A STUDY ON AIR CARGO REVENUE MANAGEMENT

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Summary

This thesis studies air cargo revenue management (RM) problems in spot market and long-term market. First, we consider a single-leg air cargo booking control problem on the spot market. The booking process is modeled as a discrete-time Markov chain and the airline's decision on accepting/rejecting booking request is based on a bid-price control policy. To avoid the complexity of high dimensionality, the bid prices are derived from maximizing a reward function of the Markov chain. Numerical experiments show that the proposed model outperforms two existing booking control policies. Second, we study the capacity allocation problem in long-term market, in which one airline serves n forwarders. We propose a capacity bundling policy (CBP) to mitigate the negative impact of seasonal imbalance between supply and demand, and model the problem as a Stackelberg game. Numerical experiments show that CBP can increase the airline's expected profit and reduce the risk under certain conditions. Last, we integrate the above two models and propose a conceptual framework for an air cargo RM system.

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Chapter 1 Introduction

Rapid globalization and intense competition has resulted in a steady increase in air cargo traffic in recent years. According to the forecasting from Boeing (2008), world air cargo traffic will increase by 5.8% annually in the following 20 years, increasing from 193.6 billion RTKs (Revenue-Ton-Kilometer) in 2007 to more than 595.9 billion RTKs in 2027. As demand for air cargo shipments grows, effective management of cargo space becomes crucial.

Revenue management (RM) had its roots in selling airline seats. In the past few decades, RM has drawn great attention from both scholars and industry practitioners and its application in airline industry has been a considerable success, particularly with the proliferation of internet booking systems. All airlines continue modifying the model of their RM system in order to enhance their revenue. In contrast, research in air cargo RM is still in its infancy. Only a few major carriers practice some form of cargo RM, and even in these cases, the systems are not comparable in sophistication to the RM system of passenger seats. Therefore, there is a need to increase knowledge in air cargo RM.

In this thesis, we propose two RM techniques for air cargo capacity management. In particular, we develop an optimal bid-price control policy based on a Markov model to control short-term capacity allocation and we propose a capacity bundling policy (CBP) to manage the long-term capacity allotment. In addition, a conceptual framework which integrates the two models to form a RM system is proposed.

To develop a successful air cargo RM system, a thorough understanding of the air cargo industry is a must. In the following section, I will introduce the market structure, characteristics and major problems of air cargo industry.

1.1 Air cargo industry backgrounds

According to Hellermann (2006), the players in air cargo industry can be divided into three groups: asset providers, shippers, and intermediaries. Asset providers are the suppliers that offer airport-to-airport transport and operate physical assets (e.g. aircraft) that provide air cargo capacity. They are represented by companies such as Lufthansa Cargo AG, Air France Cargo, and Singapore Airlines Cargo. Shippers are the senders of air freight. Shippers can be large manufacturers such as HP, DELL, IBM, etc, or companies that sell perishable products such as flowers, apparels, etc. Normally, shippers do not send freight directly to asset providers. For the major part of freight, shippers leave it to intermediaries to organize and perform transportation. These intermediaries can be freight forwarding companies that operate trucks to cover door-to-airport and airport-to-door sections of air cargo transportation. Besides, intermediaries also provide other value-added services like cargo consolidation, packing and even third-party logistics.

Typically, the capacity for air cargo transportation is sold on two bases (Slager and Kapteijns, 2004):

1. Guaranteed capacity contract: i.e. agreement between airlines and customers involving guaranteed capacity (defined in weight and volume) on a specific flight/weekday;

2. Free-sale: i.e. no capacity guarantee, usually based on specific order. Airlines can accept a booking request or reserve the space for a more profitable booking that may arrive in the future.

The market structure in air cargo industry is shown in Figure 1.1.

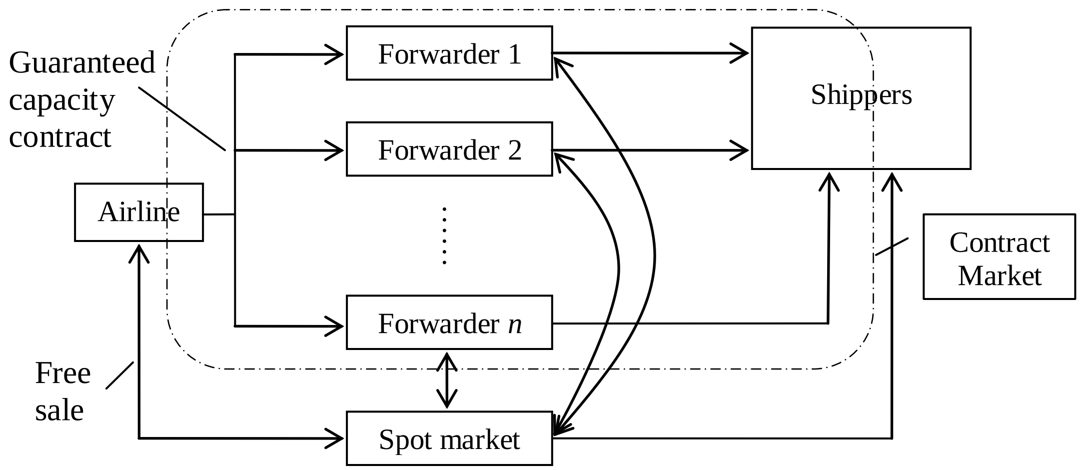


Figure 1.1 Market structure in air cargo industry

According to Hellermann (2006), it is a standard industry practice that airfreight carriers and forwarders close long-term capacity agreements upfront. In particular, forwarders order certain capacity between a certain origin-destination (O-D) pair in a certain time period, and resell the capacity to shippers. The price per unit capacity under the long-term contract is called contract rate, which is usually determined based on the negotiation between forwarders and the airline. The long-term contract is often signed months before the departure of the flight. Forwarders will decide the order quantity in the long-term contract according to the forecasting of the future demand. The order quantity in long-term capacity agreement is also called guaranteed capacity. If the actual demand is less than the order quantity (guaranteed capacity), the forwarder has to pay contract rate for used capacity and penalty rate for unused capacity. If the actual demand is larger than the order quantity, part of the demand will be lost, but no penalty is incurred. Usually, the penalty rate is a fraction of the contract rate. This market is called contract market, and the majority of capacity in air cargo industry is sold on this market. Forwarders benefit from signing capacity agreement because they can lock in certain capacity in the future, especially in those periods with high demand from shippers. The airline benefits from signing capacity agreements because it can reduce the capacity utilization risk, increase load factor and attract more forwarders. Also, long-term capacity agreements can be viewed as a hedge against the uncertainty in cargo rate for both airlines and forwarders, and thus, successfully reduce the fluctuation of revenue in the industry. In addition, long-term capacity agreements improve the communication and information sharing between airlines and forwarders, and thus, increase the efficiency in the industry.

Besides selling capacity to forwarders via long-term capacity agreements, airlines can also sell capacity directly on spot market (Free-sale). Unlike contract market in which the capacity is sold several months before departure, the demand in spot market usually arrives several days before departure. Most of the customers on spot market are shippers and forwarders that need emergency capacity. Therefore, the spot rate is expected to be higher than the contract rate. Forwarders can purchase additional capacity on the spot market, if the total capacity it ordered in the guaranteed capacity agreement is not enough to satisfy all demand. Forwarders can also sell capacity on the spot market, if there is leftover guaranteed capacity after satisfying all contractual demand from shippers. Occasionally, an airline can also purchase capacity from the spot market. Airlines will intentionally accept more orders than it can accommodate to mitigate the effect of cancellations and no-shows. This practice is known as overbooking. If the total accepted demand from long-term contract exceeds the airline's capacity, the airline may need to purchase capacity from spot market.

For any demand in the spot market, the transportation price charged by the airline is denoted as

$$\text{Revenue}\left(d_w, d_v\right) = p \max\left(d_w, d_v / \theta_s\right) \tag{1.1}$$

where d_w and d_v are weight and volume of the cargo respectively; p is the spot rate for the type of this cargo; and θ_s is a constant defined by International Air Transportation Association (IATA) volumetric standard. The quantity d_v / θ_s is called dimensional weight. If the density of a cargo is larger than $1/\theta_s$, it will be charged according to its weight. Otherwise, it will be charged according to its dimensional weight. Different types of cargos may have different spot rates. For example, airline may charge a higher cargo rate for live animals or precious cargos because they need special handling or security. When demand arrives in the spot market, the airline has to decide whether to accept the current booking or reserve the capacity for a more profitable booking that may arrive in the future. The acceptance/rejection decision will be based on the rate of the cargo, the forecasting of future demand and the current sales profile.

The contract market is very different from the spot market. The contract market is a business-to-business market, in which airlines work closely with a few important forwarders who ship large volumes. Therefore, the forwarders have strong market power. The implication of this market structure is that the customer relationship takes priority in long-term air cargo RM. An essential characteristic of a successful air cargo RM system is that it must be able to better align the interest of both carriers and forwarders and create a win-win situation. In contrast, airlines have strong market power in spot market, whereas shippers and forwarders act as price takers. As a result, the RM system for the short-term capacity allocation is somewhat similar to the RM system for airline seats allocation.

1.2 Air cargo RM vs. passenger RM

Air cargo RM differs from passenger RM in several ways.

1. Air cargo RM is a two dimensional problem. First, cargo consumes multidimensional capacity: weight and volume. Second, not only the revenue from the cargo depends on the price, but also depends on the weight and volume capacity it consumes. With two-dimensional capacity, dynamic programming, which is widely used in passenger RM, may not be suitable to solve air cargo RM problem because of the curse of dimensionality. This difference has been discussed in more details in Billings et al. (2003).

2. Customer relationship is very important in air cargo industry. As explained in the previous section, the long-term relationship with forwarders is crucial for airlines. Thus, the air cargo RM system must be customer-oriented. In contrast, long-term relationship with a single customer is not crucial for a passenger RM system, since each customer only contribute a tiny part to the entire revenue of the airline.

3. The forwarders have detailed information of demand and supply in the contract market. They behave strategically. Therefore, the air cargo RM system may need to

apply some techniques in game theory. In contrast, most RM models assume myopic passengers.

4. The market structure of air cargo RM is more complex than that of passenger RM. There are two separate markets in air cargo industry, e.g. contract market and spot market. The two markets have different types of customers and distinctive sales channels. To make things more complicated, the two markets are correlated. As explained in the previous section, forwarders can be the customers or suppliers in the spot market, depending on their order quantities in the contract market. In other words, the demand and supply in contract market can affect the spot market. An air cargo RM system should be able to take these characteristics into consideration and jointly allocate capacity in the two markets.

5. Unlike in passenger RM, there may be many different routes that cargo can take between its origin and destination and it is largely up to the carrier to choose a route. Therefore, the air cargo RM system should make good use of this flexibility and incorporate the network effect into considerations when making decisions on capacity allocation, pricing and overbooking.

6. The capacity for air cargo transportation may depend on passenger boarding, since some capacity for air cargo comes from the belly space of combination flights. Uncertainty of capacity adds to the complexity of air cargo RM and requires special attentions.

Due to these differences, the techniques used in passenger RM cannot be applied in cargo RM directly.

1.3 Motivation of the study

In the spot market, the airline has to make decision on acceptance/rejection of arrival demand. This decision is somewhat similar to the seat allocation problem in passenger RM. However, the existing RM models in passenger RM cannot be applied in air cargo industry due to the differences between air cargo RM and passenger RM as discussed above. In modeling the free sales of capacity on the spot market, the stochastic nature of cargo demand has to be considered, because of the following two reasons. First, the way that the airline charges a cargo booking provides the opportunity to increase revenue from the stochastic nature of the weight and volume of a demand. Recall that the revenue from accepting a booking request is $\text{Revenue}\left(d_w, d_v\right) = p\max\left(d_w, d_v / \theta_s\right)$. Dense cargo is charged according to its weight, while light cargo is charged according to its volume. Suppose the expected weight and volume of cargo demand are $\bar{d}_w$ and $\bar{d}_v$. The sum of the revenue from two bookings $\left(0.5\bar{d}_w, 1.5\bar{d}_v\right)$ and $\left(1.5\bar{d}_w, 0.5\bar{d}_v\right)$ will be higher than the revenue from two $\left(\bar{d}_w, \bar{d}_v\right)$ bookings, though they consume the same capacity. As a result, the expected revenue will be distorted and the decision will be non-optimal, if the stochastic nature is not captured in the decision model. Second, the cost of rejecting a cargo demand due to lack of capacity is different from the opportunity cost of unused capacity. Therefore, the stochastic demand needs to be modeled so that the total cost is minimized. There are several literature focusing on the short-term air cargo RM problem, including Karaesmen (2001), Pak and Dekker (2004), Amaruchkul et al. (2005), Huang and Hsu (2005), Chew et al. (2006), and Sandhu and Klabjan (2006). Among the above literatures, Pak and Dekker (2004) is the only one that fully captures the two-dimensionality of cargo and stochastic nature in short-term booking process. However, the algorithm proposed in Pak and Dekker (2004) is not highly efficient and the optimality of the algorithm is not guaranteed. Therefore, more research effort is needed in this area. A more detailed literature review will be given in the next chapter.

In the long-term contract market, a year can be divided into several periods. The airline has to decide the contract rate in each period, and the forwarders have to decide the order quantity in each period. The demand in air cargo industry has strong seasonality. Usually, there will be a peak period from the beginning of November till the end of December. During this period, the total demand from shippers is significantly higher than the demand in other periods. The forwarders often face difficulties to lock in enough capacity in peak season. In low season, however, the total demand from shippers is often less than airlines' capacity and airlines often face difficulties to attract sufficient loads from forwarders. The strong seasonality in demand and the relatively fixed supply create an acute seasonal imbalance between the supply (airline) and the demand (forwarder). The airline cannot charge a very high contract rate in the peak period to mitigate the seasonal imbalance, since it will negatively impact the long-term relationship with forwarders. The traditional long-term contract cannot address this seasonal imbalance, and thus, a new business model is needed. To the best of our knowledge, Hellermann (2006) is the only literature that analyzes the long-term air cargo RM problem. However, it focuses on the design of options contract in order to solve the problem of forwarders' default on penalties for unused capacity. The seasonal imbalance between supply and demand in air cargo industry was not addressed and the correlation between different seasons was not considered.

1.4 Objectives and scope

In view of the contrast between fast growth of air cargo industry and lack of effective RM methodologies, there is an intense need of further studies in air cargo RM. Hence, we conduct this research and hope to contribute to the growth of air cargo RM. The specific objectives of this thesis are:

1. To study the optimal control of short-term capacity allocation. In particular, a bid-price control policy is adopted to control short-term capacity allocation. At the beginning of selling season, the optimal bid prices are calculated based on a Markov model provided in this thesis. When demand arrives, the optimal bid prices are used as the basis of deciding whether to accept or reject the demand.

2. To investigate the management of long-term guaranteed capacity. In particular, a capacity bundling policy is proposed to solve the seasonal imbalance between the supply and demand in contract market. The optimal design of the capacity bundling policy is studied. Furthermore, the performance of capacity bundling policy is analyzed under various market conditions.

3. To develop a systematic framework of air cargo RM system based on the integration of short-term capacity allocation and long-term capacity allocation.

Nevertheless, air cargo RM system can be a very complicated system which includes forecasting, scheduling, overbooking, capacity allocation, and pricing. The present thesis mainly focuses on capacity allocation and pricing. Also, the network effect in air cargo RM is not considered in this thesis.

The insights obtained from this thesis may help air carriers make capacity allocation and pricing decisions effectively, and thus increase their profit. The techniques developed in this thesis may also be applied in other RM areas, or supply chain management problems with stochastic demand and perishable supply.

1.5 Organization

This thesis contains 6 chapters. In chapter 2, literatures related to this study will be reviewed. The topics covered in the literature review include passenger RM and air cargo RM. The literatures in air cargo RM are further categorized into 4 subgroups: qualitative overviews, overbooking, short-term booking control and long-term booking control.

Chapter 3 focuses on the short-term booking control. We considered an air cargo industry with one airline. Booking requests of air cargo capacity arrive several days before flight departure. We assume that each booking request is endowed with random weight, volume and profit rate. Then, we propose a discrete-time Markovian chain to model the booking request acceptance/rejection process. The decision on whether to accept the booking request or to reserve the capacity for future bookings follows a bid-price control policy. In particular, the cargo will be accepted only when the revenue from accepting it exceeds the opportunity cost, which is calculated based on bid prices. Optimal solutions are derived by maximizing a reward function of the Markov chain. Then, numerical comparisons between the proposed approach and two existing static single-leg air cargo capacity allocation policies are presented.

Chapter 4 focuses on the long-term control of air cargo capacity. To mitigate the negative impact of seasonal imbalance between supply and demand, we propose a capacity bundling policy (CBP), in which the guaranteed capacity that each forwarder can get in the peak season depends on its order quantity in the low season. Then, we model the sales of long-term capacity as a Stackelberg game and the airline as the Stackelberg leader. The problem is solved under a general CBP and under a linear CBP, respectively. Numerical experiments of the performance of CBP under various market conditions are presented.

Chapter 5 focuses on the design of a conceptual framework for an air cargo RM system. The spot market and contract market are correlated, and thus, the capacity allocation decision in one market affects the performance of the other market. We propose a conceptual model to jointly manage the capacity in the two markets so that the total revenue from air cargo business is maximized. Besides, we also highlight several important issues in using RM tools and analyze the implications from these issues.

Chapter 6 summarizes the studies covered in this thesis and gives some directions for future works.

Chapter 2 Literature Review

2.1 Airline passenger RM

Revenue management (RM) is the process of understanding, anticipating and influencing consumer behavior in order to maximize revenue or profits from a fixed, perishable resource. The research on RM originates from the airline industry and can be traced back 40 years ago. Before 1972, almost all quantitative research in reservations control focused on controlled overbooking. The overbooking calculations depended on predictions of the probability distributions of the number of passengers who appeared for boarding at flight time, so overbooking research also stimulated useful research on disaggregate forecasting of passenger cancellations, and no-shows. Both forecasting and controlled overbooking achieved a moderate degree of success and established a degree of credibility for scientific approaches to reservations control (McGill and Van Ryzin 1999).

After the enactment of Airline Deregulation Act in 1978, regulators loosened control of airline prices and led to a rapid change and rush innovation in the industry. Established carriers were free to change prices, schedules, and service. At the same time, new low-cost and charter airlines entered the market. They were able to profit from a much lower price because of their lower labor costs and simpler operations. These developments resulted in more price-sensitive customers and also a surge in the demand in airline industry. To survive and develop in the new environment, some airlines began offering discount fare product which mixed the discount fare customers and regular fare customers in the same flight. To get a ticket under discount fare, a customer had to book the ticket several weeks before departure. This innovation offered the airline the potential to attract more price-sensitive customers and profit from those seats that would otherwise fly empty. This innovation also raised a new problem of determining the number of seats that should be protected for full fare passengers. Littlewood (1972) proposed that discount fare bookings should be accepted as long as their revenue value exceeded the expected revenue of future full fare bookings, which was known as the Littlewood's rule. This simple, two-fare, seat inventory control rule was the first quantitative method to solve the seat allocation problem. Following the Littlewood's rule, Belobaba (1987) considered a single-leg seat allocation problem with multiple fare classes, and developed the Expected Marginal Seat Revenue (EMSR) model to solve it. The EMSR could be viewed as another breakthrough in the airline RM after the Littlewood's rule. A later refinement of EMSR, which was known as EMSRb, had been widely used in RM systems. Other methods for obtaining optimal booking limits for single-leg flights were provided in Curry (1990), Wollmer (1992), Brumelle and McGill (1993).

All the literature introduced above relied on some restrictive assumptions: 1) single-leg flight, no network effect was considered; 2) the demand for different fare classes were stochastically independent; 3) demand for low fare class arrived before demand for full fare class; 4) cancellations and no-shows were not considered; 5) no batch bookings. These assumptions created various problems in the implementation of RM techniques. Therefore, a large proportion of later research in RM aimed to release these assumptions. Lee and Hersh (1993) released the assumption of low before high arrival pattern and used a discrete-time dynamic programming model to find the optimal booking control policy.

This research work also incorporated group bookings. Besides releasing the arrival pattern assumption, Zhao and Zheng (2001) considered the dependence of demands in different fare class. They assumed that a fractional of the customers were flexible, i.e. while willing to pay the full fare, they would buy low fare tickets if available. Then, they showed that the optimal booking policy was a threshold policy: the discount fare should be closed as soon as the number of remaining seats reached a predetermined threshold. Other dynamic programming formulations of single-leg RM problem were given in Lautenbacher and Stidham (1999), Subramanian et al. (1999), and Liang (1999).

Since the 1980s, network effects in revenue management had become increasingly significant. The expansion of hub-and-spoke system dramatically increased the number of customers that involved connections to multiple flight legs. The lack of seats in one flight-leg might affect the sales of other flights. This created interdependence among the resources, and hence, there was an increasing demand for RM techniques that jointly managed the capacity controls on the entire transportation network. This type of problem was called Origin-Destination (O-D) control. Glover et al. (1982) formulated the O-D control problem as a minimum cost network flow problem, in which passenger demands were assumed deterministic. This model was implemented at Frontier Airlines. Curry (1990) combined the marginal seat revenue approach for single-leg RM and the mathematical programming approach for O-D control problem, and developed a LP that obtained distinct bucket allocations for an O-D control problem. Wong (1993) developed a network formulation for a single fare class, multi-leg itinerary capacity allocation problem. This work provided a flexible assignment approach which assigned some seats exclusively to each single or multi-leg itinerary as in fixed assignment and assigned the remaining seats to group of seats as in bucket control. Feng and Xiao (2001) considered an airline seat allocation problem with multiple origins, one hub, and one destination. They proposed a stochastic control model to allocate seats among competing O-D routes, and developed optimal control rules. Other contributions in O-D control problem were provided in Talluri (2001), Bertsimas and Popescu (2003), and Möller et al. (2004).

