MODELLING LONG MEMORY IN EXCHANGE RATE VOLATILITY

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NATIONAL UNIVERSITY OF SINGAPORE
2003

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A THESIS SUBMITTED

FOR THE DEGREE OF MASTER OF SOCIAL SCIENCES

DEPARTMENT OF ECONOMICS

NATIONAL UNIVERSITY OF SINGAPORE

2003

ACKNOWLEDGEMENTS

I wish to thank the following people:

- My supervisor/advisor, Associate Professor Albert Tsui;
- Professor Parkash Chander;
- Associate Professor Peter Wilson;
- Assistant Professor Mark Donoghue;
- Assistant Professor Lee Jin;
- Professor John Dalle Molle;
- Professor James Ramsey, President of the Society for Nonlinear Dynamics and Econometrics (SNDE), Ulrich Mueller of Converium, and other participants of the 11th Annual Symposium of the SNDE;
- The administrative officers of the Dean's Office at the Faculty of Arts and Social Sciences;
- My parents.

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SUMMARY

Understanding exchange rate dynamics has been an important research topic in both finance and economics. For example, international capital asset pricing models often require specific assumptions of exchange rate dynamics; whereas in economics, there is a need to link exchange rate behaviour to changes in key macroeconomic variables in order to establish a framework to assess government policies. Among others, several empirical regularities in exchange rate dynamics are observed by such researchers as Hsieh (1989a, 1989b, and 1993), Tse and Tsui (1997), and Tse (1998). They include: [1] exchange rate changes may not be IID; [2] there is little serial correlation in the exchange rate return series; [3] exchange rate changes indicate volatility clustering and leptokurtosis; [4] asymmetric effects in exchange rate volatility may not be present; [5] exchange rate changes may exhibit significant persistence and dependence between distant observations, which is conveniently described as “long range dependence” or “long memory”. Most studies that highlight such empirical regularities are based on the univariate version of Engle’s (1982) autoregressive conditional heteroskedasticity (ARCH) and Bollerslev’s (1986) generalised ARCH (GARCH) models and their extensions. These studies generally find that the class of univariate ARCH and GARCH models is capable of characterising the non-linear dynamics in exchange rates.

One major drawback of the univariate GARCH framework is that it does not capture the co-movements of several time series variables, which may be

influenced by the same set of events. Hence, a natural extension is to consider the multivariate GARCH set-up. Several works on modelling exchange rate volatility in multivariate contexts include Diebold and Nerlove (1989), Bollerslev (1990), Engle and Gau (1997), and Engle (2000), among others. However, the multivariate GARCH approach inevitably increases the number of parameters to be estimated and complicates the specification of the variance-covariance matrix. More specifically, it can be difficult to ensure that the variance-covariance matrix is positive-definite, let alone imposing it during estimation. To circumvent these problems, Bollerslev (1990) proposes the constant correlations (CC)-GARCH model. Although this model is relatively tractable, its validity has been questioned in certain contexts (Tsui and Yu (1999), Tse (2000), Engle and Sheppard (2001), Engle (2002), and Tse and Tsui (2002)). Indeed, there is some empirical evidence suggesting that exchange rate correlations may be significantly time-varying (Engle (2002), and Tse and Tsui (2002)). As such, it is more apposite to consider multivariate models that include time-varying correlations.

Due to the computational complexities involved, there are only a few studies on modelling long-memory in volatility using the multivariate GARCH framework. They include Teyssiere (1997, 1998), and Brunetti and Gilbert (2000). These studies have mainly applied the multivariate version of the fractionally integrated GARCH (FIGARCH) model of Baillie, Bollerslev, and Mikkelsen (1996) to stock market and exchange rate data. However, the issue of asymmetric conditional volatility has been excluded. Furthermore, such studies mostly assume the bivariate constant conditional correlation structure (Bollerslev (1990)) for convenience. Although Teyssiere (1998) also considers a trivariate unrestricted FIGARCH model based on the vech structure, his model requires implementing a set of conditions to ensure that the variance-covariance matrix is positive-definite. In this paper, we propose a family of multivariate (bivariate, trivariate, and tetravariate) asymmetric GARCH models to analyse the volatility dynamics of exchange rates. The proposed models can capture the stylised features of long-memory, persistence, asymmetric conditional volatility, and time-varying correlations commonly found in financial time series data. Furthermore, our model automatically satisfies the positive-definite condition when convergence is achieved. A total of 26 different models (see Appendix I) are applied to four currencies: the Canadian dollar, the British pound, the Singapore dollar, and the Malaysian ringgit. We want to point out that previous studies on exchange rate behaviour have predominantly concentrated on the bilateral exchange rate against the US dollar. Our approach departs from this widely accepted convention. Instead, we also choose the Japanese yen as an alternative numeraire currency to the US dollar. It is interesting to investigate the dynamics of exchange rates against this alternative as it has important implications for asset allocation, such as the construction of a diversified Asian asset portfolio.

Our main findings are as follows. First, evidence of long-memory and persistence in volatility is detected in the individual exchange rate return series, regardless of the choice of the numeraire currency. Furthermore, some exchange rates, such as the JPY rates, apparently share the same degree of long memory in their conditional variances. In addition, by comparing the log-likelihood values, the multivariate fractionally integrated models generally outperform those models without long-memory structures in the conditional variance.

Second, consistent with previous studies, such as Hsieh (1993), and Tse and Tsui (1997), the Canadian dollar, the British pound, and the Singapore dollar vis-à-vis the US dollar do not exhibit asymmetric effects in the conditional volatility. In contrast, we detect reasonably stronger evidence of asymmetric conditional volatility for these currencies when the numeraire currency is the Japanese yen. Additionally, the depreciation shocks of the Malaysian ringgit have a greater impact on future volatilities compared with the appreciation shocks of the same magnitude, and this result is robust to the choice of the numeraire currency. Furthermore, the significance and the magnitude of asymmetric effects can vary according to how the conditional variance equation is specified.

Third, we find new evidence of time-varying correlations among the currencies when they are measured against the Japanese yen. For example, similar to results in Tse (2000), we do not find strong evidence of time-varying correlations between the Singapore dollar and the Malaysian ringgit when the numeraire currency is the US dollar. However, time-varying correlations between these currencies are detected when the Japanese yen is used.

CHAPTER 1

INTRODUCTION: MODELLING LONG MEMORY PROCESSES

1.1 An Overview of Long Memory

The phenomenon of long memory (or long range dependence) has been known long before suitable statistical models are developed to characterise it. Since ancient times, Egyptian hydrologists who studied the flows of the Nile River had noted that long periods of dryness were followed by long periods of yearly returning floods. This behaviour was described in the Bible (Genesis 41, 29-30): “Seven years of great abundance are coming throughout the land of Egypt, but seven years of famine will follow them.” Mandelbrot called this behaviour the “Joseph effect” (Mandelbrot 1977, 1983). See also Hurst (1951, 1956), Lawrance and Kottegoda (1977), and McLeod and Hipel (1978). Indeed, the origin of interest in long memory processes appears to have come from the examination of data in the physical sciences such as hydrology and climatology and preceded interest from econometricians.

The presence of long memory can be formally analysed from an empirical, data-oriented approach in terms of the persistence of observed autocorrelations. The degree of persistence is consistent with an essentially stationary process; however, the autocorrelations take far longer to decay than the exponential rate associated with Box and Jenkin’s (1970) autoregressive-moving average (ARMA) class of models. When the phenomenon of long memory is viewed as the time-series realisation of a stochastic process, the autocorrelation function exhibits persistence that is neither consistent with an I(1) process nor an I(0) process. Indeed, there is no conceptual reason for restricting attention to exponential rates of decay; slower rates of decay, such as hyperbolic decay, are worth exploring.

There is considerable evidence on the success of applying long memory models to time series data in the physical sciences, and rather less to macroeconomics where it often seems difficult to distinguish I(d) processes from I(1) behaviour. However, there is substantial evidence that long memory processes describe financial data, such as exchange rates, interest rates, and stock market indices, rather adequately. Perhaps the most dramatic empirical success of long memory processes has been in the volatility of asset prices and power transformations of returns. See, amongst others, Baillie, Bollerslev, and Mikkelsen (1996), and Tse (1998).

1.2 Definitions and Theoretical Models of Long Memory

There are several possible definitions of the property of long memory. According to McLeod and Hipel (1978), given a discrete time series process y_t, with autocorrelation function at lag j, the process possesses long memory if the autocorrelations are characterised by the following property:

$$\lim_{n\to\infty}\sum_{j=-n}^{n}|\rho_j| = \infty \tag{1.1}$$

Alternatively, according to Granger and Ding (1996), a series is said to have long memory if it displays a slowly declining autocorrelation function (ACF) and an infinite spectrum at zero frequency. Put differently, the spectral density $f(\omega)$ will be unbounded at low frequencies. A stationary and invertible ARMA process has autocorrelations which are geometrically bounded, that is, $|\rho(k)| \le cm^{-k}$, for large k, where 0 < m < 1 and is hence a short memory process. On the other hand, the series y_t is a stationary long memory process if the ACF, $\rho(k)$ behaves as,

$\rho(k) \approx c\,|k|^{2d-1}$, as $|k| \to \infty$ (1.2)

where 0 < d < 0.5 and c is some positive constant. The ACF in (1.2) manifests a very slow rate of decay to zero as k goes to infinity and $\sum_{-\infty}^{\infty} |\rho(k)| = \infty$. For details on additional definitions, see Beran (1994), Baillie (1996), and Maheu (2002).

An example of long memory is the fractionally integrated white noise process, I(d), with 0 < d < 1:

$$(1-L)^d y_t = \varepsilon_t \qquad (1.3)$$

where L is the lag operator, and ε_t is a stationary and ergodic process with a bounded and positively valued spectrum at all frequencies. The fractional difference operator $(1-L)^d$ is well-defined for a fractional d and the ACF of this process exhibits a hyperbolic rate of decay consistent with (1.1). A model that incorporates the fractional difference operator is one natural starting point of characterising long-memory, and it provides the motivation for the autoregressive fractionally-integrated moving-average (ARFIMA) class of models. In particular, the process is weakly stationary for d < 0.5 and is invertible for d > -0.5. The infinite-order AR(∞) representation of (1.3) is given by

$$y_t = \sum_{k=0}^{\infty} \xi_k y_{t-k} + \varepsilon_t \qquad (1.4)$$

where the weights ξ_k are obtained from the MaClaurin series expansion,

$$(1-L)^d = 1 - dL + \frac{d(d-1)L^2}{2!} - \frac{d(d-1)(d-2)L^3}{3!} + \ldots \qquad (1.5)$$

for any real d > -1. The expansion can also be represented in terms of the hypergeometric function:

$$(1-L)^d = \sum_{b=0}^{\infty} \frac{\Gamma(k-d)L^b}{\Gamma(k+1)\Gamma(-d)} = F(-d,1,1;L) \qquad (1.6)$$

for d > 0, and where $F(a,b;c;z)$ is the hypergeometric function defined as

$$F(a;b;c;z)=\frac{\Gamma(c)}{\Gamma(a)\Gamma(b)}\times\sum_{i=1}^{\infty}\frac{z^{i}\Gamma(a+i)\Gamma(b+i)}{\Gamma(c+i)\Gamma(i+1)} \quad (1.7)$$

The typical AR coefficient at lag k, given by ξ_k can be expressed as follows:

$$\xi_k=\frac{d(d-1)(d-2)...(d-k+1)(-1)^k}{k!}=\frac{(-d)(1-d)(2-d)...(k-1-d)}{k!} \quad (1.8)$$

Since the gamma function is given by

$$\Gamma(k-d)=\{(k-d-1)(k-d-2)...(2-d)(1-d)(-d)\}\Gamma(-d)\text{ ,} \quad (1.9)$$

we can re-express the infinite AR representation coefficients as

$$\xi_k=\frac{\Gamma(k-d)}{\Gamma(-d)\Gamma(k+1)} \quad (1.10)$$

Likewise, we may express the fractional white noise process in (1.3) in terms of an infinite-order MA(∞) representation:

$$y_t=(1-L)^{-d}\varepsilon_t=\sum_{k=0}^{\infty}\vartheta_k\varepsilon_{t-k} \quad (1.11)$$

where the weights ϑ_k are given by

$$(1-L)^{-d}=1+dL+\frac{d(d+1)L^2}{2!}+\frac{d(d+1)(d+2)L^3}{3!}+... \quad (1.12)$$

Since

$$\Gamma(k+d)=\frac{(k+d-1)(k+d-2)...(2+d)(1+d)(d)}{\Gamma(d)} \quad (1.13)$$

it follows that the weights can be written as

$$\vartheta_k=\frac{\Gamma(k+d)}{\Gamma(d)\Gamma(k+1)} \quad (1.14)$$

Brockwell and Davis (1987) demonstrate that y_t is convergent in mean square through its spectral representation. Furthermore, as $\sum_{j=0}^{\infty}\vartheta_j^2<\infty$, the fractional white noise process is mean square summable and stationary for d < 0.5. When d = 0.5, the ARFIMA(0,0.5,0) process is non-stationary, since $\vartheta_k \approx k^{-1/2}$; therefore, $\sum_{j=0}^{\infty} \vartheta_j^2$ just fails to converge.

To construct a class of long memory process that is more flexible than that given in (1.3), we apply the fractional differencing operator $(1-L)^d$ to the ARMA(p,q) model (see Granger and Joyeux (1980), Granger (1980, 1981), and Hosking (1981)):

$$\Lambda(L)(1-L)^d y_t = \Theta(L)\varepsilon_t \tag{1.15}$$

where d is the fractional differencing parameter, the characteristic roots of the lag polynomials $\Lambda(L)$ and $\Theta(L)$ lie outside the unit circle, and ε_t is white noise. This is the ARFIMA(p,d,q) process. The y_t process in (1.15) is then said to be I(d), for $d \neq 0$. Similar to the process in (1.3), the autocorrelation coefficients of this process will display a slow rate of hyperbolic decay. In particular, for -0.5 < d < 0.5, the process is covariance stationary, while d < 1 implies mean reversion. For an I(d) process, the spectral density is such that $f(0) = 0$ for d < 0 and $f(0) = \infty$ for d > 0. For small frequencies, ω, an approximation for d > 0 is given by $f(\omega) \approx \omega^{-2d}$, while the process has infinite variance for d > 0.5 (and hence is non-stationary). For a more detailed exposition of the statistical properties of the ARFIMA(p,d,q) model, see Sowell (1986, 1992), and Chung (1994).

1.3 Empirical Applications of Long Memory Models

We turn to the empirical applications of long memory models in macroeconomics and finance. In macroeconomics, long memory models have contributed to the debate as to whether aggregate output (real GNP) is difference-stationary or trend-stationary. For instance, Diebold and Rudebusch (1989) apply the Geweke-Porter-Hudak (GPH) (Geweke and Porter-Hudak (1983)) approach to post-war US real GNP data, and find that the fractional differencing parameter d has an estimated value and asymptotic standard error of d = -0.50 (0.27). On the other hand, Sowell (1992) estimates an ARFIMA(3,d,2) model of the first-differenced US real GNP from 1947Q1 through 1989Q4 and obtains a value of d = -0.59 (0.35). Other than aggregate output, price series, particularly over long periods of history, also frequently appear to possess persistence and long memory. Baillie (1996) applies the ARFIMA model to the Wheat Price Index, and finds that d is in the approximate range of (0.40, 0.60). See also Baillie, Chung, and Tieslau (1995), who apply the ARFIMA(0,d,12)-GARCH(1,1) models to the inflation rates series of nine industrialised countries.

The phenomenon of long memory in financial time series is also investigated by several researchers. For instance, Lo (1991) uses the modified rescaled range statistic on returns from value and equal weighted Centre for Research on Security Prices (CRSP) indices from July 1962 to December 1987. Lo (1991), in particular, detects significant results from using the regular rescaled range statistic; however, insignificant results emerge when he employs the modified rescaled range statistic. He ascribes this disparity in the test results to the short-term persistence within the returns series. Additionally, he notes that there is a lack of long-range persistence in the annual returns from 1872-1986.

Other than stock market data, long memory models are also used to characterise the behaviour of exchange rate series. Baillie and Bollerslev (1994a), in their study of seven nominal spot exchange rates, note that the cointegrating relationship among these exchange rates possesses long memory and may possibly be well described as a fractionally integrated process. In other words, these exchange rates are fractionally cointegrated. Using the time domain approximate maximum likelihood estimator, they estimate a simple fractional white noise model and find that the fractional differencing parameter d = 0.89 with an asymptotic standard error of 0.02. On the other hand, Diebold, Husted, and Rush (1991) use the approximate maximum likelihood estimation method of Fox and Taqqu (1986) to estimate ARFIMA models for annual real exchange rate data from the mid-19th century until the present time. Their findings corroborate the purchasing power parity (PPP) doctrine, with shocks reverting to the long-run equilibrium after a very long time. See also Cheung and Lai (1993), who test for fractional cointegration between relative prices and the nominal exchange rates for annual data from 1914 through 1972.

The aforementioned applications of long memory models to financial time series have focused largely on modelling the phenomenon in the conditional mean. However, this phenomenon has also been detected in the volatility of asset returns. In particular, the fractional differencing operator $(1-L)^d$ is applied to the conditional variance structure to capture long-memory dynamics. One prominent example would be the fractionally integrated generalised autoregressive conditional heteroskedasticity (FIGARCH) model of Baillie and Bollerslev (1996a). See also Harvey (2002), who develops a long-memory stochastic volatility model. The issue of long memory in the conditional variance shall be the focus of the next chapter.

CHAPTER 2

LONG MEMORY IN VOLATILITY: A MULTIVARIATE ASYMMETRIC GARCH APPROACH

2.1 Multivariate GARCH Models

The success of Engle's (1982) Autoregressive Conditional Heteroskedastic (ARCH) and Bollerslev's (1986) Generalised ARCH (GARCH) models in characterising the volatility dynamics of economic variables in univariate applications has motivated researchers to extend these models to multivariate contexts. Since many economic variables are expected to react to similar information, it is anticipated that these variables have non-zero covariances conditional on the given information set (see Franses and Dijk (2000)). As such, a multivariate set-up is deemed more appropriate. In addition, by jointly estimating the conditional volatilities of two or more variables, it is possible to gain efficiency in parameter estimation (see Bera and Higgins (1992)). Bollerslev, Chou and Kroner (1992), Bera and Higgins (1993), and Bollerslev, Engle and Nelson (1994) have provided excellent surveys on the applications of multivariate GARCH models to equities, interest rates and exchange rates. See also Bollerslev (1990), and Kroner and Claessens (1991).

The detection and modelling of ARCH effects are important from both theoretical and practical points of view (see Hong and Lee (2001)). Neglecting ARCH effects may lead to loss in asymptotic efficiency of parameter estimation (see Engle (1982), and Wooldridge (2000)). Also, ignoring the presence of ARCH effects may cause over-rejection of conventional tests for serial correlation (see Milhoj (1985) and Diebold (1987)) and result in the identification of ARMA models that are overparameterised (Weiss (1984) and Mills (1991)). In general, as Patterson and Ashley (1999) have argued, a failure to recognise and deal with the presence of nonlinearities such as

ARCH in the generating mechanism of a time series can often lead to poorly behaved parameter estimates and to models which miss important serial dependencies altogether. This point is concurred by Franses and Dijk (2000), who show that financial time series in particular display typical non-linear features like ARCH, and linear time series models fitted to these series do not yield reliable forecasts. It is therefore important to model ARCH effects if they are detected.

The problem with multivariate generalisations of ARCH models is that they inevitably increase the number of parameters to be estimated and complicate the specifications of the conditional variance-covariance matrix. It could be difficult to verify the condition of positive-definiteness for the conditional variance-covariance matrix of an estimated multivariate GARCH model (MGARCH), not to mention impose this condition during the optimisation of the log-likelihood function. Details are discussed in Bera and Higgins (1993), Gourieroux (1997), Franses and Dijk (2000), and Tse (2000). To ensure the condition of positive-definiteness, Engle and Kroner (1995) recently proposed the BEKK model. One main disadvantage of this model is that the parameters cannot be easily interpreted, and their net effects on the future variances and co-variances are not readily apparent. The intuition associated with the GARCH equation is therefore undermined. In addition, difficulties in getting convergence when the BEKK model is used to estimate the conditional-variance structure of spot and future prices have been noted (see Lien et al (2001), and Tse and Tsui (2002)).

A more tractable alternative to the BEKK model is Bollerslev's (1990) constant conditional correlations (CC-MGARCH) model, which has proven to be very popular with empirical researchers. Under the assumption of constant correlations, the maximum likelihood estimate (MLE) of the correlation matrix is equal to the sample correlation matrix, which is always positive definite. It is apparent, however, that the assumption of a constant correlation coefficient is a strong one, and its validity in many contexts has been rejected in recent studies (see, for instance, Bera and Kim (2002), Tsui and Yu (1999), and Tse (2000)). The over-rejection may have implications on the robustness of the parameter estimates in the conditional variance equation.

To incorporate dynamic correlations in the MGARCH model and yet satisfy the positive-definite condition, Tse and Tsui (2002) have recently developed the Varying-Correlation-MGARCH (VC-MGARCH) model, which adopts the vech representation. Specifically, they assume a vech-diagonal structure in which each conditional-variance equation adheres to a univariate GARCH formulation whilst the conditional-correlation matrix is postulated to follow an autoregressive moving average (ARMA) type of analogue (see section 2.3). One main advantage of VC-MGARCH is that it retains the usual interpretation of the univariate GARCH equation, and yet satisfies the positive-definite condition. Moreover, estimation of the VC-MGARCH model is more manageable than that of the BEKK model. Another advantage of the VC-MGARCH model is that it nests the constant correlations model and therefore facilitates an indirect way of verifying the constant correlations hypothesis. According to Tse and Tsui (2002), who has applied the VC-MGARCH and BEKK models to the Singapore and Hong Kong stock markets and the Hang Seng sectoral indices, the VC-MGARCH compares favourably against the BEKK model.

However, there are two major drawbacks with Tse and Tsui’s (2002) varying correlations approach: the lack of asymmetric effects in the volatility process, and the failure to capture long-memory dynamics. First, Tse and Tsui (2002) have not explicitly modelled the possible existence of volatility asymmetry, whereby a negative return shock generated by negative news has greater impact on future volatilities compared with a positive shock of the same magnitude. Many empirical finance researchers, such as Nelson (1991), Ding, Engle, and Granger (1993), Tse and Tsui (1997), and Tse (1998) have noted this effect, which is now commonly called the “leverage effect” (see Black (1976) and Christie (1982)).

The examination of volatility asymmetry, however, has predominantly been conducted in univariate set-up; as noted by Franses and Dijk (2000), multivariate asymmetric GARCH models have only recently been considered. By and large, most of the multivariate asymmetric GARCH models conveniently assume constant conditional correlations. See Koutmos and Booth (1995), Ostermark and Hoglund (1997), Tse (1999), Ostermark (2001), Bhar (2001) and Reyes (2001); as such, the time history associated with dynamic conditional correlations is lost.

Other than the drawback of constant conditional correlations, most of the studies that apply multivariate GARCH models have generally ignored the issue of long memory persistence in the conditional volatility. In fact, this issue is predominantly examined in the univariate framework; the only few exceptions are Teyssiere (1997, 1998) and Brunneti and Gilbert (2000). These studies, however, have not incorporated asymmetric effects in the conditional variance structure. To overcome these drawbacks, this dissertation proposes a class of multivariate (bivariate, trivariate, tetravariate) asymmetric long-memory GARCH models that can capture the features of long memory, asymmetric volatility and time-varying conditional correlations. These models are explicated later. Before that, however, the long-memory property of volatility warrants further attention, as this issue has important implications for financial volatility modelling and risk management.

2.2 Long-Memory GARCH Models

The issue of long memory and persistence has recently attracted considerable attention in the modelling of the conditional variance. Many of the obvious examples of long memory processes have emerged in studies of financial time series, such as exchange rate data, and these will be described further in Chapter 3. Taylor (1986) was one of the first to notice an empirical regularity that the absolute values of stock returns tend to have very slowly decaying autocorrelations. See also Ding, Engle, and Granger (1993). Furthermore, shocks to the conditional variance process are extremely persistent. This observation has motivated the formulation of the Integrated GARCH, or IGARCH, class of models by Engle and Bollerslev (1986). One feature of this class of models is that the implied effect of a shock for the optimal forecast of the future conditional variance will be to make the corresponding cumulative impulse response weights tend to a nonzero constant, so that the forecasts will increase linearly with the forecast horizon. As noted by Baillie, Bollerslev, and Mikkelsen (1996), this implies that the pricing of risky securities, including long-term options and futures contracts, may show extreme dependence on the initial conditions. See also Lumsdaine (1995), for further applications of the IGARCH model.

However, this extreme degree of dependence seems to run counter to observed pricing behaviour. Furthermore, there are recent studies, such as Breidt, Crato, and de Lima (1998), Liu (2000), Bollerslev and Wright (2000), and Harvey (2002), which indicate the existence of long-memory in the autocorrelations of squared returns of assets. Motivated by this observation, Baillie, Bollerslev, and Mikkelsen (1996), and Bollerslev and Mikkelsen (1996) have proposed the fractionally integrated GARCH (FIGARCH) and exponential GARCH (FIEGARCH) models. These long-memory models are found to outperform both the stable GARCH and the IGARCH class of models.

Some researchers have ventured to provide economic justification for the use of long-memory volatility models, such as FIGARCH. Following the results of Granger (1980), Andersen and Bollerslev (1998) have suggested that long memory can arise from the cross-sectional aggregation of a large number of volatility components or news information arrival processes with different degrees of persistence. More specifically, by interpreting the volatility as a mixture of numerous heterogeneous short-run information arrivals, the observed volatility process may exhibit long-run dependence. As such, the long-memory characteristics constitute an intrinsic feature of the return generating process, rather than the manifestation of occasional structural shifts. This is borne out of the high-frequency Deutschemark-US exchange rate. That long memory is an intrinsic feature of the return generating process is further corroborated by Baillie, Cecen, and Han (2000), who conclude that the Deutschemark-US exchange rate returns are probably being generated by a self-similar process, as similar values of the long memory volatility parameter are obtained for different frequencies. On the whole, the FIGARCH model is found to be adequate for the Deutschemark-US exchange rate.

2.3 Econometric Methodology

We now turn to a detailed exposition of our multivariate asymmetric long-memory GARCH model that is capable of characterising the stylised features of asymmetries, long memory, and time-varying correlations. Let $y_t = (y_{1t}, y_{2t}, y_{3t} \ldots y_{kt})'$ be the k-variate vector of interest with time-varying variance-covariance matrix H_t, and let $\mu_{it}(\xi_i)$ be the arbitrary mean functions which depend on ξ_i, a column vector of parameters. A typical k-variate GARCH model may be specified as follows:

$$y_{it} = \mu_{it}(\xi_i) + \varepsilon_{it}, \qquad i = 1,2,3,...,k \tag{2.1}$$

where $(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, ..., \varepsilon_{kt})' \; | \Phi_{t-1} \sim MN(O, H_t)$ (2.2)

Note that Φ_{t-1} is the σ-algebra generated by all the available information up to time (t-1). The random disturbance terms ε_{it} and the conditional variance equations h_{iit} are modelled as follows:

$\varepsilon_{it} = \sqrt{h_{iit}} e_{it}$, where $e_{it} \sim N(0,1)$ (2.3)

$$h_{iit} = \eta_i + \alpha_i \varepsilon_{it-1}^2 + \beta_i h_{iit-1} \tag{2.4}$$

Equation (2.4) is Bollerslev's (1986) univariate GARCH(1,1) model. Denoting the ij-th element (i, j = 1, 2, 3…k) in H_t by h_{ijt}, the conditional correlation coefficients are given by $\rho_{ijt} = \frac{h_{ijt}}{\sqrt{h_{iit} h_{jjt}}}$. Tse and Tsui (2002) assume that the time-varying conditional correlation matrix $\Gamma_t = \{\rho_{ijt}\}$ is generated by the following recursion

$$\mathbf{\Gamma}_t = (1 - \pi_1 - \pi_2)\mathbf{\Gamma} + \pi_1 \mathbf{\Gamma}_{t-1} + \pi_2 \mathbf{\Psi}_{t-1} \tag{2.5}$$

where $\Gamma = \{\rho_{ij}\}$ is a (time-invariant) positive-definite correlation matrix, π_1 and π_2 are assumed to be nonnegative and sum up to less than 1, and Ψ_t is a function of standardised residuals. Denoting $\Psi_t = \{\psi_{ijt}\}$, the elements of Ψ_{t-1} are specified as

$$\psi_{ij,t-1} = \frac{\sum_{a=1}^{M} e_{i,t-a} e_{j,t-a}}{\sqrt{(\sum_{a=1}^{M} e_{i,t-a}^2)(\sum_{a=1}^{M} e_{j,t-a}^2)}} \tag{2.6}$$

where M is set equal to k. The conditional log likelihood function (ignoring constant term) of the vector of parameters $\theta = \left(\xi_1, \xi_2, ..., \xi_k, \eta_1, \eta_2, ..., \eta_k, \alpha_1, ..., \alpha_k, \beta_1, ..., \beta_k, \rho_{ij}, \pi_1, \pi_2\right)$ is specified as

$$l_t(\theta) = -\frac{1}{2}\log|H_t| - \frac{1}{2}(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, ..., \varepsilon_{kt}) H_t^{-1} (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, ..., \varepsilon_{kt})' \quad (2.7)$$

where the conditional variance-covariance matrix H_t can be defined as

$H_t = \{h_{ijt}\} = D_t \Gamma D_t$, $D_t = diag\{\sqrt{h_{iit}}\}$, and $\Gamma_t = \{\rho_{ijt}\}$

Consequently, the log likelihood can be rewritten as

$$l_t(\theta) = -\frac{1}{2}\log|D_t \Gamma_t D_t| - \frac{1}{2}(\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, ..., \varepsilon_{kt}) D_t^{-1} \Gamma_t^{-1} D_t^{-1} (\varepsilon_{1t}, \varepsilon_{2t}, \varepsilon_{3t}, ..., \varepsilon_{kt})' \quad (2.8)$$

where Γ_t is defined by the recursion in (2.5).

