \quad (2.36c)$$

where  $\mathcal{K}$  represents a set containing relatively large samples of the problem data.

**3. Worst-case Approach:** In this approach the objective and constraint functions are replaced with their least favorable representations:

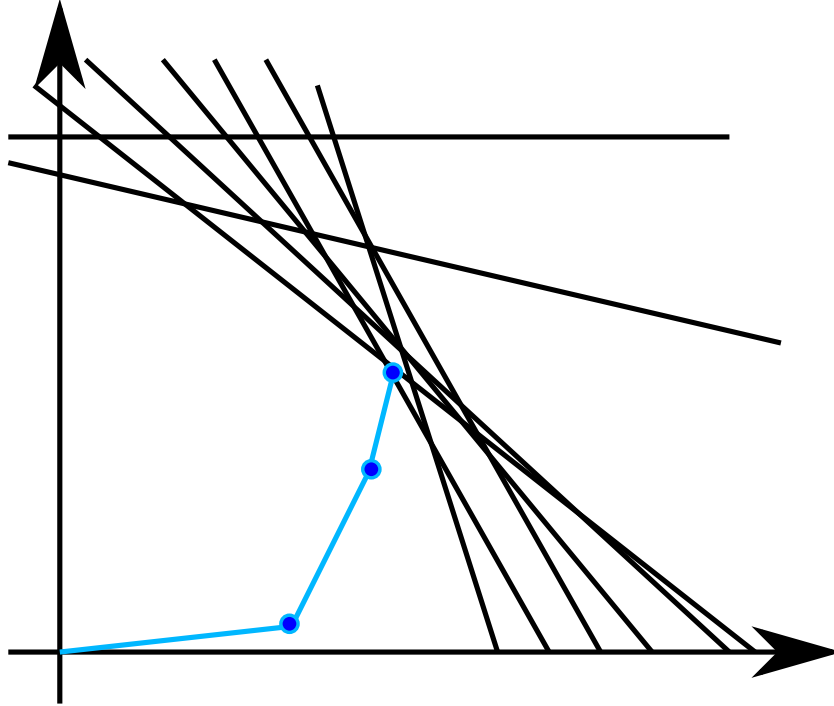
$$\underset{\mathbf{x}}{\text{minimize}} \quad \max_{D_0 \in \mathcal{D}_0} f_0(\mathbf{x}, D_0) \quad (2.37a)$$

$$\text{subject to} \quad \max_{D_i \in \mathcal{D}_i} f_i(\mathbf{x}, D_i) \leq 0, \quad i = 1, \dots, m. \quad (2.37b)$$

It is generally understood that starting from a SOCP problem with uncertain data, would lead to a SDP. To convert the original SOCP to its SDP representation the Nemirovski lemma, or its generalization are frequently used. Finally it should be mentioned that in this thesis we only resort to stochastic and worst-case robust counter parts, as the sampling approach requires many ad hoc parameter selections and is not well-understood or well-appreciated in the literature.

## 2.2.4 Interior Point Methods

Unlike the conventional way of solving a linear programming problem, which scans over the vertexes of the feasible region of the problem, an interior point algorithm



**Fig. 2.1:** Overview of an interior point problem

takes a path toward the optimal point that crosses this region. For example, in Fig. 2.1 an illustration of such an algorithm is given. In this example, the feasible region of the problem is described using a set of linear equations leading to a polytope with possibly many vertexes. Using the simplex-like algorithms [119], to find the optimal point, the algorithm should check the vertexes of this region in a smart way. But for large problems, this means that the algorithm should should meet with thousands of vertexes before reaching the optimal solution. In this case, it is better to find an algorithm which does not depend on the geometry of the feasible region. Interior point algorithms are such algorithms which relax this dependence. Among many proposed algorithms, path-following interior point algorithms, potential reduction methods, predictor-corrector methods, and self-dual methods are subject to a polynomial worst-case and average-case complexity issue, and are mostly implemented in software packages. For more information about these methods and their complexity please see [120]-[122].

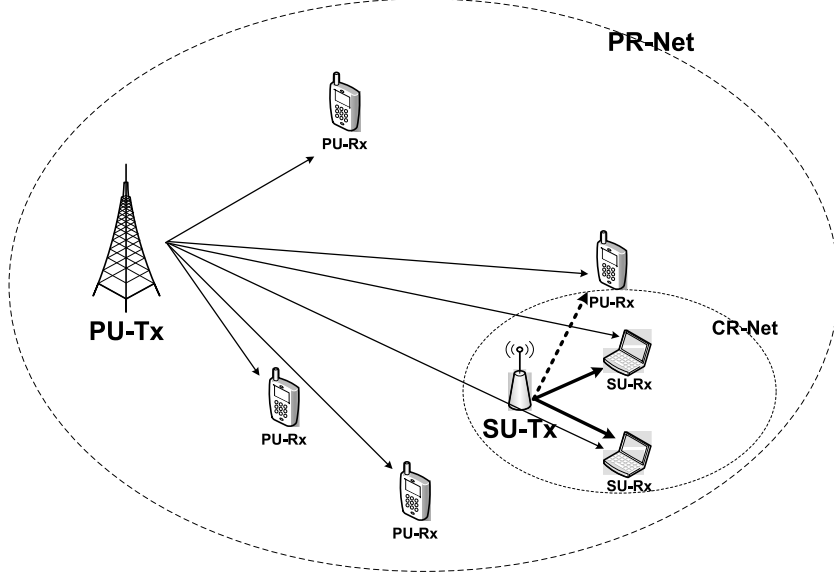
# Chapter 3

## Robust Downlink Beamforming in MU-MISO CR-Nets

### 3.1 Introduction

Cognitive Radio (CR) is a promising solution to solve the spectrum scarcity problem [2], [123]. In a Cognitive Radio Network (CR-Net), the Secondary Users (SUs) are allowed to operate within the service range of the Primary Users (PUs), though the PUs have higher priority in utilizing the spectrum. There are two types of CR-Nets: opportunistic CR-Nets for which the SUs sense the spectrum and try to utilize the channels when they are not occupied by PUs; and concurrent CR-Nets in which SUs are allowed to use the spectrum even when PUs are active, provided that the amount of interference power to each PU receiver is kept below a certain threshold [123]. In this chapter, we are interested in concurrent CR-Nets.

Fig. 3.1 illustrates the downlink scenario of a single-cell multiuser multiple-input single-output (MISO) CR-Net with  $K$  SUs coexisting with  $L$  PUs. In this configuration, the CR-Net is installed far enough from the PU-Transmitter (PU-Tx). Although the PU-Tx is interfering with normal operations of CR-Net, the power received from the intended transmitter SU-Tx is much larger, i.e., the interfering power from PU-Tx can be accumulated as a part of its noise term. Therefore, there are only two sets of relevant channel state information (CSI) which play important roles in the system design: one set describes the channels between SU-Transmitter (SU-Tx) and SU-Receiver (SU-Rx's) while the other set describes the channels between SU-Tx and PU-Rxs. For simplicity, we term the first set of



**Fig. 3.1:** Overview of a single-cell CR-Net coexisting with a single-cell PR-Net

CSI as SU-link CSI and the second set as PU-link CSI. When PUs are inactive, the system becomes a conventional multiuser MISO system, and SU-link CSI is needed for transmission design. This knowledge is usually acquired through transmitting pilot symbols from SU-Tx to SU-Rx's, and feeding back the estimated CSI from SU-Rxs to SU-Tx. In practice, however, because of the time-varying nature of wireless channels, it is not possible to acquire the CSI perfectly, either due to channel variation and/or channel estimation error and/or feedback error. On the other hand, when PUs are active, PU-link CSI is further needed at SU-Tx for the purpose of controlling interferences at the PU-Rx's. This CSI knowledge has to be acquired by SU-Tx through environmental learning [124], which again may have errors. In this chapter we consider the transmit design for a multiuser MISO CR-Net with uncertain CSI in both SU-link and PU-link.

Previously in conventional radio network design, ad-hoc methods, such as diagonal loading [125], were proposed to design robust beamforming systems. Quite recently these designs are based on well-known mathematical methodologies, such as the systematical worst case design [9]-[14]. These methods consider a Minimum Variance Distortionless Response (MVDR) problem in the signal processing domain and show that the problem may be recast as a Second-Order Cone Program (SOCP) [92]. Also, it was shown that this worst case design scenario is

equivalent to an adaptive diagonal loading [9]. One of the first worst case designs was published by Bengtsson and Ottersten [7]-[8]. They showed that the robust maximization of SINR would lead to a Semi-Definite Program (SDP) [92], after a simple Semidefinite Relaxation (SDR). Sharma *et al.*, [128] developed a model to cover the Positive Semi-definiteness (PSD) of the channel covariance matrix. They proposed two SDPs, a conventional SDP and a SDP based on an iterative algorithm. Also Mutapcic *et al.*, [129] proposed a new tractable method to solve the robust downlink beamforming. Their method is based on the cutting set algorithm which is also an iterative method. Also [38],[130]-[131] target the robust design of a beamforming system using the worst case scenarios for Quality-of-Service (QoS) constraints.

Quite a few works are published on the robust design for CR-Nets [71], [72] and [74]. Zhang *et al.* [71] have studied such a CR-Net from an information theoretic perspective. The CR-Net considered in [71] consists of one PU-Rx and one SU-Rx, and the SU-link CSI is assumed to be perfectly known, but the PU-link CSI has uncertainty. A duality theory was developed to cope with the CSI imperfectness. Additionally, the authors proposed an analytic solution for this case. Also, Zhi *et al.* [72] designed a robust beamformer for a CR-Net, where the system setup is the same as in [71], however there may be some uncertainty in both the channel covariance matrix as well as the antenna manifold. Finally, Cumanan *et al.* [74] considered a CR-Net having multiple PUs and only one SU. In this work, both channels are assumed to be imperfect. They also used the worst case design method to come up with a convex problem that can be solved efficiently.

In this chapter, we consider a downlink system of a CR-Net with multiple SU-Rx's coexisting with multiple PU-Rx's whose relevant CSI is imperfectly known. The imperfectness of the CSI is modeled using an Euclidean ball. Our design objective is to minimize the transmit power of the SU-Tx while simultaneously targeting a lower bound on the received Signal-to-Interference-plus-Noise-Ratio (SINR) for the SUs, and imposing an upper limit on the Interference-Power (IP)

at the PUs. The design parameters at the SU-Tx are the beamforming weights, i.e. the precoder matrix. The proposed methodology is based on a worst case design scenario through which the performance metrics of the design are immune to variations in the channels. To solve the problem, we reformulate our initial design problem and translate the uncertainty in CSI to the uncertainty in its covariance matrix. We propose three approaches based on convex programming for which efficient numerical solutions exist. In the first approach, the worst case SINR is derived by using loose upper and lower bounds on the terms appearing in the numerator and denominator of the SINR. Working in this line, a SDP is developed which provides us the robust beamforming coefficients. In the second approach, the minimum SINR is found through minimizing its numerator while maximizing its denominator. Different from the first method, we chose exact upper and lower bounds on the previously mentioned terms. This approach does not lead to a SDP, but the resulting problem is still convex and may be solved efficiently. Finally, in our third approach, we find the exact minimum of SINR directly, and this method is also a general convex optimization problem.

Fig. 3.1 illustrates the downlink scenario of a single-cell MU-MISO CR-Net with  $K$  SUs coexisting with  $L$  PUs. In this configuration, the CR-Net is installed far enough from the PU-Tx. Although the PU-Tx is interfering with normal operations of CR-Net, the power received from the intended transmitter SU-Tx is much larger, i.e., the interfering power from PU-Tx can be accumulated as a part of its noise term. Therefor, there are only two sets of relevant CSI which play important roles in the system design: one set describes the channels between SU-Tx and SU-Rxs while the other set describes the channels between SU-Tx and PU-Rxs. For simplicity, we term the first set of CSI as SU-link CSI and the second set as PU-link CSI. When PUs are inactive, the system becomes a conventional multiuser MISO system, and SU-link CSI is needed for transmission design. This knowledge is usually acquired through transmitting pilot symbols from SU-Tx to SU-Rx's, and feeding back the estimated CSI from SU-Rxs to SU-Tx. In practice, however, because of the time-varying nature of wireless channels, it is

not possible to acquire the CSI perfectly, either due to channel variation and/or channel estimation error and/or feedback error. On the other hand, when PUs are active, PU-link CSI is further needed at SU-Tx for the purpose of controlling interferences at the PU-Rx's. This CSI knowledge has to be acquired by SU-Tx through environmental learning [124], which again may have errors. In this chapter we consider the transmit design for a MU-MISO CR-Net with uncertain CSI in both SU-link and PU-link.

In this chapter, we consider a downlink system of a CR-Net with multiple SU-Rx's coexisting with multiple PU-Rx's whose relevant CSI is imperfectly known. The imperfectness of the CSI is modeled using an Euclidean ball. Our design objective is to minimize the transmit power of the SU-Tx while simultaneously targeting a lower bound on the received SINR for the SUs, and imposing an upper limit on the Interference-Power (IP) at the PUs. The design parameters at the SU-Tx are the beamforming weights, i.e. the precoder matrix. The proposed methodology is based on a worst case design scenario through which the performance metrics of the design are immune to variations in the channels. To solve the problem, we reformulate our initial design problem and translate the uncertainty in CSI to the uncertainty in its covariance matrix.

We propose three approaches based on convex programming for which efficient numerical solutions exist. It should be stressed that the first two approaches are well-known in the literature [7]- [12], and we mentioned them here to provide a unified approach which considers CR-Net scenarios as well. Our contribution in this chapter comes in the form of the third method, namely Exact Robust Solution (ExRS), which outperforms the first two methods as can be seen in the simulation results. In the first approach, the worst case SINR is derived by using loose upper and lower bounds on the terms appearing in the numerator and denominator of the SINR. Working in this line, a SDP is developed which provides us the robust beamforming coefficients. In the second approach, the minimum SINR is found through minimizing its numerator while maximizing its denominator. Different from the first method, we chose tight upper and lower bounds on the previously



mentioned terms. This approach does not lead to a SDP, but the resulting problem is still convex and may be solved efficiently. Finally, in our third approach, we find the exact minimum of SINR directly, and this method is also a general convex optimization problem.

The rest of the chapter is organized as follows. In Section 3.2, the model of under-study system is described and the robust design of a MU-MISO CR-Net with multiple SUs and multiple PUs is considered. In Section 3.3, we show that the resulting optimization problem, using loose upper and lower bounds, is a SDP. In Sections 3.4 and 3.5 we propose two more general problem formulations based on a stricter bound and an exact bound on the minimum value of SINR, respectively. In Section 3.6 the simulation results that demonstrate the robustness of the proposed schemes are presented. Finally, Section 3.7 concludes the chapter.

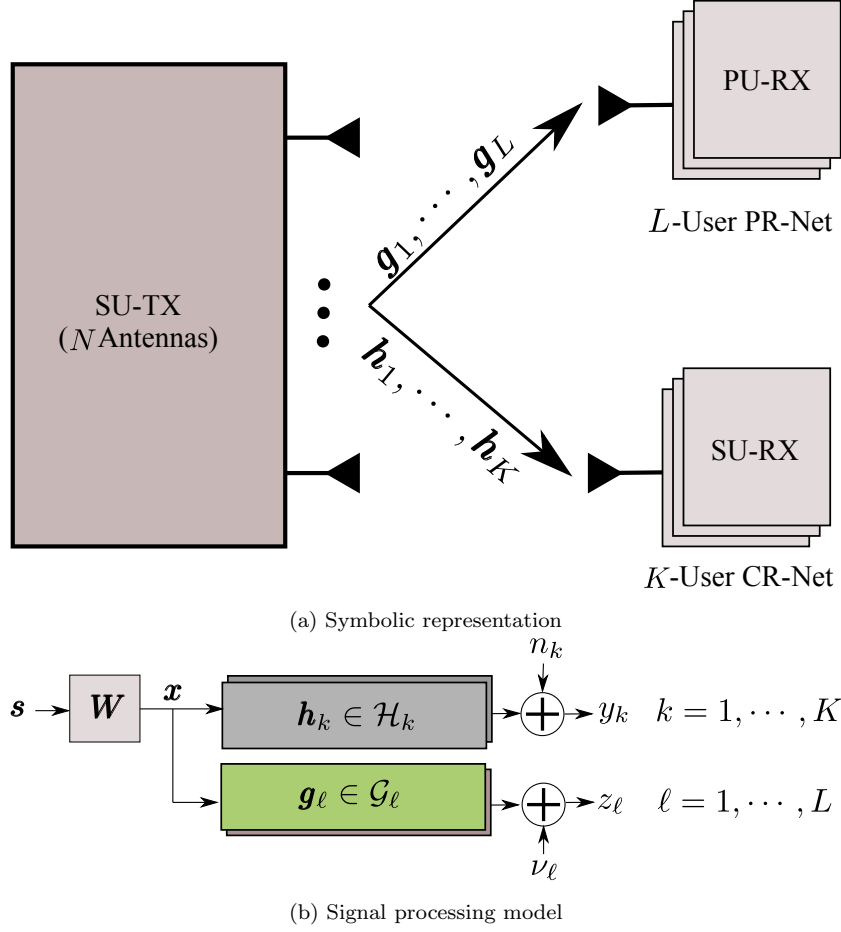
## 3.2 System Model and Problem Formulation

Fig. 3.2 shows the downlink scenario of a MU-MISO CR-Net coexisting with a Primary-Radio Network (PR-Net) having  $L$  PUs each equipped with a single antenna. The SU-Tx which is equipped with  $N$  antennas, transmitting independent symbols,  $s_k$ , to  $K$  different single antenna SU-Rxs. It is assumed that the transmitted symbols are all zero-mean, independent and identically distributed. Each symbol is precoded by a weight vector,  $\{\mathbf{w}_k \in \mathbb{C}^{N \times 1}\}_{k=1}^K$ , resulting in a vector signal,  $\{\mathbf{s}_k = \mathbf{w}_k s_k\}_{k=1}^K$ , for each one. The channel from SU-Tx to each SU-Rx is determined using a complex-valued vector,  $\{\mathbf{h}_k \in \mathbb{C}^{N \times 1}\}_{k=1}^K$  which is not perfectly known and there is some kind of uncertainty in channel gains. This uncertainty is described using an uncertainty set,  $\mathcal{H}_k$ , which is defined as an Euclidean ball

$$\mathcal{H}_k = \{\mathbf{h} \mid \|\mathbf{h} - \tilde{\mathbf{h}}_k\| \leq \delta_k\}. \quad (3.1)$$

In this definition, the ball is centered around the nominal value of the channel vector,  $\tilde{\mathbf{h}}_k$ , and the radius of the ball is determined by  $\delta_k$  which is a positive constant. Using this notion, the channel is modeled as

$$\mathbf{h}_k = \tilde{\mathbf{h}}_k + \mathbf{a}_k, \quad (3.2)$$



**Fig. 3.2:** A Typical multiuser MISO CR-Net system with uncertain CSI

where  $\mathbf{a}_k$  is a norm-bounded uncertainty vector,  $\|\mathbf{a}_k\| \leq \delta_k$ . In our notation,  $\mathbf{h}_k$  is the actual channel which is subject to uncertainty and is not known perfectly. We only know the nominal value of the channel and some information about its perturbation. This information is given by the chosen model of the uncertainty.

The SU-Tx combines the signals and transmits the combination,  $\mathbf{x}$ ,

$$\mathbf{x} = \sum_{k=1}^K \mathbf{s}_k = \mathbf{W} \mathbf{s}, \quad (3.3)$$

where  $\mathbf{s} = [s_1, \dots, s_K]^T \in \mathbb{C}^{K \times 1}$  contains the transmitted symbols and  $\mathbf{W} = [\mathbf{w}_1, \dots, \mathbf{w}_K] \in \mathbb{C}^{N \times K}$ , is called the precoding matrix. The design objective is to determine this precoding matrix  $\mathbf{W}$  based on certain criteria that is discussed in the next few paragraphs.

The channel from the SU-Tx to a PU-Rx is also defined using a complex valued vector, i.e.,  $\{\mathbf{g}_\ell \in \mathbb{C}^{N \times 1}\}_{\ell=1}^L$ . Here it is assumed that the CSI for these users is

also uncertain. We use the same notation to describe the uncertainty for these channels. The uncertainty is defined using a set,  $\mathcal{G}_\ell$ , which is

$$\mathcal{G}_\ell = \{\mathbf{g} \mid \|\mathbf{g} - \tilde{\mathbf{g}}_\ell\| \leq \eta_\ell\}. \quad (3.4)$$

Equivalently, we may write

$$\mathbf{g}_\ell = \tilde{\mathbf{g}}_\ell + \mathbf{b}_\ell, \quad (3.5)$$

where  $\mathbf{b}_\ell$  is a norm-bounded uncertain vector,  $\|\mathbf{b}_\ell\| \leq \eta_\ell$  and  $\tilde{\mathbf{g}}_\ell$  is the nominal value of the channel.

For the system depicted in Fig.3.2 the total transmitted power, TxP, is given by

$$\text{TxP} \triangleq \mathbb{E}_{\mathbf{x}} [\|\mathbf{x}\|^2] = \sum_{k=1}^K \|\mathbf{w}_k\|^2. \quad (3.6)$$

The received signal at the  $k$ th SU-Rx is

$$y_k = \mathbf{h}_k^\dagger \mathbf{w}_k s_k + \sum_{i=1 \neq k}^K \mathbf{h}_k^\dagger \mathbf{w}_i s_i + n_k. \quad (3.7)$$

The right-hand side of (3.7) has three terms. The first term is the received signal of the intended message, while the second and the third terms show the interference from other messages and noise, which is white and Gaussian, i.e.  $n_k \sim \mathcal{CN}(0, \sigma_n^2)$ , respectively. The SINR at  $k$ th SU-Rx,  $\text{SINR}_k$ , is given by

$$\text{SINR}_k \triangleq \frac{|\mathbf{w}_k^\dagger \mathbf{h}_k|^2}{\sigma_n^2 + \sum_{i=1 \neq k}^K |\mathbf{w}_i^\dagger \mathbf{h}_k|^2}. \quad (3.8)$$

Also, the received signal at the  $\ell$ th PU is

$$z_\ell = \sum_{k=1}^K \mathbf{g}_\ell \mathbf{w}_k s_k + \nu_\ell, \quad (3.9)$$

where  $\nu_\ell$  is the received noise. The interference power,  $\text{IP}_\ell$ , to this PU-Rx is

$$\text{IP}_\ell \triangleq \sum_{k=1}^K |\mathbf{w}_k^\dagger \mathbf{g}_\ell|^2. \quad (3.10)$$

Our design objective is to minimize the transmitted power, TxP, while guaranteeing that the SINR at each SU-Rx for all the channel realizations is higher than the QoS-constrained threshold,  $\{\text{SINR}_k \geq \gamma_i\}_{k=1}^K$ , and simultaneously IP at each

PU-Rx is less than the PR-Net-imposed threshold,  $\{\text{IP}_\ell \leq \kappa_\ell\}_{\ell=1}^L$ , respectively. Mathematically, this problem can be described as

$$\begin{aligned}
& \underset{\{\mathbf{w}_k\}_{k=1}^K}{\text{minimize}} && \text{TxP} \\
& \text{subject to} && \text{SINR}_k \geq \gamma_k, \quad k = 1, \dots, K \\
& && \forall \mathbf{h}_k \in \mathcal{H}_k \\
& && \text{IP}_\ell \leq \kappa_\ell, \quad \ell = 1, \dots, L. \\
& && \forall \mathbf{g}_\ell \in \mathcal{G}_\ell
\end{aligned} \tag{3.11}$$

The above problem is a problem with an infinite number of constraints. To deal with such a problem one well-known method is to find the minimum and maximum values of  $\text{SINR}_k$  and  $\text{IP}_\ell$ , respectively, related to those realizations of the channels which are claimed as the “worst ones”. The worst channel realizations for SINR and IP would lead to the minimum and maximum value of SINR and IP, respectively. In that case, the problem will guarantee that the smallest possible SINR and largest possible IP also satisfy the constraints. Using this worst case design methodology, we could recast (3.11) to a simpler problem set as follows:

$$\begin{aligned}
& \underset{\{\mathbf{w}_k\}_{k=1}^K}{\text{minimize}} && \text{TxP} \\
& \text{subject to} && \min_{\mathbf{h}_k \in \mathcal{H}_k} \text{SINR}_k \geq \gamma_k, \quad k = 1, \dots, K \\
& && \max_{\mathbf{g}_\ell \in \mathcal{G}_\ell} \text{IP}_\ell \leq \kappa_\ell, \quad \ell = 1, \dots, L.
\end{aligned} \tag{3.12}$$

In this problem quadratic expressions with the form of  $|\mathbf{w}_k^\dagger \mathbf{h}_k|^2$  are frequently used. These expressions contain the design variables and parameters as vectors for which it is mathematically appealing to write them in covariance matrix form.

We can write

$$\begin{aligned}
|\mathbf{w}_k^\dagger \mathbf{h}_k|^2 &= \mathbf{w}_k^\dagger (\tilde{\mathbf{h}}_k + \mathbf{a}_k) (\tilde{\mathbf{h}}_k + \mathbf{a}_k)^\dagger \mathbf{w}_k \\
&= \mathbf{w}_k^\dagger (\tilde{\mathbf{H}}_k + \mathbf{\Delta}_k) \mathbf{w}_k,
\end{aligned} \tag{3.13}$$

where  $\tilde{\mathbf{H}}_k = \tilde{\mathbf{h}}_k \tilde{\mathbf{h}}_k^\dagger$  is the constant covariance matrix of the nominal CSI and  $\mathbf{\Delta}_k$ , which shows the uncertainty in this matrix, is given by

$$\mathbf{\Delta}_k = \tilde{\mathbf{h}}_k \mathbf{a}_k^\dagger + \mathbf{a}_k \tilde{\mathbf{h}}_k^\dagger + \mathbf{a}_k \mathbf{a}_k^\dagger. \tag{3.14}$$

Note that  $\Delta_k$  is a norm bounded matrix,  $\|\Delta_k\| \leq \epsilon_k$ . It is straightforward to find the following relation:

$$\begin{aligned}
\epsilon_k &\geq \|\Delta_k\| = \|\tilde{\mathbf{h}}_k \mathbf{a}_k^\dagger + \mathbf{a}_k \tilde{\mathbf{h}}_k^\dagger + \mathbf{a}_k \mathbf{a}_k^\dagger\| \\
&\leq \|\tilde{\mathbf{h}}_k \mathbf{a}_k^\dagger\| + \|\mathbf{a}_k \tilde{\mathbf{h}}_k^\dagger\| + \|\mathbf{a}_k \mathbf{a}_k^\dagger\| \\
&\leq \|\tilde{\mathbf{h}}_k\| \|\mathbf{a}_k^\dagger\| + \|\mathbf{a}_k\| \|\tilde{\mathbf{h}}_k^\dagger\| + \|\mathbf{a}_k\|^2 \\
&= \delta_k^2 + 2\delta_k \|\tilde{\mathbf{h}}_k\|.
\end{aligned} \tag{3.15}$$

Using (3.15) it is possible to choose  $\epsilon_k = \delta_k^2 + 2\delta_k \|\tilde{\mathbf{h}}_k\|$ . We use the identity  $\mathbf{x}^\dagger \mathbf{A} \mathbf{x} = \text{tr} [\mathbf{A} \mathbf{x} \mathbf{x}^\dagger]$  to further simplify expression and obtain

$$|\mathbf{w}_k^\dagger \mathbf{h}_k|^2 = \text{tr} \left[ \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{W}_k \right]. \tag{3.16}$$

where  $\mathbf{W}_k = \mathbf{w}_k \mathbf{w}_k^\dagger$ . It is noted that, from now on, similar expressions will be used for the other terms of SINR<sub>k</sub>. Again using similar formulation as  $|\mathbf{w}_k^\dagger \mathbf{h}_k|^2$ , we get

$$|\mathbf{w}_k^\dagger \mathbf{g}_\ell|^2 = \text{tr} \left[ \left( \tilde{\mathbf{G}}_\ell + \Lambda_\ell \right) \mathbf{W}_k \right], \tag{3.17}$$

where  $\tilde{\mathbf{G}}_\ell$  is a constant matrix,  $\tilde{\mathbf{G}}_\ell = \tilde{\mathbf{g}}_\ell \tilde{\mathbf{g}}_\ell^\dagger$  and  $\Lambda_\ell$  is the norm bounded uncertainty matrix,  $\|\Lambda_\ell\| \leq \xi_\ell$ . Similarly we know that  $\xi_\ell = \eta_\ell^2 + 2\eta_\ell \|\tilde{\mathbf{g}}_\ell\|$ .

Adopting the above notations, we rewrite (3.12) as

$$\begin{aligned}
&\underset{\{\mathbf{W}_k\}_{k=1}^K}{\text{minimize}} && \sum_{k=1}^K \text{tr} [\mathbf{W}_k] \\
&\text{subject to} && \min_{\|\Delta_k\| \leq \epsilon_k} \frac{\text{tr} \left[ \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{W}_k \right]}{\sigma_n^2 + \sum_{i=1 \neq k}^K \text{tr} \left[ \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{W}_i \right]} \geq \gamma_k, \\
&&& k = 1, \dots, K \\
&&& \max_{\|\Lambda_\ell\| \leq \eta_\ell} \sum_{k=1}^K \text{tr} \left[ \left( \tilde{\mathbf{G}}_\ell + \Lambda_\ell \right) \mathbf{W}_k \right] \leq \kappa_\ell, \quad \ell = 1, \dots, L.
\end{aligned} \tag{3.18}$$

It should be mentioned that the aforementioned steps are indispensable due to the form of our primary problem (3.12). Even for the non-robust case, (3.12) is a separable homogeneous Quadratically Constrained Quadratic Problem (QCQP) which is a NP-Hard problem [132]. Regardless of the nature of its variables (real

or complex) (3.12) is NP-hard and should be transformed to a more proper form to be solved. To the best of our knowledge, SDP relaxation is the most well-studied way to overcome this ill-conditioning. The uncertainty regions in (3.18) are the result of the direct application of a quadratic transformation of the uncertainty regions of (3.12) as summarized before. The bounds of this uncertainty are derived by triangle inequality and multiplicity of the second norm, and are tight enough. Although the uncertainty regions in (3.12) and (3.18) are different in nature but they have a nice interrelation summarized in (3.15). In the next sections, we will solve the robust problem (3.18) and will show that this problem can be recast as a series of simple optimization problems.

### 3.3 Loosely Bounded Robust Solution (LBRS)

In this section we will deal with the problem of (3.18). In [7] and [8], it is suggested to minimize the SINR through minimizing the numerator while maximizing its denominator. So the first constraint of (3.18) is equivalent to

$$\min_{\|\Delta_k\| \leq \epsilon_k} \text{tr} \left[ (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{W}_k \right] - \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K \max_{\|\Delta_k\| \leq \epsilon_k} \text{tr} \left[ (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{W}_i \right] \geq \gamma_k \sigma_n^2. \quad (3.19)$$

As it is known, this method is a loose approximate way to find the minimum of the SINR.

