RADIO RESOURCE ALLOCATION IN WIRELESS OFDM SYSTEMS

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Contents

Summary

In this thesis, we study radio resource allocation problems in wireless orthogonal frequency division multiplexing (OFDM) systems using both centralized optimization and game theoretic approaches. Unlike many other works that use real numbers for bit-loading from the information theoretic approach, we consider only integer numbers for this purpose. Firstly, the subcarrier-and-bit allocation (SBA) problem in single-cell OFDM system with quality of service (QoS) support is formulated as a mixed integer non-linear programming (MINLP) with nonlinearities in both the objective function and constraints. We propose a method to convert the MINLP to an equivalent binary linear programming (BLP), thus drastically reducing the time required to find the optimal solution. Then we extend our study to subcarrier, bit and power allocation in multi-cell OFDM system with QoS support, a problem that can also be formulated as a MINLP with much higher complexity due to the co-channel interference (CCI) among the cells. We manage to convert the MINLP to a BLP, again making it possible to find the optimal solution much easier and faster. The optimal solution can be used as a performance bound to benchmark existing heuristic algorithms, as well as distributed decision-making methods such as game theoretic approaches. Investigations on the optimal solution also give us the inspiration to find a way to improve the system performance when resource

allocation is made in a distributed manner.

In order to reduce the computational complexity and information exchange required by the centralized optimization in wireless systems, distributed decision-making is introduced together with game theory to be used as a strong and powerful tool to analyse the problem. Spectrum sharing games with equal rights are formulated on distributed wireless systems with BER requirements and fixed modulation. We start our study on a simple 2-player non-cooperative game with a single carrier by analysing the impact of the payoff function and the effect of channel conditions on the existence of Nash equilibrium (NE). It is shown that there is always at least one NE that exists in the game. The probabilities of having one or two NEs can also be estimated with a numerical method. The existence of NE is shown to be applicable to N-player games with a simple assumption that the payoff functions are non-negative when a player chooses to transmit. With the optimal solution obtained from centralized optimization, we calculate the price of anarchy (PoA) for the games using computer simulations. Our analysis is extended to multi-carrier OFDM systems to show that a NE need not always exists. We also study the repeated play of spectrum sharing games and convergence of games based on potential games with coupled constraints, which have at least a NE so that the game-play will always converge. Then we propose an algorithm to ensure a stable solution for the games albeit suboptimal solutions may result.

Lastly, we study resource allocation games with adaptive modulation in multi-cell OFDMA systems, where we show that at least one NE exists for the 2-player single-carrier case. However, in more general scenarios with multiple players and multiple subcarriers, the existence of NE cannot be guaranteed. Next we study the

myopic play of repeated adaptive modulation games and propose an algorithm to make sure that the games will converge. Finally, interference avoidance is introduced by modifying the payoff function to mitigate CCI and improve performance in the multi-cell case.

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List of Notations

Notation	Definition
$\mathcal{N}$ and N	The set of BSs and number of BSs, respectively.
	Or in game theory, the set of players and number of players, respectively.
	So $\mathcal{N} = \{1, \ldots, N\}$ and $\lvert\mathcal{N}\rvert = N$.
$\mathcal{L}_n$ and L_n	The set of users in BS n and number of users in BS n.
	n is omitted in single-cell systems.
L_A and L_B	Number of users for service class A and class B, respectively.
$\mathcal{K}$ and K	The set of frequency subcarriers and number of frequency subcarriers.
$\mathcal{Q}$ and Q	The set of modulation indexes and total number of modulations available.
G_{ln}^{kj}	Channel gain from BS j to user l in BS n on subcarrier k.
	n and j are omitted in single-cell systems.
G_l^k	Channel gain on subcarrier k as seen by user l.
$G_{n,j}$	Channel gain from transmitter j to receiver n.
R_A and R_B	User rate requirement for service class A and class B, respectively.
R_{ln}	Data rate requirement for user l in BS n.

Notation	Definition
P_{e_A} and P_{e_B}	SER requirement for service class A and B, respectively.
P_{b_A} and P_{b_B}	BER requirement for service class A and B, respectively.
$P_{\max}$	Maximum power constraint.
α	Path loss exponent.
γ_{ln}^{q}	SINR threshold for user l in BS n with modulation index q.
N_0	Power spectral density of additive white Gaussian noise.
$\mathbb{R}^{+}$ and $\mathbb{R}^{++}$	The sets of non-negative and positive real numbers, respectively.
a_{ln}^{kq}	Assignment variable for user l in BS n using modulation q on subcarrier k. n is omitted in single-cell or distributed systems; k is omitted in single carrier systems; q is omitted in systems using fixed modulation.
p_{ln}^{kq}	Transmit power for user l in BS n using modulation q on subcarrier k.

List of Abbreviations

AWGN	Additive White Gaussian Noise
BER	Bit Error Rate
BLP	Binary Linear Programming
BS	Base Station
CCI	Co-channel Interference
CDF	Cumulative Distribution Function
CDMA	Code Division Multiple Access
CNRAG	Convergent Non-cooperative Resource Allocation Game
CNRAG-IA	Convergent Non-cooperative Resource Allocation Game with Interference Avoidance
CR	Cognitive Radio
CSI	Channel State Information
DSP	Digital Signal Processing
FDMA	Frequency Division Multiple Access
FFT	Fast Fourier Transform
FRF	Frequency Reuse Factor

IA	Interference Avoidance
ICI	Inter-cell Interference
IFFT	Inverse Fast Fourier Transform
ISI	Intersymbol Interference
MA	Multiple Access
MAC	Medium Access Control
MC-CDMA	Multi-carrier Code Division Multiple Access
MINLP	Mixed Integer Non-linear Programming
MIP	Mixed Integer Programming
NE	Nash Equilibrium
NOA	Number-of-Attempts
NPAG	Non-cooperative Power Allocation Game
NRAG	Non-cooperative Resource Allocation Game
NRAG-$N\{L\}/K$	NRAG consisting of N BSs with L users in each BS and K subcarriers
OFDM	Orthogonal Frequency Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
PA	Power Allocation
PoA	Price of Anarchy
PSD	Power Spectral Density
QAM	Quadrature Amplitude Modulation
QF	Quadratic Fitting
QoS	Quality of Service

RNC	Radio Network Controller
RRM	Radio Resource Management
SBA	Subcarrier-and-Bit Allocation
SBPA	Subcarrier-Bit-and-Power Allocation
SDR	Software Defined Radio
SER	Symbol Error Rate
SINR	Signal-to-Interference-plus-Noise Ratio
SNR	Signal-to-Noise Ratio
SW	Social Welfare
TDMA	Time Division Multiple Access
TS	Two-Step
WF	Water Filling

Chapter 1

Introduction

Many upcoming wireless applications such as audio/video streaming, mobile Internet and video conferencing are demanding for higher and higher data rates. Despite the fact that the radio spectrum is scarce, the explosive increase in the number of users in wireless and mobile networks around the world further strains on the need for higher network capacities. As a result, the question of how to improve the spectrum utilization efficiency of wireless communication networks has imposed a great challenge on current technologies. In the meantime, the emerging heterogeneous services have brought up another question on how the diverse quality of service (QoS) can be fulfilled, in the most effective and efficient way to the network operator.

As a leading candidate for the next generation mobile cellular networks and other wireless networks, Orthogonal Frequency Division Multiplexing (OFDM) has attracted very much attention from the academia and industries. By dividing a very broad bandwidth into tens or even thousands of narrow bands, OFDM can transform the whole channel that is subject to frequency-selective fading into many

subcarriers where each of them is subject to flat-fading. In a multiuser OFDM network where the so-called 'multiuser diversity' exists, almost all of the subcarriers can be fully utilized by assigning them to those users who see that their assigned subcarriers are having good channel conditions. Various techniques and algorithms have been proposed for such resource allocation in radio networks. More details of OFDM and resource allocation and management techniques are discussed in this chapter.

1.1 Orthogonal Frequency Division Multiplexing

1.1.1 Advantages of OFDM

OFDM is a multi-carrier modulation scheme which can achieve high spectral efficiency near to the Nyquist rate. In OFDM, subcarriers are placed together as densely and closely as they can while still maintaining orthogonality among them, thus resulting in very high spectrum utilization of the whole frequency band. In an OFDM system with K subcarriers, the low-pass equivalent OFDM signal is expressed as

$$c(t) = \sum_{k=1}^{K} s_k(t) e^{i2\pi kt/T}, \quad 0 \leq t < T, \tag{1.1}$$

where $\{s_k(t)\}$ are the data symbols and T is the OFDM symbol duration.

With a subcarrier spacing of $1/T$, two arbitrary subcarriers k_i and k_j are orthogonal over each symbol period. Such a property of orthogonality can be shown

as:

$$
\begin{aligned}
&\frac{1}{T}\int_0^T \left(e^{i2\pi k_i t/T}\right)^* \left(e^{i2\pi k_j t/T}\right)\,\mathrm{d}t \\
&= \frac{1}{T}\int_0^T \left(e^{i2\pi (k_j-k_i) t/T}\right)\,\mathrm{d}t \\
&= \delta_{k_i k_j},
\end{aligned}
$$

where $(\cdot)^*$ denotes the complex conjugate operator and $\delta_{k_i k_j}$ is the Kronecker delta defined as

$$
\delta_{k_i k_j} = \begin{cases} 1, & \text{if } k_i = k_k \\ 0. & \text{if } k_i \neq k_j \end{cases} \tag{1.2}
$$

A diagram showing five orthogonal subcarriers of OFDM is illustrated in Fig. 1.1.

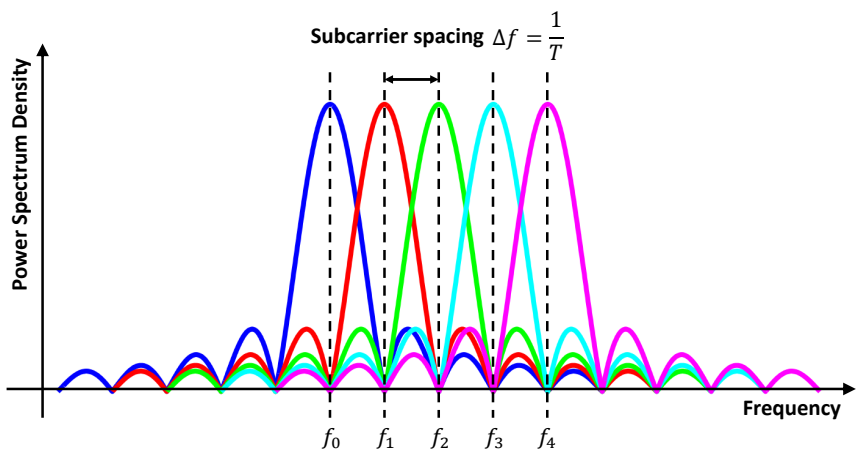

Figure 1.1: An illustration of five OFDM subcarriers.

Broadband radio signals are generally subject to frequency-selective fading, where the frequency components at different frequency bands will experience different levels of attenuation. Such fading will result in intolerable distortions in single-carrier systems, whereas OFDM has the inherent capability to mitigate this. The division of a broadband to many narrow bands can effectively transform the

frequency-selective fading to flat-fading on each subcarrier, so that the data symbols transmitted on a subcarrier can be more easily recovered.

Delay spread can cause intersymbol interference (ISI) to radio signals, especially when the length of the spread is comparable to the duration of symbols. In single carrier systems, when the data rate gets higher and higher, the symbol duration becomes shorter and shorter and ISI can be more and more severe. The parallel transmission of date symbols over the many subcarriers in OFDM, as a contrast, results in much longer symbol duration. Together with the use of cyclic prefix as a guard interval, which should have a length not less than the delay spread, ISI can be completely eliminated in OFDM systems.

Modulation and demodulation of OFDM signals can be efficiently implemented with inverse fast Fourier transform (IFFT) and FFT blocks, respectively. Meanwhile, frequency-flat fading on a subcarrier requires only simple frequency domain equalizer at the receiving end. With the technological advancements in digital signal processing (DSP) and emergence of low cost DSP components, OFDM has become a popular technology for broadband wireless communications. Besides its use in wireline communications, OFDM has also been adopted in several wireless standards such as IEEE 802.11 a/g/n, IEEE 802.16 (WiMAX) and 3GPP-LTE (Long Term Evolution). More and more use of OFDM are anticipated in the near future.

1.1.2 Multiple Access Techniques in OFDM

To allow more than one user to have access to the wireless medium at the same time, several multiple access (MA) techniques have been developed and deployed

in radio networks. These techniques can also be used in OFDM systems to support multiple mobile terminals.

With many subcarriers available in OFDM systems, an intuitive way is dividing the subcarriers into several groups and assigning a group of subcarriers to a user. As different portions of the frequency band are allocated to different users, this method is referred to as Frequency Division Multiple Access (FDMA). An example of OFDM with FDMA is illustrated in Fig. 1.2(a). If the allocation of subcarriers to a user is fixed and when the subcarriers are experiencing deep fades, the corresponding subcarriers are wasted.

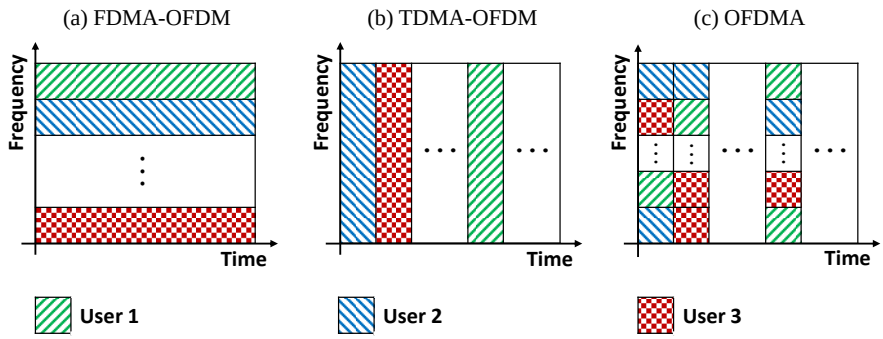

Figure 1.2: Different MA techniques in OFDM systems.

As contrast to the division of the radio spectrum in frequency domain in FDMA, Time Division Multiple Access (TDMA) divides the spectrum in time domain. With the division of time into many small intervals called time slots, the whole OFDM symbol consisting of all subcarriers is assigned to one user at a time, and the users take turn to gain access to the channel by transmitting at different OFDM symbols. Fig. 1.2(b) shows an example to illustrate TDMA-OFDM scheme. Similarly, fixed and exclusive allocation of a time slot to a single user will result in those subcarriers which are in deep fades being underutilized.

To combine the advantages of FDMA and TDMA, a combinatorial MA scheme

was invented for OFDM systems. With partitions in both frequency and time dimensions, orthogonal frequency division multiple access (OFDMA) assigns slots to users along the OFDM subcarrier index as well as OFDM symbol index. Adaptive subcarrier-to-user assignment can be achieved based on the feedback of channel conditions. Meanwhile, by assigning different numbers of subcarriers to different users, various data rates can be supported in view of fulfilling the differentiated QoS requirements. An illustrative example of OFDMA is shown in Fig. 1.2(c). An advantage of OFDMA is that it can exploit the so-called "multiuser diversity", which will be introduced in the next section.

Code division multiple access (CDMA) can also be combined with OFDM and is known as Multi-carrier CDMA (MC-CDMA) or OFDM-CDMA. It allows multiple users to access the same subcarriers at the same time, where the co-channel interference (CCI) can be mitigated with the use of orthogonal codes among the users. Therefore in OFDM-CDMA, dynamic channel allocation could be simplified to fixed channel allocation without much performance loss.

1.2 Resource Allocation in Wireless Networks

Hostile wireless environment imposes a great deal of challenges on how to efficiently utilize the radio spectrum for reliable high-speed, high-capacity communications. On the other hand, variations in channel conditions among different users provide the opportunity for higher throughput by exploiting multiuser diversity gain. In order to achieve such an increase in throughput, radio resources need to be managed in an efficient way by adapting to the instantaneous conditions of radio links. Throughout this thesis, we refer to the transmitter schemes that adapt to channel

variations as *dynamic*. In contrast, schemes that do not adapt to channel variations are referred to as *fixed*.

1.2.1 Single-Cell System

We first consider a single-cell system in cellular networks, in which there exists only one base station (BS). A fully connected centralized network can also be treated as a single-cell system. As depicted in the previous sections, subcarriers of an OFDM system generally experience different channel conditions, as long as their spacing in frequency is larger than the coherence bandwidth. Assuming such a frequency-selective behaviour remains constant for some time span, e.g. a few OFDM symbol periods, we can make use of the channel state information (CSI) to adaptively manage radio resources.

A. Point-to-Point Scenario

A point-to-point communication consists of a single transmitter and a single receiver, which corresponds to a single link in an ad hoc network, or a single-cell system with only one user. In this case, all the subcarriers are available to the receiver, and the optimal solution of resource allocation is provided by the water filling (WF) theorem in information theory [1]. To achieve the channel capacity with a given power budget, the transmission power is adapted to the transfer function of the channel, in such a way that more power is applied to frequencies with better channel conditions and less power to the frequencies undergoing deep fading. Despite its computational complexity, WF assumes continuous frequency attenuation functions, as well as continuous relationship between the allocated power and achievable capacity.

To leverage the WF benefits in OFDM systems, a discrete scheme called *finite tones water filling* is formulated as a non-linear continuous optimization problem. It can be solved analytically by applying the technique of Lagrangian multipliers, which delivers solutions with continuous rates for discrete subcarriers [2]. To achieve discrete WF in practice, continuous rates need to be replaced by bit assignment with integer values.

For realistic communication systems, only a fixed number of modulation types are available for data transmission. According to the channel states, the number of bits to be transmitted on a subcarrier can be determined by choosing the most suitable modulation assignment from a finite set. This process is called *bit loading*, while the process of deciding the corresponding transmission power is called *power loading*. The combinatorial process of bit-and-power loading can be formulated as an mixed integer programming (MIP) problem, with an objective to maximize data rate and a constraint on the power limit. Although MIP problems are generally difficult to solve, simple greedy algorithms can yield optimal solution for single-user systems.

B. Point-to-Multipoint Scenario

This scenario corresponds to a single-cell system with multiple users. As the available subcarriers need to be shared by multiple terminals, MA schemes are necessary for these systems. Multiuser diversity can also be exploited due to the fact that the fading process is statistically independent for different terminals, if their antennas are physically separated by a minimum spacing of several wavelengths, which is generally true in reality. Since the subcarriers are likely to be in different channel states for different users, a subcarrier seemed to be in deep fading to a user could

be assigned to another user seeing it with good channel condition.

With adaptive modulation and multiuser diversity, resource allocation problem can similarly be formulated as an MIP to maximize overall transmission rate with power limit constraint. Although the optimal solution can be found by using greedy algorithm again [3], the fairness issue arises as terminals with better average channel conditions are always favourable, and those terminals subject to greater path loss will suffer from higher transmission delay. To fulfil different QoS requirements, constraints are added to ensure minimum data rates for different users accordingly, resulting in the so-called *rate adaptive optimization*. If the objective function is changed to minimize total transmission power, while ensuring each terminal's specific data rates with constraints, the problem becomes the *margin adaptive optimization*.

Both rate and margin adaptive optimizations belong to the group of mixed integer non-linear programming (MINLP) problems, which are in general known to be difficult and have been claimed to be NP-hard [2]. Despite its intensive computational requirement, the large performance gain of dynamic OFDMA has attracted a lot of research interests and many suboptimal schemes are proposed to deliver solutions at reduced complexity.

To obtain the optimal solution more easily, especially when the number of variables are large, we present a method to convert the MINLP to an equivalent binary linear programming (BLP), by exploiting some properties of the subcarrier-and-bit allocation (SBA) in OFDM systems. The BLP can reduce CPU runtime by a factor of 10^2–10^5 compared to some heuristic algorithms, while preserving optimality of the solution. Details of the BLP will be presented in Chapter 2.

1.2.2 Multi-Cell System

In practical systems, a cellular network usually consists of multiple cells each serving various numbers of users. However, due to inevitable CCI among neighbouring cells, a technique called *frequency partitioning* is normally used in conventional networks to overcome the CCI problem. Frequency partitioning is achieved by grouping several neighbouring cells into a cluster, and no frequency reuse is allowed among those cells in the same cluster. Thus the whole network can be partitioned into many clusters and CCI from neighbouring cells in the same cluster is eliminated. An illustration of cellular networks partitioned with different cluster sizes are shown in Fig. 1.3(a)-1.3(c). Depending on the size of clusters, efficiency of spectral reuse is inversely proportional to the number of cells in a cluster. As higher efficiency figures become more and more desirable in future data-centric networks, aggressive spectral reuse with cluster size equal to one has emerged, e.g. in WiMax networks [5].

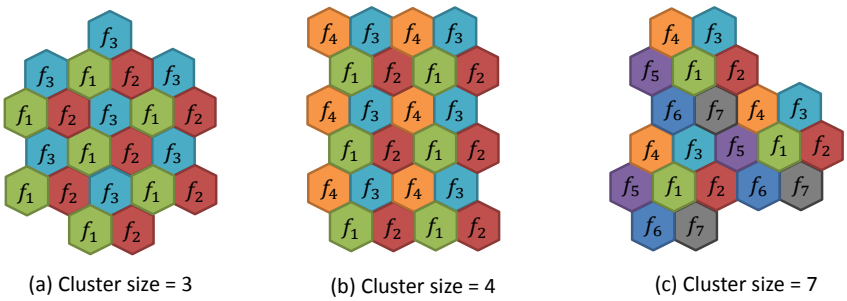

Figure 1.3: Cellular networks partitioned with different cluster sizes.

A. Centralized Optimization

In multi-cell systems with central units, such as core networks, optimal system performance can be obtained by managing all resources of the cells in the network together using a joint optimization. In such a centralized optimization for multi-cell

resource allocation, there are enormous numbers of resource variables depending on the number of cells, number of users in a cell, number of frequency subcarriers in the system, number of modulation levels available and so on. Due to CCI among the cells, however, the process of allocating so many resources is intertwined and the optimal solution is very difficult to find.

Optimization of multi-cell resource allocation can be formulated as a MINLP problem in a way similar to the single-cell scenario. However, CCI existing among the cells introduces highly non-linear constraints to the problem, which makes the MINLP much more difficult to solve than that of the single-cell scenario. By decoupling the power-loading process from SBA, we introduce a method to convert the MINLP to an equivalent BLP to reduce the complexity of searching. Although the BLP problem is still not simple enough to implement in real networks, optimal solution can be found in much shorter time than MINLP, if it were not impossible in the MINLP case. With optimal solution available as the benchmark, performance of heuristic and suboptimal algorithms can also be compared and further improved. We present in Chapter 3 the details of this method.

B. Game Theoretic Approach

Centralized optimization for multi-cell systems hinges on several practical challenges, such as frame level synchronization needed for all radios in the network area, significant computational power expected at the central unit as well as huge signalling overheads required to feedback all CSI from every network node to the central unit. Potential processing delays and information exchange will also obstruct the achievement of diversity gains especially in fast-fading channels. Some of these problems can be avoided by using distributed resource allocation schemes.

Here *distributed* means that each cell individually manages its own resources based on its locally observed channel conditions, and possibly also on it locally measured noise and interference levels.

However, due to strong coupling between locally allocated resources and interference created elsewhere in the network, locally maximizing the capacity of individual cells will not in general lead to the best overall network capacity. To investigate how multiple cells compete for common radio resources, game theory has been explored and applied. In its non-cooperative setting, game theory models the conflicts among a set of rational players, each seeking to maximize his own utility or payoff by selecting the best strategy available. Every BS in a multi-cell network can be considered as a player of the game, while the utility can be a function related to the cell capacity and the strategies determine how radio resources are allocated. More details on game theory, utility function, Nash equilibrium and resource allocation algorithms will be presented in Chapters 4–6.

1.3 Contributions

The contributions of our works are summarized as follows:

- In single-cell systems, a method to convert the MINLP to an equivalent BLP is presented so that the optimal solution can be obtained with much shorter time. This study was reported in a conference paper published on IEEE VTC 2007 Spring.

- In multi-cell systems with CCI, the MINLP can also be converted to a BLP in order to obtain the optimal solution more easily. This study was reported in a conference paper published on IEEE WCNC 2008.

- The optimal solution from centralized optimization is used as the performance bound to benchmark the results obtained from heuristic and game theoretic algorithms.

- Non-cooperative games on opportunistic access in distributed wireless systems are formulated. Fixed modulation with integer bits are used in the games. Existence of NE is studied and an algorithm is proposed to ensure convergence for the game-play. This study was reported in a journal paper for possible publication on IEEE Transactions on Communications. It is currently under the third revision.

- Resource allocation in multi-cell OFDMA systems are formulated as non-cooperative games with adaptive modulation and integer bit-loading. The existence of NEs and convergence of game-play are investigated. Some of this study were reported in two conference papers published on IEEE PIMRC

2009 and IEEE Globecom 2009. The complete study was submitted to IEEE Transactions on Vehicular Technology.

- A new utility function facilitating interference avoidance (IA) is proposed for the game on multi-cell system, and it is shown to achieve better performance for the overall system. This study was reported in a conference paper published on IEEE MILCOM 2008.

1.4 Thesis Outline

The thesis is organized as follows: Centralized optimization of resource allocation in OFDMA systems with a single cell is presented in Chapter 2, and the study on multi-cell systems follows in Chapter 3. As a useful tool for analysing distributed decision-making, game theory is introduced in Chapter 4. Applications of game theory in wireless communications and the motivation to our work are also discussed in this chapter. In Chapter 5, spectrum sharing games on a distributed wireless system with QoS constraints are formulated and investigated. Then resource allocation games in multi-cell networks with adaptive modulation are studied in Chapter 6. Lastly, concluding remarks are presented in Chapter 7.

Chapter 2

Single-Cell OFDMA Systems

The idea on adaptive SBA for multiuser OFDM systems has been extensively studied and many algorithms were proposed. Among these proposals, some aimed to minimize the total power consumption of the system [6] [7], while others tried to solve the dual problem of maximizing the overall throughput of the system [8]–[10]. Due to nonlinearities of the objective functions and constraints, however, finding the optimal solutions is computation intensive and time consuming. As a result, several suboptimal algorithms were proposed. Some of these algorithms relaxed the integer constraints into floating points, and others decoupled the combinatorial problem into two or more intermediate steps.

Mobile communication systems must be able to provide various services to users with QoS. Therefore optimal allocation of radio resources in OFDM systems while satisfying respective QoS requirements is essential, which was discussed in [11] and [12]. Although in these reported works, convexity of the objective function can be ensured through appropriate substitution, the resulting MINLP still have a complexity exponentially increasing with the product between the number of sub-

carriers and number of users in the system. Two heuristic suboptimal algorithms were proposed to reduce the complexity but solving the problem still requires a time too long to adapt the system to the change of channels in time.

By exploiting some properties of SBA in OFDM systems, we propose an approach to convert the MINLP optimization problem to a BLP problem. Firstly we notice that only discrete values are taken for the bit-loading process in practical systems, thus binary variables can be used to represent the selections of these values. Secondly, by making use of the exclusive allocation of a subcarrier to a single user, these binary variables can further be made use of in the allocation of subcarriers. The resulting BLP problem has a drastically reduced complexity and the optimization problem can be solved much faster than the two algorithms proposed in [12]. Also note that the optimality of solution is preserved, as no relaxation or assumption is made during the conversion.

The transmission power required for a certain class of QoS can be expressed as a function of SER and channel gain [6]. However, since BER is usually specified as one of the QoS parameters instead of SER, it has been used in the calculations as a lower bound for SER [6] [11] [12]. As constellation sizes increase, more and more data bits will be loaded in one OFDM symbol, and such use of BER as direct substitution for SER can result in increased excess of transmission power than what is actually required. This approximation can be accepted by users as it results in better performance than expected. However, to service providers, it not only leads to unnecessary waste of radio resources, but also may cause unfairness when allocating power among those users who are using different constellation sizes or having services of different BER requirements. We demonstrate that our

approach can easily obtain solution which is closer to what is actually desired with the use of two approximations between BER and SER.

2.1 Problem Formulation

The optimization problem of SBA in multiclass multiuser OFDM systems has been formulated as an MINLP problem in [12]. Let us consider the downlink of a rate-adaptive OFDM system which has K subcarriers in total. Two classes of service are supported, where Class A service provides constant data rate of R_A bits per OFDM symbol and target BER of P_{b_A}, and Class B service provides minimum data rate of R_B bits per OFDM symbol and target BER of P_{b_B}. The number of users is L_A for Class A and L_B for Class B, respectively. With a target to minimize the overall transmission power while satisfying all data rate and BER constraints for both Class A and Class B users, the optimization problem is formulated as:

$$\min_{a_l^k, r_l^k} \rho_A \sum_{l=1}^{L_A} \sum_{k=1}^{K} \frac{2^{a_l^k r_l^k} - 1}{G_l^k} + \rho_B \sum_{l=L_A+1}^{L_A+L_B} \sum_{k=1}^{K} \frac{2^{a_l^k r_l^k} - 1}{G_l^k}, \tag{2.1}$$

subject to

$$\sum_{k=1}^{K} a_l^k r_l^k = R_A, \qquad \text{for } l = 1, \ldots, L_A, \tag{2.2}$$

$$\sum_{k=1}^{K} a_l^k r_l^k \geq R_B, \qquad \text{for } l = L_A + 1, \ldots, L_A + L_B, \tag{2.3}$$

$$\sum_{l=1}^{L_A+L_B} a_l^k = 1, \qquad \forall k, \tag{2.4}$$

$$a_l^k \in \{0, 1\} \text{ and } r_l^k \in \{0, 2, 4, 6\}, \quad \forall l \text{ and } \forall k, \tag{2.5}$$

where constants

$$\rho_A \approx \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_{b_A}}{4}\right)\right]^2, \quad \rho_B \approx \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_{b_B}}{4}\right)\right]^2 \tag{2.6}$$

correspond to service class A and B, respectively. And we recall that $Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^\infty e^{-\frac{t^2}{2}}\,\mathrm{d}t$.

Take note that since BER is upper bounded by SER, P_{b_A} and P_{b_B} are used in place of the actual SER, P_{e_A} and P_{e_B}, in (2.6). If the exact values of P_{e_A} and P_{e_B} are to be used, ρ_A and ρ_B are no longer constants but rather depending on the values of r_l^k chosen. This will result in highly nonlinear objective function where in each term, both the exponent and coefficient contain the optimization variables, thus adding more load to the already intensive computation for solving the problem. Although such a widely accepted approximate use of BER in place of SER is first adopted, more accurate approaches to SER approximation will be discussed in Section 2.3.

In the above formulation, a_l^k denotes the assignment indicator which equals 1 when subcarrier k is assigned to user l and otherwise 0. Assuming no subcarrier sharing among different users, then for any given $a_l^k = 1$, $a_{l'}^k = 0$ for all $l' \neq l$. The number of bits modulated in one OFDM symbol on subcarrier k for user l is denoted as r_l^k. We consider M-QAM with square signal constellations in our system, where $M = 2^{r_l^k}$ denotes the constellation size. With $M = 4, 16$ and 64, r_l^k takes on a value of 2, 4 and 6, respectively. $r_l^k = 0$ means that no information bit is to be transmitted on subcarrier k by user l. Also, G_l^k denotes the channel gain of subcarrier k as seen by user l. Flat fading on each subcarrier is ensured by carefully designing the OFDM signal using a cyclic prefix, which should be longer

than the maximum delay of the multipath channel to mitigate ISI. The power spectral density (PSD) of additive white Gaussian noise (AWGN), N_0, is assumed to be identical for all users on any subcarrier.