The foregoing equations (2.1) - (2.8) summarize the gist of the Varying-Correlations (VC-GARCH) model of Tse and Tsui (2002). In particular, when k = 2, it becomes a bivariate VC-GARCH model and the corresponding equations (2.5)-(2.7) are simplified as follows:

$$\rho_{12t} = (1 - \pi_1 - \pi_2)\rho_{12} + \pi_1 \rho_{12,t-1} + \pi_2 \psi_{12,t-1} \quad (2.5')$$

$$\psi_{12,t-1} = \frac{\sum_{a=1}^{2} e_{1,t-a} e_{2,t-a}}{\sqrt{(\sum_{a=1}^{2} e_{1,t-a}^2)(\sum_{a=1}^{2} e_{2,t-a}^2)}} \quad (2.6')$$

$$l(\theta) = -\frac{1}{2}\sum_{i=1}^{2} \log h_{iit} - \frac{1}{2}\log(1 - \rho_{12t}^2) - \frac{e_{1t}^2 + e_{2t}^2 - 2\rho_{12t} e_{1t} e_{2t}}{2(1 - \rho_{12t}^2)} \quad (2.7')$$

Note that VC-GARCH nests Bollerslev's (1990) Constant-Correlations (CC-GARCH) model when $\pi_1 = \pi_2 = 0$. As such, we can apply the likelihood ratio test to compare the performance of both models.

To incorporate asymmetric volatility in the VC-GARCH model, we have to modify the conditional variance equation in (2.4). In what follows we demonstrate that the modifications are less restrictive and nest several popular GARCH models in the literature. Details can be found in Ding, Engle, and Granger (1993), and Sentana (1995). In particular, we follow one modification advocated by Sentana, which is the quadratic GARCH(1,1) (QGARCH(1,1)) model:

$$h_{iit} = \eta_i + \gamma_i \varepsilon_{it-1} + \alpha_{it-1}\varepsilon_{it-1}^2 + \beta_i h_{iit-1} \tag{2.9}$$

where γ_i is the asymmetric coefficient. As highlighted by Sentana (1995), a piecewise-quadratic spline approximation to the unknown conditional variance function would encompass such a QGARCH specification as a trivial smooth example, as well as the models of Taylor/Schwert (1986/1989), Glosten, Jaganathan, and Runkle (1993), and Zakoian (1994). Hence, QGARCH may be regarded as a useful benchmark to assess the relative performance of these models.

Another modification to (2.4) is by adopting the asymmetric power ARCH(1,1) (APARCH (1,1)) model of Ding, Engle, and Granger (1993):

$$\begin{aligned} \varepsilon_{it} &= \sqrt{h_{iit}}\, e_{it}, \;\; e_{it} \sim N(0,1) \\ h_{iit}^{\delta/2} &= \eta_i + \alpha_i (|\varepsilon_{it-1}| - \gamma\varepsilon_{it-1})^{\delta} + \beta_i h_{iit-1}^{\delta/2} \end{aligned} \tag{2.10}$$

Note that when $\delta = 2$, equation (2.10) becomes the leveraged GARCH (LGARCH(1,1)) model, and it can be shown that this nests the GJR model of Glosten, Jaganathan and Runkle (1993). When $\delta = 1$, it becomes the Threshold GARCH(1,1) (TGARCH(1,1)) model, which incorporates an asymmetric version of the Taylor/Schwert (1986/1989) model and Zakoian's (1994) Threshold ARCH (TARCH) model. Moreover, as shown in Ding, Engle, and Granger (1993), when δ approaches 0, the logarithmic GARCH(1,1)

(LOGGARCH(1,1)) model is obtained, and it incorporates an asymmetric version of the Geweke/Pantula (1986) model which takes the following form:

$$\log h_{iit} = c_i + \alpha_i \log(|\varepsilon_{it-1}| - \gamma \varepsilon_{it-1}) + \beta_i \log h_{iit-1} \tag{2.11}$$

where

$c_i = \eta_{i0}^{*} \log \varpi_i - \alpha_i \log \sqrt{2/\pi}$,

$\eta_{i0}^{*} = \{1 - \alpha_i \lim_{\delta \to 0} E(|e_{it-1}| - e_{it-1})^{\delta} - \beta_i\} = \{1 - \alpha_i - \beta_i\}$, since $\lim_{\delta \to 0} E(|e_{it-1}| - e_{it-1})^{\delta} = 1$.

Note that η_{i0}^{*} is derived from the decomposition of η_i in equation (2.10):

$$\eta_i = \{1 - \alpha_i E(|e_{it-1}| - e_{it-1})^{\delta} - \beta_i\} \varpi^{\delta} = \eta_{i0}^{*} \varpi^{\delta} \tag{2.12}$$

Note that when δ is not restricted to any positive value, the specification in equation (2.10) is equivalent to Ding, Engle, and Granger's (1993) APARCH(1,1) model.

In order to capture the long-memory dynamics in volatility, we may generalise the conditional variance equations (2.4), (2.9), (2.10), and (2.11) such that they are fractionally integrated. First, consider the GARCH(p,q) extension of (2.4) using lag polynomials:

$$h_{iit} = \eta_i + \alpha_i(L) \varepsilon_{it}^2 + \beta_i(L) h_{iit} \tag{2.13}$$

where $\alpha_i(L)$ and $\beta_i(L)$ are lag polynomials of order q and p, respectively. Equation (2.13) may be rewritten in terms of an ARMA(m,p) process in ε_{iit}^2:

$$[1 - \beta_i(L) - \alpha_i(L)] \varepsilon_{it}^2 = \eta_i + [1 - \beta_i(L)] \upsilon_{it} \tag{2.14}$$

where m = max(q,p) and $\upsilon_{it} = \varepsilon_{it}^2 - h_{iit}$ is the innovation to the variance process. The GARCH(p,q) model is covariance-stationary if all the roots of 1 - $\beta_i(L)$ - $\alpha_i(L)$ lie outside the unit circle. If there exists a unit root in 1 - $\beta_i(L)$ - $\alpha_i(L)$ = 0 then (2.13) becomes the Integrated GARCH (IGARCH) model, which implies that there exists a polynomial $\phi_i(L)$

such that $1-\beta_i(L)-\alpha_i(L)=(1-L)\phi_i(L)$, where the characteristic equation $\phi_i(L)=0$ has all the roots outside the unit circle. The IGARCH model represents an extreme situation of persistence in the conditional variance. This situation might be alleviated by replacing the first difference operator (1-L) in the factorisation of 1 - $\beta_i(L)$ - $\alpha_i(L)$ with a fractional difference operator. The result is the FIGARCH(p,d,q) model proposed by Baillie, Bollerslev, and Mikkelsen (1996) with $1-\beta_i(L)-\alpha_i(L)=(1-L)^d\phi_i(L)$, where $0 \le d \le 1$:

$$(1-L)^d\phi_i(L)\varepsilon_{it}^2=\eta_i+[1-\beta_i(L)]\upsilon_{it} \tag{2.15}$$

Clearly, the FIGARCH(p,d,q) model has a more general structure in that it nests the usual stable GARCH and the IGARCH models. Alternatively, (2.14) may be expressed as the following infinite ARCH process:

$$h_{iit}=\frac{\eta_i}{1-\beta_i(1)}+[1-(1-\beta_i(L))^{-1}\phi_i(L)(1-L)^d]\varepsilon_{it}^2 \tag{2.16}$$

In the special case when both 1 - $\beta_i(L)$ and $\phi_i(L)$ are polynomials of degree 1, we let $\phi_i(L)$ = 1 - $\phi_i L$ and $\beta_i(L)$ = $\beta_i L$. Then the FIGARCH(1,d,1) model is obtained with

$$h_{iit}=\frac{\eta_i}{1-\beta_i}+\lambda_i(L)\varepsilon_{it}^2 \tag{2.17}$$

where $\lambda_i(L)=\sum_{a=1}^{\infty}\lambda_a L^a=1-(1-\beta_i L)^{-1}(1-\phi_i L)(1-L)^d$. However, this model does not capture the phenomenon of asymmetric or non-linear volatility, whereby negative shocks have a different impact on future volatility compared with positive shocks of the same magnitude. To remedy the shortcoming, we may apply the fractionally integrated process to the conditional variance equations specified in (2.9), (2.10), and (2.11).

We now turn to deriving the fractionally integrated process for a general QGARCH(p,q) model. The derivation is rather complicated as the general QGARCH(p,q) model involves cross-products of the random disturbance terms:

$$h_{iit} = \eta_i + \sum_{a=1}^{q} \gamma_{ia} \varepsilon_{it-a} + 2\sum_{a=1}^{q} \sum_{b=a+1}^{q} \alpha_{iab} \varepsilon_{it-a} \varepsilon_{it-b} \sum_{a=1}^{q} \alpha_{iaa} \varepsilon_{it-a}^2 + \sum_{a=1}^{p} \beta_{ia} h_{iit-a} \qquad (2.18)$$

Nonetheless, the fractionally integrated process can be straightforwardly applied to the QGARCH(1,1) model by rewriting equation (2.9) as follows:

$$h_{iit} = \eta_i - \frac{\gamma_i^2}{4\alpha_i} + \alpha_i (\varepsilon_{it-1} + \frac{\gamma_i}{2\alpha_i})^2 + \beta_i h_{iit-1} \qquad (2.19)$$

This is similar to Engle's (1990) asymmetric GARCH(1,1) (AGARCH(1,1)) model, if we redefine $\omega_i = \eta_{it} - \frac{\gamma_i^2}{4\alpha_i}$ and $\gamma_i^* = \frac{\gamma_i}{2\alpha_i}$:

$$h_{iit} = \omega_i + \alpha_i (\varepsilon_{iit-1} + \gamma_i^*)^2 + \beta_i h_{iit-1} \qquad (2.20)$$

Further define $g(\varepsilon_{it}) = \varepsilon_{it} + \frac{\gamma_i}{2\alpha_i}$ and $\tau_{it} = g(\varepsilon_{it})^2 - h_{iit}$, equation (2.19) becomes:

$$[1 - \beta_i L - \alpha_i L] g(\varepsilon_{it})^2 = \eta_i - \frac{\gamma_i^2}{4\alpha_i} + (1 - \beta_i L)\tau_{it} \qquad (2.21)$$

Here, the lag polynomial 1 - β_iL - α_iL can be factorised as $(1-L)^d(1-\phi_i L)$. Note that (2.21) can be rewritten as an infinite ARCH operation applied to $g(\varepsilon_{it}) = \varepsilon_{it} + \frac{\gamma_i}{2\alpha_i}$:

$$h_{iit} = \frac{[\eta_i - \frac{\gamma_i^2}{4\alpha_i}]}{1 - \beta_i} + \lambda_i(L)(\varepsilon_{it} + \frac{\gamma_i}{2\alpha_i})^2 \qquad (2.22)$$

where $\lambda_i(L) = \sum_{a=1}^{\infty} \lambda_a L^a = 1 - (1 - \beta_i L)^{-1} (1 - \phi_i L)(1 - L)^d$.

Given the highly non-linear combination of the parameters $\frac{[\eta_i - \frac{\gamma_i^2}{4\alpha_i}]}{1 - \beta_i}$ and $\frac{\gamma_i}{2\alpha_i}$ in equation (2.22), we avoid problems of convergence by modifying (2.20) to take the following form:

$$h_{iit} = \frac{\omega_i}{1-\beta_i} + \lambda_i(L)(\varepsilon_{it} + \gamma_i^*)^2 \tag{2.23}$$

where $\omega_i = \eta_{it} - \frac{\gamma_i^2}{4\alpha_i}$ and $\gamma_i^* = \frac{\gamma_i}{2\alpha_i}$. We shall denote (2.23) as the FIAGARCH(1,d,1) model, which is similar to the FIGARCH(1,d,1) model in (2.17), except that the formulation allows past return shocks to have asymmetric effects on the conditional volatility.

As for the FIAPARCH(p,d,q) model, consider a general APARCH(p,q) model represented by the following variance equation:

$$\begin{aligned} &\varepsilon_{it} = \sqrt{h_{iit}}\, e_{it}, \;\; e_{it} \sim N(0,1) \\ &h_{iit}^{\delta/2} = \eta_i + \alpha_i\,(L)(|\,\varepsilon_{it}\,| - \gamma\varepsilon_{it})^{\delta} + \beta_i\,(L) h_{iit}^{\delta/2} \end{aligned} \tag{2.24}$$

Further define $g(\varepsilon_{it}) = |\,\varepsilon_{it}\,| - \gamma_i \varepsilon_{it}$, and $\tau_{it} = g(\varepsilon_{it})^{\delta} - h_{it}^{\delta/2}$, equation (2.24) becomes:

$$[1 - \beta_i(L) - \alpha_i(L)] g(\varepsilon_{it})^{\delta} = \eta_i + (1 - \beta_i(L))\tau_{it} \tag{2.25}$$

Again, the lag polynomial 1 - β_i(L) - α_i(L) can be factorised as $(1-L)^d\phi_i(L)$. Equation (2.25) can be rewritten as an infinite ARCH operation applied to $g(\varepsilon_{it}) = |\,\varepsilon_{it}\,| - \gamma_i \varepsilon_{it}$, and the FIAPARCH(p,d,q) takes the following form:

$$h_{iit}^{\delta/2} = \frac{\eta_i}{1 - \beta_i(1)} + [1 - (1 - \beta_i(L))^{-1} \phi_i(L)(1-L)^d] g(\varepsilon_{it})^{\delta} \tag{2.26}$$

In the special case when both 1 - β_i(L) and ϕ_i(L) are polynomials of degree 1, the FIAPARCH(1,d,1) specification becomes:

$$h_{iit}^{\delta/2} = \frac{\eta_i}{1-\beta_i} + \lambda_i(L)(|\,\varepsilon_{it}\,| - \gamma_i \varepsilon_{it})^{\delta} \tag{2.27}$$

where $\lambda_i(L)$ is defined as in (2.22). Like the FIAGARCH(1,d,1) model in (2.23), equation (2.27) allows past shocks to have asymmetric effects on the conditional volatility. We note in passing that FILOGGARCH(1,d,1), FITGARCH(1,d,1), and FILGARCH(1,d,1) specifications are nested by the structure in equation (2.27).

The parameters of various multivariate fractionally integrated GARCH models can be estimated by adopting Bollerslev-Wooldridge's (1992) quasi-maximum likelihood estimation (QMLE) approach. Appropriate assumptions are made for the start-up conditions, including the computation of $\lambda_i(L)$, the number of lags, and the initial values. To compute the response coefficients, $\lambda_i(L)=\sum_{a=1}^{\infty}\lambda_a L^a = 1-(1-\beta_i L)^{-1}(1-\phi_i L)(1-L)^d$, we adopt the following infinite recursions given in Bollerslev and Mikkelsen (1996):

$$\begin{aligned}\lambda_{i1} &= \phi_i - \beta_i + d,\\ \lambda_{ib} &= \beta_i \lambda_{b-1} + [(b-1-d)/b - \phi_i]\zeta_{b-1}, b = 2,\ldots,\infty\end{aligned} \qquad (2.28)$$

where $\zeta_b = \zeta_{b-1}(b-1-d)/b$, with $\zeta_1 = d$.

As *b* goes to infinity, a finite truncation must be imposed during estimation. Too short a truncation may fail to capture the long-memory dynamics. As such, in our calibration, we truncate the infinite series $\lambda_i(L)$ at 1000 lags. We have also experimented with 2000 lags, and it is found that the parameter estimates are reasonably close to those based on 1000 lags. To save computational time, all the estimation results of the long-memory process reported in this paper are based on 1000 lags.

Some discussions on setting the initial values according to various multivariate GARCH models is in order. For the FIGARCH(1,d,1) model, we set the presample observations ε_{it}^2 to the unconditional sample variance. This assumption, however, is inappropriate for the other models, as the infinite ARCH representation operates on $g(\varepsilon_{it})$. As such, for the FIAGARCH(1,d,1) model, we equate the presample observations of

$g(\varepsilon_{it})^2 = (\varepsilon_{it} + \gamma_i^*)^2$ to the sample mean of $(\varepsilon_{it} + \hat{\gamma}_i^*)^2$, where $\hat{\gamma}_i^*$ is the estimate of γ based on the univariate FIAGARCH(1,1) model. As for the FIAPARCH(1,d,1) model, the presample observations of $g(\varepsilon_{it})^\delta = (|\varepsilon_{it}| - \gamma_i \varepsilon_{it})^\delta$ are assumed to be equal to the sample mean of $(|\varepsilon_{it}| - \hat{\gamma}_i \varepsilon_{it})^\delta$, where $\hat{\gamma}_i$ is the estimate of γ based on the univariate FIAPARCH(1,1) model. A similar procedure is applied to FILOGGARCH(1,d,1), FITGARCH(1,d,1), and FILGARCH(1,d,1) models, respectively. For all the fractionally integrated models to be properly defined, all the response coefficients $\lambda_i(L)$ must be non-negative. Bollerslev and Mikkelsen (1996) provided the following sufficient conditions to ensure the non-negativity of the coefficients:

$$\begin{aligned} &\beta_i - d \le \phi_i \le (2-d)/3; \quad and \\ &d[\phi_i - (1-d)/2] \le \beta_i(\phi_i - \beta_i + d) \end{aligned} \tag{2.29}$$

CHAPTER 3

MODELLING LONG MEMORY IN EXCHANGE RATE VOLATILITY

3.1 Stylised Facts of Exchange Rate Data

Understanding exchange rate dynamics has been an important research topic in both finance and economics. For example, international capital asset pricing models often require specific assumptions of exchange rate dynamics; whereas in economics, there is a need to link exchange rate behaviour to changes in key macroeconomic variables in order to establish a framework to assess government policies. Among others, several empirical regularities in exchange rate dynamics are observed by such researchers as Baillie and Bollerslev (1989, 1990, 1994), Hsieh (1989a, 1989b, and 1993), Tse and Tsui (1997), Andersen and Bollerslev (1998), and Tse (1998). They include: [1] exchange rate changes may not be IID; [2] there is little serial correlation in the exchange rate return series; [3] exchange rate changes indicate volatility clustering and leptokurtosis; [4] unlike stock market volatility, exchange rate volatility may not be asymmetric; [5] exchange rate changes may exhibit significant persistence and dependence between distant observations, which is conveniently described as "long range dependence" or "long memory". Most studies that highlight such empirical regularities are based on the univariate version of Engle's (1982) ARCH and Bollerslev's (1986) GARCH models and their extensions. These studies generally find that the class of ARCH and GARCH models is capable of characterising the non-linear dynamics in exchange rates.

One major drawback of the univariate GARCH framework is that it does not capture the co-movements of several time series variables, which may be influenced by the same set of events. Hence, a natural extension is to consider the multivariate GARCH set-up. Several works on modelling exchange rate volatility in multivariate contexts include

Diebold and Nerlove (1989), Bollerslev (1990), Engle and Gau (1997), and Engle (2000), among others. By and large, these multivariate analyses of exchange rate volatility fail to model long-memory dynamics. Indeed, due to the computational complexities involved, there are very few studies on modelling long-memory in exchange rate volatility using the multivariate GARCH framework. Teyssiere (1997, 1998) has applied the multivariate version of the fractionally integrated GARCH (FIGARCH) model of Baillie, Bollerslev, and Mikkelsen (1996) to high-frequency exchange rate data. However, the issue of asymmetric conditional volatility has been excluded. Although Teyssiere (1997, 1998) has considered a trivariate unrestricted FIGARCH model based on the vech structure, his model requires implementing a set of conditions to ensure that the variance-covariance matrix is positive-definite. One other drawback of these studies is the lop-sided focus on the bilateral exchange rate against the US dollar. In fact, most of the stylised facts of exchange rate data are established based on this bilateral rate.

In what follows, we shall analyse the volatility dynamics of exchange rates by applying a family of multivariate (bivariate, trivariate, and tetravariate) asymmetric GARCH models. The proposed models can capture the stylised features of long-memory, persistence, asymmetric conditional volatility, and time-varying correlations commonly found in financial time series data. Furthermore, our model automatically satisfies the positive-definite condition when convergence is achieved. A total of 26 different models (see Appendix I) are applied to four currencies: the Canadian dollar, the British pound, the Singapore dollar, and the Malaysian ringgit. In addition, we shall depart from the widely accepted convention of concentrating on the analysis the bilateral exchange rate against the US dollar. Instead, we also choose the Japanese yen as an alternative numeraire currency to the US dollar. It is interesting to investigate the dynamics of exchange rates against this alternative as it has important implications for asset allocation, such as the construction of a diversified Asian asset portfolio.

3.2 Data

Our data sets comprise 2998 daily observations of the Canadian dollar (CND), the British pound (GBP), the Singapore dollar (SGD), and the Malaysian ringgit (MYR), covering the period from 2 January 1986 to 30 June 1997. More recent observations are excluded to avoid possible distortions caused by the outbreak of the 2-year Asian financial crisis since July 1997. The exchange rates against the US dollar (USD) are culled directly from DataStream International and details of these series can be found in Appendix II. Owing to the non-availability of the bilateral Japanese yen (JPY) exchange rates for the period under study, we use the implied cross rates instead. They are obtained by dividing the exchange rate of a nation's currency against the US dollar with the Japanese yen-US dollar (JPY/USD) exchange rate.

[Insert Tables 3.1-3.2 and Figures 3.1-3.2 here]

The daily nominal exchange rate returns (in percentage) are computed on a continuously compounding basis as:

$$y_t = \log(\frac{S_t}{S_{t-1}}) \times 100 \qquad (3.1)$$

where S_t is the daily exchange rate. We assume that the conditional mean equation is captured by an AR(p) filter:

$$y_{it} = \xi_0 + \sum_{a=1}^{p} \xi_{iat} y_{it-a} + \varepsilon_{it}, i = 1,..., k \qquad (3.2)$$

As shown in Tables 3.1 and 3.2, excess kurtosis is manifested in all the series. In particular, the MYR and SGD exhibit higher kurtosis when they are measured against the US dollar. Additionally, as indicated by the BDS test statistics, non-linear dependencies are detected in all currencies. Indeed, plots of the return series in Figures 3.1 and 3.2 display periods of tranquillity and volatility. As observed by Hsieh (1993), the existence of non-linear dependencies may be due to the conditional heteroskedasticity in the return series.

3.3 Estimation Results

Before delving into the empirical results, we briefly discuss several estimation issues. First, the augmented Dickey-Fuller (ADF) and the Phillips-Perron (PP) tests are used to check for stationarity of the return series. As shown in Table 3.2, both tests suggest that all the series are stationary. The diagnostic checking of residuals obtained from the ADF specification indicates that they are approximately white noise. Second, we find that the parsimonious AR(1) model is a reasonably adequate autoregressive filter for the conditional mean equation, taking into consideration the significance of individual parameters, the log-likelihood values, and the residual diagnostics. Third, the conditional mean, the variance/covariance matrix, and the correlation equations are estimated simultaneously using the Bollerslev and Wooldridge's (1992) QMLE method. To save space, we shall only report estimates of the conditional variance and correlation equations. In addition, estimation results of the variance equations from the constant-correlations model are not reported as they are similar to those from the varying-correlations model. Moreover, results from the trivariate models are not reported as they are relatively close to those obtained from the bivariate and tetravariate models. Instead, other than the estimation results from the tetravariate models, we shall only report some of the results from the bivariate models. All programmes are coded in Gauss Version 5.0.2, with the convergence level set to 10^{-5}.

[Insert Tables 3.3-3.11 here]

3.3.1 Asymmetric Conditional Volatility

We first discuss the evidence of asymmetric volatility. As noted in Tables 3.4-3.6 and 3.8-3.9, when the USD is the numeraire, we find that only the exchange rate of MYR consistently exhibits asymmetric volatility for all the models except AGARCH and QGARCH. The exchange rates of the GBP, CND, and SGD against the USD do not display such asymmetric effects. Our results are consistent with studies of asymmetries in exchange rate volatility by Hsieh (1989b, 1993), in which CND/USD and GBP/USD do not exhibit volatility asymmetry under the univariate EGARCH(1,1) model. However, Tse and Tsui (1997) report that the depreciation shocks of MYR/USD generate greater future volatilities compared to appreciation shocks of the same magnitude. This is not the case for the exchange rate of SGD/USD. Similar results are observed for the multivariate GARCH models we have estimated. For instance, for the bivariate VC-TGARCH model involving MYR/USD and SGD/USD (see Table 3.10), the estimated value of the asymmetric coefficient ( ) for MYR/USD is -0.2038, which is significant at the 5% level. However, the evidence of volatility asymmetry for the MYR/USD is also sensitive to the specification of the conditional volatility. For the VC-AGARCH and VC-QGARCH models (see Tables 3.5 and 3.6), the estimated values of γ for MYR/USD are insignificant at the 5% level. Indeed, the choice of the GARCH model can affect the statistical significance of volatility asymmetry. In the case of GBP/JPY (see Table 3.10), only VC-TGARCH suggests that asymmetric volatility exists; for the other models, the coefficient γ is insignificant at the 5% level.

In addition, the magnitude of the asymmetry varies substantially according to the model specified. In the case of the tetravariate VC-FILOGGARCH (VC-LOGGARCH) model (see Table 3.10), the absolute values of the estimated asymmetric coefficient γ of CND/JPY, GBP/JPY, MYR/JPY, and SGD/JPY are 0.9919 (0.9663), 0.3271 (0.2999), 0.9889 (0.8542), and 0.9751 (0.8710), respectively. However, the magnitude of the asymmetry is lower for the rest of the models. The corresponding absolute values of estimated in the VC-FITGARCH (VC-TGARCH) model are 0.3843 (0.3723), 0.2612 (0.2766), 0.3053 (0.2938), and 0.3276 (0.3264). The estimated values are even lower in the case of the VC-FILGARCH and the VC-FIAPARCH models.

Evidence of volatility asymmetry may depend on the choice of the "numeraire currency". As shown in Tables 3.4-3.6 and 3.8-3.9, we note that CND and SGD do not exhibit asymmetric volatility when they are measured against the US dollar. In contrast, when the Japanese yen is used as the numeraire currency, the exchange rates of CND, MYR, and SGD exhibit highly significant asymmetric volatility in all of the models. However, as for the exchange rate of GBP/JPY, significant asymmetric effects are detected only in the TGARCH model. In all these cases, depreciation shocks are found to have greater impact on the variance process compared to appreciation shocks of the same magnitude. These findings have important implications for the formulation of currency hedging strategies and international investment portfolios, as asymmetric effects in the exchange rate volatility can affect the pricing of currency options.

