DETECTION AND IDENTIFICATION OF MEAN SHIFTS IN MULTIVARIATE AUTOCORRELATED PROCESSES: A COMPARATIVE STUDY

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Abstract

A common problem existing in any business or industry processes is variability. Reduced variability means more consistency, thus more reliable and better products and services. Statistical process control (SPC) has been one of the widely used methods to monitor processes and to aid in reducing variability and improving process consistency. A basic assumption in traditional statistical quality control is that the observations are independently and identically distributed; however, this assumption may not be valid in many business/industry processes. Observations are often serially correlated; moreover, these processes involve multiple variables. Limited research has been done in multivariate autocorrelated SPC.

In this thesis, a neural-network-based control scheme is proposed for monitoring and controlling multivariate autocorrelated processes. The network utilizes the effective Extended Delta-Bar-Delta learning rule and is trained with the powerful Back-Propagation algorithm. To illustrate the power of the proposed control scheme, its Average Run Length (ARL) performance is evaluated against three statistical control charts, namely, the Hotelling T^2 chart, the MEWMA chart, and the Z chart, in bivariate autocorrelated processes. It is shown that the NN-based control scheme performs better than the Hotelling T^2 chart and the Z chart when it is used to detect small to moderate shifts, i.e., shift size < 2σ. Also, the NN-based control scheme is better than the MEWMA chart in detecting small to moderate shifts in the processes with high correlation or high autocorrelation.

Unlike most of the conventional control charts, a salient feature of the proposed control scheme is its ability to identify the source(s) of process mean shifts. This First-Detection capability greatly enhances the process-improvement ability in a

business/industry environment where processes are multivariate and autocorrelated. The proposed control scheme is also shown to be effective in more complex surroundings, that is, it can detect and identify mean shift in the multivariate autocorrelated processes where the number of interested variables is more than 2. Illustrative examples and a case study are given to show the application of the proposed NN-based control scheme in practice.

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Chapter 1

Introduction

1.1 Background

Increasing global competition among companies puts high pressure on organizations to lower production costs and increase product quality. Statistical process control (SPC) is a powerful tool to improve product quality by using statistical tools and techniques to monitor, control and improve processes. The control chart is the main tool associated with statistical process control. A control chart is a plot of a process characteristic, usually over time with statistically determined limits. When used for process monitoring, it helps the user to determine the appropriate type of action to take on the process.

Statistical process control can be used in a wide range of organizations and applications. For example, SPC can be used to control the delivery time in express delivery companies, such as DHL, to improve their level of service. DHL has a service called “StartDay Express” which guarantees next day door-to-door delivery by 9am; however, the delivery time varies. Since some tasks take less time while some tasks are delayed, there is a need for service process control. A control chart can be built to monitor the delivery time. When an out-of-control point appears, investigations of the process are needed and corrective actions should be taken. In this way, the service level can be maintained or even improved. As a result, the express delivery company may gain competitive advantage in international competition.

A basic assumption in traditional statistical process control is that the observations are independently and identically distributed; however, this assumption may not be valid in many industrial processes. In supply chains, some of the suppliers are manufacturing organizations whose observations are often serially correlated. For instance, measured variables from a tank, and reactors and recycle streams in chemical processes show significant serial correlation (Harris and Ross, 1991). When autocorrelation is present in the processes, traditional SPC procedures may be ineffective, indeed inappropriate, for monitoring, controlling and improving process quality. Alwan and Roberts (1988), Wardell, Moskowitz and Plante (1992), Lu and Reynolds (1999), Hwarng (2004a, 2005a) etc. proposed interesting statistical or neural-network-based approaches to controlling autocorrelated processes.

In many quality control settings the product under examination may have more than one quality characteristic, and correlations exist among these quality characteristics. One such example can be found in the automotive industry where correlation exists among different measurements taken from the rigid body of an automobile -- distortion of the body results in correlated deviations in these measurements. To control product quality in multivariate processes, multivariate statistical methods are very much desired. One important condition for multivariate analysis to be effective is that several correlated variables must be analyzed jointly. The Hotelling T^2 chart, the MEWMA and the MCUSUM control charts emerge as the times require.

With the development of information technology, data collection has become more and more accurate and convenient. It is evident that complex processes which have autocorrelated multivariate quality characteristics often exist in manufacturing (Nokimos and MacGregor, 1995). Kalgonda & Kulkarni (2004) proposed a Z chart to control product quality in such processes; however, the power of the Z chart has not been extensively studied in their paper. The Z chart is only shown to be efficient in the specified cases. West et al. (1999) recommended the use of a Radial Basis Function neural network (RBFN) to control multivariate autocorrelated manufacturing processes. Nevertheless, the performance evaluation in the RBFN method is not convincing because the criterion (Average Run Length) is only obtained from 25 runs. Moreover, the specified method can not be used to identify the source of shift. The gap in the literature requires a more convincing and reasonable approach to detecting and identifying mean shift in multivariate autocorrelated processes.

1.2 Purpose of the Research

The purpose of this research is to develop a neural-network-based control scheme to enhance process-troubleshooting capabilities in a multivariate autocorrelated environment. Specifically, there are four major objectives.

a) To propose a neural-network-based control scheme to detect and identify the mean shift in multivariate autocorrelated processes.

b) To evaluate the performance of the proposed control scheme based on the criteria of Average Run Length (ARL) and the First-Detection rate.

c) To compare the performance of the proposed control scheme with other statistical control schemes.

d) To demonstrate how to apply the proposed control scheme in practice.

1.3 Structure of the Thesis

The structure of the thesis is as follows. In Chapter 2, a literature review is conducted on the existing process control schemes. Chapter 3 illustrates the proposed methodology. In Chapter 4, the performance of the proposed control scheme on bivariate autocorrelated processes is evaluated through comparison with three statistical control charts. In Chapter 5, illustrative examples and a case study are given to show the application of the proposed NN-based control scheme in practice. The extension of the application of the proposed control scheme in multivariate autocorrelated process, where the number of interested variables is larger than 2, is also studied in Chapter 5. Chapter 6 summarizes the contributions and the limitations of the proposed control scheme and future researches are also pointed out in Chapter 6.

Chapter 2

Literature Review

2.1 Statistical Control Schemes

A primary tool used for SPC is the control chart. A control chart is a graphical representation of certain descriptive statistics for specific quantitative measurements of the process. In the following subsections, some widely used control charts will be reviewed.

2.1.1 Classical Statistical Control Schemes

The Shewhart $\overline{X}$ control chart, Cumulative Sum (CUSUM) control chart, and Exponentially Weighted Moving Average (EWMA) control chart are regarded as classical control schemes. Classical statistical control techniques focus on the monitoring of one quality variable at a time. And in classical control schemes, an assumption is made that the values of the process mean and variance are known prior to the start of process monitoring.

A general model for the $\overline{X}$ control chart is given as follows. Let x be a sample statistic that measures some quality characteristic of interest, and suppose that the mean of x is μ_x and the standard deviation of x is δ_x. Then the control limits of the $\overline{X}$ control chart are $\mu_x \pm L\delta_x$ where L is defined as the "distance" of the control limits from the in-control mean, expressed in standard deviation units. If any point exceeds the control limits, the process will be deemed out-of-control. Investigation and corrective action are required to find and eliminate the assignable cause.

A major disadvantage of the $\overline{X}$ control chart is that it can only use recent information, making it relatively insensitive to small to moderate shifts. Two control charts are proposed as excellent alternatives to the $\overline{X}$ control chart when small to moderate shifts are of primary interest. They are the CUSUM and EWMA control charts.

The CUSUM chart incorporates all information in the sequence of sample values by plotting the cumulative sums of the deviations of the sample values from a target value. There are two ways to represent cusums: the tabular cusum and the V-mask form of the cusum. Among these two cusums, as pointed out by Montgomery (2005), tabular cusum is preferable. The mechanics of the tabular cusum are as follows.

Let x_i be the ith observation of the process. If the process is in control, then x_i follows a normal distribution with mean μ_0 and variance σ. Assume σ is known or can be estimated. Accumulate deviations from the target μ_0 above the target with one statistic, $C+$. Accumulate deviations from the target μ_0 below the target with another statistic, $C-$. $C+$ and $C-$ are one-sided upper and lower cusums, respectively. The statistics are computed as follows:

$$\begin{aligned} C_i^+ &= \max(0, x_i - (\mu_0 + k) + C_{i-1}^+) \\ C_i^- &= \max(0, -x_i + (\mu_0 - k) + C_{i-1}^-) \end{aligned} \tag{2.1}$$

where starting values are $C_0^+ = C_0^- = 0$ and k is the reference value. If either statistic (C_0^+ or C_0^-) exceeds a decision interval H, the process is considered to be out-of-control.

The Exponentially Weighted Moving Average (EWMA) control chart is another control scheme useful for detecting small to moderate shifts. It is defined as

$$z_i = \lambda x_i + (1-\lambda) z_{i-1} \tag{2.2}$$

where $0 < \lambda \leq 1$ is a constant and the starting value is the process target, i.e., $z_0 = \mu_0$.

The control limits are

$$\mu_0 \pm L\delta \sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \tag{2.3}$$

where L is the width of the control limits. If any observation exceeds control limits, an out-of-control condition happens.

2.1.2 Statistical Autocorrelated Process Control

The standard application of statistical process control is based on the assumption that the observations are independently and identically distributed; however, this assumption is often violated. Observations are often autocorrelated in industrial processes. Under such conditions, traditional SPC procedures may be inappropriate for statistical process control.

Alwan and Roberts (1988) proposed a Special-Cause Control (SCC) chart to detect mean shift in autocorrelated process. To proceed, one needs to model the process first. Barring any special causes, the residuals should be independently and identically distributed, and hence the assumption of traditional quality control holds. The SCC chart is a standard control chart constructed for the residuals. Meanwhile, the Common-Cause Control (CCC) chart, which is a chart of fitted values, is also proposed to give a view of the current level of the process and its evolution through time.

Wardell, Moskowitz and Plante (1992) compared the Average Run Length performance of the Shewhart chart, EWMA chart, SCC chart and CCC chart when they are used to control ARMA(1,1) processes. They show that SCC and CCC charts perform better when the shift size exceeds 2 standard deviations in ARMA(1,1) processes; the performance of the EWMA chart is not affected much by the presence of data correlation; and the Shewhart chart performs worst in most cases.

Since early detection is helpful to improve the quality of the product, Wardell, Moskowitz and Plante (1994) derived the distributions of run length of the SCC chart for general ARMA(p,q) processes to study whether the SCC chart can detect shift earlier than traditional control charts. After investigating the shape of the probability mass function of run length, the authors conclude that the probability of detecting shifts very early for the SCC chart is actually higher.

Lu and Reynolds (1999) extensively studied the performance of the EWMA chart based both on the residuals and on the original observations of the AR(1) process with a random error. Lu and Reynolds compare the EWMA chart based on the residuals with the EWMA chart based on the original observations and the Shewhart chart. Results show that the EWMA chart based on the residuals is comparable to the EWMA chart based on the original observations when the autocorrelation is low to medium, and the EWMA of the residuals is slightly better when the autocorrelation is high and the shift is large.

Residual-based control charts are limited and require more sophisticated process modeling skill and an initial data set larger than independent case (Lu and Reynolds, 1999). Research has also been done on controlling autocorrelated process without process-modeling first. Zhang (1998) proposed a EWMAST chart to detect the mean shift under autocorrelated data set, in which no modeling effort is required. The control limits of the new chart are analytically determined by the process variance and autocorrelation, and are wider than those of an ordinary EWMA chart when positive autocorrelation is presented. Through simulation, Zhang shows that the proposed method performs better than the Shewhart $\overline{X}$ chart, SCC chart and M–M chart when the process autocorrelation is not very strong and the mean changes are not large.

However, these new control limits can be troublesome to obtain and these limits are only for selective processes.

Jiang et al. (2000) proposed an ARMA chart based on the ARMA statistic of the original observations. They show that both the SCC chart and EWMAST chart are just special cases of this new chart. Simulations show that the ARMA chart is competitive with the optimal EWMA chart for independently and identically distributed observations and performs better than the SCC chart and EWMAST chart for autocorrelated data.

2.1.3 Statistical Multivariate Process Control

In practice, many process monitoring and control scenarios involve several related variables, thus multivariate control schemes are required. The most familiar multivariate process-monitoring and control procedure is the Hotelling T^2 control chart for monitoring the mean vector of the process. The Hotelling T^2 chart was proposed by Hotelling H. in 1947. There are two versions of the Hotelling T^2 chart: one for subgrouped data and the other for individual observations. Since the process with individual observations occurs frequently in the chemical and process industries, the Hotelling T^2 method for individual observations will be introduced in the following.

Suppose that m samples, each of size $n = 1$, are available and that p is the number of quality characteristics observed in each sample. Let $\bar{x}$ and S be the sample mean vector and covariance matrix of these observations respectively. The Hotelling T^2 statistic is defined as

$$T^2 = (x - \bar{x})' S^{-1} (x - \bar{x}) \tag{2.4}$$

The Upper control limit (UCL) and Lower control limit (LCL) for monitoring processes are

$$\mathrm{UCL} = \frac{p(m+1)(m-1)}{m^2 - mp} F_{\alpha,p,m-p}$$

$$\mathrm{LCL} = 0 \tag{2.5}$$

where $F_{\alpha,p,m-p}$ is the upper α percentage point of an F distribution with parameters p and m - p.

The Hotelling T^2 chart is a Shewhart-type control chart. It only uses information from the current sample; consequently, it is relatively insensitive to small and moderate shifts in the mean vector. The MCUSUM control chart and MEWMA control chart, which are sensitive to small and moderate shifts, appear as alternatives to the Hotelling T^2 chart. Crosier (1988) proposed two multivariate CUSUM procedures. The one with the best ARL performance is based on the statistic

$$C_i = \{(S_{i-1} + X_i)'\Sigma^{-1}(S_{i-1} + X_i)\}^{1/2} \tag{2.6}$$

where

$$S_i = \begin{cases} 0, & if\ C_i \le k \\ (S_{i-1} + X_i)(1 - k / C_i), & if\ C_i > k \end{cases} \tag{2.7}$$

with $S_0 = 0$, and $k > 0$. An out-of-control signal is generated when

$$Y_i = (S_i'\Sigma^{-1}S_i)^{1/2} > H \tag{2.8}$$

where k and H are the reference value and decision interval for the procedure, respectively.

Two different forms of the multivariate CUSUM were proposed by Pignatiello and Runger (1990). Their best-performing control chart is based on the following vectors of cumulative sums:

$$D_i = \sum_{j=i-l_i+1}^{i} X_j \tag{2.9}$$

and

$$MC_i = \max\{0, (D_i'\Sigma^{-1}D_i)^{1/2} - kl_i\} \tag{2.10}$$

where $k > 0$, $l_i = l_{i-1} + 1$ if $MC_{i-1} > 0$ and $l_i = 1$ otherwise. An out-of-control signal is generated if $MC_i > H$.

The EWMA control charts were developed to provide more sensitivity to small shifts in the univariate case, and they can be extended to multivariate quality control problems. Lowry et al. (1992) and Prabhu and Runger (1997) developed a multivariate version of the EWMA control chart (MEWMA chart). The MEWMA chart is a logical extension of the univariate EWMA and is defined as follows:

$$Z_i = \lambda x_i + (1-\lambda)Z_{i-1} \tag{2.11}$$

where $0 < \lambda \leq 1$ and $Z_0 = 0$.

The MEWMA statistic is

$$T_i^2 = Z_i'\Sigma_{z_i}^{-1}Z_i \tag{2.12}$$

where the covariance matrix is as follows.

$$\Sigma_{Z_i} = \frac{\lambda}{2-\lambda}\left[1-(1-\lambda)^{2i}\right]\Sigma \tag{2.13}$$

Montgomery (2005) points out that the MEWMA and MCUSUM control charts have very similar ARL performance; however, the MEWMA control chart is much easier to implement in practice. So in this research the MEWMA chart is used as a comparison scheme.

The Hotelling T^2 chart, the MEWMA chart, and the MCUSUM chart summarize the behavior of multiple variables of interest in one single statistic. This does not relieve the need for pinpointing the source of the out-of-control signal. Jackson (1980, 1985) reports some of the earlier attempts to interpret out-of-control signals in multivariate processes. He suggests the use of principal components analysis to decompose T^2 into various independent components. You must examine these components to understand why the process is out-of-control. The disadvantage of this approach is that the principal components do not always provide a clear interpretation of the situation with respect to the original variables.

Another very useful approach to interpreting assignable reasons in multivariate environments, is to decompose the T^2 statistic into components that reflect the contribution of each individual variable. Murphy (1987) used a discriminant analysis approach to separate the suspect variables from the non-suspect variables. Murphy separated the *p* quality characteristics into two subsets, one being the subset that is intuitively suspected to be directly related to the cause of the out-of-control signal. The corresponding T^2 values for two subgroups are calculated and then compared with certain cut-offs to decide the out-of-control variables. A limitation of this procedure is that the more variables in the process, the more ambiguity is introduced in the identification process, which sometimes leads to erroneous conclusions.

Chua and Montgomery (1992) designed a system which tests every possible subset of interested process variables to improve Murphy (1987)'s procedure. However, the all-possible-subsets method can be very computer intensive and therefore may not be practical in some applications.

Mason, Tracy and Young (1995) proposed an alternative method to decompose T^2 for diagnostic purposes. They decompose T^2 into independent parts, each of which is similar to an individual T^2 variate. Given *p* multivariate characteristics, they decompose T^2 into *p* parts, one of which is a T^2 value for a single variable and those left are conditional T^2 values. Thereafter, each component in the decomposition can be compared to a critical value as a measure of largeness of contribution to the signal. However, one overall T^2 statistic can be yielded by *p!* different partitions. The computation will be huge when *p* is large.

To circumvent the problem of large computations in Mason, Tracy and Young (1995), Runger, Alt, and Montgomery (1996) proposed a similar method which requires fewer computations. They define T^2 as the current value of the statistic and $T^2_{(i)}$ as the value of the statistic for all process variables except the i-th one. Then $d_i = T^2 - T^2_{(i)}$ is defined as the indicator of the relative contribution of the ith variable to the overall statistic. When an out-of-control signal is generated, they recommend computing the values of d_i ($i = 1, 2, \ldots, p$) and focusing attention on the variables for which d_i is relatively large. Mason, Tracy and Young (1997) also put forward a new method to make the approach in Mason et al. (1995) more practical. They provide a faster sequential computation scheme for the decomposition.

Different from PCA and decomposition of the T^2 statistic, Hayter and Tsui (1994) proposed a simultaneous-confidence-intervals method to identify the source of out-of-control signal. It operates by calculating a set of simultaneous confidence intervals for the variable means μ_i with an exact simultaneous coverage probability of $1-\alpha$. The process is considered to be in control as long as each of these confidence intervals contains the respective standard value μ_i^0. And the process is deemed to be out of control whenever any of these confidence intervals does not contain its respective control value μ_i^0. However, when using the parametric method, it is hard to obtain the critical point for p-dimensional variables where $p \geq 3$.

2.1.4 Statistical Multivariate Autocorrelated Process Control

With the development of information technology, data collection has become more accurate. In many types of manufacturing processes, the assumption of independence of observation vectors is violated. This will have a profound effect on the performance of ordinary multivariate control charts. Control schemes which are designed for controlling quality in multivariate autocorrelated processes are required.

Mastrangelo and Forrest (2002) present a program to generate data for multivariate autocorrelated processes. In this program, the shift of the process is applied to the mean vector of the noise series while the covariance structure of the data is maintained.

Kalgonda & Kulkarni (2004) proposed a *Z* chart which is used to monitor the mean of multivariate autocorrelated processes. The shifts of the process mean in this paper are additive shifts. The *Z* chart extends Hayter and Tsui's (1994) idea to multivariate autocorrelated environments. It can be illustrated as follows.

The proposed *Z* statistic is given by:

$$Z_{it} = \frac{y_{it} - \mu_{i0}}{r_i(0)}, i = 1,2,\cdots,p \tag{2.14}$$

where y_{it} is the *t*th observation of the *i*th variable, $r_i(0)$ is the standard deviation of the *i*th variable and μ_{io} is the target mean of the *i*th variable. And

$$Z_t = \max(|Z_{1t}|,\cdots,|Z_{pt}|) \tag{2.15}$$

When $Z_t \leq C_{\rho,\alpha}$, the process is considered in-control. This $C_{\rho,\alpha}$ depends on the cross-correlation structure of multiple variables and is chosen to achieve a specified in-control ARL; ρ is the correlation between two variables and α is the type I error. The authors claim that this *Z* chart can not only detect an out-of-control status but also can help identify variable(s) responsible for the out- of-control situation. However, the power of the *Z* chart has not been extensively studied in their paper; the *Z* chart is only shown to be efficient in the specified cases.

Besides statistical process control techniques, neural-network-based control techniques have also been developed to perform process control. In the following subsection, literature on the application of neural network in process control will be reviewed.

2.2 Neural-Network Control Schemes

A neural network consists of a number of interconnected nodes called neurons and is considered a computational algorithm to process information. A neural network can be designed to perform process control. Compared with statistical process control methods, Neural-network-based control schemes are more flexible and adaptive. The neural network application to process control can be generally classified into two types: pattern recognition and shift detection.

2.2.1 Pattern Recognition

A process exhibits random behavior when it is only affected by common causes. Random behavior is regarded as a natural pattern. On the contrary, assignable causes trigger nonrandom behavior. Nonrandom behavior is sometimes referred as an unnatural pattern. To manage and improve quality, manufacturing industries need to find unnatural patterns and correspondingly take corrective actions.

Hwarng and Hubele (1993) developed a pattern recognizer based on back-propagation algorithm (BPPR). In order to identify unnatural patterns which are likely to be exhibited by sampled averages, BPPR is trained on all those interested pattern classes simultaneously. Using average run length index as a performance criterion, they show that the proposed pattern recognizer is capable of detecting most target patterns within two or three successive classification attempts with an acceptable Type I error.

Pham and Oztemel (1994) proposed an LVQ-based (Learning Vector Quantization) neural network to recognize unnatural patterns. Pham and Oztemel extend the existing LVQ methods which update one weight vector at one learning iteration, to update two reference vectors in most iterations. In this way, the learning time is decreased and the generalization capability is increased. Using classification accuracy (%) as the performance criteria, Pham and Oztemel conclude that the proposed new method enables the network to perform classification with almost 98% accuracy.

Hwarng and Chong (1995) developed a pattern recognizer based on adaptive resonance theory. The new pattern recognizer adopts a quasi-supervised training strategy and inserts a synthesis layer into the traditional ART network structure. By comparing with BPPR, Hwarng and Chong show that the new pattern recognizer performs better in detecting cyclic pattern, inferior on mixture patterns, and comparable on other patterns.

Cheng (1997) proposed two neural network pattern recognizers. The first one is based on the back-propagation neural network and the other is based on the modular neural network. Different from Hwarng and Hubele (1993), Cheng studied the situations where in-control data occurred before the pattern. Through Monte Carlo simulations, Cheng showed that the proposed pattern recognizers could recognize multiple unnatural patterns for which they were trained, and the proposed modular neural network could provide better recognition accuracy than back-propagation network when there would be strong interference effects.

2.2.2 Shift Detection

Another utility of the neural network method in SPC is shift detection. Pugh (1989) appears as one of the earliest researchers using neural networks in the field of shift detection. Pugh (1989) successfully trained back-propagation networks for detecting process mean shifts with subgrouping sizes of five. Pugh concluded that the proposed method performed comparably to the $\overline{X}$ control chart when average run length is used as the performance criterion.

Smith (1994) trained back-propagation networks to detect both mean and variance shifts in independently and identically distributed univariate processes. He demonstrated that neural networks could be comparable with $\overline{X}$ and R control charts for large shifts in mean or variance and would outperform them for small shifts.

Cheng (1995) developed a neural-network-based method to detect gradual trends and sudden shifts in the process mean. The network was trained by the back-propagation algorithm. The combined Shewhart-CUSUM scheme proposed by Lucas (1982) is regarded as a benchmark. Through simulation, Cheng showed that the proposed method performed superior to the combined Shewhart-CUSUM control schemes in ARL performance.

Chang and Aw (1996) proposed a neural fuzzy control chart for not only identifying univariate process mean shifts but also for classifying their magnitudes. This proposed neural network was trained by the back-propagation algorithm, then fuzzy set theory was adopted to analyze neural network outputs. Chang and Aw divide the neural network outputs into nine fuzzy decision sets, some of which may overlap with each other. Compared with the performance of the conventional $\overline{X}$ chart and CUSUM chart in terms of the average run lengths, the proposed chart is superior.

Ho and Chang (1999) conducted a relatively extensive comparative study, simultaneously monitoring process mean and variance shifts using neural networks in independently and identically distributed univariate processes. In this study, they proposed a combined neural network control scheme which consisted of one neural network for monitoring process mean and another neural network for monitoring process variability. Compared with the performance of other traditional SPC charts, such as the $\overline{X}$ and R, CUSUM and EWMA charts, and other neural networks and Bayesian classification techniques in terms of average run length (ARL) and percentage of correct classifications; the proposed combined control scheme is superior or comparable when detecting small mean shifts, except for the ARL.

Since observations are often autocorrelated in industrial processes, neural network methods are extended to the field of detecting mean shifts in autocorrelated univariate processes. Cook and Chiu (1998) developed radial basis function neural networks to identify shifts in two specified autocorrelated processes. They show that the proposed networks were successful at detecting shifts in these specified cases. In Chiu, Chen and Lee (2001), a back-propagation neural network is adopted to identify shifts in AR(1) processes which have different autocorrelation coefficients. Through simulation, they show that their networks were successful at separating data that were shifted one, two and three standard deviations from non-shifted data for generated process data. However, as Hwarng (2004) points out, the way of representing data and the performance criteria in Cook and Chiu (1998) and Chiu, Chen and Lee (2001) are not robust and the result, when compared with traditional methods, is not convincing. Hwarng (2004) proposes a back-propagation neural network which uses the Extended Delta-Bar-Delta learning rule to detect process mean shift in AR(1) processes. Using ARL as the performance criterion, through comparative study, Hwarng shows that the performance of this neural-network-based monitoring scheme is superior to that of the SCC, $\overline{X}$, EWMA, EWMAST and ARMAST control charts in most instances.

Hwarng (2005) extends his study in 2004 to identify mean shift and correlation parameter change simultaneously in AR(1) processes. This back-propagation neural network also uses the Extended Delta-Bar-Delta learning rule. Various magnitudes of process mean shift and various levels of autocorrelation are considered in this research. Hwarng shows that the proposed identifier, when it is properly trained, is capable of simultaneously indicating whether the process change is due to mean shift, correlation change, or both.

Neural network method can also be used to detect mean shift in bivariate processes. In Hwarng's (2004b, 2005b), neural-network-based control schemes are proposed to control bivariate processes. Hwarng proposes a back-propagation neural network which is capable of detecting process mean shift and identifying the sources of shifts. In these two papers, various network configurations and training strategies are investigated. Taking ARL as the performance criterion, Hwarng shows that the proposed method is superior to the Hotelling T^2 chart for small to medium shifts.

West et al. (1999) appears as the only research that has been done using the neural network method to control mean shift in multivariate autocorrelated processes. They develop a control scheme which utilizes radial basis function neural networks to capture process mean shift in multivariate autocorrelated processes. The data in West et al. (1999) are generated in a way similar to what Mastrangelo and Forrest (2002) described. The radial function employed in this article is the Gaussian function. Through experiment design, they claim that the radial basis function network is superior to three other control models—the multivariate Shewhart control chart, the multivariate EWMA control chart and a back-propagation neural network. However, there are several limitations in this paper. Firstly, the ARL results in this paper are obtained from only 25 runs, which is not convincing. Secondly, in multivariate processes, it is important to know the source of shift. This paper, however, does not consider this topic.

2.3 Gaps in the Literature

The Z chart proposed by Kalgonda & Kulkarni (2004) and the neural network method proposed by West et al. (1999) are the existing methods in detecting and identifying process mean shifts in multivariate autocorrelated processes. The Z chart, however, only considers certain cases of process mean shift and the power of this method in general cases is not clear. The neural network method, which is based on radial basis function, suffers from the disadvantage of not identifying the source of mean shift. Moreover, its performance criterion, the ARL, is obtained from 25 runs, which is relatively small and thus unconvincing. In this thesis, a new neural-network-based control scheme which is based on the back-propagation algorithm is proposed. The advantage of the proposed control scheme is that it can efficiently detect small to moderate mean shifts and identify the source of the shifts. The Z chart is also extended to a general case and its power is evaluated.

Chapter 3

Methodology

The proposed control scheme is based on the theory of neural computing. There are three major steps in this control scheme: the data generation step, the network training step, and the testing step which is used to investigate the capabilities of the proposed network. To facilitate the understanding of the proposed control scheme, a schematic diagram is given in Figure 3.1.

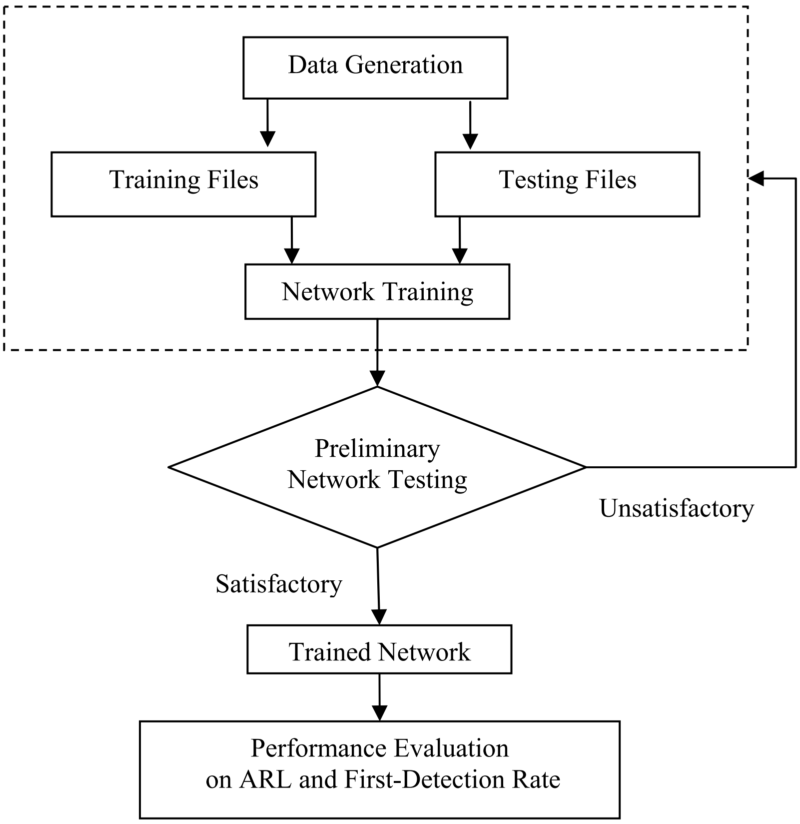


Figure 3.1 A schematic diagram of the proposed methodology

3.1 Model of Interest: Vector Autoregressive Model

The interest of this research is to detect and identify mean shifts in multivariate autocorrelated processes. A multivariate autocorrelated process can be expressed as a Vector Autoregressive model. A VAR(p) model is defined in the following way:

$$Y_t - \mu_t = \Phi_1(Y_{t-1} - \mu_{t-1}) + \Phi_2(Y_{t-2} - \mu_{t-2}) + \cdots + \Phi_p(Y_{t-p} - \mu_{t-p}) + \varepsilon_t \qquad (3.1)$$

where μ_t is the vector of mean values at time t, ε_t is an independent multivariate normal random vector with the mean vector of zeros and covariance matrix Σ, and Φ_i ($i = 1, 2, \ldots, p$) is a matrix of autocorrelation parameters.

The simplest case in the vector autoregressive model is the bivariate VAR(1) model, which is given as follows.

$$Y_t = \mu_t + \Phi(Y_{t-1} - \mu_{t-1}) + \varepsilon_t \qquad (3.2)$$

where μ_t and ε_t are the same as those in equation (3.1). Here Φ is a 2×2 matrix of autocorrelation parameters. It is assumed that $\boldsymbol{Y}_t$ is stationary in this research; therefore, μ_t is constant over time.

$$Y_t = \mu + \Phi(Y_{t-1} - \mu) + \varepsilon_t \qquad (3.3)$$

The covariance matrix of Y_t can be obtained as follows.

$$\Sigma_{Y_t} = \Phi \Sigma_{Y_t} \Phi' + \Sigma \qquad (3.4)$$

3.2 Neural Network

The purpose of this research is to propose a control scheme to monitor process mean in multivariate autocorrelated process based on the theory of neural computing. In this subsection, knowledge about neural network is explained.

A neural network consists of a number of simple, highly interconnected processing elements. The interconnections are weights that are adaptively updated according to specified input and output pairs. Processing requirements in neural computing are not programmed explicitly but encoded in the internal connection weights. A neural network does not store the information in a particular location but stores the knowledge both in the way the processing elements are connected and in the importance of each connection between processing elements. There are four basic components in a neural network: processing elements, connections, the transfer function, and the learning rule. Figure 3.2 is a schematic diagram which shows the relationship between these components.

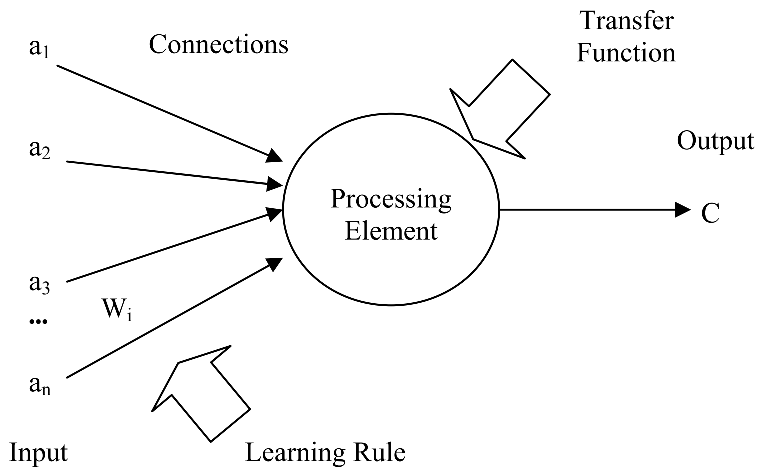


Figure 3.2 A schematic diagram of a neural network

3.2.1 Training Algorithm

In order to train the network, a proper training algorithm needs to be chosen. Back-propagation is a general purpose network paradigm that can be used for system modeling, prediction, classification, filtering and many other general types of problems.

The back-propagation network is a multilayer feed-forward network with a transfer function in the artificial neuron and a powerful learning rule. Figure 3.3 illustrates a typical back-propagation network.

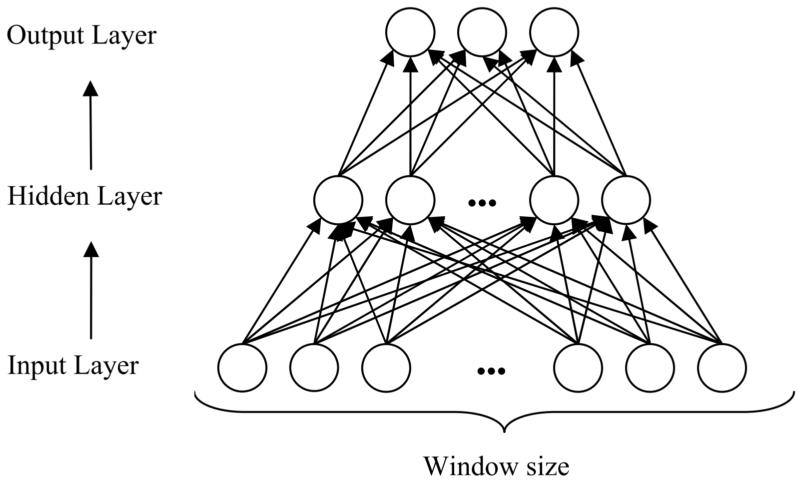


Figure 3.3 A typical back-propagation network

Back-propagation learns by calculating an error between desired and actual output and propagating this error information back to each node in the network. This backpropagated error is used to drive the learning at each node. The rate at which these errors modify the weights is referred to as the learning rate or learning coefficient. Momentum is a term added to the standard weight change which is proportional to the previous weight change. The momentum coefficient is another parameter which controls learning; it says that if weights are changing in a certain direction, there should be a tendency for them to continue changing in that direction.

Based on experiments with the Radial Basis Function network and the back-propagation network, it is found that the back-propagation algorithm **(**Rumelhart et al. 1986**)** is still the best to adopt in this research based on the Root Mean Square (RMS) of errors.

3.2.2 Learning Rule

An essential characteristic of a network is its learning rule, which specifies how weights adapt in response to a learning example. Standard back-propagation uses a generalized delta rule (Rumelhart et al. 1986) that updates network connection weights without adapting its learning coefficient or momentum coefficient over time. The standard Delta-Rule weight update is given by:

$$w[k+1] = w[k] + \alpha\delta[k] + \mu\Delta w[k] \tag{3.5}$$

where $w[k]$ is the connection weight at time k, α is the learning rate, μ is the momentum coefficient, $\delta[k]$ is the gradient component of the weight change at time k, *and* $\Delta w[k]$ is the weight change at time k. Here α and μ are fixed constants. In standard back-propagation, the gradient component is calculated as follows:

$$\delta[\mathrm{k}] \quad = \quad \frac{\partial \mathrm{E}[\mathrm{k}]}{\partial \mathrm{w}[\mathrm{k}]} \tag{3.6}$$

where E[k] is the value of the error at time k and w[k] is the connection weight at time k. The drawback of the Delta-Rule is that the learning may be tremendously slowed down or even stuck at some local minima without ever reaching convergence.

Jacobs (1988) proposed the Delta-Bar-Delta (DBD) learning rule which tries to address the speed of convergence issue via the heuristic route. DBD speeds up the learning by adapting the learning coefficient over time, which can be written as:

$$w[k+1] = w[k] + \alpha[k]\delta[k] + \mu\Delta w[k] \tag{3.7}$$

where $\alpha[k]$ is the connection learning rate at time k. $\alpha[k]$ is calculated by the following equation:

$$\Delta\alpha[\mathrm{k}] = \begin{cases} \mathrm{k} & \text{if } \bar{\delta}[\mathrm{k-1}]\delta[\mathrm{k}] > 0 \\ -\varphi\alpha[\mathrm{k}] & \text{if } \bar{\delta}[\mathrm{k-1}]\delta[\mathrm{k}] < 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.8}$$

where $\bar{\delta}[k]$ is the weighted, exponential average of previous gradient components at time *k*. It is defined as:

$$\bar{\delta}[k] = (1-\theta)\delta[k] + \theta\delta[k-1] \tag{3.9}$$

Minai and Williams (1990) proposed a new learning rule which incorporates momentum adjustment, based on heuristics, in an attempt to increase the rate of learning. This new rule is called the Extended-Delta-Bar-Delta (EDBD) learning rule. For EDBD, the variable learning rate and variable momentum rate yield

$$w[k+1] = w[k] + \alpha[k]\delta[k] + \mu[k]\Delta w[k] \tag{3.10}$$

where $\mu[k]$ is the connection momentum rate at time *k*. Similar to the DBD rule, $\alpha[k]$ is calculated as follows.

$$\alpha[k] = MIN[\alpha_{\max,} \alpha[k-1] + \Delta\alpha[k-1]]$$

$$\Delta\alpha[\mathrm{k}] = \begin{cases} \mathrm{k}_\alpha \exp(-\gamma_\alpha \left|\bar{\delta}[\mathrm{k}]\right|) & \text{if } \bar{\delta}[\mathrm{k}-1]\delta[\mathrm{k}] > 0 \\ -\varphi_\alpha \alpha[\mathrm{k}] & \text{if } \bar{\delta}[\mathrm{k}-1]\delta[\mathrm{k}] < 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.11}$$

where

$$\bar{\delta}[k] = (1-\theta)\delta[k] + \theta\delta[k-1] \tag{3.12}$$

and k_α is a constant learning rate scale factor, γ_α is a constant learning rate exponential factor, φ_α is a constant learning rate decrement factor, and α_{max} is the upper bound on the learning rate.