#### 3.3.1 Minimization of SINR

To minimize the numerator,

$$\min_{\|\Delta_k\| \leq \epsilon_k} \text{tr} \left[ \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{W}_k \right], \quad (3.20)$$

we adopt a loose lower bound, proposed by [7], [8]. Using this lower bound, we have

$$\min_{\|\Delta_k\| \leq \epsilon_k} \text{tr} \left[ \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{W}_k \right] \approx \text{tr} \left[ \left( \tilde{\mathbf{H}}_k - \epsilon_k \mathbf{I}_N \right) \mathbf{W}_k \right], \quad (3.21)$$

and to maximize the denominator, the following term should be maximized

$$\max_{\|\Delta_k\| \leq \epsilon_k} \text{tr} \left[ \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{W}_i \right]. \quad (3.22)$$

Using a similar approximation, we have

$$\max_{\|\Delta_k\| \leq \epsilon_k} \text{tr} \left[ \left( \tilde{\mathbf{H}}_k + \Delta_k \right) \mathbf{W}_i \right] \approx \text{tr} \left[ \left( \tilde{\mathbf{H}}_k + \epsilon_k \mathbf{I}_N \right) \mathbf{W}_i \right]. \quad (3.23)$$

Using these results, the problem of SINR minimization as the first constraint of (3.18), is recast as

$$\text{tr} \left[ \left( \tilde{\mathbf{H}}_k - \epsilon_k \mathbf{I}_N \right) \mathbf{W}_k \right] - \gamma_k \sum_{i=1 \neq k}^K \text{tr} \left[ \left( \tilde{\mathbf{H}}_k + \epsilon_k \mathbf{I}_N \right) \mathbf{W}_i \right] \geq \sigma_n^2 \gamma_k, \quad \forall k, \quad (3.24)$$

and by regrouping the left hand side of this equation we find

$$\text{tr} \left[ \tilde{\mathbf{H}}_k \left( \mathbf{W}_k - \gamma_k \sum_{i=1 \neq k}^K \mathbf{W}_i \right) \right] - \epsilon_k \text{tr} \left[ \mathbf{W}_k + \gamma_k \sum_{i=1 \neq k}^K \mathbf{W}_i \right] \geq \sigma_n^2 \gamma_k, \quad \forall k. \quad (3.25)$$

### 3.3.2 The Whole Conventional Program

Using the same methodology, IP maximization in the second constraint of (3.18) leads to the following problem

$$\sum_{k=1}^K \text{tr} \left[ \left( \tilde{\mathbf{G}}_\ell + \xi_\ell \mathbf{I}_N \right) \mathbf{W}_k \right] \leq \kappa_\ell, \quad \ell = 1, \dots, L. \quad (3.26)$$

Then the whole program targeting to solve the robust downlink optimization in MISO CR-Nets becomes

$$\begin{aligned} & \underset{\{\mathbf{W}_k\}_{k=1}^K}{\text{minimize}} && \sum_{k=1}^K \text{tr} [\mathbf{W}_k] \\ & \text{subject to} && \text{tr} \left[ \tilde{\mathbf{H}}_k \left( \mathbf{W}_k - \gamma_k \sum_{i=1 \neq k}^K \mathbf{W}_i \right) \right] - \\ & && \epsilon_k \text{tr} \left[ \mathbf{W}_k + \gamma_k \sum_{i=1 \neq k}^K \mathbf{W}_i \right] \geq \sigma_n^2 \gamma_k, \quad k = 1, \dots, K \\ & && \text{tr} \left[ \left( \tilde{\mathbf{G}}_\ell + \xi_\ell \mathbf{I}_N \right) \sum_{k=1}^K \mathbf{W}_k \right] \leq \kappa_\ell, \quad \ell = 1, \dots, L, \\ & && \mathbf{W}_k = \mathbf{W}_k^\dagger, \quad k = 1, \dots, K, \\ & && \mathbf{W}_k \succeq 0, \quad k = 1, \dots, K. \end{aligned} \quad (3.27)$$

Please note the fact that the last two constraints are inherent in the structure of the problem formulation. Also note that to come up with a convex problem

formulation, a non-convex constraint,  $\text{rank}\{\mathbf{W}_k\} = 1$ , is eliminated [126], [7], [128]. This final form of the problem is an SDP and can be solved using efficient numerical methods [133]. Finally it should be noted that unlike [128] the beamforming weights are not exactly<sup>1</sup> the principal eigenvector<sup>2</sup> of the matrix solution. To get the beamforming weights, the eigen decomposition of the  $\mathbf{W}_k$  is used. In this decomposition,  $\mathbf{W}_k$  may be decomposed to a series of rank one matrices, i.e.,

$$\mathbf{W}_k = \sum_{n=1}^N \lambda_{n,k} \mathbf{e}_{n,k} \mathbf{e}_{n,k}^\dagger, \quad (3.28)$$

where in this expansion,  $\lambda_{n,k}$  denotes the  $n$ th eigenvalue and  $\mathbf{e}_{n,k}$  is its corresponding eigenvector. The solution matrix of  $\mathbf{W}_k$  itself is a rank one matrix, then all the eigenvalues are equal to zero except one, let's say  $\lambda_{N,k}$ . Therefore the above mentioned equation may be written as

$$\begin{aligned} \mathbf{W}_k &= \lambda_{N,k} \mathbf{e}_{N,k} \mathbf{e}_{N,k}^\dagger \\ &= (\sqrt{\lambda_{N,k}} \mathbf{e}_{N,k})(\sqrt{\lambda_{N,k}} \mathbf{e}_{N,k})^\dagger \\ &= \mathbf{w}_k \mathbf{w}_k^\dagger, \end{aligned} \quad (3.29)$$

where  $\mathbf{w}_k = \sqrt{\lambda_{N,k}} \mathbf{e}_{N,k}$ .

Recently a rigorous proof in [132] shows that for the case of Vandermonde CSI, which is the usual case for MISO beamforming using Uniform Linear Array (ULA), the matrix solution is always rank one and the proposed relaxed SDP is equivalent to our original problem [132]. If the solution of this problem is not rank-1, standard randomization techniques are available to produce the beamforming vectors from the matrix solution (Please refer to [9] and the references therein). The next two sections consider the same problem but in different ways. It should be noted that the next two problems are not SDP and are generally convex problems, but, for these problems we use the same formula to acquire the beamforming weights from the solution matrix.

---

<sup>1</sup>A simple but important scaling is needed.

<sup>2</sup>The principal eigenvector of a rank one matrix is the eigenvector corresponding to the only non-zero eigenvalue.



### 3.4 Strictly Bounded Robust Solution (SBRS)

In the previous section, the minimum of SINR was found using loose upper and lower bounds for its constituent terms. In this section, we try to minimize the SINR using the same method: we minimize the numerator and maximize the denominator. But here, we try to find the exact maximum and the exact minimum for each term respectively:

$$\min_{\|\Delta_k\| \leq \epsilon_k} \text{tr} \left[ (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{W}_k \right] - \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K \max_{\|\Delta_k\| \leq \epsilon_k} \text{tr} \left[ (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{W}_i \right] \geq \gamma_k \sigma_n^2. \quad (3.30)$$

Our main tool, is the Lagrangian Multiplier method.

#### 3.4.1 Minimization of SINR

We start with the first minimization problem.

**Proposition 3.1.** *For the terms  $\text{tr} \left[ (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{W}_k \right]$ , using a norm-bounded variable  $\Delta_k$ ,  $\|\Delta_k\| \leq \epsilon_k$ , the minimizer and maximizer would be*

$$\Delta_k^{min} = -\epsilon_k \frac{\mathbf{W}_k^\dagger}{\|\mathbf{W}_k\|}, \quad (3.31)$$

and

$$\Delta_k^{max} = \epsilon_k \frac{\mathbf{W}_k^\dagger}{\|\mathbf{W}_k\|}, \quad (3.32)$$

respectively.

*Proof.* The Lagrangian function, using an arbitrary positive multiplier,  $\lambda \geq 0$ , is

$$\begin{aligned} L(\Delta_k, \lambda) &= \text{tr} \left[ (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{W}_k \right] + \lambda (\|\Delta_k\|^2 - \epsilon_k^2) \\ &= \text{tr} \left[ (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{W}_k \right] + \lambda (\text{tr} [\Delta_k \Delta_k^\dagger] - \epsilon_k^2). \end{aligned} \quad (3.33)$$

By differentiating this Lagrangian function with respect to  $\Delta_k^*$  and equating it to zero [134],

$$\nabla_{\Delta_k^*} L(\Delta_k, \lambda) = \mathbf{W}_k^\dagger + \lambda \Delta_k = 0, \quad (3.34)$$

we will find the optimal solution  $\Delta_k$ , which is denoted by  $\Delta_k^{opt}$ ,

$$\Delta_k^{opt} = -\frac{1}{\lambda} \mathbf{W}_k^\dagger. \quad (3.35)$$

To eliminate the role of arbitrary parameter of  $\lambda$ , again, we differentiate the Lagrangian function with respect to this unknown parameter and then equate it to zero

$$\nabla_{\lambda} L(\Delta_k, \lambda) = 0, \quad (3.36)$$

to get the optimal solution for  $\lambda$ , denoted as  $\lambda^{opt}$ ,

$$\lambda^{opt} = \frac{1}{\epsilon_k} \|\mathbf{W}_k^{\dagger}\|. \quad (3.37)$$

By combining these results, finally, we come up with

$$\Delta_k^{opt} = -\epsilon_k \frac{\mathbf{W}_k^{\dagger}}{\|\mathbf{W}_k\|}. \quad (3.38)$$

To test if this solution is a minimum, we should observe that the second derivative at the optimal solution point should be positive semi-definite,

$$\nabla_{\Delta_k}^2 L(\Delta_k^{opt}, \lambda^{opt}) = \lambda^{opt} (\text{vec}[\mathbf{I}_N] \text{vec}[\mathbf{I}_N])^T \succeq 0. \quad (3.39)$$

which completes our claim. To find the maximum of such a term, a similar procedure is undergone again, using a positive arbitrary Lagrangian multiplier, we build a Lagrangian function:

$$\begin{aligned} L(\Delta_k, \lambda) &= \text{tr} \left[ (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{W}_i \right] - \lambda (\|\Delta_k\|^2 - \epsilon_k^2) \\ &= \text{tr} \left[ (\tilde{\mathbf{H}}_k + \Delta_k) \mathbf{W}_i \right] - \lambda (\text{tr} [\Delta_k \Delta_k^{\dagger}] - \epsilon_k^2). \end{aligned} \quad (3.40)$$

By differentiating it with respect to  $\Delta_k$  and equating it to zero

$$\nabla_{\Delta_k} L(\Delta_k, \lambda) = \mathbf{W}_i^{\dagger} - \lambda \Delta_k = 0, \quad (3.41)$$

we will get

$$\Delta_k^{opt} = \frac{1}{\lambda} \mathbf{W}_i^{\dagger}. \quad (3.42)$$

Again, by differentiating the Lagrangian function with respect to  $\lambda$  and equating it to zero,

$$\nabla_{\lambda} L(\Delta_k, \lambda) = 0, \quad (3.43)$$

we are able to get the optimizer.

$$\lambda^{opt} = \frac{1}{\epsilon_k} \|\mathbf{W}_i\|, \quad (3.44)$$

$$\Delta_k^{opt} = \epsilon_k \frac{\mathbf{W}_i^\dagger}{\|\mathbf{W}_i\|}. \quad (3.45)$$

To prove if this solution is a maximum, we should observe that its second derivative is negative semi-definite:

$$\nabla_{\Delta_k^*}^2 L(\Delta_k^{opt}, \lambda^{opt}) = -\lambda^{opt} (\text{vec}[\mathbf{I}_N] \text{vec}[\mathbf{I}_N])^T \preceq 0. \quad (3.46)$$

□

Using the above results, we have

$$\begin{aligned} \min_{\|\Delta_k\| \leq \epsilon_k} \text{tr}[(\tilde{\mathbf{H}}_k + \Delta_k)\mathbf{W}_k] &= \text{tr}[\tilde{\mathbf{H}}_k \mathbf{W}_k] - \epsilon_k \|\mathbf{W}_k\|, \\ \max_{\|\Delta_k\| \leq \epsilon_k} \text{tr}[(\tilde{\mathbf{H}}_k + \Delta_k)\mathbf{W}_i] &= \text{tr}[\tilde{\mathbf{H}}_k \mathbf{W}_i] + \epsilon_k \|\mathbf{W}_i\|. \end{aligned}$$

So we may rewrite (3.30) as

$$\text{tr}\left[\tilde{\mathbf{H}}_k \left(\mathbf{W}_k - \gamma_k \sum_{i=1 \neq k} \mathbf{W}_i\right)\right] - \epsilon_k \left(\|\mathbf{W}_k\| + \gamma_k \sum_{i=1 \neq k} \|\mathbf{W}_i\|\right) \geq \gamma_k \sigma_n^2. \quad (3.47)$$

### 3.4.2 The Whole Program

Similarly, the IP constraints may be written as:

$$\max_{\|\Lambda_\ell\| \leq \xi_k} \sum_{k=1}^K \text{tr}[(\tilde{\mathbf{G}}_\ell + \Lambda_\ell)\mathbf{W}_k] = \sum_{k=1}^K \left(\text{tr}[\tilde{\mathbf{G}}_\ell \mathbf{W}_k] + \xi_\ell \|\mathbf{W}_k\|\right) \leq \kappa_\ell. \quad (3.48)$$

Finally, the whole program is summarized as follows.

$$\begin{aligned} &\underset{\{\mathbf{W}_k\}_{k=1}^K}{\text{minimize}} && \sum_{k=1}^K \text{tr}[\mathbf{W}_k] \\ &\text{subject to} && \text{tr}\left[\tilde{\mathbf{H}}_k \left(\mathbf{W}_k - \gamma_k \sum_{i=1 \neq k} \mathbf{W}_i\right)\right] - \\ &&& \epsilon_k \left(\|\mathbf{W}_k\| + \gamma_k \sum_{i=1 \neq k} \|\mathbf{W}_i\|\right) \geq \sigma_n^2 \gamma_k, \\ &&& k = 1, \dots, K; \\ &&& \sum_{k=1}^K \left(\text{tr}[\tilde{\mathbf{G}}_\ell \mathbf{W}_k] + \xi_\ell \|\mathbf{W}_k\|\right) \leq \kappa_\ell, \quad \ell = 1, \dots, L; \\ &&& \mathbf{W}_k = \mathbf{W}_k^\dagger, \quad k = 1, \dots, K \\ &&& \mathbf{W}_k \succeq 0, \quad k = 1, \dots, K. \end{aligned} \quad (3.49)$$

Although this final problem is not an SDP, it is in fact convex, because its objective function and constraints are sum of matrix traces and norms which are convex themselves, and this problem can be solved using standard numerical optimization packages, like CVX [133].

### 3.5 Exact Robust Solution (ExRS)

The last two sections considered the problem of minimizing the uncertain SINR using two conservative methods. In this section the exact worst-case channel realization is considered instead. We start again with the problem of (3.18), but with a simple alteration. This problem is stated as:

$$\begin{aligned}
& \underset{\{\mathbf{W}_k\}_{k=1}^K}{\text{minimize}} && \sum_{k=1}^K \text{tr}[\mathbf{W}_k] \\
& \text{subject to} && \min_{\|\Delta_k\| \leq \epsilon_k} \text{tr}[(\tilde{\mathbf{H}}_k + \Delta_k)\mathbf{W}_k] - \\
& && \gamma_k \sum_{i=1 \neq k}^K \text{tr}[(\tilde{\mathbf{H}}_k + \Delta_k)\mathbf{W}_i] \geq \sigma_n^2 \gamma_k, \quad k = 1, \dots, K; \\
& && \max_{\|\Lambda_\ell\| \leq \xi_\ell} \sum_{k=1}^K \text{tr}[(\tilde{\mathbf{G}}_\ell + \Lambda_\ell)\mathbf{W}_k] \leq \kappa_\ell, \quad \ell = 1, \dots, L.
\end{aligned} \tag{3.50}$$

In the above problem, we try to minimize the SINR directly without using conservative assumptions. First we have the following proposition:

**Proposition 3.2.** *The minimizer of the first constraint of (3.50) has the form of*

$$\Delta_k^{\min} = -\epsilon_k \frac{\left( \mathbf{W}_k - \gamma_k \sum_{i=1 \neq k}^K \mathbf{W}_i \right)^\dagger}{\|\mathbf{W}_k - \gamma_k \sum_{i=1 \neq k}^K \mathbf{W}_i\|}. \tag{3.51}$$

*Proof.* The Lagrangian multiplier is adopted again:

$$\begin{aligned}
L(\Delta_k, \lambda) = & \text{tr}[(\tilde{\mathbf{H}}_k + \Delta_k)\mathbf{W}_k] - \\
& \gamma_k \sum_{i=1 \neq k}^K \text{tr}[(\tilde{\mathbf{H}}_k + \Delta_k)\mathbf{W}_i] + \lambda(\text{tr}[\Delta_k \Delta_k^\dagger] - \epsilon_k^2).
\end{aligned} \tag{3.52}$$

By differentiating this function and equating it with zero,

$$\nabla_{\Delta_k^*} L(\Delta_k, \lambda) = \mathbf{W}_k^\dagger - \gamma_k \sum_{i=1 \neq k}^K \mathbf{W}_i^\dagger + \lambda \Delta_k = 0, \tag{3.53}$$

we will come up with

$$\Delta_k^{opt} = -\frac{1}{\lambda} \left( \mathbf{W}_k - \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{W}_i \right)^\dagger, \quad (3.54)$$

and to eliminate the  $\lambda$ ,

$$\nabla_\lambda L(\Delta_k, \lambda) = \|\Delta_k\| - \epsilon_k = 0, \quad (3.55)$$

we will get

$$\lambda^{opt} = \frac{1}{\epsilon_k} \|\mathbf{W}_k - \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{W}_i\|, \quad (3.56)$$

$$\Delta_k^{opt} = -\epsilon_k \frac{\left( \mathbf{W}_k - \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{W}_i \right)^\dagger}{\|\mathbf{W}_k - \gamma_k \sum_{\substack{i=1 \\ i \neq k}}^K \mathbf{W}_i\|}. \quad (3.57)$$

The second order differential test to prove that this solution belongs to a minimum, in this case, is also straight forward and is not included here.  $\square$

Finally the final and general problem is formulated:

$$\begin{aligned} & \underset{\{\mathbf{W}_k\}_{k=1}^K}{\text{minimize}} && \sum_{k=1}^K \text{tr}[\mathbf{W}_k] \\ & \text{subject to} && \text{tr} \left[ \tilde{\mathbf{H}}_k \left( \mathbf{W}_k - \gamma_k \sum_{i=1 \neq k}^K \mathbf{W}_i \right) \right] - \\ & && \epsilon_k \|\mathbf{W}_k - \gamma_k \sum_{i=1 \neq k}^K \mathbf{W}_i\| \geq \sigma_n^2 \gamma_k, \quad k = 1, \dots, K; \\ & && \sum_{k=1}^K \left( \text{tr} \left[ \tilde{\mathbf{G}}_\ell \mathbf{W}_k \right] + \xi_\ell \|\mathbf{W}_k\| \right) \leq \kappa_\ell, \quad \ell = 1, \dots, L; \\ & && \mathbf{W}_k = \mathbf{W}_k^\dagger, \quad k = 1, \dots, K \\ & && \mathbf{W}_k \succeq 0, \quad k = 1, \dots, K. \end{aligned} \quad (3.58)$$

The above problem is the most general form of the original problem. It should be noted that the beamforming weights are also the principal eigenvector of the solutions of this problem. Also it should be mentioned that the IP part of these two last problems, (3.49) and (3.58), are the same, i.e., these two problems have the same performance in IPs.

### 3.6 Simulation Results and Discussions

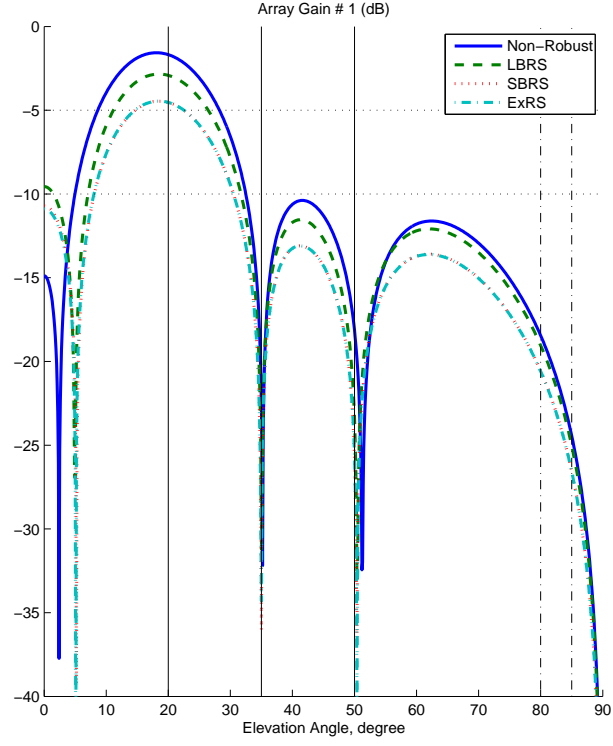
Simulations are carried out to validate the performance of the developed methods in this chapter. It is assumed that the BS is equipped with a Uniform Linear Array (ULA) with 8 elements with a spacing of half wave length. Three SU-Rx's ( $K = 3$ ) are served and the CR-Net should protect two PU-Rx's ( $L = 2$ ). The SU-Rx's are located in the directions of  $\theta_1 = 20^\circ$ ,  $\theta_2 = 35^\circ$  and  $\theta_3 = 50^\circ$  relative to the antenna boreside, respectively. The PU-Rx's are located at the directions of  $80^\circ$  and  $85^\circ$  from boresight, respectively. It is assumed that the change in Direction of Arrival (DoA) of input waves to the SU-Tx may be up to  $\pm 5^\circ$  arbitrarily. The noise power is assumed to be  $\sigma_n^2 = 0.01$ , and identical for all of the users. Also, a constant SINR level of 10dB is targeted for all the SUs, while the interference threshold of 0.01 is used to protect the PUs. The channel for PUs and SUs are modeled by

$$[\mathbf{h}_k(\theta_k)]_i = e^{j\pi(i-1)\cos(\theta_k)}, \quad i, k = 1, \dots, K, \quad (3.59)$$

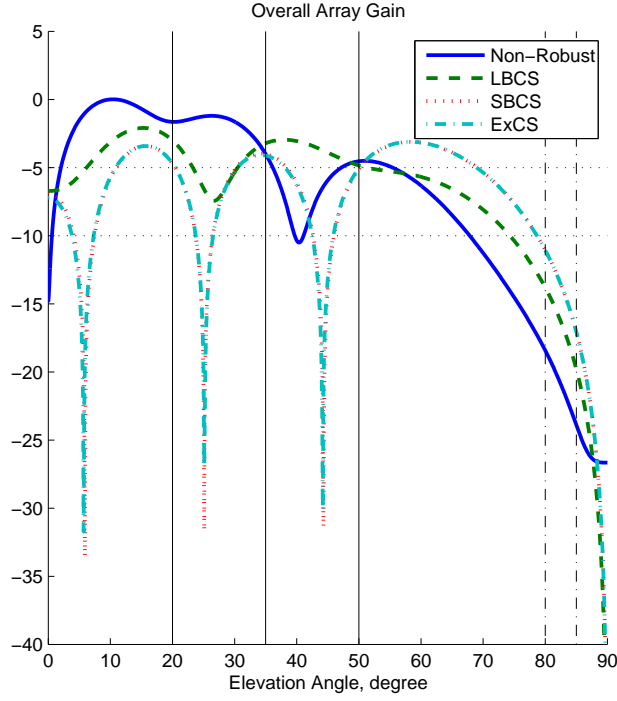
$$[\mathbf{g}_\ell(\phi_\ell)]_i = e^{j\pi(i-1)\cos(\phi_\ell)}, \quad i, \ell = 1, \dots, L. \quad (3.60)$$

The uncertainty sets are characterized with  $\epsilon_k = \eta_\ell = 0.05$ . We have used the CVX Software Package [133] to solve the proposed problems numerically. On a personal computer each run takes about 2-3 seconds on average. For rapidly changing environments like urban areas, this is not fast enough. However, when implemented with optimized code in a dedicated DSP chip the computation time can be significantly reduced.

In the subsequent figures, LBRS, SBRS and ExRS denote Loosely Bounded Robust Solution, Strictly Bounded Robust Solution, and Exact Robust Solution, respectively. The “Non-Robust” beamformer is a beamformer that assumes no uncertainty in CSI. This beamformer is designed based on the nominal value of CSI. During the testing phase of Monte Carlo simulations, these beamformers are used to transmit over channels with some variations. These variations are bounded by the limitations of the simulation scenario. For the non-robust case, this would lead to a severe deficiency in the performance of the link, especially when we are



(a) Array gain for SU#1



(b) Overall array gain

Fig. 3.3: Array gain for different users

concerned about violation of constraints. In Fig. 3.3, the vertical solid lines show the DoA corresponding to different SU-Rx's, while the vertical dashed lines show the DoA of PU-Rx's. From the figure, it is clear that the proposed method can transmit the desired data to the SU-Rx's while protecting the PU-Rx's. It is also apparent that all three approaches produce similar results. The array pattern of non-robust beamformers are calculated using conventional array design to form the main lobe toward the direction of interest and preserve deep nulls toward the known directions of other users. As can be seen, when the main lobe is toward the user of interest, the other users are in deep nulls with at least 20 dB of attenuation. For the first SU-Rx, the difference between the array gains is about 3 dB while for the second and third users the array gains are almost the same for different methods. Overall Fig. 3.3-b shows that the proposed solutions can cover the SU-Rx's while protecting PU-Rx's, i.e., the beamforming gains are less than -10 dB.

In Fig. 3.4, the histogram of normalized constraints for the SU-Rx is plotted. The x-axis shows the normalized SINR resulted from optimization of the network and y-axis shows the number of occurrences of each SINR. The normalized constraints of each SU,  $C_k^{(\text{sinr})}$ , defined as  $\text{SINR}_k/\gamma_k$ , is equivalent to

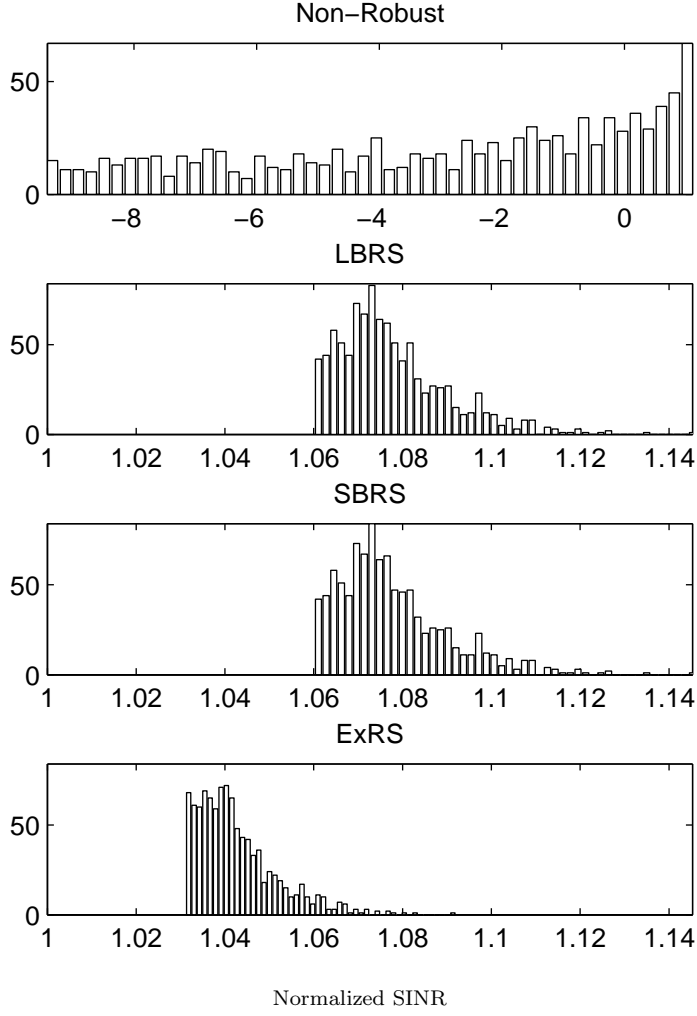
$$C_k^{(\text{sinr})} = \frac{1}{\sigma_n^2 \gamma_k} \mathbf{w}_k^\dagger \mathbf{H}_k \mathbf{w}_k - \frac{1}{\sigma_n^2} \sum_{i=1 \neq k}^K \mathbf{w}_i^\dagger \mathbf{H}_k \mathbf{w}_i,$$

where  $\mathbf{H}_k = \mathbf{h}_k \mathbf{h}_k^\dagger$ , and the normalized PU constraint,  $C_k^{(\text{ip})}$ , is defined as

$$C_\ell^{(\text{ip})} = \frac{1}{\kappa_\ell} \sum_{k=1}^K \mathbf{g}_\ell^\dagger \mathbf{H}_k \mathbf{g}_\ell.$$

Unlike the normalized SINR constraints for a SU-Rx, when a normalized IP constraint is less than one, this constraint is considered to be satisfied. In Fig. 3.4 the normalized SINR histograms for different methods are depicted. In this figure, the uncertainty sets are chosen to be  $\epsilon_k = \xi_\ell = 0.05$ . It is clear that ExRS is outperforming the other two schemes due to its exact bounds on SINR. For brevity, the IP diagrams are not shown here. As it can be seen, the robust design is immune to the variation of the channels, whereas the non-robust design fails





**Fig. 3.4:** Normalized SINR constraints for different methods for SU#1

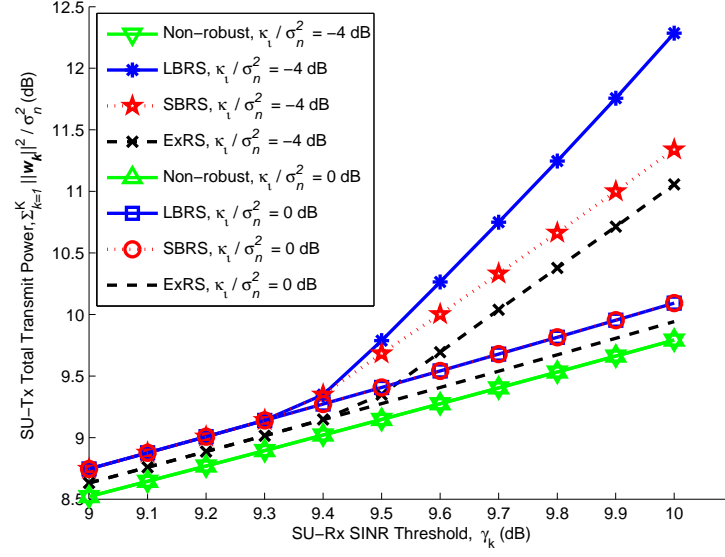
in such situations. It is also apparent that for the robust case, the variation of normalized constraints is much less than that for the non-robust case. It is also clear that SBRS and ExRS are more efficient in terms of handling the SINR. The ExRS method not only can satisfy all the constraints, but also requires less transmit power to achieve the same SINR because it uses exact minimum of SINR. Additionally, it should be mentioned that they are slightly better than the LBRS.

In Fig. 3.5 the normalized total transmit power versus the SINR thresholds for different amounts of allowed normalized IP is shown. The normalized total transmit power is the ratio of total transmit power to the noise power and the normalized IP is defined in the same manner. Both quantities are dimension-less and for better clarity are displayed in dBs. As expected, ExRS is better than

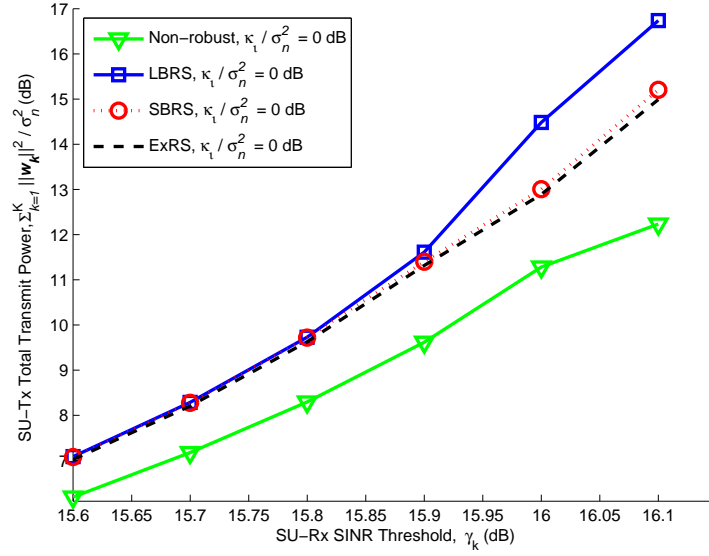
the other two methods. In Fig. 3.5-a, it is clear that for a relative IP level of  $\kappa_\ell = -4$  dB, ExRS transmits the lowest amount of power while SBRS requires to transmit a modest amount of power relative to LBRS, and finally LBRS requires the largest amount of power. In this figure, it is also observed that for a relative IP level of  $\kappa_\ell = 0$  dB, the proposed ExRS is the best scheme to use to transmit. In Fig. 3.5-b, we have plotted the same graph but in the higher SINR values. In this range of SINR thresholds, all of the optimization problems with a relative IP level of  $\kappa_\ell = -4$  dB would be infeasible, so the graph is only provided for the relative IP level of  $\kappa_\ell = 0$  dB. It is clear that in such scenarios, ExRS performs the best, although it should be noted that the performance of SBRS and ExRS are very close to each other.

