UNCERTAIN LIFETIME, BEQUEST, ANNUITY AND CAPITAL ACCUMULATION UNDER DIFFERENT MOTIVES OF BEQUESTS

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Table of Contents

Summary

Joy of giving and dynastic altruism are considered as two motives for bequests. This paper studies a lifecycle model with lifetime uncertainty under these two motives. We find that accidental bequests and planned bequests are equal under both motives, which allows us to track down family decisions across generations that are independent of the mortality history in the family. However, the allocations of bequests, annuity savings, non-annuity savings and consumptions are different between models with either of the two motives. Under the dynastic altruism model, bequests are compensatory and transfers from children to parents are also possible. More importantly, rising longevity has no impact on capital accumulation per capita in the dynastic model unlike a positive effect on capital accumulation in the joy-of-giving model. These results with dynastic altruism are consistent with some existing empirical results, supporting the validity of the dynastic model.

Key words: bequests, joy of giving, dynastic altruism, capital accumulation.

List of Tables

1. Introduction

Capital accumulation has been at the center of studies of economic growth for decades, since the advent of the neoclassical growth model in the 1950s. However, it remains a challenging subject once considering such important factors as uncertain survival to old age, intergenerational transfers, life-cycle savings and the forms of assets carried to old age or to children. It involves controversies about how rising life expectancy affects capital accumulation and growth, about why private annuity purchases are very small despite higher annuity returns, about what motivates bequests and so on. The different results emerge typically from the primitive assumptions about the availability of annuity markets and the presence and the form of altruism motivating bequests. There are generally three kinds of bequest motives in the literature: joy of giving, exchange for better behavior, and dynastic altruism.

A pioneering paper by Yaari (1965) has aroused enormous interest in a variety of topics related with the annuity puzzle in a life-cycle model with or without bequests motivated by joy-of-giving. Without bequests, all savings are only made for later life by purchasing annuities that have greater returns than the market interest rate. This complete annuitization result contradicts the fact that most elderly US individuals maintain a flat age-wealth profile rather than buy individual life annuities as documented in Friedman and Warshawsky (1990). It is also inconsistent with the fact that bequests account for a significant portion of capital in the United States as found in Kotlikoff and Summers (1981). With a joy-of-giving bequest motive in Yaari (1965), however, part of savings is held in bequeathable, non-annuity forms. One additional implication of the joy-of-giving motive is that the amount of bequests should be equal to all children in a family. Moreover, rising life expectancy raises the annuity portion for old-age consumption but reduces the non-annuity portion for bequests; overall, rising life expectancy tends to increase the total saving. The Yaari model has been extended to incorporate neoclassical production in Abel (1986) among others. In the extended models, rising longevity increases aggregate saving and hence promotes capital accumulation and growth.

Different from the Yaari model, some papers assume the absence of annuity markets; see Abel (1985) and Zhang, Zhang and Lee (2003). Without annuity markets, life-cycle savings by those who fail to survive into old age become accidental bequests that are equally shared by all surviving children in the family. The amount of bequests to a child in this case is dependent on family mortality histories; it is increasing with the number of consecutive preceding generations in their families who died before consuming their savings. As in models with annuity markets, rising life expectancy raises aggregate savings and thus promotes capital accumulation as shown in Zhang et al. (2003).

However, at least two of the implications of the models with accidental bequests or with bequests derived from joy-of-giving are not well supported by available empirical evidence. First, the implied equal bequest among siblings is inconsistent with the negative relationship between bequests to children and their earnings within families as found by Light and Kathleen (2004). Second, the empirical evidence on the effect of rising life expectancy on savings and growth is mixed. Some empirical studies claim a positive effect of life expectancy on savings and growth; see Zhang and Zhang (2005), whose use of the investment/GDP ratio as a proxy for the saving rate is questionable in open economies. In contrast to the findings mentioned above, Acemoglu and Johnson (2007) claim little effect of life expectancy on GDP and a negative effect of rising life expectancy on GDP per capita due to population aging.

In this paper, we study how individuals allocate income to consumption, annuity savings, non-annuity savings and bequests in two versions of a two-period life cycle model with the joy-of-giving motive and the dynastic altruism motive respectively. Comparisons can be easily observed in the key implications of the two versions. In the joy-of-giving model, agents derive utility from the size of bequests to children and from their own consumption. In the dynastic altruism model, agents derive utility from their own consumption and from children's welfare. Previously, we did not consider the altruism model until we found a similar paper by Abel (1986). In Abel (1986) with a joy-of-giving bequest motive, agents derive utility from the size of bequests. It argues that accidental bequests (when parents die young) and planned bequests (when parents survive to old age) are equal to each other. We also obtained the same result from the joy-of-giving model before knowing the work of Abel (1986). Compared with the joy-of-giving model, we will show how the dynastic model (with altruism toward the welfare of children) generates different results that are more consistent with the aforementioned empirical evidence.

Unlike the joy-of-giving model that assumes a known function via which utility depends on bequests, the function linking utility to bequests is unknown and thus has to be found in the dynastic model.[1] One implication of the dynastic model is that bequests are inversely related to children's wage as shown in Tomes (1981) both theoretically and empirically. This implication is also in line with the empirical evidence in Light and Kathleen (2004). The most important result that we obtain from comparing these two models is the impact of life expectancy on capital accumulation and economic growth. In the joy-of-giving model, there is a positive effect of rising life expectancy on capital accumulation and economic growth. However, in the dynastic altruism model, rising life expectancy has no effect on aggregate capital accumulation and aggregate output, thereby leading to a negative effect of life expectancy on per capita output due to population aging. By comparing the different implications to the empirical evidence from Acemoglu and Johnson (2007), it

[1] In the joy of giving model, bequests are treated similar to consumption, so the utility from bequests can be easily assumed as a primitive. While in dynastic altruism model, the function linking utility to bequests is an unknown welfare function, which cannot be assumed in priori.

is clear that the implication from the dynastic altruism model is more empirically plausible than that from the joy-of-giving model.

The rest of paper is organized as follows. In section two, we review the previous literature. Section three has two parts. In the first part, we derive results in the joy-of-giving model with uncertain survival. In part two, we derive results from the dynastic altruism model. Section four concludes the paper.

2. Literature review

We now provide more details about the related literatures concerning uncertain lifetime, the various bequest motives and the annuity puzzle.

2.1 Uncertain lifetime

The effects of uncertain lifetimes on individuals' savings decisions were first examined formally in the paper by Yaari (1965) with or without annuity markets. It considers a Fisher-type utility function and a Marshall utility function respectively. The former is the typical life-cycle saving model in which agents derive utility only from their own lifetime consumption. The latter is an extension of the former to the inclusion of separable utility derived from the amount of bequests to children. It studies four cases differentiated by the availability of bequests and annuity insurance. First, with no bequest motivation and no annuity market, survival uncertainty causes consumers to discount the future more heavily. Second, with no bequest motivation but with annuity markets available, the consumer is better off by holding annuity as the only form of savings, because under the survival uncertainty the annuity market gives higher return than non-annuity savings. Third, with bequest motivation but with no annuity market, the effect of uncertainty on consumers' degree of impatience depends on the difference between the marginal utility of consumption and that of bequests. Last but not least, with both bequest motivation and annuity markets available, the optimal saving plan is to use annuity savings to meet the need of future consumption and use conventional, non-annuitized savings for bequests to children. Yaari's model provides a fundamental theory for the subsequent studies of consumer savings when consumers are faced with uncertain lifetimes.

Elaborating the case with no bequest motive and no annuity market, Abel (1985) attempted to characterize the distribution and evolution of accidental bequests. Absent the annuity market, life-cycle savings become accidental bequests when consumers die young. Accidental bequests have been shown in Abel (1985) to play an important role in “causing the intergenerational wealth transfers as well as in the intra-generational wealth variation". More specifically, accidental bequests are a function of the family mortality history: those who have more consecutive preceding generations died young will receive more such bequests, while children in the same family receive equal bequests. The mortality history dependence makes it extremely difficult to analyze the distribution and evolution of capital across families and overtime. But the implication of equal bequests for all siblings is inconsistent with empirical evidence, as we pointed out earlier. The extension of the model of Abel (1985) to consider physical and human capital accumulation in Zhang et al. (2003) predicts a positive effect of rising life expectancy on physical capital accumulation and on economic growth, which is also inconsistent with the recent empirical evidence in Acemoglus and Johnson (2007).