2.2 Linearization and Simplification

The problem formulated in (2.1)–(2.5) has an exponential objective function and $2(L_A + L_B)K$ integer optimization variables on discrete set. Generally, global minimum will not be guaranteed for this objective function. A polynomial of order six is used to replace the exponential function [12]. However, the computation time to obtain optimal solution is still prohibitively long when $(L_A+L_B)K$ is large (> 10).

By noticing some special characteristics of the MINLP problem, however, we can linearise and simplify the problem into an equivalent BLP problem, without compromising optimality. First we observe that function $g(a_l^k, r_l^k) = 2^{a_l^k r_l^k} - 1$ can only take on discrete values of 0, 3, 15 and 63, therefore it can be replaced by a new function

$$f(b_{l,i}^k) = 3\left(b_{l,1}^k + 5b_{l,2}^k + 21b_{l,3}^k\right), \tag{2.7}$$

with additional constraints:

$$\sum_{i=1}^{3} b_{l,i}^k \leq 1, \quad \forall l \text{ and } \forall k, \tag{2.8}$$

where $b_{l,i}^k \in \{0, 1\}$, $i = 1, 2$ and 3 are three new binary variables for any given l and k. This additional constraint ensures that either all $b_{l,i}^k$ equal to 0 or only one of them equals to 1, so that outputs of functions $g(a_l^k, r_l^k)$ and $f(b_{l,i}^k)$ are equivalent.

Furthermore, notice that when $a_l^k r_l^k$ takes on a value of 2, 4 or 6, it corresponds to $2b_{l,1}^k$, $4b_{l,2}^k$ and $6b_{l,3}^k$ respectively, given $b_{l,i}^k = 1$ for $i = 1, 2$ and 3. Therefore we can replace $a_l^k r_l^k$ with $\sum_{i=1}^{3}(2i)b_{l,i}^k$ and consequently the nonlinear constraints in (2.2) and (2.3) are converted to their linear counterparts as shown in (2.10) and (2.11).

At this stage, we have changed the nonlinear objective function and constraints to be linear, at an expense of increased numbers of variables and constraints. However, since no subcarrier sharing is allowed among the users, we notice that when $a_l^k = 0$ or 1, $\sum_{i=1}^{3} b_{l,i}^k$ takes on a corresponding value of 0 or 1. Hence a_l^k can be eliminated and constraints (2.4) and (2.8) can be combined into (2.12). The original MINLP problem is further simplified and reduced to a BLP problem as follows:

$$\min_{b_{l,i}^k} \rho_A \sum_{l=1}^{L_A} \sum_{k=1}^{K} \frac{f(b_{l,i}^k)}{G_l^k} + \rho_B \sum_{l=L_A+1}^{L_A+L_B} \sum_{k=1}^{K} \frac{f(b_{l,i}^k)}{G_l^k}, \tag{2.9}$$

subject to

$$\sum_{k=1}^{K} \sum_{i=1}^{3} (2i) b_{l,i}^k = R_A, \quad \text{for } l = 1, 2, \ldots, L_A, \tag{2.10}$$

$$\sum_{k=1}^{K} \sum_{i=1}^{3} (2i) b_{l,i}^k \geq R_B, \quad \text{for } l = L_A + 1, \ldots, L_A + L_B, \tag{2.11}$$

$$\sum_{l=1}^{L_A+L_B} \sum_{i=1}^{3} b_{l,i}^k \leq 1, \quad \forall k, \tag{2.12}$$

$$b_{l,i}^k \in \{0, 1\}, \quad \text{for } i = 1, 2 \text{ and } 3, \forall l \text{ and } \forall k, \tag{2.13}$$

where $f(b_{l,i}^k)$ is given by (2.7), ρ_A and ρ_B are constants shown in (2.6).

Computation complexity of the newly formulated BLP problem is drastically reduced due to its linearity, even though the number of variables is increased from $2(L_A + L_B)K$ to $3(L_A + L_B)K$. Also note that no relaxations or approximations

are made during the linearisation and simplification process, therefore optimality of the problem is preserved. With the original optimization problem converted to a standard BLP problem, it can be readily solved by standard packages such as CPLEX with greatly reduced time.

2.3 Approximate Relationships between SER and BER

We notice that the equation to calculate required transmit power to support a certain bit rate and BER was formulated in [6] for M-ary QAM. This formulation, using BER in place of SER, have been subsequently adopted by many literatures including [11] and [12], as well as in (2.6) of this thesis. The reason to make such an approximation is that BER is normally specified as one of the QoS parameters, while its relationship with SER is generally not so straightforward. Even though approximate relationships between SER and BER exists, they are related to constellation size which is yet to be decided. This makes the objective function more complicated and increases computation complexity.

With the inverse Q-function being monotonically decreasing, a loose lower bound for SER results in unnecessary higher transmission power than what is actually needed, and hence better performance will be achieved than expected. In single-class multiuser OFDM systems, those users who are using higher order constellation will be given much higher extra power than those using lower order constellation. On the other hand, in multiclass systems, unfairness also exists when distributing power to different service classes which are having different BER

requirements. As unfairness and extra waste of power are undesirable for service providers, it is important to look for a solution which can closely approximate what is actually required.

An exact expression of BER related to average signal-to-noise ratio (SNR) per symbol was given in [14], with Gray-coded M-QAM. The reverse relationship of getting the instantaneous transmission power with a given BER, however, is not easily obtainable. Therefore in this section, we consider two approximate relationships between SER and BER and then make use of (2.6) to calculate the required transmission power. We show that constants ρ_A and ρ_B can be integrated in the respective constellation assignment parameters during formulation if some approximate relationships between SER and BER are used. Earlier approaches in [11] [12] will rely on whether a new convex function can be identified. Even if such a function can be found, if it is not simple enough, prohibitively high computation complexity is still unavoidable.

The first approximation is using the lower bound of SER for M-ary QAM modulation given as [13]

$$P_e \geq \frac{M-1}{(M/2)} P_b, \tag{2.14}$$

where P_e and P_b represent SER and BER, respectively. Taking equality in (2.14) as the lower bound and substituting it into (2.6), we have the following new constants:

$$\begin{aligned}
\rho_{A,1} &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{3}{8}P_{b_A}\right)\right]^2, & \rho_{B,1} &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{3}{8}P_{b_B}\right)\right]^2; \\
\rho_{A,2} &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{15}{32}P_{b_A}\right)\right]^2, & \rho_{B,2} &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{15}{32}P_{b_B}\right)\right]^2; \\
\rho_{A,3} &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{63}{128}P_{b_A}\right)\right]^2, & \rho_{B,3} &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{63}{128}P_{b_B}\right)\right]^2,
\end{aligned} \tag{2.15}$$

where $\rho_{j,i}$ denotes the constant for service class j, $j = A, B$, with M-QAM modulation such that M=2^{2i}.

The objective function of the BLP problem now becomes

$$\min_{b_{l,i}^k} \sum_{l=1}^{L_A} \sum_{k=1}^{K} \frac{f(\rho_{A,i}, b_{l,i}^k)}{G_l^k} + \sum_{l=L_A+1}^{L_A+L_B} \sum_{k=1}^{K} \frac{f(\rho_{B,i}, b_{l,i}^k)}{G_l^k}, \tag{2.16}$$

where

$$f(\rho_{j,i}, b_{l,i}^k) = 3\left(\rho_{j,1} b_{l,1}^k + 5\rho_{j,2} b_{l,2}^k + 21\rho_{j,3} b_{l,3}^k\right), \quad \text{for } j = A, B. \tag{2.17}$$

Constraints (2.10)–(2.13) remain unchanged.

For the second approximation, under usual operating conditions of low BER, say $P_b < 10^{-3}$, errors are usually made in such a way that the error symbol selected is the "nearest neighbour" to the correct symbol on the signal constellation. Assuming Gray code on the bit-to-symbol mapping, there is only one bit change of error for the nearest neighbour symbol. The relationship between BER and SER can be approximated by [13]

$$P_e \approx cP_b, \tag{2.18}$$

where $c = \log_2 M$ equals to the number of bits per OFDM symbol. Note that since one symbol error could comprises of more than one bit error, the more precise relationship is given by $P_e \leq cP_b$. By substituting (2.18) into (2.6), we have the

following new set of constants:

$$\begin{aligned}
\rho'_{A,1} &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_{b_A}}{2}\right)\right]^2, & \rho'_{B,1} &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{P_{b_B}}{2}\right)\right]^2; \\
\rho'_{A,2} &= \frac{N_0}{3}\left[Q^{-1}\left(P_{b_A}\right)\right]^2, & \rho'_{B,2} &= \frac{N_0}{3}\left[Q^{-1}\left(P_{b_B}\right)\right]^2; \\
\rho'_{A,3} &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{3}{2}P_{b_A}\right)\right]^2, & \rho'_{B,3} &= \frac{N_0}{3}\left[Q^{-1}\left(\frac{3}{2}P_{b_B}\right)\right]^2.
\end{aligned} \tag{2.19}$$

These constants (2.19) can be directly applied to (2.17) as $f(\rho'_{j,i}, b^k_{l,i})$ for $j = A, B$ and $i = 1, 2, 3$, while the objective function (2.16) and constraints (2.10)–(2.13) need not change. We should also note that since the different approximate relationships between SER and BER only affect values of the constants which can be pre-calculated, complexity of the BLP problem remains the same.

2.4 Numerical Results

Computer simulation results are presented in Fig. 2.1 and Fig. 2.2 to compare the performance of the converted BLP problem with the two heuristic suboptimal algorithms proposed in [12], namely the quadratic fitting (QF) and two-step (TS) approaches. The solution using the optimal method in [12] will not be considered since its computation time is very much longer. The computation time are obtained with the simulations run on a personal computer equipped with an Intel Pentium 4 2.8GHz processor and 1GB memory.

Fig. 2.1 shows the computation complexity in terms of CPU time for the three methods, averaged over 100 simulations. K is set to range from 4 to 20 in a step of 4 with $L_A = 2$, $L_B = 1$ and $R_A = 4$, $R_B = 3$ bits/OFDM symbol. It is evident that the BLP approach requires much less computation time than the other two

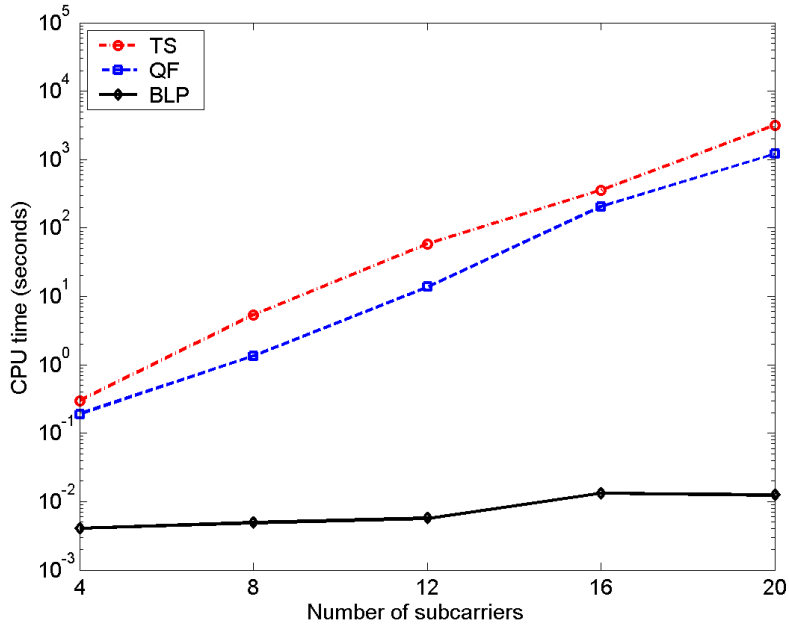

Figure 2.1: Comparison of computation complexity with $L_A = 2$, $L_B = 1$ and $R_A = 4$, $R_B = 3$ bits per OFDM symbol.

algorithms, due to the linearised objective function and constraints used. Although the CPU time for BLP increases with the number of subcarriers, the increment is too small to be obviously reflected on the log-scale Y-axis.

In Fig. 2.2, a plotting of total power consumption allocated by the three methods are shown, in which 25 instances of randomly generated channel conditions are used. The system parameters are set with $K = 8$, $L_A = 2$, $L_B = 1$ and $R_A = 12$, $R_B = 13$ bits/OFDM symbol. With its solution being optimal, power consumption allocated by the BLP approach is always less than or equal to those done by the suboptimal QF and TS algorithms under same channel conditions.

We compare the average power savings of the two SER and BER relationships given in (2.14) and (2.18) with reference to the result obtained using $P_e \approx P_b$, under various conditions of system traffic load in Fig. 2.3. The system traffic load is defined as the percentage of minimum total rate requirement to the realizable system capacity over an OFDM symbol period. With a maximum number of K

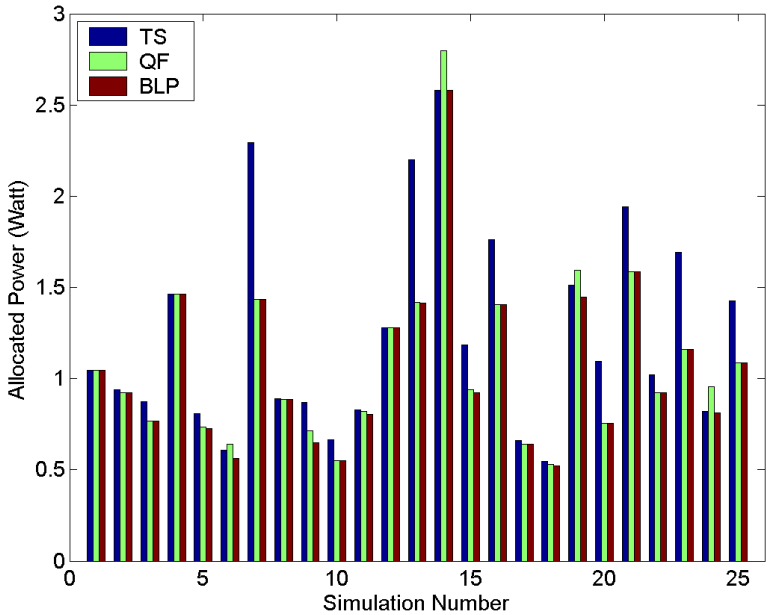

Figure 2.2: Comparison of total power consumption with $K = 8$, $L_A = 2$, $L_B = 1$ and $R_A = 12$, $R_B = 13$ bits per OFDM symbol.

subcarriers and at most 6 bits can be loaded on each subcarrier, the realizable system capacity is $6K$ and the traffic load is given by

$$\eta = \frac{L_A R_A + L_B R_B}{6K} \times 100\%. \tag{2.20}$$

From Fig. 2.3, we can see that the approximate relationship in (2.14) results in around 6% power saving and close to 12% under low and high traffic load, respectively. On the other hand, the approximation in (2.18) can lead to more than 10% power saving under low load and close to 30 % under high load. Since the approximation in (2.18) is using BER as upper bound for SER, it could lead to slight degradation in the actual BER performance. However, such a degradation is negligible when the system is operating under low BER ($P_b < 10^{-3}$) requirement, whereas the approximation in (2.18) can provide a significant saving in total transmission power under heavy traffic condition, or equivalently a considerable increase

in system throughput under a given power constraint.

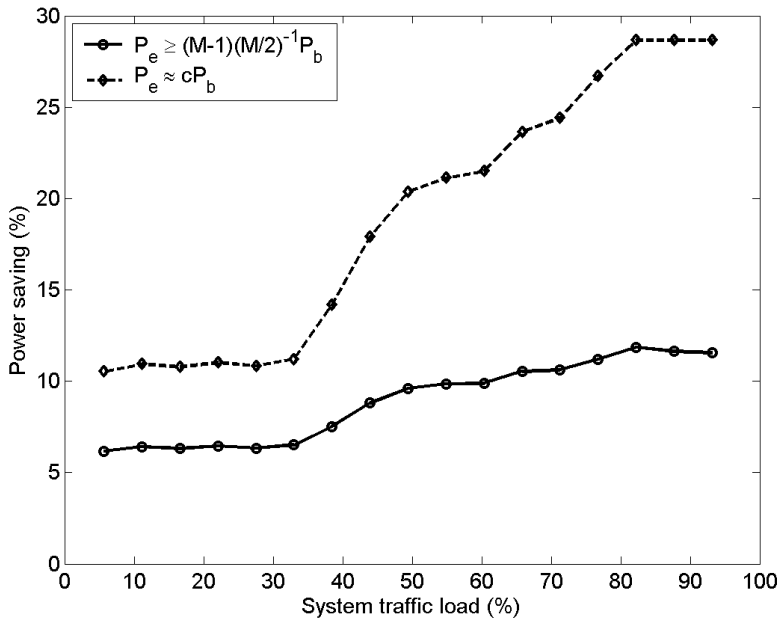

Figure 2.3: Average transmission power saved using different SER approximations over $P_e \approx P_b$, with $K = 512$, $L_A = 12$ and $L_B = 6$.

To understand the process of bit-loading under different traffic loads, the average numbers of subcarriers loaded with $c = 2, 4$ and 6 bits/OFDM symbol are illustrated in Fig. 2.4. As we can see from the figure, when system traffic is relatively low, all the allocated subcarriers are loaded with only 2 bits/OFDM symbol. More and more allocated subcarriers are loaded with 4 bits/OFDM symbol and eventually 6 bits/OFDM symbol as the traffic increases to medium and high load, until the realizable system capacity is reached. Also note that the total number of allocated subcarriers always equals to K ($K = 512$ in our simulation) when the traffic load is larger than 40%. This is intuitively reasonable as the power increment for loading more bits to an already allocated subcarrier, as indicated in (2.7), is generally higher than spreading the data bits to other available subcarriers which is not undergoing deep fade. Therefore the process of allocating the available subcarriers of the OFDM system takes precedence over that of loading more

bits to a subcarrier. Such information could be made use of to further reduce the complexity of the optimization problem.

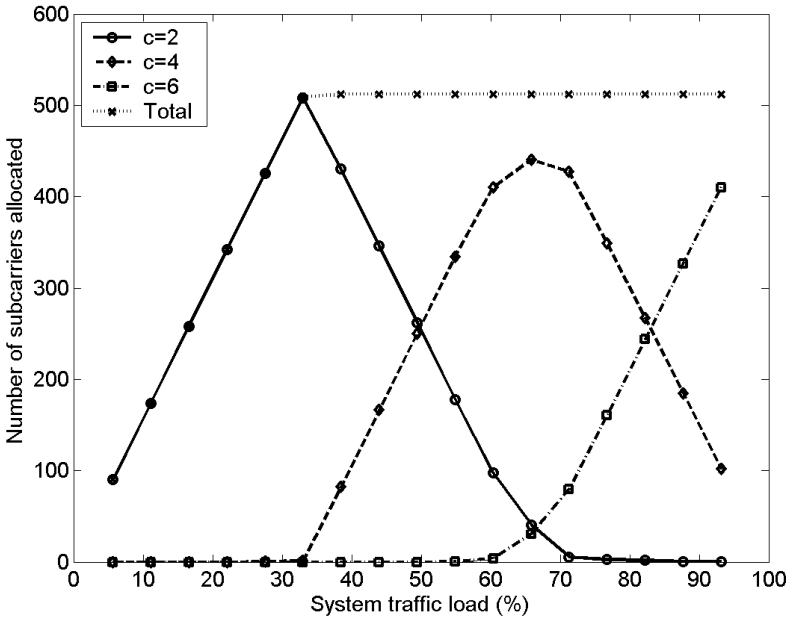

Figure 2.4: Average number of subcarriers loaded with $c = 2, 4$ and 6 bits per OFDM symbol under different conditions of system traffic load, with $K = 512$, $L_A = 12$ and $L_B = 6$.

2.5 Conclusion

In this chapter, we considered the optimization of resource allocation in single-cell OFDM systems with multiclass QoS requirements. To reduce the exponential complexity of the original MINLP problem, we propose a method to convert it to a BLP problem. As a result, the linear optimization problem has a drastically reduced complexity and maintains the optimality without relaxations applied. Since the equivalent BLP problem can be solved much faster than MINLP, a larger number of subcarriers and users can be taken into consideration. With linearity and optimality preserved, the BLP is highly effective and efficient for the optimization of resource allocation in multiclass multiuser OFDM systems. On the other

hand, using better approximations for SER has been shown to reduce the unwanted transmission power wastage and hence the fairness among users of different service classes can be better maintained.

Chapter 3

Multi-Cell OFDMA Systems

In multi-cell OFDMA systems, radio resource allocation can be made by a central unit located at the radio network controller (RNC) and the problem can be similarly formulated as MINLP. With a frequency reuse factor (FRF) equal to one, the high CCI interrelates the assignment of subcarriers in all the cells which results in nonlinear objective functions and constraints. Some algorithms are proposed to use interference avoidance techniques to prevent serious CCI among the users [15] [16], while many others are proposed with the concept of reuse partitioning, which assigns various FRF values to different groups of subcarriers to increase system capacity [17] [18]. In [19] and [20], the issue of power control and subcarrier assignment in a sectorized two-cell downlink OFDMA system was studied, where the optimal allocation was investigated, a distributed practical resource allocation algorithm with low complexity was proposed and the optimal value of the FRF was characterised. To our best knowledge, however, no optimal solution to the MINLP has been reported and hence there is a lack of performance bound to benchmark different heuristic algorithms.

In this chapter, we present a method to convert the original MINLP problem to an equivalent BLP, so that the complexity is drastically reduced and the optimal solutions can be obtained with relatively small amount of time. With these optimal solutions, we next investigate the insights of resource allocation process that lead to the best system performance. Results of the investigation inspired us on a way to improve the performance when resource allocation is being made in a distributed manner, details of which will be presented in the subsequent chapters related to game theory. Finally, unlike those existing works using the information theoretic approach [15]–[18], discrete values are used in our bit-loading process and hence our results are more readily applicable to practical systems.

3.1 System Model And Notations

Consider a multi-cell OFDMA system with N BSs, and the number of randomly distributed users in BS n is L_n, $n \in \mathcal{N}$, where $\mathcal{N} = \{1, \ldots, N\}$ denotes the set of BSs. In the centralized model, radio resources of all N BSs are being allocated through a central unit located at RNC. The total available bandwidth is equally divided into K OFDM subcarriers, in such a way that subcarriers are subject to flat and uncorrelated fading. We assume that intersymbol interference resulting from multipath can be removed through the use of cyclic prefix. The maximum total transmission power of each BS over all subcarriers is limited to $P_{\max}$.

Consider user l in BS n on subcarrier k, $l \in \mathcal{L}_n$, $n \in \mathcal{N}$ and $k \in \mathcal{K}$, where $\mathcal{L}_n = \{1, \ldots, L_n\}$ denotes the set of users in BS n and $\mathcal{K} = \{1, \ldots, K\}$ denotes the set of subcarriers. Assuming FRF equal to one, all K subcarriers are available to each BS. Depending on interference level and channel condition, the effective

bit rate on a subcarrier can be achieved by selecting the most suitable modulation level from one of M-QAM's, where $M \in \{4, 16, 64\}$. Instead of separately defining the subcarrier assignment and modulation index variables, we integrate them into a group of assignment variables denoted as a_{ln}^{kq}, where $q \in \mathcal{Q} = \{1, \ldots, Q\}$ is the set of modulation indexes. $a_{ln}^{kq} = 1$ if subcarrier k is allocated to user l in BS n with modulation index q, and $a_{ln}^{kq} = 0$ otherwise. Assume $Q = 3$, then $q = 1, 2$ and 3 correspond to the cases where 4-QAM, 16-QAM and 64-QAM are chosen, respectively. Each user therefore transmits $2q$ bits per OFDM symbol on the assigned subcarrier. Different subcarriers assigned to a user can use different modulation indexes, while each subcarrier in each BS can only use one modulation index.

The channel gain of subcarrier k from BS j ($j \in \mathcal{N}$) to user l located in BS n is denoted as G_{ln}^{kj}. The aggregate CCI imposed on user l can be either measured or computed if all channel conditions are known. p_{ln}^{kq} is the transmission power of BS n on subcarrier k required for modulation index q, so that user l in BS n can recover the signal with a specified BER. We further assumed that there is no sharing of any subcarrier among the users in a BS. CCI on subcarrier k experienced by user l in BS n is thus given by

$$I_{ln}^{k} = \sum_{\substack{j=1,\\ j\neq n}}^{N} \left(G_{ln}^{kj} \sum_{l'=1}^{L_j} \sum_{q=1}^{Q} a_{l'j}^{kq} p_{l'j}^{kq} \right), \quad \forall l, \forall n \text{ and } \forall k. \tag{3.1}$$

Fig. 3.1 shows a 3-cell system where each cell has two users. We denote user l in BS n as m_{ln}. The solid line represents received signal for the designated receiver, while those dotted lines represent interfering signals from adjacent BSs.

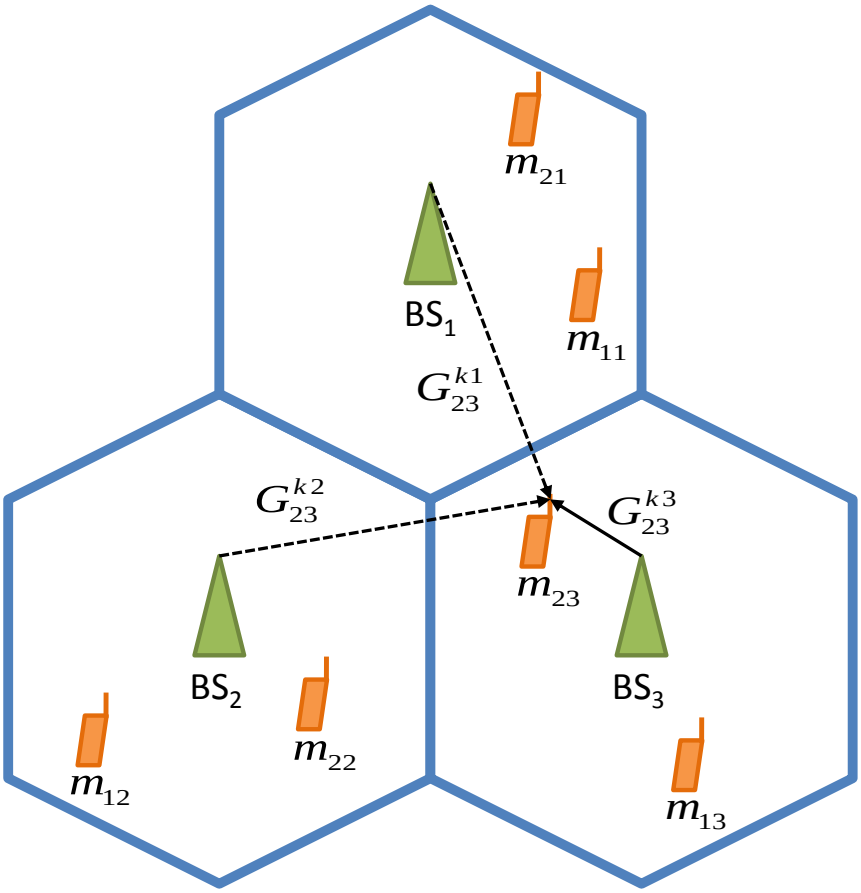

Figure 3.1: Example of a 3-cell OFDMA system.

To support multiple service classes, QoS requirements are specified by $\{R_{ln}, B_{ln}^{ER}\}$, which correspond to the minimum data rate and BER requirement for user l in BS n, respectively. For a particular q, the signal-to-interference-plus-noise ratio (SINR) threshold is a function of symbol error rate (SER) [29]. Under usual operating conditions of low BER ($B_{ln}^{ER} < 10^{-3}$), we have $S_{ln}^{ER} \approx 2q \cdot B_{ln}^{ER}$ [13]. The average SINR required to achieve B_{ln}^{ER} is thus given by

$$\frac{G_{ln}^{kn} p_{ln}^{kq}}{I_{ln}^{k} + N_0} \geq \gamma_{ln}^{q}, \tag{3.2}$$

where γ_{ln}^{q} denotes the SINR threshold for user l in BS n with modulation index q. The PSD of AWGN, N_0, is assumed to be identical on all subcarriers for all users.

3.2 Solution To Centralized Optimization

3.2.1 Direct Formulation as MINLP

Utility functions are widely used in economics to quantify the satisfaction level of users, or the benefits of resource usage. In wireless communications, utility functions have also been used to bridge across different protocol layers to optimize resource utilization [70]. For simplicity, we define the utility function to be the reward minus the cost, where the sum of data rates can be considered as rewards and the total transmit power as costs. The pricing on the power consumption helps to regulate the CCI among the BSs in multi-cell systems. This is also the adopted objective function in some of the reported works, for example [68]. Using the variables defined in the last section, the utility function of BS n is given by

$$u_n = \sum_{l=1}^{L_n} r_{ln} - c \sum_{k=1}^{K} p_n^k, \quad \forall n, \tag{3.3}$$

where $r_{ln} = \sum_{k=1}^{K} \sum_{q=1}^{Q} 2q \cdot a_{ln}^{kq}$ represents the total data rate of user l in BS n and $p_n^k = \sum_{l=1}^{L_n} \sum_{q=1}^{Q} a_{ln}^{kq} p_{ln}^{kq}$ is the transmission power on subcarrier k in BS n. The power cost factor, c, is used to make the throughput and power which have different units to have the basis to sum together. The assignment parameters, a_{ln}^{kq}'s, and power terms, p_{ln}^{kq}'s, are the unknowns to be solved.

From the system's perspective, our aim is to maximize the total utility from all BSs yet satisfying the rate and BER requirements of all users. Therefore a centralized optimization can be formed:

$$\max_{a_{ln}^{kq}, p_{ln}^{kq}} U = \sum_{n=1}^{N} u_n, \tag{3.4}$$

subject to

$$\sum_{k=1}^{K}\sum_{q=1}^{Q} 2q \cdot a_{ln}^{kq} \geq R_{ln}, \qquad \forall n \in \mathcal{N} \text{ and } \forall l \in \mathcal{L}_n, \tag{3.5}$$

$$\frac{G_{ln}^{kn} p_{ln}^{kq}}{I_{ln}^{k} + N_0} \geq a_{ln}^{kq} \gamma_{ln}^{q}, \qquad \forall n \in \mathcal{N}, \forall l \in \mathcal{L}_n, \forall k \in \mathcal{K} \text{ and } \forall q \in \mathcal{Q}, \tag{3.6}$$

$$\sum_{k=1}^{K}\sum_{l=1}^{L_n}\sum_{q=1}^{Q} a_{ln}^{kq} p_{ln}^{kq} \leq P_{\max}, \qquad \forall n \in \mathcal{N}, \tag{3.7}$$

$$\sum_{l=1}^{L_n}\sum_{q=1}^{Q} a_{ln}^{kq} \leq 1, \qquad \forall n \in \mathcal{N} \text{ and } \forall k \in \mathcal{K}, \tag{3.8}$$

$$a_{ln}^{kq} \in \{0,1\} \text{ and } p_{ln}^{kq} \in \mathbb{R}^{+}, \qquad \forall n \in \mathcal{N}, \forall l \in \mathcal{L}_n, \forall k \in \mathcal{K} \text{ and } \forall q \in \mathcal{Q}. \tag{3.9}$$

In this formulation, (3.5) and (3.6) ensure that the data rate and BER requirements are fulfilled, respectively. The maximum transmission power of each BS is constrained by (3.7). The exclusive assignment of any subcarrier in each BS to not more than one user within that BS is enforced by (3.8), where at most one a_{ln}^{kq}, $\forall l$ and $\forall q$ for any given n and k, could be set to one and the remaining are all zero. Solving such a MINLP is prohibitive due to its nonlinear objective function (3.4) and non-linear constraints (3.6) and (3.7). For example, (3.6) consist of summations of terms where each term is the product of two binary variables and an unknown power which takes a real number.