In this figure the transmit power for the non-robust case is plotted which is smaller than the transmit power of robust cases, which is the price of robustness. If the beamformers are designed to act robustly, they will send more power than required to guarantee the performance. For low SINR regime, non-robust methods transmit about 0.1 dB less power relative to robust methods, while in high SINR regime, this gap widens, i.e., the transmit power of a non-robust method is about 1 dB less than the robust methods.

Before concluding this chapter, it is noteworthy to compare the proposed methods in terms of the complexity and their computational burden. Since we have found the analytical solutions for the worst channel realization with different approximation levels, although our contribution outperforms the other first two methods, they are performing similarly in terms of the actual problem to be solved. All three problem formulations lead to three SDP problems with similar linear objectives and  $K + L$  linear/normed constraints in  $K$  symmetric positive definite matrices. All three problems lead to rank-1 solutions which waive the need to a randomization process. The last two methods contain matrix norms in their constraints with negligible computational effects comparing to internal iterations used by the solvers to solve the problems numerically.



(a) Low SINR regime



(b) High SINR regime

Fig. 3.5: The total Tx power vs. SINR thresholds

### 3.7 Conclusion

The design of robust downlink beamforming in multiuser MISO cognitive radio networks is studied. Particularly, a set up of  $K$  SU-Rx's and  $L$  PU-Rx's, all equipped with a single antenna is considered, and the SU-Tx has  $N$  transmit antennas. It is assumed that the relevant CSI is not perfectly known for both sets of users and uncertainty in the CSI is modeled using an Euclidean ball notation. Three different approaches, namely LBRS, SBRS and ExRS, are presented which can be implemented efficiently. The first solution is a SDP, while the later two solutions are the convex optimization problems. Simulation results have evaluated the robustness of proposed methods.

# Chapter 4

## Robust Transceiver Design in MIMO Ad Hoc CR-Nets

### 4.1 Introduction

To incorporate the imperfect CSI and to provide robust designs, most works use the worst case design method in which the beamformers, or precoders/decoders, are designed under the worst possible operating circumstances of the system. For example, narrow-band transmit beamforming in broadcast channels is the focus of [7] and [8]. After introducing a relaxation scheme, the authors converted the problem into a Semi-Definite Program (SDP).

The robust design for Broadcast Channels (BC) is discussed in [135] and [38]. In [135], the authors formulated the problem using different objectives and proposed an iterative solution to find the precoder and decoder in a MIMO configuration. They formulated Linear Matrix Inequalities (LMI), which lead to SDP problems. Additionally, they formulated the uncertainty of the channel using the SE model as well. In [38], the authors use a similar method to solve the same problem. It should be mentioned that this problem using nonlinear precoders, is also considered in [136]. For a single user system with uncertain CSI in the MIMO configuration, the Mean Square Error (MSE) performance measure is considered in [17] and [18]. Solving the problem of robust cognitive beamforming with partial CSI is the aim of [71]. There, a single SU-Tx—SU-Rx network interfering with a single PU-Rx in which both receivers are equipped with a single antenna is assumed. In [137] the transceivers are equipped with multiple antennas, but it is

assumed that there is only one receiver in the CR-Net which limits the practicality of this work. Finally, [138] aims to design a MSE based broadcast channel impaired by imperfect CSI using a duality result by solving the equivalent multiple access channel network.

The MIMO ad hoc network is discussed in [82],[79]. In [82] optimal resource allocation for MIMO ad hoc cognitive radio networks is presented. The objective function is the weighted sum-rate capacity of the secondary users equipped with MIMO transmitters and receivers. A semi-distributed algorithm for the optimum design of such a system is presented as well. Targeting the same problem with a very similar formulation, a game theoretic approach is proposed in [81]. The authors propose a strategic non-cooperative game to optimize the MIMO CR-net. Finally, weighted sum-rate scheduling for MIMO ad hoc networks is the focus of [79]. The authors use a duality-optimization based framework to design the network.

The major contribution of this chapter is the robust transceiver design of a MIMO ad hoc CR-Net with uncertain CSI for all important links. To the best of the authors' knowledge this has not been published before. The formulated design problem minimizes the Sum Mean Square Error (SMSE) of the interfering links while power budget and interfering power constraints are applied to the system. Both SE and NBE error models are considered. For the first case, the design problem is a Second Order Cone Program (SOCP) while for the second case, the problem is recast as a SDP [92]-[93]. The problem is difficult to solve regardless of the choice of error model, because the objective function is not simultaneously convex in the design variables and also there are infinitely many constraints. We propose an iterative solution to solve the relaxed version of the original problem. The introduced SOCP and SDP problems are solved using the standard numerical programming package, YALMIP [139], using the general solver SDPT3 [140].

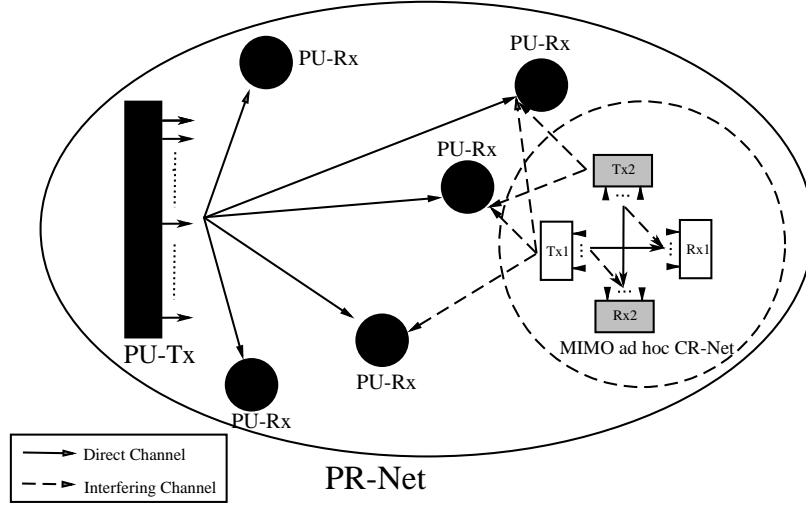
This chapter addresses the problem of joint design of optimal precoders and decoders of a MIMO ad hoc CR-Net in which each node is equipped with multiple antennas. An ad hoc network is depicted in Fig. 4.1. In this network a set of

$K$  independently interfering links coexist. It is assumed that the Tx and the Rx sides of each link is equipped with multiple antennas. It is also assumed that the CSI for all relevant links is not known perfectly. The imperfectness of the CSI is modeled using two different models: the SE model, and the NBE model. In the SE model, it is assumed that the perturbation of the CSI is a sample of a stochastic process which is defined using its first and second moments, while in the NBE model, which is a deterministic model of CSI uncertainty, the perturbation is also deterministic and is appropriately norm-bounded.

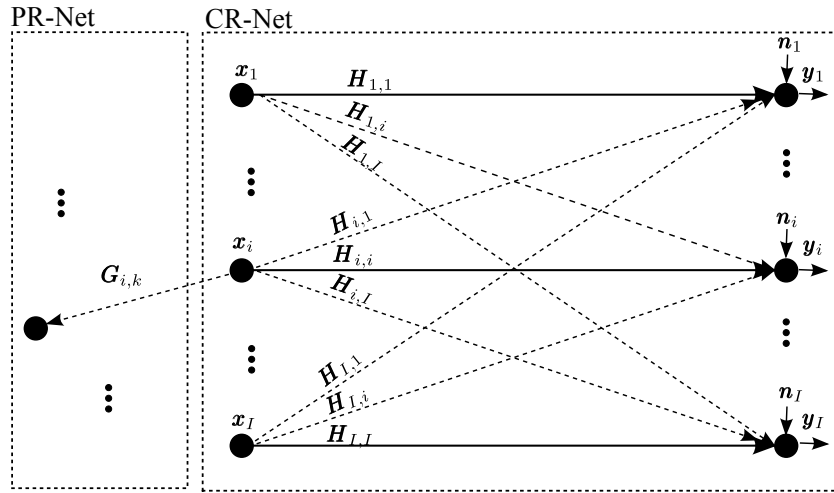
The rest of this chapter is organized as follows: our system model and problem formulation are introduced in Sections 4.2 and 4.3, respectively. A general framework is provided to model the problem regardless of the type of uncertainty. In Section 4.4 it is showed that using the SE model, our problem would be a SOCP. Also in Section 4.5 our problem is tackled focusing on NBE model and it is showed that this problem is a SDP. In Section 4.6 the simulation results are depicted and discussed while finally the chapter is concluded in Section 4.7.

## 4.2 System Model

We consider a MIMO ad hoc CR-Net which consists of  $K$  independent but interfering links (Fig. 4.1). This network is installed within the coverage range of a PR-Net. It is assumed that the studied CR-Net is installed far enough from PU-Tx to gain the spectrum opportunity. In this case, the interfering power of PU-Tx for SU-Rxs is considered as accumulated with their receive noise. This study only covers a single cell PR-Net. For multi-cell PR-Nets this assumption should be valid for all the cells under consideration. In this network, in each link both Tx and Rx sides are equipped with multiple antennas, i.e., the  $i$ th SU-Tx and SU-Rx have  $T_i$  and  $R_i$  transmit and receive antennas, respectively. In each link the SU-Tx should transmit a stream of  $t_i$  symbols to its intended receiver, SU-Rx ( $t_i \leq \min(T_i, R_i)$ ). The signal vector of  $i$ th link is denoted as  $\mathbf{s}_i \in \mathbb{C}^{t_i}$ . The transmit symbols are assumed to be complex, zero mean, independent and



(a) Network Configuration Perspective



(b) Signal Flow Graph Perspective

Fig. 4.1: Overall diagram of an ad hoc MIMO cognitive radio network.



identically distributed

$$\mathbf{E}_{\mathbf{s}_i} [\mathbf{s}_i] = \mathbf{0}, \quad (4.1)$$

$$\mathbf{E}_{\mathbf{s}_i} [\mathbf{s}_i \mathbf{s}_j^*] = \begin{cases} \sigma_{s_i}^2 \mathbf{I}_{t_i}, & i = j, \\ \mathbf{0}, & i \neq j. \end{cases} \quad (4.2)$$

At the receiver, the received signal vector is

$$\mathbf{y}_i = \mathbf{H}_{i,i} \mathbf{x}_i + \sum_{j=1 \neq i}^K \mathbf{H}_{j,i} \mathbf{x}_j + \mathbf{n}_i \in \mathbb{C}^{R_i}, \quad (4.3)$$

where  $\mathbf{H}_{i,j} \in \mathbb{C}^{R_j \times T_i}$  is CSI from  $i$ th SU-Tx to  $j$ th SU-Rx and  $\mathbf{n}_i \in \mathbb{C}^{R_i}$  is the additive white Gaussian noise samples,

$$\mathbf{E}_{\mathbf{n}_i} [\mathbf{n}_i] = \mathbf{0}, \quad (4.4)$$

$$\mathbf{E}_{\mathbf{n}_i} [\mathbf{n}_i \mathbf{n}_i^*] = \sigma_n^2 \mathbf{I}_{R_i}. \quad (4.5)$$

This network is installed within the service range of a  $L$ -user PR-Net, for which the CR-Net should provide some kind of protection to the PR-Net. Each PU-Rx is equipped with  $R'_k$  receive antennas. The received signal at the  $k$ th PU-Rx is then

$$\mathbf{w}_k = \sum_{i=1}^K \mathbf{G}_{i,k} \mathbf{x}_i, \quad (4.6)$$

where  $\mathbf{G}_{i,k} \in \mathbb{C}^{R'_k \times T_i}$  is the CSI from  $i$ th SU-Tx to  $k$ th PU-Rx.

The CSI for both SU-Tx-SU-Rx and SU-Tx-PU-Rx is assumed to be imperfectly known, i.e., there are some kind of uncertainty in CSI for both sets of links.

$$\mathbf{H}_{i,j} = \tilde{\mathbf{H}}_{i,j} + \mathbf{\Delta}_j, \quad (4.7)$$

$$\mathbf{G}_{i,k} = \tilde{\mathbf{G}}_{i,k} + \mathbf{\Lambda}_k, \quad (4.8)$$

where  $\tilde{\mathbf{H}}_{i,j}$ ,  $\tilde{\mathbf{G}}_{i,k}$ ,  $\mathbf{\Delta}_i$  and  $\mathbf{\Lambda}_k$  represent the nominal value of the CSI and the perturbations, respectively. Here we model these uncertainty with the SE and NBE models using the following parameters. In NBE model,

$$\|\mathbf{\Delta}_j\|_F \leq \delta_j, \quad (4.9)$$

$$\|\mathbf{\Lambda}_k\|_F \leq \lambda_k, \quad (4.10)$$

where  $\delta_i$  and  $\lambda_k$  represent the uncertainty bounds, respectively. The SE model is also characterized as follows:

$$\begin{cases} \mathbb{E}_{\Delta_j} [\Delta_j] = 0, \\ \mathbb{E}_{\Delta_j} [\text{vec} [\Delta_j] \text{vec} [\Delta_j]^*] = \Sigma_{\Delta_j}, \end{cases} \quad (4.11)$$

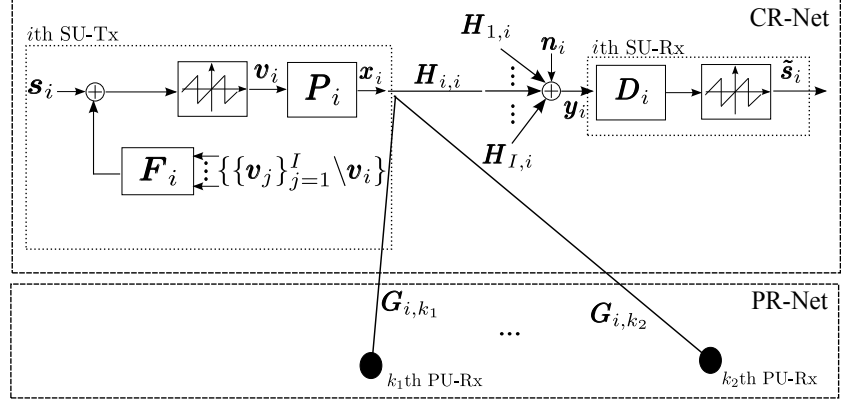
$$\begin{cases} \mathbb{E}_{\Lambda_k} [\Lambda_k] = 0, \\ \mathbb{E}_{\Lambda_k} [\text{vec} [\Lambda_k] \text{vec} [\Lambda_k]^*] = \Sigma_{\Lambda_k}. \end{cases} \quad (4.12)$$

Here we assume that all the entries of these matrices are independent, i.e.,  $\Sigma_{\Delta_k} = \sigma_{\Delta}^2 \mathbf{I}_{R_k T_i}$  and  $\Sigma_{\Lambda_k} = \sigma_{\Lambda}^2 \mathbf{I}_{R'_k T_i}$ .

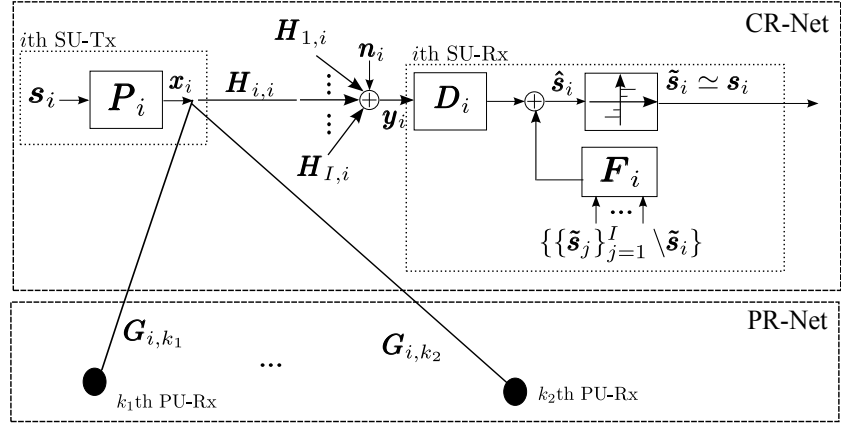
In this chapter we use a MSE measure to design the transceivers. Upper limits on both transmit power of SU-Tx's and interfering power of PU-Rx's are also imposed.

### 4.2.1 Beamforming

For the beamforming we consider both THP and DFE schemes, as well as the linear beamforming [15],[141]. To the best of our knowledge, these two non-linear configurations outperform the conventional linear precoder and decoders. It is also found in the simulation results section. Based on implementation complexity, we can compare these three beamforming methods. The linear beamforming is the simplest way of beamforming in both BS and MS while in terms of the system performance it is usually treated as a suboptimal solution to the transmission/detection problem. The THP scheme is designed to approximate the dirty paper coding scheme of Costa [142] which is known to be an optimum solution in interference-limited scenarios. In this scheme, the complexity and computational burden is on the BS side, which is assumed to have enough processing power in the system. While the DFE performs the best among the others, it has a severe drawback. In the DFE scheme, the computational burden is on the receiver side, i.e., the MS. Usually, it is hard to assume that the MS has enough processing power to do the interference cancellation. Unless the receivers themselves are relaying base stations or access points with enough processing power. Firstly, we focus on THP and DFE and then we will present the linear beamforming as a special case of the DFE. For a general schematic view of each of them please refer to Fig. 4.2.



(a) THP Scheme



(b) DFE Scheme

**Fig. 4.2:** Signal flow graph in an ad hoc MIMO cognitive radio network using THP and DFE.

The detail of each treatment is as follows.

### The Combination of THP and the Linear Equalizer

In this scheme, we use a nonlinear transmitter with pre-cancellation involved in terms of a linear feedback matrix. Relying on the known feedback signals from the other transmitters, the amount of the imposed interference on the other receivers are approximated and canceled subsequently using a linear filter, which we optimize here. The signal flow graph of this scheme is described as follows. For a detailed treatment of this method, the interested reader should consult standard texts like [141]. In this scheme, the transmit symbol,  $\mathbf{s}_i$  is linearly combined with the feedback signals from the other links, i.e.,  $\{\mathbf{v}_j \in \mathbb{C}^{t_j}\}_{j=1 \neq i}^K$ . The resultant signal would be the feed back to the other links.

$$\mathbf{v}_i = \mathbf{s}_i - \sum_{j=1 \neq i}^K \mathbf{F}_{i,j} \mathbf{v}_j \quad (4.13)$$

where  $\mathbf{F}_{i,j} \in \mathbb{C}^{t_i \times t_j}$  is the feedback matrix between the  $i$ th and  $j$ th links. It is possible to rewrite the aforementioned equation to get a straightforward one which states  $\mathbf{s}_i$  based on all feedback signals  $\{\mathbf{v}_j\}_{j=1}^K$ .

$$\mathbf{s}_i = \mathbf{v}_i + \sum_{j=1 \neq i}^K \mathbf{F}_{i,j} \mathbf{v}_j \quad (4.14)$$

To get the transmit symbol, i.e.,  $\mathbf{x}_i \in \mathbb{C}^{T_i}$ ;  $\mathbf{v}_i$  is linearly precoded by  $\mathbf{P}_i \in \mathbb{C}^{T_i \times t_i}$ , i.e.,

$$\mathbf{x}_i = \mathbf{P}_i \mathbf{v}_i. \quad (4.15)$$

Interestingly, after imposing some mild conditions on the constellation of  $\mathbf{s}_i$  and its distribution, it is possible to assume that the feedback signals as a result of the modulo operation are also zero-mean and independent of each other over the bounded signaling space [141], with a slightly higher energy relative to the original symbols [141].

$$\mathbb{E}_{\mathbf{v}_i} [\mathbf{v}_i \mathbf{v}_j^*] = \begin{cases} \sigma_{\mathbf{v}_i}^2 \mathbf{I} & i = j \\ \mathbf{0} & \text{otherwise,} \end{cases} \quad (4.16)$$

where by assumption  $\sigma_{\mathbf{v}_i}^2 \geq \sigma_{\mathbf{s}_i}^2$ .

In the receiver, the received signal vector,  $\mathbf{y}_i$ , is linearly equalized using a matrix  $\mathbf{D}_i \in \mathbb{C}^{t_i \times R_i}$  to get the finally estimated vector:

$$\mathbf{z}_i = \mathbf{D}_i \mathbf{y}_i \quad (4.17)$$

### The Combination of the Linear Precoding and DFE

In this scheme, the complexity comes in terms of a more sophisticated receiver. Unlike the THP scheme, in DFE there is no attempt to pre-cancel the imposed interference. In DFE scheme, the interference is actively canceled at the receiver side. Receivers in this scheme, cooperate by changing their detected signals, and each of them uses these received signals to cancel the amount of the received interference. In this case also, interested readers should consult standard texts like [141]. As expected from the point to point systems, the DFE scheme outperforms the THP one. The signal flow description of this scheme is as follows. In this scheme it is assumed that the precoder has a linear structure. The transmit symbols for  $i$ th SU-Tx,  $\mathbf{x}_i$ , is

$$\mathbf{x}_i = \mathbf{P}_i \mathbf{s}_i \quad (4.18)$$

where  $\mathbf{P}_i \in \mathbb{C}^{T_i \times t_i}$  is the linear precoder matrix. The received signal at the receiver is also first linearly equalized using  $\mathbf{D}_i \in \mathbb{C}^{t_i \times R_i}$  and then the resultant signal is fed to a non-linear block having a series of feed-back filters,  $\mathbf{F}_i = [\mathbf{F}_{i,1}^T, \dots, \mathbf{F}_{i,K}^T]^T$ , resulting in a temporary signal  $\hat{\mathbf{s}}_i$  where,

$$\hat{\mathbf{s}}_i = \mathbf{D}_i \mathbf{y}_i + \sum_{j=1 \neq i}^K \mathbf{F}_{i,j} \tilde{\mathbf{s}}_j. \quad (4.19)$$

where  $\tilde{\mathbf{s}}_i$  is the output signal of the equalizer. As it is clear, to have the feedback signals for both THP and DFE cases, some sort of collaboration between users is needed and assumed.

To characterize the system, and regardless of its configuration, the following

performance measures for each link are required:

$$\text{TxP}_i = \mathbb{E}_{\mathbf{s}_i} [\mathbf{x}_i^* \mathbf{x}_i], \quad \forall i \quad (4.20)$$

$$\text{MSE}_i = \begin{cases} \mathbb{E}_{\mathbf{s}_i} [\|\hat{\mathbf{s}}_i - \mathbf{s}_i\|^2] & \text{for DFE} \\ \mathbb{E}_{\mathbf{s}_i} [\|\tilde{\mathbf{s}}_i - \mathbf{s}_i\|^2] & \text{for THP} \end{cases} \quad \forall i \quad (4.21)$$

$$\text{IP}_k = \mathbb{E}_{\mathbf{s}_i} [\mathbf{w}_k^* \mathbf{w}_k], \quad \forall k \quad (4.22)$$

where all the mathematical expectation are calculated considering the signal domain. In the following proposition, the mathematical expressions of these entities are revealed.

**Proposition 4.1.** *For the aforementioned system, the transmit power of  $i$ th link, and its MSE, and the interfering power on  $k$ th PU-Rx, is represented as*

$$\text{TxP}_i = \zeta_i \|\mathbf{P}_i\|_F^2, \quad (4.23)$$

$$\text{IP}_k = \sum_{i=1}^K \zeta_i \|\mathbf{G}_{i,k} \mathbf{P}_i\|_F^2, \quad (4.24)$$

$$\text{MSE}_i = \zeta_i \|\mathbf{D}_i \mathbf{H}_{i,i} \mathbf{P}_i - \mathbf{I}\|_F^2 + \sum_{j=1 \neq i}^K \zeta_j \|\mathbf{D}_i \mathbf{H}_{j,i} \mathbf{P}_j - \mathbf{F}'_{i,j}\|_F^2 + \sigma_{\mathbf{n}_i}^2 \|\mathbf{D}_i\|_F^2, \quad (4.25)$$

where

$$\zeta_i = \begin{cases} \sigma_{\mathbf{s}_i}^2 & \text{for DFE and Linear Precoding/Decoding} \\ \sigma_{\mathbf{v}_i}^2 & \text{for THP} \end{cases} \quad (4.26)$$

and

$$\mathbf{F}'_{i,j} = \begin{cases} \mathbf{0} & \text{for Linear Precoding/Decoding} \\ \mathbf{F}_{i,j} & \text{for DFE and THP} \end{cases} \quad (4.27)$$

*Proof.* We will study the proof for the DFE case. The proof for the THP is similar and will not be repeated here. For the  $i$ th link, the MSE is

$$\text{MSE}_i = \mathbb{E}_{\mathbf{s}_i} [\|\hat{\mathbf{s}}_i - \mathbf{s}_i\|^2] \quad (4.28)$$

$$= \mathbb{E}_{\mathbf{s}_i} \left[ \left\| \mathbf{D}_i \mathbf{H}_{i,i} \mathbf{P}_i \mathbf{s}_i + \sum_{j=1 \neq i}^K (\mathbf{D}_i \mathbf{H}_{j,i} \mathbf{P}_j - \mathbf{F}'_{i,j}) \mathbf{s}_j + \mathbf{D}_i \mathbf{n}_i - \mathbf{s}_i \right\|^2 \right] \quad (4.29)$$

$$= \mathbb{E}_{\mathbf{s}_i} \left[ \left( (\mathbf{D}_i \mathbf{H}_{i,i} \mathbf{P}_i - \mathbf{I}_{t_i}) \mathbf{s}_i + \sum_{j=1 \neq i}^K (\mathbf{D}_i \mathbf{H}_{j,i} \mathbf{P}_j - \mathbf{F}'_{i,j}) \mathbf{s}_j + \mathbf{D}_i \mathbf{n}_i \right)^* \times \right. \\ \left. \left( (\mathbf{D}_i \mathbf{H}_{i,i} \mathbf{P}_i - \mathbf{I}_{t_i}) \mathbf{s}_i + \sum_{j=1 \neq i}^K (\mathbf{D}_i \mathbf{H}_{j,i} \mathbf{P}_j - \mathbf{F}'_{i,j}) \mathbf{s}_j + \mathbf{D}_i \mathbf{n}_i \right) \right]. \quad (4.30)$$

This expression has nine terms. Those six terms which contains  $\mathbf{E}_{\mathbf{s}_i} [\mathbf{s}_i^* \mathbf{s}_j], i \neq j$  and  $\mathbf{E}_{\mathbf{s}_j, \mathbf{n}_i} [\mathbf{s}_j^* \mathbf{n}_i]$  would be zero. So it is possible to continue the aforementioned expressions as it follows:

$$\begin{aligned} \text{MSE}_i = & \mathbf{E}_{\mathbf{s}_i} [\mathbf{s}_i^* (\mathbf{D}_i \mathbf{H}_{i,i} \mathbf{P}_i - \mathbf{I}_{t_i})^* (\mathbf{D}_i \mathbf{H}_{i,i} \mathbf{P}_i - \mathbf{I}_{t_i}) \mathbf{s}_i] + \mathbf{E}_{\mathbf{n}_i} [\mathbf{n}_i^* \mathbf{B}_i^* \mathbf{B}_i \mathbf{n}_i] + \\ & \sum_{j=1 \neq i}^K \sum_{k=1 \neq i}^K \mathbf{E}_{\mathbf{s}_j} [\mathbf{s}_j^* (\mathbf{D}_i \mathbf{H}_{j,i} \mathbf{P}_j - \mathbf{F}'_{i,j})^* (\mathbf{D}_i \mathbf{H}_{k,i} \mathbf{P}_k - \mathbf{F}'_{i,k}) \mathbf{s}_j]. \end{aligned} \quad (4.31)$$

To simplify this result, Lemma 2.5 is used. Knowing the statistics of  $\mathbf{s}_i$  and applying this lemma bearing in mind that the double summation has only a non-zero value when  $k = j$ , we would have

$$\begin{aligned} \text{MSE}_i = & \sigma_{\mathbf{s}_i}^2 \text{tr} [(\mathbf{D}_i \mathbf{H}_{i,i} \mathbf{P}_i - \mathbf{K}_{t_i})^* (\mathbf{D}_i \mathbf{H}_{i,i} \mathbf{P}_i - \mathbf{K}_{t_i})] + \sigma_{\mathbf{n}_i}^2 \text{tr} [\mathbf{D}_i^* \mathbf{D}_i] + \\ & \sum_{j=1 \neq i}^K \sigma_{\mathbf{s}_j}^2 \text{tr} [(\mathbf{D}_i \mathbf{H}_{j,i} \mathbf{P}_j - \mathbf{F}'_{i,j})^* (\mathbf{D}_i \mathbf{H}_{j,i} \mathbf{P}_j - \mathbf{F}'_{i,j})] \quad (4.32) \\ \triangleq & \sigma_{\mathbf{s}_i}^2 \|\mathbf{D}_i \mathbf{H}_{i,i} \mathbf{P}_i - \mathbf{I}_{t_i}\|_F^2 + \sum_{j=1 \neq i}^K \sigma_{\mathbf{s}_j}^2 \|\mathbf{D}_i \mathbf{H}_{j,i} \mathbf{P}_j - \mathbf{F}'_{i,j}\|_F^2 + \sigma_{\mathbf{n}_i}^2 \|\mathbf{D}_i\|_F^2. \end{aligned} \quad (4.33)$$

Using similar procedures would result the following expressions for  $\text{TxP}_i$  and  $\text{IP}_k$ .

$$\text{TxP}_i = \sigma_{\mathbf{s}_i}^2 \|\mathbf{P}_i\|_F^2, \quad (4.34)$$

$$\text{IP}_k = \sum_{i=1}^K \sigma_{\mathbf{s}_i}^2 \|\mathbf{G}_{i,k} \mathbf{P}_i\|_F^2. \quad (4.35)$$

□

## Linear Beamforming

The linear precoding and decoding scheme is a special case of the DFE scheme and occurs when the feedback filter at the receiver is assumed to be zero, i.e.,  $\mathbf{F}_{i,j} = \mathbf{0}$ . So we will not present it here separately, however, since this model is a simpler model, the simulation results are mostly relying the linear beamforming.

