## **SUB-CARRIER ALLOCATION SCHEMES FOR**

### **MC-DS-CDMA SYSTEMS**

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#### **A THESIS SUBMITTED**

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## Summary

Multicarrier CDMA (MC-CDMA) has attracted many interests due to its capabilities of achieving high data rate transmission while overcoming the problem of Inter-Symbol Interference (ISI). It inherits the substantial advantages from both the orthogonal frequency division multiplexing (OFDM) and CDMA systems. To further improve energy efficiency, accurate adaptive sub-carrier allocation techniques are needed. This thesis focuses on obtaining simple and efficient sub-carrier allocation schemes. Two sub-optimal sub-carrier allocation schemes that take into consideration the effect of Multiple Access Interference (MAI) for MC-DS-CDMA system are proposed.

The MC-DS-CDMA system models for our algorithms are given, which include both downlink and uplink transmissions. The algorithms developed are suitable for both links. Instead of minimizing the overall BER as in optimal solution to the sub-carrier allocation problem, the first sub-optimal algorithm is proposed with the objective to minimize the respective user BER performance while taking the effect of MAI into consideration. The algorithm to solve the problem is presented. The expression to compute the average BER that will be used in the objective function to iteratively compute the sub-carriers allocation coefficients is derived. Simulation results are presented and discussed compared with conventional sub-carrier allocation scheme reported in [9] and those without using any adaptive sub-carrier allocation scheme.

Our first proposed scheme is a promising scheme and converges much faster than the optimal solution. But it will show discrepancy when the amount of MAI present in the system is small. The discrepancy will propagate so that the subsequent allocation will be affected. To overcome the limitation of our first scheme, we formulate the objective function using Quadratic Programming (QP). By using QP, the sub-carriers are allocated using the actual values of the channel gains and jointly considering the effect of MAI. This scheme is not as simple as the first one, but it gives more optimal results. The discrepancy in the previous allocation will not propagate to affect the subsequent allocation. The formulated QP problem is shown can be converted to a linear programming problem by applying Kuhn-Tucker conditions, which is much easier to solve. Simulations results are presented to compare with the scheme presented in [9] and our first proposed scheme.

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## Nomenclature

AWGN	additive white Gaussian Noise
BER	Bit Error Rate
BPSK	Binary Phase Shift Keying
BS	basestation
CCI	Co-Channel Interference
c.d.f.	cumulative distribution function
CDMA	Code Division Multiple Access
CLT	Central Limit Theory
DS	Direct Sequence
FDM	Frequency Division Multiplexing
FH	Frequency Hopped
FFT	Fast Fourier Transform
ICI	Inter-Channel Interference
IDFT	Inverse Discrete Fourier Transform
IFFT	Inverse Fast Fourier Transform
IP	Integer Programming
ISI	intersymbol interference
KT	Kuhn-Tucker
LANs	local area networks
LOS	line-of-sight
LP	linear programming
MAI	Multiple Access Interference

#### MC-CDMA Multi-Carrier CDMA

NTT D	1.	
NLP	non-linear progra	amming
1,21		B

- OFDM Orthogonal Frequency Division Multiplexing
- OFDMA Orthogonal Frequency Division Multiplexing multiple access
- p.d.f. probability density function
- PG processing gain
- QP Quadratic Programming
- SI Self Interference
- SNR signal-to-noise ratio
- S/P Serial-to-Parallel
- SS spread spectrum
- TDD time division duplex

## **Chapter 1**

### Introduction

#### 1.1 Background

Code Division Multiple Access (CDMA) is a multiple access technique where a number of users can simultaneously and asynchronously access a channel by spreading their information-bearing signals with pre-assigned signature sequences. Recently, CDMA systems have been proposed to be a candidate for high data rate transmission in wireless communication networks [1]. The wireless communication channel is characterized by multipath reception: the signal arrived at the receiver normally contains not only a direct line-of-sight (LOS) radio wave, but also a large number of reflected radio waves and each arrives at the receiver at different times. Delayed signals are the result of reflections from terrain features such as trees, hills, mountains, vehicles or buildings. These reflected delayed waves interfere with each other will cause fading if paths are non-resolvable and Intersymbol Interference (ISI) if paths are resolvable, which in turn causes significant degradation of network performance. Hence the protocol and physical layer of a wireless network should be

designed so that the degradation in the system performance due to the presence of fading and ISI is minimized.

