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MODELING AND CONTROL OF MARINE FLEXIBLE SYSTEMS

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Contents

Summary

Modeling and control of marine flexible systems under the time-varying ocean disturbances is a challenging task and has received increasing attention in recent years with growing offshore engineering demands involving varied applications. There is a need to develop a general control framework to achieve the performance for the concerned systems. The main purpose of the research in this thesis is to develop advance strategies for the control of marine flexible systems with guaranteed stability. By investigating the characteristics of these flexible models, boundary control combining with the robust adaptive approaches are presented for three classes of marine flexible systems, i.e., mooring systems, installation systems, and riser systems. Numerical simulations are extensively carried out to illustrate the effectiveness of the proposed control.

Firstly, for the control of a thruster assisted position mooring system, the mathematical model of the flexible mooring lines is modeled as a distributed parameter system by using the Hamilton's method. Exact model based boundary control is applied at the top boundary of the mooring lines to suppress the vessel's vibrations. Adaptive control is designed to handle the system parametric uncertainties. With the proposed boundary control, uniform boundedness of the system under the ocean current disturbances is achieved. The proposed control is implementable with actual instrumentations since all the signals in the control can be measured by sensors or calculated by using of a backward difference algorithm.

Furthermore, robust adaptive boundary control of a marine installation system is developed to position the subsea payload to the desired set-point and suppress the cable's vibration. The flexible cable coupled with vessel and payload dynamics is described by a distributed parameter system with one partial differential equation (PDE) and two ordinary differential equations (ODEs). Boundary control is proposed at the top and bottom boundary of the cable based on the Lyapunov's direct method. Considering the system parametric uncertainties and the unknown ocean disturbances, the developed adaptive boundary control schemes achieve uniform boundedness of the steady state error between the boundary payload and the desired position. The control performance of the closed-loop system is guaranteed by suitably choosing the design parameters.

Thirdly, a coupled nonlinear flexible marine riser is investigated. Using the Hamilton's principle, we derive the dynamic behavior of the flexible riser represented by a set of nonlinear PDEs. After further investigation of the properties of the riser, we propose the boundary control at the top boundary of the riser based on the Lyapunov's direct method to regulate the riser's vibrations. The boundary control is implemented by two actuators in transverse and longitudinal directions. With the proposed boundary control, uniform boundedness of the riser system under the ocean current disturbances and exponential stability under the free vibration condition are achieved. The proposed control is independent of system parameters, which ensures the robustness of the system to variations in parameters.

Finally, boundary control of a flexible marine riser with the vessel dynamics is studied. Both the dynamics of the vessel and the vibration of the riser are considered in the dynamic analysis, which make the system more difficult to control. Boundary control is proposed at the top boundary of the riser to suppress the riser's vibration. Adaptive control is designed when the system parametric uncertainties exist. With the proposed robust adaptive boundary control, uniform boundedness of the system under the ocean current disturbances can be achieved. The state of the system is proven to converge to a small neighborhood of zero by appropriately choosing the design parameters.

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List of Symbols

Throughout this thesis, the following notations and conventions have been adopted:

t	temporal variable
x	spatial variable
L	length of flexible structure
$M_s,\ M$	mass of surface vessel
m	mass of payload
d_s	surface vessel motion damping
$w(L,t)$	position of the vessel
$\dot{w}(L,t)$,	velocity of the vessel
$\ddot{w}(L,t)$	acceleration of the vessel
ρ	uniform mass per unit length of the flexible structure
EI	uniform flexural rigidity of riser
T	tension of the flexible structures
E	Young's modulus
A	cable cross section area
$f(x,t)$	time-varying distributed disturbance
$d(t)$	boundary disturbance
$\bar{f}$	upper bound of the time-varying distributed disturbance
$\bar{d}$	upper bound for the boundary disturbance

$c,\ c_1,\ c_2$	damping coefficient
$u(t)$	control input
$u_1(t)$, $u_2(t)$	control input at $x = 0, L$ respectively
E_k	kinetic energy
E_p	potential energy
W_f	work done by ocean disturbances
W_d	work done by damping
W_m	work done by control input
W	work done by nonconservative force
δ	variation operator
$w(x,t)$	elastic transverse displacement
$v(x,t)$	elastic longitudinal displacement
$y(x,t)$	position of the cable
$U(t)$	ocean surface current velocity
$U(x,t)$	current profile
$\bar{U}$	mean current
D	external diameter of the flexible systems
$C_D(x,t)$	drag coefficient
$C_L(x,t)$	lift coefficient
f_v	vortex shedding frequency
S_t	Strouhal number
ρ_s	density of seawater
θ	phase angle
A_D	the amplitude of the oscillatory part of the drag force
p_d	desired set-point
$d_1,\ d_2$	damping coefficient at $x = 0,\ L$
n	the total number of the mooring lines
D_i	the distance between the ith mooring line and the coordinate point
k, k_p, k_v	control gains

$\lambda,\ \lambda_1$ to λ_3	positive constants
δ_1 to δ_5	small positive constants
$X - Y$	the fixed inertia frame
$x - y$	the local reference frame
$\alpha,\ \beta$	positive weighting constants
Γ	adaptation gain
r	positive constant
R	the set of all real numbers
$\|\|A\|\|$	the Euclidean norm of vector A or the induced norm of matrix A
A^T	the transpose of vector or matrix A
$e(t)$	the error signal
Ω	compact set
Δt	temporal step size
Δx	spatial step size
N_1	the subdivisions of the length
N_2	the subdivisions of the time interval
t_f	the duration of the simulation
w_i	the frequencies of sinusoids singal
$\lambda_{\min}(A)$	minimum eigenvalue of the matrix A where all eigenvalues are real
$\lambda_{\max}(A)$	maxmum eigenvalue of the matrix A where all eigenvalues are real
$(\hat{*})$	estimate of $(*)$
$(\tilde{*})$	estimate error of $(*)$
$(*)',\ (*)''$	first, second order derivatives of $(*)$ with respect to x
$(*)''',\ (*)''''$	third, fourth order derivatives of $(*)$ with respect to x
$(\dot{*}),\ (\ddot{*})$	first, second order derivatives of $(*)$ with respect to t

Chapter 1

Introduction

1.1 Background and Motivation

In recent decades, dealing with the vibration problem of flexible systems has become an important research topic, driven by practical needs and theoretical challenges. Lightweight mechanical flexible systems possess many advantages over conventional rigid ones, such as lower cost, better energy efficiency, higher operation speed, and improved mobility. These advantages greatly motivate the applications of the mechanical flexible systems in industry. A large number of systems can be modeled as mechanical flexible systems such as telephone wires, conveyor belts, crane cables, helicopter blades, robotic arms, mooring lines, marine risers and so on. However, unwanted vibrations due to the flexibility property and the time-varying disturbances restrict the utility of these flexible systems in different engineering applications.

Offshore engineering is concerned with the design and operation of the systems both above and below the water. With the increased focus on offshore oil and gas development in deeper and harsher environments, researches on offshore engineering have gained increasing attention. Modeling and control of marine flexible systems compatible with the extreme marine environmental conditions is a most challenging task in offshore engineering. Development of a general frame for control of the marine flexible systems in the presence of the unknown ocean disturbances is a quite challenging research topic. The mooring system, installation system and riser system can be modeled as a set of PDEs with the infinite dimensionality, which are the key components of the modern offshore engineering, and serve a variety of functions. These marine applications are characterized by the time-varying environmental disturbances and the sea conditions. Vibration and deformation of the flexible structures in offshore engineering due to the ocean current disturbances and the tension exerted at the top can produce premature fatigue problems, which require inspections and costly repairs. The proper control technique is desirable and available for preventing damage and improving the lifespan of these structures.

In comparison with the dynamic positioning system, the thruster assisted position mooring system for the anchored vessel is an economical solution in deep waters due to the long operational period in harsh environmental conditions. Floating concepts such as the use of Floating Production Offloading and Storage (FPSO) vessels in combination with subsea systems and shuttle tankers have become possible with the use of sophisticated positioning systems for precise and safe positioning. The two main types of positioning systems are the dynamic positioning systems for free floating vessels and the thruster assisted position mooring system for anchored vessels. Many results have been obtained for control of dynamic positioning systems in recent years by using model based approach [1,2] and backstepping based approaches [3,4]. In [5], the problem of tracking a desired trajectory is discussed for a fully actuated ocean vessel with dynamic positioning system, in the presence of parametric uncertainties and the unknown disturbances. In [6], a hybrid controller is developed to extend the operability and performance of the dynamic positioning system. Station keeping means maintaining the vessel within a desired position in the horizontal-plane, which has been identified as one of the typical problems in offshore engineering. A typical thruster assisted position mooring system consists of an ocean surface vessel and several flexible mooring lines. The surface vessel, to which the top boundary of the mooring lines is connected, is equipped with a dynamic positioning system with active thrusters. The bottom boundary of the mooring lines is fixed in the ocean floor by the anchors. Station keeping for the mooring system is hard to achieve due to the complicated system model and the unknown time-varying ocean disturbances including the ocean current, wave, and wind. The mooring lines spanning a long distance can produce large vibrations under the ocean disturbances, which can degrade the performance of the system and result in a larger offset from the target position of the vessel.

Marine installation system is used as the accurate position control for marine installation operation in offshore engineering. Accurate position control for marine installation operations has gained increasing attention in recent years [7, 8]. Due to the requirements for high accuracy and efficiency arising from the modern ocean industry, improving reliability and efficiency of installation operations during oil and gas production in the ocean environment is an active research topic that has received much attention in offshore engineering. A typical marine installation system consists of an ocean surface vessel, a flexible string-type cable and a subsea payload to be positioned for installation on the ocean floor. The surface vessel, to which the top boundary of the cable is connected, is equipped with a dynamic positioning system with an active thruster. The bottom boundary of the cable is a payload with an end-point thruster attached. This thruster is used for dynamic positioning of the payload. The total marine installation system is subjected to the environmental disturbances including the ocean current, wave, and wind. Taking into account the unknown time-varying ocean disturbances of the cable leads to the appearance of oscillations, which make the control problem of the marine installation system relatively difficult. Vibration suppression and position control by proper control technique is desirable and feasible for the marine installation system.

The marine riser is used as a fluid-conveyed curved pipe drilling crude oil, natural gas, hydrocarbon, petroleum materials, mud, and other undersea economic resources, and then transporting those resources in the ocean floor to the production vessel or platform in the ocean surface [9]. A drilling riser is used for drilling pipe protection and transportation of the drilling mud, while a production riser is a pipe used for oil transportation [10]. The stiffness of a flexible marine riser depends on its tension and length, thus a riser that spans a long distance can produce large vibrations under the relatively small disturbances. In marine environment, vibrations excited by vortices can degrade the performance of the flexible marine riser. Vibrations of the riser due to the ocean current disturbances and the tension exerted at the top can produce premature fatigue problems, which requires inspections and costly repairs, and as a worst case, environmental pollution due to leakage from damaged areas. Vibration suppression by proper control techniques is desirable for preventing damage and improving the lifespan of the riser.

The remainder of this chapter is organized as follows. In Section 1.1.1, a brief introduction of the control techniques for flexible mechanical systems, especially for flexible string and beam systems, is presented. Background knowledge of flexible systems is given first, and then the recent researches on boundary control of flexible systems are discussed. Some research problems to be studied in this thesis are highlighted, such as boundary control and robust adaptive control, which are both theoretically challenging and practically meaningful. In Section 1.1.2, control methods for flexible marine systems are briefly reviewed, where the researches on control of mooring systems, installation systems and riser systems are discussed. Finally, in Section 1.2, the objectives, scope, as well as the organization of the thesis are presented.

1.1.1 Flexible Mechanical Systems

Many physical processes, cannot be modeled by ODEs since the state of the system depends on more than one independent variable [11]. The state of a given physical system such as flexible structure, fluid dynamics and heat transfer may depend on the time t and the location x. The flexible mechanical systems are independent of the spatial and temporal variables, which can be modeled as the distributed parameter systems. The model are represented by a set of infinite dimensional equations (i.e., PDEs describing the dynamics of the flexible bodies) coupled with a set of finite dimensional equations (i.e., ODEs describing the boundary conditions). The dynamics of the flexible mechanical system modeled by a set of PDEs is difficult to control due to the infinite dimensionality of the system, since many control strategies for the conventional rigid body system cannot be directly applied to solve the control problem of the flexible system.

The most popular control approaches for the distributed parameter systems are modal control based on the truncated discredited system model, distributed control by using distributed sensors and actuators, and boundary control. Modal control for the distributed parameter systems is based on truncated finite dimensional modes of the system, which are derived from element method, Galerkin's method or assumed modes method [12–20]. For these finite dimensional models, many control techniques developed for ODE systems in [21–25] can be applied. The truncated models are obtained via the model analysis or spatial discretization, in which the flexibility is represented by a finite number of modes by neglecting the higher frequency modes. The problems arising from the truncation procedure in the modeling need to be carefully treated in practical applications. A potential drawback in the above control design approaches is that the control can cause the actual system to become unstable due to excitation of the unmodeled, high-frequency vibration modes (i.e., spillover effects) [26]. Spillover effects which result in instability of the system have been investigated in [27,28] when the control of the truncated system is restricted to a few critical modes. The control order needs to be increased with the number of flexible modes considered to achieve high accuracy of performance and the control may also be difficult to implement from the engineering point of view since full states measurements or observers are often required. In an attempt to overcome the above shortcomings of the truncated model based modal control, boundary control where the actuation and sensing are applied only through the boundary of the system utilizes the distributed parameter model with PDEs to avoid control spillover instabilities. Boundary control combining with other control methodologies such as variable structure control [29], sliding model control [30], energy-based robust control [31,32], model-free control [33], the averaging method [34–38], and robust adaptive control [39, 40] have been developed. In these approaches, system dynamics analysis and control design are carried out directly based on the PDEs of the system.

Distributed control [41–45] requires relative more actuators and sensors, which makes the distributed controller relatively difficult to implement. Compared with distributed controllers, boundary control is an economical method to control the distributed parameter system without decomposing the system into the finite dimensional space. Boundary control is considered to be more practical in a number of research fields including the vibration control of flexible structures, fluid dynamics and heat transfer, which requires few sensors and actuators. In addition, the kinetic energy, the potential energy, and the work done by the nonconservative forces in the process of modeling can be directly used to design the Lyapunov function of the closed loop system.

The relevant applications for boundary control approaches in mechanical flexible structures consist of second order structures (strings, and cables) and fourth order structures (beams and plates) [46]. The Lyapunov's direct method is widely used since the Lyapunov functionals for control design closely relate to kinetic, potential and work energies of the distributed parameter systems. Based on the Lyapunovs direct method, the authors in [10,20,26,29–33,39,40,47–78] have presented the results for the boundary control of the flexible mechanical systems. In [39], robust adaptive boundary control is investigated to reduce the vibration for a moving string with the spatiotemporally varying tension. In [56], robust and adaptive boundary control is developed to stabilize the vibration of a stretched string on a moving transporter. In [59], a boundary controller for a linear gantry crane model with a flexible string-type cable is developed and experimentally implemented. An active boundary control system is introduced in [60] to damp undesirable vibrations in a cable. In [63], the asymptotic and exponential stability of an axially moving string is proved by using a linear and nonlinear state feedback. In [79], a flexible rotor with boundary control has been illustrated and the experimental implementation of the flexible rotor controller is also presented. Boundary control has been applied to beams in [80] where boundary feedback is used to stabilize the wave equations and design active constrained layer damping. Active boundary control of an Euler-Bernoulli beam which enables the generation of a desired boundary condition at any designators position of a beam structure has been investigated in [81]. In [65], a nonlinear control law is constructed to exponentially stabilize a free transversely vibrating beam via boundary control. In [72, 73], a boundary controller for the flexible marine riser with actuator dynamics is designed based on the Lyapunov's direct method and the backstepping technique. In [76], a linear boundary velocity feedback control is designed to ensure exponential stabilization of the vibration of a nonlinear moving string. In [61], boundary control of a nonlinear string has been investigated where feedback from the velocity at the boundary of a string is proposed to stabilize the vibrations. It is notable that robust and adaptive control schemes have been applied to the boundary control design in [39, 40, 56]. By using Laplace transform to derive the exact solution of the wave equation, boundary impedance control for a string system is investigated in [62]. Recently, by combining the backstepping method with adaptive control design, a novel boundary controller and observer are designed to stabilize the string and beam model and tracking the target system. Many remarkable results in this area have been obtained in [74, 82–94]. However, this boundary control method is hard to be applied to the marine flexible systems due to difficulties in finding a proper gain kernel. For example, it is hard to find a gain kernel for the model of the mooring system subjected to the unknown ocean disturbances.

In the literatures of boundary control for the distributed parameter systems, functional analysis and semigroup theory are usually used for the stability analysis and the proof of the existence and uniqueness of PDEs, for example [95–103]. Such distributed parameter systems are described by operator equations on an infinite dimensional Hilbert or Banach space [104–106]. The stability analysis and the solution existence are based on the theory of semigroup on the infinite dimensional state space. In [72], the proof of existence and uniqueness of the control system is carried out by using the infinite dimensional state space. In [39], the asymptotic stability the system with proposed control is proved through the use of semigroup theory. In [95], stability and stabilization of different infinite dimensional systems are studied based on semigroup theory. In [94], semigroup theory is utilized to prove the strong stability of a one-dimensional wave equation with proposed boundary control. In [100], stabilization of a second order PDE system under non-collocated control and observations is investigated in Hilbert spaces. In [107], a non-collocated boundary control is developed to stabilize two connected strings with the joint anti-damping, and the exponentially stability is proved by using the semigroup theory. With control at one end and noncollocated observation at another end, the exponential stability of the closed-loop system is proved in [101]. In [102, 103], a uniformly exponentially stable observer is designed for a class of second-order distributed parameter systems, and the uniqueness and stability of the system are proved based on semigroup theory.

Compared with the functional analysis based methods, the Lyapunov's direct method for the distributed parameter systems requires little background beyond calculus for users to understand the control design and the stability analysis. In addition, the Lyapunov's direct method provides a convenient technique for PDEs by using well-understood mathematical tools such as algebraic and integral inequalities, and integration by parts.

1.1.2 Marine Flexible Systems

The three most common marine flexible systems, mooring systems, installation systems, and riser systems, are consisted by different flexible mechanical systems such as beam and string. Many good results [108–112] for control design of the mooring system in the literatures rely on the ODE model with neglecting the dynamics of the mooring lines. These works on the control of the thruster assisted position mooring systems mainly focus on the dynamics of the vessel, and the dynamics of the mooring lines are usually ignored for the convenience of the control design. In earlier research [108], a nonlinear passive observer for thruster assisted position moored ships has been developed, where the force from the mooring lines are regarded as external forces and mooring system is modeled as an ODE system. A finite element model of a single mooring line is derived in [113], but the control is not proposed for the system. More recently, by using a structural reliability measure for the mooring lines, the paper [109] proposes the control to maintain the probability of the mooring line failure below an acceptable level regardless of changing weather conditions. In [110], the switching control is designed for a positioning mooring system which allows the thrusters to assist the mooring system in the varying environmental conditions. In [112], the modeling and control of a positioning mooring system with a drilling riser is investigated. In these works, the dynamics of the mooring lines is considered as an external force term to the vessel dynamics. These kind of model can influence the dynamic response of the whole mooring system due to the neglect of the coupling between the vessel and the mooring lines. To overcome this shortcoming, in this thesis, the mooring system is represented by a number of PDEs describing the dynamics of the mooring lines coupled with four ODEs describing the lumped vessel dynamics. The paper [114] investigates the station keeping and tension problem in order to avoid line tensions rising for the multi-cable mooring systems, in which the dynamics of the mooring lines are modeled as PDEs. But the paper does not provide the detailed discussion for the control design. Considering a mooring system with arbitrary mooring lines, the system is governed by nonhomogeneous hyperbolic PDEs, which makes the system model quite different compared with the previous works due to the coupling between the mooring lines and the vessel.

Traditional marine installation systems consist of the vessel dynamic positioning and crane manipulation to obtain the desired position and heading for the payload [115, 116]. Such methods become difficult in deeper waters due to the longer cable between the surface vessel and payload. The longer cable increases the natural period of the cable and payload system which in turn increase the effects of oscillations. One solution to alleviate the precision installation problem is the addition of thrusters attached the payload for the installation operation [7,117,118]. Such marine installation system consists of an ocean surface vessel, a flexible string-type cable and a subsea payload to be positioned for installation on the ocean floor. The control for the dynamic positioning of the payload is challenging due to the unpredictable exogenous disturbances such as fluctuating currents and transmission of motions from the surface vessel through the lift cable. The unknown time-varying ocean disturbances along the cable lead to the appearance of oscillations. Current researches [7, 8] on the control of the marine installation systems mainly focus on the dynamics of the payload, where the dynamics of the cable is ignored for the convenience of the control design. The dynamics of the cable is considered as an external force term to the payload. One drawback of the model is that it can influence the dynamic response of the whole marine installation system due to the neglect of the coupling between the vessel, the cable and the payload. To overcome this shortcoming, the flexible marine installation system with cable, vessel and payload dynamics is represented by a set of infinite dimensional equations, (i.e., PDEs describing the dynamics of the flexible cable) coupled with a set of finite dimensional equations, (i.e., ODEs describing the lumped vessel and payload dynamics).

In earlier works of marine flexible risers [119–121], the modeling of the riser systems is investigated, and the simulations with different numerical methods are provided to verify the effectiveness of the models. In [122, 123], distributed parameter models with PDEs have been used to analyze and investigate the dynamic response of the flexible marine riser under the ocean current disturbances. But the stability and control design are not mentioned in these works. The Timoshenko model also can provide an accurate beam model, which takes into account the rotary inertial energy and the deformation owing to shear. Compared with the Euler-Bernoulli model, the Timoshenko model is more accurate at predicting the beam's response. However, the Timoshenko model is more difficult to implement for control design due to its higher order. For this reason, most of the flexible marine risers with boundary control are based on the Euler-Bernoulli model [124]. In [73], boundary control for the flexible marine riser with actuator dynamics is designed based on the Lyapunov's direct method and the backstepping technique. In [72], the boundary control problem of a three-dimensional nonlinear inextensible riser system is considered via the same method as [73]. In [10], a torque actuator is introduced at the top boundary of the riser to reduce the angle and transverse vibration of the riser with guaranteed closed-loop stability. In [78], boundary control for a coupled nonlinear flexible marine riser with two actuators in transverse and longitudinal directions has been designed to suppress the riser's vibration. However, in these works, only the riser dynamics is considered and the coupling between the riser and the vessel is neglected, which can influence the dynamic response of the riser system and lead to an imprecise model. For the purpose of dynamic analysis, the riser is modeled as an Euler-Bernoulli beam structure with PDEs since the diameter-to-length of the riser is small. Based on the distributed parameter model, various kinds of control methods integrating computer software and hardware with sensors and actuators have been investigated to design control to suppress the riser's vibration.

1.2 Thesis Objectives and Organization

The general objectives of the thesis are to develop constructive and systematic methods of designing boundary control for marine flexible systems with guaranteed stability. By investigating the characteristics of several different flexible marine models, boundary control fused with robust adaptive approaches is proposed to achieve the performance for the concerned systems and mitigate the effects of spillover without truncating the continue system models.

The remainder of the thesis is organized as follows. In Chapter 2, some necessary mathematical preliminaries are given. We will provide the brief introduction of the Hamilton's principle, the models of the ocean disturbances and some useful inequalities, which will be used throughout the thesis.

In Chapter 3, we start with the study of modeling and control of a thruster assisted position mooring system. In the first place, the mathematical model of the flexible mooring lines is modeled as a distributed parameter system by using the Hamilton's method. Then, exact model based boundary control is applied at the top boundary of the mooring lines based on the Lyapunov's direct method to regulate the vessel's vibrations. In addition, adaptive control is designed to handle the system parametric uncertainties. With the proposed boundary control, uniform boundedness of the system under the ocean current disturbances is achieved. The proposed control is implementable with actual instrumentations since all the signals in the control can be measured by sensors or calculated by using of a backward difference algorithm.

In Chapter 4, robust adaptive boundary control of a marine installation system is developed to position the subsea payload to the desired set-point and suppress the cable's vibration. The flexible cable coupled with vessel and payload dynamics is described by a distributed parameter system with one partial differential equation (PDE) and two ordinary differential equations (ODEs). Boundary control is proposed at the top and bottom boundary of the cable based on the Lyapunov's direct method. Considering the system parametric uncertainty, the developed adaptive boundary control schemes achieve uniform boundedness of the steady state error between the boundary payload and the desired position. The control performance of the closed-loop system is guaranteed by suitably choosing the design parameters. Simulations are provided to illustrate the applicability and effectiveness of the proposed control.

Chapter 5 studies the modeling and control of a coupled nonlinear flexible marine riser subjected to the ocean current disturbances. Using the Hamilton's principle, we derive the dynamic behavior of the flexible riser represented by a set of nonlinear PDEs. After further investigation of the properties of the riser, we propose the boundary control at the top boundary of the riser based on the Lyapunov's direct method to regulate the riser's vibrations. The boundary control is implemented by two actuators in transverse and longitudinal directions. With the proposed boundary control, uniform boundedness under ocean current disturbances and exponential stability under free vibration condition are achieved. The proposed control is independent of system parameters, which ensures the robustness of the system to variations in parameters.

Chapter 6 further investigates the control problem of a flexible marine riser with considering the vessel dynamics. Compared with the model in Chapter 5, both the dynamics of the vessel and the vibration of the riser are considered in the dynamic analysis, which make the system more difficult to control. Boundary control is proposed at the top boundary of the riser suppress the riser's vibration. Adaptive control is designed when the system parametric uncertainty exists. With the proposed robust adaptive boundary control, uniform boundedness under ocean current disturbances can be achieved. The state of the system is proven to converge to a small neighborhood of zero by appropriately choosing design parameters.

Finally, Chapter 7 concludes the contributions of the thesis and makes recommendation on future research works.

Chapter 2

Mathematical Preliminaries

In this Chapter, we provide some mathematical preliminaries, useful technical lemmas, properties, the model of ocean disturbance which will be extensively used throughout this thesis. The chapter is organized as follows. Firstly, the Hamilton's principle is introduced in Section 2.1. Then, a brief introduction of the ocean disturbance on marine flexible structures is given in Section 2.2, followed by Section 2.3 about some useful technical lemmas for completeness.

2.1 The Hamilton's Principle

As opposed to lumped mechanical systems, flexible mechanical systems have an infinite number of degrees of freedom and the model of the system is described by using continuous functions of space and time. The Hamilton's principle permits the derivation of equations of motion from energy quantities in a variational form and generates the motion equations of the flexible mechanical systems. The Hamilton's principle [125, 126] is represented by

$$\int_{t_1}^{t_2} \delta(E_k - E_p + W)dt \quad = \quad 0, \tag{2.1}$$

where t_1 and t_2 are two time instants, $t_1 < t < t_2$ is the operating interval and δ denotes the variational operator, E_k and E_p are the kinetic and potential energies of the system respectively, W denotes work done by the nonconservative forces acting on the system, including internal tension, transverse load, linear structural damping and external disturbance. The principle states that the variation of the kinetic and potential energy plus the variation of work done by loads during any time interval $[t_1, t_2]$ must equal to zero.

There are some advantages using the Hamilton's principle to derive the mathematical model of the flexible mechanical systems. Firstly, this approach is independent of the coordinates and the boundary conditions can be automatically generated by this approach [46]. In addition, the kinetic energy, the potential energy, and the work done by the nonconservative forces in the Hamilton's principle can be directly used to design the Lyapunov function of the closed loop system.

2.2 The Ocean Disturbance on Marine Flexible Structures

Vortex-induced vibration (VIV) is a direct consequence of lift and drag oscillations due to the vortex shedding formation behind bluff bodies [127]. The marine flexible structures used in offshore production system may get out of control when the structural natural frequency of the risers and cables equals frequency of vortex shedding.

The effects of a time-varying ocean current, $U(x,t)$, on a riser or a cable can be modeled as a vortex excitation force [128, 129]. The current profile $U(x,t)$ is a function which relates the depth to the ocean surface current velocity $U(t)$. The distributed load on a marine flexible structure, $f(x,t)$, can be expressed as a combination of the in-line drag force, $f_D(x,t)$, consisting of a mean drag and an oscillating drag about the mean modeled as

$$f_D(x,t) = \frac{1}{2}\rho_s C_D(x,t)U(x,t)^2 D + A_D \cos(4\pi f_v(x,t)t + \theta), \tag{2.2}$$

and an oscillating lift force $f_L(x,t)$, perpendicular to $f_D(x,t)$, about a mean deflected profile,

$$f_L(x,t) = \frac{1}{2}\rho_s C_L(x,t)U(x,t)^2 D \cos(2\pi f_v(x,t)t + \vartheta), \tag{2.3}$$

where ρ_s is the sea water density, $C_D(x,t)$ and $C_L(x,t)$ are the time and spatially varying drag and lift coefficient respectively, D is the outer diameter of the flexible structures, $f_v(x,t)$ is the shedding frequency, θ and ϑ are the phase angles, and A_D is the amplitude of the oscillatory part of the drag force, typically 20% of the first term in $f_D(x,t)$ [129]. The non-dimensional vortex shedding frequency [10] can be expressed as

$$f_v(x,t) = \frac{S_t U(x,t)}{D}, \tag{2.4}$$

where S_t is the Strouhal number.

In this thesis, we consider the deflection of the marine flexible structures in transverse and longitudinal directions. Hence, the distributed load can be expressed as

$$f(x,t) = f_D(x,t)\frac{1}{2}\rho_s C_D(x,t)U(x,t)^2 D + A_D \cos(4\pi f_v(x,t)t + \theta), \tag{2.5}$$

The transverse vortex-induced vibration (VIV) from the lift component is not considered in this thesis but the proposed method can be similarly applied without any loss of generality if only the lift component is considered.

2.3 Lemmas

Lemma 2.1. *[130] Let $\phi_1(x,t) \in R$ and $\phi_1(x,t) \in R$ be functions defined on $x \in [0,L]$ and $t \in [0,\infty)$, the Cauchy-Schwarz inequality is:*

$$\int_0^L \phi_1\phi_2 dx \leq \left(\int_0^L \phi_1^2 dx\right)^{\frac{1}{2}} \left(\int_0^L \phi_2^2 dx\right)^{\frac{1}{2}} \tag{2.6}$$

Lemma 2.2. *[46] Let $\phi_1(x,t)$, $\phi_2(x,t) \in R$, the following inequalities hold:*

$$\phi_1\phi_2 \leq |\phi_1\phi_2| \leq \phi_1^2 + \phi_2^2, \quad \forall \phi_1, \phi_2 \in R. \tag{2.7}$$

Lemma 2.3. *[46] Let $\phi_1(x,t)$, $\phi_2(x,t) \in R$, the following inequalities hold:*

$$|\phi_1\phi_2| \quad = \quad \left|\left(\frac{1}{\sqrt{\delta}}\phi_1\right)(\sqrt{\delta}\phi_2)\right| \leq \frac{1}{\delta}\phi_1^2 + \delta\phi_2^2, \quad \forall \phi_1, \phi_2 \in R \quad \textit{and} \quad \delta > 0. \tag{2.8}$$

Lemma 2.4. *[131] Let $\phi(x,t) \in R$ be a function defined on $x \in [0,L]$ and $t \in [0,\infty)$*

that satisfies the boundary condition

$$\phi(0,t) = 0, \quad \forall t \in [0,\infty), \tag{2.9}$$

then the following inequalities hold:

$$\int_0^L \phi^2 dx \le L^2 \int_0^L [\phi']^2 dx, \tag{2.10}$$

$$\phi^2 \le L \int_0^L [\phi']^2 dx. \tag{2.11}$$

If in addition to Eq. (2.9)*, the function* $\phi(x,t)$ *satisfies the boundary condition*

$$\phi'(0,t) = 0, \quad \forall t \in [0,\infty), \tag{2.12}$$

then the following inequalities also hold:

$$[\phi']^2 \le L \int_0^L [\phi'']^2 dx. \tag{2.13}$$

Lemma 2.5. *Let* $\phi(x,t) \in R$ *be a function defined on* $x \in [0,L]$ *and* $t \in [0,\infty)$ *that satisfies the boundary condition*

$$\phi(0,t) = C, \quad \forall t \in [0,\infty), \tag{2.14}$$

where C *is a constant. Then the following inequality hold:*

$$(\phi - C)^2 \le L \int_0^L [\phi']^2 dx, \quad \forall x \in \forall (x,t) \in [0,L] \times [0,\infty). \tag{2.15}$$

Proof: Define $\phi_1(x,t) = \phi'(x,t)$ and $\phi_2(x,t) = \chi(s-x) = \begin{cases} 1, & x \leq s \\ 0, & x > s \end{cases}$, where $s \in [0,\ L)$ is a constant. Utilizing the Cauchy-Schwarz inequality, we have

$$\begin{aligned} \int_0^L \phi_1\phi_2 dx = \int_0^L \phi'(x,t)\chi(s-x)dx = \phi(s,t) - C &\leq s^{\frac{1}{2}} \left(\int_0^L [\phi']^2 dx\right)^{\frac{1}{2}} \\ &\leq L^{\frac{1}{2}} \left(\int_0^L [\phi']^2 dx\right)^{\frac{1}{2}} \end{aligned} \tag{2.16}$$

Therefore, we have

$$(\phi - C)^2 \leq L \int_0^L [\phi']^2 dx, \quad \forall x \in \forall (x,t) \in [0,L] \times [0,\infty). \tag{2.17}$$

■

Lemma 2.6. *[132] Rayleigh-Ritz theorem: Let $A \in R^{n\times n}$ be a real, symmetric, positive-definite matrix; therefore, all the eigenvalues of A are real and positive. Let $\lambda_{\min}$ and $\lambda_{\max}$ denote the minimum and maximum eigenvalues of A, respectively; then for $\forall x \in R^n$, we have*

$$\lambda_{\min}||x||^2 \leq x^T A x \leq \lambda_{\max}||x||^2, \tag{2.18}$$

where $||\cdot||$ denotes the standard Euclidean norm.

Chapter 3

Mooring System

3.1 Introduction

Recent years, with the increasing trend towards oil and gas exploitation in deep water ($> 500m$), fixed platforms based on the seabed have become impractical. Instead, floating platforms such as anchored Floating Production Storage and Offloading (FPSO) vessels with thruster assisted position mooring systems have been used widely. A thruster-assisted moored vessel is an economical solution for station keeping in deep water due to the long operational period in harsh environmental conditions. Station keeping means maintaining the vessel within a desired position in the horizontal-plane, which has been identified as one of the most typical problems in offshore engineering. The thruster assistance is required in harsh environmental conditions to avoid mooring line failure. A typical thruster assisted position mooring system consisting of an ocean surface vessel and a number of flexible mooring lines is shown in Fig. 3.1. The surface vessel, to which the top boundary of the mooring lines is connected, is equipped with a dynamic positioning system with active thrusters. The bottom boundary of the mooring lines is fixed in the ocean floor by the anchors. The total mooring system is subjected to environmental disturbances including ocean current, wave, and wind. The mooring lines that span a long distance can produce large vibrations under relatively small disturbances, which can degrade the performance of the system and result in a larger offset from the target position of the vessel. Taking into account the unknown time-varying ocean disturbances of the mooring lines leads to the appearance of oscillations, which make the control problem of the mooring system relatively difficult.

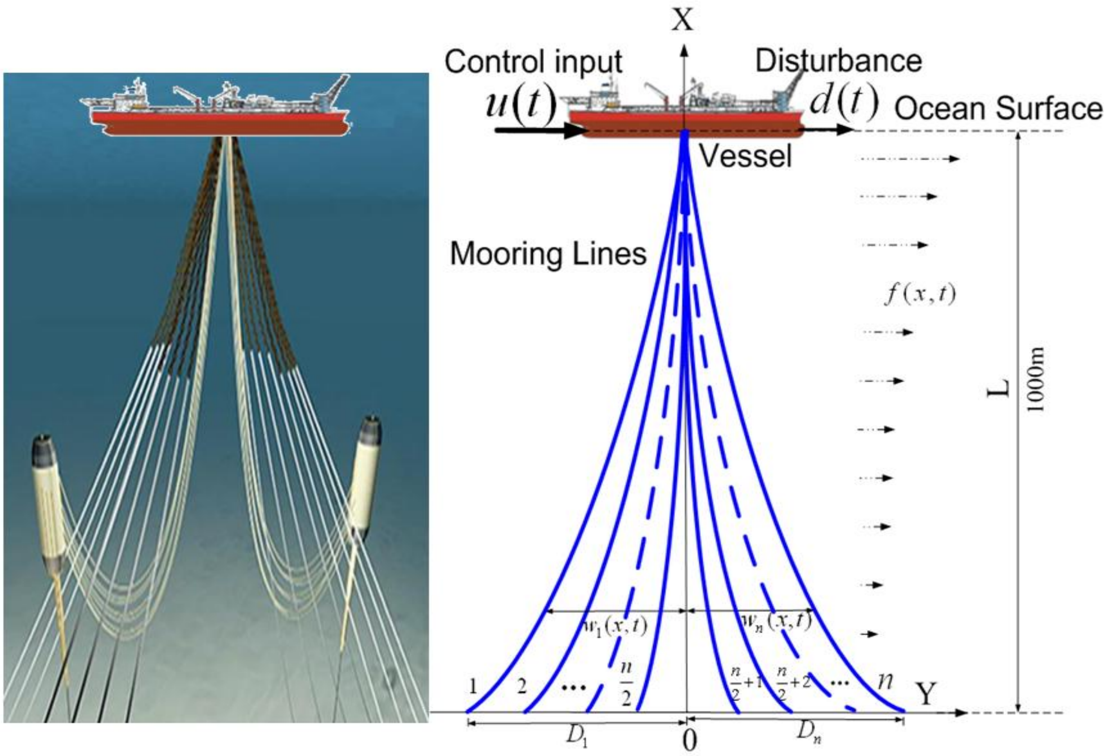


Fig. 3.1: A FPSO vessel with the thruster assisted position mooring system.

Earlier research on the control of the thruster assisted position mooring systems mainly focus on the dynamics of the vessel, where the dynamics of the mooring lines is usually ignored for the convenience of the control design. In [109–111], the dynamics of the mooring lines is considered as an external force term to the vessel dynamics.

One drawback of the model is that it can influence the dynamic response of the whole mooring system due to the neglect of the coupling between the vessel and the mooring lines. To overcome this shortcoming, in this chapter, the mooring system is represented by PDEs describing the dynamics of the mooring lines coupled with ODEs representing the lumped vessel dynamics. We design the boundary control based on the distributed parameter model of the mooring system. The stability analysis of the closed-loop system is based on the Lyapunov's direct method without resorting to semigroup theory or functional analysis. The main contributions of this chapter include:

(i) The dynamic model of a thruster assisted position mooring system with arbitrary mooring lines subjected to ocean current disturbance is derived for vibration suppression. The governing equation of the system is represented as nonhomogeneous hyperbolic PDEs.

(ii) Robust adaptive boundary control at the top boundary of the mooring lines is developed for station keeping of the vessel. Adaptation laws are designed to compensate for the system parametric uncertainties.

(iii) With the proposed boundary control, uniform boundedness of the mooring system under ocean disturbance is proved via Lyapunov synthesis. The control performance of the system is guaranteed by suitably choosing the design parameters.

The rest of the chapter is organized as follows. The governing equations (PDEs) and boundary conditions (ODEs) of the flexible mooring system are derived by use of the Hamilton's principle in Section 3.2. The boundary control design via the Lyapunov's direct method is discussed separately for both exact model case and system parametric uncertainty case in Section 3.3, where it is shown that the uniform boundedness of the closed-loop system can be achieved by the proposed control. Simulations are carried out to illustrate performance of the proposed control in Section 3.4. The conclusion of this chapter is presented in Section 3.5.

3.2 Problem Formulation

In this chapter, we assume that the vessel is at the top boundary of the mooring lines and all the mooring lines are filled with seawater. The flexible mooring lines with uniform density and flexural rigidity are modeled as the mechanical string structure. For the practical application of the thruster assisted position mooring system, there are a total of n (n is a even number) mooring lines in the system, in which $\frac{n}{2}$ mooring lines are located at the left and right hand sides of the vessel respectively. As shown in Fig. 3.1, the numbers of mooring lines in the the left hand side of the vessel are $1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}$, and the numbers of the mooring lines in the the right hand side of the vessel are $\frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n$.

The dynamics of the surface vessel can be modeled as

$$M\frac{\partial^2 w(L,t)}{\partial t^2} + d_s\frac{\partial w(L,t)}{\partial t} = u(t) - \tau(t) + d(t), \tag{3.1}$$

where $w(L,t)$, $\frac{\partial w(L,t)}{\partial t}$ and $\frac{\partial^2 w(L,t)}{\partial t^2}$ are the position, velocity and acceleration of the vessel respectively, M the mass of the surface vessel, d_s the damping, $u(t)$ the control force from controller actuation, $\tau(t)$ the tension force exerted on the vessel from the mooring lines, and $d(t)$ the unknown disturbance on the vessel due to the ocean wave, wind and current.