## 4.3 Problem Formulation

As mentioned earlier, a MSE measure is used in the design procedure of our network. But depending on the type of uncertainty, the MSE of each link itself as

well as interference power of each PU-Rx, may be a stochastic or a deterministic process. To overcome this ambiguity, in the following formulation we will use a functional of the  $\text{MSE}_i$  and  $\text{IP}_k$  for each link. For either case of SE or NBE models, we will explicitly define this functional. For the SE model, the mathematical expectation is a good example while for the NBE model, the identity function is an appropriate example.

$$\begin{cases} \mathfrak{F}[x] = \mathbb{E}_x[x], & (\text{SE model}), \\ \mathfrak{F}[x] = x, & (\text{NBE model}). \end{cases} \quad (4.36)$$

To design this network, we put the functional of MSE of the independent links in a vector, say  $\mathbf{MSE}$ , i.e.,

$$\mathbf{MSE} = [\mathfrak{F}[\text{MSE}_1], \dots, \mathfrak{F}[\text{MSE}_K]]^T. \quad (4.37)$$

Our goal is to minimize this vector bearing in mind its uncertainty. To design the transceiver it is formulated as

$$\underset{\{\mathbf{A}_i, \mathbf{B}_i\}_{i=1}^K}{\text{minimize}} \quad \max_{\forall \mathbf{H}_{i,j}} \quad \mathbf{MSE} \quad (4.38a)$$

$$\text{subject to} \quad \text{TxP}_i \leq P_i, \quad \forall i, \quad (4.38b)$$

$$\mathfrak{F}[\text{IP}_k] \leq \kappa_k, \quad \forall \mathbf{G}_{i,k}, \quad \forall k, \quad (4.38c)$$

where  $P_i$  is the capped amount of power assigned to  $i$ th transmitter and  $\kappa_k$  is the upper bound of interference power which allowed to be imposed on PU-Rx. It is also possible to combine these two kinds of uncertainty for different constraints, but here we will be consistent in choosing either of them. This vector optimization problem can be treated by a mathematically standard technique, i.e., *Scalarization* [92]. Using this technique, the objective function of (4.38) would be  $\boldsymbol{\alpha}^T \mathbf{MSE} = \sum_{i=1}^K \alpha_i \mathfrak{F}[\text{MSE}_i]$  where  $\alpha_i$  is the  $i$ th element of  $\boldsymbol{\alpha}$ . Here again, for the sake of having a tractable mathematical formulation and also for simplicity, we assume that  $\boldsymbol{\alpha} = \mathbf{1}$ , where  $\mathbf{1}$  is vector having all its element equal to 1. So the objective function is simply the sum MSE of all the links. This problem finally would be

$$\underset{\{\mathbf{A}_i, \mathbf{B}_i\}_{i=1}^K}{\text{minimize}} \quad \max_{\forall \mathbf{H}_{i,j}} \quad \mathbf{1}^T \mathbf{MSE} \quad (4.39a)$$

$$\text{subject to} \quad \text{TxP}_i \leq P_i, \quad \forall i, \quad (4.39b)$$

$$\mathfrak{F}[\text{IP}_k] \leq \kappa_k, \quad \forall \mathbf{G}_{i,k}, \quad \forall k, \quad (4.39c)$$



Using epigraph form and introducing slack variables  $\tau_i$ , (4.39) is rewritten as

$$\begin{aligned} & \underset{\{\mathbf{A}_i, \mathbf{B}_i\}_{i=1}^K}{\text{minimize}} && \sum_{i=1}^K \tau_i && (4.40a) \end{aligned}$$

$$\text{subject to} \quad \text{TxP}_i \leq P_i, \quad \forall i, \quad (4.40b)$$

$$\mathfrak{F}[\text{IP}_k] \leq \kappa_k, \quad \forall \mathbf{G}_{i,j}, \quad \forall k, \quad (4.40c)$$

$$\mathfrak{F}[\text{MSE}_i] \leq \tau_i, \quad \forall \mathbf{H}_{i,k}, \quad \forall i. \quad (4.40d)$$

In subsequent sections, we will deal with this problem for SE and NBE models.

### 4.3.1 Conventional Problem Formulation

To the best of our knowledge, there is no published work on the minimum MSE joint design of the interfering networks, except [143]. There, a minimum MSE joint design is presented and, although is not supposed to cover the cognitive radio setup, it is used as a way to assess the performance of the robust methods. This problem formulation, using our notation, is simply reviewed as (refer to (27) in [143])

$$\begin{aligned} & \underset{}{\text{minimize}} && \sum_{i=1}^K \text{MSE}_i && (4.41a) \end{aligned}$$

$$\text{subject to} \quad \text{TxP}_i \leq P_i \quad \forall i. \quad (4.41b)$$

To cover the cognitive radio setup, the interfering power constraints, i.e.,  $\text{IP}_k \leq \kappa_k, \forall k$  should be added to this problem.

## 4.4 Robust Iterative Solution for SE model

Using the SE model, both  $\text{IP}_k$  and  $\text{MSE}_i$  are stochastic quantities. To come up with the an appropriate problem formulation and elimination of stochastic uncertainty we choose *mathematical expectation* as the aforementioned functional of

these quantities, i.e.,  $\mathfrak{F}[\cdot] \equiv \mathbb{E}[\cdot]$ . So our problem for SE model would be

$$\begin{aligned} & \underset{\{\mathbf{A}_i, \mathbf{B}_i\}_{i=1}^K}{\text{minimize}} && \sum_{i=1}^K \tau_i \end{aligned} \quad (4.42a)$$

$$\text{subject to} \quad \text{TxP}_i \leq P_i \quad \forall i, \quad (4.42b)$$

$$\mathbb{E}_{\mathbf{\Lambda}_k} [\text{IP}_k] \leq \kappa_k \quad \forall k, \quad (4.42c)$$

$$\mathbb{E}_{\Delta_i} [\text{MSE}_i] \leq \tau_i \quad \forall i. \quad (4.42d)$$

To start with this problem, we should find the exact expressions for  $\mathbb{E}_{\mathbf{\Lambda}_k} [\text{IP}_k]$  and  $\mathbb{E}_{\Delta_i} [\text{MSE}_i]$ .

**Proposition 4.2.** *For the aforementioned system using SE model for the uncertainty, and averaging over the CSI,  $\mathbb{E}_{\mathbf{s}_i} [\text{MSE}_i]$ ,  $\mathbb{E}_{\mathbf{s}_i} [\text{IP}_k]$  and have SOC structure:*

$$\mathbb{E}_{\mathbf{\Lambda}_k} [\text{IP}_k] \triangleq \|\boldsymbol{\nu}_k(\mathbf{P}_i)\|^2 \quad (4.43a)$$

$$\mathbb{E}_{\Delta_i} [\text{MSE}_i] \triangleq \|\boldsymbol{\mu}_i(\mathbf{D}_i, \mathbf{P}_i, \mathbf{F}_{i,j})\|^2 \quad (4.43b)$$

where

$$\boldsymbol{\nu}_k = \begin{bmatrix} \text{MAT} \left[ \left\{ \zeta_i \text{vec} \left[ \tilde{\mathbf{G}}_{i,k} \mathbf{P}_i \right] \right\}_{i=1}^K \right] \\ \text{MAT} \left[ \left\{ \zeta_i \sigma_{\mathbf{\Lambda}_k} \text{vec} \left[ \mathbf{P}_i^T \otimes \mathbf{I}_{R'_k} \right] \right\}_{i=1}^K \right] \end{bmatrix} \quad (4.44a)$$

$$\boldsymbol{\mu}_i = \begin{bmatrix} \zeta_i \text{vec} \left[ \mathbf{D}_i \tilde{\mathbf{H}}_{i,i} \mathbf{P}_i - \mathbf{I} \right] \\ \zeta_i \sigma_{\Delta_i} \text{vec} \left[ \mathbf{P}_i^T \otimes \mathbf{D}_i \right] \\ \text{MAT} \left[ \left\{ \zeta_j \text{vec} \left[ \mathbf{D}_i \tilde{\mathbf{H}}_{j,i} \mathbf{P}_j - \mathbf{F}_{i,j} \right] \right\}_{j=1 \neq i}^K \right] \\ \text{MAT} \left[ \left\{ \zeta_j \sigma_{\Delta_j} \text{vec} \left[ \mathbf{P}_j^T \otimes \mathbf{D}_i \right] \right\}_{j=1 \neq i}^K \right] \\ \sigma_{n_i} \text{vec} \left[ \mathbf{D}_i \right] \end{bmatrix} \quad (4.44b)$$

*Proof.* Using Proposition 4.1 and Lemma 2.4 the proof is straight forward.  $\square$

**Proposition 4.3.** *The problem of (4.42) may be expressed as a SOCP.*

*Proof.* Using Proposition 4.1 and the trick (2.29c), it is clear that all the constraints are in SOC structure. Having a linear objective function with SOC constraints would lead us to this conclusion that (4.42) is a SOCP. This problem is:

$$\begin{aligned} & \underset{\{\mathbf{A}_i, \mathbf{B}_i\}_{i=1}^K}{\text{minimize}} && \sum_{i=1}^K \tau_i \end{aligned} \quad (4.45a)$$

$$\text{subject to} \quad \|\text{vec}[\mathbf{P}_i]\|^2 \leq P_i, \quad \forall i, \quad (4.45b)$$

$$\|\boldsymbol{\nu}_k(\mathbf{P}_i)\|^2 \leq \kappa_k, \quad \forall k, \quad (4.45c)$$

$$\|\boldsymbol{\mu}_i(\mathbf{P}_i, \mathbf{D}_i, \mathbf{F}'_{i,j})\|^2 \leq \tau_i, \quad \forall i. \quad (4.45d)$$

□

The cost function in the epigraph form is composed of only the slack variables and is linear in them, so it is convex. The MSE, TxP and also IP, are all the norm of a multi-linear form composed of the filter variables, so they are also bi-convex as well. This problem, despite its appealing structure, is not simultaneously convex in both variables. To overcome this, an iterative procedure is proposed. This method has a known deficiency. To update the transmit side matrices we need the receive side information and vice versa. This deficiency is subject to both robust and conventional methods and cannot be solved in the physical layer due to non-causalities. This problem may be solved in higher layers and gives rise to the concept of the cooperation in communication [143]. In the following algorithm we will iterate on all of the variables. We will divide them into two disjoint sets, at each iteration, one set is fixed and the resultant convex problem (in terms of the remaining set of variables) is solved numerically as a sub-problem and in the next sub-problem the role of the variables are changed.

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**Algorithm 4.1.** [Iterative beamformer design for the SE model]

1. Let  $k \leftarrow 0$ .

Initialize the beamformer matrices  $\mathbf{P}_i^{[0]}$ ,  $\mathbf{D}_i^{[0]}$  and  $\mathbf{F}'_{i,j}^{[0]}$ .

Compute the initial MSE,  $\text{MSE}^{[0]} = \sum_{i=1}^2 \|\boldsymbol{\mu}_i(\mathbf{P}_i^{[0]}, \mathbf{D}_i^{[0]}, \mathbf{F}'_{i,j}^{[0]})\|^2$ .

2. Let  $k \leftarrow k + 1$ .

Update beamforming matrices using the following procedures:

(I) For The THP Scheme:

$$\{\mathbf{P}_i^{[n+1]}, \mathbf{F}'_{i,j}^{[n+1]}\} = \arg \min_{\mathbf{P}_i, \mathbf{F}_i} \sum_{i=1}^K w_i \tau_i \quad (4.46a)$$

$$\text{subject to } \sigma_{v_i}^2 \|\mathbf{P}_i\|^2 \leq \rho_i, \quad \forall i, \quad (4.46b)$$

$$\|\boldsymbol{\iota}_k(\mathbf{P}_i)\|^2 \leq \kappa_k, \quad \forall k, \quad (4.46c)$$

$$\left\| \boldsymbol{\mu}_i \left( \mathbf{P}_i, \mathbf{F}'_{i,j} \mid \mathbf{D}_i^{[n]} \right) \right\|^2 \leq \tau_i, \quad \forall i, \quad (4.46d)$$

and

$$\mathbf{D}_i^{[n+1]} = \arg \min_{\mathbf{D}_i} \sum_{i=1}^K w_i \tau_i \quad (4.47a)$$

$$\text{subject to } \left\| \boldsymbol{\mu}_i \left( \mathbf{D}_i \mid \mathbf{P}_i^{[n]}, \mathbf{F}'_{i,j}^{[n]} \right) \right\|^2 \leq \tau_i, \quad \forall i. \quad (4.47b)$$

(II) for the DFE Scheme:

$$\mathbf{P}_i^{[n+1]} = \arg \min_{\mathbf{P}_i} \sum_{i=1}^K w_i \tau_i \quad (4.48a)$$

$$\text{subject to } \sigma_{s_i}^2 \|\mathbf{P}_i\|^2 \leq \rho_i, \quad \forall i, \quad (4.48b)$$

$$\|\boldsymbol{\iota}_k(\mathbf{P}_i)\|^2 \leq \kappa_k, \quad \forall k, \quad (4.48c)$$

$$\left\| \boldsymbol{\mu}_i \left( \mathbf{P}_i \mid \mathbf{D}_i^{[n]}, \mathbf{F}'_{i,j}^{[n]} \right) \right\|^2 \leq \tau_i, \quad \forall i, \quad (4.48d)$$

and

$$\{\mathbf{D}_i^{[n+1]}, \mathbf{F}'_{i,j}^{[n+1]}\} = \arg \min_{\mathbf{D}_i, \mathbf{F}_i} \sum_{i=1}^K w_i \tau_i \quad (4.49a)$$

$$\text{subject to } \left\| \boldsymbol{\mu}_i \left( \mathbf{D}_i, \mathbf{F}'_{i,j} \mid \mathbf{P}_i^{[n]} \right) \right\|^2 \leq \tau_i, \quad \forall i, \quad (4.49b)$$

Update the MSE at this stage:  $\text{MSE}^{[k]} = \sum_{i=1}^2 \|\bar{\boldsymbol{\mu}}_i(\mathbf{P}_i^{[k]}, \mathbf{D}_i^{[k]}, \mathbf{F}'_{i,j})\|^2$ .

**3. Repeat step (2) until reaching a steady MSE, i.e.,  $|\text{MSE}^{[k]} - \text{MSE}^{[k-1]}| \leq \varepsilon$  or until the maximum number of iterations ( $K_{max}$ ) is reached.**

---

Since this algorithm is based on the ACS algorithm, it will converge, but the convergence is not proved here.

## 4.5 Robust Iterative Solution for NBE Model

For NBE model  $\text{MSE}_i$  and  $\text{IP}_k$  are deterministic. In this case we use the *identity function* as the functional of these quantities, i.e.,  $\mathfrak{F}[\text{MSE}_i] = \text{MSE}_i$  and  $\mathfrak{F}[\text{IP}_k] = \text{IP}_k$ .

The next proposition shows that these two quantities, although uncertain, can be expressed as the squared norm of two vectors.

**Proposition 4.4.** *The above mentioned quantities may be represented as quantities with SOC structure having infinitely many realizations:*

$$\text{IP}_k \triangleq \|\mathfrak{J}_k\|^2 \triangleq \|\tilde{\mathfrak{J}}_k + \mathfrak{J}_{\Lambda_k} \text{vec}[\Lambda_k]\|^2 \quad (4.50a)$$

$$\text{MSE}_i \triangleq \|\mathfrak{M}_i\|^2 \triangleq \|\tilde{\mathfrak{M}}_i + \mathfrak{M}_{\Delta_i} \text{vec}[\Delta_i]\|^2 \quad (4.50b)$$

where

$$\tilde{\mathfrak{J}}_k = \text{MAT} \left[ \left\{ \sigma_{s_i} \text{vec} \left[ \tilde{\mathbf{G}}_{i,k} \mathbf{P}_i \right] \right\}_{i=1}^K \right], \quad (4.51)$$

$$\mathfrak{J}_{\Lambda_k} = \text{MAT} \left[ \left\{ \sigma_{s_i} (\mathbf{P}_i^T \otimes \mathbf{I}_{R'}) \right\}_{i=1}^K \right], \quad (4.52)$$

$$\tilde{\mathfrak{M}}_i = \begin{bmatrix} \sigma_{s_i} \text{vec} \left[ \mathbf{D}_i \tilde{\mathbf{H}}_{i,i} \mathbf{P}_i - \mathbf{I} \right] \\ \text{MAT} \left[ \left\{ \sigma_{s_j} \text{vec} \left[ \mathbf{D}_i \tilde{\mathbf{H}}_{j,i} \mathbf{P}_j - \mathbf{F}'_{i,j} \right] \right\}_{j=1 \neq i}^K \right] \\ \sigma_{n_i} \text{vec}[\mathbf{D}]_i \end{bmatrix}, \quad (4.53)$$

$$\mathfrak{M}_{\Delta_i} = \begin{bmatrix} \sigma_{s_i} \mathbf{P}_i^T \otimes \mathbf{D}_i \\ \text{MAT} \left[ \left\{ \sigma_{s_j} \mathbf{P}_j^T \otimes \mathbf{D}_i \right\}_{j=1 \neq i}^K \right] \\ \mathbf{0} \end{bmatrix}. \quad (4.54)$$

*Proof.* Using Proposition 4.1 and Lemma 2.4 the proof is straight forward.  $\square$

After these preliminaries, we may formulate our design problem for NBE model of uncertainty as

$$\begin{aligned} & \underset{\{\mathbf{A}_i, \mathbf{B}_i\}_{i=1}^K}{\text{minimize}} && \sum_{i=1}^K \tau_i \end{aligned} \quad (4.55a)$$

$$\text{subject to} \quad \|\boldsymbol{\pi}_i\|^2 \leq P_i, \quad \forall i, \quad (4.55b)$$

$$\|\mathfrak{J}_k\|^2 \leq \kappa_k, \quad \forall \Lambda_k : \|\Lambda_k\|_F \leq \lambda_k, \quad \forall k, \quad (4.55c)$$

$$\|\mathfrak{M}_i\|^2 \leq \tau_i, \quad \forall \Delta_i : \|\Delta_i\|_F \leq \delta_i, \quad \forall i. \quad (4.55d)$$

We will show that this semi-infinite problem can be recast as a SDP. To further simplify these constraints we use the Schur Complement Lemma to express these constraints in appropriate LMI form.

$$\begin{aligned} & \underset{\{\mathbf{A}_i, \mathbf{B}_i\}_{i=1}^K}{\text{minimize}} && \sum_{i=1}^K \tau_i \end{aligned} \quad (4.56a)$$

$$\text{subject to} \quad \|\boldsymbol{\pi}_i\|^2 \leq P_i, \quad \forall i, \quad (4.56b)$$

$$\begin{bmatrix} \kappa_k & \mathfrak{J}_k^* \\ \mathfrak{J}_k & \mathbf{I} \end{bmatrix} \succeq 0, \quad \forall \boldsymbol{\Lambda}_k : \|\boldsymbol{\Lambda}_k\|_F \leq \lambda_k, \quad \forall k, \quad (4.56c)$$

$$\begin{bmatrix} \tau_i & \mathfrak{M}_i^* \\ \mathfrak{M}_i & \mathbf{I} \end{bmatrix} \succeq 0, \quad \forall \boldsymbol{\Delta}_i : \|\boldsymbol{\Delta}_i\|_F \leq \delta_i, \quad \forall i. \quad (4.56d)$$

To further simplify this problem, it is needed to relax the semi-infiniteness of the last two constraints. After employing Lemma 2.8, it is possible to use the following algorithm to find the best beamforming matrices.

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**Algorithm 4.2.** [Iterative beamformer design for the NBE model]

1. Let  $k \leftarrow 0$ .

Initialize the beamformer matrices  $\mathbf{P}_i^{[0]}$ ,  $\mathbf{D}_i^{[0]}$  and  $\mathbf{F}_{i,j}^{[0]}$ .

Compute the initial MSE,  $\text{MSE}^{[0]} = \sum_{i=1}^2 \|\mathfrak{M}_i(\mathbf{P}_i^{[0]}, \mathbf{D}_i^{[0]}, \mathbf{F}_{i,j}^{[0]})\|^2$ .

2. Let  $k \leftarrow k + 1$ .

Update beamforming matrices using the following procedures:

(I) For the DFE Scheme:

$$\{\mathbf{D}_i^{[n+1]}, \mathbf{F}_{i,j}^{[n+1]}\} = \arg \min_{\mathbf{D}_i, \mathbf{F}_{i,j}} \sum_{i=1}^K w_i \tau_i \quad (4.57a)$$

$$\text{subject to} \quad \boldsymbol{\mathfrak{D}}_i \succeq 0, \quad \forall i, \quad (4.57b)$$

$$\tau_i, \eta_i \geq 0, \quad \forall i, \quad (4.57c)$$

and

$$\mathbf{P}_i^{[n+1]} = \arg \min_{\mathbf{P}_i} \sum_{i=1}^K w_i \tau_i \quad (4.58a)$$

$$\text{subject to } \|\boldsymbol{\pi}_i\|^2 \leq \rho_i, \quad \forall i, \quad (4.58b)$$

$$\mathfrak{J}_k \succeq 0, \quad \forall k, \quad (4.58c)$$

$$\mathfrak{M}_i \succeq 0, \quad \forall i, \quad (4.58d)$$

$$\zeta_k \geq 0, \quad \forall k, \quad (4.58e)$$

$$\tau_i, \eta_i \geq 0, \quad \forall i, \quad (4.58f)$$

where

$$\mathfrak{D}_i = \begin{bmatrix} \tau_i - \eta_i & \tilde{\mathfrak{M}}_i^*(\mathbf{P}_i^{[n]}) & \mathbf{0} \\ \tilde{\mathfrak{M}}_i(\mathbf{P}_i^{[n]}) & \mathbf{I} & -\delta_i \mathfrak{M}_{\Delta_i}(\mathbf{P}_i^{[n]}) \\ \mathbf{0} & -\delta_i \mathfrak{M}_{\Delta_i}^*(\mathbf{P}_i^{[n]}) & \eta_i \mathbf{I} \end{bmatrix}, \quad (4.59a)$$

$$\mathfrak{J}_k = \begin{bmatrix} \kappa_k - \zeta_k & \tilde{\mathfrak{J}}_k^* & \mathbf{0} \\ \tilde{\mathfrak{J}}_k & \mathbf{I} & -\lambda_k \mathfrak{J}_{\Lambda_k} \\ \mathbf{0} & -\lambda_k \mathfrak{J}_{\Lambda_k}^* & \zeta_k \mathbf{I} \end{bmatrix}, \quad (4.59b)$$

$$\mathfrak{M}_i = \begin{bmatrix} \tau_i - \eta_i & \tilde{\mathfrak{M}}_i^*(\mathcal{U}_{i,j}) & \mathbf{0} \\ \tilde{\mathfrak{M}}_i(\mathcal{U}_{i,j}) & \mathbf{I} & -\delta_i \mathfrak{M}_{\Delta_i}(\mathcal{U}_{i,j}) \\ \mathbf{0} & -\delta_i \mathfrak{M}_{\Delta_i}^*(\mathcal{U}_{i,j}) & \eta_i \mathbf{I} \end{bmatrix}, \quad (4.59c)$$

$$\mathcal{U}_{i,j} = \{\mathbf{D}_i^{[n]}, \mathbf{F}_{i,j}^{[n]}\}_{\forall i,j}. \quad (4.59d)$$

(II) For the THP Scheme

$$\{\mathbf{P}_i^{[n+1]}, \mathbf{F}_{i,j}^{[n+1]}\} = \arg \min_{\mathbf{P}_i, \mathbf{F}_{i,j}} \sum_{i=1}^K w_i \tau_i \quad (4.60a)$$

$$\text{subject to } \|\boldsymbol{\pi}_i\|^2 \leq \rho_i, \quad \forall i, \quad (4.60b)$$

$$\mathfrak{J}_k \succeq 0, \quad \forall k, \quad (4.60c)$$

$$\mathfrak{M}_i \succeq 0, \quad \forall i, \quad (4.60d)$$

$$\zeta_k \geq 0, \quad \forall k, \quad (4.60e)$$

$$\tau_i, \eta_i \geq 0, \quad \forall i, \quad \tau_i \geq 0, \quad \forall i. \quad (4.60f)$$

and

$$\mathbf{D}_i^{[n+1]} = \arg \min_{\mathbf{D}_i} \sum_{i=1}^K w_i \tau_i \quad (4.61a)$$

$$\text{subject to } \mathfrak{D}_i \succeq 0, \quad \forall i, \quad (4.61b)$$

$$\eta_i, \tau_i \geq 0, \quad \forall i. \quad (4.61c)$$

where

$$\mathfrak{J}_k = \begin{bmatrix} \kappa_k - \zeta_k & \tilde{\mathfrak{J}}_k^* & \mathbf{0} \\ \tilde{\mathfrak{J}}_k & \mathbf{I} & -\lambda_k \mathfrak{J}_{\Lambda_k} \\ \mathbf{0} & -\lambda_k \mathfrak{J}_{\Lambda_k}^* & \zeta_k \mathbf{I} \end{bmatrix}, \quad (4.62)$$

$$\mathfrak{M}_i = \begin{bmatrix} \tau_i - \eta_i & \tilde{\mathfrak{M}}_i^*(\mathbf{D}_i^{[n]}) & \mathbf{0} \\ \tilde{\mathfrak{M}}_i(\mathbf{D}_i^{[n]}) & \mathbf{I} & -\delta_i \mathfrak{M}_{\Delta_i}(\mathbf{D}_i^{[n]}) \\ \mathbf{0} & -\delta_i \mathfrak{M}_{\Delta_i}^*(\mathbf{D}_i^{[n]}) & \eta_i \mathbf{I} \end{bmatrix}, \quad (4.63)$$

$$\mathfrak{D}_i = \begin{bmatrix} \tau_i - \eta_i & \tilde{\mathfrak{M}}_i^*(\mathbf{U}_{i,j}) & \mathbf{0} \\ \tilde{\mathfrak{M}}_i(\mathbf{U}_{i,j}) & \mathbf{I} & -\delta_i \mathfrak{M}_{\Delta_i}(\mathbf{U}_{i,j}) \\ \mathbf{0} & -\delta_i \mathfrak{M}_{\Delta_i}^*(\mathbf{U}_{i,j}) & \eta_i \mathbf{I} \end{bmatrix}, \quad (4.64)$$

$$\mathbf{U}_{i,j} = \{\mathbf{P}_i^{[n]}, \mathbf{F}_{i,j}^{[n]}\}_{\forall i,j}. \quad (4.65)$$

Update the MSE at this stage:  $\text{MSE}^{[k]} = \sum_{i=1}^2 \|\bar{\boldsymbol{\mu}}_i(\mathbf{W}^{[k]}, \mathbf{D}_i^{[k]})\|^2$ .

3. Repeat step (2) until reaching a steady MSE, i.e.,  $|\text{MSE}^{[k]} - \text{MSE}^{[k-1]}| \leq \varepsilon$  or until the maximum number of iterations ( $K_{max}$ ) is reached.
- 

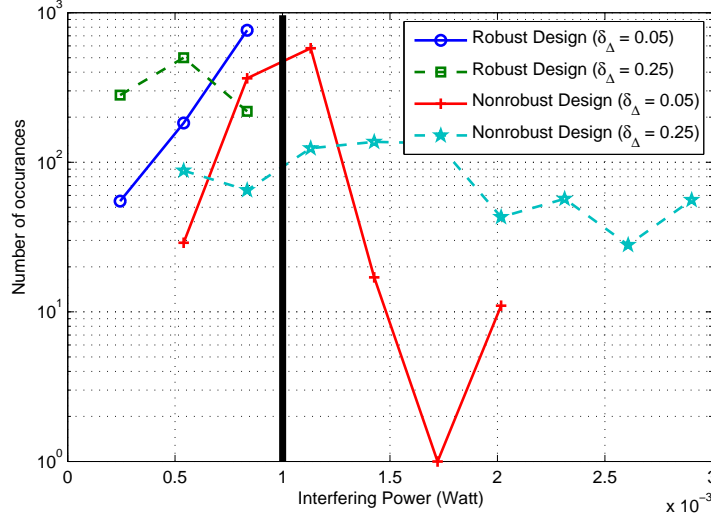
## 4.6 Simulation Results

To demonstrate the effectiveness of the developed methods, a simple MIMO ad hoc cognitive radio network is simulated. The simulation setup is as follows: a set of  $K = 2$  interfering links each having  $T = 2$  transmit antennas and  $R = 2$  receive antennas, transfer  $t = 2$  complex valued symbols each at either time instant. This network is installed within the service range of a primary network having a single user, i.e.,  $L = 1$ , which is also equipped with  $R' = 2$  receive antennas. The transmit power budget of each SU-Tx is set to  $P = 1$  mW and the maximum allowed interfering power to the PU-Rx is assumed to be  $\kappa = -20$  dBm. To solve the problem the optimization package YALMIP/SDPT3 is used in which YALMIP is our wrapper package that uses SDPT3 as its internal solver. The tolerance of the proposed iterative algorithm is set to  $10^{-4}$  and the maximum number of iterations is set to  $100^1$ . For the NBE model simulations, it is assumed

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<sup>1</sup>For the SE model, the design of the precoders and decoders for each channel realization takes about 15-20 seconds. For the NBE model, for which the number of iterations depends on the problem data, this time is between 20-50 seconds using a normal desktop PC. The proposed methods and the underlying processing demand may be useful for realistic channels with only slow variations.



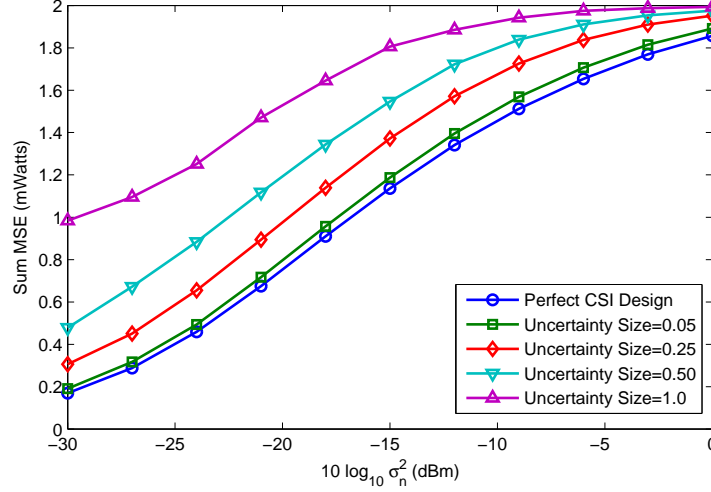


**Fig. 4.3:** Histogram of interfering power occurrences

that  $\delta_i = \lambda_k$ ,  $\forall i, k$ , and the uncertainty size varies from 0 to 1 taking values as  $[0, 0.05, 0.25, 0.50, 1.0]^T$ . For the SE model simulations, it is assumed that the variances of the vector channel variations are  $\sigma_{\Delta}^2 = \sigma_{\Lambda}^2 \in [0.0, 0.05, 0.25, 0.50]$ . To be consistent during the presentation, these quantities are uniquely named as the “Uncertainty Size.” Note that the cases with uncertainty size zero, are equal to the results of the conventional design summarized in [143].

The results depicted in this chapter are based on an average of 500 simulation runs using different channel realizations. In the following figures, except Fig. 4.3, the Perfect CSI case refers to a simulation scenario when the uncertainty has zero norm for the NBE model and zero variance for the SE model as stated in (4.41). In this case, the nominal value of the channel is the actual channel realization which is the only quantity describing the channel.