Pecchenino and Pollard (1997) introduced actuarially fair annuities sponsored by the government together with a pay-as-you-go social security system into an over-lapping generation model populated by fully selfish agents without a bequest motive. The amount of bequests is assumed to be equal to the unannuitized savings plus interest left by those parents who die at the onset of old age. It argues that complete annuitization of consumers' wealth is not dynamically optimal, and it recommends that the government should move the economy from the current pay-as-you-go social security system to a government sponsored, actuarially fair social security. Thus the government has to restrict the availability of actuarially fair annuity contracts by either setting a maximum purchasing limit or requiring a minimum mandatory amount of annuity. Their justification for government intervention hinges on their assumption of the existence of a positive externality of aggregate capital in final production.

2.2 Bequest motives

Kotlikoff and Summers (1981) empirically studied the role of bequests in aggregate saving. They applied historical U.S. data to estimate the contribution of intergenerational transfers to capital accumulation and reported the evidence that bequests account for as much as four fifths of U.S. capital stock while life cycle savings accounts “only a negligible fraction”. They show that the simple life cycle model without bequest motives is inadequate in explaining the saving behavior in the United States.

Kuehlwein (1993) used the Retirement History Survey and examined a parameterized life-cycle model with uncertainty and bequest motivation. The estimated bequest parameters for households with and without children are both significant and close to each other. This means that households value bequests as much as their own consumption. Such a strong bequest motive can be seen to mute the effects of lifetime uncertainty on consumption growth, casting doubt on models without a bequest motive.

There are different schools of thoughts on why individuals want to leave bequests to their offspring. One of them is called the joy of giving. That is, utility derived from bequests is only dependent on the size of the bequests via an assumed function. Abel (1986) uses it in an overlapping-generations model with an actuarially fair social security. One result that coincides with one of our results is that the accidental bequests and planned bequests are equal. We obtained this result before knowing Abel's work. However, this model with joy-of-giving predicts a positive effect of rising life expectancy on aggregate savings and GDP and implies equal bequests to all siblings, both of which are inconsistent with recent empirical evidence. We will therefore focus on a different model and attempt to obtain different results that can better explain empirical evidence.

Sheshinski and Weiss (1981) also introduced a joy-of-giving bequest motive into an overlapping-generation model with uncertain lifetime and with two kinds of social security. The two social security programs are a fully-funded system and a pay-as-you-go system, respectively. They show that these two social security systems are equivalent in terms of all real aggregates and have the same optimal level. They also propose a well-know "segmentation": at the optimum level, private savings provide bequests to next generation, while social security with annuity benefits is used solely to sustain future consumption in old age.

Bernheim et al. (1985) proposed a model with "strategic" bequests. In their theoretical formulation, individuals, though altruistic, are considered to have bequeathable wealth intentionally to manipulate their offspring's behavior. They present empirical support for a scenario that attention from children is positively correlated with bequeathable wealth. An essential assumption for the strategic behavior is that the number of children exceeds one. Our model with unisex and with just one child per parent bypasses such a strategic consideration.

Tomes (1981) assumed that all bequests are intentional and motivated by dynastic altruism. It used empirical tests and strongly confirmed that bequests to children were negatively related to their earnings. That is, bequests were compensatory according the data, which means that children within a family or from the families with the same income level inherited more bequests if they have lower earnings. The compensatory effect reduced the variance of bequests by 30 percent and reduced the correlation between bequests and income to 0.12. His paper does not consider uncertain survival as was typically the case in dynastic models, however.

Toshihiro (1993) added three alternative bequest motives into an overlapping-generations model and studied their effects on economic growth. Three bequest motives are: the altruistic bequest motive, the bequest-as-consumption (joy of giving) motive, and the bequest-as-exchange (strategic) motive. In the altruistic bequest model, parents concern their children's wellbeing, so the utility that parents get from giving bequests is related with their children's total utility. In the bequest-as-exchange model, parents use bequests as payment for their children's actions that they wish them to undertake such as attention to them when they get old. In the bequest-as-consumption model, parents care about their children's bequests instead of children's wellbeing. The paper studies the three bequest motives' long-run effects on economic growth. The result shows that the effects of the three bequest motives on economic growth are qualitatively the same.

However, survival is certain in his model.

2.3 Annuity puzzle

“Annuity puzzle” is the contradiction between the theoretical prediction and empirical evidence. Theoretically, individuals would choose annuity as the sole means against the uncertainty in the form of life expectancy risk since annuities yield higher returns than unannuitized savings as shown in Yaari (1965). However, empirical evidence indicates that the demand for private annuities is very low.

Bernheim (1991) concluded three different schools of thoughts to explain the “annuity puzzle”. The first and most obvious reason is that most people save to leave a bequest to their heirs. Without bequest motive, the allocation of individuals' wealth simply depends on whether the annuity market’s rate of return exceeds the market interest rate. The second explanation is the existence of social security and pension plans. The third explanation is that the annuity market is not priced fairly. All the transaction costs, monopoly profits and the adverse selection problem can discourage people from purchasing annuities. Bernheim (1991) presented new empirical evidence that individuals choose bequeathable forms of savings over annuity purchasing even if the annuity market is perfectly fair. He also argues that social security benefits depress annuity holdings and induce buying life insurance instead.

Inkmann, Lopes and Michaelides (2008) used U.K. microeconomic data to rationalize the observed annuity rates, as well as to empirically analyze the determinants of the demand for voluntary annuities. Among their results, a strong bequest motive is found out to play an important role in accounting for the low accumulation and low annuity demand, as opposed to the opinion of Vidal-Melia and Lejarraga-Garcia (2005).

3. The model

In this economy, time is discrete expanding from the initial period to infinity, $t = 1, 2, 3, \dots \infty$. Agents are unisexual and live for a maximum of two periods in lifetime, working in the first period and living in retirement in the second. Their survival rate to old age is exogenously given by $p \in (0,1)$. Each young agent gives birth to exactly one child.

Agents are allowed to save either in the form of annuity A or non-annuity S. In period t, each worker earns a wage income W_t and receives a bequest B_t^j from the last generation. The amount of the received bequest equals what the parent gives, denoted as B_t^S, if he/she survives to old age; otherwise it is denoted as B_t^D which equals the non-annuity saving plus interest:

$$B_t^j = \begin{cases} B_t^S & \text{if parent survives} \\ B_t^D = (1 + r_t)S_{t-1} & \text{otherwise} \end{cases} \tag{1}$$

They divide their resource between period-t consumption C_{1t}, annuity purchasing A_t and savings S_t. Annuity savings earn a higher rate of return a_{t+1}, conditional on survival to old age, than the market rate r_{t+1} that applies to non-annuity savings. If they are alive in old age in period $t+1$, they consume C_{2t+1} that depends on the return to different forms of assets they purchased in period t, and they leave bequests to their children B^{S}_{t+1} ; If they die accidentally at the end of the first period in life, non-annuity savings with returns are given to their offspring as accidental bequests in the second period in life. Suppose that the annuity market is a perfectly competitive market. Thus, we expect

$$1+a_{t+1}=\frac{1+r_{t+1}}{p} \tag{2}$$

The household budget constraint is given as

$$C_{1t}=B^{j}_{t}+W_{t}-A_{t}-S_{t}, \tag{3}$$

$$C_{2t+1}=\left(\frac{1+r_{t+1}}{p}\right)A_{t}+(1+r_{t+1})S_{t}-\ B^{S}_{t+1} \tag{4}$$

Two motives for bequests are considered: joy of giving and dynastic altruism. With the joy of giving bequest motive, agents derive utility from the size of the bequests that they give to their offspring. Bequests are treated like consumption.