The kinetic energy of the mooring system E_k can be represented as

$$\begin{aligned} E_k &= \frac{1}{2}M\left[\frac{\partial w(L,t)}{\partial t}\right]^2 + \frac{1}{2}\sum_{i=1}^{\frac{n}{2}}\rho\int_0^L\left[\frac{\partial[w_i(x,t)+D_i]}{\partial t}\right]^2 dx \\ &+\frac{1}{2}\sum_{i=\frac{n}{2}+1}^{n}\rho\int_0^L\left[\frac{\partial[w_j(x,t)-D_i]}{\partial t}\right]^2 dx, \end{aligned} \tag{3.2}$$

where x and t represent the independent spatial and time variables respectively, $w_i(x,t)$ is the position of the ith mooring line at the position x for time t, $\rho > 0$ is the uniform mass per unit length of the mooring lines, L is the length of the mooring line, and D_i is the distance between the ith mooring line and the coordinate point at Y direction respectively. The actual vibration displacements of the mooring lines are $w_i(x,t)+D_i$ $(i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2})$ and $w_i(x,t)-D_i$ $(i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n)$ with respect to their equilibrium positions.

Remark 3.1. *From the Fig. 1, we can obtain the top positions of the mooring lines are equal to the position of the vessel, i.e.,*

$$w_i(L,t) = w(L,t), \tag{3.3}$$

$$\frac{\partial w_i(L,t)}{\partial t} = \frac{\partial w(L,t)}{\partial t}. \tag{3.4}$$

The potential energy of the mooring system E_p can be represented as

$$E_p = \frac{1}{2}\sum_{i=1}^{\frac{n}{2}}T\int_0^L\left[\frac{\partial[w_i(x,t)+D_i]}{\partial x}\right]^2 dx + \frac{1}{2}\sum_{i=\frac{n}{2}+1}^{n}T\int_0^L\left[\frac{\partial[w_i(x,t)-D_i]}{\partial x}\right]^2 dx, \tag{3.5}$$

The virtual work done by ocean current disturbances on the mooring lines and the vessel is given by

$$\delta W_f = \sum_{i=1}^{\frac{n}{2}} \int_0^L f(x,t)\delta[w_i(x,t) + D_i]dx + \sum_{i=\frac{n}{2}+1}^{n} \int_0^L f(x,t)\delta[w_i(x,t) - D_i]dx, \\ +d(t)w(L,t) \quad (3.6)$$

where $f(x,t)$ is the distributed transverse load on the mooring lines due to the hydrodynamic effects of the ocean current, and $d(t)$ denotes the environmental disturbance on the vessel. The virtual work done by damping on the mooring lines and the vessel is represented by

$$\delta W_d = -\sum_{i=1}^{\frac{n}{2}} \int_0^L c\left[\frac{\partial[w_i(x,t) + D_i]}{\partial t}\right]\delta[w_i(x,t) + D_i]dx \\ -\sum_{i=\frac{n}{2}+1}^{n} \int_0^L c\left[\frac{\partial[w_2(x,t) - D_2]}{\partial t}\right]\delta[w_i(x,t) - D_i]dx - d_s\frac{\partial w(L,t)}{\partial t}\delta w(L,t), \quad (3.7)$$

where $c > 0$ is the distributed damping coefficient for the mooring lines, and d_s denotes the damping for the vessel. We introduce the boundary control $u(t)$ at the top boundary of the mooring lines to produce a transverse motion for the vibration reduction. The virtual work done by the vessel can be written as

$$\delta W_m = u(t)\delta w(L,t), \quad (3.8)$$

and the total virtual work done on the system is given by

$$\begin{aligned}
\delta W &= \delta W_f + \delta W_d + \delta W_m \\
&= \sum_{i=1}^{\frac{n}{2}} \int_0^L \left[f(x,t) - c\frac{\partial[w_i(x,t)+D_i]}{\partial t} \right] \delta[w_i(x,t)+D_i]dx \\
&\quad + \sum_{i=\frac{n}{2}+1}^{n} \int_0^L \left[f(x,t) - c\frac{\partial[w_i(x,t)-D_i]}{\partial t} \right] \delta[w_i(x,t)-D_i]dx \\
&\quad + \left[u(t) + d(t) - d_s\frac{\partial w(L,t)}{\partial t} \right] \delta w(L,t). \qquad (3.9)
\end{aligned}$$

Substituting Eqs. (3.2), (3.5), and (3.9) into the Hamilton's principle Eq. (2.1), we obtain the governing equations of the system as

$$\rho\ddot{w}_i - Tw_i'' - f + c\dot{w}_i = 0,\ (i = 1,\ 2,\ 3,\ \ldots,\ n), \qquad (3.10)$$

$\forall(x,t) \in (0,L) \times [0,\infty)$, and the boundary conditions of the system as

$$\begin{aligned}
w_i(0,t) &= -D_i,\ (i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), && (3.11)\\
w_i(0,t) &= D_i,\ (i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n), && (3.12)\\
w_i(L,t) &= w(L,t),\ (i = 1,\ 2,\ 3,\ \ldots,\ n), && (3.13)\\
\sum_{i=1}^{n} Tw_i'(L,t) &= u(t) + d(t) - d_s\dot{w}(L,t) - M\ddot{w}(L,t),\ (i = 1,\ 2,\ 3,\ \ldots,\ n), && (3.14)
\end{aligned}$$

$\forall t \in [0,\infty)$.

Remark 3.2. *The notations $(*)'$, $(*)''$, $(*)'''$ and $(*)''''$ representing the first, second, third and forth order derivatives of $(*)$ with respect to x respectively, $(\dot{*})$ and $(\ddot{*})$ denoting the first and second order derivative of $(*)$ with respect to time t, respectively* *are used for clarity.*

Remark 3.3. *Combining the Eqs.* (3.1) *and* (3.14)*, we obtain the total tension force exerted on the vessel from all the mooring lines,* $\tau(t) = -\sum_{i=1}^{n} T w_i'(L, t)$.

Assumption 3.1. *For the distributed load* $f(x,t)$ *on the mooring lines and the disturbance* $d(t)$ *on the vessel, we assume that there exist constants* $\bar{f} \in R^+$ *and* $\bar{d} \in R^+$*, such that* $|f(x,t)| \leq \bar{f}$, $\forall(x,t) \in [0,L] \times [0,\infty)$ *and* $|d(t)| \leq \bar{d}$, $\forall(t) \in [0,\infty)$*. This is a reasonable assumption as the time-varying disturbances* $f(x,t)$ *and* $d(t)$ *have finite energy and hence are bounded, i.e.,* $f(x,t) \in \mathcal{L}_\infty([0,L])$ *and* $d(t) \in \mathcal{L}_\infty$.

Remark 3.4. *For control design in Section 3.3, only the assertion that there exist upper bounds on the disturbances in Assumption 1,* $|f(x,t)| < \bar{f}$, $|d(t)| \leq \bar{d}$*, is necessary. The knowledge of the exact values for* $f(x,t)$ *and* $d(t)$ *is not required. As such, different distributed load models up to various levels of fidelity, such as those found in [127, 128, 133–135], can be applied without affecting the control design or analysis.*

Property 3.1. *[136]:If the kinetic energy of the system* (3.10) - (3.14)*, given by Eq.* (3.2) *is bounded* $\forall t \in [0,\infty)$*, then* $\dot{w}(x,t)$ *and* $\dot{w}'(x,t)$ *are bounded* $\forall(x,t) \in [0,L] \times [0,\infty)$.

Property 3.2. *[136]: If the potential energy of the system* (3.10) - (3.14)*, given by Eq.* (3.5) *is bounded* $\forall t \in [0,\infty)$*, then* $w'(x,t)$ *and* $w''(x,t)$ *are bounded* $\forall(x,t) \in [0,L] \times [0,\infty)$.

3.3 Control Design

The control objective is to keep the marine vessel within an envelope around the desired position and stabilize the mooring lines at the small neighborhood of their equilibrium positions in the presence of the distributed transverse load $f(x,t)$ and the disturbance $d(t)$. In this section, the Lyapunov's direct method is used to construct a boundary control law $u(t)$ at the top boundary of the mooring lines and to analyze the close-loop stability of the system.

In this chapter, two cases are investigated for the mooring system: (i) exact model-based control, i.e. T, M and d_s are all known; and (ii) adaptive control for the system parametric uncertainty, i.e. T, M and d_s are all unknown. For the first case, boundary control is introduced for the exact model of the mooring system subjected to the ocean disturbances. For second case where the system parameters cannot be directly measured, the adaptive control is designed to compensate the system parametric uncertainties.

3.3.1 Boundary control based on exact model of the mooring system

To stabilize the system given by governing Eqs. (3.10) and boundary conditions Eqs. (3.11) - (3.14), we propose the following boundary control:

$$u = \sum_{i=1}^{n} T w_i'(L,t) - \sum_{i=1}^{n} M \dot{w}_i{}'(L,t) + d_s \dot{w}(L,t) - \operatorname{sgn}(u_a)\bar{d} - k_p w(L,t) - k u_a, \tag{3.15}$$

where $\operatorname{sgn}(\cdot)$ denotes the sign function, k_p and k are the control gains and the auxiliary signal u_a is defined as

$$u_a = \dot{w}(L,t) + \sum_{i=1}^{n} w_i'(L,t). \tag{3.16}$$

After differentiating the auxiliary signal Eq. (3.16), multiplying the resulting equation by M, and substituting Eq. (3.14), we obtain

$$M\dot{u}_a = -\sum_{i=1}^{n} T w_i'(L,t) + \sum_{i=1}^{n} M\dot{w}_i'(L,t) - d_s\dot{w}(L,t) + d + u. \tag{3.17}$$

We substitute Eq. (3.15) into Eq. (3.17), we have

$$M\dot{u}_a = -ku_a + d - \mathrm{sgn}(u_a)\bar{d} - k_p w(L,t). \tag{3.18}$$

Consider the Lyapunov function candidate

$$V(t) \;=\; V_1(t) + V_2(t) + \eta(t), \tag{3.19}$$

where the energy term $V_1(t)$, the auxiliary term $V_2(t)$ and the small crossing term $\eta(t)$ are defined as

$$V_1(t) = \frac{1}{2}\sum_{i=1}^{n}\beta\rho\int_0^L [\dot{w}_i(x,t)]^2dx + \frac{1}{2}\sum_{i=1}^{n}\beta T\int_0^L [w_i'(x,t)]^2dx + \frac{\beta}{2}k_p[w(L,t)]^2, \tag{3.20}$$

$$V_2(t) = \frac{1}{2}Mu_a^2(t), \tag{3.21}$$

$$\eta(t) = \sum_{i=1}^{n}\alpha\rho\int_0^L x\dot{w}_i(x,t)w_i'(x,t)dx, \tag{3.22}$$

where α and β are two positive weighting constants.

Lemma 3.1. *The Lyapunov function candidate given by Eq.* (3.19)*, can be upper and lower bounded as*

$$0 \le \lambda_1(V_1(t)+V_2(t)) \le V(t) \le \lambda_2(V_1(t)+V_2(t)), \tag{3.23}$$

where λ_1 *and* λ_2 *are two positive constants defined as*

$$\lambda_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}, \tag{3.24}$$

$$\lambda_2 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}, \tag{3.25}$$

provided

$$0 < \alpha < \frac{\min(\beta\rho, \beta T)}{2\rho L}. \tag{3.26}$$

Proof: Substituting of Ineq. (2.7) into Eq. (3.22) yields:

$$\begin{aligned} |\eta(t)| &\leq \sum_{i=1}^{n} \alpha\rho L \int_0^L ([\dot{w}_i(x,t)]^2 + [w_i'(x,t)]^2)dx \\ &\leq \alpha_1 V_1(t), \end{aligned} \tag{3.27}$$

where

$$\alpha_1 = \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}. \tag{3.28}$$

Then, we obtain

$$-\alpha_1 V_1(t) \leq \eta(t) \leq \alpha_1 V_1(t). \tag{3.29}$$

Considering α is a small positive weighting constant satisfying $0 < \alpha < \frac{\min(\beta\rho, \beta T)}{2\rho L}$, we can obtain

$$\alpha_2 = 1 - \alpha_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)} > 0, \tag{3.30}$$

$$\alpha_3 = 1 + \alpha_1 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)} > 1. \tag{3.31}$$

Then, we further have

$$0 \le \alpha_2 V_1(t) \le V_1(t) + \eta(t) \le \alpha_3 V_1(t). \tag{3.32}$$

Given the Lyapunov function candidate Eq. (3.19), we obtain

$$0 \le \lambda_1(V_1(t) + V_2(t)) \le V_1(t) + V_2(t) + \eta(t) \le \lambda_2(V_1(t) + V_2(t)), \tag{3.33}$$

where $\lambda_1 = \min(\alpha_2, 1) = \alpha_2$ and $\lambda_2 = \max(\alpha_3, 1) = \alpha_3$ are positive constants. ■

Lemma 3.2. *The time derivative of the Lyapunov function candidate Eq.* (3.19) *can be upper bounded with*

$$\dot{V}(t) \le -\lambda V(t) + \varepsilon, \tag{3.34}$$

where $\lambda > 0$ and $\varepsilon > 0$.

Proof: Differentiating Eq. (3.19) with respect to time leads to

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{\eta}(t). \tag{3.35}$$

The first term of the Eq. (3.35)

$$\dot{V}_1(t) = A_1 + A_2 + \beta k_p w(L,t)\dot{w}(L,t), \tag{3.36}$$

where

$$A_1 = \sum_{i=1}^{n} \beta\rho \int_0^L \dot{w}_i(x,t)\ddot{w}_i(x,t)dx, \tag{3.37}$$

$$A_2 = \sum_{i=1}^{n} \beta T \int_0^L w_i'(x,t)\dot{w}_i'(x,t)dx. \tag{3.38}$$

Substituting the governing equation (3.10) into A_1, we obtain

$$A_1 = \sum_{i=1}^{n} \beta \int_0^L \dot{w}_i(x,t)\left(Tw_i''(x,t) - c\dot{w}_i(x,t) + f(x,t)\right)dx. \tag{3.39}$$

Using the boundary conditions and integrating Eq. (3.38) by part, we obtain

$$\begin{aligned} A_2 &= \sum_{i=1}^{n} \beta T \int_0^L w_i'(x,t)d(\dot{w}_i(x,t)) \\ &= \sum_{i=1}^{n} \beta T w_i'(L,t)\dot{w}_i(L,t) - \sum_{i=1}^{n} \beta T \int_0^L \dot{w}_i(x,t)w_i''(x,t)dx. \end{aligned} \tag{3.40}$$

Substituting Eqs. (3.39) and (3.40) into Eq. (3.36), we have

$$\begin{aligned} \dot{V}_1(t) &= \sum_{i=1}^{n} \beta T w_i'(L,t)\dot{w}_i(L,t) - \sum_{i=1}^{n} \beta \int_0^L c[\dot{w}_i(x,t)]^2 dx \\ &\quad + \sum_{i=1}^{n} \beta \int_0^L f(x,t)\dot{w}_i(x,t)dx + \beta k_p w(L,t)\dot{w}(L,t). \end{aligned} \tag{3.41}$$

Substituting the Eq. (3.16) into Eq. (3.41), we obtain

$$\begin{aligned}\dot{V}_1(t) &= -\frac{\beta T}{2}\left[[\dot{w}(L,t)]^2+\sum_{i=1}^{n}[w_i'(L,t)]^2\right]+\frac{\beta T}{2}u_a^2(t)-\beta T\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}w_i'(L,t)w_j'(L,t)\\ &\quad -\sum_{i=1}^{n}\beta\int_0^L c[\dot{w}_i(x,t)]^2dx+\sum_{i=1}^{n}\beta\int_0^L f(x,t)\dot{w}_i(x,t)dx+\beta k_p w(L,t)\dot{w}(L,t).\end{aligned} \tag{3.42}$$

Using Ineq. (2.8), we obtain

$$\begin{aligned}\dot{V}_1(t) &\leq -\frac{\beta T}{2}\left[[\dot{w}(L,t)]^2+\sum_{i=1}^{n}[w_i'(L,t)]^2\right]+\frac{\beta T}{2}u_a^2(t)\\ &\quad +\sum_{i=1}^{n}(n-1)\beta T\left(\delta_1+\frac{1}{\delta_1}\right)[w_i'(L,t)]^2-\sum_{i=1}^{n}\beta\int_0^L c[\dot{w}_i(x,t)]^2dx\\ &\quad +\sum_{i=1}^{n}\beta\delta_2\int_0^L[\dot{w}_i(x,t)]^2dx+\frac{n\beta}{\delta_2}\int_0^L f^2(x,t)dx+\frac{\beta k_p}{\delta_3}[w(L,t)]^2\\ &\quad +\beta k_p\delta_3[\dot{w}(L,t)]^2,\end{aligned} \tag{3.43}$$

where δ_1, δ_2 and δ_3 are positive constants.

Substituting Eq. (3.18) into the second term of the Eq. (3.35), we have

$$\begin{aligned}\dot{V}_2(t) &= Mu_a(t)\dot{u}_a(t)\\ &= -ku_a^2(t)+d(t)u_a(t)-\mathrm{sgn}(u_a(t))\bar{d}u_a(t)-k_p w(L,t)u_a(t)\\ &\leq -ku_a^2(t)+k_p[w(L,t)]^2+k_p u_a^2(t)\\ &= -ku_a^2(t)-k_p[w(L,t)]^2+2k_p[w(L,t)]^2+k_p u_a^2(t).\end{aligned} \tag{3.44}$$

Using the Ineq. (2.17) and the boundary condition Eq. (3.11), we have

$$(w(L,t)+D_1)^2 \leq L\int_0^L [w_1'(x,t)]^2dx. \tag{3.45}$$

Since

$$|w(L,t)| - |D_1| \leq |w(L,t) + D_1| \leq \sqrt{L\int_0^L [w_1'(x,t)]^2 dx}, \tag{3.46}$$

we obtain

$$|w(L,t)|^2 \leq \left(\sqrt{L\int_0^L [w_1'(x,t)]^2 dx} + |D_1|\right)^2 \leq 2L\int_0^L [w_1'(x,t)]^2 dx + 2D_1^2. \tag{3.47}$$

Substituting the above equation into Eq. (3.44), we have

$$\dot{V}_2(t) \leq -ku_a^2(t) - k_p[w(L,t)]^2 + 4k_pD_1^2 + 4k_pL\int_0^L [w_1'(x,t)]^2 + k_pu_a^2(t). \tag{3.48}$$

The third term of the Eq. (3.35) is rewritten as

$$\begin{aligned} \dot{\eta}(t) &= \sum_{i=1}^n \alpha\rho\int_0^L [x\ddot{w}_i(x,t)w_i'(x,t) + x\dot{w}_i(x,t)\dot{w}_i'(x,t)]dx \\ &= \sum_{i=1}^n \alpha\int_0^L xw_i'(x,t)\left[Tw_i''(x,t) + f(x,t) - c\dot{w}_i(x,t)\right]dx \\ &\quad + \sum_{i=1}^n \alpha\rho\int_0^L x\dot{w}_i(x,t)\dot{w}_i'(x,t)dx \\ &= B_1 + B_2 + B_3 + B_4, \end{aligned} \tag{3.49}$$

where

$$
\begin{aligned}
B_1 &= \sum_{i=1}^{n} \alpha T \int_0^L x w_i'(x,t) w_i''(x,t) dx, && (3.50)\\
B_2 &= \sum_{i=1}^{n} \alpha \int_0^L f(x,t) x w_i'(x,t) dx, && (3.51)\\
B_3 &= \sum_{i=1}^{n} \alpha\rho \int_0^L x \dot{w}_i(x,t) \dot{w}_i'(x,t) dx, && (3.52)\\
B_4 &= -\sum_{i=1}^{n} \alpha c \int_0^L x w_i'(x,t) \dot{w}_i(x,t) dx. && (3.53)
\end{aligned}
$$

After integrating Eq. (3.50) by parts and using the boundary conditions, we obtain

$$
B_1 = \sum_{i=1}^{n} \alpha T L [w_i'(L,t)]^2 - \sum_{i=1}^{n} \alpha T \int_0^L \{[w_i'(x,t)]^2 + x w_i'(x,t) w_i''(x,t)\} dx.
$$

Combining Eq. (3.50) and Eq. (3.54), we obtain

$$
B_1 = \sum_{i=1}^{n} \frac{\alpha T L}{2} [w_i'(L,t)]^2 - \sum_{i=1}^{n} \frac{\alpha T}{2} \int_0^L [w_i'(x,t)]^2 dx. \tag{3.54}
$$

Using Ineq. (2.8), we obtain

$$
B_2 \le \frac{n\alpha L}{\delta_4} \int_0^L f^2(x,t) dx + \sum_{i=1}^{n} \alpha L \delta_4 \int_0^L [w_i'(x,t)]^2 dx, \tag{3.55}
$$

where δ_4 is a positive constant. Integrating Eq. (3.52) by parts, we obtain

$$
B_3 = n\alpha\rho L [\dot{w}(L,t)]^2 - \sum_{i=1}^{n} \alpha\rho \int_0^L \{[\dot{w}_i(x,t)]^2 + x \dot{w}_i(x,t) \dot{w}_i'(x,t)\} dx. \tag{3.56}
$$

The last term in Eq. (3.56) is equal to B_3, and we have

$$B_3 = \frac{n\alpha\rho L}{2}[\dot{w}(L,t)]^2 - \sum_{i=1}^{n}\frac{\alpha\rho}{2}\int_0^L [\dot{w}_i(x,t)]^2 dx. \tag{3.57}$$

Applying Ineq. (2.8), we obtain

$$B_4 \leq \frac{n\alpha c L}{\delta_5}\int_0^L [\dot{w}_i(x,t)]^2 dx + \sum_{i=1}^{n}\alpha c L\delta_5 \int_0^L [w_i'(x,t)]^2 dx, \tag{3.58}$$

where δ_5 is a positive constant. Substituting Eqs. (3.54), (3.55), (3.57) and (3.58) into Eq. (3.49) and using the boundary conditions, we obtain

$$\begin{aligned}
\dot{\eta}(t) \leq{}& \sum_{i=1}^{n}\frac{\alpha T L}{2}[w_i'(L,t)]^2 - \sum_{i=1}^{n}\frac{\alpha T}{2}\int_0^L [w_i'(x,t)]^2 dx \\
&+ \frac{n\alpha\rho L}{2}[\dot{w}(L,t)]^2 + \int_0^L (\alpha L\delta_4 + \alpha c L\delta_5)[w_i'(x,t)]^2 dx \\
&+ \frac{n\alpha L}{\delta_4}\int_0^L f^2(x,t)dx - \sum_{i=1}^{n}\left(\frac{\alpha\rho}{2} - \frac{n\alpha c L}{\delta_5}\right)\int_0^L [\dot{w}_i(x,t)]^2 dx. \qquad (3.59)
\end{aligned}$$

Substituting Eqs. (3.43), (3.48) and (3.59) into Eq. (3.35), we obtain

$$\begin{aligned}
\dot{V}(t) \leq{}& -\left(k - k_p - \frac{\beta T}{2}\right)u_a^2(t) - \sum_{i=1}^{n}\left(\frac{\alpha\rho}{2} + c\beta - \beta\delta_2 - \frac{n\alpha c L}{\delta_5}\right)\int_0^L [\dot{w}_i(x,t)]^2 dx \\
&- \left(\frac{\alpha T}{2} - \alpha L\delta_4 - \alpha c L\delta_5 - 4k_p L\right)\int_0^L [w_1'(x,t)]^2 dx - k_p\left(1 - \frac{\beta}{\delta_3}\right)[w(L,t)]^2 \\
&- \sum_{i=2}^{n}\left(\frac{\alpha T}{2} - \alpha L\delta_4 - \alpha c L\delta_5\right)\int_0^L [w_i'(x,t)]^2 dx + \left(\frac{n\beta}{\delta_2} + \frac{n\alpha L}{\delta_4}\right)\int_0^L f^2(x,t)dx \\
&- \sum_{i=1}^{n}\left[\frac{\beta T}{2} - (n-1)\beta T\left(\delta_1 + \frac{1}{\delta_1}\right) - \frac{\alpha T L}{2}\right][w_i'(L,t)]^2 \\
&- \left(\frac{\beta T}{2} - \frac{n\alpha\rho L}{2} - \beta k_p\delta_3\right)[\dot{w}(L,t)]^2 + 4k_p D_1^2 \\
\leq{}& -\lambda_3(V_1(t) + V_2(t)) + \varepsilon, \qquad (3.60)
\end{aligned}$$

where the constants k_p, k, α, β, δ_1, δ_2, δ_3, δ_4, and δ_5 are chosen to satisfy the following conditions:

$$\begin{aligned}
\alpha &< \frac{\min(\beta\rho, \beta T)}{2\rho L}, && (3.61)\\
\frac{\beta T}{2} &- \frac{n\alpha\rho L}{2} - \beta k_p \delta_3 \geq 0, && (3.62)\\
\frac{\beta T}{2} &- (n-1)\beta T\left(\delta_1 + \frac{1}{\delta_1}\right) - \frac{\alpha T L}{2} \geq 0, && (3.63)\\
\sigma_1 &= k - k_p - \frac{\beta T}{2} > 0, && (3.64)\\
\sigma_2 &= \frac{\alpha\rho}{2} + c\beta - \beta\delta_2 - \frac{n\alpha c L}{\delta_5} > 0, && (3.65)\\
\sigma_3 &= \frac{\alpha T}{2} - \alpha L\delta_4 - \alpha c\delta_5 L - 4k_p L > 0, && (3.66)\\
\sigma_4 &= \frac{\alpha T}{2} - \alpha L\delta_4 - \alpha c\delta_5 L > 0, && (3.67)\\
\sigma_5 &= 1 - \frac{\beta}{\delta_3} > 0, && (3.68)\\
\lambda_3 &= \min\left(\frac{2\sigma_1}{M}, \frac{2\sigma_2}{\beta\rho}, \frac{2\sigma_3}{\beta T}, \frac{2\sigma_4}{\beta T}, \frac{2\sigma_5}{\beta}\right) > 0, && (3.69)\\
\varepsilon &= \left(\frac{n\beta}{\delta_2} + \frac{n\alpha L}{\delta_4}\right)\int_0^L \bar{f}^2 dx + 4k_p D_1^2 > 0. && (3.70)
\end{aligned}$$

Combining Ineqs. (3.33) and (3.60), we have

$$\dot{V}(t) \leq -\lambda V(t) + \varepsilon, \tag{3.71}$$

where $\lambda = \lambda_3/\lambda_2 > 0$ and $\varepsilon > 0$. ■

Remark 3.5. *It is not difficult to find the proper δ_1, α and β to satisfy* (3.61) *and* (3.62)*. No matter what values of δ_1, α and β have been chosen, we can always find proper δ_2, δ_3, δ_4, δ_5, k_p, k to satisfy Ineqs.* (3.63)*,* (3.64) (3.65)*,* (3.66)*,* (3.67)*,* (3.68) *respectively. Since λ_3 and ε are positive definite, Ineqs.* (3.69) *and* (3.70) *always hold. Therefore, we can conclude that a set of values for constants k_p, k, α, β, δ_1, δ_2,*

δ_3, δ_4, δ_5 *can be found to satisfy the Ineqs.* (3.61)-(3.70).

With the above lemmas, we are ready to present the following stability theorem of the closed-loop mooring system.

Theorem 3.1. *For the system dynamics described by* (3.10)-(3.14)*, under Assumption 3.1, and the boundary control Eq.* (3.15)*, given that the initial conditions are bounded, we can conclude that*

(i) uniform boundedness (UB): the position of the vessel, $w(L,t)$, will remain within the compact set defined by

$$\Omega_1 := \left\{ w(L,t) \in R \middle| \quad |w(L,t)| \leq H_1,\ \forall (x,t) \in [0,L] \times [0,\infty) \right\}, \tag{3.72}$$

where the constant $H_1 = \sqrt{\frac{2}{\beta k_p \lambda_1}\left(V(0) + \frac{\varepsilon}{\lambda}\right)}$. *The vibration displacements of the mooring lines,* $w_i(x,t) + D_i$ $(i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2})$ *and* $w_i(x,t) - D_i$ $(i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n)$*, will remain in the compact sets defined by*

$$\begin{aligned}
\Omega_2 : &= \left\{ w_i(x,t) \in R \middle| \quad |w_i(x,t) + D_i| \leq H_2,\ \forall (x,t) \in [0,L] \times [0,\infty) \right\}, \\
& \quad (i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), \qquad (3.73) \\
\Omega_3 : &= \left\{ w_i(x,t) \in R \middle| \quad |w_i(x,t) - D_i| \leq H_3,\ \forall (x,t) \in [0,L] \times [0,\infty) \right\}, \\
& \quad (i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n), \qquad (3.74)
\end{aligned}$$

where the constants $H_2 = H_3 = \sqrt{\frac{2L}{T\lambda_1}\left(V(0) + \frac{\varepsilon}{\lambda}\right)}$.

(ii) uniform ultimate boundedness (UUB): the position of the vessel, $w(L,t)$, will eventually converge to the compact set defined by

$$\Omega_4 := \left\{ w(L,t) \in R \middle| \quad \lim_{t\to\infty} |w(L,t)| \leq H_4,\ \forall (x,t) \in [0,L] \times [0,\infty) \right\}, \tag{3.75}$$

where the constant $H_4 = \sqrt{\frac{2\varepsilon}{\beta k_p \lambda_1 \lambda}}$. *The vibration displacements of the mooring lines,* $w_i(x,t) + D_i$ $(i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2})$ *and* $w_i(x,t) - D_i$ $(i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n)$, *will eventually converge to the compact sets defined by*

$$\begin{aligned}
\Omega_5 : &= \left\{ w_i(x,t) \in R \middle| \quad \lim_{t\to\infty} |w_i(x,t) + D_i| \le H_5,\ \forall x \in [0, L) \right\}, \\
&\quad (i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), \qquad (3.76) \\
\Omega_6 : &= \left\{ w_i(x,t) \in R \middle| \quad \lim_{t\to\infty} |w_i(x,t) - D_i| \le H_6,\ \forall x \in [0, L) \right\}, \\
&\quad (i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n), \qquad (3.77)
\end{aligned}$$

where the constants $H_5 = H_6 = \sqrt{\frac{2L\varepsilon}{T\lambda_1\lambda}}$.

Proof: Multiplying Eq. (3.34) by $e^{\lambda t}$ yields

$$\frac{\partial}{\partial t}(V(t)e^{\lambda t}) \ \le\ \varepsilon e^{\lambda t}. \tag{3.78}$$

Integrating of the above inequality, we obtain

$$V(t) \le \left(V(0) - \frac{\varepsilon}{\lambda} \right) e^{-\lambda t} + \frac{\varepsilon}{\lambda} \le V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda} \in \mathcal{L}_\infty. \tag{3.79}$$

which implies $V(t)$ is bounded. Combining Eq. (3.20) and Ineq. (3.33) yields

$$\frac{\beta k_p}{2}[w(L,t)]^2 \le V_1(t) \le V_1(t) + V_2(t) \le \frac{1}{\lambda_1} V(t) \in \mathcal{L}_\infty. \tag{3.80}$$

Then, we have

$$|w(L,t)| \le \sqrt{\frac{2}{\beta k_p \lambda_1}\left(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda} \right)} \le \sqrt{\frac{2}{\beta k_p \lambda_1}\left(V(0) + \frac{\varepsilon}{\lambda} \right)},\ \forall t \in [0, \infty). \tag{3.81}$$

From Eq. (3.119), we have

$$\lim_{t\to\infty} |w(L,t)| \le \sqrt{\frac{2\varepsilon}{\beta k_p \lambda_1 \lambda}},\ \forall t \in [0,\infty). \tag{3.82}$$

Utilizing Ineq. (2.17) and Eq. (3.20), we have

$$\frac{1}{2L}T|w_i(x,t)+D_i|^2 \le \frac{1}{2}T\int_0^L [w_i'(x,t)]^2 dx \le V_1(t) \le V_1(t)+V_2(t) \le \frac{1}{\lambda_1}V(t) \in \mathcal{L}_\infty,$$
$$(i=1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), \tag{3.83}$$
$$\frac{1}{2L}T|w_i(x,t)-D_i|^i \le \frac{1}{2}T\int_0^L [w_i'(x,t)]^2 dx \le V_1(t) \le V_1(t)+V_2(t) \le \frac{1}{\lambda_1}V(t) \in \mathcal{L}_\infty,$$
$$(i=\frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n). \tag{3.84}$$

Appropriately rearranging the terms of the above inequality, we obtain $w_1(x,t)+D_1$ and $w_2(x,t)-D_2$ are uniformly bounded as follows

$$|w_i(x,t)+D_i| \le \sqrt{\frac{2L}{T\lambda_1}\left(V(0)e^{-\lambda t}+\frac{\varepsilon}{\lambda}\right)} \le \sqrt{\frac{2L}{T\lambda_1}\left(V(0)+\frac{\varepsilon}{\lambda}\right)},$$
$$\forall (x,t) \in [0,L]\times[0,\infty), (i=1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), \tag{3.85}$$
$$|w_i(x,t)-D_i| \le \sqrt{\frac{2L}{T\lambda_1}\left(V(0)e^{-\lambda t}+\frac{\varepsilon}{\lambda}\right)} \le \sqrt{\frac{2L}{T\lambda_1}\left(V(0)+\frac{\varepsilon}{\lambda}\right)},$$
$$\forall (x,t) \in [0,L]\times[0,\infty), (i=\frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n). \tag{3.86}$$

From Eqs. (3.123) and (3.124), we have

$$\lim_{t\to\infty} |w_i(x,t)+D_i| \le \sqrt{\frac{2L\varepsilon}{T\lambda_1\lambda}},\ \forall x \in [0,L], (i=1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), \tag{3.87}$$
$$\lim_{t\to\infty} |w_i(x,t)-D_i| \le \sqrt{\frac{2L\varepsilon}{T\lambda_1\lambda}},\ \forall x \in [0,L], (i=\frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n). \tag{3.88}$$

■

Remark 3.6. *It is seen that the increase in the control gain k will result in a larger σ_1, which will lead a greater λ_3. Then the value of λ will increase, which will reduce the size of Ω_1, Ω_2, Ω_3, Ω_4, Ω_5, Ω_6 and produce a good control performance.*

Remark 3.7. *In the above analysis, it is clear that the steady vessel position and the vibration displacements of the mooring lines can be made arbitrarily small the design control parameters k are appropriately selected. However, increasing k will bring a high gain control scheme, which should be avoided in practical applications.*

Remark 3.8. *From Eqs.* (3.121) *and* (3.122)*, we can state that V_1 and V_2 are bounded $\forall t \in [0, \infty)$. Since V_1 and V_2 are bounded, $\dot{w}_i(x,t)$ and $w_i'(x,t)$ are bounded $\forall (x,t) \in [0, L] \times [0, \infty)$ and u_a is bounded $\forall t \in [0, \infty)$. Then, we can obtain that the kinetic energy Eq.* (3.2) *and the potential energy Eq.* (3.5) *are bounded. Using Property 3.1, we can obtain $\dot{w}_i(x,t)$, and $\dot{w}_i'(x,t)$ are also bounded $\forall (x,t) \in [0, L] \times [0, \infty)$. Using Property 3.2, we can further obtain that $w_i'(x,t)$ and $w_i''(x,t)$ are bounded. Applying Assumption 3.1, Eqs.* (3.10) *and the above statements, we can state that $\ddot{w}_i(x,t)$ and $\ddot{w}_i(x,t)$ are also bounded $\forall (x,t) \in [0, L] \times [0, \infty)$. From the above information, it is shown that the proposed control Eq.* (3.15) *ensures all internal system signals including $w_i(x,t)$, $w_i'(x,t)$, $\dot{w}_i(x,t)$, $\dot{w}_i'(x,t)$ and $\ddot{w}_i(x,t)$ are uniformly bounded. Since $w_i'(x,t)$, $\dot{w}_i(x,t)$, and $\dot{w}_i'(x,t)$ are all bounded $\forall (x,t) \in [0, L] \times [0, \infty)$, and we can conclude the boundary control Eq.* (3.15) *is also bounded $\forall t \in [0, \infty)$.*

3.3.2 Robust adaptive boundary control for system parametric uncertainty

The previous exact model-based boundary control Eq. (3.15) requires the parametric knowledge of the mooring system. Robust adaptive boundary control is designed to improve the performance of the system via parameter estimation when the system parameters are unknown. The exact model-based boundary control provides a stepping stone towards the adaptive control, which is designed to deal with the system parametric uncertainty. In this section, the previous boundary control is redesigned by using adaptive control scheme when M, T and d_s are all unknown. We redesign the following robust adaptive boundary control:

$$u(t) = -P(t)\hat{\Phi}(t) - ku_a(t) - \text{sgn}(u_a(t))\bar{d} - k_p w(L,t), \tag{3.89}$$

where the vector $P(t) = [-\sum_{i=1}^{n} w_i'(L,t) \quad \sum_{i=1}^{n} \dot{w}_i'(L,t) \quad -\dot{w}(L,t)]$, the parameter estimate vector $\hat{\Phi}(t) = [\widehat{T}(t) \quad \widehat{M}(t) \quad \widehat{d_s}(t)]^T$, $\text{sgn}(\cdot)$ denotes the signum function, k_p and k are the control gains, and the auxiliary signal $u_a(t)$ is defined as Eq. (3.16). We define the parameter vector Φ and the parameter estimate error vector $\tilde{\Phi}(t)$ as

$$\Phi = [T \quad M \quad d_s]^T, \tag{3.90}$$

$$\tilde{\Phi}(t) = \Phi - \hat{\Phi}(t) = [\widetilde{T}(t) \quad \widetilde{M}(t) \quad \widetilde{d_s}(t)]^T. \tag{3.91}$$

After differentiating the auxiliary signal Eq. (3.16), multiplying the resulting equation by M, and substituting Eq. (3.12), we obtain

$$\begin{aligned} M\dot{u}_a(t) &= -\sum_{i=1}^{n} Tw_i'(L,t) + \sum_{i=1}^{n} M\dot{w}_i'(L,t) - d_s\dot{w}(L,t) + d(t) + u(t) \\ &= P(t)\Phi + d(t) + u(t). \end{aligned} \tag{3.92}$$

Substituting Eq. (3.89) into Eq. (3.92), we have

$$M\dot{u}_a(t) = P(t)\tilde{\Phi}(t) - ku_a(t) + d(t) - \mathrm{sgn}(u_a(t))\bar{d} - k_p w(L,t). \tag{3.93}$$

The adaptation law is designed as

$$\dot{\hat{\Phi}}(t) = \Gamma P^T(t)u_a(t) - r\Gamma\hat{\Phi}(t). \tag{3.94}$$

where $\Gamma \in R^{3\times 3}$ is a diagonal positive-definite matrix and r is a positive constant. We define the maximum and minimum eigenvalue of matrix Γ as $\lambda_{\max}$ and $\lambda_{\min}$ respectively. From Eq. (3.91), we have

$$\dot{\tilde{\Phi}}(t) = -\Gamma P^T(t)u_a(t) + r\Gamma\hat{\Phi}(t). \tag{3.95}$$

Remark 3.9. *For the proposed control* (3.89)*, a parameter estimation term, a signum term and an auxiliary signal term are introduced to compensate for the system parametric uncertainties and the effect of unknown time-varying disturbance. The control is independent of system parameters and the knowledge of the exact values for disturbance* $f(x,t)$ *and* $d(t)$ *are not required, thus possessing stability robustness to variations in system parameters and unknown disturbances.*

Remark 3.10. *Both controllers* (3.15) *and* (3.89) *do not require distributed sensing* *and all the signals in the boundary control can be measured by sensors or obtained by a backward difference algorithm.* $w_i(L,t)$ *can be sensed by a laser displacement sensor at the right boundary of the mooring, and* w_i' *can be measured by an inclinometer. In practice, the effect of measurement noise from sensors is unavoidable, which will affect the control implementation, especially when the high order differentiating terms with respect to time exist. In our proposed controllers* (3.15) *and* (3.89), $\dot{w}(L,t)$ *with only one time differentiating with respect to time can be calculated with a backward difference algorithm. It is noted that differentiating twice and three times position* $w(L,t)$ *with respect to time to get* $\ddot{w}(L,t)$ *and* $\dddot{w}(L,t)$ *respectively, are undesirable in practice due to noise amplification. For these cases, observers are needed to design to estimate the states values according to the boundary conditions.*

Remark 3.11. *Both controllers* (3.15) *and* (3.89) *are based on the distributed parameter model Eqs.* (3.10) *to* (3.14)*, and the spillover problems associated with traditional truncated model-based approaches caused by ignoring high-frequency modes in controller and observer design are avoided.*

Consider a new Lyapunov function candidate

$$V_a(t) \quad = \quad V(t) + \frac{1}{2}\tilde{\Phi}^T(t)\Gamma^{-1}\tilde{\Phi}(t), \tag{3.96}$$

where $V(t)$ is defined as Eq. (3.19) and $\tilde{\Phi}(t)$ is the parameter estimate error vector.

