INVENTORY CONSIDERATION AND MANAGEMENT IN TWO SUPPLY CHAIN PROBLEMS

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SUMMARY

Inventory management has become increasingly important in various logistics and supply chain problems and it has received much attention from both researchers and practitioners in recent decades. This thesis studies both strategic and operational supply chain problems that incorporate inventory consideration and management.

The strategic supply chain problem studied is a joint facility location-allocation and inventory problem that incorporates multiple sources. The problem is motivated by a real situation faced by a multinational applied chemistry company. In this problem, multiple products are produced in several plants. A warehouse can be replenished by several plants together because of capabilities and capacities of plants. Each customer in this problem has stochastic demand and a certain amount of safety stock must be maintained in warehouses so as to achieve a certain customer service level. The problem is to determine the number and locations of warehouses, allocation of customers demand and inventory levels of warehouses so as to minimize the expected total cost with the satisfaction of desired demand weighted average customer lead time and desired cycle service level. The problem is formulated as a mixed integer nonlinear programming model. Utilizing approximation and transformation techniques, we develop an iterative heuristic method for the problem. An experiment study shows that the proposed procedure performs well in comparison with a lower bound.

The operational supply chain problem considered is a multi-channel component replenishment problem in an assemble-to-order system. It is motivated by real situations faced by some contract manufacturers. The assemble-to-order manufacturer

faces a single period stochastic demand of a single product consisting of multiple components. Before product demand is realized, the manufacturer needs to decide on initial ordering quantities of components (called pre-stocked components). After the demand is realized, the needed components which cannot be filled from inventory can be replenished through multiple sourcing channels with different prices and lead times. The manufacturer then needs to decide on timing, quantities and sourcing channels of additional components to order, as well as final product delivery schedule. We show some good properties according to the structure of the problem. Based on the properties, we formulate the problem as a stochastic programming model and we solve a restricted version of our problem in which the quantities of pre-stocked components follow a certain fixed rank order. We then provide a closed-form optimal solution for dual-channel two-component problem and we develop a branch and bound method for multi-channel multi-component problem to search over all permutations to obtain the optimal solution. We also present a greedy heuristic procedure. We finally offer a computational experiment to demonstrate the efficiency of our solution methods and to compare the performance of assemble-to-order systems with single and dual procurement channels, respectively.

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LIST OF NOTATIONS

b_i Unit salvage value of component i (without loss of generality, we assume that $b_i < c_i^1$, since it is optimal to order infinite units of component i if $b_i \geq c_i^1$).

$c_i^{e_i}$ Unit purchasing cost of component i using procurement channel e_i, where e_i = 1, 2, …, m_i (we assume $c_i^1 < c_i^2 < \ldots < c_i^{m_i}$).

cpc_{ikf} Unit transportation cost of product type f from plant i to customer k.

cpw_{ijf} Unit transportation cost of product type f from plant i to warehouse j.

cwc_{jkf} Unit transportation cost of product type f from warehouse j to customer k.

D Stochastic demand.

d_{kf} Mean annual demand of product type f at customer k.

dt Desired weighted average customer lead time.

f Product type index, f = 1, 2, …, F.

f_j Annual fixed cost for leasing warehouse j.

$f(x)$ Probability density function of D.

$F(x)$ Cumulative distribution function of D.

h_j Unit holding cost per year at warehouse j.

$l_i^{e_i}$ Procurement lead time of component i using procurement channel e_i, where e_i = 1, 2, …, m_i (we assume $l_i^1 > l_i^2 > \ldots > l_i^{m_i}$).

p_{if} Annual amount of product type f produced at plant i.

P_{ik} = 1 if customer k is directly served by plant i, 0 otherwise.

$P(t)$ Unit price for final product delivered at time t, a decreasing function of t.

Q_i Initial ordering quantity of component i which is acquired before time 0.

r_j Review period of warehouse j.

T_k Lead time for customer k (depending on the source of shipment).

tpc_{ik} Replenishment lead time from plant i to customer k.

tpw_{ij} Replenishment lead time from plant i to warehouse j.

twc_{jk} Replenishment lead time from warehouse j to customer k.

W_{jk} = 1 if customer k is served through warehouse j, 0 otherwise.

X_{ijf} Annual amount of product type f shipped from plant i to warehouse j.

z Desired safety factor (it is the standard normal value corresponding to the desired cycle service level).

Z_j = 1 if warehouse j is leased, 0 otherwise.

σ_{kf} Standard deviation of annual demand of product type f at customer k.

Chapter 1 INTRODUCTION

With the rapid development of logistics and supply chain management in recent decades, inventory management has become more and more important in various logistics and supply chain problems. Inventory management has received much attention from both researchers and practitioners. In the research society, there is a huge amount of literature on inventory management. From an industrial perspective, there is an increasing need of inventory management software in industry and the inventory management software market has drastically expanded in recent years. Researchers and practitioners have considered inventory management not only in operational supply chain problems, but also in strategic supply chain problems. As the main facility in which inventory management plays an important role is the warehouse, this thesis first studies a multi-source facility (warehouse) location-allocation and inventory problem, which belongs to a strategic level supply chain problem. Also note that nowadays warehouse does not only act as a storage facility but adds value by doing some light assembly for some assemble-to-order manufacturers. We therefore consider another warehouse inventory and assembly problem, multi-channel component replenishment problem in an assemble-to-order system, which belongs to an operational level supply chain problem.

The rest of Chapter 1 is organized as follows. Section 1.1 presents the research scope and objective of this thesis. Section 1.2 provides background on the facility location-allocation problem and the component replenishment problem in assemble-to-order systems. The organization of this thesis is given in Section 1.3.

1.1 Research scope and objective

This thesis studies inventory consideration and management in two different supply chain problems: one is the strategic multi-source facility location-allocation and inventory problem; another is the operational multi-channel component replenishment problem in an ATO system.

The specific objectives for studying the multi-source facility location-allocation and inventory problem are:

- To present a multi-source facility location-allocation and inventory problem;
- To formulate the problem as a mixed integer nonlinear programming model;
- To develop an effective solution procedure to solve the proposed model and generate a lower bound for comparison;
- To generate a series of problems to test the performance of proposed solution procedure;
- To apply the proposed model and solving method to a case study.

The specific objectives for studying the multi-channel component replenishment problem in an ATO system are:

- To find some good properties of the dual-channel two-component problem;
- To develop a stochastic programming model for the dual-channel two-component problem on the basis of these properties;
- To solve the dual-channel two-component problem to optimality;

- To extend the properties and model of dual-channel two-component problem to multi-channel multi-component problem;
- To propose an optimal branch-and-bound solution procedure and a heuristic solution procedure to solve the multi-channel multi-component problem;
- To provide some computational studies to demonstrate the efficiency of our solution methods and to compare the performance of assemble-to-order systems with single and dual procurement channels, respectively.

1.2 Background

1.2.1 Facility location-allocation problem

The study of the facility location-allocation problem has a relatively long history. Cooper (1963) presented the basic facility location-allocation problem, which is to decide locations of warehouses and allocations of customer demand given the locations and demand of customers. He described a heuristic method to solve certain classes of facility location-allocation problem. Since then, the problem has received a great deal of attention from other researchers and it has been analyzed in a number of different ways.

Although a large number of facility location-allocation problem extensions have been studied, a limitation of most existing literature on plant/warehouse location-allocation problem is that customer demand is usually assumed to be deterministic, warehouse/distribution center (DC) is assumed to be sourced by single plant, and therefore a linear warehouse/DC inventory holding cost is adopted; or warehouse/DC inventory holding cost is totally neglected. Although this simple way of modeling inventory holding cost has sharply reduced the complexity of the modeling of the facility location-allocation problem, the usefulness of these models may be questioned, especially in real-world applications. Therefore, there is a need to study multi-source facility location-allocation and inventory problem.

1.2.2 Component replenishment problem in assemble-to-order systems

With rapid development of global supply chain management in recent decades, production outsourcing has been widely adopted by many companies in the western countries. These companies outsource their production to assemble-to-order (ATO) contract manufacturers to achieve lower total manufacturing and distribution cost. In order to win production contracts from their clients, the manufacturers must be competitive in offering both low costs and short delivery times. However, to achieve such competitiveness is challenging. On the one hand, their clients often delay their confirmation of order quantities to allow themselves to mitigate market uncertainties. On the other hand, the long lead times for acquiring some components will affect the manufacturer's ability to deliver the final products in a timely fashion. Under pressure from competition, many ATO manufacturers in the regions such as China and Singapore would adopt the strategy of keeping an appropriate amount of the required components in stock before their demands are confirmed in order to gain higher profit through quicker response in delivering the final product, while at the same time trying to minimize the obsolescence costs of excess components. The component replenishment problem in an ATO system is motivated from this business situation and we consider the multi-channel component replenishment problem in an ATO system.

1.3 Organization of thesis

The thesis consists of six chapters. The rest of this thesis is organized as follows.

Chapter 2 introduces relevant works on the facility location-allocation problem and the component replenishment problem in ATO systems.

In Chapter 3, a multi-source facility location-allocation and inventory problem is described and a mixed integer nonlinear programming model is developed to formulate this problem. A heuristic method is then presented to solve the proposed model and a series of problems are generates to test the performance (in comparison with a lower bound generated) of proposed heuristic procedure.

Chapter 4 presents a dual-channel two-component replenishment problem in an ATO system. Some good properties and a stochastic programming model are developed. A closed-form optimal solution is also presented.

Chapter 5 extends the study of dual-channel two-component problem to multi-channel multi-component problem. An optimal branch-and-bound solution procedure and a heuristic solution procedure are developed. Computational studies are also provided.

The final chapter, Chapter 6, concludes this thesis and presents several directions for future research.

Chapter 2 LITERATURE REVIEW

In this chapter, detailed reviews on the facility location-allocation problem and the assemble-to-order problem are presented.

2.1 Facility location-allocation problem

Facility location-allocation problem is reviewed in terms of four categories in this section. They are continuous facility location-allocation problem, discrete facility location-allocation problem, multi-objective facility location-allocation problem and joint facility location-allocation and inventory problem. Literature reviews on facility location-allocation problem can also be found in Drezner (1995), Hamacher and Nickel (1998), Owen and Daskin (1998), Drezner and Hamacher (2002), Klose and Drexl (2005) and Shen (2007), etc.

2.1.1 Continuous facility location-allocation problem

According to the solution space of the sites of facilities, the facility location-allocation problem can be divided into two parts. If the solution space of the sites of facilities is continuous, that is, it is feasible to locate facilities on every point in the plane, the problem is called continuous facility location-allocation problem; if the solution space of the sites of facilities is restricted to some potential locations, the problem is called discrete facility location-allocation problem.

Cooper (1963) firstly presented a continuous facility location-allocation problem and he then described several heuristic methods to solve the continuous facility location-allocation problem in a later study (Cooper, 1964). Since then, the continuous facility location-allocation problem has received a great deal of attention from other researchers and it has been analyzed in a number of different ways. Drezner and Wesolowsky (1978) developed a trajectory optimization method for a continuous multi-facility location-allocation problem. Drezner (1984) introduced a minisum algorithm and a minimax algorithm for a two-median and a two-center facility location-allocation problems respectively. Bhaskaran and Turnquist (1990) studied a multi-facility location-allocation problem incorporating multiple objectives. Brandeau (1992) characterized the trajectory of a stochastic queue median location problem in a planar region. Rosing (1992) presented an optimal method for solving the generalized multi-Weber problem. Hamacher and Nickel (1994) provided several combinatorial algorithms for some single facility median problems. Klamroth (2001) considered a problem of locating one new facility in the plane with respect to a given set of existing facilities where a set of polyhedral barriers restricts traveling, and he provided an exact algorithm and a heuristic solution procedure to solve the problem. Hsieh and Tien (2004) studied a continuous facility location-allocation problem incorporating rectilinear distances and they provided a solution method based on Kohonen self-organizing feature maps. Jiang and Yuan (2008) presented a variational inequality approach to solve a constrained multi-source Weber problem. Wen and Iwamura (2008) studied a fuzzy facility location-allocation problem, which can accommodate satisfactorily various customer demands.

2.1.2 Discrete facility location-allocation problem

Wesolowsky and Truscott (1975) studied a discrete facility location-allocation problem incorporating multiple periods and relocation of facilities. Erlenkotter (1977) incorporated price-sensitive demands in a discrete facility location-allocation problem. Beasley (1993) presented a framework for developing Lagrangean heuristic method for discrete facility location-allocation problems. Revelle (1993) studied integer-friendly programming for discrete facility location-allocation problems. Chandra and Fisher (1994), Dogan and Goetschalckx (1999) and Jayaraman and Pirkul (2001) considered coordination of discrete facility location-allocation problems and production problems. Revelle and Laporte (1996) presented several extensions of the general discrete facility location-allocation problem: with different objectives, with multiple products and multiple machines in which new models of production are considered, and with spatial interactions. Ross (2000) incorporated some operationally-based decisions in a discrete facility location-allocation problem. Amiri (2006) and Ravi and Sinha (2006) studied integrated logistic problems that combine facility location-allocation problems and transport network design problems. Aboolian et al. (2007) studied a competitive facility location-allocation problem where the facilities compete for customer demand with pre-existing competitive facilities and with each other. Averbakh et al. (2007) incorporated demand-dependent setup and service costs in a discrete facility location-allocation problem. Marin (2007) studied a facility location-allocation problem incorporating both plant location and warehouse location. Melachrinoudis and Min (2007) considered a warehouse network redesigning problem. Sankaran (2007) studied a discrete facility location-allocation problem considering large instances.

2.1.3 Multi-objective facility location-allocation problem

It is important to study the facility location-allocation problem from a multi-objective perspective as decision makers in the real-world often consider multiple objectives simultaneously. However, there are a few studies considering multiple objectives in the facility location-allocation problem. Reviews of these studies are given below.

Lee and Franz (1979) studied a facility location-allocation problem with the consideration of multiple conflicting goals and they proposed a branch and bound integer goal programming approach to solve their problem. Lee et al. (1981) presented a model with multiple conflicting objectives for facility location-allocation problem and they considered a single product in a two-echelon system (plant and distribution center). Fortenberry and Mitra (1986) developed a facility location-allocation model with weighted objective function. However, it is hard to assign weights for different qualitative and quantitative factors that are considered in their model. Current et al. (1990) asserted that the objectives of facility location-allocation problem can be classified into four broad categories: cost minimization, demand coverage and assignment, profit maximization and environment concerns. Bhaskaran and Turnquist (1990) studied how to locate multiple facilities in the continental U.S. with simultaneous consideration of transportation cost and customer coverage, and they achieved some “trade-off” solutions. However, their solutions are based on an empirical study on the continuous set location problem. Pappis and Karacapilidis (1994) presented a decision support system to solve the facility location-allocation problem with both cost and service level considerations. The service level in their model was defined as the distance limit between supplying centers and customers. Revelle and Laporte (1996) proposed a two-objective facility location-allocation decision model: one objective is to minimize total cost of transportation and manufacturing, and the other is to maximize demand that can be fulfilled by shipment within 24 hours. However, they did not provide any method for solving their problem. Sabri and Beamon (2000) developed a multi-objective supply chain model that integrates decisions on facility location-allocation, customer service level and flexibility. They used two sub-models (strategic level sub-model and operational level sub-model) and “strategic-operational optimization solution algorithm” to find their solution. Fernandez and Puerto (2003) considered a general multi-objective uncapacitated plant location problem. They presented both exact and approximation methods to obtain non-dominated solutions. Caballero et al. (2007) presented a multi-objective facility location-allocation-routing problem and they developed a multi-objective metaheuristic solution procedure. The objectives in their study include economic objectives (start-up, maintenance, and transportation costs) and social objectives (social rejection by towns on the truck routes, maximum risk as an equity criterion, and the negative implications for towns close to the plant).

According to Klose and Drexl (2005), although a large number of facility location-allocation problem extensions have been studied, there are still fewer studies on the multi-objective facility location-allocation problem. In our study, we consider three objectives and we set minimizing the expected total cost as the main objective and convert the other two objectives to constraints.

2.1.4 Joint facility location-allocation and inventory problem

A limitation of most existing studies on facility location-allocation problem is that customer demand is usually assumed to be deterministic and therefore a linear inventory holding cost is adopted; or inventory holding cost is totally neglected. Without consideration of customer demand uncertainty and warehouse/distribution center (DC) inventory policy, those models usually lead to sub-optimality in terms of total cost/profit. According to Ballou (2001), there appears to be no standard way to handle the "inventory consolidation effect" in location analysis and uncertainty of customer demand in a location problem is rarely a consideration in model building. However, this situation has changed in recent years, and there are increasing studies considering stochastic demand and incorporating inventory policy into the facility location-allocation problem.

Ballou (1984) developed a large-scale computer model "DISPLAN" which considers nonlinear inventory holding cost in plant/warehouse location problem, and he presented a heuristic procedure that uses the three-dimensional transportation algorithm of linear programming in an iterative fashion. However, the solution quality of the heuristic procedure is not shown in Ballou's study. Sabri and Beamon (2000) incorporated customer demand uncertainty and inventory policy in the facility location-allocation model. Although a small-scale example was provided in the numerical study, their model may not be applicable for large-scale real-world problems. Erlebacher and Meller (2000) developed a model for a joint facility location-allocation and inventory problem and their model is only applicable for continuous customer locations approximation and continuous-review inventory policy. Teo et al. (2001) incorporated inventory cost in the “location-inventory” model, in which they focused on consolidation effect on inventory cost but ignored transportation cost. Daskin et al. (2002) studied a distribution center location model that incorporates working inventory and safety stock inventory costs at the distribution centers. However, their model is only applicable for the case that the plant to DC lead time is the same for all plant/DC combinations and the demand variance-to-mean ratio at each retailer is identical for all retailers. Shen et al. (2003) studied a joint location-inventory problem involving risk-pooling effect. They solved a set-covering integer-programming model which is restructured from the original mixed integer nonlinear location-allocation model. Their model only considered single supplier and some retailers. Shen and Daskin (2005) and Shu et al. (2005) both extended the work of Shen et al. (2003) by incorporating a customer service element and considering a stochastic version of transportation-inventory network design problem respectively. Miranda and Garrido (2004) incorporated both economic order quantity and safety stock as decision variables in a joint location-allocation and inventory problem. They solved their mixed integer nonlinear programming model using Lagrangian relaxation and sub-gradient method. Their study is only applicable for the (order point, order quantity) inventory policy. They later extended their study by incorporating optimization of service level using a sequential heuristic approach (Miranda and Garrido, 2009). Teo and Shu (2004) studied a joint facility location-allocation and inventory problem which incorporates infinite horizon multi-echelon inventory cost function. They formulated the problem as a set-partitioning integer programming model and solved it using column generation. However, their approach may not be efficient for large-scale real-world problems. Gabor and Ommeren (2006) proposed a “2-approximation” method for a facility location-allocation problem that incorporated stochastic demand, which is only applicable for simple two-echelon problems. Shen and Qi (2007) incorporated routing cost in the joint location-allocation and inventory problem, while Ambrosino and Scutella (2005) and Javid and Azad (2009) considered routing decisions in the joint location-allocation and inventory problem. Snyder et al. (2007) presented a stochastic location model with risk pooling under random parameters described by discrete scenarios, in which the objective is to minimize the expected total cost across all scenarios. Wang et al. (2007) studied a joint location-allocation and inventory problem incorporating reverse logistics, which was applied to B2C e-markets of China. Miranda and Garrido (2008) and Ozsen et al. (2008, 2009) studied joint location-allocation and inventory problem incorporating warehouse capacity constraint, while Mak and Shen (2009) considered both limited manufacturing processing capacity and storage capacity in a joint location-inventory problem. Hinojosa et al. (2008) and Gebennini et al. (2009) studied the dynamic version of the facility location-allocation and inventory problem.

However, none of the above mentioned location-inventory studies considered multiple sources of warehouse/DC in their models. Multiple sources of warehouse/DC are not uncommon in real-world applications. An example is encountered by the SOLUTIA (Singapore) company. SOLUTIA is a multinational applied chemistry company which produces a homogeneous product (Laminated Glass Interlayer) with different forms at several plants. Currently, the warehouses they leased in Asia-Pacific are mainly initiated by customers. They want to adopt distribution network optimization strategy to choose some third-party logistics service providers' warehouses from many potential locations so as to meet Asia-Pacific customers' demand. Due to the characteristics of the product, the inventory holding cost of the product is relatively high. The inventory holding cost therefore plays an important role in the warehouse location and customer allocation decisions. The chosen warehouses may be replenished by several plants due to the capabilities and capacities of plants. Therefore, it would be necessary and interesting to take into account multiple sources of warehouse/DC in joint facility location-allocation and inventory problem.

2.2 Assemble-to-order (ATO) problem

The study on ATO systems has attracted immense interests in recent decades. Song and Zipkin (2003) provided an excellent and detailed review on a wide variety of ATO models and applications. They classified most ATO research works into three main categories: one-period models, multi-period discrete time models and continuous-time models. In the following, we review some literature in recent years according to the above mentioned three categories. For those literature published before 2003, authors are referred to Song and Zipkin (2003).

2.2.1 One-period models

One-period model is mainly applicable to two situations: the products assembled have short market life or each time period in the system can be treated in isolation. Hsu et al. (2006, 2007) considered an optimal component stocking problem for an ATO system in which both the price for final product and the costs of components depend on their delivery lead times. Fu et al. (2006) studied an inventory and production planning problem for an ATO system with limited assembly capacity. Fang et al. (2008) also studied an ATO system with time-dependent pricing in a decentralized setting. Their focus is on the contractual arrangement between the assembler and the component suppliers. Zhang et al. (2008) examined an ATO system involving coordination of stocking decisions for two components which are used in two different configurations of a product. The components can either be produced internally or procured from external suppliers. Shao and Ji (2009) addressed pricing and coordination decisions in an ATO system with two substitutable products, three components and price-sensitive demand. They study both centralized and decentralized decision-making mechanisms. Fu et al. (2009) proposed an optimal component acquisition problem for an ATO system with the consideration of expediting the procurement of components. Xiao et al. (2010) also considered emergency replenishment of components in an ATO System.

They consider uncertain assembly capacity and assembly-in-advance operations.

