SUPERCORE AND STRONG NASH EQUILIBRIUM

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Summary

This paper studies the relation between Roth's (1976) notion of "supercore" and Aumann's (1959) notion of "strong Nash equilibrium" in normal-form games. Inarra et al. (2007) studied the relation between the supercore and Nash equilibrium; in particular, they offered a procedure to find the supercore in normal-form games. This paper extends Inarra et al.'s procedure to complex social interactions. This paper shows that the supercore under social interactions coincides with the set of strong Nash equilibria in the final game defined in the procedure. This study provides a valuable and useful insight into the equilibrium strategic behavior.

Key words: supercore, subsolution, strong Nash equilibrium, normal form game.

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1 Introduction

The concept of supercore introduced by Roth (1976) is an interesting solution concept in game theory. It is identified as the intersection of subsolutions in the context of abstract games. Subsolution is a generalization of "solution" which is interpreted by von Neumann and Morgenstern (1953) as a self-reinforcing standard of behavior. Subsolution characterizes the set of solutions as internally stable and self-protecting. Roth (1976) argued that once a subsolution is generally accepted by the players, it creates expectations reinforcing the notion that only the outcomes in subsolution are considered "sound".

Inarra et al. (2007) studied the relationship between Nash equilibrium and supercore. Specifically, they defined a procedure to identify the supercore. They showed that the supercore coincides with the set of Nash equilibria of the final game defined in the procedure.

However, the concept of Nash equilibrium is based on the idea of stability against any unilateral deviation (Nash 1951). Aumann (1959) proposed the notion of strong Nash equilibrium (SNE) ensuring a more restrictive stability, which is immune to any coalitional deviation. A SNE is defined as a strategy profile for which no subset of players has a joint deviation that strictly benefits all of them. Thus, a SNE is a Nash equilibrium and is weakly Pareto efficient among the Nash equilibria.

In order to look into the relationship between strong Nash equilibrium and supercore, this paper extends Inarra et al.'s procedure to complex social interactions, refines the binary relation defined by Kalai and Schmeidler (1977), and shows that the supercore under the new binary relation coincides with the set of strong Nash equilibria in the final game defined in the procedure.[1]

The remainder of the paper is organized as follows. Section 2 provides the preliminaries. Section 3 establishes an abstract system relative to a normal form game with a newly defined binary relation. Section 4 presents the procedure characterizing the relationship between SNE and supercore. Section 5 is dedicated to a formal proof of the validity of the procedure. Concluding remarks are presented in Section 6.

[1] Greenberg (1990) considered different binary relations that associate normal form games with abstract systems, including the one discussed in this paper.

2 Preliminaries

In this section, we introduce the notation and definitions used in this paper.

According to von Neumann and Morgenstern (1953), $(X, \succ)$ is an **abstract system** where X is a set of outcomes and $\succ$ is the binary preference relation defined on X. For two outcomes $x, y \in X$, we interpret $x \succ y$ as "x dominates y." Given an outcome $x \in X$, its dominion (the set of outcomes dominated by x) is defined as:

$$\mathcal{D}(x) = \{y \in X : x \succ y\}.$$

For a non-empty subset $A \subset X$,[2] its dominion is defined as:

$$\mathcal{D}(A) = \bigcup_{x \in A} \mathcal{D}(x),$$

i.e., the set of outcomes dominated by some outcome of A. Let

$$\mathcal{U}(A) = X - \mathcal{D}(A)$$

denote the set of outcomes undominated by any outcome of A.

A subset $A \subset X$ is the **core** (Gillies 1959) for $(X, \succ)$ if $A = \mathcal{U}(X)$.

We call a subset $A \subset X$ a **vN & M stable set** (von Neumann and Morgen-

[2]All the inclusions used in this paper are weak.

stern 1953) of $(X, \succ)$, if $A = \mathcal{U}(A)$. That is, a vN & M stable set is defined as a subset $A \subset X$ that satisfies:

1. [internal stability] $A \subset \mathcal{U}(A)$, i.e., no element in A dominates another element in A, and

2. [external stability] $\mathcal{U}(A) \subset A$, i.e., every element not in A is dominated by some element in A.

von Neumann and Morgenstern (1953) interpreted a vN & M stable set as a "standard of behavior" in a society, which describes "how things are in actual social organizations."