Lemma 3.3. *The Lyapunov function candidate given by Eq.* (3.104)*, can be upper and lower bounded as*

$$0 \leq \lambda_{1a}(V_1(t) + V_2(t) + ||\tilde{\Phi}(t)||^2) \leq V(t) \leq \lambda_{2a}(V_1(t) + V_2(t) + ||\tilde{\Phi}(t)||^2), \tag{3.97}$$

where λ_{1a} *and* λ_{2a} *are two positive constants defined as*

$$\lambda_{1a} = \min(1 - \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}, \frac{1}{2\lambda_{\max}}), \tag{3.98}$$

$$\lambda_{2a} = \max(1 + \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}, \frac{1}{2\lambda_{\min}}), \tag{3.99}$$

provided

$$0 < \alpha < \frac{\min(\beta\rho, \beta T)}{2\rho L}. \tag{3.100}$$

Proof: From the properties of matrix Γ and utilizing Lemma 2.6, we have

$$\frac{1}{2\lambda_{\max}}||\tilde{\Phi}(t)||^2 \leq \frac{1}{2}\tilde{\Phi}^T(t)\Gamma^{-1}\tilde{\Phi}(t) \leq \frac{1}{2\lambda_{\min}}||\tilde{\Phi}(t)||^2. \tag{3.101}$$

Combining Ineqs. (3.23) and (3.101), we have

$$0 \leq \lambda_{1a}(V_1(t) + V_2(t) + ||\tilde{\Phi}(t)||^2) \leq V_a(t) \leq \lambda_{2a}(V_1(t) + V_2(t) + ||\tilde{\Phi}(t)||^2), \tag{3.102}$$

where $\lambda_{1a} = \min(\alpha_2,\ \frac{1}{2\lambda_{\max}})$ and $\lambda_{2a} = \max(\alpha_3,\ \frac{1}{2\lambda_{\min}})$ are two positive constants. ■

Lemma 3.4. *The time derivative of the Lyapunov function in* (3.104) *can be upper bounded with*

$$\dot{V}_a(t) \leq -\lambda_a V_a(t) + \psi, \tag{3.103}$$

where λ_a and ψ are two positive constants.

Proof: Differentiating Eq. (3.104), we have

$$\dot{V}_a(t) \;\; = \;\; \dot{V}(t) + \tilde{\Phi}^T(t)\Gamma^{-1}\dot{\tilde{\Phi}}(t). \tag{3.104}$$

According to the similar derivation of Eq. (3.48), we obtain

$$\begin{aligned} \dot{V}_2(t) &= Mu_a(t)\dot{u}_a(t) \\ &\leq -ku_a^2(t) + P(t)\tilde{\Phi}(t)u_a(t) - k_p[w(L,t)]^2 + 4k_pD_1^2 + 4k_pL\int_0^L [w_1'(L,t)]^2 \\ &\quad +k_pu_a^2(t). \end{aligned} \tag{3.105}$$

Applying the results of Lemma 3.2 and substituting Eqs. (3.43), (3.105) and (3.59) into $\dot{V}$, we obtain

$$\dot{V}_a(t) \leq -\lambda_3(V_1(t) + V_2(t)) + P(t)\tilde{\Phi}(t)u_a(t) + \varepsilon, \tag{3.106}$$

where λ_3 is defined in Eq. (3.69) and ε is defined in Eq. (3.66). Substituting of Ineq. (3.106) into Eq. (3.104) yields

$$\dot{V}_a(t) \;\; \leq \;\; -\lambda_3(V_1(t) + V_2(t)) + \tilde{\Phi}^T(t)\left(P^T(t)u_a(t) + \Gamma^{-1}\dot{\tilde{\Phi}}(t)\right) + \varepsilon. \tag{3.107}$$

Substituting Eq. (3.95) into Eq. (3.107), we have

$$\begin{aligned} \dot{V}_a(t) &\leq -\lambda_3(V_1(t) + V_2(t)) + r\tilde{\Phi}^T(t)\hat{\Phi}(t) + \varepsilon \\ &\leq -\lambda_3(V_1(t) + V_2(t)) - \frac{r}{2}||\tilde{\Phi}(t)||^2 + \frac{r}{2}||\Phi||^2 + \varepsilon \\ &\leq -\lambda_{3a}(V_1(t) + V_2(t) + ||\tilde{\Phi}(t)||^2) + \frac{r}{2}||\Phi||^2 + \varepsilon, \end{aligned} \tag{3.108}$$

where $\lambda_{3a} = \min(\lambda_3, \frac{r}{2})$ is a positive constant. From Ineqs. (3.102) and (3.108), we have

$$\dot{V}_a(t) \leq -\lambda_a V_a(t) + \psi, \tag{3.109}$$

where $\lambda_a = \lambda_{3a}/\lambda_{2a}$ and $\psi = \frac{r}{2}||\Phi||^2 + \varepsilon > 0$. ∎

With the above lemmas, the adaptive control design for the mooring system subjected to the unknown disturbances can be summarized in the following theorem.

Theorem 3.2. *For the system dynamics described by* (3.10)-(3.14)*, under Assumption 3.1, and the boundary control Eq.* (3.89)*, given that the initial conditions are bounded, we can conclude that*

(i) uniform boundedness (UB): the position of the vessel, $w(L,t)$, will remain within the compact set defined by

$$\Omega_7 := \left\{ w(L,t) \in R \middle| \quad |w(L,t)| \leq H_7,\ \forall (x,t) \in [0,L] \times [0,\infty) \right\}, \tag{3.110}$$

where the constant $H_7 = \sqrt{\frac{2}{\beta k_p \lambda_{1a}} \left(V_a(0) + \frac{\psi}{\lambda_a} \right)}$. The vibration displacements of the mooring lines, $w_i(x,t) + D_i$ ($i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}$) and $w_i(x,t) - D_i$ ($i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n$), will remain in the compact sets defined by

$$\begin{aligned}
&\Omega_8 := \left\{ w_i(x,t) \in R \middle| \quad |w_i(x,t) + D_i| \leq H_8,\ \forall (x,t) \in [0,L] \times [0,\infty) \right\}, \\
&(i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), \qquad (3.111) \\
&\Omega_9 := \left\{ w_i(x,t) \in R \middle| \quad |w_i(x,t) - D_i| \leq H_9,\ \forall (x,t) \in [0,L] \times [0,\infty) \right\}, \\
&(i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n), \qquad (3.112)
\end{aligned}$$

where the constants $H_8 = H_9 = \sqrt{\frac{2L}{T\lambda_{1a}} \left(V_a(0) + \frac{\psi}{\lambda_a} \right)}$.

(ii) uniform ultimate boundedness (UUB): the position of the vessel, $w(L,t)$, will eventually converge to the compact set defined by

$$\Omega_{10} := \left\{ w(L,t) \in R \middle| \quad \lim_{t\to\infty} |w(L,t)| \leq H_{10},\ \forall (x,t) \in [0,L] \times [0,\infty) \right\}, \quad (3.113)$$

where the constant $H_{10} = \sqrt{\frac{2\psi}{\beta k_p \lambda_{1a}\lambda_a}}$. The vibration displacements of the mooring lines, $w_i(x,t) + D_i$ ($i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}$) and $w_i(x,t) - D_i$ ($i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n$), will eventually converge to the compact sets defined by

$$\Omega_{11} := \left\{ w_i(x,t) \in R \middle| \quad \lim_{t\to\infty} |w_i(x,t) + D_i| \leq H_{11},\ \forall x \in [0,L) \right\},$$
$$(i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), \quad (3.114)$$
$$\Omega_{12} := \left\{ w_i(x,t) \in R \middle| \quad \lim_{t\to\infty} |w_i(x,t) - D_i| \leq H_{12},\ \forall x \in [0,L) \right\},$$
$$(i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n), \quad (3.115)$$

where the constants $H_{11} = H_{12} = \sqrt{\frac{2L\psi}{T\lambda_{1a}\lambda_a}}$.

Proof: Multiplying Eq. (3.103) by $e^{\lambda_a t}$ yields

$$\frac{\partial}{\partial t}(V_a(t)e^{\lambda_a t}) \quad \leq \quad \psi e^{\lambda_a t}. \quad (3.116)$$

Integrating of the above inequality, we obtain

$$V_a(t) \leq \left(V_a(0) - \frac{\psi}{\lambda_a} \right) e^{-\lambda_a t} + \frac{\psi}{\lambda_a} \leq V_a(0)e^{-\lambda_a t} + \frac{\psi}{\lambda_a} \in \mathcal{L}_\infty, \quad (3.117)$$

which implies V_a is bounded. Combining Eq. (3.20) and Ineq. (3.102) yields

$$\frac{\beta k_p}{2}[w(L,t)]^2 \leq V_1(t) \leq V_1(t) + V_2(t) + ||\tilde{\Phi}(t)||^2 \leq \frac{1}{\lambda_{1a}} V_a(t) \in \mathcal{L}_\infty. \quad (3.118)$$

Then, we have

$$|w(L,t)| \leq \sqrt{\frac{2}{\beta k_p \lambda_{1a}}\left(V_a(0)e^{-\lambda t} + \frac{\psi}{\lambda_a}\right)} \leq \sqrt{\frac{2}{\beta k_p \lambda_{1a}}\left(V_a(0) + \frac{\psi}{\lambda_a}\right)},\ \forall t \in [0,\infty). \tag{3.119}$$

From Eq. (3.119), we have

$$\lim_{t\to\infty}|w(L,t)| \leq \sqrt{\frac{2\psi}{\beta k_p \lambda_1 \lambda}},\ \forall t \in [0,\infty). \tag{3.120}$$

Utilizing Ineq. (2.17) and Eq. (3.20), we have

$$\frac{1}{2L}T|w_i(x,t)+D_i|^2 \leq \frac{1}{2}T\int_0^L [w_i'(x,t)]^2 dx \leq V_1(t) \leq V_1(t)+V_2(t)+||\tilde{\Phi}(t)||^2$$
$$\leq \frac{1}{\lambda_{1a}}V_a(t) \in \mathcal{L}_\infty, (i=1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), \tag{3.121}$$
$$\frac{1}{2L}T|w_i(x,t)-D_i|^2 \leq \frac{1}{2}T\int_0^L [w_i'(x,t)]^2 dx \leq V_1(t) \leq V_1(t)+V_2(t)+||\tilde{\Phi}(t)||^2$$
$$\leq \frac{1}{\lambda_{1a}}V_a(t) \in \mathcal{L}_\infty, (i=\frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n). \tag{3.122}$$

Appropriately rearranging the terms of the above inequality, we obtain $w_i(x,t)+D_i$ $(i=1,\ 2,\ 3,\ \ldots,\ \frac{n}{2})$ and $w_i(x,t)-D_i$ $(i=\frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n)$ are uniformly bounded as follows

$$|w_i(x,t)+D_i| \leq \sqrt{\frac{2L}{T\lambda_{1a}}\left(V_a(0)e^{-\lambda_a t}+\frac{\psi}{\lambda_a}\right)} \leq \sqrt{\frac{2L}{T\lambda_{1a}}\left(V_a(0)+\frac{\psi}{\lambda_a}\right)},$$
$$\forall (x,t) \in [0,L]\times[0,\infty), (i=1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), \tag{3.123}$$
$$|w_i(x,t)-D_i| \leq \sqrt{\frac{2L}{T\lambda_{1a}}\left(V_a(0)e^{-\lambda t}+\frac{\psi}{\lambda_a}\right)} \leq \sqrt{\frac{2L}{T\lambda_{1a}}\left(V_a(0)+\frac{\psi}{\lambda_a}\right)},$$
$$\forall (x,t) \in [0,L]\times[0,\infty), (i=\frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n). \tag{3.124}$$

From Eqs. (3.123) and (3.124), we have

$$\lim_{t\to\infty} |w_i(x,t) + D_i| \leq \sqrt{\frac{2L\psi}{T\lambda_{1a}\lambda_a}},\ \forall x \in [0,L], (i = 1,\ 2,\ 3,\ \ldots,\ \frac{n}{2}), \tag{3.125}$$

$$\lim_{t\to\infty} |w_i(x,t) - D_i| \leq \sqrt{\frac{2L\psi}{T\lambda_{1a}\lambda_a}},\ \forall x \in [0,L], (i = \frac{n}{2}+1,\ \frac{n}{2}+2,\ \frac{n}{2}+3,\ \ldots,\ n). \tag{3.126}$$

■

Remark 3.12. *From the similar analysis of Remark 3.6 and Remark 3.7, we can conclude that the steady vessel position and the vibration displacements of the mooring lines can be made arbitrarily small by choosing control gain k in Eq.* (3.89) *appropriately.*

Remark 3.13. *From Eq.* (3.117)*, we can obtain the parameter estimate error $\tilde{\Phi}$ is bounded $\forall t \in [0,\infty)$. Using the derivation similar to those employed in Remark 3.8, we can state the proposed control Eq.* (3.89) *ensures all internal system signals including $w_i(x,t)$, $w_i'(x,t)$, $\dot{w}_i(x,t)$, $\dot{w}_i'(x,t)$, and $\ddot{w}_i(x,t)$ are uniformly bounded. Since $\hat{\Phi}$, $w_i'(x,t)$ and $\dot{w}_i(x,t)$ are all bounded $\forall(x,t) \in [0,L]\times[0,\infty)$, and we can conclude the robust adaptive boundary control Eq.* (3.89) *is also bounded $\forall t \in [0,\infty)$.*

3.4 Numerical Simulations

Simulations for a mooring system with four mooring lines under the ocean current disturbances are carried out to demonstrate the effectiveness of the proposed control (3.15) and (3.89). The eigenfunction expansion method based on Fourier series in [11] cannot be used to solve the model of the mooring system (3.10)-(3.14) due to the unknown term $f(x,t)$. Numerical methods are applied to get the approximate solution of the system (3.10)-(3.14), when there is no obtainable analytical solution. Several numerical methods such as finite difference (FD), assumed mode method (AMM), finite element method (FEM), and Galerkin method can be used to discretize the system for simulations. In this chapter, we select the finite difference scheme to simulate the system performance with the proposed boundary control.

The vessel and the mooring lines, initially at rest, are excited by the disturbance $d(t)$ and the distributed transverse load $f(x,t)$. The corresponding initial conditions of the mooring system are given as

$$\begin{aligned}
w(L,t) &= \dot{w}(L,t) = 0, && (3.127)\\
w_1(x,0) &= \sqrt{\frac{D_1^2}{L}x} - D_1, w_2(x,0) = \sqrt{\frac{D_2^2}{L}x} - D_2, && (3.128)\\
w_3(x,0) &= -\sqrt{\frac{D_3^2}{L}x} + D_3, w_4(x,0) = -\sqrt{\frac{D_4^2}{L}x} + D_4, && (3.129)\\
\dot{w}_1(x,0) &= \dot{w}_2(x,0) = \dot{w}_4(x,0) = \dot{w}_4(x,0) = 0. && (3.130)
\end{aligned}$$

Detailed parameters of the mooring system are given in the following table:

Table 1: Parameters of the thruster assisted position mooring system

Parameter	Description	Value
L	Length of the mooring lines	$1000.00m$
T	Tension of the mooring lines	$4.00 \times 10^6 N$
D	Diameter of the mooring lines	$0.05m$
D_1	Distance between line 1 and coordinate point	$50.00m$
D_2	Distance between line 2 and coordinate point	$100.00m$
D_3	Distance between line 3 and coordinate point	$50.00m$
D_4	Distance between line 4 and coordinate point	$100.00m$
M	Mass of the vessel	$9.60 \times 10^7 kg$
d_s	Damping coefficient of the vessel	$9.00 \times 10^7 NS/m$
ρ	Mass per unit length of the mooring lines	$8.02kg/m$
ρ_s	Sea water density	$1024.00kg/m^3$
c	Damping coefficient of the mooring lines	$1.00NS/m^2$

Large vibrational stresses are normally associated with a resonance that exists when the frequency of the imposed force is tuned to one of the natural frequencies [137]. In our simulation experiments, the ocean surface current velocity $U(t)$ is modeled as a mean flow with worst case sinusoidal components to simulate the mooring systems with a mean deflected profile. The sinusoids have frequencies of $\omega_j = \{0.867, 1.827, 2.946, 4.282\}$, for $j = 1$ to 4, corresponding to the four natural modes of vibration of the mooring lines. The current $U(t)$ is expressed as

$$U(t) = \bar{U} + U' \sum_{j=1}^{N} \sin(\omega_i t), (j = 1, 2, \ldots, N), \tag{3.131}$$

where $\bar{U} = 2ms^{-1}$ is the mean flow current and $U' = 0.2$ is the amplitude of the oscillating flow. The full current load is applied from $x = 1000m$ to $x = 0m$ and thereafter linearly decline to zero at the ocean floor, $x = 0$, to obtain a depth dependent ocean current profile $U(x,t)$. The distributed load $f(x,t)$ is generated by Eq. (2.5) with $C_D = 1$, $\theta = 0$, $S_t = 0.2$ and $f_v = 2.625$. The disturbance $d(t)$ on the vessel is generated by the following equation.

$$d(t) = [1 + 0.1\sin(0.1t) + 0.3\sin(0.3t) + 0.5\sin(0.5t)] \times 10^6 \qquad (3.132)$$

With increasing time in the sequences for the duration $t_f = 1500s$, the snapshots of the mooring system movements without control inputs are shown in Fig. 3.2. The snapshots of the mooring system movements with the proposed control (3.15), by choosing $k_p = 1 \times 10^7$, $k = 10$, are presented in Fig. 3.3. The snapshots of the mooring system movements with the proposed control (3.89), by choosing $k_p = 1\times10^7$, $k = 10$, $r = 0.001$, $\Gamma = \mathrm{diag}\{1,1,1\}$, are presented in Fig. 3.4.

It is obvious that both the proposed control (3.15) and (3.89) are effective and are able to achieve a good performance for the mooring system. In the closed-loop system, all the mooring lines is stabilized at the small neighborhood of their equilibrium positions by appropriately choosing design parameters. The surface vessel's position $w(1000,t)$ for controlled and uncontrolled responses is shown in Figs. 3.5 and 3.6, where it can be observed that the proposed control is able to position the vessel near to its desired position at the origin. The corresponding boundary control $u(t)$ is shown in Fig. 3.7.

3.5 Conclusion

Modeling and control design for a thruster assisted position mooring system has been investigated in this chapter. The mathematical model of the mooring system has been derived by using the Hamilton's principle. For this PDE model, both exact model based boundary control and adaptive boundary control have been proposed based on the Lyapunov's direct method. With proposed control, all the signals of the closed-loop system are proved to be uniformly bounded despite the presence of unknown system parameters. The proposed control strategy only requires measurements of the boundary displacement and slope of the mooring lines and the time derivatives of these quantities. Numerical simulations for a mooring system with four mooring lines have been presented to verify the effectiveness of the presented boundary control. It can be concluded that the proposed boundary control has provided a good control performance for the thruster assisted position mooring system in the presence of unknown environmental disturbances.

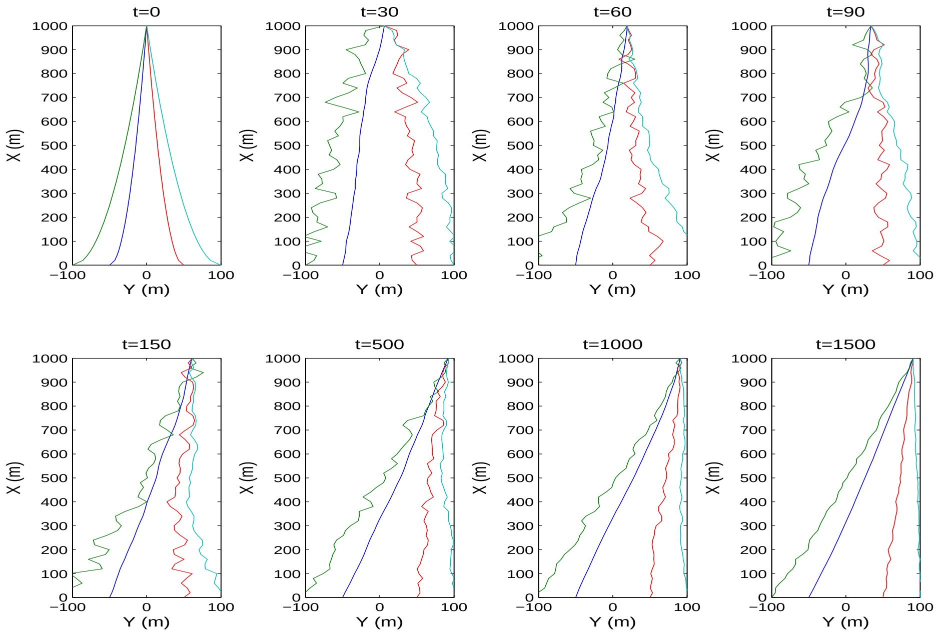


Fig. 3.2: Snapshots of the mooring system movements without control.

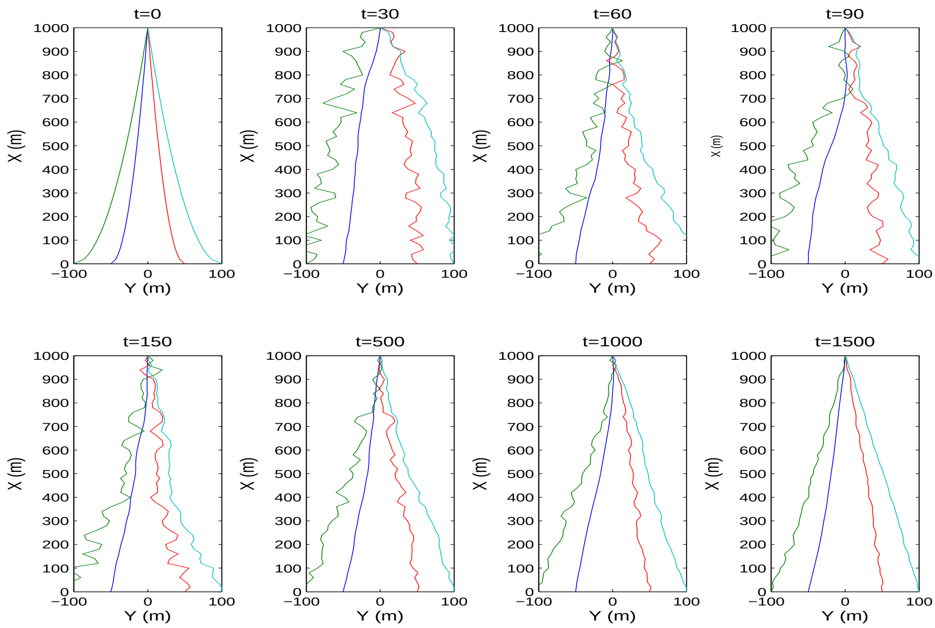


Fig. 3.3: Snapshots of the mooring system movements with the proposed exact model based boundary control.

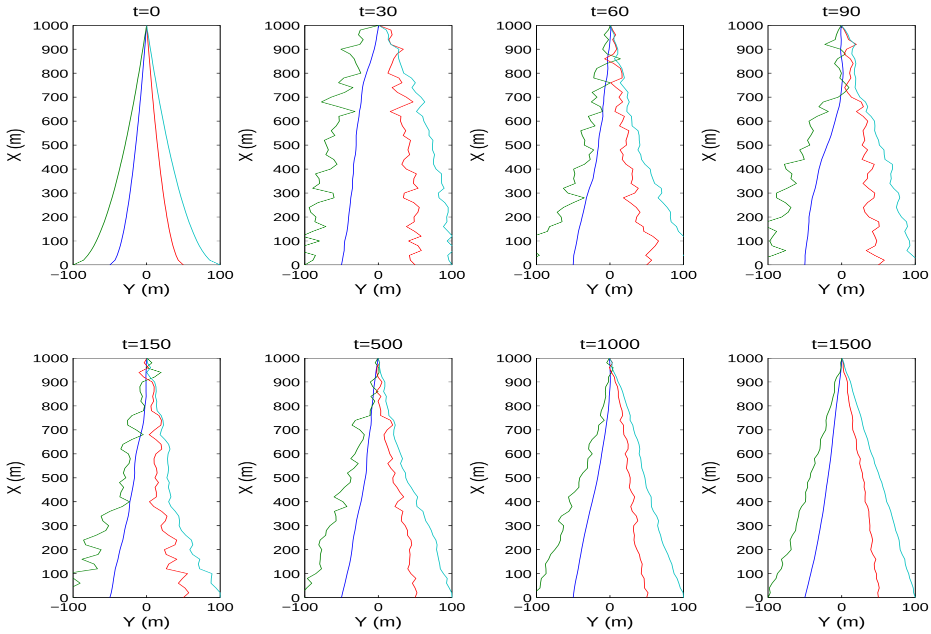


Fig. 3.4: Snapshots of the mooring system movements with the proposed adaptive boundary control.

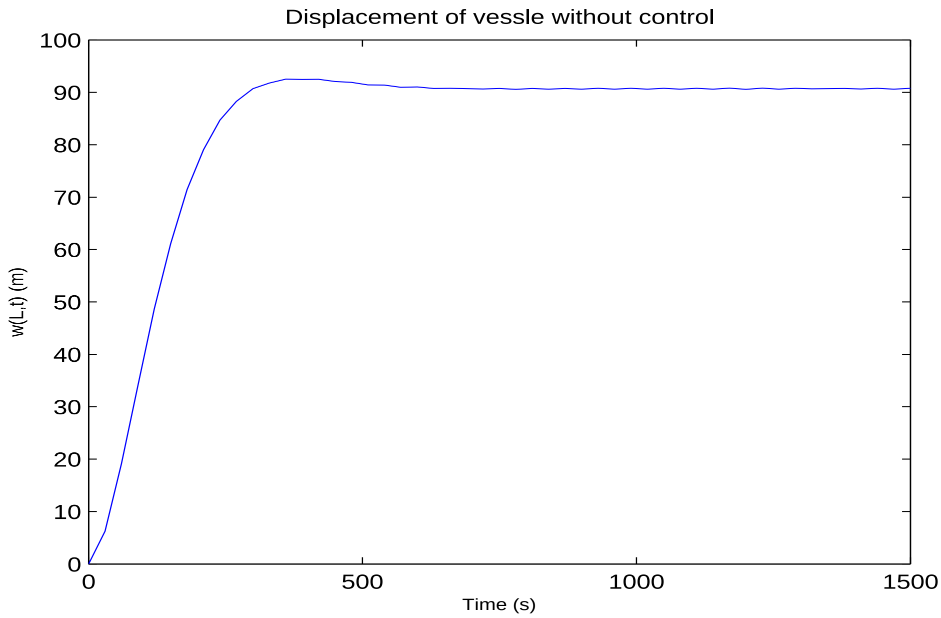


Fig. 3.5: Displacement of the vessel, w(1000,t), without control.

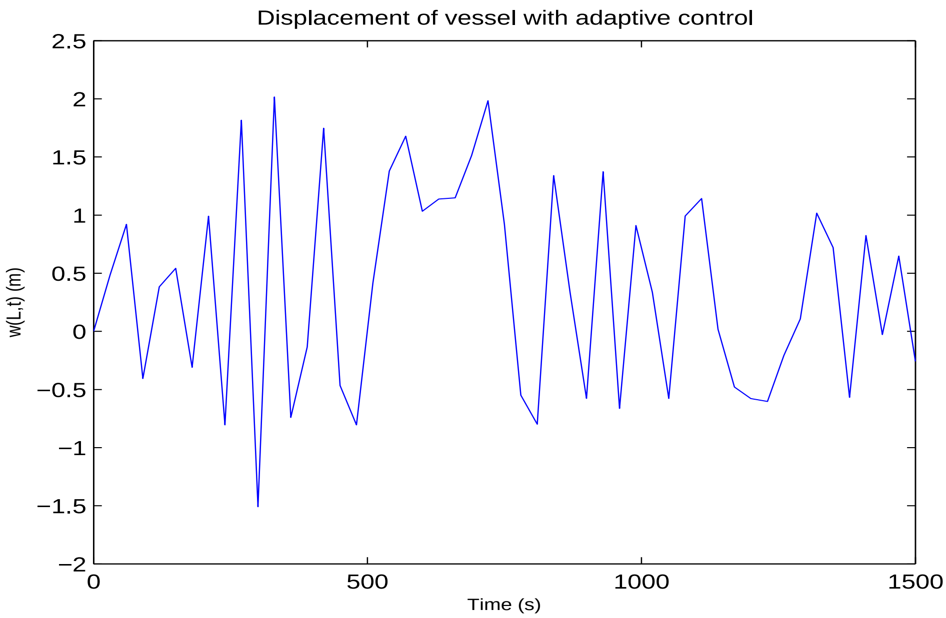


Fig. 3.6: Displacement of the vessel, w(1000,t), with the proposed control (3.89).

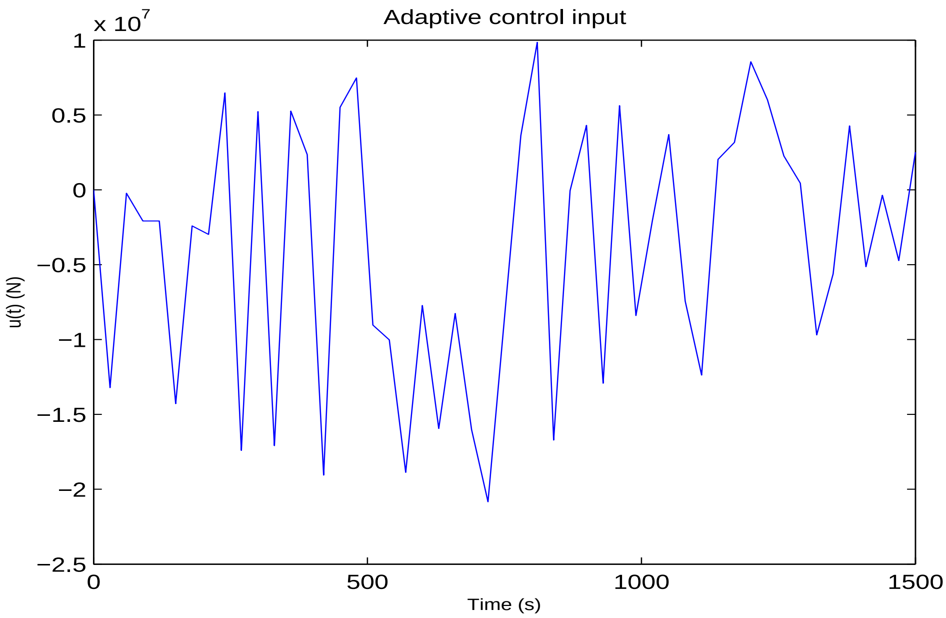


Fig. 3.7: Adaptive control input (3.89).

Chapter 4

Marine Installation System

4.1 Introduction

The accurate position control for marine installation operations has gained increasing attention when the trend in the offshore industry is towards the deep water. Due to the requirements for high accuracy and efficiency arising from the modern ocean industry, improving reliability and efficiency of installation operations during oil and gas production in the ocean environment is a challenging research topic in offshore engineering. Traditional marine installation systems consist of the vessel dynamic positioning and crane manipulation to obtain the desired position and heading for the payload [115, 116]. Such methods become difficult in deeper waters due to the longer cable between the surface vessel and payload. The longer cable increases the natural period of the cable and payload system which in turn increase the effects of oscillations. One solution to alleviate the precision installation problem is the addition of thrusters attached the payload for the installation operation [7].

Such marine installation system consists of an ocean surface vessel, a flexible string-type cable and a subsea payload to be positioned for installation on the ocean floor is depicted in Fig. 4.1. The surface vessel, to which the top boundary of the cable is connected, is equipped with a dynamic positioning system with an active thruster. The bottom boundary of the cable is a payload with an end-point thruster attached. This thruster is used for dynamic positioning of th e payload. The total marine installation system is subjected to environmental disturbances including ocean current, wave, and wind. A cable that spans a long distance can produce large vibrations under relatively small disturbances, which can degrade the performance of the system and result in a larger offset from the target installation site. The control for the dynamic positioning of the payload is challenging due to the unpredictable exogenous disturbances such as fluctuating currents and transmission of motions from the surface vessel through the lift cable. Taking into account the unknown time-varying ocean disturbances of the cable leads to the appearance of oscillations, which make the control problem of the marine installation system relatively difficult. Current research [7] on the control of the marine installation systems focuses on the dynamics of the payload, where the dynamics of the cable is ignored for the convenience of the control design. The dynamics of the cable is considered as an external force term to the payload. In this chapter, all the dynamics of the vessel, the cable and the payload are considered. The flexible marine installation system with cable, vessel and payload dynamics is represented by a set of infinite dimensional equations, (i.e., PDEs describing the dynamics of the flexible cable) coupled with a set of finite dimensional equations, (i.e., ODEs describing the lumped vessel and payload dynamics).

For the marine installation system, the dynamic position control of the payload is as vital as the vibration suppression of the cable. It is therefore necessary to consider both vibration suppression and the dynamic positioning in the control design. In the framework of boundary control, we are going to investigate the robust adaptive boundary control problem for the string-type model with system parametric uncertainty and under unknown time-varying ocean disturbance. The adaptive control design aims to compensate for the effects of both parametric and disturbance uncertainties and achieve uniform ultimate boundedness. In this chapter, we design the boundary control based on the distributed parameter model of the flexible marine installation system. The stability analysis of the closed-loop system is based on Lyapunov's direct method without resorting to semigroup theory or functional analysis. Although a flexible marine installation system is being considered in this chapter specifically, the analysis and control design can be extended and applied for position control and vibration suppression for a class of mechanical string-type system exposed to undesirable distributed transverse loads. In this chapter, both the dynamics of the vessel, payload and the vibration of the cable are considered in the dynamic analysis. The main contributions of this chapter include:

(i) The dynamic model of a flexible marine installation system subjected to ocean current disturbance is derived based on the Hamilton's principle. The governing equation of the system is represented as a nonhomogeneous hyperbolic PDE with the unknown disturbance term $f(x,t)$.

(ii) Two implementable boundary controllers at the top and bottom boundary of the cable are designed to position the subsea payload to the desired set-point and suppress the cable's vibration. Robust adaptive boundary control is designed to compensate for the system parametric uncertainty and the effect of unknown time-varying distributed disturbance $f(x,t)$.

(iii) With the proposed boundary control, uniform boundedness of the installation system under ocean disturbance is proved via Lyapunov synthesis. The control performance of the system is guaranteed by suitably choosing the design parameters.

The rest of the chapter is organized as follows. The governing equation (PDE) and boundary conditions (ODEs) of the flexible marine installation system are introduced by use of the Hamilton's principle in Section 4.2. The boundary control design via Lyapunov's direct method is discussed separately for both exact model case and system parametric uncertainty case in Section 4.3, where it is shown that the uniform boundedness of the closed-loop system can be achieved by the proposed control. Simulations are carried out to illustrate performance of the proposed control in Section 4.4. The conclusion of this chapter is shown in Section 4.5.

4.2 Problem Formulation

For the marine installation system shown in Fig. 1, frame $X - Y$ is the fixed inertia frame, and frame $x - y$ is the local reference frame fixed along the vertical direction of the surface vessel. The top boundary of the cable is at the vessel and the bottom boundary of the cable is at the underwater payload. Forces from thrusters on vessel and payload are the control inputs of the system, and the boundary position and slope of the cable are used as the feedback signals in the control design. p_d is the desired target position, $p(t)$ is the position of the vessel, $w(x,t)$ is the elastic transverse reflection with respect to frame $x - y$ at the position x for time t, and $y(x,t) := p(t) + w(x,t)$ is the position of the cable with respect to frame $X - Y$ at the position x for time t. Note that $w(L,t) = 0$ due to the connection between the vessel and the top boundary of the cable.

In this chapter, we consider the transverse degree of freedom only. We assume that the original position of the vessel is directly above the subsea payload with no horizontal offset, and the payload is filled with seawater.

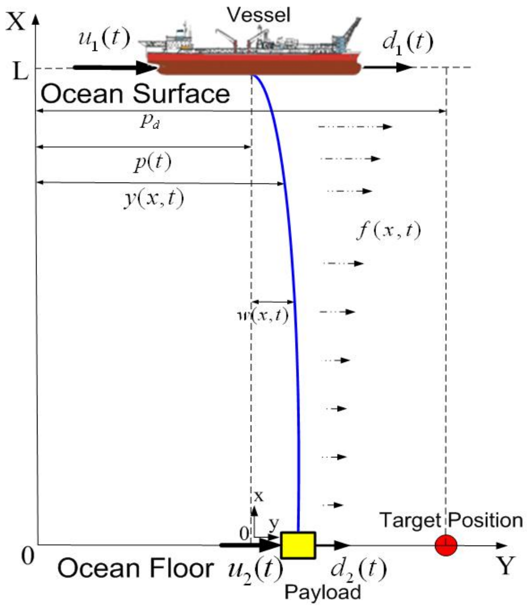


Fig. 4.1: A typical flexible marine installation system.

The kinetic energy of the installation system E_k can be represented as

$$E_k = \frac{1}{2}M\left[\frac{\partial y(L,t)}{\partial t}\right]^2 + \frac{1}{2}\rho\int_0^L \left[\frac{\partial y(x,t)}{\partial t}\right]^2 dx + \frac{1}{2}m\left[\frac{\partial y(0,t)}{\partial t}\right]^2, \quad (4.1)$$

where x and t represent the independent spatial and time variables respectively, M denotes the mass of the surface vessel, m denotes the mass of bottom payload, $y(L,t) = p(t)$, $\frac{\partial y(L,t)}{\partial t} = \frac{\partial p(t)}{\partial t}$ and $\frac{\partial^2 y(L,t)}{\partial t^2} = \frac{\partial^2 p(t)}{\partial t^2}$ are the position, velocity and acceleration of the vessel respectively, $\rho > 0$ is the uniform mass per unit length of the cable, and L is the length of the cable.

The potential energy E_p due to the strain energy of the cable can be obtained from

$$E_p = \frac{1}{2}T\int_0^L \left[\frac{\partial w(x,t)}{\partial x}\right]^2 dx, \tag{4.2}$$

where T is the tension of the cable. Definition of $y(x,t)$ yields $\frac{\partial y(x,t)}{\partial x} = \frac{\partial w(x,t)}{\partial x}$. Then we have

$$E_p = \frac{1}{2}T\int_0^L \left[\frac{\partial y(x,t)}{\partial x}\right]^2 dx. \tag{4.3}$$

The virtual work done by ocean current disturbance on the vessel, the cable and the payload is given by

$$\delta W_f = \int_0^L f(x,t)\delta y(x,t)dx + d_1(t)\delta y(L,t) + d_2(t)\delta y(0,t), \tag{4.4}$$

where $f(x,t)$ is the distributed transverse load on the cable due to the hydrodynamic effects of the ocean current, wave and wind, $d_1(t)$ denotes the environmental disturbances on the vessel, and $d_2(t)$ denotes the environmental disturbances on the paylaod. The virtual work done by damping on the vessel, the cable and the payload is represented by

$$\delta W_d = -\int_0^L c\frac{\partial y(x,t)}{\partial t}\delta y(x,t)dx - c_1\frac{\partial y(L,t)}{\partial t}\delta y(L,t) - c_2\frac{\partial y(0,t)}{\partial t}\delta y(0,t), \tag{4.5}$$

where c is the distributed viscous damping coefficient of the cable, c_1 denotes the damping coefficient of the vessel, and c_2 denotes the damping coefficient for the payload. We introduce the control u_1 applied to the top boundary of the cable from the thruster attached in the vessel, and the control u_2 applied to the bottom boundary of the cable from the thruster attached in the payload. The virtual work done by the boundary control is written as

$$\delta W_m = u_1(t)\delta w(L,t) + u_2(t)\delta w(0,t). \tag{4.6}$$

Then, we have the total virtual work done on the system as

$$\begin{aligned}\delta W &= \delta W_f + \delta W_d + \delta W_m \\ &= \int_0^L \left[f(x,t) - c\frac{\partial y(x,t)}{\partial t}\right]\delta y(x,t)dx + \left[u_1(t) + d_1(t) - c_1\frac{\partial y(L,t)}{\partial t}\right]\delta y(L,t) \\ &\quad + \left[u_2(t) + d_2(t) - c_2\frac{\partial y(0,t)}{\partial t}\right]\delta y(0,t).\end{aligned} \tag{4.7}$$

Applying the variation operator and integrating Eqs. (4.1), (4.3), and (4.7) by parts respectively and substituting them into the Hamilton's principle, we obtain the governing equation of the system as

$$\rho\ddot{y}(x,t) - Ty''(x,t) + c\dot{y}(x,t) = f(x,t), \tag{4.8}$$

and the boundary conditions of the system as

$$u_1(t) + d_1(t) - c_1\dot{y}(L,t) - M\ddot{y}(L,t) - Ty'(L,t) = 0, \tag{4.9}$$

$$u_2(t) + d_2(t) - c_2\dot{y}(0,t) - m\ddot{y}(0,t) + Ty'(0,t) = 0, \tag{4.10}$$

$\forall t \in [0,\infty)$.

