Elasticity of Substitution and Growth Effects of Taxation

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Contents

SUMMARY

In this thesis, we study the growth effects of taxation in a general two-sector endogenous growth model. We examines in particular how these effects depend on the elasticities of substitution between the factors in the final goods and human capital production. We find that the negative effects of taxation on economic growth are stronger when the elasticities of substitution between inputs are higher in both sectors. Under reasonable parameterization, for equal percentage changes, the labor income tax has a larger effect on growth than the capital income tax because the former has a larger tax base than the later. For revenue-equivalent changes, the magnitudes of the growth effects of the taxes depend on the elasticities of substitution: (i) when the elasticities are low, the capital income tax (the consumption tax) has a larger (smaller) effect on growth than the labor income tax; (ii) when the elasticities are high, the labor income tax (the consumption tax) has a larger (smaller) effect on growth than the capital income tax.

List of Tables

1 Introduction

The effect of taxation on economic growth is a key issue in the literature on taxation and growth. A large number of papers in the literature have investigated the growth effect of tax policies/reforms in the context of neoclassical models and endogenous growth models. Using neoclassical growth models, in which physical capital is the only accumulable factor, many studies (e.g.,Judd (1987); Auerbach and Kotlikoff (1987); Lucas (1990)) find that both capital and labor income taxes reduce the steady-state level of income but have only transitory effects on its rate of growth. Compared with labor income taxes, capital income taxes are more distortionary. A capital income tax creates an inter-temporal distortion by reducing the return to savings, because it effectively taxes future consumption at an increasing rate, while a labor income tax affects only the allocation of time between labor and leisure that is an intratemporal distortion. Lucas (1990) provides an analytical review of research on growth effects of capital taxation using neoclassical growth models. He finds that eliminating the capital tax and raising the labor tax in revenue-neutral way would have a trivial effect on the US growth rate.

The more recent literature on taxation and growth reconsider the impact of taxation on economic growth in endogenous growth models where both physical and human capital can be accumulated (e.g., King and Rebelo (1990); Rebelo (1991); Pecorino (1993); Devereux and Love (1994, 1995); Stokey and Rebelo (1995)). King and Rebelo (1990) show that in a two-sector endogenous growth model, national policies, such as taxation policies, can affect long-run growth rate. They argue that, modest variations in tax rates are associated with large variations in long-run growth rates. Calibrating their model using US data, they claim that taxes can easily shut down the growth process, leading to development traps in which countries stagnate or even regress for lengthy periods. Another paper by

Rebelo (1991) analyzes a class of endogenous growth models with constant-return-to-scale production function and concludes that the growth rate would be relatively low for countries in which there are high income tax rates. An increase in the income tax rate decreases the rate of return to the investment activities of the private sector and leads to a permanent decline in the rate of capital accumulation and in the rate of growth.

Devereux and Love (1994) examine the effects of factor taxation in a discrete two sector model. They look at the effects of taxes both in the steady state and during the transition to the steady state. They find that both consumption and factor income taxes lower the growth rate and for equal percentage changes, labor income taxes have larger effects on growth than capital income taxes; however, for revenue-equivalent changes, the differences in the growth rate for different taxes are negligible.

In endogenous growth models with human capital accumulation, the impacts of taxation on economic growth are closely related to two factors. The first one is whether physical inputs are required in producing human capital. The second one is whether human capital accumulation is considered as a market activity (and thus these physical inputs are taxed) or a nonmarket activity (and thus these physical inputs are untaxed). Assuming physical inputs are used and taxed in human capital production, King and Rebelo (1990) find that the effects of taxation depend critically on the production technology for new human capital. Pecorino (1993) considers the mixes of taxes on physical and human capital in a growth model with human capital accumulation and finds that factor intensities play an important role in determining the effect of the tax structure on growth. Devereux and Love (1994) assume that physical capital used in human capital production is untaxed and examine the effects of taxation in a model with joint accumulation of physical and human capital. They find that all three types of taxes (consumption, labor income and capital income taxes) reduce the growth rate. Milesi-Ferretti and Roubini (1998) show that factor taxation hurts long-run growth regardless of whether human capital production is taxable.

One limitation of many studies in the literature is the assumption that the elasticity of substitution equals to unity and thus the production function takes the Cobb-Douglas form. In previous studies, the Cobb-Douglas production functions have been widely used for goods and human capital production. The Cobb-Douglas production function is extremely restrictive as it sets the elasticity of substitution between factors to unity. However, many empirical studies find that the elasticities of substitution between production factors is less than unity. At the same time, there also exists empirical evidence that shows that this elasticity is greater than unity.[1]

The elasticity of substitution is central to many questions in growth theory. It is one of the determinants of economic growth, and it also affects the speed of convergence as well as the aggregate income distribution (see, e.g., Grandville (1989), Klump and Grandville (2000), Klump and Preissler (2000) and Hick (1932)). Several studies in the literature have investigated the connection between economic growth and the constant elasticity of substitution (CES) production technology, which allows the elasticity of substitution to take constant values that are either greater or lower than one. For example, Klump and Preissler (2000) use different variants of the CES function in a neoclassical growth model to examine how economic growth is related to the elasticity of substitution. They show that a higher elasticity of substitution makes the emergence of permanent growth more probable

[1]See Section 4 for detailed discussion.

and can lead to a higher long-run growth rate. However, they do not consider the taxation effects on growth under different variants of the CES function.

The contributions in the literature on taxation with implications of production structures includes Lucas (1990) and Stokey and Rebelo (1995), among others. Lucas (1990) considers the CES production function in physical capital production (with substitution elasticity equal 0.6) to discuss the growth effects of taxation. His focus is not on the elasticity of substitution but on other parameters. Stokey and Rebelo (1995) show that the factor shares, depreciation rates, the elasticity of inter-temporal substitution and elasticity of labor supply are important for determining the quantitative impact of taxes, but the tax reform have little or no effects on the US growth rate. They claim the elasticity of substitution in production is relatively unimportant.[2]

The objective of this thesis is to develop a two-sector endogenous growth model to examine the long-run growth effects of the three commonly used taxes (consumption, capital and labor income taxes) under different assumptions concerning the elasticities of substitution between inputs in final goods and human capital production.

We show the following results through numerical simulations: First, the negative effects of factor income taxes on economic growth are stronger when the elasticities of substitution between inputs in final goods and human capital production are higher, because, with high elasticities of substitution, taxation will have a lager distortionary effect on the economy. Second, for equal percentage changes, the labor income tax has a larger effect on growth than the capital income tax, be-

[2] In the previous studies, very few attempts have been made to consider the implications of non-Cobb-Douglas human capital production functions for the growth effects of taxation.

cause the labor's share of income exceeds the capital's share, which is reminiscent of the result in Devereux and Love (1994). Third, for revenue-equivalent changes, the magnitudes of the growth effects of the taxes depend on the elasticities of substitution: (i) when the elasticities are low, the ranking of the taxes in terms of the growth effects (starting with the largest effect) is: the capital income tax, the labor income tax and the consumption tax; (ii) when the elasticities are high, the ranking of the taxes is: the labor income tax, the capital income tax and the consumption tax. This is different from the results in Stokey and Rebelo (1995). They argue that the elasticity of substitution in production is relatively unimportant.

This thesis is organized as follows. Section 2 develops a two sector endogenous growth model. The model extends the framework in Ramsey model by considering endogenous labor supply and constant elasticity of substitution (CES) production technology. The two sectors are final goods production and human capital production. In the model, both labor (human capital) and physical capital are used in production in the two sectors. Section 3 characterizes the competitive equilibrium and derive two key equilibrium conditions. Section 4 numerically investigates how taxes affect the equilibrium growth rate under different assumptions concerning the elasticities of substitution in the two sectors. The main findings and conclusions are given in section 5.