A **subsolution** (Roth 1976) of $(X, \succ)$ is a subset A of X such that

1. $A \subset \mathcal{U}(A)$, and

2. $A = \mathcal{U}^2(A)$, where $\mathcal{U}^2(A) = \mathcal{U}(\mathcal{U}(A))$.

Let $\mathcal{P}(A) = \mathcal{U}(A) - A$. Given a subsolution A, the set X is partitioned into three sets: A, $\mathcal{D}(A)$ and $\mathcal{P}(A)$ (see Figure 1). Note that if A is a vN & M stable set then A is a subsolution with $\mathcal{P}(A) = \phi$.

Given $A \subset \mathcal{U}(A)$, we can prove that $\mathcal{U}^2(A) = A \Leftrightarrow \mathcal{P}(A) \subset \mathcal{D}(\mathcal{P}(A))$.

The reason is as follows (see Inarra et al. 2009):

$$
\begin{aligned}
\mathcal{U}(\mathcal{U}(A)) &= \mathcal{U}(A \cup \mathcal{P}(A)) \\
&= X - \mathcal{D}(A \cup \mathcal{P}(A)) \\
&= A \cup \mathcal{P}(A) - \mathcal{D}(\mathcal{P}(A)) \\
&= A \Leftrightarrow \mathcal{P}(A) \subseteq \mathcal{D}(\mathcal{P}(A)) \text{ and } A \bigcap \mathcal{D}(\mathcal{P}(A)) = \phi.
\end{aligned}
$$

Such a property of subsolution can be interpreted by Roth (1976):

> Every point in $\mathcal{U}(A) - A$ is dominated by some other point in the same set, and the entire set thus 'overrules' itself leaving only the set A as 'sound'.

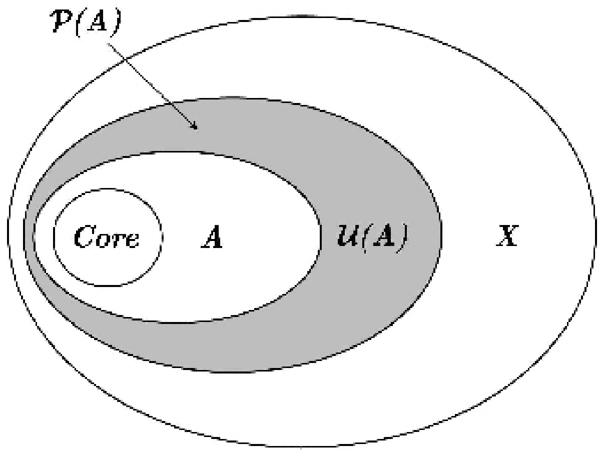

Figure 1: Partition of X

Supercore (Roth 1976) is identified as the most significant subsolution, *i.e.* a supercore of $(X, \succ)$ is a subset S of X such that

$$S = \bigcap_{A \text{ is a subsolution}} A.$$

Observe that: (1) Every vN & M stable set includes the core, (2) Every vN & M stable set is a subsolution, and (3) The supercore includes the core.

A (finite) **normal form game** Γ^N is a triple

$$< N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N} >,$$

where $N = \{1, ..., n\}$ is a finite set of players, S_i is the finite set of strategies for player i and $u_i : S = \times_{i \in N} S_i \longrightarrow \mathbb{R}$ is player i's payoff function.

An N-tuple of strategies, $s \in S$, is a **Nash equilibrium** (Nash 1951) for Γ^N if there do not exist $i \in N$ and $s_i^{'} \in S_i$ such that $u_i(s_i^{'}, s_{-i}) > u_i(s)$.

Example 1 (supercore for 3-person prisoner's dilemma)

Table 1: 3-person prisoner's dilemma

	C	D	C	D
C	1, 1, 1	-1, 3, -1	-1, -1, 3	-2, 2, 2
D	3,-1,-1	2, 2, -2	2, -2, 2	0, 0, 0
	C		D	

Inarra et al. (2007) studied this game in individual situation and solved its supercore: $\Sigma = \{(D, D, D), (D, C, C), (C, C, D), (C, D, C)\}$. Observe that it is formed by the unique Nash equilibrium (D, D, D) and the strategy profiles that any 2 players choose C. Particularly, Inarra et al. proved that the supercore for the n-person prisoner's dilemma is the unique *vN&M* stable set of its associated system. We can verify, in this 3-person prisoner's dilemma, that $\Sigma = \mathcal{U}(\Sigma)$.