Remark 4.1. *The notations $(*)'$, $(*)''$, $(*)'''$ and $(*)''''$ representing the first, second, third, forth order derivatives of $(*)$ with respect to x respectively, $(\dot{*})$ and $(\ddot{*})$ denoting the first and second order derivative of $(*)$ with respect to time t, respectively, are used for clarity.*

Remark 4.2. *With consideration of the distributed transverse load $f(x,t)$, the governing equation of the installation system Eq.* (4.8) *is represented by a nonhomogeneous hyperbolic PDE. This model differs from the string system governed by a homogeneous PDE in [39, 56, 59–61, 63, 66, 68, 74, 76].*

Assumption 4.1. *For the distributed load $f(x,t)$ on the cable, the disturbance $d_1(t)$ on the vessel, the disturbance $d_2(t)$ on the payload, we assume that there exist constants $\bar{f} \in R^+$, $\bar{d}_1 \in R^+$ and $\bar{d}_2 \in R^+$, such that $|f(x,t)| \leq \bar{f}$, $\forall(x,t) \in [0,L] \times [0,\infty)$, $|d_1(t)| \leq \bar{d}_1$, $\forall(t) \in [0,\infty)$ and $|d_2(t)| \leq \bar{d}_2$, $\forall t \in [0,\infty)$. This is a reasonable assumption as the time-varying disturbances $f(x,t)$, $d_1(t)$ and $d_2(t)$ have finite energy and hence are bounded, i.e. $f(x,t) \in \mathcal{L}_\infty([0,L])$, $d_1(t) \in \mathcal{L}_\infty$ and $d_2(t) \in \mathcal{L}_\infty$.*

Remark 4.3. *For control design in Section 4.3, only the assertion that there exist upper bounds on the disturbance in Assumption 1, $|f(x,t)| < \bar{f}$, $|d_1(t)| \leq \bar{d}_1$ and $|d_2(t)| \leq \bar{d}_2$, is necessary. The knowledge of the exact values for $f(x,t)$, $d_1(t)$ and $d_2(t)$ is not required. As such, different distributed load models up to various levels of fidelity, such as those found in [127, 128, 133–135], can be applied without affecting the control design or analysis.*

Property 4.1. *[136]:If the kinetic energy of the system* (4.8) - (4.10)*, given by Eq.* (4.1) *is bounded $\forall t \in [0,\infty)$, then $\dot{y}(x,t)$, $\dot{y}'(x,t)$ and $\dot{y}''(x,t)$ are bounded $\forall(x,t) \in [0,L] \times [0,\infty)$.*

Property 4.2. *[136]: If the potential energy of the system* (4.8) - (4.10)*, given by Eq.* (4.3) *is bounded $\forall t \in [0,\infty)$, then $y'(x,t)$ and $y''(x,t)$ are bounded $\forall(x,t) \in [0,L] \times [0,\infty)$.*

4.3 Control Design

The control objective is to design boundary control to position the subsea payload to the desired set-point p_d and simultaneously suppress the vibrations of the cable in the presence of the time-varying ocean disturbance. The control forces $u_1(t)$ and $u_2(t)$ are from the thruster in the vessel and the thruster attached in the subsea payload respectively. In this section, the Lyapunov's direct method is used to construct boundary control $u_1(t)$ and $u_2(t)$ at the top and bottom boundary of the cable and to analyze the stability of the closed-loop system.

In this chapter, we analyze two cases for the flexible marine installation system: (i) exact model-based control, i.e. T, m, and c_2 are all known; and (ii) adaptive control for the system parametric uncertainty, i.e. T, m and c_2 are unknown. For the first case, robust boundary control is introduced for the exact model of the installation system subjected to ocean disturbance. For second case where the system parameters cannot be directly measured, the adaptive control is designed to compensate the system parametric uncertainty.

4.3.1 Exact model-based boundary control of the installation system

To stabilize the system given by governing Eq. (4.8) and boundary condition Eqs. (4.9) and (4.10), we propose the following boundary control

$$
\begin{aligned}
u_1(t) &= -k_q y(L,t) - k_v \dot{y}(L,t) - \mathrm{sgn}[\dot{y}(L,t)]\bar{d}_1, && (4.11)\\
u_2(t) &= -k_p(y(0,t) - p_d) - k_s u_a(t) - Ty'(0,t) + m\dot{y}'(0,t) + c_2\dot{y}(0,t) - \mathrm{sgn}(u_a)\bar{d}_2, && (4.12)
\end{aligned}
$$

where sgn$(\cdot)$ denotes the signum function, k_q, k_v, k_p and k_s are the positive control gains and the auxiliary signal u_a is defined as

$$u_a(t) = \dot{y}(0,t) - y'(0,t). \tag{4.13}$$

After differentiating the auxiliary signal Eq. (4.13), multiplying the resulting equation by m, and substituting Eq. (4.10), we obtain

$$m\dot{u}_a(t) = Ty'(0,t) + d_2(t) - m\dot{y}'(0,t) - c_2\dot{y}(0,t) + u_2(t). \tag{4.14}$$

Substituting Eq. (4.12) into Eq. (4.14), we have

$$m\dot{u}_a(t) = -k_s u_a(t) - k_p(y(0,t) - p_d) + d_2(t) - \mathrm{sgn}(u_a)\bar{d}_2. \tag{4.15}$$

Remark 4.4. *The proposed boundary control does not require distributed sensing and all the signals in the boundary control can be measured by sensors or obtained by a* *backward difference algorithm.* $y(L,t)$ *and* $y(0,t)$ *can be sensed by two the global positioning systems (GPS) located in the vessel and the end-point thruster respectively.* $y'(0,t)$ *can be measured by an inclinometer at the bottom boundary of the cable. For the exact model based boundary control* (4.12)*, the tension of the cable can be measured via a force sensor. In practice, the effect of measurement noise from sensors is unavoidable, which will affect the controller implementation, especially when the high order differentiating terms with respect to time exist. In our proposed controller* (4.11) *and* (4.12)*,* $\dot{y}(L,t)$*,* $\dot{y}(0,t)$ *and* $\dot{y}'(0,t)$ *with only one time differentiating with respect to time can be calculated with a backward difference algorithm.*

Remark 4.5. *The control design is based on the distributed parameter model Eqs.* (4.8) *to* (4.10)*, and the spillover problems associated with traditional truncated model-based approaches caused by ignoring high-frequency modes in controller and observer design are avoided.*

Consider the Lyapunov function candidate

$$V = V_1 + V_2 + \Delta, \tag{4.16}$$

where the energy term V_1 and an auxiliary term V_2 and a small crossing term Δ are defined as

$$V_1 = \frac{\beta}{2}\rho\int_0^L [\dot{y}]^2 dx + \frac{\beta}{2}T\int_0^L [y']^2 dx + \frac{\beta}{2}M[\dot{y}(L,t)]^2 + \frac{\beta k_p}{2}[y(0,t)-p_d]^2 + \frac{\beta k_q}{2}[y(L,t)]^2, \tag{4.17}$$

$$V_2 = \frac{1}{2}mu_a^2, \tag{4.18}$$

$$\Delta = \alpha\rho\int_0^L (x-L)\dot{y}y' dx, \tag{4.19}$$

where α and β are two positive weighting constants.

Lemma 4.1. *The Lyapunov function candidate given by* (4.16)*, can be upper and lower bounded as*

$$0 \leq \lambda_1(V_1 + V_2) \leq V \leq \lambda_2(V_1 + V_2), \tag{4.20}$$

where λ_1 *and* λ_2 *are two positive constants defined as*

$$\lambda_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)} > 0, \tag{4.21}$$

$$\lambda_2 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)} > 1, \tag{4.22}$$

provided

$$\alpha < \frac{\min(\beta\rho, \beta T)}{2\rho L}. \tag{4.23}$$

Proof: Substituting of Ineq. (2.7) into Eq. (4.19) yields

$$\begin{aligned} |\Delta| &\leq \alpha\rho L \int_0^L ([y']^2 + [\dot{y}]^2)dx \\ &\leq \alpha_1 V_1, \end{aligned} \tag{4.24}$$

where

$$\alpha_1 = \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}. \tag{4.25}$$

Then, we obtain

$$-\alpha_1 V_1 \leq \Delta \leq \alpha_1 V_1. \tag{4.26}$$

Considering α is a small positive weighting constant satisfying $0 < \alpha < \frac{\min(\beta\rho, \beta T)}{2\rho L}$, we can obtain

$$\alpha_2 = 1 - \alpha_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)} > 0, \quad (4.27)$$

$$\alpha_3 = 1 + \alpha_1 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)} > 1. \quad (4.28)$$

Then, we further have

$$0 \leq \alpha_2 V_1 \leq V_1 + \Delta \leq \alpha_3 V_1, \quad (4.29)$$

Given the Lyapunov function candidate in Eq. (4.16), we obtain

$$0 \leq \lambda_1(V_1 + V_2) \leq V \leq \lambda_2(V_1 + V_2), \quad (4.30)$$

where $\lambda_1 = \min(\alpha_2, 1) = \alpha_2$ and $\lambda_2 = \max(\alpha_3, 1) = \alpha_3$ are positive constants. ∎

Lemma 4.2. *The time derivative of the Lyapunov function in* (4.16) *can be upper bounded with*

$$\dot{V} \leq -\lambda V + \varepsilon, \quad (4.31)$$

where λ and ε are two positive constants.

Proof: Differentiating Eq. (4.16) with respect to time leads to

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{\Delta}. \quad (4.32)$$

The first term of the Eq. (4.32)

$$\dot{V}_1 = A_1 + A_2 + \beta M \ddot{y}(L,t)\dot{y}(L,t) + \beta k_p(y(0,t) - p_d)\dot{y}(0,t), \tag{4.33}$$

where

$$A_1 = \beta\rho \int_0^L \dot{y}\ddot{y}dx, \tag{4.34}$$

$$A_2 = \beta T \int_0^L y'\dot{y}'dx. \tag{4.35}$$

Substituting the governing equation (4.8) into A_1, we obtain

$$A_1 = \beta \int_0^L \dot{y}\left(Ty'' + f - c\dot{y}\right)dx. \tag{4.36}$$

Using the boundary conditions and integrating Eq. (4.35) by parts, we obtain

$$\begin{aligned} A_2 &= \beta T \int_0^L y'd(\dot{y}) \\ &= \beta T y'(L,t)\dot{y}(L,t) - \beta T y'(0,t)\dot{y}(0,t) - \beta T \int_0^L \dot{y}y''dx. \end{aligned} \tag{4.37}$$

Substituting Eqs. (4.36) and (4.37) into Eq. (4.33), we have

$$\begin{aligned} \dot{V}_1 &= \beta T y'(L,t)\dot{y}(L,t) - \beta T y'(0,t)\dot{y}(0,t) - \beta c \int_0^L [\dot{y}]^2 dx + \beta \int_0^L f\dot{y}dx \\ &\quad + \beta M \ddot{y}(L,t)\dot{y}(L,t) + \beta k_p[y(0,t) - p_d]\dot{y}(0,t) + \beta k_q y(L,t)\dot{y}(L,t). \end{aligned} \tag{4.38}$$

Substituting the Eqs. (4.9) and (4.13) into Eq. (4.38), we obtain

$$\begin{aligned}\dot{V}_1 &= -\frac{\beta T}{2}\left[[\dot{y}(0,t)]^2+[y'(0,t)]^2\right]+\frac{\beta T}{2}u_a^2+\beta[u_1+d_1-c_1\dot{y}(L,t)]\dot{y}(L,t)\\ &\quad -\beta c\int_0^L[\dot{y}]^2dx+\beta\int_0^L f\dot{y}dx+\beta k_p(y(0,t)-p_d)\dot{y}(0,t)+\beta k_q y(L,t)\dot{y}(L,t).\end{aligned}\tag{4.39}$$

Substituting Eq. (4.11) and using Ineq. (2.8), we obtain

$$\begin{aligned}\dot{V}_1 &\le -\frac{\beta T}{2}\left[[\dot{y}(0,t)]^2+[y'(0,t)]^2\right]+\frac{\beta T}{2}u_a^2-\beta(k_v+c_1)[\dot{y}(L,t)]^2-\beta(c-\delta_2)\int_0^L[\dot{y}]^2dx\\ &\quad +\frac{\beta}{\delta_2}\int_0^L f^2dx+\frac{\beta k_p}{2\delta_1}[y(0,t)-p_d]^2+\frac{\beta k_p\delta_1}{2}[\dot{y}(0,t)]^2,\end{aligned}\tag{4.40}$$

where δ_1 and δ_2 are two positive constants.

The second term of the Eq. (4.32)

$$\begin{aligned}\dot{V}_2 &= mu_a\dot{u}_a,\\ &\le -k_su_a^2+k_pu_a^2-k_p[y(0,t)-p_d]^2+8k_pL\int_0^L[y']^2dx+8k_p[y(L,t)]^2+4k_pp_d^2,\end{aligned}\tag{4.41}$$

where Ineqs. (2.7) and (2.11) are employed. We obtain the third term of the Eq. (4.32) as

$$\begin{aligned}\dot{\Delta} &= \alpha\rho\int_0^L((x-L)\ddot{y}y'+(x-L)\dot{y}\dot{y}')dx\\ &= \alpha\int_0^L(x-L)y'\left[Ty''+f-c\dot{y}\right]dx+\alpha\rho\int_0^L(x-L)\dot{y}\dot{y}'dx\\ &= B_1+B_2+B_3+B_4,\end{aligned}\tag{4.42}$$

where

$$B_1 = \alpha \int_0^L T(x-L)y'y''dx, \quad (4.43)$$

$$B_2 = \alpha \int_0^L (x-L)fy'dx, \quad (4.44)$$

$$B_3 = -\alpha \int_0^L c(x-L)y'\dot{y}dx, \quad (4.45)$$

$$B_4 = \alpha\rho \int_0^L (x-L)\dot{y}\dot{y}'dx. \quad (4.46)$$

After integrating Eq. (4.43) by parts and using the boundary conditions, we obtain

$$B_1 = \alpha TL[y'(0,t)]^2 - \alpha T \int_0^L \left([y']^2 + (x-L)y'y''\right) dx. \quad (4.47)$$

Combining Eq. (4.43) and Eq. (4.47), we obtain

$$B_1 = \frac{\alpha TL}{2}[y'(0,t)]^2 - \frac{\alpha T}{2} \int_0^L [y']^2 dx. \quad (4.48)$$

Using Ineq. (2.8), we obtain

$$B_2 \le \frac{\alpha L}{\delta_3} \int_0^L f^2 dx + \alpha L \delta_3 \int_0^L [y']^2 dx, \quad (4.49)$$

$$B_3 \le \frac{\alpha c L}{\delta_4} \int_0^L [\dot{y}]^2 dx + \alpha c L \delta_4 \int_0^L [y']^2 dx, \quad (4.50)$$

where δ_3 and δ_4 are two positive constants. Integrating Eq. (4.46) by parts, we obtain

$$B_4 = \alpha\rho L[\dot{y}(0,t)]^2 - \alpha\rho \int_0^L \left([\dot{y}]^2 + x\dot{y}\dot{y}'\right) dx. \quad (4.51)$$

The last term in Eq. (4.51) equals B_4, and we have

$$B_4 = \frac{\alpha\rho L}{2}[\dot{y}(0,t)]^2 - \frac{\alpha\rho}{2}\int_0^L [\dot{y}]^2 dx. \tag{4.52}$$

Substituting Eqs. (4.48), (4.49), (4.50) and (4.52) into Eq. (4.42) and using the boundary conditions, we obtain

$$\begin{aligned}\dot{\Delta} \leq\ & \frac{\alpha TL}{2}[y'(0,t)]^2 - \frac{\alpha T}{2}\int_0^L [y']^2 dx + \frac{\alpha L}{\delta_3}\int_0^L f^2 dx + \alpha L\delta_3\int_0^L [y']^2 dx + \frac{\alpha cL}{\delta_4}\int_0^L [\dot{y}]^2 dx \\ & + \alpha cL\delta_4\int_0^L [y']^2 dx + \frac{\alpha\rho L}{2}[\dot{y}(0,t)]^2 - \frac{\alpha\rho}{2}\int_0^L [\dot{y}]^2 dx. \end{aligned}\tag{4.53}$$

Substituting Eqs. (4.40), (4.41) and (4.53) into Eq. (4.16), we obtain

$$\begin{aligned}\dot{V} \leq\ & -\left(\beta c + \frac{\alpha\rho}{2} - \beta\delta_2 - \frac{\alpha cL}{\delta_4}\right)\int_0^L [\dot{y}]^2 dx - \left(\frac{\alpha T}{2} - 16k_pL - \alpha L\delta_3 - \alpha cL\delta_4\right)\int_0^L [y']^2 dx \\ & -\beta(k_v + c_1)[\dot{y}(L,t)]^2 - \left(k_s - k_p - \frac{\beta T}{2}\right)u_a^2 - \left(\frac{\beta T}{2} - \frac{\alpha\rho L}{2} - \frac{\beta k_p\delta_1}{2}\right)[\dot{y}(0,t)]^2 \\ & -\left(\frac{\beta T}{2} - \frac{\alpha TL}{2}\right)[y'(0,t)]^2 - k_p(1 - \frac{\beta}{2\delta_1})[y(0,t) - p_d]^2 + \left(\frac{\beta}{\delta_2} + \frac{\alpha L}{\delta_3}\right)\int_0^L \bar{f}^2 dx \\ & +4k_p p_d^2 + 8k_p[y(L,t)]^2 \\ \leq\ & -\lambda_3(V_1 + V_2) + \varepsilon, \end{aligned}\tag{4.54}$$

where the constants k_s, k_v, k_p, k_q, α, β, δ_1, δ_2, δ_3 and δ_4 are chosen to satisfy the following conditions:

$$\begin{aligned}
\alpha &< \frac{\min(\beta\rho, \beta T)}{2\rho L}, && (4.55)\\
\frac{\beta T}{2} &- \frac{\alpha\rho L}{2} - \frac{\beta k_p \delta_1}{2} \geq 0, && (4.56)\\
\frac{\beta T}{2} &- \frac{\alpha T L}{2} \geq 0, && (4.57)\\
\sigma_1 &= \beta c + \frac{\alpha\rho}{2} - \beta\delta_2 - \frac{\alpha c L}{\delta_4} > 0, && (4.58)\\
\sigma_2 &= \frac{\alpha T}{2} - 16 k_p L - \alpha L \delta_3 - \alpha c L \delta_4 > 0, && (4.59)\\
\sigma_3 &= \beta(k_v + c_1) > 0, && (4.60)\\
\sigma_4 &= 1 - \frac{\beta}{2\delta_1} > 0, && (4.61)\\
\sigma_5 &= k_s - k_p - \frac{\beta T}{2} > 0, && (4.62)\\
\sigma_6 &= 8k_p > 0, && (4.63)\\
\lambda_3 &= \min\left(\frac{2\sigma_1}{\beta\rho}, \frac{2\sigma_2}{\beta T}, \frac{2\sigma_3}{\beta M}, \frac{2\sigma_4}{\beta}, \frac{2\sigma_5}{m}, \frac{2\sigma_6}{\beta k_q}\right) > 0, && (4.64)\\
\varepsilon &= \left(\frac{\beta}{\delta_2} + \frac{\alpha L}{\delta_3}\right) \int_0^L \bar{f}^2 dx + 4k_p p_d^2 \in \mathcal{L}_\infty. && (4.65)
\end{aligned}$$

From Ineqs. (4.30) and (4.54) we have

$$\dot{V} \leq -\lambda V + \varepsilon, \quad (4.66)$$

where $\lambda = \lambda_3/\lambda_2$ and ε are two positive constants. ■

With the above lemmas, the exact model-based control design for the flexible marine installation system subjected to ocean current disturbance can be summarized in the following theorem.

Theorem 4.1. *For the system dynamics described by* (4.8) *and boundary conditions* (4.9) - (4.10), *under Assumption 4.1, and the boundary control* (4.11) *and* (4.12), *given that the initial conditions are bounded, the transverse reflection* $w(x,t)$ *of the closed loop system is uniformly bounded, and the system boundary error signal* $e(t) = y(0,t) - p_d$ *will remain within the compact set* Ω *defined by*

$$\Omega := \{e \in R| \quad |e| \leq D\} \tag{4.67}$$

where $D = \sqrt{\frac{2}{\beta k_p \lambda_1}\left(V(0) + \frac{\varepsilon}{\lambda}\right)}$.

Proof: Multiplying Eq. (4.31) by $e^{\lambda t}$ yields

$$\frac{\partial}{\partial t}(Ve^{\lambda t}) \;\; \leq \;\; \varepsilon e^{\lambda t}. \tag{4.68}$$

Integration of the above inequality, we obtain

$$V \leq \left(V(0) - \frac{\varepsilon}{\lambda}\right) e^{-\lambda t} + \frac{\varepsilon}{\lambda} \leq V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda} \in \mathcal{L}_\infty, \tag{4.69}$$

which implies V is bounded. Utilizing Ineq. (2.11) and Eq. (4.17), we have

$$\frac{\beta}{2L}Tw^2(x,t) \leq \frac{\beta}{2}T\int_0^L [w'(x,t)]^2 dx = \frac{\beta}{2}T\int_0^L [y'(x,t)]^2 dx \leq V_1 \leq V_1 + V_2 \leq \frac{1}{\lambda_1}V. \tag{4.70}$$

Appropriately rearranging the terms of the above Ineq. (4.70), we obtain $w(x,t)$ is uniformly bounded as follows

$$|w(x,t)| \;\; \leq \;\; \sqrt{\frac{2L}{\beta T \lambda_1}\left(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda}\right)}, \; \forall (x,t) \in [0,L] \times [0,\infty). \tag{4.71}$$

Combining Eq. (4.17) and Ineq. (4.70) yields

$$\frac{\beta k_p}{2}[y(0,t)-p_d]^2 \leq V_1 \leq V_1 + V_2 \leq \frac{1}{\lambda_1} V \in \mathcal{L}_\infty, \tag{4.72}$$

$$|y(0,t)-p_d| \ \leq \ \sqrt{\frac{2}{\beta k_p \lambda_1}\left(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda}\right)}, \ \forall t \in [0,\infty). \tag{4.73}$$

■

Remark 4.6. *In the above analysis, the deflection of the cable $w(x,t)$ can be made arbitrarily small provided that the design control parameters are appropriately selected. By choosing the proper values of α and β, it is shown that the increase in the control gains k_v and k_s will result in a larger σ_3 and σ_5, which will lead a greater λ_3. Then the value of λ will increase, which will reduce the size of Ω and bring a better vibration suppression performance.*

Remark 4.7. *Even though the $y(0,t)$ may be far from the desired position p_d, it is guaranteed that the steady bottom boundary state error $y(0,\infty) - p_d$ can be made arbitrarily small provided that the design parameters are appropriately selected. It is easily seen that the increase in the control gains k_v and k_s will result in a better tracking performance. However, increasing k_v and k_s will lead a high gain control scheme. Therefore, in practical applications, the design parameters should be adjusted carefully for achieving suitable transient performance and control action.*

Remark 4.8. *From Eq.* (4.70)*, we can state that V_1 and V_2 are bounded $\forall t \in [0,\infty)$. Use of boundedness of V_1 and V_2 produces $\dot{y}(x,t)$, $y'(x,t)$ are bounded $\forall (x,t) \in [0,L] \times [0,\infty)$ and u_a is bounded $\forall t \in [0,\infty)$. Then, we can obtain that potential energy Eq.* (4.3) *is bounded. Using Property 4.2, we can further obtain that $y''(x,t)$ is bounded.*

From the boundedness of $\dot{y}(x,t)$*, we can state that* $\dot{y}(0,t)$ *and* $\dot{y}(L,t)$ *are bounded. Therefore, we can conclude that the kinetic energy of the system Eq.* (4.1) *is also bounded. Using Property 4.1, we can obtain* $\dot{y}(x,t)$ *and* $\dot{y}'(x,t)$ *are also bounded* $\forall(x,t) \in [0,L]\times[0,\infty)$*. Applying Assumption 4.1, Eq.* (4.8) *and the above statements, we can state that* $\ddot{y}(x,t)$ *is also bounded* $\forall(x,t) \in [0,L] \times [0,\infty)$*. From the above information, it is shown that the proposed control* (4.11) *and* (4.12) *ensure all internal system signals including* $w(x,t)$*,* $y'(x,t)$*,* $\dot{y}(x,t)$*,* $\dot{y}'(x,t)$ *and* $\ddot{y}(x,t)$ *are uniformly bounded. Since* $y'(x,t)$*,* $\dot{y}(x,t)$ *and* $\dot{y}'(x,t)$ *are all bounded* $\forall(x,t) \in [0,L]\times[0,\infty)$*, and we can conclude the boundary control* (4.11) *and* (4.12) *are also bounded* $\forall t \in [0,\infty)$*.*

4.3.2 Robust adaptive boundary control for system parametric uncertainty

The previous exact model-based boundary control Eq. (4.11) requires the exact knowledge of the marine installation system. Adaptive boundary control is designed to improve the performance of the system via parameter estimation when the system parameters are unknown. The exact model-based boundary control provides a stepping stone towards the adaptive control, which is designed to deal with the system parametric uncertainty. In this section, the previous boundary control is redesigned by using adaptive control when T, m, and c_2 are all unknown. We rewrite Eq. (4.14) as the following form

$$m\dot{u}_a = P\Phi + d_2 + u_2, \tag{4.74}$$

where vectors P and Φ are defined as

$$P = [y'(0,t) \quad -\dot{y}'(0,t) \quad -\dot{y}(0,t)], \tag{4.75}$$

$$\Phi = [T \quad m \quad c_2]^T. \tag{4.76}$$

We propose the following adaptive boundary control for system

$$u_1 = -k_q y(L,t) - k_v \dot{y}(L,t) - \mathrm{sgn}[\dot{y}(L,t)]\bar{d}_1, \tag{4.77}$$

$$u_2 = -P\hat{\Phi} - k_s u_a - \mathrm{sgn}(u_a)\bar{d}_2 - k_p(y(0,t) - p_d), \tag{4.78}$$

where parameter estimate vector $\hat{\Phi}$ is defined as

$$\hat{\Phi} = [\widehat{T} \quad \widehat{m} \quad \widehat{c_2}]^T. \tag{4.79}$$

The adaptation law is designed as

$$\dot{\hat{\Phi}} = \Gamma P^T u_a - r\Gamma\hat{\Phi}, \tag{4.80}$$

where $\Gamma \in R^{3\times 3}$ is a diagonal positive-definite matrix and r is a positive constant. We define all the eigenvalues of Γ are real and positive, and the maximum and minimum eigenvalue of matrix Γ as $\lambda_{\max}$ and $\lambda_{\min}$ respectively. The parameter estimate error vector $\tilde{\Phi} \in R^3$ is defined as

$$\tilde{\Phi} = \Phi - \hat{\Phi}. \tag{4.81}$$

Substituting Eq. (4.78) into Eq. (4.74) and substituting Eq. (4.81) into Eq. (4.80), we have

$$m\dot{u}_a = P\tilde{\Phi} - k_s u_a + d_2 - \mathrm{sgn}(u_a)\bar{d}_2 - k_p(y(0,t) - p_d), \tag{4.82}$$

$$\dot{\tilde{\Phi}} = -\Gamma P^T u_a + r\Gamma\hat{\Phi}. \tag{4.83}$$

Remark 4.9. *For the proposed adaptive control* (4.78)*, a parameter estimation term, a signum term and an auxiliary signal term are introduced to compensate for the system parametric uncertainty and the effect of unknown time-varying distributed disturbance. The control* (4.77) *and* (4.78) *are independent of system parameters and the knowledge of the exact values for disturbance $f(x,t)$, $d_1(t)$ and $d_2(t)$ is not required, thus possessing stability robustness to variations in system parameters and unknown disturbance.*

Consider the Lyapunov function candidate

$$V_a = V + \frac{1}{2}\tilde{\Phi}^T\Gamma^{-1}\tilde{\Phi}, \tag{4.84}$$

where V is defined as Eq. (4.16), and $\tilde{\Phi}$ is the parameter estimate error vector.

Lemma 4.3. *The Lyapunov function candidate given by* (4.84)*, can be upper and lower bounded as*

$$0 \leq \lambda_{1a}(V_1 + V_2 + ||\tilde{\Phi}||^2) \leq V_a \leq \lambda_{2a}(V_1 + V_2 + ||\tilde{\Phi}||^2), \tag{4.85}$$

where λ_{1a} and λ_{2a} are two positive constants defined as

$$\lambda_{1a} = \min(1 - \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}, \frac{1}{2\lambda_{\max}}), \tag{4.86}$$

$$\lambda_{2a} = \max(1 + \frac{2\alpha\rho L}{\min(\beta\rho, \beta T)}, \frac{1}{2\lambda_{\min}}). \tag{4.87}$$

Proof: From Ineq. (4.20), we have

$$\lambda_1(V_1 + V_2) \leq V \leq \lambda_2(V_1 + V_2), \tag{4.88}$$

where λ_1 and λ_2 are two positive constants defined in Eqs. (4.21) and (4.22). Utilizing the properties of matrix Γ and Lemma 2.6, we have

$$\frac{1}{2\lambda_{\max}}||\tilde{\Phi}||^2 \leq \frac{1}{2}\tilde{\Phi}^T\Gamma^{-1}\tilde{\Phi} \leq \frac{1}{2\lambda_{\min}}||\tilde{\Phi}||^2. \tag{4.89}$$

Combining Ineqs. (4.88) and (4.89), we have

$$0 \leq \lambda_{1a}(V_1 + V_2 + ||\tilde{\Phi}||^2) \leq V_a \leq \lambda_{2a}(V_1 + V_2 + ||\tilde{\Phi}||^2), \tag{4.90}$$

where $\lambda_{1a} = \min(\alpha_2,\ \frac{1}{2\lambda_{\max}})$ and $\lambda_{2a} = \max(\alpha_3,\ \frac{1}{2\lambda_{\min}})$ are two positive constants. ∎

Lemma 4.4. *The time derivative of the Lyapunov function in* (4.84) *can be upper bounded with*

$$\dot{V}_a \ \leq \ -\lambda_a V_a + \psi, \tag{4.91}$$

where λ_a and ψ are two positive constants.

Proof: We obtain the time derivation of the Lyapunov function candidate Eq. (4.84) as

$$\dot{V}_a \ = \ \dot{V} + \tilde{\Phi}^T\Gamma^{-1}\dot{\tilde{\Phi}}. \tag{4.92}$$

Substituting Eq. (4.82) into the second term of the Eq. (4.32), we have

$$\begin{aligned}\dot{V}_2 &= mu_a\dot{u}_a \\ &= -k_s u_a^2 - k_p u_a[y(0,t) - p_d] + d_2 u_a - \operatorname{sgn}(u_a)\bar{d}_2 u_a + P\tilde{\Phi}u_a \\ &\le -k_s u_a^2 + k_p u_a^2 - k_p[y(0,t) - p_d]^2 + 16k_p L\int_0^L [y']^2 dx \ + 4k_p p_d^2 + P\tilde{\Phi}u_a.\end{aligned} \tag{4.93}$$

Applying the results of Lemma 4.2 and substituting Eqs. (4.40), (4.93) and (4.53) into Eq. (4.16), we obtain

$$\dot{V} \le -\lambda_3(V_1 + V_2) + P\tilde{\Phi}u_a + \varepsilon, \tag{4.94}$$

where λ_3 is defined in Eq. (4.64) and ε is defined in Eq. (4.62). Substituting of Ineq. (4.94) into Eq. (4.92) yields

$$\dot{V}_a \ \le \ -\lambda_3(V_1 + V_2) + \tilde{\Phi}^T\left(P^T u_a + \Gamma^{-1}\dot{\tilde{\Phi}}\right) + \varepsilon. \tag{4.95}$$

Substituting Eq. (4.83) into Eq. (4.95), we have

$$\begin{aligned}\dot{V}_a &\le -\lambda_3(V_1 + V_2) + r\tilde{\Phi}^T\hat{\Phi} + \varepsilon \\ &\le -\lambda_3(V_1 + V_2) - \frac{r}{2}||\tilde{\Phi}||^2 + \frac{r}{2}||\Phi||^2 + \varepsilon \\ &\le -\lambda_{3a}(V_1 + V_2 + ||\tilde{\Phi}||^2) + \frac{r}{2}||\Phi||^2 + \varepsilon,\end{aligned} \tag{4.96}$$

where $\lambda_{3a} = \min(\lambda_3, \frac{r}{2})$ is a positive constant. From Ineqs. (4.90) and (4.96), we have

$$\dot{V}_a \ \le \ -\lambda_a V_a + \psi, \tag{4.97}$$

where $\lambda_a = \lambda_{3a}/\lambda_{2a}$ and $\psi = \frac{r}{2}||\Phi||^2 + \varepsilon > 0$.

With the above lemmas, the adaptive control design for the marine installation system subjected to ocean current disturbance can be summarized in the following theorem.

Theorem 4.2. *For the system dynamics described by* (4.8) *and boundary conditions* (4.9) - (4.10)*, under Assumption 4.1, and the boundary control* (4.77) *and* (4.78)*, given that the initial conditions are bounded, the closed loop system is uniformly bounded, and the system boundary error signal $e(t) = y(0,t) - p_d$ will remain within the compact set Ω_a defined by*

$$\Omega_a := \left\{ e \in R \middle| \quad |e| \leq D_a \right\}, \tag{4.98}$$

where $D_a = \sqrt{\frac{2}{\beta k_p \lambda_{1a}} \left(V_a(0) + \frac{\psi}{\lambda_a} \right)}$.

Proof: Multiplying Eq. (4.91) by $e^{\lambda_a t}$ yields

$$\frac{\partial}{\partial t}(V_a e^{\lambda_a t}) \quad \leq \quad \delta e^{\lambda_a t}. \tag{4.99}$$

Integrating of the above inequality, we obtain

$$V_a \leq \left(V_a(0) - \frac{\psi}{\lambda_a} \right) e^{-\lambda_a t} + \frac{\psi}{\lambda_a} \leq V_a(0) e^{-\lambda_a t} + \frac{\psi}{\lambda_a} \in \mathcal{L}_\infty, \tag{4.100}$$

which implies V_a is bounded. Utilizing Ineq. (2.11) and Eq. (4.17), we have

$$\frac{\beta}{2L} T w^2(x,t) \leq \frac{\beta}{2} T \int_0^L [w'(x,t)]^2 dx = \frac{\beta}{2} T \int_0^L [y'(x,t)]^2 dx \leq V_1 \leq V_1 + V_2 \leq \frac{1}{\lambda_{1a}} V_a, \tag{4.101}$$

Appropriately rearranging the terms of the above inequality, we obtain $w(x,t)$ is uniformly bounded as follows

$$|w(x,t)| \leq \sqrt{\frac{2L}{\beta T \lambda_{1a}}\left(V_a(0)e^{-\lambda_a t} + \frac{\psi}{\lambda_a}\right)}, \ \forall (x,t) \in [0,L]\times[0,\infty). \quad (4.102)$$

Combining Eq. (4.17) and Ineq. (4.66) yields

$$\frac{\beta k_p}{2}[y(0,t) - p_d]^2 \leq V_1 \leq V_1 + V_2 \leq \frac{1}{\lambda_{1a}} V_a \in \mathcal{L}_\infty, \quad (4.103)$$

$$|y(0,t) - p_d| \leq \sqrt{\frac{2}{\beta k_p \lambda_{1a}}\left(V_a(0)e^{-\lambda_a t} + \frac{\psi}{\lambda_a}\right)}, \ \forall t \in [0,\infty). \quad (4.104)$$

■

Remark 4.10. *From the similar analysis of Remark 4.6 and Remark 4.7, we can conclude that both steady bottom boundary state error* $y(0,\infty) - p_d$ *and the deflection of the cable* $w(x,t)$ *can be made arbitrarily small by choosing control gains* k_p, k_q, k_v *and* k_s *appropriately.*

Remark 4.11. *From Eq.* (4.100)*, we can obtain the parameter estimate error* $\tilde{\Phi}$ *is bounded* $\forall t \in [0,\infty)$*. Using the derivation similar to those employed in Remark 4.8, we can state the proposed control Eqs.* (4.77) *and* (4.78) *ensure all internal system signals including* $y(x,t)$, $y'(x,t)$, $\dot{y}(x,t)$, $\dot{y}'(x,t)$ *and* $\ddot{y}(x,t)$ *are uniformly bounded. Since* $\hat{\Phi}$, $y'(x,t)$ *and* $\dot{y}(x,t)$ *are all bounded* $\forall (x,t) \in [0,L]\times[0,\infty)$*, and we can conclude the robust adaptive boundary control Eqs.* (4.77) *and* (4.78) *are also bounded* $\forall t \in [0,\infty)$.

4.4 Numerical Simulations

Simulations for a marine installation system under ocean disturbance are carried out to demonstrate the effectiveness of the proposed boundary control Eq. (4.11) and Eq. (4.12). Numerical methods are applied to obtain the approximate solution of the system (4.8)-(4.10), when there is no obtainable analytical solution. In this chapter, we select the finite difference method to simulate the system performance with boundary control.

The cable, initially at rest, is excited by a distributed transverse disturbance due to ocean current. The corresponding initial conditions of the marine installation system system are given as

$$y(x,0) = 0, \tag{4.105}$$

$$\frac{\partial y(x,0)}{\partial t} = 0. \tag{4.106}$$

The system parameters are given in Table 1.

Table 1: Parameters of the flexible marine installation system

Parameter	Description	Value
L	Length of the cable	$1000.00m$
D	Diameter of the cable	$0.05m$
M	Mass of the vessel	$9.60 \times 10^7 kg$
m	Mass of the payload	$4 \times 10^5 kg$
c_1	Damping coefficient of the vessel	$9.00 \times 10^7 NS/m$
c_2	Damping coefficient of the payload	$2.00 \times 10^5 NS/m$
T	Tension	$4.00 \times 10^6 N$
ρ	Mass per unit length of the cable	$8.02kg/m$
ρ_s	Sea water density	$1024.00kg/m^3$
c	Distributed damping coefficient of the cable	$1.00NS/m^2$
p_d	Desired set-point	$50.00m$

In the simulation, the ocean surface current velocity $U(t)$ is given by Eq. (3.131). We assume that the full current load is applied from $x = 1000m$ to $x = 0m$ and thereafter linearly decline to zero at the ocean floor, $x = 0$, to obtain a depth dependent ocean current profile $U(x,t)$ as in Chapter 3. The distributed load $f(x,t)$ is generated by Eq. (2.5) with $C_D = 1$, $\theta = 0$, $S_t = 0.2$ and $f_v = 2.625$. The distributed load at the top boundary of the cable is shown in Fig. 4.2. The disturbance $d_1(t)$ on the vessel is generated by the following equation

$$d_1(t) = [3 + 0.8\sin(0.7t) + 0.2\sin(0.5t) + 0.2\sin(0.9t)] \times 10^6. \tag{4.107}$$

The disturbance $d_2(t)$ on the payload is given by the following equation

$$d_2(t) = [3 + 0.8\sin(0.7t) + 0.2\sin(0.5t) + 0.2\sin(0.9t)] \times 10^4. \tag{4.108}$$

Position of the cable for free vibration, i.e., $u_1(t) = u_2(t) = 0$, exposed to ocean disturbance is shown in Fig. 4.3. The boundary position of the cable is given in Fig. 4.4. It is clear that the system is unstable and the vibration of the cable is quite large. Position of the cable with exact model-based control Eqs. (4.11) and (4.12), by choosing $k_v = 2 \times 10^7$, $k_p = 4 \times 10^2$, $k_s = 2 \times 10^{10}$, under ocean disturbance is shown in Fig. 4.5. The corresponding boundary position of the cable and boundary control input are shown in Figs. 4.6 and 4.7. When the system parameters T, m and c_2 are unknown, position of the cable with adaptive control Eqs. (4.77) and (4.78), by choosing $k_v = 2 \times 10^7$, $k_p = 4 \times 10^2$, $k_s = 2 \times 10^{10}$, $r = 0.001$ and $\Gamma = \text{diag}\{5 \times 10^6, 1 \times 10^4, 5 \times 10^6\}$, under ocean disturbance is shown in Fig. 4.8. The corresponding boundary position of the cable and boundary control input are shown in Figs. 4.9 and 4.10.

Figs. 4.5 and 4.8 illustrate that both the model-based boundary control and the adaptive boundary control are able to bring the subsea payload to the desired position $p_d = 50m$ and stabilize the cable at the small neighborhood of its equilibrium position.

4.5 Conclusion

In this chapter, both position control and vibration suppression have been investigated for a flexible marine installation system subjected to the ocean disturbance. Two cases for the flexible marine installation system are studied: (i) exact model-based control, and (ii) adaptive control for the system parametric uncertainty. For the first case, a boundary controller is introduced for the exact model of the installation system. For second case where the system parameters cannot be directly measured, to fully compensate for the effect of unknown system parameters, a signum term and an auxiliary signal term are introduced to develop a robust adaptive boundary control law. Both two types of boundary control are designed based on the original infinite dimensional model (PDE), and thus the spillover instability phenomenon is eliminated. All the signals of the closed-loop system are proved to be uniformly bounded by using the Lyapunov's direct method. The exact model based boundary control (4.11) and (4.12) require the measurement of the tension, the top position and slope of the cable. While the robust adaptive boundary control (4.77) and (4.78) only require measurements of the top position and slope of the cable. The proposed schemes offer implementable design procedures for the control of marine installation systems since all the signals in the control can be measured by sensors or calculated by a backward difference algorithm. The simulation results illustrate that the proposed control is able to position the payload to the desired set-point and suppress the vibration of the cable with a good performance.