The above literature review focuses on the seat allocation problem as it closely relates to our research. Due to space constraint, only some representative literature is reviewed. Other research areas in RM, including forecasting, overbooking, pricing, and implementation issues, are not covered. For more detailed overviews, please refer to McGill and Van Ryzin (1999), Boyd and Bilegan (2003) and Chiang et al. (2007).

2.2 Air cargo RM

The development of air cargo RM followed a similar pattern as the development of passenger RM. The literature started from qualitative overview of the problems in the air cargo industry, followed by quantitative analysis of air cargo overbooking, and then studies on capacity control problems. The capacity control problem can be further classified as short-term capacity control and long-term capacity control. The literature in these four areas will be reviewed in detail in this section.

2.2.1 Qualitative overview

Kasilingam (1996) described the characteristics and complexities of air cargo RM. The differences between passenger RM and air cargo RM were discussed and the major components of air cargo RM system were analyzed in this paper. He also proposed a simple overbooking model, in which the probability distributions of capacity and final show-up rate were assumed known and the overage cost and spoilage cost were assumed known. Billings et al. (2003) compared the characteristics of air cargo RM and passenger RM. It pointed out several fundamental issues in an air cargo RM system, i.e. cargo product definition, contract pricing, short-term booking controls and medium-term allocations. Slager and Kapteijns (2004) introduced experience at KLM Cargo in implementing cargo RM system and emphasized several key factors for a successful air cargo RM system. Froehlich (2004) summarized several key factors to the success of revenue management at Lufthansa cargo.

2.2.2 Overbooking

Air cargo overbooking is the practice of intentionally selling more cargo space than the available capacity to compensate for cancellations and no-shows. Besides, air cargo overbooking must also address the stochastic nature of the capacity. Kasilingam (1997) solved an air cargo overbooking problem by minimizing the overage cost and underage cost. The capacity was assumed to be a stochastic variable. However, the two-dimensional nature of air cargo overbooking was not addressed. In the air cargo industry, offloading of cargo can result from violation of any one of the two capacity constraints. To consider the two dimensional nature in cargo overbooking decision, the decision model must be able to reflect the dependency between showing up volume and weight. Luo et al. (2008) presented the first two-dimensional model for cargo overbooking. They introduced the concept of an overbooking curve and obtained the optimal solution in two cases respectively, i.e. a booking curve with general shape and a booking curve with rectangular shape. Moussawi and Cakanyildirim (2005) developed another two-dimensional model for cargo overbooking, whose objective was profit maximization instead of cost minimization. They adopted the concept of an overbooking curve, but restricted the curve to be a box defined by two control parameters. Therefore, this approach was easier to implement in air cargo RM practices.

2.2.3 Short-term booking control

As explained in the previous chapter, customers may order capacity from airline a short period, usually days or a week, before flight departure. Since the capacity ordered by these customers is not guaranteed, the airline has to decide whether to accept the booking request or not according to current remaining capacity and the type, weight and volume of the cargo. This decision problem is called the short-term booking control problem. Short-term booking control problem is very important to airlines, especially during the peak season for air cargo transportation. If airlines can make this decision correctly, they can serve the most profitable demands, and thus earn greater profit with the limited capacity. Despite the importance of the short-term booking control problem, only a few studies focus on this problem. For the rest of this section, we will review these studies in detail.

As mentioned in section 2.1, Lee and Hersh (1993) developed a dynamic programming model for a single-leg seat allocation problem. Huang and Hsu (2005) extended the dynamic programming model in Lee and Hersh (1993) and developed a model for single-leg short-term booking control problem. They assumed that there were finite discrete sizes of cargo without considering the nature of two-dimensionality in air cargo revenue management. As a result, the model was similar to a passenger revenue management model allowing for group booking and the complexity and practicality of the research were reduced.

Sandhu and Klabjan (2006) integrated fleeting and bid-price based Origin-Destination revenue management approach and formulated a deterministic model that captured both passenger and cargo revenue for a network revenue management problem. In the cargo booking control section, the three dimensional capacities, (i.e. weight, volume and containers), and time constraint, (i.e. standard and express), are considered. However, they used expected values of cargo demands rather than stochastic demands and therefore the resulting model was deterministic.

Chew et al. (2006) considered a short-term air cargo capacity planning problem from freight forwarders' perspective. They assumed that a freight forwarder could backlog the unsatisfied demands to the next flight with cost or purchase additional ad hoc space from the airline, if the guaranteed capacity was not enough to satisfy all demands. The forwarder had to balance the cost of backlogged shipment and the cost of acquiring additional cargo space. For a given amount of long-term contract space, the decision for each stage was the quantity of additional space required so that the total cost was minimized. Then, they formulated the problem as a stochastic DP and derived optimal solution.

Karaesmen (2001) formulated the single-leg short-term booking control problem as a continuous linear programming and showed that bid-price control policy can be used in short-term booking control. To the best of my knowledge, this is the first study that established the feasibility of using bid-price control policy to solve short-term booking control problem. However, it was impractical to solve this continuous linear programming directly and thus, Karaesmen (2001) had to rely on some approximation schemes. In particular, weight and volume were discretized to form a number of regions and the demand arrival rate of a region was approximated by the average demand arrival rate. With these approximations, Karaesmen (2001) developed three methods to obtain the bid prices. It was shown that the methods outperformed the First Come First Serve (FCFS) policy. Due to the approximations, however, the short-term booking control problem solved by Karaesmen (2001) was more of a deterministic problem than a stochastic one.

Amaruchkul et al. (2005) formulated the single-leg short-term air cargo booking control problem as a two-dimensional dynamic programming and developed three heuristics to solve it. They used the same revenue function as in Moussawi and Cakanyildirim (2005) and a linear offload cost function as in Luo et al. (2008). It is shown that their heuristics outperformed the FCFS policy. Compared to Karaesmen (2001), the stochastic nature of demand arrival was captured in the heuristics in Amaruchkul et al. (2005). Unfortunately, the weight and volume of demand were approximated by average values in the heuristics to avoid the curse of dimensionality. As a result, the stochastic nature of short-term air cargo booking control problem was still not fully captured.

Pak and Dekker (2004) viewed short-term booking control problem as a static multidimensional knapsack problem and applied the greedy algorithm in Kan et al. (1993) to solve it. Extensive simulations under different demand scenarios were then used to solve for bid prices and the final bid prices were obtained by computing the average bid prices over all demand scenarios. Pak and Dekker (2004) also showed that bid-price control policy was asymptotically optimal for the short-term booking control problem, which established the basis for the use of bid-price control policy in this thesis. A problem of Pak and Dekker (2004) is that extensive simulations are extremely time-consuming. Thousands of runs of simulations are needed to obtain a stable result for a practical scale problem. In addition, the optimality of bid prices obtained by Pak and Dekker (2004) is not guaranteed since the bid prices are calculated as the simple average of the results from all simulations.

Among the above literature, Karaesmen (2001), Amaruchkul et al. (2005) and Pak and Dekker (2004) are the only studies that consider both the stochastic nature and two-dimensionality of the problem. Among the above three studies, Pak and Dekker (2004) is the only study which fully captures the stochastic nature in short-term booking process. However, the algorithm provided by Pak and Dekker (2004) is not highly efficient and the optimality of the algorithm is not guaranteed. In view of this, we believe that there is plenty of space for the improvement of research in short-term air cargo booking control problem.

2.2.4 Long-term booking control

Hellermann (2006) proposed an options contract for the long-term allotment of air cargo capacity. Under this contract, each forwarder had to decide its order capacity and paid reservation fee for the capacity at the beginning of the planning horizon. After the demand was realized, each forwarder reported the actual capacity it needed, which should be less than the initial reserved quantity, and paid execution fee for the used capacity. This contract shifted part of the risk from airlines to forwarders. To the best of our knowledge, Hellermann (2006) was the only literature that addresses the long-term air cargo capacity allocation problem. However, Hellermann (2006) focused on the design of options contract in order to solve the problem of forwarders' default on penalties for unused capacity. The seasonal imbalance between supply and demand in air cargo industry was not addressed and the correlation between different seasons was not considered.

The long-term booking control problem is similar to the problem considered in supply chain management (SCM). The airline acts as the manufacturer and forwarders act as distributors. The airline decides the pricing of its product, and forwarders decide their order quantity in each period. The airline's product, i.e. air cargo capacity, is perishable without any salvage value. These are similar to the market dynamics in a SCM problem. However, the long-term booking control problem has its own distinction, which differentiates this problem from SCM. The difference will be discussed in Section 4.1. There are vast amounts of literature in SCM. A comprehensive review in this area is given in Tsay et al. (1999) and Cachon (2003).

Chapter 3 Air cargo booking control in spot market

As introduced in the first chapter, the air cargo industry can be classified into two markets, spot market and contract market. In this chapter, we focus on the single-leg air cargo booking control problem on the spot market. In section 3.1, a problem description and a large-scale mathematical integer programming formulation of the problem will be given. In section 3.2, a Markovian model based on a bid-price control policy is developed to model the booking process. Then, the optimal bid prices are obtained by maximizing a reward function of the Markov model. In section 3.3, numerical comparisons between the proposed approach and two existing static single-leg air cargo capacity allocation policies are presented.

3.1 Preliminary framework

Notations:

n and N --- Decision period with n denoting any period along the process and N denoting the time of departure;

W_n --- Cumulative weight of accepted booking requests until period n;

V_n --- Cumulative volume of accepted booking requests until period n;

c_w --- Weight capacity for air cargo;

c_v --- Volume capacity for air cargo;

d_w --- Weight of an individual demand;

d_v --- Volume of an individual demand;

$f_{wv}(d_w, d_v)$ --- Joint probability density function (pdf) of demand's weight and volume;

θ_s --- Standard inverse density defined by IATA, which is a constant;

λ --- Constant arrival rate;

p --- spot rate from accepting a certain type of cargo;

Prob --- Probability mass function (pmf) of discrete variables or probability of the happening of a certain event;

3.1.1 Problem description

We consider a single-leg flight with weight capacity c_w and volume capacity c_v. During a given booking period, demands with different type, weight and volume arrive at a constant rate λ. When a booking request is made, the airline has to decide whether to accept it or not according to the characteristic of the demand and the current selling profile. If the booking request is accepted, airline will receive revenue:

$$\text{Revenue}(d_w, d_v) = p \max(d_w, d_v / \theta_s) \tag{3.1}$$

where d_w and d_v are the weight and volume of the demand respectively, which follow a joint distribution $f_{wv}(d_w, d_v)$; p is the spot rate for the type of this cargo; and θ_s is a constant defined by the International Air Transportation Association (IATA) volumetric standard. The quantity d_v / θ_s is called dimensional weight. If the density of a cargo is larger than $1/\theta_s$, it will be charged according to its weight. Otherwise, it will be charged according to its dimensional weight. Different types of cargos may have different spot rates. For example, airline may charge a higher spot rate for live animals or precious cargos because they need special handling or security. As a result, we assume that p follows a discrete distribution with a support $\{p_1, p_2, ..., p_a\}$. It is assumed that p is independent of d_w and d_v and that when a booking request is rejected, no penalty is incurred.

The booking period is divided into N time periods, indexed by $0, 1, 2, \ldots, N$. Period 0 corresponds to the beginning of booking period and period N corresponds to the departure of flight. We can choose a large N so that one and only one booking request may arrive in one time period, i.e. the arrival rate $\lambda << 1$. As a result, the probability of a demand arriving in a period is λ and the probability of null event is $1-\lambda$ approximately. A bid price policy similar to that of Pak and Dekker (2004) is adopted to manage the booking requests. A booking request is accepted if

$$p \max\left(d_w, d_v / \theta_s\right) \geq h_w d_w + h_v d_v \tag{3.2}$$

$$\text{and} \qquad W_n + d_w \leq c_w,\ V_n + d_v \leq c_v \tag{3.3}$$

where h_w and h_v are bid prices for weight and volume respectively; W_n and V_n are cumulative weight and volume of all accepted cargos until period n; c_w and c_v are weight and volume capacity respectively.

The left hand side of the inequality (3.2) represents the revenue from accepting the cargo. Once the booking arrives, the weight, volume and type are known and the revenue is determined. The right hand side of the inequality (3.2) represents the opportunity cost of accepting the cargo, which depends on the bid prices h_w, h_v and the capacities d_w, d_v it consumes. Inequality equation (3.3) represents capacity constraints.

Our objective is to find the optimal bid prices so that the total revenue from booking requests is maximized.

3.1.2 A Utopia formulation – large-scale MIP

Suppose we are clairvoyant and know information of all demands that will show up in the future. The information includes weight, volume and profit rate of each individual demand and also their chronological sequence, which was denoted as $S=\left\{\left(d_{w_1},d_{v_1},p_{d_1}\right),\left(d_{w_2},d_{v_2},p_{d_2}\right),...,\left(d_{w_n},d_{v_n},p_{d_n}\right)\right\}$, where $\left(d_{w_j},d_{v_j},p_{d_j}\right)$ provides the weight, volume and profit rate information of the *j*th demand; p_{d_j} follows the discrete distribution assumed in the last section. Demand $\left(d_{w_j},d_{v_j},p_{d_j}\right)$ arrives earlier than demand $\left(d_{w_l},d_{v_l},p_{d_l}\right)$, if $j<l$. A collection of $\left\{\left(d_{w_j},d_{v_j},p_{d_j}\right);j=1,2,...,n\right\}$, S is called a possible demand scenario that may realize in the future. Based on the arrival rate, joint distribution of weight and volume and the discrete distribution of price rate, a brute-force approach in solving the problem is to exhaustively enumerate all the possible demand scenarios that may realize in the future. We assume that scenarios are independent of each other. Let $\xi=\left\{S_i=\left\{\left(d^i_{w_1},d^i_{v_1},p^i_{d_1}\right),\left(d^i_{w_2},d^i_{v_2},p^i_{d_2}\right),...,\left(d^i_{w_{n_i}},d^i_{v_{n_i}},p^i_{d_{n_i}}\right)\right\},i=1,2,...,m\right\}$, where S_i denotes the *i*th scenario; n_i is the number of booking requests in this scenario S_i; m denotes the number of possible demand scenarios. For each scenario, the acceptance/rejection decision on each demand will be made according to decision rules (3.2) and (3.3). Then the revenue from each scenario can be calculated based on the decisions. Once the probability that scenario i will realize in the future is known, the

expected revenue over all scenarios can be calculated. A mixed integer programming (MIP) model can then be formulated to find the optimal bid prices $\left(h_w, h_v\right)$ under which the expected revenue is maximized. Assume that a dummy demand $\left(d_{w_0}^i = 1, d_{v_0}^i = 0, p_{d_0}^i = -1\right)$ arrives at the beginning of each scenario. The formulation is as follows,

$$\max \quad E_\xi \left[\sum_{j=0}^{n_i} p_{d_j}^i \delta_j^i \max\left(d_{w_j}^i, d_{v_j}^i / \theta_s\right) \right]$$

s.t. for $i = 1, 2, ..., m$ (flight capacity constraints)

for $j = 1, 2, ..., n_i$

$$\sum_{k=0}^{j-1} d_{w_k}^i \delta_k^i + d_{w_j}^i - c_w \le M_1 \left(1 - \delta_j^i\right) \tag{3.4}$$

$$\sum_{k=0}^{j-1} d_{v_k}^i \delta_k^i + d_{v_j}^i - c_v \le M_2 \left(1 - \delta_j^i\right) \tag{3.5}$$

for $i = 1, 2, ..., m$ (bid-price control constraints)

for $j = 0, 1, 2, ..., n_i$

$$h_w d_{w_j}^i + h_v d_{v_j}^i - p_{d_j}^i \max\left(d_{w_j}^i, d_{v_j}^i / \theta_s\right) \le M_3 \left(1 - \delta_j^i\right) \tag{3.6}$$

for $i = 1, 2, ..., m$ (sequential accepting constraints)

for $j = 0, 1, 2, ..., n_i$

$$c_w - \sum_{k=0}^{j-1} d_{w_k}^i \delta_k^i - d_{w_j}^i + \varepsilon \le M_1 x_j^i \tag{3.7}$$

$$c_v - \sum_{k=0}^{j-1} d_{v_k}^i \delta_k^i - d_{v_j}^i + \varepsilon \le M_2 y_j^i \tag{3.8}$$

$$p_{d_j}^i \max\left(d_{w_j}^i, d_{v_j}^i / \theta_s\right) - h_w d_{w_j}^i - h_v d_{v_j}^i + \varepsilon \le M_3 z_j^i \tag{3.9}$$

$$\delta_j^i + 2 \ge x_j^i + y_j^i + z_j^i \tag{3.10}$$

x_j^i, y_j^i, z_j^i are binary variables

$h_w \ge 0, h_v \ge 0$

Decision variables:

h_w – bid price for weight capacity;

h_v – bid price for volume capacity;

$\delta_j^i = \begin{cases} 1, & \text{if the } j\text{th demand in scenario } i \text{ is accepted} \\ 0, & \text{if it is rejected} \end{cases}$ $j = 0,1,2,\ldots,n_i$ and $i = 1,2,\ldots,m$;

x_j^i, y_j^i, z_j^i – binary variables, $j = 1,2,\ldots,n_i$ and $i = 1,2,\ldots,m$.

Parameters:

$d_{w_j}^i$ – the weight of jth demand in scenario i, $j = 0,1,2,\ldots,n_i$ and $i = 1,2,\ldots,m$;

$d_{v_j}^i$ – the volume of jth demand in scenario i, $j = 0,1,2,\ldots,n_i$ and $i = 1,2,\ldots,m$;

$p_{d_j}^i$ – the profit rate of jth demand in scenario i, $j = 0,1,2,\ldots,n_i$ and $i = 1,2,\ldots,m$;

ε – a very small number;

M_1, M_2, M_3 – large numbers.

The first set of constraints is capacity constraints. If the left hand side of the inequality (3.4) or (3.5) is positive, i.e. the cumulative weight/volume exceeds the capacity limit, the decision variable will be equal to zero, i.e. the demand is rejected. The second set of constraints represents the bid price control policy. If the left hand side of inequality (3.6) is positive, i.e. the opportunity cost of accepting the demand is greater than its revenue, the demand is rejected. The third set of constraints ensures that a booking request will be accepted if it satisfies the bid price control policy and capacity constraints. If the left hand sides of inequality (3.7) and (3.8) are positive, i.e. accepting the current booking request will not violate capacity constraints, the binary variable x_j^i and y_j^i will be equal to 1. A small number ε is added so as to ensure that the binary variables x_j^i will be equal to 1 when $c_w - \sum_{k=0}^{j-1} d_{w_k}^i \delta_k^i - d_{w_j}^i = 0$, i.e. the current demand is allowed to be accepted, if the acceptance of this demand will use up the remaining weight capacity. Similarly, binary variable z_j^i will equal to 1 as long as the profit from accepting the current booking request is greater or equal to the opportunity cost. Then inequality (3.10) ensures that the current booking request is accepted, when all the criteria are satisfied. Since the maximum payload of Boeing 747 is around 60 tons, M_1 is set to be 60000, which is an upper bound of what we can expect from the left hand side of inequality (3.4). Similarly, M_2 and M_3 can be set to the corresponding upper bounds of inequality (3.5) and (3.6) respectively. In conclusion, a booking request is rejected if it violates any of the capacity constraints and bid price criterion. Otherwise, it will be accepted. Therefore, the decision for each booking request is fixed once the bid prices are fixed. A dummy demand is created at the beginning of each scenario in order to express the constraints in a neater way. It will not affect the structure of problem.

To further illustrate the MIP, a simple numerical example is provided as follows.

Example:

Suppose the capacity of the flight is $(c_w, c_v) = (100,10)$. The selling season is divided into 1000 small decision periods and the arrival rate is estimated as 0.003. Also, the weight, volume and spot rate information can be estimated according to the historical information and the forecasting of future demand. Based on the estimation of future demand parameters, we can generate future demand scenarios via simulation. For the simplicity of illustration, we assume that there are 3 possible scenarios, and they are represented as $S_1 = \{(55,5.5,1.2),(45,4.5,1.1),(45,4.5,1.3)\}$, $S_2 = \{(70,8.5,1.2),(45,4,1.4)\}$, and $S_3 = \{(40,3.5,1.2),(25,3,1.4),(50,6,1.1)\}$. Each scenario has the same probability to realize. Assume that the standard inverse density $\theta_s = 0.1$. Then the problem can be solved by the MIP. The optimal (h_w^*, h_v^*) falls in a region, which is characterized as $h_w^* + 0.1h_v^* \geq 1.1$, $0.82h_w^* + 0.1h_v^* \leq 1.2$, and $h_w^* + 0.0875h_v^* \leq 1.2$. The optimal acceptance/rejection decisions are {(1,0,1); (1,0); (1,1,0)}. The second demand in S_1 is rejected because this demand does not satisfy the bid-price control policy, i.e. $1.1 \times 45 \leq 45h_w^* + 4.5h_v^*$. The second demand in S_2 and the third demand in S_3 are rejected because it violates the capacity constraints.

The above example is designed to illustrate how to use the MIP. There are only three possible scenarios and no more than 3 demands in each scenario. As a result, the optimal bid prices are characterized by a region rather than accurate solutions. To solve a real problem with satisfactory precision level, we may have to generate thousands of demand scenarios and each demand scenario may consist of dozens of booking requests. Then, there can be more than one million constraints and hundreds of thousands of integer decision variables. Therefore, the MIP is intractable for a real problem. Two general approaches can be adopted to address this. One is to find the optimal bid prices for each scenario and then combine the result. The other approach is to obtain the expected revenue as a function of the bid prices and then solve for the optimal bid prices. Pak and Dekker (2004) adopted the former via extensive simulation. Here, we shall pursue the latter via a Markovian model.