The momentum rate change is, similarly,

$$\mu[k] = MIN[\mu_{\max}, \mu[k-1] + \Delta\mu[k-1]]$$

$$\Delta\mu[\mathrm{k}] = \begin{cases} \mathrm{k}_\mu \exp(-\gamma_\mu \left|\bar{\delta}[\mathrm{k}]\right|) & \text{if } \bar{\delta}[\mathrm{k}-1]\delta[\mathrm{k}] > 0 \\ -\varphi_\mu \mu[\mathrm{k}] & \text{if } \bar{\delta}[\mathrm{k}-1]\delta[\mathrm{k}] < 0 \\ 0 & \text{otherwise} \end{cases} \tag{3.13}$$

where k_μ is a constant momentum rate scale factor, γ_μ is the constant momentum rate exponential factor, φ_μ is a constant momentum rate decrement factor, and μ_{max} is the upper bound on the momentum rate. Note that an additional tolerance parameter, λ, is used to recover the best connection weights learned if $E[k] > E_{min}\lambda$ at the end of each learning epoch where E_{min} is the minimum previous error. In this research, the EDBD rule is found to be the most effective and efficient learning rule that guarantees convergence.

3.2.3 Transfer Function

The transfer function is a method of transforming the input. It transfers the internally generated sum for each processing element to a potential output value. Usually, non-linear functions, such as the hyperbolic tangent function (TanH) or sigmoid function, are recommended.

The sigmoid function is a continuous monotonic mapping of the input into a value between 0.00 and 1.00. The sigmoid function is defined as

$$f(z) = (1 + e^{-z})^{-1} \tag{3.14}$$

The hyperbolic tangent function (TanH) is just a bipolar version of the sigmoid function. The sigmoid is a smooth version of a {0, 1} step function, whereas the hyperbolic tangent is a smooth version of a {-1, 1} step function.

The TanH is defined by

$$f(z) = \frac{e^{z} - e^{-z}}{e^{z} + e^{-z}} \tag{3.15}$$

By experiment, the sigmoid function is found to perform better than the TanH in this research.

3.3 Data Generation

3.3.1 Data Representation

In the data generation step, the most important topic is how to represent training and testing data. The data representation in the training set has critical influence on the performance of neural networks. Data representation consists of the following parts: the selection of parameters for training and testing data sets and the determination of window size.

3.3.1.1 Selection of Parameters

For easy demonstration, the mean shifts in the bivariate VAR(1) process are studied. The bivariate VAR(1) model is given in Equation (3.3). There are two process variables, X and Y, in the bivariate autocorrelated process; consequently, five parameters are required to be specified. They are the mean shift size of X (δ_x), the mean shift size of Y (δ_y), autocorrelation of X (ϕ_x), autocorrelation of Y (ϕ_y) and correlation (ρ_{xy}) between X and Y.

The purpose of this research is to detect and identify mean shifts in multivariate autocorrelated processes. For this study, various magnitudes of shift in X and Y, various levels of autocorrelation of X and Y, and correlation between X and Y should be investigated. The shift sizes in X and Y are set to 0, 0.5, 1, 2 and 3. Further, the shift can happen on either variable or on both together. Levels of autocorrelation are set as 0, 0.2, and 0.7 to cover the whole range of permissible positive parameter space. Next, the correlation between X and Y is set to 0, 0.4 or 0.7 where 0 stands for no correlation, 0.4 means moderate correlation and 0.7 is high correlation between X and Y. For convenient reference, all parameter values selected are listed in Table 3.1.

Table 3.1 Mean shift magnitude, autocorrelation level and correlation level ("--" means that cell is intended to be blank.)

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}
0	0	0	0	0
0.5	0.5	0.2	0.2	0.4
1	1	0.7	0.7	0.7
2	2	--	--	--
3	3	--	--	--

It is clear that there are a total of 675 ($5\times5\times3\times3\times3$) combinations of parameter values. Without loss of generality, the variances of error terms are set to 1 in the simulation analysis.

3.3.1.2 Window Size

The input data file for a neural network should be in a row and column format. Each logical row contains the inputs and (optionally) desired outputs for one example. One logical row of data is defined as one record. For instance, if there were 4 inputs, and 3 possible outputs, there will be 7 numbers (or fields) for each logical row. That is, this record contains 7 numbers. Each number (field) would be separated from the others with at least one space or a comma. The number of inputs each record contains is defined as the window size. Box et al. (1994) pointed out that at least 50 observations are required to obtain a useful estimate of the autocorrelation function. Likewise, to present autocorrelation structure adequately, there is a need to have a sufficiently large window size of input data. Since there are two variables in the studied process, the input should be in the form of long rows of (X, Y) data. The window size is set to 100, i.e., a window includes 50 pairs of inputs.

3.3.2 Generation of Training and Testing Files

In the back-propagation network, the inputs are presented along with desired outputs during the training phase. In this research, five output nodes are used. They are mean shift size of X (δ_x), mean shift size of Y (δ_y), autocorrelation of X (ϕ_x), autocorrelation of Y (ϕ_y) and correlation between the error terms of X and Y (ρ_{xy}). The desired outputs, δ_x and δ_y, represent the magnitude of the shift by a real value between 0.00 and 1.00 based on the classification of the input window. Five real values are assigned to the five levels of shift magnitude, namely, 0.00, 0.25, 0.50, 0.75 and 1.00 for shift size 0.0, 0.5, 1.0, 2.0 and 3.0, respectively. Figure 3.4 demonstrates the corresponding relationship between shift magnitude and the real value representation. The desired outputs, ϕ_x and ϕ_y, represent the level of autocorrelation. Their real values are used as the desired outputs. For correlation between the error terms of X and Y, ρ_{xy}, similar to the autocorrelation case, its real value is used as the desired output.

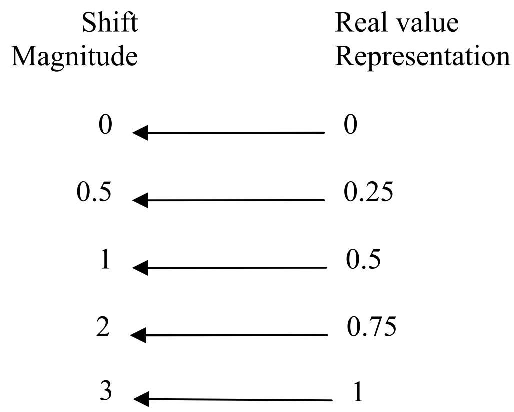


Figure 3.4 Relationship between shift magnitude and real value representation

Before generating training and testing files, the point from which the process shifts needs to be decided. This point is defined as the point of shift. For the training data set, the point of shift is set at the very beginning for each set of parameter values. When the point of shift starts from the beginning, the whole window is shifted data. In this way, ambiguity is prevented and the network can be trained more efficiently. Figure 3.5 shows how the training data is configured.

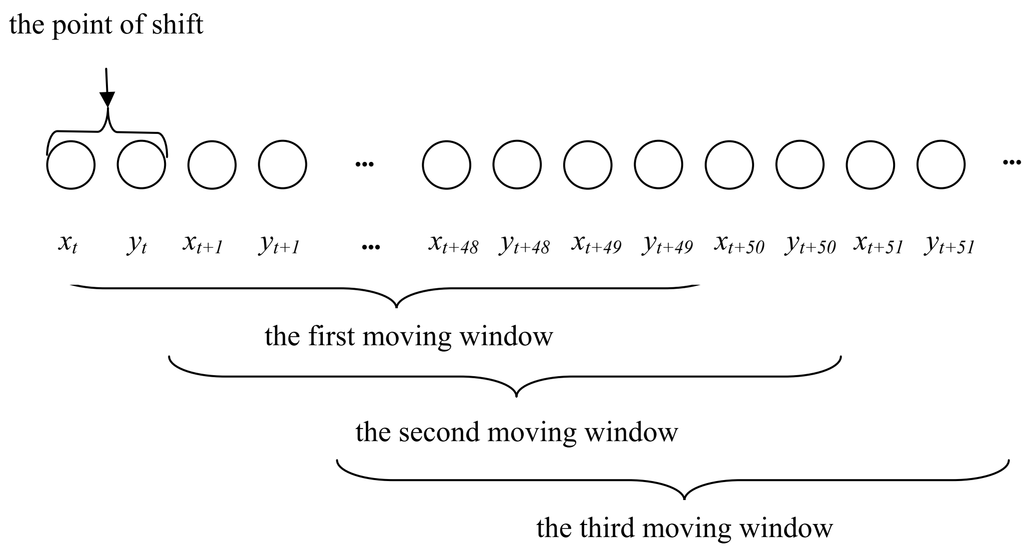


Figure 3.5 Configuration of the training data

As mentioned above, there are 675 combinations of parameter values in total. In order to ensure the effectiveness of network learning, a sufficient number of records for each combination should be generated. However, the number of records for each combination had better not be larger than enough because it will waste more time to generate and require more data storage space. Equation (3.3) is used to generate training data sets. Five training files of different sizes are generated. 27000, 39500, 54000, 67500 and 135000 records are included in these five training files. In order to train the network effectively, the records for each combination of parameter value should be represented evenly. 675 combinations of parameter values are studied in this research, so the records in training data are multiples of 675. In actual implementations, it may be hard to have such number of training data; however, it is easy to train the network with simulated data.

For preliminary study, four different testing files are generated in the same way as generating training files to assess the adequacy of network training. This allows us to confirm if the training has reached a stable situation whereby its performance against new data would not significantly be subject to different data sets. For the testing data of the performance evaluation stage, the shift is fixed at the last pair of observations of the first window, i.e., the point of shift equals 50, as if the process always begins with an in-control state. In other words, there is one shifted pair in the first moving window and two shifted pairs in the second window and so on. Figure 3.6 demonstrates the configuration of the testing data. This setting allows us to test shifts occurring at any point within an observation window. This also permits the measure of average run length (ARL) to be consistent with the conventional ARL definition.

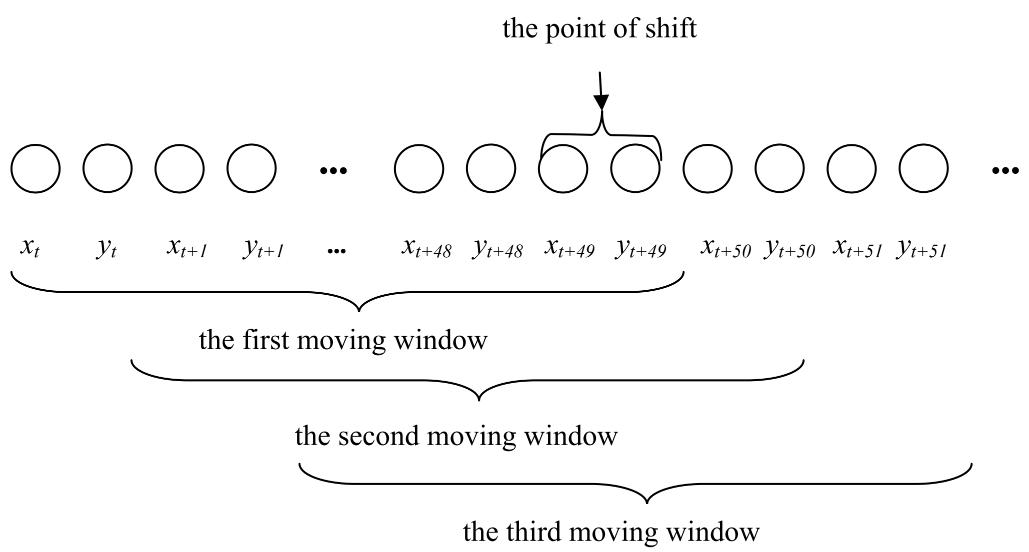


Figure 3.6 Configuration of the testing data

After deciding how to represent data, the next step is how to generate data. A program is written using software S-PLUS (Insightful, 1988) to generate multivariate autocorrelated data with a shift. Mastrangelo and Forrest (2002) pointed out that multivariate process disturbances can occur as additive shifts and innovational shifts. Kalgonda & Kulkarni (2004) studied additive shifts in VAR(1) processes. For easy comparison, additive shifts in VAR processes are studied in this research. The additive shifts are added to the process after fixing the autocorrelation and the correlation.

3.4 Network Training and Testing

With 100 input nodes and 5 output nodes confirmed, the fundamental question to address then is the number of hidden layers and the number of hidden nodes. It is difficult to determine in advance the number of hidden layers and number of processing elements in each hidden layer. This, coupled with slow learning, can lead to very time-consuming trials to achieve the optimal architecture. Although techniques such as cascade-correlation learning (Fahlmann 1988) may be used to determine systematically the number of hidden nodes, they are found to be cumbersome and inefficient for the size of the network and the data set employed in this research. Therefore, a series of experiments with a number of different network structures are carried out to find the proper network configuration. Among them were 100-20-5, 100-10-5, 100-0-5, 100-10-10-5 and 100-5-5-5 where *L1-L2-L3-L4* represents two hidden layers in a network and *Li* denotes the number of nodes in layer *i*. Preliminary studies showed that using two hidden layers provided no advantage at all and 0, 10 and 20 hidden nodes performs almost the same when the performance is measured by ARL, so a network with the structure of 100-0-5 is employed. Figure 3.7 is the neural network employed in this research.

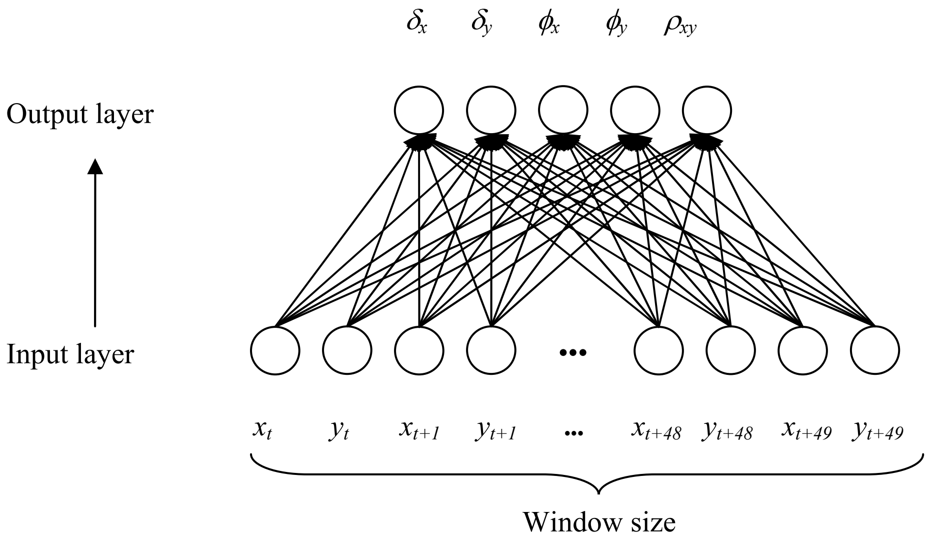


Figure 3.7 The proposed network structure

Based on the outcome of the preliminary investigation, the network is trained with the EDBD rule and the sigmoid transfer function for output layers. The software package used to develop the back propagation neural network models is NeuralWorks Professional II/PLUS (NeuralWare, 2003). All input values are scaled and mapped to the range (-1.0, 1.0). The output values are in the range of (0, 1). One of the problems that can occur with the back-propagation network is over-training. The symptom of this is that the network is performing well on the training data, but poorly on independent test data. A modeling feature, termed Save-Best, in NeuralWorks Professional II allows the user to prevent this problem. Thus, SaveBest is used to train the network and circumvent overtraining. In subsection 3.3.2, five training files with different sizes and four different testing files have already been generated. The preliminary results show that networks trained with different training file sizes achieve different performances. The training file with 135000 records performs best among all 5 cases. When using a training file with 135000 rows to test four different testing files, the network performs almost equally well. This shows that the training had reached a stable situation since its performance against new data would not significantly be subject to different data sets. So this training file is selected as the desired one. In addition, the best testing result of the proposed network was obtained within 2 million training records when using RMS as the performance criterion, thus the learning speed is not slow.

3.5 Output Interpretation

The output values from 5 network output nodes will be real values falling between 0.00 and 1.00. The purpose of this research is to detect and identify process mean shifts, therefore the outputs of mean shift size X (δ_x) and mean shift size Y (δ_y) should be of primary focus. The desired value was assigned proportionally to indicate the magnitude of the classified shift with 0.00 indicating no shift and 1.00 indicating the largest shift (shift = 3σ). This means any output value greater than 0.00 signals a certain extent of deviation from in-control state. It is obvious that the strength of the evidence of deviation is proportional to the magnitude of actual output. The threshold cut-off value is determined by tuning the in-control ARL to a reference length, i.e. 185.4.

Chapter 4

Performance Evaluation

4.1 Performance Measure -- Average Run Length

Conventionally, the average run length (ARL) serves as a very useful and standard criterion for measuring the effectiveness of a control chart scheme. ARL is the expected number of data points collected before an out-of-control situation is signaled. In the neural network context, ARL is defined as the average number of moving windows which is required by the neural network to single a shift.

When there is no shift in both variables, these kinds of processes are deemed in-control. For in-control processes, the ideal performance of control schemes should be that the control schemes can’t find any shift. However, this is impossible in reality since the type I error exists. The probability of type I error in this research is defined as the probability that a control scheme detects a shift when no shift happens in the process. A good control scheme should have small probability of type I error. In-control ARL is related to the measure of the probability of type I error. The smaller the probability of type I error is, the longer the in-control ARL is; in other words, a good control scheme should have long in-control ARL. When any shift happens on any of the process variables, the process is regarded as an out-of-control process. When a process is out-of-control, there is a probability that the control scheme deems it as in-control. This probability is defined as the probability of type II error. A good monitoring scheme should have small probability of type II error. The out-of-control ARL is related to the measure of the probability of type II error. The smaller the probability of type II error is, the shorter the out-of-control ARL is. And the shorter the out-of-control ARL is, the better the control scheme is. In general, a good control scheme should have long in-control ARL and short out-of-control ARL.

For an independently and identically distributed univariate process, the 3-sigma in-control ARL is about 370. The corresponding probability of the type I error is 0.0027. In this research, bivariate autocorrelated processes are considered. When shifts, autocorrelation and correlation are not present, the in-control ARL is calculated to be around 185.4. At this time, the probability of the type I error is 0.00549. In order to tune the in-control ARL to this desired value, several computer programs were written to analyze the network output by using the statistical software named S-PLUS from Insightful (1988). The threshold cut-off value is obtained and is equal to 0.190768.

The testing data were generated as described in subsection 3.3.2 and with parameters listed in Table 3.1. As pointed out in subsection 3.3.2, for the testing data of the performance evaluation stage, the shift is fixed at the last pair of observations of the first window, i.e., the shift point equals 50. (Please refer to Figure 3.6 in subsection 3.3.2.) In this way, there is one shifted pair in the first moving window and two shifted pairs in the second window and so on. Consequently, the ARL in the neural network context can be measured in a way consistent with the conventional practice. Each ARL is computed from 1000 independent simulation runs.

Different from the conventional control charts, the NN-based control scheme has a salient feature that it can indicate which variable is responsible for the shift. In this research, this feature is evaluated by a criterion named First-Detection rate. It is defined as the percentage rate that the NN-based control scheme detects the shifts of the 1000 simulation runs in the variable X, Y, or both variables X and Y. X% is used to represent the First-Detection rate of X, Y% is used to represent the First-Detection rate of the variable Y, and XY% is used to represent the First-Detection rate of both X and Y. The First-Detection rate is calculated from the following equations.

$$
\begin{aligned}
X\% &= \frac{D_X}{SR} \times 100\% \\
Y\% &= \frac{D_Y}{SR} \times 100\% \\
XY\% &= \frac{D_{XY}}{SR} \times 100\%
\end{aligned}
\tag{4.1}
$$

In Equation 4.1, D_X is defined as the No. of runs in which a shift is first detected in the variable X, D_Y is defined as the No. of runs in which a shift is first detected in the variable Y while D_{XY} is defined as the No. of runs in which a shift is first detected in both variables. *SR* is defined as the number of simulation runs from which the ARL is calculated.

When shifts and autocorrelation on both variables are of the same magnitude, an ideal control scheme should have small differences between the First-Detection rate of X and the First-Detection rate of Y. When shifts on both variables are of different magnitude, an ideal control scheme should have a larger First-Detection rate on the variable with larger shift.

4.2 The Performance of the NN-based Control Scheme

Table 4.1 summarizes ARL, the Standard deviation of Run Length (SRL) and First-Detection rate (X%, Y% and XY%) derived from the proposed network. From Table 4.1, it is known that the proposed NN-based control scheme is capable of detecting process mean shift effectively, especially for small to moderate shifts. It can also identify the source of shift correctly in most cases.

4.2.1 No-Shift Processes

When δ_x=0 and δ_y=0, the processes are regarded as no-shift processes. It is observed that the ARL decreases as the autocorrelation increases for the no-shift processes.

This implies that a larger probability of type I error is generated if autocorrelation is not considered. Consequently, more false alarms will be generated. In univariate process control, it is shown that the probability of type I error increases as the autocorrelation increases. So in multivariate autocorrelated process control, a similar observation with univariate autocorrelated process control could be obtained. In no-shift processes, the ARL increases with the increase of the correlation in the error terms. As mentioned in section 4.1, one salient feature of the NN-based control scheme is that it can identify the source of shift. First-Detection rate can be observed to evaluate the First-Detection performance. For no-shift processes, it is clear that autocorrelation does affect the First-Detection capability of the NN-based control scheme. When autocorrelation is present in one of the variables, although no shifts are present on both variables, the First-Detection rate shows that more shifts are detected on the variable with autocorrelation.

4.2.2 Single-Shift Processes

When $\delta_x>0$ and $\delta_y=0$ or $\delta_x=0$ and $\delta_y>0$, the processes are regarded as single-shift processes. In single-shift processes, the ARL increases as the autocorrelation in the shifted variable increases and the ARL decreases as the autocorrelation in the variable without shift increases. For example, when there is no shift on the variable X and small shift is present on the variable Y, it is observed that the ARL changes from 25.29 to 28.86 when autocorrelation on the variable Y increases from 0.2 to 0.7, and the ARL decreases from 24.56 to 22.41 when the autocorrelation on the variable X increases from 0.2 to 0.7. Another observation is that the ARL increases with the increasing of the correlation in some of the single-shift processes. For the First-Detection capability, the NN-based control scheme can detect the true source of shifts in most of the single-shift processes. This is reflected in the First-Detection rate (X%

or Y%). Similar to the observation in the no-shift processes, the autocorrelation affects the First-Detection capability of the NN-based control scheme in single-shift processes. For instance, when there is no shift on the variable X and there is small shift on the variable Y, it is observed that when the autocorrelation of X increases from 0.2 to 0.7, the First-Detection rate of Y decreases from 90.8% to 80.3%.

4.2.3 Double-Shift Processes

When δ_x>0 and δ_y>0, the processes are regarded as double-shift processes. Compared with no-shift processes and single-shift processes, the double-shift processes have smaller ARL when one of the shifts is of the same magnitude as the shift in the single-shift processes. When shifts in double-shift processes are of different magnitudes, the First-Detection rate of the variable with larger shift is larger than that of the variable with smaller shift. It is observed that the ARL increases as the autocorrelation on the larger shift variable increases and the ARL decreases as the autocorrelation on the smaller variable increases. When shifts in the double-shift processes are of the same magnitude, the ARL increases as the correlation increases and the First-Detection rate of the variable X and the First-Detection rate of the variable Y are generally larger than 40%.

Generally speaking, the out-of-control ARL decreases with the increase of the shift magnitude. This is because it is logically easier to identify a larger shift than a smaller shift. When large shifts happen, higher First-Detection rate will also be observed on the variable with large shifts.

Table 4.1 ARL、SRL and First-Detection rate of the proposed NN-based control scheme

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0	0	0	0	0	185.24	203.30	49.1	50.9	0.0
0	0	0	0	0.4	210.21	216.69	46.7	53.3	0.0
0	0	0	0	0.7	253.71	277.74	45.7	53.9	0.4
0	0	0	0.2	0	139.05	134.23	32.9	67.1	0.0
0	0	0	0.2	0.4	169.40	179.15	33.7	66.1	0.2
0	0	0	0.2	0.7	191.13	196.12	29.4	70.2	0.4
0	0	0	0.7	0	62.92	63.19	18.2	81.8	0.0
0	0	0	0.7	0.4	69.34	76.81	15.3	84.6	0.1
0	0	0	0.7	0.7	73.41	80.26	11.9	88.0	0.1
0	0	0.2	0	0	158.57	154.40	57.8	42.2	0.0
0	0	0.2	0	0.4	172.29	184.82	60.5	39.3	0.2
0	0	0.2	0	0.7	199.44	209.51	64.8	34.5	0.7
0	0	0.2	0.2	0	123.80	122.02	44.3	55.6	0.1
0	0	0.2	0.2	0.4	135.42	130.76	47.9	51.9	0.2
0	0	0.2	0.2	0.7	160.79	165.54	45.9	53.5	0.6
0	0	0.2	0.7	0	58.80	59.50	22.6	77.0	0.4
0	0	0.2	0.7	0.4	65.76	70.65	21.3	78.4	0.3
0	0	0.2	0.7	0.7	71.95	78.73	16.4	83.2	0.4
0	0	0.7	0	0	73.65	67.05	74.6	25.2	0.2
0	0	0.7	0	0.4	73.93	67.71	80.2	19.7	0.1
0	0	0.7	0	0.7	83.16	76.67	84.7	15.1	0.2
0	0	0.7	0.2	0	66.80	60.47	67.5	32.1	0.4
0	0	0.7	0.2	0.4	69.07	64.31	72.8	27.1	0.1
0	0	0.7	0.2	0.7	79.51	74.85	78.3	21.2	0.5
0	0	0.7	0.7	0	43.34	42.86	41.4	58.4	0.2
0	0	0.7	0.7	0.4	47.83	48.73	45.1	54.3	0.6
0	0	0.7	0.7	0.7	56.16	57.61	44.6	54.0	1.4
0	0.5	0	0	0	24.86	14.43	7.4	92.5	0.1
0	0.5	0	0	0.4	24.23	14.27	7.7	92.3	0.0
0	0.5	0	0	0.7	24.47	14.36	7.0	93.0	0.0
0	0.5	0	0.2	0	25.29	15.88	7.8	92.2	0.0
0	0.5	0	0.2	0.4	24.97	16.41	7.4	92.6	0.0
0	0.5	0	0.2	0.7	25.18	16.35	6.8	93.2	0.0
0	0.5	0	0.7	0	28.86	25.92	9.1	90.9	0.0
0	0.5	0	0.7	0.4	29.00	28.48	7.9	92.1	0.0
0	0.5	0	0.7	0.7	29.11	27.94	6.5	93.4	0.1
0	0.5	0.2	0	0	24.56	14.27	9.1	90.8	0.1
0	0.5	0.2	0	0.4	24.26	14.55	8.3	91.7	0.0
0	0.5	0.2	0	0.7	24.35	14.40	8.0	91.9	0.1
0	0.5	0.2	0.2	0	24.90	15.58	9.2	90.8	0.0
0	0.5	0.2	0.2	0.4	24.85	16.31	8.2	91.8	0.0
0	0.5	0.2	0.2	0.7	25.10	16.21	7.6	92.4	0.0
0	0.5	0.2	0.7	0	28.23	25.13	10.7	89.2	0.1
0	0.5	0.2	0.7	0.4	28.75	28.17	9.1	90.6	0.3
0	0.5	0.2	0.7	0.7	29.22	28.08	6.9	92.9	0.2
0	0.5	0.7	0	0	22.41	13.94	19.2	80.3	0.5
0	0.5	0.7	0	0.4	22.48	14.25	20.2	79.2	0.6
0	0.5	0.7	0	0.7	23.16	14.43	17.3	82.2	0.5
0	0.5	0.7	0.2	0	22.62	14.89	19.0	80.4	0.6
0	0.5	0.7	0.2	0.4	23.03	15.88	20.0	79.4	0.6
0	0.5	0.7	0.2	0.7	23.75	15.91	16.8	82.4	0.8
0	0.5	0.7	0.7	0	24.48	21.34	20.9	78.5	0.6
0	0.5	0.7	0.7	0.4	26.32	26.19	20.9	78.5	0.6
0	0.5	0.7	0.7	0.7	28.16	27.40	15.4	83.6	1.0
0	1	0	0	0	13.89	7.07	5.9	94.0	0.1
0	1	0	0	0.4	13.44	6.85	5.8	94.0	0.2
0	1	0	0	0.7	13.48	6.93	6.0	93.9	0.1
0	1	0	0.2	0	14.08	7.61	6.0	93.9	0.1
0	1	0	0.2	0.4	13.62	7.26	6.1	93.8	0.1
0	1	0	0.2	0.7	13.66	7.27	5.9	94.0	0.1
0	1	0	0.7	0	16.94	14.28	6.2	93.8	0.0
0	1	0	0.7	0.4	16.39	13.91	6.7	93.0	0.3
0	1	0	0.7	0.7	16.60	13.84	5.8	93.9	0.3
0	1	0.2	0	0	13.83	7.02	6.6	93.4	0.0
0	1	0.2	0	0.4	13.38	6.89	6.6	93.1	0.3
0	1	0.2	0	0.7	13.46	6.96	6.2	93.7	0.1
0	1	0.2	0.2	0	14.06	7.63	7.0	93.0	0.0
0	1	0.2	0.2	0.4	13.60	7.30	6.7	93.1	0.2
0	1	0.2	0.2	0.7	13.65	7.30	6.1	93.8	0.1
0	1	0.2	0.7	0	16.85	14.30	7.0	92.9	0.1
0	1	0.2	0.7	0.4	16.36	13.89	7.3	92.2	0.5
0	1	0.2	0.7	0.7	16.66	13.87	6.1	93.6	0.3
0	1	0.7	0	0	13.24	7.05	13.1	86.1	0.8
0	1	0.7	0	0.4	12.82	6.84	14.1	85.1	0.8
0	1	0.7	0	0.7	13.04	6.97	12.9	86.4	0.7
0	1	0.7	0.2	0	13.58	7.69	12.8	86.2	1.0
0	1	0.7	0.2	0.4	13.11	7.30	13.9	85.3	0.8
0	1	0.7	0.2	0.7	13.24	7.37	12.9	86.6	0.5
0	1	0.7	0.7	0	15.57	12.85	14.7	84.8	0.5
0	1	0.7	0.7	0.4	15.93	14.11	13.1	86.1	0.8
0	1	0.7	0.7	0.7	16.19	13.61	10.3	88.8	0.9
0	2	0	0	0	7.68	3.47	4.3	95.6	0.1
0	2	0	0	0.4	7.78	3.51	4.3	95.5	0.2
0	2	0	0	0.7	7.79	3.54	4.3	95.3	0.4
0	2	0	0.2	0	7.74	3.61	4.2	95.6	0.2
0	2	0	0.2	0.4	7.87	3.62	4.3	95.5	0.2
0	2	0	0.2	0.7	7.87	3.66	4.4	95.2	0.4
0	2	0	0.7	0	8.46	5.34	3.9	95.8	0.3
0	2	0	0.7	0.4	8.73	5.37	4.5	95.3	0.2
0	2	0	0.7	0.7	8.76	5.48	4.6	95.0	0.4
0	2	0.2	0	0	7.68	3.48	4.3	95.5	0.2
0	2	0.2	0	0.4	7.76	3.50	4.8	94.9	0.3
0	2	0.2	0	0.7	7.79	3.53	4.6	94.9	0.5
0	2	0.2	0.2	0	7.76	3.61	4.2	95.4	0.4
0	2	0.2	0.2	0.4	7.84	3.62	4.7	95.0	0.3
0	2	0.2	0.2	0.7	7.87	3.67	4.7	94.9	0.4
0	2	0.2	0.7	0	8.46	5.31	4.4	95.3	0.3
0	2	0.2	0.7	0.4	8.73	5.37	4.8	94.9	0.3
0	2	0.2	0.7	0.7	8.78	5.49	4.6	95.1	0.3
0	2	0.7	0	0	7.52	3.46	7.4	92.3	0.3
0	2	0.7	0	0.4	7.66	3.47	6.6	92.3	1.1
0	2	0.7	0	0.7	7.66	3.49	6.7	92.6	0.7
0	2	0.7	0.2	0	7.54	3.57	7.7	92.0	0.3
0	2	0.7	0.2	0.4	7.74	3.59	6.8	92.1	1.1
0	2	0.7	0.2	0.7	7.76	3.63	6.6	92.8	0.6
0	2	0.7	0.7	0	8.27	5.23	7.7	91.8	0.5
0	2	0.7	0.7	0.4	8.61	5.27	6.6	92.4	1.0
0	2	0.7	0.7	0.7	8.71	5.47	5.8	93.5	0.7

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0	3	0	0	0	5.73	2.39	3.6	96.1	0.3
0	3	0	0	0.4	5.73	2.43	3.4	96.3	0.3
0	3	0	0	0.7	5.73	2.43	3.5	96.2	0.3
0	3	0	0.2	0	5.72	2.43	3.6	96.1	0.3
0	3	0	0.2	0.4	5.74	2.48	3.3	96.3	0.4
0	3	0	0.2	0.7	5.73	2.48	3.5	96.4	0.1
0	3	0	0.7	0	5.97	3.14	3.7	96.1	0.2
0	3	0	0.7	0.4	6.10	3.08	4.1	95.8	0.1
0	3	0	0.7	0.7	6.15	3.15	3.8	96.0	0.2
0	3	0.2	0	0	5.73	2.38	3.6	96.1	0.3
0	3	0.2	0	0.4	5.72	2.42	3.6	96.0	0.4
0	3	0.2	0	0.7	5.73	2.43	3.5	96.2	0.3
0	3	0.2	0.2	0	5.73	2.43	3.6	96.1	0.3
0	3	0.2	0.2	0.4	5.74	2.47	3.7	95.9	0.4
0	3	0.2	0.2	0.7	5.73	2.48	3.9	96.0	0.1
0	3	0.2	0.7	0	5.97	3.13	3.9	96.0	0.1
0	3	0.2	0.7	0.4	6.12	3.07	3.9	95.7	0.4
0	3	0.2	0.7	0.7	6.14	3.14	3.8	96.1	0.1
0	3	0.7	0	0	5.64	2.39	5.4	94.2	0.4
0	3	0.7	0	0.4	5.67	2.41	4.8	94.2	1.0
0	3	0.7	0	0.7	5.67	2.43	4.7	94.2	1.1
0	3	0.7	0.2	0	5.66	2.45	5.5	93.9	0.6
0	3	0.7	0.2	0.4	5.69	2.46	5.0	94.3	0.7
0	3	0.7	0.2	0.7	5.69	2.48	4.6	94.5	0.9
0	3	0.7	0.7	0	5.86	3.09	5.6	93.9	0.5
0	3	0.7	0.7	0.4	6.06	3.07	4.7	94.7	0.6
0	3	0.7	0.7	0.7	6.10	3.14	4.3	95.3	0.4
0.5	0.5	0	0	0	19.84	11.42	42.1	56.7	1.2
0.5	0.5	0	0	0.4	20.06	11.88	45.2	53.7	1.1
0.5	0.5	0	0	0.7	20.63	12.64	45.5	53.4	1.1
0.5	0.5	0	0.2	0	19.70	11.62	42.6	56.0	1.4
0.5	0.5	0	0.2	0.4	20.13	12.48	46.6	52.4	1.0
0.5	0.5	0	0.2	0.7	20.77	13.22	46.9	51.8	1.3
0.5	0.5	0	0.7	0	18.52	12.37	44.3	55.1	0.6
0.5	0.5	0	0.7	0.4	18.89	13.70	48.2	50.9	0.9
0.5	0.5	0	0.7	0.7	19.66	14.71	49.0	50.3	0.7
0.5	0.5	0.2	0	0	19.77	11.69	42.1	56.5	1.4
0.5	0.5	0.2	0	0.4	20.07	12.30	44.6	54.4	1.0
0.5	0.5	0.2	0	0.7	21.27	13.40	45.4	53.1	1.5
0.5	0.5	0.2	0.2	0	19.64	12.01	42.6	56.3	1.1
0.5	0.5	0.2	0.2	0.4	20.26	13.00	45.9	53.4	0.7
0.5	0.5	0.2	0.2	0.7	21.09	13.92	44.7	53.7	1.6
0.5	0.5	0.2	0.7	0	18.70	13.14	44.1	55.0	0.9
0.5	0.5	0.2	0.7	0.4	19.29	14.64	46.4	52.7	0.9
0.5	0.5	0.2	0.7	0.7	20.48	16.26	48.0	50.9	1.1
0.5	0.5	0.7	0	0	18.86	12.51	41.5	57.2	1.3
0.5	0.5	0.7	0	0.4	19.36	13.30	42.0	57.5	0.5
0.5	0.5	0.7	0	0.7	20.44	14.31	41.1	57.9	1.0
0.5	0.5	0.7	0.2	0	18.82	12.99	42.8	56.6	0.6
0.5	0.5	0.7	0.2	0.4	19.63	14.34	42.1	57.2	0.7
0.5	0.5	0.7	0.2	0.7	20.99	15.77	41.3	58.1	0.6
0.5	0.5	0.7	0.7	0	18.81	16.02	43.6	55.4	1.0
0.5	0.5	0.7	0.7	0.4	20.74	19.81	43.9	55.2	0.9
0.5	0.5	0.7	0.7	0.7	23.31	22.87	43.1	54.8	2.1