2 The Model

The economy is closed and populated by many infinitely-lived, rational, and identical households with homothermic preferences, many competitive firms with identical technology and a government. Population remains fixed over time. There are two sectors in which production takes place: final goods and human capital; and two factors of production: physical capital and human capital. Both factors are necessary for production in both sectors.

A single consumption good is produced in this economy from a technology that combines physical capital (K) and (effective) labor ($H(1-l)\phi$). Physical capital is obtained from unconsumed final goods.

Human capital is embodied within individuals, so that it is useful only if combined with time spent at work by households. Human capital is produced in the human capital sector. Both human capital and physical capital are assumed to be able to grow without bound.

Each household has one unit of time endowment and allocates it to leisure l, labor $(1-l)\phi$, and education $(1-l)(1-\phi)$. That is, the household allocates its time between income-generating activities and all other activities (leisure), also allocates income-generating time between production of goods and accumulation of human capital (learning) and distributes income between consumption and investment (saving).

All markets are perfectly competitive.

2.1 Households

Households deriving their utility from consuming a single produced good and leisure, over an infinite time horizon. The discounted sum of future utilities of the representative household is given by:

$$U = \int_0^\infty e^{-\rho t} \left[\frac{(C_t l_t^\eta)^{1-\theta}}{1-\theta} \right] dt,$$
$$\theta > 0 \ (\neq 1), \quad \rho > 0, \quad \eta > 0, \tag{1}$$

where C_t and l_t are consumption and leisure at time t; ρ is the rate of time preference; $1/\theta$ is the intertemporal elasticity of substitution; and η reflects the household's preferences for leisure. The household is endowed with one unit of time, so $(1-l)$ is the amount of time spent on the income-generating activities.

It is assumed that any taxes are levied on the final goods sector only. Hours not consumed in leisure may be devoted to either production in the market sector or human capital formation. Hours supplied to the market earn a direct market wage. If hours are devoted to human capital formation, they generate a return in the future when wages per hour are augmented by a higher productivity of time.

Assume that households directly save in terms of capital, renting out capital to firms at competitive interest rates. In choosing among saving, consumption and hours supplied to the market, households face the following constraint:

$$(1+\tau_c)\, C = (1-\tau_l)\, w\, (1-l)\, H\phi + (1-\tau_k)\, rK - E - \dot{K} - \delta_k K + T, \tag{2}$$

and

$$l < 1,$$

where τ_c τ_l , τ_k , and T are, respectively, a consumption tax, a wage tax, a capital income tax, and a lump-sum transfer from the government. The tax on human capital is a wage tax. The taxing authority does not distinguish between the returns to raw labor and human capital. The wage w represents the return to hours measured in the efficiency units. r is the rental rate on physical capital. H is the total stock of human capital. $(1-l)$ is the time spent in the income-generating activities and ϕ is the fraction of this time supplied to the market. $(1-l)\,\phi$ represents hours supplied to the final goods sector. E is the physical inputs in the human capital sector.

Human capital production requires the use of both physical inputs and (effective) labor. Human capital is produced according to:

$$\dot{H} = B\left[\beta\left(H\left(1-l\right)\left(1-\phi\right)\right)^{\frac{\epsilon-1}{\epsilon}} + \left(1-\beta\right)E^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} - \delta_h H. \tag{3}$$

where ϵ is the elasticity of substitution; δ_h is the human capital depreciation rate; E is the household's investment of physical inputs; B is the productivity parameter; and β is the share parameter that measures the importance of physical inputs relative to the effective units of time inputs.

2.2 Final Goods Production

Firms in the market sector simply rent capital and employ labor to maximize profits. The technology for final goods production is assumed to take the following form:

$$Y = A\left[\alpha\left(H\left(1-l\right)\phi\right)^{\frac{\xi-1}{\xi}} + \left(1-\alpha\right)K^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}. \tag{4}$$

This technology for final goods production is a CES (constant elasticity of substitution) production function, which allows physical capital and human capital to be either complements or substitutes depending upon the value of the elasticity of substitution.[3] In (4), Y is output; A is a productivity parameter; K is physical

[3] One useful property of the CES production function is that it nests a number of famous simple cases. When the elasticity of substitution ξ converges to 0, we have the Leontief production function $Y = A\min\{\frac{K}{\alpha}, \frac{L}{(1-\alpha)}\}$. When the elasticity of substitution ξ is 1, we have the Cobb-Douglas production function $Y = AK^{\alpha}L^{(1-\alpha)}$. When the elasticity of substitution ξ goes to infinity, we have the linear production function $Y = A\left(\alpha K + \left(1-\alpha\right)L\right)$.

capital; α is the share parameter that measures the importance of physical capital relative to effective labor; and ξ determines the elasticity of substitution of inputs, labor and capital are complements if $\xi < 1$ and substitutes if $\xi > 1$.

Assuming perfect competition in final goods production, profit maximization therefore implies:

$$w = \frac{\partial Y}{\partial \left(H \left(1-l\right) \phi\right)} = A^{\frac{\xi-1}{\xi}} \alpha \left(\frac{Y}{H\left(1-l\right)\phi}\right)^{\frac{1}{\xi}}, \tag{5}$$

$$r = \frac{\partial Y}{\partial K} = A^{\frac{\xi-1}{\xi}} \left(1-\alpha\right) \left(\frac{Y}{K}\right)^{\frac{1}{\xi}}, \tag{6}$$

where w and r are the wage rate and the interest rate, respectively.

The market clearing condition for the final good gives the law of motion for physical capital stock:

$$\dot{K} = Y - C - E - \delta_k K, \tag{7}$$

where δ_k is the physical capital depreciation rate.

2.3 Government

Government is introduced into this economy in a very minimal fashion. We abstract from government spending and simply assume that all tax revenue is rebated in a lump-sum transfer. Letting T denote this transfer, we have

$$T = \tau_k r K + \tau_l w \left(1-l\right) H\phi + \tau_c C. \tag{8}$$

We assume that the government only has access to distortionary taxes (at flat rates): a capital income tax τ_k, a labor income tax τ_l and a consumption tax τ_c. In the literature on taxation, it is generally assumed that the tax revenue is used for a single type of government expenditures (normally lump-sum transfers).

For simplicity, we assume that the government balances its budget at each point in time, thus avoiding any unnecessary notational burden associated with government debt.

3 Equilibrium

In this section, we will first characterize the competitive equilibrium. We will then numerically investigate the growth effects of various taxes and compare the magnitudes of these growth effects under different tax policies and different values of the elasticity of substitution.

Without distortions, the competitive equilibrium under perfect foresight is Pareto optimal. In this thesis, we focus on the equilibrium conditions that determine the steady-state growth rate.

To characterize the competitive equilibrium, in this case, we simply think of the economy as having prefect competitive markets for all goods and factors. Firms make their production decisions seeking to maximize profits, while households rent the two factors of production to firms, make learning decisions and choose their leisure time and consumption so as to maximize their lifetime utility.