Future broadband multimedia mobile communication systems need to transmit at high bit-rate of at least several megabits per second [19][21][22][23]. However, if digital information is transmitted at the rates of several megabits per second, the delay times of the delayed waves are generally greater than one symbol time and result in severe ISI. The use of adaptive equalization techniques at the receiver is one conventional method to overcome ISI. However, there are practical complexities in implementing the equalizers at a transmission rate of several megabits per second using compact, low-cost hardware.

To achieve reliable high data rate transmission in such a multipath fading environment using a lower complexity receiver, the multi-carrier modulation (MCM) scheme, often called Orthogonal Frequency Division Multiplexing (OFDM), has been proposed. OFDM system divides the available bandwidth into a number of sub-carriers. Data is transmitted in parallel over these sub-carriers. Each sub-carrier thus has a lower bit rate stream and encounters only flat fading. We will introduce OFDM in detail in Chapter 2.

Recently, MCM has been integrated with CDMA systems. This is because the MCM-based CDMA system owns the capabilities to cope with the concurrent asynchronous transmission of multimedia data traffic in any of the sub-carriers and inherits advantages from both types of the systems. This new combination of OFDM

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and CDMA technique starts an epoch of CDMA application, i.e., Multi-Carrier CDMA (MC-CDMA). We will introduce MC-CDMA systems in detail in Chapter 2.

#### **1.2** Problem of Adaptive Sub-carrier Allocation

The MC-CDMA system has the ability to allocate sub-carriers adaptively which is inherited from OFDM [20] and this ability is important. For a multiuser system using a static allocation scheme, i.e., each user is allocated with a number of predetermined sub-carriers to transmit data, the sub-carriers that are being allocated may not be always suitable for carrying information bits. This is because in the system, some of the sub-carriers may be in deep fade and hence are not power efficient to carry any information bit over some periods of time. Such a momentarily deep fade can cause the lost of a burst of data bits that cannot be corrected by the error correction code used in the receiver properly. Furthermore, the sub-carriers which appear in deep fade to one user may not be in deep fade to other users, as the fading parameters for different users are mutually independent. This motivates us to consider an adaptive multiuser sub-carrier allocation scheme where the sub-carriers are allocated to the users based on instantaneous channel information. This approach will allow all the sub-carriers to be used more efficiently because a sub-carrier will be left unused only if it appears to be in deep fade to all users.

#### **1.3** Literature Review of Adaptive Subcarrier Allocation Schemes

Because of the reasons we have mentioned in the previous section, the optimum allocation of sub-carriers should be performed. In this section, we will carry out a brief review of previous work on adaptive sub-carrier allocation schemes. We categorize the available schemes into two types: the sub-carrier allocation schemes for OFDM systems and sub-carrier allocation schemes for MC-CDMA systems.