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0.5	0	0	0	0	27.53	15.58	89.4	10.5	0.1
0.5	0	0	0	0.4	27.28	15.38	89.7	10.3	0.0
0.5	0	0	0	0.7	27.56	15.42	90.5	9.4	0.1
0.5	0	0	0.2	0	27.05	15.29	86.9	12.9	0.2
0.5	0	0	0.2	0.4	26.88	15.18	87.4	12.6	0.0
0.5	0	0	0.2	0.7	27.28	15.39	88.7	11.2	0.1
0.5	0	0	0.7	0	23.38	14.11	70.5	29.0	0.5
0.5	0	0	0.7	0.4	23.26	14.99	72.0	27.8	0.2
0.5	0	0	0.7	0.7	24.11	15.34	75.4	24.4	0.2
0.5	0	0.2	0	0	28.47	17.53	89.1	10.8	0.1
0.5	0	0.2	0	0.4	28.18	16.82	90.0	10.0	0.0
0.5	0	0.2	0	0.7	29.00	18.05	90.6	9.3	0.1
0.5	0	0.2	0.2	0	27.89	17.02	86.3	13.5	0.2
0.5	0	0.2	0.2	0.4	28.08	17.52	87.5	12.5	0.0
0.5	0	0.2	0.2	0.7	28.73	18.00	89.1	10.8	0.1
0.5	0	0.2	0.7	0	24.07	15.83	69.9	29.6	0.5
0.5	0	0.2	0.7	0.4	24.16	16.65	71.6	28.2	0.2
0.5	0	0.2	0.7	0.7	25.30	17.20	75.4	24.5	0.1
0.5	0	0.7	0	0	33.79	29.04	88.1	11.8	0.1
0.5	0	0.7	0	0.4	35.15	31.36	89.7	10.2	0.1
0.5	0	0.7	0	0.7	35.88	31.43	92.1	7.9	0.0
0.5	0	0.7	0.2	0	32.80	28.15	84.3	15.2	0.5
0.5	0	0.7	0.2	0.4	34.48	31.23	86.9	12.8	0.3
0.5	0	0.7	0.2	0.7	35.39	30.85	90.1	9.8	0.1
0.5	0	0.7	0.7	0	26.62	22.95	67.1	32.4	0.5
0.5	0	0.7	0.7	0.4	28.21	26.55	69.1	30.4	0.5
0.5	0	0.7	0.7	0.7	31.49	28.99	74.8	24.5	0.7
0.5	1	0	0	0	12.87	6.74	20.3	78.8	0.9
0.5	1	0	0	0.4	12.69	6.75	18.1	80.8	1.1
0.5	1	0	0	0.7	12.78	6.89	19.2	79.9	0.9
0.5	1	0	0.2	0	12.98	7.14	20.8	78.4	0.8
0.5	1	0	0.2	0.4	12.81	7.07	19.3	79.6	1.1
0.5	1	0	0.2	0.7	12.95	7.23	19.2	80.1	0.7
0.5	1	0	0.7	0	13.81	9.54	26.6	71.8	1.6
0.5	1	0	0.7	0.4	14.33	10.78	27.5	71.2	1.3
0.5	1	0	0.7	0.7	14.89	11.62	26.9	71.8	1.3
0.5	1	0.2	0	0	12.77	6.77	20.7	78.4	0.9
0.5	1	0.2	0	0.4	12.60	6.75	19.2	79.8	1.0
0.5	1	0.2	0	0.7	12.75	6.93	19.2	79.9	0.9
0.5	1	0.2	0.2	0	12.89	7.20	20.8	77.9	1.3
0.5	1	0.2	0.2	0.4	12.73	7.10	19.3	79.5	1.2
0.5	1	0.2	0.2	0.7	12.93	7.29	19.0	80.3	0.7
0.5	1	0.2	0.7	0	13.84	9.74	27.2	71.3	1.5
0.5	1	0.2	0.7	0.4	14.42	11.22	27.4	71.4	1.2
0.5	1	0.2	0.7	0.7	15.24	12.55	25.7	72.1	2.2
0.5	1	0.7	0	0	12.11	6.79	26.1	72.3	1.6
0.5	1	0.7	0	0.4	11.98	6.72	24.6	74.6	0.8
0.5	1	0.7	0	0.7	12.35	7.03	21.9	76.6	1.5
0.5	1	0.7	0.2	0	12.30	7.19	26.6	72.3	1.1
0.5	1	0.7	0.2	0.4	12.25	7.15	24.7	74.5	0.8
0.5	1	0.7	0.2	0.7	12.61	7.46	21.7	76.9	1.4
0.5	1	0.7	0.7	0	13.40	10.60	30.1	69.0	0.9
0.5	1	0.7	0.7	0.4	14.46	12.79	27.0	71.7	1.3
0.5	1	0.7	0.7	0.7	15.49	13.57	22.4	75.2	2.4

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0.5	2	0	0	0	7.47	3.47	9.5	89.2	1.3
0.5	2	0	0	0.4	7.54	3.52	10.1	88.9	1.0
0.5	2	0	0	0.7	7.58	3.56	9.7	89.2	1.1
0.5	2	0	0.2	0	7.51	3.59	9.8	89.0	1.2
0.5	2	0	0.2	0.4	7.64	3.65	10.3	88.8	0.9
0.5	2	0	0.2	0.7	7.65	3.68	9.7	89.0	1.3
0.5	2	0	0.7	0	8.13	5.10	10.9	87.6	1.5
0.5	2	0	0.7	0.4	8.39	5.20	11.5	87.8	0.7
0.5	2	0	0.7	0.7	8.48	5.35	11.1	87.9	1.0
0.5	2	0.2	0	0	7.42	3.48	9.9	88.9	1.2
0.5	2	0.2	0	0.4	7.54	3.53	10.0	88.6	1.4
0.5	2	0.2	0	0.7	7.57	3.58	10.1	88.7	1.2
0.5	2	0.2	0.2	0	7.49	3.60	9.8	89.0	1.2
0.5	2	0.2	0.2	0.4	7.63	3.66	10.3	88.5	1.2
0.5	2	0.2	0.2	0.7	7.65	3.69	9.8	89.0	1.2
0.5	2	0.2	0.7	0	8.12	5.15	10.8	87.8	1.4
0.5	2	0.2	0.7	0.4	8.41	5.22	11.5	87.4	1.1
0.5	2	0.2	0.7	0.7	8.51	5.42	10.8	88.2	1.0
0.5	2	0.7	0	0	7.28	3.45	13.1	84.9	2.0
0.5	2	0.7	0	0.4	7.45	3.54	11.3	86.3	2.4
0.5	2	0.7	0	0.7	7.47	3.58	10.7	86.7	2.6
0.5	2	0.7	0.2	0	7.32	3.56	13.3	84.8	1.9
0.5	2	0.7	0.2	0.4	7.54	3.65	11.4	86.2	2.4
0.5	2	0.7	0.2	0.7	7.57	3.73	11.0	86.6	2.4
0.5	2	0.7	0.7	0	7.90	5.04	14.5	83.9	1.6
0.5	2	0.7	0.7	0.4	8.39	5.27	12.1	85.5	2.4
0.5	2	0.7	0.7	0.7	8.54	5.54	10.4	87.4	2.2
1	0	0	0	0	15.84	8.55	92.3	7.6	0.1
1	0	0	0	0.4	15.51	8.48	92.5	7.3	0.2
1	0	0	0	0.7	15.62	8.52	93.2	6.7	0.1
1	0	0	0.2	0	15.68	8.51	90.6	9.3	0.1
1	0	0	0.2	0.4	15.46	8.49	91.5	8.3	0.2
1	0	0	0.2	0.7	15.57	8.57	92.1	7.7	0.2
1	0	0	0.7	0	14.49	8.08	77.9	21.5	0.6
1	0	0	0.7	0.4	14.41	8.32	81.3	18.5	0.2
1	0	0	0.7	0.7	14.58	8.45	82.6	17.0	0.4
1	0	0.2	0	0	16.09	8.93	92.2	7.7	0.1
1	0	0.2	0	0.4	15.86	8.95	92.5	7.3	0.2
1	0	0.2	0	0.7	15.92	8.97	93.1	6.9	0.0
1	0	0.2	0.2	0	15.89	8.88	90.3	9.6	0.1
1	0	0.2	0.2	0.4	15.78	8.95	91.6	8.3	0.1
1	0	0.2	0.2	0.7	15.88	9.03	92.3	7.7	0.0
1	0	0.2	0.7	0	14.69	8.50	77.5	21.7	0.8
1	0	0.2	0.7	0.4	14.68	8.76	80.9	18.7	0.4
1	0	0.2	0.7	0.7	14.91	8.93	82.8	16.7	0.5
1	0	0.7	0	0	19.21	15.64	91.5	8.3	0.2
1	0	0.7	0	0.4	19.25	15.40	92.0	7.8	0.2
1	0	0.7	0	0.7	19.15	15.37	93.0	6.9	0.1
1	0	0.7	0.2	0	18.80	15.11	88.9	10.8	0.3
1	0	0.7	0.2	0.4	19.09	15.32	90.8	8.9	0.3
1	0	0.7	0.2	0.7	19.26	15.78	92.6	7.3	0.1
1	0	0.7	0.7	0	16.38	12.93	75.6	22.8	1.6
1	0	0.7	0.7	0.4	17.48	14.44	80.4	18.9	0.7
1	0	0.7	0.7	0.7	18.21	15.29	83.6	15.2	1.2

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0.5	3	0	0	0	5.63	2.40	6.4	92.3	1.3
0.5	3	0	0	0.4	5.60	2.42	7.1	91.9	1.0
0.5	3	0	0	0.7	5.62	2.45	7.1	92.2	0.7
0.5	3	0	0.2	0	5.62	2.45	6.4	92.2	1.4
0.5	3	0	0.2	0.4	5.62	2.47	7.4	91.9	0.7
0.5	3	0	0.2	0.7	5.62	2.48	7.1	92.0	0.9
0.5	3	0	0.7	0	5.84	3.07	6.3	92.7	1.0
0.5	3	0	0.7	0.4	5.99	3.08	7.3	91.2	1.5
0.5	3	0	0.7	0.7	6.03	3.15	6.9	91.9	1.2
0.5	3	0.2	0	0	5.61	2.40	6.7	92.1	1.2
0.5	3	0.2	0	0.4	5.59	2.41	7.2	92.1	0.7
0.5	3	0.2	0	0.7	5.62	2.45	7.0	92.0	1.0
0.5	3	0.2	0.2	0	5.61	2.46	6.7	91.9	1.4
0.5	3	0.2	0.2	0.4	5.61	2.47	7.3	92.0	0.7
0.5	3	0.2	0.2	0.7	5.62	2.50	7.1	91.9	1.0
0.5	3	0.2	0.7	0	5.83	3.08	6.5	92.4	1.1
0.5	3	0.2	0.7	0.4	5.99	3.09	7.5	91.1	1.4
0.5	3	0.2	0.7	0.7	6.03	3.16	7.0	92.0	1.0
0.5	3	0.7	0	0	5.52	2.41	8.4	89.8	1.8
0.5	3	0.7	0	0.4	5.56	2.42	7.8	89.9	2.3
0.5	3	0.7	0	0.7	5.57	2.46	8.0	90.2	1.8
0.5	3	0.7	0.2	0	5.54	2.45	8.1	90.0	1.9
0.5	3	0.7	0.2	0.4	5.59	2.48	7.7	90.7	1.6
0.5	3	0.7	0.2	0.7	5.59	2.51	7.8	90.5	1.7
0.5	3	0.7	0.7	0	5.75	3.08	8.4	89.6	2.0
0.5	3	0.7	0.7	0.4	5.96	3.12	7.9	90.5	1.6
0.5	3	0.7	0.7	0.7	6.02	3.20	7.3	91.0	1.7
1	0.5	0	0	0	14.00	7.72	70.5	28.1	1.4
1	0.5	0	0	0.4	13.96	7.80	74.7	24.4	0.9
1	0.5	0	0	0.7	14.20	8.07	76.0	23.0	1.0
1	0.5	0	0.2	0	13.64	7.53	67.3	31.4	1.3
1	0.5	0	0.2	0.4	13.79	7.79	73.9	25.2	0.9
1	0.5	0	0.2	0.7	14.10	8.16	75.7	23.3	1.0
1	0.5	0	0.7	0	12.87	7.51	61.5	36.9	1.6
1	0.5	0	0.7	0.4	12.96	0.98	67.3	31.4	1.3
1	0.5	0	0.7	0.7	13.28	8.27	69.6	29.4	1.0
1	0.5	0.2	0	0	13.97	7.82	68.6	30.0	1.4
1	0.5	0.2	0	0.4	14.23	8.14	73.4	25.2	1.4
1	0.5	0.2	0	0.7	14.44	8.46	75.4	23.6	1.0
1	0.5	0.2	0.2	0	13.77	7.79	67.5	31.1	1.4
1	0.5	0.2	0.2	0.4	14.05	8.19	73.1	25.7	1.2
1	0.5	0.2	0.2	0.7	14.38	8.62	75.7	23.3	1.0
1	0.5	0.2	0.7	0	12.95	7.73	60.4	37.4	2.2
1	0.5	0.2	0.7	0.4	13.24	8.44	67.1	32.0	0.9
1	0.5	0.2	0.7	0.7	13.63	8.79	70.0	29.1	0.9
1	0.5	0.7	0	0	14.51	9.70	64.1	34.6	1.3
1	0.5	0.7	0	0.4	15.39	10.90	66.1	32.6	1.3
1	0.5	0.7	0	0.7	15.98	11.91	67.1	31.0	1.9
1	0.5	0.7	0.2	0	14.44	9.96	63.4	35.4	1.2
1	0.5	0.7	0.2	0.4	15.41	11.27	66.0	32.5	1.5
1	0.5	0.7	0.2	0.7	16.30	12.59	68.2	30.0	1.8
1	0.5	0.7	0.7	0	13.56	10.31	57.7	40.6	1.7
1	0.5	0.7	0.7	0.4	15.11	12.65	62.5	35.1	2.4
1	0.5	0.7	0.7	0.7	16.39	14.29	67.2	30.1	2.7

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
1	1	0	0	0	11.09	5.75	41.9	55.1	3.0
1	1	0	0	0.4	10.90	5.83	43.4	54.6	2.0
1	1	0	0	0.7	11.08	6.07	44.0	53.9	2.1
1	1	0	0.2	0	11.06	5.92	42.2	54.9	2.9
1	1	0	0.2	0.4	10.96	5.96	44.9	53.5	1.6
1	1	0	0.2	0.7	11.17	6.29	44.6	53.1	2.3
1	1	0	0.7	0	11.05	6.75	44.5	53.4	2.1
1	1	0	0.7	0.4	11.06	6.98	49.5	48.4	2.1
1	1	0	0.7	0.7	11.51	7.47	51.0	46.8	2.2
1	1	0.2	0	0	11.04	5.86	41.4	56.1	2.5
1	1	0.2	0	0.4	10.94	5.89	42.5	55.7	1.8
1	1	0.2	0	0.7	11.18	6.22	42.6	54.9	2.5
1	1	0.2	0.2	0	11.02	5.98	41.9	55.4	2.7
1	1	0.2	0.2	0.4	10.99	6.11	43.6	54.4	2.0
1	1	0.2	0.2	0.7	11.31	6.53	43.5	54.7	1.8
1	1	0.2	0.7	0	11.07	6.84	44.8	53.5	1.7
1	1	0.2	0.7	0.4	11.24	7.35	48.3	49.3	2.4
1	1	0.2	0.7	0.7	11.76	7.98	49.7	47.8	2.5
1	1	0.7	0	0	10.76	6.12	41.9	56.5	1.6
1	1	0.7	0	0.4	10.86	6.30	39.5	57.8	2.7
1	1	0.7	0	0.7	11.14	6.68	38.4	58.2	3.4
1	1	0.7	0.2	0	10.86	6.40	42.9	56.0	1.1
1	1	0.7	0.2	0.4	11.11	6.74	39.7	57.7	2.6
1	1	0.7	0.2	0.7	11.38	7.06	39.8	57.2	3.0
1	1	0.7	0.7	0	11.19	7.89	44.4	53.4	2.2
1	1	0.7	0.7	0.4	12.11	9.72	43.9	52.9	3.2
1	1	0.7	0.7	0.7	13.12	11.01	45.0	52.1	2.9
1	3	0	0	0	5.47	2.39	10.7	86.8	2.5
1	3	0	0	0.4	5.46	2.42	11.5	85.6	2.9
1	3	0	0	0.7	5.48	2.45	11.2	85.9	2.9
1	3	0	0.2	0	5.47	2.44	10.7	87.2	2.1
1	3	0	0.2	0.4	5.46	2.45	11.6	86.2	2.2
1	3	0	0.2	0.7	5.48	2.48	11.3	86.0	2.7
1	3	0	0.7	0	5.67	3.03	11.4	85.9	2.7
1	3	0	0.7	0.4	5.79	2.99	13.3	84.3	2.4
1	3	0	0.7	0.7	5.83	3.08	12.6	85.2	2.2
1	3	0.2	0	0	5.45	2.40	11.1	86.5	2.4
1	3	0.2	0	0.4	5.47	2.42	11.1	85.8	3.1
1	3	0.2	0	0.7	5.49	2.45	11.0	85.7	3.3
1	3	0.2	0.2	0	5.45	2.45	11.1	86.5	2.4
1	3	0.2	0.2	0.4	5.48	2.45	11.4	86.3	2.3
1	3	0.2	0.2	0.7	5.48	2.48	11.2	85.9	2.9
1	3	0.2	0.7	0	5.66	3.03	11.2	85.6	3.2
1	3	0.2	0.7	0.4	5.81	3.01	12.9	84.5	2.6
1	3	0.2	0.7	0.7	5.86	3.11	12.4	84.9	2.7
1	3	0.7	0	0	5.37	2.41	13.3	83.4	3.3
1	3	0.7	0	0.4	5.42	2.43	12.5	83.9	3.6
1	3	0.7	0	0.7	5.44	2.47	12.0	84.4	3.6
1	3	0.7	0.2	0	5.38	2.43	13.7	83.1	3.2
1	3	0.7	0.2	0.4	5.44	2.49	12.6	83.9	3.5
1	3	0.7	0.2	0.7	5.46	2.53	12.0	84.4	3.6
1	3	0.7	0.7	0	5.56	2.97	13.4	83.5	3.1
1	3	0.7	0.7	0.4	5.81	3.12	13.6	83.7	2.7
1	3	0.7	0.7	0.7	5.91	3.21	11.6	85.3	3.1

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
1	2	0	0	0	7.10	3.41	16.8	80.0	3.2
1	2	0	0	0.4	7.19	3.45	18.8	78.2	3.0
1	2	0	0	0.7	7.22	3.49	18.5	78.5	3.0
1	2	0	0.2	0	7.15	3.52	17.5	79.7	2.8
1	2	0	0.2	0.4	7.26	3.55	19.3	77.9	2.8
1	2	0	0.2	0.7	7.29	3.60	18.5	78.3	3.2
1	2	0	0.7	0	7.52	4.47	20.5	76.8	2.7
1	2	0	0.7	0.4	7.80	4.64	22.0	75.5	2.5
1	2	0	0.7	0.7	7.95	4.94	21.7	75.6	2.7
1	2	0.2	0	0	7.10	3.41	17.2	79.7	3.1
1	2	0.2	0	0.4	7.20	3.46	18.4	78.2	3.4
1	2	0.2	0	0.7	7.25	3.52	18.4	78.6	3.0
1	2	0.2	0.2	0	7.13	3.52	17.3	79.4	3.3
1	2	0.2	0.2	0.4	7.28	3.56	19.1	77.4	3.5
1	2	0.2	0.2	0.7	7.33	3.63	18.2	78.5	3.3
1	2	0.2	0.7	0	7.52	4.52	20.8	76.4	2.8
1	2	0.2	0.7	0.4	7.86	4.71	22.2	75.4	2.4
1	2	0.2	0.7	0.7	7.96	5.03	21.1	76.2	2.7
1	2	0.7	0	0	6.96	3.36	20.6	75.6	3.8
1	2	0.7	0	0.4	7.14	3.53	20.5	75.4	4.1
1	2	0.7	0	0.7	7.20	3.60	19.2	76.5	4.3
1	2	0.7	0.2	0	6.99	3.47	21.1	75.7	3.2
1	2	0.7	0.2	0.4	7.24	3.65	20.1	75.3	4.6
1	2	0.7	0.2	0.7	7.28	3.72	19.5	76.9	3.6
1	2	0.7	0.7	0	7.35	4.51	24.3	72.8	2.9
1	2	0.7	0.7	0.4	7.93	5.01	22.0	74.1	3.9
1	2	0.7	0.7	0.7	8.21	5.43	19.3	76.4	4.3
2	0	0	0	0	8.28	4.21	94.2	5.8	0.0
2	0	0	0	0.4	8.24	4.25	94.1	5.7	0.2
2	0	0	0	0.7	8.29	4.28	94.0	5.8	0.2
2	0	0	0.2	0	8.31	4.27	94.5	5.5	0.0
2	0	0	0.2	0.4	8.22	4.21	93.9	5.9	0.2
2	0	0	0.2	0.7	8.25	4.26	94.1	5.7	0.2
2	0	0	0.7	0	8.07	4.18	88.4	10.7	0.9
2	0	0	0.7	0.4	8.10	4.22	89.7	9.7	0.6
2	0	0	0.7	0.7	8.16	4.28	89.8	9.6	0.6
2	0	0.2	0	0	8.36	4.35	94.2	5.8	0.0
2	0	0.2	0	0.4	8.38	4.43	94.3	5.5	0.2
2	0	0.2	0	0.7	8.37	4.43	93.9	5.9	0.2
2	0	0.2	0.2	0	8.35	4.33	93.8	6.2	0.0
2	0	0.2	0.2	0.4	8.33	4.40	93.9	5.9	0.2
2	0	0.2	0.2	0.7	8.37	4.45	94.0	5.8	0.2
2	0	0.2	0.7	0	8.15	4.29	88.2	10.8	1.0
2	0	0.2	0.7	0.4	8.19	4.34	89.6	9.5	0.9
2	0	0.2	0.7	0.7	8.26	4.38	89.8	9.4	0.8
2	0	0.7	0	0	9.37	6.37	93.9	6.0	0.1
2	0	0.7	0	0.4	9.35	6.38	94.5	5.4	0.1
2	0	0.7	0	0.7	9.43	6.56	94.1	5.2	0.7
2	0	0.7	0.2	0	9.36	6.37	93.4	6.5	0.1
2	0	0.7	0.2	0.4	9.40	6.40	94.5	5.2	0.3
2	0	0.7	0.2	0.7	9.43	6.58	94.2	5.3	0.5
2	0	0.7	0.7	0	9.00	6.05	86.3	12.7	1.0
2	0	0.7	0.7	0.4	9.25	6.31	89.7	9.0	1.3
2	0	0.7	0.7	0.7	9.39	6.45	90.9	7.4	1.7

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
2	0.5	0	0	0	7.96	4.04	87.9	11.5	0.6
2	0.5	0	0	0.4	7.97	4.14	88.9	10.4	0.7
2	0.5	0	0	0.7	8.05	4.19	89.1	10.1	0.8
2	0.5	0	0.2	0	7.94	4.05	87.3	12.0	0.7
2	0.5	0	0.2	0.4	7.93	4.12	87.7	11.6	0.7
2	0.5	0	0.2	0.7	8.00	4.20	88.1	11.0	0.9
2	0.5	0	0.7	0	7.64	3.98	78.1	20.5	1.4
2	0.5	0	0.7	0.4	8.92	5.94	87.0	11.8	1.2
2	0.5	0	0.7	0.7	7.80	4.19	82.0	16.3	1.7
2	0.5	0.2	0	0	8.06	4.18	87.9	11.7	0.4
2	0.5	0.2	0	0.4	8.07	4.24	88.3	11.0	0.7
2	0.5	0.2	0	0.7	8.15	4.34	89.0	10.1	0.9
2	0.5	0.2	0.2	0	8.03	4.17	87.0	12.3	0.7
2	0.5	0.2	0.2	0.4	8.06	4.28	87.0	12.1	0.9
2	0.5	0.2	0.2	0.7	8.10	4.37	88.0	11.1	0.9
2	0.5	0.2	0.7	0	7.68	4.04	78.4	20.0	1.6
2	0.5	0.2	0.7	0.4	7.83	4.19	82.0	16.2	1.8
2	0.5	0.2	0.7	0.7	7.93	4.29	82.3	15.6	2.1
2	0.5	0.7	0	0	8.78	5.62	84.2	14.6	1.2
2	0.5	0.7	0	0.4	8.92	5.94	87.0	11.8	1.2
2	0.5	0.7	0	0.7	9.08	6.21	88.0	10.5	1.5
2	0.5	0.7	0.2	0	8.77	5.65	83.2	15.6	1.2
2	0.5	0.7	0.2	0.4	8.90	5.92	86.0	12.6	1.4
2	0.5	0.7	0.2	0.7	9.08	6.29	87.7	10.6	1.7
2	0.5	0.7	0.7	0	8.27	5.34	76.3	21.6	2.1
2	0.5	0.7	0.7	0.4	8.77	5.99	81.3	16.0	2.7
2	0.5	0.7	0.7	0.7	9.07	6.45	84.0	13.6	2.4
2	2	0	0	0	6.05	2.89	42.3	53.0	4.7
2	2	0	0	0.4	6.15	3.01	45.3	48.5	6.2
2	2	0	0	0.7	6.18	3.07	44.7	48.9	6.4
2	2	0	0.2	0	6.06	2.92	42.0	52.6	5.4
2	2	0	0.2	0.4	6.17	3.08	46.3	47.3	6.4
2	2	0	0.2	0.7	6.20	3.13	45.3	48.4	6.3
2	2	0	0.7	0	6.08	3.23	43.9	50.1	6.0
2	2	0	0.7	0.4	6.29	3.34	48.2	46.1	5.7
2	2	0	0.7	0.7	6.35	3.47	48.4	45.7	5.9
2	2	0.2	0	0	6.04	2.93	42.2	53.0	4.8
2	2	0.2	0	0.4	6.17	3.04	44.8	48.9	6.3
2	2	0.2	0	0.7	6.22	3.12	44.6	48.9	6.5
2	2	0.2	0.2	0	6.06	2.95	42.1	52.3	5.6
2	2	0.2	0.2	0.4	6.19	3.11	45.4	48.6	6.0
2	2	0.2	0.2	0.7	6.24	3.19	44.8	48.8	6.4
2	2	0.2	0.7	0	6.10	3.26	43.4	49.9	6.7
2	2	0.2	0.7	0.4	6.31	3.41	47.9	46.5	5.6
2	2	0.2	0.7	0.7	6.45	3.62	48.0	45.9	6.1
2	2	0.7	0	0	6.08	3.09	39.5	54.3	6.2
2	2	0.7	0	0.4	6.22	3.29	42.3	51.5	6.2
2	2	0.7	0	0.7	6.33	3.40	41.5	51.6	6.9
2	2	0.7	0.2	0	6.08	3.14	39.5	54.4	6.1
2	2	0.7	0.2	0.4	6.27	3.37	42.1	51.8	6.1
2	2	0.7	0.2	0.7	6.37	3.48	42.0	51.6	6.4
2	2	0.7	0.7	0	6.18	3.57	41.8	52.5	5.7
2	2	0.7	0.7	0.4	6.64	4.16	44.4	50.1	5.5
2	2	0.7	0.7	0.7	6.94	4.60	44.0	49.5	6.5

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
2	1	0	0	0	7.39	3.68	73.5	24.1	2.4
2	1	0	0	0.4	7.46	3.79	75.2	22.3	2.5
2	1	0	0	0.7	7.50	3.86	75.1	22.2	2.7
2	1	0	0.2	0	7.39	3.70	72.5	24.7	2.8
2	1	0	0.2	0.4	7.44	3.85	75.3	22.1	2.6
2	1	0	0.2	0.7	7.49	3.89	75.5	22.1	2.4
2	1	0	0.7	0	7.15	3.77	68.9	28.6	2.5
2	1	0	0.7	0.4	7.33	3.91	72.9	24.8	2.3
2	1	0	0.7	0.7	7.43	4.07	73.8	24.1	2.1
2	1	0.2	0	0	7.44	3.77	73.8	24.0	2.2
2	1	0.2	0	0.4	7.53	3.88	74.8	22.5	2.7
2	1	0.2	0	0.7	7.60	4.01	74.6	22.3	3.1
2	1	0.2	0.2	0	7.43	3.77	72.9	24.6	2.5
2	1	0.2	0.2	0.4	7.52	3.95	75.2	22.2	2.6
2	1	0.2	0.2	0.7	7.61	4.04	75.6	21.5	2.9
2	1	0.2	0.7	0	7.19	3.84	68.1	28.6	3.3
2	1	0.2	0.7	0.4	7.42	4.05	73.0	24.8	2.2
2	1	0.2	0.7	0.7	7.54	4.21	73.7	24.1	2.2
2	1	0.7	0	0	7.76	4.45	69.3	28.3	2.4
2	1	0.7	0	0.4	7.99	4.79	71.3	25.6	3.1
2	1	0.7	0	0.7	8.14	5.07	70.8	25.5	3.7
2	1	0.7	0.2	0	7.78	4.51	69.2	28.3	2.5
2	1	0.7	0.2	0.4	8.01	4.88	70.7	25.9	3.4
2	1	0.7	0.2	0.7	8.22	5.38	70.7	25.3	4.0
2	1	0.7	0.7	0	7.58	4.84	64.4	32.5	3.1
2	1	0.7	0.7	0.4	8.16	5.56	71.3	25.5	3.2
2	1	0.7	0.7	0.7	8.49	6.11	73.8	23.3	2.9
2	3	0	0	0	5.01	2.30	26.0	68.4	5.6
2	3	0	0	0.4	5.06	2.35	25.3	67.2	7.5
2	3	0	0	0.7	5.06	2.37	25.6	66.9	7.5
2	3	0	0.2	0	5.01	2.33	25.8	68.2	6.0
2	3	0	0.2	0.4	5.05	2.37	26.2	67.0	6.8
2	3	0	0.2	0.7	5.06	2.40	25.8	67.1	7.1
2	3	0	0.7	0	5.09	2.57	26.7	66.8	6.5
2	3	0	0.7	0.4	5.20	2.63	30.4	63.9	5.7
2	3	0	0.7	0.7	5.24	2.74	30.9	63.4	5.7
2	3	0.2	0	0	4.99	2.29	26.2	68.1	5.7
2	3	0.2	0	0.4	5.05	2.34	25.1	67.6	7.3
2	3	0.2	0	0.7	5.07	2.39	24.5	67.5	8.0
2	3	0.2	0.2	0	5.00	2.32	26.0	67.9	6.1
2	3	0.2	0.2	0.4	5.06	2.37	25.9	67.2	6.9
2	3	0.2	0.2	0.7	5.07	2.42	25.0	68.0	7.0
2	3	0.2	0.7	0	5.07	2.56	26.3	66.9	6.8
2	3	0.2	0.7	0.4	5.23	2.67	30.1	64.1	5.8
2	3	0.2	0.7	0.7	5.29	2.83	31.0	63.1	5.9
2	3	0.7	0	0	4.93	2.29	25.9	69.0	5.1
2	3	0.7	0	0.4	4.99	2.38	26.1	68.3	5.6
2	3	0.7	0	0.7	5.03	2.44	25.3	68.6	6.1
2	3	0.7	0.2	0	4.95	2.31	26.0	68.5	5.5
2	3	0.7	0.2	0.4	5.02	2.43	25.9	68.9	5.2
2	3	0.7	0.2	0.7	5.06	2.48	25.2	68.8	6.0
2	3	0.7	0.7	0	5.06	2.71	26.5	68.4	5.1
2	3	0.7	0.7	0.4	5.27	2.87	28.6	66.5	4.9
2	3	0.7	0.7	0.7	5.43	3.05	27.1	67.4	5.5

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
3	0	0	0	0	5.76	2.81	94.9	4.8	0.3
3	0	0	0	0.4	5.72	2.74	94.8	5.1	0.1
3	0	0	0	0.7	5.71	2.78	94.5	5.1	0.4
3	0	0	0.2	0	5.76	2.80	94.8	4.8	0.4
3	0	0	0.2	0.4	5.71	2.75	94.7	5.1	0.2
3	0	0	0.2	0.7	5.71	2.78	94.5	5.2	0.3
3	0	0	0.7	0	5.69	2.79	91.9	7.2	0.9
3	0	0	0.7	0.4	5.67	2.75	92.3	6.7	1.0
3	0	0	0.7	0.7	5.67	2.76	92.1	6.5	1.4
3	0	0.2	0	0	5.79	2.88	94.9	4.7	0.4
3	0	0.2	0	0.4	5.77	2.82	94.8	5.1	0.1
3	0	0.2	0	0.7	5.76	2.85	94.4	5.1	0.5
3	0	0.2	0.2	0	5.79	2.87	94.9	4.7	0.4
3	0	0.2	0.2	0.4	5.77	2.83	94.7	5.1	0.2
3	0	0.2	0.2	0.7	5.75	2.87	94.5	5.0	0.5
3	0	0.2	0.7	0	5.72	2.84	91.8	7.3	0.9
3	0	0.2	0.7	0.4	5.74	2.82	92.5	6.5	1.0
3	0	0.2	0.7	0.7	5.72	2.83	92.1	6.3	1.6
3	0	0.7	0	0	6.29	3.75	94.8	4.7	0.5
3	0	0.7	0	0.4	6.24	3.78	94.9	4.4	0.7
3	0	0.7	0	0.7	6.27	3.79	94.6	4.7	0.7
3	0	0.7	0.2	0	6.29	3.73	94.6	4.9	0.5
3	0	0.7	0.2	0.4	6.24	3.79	94.8	4.5	0.7
3	0	0.7	0.2	0.7	6.26	3.79	94.7	4.6	0.7
3	0	0.7	0.7	0	6.20	3.67	92.1	7.2	0.7
3	0	0.7	0.7	0.4	6.22	3.76	93.1	6.1	0.8
3	0	0.7	0.7	0.7	6.25	3.76	93.2	5.6	1.2
3	1	0	0	0	5.51	2.71	83.9	13.7	2.4
3	1	0	0	0.4	5.51	2.64	84.3	12.5	3.2
3	1	0	0	0.7	5.51	2.68	84.8	12.5	2.7
3	1	0	0.2	0	5.51	2.70	83.9	13.8	2.3
3	1	0	0.2	0.4	5.48	2.64	84.3	12.9	2.8
3	1	0	0.2	0.7	5.49	2.67	84.6	12.8	2.6
3	1	0	0.7	0	5.35	2.67	77.4	18.8	3.8
3	1	0	0.7	0.4	5.43	2.65	82.3	15.0	2.7
3	1	0	0.7	0.7	5.43	2.71	82.3	15.0	2.7
3	1	0.2	0	0	5.54	2.75	83.3	14.0	2.7
3	1	0.2	0	0.4	5.55	2.73	83.9	13.1	3.0
3	1	0.2	0	0.7	5.56	2.76	84.8	12.1	3.1
3	1	0.2	0.2	0	5.54	2.75	83.0	14.2	2.8
3	1	0.2	0.2	0.4	5.54	2.73	84.1	13.2	2.7
3	1	0.2	0.2	0.7	5.54	2.75	84.5	12.8	2.7
3	1	0.2	0.7	0	5.39	2.72	77.0	18.6	4.4
3	1	0.2	0.7	0.4	5.50	2.73	82.1	15.5	2.4
3	1	0.2	0.7	0.7	5.49	2.79	82.1	14.7	3.2
3	1	0.7	0	0	5.83	3.17	81.0	16.4	2.6
3	1	0.7	0	0.4	5.90	3.36	83.7	14.0	2.3
3	1	0.7	0	0.7	5.98	3.51	84.1	12.9	3.0
3	1	0.7	0.2	0	5.82	3.17	80.9	16.6	2.5
3	1	0.7	0.2	0.4	5.90	3.37	83.8	13.7	2.5
3	1	0.7	0.2	0.7	6.00	3.58	85.0	12.5	2.5
3	1	0.7	0.7	0	5.65	3.23	75.4	21.2	3.4
3	1	0.7	0.7	0.4	5.91	3.54	82.3	14.8	2.9
3	1	0.7	0.7	0.7	6.01	3.74	83.5	13.2	3.3

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
3	0.5	0	0	0	5.68	2.76	91.1	8.1	0.8
3	0.5	0	0	0.4	5.64	2.70	91.6	7.4	1.0
3	0.5	0	0	0.7	5.66	2.76	91.7	7.0	1.3
3	0.5	0	0.2	0	5.68	2.76	90.6	8.1	1.3
3	0.5	0	0.2	0.4	5.65	2.73	91.2	7.4	1.4
3	0.5	0	0.2	0.7	5.64	2.74	91.2	7.5	1.3
3	0.5	0	0.7	0	5.55	2.76	85.7	12.6	1.7
3	0.5	0	0.7	0.4	5.55	2.70	88.0	9.9	2.1
3	0.5	0	0.7	0.7	5.54	2.74	87.9	10.4	1.7
3	0.5	0.2	0	0	5.71	2.84	91.1	8.0	0.9
3	0.5	0.2	0	0.4	5.70	2.80	91.5	7.4	1.1
3	0.5	0.2	0	0.7	5.73	2.84	91.9	6.8	1.3
3	0.5	0.2	0.2	0	5.71	2.84	90.8	7.9	1.3
3	0.5	0.2	0.2	0.4	5.69	2.80	91.0	7.7	1.3
3	0.5	0.2	0.2	0.7	5.70	2.82	91.1	7.2	1.7
3	0.5	0.2	0.7	0	5.58	2.81	85.3	12.6	2.1
3	0.5	0.2	0.7	0.4	5.61	2.77	88.3	9.9	1.8
3	0.5	0.2	0.7	0.7	5.60	2.81	88.0	10.1	1.9
3	0.5	0.7	0	0	6.12	3.58	89.8	9.2	1.0
3	0.5	0.7	0	0.4	6.15	3.69	91.5	7.2	1.3
3	0.5	0.7	0	0.7	6.22	3.77	92.2	6.4	1.4
3	0.5	0.7	0.2	0	6.13	3.58	89.7	9.1	1.2
3	0.5	0.7	0.2	0.4	6.15	3.72	91.1	7.4	1.5
3	0.5	0.7	0.2	0.7	6.20	3.77	91.6	6.9	1.5
3	0.5	0.7	0.7	0	5.95	3.44	83.4	14.2	2.4
3	0.5	0.7	0.7	0.4	6.08	3.69	88.1	9.6	2.3
3	0.5	0.7	0.7	0.7	6.15	3.76	89.6	8.6	1.8
3	2	0	0	0	4.99	2.37	61.7	31.9	6.4
3	2	0	0	0.4	5.02	2.40	63.7	30.1	6.2
3	2	0	0	0.7	5.05	2.44	65.1	29.2	5.7
3	2	0	0.2	0	4.99	2.39	61.6	31.8	6.6
3	2	0	0.2	0.4	5.02	2.43	64.9	29.6	5.5
3	2	0	0.2	0.7	5.06	2.46	66.3	28.7	5.0
3	2	0	0.7	0	4.92	2.48	61.2	32.4	6.4
3	2	0	0.7	0.4	5.05	2.47	66.1	29.1	4.8
3	2	0	0.7	0.7	5.07	2.53	67.7	27.7	4.6
3	2	0.2	0	0	5.00	2.40	61.2	32.0	6.8
3	2	0.2	0	0.4	5.04	2.45	63.1	31.2	5.7
3	2	0.2	0	0.7	5.08	2.50	64.0	29.4	6.6
3	2	0.2	0.2	0	5.01	2.41	61.0	32.0	7.0
3	2	0.2	0.2	0.4	5.05	2.50	63.7	30.6	5.7
3	2	0.2	0.2	0.7	5.09	2.52	65.2	29.2	5.6
3	2	0.2	0.7	0	4.94	2.52	60.5	32.8	6.7
3	2	0.2	0.7	0.4	5.08	2.53	66.0	29.9	4.1
3	2	0.2	0.7	0.7	5.12	2.62	67.2	28.2	4.6
3	2	0.7	0	0	5.12	2.57	58.8	34.8	6.4
3	2	0.7	0	0.4	5.23	2.76	61.3	31.7	7.0
3	2	0.7	0	0.7	5.28	2.79	62.0	31.3	6.7
3	2	0.7	0.2	0	5.11	2.57	58.6	34.8	6.6
3	2	0.7	0.2	0.4	5.24	2.80	61.5	32.0	6.5
3	2	0.7	0.2	0.7	5.30	2.86	62.5	31.3	6.2
3	2	0.7	0.7	0	5.10	2.79	58.1	35.1	6.8
3	2	0.7	0.7	0.4	5.40	3.11	63.7	29.7	6.6
3	2	0.7	0.7	0.7	5.54	3.35	65.1	27.1	7.8

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
3	3	0	0	0	4.42	2.05	43.7	48.9	7.4
3	3	0	0	0.4	4.44	2.06	43.9	48.0	8.1
3	3	0	0	0.7	4.45	2.09	45.1	47.1	7.8
3	3	0	0.2	0	4.43	2.07	43.1	48.3	8.6
3	3	0	0.2	0.4	4.44	2.07	44.2	47.9	7.9
3	3	0	0.2	0.7	4.44	2.09	44.9	47.7	7.4
3	3	0	0.7	0	4.40	2.18	42.9	48.4	8.7
3	3	0	0.7	0.4	4.48	2.15	47.5	45.5	7.0
3	3	0	0.7	0.7	4.53	2.25	48.4	44.0	7.6
3	3	0.2	0	0	4.42	2.06	43.4	49.1	7.5
3	3	0.2	0	0.4	4.44	2.06	43.9	48.6	7.5
3	3	0.2	0	0.7	4.44	2.11	44.9	47.6	7.5
3	3	0.2	0.2	0	4.41	2.06	43.0	48.8	8.2
3	3	0.2	0.2	0.4	4.44	2.09	44.0	48.4	7.6
3	3	0.2	0.2	0.7	4.46	2.13	44.5	47.9	7.6
3	3	0.2	0.7	0	4.40	2.18	42.6	49.1	8.3
3	3	0.2	0.7	0.4	4.50	2.18	46.8	46.3	6.9
3	3	0.2	0.7	0.7	4.55	2.29	47.9	44.3	7.8

δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
3	3	0.7	0	0	4.45	2.11	41.2	50.2	8.6
3	3	0.7	0	0.4	4.48	2.17	42.6	50.4	7.0
3	3	0.7	0	0.7	4.53	2.20	41.6	50.1	8.3
3	3	0.7	0.2	0	4.45	2.12	41.3	50.2	8.5
3	3	0.7	0.2	0.4	4.49	2.22	42.3	50.5	7.2
3	3	0.7	0.2	0.7	4.55	2.25	41.6	50.4	8.0
3	3	0.7	0.7	0	4.46	2.29	40.8	50.9	8.3
3	3	0.7	0.7	0.4	4.66	2.52	44.2	47.4	8.4
3	3	0.7	0.7	0.7	4.77	2.66	45.8	44.8	9.4

4.3 Comparison with Other Control Schemes

To illustrate the power of the proposed NN-based control scheme, its Average Run Length (ARL) performance is evaluated against three statistical control charts, namely, the Hotelling T^2 chart, the MEWMA chart, and the Z chart. As pointed out by Montgomery (2005), the MEWMA and MCUSUM control charts have very similar ARL performance; however, the MEWMA control chart is much easier to implement in practice. So the MEWMA chart, instead of the MCUSUM chart, is employed as a comparison control scheme in this research.