3.3.2 Long Memory and Persistence in Volatility

We turn to the issue of volatility persistence. From the estimated values of ($\alpha + \beta$) in all the VC-GARCH models (Table 3.3), we observe that all exchange rates of the four currencies exhibit a high degree of volatility persistence. The estimated values of ($\alpha + \beta$) are very close to unity for CND and GBP, which is suggestive of IGARCH behaviour. Indeed it may prompt a researcher working within the conventional paradigm to conclude that the IGARCH model provides a satisfactory description of exchange rate volatility. However, based on the possible inference from various fractionally integrated models, the long-run volatility dynamics are better characterised by the fractional differencing parameter d.

Several interesting findings from the fractionally integrated models deserve mention. First, as indicated in Tables 3.7-3.9 and Table 3.11, the fractional differencing parameters of various models are generally different from 0 and 1, suggesting that the impact of shocks to the conditional variance exhibits a hyperbolic rather than exponential rate of decay. This result is applicable to both types of exchange rates.

Second, the fractional differencing parameters for some groups of currencies are quite similar. For instance, for the bivariate VC-FIGARCH(1,d,1) model (see Table 3.7), the estimated values of d for CND/USD and GBP/USD are 0.3458 and 0.3515 respectively; but higher values of 0.4583 and 0.4428 are obtained for MYR/USD and SGD/USD. However, for the exchange rates vis-à-vis the JPY, all four currencies have similar estimated values for d at around 0.23-0.25. These results indicate that some groups of currencies share a common degree of long-memory in their conditional variances. Indeed, this is borne out by a series of likelihood ratio tests for the restriction that values of d in the model are equal. For instance, when we estimate the bivariate VC-FIGARCH(1,d,1) model by imposing the constraint that the value of d for CND/USD is equal to that of GBP/USD, the log-likelihood value of the restricted model is found to be statistically insignificant from that of the unrestricted model. The full set of test results are not reported here, but they are available from the author upon request.

Third, there are substantial differences in the estimated values of d across different models for the same exchange rate. For instance, estimated values of d for the CND/JPY, GBP/JPY, MYR/JPY, and SGD/JPY are 0.0565, 0.1591, 0.0580, and 0.0647, respectively under the tetravariate VC-FILOGGARCH(1,d,1) model (see Table 3.10). These values are the lowest among all the models. In contrast, the estimated values of d under the tetravariate VC-FITGARCH(1,d,1) and the tetravariate VC-FIAPARCH(1,d,1) models are much greater (see Table 3.10). For example, estimated values of d for CND/JPY, GBP/JPY, MYR/JPY, and SGD/JPY are 0.2917 (0.2823), 0.3448 (0.2681), 0.2856 (0.2888), and 0.3208 (0.3243), respectively in the TGARCH (APARCH) model. These imply that the rate of decay of the impact of shocks to the variance process is slower for TGARCH and APARCH. Such discrepancies could be due to the power transformation of the random disturbance terms ε_{it} in the conditional standard deviation.

3.3.3 Dynamic Conditional Correlations

[Insert Figures 3.3-3.6]

We now discuss the correlation dynamics of the four currencies (see Figures 3.3-3.6). The likelihood ratio test is applied to examine if the restriction $\pi_1 = \pi_2 = 0$ in equation (2.5) can be rejected (see Tables 3.10-3.11). Also, both π_1 and π_2 are checked to see if they are individually significant. We find that the evidence of time-varying correlations is much stronger when the exchange rates are based on the JPY than the USD. Specifically, for the bivariate VC-GARCH(1,1) model with different pair of currencies against the JPY (see Table 3.3), all the likelihood ratio test statistics are highly significant, and the estimated parameters are individually significant at least at the 5% level. Similar likelihood ratio tests are performed for the currencies against the US dollar. We find that although most likelihood ratio test statistics are quite significant, π_1 and π_2 are individually significant at the 5% level only for the model involving the (CND/USD, GBP/USD) pair. For the bivariate VC-GARCH(1,1) model involving the (MYR/USD, SGD/USD) pair, both π_1 and π_2 are individually insignificant even at the 10% level. This implies that the evidence of time-varying correlations between MYR/USD and SGD/USD is relatively weaker, and it is consistent with Tse's (2000) conclusion that the hypothesis of constant conditional correlation cannot be rejected for MYR/USD and SGD/USD. However, this finding is dependent on the choice of numeraire currency, as we have detected significantly time-varying correlations between MYR and SGD when the Japanese yen is used as the numeraire currency in the same models.

We note in passing that there is substantial evidence that the correlation dynamics exhibit slow mean-reversion. In many cases, the sum of the parameters π_1 and π_2 is quite close to unity, suggesting that shocks to the correlation process can be highly persistent (see Tables 3.10-3.11). The intuition is that since ρ_t in the correlation equation is a function of standardised residuals, a large shock arising from one of the foreign exchange markets would induce a substantial deviation of the conditional correlation from the long-run value, which tends to persist in future periods.

Another noteworthy finding is that the magnitude of the time-invariant component of the correlation equation, $\Gamma = \{\rho_{ij}\}$, is much higher when the Japanese yen is the numeraire (see Tables 3.10-3.11). For example, under the tetravariate VC-APARCH(1,1) model for CND/JPY, GBP/JPY, MYR/JPY, and SGD/JPY, the estimated values of $\Gamma = \{\rho_{ij}\}$ are 0.6170 (CND-GBP), 0.9104 (CND-MYR), 0.9025 (CND-SGD), 0.6308 (GBP-MYR), 0.6383 (GBP-SGD), and, 0.9340 (MYR-SGD), and these estimated correlations are significant at the 1% level. However, for the same model with the currencies measured against the USD, the estimated values of $\Gamma = \{\rho_{ij}\}$ are 0.0727 (CND-GBP), 0.0324 (CND-MYR), -0.0090 (CND-SGD), 0.2181 (GBP-MYR), 0.3435 (GBP-SGD), and, 0.3230 (MYR-SGD), and three out of these four correlations are insignificant at the 5% level.

3.3.4 Residual Diagnostics

[Insert Tables 3.16-3.23 here]

Finally, we examine performance of the residual diagnostics for all the models. To save space, we only include the residual diagnostics of the VC-APARCH and VC-FIAPARCH models. As can be observed from Tables 3.16-3.21, most of the Ljung-Box Q-statistics (Ljung and Box (1978)) and McLeod-Li (McLeod and Li (1983)) test statistics are insignificant at the 5% levels. However, some of the BDS (Brock et al (1996)) test statistics for the tetravariate VC-APARCH model are still significant at the 5% level (see

Tables 3.20-3.21). In particular, the statistics suggest that dependencies are detected for the CND/JPY, MYR/JPY, and SGD/JPY series. However, the results are considerably better when we consider the same series in the VC-FIAPARCH model. Apparently, the fractionally integrated model is a better variance filter.

We also apply the diagnostic tests to the cross-product of the standardised residuals (see Tables 3.22-3.33). Under the null hypothesis of constant correlations, the cross product of the standardised residuals should be serially uncorrelated (Bollerslev (1990)). We focus on the Ljung-Box Q-statistic (Ljung and Box (1978)) to test for serial correlation as this test is found to have reliable empirical size and reasonably good power compared to the Ling-Li (1997) test. See Tse and Tsui (1999) for details of their Monte Carlo study of various diagnostic tests for multivariate conditional heteroskedasticity models. As shown in Tables 3.22 and 3.23, most Ljung-Box Q-statistics based on the cross product of the standardised residuals are insignificant at the 5% level, thereby suggesting the absence of serial correlation. This is corroborated by the BDS test results. We mention in passing that the diagnostic results for the constant-correlation models are less favourable compared with those results in Tables 3.22 and 3.23. In particular, there is some evidence of serial correlation in the cross product of the standardised residuals.

Table 3.1 Summary Statistics of Exchange Rates against the Japanese Yen and the US Dollar

Variable	CND/JPY	GBP/JPY	MYR/JPY	SGD/JPY	CND/USD	GBP/USD	MYR/USD	SGD/USD
Panel A: Moments, Maximum, Minimum								
Mean	0.0178	0.0140	0.0198	0.0053	-0.0005	-0.0044	0.0015	-0.0130
Median	0.0000	0.0000	0.0000	-0.0058	0.0000	0.0000	0.0000	0.0000
Maximum	6.6218	3.6509	4.9830	4.6842	1.4479	4.2862	2.3736	2.0232
Minimum	-4.8299	-3.4139	-3.9626	-3.8903	-1.8830	-3.4165	-2.9363	-2.2501
Std. Dev.	0.7636	0.6868	0.6822	0.6340	0.2781	0.6763	0.2555	0.2538
Skewness	0.4441	0.4619	0.4145	0.3571	0.0450	0.2402	-0.2254	-0.3627
Kurtosis	8.8451	6.1582	6.9749	6.6848	7.1992	6.0772	24.1763	10.3654
Observations	2997	2997	2997	2997	2997	2997	2997	2997
Panel B: Ljung-Box Q-statistic								
5 lags	2.8695	4.3030	9.1017	10.6973	2.7845	3.9410	49.1369	68.9244
10 lags	25.5270	23.1934	40.4815	37.4101	9.4522	15.3633	88.6244	79.6134
Panel C: McLeod-Li Test								
5 lags	29.8625	37.1868	158.1128	27.4757	107.3969	104.4159	651.4362	181.8517
10 lags	50.9919	54.5679	229.7626	38.9931	125.9072	200.9949	1196.7779	196.8995
Panel D: ARCH LM Test								
5 lags	24.7471	112.5506	32.9668	27.4292	100.7821	82.7207	447.1970	152.7907
10 lags	32.4967	140.2620	43.1101	42.0719	108.6236	134.0420	592.3810	159.6123
Panel E: QARCH LM Test								
1 lag	15.4041	33.3831	12.7350	15.4606	115.4567	17.4335	468.6607	144.2329
4 lags	55.0151	170.2917	52.2993	53.0421	134.5537	117.3002	710.5332	200.5036
Panel F: BDS Test								
e=3,l=1.5	6.9043	7.0926	7.5261	6.3680	8.2210	6.3342	16.2961	14.0798
e=4,l=1.5	7.8323	8.4054	9.0158	7.5216	8.8971	7.6883	16.7226	14.8928
e=5,l=1.5	8.2707	9.7585	9.6963	8.0265	10.0108	8.9399	16.9930	15.4110
e=3,l=1.0	7.4951	7.0005	7.8580	6.5282	8.1421	5.7832	15.8300	14.0291
e=4,l=1.0	8.3506	8.3036	9.1719	7.5098	9.3835	7.1492	16.8401	15.7770
e=5,l=1.0	9.0951	9.8864	10.0651	8.1987	11.1358	8.9102	17.7326	17.1760

Variable	CND/JPY	GBP/JPY	MYR/JPY	SGD/JPY	CND/USD	GBP/USD	MYR/USD	SGD/USD
Panel G: Runs Test								
R_1	1.4645	0.6375	3.8623	2.9018	-0.3293	1.3379	0.4672	3.3028
R_2	-5.2683	-3.6669	-5.3707	-5.0340	-3.1465	-3.8528	-8.4397	-7.9397
R_3	-3.8779	-5.2489	-4.2167	-2.1550	-3.5650	-2.9981	-10.5792	-7.6008

Notes:

1. CND = Canadian dollar, GBP = British pound, JPY = Japanese Yen, MYR = Malaysian ringgit, SGD = Singapore dollar, USD = US dollar
2. QARCH LM test statistic is due to Sentana (1995) and it is distributed as chi-squared with q(q+3)/2 degrees of freedom, where q is the number of lags.
3. For the BDS Test, e represents the embedding dimension whereas l represents the distance between pairs of consecutive observations, measured as a multiple of the standard deviation of the series. Under the null hypothesis of independence, the test statistic is asymptotically distributed as standard normal.
4. For the Runs Test, R_i for i = 1, 2, 3 denote the runs tests of the series R_t, $|R_t|$, and R_t^2 respectively. Under the null hypothesis that successive observations in the series are independent, the test statistic is asymptotically standard normal.

Table 3.2 Unit Root Tests

Exchange Rate	ADF Model	ADF Test Statistic	Q-statistic (20 lags)	PP Test Statistic
CND/JPY	Case 3 (20)	-10.9200	0.2942	-56.3189
GBP/JPY	Case 3 (20)	-11.6592	0.5773	-53.5362
MYR/JPY	Case 3 (20)	-10.2777	0.4246	-57.8322
SGD/JPY	Case 3 (20)	-10.7728	0.4151	-58.1493
CND/USD	Case 3 (20)	-12.2423	0.4132	-53.6379
GBP/USD	Case 3 (20)	-11.4821	0.1597	-54.7540
MYR/USD	Case 3 (20)	-11.1393	0.9246	-58.1271
SGD/USD	Case 3 (20)	-10.7799	0.5067	-63.4835

Notes:

1. ADF Model: Case 1 refers to the regression equation without any deterministic regressors; Case 2 refers to the equation with intercept; Case 3 refers to the equation with both the intercept and the deterministic time trend. The figure in parenthesis highlights the number of lagged difference terms.
2. For the PP test, both intercept and time trend are included and 8 truncation lags are chosen. It is found that the results are robust to different lag lengths.
3. Q-statistic: this refers to the Ljung-Box Q-statistic with 20 degrees of freedom.

Table 3.3 Estimation Results of Bivariate VC-GARCH(1,1) Model: $h_t = \eta + \alpha\varepsilon^2_{t-1} + \beta h_{t-1}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$

Variable	η	β	α	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
CND/USD	0.0019 (0.0013)	0.9226 (0.0327)	0.055 (0.0206)	0.2774 (0.1470)	0.9813 (0.0041)	0.0105 (0.0039)	2339.1540	0.0635 (0.0213)	2313.9097	50.4886
GBP/USD	0.0025 (0.0014)	0.9605 (0.0092)	0.0345 (0.0078)							
CND/USD	0.0018 (0.0013)	0.9226 (0.0319)	0.0555 (0.0203)	0.0267 (0.0252)	0.9443 (0.0330)	0.0086 (0.0067)	5703.6650	0.0291 (0.0212)	5702.5265	2.2770
MYR/USD	0.0034 (0.0013)	0.7751 (0.0460)	0.1771 (0.0351)							
CND/USD	0.0018 (0.0012)	0.9231 (0.0310)	0.0550 (0.0197)	-0.0168 (0.0277)	0.7658 (0.3263)	0.0631 (0.0544)	5319.7875	-0.0189 (0.0231)	5305.4067	28.7616
SGD/USD	0.0022 (0.0013)	0.8771 (0.0503)	0.0933 (0.0354)							
GBP/USD	0.0027 (0.0015)	0.9586 (0.0096)	0.0358 (0.0081)	0.2548 (0.0386)	0.9624 (0.0284)	0.0159 (0.0088)	3197.1514	0.2331 (0.0220)	3186.1262	22.0506
MYR/USD	0.0032 (0.0012)	0.7784 (0.0463)	0.1791 (0.0367)							
GBP/USD	0.0030 (0.0015)	0.9584 (0.0096)	0.0349 (0.0079)	0.3867 (0.0317)	0.9658 (0.0236)	0.0142 (0.0079)	2907.7169	0.3479 (0.0183)	2899.4000	16.6338
SGD/USD	0.0023 (0.0013)	0.8709 (0.0531)	0.0992 (0.0376)							
MYR/USD	0.0031 (0.0012)	0.7872 (0.0453)	0.1705 (0.0349)	0.3782 (0.0263)	0.4309 (0.7034)	0.1129 (0.0744)	6327.2524	0.3591 (0.0229)	6305.3816	43.7416
SGD/USD	0.0029 (0.0016)	0.8595 (0.0562)	0.1011 (0.0381)							
CND/JPY	0.0130 (0.0052)	0.9453 (0.0159)	0.0322 (0.0096)	0.7282 (0.0469)	0.9479 (0.0146)	0.0321 (0.0086)	-296.7859	0.5083 (0.0225)	-380.8575	168.1433
GBP/JPY	0.0089 (0.0040)	0.9380 (0.0164)	0.0422 (0.0099)							
CND/JPY	0.0144 (0.0068)	0.9383 (0.0212)	0.0378 (0.0120)	0.9186 (0.0134)	0.8704 (0.0810)	0.0344 (0.0171)	1394.8597	0.8724 (0.0083)	1283.2287	223.2619
MYR/JPY	0.0135 (0.0073)	0.9261 (0.0290)	0.0456 (0.0158)							
CND/JPY	0.0145 (0.0050)	0.9393 (0.0159)	0.0361 (0.0101)	0.9147 (0.0122)	0.9268 (0.0162)	0.0180 (0.0042)	1468.7560	0.8674 (0.0069)	1433.5560	70.4000
SGD/JPY	0.0117 (0.0040)	0.9303 (0.0172)	0.0415 (0.0098)							

Variable	η	β	α	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
GBP/JPY	0.0102 (0.0044)	0.9310 (0.0176)	0.0467 (0.0107)	0.7781 (0.0619)	0.9548 (0.0113)	0.0305 (0.0076)	63.0337	0.5037 (0.0226)	-44.6644	215.3962
MYR/JPY	0.0111 (0.0051)	0.9361 (0.0200)	0.0398 (0.0109)							
GBP/JPY	0.0103 (0.0047)	0.9321 (0.0184)	0.0451 (0.0110)	0.8000 (0.0548)	0.9532 (0.0113)	0.0310 (0.0073)	316.6233	0.5290 (0.0204)	213.0890	207.0685
SGD/JPY	0.0097 (0.0041)	0.9388 (0.0178)	0.0368 (0.0099)							
MYR/JPY	0.0186 (0.0236)	0.9013 (0.0952)	0.0601 (0.0502)	0.9408 (0.0087)	0.8715 (0.0363)	0.0351 (0.0077)	2372.3197	0.9044 (0.9044)	2214.5174	315.6046
SGD/JPY	0.0147 (0.0133)	0.9113 (0.0586)	0.0540 (0.0293)							

Notes:

1. All standard errors (in parenthesis) are the heteroskedastic-consistent Bollerslev-Wooldridge standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.
2. Log-likelihood value (VC) and Log-likelihood value (CC) refer to the likelihood values obtained from the VC-GARCH(1,1) and CC-GARCH(1,1) models respectively.
3. Correlations (CC) refer to the conditional correlation coefficient obtained from the CC-GARCH(1,1) model.
4. LR is the likelihood ratio statistic for H_0: $\pi_1 = \pi_2 = 0$ in the VC-GARCH(1,1) model. It is distributed as chi-squared with 2 degrees of freedom under H_0.

Table 3.4 Estimation Results of Bivariate VC-APARCH(1,1) Model: $h_t^{\delta/2} = \eta + \alpha(|\varepsilon_{t-1}| - \gamma\varepsilon_{t-1})^{\delta} + \beta h^{\delta/2}_{t-1}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$

Variable	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
CND/USD	0.0054	0.9061	0.0711	-0.1979	1.4806	0.2664	0.9894	0.0104	2344.870332	0.0638	2320.225	49.290192
	(0.0080)	(0.0619)	(0.0364)	(0.1604)	(0.4164)	(0.1542)	(0.0043)	(0.0040)		(0.0211)		
GBP/USD	0.0027	0.9599	0.0356	-0.0203	1.9395							
	(0.0021)	(0.0104)	(0.0114)	(0.0843)	(0.3501)							
CND/USD	0.0058	0.9052	0.0729	-0.2100	1.4357	0.0266	0.9424	0.0089	5717.02712	0.0295	5715.799	2.45672
	(0.0084)	(0.0607)	(0.0353)	(0.1719)	(0.4083)	(0.0248)	(0.0375)	(0.0070)		(0.0209)		
MYR/USD	0.0039	0.7916	0.1675	-0.1357	1.8826							
	(0.0020)	(0.0484)	(0.0357)	(0.0660)	(0.2698)							
CND/USD	0.0052	0.9090	0.0697	-0.2030	1.4770	-0.0152	0.7471	0.0670	5330.9326	-0.0172	5316.282	29.30088
	(0.0069)	(0.0539)	(0.0322)	(0.1542)	(0.3920)	(0.0276)	(0.2642)	(0.0437)		(0.0231)		
SGD/USD	0.0057	0.8763	0.1102	-0.0121	1.4244							
	(0.0038)	(0.0446)	(0.0329)	(0.0840)	(0.2853)							
GBP/USD	0.0033	0.9574	0.0385	-0.0431	1.8464	0.2555	0.9636	0.0155	3204.5216	0.2335	3193.586	21.8708
	(0.0024)	(0.0111)	(0.0118)	(0.0873)	(0.3231)	(0.0390)	(0.0317)	(0.0096)		(0.0219)		
MYR/USD	0.0036	0.7922	0.1714	-0.1302	1.8885							
	(0.0020)	(0.0485)	(0.0372)	(0.0657)	(0.2768)							
GBP/USD	0.0036	0.9573	0.0368	-0.0506	1.8824	0.3879	0.9654	0.0142	2913.5351	0.3490	2905.467	16.136456
	(0.0023)	(0.0107)	(0.0108)	(0.0864)	(0.2981)	(0.0320)	(0.0254)	(0.0082)		(0.0182)		
SGD/USD	0.0060	0.8691	0.1169	-0.0135	1.4316							
	(0.0045)	(0.0480)	(0.0358)	(0.0812)	(0.3039)							
MYR/USD	0.0035	0.8010	0.1627	-0.1106	1.8861	0.3784	0.4815	0.1069	6338.1878	0.3585	6316.497	43.38208
	(0.0019)	(0.0470)	(0.0358)	(0.0656)	(0.2646)	(0.0263)	(0.9150)	(0.1008)		(0.0228)		
SGD/USD	0.0075	0.8619	0.1178	0.0176	1.3827							
	(0.0054)	(0.0512)	(0.0359)	(0.0921)	(0.3099)							
CND/JPY	0.0141	0.9496	0.0334	-0.3242	1.5189	0.7232	0.9464	0.0326	-285.786642	0.5103	-367.0819	162.5905232
	(0.0077)	(0.0193)	(0.0125)	(0.1783)	(0.2883)	(0.0455)	(0.0152)	(0.0087)		(0.0218)		
GBP/JPY	0.0112	0.9385	0.0448	-0.1562	1.6475							
	(0.0052)	(0.0156)	(0.0122)	(0.1125)	(0.3111)							
CND/JPY	0.0194	0.9388	0.0432	-0.3606	1.2878	0.9183	0.8569	0.0366	1419.404734	0.8738	1306.46	225.8900112
	(0.0088)	(0.0216)	(0.0131)	(0.1514)	(0.2162)	(0.0120)	(0.0793)	(0.0164)		(0.0078)		
MYR/JPY	0.0236	0.9264	0.0536	-0.3210	0.9902							
	(0.0115)	(0.0288)	(0.0171)	(0.1317)	(0.2546)							
CND/JPY	0.0173	0.9378	0.0408	-0.1807	1.6211	0.9126	0.9216	0.0187	1477.860888	0.8671	1443.521	68.680304
	(0.0067)	(0.0175)	(0.0113)	(0.0998)	(0.2098)	(0.0118)	(0.0194)	(0.0046)		(0.0068)		
SGD/JPY	0.0183	0.9262	0.0519	-0.1827	1.3309							
	(0.0075)	(0.0194)	(0.0119)	(0.0995)	(0.2336)							

Variable	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
GBP/JPY	0.0146 (0.0062)	0.9289 (0.0169)	0.0514 (0.0125)	-0.1946 (0.1209)	1.5424 (0.2733)	0.7671 (0.0610)	0.9537 (0.0119)	0.0307 (0.0077)	75.7996988	0.5084 (0.0219)	-27.4504	206.5001988
MYR/JPY	0.0157 (0.0083)	0.9414 (0.0209)	0.0428 (0.0129)	-0.3076 (0.1761)	1.2286 (0.3307)							
GBP/JPY	0.0136 (0.0059)	0.9322 (0.0171)	0.0485 (0.0128)	-0.1667 (0.1145)	1.5932 (0.2885)	0.7941 (0.0542)	0.9522 (0.0119)	0.0315 (0.0075)	329.443156	0.5305 (0.0199)	227.8949	203.0964432
SGD/JPY	0.0144 (0.0080)	0.9441 (0.0210)	0.0416 (0.0136)	-0.3495 (0.1592)	1.1407 (0.2658)							
MYR/JPY	0.0287 (0.0249)	0.9031 (0.0659)	0.0713 (0.0377)	-0.1943 (0.1117)	1.1206 (0.2053)	0.9408 (0.0089)	0.8670 (0.0383)	0.0356 (0.0080)	2394.499072	0.9045 (0.0065)	2237.167	314.663888
SGD/JPY	0.0244 (0.0163)	0.9123 (0.0425)	0.0646 (0.0235)	-0.2139 (0.1132)	1.1065 (0.1713)							

Notes:

1. All standard errors (in parenthesis) are the heteroskedastic-consistent Bollerslev-Wooldridge standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.
2. Log-likelihood value (VC) and Log-likelihood value (CC) refer to the likelihood values obtained from the VC-APARCH(1,1) and CC-APARCH(1,1) models respectively.
3. Correlations (CC) refer to the conditional correlation coefficient obtained from the CC-APARCH(1,1) model.
4. LR is the likelihood ratio statistic for H_0: $\pi_1 = \pi_2 = 0$ in the VC-APARCH(1,1) model. It is distributed as chi-squared with 2 degrees of freedom under H_0.

Table 3.5 Estimation Results of Bivariate VC-QGARCH(1,1) Model: $h_t = \eta + \alpha\varepsilon^2_{t-1} + \beta h_{t-1} + \gamma\varepsilon_{t-1}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$

Variable	η	β	α	γ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
CND/USD	0.0022 (0.0020)	0.9176 (0.0481)	0.0553 (0.0257)	0.0061 (0.0070)	0.2718 (0.1519)	0.9895 (0.0039)	0.0102 (0.0037)	2342.38964	0.0638 (0.0213)	2317.37904	50.021216
GBP/USD	0.0019 (0.0014)	0.9631 (0.0095)	0.0332 (0.0079)	-0.0059 (0.0058)							
CND/USD	0.0022 (0.0021)	0.9167 (0.0488)	0.0561 (0.0260)	0.0067 (0.0075)	0.0276 (0.0250)	0.9425 (0.0342)	0.0088 (0.0069)	5708.33872	0.0300 (0.0210)	5707.17028	2.33688
MYR/USD	0.0034 (0.0013)	0.7800 (0.0465)	0.1711 (0.0355)	0.0082 (0.0077)							
CND/USD	0.0022 (0.0019)	0.9180 (0.0451)	0.0551 (0.0242)	0.0065 (0.0070)	-0.0154 (0.0275)	0.7592 (0.2988)	0.0644 (0.0497)	5322.63368	-0.0177 (0.0231)	5308.19296	28.88144
SGD/USD	0.0022 (0.0012)	0.8795 (0.0495)	0.0924 (0.0347)	-0.0029 (0.0049)							
GBP/USD	0.0023 (0.0015)	0.9605 (0.0101)	0.0349 (0.0082)	-0.0041 (0.0060)	0.2554 (0.0395)	0.9636 (0.0284)	0.0158 (0.0090)	3200.3272	0.2324 (0.0219)	3188.91244	22.82952
MYR/USD	0.0032 (0.0012)	0.7823 (0.0464)	0.1740 (0.0367)	0.0085 (0.0077)							
GBP/USD	0.0026 (0.0016)	0.9601 (0.0102)	0.0341 (0.0081)	-0.0039 (0.0060)	0.3872 (0.0323)	0.9671 (0.0227)	0.0141 (0.0078)	2908.68158	0.3472 (0.0183)	2900.10703	17.149104
SGD/USD	0.0023 (0.0013)	0.8729 (0.0519)	0.0983 (0.0367)	-0.0028 (0.0052)							
MYR/USD	0.0029 (0.0011)	0.7956 (0.0442)	0.1642 (0.0346)	0.0051 (0.0074)	0.1430 (0.1712)	0.9885 (0.0088)	0.0118 (0.0077)	6319.94216	0.3587 (0.0227)	6307.53872	24.80688
SGD/USD	0.0022 (0.0012)	0.8771 (0.0479)	0.0932 (0.0333)	-0.0044 (0.0049)							
CND/JPY	0.0134 (0.0060)	0.9475 (0.0181)	0.0287 (0.0096)	0.0243 (0.0104)	0.7187 (0.0454)	0.9452 (0.0155)	0.0332 (0.0088)	-283.319736	0.5096 (0.0216)	-363.750352	160.861232
GBP/JPY	0.0095 (0.0042)	0.9375 (0.0163)	0.0412 (0.0097)	0.0099 (0.0091)							
CND/JPY	0.0166 (0.0081)	0.9347 (0.0245)	0.0370 (0.0126)	0.0280 (0.0111)	0.9158 (0.0108)	0.8517 (0.0704)	0.0369 (0.0147)	1417.75813	0.8732 (0.0080)	1308.43409	218.64808
MYR/JPY	0.0167 (0.0092)	0.9171 (0.0345)	0.0471 (0.0173)	0.0232 (0.0116)							
CND/JPY	0.0145 (0.0051)	0.9418 (0.0156)	0.0328 (0.0092)	0.0186 (0.0080)	0.9100 (0.0117)	0.9217 (0.0198)	0.0179 (0.0045)	1479.15216	0.8671 (0.0068)	1447.53238	63.239568
SGD/JPY	0.0122 (0.0044)	0.9306 (0.0181)	0.0391 (0.0096)	0.0141 (0.0065)							

Variable	η	β	α	γ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
GBP/JPY	0.0114 (0.0048)	0.9285 (0.0179)	0.0460 (0.0105)	0.0125 (0.0099)	0.7657 (0.0616)	0.9536 (0.0121)	0.0309 (0.0078)	74.0692092	0.5053 (0.0218)	-29.4223378	206.983094
MYR/JPY	0.0120 (0.0055)	0.9360 (0.0213)	0.0373 (0.0111)	0.0190 (0.0083)							
GBP/JPY	0.0109 (0.0047)	0.9322 (0.0179)	0.0434 (0.0107)	0.0108 (0.0093)	0.7925 (0.0545)	0.9528 (0.0120)	0.0311 (0.0076)	326.887568	0.5292 (0.0199)	226.074565	201.626006
SGD/JPY	0.0096 (0.0042)	0.9430 (0.0187)	0.0324 (0.0097)	0.0180 (0.0069)							
MYR/JPY	0.0166 (0.0162)	0.9132 (0.0656)	0.0513 (0.0342)	0.0158 (0.0126)	0.9400 (0.0089)	0.8708 (0.0360)	0.0354 (0.0079)	2379.86661	0.9041 (0.0071)	2221.51303	316.70716
SGD/JPY	0.0138 (0.0099)	0.9199 (0.0438)	0.0467 (0.0220)	0.0141 (0.0079)							

Notes:

1. All standard errors (in parenthesis) are the heteroskedastic-consistent Bollerslev-Wooldridge standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.
2. Log-likelihood value (VC) and Log-likelihood value (CC) refer to the likelihood values obtained from the VC-QGARCH(1,1) and CC-QGARCH(1,1) models respectively.
3. Correlations (CC) refer to the conditional correlation coefficient obtained from the CC-QGARCH(1,1) model.
4. LR is the likelihood ratio statistic for H_0: $\pi_1 = \pi_2 = 0$ in the VC-QGARCH(1,1) model. It is distributed as chi-squared with 2 degrees of freedom under H_0.