#### 1.3.1 Sub-carrier Allocation Schemes for OFDM Systems

In order to combat the temporally deep fading of a wireless communication channel, adaptive transmission schemes are proposed. Ref. [24] [25] [26] have demonstrated that significant performance improvement can be achieved if adaptive modulation is used for OFDM. This motivated Wong et al. to propose a multiuser OFDM sub-carrier, bit, and power allocation algorithm to minimize the total transmit power [20]. They applied Lagrangian relaxation to solve the optimization problem. The iterative algorithm first performed the multiuser sub-carrier allocation, then the bit and power allocations were applied to each user on its allocated sub-carriers. This two-step algorithm was further developed by Kivanc et al. [27]. They first use average signal-to-noise ratio (SNR) for each user to decide the number of sub-carriers to be allocated to each user. Once the number of sub-carriers is determined, the algorithm moves on to assign the most suitable sub-carriers to users. In order to reduce the computation complexity, Kim et al. at [28] converted the non-linear optimizations in [20] into a linear one and solved it by using integer programming (IP) method. A sub-optimal approach that separately performs sub-carrier allocation and bit loading was proposed to further reduce the computational load of the IP problem. In [29], Wong, et al. also proposed an improved algorithm for fixed modulation systems. The solution was obtained by the known Hungarian method. Bakhtiari et al. proposed an algorithm for a given rate distribution among users in [30]. The algorithm tried to optimize the trade off between the transmit power and computational complexity. It also used an efficient re-allocation scheme to achieve the required performance with minimum number of computations.

The concept of grouping the sub-carriers and the use of group allocation was used in [31] [32] in the algorithms for sub-carrier allocation. The computational complexity by using group allocation can be reduced further. Algorithm in [33] applied utility functions to quantify the level of users' satisfaction derived from the radio resources they occupied. They formulated optimization problem to maximize the sum of the utilities over all active users subject to the feasible rate, which is determined by adaptive resource allocation schemes deployed and its channel conditions.

Sub-carrier allocation algorithms have also been developed in more practical situation. Ref. [34] and [35] proposed adaptive sub-carrier and bit allocation algorithms for multi-cell systems in the presence of Co-Channel Interference (CCI). The work was aimed to maximize the throughput of the OFDM systems. Ref. [36] proposed an adaptive sub-carrier allocation scheme for OFDM multiple access/ time division duplex (OFDMA/TDD) system in wireless local area networks (LANs).

#### 1.3.2 Sub-carrier Allocation Schemes for MC-CDMA Systems

Compared with OFDM systems, only a limited number of sub-carrier allocation schemes were proposed for MC-CDMA systems. In [9], a simple adaptive sub-carrier allocation scheme was proposed for MC-DS-CDMA systems. Each user transmits

over the user's favorite sub-carrier which has the largest channel gain among all the sub-carriers. The mobile estimates the channel gains of all sub-carriers and feeds back the index of the sub-carriers to the basestation (BS). With the index information, the BS allocates the sub-carrier that has the largest channel gain to the respective user. This will be the optimal policy if orthogonal sequences are used as the signature sequence. When random sequences are used, the system still shows some performance improvement when compared to the MC-CDMA systems without adaptive sub-carrier allocation scheme. The author also investigated how the performance is affected when the transmit data is not perfectly allocated to the best sub-carrier. However, the scheme did not consider the influence of the Multiple Access Interference (MAI) when multiple users are allocated to use the same sub-carrier. That is, the scheme just simply allocates the sub-carrier with the largest channel gain of each user, but the sub-carrier that is the largest to one user may also be the largest to some other users. Hence there is possibility that many users select one sub-carrier at the same time, and some sub-carriers which also have reasonably good channel conditions may not be selected. When random signature sequences are used, the MAI will degrade the performance of the system and those unselected sub-carriers may be a waste to the system. So generally, this scheme is non-optimal because it does not jointly consider the effect of MAI in the process of allocating sub-carriers.

To increase the data rate of the system, a certain number of sub-carriers that have relatively larger channel gains are chosen to transmit the data of a user in [10]. That is, the BS selects L sub-carriers for a mobile to transmit data out of total N sub-carriers. The remaining (*N*-*L*) sub-carriers transmit reference data with a small power. These

reference data are used at the BS to estimate the channel gains of the sub-carriers, which are used for the adaptive determination of sub-carrier allocation. On these selected L sub-carriers, data rate, power allocation and the processing gain (PG) are adjusted according to the value of L. For example, if the data rate is  $r_d$  for an N/Nsystem, the data rate of an L/N system will be  $r_d \times N/L$  and a smaller PG  $G' = G \times L/N$  should be used to support the same amount of data as the N/Nsystem. As to the power allocation, if  $P_1$  is the power allocated to the selected sub-carriers, a suppression factor will be used to reduce the transmitting power allocated to the unselected sub-carriers. The suppression factor is much less than 1 to avoid excessive interference to other users who are using these sub-carriers. The scheme can achieve better bit-error-rate (BER) than MC-CDMA system that without using adaptive sub-carrier allocation scheme while keeping the same amount of data transmitted over the same bandwidth and the same amount of power. This scheme is a good alternative of power control scheme to compensate the fading effect. However, their proposal also does not consider the effect of MAI when random signature sequences are used.