3.1 Competitive Equilibrium: Definition

A competitive equilibrium for the economy constructed above consists of the sequences of consumption, leisure, physical capital, fraction of time spent on working, human capital, investment in education, lump-sum transfer, tax rates, wages, and rental rates $\{C(t), l(t), K(t),\ \phi(t),$ H(t), E(t), T(t), $\tau_k(t),\ \tau_l(t),\ \tau_c(t),$ $w(t), r(t)\}_{t=0}^{\infty}$ that satisfy the following conditions:

(i). Household utility maximization

Maximize (1), subject to (2), (3), $C_t > 0$, $l_t < 1$ and relevant transversality conditions: $\lim_{t\to\infty} e^{-\rho t}\lambda_t K_t = 0$ and $\lim_{t\to\infty} e^{-\rho t}\mu_t H_t = 0$;

(ii). Profits maximization;

(iii). Government budget constraint;

(iv). Market clearing condition (7)

$$\dot{K} = Y - C - E - \delta_k K.$$

3.2 Competitive Equilibrium: Characterization

We now characterize the competitive equilibrium, starting with constructing the following current-value Hamiltonian for the household's utility maximization problem:

$$\mathcal{H} = \frac{(Cl^{\eta})^{1-\theta}}{1-\theta} + \lambda\left[(1-\tau_l)\, w\,(1-l)\, H\phi + (1-\tau_k)\, rK - E - \delta_k K + T - (1+\tau_c)\, C\right]$$

$$+\mu\left[B\left[\beta\left(H\left(1-l\right)\left(1-\phi\right)\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\beta)\, E^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} - \delta_h H\right], \tag{9}$$

where λ and μ are the co-state variables associated with the household's budget constraint (2) and human capital production technology (3).

The first-order conditions for this optimization problem are:

$$\frac{\partial\mathcal{H}}{\partial C} = (Cl^{\eta})^{-\theta}\, l^{\eta} - \lambda\,(1+\tau_c) = 0, \tag{10}$$

$$\frac{\partial\mathcal{H}}{\partial l} = (Cl^{\eta})^{-\theta}\, C\eta l^{\eta-1} - \lambda\,(1-\tau_l)\, wH\phi - \mu B\beta H\,(1-\phi)$$

$$\times\left[\beta\left(H\left(1-l\right)\left(1-\phi\right)\right)^{\frac{\epsilon-1}{\epsilon}} + (1-\beta)\, E^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}}\left[H\,(1-l)\,(1-\phi)\right]^{-\frac{1}{\epsilon}} = 0, \tag{11}$$

$$\frac{\partial \mathcal{H}}{\partial \phi} = \lambda\left(1-\tau_l\right)w\left(1-l\right)H - \mu B\beta H\left(1-l\right)$$

$$\times\left[\beta\left(H\left(1-l\right)\left(1-\phi\right)\right)^{\frac{\epsilon-1}{\epsilon}} + \left(1-\beta\right)E^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}}\left[H\left(1-l\right)\left(1-\phi\right)\right]^{-\frac{1}{\epsilon}} = 0, \quad (12)$$

$$\frac{\partial \mathcal{H}}{\partial E} = -\lambda + \mu B\left(1-\beta\right)\left[\beta\left(H\left(1-l\right)\left(1-\phi\right)\right)^{\frac{\epsilon-1}{\epsilon}} + \left(1-\beta\right)E^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}} E^{-\frac{1}{\epsilon}} = 0, \quad (13)$$

$$\dot{\lambda} = -\lambda\left(1-\tau_k\right)r + \lambda\delta_k + \rho\lambda, \quad (14)$$

$$\dot{\mu} = -\lambda\left(1-\tau_l\right)w\left(1-l\right)\phi - \mu B\beta\left(1-l\right)\left(1-\phi\right)$$

$$\times\left[\beta\left(H\left(1-l\right)\left(1-\phi\right)\right)^{\frac{\epsilon-1}{\epsilon}} + \left(1-\beta\right)E^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{1}{\epsilon-1}}\left[H\left(1-l\right)\left(1-\phi\right)\right]^{-\frac{1}{\epsilon}} + \mu\delta_h + \rho\mu. \quad (15)$$

The system of equations (3) and (7), along with (10)-(15), implicitly determines the solution sequence for the model's variables.

Equation(10) represents the household's choice of consumption. The first term is the marginal benefit of consumption, and the second term is the marginal cost of consumption. The solution for the household's consumption decision must satisfy the condition that the marginal benefit of consumption equals to the marginal cost of consumption. According to equation (11), the optimal allocation of the household's time is such that the gain in utility from leisure equals to the loss in utility from the time spent on working and learning. The first term of equation (11) is the marginal utility of leisure. The second and third terms represent the marginal benefit from working and learning through earning more income for private consumption (the disutility of labor supply). Equation(12) is a condition that determines the optimal allocation of labor between production of final goods and accumulation of human capital; The first term is the marginal benefit of working in the final good sector while the second term is the marginal cost of working in the final good sector (in terms of the benefit of education). Equation (13) gives the optimal investment in education; The first term is the marginal cost of investment (the utility foregone) while the second term is the marginal benefit of investment in the human capital sector.

Equations (14) and (15) are the conditions that determine the optimal paths of the shadow prices of physical and human capital (dynamic efficiency of resource allocation). The λ and μ are shadow prices of physical capital and human capital respectively. These are the Euler conditions determining the optimal accumulation of physical and human capital as functions of their separate returns.

From (10), (11) and (12), we have

$$\frac{l}{C\eta} = \frac{(1+\tau_c)}{(1-\tau_l)\,wH}. \tag{16}$$

This equation represents the trade-off between goods consumption and leisure. The left hand side is the marginal rate of substitution between goods consumption and leisure, while the right hand side is the inverse of the real wage after adjustment for consumption and wage taxes.

From (5), (6), (10),(12) and (13), we obtain

$$\left[\frac{E}{H\,(1-l)\,(1-\phi)}\right]^{-\frac{1}{\epsilon}}\left[\frac{(1-\beta)}{\beta}\right]$$

$$= \left[\frac{K}{H\,(1-l)\,\phi}\right]^{\xi}\left[\frac{(1-\alpha)}{\alpha}\right]\left[\frac{(1-\tau_k)}{(1-\tau_l)\,(\rho+\theta g+\delta_k)}\right]. \tag{17}$$

This equation says that the allocation of factors between the two sectors is optimal when the after-tax marginal rates of technical substitution between the two factors are equalized across sector.

From the above equations, we can see that all the three types of taxes (consumption taxes, wage taxes and capital income taxes) affect the allocation of resources. A consumption tax drives a wedge between the marginal rates of substitution of consumption for leisure and the real wage rate. A wage tax has the same effect as the consumption tax. In addition, from equations (12) and (13), we can see that the wage tax also affects the returns to human capital accumulation by distorting the inter-sectoral allocation of time and physical capital. A tax on capital income affects the incentive to invest in final goods production. As shown in (17), the capital income tax affects the returns to both human capital accumulation and physical capital accumulation by causing the re-allocation of physical capital between sectors as described in equations (12) and (15).

Now we follow the approach used in the literature (e.g., King and Rebelo (1990), Rebelo (1991) and Devereux and love (1994)) to investigate the properties of the steady-state equilibrium.

To examine the properties of the steady-state equilibrium in this two-sector economy, we use the equilibrium conditions (10)-(15) to derive two conditions that determine the equilibrium growth rate (g) and labor supply ($1-l$). We deal with this in three steps.

First, along a balanced growth path, (l, ϕ, r, w) are all constant, and

$$\frac{\dot{K}}{K} = \frac{\dot{H}}{H} = \frac{\dot{C}}{C} = \frac{\dot{E}}{E} = \frac{\dot{Y}}{Y} = g,$$

that is, in steady state, physical capital, human capital, consumption, physical investment in human accumulation and output all grow at the same rate.