2.2.2 Multi-period discrete-time models

The studies on multi-period discrete-time ATO systems are limited in recent years. Akcay and Xu (2004) studied a periodic-review ATO system in which they jointly considered inventory replenishment problem and component allocation problem. They developed a two-stage stochastic integer program and proposed an order-based component allocation heuristic, and they used both sample average approximation method and equal fractile heuristics to determine the optimal base-stock levels. Bollapragada et al. (2004) studied an ATO system with uncertain supply capacity and uncertain demand, and they proposed a decomposition approach to solve industrial-sized assembly problems. Mohebbi and Choobineh (2005) provided an extensive simulation study of an ATO system with a two-level bill-of-material and they studied the combined effects of component commonality, demand uncertainty and supply uncertainty on the system's performance. Louly and Dolgui (2009) developed an inventory control model for an ATO system with random component procurement lead times and they developed a branch and bound algorithm to calculate component safety stock.

2.2.3 Continuous-time models

In contrast to few studies on multi-period discrete-time models in recent years, there are quite a lot of studies on continuous-time models. Dayanik et al. (2003) developed several computationally efficient performance estimates for an ATO system incorporating capacitated production and partial order service. Lu et al. (2003) studied an ATO system incorporating stochastic lead times for component replenishment. Betts and Johnston (2005) considered just-in-time component replenishment decisions for an ATO system under stochastic demand and limited capital investment. Lu et al. (2005) and Lu (2007) studied approximations for expected number of backorders in ATO systems. Lu and Song (2005) developed a cost-minimization model incorporating order-based backorder costs to determine the optimal base-stock level for each component in an ATO system. Benjaafar and ElHafsi (2006) studied an optimal production control and inventory allocation problem for a single-product ATO system with multiple customer classes. Fu (2006) proposed two approximation methods to evaluate performance measures (e.g. fill rate, average waiting time and average number of backorders) in a capacitated continuous time ATO system. Plambeck and Ward (2006, 2008) studied optimal control of product prices, component production capacities and policy of sequencing customer orders for assembly for an ATO system with a high volume of prospective customers' orders arriving per unit time. Zhao and Simchi-Levi (2006) considered an ATO system with stochastic sequential component replenishment lead times. They studied performance analysis and evaluation of such an ATO system under two different component inventory control policies: continuous-time base-stock policy and continuous-time batch-ordering policy. Plambeck and Ward (2007) identified a separation principle for a class of ATO systems, in which they considered three controls: sequencing orders for assembly, component production planning and component expediting. ElHafsi et al. (2008) studied an ATO system with “nested-product” (product *i* has only one additional component more than product *i*-1). Feng et al. (2008) studied an optimal component production and product pricing problem in an ATO system. Lu (2008) studied performance analysis of an ATO system where demand and replenishment lead time follows renewal process and general distribution respectively. Plambeck (2008) examined an ATO system with capacitated component production and fixed transport costs. DeCroix et al. (2009) considered an ATO system incorporating component returns. ElHafsi (2009) considered an integrated production and inventory problem in an ATO system with multiple demand classes where customer orders arrive according to a compound Poisson process. Song and Zhao (2009) studied the value of component commonality in an ATO system with positive lead times. Zhao (2009) studied an ATO system considering demand that follows compound Poisson processes and continuous-time batch ordering policies of components. Lu et al. (2010) considered no-holdback allocation rules in continuous-time ATO systems.

Among studies of ATO systems in the literature, recent works by Hsu et al. (2006) and Fu et al. (2009) are closely related to the problems investigated in this study. The ATO problems considered in these two papers include time-sensitive pricing for the final product which can be delivered in partial quantities of the entire order; both also allow for the replenishment of each component through a single procurement channel after the demand is realized. The difference is that Hsu at al. (2006) requires that each component can only be replenished through a single “regular” purchase channel, i.e., the unit price for replenishing a component is the same as the regular price paid before demand realization. Fu et al. (2009) assume that the replenishment for each component can only be obtained through a single “expediting” channel and the expediting price is equal to or greater than the regular price.

The single-channel settings of Hsu at al. (2006) and Fu et al. (2009) may be justifiable in situations where only a single replenishment price is available; for example, the urgency of meeting a demand can be handled only through the expedition of component acquisition. However, the restriction on a single replenishment channel for every component does limit their applicability in reality. For example, in many applications we observed in practice, a manufacturer may be able to purchase a component from different vendors who offer differentiated prices and supply lead times. He may even pay different prices for a component delivered from a single supplier using different shipping modes. Thus, the manufacturer would be very interested in understanding his optimal component pre-stocking decisions in a multiple replenishment channels ATO system; for example, when he faces a dual-channel system which consists of a regular channel and an expediting channel.

Our study extends the works of Hsu at al. (2006) and Fu et al. (2009) to include a more realistic environment in which each component can be replenished through multiple acquisition channels, each offering a unique combination of unit price and lead time. The most general version of our problem allows any arbitrary fixed number of replenishment channels for each component. It turns out that this assumption makes our model much more challenging to formulate and to solve. The approach developed in Hsu at al. (2006) and Fu et al. (2009) cannot be modified and applied to our problem.

We have reviewed existing studies on the facility location-allocation problem and the assemble-to-order problem in this chapter. In the following three chapters, we study the two problems we reviewed. In the next chapter, we describe a multi-source facility location-allocation and inventory problem and develop a mixed integer nonlinear programming model to formulate this problem. A heuristic method is presented to solve the proposed model and a series of problems are generates to test the performance (in comparison with a lower bound generated) of proposed heuristic procedure.

Chapter 3 MULTI-SOURCE FACILITY LOCATION-ALLOCATION AND INVENTORY PROBLEM

3.1 Problem description

We study a multi-source facility location-allocation and inventory problem, which is motivated by a research project with SOLUTIA (Singapore). As mentioned, SOLUTIA is a multinational applied chemistry company which produces a homogeneous product (Laminated Glass Interlayer) with different forms at several plants. Currently, the warehouses they leased in Asia-Pacific are mainly initiated by customers. They want to adopt distribution network optimization strategy to choose some third-party logistics service providers' warehouses from many potential locations so as to meet Asia-Pacific customers' demand. Their objective is to minimize the expected total cost while keeping certain customer service level. Due to the characteristics of the product, the inventory holding cost of the product is relatively high. The inventory holding cost therefore plays an important role in the warehouse location and customer allocation decisions. The chosen warehouses may be replenished by several plants due to the capabilities and capacities of plants.

In this problem, we consider several plants, some potential warehouses, a set of customers and multiple types of products. A warehouse can be replenished by several plants together because of limited capabilities and capacities of plants. The demand of customers is stochastic. Customers can be either served by warehouse or replenished by plant directly. The problem is described as follows. Given distribution of customers' demand, capacities of plants, and locations of plants, customers and potential warehouses, the problem is to determine where to locate warehouses, how to allocate customers to warehouses or plants, and how much inventory should be held in each warehouse. The objective is to minimize the expected total cost with the satisfaction of desired demand weighted average customer lead time and desired cycle service level. The satisfaction of desired demand weighted average customer lead time is motivated by the fact that the company wants to make sure that the strategic customers (with big demand) have short lead time and other customers can have relatively long lead time. The total cost includes transportation cost (transportation cost from plants to warehouses, transportation cost from plants to customers and transportation cost from warehouses to customers), fixed cost of warehouses, and inventory holding cost (inventory consists of working inventory and safety stock) of warehouses. The proposed problem is formulated by a mixed integer nonlinear programming model. Utilizing approximation and transformation techniques, we develop an iterative heuristic method for the problem. An experiment study shows that the proposed procedure performs well in comparison with a lower bound. We also present an example study from a research project.

3.2 Model development

The problem we described is a multi-source facility location-allocation and inventory problem. In order to formulate the proposed problem, a mixed integer nonlinear programming model is developed.

3.2.1 Modeling assumptions

The following assumptions are necessary in developing the mathematical formulation for the problem:

(1) Customer demand follows independent normal distribution with known mean and standard deviation;

(2) Each customer is directly served by only one facility (warehouse or plant);

(3) Transportation cost is proportional to shipment amount;

(4) No capacity is considered for warehouse but there is capacity of plant;

(5) Periodic review, order-up-to-level (r, S) inventory policy is adopted for each warehouse, and review period is known for each warehouse;

(6) All warehouses use the same cycle service level.

The reason that we do not consider capacity constraints at the warehouses is motivated by the real problem faced by SOLUTIA (Singapore). All warehouses used by the company are rented from third-party logistics service providers, thus the capacities of warehouses can be easily extended. Note that including the warehouse capacity constraints will not make the problem much more difficult. One may observe that the solution procedure presented in the thesis can still be used for the case that considers the warehouse capacity constraints.

3.2.2 Notations

Notations we adopted in proposed model are given as follows:

Indices

i Plant index, $i = 1, 2, \ldots, I$;

j Potential warehouse index, $j = 1, 2, \ldots, J$;

k Customer index, $k = 1, 2, \ldots, K$;

f Product type index, $f = 1, 2, \ldots, F$.

Parameters

p_{if} Annual amount of product type f produced at plant i;

d_{kf} Mean annual demand of product type f at customer k;

σ_{kf} Standard deviation of annual demand of product type f at customer k;

cpw_{ijf} Unit transportation cost of product type f from plant i to warehouse j;

cpc_{ikf} Unit transportation cost of product type f from plant i to customer k;

cwc_{jkf} Unit transportation cost of product type f from warehouse j to customer k;

f_j Annual fixed cost for leasing warehouse j;

h_j Unit holding cost per year at warehouse j;

r_j Review period of warehouse j;

tpw_{ij} Replenishment lead time from plant i to warehouse j;

tpc_{ik} Replenishment lead time from plant i to customer k;

twc_{jk} Replenishment lead time from warehouse j to customer k;

dt Desired weighted average customer lead time;

z Desired safety factor (it is the standard normal value corresponding to the desired cycle service level).

Decision variables

X_{ijf} Annual amount of product type *f* shipped from plant *i* to warehouse *j*;

T_k Lead time for customer *k* (depending on the source of shipment);

P_{ik} = 1 if customer *k* is directly served by plant *i*, 0 otherwise;

W_{jk} = 1 if customer *k* is served by warehouse *j*, 0 otherwise;

Z_j = 1 if warehouse *j* is leased, 0 otherwise;

3.2.3 Model formulation

Our proposed model differs from those in previous plant/warehouse location-allocation literature in three main aspects. Firstly, in most existing models, customers are all served by warehouses or distribution centers, while in our model, the customer can be either replenished by single warehouse or served by single plant directly. This difference is mainly due to the fact that some companies lease warehouses from third-party logistics service providers instead of building their own warehouses. In this case, some customers can be directly served by a plant rather than through a warehouse. Secondly, we take into account a constraint for desired demand weighted average customer lead time so as to make sure that weighted average customer lead time is at an acceptable level. This constraint is motivated by the fact that some companies want to ensure that they can provide short lead times to the strategic customers (with large demand) by opening/hiring nearby warehouses and they can serve those customers with small demand directly by plants without going through warehouses (which means relatively long lead time). Thirdly, in most existing models, customer demand is assumed to be deterministic and warehouse/DC is assumed to be sourced by single plant and therefore a linear inventory holding cost is adopted; or inventory holding cost is totally neglected. In this study, we incorporate stochastic customer demand and periodic review, order-up-to-level (r, S) inventory policy into the facility location-allocation problem and we consider multiple sources for each warehouse.

As we consider multiple sources (plants) for each warehouse because of limited capabilities and capacities of sources, the products ordered by a specific warehouse may come from several different sources (with different lead times). Also, for a given warehouse, the proportion of quantity ordered from each source may vary from one order period to another. This complicates the determination of an appropriate safety stock level. Our idea is to provide an approximation of safety stock level that is simple while taking into accounts the lead times and order quantities from each source. Here, we treat multiple sources as a single source and use an order quantity weighted lead time in the computation of safety stock level. The approximated safety stock level of product type f at warehouse j is given by

$$SS_{jf} = \begin{cases} z\sqrt{\sum_k (\sigma_{kf}^2 W_{jk})}\sqrt{r_j + \dfrac{\sum_i (tpw_{ij} X_{ijf})}{\sum_i X_{ijf}}} & \text{if } \sum_i X_{ijf} > 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{for all } j \text{ and } f \qquad (3.1)$$

In order to use the proposed safety stock in our model formulation, we replace the formula (3.1) by its approximation.

$$SS_{jf} \approx z\sqrt{\sum_k(\sigma_{kf}^2 W_{jk})}\sqrt{r_j + \frac{\sum_i(tpw_{ij}X_{ijf})}{\varepsilon + \sum_i X_{ijf}}} \quad \text{for all } j \text{ and } f \tag{3.2}$$

where ε is a very small positive value. For the case that $\sum_i X_{ijf} > 0$, it is easy to see that (3.2) approximates SS_{jf}. On the other hand, if $\sum_i X_{ijf} = 0$, then $\sum_k(\sigma_{kf}^2 W_{jk}) = 0$. Thus, $SS_{jf} = 0$. In the Appendix A, we demonstrate that the proposed safety stock level is quite reasonable regardless of how an actual ordering policy is implemented.

Recall that our objective is to minimize the expected annual total cost with the satisfaction of desired weighted average customer lead time and desired cycle service level. Our problem therefore can be formulated as a mixed integer nonlinear programming model P as follows.

Model *P*:

$$\begin{aligned}
\min TC &= \sum_{ijf}(cpw_{ijf}X_{ijf}) + \sum_{ikf}(cpc_{ikf}d_{kf}P_{ik}) + \sum_{jkf}(cwc_{jkf}d_{kf}W_{jk}) + \sum_j f_j Z_j \\
&\quad + \sum_j\left[h_j \sum_f\left(\frac{r_j\sum_k(d_{kf}W_{jk})}{2} + z\sqrt{\sum_k(\sigma_{kf}^2 W_{jk})}\sqrt{r_j + \frac{\sum_i(tpw_{ij}X_{ijf})}{\varepsilon + \sum_i X_{ijf}}}\right)\right] \\
&= \sum_{ijf}(cpw_{ijf}X_{ijf}) + \sum_{ikf}(cpc_{ikf}d_{kf}P_{ik}) + \sum_{jkf}\left[\left(cwc_{jkf} + \frac{h_j r_j}{2}\right)d_{kf}W_{jk}\right] + \sum_j f_j Z_j \\
&\quad + \sum_{jf}\left(h_j z\sqrt{\sum_k(\sigma_{kf}^2 W_{jk})}\sqrt{r_j + \frac{\sum_i(tpw_{ij}X_{ijf})}{\varepsilon + \sum_i X_{ijf}}}\right)
\end{aligned}$$

Subject to

$$\frac{\sum_{kf}(d_{kf}T_k)}{\sum_{kf}d_{kf}} \le dt \tag{3.3}$$

$$\sum_j X_{ijf} + \sum_k(d_{kf}P_{ik}) \le p_{if} \quad \text{for all } i \text{ and } f \tag{3.4}$$

$$\sum_i X_{ijf} = \sum_k (d_{kf} W_{jk}) \quad \text{for all } j \text{ and } f \tag{3.5}$$

$$\sum_i P_{ik} + \sum_j W_{jk} = 1 \quad \text{for all } k \tag{3.6}$$

$$W_{jk} \le Z_j \quad \text{for all } j \text{ and } k \tag{3.7}$$

$$T_k \ge \sum_j (twc_{jk} W_{jk}) \quad \text{for all } k \tag{3.8}$$

$$T_k \ge \sum_i (tpc_{ik} P_{ik}) \quad \text{for all } k \tag{3.9}$$

$$P_{ik} = \{0,1\} \quad \text{for all } i \text{ and } k \tag{3.10}$$

$$W_{jk} = \{0,1\} \quad \text{for all } j \text{ and } k \tag{3.11}$$

$$Z_j = \{0,1\} \quad \text{for all } j \tag{3.12}$$

$$X_{ijf} \ge 0 \quad \text{for all } i, j \text{ and } f \tag{3.13}$$

$$T_k \ge 0 \quad \text{for all } k \tag{3.14}$$

The objective function shows the expected annual total cost to be minimized. The first two terms represent the transportation costs from plants to warehouses and from plants to customers respectively. The third term includes transportation costs from warehouses to customers and cycle stock holding costs of warehouses. The fourth term denotes the fixed costs of warehouses and the last term represents safety stock holding costs of warehouses. Note that we intentionally exclude inventory holding costs at the plants because SOLUTIA (Singapore) is not accountable for the costs.

Constraint (3.3) assures that the demand weighted average customer lead time is at a satisfactory level. Constraint (3.4) assures that the capacity for each product type at each plant cannot be violated. Constraint (3.5) ensures the flow conservation at each warehouse. Constraint (3.6) states that each customer must be served by either a plant or a warehouse. Constraint (3.7) ensures that customers can be served by a warehouse only when it is opened. Constraints (3.8) and (3.9) determine the lead time for each customer. Constraints (3.10), (3.11) and (3.12) are binary constraints and constraints (3.13) and (3.14) are non-negativity constraints.

To simplify the notation, define:

$$LTC = \sum_{ijf}(cpw_{ijf}X_{ijf}) + \sum_{ikf}(cpc_{ikf}d_{kf}P_{ik}) + \sum_{jkf}\left[\left(cwc_{jkf} + \frac{h_j r_j}{2}\right)d_{kf}W_{jk}\right] + \sum_j f_j Z_j$$

and

$$V_{jf} = \left(h_j z\right)^2 \sum_k (\sigma_{kf}^2 W_{jk}) \qquad \text{for all } j \text{ and } f.$$

These two notations will replace all linear terms in objective function *TC*.

3.3 Heuristic method for solving *P*

The difficulty for solving the model *P* comes from the complicated nonlinearity term $\sqrt{V_{jf}}\sqrt{r_j + \sum_i (tpw_{ij}X_{ijf})\big/\left(\varepsilon + \sum_i X_{ijf}\right)}$ in its objective function *TC*. Let

$$ST_{jf} = \sqrt{r_j + \sum_i (tpw_{ij}X_{ijf})/(\varepsilon + \sum_i X_{ijf})} \quad \text{for all } j \text{ and } f \tag{3.15}$$

denote the square root of the sum of review period and weighted lead time for product type *f* at warehouse *j*. Suppose that we replace the whole term ST_{jf} in objective function *TC* by a given value α_{jf} and add the constraints $ST_{jf} = \alpha_{jf}$ for all *j* and *f*. For all *j* and *f*, observe that the constraint $ST_{jf} = \sqrt{r_j + \sum_i (tpw_{ij}X_{ijf})\big/\left(\varepsilon + \sum_i X_{ijf}\right)} = \alpha_{jf}$ can be converted to a linear constraint $r_j\left(\varepsilon + \sum_i X_{ijf}\right) + \sum_i (tpw_{ij}X_{ijf}) = \alpha_{jf}^2\left(\varepsilon + \sum_i X_{ijf}\right)$. Let $\boldsymbol{\alpha}$ be the matrix of α_{jf}'s. For given values of $\boldsymbol{\alpha}$, the above mentioned model is denoted as model $PA(\boldsymbol{\alpha})$.

Model $PA(\alpha)$:

Min $\quad TCA(\boldsymbol{\alpha}) = LTC + \sum_{jf} \alpha_{jf} \sqrt{V_{jf}}$

Subject to

Constraints (3.3) – (3.14)

$$r_j\left(\varepsilon + \sum_i X_{ijf}\right) + \sum_i (tpw_{ij} X_{ijf}) = \alpha_{jf}^2 \left(\varepsilon + \sum_i X_{ijf}\right) \qquad \text{for all } j \text{ and } f \tag{3.16}$$

$$\begin{aligned} LTC = & \sum_{ijf} (cpw_{ijf} X_{ijf}) + \sum_{ikf} (cpc_{ikf} d_{kf} P_{ik}) \\ & + \sum_{jkf} \left[\left(cwc_{jkf} + \frac{h_j r_j}{2} \right) d_{kf} W_{jk} \right] + \sum_j f_j Z_j \end{aligned} \tag{3.17}$$

$$V_{jf} = \left(h_j z\right)^2 \sum_k (\sigma_{kf}^2 W_{jk}) \qquad \text{for all } j \text{ and } f \tag{3.18}$$

Note that all constraints of model $PA(\alpha)$ are linear constraints. If we were able to solve the model $PA(\alpha)$ for all possible values of α, then the best solution obtained would be an optimal solution to the model P. However, it is extremely time-consuming to try all the values of α because the number of combinations of α_{jf}'s can be very large even when we discretize the range for α. Moreover, for a given α, the model $PA(\alpha)$ might be infeasible because of the equality constraint (3.16). Note that even if we know the set of all feasible values of α, there is no easy way to determine the optimal value of α because of nonlinearity of the objective function. With these reasons, we will propose a heuristic method to find a solution for the original model P. For comparison purpose, we generate a lower bound of P (see Section 3.4). We also report the computational results that highlight effectiveness of proposed solution approach (see Section 3.5).

Our idea is how to find a feasible value of α which will result in a good objective value. As mentioned earlier, given the value of α, the model $PA(\alpha)$ may be infeasible due to the equality constraint (3.16). However, it can be observed that, given a feasible value of X_{ijf}'s, a corresponding feasible value of α can be easily determined using equation (3.15). Our approach is to relax the equality constraint (3.16) so that the relaxation of model $PA(\alpha)$ will always be feasible for any given value of α. Although constraint (3.16) does not hold for the solution obtained from the relaxed problem, we can use this solution to obtain another feasible value of α. Then, we can repeatedly solve the relaxation of model $PA(\alpha)$ using the newly obtained value of α. If the solutions converge (i.e., new α is the same as old α), it can be easily seen that constraint (3.16). holds (i.e., also feasible to the original model $PA(\alpha)$). Unfortunately, it is very likely that the solutions for the relaxation of model $PA(\alpha)$ do not converge. With this reason, we will present an approach that will force its convergence.

Let L_{jf} and U_{jf}, respectively, be the given lower and upper limits for ST_{jf}. That is, we will replace (3.16) with the constraints $L_{jf} \le ST_{jf} \le U_{jf}$ for all j and f. Observe that the constraints $L_{jf} \le ST_{jf} = \sqrt{r_j + \sum_i (tpw_{ij} X_{ijf}) / (\varepsilon + \sum_i X_{ijf})} \le U_{jf}$ can be converted to linear constraints $L_{jf}^2 (\varepsilon + \sum_i X_{ijf}) \le r_j (\varepsilon + \sum_i X_{ijf}) + \sum_i (tpw_{ij} X_{ijf}) \le U_{jf}^2 (\varepsilon + \sum_i X_{ijf})$.

For given values of α, our relaxed problem is denoted as model $PB(\alpha)$.