3 The abstract system $(S, \triangleright)$

In this section we define a new binary relation $\triangleright$ on an abstract system relative to a normal form game by refining the conventional relation which only accounts for individual profitable deviations.[3]

Definition 1 *The abstract system $(S, \triangleright)$ associated to the normal-form game $< N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N} >$ is defined as follows:*

1. $S = \times_{i \in N} S_i$;

2. *The binary relation $\triangleright$ on S is defined as: for $s^{'}, s \in S$, $s^{'} \triangleright s$ iff $\exists T \subseteq N$ s.t. $u_i(s^{'}) > u_i(s)$ $\forall i \in T,$ where $s^{'} = (s^{'}_T, s_{-T})$ and $s^{'}_T \in S_T = \times_{i \in T} S_i$.*

Different from Inarra et al.'s (2007) system, the binary relation defined in the present paper is based on coalitional deviations. In this case, Nash equilibrium is no longer considered stable as there may exist opportunities for coalitional deviations that strictly benefit all the members within the coalition. As an example, consider the following game:

[3] See Kalai and Schmeidler (1977); adopted by Inarra et al. (2007).

Example 2

Table 2: A two-person game

	L	R
U	2,2	0,0
D	0,0	1,1

The Nash equilibria in this game are (U, L) and (D, R). But notice that (D, R) can be improved upon by both players jointly agreeing to play (U, L). This led Aumann (1959) to propose the idea of "strong Nash equilibrium":

An N-tuple of strategies, $s \in S_N$, is a **strong Nash equilibrium (SNE)** for Γ^N if there do not exist $T \subset N$ and $s'_T \in S_T$ such that $u_i(s'_T, s_{-T}) > u_i(s)$ for all $i \in T$. The unique strong Nash equilibrium in the above game is (U, L).

Observe that: since no strong Nash equilibrium is dominated by any strategy profile, the set of strong Nash equilibria coincides with the core, i.e., $\text{SNE}(\Gamma^N) = \mathcal{U}(S)$.

4 The supercore for $(S, \triangleright)$ and SNE

In this section, we give an example to illustrate the procedure of deriving the supercore for $(S, \triangleright)$ defined in Section 3. Then we offer the formal procedure for normal form game.

Example 3 *Consider the following game* Γ_0^N *in Table* 3.

Table 3: Game Γ_0^N at stage 0

	b_1	b_2	b_3	b_4
a_1	6,5	6,4	1,3	2,2
a_2	3,7	5,6	6,2	1,3
a_3	6,4	3,3	5,5	6,0
a_4	5,3	2,4	7,2	0,6

At stage 0, **we find out the set of strong Nash equilibria** of Γ_0^N. Obviously, (a_1, b_1) is the unique Nash equilibrium for this game, and it is also the unique strong Nash equilibrium. Therefore,

$$S_0^* = \text{SNE}(\Gamma_0^N) = \{(a_1, b_1)\}.$$

Then **we replace the payoffs to the profiles that are dominated by SNE profiles, with the corresponding players' lowest payoffs in the game (Γ_0^N) at stage 0** – payoffs $(0, 0)$. In particular, we identify the strategic profiles dominated by S_0^*:

$$(a_1,b_2),(a_1,b_3),(a_1,b_4),(a_2,b_1),(a_4,b_1),(a_3,b_2),(a_4,b_2),(a_2,b_4).$$

Replace the payoffs from them by $(0,0)$ and obtain a new game Γ_1^N (see Table 4).

Table 4: Game Γ_1^N at stage 1

	b_1	b_2	b_3	b_4
a_1	6,5	0,0	0,0	0,0
a_2	0,0	5,6	6,2	0,0
a_3	6,4	0,0	5,5	6,0
a_4	0,0	0,0	7,2	0,6

At stage 1, we carry on a similar procedure conducted in stage 0. First, **we find the set of SNE profiles** for Γ_1^N. In order to do so, we firstly identify the set of Nash equilibria, which is $\{(a_1,b_1),(a_2,b_2)\}$. As the two NE profiles do not dominate each other in a coalitional situation, they are both SNE profiles. Thus,

$$S_1^* = \{(a_1,b_1),(a_2,b_2)\}.$$

Second, **replace the payoffs to the profiles that are dominated by SNE with the lowest ones in $\mathbf{\Gamma_0^N}$**. As the profiles dominated by S_0^* are all dominated at stage 1 and already replaced, we only need to replace the payoffs to the newly dominated profile, which is (a_2, b_3). Then we obtain Γ_2^N and proceed to the next stage (see Table 5):