Table 3.6 Estimation Results of VC-AGARCH(1,1) Model: $h_t = \eta + \alpha(\varepsilon_{t-1} + \gamma)^2 + \beta h_{t-1}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$

Variable	η	β	α	γ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
CND/USD	0.0020 (0.0018)	0.9176 (0.0481)	0.0553 (0.0257)	0.0549 (0.0495)	0.2718 (0.1519)	0.9895 (0.0039)	0.0102 (0.0037)	2342.39	0.0638 (0.0213)	2317.37904	50.021216
GBP/USD	0.0017 (0.0017)	0.9631 (0.0095)	0.0332 (0.0079)	-0.0893 (0.0952)							
CND/USD	0.0020 (0.0018)	0.9167 (0.0488)	0.0561 (0.0260)	0.0595 (0.0508)	0.0276 (0.0250)	0.9425 (0.0342)	0.0088 (0.0068)	5708.339	0.0300 (0.0210)	5707.17028	2.33688
MYR/USD	0.0033 (0.0013)	0.7800 (0.0465)	0.1711 (0.0355)	0.0241 (0.0236)							
CND/USD	0.0020 (0.0017)	0.9180 (0.0451)	0.0551 (0.0242)	0.0591 (0.0495)	-0.0154 (0.0275)	0.7592 (0.2988)	0.0644 (0.0497)	5322.634	-0.0177 (0.0229)	5308.19296	28.88144
SGD/USD	0.0021 (0.0013)	0.8795 (0.0495)	0.0924 (0.0347)	-0.0157 (0.0293)							
GBP/USD	0.0021 (0.0017)	0.9605 (0.0101)	0.0349 (0.0082)	-0.0590 (0.0908)	0.2554 (0.0395)	0.9636 (0.0284)	0.0158 (0.0090)	3200.327	0.2324 (0.0219)	3188.91244	22.82952
MYR/USD	0.0031 (0.0013)	0.7823 (0.0464)	0.1740 (0.0367)	0.0243 (0.0231)							
GBP/USD	0.0025 (0.0018)	0.9601 (0.0103)	0.0341 (0.0081)	-0.0573 (0.0931)	0.3872 (0.0323)	0.9671 (0.0227)	0.0141 (0.0078)	2908.682	0.3472 (0.0183)	2900.10703	17.149104
SGD/USD	0.0022 (0.0013)	0.8729 (0.0519)	0.0983 (0.0367)	-0.0142 (0.0285)							
MYR/USD	0.0030 (0.0013)	0.7900 (0.0459)	0.1670 (0.0355)	0.0136 (0.0236)	0.3778 (0.0258)	0.4186 (0.6992)	0.1139 (0.0734)	6329.32	0.3587 (0.0227)	6307.53872	43.56184
SGD/USD	0.0027 (0.0015)	0.8638 (0.0535)	0.0997 (0.0367)	-0.0225 (0.0306)							
CND/JPY	0.0083 (0.0051)	0.9475 (0.0181)	0.0287 (0.0096)	0.4228 (0.1450)	0.7187 (0.0454)	0.9452 (0.0155)	0.0332 (0.0088)	-283.3197	0.5096 (0.0216)	-363.750352	160.861232
GBP/JPY	0.0089 (0.0039)	0.9375 (0.0163)	0.0412 (0.0098)	0.1201 (0.1140)							
CND/JPY	0.0113 (0.0068)	0.9347 (0.0244)	0.0370 (0.0126)	0.3788 (0.1039)	0.9158 (0.0108)	0.8517 (0.0704)	0.0369 (0.0147)	1417.758	0.8732 (0.0080)	1308.43409	218.64808
MYR/JPY	0.0139 (0.0077)	0.9171 (0.0344)	0.0471 (0.0172)	0.2461 (0.0727)							
CND/JPY	0.0119 (0.0046)	0.9418 (0.0156)	0.0328 (0.0092)	0.2827 (0.1089)	0.9100 (0.0117)	0.9217 (0.0198)	0.0179 (0.0045)	1479.152	0.8671 (0.0068)	1447.53238	63.239568
SGD/JPY	0.0110 (0.0041)	0.9306 (0.0181)	0.0391 (0.0095)	0.1806 (0.0762)							

Variable	η	β	α	γ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
GBP/JPY	0.0106 (0.0045)	0.9285 (0.0179)	0.0460 (0.0105)	0.1362 (0.1110)	0.7657 (0.0616)	0.9536 (0.0121)	0.0309 (0.0078)	74.06921	0.5053 (0.0218)	-29.4223378	206.9830941
MYR/JPY	0.0096 (0.0049)	0.9360 (0.0213)	0.0373 (0.0111)	0.2553 (0.0951)							
GBP/JPY	0.0102 (0.0045)	0.9322 (0.0179)	0.0434 (0.0107)	0.1246 (0.1128)	0.7925 (0.0545)	0.9528 (0.0120)	0.0311 (0.0076)	326.8876	0.5292 (0.0199)	226.074565	201.6260064
SGD/JPY	0.0071 (0.0041)	0.9430 (0.0187)	0.0324 (0.0097)	0.2779 (0.0991)							
MYR/JPY	0.0154 (0.0154)	0.9132 (0.0653)	0.0513 (0.0341)	0.1537 (0.0770)	0.9400 (0.0089)	0.8708 (0.0360)	0.0354 (0.0079)	2379.867	0.9041 (0.0070)	2221.51303	316.70716
SGD/JPY	0.0127 (0.0097)	0.9199 (0.0436)	0.0467 (0.0219)	0.1503 (0.0774)							

Notes:

1. All standard errors (in parenthesis) are the heteroskedastic-consistent Bollerslev-Wooldridge standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.
2. Log-likelihood value (VC) and Log-likelihood value (CC) refer to the likelihood values obtained from the VC-AGARCH(1,1) and CC-AGARCH(1,1) models respectively.
3. Correlations (CC) refer to the conditional correlation coefficient obtained from the CC-AGARCH(1,1) model.
4. LR is the likelihood ratio statistic for H_0: $\pi_1 = \pi_2 = 0$ in the VC-AGARCH(1,1) model. It is distributed as chi-squared with 2 degrees of freedom under H_0.

Table 3.7 Estimation Results of Bivariate VC-FIGARCH(1,d,1) Model

Variable	η	ϕ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
CND/USD	0.0035 (0.0024)	0.4432 (0.0886)	0.6813 (0.1063)	0.3458 (0.1056)	0.2713 (0.1558)	0.9893 (0.0045)	0.0106 (0.0043)	2345.404	0.0652 (0.0211)	2321.606	47.59446
GBP/USD	0.0201 (0.0099)	0.2298 (0.0711)	0.5720 (0.0960)	0.3515 (0.0576)							
CND/USD	0.0033 (0.0023)	0.4386 (0.0880)	0.6851 (0.1058)	0.3531 (0.1077)	0.0275 (0.0258)	0.9450 (0.0313)	0.0079 (0.0063)	5751.901	0.0294 (0.0220)	5750.882	2.03728
MYR/USD	0.0027 (0.0012)	0.3659 (0.1223)	0.5356 (0.1111)	0.4435 (0.1652)							
CND/USD	0.0032 (0.0022)	0.4420 (0.0842)	0.6923 (0.0981)	0.3558 (0.1074)	-0.0183 (0.0299)	0.7987 (0.4242)	0.0567 (0.0740)	5343.666	-0.0186 (0.0241)	5329.914	27.50328
SGD/USD	0.0017 (0.0009)	0.5223 (0.1330)	0.7794 (0.1138)	0.4897 (0.2306)							
GBP/USD	0.0217 (0.0101)	0.1995 (0.0707)	0.5451 (0.0958)	0.3460 (0.0533)	0.2676 (0.0378)	0.9526 (0.0243)	0.0184 (0.0078)	3243.589	0.2417 (0.0220)	3231.935	23.30888
MYR/USD	0.0026 (0.0011)	0.3721 (0.1228)	0.5612 (0.1055)	0.4766 (0.1610)							
GBP/USD	0.0199 (0.0096)	0.2459 (0.0695)	0.5829 (0.0927)	0.3466 (0.0574)	0.3936 (0.0340)	0.9672 (0.0290)	0.0138 (0.0094)	2925.639	0.3523 (0.0192)	2917.283	16.71169
SGD/USD	0.0017 (0.0010)	0.5214 (0.1612)	0.7851 (0.1363)	0.5146 (0.2908)							
MYR/USD	0.0026 (0.0012)	0.3595 (0.1215)	0.5467 (0.1090)	0.4583 (0.1481)	0.3920 (0.0257)	0.0993 (0.3147)	0.1482 (0.0409)	6392.685	0.3694 (0.0236)	6367.309	50.75224
SGD/USD	0.0020 (0.0010)	0.5869 (0.1357)	0.7797 (0.1093)	0.4428 (0.2290)							
CND/JPY	0.0650 (0.0526)	0.4398 (0.2412)	0.5792 (0.2506)	0.2146 (0.0705)	0.7345 (0.0470)	0.9481 (0.0155)	0.0325 (0.0093)	-299.7138	0.5065 (0.0221)	-386.8765	174.3253
GBP/JPY	0.0531 (0.0212)	0.1645 (0.0981)	0.3843 (0.1102)	0.2530 (0.0441)							
CND/JPY	0.0625 (0.0265)	0.4134 (0.1054)	0.5694 (0.1175)	0.2323 (0.0505)	0.9168 (0.0114)	0.8529 (0.0814)	0.0362 (0.0164)	1398.356	0.8750 (0.0079)	1296.052	204.6088
MYR/JPY	0.0578 (0.0248)	0.3286 (0.1383)	0.4721 (0.1498)	0.2308 (0.0465)							
CND/JPY	0.0623 (0.0384)	0.4608 (0.1695)	0.5920 (0.1861)	0.2179 (0.0565)	0.9123 (0.0120)	0.9240 (0.0199)	0.0173 (0.0045)	1470.086	0.8690 (0.0070)	1437.708	64.75554
SGD/JPY	0.0372 (0.0170)	0.4670 (0.0929)	0.6148 (0.1117)	0.2386 (0.0577)							

Variable	η	ϕ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
GBP/JPY	0.0537 (0.0216)	0.1506 (0.0966)	0.3684 (0.1093)	0.2565 (0.0461)	0.7854 (0.0613)	0.9552 (0.0116)	0.0305 (0.0077)	63.01846	0.5036 (0.0221)	-48.13314	222.3032
MYR/JPY	0.0695 (0.0366)	0.1961 (0.1869)	0.3584 (0.2138)	0.2235 (0.0520)							
GBP/JPY	0.0517 (0.0215)	0.1773 (0.0929)	0.3980 (0.1058)	0.2578 (0.0489)	0.8082 (0.0553)	0.9540 (0.0118)	0.0307 (0.0076)	316.4405	0.5286 (0.0202)	209.9558	212.9695
SGD/JPY	0.0388 (0.0206)	0.3929 (0.1056)	0.5689 (0.1312)	0.2426 (0.0662)							
MYR/JPY	0.0786 (0.0365)	0.2130 (0.2298)	0.3018 (0.2411)	0.2153 (0.0439)	0.9409 (0.0080)	0.8659 (0.0349)	0.0345 (0.0079)	2394.589	0.9058 (0.0063)	2243.902	301.3736
SGD/JPY	0.0365 (0.0143)	0.5094 (0.1093)	0.6175 (0.0993)	0.2369 (0.0574)							

Notes:

1. All standard errors (in parenthesis) are the heteroskedastic-consistent Bollerslev-Wooldridge standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.
2. Log-likelihood value (VC) and Log-likelihood value (CC) refer to the likelihood values obtained from the VC-FIGARCH(1,d,1) and CC-FIGARCH(1,d,1) models respectively.
3. Correlations (CC) refer to the conditional correlation coefficient obtained from the CC-FIGARCH(1,d,1) model.
4. LR is the likelihood ratio statistic for H_0: $\pi_1 = \pi_2 = 0$ in the VC-FIGARCH(1,d,1) model. It is distributed as chi-squared with 2 degrees of freedom under H_0.

Table 3.8 Estimation Results of Bivariate VC-FIAPARCH(1,d,1) Model

Variable	η	ϕ	γ	δ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
CND/USD	0.0087 (0.0107)	0.4142 (0.1413)	-0.2162 (0.1263)	1.7472 (0.3928)	0.6132 (0.1560)	0.3046 (0.1198)	0.2533 (0.1578)	0.9895 (0.0044)	0.0103 (0.0042)	2353.124	0.0661 (0.0211)	2329.504	47.24093
GBP/USD	0.0186 (0.0201)	0.2336 (0.0753)	-0.0461 (0.0829)	2.0464 (0.3369)	0.5540 (0.1255)	0.3312 (0.0898)							
CND/USD	0.0089 (0.0115)	0.4057 (0.1467)	-0.2230 (0.1369)	1.7361 (0.4089)	0.6096 (0.1654)	0.3087 (0.1221)	0.0276 (0.0253)	0.9419 (0.0350)	0.0084 (0.0068)	5770.206	0.0299 (0.0215)	5769.128	2.15712
MYR/USD	0.0034 (0.0026)	0.3607 (0.1153)	-0.1631 (0.0626)	1.9014 (0.1837)	0.5328 (0.1108)	0.4144 (0.1434)							
CND/USD	0.0073 (0.0092)	0.4275 (0.1317)	-0.2124 (0.1267)	1.7878 (0.3970)	0.6313 (0.1422)	0.3071 (0.1256)	-0.0151 (0.0282)	0.7591 (0.3243)	0.0632 (0.0528)	5358.406	-0.0159 (0.0232)	5344.564	27.68304
SGD/USD	0.0042 (0.0034)	0.4332 (0.2914)	-0.0897 (0.0802)	1.6027 (0.2137)	0.8262 (0.1739)	0.6322 (0.4724)							
GBP/USD	0.0227 (0.0213)	0.2035 (0.0714)	-0.0538 (0.0820)	1.9914 (0.3517)	0.5394 (0.1242)	0.3382 (0.0899)	0.2652 (0.0370)	0.9536 (0.0268)	0.0180 (0.0082)	3254.495	0.2403 (0.0212)	3243.17	22.64976
MYR/USD	0.0033 (0.0024)	0.3648 (0.1168)	-0.1537 (0.0622)	1.9111 (0.1792)	0.5526 (0.1059)	0.4452 (0.1437)							
GBP/USD	0.0242 (0.0179)	0.2506 (0.0663)	-0.0706 (0.0832)	1.9182 (0.3145)	0.5911 (0.1083)	0.3530 (0.0905)	0.3914 (0.0319)	0.9613 (0.0357)	0.0150 (0.0103)	2936.991	0.3535 (0.0187)	2929.024	15.93273
SGD/USD	0.0015 (0.0012)	0.0992 (0.1513)	-0.0742 (0.0755)	1.5348 (0.1633)	0.9492 (0.0428)	1.0849 (0.1703)							
MYR/USD	0.0032 (0.0023)	0.3598 (0.1167)	-0.1400 (0.0636)	1.9157 (0.1685)	0.5401 (0.1080)	0.4217 (0.1332)	0.3856 (0.0254)	0.0899 (0.3452)	0.1441 (0.0406)	6409.403	0.3644 (0.0232)	6385.195	48.41536
SGD/USD	0.0057 (0.0036)	0.4877 (0.1662)	-0.0610 (0.0814)	1.5333 (0.1803)	0.8088 (0.1102)	0.5690 (0.2592)							
CND/JPY	0.1125 (0.0850)	0.3744 (0.2257)	-0.3913 (0.2068)	1.3673 (0.2553)	0.5443 (0.2584)	0.2396 (0.0753)	0.7267 (0.0459)	0.9464 (0.0160)	0.0332 (0.0093)	-285.59	0.5078 (0.0215)	-369.26	167.3392
GBP/JPY	0.0644 (0.0289)	0.1962 (0.1069)	-0.1143 (0.1039)	1.8140 (0.2723)	0.4153 (0.1220)	0.2594 (0.0671)							
CND/JPY	0.1140 (0.0458)	0.3646 (0.1160)	-0.4190 (0.1584)	1.2961 (0.1940)	0.5484 (0.1374)	0.2576 (0.0513)	0.9148 (0.0101)	0.8314 (0.0771)	0.0398 (0.0154)	1430.479	0.8750 (0.0077)	1326.677	207.6048
MYR/JPY	0.1280 (0.0447)	0.2920 (0.1146)	-0.3702 (0.1262)	1.0909 (0.2026)	0.4823 (0.1329)	0.2746 (0.0499)							
CND/JPY	0.0942 (0.0558)	0.4110 (0.1740)	-0.2188 (0.1119)	1.4921 (0.1765)	0.5786 (0.2110)	0.2560 (0.0741)	0.9085 (0.0117)	0.9170 (0.0256)	0.0181 (0.0051)	1484.662	0.8678 (0.0068)	1453.86	61.60375
SGD/JPY	0.0715 (0.0294)	0.4257 (0.0739)	-0.2207 (0.1018)	1.2549 (0.1774)	0.6337 (0.1121)	0.3097 (0.0787)							

Variable	η	ϕ	γ	δ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
GBP/JPY	0.0754 (0.0292)	0.1818 (0.1028)	-0.1460 (0.1080)	1.6949 (0.2375)	0.4041 (0.1176)	0.2695 (0.0671)	0.7733 (0.0606)	0.9550 (0.0123)	0.0302 (0.0080)	78.0467	0.5071 (0.0215)	-27.6391	211.3716
MYR/JPY	0.1243 (0.0570)	0.2197 (0.1622)	-0.3225 (0.1602)	1.3020 (0.2973)	0.4146 (0.1937)	0.2552 (0.0588)							
GBP/JPY	0.0670 (0.0267)	0.2096 (0.1000)	-0.1157 (0.1033)	1.7433 (0.2465)	0.4386 (0.1122)	0.2740 (0.0719)	0.8011 (0.0548)	0.9533 (0.0124)	0.0311 (0.0079)	332.2025	0.5291 (0.0198)	228.4639	207.4772
SGD/JPY	0.0810 (0.0321)	0.3772 (0.0726)	-0.3406 (0.1521)	1.1239 (0.2411)	0.6174 (0.1055)	0.3155 (0.0796)							
MYR/JPY	0.1427 (0.0720)	0.2180 (0.2247)	-0.1880 (0.1004)	1.2189 (0.1731)	0.3669 (0.2517)	0.2675 (0.0532)	0.9398 (0.0080)	0.8570 (0.0402)	0.0362 (0.0087)	2419.755	0.9056 (0.0063)	2270.288	298.9349
SGD/JPY	0.0790 (0.0295)	0.4527 (0.0802)	-0.2304 (0.1012)	1.1728 (0.1549)	0.6241 (0.1004)	0.3007 (0.0755)							

Notes:

1. All standard errors (in parenthesis) are the heteroskedastic-consistent Bollerslev-Wooldridge standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.
2. Log-likelihood value (VC) and Log-likelihood value (CC) refer to the likelihood values obtained from the VC-FIAPARCH(1,d,1) and CC-FIAPARCH(1,d,1) models respectively.
3. Correlations (CC) refer to the conditional correlation coefficient obtained from the CC-FIAPARCH(1,d,1) model.
4. LR is the likelihood ratio statistic for H_0: $\pi_1 = \pi_2 = 0$ in the VC-FIAPARCH(1,d,1) model. It is distributed as chi-squared with 2 degrees of freedom under H_0.

Table 3.9 Estimation Results of Bivariate VC-FIAGARCH(1,d,1) Model

Variable	η	ϕ	γ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
CND/USD	0.0035 (0.0036)	0.4087 (0.1772)	0.0918 (0.0541)	0.5603 (0.2253)	0.2579 (0.0954)	0.2542 (0.1673)	0.9895 (0.0044)	0.0103 (0.0042)	2351.204	0.0657 (0.0211)	2327.877	46.65371
GBP/USD	0.0188 (0.0102)	0.2284 (0.0700)	-0.0377 (0.0728)	0.5762 (0.0979)	0.3557 (0.0588)							
CND/USD	0.0032 (0.0036)	0.3943 (0.1886)	0.0967 (0.0542)	0.5476 (0.2389)	0.2588 (0.0951)	0.0295 (0.0256)	0.9423 (0.0329)	0.0085 (0.0067)	5760.439	0.0313 (0.0218)	5759.331	2.21704
MYR/USD	0.0026 (0.0014)	0.3582 (0.1220)	0.0219 (0.0244)	0.5218 (0.1201)	0.4297 (0.1768)							
CND/USD	0.0031 (0.0033)	0.4237 (0.1586)	0.0914 (0.0553)	0.5837 (0.2094)	0.2655 (0.1002)	-0.0153 (0.0292)	0.7799 (0.4205)	0.0595 (0.0699)	5349.598	-0.0165 (0.0239)	5336.056	27.08384
SGD/USD	0.0016 (0.0009)	0.5339 (0.1439)	0.0133 (0.0315)	0.7850 (0.1173)	0.4891 (0.2313)							
GBP/USD	0.0211 (0.0103)	0.1977 (0.0705)	-0.0262 (0.0684)	0.5465 (0.0970)	0.3483 (0.0537)	0.2689 (0.0383)	0.9540 (0.0240)	0.0181 (0.0077)	3246.286	0.2419 (0.0220)	3234.512	23.54856
MYR/USD	0.0026 (0.0013)	0.3612 (0.1235)	0.0212 (0.0235)	0.5485 (0.1123)	0.4680 (0.1710)							
GBP/USD	0.0194 (0.0097)	0.2444 (0.0692)	-0.0208 (0.0682)	0.5840 (0.0938)	0.3486 (0.0580)	0.3923 (0.0334)	0.9665 (0.0307)	0.0138 (0.0096)	2926.166	0.3526 (0.0193)	2918.143	16.04658
SGD/USD	0.0017 (0.0010)	0.5303 (0.1685)	0.0127 (0.0306)	0.7913 (0.1390)	0.5171 (0.2935)							
MYR/USD	0.0026 (0.0013)	0.3532 (0.1215)	0.0138 (0.0238)	0.5371 (0.1118)	0.4497 (0.1538)	0.3915 (0.0257)	0.0973 (0.3162)	0.1478 (0.0411)	6393.943	0.3692 (0.0236)	6368.837	50.21296
SGD/USD	0.0019 (0.0011)	0.5911 (0.1392)	0.0082 (0.0312)	0.7848 (0.1190)	0.4470 (0.2368)							
CND/JPY	0.0325 (0.0457)	0.2961 (0.2648)	0.4110 (0.1217)	0.4236 (0.2767)	0.1802 (0.0444)	0.7271 (0.0461)	0.9458 (0.0162)	0.0337 (0.0094)	-282.922	0.5061 (0.0212)	-367.597	169.3513
GBP/JPY	0.0517 (0.0216)	0.1632 (0.1039)	0.0900 (0.0995)	0.3730 (0.1144)	0.2445 (0.0462)							
CND/JPY	0.0268 (0.0245)	0.3471 (0.1480)	0.3864 (0.0829)	0.4779 (0.1505)	0.1980 (0.0342)	0.9134 (0.0098)	0.8290 (0.0762)	0.0394 (0.0150)	1431.357	0.8752 (0.0076)	1330.922	200.8698
MYR/JPY	0.0448 (0.0203)	0.2321 (0.1493)	0.2775 (0.0641)	0.3471 (0.1529)	0.1937 (0.0339)							
CND/JPY	0.0551 (0.0375)	0.3679 (0.2386)	0.2842 (0.0977)	0.4880 (0.2499)	0.1886 (0.0430)	0.9061 (0.0116)	0.9154 (0.0281)	0.0175 (0.0052)	1484.665	0.8680 (0.0068)	1456.245	56.84011
SGD/JPY	0.0350 (0.0196)	0.4383 (0.1328)	0.2063 (0.0760)	0.5554 (0.1528)	0.2008 (0.0489)							

Variable	η	ϕ	γ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
GBP/JPY	0.0522 (0.0219)	0.1409 (0.1040)	0.1100 (0.0954)	0.3445 (0.1142)	0.2433 (0.0462)	0.7752 (0.0611)	0.9546 (0.0123)	0.0307 (0.0080)	78.37506	0.5042 (0.0214)	-29.1535	215.0572
MYR/JPY	0.0428 (0.0282)	0.1229 (0.1407)	0.2798 (0.0839)	0.2797 (0.1540)	0.2014 (0.0382)							
GBP/JPY	0.0496 (0.0211)	0.1800 (0.0989)	0.0912 (0.0963)	0.3909 (0.1078)	0.2485 (0.0494)	0.8036 (0.0553)	0.9537 (0.0124)	0.0310 (0.0079)	328.6223	0.5277 (0.0198)	223.8704	209.5037
SGD/JPY	0.0213 (0.0215)	0.3475 (0.1165)	0.2855 (0.0939)	0.5055 (0.1359)	0.2092 (0.0478)							
MYR/JPY	0.0793 (0.0316)	0.1602 (0.2027)	0.1289 (0.0703)	0.2481 (0.2070)	0.1968 (0.0387)	0.9394 (0.0081)	0.8611 (0.0388)	0.0351 (0.0084)	2405.644	0.9053 (0.0063)	2257.234	296.8197
SGD/JPY	0.0321 (0.0151)	0.5403 (0.1340)	0.1638 (0.0692)	0.6275 (0.1218)	0.2066 (0.0489)							

Notes:

1. All standard errors (in parenthesis) are the heteroskedastic-consistent Bollerslev-Wooldridge standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.
2. Log-likelihood value (VC) and Log-likelihood value (CC) refer to the likelihood values obtained from the VC-FIAGARCH(1,d,1) and CC-FIAGARCH(1,d,1) models respectively.
3. Correlations (CC) refer to the conditional correlation coefficient obtained from the CC-FIAGARCH(1,d,1) model.
4. LR is the likelihood ratio statistic for H_0: $\pi_1 = \pi_2 = 0$ in the VC-FIAGARCH(1,d,1) model. It is distributed as chi-squared with 2 degrees of freedom under H_0.