In [11], Fan et al. set a prefixed threshold of the channel gain that can satisfy the user's requirement on BER so that the scheme reduces the number of sub-carriers that need to be considered. Method that uses grouping of sub-carriers is also used in MC-CDMA systems. Tabulo et al. proposed in [12] a linear programming based approach to the grouping and sub-carrier allocation problem for a MC-CDMA system. The sub-carriers are first grouped and then users are assigned to a group of sub-carriers using a linear programming (LP) algorithm. This algorithm can improve

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BER performance of the system compared to the algorithm reported in [9], as well as those static sub-carrier allocation algorithms. To make the algorithm much more practical, Al-susa et al. proposed a practical fast-converging algorithm in [13] based on priority swapping allocation. The algorithm can achieve near equal diversity gain as the method uses the maximum likelihood allocation approach but at a much lower computational complexity.

In [14], Chen et al. proposed a sub-carrier allocation scheme for uplink communication. The bit stream is split into sub-streams and transmitted over a few chosen sub-carriers simultaneously. While every sub-stream must have constant rate, users can use a different number of sub-streams up to a maximum of N, which offers the potential to provide multi-rate services without any increase in the system complexity. Also, instead of random or deterministic hopping patterns as adopted in conventional frequency hopped (FH) systems, the hopping pattern in the proposed system is adaptively determined by the channel fading conditions. The scheme outperforms the conventional DS-CDMA system with RAKE reception without incurring any bandwidth expansion associated with coding or diversity schemes. It is especially suitable for operating in channels with very slow time variations which are often encountered by slow moving vehicles or pedestrians. However, this scheme needs a large amount of signature sequences. This is because it is possible for more than one sub-stream belonging to the same user to hop to the same sub-carrier. Each user will need N (the number of sub-carriers) signature sequences so that every data sub-stream can be uniquely identified.

Other works such as [15] [16] investigated various adaptive modulation and power allocation algorithms for MC-CDMA systems. Adaptive sub-carrier allocation algorithms are proposed in [17] [18] for discrete multitone systems can also be useful references to develop algorithms for MC-CDMA systems.

#### **1.4** Motivations and Contributions of the Thesis

CDMA systems can distinguish different users with signature sequences assigned to each user. This works well if the signature sequences are orthogonal to each other. However, if random signature sequences are used, there will be interference from other users, which is called MAI. The MAI problem will become severe if the number of users is large. In MC-CDMA systems, if there are too many users using the same sub-carrier for transmission, the system BER performance of the sub-carrier will degrade significantly if MAI becomes dominant.

From the literature review conducted on sub-carrier allocation schemes of MC-CDMA systems presented in Section 1.3, we found that the existing schemes generally did not consider MAI when allocating sub-carriers to users. Hence, our aim is to design a sub-carrier allocation scheme for MC-CDMA system with MAI taken into consideration. However, this cannot be done easily. The fact that sub-carrier allocation process and the computation of MAI and hence BER intertwined to each other makes the calculation very difficult. We explain this using a simple illustration.