Also, along a balanced growth path, we have $\dot{\mu}/\mu = \dot{\lambda}/\lambda$. That is, in equilibrium the (shadow) prices of physical capital and human capital must change at the same rate. As a result, we have the following equation (from equations (14) and (15)):[4]

$$(1-\tau_k)r - \rho - \delta_k$$
$$= B\beta(1-l)[\beta(H(1-l)(1-\phi))^{\frac{\epsilon-1}{\epsilon}} + (1-\beta)E^{\frac{\epsilon-1}{\epsilon}}]^{\frac{1}{\epsilon-1}}[H(1-l)(1-\phi)]^{-\frac{1}{\epsilon}} - \rho - \delta_h. \quad (18)$$

The left hand side is the after-tax return on physical capital accumulation. It is the marginal product of capital in the final goods production sector. To satisfy inter-temporal efficiency, it must be equal to the return on human capital accumulation, which is given by the right hand side. The return on human capital should be $\frac{[(1-\tau_l)w(1-l)+(-\rho-\delta_h)P]}{P}$, where P is the relative price of human capital (in terms of the price of physical capital). That is after-tax wage rate, multiplied by total labor supply, eliminating the value of depreciated stock of human capital and time amortization, all divided by the relative price of human capital (i.e. $P = \mu/\lambda$).

We can get the relative price P of human capital from equation (12). Dividing both sides of equation (13) by μ and using the expression for P, we have the atemporal efficiency condition:

$$(1-\tau_l)\,w = \frac{\beta\,[H\,(1-l)\,(1-\phi)]^{-\frac{1}{\epsilon}}}{(1-\beta)\,E^{-\frac{1}{\epsilon}}}, \quad (19)$$

that is, the after-tax wage equals to the marginal product of effective labor in human capital production. Substituting this condition and the expression for P into the right hand side of equation (15) and dividing it by μ gives the right hand

[4]The detailed derivations of the right side of this equation will be explained later.

side of (18).

We now differentiate equation (10) with respect to time to obtain:

$$\frac{\dot{\lambda}}{\lambda} = -\theta \frac{\dot{C}}{C} = -\theta g.$$

Using this condition, along with the human capital production technology (3), we can rewrite equation (18) as

$$\rho + \theta g + \delta_h = B\beta(1-l)\left[\frac{\dot{H} + H\delta_h}{BH(1-l)(1-\phi)}\right]^{\frac{1}{\epsilon}} = B\beta(1-l)\left[\frac{g+\delta_h}{B(1-l)(1-\phi)}\right]^{\frac{1}{\epsilon}}, \tag{20}$$

which gives the time allocation (ϕ) in terms of the growth rate (g) and labor supply ($1-l$):

$$\phi = 1 - \left(\frac{\beta}{\rho + \theta g + \delta_h}\right)^{\epsilon} B^{\epsilon - 1}\left(g + \delta_h\right)\left(1-l\right)^{\epsilon-1}. \tag{21}$$

Second, we derive the first equation for the relationship between growth and total labor supply from the market clearing condition. From (14), we have

$$g = \frac{\left(1-\tau_k\right) r - \rho - \delta_k}{\theta}. \tag{22}$$

This is a familiar growth equation in the literature. The steady-state growth rate depends positively on the after-tax rate of return to investment in physical capital ($(1-\tau_k)r$) and the elasticity of intertemproal substitution ($1/\theta$) and negatively on the household's subjective discount rate (ρ) and the capital depreciation rate (δ_k).

Define $y \equiv \frac{Y}{H(1-l)}$, $k \equiv \frac{K}{H(1-l)}$, $c \equiv \frac{C}{H(1-l)}$ and $e \equiv \frac{E}{H(1-l)}$, where y is output per unit of effective labor, k is physical capital per unit of effective labor, c is consumption per unit of effective labor, and e is education investment per unit of effective labor. From the final goods production technology, we get

$$y = A\left[\alpha\phi^{\frac{\xi-1}{\xi}} + (1-\alpha)\,k^{\frac{\xi-1}{\xi}}\right]^{\frac{\xi}{\xi-1}}.$$

Using (6), along with $\frac{y}{k} = \frac{Y}{K}$, we have

$$k = A^{\xi}\left[\frac{\alpha}{\left(\frac{r}{(1-\alpha)}\right)^{(\xi-1)} - A^{\xi-1}(1-\alpha)}\right]^{\frac{\xi}{\xi-1}}\phi \equiv \Gamma(g)\phi,$$

where

$$\Gamma(g) \equiv A^{\xi}\left[\frac{\alpha}{\left(\frac{r}{(1-\alpha)}\right)^{(\xi-1)} - A^{\xi-1}(1-\alpha)}\right]^{\frac{\xi}{\xi-1}}$$

$$= \left\{\frac{1-\alpha}{\alpha}\left[\left(\frac{r}{A}\right)^{\xi-1}(1-\alpha)^{-\xi} - 1\right]\right\}^{\frac{\xi}{\xi-1}}.$$

Let

$$\Omega(g) \equiv \frac{y}{\phi} = A\left[\frac{r}{(1-\alpha)}\right]^{\xi}\left[\frac{\alpha}{\left(\frac{r}{(1-\alpha)}\right)^{(\xi-1)} - A^{\xi-1}(1-\alpha)}\right]^{\frac{\xi}{\xi-1}}$$

$$= A\left\{\alpha + (1-\alpha)\left[\Gamma(g)\right]^{\frac{\xi-1}{\xi}}\right\}^{\frac{\xi}{\xi-1}}. \tag{23}$$

Rearranging (16), we have

$$C = \frac{(1-\tau_l)\,wHl}{(1+\tau_c)\,\eta}. \tag{24}$$

Substituting (5) into (24), we obtain

$$c = \frac{C}{H(1-l)} = \frac{A^{\frac{\xi-1}{\xi}}(1-\tau_l)\alpha[\Omega(g)]^{\frac{1}{\xi}}l}{(1+\tau_c)\eta(1-l)},$$

which gives the ratio of consumption to physical capital stock

$$\frac{c}{k} = \frac{l(1-\tau_l)\alpha(A^{1-\frac{1}{\xi}})[\Omega(g)]^{\frac{1}{\xi}}}{\eta\phi\Gamma(g)(1+\tau_c)(1-l)}. \tag{25}$$

From equations (5) and (19), we have

$$e = \frac{\alpha^{\epsilon}(1-\beta)^{\epsilon}(1-\tau_l)^{\epsilon}A^{\frac{(\xi-1)\epsilon}{\xi}}[\Omega(g)]^{\frac{\epsilon}{\xi}}(1-\phi)}{\beta^{\epsilon}},$$

which gives the ratio of education investment to physical capital stock

$$\frac{e}{k} = \left[\frac{1-\phi}{\phi\Gamma(g)}\right]\left[\frac{(1-\beta)\alpha(1-\tau-l)A^{1-\frac{1}{\xi}}}{\beta}\right]^{\epsilon}[\Omega(g)]^{\frac{\epsilon}{\xi}}. \tag{26}$$

From (6), we have

$$\frac{y}{k} = \left[\frac{r(g)}{1-\alpha}\right]^{\xi}A^{1-\xi}. \tag{27}$$

We now rewrite the market clearing condition ($\dot{K} = Y - C - E - \delta_k K$) as

$$g + \delta_k = \frac{Y}{K} - \frac{C}{K} - \frac{E}{K} = \frac{y}{k} - \frac{c}{k} - \frac{e}{k},$$

and substitute (25), (26) and (27) into it to obtain the first condition that relates the growth rate to labor supply:

$$\left[\frac{1-\phi}{\phi\Gamma(g)}\right]\left[\frac{(1-\beta)\alpha(1-\tau_l)A^{1-\frac{1}{\xi}}}{\beta}\right]^{\epsilon}[\Omega(g)]^{\frac{\epsilon}{\xi}}$$

$$= \left[\frac{r(g)}{1-\alpha}\right]^{\xi}A^{1-\xi} - \delta_k - g - \frac{l(1-\tau_l)\alpha(A^{1-\frac{1}{\xi}})[\Omega(g)]^{\frac{1}{\xi}}}{\eta_{\phi}\Gamma(g)(1+\tau_c)(1-l)}. \tag{28}$$