Table 3.10 Estimation Results of Tetravariate Varying-Correlations (VC) Models

Variable	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
VC-GARCH: $h_t = \eta + \alpha\varepsilon^2_{t-1} + \beta h_{t-1}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$												
CND/USD	0.0018 (0.0013)	0.9234 (0.0329)	0.0540 (0.0205)	-	-	ρ_{CG} = 0.0707 (0.0375)	0.9730 (0.0228)	0.0124 (0.0072)	8884.6979	ρ_{CG} = 0.0628 (0.0213)	8839.1887	91.0185
GBP/USD	0.0030 (0.0015)	0.9582 (0.0098)	0.0350 (0.0079)	-	-	ρ_{CM} = 0.0318 (0.0376)				ρ_{CM} = 0.0295 (0.0211)		
MYR/USD	0.0029 (0.0012)	0.7908 (0.0444)	0.1705 (0.0350)	-	-	ρ_{CS} = -0.0145 (0.0376)				ρ_{CS} = -0.0201 (0.0233)		
SGD/USD	0.0027 (0.0014)	0.8600 (0.0532)	0.1047 (0.0372)	-	-	ρ_{GM} = 0.2170 (0.0399)				ρ_{GM} = 0.2349 (0.0220)		
						ρ_{GS} = 0.3408 (0.0320)				ρ_{GS} = 0.3499 (0.0183)		
						ρ_{MS} = 0.3212 (0.0510)				ρ_{MS} = 0.3591 (0.0229)		
CND/JPY	0.0125 (0.0045)	0.9427 (0.0139)	0.0358 (0.0084)	-	-	ρ_{CG} = 0.6180 (0.0352)	0.9098 (0.0195)	0.0393 (0.0073)	4656.3233	ρ_{CG} = 0.5094 (0.0228)	4279.6961	753.2543
GBP/JPY	0.0107 (0.0045)	0.9313 (0.0176)	0.0465 (0.0108)	-	-	ρ_{CM} = 0.9097 (0.0108)				ρ_{CM} = 0.8731 (0.0083)		
MYR/JPY	0.0118 (0.0053)	0.9325 (0.0212)	0.0416 (0.0120)	-	-	ρ_{CS} = 0.9034 (0.0110)				ρ_{CS} = 0.8679 (0.0070)		
SGD/JPY	0.0102 (0.0037)	0.9348 (0.0158)	0.0406 (0.0090)	-	-	ρ_{GM} = 0.6324 (0.0353)				ρ_{GM} = 0.5061 (0.0225)		
						ρ_{GS} = 0.6408 (0.0317)				ρ_{GS} = 0.5308 (0.0204)		
						ρ_{MS} = 0.9343 (0.0080)				ρ_{MS} = 0.9033 (0.0067)		
VC-LOGGARCH: $\log h_t^{1/2} = \eta + \alpha \log(\lvert\varepsilon_{t-1}\rvert - \gamma\varepsilon_{t-1}) + \beta \log h_{t-1}^{1/2}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$												
CND/USD	-0.0343 (0.0464)	0.9117 (0.0647)	0.0323 (0.0146)	-0.6612 (0.1834)	-	ρ_{CG} = 0.0674 (0.0344)	0.9614 (0.0218)	0.0159 (0.0064)	8406.4464	ρ_{CG} = 0.0632 (0.0210)	8362.4352	88.0225
GBP/USD	0.0171 (0.0033)	0.9720 (0.0083)	0.0199 (0.0048)	0.3783 (0.2279)	-	ρ_{CM} = 0.0108 (0.0327)				ρ_{CM} = 0.0248 (0.0203)		
MYR/USD	0.0150 (0.0063)	0.9764 (0.0127)	0.0150 (0.0069)	-0.7620 (0.2479)	-	ρ_{CS} = -0.0118 (0.0361)				ρ_{CS} = -0.0173 (0.0222)		
SGD/USD	-0.0342	0.8890	0.0496	0.3241	-	ρ_{GM} = 0.1911 (0.0358)				ρ_{GM} = 0.2118 (0.0207)		
						ρ_{GS} = 0.3330 (0.0309)				ρ_{GS} = 0.3437 (0.0190)		

Variable	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
	(0.0357)	(0.0469)	(0.0154)	(0.2333)		ρ_{MS} = 0.3142 (0.0422)				ρ_{MS} = 0.3453 (0.0214)		
CND/JPY	0.0118 (0.0021)	0.9823 (0.0056)	0.0063 (0.0021)	-0.9663 (0.0383)	-	ρ_{CG} = 0.6099 (0.0343)	0.9232 (0.0173)	0.0321 (0.0062)	4541.3068	ρ_{CG} = 0.5034 (0.0210)	4227.7155	627.1826
						ρ_{CM} = 0.9092 (0.0101)				ρ_{CM} = 0.8775 (0.0072)		
GBP/JPY	0.0123 (0.0029)	0.9584 (0.0067)	0.0207 (0.0029)	-0.2999 (0.2131)	-	ρ_{CS} = 0.8978 (0.0111)				ρ_{CS} = 0.8653 (0.0068)		
MYR/JPY	0.0095 (0.0019)	0.9770 (0.0044)	0.0095 (0.0020)	-0.8542 (0.0935)	-	ρ_{GM} = 0.6195 (0.0336)				ρ_{GM} = 0.5014 (0.0211)		
						ρ_{GS} = 0.6301 (0.0315)				ρ_{GS} = 0.5257 (0.0200)		
SGD/JPY	0.0085 (0.0019)	0.9822 (0.0040)	0.0080 (0.0019)	-0.8710 (0.0771)	-	ρ_{MS} = 0.9304 (0.0078)				ρ_{MS} = 0.9044 (0.0061)		
VC-TGARCH: $h_t^{1/2} = \eta + \alpha(\lvert\varepsilon_{t-1}\rvert - \gamma\varepsilon_{t-1}) + \beta h^{1/2}_{t-1}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$												
CND/USD	0.0149 (0.0125)	0.8843 (0.0650)	0.0842 (0.0304)	-0.3241 (0.1716)	-	ρ_{CG} = 0.0694 (0.0368)	0.9715 (0.0224)	0.0130 (0.0071)	8855.2173	ρ_{CG} = 0.0616 (0.0212)	8808.8692	92.6962
						ρ_{CM} = 0.0281 (0.0360)				ρ_{CM} = 0.0276 (0.0204)		
GBP/USD	0.0077 (0.0035)	0.9536 (0.0111)	0.0478 (0.0098)	-0.0444 (0.1184)	-	ρ_{CS} = -0.0068 (0.0366)				ρ_{CS} = -0.0171 (0.0232)		
MYR/USD	0.0133 (0.0050)	0.8363 (0.0368)	0.1581 (0.0277)	-0.2038 (0.0889)	-	ρ_{GM} = 0.2171 (0.0391)				ρ_{GM} = 0.2329 (0.0215)		
						ρ_{GS} = 0.3434 (0.0319)				ρ_{GS} = 0.3502 (0.0184)		
SGD/USD	0.0132 (0.0065)	0.8617 (0.0464)	0.1222 (0.0321)	0.0451 (0.0934)	-	ρ_{MS} = 0.3253 (0.0493)				ρ_{MS} = 0.3569 (0.0225)		
CND/JPY	0.0161 (0.0064)	0.9500 (0.0140)	0.0395 (0.0091)	-0.3723 (0.1024)	-	ρ_{CG} = 0.6186 (0.0342)	0.9119 (0.0196)	0.0380 (0.0072)	4680.8006	ρ_{CG} = 0.5117 (0.0218)	4309.2666	743.0679
						ρ_{CM} = 0.9108 (0.0103)				ρ_{CM} = 0.8739 (0.0080)		
GBP/JPY	0.0183 (0.0063)	0.9337 (0.0147)	0.0533 (0.0106)	-0.2766 (0.1282)	-	ρ_{CS} = 0.9023 (0.0109)				ρ_{CS} = 0.8670 (0.0069)		
MYR/JPY	0.0168 (0.0078)	0.9420 (0.0191)	0.0449 (0.0117)	-0.2938 (0.0855)	-	ρ_{GM} = 0.6316 (0.0337)				ρ_{GM} = 0.5099 (0.0216)		
						ρ_{GS} = 0.6390 (0.0307)				ρ_{GS} = 0.5318 (0.0197)		
SGD/JPY	0.0152	0.9445	0.0433	-0.3264	-							

Variable	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
	(0.0065)	(0.0165)	(0.0102)	(0.0929)	-	ρ_{MS} = 0.9339 (0.0080)				ρ_{MS} = 0.9035 (0.0065)		
VC-LGARCH: $h_t = \eta + \alpha(\lvert\varepsilon_{t-1}\rvert - \gamma\varepsilon_{t-1})^2 + \beta h_{t-1}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$												
CND/USD	0.0020 (0.0018)	0.9238 (0.0438)	0.0499 (0.0229)	-0.1385 (0.0915)	-	ρ_{CG} = 0.0733 (0.0374) ρ_{CM} = 0.0324 (0.0376)	0.9728 (0.0220)	0.0124 (0.0070)	8894.1353	ρ_{CG} = 0.0634 (0.0212) ρ_{CM} = 0.0300 (0.0209)	8848.5062	91.2582
GBP/USD	0.0032 (0.0018)	0.9578 (0.0103)	0.0346 (0.0079)	-0.0498 (0.0829)	-	ρ_{CS} = -0.0117 (0.0366)				ρ_{CS} = -0.0191 (0.0244)		
MYR/USD	0.0028 (0.0012)	0.8000 (0.0455)	0.1604 (0.0359)	-0.1131 (0.0623)	-	ρ_{GM} = 0.2178 (0.0395) ρ_{GS} = 0.3421 (0.0318)				ρ_{GM} = 0.2353 (0.0220) ρ_{GS} = 0.3505 (0.0183)		
SGD/USD	0.0027 (0.0015)	0.8590 (0.0557)	0.1053 (0.0385)	-0.0220 (0.0607)	-	ρ_{MS} = 0.3204 (0.0500)				ρ_{MS} = 0.3578 (0.0228)		
CND/JPY	0.0118 (0.0048)	0.9456 (0.0155)	0.0326 (0.0088)	-0.1363 (0.0646)	-	ρ_{CG} = 0.6157 (0.0341) ρ_{CM} = 0.9087 (0.0104)	0.9069 (0.0204)	0.0399 (0.0075)	4666.5097	ρ_{CG} = 0.5102 (0.0223) ρ_{CM} = 0.8732 (0.0082)	4292.9085	747.2024
GBP/JPY	0.0109 (0.0047)	0.9333 (0.0179)	0.0422 (0.0110)	-0.1177 (0.1803)	-	ρ_{CS} = 0.9015 (0.0108)				ρ_{CS} = 0.8677 (0.0070)		
MYR/JPY	0.0117 (0.0058)	0.9339 (0.0235)	0.0389 (0.0124)	-0.1050 (0.0600)	-	ρ_{GM} = 0.6292 (0.0341) ρ_{GS} = 0.6372 (0.0308)				ρ_{GM} = 0.5072 (0.0222) ρ_{GS} = 0.5312 (0.0202)		
SGD/JPY	0.0103 (0.0042)	0.9363 (0.0178)	0.0374 (0.0093)	-0.1207 (0.0670)	-	ρ_{MS} = 0.9337 (0.0080)				ρ_{MS} = 0.9030 (0.0067)		
VC-APARCH: $h_t^{\delta/2} = \eta + \alpha(\lvert\varepsilon_{t-1}\rvert - \gamma\varepsilon_{t-1})^{\delta} + \beta h_{t-1}^{\delta/2}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$												
CND/USD	0.0054 (0.0080)	0.9070 (0.0624)	0.0696 (0.0363)	-0.2056 (0.1639)	1.4828 (0.4176)	ρ_{CG} = 0.0727 (0.0377) ρ_{CM} = 0.0324 (0.0377)	0.9728 (0.0231)	0.0125 (0.0073)	8903.2432	ρ_{CG} = 0.0629 (0.0211) ρ_{CM} = 0.0298 (0.0208)	8857.4343	91.6177
GBP/USD	0.0036 (0.0023)	0.9573 (0.0108)	0.0366 (0.0108)	-0.0519 (0.0862)	1.8959 (0.2935)	ρ_{CS} = -0.0090 (0.0397)				ρ_{CS} = -0.0180 (0.0236)		
MYR/USD	0.0035 (0.0019)	0.8055 (0.0466)	0.1633 (0.0356)	-0.1188 (0.0651)	1.8610 (0.2673)	ρ_{GM} = 0.2181 (0.0397)				ρ_{GM} = 0.2353 (0.0220)		

Variable	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
SGD/USD	0.0069 (0.0050)	0.8620 (0.0485)	0.1215 (0.0351)	0.0108 (0.0984)	1.3896 (0.3138)	ρ_{GS} = 0.3435 (0.0318) ρ_{MS} = 0.3230 (0.0505)				ρ_{GS} = 0.3509 (0.0182) ρ_{MS} = 0.3584 (0.0228)		
CND/JPY	0.0141 (0.0055)	0.9481 (0.0142)	0.0386 (0.0091)	-0.2673 (0.1120)	1.3966 (0.1794)	ρ_{CG} = 0.6170 (0.0343) ρ_{CM} = 0.9104 (0.0103) ρ_{CS} = 0.9025 (0.0108) ρ_{GM} = 0.6308 (0.0339) ρ_{GS} = 0.6383 (0.0309) ρ_{MS} = 0.9340 (0.0080)	0.9102 (0.0199)	0.0387 (0.0073)	4690.0582	ρ_{CG} = 0.5111 (0.0220) ρ_{CM} = 0.8738 (0.0080) ρ_{CS} = 0.8673 (0.0069) ρ_{GM} = 0.5091 (0.0217) ρ_{GS} = 0.5319 (0.0199) ρ_{MS} = 0.9036 (0.0065)	4318.0150	744.0866
GBP/JPY	0.0133 (0.0058)	0.9322 (0.0174)	0.0475 (0.0122)	-0.1677 (0.1110)	1.6769 (0.2818)							
MYR/JPY	0.0168 (0.0079)	0.9387 (0.0205)	0.0473 (0.0125)	-0.2526 (0.1090)	1.1015 (0.1913)							
SGD/JPY	0.0148 (0.0061)	0.9410 (0.0170)	0.0454 (0.0104)	-0.2692 (0.1092)	1.1543 (0.1644)							
VC-AGARCH: $h_t = \eta + \alpha(\varepsilon_{t-1} + \gamma)^2 + \beta h_{t-1}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$												
CND/USD	0.0020 (0.0019)	0.9181 (0.0493)	0.0543 (0.0260)	0.0588 (0.0499)	-	ρ_{CG} = 0.0724 (0.0384) ρ_{CM} = 0.0334 (0.0383) ρ_{CS} = -0.0123 (0.0391) ρ_{GM} = 0.2157 (0.0411) ρ_{GS} = 0.3398 (0.0326) ρ_{MS} = 0.3188 (0.0528)	0.9743 (0.0226)	0.0121 (0.0073)	8889.7612	ρ_{CG} = 0.0632 (0.0213) ρ_{CM} = 0.0303 (0.0212) ρ_{CS} = -0.0189 (0.0237) ρ_{GM} = 0.2345 (0.0220) ρ_{GS} = 0.3493 (0.0183) ρ_{MS} = 0.3586 (0.0227)	8843.5029	92.5165
GBP/USD	0.0026 (0.0018)	0.9597 (0.0104)	0.0343 (0.0082)	-0.0492 (0.0912)	-							
MYR/USD	0.0028 (0.0012)	0.7941 (0.0448)	0.1663 (0.0353)	0.0178 (0.0233)	-							
SGD/USD	0.0025 (0.0014)	0.8634 (0.0516)	0.1034 (0.0362)	-0.0184 (0.0290)	-							
CND/JPY	0.0090 (0.0043)	0.9477 (0.0146)	0.0309 (0.0083)	0.3167 (0.0965)	-	ρ_{CG} = 0.6164 (0.0342) ρ_{CM} = 0.9090 (0.0104) ρ_{CS} = 0.9015 (0.0108)	0.9077 (0.0204)	0.0396 (0.0075)	4679.2427	ρ_{CG} = 0.5104 (0.0218) ρ_{CM} = 0.8735 (0.0081) ρ_{CS} = 0.8675 (0.0069)	4305.3718	747.7417
GBP/JPY	0.0105 (0.0046)	0.9315 (0.0183)	0.0446 (0.0109)	0.1270 (0.1140)	-							

Variable	η	β	α	γ	δ	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
MYR/JPY	0.0102 (0.0050)	0.9360 (0.0218)	0.0372 (0.0117)	0.2138 (0.0688)	-	ρ_{GM} = 0.6287 (0.0341) ρ_{GS} = 0.6374 (0.0310)				ρ_{GM} = 0.5072 (0.0218) ρ_{GS} = 0.5310 (0.0199)		
SGD/JPY	0.0087 (0.0040)	0.9388 (0.0176)	0.0359 (0.0093)	0.2054 (0.0731)	-	ρ_{MS} = 0.9335 (0.0081)				ρ_{MS} = 0.9025 (0.0067)		
VC-QGARCH: $h_t = \eta + \alpha\varepsilon_{t-1}^2 + \gamma\varepsilon_{t-1} + \beta h_{t-1}$; $\Gamma_t = (1 - \pi_1 - \pi_2)\Gamma + \pi_1\Gamma_{t-1} + \pi_2\Psi_{t-1}$												
CND/USD	0.0022 (0.0021)	0.9181 (0.0494)	0.0543 (0.0260)	0.0064 (0.0071)	-	ρ_{CG} = 0.0724 (0.0384) ρ_{CM} = 0.0334 (0.0384)	0.9743 (0.0226)	0.0121 (0.0073)	8889.7612	ρ_{CG} = 0.0632 (0.0212) ρ_{CM} = 0.0303 (0.0214)	8843.5029	92.5165
GBP/USD	0.0027 (0.0016)	0.9597 (0.0104)	0.0343 (0.0082)	-0.0034 (0.0060)	-	ρ_{CS} = -0.0123 (0.0388)				ρ_{CS} = -0.0189 (0.0238)		
MYR/USD	0.0029 (0.0012)	0.7941 (0.0448)	0.1663 (0.0353)	0.0059 (0.0074)	-	ρ_{GM} = 0.2157 (0.0411) ρ_{GS} = 0.3398 (0.0326)				ρ_{GM} = 0.2345 (0.0220) ρ_{GS} = 0.3493 (0.0183)		
SGD/USD	0.0025 (0.0014)	0.8634 (0.0517)	0.1034 (0.0362)	-0.0038 (0.0054)	-	ρ_{MS} = 0.3188 (0.0528)				ρ_{MS} = 0.3586 (0.0227)		
CND/JPY	0.0121 (0.0054)	0.9477 (0.0169)	0.0309 (0.0094)	0.0196 (0.0077)	-	ρ_{CG} = 0.6164 (0.0340) ρ_{CM} = 0.9090 (0.0104)	0.9077 (0.0209)	0.0396 (0.0075)	4679.2427	ρ_{CG} = 0.5104 (0.0219) ρ_{CM} = 0.8735 (0.0081)	4305.3718	747.7417
GBP/JPY	0.0112 (0.0055)	0.9315 (0.0199)	0.0446 (0.0112)	0.0113 (0.0108)	-	ρ_{CS} = 0.9015 (0.0108)				ρ_{CS} = 0.8675 (0.0069)		
MYR/JPY	0.0119 (0.0063)	0.9360 (0.0248)	0.0372 (0.0131)	0.0159 (0.0080)	-	ρ_{GM} = 0.6287 (0.0339) ρ_{GS} = 0.6374 (0.0309)				ρ_{GM} = 0.5072 (0.0220) ρ_{GS} = 0.5310 (0.0200)		
SGD/JPY	0.0103 (0.0047)	0.9388 (0.0204)	0.0359 (0.0107)	0.0147 (0.0059)	-	ρ_{MS} = 0.9335 (0.0081)				ρ_{MS} = 0.9025 (0.0067)		

Notes:

1. All standard errors (in parenthesis) are the heteroskedastic-consistent Bollerslev-Wooldridge (1992) standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.
2. LL (VC) and LL (CC) refer to the likelihood values obtained from the VC and CC models respectively.
3. Corr (CC) refer to the conditional correlation coefficient obtained from the CC model. The symbol ρ_{ij} denotes the (time-invariant) correlation between the variables i and j. Canadian dollar (CND) = C, British pound (GBP) = G, Malaysian ringgit (MYR) = M, and Singapore dollar (SGD) = S.
4. LR is the likelihood ratio statistic for H_0: $\pi_1 = \pi_2 = 0$ in the VC model. It is distributed as chi-squared with 2 degrees of freedom under H_0.

Table 3.11 Estimation Results of Tetravariate Varying-Correlations Fractionally Integrated (VC-FI) Models

Variable	η	ϕ	γ	δ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
VC-FIGARCH													
CND/USD	0.0034 (0.0024)	0.4450 (0.0867)	-	-	0.6819 (0.1037)	0.3424 (0.1046)	ρ_{CG} = 0.0705 (0.0383)	0.9767 (0.0346)	0.0109 (0.0112)	8954.95412	ρ_{CG} = 0.0645 (0.0213)	8911.213	87.4832
GBP/USD	0.0201 (0.0098)	0.2369 (0.0693)	-	-	0.5722 (0.0942)	0.3410 (0.0560)	ρ_{CM} = 0.0327 (0.0398)				ρ_{CM} = 0.0295 (0.0219)		
							ρ_{CS} = -0.0161 (0.0404)				ρ_{CS} = -0.0188 (0.0240)		
MYR/USD	0.0026 (0.0011)	0.3382 (0.1168)	-	-	0.5597 (0.1065)	0.4929 (0.1523)	ρ_{GM} = 0.2331 (0.0431)				ρ_{GM} = 0.2456 (0.0223)		
SGD/USD	0.0019 (0.0011)	0.5505 (0.1468)	-	-	0.7691 (0.1318)	0.4694 (0.2536)	ρ_{GS} = 0.3465 (0.0340)				ρ_{GS} = 0.3554 (0.0193)		
							ρ_{MS} = 0.3374 (0.0529)				ρ_{MS} = 0.3699 (0.0237)		
CND/JPY	0.0569 (0.0229)	0.4383 (0.0857)	-	-	0.5920 (0.0962)	0.2334 (0.0482)	ρ_{CG} = 0.6134 (0.0339)	0.9037 (0.0205)	0.0400 (0.0074)	4668.30728	ρ_{CG} = 0.5061 (0.0222)	4310.315	715.98408
GBP/JPY	0.0551 (0.0206)	0.1576 (0.0855)	-	-	0.3905 (0.0970)	0.2639 (0.0468)	ρ_{CM} = 0.9099 (0.0101)				ρ_{CM} = 0.8750 (0.0080)		
							ρ_{CS} = 0.9028 (0.0107)				ρ_{CS} = 0.8692 (0.0070)		
MYR/JPY	0.0574 (0.0256)	0.3208 (0.1371)	-	-	0.4645 (0.1511)	0.2249 (0.0420)	ρ_{GM} = 0.6286 (0.0335)				ρ_{GM} = 0.5034 (0.0221)		
SGD/JPY	0.0362 (0.0135)	0.4538 (0.0658)	-	-	0.6122 (0.0799)	0.2480 (0.0539)	ρ_{GS} = 0.6374 (0.0310)				ρ_{GS} = 0.5287 (0.0201)		
							ρ_{MS} = 0.9354 (0.0076)				ρ_{MS} = 0.9058 (0.0064)		
VC-FILOGGARCH													
CND/USD	0.0607 (0.1182)	0.2774 (0.1134)	-0.9051 (0.0876)	-	0.3343 (0.1131)	0.0916 (0.0313)	ρ_{CG} = 0.0696 (0.0341)	0.9581 (0.0172)	0.0168 (0.0052)	8422.83456	ρ_{CG} = 0.0669 (0.0215)	8378.973	87.72288
GBP/USD	0.2753 (0.0397)	0.3605 (0.0455)	0.4697 (0.1891)	-	0.5988 (0.0574)	0.2340 (0.0305)	ρ_{CM} = 0.0026 (0.0330)				ρ_{CM} = 0.0201 (0.0218)		
							ρ_{CS} = -0.0125 (0.0351)				ρ_{CS} = -0.0164 (0.0222)		
MYR/USD	0.3350 (0.0925)	0.5667 (0.0715)	-0.9867 (0.0193)	-	0.6583 (0.0659)	0.1262 (0.0317)	ρ_{GM} = 0.1846 (0.0345)				ρ_{GM} = 0.2089 (0.0204)		
SGD/USD	0.1028 (0.0922)	0.3207 (0.2266)	0.2827 (0.2357)	-	0.4012 (0.2483)	0.1515 (0.0374)	ρ_{GS} = 0.3318 (0.0308)				ρ_{GS} = 0.3439 (0.0193)		
							ρ_{MS} = 0.3108 (0.0380)				ρ_{MS} = 0.3384 (0.0214)		
CND/JPY	0.3485 (0.0627)	0.4420 (0.0931)	-0.9919 (0.0063)	-	0.4902 (0.0974)	0.0565 (0.0115)	ρ_{CG} = 0.6095 (0.0336)	0.9087 (0.0208)	0.0357 (0.0070)	4511.976	ρ_{CG} = 0.5095 (0.0216)	4184.843	654.26648

Variable	η	ϕ	γ	δ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
							ρ_{CM} = 0.9082 (0.0095)				ρ_{CM} = 0.8789 (0.0072)		
GBP/JPY	0.2225 (0.0380)	0.3812 (0.0615)	-0.3271 (0.2830)	-	0.5291 (0.0841)	0.1591 (0.0376)	ρ_{CS} = 0.8973 (0.0107)				ρ_{CS} = 0.8685 (0.0068)		
MYR/JPY	0.2953 (0.0546)	0.4123 (0.1055)	-0.9889 (0.0086)	-	0.4635 (0.1103)	0.0580 (0.0126)	ρ_{GM} = 0.6184 (0.0326)				ρ_{GM} = 0.5062 (0.0214)		
SGD/JPY	0.2071 (0.0427)	0.4813 (0.0743)	-0.9751 (0.0209)	-	0.5366 (0.0838)	0.0647 (0.0191)	ρ_{GS} = 0.6288 (0.0307) ρ_{MS} = 0.9317 (0.0076)				ρ_{GS} = 0.5304 (0.0201) ρ_{MS} = 0.9049 (0.0063)		
VC-FITGARCH													
CND/USD	0.0438 (0.0219)	0.3267 (0.1085)	-0.3821 (0.1566)	-	0.5471 (0.1642)	0.3299 (0.1116)	ρ_{CG} = 0.0679 (0.0346) ρ_{CM} = 0.0246 (0.0341)	0.9673 (0.0228)	0.0136 (0.0068)	8903.0634	ρ_{CG} = 0.0637 (0.0210) ρ_{CM} = 0.0273 (0.0204)	8859.591	86.94392
GBP/USD	0.0574 (0.0222)	0.1923 (0.0519)	-0.0647 (0.1137)	-	0.7032 (0.0975)	0.5171 (0.1140)	ρ_{CS} = -0.0089 (0.0353)				ρ_{CS} = -0.0152 (0.0227)		
MYR/USD	0.0272 (0.0072)	0.2875 (0.0856)	-0.2283 (0.0930)	-	0.6274 (0.0849)	0.5368 (0.1059)	ρ_{GM} = 0.2193 (0.0361) ρ_{GS} = 0.3381 (0.0305)				ρ_{GM} = 0.2339 (0.0215) ρ_{GS} = 0.3517 (0.0188)		
SGD/USD	0.0174 (0.0176)	0.4079 (0.1923)	-0.0275 (0.0981)	-	0.7769 (0.2111)	0.5837 (0.3864)	ρ_{MS} = 0.3242 (0.0424)				ρ_{MS} = 0.3551 (0.0221)		
CND/JPY	0.1159 (0.0438)	0.3897 (0.0817)	-0.3843 (0.1169)	-	0.6055 (0.1137)	0.2917 (0.0615)	ρ_{CG} = 0.6071 (0.0321) ρ_{CM} = 0.9084 (0.0097)	0.9003 (0.0207)	0.0403 (0.0073)	4693.41376	ρ_{CG} = 0.5081 (0.0215) ρ_{CM} = 0.8748 (0.0077)	4343.691	699.44616
GBP/JPY	0.1092 (0.0311)	0.2356 (0.0781)	-0.2612 (0.1316)	-	0.5300 (0.0940)	0.3448 (0.0695)	ρ_{CS} = 0.8985 (0.0102)				ρ_{CS} = 0.8672 (0.0068)		
MYR/JPY	0.1227 (0.0448)	0.3098 (0.1072)	-0.3053 (0.0922)	-	0.5183 (0.1365)	0.2856 (0.0526)	ρ_{GM} = 0.6209 (0.0315) ρ_{GS} = 0.6284 (0.0294)				ρ_{GM} = 0.5068 (0.0214) ρ_{GS} = 0.5286 (0.0197)		
SGD/JPY	0.0823 (0.0237)	0.4188 (0.0455)	-0.3276 (0.0947)	-	0.6494 (0.0731)	0.3208 (0.0656)	ρ_{MS} = 0.9330 (0.0076)				ρ_{MS} = 0.9051 (0.0063)		
VC-FILGARCH													
CND/USD	0.0045 (0.0036)	0.4520 (0.1178)	-0.2002 (0.1070)	-	0.6132 (0.1488)	0.2590 (0.0952)	ρ_{CG} = 0.0738 (0.0384) ρ_{CM} = 0.0341 (0.0399)	0.9754 (0.0348)	0.0113 (0.0111)	8971.76168	ρ_{CG} = 0.0653 (0.0215) ρ_{CM} = 0.0307 (0.0219)	8928.11	87.30344
GBP/USD	0.0208 (0.0100)	0.2431 (0.0704)	-0.0669 (0.0810)	-	0.5657 (0.0930)	0.3303 (0.0578)	ρ_{CS} = -0.0103				ρ_{CS} = -0.0164		