Model $PB(\alpha)$:

Min $TCB(\boldsymbol{\alpha}) = LTC + \sum_{jf} \alpha_{jf} \sqrt{V_{jf}}$

Subject to

Constraints (3.3) – (3.14) and (3.17) – (3.18)

$$r_j(\varepsilon + \sum_i X_{ijf}) + \sum_i (tpw_{ij} X_{ijf}) \geq L_{jf}^2 (\varepsilon + \sum_i X_{ijf}) \qquad \text{for all } j \text{ and } f \qquad (3.19)$$

$$r_j(\varepsilon + \sum_i X_{ijf}) + \sum_i (tpw_{ij} X_{ijf}) \leq U_{jf}^2 (\varepsilon + \sum_i X_{ijf}) \qquad \text{for all } j \text{ and } f \qquad (3.20)$$

The model $PB(\alpha)$ is still a mixed integer nonlinear program because of the terms $\sqrt{V_{jf}}$ in its objective function. It is worth noting that some commercial solvers may be able to solve model $PB(\alpha)$; however, its optimality is not guaranteed. In this thesis, we adopt the standard piecewise linear approximation technique to approximate $\sqrt{V_{jf}}$, which is to replace the term $\sqrt{V_{jf}}$ in $TCB(\alpha)$ by a piecewise linear function of V_{jf}.

After solving the model $PB(\alpha)$, we can obtain location-allocation solution (X_{ijf}'s, P_{ik}'s, W_{jk}'s, and Z_j's) which is also feasible to the model P. We can use the location-allocation solution to calculate the objective function TC of the model P and its corresponding values of ST_{jf}'s (using equation (3.15)); we denote these ST_{jf}'s as β_{jf}'s.

As described earlier, we use an iterative approach (with a good convergence) to find a good solution of model P. Our idea is to iteratively update the values of α, L_{jf} and U_{jf} used in constraints (3.19) and (3.20) and narrow the search space of α until it results in a solution such that $\beta_{jf} = \alpha_{jf}$ for all j and f. It is worth noting that if $\beta_{jf} = \alpha_{jf}$ for all j and f, then the objective value $TCB(\alpha) = TC$. Recall that this solution is not an optimal solution to the original problem. We will next discuss how to update the values of α, L_{jf}'s, U_{jf}'s and search space of α.

3.3.1 Initialization

According to equation (3.15), we can easily compute the lower bound of ST_{jf} by setting X_{ijf} = 0 for all i, j and f. Also we can compute the upper bound of ST_{jf} by setting the weighted lead time $\sum_i (tpw_{ij} X_{ijf}) / (\varepsilon + \sum_i X_{ijf})$ as the largest lead time $\max_i (tpw_{ij})$ for all j and f. Therefore, the lower bound and upper bound of ST_{jf} are $\sqrt{r_j}$ and $\sqrt{r_j + \max_i (tpw_{ij})}$, respectively.

Let $\boldsymbol{\alpha}^L$ and $\boldsymbol{\alpha}^U$ define the lower and upper boundaries of the search space of $\boldsymbol{\alpha}$.respectively. Clearly, we can initially let $\alpha_{jf}^L = \sqrt{r_j}$ and $\alpha_{jf}^U = \sqrt{r_j + \max_i (tpw_{ij})}$. We can also set $L_{jf} = \sqrt{r_j}$ and $U_{jf} = \sqrt{r_j + \max_i (tpw_{ij})}$ as initial values of lower and upper limits used in constraints (3.19) and (3.20) respectively. With these, we can solve model $PB(\boldsymbol{\alpha})$ for $\boldsymbol{\alpha} = \boldsymbol{\alpha}^L$ and $\boldsymbol{\alpha} = \boldsymbol{\alpha}^U$. For all j and f, let β_{jf}^L and β_{jf}^U denote the corresponding values of β_{jf} given that $\boldsymbol{\alpha} = \boldsymbol{\alpha}^L$ and $\boldsymbol{\alpha} = \boldsymbol{\alpha}^U$, respectively. Note that the values of β_{jf}^L and β_{jf}^U are between the lower bound and upper bound of ST_{jf}, and recall that the initial values of α_{jf}^L and α_{jf}^U are lower bound and upper bound of ST_{jf} respectively. Therefore, we know that $\beta_{jf}^L \geq \alpha_{jf}^L$ and $\beta_{jf}^U \leq \alpha_{jf}^U$.

3.3.2 Selecting new α and updating limits

We now discuss how to select the next value of $\boldsymbol{\alpha}$ to explore in the next iteration. The discussion will be based on particular value of j and f; henceforth, we drop subscripts j and f from α_{jf}, β_{jf}, α_{jf}^L, α_{jf}^U, β_{jf}^L, β_{jf}^U, L_{jf} and U_{jf} in the remainder of this section.

Recall that our aim is to force the value of β such that it will eventually be equal to

α. Given that we have found two points (α^L, β^L) and (α^U, β^U) from initialization, we would like to find a new α which is between the α^L and α^U. Our idea is to seek the coordinate (α, β) in the line joining points (α^L, β^L) and (α^U, β^U) such that $\alpha = \beta$. We arbitrarily choose the case $\beta^L \geq \beta^U$ in Figure 3.1 as an example (same results hold for the case $\beta^L < \beta^U$). That is, α is the solution of the following system of equations

$$\alpha = \beta \tag{3.21}$$

$$\beta = \frac{\beta^L - \beta^U}{\alpha^L - \alpha^U}\alpha + \frac{\beta^U \alpha^L - \beta^L \alpha^U}{\alpha^L - \alpha^U} \tag{3.22}$$

Recall that $\beta^L \geq \alpha^L$ and $\beta^U \leq \alpha^U$. Thus, there is a solution (denoted as α^C) to the above system of equations and it is given by

$$\alpha^C = \frac{\alpha^U \beta^L - \alpha^L \beta^U}{\alpha^U - \alpha^L + \beta^L - \beta^U} \tag{3.23}$$

We choose this α^C value as the next value of α to explore.

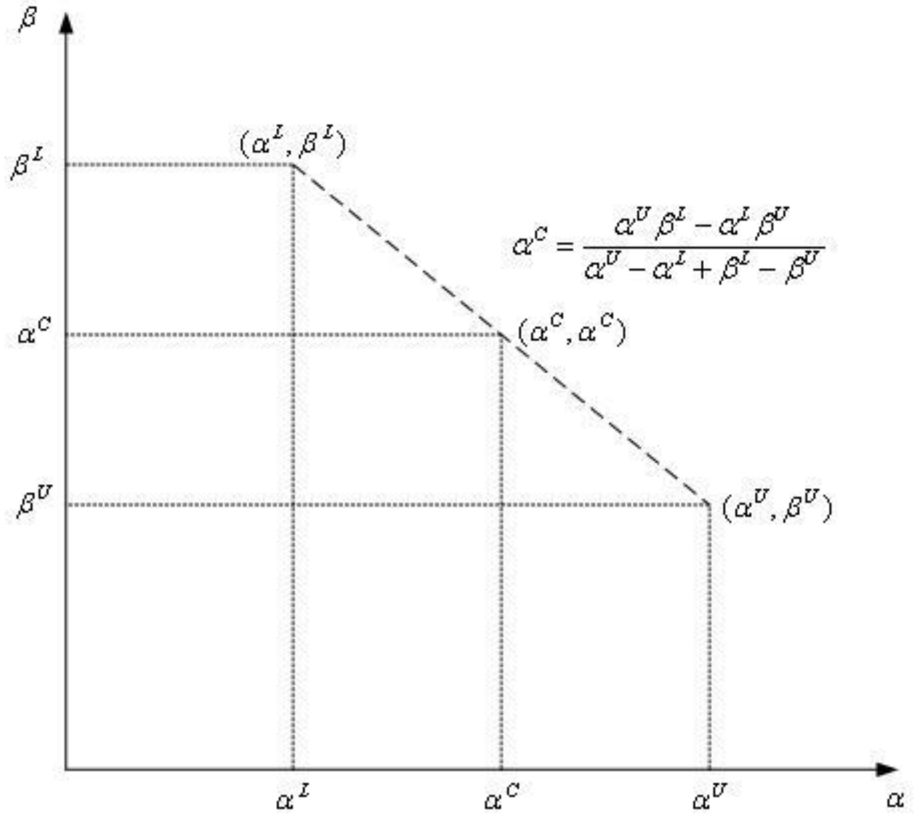

Figure 3.1 Selecting new α

We now discuss how to update lower and upper limits used in constraints (3.19) and (3.20). Let β^C denote the corresponding value of β given that $\boldsymbol{\alpha} = \boldsymbol{\alpha}^C$. Our idea is to ensure that β^C will be between the two boundary points (i.e., between β^L and β^U). Hence, the new lower and upper limits used in solving the model $PB(\boldsymbol{\alpha})$ for $\boldsymbol{\alpha} = \boldsymbol{\alpha}^C$ are given by

$$L = \min\{\beta^L, \beta^U\} \text{ and } U = \max\{\beta^L, \beta^U\}. \tag{3.24}$$

With bounded constraints (3.19) and (3.20) and the way of updating lower and upper limits L and U, the two solutions corresponding to β^L and β^U are feasible for the model $PB(\boldsymbol{\alpha})$ for $\boldsymbol{\alpha} = \boldsymbol{\alpha}^C$.

3.3.3 Updating search space

We now discuss how to update the search space. Suppose that we have found the new point (α^C, β^C). Recall that α^L and α^U are the lower and upper boundaries of the search space of α, respectively. As mentioned, we iteratively narrow the search space of α until it results in a solution such that $\beta = \alpha$. If $\beta^C \geq \alpha^C$, we let $(\alpha^L, \beta^L) = (\alpha^C, \beta^C)$; otherwise, if $\beta^C < \alpha^C$, we let $(\alpha^U, \beta^U) = (\alpha^C, \beta^C)$. The way we set the lower and upper limits in (3.24) will shrink the feasible region of ST_{jf} continuously at each iteration. As a consequence, the gap between α and β will become smaller and smaller; and β will eventually be equal to α. In addition, observe that the way we update the boundary points always guarantees that there exists a solution for the system of equations (3.21) and (3.22). That is, we can always find the next α^C to explore.

3.3.4 Solution procedure

The procedure of proposed heuristic method is given as follows.

Step 1: **(Initialization)** Set n = 1, $\alpha^L = \sqrt{r_j}$, $\alpha^U = \sqrt{r_j + \max_i(tpw_{ij})}$, $L = \sqrt{r_j}$ and $U = \sqrt{r_j + \max_i(tpw_{ij})}$. Determine β^L and β^U using equation (3.15) and the solutions obtained from model $PB(\boldsymbol{\alpha})$ for $\boldsymbol{\alpha} = \boldsymbol{\alpha}^L$ and $\boldsymbol{\alpha} = \boldsymbol{\alpha}^U$.

Step 2: **(Selecting new α and updating limits)** Determine the new value of α to explore using equation (3.23) and set the new lower and upper limits using (3.24). Solve the corresponding model $PB(\boldsymbol{\alpha})$ for $\boldsymbol{\alpha} = \boldsymbol{\alpha}^C$ obtaining β^C.

Step 3: **(Stopping criteria)** If $n = N$ or $|\beta^C - \alpha^C| < \delta$ for all product types and warehouses, then stop. Otherwise, go to Step 4.

Step 4: **(Updating search space)** If $\beta^C \geq \alpha^C$, we let $(\alpha^L, \beta^L) = (\alpha^C, \beta^C)$; otherwise, if $\beta^C < \alpha^C$, we let $(\alpha^U, \beta^U) = (\alpha^C, \beta^C)$. Set $n = n + 1$ and go to Step 2.

In the above procedure, n is the number of iterations, N is the maximum number of iterations allowed, and δ is a small positive value representing allowable tolerance. Note that the value of N depends on how much computing effort can be put into the heuristic procedure, and the value of δ can be chosen according to the values of α^C and β^C. The final solution of the proposed heuristic method is given by location-allocation solution obtained from the last iteration. With this solution, the final objective value of the heuristic is given by TC of the model P. Note that our heuristic method can still be used for the case that considers the warehouse capacity constraints as we still can find two initial solutions using model $PB(\alpha^L)$ and $PB(\alpha^U)$ and we can continuously update α, limits and search space using the same procedure introduced above.

3.4 Lower bound generation

For comparison purpose, we establish a lower bound of the objective value of P. We use the following underestimate function to replace the original safety stock function as follows.

$$\begin{aligned}
SS_{jf} &= z\sqrt{\sum_k(\sigma_{kf}^2 W_{jk})}\sqrt{r_j+\sum_i(tpw_{ij}X_{ijf})/(\varepsilon+\sum_i X_{ijf})} \\
&\geq z\sqrt{\sum_k(\sigma_{kf}^2 W_{jk})}\sqrt{r_j+\min_i(tpw_{ij})\sum_i X_{ijf}\Big/\left(\varepsilon+\sum_i X_{ijf}\right)} \\
&\approx z\sqrt{\sum_k(\sigma_{kf}^2 W_{jk})}\sqrt{r_j+\min_i(tpw_{ij})}
\end{aligned}$$

Note that above underestimate function is also true for the case that $\sum_i X_{ijf}=0$ because $\sum_k(\sigma_{kf}^2 W_{jk})=0$ if $\sum_i X_{ijf}=0$.

Recall that we can adopt the standard piecewise linear approximation technique to approximate $\sqrt{\sum_k(\sigma_{kf}^2 W_{jk})}$, which is to replace the term $\sqrt{\sum_k(\sigma_{kf}^2 W_{jk})}$ by a piecewise linear function of $\sum_k(\sigma_{kf}^2 W_{jk})$. Note that this approximation is also an underestimate function of $\sqrt{\sum_k(\sigma_{kf}^2 W_{jk})}$. Therefore, by means of above underestimated function and the standard piecewise linear underestimate approximation, objective function TC can be changed to an underestimate linear objective function (we denote it as $TCUL$). Therefore, model P can be converted to a mixed integer linear programming model PUL.

Model *PUL*:

Min $TCUL$

Subject to

Constraints (3.3) – (3.14).

A lower bound of the objective value of P can thus be obtained by solving PUL.

3.5 Computational results

In this section, we use the proposed model and method to perform a number of computational studies. We first generate twenty-seven data sets to examine the performance of our proposed method. We then apply our model and method to the example study motivated by our interaction with SOLUTIA Company.

3.5.1 Computational studies

The computational experiments described in this section are designed to evaluate the performance of our proposed method with respect to a series of test problems. In order to show the effectiveness of our proposed model and method, we make the following two comparisons. Firstly, we compare our solution with the solution obtained by simple and normal two-stage procedure (at first stage, the location-allocation solutions are solved while ignoring warehouse inventory holding cost, at second stage, the warehouse inventory holding cost is computed based on known warehouse location and customer allocation). Secondly, we compare our solution with the lower bound which is obtained by the model *PUL* described in Section 3.4.

The heuristic is coded in C++. Both mixed integer linear programming models *PB*(α) and *PUL* are solved using CPLEX 11.0 with Concert Technology. The computer configuration is as follows: processor is Intel (R) Core (TM) 2 Duo CPU E6750 @ 2.66 GHz 2.67 GHz, memory is 4 GB and operating system is 32-bit Microsoft Windows Vista. Due to limited computer memory, both models (*PB*(α) and *PUL*) are written into two MPS files respectively and the code calls CPLEX to read the models from the MPS files.

Twenty-seven problem sets are generated. The number of plants, customers, potential warehouse locations and product types vary from 2 to 6, from 10 to 200, from 2 to 20, and from 2 to 10 respectively. The mean demand requirement of each product type for each customer is drawn from a uniform distribution between 500 and 5000. The standard deviation of demand varies from 10% to 50% of mean demand. The production capacities of plants are drawn from a uniform distribution corresponding to total customer demand requirements. Plants, potential warehouses and customer sites are generated from a uniform distribution over a square with side 1000. The unit transportation cost and lead time have been computed as being proportional to the Euclidean distance among locations of plants and warehouses, plants and customers, and warehouses and customers respectively, and the proportional rate is based on the case in practice. Fixed cost, inventory holding cost rate and review period of warehouses are assumed according to the case in practice. According to real cases, some potential warehouses can only serve certain customers (e.g. warehouse in Japan can only serve customers in Japan, it cannot serve customers in China), we also take into account this restriction in the generating of test problems. Table 3.1 provides relevant parameters for the test problems we generated.

Table 3.1 Parameters for test problems

Parameter	Value
Number of plants	2 4 6
Number of potential warehouses	2 5 10 15 20
Number of customers	10 50 100 150 200
Number of product types	2 5 10
Mean demand	U(500, 5000)[a]
Coefficient of variation of demand	0.1 0.2 0.5
Plant capacity	Corresponding to total customer demand
Lead time	0.0001[b] $\times$ Euclidean distance
Unit transportation cost	0.0004[c] $\times$ Euclidean distance
Fixed cost of warehouse	U(1000, 3000)
Inventory holding cost rate of warehouse	1 2 3 4 5
Review period of warehouse	U(0.05, 0.1)
Very small positive value ε	0.00001
Allowable tolerance δ	0.001
Maximum number of iterations allowed N	20

[a]U(500, 5000): Uniform distribution between 500 and 5000.
[b]0.0001: The proportional rate is computed based on the real case data.
[c]0.0004: The proportional rate is computed based on the real case data.

Table 3.2 presents the gap between our solution and the solution obtained by simple two-stage procedure under different settings of inventory holding cost rate for twenty-seven problem sets. Figure 3.2 shows the trend of the average gap of the twenty-seven problem sets with the increase of inventory holding cost rate of

Table 3.2 Comparison between our solution and two-stage solution under different inventory holding cost rates

#	# of plants	# of potential warehouses	# of customers	# of product types	Gap[a] hcr[b] = 1	hcr = 2	hcr = 3	hcr = 4	hcr = 5
1	2	2	10	2	0.41%	0.98%	1.50%	1.96%	2.24%
2	2	2	10	5	1.76%	4.33%	6.80%	8.97%	10.78%
3	2	2	10	10	1.01%	2.57%	4.47%	6.31%	8.06%
4	2	5	50	2	0.17%	0.45%	0.69%	0.88%	0.98%
5	2	5	50	5	0.26%	0.63%	1.14%	1.68%	2.15%
6	2	5	50	10	1.53%	3.36%	4.78%	5.84%	6.88%
7	2	5	100	2	0.97%	1.87%	2.70%	3.54%	4.16%
8	2	5	100	5	0.52%	1.15%	1.76%	2.32%	2.86%
9	2	5	100	10	0.56%	1.05%	1.54%	2.32%	2.92%
10	2	10	100	2	0.63%	1.20%	1.70%	2.23%	2.60%
11	2	10	100	5	0.43%	1.22%	1.95%	2.61%	3.19%
12	2	10	100	10	1.26%	2.97%	4.27%	5.32%	6.23%
13	2	10	150	2	0.59%	1.27%	1.92%	2.43%	2.84%
14	2	10	150	5	0.60%	1.53%	2.31%	2.93%	3.51%
15	2	10	150	10	0.91%	2.15%	3.31%	4.28%	5.12%
16	4	15	150	2	1.20%	2.76%	4.28%	5.59%	6.81%
17	4	15	150	5	0.59%	2.67%	4.73%	6.49%	8.20%
18	4	15	150	10	1.87%	4.50%	7.00%	9.08%	10.94%
19	4	15	200	2	0.62%	1.55%	2.66%	3.40%	4.05%
20	4	15	200	5	1.29%	5.02%	8.07%	10.48%	12.50%
21	4	15	200	10	1.57%	3.75%	5.72%	7.27%	8.91%
22	6	15	150	2	0.85%	2.34%	3.86%	5.37%	6.66%
23	6	15	150	5	1.66%	4.18%	6.64%	8.90%	11.07%
24	6	15	150	10	2.26%	5.05%	7.82%	10.30%	13.08%
25	6	20	200	2	2.22%	4.53%	6.42%	7.90%	9.68%
26	6	20	200	5	2.28%	5.45%	8.21%	10.51%	12.52%
27	6	20	200	10	1.96%	5.00%	7.78%	9.93%	11.87%

[a]Gap: The gap is the average gap ((two-stage solution – our solution) / two-stage solution * 100%) under different cycle service levels (80% and 90%), different demand weighted average customer lead times (5 and 10 days) and different coefficients of variance of demand (0.1, 0.2 and 0.5) for the same problem set.
[b]hcr: inventory holding cost rate of warehouse.

warehouse. According to these results, the gap between our solution and two-stage solution increases with the increase of inventory holding cost rate, and the gap is quite significant under high inventory holding cost rate. The reason for this is obvious. With high inventory holding cost rate, the inventory holding cost has high weight in total cost. Considering inventory holding cost with transportation cost simultaneously can provide a better solution compared to the solution obtained by the simple two-stage procedure.

Figure 3.2 Average gap between our solution and the solution obtained by the two-stage procedure at different inventory holding cost rates

Table 3.3 presents the gap between our solution and the solution obtained by simple two-stage procedure under different settings of coefficient of variance of demand for twenty-seven problem sets. Figure 3.3 shows the trend of the average gap of the twenty-seven problem sets with the increase of the coefficient of variance of demand. According to the results, the gap between our solution and two-stage solution

Table 3.3 Comparison between our solution and two-stage solution under different coefficients of variance of demand

#	# of plants	# of potential warehouses	# of customers	# of product types	Gap[a] CV[b] = 0.1	Gap[a] CV = 0.2	Gap[a] CV = 0.5
1	2	2	10	2	0.84%	1.15%	2.26%
2	2	2	10	5	4.46%	5.72%	9.41%
3	2	2	10	10	2.52%	3.73%	7.20%
4	2	5	50	2	0.41%	0.53%	0.96%
5	2	5	50	5	1.01%	1.09%	1.42%
6	2	5	50	10	3.32%	4.14%	5.97%
7	2	5	100	2	2.21%	2.45%	3.29%
8	2	5	100	5	1.33%	1.57%	2.27%
9	2	5	100	10	1.51%	1.56%	1.97%
10	2	10	100	2	1.29%	1.49%	2.24%
11	2	10	100	5	1.70%	1.87%	2.06%
12	2	10	100	10	3.53%	3.83%	4.67%
13	2	10	150	2	1.61%	1.77%	2.05%
14	2	10	150	5	1.78%	2.06%	2.69%
15	2	10	150	10	2.71%	2.99%	3.76%
16	4	15	150	2	3.68%	3.99%	4.72%
17	4	15	150	5	4.01%	4.38%	5.21%
18	4	15	150	10	5.99%	6.44%	7.60%
19	4	15	200	2	2.26%	2.35%	2.76%
20	4	15	200	5	7.12%	7.32%	7.97%
21	4	15	200	10	4.80%	5.17%	6.36%
22	6	15	150	2	2.53%	3.36%	5.55%
23	6	15	150	5	5.27%	6.11%	8.09%
24	6	15	150	10	6.41%	7.26%	9.44%
25	6	20	200	2	5.38%	5.97%	7.10%
26	6	20	200	5	6.29%	7.30%	9.80%
27	6	20	200	10	5.99%	6.91%	9.03%

[a]Gap: The gap is the average gap ((two-stage solution – our solution) / two-stage solution * 100%) under different cycle service levels (80% and 90%), different demand weighted average customer lead times (5 and 10 days) and different inventory holding cost rates of warehouse (1, 2, 3, 4 and 5) for the same problem set.
[b]CV: Coefficient of variance of demand (standard deviation over mean).

increases slightly with the increase of coefficient of variance of demand. The reason can be explained as follows. With the increase of coefficient of variance of demand, the safety stock holding cost increases. Consequently, the total inventory holding cost increases. Simultaneously considering both inventory holding cost and transportation cost can achieve better solution than the solution obtained by two-stage method.