We define the sub-carrier allocation coefficients to be

$$s_{k,n} = \begin{cases} 1 & \text{if } k \text{th user's } n \text{th subcarrier is selected} \\ 0 & \text{if } k \text{th user's } n \text{th subcarrier is not selected} \end{cases}$$
 (1.1)

so that the sub-carrier allocation coefficients of the *k*th user will be in the form of  $[s_{k,1}, s_{k,2}, ..., s_{k,N}] = [1,0,0,1...,1]$ , where the dimension of the vector is equal to the number of sub-carriers of the system. In a system with *K* users and 4 sub-carriers, we first allocate sub-carriers to the 1<sup>st</sup> user. Supposing sub-carrier #1 and #4 are the two sub-carriers having the largest two channel gains for the 1<sup>st</sup> user, then its allocate sub-carriers to the 2<sup>nd</sup> user using the same rules. We can see that both the users are allocated with sub-carrier #4 as indicated in Figure 1.1 (b). If the effect of MAI is taken into consideration, sub-carrier #4 may not be the most suitable choice to support the transmission of the two users. If this happens, either one of the users may have to change the sub-carrier sallocated if the effect of MAI resulting in degradation in system performance is larger than that when either one of the two users selects the sub-carrier of the next better channel to transmit.

$$1^{st} user \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}$$

$$2^{nd} user \begin{bmatrix} 1 & 0 & 0 & 1 \end{bmatrix}$$

$$2^{nd} user \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$$

$$(a) \qquad (b)$$

$$1^{st} user \begin{bmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$1^{st} user \begin{pmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$2^{nd} user \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$$

$$2^{nd} user \begin{bmatrix} 0 & 1 & 0 & 1 \end{bmatrix}$$

$$3^{rd} user \begin{pmatrix} 1 & 0 & 1 & 0 \end{bmatrix}$$

$$(c) \qquad (d)$$

Figure 1.1 Example of the sub-carrier allocation problem

Supposing the 1<sup>st</sup> user and the 2<sup>nd</sup> user have re-allocated sub-carriers as shown in Figure 1.1 (c), we continue to allocate sub-carriers to the 3<sup>rd</sup> user, as shown in Figure 1.1 (d). This time, the sub-carrier #1 is allocated simultaneously to the 1<sup>st</sup> user and the 3<sup>rd</sup> user, while the sub-carrier #2 is allocated to the 2<sup>nd</sup> and the 3<sup>rd</sup> user, respectively. Under the consideration of MAI, the sub-carrier allocation coefficients may need to change again. This process may have to continue for some time before a reasonably good solution can be obtained. From the above simple illustration, we find that allocating sub-carriers while considering MAI is not simple. Especially when the number of users in the system is large, the solution to this problem may become NP-hard.

In this thesis, we devise two new sub-optimal algorithms to solve the problem. In our first algorithm, we define the MAI as interference from other users who are using the same sub-carrier as the desired user. We assume that the channel conditions of all users are slow time varying and independent to each other. All the sub-carriers of each user are to be ordered according to their respective channel gains at every updating stage. The sub-carrier allocation coefficients of the previous updating stage together with the ordering index vector of the sub-carriers of the current updating stage are used to calculate the average BER of the respective sub-carrier. The average BER will be used in the objective function to perform the sub-carrier allocation. The use of the previous allocation coefficients and the ordering index vector of the previous allocation coefficients and the ordering index vector of the previous allocation coefficients and the ordering index vector of the previous allocation coefficients and the ordering index vector of the previous allocation coefficients and the ordering index vector of the previous allocation coefficients and the ordering index vector of the previous allocation coefficients and the ordering index vector of the previous allocation coefficients avoid computing MAI concurrently with the allocation process, so that it will reduce the complexity of the design, but at the expense of slight deviation from the optimal solution. The ordering index vector is

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used to compute the average BER of respective sub-carriers of the user, which is evaluated by taking MAI into consideration.

This method works very well when MAI is significantly large but may show discrepancy when the effect due to MAI is small, i.e. small number of users share the same sub-carrier. The discrepancy in each update will further propagate and affect the subsequent decision. In the second algorithm, we formulate the problem without using the sub-carrier allocation coefficients of the previous updating stage. Although the objective function is much complex and will be non-linear, it will be more accurate compared with the first algorithm. The objective function is formulated as a quadratic programming (QP) problem. We then use the Kuhn-Tucker (KT) conditions to convert the problem to a solvable LP problem.