Variable	η	ϕ	γ	δ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
							(0.0387)				(0.0242)		
MYR/USD	0.0025	0.3346	-0.1341	-	0.5457	0.4670	ρ_{GM} = 0.2361				ρ_{GM} = 0.2463		
	(0.0012)	(0.1142)	(0.0601)		(0.1111)	(0.1574)	(0.0415)				(0.0224)		
							ρ_{GS} = 0.3494				ρ_{GS} = 0.3567		
SGD/USD	0.0018	0.5557	-0.0688	-	0.7833	0.4859	(0.0335)				(0.0195)		
	(0.0011)	(0.1515)	(0.0666)		(0.1369)	(0.2745)	ρ_{MS} = 0.3389				ρ_{MS} = 0.3689		
							(0.0509)				(0.0236)		
CND/JPY	0.0587	0.4231	-0.1521	-	0.5618	0.2090	ρ_{CG} = 0.6090	0.8998	0.0408	4682.38848	ρ_{CG} = 0.5055	4328.261	708.2544
	(0.0238)	(0.1001)	(0.0694)		(0.1075)	(0.0424)	(0.0330)	(0.0215)	(0.0076)		(0.0218)		
							ρ_{CM} = 0.9085				ρ_{CM} = 0.8751		
GBP/JPY	0.0563	0.1865	-0.0971	-	0.3895	0.2398	(0.0099)				(0.0078)		
	(0.0214)	(0.1065)	(0.1034)		(0.0975)	(0.0554)	ρ_{CS} = 0.9001				ρ_{CS} = 0.8685		
							(0.0104)				(0.0069)		
MYR/JPY	0.0641	0.2861	-0.1294	-	0.4087	0.1969	ρ_{GM} = 0.6235				ρ_{GM} = 0.5032		
	(0.0284)	(0.1521)	(0.0678)		(0.1666)	(0.0387)	(0.0325)				(0.0218)		
							ρ_{GS} = 0.6317				ρ_{GS} = 0.5275		
SGD/JPY	0.0393	0.4659	-0.1409	-	0.5931	0.2120	(0.0302)				(0.0199)		
	(0.0154)	(0.0753)	(0.0709)		(0.0866)	(0.0499)	ρ_{MS} = 0.9344				ρ_{MS} = 0.9054		
							(0.0076)				(0.0064)		
VC-FIAPARCH													
CND/USD	0.0084	0.4225	-0.2229	1.7594	0.6159	0.2969	ρ_{CG} = 0.0724	0.9723	0.0122	8980.6598	ρ_{CG} = 0.0651	8937.457	86.40464
	(0.0105)	(0.1400)	(0.1291)	(0.4030)	(0.1532)	(0.1200)	(0.0367)	(0.0334)	(0.0102)		(0.0213)		
							ρ_{CM} = 0.0324				ρ_{CM} = 0.0302		
GBP/USD	0.0228	0.2430	-0.0677	1.9492	0.5761	0.3411	(0.0377)				(0.0218)		
	(0.0182)	(0.0665)	(0.0838)	(0.3201)	(0.1143)	(0.0912)	ρ_{CS} = -0.0099				ρ_{CS} = -0.0163		
							(0.0380)				(0.0239)		
MYR/USD	0.0033	0.3380	-0.1430	1.8988	0.5540	0.4575	ρ_{GM} = 0.2313				ρ_{GM} = 0.2427		
	(0.0024)	(0.1091)	(0.0625)	(0.1750)	(0.1044)	(0.1373)	(0.0390)				(0.0215)		
							ρ_{GS} = 0.3438				ρ_{GS} = 0.3532		
SGD/USD	0.0049	0.4399	-0.0643	1.5640	0.8182	0.6272	(0.0326)				(0.0190)		
	(0.0055)	(0.3965)	(0.0784)	(0.2073)	(0.2489)	(0.6634)	ρ_{MS} = 0.3349				ρ_{MS} = 0.3647		
							(0.0495)				(0.0233)		
CND/JPY	0.0874	0.4008	-0.2703	1.4062	0.6057	0.2823	ρ_{CG} = 0.6070	0.8998	0.0408	4707.28524	ρ_{CG} = 0.5066	4353.997	706.57664
	(0.0315)	(0.0797)	(0.1076)	(0.1540)	(0.1064)	(0.0588)	(0.0325)	(0.0210)	(0.0074)		(0.0218)		
							ρ_{CM} = 0.9085				ρ_{CM} = 0.8749		
GBP/JPY	0.0676	0.2008	-0.1290	1.7827	0.4279	0.2681	(0.0097)				(0.0078)		
	(0.0259)	(0.1044)	(0.1108)	(0.2445)	(0.1138)	(0.0694)	ρ_{CS} = 0.8993				ρ_{CS} = 0.8675		
							(0.0103)				(0.0069)		
MYR/JPY	0.1053	0.3069	-0.2444	1.2108	0.5147	0.2888	ρ_{GM} = 0.6214				ρ_{GM} = 0.5052		
	(0.0394)	(0.1083)	(0.0955)	(0.1550)	(0.1363)	(0.0523)	(0.0319)				(0.0217)		
							ρ_{GS} = 0.6294				ρ_{GS} = 0.5281		

Variable	η	ϕ	γ	δ	β	d	Γ	π_1	π_2	LL (VC)	Corr (CC)	LL (CC)	LR
SGD/JPY	0.0707	0.4191	-0.2639	1.1862	0.6489	0.3243	(0.0298)				(0.0199)		
	(0.0213)	(0.0477)	(0.0967)	(0.1313)	(0.0741)	(0.0681)	ρ_{MS} = 0.9334				ρ_{MS} = 0.9051		
							(0.0076)				(0.0063)		
VC-FIAGARCH													
CND/USD	0.0033	0.4177	0.0956	-	0.5647	0.2526	ρ_{CG} = 0.0724	0.9772	0.0108	8962.71376	ρ_{CG} = 0.0651	8918.882	87.66296
	(0.0036)	(0.1730)	(0.0542)		(0.2207)	(0.0945)	(0.0388)	(0.0324)	(0.0106)		(0.0213)		
							ρ_{CM} = 0.0354				ρ_{CM} = 0.0313		
GBP/USD	0.0197	0.2354	-0.0177	-	0.5735	0.3433	(0.0402)				(0.0218)		
	(0.0099)	(0.0691)	(0.0702)		(0.0952)	(0.0567)	ρ_{CS} = -0.0120				ρ_{CS} = -0.0166		
							(0.0395)				(0.0245)		
MYR/USD	0.0026	0.3288	0.0160	-	0.5503	0.4868	ρ_{GM} = 0.2340				ρ_{GM} = 0.2461		
	(0.0012)	(0.1160)	(0.0236)		(0.1094)	(0.1588)	(0.0435)				(0.0224)		
							ρ_{GS} = 0.3469				ρ_{GS} = 0.3559		
SGD/USD	0.0018	0.5548	0.0072	-	0.7740	0.4728	(0.0342)				(0.0194)		
	(0.0011)	(0.1534)	(0.0300)		(0.1391)	(0.2592)	ρ_{MS} = 0.3372				ρ_{MS} = 0.3697		
							(0.0523)				(0.0237)		
CND/JPY	0.0393	0.4016	0.3020	-	0.5390	0.2029	ρ_{CG} = 0.6081	0.8991	0.0409	4697.728	ρ_{CG} = 0.5052	4344.679	706.09728
	(0.0203)	(0.1048)	(0.0810)		(0.1089)	(0.0353)	(0.0325)	(0.0217)	(0.0076)		(0.0217)		
							ρ_{CM} = 0.9084				ρ_{CM} = 0.8752		
GBP/JPY	0.0526	0.1664	0.1015	-	0.3832	0.2513	(0.0097)				(0.0077)		
	(0.0215)	(0.0997)	(0.1054)		(0.1057)	(0.0484)	ρ_{CS} = 0.8994				ρ_{CS} = 0.8682		
							(0.0103)				(0.0068)		
MYR/JPY	0.0528	0.2679	0.2081	-	0.3935	0.1945	ρ_{GM} = 0.6221				ρ_{GM} = 0.5032		
	(0.0218)	(0.1406)	(0.0625)		(0.1494)	(0.0336)	(0.0321)				(0.0218)		
							ρ_{GS} = 0.6305				ρ_{GS} = 0.5271		
SGD/JPY	0.0310	0.4623	0.2105	-	0.5871	0.2078	(0.0299)				(0.0199)		
	(0.0144)	(0.0799)	(0.0679)		(0.0921)	(0.0451)	ρ_{MS} = 0.9337				ρ_{MS} = 0.9050		
							(0.0076)				(0.0063)		

Notes:

1. All standard errors (in parenthesis) are the heteroskedastic-consistent Bollerslev-Wooldridge (1992) standard errors computed based on the Quasi-Maximum Likelihood Estimation (QMLE) technique.
2. LL (VC) and LL (CC) refer to the likelihood values obtained from the VC-FI and CC-FI models respectively.
3. Corr (CC) refer to the conditional correlation coefficient obtained from the CC-FI model. The symbol ρ_{ij} denotes the (time-invariant) correlation between the variables i and j. Canadian dollar (CND) = C, British pound (GBP) = G, Malaysian ringgit (MYR) = M, and Singapore dollar (SGD) = S.
4. LR is the likelihood ratio statistic for H_0: $\pi_1 = \pi_2 = 0$ in the VC model. It is distributed as chi-squared with 2 degrees of freedom under H_0.

Table 3.12 Likelihood Ratio Test: Bivariate VC and VC-FI Models

Exchange Rate	VC-GARCH	VC-FIGARCH	LR	VC-LOGGARCH	VC-FILOGGARCH	LR	VC-TGARCH	VC-FITGARCH	LR
CND/USD GBP/USD	2339.1540	2345.4036	12.4993	2278.2213	2283.1587	9.8748	2334.8996	2341.9103	14.0213
CND/USD MYR/USD	5703.6650	5751.9006	96.4712	5337.9133	5358.9752	42.1238	5682.6031	5714.6303	64.0545
CND/USD SGD/USD	5319.7875	5343.6656	47.7562	5225.8629	5229.2484	6.7710	5323.5624	5347.0810	47.0372
GBP/USD MYR/USD	3197.1514	3243.5894	92.8760	2826.8818	2844.8758	35.9880	3165.4538	3193.1668	55.4260
GBP/USD SGD/USD	2907.7169	2925.6389	35.8441	2817.3305	2823.0409	11.4208	2902.5008	2922.3224	39.6431
MYR/USD SGD/USD	6327.2524	6392.6850	130.8653	5922.1333	5928.4548	12.6431	6300.2884	6344.1798	87.7828
CND/JPY GBP/JPY	-296.7859	-299.7138	-5.8560	-377.1784	-387.1311	-19.9054	-293.0370	-295.6468	-5.2196
CND/JPY MYR/JPY	1394.8597	1398.3560	6.9927	1384.5115	1376.5751	-15.8728	1417.0391	1427.9595	21.8408
CND/JPY SGD/JPY	1468.7560	1470.0863	2.6604	1402.5774	1405.3577	5.5606	1469.7687	1478.3163	17.0952
GBP/JPY MYR/JPY	63.0337	63.0185	-0.0306	-12.8893	-29.7790	-33.7795	71.8791	70.1412	-3.4760
GBP/JPY SGD/JPY	316.6233	316.4405	-0.3655	237.9366	228.3563	-19.1606	325.3806	324.3829	-1.9953
MYR/JPY SGD/JPY	2372.3197	2394.5890	44.5385	2332.1134	2321.0431	-22.1404	2394.0107	2418.5180	49.0146

Note: LR is the likelihood ratio test statistics.

Table 3.13 Likelihood Ratio Test: Bivariate VC and VC-FI Models

Exchange Rate	VC-LGARCH	VC-FILGARCH	LR	VC-APARCH	VC-FIAPARCH	LR	VC-AGARCH	VC-FIAGARCH	LR
CND/USD GBP/USD	2341.9972	2351.2698	18.5452	2344.8703	2353.1243	16.5080	2342.3896	2351.2039	17.6285
CND/USD MYR/USD	5713.2821	5767.2401	107.9159	5717.0271	5770.2061	106.3580	5708.3387	5760.4392	104.2009
CND/USD SGD/USD	5323.2928	5351.8447	57.1038	5330.9326	5358.4059	54.9466	5322.6337	5349.5977	53.9280
GBP/USD MYR/USD	3204.1321	3253.5661	98.8680	3204.5216	3254.4949	99.9466	3200.3272	3246.2858	91.9173
GBP/USD SGD/USD	2908.7595	2929.3270	41.1351	2913.5351	2936.9908	46.9114	2908.6816	2926.1662	34.9693
MYR/USD SGD/USD	6332.3756	6402.3322	139.9132	6338.1878	6409.4027	142.4298	6329.3196	6393.9434	129.2474
CND/JPY GBP/JPY	-288.6113	-290.2261	-3.2297	-285.7866	-285.5904	0.3925	-283.3197	-282.9216	0.7963
CND/JPY MYR/JPY	1404.2671	1415.1097	21.6850	1419.4029	1430.4791	22.1524	1417.7581	1431.3570	27.1977
CND/JPY SGD/JPY	1472.5819	1475.8116	6.4594	1477.8609	1484.6618	13.6018	1479.1522	1484.6648	11.0253
GBP/JPY MYR/JPY	69.6648	71.6754	4.0212	75.7997	78.0467	4.4940	74.0692	78.3751	8.6117
GBP/JPY SGD/JPY	322.2887	322.4115	0.2457	329.4432	332.2025	5.5186	326.8876	328.6223	3.4694
MYR/JPY SGD/JPY	2375.4984	2401.2371	51.4773	2394.4991	2419.7554	50.5126	2379.8666	2405.6442	51.5552

Note: LR is the likelihood ratio test statistics.

Table 3.14 Likelihood Ratio Tests of Tetravariate Models

Exchange Rate	VC-GARCH	VC-FIGARCH	LR	VC-LOGGARCH	VC-FILOGGARCH	LR	VC-TGARCH	VC-FITGARCH	LR
CND/USD GBP/USD MYR/USD SGD/USD	8884.6979	8954.9541	140.5124	8406.4464	8422.8346	32.7762	8855.2173	8903.0634	95.6922
CND/JPY GBP/JPY MYR/JPY SGD/JPY	4656.3233	4668.3073	23.9680	4541.3068	4511.9760	-58.6617	4680.8006	4693.4138	25.2263
Exchange Rate	CC-GARCH	CC-FIGARCH	LR	CC-LOGGARCH	CC-FILOGGARCH	LR	CC-TGARCH	CC-FITGARCH	LR
CND/USD GBP/USD MYR/USD SGD/USD	8839.1887	8911.2125	144.0477	8362.4352	8378.9731	33.0758	8808.8692	8859.5914	101.4446
CND/JPY GBP/JPY MYR/JPY SGD/JPY	4279.6961	4310.3152	61.2382	4227.7155	4184.8428	-85.7455	4309.2666	4343.6907	68.8481
Exchange Rate	CC-GARCH	VC-GARCH	LR	CC-LOGGARCH	VC-LOGGARCH	LR	CC-TGARCH	VC-TGARCH	LR
CND/USD GBP/USD MYR/USD SGD/USD	8839.1887	8884.6979	91.0185	8362.4352	8406.4464	88.0225	8808.8692	8855.2173	92.6962
CND/JPY GBP/JPY MYR/JPY SGD/JPY	4279.6961	4656.3233	753.2543	4227.7155	4541.3068	627.1826	4309.2666	4680.8006	743.0679
Exchange Rate	CC-FI-GARCH	VC-FIGARCH	LR	CC-FI-LOGGARCH	VC-FILOGGARCH	LR	CC-FI-TGARCH	VC-FITGARCH	LR
CND/USD GBP/USD MYR/USD SGD/USD	8911.2125	8954.9541	87.4832	8378.9731	8422.8346	87.7229	8859.5914	8903.0634	86.9439
CND/JPY GBP/JPY MYR/JPY SGD/JPY	4310.3152	4668.3073	715.9841	4184.8428	4511.9760	654.2665	4343.6907	4693.4138	699.4462

Note: LR is the likelihood ratio test statistics.

Table 3.15 Likelihood Ratio Tests of Tetravariate Models

Exchange Rate	VC-LGARCH	VC-FILGARCH	LR	VC-APARCH	VC-FIAPARCH	LR	VC-AGARCH	VC-FIAGARCH	LR
CND/USD GBP/USD MYR/USD SGD/USD	8894.1353	8971.7617	155.2527	8903.2432	8980.6598	154.8333	8889.7612	8962.7138	145.9052
CND/JPY GBP/JPY MYR/JPY SGD/JPY	4666.5097	4682.3885	31.7576	4690.0582	4707.2852	34.4540	4679.2427	4697.7280	36.9706
Exchange Rate	CC-LGARCH	CC-FILGARCH	LR	CC-APARCH	CC-FIAPARCH	LR	CC-AGARCH	CC-FIAGARCH	LR
CND/USD GBP/USD MYR/USD SGD/USD	8848.5062	8928.1100	159.2074	8857.4343	8937.4575	160.0463	8843.5029	8918.8823	150.7587
CND/JPY GBP/JPY MYR/JPY SGD/JPY	4292.9085	4328.2613	70.7056	4318.0150	4353.9969	71.9639	4305.3718	4344.6794	78.6150
Exchange Rate	CC-LGARCH	VC-LGARCH	LR	CC-APARCH	VC-APARCH	LR	CC-AGARCH	VC-AGARCH	LR
CND/USD GBP/USD MYR/USD SGD/USD	8848.5062	8894.1353	91.2582	8857.4343	8903.2432	91.6177	8843.5029	8889.7612	92.5165
CND/JPY GBP/JPY MYR/JPY SGD/JPY	4292.9085	4666.5097	747.2024	4318.0150	4690.0582	744.0866	4305.3718	4679.2427	747.7417
Exchange Rate	CC-FI-LGARCH	VC-FILGARCH	LR	CC-FI-APARCH	VC-FIAPARCH	LR	CC-FI-AGARCH	VC-FIAGARCH	LR
CND/USD GBP/USD MYR/USD SGD/USD	8928.1100	8971.7617	87.3034	8937.4575	8980.6598	86.4046	8918.8823	8962.7138	87.6630
CND/JPY GBP/JPY MYR/JPY SGD/JPY	4328.2613	4682.3885	708.2544	4353.9969	4707.2852	706.5766	4344.6794	4697.7280	706.0973

Note: LR is the likelihood ratio test statistics.

Table 3.16 Standardised Residuals of Bivariate VC-APARCH(1,1) Model: USD Rates

Variable	CND/USD-GBP/USD		CND/USD-MYR/USD		CND/USD-SGD/USD		GBP/USD-MYR/USD		GBP/USD-SGD/USD		MYR/USD-SGD/USD	
Panel A: Moments, Maximum, Minimum												
Mean	0.0092	0.0002	0.0091	-0.0098	0.0091	-0.0049	0.0002	-0.0098	0.0005	-0.0050	-0.0097	-0.0047
Median	0.0087	0.0132	0.0087	-0.0157	0.0087	0.0378	0.0132	-0.0157	0.0133	0.0377	-0.0156	0.0380
Maximum	7.3407	5.6400	7.3672	10.5591	7.4067	7.8939	5.6690	10.6450	5.6446	7.9215	10.6096	7.7735
Minimum	-6.3556	-4.2421	-6.3863	-7.3187	-6.3705	-11.6602	-4.2317	-7.4203	-4.1681	-11.7521	-7.4056	-11.5516
Std. Dev.	0.9999	0.9993	1.0008	1.0002	1.0009	1.0000	1.0012	1.0011	1.0028	1.0006	1.0012	0.9944
Skewness	0.3966	0.2361	0.4001	0.1241	0.4021	-0.6263	0.2378	0.1289	0.2397	-0.6422	0.1289	-0.6375
Kurtosis	7.6766	5.3692	7.7192	12.4297	7.7184	13.0496	5.3712	12.5725	5.3610	13.2481	12.4883	12.9543
Panel B: Ljung-Box Q-statistic												
5 lags	0.8200	4.4639	0.7905	5.0739	0.7855	7.3954	4.4422	5.0817	4.3657	7.4622	5.0339	7.4620
10 lags	6.7148	11.5213	6.6732	12.0425	6.6534	17.7575	11.6796	11.8924	11.6913	17.8131	12.0509	17.7183
Panel C: McLeod-Li Test												
5 lags	23.1772	7.9143	23.5157	1.7965	24.0914	4.9157	7.0850	1.7043	7.3174	4.4026	2.0939	4.5649
10 lags	24.6002	16.2987	24.9810	4.8011	25.5258	6.2406	15.3282	4.6778	15.7552	5.7493	5.1261	5.8203
Panel D: Runs Test												
R_1	1.1323	0.8817	1.1323	-0.7048	1.1323	-2.7684	0.8817	-0.7048	0.8817	-2.7684	-0.7048	-2.7684
R_2	0.0740	-0.1795	0.1927	-1.9477	0.0147	-2.4656	-0.4130	-2.0360	-0.3352	-2.2089	-2.2749	-2.4425
R_3	-0.7560	0.2733	-0.7546	-3.4619	-1.0037	-3.4832	0.1284	-3.2527	0.3206	-3.1205	-3.5200	-2.9097
Panel E: BDS Test												
e=3,l=1.5	1.5261	0.2237	1.4344	2.3507	1.5723	4.4229	0.2237	2.2101	0.2189	4.1324	2.4728	4.2051
e=4,l=1.5	1.1173	0.3360	1.0161	1.7193	1.1600	4.2937	0.3360	1.5372	0.3156	3.9682	1.8158	4.0388
e=5,l=1.5	1.3165	0.4460	1.2098	1.0707	1.3572	3.9446	0.4460	0.8634	0.4161	3.5961	1.1554	3.6762
e=3,l=1.0	0.8372	-0.3040	0.7380	2.9179	0.8664	4.1158	-0.4138	2.7708	-0.3563	3.8583	3.1137	3.9397
e=4,l=1.0	0.5686	-0.3364	0.4569	2.3190	0.6010	4.1411	-0.4631	2.1597	-0.3941	3.8446	2.4988	3.9410
e=5,l=1.0	0.8441	0.0246	0.7066	1.5477	0.8657	3.8528	-0.1173	1.3722	-0.0326	3.5220	1.7179	3.6480
Panel F: Jarque-Bera Test												
Statistic	2810.594	729.0238	2861.987	11115.19	2861.845	12811.89	730.6107	11454.76	725.0345	13325.27	11254.28	12580.82

See Notes to Table 1.

Table 3.17 Standardised Residuals of Bivariate VC-APARCH(1,1) Model: JPY Rates

Variable	CND/JPY-GBP/JPY		CND/JPY-MYR/JPY		CND/JPY-SGD/JPY		GBP/JPY-MYR/JPY		GBP/JPY-SGD/JPY		MYR/JPY-SGD/JPY	
Panel A: Moments, Maximum, Minimum												
Mean	0.0154	0.0138	0.0149	0.0152	0.0156	0.0128	0.0134	0.0152	0.0136	0.0122	0.0160	0.0128
Median	-0.0177	-0.0035	-0.0177	-0.0186	-0.0176	-0.0286	-0.0035	-0.0184	-0.0035	-0.0281	-0.0193	-0.0282
Maximum	7.9189	7.7692	7.7650	7.0942	7.8411	7.3563	7.7273	7.3233	7.7029	7.3859	6.9230	7.2090
Minimum	-5.2220	-4.5897	-5.2348	-5.0048	-5.3683	-5.5330	-4.6871	-5.1175	-4.6402	-5.3524	-5.1439	-5.3940
Std. Dev.	1.0048	1.0050	0.9986	1.0001	1.0014	0.9979	1.0046	1.0074	1.0038	1.0064	1.0019	0.9966
Skewness	0.5337	0.5329	0.5178	0.4426	0.5317	0.4307	0.5227	0.4489	0.5247	0.4223	0.4613	0.4241
Kurtosis	7.8639	6.0880	7.7643	6.4509	7.9266	6.2830	6.0590	6.4506	6.0527	6.2266	6.5400	6.2097
Panel B: Ljung-Box Q-statistic												
5 lags	1.1335	1.9695	1.2059	2.1376	1.2453	1.6315	1.6987	2.1825	1.8298	1.6136	2.0980	1.6263
10 lags	20.6393	14.6274	20.5247	31.4105	20.5553	24.2606	14.4044	32.0749	14.5957	24.0060	31.2486	23.5720
Panel C: McLeod-Li Test												
5 lags	5.0628	7.0189	5.0851	8.1184	4.6511	4.2833	5.1980	8.9314	6.0864	5.0994	7.0133	4.3923
10 lags	6.3462	11.7864	6.5258	9.1613	5.8771	7.8129	10.2055	10.0045	10.9610	8.1275	8.9009	8.1274
Panel D: Runs Test												
R_1	-0.4546	2.4763	-0.4546	-0.8410	-0.4546	-1.6878	2.4763	-0.8410	2.4763	-1.6878	-0.8410	-1.6878
R_2	-3.1374	-1.7148	-2.8443	-2.0938	-3.0360	-2.6375	-1.8934	-2.2287	-1.8934	-2.7846	-1.8976	-1.8385
R_3	-0.8901	-1.1033	-0.6489	-1.6941	-1.4478	-0.8568	-1.0169	-1.8888	-1.0169	-0.8572	-1.4495	-0.3518
Panel E: BDS Test												
e=3,l=1.5	2.2508	1.3625	1.9307	1.6814	1.7917	1.7906	1.1090	2.1455	1.2620	2.3644	0.8315	1.3665
e=4,l=1.5	2.6012	1.5429	2.2432	2.4314	2.0672	2.1336	1.2477	2.9404	1.4295	2.7913	1.4674	1.6394
e=5,l=1.5	2.5436	1.9915	2.1572	2.4617	1.9632	2.0190	1.6672	3.0109	1.8699	2.7413	1.4092	1.4690
e=3,l=1.0	2.9604	1.1405	2.5980	2.0728	2.6027	2.0362	0.9313	2.5785	1.0441	2.4469	1.2854	1.5315
e=4,l=1.0	3.1121	1.2992	2.7042	2.5256	2.6913	2.1468	1.0375	3.1007	1.1791	2.6108	1.6367	1.5639
e=5,l=1.0	3.1675	1.7362	2.7090	2.4894	2.7192	2.0094	1.4587	3.1286	1.6151	2.5141	1.5009	1.3723
Panel F: Jarque-Bera Test												
Statistic	3097.542	1333.07	2969.397	1585.479	3173.159	1439.052	1305.422	1588.026	1301.659	1389.61	1671.734	1376.784

See Notes to Table 1.