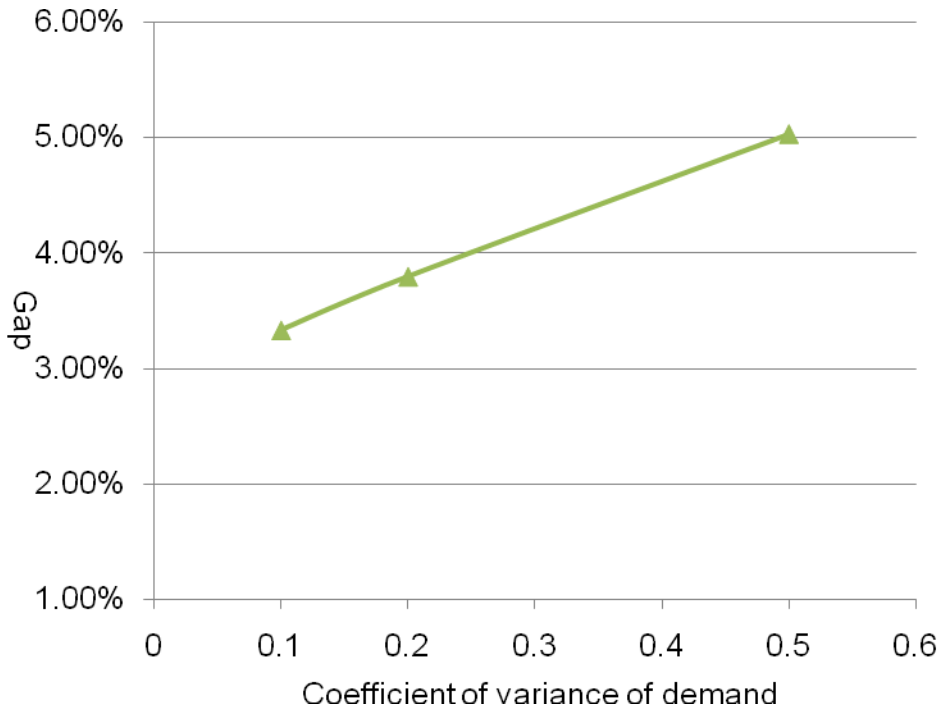

Figure 3.3 Average gap between our solution and the solution obtained by the two-stage procedure at different coefficients of variance of demand

Table 3.4 presents the gap between our solution and the lower bound obtained by solving model *PUL* described in Section 3.4 for twenty-seven problem sets. We can see from Table 3.4 that the gap ranges from 0.78% to 8.93%. Most gaps between the solution obtained by our solving procedure and the lower bound are relatively small. Note that safety stock is a part (about 10% - 30%) of *TC*, and some gaps are a little bit high. The reason is mainly due to the using of underestimated linear functions of safety stock in the model *PUL*. With the increase of the coefficient of variance of demand, the

Table 3.4 Comparison between our solution and lower bound

#	# of plants	# of potential warehouses	# of customers	# of product types	Gap[a]
1	2	2	10	2	2.38%
2	2	2	10	5	3.17%
3	2	2	10	10	1.29%
4	2	5	50	2	1.06%
5	2	5	50	5	2.40%
6	2	5	50	10	2.50%
7	2	5	100	2	0.78%
8	2	5	100	5	1.85%
9	2	5	100	10	2.16%
10	2	10	100	2	2.51%
11	2	10	100	5	2.60%
12	2	10	100	10	4.53%
13	2	10	150	2	2.23%
14	2	10	150	5	3.12%
15	2	10	150	10	5.17%
16	4	15	150	2	2.82%
17	4	15	150	5	6.87%
18	4	15	150	10	8.21%
19	4	15	200	2	4.26%
20	4	15	200	5	7.22%
21	4	15	200	10	8.40%
22	6	15	150	2	2.84%
23	6	15	150	5	6.33%
24	6	15	150	10	8.29%
25	6	20	200	2	3.00%
26	6	20	200	5	5.42%
27	6	20	200	10	8.93%

[a]Gap: The gap is the average gap ((our solution – lower bound) / lower bound * 100%) under different cycle service levels (80% and 90%), different demand weighted average customer lead times (5 and 10 days), different coefficients of variance of demand (0.1, 0.2 and 0.5) and different inventory holding cost rates (1, 2, 3, 4 and 5) for the same problem set.

inventory holding cost rate and the variety of replenishment lead times from plants to warehouses, the gap between our underestimated linear expression of safety stock and the original nonlinear form of safety stock increases. The lower bound therefore becomes not very tight. As a consequence, the gap between our solution and the lower bound increases.

3.5.2 Example study

We now apply our model and method to the problem faced by SOLUTIA Company. As described in the introduction section, the multinational applied chemistry company SOLUTIA plans to choose some third-party logistics service providers' warehouses from many potential locations so as to meet Asia-Pacific customers' demand request for the product (Laminated Glass Interlayer) with some different forms.

In the current situation of SOLUTIA, they have 23 potential warehouses locations. After applying our model and method, the result shows that they need to lease 16 warehouses in Asia-Pacific. Compared to the current situation, the total cost can decrease 5.8% while keeping the original 95% cycle service level and 8 days demand weighted average customer lead time.

We also provide tradeoff solutions between total cost and cycle service level and between total cost and demand weighted average customer lead time for the decision makers. Figure 3.4 shows the total cost at different demand weighted average customer lead times and cycle service levels (In order to protect confidential data, all cost values are normalized). We can find from Figure 3.4 that the total cost increases when one of the following two situations occurs: (1) desired cycle service level increases, (2)

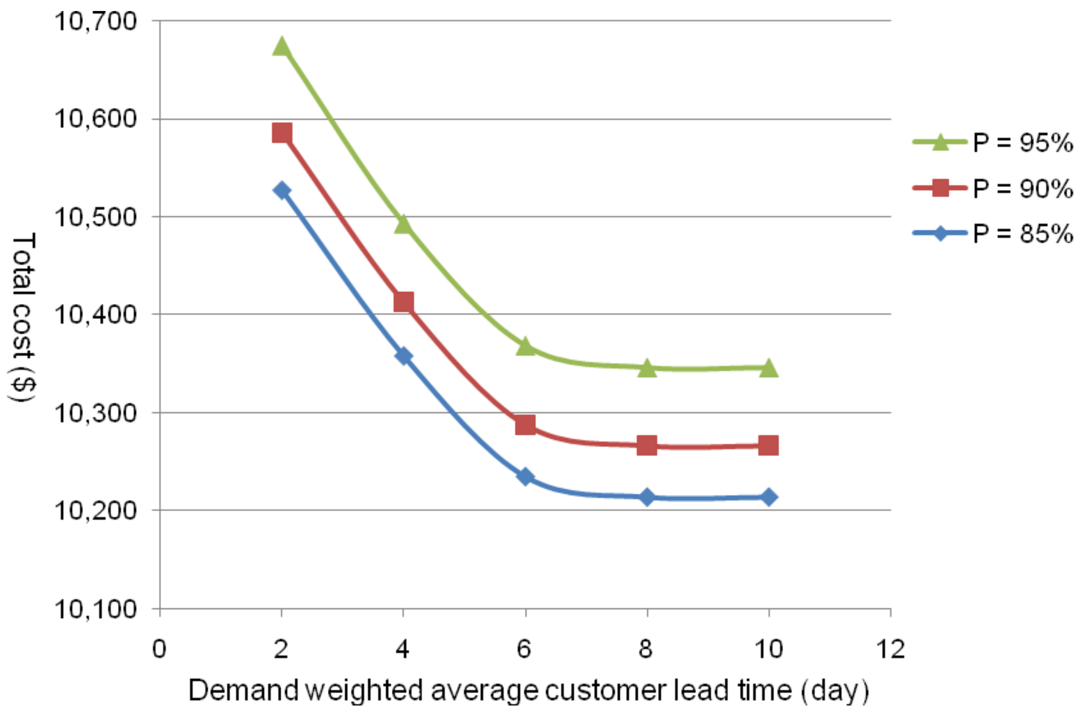

Figure 3.4 Total cost at different lead times and cycle service levels

desired demand weighted average customer lead time decreases. The reason for the increase of the total cost when situation (1) happens is obvious. If the desired cycle service level increases, the inventory levels in warehouses will inevitably increase, which leads to an increase in inventory holding cost. The reason that the total cost increases when situation (2) happens can be explained as follows. If the desired demand weighted average customer lead time decreases, more customers have to be served by local warehouses (with short lead time) instead of being replenished by plants directly (with long lead time). This will increase inventory holding cost of warehouses and as a consequence, total cost increases.

3.6 Summary

In this chapter, we first present a multi-source facility location-allocation and inventory problem. We then develop a mixed integer nonlinear programming model to formulate the problem. In order to solve the proposed model *P*, we develop an iterative heuristic method. We then generate a lower bound of *P* for comparison purpose. We also report the computational results that highlight effectiveness of proposed solution approach and apply our approach to an example study.

We have discussed the study of multi-source facility location-allocation and inventory problem in this chapter. In the following two chapters, we will study dual-channel two-component replenishment problem in an assemble-to-order system in Chapter 4 and then extend our study to multi-channel multi-component problem in Chapter 5.

Chapter 4 DUAL-CHANNEL TWO-COMPONENT REPLENISHMENT PROBLEM IN AN ASSEMBLE-TO-ORDER SYSTEM

4.1 Problem description

We consider an ATO contract manufacturer who will receive a single order of a single product (perhaps a product such as toy, fashion product and certain electronic product with short market life). The quantity of the order will be confirmed at a time 0. However, the customer will share the bill-of-materials and the demand forecast for the product before time 0 to allow the manufacturer sufficient time to identify suppliers of the components needed for the assembly of the final product; or even purchase some components in advance of the final order quantity confirmation. Upon confirmation of the stochastic demand D (with cumulative distribution function $F(x)$ and probability density function $f(x)$) at time 0, the customer will accept partial delivery of the entire order at a unit price of $P(t)$, which is a decreasing function of the delivery time t. No shortage is allowed.

We assume that each unit of the final product requires two components. When both components are available at the manufacturer's facility, the assembly time of final product is assumed to be zero. In other words, we assume unlimited assembly capacity. Due to decreasing final product price, the manufacturer has the incentive to purchase some initial quantity of each component (called pre-stocked components) before time 0 so that he can deliver the first batch of the final product at the highest unit price $P(0)$.

If the confirmed demand is not met with the pre-stocked components, the manufacturer has a chance to procure additional components through two channels with different prices and different guaranteed lead-times. We assume that component *i*, i=1, 2 can be purchased at unit price $c_i^{e_i}$ with a deterministic delivery lead-time $l_i^{e_i}$, where e_i = 1, 2. To avoid any channel dominating another, we assume that a purchase channel with longer lead time has a lower price. In other words, we assume $c_i^1 < c_i^2$, and $l_i^1 > l_i^2$. It is also reasonable to assume that the unit price for the pre-stocked component *i* is c_i^1 as we assume that there is ample time for the procurement of components before time 0.

Assume that the excess component *i* can be salvaged at a price b_i, $b_i < c_i^1$. The manufacturer needs to make a tradeoff between missing higher selling price due to insufficient pre-stocked components and incurring cost of overstocking inventory of components. The manufacturer makes decisions in two stages. At the first stage before time 0, the manufacturer decides the pre-stocking quantity Q_i of each component *i*. At the second stage when the final product demand is realized, if the product demand cannot be satisfied by the pre-stocked inventory, the manufacturer needs to decide the quantities, channels and the timing of acquiring additional components, and the delivery schedule of the final product to satisfy the unmet demand.

4.2 Problem formulation

We now present a mathematical formulation for the problem for which the manufacturer wishes to find a solution of the problem with the highest expected profit. Note that it is optimal to restrict the delivery time of the final product for the two-component problem to a set of finite times $\{ l_i^{e_i} \mid i$ = 1, 2 and e_i = 1, 2$\}$ besides time 0 (based on the insight similar to that of proposition 1 in Fu et al., 2009). Let $q_i^{e_i}$ (i = 1, 2 and e_i = 1, 2) denotes the quantity of additional component i ordered by procurement channel e_i at time 0, $w_i^{e_i}$ (i = 1, 2 and e_i = 1, 2) denotes the quantity of the final product delivered at time $l_i^{e_i}$ and w_0 denotes the quantity of the final product delivered at time 0. The problem can be represented by the following profit maximization problem, denoted as problem **A**.

(**A**)

$$Z_{\mathbf{A}} = \max E_D \left[w_0 P(0) + \sum_{i=1}^{2} \sum_{e_i=1}^{2} w_i^{e_i} P(l_i^{e_i}) - \sum_{i=1}^{2} Q_i c_i^1 - \sum_{i=1}^{2} \sum_{e_i=1}^{2} q_i^{e_i} c_i^{e_i} + \sum_{i=1}^{2} \left(Q_i - D \right)^+ b_i \right]$$

subject to

$$w_0 + \sum_{i=1}^{2} \sum_{e_i=1}^{2} w_i^{e_i} = D \tag{4.1}$$

$$w_0 \le Q_i \qquad i = 1, 2 \tag{4.2}$$

$$w_0 + \sum_{k=1}^{2} \sum_{e_k : l_k^{e_k} \le l_j^{e_j}} w_k^{e_k} \le Q_i + \sum_{e_i : l_i^{e_i} \le l_j^{e_j}} q_i^{e_i} \qquad i = 1, 2, \quad j = 1, 2, \quad e_j = 1, 2 \tag{4.3}$$

$$Q_i, q_i^{e_i}, w_0, w_i^{e_i} \ge 0 \qquad i = 1, 2, \quad e_i = 1, 2 \tag{4.4}$$

where $(\cdot)^+ = \max(0, \cdot)$.

The objective function is to maximize the expected total profit. The first two terms of the objective function represent the revenue received from all shipments of final product, the third and fourth term denote the procurement costs of pre-stocking and additional components and the fifth term represents the salvage values of excess pre-stocking components. Constraint (4.1) ensures that the demand is satisfied. Constraints (4.2) and (4.3) make sure that both components are available for each delivery of final product. Constraints (4.4) are nonnegativity constraints.

Note that above formulation can be directly reduced to the formulation presented in Fu et al. (2009) and therefore can be solved by their solution approach if there is only one expediting channel available for each component after the demand is realized. However, with dual-channel sourcing, the contract manufacturer faces more options than that of single-channel sourcing after product demand is realized. Besides, for any given **Q** and *D*, the expressions of $q_i^{e_i}$ and $w_i^{e_i}$ in above formulation vary with different lead time sequences (i.e. the time sequence of $l_i^{e_i}$) and different parameter values, it is very difficult to provide closed-form expressions for $q_i^{e_i}$ and $w_i^{e_i}$. For example, above formulation has to be divided into large numbers of sub-cases (in terms of different lead time sequences) even for two-component problem, and the number of sub-cases increases exponentially with the number of components. Therefore, the formulation of problem **A** is expected to be much more difficult to solve than the single channel sourcing problems with a normal or emergency channel considered by Hsu et al. (2006) and Fu et al. (2009), respectively.

Instead of solving the above formulation of the problem **A** directly, we will approach the problem from a different angle. Let $R(\mathbf{Q}, D)$ denotes the maximum profit for a given first-stage decision **Q** and a given realization of the demand *D*. Thus, we can also model the problem **A** as

$$(\mathbf{A}) \quad Z_{\mathbf{A}} = \max_{\mathbf{Q}\in\mathbf{R}_+^2} E_D\left(R(\mathbf{Q}, D)\right) \tag{4.5}$$

Where $\mathbf{R}_+^2$ is the set of 2-dimensional arrays of non-negative real numbers.

The problem **A** will be solved analytically. We will first present a restricted version of problem **A** where the pre-stocked quantities **Q** follow a given rank order $Q_{[1]} \le Q_{[2]}$ ([1] = 1 or 2). We will then obtain the optimal solution to the general problem **A**.

For ease of exposition, we will focus our discussions in this section on a special rank order of Q_i where [*i*] = *i*. In other words, we will focus on solving the restricted problem whose optimal solution satisfies a rank order requiring $Q_1 \le Q_2$. We will establish some structural properties for the restricted problems which allow us to solve them efficiently. Specifically, for each restricted problem with a fixed rank order of the pre-stocked inventories, we will show that there are at most three batches of deliveries of the final product in the optimal solution. We can then determine the optimal delivery quantity and highest unit profit of the final product for each batch.

Note that the unit price for the final product is a decreasing function of the delivery time and the manufacturer is permitted to deliver partial quantity of the entire order. Thus, it is optimal for the manufacture to assemble and deliver final products as soon as both components are available. It can be easily seen that the manufacturer will deliver the first batch (i.e. batch 0) of the final product at time 0 when the demand *D* is confirmed. Due to the fact that the pre-stocking quantities **Q** may be smaller that the confirmed demand *D*, some final products must be assembled using some newly procured components. Since $Q_1 \le Q_2$ by definition of the restricted problem, it can be easily seen that, the manufacturer has the opportunity to deliver up to three batches

(batches 0, 1, 2) of the final product to fulfill the entire demand D, where batch k is produced from additional purchases of components 1, ..., k and the pre-stocked components k + 1, ..., 2. Without loss of generality, we can simplify the problem by imposing that there are exactly three delivery batches, where the quantity produced for some batches may be equal to zero. It will be clear later that the quantity produced for batch k is equal to zero when (i) confirmed demand does not exceed the pre-stocking quantity of component k, or (ii) the pre-stocking quantity of component k is equal to that of component k + 1, i.e., $Q_k = Q_{k+1}$.

We denote $UP_k(\mathbf{Q}, D)$ as the highest unit profit for a final product delivered at batch k (k = 0, 1, 2), $t_k(\mathbf{Q}, D)$ as the optimal delivery time of the final product delivered at batch k and $W_k(\mathbf{Q}, D)$ as the optimal delivery quantity of the final product delivered at batch k. We then can formulate the restricted problem $\mathbf{A}(Q_1 \le Q_2)$ as

$$(\mathbf{A}(Q_1 \le Q_2))$$
$$Z_{\mathbf{A}(Q_1 \le Q_2)} = \max_{Q_1 \le Q_2} E_D\left[\sum_{k=0}^{2} UP_k(\mathbf{Q}, D) \cdot W_k(\mathbf{Q}, D) - \sum_{i=1}^{2} OC_i (Q_i - D)^+\right] \tag{4.6}$$

where $OC_i = c_i^1 - b_i$, $1 \le i \le 2$, is the overstocking cost for each excess unit of pre-stocked component i. Inside the expectation of the overall profit function, the first term is the total profit generated from the three batches of final product delivery. The second term represents the total loss from overstocking the initial components. Note that the formulation of a restricted problem $\mathbf{A}(Q_2 \le Q_1)$ is almost identical to what we have presented in this section for the rank order $Q_1 \le Q_2$.

It is clear that the optimal delivery time of the final product at batch 0 is $t_0(\mathbf{Q}, D) = 0$ and the highest unit profit for a final product delivered in batch 0 is

$UP_0(\mathbf{Q}, D) = P(0) - \sum_{i=1}^{2} c_i^1$. To obtain the highest profit for this second batch (batch 1) of delivery, the manufacturer will choose a sourcing channel with lead time $l_1^{e_1}$ so that batch 1 will be delivered at time $t_1 = l_1^{e_1}$ to command the highest unit profit $UP_1(\mathbf{Q}, D)$ which is given by $UP_1(\mathbf{Q}, D) = \max_{1 \le e_1 \le 2} \{P(l_1^{e_1}) - c_1^{e_1} - c_2^1\}$. The highest unit profit $UP_2(\mathbf{Q}, D)$ and the optimal delivery time t_2 for delivering batch 2 is determined by the following unit-profit-maximization problem:

$$UP_2(\mathbf{Q}, D) = \max_{\{t, e_i\}} \left\{ P(t) - \sum_{i=1}^{2} c_i^{e_i} \middle| l_1^{e_1} \le t, l_2^{e_2} \le t, 1 \le e_1 \le 2, 1 \le e_2 \le 2 \right\}$$

We have the following remarks on the highest unit profit $UP_k(\mathbf{Q}, D)$ and the optimal delivery time $t_k(\mathbf{Q}, D)$. Firstly, these values depend only on the rank order of **Q** and *not* on any specific pre-stocked inventory decision and the realization of demand under problem $\mathbf{A}(Q_1 \le Q_2)$, thus, $UP_k(\mathbf{Q}, D)$ and $t_k(\mathbf{Q}, D)$ can be reduced to UP_k and t_k. Secondly, since the unit price for the final product decreases in delivering time and each subsequent batch in the order of 0, 1, 2 requires the purchase of increasing number of additional components, it is intuitive to expect the following result:

Lemma 4.1. *The highest unit profit for the final product delivered in batch k is non-increasing with k, i.e., we have* $UP_0 > UP_1 \ge UP_2$.

Third, we note that for any unit profit for batch *k*+1 at the time point which is earlier than t_k (the optimal delivery time of the final product at batch *k*), we can always find a higher or at least same unit profit for batch *k*+1 at time t_k. Therefore, we can obtain the following result:

Lemma 4.2. *The optimal delivery time of the final product in batch k is non-decreasing* *with k, i.e., we have* $t_0 < t_1 \le t_2$.