We show that both of our proposed algorithms have a much better performance than those reported in [9], where MAI has not been taken into consideration. The second proposed method works much better than the first one, and no discrepancy is shown at lower MAI.

#### **1.5** Organization of the Thesis

The organization of the thesis is as follows:

**Chapter 1** – Because the severe ISI problem that exists in high rate wireless communication, OFDM that uses parallel transmission to overcome the effect of ISI is proposed. CDMA is then proposed to combine with OFDM to make the system to

have the advantages of both. As a result, the problem of adaptive sub-carrier allocation for MC-CDMA systems arises. We then perform a literature review on the adaptive sub-carrier allocation schemes for both OFDM and MC-CDMA systems. Our main contributions in this thesis are then given, which is to solve the adaptive sub-carrier allocation problem for MC-CDMA systems in the presence of MAI.

**Chapter 2** – This chapter gives a review on MC-CDMA systems. We first discuss the characteristics of the small scale fading channels. This leads to the discussion on why MCM is needed to overcome ISI for CDMA and OFDM system. CDMA can increase system capacity and reject narrow band interference. But it is sensitive to the frequency selective fading, which causes severe ISI problem in wireless communication. Thus OFDM is proposed to overcome this problem. OFDM can also be implemented with Fast Fourier Transform (FFT) easily compared with adaptive equalization techniques in high data rate transmission systems. MC-CDMA is the combination of these two schemes which inherits advantages from both. We review the three types of MC-CDMA systems in this chapter and discuss their advantages and disadvantages in terms of transmitter, receiver structures and spectral efficiency.

**Chapter 3** – The description of MC-DS-CDMA system used in this thesis is given in this chapter. The data stream is spread in the time domain. Then the serial-to-parallel (S/P) converted data will pass through a "Subcarrier Selector" block. The sub-carrier allocation procedure will be carried out in this block. After the appropriate sub-carriers are allocated to users, the data is modulated on the corresponding sub-carriers. We then give the receiver structure of our system model. Expression to evaluate BER for MC-CDMA systems in flat fading channel for a single user case is then given.

**Chapter 4** – A sub-optimal adaptive sub-carrier allocation scheme for MC-DS-CDMA systems is proposed. We first formulate the problem with the designed objectives and constraints. Then we describe the algorithm used to solve the problem. The expression to compute the average BER that will be used in the algorithm is derived. Then we will compare our scheme with some existing schemes using computer simulation. The simulation results are analyzed and discussed.

**Chapter 5** – From the discussion in Chapter 4, we find that we can still improve the scheme proposed in Chapter 4 by using the actual values of the channel gains while considering the MAI, rather than using the allocation coefficients of the previous updating stage. Although such scheme is not as simple as our scheme in Chapter 4, it improves the system performance further. Besides, it can avoid some situations where the earlier proposed method failed. In this Chapter, we use QP to formulate the objective function. The reason why QP is used is because it can express both the interference and the allocation coefficients in the same objective function. From the mathematical point of view, QP problem is solvable although it belongs to non-linear programming (NLP). We will show how to use the KT conditions to convert our formulation to a QP problem. Simulation results then presented.

**Chapter 6** – Concludes the findings in this thesis and gives some advices on future research areas.

## Chapter 2

### **Multicarrier CDMA Schemes**

Multicarrier CDMA (MC-CDMA) are robust against frequency selective fading, which is a severe problem in mobile radio communications. The robustness is because MC-CDMA systems are the combination of OFDM and CDMA and hence inherit advantages from both of the systems.

In this chapter, we will review MC-CDMA systems. The characteristics of wireless communication fading channels is first introduced. CDMA and OFDM form the basis of MC-CDMA systems, so we will first introduce these two systems. In the literature, three types of MC-CDMA systems are proposed, which they are categorized into two groups. The first spreads the original data stream using a given signature sequence, and then modulates a different sub-carrier with one chip (i.e., spreading is made in the frequency domain) [2][3][4]. The other spreads the serial-to-parallel converted data streams using a given spreading code (i.e., the spreading in the time domain) [5][6]. The comparison of different types of MC-CDMA systems is given at the end of this chapter.