3.2 A Discrete-Time Markov Chain Formulation with Bid Price Control Policy

The problem is solved in two phases. First, the expected revenue from the cargo bookings is expressed as a function of the bid prices h_w and h_v. Then, the optimal bid prices h_w^* and h_v^* is obtained by maximizing the expected revenue.

To simplify the modeling of booking process, the demand size d_w, d_v, state variables W_n , V_n and capacity c_w, c_v are discretized.

$$d_w = \begin{cases} x & \text{if } (x-0.5)SS_w \le D_w \le (x+0.5)SS_w \text{ and } x \in \{2,3,...,H_w - 1\} \\ H_w & \text{if } D_w > (H_w - 0.5)SS_w \\ 1 & \text{if } D_w < 1.5SS_w \end{cases} \tag{3.11}$$

where

d_w denotes the weight of cargo after discretization; D_w denotes the weight of cargo before discretization; SS_w is the step size for weight discretization; H_w is the maximum weight of an individual demand after discretization.

Similarly, d_v, W_n, V_n, c_w and c_v are discretized using the same scheme.

Let

d_v denotes the volume of a demand after discretization, taking value from $\{1,2,...,H_v\}$ and H_v is the maximum volume of individual demand after discretization;

$c_w = \lfloor C_w / SS_w \rfloor$ denotes the weight capacity after discretization, where C_w denotes the weight capacity before discretization;

W_n denotes the cumulative weight of accepted cargos until period n after discretization, taking value from $\{0,1,2,...,c_w\}$;

$c_v = \lfloor C_v / SS_v \rfloor$ denotes the volume capacity after discretization, where C_v denotes the volume capacity before discretization and SS_v is the step size for volume discretization;

V_n denotes the cumulative volume of accepted cargos until period n after discretization, taking value from $\{0,1,2,...,c_v\}$.

Although we use the same notations, i.e. d_v, W_n, V_n, c_w and c_v, we refer to the weight and volume after discretization in the remainder of this chapter. The joint pmf of d_w and d_v after discretization can be derived from $f_{wv}\left(d_w, d_v\right)$, i.e. the joint pdf of individual demand's weight and volume before discretization.

3.2.1 Phase I – Evolvement of Cumulative Weight and Volume

Let $W=\{W_n; n=0,1,...,N\}$ be the process of cumulative weight with a state space $E_w=\{0,1,...,c_w\}$ and $V=\{V_n; n=0,1,2,...,N\}$ be the process of cumulative volume with a state space $E_v=\{0,1,...,c_v\}$. Recall that the probability of a booking request in one period is λ and the probability of null event is $1-\lambda$. A booking request has to satisfy bid price control criterion and capacity constraints before it can be accepted. These two criteria are represented by inequality equation (3.2) and (3.3) respectively. There are three possible events in each period:

1. a demand arrives and is accepted
2. a demand arrives but is rejected because it violates any of the two criteria
3. no demand arrives

The three possible events and their effects on state transition are illustrated in the following graph.

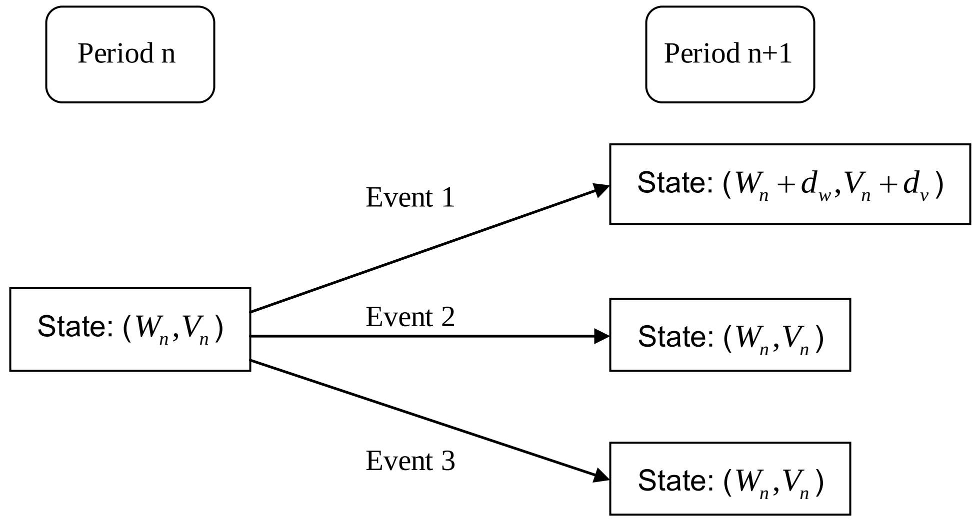


Figure 3.1 Three transitions in booking process

The first event will increase the cumulative weight and volume and thus, it is called acceptance transition. The second and third event will not change the cumulative weight and volume and thus, it is called no-change transition.

Note that we do not consider cancellations and no-shows in the current research. In the case of air cargo, to model cancellations and no-shows, one needs to keep track of not only the total accepted cargos but also each single order as the size and weight of each cancellation or no-show is tied to a particular booking and whether partial fulfillment is allowed. This will only dramatically complicate the model. A simple way around this is to adjust for the effect of cancellations and no-shows by an appropriate overbooking limit. This idea is proposed by Belobaba (1987) and has been used in practice for passenger revenue management. Since cancellations and no-shows are not considered in this research, W_n and V_n are non-decreasing over *n*.

Let $S=\left\{\left(W_n,V_n\right),n=0,...,N\right\}$, and $Q_{ij}\left(k,l\right)=\text{Prob}\left\{\left(W_n,V_n\right)=\left(k,l\right)|\left(W_{n-1},V_{n-1}\right)=\left(i,j\right)\right\}$.

We have the following.

Lemma 1 *The process S is a discrete-time Markov chain with state* $E_w \times E_v$ *and transition probability*

$$Q_{ij}\left(k,l\right)=\begin{cases}1-\lambda+\lambda\left[1-\text{Prob}\left(i+d_w\le c_w,\ j+d_v\le c_v,h_w d_w+h_v d_v\le p\max\left(d_w,d_v/\theta_s\right)\right)\right] & \text{if } i=k \text{ and } j=l\\ \lambda\text{Prob}\left(d_w=k-i,d_v=l-j\right)\text{Prob}\left(p\ge\dfrac{h_w\left(k-i\right)+h_v\left(l-j\right)}{\max\left(k-i,\left(l-j\right)/\theta_s\right)}\right) & \text{if } i<k \text{ and } j<l\\ 0 & \text{otherwise}\end{cases} \tag{3.12}$$

Proof:

Let

$$I_n=\begin{cases}1 & \text{if a demand arrives in period n}\\ 0 & \text{if no demand arrives in period n}\end{cases}.$$

Without loss of generality, let the size of demand in period n equal to $\left(d_w,d_v\right)$, if $I_n=1$.

$$\begin{aligned}&\text{Prob}\left(\left(W_n,V_n\right)=\left(k,l\right)|\left(W_{n-1},V_{n-1}\right)=\left(i,j\right),...,\left(W_0,V_0\right)\right)\\ &=\text{Prob}\left(\left(W_n,V_n\right)=\left(k,l\right),I_{n-1}=0|\left(W_{n-1},V_{n-1}\right)=\left(i,j\right),...,\left(W_0,V_0\right)\right)\\ &\quad+\text{Prob}\left(\left(W_n,V_n\right)=\left(k,l\right),I_{n-1}=1|\left(W_{n-1},V_{n-1}\right)=\left(i,j\right),...,\left(W_0,V_0\right)\right)\\ &=\text{Prob}\left(\left(W_n,V_n\right)=\left(k,l\right),I_{n-1}=0|\left(W_{n-1},V_{n-1}\right)=\left(i,j\right)\right)\\ &\quad+\text{Prob}\left(\left(W_n,V_n\right)=\left(k,l\right),I_{n-1}=1|\left(W_{n-1},V_{n-1}\right)=\left(i,j\right)\right)\end{aligned}$$

The process S possesses Markovian property.

Case 1: No-change transition

The probability of no-change transition consists of two parts. One is that no demand arrives and the other is that a demand arrives but is rejected. Given that a demand arrives, the rejection decision is due to the fact that the cargo fails to meet the two criteria. Since the arrival rate λ is independent of the sales profile,

$$
\begin{aligned}
&Q_{ij}(i,j)\\
&=1-\lambda+\lambda\left[\text{Prob}\left((W_n,V_n)=(i,j)\mid I_{n-1}=1,(W_{n-1},V_{n-1})=(i,j)\right)\right]\\
&=1-\lambda+\lambda\left[1-\text{Prob}\left(i+d_w\le c_w,\ j+d_v\le c_v, h_w d_w+h_v d_v\le p\max\left(d_w,d_v/\theta_s\right)\right)\right] \qquad (3.13)
\end{aligned}
$$

Case 2: Acceptance transition

In any period, the happening of all following three events will result in a system acceptance transition from $(W_{n-1}=i,V_{n-1}=j)$ to $(W_n=k,V_n=l)$:

(1) a demand with weight $d_w=k-i$ and volume $d_v=l-j$ arrives in period *n*;

(2) there is enough capacity for the demand

(3) the revenue of this demand is larger than the opportunity cost

The second event will definitely happen since $i,k\in E_w$ and $j,l\in E_v$. Therefore, the acceptance transition probability is

$$
\begin{aligned}
&Q_{ij}(k,l)\\
&=\text{Prob}\left(I_n=1,d_w=k-i,d_v=l-j,h_w d_w+h_v d_v\le p\max\left(d_w,d_v/\theta_s\right)\mid(W_{n-1},V_{n-1})=(i,j)\right)\\
&=\lambda\cdot\text{Prob}\left(d_w=k-i,d_v=l-j\right)\text{Prob}\left(p\ge\frac{h_w(k-i)+h_v(l-j)}{\max\left(k-i,(l-j)/\theta_s\right)}\right) \qquad (3.14)
\end{aligned}
$$

Therefore, S is a discrete-time Markov chain with transition probability given in equation (3.12). □

Let $\mathbf{Q} = \left\{ Q_{ij}\left(k,l\right); i,k \in E_w, j,l \in E_v \right\}$ denote the transition matrix of S. Since we do not consider cancellations and no-shows, $\mathbf{Q}$ is an upper-triangular matrix.

Let $P_{kl}^{(n)} = \text{Prob}\left(\left(W_n, V_n\right) = \left(k,l\right)\right)$, $k \in E_w$, $l \in E_v$, denote the probability that the state of process S is $\left(k,l\right)$ in period n. Therefore, the state transition of S can be described by the following recursive function:

$$P_{kl}^{(n)} = \sum_{i,j} P_{ij}^{(n-1)} \cdot Q_{ij}\left(k,l\right) \tag{3.15}$$

Denote vector $\mathbf{P}^{(n)} = \left\{ P_{kl}^{(n)}; k \in E_w, l \in E_v \right\}$. Then equation (3.15) can be expressed in a matrix form:

$$\mathbf{P}^{(n)} = \mathbf{P}^{(n-1)}\mathbf{Q} \tag{3.16}$$

It is obvious that the initial state of S is $\left(W_0, V_0\right) = \left(0,0\right)$ i.e. $P_{00}^{(0)} = 1$ and $P_{kl}^{(0)} = 0$; $\forall k \in E_w, k \neq 0, \forall l \in E_v, l \neq 0$. Therefore, we can predict the state of S in any period based on equation (3.16).

To better illustrate the model, we provide a small numerical example as follows.

Example:

Suppose $c_v = 5$, $c_w = 5$. The arrival rate λ is estimated as λ = 0.01. Suppose there are two types of cargo, ordinary cargo and precious good. The spot rate for ordinary cargo is 1.1 and the spot rate for precious good is 1.3. Assume $\text{Prob}\left(p = 1.1\right) = 0.7$ and

$\text{Prob}(p=1.3)=0.3$. The weight and volume of each individual cargo follows a join discrete distribution. $\text{Prob}(d_w=1,d_v=1)=0.4$, $\text{Prob}(d_w=2,d_v=2)=0.3$, $\text{Prob}(d_w=2,d_v=3)=0.2$ and $\text{Prob}(d_w=3,d_v=2)=0.1$. Assume that the distributions are the same for the two types of cargo. Let $\theta_s=1$. Also, suppose the bid prices are $h_w=0.5$ and $h_v=0.7$. Then, the one-step transition probability can be calculated from equation (3.12). For example,

$$Q_{00}(1,1)=0.01\times\text{Prob}(d_w=1,d_v=1)\times\text{Prob}\left(p\geq\frac{0.5+0.7}{1}\right)=0.0012$$

$$Q_{00}(0,0)=0.99+0.01\left[1-\text{Prob}\left(0.5d_w+0.7d_v\leq p\max(d_w,d_v)\right)\right]$$

If $(d_w,d_v)=(1,1)$, $\text{Prob}\left(0.5d_w+0.7d_v\leq p\max(d_w,d_v)\right)=\text{Prob}(p\geq1.2)=0.3$;

If $(d_w,d_v)=(2,2)$, $\text{Prob}\left(0.5d_w+0.7d_v\leq p\max(d_w,d_v)\right)=\text{Prob}(p\geq1.2)=0.3$;

If $(d_w,d_v)=(2,3)$, $\text{Prob}\left(0.5d_w+0.7d_v\leq p\max(d_w,d_v)\right)=\text{Prob}(3p\geq3.1)=1$;

If $(d_w,d_v)=(2,3)$, $\text{Prob}\left(0.5d_w+0.7d_v\leq p\max(d_w,d_v)\right)=\text{Prob}(3p\geq2.9)=1$.

Therefore, $Q_{00}(0,0)=0.9949$. Also, we have $Q_{00}(2,2)=0.0009$, $Q_{00}(2,3)=0.002$, $Q_{00}(3,2)=0.001$, and other transition probabilities with initial state (0, 0) equal to zero. Other probabilities in the transition matrix can be calculated likewise. Based on the one-step transition matrix, which is a 25×25 matrix in our example, we can calculate the state of the process in any decision period.

3.2.2 Phase I – Evolvement of Expected Revenue

The evolvement of cumulative weight and volume of accepted cargos during booking season can be monitored based on stochastic process *S*. To meet our objective of maximizing the expected revenue, a model for tracking the expected revenue is needed as the revenue from each booking request is a nonlinear function of d_w and d_v.

Let $\{R_n; n = 0,1,...,N\}$ denote the airline's expected revenue until period *n*. It is obvious that $R_0 = 0$;

Let $Q_{ij}\left(k,l \mid p_m\right)$ denotes the probability that S transits from state $\left(W_{n-1},V_{n-1}\right) = \left(i,j\right)$ to state $\left(W_n,V_n\right) = \left(k,l\right)$ given that cargo rate is p_m;

Let $r\left(i,j,k,l \mid p_m\right) = p_m \max\left(k-i,\left(l-j\right)/\theta_s\right)$ denotes the incremental revenue received by the airline if S transits from state $\left(W_{n-1},V_{n-1}\right) = \left(i,j\right)$ to state $\left(W_n,V_n\right) = \left(k,l\right)$ given that cargo rate is p_m.

Then we have, for $n \in \{1,2,...,N\}$,

Lemma 2 *Airline's expected revenue until period n can be expressed as follows*

$$R_n = \sum_{k,l}\sum_{i,j}\left(P_{ij}^{(n-1)} + P_{ij}^{(n-2)} + ... + P_{ij}^{(0)}\right)\Pi\left(i,j,k,l\right) \tag{3.17}$$

where

$$\Pi\left(i,j,k,l\right) = \sum_{m=1}^{a} Q_{ij}\left(k,l \mid p_m\right)\cdot r\left(i,j,k,l \mid p_m\right)\cdot \mathrm{Prob}\left(p = p_m\right) \tag{3.18}$$

Proof: According to Lemma 1,

$$R_n = \text{expected revenue until period } n-1 + \text{expected revenue accepted in period } n$$

$$= R_{n-1} + \sum_{i,j} P_{ij}^{(n-1)} \cdot \sum_{k,l} \sum_{m=1}^{a} Q_{ij}\left(k,l \mid p_m\right) \cdot r\left(i,j,k,l \mid p_m\right) \cdot \text{Prob}\left(p = p_m\right)$$

$$= R_{n-1} + \sum_{i,j} \sum_{k,l} P_{ij}^{(n-1)} \cdot \sum_{m=1}^{a} Q_{ij}\left(k,l \mid p_m\right) \cdot r\left(i,j,k,l \mid p_m\right) \cdot \text{Prob}\left(p = p_m\right)$$

$$= R_{n-1} + \sum_{k,l} \sum_{i,j} P_{ij}^{(n-1)} \cdot \Pi\left(i,j,k,l\right)$$

$$= \sum_{k,l} \sum_{i,j} \left(P_{ij}^{(n-1)} + P_{ij}^{(n-2)} + \ldots + P_{ij}^{(0)}\right) \Pi\left(i,j,k,l\right) \qquad \square$$

Remark. From equation (3.18),

$$\Pi\left(i,j,k,l\right)$$

$$= \begin{cases} \lambda \sum_{m=1}^{a} \text{Prob}\left(d_w = k-i, d_v = l-j\right) \cdot \text{Prob}\left(p_m \max\left(k-i, \left(l-j\right)/\theta_s\right) \geq h_w d_w + h_v d_v\right) & \\ \quad \cdot p_m \max\left(k-i, \left(l-j\right)/\theta_s\right) \cdot \text{Prob}\left(p = p_m\right) & \text{for } \forall i,k \in E_w, i<k \text{ and } j,l \in E_v, j<l \\ 0 & \text{otherwise} \end{cases} \tag{3.19}$$

Let matrix $\mathbf{\Pi} = \left\{\Pi\left(i,j,k,l\right); i,k \in E_w \text{ and } j,l \in E_v\right\}$. The sum operator $\sum_{i,j}$ in equation (3.17) can be recognized as a matrix multiplication operation, and thus,

$$\sum_{i,j} \left(P_{ij}^{(n-1)} + P_{ij}^{(n-2)} + \ldots + P_{ij}^{(0)}\right) \Pi\left(i,j,k,l\right)$$

$$= \left(\mathbf{P}^{(n-1)} + \mathbf{P}^{(n-2)} + \ldots + \mathbf{P}^{(0)}\right) \mathbf{\Pi} = \left(\mathbf{P}^{(0)} \mathbf{Q}^{n-1} + \mathbf{P}^{(0)} \mathbf{Q}^{n-2} + \ldots + \mathbf{P}^{(0)} \mathbf{I}\right) \mathbf{\Pi} \tag{3.20}$$

where $\mathbf{I}$ is an identity matrix with corresponding dimension.

The sum operator $\sum_{k,l}$ in equation (3.17) can be recognized as a matrix multiplication operation as well and thus, equation (3.17) becomes

$$R_n = \left(\mathbf{P}^{(0)}\mathbf{Q}^{n-1} + \mathbf{P}^{(0)}\mathbf{Q}^{n-2} + \ldots + \mathbf{P}^{(0)}\mathbf{I}\right)\mathbf{\Pi}\cdot\mathbf{u} \tag{3.21}$$

where $\mathbf{u}$ is a column vector with corresponding dimension and all its elements are equal to one, i.e. $\mathbf{u} = \{1,1,\ldots,1\}^{\mathrm{T}}$.

Since $\left(\mathbf{Q}^{n-1} + \mathbf{Q}^{n-2} + \ldots + \mathbf{I}\right)\cdot\left(\mathbf{I}-\mathbf{Q}\right) = \mathbf{I}-\mathbf{Q}^n$, the right-hand side of equation (3.21) can be further simplified as $\mathbf{P}^{(0)}\left(\mathbf{I}-\mathbf{Q}^n\right)\left(\mathbf{I}-\mathbf{Q}\right)^{-1}\mathbf{\Pi}\cdot\mathbf{u}$, if $\mathbf{I}-\mathbf{Q}$ is a nonsingular matrix. Unfortunately, states $\left\{(W,V); \forall W = c_w \text{ or } V = c_v\right\}$ of Markov chain S are absorbing states, and thus, $\mathbf{I}-\mathbf{Q}$ is a singular matrix. Therefore, we have to partition the matrices in order to calculate R_n.

Let $E_{\tilde{w}} = \{0,1,\ldots,c_w-1\}$ and $E_{\tilde{v}} = \{0,1,\ldots,c_v-1\}$ denote two sets. The states of S can be classified into two categories. States $\left\{(W,V); W \in E_{\tilde{w}}, V \in E_{\tilde{v}}\right\}$ are transient states, denoted as T, and other states are absorbing states, denoted as T^c. Partition the transition matrix $\mathbf{Q}$ as

$$\mathbf{Q} = \begin{pmatrix} \tilde{\mathbf{Q}} & \hat{\mathbf{Q}} \\ \mathbf{0} & \bar{\mathbf{Q}} \end{pmatrix} \tag{3.22}$$

where $\tilde{\mathbf{Q}} = \left\{Q_{ij}(k,l); (i,j)\in \mathrm{T}, (k,l)\in \mathrm{T}\right\}$; $\hat{\mathbf{Q}} = \left\{Q_{ij}(k,l); (i,j)\in \mathrm{T}, (k,l)\in \mathrm{T}^c\right\}$; and $\bar{\mathbf{Q}} = \left\{Q_{ij}(k,l); (i,j)\in \mathrm{T}^c, (k,l)\in \mathrm{T}^c\right\}$.

Partition matrix $\mathbf{\Pi}$ as

$$\mathbf{\Pi} = \begin{pmatrix} \tilde{\mathbf{\Pi}} \\ \bar{\mathbf{\Pi}} \end{pmatrix} \tag{3.23}$$

where $\tilde{\mathbf{\Pi}} = \left\{ \Pi\left(i, j, k, l\right); \left(i, j\right) \in \mathrm{T} \right\}$; and $\bar{\mathbf{\Pi}} = \left\{ \Pi\left(i, j, k, l\right) = 0; \left(i, j\right) \in \mathrm{T}^c \right\}$.