We then determine $W_k(\mathbf{Q}, D)$. The quantity delivered by batch 0 is the smaller of D and the number of units of the final product that can be assembled from complete sets of the pre-stocked components at time 0, which is given by $\min\{Q_1, Q_2\} = Q_1$. In other words, $W_0(\mathbf{Q}, D) = \min\{D, Q_1\}$. If the demand is not yet fully satisfied at time 0, i.e., if $D - Q_1 > 0$, the next delivery will not occur until some additional units of component 1 arrive to match with the remaining $Q_2 - Q_1$ pre-stocked component 2. In this case, the manufacturer will order additional $W_1(\mathbf{Q}, D) = \min\{D - Q_1, Q_2 - Q_1\}$ units of component 1. We see now that in general, each batch k, k = 1, 2 will deliver

$$W_k(\mathbf{Q}, D) = \min\{(D - Q_k)^+, Q_{k+1} - Q_k\}$$

units of the final product which are produced from additional purchases of components 1, …, k and the pre-stocked components k+1, …, 2 (where $Q_3 \equiv +\infty$). The last batch, batch 2, will be assembled from additional orders of both two components.

4.3 Result and analysis

Note that $\min\{x, y\} = x - (x - y)^+$ and if $y \ge 0$, $\min\{x^+, y\} = x^+ - (x - y)^+$. We can re-arrange the terms of the objective function (inside the expectation) of the restricted problem $\mathbf{A}(Q_1 \le Q_2)$ as follows:

$$\sum_{k=0}^{2} UP_k \cdot W_k(\mathbf{Q}, D) - \sum_{i=1}^{2} OC_i (Q_i - D)^+$$

$$= UP_0 \cdot \left[D - (D - Q_1)^+\right] + UP_1 \cdot [(D - Q_1)^+ - (D - Q_2)^+] + UP_2 \cdot (D - Q_2)^+ - \sum_{i=1}^{2} OC_i (Q_i - D)^+$$

$$= UP_0 \cdot D - \sum_{k=1}^{2} \Big[(UP_{k-1} - UP_k) \cdot (D - Q_k)^+ + OC_k \cdot (Q_k - D)^+ \Big].$$

Based on the new expression of objective function of $\mathbf{A}(Q_1 \le Q_2)$, we can solve the general problem A to optimality (see Appendix B.1) and obtain the following closed-form optimal solution.

Closed-form optimal solution for the dual-channel two-component problem:

(a) The optimal pre-stocked quantities of components for dual-channel two-component problem are given as follows.

If $\dfrac{OC_1}{OC_2} > \left.\dfrac{UP_0 - UP_1}{UP_1 - UP_2}\right|_{[1]=1}$

$$Q_1 < Q_2 \text{ and } Q_1 = F^{-1}\left(\frac{UP_0 - UP_1}{UP_0 - UP_1 + OC_1}\right)\bigg|_{[1]=1}, \quad Q_2 = F^{-1}\left(\frac{UP_1 - UP_2}{UP_1 - UP_2 + OC_2}\right)\bigg|_{[1]=1}$$

If $\dfrac{OC_1}{OC_2} < \left.\dfrac{UP_1 - UP_2}{UP_0 - UP_1}\right|_{[1]=2}$

$$Q_1 > Q_2 \text{ and } Q_1 = F^{-1}\left(\frac{UP_1 - UP_2}{UP_1 - UP_2 + OC_1}\right)\bigg|_{[1]=2}, \quad Q_2 = F^{-1}\left(\frac{UP_0 - UP_1}{UP_0 - UP_1 + OC_2}\right)\bigg|_{[1]=2}$$

If $\left.\dfrac{UP_0 - UP_1}{UP_1 - UP_2}\right|_{[1]=1} \ge \dfrac{OC_1}{OC_2} \ge \left.\dfrac{UP_1 - UP_2}{UP_0 - UP_1}\right|_{[1]=2}$

$$Q_1 = Q_2 = F^{-1}\left(\frac{UP_0 - UP_2}{UP_0 - UP_2 + OC_1 + OC_2}\right).$$

where $UP_1 = \max_{1 \le e_1 \le 2}\{P(l_1^{e_1}) - c_1^{e_1} - c_2^1\}$ when [1] = 1 and [2] = 2;

$UP_1 = \max_{1 \le e_2 \le 2}\{P(l_2^{e_2}) - c_2^{e_2} - c_1^1\}$ when [1] = 2 and [2] = 1.

(b) Optimal final product delivery schedule is given as follows.

First delivery time $= t_0$, delivery quantity $= W_0(\mathbf{Q}, D) = \min\{D, Q_{[1]}\}$

Second delivery time $= t_1$, delivery quantity $= W_1(\mathbf{Q}, D) = \min\{(D - Q_{[1]})^+, Q_{[2]} - Q_{[1]}\}$

Third delivery time $= t_2$, delivery quantity $= W_2(\mathbf{Q}, D) = (D - Q_{[2]})^+$

Proof. See Appendix B.1. □

The above result of is similar to the result of the newsvendor problem. For example, if $\frac{OC_1}{OC_2} > \left.\frac{UP_0 - UP_1}{UP_1 - UP_2}\right|_{[1]=1}$, we can regard OC_1 as overage cost of component 1, OC_2 as overage cost of component 2, $UP_0 - UP_1$ as underage cost of component 1 and $UP_1 - UP_2$ as underage cost of component 2. This inequality then means overage cost ratio between component 1 and component 2 is greater than underage cost ratio between components 1 and 2. Therefore, the initial order quantity of component 1 is less than the initial order quantity of component 2 and the order quantities are given by $F^{-1}\left(\frac{\text{underage cost}}{\text{underage cost} + \text{overage cost}}\right)$. If $\frac{OC_1}{OC_2} < \left.\frac{UP_1 - UP_2}{UP_0 - UP_1}\right|_{[1]=2}$, the initial order quantity of component 2 is less than the initial order quantity of component 1. If $\frac{OC_1}{OC_2}$ is in-between the above two terms, we should order equal quantities of components 1 and 2, and $UP_0 - UP_2$ (i.e. $UP_0 - UP_1 + UP_1 - UP_2$) is the underage cost of components 1 and 2, $OC_1 + OC_2$ is the overage cost of components 1 and 2.

4.4 Summary

In this chapter, we present an optimal dual-channel two-component replenishment problem in an assemble-to-order system. We investigate the structure of the problem to gain some good properties that help us to develop a good formulation of the problem.

We finally provide the closed-form optimal solution of the dual-channel two-component problem. In the next chapter, we will extend the study of dual-channel two-component problem to multi-channel multi-component problem.

Chapter 5 MULTI-CHANNEL MULTI-COMPONENT PROBLEM

In this chapter, we extend our analysis on dual-channel two-component problem to multi-channel multi-component problem. We will still solve the problem analytically. We will first formulate and solve a restricted version of our problem in which the quantities of pre-stocked components follow a certain fixed rank order. Then, we will develop a branch-and-bound method to solve the general problem by searching over all rank orders of pre-stocked components and solving the corresponding restricted problems. We also present a greedy heuristic procedure. Finally, computational studies are provided at the end of this chapter.

5.1 Description and solution approach for the general problem

The problem is the same as the one described in Chapter 4 except that each unit of the final product now requires n components indexed as i = 1, 2, …, n. If the confirmed demand is not met with the pre-stocked components, the manufacturer has a chance to procure additional components through various suppliers who offer different prices with different guaranteed lead times. We assume that component i, i = 1, 2, …, n, can be purchased at unit price $c_i^{e_i}$ with a deterministic delivery lead time $l_i^{e_i}$, where e_i = 1, 2, …, m_i. In other words, component i can be purchased through m_i purchase channels (possibly through different delivery modes and/or from different suppliers). Similarly, to avoid any channel dominating another, we assume that a purchase channel with longer lead time has a lower price. In other words, we assume $c_i^1 < c_i^2 < \cdots < c_i^{m_i}$, and $l_i^1 > l_i^2 > \cdots > l_i^{m_i}$.

Similarly, Let $R(\mathbf{Q}, D)$ denotes the maximum profit for a given first-stage decision $\mathbf{Q}$ = $(Q_1, Q_2, ..., Q_n)$ and a given realization of the demand D. Thus, we can model the problem as

$$(\mathbf{A}) \quad Z_{\mathbf{A}} = \max_{\mathbf{Q} \in \mathbf{R}_+^n} E_D\left(R(\mathbf{Q}, D)\right)$$

Where $\mathbf{R}_+^n$ is the set of n-dimensional arrays of non-negative real numbers.

We now reformulate the problem A by imposing a certain rank order on the first-stage decision $\mathbf{Q}$. Let $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, ..., \sigma_n)$ be a permutation of the component indices 1,…, n. Define

$$S(\boldsymbol{\sigma}) = \{\mathbf{Q} \in \mathbf{R}_+^n \big| Q_{\sigma_1} \le Q_{\sigma_2} \le \cdots \le Q_{\sigma_n}\}$$

as the set of all pre-stocked quantities that satisfy the given rank order defined by the permutation $\boldsymbol{\sigma}$. We define the restricted version of problem **A** where the maximization of expected profit is taken over all decision variables which satisfy a rank order defined by the permutation $\boldsymbol{\sigma}$ as

$$(\mathbf{A}(\boldsymbol{\sigma})) \quad Z_{\mathbf{A}(\boldsymbol{\sigma})} = \max_{\mathbf{Q} \in S(\boldsymbol{\sigma})} E_D\left(R(\mathbf{Q}, D)\right).$$

Let Ω be the set of all possible permutation operators $\boldsymbol{\sigma}$, Clearly, the optimal objective function value of the overall problem **A** is given by

$$Z_{\mathbf{A}} = \max_{\boldsymbol{\sigma} \in \Omega} Z_{\mathbf{A}(\boldsymbol{\sigma})}.$$

Instead of solving the restricted problems for all possible permutations, we will develop a branch-and-bound procedure to find an optimal solution of the original problem as well as an efficient heuristic procedure. In the next two sections, we will present the formulation and the solution method for the restricted problem which is an essential part for both the branch-and-bound and the heuristic procedures.

5.2 Formulation for the restricted problem $\mathbf{A(\sigma)}$

For ease of exposition, we will focus our discussions in this section on a special permutation $\boldsymbol{\sigma}^0$ where $\sigma_i^0 \equiv i$. In other words, we will focus on solving the restricted problem whose optimal solution satisfies a rank order requiring $Q_1 \le Q_2 \le \cdots \le Q_n$. We will establish some structural properties for the restricted problems which allow us to solve them efficiently. Specifically, for each restricted problem with a fixed rank order of the pre-stocked inventories, we will show that there are at most n + 1 batches of deliveries of the final product in the optimal solution. We can then determine the optimal delivery quantity and highest unit profit of the final product for each batch.

Note that the unit price for the final product is a decreasing function of the delivery time and the manufacturer is permitted to deliver partial quantity of the entire order. Thus, it is optimal for the manufacture to assemble and deliver final products as soon as all components are available. It can be easily seen that the manufacturer will deliver the first batch (i.e. batch 0) of the final product at time 0 when the demand D is confirmed. Due to the fact that the pre-stocking quantities $\mathbf{Q}$ may be smaller that the confirmed demand D, some final products must be assembled using some newly procured components. Since $Q_1 \le Q_2 \le \cdots \le Q_n$ by definition of the restricted problem, it can be easily seen that, the manufacturer has the opportunity to deliver up to $n + 1$ batches (batches 0, 1, ..., n) of the final product to fulfill the entire demand D, where batch k is produced from additional purchases of components 1, 2, ..., k and the pre-stocked components $k + 1$, ..., n. Without loss of generality, we can simplify the problem by imposing that there are exactly $n + 1$ delivery batches, where the quantity produced for some batches may be equal to zero. Similarly, the quantity produced for batch k is equal to zero when (i) confirmed demand does not exceed the pre-stocking quantity of component k, or (ii) the pre-stocking quantity of component k is equal to that of component $k + 1$, i.e., $Q_k = Q_{k+1}$.

We still denote $UP_k(\mathbf{Q}, D)$ as the highest unit profit for a final product delivered at batch k ($k = 0, 1, \ldots, n$), $t_k(\mathbf{Q}, D)$ as the optimal delivery time of the final product delivered at batch k and $W_k(\mathbf{Q}, D)$ as the optimal delivery quantity of the final product delivered at batch k. We then can formulate the restricted problem $\mathbf{A}(\boldsymbol{\sigma}^0)$ as

$$(\mathbf{A}(\boldsymbol{\sigma}^0)) \quad Z_{\mathbf{A}(\boldsymbol{\sigma}^0)} = \max_{\mathbf{Q}\in S(\boldsymbol{\sigma}^0)} E_D\left[\sum_{k=0}^{n} UP_k(\mathbf{Q},D)\cdot W_k(\mathbf{Q},D) - \sum_{i=1}^{n} OC_i(Q_i - D)^+\right] \qquad (5.1)$$

where $OC_i = c_i^1 - b_i$, $1 \le i \le n$, is the overstocking cost for each excess unit of pre-stocked component i. Inside the expectation of the overall profit function, the first term is the total profit generated from the $n + 1$ batches of final product delivery. The second term represents the total loss from overstocking the initial components.

It is clear that the optimal delivery time of the final product at batch 0 is $t_0(\mathbf{Q}, D) = 0$ and the highest unit profit for a final product delivered in batch 0 is $UP_0(\mathbf{Q}, D) = P(0) - \sum_{i=1}^{n} c_i^1$. We now discuss how to determine the values of $UP_k(\mathbf{Q}, D)$

and $t_k(\mathbf{Q}, D)$ for $k = 1, 2, ..., n$.

Let $e_i(t)$ be the channel selected to procure additional units of component i required to produce final products at time $t > 0$. It is easy to see that any additional components required for any batch 1, ..., n must be procured using the cheapest/slowest channel with a lead time that is no later than the delivery/production time of that batch. That is,

$$e_i(t) = \arg\max_{e_i=1,\dots,m_i} \left\{ l_i^{e_i} \middle| l_i^{e_i} \le t \right\}.$$

Consider the delivery of batch k. Recall that each final product in batch k is produced from additional purchases of components 1, ..., k and the pre-stocked components k + 1, ..., n. Since additional unit of component k must be procured for batch k, we must have $t_k(\mathbf{Q}, D) \ge l_k^{m_k}$. Due to the fact that the unit price of the final product is decreasing over time, each batch must be produced/delivered immediately after all components are available. Therefore, we only need to consider the lead times of components as potential delivery/production times. Let Δ_k, $k = 1, ..., n$, be the set of potential delivery/production times for batch k; formally, $\Delta_k = \left\{ l_i^{e_i} \middle| e_i = 1,\dots,m_i, i = 1,\dots,k \right\}$. Hence, the highest unit profit for a final product delivered in batch k is

$$UP_k(\mathbf{Q}, D) = \max_{t \in \Delta_k, t \ge l_k^{m_k}} \left\{ P(t) - \sum_{i=1}^{k} c_i^{e_i(t)} - \sum_{i=k+1}^{n} c_i^1 \right\},$$

and $t_k(\mathbf{Q}, D)$ is the time corresponding to $UP_k(\mathbf{Q}, D)$.

Similarly, we have the following results, which are extensions of Lemma 4.1 and Lemma 4.2 respectively.

Lemma 5.1. *The highest unit profit for the final product delivered in batch k is non-increasing with k, i.e., we have* $UP_0 > UP_1 \ge \cdots \ge UP_n$.

Lemma 5.2. *The optimal delivery time of the final product in batch k is non-decreasing with k, i.e., we have* $t_0 < t_1 \le \cdots \le t_n$.

With this property, we can make our UP_k searching procedure more efficient by including the constraint $t_{k-1} \le t_k$ (reduce the search space of *t*).

Similarly, we can determine $W_k(\mathbf{Q}, D)$ as follows. The quantity delivered by batch 0 is the smaller of *D* and the number of the final product that can be assembled from complete sets of the pre-stocked components at time 0, which is given by $\min\{Q_1, Q_2, ..., Q_n\} = Q_1$. In other words, $W_0(\mathbf{Q}, D) = \min\{D, Q_1\}$. If the demand is not yet fully satisfied at time 0, i.e., if $D - Q_1 > 0$, the next delivery will not occur until some additional component 1 arrive to match with the remaining $Q_2 - Q_1$ complete sets of the pre-stocked components 2, …, *n*. In this case, the manufacturer will order additional $W_1(\mathbf{Q}, D) = \min\{D - Q_1, Q_2 - Q_1\}$ units of component 1. We see now that in general, each batch *k*, *k* = 1, 2, …, *n*, will deliver

$$W_k(\mathbf{Q}, D) = \min\{(D - Q_k)^+, Q_{k+1} - Q_k\}$$

units of the final product which are produced from additional purchases of components 1, …, *k* and the pre-stocked components *k*+1, …, *n* (where $Q_{n+1} \equiv +\infty$). The last batch, batch *n*, will be assembled from additional orders of all *n* components.

Similarly, we can re-arrange the terms of the objective function (5.1) (inside the expectation) of the restricted problem $\mathbf{A}(\boldsymbol{\sigma}^0)$ as follows:

$$\sum_{k=0}^{n} UP_k \cdot W_k(\mathbf{Q}, D) - \sum_{i=1}^{n} OC_i (Q_i - D)^+$$

$$= UP_0 \cdot \left[D - (D - Q_1)^+ \right] + \sum_{k=1}^{n-1} \left\{ UP_k \cdot [(D - Q_k)^+ - (D - Q_{k+1})^+] \right\} + UP_n \cdot (D - Q_n)^+ - \sum_{i=1}^{n} OC_i (Q_i - D)^+$$

$$= UP_0 \cdot D - \sum_{k=1}^{n} \left[(UP_{k-1} - UP_k) \cdot (D - Q_k)^+ + OC_k \cdot (Q_k - D)^+ \right].$$

For all k ($1 \le k \le n$), define $UC_k = UP_{k-1} - UP_k$. Also note that UP_0 and $E_D(D)$ are constants. We see that $\mathbf{A}(\boldsymbol{\sigma}^0)$ is equivalent to the following problem $\mathbf{B}(\boldsymbol{\sigma}^0)$

$$(\mathbf{B}(\boldsymbol{\sigma}^0)) \quad Z_{\mathbf{B}(\boldsymbol{\sigma}^0)} = \min_{\mathbf{Q} \in S(\boldsymbol{\sigma}^0)} E_D \left\{ \sum_{k=1}^{n} \left[UC_k \cdot (D - Q_k)^+ + OC_k \cdot (Q_k - D)^+ \right] \right\} \tag{5.2}$$

To conclude this section, we note that the formulation of a restricted problem $\mathbf{A}(\boldsymbol{\sigma})$ for any given permutation $\boldsymbol{\sigma}$ will be almost identical to what we have presented in this section for the special permutation $\boldsymbol{\sigma}^0$, except that the index for a component $\sigma_i^0 \equiv i$ in problem $\mathbf{A}(\boldsymbol{\sigma}^0)$ will be replaced by σ_i in problem $\mathbf{A}(\boldsymbol{\sigma})$. The solution method used for solving any of these restricted problems will be similar too. We will therefore present the solution method for problem $\mathbf{B}(\boldsymbol{\sigma}^0)$ in the next section.

5.3 Solution method for the restricted problem $\mathbf{B}(\boldsymbol{\sigma}^0)$

Note that the structure of problem $\mathbf{B}(\boldsymbol{\sigma}^0)$ is similar to the problem defined by (11) in Hsu at al. (2006). Both problems minimize (maximize) the summation of n single variable, convex (concave) functions subject to a certain rank order of the n decision variables. However, the Decompose-and-Combine procedure developed in Hsu at al. (2006) cannot be used directly to solve $\mathbf{B}(\boldsymbol{\sigma}^0)$ which is more general. Next, we will establish a series of structural properties of the optimal solutions and use them to develop a new procedure to solve $\mathbf{B}(\boldsymbol{\sigma}^0)$.

For $1 \le i \le j \le n$, define problem $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$ as follows:

$$(\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)) \qquad Z_{\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)} = \min_{\mathbf{Q}\in S(\boldsymbol{\sigma}^0)} E_D\left\{\sum_{k=i}^{j}\left[UC_k\cdot(D-Q_k)^+ + OC_k\cdot(Q_k-D)^+\right]\right\} \tag{5.3}$$

Note that $\mathbf{B}(\boldsymbol{\sigma}^0)$ is identical to $\mathbf{B}_{1,n}(\boldsymbol{\sigma}^0)$. We can develop a lemma which gives structural property of the optimal solution of the problem $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$.

Lemma 5.3. Suppose that $\tilde{Q}_i,...,\tilde{Q}_k$ is an optimal solution to $\mathbf{B}_{i,k}(\boldsymbol{\sigma}^0)$, $1 \le i \le k \le n-1$, and $\tilde{Q}_{k+1},...,\tilde{Q}_j$ is an optimal solution to $\mathbf{B}_{k+1,j}(\boldsymbol{\sigma}^0)$, $k+1 \le j \le n$. If $\tilde{Q}_k \le \tilde{Q}_{k+1}$, then $\tilde{Q}_i,...,\ \tilde{Q}_j$ is an optimal solution to $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$.

Proof. Since $\tilde{Q}_k \le \tilde{Q}_{k+1}$ satisfies the permutation $\boldsymbol{\sigma}^0$ and note that $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0) = \mathbf{B}_{i,k}(\boldsymbol{\sigma}^0) + \mathbf{B}_{k+1,j}(\boldsymbol{\sigma}^0)$, the optimal solution of $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$ is the combination of the optimal solution of $\mathbf{B}_{i,k}(\boldsymbol{\sigma}^0)$ and the optimal solution of $\mathbf{B}_{k+1,j}(\boldsymbol{\sigma}^0)$. □

Lemma 5.3 shows that the optimal solution of $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$ is the combination of the optimal solutions of two separate problems $\mathbf{B}_{i,k}(\boldsymbol{\sigma}^0)$ and $\mathbf{B}_{k+1,j}(\boldsymbol{\sigma}^0)$ provided that the boundaries of the two separate optimal solutions, $\tilde{Q}_k$ and $\tilde{Q}_{k+1}$, satisfy the permutation $\boldsymbol{\sigma}^0$.

We now define $r_{i,j} = \dfrac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} OC_k}$ $(1 \le i \le j \le n)$. The following lemma shows a property of the relationship of $r_{i,j}$.