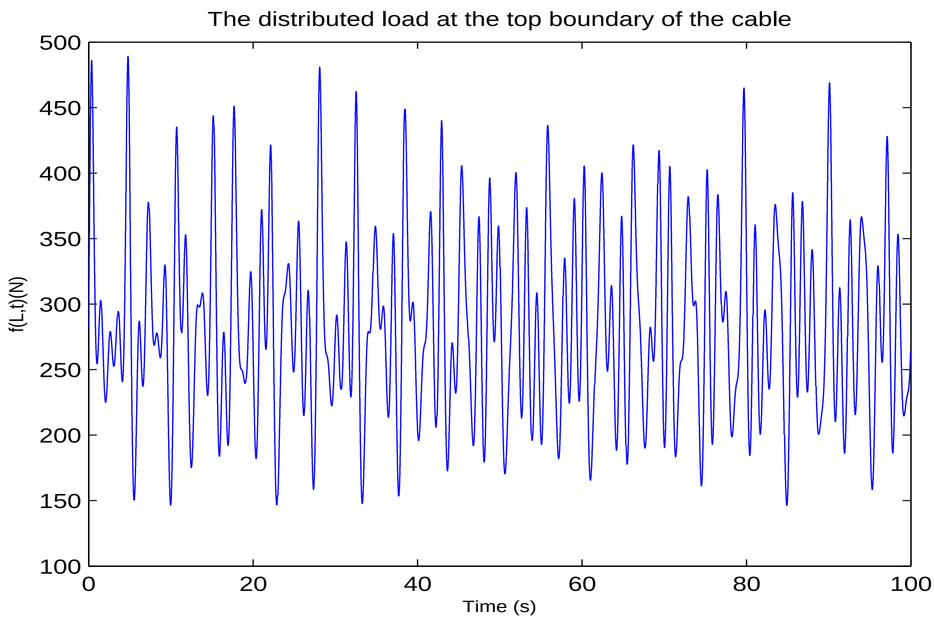


Fig. 4.2: The distributed load at the top boundary of the cable $f(L,t)$.

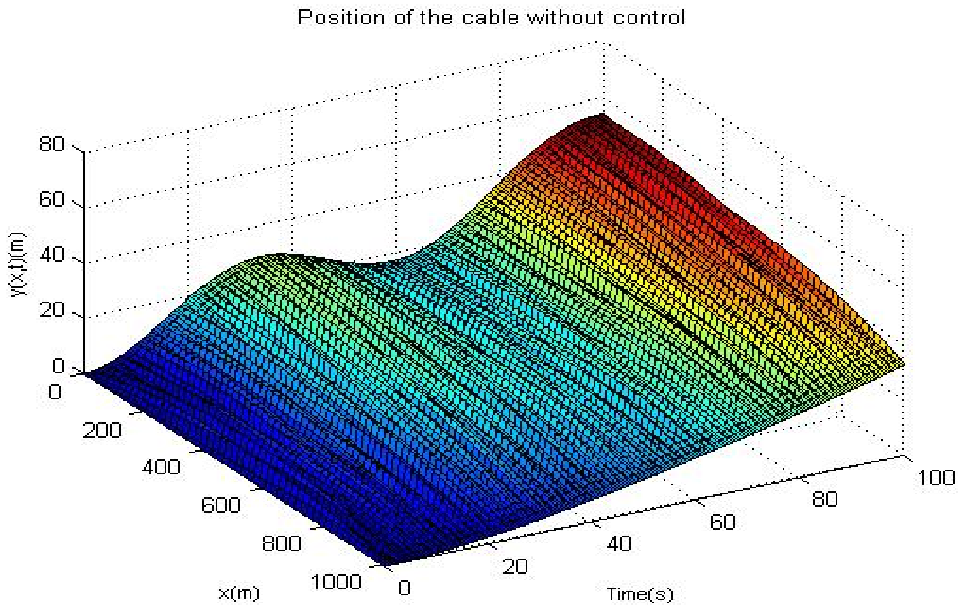


Fig. 4.3: Position of the cable without control.

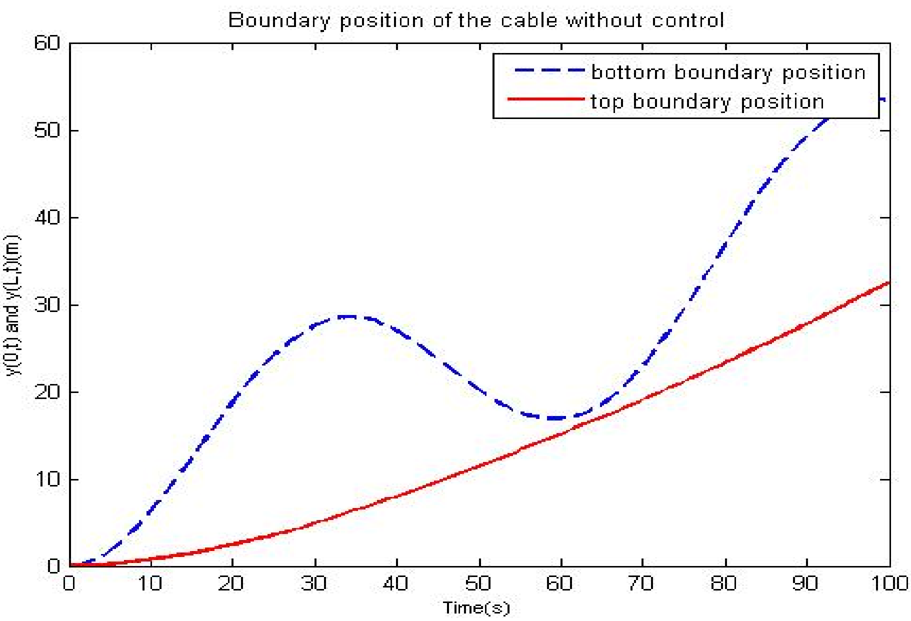


Fig. 4.4: Boundary position of the cable without control.

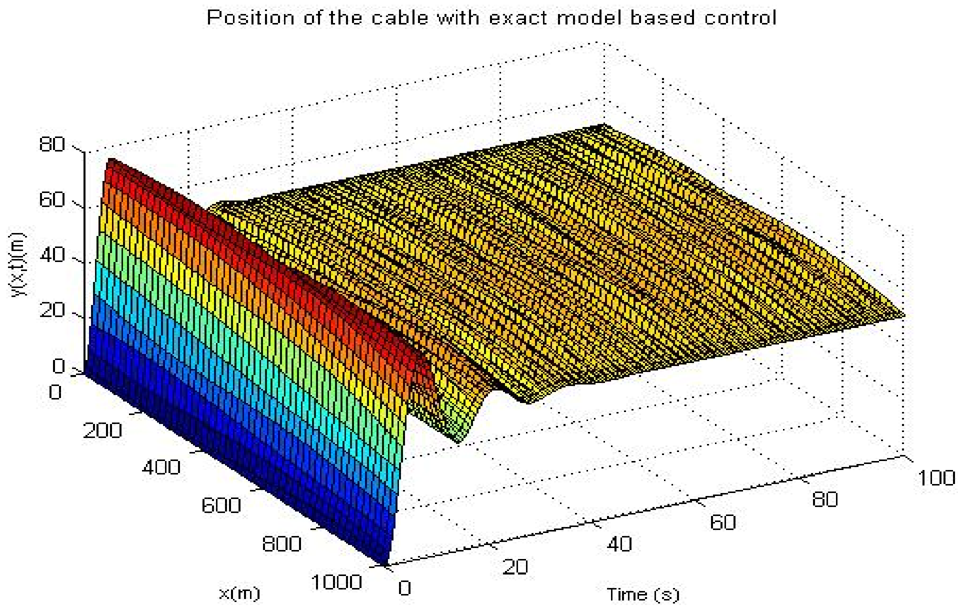


Fig. 4.5: Position of the cable with model based boundary control.

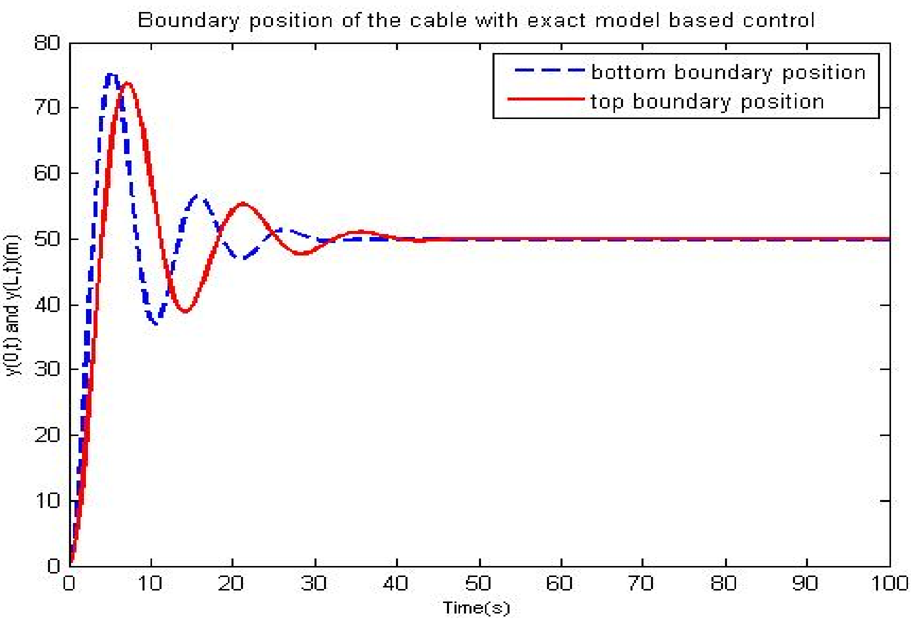


Fig. 4.6: Boundary position of the cable with model based control.

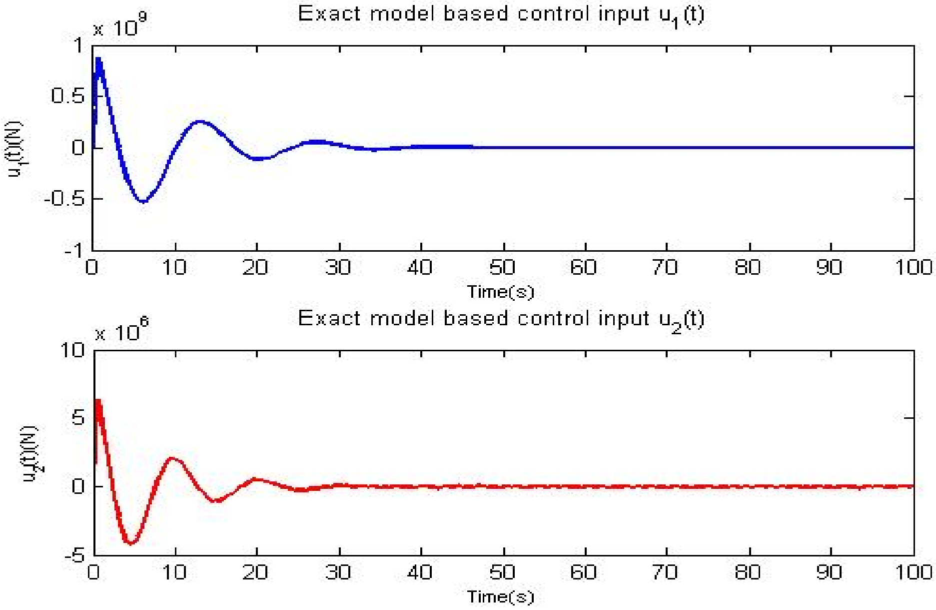


Fig. 4.7: Model-based control input $u_1(t)$ and $u_2(t)$.

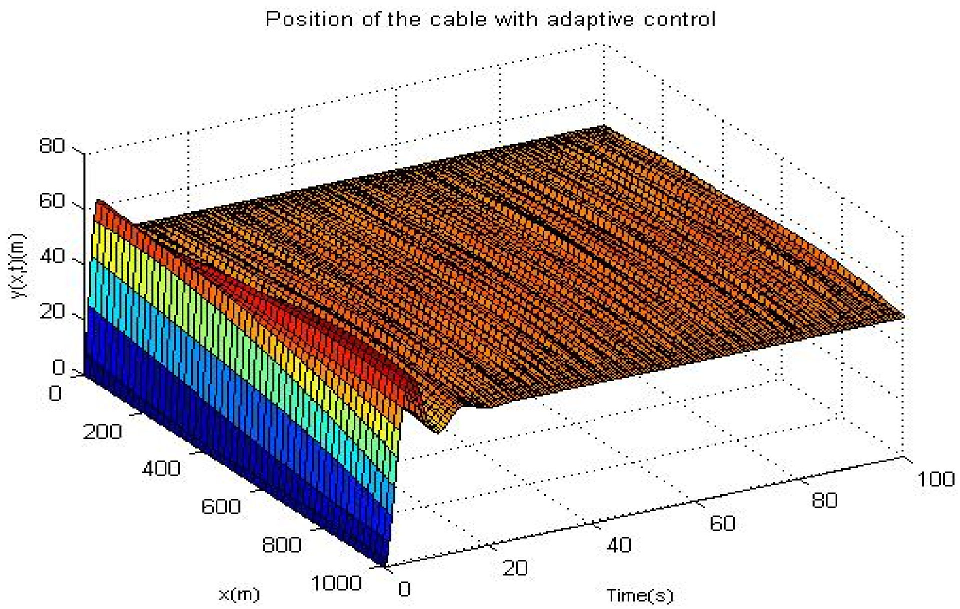


Fig. 4.8: Position of the cable with robust adaptive boundary control.

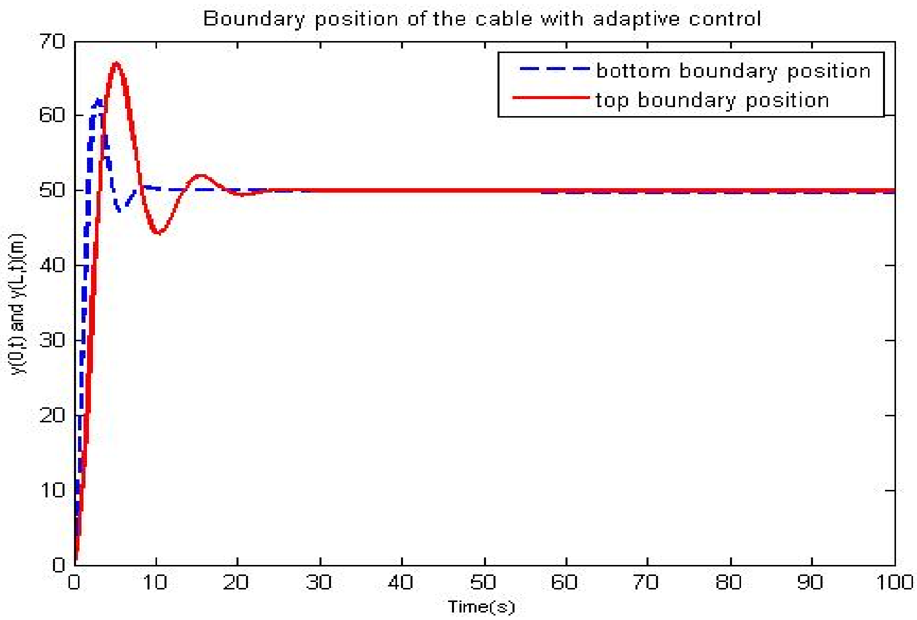


Fig. 4.9: Boundary position of the cable with robust adaptive control.

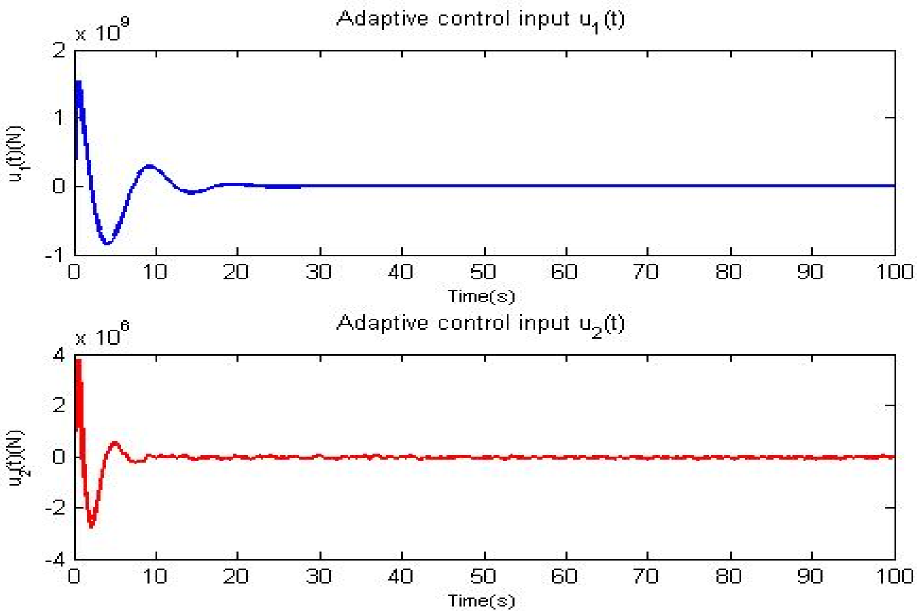


Fig. 4.10: Adaptive control input $u_1(t)$ and $u_2(t)$.

Chapter 5

Flexible Marine Riser

5.1 Introduction

Vibration problems of slender bodies in ocean engineering such as oil drilling and gas exploration have received increasing attention. Improving reliability and efficiency of operations during oil and gas production in the ocean environment is a challenging research topic in offshore engineering. With the trends towards exploiting resources in deep waters and harsher environments, the vibration problem of riser becomes more and more significant [138]. A typical marine riser system depicted in Fig. 5.1 is the connection between a vessel on the ocean surface and a well head on the ocean floor. A drilling riser is used for drilling pipe protection and transportation of the drilling mud, while a production riser is a pipe used for oil transportation. The stiffness of a flexible marine riser depends on its tension and length, thus a riser that spans a long distance can produce large vibrations under relatively small disturbances. In marine environment, vibrations excited by vortices can degrade the performance of the flexible marine riser. Vibrations of the riser due to the ocean current disturbance and tension exerted at the top can produce premature fatigue problems, which requires inspections and costly repairs, and as a worst case, environmental pollution due to leakage from damaged areas. Vibration reduction to minimize the bending stresses is desirable for preventing damage and improving lifespan.

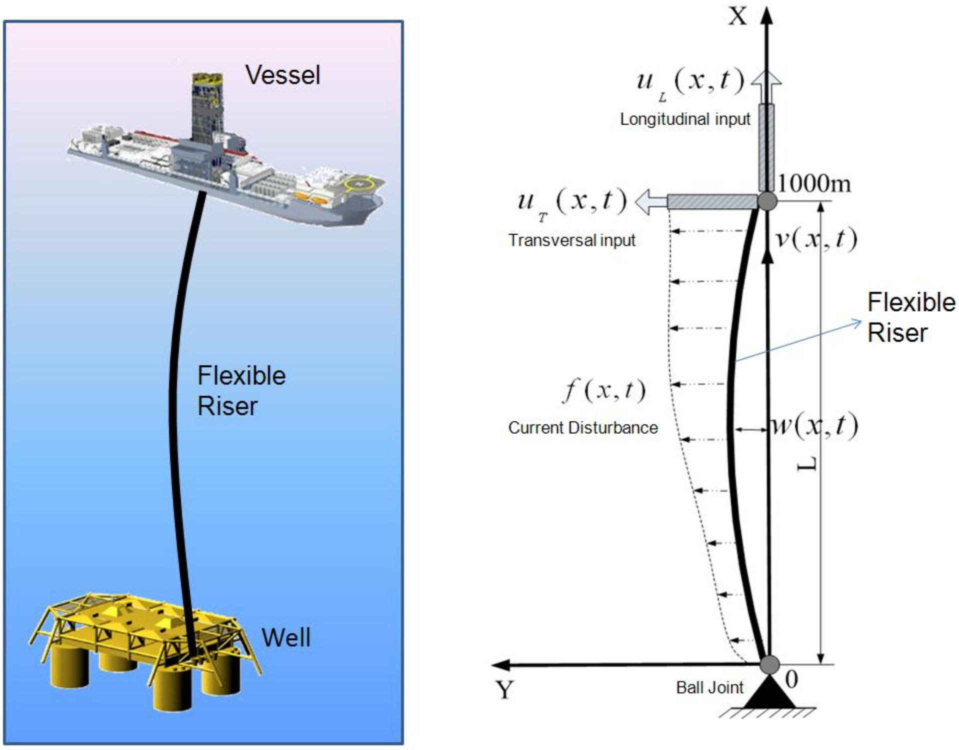


Fig. 5.1: A typical marine riser system.

For purpose of dynamic analysis, the flexible riser is regarded as a distributed parameter system which is infinite dimensional and mathematically represented by PDEs with various boundary conditions involving functions of space and time. The riser system can be modeled as an Euler-Bernoulli beam structure since the diameter-to-length of the riser is small from the ocean surface to the ocean floor. In practice, dynamics of flexible risers are usually represented by a set of PDEs with appropriate boundary equations or approximated by ordinary differential equations (ODEs).

In [9,122,123], PDEs based on the Euler-Bernoulli beam model have been used to analyze the dynamic response of the flexible marine riser system under the ocean current disturbance. In [72], a boundary controller for the flexible marine riser with actuator dynamics is designed based on Lyapunov's direct method and the backstepping technique. In [10], a boundary control law is designed to generate the required signal for riser angle control and transverse vibration reduction with guaranteed closed-loop stability and the exponential stability of the system is proved under the free vibration conditions. The dynamics of the flexible mechanical system is modeled by a set of PDEs with infinite number of dimensions which makes it difficult to control.

In this chapter, we design the boundary control law based on the distributed parameter system model of the flexible riser system. As shown in Fig. 5.1, the control is implemented at the top of the riser through two actuators respectively in transverse and longitudinal directions. The control objective is to design a controller to reduce both transverse and longitudinal vibrations of the riser. The control inputs from the two actuators in the vessel are designed via Lyapunov's direct method and the required measurements for feedback are the displacement in the transverse and longitudinal directions at the top of the riser. Although a flexible riser is being considered in this chapter specifically, the analysis and control design can be extended and applied for vibration control for a class of tensioned beams exposed to undesirable distributed transverse loads. Other examples of practical application in the marine environment include free hanging underwater pipelines or drill strings.

In former marine flexible riser research, the axial deformation of the riser is usually ignored for the convenience of dynamic analysis. Only the transverse dynamics of the riser is considered and the coupling between transverse and longitudinal displacements is neglected, which can influence the dynamic response of the riser system and lead to an imprecise model. In this chapter, both the axial deformation and transverse displacement of the riser are considered in the dynamic analysis. To the best of our knowledge, this is the first application of boundary control to a flexible marine riser for transverse and longitudinal vibrations reduction through two actuators. The main contributions of this chapter include:

(i) A coupled nonlinear dynamic model of the marine flexible riser for transverse and longitudinal vibrations reduction is derived under the distributed ocean current disturbance.

(ii) An implementable boundary control with two actuators in transverse and longitudinal directions is designed to reduce both transverse and longitudinal vibrations of the marine flexible riser.

(iii) Uniform boundedness under ocean current disturbance and exponential stability under free vibration condition are proved via Lyapunov's direct method.

(iv) Numerical simulations via finite difference method are used to verify the effectiveness and performance of the proposed controller.

The rest of the chapter is organized as follows. Section 5.2 illustrates the dynamic equations (PDEs) of the flexible riser and boundary conditions by analyzing the dynamics of this flexible structure with fluctuant environmental disturbances. In Section 5.3, the boundary control design via Lyapunov's direct method is discussed for this coupled nonlinear flexible beam, where it is shown that the uniform boundedness of the closed-loop system can be guaranteed under the distributed ocean current disturbance and the exponential stability can be achieved under free vibration condition. The numerical simulation with finite difference method is presented in Section 5.4 to verify the performance of the proposed controller. The conclusion of this chapter is shown in Section 5.5.

5.2 Problem Formulation

In this chapter, we assume that the vessel is directly above the subsea well head with no horizontal offset and the riser is filled with seawater. The flexible marine riser with uniform density and flexural rigidity is modeled as the Euler-Bernoulli beam structure since the diameter-to-length of the riser is small.

The kinetic energy of the riser system E_k can be represented as

$$E_k = \frac{1}{2}\rho \int_0^L \left[\left(\frac{\partial w(x,t)}{\partial t} \right)^2 + \left(\frac{\partial v(x,t)}{\partial t} \right)^2 \right] dx, \tag{5.1}$$

where x and t represent the independent spatial and time variables respectively, $w(x,t)$ and $v(x,t)$ are the displacement in the transverse and longitudinal directions of the riser at the position x for time t, $\rho > 0$ is the uniform mass per unit length of the riser, and L is the length of the beam.

The potential energy E_p due to the bending and the axial deformation [18] can be obtained from

$$\begin{aligned} E_p = & \frac{1}{2}EI \int_0^L \left[\frac{\partial^2 w(x,t)}{\partial x^2} \right]^2 dx + \frac{1}{2}T \int_0^L \left[\frac{\partial w(x,t)}{\partial x} \right]^2 dx \\ & + \frac{1}{2}EA \int_0^L \left\{ \frac{\partial v(x,t)}{\partial x} + \frac{1}{2} \left[\frac{\partial w(x,t)}{\partial x} \right]^2 \right\}^2 dx, \end{aligned} \tag{5.2}$$

where T is the tension of the riser, EI is the bending stiffness, and EA is the axial stiffness. Both EI and EA are assumed to be constant throughout this chapter. The first term of Eq. (5.2) is due to the bending, the second term is due to axial force, and the third term is due to the strain energy of the beam.

The work done by ocean current disturbance on the riser is given by

$$W_f = \int_0^L f(x,t)w(x,t)dx, \tag{5.3}$$

where $f(x,t)$ is the distributed transverse load due to the hydrodynamic effects of the ocean current. The work done by linear structure damping is represented by

$$W_d = -\int_0^L c_1 \left[\frac{\partial w(x,t)}{\partial t}\right] w(x,t)dx - \int_0^L c_2 \left[\frac{\partial v(x,t)}{\partial t}\right] v(x,t)dx, \tag{5.4}$$

where $c_1, c_2 > 0$ are the structural distributed transverse and longitudinal damping coefficients respectively. Both c_1 and c_2 are assumed to be constant in this chapter. We introduce the boundary control at the top boundary of the riser to produce a transverse motion u_T and a longitudinal motion u_L for vibration reduction. The work done by the two actuators can be written as

$$W_m = u_T w(L,t) + u_L v(L,t), \tag{5.5}$$

and the total work done on the system, i.e. W, is given by

$$\begin{aligned} W &= W_f + W_d + W_m \\ &= \int_0^L \left\{ \left[f(x,t) - c_1 \frac{\partial w(x,t)}{\partial t}\right] w(x,t) - c_2 \left[\frac{\partial v(x,t)}{\partial t}\right] v(x,t) \right\} dx \\ &\quad + u_T w(L,t) + u_L v(L,t), \end{aligned} \tag{5.6}$$

where W_f is the work done by the distributed transverse load $f(x,t)$ due to the hydrodynamic effects of the ocean current and W_d is the work done by linear structural damping with the damping coefficients, c_1, $c_2 > 0$.

Substituting Eqs. (5.1), (5.2) and (5.6) into the Hamilton's principle Eq. (2.1), we obtain the governing equations of the system as

$$\rho\ddot{w} + EIw'''' - Tw'' - f + c_1\dot{w} - EAv''w' - EAv'w'' - \frac{3}{2}EA\,(w')^2\,w'' = 0, \quad (5.7)$$

$$\rho\ddot{v} + c_2\dot{v} - EAv'' - EAw'w'' = 0, \quad (5.8)$$

and the boundary conditions of the system as

$$w''(0,t) = w''(L,t) = w(0,t) = v(0,t) = 0, \quad (5.9)$$

$$-EIw'''(L,t) + Tw'(L,t) + EAv'(L,t)w'(L,t) + \frac{1}{2}EA\,[w'(L,t)]^3 = u_T(t), \quad (5.10)$$

$$\frac{1}{2}EA\,[w'(L,t)]^2 + EAv'(L,t) = u_L(t), \quad (5.11)$$

$\forall t \in [0,\infty)$.

Remark 5.1. *The notations* $w'(x,t) = \frac{\partial w(x,t)}{\partial x}$, $w''(x,t) = \frac{\partial^2 w(x,t)}{\partial x^2}$, $w'''(x,t) = \frac{\partial^3 w(x,t)}{\partial x^3}$, $w''''(x,t) = \frac{\partial^4 w(x,t)}{\partial x^4}$, $v'(x,t) = \frac{\partial v(x,t)}{\partial x}$, $v''(x,t) = \frac{\partial^2 v(x,t)}{\partial x^2}$, $\ddot{w}(x,t) = \frac{\partial^2 w(x,t)}{\partial t^2}$, $\dot{w}(x,t) = \frac{\partial w(x,t)}{\partial t}$, *and* $\dot{v}(x,t) = \frac{\partial v(x,t)}{\partial t}$ *are used to reduce the notational complexity.*

Property 5.1. *[10, 136]: If the kinetic energy of the system* (5.7) - (5.11)*, given by Eq.* (5.1) *is bounded* $\forall(x,t) \in [0,L] \times [0,\infty)$*, then* $\dot{w}'(x,t)$*,* $\dot{w}''(x,t)$*,* $\dot{v}'(x,t)$ *and* $\dot{v}''(x,t)$ *are bounded* $\forall(x,t) \in [0,L] \times [0,\infty)$*.*

Property 5.2. *[10, 136]: If the potential energy of the system* (5.7) - (5.11)*, given by Eq.* (5.2) *is bounded* $\forall(x,t) \in [0,L] \times [0,\infty)$*, then* $w''(x,t)$*,* $w'''(x,t)$*,* $w''''(x,t)$ *and* $v''(x,t)$ *are bounded* $\forall(x,t) \in [0,L] \times [0,\infty)$*.*

Assumption 5.1. *For the distributed disturbance $f(x,t)$, we assume that there exists a constant $\bar{f} \in R^+$, such that $||f(x,t)|| \leq \bar{f}$, $\forall(x,t) \in [0,L] \times [0,\infty)$. This is a reasonable assumption as the effects of the time-varying current $f(x,t)$ have finite energy and hence are bounded, i.e., $f(x,t) \in \mathcal{L}_\infty([0,L])$.*

Remark 5.2. *For control design in Section 5.3, only the assertion that there exist an upper bound on the disturbance in Assumption 1, $||f(x,t)|| < \bar{f}$, is necessary. The knowledge of the exact value for $f(x,t)$ is not required $\forall(x,t) \in [0,L] \times [0,\infty)$. As such, different VIV models up to various levels of fidelity, such as those found in [127, 128, 133–135], can be applied without affecting the control design or analysis.*

Remark 5.3. *The VIV problem can be separated into the drag and the lift components, perpendicular to each other. The vector sum results in a force with oscillating magnitude and direction, thereby producing of figure of "8" response in the riser. Under Assumption 5.1, it is possible that control applied to these two cases in separate axis may be sufficient for vibration reduction of the VIV problem. The combination of drag and oscillating lift will be treated in future analysis using a 3D riser model.*

5.3 Control Design

The control objective is to reduce the vibrations of the riser, i.e. $w(x,t)$ and $v(x,t)$, under the time-varying distributed transverse load $f(x,t)$ from the ocean current. In this section, Lyapunov's direct method is used to construct boundary control laws $u_T(t)$ and $u_L(t)$ at the top boundary of the flexible marine riser and to analyze the closed-loop stability of the system.

To stabilize the system given by Eqs. (5.7) and (5.8), we propose the following control laws:

$$u_T = -k_1\dot{w}(L,t) - k_2 w(L,t), \tag{5.12}$$

$$u_L = -k_3\dot{v}(L,t) - k_4 v(L,t), \tag{5.13}$$

where k_i, $i = 1,2,3,4$, are positive constants.

Remark 5.4. *The control is independent of system parameters, thus possessing stability robustness to variations in system parameters. The control design is based on the distributed parameter system model Eqs.* (5.7) *and* (5.8)*, and the spillover problems associated with traditional truncated model-based approaches caused by ignoring high-frequency modes in controller and observer design are avoided. For recent results on model-based control of distributed system which are helpful in avoiding spillover effects, the readers can refer to [16, 17].*

Remark 5.5. *In the proposed controller* (5.12) *and* (5.13)*, $w(L,t)$ and $v(L,t)$ can be measured through position sensors at the top boundary of the riser. In practice, the effect of measurement noise from sensors is unavoidable, which will affect the the controller implementation, especially when the high order differentiating terms with respect to time exist. In our proposed controller* (5.12) *and* (5.13)*, $\dot{w}(L,t)$ and $\dot{v}(L,t)$ with only one time differentiating with respect to time can be obtained through a backwards difference algorithm of the values of $w(L,t)$ and $v(L,t)$. It is noted that differentiating twice and three times positions $w(L,t)$ and $v(L,t)$ with respect to time to get $\ddot{w}(L,t)$, $\dddot{w}(L,t)$, $\ddot{v}(L,t)$, and $\dddot{v}(L,t)$ respectively, are undesirable in practice due to noise amplification. For these cases, observers are needed to design to estimate the states values according to the boundary conditions.*

5.3.1 Uniformly stable control under ocean current disturbance

Lemma 5.1. *[139, 140] For bounded initial conditions, $\forall x$ and $\forall t \geq 0$, if there exists a C^1 continuous and positive definite Lyapunov function $V(x,t) : \Re^n \times \Re_+ \longrightarrow \Re$ satisfying $\kappa_1(\|x\|) \leq V(x,t) \leq \kappa_2(\|x\|)$, such that $\dot{V}(x,t) \leq -\lambda V(x,t) + c$, where κ_1, $\kappa_2 : R^n \rightarrow R$ are class K functions and c is a positive constant, then the equilibrium point $x = 0$ of the system $\dot{x} = f(x,t)$ is uniformly bounded.*

Consider the Lyapunov function candidate

$$V(t) = E_b(t) + E_c(t) + E_d(t). \tag{5.14}$$

The energy term $E_b(t)$ and an auxiliary term $E_c(t)$ and a small crossing term $E_d(t)$ are defined as

$$\begin{aligned}
E_b &= \frac{1}{2}\rho\int_0^L \left(\dot{w}^2 + \dot{v}^2\right)dx + \frac{1}{2}EI\int_0^L [w'']^2 dx + \frac{1}{2}EA\int_0^L \left(v' + \frac{1}{2}[w']^2\right)^2 dx \\
&\quad + \frac{1}{2}T\int_0^L [w']^2 dx, &(5.15)\\
E_c &= \frac{k_2 + \beta_1 k_1}{2} w^2(L,t) + \frac{k_4 + \beta_2 k_3}{2} v^2(L,t), &(5.16)\\
E_d &= \beta_1\rho\int_0^L w\dot{w}dx + \beta_2\rho\int_0^L v\dot{v}dx, &(5.17)
\end{aligned}$$

where k_i, $i = 1,2,3,4$, are positive control parameters, and β_j, $j = 1,2$, are two small positive weighting constants.

Lemma 5.2. *The Lyapunov candidate function given by* (5.14)*, can be upper and* *lower bounded as*

$$0 \leq \lambda_1(E_b(t) + E_c(t)) \leq V(t) \leq \lambda_2(E_b(t) + E_c(t)), \tag{5.18}$$

where λ_1 and λ_2 are positive constants.

Proof: Substituting Ineqs. (2.7) and (2.10) into Eq. (5.17) yields:

$$\begin{aligned} |E_d| &\leq \beta_1\rho \int_0^L \dot{w}^2 dx + \beta_1\rho L^2 \int_0^L [w']^2 dx + \beta_2\rho \int_0^L \dot{v}^2 dx + \beta_2\rho L^2 \int_0^L [v']^2 dx \\ &\leq \alpha E_b, \end{aligned} \tag{5.19}$$

where

$$\alpha = 2\rho \frac{\max(\beta_1, \beta_1 L^2, \beta_2, \beta_2 L^2)}{\min(\rho, T, EI, EA)}, \tag{5.20}$$

Then, we obtain

$$-\alpha E_b \leq E_d \leq \alpha E_b, \tag{5.21}$$

Considering β_1 and β_2 are two small positive weighting constants, and by choosing β_1 and β_2 properly, we can obtain

$$\alpha_1 = 1 - \alpha = 1 - 2\rho \frac{\max(\beta_1, \beta_1 L^2, \beta_2, \beta_2 L^2)}{\min(\rho, T, EI, EA)} > 0, \tag{5.22}$$

$$\alpha_2 = 1 + \alpha = 1 + 2\rho \frac{\max(\beta_1, \beta_1 L^2, \beta_2, \beta_2 L^2)}{\min(\rho, T, EI, EA)} > 1, \tag{5.23}$$

Then, we further have

$$0 \leq \alpha_1 E_b \leq E_b + E_d \leq \alpha_2 E_b, \tag{5.24}$$

Given the Lyapunov function candidate in Eq. (5.14), we obtain

$$0 \leq \lambda_1(E_b(t) + E_c(t)) \leq V(t) \leq \lambda_2(E_b(t) + E_c(t)), \tag{5.25}$$

where $\lambda_1 = \min(\alpha_1, 0.5(k_2+\beta_1 k_1), 0.5(k_4+\beta_2 k_3))$ and $\lambda_2 = \max(\alpha_1, 0.5(k_2+\beta_1 k_1), 0.5(k_4+\beta_2 k_3))$ are positive constants. ■

Lemma 5.3. *The time derivative of the Lyapunov function in* (5.14) *can be upper bounded with*

$$\dot{V}(t) \;\; \leq \;\; -\lambda V(t) + \varepsilon, \tag{5.26}$$

where λ and ε are two positive constants.

Proof: We differentiate Eq. (5.14) with respect to time to obtain

$$\dot{V}(t) = \dot{E}_b + \dot{E}_c + \dot{E}_d, \tag{5.27}$$

The first term of the Eq. (5.27)

$$\dot{E}_b = B_1 + B_2 + B_3 + B_4, \tag{5.28}$$

where

$$\begin{aligned}
B_1 &= \rho \int_0^L \dot{w}\ddot{w}dx + \rho \int_0^L \dot{v}\ddot{v}dx, && (5.29)\\
B_2 &= EI \int_0^L w''\dot{w}''dx, && (5.30)\\
B_3 &= EA \int_0^L \left(v' + \frac{1}{2}[w']^2\right)(\dot{v}' + w'\dot{w}')\,dx, && (5.31)\\
B_4 &= T \int_0^L w'\dot{w}'dx. && (5.32)
\end{aligned}$$

Using governing equation in the expression for B_1, we obtain

$$\begin{aligned}
B_1 &= \int_0^L \dot{w}\left(-EIw'''' + Tw'' + \frac{3}{2}EA[w']^2w'' + EAw''v' + EAw'v'' + f - c_1\dot{w}\right)dx\\
&\quad + \int_0^L \dot{v}\,(EAv'' + EAw'w'' - c_2\dot{v})\,dx, \qquad (5.33)
\end{aligned}$$

Applying the boundary conditions and integrating Eq.(5.30) by parts, we obtain

$$\begin{aligned}
B_2 &= EI \int_0^L w''d(\dot{w}')\\
&= -EIw'''(L,t)\dot{w}(L,t) + EI \int_0^L \dot{w}w''''dx, \qquad (5.34)
\end{aligned}$$

Applying the boundary conditions and integrating Eq. (5.31) by parts, we obtain

$$\begin{aligned}
B_3 &= EA \int_0^L v'\dot{v}'dx + EA \int_0^L v'w'\dot{w}'dx + \frac{1}{2}EA \int_0^L [w']^2\dot{v}'dx + \frac{1}{2}EA \int_0^L [w']^3\dot{w}'dx\\
&= EAv'(L,t)\dot{v}(L,t) - EA \int_0^L \dot{v}v''dx + EAv'(L,t)w'(L,t)\dot{w}(L,t)\\
&\quad -EA \int_0^L \dot{w}(v''w' + v'w'')dx + \frac{1}{2}EA[w'(L,t)]^2\dot{v}(L,t) - EA \int_0^L \dot{v}w'w''dx\\
&\quad +\frac{1}{2}EA[w'(L,t)]^3\dot{w}(L,t) - \frac{3}{2}EA \int_0^L \dot{w}[w']^2w''dx, \qquad (5.35)
\end{aligned}$$

Using the boundary conditions and integrating Eq. (5.32) by part, we obtain

$$\begin{aligned} B_4 &= T\int_0^L w' d(\dot{w}) \\ &= Tw'(L,t)\dot{w}(L,t) - T\int_0^L \dot{w}w'' dx, \end{aligned} \tag{5.36}$$

Substituting Eqs. (5.33), (5.34), (5.35), (5.36) into Eq. (5.28), we have

$$\begin{aligned} \dot{E}_b &= \left(-EIw'''(L,t) + Tw'(L,t) + EAw'(L,t)v'(L,t) + \frac{1}{2}EA[w'(L,t)]^3\right)\dot{w}(L,t) \\ &+ \left(\frac{1}{2}EA[w'(L,t)]^2 + EAv'(L,t)\right)\dot{v}(L,t) - c_1\int_0^L \dot{w}^2 dx - c_2\int_0^L \dot{v}^2 dx \\ &+ \int_0^L f\dot{w}dx. \end{aligned} \tag{5.37}$$

Substituting the boundary conditions Eqs. (5.10) and (5.11) into Eq. (5.37), we obtain

$$\dot{E}_b = u_T\dot{w}(L,t) + u_L\dot{v}(L,t) - c_1\int_0^L \dot{w}^2 dx - c_2\int_0^L \dot{v}^2 dx + \int_0^L f\dot{w}dx, \tag{5.38}$$

Using Ineq. (2.8) , we obtain

$$\dot{E}_b \leq u_T\dot{w}(L,t) + u_L\dot{v}(L,t) - (c_1-\delta_1)\int_0^L \dot{w}^2 dx - c_2\int_0^L \dot{v}^2 dx + \int_0^L \frac{1}{\delta_1}f^2 dx, \tag{5.39}$$

where $\delta_1 > 0$ is a positive constant.