With the dynastic altruism bequest motive, agents care about their children's welfare instead of the bequests' size itself: The utility from giving bequests is the discounted welfare of their children. In other words, agents' welfare function comes from the utility from their own consumptions and their next generation' welfare. There is thus a tradeoff between the current generation's consumption and the next generation's. Agents can choose to either consume or save as bequests for the next generation's consumption. [2]

3.1 Joy-of-giving model

Suppose the preference of agents is defined over their own lifecycle consumption and the joy of giving bequests to their children:

$$U(C_{1t}) + \beta p U(C_{2t+1}) + pV(B_t^S) + (1-p)V(B_t^D) \tag{5}$$

where β is discount factor, $0 < \beta < 1$. Both $U(\cdot)$ and $V(\cdot)$ are strictly increasing and strictly concave and satisfy the Inada conditions.

Production is neoclassical $Y_t = F(K_t, L_t)$, where K_t is the society's total capital, and L_t is the total labor force, where $F(K_t, L_t)$ is increasing, concave and homogenous of degree one. It also meets the Inada conditions for interior solution. In this model with one unit of inelastic labor supply, the production function can be described as $y_t = f(k_t)$ in terms of per worker units.

Suppose that firms earn zero profit and that all markets are competitive, with a 100% depreciation rate. Then, production factors are compensated by their marginal products: $1 + r_t = f'(k_t)$ and $W_t = f(k_t) - k_t f'(k_t)$. The initial stock of capital k_0 is owned by old people.

The young agents maximize their lifelong utility,

$$\max_{A,S,B} \; U_t = U(C_{1t}) + \beta p U(C_{2t+1}) + pV(B_{t+1}^S) + (1-p)V(B_{t+1}^D)$$

$$\text{s.t. } C_{1t} = B_t^j + W_t - A_t - S_t \,, \tag{6}$$

$$C_{2t+1} = \left(\frac{1+r_{t+1}}{p}\right) A_t + (1 + r_{t+1}) S_t - B_{t+1}^S \tag{7}$$

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[2] In the dynastic altruism model, the size of bequests agents leave to their children depends not on their own preference like in the joy of giving model, but on their expectation on their children's living standard.

The first-order conditions are derived below:

$$A_t: \quad U'(C_{1t}) = \beta(1 + r_{t+1})U'(C_{2t+1}), \tag{8}$$

$$B_{t+1}^S: \quad \beta U'(C_{2t+1}) = V'(B_{t+1}^S), \tag{9}$$

$$S_t: U'(C_{1t}) = (1 - p)(1 + r_{t+1})V'(B_{t+1}^D) + \beta p U'(C_{2t+1})(1 + r_{t+1}). \tag{10}$$

Equation (8) is the optimal condition for the annuity purchasing and states that the loss of utility for buying one unit annuity in period t is equal to the present value of the expected utility in period $t + 1$ from the returns of the one unit bought in period t. Equation (9) is the optimal condition for the planned bequest given to the next generation. The present loss of utility from saving one unit of consumption for bequests is equal to the increased utility from giving bequests. Equation (10) is the optimal condition for non-annuity savings. It states that the loss of utility form saving one extra unit of consumption is equal to the sum of the expected utility from giving accidental bequests if failing to survive or from the consumption in period $t + 1$ if surviving.

Proposition 1: *Accidental bequests and planned bequests are equal to each other:* $B_{t+1}^S = B_{t+1}^D = (1 + r_{t+1})S_t$.

Proof: Equations (8) and (9) imply

$$U'(C_{1t}) = V'(B_{t+1}^S)\,(1 + r_{t+1}). \tag{11}$$

Substituting (9) and (11) into equation (10) yields

$$V'(B_{t+1}^{S}) = V'(B_{t+1}^{D}) \tag{12}$$

Since $V(\cdot)$ is strictly increasing and strictly concave, this gives that

$$B_{t+1}^{S} = B_{t+1}^{D} = (1 + r_{t+1})S_t \tag{13}$$

Q.E.D.

This result allows us to assume that agents start with the same amount of bequests regardless of whether their parents survive to old age or not, i.e. $B_t^j = B_t^D = B_t^S = (1 + r_t)S_{t-1}$.[3] Consequently, the decisions are independent of the mortality history of a family. This allows us to focus on the two periods of generations in dealing with the asset transfers from generation to generation under the circumstances that annuity markets exist. This differs from Abel (1985)'s result that accidental bequests cause bequests' intra-cohort variation due to the different mortality histories of their families. Moreover, it is worth noting that in doing so we do not assume that all assets must be held in annuities as opposed to some related literature on the evolution of wealth across generations with annuity markets.

Abel (1986) has proved this result in a different way by deriving utility from the size of the bequests. We find that there are similarities between this joy-of-giving model and Abel (1986)'s model, although Abel (1986) focused on the social security's influence on capital accumulation. This finding forced us to think further on the limitation of this model and to analyze the dynastic

[3] Without this result, we will face a lot of difficulty in modeling intergenerational transfers when survival is not for certain,

altruism model for more empirically plausible predictions in the next part.

We now assume constant-relative-risk-aversion utility as an important example:

$$U(C) = \frac{C^{1-\sigma}-1}{1-\sigma} \text{ and } V(b) = \emptyset \frac{b^{1-\sigma}-1}{1-\sigma} \qquad 0 < \sigma;\ 0 < \emptyset < \beta < 1$$

where $\emptyset$ is the paremeter that reflects how people value giving bequests. Vidal-Melia and Lejarraga-Garcia (2005) consider this parameter as increasing with age because agents are strategic in order to encourage children to take care of them. Since we are studying a joy-of-giving motive which is not related with age, we assume $\emptyset$ is constant. The restriction $0 < \emptyset < \beta < 1$ means that people value more of their own consumption in their second period of life than bequests given to their next generation.

Proposition 2: *With a CRRA utility and an exogenous survival rate and bequest motive, the ratio of annuity to non-annuity savings is increasing with the survival rate but decreasing with the joy of giving bequests to children.*

Proof: With the result of proposition 1, we can use the utility function that we assumed previously and rewrite equation (12)

$$\beta C_{2t+1}^{-\sigma} = \emptyset[(1 + r_{t+1})S_t]^{-\sigma}$$

It gives

$$\left(\frac{C_{2t+1}}{(1+r_{t+1})S_t}\right)^{\sigma} = \frac{\beta}{\emptyset} \tag{14}$$

Equation (7) implies that

$$C_{2t+1} = \frac{1+r_{t+1}}{p} A_t \tag{15}$$

Substitute (15) into (14) and we can get

$$\frac{A_t}{s_t} = p\left(\frac{\beta}{\emptyset}\right)^{\frac{1}{\sigma}} \tag{16}$$

The claims follow. Q.E.D.

This result shows how people allocate their income between non-annuity savings and annuities. Under the condition that $\emptyset$ and β are constant, the ratio of annuity savings to non-annuity savings are increasing in the survival rate p. That happens when agents expecting a greater probability of survival save less in non-annuity forms and buy more annuities to support their second period's consumption. If p and β are constant, the higher the taste for giving bequests, the greater the non-annuity savings relative to annuity savings, because non-annuity savings are left for bequests. When β, $\emptyset$, p, and σ are all exogenously determined, the ratio of annuity savings to non annuity savings is constant. There is a balance between annuity purchasing and non annuity savings. This helps to explain the annuity puzzle. Agents with a bequest motive tend to save a certain portion of their income for the joy of giving bequests in case they die young.