Third, the human capital production function gives the following equation

$$\frac{\dot{H}}{H\,(1-l)} = B\left[\beta\,(1-\phi)^{\frac{\epsilon-1}{\epsilon}} + (1-\beta)\,e^{\frac{\epsilon-1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon-1}} - \frac{\delta_h}{(1-l)},$$

which gives

$$\left[\frac{g+\delta_h}{(1-l)}\right]^{\frac{\epsilon-1}{\epsilon}} - B^{\frac{\epsilon-1}{\epsilon}}\beta\,(1-\phi)^{\frac{\epsilon-1}{\epsilon}} = B^{\frac{\epsilon-1}{\epsilon}}\,(1-\beta)\,e^{\frac{\epsilon-1}{\epsilon}}. \tag{29}$$

From (5) and (19), we have

$$e^{\frac{\epsilon-1}{\epsilon}} = \frac{\beta^{1-\epsilon}(1-\phi)^{\frac{\epsilon-1}{\epsilon}}\left[\frac{\alpha}{\left(\frac{r}{1-\alpha}\right)^{\xi-1}-A^{\xi-1}(1-\alpha)}\right]^{\frac{\epsilon-1}{\xi-1}}}{A^{1-\epsilon}\alpha^{1-\epsilon}(1-\beta)^{1-\epsilon}(1-\tau_l)^{1-\epsilon}\left(\frac{r}{1-\alpha}\right)^{1-\epsilon}}. \tag{30}$$

Substituting (29) into (30), we obtain the second condition that relates the growth rate to labor supply:

$$(1-\beta)^{\frac{\epsilon}{1-\epsilon}}\left[\left(\frac{\theta g+\delta_h+\rho}{B(1-l)}\right)^{\epsilon-1}-\beta^{\epsilon}\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$= \left[(1-\beta)\,\alpha\,(1-\tau_l)\,A^{1-\frac{1}{\xi}}\right]^{\epsilon}[\Omega(g)]^{\frac{\epsilon}{\xi}}. \tag{31}$$

The above two equilibrium conditions, (28) and (31), determine the equilibrium values of the growth rate (g) and labor supply ($1-l$). In what follows, we use these two conditions to investigate the following two issues: (i) How do tax policies affect the growth rate? and (ii) How do the growth effects of taxes respond to the elasticities of substitution between production factors in the final good and human capital production sectors? Since the two equations are highly non-linear, we will not attempt to examine these issues analytically. Instead, we will perform numerical simulations to answer these questions.

4 Growth Effects of Taxation: Numerical Results

4.1 Parameterization

To perform numerical simulations, we need to choose the values of the model's parameters for the benchmark economy. There are three types of parameters:

(i) preference parameters: η, ρ and θ;

(ii) technology parameters: A, B, α, β, δ_k, δ_h, ϵ and ξ;

(iii) tax parameters, τ_c, τ_k and τ_l.

We choose the values of the model's parameters to mimic an economy similar to the US economy. We first choose the values of the model's parameters for a benchmark economy, and then vary the values of some parameters such as the tax policies and the elasticities of substitution to see how the tax policies influence the growth rate under different assumptions concerning the final good and human capital production technologies.

We set $\alpha = 2/3$, $\beta = 0.7$, $\rho = 0.05$, $\theta = 2$, and $\delta_k = \delta_h = 0.08$. (These parameters are summarized in Table 1.) The value of the capital's share in final goods production ($\alpha = 2/3$) and the value of the household's subjective discount rate ($\rho = 0.05$) are very close to those used in the literature (e.g., King and Rebelo (1990)). The value of the elasticity of marginal utility ($\theta = 2$) and the depreciation rates of physical and human capital ($\delta_k = \delta_h = 0.08$) are also in the range of values used in the literature (e.g., Lucas (1990), Pecorino (1993), Kydland and Prescott (1982), Jones et al (1993)).[5] Since there is little precise information about the human capital technology, we assume that $\beta = 0.7$, larger than the value used in the King and Rebelo (1990) model.

[5]For the depreciation rate of physical capital, Kydland and Prescott (1982) and Jones et al (1993) use $\delta_k = 0.1$ as their estimate. Judd (1987) estimates $\delta_k = 0.12$, while Jorgenson and Yun (1991) suggest a smaller value for δ_k, near 0.06. The value of the depreciation rate of human capital has been estimated in the applied labor economics literature. There is a variety of evidence on the magnitude of the depreciation rate of human capital: Mincer's (1974) estimate of $\delta_h = 0.012$ for individuals is the lowest one that we ever found. Haley (1976) reports the estimates in the range of 3-4 percent. Heckman's (1976) estimates of δ_h range from 4 percent to 9 percent (but these estimates seem very sensitive to the specification of the model). Rosen's (1976) estimates vary from 5 percent (high school graduates in 1960) to 19 percent (college graduates in 1970). King and Rebelo (1990) use $\delta_h = 0.1$ as their estimate. Devereux and Love (1994) assume the depreciation rates of physical and human capital to be the same across and set them equal to 0.1. Because of the wide variance in these estimates, we set δ_k=δ_h=0.08.

Table 1: Benchmark Parameter Values

labor share in final goods production	α	2/3
labor share in human capital production	β	0.7
time preference	ρ	0.05
elasticity of marginal utility	θ	2
depreciation rate of physical capital	δ_k	0.08
depreciation rate of human capital	δ_h	0.08

Since our focus is on how the growth effects of taxation respond to the elasticities of substitution between factors in the final good and human capital sectors, we need to carefully consider the values of these two elasticities (ξ and ϵ). A wide range of values for these elasticities have been found and used in the literature.

Several empirical studies found that the elasticity of substitution (ξ) in goods production is *close to unity* (e.g., Nadiri (1970), Griliches (1967)). Many theoretical papers that examine the growth effects of taxation assume that the elasticity of substitution is unity and thus employ Cobb-Douglas production functions (e.g., King and Rebelo (1990), Jones et al (1993)).

However, many empirical studies found that the elasticity of substitution is *less than unity*. Kravis (1959) estimated $\xi = 0.64$ over the period 1900-1957. Arrow et al. (1961) estimated $\xi = 0.57$ for 1909-1949 and Kendrick and Sato (1963) estimated $\xi = 0.58$ for 1919-1960. Yuhn (1991) reports estimates in the range of 0.078-0.763. Chirinko et al (2004) suggest a smaller value, near 0.44. Antras (2004) uses $\xi = 0.8$ as their estimate. Klump et al (2007) estimate $\xi = 0.6$. Devesh Raval (2011) estimates that ξ ranges from 42 percent to 67 percent. A number of theoretical papers assume that the elasticity of substitution is less than one. For example, $\xi = 0.6$ is used in Lucas (1990).

Also, quite a few studies show that the elasticity of substitution is *greater than one*. For example, using cross-state data, Chiswick (1979) estimated both pair-wise elasticities range from 2.2 to 2.9 for 1909 and 1919. Berndt and Christensen (1974) showed that the "shadow" elasticities of substitution varies from 1.2 to 2.7 for 1929, 1939, 1949, 1959, and 1968. More recently, Karagiannis et al (2004) found that the elasticity is greater than one.