Let matrix $\tilde{\mathbf{P}}^{(n)} = \left\{ P_{ij}^{(n)}; \left(i, j\right) \in \mathrm{T} \right\}$. We then have,

Theorem: *Airline's expected revenue until period n can be expressed as follows*

$$R_n = \tilde{\mathbf{P}}^{(0)} \left(\mathbf{I} - \tilde{\mathbf{Q}}^n \right) \left(\mathbf{I} - \tilde{\mathbf{Q}} \right)^{-1} \cdot \tilde{\mathbf{\Pi}} \cdot \mathbf{u} \tag{3.24}$$

Proof: Since $\Pi\left(i, j, k, l\right) = 0$ for any $i \notin E_{\tilde{w}}$ or $j \notin E_{\tilde{v}}$, according to Lemma 2,

$$R_n = \sum_{k,l}^{k \in E_w, l \in E_v} \sum_{i,j}^{i \in E_{\tilde{w}}, j \in E_{\tilde{v}}} \left(P_{ij}^{(n-1)} + P_{ij}^{(n-2)} + \ldots + P_{ij}^{(0)} \right) \Pi\left(i, j, k, l\right) \tag{3.25}$$

Again, the sum operator $\sum_{i,j}^{i \in E_{\tilde{w}}, j \in E_{\tilde{v}}}$ and $\sum_{k,l}^{k \in E_w, l \in E_v}$ in equation (3.25) can be recognized as matrix multiplication operations. Therefore,

$$R_n = \left(\tilde{\mathbf{P}}^{(n-1)} + \tilde{\mathbf{P}}^{(n-2)} + \ldots + \tilde{\mathbf{P}}^{(0)} \right) \tilde{\mathbf{\Pi}} \cdot \mathbf{u} = \left(\tilde{\mathbf{P}}^{(0)} \tilde{\mathbf{Q}}^{n-1} + \tilde{\mathbf{P}}^{(0)} \tilde{\mathbf{Q}}^{n-2} + \ldots + \tilde{\mathbf{P}}^{(0)} \right) \cdot \tilde{\mathbf{\Pi}} \cdot \mathbf{u} \tag{3.26}$$

The matrix $\left(\mathbf{I} - \tilde{\mathbf{Q}} \right)^{-1}$ arises frequently in absorption calculations and is known as the fundamental matrix (Resnick, 1992). Since the state space is finite, $\mathbf{I} - \tilde{\mathbf{Q}}$ is a nonsingular matrix and has an inverse. The above equation can be simplified as

$$R_n = \tilde{\mathbf{P}}^{(0)} \left(\mathbf{I} - \tilde{\mathbf{Q}}^n \right) \left(\mathbf{I} - \tilde{\mathbf{Q}} \right)^{-1} \cdot \tilde{\mathbf{\Pi}} \cdot \mathbf{u} \qquad \square$$

Recursive function (3.21), which tracks the evolvement of expected revenue over time, is a reward function built on discrete-time Markov chain *S*. *S* describes how the capacity of aircraft is consumed over time and *R* represents the reward that airline receives from the consumption of capacity. Because of this modeling technique, the use of high dimension

Markov chain is avoided and thus, the problem is more tractable. The transition can be shown in the following chart.

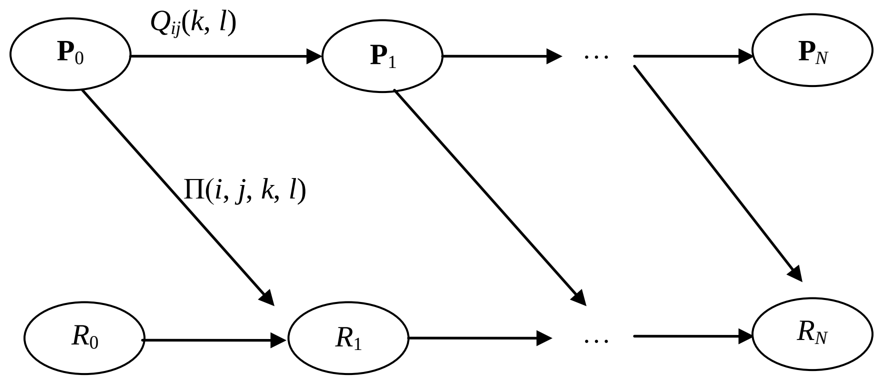


Figure 3.2 Transition diagram of capacity and expected revenue

3.2.3 Phase II – Optimizing control parameters:

According to the theorem, we can develop the final expected revenue of system:

$$R_N = \tilde{\mathbf{P}}^{(0)}\left(\mathbf{I}-\tilde{\mathbf{Q}}^N\right)\left(\mathbf{I}-\tilde{\mathbf{Q}}\right)^{-1}\cdot\tilde{\mathbf{\Pi}}\cdot\mathbf{u} \tag{3.27}$$

The final expected revenue of the system R_N is a function of bid prices h_w and h_v. Once h_w and h_v are determined, the final expected revenue of the system can be obtained.

So far, we have successfully solved phase I problem. In phase II problem, the optimal bid prices h_w^* and h_v^* is to be determined so that the expected revenue is maximized. However, it is very difficult to develop close form solutions to h_w^* and h_v^* since the structure of problem is very complex. Fortunately, Figure 3.3 shows that the surface of $R_N\left(h_w, h_v\right)$ is unimodal and thus the optimal bid prices can be easily found by some unconstrained optimization methods. From the MIP model, it is clear that the expected revenue is a discrete function of h_w and h_v, but the discrete function can be "smoothed" by precise discretization so that the numerical unconstrained optimization methods can provide satisfactory results. The direct searching algorithm instead of gradient based searching algorithm is chosen so that the "spiky" surface will render less negative effects on our searching. In the numerical analysis, simplex searching method (Murray (1972) p24-28) with nonnegative restriction is applied to find the optimal bid prices.

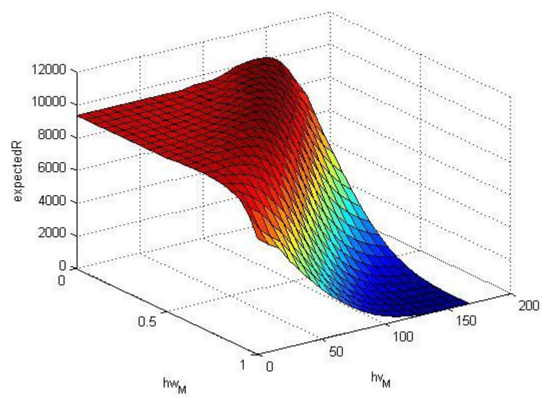


Figure 3.3 Surfaces of expected revenue with respect to bid prices

3.3 Numerical Analysis

As mentioned in the literature review, Pak and Dekker (2004) provided an algorithm for a single-leg short-term air cargo booking control problem with two-dimensional capacity constraints. First-Come-First-Booked (FCFB) can be viewed as another policy for air cargo booking control problem. In this section, simulation runs are conducted in order to compare the performance of the algorithm proposed in this thesis (named algorithm A in the remainder of this chapter) with the performance of the algorithm in Pak and Dekker

(2004) (named algorithm B) and the FCFB policy. These three policies are all static bid price control policies since the FCFB policy can be viewed as a bid-price control policy with two zero bid prices.

The simulation procedures are as follows:

Step 1. Set the capacities of the aircraft, demand distributions and the length of booking period. Solve the corresponding booking control problem using algorithm A and B and record the resulting control parameters respectively. The flow chart of step 1 is shown in Figure 3.4.

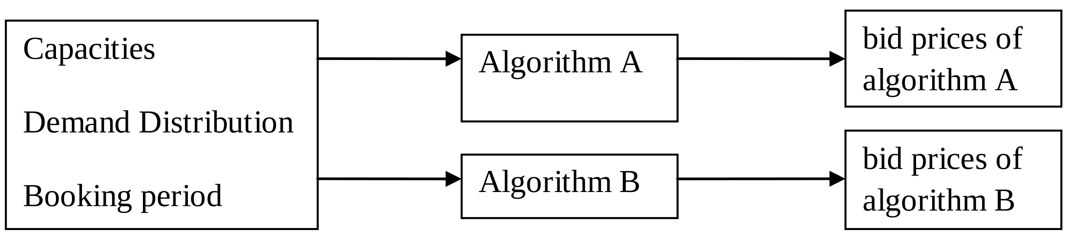


Figure 3.4 Flow chart for step 1 of simulation

Step 2. Generate a demand scenario based on the demand distribution and the length of booking period as assumed in step 1. Accept demands in this demand scenario under different bid prices of different policies and record the corresponding revenue respectively. The flow chart of step 2 is shown in Figure 3.5.

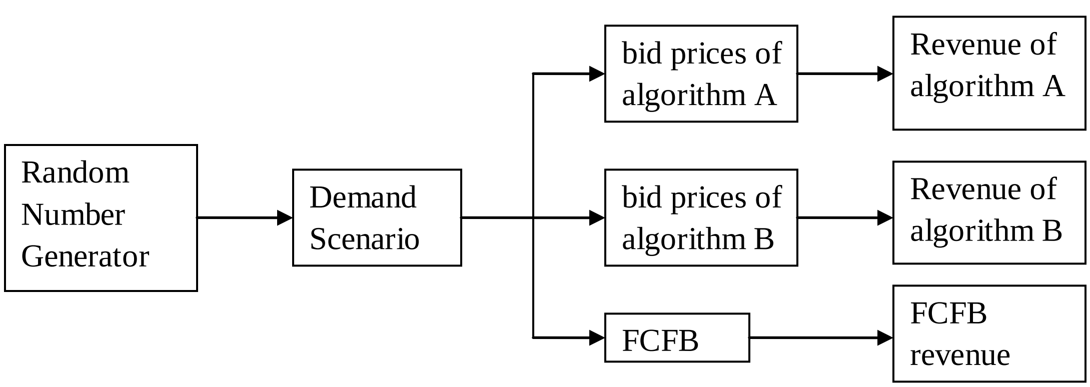


Figure 3.5 Flow chart for step 2 of simulation

Step 3. Find the difference between the revenue of algorithm A and algorithm B, and also the difference between the revenue of algorithm A and the FCFB policy.

Step 4. Repeat step 2 and step 3. Record the results and plot histograms.

Step 5. Repeat step 1 ~ step 4 under different capacities, demand distribution and length of booking period. Compare the performance of the three policies under different situations.

Boeing 747, which is commonly used combi-aircraft, is chosen as the aircraft in the simulation. The technical data of Boeing 747 is shown in Table 3.1.

Table 3.1 Technical data of Boeing 747

Type	Boeing 747
Max. Payload (kg)	63,917
Seat Num.	400
passenger weight (include luggage) (kg)	40,000
cargo weight (kg)	23,917
total cargo volume (m^3)	157
available volume (m^3)	109
Short-term booking percentage	50%
cargo weight for short-term booking (kg)	11,958.5
cargo volume for short-term booking (m^3)	54.5

We assume that each passenger (including luggage) weights 100 kg, and the volume for the luggage of each passenger is 0.12 m^3. Therefore, under full customers' loading, the weight available for cargo transportation is $\text{Max. payload} - 100 \times \text{seat number} = 23{,}917 \text{ kg}$, and the available volume for cargo transportation is total volume capacity – 0.12 $\times$ seat number = 109 m^3. We further assume that 50% of the capacity for cargo transportation is reserved for long-term contracts. Therefore, the available capacities for ad-hoc sales are 11958.50 kg and 54.50 m^3.

The free-sale demand distribution is shown in Table 3.2.

Table 3.2 Parameters of demand distribution

Demand	Distribution	Parameters	Mean	Standard deviation
Weight	Lognormal	μ=6.2365, σ=0.9380	793.474 (kg)	942.370 (kg)
Inverse density	Lognormal	μ=-5.2939, σ=0.5399	0.00581 (m^3/kg)	0.00338 (m^3/kg)
Volume	Lognormal	μ=0.9426, σ=1.0823	4.61 (m^3)	6.8791 (m^3)

The weight of each cargo follows a lognormal distribution with μ=6.2365 and σ=0.9380. The inverse density of each cargo follows a lognormal distribution with μ=-5.2939 and σ=0.5399. It is assumed that the cargo weight is independent of cargo density. Therefore, the volume of each cargo also follows a lognormal distribution with parameters μ=0.9426, σ=1.0823. These assumptions are adopted from Pak and Dekker (2004), in which it is claimed that these assumptions were derived from real data. Denote the joint pdf of the weight and volume of each cargo as $f_{wv}(D_w, D_v)$, where D_w and D_v are the weight and volume of cargo before discretization. The joint probability mass (pmf) function $\text{Prob}(d_w, d_v)$ needs to be derived from $f_{wv}(D_w, D_v)$, where d_w and d_v are the weight and volume of cargo after discretization. Suppose the discretization method described in equation (3.11) is used. Then, the joint pmf can be calculated by a numerical integration. For example, given that a demand arrives, the probability of $(d_w = x, d_v = y)$ is $\text{Prob}(x, y) = \int_{(x-0.5)SS_w}^{(x+0.5)SS_w} \int_{(y-0.5)SS_v}^{(y+0.5)SS_v} f_{wv}(D_w, D_v) dD_w dD_v$, where $x < H_w$ and $y < H_v$.

As mentioned in section 2.1, different types of cargos may have different profit rates *p*. In our numerical experiments, we assume that there are ten types of cargos. The profit rates and the corresponding probabilities that a cargo belongs to a certain type are listed in Table 3.3.

Table 3.3 Profit rates and corresponding probabilities of cargos

Type	1	2	3	4	5	6	7	8	9	10
profit rates	1.1	1.02	0.9	0.8	0.78	0.85	0.97	0.7	0.68	0.53
probabilities	0.0835	0.0307	0.1241	0.0993	0.0699	0.0767	0.1399	0.1931	0.1151	0.0677

Based on the initial condition introduced above, the performances of the three policies are simulated under different demand rates, which are shown in Table 3.4.

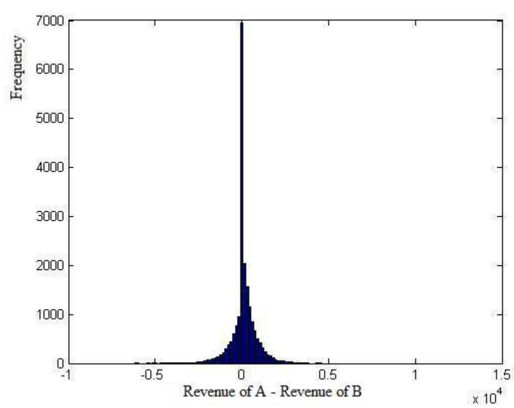
Table 3.4 Demand rates of different simulation runs

Parameters	simulation 1	simulation 2	simulation 3	simulation 4
decision period T (10 days)	1000	1500	2000	4000
demand rate	0.0151	0.0201	0.02262	0.01885
demand/capacity ratio (weight)	1	2	3	5
sample size	20000	20000	20000	20000

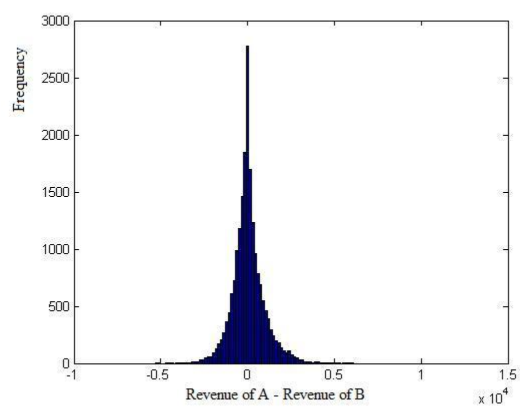
In simulation 1, the booking period is divided into 1000 decision periods. The demand follows a homogeneous Poisson process with demand arrival rate $\lambda = 0.0151$ and thus, the expected total weight of demands is approximately equal to the capacity of aircraft. The sample size for each simulation run is 20000 so that histograms are stable. We change demand rates of the other 3 simulation runs so that the expected demand/capacity ratios are 2, 3 and 5 in terms of weight respectively. In doing so, the performances of the three policies under peak demand period or non-peak demand period can be examined. The results of the four simulation runs are shown in the following Table and graphs.

Table 3.5 Simulation results under different demand/capacity ratio

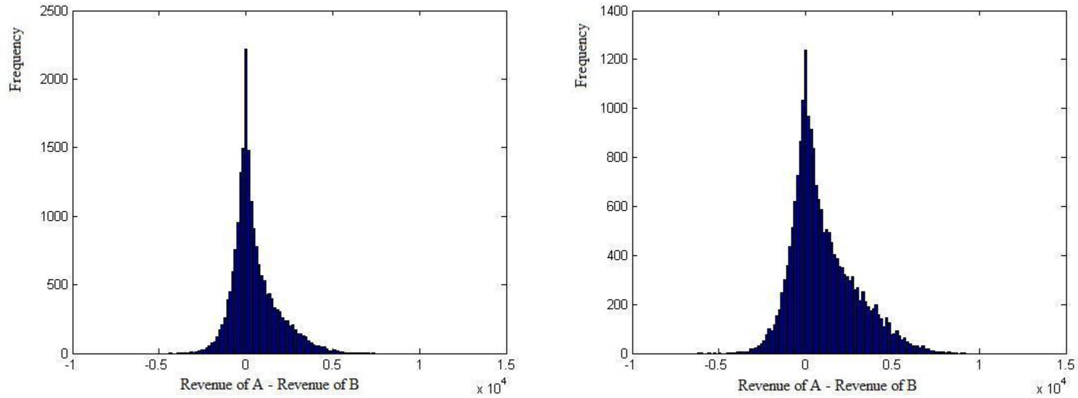
	simulation 1	simulation 2	simulation 3	simulation 4
A – B > 0	9290 (46.45%)	9516 (47.58%)	11450 (57.25%)	13264 (66.32%)
A – B = 0	5554 (27.77%)	928 (4.64%)	612 (3.06%)	130 (0.65%)
A – B < 0	5156 (25.78%)	9556 (47.78%)	7938 (39.69%)	6606 (33.03%)
A – FCFB > 0	0	15592 (77.96%)	18188 (90.94%)	19172 (95.86%)
A – FCFB = 0	20000 (100%)	166 (0.83%)	8 (0.08%)	2 (0.02%)
A – FCFB < 0	0	4242 (21.21%)	1804 (9.02%)	826 (4.13%)
Std of A	1666	1243.4	1163.8	1125.4
Std of B	1900.4	1492.2	1866.6	2228.1
Std of FCFB	1666	1030.9	1032	1035



(a) simulation 1

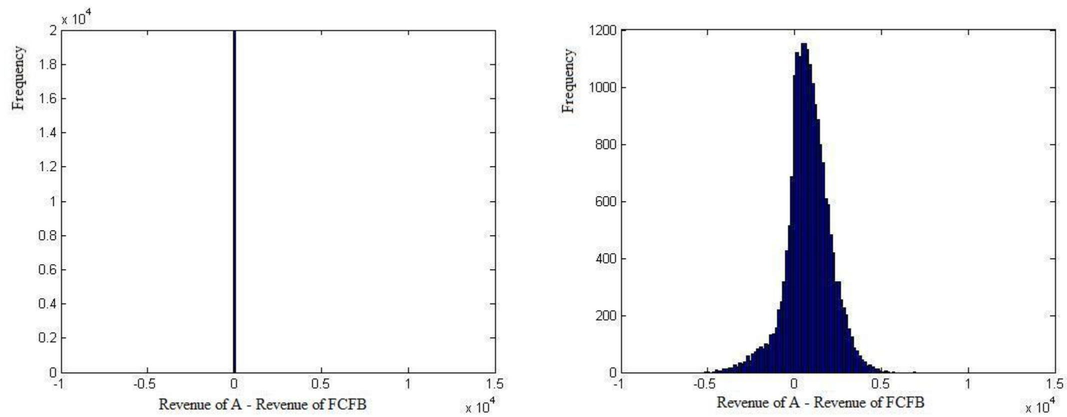


(b) simulation 2

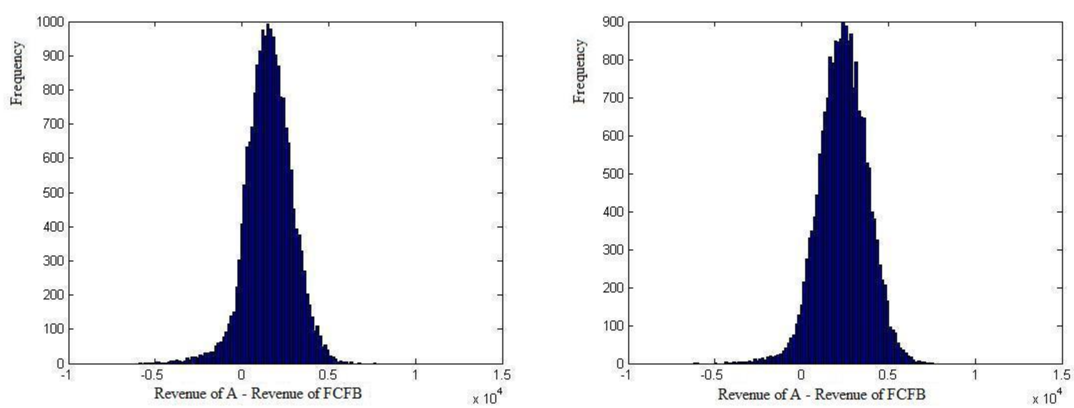


(c) simulation 3 (d) simulation 4

Figure 3.6 Histogram of the difference between the revenue of A and B



(a) simulation 1 (b) simulation 2



(c) simulation 3 (d) simulation 4

Figure 3.7 Histogram of the difference between the revenue of A and FCFB

The second row in Table 3.5 records the number of samples (and the percentage of samples) for which algorithm A generates a higher revenue than algorithm B. The third row in Table 3.5 records the number of samples (and the percentage of samples) for which algorithm A generates the same revenue as algorithm B. The fourth row in Table 3.5 records the number of samples (and the percentage of samples) for which algorithm B generates a higher revenue than algorithm A. In simulation 1, A outperforms B in 46.45% of all scenarios, whereas B outperforms A in 25.78% of all scenarios. As a result, A outperforms B in simulation 1. In simulation 2, A outperforms B in 47.58% of all scenarios, whereas B outperforms A in 47.78% of all scenarios. As a result, algorithm A is as good as algorithm B in simulation 2. Following the same way of comparison, we can see from Table 3.5 that algorithm A outperforms algorithm B in simulation 3 and 4. The following three rows in Table 3.5 are the comparisons between algorithm A and FCFB policy. It is clear that A significantly outperforms FCFB policy in simulation 2 ~ 4. The last three rows of Table 3.5 record the standard deviations of revenue under different policies respectively. From the results, we can see that the standard deviation of revenue obtained by algorithm A is smaller than that of algorithm B, though slightly higher than that of FCFB policy.