Lemma 5.4. *(a) If* $r_{i,x} > r_{x+1,j}$ $(1 \le i \le x < j \le n)$*, then* $r_{i,x} > r_{i,j} > r_{x+1,j}$.

(b) If $r_{i,x} \le r_{x+1,j}$ $(1 \le i \le x < j \le n)$*, then* $r_{i,x} \le r_{i,j} \le r_{x+1,j}$.

Proof. We first prove (a).

We first prove $r_{i,x} > r_{i,j}$. If $r_{i,x} > r_{x+1,j}$ $(i \le x \le j-1)$, then $\frac{\sum_{k=i}^{x} UC_k}{\sum_{k=i}^{x} OC_k} > \frac{\sum_{k=x+1}^{j} UC_k}{\sum_{k=x+1}^{j} OC_k}$

$$\sum_{k=i}^{x} UC_k \sum_{k=x+1}^{j} OC_k > \sum_{k=x+1}^{j} UC_k \sum_{k=i}^{x} OC_k$$

$$\sum_{k=i}^{x} UC_k \sum_{k=x+1}^{j} OC_k + \sum_{k=i}^{x} UC_k \sum_{k=i}^{x} OC_k > \sum_{k=x+1}^{j} UC_k \sum_{k=i}^{x} OC_k + \sum_{k=i}^{x} UC_k \sum_{k=i}^{x} OC_k$$

$$\sum_{k=i}^{x} UC_k \left(\sum_{k=x+1}^{j} OC_k + \sum_{k=i}^{x} OC_k \right) > \sum_{k=i}^{x} OC_k \left(\sum_{k=x+1}^{j} UC_k + \sum_{k=i}^{x} UC_k \right)$$

$$\sum_{k=i}^{x} UC_k \sum_{k=i}^{j} OC_k > \sum_{k=i}^{x} OC_k \sum_{k=i}^{j} UC_k$$

$$\frac{\sum_{k=i}^{x} UC_k}{\sum_{k=i}^{x} OC_k} > \frac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} OC_k}, \text{ i.e. } r_{i,x} > r_{i,j}$$

We now prove $r_{i,j} > r_{x+1,j}$. If $r_{i,x} > r_{x+1,j}$ $(i \le x \le j-1)$, then $\frac{\sum_{k=i}^{x} UC_k}{\sum_{k=i}^{x} OC_k} > \frac{\sum_{k=x+1}^{j} UC_k}{\sum_{k=x+1}^{j} OC_k}$

$$\sum_{k=i}^{x} UC_k \sum_{k=x+1}^{j} OC_k > \sum_{k=x+1}^{j} UC_k \sum_{k=i}^{x} OC_k$$

$$\sum_{k=i}^{x} UC_k \sum_{k=x+1}^{j} OC_k + \sum_{k=x+1}^{j} UC_k \sum_{k=x+1}^{j} OC_k > \sum_{k=x+1}^{j} UC_k \sum_{k=i}^{x} OC_k + \sum_{k=x+1}^{j} UC_k \sum_{k=x+1}^{j} OC_k$$

$$\sum_{k=x+1}^{j} OC_k \left(\sum_{k=i}^{x} UC_k + \sum_{k=x+1}^{j} UC_k \right) > \sum_{k=x+1}^{j} UC_k \left(\sum_{k=i}^{x} OC_k + \sum_{k=x+1}^{j} OC_k \right)$$

$$\sum_{k=x+1}^{j} OC_k \sum_{k=i}^{j} UC_k > \sum_{k=x+1}^{j} UC_k \sum_{k=i}^{j} OC_k$$

$$\frac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} OC_k} > \frac{\sum_{k=x+1}^{j} UC_k}{\sum_{k=x+1}^{j} OC_k}, \text{ i.e. } r_{i,j} > r_{x+1,j}$$

Proof of (b) is similar to the proof of (a), so we omit the details here. □

Lemma 5.4 is used in the proof of Lemma 5.5 and Lemma 5.6.

Lemma 5.5. *If* $r_{i,x} > r_{x+1,j}$ ($1 \le i \le j \le n$) *for all* x = *i*, ..., $j-1$, *then*

$$\tilde{Q}_i = \cdots = \tilde{Q}_j = F^{-1}\left(\frac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} UC_k + \sum_{k=i}^{j} OC_k}\right)$$ *is an optimal solution to* $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$.

Proof. See Appendix B.2. □

Lemma 5.5 indicates the optimal solutions of restricted problems $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$ under certain { $r_{i,j}$ } relationship. Note that if *i* = *j*, Lemma 5.5 shows that $\tilde{Q}_i = F^{-1}(\frac{UC_i}{UC_i + OC_i})$ is the optimal solution to the problem $\mathbf{B}_{i,i}(\boldsymbol{\sigma}^0)$, which is the result of Newsvendor model.

Lemma 5.6. *If* $r_{i,x} > r_{x+1,k}$ *for all x = i, ..., k - 1,* $r_{k+1,x} > r_{x+1,j}$ *for all x = k + 1, ..., j - 1, and* $r_{i,k} > r_{k+1,j}$, *then* $r_{i,x} > r_{x+1,j}$ *holds for all x = i, ..., j - 1.*

Proof.

As $r_{i,x} > r_{x+1,k}$ for all *x* = *i*, ..., *k* - 1, according to Lemma 5.4 (a), $r_{i,x} > r_{i,k}$ for all *x* = *i*, ..., *k* - 1. Note that $r_{i,k} > r_{k+1,j}$, according to Lemma 5.4 (a), $r_{i,k} > r_{i,j}$. Thus, $r_{i,x} > r_{i,j}$ for all *x* = *i*, ..., *k* - 1. Therefore we know that $r_{i,x} > r_{x+1,j}$ for all *x* = *i*, ..., *k* - 1 (otherwise, if $r_{i,x} \le r_{x+1,j}$ for any *x* = *i*, ..., *k* - 1, then $r_{i,x} \le r_{i,j}$ for the *x* according to Lemma 5.4 (b)).

As $r_{k+1,x} > r_{x+1,j}$ for all *x* = *k* + 1, ..., *j* - 1, according to Lemma 5.4 (a), $r_{k+1,j} > r_{x+1,j}$ for all *x* = *k* + 1, ..., *j* - 1. Note that $r_{i,k} > r_{k+1,j}$, according to Lemma 5.4

(a), $r_{i,j} > r_{k+1,j}$. Thus $r_{i,j} > r_{x+1,j}$ for all $x = k + 1, ..., j - 1$. Therefore we know that $r_{i,x} > r_{x+1,j}$ for all $x = k + 1, ..., j - 1$ (otherwise, if $r_{i,x} \le r_{x+1,j}$ for any $x = k + 1, ..., j - 1$, then $r_{i,j} \le r_{x+1,j}$ for the x according to Lemma 5.4 (b)).

We therefore can conclude that $r_{i,x} > r_{x+1,j}$ holds for all $x = i, ..., k - 1, k + 1, ..., j - 1$. Also note that $r_{i,k} > r_{k+1,j}$, Thus $r_{i,x} > r_{x+1,j}$ holds for all $x = i, ..., j - 1$. □

Lemma 5.6 shows an important property of the relationship of { $r_{i,j}$ }, which is used to derive the optimal solution procedure.

The following lemma shows that optimal Q values follow the same order with the values of $r_{i,j}$.

Lemma 5.7. *If* $r_{i,t} \le r_{t+1,j}$ $(1 \le i \le t < j \le n)$*, then*

$$F^{-1}\left(\frac{\sum_{k=i}^{t} UC_k}{\sum_{k=i}^{t} UC_k + \sum_{k=i}^{t} OC_k}\right) \le F^{-1}\left(\frac{\sum_{k=t+1}^{j} UC_k}{\sum_{k=t+1}^{j} UC_k + \sum_{k=t+1}^{j} OC_k}\right).$$

Proof. According to the definition of $r_{i,j}$, $r_{i,t} \le r_{t+1,j}$ means $\frac{\sum_{k=i}^{t} UC_k}{\sum_{k=i}^{t} OC_k} \le \frac{\sum_{k=t+1}^{j} UC_k}{\sum_{k=t+1}^{j} OC_k}$,

which implies $\frac{\sum_{k=i}^{t} UC_k}{\sum_{k=i}^{t} UC_k + \sum_{k=i}^{t} OC_k} \le \frac{\sum_{k=t+1}^{j} UC_k}{\sum_{k=t+1}^{j} UC_k + \sum_{k=t+1}^{j} OC_k}$.

Thus $F^{-1}(\frac{\sum_{k=i}^{t} UC_k}{\sum_{k=i}^{t} UC_k + \sum_{k=i}^{t} OC_k}) \le F^{-1}(\frac{\sum_{k=t+1}^{j} UC_k}{\sum_{k=t+1}^{j} UC_k + \sum_{k=t+1}^{j} OC_k})$. □

By using Lemma 5.7, we can decide the rank order of optimal-Q based on the order of $r_{i,j}$, which can be used to show the condition in Lemma 5.3.

Based on Lemmas 5.3, 5.5, and 5.7, we can obtain an optimal solution to the restricted problem if we are able to group the $r_{i,j}$'s in the form of $r_{k_0+1,k_1} \le r_{k_1+1,k_2} \le \cdots \le r_{k_{m-1}+1,k_m}$ for some m where $k_0 = 0$ and $k_m = n$ such that $r_{k_{j-1}+1,x} > r_{x+1,k_j}$ for all $x = k_{j\text{-}1} + 1, \ldots, k_j - 1$ and $j = 1, \ldots, m$. We therefore can develop the following optimal solution procedure for solving problem $\mathbf{B}(\boldsymbol{\sigma}^0)$.

Step 1: Calculate all $r_{k,k}$ ($k = 1, ..., n$), let $R = \{r_{1,1}, r_{2,2}, ..., r_{n,n}\}$.

Step 2: Check the r values in R. If the r values are non-decreasing, go to Step 4.

Step 3: Suppose that the $r_{i,k}$ and $r_{k+1,j}$ are the first adjacent pair such that $r_{i,k} > r_{k+1,j}$, update R by replacing the pair $r_{i,k}$ and $r_{k+1,j}$ by $r_{i,j}$ and go back to Step 2.

Step 4: With $R = \{r_{k_0+1,k_1}, r_{k_1+1,k_2}, ..., r_{k_{m-1}+1,k_m}\}$ for some m where $k_0 = 0$ and $k_m = n$, the optimal solution is $\tilde{Q}_{k_{i-1}+1} = \cdots = \tilde{Q}_{k_i} = F^{-1}\left(\dfrac{\sum_{k=k_{i-1}+1}^{k_i} UC_k}{\sum_{k=k_{i-1}+1}^{k_i} UC_k + \sum_{k=k_{i-1}+1}^{k_i} OC_k} \right)$, $i = 1, ..., m$.

Above procedure is to find a non-decreasing $r_{i,j}$ values, i.e., $r_{k_0+1,k_1} \le r_{k_1+1,k_2} \le \cdots \le r_{k_{m-1}+1,k_m}$ so as to obtain the optimal solution. When $r_{i,k}$ and $r_{k+1,j}$ are combined to $r_{i,j}$ in Step 3, Lemma 5.6 guarantees that the conditions $r_{i,x} > r_{x+1,j}$ hold for all $x = i, \ldots, j - 1$. It follows from Lemmas 5.3, 5.5, and 5.7 that the solution obtained from Step 4 is an optimal solution for the problem $\mathbf{B}(\boldsymbol{\sigma}^0)$. Recall that the problem $\mathbf{B}(\boldsymbol{\sigma}^0)$ is equivalent to the problem $\mathbf{A}(\boldsymbol{\sigma}^0)$, so the optimal solution of $\mathbf{B}(\boldsymbol{\sigma}^0)$ is also the optimal solution of $\mathbf{A}(\boldsymbol{\sigma}^0)$.

In the next section, we will present a branch-and-bound algorithm to search over all possible permutations to obtain the optimal solution to the general problem **A**. We will also develop a more efficient greedy heuristic procedure for problem **A**.

5.4 Branch-and-bound algorithm and heuristic procedure for the problem A

In this section, we will present a branch-and-bound procedure to solve problem **A**. We will also outline a greedy heuristic procedure. We begin by describing our branching.

5.4.1 Branching

Our branching is done on the $\sigma_i \ (i = 1, ..., n)$ values. The search tree has n levels, which corresponds to the n values $\sigma_1, \sigma_2, ..., \sigma_n$ of any permutation $\boldsymbol{\sigma}$. Several fathoming rules (see Section 5.4.2) are used in narrowing the selection of σ_k at level k ($k = 1, ..., n-1$) when we expand the tree. We can also generate an upper bound and a lower bound (feasible solution) for every node at level k (see Section 5.4.3). The search terminates after all nodes have been explored. After the procedure is terminated, the best lower bound obtained is the optimal value to problem **A**.

5.4.2 Fathoming rules

Before we discuss the fathoming rules, we first present a preprocessing rule, which is described in the following lemma.

Lemma 5.8. *For component i = 1, ..., n, if $c_i^{e_i} - c_i^1 > P(l_i^{e_i}) - P(l_i^1)$, e_i = 2, ..., m_i, then procurement channel e_i is never used in the optimal solution of the problem* **A**.

Proof. On the one hand, the increase of selling price of final product is less than (if there is no delivery of final product at time $l_i^{e_i}$) or equal to (if there is delivery of final product at time $l_i^{e_i}$) $P(l_i^{e_i}) - P(l_i^1)$ if we use procurement channel $e_i\ (=2,...,m_i)$ instead of procurement channel 1 to procure additional component i. On the other hand, the increase of purchasing cost of component i is equal to $c_i^{e_i} - c_i^1$ if procurement channel e_i rather than procurement channel 1 is adopted for procurement of component i. Channel e_i for procurement of component i therefore is not cost effective if the increase of purchasing cost of component i outweighs the increase of selling price of final product. □

The preprocessing rule shown in Lemma 5.8 can be used to exclude some procurement channels from consideration. We now develop the following lemma which shows that optimal permutation of pre-stocked quantities follows certain rank order under certain parameter conditions.

Lemma 5.9. *If* $l_i^1 \le l_j^{m_j}$, *then* $\tilde{Q}_i \le \tilde{Q}_j$ *in the optimal solution.*

Proof. If $l_i^1 \le l_j^{m_j}$, arrival time of additional component *i* ordered by procurement channel 1 is earlier than the arrival time of additional component *j* (ordered by any procurement channel 1, 2, …, m_j). Therefore, it is not cost efficient to order more pre-stocked component *i* than pre-stocked component *j* in the initial ordering. □

Note that if we know the optimal solution satisfies $\tilde{Q}_i \le \tilde{Q}_j$ according to Lemma 5.9, then component *j* cannot be chosen as σ_k at level *k* unless component *i* has been chosen at an earlier level. This can be used as fathoming rule when we branch the tree. Besides, the upper bound and lower bound (feasible solution) we generated for every node can also be used as fathoming rules as follows:

(i) When the upper bound of a node is equal to the lower bound of the same node, the sub-tree whose root is this node can be discarded;

(ii) When the upper bound of a node is less than or equal to an existing best lower bound, this node can be discarded.

5.4.3 Bounding

To compute the upper bound and the lower bound for every node at each level, we will utilize the solution procedure for the restricted problem which requires a complete permutation $\boldsymbol{\sigma}$. Note that for any node at level k $(k=1,...,n-1)$, components $\sigma_1,\sigma_2,...,\sigma_k$ have been assigned from level 1 to level *k*. That is, we only have a partial permutation of $\boldsymbol{\sigma}$. Thus, we will build upon this partial permutation to obtain a complete permutation necessary for our solution procedure. Based on this complete permutation, we will use two different cost settings to obtain its respected upper and lower bounds.

Observe that, for each component, if we set its cheapest procurement cost to any faster procurement channels, then the optimal solution to this problem provides an upper bound to the original restricted problem. With this idea, we will modify the problem such that each unassigned component will have only one procurement channel with the shortest lead time at the cheapest cost.

Note that t_k can be computed for given $\sigma_1,\sigma_2,...,\sigma_k$ which are independent of the cost and lead time of the unassigned components. According to Lemma 5.2, all unassigned components will not used to assemble products delivered at batch $k + 1$ before time t_k. Thus, for any component that has more than one procurement channels prior to t_k, only the slowest one needs to be considered in the modified problem for generating the upper bound. Denote the set of all unassigned components at level k as Λ_k . For each $i \in \Lambda_k$, the lead time of its single procurement channel for the modified problem is given by

$$\tilde{l}_i^1 = \begin{cases} l_i^{m_i} & \text{if } l_i^{m_i} \geq t_k \\ \max\left\{ l_i^{e_i} \middle| l_i^{e_i} < t_k \right\} & \text{if } l_i^{m_i} < t_k \end{cases}$$

with the procurement cost $\tilde{c}_i^1 = c_i^1$.

It follows from Lemma 5.9 that, for the modified problem, it is optimal to assign all unassigned components according to their non-decreasing order of lead times to form a complete permutation. Therefore, with a complete permutation, we can then use the solution procedure presented in Section 5.3 to obtain an upper bound of the node. On the other hand, we can obtain the lower bound solution for the node by using the original procurement costs associated with each of the procurement channels considered under this modified problem. Note that this lower bound solution is also a feasible solution.

5.4.4 Heuristics

In this section, we describe a solution procedure based on a greedy heuristic. We attempt to sequentially assign components until a complete permutation is obtained. At each step, we use the same technique utilized in our branch-and-bound algorithm to assign a component to a partial permutation. Fathoming rules provided in Section 5.4.2 are used to reduce the number of possible candidates for the next assignment. The component which has the largest lower bound (see Section 5.4.3) will be chosen as the next assignment in the partial permutation. We keep repeating the procedure until all components have been assigned. With a complete permutation, we use the procedure described in Section 5.3 to obtain the solution of this heuristic procedure.

5.5 Computational studies

In this section, we apply our solution approach to a dual-channel case, i.e. $m_i = 2$ for all i. We call channel 1 and 2 as normal and expediting procurement channel respectively. We first compare dual-channel solution with two single-channel solutions (set $m_i = 1$ for all i), i.e. either only normal channels (studied in Hsu et al. 2006) or only expediting channels (studied in Fu et al. 2009) are allowed for replenishing all components. We then compare the performance of the optimal branch-and-bound procedure and that of the heuristic procedure in terms of the solution quality and number of nodes explored.

5.5.1 Comparison between dual-channel solution and single-channel solution

We use a normal distribution to generate three demand patterns, where the demand mean and coefficient of variance are shown in Table 5.1. Number of components, lead times, costs and salvage values settings are also listed in Table 5.1. The price function for the final product has the following three forms: quadratic function of time $P_1(t) = a - b_1 \cdot t^2$ (representing a non-increasing slope price function), linear function of time $P_2(t) = a - b_2 \cdot t$ and square root function of time $P_3(t) = a - b_3 \cdot \sqrt{t}$ (representing a non-decreasing slope price function). The constant part a, which is the price for delivering the final product upon realization of the demand (i.e. $t = 0$), is generated as a percentage of the total component normal channel procurement cost. b_1, b_2 and b_3 are generated to make sure that $P_1(50) = P_2(50) = P_3(50) = 0$. The values a, b_1, b_2 and b_3 are shown in Table 5.1. The forms of three price functions are depicted in Figure 5.1.

Table 5.1 Parameters for test problems

Parameter	Value
Number of components	4 8
Demand mean	U(100, 200)[a]
Demand coefficient of variance	0.5 1 2
Normal channel lead time	U(2, 50)
Expediting channel lead time	normal channel lead time $\times$U(0.2, 0.8)
Normal channel cost	U(10, 100)
Expediting channel cost	normal channel cost $\times$U(1.2, 2)
Salvage value	normal channel cost $\times$U(0.1, 0.9)
Price function	$P_1(t) = a - b_1 \cdot t^2$, $P_2(t) = a - b_2 \cdot t$, $P_3(t) = a - b_3 \cdot \sqrt{t}$ $a = \sum_i c_i^1 \times$U(1.5, 3.5) $b_1 = a \times 0.04\%$, $b_2 = a \times 2\%$, $b_3 = a \times 1/\sqrt{50}$

[a]U(100, 200): Uniform distribution between 100 and 200

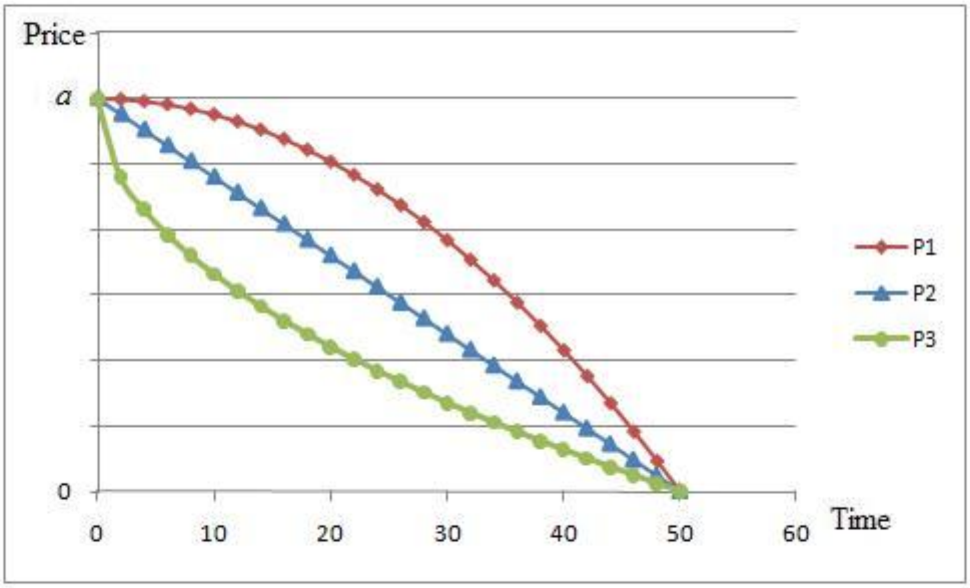

Figure 5.1 Three forms of price functions

Note that the overstocking cost OC_i for each excess unit of pre-stocked component *i*, the maximum unit profit UP_k at each batch *k* and the optimal delivery time t_k at each batch *k* are all independent of product demand *D*. Thus, only changing the mean of demand doesn't affect quantity-independent optimal decisions, which include the selection of procurement channels and final product delivery schedule. In other words, mean demand does not affect the gap between the dual-channel solution and the single-channel solution. Thus, we only consider one demand mean in the experiment studies of this section.