Table 3.18 Standardised Residuals from VC-FIAPARCH(1,d,1) Model: USD Rates

Variable	CND/USD-GBP/USD		CND/USD-MYR/USD		CND/USD-SGD/USD		GBP/USD-MYR/USD		GBP/USD-SGD/USD		MYR/USD-SGD/USD	
Panel A: Moments, Maximum, Minimum												
Mean	0.0105	0.0013	0.0104	-0.0116	0.0105	-0.0071	0.0014	-0.0117	0.0016	-0.0068	-0.0115	-0.0068
Median	0.0091	0.0129	0.0091	-0.0155	0.0090	0.0410	0.0129	-0.0155	0.0129	0.0396	-0.0155	0.0402
Maximum	7.2606	5.6966	7.2944	6.6902	7.3009	8.0762	5.6798	6.8315	5.6682	8.3512	6.6561	7.9431
Minimum	-6.0997	-4.0431	-6.1170	-8.6008	-6.1104	-11.0553	-4.0763	-8.6441	-4.0131	-10.8060	-8.6264	-10.9130
Std. Dev.	1.0002	1.0012	1.0010	0.9946	1.0007	0.9960	1.0031	0.9950	1.0028	0.9930	0.9942	0.9901
Skewness	0.4403	0.2050	0.4432	-0.2581	0.4453	-0.5506	0.2040	-0.2552	0.2109	-0.5129	-0.2557	-0.5541
Kurtosis	7.4949	5.2543	7.5217	9.9531	7.5038	11.9971	5.2286	10.0251	5.2636	11.8534	9.9572	11.8104
Panel B: Ljung-Box Q-statistic												
5 lags	1.0876	3.7851	1.0776	6.8502	1.0750	7.2447	3.7811	6.8120	3.8355	6.9212	6.7715	7.2881
10 lags	6.8931	10.9033	6.8941	13.9877	6.8541	19.1832	10.9431	13.7522	11.0773	18.9090	13.9671	19.0211
Panel C: McLeod-Li Test												
5 lags	3.8685	2.1403	4.0269	1.9778	3.6516	0.8801	1.8562	1.9657	3.0226	0.9522	2.0724	0.7995
10 lags	5.0297	9.8417	5.2286	7.6599	4.8249	1.6835	9.5097	7.4384	10.9024	1.9050	7.7798	1.6236
Panel D: Runs Test												
R_1	1.1323	0.8817	1.1323	-0.7048	1.1323	-2.7684	0.8817	-0.7048	0.8817	-2.7684	-0.7048	-2.7684
R_2	1.0778	-0.5705	1.0187	-0.9785	1.0235	0.2922	-0.6486	-0.7049	-0.6094	-0.1018	-1.0582	0.2922
R_3	0.2791	-0.8711	0.1828	-2.5878	0.2817	-0.0418	-1.3896	-2.3060	-0.7295	-0.0298	-2.9671	-0.0418
Panel E: BDS Test												
e=3,l=1.5	0.2719	1.2226	0.2742	0.4331	0.3839	1.5066	1.5689	0.0968	1.3170	1.2653	0.4749	1.2663
e=4,l=1.5	0.1796	0.9706	0.1635	0.2697	0.3214	2.3592	1.2158	-0.0977	1.1370	1.8269	0.2658	2.0957
e=5,l=1.5	0.6010	0.7225	0.5669	-0.0417	0.7612	2.7121	0.9114	-0.4148	0.9320	2.0148	-0.0683	2.4729
e=3,l=1.0	-0.1029	0.6229	-0.1249	1.7637	0.0157	1.5109	0.8930	1.4074	0.6891	1.2033	1.7631	1.3119
e=4,l=1.0	-0.0684	0.2826	-0.1108	1.5862	0.0904	2.5602	0.4635	1.2236	0.4197	1.9720	1.5603	2.3593
e=5,l=1.0	0.3834	0.3136	0.3224	1.1016	0.5613	2.9559	0.4349	0.7371	0.4969	2.2060	1.0696	2.7873
Panel F: Jarque-Bera Test												
Statistic	2620.699	655.8086	2652.165	6072.457	2632.916	10263.21	641.2125	6197.433	662.2832	9922.75	6078.965	9849.836

See Notes to Table 1.

Table 3.19 Standardised Residuals from Bivariate VC-FIAPARCH(1,d,1) Model: JPY Rates

Variable	CND/JPY-GBP/JPY		CND/JPY-MYR/JPY		CND/JPY-SGD/JPY		GBP/JPY-MYR/JPY		GBP/JPY-SGD/JPY		MYR/JPY-SGD/JPY	
Panel A: Moments, Maximum, Minimum												
Mean	0.0161	0.0129	0.0159	0.0156	0.0166	0.0130	0.0127	0.0160	0.0129	0.0128	0.0166	0.0130
Median	-0.0178	-0.0035	-0.0178	-0.0180	-0.0176	-0.0280	-0.0035	-0.0178	-0.0035	-0.0281	-0.0180	-0.0279
Maximum	7.5819	7.6962	7.5159	6.6630	7.7652	7.3723	7.7078	6.7075	7.7177	7.3113	6.8979	7.3548
Minimum	-5.2665	-4.5314	-5.1779	-5.1224	-5.5458	-5.5917	-4.6024	-5.2472	-4.5436	-5.4317	-5.3397	-5.5711
Std. Dev.	1.0024	1.0025	1.0000	1.0012	1.0006	0.9974	1.0030	1.0051	1.0013	1.0034	1.0045	0.9989
Skewness	0.5164	0.5122	0.5133	0.4512	0.5326	0.4246	0.5089	0.4565	0.5128	0.4215	0.4678	0.4175
Kurtosis	7.6048	5.9873	7.5526	6.3692	7.7758	6.2397	5.9884	6.3522	6.0028	6.1626	6.4724	6.2078
Panel B: Ljung-Box Q-statistic												
5 lags	1.0280	1.3901	1.0560	1.6684	1.1599	1.2849	1.2535	1.6976	1.4126	1.3388	1.5502	1.1439
10 lags	20.7823	14.4439	20.7366	31.6063	20.8050	23.6194	14.3848	31.9183	14.4485	23.0292	32.6554	23.2928
Panel C: McLeod-Li Test												
5 lags	4.3492	1.2428	4.4397	6.8978	4.5728	3.7947	0.9651	6.2606	1.4503	3.8558	7.7480	4.4647
10 lags	5.7418	6.4022	5.8890	7.6530	5.7656	6.8601	6.0346	7.1389	6.6378	6.7258	8.6254	7.6513
Panel D: Runs Test												
R_1	-0.4546	2.4763	-0.4546	-0.8410	-0.4546	-1.6878	2.4763	-0.8410	2.4763	-1.6878	-0.8410	-1.6878
R_2	-2.2849	-2.0474	-2.0726	-0.9608	-1.9883	-0.7658	-1.7728	-1.7605	-1.9369	-1.9375	-0.1574	0.0398
R_3	-0.3941	-0.7525	-0.1414	-0.8081	-0.3421	0.6743	-0.8076	-1.1985	-0.7076	-0.0152	0.3807	1.1253
Panel E: BDS Test												
e=3,l=1.5	0.9795	1.0865	0.8605	0.3499	0.2576	0.3440	0.7917	1.0037	0.9929	1.2984	-1.0824	-0.6114
e=4,l=1.5	1.3688	0.9726	1.2302	1.0693	0.6845	0.9564	0.6457	1.5502	0.8987	1.8250	-0.4456	0.0245
e=5,l=1.5	1.3082	1.1179	1.1474	1.0414	0.6886	0.9935	0.7771	1.4276	1.0543	1.7877	-0.4851	0.0831
e=3,l=1.0	1.6781	0.9637	1.5373	0.8021	1.0253	0.5207	0.7101	1.5634	0.8826	1.4160	-0.5694	-0.4780
e=4,l=1.0	1.8437	0.8322	1.6728	1.2325	1.2452	0.8952	0.5336	1.8453	0.7630	1.6896	-0.1714	-0.0793
e=5,l=1.0	1.8731	1.0433	1.6717	1.1730	1.3329	0.8997	0.7159	1.7018	0.9800	1.6013	-0.2162	-0.0389
Panel F: Jarque-Bera Test												
Statistic	2782.001	1245.838	2720.694	1519.715	2990.869	1401.165	1244.976	1507.846	1257.744	1338.193	1615.536	1372.485

See Notes to Table 1.

Table 3.20 Standardised Residuals from Tetravariate VC-APARCH(1,1) and VC-FIAPARCH(1,d,1) Models: JPY Rates

Model	VC-APARCH				VC-FIAPARCH			
Variable	CND/JPY	GBP/JPY	MYR/JPY	SGD/JPY	CND/JPY	GBP/JPY	MYR/JPY	SGD/JPY
Panel A: Moments, Minimum, Maximum								
Mean	0.0156	0.0135	0.0157	0.0125	0.0164	0.0128	0.0164	0.0129
Median	-0.0178	-0.0035	-0.0186	-0.0281	-0.0178	-0.0035	-0.0180	-0.0279
Maximum	7.9935	7.6151	7.4019	7.3499	7.8702	7.6199	6.9287	7.3136
Minimum	-5.2380	-4.6019	-5.1023	-5.3733	-5.3879	-4.5038	-5.2841	-5.5309
Std. Dev.	1.0070	0.9954	1.0095	1.0003	1.0064	0.9944	1.0093	0.9992
Skewness	0.5430	0.5247	0.4568	0.4280	0.5387	0.5115	0.4703	0.4208
Kurtosis	7.9304	6.0491	6.5225	6.2557	7.7879	5.9829	6.4788	6.1984
Panel B: Ljung-Box Q-statistic								
5 lags	1.2321	1.7836	2.2553	1.6948	1.2056	1.3770	1.7390	1.3167
10 lags	20.5202	14.5340	32.1358	24.3823	20.5034	14.4397	31.9495	23.3939
Panel C: McLeod-Li Test								
5 lags	5.3973	5.7057	8.8961	4.8403	5.0005	1.2984	7.2546	3.8705
10 lags	6.5536	10.6261	10.0821	8.1168	6.2644	6.4970	8.2348	6.8932
Panel D: Runs Test								
R_1	0.8140	1.0995	0.0972	-1.1641	0.8205	1.0995	0.1751	-1.1641
R_2	-2.8388	-1.8498	-1.8402	-2.5742	-2.0511	-2.0307	-1.1171	-1.0195
R_3	-0.9934	-1.1535	-1.6923	-0.5602	-0.4944	-0.8526	-0.7583	0.2817
Panel E: BDS Test								
e=3,l=1.5	2.0260	1.2707	1.9381	2.1848	0.7773	1.1274	0.4936	0.6477
e=4,l=1.5	2.3421	1.4490	2.7028	2.5747	1.2183	1.0135	1.1996	1.2957
e=5,l=1.5	2.2647	1.8865	2.7449	2.4930	1.2022	1.1569	1.1732	1.3437
e=3,l=1.0	2.7402	1.0865	2.3713	2.2984	1.4831	0.9936	1.0438	0.8112
e=4,l=1.0	2.8581	1.2338	2.8655	2.4369	1.7207	0.8649	1.4918	1.1972
e=5,l=1.0	2.8862	1.6773	2.8638	2.3295	1.7919	1.0810	1.4297	1.1891
Panel F: Jarque-Bera Test								
Statistic	3183.906	1298.915	1654.23	1415.595	3008.59	1242.199	1622.265	1366.345

See Notes to Table 1.

Table 3.21 Standardised Residuals from Tetravariate VC-APARCH(1,1) and VC-FIAPARCH(1,d,1) Models: USD Rates

Model	VC-APARCH				VC-FIAPARCH			
Variable	CND/USD	GBP/USD	MYR/USD	SGD/USD	CND/USD	GBP/USD	MYR/USD	SGD/USD
Panel A: Moments, Minimum, Maximum								
Mean	0.0092	0.0005	-0.0099	-0.0048	0.0105	0.0016	-0.0116	-0.0069
Median	0.0087	0.0133	-0.0156	0.0379	0.0091	0.0129	-0.0156	0.0409
Maximum	7.3514	5.6566	10.6914	7.9331	7.2677	5.6816	6.8670	8.1392
Minimum	-6.3585	-4.1726	-7.5165	-11.7469	-6.0964	-4.0452	-8.6688	-11.1170
Std. Dev.	1.0022	1.0040	1.0024	1.0025	1.0025	1.0056	0.9954	0.9980
Skewness	0.3967	0.2398	0.1311	-0.6428	0.4407	0.2103	-0.2470	-0.5555
Kurtosis	7.6722	5.3626	12.6326	13.2078	7.4825	5.2545	10.0202	12.1144
Panel B: Ljung-Box Q-statistic								
5 lags	0.8146	4.3620	0.3983	7.4530	1.0854	3.8348	6.8014	7.2441
10 lags	6.7239	11.6837	0.2819	17.6881	6.8940	10.9827	13.8155	18.9690
Panel C: McLeod-Li Test								
5 lags	23.4704	7.2577	2.1002	4.2252	3.7959	2.7957	2.0667	0.8724
10 lags	24.8634	15.6651	5.0843	5.5689	4.9269	10.5816	7.3204	1.6994
Panel D: Runs Test								
R_1	1.9005	0.8031	-2.5777	-2.5673	2.3439	0.5733	-2.8171	-2.3906
R_2	0.0919	-0.3352	-2.0360	-1.9895	0.9824	-0.6876	-0.8865	0.1413
R_3	-0.7560	0.3206	-3.5705	-2.8490	0.3273	-0.8253	-2.9022	0.3886
Panel E: BDS Test								
e=3,l=1.5	1.5770	0.2126	2.4562	3.9886	0.3442	1.4610	0.4586	1.1806
e=4,l=1.5	1.1745	0.3083	1.7931	3.7985	0.2688	1.2480	0.2005	1.9925
e=5,l=1.5	1.3817	0.4090	1.1205	3.4107	0.7044	1.0211	-0.1689	2.3171
e=3,l=1.0	0.8810	-0.3589	3.0467	3.7060	-0.0363	0.8407	1.7299	1.1547
e=4,l=1.0	0.6207	-0.3975	2.4317	3.6601	0.0189	0.5357	1.4954	2.1659
e=5,l=1.0	0.9020	-0.0392	1.6344	3.3223	0.4810	0.5918	0.9578	2.5344
Panel F: Jarque-Bera Test								
Statistic	2805.495	726.0025	11599.23	13222.67	2606.97	657.021	6186.778	10531.3

See Notes to Table 1.

Table 3.22 Cross-Product of Standardised Residuals from Tetravariate VC-APARCH(1,1) and VC-FIAPARCH(1,d,1) Models: JPY Rates

	VC-APARCH						VC-FIAPARCH					
	CND/JPY & GBP/JPY	CND/JPY & MYR/JPY	CND/JPY & SGD/JPY	GBP/JPY & MYR/JPY	GBP/JPY & SGD/JPY	MYR/JPY & SGD/JPY	CND/JPY & GBP/JPY	CND/JPY & MYR/JPY	CND/JPY & SGD/JPY	GBP/JPY & MYR/JPY	GBP/JPY & SGD/JPY	MYR/JPY & SGD/JPY
Panel A: Ljung-Box Q-statistic												
5 lags	4.7734	6.1795	4.5308	4.8676	6.0977	6.1675	4.3622	5.1758	4.3262	4.2627	5.0145	5.0790
10 lags	6.2917	7.3221	5.6795	6.7921	7.8322	7.6301	5.6508	6.4299	5.6797	6.1981	6.5906	6.5517
Panel B: BDS Test												
e=3,l=1.5	-0.9725	-0.8618	-1.2546	-0.7161	-0.2288	-0.7854	-1.0425	-0.8947	-1.6236	-1.2493	-0.6201	-1.4052
e=4,l=1.5	-1.3493	-0.0115	-0.5568	-0.7967	-0.3784	0.4087	-1.4070	0.0360	-0.8204	-1.3918	-0.8847	-0.3200
e=5,l=1.5	-1.2848	0.2438	-0.2407	-0.6990	-0.3535	0.7775	-1.3701	0.2209	-0.5184	-1.3711	-0.9482	0.0574
e=3,l=1.0	-0.0127	0.6849	0.7321	-0.0949	0.5749	0.4596	-0.2662	-0.3898	-0.1115	-0.2412	0.1996	-0.6924
e=4,l=1.0	-0.2018	1.4823	1.3228	-0.2030	0.5618	1.1647	-0.4791	0.4581	0.6933	-0.5385	0.1911	0.0956
e=5,l=1.0	-0.1860	1.7128	1.5535	-0.2319	0.6462	1.3503	-0.5639	0.6743	1.0496	-0.7632	0.2010	0.3582

Table 3.23 Cross-Product of Standardised Residuals from Tetravariate VC-APARCH(1,1) and VC-FIAPARCH(1,d,1) Models: USD Rates

	VC-APARCH						VC-FIAPARCH					
	CND/USD & GBP/USD	CND/USD & MYR/USD	CND/USD & SGD/USD	GBP/USD & MYR/USD	GBP/USD & SGD/USD	MYR/USD & SGD/USD	CND/USD & GBP/USD	CND/USD & MYR/USD	CND/USD & SGD/USD	GBP/USD & MYR/USD	GBP/USD & SGD/USD	MYR/USD & SGD/USD
Panel A: Ljung-Box Q-statistic												
5 lags	49.5922	2.3890	13.7625	6.9345	6.3727	8.0007	44.2067	3.1016	12.6564	6.1144	5.2529	4.2370
10 lags	55.3582	8.3068	15.1137	8.6514	14.4946	15.0780	49.8672	9.8460	13.9631	7.9997	14.7073	16.0652
Panel B: BDS Test												
e=3,l=1.5	2.1964	0.9121	-0.3980	2.8791	1.8080	0.7607	2.2447	0.5155	-1.2243	2.4096	1.0438	-0.6646
e=4,l=1.5	2.8240	0.9841	-0.2071	2.8435	1.7585	0.7357	2.8645	0.8109	-0.6768	2.2633	1.1543	-0.3566
e=5,l=1.5	3.0955	1.1726	-0.2201	2.5806	1.7546	0.6068	3.0655	1.1280	-0.2758	1.9930	1.2734	-0.2871
e=3,l=1.0	0.8609	1.2682	-0.1288	2.4278	1.9293	1.8120	0.9532	0.5700	-1.3887	1.7585	1.6164	0.5311
e=4,l=1.0	1.5216	1.3296	0.1439	2.0424	1.8318	1.9148	1.6503	0.8983	-0.4825	1.2952	1.8600	1.0022
e=5,l=1.0	1.7037	1.2622	0.4135	1.6078	1.6336	1.6215	1.8281	0.9772	0.1645	0.9094	1.8623	1.0368

Figure 3.1

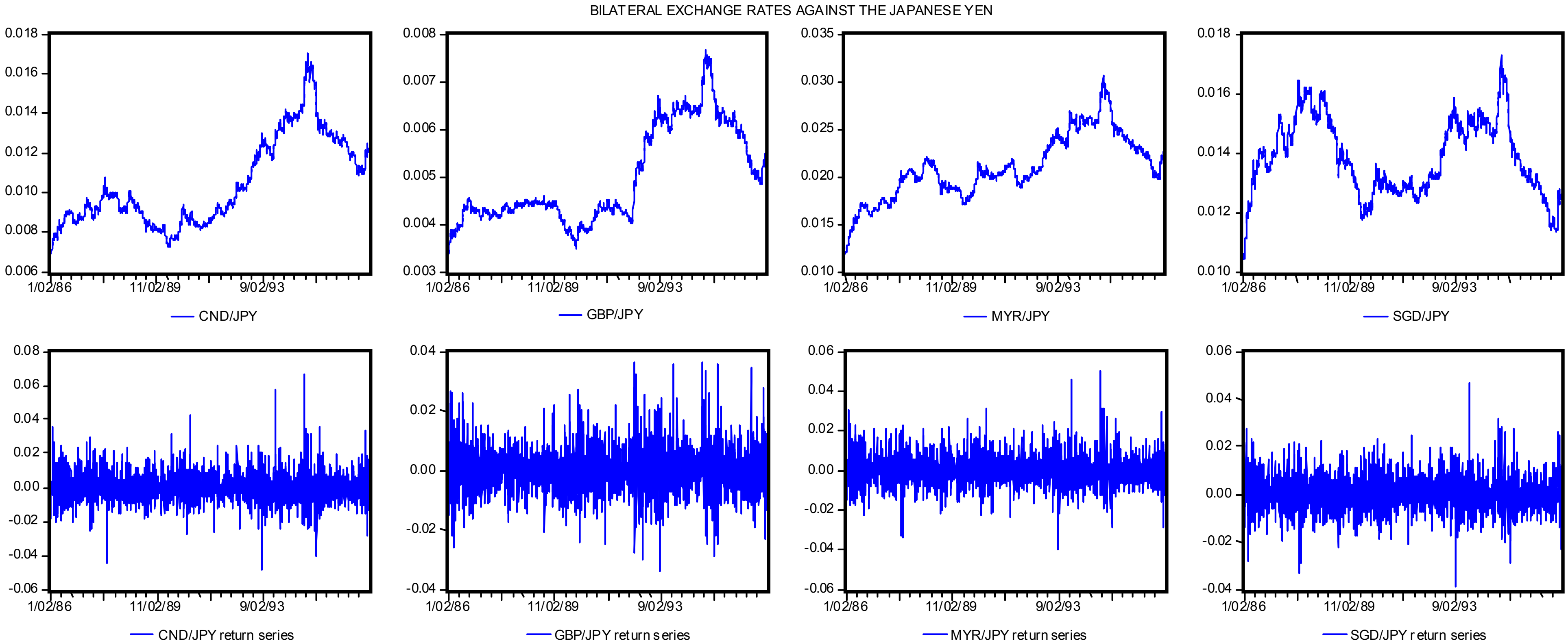

BILATERAL EXCHANGE RATES AGAINST THE JAPANESE YEN
1/02/86
11/02/89
9/02/93
CND/JPY
GBP/JPY
MYR/JPY
SGD/JPY
CND/JPY return series
GBP/JPY return series
MYR/JPY return series
SGD/JPY return series

Figure 3.2

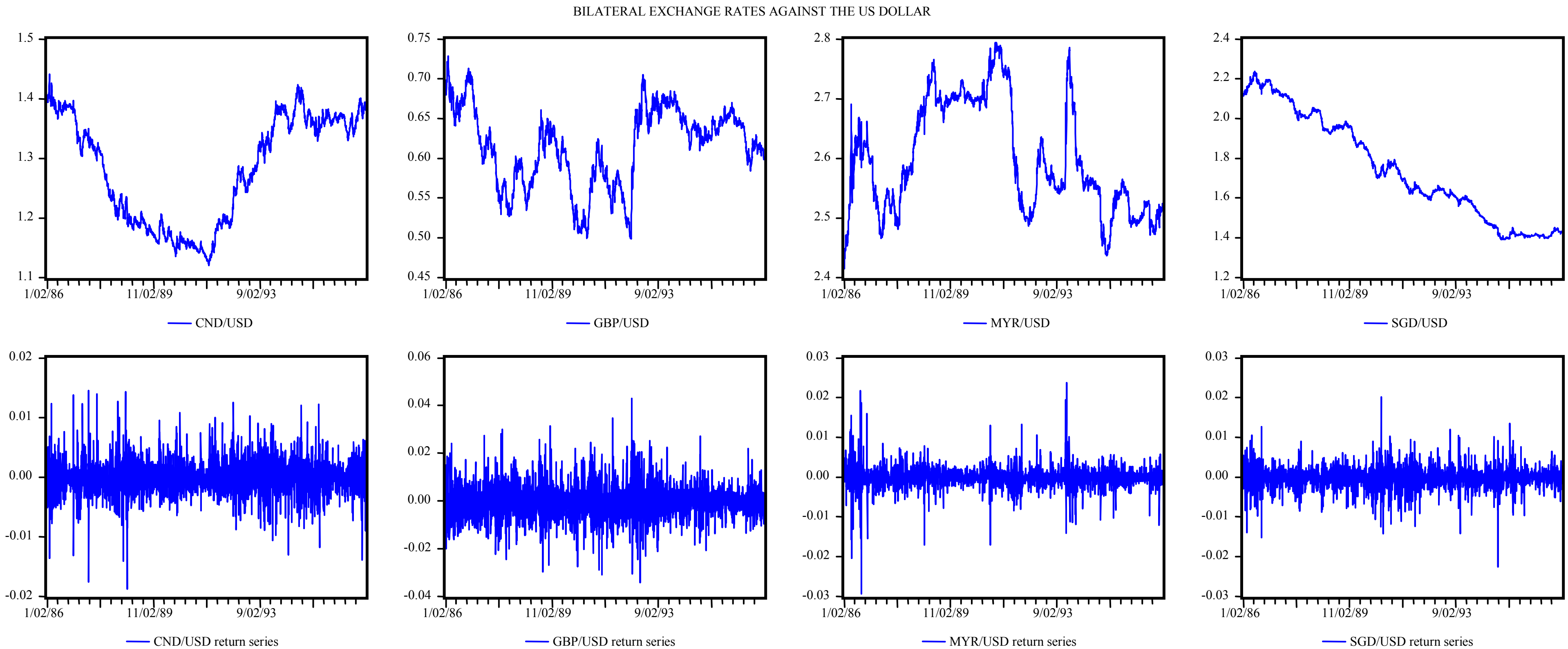

BILATERAL EXCHANGE RATES AGAINST THE US DOLLAR
1.5
1.4
1.3
1.2
1.1
1/02/86
11/02/89
9/02/93
CND/USD
0.75
0.70
0.65
0.60
0.55
0.50
0.45
GBP/USD
2.8
2.7
2.6
2.5
2.4
MYR/USD
2.4
2.2
2.0
1.8
1.6
1.4
1.2
SGD/USD
0.02
0.01
0.00
-0.01
-0.02
CND/USD return series
0.06
0.04
0.02
0.00
-0.02
-0.04
GBP/USD return series
0.03
0.02
0.01
0.00
-0.01
-0.02
-0.03
MYR/USD return series
SGD/USD return series

Figure 3.3

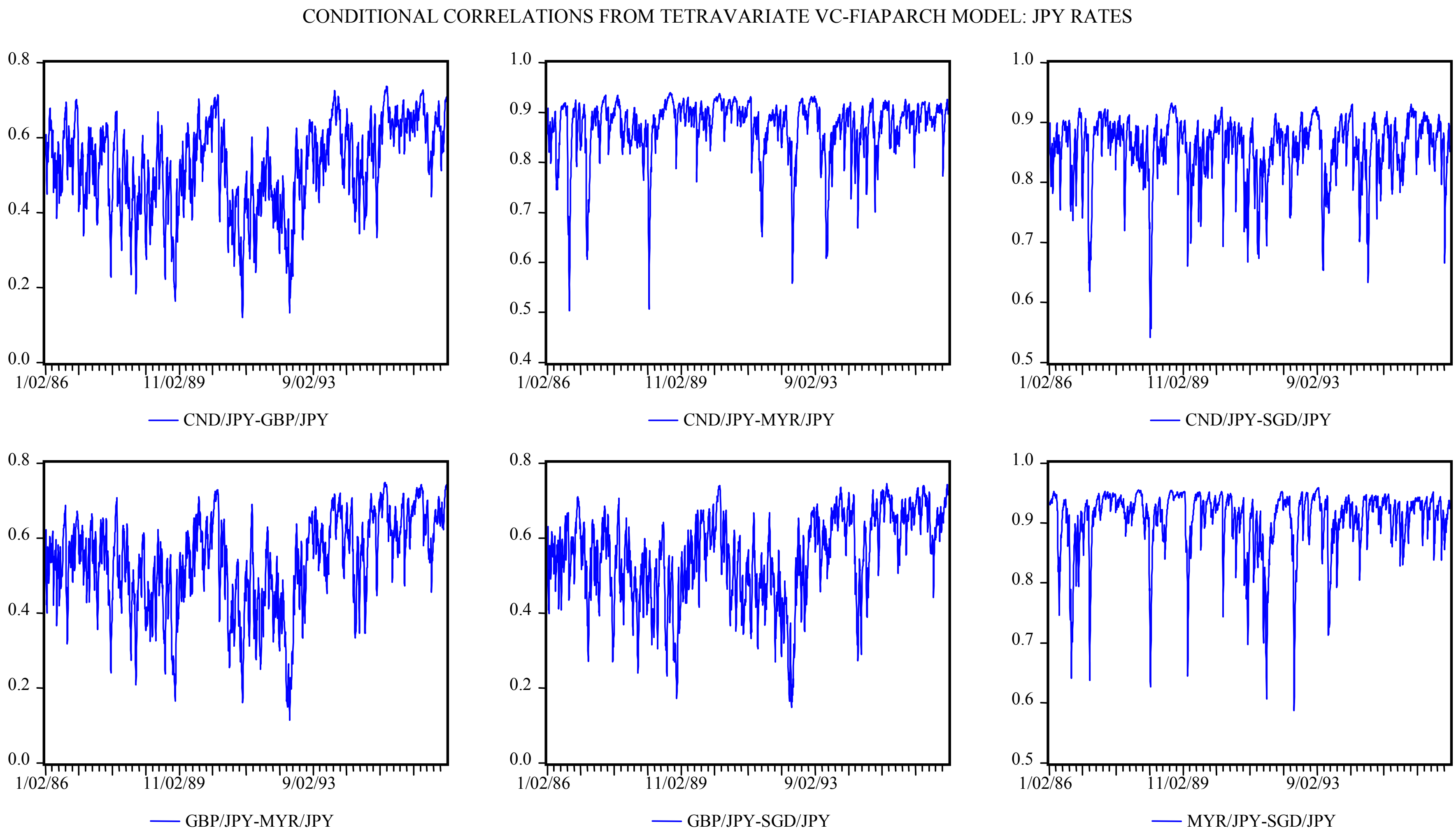

CONDITIONAL CORRELATIONS FROM TETRAVARIATE VC-FIAPARCH MODEL: JPY RATES
CND/JPY-GBP/JPY
CND/JPY-MYR/JPY
CND/JPY-SGD/JPY
GBP/JPY-MYR/JPY
GBP/JPY-SGD/JPY
MYR/JPY-SGD/JPY
1/02/86
11/02/89
9/02/93