Histograms in Figure 3.6 and 3.7 also show that A outperforms B and FCFB policy. The horizontal axis of Figure 3.6 is the difference between the revenue from A and B, and the vertical axis of Figure 3.6 is the number of replications/scenarios. The horizontal axis of Figure 3.7 is the difference between the revenue from A and FCFB, and the vertical axis of Figure 3.7 is the number of replications/scenarios. From these two figures, we can see that the scenarios located to the right of zero point are more than those to the left of zero point in most cases. Therefore, it is clear that the algorithm proposed in this thesis outperforms Pak and Dekker's algorithm and FCFB policy in most cases in terms of expected revenue.

Table 3.5 and Figure 3.6 and 3.7 also show that the improvement from algorithm A becomes more significant as the demand/capacity ratio increases. The reason can be illustrated in Figure 3.8 and 3.9.

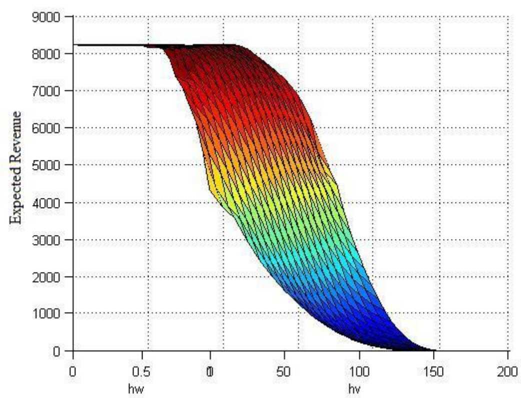


Fig. 3.8 (a) surface of expected revenue

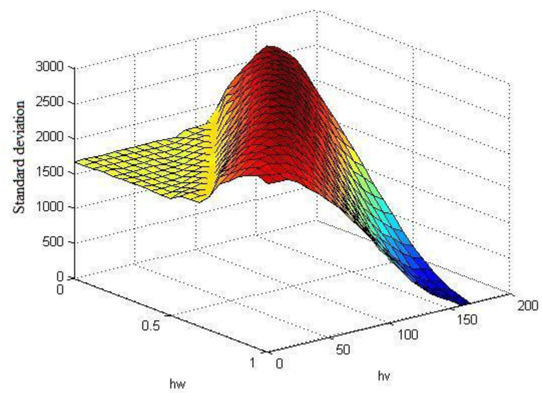


Fig. 3.8 (b) surface of standard deviation

Figure 3.8 Surfaces with demand/capacity ratio equal to 1

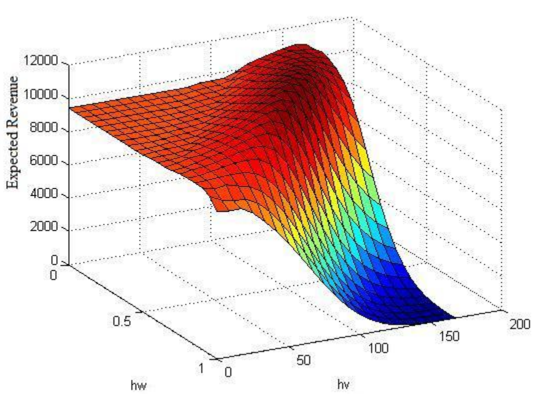


Fig. 3.9 (a) surface of expected revenue

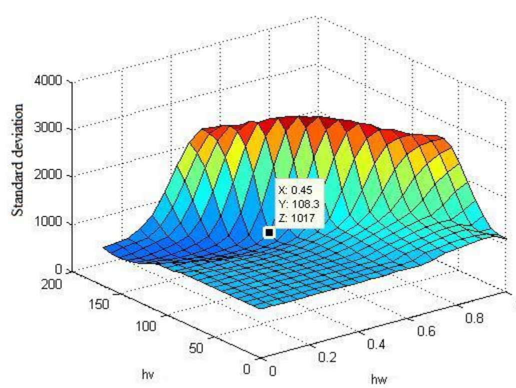


Fig. 3.9 (b) surface of standard deviation

Figure 3.9 Surfaces with demand/capacity ratio equal to 5

The demand/capacity ratio of Figure 3.8 is approximately equal to 1. In this case, most booking requests will be accepted due to the low demand. The results of different control policies are about the same. From Figure 3.8, one can see that the response surface around the peak is very flat. The difference between the expected revenue of peak point and the expected revenue of a point around the peak is very small compared to the standard deviation. As a result, the signal (difference between algorithm A and algorithm B) is overwhelmed by noise (variance of revenue). In contrast, the airline has much more flexibility to choose more “profitable” booking requests during the peak period of demand. Whether the optimal bid prices are chosen can greatly affect the expected revenue. From Figure 3.9, one can see that the response surface around the peak is much steeper when demand/capacity ratio is about 5. The difference between the expected revenue of peak point and the expected revenue of a point around the peak is around 2000, which is larger than the standard deviation. Therefore, the improvement from algorithm A is more significant during the peak period of demand.

In conclusion, the algorithm proposed in this thesis can generate higher revenue than both Pak and Dekker’s algorithm and FCFB policy while providing consistent results.

Chapter 4 Long-Term Capacity Control in Contract Market

In this chapter, we focus on the capacity allocation in the contract market. In section 4.1, a detailed description of the problem is given and a capacity bundling policy (CBP) is proposed to solve the problem. In section 4.2, some preliminaries and a business model will be introduced. Also, the mathematical model for airline's problem and forwarder's problem will be proposed and solved respectively. In section 4.3, it is assumed that the CBP takes a linear form. With this assumption, the forwarder's problem and the airline's problem are re-solved and some properties are explained. In section 4.4, some numerical experiments are conducted to further investigate the effects of this business model under various conditions.

4.1 Introduction and problem description

Here we focus on managing capacity allocation in contract market. In long-term contract market, it is a standard industry practice that air carriers and forwarders close long-term capacity agreements. In particular, forwarders order certain capacity between a certain origin-destination (O-D) pair in a certain time period, and resell the capacity to shippers. The demand in air cargo industry has strong seasonality. Usually, there will be a peak period from the beginning of November till the end of December. During this period, the total demand from shippers is significantly higher than the demand in other periods. The forwarders often face difficulties to lock in enough capacity in the peak season. In low season, however, the total demand from shippers is often less than airlines' capacity and airlines often face difficulties to sell out all capacity. The strong seasonality in demand and the relatively fixed supply create a mismatch between the supply (airline) and the demand (forwarder).

Airlines also face a similar problem in passenger service sector, and a common way to solve this problem in RM is to charge a high price during the peak season while giving discount during the low season. However, this method may not be appropriate in air cargo industry, because the concerns in managing the guaranteed capacity contract are different from the concerns in passenger RM. In the sale of airline seats, each passenger booking only contributes a tiny part of the total revenue of the airline. Unlike passenger sales, which are anonymous and numerous, air cargo carriers work closely with a few important customers who ship large volumes. Long-term customer relations take priority. Therefore, most airlines do not charge a very high rate for the capacity in peak season to keep good relationships with forwarders.

Besides the relationship considerations, the airline may also face the pressure from regulators. This can happen because the regulators are afraid that the high rate in the peak season will harm the business of large manufacturers and create negative effect on the country's economic development. To attract the high-value manufacturers, the regulators may impose "invisible" restrictions on the rate of air cargo transportation. As a result, though the rate during the peak season is expected to be higher than the rate during low season, there will be an implicit upper bound for the rate in peak season.

To make things more complicated, the long-term capacity agreements are signed several months in advance. As a result, the forecasting of the future demand can be very inaccurate. Because of the uncertainty in the forecasting, forwarders are not willing to make any fixed commitment in low seasons, if there are no substantial benefits from the commitment. In turn, forwarders' unwillingness to make fixed commitment creates a lot of uncertainty in the airline's revenue during low seasons, and thus, the airline relies more on the revenue from peak season. As a result, it is more difficult for forwarders to get enough capacity at a reasonable rate during the peak season.

To deal with the seasonality, many airlines in air cargo industry adopt a capacity bundling policy (CBP). In particular, the capacity during the peak season that each forwarder can lock in depends on its order quantity during the low season. The more capacity the forwarder orders during the low season, the more guaranteed capacity it can get during the peak season. As a result, the forwarder has incentive to book more capacity during the low season as a support to the airline, and expect for the reciprocation from the airline during the peak season.

The CBP has several benefits. First, this policy can motivate forwarders to market more aggressively during the low season and save them the efforts to secure capacity during the peak season. Second, this policy may increase the airline's load factor during the low season and smooth the revenue over a year. Last but not the least, the airline adopting the CBP can have an advantage over those airlines who do not adopt the policy. After years operations, airlines and forwarders can form strong strategic alliance, and thus, can achieve better risk sharing. Because of these advantages, many airlines adopt such a policy nowadays.

Although the CBP is widely adopted by airlines, there is a lack of quantitative analysis of its effect on the expected profit and risk of the airline. Here, risk is defined as the probability that the airline's profit over a year is less than a certain sales target. Also, whether the policy is not as effective as expected under some conditions is still an open question. Therefore, we would like to study these questions in this research.

As explained in chapter 2, Hellermann (2006) is the only literature that addressed the long-term capacity allocation problem in air cargo industry. However, Hellermann (2006) focused on the design of options contract in order to mitigate the effect of forwarders' defaulting on penalties for unused capacity. The mismatch between supply and demand in air cargo industry was not addressed and the correlation between different seasons was not considered.

It seems that the long-term capacity control problem is similar to the problem considered in supply chain management (SCM). The problem considered in this thesis is a two-echelon, multi-period, static pricing SCM problem with perishable product and no backlog of demand. Since no demand and capacity can be backlogged into the next period, it seems that the multi-period problem can be decomposed into several independent single-period SCM problems. However, to mitigate the mismatch between demand and supply in different periods, the CBP is adopted and it links the revenue in peak seasons with the decisions in the low seasons. Thus, the long-term capacity control problem is different from the commonly considered SCM problems.

4.2 Long-term capacity allocation problem

4.2.1 Preliminaries and the business model

Notations:

Q_{it} – forwarder *i*'s order quantity in period *t*;

w_t – contract rate in period *t*;

x_t – penalty rate for the unused capacity in period *t*;

C_t – airline's capacity in period *t*;

p_{it} – forwarder *i*'s resale rate in period *t*;

$\tilde{D}_{it}$ – stochastic demand faced by forwarder *i* in period *t*;

d_{it} – deterministic part of the demand faced by forwarder *i* in period *t*;

$\tilde{\varepsilon}_{it}$ – stochastic noise of the demand faced by forwarder *i* in period *t*;

$f_{it}\left(\varepsilon_{it}\right)$ and $F_{it}\left(\varepsilon_{it}\right)$ – pdf and cdf of stochastic noise respectively.

We consider an air cargo industry, which includes one major airline serving *n* forwarders. The planning horizon is one year. We assume that the planning horizon can be divided into *m* periods. The capacity that the airline will provide in period *t* is denoted as C_t. It is assumed that this capacity is given. At the beginning of the year, the airline announces the contract rates w_t in each period. Then, each forwarder closes long-term contracts with the airline to reserve certain capacity between an O-D pair in each period. The order quantity from forwarder *i* in period *t* is denoted as Q_{it}. The forwarder decides the order quantity according to the contract rate w_t and its forecasting of the future demand in the period. Forwarder *i*'s estimation of future demand from shippers in period *t* is denoted as

$$\tilde{D}_{it} = d_{it} + \tilde{\varepsilon}_{it} = a_{it} - b_{it} p_{it} + \tilde{\varepsilon}_{it},\ t = 1, 2, ..., m,$$

where a_{it} and b_{it} are parameters of the linear demand function, $a_{it} > 0$ and $b_{it} > 0$;

p_{it} is forwarder *i*'s resale rate in period *t*;

$\tilde{\varepsilon}_{it} \in \left[\underline{\varepsilon}_{it}, \overline{\varepsilon}_{it}\right]$ is the noise of demand, which follows a cdf $F_{it}\left(\varepsilon_{it}\right)$.

This linear additive demand function is widely used in newsboy problems and is studied by both Lau and Lau (1988) and Polatoglu (1991). Here, it is assumed that the resale rate p_{it} is not a decision variable of forwarder *i*. According to Hellermann (2006), forwarders usually do not decide the resale rate analytically. Typically, they will add a markup to the contract rate to cover their cost and profit. As a result, we assume that $p_{it} = w_t \cdot \left(1 + \gamma_{it}\right)$, where γ_{it} denotes the markup of forwarder *i* in period *t* and it is known by the airline. If the actual demand from its customers is less than its ordered capacity, the forwarder has to pay penalty $x_t = \beta_t w_t$ to the airline for each unit of unused capacity, where β_t is the penalty ratio in period *t*.

For the ease of presentation, we assume that periods 1 to *k* are low seasons, while periods $k+1$ to *m* are peak seasons. Any change in the sequence of the low/peak seasons will not affect the mathematical formulation and the results. During low seasons, the total demand from shippers is generally less than the capacity of the airline. Although the demand from shippers can be close to the airline's capacity in some periods during low seasons, the airline has no incentive to encourage the forwarders to book more capacity in these periods by sacrificing some profit in peak season. In such a case, the airline's problem and the forwarders problem are not related to the problems in other periods and they can be solved independently. Therefore, it is natural to assume that $\sum_{i=1}^{n} Q_{it} < C_t$ and the airline will accept all orders in the low seasons. Also, it is assumed that the unsold capacity of the airline in the low season has no salvage value. In contrast, the demand from forwarders will always exceed the airline's capacity during the peak period. It results from both seasonality and the restriction on airline's pricing.

As mentioned in the previous chapter, the airline adopts a CBP that the guaranteed capacity that each forwarder can get during the peak season depends on its order quantities during the low season. Let c_{it} denote forwarder i's guaranteed capacity during the peak season.

$$c_{it} = \varphi_t\left(Q_{i1}, Q_{i2}, \ldots, Q_{ik}\right), \qquad t = k+1, \ldots, m$$

where $\varphi_t(\bullet)$ is a given function and it is known by both the airline and forwarders.

The airline's problem is to decide the contract rate w_t in each period so that the expected profit over a year is maximized. The forwarder's problem is to decide the order quantity Q_{it} in each period so that the expected profit over a year is maximized.

4.2.2 Forwarder's problem

Let $\pi_{it}\left(Q_{it}\right)$ denote forwarder i's expected profit in period t in low season, given its order quantity Q_{it}. Then, $\pi_{it}\left(Q_{it}\right)$ can be expressed as follows.

$$\pi_{it}\left(Q_{it}\right)=\begin{cases}\int_{\underline{\varepsilon}_{it}}^{Q_{it}-d_{it}}\left[\left(p_{it}-w_t\right)\left(d_{it}+\varepsilon_{it}\right)-x_t\left(Q_{it}-d_{it}-\varepsilon_{it}\right)\right]\cdot f_{it}\left(\varepsilon_{it}\right)d\varepsilon_{it} & \\ \quad+\int_{Q_{it}-d_{it}}^{\bar{\varepsilon}_{it}}\left(p_{it}-w_t\right)Q_{it}\cdot f_{it}\left(\varepsilon_{it}\right)d\varepsilon_{it} & \text{if } \underline{D}_{it}<Q_{it}<\bar{D}_{it} \\ \int_{\underline{\varepsilon}_{it}}^{\bar{\varepsilon}_{it}}\left[\left(p_{it}-w_t\right)\left(d_{it}+\varepsilon_{it}\right)-x_t\left(Q_{it}-d_{it}-\varepsilon_{it}\right)\right]\cdot f_{it}\left(\varepsilon_{it}\right)d\varepsilon_{it} & \text{if } Q_{it}\geq\bar{D}_{it} \\ \int_{\underline{\varepsilon}_{it}}^{\bar{\varepsilon}_{it}}\left(p_{it}-w_t\right)Q_{it}\cdot f_{it}\left(\varepsilon_{it}\right)d\varepsilon_{it} & \text{if } Q_{it}\leq\underline{D}_{it}\end{cases}\quad t=1,2,...,k \tag{4.1}$$

During the peak season, the total demand from the shippers is considerably higher than the capacity of the airline, and thus, we assume that forwarders can always sell out all the guaranteed capacity. Therefore, the expected profit of forwarder *i* over period *t* in peak season can be expressed as

$$\pi_{it}=\left(p_{it}-w_t\right)\cdot c_{it},\quad t=k+1,...,m \tag{4.2}$$

where $c_{it}=\varphi_t\left(Q_{i1},Q_{i2},...,Q_{ik}\right)$, $t=k+1,...,m$.

The objective of the forwarder is to find the optimal order quantity in each period so that the expected profit over a year is maximized. This can be modeled as follows.

$$\max_{Q_{i1},...Q_{ik}}\sum_{t=1}^{k}\pi_{it}\left(Q_{it}\right)+\sum_{t=k+1}^{m}\pi_{it} \tag{4.3}$$

Let $U_t\left(\mathbf{Q}_{it-1}\right)$ denotes the forwarder *i*'s maximal expected profit over periods *t*, *t*+1, …, *m*, given that the forwarder's order quantity over period 1, 2, …, $t-1$ is $\mathbf{Q}_{it-1}=\left\{Q_{i1},...,Q_{it-1}\right\}$. Then $U_t\left(\mathbf{Q}_{it-1}\right)$ can be expressed as the following recursive function.

$$U_t\left(\mathbf{Q}_{it-1}\right)=\max_{Q_{it}}\left(U_{t+1}\left(\mathbf{Q}_{it-1},Q_{it}\right)+\pi_{it}\left(Q_{it}\right)\right),\quad t=1,...,k \tag{4.4}$$

$$U_{k+1}\left(\mathbf{Q}_{ik}\right)=\sum_{t=k+1}^{m}\pi_{it}=\sum_{t=k+1}^{m}\left(p_{it}-w_t\right)\cdot\varphi_t\left(\mathbf{Q}_{ik}\right) \tag{4.5}$$

The forwarder's problem can be solved via dynamic programming, and the optimal order quantity Q_{it}^{*} can be obtained.

4.2.3 Airline's problem

Once we understand how the forwarders will response to the airline's pricing, we can start to analyze the airline's problem. The airline has a crude estimate of the demand $\tilde{D}_{it}^{A}$ that the forwarder *i* will face in period *t*, which is denoted as

$$\tilde{D}_{it}^{A} = a_{it}^{A} - b_{it}^{A} p_{it} + \tilde{\varepsilon}_{it}^{A}, t = 1, 2, ..., m \tag{4.6}$$

where a_{it}^{A} and b_{it}^{A} are airline's estimate of the parameters in the demand function, $a_{it}^{A} > 0$ and $b_{it}^{A} > 0$;

p_{it} is forwarder *i*'s resale rate in period *t*;

$\tilde{\varepsilon}_{it}^{A}$ is the airline's estimate of the noise of demand, which follows a cdf $F_{it}^{A}\left(\varepsilon_{it}^{A}\right)$.

The airline's estimate cannot be as accurate as forwarder's forecasting, and thus, the parameters in equation (4.6) can be different from the parameters forecasted by the forwarder.

Based on its estimation of forwarders' future demands, the airline can predict forwarders' responses to the contract rates in each period. Let $\left(Q_{i1}^{A*}, Q_{i2}^{A*}, ..., Q_{ik}^{A*}\right)$ denote forwarder *i*'s optimal order quantities, and let π_{t}^{A} denote the expected profit of the airline in period *t*. Then, in low seasons,

$$\pi_{t}^{A} = \sum_{i=1}^{n} \left\{ \int_{\underline{\varepsilon}_{it}^{A}}^{Q_{it}^{A*} - d_{it}^{A}} \left[w_{t} \cdot \left(d_{it}^{A} + \varepsilon_{it}^{A}\right) + x_{t} \cdot \left(Q_{it}^{A*} - d_{it}^{A} - \varepsilon_{it}^{A}\right) \right] \cdot f_{it}^{A}\left(\varepsilon_{it}^{A}\right) d\varepsilon_{it}^{A} + \int_{Q_{it}^{A*} - d_{it}^{A}}^{\bar{\varepsilon}_{it}^{A}} w_{t} \cdot Q_{it}^{A*} \cdot f_{it}^{A}\left(\varepsilon_{it}^{A}\right) d\varepsilon_{it}^{A} \right\} \tag{4.7}$$

In peak season, c_{it} becomes the actual used capacity, since all forwarders will use up the guaranteed capacity. If $\sum_{i=1}^{n} c_{it} > C_t$, the airline has to purchase additional capacity from the spot market in that period. On the other hand, if $\sum_{i=1}^{n} c_{it} < C_t$, the remaining capacity can be sold in the spot market. Therefore, π_t^A in the peak season can be expressed as the following equation.

$$\pi_t^A = w_t \cdot \sum_{i=1}^{n} c_{it}^{A*} + s_t \left(C_t - \sum_{i=1}^{n} c_{it}^{A*} \right), \quad t = k+1, \ldots, m \tag{4.8}$$

where s_t is the expected spot rate in period *t* in peak season.