Proposition 3: *In period t, young people allocate their income depending on the survival rate. The amount of annuity savings that they purchase is increasing in the survival rate; non-annuity savings and young-age consumption are both decreasing in the survival rate.*

Proof: Use the utility function that we assumed previously and rewrite equation (8) as below,

$$C_{1t}^{-\sigma} = \beta(1 + r_{t+1})C_{2t+1}^{-\sigma}. \quad (17)$$

Substitute equation (17) to equation (15),

$$C_{1t} = \frac{(1+r_{t+1})^{1-\frac{1}{\sigma}}}{p\beta^{\frac{1}{\sigma}}} A_t \quad (18)$$

Equation (16) implies that

$$S_t = \frac{\emptyset^{\frac{1}{\sigma}}}{p\beta^{\frac{1}{\sigma}}} A_t \quad (19)$$

Substitute equation (18) and (19) into the first constraint and get

$$A_t = \frac{p\beta^{\frac{1}{\sigma}}}{(1+r_{t+1})^{1-\frac{1}{\sigma}}+\emptyset^{\frac{1}{\sigma}}+p\beta^{\frac{1}{\sigma}}} (B_t + W_t) \quad (20)$$

$$S_t = \frac{\emptyset^{\frac{1}{\sigma}}}{(1+r_{t+1})^{1-\frac{1}{\sigma}}+\emptyset^{\frac{1}{\sigma}}+p\beta^{\frac{1}{\sigma}}} (B_t + W_t) \quad (21)$$

$$C_{1t} = \frac{(1+r_{t+1})^{1-\frac{1}{\sigma}}}{(1+r_{t+1})^{1-\frac{1}{\sigma}}+\emptyset^{\frac{1}{\sigma}}+p\beta^{\frac{1}{\sigma}}} (B_t + W_t) \quad (22)$$

The claims now become obvious. Q.E.D.

From equations (20), (21) and (22), obviously A_t is increasing in the survival rate p while S_t and C_{1t} are decreasing in p. The economic implication is as follows. When the survival rate increases, agents would concern more about their consumption in the second period of life. They will consume less when young and hold more savings for old age. Since the return of annuity savings is larger than the return of non-annuity savings, and since the risk of losing annuity savings in the case of death is decreased, they are more willing to increase annuity savings rather than non-annuity savings.

In a closed economy, the equilibrium condition for the capital market in our model is given below.

$$K_{t+1} = k_{t+1} = A_t + S_t \tag{23}$$

Substituting equations (20) and (21) into (23) gives,

$$k_{t+1} = \frac{\emptyset^{\frac{1}{\sigma}} + p\beta^{\frac{1}{\sigma}}}{(1+r_{t+1})^{1-\frac{1}{\sigma}} + \emptyset^{\frac{1}{\sigma}} + p\beta^{\frac{1}{\sigma}}}(B_t + W_t) \tag{24}$$

From Proposition 1, we know that

$$B_t = (1+r_t)S_{t-1} = (1+r_t)\frac{\emptyset^{\frac{1}{\sigma}}}{\emptyset^{\frac{1}{\sigma}} + p\beta^{\frac{1}{\sigma}}}k_t \tag{25}$$

From firms' behavior,

$$1 + r_t = f'(k_t) \tag{26}$$

$$W_t = f(k_t) - k_t f'(k_t) \tag{27}$$

Substituting (25), (26) and (27) into (24), we get the law of motion of k,

$$k_{t+1} = \frac{\emptyset^{\frac{1}{\sigma}} + p\beta^{\frac{1}{\sigma}}}{(f'(k_{t+1}))^{1-\frac{1}{\sigma}} + \emptyset^{\frac{1}{\sigma}} + p\beta^{\frac{1}{\sigma}}}[(f'(k_t))\frac{\emptyset^{\frac{1}{\sigma}}}{\emptyset^{\frac{1}{\sigma}} + p\beta^{\frac{1}{\sigma}}}k_t + f(k_t) - k_t f'(k_t)] \tag{28}$$

This is an implicit function where k_{t+1} is determined by k_t. That is, given k_0, this function will determine the capital stock in every future period implicitly. But it cannot provide a reduced form solution for the sequence of capital stock explicitly. To this end, we assume logarithmic utility and Cobb-Douglas production. Under such conditions, we have $\sigma = 1$, $f(k) = k^{\alpha}$, $1 + r_t = f'(k_t) = \alpha {k_t}^{\alpha-1}$, and $W_t = (1-\alpha)\,{k_t}^{\alpha}$.

Equation (28) becomes

$$k_{t+1} = \frac{\emptyset+p\beta}{1+\emptyset+p\beta}[(\alpha k_t{}^{\alpha-1})\frac{\emptyset}{\emptyset+p\beta}k_t + k_t{}^{\alpha} - \alpha k_t{}^{\alpha}]$$

Rewrite the above equation, we get

$$k_{t+1}(k_t) = \frac{p\beta(1-\alpha)+\emptyset}{1+\emptyset+p\beta}k_t{}^{\alpha} \quad (29)$$

Proposition 4: *The economy converges to a unique steady state* k^*, *and* $k^* = [\frac{p\beta(1-\alpha)+\emptyset}{1+\emptyset+p\beta}]^{1/(1-\alpha)}$.

Proof: It is easy to find the unique steady state level of capital from (29). Take the first derivative of equation (29) and get

$$\frac{dk_{t+1}}{dk_t} = \alpha\frac{p\beta(1-\alpha)+\emptyset}{1+\emptyset+p\beta}k_t{}^{\alpha-1} > 0 \quad (30)$$

which is greater than 1 at the origin with k near zero but less than 1 at the steady state k^*. Take the second derivative of equation (29) and get

$$\frac{dk_{t+1}}{dk_t{}^2} = \alpha\frac{p\beta(1-\alpha)+\emptyset}{1+\emptyset+p\beta}(\alpha-1)\,k_t{}^{\alpha-2} < 0 \quad (31)$$

Therefore equation (29) is a concave function. Also note that at the origin point k_{t+1} is divergent. So k_{t+1} is increasing in k_t at a diminishing rate and globally convergent to the unique steady state. Q.E.D.

From the first derivative in (30), we can get

$$\lim_{k_t\to 0}\frac{dk_{t+1}}{dk_t} = \infty \quad and \quad \lim_{k_t\to\infty}\frac{dk_{t+1}}{dk_t} = 0$$

Graphically, $k_{t+1}(k_t)$ starts above the 45-degree line and then intersects it. Thus the economy converges to its balanced growth path as shown in Figure 1.

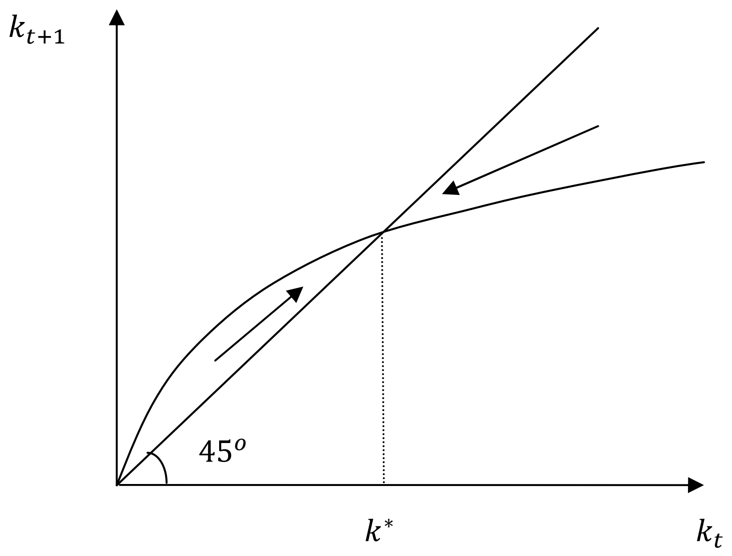

Figure 1

In Figure 1, k^* is the steady state, which is the point where k_{t+1} function intersects the 45-degree line. From equation (29), when $k_{t+1} = k_t$, we can get

$$k^* = [\frac{p\beta(1-\alpha)+\emptyset}{1+\emptyset+p\beta}]^{1/(1-\alpha)} \tag{32}$$

If $k_0 > k^*$, then $k_{t+1} < k_t$, thus $k_1 < k_0$, as shown in Figure 1, that is, k_t starts to decreasing until it converges to k^* and becomes stable. If $k_0 < k^*$, then $k_{t+1} > k_t$, thus $k_1 > k_0$, as shown in Figure 1, that is, k_t starts to increasing until it converges to k^* and becomes stable.