The second term of the Eq. (5.27)

$$\dot{E}_c = (k_2+\beta_1 k_1)w(L,t)\dot{w}(L,t) + (k_4+\beta_2 k_3)v(L,t)\dot{v}(L,t), \tag{5.40}$$

The third term of the Eq. (5.27)

$$\begin{aligned}
\dot{E}_d &= \beta_1\rho\int_0^L(\dot{w}^2+w\ddot{w})dx+\beta_2\rho\int_0^L(\dot{v}^2+v\ddot{v})dx \\
&= \beta_1\int_0^L\Big[-EIww''''+Tww''+fw-c_1w\dot{w}+\frac{3}{2}EAww'^2w''+EAww''v' \\
&\quad +EAww'v''+\rho\dot{w}^2\Big]dx+\beta_2\int_0^L\left[EAvv''+EAvw'w''+\rho\dot{v}^2-c_2v\dot{v}\right]dx \\
&= D_1+D_2+D_3+D_4+D_5+D_6+D_7+D_8+D_9+D_{10}+D_{11}+D_{12},
\end{aligned} \tag{5.41}$$

where

$$D_1 = -\beta_1\int_0^L EIww''''dx, \tag{5.42}$$

$$D_2 = \beta_1\int_0^L Tww''dx, \tag{5.43}$$

$$D_3 = \beta_1\int_0^L fwdx, \tag{5.44}$$

$$D_4 = -\beta_1\int_0^L c_1w\dot{w}dx, \tag{5.45}$$

$$D_5 = \beta_1\int_0^L \frac{3}{2}EAww'^2w''dx, \tag{5.46}$$

$$D_6 = \beta_1\int_0^L EAww''v'dx, \tag{5.47}$$

$$D_7 = \beta_1\int_0^L EAww'v''dx, \tag{5.48}$$

$$D_8 = \beta_1\int_0^L \rho\dot{w}^2dx, \tag{5.49}$$

$$D_9 = \beta_2\int_0^L EAvv''dx, \tag{5.50}$$

$$D_{10} = \beta_2\int_0^L EAvw'w''dx, \tag{5.51}$$

$$D_{11} = \beta_2\int_0^L \rho\dot{v}^2dx, \tag{5.52}$$

$$D_{12} = \beta_2 \int_0^L c_2 v\dot{v} dx. \tag{5.53}$$

After integrating Eqs. (5.42) and (5.43) by parts and using the boundary conditions, we obtain

$$\begin{aligned} D_1 &= -\beta_1 EIw(L,t)w'''(L,t) - \beta_1 EI \int_0^L [w'']^2 dx, \\ D_2 &= \beta_1 Tw(L,t)w'(L,t) - \beta_1 T \int_0^L [w']^2 dx, \end{aligned}$$

Using the Ineqs. (2.8) and (2.10), we obtain

$$\begin{aligned} D_3 &\le \frac{\beta_1}{\delta_2} \int_0^L f^2 dx + \beta_1 \delta_2 \int_0^L w^2 dx \\ &\le \frac{\beta_1}{\delta_2} \int_0^L f^2 dx + \beta_1 \delta_2 L^2 \int_0^L [w']^2 dx \qquad (5.54) \\ D_4 &\le \beta_1 \frac{c_1}{\delta_3} \int_0^L \dot{w}^2 dx + \beta_1 c_1 \delta_3 L^2 \int_0^L [w']^2 d, \qquad (5.55) \end{aligned}$$

where $\delta_2, \delta_3 > 0$. Integrating Eq. (5.46) by parts, we obtain

$$\begin{aligned} D_5 &= \beta_1 \int_0^L \frac{3}{2} EAww'^2 d(w') \\ &= \frac{3\beta_1}{2} EAw[w']^3 \Big|_0^L - \frac{3\beta_1}{2} EA \int_0^L w'(w'[w']^2 + 2ww'w'') dx, \qquad (5.56) \end{aligned}$$

The polynomial of the last term in Eq. (5.56) is same as D_5. Rewriting Eq. (5.56), we obtain

$$D_5 = \frac{\beta_1}{2} EAw(L,t)[w'(L,t)]^3 - \frac{\beta_1}{2} EA \int_0^L [w']^4 dx, \tag{5.57}$$

After integrating Eqs. (5.47), (5.50) and (5.51) by parts and using the boundary conditions, we obtain

$$D_6 = \beta_1 EAw(L,t)w'(L,t)v'(L,t) - \beta_1 EA\int_0^L [w']^2 v' dx - D_7, \quad (5.58)$$

$$D_9 = \beta_2 EAv(L,t)v'(L,t) - \beta_2 EA\int_0^L [v']^2 dx, \quad (5.59)$$

$$D_{10} = \beta_2 EAv(L,t)[w'(L,t)]^2 - \beta_2 EA\int_0^L v'[w']^2 dx - D_{10}, \quad (5.60)$$

The last term in Eq. (5.60) is same as D_{10}. Rewriting Eq. (5.60), we obtain

$$D_{10} = \frac{\beta_2}{2} EAv(L,t)[w'(L,t)]^2 - \frac{\beta_2}{2} EA\int_0^L v'[w']^2 dx, \quad (5.61)$$

Using the Ineq. (2.8), we obtain

$$D_{12} \leq \beta_2 \frac{c_2}{\delta_4}\int_0^L \dot{v}^2 dx + \beta_2 c_2 \delta_4 L^2 \int_0^L [v']^2 dx, \quad (5.62)$$

where $\delta_4 > 0$. Combining the above expressions $D_1 - D_{12}$ and utilizing the boundary conditions Eqs. (5.10), (5.11) , we obtain

$$\begin{aligned} \dot{E}_d(t) \leq\ & \beta_1 w(L,t)u_T + \beta_2 v(L,t)u_L - \beta_1 EI\int_0^L [w'']^2 dx - \beta_1 T\int_0^L [w']^2 dx + \frac{\beta_1}{\delta_2}\int_0^L f^2 dx \\ & + \beta_1 \delta_2 L^2 \int_0^L [w']^2 dx + \beta_1 \frac{c_1}{\delta_3}\int_0^L \dot{w}^2 dx + \beta_1 c_1 \delta_3 L^2 \int_0^L [w']^2 dx - \frac{\beta_1}{2} EA\int_0^L [w']^4 dx \\ & - \beta_1 EA\int_0^L [w']^2 v' dx + \beta_1 \int_0^L \rho \dot{w}^2 dx - \beta_2 EA\int_0^L [v']^2 dx - \frac{\beta_2}{2} EA\int_0^L v'[w']^2 dx \\ & + \beta_2 \int_0^L \rho \dot{v}^2 dx + \beta_2 \frac{c_2}{\delta_4}\int_0^L \dot{v}^2 dx + \beta_2 c_2 \delta_4 L^2 \int_0^L [v']^2 dx, \end{aligned} \quad (5.63)$$

Substituting Eqs. (5.39), (5.40) and (5.63) into Eq. (5.27), we obtain

$$\begin{aligned}
\dot{V}(t) \;\le\; & [\dot{w}(L,t)+\beta_1 w(L,t)]u_T + [\dot{v}(L,t)+\beta_2 v(L,t)]u_L \\
& -\left(c_1-\delta_1-\beta_1\rho-\beta_1\frac{c_1}{\delta_3}\right)\int_0^L \dot{w}^2 dx - \left(c_2-\beta_2\rho-\beta_2\frac{c_2}{\delta_4}\right)\int_0^L \dot{v}^2 dx \\
& -\beta_1 EI\int_0^L [w'']^2 dx - \frac{\beta_1}{2}EA\int_0^L [w']^4 dx - \left(\beta_1-\frac{\beta_2}{2}\right)EA\int_0^L [w']^2 v' dx \\
& -(\beta_2 EA - \beta_2 c_2\delta_4 L^2)\int_0^L [v']^2 dx - \left(\beta_1 T - \beta_1\delta_2 L^2 - \beta_1 c_1\delta_3 L^2\right)\int_0^L [w']^2 dx \\
& +\left(\frac{1}{\delta_1}+\frac{\beta_1}{\delta_2}\right)\int_0^L f^2 dx + (k_2+\beta_1 k_1)w(L,t)\dot{w}(L,t) + (k_4+\beta_2 k_3)v(L,t)\dot{v}(L,t),
\end{aligned} \tag{5.64}$$

Then substituting the control law Eqs. (5.12) and (5.13) into Eq. (5.64), we obtain

$$\begin{aligned}
\dot{V}(t) \;\le\; & -k_1[\dot{w}(L,t)]^2 - k_2\beta_1[w(L,t)]^2 - k_3[\dot{v}(L,t)]^2 - k_4\beta_2[v(L,t)]^2 \\
& -\left(c_1-\delta_1-\beta_1\rho-\beta_1\frac{c_1}{\delta_3}\right)\int_0^L \dot{w}^2 dx - \left(c_2-\beta_2\rho-\beta_2\frac{c_2}{\delta_4}\right)\int_0^L \dot{v}^2 dx \\
& -\beta_1 EI\int_0^L [w'']^2 dx - \left(\beta_1-\frac{\beta_2}{2}\right)EA\int_0^L \left(v'+\frac{1}{2}[w']^2\right)^2 dx \\
& -\left(\beta_1 T - \beta_1\delta_2 L^2 - \beta_1 c_1\delta_3 L^2\right)\int_0^L [w']^2 dx + \left(\frac{1}{\delta_1}+\frac{\beta_1}{\delta_2}\right)\int_0^L f^2 dx \\
\le\; & -\lambda_3(E_b+E_c)+\varepsilon,
\end{aligned} \tag{5.65}$$

where

$$\begin{aligned}
\lambda_3 &= \min\left(\frac{2k_2\beta_1}{k_2+\beta_1 k_1}, \frac{2k_4\beta_2}{k_4+\beta_2 k_3}, \frac{2\sigma_1}{m_z}, \frac{2\sigma_2}{m_z}, \frac{2\sigma_3}{EI}, \frac{2\sigma_5}{EA}, \frac{2\sigma_7}{T}\right) > 0, && (5.66)\\
\sigma_1 &= c_1 - \delta_1 - \beta_1\rho - \beta_1\frac{c_1}{\delta_3} > 0, && (5.67)\\
\sigma_2 &= c_2 - \beta_2\rho - \beta_2\frac{c_2}{\delta_4} > 0, && (5.68)\\
\sigma_3 &= \beta_1 EI > 0, && (5.69)\\
\sigma_4 &= \frac{\beta_1}{2}EA > 0, && (5.70)\\
\sigma_5 &= \left(\beta_1 - \frac{\beta_2}{2}\right)EA > 0, && (5.71)\\
\sigma_6 &= \beta_2 EA - \beta_2 c_2\delta_4 L^2 > 0, && (5.72)\\
\sigma_5 &\le \frac{\min(4\sigma_4, \sigma_6)}{4}, && (5.73)\\
\sigma_7 &= \beta_1 T - \beta_1\delta_2 L^2 - \beta_1 c_1\delta_3 L^2 > 0, && (5.74)\\
\varepsilon &= \left(\frac{1}{\delta_1} + \frac{\beta_1}{\delta_2}\right)\int_0^L f^2 dx \le \left(\frac{1}{\delta_1} + \frac{\beta_1}{\delta_2}\right)\int_0^L \bar{f}^2 dx < \infty. && (5.75)
\end{aligned}$$

From Ineqs. (5.25) and (5.65) we have

$$\dot{V}(t) \;\le\; -\lambda V(t) + \varepsilon, \tag{5.76}$$

where $\lambda = \lambda_3/\lambda_2 > 0$ and $\varepsilon > 0$. ■

With the above lemmas, we are ready to present the following stability theorem of the closed-loop riser system subject to ocean current disturbance.

Theorem 5.1. *For the system dynamics described by* (5.7) *and* (5.8) *and boundary conditions* (5.9) *to* (5.11)*, under Assumption 5.1, and the control laws* (5.12) *and* (5.13)*, given that the initial conditions are bounded, and that the required state information* $w(L,t)$*,* $v(L,t)$*,* $\dot{w}(L,t)$ *and* $\dot{v}(L,t)$ *are available, the closed loop system is* *uniformly bounded as follows:*

$$|w(x,t)| \le \sqrt{\frac{2L}{T\lambda_1}\Big(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda}\Big)}, \ \forall x \in [0, L], \tag{5.77}$$

$$|v(x,t)| \le \sqrt{\frac{2L}{EA\lambda_1}\Big(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda}\Big)}, \ \forall x \in [0, L], \tag{5.78}$$

where λ and ε are two positive constants.

Proof: Multiplying Eq.(5.26) by $e^{\lambda t}$ yields

$$\frac{\partial}{\partial t}(Ve^{\lambda t}) \le \varepsilon e^{\lambda t}. \tag{5.79}$$

Integration of the above inequalities, we obtain

$$V(t) \le \Big(V(0) - \frac{\varepsilon}{\lambda}\Big) e^{-\lambda t} + \frac{\varepsilon}{\lambda} \le V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda} \in \mathcal{L}_\infty, \tag{5.80}$$

which implies $V(t)$ is bounded. Utilizing Ineq. (2.11) and Eq. (5.15), we have

$$\frac{1}{2L}Tw^2(x,t) \le \frac{1}{2}T\int_0^L [w'(x,t)]^2 dx \le E_b(t) \le \frac{1}{\lambda_1}V(t) \in \mathcal{L}_\infty, \tag{5.81}$$

$$\frac{1}{2L}EAv^2(x,t) \le \frac{1}{2}EA\int_0^L [v'(x,t)]^2 dx \le E_b(t) \le \frac{1}{\lambda_1}V(t) \in \mathcal{L}_\infty. \tag{5.82}$$

Rearranging the terms of the above two inequalities, we obtain

$$|w(x,t)| \le \sqrt{\frac{2L}{T\lambda_1}\Big(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda}\Big)}, \ \forall x \in [0, L], \tag{5.83}$$

$$|v(x,t)| \le \sqrt{\frac{2L}{EA\lambda_1}\Big(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda}\Big)}, \ \forall x \in [0, L]. \tag{5.84}$$

From Eqs. (5.81) and (5.82), we can state the $E_b(t)$ is bounded $\forall t \in [0, \infty)$. Since $E_b(t)$ is bounded, $\dot{w}(x,t)$, $w'(x,t)$, $w''(x,t)$, $\dot{v}(x,t)$ and $v'(x,t)$ are bounded $\forall (x,t) \in$

$[0, L] \times [0, \infty)$. From Eq. (5.1), the kinetic energy of the system is bounded and using Property 5.1, $\dot{w}'(x,t)$ and $\dot{v}'(x,t)$ are bounded $\forall(x,t) \in [0, L] \times [0, \infty)$. From the boundedness of the potential energy Eq. (5.2), we can use Property 5.2 to conclude that $w''''(x,t)$ and $v''(x,t)$ are bounded $\forall(x,t) \in [0, L] \times [0, \infty)$. Finally, using Assumption 5.1, Eqs. (5.7) through (5.11), and the above statements, we can conclude that $\ddot{w}(x,t)$ and $\ddot{v}(x,t)$ are also bounded $\forall(x,t) \in [0, L] \times [0, \infty)$. From Lemma 5.1 and the above proof, it is shown the deflection $w(x,t)$ and $v(x,t)$ are uniformly bounded $\forall(x,t) \in [0, L] \times [0, \infty)$. ■

Remark 5.6. *From above stability analysis, $\dot{w}(x,t)$, $\dot{v}(x,t)$, $w(x,t)$ and $v(x,t)$ are all bounded $\forall(x,t) \in [0, L] \times [0, \infty)$, and we can conclude the control inputs of u_T and u_L are bounded $\forall t \in [0, \infty)$.*

5.3.2 Exponentially stable control without disturbance

In this section, by using the same Lyapunov function candidate Eq. (5.14) and control law Eqs. (5.12) and (5.13) of Section 5.3.1, we analyze the free vibration case of the flexible riser system, i.e. ocean current disturbance $f(x,t) = 0$, and the exponentially stability is proved.

Lemma 5.4. *[140] For bounded initial conditions, $\forall x$ and $\forall t \geq 0$, if there exists a C^1 continuous and positive definite Lyapunov function $V(x,t) : \Re^n \times \Re_+ \longrightarrow \Re$ satisfying $\kappa_1(\|x\|) \leq V(x,t) \leq \kappa_2(\|x\|)$, such that $\dot{V}(x,t) \leq -\lambda V(x,t)$, where κ_1, $\kappa_2 : R^n \to R$ are class K functions, then the equilibrium point $x = 0$ of the system $\dot{x} = f(x,t)$ is an exponentially stable.*

Theorem 5.2. *For the system dynamics described by (5.7) and (5.8) and boundary conditions (5.9) to (5.11), if the free vibration case is considered, i.e. $f(x,t) = 0$, the* *exponential stability under free vibration condition can be achieved with the proposed boundary controllers* (5.12) *and* (5.13) *as follows,*

$$|w(x,t)| \leq \sqrt{\frac{2L}{T\lambda_1}V(0)e^{-\lambda t}}, \ \forall x \in [0,L], \tag{5.85}$$

$$|v(x,t)| \leq \sqrt{\frac{2L}{EA\lambda_1}V(0)e^{-\lambda t}}, \ \forall x \in [0,L]. \tag{5.86}$$

where λ *and* λ_1 *are positive constants.*

Proof: From Ineq. (5.26), under the free vibration condition, i.e. $f(x,t)=0$, we obtain the time derivation of the Lyapunov function candidate (5.14) as

$$\dot{V}(t) \leq -\lambda V(t), \tag{5.87}$$

where $\lambda = \lambda_3/\lambda_2$. Multiplying Eq.(5.87) by $e^{\lambda t}$ yeilds

$$\frac{\partial}{\partial t}(Ve^{\lambda t}) \leq 0. \tag{5.88}$$

Integration of the above inequality, we obtain

$$V(t) \leq V(0)e^{-\lambda t} \in \mathcal{L}_\infty, \tag{5.89}$$

which implies $V(t)$ is bounded. Similarly, utilizing Ineq. (2.11) and Eq. (5.15), we have

$$\frac{1}{2L}Tw^2(x,t) \leq \frac{1}{2}T\int_0^L [w'(x,t)]^2 dx \leq E_b(t) \leq \frac{1}{\lambda_1}V(t) \in \mathcal{L}_\infty, \tag{5.90}$$

$$\frac{1}{2L}EAv^2(x,t) \leq \frac{1}{2}EA\int_0^L [v'(x,t)]^2 dx \leq E_b(t) \leq \frac{1}{\lambda_1}V(t) \in \mathcal{L}_\infty. \tag{5.91}$$

Rearranging the terms of the above two inequalities, we obtain

$$|w(x,t)| \leq \sqrt{\frac{2L}{T\lambda_1}V(0)e^{-\lambda t}}, \ \forall x \in [0,L], \quad (5.92)$$

$$|v(x,t)| \leq \sqrt{\frac{2L}{EA\lambda_1}V(0)e^{-\lambda t}}, \ \forall x \in [0,L]. \quad (5.93)$$

From Lemma 5.4 and the above proof, we have the conclusion that the free vibration riser system under the control law is exponentially stable. ■

Remark 5.7. *For the free vibration case of the flexible riser system, the displacements $w(x,t)$ and $v(x,t)$ exponentially converge to zero at the rate of convergence λ as $t \longrightarrow \infty$.*

5.4 Numerical Simulations

Simulations for a 1000m riser under ocean current disturbance are carried out to demonstrate the effectiveness of the proposed control laws (5.12) and (5.13). Detailed parameters of the riser system are shown in Table 1. Numerical methods are applied to get the approximate solution of the system (5.7)-(5.11), when there is no obtainable analytical solution. In this chapter, the finite difference (FD) scheme is chosen to simulate the system performance.

Table 1: parameters of the riser system

Parameter	Description	Value
L	Length of riser	$1000.00m$
d	Riser inner diameter	$76.2mm$
D	Riser external diameter	$152.40mm$
EI	Bending stiffness of the riser	$1.22 \times 10^5 Nm^2$
EA	Axial stiffness of the riser	$3.92 \times 10^8 N$
T	Tension	$1.11 \times 10^8 N$
ρ	Mass per unit length of the flexible riser	$108.00kg/m$
ρ_s	Sea water density	$1024.00kg/m^3$
c_1	Structural transverse damping coefficient	$5.00Ns/m^2$
c_2	Structural longitudinal damping coefficient	$1.00Ns/m^2$

The riser, initially at rest, is excited by a distributed transverse disturbance due to the ocean current. The corresponding initial conditions of the riser system are given as

$$w(x,0) = \dot{w}(x,0) = v(x,0) = \dot{v}(x,0) = 0. \tag{5.94}$$

In the simulation, the ocean surface current velocity $U(t)$ given by Eq. (3.131) is shown in Fig. 5.2. We assume that the full current load is applied from $x = 1000m$ to $x = 0m$ and thereafter linearly decline to zero at the ocean floor, $x = 0$, to obtain a depth dependent ocean current profile $U(x,t)$ as in Chapter 3. The distributed load $f(x,t)$ is generated by Eq. (2.5) with $C_D = 1$, $\theta = 0$, $S_t = 0.2$ and $f_v = 2.625$. In this chapter, we consider four simulation cases with different control inputs.

(i) The transverse and longitudinal displacements of the riser for free vibration (i.e. without control input, $k_1 = k_2 = k_3 = k_4 = 0$) under ocean current disturbance are shown in Fig. 5.3.

(ii) With only transverse control input, i.e. $k_1 = k_2 = 1 \times 10^6$, $k_3 = k_4 = 0$, the transverse and longitudinal displacements of the riser are shown in Fig. 5.4.

(iii) With only longitudinal control input, i.e. $k_1 = k_2 = 0$, $k_3 = k_4 = 1 \times 10^8$, the transverse and longitudinal displacements of the riser are shown in Fig. 5.5.

(iv) With both transverse and longitudinal control inputs, i.e. $k_1 = k_2 = 1 \times 10^6, k_3 = k_4 = 1 \times 10^8$, the transverse and longitudinal displacements of the riser are shown in Fig. 5.6.

From Fig. 5.4 to 5.6, it is observed that there is a significant reduction of the riser's transverse displacement when the transverse control is applied. Similarly, when the longitudinal control is applied, a significant reduction of the riser's longitudinal displacement is observed. When control inputs in transverse and longitudinal directions are applied, the riser's displacements in both transverse and longitudinal directions are reduced. Peak displacement reduction in the middle and bottom of riser is observed although the actuators are not located at these positions. The corresponding control inputs $u_T(t)$ and $u_L(t)$ are shown in Fig. 5.7 and Fig. 5.8 respectively. It is shown that transverse control input is a negative value, which means the actual transverse control input is exerted in the opposite direction of the ocean disturbance $f(x,t)$. The transverse control input varies between 0 and 2.5×10^4N, and the longitudinal control input varies between 0 and 1200N, which are implementable in practice.

Vibration displacements of the riser are examined at $x = 1000m$ and $x = 500m$, and the results for controlled and uncontrolled responses are shown in Fig. 5.9 and Fig. 5.10 respectively. With the two control inputs, it can be observed that the vibration displacements are reduced at both locations, which brings the top displacements of the riser close to zero.

5.5 Conclusion

Vibration regulation of a distributed parameter marine flexible riser subject to the ocean current disturbance has been investigated in this chapter. The boundary control has been developed with two actuators in transverse and longitudinal directions based on the distributed parameter system model with PDEs, and the problems associated with traditional truncated-model-based design are overcome. With proposed control, closed-looped stability under external disturbance and exponential stability under free vibration condition have been proven based on Lyapunov's direct method. The control is easy to implement since they are independent of the system parameters and only two sensors and actuators are required. Numerical simulations have been provided to verify the effectiveness of the presented boundary control.

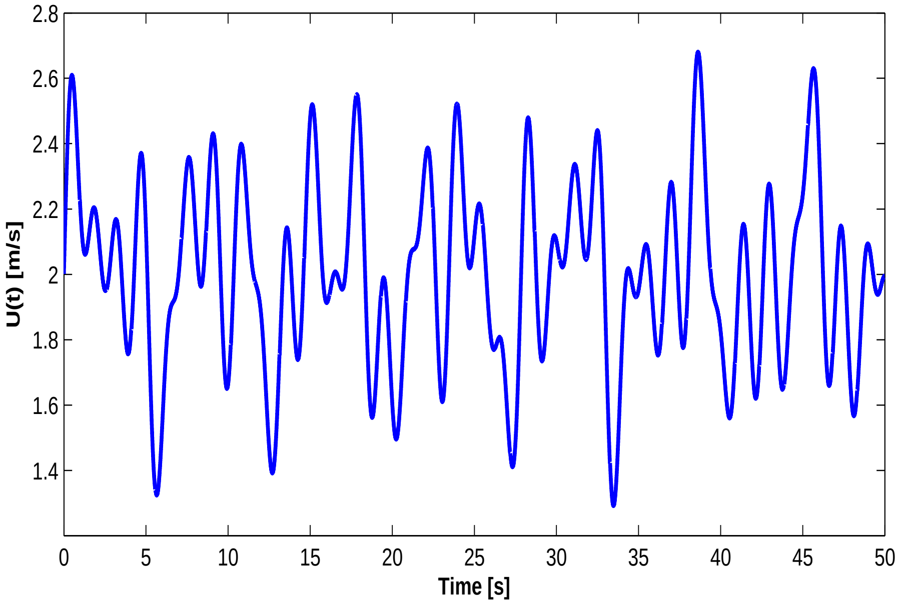
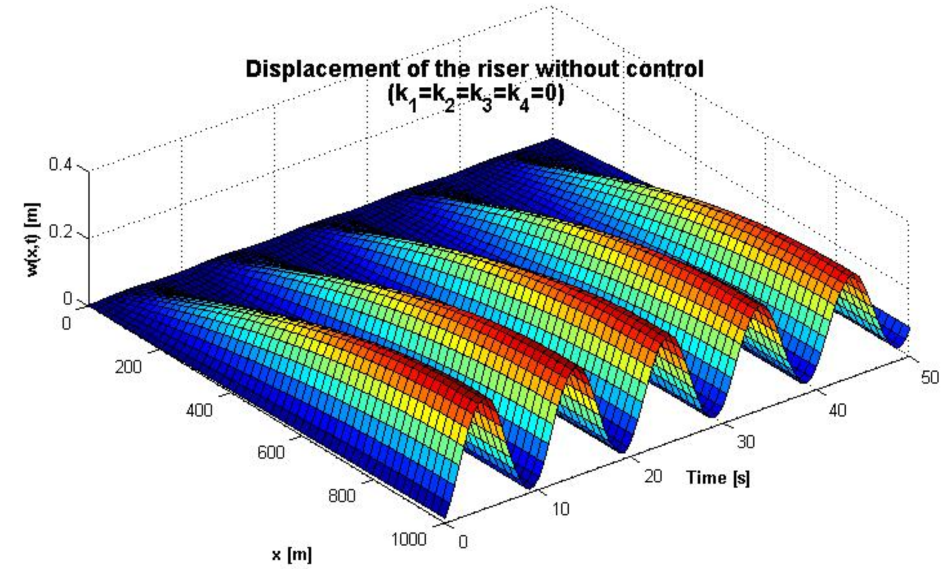
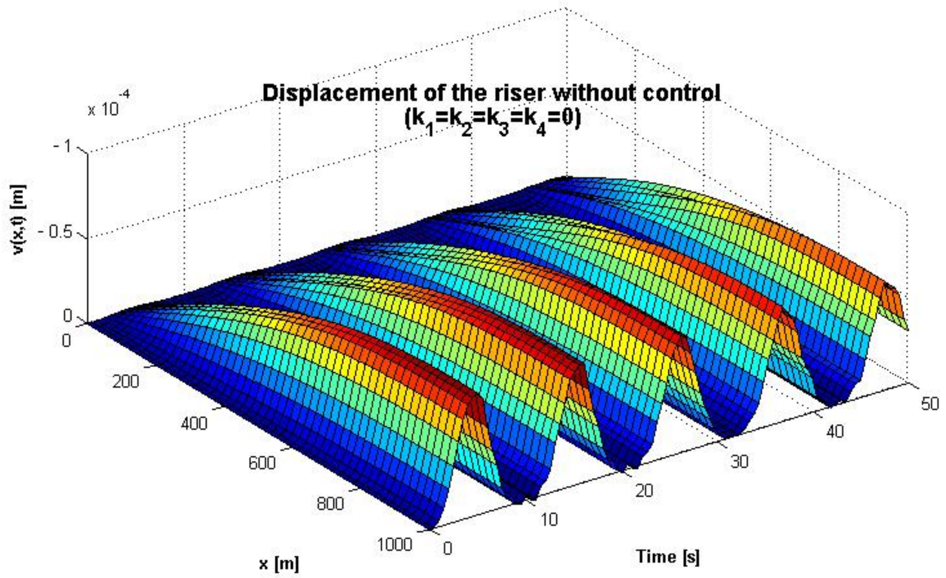


Fig. 5.2: Surface current $U(t)$.

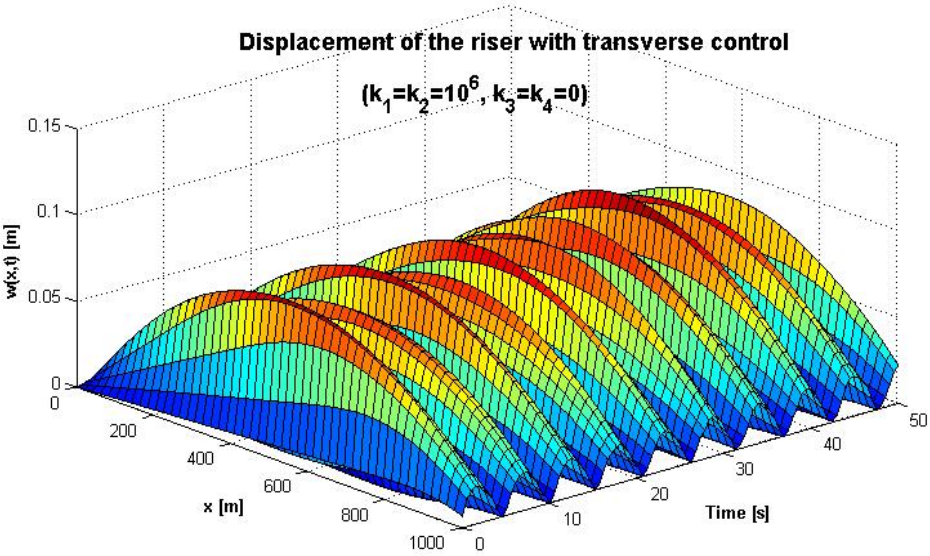


(a)

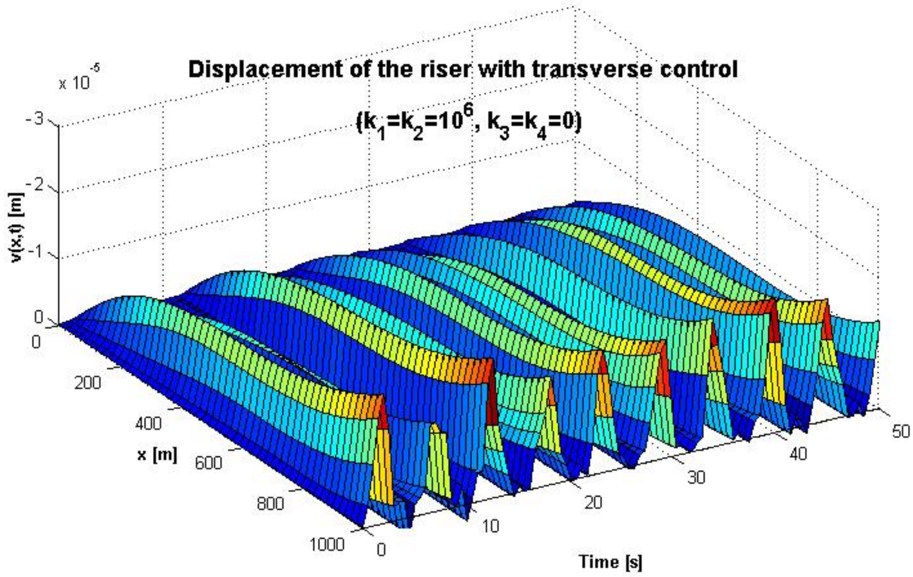


(b)

Fig. 5.3: (a) Transverse displacement $w(x,t)$ and (b) longitudinal displacement $v(x,t)$.

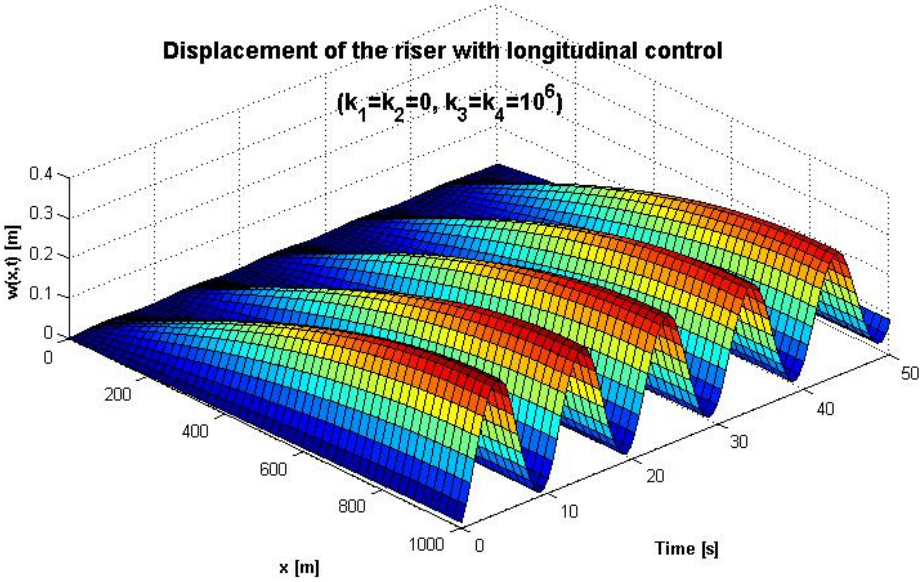


(a)

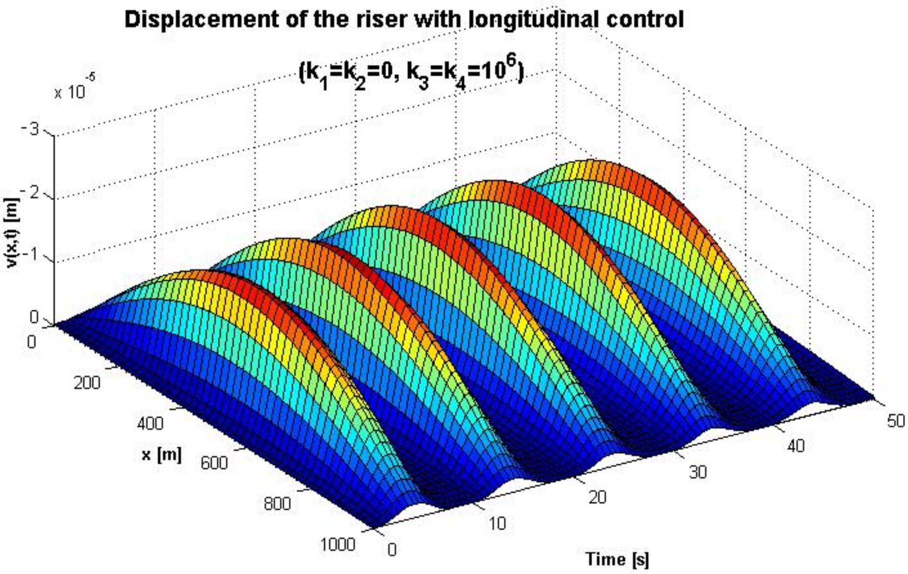


(b)

Fig. 5.4: (a) Transverse displacement $w(x,t)$ and (b) longitudinal displacement $v(x,t)$.

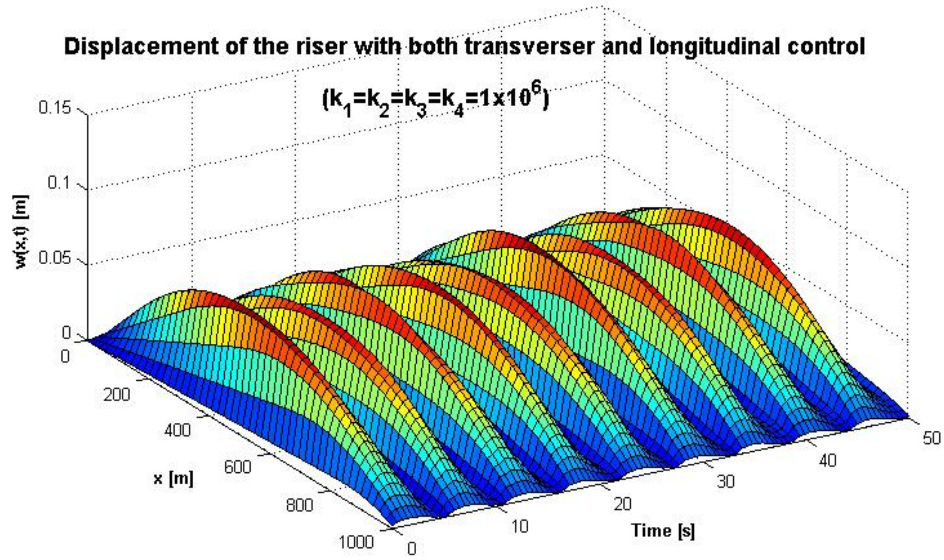


(a)

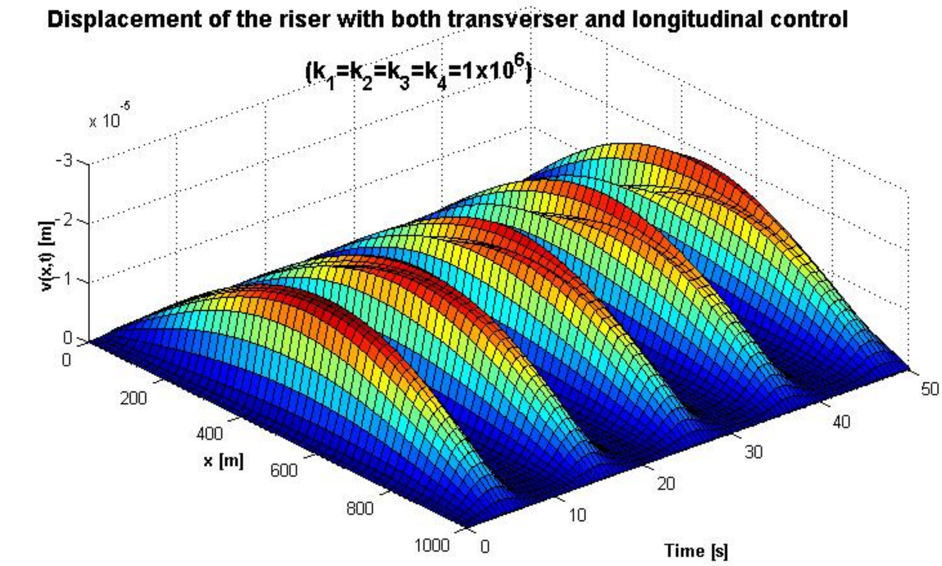


(b)

Fig. 5.5: (a) Transverse displacement $w(x,t)$ and (b) longitudinal displacement $v(x,t)$.



(a)



(b)

Fig. 5.6: (a) Transverse displacement $w(x,t)$ and (b) longitudinal displacement $v(x,t)$.