3.2.2 Conversion to BLP

We take some measures to reduce the computation complexity so that optimal solution becomes possible. Through some manipulations, the interdependency between SBA and PA (power allocation) can be decoupled. The required power for every possible SBA combination can be first computed. Then with a change of

variables, the MINLP can be converted to a BLP so that the optimal solution can be found more efficiently.

We first consider a single subcarrier in BS n, where the superscript k is omitted for simplicity. Since no subcarrier sharing among the users in the same BS is allowed, i.e. to fulfil (3.8), either one out of L_n users in BS n will be selected to use this subcarrier with a suitable q, or none of the users is selected. Mathematically, this means that for BS n, either only one out of $Q \times L_n$ assignment variables a_{ln}^q, $\forall l$, $\forall q$, can take on value of 1, or all of them equal to 0 which corresponds to the case where no user is transmitting on the subcarrier. For example, if user $\tilde{l}$ is selected with modulation index $\tilde{q}$, then $a_{\tilde{l}n}^{\tilde{q}} = 1$ and $a_{ln}^q = 0$ for all $q \neq \tilde{q}$ or $l \neq \tilde{l}$. It is therefore more convenient to use a symbolic notation $b_{ln}^{\tilde{q}}$ to denote such a SBA choice when the subcarrier is allocated, and b_n^0 to denote the case where the subcarrier is not allocated to any user in BS n.

All possible choices of SBA for all the users in BS n are grouped in a set $\mathcal{B}_n = \{b_n^0, b_{ln}^q \mid q \in \mathcal{Q};\ l \in \mathcal{L}_n\}$ with cardinality $QL_n + 1$. For example, in a 2-cell system where each cell has 2 users and the modulation index can be chosen from $\mathcal{Q}$, we have $\mathcal{B}_1 = \{b_1^0, b_{11}^1, b_{11}^2, b_{11}^3, b_{21}^1, b_{21}^2, b_{21}^3\}$ and $\mathcal{B}_2 = \{b_2^0, b_{12}^1, b_{12}^2, b_{12}^3, b_{22}^1, b_{22}^2, b_{22}^3\}$ for BS 1 and 2, respectively.

For any BS n, we can select one element from $\mathcal{B}_n$ independently. Denoting the n-ary Cartesian product of all $\mathcal{B}_n$'s as $\mathcal{B} = \times_{n \in \mathcal{N}} \mathcal{B}_n$, we can see that each element of $\mathcal{B}$ is a N-tuple vector $\mathbf{b} = [b_1, \ldots, b_n, \ldots, b_N]$, where b_n is an element selected from $\mathcal{B}_n$. Since every $b_n \in \mathcal{B}_n$ corresponds to a SBA choice in BS n, a given vector $\mathbf{b} \in \mathcal{B}$ represents a particular SBA combination of all N BSs of the whole system. Using the 2-cell system for illustration again, if $\tilde{\mathbf{b}} = [b_{21}^3, b_{12}^2]$, the first element indicates

Table 3.1: An example of the mapping between $\mathbf{b}$ and combinations of a_{ln}^{q}.

$\mathbf{b}$	a_{11}^{1}	a_{11}^{2}	a_{11}^{3}	a_{21}^{1}	a_{21}^{2}	a_{21}^{3}	a_{12}^{1}	a_{12}^{2}	a_{12}^{3}	a_{22}^{1}	a_{22}^{2}	a_{22}^{3}
$[b_1^0, b_2^0]$	0	0	0	0	0	0	0	0	0	0	0	0
$[b_{11}^1, b_2^0]$	1	0	0	0	0	0	0	0	0	0	0	0
$[b_{11}^2, b_2^0]$	0	1	0	0	0	0	0	0	0	0	0	0
$\vdots$	$\vdots$					$\ddots$						$\vdots$
$[b_{21}^3, b_2^0]$	0	0	0	0	0	1	0	0	0	0	0	0
$[b_1^0, b_{12}^1]$	0	0	0	0	0	0	1	0	0	0	0	0
$\vdots$	$\vdots$					$\ddots$						$\vdots$
$[b_{21}^2, b_{22}^3]$	0	0	0	0	1	0	0	0	0	0	0	1
$[b_{21}^3, b_{22}^3]$	0	0	0	0	0	1	0	0	0	0	0	1

that user 2 ($l = 2$) from cell 1 ($n = 1$) transmits at 64-QAM ($q = 3$), whilst the second element indicates that user 1 in cell 2 transmits at 16-QAM on the same subcarrier. Therefore $\mathcal{B}$ represents the space of all possible SBA combinations for all the N BSs, with $\prod_{n=1}^{N}(QL_n + 1)$ combinations in total. From our definition, given $\tilde{\mathbf{b}} = [b_{\tilde{l}_1 1}^{\tilde{q}_1}, \ldots, b_{\tilde{l}_n n}^{\tilde{q}_n}, \ldots, b_{\tilde{l}_N N}^{\tilde{q}_N}]$, we have $a_{\tilde{l}_n n}^{\tilde{q}_n} = 1$, and $a_{l_n n}^{q_n} = 0$ for any $l_n \neq \tilde{l}_n$ or $q_n \neq \tilde{q}_n$, $\forall n \in \mathcal{N}$. Hence when $\mathbf{b}$ is given, it is sufficient to derive the values of a_{ln}^{q}, $\forall l, n, q$. The mapping between $\mathbf{b}$ and values of a_{ln}^{q}'s is illustrated in Table 3.1 for the 2-cell example.

With each combination of SBA on one subcarrier being independent of that on another subcarrier, we introduce the choice indicators $e_{\mathbf{b}}^{k} \in \{0, 1\}$, where $e_{\tilde{\mathbf{b}}}^{k} = 1$ indicates that a particular combination vector $\tilde{\mathbf{b}}$ is selected on subcarrier k, and $e_{\mathbf{b}}^{k} = 0$ for all $\mathbf{b} \neq \tilde{\mathbf{b}}$. Since only one combination will be selected on every subcarrier, constraints (3.8) are equivalent to

$$\sum_{\mathbf{b} \in \mathcal{B}} e_{\mathbf{b}}^{k} = 1, \quad \forall k \in \mathcal{K}. \tag{3.10}$$

Conditioned on $e^k_{\tilde{\mathbf{b}}} = 1$, the corresponding values of a^{kq}_{ln} are known. Representing the non-zero variables as $a^{k\tilde{q}_n}_{\tilde{l}_n n} = 1$, $\forall n$, we substitute them into (3.6) for BS $\tilde{n}$ to get $G^{k\tilde{n}}_{\tilde{l}_{\tilde{n}}\tilde{n}} p^{k\tilde{q}_{\tilde{n}}}_{\tilde{l}_{\tilde{n}}\tilde{n}} = (I^k_{\tilde{l}_{\tilde{n}}\tilde{n}} + N_0)\gamma^{\tilde{q}_{\tilde{n}}}_{\tilde{l}_{\tilde{n}}\tilde{n}}$. Using (3.1), constraints (3.6) for all BSs reduce to a set of linear equations which can be written as

$$\mathbf{G}^k_{\tilde{\mathbf{b}}} \mathbf{p}^k_{\tilde{\mathbf{b}}} = N_0 \boldsymbol{\gamma}_{\tilde{\mathbf{b}}}, \tag{3.11}$$

where

$$\mathbf{G}^k_{\tilde{\mathbf{b}}} = \begin{bmatrix} G^{k1}_{\tilde{l}_1 1} & -\gamma^{\tilde{q}_1}_{\tilde{l}_1 1} G^{k2}_{\tilde{l}_1 1} & \cdots & -\gamma^{\tilde{q}_1}_{\tilde{l}_1 1} G^{kN}_{\tilde{l}_1 1} \\ -\gamma^{\tilde{q}_2}_{\tilde{l}_2 2} G^{k1}_{\tilde{l}_2 2} & G^{k2}_{\tilde{l}_2 2} & \cdots & -\gamma^{\tilde{q}_2}_{\tilde{l}_2 2} G^{kN}_{\tilde{l}_2 2} \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma^{\tilde{q}_N}_{\tilde{l}_N N} G^{k1}_{\tilde{l}_N N} & -\gamma^{\tilde{q}_N}_{\tilde{l}_N N} G^{k2}_{\tilde{l}_N N} & \cdots & G^{kN}_{\tilde{l}_N N} \end{bmatrix}_{N \times N},$$

and $\mathbf{p}^k_{\tilde{\mathbf{b}}} = [p^{k\tilde{q}_1}_{\tilde{l}_1 1}, \ldots, p^{k\tilde{q}_n}_{\tilde{l}_n n}, \ldots, p^{k\tilde{q}_N}_{\tilde{l}_N N}]$ and $\boldsymbol{\gamma}_{\tilde{\mathbf{b}}} = [\gamma^{\tilde{q}_1}_{\tilde{l}_1 1}, \ldots, \gamma^{\tilde{q}_n}_{\tilde{l}_n n}, \ldots, \gamma^{\tilde{q}_N}_{\tilde{l}_N N}]$ are two column vectors of length N. The elements $G^{kj}_{\tilde{l}_n n}$'s, $j \in \mathcal{N}$, $p^{k\tilde{q}_k}_{\tilde{l}_n n}$'s and $\gamma^{\tilde{q}_n}_{\tilde{l}_n n}$'s take the subscripts $\tilde{l}_n$'s and superscripts $\tilde{q}_n$'s given by $\tilde{\mathbf{b}}$. Assuming that $\mathbf{G}^k_{\tilde{\mathbf{b}}}$ is non-singular, the power vector $\mathbf{p}^k_{\tilde{\mathbf{b}}}$ is given by

$$\mathbf{p}^k_{\tilde{\mathbf{b}}} = (\mathbf{G}^k_{\tilde{\mathbf{b}}})^{-1} N_0 \boldsymbol{\gamma}_{\tilde{\mathbf{b}}}, \tag{3.12}$$

where $\mathbf{X}^{-1}$ denotes inverse of matrix $\mathbf{X}$. Using the same 2-cell example, given $\tilde{\mathbf{b}} = [b^3_{21}, b^2_{12}]$, we have

$$\mathbf{G}^k_{\tilde{\mathbf{b}}} = \begin{bmatrix} G^{k1}_{21} & -\gamma^3_{21} G^{k2}_{21} \\ -\gamma^2_{12} G^{k1}_{12} & G^{k2}_{12} \end{bmatrix},$$

$\mathbf{p}^k_{\tilde{\mathbf{b}}} = [p^{k3}_{21}, p^{k2}_{12}]$ and $\boldsymbol{\gamma}_{\tilde{\mathbf{b}}} = [\gamma^3_{21}, \gamma^2_{12}]$.

We next define

$$\rho^k_{\tilde{\mathbf{b}}} = \sum_{p^{k\tilde{q}_n}_{\tilde{l}_n n} \in \mathbf{p}^k_{\tilde{\mathbf{b}}}} p^{k\tilde{q}_n}_{\tilde{l}_n n} = \sum_{n=1}^{N} \sum_{l=1}^{L_n} \sum_{q=1}^{Q} a^{kq}_{ln} p^{kq}_{ln}, \quad \text{given any } e^k_{\tilde{\mathbf{b}}} = 1, \tag{3.13}$$

then the objective function (3.4) can be converted to

$$\max_{a^{kq}_{ln}, e^k_{\mathbf{b}}} \sum_{n=1}^{N} \sum_{l=1}^{L_n} r_{ln} - c \sum_{k=1}^{K} \sum_{\mathbf{b} \in \mathcal{B}} \rho^k_{\mathbf{b}} e^k_{\mathbf{b}}, \tag{3.14}$$

where $\rho^k_{\mathbf{b}}$'s, $\forall e^k_{\mathbf{b}} = 1$, are constants. It can be seen that the new objective function (3.14) is linear, and the constraints (3.6) can be removed since the required SINR have been guaranteed by (3.12). Constraints (3.7) for maximum power limit of each station are then converted to

$$\sum_{k=1}^{K} \sum_{\mathbf{b} \in \mathcal{B}} e^k_{\mathbf{b}} \mathbf{p}^k_{\mathbf{b}} \leq P_{\max} \cdot \mathbf{1}, \tag{3.15}$$

where $\mathbf{1}$ is a vector of length N with all elements having values 1.

In a way similar to how $\rho^k_{\mathbf{b}}$ is introduced, we further define two sets of constants:

$$\sigma^k_{\tilde{\mathbf{b}}} = \sum_{n=1}^{N} \sum_{l=1}^{L_n} \sum_{q=1}^{Q} 2q \cdot a^{kq}_{ln}, \quad \text{given any } e^k_{\tilde{\mathbf{b}}} = 1, \tag{3.16}$$

$$\delta^k_{\tilde{\mathbf{b}},ln} = \sum_{q=1}^{Q} 2q \cdot a^{kq}_{ln}, \quad \text{given any } e^k_{\tilde{\mathbf{b}}} = 1, \forall l \in \mathcal{L}_n \text{ and } \forall k \in \mathcal{K}. \tag{3.17}$$

These values actually give the total data bits on a subcarrier in all BSs, and the data bits for every single user on a subcarrier, respectively, for each SBA combination. The objective function (3.14) and constraints (3.5) can then be converted equivalently to the forms involving only the choice indicators, $e^k_{\tilde{\mathbf{b}}}$'s, as optimization

variables. The original MINLP is finally transformed to an equivalent BLP given by

$$\max_{e_{\mathbf{b}}^k} \sum_{k=1}^{K} \sum_{\mathbf{b}\in\mathcal{B}} \left(\sigma_{\mathbf{b}}^k - c\rho_{\mathbf{b}}^k\right) e_{\mathbf{b}}^k, \tag{3.18}$$

subject to

$$\sum_{k=1}^{K} \sum_{\mathbf{b}\in\mathcal{B}} \delta_{\mathbf{b},ln}^k e_{\mathbf{b}}^k \geq R_{ln}, \qquad \forall n \in \mathcal{N}, \forall l \in \mathcal{L}_n \text{ and} \tag{3.19}$$

$$\sum_{k=1}^{K} \sum_{\mathbf{b}\in\mathcal{B}} e_{\mathbf{b}}^k \mathbf{p}_{\mathbf{b}}^k \leq P_{\max} \cdot \mathbf{1}, \tag{3.20}$$

$$\sum_{\mathbf{b}\in\mathcal{B}} e_{\mathbf{b}}^k = 1, \qquad \forall k \in \mathcal{K}, \tag{3.21}$$

$$e_{\mathbf{b}}^k \in \{0, 1\}, \qquad \forall \mathbf{b} \in \mathcal{B} \text{ and } \forall k \in \mathcal{K}. \tag{3.22}$$

We can see that the objective function has been converted to a linear form and the nonlinearities in constraints are eliminated, thus the complexity of optimization is drastically reduced, even though at the expense of an increased number of variables from $2KQ\sum_{n=1}^{N} L_n$ to $K\prod_{n=1}^{N}(QL_n + 1)$. Compared with the MINLP, the BLP requires potentially large numbers of constants to be calculated before it can be solved. Among these constants, however, $\sigma_{\mathbf{b}}^k$'s and $\delta_{\mathbf{b},ln}^k$'s need to be calculated only once and can be stored in lookup tables, while $\rho_{\mathbf{b}}^k$'s are periodically calculated at the beginning of the cycles where channel conditions of the subcarriers are updated. With all variables having binary values, the BLP in (3.18)-(3.22) can be efficiently solved by an algorithm which selects one out of the $\prod_{n=1}^{N}(QL_n + 1)$ combinations on a subcarrier, and repeats K times over all the subcarriers to decide the optimal solution. Since quite a number of SBA combinations may not be feasible, some measures could be taken to reduce the search space.

To check for the feasibility of a SBA combination, (3.11) can be rewritten in the following form

$$\left[\mathbf{I} - \mathbf{F}_{\mathbf{b}}^{k}\right] \mathbf{p}_{\mathbf{b}}^{k} = \mathbf{v}_{\mathbf{b}}^{k}, \quad \text{given any } e_{\mathbf{b}}^{k} = 1, \forall \mathbf{b} \text{ and } \forall k, \tag{3.23}$$

where $\mathbf{I}$ is the identity matrix of size $N \times N$, $\mathbf{v}_{\mathbf{b}}^{k} = [v_{l1}^{kq}, \ldots, v_{ln}^{kq}, \ldots, v_{lN}^{kq}]$ with $v_{ln}^{kq} = N_0 \gamma_{ln}^{q} / G_{ln}^{kn}$, and $\mathbf{F}_{\mathbf{b}}^{k}$ is also a $N \times N$ matrix with its $(i,j)^{th}$ element given by

$$\left[\mathbf{F}_{\mathbf{b}}^{k}\right]_{ij} = \begin{cases} 0, & \text{if } j = i; \\ \gamma_{li}^{q} G_{li}^{kj} / G_{li}^{ki}, & \text{if } j \neq i. \end{cases} \tag{3.24}$$

By Perron-Frobenius theorem for non-negative matrices, there exists a non-negative power vector, $\mathbf{p}_{\mathbf{b}}^{k}$, if and only if the maximum eigenvalue of $\mathbf{F}_{\mathbf{b}}^{k}$ does not lie outside the unit circle, or the spectral radius of $\mathbf{F}_{\mathbf{b}}^{k}$ is not larger than one [67]. Without considering data rate constraints, the feasibility region Ω is defined over all $\mathbf{b} \in \mathcal{B}$, $k \in \mathcal{K}$, where non-negative solutions exist for the power vector. As a result, any $e_{\mathbf{b}}^{k}$ which corresponds to a region outside Ω can immediately be set to zero to reduce search time. In the implementation, when the power vector $\mathbf{p}_{\tilde{\mathbf{b}}}^{k}$ cannot be solved for its corresponding $e_{\tilde{\mathbf{b}}}^{k}$, all those power vectors on subcarrier k that have more users than those existing ones in $\tilde{\mathbf{b}}$ are infeasible as well and need not to be calculated.

3.3 Numerical Results

Computer simulations for a 3-cell OFDMA system are conducted, where each cell has a radius of 100 and is separated by $100\sqrt{3}$ among each other. BSs are located at the centre of the cells, and locations of the two users in each cell are

randomly generated. The propagation model takes into consideration the path loss, shadowing and fast fading. The path loss (in dB) at a distance d from a BS is taken as $L(d) = L(d_0) + 10\alpha \log_{10}(d/d_0)$, with $d_0 = 10$ being the reference point ($L(d_0) = 0$dB) and $\alpha = 3.8$. The shadowing effect is modelled as a lognormal random variable with 10dB standard deviation. The four-path Rayleigh model is used to model frequency selective fading with an exponential power profile. We consider a multi-cell OFDMA system with 16 subcarriers. The receiver thermal noise is -70dBm, and the required BER is 10^{-5} for every user. The maximum total transmission power for each BS is 40dBm.

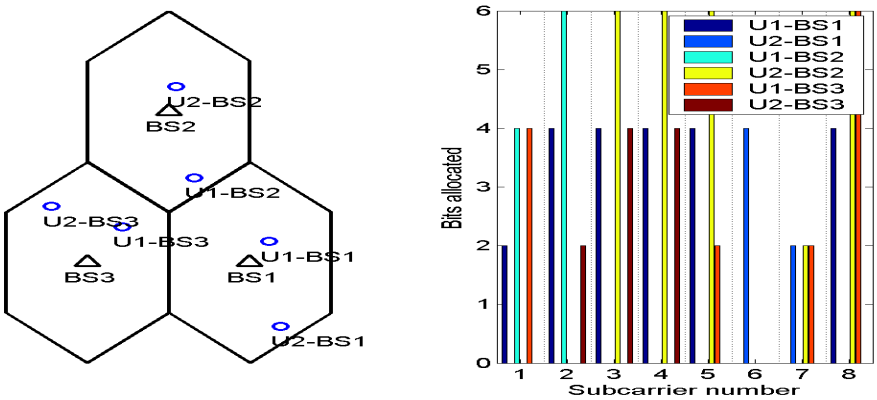

Figure 3.2: Example 1 of the optimal solution on subcarrier-and-bit allocation.

Consider a snapshot of user distribution and channel conditions shown in Fig. 3.2. The optimal SBA solution for such a scenario is also presented in the histogram shown in Fig. 3.2. It is observed that every subcarrier except subcarrier 6 has been occupied by all three BSs, which corresponds to FRF = 1 on these subcarriers. Such an allocation can fully utilize the available radio bandwidth in all cells and help to increase total capacity of the whole system. In the exceptional case where subcarrier 6 is exclusively assigned to user 2 in BS 1, it is due to the fact that this user is located very closely to the cell boundary of BS 1, and the user may be

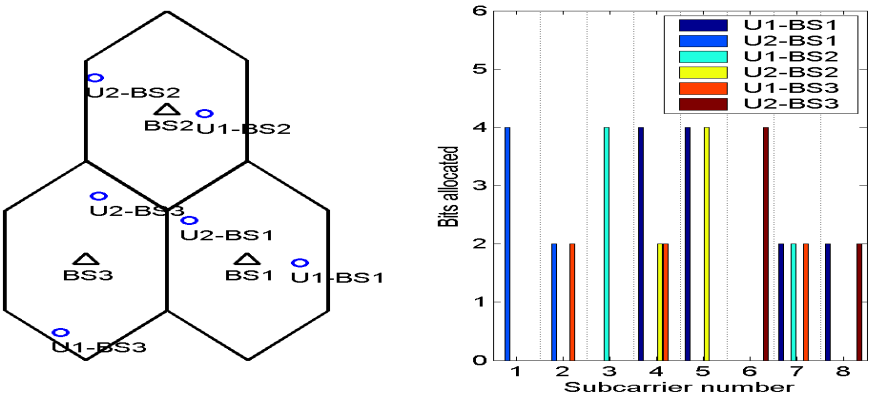

Figure 3.3: Example 2 of the optimal solution on subcarrier-and-bit allocation.

subject to strong attenuation in path loss, therefore reusing this subcarrier in any other BSs must be avoided to prevent causing severe interference which can lead to failure of meeting the user's SINR requirement.

In Fig. 3.3, optimal solution of Example 2 is shown for another snapshot of user distribution and channel conditions. In this case the number of subcarriers with RFR=1 has reduced to only two, whilst three other subcarriers are shared by two BSs and the rest are exclusively allocated to only one BS. In this scenario, we can see that most of the users are located in the outer part of the cells, and some are very close to the border of their corresponding cell and the neighbouring cells, e.g. user 2 in BS 1 and user 2 in BS 2. Such users will usually suffer from strong CCI and low SINR. To overcome this problem, the optimal solution is to allocate exclusive subcarriers to these two users, in order to eliminate CCI and to ensure SINR fulfilment, resulting in only a single use on these subcarriers over the 3 cells. Also we notice that there are three subcarriers which are allocated to two cells, therefore the optimal solution has adaptively assigned various reuse factors to the subcarriers according to instantaneous path losses of the whole system.

In Fig. 3.4(a), we show the average utility of the system under different mini-

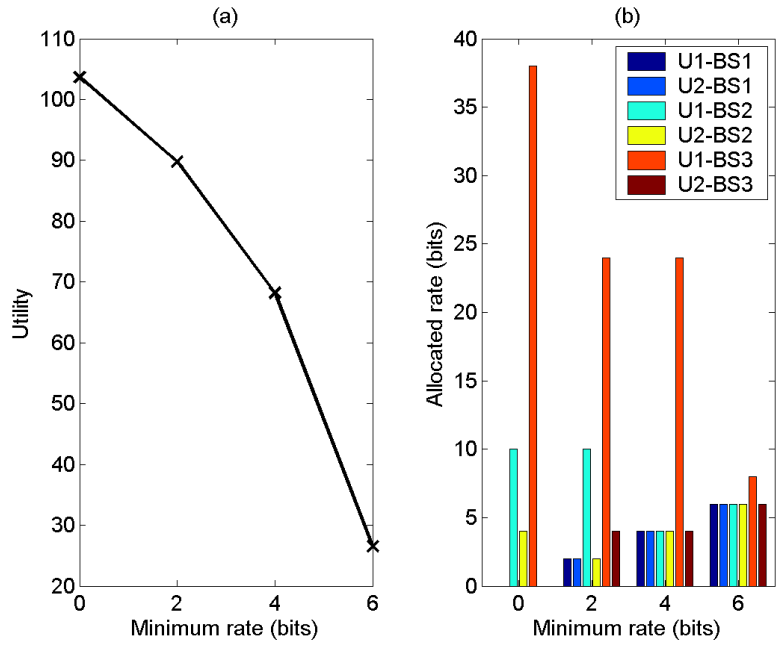

Figure 3.4: Under various minimum rate requirements, (a) average utility (b) user data rate.

mum rate requirements. As the minimum rate required by the users increases, we can see that the utility of the system decreases. The difference in the allocated data rate among the users, however, will reduce as the minimum data rate increases, which can also be considered as a form of increased fairness among the users. The result of a simulation instance is illustrated in Fig. 3.4(b), which clearly shows that the fairness among users increases at the expense of reduced data rates for user 1 in BS 2 and user 1 in BS 3. We can tradeoff between the system utility and fairness by setting appropriate minimum data rates for the users, and the optimal solution will achieve the best total utility while maintaining the fairness.

3.4 Conclusion

Adaptive subcarrier-bit-and-power allocation (SBPA) problem in the downlink of multi-cell OFDMA systems with FRF equal to one is considered in this chapter. We presented a method to convert the MINLP problem to an equivalent BLP in multi-cell OFDMA systems, by which the centralized optimal solution can be obtained without relaxation and approximation. The optimal solution can serve as the benchmark for comparing the performance of heuristic algorithms developed using interference avoidance or reuse partitioning techniques.

Chapter 4

Game Theory in Wireless Communications

Game theory is a mathematical tool developed to model situations where multiple decision makers have conflicts or competitions among each, and to analyse how such decision makers interact strategically to achieve their objectives. Depending on whether cooperative behaviour may be enforced among groups of players, games can be broadly classified into cooperative and non-cooperative games. The players in a non-cooperative game are assumed to be selfish and rational, and they make decisions independently. The selfishness and rationality assumptions of non-cooperative games imply that every player will always adopts a strategy that will maximize his own payoff, which makes the non-cooperative game theory a very powerful tool to model wireless systems consisting of independent agents that compete for radio resources among themselves. Therefore our work in this thesis focuses on applying non-cooperative games to model radio resource allocation in wireless systems.

In this chapter, we will first introduce the basic concepts of non-cooperative games, then review the applications of game theory on wireless systems, and finally present our motivation on using integer bit numbers in game studies.

4.1 Introduction to Non-cooperative Games

4.1.1 Strategic Form Games and Pure Strategies

In this subsection, we first introduce the *static* game which has a single stage and the players have only one move. The players are assumed to make their moves simultaneously without knowing what others choose. A static non-cooperative game can be represented in strategic form as

$$\Gamma = \langle \mathcal{N}, \mathcal{S}, \{u_n\}_{n \in \mathcal{N}} \rangle \,,$$

where the three major components are represented by:

- $\mathcal{N}$ – a finite set of **players**;
- $\mathcal{S}$ – a **strategy** space of the game; and
- u_n – **payoff/utility** function of player n, for $n \in \mathcal{N}$.

The opponents of player n refers to all the players belonging to $\mathcal{N}$ except n himself, which is often designated by subscript $-n$ for convenience. The pure strategy space of an individual player is denoted as $\mathcal{S}_n$, for $n \in \mathcal{N}$, where a *pure* strategy, $s_n \in \mathcal{S}_n$, assigns zero probability to all moves except one, i.e. a player's move is deterministic.

If the set of eligible strategies for a player is independent on the strategies chosen by other players, the strategy space of the game is simply the Cartisian product

of all strategy spaces of the players, given as $\mathcal{S} = \times_{n \in \mathcal{N}} \mathcal{S}_n$. This is likely to be the case if the game has no constraints, or if the payoff function of one player contains the variables of other players but the constraints are free from variables of other players. However, in games formulated with constraints that contain other players' variables, such as resource allocation games with QoS requirement in wireless networks, the set of eligible strategies of a player changes as the players change their strategies, hence $\mathcal{S} \neq \times_{n \in \mathcal{N}} \mathcal{S}_n$.

Definition 1. A *joint strategy* of Γ, $\hat{s}$, refers to any possible combination of the strategies chosen by more than one players.

Definition 2. A *strategy profile* of Γ, $s = (s_1, \ldots, s_N)$, refers to a joint strategy which consists of the selected strategies of every player and is eligible to the game such that all the constraints are satisfied.

A joint strategy does not necessarily satisfy all constraints and may not be eligible to the game as a solution. Therefore a joint strategy may or may not be a strategy profile, depending on whether all constraints can be satisfied. We refer to the collective strategies of the opponents as $\hat{s}_{-k} \in \mathcal{S}_{-k}$, where $\mathcal{S}_{-k} = \mathcal{S} \setminus \mathcal{S}_k$. For any given strategy $s_n \in \mathcal{S}_n$ and an arbitrary joint strategy from the opponents $s_{-n} \in \mathcal{S}_{-n}$, there is no guarantee that all players' constraints will be satisfied for $(s_n, s_{-n}$. Hence, it is more appropriate to define the strategy space, $\mathcal{S}$, of the game as the set of all strategy profiles with all constraints being satisfied, which also implies that s is feasible, $\forall s \in \mathcal{S}$.

The payoff function, $u_n(s)$, quantifies the payoff of player n in the game for a given strategy profile s, hence is a scalar-valued function $u_n(s) : \mathcal{S} \mapsto \mathbb{R}$, where $\mathbb{R}$ denotes the set of real numbers. By convention, u_n can also be denoted as

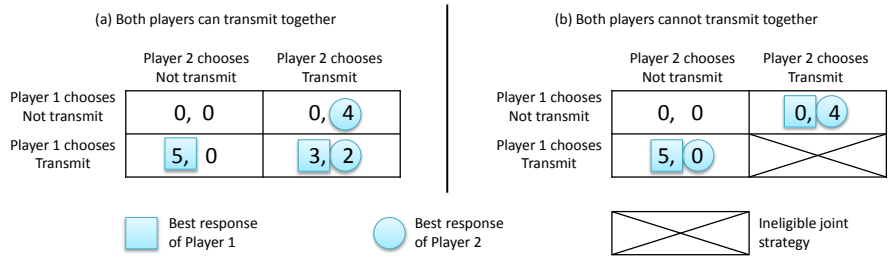

Figure 4.1: A non-cooperative wireless transmission game.

$u_n(s'_n, s_{-n}) = u_n(s_1, \ldots, s_{n-1}, s'_n, s_{n+1}, \ldots, s_N)$ to emphasize the payoff for different strategies of player n with any given joint strategy of his opponents.

Let's look at a non-cooperative wireless transmission game as a simple example. In this game, there are two transmission pairs (players) which are deciding to transmit or not on a common frequency channel. Assuming that BER requirements can be fulfilled when both players transmit, the payoff table of the game is shown in Fig. 4.1(a). Player 1 chooses the row and Player 2 chooses the column, where each player has two strategies specified by the number of rows or number of columns. The payoff received by Player 1 is represented by the first number in the interior of a table cell, and the payoff for Player 2 is provided by the second number. Suppose that Player 1 transmits and Player 2 does not transmit, then Player 1 gets a payoff of 5 and Player 2 gets 0. If both player choose to transmit, they will get a payoff of 3 and 2, respectively. The payoff of Player 1 is reduced when Player 2 transmits because higher transmission power is required which results in higher cost.

On the other hand, if BER requirements can not be met when both players transmit due to poor channel condition or power limit, the joint strategy consisting of both players transmitting is ineligible in the game. This is shown in Fig. 4.1(b). Such a change in strategy space of the game could lead to different outcomes for the game, which will be discussed in more details in the next subsection.