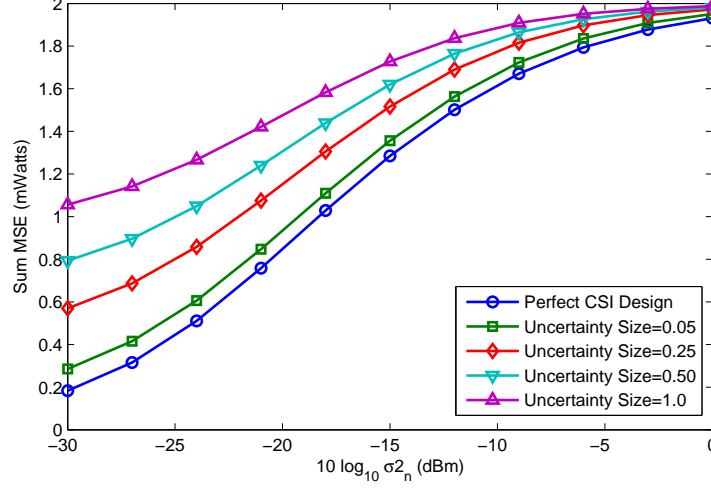
In Fig. 4.3 the histogram of achieved interfering powers to PU-Rx in different cases is depicted. To obtain this figure, 1000 random channel realizations are examined. For each realization, the precoder/equalizer filters based on uncertainty sizes of 0.0, 0.05 and 0.25 are generated and the resultant systems are used to transmit over channels with the appropriate uncertain CSI. This graph demonstrates that the cases designed to be robust against the uncertainty will not violate the interfering power constraints while the perfect-CSI-case (non-robust



**Fig. 4.4:** Sum mean square error of symbol estimation in the cognitive radio system using NBE model

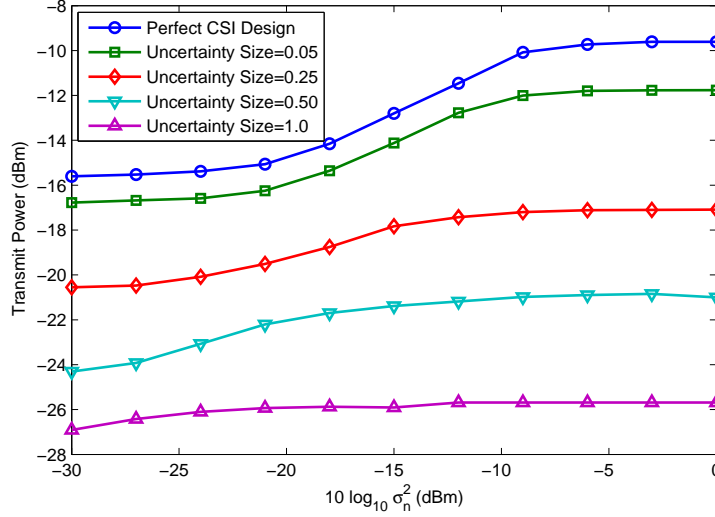
case) filters will violate these constraints for most of the times. As explained before, the maximum allowable interfering power is equal to 1 mW, which is depicted in the graph using a vertical bold solid line. As expected, the non-robust designs, which face a larger uncertainty, will violate the constraints more often relative to the cases facing a smaller uncertainty. It should be mentioned that the range of achieved interfering powers for the non-robust case with an uncertainty size of 0.25 is much larger than the others, though these cases occur less frequently. The graph is further magnified to highlight this.

In Figs. 4.4 and 4.5 the average sum MSE for different cases using NBE and SE uncertainty models, are depicted. As expected, both graphs have an increasing trend with noise power at the receiver, i.e., the higher the noise power, the larger the sum-MSE. This relation is linear in nature, but here the graph shows sum-MSE versus the  $10 \log_{10}(\sigma_n^2)$  which results in a logarithmic curve. Based on this figure, the non-robust case has the least amount of sum-MSE and the case with the largest uncertainty size has the highest sum-MSE, although the difference is not so large for both models which may be described based on the amount of the transmit power. In case of perfect CSI, the transmit power is higher and it results in a better Signal to Interference plus Noise Ratio (SINR) at the receive side, which results in a better sum-MSE performance. The transmit power is lower



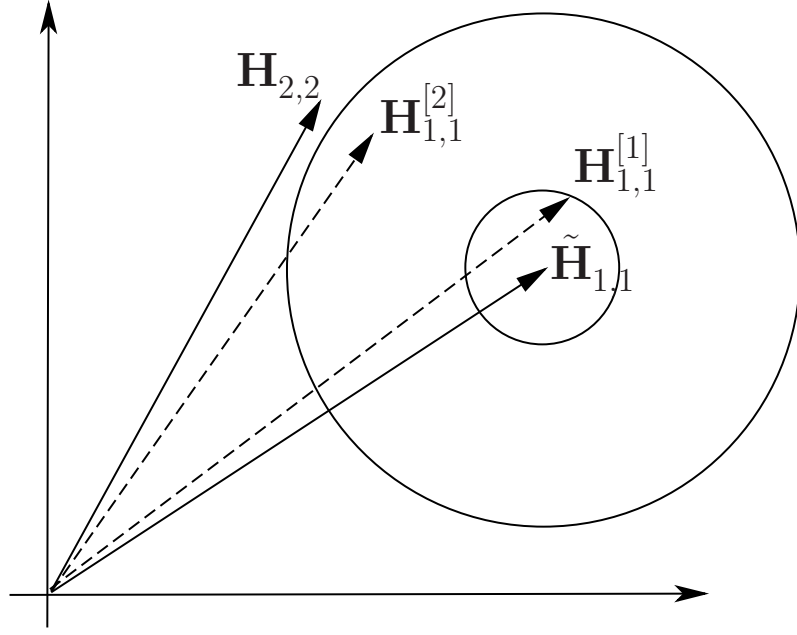
**Fig. 4.5:** Sum mean square error of symbol estimation in the cognitive radio system using SE model

in the uncertain scenarios leading to a smaller SINR and, as a result, a worse sum-MSE performance. Interestingly, at the right most part of the graph, when the noise power is 0 dBm, the performance of all scenarios converges and it seems that noise dominates the received signal and the performance is independent of the uncertainty or precoder/decoder choice. In Fig. 4.5 the sum MSE is depicted for the SE model. Again, like in the case of the NBE model, the sum-MSE for the non-robust case has a better performance relative to uncertain cases. As expected, when the noise power of the users increases the sum-MSE also increases. This is also found when the uncertainty size increases from  $\sigma_{\Delta}^2 = 0$ , which indicates a perfect CSI case, to  $\sigma_{\Delta}^2 = 1.0$ , which has the highest uncertainty size studied here. Using the SE model for uncertainty and in the perfect CSI case, the  $\mathbf{E}_{s_i}[\text{MSE}_i]$  has fewer terms which results in a smaller sum MSE, especially relative to these cases with large uncertainty size. This is due to the nature of  $\mathbf{E}_{s_i}[\text{MSE}_i]$ . From now on and to be concise, as the transmit power, interfering power and required iteration number to converge for both models have similar trends, only one figure for each metric is included. As can be seen in Fig. 4.6, TxP varies less relative to the sum MSE. This can be expected, because it is a quantity related to the transmitter and should be less dependent on the receiver parameters such as the receiver side noise variance. But because the transmit filters depend through the



**Fig. 4.6:** Transmit power of a typical secondary transmitter using NBE model.

sum MSE to the noise power, there is a variation in TxP. In the larger noise power region, the TxP does not increase monotonically, because it is limited by the IP constraint. Interestingly, there is a large difference between the transmit power of different scenarios. For the perfect CSI case when the noise power is equal to 0 dBm, TxP is almost  $-10$  dBm while for robust cases with an uncertainty size of 0.05, 0.25, 0.50 and 1.0 it is about  $-12$  dBm,  $-17$  dBm,  $-21$  dBm and  $-26$  dBm respectively. To have a better understanding about this phenomenon, it may be interpreted that the design procedure does not consider the nominal value of the channels with large variations as a fair representation of a typical channel realization. In these cases, the designed precoder transmits less power over the uncertain channels. This is further explained using Fig. 4.7. In this figure,  $\tilde{\mathbf{H}}_{1,1}$  shows the vectorized representation of the nominal value of the channel between the first set of transmitters and receivers. The circles show the uncertainty bounds for two different cases. In this figure  $\mathbf{H}_{1,1}^{[1]}$  is one of the realizations of the channel with a fairly small uncertainty, while  $\mathbf{H}_{1,1}^{[2]}$  belongs to a channel realization with the same nominal value but with a larger uncertainty. Initially  $\mathbf{H}_{2,2}$  and  $\tilde{\mathbf{H}}_{1,1}$  are far apart and the transmit power towards them will have a small interference, but this is not true for  $\mathbf{H}_{2,2}$  and  $\mathbf{H}_{1,1}^{[2]}$ . The transmit power in either of them will interfere more on the other one and therefore it is not a good idea to transmit



**Fig. 4.7:** Channel uncertainty concept illustration.

using high power when the uncertainty is large enough to avoid the interference with adjacent channels. This phenomenon is automatically taken into account by the design method and may be used to describe the behavior of  $\text{IP}_k$  as well. It should be mentioned that the contrasting behavior of the transmit power in this chapter with the previous one is a result of the different problem formulations. In the previous chapter we aimed to minimize the transmit power while a bounded SINR was targeted, but in this chapter our goal is to minimize the sum MSE while having a bounded power. The difference in the trend of the transmit power resulted from the problem formulation of chapter 3 and subsequent chapters is inevitable and is due to the problem structure. It is clear that in Chapter 3, forgetting the  $\text{IP}_\ell$  constraint, the problem formulation would be something like

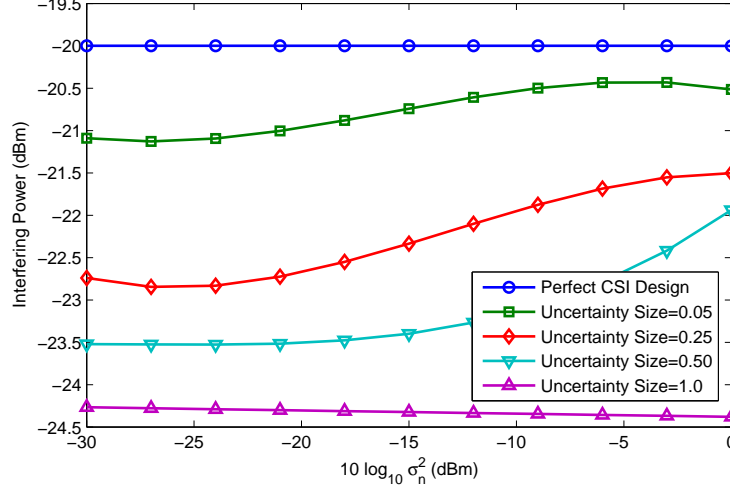
$$\min \quad \text{TxP} \quad (\text{P1})$$

$$S.t. \quad \text{SINR}_k \geq \gamma_k$$

while in Chapters 4, 5, and 6, the problem formulation is as follows:

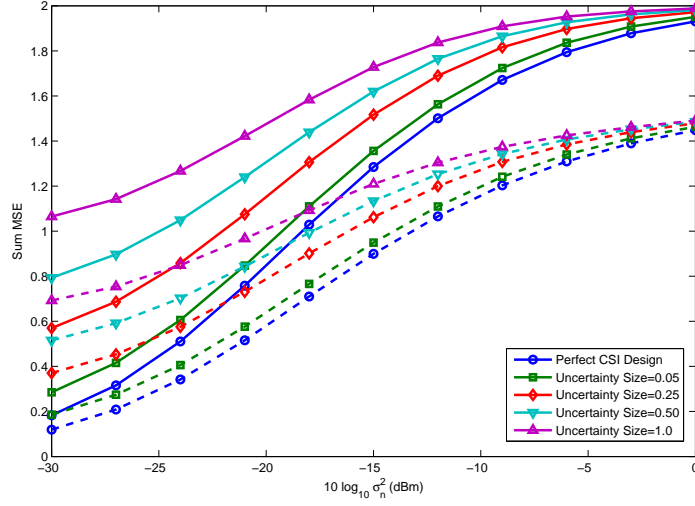
$$\min \quad \sum \text{MSE}_i \quad (\text{P2})$$

$$S.t. \quad \text{TxP}_k \leq P_k$$

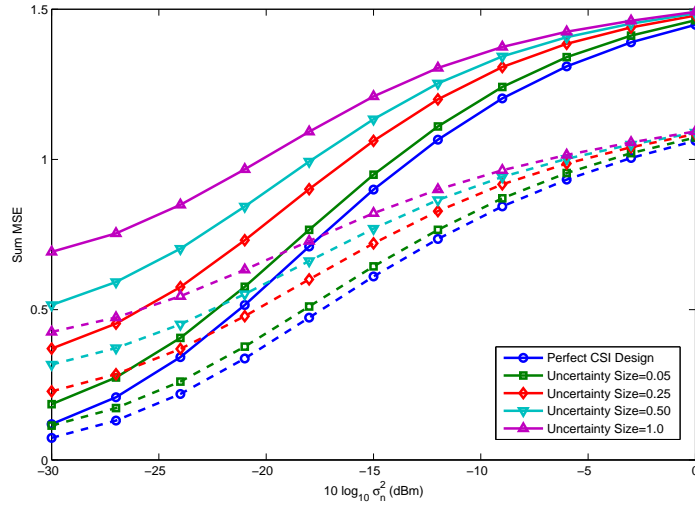


**Fig. 4.8:** Interference power received at a typical primary receiver.

It is clear that for perfect CSI case, (P2) will result in lower sum MSE relative to partial CSI cases, which makes it possible for the Tx side to transmit with more power although bounded, contributing to even better sum MSE. And because of that (P2) will result in more transmit power for perfect CSI case which is contradicting to the behaviour of (P1), aiming to minimize the transmit power itself. In (P1), for perfect CSI case, the SINR is larger relative to partial CSI cases, which makes it possible to reduce the transmit power further resulting in lower transmit power for perfect CSI case. In Fig. 4.8 the interfering power to PU-Rx for different scenarios is shown. As can be seen from this figure, for the perfect CSI case, the interfering power is equal to its design limit, i.e.  $-20$  dBm while for robust cases, the interfering power is more or less smaller than this non-robust case. It is observed that the interfering power for uncertain cases is more or less constant and follows the TxP trends. For the largest uncertainty size of  $\delta = \lambda = 1.0$ , the amount of  $IP_k$  is about 4 dBm less than the non-robust case. In terms of satisfying the PR-Net interfering power requirements, the designed network for the largest uncertainty size has clearly a better performance relative to other cases. As the perfect CSI design transmits more power relative to uncertain cases, it is expected to have more interfering power for the perfect CSI case which is explained in Fig. 4.8. Although the interfering power to each



(a) Linear (solid lines) vs. THP scheme (Dashed lines)



(b) THP scheme (solid lines) vs. DFE scheme (Dashed lines)

**Fig. 4.9:** Comparison of the sum MSE of the system for linear and nonlinear designs.

PU-Rx does not depend directly on noise power, there is a relation between them through the precoder matrices. In the case of higher noise powers, to support the system appropriately, the transmitters are transmitting more power resulting in the precoder matrices with larger norms, which results in higher interfering powers.

In Fig. 4.9 the comparison of different linear and nonlinear schemes is displayed. As expected, the nonlinear designs outperform the linear designs, as they oppose more complicated structure for the precoder and decoder subsystems, resulting

in a better sum MSE. Also as expected, the DFE scheme outperforms the THP design. It can be observed that the effective signal variance is larger in the THP scheme, resulting in larger sum MSE. The other trends of the sum MSE which are explained for Figs. 4.4 and 4.5 are preserved here as well.

## 4.7 Conclusion

The design of robust optimized precoder/decoders for MIMO ad hoc interfering networks is studied in this chapter. It is assumed that the CSI for all important matrix channel gains is not known perfectly, i.e., the CSI has uncertainty. This uncertainty is described into two different ways: using the NBE model which is a deterministic model and using the SE model which is a purely stochastic model. Both scenarios are studied leading to the following conclusions: for the SE model, the aforementioned problem can be recast as SOCP while for the NBE model, the same problem can be a SDP. These two programming problems are efficiently solved using general numerical optimization packages. Simulations are used to demonstrate the behavior of the designed precoder/decoders.



# Chapter 5

## Robust Linear Beamforming for MIMO One-Way Relay Channel

### 5.1 Introduction

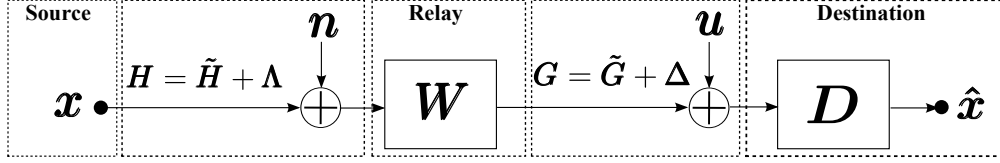
The problem of non-regenerative MIMO relay design with or without a direct link is introduced in [83] where the authors optimize the capacity between the source and the destination. Both of the optimum canonical coordinates of the relay matrix and the upper and the lower bounds of the optimal system capacity are discussed respectively. The problem of joint Multiple-Input Multiple-Output (MIMO) beamforming and power adaptation at source and relay with QoS constraints is discussed in [85],[86]. In these papers, a broadcasting relay scheme with a MIMO architecture in a source-to-relay link is studied. Both receive and transmit beamforming is accomplished in the relay station. As a result, optimum beamforming weights and power adaptation are calculated. The linear relay beamformer for a MIMO relay broadcast channel with limited feedback with zero forcing and minimum mean square error measures is described in [87]. There it is concluded that only Channel Direction Information (CDI) feedback is sufficient to determine the beamforming vector in the proposed scheme which highly reduces the amount of the required feedback. A similar treatment is also covered in [88] for the Single-Input Multiple-Output (SIMO) / Multiple-Input Single-Output (MISO) case with only one set of CSI imperfectly known. In this paper both systems are designed iteratively. The iterative design of the linear relay precoder and the destination equalizer is considered in [89]. The joint power and time allocation for multi-hop

relay networks are addressed in [90],[91] based on the Second Order Cone Program (SOCP) and Semidefinite Program (SDP) [92],[93]. In [94] the problem of collaborative uplink transmit beamforming with robustness against channel estimation errors for a DF relay is addressed. The CSI has a stochastic error model and the proposed algorithms can be applied to both line of sight propagation and fading channels.

In this paper a MIMO AF relay scheme with imperfect CSI is studied. It is assumed that the CSI for both source-to-relay and relay-to-destination is not perfectly known. The problem formulation is based on the minimization of the Mean Square Error (MSE) of symbol estimation with a constraint on the transmission power of the relay station.

The aforementioned problems are efficiently solved numerically using the packages YALMIP [139] and SDPT3 [140].

Future wireless networks are expected to provide large bandwidth for high data rate services like mobile TV or video on demand services. Unfortunately the achievable data rate of wireless systems decreases dramatically when the distance between the source and the destination increases. In order to increase the transmission distance, relay networks are required. The relay station sits in between the two ends of the communication link and extends the range of the wireless transmission. This network, also saves the BS power by preventing it from transmitting a fairly large amount of power to provide the required QoS for the distant mobile users. Simultaneously exploiting the relay concept and multiple antennas in the system will improve the reliability of the transmission. A two-hop AF relaying scheme is considered in this chapter. In this scheme, the source first transmits information to the relay station and then, in the subsequent time slot, this information is sent to the destination from the relay station. Based on this definition, a half-duplex transmission needs two time slots for any transmission of data from source to destination. Similar to the conventional wireless communication system design, the CSI plays an important role in the design of the system. Practically the CSI is imperfect due to rapidly changing urban environments and channel es-



**Fig. 5.1:** Signal Flow Graph of a Point to Point MIMO Relay Channel.

timization errors, for example. In this chapter, the problem of linear beamforming design for a two-hop AF relay station with imperfect CSI for both source-to-relay and relay-to-destination is solved. The problem formulation is based on the minimization of the system wide MSE of symbol estimation, with a constraint on the transmission power of the relay station.

The rest of the chapter is structured as follows. In Section 5.2 the model of the MIMO relay system is defined and analyzed. The problem formulation for both SE and NBE models are also described in this section. Section 5.3 contains the solutions to the problems. Simulation results are shown in Section 5.4. Finally Section 5.5 concludes the chapter.

## 5.2 System Model

A wireless MIMO relay system in which source, relay and destination stations are equipped with multiple antennas is shown in Fig. 5.1. It is assumed that source and destination are equipped with  $T$  transmit and receive antennas, respectively, while the relay has  $R$  antennas. The input signal samples,  $\mathbf{x} \in \mathbb{C}^T$ , are drawn from a stochastic source with the following characteristics:

$$\mathbf{E}_{\mathbf{x}}[\mathbf{x}] = \mathbf{0}, \quad (5.1a)$$

$$\mathbf{E}_{\mathbf{x}}[\mathbf{x}\mathbf{x}^*] = \mathbf{I}. \quad (5.1b)$$

This signal is sent over a channel,  $\mathbf{H} \in \mathbb{C}^{R \times T}$ , and received by the relay station. The received signal at the relay station is linearly precoded using  $\mathbf{W} \in \mathbb{C}^{R \times R}$  and sent again to the destination over channel  $\mathbf{G} \in \mathbb{C}^{T \times R}$ . It is assumed that both channels are not known perfectly at both the relay station and the destination. It is also assumed that the CSI is constant over the time period of the transmission

of each frame, i.e., both channels are in slow block fading mode. The uncertainty model for both  $\mathbf{H}$  and  $\mathbf{G}$  is defined as

$$\mathbf{H} = \tilde{\mathbf{H}} + \mathbf{\Lambda}, \quad (5.2a)$$

$$\mathbf{G} = \tilde{\mathbf{G}} + \mathbf{\Delta}, \quad (5.2b)$$

where  $\tilde{\mathbf{H}}$  and  $\tilde{\mathbf{G}}$  are the nominal values of the CSI and  $\mathbf{\Lambda}$  and  $\mathbf{\Delta}$  are the uncertainty matrix. Uncertainty is mainly modeled using the SE or the NBE models. In the SE model, it is assumed that

$$\begin{cases} \mathbb{E}_{\mathbf{\Lambda}} [\mathbf{\Lambda}] = \mathbf{0}, \\ \mathbb{E}_{\mathbf{\Lambda}} [\text{vec} [\mathbf{\Lambda}] \text{vec} [\mathbf{\Lambda}]^*] = \sigma_{\mathbf{\Lambda}}^2, \end{cases} \quad (5.3a)$$

$$\begin{cases} \mathbb{E}_{\mathbf{\Delta}} [\mathbf{\Delta}] = \mathbf{0}, \\ \mathbb{E}_{\mathbf{\Delta}} [\text{vec} [\mathbf{\Delta}] \text{vec} [\mathbf{\Delta}]^*] = \sigma_{\mathbf{\Delta}}^2, \end{cases} \quad (5.3b)$$

while for the NBE model, the uncertainty matrix has the following property:

$$\|\mathbf{\Lambda}\|_F^2 \leq \delta_{\mathbf{\Lambda}}, \quad (5.4a)$$

$$\|\mathbf{\Delta}\|_F^2 \leq \delta_{\mathbf{\Delta}}. \quad (5.4b)$$

The received signal is then equalized at the destination using a linear receive filter  $\mathbf{D} \in \mathbb{C}^{T \times T}$ . The output of the equalizer is given by

$$\hat{\mathbf{x}} = \mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{D}\mathbf{G}\mathbf{W}\mathbf{n} + \mathbf{D}\mathbf{u}, \quad (5.5)$$

where  $\mathbf{n} \in \mathbb{C}^R$  and  $\mathbf{u} \in \mathbb{C}^T$  are the zero-mean white Gaussian noise in the relay station and at the destination respectively, i.e.,  $\mathbf{n} \sim \mathcal{N}(\mathbf{0}, \sigma_{\mathbf{n}}^2 \mathbf{I})$  and  $\mathbf{u} \sim \mathcal{N}(\mathbf{0}, \sigma_{\mathbf{u}}^2 \mathbf{I})$ .

A MSE-based design procedure is used in this chapter to design the relay station precoder matrix.

**Proposition 5.1.** *For the aforementioned system, the MSE of symbol estimation is defined as follows:*

$$\text{MSE}(\mathbf{W}, \mathbf{D}) \triangleq \mathbb{E}_{\mathbf{x}} [\|\hat{\mathbf{x}} - \mathbf{x}\|^2] \quad (5.6)$$

$$= \|\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I}\|_F^2 + \sigma_{\mathbf{n}}^2 \|\mathbf{D}\mathbf{G}\mathbf{W}\|_F^2 + \sigma_{\mathbf{u}}^2 \|\mathbf{D}\|_F^2. \quad (5.7)$$

*Proof.* Let

$$\text{MSE}(\mathbf{W}, \mathbf{D}) = \mathbb{E}_{\mathbf{x}} [\|\hat{\mathbf{x}} - \mathbf{x}\|^2] \quad (5.8)$$

$$= \mathbb{E}_{\mathbf{x}} [\|\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{D}\mathbf{G}\mathbf{W}\mathbf{n} + \mathbf{D}\mathbf{u} - \mathbf{x}\|^2] \quad (5.9)$$

$$= \mathbb{E}_{\mathbf{x}} \{((\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I})\mathbf{x} + \mathbf{D}\mathbf{G}\mathbf{W}\mathbf{n} + \mathbf{D}\mathbf{u})^* \times \\ ((\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I})\mathbf{x} + \mathbf{D}\mathbf{G}\mathbf{W}\mathbf{n} + \mathbf{D}\mathbf{u})\}. \quad (5.10)$$

The above expression has six terms. These terms containing  $\mathbf{x}$  &  $\mathbf{n}$ ,  $\mathbf{x}$  &  $\mathbf{u}$  and  $\mathbf{n}$  &  $\mathbf{u}$  are zeros, due to this fact that they are mutually independent. Working with the three remaining terms,

$$\text{MSE}(\mathbf{W}, \mathbf{D}) = \mathbb{E}_{\mathbf{x}} [\mathbf{x}^*(\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I})^*(\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I})\mathbf{x}] + \\ \mathbb{E}_{\mathbf{n}} [\mathbf{n}^*(\mathbf{D}\mathbf{G}\mathbf{W})^*(\mathbf{D}\mathbf{G}\mathbf{W})\mathbf{n}] + \mathbb{E}_{\mathbf{u}} [\mathbf{u}^*\mathbf{D}^*\mathbf{D}\mathbf{u}]. \quad (5.11)$$

Using Lemma 2.5 and using the statistics of the previously mentioned signals, the MSE is

$$\text{MSE}(\mathbf{W}, \mathbf{D}) = \text{tr}[(\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I})^*(\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I})] + \\ \sigma_{\mathbf{n}}^2 \text{tr}[(\mathbf{D}\mathbf{G}\mathbf{W})^*(\mathbf{D}\mathbf{G}\mathbf{W})] + \sigma_{\mathbf{u}}^2 \text{tr}[\mathbf{D}^*\mathbf{D}]. \quad (5.12)$$

Finally using Lemma 2.2, the result is concluded.

$$\text{MSE}(\mathbf{W}, \mathbf{D}) = \|\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I}\|_F^2 + \sigma_{\mathbf{n}}^2 \|\mathbf{D}\mathbf{G}\mathbf{W}\|_F^2 + \sigma_{\mathbf{u}}^2 \|\mathbf{D}\|_F^2. \quad (5.13)$$

□

In the design procedure, the amount of transmit power of the relay station is constrained and therefore this power is of great importance.

**Proposition 5.2.** *The transmit power of the relay station is given by:*

$$\text{TxP}(\mathbf{W}) \triangleq \mathbb{E}_{\mathbf{x}} [\|\mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{W}\mathbf{n}\|^2] \quad (5.14)$$

$$= \|\mathbf{W}\mathbf{H}\|_F^2 + \sigma_{\mathbf{n}}^2 \|\mathbf{W}\|_F^2. \quad (5.15)$$

*Proof.* Using the Lemma 2.2 and 2.5, it is clear that

$$\text{TxP}(\mathbf{W}) = \mathbb{E}_{\mathbf{x}} [\|\mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{W}\mathbf{n}\|^2] \quad (5.16)$$

$$= \mathbb{E}_{\mathbf{x}} [(\mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{W}\mathbf{n})^*(\mathbf{W}\mathbf{H}\mathbf{x} + \mathbf{W}\mathbf{n})] \quad (5.17)$$

$$= \mathbb{E}_{\mathbf{x}} [\mathbf{x}^*(\mathbf{W}\mathbf{H})^*(\mathbf{W}\mathbf{H})\mathbf{x}] + \mathbb{E}_{\mathbf{x}} [\mathbf{n}^*\mathbf{W}^*\mathbf{W}\mathbf{n}] \quad (5.18)$$

$$= \text{tr}[(\mathbf{W}\mathbf{H})^*(\mathbf{W}\mathbf{H})] + \sigma_{\mathbf{n}}^2 \text{tr}[\mathbf{W}^*\mathbf{W}] \quad (5.19)$$

$$= \|\mathbf{W}\mathbf{H}\|_F^2 + \sigma_{\mathbf{n}}^2 \|\mathbf{W}\|_F^2. \quad (5.20)$$

□

For a given set of  $(\mathbf{W}, \mathbf{D})$ , both  $\text{MSE}(\mathbf{W}, \mathbf{D})$  and  $\text{TxP}(\mathbf{W})$  depend on  $\mathbf{H}$ ,  $\mathbf{G}$  and consequently on  $\mathbf{\Lambda}$  and  $\mathbf{\Delta}$ , and thus they are random variables. The design formulation based on the worst-case scenario is to solve:

$$\underset{\mathbf{W}, \mathbf{D}}{\text{minimize}} \quad \max_{\mathbf{\Lambda}, \mathbf{\Delta}} \text{MSE}(\mathbf{W}, \mathbf{D}) \quad (5.21a)$$

$$\text{subject to} \quad \max_{\mathbf{\Lambda}} \text{TxP}(\mathbf{W}) \leq P. \quad (5.21b)$$

Using the epigraph form, this problem is equivalent to the following:

$$\underset{\mathbf{W}, \mathbf{D}, \tau \geq 0}{\text{minimize}} \quad \tau \quad (5.22a)$$

$$\text{subject to} \quad \max_{\mathbf{\Lambda}} \text{TxP}(\mathbf{W}) \leq P, \quad (5.22b)$$

$$\max_{\mathbf{\Lambda}, \mathbf{\Delta}} \text{MSE}(\mathbf{W}, \mathbf{D}) \leq \tau. \quad (5.22c)$$

### 5.2.1 Conventional Problem Formulation

To compare the robust solutions with conventional solutions, the problem formulation for a system with perfect CSI is presented. This problem formulation is addressed in [144] as the “ZF-Based Design,” and for half-duplex relays, it is also mentioned in [145]. It is straightforward to show that the conventional design is a SOCP.

**Proposition 5.3.** *Conventionally, the design of a MIMO relay channel with per-*

fect CSI is a SOCP:

$$\underset{\mathbf{W}, \mathbf{D}, \tau \geq 0}{\text{minimize}} \quad \tau \quad (5.23a)$$

$$\text{subject to} \quad \|\mathbf{W}\mathbf{H}\|_F^2 + \sigma_n^2 \|\mathbf{W}\|_F^2 \leq P, \quad (5.23b)$$

$$\|\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I}\|_F^2 + \sigma_n^2 \|\mathbf{D}\mathbf{G}\mathbf{W}\|_F^2 + \sigma_u^2 \|\mathbf{D}\|_F^2 \leq \tau. \quad (5.23c)$$

It is clear that (5.23) is not convex simultaneously in design variables  $\mathbf{W}$  and  $\mathbf{D}$ , and we should resort to an iterative procedure to find the optimal values of the filters.

### 5.2.2 SE Model for Uncertain CSI

In the SE model, both  $\text{MSE}(\mathbf{W}, \mathbf{D})$  and  $\text{TxP}(\mathbf{W})$  are stochastic variables, we take their expectations in the design formulation.