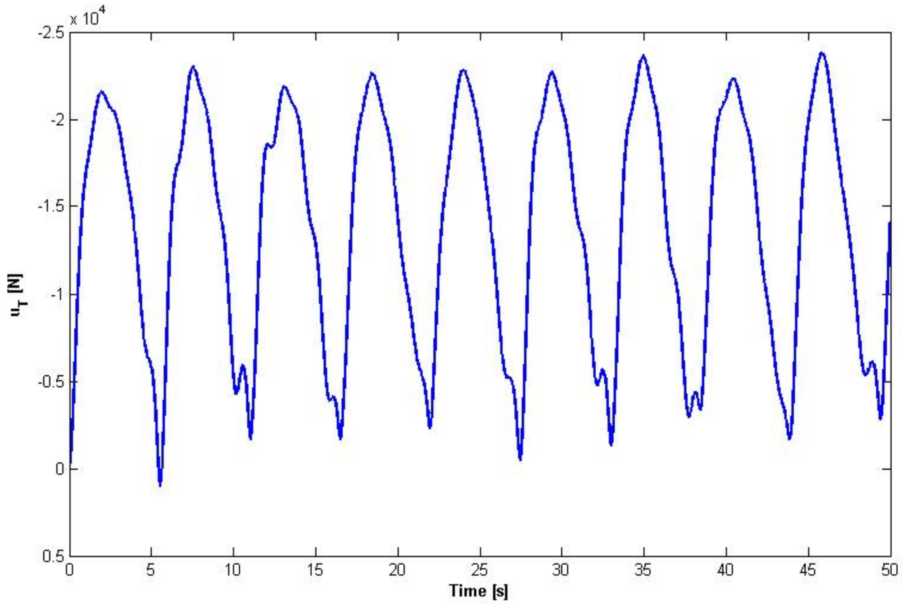


Fig. 5.7: Transverse control input $u_T(t)$.

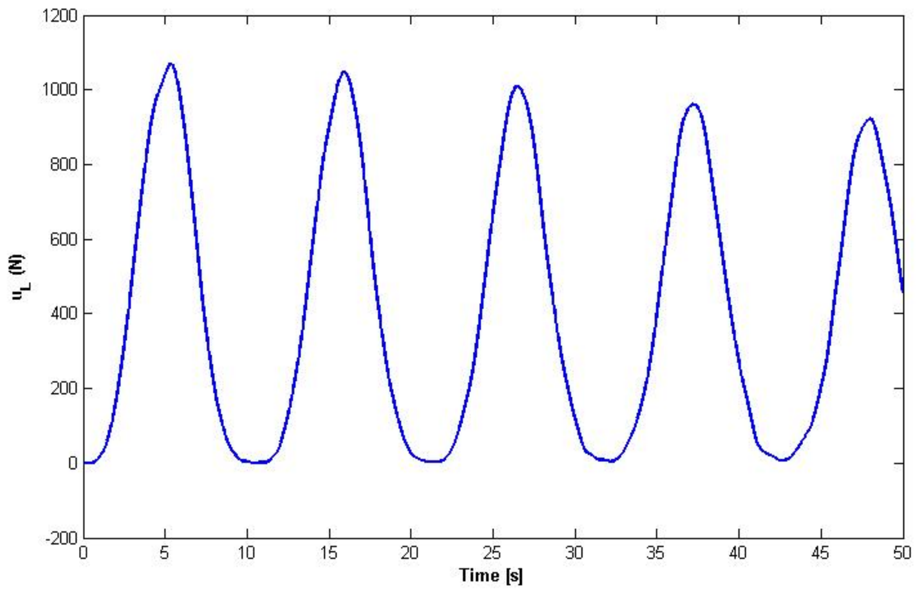
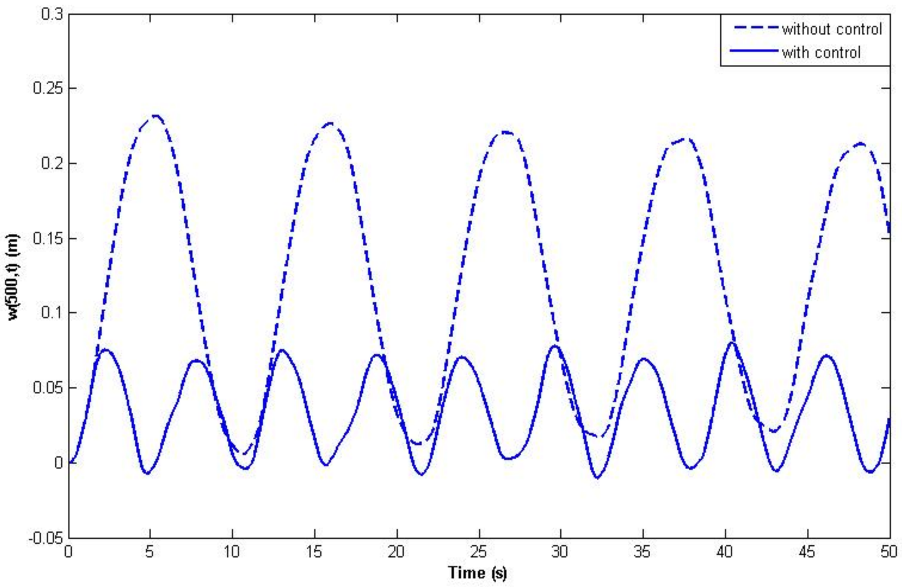
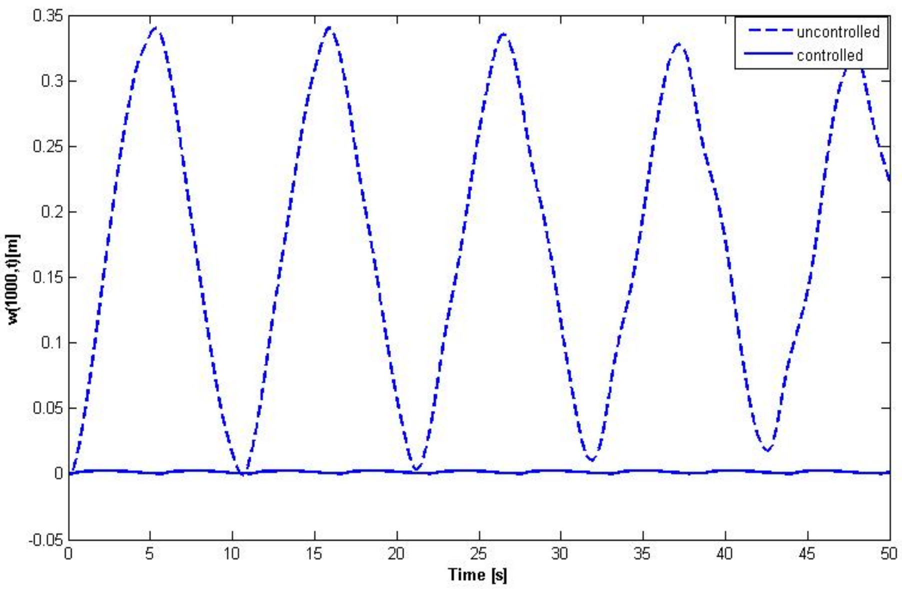


Fig. 5.8: Longitudinal control input $u_L(t)$.

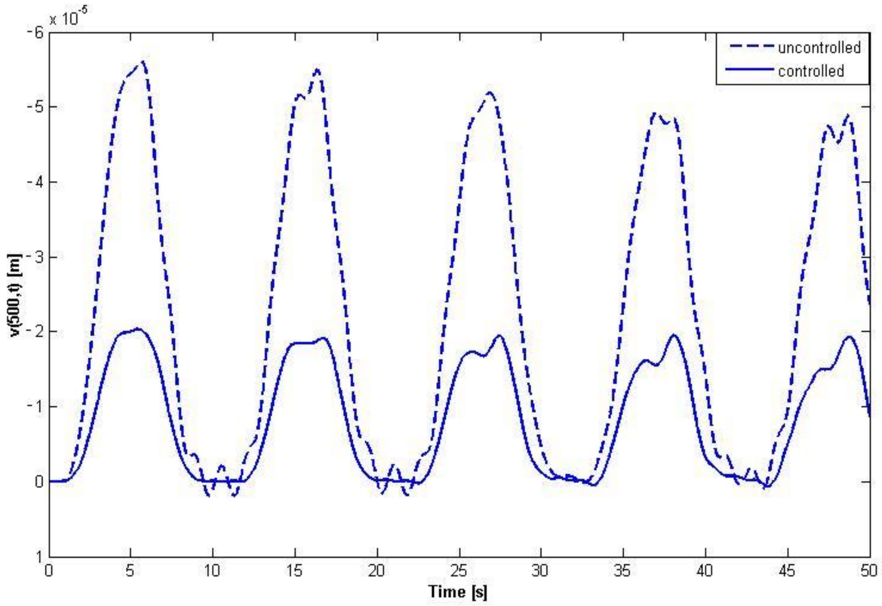


(a)

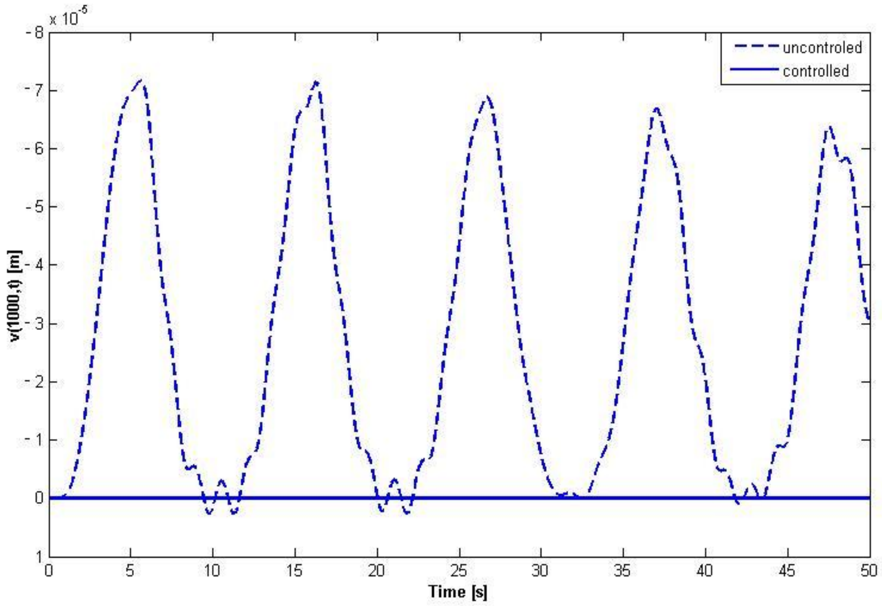


(b)

Fig. 5.9: Transverse displacements: (a) transverse displacement at $x = 500m$, $w(500, t)$ for controlled (solid) and uncontrolled (dashed) and (b) transverse displacement at $x = 1000m$, $w(1000, t)$ for controlled (solid) and uncontrolled (dashed).



(a)



(b)

Fig. 5.10: Longitudinal displacements: (a) longitudinal displacement at $x = 500m$, $v(500, t)$ for controlled (solid) and uncontrolled (dashed) and (b) longitudinal displacement at $x = 1000m$, $v(1000, t)$ for controlled (solid) and uncontrolled (dashed).

Chapter 6

Flexible Marine Riser with Vessel Dynamics

6.1 Introduction

With the increased focus on offshore oil and gas development in deeper and harsher environments, vibration control of the flexible marine risers has gained increasing attention. The marine riser is used as a fluid-conveyed curved pipe drilling crude oil, natural gas, hydrocarbon, petroleum materials, mud, and other undersea economic resources, and then transporting those resources in the ocean floor to the production vessel or platform in the ocean surface [9]. Vibration and deformation of the riser due to the ocean current disturbance and tension exerted at the top can produce premature fatigue problems, which require inspections and costly repairs. Recent advances in computer and electronics technology have allowed the development of complex electromechanical control system to suppress the riser's vibration. Vibration suppression by proper control techniques is desirable and available for preventing the damage and improving the lifespan of the riser.

For the purpose of dynamic analysis, the riser is modeled as an Euler-Bernoulli beam structure with PDEs since the diameter-to-length of the riser is small. Based on the distributed parameter model, various kinds of control methods integrating computer software and hardware with sensors and actuators have been investigated to suppress the riser's vibration. In [72], boundary control for the flexible marine riser with actuator dynamics is designed based on the Lyapunov's direct method and the backstepping technique. In [10], a torque actuator is introduced at the top boundary of the riser to reduce the angle and transverse vibration of the riser with guaranteed closed-loop stability. In [78], boundary control for a coupled nonlinear flexible marine riser with two actuators in transverse and longitudinal directions has been designed to suppress the riser's vibration. However, in these works, only the riser dynamics is considered and the coupling between riser and vessel is neglected, which can influence the dynamic response of the riser system and lead to an imprecise model.

Mathematically, the flexible marine riser with vessel dynamics is represented by a set of infinite dimensional equations (i.e., PDEs describing the dynamics of the flexible riser) coupled with a set of finite dimensional equations (i.e., ODEs describing the vessel dynamics). The dynamics of the flexible mechanical system modeled by a set of PDEs is difficult to control due to the infinite dimensionality of the system. Modal control method for the control design of PDE is based on truncated finite dimensional modes of the system, which are derived from finite element method, Galerkin's method or assumed modes method [12, 13, 16–19]. The truncated models are obtained via the model analysis or spatial discretization, in which the flexibility is represented by a finite number of modes by neglecting the higher frequency modes. The problems from the truncation procedure in the modeling need to be carefully treated in practical applications. A potential drawback in the above control design approaches is that the control can cause the actual system to become unstable due to excitation of the unmodeled, high-frequency vibration modes (i.e., spillover effects) [26]. Spillover effects which result in instability of the system have been investigated in [27,28] when the control of the truncated system is restricted to a few critical modes. The control order needs to be increased with the number of flexible modes considered to achieve high accuracy of performance and the control may also be difficult to implement from the engineering point of view since full states measurements or observers are often required. In an attempt to overcome the above shortcomings of the truncated model based control, control methodologies such as method based on bifurcation theory and the application of Poincare maps [141], variable structure control [29], sliding model control [30], energy-based robust control [31,32], model-free control [33] and boundary control [8, 39, 40, 51, 55, 68, 69, 74, 76, 77] have been developed. In these approaches, system dynamics analysis and control design are carried out directly based on the PDEs of the system. In contrast, boundary control where the actuation and sensing are applied only through the boundary of the system utilizes the distributed parameter model with PDEs to avoid control spillover instabilities.

Boundary control is considered to be more practical in a number of research fields including vibration control of flexible structures, fluid dynamics and heat transfer, which requires relatively few sensors and actuators. The relevant applications for this approach in mechanical flexible structures consist of second order structures (strings, and cables) and fourth order structures (beams and plates) [46]. In [56], robust and adaptive boundary control laws based on the Lyapunov synthesis are developed to reduce the vibration of a stretched string on a moving transporter. In [39], adaptive boundary control is designed for an axially moving string with a spatiotemporally varying tension, where the system is proved to be asymptotically stable. In [63], a boundary control law based on the Lyapunov method with sliding mode is employed to guarantee the asymptotic and exponential stability of an axially moving string. In [59], boundary control for a linear gantry crane model with a flexible cable is developed and experimentally implemented. In [74, 89], backstepping boundary controller and observer are designed to stabilize the string and beam model respectively. In [80], boundary control is presented to stabilize beams by using active constrained layer damping. In [65], nonlinear boundary control is constructed to exponentially stabilize a free transversely vibrating beam.

In this chapter, both the dynamics of the vessel and the vibration of the riser are considered in the dynamic analysis based on the Lyapunov's direct method. The main contributions of this chapter include:

(i) A dynamic model of the marine flexible riser with vessel dynamics subjected to the ocean current disturbance is derived for vibration suppression.

(ii) An implementable robust adaptive boundary control at the top boundary of the riser is designed to suppress the riser's vibration.

(iii) With proposed boundary control, uniform boundedness of the riser system under the ocean current disturbance is proved via the Lyapunov synthesis.

The rest of the chapter is organized as follows. The governing equation (PDE) and boundary conditions (ODEs) of the flexible riser system are introduced by use of Hamilton's principle in Section 6.2. The boundary control design via the Lyapunov's direct method is discussed separately for both exact model case and system parametric uncertainty case in Section 6.3, where it is shown that the uniform boundedness of the closed-loop system can be achieved by the proposed control. Simulations are carried out to illustrate performance of the proposed control in Section 6.4. The conclusion of this chapter is shown in Section 6.5.

6.2 Problem Formulation

Remark 6.1. *For clarity, the notations, the notations* $w'(x,t) = \frac{\partial w(x,t)}{\partial x}$, $w''(x,t) = \frac{\partial^2 w(x,t)}{\partial x^2}$, $w'''(x,t) = \frac{\partial^3 w(x,t)}{\partial x^3}$, $w''''(x,t) = \frac{\partial^4 w(x,t)}{\partial x^4}$, $\dot{w}(x,t) = \frac{\partial w(x,t)}{\partial t}$, *and* $\ddot{w}(x,t) = \frac{\partial^2 w(x,t)}{\partial t^2}$ *are introduced throughout the chapter.*

A typical marine riser system for crude oil transportation depicted in Fig. 6.1 is the connection between a production vessel on the ocean surface and a well head on the ocean floor. As shown in Fig. 6.1, the control is implemented from the actuator in the vessel, i.e., the top boundary of the riser. In this chapter, we assume that the original position of the vessel is directly above the subsea well head with no horizontal offset and the riser is filled with seawater.

The kinetic energy of the riser system E_k can be represented as

$$E_k = \frac{1}{2}M_s[\dot{w}(L,t)]^2 + \frac{1}{2}\rho\int_0^L [\dot{w}(x,t)]^2 dx, \tag{6.1}$$

where x and t represent the independent spatial and time variables respectively, M_s denotes the mass of the surface vessel, $w(L,t)$ and $\dot{w}(L,t)$ are the position and velocity of the vessel respectively, $w(x,t)$ is the displacement of the riser at the position x for time t, $\rho > 0$ is the uniform mass per unit length of the riser, and L is the length of the riser.

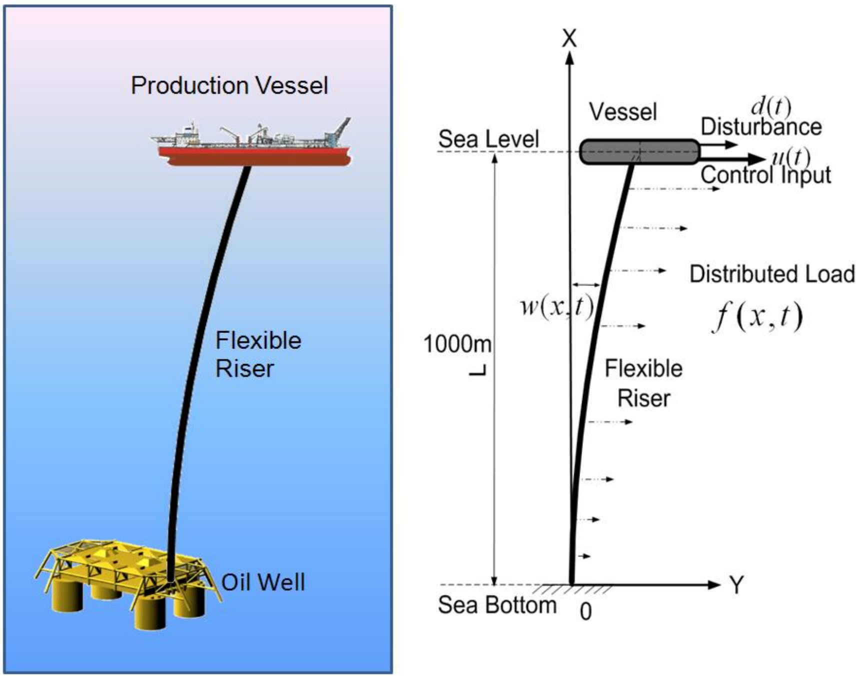


Fig. 6.1: A typical flexible marine riser system.

The potential energy E_p of the riser system can be obtained from

$$E_p = \frac{1}{2}EI\int_0^L [w''(x,t)]^2 dx + \frac{1}{2}T\int_0^L [w'(x,t)]^2 dx, \tag{6.2}$$

where EI is the bending stiffness of the riser and T is the tension of the riser. The first term of Eq. (5.2) is due to the bending, the second term is due to the strain energy of the riser.

The virtual work done by the ocean current disturbance on the riser and the vessel is given by

$$\delta W_f = \int_0^L f(x,t)\delta w(x,t)dx + d(t)\delta w(L,t), \tag{6.3}$$

where $f(x,t)$ is the distributed transverse load on the riser due to the hydrodynamic effects of the ocean current, and $d(t)$ denotes the environmental disturbances on the vessel. The virtual work done by damping on the riser and the vessel is represented by

$$\delta W_d = -\int_0^L c\dot{w}(x,t)\delta w(x,t)dx - d_s\dot{w}(L,t)\delta w(L,t), \tag{6.4}$$

where c is the damping coefficient of the riser, and d_s denotes the damping coefficient of the vessel. We introduce the boundary control u from the actuator in the vessel, i.e., the top boundary of the riser, to produce a transverse force for vibration suppression. The virtual work done by the boundary control is written as

$$\delta W_m = u(t)\delta w(L,t). \tag{6.5}$$

Then, we have the total virtual work done on the system as

$$\begin{aligned}\delta W &= \delta W_f + \delta W_d + \delta W_m \\ &= \int_0^L \left[f(x,t) - c\dot{w}(x,t)\right]\delta w(x,t)dx + \left[u(t) + d(t) - d_s\dot{w}(L,t)\right]\delta w(L,t).\end{aligned} \tag{6.6}$$

Substituting Eqs. (6.1), (6.2), and (6.6) into the Hamilton's principle Eq. (2.1), we obtain the governing equations of the system as

$$\rho\ddot{w}(x,t) + EIw''''(x,t) - Tw''(x,t) - f(x,t) + c\dot{w}(x,t) = 0, \tag{6.7}$$

and the boundary conditions of the system as

$$w'(0,t) = 0, \tag{6.8}$$

$$w''(L,t) = 0, \tag{6.9}$$

$$w(0,t) = 0, \tag{6.10}$$

$$-EIw'''(L,t) + Tw'(L,t) = u(t) + d(t) - d_s\dot{w}(L,t) - M_s\ddot{w}(L,t), \tag{6.11}$$

$\forall t \in [0, \infty)$.

Assumption 6.1. *For the distributed load $f(x,t)$ on the riser and the environmental disturbance $d(t)$ on the vessel, we assume that there exist constants $\bar{f} \in R^+$ and $\bar{d} \in R^+$, such that $|f(x,t)| \leq \bar{f}$, $\forall(x,t) \in [0,L] \times [0,\infty)$ and $|d(t)| \leq \bar{d}$, $\forall(t) \in [0,\infty)$. This is a reasonable assumption as the time-varying disturbances $f(x,t)$ and $d(t)$ have finite energy and hence are bounded, i.e., $f(x,t) \in \mathcal{L}_\infty([0,L])$ and $d(t) \in \mathcal{L}_\infty$.*

Remark 6.2. *For control design in Section 6.3, only the assertion that there exist an upper bound on the disturbance in Assumption 1, $|f(x,t)| < \bar{f}$ and $|d(t)| \leq \bar{d}$, is necessary. The knowledge of the exact values for $f(x,t)$ and $d(t)$ is not required. As such, different distributed load models up to various levels of fidelity, such as those found in [127, 128, 133–135], can be applied without affecting the control design or analysis.*

For the convenience of stability analysis, we present the following properties for the subsequent development.

Property 6.1. *[136]: If the kinetic energy of the system* (6.7) - (6.11), *given by Eq.* (6.1) *is bounded* $\forall t \in [0,\infty)$, *then $\dot{w}(x,t)$, $\dot{w}'(x,t)$, $\dot{w}''(x,t)$ and $\dot{w}'''(x,t)$ are bounded* $\forall(x,t) \in [0,L] \times [0,\infty)$.

Property 6.2. *[136]: If the potential energy of the system* (6.7) - (6.11)*, given by Eq.* (6.2) *is bounded* $\forall t \in [0, \infty)$*, then* $w''(x,t)$*,* $w'''(x,t)$ *and* $w''''(x,t)$ *are bounded* $\forall (x,t) \in [0, L] \times [0, \infty)$.

6.3 Control Design

The control objective is to suppress the vibration of the riser and stabilize the riser at the small neighborhood of its original position in the presence of the time-varying distributed load $f(x,t)$ and the disturbance $d(t)$ due to the ocean current. In this section, the Lyapunov's direct method is used to construct a boundary control law $u(t)$ at the top boundary of the riser and to analyze the closed-loop stability of the system.

In this chapter, we analyze two cases for the riser system: (i) exact model-based control, i.e., EI, T, M_s and d_s are all known; and (ii) adaptive control for the system parametric uncertainty, i.e., EI, T, M_s and d_s are unknown. For the first case, robust boundary control is introduced for the exact model of the riser system subject to the ocean disturbance. For second case where the system parameters cannot be directly measured, the adaptive control is designed to compensate the system parametric uncertainty.

6.3.1 Exact model based boundary control of the riser system

To stabilize the system given by governing Eq. (6.7) and boundary condition Eqs. (6.8) - (6.11), we propose the following control law:

$$\begin{aligned} u &= -EIw'''(L,t) + Tw'(L,t) - \text{sgn}(u_a)\bar{d} + d_s\dot{w}(L,t) - k_1 M_s \dot{w}'(L,t) \\ &\quad + k_2 M_s \dot{w}'''(L,t) - k u_a, \end{aligned} \tag{6.12}$$

where $\text{sgn}(\cdot)$ denotes the signum function, k, k_1, k_2 are the control gains and the auxiliary signal u_a is defined as

$$u_a = \dot{w}(L,t) + k_1 w'(L,t) - k_2 w'''(L,t). \tag{6.13}$$

After differentiating the auxiliary signal Eq. (6.13), multiplying the resulting equation by M_s, and substituting Eq. (6.11), we obtain

$$M_s\dot{u}_a = EIw'''(L,t) - Tw'(L,t) + d - d_s\dot{w}(L,t) + k_1 M_s \dot{w}'(L,t) - k_2 M_s \dot{w}'''(L,t) + u \tag{6.14}$$

Substituting Eq. (6.12) into Eq. (6.14), we have

$$M_s\dot{u}_a = -k u_a + d - \text{sgn}(u_a)\bar{d}. \tag{6.15}$$

Remark 6.3. *All the signals in the boundary control can be measured by sensors or obtained by a backward difference algorithm.* $w(L,t)$ *can be sensed by a laser displacement sensor at the top boundary of the riser,* $w'(L,t)$ *can be measured by an inclinometer and* $w'''(L,t)$ *can be obtained by a shear force sensor. In practice, the* *effect of measurement noise from sensors is unavoidable, which will affect the control implementation, especially when the high order differentiating terms with respect to time exist. In our proposed control* (6.12), $\dot{w}(L,t)$, $\dot{w}'(L,t)$ *and* $\dot{w}'''(L,t)$ *with only one time differentiating with respect to time can be calculated with a backward difference algorithm. It is noted that differentiating twice and three times position* $w(L,t)$ *with respect to time to get* $\ddot{w}(L,t)$ *and* $\dddot{w}(L,t)$ *respectively, are undesirable in practice due to noise amplification. For these cases, observers are needed to design to estimate the states values according to the boundary conditions.*

Remark 6.4. *The control design is based on the distributed parameter model Eqs.* (6.7) *to* (6.11)*, and the spillover problems associated with traditional truncated model-based approaches caused by ignoring high-frequency modes in controller and observer design are avoided. For results on model-based control of distributed parameter system which is helpful in avoiding spillover effects, the readers can refer to [16, 17].*

Consider the Lyapunov function candidate

$$V = V_1 + V_2 + V_3, \tag{6.16}$$

where the energy term V_1 and an auxiliary term V_2 and a small crossing term V_3 are defined as

$$\begin{aligned} V_1 &= \frac{\beta k_2}{2}\rho\int_0^L [\dot{w}]^2 dx + \frac{\beta k_2}{2}EI\int_0^L [w'']^2 dx + \frac{\beta k_2}{2}T\int_0^L [w']^2 dx, && (6.17)\\ V_2 &= \frac{1}{2}M_s u_a^2, && (6.18)\\ V_3 &= \alpha\rho\int_0^L x\dot{w}w' dx, && (6.19) \end{aligned}$$

where k_2 is the control gain, and α and β are two positive weighting constants.

Lemma 6.1. *The Lyapunov function candidate given by* (6.16) *is upper and lower bounded as*

$$0 \leq \lambda_1(V_1 + V_2) \leq V \leq \lambda_2(V_1 + V_2), \tag{6.20}$$

where λ_1 *and* λ_2 *are two positive constants defined as*

$$\lambda_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)} \quad \text{and} \quad \lambda_2 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)}. \tag{6.21}$$

Proof: Applying Ineq. (2.7) in Eq. (6.19) yields

$$\begin{aligned} |V_3| &\leq \alpha\rho L \int_0^L ([w']^2 + [\dot{w}]^2)dx \\ &\leq \alpha_1 V_1, \end{aligned} \tag{6.22}$$

where

$$\alpha_1 = \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)}. \tag{6.23}$$

Then, we obtain

$$-\alpha_1 V_1 \leq V_3 \leq \alpha_1 V_1. \tag{6.24}$$

Considering α is a small positive weighting constant satisfying $0 < \alpha < \frac{\min(\beta\rho k_2, \beta T k_2)}{2\rho L}$, we can obtain

$$\alpha_2 = 1 - \alpha_1 = 1 - \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)} > 0, \tag{6.25}$$

$$\alpha_3 = 1 + \alpha_1 = 1 + \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)} > 1. \tag{6.26}$$

Then, we further have

$$0 \le \alpha_2 V_1 \le V_1 + V_3 \le \alpha_3 V_1. \tag{6.27}$$

Given the Lyapunov function candidate in Eq. (6.16), we obtain

$$0 \le \lambda_1(V_1 + V_2) \le V \le \lambda_2(V_1 + V_2), \tag{6.28}$$

where $\lambda_1 = \min(\alpha_2, 1) = \alpha_2$ and $\lambda_2 = \max(\alpha_3, 1) = \alpha_3$ are two positive constants. ■

Lemma 6.2. *The time derivative of the Lyapunov function candidate* (6.16) *is upper bounded with*

$$\dot{V} \quad \le \quad -\lambda V + \varepsilon, \tag{6.29}$$

where λ and ε are positive constants.

Proof: Differentiating Eq. (6.16) with respect to time leads to

$$\dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3. \tag{6.30}$$

The first term of the Eq. (6.30)

$$\dot{V}_1 = A_1 + A_2 + A_3, \tag{6.31}$$

where

$$A_1 = \beta\rho k_2 \int_0^L \dot{w}\ddot{w}dx, \quad (6.32)$$

$$A_2 = \beta EIk_2 \int_0^L w''\dot{w}''dx, \quad (6.33)$$

$$A_3 = \beta Tk_2 \int_0^L w'\dot{w}'dx. \quad (6.34)$$

Substituting the governing equation (6.7) into A_1, we obtain

$$A_1 = \beta k_2 \int_0^L \dot{w}\left(-EIw'''' + Tw'' + f - c\dot{w}\right)dx. \quad (6.35)$$

Using the boundary conditions and integrating Eq. (6.33) by parts, we obtain

$$\begin{aligned} A_2 &= \beta EIk_2 \int_0^L w''d(\dot{w}') \\ &= \beta EIw''\dot{w}'\Big|_0^L - \beta EI \int_0^L \dot{w}'w'''dx \\ &= -\beta EIk_2 w'''(L,t)\dot{w}(L,t) + \beta EIk_2 \int_0^L \dot{w}w''''dx. \end{aligned} \quad (6.36)$$

Using the boundary conditions and integrating Eq. (6.34) by parts, we obtain

$$\begin{aligned} A_3 &= \beta Tk_2 \int_0^L w'd(\dot{w}) \\ &= \beta Tk_2 w'(L,t)\dot{w}(L,t) - \beta Tk_2 \int_0^L \dot{w}w''dx. \end{aligned} \quad (6.37)$$

Substituting Eqs. (6.35), (6.36), and (6.37) into Eq. (6.31), we have

$$\dot{V}_1 = \beta k_2\left[-EIw'''(L,t) + Tw'(L,t)\right]\dot{w}(L,t) - \beta ck_2 \int_0^L [\dot{w}]^2dx + \beta k_2 \int_0^L f\dot{w}dx. \quad (6.38)$$

Substituting the Eq. (6.13) into Ineq. (6.38), we obtain

$$\begin{aligned}
\dot{V}_1 &= -\frac{\beta EI}{2}\left[[\dot{w}(L,t)]^2 + k_2^2[w'''(L,t)]^2 + k_1^2[w'(L,t)]^2\right] + \frac{\beta EI}{2}u_a^2 \\
&\quad +\beta(Tk_2 - EIk_1)w'(L,t)\dot{w}(L,t) + \beta EIk_1k_2w'''(L,t)w'(L,t) \\
&\quad -\beta ck_2\int_0^L[\dot{w}]^2dx + \beta k_2\int_0^L f\dot{w}dx. \qquad (6.39)
\end{aligned}$$

Using Ineq. (2.8), we obtain

$$\begin{aligned}
\dot{V}_1 &\leq -\frac{\beta EI}{2}\left[[\dot{w}(L,t)]^2 + k_2^2[w'''(L,t)]^2 + k_1^2[w'(L,t)]^2\right] + \frac{\beta EI}{2}u_a^2 \\
&\quad +\beta|Tk_2 - EIk_1|\delta_1[w'(L,t)]^2 + \frac{\beta}{\delta_1}|Tk_2 - EIk_1|[\dot{w}(L,t)]^2 + \beta EIk_1k_2w'''(L,t)w'(L,t) \\
&\quad -\beta(c-\delta_2)k_2\int_0^L[\dot{w}]^2dx + \frac{\beta k_2}{\delta_2}\int_0^L f^2dx, \qquad (6.40)
\end{aligned}$$

where δ_1 and δ_2 are two positive constants.

The second term of the Eq. (6.30)

$$\begin{aligned}
\dot{V}_2 &= M_su_a\dot{u}_a, \\
&= -ku_a^2 + du_a - \mathrm{sgn}(u_a)u_a\bar{d} \\
&= -ku_a^2 + du_a - |u_a|\bar{d} \\
&\leq -ku_a^2. \qquad (6.41)
\end{aligned}$$

The third term of the Eq. (6.30)

$$\begin{aligned}
\dot{V}_3 &= \alpha\rho\int_0^L(x\ddot{w}w' + x\dot{w}\dot{w}')dx \\
&= \alpha\int_0^L xw'\left[-EIw'''' + Tw'' + f - c\dot{w}\right]dx + \alpha\rho\int_0^L x\dot{w}\dot{w}'dx \\
&= B_1 + B_2 + B_3 + B_4 + B_5, \qquad (6.42)
\end{aligned}$$

where

$$B_1 = -\alpha \int_0^L EIxw'w''''dx, \quad (6.43)$$

$$B_2 = \alpha \int_0^L Txw'w''dx, \quad (6.44)$$

$$B_3 = \alpha \int_0^L fxw'dx, \quad (6.45)$$

$$B_4 = -\alpha \int_0^L cxw'\dot{w}dx, \quad (6.46)$$

$$B_5 = \alpha\rho \int_0^L x\dot{w}\dot{w}'dx. \quad (6.47)$$

After integrating Eq. (6.43) by parts and using the boundary conditions, we obtain

$$B_1 = -\alpha EILw'(L,t)w'''(L,t) + \alpha EI \int_0^L w'w'''dx + \alpha EI \int_0^L xw''w'''dx. \quad (6.48)$$

By integrating the second term of Eq. (6.48), we have

$$B_1 = -\alpha EILw'(L,t)w'''(L,t) - \alpha EI \int_0^L [w'']^2dx + \alpha EI \int_0^L xw''w'''dx. \quad (6.49)$$

By integrating the last term of Eq. (6.49), we have

$$B_1 = -\alpha EILw'(L,t)w'''(L,t) - \alpha EI \int_0^L [w'']^2dx - \alpha EI \int_0^L \left([w'']^2 + xw''w'''\right) dx. \quad (6.50)$$

Combining the Eq. (6.49) and Eq. (6.50), we obtain

$$B_1 = -\alpha EILw'(L,t)w'''(L,t) - \frac{3\alpha EI}{2} \int_0^L [w'']^2dx. \quad (6.51)$$

After integrating Eq. (6.44) by parts and using the boundary conditions, we obtain

$$B_2 = \alpha TL[w'(L,t)]^2 - \alpha T \int_0^L \left([w']^2 + xw'w''\right) dx. \tag{6.52}$$

Combining Eq. (6.44) and Eq. (6.52), we obtain

$$B_2 = \frac{\alpha TL}{2}[w'(L,t)]^2 - \frac{\alpha T}{2}\int_0^L [w']^2 dx. \tag{6.53}$$

Using Ineq. (2.8), we obtain

$$B_3 \leq \frac{\alpha L}{\delta_3}\int_0^L f^2 dx + \alpha L\delta_3 \int_0^L [w']^2 dx, \tag{6.54}$$

$$B_4 \leq \frac{\alpha cL}{\delta_4}\int_0^L [\dot{w}]^2 dx + \alpha cL\delta_4 \int_0^L [w']^2 dx. \tag{6.55}$$

where δ_3 and δ_4 are two positive constants. Integrating Eq. (6.47) by parts, we obtain

$$B_5 = \alpha\rho L[\dot{w}(L,t)]^2 - \alpha\rho \int_0^L \left([\dot{w}]^2 + x\dot{w}\dot{w}'\right) dx. \tag{6.56}$$

The last term in Eq. (6.56) equals B_5, and we have

$$B_5 = \frac{\alpha\rho L}{2}[\dot{w}(L,t)]^2 - \frac{\alpha\rho}{2}\int_0^L [\dot{w}]^2 dx. \tag{6.57}$$

Applying Eqs. (6.51), (6.57) and Ineqs. (6.53), (6.54), (6.55) in Eq. (6.42), we obtain

$$\begin{aligned}\dot{V}_3 \leq\ & -\alpha EILw'(L,t)w'''(L,t) - \frac{3\alpha EI}{2}\int_0^L [w'']^2 dx + \frac{\alpha TL}{2}[w'(L,t)]^2 - \frac{\alpha T}{2}\int_0^L [w']^2 dx \\ & + \frac{\alpha L}{\delta_3}\int_0^L f^2 dx + \alpha L\delta_3 \int_0^L [w']^2 dx + \frac{\alpha cL}{\delta_4}\int_0^L [\dot{w}]^2 dx + \alpha cL\delta_4 \int_0^L [w']^2 dx \\ & + \frac{\alpha\rho L}{2}[\dot{w}(L,t)]^2 - \frac{\alpha\rho}{2}\int_0^L [\dot{w}]^2 dx.\end{aligned} \tag{6.58}$$

Applying Ineqs. (6.40), (6.41) and (6.58) into Eq. (6.16), and utilizing the Ineq. (2.8), we obtain

$$\begin{aligned}
\dot{V} &\leq -\left(\beta c k_2 + \frac{\alpha\rho}{2} - \beta\delta_2 k_2 - \frac{\alpha c L}{\delta_4}\right)\int_0^L [\dot{w}]^2 dx - \left(\frac{\alpha T}{2} - \alpha L\delta_3 - \alpha c L\delta_4\right)\int_0^L [w']^2 dx \\
&\quad -\frac{3\alpha EI}{2}\int_0^L [w'']^2 dx - \left(\frac{\beta EI}{2} - \frac{\beta}{\delta_1}|Tk_2 - EIk_1| - \frac{\alpha\rho L}{2}\right)[\dot{w}(L,t)]^2 \\
&\quad -\left(\frac{\beta EIk_1^2}{2} - \frac{\alpha TL}{2} - \beta|Tk_2 - EIk_1|\delta_1 - |\beta EIk_1k_2 - \alpha EIL|\delta_5\right)[w'(L,t)]^2 \\
&\quad -\left(\frac{\beta EIk_2^2}{2} - \frac{|\beta EIk_1k_2 - \alpha EIL|}{\delta_5}\right)[w'''(L,t)]^2 - \left(k - \frac{\beta EI}{2}\right)u_a^2 \\
&\quad +\left(\frac{\beta k_2}{\delta_2} + \frac{\alpha L}{\delta_3}\right)\int_0^L \bar{f}^2 dx \\
&\leq -\lambda_3(V_1 + V_2) + \varepsilon, \qquad (6.59)
\end{aligned}$$

where $\varepsilon = \left(\frac{\beta k_2}{\delta_2} + \frac{\alpha L}{\delta_3}\right)\int_0^L \bar{f}^2 dx = \left(\frac{\beta k_2}{\delta_2} + \frac{\alpha L}{\delta_3}\right)L\bar{f}^2$, the constants k, k_1, k_2, α, β, δ_1, δ_2, δ_3, δ_4 and δ_5 are chosen to satisfy the following conditions:

$$\alpha < \frac{\min(\beta\rho k_2, \beta T k_2)}{2\rho L}, \qquad (6.60)$$

$$\frac{\beta EIk_1^2}{2} - \frac{\alpha TL}{2} - \beta|Tk_2 - EIk_1|\delta_1 - |\beta EIk_1k_2 - \alpha EIL|\delta_5 \geq 0, \qquad (6.61)$$

$$\frac{\beta EI}{2} - \frac{\beta}{\delta_1}|Tk_2 - EIk_1| - \frac{\alpha\rho L}{2} \geq 0, \qquad (6.62)$$

$$\frac{\beta EIk_2^2}{2} - \frac{|\beta EIk_1k_2 - \alpha EIL|}{\delta_5} \geq 0, \qquad (6.63)$$

$$\sigma_1 = \beta c k_2 + \frac{\alpha\rho}{2} - \beta\delta_2 k_2 - \frac{\alpha c L}{\delta_4} > 0, \qquad (6.64)$$

$$\sigma_2 = \frac{3\alpha EI}{2} > 0, \qquad (6.65)$$

$$\sigma_3 = \frac{\alpha T}{2} - \alpha L\delta_3 - \alpha c L\delta_4 > 0, \qquad (6.66)$$

$$\sigma_4 = k - \frac{\beta EI}{2} > 0, \qquad (6.67)$$

$$\lambda_3 = \min\left(\frac{2\sigma_1}{\beta\rho}, \frac{2\sigma_2}{\beta EI}, \frac{2\sigma_3}{\beta T}, \frac{2\sigma_4}{M_s}\right) > 0. \qquad (6.68)$$

From Ineqs. (6.28) and (6.59) we have

$$\dot{V} \leq -\lambda V + \varepsilon, \tag{6.69}$$

where $\lambda = \lambda_3/\lambda_2$ and ε are two positive constants. ■

With the above lemmas, the exact model-based control design for riser system subjected to the ocean current disturbance can be summarized in the following theorem.