#### 2.1 Rayleigh Fading Channel

There are two types of fading that are used to characterize mobile channels: large-scale fading and small-scale fading [37]. Large-scale fading represents the average signal power attenuation or path loss due to motion over large areas. Such phenomenon is affected by prominent terrain features, such as hills, forests, high-rise buildings and so on, between the transmitter and the receiver. The receiver is often represented as being shadowed by these terrain features. The statistics of large-scale fading provide a way of computing an estimate of path loss as a function of distance. This is described in terms of a mean path loss and a log-normally distributed variation about the mean.

Small-scale fading refers to the rapid fluctuation of the amplitude of a radio signal over a short period of time or travel distance, so that the large-scale path loss effects may be neglected. Such fading is caused by interference between two or more versions of the transmitted signal which arrive at the receiver at slightly different times [37]. These waves will combine at the receiver to give a resultant signal which can vary widely in amplitude and phase, depending on the distribution of the intensity and relative propagation time of the waves and the bandwidth of the transmitted signal.

In this thesis, we only consider small-scale fading with the assumption that the communications between the mobile users and the BS occur within a relatively small area only. Thus no large-scale path loss has to be taken into account. Alternatively, this can be viewed as a power control mechanism exists to remove the path loss and lognormal shadowing.

#### 2.1.1 Doppler Shift

Consider a mobile moving at a constant velocity v, along a path segment having length d between points X and Y, while it receives signals from a remote source S, as illustrated in Figure 2.1. The difference in path lengths traveled by the wave from source S to the mobile at points X and Y is given by  $\Delta l = d \cos \theta = v \Delta t \cos \theta$ , where  $\Delta t$  is the time required for the mobile to travel from X to Y, and  $\theta$  is assumed to be the same at points X and Y since the source is assumed to be very far away. The phase change in the received signal due to the difference in path lengths is therefore given by

$$\Delta \phi = \frac{2\pi \Delta l}{\lambda} = \frac{2\pi v \Delta t}{\lambda} \cos \theta , \qquad (2.1a)$$

and hence the apparent change in frequency, or Doppler shift, is given by

$$f_d = \frac{1}{2\pi} \cdot \frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda} \cdot \cos\theta, \qquad (2.1b)$$

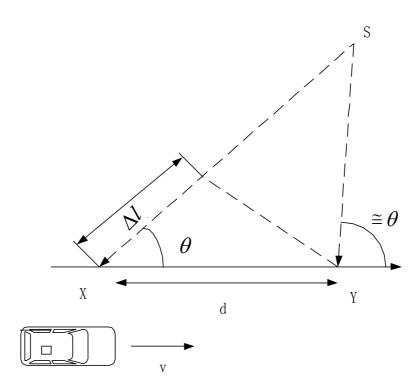


Figure 2.1 Illustration of Doppler Effect

Eq. (2.1b) relates the Doppler shift to the mobile velocity and the spatial angle between the direction of motion of the mobile and the direction of arrival of the wave. It can be seen from Eq. (2.1b) that if the mobile is moving toward the direction of arrival of the wave, the Doppler shift is positive (i.e. the apparent received frequency is increased), and if the mobile is moving away from the direction of arrival of the wave, the Doppler shift is negative (i.e. the apparent received frequency is decreased). Multipath components that arrive from different directions contribute to the Doppler spreading of the received signal, thus increasing the signal bandwidth.

#### 2.1.2 Fading Effects Due to Doppler Spread

Due to the relative motion between the mobile and the BS, each multipath wave experiences an apparent shift in frequency. The shift in received signal frequency due to motion is called the Doppler shift and is directly proportional to the speed and direction of motion of the mobile with respect to the direction of arrival of the received multipath wave [37]. Doppler spread is a measure of the range of frequencies over which the received Doppler spectrum is essentially non-zero. Its corresponding time domain dual is called the coherence time. The coherence time of a channel is actually a statistical measure of the time duration over which the channel impulse response is essentially invariant, and quantifies the similarity of the channel response at different times.