The airline's objective is to find the optimal contract rate in each period so that the expected profit over a year is maximized. This can be modeled as follows.

$$\max_{w_1,\ldots,w_m} \sum_{t=1}^{m} \pi_t^A$$

$$\text{s.t.} \qquad w_t \le ub_t, \; t = 1, \ldots, m$$

where ub_t is the upper bound on airline's contract rate in period *t*.

Due to the complexity of Q_{it}^{A*}, it is very difficult to derive the close form solution to the airline's problem. Some numerical searching algorithm, such as genetic algorithm, can be applied to find the optimal pricing strategy of the airline.

4.3 Long-term capacity allotment under linear $\varphi_t(\bullet)$

In the long-term contract market, the guaranteed capacity not only depends on the order quantities during the low seasons, but also depends on the bargain power of the forwarder and some other market conditions. The function that reflects the relationship between the guaranteed capacity and low seasons order quantities can take many different forms. Therefore, the guaranteed capacity in the peak season is calculated by a function $\varphi_t(\bullet)$ without a specific expression in the previous section. This provides the most general model for the long-term capacity allocation. Because of the general function $\varphi_t(\bullet)$, however, we cannot do much analysis on the structure of the problem in the previous section.

Among the many forms of $\varphi_t(\bullet)$, the most commonly used form is a linear function, i.e. $c_{it} = \sum_{t=1}^{k} \alpha_t Q_{it}$, $0 \le \alpha_t \le 1$. In this section, we will analyze the structure of the long-term capacity allocation problem in details under a linear $\varphi_t(\bullet)$. To simplify the problem, we assume that period 1 ~ $m-1$ are low seasons and the last period m is the peak season. Then, the forwarder's problem and the airline's problem will be resolved, and some properties will be explained.

4.3.1 Forwarder's problem

The forwarder i's objective function is

$$\sum_{t=1}^{m-1} \pi_{it}\left(Q_{it}\right) + \pi_{im} = \sum_{t=1}^{m-1} \pi_{it}\left(Q_{it}\right) + \left(p_{im} - w_m\right) \sum_{t=1}^{m-1} \alpha_t Q_{it} \tag{4.9}$$

Proposition 1: The optimal order quantity of forwarder i in period t is

$$Q_{it}^{*} = F_{it}^{-1}\left(\frac{p_{it} - w_t + \alpha_t\left(p_{im} - w_m\right)}{p_{it} + x_t - w_t}\right) + d_{it} \qquad t = 1, \ldots, m-1$$

Proof: The objective function can be reorganized in a way that the second $\sum$ in equation (4.9) is opened and the profit $\left(p_{im} - w_m\right)\alpha_t Q_{it}$ is associated with $\pi_{it}\left(Q_{it}\right)$ in the corresponding period. Then, the new mathematical model is shown as follows.

$$\max_{Q_{i1},\ldots Q_{im-1}} \sum_{t=1}^{m-1} \Pi_{it}\left(Q_{it}\right)$$

s.t.

$$\Pi_{it}\left(Q_{it}\right) = \begin{cases} \int_{\underline{\varepsilon}_{it}}^{Q_{it}-d_{it}} \left[\left(p_{it} - w_t\right)\left(d_{it} + \varepsilon_{it}\right) - x_t\left(Q_{it} - d_{it} - \varepsilon_{it}\right)\right] \cdot f_{it}\left(\varepsilon_{it}\right) d\varepsilon_{it} & \\ \quad + \int_{Q_{it}-d_{it}}^{\bar{\varepsilon}_{it}} \left(p_{it} - w_t\right) Q_{it} \cdot f_{it}\left(\varepsilon_{it}\right) d\varepsilon_{it} + \alpha_t Q_{it}\left(p_{im} - w_m\right) & \text{if } d_{it} + \underline{\varepsilon}_{it} < Q_{it} < d_{it} + \bar{\varepsilon}_{it} \\ \int_{\underline{\varepsilon}_{it}}^{\bar{\varepsilon}_{it}} \left[\left(p_{it} - w_t\right)\left(d_{it} + \varepsilon_{it}\right) - x_t\left(Q_{it} - d_{it} - \varepsilon_{it}\right)\right] \cdot f_{it}\left(\varepsilon_{it}\right) d\varepsilon_{it} + \alpha_t Q_{it}\left(p_{im} - w_m\right) & \text{if } Q_{it} \geq d_{it} + \bar{\varepsilon}_{it} \\ \int_{\underline{\varepsilon}_{it}}^{\bar{\varepsilon}_{it}} \left(p_{it} - w_t\right) Q_{it} \cdot f_{it}\left(\varepsilon_{it}\right) d\varepsilon_{it} + \alpha_t Q_{it}\left(p_{im} - w_m\right) & \text{if } Q_{it} \leq d_{it} + \underline{\varepsilon}_{it} \end{cases}$$

After the reorganization, $\Pi_{it}\left(Q_{it}\right)$ only depends on the order quantity in the current period *t*. Therefore, the forwarder's problem can be separated as *m* – 1 independent optimization problems. The first derivatives of $\Pi_{it}\left(Q_{it}\right)$ is

$$\frac{d\Pi_{it}}{dQ_{it}} = \begin{cases} p_{it} - w_t - \left(p_{it} + x_t - w_t\right) F_{it}\left(Q_{it} - d_{it}\right) + \alpha_t\left(p_{im} - w_m\right) & \text{if } d_{it} + \underline{\varepsilon}_{it} < Q_{it} < d_{it} + \bar{\varepsilon}_{it} \\ -x_t + \alpha_t\left(p_{im} - w_m\right) & \text{if } Q_{it} \geq d_{it} + \bar{\varepsilon}_{it} \\ p_{it} - w_t + \alpha_t\left(p_{im} - w_m\right) & \text{if } Q_{it} \leq d_{it} + \underline{\varepsilon}_{it} \end{cases} \tag{4.10}$$

From equation (4.10), we can see that $\frac{d\Pi_{it}}{dQ_{it}} > 0$, if $Q_{it} \leq d_{it} + \underline{\varepsilon}_{it}$. Therefore, $Q_{it}^{*} \geq d_{it} + \underline{\varepsilon}_{it}$.

If $\alpha_t\left(p_{im} - w_m\right) > x_t$, the forwarder will book as much as possible during the low season to increase the profit during the peak season. However, this case will never happen. The airline cannot have unlimited access to capacity in spot market during the peak season, and thus, the unlimited guaranteed capacity will result in great loss to the airline. The airline will avoid such loss by setting $x_t \geq \alpha_t \left(p_{im} - w_m \right)$. Thus, the optimal order quantity of forwarder *i* in period t is

$$Q_{it}^* = F_{it}^{-1}\left(\frac{p_{it} - w_t + \alpha_t \left(p_{im} - w_m \right)}{p_{it} + x_t - w_t} \right) + d_{it} \qquad t = 1, \ldots, m-1 \ \square$$

4.3.2 Airline's problem

The airline's expected profit in period *m* is

$$\pi_m^A = w_m \cdot \sum_{i=1}^{n} c_{im}^{A*} + s_m \left(C_m - \sum_{i=1}^{n} c_{im}^{A*} \right) = \left(w_m - s_m \right) \cdot \sum_{i=1}^{n} \sum_{t=1}^{m-1} \alpha_t Q_{it}^{A*} + s_m \cdot C_m \tag{4.11}$$

where $Q_{it}^{A*} = F_{it}^{A^{-1}}\left(\dfrac{p_{it} - w_t + \alpha_t \left(p_{im} - w_m \right)}{p_{it} + x_t - w_t} \right) + d_{it}^A$ is the airline's prediction of the forwarder *i*'s optimal order quantity in period *t*.

Using the same technique as in the above proof, we can reorganize the airline's profit in each period so that the airline's problem can be separated as *m* sub-problems. Here, the airline's capacity during the peak season is denoted as *C*.

Let

$$\begin{aligned} \Pi_t^A = \sum_{i=1}^{n} \Bigg\{ & \int_{\underline{\varepsilon}_{it}^A}^{Q_{it}^{A*} - d_{it}^A} \left[w_t \cdot \left(d_{it}^A + \varepsilon_{it}^A \right) + x_t \cdot \left(Q_{it}^{A*} - d_{it}^A - \varepsilon_{it}^A \right) \right] \cdot f_{it}^A \left(\varepsilon_{it}^A \right) d\varepsilon_{it}^A \\ & + \int_{Q_{it}^{A*} - d_{it}^A}^{\bar{\varepsilon}_{it}^A} w_t \cdot Q_{it}^{A*} \cdot f_{it}^A \left(\varepsilon_{it}^A \right) d\varepsilon_{it}^A + \alpha_t \left(w_m - s_m \right) Q_{it}^{A*} \Bigg\}, \quad t = 1, \ldots, m-1 \end{aligned} \tag{4.12}$$

$$\Pi_m^A = s_m \cdot C \tag{4.13}$$

Therefore, the airline's problem is

$$\max_{w_1,\dots,w_m} \sum_{t=1}^{m-1} \Pi_t^A + \Pi_m^A$$

$$\text{s.t.} \quad w_t \le ub_t,\ t = 1,\dots,m$$

In doing so, the Π_t^A only depends on the contract rate in the current period w_t and the contract rate at the peak season w_m.

Proposition 2: The airline's profit over a year is monotonically increasing with w_m.

Proof: $$\Pi_t^A = \sum_{i=1}^{n}\left\{\int_{\underline{\varepsilon}_{it}^A}^{Q_{it}^{A*}-d_{it}^A}\left[w_t\cdot\left(d_{it}^A+\varepsilon_{it}^A\right)+x_t\cdot\left(Q_{it}^{A*}-d_{it}^A-\varepsilon_{it}^A\right)\right]\cdot f_{it}^A\left(\varepsilon_{it}^A\right)d\varepsilon_{it}^A\right.$$
$$\left.+\int_{Q_{it}^{A*}-d_{it}^A}^{\bar{\varepsilon}_{it}^A} w_t\cdot Q_{it}^{A*}\cdot f_{it}^A\left(\varepsilon_{it}^A\right)d\varepsilon_{it}^A+\alpha_t\left(w_m-s_m\right)Q_{it}^{A*}\right\},\quad t=1,\dots,m-1$$

Let $Q_{it}^{A*}{}'$ denote $\frac{\partial Q_{it}^{A*}}{\partial w_m}$. $Q_{it}^{A*}{}' = \frac{\alpha_t\gamma_{im}}{\left(p_{it}+x_t-w_t\right)f\left(Q_{it}^{A*}-d_{it}^A\right)} > 0$

$$\frac{\partial \Pi_t^A}{\partial w_m} = \sum_{i=1}^{n}\left\{w_t d_t^A f_{it}^A\left(Q_{it}^{A*}-d_{it}^A\right)Q_{it}^{A*}{}'+\left(w_t-x_t\right)\left(Q_{it}^{A*}-d_{it}^A\right)f\left(Q_{it}^{A*}-d_{it}^A\right)Q_{it}^{A*}{}'\right.$$
$$+x_t Q_{it}^{A*}{}'\frac{p_{it}-w_t+\alpha_{it}\left(p_{im}-w_m\right)}{p_{it}+x_t-w_t}+x_t\left(Q_{it}^{A*}-d_{it}^A\right)\cdot f\left(Q_{it}^{A*}-d_{it}^A\right)Q_{it}^{A*}{}'$$
$$\left.+w_t Q_{it}^{A*}{}'\frac{x_t-\alpha_t\left(p_{im}-w_m\right)}{p_{it}+x_t-w_t}-w_t Q_{it}^{A*} f\left(Q_{it}^{A*}-d_{it}^A\right)Q_{it}^{A*}{}'+\alpha_t Q_{it}^{A*}+\alpha_t\left(w_m-s_m\right)Q_{it}^{A*}{}'\right\}$$

$$=\sum_{i=1}^{n}\left\{x_t Q_{it}^{A*}{}'\frac{p_{it}-w_t+\alpha_t\left(p_{im}-w_m\right)}{p_{it}+x_t-w_t}+w_t Q_{it}^{A*}{}'\frac{x_t-\alpha_t\left(p_{im}-w_m\right)}{p_{it}+x_t-w_t}+\alpha_t Q_{it}^{A*}+\alpha_t\left(w_m-s_m\right)Q_{it}^{A*}{}'\right\}$$

$$=\sum_{i=1}^{n}\left\{x_t Q_{it}^{A*}{}'\frac{\gamma_{it}w_t+\alpha_t\gamma_{it}w_m}{\gamma_{it}w_t+x_t}+w_t Q_{it}^{A*}{}'\frac{x_t-\alpha_t\gamma_{im}w_m}{\gamma_{it}w_t+x_t}+\alpha_t Q_{it}^{A*}+\alpha_t\left(w_m-s_m\right)Q_{it}^{A*}{}'\right\}$$

$$=\sum_{i=1}^{n}\left\{\frac{x_t\gamma_{it}w_t+x_t\alpha_t\gamma_{it}w_m+w_t x_t-w_t\alpha_t\gamma_{im}w_m+\alpha_t w_m\gamma_{it}w_t+\alpha_t w_m x_t-\alpha_t s_m\gamma_{it}w_t-\alpha_t s_m x_t}{\gamma_{it}w_t+x_t}Q_{it}^{A*}{}'+\alpha_t Q_{it}^{A*}\right\}$$

Let $\Omega_{it} = x_t\gamma_{it}w_t + x_t\alpha_t\gamma_{it}w_m + w_t x_t - w_t\alpha_t\gamma_{im}w_m + \alpha_t w_m\gamma_{it}w_t + \alpha_t w_m x_t - \alpha_t s_m\gamma_{it}w_t - \alpha_t s_m x_t$.

Then, $\Omega_{it} = w_t\left(x_t - \alpha_t\gamma_{im}w_m\right) + \gamma_{it}w_t\left(x_t + \alpha_t w_m - \alpha_t s_m\right) + \alpha_t x_t\left(p_{im} - s_m\right)$

$$= w_t\left(x_t + \alpha_t w_m - \alpha_t s_m\right) + \gamma_{it}w_t\left(x_t + \alpha_t w_m - \alpha_t s_m\right) + w_t\alpha_t\left(s_m - p_{im}\right) + \alpha_t x_t\left(p_{im} - s_m\right)$$

$$= w_t\left(x_t + \alpha_t w_m - \alpha_t s_m\right) + \gamma_{it}w_t\left(x_t + \alpha_t w_m - \alpha_t s_m\right) + \alpha_t w_t\left(1 - \beta_t\right)\left(s_m - p_{im}\right)$$

Because of the CBP, forwarder *i* can get α_t unit of capacity in the peak season for each unit of capacity ordered during period *t* in the low season. If $x_t + \alpha_t w_m - \alpha_t s_m < 0$, forwarder *i* will order infinite capacity during the low season in order to exploit the profit in the spot market in the peak season. This is obviously not optimal for the airline. Therefore, $x_t + \alpha_t w_m - \alpha_t s_m \geq 0$. Similarly, $x_t - \alpha_t\gamma_{im}w_m \geq 0$. Therefore, $\Omega_{it} \geq 0$ for $i = 1, \ldots, n$. Also, $Q_{it}^{A*}{}' = \dfrac{\alpha_t\gamma_{im}}{f\left(Q_{it}^{A*} - d\right)\left(p_{it} + x_t - w_t\right)} > 0$. Therefore, $\dfrac{\partial\Pi_t^A}{\partial w_m} \geq 0$. The airline's profit over a year is monotonically increasing with w_m. □

Therefore, the optimal contract rate in the peak season is equal to the upper bound of airline's pricing. Once we obtained the w_m^*, we can solve for the optimal w_t. The easiest way to do so is to exhaustively search all possible w_t in each period.

Besides the contract rate and order quantity, the guaranteed ratio α_t also plays an important role in the long-term contract. It represents the airline's attitude towards the CBP. A high α_t shows the airline has strong incentive to exchange the capacity during the peak season with the profit in low season *t*. In contrast, a guaranteed ratio $\alpha_t = 0$ shows that the airline has no incentive to adopt the CBP in period *t*. Therefore, the value of guaranteed ratio shows whether the CBP is effective from a certain angle. A way to calculate the optimal guaranteed ratio in each period is needed. Given that one decision period in our model usually represents two months or a quarter in the real world, the number of guaranteed ratios is limited and thus, exhaustively searching for the optimal guaranteed ratios is acceptable.

At the end of this section, we provide a flow chart to further illustrate the algorithm proposed in this thesis to solve the long-term capacity allocation model.

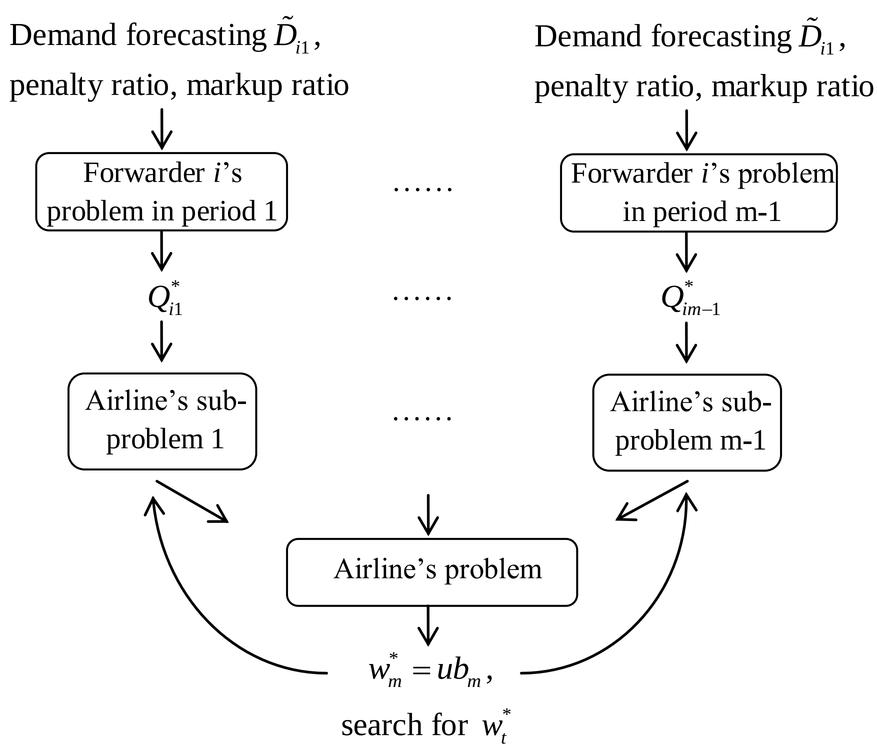


Figure 4.1 Flow chart of the long-term capacity allocation model

At the beginning, forwarder's problem is decomposed into several sub-problems, and the sub-problem in each decision period is solved based on the forecasting of future demand, the penalty ratio and markup ratio to obtain the optimal order quantity. These order quantities are functions of the airline's contract rate in the current period w_t and the contract rate in the peak season w_m. By feeding these order quantities into the airline's sub-problem in each period, we can obtain the objective function Π_t^A. Then, the optimal contract rate in each period in low season is obtained by maximizing Π_t^A. According to proposition 2, $w_m^* = ub_m$. Next, all w_t^* will be obtained via exhaustive search.

4.4 Numerical Experiments

As explained in the beginning of this chapter, one reason for the airline's adopting of the CBP is that the airline cannot charge a very high rate in the peak season. Intuitively, the airline has no incentive to adopt the CBP if the expected spot rate is very high during the peak season. Therefore, the performance of CBP depends on the upper bound of airline's contract rate and the expected spot rate during the peak. Also, the CBP can encourage forwarders to market more aggressively in low seasons. However, if forwarders' demand forecasting is perfectly accurate, they will only book capacity equal to the future demand. Therefore, it is expected that the performance of CBP also depends on the variation of the future demand. In this section, we are going to analyze the performance of the CBP under various market conditions.

In this section, it is assumed that 1 airline serves 3 forwarders in the market and one fiscal year can be divided into 6 periods. The first 5 periods are low seasons and the last period is peak season. It is also assumed that $c_{it} = \alpha \sum_{t=1}^{m-1} Q_{it}$. With a single guaranteed ratio, we can clearly observe how the optimal ratio changes in different market conditions. The capacity in each period is set to be 100. Some other parameters are chosen arbitrarily as follows.

Table 4.1 The parameters used in the numerical experiments

	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
a_1	38	30	37	41	38	78
a_2	40	32	41	45	38	84
a_3	35	29	36	42	37	74
b_1	6	6.2	6.1	6.4	5.5	5
b_2	6	6	6.1	6.3	5.7	5
b_3	6	6.1	6.1	6.5	5.5	5
cv	0.25	0.2	0.25	0.3	0.3	0.15
γ	20%	20%	20%	20%	20%	20%
β	50%	50%	50%	50%	50%	70%

In table 4.1, a_1, a_2, a_3 and b_1, b_2, b_3 are the parameters in the demand functions faced by forwarder 1, 2 and 3, respectively. The residue in the demand function for forwarder i follows a truncated normal distribution, i.e. $\tilde{\varepsilon}_{it} \sim N\left(0, \sigma_{it}\right)$ and $\tilde{\varepsilon}_{it} \in [\max\left(0, d_{it} - 2\sigma_{it}\right), 2\sigma_{it}]$, where $d_{it} = a_{it} - b_{it} p_{it}$; $\sigma_{it} = cv_t \cdot d_{it}$; cv_t is the coefficient of variation. In this section, it is assumed that the 3 forwarders have the same markup ratio γ, which is 20%. The penalty ratio β is set to be 50% in the low season and 70% in the peak season. The upper bound for the airline's contract rate is set to 3.5. It is assumed that the spot rate during the peak season follows a truncated normal distribution, i.e. $\tilde{s}_m \sim N\left(s_m, \sigma_m^s\right)$ and $\tilde{s}_m \in \left[s_m - 2\sigma_m^s, s_m + 2\sigma_m^s\right]$, where s_m is the expected spot rate and is up to change in the following experiments; σ_m^s is the standard deviation of the spot rate and is set to 0.7. Under the parameters listed in Table 1, the sum of the optimal order quantities in a low season period is around 50, given that the airline does not adopt the CBP. Whereas, the sum of the demand faced by forwarders in the peak season is larger than 100, even if the airline charges the highest contract rate. Therefore, the market settings correspond to the assumptions in this thesis.