Proposition 5: *When there is an increase in* $\emptyset$, $k_{t+1}(k_t)$ *shift upwards, which leads to an increase in the steady state* k^*. *With plausible numerical values of* α *and* $\emptyset$, *the steady state* k^* *is increasing in the survival rate.*

Proof: Take the first derivative of equation (29) in $\emptyset$ and get

$$\frac{\partial k_{t+1}}{\partial \emptyset} = \frac{1+p\beta\alpha}{(1+\emptyset+p\beta)^2} {k_t}^{\alpha} > 0$$

Thus k_{t+1} is increasing in $\emptyset$, which means that k_{t+1} shifts upwards as shown in figure 2. As we can see in Figure 2, the steady state k^* is also increased to $k^{*\prime}$.

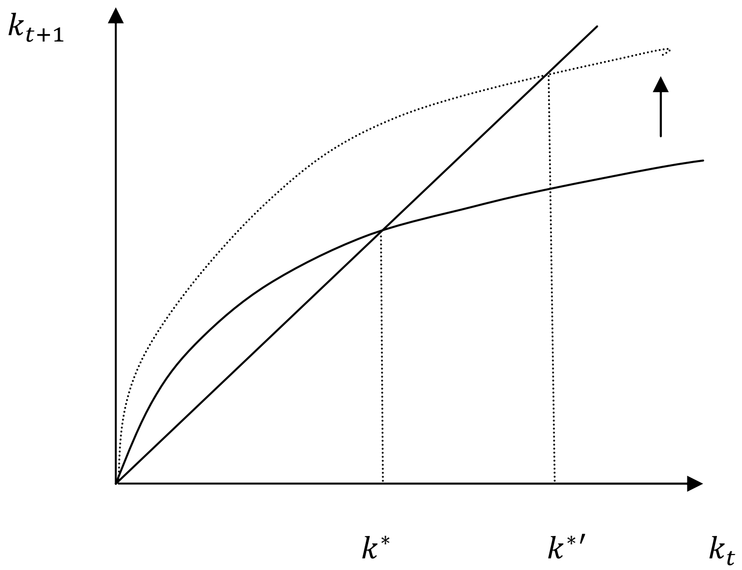

Figure 2

Starting from an initial balanced growth path, when there is an increase in $\emptyset$ (i.e. with more joy of giving), agents will save more capital as bequests to their offspring. Thus, the k_{t+1} curve shifts upwards and the steady state k^* is increased.

Take the first derivative of equation (29) in p and get

$$\frac{\partial k_{t+1}}{\partial p} = \frac{\beta(1-\alpha-\alpha\emptyset)}{(1+\emptyset+p\beta)^2} {k_t}^{\alpha}$$

If $\emptyset < \frac{1-\alpha}{\alpha}$, k_{t+1} is increasing in p. This means that when the survival rate increases, the k_{t+1} curve shifts upwards and leads to a higher steady state level of capital if the taste for bequest giving is less than the ratio of the labor share to the capital share in output. If $\emptyset > \frac{1-\alpha}{\alpha}$, k_{t+1} is decreasing in p. This means that when survival rate increases, the k_{t+1} curve shifts downwards and leads to a lower steady state level of capital. Q.E.D.

The reason why it is ambiguous about the influence of p on k^* is as follows: when the survival rate increases, agents concern more about old-age consumption and less about bequests to the next generation. From Proposition 3, agents respond to the increase in the survival rate by increasing annuity savings for their own consumption in old age but decreasing non-annuity savings for bequests to children. According to Proposition 4, the net change in the aggregate saving then depends on whether the taste for bequest giving is below or above the ratio of the labor share to the capital share in output. If the taste for bequest giving is below (above) the ratio of the labor share to the capital share in output, the decline in the non-annuity savings is smaller (larger) than the increase in annuity savings, leading to a net increase (decrease) in the total saving.

In the real world, the labor share exceeds the capital share in output, with a standard value of α being equal to 1/3. Also, $\emptyset$ is less than β <1 under the plausible postulation that agents are mainly concerned about their own consumption. So $\emptyset$ is less than $\frac{1-\alpha}{\alpha}$. Therefore the overall influence of p on economic growth is positive. The higher total saving rate can compensate for the increased old-aged population's consumption and lead to a higher steady state capital per worker.

There are three limitations of the joy-of-giving model. Firstly, the assumption of the bequest motive is only one of several possible motives for bequests. From the previous literature, there are generally three ways to assume the utility from bequests. One is from the size of the bequests like Abel (1986) under the joy of giving motive; one is from the total bequeathable assets that agents are holding like Vidal-Melia and Ana Lejarraga-Garcia (2005) under the joy of giving motive and strategic motive; the other one is from the next generations' total income like Lambrecht, Michel and Vidal (2005) under the dynastic altruism. But there is no direct evidence showing which assumption should be selected. Therefore, it is not convincing enough that the utility function is only derived from the size of bequests.

Second, agents may care children's welfare rather than the bequests only. From previous literature, children's income is also taken into consideration when agents make decisions about the size of the bequests they give to their children. Light and McGarry (2004) argued that bequests are compensatory: The children with lower income tend to get more bequests from their parents. This is indirect evidence again the joy-of-giving assumption because it implies equal bequests to children regardless of their relative earnings.

Third, under the joy of giving bequest motive, the implication that capital accumulation is increasing in the survival rate is inconsistent with empirical studies such as Acemoglu and Johnson (2007).

To overcome the limitations, we further introduce an altruism model. The major difference between the joy-of-giving model and the altruism model is the different notions of marginal utility these two models are trying to equalize with respect to bequest giving. The joy-of-giving model equalizes the marginal utility of one's own consumption with the marginal utility of giving bequests to children, whereby children's earnings do not matter. By contrast, the altruism model equalizes the marginal utility of one's own consumption with the marginal utility of children's consumption symmetrically and recursively, whereby children's earnings do matter.

3.2 Dynastic altruism model

In this altruism model, a Bellman equation is set up and B_t^j is the state variable. S_t, A_t and B_{t+1}^S are the control variables. $V(B+W)$ is the total welfare of one generation.[4] The form of the function is unknown and has to be solved. Agents' welfare includes not only the utilities from two periods' life-cycle consumption but also the discounted expectation value of their next generation's welfare. Agents care their own life-cycle consumption more than their children's. Let δ be the discounted factor on child welfare, with $0 < \delta < 1$.[5] Firms' behavior is the same as that in the joy-of-giving model.

[4] B+W is the total income of one generation, which is the major determinants of welfare. Therefore, their welfare is the function of B+W as we will see.

[5] This discounted factor may be different from β, because the degrees of how agents care for the second period consumption and their children's welfare may not be the same.