**Proposition 5.4.** *For the system:*

$$\begin{aligned} \mathbb{E}_{\Lambda, \Delta} [\text{MSE}(\mathbf{W}, \mathbf{D})] &= \|\mathbf{D}\tilde{\mathbf{G}}\mathbf{W}\tilde{\mathbf{H}} - \mathbf{I}\|_F^2 + \sigma_\Delta^2 \|(\mathbf{W}\tilde{\mathbf{H}})^T \otimes \mathbf{D}\|_F^2 + \sigma_\Lambda^2 \|\mathbf{I} \otimes \mathbf{D}\tilde{\mathbf{G}}\mathbf{W}\|_F^2 \\ &\quad + \sigma_n^2 \|\mathbf{D}\tilde{\mathbf{G}}\mathbf{W}\|_F^2 + \sigma_n^2 \sigma_\Delta^2 \|\mathbf{W}^T \otimes \mathbf{D}\|_F^2 + \sigma_u^2 \|\mathbf{D}\|_F^2, \end{aligned} \quad (5.24a)$$

$$\mathbb{E}_\Lambda [\text{TxP}(\mathbf{W})] = \|\mathbf{W}\tilde{\mathbf{H}}\|_F^2 + \sigma_\Lambda^2 \|\mathbf{I} \otimes \mathbf{W}\|_F^2 + \sigma_n^2 \|\mathbf{W}\|_F^2. \quad (5.24b)$$

*Proof.* Using Lemma 2.2 and 2.3 it is possible to prove this proposition.

$$\begin{aligned} \mathbb{E}_\Delta [\text{MSE}(\mathbf{W}, \mathbf{D})] &= \mathbb{E}_\Delta [\text{vec} [\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I}]^* \text{vec} [\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I}]] + \\ &\quad \sigma_n^2 \mathbb{E}_\Delta [\text{vec} [\mathbf{D}\mathbf{G}\mathbf{W}]^* \text{vec} [\mathbf{D}\mathbf{G}\mathbf{W}]] + \sigma_u^2 \|\mathbf{D}\|_F^2. \end{aligned} \quad (5.25)$$

It is possible to write  $\text{vec} [\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I}]$  as

$$\text{vec} [\mathbf{D}\mathbf{G}\mathbf{W}\mathbf{H} - \mathbf{I}] = \text{vec} \left[ \mathbf{D}(\tilde{\mathbf{G}} + \Delta)\mathbf{W}(\tilde{\mathbf{H}} + \Lambda) - \mathbf{I} \right] \quad (5.26)$$

$$\begin{aligned} &= \text{vec} \left[ \mathbf{D}\tilde{\mathbf{G}}\mathbf{W}\tilde{\mathbf{H}} - \mathbf{I} \right] + (\mathbf{I} \otimes (\mathbf{D}\tilde{\mathbf{G}}\mathbf{W}))\text{vec} [\Lambda] + \\ &\quad ((\mathbf{W}\tilde{\mathbf{H}})^T \otimes \mathbf{D})\text{vec} [\Delta] + \text{vec} [\mathbf{D}\Delta\mathbf{W}\Lambda]. \end{aligned} \quad (5.27)$$

Neglecting the last term which is a bilinear function of both uncertainty matrices (to have a mathematically tractable formulation), and conjugating the remaining

terms, and performing the multiplication process of the first term of (5.25), a nine-term expression remains. Applying the expectation, only three self multiplication terms remain and the other six mutual multiplication terms become zero. The same procedure can be applied to the second term of (5.25). Using Lemma 2.5, will give the result. A similar but simpler procedure will prove the proposition for  $E_{\Lambda} [\text{TxP}(\mathbf{W})]$ .  $\square$

Using this proposition, it is possible to formulate the design based on the following problem:

$$\underset{\mathbf{W}, \mathbf{D}, \tau \geq 0}{\text{minimize}} \quad \tau \quad (5.28a)$$

$$\text{subject to} \quad E_{\Lambda} [\text{TxP}(\mathbf{W})] \leq P, \quad (5.28b)$$

$$E_{\Lambda, \Delta} [\text{MSE}(\mathbf{W}, \mathbf{D})] \leq \tau. \quad (5.28c)$$

Again, this problem is not a convex problem. In the following section, we will propose an iterative solution for this problem.

### 5.2.3 NBE Model

In the NBE model, both  $\text{MSE}(\mathbf{W}, \mathbf{D})$  and  $\text{TxP}(\mathbf{W})$  are not stochastic processes, but there are infinitely many realizations of them, because they directly depend on  $\mathbf{H}$  and  $\mathbf{G}$  which have infinitely many realizations. Therefore the problem formulation in this case is:

$$\underset{\mathbf{W}, \mathbf{D}, \tau \geq 0}{\text{minimize}} \quad \tau \quad (5.29a)$$

$$\text{subject to} \quad \text{TxP}(\mathbf{W}) \leq P, \quad \forall \mathbf{\Lambda} : \|\mathbf{\Lambda}\|_F^2 \leq \delta_{\mathbf{\Lambda}}; \quad (5.29b)$$

$$\text{MSE}(\mathbf{W}, \mathbf{D}) \leq \tau, \quad \begin{cases} \forall \mathbf{\Delta} : \|\mathbf{\Delta}\|_F^2 \leq \delta_{\mathbf{\Delta}}, \\ \forall \mathbf{\Lambda} : \|\mathbf{\Lambda}\|_F^2 \leq \delta_{\mathbf{\Lambda}}. \end{cases} \quad (5.29c)$$

## 5.3 Solutions for the Relay Design

The solutions of the relay beamformer design highly depend on the nature of the uncertainty. The following proposition gives a very important property of the design-involved entities.



**Proposition 5.5.** *For the MIMO relay system,  $\text{TxP}(\mathbf{W})$  and  $\text{MSE}(\mathbf{W}, \mathbf{D})$  or their expectations have a SOC structure: in particular for the SE model,*

$$\mathbb{E}_{\Lambda} [\text{TxP}(\mathbf{W})] \triangleq \|\mathbf{p}(\mathbf{W})\|_2^2, \quad (5.30a)$$

$$\mathbb{E}_{\Lambda, \Delta} [\text{MSE}(\mathbf{W}, \mathbf{D})] \triangleq \|\mathbf{m}(\mathbf{W}, \mathbf{D})\|_2^2, \quad (5.30b)$$

and for the NBE model,

$$\text{TxP}(\mathbf{W}) = \|\tilde{\boldsymbol{\pi}}(\mathbf{W}) + \boldsymbol{\pi}_{\Lambda}(\mathbf{W})\text{vec}[\Lambda]\|_2^2 \quad (5.31a)$$

$$\triangleq \|\boldsymbol{\pi}(\mathbf{W})\|_2^2, \quad (5.31b)$$

$$\text{MSE}(\mathbf{W}, \mathbf{D}) = \|\tilde{\boldsymbol{\mu}}(\mathbf{W}, \mathbf{D}) + \boldsymbol{\mu}_{\Lambda}(\mathbf{W}, \mathbf{D})\text{vec}[\Lambda] + \boldsymbol{\mu}_{\Delta}(\mathbf{W}, \mathbf{D})\text{vec}[\Delta]\|_2^2 \quad (5.31c)$$

$$\triangleq \|\boldsymbol{\mu}(\mathbf{W}, \mathbf{D})\|_2^2. \quad (5.31d)$$

*Proof.* Using Lemma 2.2 and 2.5 and neglecting the bilinear terms:

$$\mathbf{m}(\mathbf{W}, \mathbf{D}) = \begin{bmatrix} \text{vec} [D\tilde{G}\mathbf{W}\tilde{H} - \mathbf{I}] \\ \sigma_{\Delta} \text{vec} [(\mathbf{W}\tilde{H})^T \otimes \mathbf{D}] \\ \sigma_{\Lambda} \text{vec} [\mathbf{I} \otimes (D\tilde{G}\mathbf{W})] \\ \sigma_n \text{vec} [D\tilde{G}\mathbf{W}] \\ \sigma_n \sigma_{\Delta} \text{vec} [\mathbf{W}^T \otimes \mathbf{D}] \\ \sigma_u \text{vec} [\mathbf{D}] \end{bmatrix}, \quad (5.32a)$$

$$\mathbf{p}(\mathbf{W}) = \begin{bmatrix} \text{vec} [\mathbf{W}\tilde{H}] \\ \sigma_{\Lambda} \text{vec} [\mathbf{I} \otimes \mathbf{W}] \\ \sigma_n \text{vec} [\mathbf{W}] \end{bmatrix}, \quad (5.32b)$$

$$\tilde{\boldsymbol{\mu}}(\mathbf{W}, \mathbf{D}) = \begin{bmatrix} \text{vec} [D\tilde{G}\mathbf{W}\tilde{H} - \mathbf{I}] \\ \sigma_n \text{vec} [D\tilde{G}\mathbf{W}] \\ \sigma_u \text{vec} [\mathbf{D}] \end{bmatrix}, \quad (5.32c)$$

$$\boldsymbol{\mu}_{\Delta}(\mathbf{W}, \mathbf{D}) = \begin{bmatrix} (\mathbf{W}\tilde{H})^T \otimes \mathbf{D} \\ \sigma_n \mathbf{W}^T \otimes \mathbf{D} \\ \mathbf{0} \end{bmatrix}, \quad (5.32d)$$

$$\boldsymbol{\mu}_{\Lambda}(\mathbf{W}, \mathbf{D}) = \begin{bmatrix} \mathbf{I} \otimes (D\tilde{G}\mathbf{W}) \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}, \quad (5.32e)$$

$$\tilde{\boldsymbol{\pi}}(\mathbf{W}) = \begin{bmatrix} \text{vec} [\mathbf{W}\tilde{H}] \\ \sigma_n \text{vec} [\mathbf{W}] \end{bmatrix}, \quad (5.32f)$$

$$\boldsymbol{\pi}_{\Lambda}(\mathbf{W}) = \begin{bmatrix} \mathbf{I} \otimes \mathbf{W} \\ \mathbf{0} \end{bmatrix}. \quad (5.32g)$$

□

Therefor problem (5.28) for the SE model becomes

$$\underset{\mathbf{W}, \mathbf{D}, \tau \geq 0}{\text{minimize}} \quad \tau \quad (5.33a)$$

$$\text{subject to} \quad \|\mathbf{p}(\mathbf{W})\|^2 \leq P, \quad (5.33b)$$

$$\|\mathbf{m}(\mathbf{W}, \mathbf{D})\|^2 \leq \tau. \quad (5.33c)$$

As this problem is a biconvex problem, we should resort to an iterative method to solve it. The solution procedure is as follows:

---

**Algorithm 5.1.** [Iterative beamformer design for SE model]

1. Let  $k \leftarrow 0$ .

Initialize the beamformer matrices  $\mathbf{W}^{[0]}$  and  $\mathbf{D}^{[0]}$ .

Compute the initial MSE,  $\text{MSE}^{[0]} = \|\mathbf{m}(\mathbf{W}^{[0]}, \mathbf{D}^{[0]})\|^2$ .

2. Let  $k \leftarrow k + 1$ .

Update beamforming matrices using the following procedures:

$$\mathbf{W}^{[k]} = \arg \min_{\mathbf{W}, \tau \geq 0} \quad \tau \quad (5.34a)$$

$$\text{S. t.} \quad \|\mathbf{p}(\mathbf{W})\|^2 \leq P, \quad (5.34b)$$

$$\|\mathbf{m}(\mathbf{W}, \mathbf{D}^{[k-1]})\|^2 \leq \tau, \quad (5.34c)$$

and

$$\mathbf{D}^{[k]} = \arg \min_{\mathbf{D}, \tau \geq 0} \quad \tau \quad (5.35a)$$

$$\text{S. t.} \quad \|\mathbf{m}(\mathbf{W}^{[k-1]}, \mathbf{D})\|^2 \leq \tau. \quad (5.35b)$$

Update the MSE at this stage:  $\text{MSE}^{[k]} = \|\mathbf{m}(\mathbf{W}^{[k]}, \mathbf{D}^{[k]})\|^2$ .

3. Repeat step (2) until a steady MSE is reached, i.e.,  $|\text{MSE}^{[k]} - \text{MSE}^{[k-1]}| \leq \varepsilon$  or until the maximum number of iterations ( $K_{max}$ ) is reached.

---

In this procedure,  $\varepsilon$  is a predefined tolerance level and  $K_{max}$  is selected large enough to let the procedure to converge. Since this algorithm practically converges after few iterations, during the simulations  $K_{max}$  is selected to be 100 but it is not limited to this value. It also should be mentioned that using some standard tricks

[93], each of (5.34) and (5.35) may be converted to a standard SOCP and then efficiently solved using interior-point methods. Since Algorithm 5.1 is based on the ACS algorithm, it will converge and the proof is not mentioned here. From the computer simulations we found on average that the SE model requires 8 iterations to converge (with  $\varepsilon = 10^{-4}$ ) while the NBE model needs 12 iterations using a similar scenario.

Unlike the SE model and its straightforward problem formulation, for NBE model it is required to manipulate the problem formulation. To keep the following equations simple, the dependency of  $\boldsymbol{\pi}$  and  $\boldsymbol{\mu}$  on  $\mathbf{W}$  and  $\mathbf{D}$  is not mentioned explicitly, though, it is implicitly presumed. Using the Schur Complement Lemma (Lemma 2.6) the following problem

$$\underset{\mathbf{W}, \mathbf{D}, \tau \geq 0}{\text{minimize}} \quad \tau \quad (5.36a)$$

$$\text{subject to} \quad \|\boldsymbol{\pi}\|^2 \leq P \quad \forall \boldsymbol{\Lambda} : \|\boldsymbol{\Lambda}\|_F^2 \leq \delta_{\boldsymbol{\Lambda}}, \quad (5.36b)$$

$$\|\boldsymbol{\mu}\|^2 \leq \tau, \quad \begin{cases} \forall \boldsymbol{\Lambda} : \|\boldsymbol{\Lambda}\|_F^2 \leq \delta_{\boldsymbol{\Lambda}}, \\ \forall \boldsymbol{\Delta} : \|\boldsymbol{\Delta}\|_F^2 \leq \delta_{\boldsymbol{\Delta}}, \end{cases} \quad (5.36c)$$

is rewritten as

$$\underset{\mathbf{W}, \mathbf{D}, \tau \geq 0}{\text{minimize}} \quad \tau \quad (5.37a)$$

$$\text{subject to} \quad \begin{bmatrix} P & \boldsymbol{\pi}^* \\ \boldsymbol{\pi} & \mathbf{I} \end{bmatrix} \succeq 0, \quad \forall \boldsymbol{\Lambda} : \|\boldsymbol{\Lambda}\|_F^2 \leq \delta_{\boldsymbol{\Lambda}}, \quad (5.37b)$$

$$\begin{bmatrix} \tau & \boldsymbol{\mu}^* \\ \boldsymbol{\mu} & \mathbf{I} \end{bmatrix} \succeq 0, \quad \begin{cases} \forall \boldsymbol{\Lambda} : \|\boldsymbol{\Lambda}\|_F^2 \leq \delta_{\boldsymbol{\Lambda}}, \\ \forall \boldsymbol{\Delta} : \|\boldsymbol{\Delta}\|_F^2 \leq \delta_{\boldsymbol{\Delta}}. \end{cases} \quad (5.37c)$$

To further simplify this problem, the relaxation of the semi-infinite constraints is done using the Theorem 2.1.

To use this theorem (5.37c) is rewritten. The LMI of (5.37c), using the notations of (5.31) is rewritten as

$$\begin{bmatrix} \tau & \tilde{\boldsymbol{\mu}}^* \\ \tilde{\boldsymbol{\mu}} & \mathbf{I} \end{bmatrix} + \begin{bmatrix} 0 & (\boldsymbol{\mu}_{\boldsymbol{\Delta}}^{\text{vec}}[\boldsymbol{\Delta}])^* \\ \boldsymbol{\mu}_{\boldsymbol{\Delta}}^{\text{vec}}[\boldsymbol{\Delta}] & \mathbf{0} \end{bmatrix} + \begin{bmatrix} 0 & (\boldsymbol{\mu}_{\boldsymbol{\Lambda}}^{\text{vec}}[\boldsymbol{\Lambda}])^* \\ \boldsymbol{\mu}_{\boldsymbol{\Lambda}}^{\text{vec}}[\boldsymbol{\Lambda}] & \mathbf{0} \end{bmatrix} \succeq 0, \quad (5.38)$$

By choosing

$$\begin{aligned} \mathbf{A} &= \begin{bmatrix} \tau & \tilde{\boldsymbol{\mu}}^* \\ \tilde{\boldsymbol{\mu}} & \mathbf{I} \end{bmatrix}, & \mathbf{Q}_1 = \mathbf{Q}_2 &= \begin{bmatrix} -1 & \mathbf{0} \end{bmatrix}, \\ \mathbf{P}_1 &= \begin{bmatrix} \mathbf{0} & \boldsymbol{\mu}_\Delta^* \end{bmatrix}, & \mathbf{P}_2 &= \begin{bmatrix} \mathbf{0} & \boldsymbol{\mu}_\Lambda^* \end{bmatrix}, \\ \mathbf{X}_1 &= \text{vec}[\boldsymbol{\Delta}], & \mathbf{X}_2 &= \text{vec}[\boldsymbol{\Lambda}], \end{aligned}$$

the relaxed version of (5.37c) is as follows:

$$\begin{bmatrix} \tau - (\epsilon_1 + \epsilon_2) & \tilde{\boldsymbol{\mu}}^* & \mathbf{0} & \mathbf{0} \\ \tilde{\boldsymbol{\mu}} & \mathbf{I} & -\delta_\Delta \boldsymbol{\mu}_\Delta & -\delta_\Lambda \boldsymbol{\mu}_\Lambda \\ \mathbf{0} & -\delta_\Delta \boldsymbol{\mu}_\Delta^* & \epsilon_1 \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\delta_\Lambda \boldsymbol{\mu}_\Lambda^* & \mathbf{0} & \epsilon_2 \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \quad (5.39)$$

$$\tau, \epsilon_1, \epsilon_2 \geq 0. \quad (5.40)$$

Using a similar procedure, (5.37b) is recast as

$$\begin{bmatrix} P - \epsilon_3 & \tilde{\boldsymbol{\pi}}^* & \mathbf{0} \\ \tilde{\boldsymbol{\pi}} & \mathbf{I} & -\delta_\Lambda \boldsymbol{\pi}_\Lambda \\ \mathbf{0} & -\delta_\Lambda \boldsymbol{\pi}_\Lambda^* & \epsilon_3 \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \quad (5.41)$$

$$\epsilon_3 \geq 0. \quad (5.42)$$

Finally, using the previous results, the design problem becomes:

$$\underset{\mathbf{W}, \mathbf{D}, \tau \geq 0}{\text{minimize}} \quad \tau \quad (5.43a)$$

$$\text{subject to} \quad \mathcal{P}(\mathbf{W}) \succeq \mathbf{0}, \quad (5.43b)$$

$$\mathcal{M}(\mathbf{W}, \mathbf{D}) \succeq \mathbf{0}, \quad (5.43c)$$

$$\epsilon_1, \epsilon_2, \epsilon_3 \geq 0, \quad (5.43d)$$

where

$$\mathcal{P}(\mathbf{W}) = \begin{bmatrix} P - \epsilon_3 & \tilde{\boldsymbol{\pi}}^* & \mathbf{0} \\ \tilde{\boldsymbol{\pi}} & \mathbf{I} & -\delta_\Lambda \boldsymbol{\pi}_\Lambda \\ \mathbf{0} & -\delta_\Lambda \boldsymbol{\pi}_\Lambda^* & \epsilon_3 \mathbf{I} \end{bmatrix}, \quad (5.44a)$$

$$\mathcal{M}(\mathbf{W}, \mathbf{D}) = \begin{bmatrix} \tau - (\epsilon_1 + \epsilon_2) & \tilde{\boldsymbol{\mu}}^* & \mathbf{0} & \mathbf{0} \\ \tilde{\boldsymbol{\mu}} & \mathbf{I} & -\delta_\Delta \boldsymbol{\mu}_\Delta & -\delta_\Lambda \boldsymbol{\mu}_\Lambda \\ \mathbf{0} & -\delta_\Delta \boldsymbol{\mu}_\Delta^* & \epsilon_1 \mathbf{I} & \mathbf{0} \\ \mathbf{0} & -\delta_\Lambda \boldsymbol{\mu}_\Lambda^* & \mathbf{0} & \epsilon_2 \mathbf{I} \end{bmatrix}. \quad (5.44b)$$

This problem also is not convex, due to the biconvex structure of  $\tilde{\boldsymbol{\mu}}$ ,  $\boldsymbol{\mu}_\Delta$  and  $\boldsymbol{\mu}_\Lambda$ .

Here, again, an iterative procedure is proposed to find the optimal value of the beamformer matrices.

---

**Algorithm 5.2.** [Iterative beamformer design for NBE model]

1. Let  $k \leftarrow 0$ .

Initialize the beamformer matrices,  $\mathbf{W}^{[0]}$  and  $\mathbf{D}^{[0]}$ .

Compute the initial MSE,  $\text{MSE}^{[0]} = \|\boldsymbol{\mu}(\mathbf{W}^{[0]}, \mathbf{D}^{[0]})\|^2$ .

2. Let  $k \leftarrow k + 1$ .

Update beamforming matrices using the following procedures:

$$\mathbf{W}^{[k]} = \arg \min_{\mathbf{W}, \tau \geq 0} \quad \tau \quad (5.45a)$$

$$\text{S. t.} \quad \mathcal{P}(\mathbf{W}) \succeq 0, \quad (5.45b)$$

$$\mathcal{M}(\mathbf{W}, \mathbf{D}^{[k-1]}) \succeq 0, \quad (5.45c)$$

$$\epsilon_1, \epsilon_2, \epsilon_3 \geq 0, \quad (5.45d)$$

and

$$\mathbf{D}^{[k]} = \arg \min_{\mathbf{D}, \tau \geq 0} \quad \tau \quad (5.46a)$$

$$\text{S. t.} \quad \mathcal{M}(\mathbf{W}^{[k-1]}, \mathbf{D}) \succeq 0, \quad (5.46b)$$

$$\epsilon_1, \epsilon_2 \geq 0, \quad (5.46c)$$

Update the MSE at this stage:  $\text{MSE}^{[k]} = \|\boldsymbol{\mu}(\mathbf{W}^{[k]}, \mathbf{D}^{[k]})\|^2$ .

3. Repeat step (2) until a steady MSE is reached, i.e.,  $|\text{MSE}^{[k]} - \text{MSE}^{[k-1]}| \leq \varepsilon$  or until the maximum number of iterations ( $K_{max}$ ) is reached.

---

The convergence of this algorithm is inherent from the convergence of the ACS algorithm.

One of most important aspects of both Algorithm 5.1 and Algorithm 5.2 is that it is possible to show that these problem formulations cover the conventional problem formulation, as conventional design procedure is a special case of these algorithms. This fact is stated and proved in the following proposition.

**Proposition 5.6.** *The robust problem formulation with uncertainty size of zero, would be equal to the conventional problem formulation.*

Table 5.1: Percentage of the Power Constraint Violations

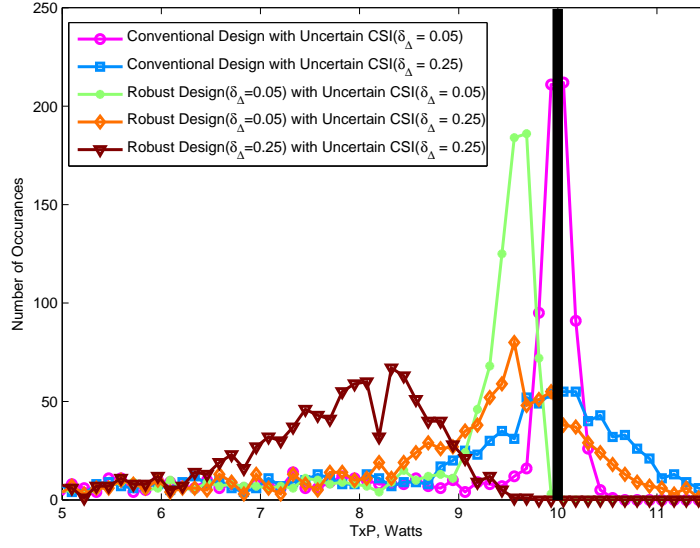
	Perfect CSI	CSI with $\delta_{\Delta} = 0.05$	CSI with $\delta_{\Delta} = 0.25$
Conventional Design	0%	32.5%	36.5%
Robust Design ( $\delta_{\Delta} = 0.05$ )	0%	0%	19.1%
Robust Design ( $\delta_{\Delta} = 0.25$ )	0%	0%	0%

*Proof.* To prove this proposition, we will focus on each individual iteration. At each iteration, for the SE model, it is clear that with no uncertainty, the mathematical expectation of both TxP and MSE are equal to their conventional counterparts. For the NBE model it is not that straightforward and it is formally proven here. In this case, with the uncertainty size equal to zero, i.e.,  $\delta_{\Delta} = \delta_{\Lambda} = 0$ , both  $\mathcal{P}$  and  $\mathcal{M}$  become block diagonal matrices. To be positive semi-definite, these block diagonal matrices should have a positive semi-definite diagonal element. The lower right elements of these matrices are composed of identity matrices that are positive semi-definite for any positive slack variables, even when  $\epsilon_i \rightarrow 0^+$ . Interestingly, the upper left elements of these matrices, using Schur's complement lemma, are equal to the constraints of (5.23) with any arbitrary precision, which completes the proof.  $\square$

In the following section the simulation results, based the numerical solution of this problem, are presented.

## 5.4 Simulation Results

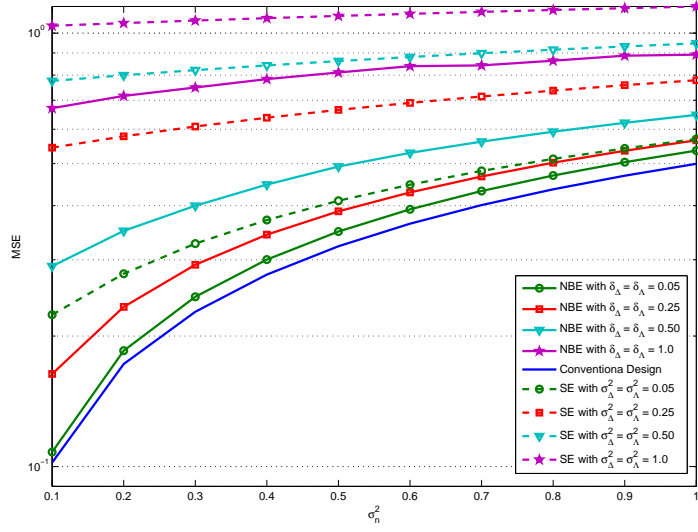
It is assumed that the source and the destination are equipped with  $T = 2$  antennas. It is also assumed that the relay station has  $R = 3$  antennas. The transmit power of the relay station is limited to  $P = 10$  Watts. The uncertainty size is varied from 0 (perfect CSI case, which is equivalent to the conventional case) to 1. The resulting graphs are the average of 100 simulation runs. In each run, the CSI (for nominal value of the channel) is generated randomly according to a zero-mean complex normal distribution with a variance equal to 1. In the following, the perfect CSI case is the conventional design procedure based on (5.23).



**Fig. 5.2:** Transmit power constraint histogram.

In Fig. 5.2, the histogram of the transmit power for different scenarios using the NBE model of uncertainty is depicted. To better quantify these graphs, the data is summarized in Table 5.1 as well. To get these results, a 10,000-time Monte Carlo simulation is executed. At each run, the beamforming matrices for three different cases, i.e., perfect CSI case and the cases designed to be robust against the uncertainty size of  $\delta_{\Delta} = 0.05$  and  $0.25$  are generated. The resulting system is faced with a channel realization with the aforementioned uncertainties. This means that all three designed systems are used against the CSI with uncertainties equal to  $0.0$ ,  $0.05$  and  $0.25$  to test the robustness of the design procedure. This is in line with our previous robustness test in previous chapters. This test shows that the robustly designed systems, are actually not violating the constraints in real systems based on a Monte Carlo simulation. It is clear that the systems that were designed to be robust against CSI variations, do not violate the transmit power constraint at all. The systems which were designed to rely on the perfect CSI only, may violate this constraint when faced with uncertain CSI. And as expected, the larger the uncertainty size, the more the transmit power constraint is violated with perfect CSI.

In Fig. 5.3 the MSE of the system for both models is displayed. As can be seen, both SE and NBE model-based MSE's exhibit an increasing trend with the



**Fig. 5.3:** MSE of the symbol detection.

increasing noise power, because MSE is proportional to the noise power. With increasing the noise power, the MSE increases as well. It is clear that the curves referring to scenarios with larger uncertainties have larger MSE. Since the MSE is also proportional to the uncertainty size, with increasing the uncertainty size, the MSE also increases.

In Fig. 5.4 the transmit power of the relay station for both models is shown. As can be observed, by increasing the noise power the transmit power of the relay station decreases slightly for both models. The amount of the transmit power is dependent on the uncertainty size. For the perfect CSI case, the required transmit power is larger than in the cases with larger variations in the CSI. It is because in perfect CSI case the resultant MSE is smaller than cases with larger uncertainty size resulting in this opportunity to transmit more power to get better signals at the destination. Also as can be seen from Fig. 5.2, the larger the uncertainty size, the smaller the transmit power.

In Fig. 5.5 the Bit Error Rate (BER) of the system under the different uncertainty models is depicted. For each channel realizations, a frame of  $2 \times 100,000$  symbols is sent over the channel (a frame of 100,000 symbols over each individual antenna). The symbols of the frame are modulated and demodulated using a standard 4-QAM modulator/demodulator and finally the BER graphs are pro-



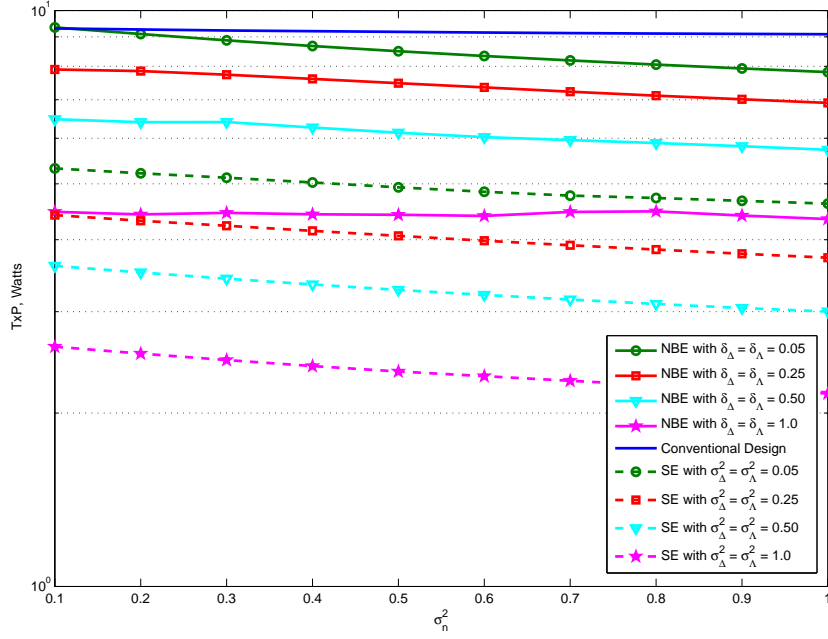


Fig. 5.4: Transmit power of the relay station.

duced. As can be seen, the BER of both models shows an increasing trend with the decrease of the SNR, i.e.,  $-10 \log(\sigma_n^2)$ . The amount of the change in BER for the perfect-CSI case is much larger relative to cases with a large uncertainty size. Also, as can be seen, the BER for both models is less sensitive to the change of the SNR in the cases with large uncertainty size.

## 5.5 Conclusion

The problem of robust linear beamforming design in the relay station is described. In this problem a MIMO point to point relay station with uncertain CSI for both links is studied. The MSE is used as the performance measure in the design problem and this problem is constrained to fulfill the transmit power requirements of the relay station. It is concluded that both MSE and the transmit power are subject to the SOC structure. It is also concluded that for the SE model of uncertainty, the problem is an iterative SOCP and in case of the NBE model, it is an iterative SDP. These two problems are solved numerically and simulation results are provided.