When adopting the CBP, the airline has to sell part of the capacity to forwarders in the peak season under contract rate, which is much lower than the expected spot rate s_m. The larger the difference between the contract rate and expected spot rate, the more the airline loses in the peak season. Intuitively, when the expected spot rate is too high, adopting the capacity bundling policy will reduce the airline's expected profit. In the first numerical experiment, we would like to compare the expected profit of the airline when the CBP is used and when it is not used under different s_m. The procedures for the experiment are explained as follows.

Step 1. Set parameters as explained above and choose an initial expected spot rate s_m. Then, solve the optimal guaranteed ratio α^*, airline's contract rate in each period, and order quantity from each forwarder in each period.

Step 2. Generate demand for each forwarder in each period according to the demand distribution. Calculate the profit of the airline under the demand series when CBP is adopted and when it is not, i.e. $\alpha = 0$.

Step 3. Simulate step 2 repeatedly and record the results. Then, calculate $PI = \dfrac{E\left(\pi^A\left(\alpha^*\right)\right) - E\left(\pi^A\left(\alpha = 0\right)\right)}{E\left(\pi^A\left(\alpha = 0\right)\right)} \times 100$, where PI is the percentage improvement on airline's expected profit; $E\left(\pi^A\left(\alpha^*\right)\right)$ is the airline's expected profit under CBP; $E\left(\pi^A\left(\alpha=0\right)\right)$ is the airline's expected profit without CBP.

Step 4. Change expected spot rate and redo the first 3 steps.

The procedures for the experiment are further illustrated in the following chart.

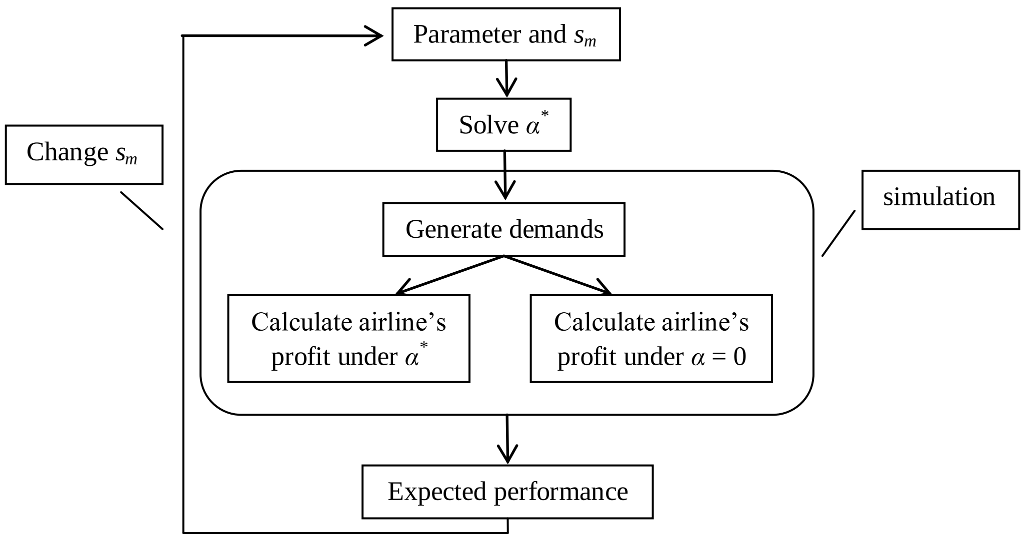
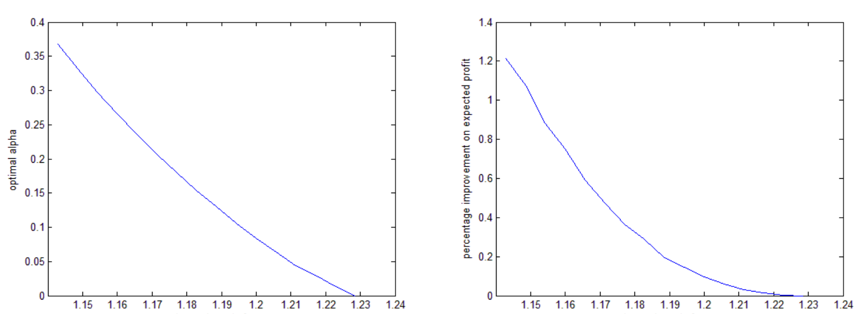


Figure 4.2 The flow chart of the procedures in experiment 1.

The results of simulation 1 are shown in Figure 4.5.



(a) (b)

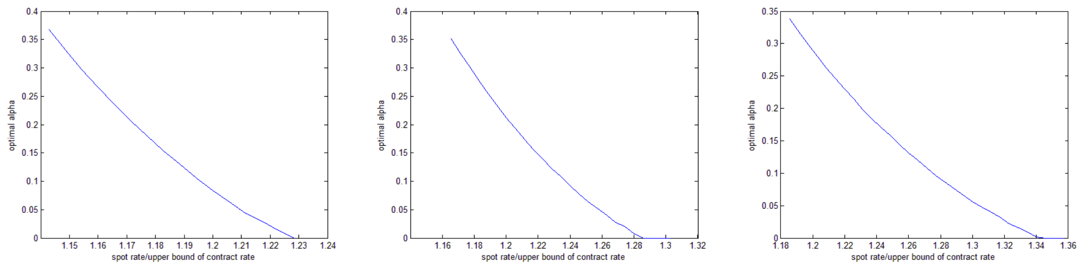
Figure 4.3 The optimal a and percentage improvement under different s_m in experiment 1

The horizontal axis in Figure 4.5 is the ratio of s_m to the upper bound of contract rate. The vertical axis in Figure 4.5(a) records the optimal guaranteed ratio under the market condition, and the vertical axis in Figure 4.5(b) records the corresponding percentage improvement. From Figure 4.5(a), we can observe that the α^* decreases with s_m. As α^* decreases, the influence of CBP decrease, and thus, the percentage improvement also decreases with s_m. When s_m is close to the forwarder's resale rate, i.e. the horizontal axis equal to 1.2, α^* is close to zero, and thus, the improvement that the bundling policy can provide is insignificant, which can be shown in Figure 4.5(b).

Experiment 1 shows that the CBP can significantly increase the expected profit of the airline when s_m is less than the forwarder's resale rate. With CBP, the capacity in the peak period is sold under a higher rate, i.e. forwarder's resale rate, rather than the spot rate. The increment in the profit is distributed between the airline and forwarders via the increased payment in low season. When s_m is slightly higher than the forwarder's resale rate, CBP can still slightly increase the expected profit of the airline. This increase results from the effect that the CBP can stimulate the forwarder to book more capacity in the low season. When the demand is unexpectedly high in any period in low season, it is more likely that the forwarder can accommodate the demand. However, CBP cannot increase the expected profit when s_m is considerably higher than forwarder's resale rate.

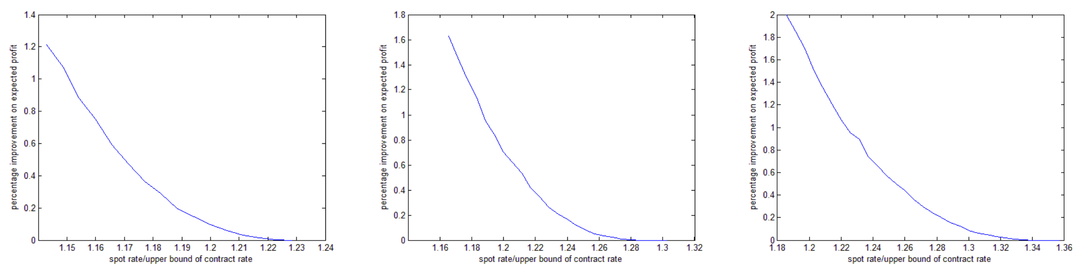
During the peak season, the capacity is scarce and the forwarder may want to earn more profit from the guaranteed capacity. Thus, it is natural that a forwarder sets a higher markup ratio in the peak season than that in the low season. In the next two experiments, the markup ratio in the peak season is increased to 25% and 30%. Then, we solve for α^*

and the corresponding percentage improvement on the expected profit of the airline, respectively. The simulation procedures are the same as in experiment 1. The results are shown in Figure 4.6 and 4.7. For the ease of comparison, the result in experiment 1 is also included.



(a) (b) (c)

Figure 4.4 The optimal a under different s_m in the three experiments



(a) (b) (c)

Figure 4.5 The percentage improvement under different s_m in the three experiments

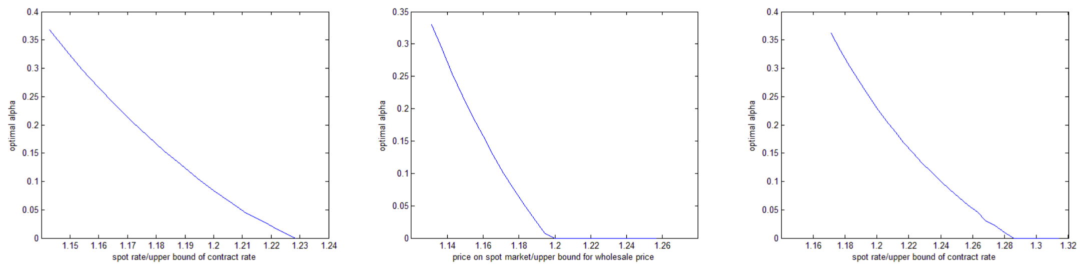
Figure 4.6 (a), (b) and (c) show the optimal α in experiment 1, 2 and 3, respectively. Figure 4.7 (a), (b) and (c) show the percentage improvement in experiment 1, 2 and 3, respectively. From Figure 4.6 and 4.7, we can see that the results in experiment 2 and 3 are similar as the result in experiment 1. Again, α^* and percentage improvement decrease with s_m. When s_m is close to forwarder's resale rate, α^* is close to zero and CBP cannot significantly increase the expected profit of the airline. The only difference is that the percentage improvement becomes more significant as the markup ratio increases. The increased markup ratio reflects the increased severity of the imbalance between supply and demand in the peak season. Adopting the CBP enables the airline to sell the capacity under a higher rate, and thus, achieve a more significant improvement in the expected profit.

The CBP can increase the airline's expected profit from two sources, increasing the selling rate in the peak season and encouraging forwarders to market more aggressively in the low season. The first source depends on s_m and forwarder's resale rate, and is analyzed in experiments 1, 2 and 3. The latter source depends on the variation of the future demand. In the following two experiments, therefore, we increase the coefficient of variation of the residue for each forwarder, and analyze its effect on the performance of the CBP. The coefficients of variation used in experiment 4 and 5 are shown in the following table.

Table 4.2 The coefficient of variation used in experiment 4 and 5

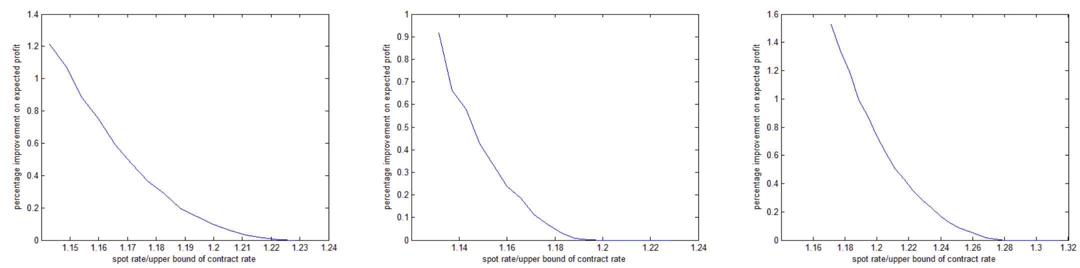
	Period 1	Period 2	Period 3	Period 4	Period 5	Period 6
Experiment 4	0.22	0.17	0.22	0.27	0.27	0.15
Experiment 5	0.3	0.25	0.3	0.35	0.35	0.15

Compared to the parameters used in experiment 1, each *cv* in the low season is decreased by 0.03 in experiment 4 and increased by 0.05 in experiment 5. The procedures for the two experiments are the same as in experiment 1. The results are shown as follows. Again, the result in experiment 1 is included for the ease of comparison.



(a) (b) (c)

Figure 4.6 The optimal *a* under different *cv* in the three experiments



(a) (b) (c)

Figure 4.7 The percentage improvement under different *cv* in the three experiments

Figure 4.8 (a), (b) and (c) show the optimal α in experiment 1, 4 and 5, respectively. Figure 4.9 (a), (b) and (c) show the percentage improvement in experiment 1, 4 and 5, respectively. It is observed that the percentage improvement in experiment 4 is the least significant whereas the percentage improvement in experiment 5 is the most significant. The reason for these results is explained as follows. With large variation of demand, aggressive marketing in low seasons can enable the forwarder to benefit from unexpected high demand, while the lost from unexpected low demand is partially compensated by the gains from the peak season. Therefore, the improvement from forwarder's aggressive marketing in the low season decrease/increase as the variation of demand decrease/increase. As an extreme case, the benefit from aggressive marketing will diminish when forwarder's forecasting of future demand is perfectly accurate. In air cargo industry, forwarder will forecast the future demand several months in advance. The forecasting can be very inaccurate. With a positive relationship between the effect of CBP and variation of demand, we believe that it will provide significant improvement on the expected profit of the airline.

In the next experiment, the effect of CBP on the risk of the airline's profit is analyzed. Generally, the risk of profit in air cargo industry is defined as the probability that the profit falls below a certain sales target. Here, the expected profit of the airline without CBP is chosen as a sales target, and thus, the risk of airline's profit is defined as $\text{risk} = \mathrm{P}\left(\pi\left(\alpha^*\right) < E\left(\pi\left(\alpha = 0\right)\right)\right)$. The procedures of experiment are generally the same as in experiment 1. However, we do not calculate the percentage improvement here. After $E\left(\pi\left(\alpha = 0\right)\right)$ is obtained based on the simulation results, the $\pi\left(\alpha^*\right)$ in each simulation scenario is compared with $E\left(\pi\left(\alpha = 0\right)\right)$, and the number of cases in which $\pi\left(\alpha^*\right)$ is less than $E\left(\pi\left(\alpha = 0\right)\right)$ is recorded. Then, the ratio of this number to the total simulation run will be the risk of airline's profit. The result is shown in Figure 4.10.

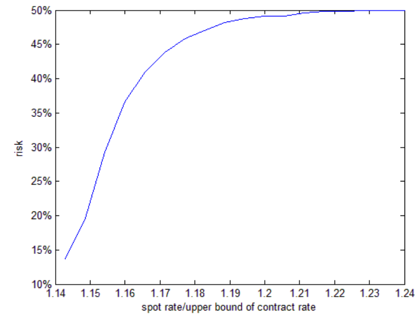
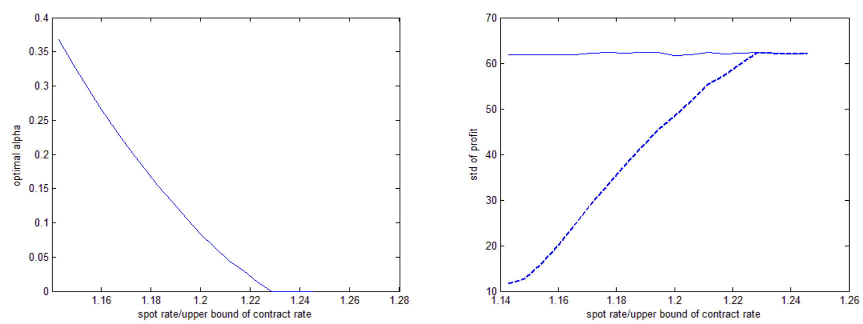


Figure 4.8 The effect of capacity bundling policy on risk

Figure 4.10 shows that the CBP can reduce the risk of the airline, when s_m is less than the forwarder's resale price. The effect can be very significant, when s_m is considerably lower than the forwarder's resale price. The reduction of risk results from two main reasons. First, CBP can improve the airline's expected profit, when s_m is less than the forwarder's resale price. As a result, the chance that $\pi\left(\alpha^*\right)$ is less than $E\left(\pi\left(\alpha=0\right)\right)$ is reduced. Second, the capacity bundling policy can reduce the standard deviation of the profit of the airline, which is shown in Figure 4.11.



(a) (b)

Figure 4.9 The effect of CBP on standard deviation of profit

In Figure 4.11(b), the dash line represents the standard deviation of airline's profit with CBP, whereas the solid line represents the standard deviation of airline's profit without CBP. It is clear that CBP can reduce the standard deviation. When s_m is lower than forwarder's resale rate, the guaranteed ratio α^* is relatively large, and the reduction on standard deviation is more obvious. As s_m increases, α^* approaches zero and the standard deviation converges to the standard deviation without CBP. Under CBP, airline will sell the guaranteed capacity to forwarder under contract rate in the peak season. The difference between the expected spot rate and contract rate is considered as the opportunity cost of the guaranteed capacity, and this opportunity cost is compensated through the higher profit in low seasons. This is equivalent to selling the capacity in peak season in advance under a more certain rate. Therefore, the volatility in the spot rate is avoided and the standard deviation of airline's profit is reduced. Since CBP can increase the expected profit and reduce the standard deviation, it is natural that CBP can reduce the airline's risk in profit, when s_m is lower than forwarder's resale rate.

In summary, the CBP can increase the airline's expected profit when the expected spot rate is close to or lower than forwarder's resale rate during the peak season. As the expected spot rate decreases, the improvement on the expected profit becomes more significant. However, the CBP cannot significantly increase the expected profit, when expected spot rate is higher than the forwarder's resale rate, because the airline loses more during the peak season than its gain during the low seasons. The policy's influence on the expected profit becomes stronger as the variation of the demand increases. Therefore, when the forwarder's future demand is highly unpredictable, the airline can benefit from CBP. Besides, the capacity bundling policy can reduce the risk of the airline's profit, when expected spot rate is lower than forwarder's resale rate.

Chapter 5 Integration of the Short-Term and Long-Term RM Models

In chapter 3 and 4, we develop RM tools for the capacity control in spot market and contract market, respectively. As explained in the introduction chapter, the two markets are correlated. A RM system for air cargo capacity control should be able to integrate the models for contract market and spot market, and jointly allocate capacity to the two markets. In section 5.1, we will provide a conceptual framework of an air cargo capacity control system, which can achieve the reasonable allocation of capacity between the two markets. In section 5.2, we will discuss several issues in the capacity control system. Again, we consider a market with one major airline serving several forwarders.

5.1 Integration of capacity control in spot market and contract market

In chapter 3, we develop a Markovian model for capacity control on spot market. The function of the model can be illustrated in Figure 5.1

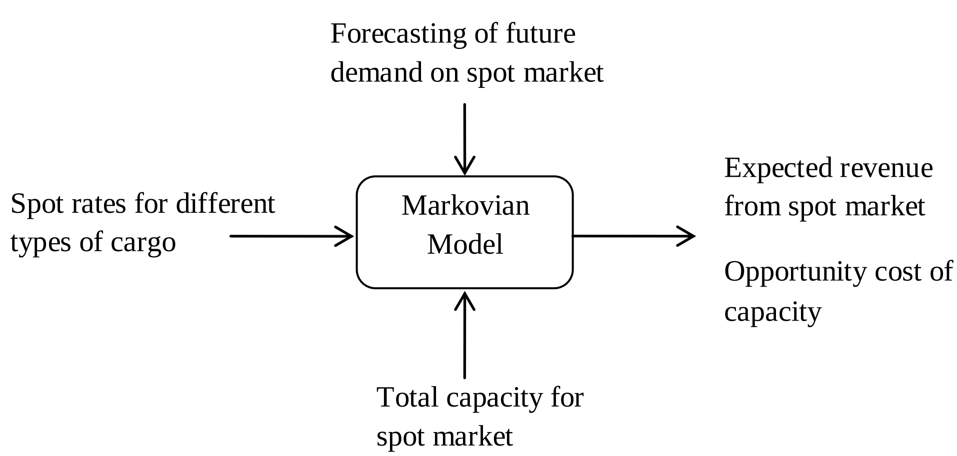


Figure 5.1 Function of the Markovian model

As shown in Figure 5.1, the inputs to the Markovian model include the forecasting of future demand on spot market, spot rates for different types of cargo, and the total capacity allocated to spot market. The last input depends on the order quantities in the long-term capacity agreements, and thus, depends on capacity control on the contract market. With all three inputs, the Markovian model can provide the expected profit from a specific flight in spot market. Let C_j^s denote the capacity on flight *j* that is allocated to the spot market. Let $E\left(R_j^s\right)$ denote the expected revenue from flight *j* in the spot market. Then, the opportunity cost of the capacity can be obtained as $oc_j^s = E\left(R_j^s\right)/C_j^s$. This opportunity cost will be used later in this section.

In chapter 4, we develop a capacity allocation model, which adopts the capacity bundling policy, for contract market. The function of the model is illustrated in the following graph.