Figure 3.4

CONDITIONAL CORRELATIONS FROM TETRAVARIATE VC-FIAPARCH MODEL: USD RATES

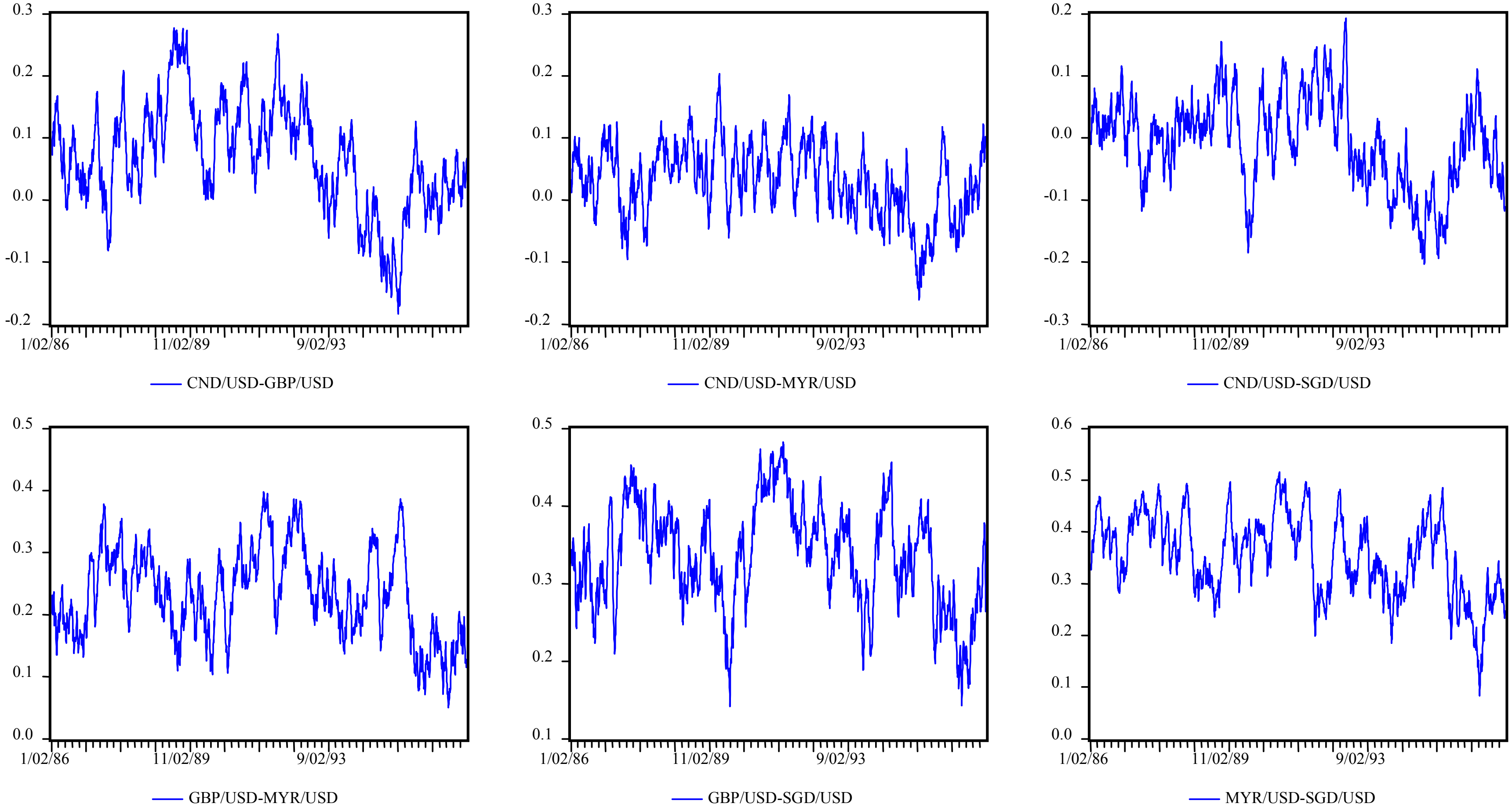


Figure 3.5

CONDITIONAL CORRELATIONS FROM TETRAVARIATE VC-APARCH MODEL: JPY RATES

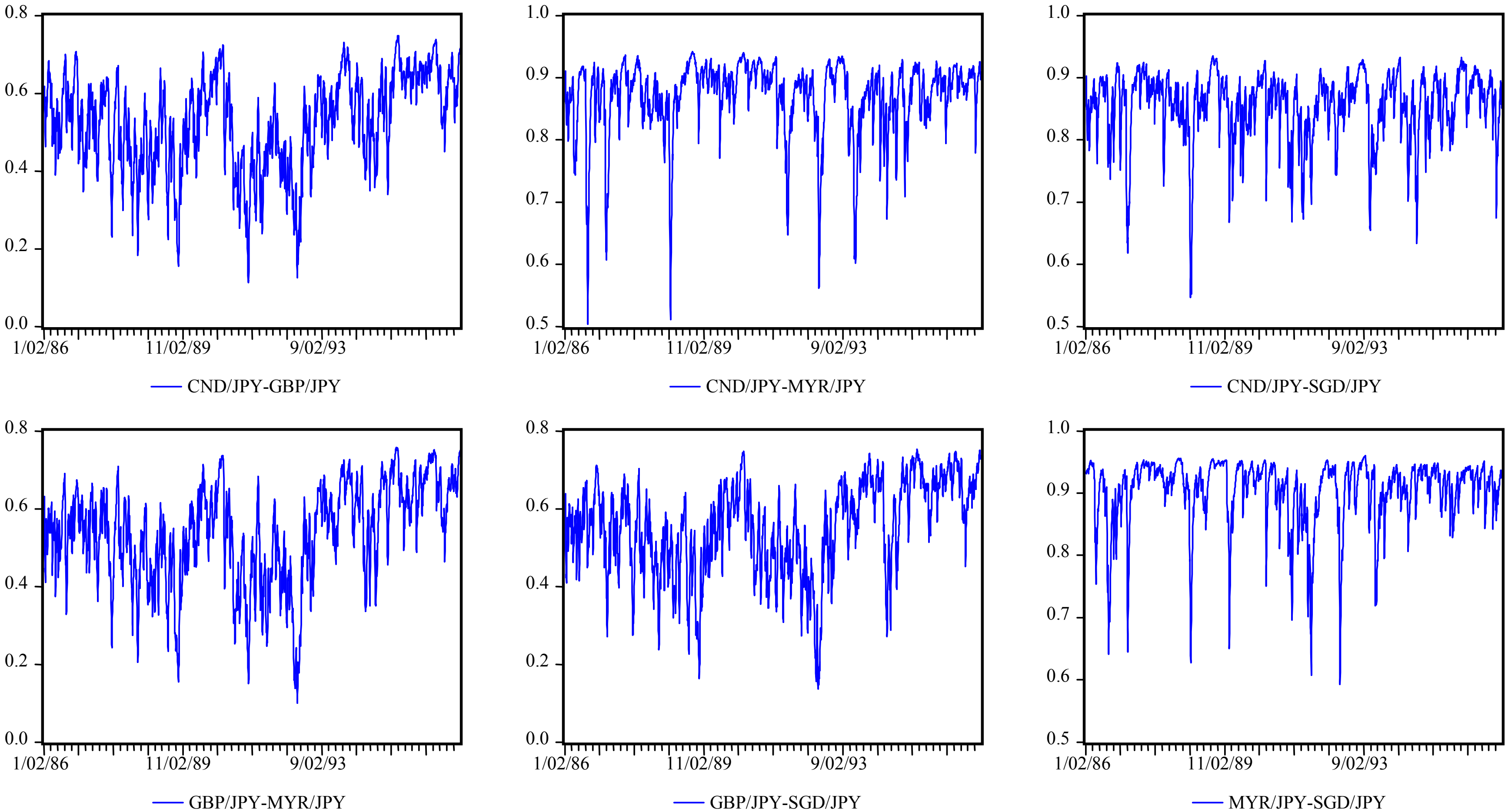


Figure 3.6

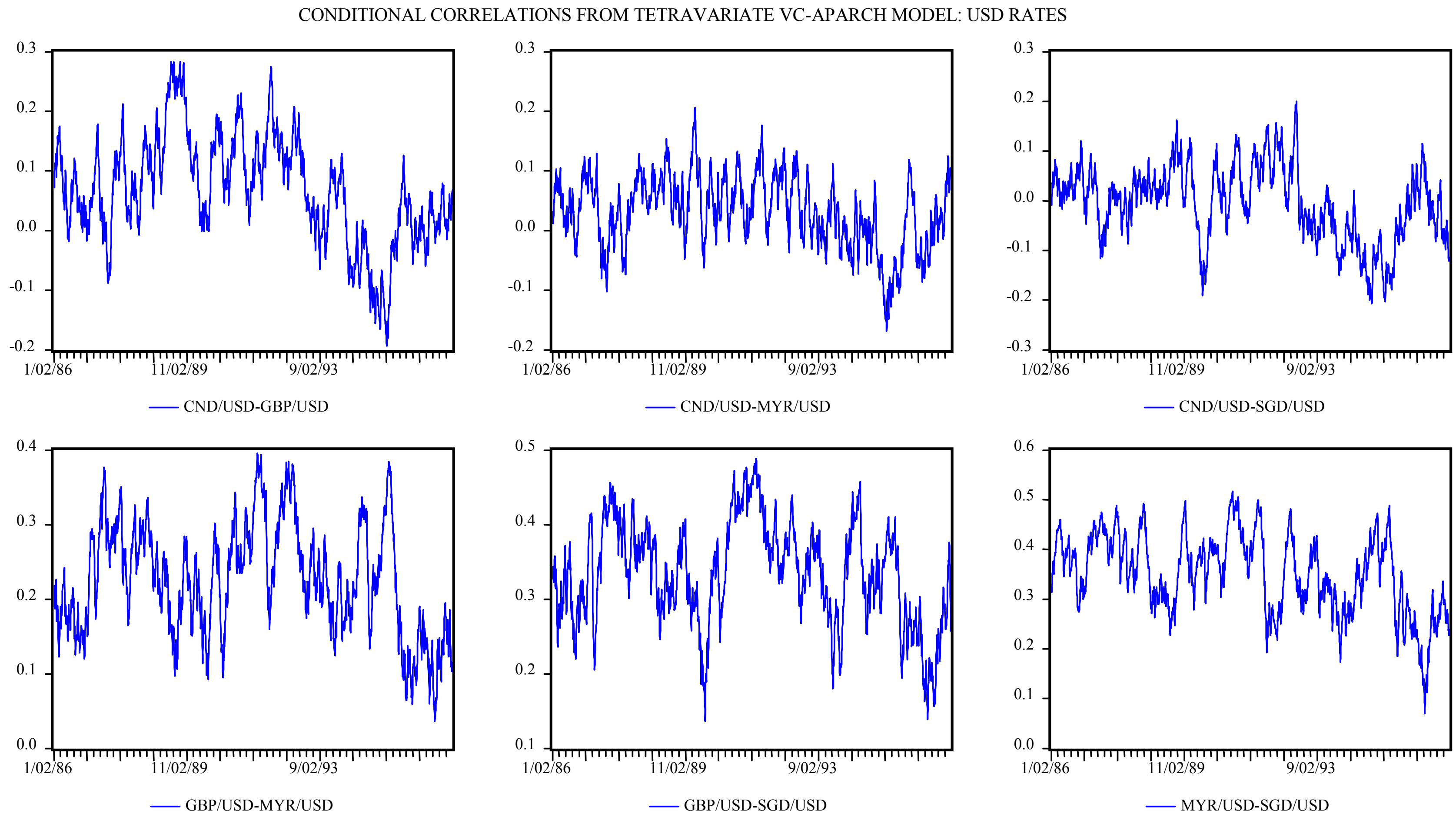

CONDITIONAL CORRELATIONS FROM TETRAVARIATE VC-APARCH MODEL: USD RATES
0.3
0.2
0.1
0.0
-0.1
-0.2
1/02/86
11/02/89
9/02/93
CND/USD-GBP/USD
CND/USD-MYR/USD
-0.3
CND/USD-SGD/USD
0.4
0.5
0.6
GBP/USD-MYR/USD
GBP/USD-SGD/USD
MYR/USD-SGD/USD

Figure 3.7

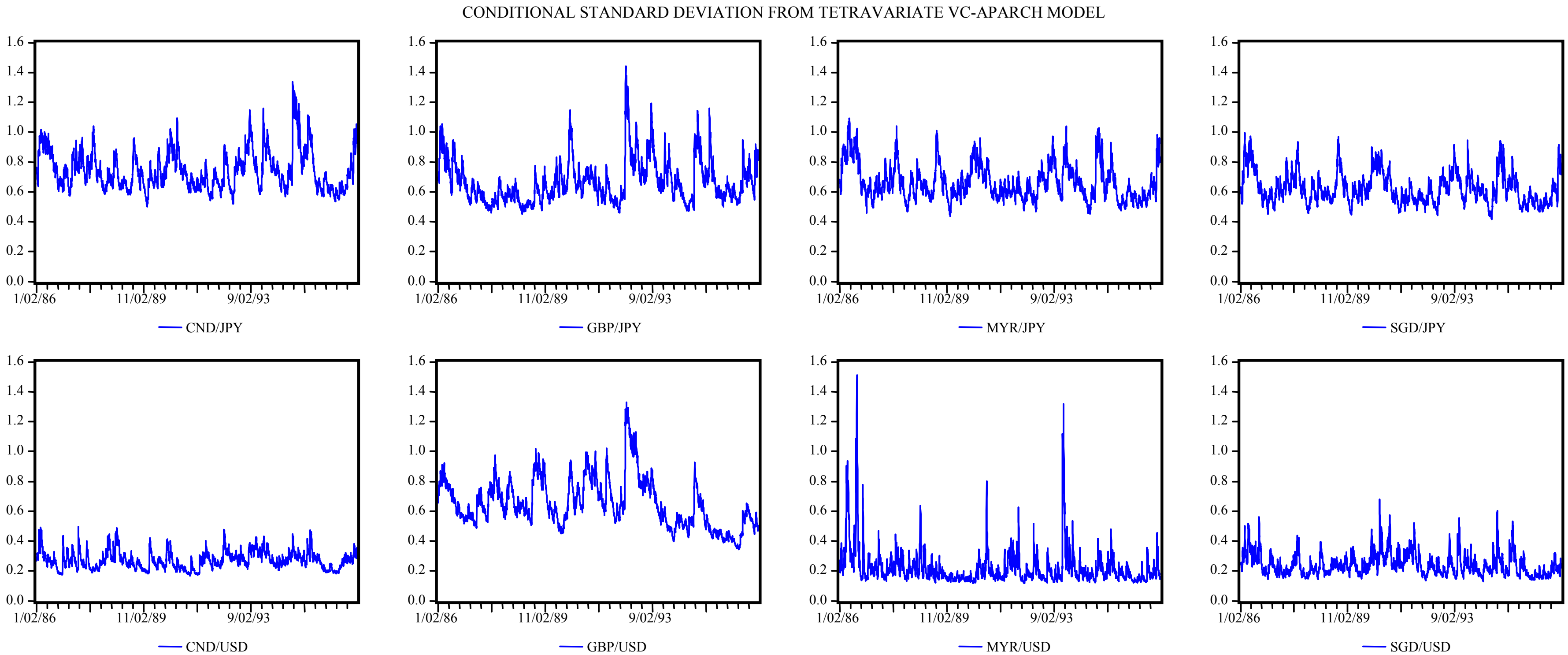
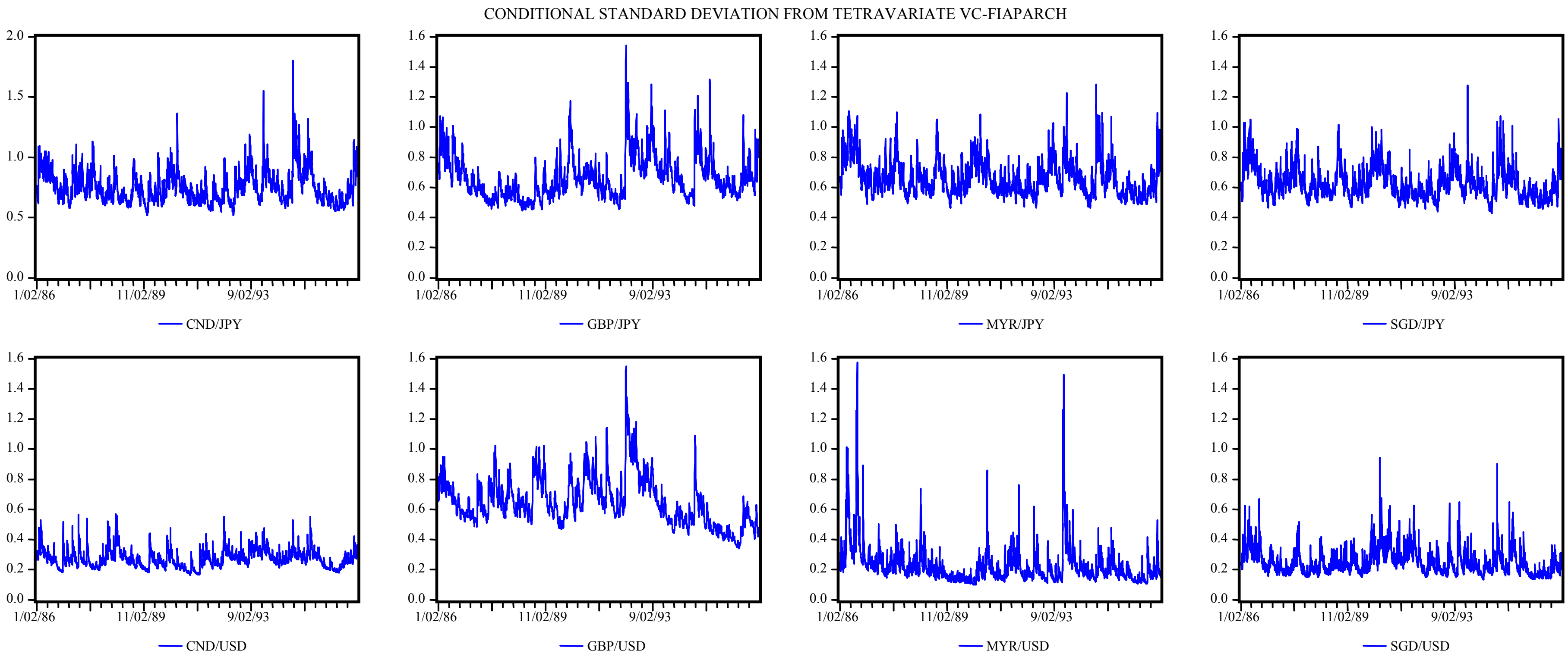

CONDITIONAL STANDARD DEVIATION FROM TETRAVARIATE VC-APARCH MODEL
CND/JPY
GBP/JPY
MYR/JPY
SGD/JPY
CND/USD
GBP/USD
MYR/USD
SGD/USD
1/02/86
11/02/89
9/02/93
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6

Figure 3.8


CONDITIONAL STANDARD DEVIATION FROM TETRAVARIATE VC-FIAPARCH
CND/JPY
GBP/JPY
MYR/JPY
SGD/JPY
CND/USD
GBP/USD
MYR/USD
SGD/USD
1/02/86
11/02/89
9/02/93

CHAPTER 4

CONCLUSION

We have proposed a family of multivariate GARCH models to investigate the volatility dynamics of exchange rates. A group of these models are capable of capturing the stylised features of long-memory, persistence, asymmetric conditional volatility, and time-varying correlations typically found in financial time series data. These models are applied to the exchange rates of the Canadian dollar (CND), the British pound (GBP), the Malaysian ringgit (MYR), and the Singapore dollar (SGD), respectively. Our approach departs from the convention of using only the US dollar (USD) as the numeraire currency. Instead, we also examine the behaviour of these currencies based on their exchange rates against the Japanese yen (JPY).

Our main results are as follows. First, we find evidence of long-memory and persistence in the conditional volatility of individual currencies, regardless of the choice of the numeraire. Furthermore, some exchange rates, such as the JPY rates, apparently share a common degree of long memory. In addition, based on a comparison of the log-likelihood values, the multivariate fractionally integrated models generally outperform those models without long-memory structures in the conditional variance.

Second, consistent with previous studies, such as Hsieh (1989b), the CND, the GBP, and the SGD vis-à-vis the USD do not exhibit asymmetries in the conditional volatility. In contrast, we detect statistically significant evidence of asymmetric volatility when these currencies are measured against the JPY. Furthermore, depreciation shocks of the MYR have a greater impact on future volatilities compared with appreciation shocks of the same magnitude, and this result is robust to the choice of the numeraire currency. In addition, the magnitude of asymmetry varies with the specification of the conditional volatility.

Third, we find stronger evidence of time-varying correlations among the exchange rates when the JPY is the numeraire. For instance, when the MYR and the SGD are measured using the JPY, the correlation between these two exchange rates are significantly time-varying. However, we do not detect conclusive evidence that the correlation is time-varying using the USD as the numeraire. In addition, the correlations among the currencies are usually stronger when they are measured using the JPY.

Several areas on the volatility dynamics of exchange rate warrant future research. First, the causes of significant asymmetric volatility in three of the four exchange rates vis-à-vis the JPY deserve greater attention. Second, it is interesting to understand why exchange rate correlations are substantially stronger when the Japanese yen is the numeraire currency. Third, it is vital to model explicitly the relationship between exchange rate volatility and conditional correlations. Are correlations stronger when foreign exchange markets are more volatile? Future study should address this issue in a multivariate framework.

Appendix I Models Estimated

The following conditional variance equations are applied to Bollerslev's (1990) constant-correlation and Tse and Tsui's (2002) varying-correlation frameworks, respectively.

Bollerslev's (1986) GARCH(1,1):

$$h_{iit} = \eta_i + \alpha_i \varepsilon_{it-1}^2 + \beta_i h_{iit-1}$$

Engle's (1990) Asymmetric GARCH(1,1):

$$h_{iit} = \omega_i + \alpha_i (\varepsilon_{iit-1} + \gamma_i^*)^2 + \beta_i h_{iit-1}$$

Sentana's (1995) Quadratic GARCH(1,1):

$$h_{iit} = \eta_i + \gamma_i \varepsilon_{it-1} + \alpha_{it-1} \varepsilon_{it-1}^2 + \beta_i h_{iit-1}$$

Ding, Engle, and Granger's (1993) Asymmetric Power ARCH(1,1):

$$\varepsilon_{it} = \sqrt{h_{iit}}\, e_{it}, \quad e_{it} \sim N(0,1)$$
$$h_{iit}^{\delta/2} = \eta_i + \alpha_i (|\varepsilon_{it-1}| - \gamma \varepsilon_{it-1})^{\delta} + \beta_i h_{iit-1}^{\delta/2}$$

Logarithmic GARCH(1,1):

$$\log h_{iit} = c_i + \alpha_i \log(|\varepsilon_{it-1}| - \gamma \varepsilon_{it-1}) + \beta_i \log h_{iit-1}$$

$$c_i = \eta_{i0}^* \log \varpi_i - \alpha_i \log \sqrt{2/\pi},$$

$$\eta_{i0}^* = \{1 - \alpha_i \lim_{\delta \to 0} E(|e_{it-1}| - e_{it-1})^{\delta} - \beta_i\} = \{1 - \alpha_i - \beta_i\}$$

$$\eta_i = \{1 - \alpha_i E(|e_{it-1}| - e_{it-1})^{\delta} - \beta_i\} \varpi^{\delta} = \eta_{i0}^* \varpi^{\delta}$$

Threshold GARCH(1,1):

$$h_{iit}^{1/2} = \eta_i + \alpha_i (|\varepsilon_{it-1}| - \gamma \varepsilon_{it-1}) + \beta_i h_{iit-1}^{1/2}$$

Leveraged GARCH(1,1):

$$h_{iit} = \eta_i + \alpha_i (|\varepsilon_{it-1}| - \gamma \varepsilon_{it-1})^2 + \beta_i h_{iit-1}$$

Fractionally Integrated (FI) GARCH(1,d,1):

$$h_{iit} = \frac{\eta_i}{1-\beta_i} + \lambda_i(L)\varepsilon_{it}^2\,,\ \lambda_i(L) = \sum_{a=1}^{\infty} \lambda_a L^a = 1-(1-\beta_i L)^{-1}(1-\phi_i L)(1-L)^d$$

FIAGARCH(1,d,1):

$$h_{iit} = \frac{\omega_i}{1-\beta_i} + \lambda_i(L)(\varepsilon_{it} + \gamma_i^*)^2$$

FIAPARCH(1,d,1):

$$h_{iit}^{\delta/2} = \frac{\eta_i}{1-\beta_i} + \lambda_i(L)(|\varepsilon_{it}| - \gamma_i \varepsilon_{it})^{\delta}$$

FILOGGARCH(1,d,1):

$$\log\sqrt{h_{iit}} = \frac{c_i}{1-\beta_i} + \lambda_i(L)\log(|\varepsilon_{it}| - \gamma_i \varepsilon_{it})$$

FITGARCH(1,d,1):

$$\sqrt{h_{iit}} = \frac{\eta_i}{1-\beta_i} + \lambda_i(L)(|\varepsilon_{it}| - \gamma_i \varepsilon_{it})$$

FILGARCH(1,d,1):

$$h_{iit} = \frac{\eta_i}{1-\beta_i} + \lambda_i(L)(|\varepsilon_{it}| - \gamma_i \varepsilon_{it})^2$$

Appendix II Data

DataStream Name	DataStream Code
Bilateral exchange rates against the US Dollar (USD)	
Canadian \$ to US \$ (GTIS)	USCDNDL
UK £ to US \$ (GTIS)	USBRITP
Malaysian Ringgit to US \$ (GTIS)	USMALAY
Singapore \$ to US \$ (GTIS)	USSINGD
Japanese Yen to US \$ (GTIS)	USJAPYN

For the bilateral exchange rates against the Japanese yen, we use the implied cross rates, which are calculated by dividing the bilateral USD exchange rate with JPY/USD. This is because most of the cross rates in DataStream International are not available starting from 2 January 1986. We have compared the summary statistics of the implied cross rates with those of the cross rates in DataStream International for the common period from the 1990s. Both sets of statistics are similar to each other.

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