Table 5: Game Γ_2^N at stage 2

	b_1	b_2	b_3	b_4
a_1	6,5	0,0	0,0	0,0
a_2	0,0	5,6	0,0	0,0
a_3	6,4	0,0	5,5	6,0
a_4	0,0	0,0	7,2	0,6

At stage 2, we find that the set of strong Nash equilibria for Γ_2^N is $\{(a_1, b_1), (a_2, b_2)\}$, which coincides with that for the previous game Γ_1^N, i.e.

$$S_2^* = S_1^*.$$

This is the condition on which the procedure stops. Thus, we claim that the supercore for the original game is $\{(a_1, b_1), (a_2, b_2)\}$, that is,

$$\text{Supercore}(\Gamma_0^N) = \{(a_1, b_1), (a_2, b_2)\}.$$

To sum up, by repeatedly conducting the "finding and replacing" process, the procedure generates a sequence of games $\{\Gamma_\ell^N\}$ such that the supercore for the original game coincides with the set of SNE profiles in the final game (note that the original game is a finite game, so the procedure is finite). Let S_ℓ^* denote the set of strong Nash equilibria of Γ_ℓ^N. To formalize the procedure, we present the following definition:

Definition 2 *We define a sequence of games* $\{\Gamma_\ell^N\}_{\ell=0}^k$ *and a sequence of systems* $\{(S, \triangleright_\ell)\}_{\ell=0}^k$ *as follows:*

1. $\Gamma_0^N = \Gamma^N$ *and* $(S, \triangleright_0)$ *is the system associated to* Γ_0^N*.*

2. *For* $\ell \geq 1$*,* $\Gamma_\ell^N =< N, \{S_i\}_{i \in N}, \{u_i^\ell\}_{i \in N} >$*, with*

$$u_i^\ell(s) = \begin{cases} v_i(\Gamma^N), & \textit{if } s \in \mathcal{D}_{\ell-1}(S_{\ell-1}^*) \\ u_i^{\ell-1}(s), & \textit{otherwise} \end{cases} ,$$

where $v_i(\Gamma^N) = \min\{u_i(s) : s \in S\}$ *and* $\mathcal{D}_{\ell-1}(S_{\ell-1}^*)$ *is the set of outcomes dominated by some outcome in* $S_{\ell-1}^*$ *in* $(S, \triangleright_{\ell-1})$*. Define* $(S, \triangleright_\ell)$ *as the system associated to* Γ_ℓ^N*.*

We are now in a position to present our main results in this paper.

Proposition 1 *If* $S_k^* = S_{k-1}^*$*, then* S_k^* *is the supercore for* $(S, \triangleright_0)$*.*

Proposition 2 *The set of strong Nash equilibria is a subset of supercore for* $(S, \triangleright_0)$*.*

5 Proofs

In order to prove Proposition 1 and 2, we need to introduce the following 2 lemmas.

Lemma 1 [4] $A \subset B$ *implies* $\mathcal{U}(B) \subset \mathcal{U}(A)$.

Proof. *$A \subset B$ yields $\mathcal{D}(A) \subset \mathcal{D}(B)$; taking complement of $\mathcal{D}(A)$ and $\mathcal{D}(B)$, we have $\mathcal{U}(A) \supset \mathcal{U}(B)$.* ■

Lemma 2 $\mathcal{D}_\ell(S^*_\ell) = \mathcal{D}_0(S^*_\ell)$, $\forall\, \ell \geq 0$.