In addition to the elasticity of substitution in final goods production (ξ), we also need to choose the value of the elasticity of substitution in human capital production (ϵ). Unfortunately, very few attempts have been made to estimate this elasticity. Gyimah-Brempong and Gyapong (1992) used a value less than one for this elasticity. Since we believe that labor and physical inputs are less substitutable in education than in final goods production, we assume that the elasticity of substitution in human capital production (ϵ) is less than that in final goods production (ξ).

To consider a wide range of final goods and human capital production technologies, we will investigate the following five cases:

(i) Both production functions are Cobb-Douglas ($\xi = 1.0$ and $\epsilon = 1.0$);

(ii) Both elasticities are less than unity ($\xi = 0.5$ and $\epsilon = 0.3$);

(iii) Both elasticities are greater than unity ($\xi = 1.5$ and $\epsilon = 1.1$);

(iv) The elasticity in final goods production is greater than unity while the human capital production function is Cobb-Douglas ($\xi = 1.4$ and $\epsilon = 1.0$);

(v) The final goods production function is Cobb-Douglas while the elasticity in human capital production is less than unity ($\xi = 1.0$ and $\epsilon = 0.6$).

The two productivity parameters, A and B, are then set so as to achieve the desired trend growth rate in the benchmark economy. We follow Devereux and Love (1994) to choose a growth rate of 2 percent after tax. Then without taxes, we set the growth rate at 4 percent.

The importance of leisure η (along with the productivity parameter A and B) should be set so as to satisfy the following two conditions (i) the trend growth rate is set equal to around 4 percent without taxation; and (ii) households spend about 40 percent of their time for leisure.[6] The values of A, B and η are reported in Table 2.

For the tax policy parameters in the benchmark economy, we follow Devereux and Love (1994) to assume that there is no consumption taxes, and that both the labor and capital income tax are 20%.

Table 2: Benchmark Parameter Values and the Equilibrium Growth Rate

elasticity of substitution	g	l	value of parameters
$\xi = 1.5$, $\epsilon = 1.1$,	0.04	0.40	$A = 2.745$, $B = 0.33$, $\eta = 0.647$
$\xi = 1.4$, $\epsilon = 1.0$,	0.04	0.40	$A = 3.445$, $B = 0.305$, $\eta = 0.724$
$\xi = 1.0$, $\epsilon = 1.0$,	0.04	0.40	$A = 3.75$, $B = 0.375$, $\eta = 1.197$
$\xi = 1.0$, $\epsilon = 0.6$,	0.04	0.40	$A = 3.875$, $B = 0.385$, $\eta = 1.22$
$\xi = 0.5$, $\epsilon = 0.3$,	0.04	0.40	$A = 4.58$, $B = 0.41$, $\eta = 1.45$

4.2 Growth Effects of Taxation

We are now in a position to numerically investigate how various tax policies affect the steady-state growth rate and how the magnitudes of the policy effects on growth respond to the two elasticities of substitution in the final goods and human capital production sectors.

[6] For households' time allocation, see, for example, Prescott (1986) and Benhabib et al (1991).

To obtain the numerical results, we first solve the equation system (28) and (31) for the balanced growth rate (g) and leisure (l) and then substitute these values into the other equilibrium conditions to obtain the values of other variables. Tables 3, 5 and 7 report the results for different tax regimes in the first three cases.[7] Tables 4, 6 and 8 compare the percentage changes in the growth rate under different tax regimes.

Part I of each table compares the balanced growth solutions for the benchmark economy with the solutions in a number of alternative tax regimes (*equal percentage changes in taxes*). Part II reports the results for three different changes in the tax regime that provide a present value of tax revenue equivalent to that provided by imposing a 10% tax (i.e., $\tau_k = \tau_l = \tau = 0.1$) on both capital and labor income (*revenue-equivalent changes in taxes*). The first row in each table illustrates the effect of eliminating all distortional taxation (the benchmark case). The subsequent rows vary the tax rates in order to see the different tax effects on the growth rate and other variables.

Table 3: Balanced Growth Solution in Case (i)

$\xi = 1.0,\ \epsilon = 1.0,\ A = 3.75,\ B = 0.375,\ \eta = 1.197$											
τ_c	τ_k	τ_l	g	l	r	ϕ	e/k	c/k	y/k	C/Y	E/Y
I Equal Percentage Changes in Taxes											
0	0	0	0.040	0.40	0.21	0.60	0.12	0.39	0.63	0.62	0.19
0	0	0.2	0.025	0.46	0.18	0.59	0.09	0.35	0.54	0.65	0.16
0	0.2	0	0.035	0.41	0.25	0.60	0.14	0.49	0.75	0.65	0.19
0	0.2	0.2	0.021	0.47	0.21	0.59	0.10	0.44	0.64	0.68	0.16
II Revenue-Equivalent Changes in Taxes (Equivalent to τ=0.1)											
0.162	0	0	0.035	0.43	0.20	0.60	0.12	0.37	0.60	0.62	0.19
0	0.300	0	0.033	0.42	0.28	0.60	0.16	0.56	0.84	0.67	0.19
0	0	0.150	0.029	0.45	0.19	0.59	0.09	0.36	0.56	0.64	0.17
0	0.1	0.1	0.031	0.44	0.21	0.60	0.11	0.41	0.64	0.65	0.17

[7]The results for the last two cases are reported in the Appendix (Tables 11-14).

Table 4: Percentage Changes in the Growth Rate in Case (i)

τ_c	τ_k	τ_l	g	Δg	$\Delta g/g$
I Equal Percentage Changes in Taxes					
0	0	0	0.04004		
0	0	0.2	0.02508	-0.01496	-0.37371
0	0.2	0	0.03519	-0.00485	-0.12107
0	0.2	0.2	0.02077	-0.01927	-0.48136
II Revenue-Equivalent Changes in Taxes (Equivalent to τ=0.1)					
0	0	0	0.04004		
0.16154	0	0	0.03484	-0.00520	-0.12989
0	0.29994	0	0.03261	-0.00743	-0.18564
0	0	0.15001	0.02898	-0.01106	-0.27618
0	0.1	0.1	0.03052	-0.00952	-0.23781

Table 3 reports the impact of a rise in the income tax to 20% as well as of separate rises in wage and capital taxes. We have the following results: (i) Both the labor and capital income taxes reduce the growth rate; (ii) For both equal percentage and revenue-equivalent changes, the labor income tax has a larger negative effect on growth than the capital income tax. Table 4 shows that, for equal percentage changes, when the labor (capital) income tax τ_l (τ_k) increases from 0 to 20%, the growth rate decreases by 37.4% (12.1%). Similarly, for revenue-equivalent changes, when the labor (capital) income tax τ_l (τ_k) increases from 0 to 15% (30%), the growth rate decreases by 27.6% (18.6%).[8]

Table 3 also reports the changes in the other variables under different tax regimes. For example, both the labor and capital income taxes increase leisure (and thus

[8]For revenue-equivalent changes, the consumption tax has the smallest negative effect on growth. When the consumption tax increases from 0 to 16.2%, the growth rate drops by 13.0%.

decrease labor supply), but the labor income tax raises leisure by more than the capital income tax; The labor income tax discourages investment in education while the capital income tax discourages investment in physical capital; Both the labor and capital income taxes encourage consumption. Also, as in Devereux and Love (1994), there are only small changes in the value of ϕ. Although labor input in human capital production is not subject to taxes, the only use of human capital is for final goods production which is taxed. That is, labor in final goods production is taxed now while labor in human capital production will be taxed in the future. As a result, taxes do have large effects on the allocation of labor between the two activities.