4.1.2 Nash Equilibrium

Due to the existence of competitions and lack of coordination among selfish players, stability of game-play is a very important issue in non-cooperative games. The concept of NE studies the existence and properties of stable outcome for non-cooperative games.

Definition 3. A (pure) *Nash equilibrium*, $s^* \in \mathcal{S}$, is a strategy profile in a non-cooperative game such that, for any $n \in \mathcal{N}$,

$$u_n(s_n^*, s_{-n}^*) \geq u_n(s_n, s_{-n}^*), \quad \forall s_n \in \mathcal{S}_n. \tag{4.1}$$

In other words, if the game is at a NE point, no player can improve his payoff with a unilateral deviation, given all other players' chosen strategies. If there exists a game-master, she would suggest a NE to the players, on which all players will agree and will not deviate in the absence of coordination, since all players can obtain maximum achievable payoff. Therefore a NE can be considered as the strategy profile such that all players could obtain their best responses simultaneously.

Definition 4. The *best response* of player n to a given joint strategy $s_{-n} \in \mathcal{S}_{-n}$ is a strategy $\beta_{s_{-n}} \in \mathcal{S}_n$ such that:

$$\beta_{s_{-n}} = \arg\max_{s_n \in \mathcal{S}_n} u_n(s_n, s_{-n}), \tag{4.2}$$

and $(\beta_{s_{-n}}, s_{-n}) \in \mathcal{S}$. In other words, given any s_{-n}, one or more of the eligible strategies of player n that would achieve maximum payoff are the best responses for the player.

We consider the examples in Fig. 4.1 to show how NEs could be found. For the game with payoff table given in Fig. 4.1(a), assuming Player 2 not transmitting, the best response of Player 1 is to transmit since $5 > 0$; similarly if the given strategy of Player 2 is to transmit, the best response of Player 1 is still to transmit since $3 > 0$. Vice versa, the best response of Player 2 is to transmit no matter Player 1 chooses to transmit or not, since the payoff received by Player 2 is always positive when he transmits. Marking each best response of Player 1 with a 'square' and each best response of Player 2 with a 'circle', respectively, the procedure to find out the NE(s) in the game is illustrated in Fig. 4.2(a). Therefore we can see that both players will choose to transmit, which is the outcome of the game, or a unique NE for this game.

Whereas for the other game with payoff table given in Fig. 4.1(b), the NEs are either Player 1 or 2 transmits, while the other player must not transmit. And the procedure to search for the NEs in this game is illustrated in Fig. 4.2(b). It also shows how NE(s) would change if the strategy space of a game has changed.

The existence of pure NE(s) is an important consideration when game theoretic approach is used, as it decides whether stable solution(s) exist. For example, for the power control games which deal with best effort transmission rate (i.e., without any rate or QoS constraint), it was shown that a unique NE solution always exists [68]. An existence theorem that has been widely applied in radio resource allocation games is called the Debreu-Fan-Glicksberg theorem [62].

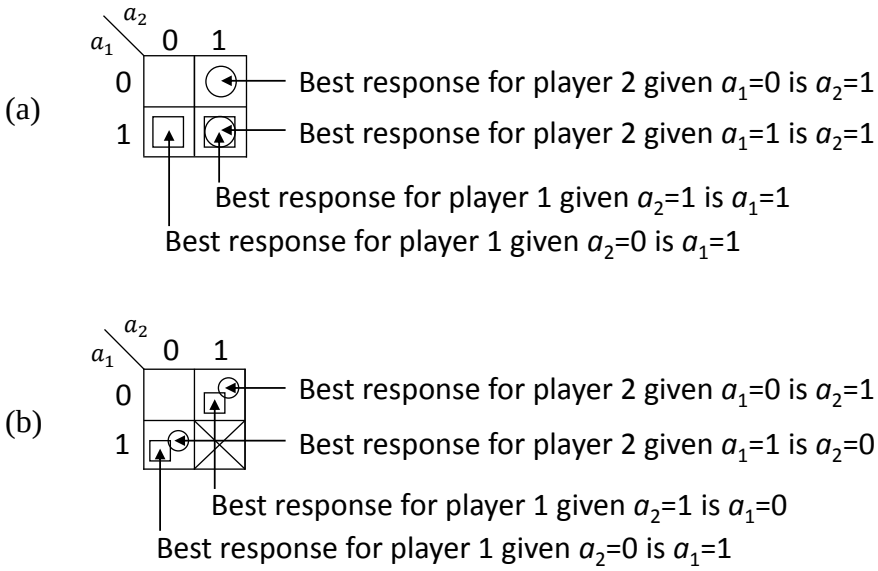

Figure 4.2: A graphical illustration to show how to find out: (a) A single NE in the game shown in Fig. 4.1(a); (b) Two NEs in the game shown in Fig. 4.1(b).

Debreu-Fan-Glicksberg Theorem. *A strategic game* $\langle \mathcal{N}, \mathcal{S}, \{u_n\}_{n\in\mathcal{N}} \rangle$ *has at least one pure NE if,* $\forall n \in \mathcal{N}$*, the following conditions hold.*

- $\mathcal{S}_n$ *is a compact and convex set;*
- *the payoff function* u_n *is continuous in* $\mathcal{S}$ *and quasi-concave in* $\mathcal{S}_n$

A real number set $\mathcal{S}_n$ is *compact* if and only if it is closed and bounded. A set $\mathcal{S}_n$ in a vector space over $\mathbb{R}$ is convex if for any pair of points in $\mathcal{S}_n$, every point on the straight line segment joining them also lies entirely inside $\mathcal{S}_n$. Illustrations of a convex set and non-convex set are shown in Fig. 4.3(a) and 4.3(b), respectively. Also, a function $f(x)$ is *quasi-concave* if $f(\lambda x + (1-\lambda)x') \geq \min\{f(x), f(x')\}$, $\forall x, x' \in \mathcal{S}_n$ and $\forall \lambda \in [0, 1]$.

4.1.3 Repeated Games

Whether one or more NEs exist in a game is important in game theory, and if a NE does exist, how the players might reach the NE without coordination is another

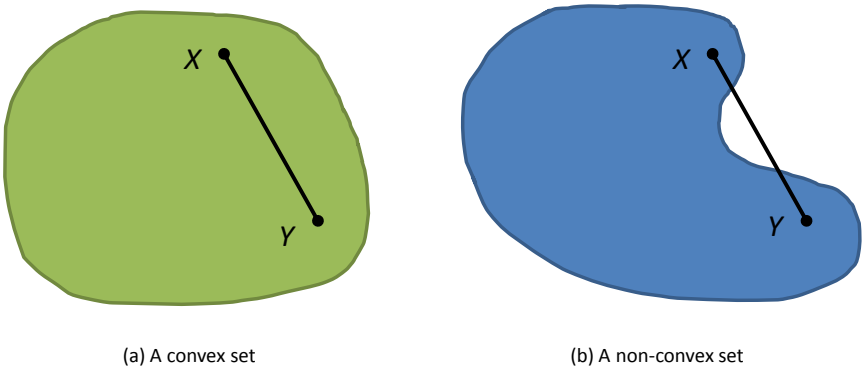

Figure 4.3: Illustrations of convex set and non-convex set.

important issue. One possible way to study the convergence to NE in a game is to have the game repeatedly played, where we assume that the players in the game know the actions of all other players in the previous interactions. Such games are formally defined as repeated games with *observable actions* and *perfect recall*, which means that all players know all the moves of others, and that each player remember the history of his own moves in all the previous stages.

In repeated games, an *action* or move refers to the decision a player makes in a given round, which should be differentiated from *strategy* that is used to refer to the rule in mapping a possible information state of the player into an action. Therefore a strategy in repeated games can be represented as $s : \mathcal{X}_{-n} \rightarrow \mathcal{X}_n$, where $\mathcal{X}_n$ and $\mathcal{X}_{-n}$ denotes the action space of user n and that of his opponents, respectively. A repeated game can be expressed in strategic form as $\Gamma = \langle \mathcal{N}, \mathcal{X}, \{u_n\}_{n \in \mathcal{N}} \rangle$, where $\mathcal{X}$ denotes the action space of the game. For a given action profile of his opponents, $x_{-n} \in \mathcal{X}_{-n}$, a player might choose the best action or one of the better actions to improve his payoff as a response, which are formally defined as the best and better responses, respectively.

Definition 5. The *better response* of player n to the action profile $x \in \mathcal{X}$ is an action x'_n such that:

$$u_n(x'_n, x_{-n}) \geq u_n(x). \tag{4.3}$$

4.2 Applications of Game Theory

4.2.1 Non-Cooperative Games

In OFDMA cellular systems, the complexity of optimization increases rapidly with the number of subcarriers, number of users and number of cells in the centralized approach. There is also a need to obtain all channel information of all the BSs. Recently there are efforts to reduce computation complexity and information exchange by performing radio resource management (RRM) at the respective BS. A suboptimal approach was proposed to have the decision of subcarrier allocation made at a RNC, followed by performing bit and power allocations at every individual BS [25]. More algorithms were proposed to have resource allocation centrally made at each BS but decentralized across the BSs [26]–[28].

As an effective tool to study these decentralized, multi-agent systems, game theory has been extensively applied in recent reported works in this context. Also due to generally little to no coordination among the agents in distributed systems, they will compete among each other for radio resources to maximize their own payoff. The steady state of such competitions can be described by the concept of Nash equilibrium (NE). Conditions for the existence and uniqueness of pure NE, as well as the convergence to one of the pure NEs if they exist, have been investigated in [67]–[69] for OFDMA systems. To improve the efficiency of game solutions, an

algorithm was proposed with a pricing function proportional to transmission power in [71] to achieve at lower power levels and higher utility than the terminals can achieve when they individually strive to maximize utility.

In the context of SBPA for OFDMA systems, various types of games have been formulated, where conditions for the existence and uniqueness of NE, as well as the convergence to one or any of the NE that exists have been investigated and derived. For instance, a potential game on the uplink of multi-cell OFDMA systems was formulated in [79]. The NE has been proved to be unique and convergence to the NE is guaranteed. In [86], a power minimizing game was proposed for OFDMA-based distributed systems. By taking the interference power minimization into consideration, the game was shown to be a potential game and the existence of NE and convergence to the NE were also proved. Game theory has also been applied to relay-based OFDMA systems [80] [81], as well as to multiuser MC-CDMA systems [72]. A recent work on OFDMA femtocell networks was also reported in [73], where two-tier resource allocation games were formulated on the macrocell and femtocell levels to achieve efficient and fair resource sharing.

However, most of the existing results were obtained by tackling the problem from an information theoretic aspect [67]–[81], whereby algorithms which relax the bit loading process to a real number is developed. The next step is to approximate the computed real values to the nearest integer so that practical modulation level can be used. Unfortunately, this can be efficiently performed only with the presence of a centralized controller. None of these reported works mentioned about how this can be achieved in a distributed system – it is of no surprise that the decisions from the distributed decision makers may contradict and further worsen

the interference of some of the subcarriers if without any coordination. This makes WF-based algorithms, which is optimal for single-user system, become inefficient under multiuser environment. To our best knowledge, there is no attempt to formulate the non-cooperative game model of OFDMA systems directly using integer number of bits.

Besides its applications in OFDMA systems, RRM can also be applied to other areas such as dynamic spectrum access which represents a new paradigm in managing scarce radio spectrum resource. Unlike conventional method where the spectrum is exclusively assigned to and used by primary licensed users, a more flexible approach using dynamic spectrum assignment is under initiative. With the forecast regulatory policy changes and radio technology advancements, opportunistic spectrum sharing is very promising to improve spectrum utilization. There are two possible ways to realize this vision. One way is to allow secondary users of lower access priority to operate in the spectrum white space while not interfering the operation of the primary network. The other way is to operate all radios at equal rights to opportunistically access the spectrum for transmission.

Reconfigurable or cognitive radios are believed to be an enabling platform to realize this ambitious vision. These distributed radios are envisioned to be equipped with the computational intelligence to sense, learn and dynamically adapt to the radio environment. Like radios in ad-hoc networks [74] [82], there are no centralized controllers such as BSs to decide which nodes to transmit. The lack of central unit imposes a challenge to system designers in making efficient use of radio resources, since global optimization may not always be possible. With competitions on radio resource for transmission, cognitive radios interact among each other in a selfish

way and their decisions are made in a distributed manner.

Game theory is again a suitable mathematical tool to analyse such interactive decision makers with conflict-of-interests. A non-cooperative game model is proposed in [83] such that the spectrum sharing problem can be formulated as an oligopoly market, where the market is dominated by a small number of sellers. The power optimization problem in decentralized cognitive radio networks, satisfying interference temperature limits at multiple measurement points, has been formulated as a potential game to ensure the existence of and convergence to a pure NE [75]. Potential games have a nice property of convergence and will be introduced in more details in Section 5.4.2. In order to have efficient cognitive medium access control (MAC), two MAC problems are proposed, one as a channel allocation game and the other as a per channel multiple access game, respectively [76]. The first game shows near optimal allocation of channels at NE solution and the second admits a NE that is both fair and Pareto-optimal.

Regardless of whether game theory is applied to RRM of distributed nodes, or as an attempt to distribute the computation among BSs, stability and convergence are two highly desirable properties in wireless system design. If a stable solution cannot be guaranteed by an algorithm, the system would not be reliable since its output state will eventually become unpredictable. Although the convergence of power control in cellular systems are extensively investigated in some works such as [60], such a study in the works using game theory still has much to explore. To enhance system stability, a virtual referee was introduced to monitor the competitions among the non-cooperative players [61], such that when the outcome of game is not desirable, the virtual referee will reduce the transmission rates of some of

the users in order to improve the overall efficiency.

Unfortunately, only a few known problems can achieve these objectives in non-cooperative games, including power control games which can be modelled as potential games [35]. For most of these works, the objective is to maximize either the individuals' capacity or SINR, and the problems are formulated from the information theoretic perspective, where real numbers of bits are obtained through applying water-filling algorithm. However, rounding the real numbers to integers for practical implementation in an uncoordinated manner would actually make the system to deviate from the equilibrium point and cause instability. Furthermore, there is no guarantee of transmission quality in a power game if each user is just to maximize its SINR. In situations where we expect individuals to meet specific QoS supports such as BER requirements, as will be seen shortly, the problem changes to a game with discrete strategy space.

4.2.2 Games with Coordination and Cooperation

The purely non-cooperative game model could be overly pessimistic in infrastructure-based networks, such as cellular networks, broadband access networks and wireless local area networks (WLAN). Since a centralized operator retains the control over common resources in these networks, the non-cooperative setting may not be able to fully capture the gain that could be obtained if coordination were made.

As an alternative to centralized optimization, meanwhile, cooperative game theory has been applied to analyse networks with spectrum sharing [41], which can help us better understand some of the issues involved in cooperation and bargaining. In a cooperative game, players coordinate to achieve a mutually desirable

solution, by bargaining with each other before the game is played. If an agreement is reached, players act according to the agreement made, otherwise players act in a non-cooperative way. The agreement reached must be compulsory so that no players are allowed to deviate from what is agreed upon, which can be enforced by a central controller. For example, Uplink scheduling in LTE systems was considered in [42], where the scheduling problem was represented as a Nash bargaining solution, and an algorithm was proposed to tradeoff between the increase of total throughput and fairness towards the different users.

In daily life, a market serves as a central gathering point, where people can exchange goods and negotiate transactions to satisfy their needs through bargaining. Cooperative game theory for resource allocation in single-cell multiuser OFDMA systems is applied in [38], where the BS serves the function of a market. The distributed users can negotiate via the BS to cooperate in making decisions on the subcarrier usage, such that each of them can operate at its optimum point and joint agreements are made about their operating points. Such a model can be extended to multi-cell cases where the RNC can act as a market and the BSs are players who can negotiate and bargain between each other.

The cooperative game model faces some challenges similar to centralized optimization, such as large signalling overhead, high computation complexity and processing delay. A semi-distributed scheme was proposed in [25] to reduce overhead and computational load by splitting the decisions between RNC and BSs. The RNC coordinates mutual interference (inter-cell) at a super-frame level, whereas each BS makes faster frame level channel assignment decision based on the resource utility values for its users.

Between the two extremes of purely noncooperative and completely cooperative settings, there could exist games with various degrees of coordination lying in. It remains to be a vastly open issue to fully exploit the coordination gain in infrastructure-based networks to maximize system performance, yet keeping the complexity to a minimal level with appropriate game models. By appropriately splitting the scopes of resource allocation between RNC and BSs, and introducing bargaining among the BSs to enhance cooperation, we could obtain some benefits from both coordination and autonomy. It it also possible to combine the use of non-cooperative and cooperative games depending on the need of system study, such as [43], where a game-theoretic framework for radio resource management in heterogeneous wireless access networks was proposed. In this framework, two non-cooperative games were formulated to obtain bandwidth allocation to a service area, as well as to an incoming connection, respectively, while a bargaining game was used to satisfy the connection-level QoS requirements.

4.2.3 Cognitive Radios and Networks

Wireless networks are evolving towards networks of small, smart devices which opportunistically utilize the radio spectrum with minimal coordination and infrastructure. This evolution is motivated by the emergence of cognitive radios (CRs) and the underlying software defined radio (SDR) technological advancements. Proposed by Mitola in [44], cognitive radios are envisioned to have the capabilities to measure and analyse their environment, and make decisions in terms of their transmission parameters to maximize their own defined payoff parameters, through a process known as cognition cycle. Naturally a network consisting of such intel-

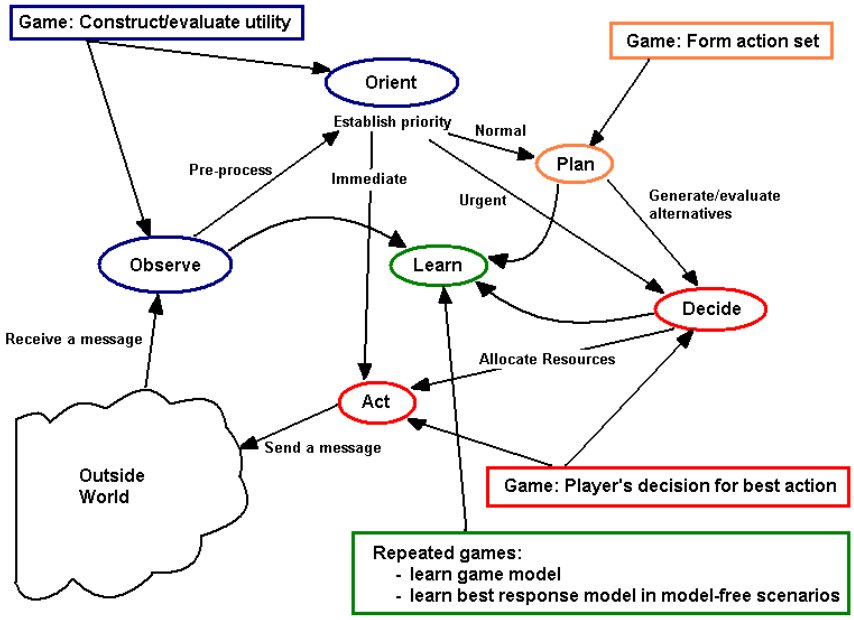

Figure 4.4: A game theoretic model for the cognition cycle [46].

ligent terminals can be modelled by a game theoretic framework, in which the players of the game are the terminals and the strategic actions they can take are the transmission parameters. Each player will rationally and independently decide their own actions to maximize his utility, which can be some performance measures depending on throughput, delay, power consumption and so on. A game theoretic model of the cognition cycle is depicted in Fig. 4.4.

The performance of such networks is ultimately limited by interferences, and therefore efficient and adaptive interference management techniques are essential elements in network design. With possible heterogeneous capabilities and requirements, the smart terminals independently measure the channels and autonomously make decisions to maximize their own benefit. Such actions affect not only their own performance but also that of the local neighbourhood or even of the entire network. Several previous works have shown that cooperation in ad hoc networks can increase the throughput per node [47] or improve spectral efficiency [48] by

exploiting some form of multiuser diversity. While sufficient evidence exists that various forms of cooperation may improve network performance ([47]–[51]), the issue on how to enable and exploit cooperation, e.g. by developing higher layer distributed protocols, has become a recent research focus.

The concept of cognitive radios is envisioned to eventually extend to cognitive networks, where each node individually adapts based on network-wide considerations to yield better overall performance to the network as a whole [52]. In this context, network nodes are expected to be aware of their environment, to be able to dynamically adapt to that environment, and to be able to learn from outcomes of past decisions. Both more sophisticated adaptation and the ability to learn are expected from future radios, to solve complex problems such as opportunistic utilization of spectrum. The study of how these radios will learn and how fast they will converge to effective solutions becomes relevant. There has been work on applying game theory to analyse the performance of cognitive networks, but there is still much to be done in this area.

4.2.4 Spectrum Sharing Games

The frequency spectrum is a fundamental resource whose management has been performed primarily through policies imposed by national regulatory authorities. To eliminate interference between different wireless technologies, current policies allocate a fixed spectrum slice to each wireless technology. This static assignment prevents devices from efficiently utilizing allocated spectrum, resulting in spectrum holes (no devices in area) and very poor utilization [40].

To achieve higher efficiency in spectrum utilization, currently there is much ef-

fort by regulators to study the use of more flexible spectrum management policies such as those using market mechanisms and liberalization. An overview of alternatives to the static spectrum management approach is presented in [39], where three broad categories of spectrum sharing are identified.

On the other hand, many researchers are currently engaged in designing spectrum sharing schemes using game theory. A broad category of spectrum sharing games consists of those games in which the players are network operators. For example, a spectrum sharing game between two cellular operators along the national border of two countries was studied in [53], while another scenario where the mobile users can freely roam across the BSs of different operators and attach to the one with the most favourable signal quality was investigated in [54]. There are also other spectrum sharing games modelling the competition between cellular and Wi-Fi operators [55], as well as among Wi-Fi operators [56] and heterogenous wireless systems [57]. Such studies have provided some insights on the competitions and interactions among the players of spectrum sharing games.

Cognitive radios can detect whether a certain radio band or channel is being occupied, sense the interference levels in the channels, as well as control the transmission power and manage spectrum usage dynamically [45]. These capabilities enable a flexible sharing of the wireless spectrum which can enhance spectrum utilization. A typical scenario is called opportunistic spectrum sharing, in which the radio spectrum is licensed to primary users (operators), while secondary users (cognitive radios) are allowed to utilize the channels as long as they do not interfere with the primary users [40], [58], [59]. Fig. 4.5 provides an example of opportunistic spectrum sharing where the unused spectrum from a TV broadcast channel is

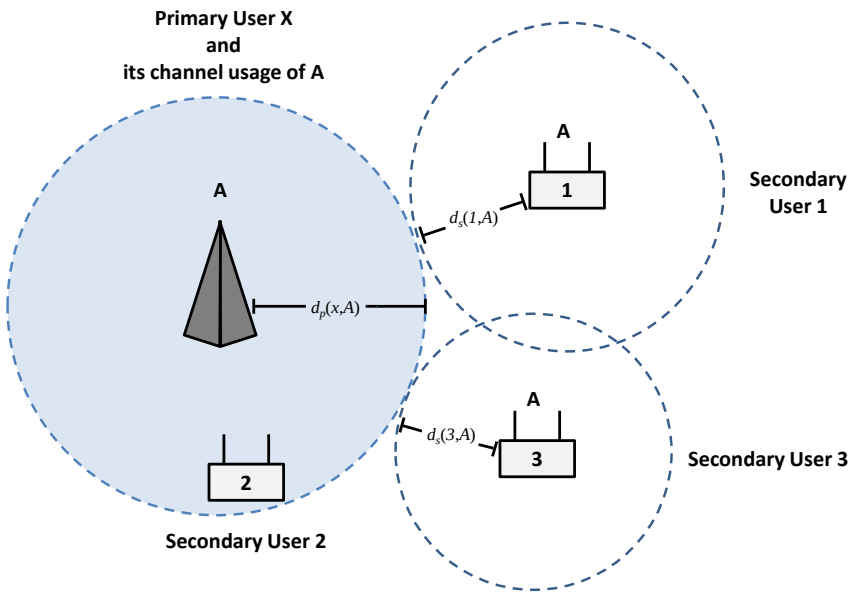

Figure 4.5: An example of opportunistic spectrum sharing.

utilized to provide Wi-Fi connections to a residential community. Located within the coverage of primary user X, secondary user 2 can not make use of channel A as it would interfere with X. Secondary users 1 and 3, however, can emit on channel A as long as they control their transmission power not to interfere further than $d_s(1, A)$ and $d_s(3, A)$, respectively.

There are also recent studies investigating spectrum sharing among cognitive radios in OFDMA networks, e.g. [61]. It can be seen that game theory has been applied to various spectrum sharing schemes, particularly with the players being the network operators and cognitive radios. Such studies provide insights on the possible consequence of greedy behaviour in wireless networks with distributed resource allocation.

4.3 Motivation

Although game theory has been extensively applied in many areas of wireless communications, there still exist several issues that need to be solved or improved, one of them being the way how existence of NE is proved. In most existing works, the Debreu-Fan-Glicksberg theorem forms the basis for the proofs of NE existence in various non-cooperative games formulated. With the assumption that the strategy spaces of all players are compact and convex, it implies that the strategy variables are continuous. When such strategy variables are representing the numbers of bits to transmit, they may no longer be appropriate since the actual numbers of bits transmitted should be integer values in reality. Hence potential issues could exist on the NE existence proved under such an assumption.

For demonstration purpose, we look at a resource allocation game with QoS constraint in the downlink of a cellular system. The system consists of N cells, where each cell has one user. Each BS attempts to transmit to its corresponding user over a common frequency channel, causing ICI among the cells. The SINR of user n can be expressed as:

$$\gamma_n = \frac{p_n G_n^n}{\sum_{j \neq n} p_j G_n^j + N_0}, \tag{4.4}$$

where p_n is the transmitted power from BS n and G_n^j is the path loss from BS j to user n, respectively.

To achieve high spectral efficiency, we assume M-QAM is used. For a specific

BER requirement, the transmission rate is approximated by [61]:

$$r_n = W \log_2(1 + c_3\gamma_n), \tag{4.5}$$

where W is the bandwidth, $c_3 = -\frac{c_2}{\ln(\mathbf{BER}_n/c_1)}$, and $c_1 \approx 0.2$ and $c_2 \approx 1.5$ with a small $\mathbf{BER}_n$.

It can be seen that each BS needs to choose how much transmit power to use. By considering the BS and its corresponding user as a player of the game, the above system can be formulated as a resource allocation game as below:

$$\langle \mathcal{N}, \mathcal{P}, \{u_n\}_{n\in\mathcal{N}} \rangle , \tag{4.6}$$

where $\mathcal{P}$ is the strategy space of transmission power for all players. The payoff function of a player is defined as the transmission rate minus the power cost, given as follows:

$$u_n = r_n - cp_n, \tag{4.7}$$

where c is the cost factor of transmission power.

Such a radio resource allocation game can be shown to have at least one NE by applying the Debreu-Fan-Glicksberg theorem. Firstly, $\mathcal{P}_n \in \mathbb{R}^+$ is compact and convex; secondly, u_n is continuous in $\mathcal{P}$ for any $n \in \mathcal{N}$; lastly, u_n is a log function in p_n minus a linear term also in p_n, which is quasi-concave in $\mathcal{P}_n$. Since all the conditions stated in Debreu-Fan-Glicksberg theorem are met, existence of NE in the game is guaranteed.

Although NE existence can be easily proved using the information theoretic approach where the payoff function is usually log-based, a certain issue could arise

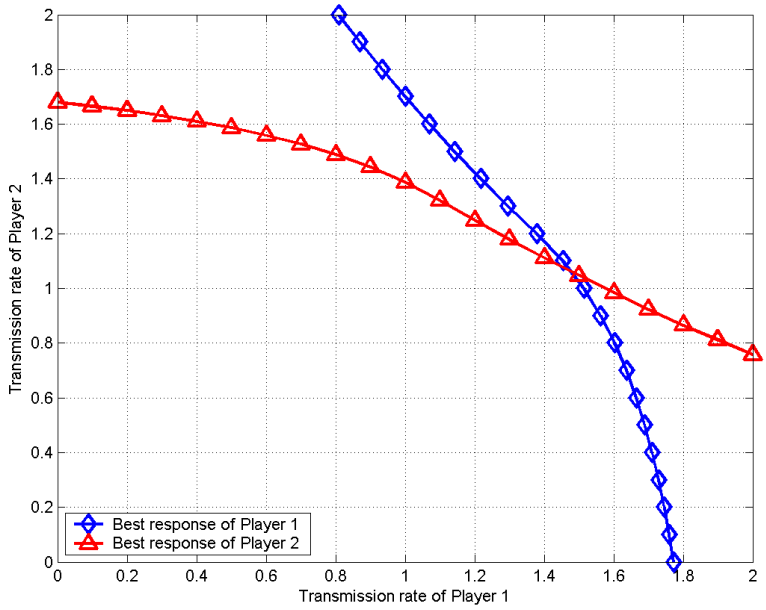

Figure 4.6: An example of NE in a radio resource allocation game.

when the results are to be implemented in practical systems. As an example, let's take a look at Fig. 4.6 which shows a single NE in a 2-player game. The NE is given by the intersection point of the two curves representing the best responses of player 1 and 2, respectively. The NE suggests that the players are to transmit at a data rate greater than 1 bits, which in practice will lead to that both players transmit at a rate of 2 bits after rounding the real number to an integer. However, if both players do transmit 2 bits at the same time, the solution is infeasible and both of their BER requirements could not be met due to high CCI.

On the other hand, if we use discrete values for the bit rates to analyse the game, we would find that the actual NEs for the system are either player 1 to transmit 2 bits and player 2 not to transmit, or vice versa. This example shows that when real numbers are used to determine the numbers of bits to transmit, the NE(s) found can be different from the reality where only integer values are practical for the numbers of bits. Therefore we are motivated to use integer values

for the bit-loading process in non-cooperative resource allocation games, resulting in discrete strategy spaces for the games formulated. Our analysis on the games hence uses an approach different from those where information theory are usually applied, and the details will be presented in the next two chapters.

Chapter 5

Spectrum Sharing Games

In this chapter, we study the existence of static NE solutions when distributed nodes are opportunistically accessing a common frequency channel for transmission, and those nodes which decide to transmit must fulfil their BER requirements. The payoff function of each node is a trade-off between the revenue generated from throughput and the cost of energy consumed. This chapter contributes in the following ways. The problem of distributed nodes accessing a common frequency channel is formulated as a non-cooperative resource allocation game (NRAG) where integer number of transmitted bits is used and BER requirement is expressed in term of SINR. To our best knowledge, some simulation works have been reported [77] but there is no theoretical proof to assure that the problem has pure NE solutions. We first prove that for NRAG with only two players, there is at least one NE solution, and we numerically compute the probabilities where there are one or two pure NEs under Rayleigh fading channel conditions. Then we extend the investigation to N-player NRAG, where a reasonable assumption is made such that a player's strategy is to transmit only if positive payoff can be received. We

prove by mathematical induction that there exists at least one pure NE with integer bit-loading and BER requirement in the presence of N players. We benchmark the performance of NRAG to that when a centralized decision maker is used. The price of anarchy is obtained based on the average performance of NE solutions, which shows that almost two-third loss in performance could occur when the number of nodes is large (i.e., higher interference). The results derived from this work are finally extended to the discussions on the existence of NE in multi-channel distributed networks.