Proof. *($\Longleftarrow$): At stage ℓ, first notice that players' payoffs to the strategy profiles in S^*_ℓ have never been replaced since stage 0; the payoffs to a strategy profile in $\mathcal{D}_0(S^*_\ell)$, on the other hand, either remain the same or have been replaced with the lowest payoffs. In either case, it is $\rhd_\ell$-dominated by S^*_ℓ. Therefore, $t \in \mathcal{D}_0(S^*_\ell)$ implies $t \in \mathcal{D}_\ell(S^*_\ell)$.*

($\Longrightarrow$): We show this by contradiction. Given a strategy profile $t' \in \mathcal{D}_\ell(S^*_\ell)$. Suppose that t' is not $\rhd_0$-dominated by any outcome in S^*_ℓ, then according to the procedure, no replacement will happen to the payoffs to t' from stage 0 to stage ℓ, because each set of SNE profiles before stage ℓ (inclusive) are subsets of S^*_ℓ. Consequently, at stage ℓ, t' is still undominated by any profile in S^*_ℓ,

[4] *This lemma was first introduced by Roth (1976).*

contradicting the condition $t^{'} \in \mathcal{D}_\ell(S_\ell^*)$. *Therefore,* $t^{'} \in \mathcal{D}_0(S_\ell^*)$ *follows from* $t^{'} \in \mathcal{D}_\ell(S_\ell^*)$. ■

5.1 Proof of Proposition 1

To show that the defined procedure can always generate the supercore, it suffices to prove that the following two conditions hold:

(i) S_k^* is a subsolution for $(S, \triangleright_0)$. That is, $S_k^* \subset \mathcal{U}_0(S_k^*)$ and $S_k^* = \mathcal{U}_0^2(S_k^*)$; [5]

(ii) Any other subsolution A for $(S, \triangleright_0)$ contains S_k^*.

Proof of condition (i). In the final game of the sequence, Γ_k^N, every player i's payoff can be written as

$$u_i^k(s) = \begin{cases} v_i(\Gamma^N), & \text{if } s \in \mathcal{D}_0(S_k^*) \\ u_i^0(s), & \text{otherwise} \end{cases} \tag{1}$$

We first prove $S_k^* \subset \mathcal{U}_0(S_k^*)$. As S_k^* is the set of strong Nash equilibria for the final game, all the strategies in S_k^* do not dominate each other in a coalitional situation, hence, $S^* \subset \mathcal{U}_k(S_k^*)$. Moreover, the payoffs to any strategy profile in S_k^* have never been replaced ever since stage 0, thus we have $S_k^* \subset \mathcal{U}_0(S_k^*)$.

Next we prove $S_k^* = \mathcal{U}_0(\mathcal{U}_0(S_k^*))$, which is equivalent to showing that $\mathcal{P}_0(S_k^*) \subset \mathcal{D}_0(\mathcal{P}_0(S_k^*))$. We show it by contradiction. Suppose there is a strategy profile

[5] $\mathcal{U}_0(S_k^*)$ is the set of outcomes undominated by any outcome in S_k^* in $(S, \triangleright_0)$.

$s^{'} \in \mathcal{P}_0(S_k^*)$ s.t. $s^{'} \notin \mathcal{D}_0(\mathcal{P}_0(S_k^*))$. By *Lemma 2*, we have $\mathcal{U}_k(S_k^*) = \mathcal{U}_0(S_k^*)$, thus $\mathcal{U}_k(S_k^*)/S_k^* = \mathcal{U}_0(S_k^*)/S_k^*$, *i.e.* $\mathcal{P}_k(S_k^*) = \mathcal{P}_0(S_k^*)$. Hence, $s^{'} \in \mathcal{P}_k(S_k^*)$. In game Γ_k^N, $s^{'}$ is a non-SNE strategy profile and is not dominated by any strong Nash equilibrium, implying that $s^{'}$ is $\triangleright_k$-dominated by some strategy profile(s) outside of S_k^*. The entire set of strategy profiles can be divided into three partitions: S_k^*, $\mathcal{D}_k(S_k^*)$ and $\mathcal{P}_k(S_k^*)$ (*i.e.* $\mathcal{U}_k(S_k^*)/S_k^*$). Thus the strategy profiles that can possibly $\triangleright_k$-dominate $s^{'}$ are in either $\mathcal{D}_k(S_k^*)$ or $\mathcal{P}_k(S_k^*)$. It is impossible to find one in $\mathcal{D}_k(S_k^*)$, because by $\mathcal{D}_k(S_k^*) = \mathcal{D}_0(S_k^*)$ (*Lemma 2*) and equation (1), we conclude that the payoffs to all strategy profiles in $\mathcal{D}_k(S_k^*)$ are replaced with the lowest payoffs, leaving no strategy profile that can possibly $\triangleright_k$-dominate $s^{'}$, hence $s^{'} \in \mathcal{U}_k(\mathcal{D}_k(S_k^*))$. Therefore, the only possible situation is $s^{'} \in \mathcal{D}_k(\mathcal{P}_k(S_k^*))$, i.e. $s^{'} \in \mathcal{D}_k(\mathcal{P}_0(S_k^*))$. It is straightforward that, $u_i^k(s^{''}) = u_i^0(s^{''})\ \forall s^{''} \in \mathcal{P}_0(S_k^*)$. This means that the payoffs for the strategy profiles in $\mathcal{P}_0(S_k^*)$ (including $s^{'}$) are never replaced. Following this argument, $s^{'} \in \mathcal{D}_k(\mathcal{P}_0(S_k^*))$ implies $s^{'} \in \mathcal{D}_0(\mathcal{P}_0(S_k^*))$, which contradicts the assumption $s^{'} \notin \mathcal{D}_0(\mathcal{P}_0(S_k^*))$. Therefore, $\mathcal{P}_0(S_k^*) \subset \mathcal{D}_0(\mathcal{P}_0(S_k^*))$. ■