Table 5: Balanced Growth Solution in Case (ii)

$\xi = 0.5$, $\epsilon = 0.3$, $A = 4.58$, $B = 0.41$, $\eta = 1.45$											
τ_c	τ_k	τ_l	g	l	r	ϕ	e/k	c/k	y/k	C/Y	E/Y
I Equal Percentage Changes											
0	0	0	0.040	0.40	0.21	0.54	0.31	1.27	1.70	0.75	0.18
0	0	0.2	0.026	0.46	0.18	0.54	0.26	1.21	1.58	0.77	0.17
0	0.2	0	0.038	0.41	0.26	0.54	0.34	1.42	1.88	0.76	0.18
0	0.2	0.2	0.024	0.47	0.22	0.55	0.29	1.35	1.75	0.77	0.17
II Revenue-Equivalent Changes (Equivalent to τ=0.1)											
0.134	0	0	0.035	0.43	0.20	0.54	0.30	1.24	1.66	0.75	0.18
0	0.809	0	0.023	0.46	0.92	0.55	0.66	2.80	3.56	0.79	0.18
0	0	0.114	0.032	0.43	0.19	0.54	0.28	1.24	1.63	0.76	0.17
0	0.1	0.1	0.032	0.43	0.22	0.54	0.30	1.31	1.72	0.76	0.17

Tables 5 and 6 report the results for Case (ii), in which both the elasticity of substitution in final goods production (ξ) and the elasticity of substitution in human capital production (ϵ) are less than unity. As in Case (i), both the labor and capital income taxes lower the growth rate and, for equal percentage change, the labor income tax reduces the growth rate by more than the capital income tax. When the labor (capital) income tax τ_l (τ_k) increases from 0 to 20%, the growth rate decreases by 36.1% (4.4%). However, unlike Case (i), for revenue-equivalent

Table 6: Percentage Changes in the Growth Rate in Case (ii)

τ_c	τ_k	τ_l	g	Δg	$\Delta g/g$
I Equal Percentage Changes					
0	0	0	0.04001		
0	0	0.2	0.02556	-0.01445	
					-0.36117
0	0.2	0	0.03824	-0.00177	
					-0.04431
0	0.2	0.2	0.02397	-0.01604	
					-0.40093
II Revenue-Equivalent Changes (Equivalent to τ=0.1)					
0	0	0	0.04001		
0.13358	0	0	0.03524	-0.00477	
					-0.11924
0	0.80885	0	0.02314	-0.01687	
					-0.42166
0	0	0.11411	0.03211	-0.00790	
					-0.19750
0	0.1	0.1	0.03235	-0.00767	
					-0.19159

changes, the capital income tax has a stronger effect on growth than the labor income tax. When the labor (capital) income tax τ_l (τ_k) increases from 0 to 11.4% (80.9%), the growth rate decreases by 19.8% (42.2%). We can see that reducing the elasticities of substitution tends to weaken the distortionary effects of taxation on growth.[9]

Tables 7 and 8 contain the results for Case (iii), in which both the elasticity of substitution in final goods production (ξ) and the elasticity of substitution in human capital production (ϵ) are greater than unity. As in Case (ii), both the labor and capital income taxes reduce the growth rate and, for both equal percentage and revenue-equivalent changes, the labor income tax reduces the growth rate by more than the capital income tax. We also find that increasing the elasticities

[9]A revenue-equivalent change in the capital income tax is an exception due to the very high tax rate resulting from a much smaller tax base.

Table 7: Balanced Growth Solution in Case (iii)

$\xi = 1.5,\ \epsilon = 1.1,\ A = 2.745,\ B = 0.33,\ \eta = 0.647$												
τ_c	τ_k	τ_l	g	l	r	ϕ	e/k	c/k	y/k	C/Y	E/Y	
I Equal Percentage Changes												
0	0	0	0.040	0.40	0.21	0.62	0.028	0.15	0.30	0.51	0.09	
0	0	0.2	0.027	0.49	0.18	0.61	0.016	0.13	0.25	0.50	0.06	
0	0.2	0	0.031	0.40	0.24	0.61	0.040	0.22	0.37	0.59	0.11	
0	0.2	0.2	0.019	0.49	0.21	0.60	0.023	0.18	0.30	0.59	0.08	
II Revenue-Equivalent Changes (Equivalent to τ=0.1)												
0.196	0	0	0.035	0.45	0.20	0.61	0.28	0.50	0.08			
0	0.144	0	0.033	0.40	0.23	0.61	0.04	0.20	0.35	0.57	0.10	
0	0	0.329	0.018	0.57	0.17	0.61	0.01	0.10	0.21	0.49	0.05	
0	0.1	0.1	0.029	0.44	0.21	0.61	0.03	0.17	0.30	0.55	0.08	

Table 8: Percentage Changes in the Growth Rate in Case (iii)

τ_c	τ_k	τ_l	g	Δg	$\Delta g/g$
I Equal Percentage Changes					
0	0	0	0.04002		
0	0	0.2	0.02734	-0.01268	
					-0.31687
0	0.2	0	0.03105	-0.00897	
					-0.22416
0	0.2	0.2	0.01862	-0.02140	
					-0.53470
II Revenue-Equivalent Changes (Equivalent to τ=0.1)					
0	0	0	0.04002		
0.19617	0	0	0.03457	-0.00545	
					-0.30872
0	0.14373	0	0.03342	-0.00660	
					-0.16486
0	0	0.32866	0.01778	-0.02224	
					-0.55576
0	0.1	0.1	0.02932	-0.01069	
					-0.26720

of substitution tends to strengthen the distortionary effects of taxation on growth.

The results for the last two cases are reported in the Appendix (Tables 11-14). These results give us more or less the same information about the impact of taxation and the role of the two elasticities of substitution as those in the first three cases. All the five cases will be discussed in next subsection.

4.3 The Role of Elasticities of Substitution

In this subsection, we will briefly discuss the connection between the elasticities of substitution and the growth effects of taxation.

Tables 9 and 10 compare the impacts on growth of equal percentage and revenue equivalent changes in taxes.

Table 9: Growth Effects of Equal Percentage Changes in Taxes

τ_c	τ_k	τ_l	$\xi = 0.5$	$\xi = 1$	$\xi = 1$	$\xi = 1.4$	$\xi = 1.5$
			$\epsilon = 0.3$	$\epsilon = 0.6$	$\epsilon = 1$	$\epsilon = 1$	$\epsilon = 1.1$
0	0	0.2	-0.36117	-0.34480	-0.37371	-0.31203	-0.31687
0	0.2	0	-0.04436	-0.10570	-0.12107	-0.21485	-0.22416
0	0.2	0.2	-0.40093	-0.44406	-0.48136	-0.51991	-0.53470

From Table 9, we can easily see that the negative effects of factor income taxes on growth are stronger when the two elasticities of substitution between inputs are higher. That is, when the elasticities of substitution are high, the factor impacts are more substitutable. As a result, the taxes have stronger distortionary effects on the inter-sectoral allocation of resources, leading to lager changes in the rate of return on investment and in the growth rate.

The labor income tax has a larger effect on growth than the capital income tax. The reason is as follows, the growth rate depends on the rate of return on investment in physical and human capital. All the taxes we consider here reduce the rate of return on investment. The relative size of the growth effect of factor taxes on the rate of return on investment is positively related to the factor share in final goods production. Under our parameterization, the share of human capital is 2/3. As a result, for equal percentage changes, the labor income tax has a larger growth effect than the capital income tax.