Table 5.2 shows comparisons between our dual-channel solutions and single-channel solutions under different scenarios. Observe that the gap between dual-channel solution and single-channel solution increases with the coefficient of variance of demand. This observation indicates that the higher variation of product demand, the more benefit is the dual-channel sourcing of components. And dual-channel sourcing of components can bring more than 10% profit increase for the ATO manufacturer who faces high uncertain product demand. The reason can be explained as follows. With the increase of the coefficient of variance of the demand, the variation of the demand increases. Accordingly, the ability of the pre-stocked quantities **Q** to match realized demand reduces; which in turn leads to the decrease of the percentage of the first delivery of final product among total deliveries of final product. In other words, more deliveries of final product will be made after time 0 and these final products are assembled by some additional components procured. In this case, the dual-channel sourcing offers more significant economic benefits than the single-channel sourcing because it gives the manufacturer more options to acquire additional components.

Table 5.2 Comparison between dual-channel and single-channel solutions

Number of components	CV[a]	Price function					
		$P_1(t)$		$P_2(t)$		$P_3(t)$	
		Gap_nor[b]	Gap_exp[c]	Gap_nor	Gap_exp	Gap_nor	Gap_exp
4	0.5	2.76%	1.56%	1.46%	1.14%	0.46%	0.96%
	1	4.86%	2.75%	2.66%	2.09%	0.85%	1.79%
	2	8.34%	4.74%	4.85%	3.81%	1.60%	3.36%
8	0.5	3.37%	1.77%	1.99%	0.94%	0.90%	0.70%
	1	5.91%	3.05%	3.60%	1.69%	1.64%	1.28%
	2	10.10%	5.02%	6.43%	3.01%	3.01%	2.33%

[a]CV: coefficient of variance of demand
[b]Gap_nor = (dual – normal)/dual $\times$ 100%
[c]Gap_exp = (dual - expediting)/dual $\times$ 100%

We now turn to investigate the effects of pricing functions on performances of various sourcing structures. Note that the three types of price functions represent three different types of products in terms of product price erosion. The non-increasing slope price function represents the product whose price decreases slowly during the time immediately after demand realization; the price then drops significantly as the time goes by. The linear price function represents the product whose price decreases at the same rate at all times. The non-decreasing slope price function represents the product whose price drops deeply initially after time 0; but the price erosion stabilizes over time. We find from Table 5.2 that the non-increasing slope price function gives the highest gap between dual-channel solution and single-channel solution, the linear price function gives medium size gap and the non-decreasing slope price function has the lowest gap (given that the prices at time 0 are the same for all three types of functions, and the prices at the largest lead time point are same too). The reasons can be explained as follows. For the non-increasing slope price function, we tend to order less pre-stocked quantity as understocking cost is less. We will be more likely to expedite more components later. That is, more deliveries of final product will be made after time 0 and these final products are assembled by some additional components procured. In this case, the dual-channel sourcing offers more significant economic benefits than the single-channel sourcing because it gives the manufacturer more options to acquire additional components. Besides, Gap_nor is higher than Gap_exp in the non-increasing slope price function case as the benefit of using the expediting sourcing to capture higher final product price is significant in the non-increasing slope price function case under given parameter settings. For the non-decreasing slope price function case, we tend to order more pre-stocked quantity as understocking cost is high. That is, less deliveries of final product will be made after time 0. Therefore, the economic benefit of the dual-channel sourcing over the single-channel sourcing is not significant. For the liner price function case, the effect is in between that of non-increasing slope price function case and non-decreasing slope price function case.

5.5.2 Comparison between the optimal branch-and-bound procedure and the heuristic procedure

We use the same test problems generated in Section 5.5.1 to compare the performance between the optimal branch-and-bound procedure and the heuristic procedure. Both solution procedures are coded in MATLAB and the branch-and-bound algorithm uses the breadth-first branching strategy. Table 5.3 reports the comparison results. Note that we use the numbers of nodes explored instead of computation times to compare the heuristic procedure and the optimal branch-and-bound procedure as we would like the results to be independent from the performance of MATLAB and computer configuration. We find that our heuristic procedure can always find the optimal solution for our 4-component problem case and the number of nodes explored for the optimal branch-and-bound procedure and the heuristic procedure are both less than 10. For 8-component problem case, our optimal branch-and-bound procedure needs to explore about 900 nodes while the heuristic procedure only needs to explore about 20 nodes. From the results, we observe that our heuristic procedure performs quite well in terms of solution quality and number of nodes explored. Especially when the number of components is large, our heuristics can explore significant fewer nodes while obtaining a sufficiently good solution.

Table 5.3 Comparison between the heuristic procedure and the optimal branch-and-bound procedure

n[a]	CV	Price function											
		$P_1(t)$				$P_2(t)$				$P_3(t)$			
		Heuristic solution status	Nodes explored			Heuristic solution status	Nodes explored			Heuristic solution status	Nodes explored		
			Opt[b]	Heu[c]	Heu/Opt		Opt	Heu	Heu/Opt		Opt	Heu	Heu/Opt
4	0.5	Optimal	5	4	80%	Optimal	5	4	80%	Optimal	5	4	80%
	1	Optimal	5	4	80%	Optimal	5	4	80%	Optimal	5	4	80%
	2	Optimal	5	4	80%	Optimal	5	4	80%	Optimal	5	4	80%
8	0.5	Gap[d] = 0.030%	853	20	2.3%	Optimal	921	22	2.4%	Optimal	921	22	2.4%
	1	Gap = 0.051%	873	20	2.3%	Optimal	921	22	2.4%	Optimal	921	22	2.4%
	2	Gap = 0.082%	873	20	2.3%	Optimal	921	22	2.4%	Optimal	921	22	2.4%

[a]n: number of components
[b]Opt: number of nodes explored for the optimal branch-and-bound procedure
[c]Heu: number of nodes explored for the heuristic procedure
[d]Gap: (optimal solution – heuristic solution)/optimal solution $\times 100\%$

5.6 Summary

In this section, we extend our study on dual-channel two-component problem to multi-channel multi-component problem. We solve our problem analytically. We first formulate and solve a restricted version of our problem in which the quantities of pre-stocked components follow a certain fixed rank order. Then, we develop a branch-and-bound method to solve the general problem by searching over all rank orders of pre-stocked components and solving the corresponding restricted problems. A simple heuristic procedure is also developed. We finally present computational studies to demonstrate the efficiency of our solution methods and to compare the performance of ATO systems with single and dual procurement channels, respectively.

Chapter 6 CONCLUSIONS AND FUTURE RESEARCH

This thesis studied inventory consideration and management in a strategic supply chain problem and in an operational supply chain problem. The strategic supply chain problem studied is a multi-source facility location-allocation and inventory problem and the operational supply chain problem studied is multi-channel component replenishment problem in an assemble-to-order system.

6.1 Multi-source facility location-allocation and inventory problem

In this thesis, we study a joint facility location-allocation and inventory problem which incorporates multiple sources of warehouses. A mixed integer nonlinear programming model is formulated and a solution procedure is developed to solve the proposed model. A lower bound of the model is also generated for comparison purpose. In order to show the quality of the solution found by the proposed solving procedure, a series of generated test problems are solved. The model and solution method are also applied to a case study.

Results show that the gap between our solution and two-stage solution increases with the increase of inventory holding cost rate, and the gap is quite significant under high inventory holding cost rate. The reason for this is obvious. With high inventory holding cost rate, the inventory holding cost has high weight in total cost. Considering inventory holding cost with transportation cost simultaneously can provide a better solution compared to the solution obtained by the simple two-stage procedure. Results also show that the gap between our solution and two-stage solution increases slightly with the increase of coefficient of variance of demand. The reason can be explained as follows. With the increase of coefficient of variance of demand, the safety stock holding cost increases. Consequently, the total inventory holding cost increases. Simultaneously considering both inventory holding cost and transportation cost can achieve better solution than the solution obtained by two-stage method.

We can obtain from the results that the proposed solution method performs well. The gap between the solution obtained by our heuristic method and the lower bound ranges from 0.78% to 8.93%. Most gaps are relatively small. Note that safety stock is a part (about 10% – 30%) of *TC*, and some gaps are a little bit high. The reason is mainly due to the using of underestimated linear functions of safety stock in the model *PUL*. With the increase of the coefficient of variance of demand, the inventory holding cost rate and the variety of replenishment lead times from plants to warehouses, the gap between our underestimated linear expression of safety stock and the original nonlinear form of safety stock increases. The lower bound therefore becomes not very tight. As a consequence, the gap between our solution and the lower bound increases.

We can also obtain from the results that there are tradeoff solutions between total cost and cycle service level and between total cost and demand weighted average customer lead time for the decision makers. The total cost increases when one of the following two situations occurs: (a) desired cycle service level increases, (b) desired demand weighted average customer lead time decreases. The reason for the increase of the total cost when situation (a) happens is obvious. If the desired cycle service level increases, the inventory levels in warehouses will inevitably increase, which leads to an increase in inventory holding cost. The reason that the total cost increases when situation (b) happens can be explained as follows. If the desired demand weighted average customer lead time decreases, more customers have to be served by local warehouses (with short lead time) instead of being replenished by plants directly (with long lead time). This will increase inventory holding cost of warehouses and as a consequence, total cost increases.

Compared to the traditional sequential decision process, in which the facility location-allocation problem is considered first and then inventory problem is studied based on given facility location-allocation decisions, this study indicates that it is quite important and meaningful to consider the inventory policy in the facility location-allocation problem. This also follows the trend that inventory management has become more and more important in various logistics and supply chain problems. Therefore, this study can be applied in distribution network design problems in various kinds of industry. It can also be used in healthcare problems (e.g. blood storage points locating problem). Similar applications can be found in other areas.

The problem, model and method we presented are valuable extensions to existing facility location-allocation research. However, there are some limitations to this study. In this study, only the (r, S) ("review period, order-up-to-level") inventory policy is used and the review period for each warehouse is fixed as this policy is easy to implement in the real-world applications. However, there are several other kinds of inventory policies in real-world applications. Another limitation is the proportional transportation cost assumption, which is adopted in order to reduce the complexity of the model. Therefore, important additional research can come from this study. Specifically, proposed future research can take the following several directions:

(1) Other inventory policies, such as (s, Q) ("order point, order quantity") inventory policy, can be considered in the future research.

(2) More practical ways of expressing real transportation cost (e.g. fixed and per-unit transportation cost) may be adopted instead of proportional transportation cost in order to make the model more realistic.

(3) In some real world problems, companies may give price discount to the customers that have long lead time. Accordingly, the revenue of the companies relates to the design of the distribution network. Therefore, we may use maximizing the expected total profit rather than minimizing the expected total cost as the main objective.

6.2 Multi-channel component replenishment problem in an assemble-to-order system

In the study of the multi-channel component replenishment problem in an assemble-to-order system, we first study the dual-channel two-component problem. A closed-form optimal solution to the dual-channel two-component problem is provided. We then extend our study to the multi-channel multi-component problem and we solve the problem analytically. We first present a restricted version of the problem where the pre-stocked components quantities follow a certain permutation and we develop an optimal solution procedure for solving the restricted problem. We then provide an optimal branch-and-bound procedure which searches over all permutations to obtain an optimal solution to the general problem. A simple greedy heuristic procedure is also developed. We finally present computational studies to demonstrate the efficiency of our solution methods and to compare the performance of ATO systems with single and dual procurement channels, respectively. Some managerial insights are obtained based on the results of computational studies.

Results show that the gap between dual-channel solution and single-channel solution increases with the coefficient of variance of demand. This observation indicates that the higher variation of product demand, the more benefit is the dual-channel sourcing of components. And dual-channel sourcing of components can bring more than 10% profit increase for the ATO manufacturer who faces high uncertain product demand. The reason can be explained as follows. With the increase of the coefficient of variance of the demand, the variation of the demand increases. Accordingly, the ability of the pre-stocked quantities **Q** to match realized demand reduces; which in turn leads to the decrease of the percentage of the first delivery of final product among total deliveries of final product. In other words, more deliveries of final product will be made after time 0 and these final products are assembled by some additional components procured. In this case, the dual-channel sourcing offers more significant economic benefits than the single-channel sourcing because it gives the manufacturer more options to acquire additional components.

Results also show that the non-increasing slope price function gives the highest gap between dual-channel solution and single-channel solution, the linear price function gives medium size gap and the non-decreasing slope price function has the lowest gap (given that the prices at time 0 are the same for all three types of functions, and the prices at the largest lead time point are same too). The reasons can be explained as follows. For the non-increasing slope price function, we tend to order less pre-stocked quantity as understocking cost is less. We will be more likely to expedite more components later. That is, more deliveries of final product will be made after time 0 and these final products are assembled by some additional components procured. In this case, the dual-channel sourcing offers more significant economic benefits than the single-channel sourcing because it gives the manufacturer more options to acquire additional components. Besides, Gap_nor is higher than Gap_exp in the non-increasing slope price function case as the benefit of using the expediting sourcing to capture higher final product price is significant in the non-increasing slope price function case under given parameter settings. For the non-decreasing slope price function case, we tend to order more pre-stocked quantity as understocking cost is high. That is, less deliveries of final product will be made after time 0. Therefore, the economic benefit of the dual-channel sourcing over the single-channel sourcing is not significant. For the liner price function case, the effect is in between that of non-increasing slope price function case and non-decreasing slope price function case.

We also can find from results that our heuristic procedure can always find the optimal solution for our 4-component problem case and the number of nodes explored for the optimal branch-and-bound procedure and the heuristic procedure are both less than 10. For 8-component problem case, our optimal branch-and-bound procedure needs to explore about 900 nodes while the heuristic procedure only needs to explore about 20 nodes. From the results, we observe that our heuristic procedure performs quite well in terms of solution quality and number of nodes explored. Especially when the number of components is large, our heuristics can explore significant fewer nodes while obtaining a sufficiently good solution.

There are several directions where future research can be conducted. Firstly, ordering setup costs and assembling setup costs are ignored in this thesis, but these setup costs do exist in real world applications although they are usually not high. Therefore, future research can incorporate the setup costs so as to make the model more accurate. Secondly, the optimal solution procedure developed for multi-channel multi-component model in this thesis is not quite efficient. Future research can consider more efficient optimal solution methods for multi-channel multi-component problem. Thirdly, only one type of product is considered in this study. In the real world, more than one product is not uncommon. Future work therefore can study an ATO system with multiple final products sharing multiple components problem which can be replenished through multiple supply channels.

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APPENDICES

APPENDIX A

A.1 Difficulty of determining multi-source safety stock level

Suppose we consider a single-product problem with one opened warehouse which is replenished by two plants simultaneously. We therefore can drop the subscript index j and f in below analysis. We let μ and σ denote the mean and standard deviation of annual demand of the warehouse respectively (note that $\mu = \sum_k (d_k W_k)$ and $\sigma = \sqrt{\sum_k (\sigma_k^2 W_k)}$); D_r denotes demand in review period r (note that it is a random variable and it is the total order quantity at each replenishment cycle); R ($0 \le R \le 1$) and $1 - R$ denote the proportions of the total annual quantities ordered from two plants respectively (which can be determined from the solution of our model). Without loss of generality, we let $tpw_1 \le tpw_2$.

Figure A.1 shows inventory levels of a warehouse replenished by two plants under a general implementation of (r, S) inventory policy, where $R_c D_{rc}$ ($0 \le R_c \le 1$) and $(1 - R_c)D_{rc}$ denote quantities ordered from two plants at replenishment cycle c respectively. Note that R_c is not fixed for each replenishment cycle c but the overall proportions of quantities ordered from two plants equal to R and 1- R respectively, i.e. $\sum_{c=1}^{N} R_c = NR$, where N is the total number of replenishment cycles. From Figure A.1, we find that it is difficult to compute the real constant safety stock level given a certain cycle service level as R_c's are not known. Even if we know the exact implementation, it is still difficult to compute the real constant safety stock level for a desired service level as R_c may vary from one replenishment cycle to another.

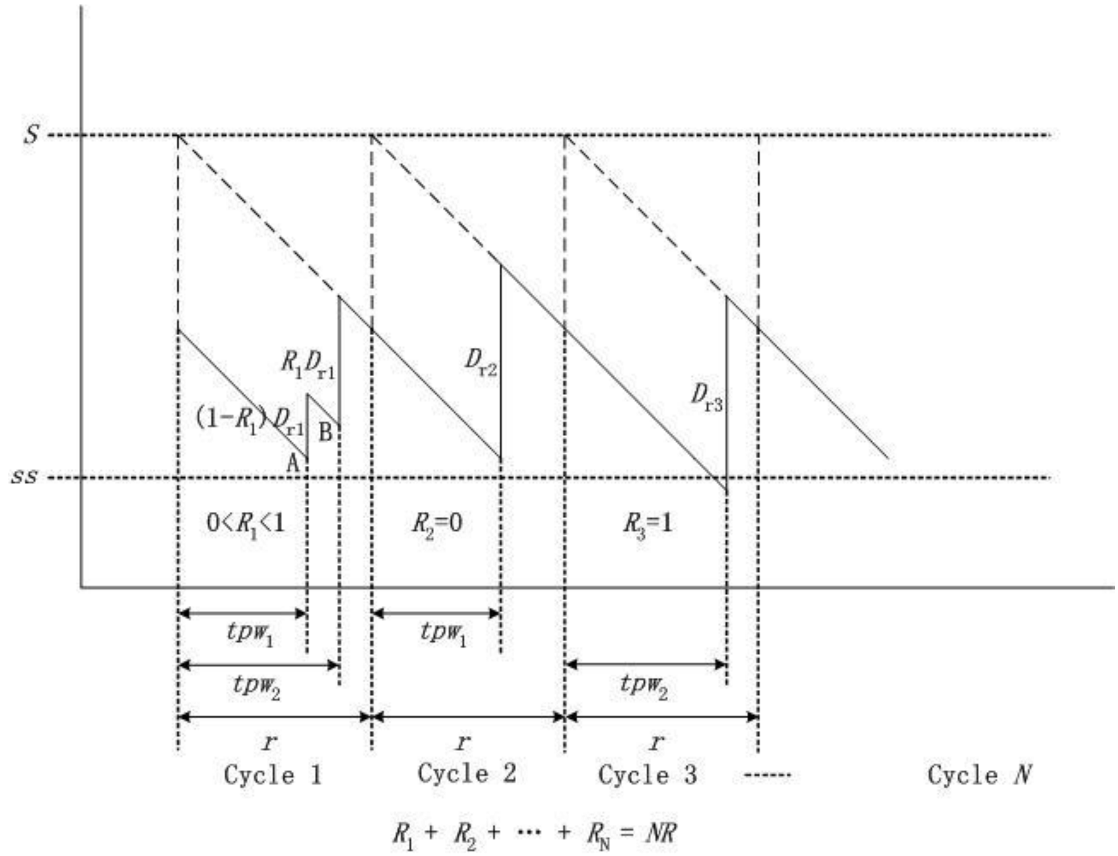

Figure A.1 Inventory position (dashed line) and on-hand inventory (solid line) of a warehouse replenished by two plants (General Implementation)

A.2 Analysis of cycle service level

As described in Section 3.2.3, our proposed safety stock level is given by $SS_{jf} = z\sqrt{\sum_k(\sigma_{kf}^2 W_{jk})}\sqrt{r_j + \sum_i(tpw_{ij}X_{ijf})\big/\sum_i X_{ijf}}$. Note that ε can be neglected here as its effect to safety stock level is very small. Also note that we do not need to analyze the case of $\sum_i X_{ijf} = 0$ as $SS_{jf} = 0$ when $\sum_i X_{ijf} = 0$, i.e., our analysis is only for a positive safety stock level.

Note that it is difficult to directly compare the real constant safety stock level with our proposed safety stock level due to the difficulty of computing the real constant safety stock level. Our idea is to compare desired cycle service level and actual cycle service level based on proposed safety stock level. Note that if at least one of the three equalities ($tpw_1 = tpw_2$, $R = 0$, $R = 1$) holds, the problem is reduced to single-source problem and actual cycle service level based on proposed safety stock level is equal to desired cycle service level under any implementations. Thus, it is a trivial problem.

We then study the case when $tpw_1 < tpw_2$ and $0 < R < 1$. We now compute actual cycle service level for cycle c ($1 \le c \le N$) given desired cycle service level. If desired cycle service level is P_z (with corresponding safety factor z), we can compute order-up-to-level S according to our proposed safety stock and it is given as follows:

$$S = \mu \cdot [r + (1-R)tpw_1 + R \cdot tpw_2] + z \cdot \sigma \cdot \sqrt{r + (1-R)tpw_1 + R \cdot tpw_2}$$

Based on this order-up-to-level S, actual cycle service level P_c for cycle c is given by

$$P_c = \begin{cases} P\{S - D_r - D_{tpw_1} \ge 0) & R_c = 0 \\ P\{S - D_r - D_{tpw_1} \ge 0,\ \ S - D_r - D_{tpw_1} + (1-R_c)D_r - D_{tpw_2 - tpw_1} \ge 0\} & 0 < R_c < 1 \\ P\{S - D_r - D_{tpw_2} \ge 0) & R_c = 1 \end{cases}$$

The average actual cycle service level P is given by $P = \sum_{c=1}^{N} P_c \Big/ N$.

Note that R_c may vary from one replenishment cycle to another. Thus, it is difficult to analyze the average actual cycle service level under the general implementation given in Figure A.1. We therefore first study two extreme ways of implementation: (1) $R_c = R$ for all replenishment cycle c, (2) we order from only one plant at each replenishment cycle (R_c = 0 or 1), and the proportion of the two different replenishment cycles equals to $(1-R)/R$. We now compute average actual cycle service level under implementations (1) and (2).