5.1 System Model and Game Formulation

We consider N communication pairs spatially distributed over a region that are competing to transmit over a common frequency band. For brevity, TX_n and RX_n are used to denote the transmitter and the receiver of communication pair n, respectively, where $n \in \mathcal{N}$, with $\mathcal{N} = \{1, \ldots, N\}$ denoting the set of communication pairs. TX_n will transmit only if there exists a transmission power p_n so that the designated RX_n can meet the QoS requirement in term of specific BER for a given transmission rate, otherwise, the transmission link is unreliable and TX_n prefers not to transmit.

When two or more communication pairs ($N \geq 2$) are transmitting at the same time, CCI exists. The total interference power experienced by RX_n can be written as

$$I_n = \sum_{\substack{j=1,\\ j\neq n}}^{N} a_j p_j \cdot G_{n,j}, \tag{5.1}$$

where $G_{n,j}$ denotes the channel gain from TX_j to RX_n, for $n, j \in \mathcal{N}$. We use a_n to

denote the choice of communication pair n: $a_n = 1$ if TX_n is transmitting, otherwise $a_n = 0$. Let γ_n be the SINR threshold to satisfy the BER requirement of pair n transmitting with M-QAM modulation, where the transmission rate $r_n = \log_2 M$. Then the SINR at RX_n must satisfy:

$$\frac{p_n \cdot G_{n,n}}{I_n + N_0} = a_n \gamma_n, \tag{5.2}$$

where N_0 is the PSD of AWGN and is assumed to be identical for all receivers. If transmitting, $a_n = 1$ and pair n has a throughput of r_n bits per symbol duration, else $a_n = 0$ which also implies $p_n = 0$.

The reward from successful transmission of r_n bits is given by $\xi_n r_n a_n$, where ξ_n denotes the reward generated per transmitted bit for pair n. The cost for transmission comes from power consumption and is given by $\zeta_n \times p_n$, where ζ_n is the cost per unit power consumed by TX_n. The payoff for each pair is given as the reward subtracted by cost. Without loss of generality, denoting $c_n = \frac{\zeta_n}{\xi_n}$ and ignoring the constant of proportionality, the payoff function for player n is given by

$$u_n = r_n a_n - c_n p_n. \tag{5.3}$$

Such a payoff function with linear pricing has also been adopted in some reported works, for example, [78] and [68]. The term c_n can be used to reflect priorities among the transmission pairs if different values are used.

Without loss of generality, we assume $\frac{r_n}{c_n} < P_{\max,n}$, where $P_{\max,n}$ denotes the maximum achievable power of TX_n. Then the individual transmission power, p_n, can be bounded in two ways:

Case 1: $0 \leq p_n < \frac{r_n}{c_n}$. This ensures that $u_n > 0$ for all pairs that choose to transmit.

Case 2: $0 \leq p_n \leq P_{\max,n}$, i.e. each transmitter is just bounded by a maximum transmission power. This gives no constraint on the payoff. Even if SINR can be met, some transmission pairs might receive negative payoffs.

For the case where $\frac{r_n}{c_n} \geq P_{\max,n}$, all pairs that choose to transmit will receive $u_n > 0$, which is the same as Case 1 except that p_n is bounded by $P_{\max,n}$ instead.

Since each transmission pair will try to maximize its own payoff, the system can be formulated as a collection of optimizations,

$$\max_{a_n} u_n, \quad \forall n \in \mathcal{N}, \tag{5.4}$$

subject to

$$\frac{p_n \cdot G_{n,n}}{I_n + N_0} = a_n \gamma_n, \qquad \forall n \in \mathcal{N}, \tag{5.5}$$

$$\begin{cases} \text{Case 1:} \quad 0 \leq p_n < \frac{r_n}{c_n}; \\ \\ \text{Case 2:} \quad 0 \leq p_n \leq P_{\max,n}, \end{cases} \qquad \forall n \in \mathcal{N}, \tag{5.6}$$

$$a_n \in \{0, 1\} \qquad \forall n \in \mathcal{N}. \tag{5.7}$$

Equations (5.5) and (5.6) represent the SINR requirement at RX_n and power constraint at TX_n, respectively. The optimization variables consist of a_n's only, as shown in (5.7). This is because the power variables, p_n's, can be computed using the method to be described in Section 5.1.2, conditioned on a given combination of a_n's.

Due to interactions among the communication pairs through I_n, the SINR constraints have nonlinear terms which consist of the products, $a_n a_j$, for $n, j \in \mathcal{N}$ and $j \neq n$. Therefore the optimizations in (5.4)–(5.7) are non-linear programming problems. The optimal choice of pair n also depends on all other transmission pairs' decisions. In the next section, we will show the necessary condition for the existence of stable solutions to the problem.

It may be good to compare the optimality of the solution obtained in the above whereby distributed decisions are made with the social welfare (SW). The SW is defined as the total payoff received by all the individuals [30],

$$u = \sum_{n=1}^{N} u_n. \tag{5.8}$$

Then the optimal solution is obtained by maximizing the SW over the union of all players' constraints. Mathematically, this can be written as

$$\text{SW}: \quad \max_{\{a_n\}_{n\in\mathcal{N}}} u, \tag{5.9}$$

subject to the same constraints (5.5) – (5.7). It can be expected that the optimal value of SW is the same as the optimal solution obtained by centralized optimization presented in Chapter 3.

5.1.1 Formulation of Non-cooperative Games

Under circumstances where distributive decisions are made and there exists conflict-of-interest among the decision-makers, game theory is a useful tool to study and analyse such systems. In our system, since the decision for TX_n whether to trans-

mit depends on maximizing its payoff which is also influenced by the decisions made by other communication pairs through the CCI generated, opportunistic transmission among the pairs can be modelled as a non-cooperative game with each communication pair being a player.

The system described in (5.4) – (5.7) can be formulated as a NRAG in the strategic form given by:

$$\text{NRAG}: \quad \Gamma = \langle \mathcal{N}, \mathcal{A}, \{u_n\}_{n \in \mathcal{N}} \rangle \,, \tag{5.10}$$

where $\mathcal{N}$ and $\mathcal{A}$ denote the set of players and the strategy space, respectively. The players are the transmission pairs, and a strategy is the choice of a player to transmit or not to transmit.

We assume that the NRAG is a game with complete information, so that every player knows the payoffs and strategies available to other players. Furthermore, the players are assumed to be rational and selfish, thus each of them will try to maximize his own payoff for a given joint strategy of his opponents.

5.1.2 Strategy Profile and Strategy Space

Since the selection of a strategy should be deterministic rather than probabilistic for implementable practical systems, throughout this chapter we shall consider only the pure strategies. In the formulated NRAG, any combination of $a_n \in \{0, 1\}$, $\forall n \in \mathcal{N}$, is a possible joint strategy to the game. A method was proposed in [66] to convert a MINLP to a BLP by solving the values of p_n for any given joint strategy. The key point is that the SINR constraints for all the N players form a set of linear

equations which can be expressed as

$$(\mathbf{A} \circ \mathbf{G})\, \mathbf{p} = (\mathbf{a} \circ \boldsymbol{\gamma})\, N_0, \tag{5.11}$$

where $\mathbf{a} = [a_1 \cdots a_N]^T$, $\mathbf{p} = [p_1 \cdots p_N]^T$, $\boldsymbol{\gamma} = [\gamma_1 \cdots \gamma_N]^T$ and

$$\mathbf{G} = \begin{bmatrix} G_{1,1} & -\gamma_1 G_{1,2} & \cdots & -\gamma_1 G_{1,N} \\ -\gamma_2 G_{2,1} & G_{2,2} & \cdots & -\gamma_2 G_{2,N} \\ \vdots & \vdots & \ddots & \vdots \\ -\gamma_N G_{N,1} & -\gamma_N G_{N,2} & \cdots & G_{N,N} \end{bmatrix}. \tag{5.12}$$

The (i, j)-th element of matrix $\mathbf{A}$ is denoted as $[\mathbf{A}]_{i,j} = a_i a_j$, $\forall i, j \in \mathcal{N}$. The operator $\circ$ denotes the Hadamard product, or entry-wise product of two matrices or vectors.

Assuming that a feasible solution of the power vector, $\mathbf{p}$, exists, it can be obtained as

$$\mathbf{p} = (\mathbf{A} \circ \mathbf{G})^{-1} (\mathbf{a} \circ \boldsymbol{\gamma})\, N_0. \tag{5.13}$$

Therefore with a given combination of a_n's, $\forall n \in \mathcal{N}$, the transmission power for all pairs can be determined. In particular, if the solution is not a positive power vector, the SINR of transmitting players cannot be met concurrently. On the other hand, even if positive power vector is obtained, in order to be in the strategy space of Γ, all the power constraints given in (5.6) must be satisfied.

5.2 2-Player Non-cooperative Game

5.2.1 Existence of NE

We first consider the simplest two-player NRAG, which can be formulated as follows:

$$\text{NRAG-2:} \quad \Gamma_2 = \langle \{1,2\}, \mathcal{A}, \{u_1, u_2\} \rangle .$$

A joint strategy of Γ_2 can thus be represented by a 2-tuple, (a_1, a_2), with $a_1, a_2 \in \{0,1\}$. Without loss of generality, we assume a fixed modulation, 4-QAM, is used if TX_n transmits, i.e., $r_n = 2$.

Using the case where both players transmit as an example, the transmission power of both players can be computed from (5.13) as

$$\begin{cases} p_1 = (G_{2,2} + \gamma_2 G_{1,2}) \frac{\gamma_1 N_0}{\Delta} \\ p_2 = (G_{1,1} + \gamma_1 G_{2,1}) \frac{\gamma_2 N_0}{\Delta} \end{cases}, \tag{5.14}$$

where

$$\Delta = G_{1,1} G_{2,2} - G_{2,1} G_{1,2} \gamma_1 \gamma_2. \tag{5.15}$$

It can be easily seen that p_1 and p_2 will take non-negative values only if $\Delta > 0$. By substituting (5.14) into the payoff functions of both players, u_n can be expressed as a function of channel gains.

For each of the five possible joint strategies, we can compute the payoffs for the two players. These values can be tabulated in a 2×2 table, as shown in Fig. 5.1. Using Fig. 5.1(e) as an example, the strategy space is given by $\{(0,0), (0,1), (1,0), (1,1)\}$ where a '0' indicates no transmission while a '1' indicates transmission with 4-QAM.

Each entry in the table is the collection of the payoffs for player 1 and player 2, denoted as $(u_1(a_1, a_2), u_2(a_1, a_2))$, respectively. It is not difficult to show that the following properties hold:

(a) $u_1(0, 0) = u_2(0, 0) = 0$, $u_2(1, 0) = u_1(0, 1) = 0$;

(b) $u_1(1, 0) > u_1(1, 1)$, $u_2(0, 1) > u_2(1, 1)$.

The reason behind (a) follows the definition of the payoff functions, where if there is no transmission, the players receive zero payoff. The reason to (b) is intuitive: if the opponent transmits, the player has to increase its transmission power to sustain its quality, and hence results in a smaller payoff. For Case 1, a joint strategy will not be a strategy profile if SINR requirement cannot be met even it is transmitting at maximum power, or if its payoff is negative even though SINR can be met. For Case 2, a joint strategy will not be a strategy profile only if SINR cannot be satisfied, whereas the payoffs for some players could be negative with a certain strategy profile. However, a rational player will always prefer a strategy with zero payoff (i.e. no transmission) over that with a negative payoff.

In Fig. 5.1, we use α, β, α^- and β^- to denote positive utility values, where $0 < \alpha^- < \alpha$, $0 < \beta^- < \beta$. The notation '$-$' is used to denote some negative utility value in Case 2. A 'cross' is used if the joint strategy is not a strategy profile. A 'do not care' indicates that there are more than one joint strategies which will not affect the choice of NE(s).

We first consider Case 1, in which a player transmits only if it can achieve a positive payoff. All possible scenarios are shown in Fig. 5.1(a)–5.1(e). For example, Fig. 5.1(d) illustrates a scenario where $(1, 1)$ is marked with a 'cross' to indicate that it is not a strategy profile due to at least one of the two reasons aforementioned.

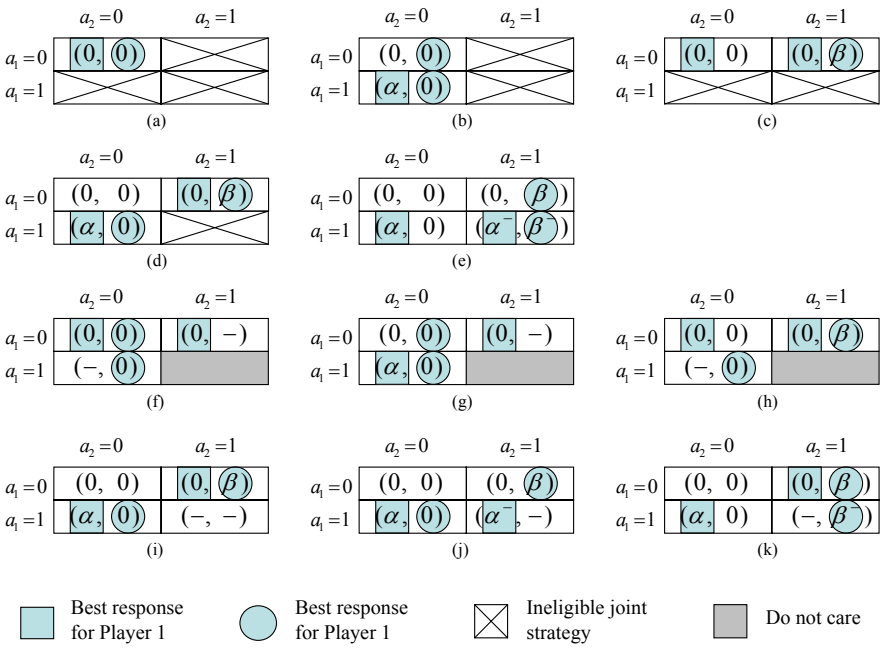

Figure 5.1: All possible scenarios of NE existence for 2-player NRAG game. Case 1: (a)-(e); Case 2: (a)-(k). $\alpha > \alpha^- > 0$, $\beta > \beta^- > 0$.

In this example, the strategy space is given by $\{(0,0),(0,1),(1,0)\}$. The players have positive payoffs, i.e. $u_1 = \alpha > 0$, $u_2 = \beta > 0$.

Using Fig. 5.1(e) as another example, we illustrate the procedure to look for the NE in NRAG-2. In this case, the best response of player 2, regardless of his opponent's (player 1) decision to transmit or not, is to transmit. The two best response solutions are marked using the circles. Similar argument holds for player 1 and the best responses are marked with squares. From Definition 3, the best response common to both gives the NE, i.e., $(a_1 = 1, a_2 = 1)$ is the NE solution. In the situation shown in Fig. 5.1(d), if we follow the same working procedure, $(a_1 = 1, a_2 = 0)$ and $(a_1 = 0, a_2 = 1)$ are found to be two NEs. Similarly, we can show that the remaining scenarios shown in Fig. 5.1(a)–(c) have only one NE.

We shall next consider Case 2, in which the payoff can take negative values as long as the transmission power is less than $P_{\max,n}$ to meet the SINR requirement. Fig. 5.1(a)–5.1(k) show all the possible scenarios. It is not difficult to verify that

for any of the scenarios in Fig. 5.1(f)–5.1(i), if we replace every cell containing negative payoff with a 'cross', then the scenario becomes one of those in Fig. 5.1(a)–5.1(e). For example, if $u_1(1,1) < 0 < u_1(1,0)$ and $u_2(1,1) < 0 < u_2(0,1)$, the corresponding payoff table shown in Fig. 5.1(i) has the same NEs as the one shown in Fig. 5.1(d). For the two exceptional cases in Fig. 5.1(j) and 5.1(k), the game has only one NE solution. But if cell (1,1) is replaced with a 'cross', the game becomes the same as that in Fig. 5.1(d), which has two NEs. More will be discussed when we referred to the NE regions in Fig. 5.2 shortly.

We have shown that in either Case 1 or 2, for two players, at least one NE exists through listing all the possible strategy space. Therefore we have the following theorem.

Theorem 5.1. *There exists at least one NE in* Γ_2.

Proof. We prove the theorem by examining the existence of NE under all possible conditions of the payoff functions, u_1 and u_2. In fact, all the possible scenarios of u_1 and u_2 have been illustrated in Fig. 5.1(a)–(e) for Case 1 and Fig. 5.1(a)–(k) for Case 2. It can be seen that there are one or two NEs corresponding to each scenario in Fig. 5.1. Since the scenarios listed are exhaustive, we can conclude that there exists at least one NE in any condition, hence Theorem 5.1 is proved. □

5.2.2 Effect of Channel Conditions

Having shown that NRAG-2 has at least one NE, we now study how the number of NEs changes as channels vary using a numerical example. The interference gains are fixed at $G_{2,1} = 0.03$, $G_{1,2} = 0.05$. The direct paths $G_{1,1}$ and $G_{2,2} \in [0, 2]$. For simplicity, the transmission power limit is removed in this study. By using the NE

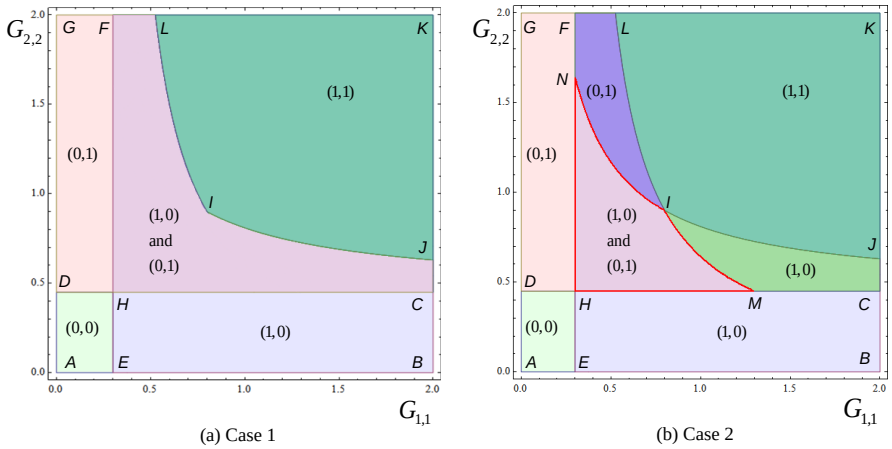

Figure 5.2: Illustration of Nash equilibrium regions for 2-player NRAG game with $G_{2,1} = 0.03$ and $G_{1,2} = 0.05$.

definition given in (4.1), the regions where various strategy profiles become the NEs of Γ_2 can be drawn, as shown in Fig. 5.2(a) and 5.2(b) for Case 1 and Case 2, respectively. The behaviours of NE for both cases are the same except at regions $IMCJ$ and $ILFN$ shown in 5.2(b), which will be explained further.

Now we consider the NE regions shown in Fig. 5.2 as follows. In region $AEHD$, both players will choose not to transmit due to poor channel conditions because $u_1(1,0) \leq 0$ and $u_2(0,1) \leq 0$, which corresponds to Fig. 5.1(a) in Case 1 and Fig. 5.1(f) in Case 2. There is only one NE at $(0,0)$ for this region. Since the area of $AEHD$ depends only on the main path gains and is usually very small, we have amplified the background noise N_0 to enlarge this region in our illustration. Next we consider region $EBCH$, which corresponds to $u_1(1,0) > 0$ and $u_2(0,1) \leq 0$, or Fig. 5.1(b) for Case 1 and Fig. 5.1(g) for Case 2. To prevent negative payoff from transmission, Player 2 always chooses not to transmit regardless of player 1's choice. On the other hand, player 1 will always choose to transmit to obtain positive payoff since player 2 is not transmitting. This results in a single NE at $(1,0)$. Similarly, region $DHFG$ represents the case where $u_1(1,0) \leq 0$ and $u_2(0,1) > 0$, or Fig. 5.1(c) for Case 1 and Fig. 5.1(h) for Case 2. Hence, this region corresponds to a

case where only one NE exists at $(0, 1)$.

Finally, region $HCKF$ corresponds to $u_1(1, 0) > 0$ and $u_2(0, 1) > 0$, which can be further divided into the following sub-regions:

1) $IJKL$: This corresponds to $u_1(1, 1) > 0$ and $u_2(1, 1) > 0$, or Fig. 5.1(e). Both players will always choose to transmit in order to have positive payoffs, and a unique NE exists at $(1, 1)$.

2) $IMCJ$: In this case, $u_1(1, 1) > 0$ and $u_2(1, 1) \leq 0$. This corresponds to Fig. 5.1(d) for Case 1 and Fig. 5.1(j) for Case 2. The NE in this region is different for both cases. Two NEs, $(1, 0)$ and $(0, 1)$, for Case 1 and a single NE exists at $(1, 0)$ for Case 2.

3) $ILFN$: In this case, $u_1(1, 1) \leq 0$ and $u_2(1, 1) > 0$. This corresponds to Fig. 5.1(d) for Case 1 and Fig. 5.1(k) for Case 2. Again the behaviour of NE is different. Two NEs, $(1, 0)$ and $(0, 1)$, for Case 1 and a single NE exists at $(0, 1)$ for Case 2.

4) $HMIN$: In this case, $u_1(1, 1) \leq 0$ and $u_2(1, 1) \leq 0$, or $\Delta \leq 0$. This corresponds to Fig. 5.1(d) for Case 1, and Fig. 5.1(d) and 5.1(i) for Case 2. As a result, two NEs exist where both $(1, 0)$ and $(0, 1)$ are the NEs for both cases.

We now explain the difference in regions $DHFG$ and $ILFN$ where both have (0,1) as the only NE, and regions $EBCH$ and $IMCJ$ where both have (1,0) as the only NE. For example, the scenarios corresponding to the regions $EBCH$ and $IMCJ$ are quite different, as reflected in Fig. 5.1(g) and Fig. 5.1(j), respectively. Region $EBCH$ corresponds to the scenario in Fig. 5.1(g), where $u_1(1, 0) > 0$ if only player 1 transmits but $u_2(0, 1) \leq 0$ if only player 2 transmits. Physically this is due to the fact that $G_{2,2}$ is small and hence high power is required for player 2 to transmit for the required BER, resulting in negative payoff value.

As a contrast, at region $IMCJ$, $u_1(1,0) > 0$ and $u_2(0,1) > 0$, however, $u_1(1,1) > 0$ and $u_2(1,1) \leq 0$. It can be seen that player 1 will always choose to transmit since a positive payoff will be received no matter what player 2's choice is. As a result, player 2 can only chose not to transmit to avoid receiving a negative payoff. Hence $(1,0)$ becomes the only NE. This corresponds to the scenario in Fig. 5.1(j), which usually happens when the interference from player 1 to player 2 is strong. A similar analysis can be applied to Fig. 5.2(a) for Case 1.

5.2.3 Probabilities of Given Strategy Profile As NE

We next examine how the probability of having a given strategy profile as the NE can be computed under all possible channel conditions at a given location. The probabilities of all NE existence scenarios can be estimated with the method introduced in this section. Without loss of generality, we use Case 2 as an example. The two boundary curves, NI and IM, in Fig. 5.2(b) can be shown to be hyperbolic with the following expressions:

$$NI: \quad G_{2,2} = f_1(G_{1,1}) = \frac{\gamma_2}{2} cN_0 + \frac{\gamma_1\gamma_2 G_{2,1}}{2G_{1,1}}(cN_0 + 2G_{1,2}), \quad \text{for } x_N \leq G_{1,1} \leq x_I, \tag{5.16}$$

$$IM: \quad G_{2,2} = f_2(G_{1,1}) = \frac{\gamma_1\gamma_2 G_{1,2}}{2G_{1,1} - cN_0\gamma_1}(cN_0 + 2G_{2,1}), \qquad \text{for } x_I \leq G_{1,1} \leq x_M. \tag{5.17}$$

Denoting a point P on plane $G_{1,1}$–$G_{2,2}$ as (x_P, y_P), the coordinates of the vertices of region $HMIN$ can be found as:

$$x_H = x_N = \frac{\gamma_1}{2} c N_0, \tag{5.18}$$

$$x_I = \frac{\gamma_1}{2}(cN_0 + 2G_{1,2}), \tag{5.19}$$

$$x_M = \frac{\gamma_1}{2}(cN_0 + 2G_{1,2} + \frac{4G_{2,1}G_{1,2}}{cN_0}), \tag{5.20}$$

$$y_H = y_M = \frac{\gamma_2}{2} c N_0. \tag{5.21}$$

The probability of having two NEs in Γ_2, conditioned on $G_{2,1}$ and $G_{1,2}$, can thus be obtained as

$$P_{2NE|G_{2,1},G_{1,2}} = \int_{x_H}^{x_I} \int_{y_H}^{f_1(x)} f_x(x) f_y(y)\, \mathrm{d}y\mathrm{d}x + \int_{x_I}^{x_M} \int_{y_H}^{f_2(x)} f_x(x) f_y(y)\, \mathrm{d}y\mathrm{d}x, \tag{5.22}$$

with $f_x(x)$ and $f_y(y)$ denoting the probability density functions (pdfs) of $G_{1,1}$ and $G_{2,2}$, respectively. The area corresponding to the integral in (5.22) is enclosed by the red lines as indicated in Fig. 5.2.

The above computation is conditioned on the given interference path gains, $G_{2,1}$ and $G_{1,2}$. Denoting the pdfs of $G_{2,1}$ and $G_{1,2}$ as $f_u(u)$ and $f_v(v)$, respectively, the overall probability of having two pure NEs in Γ_2 is given by

$$P_{2NE} = \int_0^\infty \int_0^\infty P_{|u,v} f_u(u) f_v(v)\, \mathrm{d}v\mathrm{d}u \tag{5.23}$$

The pdfs of channel gains in (5.22) and (5.23) are in general forms. In practice, for example, they can be log-normal or Rayleigh distributions for slow or fast fading, respectively, where the mean values are related to the path losses. However, the

probability of having two pure NEs in Γ_2 cannot be obtained in a closed form for the given pdfs of channel gains. Similarly we can compute the probabilities that there is only one NE (corresponding to either (0,0), or (1,1), or (1,0), or (0,1)).

We present a method to estimate these probabilities. Theoretically the value of a channel gain ranges from 0 to ∞. We divide the channel x into ten ranges, with each range having a probability of occurrence equal to 0.1. Denote the index of the ranges as i, $i = 1, \ldots, 10$. The mean value of the ith channel gain, $\bar{x}_i$, is used to represent the overall channel gain over this range, where

$$\int_0^{\bar{x}_i} f_x(x)\,\mathrm{d}x = 0.1i - 0.05 \tag{5.24}$$

Repeat the same procedure for the other channels y, u and v. After getting the mean values of the ten ranges for all the channels, there are 10^4 possible combinations of channel gains, each having a probability of occurrence equal to 10^{-4}. For each channel gain combination, we can calculate the payoff values for both players using all joint strategies, and determine the NE(s). Finally, by counting the number of NE occurrences and normalizing it with the total population 10^4, we can obtain the estimated probability of having two pure NEs (or one pure NE) in Γ_2, for the given pdfs of the main and interference paths.

We use the numerical method to compute the probabilities where the game has two NEs and only a unique NE at $(1, 1)$, respectively, for the system shown in Fig. 5.3. The propagation model of each transmission link comprises of path loss and multipath fading. Assuming a path loss exponent, $\alpha = 3.8$, the path loss (in dB) at a distance d from the transmitter can be taken as $L(d) = L(d_0) + 10\alpha \log_{10}(d/d_0)$, with $d_0 = 1$ being the reference distance ($L(d_0) = 0$dB). The small-scale fading

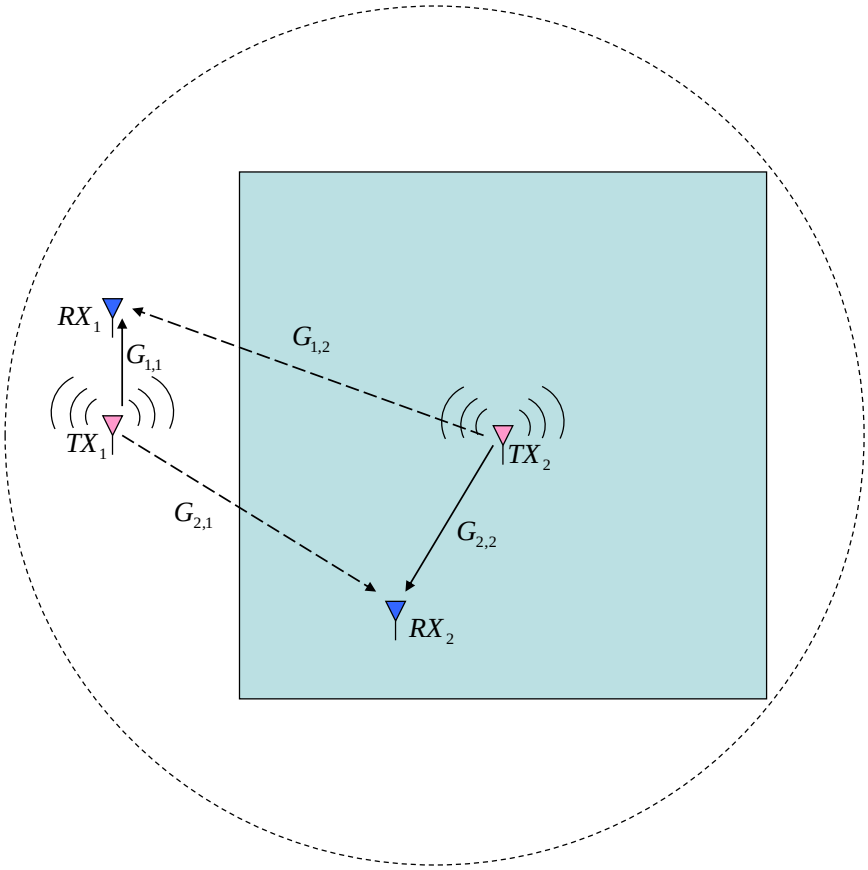

Figure 5.3: Model of the ad hoc system used to estimate the probability of two NEs in Γ_2. The locations of TX_1, TX_2 and RX_1 are fixed as shown. The distances between TX_1 and TX_2, as well as between TX_1 and RX_1, are 150 and 50, respectively. RX_2 is randomly distributed in the shaded square with equal probability, where the edge of the square has a length of 200.

superimposed upon the path loss is assumed to have Rayleigh distribution with unit mean power. For simplicity, we assume no power limit for the transmitters. Thermal noise power at the receivers is taken to be -70dBmW.

Simulation results of the above system with two different sets of parameters are shown in Fig. 5.4 and 5.5. It can be seen that when RX_2 is located around TX_2, it has a higher probability for $(1, 1)$ to become a unique NE. As a contrast, the probability of having two NEs is relatively low in this area. However, when the location of RX_2 is moving farther away from TX_2, the probability of having two NEs will first increase and then decrease, while it becomes less and less possible

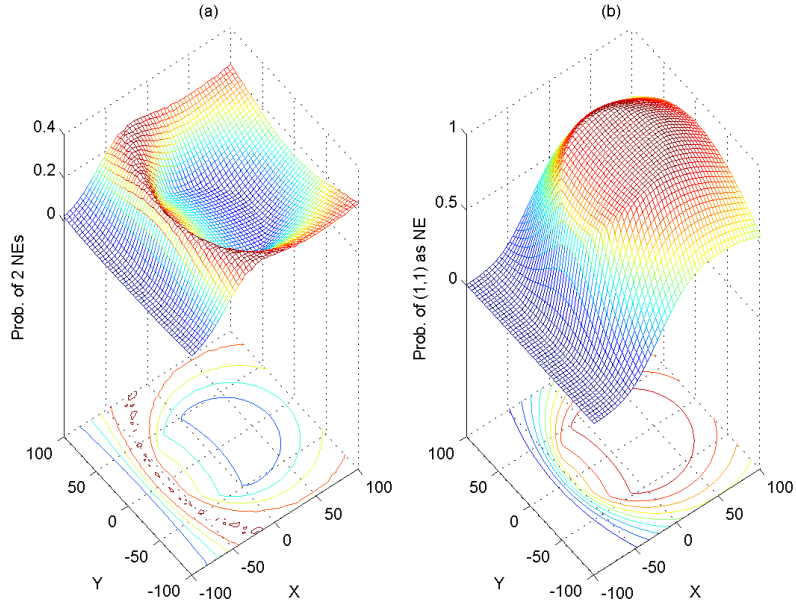

Figure 5.4: Results of the system in Fig. 5.3 with $\gamma_1 = 20$, $\gamma_2 = 20$ and $c = 50$. (a) Probability of two NEs in Γ_2; (b) Probability of having (1,1) as NE in Γ_2.

for $(1, 1)$ to be a NE. The isolines shown underneath the three-dimensional graphs represent the loci of RX_2 where along the contour, the probability of having a given number of NEs is the same. As γ_1, γ_2 and c increase, the area where Γ_2 has $(1, 1)$ as a unique NE will reduce.