Table 10: Growth Effects of Revenue-Equivalent Changes in Taxes

$\tau = 0.1$	$\xi = 0.5$	$\xi = 1$	$\xi = 1$	$\xi = 1.4$	$\xi = 1.5$
	$\epsilon = 0.3$	$\epsilon = 0.6$	$\epsilon = 1$	$\epsilon = 1$	$\epsilon = 1.1$
τ_c	-0.11924	-0.12642	-0.12989	-0.13360	-0.13615
τ_k	-0.42166	-0.16274	-0.18564	-0.16558	-0.16486
τ_l	-0.19750	-0.25336	-0.27618	-0.47402	-0.55576
$\tau_k = \tau_l = 0.1$	-0.19159	-0.21593	-0.23781	-0.25905	-0.26720

As in Table 9, Table 10 also shows that, with higher elasticites of substitution, factor income taxes have stronger impacts on growth for the same reason as explained above.

For revenue-equivalent changes, the ranking of the taxes in terms of the growth effects depends on the elasticities of substitution. When the elasticities are low, the capital income tax has a larger effect on growth than the labor income tax; when the elasticities are high, the opposite is true. The consumption tax always has the smallest negative effect on growth.

For revenue-equivalent changes, the growth effect of a tax depends on two factors: (i) the relative size of the negative growth effect of the tax. (The labor income tax has a larger negative effect than the capital income tax.) (ii) the relative size of the tax base. (The capital income tax base is in general smaller than the labor income tax base; As a result,the capital income tax rate has to be higher than the labor income tax rate.) For revenue-equivalent changes, when the elasticities are low, the second factor dominates, the smaller base of the capital income tax leads to the capital income tax being more harmful to growth than the labor income tax. When the elasticities are high, the first factor dominates, therefore the labor income tax reduces the growth rate by more than the capital income tax.

5 Conclusions

In the thesis, we have developed a general two sector endogenous growth model and numerically investigated how various tax policies affect economic growth and how these growth effects respond to changes in the elasticities of substitution in the two sectors.

Under reasonable parameterization, we find that the negative effects of taxation on economic growth are stronger when the elasticities of substitution between inputs in the two sectors are higher. When the elasticities of substitution are low, the production factors are less substitutable. As a result, the distortions created by taxes tend to be relatively small. When the elasticities of substitution are high, the production factors are more substitutable. therefore, taxes generate relatively large distortions. These results are in sharp contrast with those in the literature. [10]

We also find that, for equal percentage changes, the labor income tax is more

[10] For example, Stokey and Rebelo (1995) argue that the elasticities of substitution in production are not critical for growth effects. Many studies empoly Cobb-Douglas production functions in all sectors to examine policy issues.

harmful to growth than the capital income tax. This result confirms the finding in Devereux and Love (1994).

Another important finding in this thesis is that, for revenue equivalent changes, the ranking of the taxes in terms of their impact on growth depends on the elasticities of substitutions in the two sectors. When the elasticities are low, the capital income tax has a lager negative effect on growth than the labor income tax; When the elasticities are high, the opposite is true.

We believe that our findings help us to further understand the impact of tax policies on economic growth and thus contribute to the literature on taxation and growth.

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Appendix

I. Numerical results for Case (iv) ($\xi = 1.4$ and $\epsilon = 1$).

Table 11: Balanced Growth Solution in Case (iv)

$\xi = 1.4$, $\epsilon = 1$, $A = 3.445$, $B = 0.305$, $\eta = 0.724$											
τ_c	τ_k	τ_l	g	l	r	ϕ	e/k	c/k	y/k	C/Y	E/Y
I Equal Percentage Changes											
0	0	0	0.040	0.40	0.21	0.60	0.03	0.17	0.32	0.53	0.10
0	0	0.2	0.028	0.48	0.19	0.59	0.02	0.14	0.27	0.53	0.07
0	0.2	0	0.031	0.41	0.25	0.60	0.04	0.23	0.39	0.60	0.11
0	0.2	0.2	0.019	0.49	0.21	0.59	0.03	0.19	0.32	0.61	0.08
II Revenue-Equivalent Changes (Equivalent to τ=0.1)											
0.190	0	0	0.035	0.45	0.20	0.60	0.03	0.15	0.30	0.52	0.09
0	0.152	0	0.033	0.41	0.23	0.60	0.04	0.21	0.37	0.58	0.11
0	0	0.292	0.021	0.52	0.17	0.59	0.01	0.13	0.24	0.52	0.06
0	0.1	0.1	0.030	0.44	0.21	0.59	0.03	0.18	0.32	0.57	0.09

Table 12: Percentage Changes in the Growth Rate in Case (iv)

τ_c	τ_k	τ_l	g	Δg	$\Delta g/g$
I Equal Percentage Changes					
0	0	0	0.04001		
0	0	0.2	0.02752	-0.01248	
					-0.31203
0	0.2	0	0.03141	-0.00860	
					-0.21485
0	0.2	0.2	0.01921	-0.02080	
					-0.51991
II Revenue-Equivalent Changes (Equivalent to τ=0.1)					
0	0	0	0.04001		
0.18998	0	0	0.03466	-0.00535	
					-0.13360
0	0.15205	0	0.03338	-0.00662	
					-0.16558
0	0	0.29211	0.02104	-0.01896	
					-0.47402
0	0.1	0.1	0.02964	-0.01036	
					-0.25905

II. Numerical results for Case (v) ($\xi = 1$ and $\epsilon = 0.6$).

Table 13: Balanced Growth Solution in Case (v)

$\xi = 1$, $\epsilon = 0.6$, $A = 3.875$, $B = 0.385$, $\eta = 1.22$											
τ_c	τ_k	τ_l	g	l	r	ϕ	e/k	c/k	y/k	C/Y	E/Y
I Equal Percentage Changes											
0	0	0	0.040	0.40	0.21	0.56	0.10	0.41	0.63	0.66	0.15
0	0	0.2	0.026	0.46	0.18	0.55	0.07	0.37	0.55	0.68	0.13
0	0.2	0	0.036	0.41	0.25	0.56	0.12	0.52	0.76	0.69	0.16
0	0.2	0.2	0.022	0.47	0.22	0.55	0.09	0.46	0.65	0.71	0.14
II Revenue-Equivalent Changes (Equivalent to τ=0.1)											
0.152	0	0	0.035	0.43	0.20	0.55	0.09	0.39	0.60	0.66	0.15
0	0.300	0	0.034	0.42	0.28	0.56	0.14	0.60	0.84	0.71	0.16
0	0	0.150	0.030	0.44	0.19	0.55	0.08	0.38	0.57	0.67	0.14
0	0.1	0.1	0.031	0.44	0.21	0.56	0.09	0.44	0.64	0.68	0.14

Table 14: Percentage Changes in the Growth Rate in Case (v)

τ_c	τ_k	τ_l	g	Δg	$\Delta g/g$
I Equal Percentage Changes					
0	0	0	0.04003		
0	0	0.2	0.02623	-0.01380	
					-0.34480
0	0.2	0	0.03580	-0.00423	
					-0.10570
0	0.2	0.2	0.02226	-0.01778	
					-0.44406
II Revenue-Equivalent Changes (Equivalent to τ=0.1)					
0	0	0	0.04003		
0.15220	0	0	0.03497	-0.00506	
					-0.12642
0	0.30006	0	0.03352	-0.00651	
					-0.16274
0	0	0.14999	0.02989	-0.01014	
					-0.25336
0	0.1	0.1	0.03139	-0.00865	
					-0.21597