

**NUMERICAL INVESTIGATION OF FLUID-
STRUCTURE INTERACTION NEAR THE SEA-BED**

SHEN LINWEI

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Summary

In this dissertation, two- and three-dimensional numerical models have been developed to study the physics of fluid-structure interactions near the sea bed. Apart from the details of flow separation and vortex generation, the models have been developed with the ability to model arbitrary moving boundaries, including the free surface and the boundary of a moving solid object, in inertial Cartesian coordinate system without the need for grid regeneration. The two-dimensional model solves the Navier-Stokes equations directly. The three-dimensional model is based on the filtered Navier-Stokes equations, in which the large eddies are solved explicitly and the subgrid-scale flow effects are modeled using a dynamic eddy viscosity model.

A numerical scheme based on immersed boundary (IB) method has been developed to capture the physics of fluid interactions with both stationary and moving objects. Based on the IB methodology, general formulae have been derived to calculate the hydrodynamic force components acting on objects of arbitrary shapes, either stationary or in motion. The formulae highlight the fact that the imposed force term introduced in the IB method contributes to both the force applied by the object on the fluid and the unsteady flow inside the virtual space occupied by the object in the IB formulation. The derived formulae are particularly useful when dealing with objects in unsteady motion. Several case studies, including the simulation of the vortex-induced vibration (VIV) of a circular cylinder, are carried out in the thesis to demonstrate the flexibility of the IB scheme.

In the integrated modeling effort, free surface motions are tracked with the Volume of Fluid (VOF) method including the use of a piecewise linear interface calculation (PLIC) scheme. The free surface boundary condition for the pressure is accurately satisfied in the computation.

Both the IB and VOF methods have been validated separately by carrying out various case studies in two and three dimensions. In addition, the combined two-dimensional IB-VOF model has also been rigorously validated by carrying out several case studies, including the simulations of a solitary wave passing over a shelf and a submerged rectangular object, progressive periodic waves propagating over a submerged trapezoid, and wave generation induced by a moving bed. The numerical results compared very well with the experimental data and numerical results reported by other researchers, including the details of free surface evolutions and the velocity field near the object. These case studies not only exhibit the model's capability in predicting the nonlinear free surface movement accurately in the presence of a submerged stationary object of regular/irregular geometry, but also demonstrate the model's ability to deal with the moving solid boundary in a flexible manner.

The well-validated IB-VOF model has been applied to simulate the progressive periodic waves propagating over a circular cylinder close to or sitting on the bed. The mechanism of the boundary layers separation, vortex shedding and advection of vortices is explored by carefully examining the instantaneous flow fields in the first two wave cycles. For the case of a small separation gap between the cylinder and the sea bed ($e = 0.1D$), the flow is characterized by a jet-like flow between the cylinder and the bed and the vortex shedding mode is found to be of the type '2P+S'. In the case of a cylinder sitting on the bed ($e = 0$), a vortex pair (or vorticity concentration pair) is formed in each half wave cycle and the vortex shedding mode is 'P+S'. In the latter case, the vortex pair rotates due to the unequal strengths of the two vortices and therefore moves towards the bed. This mechanism, together with the bed-shear stress distributions, clearly explains the possible occurrence of local scouring which is also observed in laboratory experiments.

The integrated three-dimensional model has also been carefully validated by simulating the cases of a solitary wave passing over a submerged rectangular body and a uniform flow over a bed-mounted short circular cylinder. The model's capability in handling three-dimensional objects of complex geometry is demonstrated. The numerical model has been applied to examine the case of regular waves propagating over a bed-mounted horizontal short circular cylinder. In this study, the Reynolds number is around 40,000 and the Keulegan-Carpenter (KC) number is about 3.5. A vortex is generated and shed behind the cylinder in each half wave cycle and the end effect is very small. Under this condition, the numerical results show that the three-dimensional model can still predict reasonable flow fields to some extent.

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List of Symbols

a	Wave amplitude
a_r	Reflected wave amplitude
A	Matrix with the coefficients a_e , a_s , etc; Cylinder oscillatory amplitude
a_e, a_w, a_s, a_n	Coefficients in PPE.
B	Matrix with the coefficient $b_{i,j}$
$b_{i,j}$	Coefficient in PPE
c	Damping coefficient
c^*	Dimensionless damping coefficient
C_D	Drag coefficient
\bar{C}_D	Mean drag coefficient
C_{Damp}	Amplitude of the periodic oscillatory coefficient C_D
C_L	Lift force coefficient
\bar{C}_L	Mean lift force coefficient
C_{Lmax}	Amplitude of the periodic oscillatory coefficient C_L
C_{Lrms}	Root-mean-square lift coefficient
C_P	Pressure coefficient
C_ρ	Variable associated with a straight line or a plane
C_{PB}	Pressure coefficient at base point
\bar{C}_{PB}	Mean pressure coefficient at base point