Proof of condition (ii). We proceed by induction. Given any subsolution A of the abstract system $(S, \triangleright_0)$, note that $S_0^* \subset A$ since the set of strong Nash equilibria of Γ_0^N is contained in any subsolution of the associated system. Suppose $S_{\ell-1}^* \subset A$, we next show that $S_\ell^* \subset A$. Because $S_{\ell-1}^* \subset S_\ell^*$, it suffices to show that $r \in A$, for any $r \in S^*_\ell / S^*_{\ell-1}$.

We claim that $r \in \mathcal{U}_0(\mathcal{U}_0(S^*_{\ell-1}))$. Notice that $r \in S^*_\ell / S^*_{\ell-1}$, it is a strong Nash equilibrium of Γ^N_ℓ, but not one for $\Gamma^N_{\ell-1}$. By the construction of Γ^N_ℓ, r can only be $\triangleright_{\ell-1}$-dominated by some strategy in $\mathcal{D}_{\ell-1}(S^*_{\ell-1})$. That is, r can never be $\triangleright_{\ell-1}$-dominated by any strategy in $\mathcal{U}_{\ell-1}(S^*_{\ell-1})$. Notice that in $\Gamma^N_{\ell-1}$, the payoffs to any strategy profile in $\mathcal{U}_{\ell-1}(S^*_{\ell-1})$ remain the same as in Γ^N_0, thus r can never be $\triangleright_0$-dominated by any strategy in $\mathcal{U}_{\ell-1}(S^*_{\ell-1})$. Hence r cannot be $\triangleright_0$-dominated by $\mathcal{U}_0(S^*_{\ell-1})$, i.e., $r \in \mathcal{U}_0(\mathcal{U}_0(S^*_{\ell-1}))$, because $\mathcal{U}_0(S^*_{\ell-1}) = \mathcal{U}_{\ell-1}(S^*_{\ell-1})$ (by *Lemma 2*).

Finally, $S^*_{\ell-1} \subset A$ implies that $\mathcal{U}_0(\mathcal{U}_0(S^*_{\ell-1})) \subset \mathcal{U}_0(\mathcal{U}_0(A))$ (by applying *Lemma1*). By the definition of subsolution, $\mathcal{U}_0(\mathcal{U}_0(A)) = A$; thus $r \in A$ follows from $r \in \mathcal{U}_0(\mathcal{U}_0(S^*_{\ell-1}))$. ■

5.2 Proof of Proposition 2

The proof is straightforward. The procedure of deriving the supercore implies that SNE(Γ^N) $\equiv S^*_0 \subset S^*_k$. And in Proposition 1 we have shown that S^*_k is the supercore of Γ^N. Therefore, SNE(Γ^N_0) $\subset$Supercore(Γ^N_0). ■

6 Concluding remarks

This paper extends Inarra et al.'s procedure to complex social interactions. By integrating coalitional deviations and refining the binary relation on an abstract system, we explore the relation between strong Nash equilibrium and supercore in normal form games. We show that the supercore under social interactions coincides with the set of strong Nash equilibria in the final game defined in the procedure. In the original game strong Nash equilibria lie in the supercore.

Inarra et al. (2009) advanced their study by investigating the relationship between supercore and Nash equilibrium in the mixed extension of normal-form game. It is interesting to study the relationship between supercore and strong Nash equilibrium in the mixed extension of normal-form game.

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