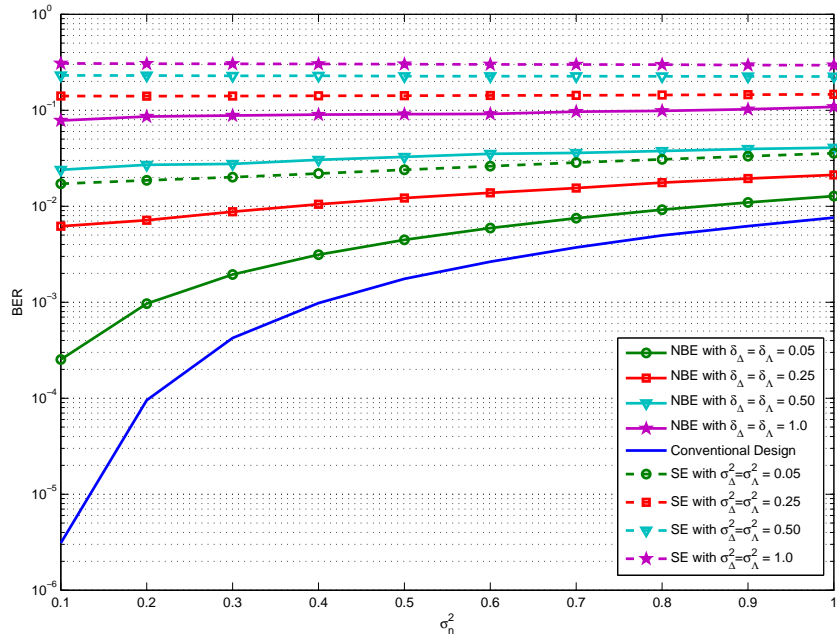


Fig. 5.5: BER of the system.

## Chapter 6

# Robust Linear Beamforming for MIMO Two-Way Relay Channel

### 6.1 Introduction

Multiuser two-way AF relay processing and power control methods for the beamforming systems is studied in [100]. The relay processing is optimized based on both zero-forcing (ZF) and minimum mean-square-error (MMSE) criteria under relay power constraints. The authors have examined various transmit and receive beamforming methods, for example, eigen beamforming, antenna selection, random beamforming, and modified equal gain beamforming. Performance bounds for two-way amplify-and-forward relaying system is discussed in [95]. In this paper, the average sum rate of an AF half-duplex TWRC system is analyzed. Optimal beamforming for two-way multi-antenna relay channel with analogue network coding is addressed in [96]. In this study only the relay station is equipped with multiple antennas. The optimal relay beamforming structure as well as an efficient algorithm to compute the optimal beamforming matrix are the main results of the aforementioned study. In [101, 103], the optimal resource allocation for a two-way relay-assisted OFDMA is explained. Here, A new transmission protocol, named hierarchical OFDMA, is proposed to support two-way communications between the base station (BS) and each mobile user (MU) with or without an assisting relay station (RS). An iterative receiver for a multi-input multi-output (MIMO) two-way wireless relay system is presented in [104]. A minimum mean square error (MMSE)-based iterative soft interference cancellation (SIC) unit and an Expec-

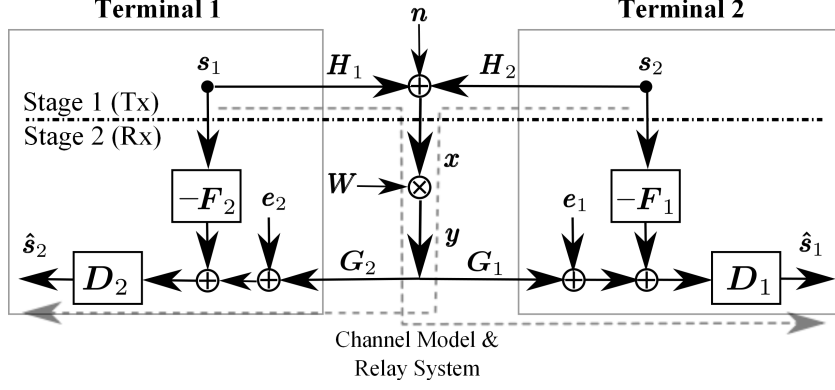
tation Conditional Maximization (ECM)-based estimation algorithm is used to build the receiver. The training-based channel estimation under the amplify-and-forward (AF) relay scheme is studied in [105]. A two-phase training protocol for channel estimation is proposed.

In this chapter the robust linear beamforming of a TWRC with imperfect CSI is considered. The imperfection of CSI is modeled using Stochastic Error (SE) and Norm-Bounded Error (NBE) models.

Relay systems have been used to enhance the coverage range and the QoS of communication systems. The information theoretic aspects of the capacity of the relay channels were studied in the 1970's and early 1980's [146, 147]. But interestingly some features of these systems only recently received considerable interest, including e.g., the capacity of and coding for the MIMO configurations in setups like the OWRC and the TWRC. In general, the relay station can operate using one of the following methods: AF, DF, and CF. In the design of MIMO relay channels the knowledge of the CSI is of great importance. Here, an AF based MIMO TWRC is studied for which our objective is to design the linear beamforming matrices of the relay station and the detectors and self cancellation filters of the destinations, particularly when the CSI is not perfectly known. In this chapter the robust linear beamforming of a MIMO TWRC with imperfect CSI is considered. The CSI is modeled using SE or NBE models. It is assumed that all the uncertainties either in the SE or the NBE model are of the same nature. The remainder is structured as follows: in Section 6.2 the model of the relay system is defined and analyzed. The problem formulation for both the SE model and the NBE model are described in Section 6.3. Section 6.4 contains the solutions to the aforementioned problems. Simulation results are shown in Section 6.5. Finally Section 6.6 concludes the chapter.

## 6.2 System Model

The signal flow graph of a MIMO TWRC is depicted in Fig. 6.1. It is assumed that both terminals and the relay station are equipped with  $N$  antennas. At each time



**Fig. 6.1:** Signal Flow Graph of a MIMO Two-Way Relay System

instant,  $\mathbf{s}_1, \mathbf{s}_2 \in \mathbb{C}^N$  are sent toward the relay station. It is assumed that these symbols are mutually independent and are drawn randomly from two independent sources with the energy of  $\sigma_{\mathbf{s}_1}^2$  and  $\sigma_{\mathbf{s}_2}^2$  respectively. These symbols are sent over two imperfectly known channels, i.e.,  $\mathbf{H}_1, \mathbf{H}_2 \in \mathbb{C}^{N \times N}$ .

The received signal at the relay station is:

$$\mathbf{x} = \mathbf{H}_1 \mathbf{s}_1 + \mathbf{H}_2 \mathbf{s}_2 + \mathbf{n}, \quad (6.1)$$

where  $\mathbf{n} \in \mathbb{C}^N$  is the zero-mean white Gaussian noise term in the relay station, i.e.,  $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \sigma_n^2 \mathbf{I})$ . It is assumed that its elements are mutually independent of each other and are also independent of the signals.

At the relay station, this signal is beamformed using a weight matrix,  $\mathbf{W} \in \mathbb{C}^{N \times N}$ . The resultant signal,  $\mathbf{y}$ , is as follows:

$$\mathbf{y} = \mathbf{W} \mathbf{x} \quad (6.2a)$$

$$= \mathbf{W} (\mathbf{H}_1 \mathbf{s}_1 + \mathbf{H}_2 \mathbf{s}_2 + \mathbf{n}). \quad (6.2b)$$

This signal is sent back to both terminals over the two different channels, i.e.,  $\mathbf{G}_1, \mathbf{G}_2 \in \mathbb{C}^{N \times N}$ . The received signal first goes through a self interference canceler followed by a linear equalizer. The outputs of the equalizers at both ends are:

$$\hat{\mathbf{s}}_i = \mathbf{D}_i (\mathbf{G}_i \mathbf{y} - \mathbf{F}_i \mathbf{s}_{-i} + \mathbf{e}_i) \quad (6.3a)$$

$$= \mathbf{D}_i (\mathbf{G}_i \mathbf{W} \mathbf{H}_i \mathbf{s}_i + (\mathbf{G}_i \mathbf{W} \mathbf{H}_{-i} - \mathbf{F}_i) \mathbf{s}_{-i}) + \mathbf{D}_i (\mathbf{G}_i \mathbf{W} \mathbf{n} + \mathbf{e}_i), \quad (6.3b)$$

where  $\mathbf{e}_1$  and  $\mathbf{e}_2$  are the received noise at each terminal and are assumed to be mutually independent having  $\mathbf{E}_{\mathbf{e}_1} [\|\mathbf{e}_1\|^2] = \sigma_{\mathbf{e}_1}^2$  and  $\mathbf{E}_{\mathbf{e}_2} [\|\mathbf{e}_2\|^2] = \sigma_{\mathbf{e}_2}^2$  respectively.

Although mentioned in the notations, it worth to mention that

$$\mathbf{s}_{-i} = \begin{cases} \mathbf{s}_1 & i = 2 \\ \mathbf{s}_2 & i = 1. \end{cases} \quad (6.4)$$

$\mathbf{F}_1 \in \mathbb{C}^{N \times N}$  and  $\mathbf{F}_2 \in \mathbb{C}^{N \times N}$  are the self-cancellation filters that will be discussed in more detail in the following subsection.

The model of uncertainty for these channels is as follows:

$$\mathbf{H}_i = \tilde{\mathbf{H}}_i + \mathbf{\Delta}_i, \quad \forall i, \quad (6.5a)$$

$$\mathbf{G}_i = \tilde{\mathbf{G}}_i + \mathbf{\Lambda}_i, \quad \forall i, \quad (6.5b)$$

where  $\tilde{\mathbf{H}}_i$  and  $\tilde{\mathbf{G}}_i$  are the nominal values of the channels and  $\mathbf{\Delta}_i$  and  $\mathbf{\Lambda}_i$  are the uncertainty matrices. The uncertainty is defined using both SE model and NBE model. In the SE model, it is assumed that

$$\begin{cases} \mathbb{E}_{\mathbf{\Delta}_i} [\mathbf{\Delta}_i] = \mathbb{E}_{\mathbf{\Lambda}_i} [\mathbf{\Lambda}_i] = \mathbf{0}, \\ \mathbb{E}_{\mathbf{\Lambda}_i} [\text{vec} [\mathbf{\Lambda}_i] \text{vec} [\mathbf{\Lambda}_i]^*] = \sigma_{\mathbf{\Lambda}_i}^2 \mathbf{I}, \\ \mathbb{E}_{\mathbf{\Delta}_i} [\text{vec} [\mathbf{\Delta}_i] \text{vec} [\mathbf{\Delta}_i]^*] = \sigma_{\mathbf{\Delta}_i}^2 \mathbf{I}. \end{cases} \quad (6.6)$$

In this model it is also assumed that the variances of the uncertainty matrices, i.e.,  $\sigma_{\mathbf{\Delta}_i}^2$  and  $\sigma_{\mathbf{\Lambda}_i}^2$ , are a priori known and fixed. In the NBE model, it is assumed that

$$\begin{cases} \|\mathbf{\Delta}_i\| \leq \delta_i, \\ \|\mathbf{\Lambda}_i\| \leq \lambda_i, \end{cases} \quad (6.7)$$

where  $\delta_i$  and  $\lambda_i$  are the known and fixed uncertainty sizes for the  $i$ th link's CSI.

### 6.2.1 Self Cancellation Filter

To avoid the corruption of the intended received signal by self-received signals in TWRC, two self cancellation filters are placed at the receiver of each terminal. Usually it is assumed that due to the perfect knowledge of the self input signals and the CSI, it is possible to cancel out the harmful effects of the self interference. However, in this chapter, with imperfect CSI knowledge it is not possible to implement ideal self cancellation filters. In the following, two possible mechanisms are addressed to mitigate the self interference effects.

### Conventional Self Cancellation (CSC)

Because of the incomplete knowledge of the CSI, the self cancellation filters are not able to cancel the self signals in each receiver. It is assumed that the self cancellation filters can only cancel the nominal values of the self interference, i.e., we choose

$$\mathbf{F}_i \triangleq \tilde{\mathbf{G}}_i \mathbf{W} \tilde{\mathbf{H}}_{-i}, \quad i = 1, 2, \quad (6.8)$$

### Strict Self Cancellation (SSC)

Unlike CSC, in SSC the self cancellation filters remain in the problem formulation as two matrices to be designed. It is possible to have a convex problem formulation to design both self cancellation filters and the beamforming filter of the relay station simultaneously.

### 6.2.2 MSE and Transmit Power

In the design procedure, a robust minimax MSE approach is employed with a constraint on the transmit power of the relay station. In this formulation, the sum of the MSE of both links, i.e., from the transmit (Tx) side of either links to the receive (Rx) side of the other link, is minimized while the transmit power of the relay system is upper bounded. The first link is the link between the Tx side of terminal 1 to the Rx side of the terminal 2 while the second link is vice versa. In the following, the structure of these quantities is discussed in detail.

**Proposition 6.1.** *MSE<sub>1</sub> for the first link and MSE<sub>2</sub> for the second link as well as the transmit power TxP of the relay station, have SOC structures, i.e.*

$$\begin{aligned} \text{MSE}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i) &\triangleq \mathbb{E}_{\mathbf{s}_1, \mathbf{s}_2} [\|\hat{\mathbf{s}}_i - \mathbf{s}_i\|^2] \\ &= \|\boldsymbol{\mu}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i)\|_2^2, \quad i = 1, 2; \end{aligned} \quad (6.9a)$$

$$\begin{aligned} \text{TxP}(\mathbf{W}) &\triangleq \mathbb{E}_{\mathbf{s}_1, \mathbf{s}_2, \mathbf{n}} [\|\mathbf{y}\|^2] \\ &= \|\boldsymbol{\pi}(\mathbf{W})\|_2^2. \end{aligned} \quad (6.9b)$$

*Proof.* The MSE of each link and the TxP of the relay station, can be calculated as follows:

$$\begin{aligned}
\text{MSE}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i) &= \mathbb{E}_{\mathbf{s}_{i,-i}} [\|\hat{\mathbf{s}}_i - \mathbf{s}_i\|^2] \triangleq \|\boldsymbol{\mu}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i)\|_2^2, \quad i = 1, 2, \\
&= \sigma_{\mathbf{s}_i}^2 \|\mathbf{D}_i \mathbf{G}_i \mathbf{W} \mathbf{H}_i - \mathbf{I}\|_F^2 + \sigma_{\mathbf{s}_{-i}}^2 \|\mathbf{D}_i (\mathbf{G}_i \mathbf{W} \mathbf{H}_{-i} - \mathbf{F}_i)\|_F^2 + \\
&\quad \sigma_n^2 \|\mathbf{D}_i \mathbf{G}_i \mathbf{W}\|_F^2 + \sigma_{e_i}^2 \|\mathbf{D}_i\|_F^2 \\
\text{TxP}(\mathbf{W}) &= \mathbb{E}_{\mathbf{s}_1, \mathbf{s}_2, \mathbf{n}} [\|\mathbf{y}\|^2] \triangleq \|\boldsymbol{\pi}(\mathbf{W})\|_2^2 \\
&= \mathbb{E}_{\mathbf{s}_1, \mathbf{s}_2, \mathbf{n}} [\|\mathbf{W} \mathbf{H}_1 \mathbf{s}_1 + \mathbf{W} \mathbf{H}_2 \mathbf{s}_2 + \mathbf{W} \mathbf{n}\|^2] \\
&= \sigma_{\mathbf{s}_1}^2 \|\mathbf{W} \mathbf{H}_1\|_F^2 + \sigma_{\mathbf{s}_2}^2 \|\mathbf{W} \mathbf{H}_2\|_F^2 + \sigma_n^2 \|\mathbf{W}\|_F^2
\end{aligned}$$

Each expectation for MSEs has 16 terms, that due to orthogonality only four self multiplication terms remain leading to aforementioned expressions. A similar procedure may be undergone for the TxP.  $\square$

## 6.3 Problem formulation

Since SE and NBE models of uncertainty impose completely different conditions on  $\text{MSE}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i)$ ,  $i = 1, 2$ ; and  $\text{TxP}(\mathbf{W})$ , the details of the problem formulation for these two uncertainty models are summarized distinctly. Using the SE model, both the sum MSE of the system and the transmit power of the relay station are stochastic variables. In this case, the average performance measures are optimized. In the NBE model, unlike the SE model, the aforementioned quantities are deterministic, having infinitely many realizations. In this case, the worst case performance measures are optimized. In the following subsections, the problem formulation is summarized for the two models. It is possible to have the power-minimization based formulations as well as the sum MSE based formulation. Both cases are addressed for each of the uncertainty models.

### 6.3.1 Non-robust Design

Conventionally the beamforming design is based on the perfect CSI which is called non-robust design. This problem formulation is in line with [100]. Using Proposi-



tion 6.1, it is found that the conventional design is a SOCP:

$$\underset{\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i}{\text{minimize}} \quad \sum_{i=1}^2 \tau_i \quad (6.11a)$$

$$\text{subject to} \quad \|\boldsymbol{\pi}(\mathbf{W})\|_2^2 \leq P, \quad (6.11b)$$

$$\|\boldsymbol{\mu}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i)\|_2^2 \leq \tau_i, \quad i = 1, 2. \quad (6.11c)$$

This problem can be solved numerically and the results can be used to compare with the performance of the robust system.

### 6.3.2 NBE-Based Problem Formulation

The problem formulation based on power minimization in the epigraph form, is

$$\underset{\mathcal{V}}{\text{minimize}} \quad \tau \quad (6.12a)$$

$$\text{subject to} \quad \max_{\boldsymbol{\Delta}_{1,2}} \text{TxP}(\mathbf{W}) \leq \tau, \quad (6.12b)$$

$$\max_{\boldsymbol{\Delta}_{1,2}, \boldsymbol{\Lambda}_i} \text{MSE}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i) \leq \epsilon_i, \quad i = 1, 2, \quad (6.12c)$$

where  $\epsilon_1$  and  $\epsilon_2$  are constants and  $\mathcal{V}$  is a set that contains the optimization variables, i.e.,

$$\mathcal{V} = \begin{cases} \{\mathbf{W}, \mathbf{F}_1, \mathbf{F}_2, \mathbf{D}_1, \mathbf{D}_2, \tau\}, & \text{in SSC mode,} \\ \{\mathbf{W}, \mathbf{D}_1, \mathbf{D}_2, \tau\}, & \text{in CSC mode.} \end{cases} \quad (6.13)$$

Also the problem formulation based on the sum MSE minimization is

$$\underset{\mathcal{X}}{\text{minimize}} \quad \tau_1 + \tau_2 \quad (6.14a)$$

$$\text{subject to} \quad \max_{\boldsymbol{\Delta}_{1,2}} \text{TxP}(\mathbf{W}) \leq P, \quad (6.14b)$$

$$\max_{\boldsymbol{\Delta}_{1,2}, \boldsymbol{\Lambda}_i} \text{MSE}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i) \leq \tau_i, \quad i = 1, 2, \quad (6.14c)$$

where  $P$  is a constant and  $\mathcal{X}$  is defined as follows:

$$\mathcal{X} = \begin{cases} \{\mathbf{W}, \mathbf{F}_1, \mathbf{F}_2, \mathbf{D}_1, \mathbf{D}_2, \tau_1, \tau_2\}, & \text{in SSC mode} \\ \{\mathbf{W}, \mathbf{D}_1, \mathbf{D}_2, \tau_1, \tau_2\}, & \text{in CSC mode.} \end{cases} \quad (6.15)$$

These two problems have almost the same structure and therefore we consider without loss of generality, the second formulation only. By changing the role of the slack variables and the constants of these two problems, either of them can be cast as the other one.

### 6.3.3 SE-Based Problem Formulation

In the presence of the SE model, the expected values of the transmit power of the relay station and the sum MSE of the whole system are optimized. The power minimization problem formulation for the SE model of uncertainty is

$$\underset{\mathcal{V}}{\text{minimize}} \quad \tau \quad (6.16a)$$

$$\text{subject to} \quad \mathbb{E}_{\Delta_{1,2}} [\text{TxP}(\mathbf{W})] \leq \tau, \quad (6.16b)$$

$$\mathbb{E}_{\Delta_{1,2}, \Lambda_i} [\text{MSE}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i)] \leq \epsilon_i, \quad i = 1, 2, \quad (6.16c)$$

where  $\epsilon_1$  and  $\epsilon_2$  are the constants and  $\mathcal{V}$  is the set defined before for the NBE model. Similarly, the sum MSE formulation is

$$\underset{\mathcal{X}}{\text{minimize}} \quad \tau_1 + \tau_2 \quad (6.17a)$$

$$\text{subject to} \quad \mathbb{E}_{\Delta_{1,2}} [\text{TxP}(\mathbf{W})] \leq P, \quad (6.17b)$$

$$\mathbb{E}_{\Delta_{1,2}, \Lambda_i} [\text{MSE}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i)] \leq \tau_i, \quad i = 1, 2, \quad (6.17c)$$

where the constant  $P$  and  $\mathcal{X}$  are as defined previously. As stated before, the sum MSE based formulation is treated in this chapter.

## 6.4 Solutions

In this section, the solution of the linear beamforming design is presented for both models of uncertainty.

### 6.4.1 SE-Based Solutions

To proceed with the SE-based solutions, simplification of the mathematical expectations of the constituent  $\text{TxP}(\mathbf{W})$ ,  $\text{MSE}_1(\mathbf{W}, \mathbf{D}_1, \mathbf{F}_1)$  and  $\text{MSE}_2(\mathbf{W}, \mathbf{D}_2, \mathbf{F}_2)$  is required, which is done in the following proposition.

**Proposition 6.2.** *For the SE model of uncertainty,  $\mathbb{E}_{\Delta_{1,2}, \Lambda_i} [\text{MSE}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i)]$ , and  $\mathbb{E}_{\Delta_{1,2}} [\text{TxP}(\mathbf{W})]$ , all have the following SOC structure:*

$$\mathbb{E}_{\Delta_{1,2}} [\text{TxP}(\mathbf{W})] = \sum_{i=1}^2 \sigma_{s_i}^2 \left( \|\mathbf{W} \tilde{\mathbf{H}}_i\|_F^2 + \sigma_{\Delta_i}^2 \|\mathbf{I} \otimes \mathbf{W}\|_F^2 \right) + \sigma_n^2 \|\mathbf{W}\|_F^2 \quad (6.18a)$$

$$\triangleq \|\bar{\boldsymbol{\pi}}(\mathbf{W})\|_2^2 \quad (6.18b)$$

$$\mathbb{E}_{\Delta_{1,2}, \Lambda_i} [\text{MSE}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i)] =$$

$$\begin{aligned} & \sigma_{s_i}^2 \left( \|\mathbf{D}_i \tilde{\mathbf{G}}_i \mathbf{W} \tilde{\mathbf{H}}_i - \mathbf{I}\|_F^2 + \sigma_{\Delta_i}^2 \|\mathbf{I} \otimes \mathbf{D}_i \tilde{\mathbf{G}}_i \mathbf{W}\|_F^2 + \sigma_{\Lambda_i}^2 \|(\mathbf{W} \tilde{\mathbf{H}}_i)^T \otimes \mathbf{D}_i\|_F^2 \right) + \\ & \sigma_{s_{-i}}^2 \left( \|\mathbf{D}_i (\tilde{\mathbf{G}}_i \mathbf{W} \tilde{\mathbf{H}}_{-i} - \mathbf{F}_i)\|_F^2 + \sigma_{\Delta_{-i}}^2 \|\mathbf{I} \otimes \mathbf{D}_i \tilde{\mathbf{G}}_i \mathbf{W}\|_F^2 + \sigma_{\Lambda_i}^2 \|(\mathbf{W} \tilde{\mathbf{H}}_{-i})^T \otimes \mathbf{D}_i\|_F^2 \right) + \\ & \sigma_n^2 \left( \|\mathbf{D}_i \tilde{\mathbf{G}}_i \mathbf{W}\|_F^2 + \sigma_{\Lambda_i}^2 \|\mathbf{W}^T \otimes \mathbf{D}_i\|_F^2 \right) + \sigma_{e_i}^2 \|\mathbf{D}_i\|_F^2 \end{aligned} \quad (6.19a)$$

$$\triangleq \|\bar{\boldsymbol{\mu}}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i)\|_2^2, \quad i = 1, 2. \quad (6.19b)$$

*Proof.* The proof is similar to the proof of Proposition 6.1 and is not repeated here. Again to have a tractable formulation, we have neglected the second order uncertain terms. It is so for the remaining analysis.  $\square$

Interestingly, because the minimum value of

$$\|\mathbf{D}_i(\tilde{\mathbf{G}}_i \mathbf{W} \tilde{\mathbf{H}}_{-i} - \mathbf{F}_i)\|_F^2 \quad (6.20)$$

is equal to zero, both self cancellation procedures lead to similar self cancellation filters:

$$\mathbf{F}_i = \tilde{\mathbf{G}}_i \mathbf{W} \tilde{\mathbf{H}}_{-i}. \quad (6.21)$$

Using this result, the expectation of individual MSE's are independent of the self cancellation filters. By using an appropriate vector notation, the SE problem formulation is a SOCP as follows:

$$\underset{\mathcal{X}}{\text{minimize}} \quad \tau_1 + \tau_2 \quad (6.22a)$$

$$\text{subject to} \quad \|\bar{\boldsymbol{\pi}}(\mathbf{W})\|^2 \leq P, \quad (6.22b)$$

$$\|\bar{\boldsymbol{\mu}}_i(\mathbf{W}, \mathbf{D}_i)\|^2 \leq \tau_i, \quad i = 1, 2. \quad (6.22c)$$

This problem can be solved numerically using interior point methods. Unfortunately, despite the appealing structure of this problem, due to the biconvexity of  $\bar{\boldsymbol{\mu}}_1$  and  $\bar{\boldsymbol{\mu}}_2$  this problem should be solved iteratively. The procedure is as follows:

---

**Algorithm 6.1.** [Iterative beamformer design for the SE model]

1. Let  $k \leftarrow 0$ . Initialize the beamformer matrices  $\mathbf{W}^{[0]}$ ,  $\mathbf{D}_1^{[0]}$  and  $\mathbf{D}_2^{[0]}$ .

Compute the initial MSE,  $\text{MSE}^{[0]} = \sum_{i=1}^2 \|\bar{\boldsymbol{\mu}}_i(\mathbf{W}^{[0]}, \mathbf{D}_i^{[0]})\|^2$ .

2. Let  $k \leftarrow k + 1$ .

Update beamforming matrices using the following procedures:

$$\mathbf{W}^{[k]} = \arg \min_{\mathbf{W}, \tau_i \geq 0} \quad \tau_1 + \tau_2 \quad (6.23a)$$

$$\text{S. t.} \quad \|\bar{\boldsymbol{\pi}}(\mathbf{W})\|^2 \leq P, \quad (6.23b)$$

$$\|\bar{\boldsymbol{\mu}}_i(\mathbf{W}, \mathbf{D}_i^{[k-1]})\|^2 \leq \tau_i, \quad (6.23c)$$

and

$$\mathbf{D}_i^{[k]} = \arg \min_{\mathbf{D}, \tau_i \geq 0} \quad \tau_1 + \tau_2 \quad (6.24a)$$

$$\text{S. t.} \quad \|\bar{\boldsymbol{\mu}}_i(\mathbf{W}^{[k-1]}, \mathbf{D}_i)\|^2 \leq \tau_i. \quad (6.24b)$$

Update the MSE at this stage:  $\text{MSE}^{[k]} = \sum_{i=1}^2 \|\bar{\boldsymbol{\mu}}_i(\mathbf{W}^{[k]}, \mathbf{D}_i^{[k]})\|^2$ .

3. Repeat step (2) until reaching a steady MSE, i.e.,  $|\text{MSE}^{[k]} - \text{MSE}^{[k-1]}| \leq \varepsilon$  or until the maximum number of iterations ( $K_{max}$ ) is reached.

---

In this procedure  $\varepsilon$  and  $K_{max}$  are a constant predefined tolerance and the maximum iteration number, respectively. It should be mentioned that using some standard tricks [93], each of (6.23) and (6.24) may be converted to a standard SOCP and then efficiently solved using interior-point methods. As this algorithm is also based on the ACS algorithm, the convergence of it is not proved here.

## 6.4.2 NBE-Based Solutions

The design problem, after substituting the appropriate terms for which to facilitate the simplified notations, in which the dependency to design variables is presumed but not mentioned explicitly, becomes  $\forall i = 1, 2$ ,

$$\underset{\mathcal{X}}{\text{minimize}} \quad \tau_1 + \tau_2 \quad (6.25a)$$

$$\text{subject to} \quad \max_{\boldsymbol{\Delta}_{1,2}} \|\boldsymbol{\pi}\|^2 \leq P, \quad \forall \boldsymbol{\Delta}_i : \|\boldsymbol{\Delta}_i\| \leq \delta_i, \quad (6.25b)$$

$$\begin{aligned} \max_{\boldsymbol{\Delta}_{1,2}, \boldsymbol{\Lambda}_i} \|\boldsymbol{\mu}_i\|^2 &\leq \tau_i \quad \forall \boldsymbol{\Delta}_i : \|\boldsymbol{\Delta}_i\| \leq \delta_i, \\ &\forall \boldsymbol{\Lambda}_i : \|\boldsymbol{\Lambda}_i\| \leq \lambda_i. \end{aligned} \quad (6.25c)$$

and using Schur's Complement Lemma, it is possible to recast the above problem to a semi-infinite LMI-based problem. The aforementioned problem is recast as

$$\underset{\mathcal{X}}{\text{minimize}} \quad \tau_1 + \tau_2 \quad (6.26a)$$

$$\text{subject to} \quad \begin{bmatrix} P & \boldsymbol{\pi}^* \\ \boldsymbol{\pi} & \mathbf{I} \end{bmatrix} \succeq 0 \quad \forall \boldsymbol{\Delta}_i : \|\boldsymbol{\Delta}_i\| \leq \delta_i, \quad (6.26b)$$

$$\begin{bmatrix} \tau_i & \boldsymbol{\mu}_i^* \\ \boldsymbol{\mu}_i & \mathbf{I} \end{bmatrix} \succeq 0 \quad \forall \boldsymbol{\Delta}_i : \|\boldsymbol{\Delta}_i\| \leq \delta_i, \\ \forall \boldsymbol{\Lambda}_i : \|\boldsymbol{\Lambda}_i\| \leq \lambda_i, \quad (6.26c)$$

This problem is a semi-infinite optimization problem. To further simplify the problem, the Theorem 2.1 is used. The key idea in applying this theorem to our problem is to use the underlying structure of MSE and the transmit power which is summarized in the following proposition.