C_{PS}	Pressure coefficient at stagnation point
\bar{C}_{PS}	Mean pressure coefficient at stagnation point
C_S	Smagorinsky constant
C_x	In-line force coefficient
CFL	Courant – Friedrichs – Lewy number
D	Cylinder size
d	Sand size
e	Gap between the cylinder and the bed
f	Imposed force term; color function; frequency
F	Hydrodynamic force
F_x, F_y, F_z	Hydrodynamic force components in x-, y- and z-direction respectively
F_L	Lift force
g	Magnitude of gravitational acceleration
G	Low-pass filter function; Wake vertical region length
h	Water depth
H	Wave height
k	Spring constant
k^*	Dimensionless spring constant
KC	Keulegan Carpenter number
L	Length scale
l_x, l_y	Numerical sponge layer sizes in x- and y-direction, respectively
m	Cylinder mass
m^*	Dimensionless cylinder mass

m_d	$= \rho\Lambda$, fluid mass occupied by the submerged solid object
n	Normal vector; number of the oscillation cycle
\bar{n}	Normal vector
p	Fluid pressure
P	Matrix with discrete pressure $p_{i,j}$
Pe	Grid Peclet number
Re	Reynolds number
\bar{S}_{ij}	Strain-rate tensor
S_{out}	Bounding surface of domain Ω'
St	Strouhal number
t	Time
T	Wave period; oscillating flow period
t_c	Bed motion duration time
U_∞	Amplitude of incoming oscillating velocity or oscillating speed
u, v, w	Fluid velocity component in x-, y-, and z-direction
$\hat{u}, \hat{v}, \hat{w}$	Fluid velocity component in \hat{x} -, \hat{y} -, and \hat{z} -direction
$\tilde{u}, \tilde{v}, \tilde{w}$	Intermediate fluid velocity component in x-, y-, and z-direction
u_b	Incoming horizontal velocity amplitude near bed
u_c	Uniform current velocity
u_w	Horizontal velocity amplitude associated with wave motion
u_τ	Friction velocity
V	Volume of a cell
x	Longitudinal coordinate in Cartesian system

x_e, x_s	Start and end points of a numerical sponge layer in x-direction
x_R, x_L	Distances from two scouring holes (on the each side of the cylinder) to the cylinder
y	Lateral coordinate in Cartesian system
z	Vertical coordinate in Cartesian system
z_b, z_f	Bottom and free surface positions of a numerical sponge layer in z-direction
$\hat{x}, \hat{y}, \hat{z}$	Coordinates in body-fixed Cartesian system

Greek Symbols

α	Sponge layer damping coefficient; stagnation angle; vortex departing angle; $\alpha = u_c/u_w$; $0.5D\omega/U_\infty$
α_f	Artificial negative coefficient in feedback forcing model
β_f	Artificial negative coefficient in feedback forcing model
Γ	Strength of vortex or vorticity concentration
Γ^*	Dimensionless parameter of Γ , $\Gamma^* = \Gamma/\pi u_b D$
δ_{ij}	Kronecker delta
$\bar{\Delta}$	Filter width
Δt	Time step
$\Delta x, \Delta y, \Delta z$	Grid size in x-, y- and z-direction respectively
ε	Internal energy; prescribed small positive value
η	Kolmogorov scale
θ	Cylinder orientation

θ_s	Flow separation angle
κ	Wave number; artificial coefficient in feedback forcing model
λ_2	Second largest eigenvalue of a symmetric tensor used to identify a vortex
Λ	Volume of the enclosed domain Ω'
μ	Dynamic viscosity coefficient
ν	Kinematic molecular viscosity coefficient
ν_T	Eddy viscosity coefficient
ξ	Distance of the sea-bed movement
ξ_0	Bed motion displacement
π	=3.141592654...partition coefficient
ρ	Fluid density
τ	Stress
ω	Flow vorticity; oscillating frequency; angular velocity of the rotating cylinder
ω_k	Weight function
ω_i	i -th mode frequency
Ω	Flow domain bounded by S_{out} and S_b
Ω'	Flow domain equals $\Omega + \Omega'$
Ω''	Virtual flow domain enclosed by S_b
Ψ	Stream function

Subscripts

b	Solid boundary
c	Cylinder

i, j	Tensor notations
i, j, k	Grid position
n	Normal direction
rms	Root-mean-square
S_b	Body surface
x, y, z	Components in x, y, z direction
w	Wall

Superscripts

n	Number of time step lengths
$+$	In wall unit