Then the problem can be formulated as:

$$V\left(B_t^j + W_t\right) = \max_{A,S,B}\{U(C_{1t}) + \beta p U(C_{2t+1}) + \delta[pV(B_{t+1}^S + W_{t+1}) + (1-p)V(B_{t+1}^D + W_{t+1})]\}$$

$$\text{s.t.} \quad C_{1t} = B_t^j + W_t - A_t - S_t \tag{33}$$

$$C_{2t+1} = \left(\frac{1+r_{t+1}}{p}\right) A_t + (1+r_{t+1})S_t - B_{t+1}^S \tag{34}$$

The first-order conditions are given below:

$$B_t^j: \ V'\left(B_t^j + W_t\right) = U'(C_{1t}), \tag{35}$$

$$A_t: \ U'(C_{1t}) = \beta(1+r_{t+1})U'(C_{2t+1}), \tag{36}$$

$$B_{t+1}^S: \ \beta U'(C_{2t+1}) = \delta V'(B_{t+1}^S + W_{t+1}), \tag{37}$$

$$S_t: \ U'(C_{1t}) = \delta(1-p)(1+r_{t+1})V'(B_{t+1}^D + W_{t+1}) + \beta p U'(C_{2t+1})(1+r_{t+1}). \tag{38}$$

These equations are similar to those of the joy-of-giving model except for having one more condition. However, the meaning is different since $V(B+W)$ is an unknown welfare function instead of an assumed utility function from giving bequests. Equation (35) is the new condition which means that an increase in bequests increases utility from consumption which can be also reflected in the increased total welfare. Equation (36) is the optimal condition for the annuity purchasing and states that the loss of the utility for buying one unit annuity in period *t* is equal to the present value of the expected utility in period *t*+1 from the returns of the one unit bought in period *t*. Equation (37) is the optimal condition for the planned bequest given to the next generation. The present loss of the utility from saving one unit instead of consumption for bequests is equal to the increased discounted utility from the increased welfare of the next generation. Equation (38) is the optimal condition for non-annuity savings. It states that the loss of utility form saving one extra unit is equal to the sum of the discounted next generation's welfare and the expected utility from the consumption in period *t*+1.

Proposition 6: *With a dynastic model, accidental bequests and planned bequests are equal to each other:* $B_{t+1}^{S} = B_{t+1}^{D} = (1 + r_{t+1})S_t$.

Proof: Equations (36) and (37) imply

$$U'(C_{1t}) = V'(B_{t+1}^{S} + W_{t+1})\delta\,(1 + r_{t+1}) \tag{39}$$

Then substituting (39) and (37) into equation (38), we can get

$$V'(B_{t+1}^{S} + W_{t+1}) = V'(B_{t+1}^{D} + W_{t+1}) \tag{40}$$

Since $V(\cdot)$ is strictly increasing and strictly concave following the primitive assumptions of $U(\cdot)$ in equations (35) and (37), this gives that

$$B_{t+1}^{S} = B_{t+1}^{D} = (1 + r_{t+1})S_t = B_{t+1} \tag{41}$$

Q.E.D.

This result is the same as that in the joy-of-giving model. This means that decisions are independent of the mortality history of a family no matter what kind of motive of the two that induces parents to give bequests. This independence is particularly useful in the dynastic model because otherwise it would be extremely difficult to work out the evolution of the state variables. In the dynastic model, the agent's welfare function becomes

$$V(B_t + W_t) = \max_{A,S,B}\{U(C_{1t}) + \beta p U(C_{2t+1}) + \delta V(B_{t+1} + W_{t+1})\}$$

All the following proofs are based on this result.

For simplicity, we assume $\sigma = 1$. Then, the constant-relative-risk-aversion utility from consumption $U(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$ becomes logarithmic utility $U(C) = ln\, C$.

Proposition 7: *With log utility and the Cobb-Douglas production function, agents allocate their annuity saving, non annuity saving and consumption in proportion to their income. Annuity savings are increasing in the survival rate; both non annuity savings and young-age consumption are decreasing in the survival rate.*

Proof: From equation (36), we can get

$$\frac{1}{C_{1t}} = \frac{\beta(1+r_{t+1})}{C_{2t+1}} \tag{42}$$

Substitute (41) into constraint (34),

$$C_{2t+1} = \left(\frac{1+r_{t+1}}{p}\right) A_t \tag{43}$$

Substitute (43) and constraint (33) into equation (42),

$$(1 + \beta p)A_t + \beta p S_t = \beta p(B_t + W_t) \tag{44}$$

From Proposition 6 we know that $B_t^S = B_t^D$,

Equation (39) can be rewritten as

$$U'(C_{1t}) = V'(B_{t+1} + W_{t+1})\delta(1 + r_{t+1}) \tag{45}$$

Equation (35) can be rewritten as

$$V'(B_t + W_t) = U'(C_{1t})$$

Forward to $t+1$ period, then

$$V'(B_{t+1} + W_{t+1}) = U'(C_{1t+1}) \tag{46}$$

Substitute equation (46) into equation (45),

$$U'(C_{1t}) = \delta\,(1 + r_{t+1})\,U'(C_{1t+1}) \tag{47}$$

Under the assumption $U(C) = ln\,C$, equation (47) becomes

$$C_{1t+1} = \delta(1 + r_{t+1})C_{1t} \tag{48}$$

As in the joy-of-giving model, when $\sigma = 1$, the ratios of annuity savings, non annuity savings and young-age consumption to income are all constant. Thus we guess for the dynastic altruism model this result applies as well. We assume $A_t = \xi_A(B_t + W_t)$, $S_t = \xi_S(B_t + W_t)$ and $C_{1t} = (1 - \xi_A - \xi_S)(B_t + W_t)$.

Then equation (44) becomes

$$(1 + \beta p)\xi_A + \beta p \xi_S = \beta p \tag{49}$$

And equation (48) becomes

$$(B_{t+1} + W_{t+1}) = \delta\,(1 + r_{t+1})\,(B_t + W_t) \tag{50}$$

Rewrite equation (50) as

$$\frac{B_{t+1}}{1+r_{t+1}} + \frac{W_{t+1}}{1+r_{t+1}} = \delta\,(B_t + W_t) \tag{51}$$

Under the assumption of the Cobb–Douglas function, $1 + r_{t+1} = \alpha k_{t+1}^{\alpha-1}$; $W_t = (1-\alpha)k_t^{\alpha}$.

$$\frac{W_{t+1}}{1+r_{t+1}} = \frac{(1-\alpha)k_{t+1}^{\alpha}}{\alpha k_{t+1}^{\alpha-1}} = \frac{1-\alpha}{\alpha}k_{t+1} \tag{52}$$

According to general equilibrium, this is the same as in the joy-of-giving model,

$$k_{t+1} = K_{t+1} = A_t + S_t \tag{53}$$

Substitute equation (53) into equation (52),

$$\frac{W_{t+1}}{1+r_{t+1}} = \frac{1-\alpha}{\alpha}(A_t + S_t) \tag{54}$$

According to equation (41) and (54), equation (51) becomes

$$\frac{1}{\alpha}S_t + \frac{1-\alpha}{\alpha}A_t = \delta\,(B_t + W_t) \tag{55}$$

Equation (44) and (55) can be used to solve for both S_t and A_t as follows:

$$A_t = \frac{\beta p(1-\alpha\delta)}{1+\alpha\beta p}(B_t + W_t)$$

$$S_t = \frac{\alpha\delta(1+\beta p)-\beta p(1-\alpha)}{1+\alpha\beta p}(B_t + W_t)$$

The ratios of annuity savings, non annuity savings and the first period's consumption to income are all constant. This result proves our guess is correct as given below:

$$\xi_A = \frac{\beta p(1-\alpha\delta)}{1+\alpha\beta p}$$

$$\xi_S = \frac{\alpha\delta(1+\beta p)-\beta p(1-\alpha)}{1+\alpha\beta p}$$

$$\xi_C = 1 - \xi_A - \xi_S = \frac{1-\alpha\delta}{1+\alpha\beta p}$$

The allocations of annuity savings, non annuity savings and young-age consumption are as follows:

$$A_t = \frac{\beta p(1-\alpha\delta)}{1+\alpha\beta p}(B_t + W_t) \tag{56}$$

$$S_t = \frac{\alpha\delta(1+\beta p)-\beta p(1-\alpha)}{1+\alpha\beta p}(B_t + W_t) \tag{57}$$

$$C_{1t} = \frac{1-\alpha\delta}{1+\alpha\beta p}(B_t + W_t) \tag{58}$$

It is obvious that A_t is increasing in p, and C_{1t} is decreasing in p.

Then take the first derivative of S_t,

$$\frac{\partial S_t}{\partial p} = \frac{(1-\alpha)(\alpha\delta-1)}{(1+\alpha\beta p)^2} < 0$$

Thus S_t is decreasing in p. Q.E.D.