5.3 N-Player Non-cooperative Game

To extend our study to NRAG with more than two players, we first take a look at the 3-player NRAG, Γ_3. The payoff table of Γ_3 can be illustrated using two 2-player 2×2 payoff tables, one for $a_3 = 0$ and one for $a_3 = 1$. An example of a simple Γ_3 where no NE exists is shown in Fig. 5.6(b) for Case 2. The strategy space of the game is $\{(0, 0, 0), (0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 0), (1, 0, 1), (0, 1, 1)\}$, and $(1, 1, 1)$ is not an eligible joint strategy. We use squares, circles and triangles to represent the best responses of player 1, 2 and 3, respectively. As we can see, there exists no

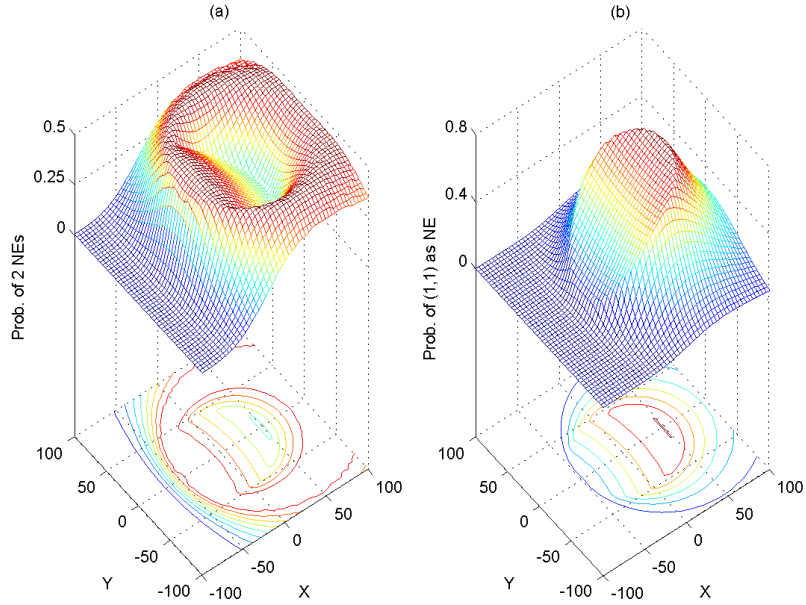

Figure 5.5: Results of the system in Fig. 5.3 with $\gamma_1 = 30$, $\gamma_2 = 40$ and $c = 100$. (a) Probability of two NEs in Γ_2; (b) Probability of having (1,1) as NE in Γ_2.

strategy profile which contains best responses for all the three players. Hence the existence of NE cannot be guaranteed for Case 2 in Γ_3.

On the other hand, for Case 1 in which any joint strategy that results in negative payoff is marked by a 'cross' as shown in Fig. 5.6(a), $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ are the three NEs. Therefore the example shows that the payoff function need to be carefully chosen to guarantee the existence of NE. If negative payoff values are allowed, they will cause instability to systems consisting of more than two players.

In this subsection, we shall generalize Γ_2 to N-player NRAG, Γ_N, and show that for Case 1, i.e. $u_n > 0$, $\forall a_n = 1$, Γ_N has at least one NE. The assumption that a player transmits only if he receives positive payoff is not unreasonable. This can be explained when deciding the best response for a given joint strategy of the opponents. If a player receives non-positive payoff when transmitting, he will inevitably diverse from transmission to no transmission, since a zero payoff is

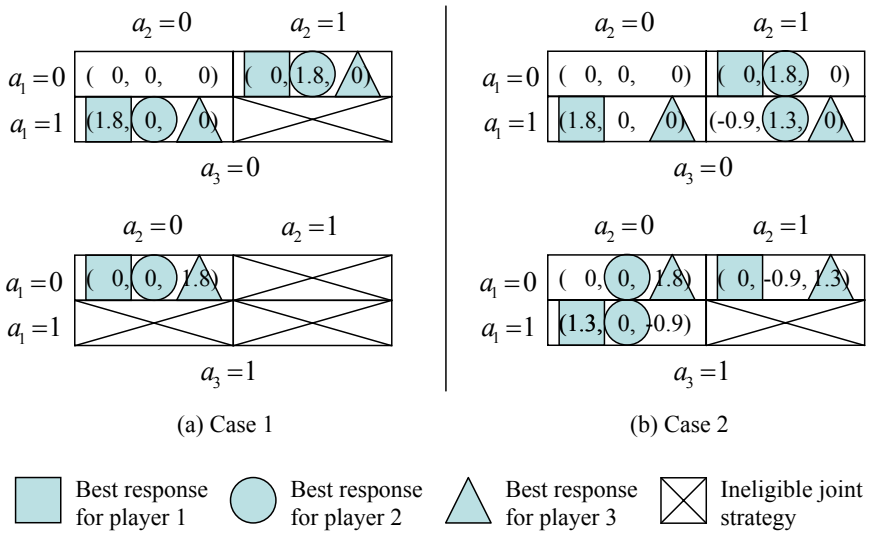

Figure 5.6: Payoff tables for a 3-Player NRAG example. (a) Case 1: three NEs exist; (b) Case 2: no NE exists.

still better than a negative one. We have seen from the example in Fig. 5.6(b) that, the existence of non-positive payoff value for a player who transmits actually causes instability in the game. However, by excluding such joint strategies from the strategy space of the game, it helps to enforce the existence of NE.

Before we proceed to prove the existence of NE in the N-player game for Case 1, Γ_N, we need to introduce the following results.

Theorem 5.2. *For any player $n \in \mathcal{N}$ in Γ_N, assuming both $u_n(a_i = 0, a_{-i}) > 0$ and $u_n(a_i = 1, a_{-i}) > 0$ for a given $i \in \mathcal{N}$ and $i \neq n$, we have $u_n(a_i = 0, a_{-i}) > u_n(a_i = 1, a_{-i})$.*

Proof. We can prove this theorem intuitively. When a player chooses to transmit, he will generate non-zero interference to all other players that are also transmitting at the same time. As a result, the aggregate interference level experienced by all other players that are transmitting will increase. In order to maintain their respective SINR thresholds, the players have to increase the transmission power. Since the payoff function for any player decreases with that player's transmission

power, the payoff values of players that are already transmitting will all decrease when a new player begins to transmit. □

Corollary 5.3. *For any player $n \in \mathcal{N}$ in Γ_N,*

(a) if $(a_n = 0, a_{-n})$ is ineligible, then $(a_n = 1, a_{-n})$ is also ineligible;

(b) for a given $i \in \mathcal{N}$ and $i \neq n$, if $\beta_{a_{-n}}(a_i = 0, a_{-i}) = 0$, then $\beta_{a_{-n}}(a_i = 1, a_{-i})$ is also 0.

Theorem 5.4. *There exists at least one NE in the N-player NRAG, Γ_N, $\forall N > 2$.*

Proof. We prove by induction. From Theorem 5.1, there exists at least one NE in Γ_2, or when $N = 2$. We assume that this holds for $N-1$, i.e., Γ_{N-1} has at least one NE, and we denote the set of NEs in Γ_{N-1} as $\mathcal{E}_{N-1}$, where $\mathcal{E}_{N-1}$ is non-empty. Next we consider the N-player game which can be divided into two subgames: $(N-1)$-player games corresponding to $a_N = 1$ and $a_N = 0$, i.e. player N chooses to or not to transmit, respectively. These two subgames are denoted as $\Gamma_{N-1}(a_N = 1)$ and $\Gamma_{N-1}(a_N = 0)$, respectively. It can be seen that the payoff table of $\Gamma_{N-1}(a_N = 0)$ is identical to that of Γ_{N-1}.

Denoting a NE in Γ_{N-1} as $a^*_{\Gamma_{N-1}} \in \mathcal{E}_{N-1}$, $(a^*_{\Gamma_{N-1}}, a_N = 1)$ and $(a^*_{\Gamma_{N-1}}, a_N = 0)$ are two possible joint strategies for Γ_N. According to $a^*_{\Gamma_{N-1}}$, we further denote the sets of players who are transmitting and not transmitting as $\mathcal{N}_1$ and $\mathcal{N}_0$, respectively, i.e. $a_i = 1$, $\forall i \in \mathcal{N}_1$, and $a_j = 0$, $\forall j \in \mathcal{N}_0$. One of the following situations applies.

(a) If $(a^*_{\Gamma_{N-1}}, a_N = 1)$ is eligible, we have $u_n > 0$ for all $n \in \mathcal{N}_1 \cup \{N\}$, and therefore $\beta_{a_n}(a^*_{\Gamma_{N-1}}, a_N = 1) = 1 = a_n$. Meanwhile, by applying Theorem 5.2, we also have $\beta_{a_j}(a^*_{\Gamma_{N-1}}, a_N = 1) = 0 = a_j$, $\forall j \in \mathcal{N}_0$. We can see that the strategy profile $(a^*_{\Gamma_{N-1}}, a_N = 1)$ consists of all the best responses of the N players. Hence it

is a NE of Γ_N.

(b) If $(a^*_{\Gamma_{N-1}}, a_N = 1)$ is an ineligible joint strategy, player N has to choose $a_N = 0$. With $a_N = 0$, u_n remains the same as those of Γ_{N-1} for all $n \in \{1, \ldots, N-1\}$. Since $a^*_{\Gamma_{N-1}} \in \mathcal{E}_{N-1}$ is a NE, $(a^*_{\Gamma_{N-1}}, a_N = 0)$ is also a NE in Γ_N.

In conclusion, for $\mathcal{E}_{N-1} \neq \varnothing$, there exists at least one $a^*_{\Gamma_{N-1}} \in \mathcal{E}_{N-1}$. If $(a^*_{\Gamma_{N-1}}, a_N = 1)$ is eligible, it is a NE of Γ_N. Otherwise, $(a^*_{\Gamma_{N-1}}, a_N = 0)$ is a NE. Hence Theorem 5.4 is proved by induction. □

5.4 Repeated Games and Convergence of Game-play

5.4.1 Repeated Games and Myopic Play

If complete information of the opponent players is assumed and only one NE exists, each player can individually decide the NE and select its transmission strategy. The problem gets more complicated if multiple NEs exist since these NEs cannot be achieved simultaneously, but only one could be selected at a time for a practical system. Although the existence of at least one NE has been proved for N-player NRAG, it provides no information on how the distributed players can achieve these stable solutions. In this section, we look into the convergence behaviour of a practical algorithm which aims at achieving one of the NEs. Moreover, the assumption that every player has the complete information of the opponent players requires a lot of information exchange overheads. We shall look into how such an assumption could be relaxed through performing measurement.

One way to achieve a NE in practice is to play the formulated NRAG repeatedly

among the players in a round-robin manner, such that in each stage the players update their actions sequentially after observing the actions of the opponent players. This introduces the concept of *repeated* game, as opposed to the single-stage *static* game discussed in Sections 5.2 and 5.3. In the context of repeated game, an *action* taken by player n refers to the decision of the player in a given round and is denoted as x_n. Although the actions of his opponents are not directly known to player n, they can be observed through measurement of the aggregate interference I_n defined in (5.1). The game is also assumed to be *myopic*, i.e. the players are short-sighted optimizers each trying to maximize the payoff for the next round only.

The classical nonlinear distributed Gauss-Seidel algorithm [89] can be modified and used in the myopic game:

1. In the initial stage, no player would transmit on the common channel. The order for the players to update their actions and the maximum number of iterations are predefined.

2. The first player in the list updates his action profile $(a_1,\ p_1)$ against the CCI obtained by measurement. In the first round, the CCI would be zero due to no transmission initially. This maximizes his payoff at the moment of update. Then the second player will update his action profile by observing the interference generated by the first player, and the third player will update his action profile based on the measurement of interference generated by the first two players, and so on. In general, at the $(t+1)$th round, the nth player in the list will update his

action profile according to:

$$x_n^{(t+1)} = \mathbf{D}\left(x_1^{(t+1)}, \ldots, x_{n-1}^{(t+1)}, x_n^{(t)}, x_{n+1}^{(t)}, \ldots, x_N^{(t)}\right), \tag{5.25}$$

where $\mathbf{D}$ returns the best response solution under the given opponents' joint action, which comprises the actions taken by player 1 to player $n-1$ at the $(t+1)$th round, and those taken by player $n+1$ to player N at the tth round. The action for player n can be determined by substituting the respective actions taken by the opponents into (5.5) to obtain p_n and by maximizing (5.4) to decide on a_n's. The process is repeated until player N makes his decision.

3. Repeat step 2 for the next round until convergence is observed or the maximum number of iterations has been reached.

Basically, myopic play of a game in some way is different from the static game. For example, in a particular round of the myopic game where the *instantaneous* actions taken by the players at round t can be denoted as $x^{(t)} = \{\mathbf{a}_x, \mathbf{p}_x\}$, the power vector, $\mathbf{p}_x$, is generally different from $\mathbf{p}$ given by (5.13) even if $\mathbf{a}_x = \mathbf{a}$, i.e. $\mathbf{p}_x \neq \mathbf{p}|_{\mathbf{a}_x}$. This is because if the game is played myopically and before the game becomes stable, the power vector will change in each round even though $\mathbf{a}_x$ may remain the same.

5.4.2 Condition of Convergence for Γ_N

Based on our simulations using the myopic play, we observe that the NRAG converges to a steady state for some of the channel realizations generated. However, there are also situations where the game cycles through a few stages and does not converge, resulting in unstable power and BER performance. Such an oscillation

could be due to lack of complete information seen by the players, which would lead to the players attempting to achieve a stage that is not feasible to satisfy all constraints. Therefore we shall see what we could do to the algorithm so that the myopic play of the game converges to a stable solution.

In the literature, myopic play of a potential game is well-known to be able to converge to its NE solution. Potential games are a group of games where the difference in the payoff due to unilaterally deviation in each player can be reflected by a global function. The details can be found in [34]–[37]. A potential game is defined as:

Definition 6. For a strategic game $\Gamma = \langle \mathcal{N}, \mathcal{X}, \{u_n\}_{n\in\mathcal{N}} \rangle$, a function $P : \mathcal{X} \mapsto \mathbb{R}$ is called

(i) an *exact* potential function if for all $n \in \mathcal{N}$ and (x_n, x_{-n}), $(x'_n, x_{-n}) \in \mathcal{X}$:

$$u_n(x'_n, x_{-n}) - u_n(x_n, x_{-n}) = P(x'_n, x_{-n}) - P(x_n, x_{-n}); \tag{5.26}$$

(ii) an *ordinal* potential function if for all $n \in \mathcal{N}$ and (x_n, x_{-n}), $(x'_n, x_{-n}) \in \mathcal{X}$:

$$u_n(x'_n, x_{-n}) > u_n(x_n, x_{-n}) \Leftrightarrow P(x'_n, x_{-n}) > P(x_n, x_{-n}). \tag{5.27}$$

A game is called an exact (or ordinal) potential game if there exists an exact (or ordinal) potential function for that game.

The formulated NRAG cannot be described as a potential game. However, using the framework developed in [35], convergence to NE for this game can be understood more easily by using the concept of potential game.

Theorem 5.5. *When a joint strategy $\hat{a}$ is fixed, the NRAG reduces to a non-cooperative power allocation game (NPAG) which is a potential game.*

Proof. The payoff function of player n can be expressed as

$$u_n(p_n) = \begin{cases} r_n - cp_n, & \text{if } \hat{a}_n = 1; \\ 0, & \text{if } \hat{a}_n = 0. \end{cases} \tag{5.28}$$

If we define a function $P = \sum_{n \in \mathcal{N}} u_n(p_n)$, it can be easily shown that

$$P(p'_n) - P(p_n) = u_n(p'_n) - u_n(p_n), \quad \forall n \in \mathcal{N}. \tag{5.29}$$

Therefore the NPAG is an exact potential game with a potential function P. □

If the given $\hat{a}$ is feasible with all the constraints, then set $\mathcal{P}_n|_{a_n}$ is compact and there is at least one NE in the NPAG. The distributed Gauss-Seidel algorithm with best response also ensures that the game will converge to a NE of the NPAG.

Let us re-examine the process of myopic game and see how we can use Theorem 5.5 to explain the convergence of NRAG. In a myopic game, each player takes turn to decide his action $x_n = \{a_n^{(t+1)}, p_n^{(t+1)}\}$ given by (5.25). In the initial stage, each player tends to transmit because the interference level is zero. As the interference level gradually builds up, a player updating his transmission power at each decision point may stop transmission if the payoff value is non-positive or some constraints are not met. Suppose after some number of plays, the joint strategy converges to a steady profile $\hat{a}$, i.e., $\hat{a}_n^{(T)} = \hat{a}_n^{(T-1)}$, $\forall n \in \mathcal{N}$, then for the subsequent play the game becomes a NPAG. Under this circumstance, from Theorem 5.5, the game is guaranteed to converge to a NE of the NPAG.

5.4.3 Heuristic Algorithm to Achieve Convergence

Although there are existing algorithms to ensure convergence in distributed systems based on continuous sets [21] [22], a practical algorithm for non-cooperative games with discrete strategy space is still lacking. Even though our analysis shows that the NRAG has at least one NE, the myopic play cannot guarantee the convergence of the game for all channel realizations. This happens because some of the players may repeatedly choose the same joint strategy after a number of game plays and results in "toggling" in the joint strategy, rather than converging to a stable joint strategy. This observation inspires us on a method to modify the myopic play of NRAG – we shall eliminate the "toggling" between transmitting and not transmitting for the players, which leads to instability in the solution.

One way to realize this is to introduce the number-of-attempts (NOA) variables which keep track of the numbers of transitions of a_n's, $\forall n \in \mathcal{N}$. A large NOA indicates that the value of a_n is frequently toggled. If the NOA of player n has reached the maximum number of allowable attempts, the common channel is deemed unsustainable to support transmission with a stable BER requirement for this player, and consequently player n will stop from further attempts of transmission on the channel. By eliminating such repeated "toggling" of a_n's in game-play, the NRAG will gradually arrive at a stable $\hat{a}$ and finally converge as a NPAG.

To explain in more details, we denote the NOA variable of player n as w_n, and the maximum value that w_n can take as W_n, $\forall n \in \mathcal{N}$. We propose a heuristic algorithm using NOA to ensure convergence for the game-play as follows:

Step 1: All w_n's are initialized to zero;

Step 2: In the mth iteration, where $m = 1, 2, \ldots,$ Player 1 chooses his best re-

sponse $\{a_1^m, p_1^m\}$ according to (5.25), player 2 chooses $\{a_2^m, p_2^m\}$ and so on, until all $\{a_n^m, p_n^m\}$'s have been chosen, $\forall n \in \mathcal{N}$;

Step 3: In each iteration, if a_n^m is updated to 0 whereas its previous value, $a_n^{m-1} = 1$, player n increments w_n by 1. If $w_n \geq W_n$, player n will no longer attempt to transmit, and a_n^m will remain 0 afterwards;

Step 4: Repeat Steps 2 and 3 until a stable $\hat{a}$ is achieved and eventually the game converges.

Hereafter we refer to the modified NRAG with NOA as the convergent-NRAG (CNRAG). Take note that the CNRAG only make use of local information on CCI measurement, and converges to a NE of the NPAG with fixed $\hat{a}$. However, the NE of the NPAG is not necessarily the same as that of the NRAG, as the $\hat{a}$ given may not satisfy $\hat{a} \in \mathcal{A}$.

5.5 Discussions

5.5.1 Simulation Results

To evaluate the efficiency of N-player NRAG in resource allocation, we run computer simulations to compare the payoff values given by various channel conditions. The optimal SW defined in (5.9) will be used to benchmark the efficiency of the NRAG. Take note that the optimal SW is usually not the same as the maximum NE values, as the optimal SW is obtained through the centralized approach and needs not to be a NE solution.

A number of players are randomly generated in a circular area with a diameter of 50 metres. The actual distance is randomly generated between 1 and d_{max}, where

d_{max} is the maximum distance between the transmitter and receiver of a player. The propagation model is the same as that in Section 5.2.3 which consists of path loss and multipath fading. The results are averaged over a total of 10^4 simulation instances. For each instance, we search for all the possible NEs and evaluate the total payoff of all players for each NE solution. The NE solutions with maximum and minimum total payoff value are denoted as NE_{max} and NE_{min}, respectively. We plot the average total payoff against various numbers of players and d_{max} in Fig. 5.7.

The optimal values of SW are not shown in Fig. 5.7 since it is too close to NE_{max}. It is found that the difference between the total payoff of NE_{max} and the optimal SW are less than 0.5%. From Fig. 5.7, it can be seen that the total payoff values of NE_{max} increase with N, which shows that the frequency channel can be more efficiently utilized by more players in the NRAG. However, when d_{max} is large ($d_{max} > 20$), the total payoff values of NE_{min} decrease with N. This is because when both d_{max} and N increase, the players have higher probability of experiencing severe interference, which could lead to deteriorated performance if there is no coordination among them.

Next we plot the price of anarchy (PoA) versus number of players for various d_{max} in Fig. 5.8. The PoA is defined as the ratio of optimal SW to NE_{min}, which is a measure of how good the result is when the game is played selfishly versus that of the social optimal when a central authority is present. It can be seen that the performance loss is very little ($< 2\%$) with $d_{max} = 5$. As d_{max} increases, however, the performance loss gradually increases and can become as large as nearly 65% when N is also large. This shows that as the interference between players increases,

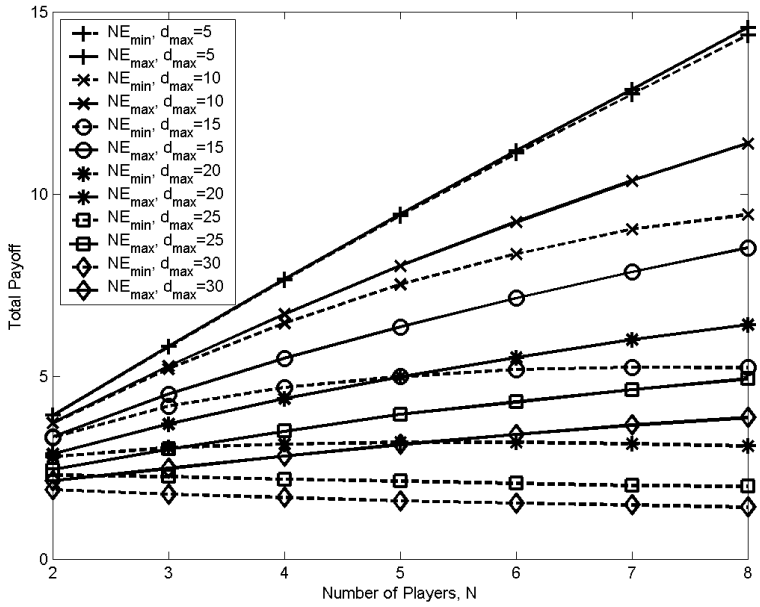

Figure 5.7: Performance of the best and worst NE solutions, versus different number of players and $d_{\max}$.

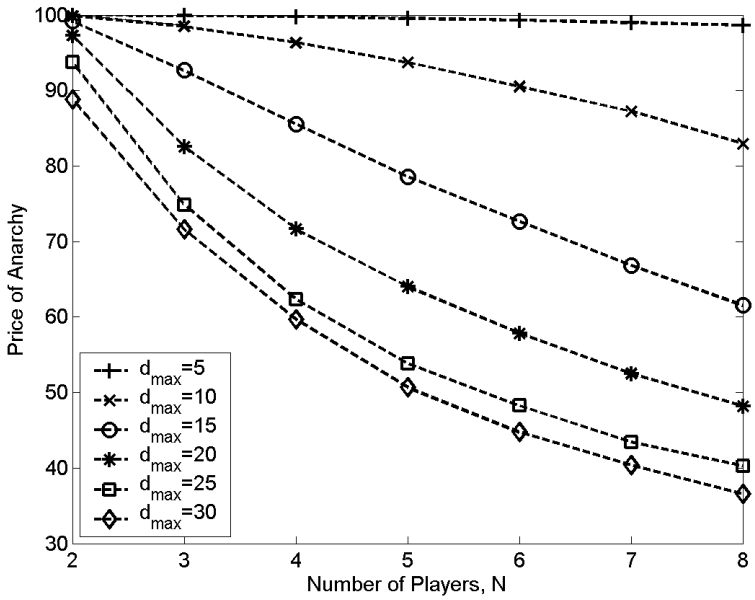

Figure 5.8: Price of anarchy for the NRAGs, versus different number of players and $d_{\max}$.

the interaction among the selfish players could lead to highly undesirable results from a system perspective.

To study the convergence of myopic play of the NRAG, we find out the numbers of stages needed for the game to converge in a total of 10^3 channel realizations.

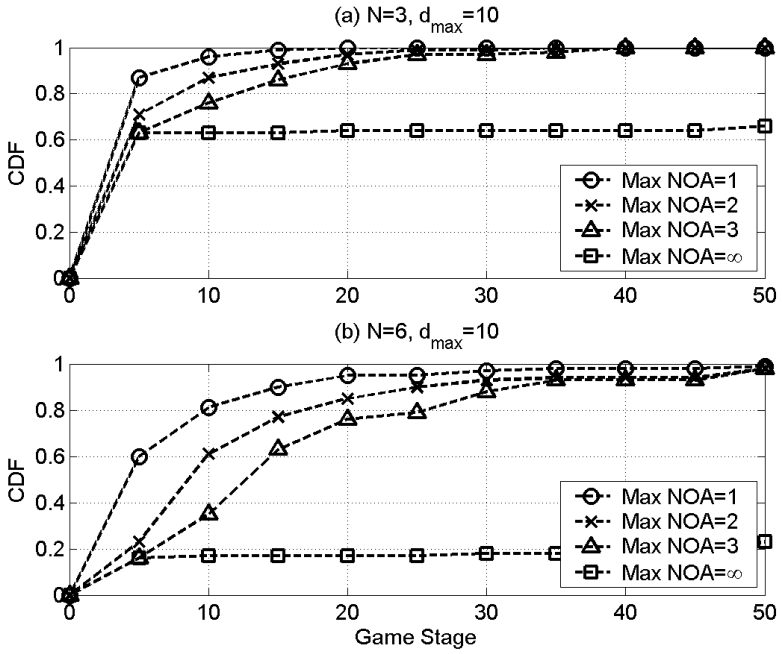

Figure 5.9: CDFs of CNRAG with different values of maximum NOA: (a) $N = 3$ (b) $N = 6$.

Then the cumulative distribution function (CDF) on the convergence of CNRAG is plotted in Fig. 5.9(a) and 5.9(b) for $N = 3$ and $N = 6$, respectively. The most important factor affecting the speed of convergence is the maximum NOA allowed. The smaller the maximum NOA, the faster the convergence of the NRAG. This conclusion is intuitive because with a larger NOA, more game stages are allowed before a player decides to stop attempting for transmission again. However, no matter how slowly it does, the game will eventually converge if the value of NOA is finite. And it can also be seen that if NOA is not used, or $W_n = \infty$, $\forall n \in \mathcal{N}$, the convergence of NRAG is not guaranteed.

In Fig. 5.10, performance of the CNRAG with different values of maximum NOA is compared to the optimal solution. It is obvious that the repeated play of NRAG without NOA is highly unstable. Although its total payoff sometimes seems to be higher than the optimal solution, it is compromised at the cost of causing instability to the system, which results in some of the constraints not

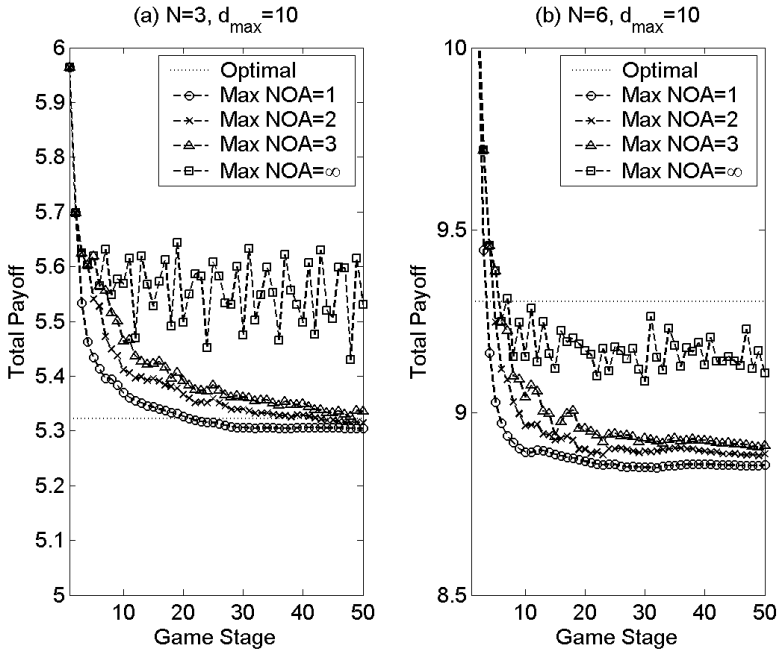

Figure 5.10: Network payoff comparison for CNRAG with different values of maximum NOA: (a) $N = 3$ (b) $N = 6$.

being satisfied. As a contrast, the CNRAG is much more stable than the NRAG, implying that an equilibrium could be achieved with all the constraints satisfied. It can be seen that the CNRAG with maximum NOA equal to 1 incurs slightly more performance loss than those with higher value of maximum NOA, as a larger number of maximum NOA allows the system to explore more possible stages with potentially higher payoff, at the expense of slower convergence.

5.5.2 Multi-channel Allocation Game

We shall see how the results for Γ_N can be applied when the problem defined in Section 5.3 where only one channel is available for transmission is extended to multi-channel systems. We refer to the case where distributed nodes each attempts to get one channel for transmission but there are K available channels. Define the variables $a_n^k = 1$ if communication pair n ($n \in \mathcal{N}$) uses channel k ($k \in \mathcal{K}$) to transmit, otherwise $a_n^k = 0$. In this case, the channel selection of player n can

be represented by a vector $\mathbf{a}_n = \{a_n^1, \ldots, a_n^k, \ldots, a_n^K\}$, which also represents the strategy of player n. The multi-channel allocation problem can be formulated as

$$\max_{\mathbf{a}_n} u_n, \quad \forall n \in \mathcal{N}, \tag{5.30}$$

subject to

$$\frac{p_n \cdot G_{n,n}^k}{I_n^k + N_0} = a_n^k \gamma_n, \quad \forall n \in \mathcal{N}, \tag{5.31}$$

$$p_n \leq P_{\max,n}, \quad \forall n \in \mathcal{N}, \tag{5.32}$$

$$\sum_{k=1}^{K} a_n^k \leq 1, \quad \forall n \in \mathcal{N}, \tag{5.33}$$

$$a_n^k \in \{0, 1\}, \quad \forall n \in \mathcal{N}, \tag{5.34}$$

where $I_n^k = \sum_{j=1, j \neq n}^{N} a_j^k p_j \cdot G_{n,j}^k$.