Table 4.2 summarizes the ARL, SRL and First-Detection rate derived from the network trained in this study, the ARL, SRL obtained from the Hotelling T^2 chart, the MEWMA chart and the Z chart, and the First-Detection rate derived from the Z chart. For the purpose of comparison, when shifts, autocorrelation and correlation are not present, the in-control ARLs of the three comparison control schemes are tuned to around 185.4. In Table 4.2, two groups of results are included which are derived from the MEWMA chart. In these two groups, one group of results are obtained by setting the parameter that controls the magnitude of smoothing equal to 0.05 and the other group of results are calculated by setting the smoothing parameter to 0.5. The reason why two smoothing parameters are employed in the MEWMA chart is that different smoothing parameters will generate different MEWMA results. For the MEWMA chart, when the smoothing parameter is small, the MEWMA chart is more sensitive to small shifts; when the smoothing parameter is equal to 1, it is equivalent to the Hotelling T^2 chart.

4.3.1 No-Shift Processes

As can be seen from Table 4.2, for no-shift processes, when high autocorrelation or high correlation is present, the NN-based control scheme performs better than the other three control schemes in ARL. Among these four control schemes, the MEWMA control scheme performs worst which is reflected in the small in-control ARL. When it comes to the First-Detection capability, among these four control schemes, only the *Z* chart and the proposed NN-based control scheme can identify the source of the shift. Compared to the *Z* chart, the proposed NN-based control scheme has a larger difference between the First-Detection rate of X and the First-Detection rate of Y when autocorrelation on both variables are of the same magnitude.

4.3.2 Single-Shift Processes

In single-shift processes, the NN-based control scheme performs much better than the Hotelling T^2 chart, the *Z* chart and the MEWMA chart (λ=0.5), and it is comparable to the MEWMA chart (λ=0.05) when the shift is small to moderate (shift magnitude is less than 2σ) and no high autocorrelation is present. When high autocorrelation is present in single-shift processes, the MEWMA charts may obtain the smallest ARLs. It is observed that when high autocorrelation is present in one of the variables, the

NN-based control scheme performs better than the Hotelling T^2 chart and the Z chart. However, when high autocorrelations are present on both variables, the Hotelling T^2 chart and the Z chart have better ARL performance than the NN-based control scheme. Except the situation where high autocorrelation is present on one of the variables, the NN-based control scheme can detect the true sources of shifts with higher percentage rates than the Z chart.

When large shift (shift size $\geq 2\sigma$) happens on one of the variables and no shift happens on the other variable, the NN-based control scheme has the largest ARL in all four control schemes. This is due to the way that the NN-based control scheme tests, which is by moving windows instead of by observation. Except the cases where high autocorrelation are present on both variables, the NN-based control scheme can detect the true source of shift with a rate of more than 94.9% when large shift happens in the processes with only one shift. However, the Z chart performs even better.

4.3.3 Double-Shift Processes

Compared with the NN-based control scheme, the Hotelling T^2 chart and the Z chart, the MEWMA (λ=0.05) chart obtains the smallest ARL in the double-shift processes where small shifts are present. When autocorrelation is high, the MEWMA (λ=0.5) control chart has smaller ARL than all the other control schemes. The NN-based control scheme performs much better than the Z chart and the Hotelling T^2 chart on ARL performance except the cases where high autocorrelations are present on both variables. For the First-Detection capability, the Z chart is very sensitive to high autocorrelation while the NN-based control scheme performs robustly when high autocorrelation is present.

In the double-shift processes where moderate shifts are present on both variables, the NN-based control scheme performs better than the Hotelling T^2 chart and the Z chart on ARL performance when high autocorrelations are not present on both variables. The MEWMA charts obtain the smallest ARLs in these processes. For the First-Detection capability, the NN-based control scheme performs better than the *Z* chart when high autocorrelation is present on one of the variables.

When one small shift and one moderate shift are present in double-shift processes, the NN-based control scheme has better ARL performance than the Hotelling T^2 chart and the *Z* chart except for the cases where high autocorrelation are present on both variables. The *Z* chart and the NN-based control scheme have similar First-Detection capability except the cases with high autocorrelation.

The NN-based control scheme achieves worst ARL performance in the double-shift processes where shifts are present on both variables and at least one of the shifts is large. What's more, it is not comparable to the *Z* chart on First-Detection capability in these kinds of processes.

4.3.4 Summary on Control Scheme Comparison

Generally speaking, the NN-based control scheme is good at detecting and identifying small to moderate shifts while the *Z* chart performs well on detecting and identifying large shifts. In the existing related literature, the NN-based control scheme is always found to be good at detecting and identifying small to moderate shifts. The results obtained from this research are consistent with the conclusion in the existing literature. The Hotelling T^2 chart can detect large shift effectively. For the MEWMA charts, although smallest ARL can be obtained in most of the out-of-control processes, they have very small in-control ARLs in the processes with high autocorrelation or high correlation. Small in-control ARL implies frequent false alarms which is a huge disaster for the production process. In the literature, it has been concluded that high false alarms are worse than large out-of-control ARL.

4.3.5 Discussion on the MEWMA Charts

To compare the performances of the MEWMA chart and the NN-based control scheme in the processes with high autocorrelation or high correlation, the in-control ARLs of these two control schemes are tuned to the same values. For in-control processes with high correlation, the in-control ARLs of both control schemes are tuned to around 253.71. The corresponding results are reported in Table 4.3. For processes with high autocorrelation, the in-control ARLs are tuned to around 73.41 and 53.16. The results are reported in Table 4.4 and Table 4.5. For Table 4.3 to Table 4.5, it is found that the NN-based control scheme is comparable with or superior to the MEWMA chart in almost all the cases with small to moderate shifts. So the NN-based control scheme is better than the MEWMA chart in detecting small to moderate shifts in high correlation or high autocorrelation processes.

For the MEWMA chart, the upper control limit H increases as the smoothing parameter λ increases. As shown in Table 4.2, when smoothing parameter λ increases from 0.05 to 0.5, the upper control limit H increases from 7.23 to 10.25. However, when autocorrelation is present in multivariate processes, the upper control limit doesn't change following the above trend. From Table 4.4 and Table 4.5, the upper control limit H is found to decrease when the smoothing parameter λ increases. The reason can be explained as follows.

a) For non-autocorrelated multivariate processes

   In the MEWMA chart, λ is the parameter that determines the depth of memory of the MEWMA. The smaller the λ is, the larger consideration about the historical data is. So for in-control processes, the MEWMA (λ=0.05) chart will take more observations to signal a shift than the MEWMA (λ=0.5) chart. That means larger ARL will be obtained by using the MEWMA (λ=0.05) chart. When the in-control

ARLs of these two charts are tuned to the same value, the upper control limit *H* in the MEWMA (λ=0.5) chart is larger.

b) For autocorrelated multivariate processes

When λ is small, more consideration is put on historical data. In this way, the autocorrelation affects the MEWMA (λ=0.05) chart more. From the literature, it is shown that the more the chart is affected by the autocorrelation, the smaller the ARL is. So the MEWMA (λ=0.05) chart may have a smaller ARL when the same control limit is used by both MEWMA charts. Consequently, the upper control limit of the MEWMA (λ=0.05) chart should be larger.

Table 4.2 ARL, SRL derived from the NN-based network, Hotelling, MEWMA and *Z* charts and First-Detection rate obtained from the NN-based network and the *Z* chart

					The NN-based control scheme					*Z chart*					*Hotelling*		*MEWMA chart*			
																	λ=0.05 H=7.23		*λ=0.5 H=10.25*	
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*
0	0	0	0	0	185.24	203.30	49.1	50.9	0.0	185.58	185.58	51.8	48.2	0.0	185.53	188.82	185.23	181.85	185.38	183.76
0	0	0	0	0.4	210.21	216.69	46.7	53.3	0.0	179.49	182.92	50.3	48.0	1.7	114.55	113.95	158.41	149.43	116.65	115.84
0	0	0	0	0.7	253.71	277.74	45.7	53.9	0.4	201.40	197.45	46.2	45.0	8.8	66.52	64.91	125.21	114.37	70.82	68.53
0	0	0	0.2	0	139.05	134.23	32.9	67.1	0.0	161.09	163.28	44.4	55.3	0.3	157.84	152.71	107.61	96.22	106.52	102.73
0	0	0	0.2	0.4	169.40	179.15	33.7	66.1	0.2	160.66	157.53	45.1	53.1	1.8	106.51	104.63	97.03	84.96	74.71	72.51
0	0	0	0.2	0.7	191.13	196.12	29.4	70.2	0.4	181.45	179.36	41.3	50.4	8.3	62.76	59.77	83.58	75.30	52.10	49.94
0	0	0	0.7	0	62.92	63.19	18.2	81.8	0.0	38.86	39.07	9.4	90.6	0.0	40.10	40.27	20.22	15.78	16.59	15.01
0	0	0	0.7	0.4	69.34	76.81	15.3	84.6	0.1	38.51	37.73	10.0	88.9	1.1	36.05	38.07	21.86	19.71	16.89	15.53
0	0	0	0.7	0.7	73.41	80.26	11.9	88.0	0.1	40.90	41.29	7.9	89.5	2.6	30.52	31.08	21.54	18.59	15.64	13.92
0	0	0.2	0.2	0	123.80	122.02	44.3	55.6	0.1	150.62	155.37	48.9	50.8	0.3	143.73	145.75	73.67	64.94	71.53	67.93
0	0	0.2	0.2	0.4	135.42	130.76	47.9	51.9	0.2	145.16	139.42	49.5	48.8	1.7	93.10	85.97	70.46	61.33	54.25	51.58
0	0	0.2	0.2	0.7	160.79	165.54	45.9	53.5	0.6	159.16	151.36	46.1	43.8	10.1	58.93	56.58	64.66	55.53	40.63	37.11
0	0	0.2	0.7	0	58.80	59.50	22.6	77.0	0.4	38.25	38.67	11.4	88.6	0.0	38.76	38.98	18.96	14.56	15.17	13.29
0	0	0.2	0.7	0.4	65.76	70.65	21.3	78.4	0.3	37.93	37.25	12.0	86.5	1.5	33.72	34.61	20.19	17.18	15.70	14.31
0	0	0.2	0.7	0.7	71.95	78.73	16.4	83.2	0.4	39.92	39.59	9.6	86.1	4.3	28.14	28.01	20.18	17.23	14.61	13.14
0	0	0.7	0.7	0	43.34	42.86	41.4	58.4	0.2	21.97	21.52	47.6	51.3	1.1	20.93	20.46	11.71	8.95	8.86	7.81
0	0	0.7	0.7	0.4	47.83	48.73	45.1	54.3	0.6	23.00	22.84	47.1	48.8	4.1	19.70	18.86	12.06	9.19	9.05	8.22
0	0	0.7	0.7	0.7	56.16	57.61	44.6	54.0	1.4	26.44	25.50	43.2	45.9	10.9	17.63	17.56	12.75	9.89	9.13	8.52
0	0.5	0	0	0	24.86	14.43	7.4	92.5	0.1	115.34	111.57	29.6	70.1	0.3	115.15	116.58	26.45	14.76	63.68	64.60
0	0.5	0	0	0.4	24.23	14.27	7.7	92.3	0.0	105.77	102.84	28.9	69.6	1.5	74.49	71.30	26.42	15.71	50.98	46.67
0	0.5	0	0	0.7	24.47	14.36	7.0	93.0	0.0	112.40	112.06	25.0	68.5	6.5	53.06	52.31	27.05	15.73	39.62	35.89
0	0.5	0	0.2	0	25.29	15.88	7.8	92.2	0.0	105.39	101.08	26.9	72.9	0.2	101.36	100.05	26.00	16.38	43.86	41.24
0	0.5	0	0.2	0.4	24.97	16.41	7.4	92.6	0.0	96.72	94.71	26.1	72.1	1.8	70.54	68.23	26.04	17.20	38.78	35.66
0	0.5	0	0.2	0.7	25.18	16.35	6.8	93.2	0.0	104.55	102.21	23.4	70.7	5.9	49.99	47.60	26.40	17.13	32.18	29.40
0	0.5	0	0.7	0	28.86	25.92	9.1	90.9	0.0	32.79	32.18	8.0	91.5	0.5	33.03	32.50	17.88	14.00	15.10	14.15
0	0.5	0	0.7	0.4	29.00	28.48	7.9	92.1	0.0	32.62	30.00	9.1	89.7	1.2	31.12	29.67	17.93	14.92	14.92	14.11
0	0.5	0	0.7	0.7	29.11	27.94	6.5	93.4	0.1	34.96	34.88	6.4	90.8	2.8	27.74	26.81	18.22	15.35	14.48	13.67

					The NN-based control scheme					Z chart					Hotelling		MEWMA chart			
																	λ=0.05 H=7.23		λ=0.5 H=10.25	
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*
0	0.5	0.2	0.2	0	24.90	15.58	9.2	90.8	0.0	103.33	104.06	30.9	68.9	0.2	96.70	95.57	24.07	15.11	35.43	34.96
0	0.5	0.2	0.2	0.4	24.85	16.31	8.2	91.8	0.0	91.44	88.47	28.0	70.1	1.9	81.26	81.19	23.44	14.90	32.18	28.74
0	0.5	0.2	0.2	0.7	25.10	16.21	7.6	92.4	0.0	99.20	96.42	26.8	66.1	7.1	46.67	44.43	23.80	14.89	26.96	24.15
0	0.5	0.2	0.7	0	28.23	25.13	10.7	89.2	0.1	32.51	32.24	9.8	89.8	0.4	32.26	32.26	16.73	12.93	13.91	13.19
0	0.5	0.2	0.7	0.4	28.75	28.17	9.1	90.6	0.3	32.35	30.04	9.8	88.7	1.5	29.99	28.66	16.95	13.99	13.75	13.06
0	0.5	0.2	0.7	0.7	29.22	28.08	6.9	92.9	0.2	34.87	34.85	7.0	89.4	3.6	25.70	25.04	17.29	14.35	13.35	12.54
0	0.5	0.7	0.7	0	24.48	21.34	20.9	78.5	0.6	20.38	21.12	43.0	55.9	1.1	18.93	19.13	10.66	7.62	8.27	7.64
0	0.5	0.7	0.7	0.4	26.32	26.19	20.9	78.5	0.6	21.26	21.28	42.0	53.7	4.3	18.01	18.35	10.85	8.07	8.49	7.92
0	0.5	0.7	0.7	0.7	28.16	27.40	15.4	83.6	1.0	23.45	23.08	37.9	50.6	11.5	16.12	16.71	11.32	8.83	8.59	8.06
0	1	0	0	0	13.89	7.07	5.9	94.0	0.1	40.32	38.65	9.4	90.2	0.4	39.19	37.63	11.19	4.30	15.09	13.29
0	1	0	0	0.4	13.44	6.85	5.8	94.0	0.2	37.73	36.52	10.1	88.9	1.0	34.03	33.74	10.84	3.81	14.95	12.66
0	1	0	0	0.7	13.48	6.93	6.0	93.9	0.1	38.77	38.30	6.4	90.7	2.9	27.00	25.01	10.90	3.77	14.74	13.03
0	1	0	0.2	0	14.08	7.61	6.0	93.9	0.1	38.21	35.93	9.2	90.5	0.3	38.41	36.23	11.55	5.37	13.68	11.66
0	1	0	0.2	0.4	13.62	7.26	6.1	93.8	0.1	36.46	35.09	9.3	89.8	0.9	33.59	31.16	11.05	4.71	13.83	12.12
0	1	0	0.2	0.7	13.66	7.27	5.9	94.0	0.1	37.47	34.48	6.5	90.9	2.6	26.56	24.15	11.09	4.67	13.48	11.56
0	1	0	0.7	0	16.94	14.28	6.2	93.8	0.0	21.34	21.66	4.6	95.3	0.1	22.26	22.47	12.70	9.88	11.17	10.76
0	1	0	0.7	0.4	16.39	13.91	6.7	93.0	0.3	22.65	23.01	5.8	93.8	0.4	21.61	21.36	12.19	9.40	10.71	10.84
0	1	0	0.7	0.7	16.60	13.84	5.8	93.9	0.3	22.42	22.62	4.0	94.6	1.4	19.51	19.73	12.07	9.02	10.70	10.79
0	1	0.2	0.2	0	14.06	7.63	7.0	93.0	0.0	37.67	36.51	11.0	88.7	0.3	37.02	35.61	11.20	5.22	12.86	11.10
0	1	0.2	0.2	0.4	13.60	7.30	6.7	93.1	0.2	35.78	34.33	10.4	88.3	1.3	32.44	29.45	10.66	4.46	12.49	10.91
0	1	0.2	0.2	0.7	13.65	7.30	6.1	93.8	0.1	36.60	33.66	8.1	88.3	3.6	26.16	24.67	10.73	4.35	12.73	11.12
0	1	0.2	0.7	0	16.85	14.30	7.0	92.9	0.1	20.94	21.18	5.8	93.9	0.3	21.64	21.86	12.27	9.38	10.50	10.34
0	1	0.2	0.7	0.4	16.36	13.89	7.3	92.2	0.5	22.35	22.85	7.1	92.2	0.7	20.89	20.80	11.67	8.75	10.04	10.07
0	1	0.2	0.7	0.7	16.66	13.87	6.1	93.6	0.3	22.20	22.17	5.0	92.4	2.6	18.77	19.26	11.59	8.61	9.92	9.75
0	1	0.7	0.7	0	15.57	12.85	14.7	84.8	0.5	14.79	15.00	32.9	65.8	1.3	14.11	14.84	8.88	6.31	6.85	6.46
0	1	0.7	0.7	0.4	15.93	14.11	13.1	86.1	0.8	15.86	16.26	32.0	65.0	3.0	14.10	15.18	8.53	6.00	6.55	5.75
0	1	0.7	0.7	0.7	16.19	13.61	10.3	88.8	0.9	16.65	17.44	26.8	64.1	9.1	13.33	13.87	8.54	5.68	6.76	6.22

					The NN-based control scheme					Z chart					Hotelling		MEWMA chart			
																	λ=0.05	*H=7.23*	*λ=0.5*	*H=10.25*
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*
0	2	0	0	0	7.68	3.47	4.3	95.6	0.1	6.19	5.52	0.9	99.1	0.0	6.60	5.75	5.22	1.26	3.46	1.83
0	2	0	0	0.4	7.78	3.51	4.3	95.5	0.2	6.13	5.53	1.0	98.8	0.2	6.72	5.90	5.24	1.25	3.59	2.04
0	2	0	0	0.7	7.79	3.54	4.3	95.3	0.4	6.31	5.98	1.2	98.3	0.5	7.00	6.26	5.22	1.20	3.64	2.06
0	2	0	0.2	0	7.74	3.61	4.2	95.6	0.2	6.59	6.05	1.3	98.6	0.1	6.85	6.23	5.30	1.54	3.61	2.13
0	2	0	0.2	0.4	7.87	3.62	4.3	95.5	0.2	6.51	5.90	0.9	98.7	0.4	6.92	6.22	5.32	1.52	3.84	2.46
0	2	0	0.2	0.7	7.87	3.66	4.4	95.2	0.4	6.70	6.41	1.1	98.3	0.6	7.15	6.41	5.30	1.47	3.93	2.63
0	2	0	0.7	0	8.46	5.34	3.9	95.8	0.3	7.31	7.95	1.3	98.7	0.0	7.67	8.15	6.13	3.82	4.67	4.33
0	2	0	0.7	0.4	8.73	5.37	4.5	95.3	0.2	8.03	9.04	1.3	98.1	0.6	8.18	8.77	6.39	4.07	5.06	4.77
0	2	0	0.7	0.7	8.76	5.48	4.6	95.0	0.4	7.93	8.43	0.9	98.2	0.9	8.22	8.73	6.36	4.02	5.17	4.96
0	2	0.2	0.2	0	7.76	3.61	4.2	95.4	0.4	6.55	6.03	1.7	98.1	0.2	6.74	6.21	5.22	1.51	3.56	2.07
0	2	0.2	0.2	0.4	7.84	3.62	4.7	95.0	0.3	6.49	5.88	1.2	98.3	0.5	6.91	6.25	5.25	1.48	3.73	2.37
0	2	0.2	0.2	0.7	7.87	3.67	4.7	94.9	0.4	6.71	6.41	1.0	98.3	0.7	7.20	6.65	5.24	1.43	3.86	2.52
0	2	0.2	0.7	0	8.46	5.31	4.4	95.3	0.3	7.23	7.74	1.8	98.0	0.2	7.44	7.83	6.07	3.75	4.51	4.11
0	2	0.2	0.7	0.4	8.73	5.37	4.8	94.9	0.3	7.98	8.98	1.7	97.5	0.8	8.01	8.51	6.31	4.01	4.89	4.52
0	2	0.2	0.7	0.7	8.78	5.49	4.6	95.1	0.3	7.90	8.34	1.2	97.7	1.1	8.18	8.63	6.23	3.77	4.98	4.76
0	2	0.7	0.7	0	8.27	5.23	7.7	91.8	0.5	6.24	6.67	12.7	85.5	1.8	6.11	6.61	5.30	3.01	3.78	3.34
0	2	0.7	0.7	0.4	8.61	5.27	6.6	92.4	1.0	6.89	7.84	12.2	84.7	3.1	6.44	7.02	5.39	2.83	3.80	3.32
0	2	0.7	0.7	0.7	8.71	5.47	5.8	93.5	0.7	6.78	7.29	11.1	85.3	3.6	6.77	7.43	5.21	2.52	3.87	3.25
0	3	0	0	0	5.73	2.39	3.6	96.1	0.3	1.96	1.35	0.1	99.8	0.1	2.07	1.46	3.50	0.68	1.92	0.74
0	3	0	0	0.4	5.73	2.43	3.4	96.3	0.3	1.91	1.33	0.3	99.7	0.0	2.10	1.54	3.53	0.69	1.92	0.79
0	3	0	0	0.7	5.73	2.43	3.5	96.2	0.3	1.90	1.34	0.3	99.5	0.2	2.17	1.66	3.53	0.68	1.89	0.77
0	3	0	0.2	0	5.72	2.43	3.6	96.1	0.3	2.12	1.69	0.1	99.8	0.1	2.22	1.73	3.56	0.76	1.97	0.81
0	3	0	0.2	0.4	5.74	2.48	3.3	96.3	0.4	2.07	1.67	0.3	99.7	0.0	2.22	1.81	3.57	0.80	1.94	0.85
0	3	0	0.2	0.7	5.73	2.48	3.5	96.4	0.1	2.09	1.67	0.3	99.5	0.2	2.37	2.01	3.56	0.79	1.95	0.85
0	3	0	0.7	0	5.97	3.14	3.7	96.1	0.2	3.12	3.60	0.4	99.6	0.0	3.23	3.60	3.90	1.77	2.43	2.06
0	3	0	0.7	0.4	6.10	3.08	4.1	95.8	0.1	3.23	3.70	0.6	99.3	0.1	3.51	4.12	4.04	1.99	2.56	2.23
0	3	0	0.7	0.7	6.15	3.15	3.8	96.0	0.2	3.29	3.83	0.6	99.1	0.3	3.61	4.24	4.02	1.92	2.59	2.30

					The NN-based control scheme					Z chart					Hotelling		MEWMA chart			
																	λ=0.05	H=7.23	λ=0.5	H=10.25
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	ARL	SRL	X%	Y%	XY%	ARL	SRL	X%	Y%	XY%	ARL	SRL	ARL	SRL	ARL	SRL
0	3	0.2	0.2	0	5.73	2.43	3.6	96.1	0.3	2.12	1.69	0.1	99.5	0.4	2.20	1.73	3.54	0.76	1.96	0.81
0	3	0.2	0.2	0.4	5.74	2.47	3.7	95.9	0.4	2.07	1.67	0.4	99.5	0.1	2.26	1.88	3.55	0.79	1.94	0.85
0	3	0.2	0.2	0.7	5.73	2.48	3.9	96.0	0.1	2.09	1.67	0.3	99.6	0.1	2.36	1.98	3.53	0.77	1.93	0.82
0	3	0.2	0.7	0	5.97	3.13	3.9	96.0	0.1	3.12	3.60	0.5	99.3	0.2	3.19	3.51	3.87	1.75	2.41	2.07
0	3	0.2	0.7	0.4	6.12	3.07	3.9	95.7	0.4	3.22	3.58	0.6	99.2	0.2	3.50	4.07	4.00	1.94	2.55	2.20
0	3	0.2	0.7	0.7	6.14	3.14	3.8	96.1	0.1	3.28	3.72	0.6	99.1	0.3	3.62	4.24	3.98	1.88	2.52	2.15
0	3	0.7	0.7	0	5.86	3.09	5.6	93.9	0.5	2.90	3.27	5.3	92.5	2.2	2.79	3.16	3.66	1.58	2.16	1.69
0	3	0.7	0.7	0.4	6.06	3.07	4.7	94.7	0.6	2.96	3.26	5.5	92.3	2.2	2.99	3.45	3.68	1.60	2.21	1.63
0	3	0.7	0.7	0.7	6.10	3.14	4.3	95.3	0.4	2.92	3.23	6.3	92.0	1.7	3.09	3.53	3.62	1.35	2.14	1.48
0.5	0.5	0	0	0	19.84	11.42	42.1	56.7	1.2	77.41	78.06	50.8	48.8	0.4	73.07	73.57	16.83	7.99	31.87	29.45
0.5	0.5	0	0	0.4	20.06	11.88	45.2	53.7	1.1	77.43	73.01	46.7	50.0	3.3	46.65	45.43	17.06	9.10	24.40	21.83
0.5	0.5	0	0	0.7	20.63	12.64	45.5	53.4	1.1	84.31	81.65	42.1	44.1	13.8	32.82	30.37	17.22	9.65	20.84	19.16
0.5	0.5	0	0.2	0	19.70	11.62	42.6	56.0	1.4	71.73	70.92	48.8	50.8	0.4	67.22	64.93	16.59	8.45	26.25	24.40
0.5	0.5	0	0.2	0.4	20.13	12.48	46.6	52.4	1.0	73.03	70.70	43.6	53.1	3.3	45.03	43.48	16.86	9.55	22.39	20.70
0.5	0.5	0	0.2	0.7	20.77	13.22	46.9	51.8	1.3	79.58	76.11	40.5	47.2	12.3	32.42	29.43	17.16	10.18	18.39	16.86
0.5	0.5	0	0.7	0	18.52	12.37	44.3	55.1	0.6	28.64	28.97	19.4	80.2	0.4	27.30	26.80	12.58	7.74	12.41	11.17
0.5	0.5	0	0.7	0.4	18.89	13.70	48.2	50.9	0.9	30.53	28.67	17.1	80.0	2.9	26.14	25.78	13.31	8.75	12.41	11.48
0.5	0.5	0	0.7	0.7	19.66	14.71	49.0	50.3	0.7	32.65	32.44	14.5	78.9	6.6	22.35	22.25	13.73	9.38	12.33	11.70
0.5	0.5	0.2	0.2	0	19.64	12.01	42.6	56.3	1.1	67.78	64.79	52.0	47.6	0.4	63.33	62.74	16.10	8.79	22.05	19.96
0.5	0.5	0.2	0.2	0.4	20.26	13.00	45.9	53.4	0.7	69.56	68.93	45.1	50.8	4.1	42.24	40.27	16.67	9.87	19.49	17.72
0.5	0.5	0.2	0.2	0.7	21.09	13.92	44.7	53.7	1.6	75.79	72.69	43.0	43.6	13.4	31.21	28.93	17.07	10.79	16.83	15.32
0.5	0.5	0.2	0.7	0	18.70	13.14	44.1	55.0	0.9	28.20	28.90	21.7	78.0	0.3	26.97	27.25	12.33	7.98	11.35	10.10
0.5	0.5	0.2	0.7	0.4	19.29	14.64	46.4	52.7	0.9	30.05	28.68	18.1	78.1	3.8	25.19	25.26	13.06	9.01	11.68	11.00
0.5	0.5	0.2	0.7	0.7	20.48	16.26	48.0	50.9	1.1	32.36	32.17	14.8	77.0	8.2	20.93	21.16	13.49	9.52	11.76	11.32
0.5	0.5	0.7	0.7	0	18.81	16.02	43.6	55.4	1.0	18.14	18.81	47.8	50.8	1.4	16.55	17.45	9.54	6.46	7.36	6.47
0.5	0.5	0.7	0.7	0.4	20.74	19.81	43.9	55.2	0.9	19.31	19.02	46.4	48.2	5.4	16.21	16.84	10.17	7.81	7.85	7.37
0.5	0.5	0.7	0.7	0.7	23.31	22.87	43.1	54.8	2.1	22.46	22.08	41.7	43.3	15.0	14.68	14.95	10.76	8.33	8.07	7.75

					The NN-based control scheme					Z chart					Hotelling		MEWMA chart			
																	λ=0.05	H=7.23	λ=0.5	H=10.25
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	ARL	SRL	X%	Y%	XY%	ARL	SRL	X%	Y%	XY%	ARL	SRL	ARL	SRL	ARL	SRL
0.5	1	0	0	0	12.87	6.74	20.3	78.8	0.9	35.11	34.78	20.9	78.8	0.3	32.57	32.34	9.74	3.40	11.85	9.99
0.5	1	0	0	0.4	12.69	6.75	18.1	80.8	1.1	34.52	33.67	18.2	78.4	3.4	22.81	22.57	9.79	3.82	10.55	8.44
0.5	1	0	0	0.7	12.78	6.89	19.2	79.9	0.9	36.76	35.12	10.3	77.8	11.9	18.02	17.18	9.94	4.11	10.04	8.32
0.5	1	0	0.2	0	12.98	7.14	20.8	78.4	0.8	33.96	33.87	20.7	79.0	0.3	31.00	29.83	9.89	4.06	11.14	9.57
0.5	1	0	0.2	0.4	12.81	7.07	19.3	79.6	1.1	33.64	32.79	17.4	79.9	2.7	22.13	20.87	9.96	4.43	10.31	8.37
0.5	1	0	0.2	0.7	12.95	7.23	19.2	80.1	0.7	36.24	34.41	11.5	78.1	10.4	18.39	17.42	10.15	4.85	10.04	8.52
0.5	1	0	0.7	0	13.81	9.54	26.6	71.8	1.6	19.19	19.62	13.0	86.7	0.3	18.73	19.42	10.24	6.52	9.46	9.11
0.5	1	0	0.7	0.4	14.33	10.78	27.5	71.2	1.3	21.73	22.16	10.9	87.4	1.7	17.96	18.24	10.43	6.96	9.27	8.97
0.5	1	0	0.7	0.7	14.89	11.62	26.9	71.8	1.3	21.77	21.62	7.7	86.7	5.6	16.14	16.52	10.83	7.75	9.18	8.97
0.5	1	0.2	0.2	0	12.89	7.20	20.8	77.9	1.3	33.19	32.47	21.7	77.7	0.6	30.29	29.19	9.70	4.13	10.15	8.79
0.5	1	0.2	0.2	0.4	12.73	7.10	19.3	79.5	1.2	33.07	32.55	19.6	76.9	3.5	21.89	20.36	9.89	4.59	10.07	8.42
0.5	1	0.2	0.2	0.7	12.93	7.29	19.0	80.3	0.7	35.38	33.04	13.4	75.7	10.9	18.17	17.30	10.04	4.84	9.70	8.48
0.5	1	0.2	0.7	0	13.84	9.74	27.2	71.3	1.5	19.26	19.84	13.0	86.4	0.6	18.22	18.32	9.97	6.46	8.84	8.33
0.5	1	0.2	0.7	0.4	14.42	11.22	27.4	71.4	1.2	21.19	21.44	12.8	84.5	2.7	17.42	17.77	10.25	6.94	8.76	8.36
0.5	1	0.2	0.7	0.7	15.24	12.55	25.7	72.1	2.2	21.81	21.72	7.7	85.2	7.1	15.33	15.42	10.73	7.95	8.66	8.15
0.5	1	0.7	0.7	0	13.40	10.60	30.1	69.0	0.9	14.20	15.06	35.7	62.6	1.7	13.09	14.19	8.21	5.69	6.42	6.12
0.5	1	0.7	0.7	0.4	14.46	12.79	27.0	71.7	1.3	15.71	16.16	35.7	57.7	6.6	13.14	13.92	8.54	6.12	6.48	5.84
0.5	1	0.7	0.7	0.7	15.49	13.57	22.4	75.2	2.4	17.37	18.21	27.9	56.2	15.9	11.79	12.26	9.02	6.74	6.96	6.46
0.5	2	0	0	0	7.47	3.47	9.5	89.2	1.3	6.02	5.38	3.0	96.5	0.5	6.23	5.68	5.04	1.20	3.28	1.76
0.5	2	0	0	0.4	7.54	3.52	10.1	88.9	1.0	6.14	5.59	1.0	97.3	1.7	5.98	5.56	5.11	1.34	3.45	2.03
0.5	2	0	0	0.7	7.58	3.56	9.7	89.2	1.1	6.40	6.08	0.2	96.5	3.3	5.82	5.48	5.13	1.38	3.51	2.10
0.5	2	0	0.2	0	7.51	3.59	9.8	89.0	1.2	6.38	5.91	3.3	96.0	0.7	6.40	6.07	5.10	1.46	3.43	2.07
0.5	2	0	0.2	0.4	7.64	3.65	10.3	88.8	0.9	6.54	6.03	1.2	97.0	1.8	6.17	5.76	5.19	1.59	3.61	2.26
0.5	2	0	0.2	0.7	7.65	3.68	9.7	89.0	1.3	6.79	6.54	0.1	96.6	3.3	6.06	5.77	5.22	1.66	3.69	2.41
0.5	2	0	0.7	0	8.13	5.10	10.9	87.6	1.5	6.95	7.45	4.0	95.1	0.9	6.98	7.47	5.73	3.21	4.37	3.95
0.5	2	0	0.7	0.4	8.39	5.20	11.5	87.8	0.7	8.00	8.97	2.9	95.6	1.5	7.25	7.57	6.19	4.06	4.76	4.51
0.5	2	0	0.7	0.7	8.48	5.35	11.1	87.9	1.0	8.00	8.53	0.9	95.3	3.8	7.09	7.34	6.30	4.34	4.91	4.84