**Proposition 6.3.** *For the NBE model of uncertainty and regardless of the type of self cancellation filter,  $\boldsymbol{\mu}_1$  and  $\boldsymbol{\mu}_2$  have a multi-linear structure in terms of the uncertainty matrices, i.e.,*

$$\boldsymbol{\pi} \triangleq \tilde{\boldsymbol{\pi}} + \sum_{i=1}^2 \boldsymbol{\pi}_{\boldsymbol{\Delta}_i} \text{vec} [\boldsymbol{\Delta}_i], \quad (6.27a)$$

$$\boldsymbol{\mu}_i \triangleq \tilde{\boldsymbol{\mu}}_i + \sum_{j=1}^2 \boldsymbol{\mu}_{i,\boldsymbol{\Delta}_j} \text{vec} [\boldsymbol{\Delta}_j] + \boldsymbol{\mu}_{i,\boldsymbol{\Lambda}_i} \text{vec} [\boldsymbol{\Lambda}_i], \quad (6.27b)$$

where

$$\tilde{\boldsymbol{\pi}} = \begin{bmatrix} \sigma_{s_1} \text{vec} [\mathbf{W} \tilde{\mathbf{H}}_1] \\ \sigma_{s_2} \text{vec} [\mathbf{W} \tilde{\mathbf{H}}_2] \\ \sigma_n \text{vec} [\mathbf{W}] \end{bmatrix}, \quad (6.28a)$$

$$\boldsymbol{\pi}_{\boldsymbol{\Delta}_1} = \begin{bmatrix} \sigma_{s_1} \mathbf{I} \otimes \mathbf{W} \\ \mathbf{0} \end{bmatrix}, \quad (6.28b)$$

$$\boldsymbol{\pi}_{\boldsymbol{\Delta}_2} = \begin{bmatrix} \mathbf{0} \\ \sigma_{s_2} \mathbf{I} \otimes \mathbf{W} \\ \mathbf{0} \end{bmatrix}, \quad (6.28c)$$

and

$$\tilde{\boldsymbol{\mu}}_i = \begin{bmatrix} \sigma_{s_i} \text{vec} [D_i \tilde{G}_i W \tilde{H}_i - I] \\ \tilde{\boldsymbol{\mu}}_{i,SC} \\ \sigma_n \text{vec} [D_i \tilde{G}_i W] \\ \sigma_{e_i} \text{vec} [D_i] \end{bmatrix}, \quad (6.28d)$$

$$\tilde{\boldsymbol{\mu}}_{i,SC} = \begin{cases} \mathbf{0}, & \text{for CSC,} \\ \sigma_{s_{-i}} \text{vec} [D_i (\tilde{G}_i W \tilde{H}_{-i} - F_i)], & \text{for SSC,} \end{cases} \quad (6.28e)$$

$$\boldsymbol{\mu}_{i,\Delta_1} = \begin{bmatrix} \sigma_{s_i} I \otimes (D_i \tilde{G}_i W) \\ \mathbf{0} \end{bmatrix}, \quad (6.28f)$$

$$\boldsymbol{\mu}_{i,\Delta_2} = \begin{bmatrix} \mathbf{0} \\ \sigma_{s_{-i}} I \otimes (D_i \tilde{G}_i W) \\ \mathbf{0} \end{bmatrix}, \quad (6.28g)$$

$$\boldsymbol{\mu}_{i,\Lambda_i} = \begin{bmatrix} \sigma_{s_i} (W \tilde{H}_i)^T \otimes D_i \\ \sigma_{s_{-i}} (W \tilde{H}_{-i})^T \otimes D_i \\ \sigma_n W^T \otimes D_i \\ \mathbf{0} \end{bmatrix}.$$

*Proof.* For  $\text{MSE}_i$  it is known that

$$\begin{aligned} \text{MSE}_i(W, D_i, F_i) &= \sigma_{s_i}^2 \|D_i G_i W H_i - I\|_F^2 + \sigma_{s_{-i}}^2 \|D_i (G_i W H_{-i} - F_i)\|_F^2 + \\ &\quad \sigma_n^2 \|D_i G_i W\|_F^2 + \sigma_{e_i}^2 \|D_i\|_F^2 \\ &\triangleq \|\boldsymbol{\mu}_i(W, D_i, F_i)\|^2. \end{aligned}$$

This expression is valid, if

$$\boldsymbol{\mu}_i(W, D_i, F_i) = \begin{bmatrix} \sigma_{s_i} \text{vec} [D_i G_i W H_i - I] \\ \sigma_{s_{-i}} \text{vec} [D_i (G_i W H_{-i} - F_i)] \\ \sigma_n \text{vec} [D_i G_i W] \\ \sigma_{e_i} \text{vec} [D_i] \end{bmatrix} \quad (6.29)$$

The above expression is gained using the following mathematical procedures: the Frobenius norm of each term may be written in terms of the second norm of the vectorized version of its matrix argument (Lemma 2.2). By applying the Lemma 2.4 to the resultant series of the second norms,  $\boldsymbol{\mu}_1$  is resulted. In the following subsections, the different self cancellation treatments are dealt to further simplify the resultant  $\boldsymbol{\mu}_1$ .  $\square$

Without loss of generality, the first constraint is dealt with in detail and a

similar procedure can be applied to the next two constraints. As it is known,

$$\begin{bmatrix} P & \pi^* \\ \pi & I \end{bmatrix} \succeq 0. \quad (6.30)$$

By appropriate selection of

$$A = \begin{bmatrix} P & \tilde{\pi}^* \\ \tilde{\pi} & I \end{bmatrix} \quad (6.31)$$

$$P_1 = [0 \quad \pi_{\Delta_1}^*], \quad P_2 = [0 \quad \pi_{\Delta_2}^*] \quad (6.32)$$

$$X_1 = \text{vec}[\Delta_1], \quad X_2 = \text{vec}[\Delta_2] \quad (6.33)$$

$$Q_1 = Q_2 = [-1 \quad 0] \quad (6.34)$$

the first constraint is equal to the following LMI:

$$\left[ \begin{array}{cc|cc} P - (\epsilon_1 + \epsilon_2) & \tilde{\pi}^* & 0 & 0 \\ \tilde{\pi} & I & -\delta_1 \pi_{\Delta_1} & -\delta_2 \pi_{\Delta_2} \\ \hline 0 & -\delta_1 \pi_{\Delta_1}^* & \epsilon_1 I & 0 \\ 0 & -\delta_2 \pi_{\Delta_2}^* & 0 & \epsilon_2 I \end{array} \right] \succeq 0, \quad (6.35)$$

$$\epsilon_1, \epsilon_2 \geq 0, \quad (6.36)$$

Applying a similar procedure to the other two constraints of the design problem,

the design problem is the following LMI:

$$\underset{\mathcal{X}'}{\text{minimize}} \quad \tau_1 + \tau_2 \quad (6.37a)$$

$$\text{subject to} \quad (6.37b)$$

$$\mathcal{P}(W) \succeq 0, \quad (6.37c)$$

$$\mathcal{M}_i(W, F_i, D_i) \succeq 0, \quad i = 1, 2, \quad (6.37d)$$

$$\varepsilon_i, \varepsilon_{\mu_i, j}, \varepsilon_{\Lambda_i}, \tau_i \geq 0, \quad i, j = 1, 2, \quad (6.37e)$$

where  $\mathcal{P}$  and  $\mathcal{M}_i$  are defined as

$$\mathcal{P} = \left[ \begin{array}{cc|cc} P - (\varepsilon_1 + \varepsilon_2) & \tilde{\pi}^* & 0 & 0 \\ \tilde{\pi} & I & -\delta_1 \pi_{\Delta_1} & -\delta_2 \pi_{\Delta_2} \\ \hline 0 & -\delta_1 \pi_{\Delta_1}^* & \epsilon_1 I & 0 \\ 0 & -\delta_2 \pi_{\Delta_2}^* & 0 & \epsilon_2 I \end{array} \right], \quad (6.38a)$$

$$\mathcal{M}_i = \left[ \begin{array}{cc|ccc} \tau_i - \sum_{j=1,2,k} \varepsilon_{\mu_i, j} & \tilde{\mu}_i^* & 0 & 0 & 0 \\ \tilde{\mu}_i & I & -\delta_1 \mu_{i, \Delta_1} & -\delta_2 \mu_{i, \Delta_2} & -\lambda_i \mu_{i, \Lambda_i} \\ \hline 0 & -\delta_1 \mu_{i, \Delta_1}^* & \varepsilon_{\mu_i, 1} I & 0 & 0 \\ 0 & -\delta_2 \mu_{i, \Delta_2}^* & 0 & \varepsilon_{\mu_i, 2} I & 0 \\ 0 & -\lambda_i \mu_{i, \Lambda_i}^* & 0 & 0 & \varepsilon_{\Lambda_i} I \end{array} \right], \quad (6.38b)$$

and

$$\mathcal{X}' = \mathcal{X} \cup \{\varepsilon_i, \varepsilon_{\mu_i, j}, \varepsilon_{\Lambda_i}\}_{i, j=1, 2}. \quad (6.39)$$

To be able to solve this biconvex problem we use the following procedure:

---

**Algorithm 6.2.** [iterative beamformer design for the NBE model]

1. Let  $k \leftarrow 0$ . Initialize the beamformer matrices  $\mathbf{W}^{[0]}, \mathbf{D}_1^{[0]}, \mathbf{D}_2^{[0]}, \mathbf{F}_1^{[0]}$  and  $\mathbf{F}_2^{[0]}$ . Compute the initial MSE,  $\text{MSE}^{[0]} = \sum_{i=1}^2 \|\boldsymbol{\mu}_i(\mathbf{W}^{[0]}, \mathbf{D}_i^{[0]}, \mathbf{F}_i^{[0]})\|^2$ .
2. Let  $k \leftarrow k + 1$ . Update beamforming matrices using the following procedures:

(I) For CSC scheme:  $\forall i, j = 1, 2$ ,

$$\mathbf{W}^{[k]} = \arg \min_{\mathbf{W}, \tau_i \geq 0} \tau_1 + \tau_2 \quad (6.40a)$$

$$\text{S. t. } \mathcal{P}(\mathbf{W}) \succeq 0, \quad (6.40b)$$

$$\mathcal{M}_i(\mathbf{W}, \mathbf{D}_i^{[k-1]}) \succeq 0, \quad (6.40c)$$

$$\varepsilon_i, \varepsilon_{\mu_i, j}, \varepsilon_{\Lambda_i}, \tau_i \geq 0, \quad (6.40d)$$

and

$$\mathbf{D}_i^{[k]} = \arg \min_{\mathbf{D}, \tau_i \geq 0} \tau_1 + \tau_2 \quad (6.41a)$$

$$\text{S. t. } \mathcal{M}_i(\mathbf{W}^{[k-1]}, \mathbf{D}_i) \succeq 0, \quad (6.41b)$$

$$\varepsilon_{\mu_i, j}, \varepsilon_{\Lambda_i}, \tau_i \geq 0. \quad (6.41c)$$

Update the MSE at this stage:

$$\text{MSE}^{[k]} = \sum_{i=1}^2 \|\boldsymbol{\mu}_i(\mathbf{W}^{[k]}, \mathbf{D}_i^{[k]})\|^2.$$

(II) For SSC scheme:  $\forall i, j = 1, 2$ ,

$$\{\mathbf{W}^{[k]}, \mathbf{F}_i^{[k]}\} = \arg \min_{\mathbf{W}, \mathbf{F}_i, \tau_i \geq 0} \tau_1 + \tau_2 \quad (6.42a)$$

$$\text{S. t. } \mathcal{P}(\mathbf{W}) \succeq 0, \quad (6.42b)$$

$$\mathcal{M}_i(\mathbf{W}, \mathbf{F}_i, \mathbf{D}_i^{[k-1]}) \succeq 0, \quad (6.42c)$$

$$\varepsilon_i, \varepsilon_{\mu_i, j}, \varepsilon_{\Lambda_i}, \tau_i \geq 0, \quad (6.42d)$$



and

$$\mathbf{D}_i^{[k]} = \arg \min_{\mathbf{D}, \tau_i \geq 0} \tau_1 + \tau_2 \quad (6.43a)$$

$$\text{S. t. } \mathcal{M}_i(\mathbf{W}^{[k-1]}, \mathbf{F}_i^{[k-1]}, \mathbf{D}_i) \succeq 0, \quad (6.43b)$$

$$\varepsilon_{\mu_i, j}, \varepsilon_{\Lambda_i}, \tau_i, \geq 0. \quad (6.43c)$$

**Update the MSE at this stage:**  $\text{MSE}^{[k]} = \sum_{i=1}^2 \|\boldsymbol{\mu}_i(\mathbf{W}^{[k]}, \mathbf{D}_i^{[k]}, \mathbf{F}_i^{[k]})\|^2$ .

**3. Repeat step (2) until reaching a steady MSE, i.e.,  $|\text{MSE}^{[k]} - \text{MSE}^{[k-1]}| \leq \varepsilon$  or until the maximum number of iterations ( $K_{max}$ ) is reached.**

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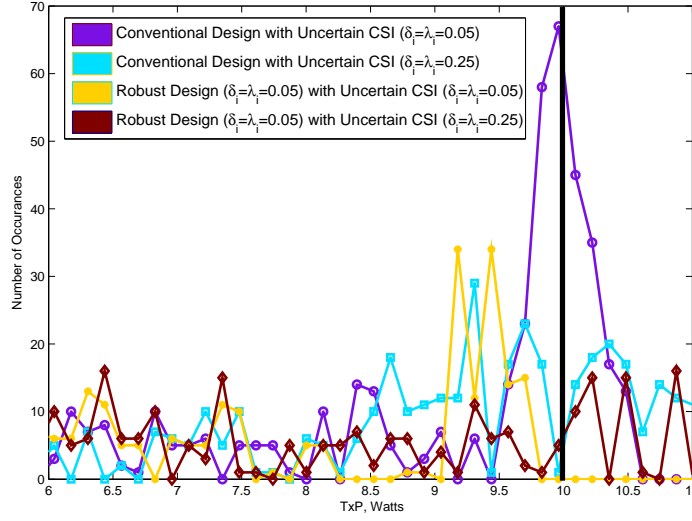
This problem is a convex optimization problem and can be solved numerically using a standard numerical package bundle such as YALMIP/SDPT3. It is possible to show that the non-robust problem formulation is a special case of these problem formulations.

**Proposition 6.4.** *The robust problem formulation with an uncertainty size of zero, would be equal to the conventional problem formulation.*

*Proof.* This proof applies to each iteration. For the SE model, it is clear that with no uncertainty, the mathematical expectation of both  $\text{TxP}(\mathbf{W})$  and  $\text{MSE}_i(\mathbf{W}, \mathbf{D}_i, \mathbf{F}_i)$  are equal to their conventional counterpart. For the NBE model it is not that straightforward and it is formally proven here. In this case, with the uncertainty size equal to zero, i.e.,  $\delta_i = 0$ , both  $\mathcal{P}$  and  $\mathcal{M}_i$  become block diagonal matrices. To be positive semi-definite, these block diagonal matrices should have positive semi-definite diagonal elements. The lower right elements of these matrices are composed of identity matrices that are positive semi-definite for any positive slack variables, even when  $\epsilon_i \rightarrow 0^+$ . Interestingly, the upper left elements of these matrices, using Schur's complement lemma, are equal to the constraints of (6.11) with any arbitrary precision, which completes the proof.  $\square$

## 6.5 Simulation Results

In this section simulation results are presented. The simulation setup is as follows: both terminals are equipped with  $N_1 = N_2 = 2$  antennas. The relay station is also



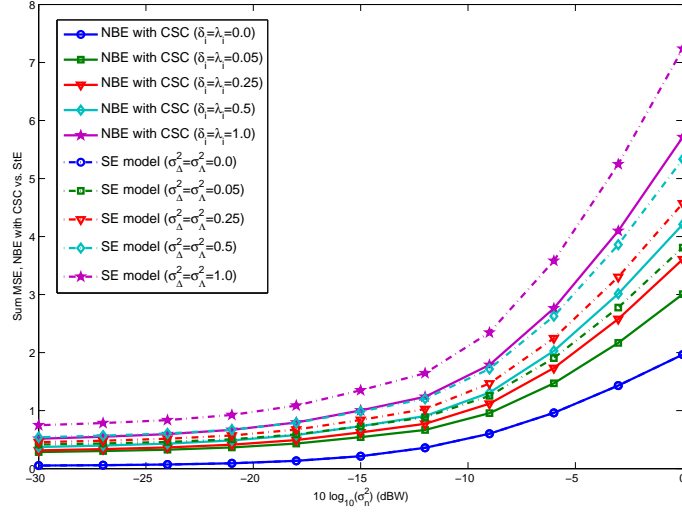
**Fig. 6.2:** Histogram of transmit power violations for different system setups

equipped with  $N_r = 2$  transmit and receive antennas. The transmit power of the relay station is limited to  $P = 10$  Watts. It is assumed that the uncertainty size of the CSI is equal for all channels for both SE and NBE models, i.e.,  $\sigma_{\Delta_i}^2 = \sigma_{\Delta_j}^2, \forall i, j$ . The transmit signal power for both terminals is assumed to be equal to  $\sigma_{s_1}^2 = \sigma_{s_2}^2 = 1$  Watt. Also, a similar assumption is made about the noise power of both terminals, i.e.,  $\sigma_{e_1}^2 = \sigma_{e_2}^2 = 10^{-3}$  Watt. The nominal CSI matrices are generated randomly according to a complex valued, zero-mean, unit variance normal random variable. The following results, except for Fig. 6.2, are the average of 300 Monte Carlo simulation runs. In each figure, the five cases are compared: the conventional linear precoder design with perfect CSI, and four cases with different uncertainty sizes from fairly small to rather large uncertainty.

In Fig. 6.2 the histogram of the transmit power realizations is depicted. This graph depicts the advantages of the robust design best. To obtain these results, a 500-run Monte Carlo simulation is performed. For the simulation, the beamforming matrices for three different cases are generated, i.e., the perfect CSI scenario and two scenarios relative to the uncertainty sizes of  $\delta_i = 0.05$  and  $\delta_i = 0.25$ . The resulting systems use channel realizations which are subject to aforementioned uncertainty sizes. The graph shows that the systems that were designed to be robust against uncertainties are not violating the TxP constraint at all, while the

Table 6.1: Percentage of the Power Constraint Violations

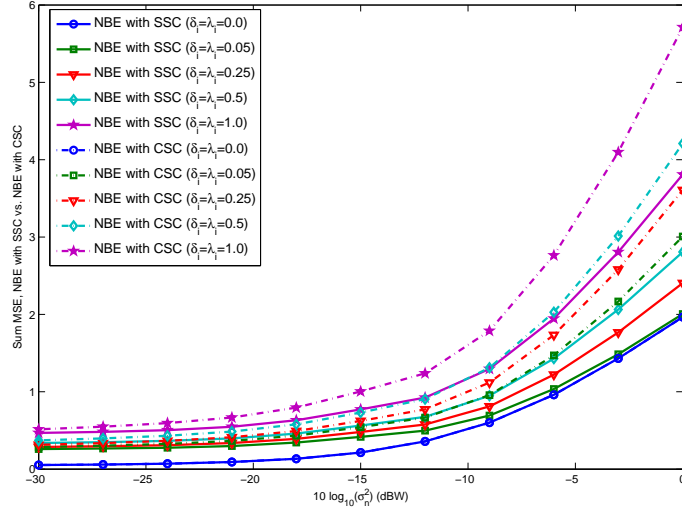
	Perfect CSI	CSI with $\delta_i = 0.05$	CSI with $\delta_i = 0.25$
Non-robust Design	0%	25.4%	32.6%
Robust Design ( $\delta_i = 0.05$ )	0%	0%	11.4%
Robust Design ( $\delta_i = 0.25$ )	0%	0%	0%



**Fig. 6.3:** Sum MSE for the system with NBE model of uncertainty having CSC vs. a system with SE model of uncertainty.

conventional system will violate it. And, as expected, the larger the size of the uncertainty, the more violations occur. These results are quantitatively summarized in Table 6.1.

In Figs. 6.3–6.5, the Sum MSE of the system with both SE and NBE models of uncertainty are shown. In Fig. 6.3, the sum MSE performance measure of a system having CSC with the NBE model is compared with the performance of a system with the SE model of uncertainty. As can be seen, using each model, the non-robust design with perfect CSI scenario will result in a better sum MSE performance relative to the robust cases, and with the increase of the uncertainty size this performance measure gets worse. This trend is irrespective of the type of the self cancellation filter (Fig. 6.5). As was expected, the system which uses SSC is outperforming the system with CSC, because in SSC mode, not only the nominal value of the self induced signals is canceled, but also, by dynamically

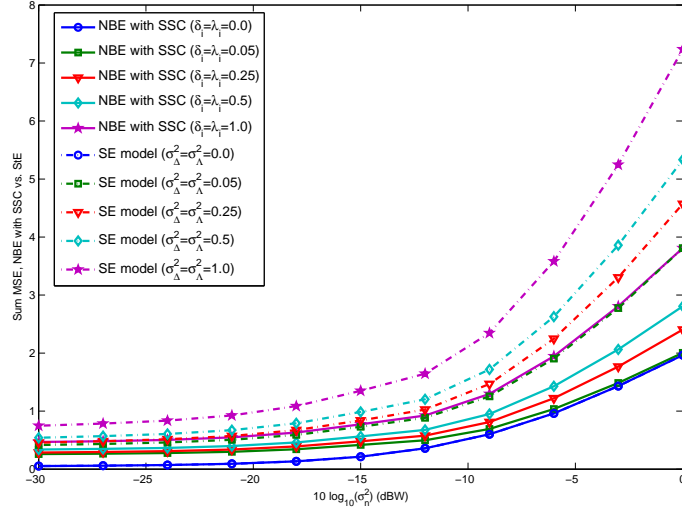


**Fig. 6.4:** Sum MSE for the system with NBE model of uncertainty having SSC vs. a system with NBE model of uncertainty having CSC.

designing the self cancellation filter, it is possible to further reduce the amount of self induced signals (Fig. 6.4).

In Fig. 6.6 the transmit power of the relay station is depicted. As can be seen, the amount of the transmit power for the non-robust design (uncertainty size equal to 0) is the highest among all, which results in a better sum MSE performance as seen in previous figures. As expected with the increase in uncertainty size, the amount of the transmit power decreases to prevent a larger sum MSE in these cases. Because in the uncertain case, it is highly possible to transmit in a wrong direction leading to increase of the interfering power to other users. Also to limit the sum MSE, with the increase of the noise power, the transmit power decreases.

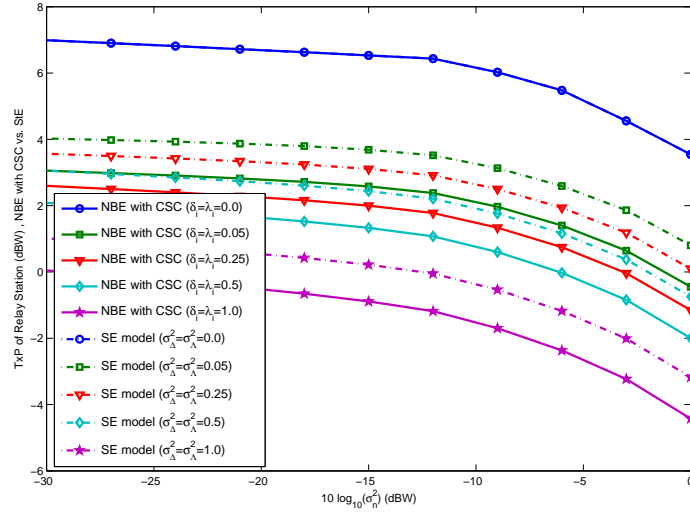
In Fig. 6.7 the BER performance of a MIMO TWRC is displayed. In this figure, the BER performance of a system with NBE model of uncertainty having SSC is compared with the performance of a system with SE model of uncertainty. As expected the BER performance of these two systems, when the uncertainty size is equal to zero leading to a non-robust design, is similar. Also as expected, by increasing the uncertainty size, the BER increases because the sum MSE of the system degrades.



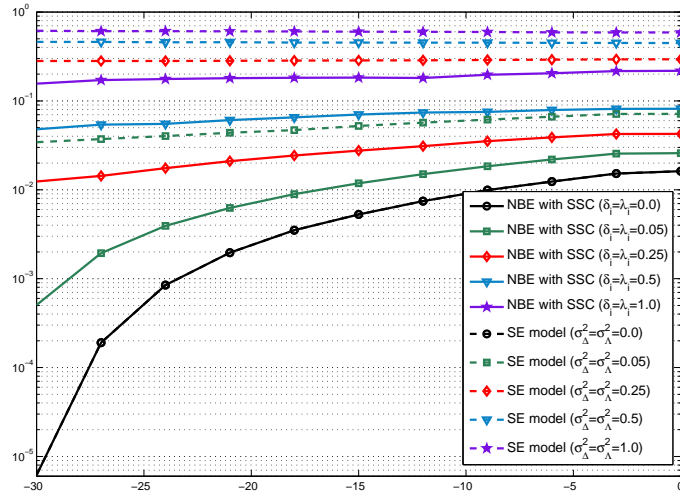
**Fig. 6.5:** Sum MSE for the system with NBE model of uncertainty having SSC vs. a system with SE model of uncertainty.

## 6.6 Conclusion

Robust linear beamforming for a MIMO TWRC is studied. The uncertainty is modeled using SE and NBE uncertainty models. Two different cancellation filters, namely, conventional self cancellation and strict self cancellation, are presented. It is shown that for the SE model, these two filters are the same. But for the NBE model, these filters perform differently. It is also shown that the MSE of each link and the transmit power of the relay station exhibit a SOC structure. For the SE uncertainty model the average performance measures are optimized and for the NBE model, the worst case performance measures are optimized. The SE model would lead to a SOCP while the NBE model leads to a SDP. Finally, simulation results are included to assess the performance of the system.



**Fig. 6.6:** Transmit power of the relay station for the system with NBE model of uncertainty.



**Fig. 6.7:** BER performance of a system with NBE model of uncertainty having SSC vs. a system with SE model of uncertainty.

# Chapter 7

## Conclusions and Recommendations

This thesis studied four problems in the area of robust beamforming for cognitive and cooperative wireless networks. We have examined the robust downlink beamforming in MU-MISO CR-Nets, robust linear and nonlinear transceiver optimization in MU-MIMO ad hoc CR-Nets, and finally robust linear beamforming for both MIMO OWRC and TWRC. In these studies, it is assumed that the CSI is imperfectly known and is impaired by an ambient uncertainty. We have used two well-known models to describe the uncertainty of CSI, i.e., both SE and NBE models are used to characterize the uncertainty. It should be mentioned that although the presented works share a lot in terms of the structure and underlying math, the complexity of the network and the number of decision variables and also uncertainty sources are increasing gradually. We started with a single variable Nemirovski lemma, and proved a more general version with two or any arbitrary number of uncertainties in each constraint. The former case appears in single-hop networks like the interfering adhoc networks, while in multi-hop networks like relaying scenarios, multiple uncertainties appear on the scene. In full-duplex multi-hop networks like the TWRC the number of decision variables are much more than the half-duplex case of OWRC. In TWRC we should deal with self-cancellation which never appears in single-hop and half-duplex cases. It is noteworthy that we have dealt with both conventional and strict self cancellation filters in previous chapter.

In the study of the robust downlink beamforming in a MU-MISO CR-Net, a BC model is central to our study. The multi-antenna BS serves  $K$  single antenna SUs while protecting  $L$  single antenna PU-Rxs. The problem formulation is based on the maximization of the received SINR at the SUs while maintaining proper IP constraints for the PUs. It is assumed that a ball-shaped uncertainty set describes the CSI imperfections. Since this problem formulation leads to a separable homogeneous quadratically constrained quadratic problem which is a NP-Hard problem, the current research usually employs loose approximate solutions. This ill-posed problem is recast as a nonconvex SDP. After relaxing the rank constraints of this SDP, we provide an exact solution for the beamforming problem. It is possible to find the actual beamforming weights using the Eigen decomposition of this SDP-relaxed version of original problem. This study provides an exact solution that maximizes the SINR of the SU-Rxs and, as expected, is less conservative than the approximate methods. It should be noted that this study does not take into account that the PUs and the SUs are equipped with multiple antennas which lead to MU-MIMO CR-Nets. It is so because generally, mobile devices in the currently operational networks are equipped with only one antenna. Additionally in MISO case, it is possible to find a closed form expression for the worst channel realization while it is not possible to find such a nice property using the MIMO case. It should be mentioned that as the MIMO case is studied by different authors in non-cognitive radio setups however, the extension of current work to cover the CR-Net is trivial. It is also of vital importance to find a mathematically appealing expression which describes the worst-channel realization in MIMO case in which S-Procedure based methods fail to do so.

In Chapter 4, robust linear and nonlinear transceiver optimization is studied in a MU-MIMO ad hoc CR-Net. In this network a set of  $I$  interfering links coexists with  $K$  PU-Rxs. The design procedure is to find the optimum precoder and equalizer filters of each SU-Tx-SU-Rx link. The problem formulation is to minimize the system-wide sum MSE while imposing two power constraints: the transmit power constraint for each SU-Tx and the interfering power constraint on



each PU-Rx. This problem, regardless of the uncertainty model, is a bi-convex problem and is hard to solve. A widely known solution for such problems is to resort to iterative procedures. The channel uncertainty is modeled using either SE or the NBE models. In case of the SE model, the mathematical expectation of the MSE of each link and the interfering power on each PU-Rx are considered in the design process. It is shown that both MSE and IP have SOC structures and the robust problem would be an iterative SOCP. For the NBE model of uncertainty, the worst case analysis reveals that both MSE and IP have SOC structure and are linear and affine in terms of the problem data and design variables. Employing this structure, and using Nemirovski lemma, it is possible to show that the worst-case problem formulation would lead to an iterative SDP. This study, to the best of our knowledge, is the only documented work that aims to jointly design the precoder and equalizer filters of a MU-MIMO ad hoc network. In this work we not only aim to have a linear design, but also consider the problem with nonlinear designs as well; namely, we study both THP and DFE schemes. It should be mentioned that since the network structure is complicated and the problem formulation is ill-posed, the proposed algorithms need collaboration of either parties of the network to pass the updated matrices to other users in the network. This fact reduces the applicability of the proposed schemes, especially for nonlinear designs which lead to non-causality in these communications. It is recommended to explore more on distributed algorithms which are more practical algorithms and need less inter-communication.

In robust linear beamforming for MIMO OWRC and TWRC, half- and full-duplex cooperative communications, a.k.a. relaying, are studied. In both networks a multi-antenna relay station sits in between the multi-antenna source and destination. It is assumed that there exists no direct link between the source and the destination. Both problems target to jointly design the relay beamformer and the destination equalizer. A problem formulation to minimize the system-wide MSE is given for both networks. These problems are constrained to limit the amount of the transmitted power of the relay station. As these problems are also biconvex

and then hard to solve, two iterative algorithms to design the beamformer and equalizer are proposed. For both networks, it is assumed that the relevant CSI is imperfectly known and is modeled using either SE or the NBE models. As before, In case of the SE model of uncertainty, the average performance measures are optimized resulting in an iterative SOCP and in case of the NBE model of uncertainty, the worst-case performance measures are optimized resulting in an iterative SDP. For MIMO TWRC, two different self cancellation filters are proposed. It is also shown that these two different mechanisms would lead to similar filters in case of SE model of uncertainty. It is noteworthy that we did not consider the direct link between the source and the destination. But it should be mentioned that the extension of current work to this case is a trivial task. Additionally, it should be mentioned that in our treatment we assumed that the relay is about to provide the connection between a source and a destination, located out of the coverage range of the source. In the literature there exists a model called GIFRC which covers both OWRC and TWRC with/without direct link. It is recommended to study this problem because the solution of this problem covers the solution of both OWRC and TWRC beamformer design.

## 7.1 Future Works

Amended to the abovementioned recommendations, it is possible to extend the scope of current study in the following directions:

- The current studies use the MSE and sum MSE as the performance measure of the systems. It is because of the appealing mathematical structure of the problem formulated using MSE. Although this measure is helpful from the signal processing viewpoint, it is more important to study the beamforming problems from pure communication system viewpoint. To do so, it is recommended to solve similar problems when the performance measure is the sum rate capacity of the mentioned systems. Unfortunately for the robust designs, to the best of our knowledge, no one has considered the problems

using capacity based formulations.

- In current treatment of the robust beamforming, we usually start from a semi-infinite SOCP problem formulation, and using S-procedure based methods, we encounter with a SDP. But there is another way of treating the semi-infinite problems proposed by Bertsimas & Sim (2006). In this treatment, the robust counterpart of any problem exhibits the same structure, i.e., the robust counterpart of SOCP based beamforming problems are also SOCPs with more constraints and variables. It is recommended to assess the performance of this new treatment as well.
- In current studies it is always assumed that there is no correlation between the input signals. It may be helpful to consider the spatio-temporal properties of the input signals in the beamformer design.
- It is usually assumed that the system is designed to act in a flat fading environment. In rapidly changing environments it is not a practical assumption. It is recommended that the beamformer be designed for non-flat fading channels.
- In the proposed algorithms, SOCP and SDP problems are central. The performance of these algorithms in real-time implementations is of great importance. It is recommended that a dedicated hardware module or software routines be designed to facilitate the implementation of real time beamforming techniques.
- Robust design gets its roots from the control theories. It is recommended that a new look be taken at the robust beamformer design from the nonlinear or the robust control theories like  $H^\infty$  control [148, 149], and employing nonlinear optimizations, like Penalty/Barrier methods.

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