P (under implementation (1))

$= P\{S - D_r - D_{tpw_1} \geq 0,\ \ S - D_r - D_{tpw_1} + (1-R)D_r - D_{tpw_2 - tpw_1} \geq 0\}$

$= P\{D_r + D_{tpw_1} \leq S,\ \ R \cdot D_r + D_{tpw_1} + D_{tpw_2 - tpw_1} \leq S\}$

Let $X_1 = D_r + D_{tpw_1}$ and $X_2 = R \cdot D_r + D_{tpw_1} + D_{tpw_2 - tpw_1}$

Actual cycle service level $= P\{X_1 \leq S,\ \ X_2 \leq S\} = \int_{-\infty}^{S} \int_{-\infty}^{S} f(x_1, x_2) dx_1 dx_2$

Note that D_r, D_{tpw_1} and $D_{tpw_2 - tpw_1}$ are independent normal random variables, therefore, the random variables X_1 and X_2 have bivariate normal distribution with mean and covariance matrix as follows:

$$\mu = (\mu_{X_1},\ \ \mu_{X_2}) \qquad \text{and} \qquad \Sigma = \begin{bmatrix} \sigma_{X_1}{}^2 & COV(X_1, X_2) \\ COV(X_1, X_2) & \sigma_{X_2}{}^2 \end{bmatrix}$$

where $\mu_{X_1} = (r + tpw_1)\mu$, $\mu_{X_2} = (R \cdot r + tpw_2)\mu$, $\sigma_{X_1} = \sigma\sqrt{r + tpw_1}$, $\sigma_{X_2} = \sigma\sqrt{R^2 r + tpw_2}$ and $COV(X_1, X_2) = (R \cdot r + tpw_1)\sigma^2$. We can easily calculate $\int_{-\infty}^{S} \int_{-\infty}^{S} f(x_1, x_2) dx_1 dx_2$ by the MATLAB function mvncdf([S, S], μ, Σ) (this function is available in versions after 7.3).

P (under implementation (2))

$= (1-R)P(S - D_r - D_{tpw_1} \geq 0) + R \cdot P(S - D_r - D_{tpw_2} \geq 0)$

$= (1-R)P(D_r + D_{tpw_1} \leq S) + R \cdot P(D_r + D_{tpw_2} \leq S)$

$$= (1-R)\phi\left(\frac{S - (r + tpw_1)\mu}{\sigma\sqrt{r + tpw_1}}\right) + R \cdot \phi\left(\frac{S - (r + tpw_2)\mu}{\sigma\sqrt{r + tpw_2}}\right)$$

$$= (1-R)\phi\left(\frac{\frac{\mu}{\sigma} \cdot R(tpw_2 - tpw_1) + z\sqrt{r + (1-R)tpw_1 + R \cdot tpw_2}}{\sqrt{r + tpw_1}}\right)$$

$$+ R \cdot \phi\left(\frac{\frac{\mu}{\sigma}(1-R)(tpw_1 - tpw_2) + z\sqrt{r + (1-R)tpw_1 + R \cdot tpw_2}}{\sqrt{r + tpw_2}}\right)$$

We then compare desired cycle service levels with average actual cycle service levels under implementations (1) and (2). We also can adopt Monte-Carlo simulation to compare desired cycle service levels with average actual cycle service levels under general implementation. From simulation results, we find that:

- Our proposed safety stock level can result in a cycle service level that is very close to and a little bit higher than desired cycle service level if we adopt implementation (1) to implement (r, S) inventory policy.
- Our proposed safety stock level can result in a cycle service level that is very close to and a little bit lower than desired cycle service level if we adopt implementation (2).
- Our proposed safety stock level can result in a cycle service level that is very close to desired cycle service level if we adopt general implementation.

Therefore, our proposed safety stock level is a good approximation that can result in a cycle service level which is very close to desired cycle service level for two replenishment sources situation. We can extend our analysis to more than two replenishment sources situation. We consider n replenishment lead times tpw_1, tpw_2, …, tpw_n ($tpw_1 \le tpw_2 \le \cdots \le tpw_n$) from n replenishment sources respectively. Let R^1, R^2, …, R^n ($R^1 + R^2 + \cdots + R^n = 1$) denote the proportions of the total annual quantities ordered from n plants respectively; $R_c^1 D_{rc}$, $R_c^2 D_{rc}$, …, $R_c^n D_{rc}$ ($R_c^1 + R_c^2 + \cdots + R_c^n = 1$) denote quantities ordered from n plants at replenishment cycle c respectively. Note that R_c^1, R_c^2, …, R_c^n are not fixed for each replenishment cycle c but the overall proportions of quantities ordered from n plants equal to R^1, R^2, …, R^n respectively, i.e.

$\sum_{c=1}^{N} R_c^i = NR^i$ (i = 1, 2,…, n), where N is the total number of replenishment cycles. We then can use similar analysis to compare desired cycle service levels with average actual cycle service levels under extreme ways of implementation and use Monte-Carlo simulation to compare desired cycle service levels with average actual cycle service levels under general implementation, and we obtain similar results. Therefore, our proposed safety stock level is quite reasonable regardless of how an actual ordering policy is implemented.

APPENDIX B

B.1 Proof for the closed-form optimal solution

Proof. As (b) has been shown in Section 4.2, we only need to prove (a). We first consider the following two cases ([1] = 1 and [1] = 2) separately.

- Case 1: [1] = 1 and [2] = 2 (i.e. $Q_1 \le Q_2$)

  According to the result of newsvendor model, we know that if $\frac{UP_0 - UP_1}{UP_0 - UP_1 + OC_1} < \frac{UP_1 - UP_2}{UP_1 - UP_2 + OC_2}$ (i.e. $\frac{OC_1}{OC_2} > \frac{UP_0 - UP_1}{UP_1 - UP_2}$), the optimal value of Q_1 and Q_2 for case 1 is $Q_1 = F^{-1}(\frac{UP_0 - UP_1}{UP_0 - UP_1 + OC_1})$ and $Q_2 = F^{-1}(\frac{UP_1 - UP_2}{UP_1 - UP_2 + OC_2})$.

  If $\frac{UP_0 - UP_1}{UP_0 - UP_1 + OC_1} \ge \frac{UP_1 - UP_2}{UP_1 - UP_2 + OC_2}$ (i.e. $\frac{OC_1}{OC_2} \le \frac{UP_0 - UP_1}{UP_1 - UP_2}$), we can prove that $Q_1 = Q_2 = F^{-1}(\frac{UP_0 - UP_2}{UP_0 - UP_2 + OC_1 + OC_2})$ is the optimal solution for case 1 by K-T conditions as follows.

Proof. As newsvendor model is a convex function, so $\mathbf{A}(Q_1 \le Q_2)$ is a concave function. The constraint $Q_1 - Q_2 \le 0$ is a convex function. We can show that $(\bar{Q}_1, \bar{Q}_2)$ $[\bar{Q}_1 = \bar{Q}_2 = F^{-1}(\frac{UP_0 - UP_2}{UP_0 - UP_2 + OC_1 + OC_2})]$ satisfies the following hypothesis, therefore $(\bar{Q}_1, \bar{Q}_2)$ is an optimal solution to $\mathbf{A}(Q_1 \le Q_2)$.

Hypothesis: We can find a multiplier λ_1 satisfying

$$\left.\frac{\partial \mathbf{A}(Q_1 \le Q_2)}{\partial Q_1}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)} - \lambda_1 \left.\frac{\partial (Q_1 - Q_2)}{\partial Q_1}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)} \le 0 \tag{B.1}$$

$$\left.\frac{\partial \mathbf{A}(Q_1 \le Q_2)}{\partial Q_2}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)} - \lambda_1 \left.\frac{\partial (Q_1 - Q_2)}{\partial Q_2}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)} \le 0 \tag{B.2}$$

$$\lambda_1 [0 - (\bar{Q}_1 - \bar{Q}_2)] = 0 \tag{B.3}$$

$$\left[\left.\frac{\partial \mathbf{A}(Q_1 \le Q_2)}{\partial Q_1}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)} - \lambda_1 \left.\frac{\partial (Q_1 - Q_2)}{\partial Q_1}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)}\right] \bar{Q}_1 = 0 \tag{B.4}$$

$$\left[\left.\frac{\partial \mathbf{A}(Q_1 \le Q_2)}{\partial Q_2}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)} - \lambda_1 \left.\frac{\partial (Q_1 - Q_2)}{\partial Q_2}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)}\right] \bar{Q}_2 = 0 \tag{B.5}$$

$$\lambda_1 \ge 0 \tag{B.6}$$

As $\bar{Q}_1 = \bar{Q}_2$, (B.3) can be satisfied. As $\bar{Q}_1 = \bar{Q}_2 \ne 0$, in order to ensure (B.1), (B.2), (B.4) and (B.5) hold, (B.7) and (B.8) must hold.

$$\left.\frac{\partial \mathbf{A}(Q_1 \le Q_2)}{\partial Q_1}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)} - \lambda_1 \left.\frac{\partial (Q_1 - Q_2)}{\partial Q_1}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)} = 0 \tag{B.7}$$

$$\left.\frac{\partial \mathbf{A}(Q_1 \le Q_2)}{\partial Q_2}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)} - \lambda_1 \left.\frac{\partial (Q_1 - Q_2)}{\partial Q_2}\right|_{(Q_1,Q_2)=(\bar{Q}_1,\bar{Q}_2)} = 0 \tag{B.8}$$

From (B.7), we can obtain the value of λ_1 as $\frac{OC_1(UP_2 - UP_1) + OC_2(UP_0 - UP_1)}{UP_0 - UP_2 + OC_1 + OC_2}$, and we can obtain the same value of λ_1 from (B.8). Recall that the given condition

$\dfrac{OC_1}{OC_2} \le \dfrac{UP_0 - UP_1}{UP_1 - UP_2}$, we can easily know that $\dfrac{OC_1(UP_2 - UP_1) + OC_2(UP_0 - UP_1)}{UP_0 - UP_2 + OC_1 + OC_2} \ge 0$, (B.6)

therefore holds. □

- Case 2: [1] = 2 and [2] = 1 (i.e. $Q_2 \le Q_1$)

  Using similar analysis with case 1, we can obtain the following result:

  If $\dfrac{UP_1 - UP_2}{UP_1 - UP_2 + OC_1} > \dfrac{UP_0 - UP_1}{UP_0 - UP_1 + OC_2}$ (i.e. $\dfrac{OC_1}{OC_2} < \dfrac{UP_1 - UP_2}{UP_0 - UP_1}$), the optimal value of

Q_1 and Q_2 for case 2 is $Q_1 = F^{-1}(\dfrac{UP_1 - UP_2}{UP_1 - UP_2 + OC_1})$ and $Q_2 = F^{-1}(\dfrac{UP_0 - UP_1}{UP_0 - UP_1 + OC_2})$.

If $\dfrac{UP_1 - UP_2}{UP_1 - UP_2 + OC_1} \le \dfrac{UP_0 - UP_1}{UP_0 - UP_1 + OC_2}$ (i.e. $\dfrac{OC_1}{OC_2} \ge \dfrac{UP_1 - UP_2}{UP_0 - UP_1}$), the optimal value of Q_1 and Q_2 for case 2 is $Q_1 = Q_2 = F^{-1}(\dfrac{UP_0 - UP_2}{UP_0 - UP_2 + OC_1 + OC_2})$.

Note that $UP_0\big|_{[1]=1} = UP_0\big|_{[1]=2}$ and $UP_2\big|_{[1]=1} = UP_2\big|_{[1]=2}$ and we can show that $UP_0 + UP_2 \ge UP_1\big|_{[1]=1} + UP_1\big|_{[1]=2}$ by analyzing all cases of dual-channel two-component problem, therefore we can show that $\left.\dfrac{UP_0 - UP_1}{UP_1 - UP_2}\right|_{[1]=1} \ge \left.\dfrac{UP_1 - UP_2}{UP_0 - UP_1}\right|_{[1]=2}$. We therefore can summarize the results of case 1 and case 2 and obtain the following solution.

If $\dfrac{OC_1}{OC_2} > \left.\dfrac{UP_0 - UP_1}{UP_1 - UP_2}\right|_{[1]=1}$

$$Q_1 < Q_2 \text{ and } Q_1 = \left.F^{-1}(\frac{UP_0 - UP_1}{UP_0 - UP_1 + OC_1})\right|_{[1]=1}, \quad Q_2 = \left.F^{-1}(\frac{UP_1 - UP_2}{UP_1 - UP_2 + OC_2})\right|_{[1]=1}$$

If $\dfrac{OC_1}{OC_2} < \left.\dfrac{UP_1 - UP_2}{UP_0 - UP_1}\right|_{[1]=2}$

$$Q_1 > Q_2 \text{ and } Q_1 = \left.F^{-1}(\frac{UP_1 - UP_2}{UP_1 - UP_2 + OC_1})\right|_{[1]=2}, \quad Q_2 = \left.F^{-1}(\frac{UP_0 - UP_1}{UP_0 - UP_1 + OC_2})\right|_{[1]=2}$$

If $\left.\dfrac{UP_0 - UP_1}{UP_1 - UP_2}\right|_{[1]=1} \ge \dfrac{OC_1}{OC_2} \ge \left.\dfrac{UP_1 - UP_2}{UP_0 - UP_1}\right|_{[1]=2}$

$$Q_1 = Q_2 = F^{-1}\left(\frac{UP_0 - UP_2}{UP_0 - UP_2 + OC_1 + OC_2}\right). \qquad \square$$

B.2 Proof of Lemma 5.5

Proof. (by using KKT condition)

If $i = j$, Lemma 5.5 shows that $\tilde{Q}_i = F^{-1}(\frac{UC_i}{UC_i + OC_i})$ is the optimal solution to the problem $\mathbf{B}_{i,i}(\boldsymbol{\sigma}^0)$, which is the result of Newsvendor model. We therefore only need to prove the case $i < j$.

If $\sum_{k=i}^{j} UC_k = 0$, note that $\tilde{Q}_i = \tilde{Q}_{i+1} = \cdots = \tilde{Q}_j = F^{-1}\left(\frac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} UC_k + \sum_{k=i}^{j} OC_k}\right) = 0$ is an optimal solution to $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$. Therefore, we only need to prove that $\tilde{Q}_i = \tilde{Q}_{i+1} = \cdots = \tilde{Q}_j = F^{-1}\left(\frac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} UC_k + \sum_{k=i}^{j} OC_k}\right)$ is an optimal solution to $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$ when $\sum_{k=i}^{j} UC_k \neq 0$.

Note that $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$ is a convex function, and all constraints $Q_t - Q_{t+1} \le 0 \ (i \le t \le j-1)$ are convex functions. It is obvious that $\tilde{Q}_i = \tilde{Q}_{i+1} = \cdots = \tilde{Q}_j = F^{-1}\left(\frac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} UC_k + \sum_{k=i}^{j} OC_k}\right)$ satisfy all constraints $Q_t - Q_{t+1} \le 0 \ (i \le t \le j-1)$ and we can show that $\tilde{Q}_i = \tilde{Q}_{i+1} = \cdots = \tilde{Q}_j =$

$F^{-1}\left(\dfrac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} UC_k + \sum_{k=i}^{j} OC_k}\right)$ satisfy the following hypothesis, therefore it is an optimal solution to $\mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)$.

Hypothesis: We can find a set of multipliers $\lambda_i, \lambda_{i+1}, ..., \lambda_{j-1}$ satisfying

$$\left.\left(\frac{\partial \mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)}{\partial Q_i} + \lambda_i \frac{\partial (Q_i - Q_{i+1})}{\partial Q_i}\right)\right|_{(\tilde{Q}_i,...,\tilde{Q}_j)} \geq 0 \tag{B.9}$$

$$\left.\left(\begin{array}{l}\dfrac{\partial \mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)}{\partial Q_t} + \lambda_{t-1} \dfrac{\partial (Q_{t-1} - Q_t)}{\partial Q_t} \\ +\lambda_t \dfrac{\partial (Q_t - Q_{t+1})}{\partial Q_t}\end{array}\right)\right|_{(\tilde{Q}_i,...,\tilde{Q}_j)} \geq 0 \quad (i+1 \leq t \leq j-1) \tag{B.10}$$

$$\left.\left(\frac{\partial \mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)}{\partial Q_j} + \lambda_{j-1} \frac{\partial (Q_{j-1} - Q_j)}{\partial Q_j}\right)\right|_{(\tilde{Q}_i,...,\tilde{Q}_j)} \geq 0 \tag{B.11}$$

$$\lambda_t[0 - (\tilde{Q}_t - \tilde{Q}_{t+1})] = 0 \quad (i \leq t \leq j-1) \tag{B.12}$$

$$\left[\left.\left(\frac{\partial \mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)}{\partial Q_i} + \lambda_i \frac{\partial (Q_i - Q_{i+1})}{\partial Q_i}\right)\right|_{(\tilde{Q}_i,...,\tilde{Q}_j)}\right] \tilde{Q}_i = 0 \tag{B.13}$$

$$\left[\left.\left(\begin{array}{l}\dfrac{\partial \mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)}{\partial Q_t} + \lambda_{t-1} \dfrac{\partial (Q_{t-1} - Q_t)}{\partial Q_t} \\ +\lambda_t \dfrac{\partial (Q_t - Q_{t+1})}{\partial Q_t}\end{array}\right)\right|_{(\tilde{Q}_i,...,\tilde{Q}_j)}\right] \tilde{Q}_t = 0 \quad (i+1 \leq t \leq j-1) \tag{B.14}$$

$$\left[\left.\left(\frac{\partial \mathbf{B}_{i,j}(\boldsymbol{\sigma}^0)}{\partial Q_j} + \lambda_{j-1} \frac{\partial (Q_{j-1} - Q_j)}{\partial Q_j}\right)\right|_{(\tilde{Q}_i,...,\tilde{Q}_j)}\right] \tilde{Q}_j \geq 0 \tag{B.15}$$

$$\lambda_t \geq 0 \quad (i \leq t \leq j-1) \tag{B.16}$$

Equations (B.9) – (B.15) are equivalent to the following equations (B.17) – (B.23) respectively.

$$(UC_i + OC_i)F(\tilde{Q}_i) - UC_i + \lambda_i \geq 0 \quad \text{(B.17)}$$

$$(UC_t + OC_t)F(\tilde{Q}_t) - UC_t - \lambda_{t-1} + \lambda_t \geq 0 \quad (i+1 \leq t \leq j-1) \quad \text{(B.18)}$$

$$(UC_j + OC_j)F(\tilde{Q}_j) - UC_j - \lambda_{j-1} \geq 0 \quad \text{(B.19)}$$

$$\lambda_t(\tilde{Q}_{t+1} - \tilde{Q}_t) = 0 \quad (i \leq t \leq j-1) \quad \text{(B.20)}$$

$$\left((UC_i + OC_i)F(\tilde{Q}_i) - UC_i + \lambda_i\right)\tilde{Q}_i = 0 \quad \text{(B.21)}$$

$$\left((UC_t + OC_t)F(\tilde{Q}_t) - UC_t - \lambda_{t-1} + \lambda_t\right)\tilde{Q}_t = 0 \quad (i+1 \leq t \leq j-1) \quad \text{(B.22)}$$

$$\left((UC_j + OC_j)F(\tilde{Q}_j) - UC_j - \lambda_{j-1}\right)\tilde{Q}_j = 0 \quad \text{(B.23)}$$

As $\tilde{Q}_i = \tilde{Q}_{i+1} = \cdots = \tilde{Q}_j = F^{-1}\left(\dfrac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} UC_k + \sum_{k=i}^{j} OC_k}\right)$, (B.20) can be satisfied. As $\tilde{Q}_i = \tilde{Q}_{i+1} = \cdots = \tilde{Q}_j = F^{-1}\left(\dfrac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} UC_k + \sum_{k=i}^{j} OC_k}\right) \neq 0$, in order to ensure that (B.17), (B.18), (B.19), (B.21), (B.22) and (B.23) hold, (B.24), (B.25) and (B.26) must hold.

$$(UC_i + OC_i)F(\tilde{Q}_i) - UC_i + \lambda_i = 0 \quad \text{(B.24)}$$

$$(UC_t + OC_t)F(\tilde{Q}_t) - UC_t - \lambda_{t-1} + \lambda_t = 0 \quad (i+1 \leq t \leq j-1) \quad \text{(B.25)}$$

$$(UC_j + OC_j)F(\tilde{Q}_j) - UC_j - \lambda_{j-1} = 0 \quad \text{(B.26)}$$

Equations (B.24), (B.25) and (B.26) are equivalent to (B.27), (B.28) and (B.29) respectively.

$$\lambda_i = UC_i - (UC_i + OC_i)F(\tilde{Q}_i) \tag{B.27}$$

$$\lambda_t = \lambda_{t-1} + UC_t - (UC_t + OC_t)F(\tilde{Q}_t) \quad (i+1 \le t \le j-1) \tag{B.28}$$

$$\lambda_{j-1} = (UC_j + OC_j)F(\tilde{Q}_j) - UC_j \tag{B.29}$$

According to (B.27) and (B.28) and recall $\tilde{Q}_i = \cdots = \tilde{Q}_j$, we can obtain the values of λ_t $(i \le t \le j-1)$ as follows:

$$\lambda_t = \sum_{k=i}^{t} UC_k - (\sum_{k=i}^{t} UC_k + \sum_{k=i}^{t} OC_k)F(\tilde{Q}_i) \qquad (i \le t \le j-1)$$

As $\tilde{Q}_i = \tilde{Q}_{i+1} = \cdots = \tilde{Q}_j = F^{-1}\left(\dfrac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} UC_k + \sum_{k=i}^{j} OC_k} \right)$, we can show that $\lambda_{j-1} = \sum_{k=i}^{j-1} UC_k - \left(\sum_{k=i}^{j-1} UC_k + \sum_{k=i}^{j-1} OC_k \right) F(\tilde{Q}_i) = \left(UC_j + OC_j \right) F(\tilde{Q}_j) - UC_j$. Therefore, (B.29) holds. We thus only need to show that (B.16) holds, which is to show the following inequalities are correct: $\sum_{k=i}^{t} UC_k - (\sum_{k=i}^{t} UC_k + \sum_{k=i}^{t} OC_k)F(\tilde{Q}_i) \ge 0 \quad (i \le t \le j-1)$.

Recall that $r_{i,x} > r_{x+1,j}$ $(1 \le i < j \le n)$ holds for all x = i, ..., $j-1$. According to Lemma 5.4 (a), $r_{i,x} > r_{i,j}$ ($1 \le i < j \le n$) holds for all x = i, ..., $j-1$, thus

$$\frac{\sum_{k=i}^{t} UC_k}{\sum_{k=i}^{t} OC_k} > \frac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} OC_k} \quad (i \le t \le j-1), \quad \frac{\sum_{k=i}^{t} UC_k}{\sum_{k=i}^{t} UC_k + \sum_{k=i}^{t} OC_k} > \frac{\sum_{k=i}^{j} UC_k}{\sum_{k=i}^{j} UC_k + \sum_{k=i}^{j} OC_k} \quad (i \le t \le j-1).$$

Therefore, $\sum_{k=i}^{t} UC_k - (\sum_{k=i}^{t} UC_k + \sum_{k=i}^{t} OC_k)F(\tilde{Q}_i) > 0 \quad (i \le t \le j-1)$. □