					The NN-based control scheme					Z chart					Hotelling		MEWMA chart			
																	λ=0.05 H=7.23		λ=0.5 H=10.25	
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*
0.5	2	0.2	0.2	0	7.49	3.60	9.8	89.0	1.2	6.35	5.89	3.8	95.5	0.7	6.34	6.01	5.04	1.45	3.38	2.04
0.5	2	0.2	0.2	0.4	7.63	3.66	10.3	88.5	1.2	6.53	5.98	1.4	96.8	1.8	6.08	5.75	5.16	1.60	3.58	2.20
0.5	2	0.2	0.2	0.7	7.65	3.69	9.8	89.0	1.2	6.79	6.54	0.1	96.8	3.1	6.06	5.80	5.19	1.65	3.65	2.39
0.5	2	0.2	0.7	0	8.12	5.15	10.8	87.8	1.4	6.94	7.44	4.1	94.9	1.0	7.01	7.57	5.67	3.16	4.27	3.83
0.5	2	0.2	0.7	0.4	8.41	5.22	11.5	87.4	1.1	7.98	8.91	2.5	95.9	1.6	7.35	7.74	6.16	4.02	4.72	4.51
0.5	2	0.2	0.7	0.7	8.51	5.42	10.8	88.2	1.0	8.00	8.52	0.6	95.5	3.9	7.07	7.29	6.28	4.27	4.89	4.75
0.5	2	0.7	0.7	0	7.90	5.04	14.5	83.9	1.6	6.02	6.65	16.2	81.7	2.1	5.88	6.59	5.12	2.85	3.63	3.13
0.5	2	0.7	0.7	0.4	8.39	5.27	12.1	85.5	2.4	7.14	7.81	10.1	83.1	6.8	6.40	6.76	5.56	3.34	4.07	3.71
0.5	2	0.7	0.7	0.7	8.54	5.54	10.4	87.4	2.2	7.29	7.68	7.0	83.6	9.4	6.56	6.86	5.56	3.21	4.21	3.84
0.5	3	0	0	0	5.63	2.40	6.4	92.3	1.3	1.95	1.33	0.7	98.5	0.8	2.04	1.39	3.45	0.66	1.88	0.72
0.5	3	0	0	0.4	5.60	2.42	7.1	91.9	1.0	1.91	1.32	0.2	99.1	0.7	2.04	1.50	3.51	0.72	1.88	0.80
0.5	3	0	0	0.7	5.62	2.45	7.1	92.2	0.7	1.91	1.35	0.0	99.1	0.9	2.14	1.70	3.50	0.74	1.90	0.82
0.5	3	0	0.2	0	5.62	2.45	6.4	92.2	1.4	2.10	1.67	0.9	98.5	0.6	2.14	1.67	3.50	0.75	1.93	0.82
0.5	3	0	0.2	0.4	5.62	2.47	7.4	91.9	0.7	2.07	1.66	0.2	99.1	0.7	2.20	1.77	3.54	0.83	1.93	0.88
0.5	3	0	0.2	0.7	5.62	2.48	7.1	92.0	0.9	2.10	1.68	0.0	99.1	0.9	2.32	1.98	3.54	0.86	1.96	0.91
0.5	3	0	0.7	0	5.84	3.07	6.3	92.7	1.0	3.03	3.42	1.7	97.8	0.5	3.07	3.32	3.82	1.73	2.34	1.98
0.5	3	0	0.7	0.4	5.99	3.08	7.3	91.2	1.5	3.26	3.76	0.4	98.5	1.1	3.26	3.79	3.99	2.00	2.51	2.18
0.5	3	0	0.7	0.7	6.03	3.15	6.9	91.9	1.2	3.33	3.91	0.1	98.2	1.7	3.37	4.04	4.02	2.05	2.56	2.42
0.5	3	0.2	0.2	0	5.61	2.46	6.7	91.9	1.4	2.09	1.66	1.0	98.4	0.6	2.17	1.70	3.48	0.75	1.93	0.81
0.5	3	0.2	0.2	0.4	5.61	2.47	7.3	92.0	0.7	2.08	1.67	0.0	99.3	0.7	2.21	1.82	3.52	0.83	1.91	0.87
0.5	3	0.2	0.2	0.7	5.62	2.50	7.1	91.9	1.0	2.10	1.68	0.0	99.0	1.0	2.35	2.00	3.52	0.85	1.95	0.88
0.5	3	0.2	0.7	0	5.83	3.08	6.5	92.4	1.1	3.06	3.50	1.8	97.7	0.5	3.07	3.32	3.77	1.60	2.31	1.90
0.5	3	0.2	0.7	0.4	5.99	3.09	7.5	91.1	1.4	3.26	3.76	0.2	98.7	1.1	3.27	3.79	3.98	2.00	2.51	2.18
0.5	3	0.2	0.7	0.7	6.03	3.16	7.0	92.0	1.0	3.33	3.91	0.0	98.4	1.6	3.42	4.09	4.01	2.04	2.52	2.22
0.5	3	0.7	0.7	0	5.75	3.08	8.4	89.6	2.0	2.83	3.12	6.5	91.0	2.5	2.74	3.04	3.58	1.45	2.11	1.59
0.5	3	0.7	0.7	0.4	5.96	3.12	7.9	90.5	1.6	3.12	3.48	3.5	92.2	4.3	3.06	3.48	3.74	1.68	2.26	1.78
0.5	3	0.7	0.7	0.7	6.02	3.20	7.3	91.0	1.7	3.13	3.53	3.2	92.0	4.8	3.27	3.76	3.74	1.60	2.25	1.69

					The NN-based control scheme					Z chart					Hotelling		MEWMA chart			
																	λ=0.05	H=7.23	λ=0.5	H=10.25
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	ARL	SRL	X%	Y%	XY%	ARL	SRL	X%	Y%	XY%	ARL	SRL	ARL	SRL	ARL	SRL
1	1	0	0	0	11.09	5.75	41.9	55.1	3.0	22.66	22.29	49.0	49.5	1.5	18.19	16.98	7.50	2.27	6.78	5.00
1	1	0	0	0.4	10.90	5.83	43.4	54.6	2.0	22.96	21.90	45.9	47.3	6.8	13.16	12.01	7.67	2.78	6.65	5.07
1	1	0	0	0.7	11.08	6.07	44.0	53.9	2.1	25.96	25.51	38.8	41.9	19.3	11.64	10.95	7.79	3.05	6.51	5.04
1	1	0	0.2	0	11.06	5.92	42.2	54.9	2.9	22.14	21.70	47.8	50.8	1.4	17.90	17.45	7.52	2.49	6.71	5.10
1	1	0	0.2	0.4	10.96	5.96	44.9	53.5	1.6	22.61	21.87	44.0	49.2	6.8	13.40	12.27	7.73	3.07	6.69	5.24
1	1	0	0.2	0.7	11.17	6.29	44.6	53.1	2.3	25.43	23.73	39.4	43.5	17.1	11.83	11.04	7.90	3.46	6.59	5.31
1	1	0	0.7	0	11.05	6.75	44.5	53.4	2.1	14.94	15.35	32.1	66.7	1.2	12.53	12.68	7.26	3.39	6.24	5.51
1	1	0	0.7	0.4	11.06	6.98	49.5	48.4	2.1	16.63	16.71	31.1	63.5	5.4	12.03	11.96	7.54	3.84	6.39	5.46
1	1	0	0.7	0.7	11.51	7.47	51.0	46.8	2.2	17.71	18.23	26.4	61.4	12.2	11.33	11.49	7.78	4.06	6.72	5.90
1	1	0.2	0.2	0	11.02	5.98	41.9	55.4	2.7	21.54	20.82	48.5	50.0	1.5	18.01	17.33	7.56	2.75	6.64	5.27
1	1	0.2	0.2	0.4	10.99	6.11	43.6	54.4	2.0	22.11	21.26	46.2	46.8	7.0	13.61	12.78	7.82	3.41	6.69	5.34
1	1	0.2	0.2	0.7	11.31	6.53	43.5	54.7	1.8	25.07	23.35	41.8	40.4	17.8	11.89	11.10	7.99	3.77	6.59	5.32
1	1	0.2	0.7	0	11.07	6.84	44.8	53.5	1.7	14.69	15.30	33.5	65.3	1.2	12.77	12.95	7.28	3.60	6.08	5.50
1	1	0.2	0.7	0.4	11.24	7.35	48.3	49.3	2.4	16.59	17.08	33.7	60.6	5.7	12.04	12.26	7.65	4.23	6.35	5.43
1	1	0.2	0.7	0.7	11.76	7.98	49.7	47.8	2.5	17.94	18.36	27.0	60.1	12.9	10.90	10.77	7.92	4.46	6.67	5.94
1	1	0.7	0.7	0	11.19	7.89	44.4	53.4	2.2	11.24	12.02	47.6	49.2	3.2	10.03	10.77	7.02	4.56	5.29	4.79
1	1	0.7	0.7	0.4	12.11	9.72	43.9	52.9	3.2	12.73	13.77	49.0	42.3	8.7	10.20	10.87	7.58	5.15	5.79	5.23
1	1	0.7	0.7	0.7	13.12	11.01	45.0	52.1	2.9	14.75	15.76	43.2	36.4	20.4	10.02	10.49	8.03	5.80	6.10	5.61
1	2	0	0	0	7.10	3.41	16.8	80.0	3.2	5.67	5.12	9.8	88.6	1.6	4.75	4.36	4.62	1.02	2.86	1.34
1	2	0	0	0.4	7.19	3.45	18.8	78.2	3.0	5.85	5.37	6.2	87.9	5.9	4.46	3.96	4.74	1.25	3.01	1.74
1	2	0	0	0.7	7.22	3.49	18.5	78.5	3.0	6.31	6.03	2.3	86.6	11.1	4.30	3.78	4.78	1.35	3.06	1.83
1	2	0	0.2	0	7.15	3.52	17.5	79.7	2.8	6.02	5.72	10.3	87.5	2.2	5.01	4.68	4.67	1.20	2.99	1.63
1	2	0	0.2	0.4	7.26	3.55	19.3	77.9	2.8	6.23	5.79	6.6	86.9	6.5	4.79	4.55	4.82	1.47	3.14	1.96
1	2	0	0.2	0.7	7.29	3.60	18.5	78.3	3.2	6.61	6.31	2.5	86.1	11.4	4.51	4.16	4.86	1.59	3.25	2.15
1	2	0	0.7	0	7.52	4.47	20.5	76.8	2.7	6.43	7.01	11.4	85.8	2.8	5.77	6.47	5.02	2.27	3.58	3.01
1	2	0	0.7	0.4	7.80	4.64	22.0	75.5	2.5	7.29	7.96	10.9	83.7	5.4	6.00	6.31	5.47	3.07	4.02	3.59
1	2	0	0.7	0.7	7.95	4.94	21.7	75.6	2.7	7.52	7.80	6.5	83.1	10.4	5.76	6.17	5.57	3.32	4.12	3.79

					The NN-based control scheme					Z chart					Hotelling		MEWMA chart			
																	λ=0.05	H=7.23	λ=0.5	H=10.25
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	ARL	SRL	X%	Y%	XY%	ARL	SRL	X%	Y%	XY%	ARL	SRL	ARL	SRL	ARL	SRL
1	2	0.2	0.2	0	7.13	3.52	17.3	79.4	3.3	5.90	5.54	11.4	86.3	2.3	5.05	4.65	4.65	1.23	2.98	1.63
1	2	0.2	0.2	0.4	7.28	3.56	19.1	77.4	3.5	6.20	5.73	6.7	86.6	6.7	4.95	4.75	4.81	1.53	3.20	2.06
1	2	0.2	0.2	0.7	7.33	3.63	18.2	78.5	3.3	6.61	6.35	2.8	85.9	11.3	4.73	4.40	4.87	1.65	3.30	2.24
1	2	0.2	0.7	0	7.52	4.52	20.8	76.4	2.8	6.37	6.91	12.0	85.6	2.4	5.85	6.61	5.00	2.36	3.54	2.95
1	2	0.2	0.7	0.4	7.86	4.71	22.2	75.4	2.4	7.29	7.58	10.0	84.1	5.9	5.99	6.35	5.44	3.14	4.02	3.63
1	2	0.2	0.7	0.7	7.96	5.03	21.1	76.2	2.7	7.61	7.96	5.8	83.3	10.9	5.83	6.26	5.62	3.52	4.17	3.86
1	2	0.7	0.7	0	7.35	4.51	24.3	72.8	2.9	5.46	6.17	23.5	71.4	5.1	5.21	6.07	4.76	2.53	3.26	2.86
1	2	0.7	0.7	0.4	7.93	5.01	22.0	74.1	3.9	6.67	7.22	16.8	71.5	11.7	5.85	6.20	5.32	3.23	3.95	3.76
1	2	0.7	0.7	0.7	8.21	5.43	19.3	76.4	4.3	7.19	7.51	10.5	72.3	17.2	5.92	6.33	5.55	3.56	4.19	4.06
1	3	0	0	0	5.47	2.39	10.7	86.8	2.5	1.92	1.30	2.0	96.5	1.5	1.87	1.23	3.33	0.62	1.79	0.70
1	3	0	0	0.4	5.46	2.42	11.5	85.6	2.9	1.91	1.32	0.3	96.1	3.6	1.87	1.29	3.38	0.71	1.81	0.78
1	3	0	0	0.7	5.48	2.45	11.2	85.9	2.9	1.91	1.35	0.0	95.9	4.1	1.96	1.45	3.39	0.74	1.84	0.82
1	3	0	0.2	0	5.47	2.44	10.7	87.2	2.1	2.06	1.64	2.4	96.3	1.3	1.98	1.44	3.37	0.69	1.82	0.77
1	3	0	0.2	0.4	5.46	2.45	11.6	86.2	2.2	2.06	1.66	0.3	96.3	3.4	2.01	1.60	3.42	0.81	1.84	0.86
1	3	0	0.2	0.7	5.48	2.48	11.3	86.0	2.7	2.09	1.68	0.1	95.9	4.0	2.11	1.73	3.44	0.86	1.88	0.91
1	3	0	0.7	0	5.67	3.03	11.4	85.9	2.7	2.92	3.24	4.4	94.0	1.6	2.62	2.75	3.60	1.38	2.17	1.62
1	3	0	0.7	0.4	5.79	2.99	13.3	84.3	2.4	3.21	3.73	1.9	94.1	4.0	2.81	3.22	3.76	1.78	2.29	1.81
1	3	0	0.7	0.7	5.83	3.08	12.6	85.2	2.2	3.30	3.88	1.1	93.8	5.1	2.88	3.30	3.83	1.91	2.34	1.99
1	3	0.2	0.2	0	5.45	2.45	11.1	86.5	2.4	2.06	1.63	2.5	95.6	1.9	1.98	1.43	3.35	0.69	1.81	0.76
1	3	0.2	0.2	0.4	5.48	2.45	11.4	86.3	2.3	2.07	1.66	0.2	96.4	3.4	2.04	1.66	3.41	0.83	1.86	0.88
1	3	0.2	0.2	0.7	5.48	2.48	11.2	85.9	2.9	2.09	1.68	0.1	95.8	4.1	2.14	1.81	3.43	0.88	1.90	0.92
1	3	0.2	0.7	0	5.66	3.03	11.2	85.6	3.2	2.93	3.26	4.3	93.5	2.2	2.64	2.79	3.58	1.38	2.17	1.64
1	3	0.2	0.7	0.4	5.81	3.01	12.9	84.5	2.6	3.23	3.74	1.5	94.3	4.2	2.89	3.31	3.77	1.83	2.30	1.88
1	3	0.2	0.7	0.7	5.86	3.11	12.4	84.9	2.7	3.31	3.89	0.7	93.7	5.6	2.94	3.37	3.85	1.94	2.38	2.14
1	3	0.7	0.7	0	5.56	2.97	13.4	83.5	3.1	2.71	2.99	9.5	84.5	6.0	2.52	2.69	3.46	1.48	2.00	1.56
1	3	0.7	0.7	0.4	5.81	3.12	13.6	83.7	2.7	3.14	3.54	4.0	86.3	9.7	3.01	3.41	3.70	1.77	2.22	1.88
1	3	0.7	0.7	0.7	5.91	3.21	11.6	85.3	3.1	3.26	3.74	1.3	87.7	11.0	3.17	3.65	3.77	1.79	2.29	1.93

					The NN-based control scheme					Z chart					Hotelling		MEWMA chart			
																	λ=0.05	H=7.23	λ=0.5	H=10.25
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	ARL	SRL	X%	Y%	XY%	ARL	SRL	X%	Y%	XY%	ARL	SRL	ARL	SRL	ARL	SRL
2	2	0	0	0	6.05	2.89	42.3	53.0	4.7	3.40	2.97	46.2	45.1	8.7	2.40	1.82	3.68	0.75	2.06	0.86
2	2	0	0	0.4	6.15	3.01	45.3	48.5	6.2	3.77	3.37	40.1	39.6	20.3	2.46	2.03	3.77	0.88	2.08	0.97
2	2	0	0	0.7	6.18	3.07	44.7	48.9	6.4	4.26	3.88	33.4	31.8	34.8	2.49	2.08	3.81	0.97	2.16	1.12
2	2	0	0.2	0	6.06	2.92	42.0	52.6	5.4	3.45	3.00	47.7	44.4	7.9	2.42	1.87	3.69	0.80	2.09	0.92
2	2	0	0.2	0.4	6.17	3.08	46.3	47.3	6.4	3.87	3.51	41.6	37.2	21.2	2.56	2.16	3.79	0.96	2.12	1.08
2	2	0	0.2	0.7	6.20	3.13	45.3	48.4	6.3	4.31	3.91	36.4	28.7	34.9	2.59	2.22	3.84	1.07	2.20	1.21
2	2	0	0.7	0	6.08	3.23	43.9	50.1	6.0	3.46	3.40	45.8	44.6	9.6	2.79	2.65	3.77	1.13	2.15	1.20
2	2	0	0.7	0.4	6.29	3.34	48.2	46.1	5.7	3.96	3.93	48.1	35.8	16.1	3.01	3.15	3.89	1.33	2.32	1.49
2	2	0	0.7	0.7	6.35	3.47	48.4	45.7	5.9	4.28	4.30	46.3	30.2	23.5	3.15	3.35	3.97	1.49	2.43	1.68
2	2	0.2	0.2	0	6.06	2.95	42.1	52.3	5.6	3.52	3.08	46.0	45.7	8.3	2.49	2.05	3.69	0.86	2.11	0.98
2	2	0.2	0.2	0.4	6.19	3.11	45.4	48.6	6.0	4.08	3.79	39.5	40.1	20.4	2.64	2.29	3.81	1.03	2.18	1.17
2	2	0.2	0.2	0.7	6.24	3.19	44.8	48.8	6.4	4.57	4.29	33.4	30.9	35.7	2.68	2.38	3.88	1.17	2.25	1.30
2	2	0.2	0.7	0	6.10	3.26	43.4	49.9	6.7	3.54	3.61	43.6	46.5	9.9	3.00	3.07	3.77	1.20	2.22	1.36
2	2	0.2	0.7	0.4	6.31	3.41	47.9	46.5	5.6	4.30	4.44	44.9	37.6	17.5	3.23	3.43	3.93	1.48	2.38	1.64
2	2	0.2	0.7	0.7	6.45	3.62	48.0	45.9	6.1	4.69	4.90	42.8	30.6	26.6	3.41	3.66	4.04	1.66	2.56	1.93
2	2	0.7	0.7	0	6.18	3.57	41.8	52.5	5.7	3.61	3.99	43.3	48.0	8.7	3.27	3.79	3.85	1.78	2.38	1.98
2	2	0.7	0.7	0.4	6.64	4.16	44.4	50.1	5.5	4.81	5.35	39.9	39.8	20.3	3.91	4.71	4.31	2.46	2.90	2.76
2	2	0.7	0.7	0.7	6.94	4.60	44.0	49.5	6.5	5.58	6.24	34.1	33.4	32.5	4.20	5.09	4.57	2.88	3.16	3.01
2	3	0	0	0	5.01	2.30	26.0	68.4	5.6	1.69	1.06	14.3	72.5	13.2	1.43	0.77	2.98	0.52	1.52	0.58
2	3	0	0	0.4	5.06	2.35	25.3	67.2	7.5	1.79	1.19	6.7	69.8	23.5	1.53	0.89	2.98	0.63	1.56	0.67
2	3	0	0	0.7	5.06	2.37	25.6	66.9	7.5	1.87	1.31	1.9	68.2	29.9	1.58	0.96	3.01	0.70	1.60	0.71
2	3	0	0.2	0	5.01	2.33	25.8	68.2	6.0	1.78	1.21	15.4	71.7	12.9	1.47	0.83	3.00	0.57	1.54	0.60
2	3	0	0.2	0.4	5.05	2.37	26.2	67.0	6.8	1.89	1.42	7.8	69.3	22.9	1.55	1.00	3.03	0.71	1.59	0.71
2	3	0	0.2	0.7	5.06	2.40	25.8	67.1	7.1	2.04	1.64	2.8	66.8	30.4	1.63	1.13	3.04	0.76	1.62	0.76
2	3	0	0.7	0	5.09	2.57	26.7	66.8	6.5	2.19	2.04	20.4	66.1	13.5	1.80	1.56	3.07	0.89	1.65	0.85
2	3	0	0.7	0.4	5.20	2.63	30.4	63.9	5.7	2.47	2.67	18.9	61.5	19.6	1.93	1.95	3.15	1.07	1.66	0.92
2	3	0	0.7	0.7	5.24	2.74	30.9	63.4	5.7	2.69	3.06	16.4	57.7	25.9	1.97	2.01	3.23	1.19	1.73	1.05

					The NN-based control scheme					Z chart					Hotelling		MEWMA chart			
																	λ=0.05 H=7.23		λ=0.5 H=10.25	
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*
2	3	0.2	0.2	0	5.00	2.32	26.0	67.9	6.1	1.79	1.28	14.6	72.0	13.4	1.47	0.91	3.69	0.86	1.54	0.61
2	3	0.2	0.2	0.4	5.06	2.37	25.9	67.2	6.9	1.94	1.53	6.3	71.4	22.3	1.58	1.07	3.81	1.03	1.60	0.74
2	3	0.2	0.2	0.7	5.07	2.42	25.0	68.0	7.0	2.05	1.65	2.2	68.8	29.0	1.68	1.26	3.88	1.17	1.64	0.78
2	3	0.2	0.7	0	5.07	2.56	26.3	66.9	6.8	2.22	2.12	20.4	66.9	12.7	1.80	1.59	3.77	1.20	1.65	0.89
2	3	0.2	0.7	0.4	5.23	2.67	30.1	64.1	5.8	2.61	2.91	16.5	63.1	20.4	1.96	1.96	3.93	1.48	1.70	0.99
2	3	0.2	0.7	0.7	5.29	2.83	31.0	63.1	5.9	2.91	3.37	12.2	58.7	29.1	2.08	2.14	4.04	1.66	1.80	1.17
2	3	0.7	0.7	0	5.06	2.71	26.5	68.4	5.1	2.18	2.30	21.5	63.8	14.7	1.88	1.90	3.06	1.06	1.70	1.10
2	3	0.7	0.7	0.4	5.27	2.87	28.6	66.5	4.9	2.76	3.25	14.0	62.4	23.6	2.35	2.83	3.36	1.67	1.94	1.67
2	3	0.7	0.7	0.7	5.43	3.05	27.1	67.4	5.5	3.08	3.59	6.8	61.0	32.2	2.59	3.14	3.50	1.87	2.08	1.91
3	3	0	0	0	4.42	2.05	43.7	48.9	7.4	1.31	0.64	35.0	31.6	33.4	1.12	0.38	2.56	0.51	1.25	0.44
3	3	0	0	0.4	4.44	2.06	43.9	48.0	8.1	1.46	0.84	25.0	26.1	48.9	1.22	0.53	2.58	0.56	1.31	0.50
3	3	0	0	0.7	4.45	2.09	45.1	47.1	7.8	1.58	0.98	17.9	18.6	63.5	1.27	0.59	2.60	0.59	1.35	0.55
3	3	0	0.2	0	4.43	2.07	43.1	48.3	8.6	1.34	0.70	35.5	30.2	34.3	1.14	0.41	2.55	0.52	1.25	0.45
3	3	0	0.2	0.4	4.44	2.07	44.2	47.9	7.9	1.48	0.92	27.4	24.6	48.0	1.23	0.56	2.58	0.57	1.31	0.52
3	3	0	0.2	0.7	4.44	2.09	44.9	47.7	7.4	1.63	1.13	21.9	17.0	61.1	1.28	0.62	2.60	0.61	1.35	0.56
3	3	0	0.7	0	4.40	2.18	42.9	48.4	8.7	1.39	0.83	39.7	28.4	31.9	1.22	0.61	2.59	0.62	1.28	0.49
3	3	0	0.7	0.4	4.48	2.15	47.5	45.5	7.0	1.51	1.05	40.4	24.1	35.5	1.30	0.79	2.64	0.67	1.35	0.57
3	3	0	0.7	0.7	4.53	2.25	48.4	44.0	7.6	1.63	1.26	38.9	17.9	43.2	1.37	0.91	2.68	0.75	1.38	0.62
3	3	0.2	0.2	0	4.41	2.06	43.0	48.8	8.2	1.36	0.74	34.8	30.9	34.3	1.16	0.46	2.55	0.54	1.26	0.46
3	3	0.2	0.2	0.4	4.44	2.09	44.0	48.4	7.6	1.51	0.96	26.8	25.4	47.8	1.26	0.63	2.59	0.60	1.33	0.55
3	3	0.2	0.2	0.7	4.46	2.13	44.5	47.9	7.6	1.65	1.17	20.8	19.7	59.5	1.32	0.75	2.62	0.66	1.36	0.58
3	3	0.2	0.7	0	4.40	2.18	42.6	49.1	8.3	1.42	0.93	39.5	29.0	31.5	1.25	0.67	2.58	0.64	1.30	0.52
3	3	0.2	0.7	0.4	4.50	2.18	46.8	46.3	6.9	1.59	1.28	39.2	22.4	38.4	1.36	0.95	2.67	0.74	1.37	0.63
3	3	0.2	0.7	0.7	4.55	2.29	47.9	44.3	7.8	1.70	1.38	37.0	17.3	45.7	1.44	1.11	2.71	0.80	1.41	0.68
3	3	0.7	0.7	0	4.46	2.29	40.8	50.9	8.3	1.63	1.59	34.1	36.7	29.2	1.45	1.29	2.65	0.82	1.37	0.79
3	3	0.7	0.7	0.4	4.66	2.52	44.2	47.4	8.4	2.01	2.34	30.3	28.7	41.0	1.69	1.83	2.83	1.17	1.52	0.98
3	3	0.7	0.7	0.7	4.77	2.66	45.8	44.8	9.4	2.42	2.88	24.9	23.9	51.2	1.85	2.10	2.98	1.46	1.63	1.18

Table 4.3 ARL, SRL derived from the NN-based network and the MEWMA chart when in-control ARL of the high correlation case is tuned to the same value

					NN-based		*MEWMA*							
							λ=0.05	*H=7.23*	*λ=0.5*	*H=10.25*	*λ=0.05*	*H=9.68*	*λ=0.5*	*H=14.21*
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*
0	0	0	0	0.7	253.71	277.74	125.21	114.37	70.82	68.53	253.93	250.49	253.80	249.98
0	0.5	0	0	0.7	24.47	14.36	27.05	15.73	39.62	35.89	37.03	22.17	120.09	114.05
0	1	0	0	0.7	13.48	6.93	10.90	3.77	14.74	13.03	13.34	4.65	33.19	29.75
0	2	0	0	0.7	7.79	3.54	5.22	1.20	3.64	2.06	6.06	1.36	5.61	3.70
0	3	0	0	0.7	5.73	2.43	3.53	0.68	1.89	0.77	4.05	0.75	2.41	0.93
0.5	0.5	0	0	0.7	20.63	12.64	17.22	9.65	20.84	19.16	21.31	11.54	48.78	45.98
0.5	1	0	0	0.7	12.78	6.89	9.94	4.11	10.04	8.32	11.78	4.60	18.85	16.48
0.5	2	0	0	0.7	7.58	3.56	5.13	1.38	3.51	2.10	5.96	1.54	5.08	3.36
0.5	3	0	0	0.7	5.62	2.45	3.50	0.74	1.90	0.82	4.02	0.81	2.41	1.02
1	1	0	0	0.7	11.08	6.07	7.79	3.05	6.51	5.04	9.14	3.40	10.90	8.89
1	2	0	0	0.7	7.22	3.49	4.78	1.35	3.06	1.83	5.51	1.48	4.22	2.72
1	3	0	0	0.7	5.48	2.45	3.39	0.74	1.84	0.82	3.89	0.82	2.28	1.00
2	2	0	0	0.7	6.18	3.07	3.81	0.97	2.16	1.12	4.35	1.09	2.76	1.42
2	3	0	0	0.7	5.06	2.37	3.01	0.70	1.60	0.71	3.45	0.73	1.90	0.82
3	3	0	0	0.7	4.45	2.09	2.60	0.59	1.35	0.55	2.96	0.61	1.58	0.66

Table 4.4 ARL, SRL derived from the NN-based network and the MEWMA chart when in-control ARL of the single high autocorrelation case is tuned to the same value

					NN-based		*MEWMA*							
							λ=0.05	*H=7.23*	*λ=0.5*	*H=10.25*	*λ=0.05*	*H=9.68*	*λ=0.5*	*H=14.21*
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*
0	0	0	0.7	0.7	73.41	80.26	21.54	18.59	15.64	13.92	73.22	65.06	73.90	72.38
0	0.5	0	0.7	0.7	29.11	27.94	18.22	15.35	14.48	13.67	48.92	39.58	56.15	52.86
0	1	0	0.7	0.7	16.60	13.84	12.07	9.02	10.70	10.79	27.09	20.26	34.61	34.20
0	2	0	0.7	0.7	8.76	5.48	6.36	4.02	5.17	4.96	11.19	6.27	11.28	10.94
0	3	0	0.7	0.7	6.15	3.15	4.02	1.92	2.59	2.30	6.90	2.93	5.26	4.74
0.5	0.5	0	0.7	0.7	19.66	14.71	13.73	9.38	12.33	11.70	35.73	26.90	46.04	43.78
0.5	1	0	0.7	0.7	14.89	11.62	10.83	7.75	9.18	8.97	22.97	16.61	28.65	28.38
0.5	2	0	0.7	0.7	8.48	5.35	6.30	4.34	4.91	4.84	10.83	6.12	10.15	9.48
0.5	3	0	0.7	0.7	6.03	3.15	4.02	2.05	2.56	2.42	6.86	3.12	4.95	4.34
1	1	0	0.7	0.7	11.51	7.47	7.78	4.06	6.72	5.90	15.82	8.50	19.62	18.52
1	2	0	0.7	0.7	7.95	4.94	5.57	3.32	4.12	3.79	9.69	5.09	8.39	7.60
1	3	0	0.7	0.7	5.83	3.08	3.83	1.91	2.34	1.99	6.50	2.80	4.47	3.85
2	2	0	0.7	0.7	6.35	3.47	3.97	1.49	2.43	1.68	6.97	2.44	5.21	4.30
2	3	0	0.7	0.7	5.24	2.74	3.23	1.19	1.73	1.05	5.50	1.89	3.21	2.44
3	3	0	0.7	0.7	4.53	2.25	2.68	0.75	1.38	0.62	4.52	1.18	2.25	1.23

Table 4.5 ARL, SRL derived from the NN-based network and the MEWMA chart when in-control ARL of the double high autocorrelation case is tuned to the same value

					NN-based		*MEWMA*							
							λ=0.05	*H=7.23*	*λ=0.5*	*H=10.25*	*λ=0.05*	*H=9.68*	*λ=0.5*	*H=14.21*
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*	*ARL*	*SRL*
0	0	0.7	0.7	0.7	56.16	57.61	12.75	9.89	9.13	8.52	56.63	47.77	56.21	54.68
0	0.5	0.7	0.7	0.7	28.16	27.40	11.32	8.83	8.59	8.06	43.14	32.58	49.32	48.57
0	1	0.7	0.7	0.7	16.19	13.61	8.54	5.68	6.76	6.22	26.31	18.30	37.02	34.59
0	2	0.7	0.7	0.7	8.71	5.47	5.21	2.52	3.87	3.25	11.98	5.56	15.06	13.74
0	3	0.7	0.7	0.7	6.10	3.14	3.62	1.35	2.14	1.48	7.64	2.74	6.59	5.55
0.5	0.5	0.7	0.7	0.7	23.31	22.87	10.76	8.33	8.07	7.75	39.08	32.59	40.90	37.31
0.5	1	0.7	0.7	0.7	15.49	13.57	9.02	6.74	6.96	6.46	26.05	20.18	30.41	29.82
0.5	2	0.7	0.7	0.7	8.54	5.54	5.56	3.21	4.21	3.84	12.26	6.62	13.82	13.22
0.5	3	0.7	0.7	0.7	6.02	3.20	3.74	1.60	2.25	1.69	7.78	3.16	6.53	5.76
1	1	0.7	0.7	0.7	13.12	11.01	8.03	5.80	6.10	5.61	20.57	15.35	23.21	23.49
1	2	0.7	0.7	0.7	8.21	5.43	5.55	3.56	4.19	4.06	11.60	6.52	11.57	11.23
1	3	0.7	0.7	0.7	5.91	3.21	3.77	1.79	2.29	1.93	7.64	3.34	6.26	5.70
2	2	0.7	0.7	0.7	6.94	4.60	4.57	2.88	3.16	3.01	9.13	5.11	7.52	6.91
2	3	0.7	0.7	0.7	5.43	3.05	3.50	1.87	2.08	1.91	6.79	3.22	5.01	4.62
3	3	0.7	0.7	0.7	4.77	2.66	2.98	1.46	1.63	1.18	5.69	2.49	3.60	3.15

4.4 Improvement on First-Detection Capability

As discussed above, the First-Detection capability of the proposed NN-based control scheme is limited in several cases. Motivated by the alternative monitoring heuristics in Hwarng (2004), alternative decision criteria are proposed which may be developed for better identifying source of shift.

The proposed alternative decision criterion may be described as follows.

1. Define the sequential decision number (γ) and corresponding boundary (Γ_c).
2. Compare the network output at time t, $X(t)$ and $Y(t)$, with the boundary (Γ_c).
   a) If all the observations $X(t+i-1)$, where $i=1,\cdots,\gamma$, are larger than or equal to α, while not all the $Y(t+i-1)$ ($i=1,\cdots,\gamma$) are larger than or equal to α, it is defined that a shift happens on the variable X and the run length is equal to $t+\gamma-1$.
   b) If all the observations $Y(t+i-1)$, where $i=1,\cdots,\gamma$, are larger than or equal to α, while not all the $X(t+i-1)$ ($i=1,\cdots,\gamma$) are larger than or equal to α, it is defined that a shift happens on the variable Y and the run length is equal to $t+\gamma-1$.
   c) If all the observations $X(t+i-1)$ and $Y(t+i-1)$, where $i=1,\cdots,\gamma$, are larger than or equal to α, it can be concluded that shifts happen on both variables and the run length is equal to $t+\gamma-1$.
   d) Otherwise, the detection process should be continued from time t to time $t+1$.

Table 4.6 summarizes ARL, SRL and the First-Detection rate performance of the proposed NN-based control scheme with alternative decision criteria, i.e., $\gamma=1,2,3,4$

for some representative cases. As can be seen, the decision heuristics alter the average run length behavior of the monitoring scheme and the percentage of identification.

From Table 4.6, it is observed that the First-Detection rate of the variable with high autocorrelation increases as the sequential decision number increases in the no-shift processes where high autocorrelation is present on one of the variables. This implies that the decision heuristic becomes more sensitive to high autocorrelation when sequential decision numbers increase.

As can be seen in Table 4.6, the First-Detection capability of the proposed neural network improves as the sequential decision number increases in the single-shift processes. The decision heuristic can detect the true source of shift with a rate up to 99.4%. When high autocorrelation is present in the single-shift processes, the First-Detection capability of the proposed neural network is affected. However, the decision heuristic can detect the true source of shift with a rate more than 79% even when high autocorrelation is present.

For the double-shift processes with two different shift magnitudes, the First-Detection rate of the variable with higher shift magnitude increases as sequential decision numbers increase. It can be inferred that the alternative monitoring heuristic is more sensitive to larger shift magnitude.

In general, the alternative decision heuristic is more valuable in improving the First-Detection capability in the single-shift processes or in the double-shift processes with two different shift magnitudes.

From Table 4.6, it is observed that the ARLs increase as the sequential decision number increases, which is true for all the parameter value combinations. This means that there is a trade-off between the ARL performance and the First-Detection capability when the alternative decision heuristic is employed. Before using the alternative decision heuristic, it is important to decide what is more important to the implementer. If the time-to-signal is more important, then the original decision criterion ($\gamma = 1$) is recommended. One the other hand, if the First-Detection capability is your primary concern, the alternative decision heuristic ($\gamma \geq 2$) is recommended in identifying the source of shift in the single-shift processes or in the double-shift processes with two different shift magnitudes.