The implications of these results are intuitive. If the survival rate is increased, then agents need to sacrifice more consumption when young for consumption when old. Therefore, young-age consumption decreases and annuity savings for old-age consumption increase but non-annuity savings for bequest giving decrease. These results are still the same as those with the joy-of-giving model. But the allocation of annuity savings, non annuity savings, and young-age consumption has been changed due to different models. We show the different implications below.

Proposition 8: *Agents not only care the size of the bequests they give to offspring, but also take into consideration of their future income. The size of bequests that agents leave is decreasing in their children's wage. Bequests are compensatory.*[6]

Proof: The claim follows equation (50):

$$B_{t+1} = \delta\,(1 + r_{t+1})(B_t + W_t) - W_{t+1}$$

[6] This result shows the difference between the joy of giving model and the dynastic altruism model. In the joy of giving model, agents do not need to consider their children's wage, because their own preference determines the size of bequests only. In the dynastic altruism model, agents care their children's total welfare. Therefore, if their children earn relatively more wages, agents would give fewer bequests.

Q.E.D.

From the equation, it is obvious that bequests are increasing in agents' own income and decreasing in their children's wage. When agents make decisions on how much bequests to give, they will compare their own income with children's earnings. Once their own income overwhelms their children's earnings, they save a portion of their income to support their children. That is, bequests are compensatory. This result has solid empirical supports. Papers like Tomes (1981) and Light and McGarry (2003) used empirical tests and strongly confirmed that children within a family or from the families with the same income level inherited more bequests if they have lower earnings. Comparing to this model, the joy-of-giving model has neglected an important variable that affects agents' decision on giving bequests.

Proposition 9: *The economy converges to a unique steady state* $k^* = (\alpha\delta)^{1/(1-\alpha)}$.

Proof: Substitute equation (56) and (57) into equation (53),

$$k_{t+1} = \frac{\alpha(\delta+\beta p)}{1+\alpha\beta p}(B_t + W_t) \tag{59}$$

Rewrite it as

$$B_t + W_t = \frac{1+\alpha\beta p}{\alpha(\delta+\beta p)} k_{t+1} \tag{60}$$

Substitute equation (60) into equation (50),

$$(B_{t+1} + W_{t+1}) = \delta\,(1 + r_{t+1})\,\frac{1+\alpha\beta p}{\alpha(\delta+\beta p)}\,k_{t+1}$$

$$= \delta\,\frac{1+\alpha\beta p}{\alpha(\delta+\beta p)}\,k_{t+1}\alpha k_{t+1}^{\alpha-1}$$

$$= \delta\,\frac{1+\alpha\beta p}{(\delta+\beta p)}\,k_{t+1}^{\alpha}$$

Back to period t, we get

$$(B_t + W_t) = \delta\,\frac{1+\alpha\beta p}{\delta+\beta p}\,k_t^{\alpha} \tag{61}$$

Substituting equation (61) into equation (59), we can get the capital accumulation function

$$k_{t+1} = \alpha\delta k_t^{\alpha} \tag{62}$$

It is a different capital accumulation function from that in the joy-of-giving model. Thus we can easily get the conclusion that

$$\lim_{k_t\to 0}\frac{dk_{t+1}}{dk_t} = \infty \quad and \quad \lim_{k_t\to\infty}\frac{dk_{t+1}}{dk_t} = 0$$

The economy converges to a steady state.

The balanced growth path is shown in Figure 3.

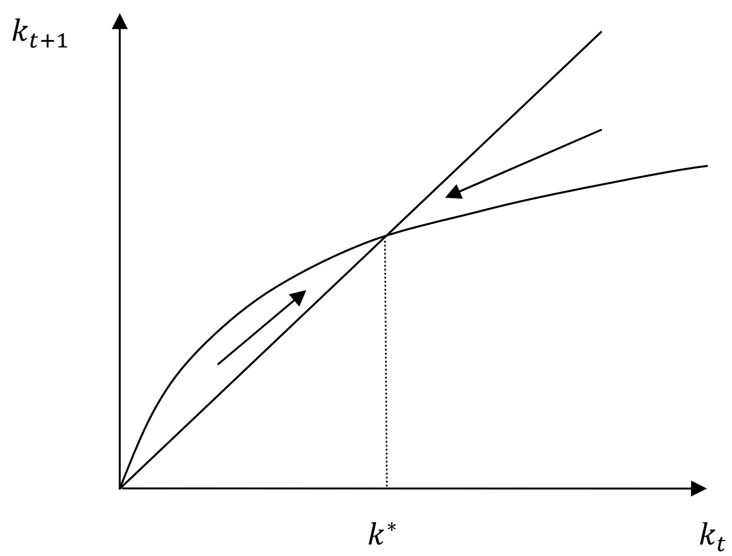

Figure 3

Set $k_{t+1} = k_t$, we get k^*,

$$k^* = (\alpha\delta)^{1/(1-\alpha)} \tag{63}$$

Q.E.D.

We can now obtain the key result in the current paper:

Proposition 10: *Since k^* is increasing in δ, the more agents care about the next generation's welfare, the more capital accumulated. There is no influence of the survival rate on capital accumulation.*

Proof: The proof follows Proposition 9. Q.E.D.

Like the joy-of-giving model, increasing in δ shifts the k_{t+1} curve upwards as shown in Figure 4. Hence, the steady state k^* is also increased to $k^{*\prime}$.

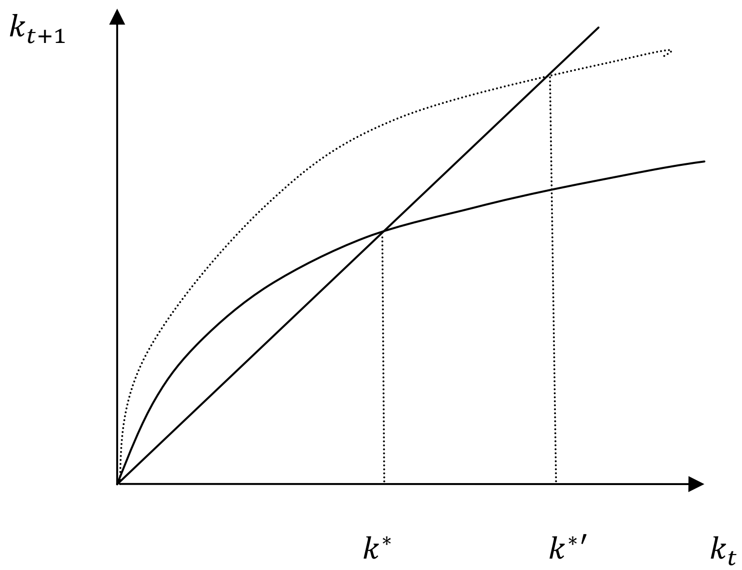

Figure 4

The implication is similar to that in the joy-of-giving model as well. Both δ and Φ represent how much agents care about their next generations. The more they care about their children, the more they save and give, which results in a higher steady state level of capital. The major difference is that the altruism model implies that no matter how much agents care about their second period life or how likely they survive to old age, capital accumulation remains unaffected. As we discussed in the joy-of-giving model, increasing in p or β will on the one hand increase annuity savings and on the other hand increase old-age consumption by cutting non-annuity savings as bequests. The total effect of the survival rate or the taste for old-age consumption on capital accumulation is positive for most plausible parameterization. In the dynastic altruism model, the capital accumulation function is only related with the capital share and the degree of how agents care about the next generation's welfare. The effects of p or β on saving and old-age consumption mutually offset each other. The capital we are discussing about is the capital per worker. Considering capital per person, then it should be divided by the total population, $1+p$, then capital per person is decreasing in the survival rate. This result helps to explain the evidence in Acemoglu and Johnson (2007) that found little relationship between rising life expectancy and total GDP and a negative effect of rising life expectancy on GDP per capita due to population aging. Their paper provides empirical support of our theoretical result. By comparing the different impacts of the survival rate on capital accumulation under two different models, we can conclude that the bequest motive of dynastic altruism is more empirically relevant.