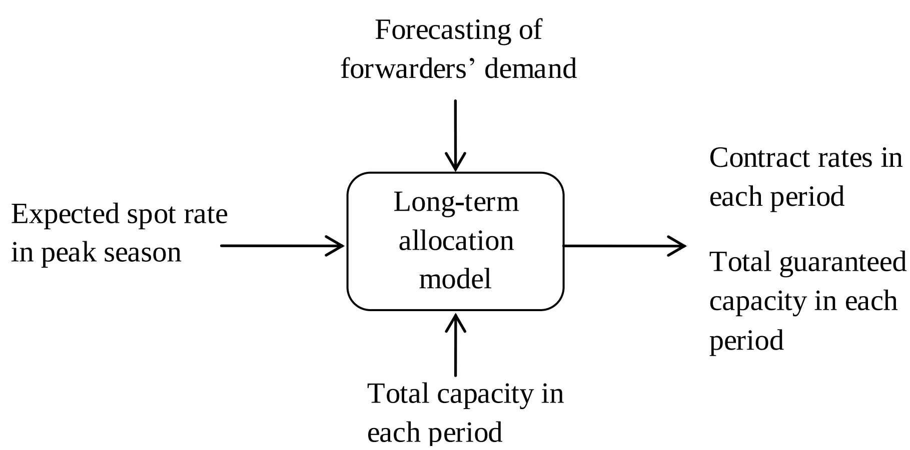


Figure 5.2 Function of the long-term capacity allocation model

As shown in Figure 5.2, the inputs to the long-term capacity allocation model include the forecasting of each forwarder's demand function, the expected spot rate in the peak season, and the total capacity available in each decision period. Here, the expected spot rate in the peak season is actually the opportunity cost of capacity in spot market during the peak season. As explained above, this opportunity cost can be derived from the Markovian model for short-term capacity allocation.

From the above analysis, we can see that the output of long-term capacity allocation model is the input of short-term capacity allocation model and the output of the short-term model is the input of long-term model. A way to integrate the two models and jointly solve the problem is to solve the two models iteratively, which is shown in the following chart.

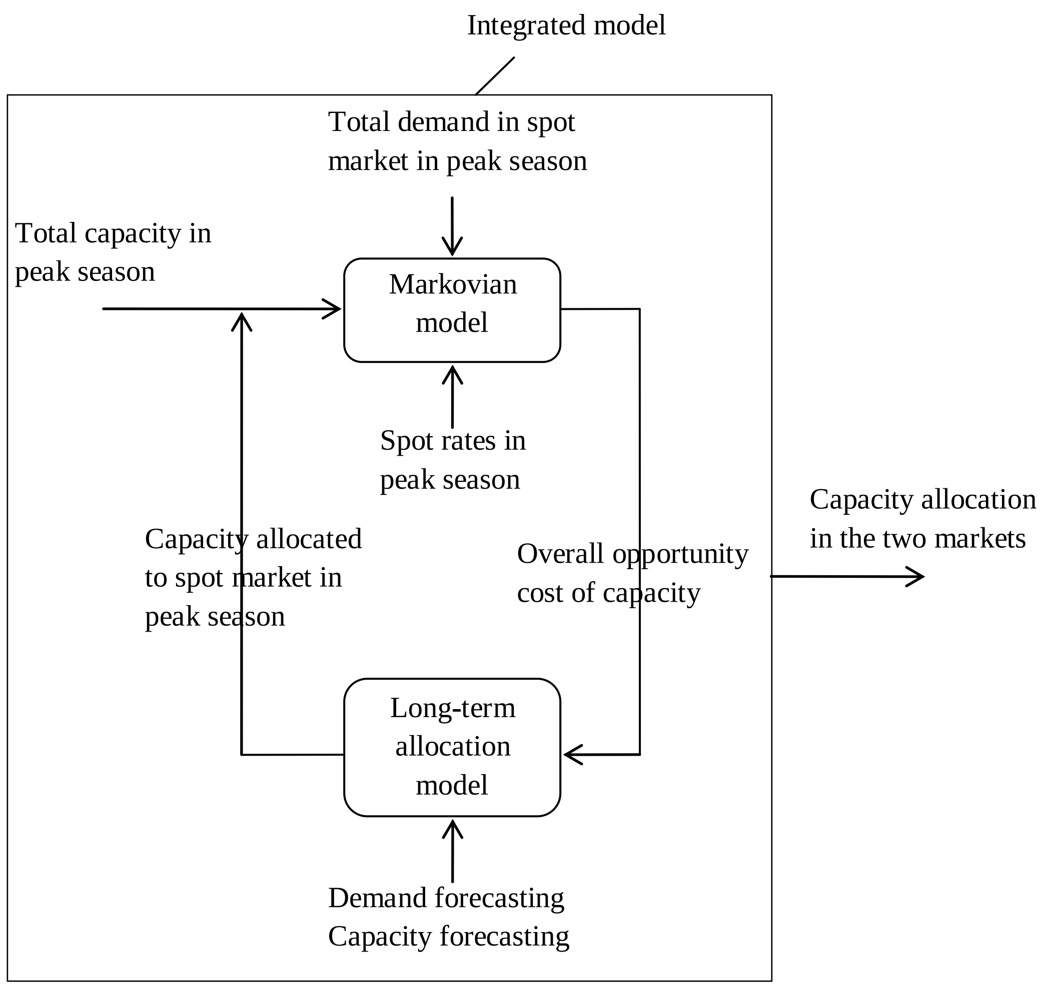


Figure 5.3 The flow chart of the integrated model

Before we introduce how the integrated model works, we need to clarify some necessary inputs to the model. First, the airline has to forecast the aggregate demand in the spot market during the peak season. Here, the aggregate demand is used rather than the demand on each specific flight during the peak season. In the long-term capacity allocation, the airline often negotiates the contract rates and order quantities with forwarders 1 year or several months before the peak season. The forecasting of spot demand on each specific flight several months in advance can be very inaccurate. Without accurate input data, the result from the Markovian model is unreliable. Also, using aggregate demand and capacity in a long period, say one month, can bring in the risk pooling effect, and thus, the result from the Markovian model is more stable. If the demand and supply in each flight is used instead, we may get a spiky rate curve because of the congestion of demand on some specific flight. Second, the airline has to forecast the aggregate capacity available in the peak season. Here, the two-dimensional characteristic of capacity is not considered, since this opportunity cost will be fed into the long-term capacity allocation model, in which the two-dimensionality can be neglected without affecting the accuracy of the result. Third, the airline has to estimate the spot rates for different types of cargo during the peak season. Here, we assume that the spot rates during the peak season are exogenously determined. As explained in chapter 4, the airline has to consider the relationship with major forwarders and shippers, and may face price regulations. Therefore, the airline cannot charge a revenue-maximizing spot rate during the peak season. Also, forwarders can be active players in spot market and affect the spot rates. In such a case, the airline can combine the forecasting of demand and historical rates to estimate the spot rates in the peak season.

After clarifying the necessary inputs to the integrated model, we can analyze how it works now. To solve the Markovian model, the total capacity allocated to spot market in the peak season is needed. At the first iteration, the total capacity available in the peak season will be allocated to the spot market. This is the optimal allocation when CBP is not adopted, since the expected spot rate is much higher than the contract rate. With the aggregate demand, aggregate capacity in spot market, and the spot rates, we can solve the Markovian model to obtain the aggregate expected profit from the spot market in peak season. Let $E\left(R_A^s\right)$ denote the aggregate expected profit and C_A denote the aggregate capacity during the peak season. The overall opportunity cost can be approximated as $OC_A = E\left(R_A^s\right)/C_A$.

Then, OC_A can be fed into the long-term capacity allocation model to determine the contract rate and the corresponding order quantity in each period. Under CBP, the airline will sell the guaranteed capacity in peak season to forwarders under contract rate. Therefore, the capacity allocated to spot market during the peak season will be equal to the total capacity less the guaranteed capacity during peak season. The opportunity cost of capacity in peak season will be compensated by the increase in revenue during low seasons. Using updated capacity for spot market, the Markovian model will be solved again to obtain the adjusted OC_A. Then, this OC_A will be fed into the long-term model to solve the updated capacity allocation again. These two models are solved iteratively and repeatedly until the solution becomes stable. The final solution will be the suggested long-term capacity allocation plan. Based on this plan, the short-term capacity allocation can be derived by solving the Markovian model as the departure of a certain flight approaches.

5.2 Several issues in the integrated model

5.2.1 Contract rate

The long-term capacity allocation model is a macro level decision model. The contract rates and order quantities obtained from the model cannot be the exact terms in the guaranteed capacity agreements, because of the following three reasons:

1. The airline's forecasting can be different from forwarder's forecasting. The contract rates derived from the model are based on the airline's forecasting, and thus, may be very different from forwarders' expectations.

2. The contract rate is based on the aggregate demand and supply in the decision period and it is assumed to be homogenous for the entire decision period. However, the real contract rate in the decision period varies depending on the day of the week, the actual supply/demand of capacity in a certain flight, and some other factors. The demand for air cargo capacity between an O-D pair fluctuates in a week. For example, some manufacturers require their suppliers to deliver raw materials before Sunday so that the production next week will not be delayed. As a result, the demand for the capacity will peak at the end of each week. The supply of capacity can also vary in a week. A large proportion of capacity comes from the belly space of passenger flights. The flight is usually more crowded during the weekends. Therefore, the supply of cargo capacity may drop in weekends. If such pattern in demand/supply exists, the

contract rate for different days of the week will vary.

3. In our model, the contract rate is the same for different forwarders. Actually, however, different forwarders can have different contract rate for the same flight. Normally, the airline will classify forwarders into several tiers according to the importance of each forwarder. Tier 1 forwarders are usually required to commit a larger guaranteed capacity in each period than forwarders in other tiers. In return, tier 1 forwarders can enjoy lower contract rate. Likewise, tier 2 forwarders have to commit to larger guaranteed capacity and can enjoy lower contract rate than tier 3 forwarders. This business strategy results from the special characteristics in air cargo industry. On the one hand, most forwarders are only able to meet the capacity commitment requirement of one airline, due to the limited demand from shippers. Under this business strategy, forwarders will choose one airline as their major business partner. On the other hand, an airline can only accommodate several tier 1 forwarders, due to its limited capacity, especially its limited capacity in peak season. As a result, each airline can have several major customers. This business strategy can help airlines establish a relatively clear territory in the market, and thus, stabilize the air cargo industry and reduce the chance of chaos situations, such as price wars.

From the above explanations, it is clear that the contract rates and order quantities obtained from the long-term capacity allocation model cannot be the final terms in guaranteed capacity agreements. Instead, the airline can use the result as a basis for the negotiation with forwarders. The final contract rates and order quantities also depend on the bargain power of players, the demand/capacity forecasting, relationship between the airline and the forwarder, and also some long-term strategic considerations. Expert judgments should be involved when making long-term capacity allocation decisions, since the capacity allocation can be more art than science.

5.2.2 Backlog or purchase additional capacity?

The integrated capacity allocation model proposed in the previous section is based on the aggregate demand and capacity over a long period. Usually, the forecasting of future demand is unreliable. Even if the model is correct, the situations that some flights are too congested while others fly empty can still happens. As departure date approaches, more information becomes available and the airline may find that the capacity on a flight is not enough to satisfy demand. If so, the airline has to decide whether to backlog some cargo to the next flight or purchase additional capacity from spot market. These decisions are called capacity re-allocation in this thesis. There are three major cases in which the capacity re-allocation is needed.

1. Order congestion on a certain flight. As explained above, the demand of capacity can vary from flight to flight. It is possible that the total demand on one flight is higher than the total capacity while the total demand on the next flight is considerably less than available capacity.

2. Result of overbooking. To mitigate the effect of cancellations and no-shows, the airline usually accepts more demand than its capacity. This practice is known as overbooking. If the volume or weight of show-up cargos is larger than the airline's capacity, re-allocation of capacity is needed.

3. Unexpected high demand in spot market. It is possible that the ad hoc demand in a certain flight is unexpectedly high, and thus exceeds the total capacity on the flight less total guaranteed capacity. Since the spot rate is usually much higher than contract rate, the airline may make more profit if it is allowed to backlog some guaranteed demand to the next flight with reasonable cost and re-allocate more capacity to spot market.

When making decision on capacity re-allocation in the first two situations discussed above, the airline has to balance the cost of backlogged cargos and the cost of purchasing additional capacity. Chew et al. (2006) considers a similar problem from forwarder's perspective. They formulate the problem as a stochastic dynamic programming, which can provide the optimal decision on the amount of capacity repurchased and the amount of cargo backlogged. The model proposed in Chew et al. (2006) can be applied to solve our problem without major revision.

If the non-guaranteed demand with high profit margin exceeds the remaining capacity, like the 3rd situation described above, the problem will be more complex. The airline has to first decide whether the demand should be accepted before the decision on backlogging/repurchasing can be made. In real operations, the cost of backlog or additional capacity is very high. Therefore, the non-guaranteed demand has to be rejected in most cases.

Gallego and Phillips (2004) discusses a special product in air cargo industry, called time-definite product, in which the airline specifies only the pick-up time and delivery time rather than the specific flight. This type of products provides flexibility by allowing the airline to allocate some cargos from a congested flight to a vacant flight as long as the pick-up time and delivery time are met. Time-definite product is only an existing example of flexible products in air cargo industry. More flexible products can be created to meet the industry's need. For example, consider an airline with three flights from Singapore to Hong Kong in the first week of August. One departs on Monday, the second departs on Wednesday, and the third departs on Friday. Forwarders can book capacity on any flights. Besides the ordinary product, the airline can offer a flexible product at a discount. Forwarders who purchase the flexible product will get a certain amount of guaranteed capacity in the first week of August, but they would not be informed which flight until later. The airline will have the right to observe the demand in each flight and decide the allocation of flexible products accordingly. If properly designed, the flexible products can solve the problems discussed in this section and increase the expected revenue of the airline.

Chapter 6 Conclusions and Future Research

The main purpose of this thesis is to develop a revenue management system to help the airline allocate the capacity on both long-term contract market and spot market so as to maximize the total revenue. This chapter concludes the study by presenting a summary of research findings and discussing the implications and limitations of this research, as well as suggesting several directions for future research.

6.1 Main findings

In the first part of the thesis (chapter 3), we consider a single-leg air cargo booking control problem on the spot market. Air cargo booking requests arrives several days before departure on the spot market. When booking request arrives, the airline has to decide whether to accept the booking or reserve the capacity for a more profitable booking that may arrive in the future. The booking process is modeled as a discrete-time Markov chain and the decision on acceptance/rejection is based on a bid-price control policy. To avoid the complexity of high dimensionality, the bid prices are derived from maximizing a reward function of the Markov chain. Numerical experiments show that the proposed model outperforms two existing static single-leg air cargo booking control policies.

In the second part of this thesis (chapter 4), we consider the long-term capacity allocation problem in air cargo industry. We assume that one major airline serves n forwarders in the industry. The airline and forwarders will close long-term contract several months in advance. The airline will decide the contract rate and the forwarder will decide the order quantity in the contract. To mitigate the negative impact of seasonal imbalance between supply and demand, we propose a capacity bundling policy, in which the guaranteed capacity that each forwarder can get during the peak season depends on its order quantity during low seasons. Then, we model the problem as a Stackelberg game and the airline as the Stackelberg leader. For a general capacity bundling policy, the forwarder's decision problem is modeled as a dynamic programming and the airline's decision problem can be solved via numerical methods. Then, a commonly used linear form capacity bundling policy is assumed. Based on this assumption, the problem is decomposed into several sub-problems and the optimal solution is obtained. Numerical experiments show that the capacity bundling policy can increase the airline's expected profit and reduce the risk, when the expected spot rate is less than forwarder's resale rate in the peak season. The policy can have a stronger effect when the future demand is highly unpredictable. Therefore, the capacity bundling policy can successfully solve the mismatch between capacity supply and demand in air cargo industry.

In the third part of this thesis (chapter 5), we propose a conceptual framework of a revenue management system for air cargo capacity allocation. The two capacity control model developed in chapter 3 and 4 are interrelated. The capacity allocation decision in the long-term contract market will affect the available capacity in spot market. The opportunity cost of capacity on spot market, in turn, affects the decision on long-term capacity allocation. In view of the relationship between the two models, we propose an integrated model that can jointly allocate capacity between spot market and contract market. The integrative capacity allocation can be obtained by solving the two models iteratively and repeatedly. Then, we highlight two issues in using the proposed air cargo revenue management system. The first issue is how to make use of the “optimal” contract rate obtained from the model. The airline should view the contract rate as a basis and guideline for the negotiation with forwarders when signing the long-term contract. The actual contract rates and order quantities will depend on the bargain power, the demand/capacity forecasting, relationship between the airline and the forwarder, and also some long-term strategic considerations. Expert judgments should be involved when making long-term capacity allocation decisions. The second issue that needs our attentions is the case when the airline faces shortage of capacity to satisfy all guaranteed demands. In such a situation, the airline has to decide the capacity that should be purchased from spot market and the quantity of cargo that should be backlogged, so that the total cost is maximized. We suggest that the model developed in Chew et al. (2006) can be used to solve this problem. We also suggest the airline design flexible products for air cargo transportation so that the capacity can be used more efficiently.

6.2 Suggestion for future research

Competing behavior among airlines

The air cargo industry considered in this thesis consists of one major airline and several forwarders. Therefore, the airline has strong power in the product pricing. This may be true for the spot market, in which the customers often act as price takers. However, the airline may not have such strong influence on the cargo rate in contract market since it may face the competition from other airlines. During the low season, especially on some route where overcapacity exists, some airlines may charge a very low contract rate, which can only cover its operating cost to attract demands and keep a good relationship with forwarders. In such a situation, other airlines may not have strong bargain power and need to consider the strategic behaviors from its competitors.

Moreover, airlines may focus on the long-term benefits from the strategic behaviors rather than maximizing the revenue in a single decision period. For example, if airline A has dominating power in a certain market, it may start a price war and try to wipe out other competing airlines. The short-term performance of the airline may be very poor under this strategy, but the long-term benefits may be maximized. In contrast, the airlines in a market may try to maintain a stable contract rate and form a relative clear market territory. By doing so, every airline may survive and make profit in the long run.

In summary, the complex strategic behaviors of all airlines need special analysis when making decisions on capacity allotment in a competitive market. Game theory may be needed to analyze the problem and the proof on the existence and uniqueness of the equilibrium may be necessary.

Multiple tiers of customer

In this thesis, we assume that the contract rate is the same for all customers. Actually, however, different forwarders can have different contract rate for the same flight. Normally, the airline will classify the forwarders into several tiers according to the importance of each forwarder. Tier 1 forwarders are usually required to commit a larger guaranteed capacity in each period than forwarders in other tiers. In return, tier 1 forwarders can enjoy lower contract rate. This business strategy is not useful when one major airline serves the entire market, as assumed in our thesis. However, it will be very important for airlines operating in a competing market. The airlines that can attract big shippers and forwarders will have stronger power in pricing the product and have greater chance to survive. Then, the research question is how to set the contract rate and minimum capacity commitment for each tier of customer so that the sales target is achieved. When making the above decision, the airline should consider not only the demand/supply relationship in the market but also strategic behaviors from its competitors. Also, the airline should not merely focus on maximizing the short-term revenue but on maximizing the long run interests. It will be very helpful for the airline if future research can solve this problem successfully.

Relationship between contract market and spot market

There are three types of demand/supply shift between contract market and spot market.

1. The more the forwarder orders in the long-term contract market, the less the chance that it cannot satisfy all demands and need to purchase additional capacity from spot market. Therefore, part of the potential demand in the spot market shift to the contract market. Vice versa.
2. If the forwarder orders a lot in the contract market but cannot sell out all the guaranteed capacity, it may sell the remaining capacity at spot price to other forwarders requiring emergency capacity. Therefore, part of the supply may shift from the contract market to the spot market.
3. The problem will be more complicated when considering the competing behavior from other airlines. Suppose two airlines operating in a region, A and B. If airline B allocates a large amount of capacity to the contract market and market aggressively, airline A will face strong competition in the contract market.

However, airline B will provide less capacity to spot market. If airline B commits more guaranteed capacity than it can provide, airline B can be the potential customer on spot market. Airline A can adjust its pricing strategy according to B's strategic behavior.

The demand/supply shift introduced above is not considered in this thesis. The demand/supply shift is based on the strategic behavior of forwarders and other competing airlines. To model such behavior the airline has to possess accurate information on forwarders' and other airlines' forecasting of future demand. Therefore, the analysis on the demand/supply shift between contract market and spot market should be based on the advance of information sharing in the air cargo industry.

Non-constant arrival rate λ in spot market

In chapter 3, the arrival rate of demand λ is assumed to be a constant throughout the selling season. However, it is expected that the arrival rate depends on the amount of time before departure. As the time approaches the departure, the demand rate may increase. Therefore, a more realistic assumption is that the arrival of demand follows a non-homogeneous Poisson process and the arrival rate is a function of time $\lambda(t)$. One possible way to incorporate the time-dependent arrival rate into the proposed Markovian model is to use several homogeneous Poisson process with different λ to approximate the non-homogeneous Poisson process. For example, suppose the selling season starts 10 days before the departure, and the airline estimates the arrival rate function $\lambda(t)$ during the selling season. It is natural to assume that the arrival rate remains constant within each day. Denote the arrival rates as $\lambda(1), \lambda(2), ..., \lambda(10)$. Then, the capacity control problem during the 10 days can be decomposed into 10 sub-problems, and each problem has similar characteristics as the capacity control problem considered in chapter 3. Similar as in chapter 3, define process $S = \{(W_n, V_n), n = 0,..., N\}$, where W_n and V_n are the total accepted weight and volume until period *n*. At the beginning of the selling season, the state of process S is known. Then, according to equation (3.16), the end state of process S in day 1 can be predicted, and it is a function of bid prices h_w and h_v. Also, the total expected revenue received in the first day can be predicted based on lemma 2. At the beginning of second day in selling season, the state of process *S* is the end state of process S in day 1. Based on the arrival rate $\lambda(2)$, the state evolvement and the expected revenue in day 2 can be predicted. Following this way, the expected revenue in each day can be predicted and the sum of all revenue will give us the total expected revenue from the flight. Numerical searching algorithms can be used to find the optimal bid prices.

Besides the time before the departure, the arrival rate may also depend on other factors, such as the spot rate on the market and the competing strategy of other airlines. By incorporating these factors into the model, the capacity control model can better reflect market dynamics, but it is out of the scope of this thesis.

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