Table 4.6 ARL, SRL and First-Detection rate derived from alternative monitoring heuristics

					$\gamma = 1$, $\alpha = 0.190768$					$\gamma = 2$, $\alpha = 0.190768$					$\gamma = 3$, $\alpha = 0.190768$					$\gamma = 4$, $\alpha = 0.190768$				
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0	0	0	0.7	0	62.92	63.19	18.2	81.8	0.0	87.85	85.61	10.1	89.9	0.0	99.65	95.95	7.2	92.7	0.1	112.28	110.59	5.8	94.2	0.0
0	0	0	0.7	0.4	69.34	76.81	15.3	84.6	0.1	88.88	89.71	7.7	92.3	0.0	99.85	98.22	5.4	94.6	0.0	111.43	106.80	3.5	96.5	0.0
0	0	0	0.7	0.7	73.41	80.26	11.9	88.0	0.1	78.29	87.49	2.7	96.6	0.7	89.52	96.26	1.6	98.2	0.2	97.38	101.76	1.2	98.6	0.2
0	0	0.2	0.2	0	123.80	122.02	44.3	55.6	0.1	216.12	204.71	48.6	51.4	0.0	312.76	291.93	50.0	49.9	0.1	419.87	378.57	49.9	49.5	0.6
0	0	0.2	0.2	0.4	135.42	130.76	47.9	51.9	0.2	252.05	245.29	51.3	48.6	0.1	369.01	344.32	48.8	50.7	0.5	474.29	411.10	47.5	51.6	0.9
0	0	0.2	0.2	0.7	160.79	165.54	45.9	53.5	0.6	306.32	305.12	50.3	48.9	0.8	422.40	396.83	47.6	51.4	0.1	503.74	433.33	46.5	52.5	0.1
0	0	0.2	0.7	0	58.80	59.50	22.6	77.0	0.4	80.10	76.89	17.5	82.3	0.2	92.36	86.20	13.6	86.2	0.2	102.66	95.70	11.7	88.2	0.1
0	0	0.2	0.7	0.4	65.76	70.65	21.3	78.4	0.3	83.97	83.29	14.1	85.7	0.2	95.91	94.95	10.4	89.6	0.0	106.53	102.00	8.5	91.5	0.0
0	0	0.2	0.7	0.7	71.95	78.73	16.4	83.2	0.4	89.76	90.09	9.3	90.4	0.3	102.24	99.90	7.0	92.9	0.1	111.62	107.67	5.3	94.7	0.0
0	0	0.7	0.7	0	43.34	42.86	41.4	58.4	0.2	53.32	47.14	43.4	56.4	0.2	59.39	49.45	44.1	55.5	0.4	64.54	52.65	45.6	54.2	0.2
0	0	0.7	0.7	0.4	47.83	48.73	45.1	54.3	0.6	58.35	53.88	46.6	52.6	0.8	64.63	56.02	45.7	53.3	1.0	70.34	58.35	46.3	52.8	0.9
0	0	0.7	0.7	0.7	56.16	57.61	44.6	54.0	1.4	67.55	62.66	45.7	52.8	1.5	75.40	67.27	44.0	54.1	1.9	82.10	72.16	44.0	54.0	2.0
0	0.5	0	0	0	24.86	14.43	7.4	92.5	0.1	32.98	15.48	3.2	96.7	0.1	37.23	16.34	1.9	98.1	0.0	41.03	17.81	1.3	98.7	0.0
0	0.5	0	0	0.4	24.23	14.27	7.7	92.3	0.0	32.91	16.41	3.3	96.7	0.0	36.95	16.81	2.1	97.9	0.0	40.54	18.50	1.6	98.4	0.0
0	0.5	0	0	0.7	24.47	14.36	7.0	93.0	0.0	32.84	15.84	3.0	97.0	0.0	36.81	16.47	1.7	98.3	0.0	40.33	17.42	1.3	98.7	0.0
0	0.5	0	0.2	0	25.29	15.88	7.8	92.2	0.0	33.70	17.66	3.3	96.7	0.0	38.31	19.25	2.3	97.7	0.0	42.01	20.86	1.5	98.5	0.0
0	0.5	0	0.2	0.4	24.97	16.41	7.4	92.6	0.0	33.36	18.52	3.4	96.6	0.0	37.31	18.97	2.0	98.0	0.0	41.05	21.06	1.8	98.2	0.0
0	0.5	0	0.2	0.7	25.18	16.35	6.8	93.2	0.0	33.35	18.26	2.9	97.0	0.1	37.42	19.20	1.7	98.3	0.0	41.21	20.91	1.3	98.7	0.0
0	0.5	0	0.7	0	28.86	25.92	9.1	90.9	0.0	36.73	30.79	4.1	95.9	0.0	40.85	32.49	2.6	97.3	0.1	44.05	33.64	1.8	98.2	0.0
0	0.5	0	0.7	0.4	29.00	28.48	7.9	92.1	0.0	35.72	31.20	3.5	96.4	0.1	39.64	32.65	1.9	98.1	0.0	43.11	34.90	1.4	98.6	0.0
0	0.5	0	0.7	0.7	29.11	27.94	6.5	93.4	0.1	35.81	30.83	2.9	96.9	0.2	39.63	32.10	1.6	98.3	0.1	42.99	34.29	0.9	99.1	0.0
0	0.5	0.2	0	0	24.56	14.27	9.1	90.8	0.1	32.67	15.39	4.4	95.4	0.2	36.89	16.10	3.0	97.0	0.0	40.74	17.88	2.0	98.0	0.0
0	0.5	0.2	0	0.4	24.26	14.55	8.3	91.7	0.0	32.67	16.26	4.4	95.6	0.0	36.62	16.55	2.9	97.1	0.0	40.18	17.57	2.5	97.5	0.0
0	0.5	0.2	0	0.7	24.35	14.40	8.0	91.9	0.1	32.62	15.79	4.3	95.7	0.0	36.58	16.46	2.5	97.4	0.1	40.05	17.23	1.7	98.3	0.0
0	0.5	0.2	0.2	0	24.90	15.58	9.2	90.8	0.0	33.34	17.44	4.6	95.2	0.2	37.79	19.03	3.3	96.7	0.0	41.57	20.67	2.3	97.7	0.0
0	0.5	0.2	0.2	0.4	24.85	16.31	8.2	91.8	0.0	32.98	18.16	4.4	95.6	0.0	37.15	19.02	3.1	96.9	0.0	40.77	20.72	2.7	97.3	0.0
0	0.5	0.2	0.2	0.7	25.10	16.21	7.6	92.4	0.0	33.08	17.97	4.2	95.8	0.0	37.01	18.67	2.5	97.5	0.0	40.77	20.54	1.6	98.4	0.0

					$\gamma = 1$ $\alpha = 0.190768$					$\gamma = 2$ $\alpha = 0.190768$					$\gamma = 3$ $\alpha = 0.190768$					$\gamma = 4$ $\alpha = 0.190768$				
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0	0.5	0.2	0.7	0	28.23	25.13	10.7	89.2	0.1	35.80	29.28	6.0	94.0	0.0	39.99	31.49	4.9	94.9	0.2	43.06	32.93	3.7	96.1	0.2
0	0.5	0.2	0.7	0.4	28.75	28.17	9.1	90.6	0.3	35.49	31.06	5.0	95.0	0.0	39.30	32.32	3.0	97.0	0.0	42.55	33.75	2.1	97.9	0.0
0	0.5	0.2	0.7	0.7	29.22	28.08	6.9	92.9	0.2	35.59	30.12	3.7	96.3	0.0	39.28	31.67	2.4	97.4	0.2	42.85	34.25	1.5	98.3	0.2
0	0.5	0.7	0	0	22.41	13.94	19.2	80.3	0.5	29.32	14.89	19.0	80.7	0.3	33.12	15.43	18.3	81.4	0.3	36.31	16.33	18.5	81.4	0.1
0	0.5	0.7	0	0.4	22.48	14.25	20.2	79.2	0.6	30.06	16.03	17.6	81.9	0.5	33.82	16.53	15.3	84.4	0.3	37.03	17.31	14.7	85.0	0.3
0	0.5	0.7	0	0.7	23.16	14.43	17.3	82.2	0.5	30.68	16.01	15.1	84.3	0.6	34.36	16.43	13.2	86.2	0.6	37.24	16.75	12.6	87.0	0.4
0	0.5	0.7	0.2	0	22.62	14.89	19.0	80.4	0.6	29.56	16.18	19.4	80.2	0.4	33.29	17.01	18.9	80.6	0.5	36.41	17.91	18.7	81.1	0.2
0	0.5	0.7	0.2	0.4	23.03	15.88	20.0	79.4	0.6	30.40	17.81	17.1	82.6	0.3	34.26	18.53	15.1	84.6	0.3	37.28	19.26	14.9	84.7	0.4
0	0.5	0.7	0.2	0.7	23.75	15.91	16.8	82.4	0.8	31.23	17.89	15.1	84.5	0.4	34.97	18.55	13.4	86.3	0.3	38.16	19.29	12.5	87.2	0.3
0	0.5	0.7	0.7	0	24.48	21.34	20.9	78.5	0.6	30.24	23.29	20.6	79.0	0.4	33.88	24.58	19.9	79.7	0.4	36.74	26.34	19.9	79.6	0.5
0	0.5	0.7	0.7	0.4	26.32	26.19	20.9	78.5	0.6	32.40	28.53	18.5	81.1	0.4	35.63	29.12	16.3	83.2	0.5	39.10	30.89	15.2	84.1	0.7
0	0.5	0.7	0.7	0.7	28.16	27.40	15.4	83.6	1.0	34.07	29.29	13.6	85.8	0.6	37.40	30.17	11.9	87.7	0.4	40.65	31.82	11.5	88.1	0.4
0	1	0	0	0	13.89	7.07	5.9	94.0	0.1	17.55	7.30	2.6	97.4	0.0	19.60	7.39	1.7	98.3	0.0	21.38	7.52	1.1	98.9	0.0
0	1	0	0	0.4	13.44	6.85	5.8	94.0	0.2	17.26	7.11	2.8	97.1	0.1	19.38	7.20	1.4	98.6	0.0	21.11	7.33	1.0	99.0	0.0
0	1	0	0	0.7	13.48	6.93	6.0	93.9	0.1	17.22	7.16	2.8	97.2	0.0	19.44	7.23	1.3	98.7	0.0	21.13	7.29	0.9	99.1	0.0
0	1	0	0.2	0	14.08	7.61	6.0	93.9	0.1	17.92	8.05	2.7	97.3	0.0	20.01	8.21	1.4	98.6	0.0	21.79	8.28	1.0	99.0	0.0
0	1	0	0.2	0.4	13.62	7.26	6.1	93.8	0.1	17.54	7.81	2.6	97.3	0.1	19.79	8.01	1.4	98.5	0.1	21.42	8.08	1.0	98.9	0.1
0	1	0	0.2	0.7	13.66	7.27	5.9	94.0	0.1	17.53	7.81	2.9	97.1	0.0	19.67	7.94	1.3	98.7	0.0	21.33	8.00	1.1	98.9	0.0
0	1	0	0.7	0	16.94	14.28	6.2	93.8	0.0	20.84	15.24	2.9	97.1	0.0	23.07	15.52	1.4	98.6	0.0	24.90	16.08	1.0	99.0	0.0
0	1	0	0.7	0.4	16.39	13.91	6.7	93.0	0.3	20.35	15.27	2.8	97.1	0.1	22.61	16.09	1.5	98.4	0.1	24.46	16.47	1.0	98.9	0.1
0	1	0	0.7	0.7	16.60	13.84	5.8	93.9	0.3	20.45	15.21	2.5	97.5	0.0	22.53	15.48	1.4	98.6	0.0	24.20	15.97	1.0	99.0	0.0
0	1	0.2	0	0	13.83	7.02	6.6	93.4	0.0	17.54	7.32	3.0	96.8	0.2	19.49	7.40	2.0	98.0	0.0	21.28	7.47	1.3	98.7	0.0
0	1	0.2	0	0.4	13.38	6.89	6.6	93.1	0.3	17.23	7.15	3.4	96.5	0.1	19.26	7.19	1.8	98.2	0.0	21.08	7.32	1.0	99.0	0.0
0	1	0.2	0	0.7	13.46	6.96	6.2	93.7	0.1	17.24	7.19	3.4	96.5	0.1	19.35	7.21	1.8	98.2	0.0	21.10	7.27	1.1	98.9	0.0
0	1	0.2	0.2	0	14.06	7.63	7.0	93.0	0.0	17.85	8.00	3.2	96.8	0.0	19.88	8.16	1.9	98.1	0.0	21.66	8.24	1.3	98.7	0.0
0	1	0.2	0.2	0.4	13.60	7.30	6.7	93.1	0.2	17.41	7.84	3.6	96.3	0.1	19.67	8.00	1.8	98.1	0.1	21.34	8.08	1.1	98.8	0.1
0	1	0.2	0.2	0.7	13.65	7.30	6.1	93.8	0.1	17.48	7.79	3.3	96.5	0.2	19.63	7.96	1.6	98.3	0.1	21.31	7.98	1.1	98.8	0.1

					$\gamma = 1$ $\alpha = 0.190768$					$\gamma = 2$ $\alpha = 0.190768$					$\gamma = 3$ $\alpha = 0.190768$					$\gamma = 4$ $\alpha = 0.190768$				
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0	1	0.2	0.7	0	16.85	14.30	7.0	92.9	0.1	20.77	15.19	3.5	96.5	0.0	22.98	15.38	2.0	97.9	0.1	24.78	15.81	1.5	98.4	0.1
0	1	0.2	0.7	0.4	16.36	13.89	7.3	92.2	0.5	20.21	15.23	3.4	96.4	0.2	22.45	16.03	2.0	97.9	0.1	24.27	16.33	1.4	98.5	0.1
0	1	0.2	0.7	0.7	16.66	13.87	6.1	93.6	0.3	20.38	15.08	3.2	96.7	0.1	22.43	15.31	1.9	98.0	0.1	24.17	15.97	1.3	98.7	0.0
0	1	0.7	0	0	13.24	7.05	13.1	86.1	0.8	16.76	7.28	11.5	88.1	0.4	18.69	7.35	9.5	90.1	0.4	20.46	7.53	8.8	90.9	0.3
0	1	0.7	0	0.4	12.82	6.84	14.1	85.1	0.8	16.57	7.26	10.4	88.5	1.1	18.61	7.23	8.5	90.7	0.8	20.41	7.41	7.5	92.0	0.5
0	1	0.7	0	0.7	13.04	6.97	12.9	86.4	0.7	16.79	7.37	9.0	89.8	1.2	18.79	7.35	7.0	92.1	0.9	20.54	7.42	6.9	92.6	0.5
0	1	0.7	0.2	0	13.58	7.69	12.8	86.2	1.0	17.01	7.91	12.0	87.6	0.4	18.98	8.11	10.1	89.5	0.4	20.78	8.26	9.0	90.8	0.2
0	1	0.7	0.2	0.4	13.11	7.30	13.9	85.3	0.8	16.80	7.85	10.7	88.4	0.9	18.99	7.99	8.4	90.8	0.8	20.70	8.14	7.3	92.1	0.6
0	1	0.7	0.2	0.7	13.24	7.37	12.9	86.6	0.5	16.99	7.91	9.0	90.2	0.8	19.15	8.03	6.9	92.5	0.6	20.76	8.03	6.4	93.2	0.4
0	1	0.7	0.7	0	15.57	12.85	14.7	84.8	0.5	19.27	13.82	13.3	85.9	0.8	21.44	14.29	12.3	87.1	0.6	23.30	14.74	12.4	87.3	0.3
0	1	0.7	0.7	0.4	15.93	14.11	13.1	86.1	0.8	19.47	14.87	10.7	88.7	0.6	21.77	15.65	8.9	90.5	0.6	23.56	15.89	7.8	91.7	0.5
0	1	0.7	0.7	0.7	16.19	13.61	10.3	88.8	0.9	20.02	15.02	8.3	90.5	1.2	22.02	15.07	5.6	93.2	1.2	23.81	15.69	4.5	94.3	1.2
0	2	0	0	0	7.68	3.47	4.3	95.6	0.1	9.77	3.45	1.8	98.2	0.0	11.07	3.41	1.0	98.9	0.1	12.17	3.37	0.7	99.2	0.1
0	2	0	0	0.4	7.78	3.51	4.3	95.5	0.2	9.83	3.48	1.7	97.9	0.4	11.10	3.45	1.2	98.8	0.0	12.20	3.45	0.8	99.2	0.0
0	2	0	0	0.7	7.79	3.54	4.3	95.3	0.4	9.80	3.47	1.7	98.1	0.2	11.07	3.43	1.1	98.9	0.0	12.20	3.43	0.7	99.3	0.0
0	2	0	0.2	0	7.74	3.61	4.2	95.6	0.2	9.83	3.60	2.0	98.0	0.0	11.13	3.59	1.0	98.9	0.1	12.26	3.57	0.7	99.2	0.1
0	2	0	0.2	0.4	7.87	3.62	4.3	95.5	0.2	9.90	3.60	1.6	98.1	0.3	11.15	3.57	1.1	98.8	0.1	12.26	3.58	0.8	99.2	0.0
0	2	0	0.2	0.7	7.87	3.66	4.4	95.2	0.4	9.89	3.62	1.8	97.9	0.3	11.14	3.59	1.1	98.9	0.0	12.25	3.58	0.7	99.3	0.0
0	2	0	0.7	0	8.46	5.34	3.9	95.8	0.3	10.59	5.61	1.8	98.0	0.2	11.97	5.71	1.1	98.9	0.0	13.17	5.80	0.8	99.2	0.0
0	2	0	0.7	0.4	8.73	5.37	4.5	95.3	0.2	10.86	5.61	1.5	98.2	0.3	12.14	5.69	1.2	98.8	0.0	13.32	5.76	1.1	98.9	0.0
0	2	0	0.7	0.7	8.76	5.48	4.6	95.0	0.4	10.91	5.72	1.4	98.4	0.2	12.18	5.78	1.0	99.0	0.0	13.38	5.85	0.9	99.1	0.0
0	2	0.2	0	0	7.68	3.48	4.3	95.5	0.2	9.74	3.44	2.2	97.8	0.0	11.06	3.41	1.3	98.6	0.1	12.17	3.38	0.9	99.0	0.1
0	2	0.2	0	0.4	7.76	3.50	4.8	94.9	0.3	9.80	3.51	1.9	97.7	0.4	11.08	3.46	1.4	98.5	0.1	12.18	3.45	1.0	98.9	0.1
0	2	0.2	0	0.7	7.79	3.53	4.6	94.9	0.5	9.79	3.49	1.9	97.8	0.3	11.08	3.44	1.4	98.6	0.0	12.20	3.42	1.2	98.8	0.0
0	2	0.2	0.2	0	7.76	3.61	4.2	95.4	0.4	9.81	3.60	2.2	97.8	0.0	11.11	3.58	1.4	98.6	0.0	12.25	3.56	1.0	99.0	0.0
0	2	0.2	0.2	0.4	7.84	3.62	4.7	95.0	0.3	9.86	3.61	2.0	97.5	0.5	11.13	3.58	1.4	98.4	0.2	12.23	3.57	1.0	98.9	0.1
0	2	0.2	0.2	0.7	7.87	3.67	4.7	94.9	0.4	9.87	3.64	2.0	97.8	0.2	11.14	3.61	1.3	98.7	0.0	12.24	3.58	1.1	98.9	0.0

					$\gamma = 1$, $\alpha = 0.190768$					$\gamma = 2$, $\alpha = 0.190768$					$\gamma = 3$, $\alpha = 0.190768$					$\gamma = 4$, $\alpha = 0.190768$				
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0	2	0.2	0.7	0	8.46	5.31	4.4	95.3	0.3	10.56	5.61	2.1	97.8	0.1	11.93	5.70	1.5	98.5	0.0	13.13	5.79	1.2	98.8	0.0
0	2	0.2	0.7	0.4	8.73	5.37	4.8	94.9	0.3	10.83	5.63	1.7	97.8	0.5	12.13	5.68	1.4	98.4	0.2	13.34	5.79	1.2	98.7	0.1
0	2	0.2	0.7	0.7	8.78	5.49	4.6	95.1	0.3	10.87	5.70	1.7	97.8	0.5	12.17	5.78	1.1	98.9	0.0	13.39	5.86	0.9	99.1	0.0
0	2	0.7	0	0	7.52	3.46	7.4	92.3	0.3	9.57	3.48	5.9	93.6	0.5	10.89	3.45	4.7	94.8	0.5	12.01	3.47	4.0	95.6	0.4
0	2	0.7	0	0.4	7.66	3.47	6.6	92.3	1.1	9.64	3.50	4.9	94.4	0.7	10.95	3.45	4.0	95.5	0.5	12.05	3.45	3.3	96.2	0.5
0	2	0.7	0	0.7	7.66	3.49	6.7	92.6	0.7	9.65	3.51	4.6	94.3	1.1	10.96	3.44	3.6	95.7	0.7	12.10	3.45	3.2	96.2	0.6
0	2	0.7	0.2	0	7.54	3.57	7.7	92.0	0.3	9.60	3.64	5.9	93.5	0.6	10.94	3.59	4.7	94.8	0.5	12.08	3.60	4.0	95.5	0.5
0	2	0.7	0.2	0.4	7.74	3.59	6.8	92.1	1.1	9.73	3.62	4.8	94.3	0.9	11.00	3.59	3.8	95.7	0.5	12.11	3.57	3.1	96.4	0.5
0	2	0.7	0.2	0.7	7.76	3.63	6.6	92.8	0.6	9.73	3.63	4.5	94.4	1.1	11.02	3.57	3.4	96.0	0.6	12.14	3.56	3.2	96.3	0.5
0	2	0.7	0.7	0	8.27	5.23	7.7	91.8	0.5	10.32	5.53	6.1	93.2	0.7	11.65	5.57	5.4	94.3	0.3	12.85	5.67	5.0	94.7	0.3
0	2	0.7	0.7	0.4	8.61	5.27	6.6	92.4	1.0	10.64	5.58	4.6	94.8	0.6	12.00	5.65	3.7	96.0	0.3	13.24	5.75	2.8	96.9	0.3
0	2	0.7	0.7	0.7	8.71	5.47	5.8	93.5	0.7	10.75	5.68	3.9	95.1	1.0	12.07	5.72	2.5	96.7	0.8	13.25	5.79	2.3	96.8	0.9
0	3	0	0	0	5.73	2.39	3.6	96.1	0.3	7.18	2.38	1.3	98.2	0.5	8.31	2.36	0.8	99.0	0.2	9.35	2.33	0.7	99.3	0.0
0	3	0	0	0.4	5.73	2.43	3.4	96.3	0.3	7.23	2.35	1.1	98.7	0.2	8.31	2.35	0.9	99.0	0.1	9.34	2.34	0.8	99.2	0.0
0	3	0	0	0.7	5.73	2.43	3.5	96.2	0.3	7.22	2.36	1.2	98.6	0.2	8.30	2.35	0.9	99.0	0.1	9.33	2.33	0.8	99.2	0.0
0	3	0	0.2	0	5.72	2.43	3.6	96.1	0.3	7.20	2.44	1.3	98.2	0.5	8.35	2.43	0.8	99.0	0.2	9.38	2.42	0.7	99.3	0.0
0	3	0	0.2	0.4	5.74	2.48	3.3	96.3	0.4	7.27	2.41	1.1	98.7	0.2	8.34	2.41	0.9	99.0	0.1	9.38	2.40	0.8	99.2	0.0
0	3	0	0.2	0.7	5.73	2.48	3.5	96.4	0.1	7.29	2.44	1.2	98.5	0.3	8.36	2.43	0.9	99.0	0.1	9.40	2.43	0.8	99.2	0.0
0	3	0	0.7	0	5.97	3.14	3.7	96.1	0.2	7.56	3.27	1.4	98.3	0.3	8.70	3.29	0.9	99.0	0.1	9.79	3.33	0.7	99.3	0.0
0	3	0	0.7	0.4	6.10	3.08	4.1	95.8	0.1	7.72	3.22	1.2	98.7	0.1	8.85	3.25	0.9	99.1	0.0	9.91	3.26	0.8	99.2	0.0
0	3	0	0.7	0.7	6.15	3.15	3.8	96.0	0.2	7.69	3.21	1.1	98.9	0.0	8.82	3.24	0.8	99.2	0.0	9.89	3.27	0.7	99.3	0.0
0	3	0.2	0	0	5.73	2.38	3.6	96.1	0.3	7.18	2.37	1.4	98.1	0.5	8.30	2.36	1.0	98.8	0.2	9.34	2.33	0.7	99.3	0.0
0	3	0.2	0	0.4	5.72	2.42	3.6	96.0	0.4	7.21	2.36	1.5	98.3	0.2	8.30	2.35	1.1	98.9	0.0	9.34	2.34	0.9	99.1	0.0
0	3	0.2	0	0.7	5.73	2.43	3.5	96.2	0.3	7.21	2.37	1.6	98.2	0.2	8.29	2.34	1.1	98.9	0.0	9.33	2.33	1.0	99.0	0.0
0	3	0.2	0.2	0	5.73	2.43	3.6	96.1	0.3	7.20	2.44	1.4	98.1	0.5	8.33	2.44	0.9	98.9	0.2	9.38	2.42	0.6	99.4	0.0
0	3	0.2	0.2	0.4	5.74	2.47	3.7	95.9	0.4	7.27	2.43	1.4	98.4	0.2	8.34	2.41	1.1	98.9	0.0	9.39	2.41	0.9	99.1	0.0
0	3	0.2	0.2	0.7	5.73	2.48	3.9	96.0	0.1	7.27	2.45	1.6	98.2	0.2	8.35	2.44	1.1	98.9	0.0	9.40	2.43	1.0	99.0	0.0

					$\gamma = 1$ $\alpha = 0.190768$					$\gamma = 2$ $\alpha = 0.190768$					$\gamma = 3$ $\alpha = 0.190768$					$\gamma = 4$ $\alpha = 0.190768$				
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0	3	0.2	0.7	0	5.97	3.13	3.9	96.0	0.1	7.54	3.25	1.7	98.1	0.2	8.69	3.27	1.1	98.8	0.1	9.80	3.32	0.6	99.4	0.0
0	3	0.2	0.7	0.4	6.12	3.07	3.9	95.7	0.4	7.71	3.22	1.3	98.6	0.1	8.84	3.25	1.0	99.0	0.0	9.90	3.24	0.7	99.2	0.1
0	3	0.2	0.7	0.7	6.14	3.14	3.8	96.1	0.1	7.67	3.21	1.4	98.6	0.0	8.81	3.23	0.9	99.1	0.0	9.89	3.25	0.7	99.3	0.0
0	3	0.7	0	0	5.64	2.39	5.4	94.2	0.4	7.11	2.36	3.6	95.2	1.2	8.24	2.35	2.9	96.1	1.0	9.27	2.34	2.6	96.7	0.7
0	3	0.7	0	0.4	5.67	2.41	4.8	94.2	1.0	7.16	2.39	2.9	96.1	1.0	8.26	2.37	2.0	97.5	0.5	9.31	2.35	1.7	97.8	0.5
0	3	0.7	0	0.7	5.67	2.43	4.7	94.2	1.1	7.15	2.39	3.0	96.0	1.0	8.24	2.35	2.1	97.3	0.6	9.28	2.34	1.8	97.5	0.7
0	3	0.7	0.2	0	5.66	2.45	5.5	93.9	0.6	7.13	2.45	3.6	95.3	1.1	8.27	2.44	2.8	96.3	0.9	9.30	2.43	2.5	97.0	0.5
0	3	0.7	0.2	0.4	5.69	2.46	5.0	94.3	0.7	7.21	2.46	3.1	96.1	0.8	8.29	2.45	2.0	97.4	0.6	9.35	2.43	1.7	97.7	0.6
0	3	0.7	0.2	0.7	5.69	2.48	4.6	94.5	0.9	7.22	2.48	3.0	96.0	1.0	8.31	2.45	1.9	97.5	0.6	9.36	2.44	1.6	97.7	0.7
0	3	0.7	0.7	0	5.86	3.09	5.6	93.9	0.5	7.46	3.24	3.9	95.5	0.6	8.60	3.25	3.2	96.2	0.6	9.68	3.31	2.7	96.6	0.7
0	3	0.7	0.7	0.4	6.06	3.07	4.7	94.7	0.6	7.61	3.21	2.7	96.9	0.4	8.77	3.24	1.8	97.9	0.3	9.84	3.24	1.5	98.1	0.4
0	3	0.7	0.7	0.7	6.10	3.14	4.3	95.3	0.4	7.63	3.22	2.4	97.3	0.3	8.77	3.22	1.7	98.1	0.2	9.87	3.25	1.3	98.4	0.3
0.5	0.5	0	0	0	19.84	11.42	42.1	56.7	1.2	26.87	11.89	43.8	55.3	0.9	30.81	12.34	44.8	54.1	1.1	34.14	12.62	45.4	53.8	0.8
0.5	0.5	0	0	0.4	20.06	11.88	45.2	53.7	1.1	27.68	13.20	46.7	52.2	1.1	31.76	13.53	46.6	52.5	0.9	35.19	14.33	46.6	52.3	1.1
0.5	0.5	0	0	0.7	20.63	12.64	45.5	53.4	1.1	28.63	14.27	47.6	51.2	1.2	32.96	15.18	46.1	52.3	1.6	36.56	16.33	46.4	52.0	1.6
0.5	0.5	0	0.2	0	19.70	11.62	42.6	56.0	1.4	26.61	12.34	44.8	54.6	0.6	30.37	12.74	45.4	53.7	0.9	33.61	12.99	45.0	53.7	1.3
0.5	0.5	0	0.2	0.4	20.13	12.48	46.6	52.4	1.0	27.26	13.78	47.9	51.0	1.1	31.34	14.11	46.7	52.2	1.1	34.70	15.43	46.6	52.7	0.7
0.5	0.5	0	0.2	0.7	20.77	13.22	46.9	51.8	1.3	28.51	14.96	48.8	49.9	1.3	32.89	16.13	47.1	51.8	1.1	36.43	17.60	46.3	52.2	1.5
0.5	0.5	0	0.7	0	18.52	12.37	44.3	55.1	0.6	24.31	13.83	44.5	54.9	0.6	27.45	14.20	43.4	56.0	0.6	30.02	14.95	42.6	56.6	0.8
0.5	0.5	0	0.7	0.4	18.89	13.70	48.2	50.9	0.9	25.60	16.39	47.7	51.8	0.5	28.12	16.09	43.9	55.4	0.7	30.99	16.94	43.7	55.6	0.7
0.5	0.5	0	0.7	0.7	19.66	14.71	49.0	50.3	0.7	25.60	16.39	47.7	51.8	0.5	29.42	17.69	45.0	54.5	0.5	32.33	18.58	46.1	53.7	0.2
0.5	0.5	0.2	0	0	19.77	11.69	42.1	56.5	1.4	26.55	12.14	44.4	54.4	1.2	30.53	12.61	45.1	54.0	0.9	33.68	13.15	45.3	54.1	0.6
0.5	0.5	0.2	0	0.4	20.07	12.30	44.6	54.4	1.0	27.50	13.69	45.5	53.4	1.1	31.60	14.05	44.8	53.6	1.6	35.10	14.86	45.5	53.1	1.4
0.5	0.5	0.2	0	0.7	21.27	13.40	45.4	53.1	1.5	28.91	15.58	46.5	51.9	1.6	33.11	16.35	44.5	53.6	1.9	36.79	18.04	43.7	54.4	1.9
0.5	0.5	0.2	0.2	0	19.64	12.01	42.6	56.3	1.1	25.68	12.65	45.5	53.8	0.7	30.09	13.00	45.4	54.1	0.5	33.22	13.44	45.4	53.9	0.7
0.5	0.5	0.2	0.2	0.4	20.26	13.00	45.9	53.4	0.7	27.23	14.52	46.5	52.8	0.7	31.38	15.16	46.4	52.7	0.9	34.57	15.91	47.1	52.4	0.5
0.5	0.5	0.2	0.2	0.7	21.09	13.92	44.7	53.7	1.6	28.78	16.05	48.3	50.4	1.3	32.96	16.89	46.0	52.7	1.3	36.35	17.81	46.0	52.5	1.5

					$\gamma = 1$					$\gamma = 2$					$\gamma = 3$					$\gamma = 4$				
					$\alpha = 0.190768$					$\alpha = 0.190768$					$\alpha = 0.190768$					$\alpha = 0.190768$				
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0.5	0.5	0.2	0.7	0	18.70	13.14	44.1	55.0	0.9	24.21	14.38	44.6	54.6	0.8	27.41	14.82	43.1	55.9	1.0	29.90	15.49	42.2	56.9	0.9
0.5	0.5	0.2	0.7	0.4	19.29	14.64	46.4	52.7	0.9	24.76	16.24	46.0	53.2	0.8	28.30	17.15	43.4	55.6	1.0	31.24	18.13	43.5	55.6	0.9
0.5	0.5	0.2	0.7	0.7	20.48	16.26	48.0	50.9	1.1	26.33	17.96	47.6	51.6	0.8	30.01	19.15	44.6	54.2	1.2	32.98	19.99	44.9	54.2	0.9
0.5	0.5	0.7	0	0	18.86	12.51	41.5	57.2	1.3	24.87	13.74	44.3	54.4	1.3	28.33	14.64	44.5	54.3	1.2	31.03	14.99	45.8	53.2	1.0
0.5	0.5	0.7	0	0.4	19.36	13.30	42.0	57.5	0.5	26.03	15.40	45.4	53.6	1.0	29.42	16.19	45.9	53.0	1.1	32.39	16.97	45.8	53.2	1.0
0.5	0.5	0.7	0	0.7	20.44	14.31	41.1	57.9	1.0	27.29	16.37	43.8	55.0	1.2	31.07	17.50	43.6	55.1	1.3	34.15	18.39	43.3	55.0	1.7
0.5	0.5	0.7	0.2	0	18.82	12.99	42.8	56.6	0.6	24.88	14.33	44.4	54.6	1.0	28.21	15.18	44.5	54.5	1.0	30.81	15.66	45.9	53.5	0.6
0.5	0.5	0.7	0.2	0.4	19.63	14.34	42.1	57.2	0.7	26.00	16.47	45.7	53.0	1.3	29.56	17.43	46.0	52.8	1.2	32.46	18.31	46.9	52.2	0.9
0.5	0.5	0.7	0.2	0.7	20.99	15.77	41.3	58.1	0.6	27.88	18.13	43.5	54.9	1.6	31.65	19.52	43.7	54.6	1.7	34.61	20.40	44.4	54.1	1.5
0.5	0.5	0.7	0.7	0	18.81	16.02	43.6	55.4	1.0	23.68	17.28	43.3	55.8	0.9	26.90	18.61	43.5	55.6	0.9	29.24	19.43	43.9	55.3	0.8
0.5	0.5	0.7	0.7	0.4	20.74	19.81	43.9	55.2	0.9	25.63	21.36	45.2	53.4	1.4	28.52	21.74	44.8	54.0	1.2	31.52	23.38	44.7	54.2	1.1
0.5	0.5	0.7	0.7	0.7	23.31	22.87	43.1	54.8	2.1	28.35	23.98	44.5	53.5	2.0	31.77	25.05	44.2	53.2	2.6	34.50	25.72	45.0	53.0	2.0
0.5	3	0	0	0	5.63	2.40	6.4	92.3	1.3	7.14	2.40	3.1	96.1	0.8	8.28	2.38	2.2	97.1	0.7	9.33	2.36	1.7	97.9	0.4
0.5	3	0	0	0.4	5.60	2.42	7.1	91.9	1.0	7.17	2.41	3.1	96.2	0.7	8.27	2.38	2.2	97.2	0.6	9.32	2.37	1.8	98.0	0.2
0.5	3	0	0	0.7	5.62	2.45	7.1	92.2	0.7	7.16	2.41	3.4	95.9	0.7	8.27	2.37	2.4	97.1	0.5	9.31	2.35	1.9	97.8	0.3
0.5	3	0	0.2	0	5.62	2.45	6.4	92.2	1.4	7.16	2.46	3.1	95.7	1.2	8.32	2.45	2.2	96.9	0.9	9.37	2.45	1.7	97.7	0.6
0.5	3	0	0.2	0.4	5.62	2.47	7.4	91.9	0.7	7.20	2.46	3.2	96.2	0.6	8.30	2.43	2.3	97.2	0.5	9.35	2.42	1.9	97.9	0.2
0.5	3	0	0.2	0.7	5.62	2.48	7.1	92.0	0.9	7.22	2.49	3.2	95.9	0.9	8.32	2.46	2.3	97.1	0.6	9.38	2.45	1.8	97.8	0.4
0.5	3	0	0.7	0	5.84	3.07	6.3	92.7	1.0	7.48	3.25	4.0	95.6	0.4	8.65	3.28	2.7	97.0	0.3	9.75	3.34	2.0	97.8	0.2
0.5	3	0	0.7	0.4	5.99	3.08	7.3	91.2	1.5	7.66	3.25	2.9	96.3	0.8	8.81	3.27	2.1	97.4	0.5	9.90	3.28	1.5	98.1	0.4
0.5	3	0	0.7	0.7	6.03	3.15	6.9	91.9	1.2	7.64	3.25	2.8	96.6	0.6	8.79	3.26	1.9	97.7	0.4	9.88	3.28	1.3	98.3	0.4
0.5	3	0.2	0	0	5.61	2.40	6.7	92.1	1.2	7.11	2.39	3.8	95.2	1.0	8.27	2.37	2.3	97.1	0.6	9.32	2.35	1.9	97.7	0.4
0.5	3	0.2	0	0.4	5.59	2.41	7.2	92.1	0.7	7.14	2.42	3.6	95.7	0.7	8.26	2.38	2.4	97.0	0.6	9.30	2.38	1.9	97.7	0.4
0.5	3	0.2	0	0.7	5.62	2.45	7.0	92.0	1.0	7.16	2.41	3.4	95.8	0.8	8.27	2.36	2.4	97.0	0.6	9.31	2.35	1.9	97.8	0.3
0.5	3	0.2	0.2	0	5.61	2.46	6.7	91.9	1.4	7.13	2.46	3.8	94.8	1.4	8.31	2.45	2.5	96.6	0.9	9.36	2.44	1.8	97.4	0.8
0.5	3	0.2	0.2	0.4	5.61	2.47	7.3	92.0	0.7	7.19	2.47	3.6	95.7	0.7	8.29	2.43	2.5	96.9	0.6	9.34	2.43	1.9	97.7	0.4
0.5	3	0.2	0.2	0.7	5.62	2.50	7.1	91.9	1.0	7.22	2.49	3.3	96.0	0.7	8.32	2.46	2.2	97.2	0.6	9.37	2.46	1.7	98.0	0.3

					$\gamma = 1$ $\alpha = 0.190768$					$\gamma = 2$ $\alpha = 0.190768$					$\gamma = 3$ $\alpha = 0.190768$					$\gamma = 4$ $\alpha = 0.190768$				
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
0.5	3	0.2	0.7	0	5.83	3.08	6.5	92.4	1.1	7.44	3.23	4.8	94.7	0.5	8.62	3.26	3.3	96.4	0.3	9.72	3.31	2.7	97.2	0.1
0.5	3	0.2	0.7	0.4	5.99	3.09	7.5	91.1	1.4	7.64	3.25	3.4	95.8	0.8	8.81	3.28	2.1	97.3	0.6	9.89	3.29	1.6	98.0	0.4
0.5	3	0.2	0.7	0.7	6.03	3.16	7.0	92.0	1.0	7.63	3.23	2.8	96.7	0.5	8.79	3.25	2.0	97.5	0.5	9.88	3.27	1.3	98.3	0.4
0.5	3	0.7	0	0	5.52	2.41	8.4	89.8	1.8	7.03	2.40	6.8	91.9	1.3	8.15	2.37	5.9	93.4	0.7	9.19	2.36	5.0	94.4	0.6
0.5	3	0.7	0	0.4	5.56	2.42	7.8	89.9	2.3	7.09	2.40	5.7	92.7	1.6	8.21	2.38	4.5	93.8	1.7	9.24	2.38	4.0	94.5	1.5
0.5	3	0.7	0	0.7	5.57	2.46	8.0	90.2	1.8	7.08	2.42	5.5	93.2	1.3	8.19	2.38	4.4	94.2	1.4	9.23	2.38	4.0	94.8	1.2
0.5	3	0.7	0.2	0	5.54	2.45	8.1	90.0	1.9	7.02	2.46	6.8	91.4	1.8	8.17	2.45	5.9	93.0	1.1	9.22	2.45	5.0	93.9	1.1
0.5	3	0.7	0.2	0.4	5.59	2.48	7.7	90.7	1.6	7.14	2.46	5.7	92.8	1.5	8.25	2.44	4.5	93.8	1.7	9.29	2.44	3.9	94.7	1.4
0.5	3	0.7	0.2	0.7	5.59	2.51	7.8	90.5	1.7	7.14	2.51	5.4	93.3	1.3	8.26	2.48	4.3	94.3	1.4	9.30	2.47	3.9	94.9	1.2
0.5	3	0.7	0.7	0	5.75	3.08	8.4	89.6	2.0	7.32	3.19	7.2	91.5	1.3	8.46	3.22	6.1	93.0	0.9	9.56	3.27	5.7	93.5	0.8
0.5	3	0.7	0.7	0.4	5.96	3.12	7.9	90.5	1.6	7.56	3.25	5.3	93.3	1.4	8.72	3.25	4.3	94.8	0.9	9.79	3.27	3.7	95.6	0.7
0.5	3	0.7	0.7	0.7	6.02	3.20	7.3	91.0	1.7	7.58	3.27	4.5	94.3	1.2	8.75	3.26	3.1	95.9	1.0	9.82	3.29	3.0	96.2	0.8
1	1	0	0	0	11.09	5.75	41.9	55.1	3.0	14.46	5.95	40.1	56.6	3.3	16.62	6.12	38.0	58.7	3.3	18.36	6.25	38.5	58.6	2.9
1	1	0	0	0.4	10.90	5.83	43.4	54.6	2.0	14.64	6.25	41.4	56.0	2.6	16.88	6.44	39.9	58.4	1.7	18.59	6.54	38.2	59.8	2.0
1	1	0	0	0.7	11.08	6.07	44.0	53.9	2.1	14.81	6.46	42.0	55.1	2.9	17.13	6.69	39.5	58.1	2.4	18.82	6.75	38.2	59.5	2.3
1	1	0	0.2	0	11.06	5.92	42.2	54.9	2.9	14.45	6.06	41.1	56.2	2.7	16.62	6.28	39.8	57.1	3.1	18.35	6.40	39.6	57.4	3.0
1	1	0	0.2	0.4	10.96	5.96	44.9	53.5	1.6	14.70	6.55	42.9	54.5	2.6	16.99	6.77	41.2	56.9	1.9	18.64	6.88	39.3	58.9	1.8
1	1	0	0.2	0.7	11.17	6.29	44.6	53.1	2.3	14.90	6.85	42.9	54.4	2.7	17.20	6.97	40.6	56.8	2.6	18.86	7.02	39.4	58.5	2.1
1	1	0	0.7	0	11.05	6.75	44.5	53.4	2.1	14.30	7.07	45.0	52.8	2.2	16.44	7.26	43.4	54.3	2.3	17.97	7.37	44.3	54.1	1.6
1	1	0	0.7	0.4	11.06	6.98	49.5	48.4	2.1	14.44	7.60	46.4	51.6	2.0	16.66	7.95	45.7	52.5	1.8	18.31	8.17	45.1	53.6	1.3
1	1	0	0.7	0.7	11.51	7.47	51.0	46.8	2.2	14.93	8.12	48.6	49.2	2.2	17.09	8.38	47.5	50.8	1.7	18.60	8.54	45.9	52.2	1.9
1	1	0.2	0	0	11.04	5.86	41.4	56.1	2.5	14.39	6.02	40.8	56.3	2.9	16.51	6.18	38.5	57.9	3.6	18.26	6.29	38.7	57.9	3.4
1	1	0.2	0	0.4	10.94	5.89	42.5	55.7	1.8	14.63	6.35	41.0	56.0	3.0	16.80	6.48	39.4	58.6	2.0	18.50	6.67	38.5	59.9	1.6
1	1	0.2	0	0.7	11.18	6.22	42.6	54.9	2.5	14.87	6.64	41.6	55.4	3.0	17.12	6.90	39.4	57.9	2.7	18.83	7.00	37.7	60.0	2.3
1	1	0.2	0.2	0	11.02	5.98	41.9	55.4	2.7	14.36	6.15	41.4	55.7	2.9	16.50	6.37	39.7	56.8	3.5	18.25	6.52	39.5	57.4	3.1
1	1	0.2	0.2	0.4	10.99	6.11	43.6	54.4	2.0	14.70	6.64	42.3	55.2	2.5	16.93	6.91	40.6	57.3	2.1	18.57	7.05	39.4	58.6	2.0
1	1	0.2	0.2	0.7	11.31	6.53	43.5	54.7	1.8	14.98	7.12	42.9	54.6	2.5	17.24	7.24	39.8	57.0	3.2	18.92	7.33	38.8	59.2	2.0