Proposition 11: *Agents' welfare function is a log-linear function of their total wealth and increases with bequests received from parents, given the logarithmic utility.*

Proof: According to equation (58) and (56), we can get

$$C_{1t} = \frac{1-\alpha\delta}{1+\alpha\beta p}(B_t + W_t) \quad (58)$$

$$C_{2t+1} = A_t = \left(\frac{1+r_{t+1}}{p}\right)A_t = \frac{1+r_{t+1}}{p} * \frac{\beta p(1-\alpha\delta)}{1+\alpha\beta p}(B_t + W_t) \quad (64)$$

Then

$$U(C_{1t}) = ln[\frac{1-\alpha\delta}{1+\alpha\beta p}(B_t + W_t)] = ln\frac{1-\alpha\delta}{1+\alpha\beta p} + ln(B_t + W_t) \quad (65)$$

According to equation (59), it follows that

$$U(C_{2t+1}) = ln\frac{\beta(1-\alpha\delta)}{1+\alpha\beta p} + ln(1 + r_{t+1}) + ln(B_t + W_t)$$

$$= ln\frac{\beta(1-\alpha\delta)}{1+\alpha\beta p} + ln(\alpha k_{t+1}^{\alpha-1}) + ln(B_t + W_t)$$

$$= ln\frac{\beta(1-\alpha\delta)}{1+\alpha\beta p} + ln\,\alpha + (\alpha - 1)\,ln\,k_{t+1} + ln(B_t + W_t)$$

$$= ln\frac{\beta(1-\alpha\delta)}{1+\alpha\beta p} + ln\,\alpha + ln(B_t + W_t) + (\alpha - 1)\,ln[\frac{\alpha(\delta+\beta p)}{1+\alpha\beta p}(B_t + W_t)]$$

$$= ln\frac{\beta(1-\alpha\delta)}{1+\alpha\beta p} + ln\,\alpha + \alpha\,ln(B_t + W_t) + (\alpha - 1)\,ln\frac{\alpha(\delta+\beta p)}{1+\alpha\beta p} \quad (66)$$

Because $U(C_{1t})$ and $U(C_{2t+1})$ are both linear function of $ln(B_t + W_t)$, we further assume $V(B_t + W_t) = E + F\,ln(B_t + W_t)$, then $V(B_{t+1} + W_{t+1}) = E + F\,ln(B_{t+1} + W_{t+1})$.

According to equation (50),

$$V(B_{t+1}+W_{t+1}) = E + F\,ln(B_{t+1}+W_{t+1})$$

$$= E + F\,ln[\delta\,(1+r_{t+1})(B_t+W_t)]$$

$$= E + Fln\delta + F\,ln(1+r_{t+1}) + Fln(B_t+W_t)$$

$$= E + Fln\delta + \ Fln(B_t+W_t) +$$

$$F\{ln\,\alpha + (\alpha-1)\ \ ln[\tfrac{\alpha(\delta+\beta p)}{1+\alpha\beta p}(B_t+W_t)]\}$$

$$= E + Fln\delta + F\,ln\,\alpha + F(\alpha-1)\,ln\tfrac{\alpha(\delta+\beta p)}{1+\alpha\beta p} +$$

$$\alpha F\ \ ln(B_t+\ W_t) \tag{67}$$

Substitute equation (65), (66) and (67) into the Bellman equation,

$$V(B_t+W_t) = max_{A,S,B}\{U(C_{1t}) + \beta pU(C_{2t+1}) + \delta V(B_{t+1}+W_{t+1})\}$$

R.H.S.= $E + F\,ln(B_t+W_t)$

L.H.S. = $ln\tfrac{1-\alpha\delta}{1+\alpha\beta p} + ln(B_t+W_t) + \beta p[ln\tfrac{\beta(1-\alpha\delta)}{1+\alpha\beta p} + ln\alpha + \alpha ln(B_t+W_t)$

$+(\alpha-1)\,ln\tfrac{\alpha(\delta+\beta p)}{1+\alpha\beta p}] + \delta[E +\ Fln\delta + F\,ln\,\alpha + \alpha Fln(B_t+W_t)$

$+F(\alpha-1)ln\tfrac{\alpha(\delta+\beta p)}{1+\alpha\beta p}]$

R.H.S.= L.H.S., then

$$Fln(B_t+W_t) = (1+\ \alpha\beta p + \alpha\delta F)ln(B_t+W_t)$$

$$E = ln\tfrac{1-\alpha\delta}{1+\alpha\beta p} +\ \beta p[ln\tfrac{\beta(1-\alpha\delta)}{1+\alpha\beta p} + ln\,\alpha + (\alpha-1)\,ln\tfrac{\alpha(\delta+\beta p)}{1+\alpha\beta p}] +\ \delta[E +$$

$$Fln\delta + F\,ln\,\alpha + F(\alpha-1)\,ln\tfrac{\alpha(\delta+\beta p)}{1+\alpha\beta p}]$$

We can solve the results for E and F,

$$F^* = \tfrac{1+\,\alpha\beta p}{1-\alpha\delta}$$

$$E^* = \tfrac{1+\beta p}{1-\delta}\,ln\tfrac{1-\alpha\delta}{1+\alpha\beta p} + \tfrac{\alpha(\delta+\beta p)}{(1-\delta)(1-\alpha\delta)}\,ln\,\alpha + \tfrac{\beta p}{1-\delta}\,ln\,\beta + \tfrac{\delta(1+\,\alpha\beta p)}{(1-\delta)(1-\alpha\delta)}\,ln\delta -$$

$$\tfrac{1+\,\alpha\beta p}{1-\alpha\delta}\,ln(1+\alpha\beta p) - \tfrac{(1-\alpha)(\delta+\beta p)}{(1-\delta)(1-\alpha\delta)}\,ln(\delta+\beta p)$$

Thus our guess is correct and the welfare function is

$$V(B_t + W_t) = E^* + F^* \, ln(B_t + W_t)$$

Q.E.D.

Agents' welfare function is a linear function of $\ln(B_t + W_t)$, and increasing in bequests and income. According the previous proofs, the bequest motive of dynastic altruism is more empirically plausible than joy of giving. Therefore, in our view the utility from bequests should be derived from children's welfare. That is, when parents give bequests, they consider the children's welfare in a form $\mathrm{V}(\mathrm{B}_{\mathrm{t+1}} + \mathrm{W}_{\mathrm{t+1}})$. This result once more proves that parents' bequests are compensatory. When children have lower income, parents will relatively give higher bequests because parents want to keep their children's welfare at certain level. The solution for the welfare function gives a theoretical reference for future assumption of the bequest function.

4. Conclusion

We have analyzed two models in comparison in this paper. The first one is an uncertain lifetime overlapping-generations model with the joy of giving bequests to children. In this model, the utility function of bequests is assumed and the size of bequests determines how much utility agents can get from giving bequests. We maximize agents' two-period lifetime utilities including lifecycle consumption and the expected utility from accidental bequests and planned bequests. We find that the accidental bequest equals the planned bequest with joy-of-giving. Agents purchase annuity to support their old-age consumption and hold non-annuity savings to give bequests. Annuity savings are increasing in the survival rate while non annuity savings are decreasing in the survival rate. The economy in the joy-of-giving model converges to a unique steady state of capital. The steady state capital is increasing in the survival rate, a result that may be inconsistent with evidence in the literature. Also inconsistent with evidence is the implication of the joy-of-giving for equal bequests among siblings.

The second model in our paper assumes dynastic altruism whereby agents derive utility from their own consumption as well as from future generations' welfare. The welfare function is unknown and has to be solved. We find that accidental bequests and planned bequests are still equal to each other as in the joy-of-giving model. This result simplifies the analysis of the distribution and evolution of capital whereby family mortality history does not matter. The steady state level of aggregate capital or total output in the dynastic model is not affected by the survival rate, which is consistent with recent empirical evidence. In this sense, the dynastic altruism bequest motive is a more plausible assumption than the joy of giving bequest motive.

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