Unlike the single-channel NRAG considered in the previous section, the existence of NE solution cannot be guaranteed when the game is played on multiple channels. To illustrate this, we present an example of the multi-channel game in which a NE solution does not exist. Let us consider the payoff tables shown in Fig. 5.11, which are obtained under the following parameters: $N = 3$, $K = 2$, $c = 2$, $P_e = 10^{-5}$, $N_0 = -70$dBmW, $P_{\max,n} = \infty$, and the channel gains are given by

$$\mathbf{G}^1 = \mathbf{G}^2 = \begin{bmatrix} 5 & 0.2 & 0.3 \\ 0.3 & 5 & 0.2 \\ 0.2 & 0.3 & 5 \end{bmatrix} \times 10^{-5}, \tag{5.35}$$

where the i, j-th element of $\mathbf{G}^k$ gives the value of $G_{i,j}^k$ for $k = 1, 2$ and $i, j = 1, 2, 3$.

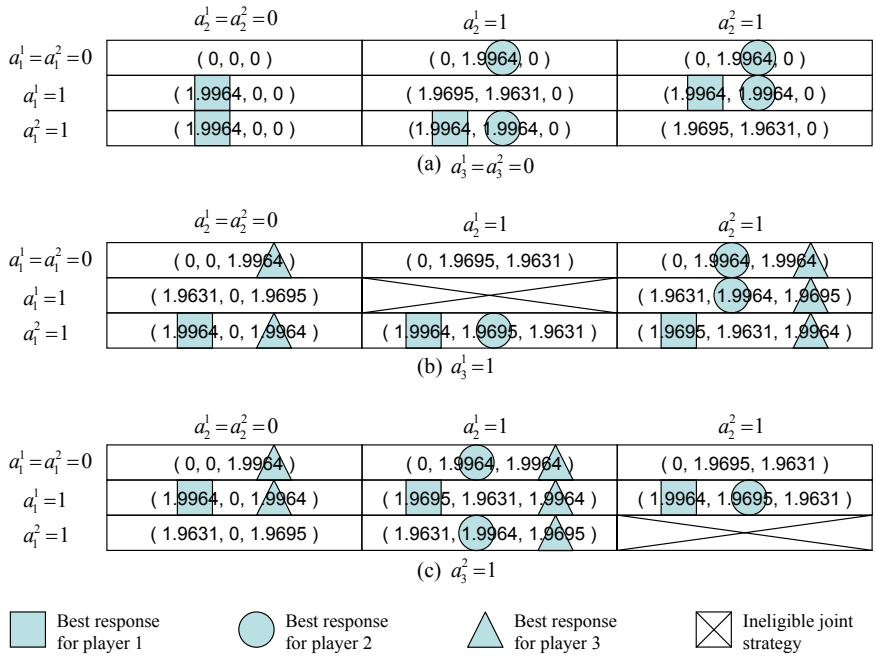

Figure 5.11: An example to show that a pure NE does not exist in the multi-channel game. In this case, $N = 3$ and $K = 2$, (a) $a_3^1 = a_3^2 = 0$; (b) $a_3^1 = 1$; (c) $a_3^2 = 1$. This example clearly shows that there exists no strategy profile which can be the best responses of all the three players.

The payoff tables corresponding to $a_3^1 = a_3^2 = 0$, $a_3^1 = 1$ and $a_3^2 = 1$ are shown in Fig. 5.11 (a)–(c), respectively. By marking the best responses of the three players with the corresponding symbols respectively ('square' for player 1, 'circle' for player 2 and 'triangle' for player 3), for all the given joint strategies of their opponents, this example clearly shows that there is no strategy profile which can simultaneously become the best responses of all three players. Hence no NE exists for this set of channel realizations.

The possibility of no NE in the multi-channel NRAG does exist due to the randomness of wireless channels. However, this problem can be overcome if we first partition the N players into K groups and make the kth group of users compete on channel k only.

Theorem 5.6. *There exists at least a pure strategy NE in the multi-channel NRAG*

provided channel assignment is fixed to each group of users.

Proof. The proof is intuitive. We can partition the set of players in $\mathcal{N}$ into K subsets, denoted as $\mathcal{N}_1$, $\mathcal{N}_2$, ..., $\mathcal{N}_K$, respectively. Since there is no interference across the channels, the group of players in each channel will play the single-channel NRAG independently. From Theorem 5.4, each single-channel NRAG has at least a NE. The joint strategies given by n-ary Cartesian product of the NEs for all single-channel NRAGs will give the NEs of the multi-channel NRAG. □

Some algorithms have to be adopted to strategically partition the players into groups with appropriate channels in order to maximize system performance. A possible solution is those transmission pairs which are well separated should be grouped together since they create the least amount of interference and hence result in overall lower power consumption or higher payoff for each other.

5.6 Conclusion

In this chapter, we studied opportunistic transmission of distributed nodes over a common channel using a non-cooperative game theoretic approach. Unlike the commonly adopted information theoretic approach, with the objective to maximize SINR and generally results in real numbers of bits to be transmitted, in our work, a transmission pair transmits only if its SINR and transmission power constraints can be satisfied. The payoff functions of the players in the game are taken as the reward due to successful data transmission minus the cost of power consumed. With the SINR constraints imposed, the strategies chosen by the players coupled with each other and a joint strategy needs to fulfil all the constraints to justify as

a strategy profile of the game.

We first showed that there always exists at least a NE solution in the 2-player single-channel NRAG under all possible channel realizations. We proposed a numerical method to estimate the probabilities of having a unique NE and two NEs, respectively, and the computed results are presented. Subsequently we extended our analysis to more general case with N players, where a reasonable assumption on the payoff function is made to ensure the existence of NE. Then we show by mathematical induction that there also exists at least a NE solution in the N-player NRAG if we ensure that a strategy profile should only have positive payoff when a player transmits.

We also study the condition to ensure convergence of the game if it is played myopically among the players. Based on this condition, we proposed a heuristic algorithm by modifying the NRAG so that myopic play of the game will always converge to a stable outcome under all channel conditions. Then the price of anarchy for the game is estimated using computer simulations with various settings, and the results on convergence and performance of the CNRAG are presented.

Finally, we took a step further and applied the results derived for single-channel NRAGs to multi-channel NRAGs. An example is used to show that NE solutions may not exist for the multi-channel game under certain channel realizations. However, the problem can be modified so that sub-optimal solutions which can ensure the existence of NEs are possible, although having more efficient and fairer algorithms to improve the optimality of sub-optimal solutions remains to be a challenging problem.

Chapter 6

Adaptive Modulation Games

OFDMA has become a promising candidate for future wireless systems due to its ability to mitigate multipath fading and its efficient implementation using IFFT and FFT blocks [23]. The dynamic radio environment, although hostile, provides different channel conditions on the subcarrier seen by users at various locations. Such variations can be exploited to achieve multiuser diversity gain which can improve the overall system performance, while how to allocate radio resources efficiently remains to be a challenging design problem.

On the other hand, adaptive modulation technique which adjusts modulation levels based on the received signal quality and channel conditions, is proven to be able to improve system capacity and coverage reliability [85]. This is also known as bit loading in the context of OFDM, and from a practical point of view, it can improve energy efficiency of OFDMA systems.

In Chapter 5 we studied on the sharing of a single channel for transmission with fixed modulation, and briefly extended our discussions to the existence of NE with multiple subcarriers in 5.5.2. In this chapter, we present our study on SBPA games

in the downlink of multi-cell OFDMA systems using adaptive modulation, where the BSs are treated as the players, for the reasons to reduce both the computation complexity and amount of channel information exchanged. We first formulate the NRAG for N-BS K-subcarrier OFDMA systems, with each BS having L users. The game is denoted as NRAG-$N\{L\}/K$ and an integer number of bits is allocated to each subcarrier according to the channel conditions.

We prove that NRAG-2{1}/1 has at least one NE under all channel conditions. However, the existence of NE solution cannot be guaranteed in general for games with $N > 2$, $L_n \geq 2$ or $K \geq 2$. Some exception to this is when there is no power constraint for the BSs. Repeated play of the game is a way to obtain a stable solution if not all channel information are available to the players. Based on our observations during simulations, we discuss how we developed an algorithm which ensures convergence to a stable solution for the modified NRAG, denoted as convergent NRAG (CNRAG).

We also modified the payoff functions so that the players receive penalties when they take up too many subcarriers. Such a formulated model inherently introduces interference avoidance mechanism to the players and is abbreviated as CNRAG-IA. We show that a better overall system payoff can be achieved in CNRAG-IA as compared to CNRAG. Although game theoretic approach is used to study the competitions and interactions among rational players, how good is the solution found remains unclear as the optimal solution is not easily available for comparison. With the methods introduced in Chapters 2 and 3, however, the optimal solution becomes much easier to obtain and can be used to benchmark the performance of those algorithms using game theoretic approach. Therefore finally, we compare the

performances of CNRAG and CNRAG-IA with the network payoff obtained from centralized optimization.

6.1 Static Game Formulation

For the multi-cell OFDMA system described in Section 3.1, although it is possible to obtain the centralized optimal solution, computation complexity remains high. Furthermore, the use of a centralized controller may not be always possible, or it is a large overhead to transmit all channel information. Under such circumstances, one may want to have resource allocation processed at every individual BS in a distributed manner. Non-cooperative game theory can therefore be used, and we shall consider only the pure strategies since the selection of a strategy is deterministic rather than probabilistic.

The SBPA problem in a multi-cell OFDMA system can be formulated as a NRAG given in the strategic form as

$$\Gamma = \langle \mathcal{N}, \mathcal{S}, \{u_n\}_{n\in\mathcal{N}} \rangle ,$$

where $\mathcal{N}$ and $\mathcal{S}$ denote the set of BSs and strategy space of the game, respectively. The payoff function of player n,

$$u_n = \sum_{l=1}^{L_n} r_{ln} - c\sum_{k=1}^{K} p_n^k = \sum_{k=1}^{K}\sum_{l=1}^{L_n}\sum_{q=1}^{Q}(2qa_{ln}^{kq} - cp_{ln}^{kq}), \quad \forall n \in \mathcal{N}, \tag{6.1}$$

is the same as that in (3.3).

We denote the game as NRAG-$N\{L\}/K$, where N, L and K are the number of

BSs, number of users in each BS, and number of subcarriers, respectively. It can also be expressed as

$$\text{NRAG:} \quad \max_{\mathbf{A}_n, \mathbf{P}_n} u_n, \quad \forall n \in \mathcal{N}, \tag{6.2}$$

subject to

$$\frac{G_{ln}^{kn} p_{ln}^{kq}}{I_{ln}^{k} + N_0} = a_{ln}^{kq} \gamma_{ln}^{q}, \qquad \forall n \in \mathcal{N}, \forall l \in \mathcal{L}_n, \forall k \in \mathcal{K} \text{ and } \forall q \in \mathcal{Q}, \tag{6.3}$$

$$\sum_{k=1}^{K} \sum_{l=1}^{L_n} \sum_{q=1}^{Q} p_{ln}^{kq} \leq P_{\max}, \qquad \forall n \in \mathcal{N}, \tag{6.4}$$

$$\sum_{l=1}^{L_n} \sum_{q=1}^{Q} a_{ln}^{kq} \leq 1, \qquad \forall n \in \mathcal{N} \text{ and } \forall k \in \mathcal{K}, \tag{6.5}$$

$$a_{ln}^{kq} \in \{0, 1\} \text{ and } p_{ln}^{kq} \in \mathbb{R}^{+}, \quad \forall n \in \mathcal{N}, \forall l \in \mathcal{L}_n, \forall k \in \mathcal{K} \text{ and } \forall q \in \mathcal{Q}. \tag{6.6}$$

A strategy for a player in NRAG comprises two components: SBA and PA matrices, i.e., $s_n = \{\mathbf{A}_n, \mathbf{P}_n\}, \forall n \in \mathcal{N}$. The elements in the SBA matrix are SBA variables which take on discrete values whilst those in the PA matrix are PA variables which take on continuous values. In NRAG, player n chooses the best response $\beta_n \in \mathcal{S}_n$ to maximize his payoff u_n. However, since the payoff function contains the joint strategy of his opponents, s_{-n}, player n is unable to maximize his own payoff without taking the strategies of his opponents into consideration. The best response for each player again involves solving a MINLP of much smaller scale, where the joint strategy of the opponents is known. The method described in Chapter 2 can thus be applied to obtain the solution.

Next we show that the NRAG is equivalent to a game with discrete strategy space, where only the SBA variables are considered. We define the strategy space, $\mathcal{A}$, as the set of strategy profiles which contains only the SBA components of all

strategy profiles in $\mathcal{S}$. The following Theorem about $\mathcal{A}$ holds.

Theorem 6.1. *In NRAG,* $\langle \mathcal{N}, \mathcal{S}, \{u_n\}_{n\in\mathcal{N}} \rangle$ *and* $\langle \mathcal{N}, \mathcal{A}, \{u_n\}_{n\in\mathcal{N}} \rangle$ *have the same NE solutions.*

Proof. The proof to this is intuitive. Conditioned on any given $\widetilde{\mathbf{A}} \in \mathcal{A}$, by substituting all the values of $\tilde{a}_{ln}^{kq} \in \widetilde{\mathbf{A}}$ into (6.3), the SINR constraints reduce to a set of linear equations. Since the number of equations is equal to that of PA variables, the PA matrix can be easily computed and is unique as the given $\widetilde{\mathbf{A}}$ is feasible. On the other hand, since $\widetilde{\mathcal{A}}$ is the binary component of the strategy space of Γ ($i.e., \mathcal{S}$), by definition, the SINR constraints are already satisfied. Therefore specifying the strategy space $\mathcal{A}$ contains the same information as specifying $\mathcal{S}$, since $\mathbf{P}|_{\widetilde{\mathbf{A}}} = \{\mathbf{P}_1, \ldots, \mathbf{P}_N\}$ can be readily obtained once $\widetilde{\mathbf{A}} = \{\widetilde{\mathbf{A}}_1, \ldots, \widetilde{\tilde{A}}_N\}$ is given. Hence, hereafter, we only need to consider the strategy space $\mathcal{A}$ rather than $\mathcal{S}$. □

6.2 Search for NEs

By definition, the solutions of NRAG, if it exists, should be given by one of the NE points. Searching for NE points should be made by directly applying (4.1) to each of the strategy profile in $\mathcal{S}$. However, as the strategy space of NRAG are not directly obtainable through n-ary Cartesian product of the individual strategy spaces of all players, in this section, we look into systematic search for the NE of the NRAG.

In the formulated NRAG, the conditions for any strategy profile in $\mathcal{S}$ to become a NE depend on whether that is an equilibrium point to all the players, so that none of the player has the intention to change his strategy. Fortunately, by making

use of Theorem 6.1, the search space for strategy profiles can be obtained quite easily and the problem might not be that intractable. A way to identify the search space and to find a NE is as follows:

1. For each player $n \in \mathcal{N}$, we can find all SBA strategies for player n which satisfy (6.5). Denote the set of SBA strategies as $\widetilde{\mathcal{A}}_n$.

2. We next list down all the possible joint SBA strategies $\widetilde{\mathcal{A}} = \times_{n\in\mathcal{N}}\widetilde{\mathcal{A}}_n$.

3. For each joint SBA strategy in $\widetilde{\mathcal{A}}$, the SINR constraints (6.3) reduce to a set of linear equations. Together with the power constraints (6.4), the required transmission power can be solved using Eq. (3.12). All transmission power computed must be non-negative and have to satisfy the maximum transmission power limit of the BS, and for those $\mathbf{A}$ which do not meet these requirements are infeasible solutions to be removed. Repeat the process and all the strategy profiles can be identified and denoted as $\mathcal{A}$. After including the corresponding PA matrix, the set of strategy profiles is denoted as $\mathcal{S}$.

4. If $\mathcal{S} \neq \emptyset$, go through all the strategy profiles in $\mathcal{S}$ to search for the NE based on its definition in (4.1). Some algorithms based on enumeration method were proposed in [90] and [91]. Other algorithms, such as those based on continuation or heuristic methods ([92]–[94]), are also available for finding NEs.

As an example, we consider the NRAG with a 2-cell system with fixed modulation (4-QAM) and two subcarriers. Each BS has four available strategies: do not transmit (N), transmit on subcarrier 1 (T_1), transmit on subcarrier 2 (T_2) and transmit on both subcarriers (T_b), i.e., $\mathcal{A}_n = \{N, T_1, T_2, T_b\}$ for $n = 1, 2$. In this example, we have also introduced a minimum data rate of 2 bits per user to illustrate how the search space for NE could be affected by different constraints.

	N	T_1	T_2	T_b
N	(0, 0)	(0, 1.6250)	(0, 1.5714)	(0, 3.1964)
T_1	(1.6667, 0)	(1.2879, 1.0909)	(1.6667, 1.5714)	(1.2879, 2.6623)
T_2	(1.7647, 0)	(1.7647,1.6250)	(1.4043, 1.1246)	(1.4043, 2.7496)
T_b	(3.4314, 0)	(3.0526, 1.0909)	(3.0709, 1.1246)	**(2.6921, 2.2155)**

(a)

NEs

	N	T_1	T_2	T_b
N	(0, 0)	(0, 1.5652)	(0, 1.5000)	(0, 3.0652)
T_1	(1.4872,0)	(0.2116, 0.0101)	**(1.4872, 1.5000)**	(0.2116, 1.5101)
T_2	(1.6364, 0)	**(1.6364, 1.5652)**	(0.4615, 0.1538)	(0.4615, 1.7191)
T_b	(3.1235, 0)	(1.8480, 0.0101)	(1.9487, 0.1538)	(0.6731, 0.1639)

(b)

Figure 6.1: Examples of NE existence in a 2-cell 2-subcarrier OFDMA system using 4-QAM, where each cell has a single user with $R_{\min} = 2$ and $P_{\max} = 0.1$. (a) A unique NE exists; (b) Two NEs exist.

At first, without considering the data rate and power constraints, strategy space of the game is simply given by the Cartesian product of the strategy spaces of the two players. Therefore there are sixteen strategy profiles in $\mathcal{A} = \mathcal{A}_1 \times \mathcal{A}_2$. The payoff tables for two sets of channel conditions are shown in Fig. 6.1(a) and 6.1(b).

When the rate and power constraints are imposed on the players in this example, some of the strategy profiles become infeasible if any of the constraints is not satisfied. Such strategy profiles that could not fulfil the rate or power requirements are represented by the grey and red cells, respectively, and are not available for the players to choose. In case (a), by removing those strategies that do not meet the rate requirement, the number of feasible strategies has been reduced from 16 to 9. Among the remaining strategies, it can been seen that strategy T_b is always preferable over T_1 and T_2 due to higher payoff for both players. Therefore there exists a unique NE in this NRAG, which is (T_b, T_b). As a contrast, the number of feasible strategies further reduces to only 4 in case (b), whereas the NRAG has two NEs instead of one.

6.3 NE in 2-player Non-cooperative Modulation Game

Existence of NEs is important when game theoretic approach is used, as it decides whether stable solutions exist. However, the study on NE existence becomes very complicated when the complexity of NRAG is high. Therefore, we first study the simplest form of our proposed game, NRAG-2{1}/1, which corresponds to a single-carrier two-cell system with one user per cell. The system under consideration here is different from what we have presented in Chapter 3, in the sense that the decisions are now being distributively made by the individual BSs (as players of the game), rather than the decision being made in a centralized manner. Each player can choose his strategy by setting one of the SBA variables in $\{a^0, a^1, \ldots, a^Q\}$ to be 1, or equivalently by selecting his modulation level from $\widehat{\mathcal{Q}} = \{0, 1, \ldots, Q\}$, where $q = 0$ means no transmission is made. In this section, we first show how the best response of a player depends on the opponent's strategy, and then prove that NRAG-2{1}/1 has at least one NE.

6.3.1 Behavior in the Best Response

Theorem 6.2. *In NRAG-2{1}/1, $\forall q_2', q_2 \in \widehat{\mathcal{Q}}$, given that $q_2' > q_2$, we have*

$$\beta_1(q_2') \leq \beta_1(q_2), \quad \beta_1(q_2'), \beta_1(q_2) \in \widehat{\mathcal{Q}}.$$

In other words, player 1's best response is a non-increasing function over the modulation level adopted by player 2.

Proof. We first remove the power constraint. Without loss of generality, we take

player 1 as the reference and observe the following:

Observation 6.3. *For $q_2' > q_2$, we define Δ as below and it can be shown that*

$$\Delta_{q_1}(q_2', q_2) := p_1(q_1, q_2') - p_1(q_1, q_2) > 0. \tag{6.7}$$

$\Delta_{q_1}(q_2', q_2)$ represents the power difference for player 1 to transmit at modulation level q_1 when player 2 changes his modulation level from q_2 to q_2'. Observation 6.3 reveals that if player 1 decides to transmit using modulation level q_1, higher transmission power is required for player 1 when his opponent transmits at higher modulation level than when his opponent transmits at lower modulation level. This is intuitive since if the opponent transmits at higher power, player 1 also needs to transmit at higher power to maintain the transmission quality due to higher interference level. To prove this is quite straightforward. Given q_2 is adopted by player 2, the transmission power of player 1 can be obtained as

$$p_1 = \frac{(G_2^2 + G_1^2\gamma_{q_2})N_0\gamma_{q_1}}{G_1^1G_2^2 - G_2^1G_1^2\gamma_{q_1}\gamma_{q_2}}, \tag{6.8}$$

where γ_{q_n} is the SINR threshold for modulation level q_n $(n = 1, 2)$, and $\gamma_{q_2'} > \gamma_{q_2}$ if $q_2' > q_2$. By substituting (6.8) into $\Delta_{q_1}(q_2', q_2)$ defined in (6.7) and after simplification, it is not difficult to show that $\Delta_{q_1}(q_2', q_2)$ is always greater than zero.

Observation 6.4. *For any $q_1' > q_1$ and $q_2' > q_2$, we have*

$$\Delta_{q_1'}(q_2', q_2) > \Delta_{q_1}(q_2', q_2). \tag{6.9}$$

The result reveals that the power difference for player 1 is increasing on q_1. By relaxing q_1, q_2 and q_2' to real numbers and taking the partial derivative of $\Delta_{q_1}(q_2', q_2)$ in the direction q_1, it is not difficult to show that

$$\frac{\partial}{\partial q_1}\Delta_{q_1}(q_2', q_2) > 0. \tag{6.10}$$

Eq. (6.10) shows a gradual increase in $\Delta_{q_1}(q_2', q_2)$ if q_1 increases, hence, (6.9) is proved.

We now proceed to prove Theorem 6.2. For any $q_1 > \beta_1(q_2)$, $q_1 \in \widehat{\mathcal{Q}}$, from the definition of best response, we have

$$u_1(\beta_1(q_2), q_2) > u_1(q_1, q_2). \tag{6.11}$$

It is not difficult to show using (6.7) that (6.11) is equivalent to

$$u_1(\beta_1(q_2), q_2') + c\Delta_{\beta_1(q_2)}(q_2', q_2) > u_1(q_1, q_2') + c\Delta_{q_1}(q_2', q_2). \tag{6.12}$$

From Observation 6.4, $\Delta_{\beta_1(q_2)}(q_2', q_2) < \Delta_{q_1}(q_2', q_2)$, this leads to

$$u_1(\beta_1(q_2), q_2') > u_1(q_1, q_2'). \tag{6.13}$$

Note that (6.13) is different from (6.11) as the opponent's strategy is q_2' and not q_2. Eq. (6.13) can be interpreted as follows. Since we have assumed that $q_1 > \beta_1(q_2)$, and player 1 gets a lower payoff when applying any strategy greater than $\beta_1(q_2)$, by definition, the best response for player 1, $\beta_1(q_2')$, cannot be anything which is

greater than $\beta_1(q_2)$. Hence, we have

$$\beta_1(q_2') \leq \beta_1(q_2),$$

and Theorem 6.2 is proved. □

Theorem 6.2 can be visualized by using a graphical method illustrated in Fig. 6.2. The strategy profiles of NRAG-2{1}/1 are represented by a $(Q+1) \times (Q+1)$ table as shown. The effect of power constraints is now becoming clear, that it only makes some of these joint strategies to become ineligible to NRAG and should not affect the validity of Theorem 6.2. The best response of player 1 to a given modulation level of player 2 is indicated by a square in the table, and that of player 2 given player 1's strategy is indicated by a circle.

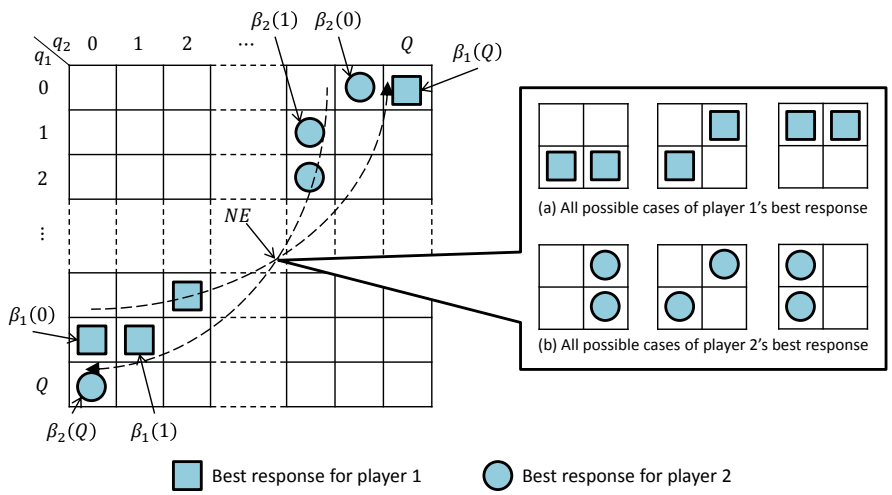

Figure 6.2: Graphical illustration of the existence of NE in NRAG-2{1}/1.

The implication of Theorem 6.2 is as follows. If we start from left to right where the modulation level used by player 2 increases, the position of the square marker gradually takes smaller q_1 (or at least remains the same). Similarly, from top to bottom of the table where the modulation level used by player 1 increases,

the position for the circle marker gradually takes smaller q_2. If a grid contains both a square and a circle, then the strategy profile represented by that grid is a NE. All possibilities which may happen at the intersection of the two trajectories are shown in the subfigure of Fig. 6.2 and it is therefore postulated that there exists at least one NE given by the intersection point.

6.3.2 Existence of NE

We use both Theorem 6.2 and Tarski's Fixed Point Theorem to show that there exists a non-empty set of fixed points in NRAG-2{1}/1, and then show each fixed point corresponds to a NE.

Tarski's Fixed Point Theorem. *[87]: Let L be a complete lattice and let $f : L \mapsto L$ be an order-preserving function. Then the set of fixed points of f in L is also a complete lattice.*

A complete lattice is a non-empty partially ordered set in which all subsets have both a supremum and an infimum. A *fixed point* is a point that is mapped to itself by the function, i.e., x is a fixed point of function f if and only if $f(x) = x$. Tarski's Fixed Point Theorem guarantees the existence of at least one fixed point of f. Readers can refer to [88] for more details.

Theorem 6.5. *There exists at least one NE in NRAG-2{1}/1.*

Proof. From Theorem 6.2, for any $q_1 \leq q_1'$, $q_1, q_1' \in \widehat{\mathcal{Q}}$,

$$\beta_2(q_1) \geq \beta_2(q_1'). \tag{6.14}$$

Therefore $\beta_2(x)$ is an order-reversing function. Applying Theorem 6.2 once more,

we then have

$$\beta_1(\beta_2(q_1)) \leq \beta_1(\beta_2(q_1')). \tag{6.15}$$

Defining the function $f = \beta_1(\beta_2(x))$, it can be seen that f is order-preserving. Since $\widehat{\mathcal{Q}} = \{0, 1, \ldots, Q\}$ where $0 < 1 < \cdots < Q$ is clearly totally ordered, hence $\widehat{\mathcal{Q}}$ is also a partially ordered set. Therefore $L = \left\langle \widehat{\mathcal{Q}}, \leq \right\rangle$ is a complete lattice. From Tarski's Fixed Point Theorem, the set of fixed points of function f in L is also a complete lattice and non-empty. This shows that there exists at least one fixed point of f in L.

We now prove that a fixed point of f is equivalent to a NE in NRAG-2{1}/1, by showing that a fixed point is a necessary and sufficient condition for a NE. Denoting a NE of NRAG-2{1}/1 as (q_1^*, q_2^*), $q_1^*, q_2^* \in \widehat{\mathcal{Q}}$, from the definition of NE,

$$\beta_1(q_2^*) = q_1^* \tag{6.16}$$

$$\text{and} \quad \beta_2(q_1^*) = q_2^*. \tag{6.17}$$

Substituting (6.17) into (6.16), we have

$$\beta_1(\beta_2(q_1^*)) = f(q_1^*) = q_1^*. \tag{6.18}$$

We can see that q_1^* is a fixed point of $f = \beta_1(\beta_2(x))$, and hence a fixed point of f is a necessary condition of a NE.

Next, denoting a fixed point of f as $\widetilde{q}_1$, if $\widetilde{q}_1$ is given as the strategy of player 1, the best response of player 2 would be

$$\beta_2(\widetilde{q}_1) = \widetilde{q}_2. \tag{6.19}$$

And the best response of player 1 to $\widetilde{q}_2$ would be

$$\beta_1(\widetilde{q}_2) = \beta_1(\beta_2(\widetilde{q}_1)) = f(\widetilde{q}_1) = \widetilde{q}_1. \tag{6.20}$$

Comparing (6.19) to (6.17) and (6.20) to (6.16), respectively, it can be seen that $(\widetilde{q}_1, \widetilde{q}_2)$ is a NE of NRAG-2{1}/1. Hence a fixed point of f is also a sufficient condition of a NE. The proof on existence of NE in NRAG-2{1}/1 is complete. □

6.4 Extensions to More Complicated Systems

Although NE always exists for NRAG-2{1}/1, the existence of NE requires further investigation when the number of subcarriers, number of BSs and number of users in a BS are large. Simple examples can be used to show that the existence of NE cannot be guaranteed for these more complicated NRAGs.

6.4.1 NRAG-2{1}/K with $K > 1$

We here extend NRAG-2{1}/1 to OFDMA systems, NRAG-2{1}/K, where there are K subcarriers and two BSs. Each player needs to determine which subcarriers to transmit and at what modulation levels.

For simplicity reason, we assume that there are no power constraints, i.e., (6.4) are removed. Under such an assumption, the strategy chosen by a player on a subcarrier will not be affected by the decisions of his opponents on other subcarriers, i.e., the players compete for a subcarrier and the decision is independent from the outcome on any other subcarriers. Hence NRAG-2{1}/K can be considered as a game consisting of K independent NRAG-2{1}/1 games, i.e., all players compete

	a_2^1=a_2^2=0	a_2^1=1	a_2^2=1
a_1^1=a_1^2=0	(0, 0, 0)	(0, 1.99, 0)	(0, 1.99, 0)
a_1^1=1	(1.99, 0, 0)	(1.97, 1.96, 0)	(1.99, 1.99, 0)
a_1^2=1	(1.99, 0, 0)	(1.99, 1.99, 0)	(1.97, 1.96, 0)

(a) a_3^1=a_3^2=0

	a_2^1= a_2^2=0	a_2^1=1	a_2^2=1
a_1^1=a_1^2=0	(0, 0, 1.99)	(0, 1.97, 1.96)	(0, 1.99, 1.99)
a_1^1=1	(1.96, 0, 1.97)		(1.96, 1.99, 1.97)
a_1^2=1	(1.99, 0, 1.99)	(1.99, 1.97, 1.96)	(1.97, 1.96, 1.99)

(b) a_3^1=1

	a_2^1=a_2^2=0	a_2^1=1	a_2^2=1
a_1^1=a_1^2=0	(0, 0, 1.99)	(0, 1.99, 1.99)	(0, 1.97, 1.96)
a_1^1=1	(1.99, 0, 1.99)	(1.97, 1.96, 1.99)	(1.99, 1.97, 1.96)
a_1^2=1	(1.96, 0, 1.97)	(1.96, 1.99, 1.97)	

(c) a_3^2=1

Best response for player 1 | Best response for player 2 | Best response for player 3 | Ineligible joint strategy

Figure 6.3: A possible case of no NE in NRAG-3{1}/2 with $Q = 1$.

with each other on every subcarrier independently. Given that each NRAG-2{1}/1 has at least one NE from Theorem 6.5, the NE space of NRAG-2{1}/K is obtained by the Cartesian product of the NEs for all K NRAG-2{1}/1 games.