					$\gamma = 1$					$\gamma = 2$					$\gamma = 3$					$\gamma = 4$				
					$\alpha = 0.190768$					$\alpha = 0.190768$					$\alpha = 0.190768$					$\alpha = 0.190768$				
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
1	1	0.2	0.7	0	11.07	6.84	44.8	53.5	1.7	14.26	7.24	44.6	53.5	1.9	16.33	7.42	44.1	53.8	2.1	17.88	7.49	44.6	53.7	1.7
1	1	0.2	0.7	0.4	11.24	7.35	48.3	49.3	2.4	14.56	7.91	44.6	52.9	2.5	16.71	8.26	45.2	53.1	1.7	18.34	8.51	45.0	53.6	1.4
1	1	0.2	0.7	0.7	11.76	7.98	49.7	47.8	2.5	15.13	8.56	47.9	49.9	2.2	17.29	8.88	47.3	50.7	2.0	18.87	9.09	45.7	52.3	2.0
1	1	0.7	0	0	10.76	6.12	41.9	56.5	1.6	13.86	6.53	42.4	55.7	1.9	15.80	6.78	40.5	58.0	1.5	17.45	7.06	40.2	58.0	1.8
1	1	0.7	0	0.4	10.86	6.30	39.5	57.8	2.7	14.20	7.02	41.0	57.0	2.0	16.23	7.22	39.2	58.3	2.5	17.94	7.45	39.1	59.1	1.8
1	1	0.7	0	0.7	11.14	6.68	38.4	58.2	3.4	14.55	7.39	39.1	58.7	2.2	16.56	7.64	38.4	58.9	2.7	18.27	7.79	37.1	60.0	2.9
1	1	0.7	0.2	0	10.86	6.40	42.9	56.0	1.1	13.88	6.74	43.6	54.5	1.9	15.85	7.08	41.1	57.0	1.9	17.51	7.41	41.2	56.9	1.9
1	1	0.7	0.2	0.4	11.11	6.74	39.7	57.7	2.6	14.33	7.41	41.1	56.9	2.0	16.45	7.80	39.9	57.7	2.4	18.08	7.92	39.8	58.1	2.1
1	1	0.7	0.2	0.7	11.38	7.06	39.8	57.2	3.0	14.81	7.90	39.2	58.0	2.8	16.94	8.22	38.9	57.6	3.5	18.55	8.35	38.1	59.2	2.7
1	1	0.7	0.7	0	11.19	7.89	44.4	53.4	2.2	14.08	8.38	44.6	53.8	1.6	15.99	8.65	43.9	54.6	1.5	17.47	8.83	44.6	53.6	1.8
1	1	0.7	0.7	0.4	12.11	9.72	43.9	52.9	3.2	15.11	10.33	44.8	53.2	2.0	17.19	10.90	45.3	52.7	2.0	18.91	11.45	44.3	53.7	2.0
1	1	0.7	0.7	0.7	13.12	11.01	45.0	52.1	2.9	16.45	11.99	44.9	52.0	3.1	18.59	12.68	44.3	52.4	3.3	20.28	13.14	43.7	53.6	2.7
3	3	0	0	0	4.42	2.05	43.7	48.9	7.4	5.89	2.04	41.6	50.4	8.0	7.04	2.05	41.3	50.4	8.3	8.08	2.04	41.2	50.5	8.3
3	3	0	0	0.4	4.44	2.06	43.9	48.0	8.1	5.94	2.06	43.2	47.7	9.1	7.08	2.05	42.0	48.4	9.6	8.11	2.04	42.0	48.4	9.6
3	3	0	0	0.7	4.45	2.09	45.1	47.1	7.8	5.97	2.12	43.2	47.9	8.9	7.10	2.09	42.3	48.3	9.4	8.14	2.09	42.2	48.4	9.4
3	3	0	0.2	0	4.43	2.07	43.1	48.3	8.6	5.89	2.06	41.0	49.8	9.2	7.04	2.06	41.0	49.9	9.1	8.08	2.05	41.0	49.9	9.1
3	3	0	0.2	0.4	4.44	2.07	44.2	47.9	7.9	5.93	2.08	43.5	48.0	8.5	7.06	2.07	42.4	48.6	9.0	8.09	2.07	42.4	48.6	9.0
3	3	0	0.2	0.7	4.44	2.09	44.9	47.7	7.4	5.96	2.13	43.6	48.4	8.0	7.09	2.10	42.8	49.0	8.2	8.13	2.11	42.7	48.9	8.4
3	3	0	0.7	0	4.40	2.18	42.9	48.4	8.7	5.89	2.24	41.8	48.4	9.8	7.03	2.25	41.7	48.2	10.1	8.07	2.24	42.1	48.2	9.7
3	3	0	0.7	0.4	4.48	2.15	47.5	45.5	7.0	6.00	2.25	46.7	45.2	8.1	7.13	2.25	46.4	45.8	7.8	8.18	2.26	46.5	45.4	8.1
3	3	0	0.7	0.7	4.53	2.25	48.4	44.0	7.6	6.06	2.33	47.2	43.8	9.0	7.19	2.32	46.5	44.3	9.2	8.24	2.35	46.7	43.9	9.4
3	3	0.2	0	0	4.42	2.06	43.4	49.1	7.5	5.88	2.04	41.6	51.0	7.4	7.02	2.06	41.5	50.7	7.8	8.07	2.06	41.3	50.9	7.8
3	3	0.2	0	0.4	4.44	2.06	43.9	48.6	7.5	5.93	2.08	43.0	48.0	9.0	7.07	2.07	42.1	48.5	9.4	8.09	2.07	42.1	48.5	9.4
3	3	0.2	0	0.7	4.44	2.11	44.9	47.6	7.5	5.97	2.13	43.0	47.9	9.1	7.10	2.10	42.0	48.4	9.6	8.13	2.10	41.9	48.5	9.6
3	3	0.2	0.2	0	4.41	2.06	43.0	48.8	8.2	5.88	2.07	41.5	50.2	8.3	7.02	2.09	41.6	50.2	8.2	8.06	2.07	41.6	50.2	8.2
3	3	0.2	0.2	0.4	4.44	2.09	44.0	48.4	7.6	5.93	2.10	43.1	48.0	8.9	7.06	2.08	42.0	48.7	9.3	8.09	2.08	42.0	48.6	9.4
3	3	0.2	0.2	0.7	4.46	2.13	44.5	47.9	7.6	5.96	2.15	43.5	48.1	8.4	7.09	2.13	42.5	48.7	8.8	8.13	2.13	42.5	48.6	8.9

					γ = 1 α = 0.190768					γ = 2 α = 0.190768					γ = 3 α = 0.190768					γ = 4 α = 0.190768				
δ_x	δ_y	ϕ_x	ϕ_y	ρ_{xy}	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*	*ARL*	*SRL*	*X%*	*Y%*	*XY%*
3	3	0.2	0.7	0	4.40	2.18	42.6	49.1	8.3	5.89	2.24	42.0	48.2	9.8	7.02	2.26	42.0	48.1	9.9	8.07	2.24	42.3	48.1	9.6
3	3	0.2	0.7	0.4	4.50	2.18	46.8	46.3	6.9	6.00	2.28	46.9	45.7	7.4	7.13	2.29	46.6	45.9	7.5	8.18	2.30	46.8	45.5	7.7
3	3	0.2	0.7	0.7	4.55	2.29	47.9	44.3	7.8	6.07	2.36	46.4	44.5	9.1	7.21	2.36	45.8	44.7	9.5	8.27	2.40	46.1	44.2	9.7
3	3	0.7	0	0	4.45	2.11	41.2	50.2	8.6	5.87	2.13	41.6	50.4	8.0	7.01	2.13	40.9	51.0	8.1	8.04	2.12	40.8	51.3	7.9
3	3	0.7	0	0.4	4.48	2.17	42.6	50.4	7.0	5.93	2.22	42.7	50.1	7.2	7.06	2.22	42.1	50.5	7.4	8.08	2.22	42.3	50.5	7.2
3	3	0.7	0	0.7	4.53	2.20	41.6	50.1	8.3	5.97	2.27	41.3	50.4	8.3	7.10	2.26	40.6	50.9	8.5	8.13	2.25	40.8	50.9	8.3
3	3	0.7	0.2	0	4.45	2.12	41.3	50.2	8.5	5.87	2.16	41.5	50.8	7.7	7.01	2.17	41.3	50.6	8.1	8.05	2.15	41.3	50.8	7.9
3	3	0.7	0.2	0.4	4.49	2.22	42.3	50.5	7.2	5.96	2.25	42.5	50.1	7.4	7.08	2.24	41.7	50.8	7.5	8.11	2.24	41.9	50.8	7.3
3	3	0.7	0.2	0.7	4.55	2.25	41.6	50.4	8.0	6.01	2.32	41.3	49.9	8.8	7.14	2.31	40.8	50.3	8.9	8.18	2.31	40.8	50.4	8.8
3	3	0.7	0.7	0	4.46	2.29	40.8	50.9	8.3	5.88	2.37	41.9	50.2	7.9	7.01	2.38	41.2	50.5	8.3	8.04	2.37	41.6	50.6	7.8
3	3	0.7	0.7	0.4	4.66	2.52	44.2	47.4	8.4	6.15	2.63	44.4	47.7	7.9	7.29	2.66	44.6	47.5	7.9	8.32	2.66	45.0	47.1	7.9
3	3	0.7	0.7	0.7	4.77	2.66	45.8	44.8	9.4	6.27	2.81	44.5	46.1	9.4	7.44	2.85	44.1	46.4	9.5	8.51	2.88	44.1	46.2	9.7

Chapter 5

Applications & Extensions

To demonstrate the application of the proposed NN-based control scheme in practice, two illustrative examples and a case study are devised.

5.1 Illustrative Examples

The two illustrative examples are devised to demonstrate how to apply the NN-based control scheme in practice. In these two examples, 300 pairs of bivariate autocorrelated observations with different combinations of parameter values are generated. The combinations of parameter values and the stages of the processes are listed in Table 5.1.

Table 5.1 Illustrative examples with 300 input pairs of (X,Y) observations

Stage	Observation No.	Corresponding Window No.	Case I					Case II				
			δ_x	δ_y	ϕ_x	ϕ_y	ρ	δ_x	δ_y	ϕ_x	ϕ_y	ρ
I	1 - 100	1-51	0	0	0	0	0	0	0	0	0	0
II	101-150	52-101	0.5	0	0	0.2	0.4	0.5	0.5	0	0.2	0.4
III	151-200	102-151	0	0.5	0.2	0	0.4	0.5	1	0.7	0	0.4
IV	201-250	152-201	0.5	0	0	0.7	0.7	1	0.5	0.2	0.7	0.7
V	251-300	202-251	0	0.5	0.7	0	0.7	1	1	0.7	0.7	0.7

Figures 5.1 and 5.3 represent the realizations of the two cases for the variable X and the variable Y. The NN-based control scheme is applied to the simulated data sets. Figures 5.2 and 5.4 are the neural network output charts of the devised cases. In Figures 5.2 and 5.4, X represents the neural network output of the mean of the variable X and Y represents the neural network output of the mean of the variable Y. In Figures 5.2 and 5.4, the neural network output falls between 0.00 and 1.00 with 0.00 indicating no shift and 1.00 indicating a large shift of 3σ. The neural network

output can be interpreted as the estimate of the shift magnitude based on the window of data presented to the NN-based neural network.

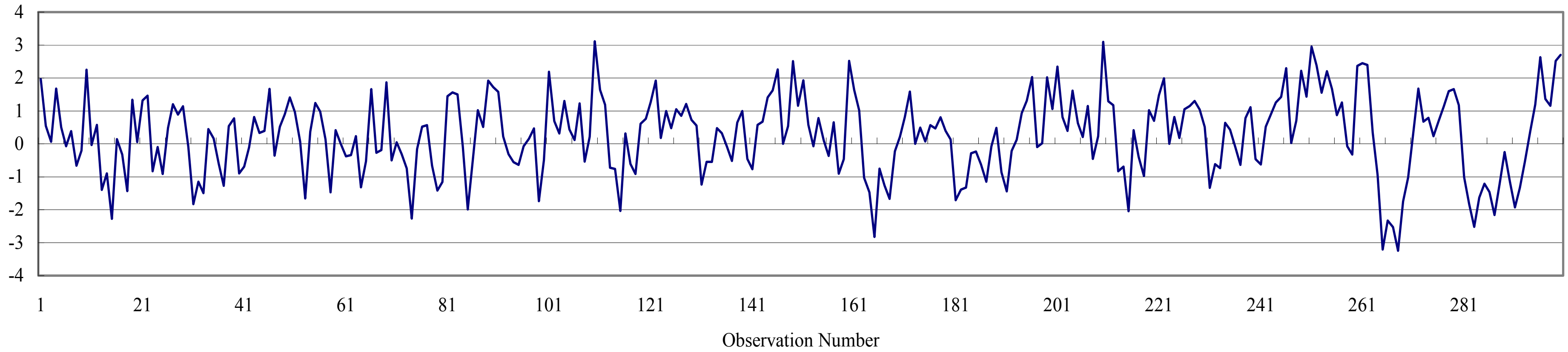
As discussed in Chapter 4, the cut-off threshold is tuned to 0.190768 if the in-control ARL of the no-autocorrelation and no-correlation process is tuned to 185.4. From Figure 5.2, it is observed that the first output value which is larger than the cut-off threshold is detected at window No. 58 on the variable Y. Note that window No. 58 includes the observations from No. 58 to No. 107. If the output on window No. 58 is treated as a shift, this is a false alarm since in Table 5.1 it is clear that a small shift starts from observation No. 101 on variable X. However, from observation No. 101 small autocorrelation is present on variable Y and moderate correlation between variable X and variable Y is also present, so the change on variable Y may be caused by the positive moderate correlation or the small autocorrelation change. When the outputs on variable Y that come after the output at window No. 58 are observed, it is found that these outputs have a decreasing trend. The next output value that is larger than 0.190768 is the output at window No. 78 on the variable X. The outputs following the output at window No. 78 have an increasing trend; a true shift is detected at this time.

From the above observations, it can be inferred that when there is a mean shift on a specific variable, the neural network outputs should have an increasing trend. It should not arbitrarily be concluded that a single output, larger than the threshold is a shift. More observations are needed to come to a more reliable conclusion.

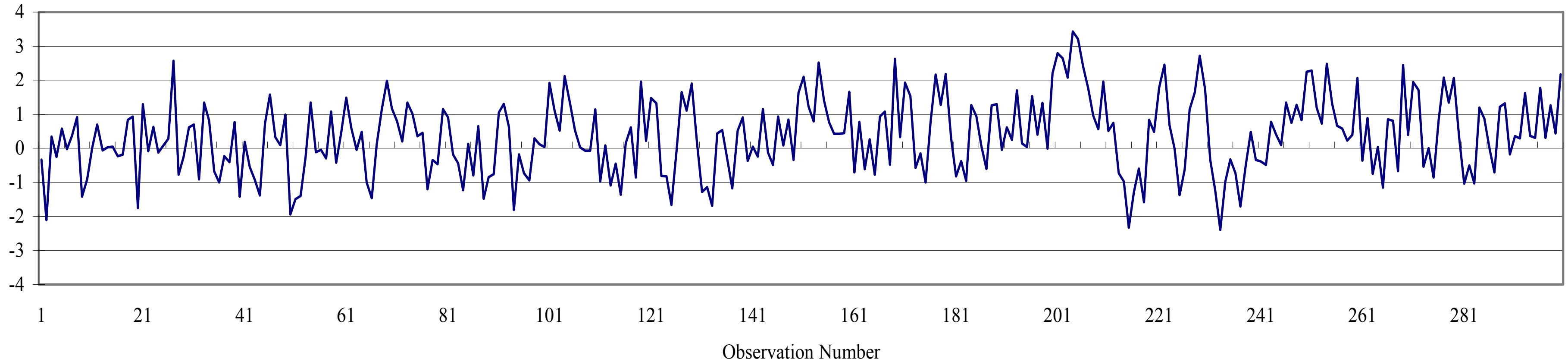
Furthermore, Figure 5.2 can be analyzed from a macro view. From Figure 5.2, it is clear that there is an increasing trend on the neural network output of variable X from window No. 52 to window No. 101 while the output of the variable Y keeps normal. From this observation, it can be inferred that there is a shift on variable X between observation No. 101 to observation No. 150. From window No. 102 to window No. 151, the neural network output of Y has an obvious increasing trend which implies a mean shift on variable Y. The output of variable X is decreasing during this period, which implies that the mean of variable X is decreasing from the mean of X of the last stage. From window No. 152 to window No. 201, an increasing trend is present on the output of the variable X and a decreasing trend is present on the output of the variable Y. From this observation in comparison with the previous stage, it can be inferred that the mean of X increases and the mean of Y decreases. For the fifth stage of Case I, Figure 5.2 shows that the output of X has a decreasing trend and the output of Y has an increasing trend. All the above observations are consistent with the change of raw data.

The interpretation of Case II is similar to the interpretation of Case I. It is obvious that the underlying shifts on the variables can be easily classified from the neural network output chart. Consequently, it is easy to point out the variable responsible for the shift. This proves that the proposed NN-based control scheme is effective in detecting and identifying process mean shift.

In the next subsection, a case study will be developed to show the power of the proposed NN-based control scheme in practice.

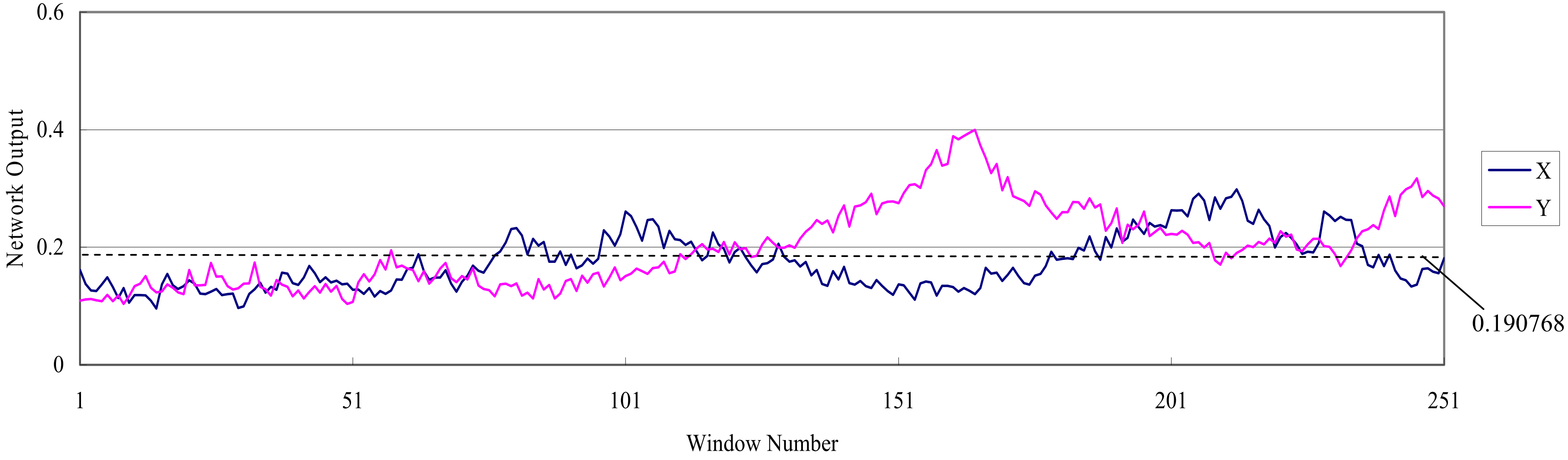


(a) X values



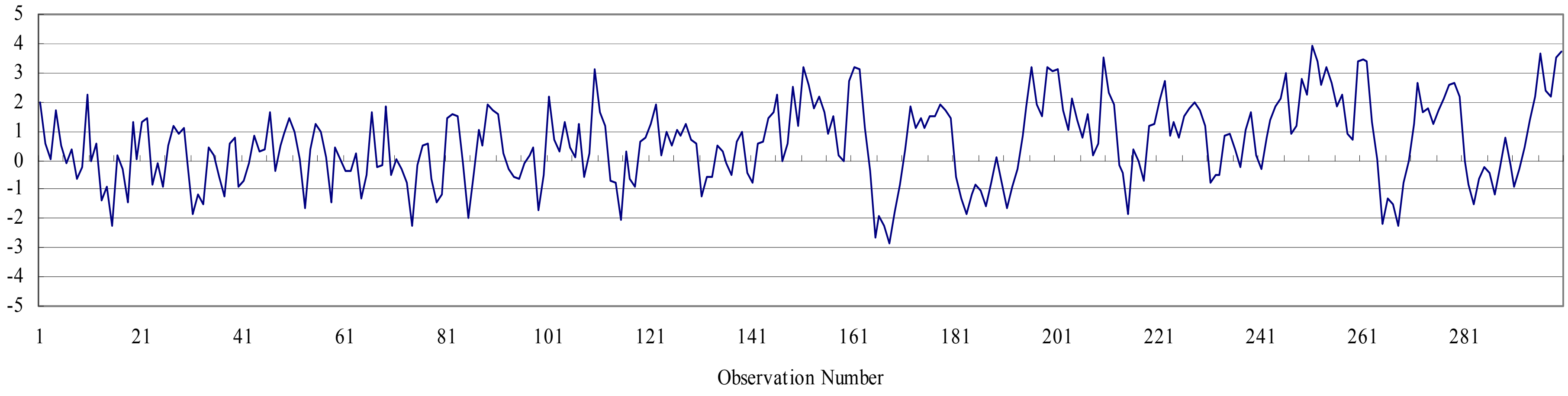
(b) Y values

Figure 5.1 The raw data of case I

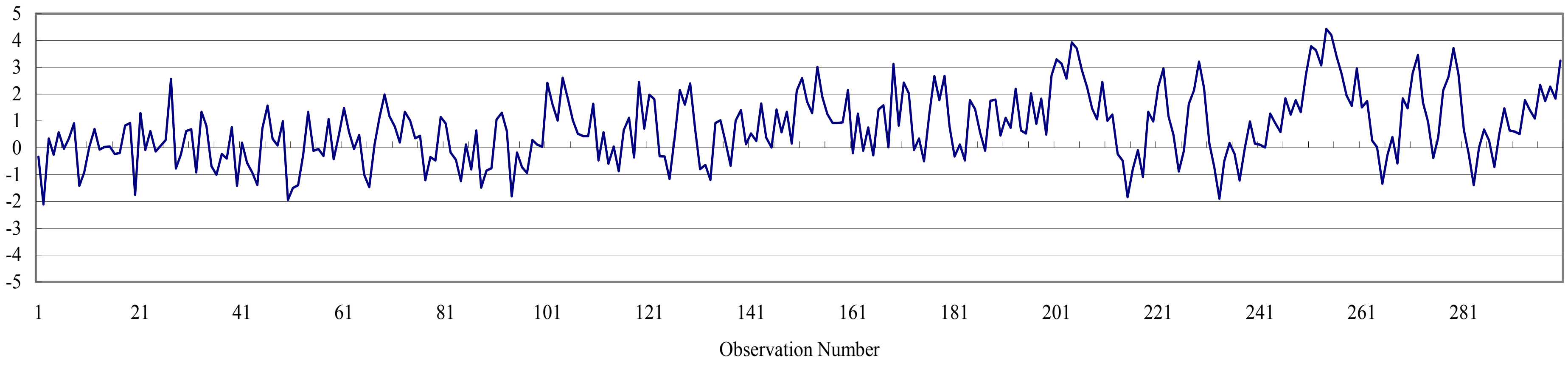


Note: X represents the neural network output of the mean of the variable X and Y represents the neural network output of the mean of the variable Y

Figure 5.2 The neural network output chart for Case I

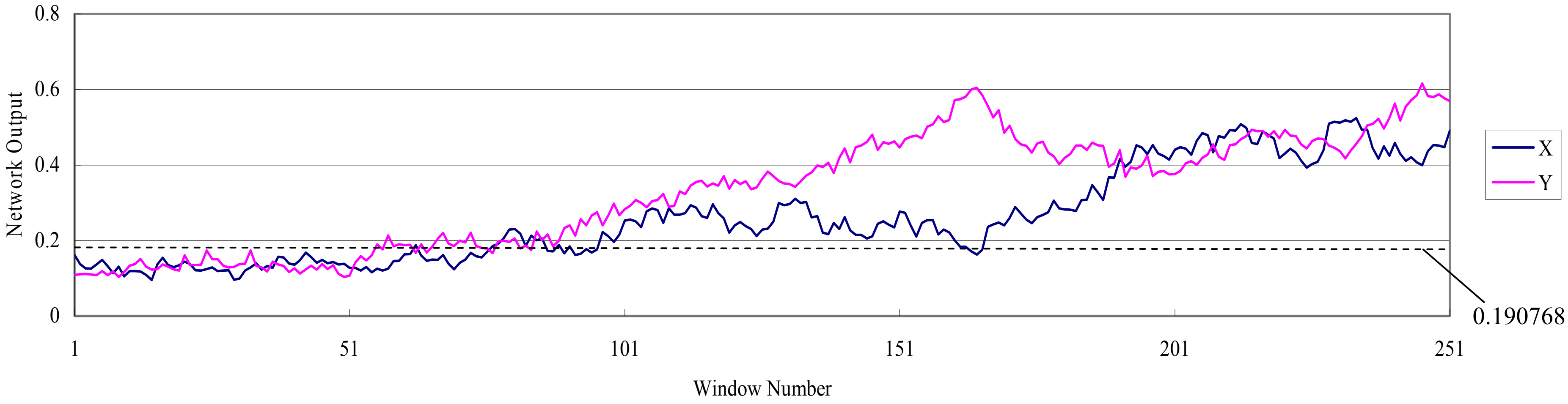


(a) X values



(b) Y values

Figure 5.3 The raw data of case II



Note: X represents the neural network output of the mean of the variable X and Y represents the neural network output of the mean of the variable Y

Figure 5.4 The neural network output chart for Case II

5.2 Case Study

Statistical process control can be applied in a wide range of organizations and applications. For example, it can be used to improve product quality in the manufacturing industry. Also, it can be used to control service time in the service industry to improve service quality. To increase sales in the retail sector, statistical process control can also be used to monitor the order quantity by the supplier.

5.2.1 Background

In the retail sector, it is important to consider the order quantity, especially for perishable products. Since perishable products decay rapidly, the order quantity should not be more than enough from the retailers' side. From the suppliers' side, the more the vendors order, the more the suppliers earn. Hence, this points out a need for order quantity control. The Campus-Bread-Control case will be considered, a perishable product order quantity.

On a university campus, there are two vendors, named V1 and V2, which sell the same kind of bread. Since bread is perishable, the vendors typically don't hold any inventory at the end of the day. Consequently, V1 and V2 need to order bread from the supplier on a daily basis. Because they sell the same brand of bread, the supplier of these two vendors is the same. The quantity ordered by V1 is defined as Q1 while the order quantity issued by V2 is defined as Q2. It is obvious that variable Q1 and variable Q2 are somehow correlated due to the shared market and the shared supplier. Moreover, the order quantity issued by each vendor is serially correlated, based on the historical data, so Q1 and Q2 follow a vector autoregressive model. To facilitate the understanding of this process, a schematic diagram is given in Figure 5.5.

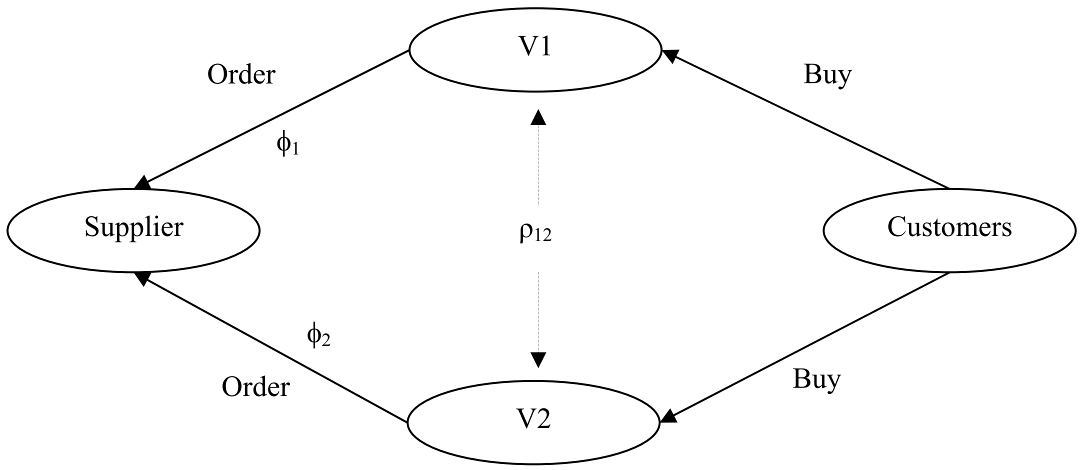


Figure 5.5 A schematic diagram of the Campus-Bread-Control case

In this Campus-Bread-Control case, the supplier has to monitor the order quantities Q1 and Q2. With the order quantity control, the supplier may find the change in the mean of order quantity. Using this information, it can trace the reason why the order quantity changes. Furthermore, the supplier could propose policies which are helpful to the sales based on the causes of changes.

5.2.2 Data Pre-processing

The order quantities* issued by both vendors were collected from January 1st, 2006 to March 31st, 2006. In total, there are 90 pairs of observations. Control schemes should be applied to monitor the order quantities. As shown in Chapter 4, the proposed NN-based control scheme is an effective control scheme for detecting and identifying process mean shift in bivariate autocorrelated process. So the proposed NN-based control scheme could be applied to detect the changes in the mean of order quantities.

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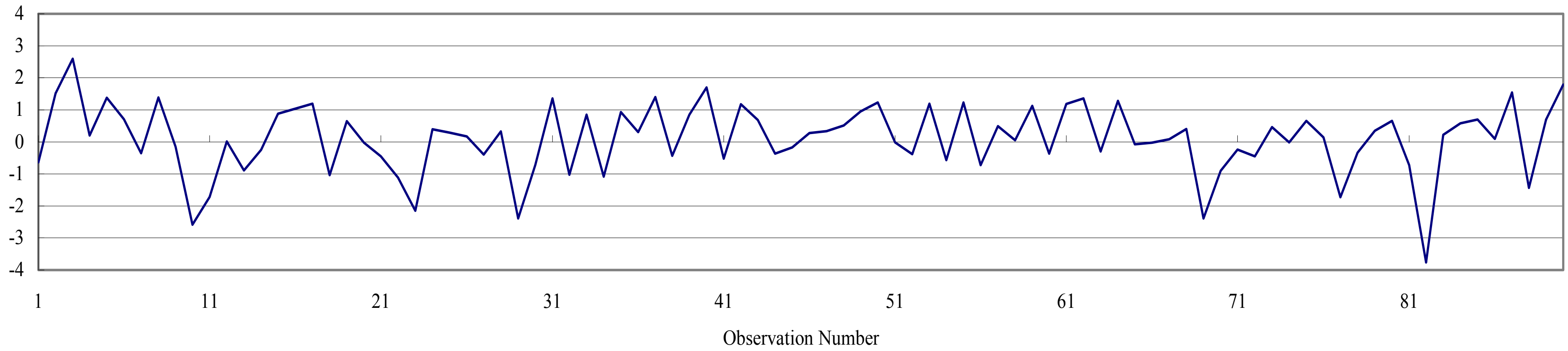
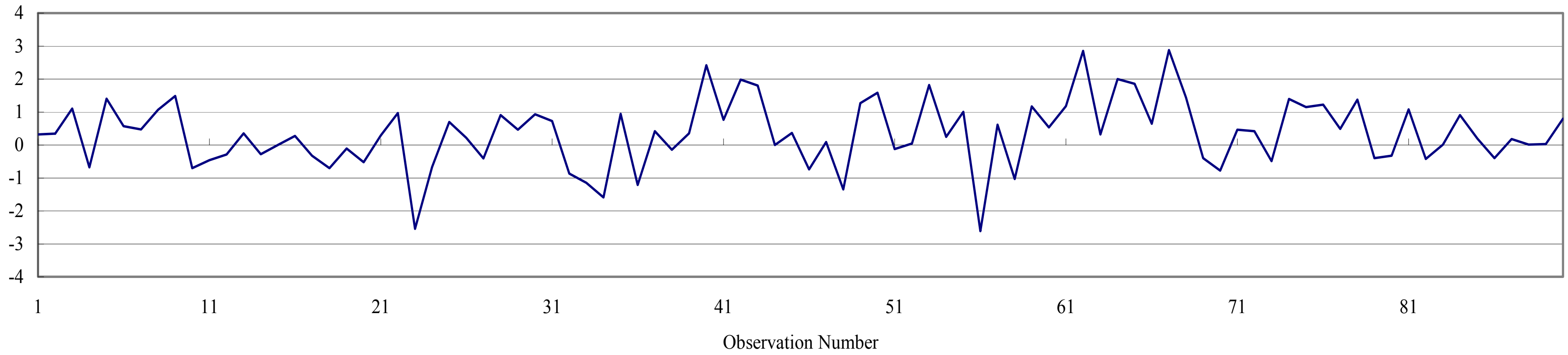
* Q1 and Q2 are simulated with moderate correlation and low autocorrelation. The correlation between both Q1 and Q2 is set to 0.4 while the autocorrelation of both variables is set to 0.2 separately. From January 1st, 2006 to February 28th, 2006, the means of both order quantities are set to 100. From March 1st, 2006, a promotion is held by V2. V2 offers two free bottles of drink with the purchase of one loaf of bread. Consequently, the demand of bread at V2 is forecasted to increase. So V2 correspondingly orders more bread starting from March 1st, 2006. The average order quantity issued by V2 increases by an amount of 0.5 standard deviation of the existing ordering process. This means that the mean of Q2 changes from observation No. 60.

Before applying the NN-based control scheme, data pre-processing is required. The collected data are first standardized and then plotted in Figure 5.6. The number of input nodes in the proposed neural network is 100. Before being tested by the proposed NN-based control scheme, the standardized data need to be tuned to follow the format of the neural network testing file. That is, the standardized data should be arranged to have 50 pairs of observations in a row and this is regarded as a record. The data from the 1st pair to the 50th pair are set as the first record and the data from the 2nd pair to the 51st pair are set as the second record, and so on. In summary, there will be 41 records in the collected data. Figure 5.7 shows how the standardized data are processed to meet the requirement of neural network input.



(a) Standardized order quantity issued by V1



(b) Standardized order quantity issued by V2

Figure 5.6 The raw data of the Campus-Bread-Control case

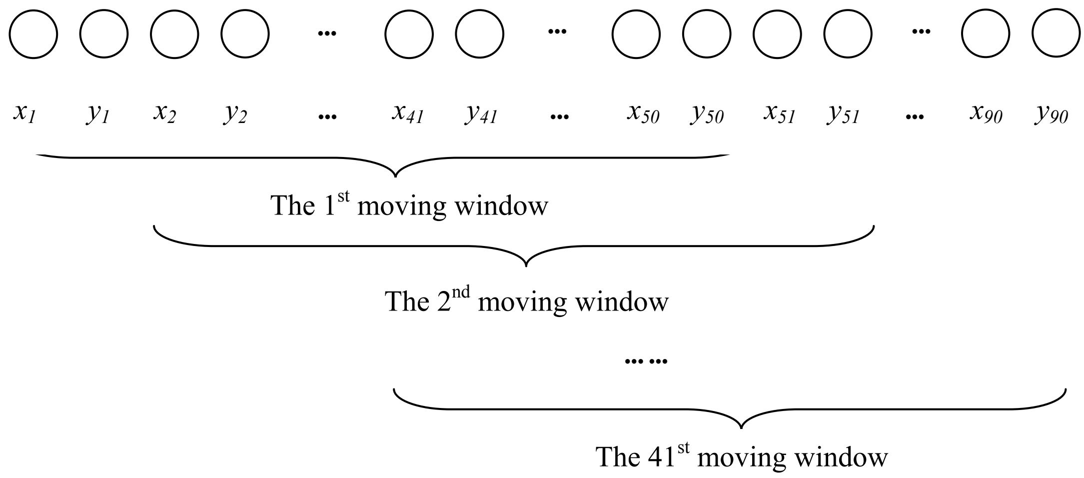


Figure 5.7 Transfer standardized data to neural network input

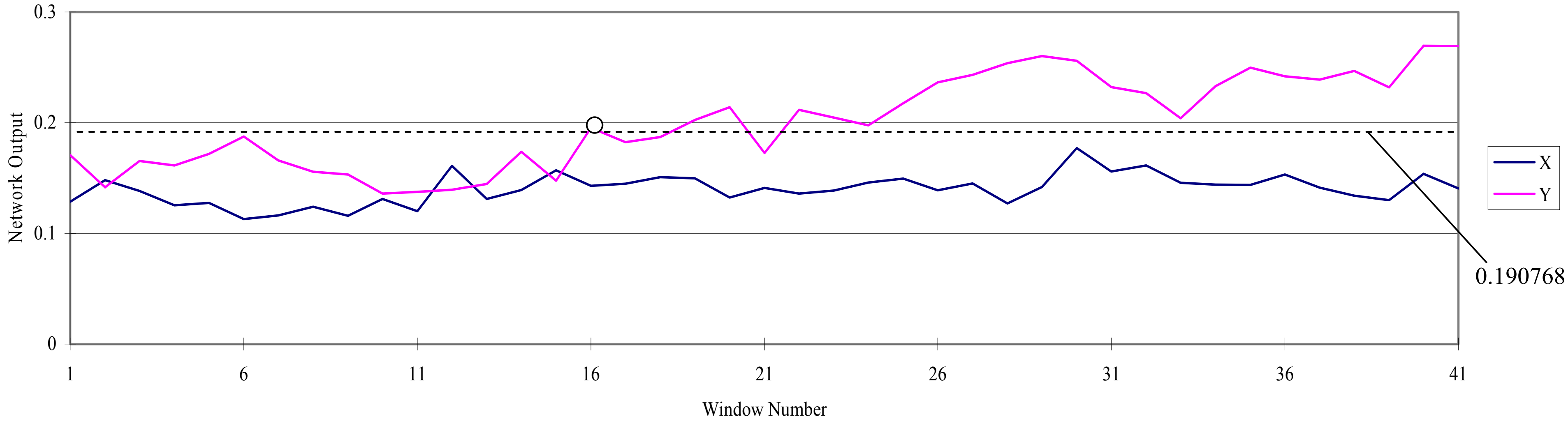
5.2.3 The Application of Control Schemes

After data pre-processing, the NN-based control scheme is applied to analyze the standardized data. Figure 5.8 is the neural network output chart of the Campus-Bread-Control case. In Figure 5.8, X represents the neural network output of the order quantity Q_1 and Y represents the neural network output of the order quantity Q_2. In Figure 5.8, the neural network output falls between 0.00 and 1.00 with 0.00 indicating no shift and 1.00 indicating a large shift of 3σ. The output can be interpreted as an estimate of the shift magnitude based on the window of data presented to the neural network.