Theorem 6.1. *For the system dynamics described by* (6.7) *and boundary conditions* (6.8) - (6.11), *under Assumption 6.1, and the control law* (6.12), *given that the initial conditions are bounded, we can conclude that uniform boundedness (UB): the state of the closed loop system $w(x,t)$ will remain in the compact set Ω defined by*

$$\Omega := \left\{ w(x,t) \in R \middle| \quad |w(x,t)| \leq H_1, \ \forall (x,t) \in [0,L] \times [0,\infty) \right\}, \tag{6.70}$$

where constant $H_1 = \sqrt{\frac{2L}{\beta T \lambda_1 k_2}\left(V(0) + \frac{\varepsilon}{\lambda}\right)}$.

Proof: Multiplying Eq. (6.29) by $e^{\lambda t}$ yields

$$\frac{\partial}{\partial t}(V e^{\lambda t}) \leq \varepsilon e^{\lambda t}. \tag{6.71}$$

Integration of the above inequality, we obtain

$$V \leq \left(V(0) - \frac{\varepsilon}{\lambda} \right) e^{-\lambda t} + \frac{\varepsilon}{\lambda} \leq V(0) e^{-\lambda t} + \frac{\varepsilon}{\lambda} \in \mathcal{L}_\infty, \tag{6.72}$$

which implies V is bounded. Utilizing Ineq. (2.11) and Eq. (6.17), we have

$$\frac{\beta k_2}{2L}Tw^2(x,t) \leq \frac{\beta k_2}{2}T\int_0^L [w'(x,t)]^2dx \leq V_1 \leq V_1 + V_2 \leq \frac{1}{\lambda_1}V \in \mathcal{L}_\infty. \qquad (6.73)$$

Appropriately rearranging the terms of the above inequality, we obtain $w(x,t)$ is uniformly bounded as follows:

$$|w(x,t)| \leq \sqrt{\frac{2L}{\beta T\lambda_1 k_2}\left(V(0)e^{-\lambda t} + \frac{\varepsilon}{\lambda}\right)} \leq \sqrt{\frac{2L}{\beta T\lambda_1 k_2}\left(V(0) + \frac{\varepsilon}{\lambda}\right)},$$
$$\forall(x,t) \in [0,L] \times [0,\infty). \qquad (6.74)$$

■

Remark 6.5. *By choosing the proper values of α and β, it is shown that the increase in the control gain k will result in a larger σ_4, which will lead a greater λ_3. Then the value of λ will increase, which will reduce the size of Ω and produce a better vibration suppression performance. We can conclude that the bound of the system state $w(x,t)$ can be made arbitrarily small provided that the design control parameters are appropriately selected. However, increasing k will bring a high gain control problem. Therefore, in practical applications, the design parameters should be adjusted carefully for achieving suitable transient performance and control action.*

Remark 6.6. *From Eq. (6.73), we can state that V_1 is bounded $\forall t \in [0,\infty)$. Since V_1 is bounded, $\dot{w}(x,t)$, $w''(x,t)$ and $w'(x,t)$ are bounded $\forall(x,t) \in [0,L] \times [0,\infty)$. From Eq. (6.1), the kinetic energy of the system is bounded and using Property 6.1, $\dot{w}'(x,t)$ and $\dot{w}'''(x,t)$ are also bounded $\forall(x,t) \in [0,L] \times [0,\infty)$. From the boundedness of the potential energy Eq. (6.2), we can use Property 6.2 to obtain that $w'''(x,t)$ and $w''''(x,t)$ are bounded. Using Assumption 5.1, Eq. (6.7) and the above statements, we can state that $\ddot{w}(x,t)$ is also bounded $\forall(x,t) \in [0,L] \times [0,\infty)$. From the above* *information, it is shown that the proposed control Eq.* (6.12) *ensures all internal system signals including* $w(x,t)$, $w'(x,t)$, $\dot{w}(x,t)$, $\dot{w}'(x,t)$, $\ddot{w}(x,t)$, $w'''(x,t)$, $\dot{w}'''(x,t)$ *and* $w''''(x,t)$ *are uniformly bounded. Since* $\dot{w}(x,t)$, $w'(x,t)$, $\dot{w}'(x,t)$, $w'''(x,t)$ *and* $\dot{w}'''(x,t)$ *are all bounded* $\forall(x,t) \in [0,L] \times [0,\infty)$*, and we can conclude the boundary control Eq.* (6.12) *is also bounded* $\forall t \in [0,\infty)$*.*

Remark 6.7. *For the system dynamics described by Eq.* (6.7) *and boundary conditions* (6.8) *to* (6.11)*, if* $f(x,t) = 0$*, the exponential stability can be achieved with the proposed boundary control* (6.12) *as follows:*

$$|w(x,t)| \leq \sqrt{\frac{2L}{\beta T \lambda_1 k_2} V(0) e^{-\lambda t}}, \ \forall (x,t) \in [0,L] \times [0,\infty). \tag{6.75}$$

6.3.2 Robust adaptive boundary control for system parametric uncertainty

In Section 3.1, the exact model-based boundary control Eq. (6.12) requires the exact knowledge of the riser system. Adaptive boundary control is designed to improve the performance of the system via parameter estimation when there are some unknown parameters. The exact model-based boundary control provides a stepping stone towards the adaptive control, which is designed to deal with the system parametric uncertainty. In this section, the boundary control Eq. (6.12) is redesigned by using the adaptive control since the EI, T, d_s and M_s are unknown. We rewrite Eq. (6.14) as the following form

$$M_s \dot{u}_a = P\Phi + d + u, \tag{6.76}$$

where vectors P and Φ are defined as

$$P = [w'''(L,t) \quad -w'(L,t) \quad -\dot{w}(L,t) \quad k_1\dot{w}'(L,t) - k_2\dot{w}'''(L,t)], \tag{6.77}$$

$$\Phi = [EI \quad T \quad d_s \quad M_s]^T. \tag{6.78}$$

We propose the following adaptive boundary control law for system

$$u = -P\hat{\Phi} - ku_a - \mathrm{sgn}(u_a)\bar{d}, \tag{6.79}$$

where the parameter estimate vector $\hat{\Phi}$ is defined as

$$\hat{\Phi} = [\widehat{EI} \quad \widehat{T} \quad \widehat{d_s} \quad \widehat{M_s}]^T. \tag{6.80}$$

The adaptation law is designed as

$$\dot{\hat{\Phi}} = \Gamma P^T u_a - r\Gamma\hat{\Phi}, \tag{6.81}$$

where $\Gamma \in R^{4\times 4}$ is a diagonal positive-definite matrix and r is a positive constant. We define the maximum and minimum eigenvalue of matrix Γ as $\lambda_{\max}$ and $\lambda_{\min}$ respectively. The parameter estimate error vector $\tilde{\Phi} \in R^4$ is defined as

$$\tilde{\Phi} = \Phi - \hat{\Phi}. \tag{6.82}$$

Substituting Eq. (6.79) into Eq. (6.76) and using Eq. (6.82) in Eq. (6.81), we have

$$M_s\dot{u}_a = P\tilde{\Phi} - ku_a + d - \mathrm{sgn}(u_a)\bar{d}, \tag{6.83}$$

$$\dot{\tilde{\Phi}} = -\Gamma P^T u_a + r\Gamma\hat{\Phi}. \tag{6.84}$$

Consider the Lyapunov function candidate

$$V_a = V + \frac{1}{2}\tilde{\Phi}^T\Gamma^{-1}\tilde{\Phi}, \tag{6.85}$$

where V is defined as Eq. (6.16), and $\tilde{\Phi}$ is the parameter estimate error vector.

Lemma 6.3. *The Lyapunov function candidate given by* (6.85) *is upper and lower bounded as*

$$0 \leq \lambda_{1a}(V_1 + V_2 + ||\tilde{\Phi}||^2) \leq V_a \leq \lambda_{2a}(V_1 + V_2 + ||\tilde{\Phi}||^2), \tag{6.86}$$

where λ_{1a} and λ_{2a} are two positive constants defined as

$$\lambda_{1a} = \min(1 - \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)}, \frac{1}{2\lambda_{\max}}), \lambda_{2a} = \max(1 + \frac{2\alpha\rho L}{\min(\beta\rho k_2, \beta T k_2)}, \frac{1}{2\lambda_{\min}}) \tag{6.87}$$

Proof: From Ineq. (6.20), we have

$$\lambda_1(V_1 + V_2) \leq V \leq \lambda_2(V_1 + V_2), \tag{6.88}$$

where λ_1 and λ_2 are two positive constants defined in Eq. (6.21) . From the properties of matrix Γ, we have

$$\frac{1}{2\lambda_{\max}}||\tilde{\Phi}||^2 \leq \frac{1}{2}\tilde{\Phi}^T\Gamma^{-1}\tilde{\Phi} \leq \frac{1}{2\lambda_{\min}}||\tilde{\Phi}||^2. \tag{6.89}$$

Combining Ineqs. (6.88) and (6.89), we have

$$0 \leq \lambda_{1a}(V_1 + V_2 + ||\tilde{\Phi}||^2) \leq V_a \leq \lambda_{2a}(V_1 + V_2 + ||\tilde{\Phi}||^2), \tag{6.90}$$

where $\lambda_{1a} = \min(\lambda_1,\ \frac{1}{2\lambda_{\max}})$ and $\lambda_{2a} = \max(\lambda_2,\ \frac{1}{2\lambda_{\min}})$ are two positive constants. ■

Lemma 6.4. *The time derivative of the Lyapunov function candidate* (6.85) *is upper bounded with*

$$\dot{V}_a \;\; \leq \;\; -\lambda_a V_a + \psi, \tag{6.91}$$

where λ_a and ψ are two positive constants.

Proof: We obtain the time derivation of the Lyapunov function candidate Eq. (6.85) as

$$\dot{V}_a \;\; = \;\; \dot{V} + \tilde{\Phi}^T \Gamma^{-1} \dot{\tilde{\Phi}}. \tag{6.92}$$

Substituting Eq. (6.83) into the second term of the Eq. (6.30), we have

$$\begin{aligned} \dot{V}_2 \;\; &= \;\; M_s u_a \dot{u}_a \\ &= \;\; -k u_a^2 + d u_a - \operatorname{sgn}(u_a) \bar{d} u_a + P \tilde{\Phi} u_a \\ &\leq \;\; -k u_a^2 + P \tilde{\Phi} u_a. \end{aligned} \tag{6.93}$$

Applying the results of Lemma 5 and utilizing Ineqs. (6.40), (6.93) and (6.58) in $\dot{V}$, we obtain

$$\dot{V} \leq -\lambda_3 (V_1 + V_2) + P \tilde{\Phi} u_a + \varepsilon, \tag{6.94}$$

where λ_3 is defined in Eq. (6.68) and ε is a positive constant. Application of Ineq.

(6.94) into Eq. (6.92) yields

$$\dot{V}_a \leq -\lambda_3(V_1+V_2)+\tilde{\Phi}^T\left(P^T u_a+\Gamma^{-1}\dot{\hat{\Phi}}\right)+\varepsilon. \tag{6.95}$$

Substituting Eq. (6.84) into Ineq. (6.95), we have

$$\begin{aligned}\dot{V}_a &\leq -\lambda_3(V_1+V_2)+r\tilde{\Phi}^T\hat{\Phi}+\varepsilon\\ &\leq -\lambda_3(V_1+V_2)-\frac{r}{2}||\tilde{\Phi}||^2+\frac{r}{2}||\Phi||^2+\varepsilon\\ &\leq -\lambda_{3a}(V_1+V_2+||\tilde{\Phi}||^2)+\frac{r}{2}||\Phi||^2+\varepsilon,\end{aligned} \tag{6.96}$$

where $\lambda_{3a}=\min(\lambda_3,\frac{r}{2})$ is a positive constant. From Ineqs. (6.90) and (6.96), we have

$$\dot{V}_a \leq -\lambda_a V_a+\psi, \tag{6.97}$$

where $\lambda_a=\lambda_{3a}/\lambda_{2a}$ and $\psi=\frac{r}{2}||\Phi||^2+\varepsilon>0$. ■

With the above lemmas, the adaptive control design for the riser system subjected to the ocean current disturbance can be summarized in the following theorem.

Theorem 6.2. *For the system dynamics described by* (6.7) *and boundary conditions* (6.8) - (6.11)*, under Assumption 6.1, and the control law* (6.79)*, given that the initial conditions are bounded, we can conclude that uniform boundedness (UB): the state of the closed loop system $w(x,t)$ will remain in the compact set Ω_a defined by*

$$\Omega_a := \left\{w(x,t)\in R\middle|\quad |w(x,t)|\leq H_2,\ \forall(x,t)\in[0,L]\times[0,\infty)\right\}, \tag{6.98}$$

where constant $H_2=\sqrt{\frac{2L}{\beta T\lambda_{1a}k_2}\left(V_a(0)+\frac{\psi}{\lambda_a}\right)}$.

Proof: Multiplying Eq. (6.91) by $e^{\lambda_a t}$ yields

$$\frac{\partial}{\partial t}(V_a e^{\lambda_a t}) \ \leq \ \psi e^{\lambda_a t}. \tag{6.99}$$

Integrating of the above inequality, we obtain

$$V_a \leq \left(V_a(0) - \frac{\psi}{\lambda_a}\right) e^{-\lambda_a t} + \frac{\psi}{\lambda_a} \leq V_a(0)e^{-\lambda_a t} + \frac{\psi}{\lambda_a} \in \mathcal{L}_\infty, \tag{6.100}$$

which implies V_a is bounded. Utilizing Ineq. (2.11) and Eq. (6.17), we have

$$\frac{\beta k_2}{2L} T w^2(x,t) \leq \frac{\beta k_2}{2} T \int_0^L [w'(x,t)]^2 dx \leq V_1 \leq V_1 + V_2 \leq \frac{1}{\lambda_{1a}} V_a \in \mathcal{L}_\infty. \tag{6.101}$$

Appropriately rearranging the terms of the above inequality, we obtain $w(x,t)$ is uniformly bounded as follows:

$$|w(x,t)| \leq \sqrt{\frac{2L}{\beta T \lambda_{1a} k_2}\left(V_a(0)e^{-\lambda_a t} + \frac{\psi}{\lambda_a}\right)} \leq \sqrt{\frac{2L}{\beta T \lambda_{1a} k_2}\left(V_a(0) + \frac{\psi}{\lambda_a}\right)},$$
$$\forall (x,t) \in [0,L] \times [0,\infty). \tag{6.102}$$

■

Remark 6.8. *From the similar analysis of Remark 6.5, we can conclude that system state $w(x,t)$ with the proposed robust adaptive boundary control can be made arbitrarily small by choosing control gain k in Eq. (6.79) appropriately.*

Remark 6.9. *From Eq. (6.100), we can obtain the parameter estimate error $\tilde{\Phi}$ is bounded $\forall t \in [0,\infty)$. Using the derivation similar to those employed in Remark 6.6, we can state the proposed control Eq. (6.79) ensures all internal system signals including $w(x,t)$, $w'(x,t)$, $\dot{w}(x,t)$, $\dot{w}'(x,t)$, $\ddot{w}(x,t)$, $w'''(x,t)$, $\dot{w}'''(x,t)$ and $w''''(x,t)$ are* *uniformly bounded. Since* $\hat{\Phi}$, $w'(x,t)$, $\dot{w}(x,t)$, $w'''(x,t)$ *and* $\dot{w}'''(x,t)$ *are all bounded* $\forall(x,t) \in [0,L]\times[0,\infty)$, *and we can conclude the boundary adaptive control Eq.* (6.79) *is also bounded* $\forall t \in [0,\infty)$.

Remark 6.10. *For the system dynamics described by Eq.* (6.7) *and boundary conditions* (6.8) *to* (6.11)*, if there is no distributed disturbance for the riser system, i.e.,* $f(x,t) = 0$*, the boundedness stability can be achieved with the proposed boundary control* (6.79) *as follows:*

$$|w(x,t)| \leq \sqrt{\frac{2L}{\beta T \lambda_{1a} k_2}\left(V_a(0)e^{-\lambda_a t} + \frac{r||\Phi||^2}{2\lambda_a}\right)}, \ \forall(x,t) \in [0,L] \times [0,\infty) \quad (6.103)$$

6.4 Numerical Simulations

Simulations for a riser of length 1000m under the ocean current disturbance are carried out to demonstrate the effectiveness of the proposed boundary control Eq. (6.12) and Eq. (6.79).

The riser, initially at rest, is excited by a distributed transverse disturbance due to the ocean current. The corresponding initial conditions of the riser system are given as

$$w(x,0) = 0, \quad (6.104)$$

$$\dot{w}(x,0) = 0. \quad (6.105)$$

The system parameters are given in Table 1.

Table 1: parameters of the riser system

Parameter	Description	Value
L	Riser Length	$1000.00m$
D	Riser external diameter	$152.40mm$
EI	Riser stiffness	$1.5 \times 10^7 Nm^2$
M_s	Vessel mass	$9.60 \times 10^6 kg$
d_s	Vessel damping	$1 \times 10^3 NS/m$
T	Riser tension	$8.11 \times 10^7 N$
ρ	Riser mass per unit	$500.00kg/m$
ρ_s	Sea water density	$1024.00kg/m^3$
c	Riser damping	$2.00NS/m^2$

In the simulation, the ocean surface current velocity $U(t)$ is generated by Eq. (3.131).The full current load is applied from $x = 1000m$ to $x = 0m$ and thereafter linearly decline to zero at the ocean floor, $x = 0$, to obtain a depth dependent ocean current profile $U(x, t)$ as in Chapter 3. The distributed load $f(x, t)$ is generated by Eq. (2.5) with $C_D = 1$, $\theta = 0$, $S_t = 0.2$ and $f_v = 2.625$. The disturbance $d(t)$ on the vessel generated by the following equation is shown in Fig. 6.2.

$$d(t) = [3 + 0.8 \sin(0.7t) + 0.2 \sin(0.5t) + 0.2 \sin(0.9t)] \times 10^6. \qquad (6.106)$$

Displacement of the riser system for free vibration, i.e., $u(t) = 0$, under the ocean disturbance is shown in Fig. 6.3. Displacement of the riser system with exact model-based control Eq. (5.12), by choosing $k = 1 \times 10^7$, under the ocean disturbance is shown in Fig. 6.4. When the system parameters EI, T, d_s and M_s are unknown, displacement of the riser system with adaptive control Eq. (6.79), by choosing $k = 1 \times 10^7$, $r = 0.0001$ and $\Gamma = \mathrm{diag}\{1, 1, 1, 1\}$, under the ocean disturbance is shown in Fig. 6.5. Figs. 6.4 and 6.5 illustrate that the proposed boundary control (6.12) and

(6.79) are able to stabilize the riser at the small neighborhood of zero by appropriately choosing design parameters. The corresponding boundary control input for the exact model-based control and the adaptive control are shown in Fig. 6.6. Both two control inputs vary between 0 and 5×10^4N, which are implementable in practice.

6.5 Conclusion

Vibration suppression for a flexible marine riser system subjected to the ocean current disturbance has been presented in this chapter. Two cases have been investigated: (i) exact model-based control, and (ii) robust adaptive control for the system parametric uncertainty. Robust boundary control has been proposed based on the exact model of the riser system, and adaptive control has been designed to compensate the system parametric uncertainty. With the proposed control, closed-looped stability under the external disturbance has been proven by using the Lyapunov's direct method. The proposed control is designed based on the original infinite dimensional model (PDE), and the spillover instability phenomenon is eliminated. The control is implementable since all the required signals in the control can be measured by sensors or obtained by a backward difference algorithm. Numerical simulations have been provided to illustrate the effectiveness of the proposed boundary control.

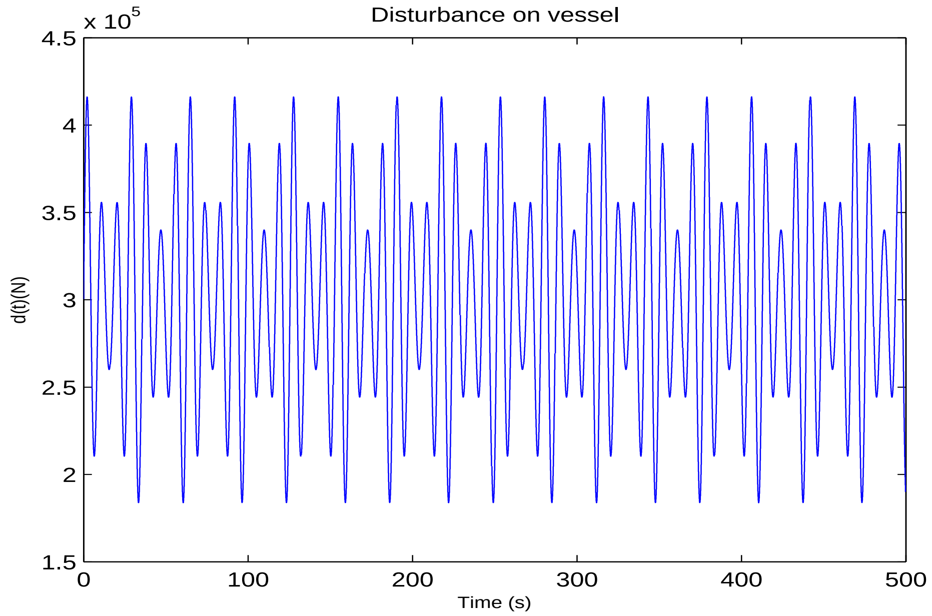


Fig. 6.2: Disturbance on the vessel $d(t)$.

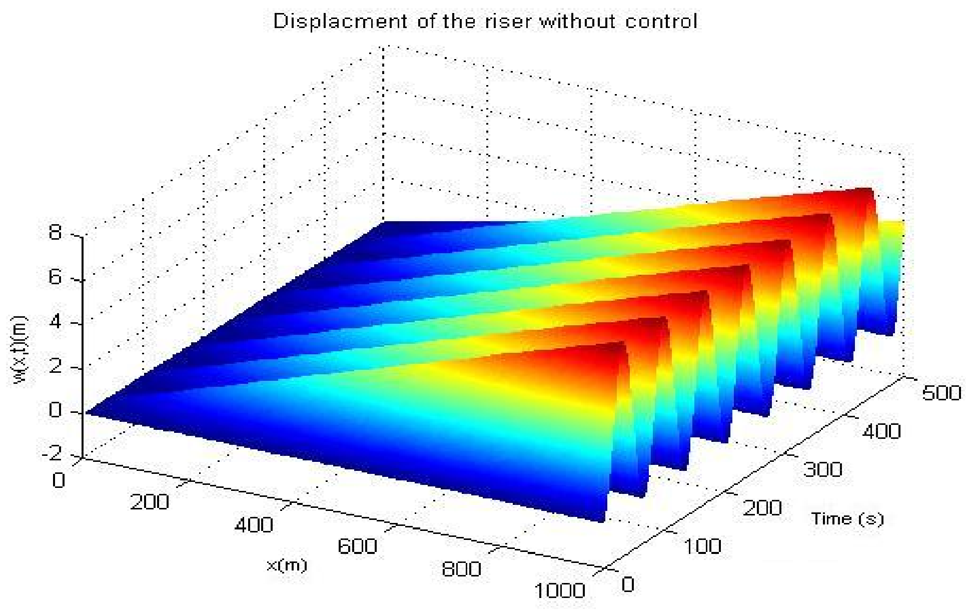


Fig. 6.3: Displacement of the riser without control.

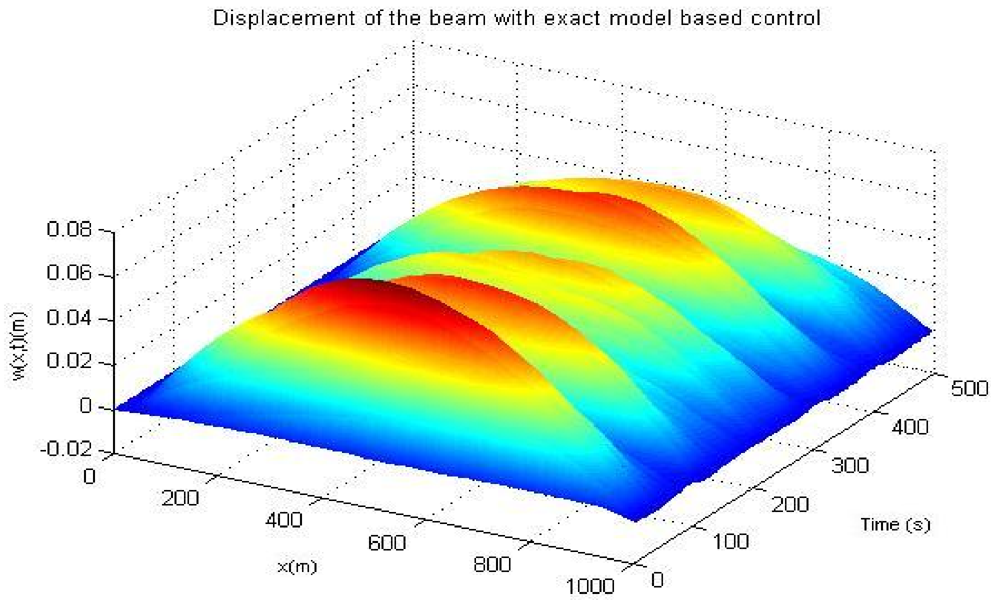


Fig. 6.4: Displacement of the riser with exact model-based control.

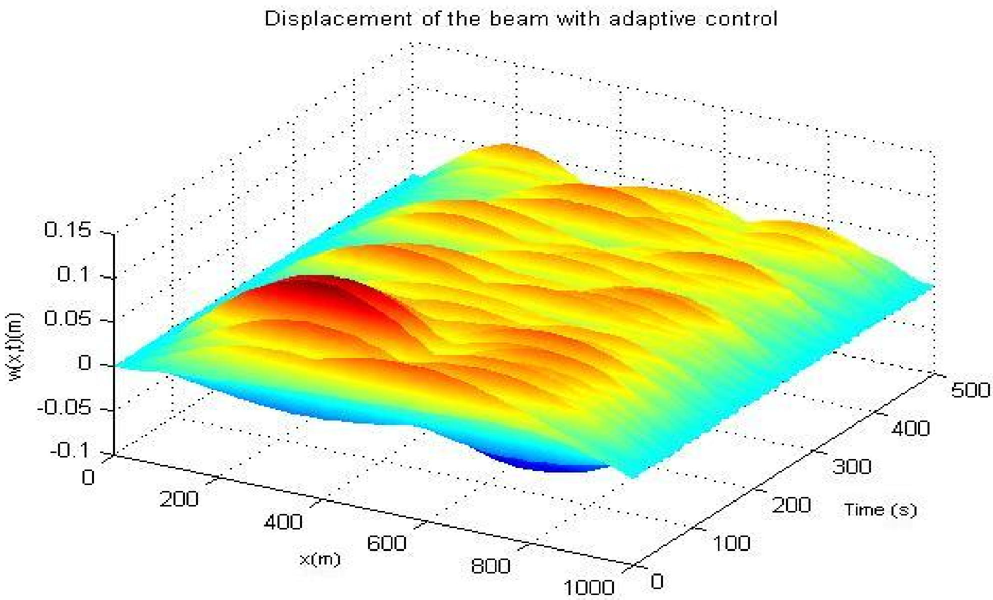


Fig. 6.5: Displacement of the riser with adaptive control.

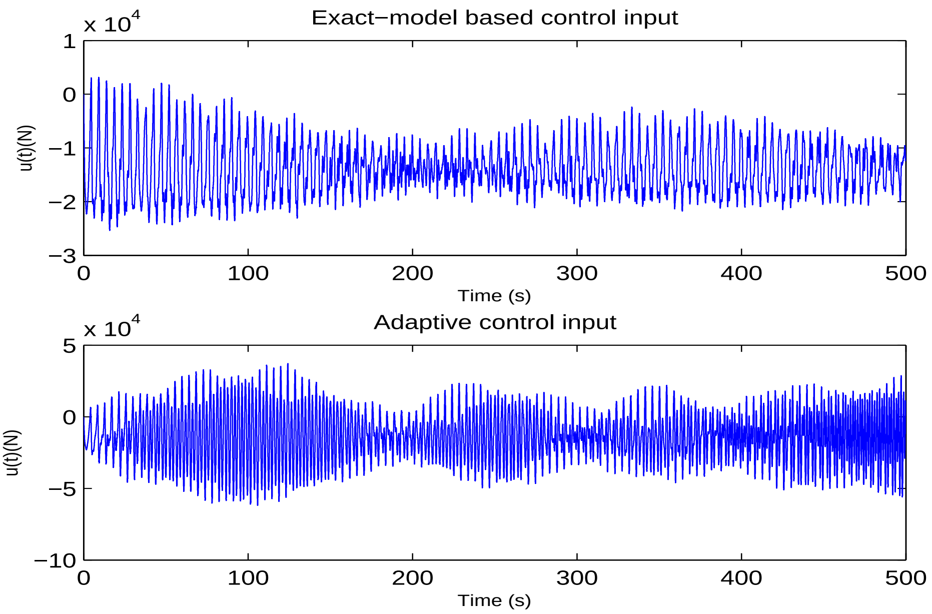


Fig. 6.6: Control input $u(t)$.

Chapter 7

Conclusions

7.1 Conclusions

The thesis has been dedicated to the modeling and control design of the marine flexible systems subjected to the environmental disturbances. In this chapter, the results of the research work conducted in this thesis are summarized and the contributions made are reviewed. Suggestions for future work are also presented. The key results are as follows:

- **Mooring System**

  We have studied the modeling and control design for a thruster assisted position mooring system with arbitrary mooring lines. The mathematical model of the mooring system has been derived by using the Hamilton's principle. For this PDE model, both exact model based boundary control and adaptive boundary control have been proposed based on the Lyapunov's direct method. With proposed control, all the signals of the closed-loop system are proved to be uniformly bounded despite the presence of unknown system parameters. The proposed control strategy only requires measurements of the boundary displacement and slope of the mooring line and the time derivatives of these quantities. The proposed boundary control has provided a good control performance for the thruster assisted position mooring system with unknown environmental disturbances. The main contributions include: (i) the dynamic model of a thruster assisted position mooring system with arbitrary mooring lines has been derived; and (ii) robust adaptive boundary control at the top boundary of the mooring lines has been developed for station keeping of the vessel.

- **Marine Installation System**

  Both position control and vibration suppression have been considered for a flexible marine installation system. Two cases for the flexible marine installation system are studied: (i) exact model-based control, and (ii) adaptive control for the system parametric uncertainty. For the first case, a boundary controller is introduced for the exact model of the installation system. For second case where the system parameters cannot be directly measured, to fully compensate for the effect of unknown system parameters, a signum term and an auxiliary signal term are introduced to develop a robust adaptive boundary control law. Both two types of boundary control are designed based on the original infinite dimensional model (PDE), and thus the spillover instability phenomenon is eliminated. All the signals of the closed-loop system are proved to be uniformly bounded by using the Lyapunov's direct method. The proposed schemes offer implementable design procedures for the control of marine installation systems since all the signals in the control can be measured by sensors or calculated by a backward difference algorithm. The main contributions include: (i) the mathematical model of the marine installation system has been described as a nonhomogeneous hyperbolic PDE; and (ii) two implementable boundary controllers at the top and bottom boundary of the cable have been designed to position the subsea payload to the desired set-point and suppress the cable's vibration.

- **Flexible Marine Riser**

  We have studied the vibration problems of a coupled nonlinear marine flexible riser subjected to the ocean disturbances. The riser system is modeled as a nonlinear PDE system via the Hamilton's principle. The difficulty of the control of the nonlinear PDE system lies in the couplings between the transverse and longitudinal vibrations. To overcome this difficulty, we have developed the boundary control with two actuators in transverse and longitudinal directions based on the distributed parameter system model, and the problems associated with traditional truncated-model-based design are overcome. With the proposed control, uniform boundedness under the ocean current disturbances and exponential stability under free vibration condition have been theoretically proved based on the Lyapunov's direct method. The control is easy to implement since they are independent of the system parameters and only two sensors and actuators are required. The main contributions are: (i) the coupled nonlinear dynamic model of the marine flexible riser for transverse and longitudinal vibrations reduction has been formulated; and (ii) the implementable boundary control with two actuators in transverse and longitudinal directions has been designed to reduce both transverse and longitudinal vibrations of the marine flexible riser.

- **Flexible Marine Riser with Vessel Dynamics**

Robust adaptive boundary control for a flexible marine riser with vessel dynamics has been designed to suppress the riser's vibration. To provide an accurate and concise representation of the riser's dynamic behavior, the flexible marine riser with vessel dynamics is described by a distributed parameter system with a partial differential equation (PDE) and four ordinary differential equations (ODEs). Two cases have been investigated: (i) exact model-based control, and (ii) robust adaptive control for the system parametric uncertainty. Robust boundary control has been proposed based on the exact model of the riser system, and adaptive control has been designed to compensate the system parametric uncertainty. With the proposed control, closed-looped stability under the external disturbances has been proven by using the Lyapunov's direct method. The state of the system is proven to converge to a small neighborhood of zero by appropriately choosing design parameters. The main contributions are: (i) the model of the marine flexible riser with vessel dynamics has been formulated; and (ii) robust adaptive boundary control at the top boundary of the riser has been developed to suppress the riser's vibration.

7.2 Recommendations for Future Research

In this section, some research topics are proposed for future investigation:

- **Experiments for the proposed control**

  In this thesis, we focused on the vibration problems of the marine flexible systems, and numerical simulations are extensively provided to illustrate the performance of the proposed control. One drawback of the current research is the lacks of the experimental results. There is no experiment for verifying the derived model and the proposal control design since the the experiments would involve a huge infrastructure investment in practice. Recent year, some researchers in Norwegian University of Science and Technology (NUNT) used the scale model to carry out the experiments in the ocean basin for the mooring system and riser system [6,109–112]. In these experiments, the feasibility of the proposed control has been well illustrated. In future, we plan to implement the proposed control strategies based on a scale vessel model in the ocean basin for demonstrating the control performance of the controllers.

- **Control of vibrations in three three-dimensional space**

  In this thesis, we focused on a specific system model in the vertical plane, and only transverse vibration is considered and controlled in the above control designs. In practice, all the marine flexible systems are located in the Earth-frame, which is a three-dimensional space including X, Y and Z axis. In the three-dimensional space, there are strong couplings between motions of a flexible marine system along the X, Y and Z axis. Due to the coupled effects, the modeling and control design for the marine flexible systems in the three-dimensional space is not a straightforward extension. These couplings make control a flexible marine system in three-dimensional space more difficulties than the one studied in this thesis. Therefore, the control problem of a flexible marine system that deforms in three-dimensional space is an interesting and challenging topic. For example, boundary control of a three-dimensional flexible marine riser has been investigated in [73]. More investigations are needed to explore the characteristics of such three-dimensional models with the available control techniques to mitigate the effects of couplings while satisfying the basic requirements for the concerned system.

- **Control of flexible systems with the time-varying distributed disturbance $f(x,t)$**

  In our control design, there is no term introduced to cope with the effect of the time-varying distributed disturbance $f(x,t)$. When the upper bound of the time-varying distributed disturbance is large, the control performance will be affected. For example, in the mooring system, H_1 and H_2 can receive a large value when the upper bound of the time-varying distributed disturbance increases. Even though the size of H_1 and H_2 can be reduced by choosing the control gains k_p, k_v and k_s appropriately, it may bring a high gain control scheme. Observer for the time-varying distributed disturbance may be regarded as one solution for such problem. State observers for distributed parameter systems has been investigated in [102, 103, 142–148], which could be used to deal with the the time-varying distributed disturbance $f(x,t)$. In the marine environment, the observer and control design is more challenging due to the complicated flexible system models coupled with the vessel's motion. Boundary control of flexible systems is currently an active research area, and how to take into account the time-varying distributed disturbance in the control design becomes an important and challenging problem.

- **Control of longitudinal vibrations and tension**

  During the lowering operation on long lines, there can be very significant dynamic effects on the lift cable and load. The excitation caused by the motions of the surface vessel can be amplified with large oscillations and high dynamic tensile loads in the lifting line which may result in breaking of the lifting cable. Considering the ship motions, control of longitudinal vibrations and tension to reduce the high dynamic tensile loads is desirable in ocean engineering. Due to the coupled effects, the control design and the direct proof for the Lyapunov stability is quite difficult. Boundary control on axially moving systems has been studied in [39, 63, 66, 76, 149], which may inspire the control design for the longitudinal vibrations of the flexible systems. In the marine environment, the control longitudinal vibrations and tension is challenging due to the unpredictable ocean disturbances such as fluctuating currents and transmission of motions from the surface vessel through the lift cable.

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Author's Publications

The contents of this thesis are based on the following papers that have been published, accepted, or submitted to the peer-reviewed journals and conferences.

Journal papers:

1. S. S. Ge, W. He, B. V. E. How, and Y. S. Choo, "Boundary Control of a Coupled Nonlinear Flexible Marine Riser," *IEEE Transactions on Control Systems Technology*, vol. 18, no. 5, pp. 1080-1091, 2010.

2. W. He, S. S. Ge, B. V. E. How, Y. S. Choo, and K.-S. Hong, "Robust Adaptive Boundary Control of a Flexible Marine Riser with Vessel Dynamics," *Automatica*, vol. 47, no. 4, pp. 722-732, 2011.

3. S. S. Ge, W. He, and S. Zhang, "Dynamic Modeling and Control Design for a Multi-cable Mooring System," *IEEE Transactions on Control Systems Technology*, under review, 2011.

4. W. He, S. S. Ge, and S. Zhang, "Adaptive Boundary Control of a Flexible Marine Installation System," *Automatica*, Accepted, 2011.

5. S. S. Ge, S. Zhang and W. He, "Vibration Control of an Euler-Bernoulli Beam under Unknown Spatiotemporally Varying Disturbance," *International Journal of Control*, vol. 84, no. 5, pp. 947-960, 2011.

Conference papers:

1. W. He, B. V. E. How, S. S. Ge, and Y. S. Choo, "Boundary Control of a Flexible Marine Riser with Vessel Dynamics", Proceedings of the 2010 American Control Conference, pp. 1532-1537, Baltimore, MD, USA, June 30-July 02, 2010.

2. S. S. Ge, W. He, B. Ren, and Y. S. Choo, "Boundary Control of a Flexible Marine Installation System", Proceedings of the 49th IEEE Conference on Decision and Control, pp. 2590-2595, Atlanta, GA, USA, December 15-17, 2010.

3. W. He, S. S. Ge, C. C. Hang, and K.-S. Hong, "Boundary Control of a Vibrating String under Unknown Time-varying Disturbance", Proceedings of the 49th IEEE Conference on Decision and Control, pp. 2584-2589, Atlanta, GA, USA, December 15-17, 2010.

4. S. S. Ge, S. Zhang, and W. He, "Modeling and Control of a Vibrating Beam under Unknown Spatiotemporally Varying Disturbance", Proceedings of the 2011 American Control Conference, pp. 2988-2993, San Francisco, CA, USA, June 29 - July 01, 2011.

5. S. Zhang, S. S. Ge, and W. He, "Modeling and Control of a Nonuniform Vibrating String under Spatiotemporally Varying Tension and Disturbance", the 2011 IFAC World Congress, Accepted, 2011.