6.4.2 NRAG-$N\{L\}/K$ with $N > 2$

The NE existence in NRAG-2{1}/K can be generalized to the case where each BS has L users, i.e. NRAG-2{L}/K, for $L \geq 1$. However, in practical systems where the number of BSs is usually greater than two, NE does not always exist in N-cell OFDMA systems ($N > 2$). To illustrate this, we consider NRAG-3{1}/2 where each BS has one user and two subcarriers. We further assume that $Q = 1$, i.e., each player has two strategies: either to transmit with a given modulation level or no transmission. A possible case where no NE exists is presented with its payoff

Table 6.1: Does a NE always exist in NRAG-$N\{L\}/K$?

	Single-carrier		Multi-carrier	
	Single user in each BS	Multiple users in each BS	Single user in each BS	Multiple users in each BS
2 BSs	Yes	Yes	Yes	Yes
>2 BSs	Yes*	Yes*	No	No

* With the assumption that $u_n \geq 0, n = 1, \ldots, N$

table shown in Fig. 6.3. This result shows that there does not exist a strategy profile which can be the best responses of all players concurrently, even there are only three cells. The results of our study on the NE existence of NRAG-$N\{L\}/K$ are summarized in Table 6.1.

6.5 Convergence of Game-play

Our study in the previous sections has provided some insights on the existence of NEs for NRAG, and showed that a NE cannot be generally guaranteed. Furthermore, the searching of NE is through exhaustive search using Definition 3, based on the assumption that every player has complete information including the payoff values of the opponent players. In distributed systems where only local information is available to an agent, the search for NE becomes more challenging even if NE does exist. In this section, we consider the myopic play of NRAG using the modified distributed Gauss-Seidel algorithm described in Subsection 5.4.1, as well as ensuring convergence of the game-play.

The examples presented in Fig. 6.3 disclosed that the formulated NRAG may not always have a NE solution under some channel conditions. If the NE is not to be obtained through exhaustive search but rather through myopic play, we have to

q_1 \ q_2	0	1	2	3
0	(0, 0, 0)	(0, 1.82, 0)	(0, 3.14, 0)	(0, 2.58, 0)
1	(1.88, 0, 0)	(1.88, 1.62, 0)	(1.88, 2.17, 0)	(1.85, -2.09, 0)
2	(3.45, 0, 0)	(3.43, 0.86, 0)	(3.33, -2.11, 0)	(2.50, -44.29, 0)
3	(3.79, 0, 0)	(3.47, -2.40, 0)	(0.50, -40.09, 0)	

(a) $q_3 = 0$

q_1 \ q_2	0	1	2	3
0	(0, 0, 0.79)	(0, 1.80, 0.67)	(0, 3.02, 0.22)	(0, 1.53, -1.79)
1	(1.86, 0, 0.76)	(1.85, 1.55, 0.49)	(1.82, 1.51, -0.68)	(1.45, -19.00, -13.70)
2	(3.31, 0, 0.64)	(3.19, 0.44, -0.28)	(1.88, -14.37, -10.21)	
3	(3.07, 0, 0.18)	(-0.76, -9.57, -7.26)		

(b) $q_3 = 1$

Ineligible joint strategy

Figure 6.4: A simulation example showing the 'cycling' in NRAG-3{1}/1 with $Q = 3$.

answer questions such as:

(1) If there is at least one NE in the NRAG, will the game eventually converge to one of the NE solutions?

(2) If there is no NE or if it does not converge to the NE, is it still possible to have the game converge to a steady state with necessary modifications?

Based on our simulations using myopic play, we observe that sometimes the NRAG converges to a steady state. However, there are also situations at which the game cycles among a few states and does not converge, resulting in unstable power and BER performance. Taking NRAG-3{1}/1 with $Q = 3$ as an example, for one set of the given channel conditions, the payoff table is computed and shown in Fig. 6.4. In this example, when $q_3 = 0$ is given, player 1 and 2 will settle on the strategy profile $(3, 0, 0)$. However, when $q_1 = 3$ and $q_2 = 0$, player 3 tends to choose q_3 to be 1 over 0. Consequently the play of the game will enter a loop as illustrated by the arrows in Fig. 6.4. Since no stable solution exists, the game has no NE.

6.5.1 Potential Games and Convergence to a NE

The formulated NRAG cannot be described as a potential game. However, using the framework developed in [35], convergence to NE for this game can be understood more easily by using the concept of potential game given in Definition 6.

Theorem 6.6. *Assuming that the SBA for all BSs are fixed and given by* $\widetilde{\mathbf{A}}_n$, $\forall n$, *the NRAG reduces to a NPAG which is a potential game.*

Proof. Given $\widetilde{\mathbf{A}}_n$, the payoff function of player n can be expressed as

$$u_n(\mathbf{p}_n, \mathbf{p}_{-n}) = \tilde{r}_n - c\sum_{k=1}^{K} p_n^k, \quad \forall n \in \mathcal{N}, \tag{6.21}$$

where $\tilde{r}_n$ is the total data rate of BS n and is related to $\tilde{a}_{ln}^{kq}$ only. If we define

$$P(\mathbf{p}_n, \mathbf{p}_{-n}) = \sum_{n\in\mathcal{N}} u_n(\mathbf{p}_n, \mathbf{p}_{-n}), \tag{6.22}$$

it can be shown that $P(\mathbf{p}_n', \mathbf{p}_{-n}) - P(\mathbf{p}_n, \mathbf{p}_{-n}) = u_n(\mathbf{p}_n', \mathbf{p}_{-n}) - u_n(\mathbf{p}_n', \mathbf{p}_{-n})$. Therefore the NPAG is an exact potential game with a potential function given by P. If the given SBA is feasible on all the subcarriers, then the set $\mathcal{P}_n|_{\widetilde{\mathbf{A}}_n}$ is compact and there exists at least one NE in the NPAG. □

Let us re-examine the process of myopic game and see how we can use Theorem 6.6 to explain the convergence of NRAG. In a myopic game, each player takes turn to decide his action $x_n = \{A_n^{(t+1)}, P_n^{(t+1)}\}$ given by (5.25). In the initial stage, each player tend to transmit at high rate using high modulation level because the interference level is zero. As the interference level gradually builds up, the player updates the SBA matrix (and hence PA matrix) at each decision point. Suppose

after some number of plays, the SBA component of the players converge to a steady profile, i.e., $A_n^{(T)} = A_n^{(T-1)}$, $\forall n \in \mathcal{N}$, then for subsequent play the game becomes a NPAG. Under this circumstance, from Theorem 6.6, NPAG is guaranteed to converge to its NE, $x^* \in \mathcal{S}$.

Hence, it is not difficult to conclude that convergence to one of the NEs is thus guaranteed by using the distributed Gauss-Seidel algorithm with best response provided: (1) There exists at least a NE in $\mathcal{S}$; (2) After NRAG has been played for a number of rounds, the SBA should show convergence to a steady SBA action profile corresponding to the NE.

6.5.2 Ensuring Convergence for NRAG

NRAG has no guarantee on the existence of NE and convergence to NE even if it exists. The myopic play of NRAG normally involves either players swapping the subcarriers occupied to avoid excessive interference or changing the modulation level used from high to low as the interference level in the system gradually builds up. In some cases, the players may have experienced transitions from high to low modulation level previously, but still repeatedly choose to increase the modulation level at a later stage when the interference level of the subcarrier is deemed to be suitable again, and this may result in "looping" in the SBA profiles chosen.

Theorem 6.6 implies that unless SBA becomes stable among the players, the game will not converge. If SBA is stabilized, NRAG becomes NPAG and will eventually converge. If the game does not converge, the SBA profiles of certain players must be repeatedly changing for some players. We use the method with NOA described in Subsection 5.4.3 once again to eliminate the "cycling" between

higher and lower modulation levels for some players which leads to instability in the solution.

To explain in more details, we define $\mathbf{W}_n$ as the NOA matrix of BS n:

$$\mathbf{W}_n := \left[\mathbf{W}_n^1 \ \cdots \ \mathbf{W}_n^Q\right]_{L_n \times KQ}, \quad \forall n \in \mathcal{N}. \tag{6.23}$$

Here $\mathbf{W}_n^q$ is the NOA matrix for modulation level $q \in \mathcal{Q}$ with its (i,j)th element given by $[\mathbf{W}_n^q]_{i,j} = w_{in}^{jq}$, for $i \in \mathcal{L}_n$ and $j \in \mathcal{K}$, where w_{in}^{jq} is the NOA variable which takes a integer value, and is used to store the number of attempts that has been made for user i in BS n, with modulation level q on subcarrier j. All the w_{in}^{jq} variables are initialized to zero and can take up to a maximum value W_{in}, for $i \in \mathcal{L}_n$. As the myopic plays progress, when the modulation level of a user in BS n is adjusted to a lower one, the NOA variable corresponding to the previous higher modulation level is incremented by one. When the maximum NOA, W_{in}, is reached, that particular modulation level is deemed unsustainable and will be forbidden for future use.

Theorem 6.7. *For any finite W_{in}, the modified NRAG using NOA always converges .*

Proof. We prove this by contradiction. Assume that the NRAG does not converge, then the SBA profile must not be stable and there exists infinite number of changes in the modulation level from low to high as well as from high to low. On the other hand, since Q, K, L_n and N are all finite, the total number of SBA combinations is also finite. With the use of NOA, any change of SBA combinations that involves the modulation level changing from high to low can occur at most W_{in} times, for

Modulation Index	Stage	1		2		3		4		5		6		7
3		3.34		-2.56		-5.00		-2.51		-4.97		-2.51		-4.97
2		**3.35**		**1.91**		1.31		**1.91**		1.31		**1.91**		1.31
1		1.87		1.57		**1.45**		1.57		**1.45**		1.57		**1.45**
0		0		0		0		0		0		0		0

(a) Without NOA

NOA Values

Modulation Index	Stage	1		2		3		4		5		6		7
3	0	3.34	0	-2.56	0	-5.00	0	-2.51	0	-4.97	0	-2.51	0	-2.26
2	0	**3.35**	0	**1.91**	1	1.31	1	**1.91**	2	1.31	2		2	
1	0	1.87	0	1.57	0	**1.45**	0	1.57	0	**1.45**	0	**1.57**	0	**1.58**
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

(b) Max NOA=2

Forbidden modulation index

Figure 6.5: An example of a NRAG-3{1}/1 played: (a) without NOA; (b) Max NOA=1. It clearly shows that the use of NOA helps to stabilize the play of the game and to settle the game at an equilibrium.

$i \in \mathcal{L}_n$. Hence the number of changes in the modulation level from high to low is finite, which contradicts to the assumption and the proof is complete. □

Hereafter we refer to the modified NRAG with NOA as CNRAG. To better illustrate the function of NOA, we use the same example in Fig. 6.5(a) where no NE exists in the original NRAG. The game play of CNRAG using NOA is shown in Fig. 6.5(b). As we can see, when the player changed from $q = 2$ to $q = 1$, the value of the corresponding element in NOA was set to one. Subsequently $q = 2$ would not be selected again since the maximum NOA (=2 in this example) had been reached. The player settled on $q = 1$ and eventually the game converged to a stable solution.

Because the CNRAG does not have the same strategy space for the players as that of the NRAG, the NE is modified if deletion of strategies has taken place.

6.6 Improving Network Payoff with IA

6.6.1 Advantage of IA over Water Filling

In NRAG, each player wants to maximize his payoff but the competitions among the players can lead to an equilibrium point that is highly inefficient. From a network perspective, it is preferable to let each player use a payoff function which also reflects the network's desires in order to achieve good overall network payoff. In Section 6.1, we choose (6.1) as the player payoff function, which is also related to the payoff function defined in (3.4) for centralized approach. This also ensures that both the centralized and game theoretic approaches have the basis to compare the performance. However, another relevant question arises: how can a network designer leads the players to reach a desirable equilibrium point in NRAG, or how to modify the game to have more preferable NEs?

Mechanism design looks into how to put in incentive mechanisms or to obtain optimal design parameters of a game, in order to achieve a more desirable outcome from the system point of view. Particularly, pricing has been used as a technique to regulate the usage of a certain resource. In NRAG, pricing on the transmission power has been incorporated in the payoff function (6.1), e.g. in [68], to lessen possible severe CCI among the BSs. We will show that such a pricing mechanism is not the most effective way in multi-cell OFDMA systems.

The process of maximizing (6.1) in a single BS is effectively the same as performing iterative water-filling over all the subcarriers [68]. In NRAG for multi-cell systems, making all BSs to operate at their own optimal solutions does not guarantee that the system global optimal can be obtained [33]. Since every player tries

to maximize his own payoff in the NRAG, a BS will choose to transmit as long as the resulting payoff value is positive on a subcarrier. It is shown that the optimal solution in a single-cell system tends to load the data bits over all the subcarriers [24]. However, multi-cell systems which apply some kind of interference avoidance mechanism can achieve better overall network payoff. A good discussion on this can be found in [33].

6.6.2 Pricing Mechanism for IA

For OFDMA multi-cell systems with FRF equal to one, IA would be a more effective way to alleviate strong CCI among the BSs. We use a 2-BS system, with each BS using 2 subcarriers to transmit 4 data bits, to illustrate this concept. If WF approach is used, both BS will transmit 2 bits on each subcarrier, resulting in CCI to each other. On the other hand, to avoid unnecessary interference, each BS may also choose a higher modulation level to transmit the 4 bits on different subcarrier and results in zero CCI. It can be seen from Fig. 6.6 that at low interference, WF outperforms IA. As the interference level increases, however, the power needed in WF eventually arrives at a crossover point, beyond which WF is outperformed by IA. Similar conclusion can be observed for 6 bits to be loaded on 3 subcarriers. The example shows that WF algorithm may not result in desirable solution from the system perspective, especially in the interference limited environments.

To discourage the BSs from excessively occupying the subcarriers in the system, we propose to include the number of subcarriers used by each BS as a cost factor in the player's payoff functions. The new payoff function is given by

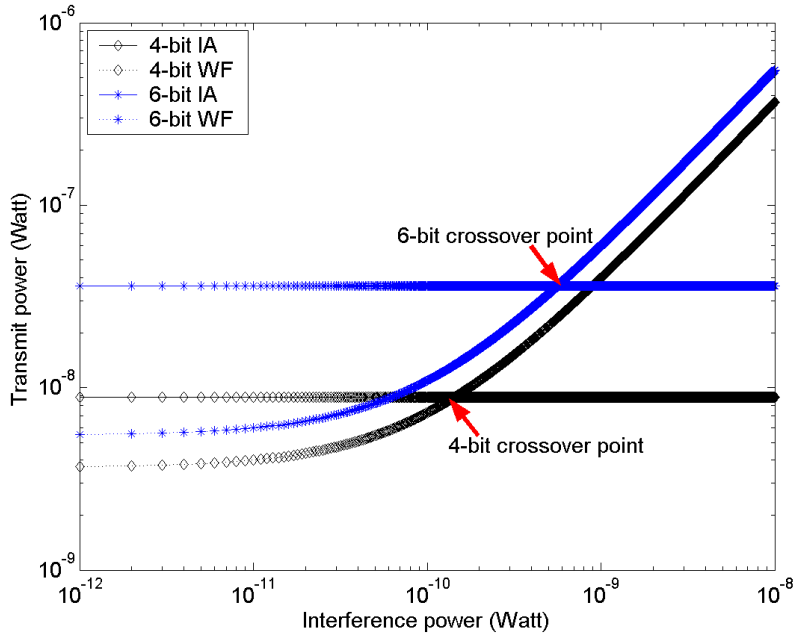

Figure 6.6: Interference Avoidance versus Water Filling.

$$v_n = u_n - \sum_{k=1}^{K}\sum_{l=1}^{L_n}\sum_{q=1}^{Q} b a_{ln}^{kq} = \sum_{k=1}^{K}\sum_{l=1}^{L_n}\sum_{q=1}^{Q}\left[(2q-b)\, a_{ln}^{kq} - c p_{ln}^{kq}\right], \quad \forall n \in \mathcal{N}. \quad (6.24)$$

where b is the spectrum cost factor, a value set to tradeoff bit rate with the number of allocated subcarriers, with a unit of bits/MHz. Since $\sum_{k=1}^{K}\sum_{l=1}^{L_n}\sum_{q=1}^{Q} a_{ln}^{kq}$ corresponds to the total number of subcarriers occupied by BS n, b can also be considered as the cost of using the spectrum.

With appropriate values of b, the new payoff function can prevent players in the NRAG from unnecessarily occupying too many subcarriers and causing strong interference to the others, thus the behaviours similar to IA are incorporated in the game. By including the number of subcarriers used by the respective BS as a cost in its utility function, we show that strong interference among the players (BSs) can be avoided and as a result better resource usage can be achieved. Simulation

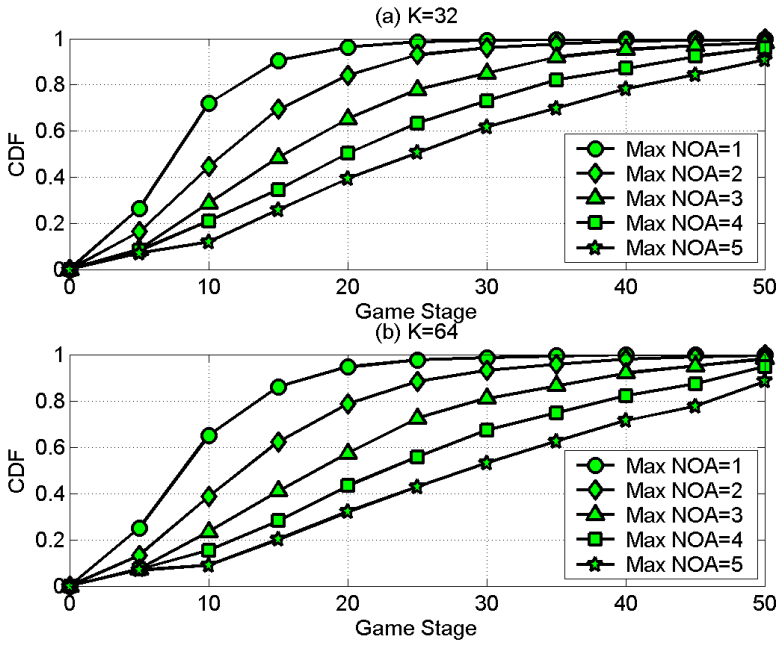

Figure 6.7: CDFs of CNRAG with different values of NOA.

results to be presented in the next section show that (6.24) is effective in improving the overall performance of the multi-cell systems.

6.7 Results and Discussions

Simulations are conducted for a 3-cell OFDMA system, where each cell has a radius of 100 and is separated by $100\sqrt{3}$ among each other. BSs are located at the centre of the cells, and the locations of the users in each cell are randomly generated with uniform distribution. The radio propagation model takes into consideration the path loss, shadowing and fast fading. The path loss (in dB) at a distance d from the BS is taken as $L(d) = L(d_0) + 10\alpha \log_{10}(d/d_0)$, with $d_0 = 10$ being the reference point ($L(d_0) = 0$dB) and $\alpha = 3.8$. The shadowing effect is modelled as a lognormal random variable with 10dB standard deviation. The four-path Rayleigh model is used to model frequency selective fading with an exponential power profile. The receiver thermal noise is -70dBmW.

For each channel and location realization, we study the number of stages needed for the solution to converge. The CDF on the convergence of CNRAG is plotted in Fig. 6.7. It can be seen that the speed of convergence only slightly decrease when the number of subcarriers increases. The most important factor which affects the speed of convergence is the maximum NOA allowed. The CNRAG converges much more slowly when the value of NOA is getting larger. This conclusion is intuitive because more number of game stages are allowed before a SBA strategy is considered to be aborted. However, no matter how slowly it does, the game will converge if the value of NOA is finite. Through intensive simulations for K ranging from 8 to 128, we find that in general NOA=2 gives the best improvement in network payoff although at the expense of sacrificing the convergence speed.

To study the average network payoff and to compare the performance of the three different games – NRAG, CNRAG and CNRAG-IA, we generate 10^3 instances of user and channel realizations and the results are shown in Fig. 6.8. The figure shows that for both $K = 64$ and $K = 128$, the performance of CNRAG is slightly better than that of the original NRAG which occasionally does not converge. More importantly, while the result of the original NRAG is not stable, CNRAG always converges to a steady state. On the other hand, CNRAG-IA shows clear improvement in network payoff over both NRAG and CNRAG, while still being able to converge and remain stable. Although the results from both CNRAG and CNRAG-IA could not reach the optimal solutions of the overall system, they do provide stable and convergent results to support the user requirements in a decentralized manner.

The performance difference cannot be easily seen from Fig. 6.8 since all different schemes transmit at different number of bits and the actual value of network

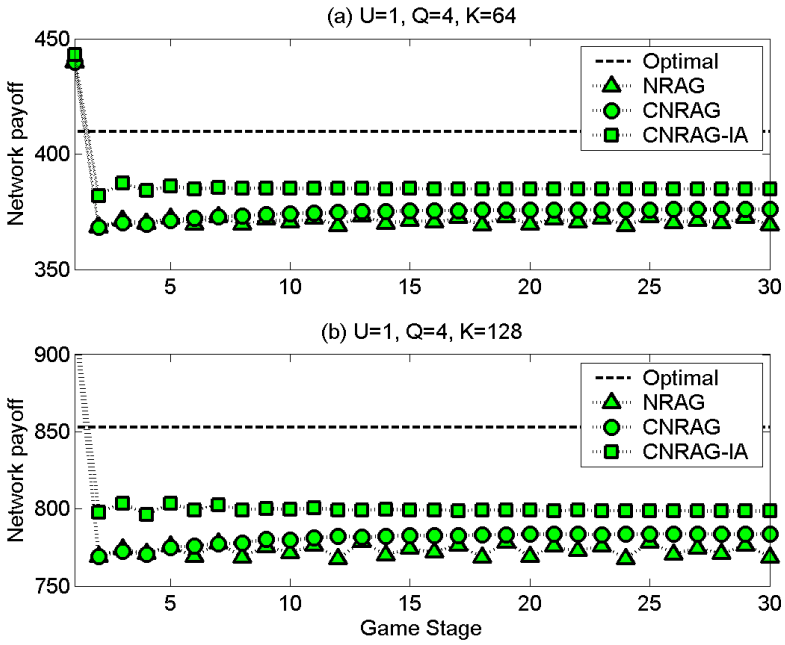

Figure 6.8: Network payoff comparison for the different games.

payoff is affected by the value of c, the power cost factor. Therefore we compare the performance of the three games in term of the transmission power required to transmit a single bit, and the results are shown in Fig. 6.9. It can be seen that NRAG requires more than 2dBm (or about 20%) than the optimal solution to transmit a single data bit. With the introduction of NOA, CNRAG not only makes sure the game will converge, but also provides a performance improvement of about 0.6dBm. And by taking interference avoidance into consideration, CNRAG-IA improves the performance further with another 0.6dBm reduction in the transmission power per bit, without increasing the complexity of the game.

An example to compare the SBA of CNRAG, CNRAG-IA, and the optimal solution is shown in Fig. 6.10. For illustrative purpose, we reduce the number of subcarriers to 3, and every BS has only one user. Results of the repeated plays are taken at the end of the tenth iteration. It can be seen that in CNRAG, the players put the bits on more than one subcarriers, as contrast to the optimal case

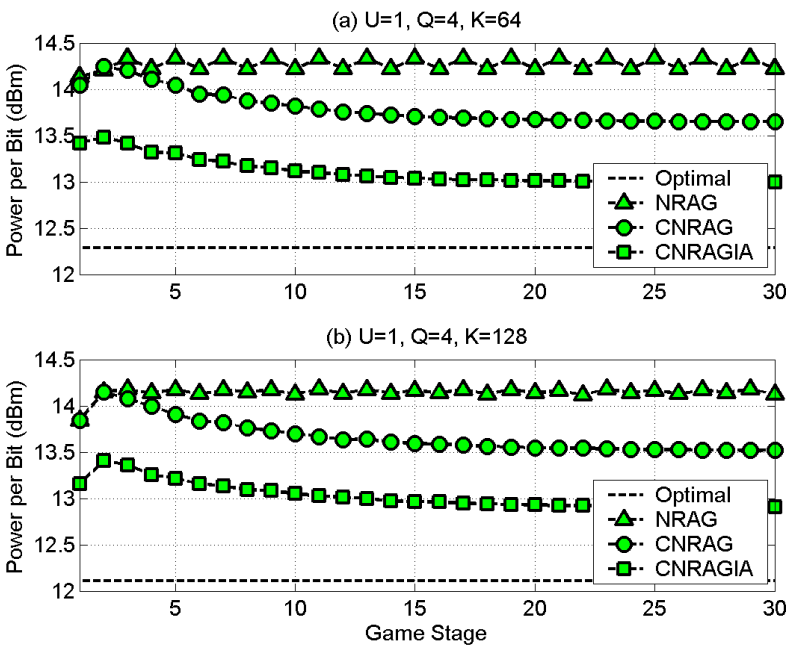

Figure 6.9: Comparison on transmission power per bit for the different games.

where the three BSs load all the bits on three distinctive subcarrier so that no interference is caused among each other. As a contrast, the outcome of CNRAG-IA using new utility function (6.24) happened to be exactly the same as the optimal case. Although it does not guarantee to result in the optimal solution every time, CNRAG-IA generally achieves better overall system utility over CNRAG.

6.8 Conclusion

The adaptive allocation of subcarrier, bit and power resources in multi-cell OFDMA systems were studied using the non-cooperative game theoretic approach. In contrast to the previous works, integer values were used in our study. The simplest NRAG-2{1}/1 was first studied, which has shown that there exists at least one NE for the game. However, as the numbers of players, users in a BS and subcarriers increase, the existence of NE cannot be guaranteed. In the case where no NE ex-

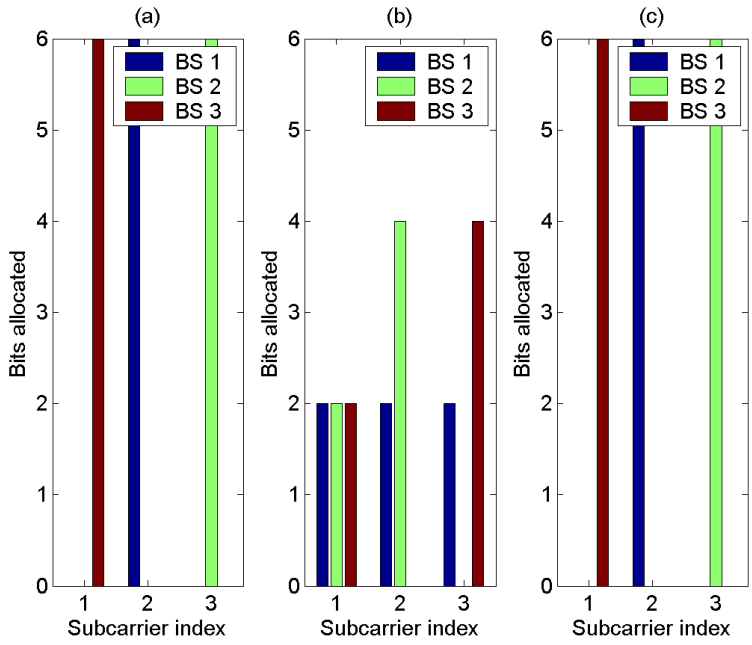

Figure 6.10: Comparison of subcarrier-and-bit allocation: (a) Optimal (b) CNRAG (c) CNRAG-IA.

ists, it was shown that the myopic play of NRAG will oscillate in a cycle of two or more stages and will not arrive at a stable outcome. Based on the framework of potential games with coupled constraints which can guarantee convergence, the procedure of the myopic play was modified to detect and remove those modulation levels which could lead to unstable outcome. As a result of removing the possible cycling of the game stages, the game would eventually converge without increasing the complexity significantly. Moreover, an additional term was introduced in the payoff function to enforce interference avoidance among neighbouring BS. The IA mechanism was proved to be effective in mitigating CCI, with CNRAG-IA able to achieve higher network payoff than CNRAG. Finally, the network payoff obtained by all the three game theoretic approaches were compared with the centralized approach.

Chapter 7

Conclusion

In this thesis, we studied radio resource allocation problems in wireless systems using both the centralized optimization and game theoretic approaches. Firstly, the SBA in single-cell multiuser multiclass OFDMA systems was formulated as a MINLP optimization problem. The MINLP is highly nonlinear and complex to solve. Thus a method was proposed to convert it into a BLP which has a drastically reduced complexity due to its linearity. As a result, the optimal solution can be obtained much more easily than before.

Secondly, the similar resource allocation problem was extended to multi-cell OFDMA systems. As the complexity of the formulated MINLP increases exponentially with the number of cells and number of users in a cell, it is much more difficult to solve the MINLP directly. Once again, a method was proposed to convert the MINLP into a BLP to obtain the optimal solution much more easily without relaxation and approximation. The optimal solutions can act as a performance bound to benchmark the results obtained from other approaches such as game theory and heuristic algorithms.

Thirdly, the opportunistic transmission of distributed nodes over a common channel was studied using a non-cooperative game theoretic approach. In the formulated NRAG, integer numbers of bits are used which results in discrete strategy spaces for the players. It was shown that there is at least one NE solution in the 2-player single-channel NRAG under all possible channel realizations. Then the N-player NRAG was also shown by mathematical induction to have at least one NE solution, with the assumption that a strategy profile should only have positive payoff when a player transmits. However, existence of NE does not guarantee convergence to one of the NEs when the game is played repeatedly. To overcome this problem, it was shown that the NRAG will become a NPAG when the subcarrier assignments are fixed, and the NPAG is a potential game which will always converge. Therefore we proposed an algorithm introducing NOA to the NRAG in order to ensure convergence of game-play without increasing the complexity significantly. The price of anarchy for the games was also estimated using computer simulations with various settings.

Lastly, the SBPA in multi-cell OFDMA systems was studied using the non-cooperative game theoretic approach. With integer numbers of bits being used, our study also dealt with discrete strategy spaces of the players. The simplest NRAG-2{1}/1 was first studied and shown that there is at least one NE for the game. However, existence of NE cannot be guaranteed for the games with more players or subcarriers, hence the myopic play of NRAG will oscillate and no stable outcome can be obtained. Based on the framework of potential games with coupled constraints, an algorithm using NOA was proposed to modify the procedure of myopic play so that those unsustainable modulation levels which could lead to

unstable outcomes would be detected and removed. As a result, the game will eventually converge without increasing the complexity significantly. Moreover, by introducing a cost factor on the spectrum usage to the payoff functions of the players, IA mechanism was proved to be effective in mitigating CCI, with CNRAG-IA being able to achieve higher network payoff than CNRAG.

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