The cut-off threshold is 0.190768, which is obtained by setting the in-control ARL of the no-autocorrelation and no-correlation process to 185.4. From Figure 5.8, it is observed that the first output value which is larger than the cut-off threshold, is observed at Window No. 16 on the variable Y. Note that Window No. 16 includes the observations from No. 16 to No. 65. As discussed in the two previous simulated illustrative examples, a single output that is larger than the threshold should not arbitrarily be concluded as a shift. More observations are required to draw a more reliable conclusion. With extended monitoring of the follow-up observations, it is found that the neural network output of variable Y after Window No. 16, has an increasing trend. Hence, it can be inferred that there is a shift in the order quantity Q_2. When looking into the case, a promotion held by V2 from March 1st, 2006 is found to be reason that caused V2 to order more bread to meet the increasing demand. The mean of Q_2 increases by an amount equivalent to 0.5 standard deviation of the existing ordering process. This means that the mean of Q2 had a 0.5σ shift from observation No. 60. The proposed control scheme detected the shift at observation No. 65, which shows that the proposed NN-based control scheme can detect the small mean shift in Q2 very quickly.

To illustrate the effectiveness of the NN-based control scheme, other statistical control schemes are also applied to the standardized data. For the purpose of comparison, the control limits of the other three control schemes are decided by setting the in-control ARL to 185.4.

The control limit in the Hotelling T^2 chart is tuned to 10.44. The Hotelling T^2 statistic is plotted in Figure 5.9. The Hotelling T^2 chart summarizes the behavior of multiple variables in one single statistic, so that it can't be used to identify the source of the out-of-control signal. In Figure 5.9, it can be observed that an out-of-control point is signaled at observation No. 23 with a value of 11.0888. When looking into the Campus-Bread-Control case, it is found that this is a false alarm since true shift starts from observation No. 60. The next out-of-control point detected by the Hotelling T^2 chart is observation No. 82 with a value of 14.3659.



Note: X represents the neural network output of the mean of the order quantity of vendor 1 and Y represents the neural network output of the mean of the order quantity of vendor 2

Figure 5.8 The neural network output chart for the Campus-Bread-Control case

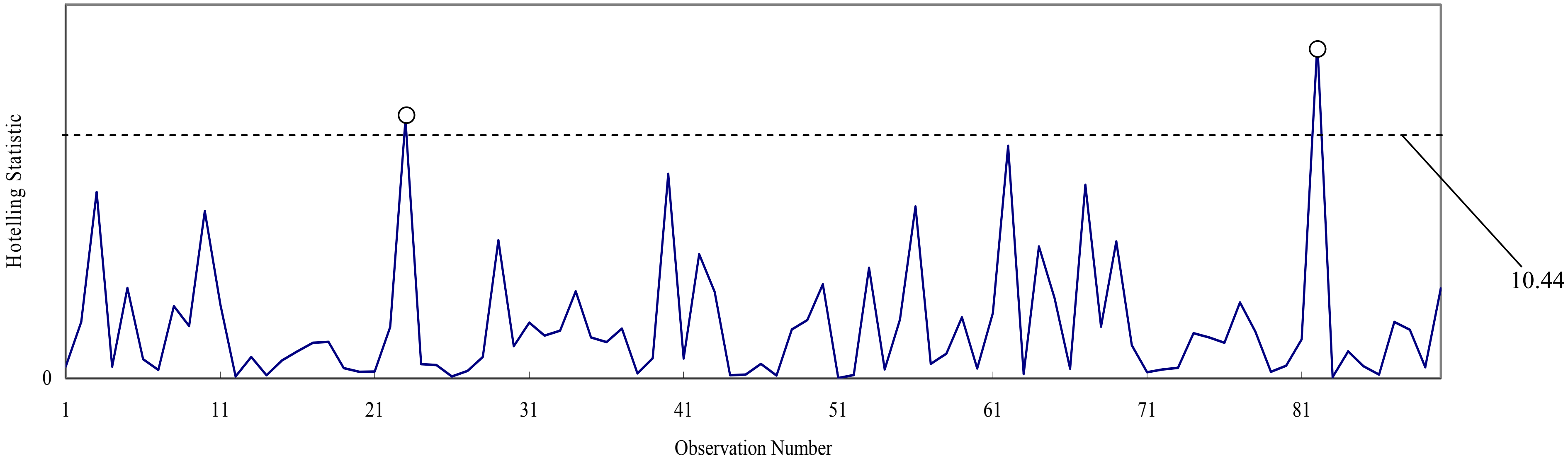


Figure 5.9 The T^2 statistic obtained from the Hotelling T^2 chart for the Campus-Bread-Control case

The MEWMA statistic is reported in Figure 5.10. When the smoothing parameter is set to 0.05, the control limit in the MEWMA chart is tuned to be 7.23. In Figure 5.10, an out-of-control point is signaled at observation No. 55 with a value of 7.2709. Since true shift starts from observation No. 60 on the variable Q2, this signal is a false alarm. The next out-of-control point is found at observation No. 62 with the value of 10.5083. Similar to the Hotelling T^2 chart, the MEWMA chart can't be used to identify the source of shift either.

Figure 5.11 reports the Z statistic obtained from the Z chart. The control limit in the Z chart is tuned to be 2.9965. In Figure 5.11, an out-of-control point is signaled at observation No. 82 with the value of 3.7662. Figure 5.12 is the plot which includes the separate Z statistics, that is, Z_1 and Z_2. When tracing the source of the shift from Figure 5.12, it is found that the source of the shift is identified to be Q1. This is a false identification.

5.2.4 Summary

Compared with the Hotelling T^2 chart and the MEWMA chart, the proposed NN-based control scheme can identify the source of the shift with a run length of 5. The Hotelling T^2 chart and the MEWMA chart generated a false alarm before they detected the true shift. The NN-based control scheme has smaller run length than the Z chart. Moreover, it can identify the true source of the shift while the Z chart identifies the shift-source wrongly. Hence, this case study reinforces the conclusion that the proposed NN-based control scheme is an effective control scheme for detecting and identifying small to moderate shifts.

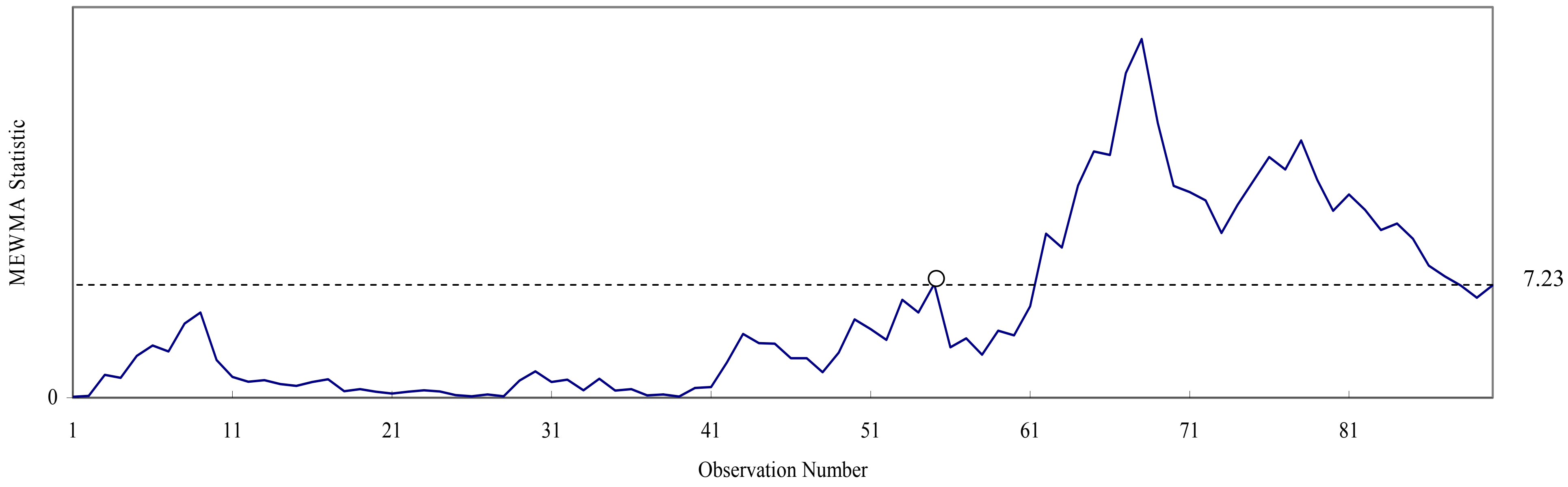


Figure 5.10 The MEWMA statistic (λ=0.05) for the Campus-Bread-Control case

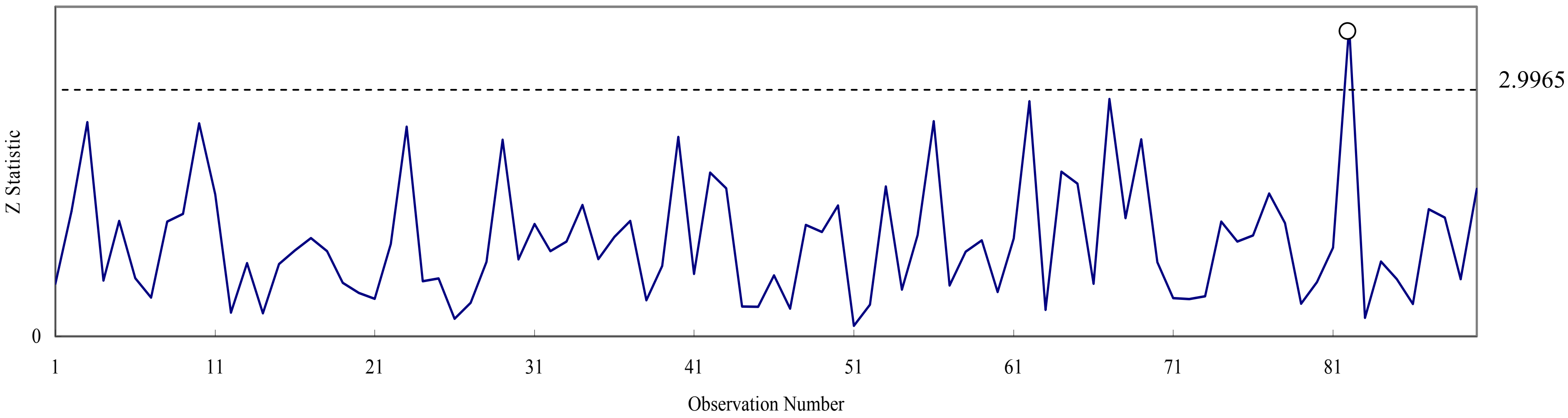
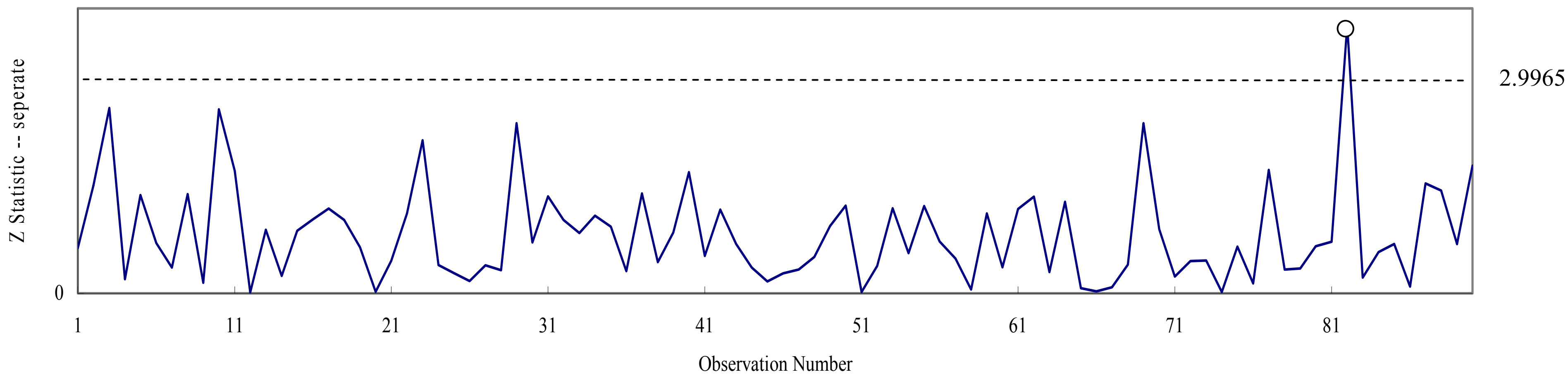
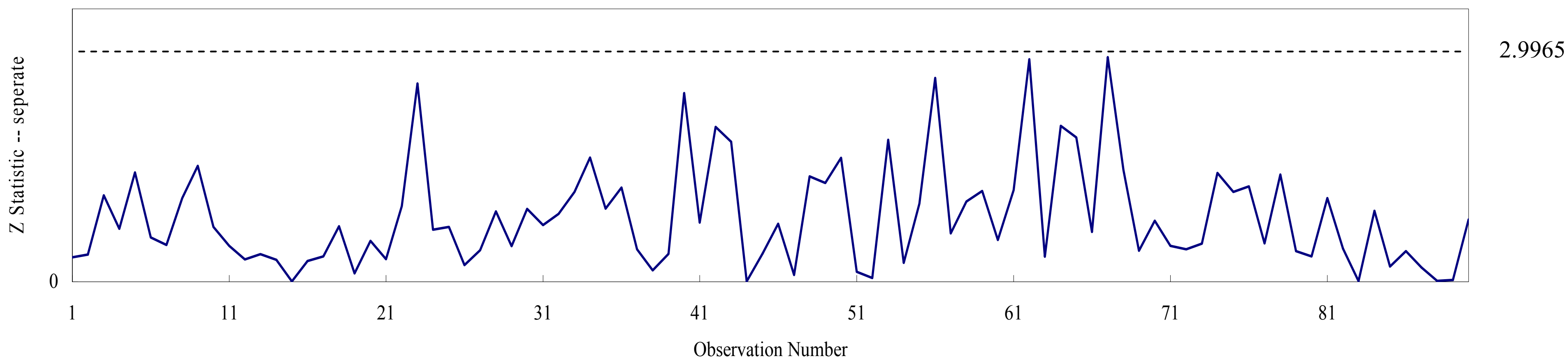


Figure 5.11 The *Z* statistic obtained from the *Z* chart for the Campus-Bread-Control case



(a) Z_1 statistic



(b) Z_2 statistic

Figure 5.12 Z statistic for separate variables

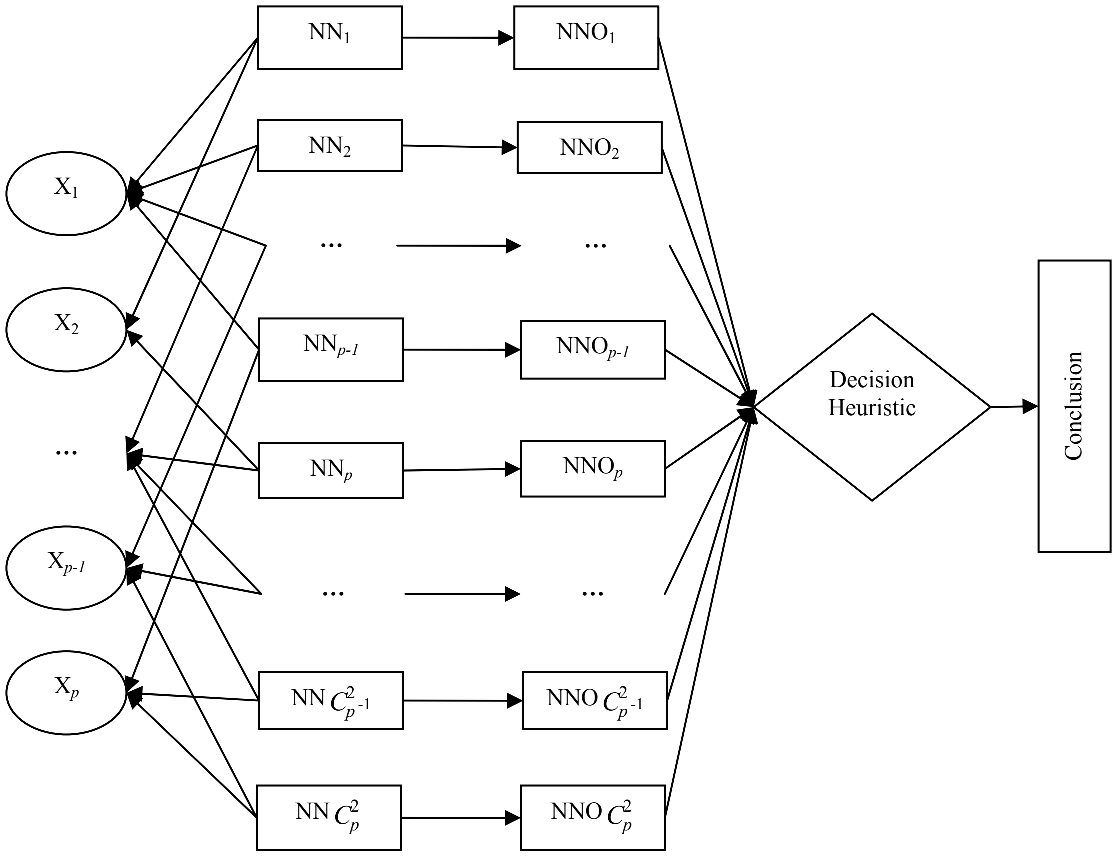
In the Campus-Bread-Control case, the supplier holds the information of order quantity. By applying the proposed control scheme, the supplier may detect the change in the mean of the quantities ordered by V1 and V2. When the supplier notices the change in the mean of order quantity, the supplier can trace the cause of the change. In the Campus-Bread-Control case, V2 ordered more from March 1st, 2006 for the reason that a promotion was held by V2. After obtaining all this information, the supplier can infer that a promotion held by the vendor increases the order quantity. Based on this inference, the supplier may constitute relevant policy to help vendors hold more promotions and thus the supplier may sell more products.

The Campus-Bread-Control case is just a simple case in which the proposed NN-based control scheme can be applied. The more valuable feature of the proposed NN-based control scheme is that it can be applied in more complex processes. In the process where it is not easy to detect the change manually, the proposed NN-based control scheme can detect the out-of-control point automatically.

5.3 Extension to Multivariate Autocorrelated Process

So far, the application of the proposed NN-based control scheme in bivariate autocorrelated process has been illustrated. In the following, the application of the proposed NN-based control scheme will be extended to multivariate autocorrelated process.

The control scheme is applied in a pair-wise manner. If there are p variables in a multivariate autocorrelated process, the control scheme is applied to each of the C_p^2 pairs of variables to monitor the process simultaneously. As correlation is defined between a pair of variables, when the control scheme is applied to a pair of variables, the correlation between those two variables is already taken into account. Pair-wise application of the scheme to all pairs of variables would ensure that all the correlations between any pairs of variables are considered. Among C_p^2 neural network applications, only ($p-1$) neural network applications are used to monitor a certain variable. For example, there are 3 variables in an interested process, namely variable X_1, variable X_2 and variable X_3. To monitor this process, 3 neural network applications are required, namely NN_1, NN_2 and NN_3. NN_1 is defined to monitor variable X_1 and variable X_2, NN_2 is defined to monitor variable X_1 and variable X_3 while NN_3 is defined to monitor variable X_2 and variable X_3. Among these three neural network applications, only NN_1 and NN_2 are used to monitor variable X_1. The decision heuristic is that variable X_i can be concluded as a shifted variable if the outputs from each application of the neural network to each of the ($p-1$) pairs of variables involving X_i indicate that X_i is shifted. Figure 5.13 is a schematic diagram of the application of the proposed NN-based control scheme in multivariate autocorrelated process ($p \geq 3$).



Note: X_i is defined as the *i*th variable. NN_j represents the *j*th neural network application while NNO_j represents the output of the *j*th network application. ($1 \le i \le p,\ \ 1 \le j \le C_p^2$)

Figure 5.13 A schematic diagram of the application of the proposed NN-based control scheme in multivariate autocorrelated processes ($p \ge 3$)

In Figure 5.13, the decision heuristic contains two parts. The first part is the same as the decision heuristic in the bivariate autocorrelated process. That is, compare the neural network output NNO_i with the cut-off value to derive which variable is responsible for the shift. The second part is the decision heuristic in the multivariate autocorrelated case. That is, variable X_i can be concluded as a shifted variable if the outputs from each application of the neural network to each of the ($p-1$) pairs of variables involving X_i indicate that X_i is shifted.

To demonstrate the application of the proposed NN-based control scheme in multivariate autocorrelated process, an illustrative example is devised. A 3-variable autocorrelated process is simulated with two different combinations of parameter values. The combinations of parameter values and the stages of the process are listed in Table 5.2.

Table 5.2 Illustrative example with 200 input of (X,Y,Z) observations

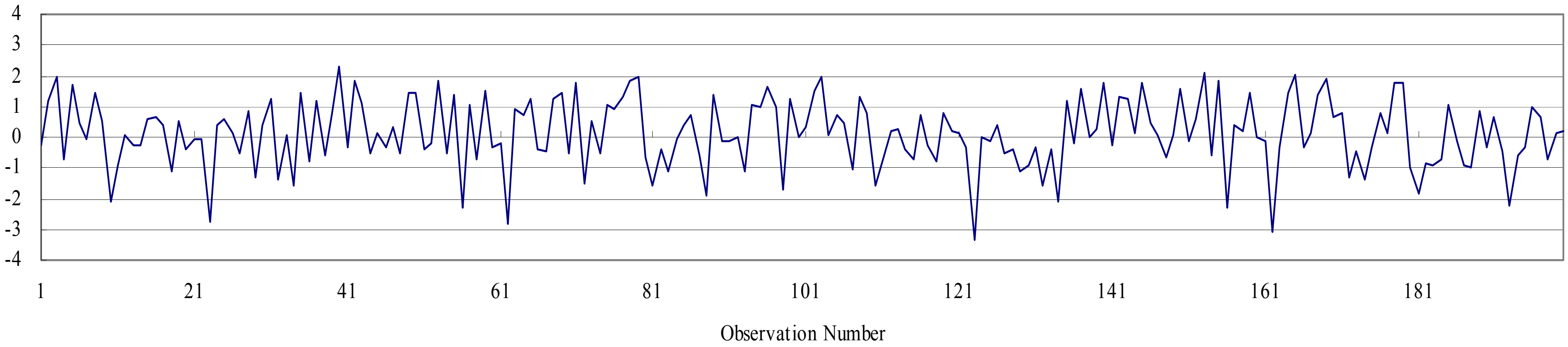
Stage	Observation No.	Window No.	δ_x	δ_y	δ_z	ϕ_x	ϕ_y	ϕ_z	ρ_{xy}	ρ_{yz}	ρ_{xz}
I	1 - 100	1-51	0	0	0	0	0	0	0	0	0
II	101-200	52-151	0	0	0.5	0.2	0.2	0.2	0.4	0.4	0.4

Figure 5.14 represents the realization of the example for the variable X, the variable Y and the variable Z. Three proposed NN-based control scheme applications are made to the simulated data sets. Figures 5.15, 5.16 and 5.17 are the neural network output charts of the devised case.

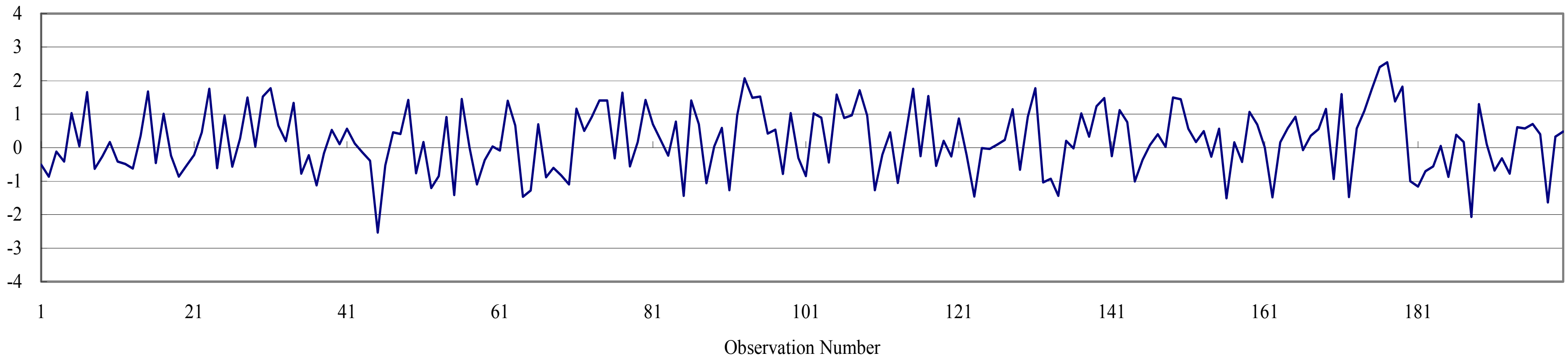
In Figures 5.15 and 5.16, X represents the neural network output of the mean of the variable X. In Figures 5.15 and 5.17, Y represents the neural network output of the mean of the variable Y. Z represents the neural network output of the mean of the variable Z in Figures 5.16 and 5.17. The interpretation of the neural network output is similar to that of the bivariate autocorrelated process. It can be interpreted as the estimate of the shift magnitude based on the window of data presented to the NN-based neural network.

The cut-off threshold is 0.190768. From Figure 5.15, it can be observed that although some of the neural network output of the variable X and the variable Y exceed the cut-off value, it can't be inferred as shifts since there is no increasing trend on the output of both variables. In Figure 5.16, it is clear that an increasing trend appears on the variable Z from Window No. 93. From Figure 5.17, it is also easy to infer that a shift exists on the variable Z. As discussed before, when the neural network output from ($p-1$) NN-based control scheme applications shows that a variable is shifted, the variable can be concluded as a shifted variable. So a conclusion can be drawn that there is a shift on the variable Z, which is consistent with the raw data.

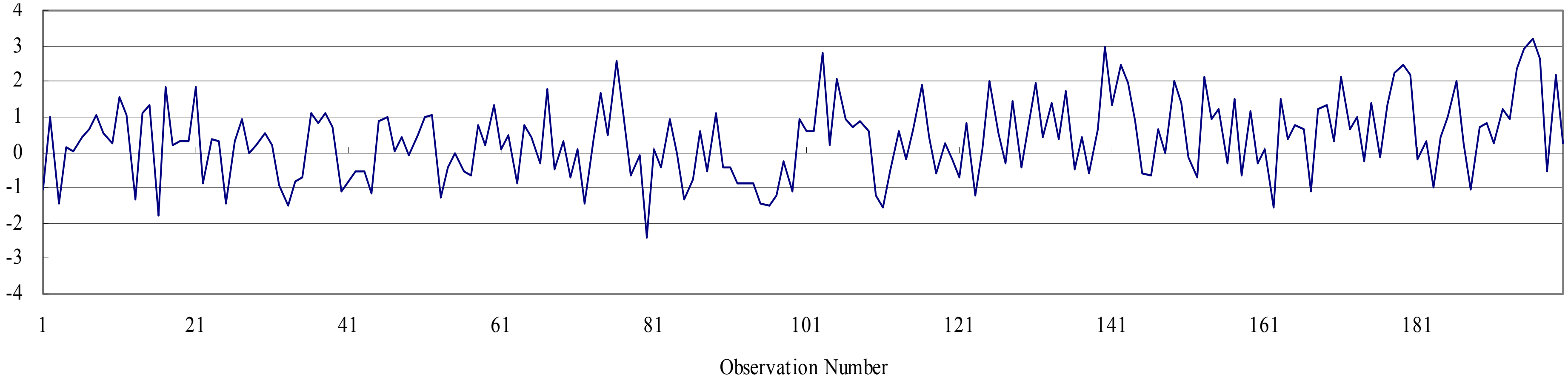
In the above example, the proposed NN-based control scheme has been successfully applied in multivariate autocorrelated processes. The results obtained clearly show that the proposed NN-based control scheme is effective and efficient when it is used to detect and identify mean shift in multivariate autocorrelated process. It is believed that this research can greatly enhance the process-improvement ability in business/industry environment where processes are multivariate and autocorrelated.



(a) X values

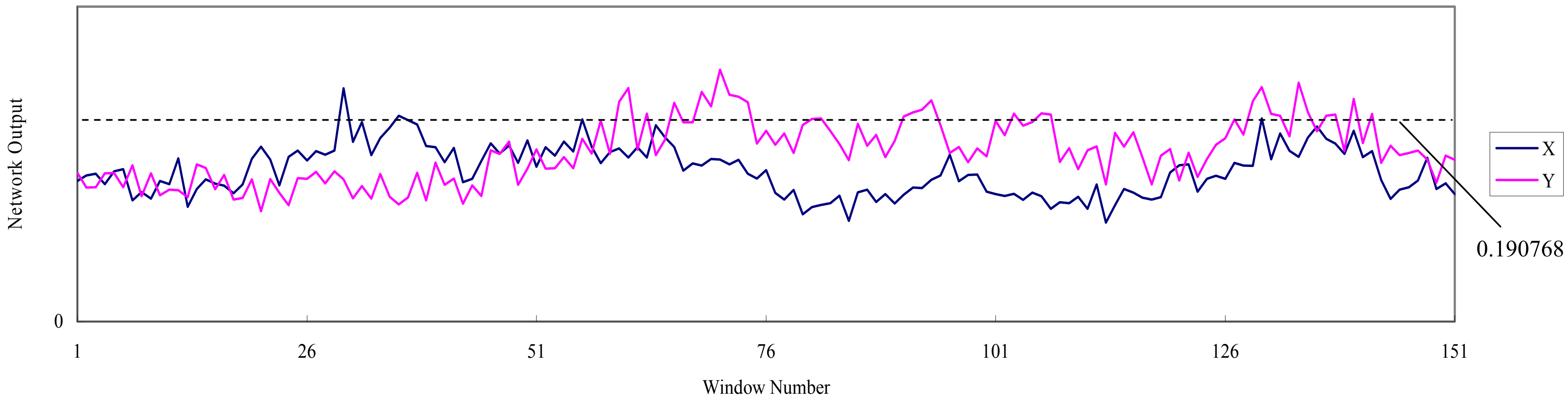


(b) Y values



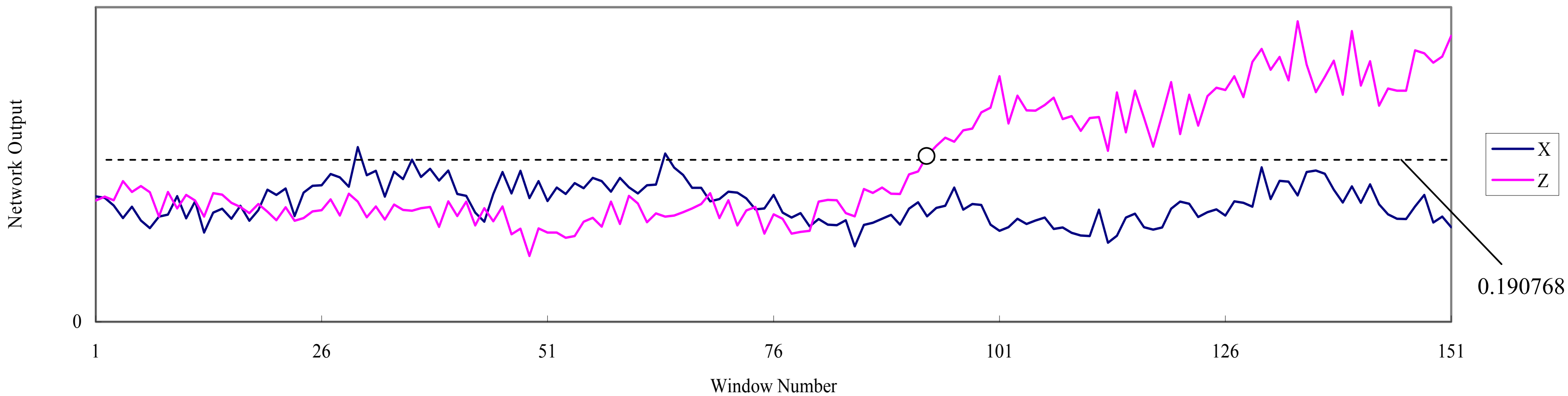
(c) Z values

Figure 5.14 The raw data of the 3-variable autocorrelated example



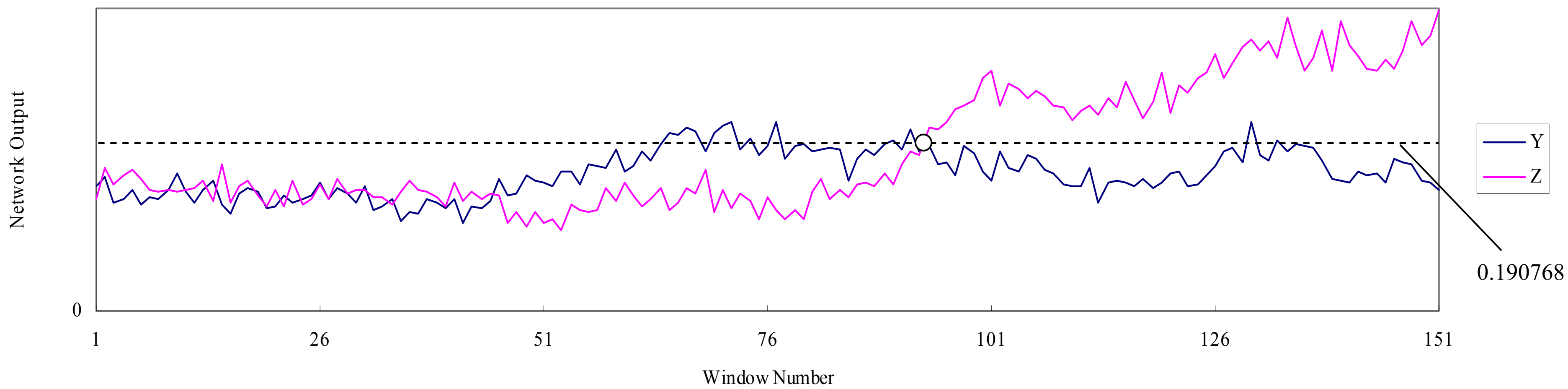
Note: X represents the neural network output of the mean of the variable X and Y represents the neural network output of the mean of the variable Y

Figure 5.15 The neural network output chart for the variable X and the variable Y



Note: X represents the neural network output of the mean of the variable X and Z represents the neural network output of the mean of the variable Z

Figure 5.16 The neural network output chart for the variable X and the variable Z



Note: Y represents the neural network output of the mean of the variable Y and Z represents the neural network output of the mean of the variable Z

Figure 5.17 The neural network output chart for the variable Y and the variable Z

Chapter 6

Conclusion

6.1 Summary

In the past few decades, few researches have been done in the field of detecting process mean shift in multivariate autocorrelated processes. This gap leads to a need for an effective and efficient method which is capable of detecting and identifying mean shifts in multivariate autocorrelated processes. In this thesis, a neural-network-based control scheme is proposed to meet this requirement.

The proposed control scheme utilized the effective and efficient Extended Delta-Bar-Delta learning rule and was trained with the powerful back-propagation algorithm. Various magnitudes of process mean shift, under the presence of various levels of autocorrelation and correlation, are considered. Extensive simulation was carried out to evaluate the network's performance.

The results show that the proposed control scheme is efficient and effective to detect and identify process mean shifts. The comparison between the proposed NN-based control scheme and the other three statistical control schemes shows that the NN-based control scheme performs better than the Hotelling T^2 chart and the Z chart when it is used to detect small to moderate shifts, i.e., shift size $< 2\sigma$. And it also shows that the NN-based control scheme is better than the MEWMA chart in detecting small to moderate shifts in high correlation or high autocorrelation processes. When used to identify the source of small to moderate shift, the proposed NN-based control scheme outperforms the Z chart. However, the Z chart performs better in identifying the source of large shift (shift size $\geq 2\sigma$). Based on the alternative monitoring heuristics in Hwarng (2004), alternative decision criteria which can identify the source of shift better, especially in the single-shift process or in the double-shift process with two different shift magnitudes, are proposed.

Illustrative examples are devised to show how the proposed NN-based control scheme can be applied in practice. In reality, the proposed control scheme should perform well if the data being monitored are similar to the data used to train the network.

6.2 Contributions of this Research

This research contributes to the literature in the following aspects.

a) By proposing an NN-based control scheme which is capable of detecting and identifying mean shift in multivariate autocorrelated processes efficiently, this research fills the gap in the literature.

b) Comprehensive comparison studies between the proposed control scheme and three statistical control schemes are carried out in this research. Through the comparison, the strengths and weaknesses of each control scheme are shown.

c) Alternative decision criteria are proposed to enhance the First-Detection capability of the proposed control scheme. Increased First-Detection capability greatly enhances the process-improvement ability in business/industry environment.

6.3 Limitations of this Research

An inherent limitation of the NN-based control scheme is that it can only be used in the processes which have similar parameter values with the training data sets. In this research, although various magnitudes of mean shift and various levels of autocorrelation and correlation parameters are covered, the results obtained need to be interpreted with caution when the proposed network is applied to the processes with parameter values out of the studied range.

Another limitation of this research is that the size of the training data set and the training time will increase when the interested parameter sets increase. This is not good for the application of the proposed control scheme in practice. However, solutions to this problem can be proposed; different networks can be developed to handle different parameter sets.

The third limitation is that the decision heuristic used in detecting and identifying multivariate autocorrelated processes is somehow strict, where the number of interested variables is larger than 2. By using this decision heuristic, sometimes it may take a long time to identify the source of the shift.

6.4 Future Research

In this research, only positive parameter values are considered; negative parameter values warrant further investigation in the future. When the number of parameter sets increases, different networks may be developed to handle different parameter subsets. Concerning source detection, decision heuristics with greater flexibility should be proposed when applied to multivariate autocorrelated processes where the number of variables is larger than 2.

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