Using Clarke's model [49], the commonly adopted Doppler spectrum of the fading channel can be given as

$$S(f) = \frac{1.5}{\pi f_m \sqrt{1 - (f/f_m)^2}},$$
(2.2)

 $f_m$  is the maximum Doppler shift. The Doppler spread spectral will determine the time domain fading waveform and dictate the temporal correlation and fade slope behaviors. Rayleigh fading simulators must use a fading spectrum such as Eq. (2.2) in order to produce realistic fading waveforms that have proper time correlation.

Both Doppler spread and coherence time describe the time varying nature of the communications channel in a small scale region. In fact, the Doppler spread is inversely proportional to the coherence time. When the Doppler spread is greater than the baseband signal bandwidth, or the coherence time is less than the symbol duration, the channel is said to be a fast fading channel. In a fast fading channel, the channel impulse response changes within the symbol duration. Thus the channel variations are

faster than the baseband signal variations. Mathematically, the fast fading channel impulse response can be represented as

$$h(\tau,t) = \sum_{l=0}^{N_m - 1} f_l(t) \delta(\tau - \tau_l) = \sum_{l=0}^{N_m - 1} \alpha_l(t) e^{j\phi_l(t)} \delta(\tau - \tau_l), \qquad (2.3)$$

where  $N_m$  is the number of received paths,  $\tau_l$  is the associated time delay,  $f_l(t)$ represents the complex-valued time-varying channel coefficients of the *l*th path,  $\alpha_l(t)$  is the Rayleigh distributed attenuation factor received on the *l*th path and  $\phi_l(t)$ is the channel phase uniformly distributed over  $[0,2\pi]$ .

On the other hand, when the Doppler spread is much lower than the signal bandwidth, or the coherence time is much greater than the symbol duration, the channel is known as a slow fading one. In a slow fading channel, the channel impulse response changes at a rate much slower than the baseband signal. Hence the channel may be assumed to be static over one or several symbol intervals. The mathematical expression for the slow fading channel impulse response within a symbol interval is given by

$$h(\tau,t) = \sum_{l=0}^{N_m - 1} f_l \delta(\tau - \tau_l) = \sum_{l=0}^{N_m - 1} \alpha_l e^{j\phi_l} \delta(\tau - \tau_l).$$
(2.4)

The difference between (2.3) and (2.4) is that the channel coefficient  $f_l$ , channel gain  $\alpha_l$  and the channel phase  $\phi_l$  in Eq. (2.4) are no longer time-varying but are constant within the symbol duration when the transmission channel is slow fading.

In our thesis, slow fading channel over multiple MC-CDMA symbols is considered.

#### 2.1.3 Fading Effects Due to Multipath Time Delay Spread

While the fast or slow fading describes the time variant nature of the communication channel, the flat or frequency selective fading characterizes the time spreading of the signal. To determine whether a channel is flat or frequency selective fading, the term delay spread has to be defined first. The delay spread is defined as the time between the first and last received component if a single transmitted signal during which the multipath signal power falls to some threshold level below that of the strongest component. The threshold level might be chosen at 10 or 20 dB below the level of the strongest component. Analogous to delay spread in the time domain, coherence bandwidth is used to characterize the channel in the frequency domain. The delay spread and the coherence bandwidth are inversely proportional to each other although their exact relationship is a function of the multipath structure.

A signal undergoes flat fading if the symbol duration is much greater than the delay spread, or in the frequency domain, the signal bandwidth is much smaller than the coherence bandwidth. In such a case, the mobile radio channel has a constant gain and linear phase response over a bandwidth which is greater than the bandwidth of the transmitted signal. The channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver. However, the strength of the received signal changes with time, due to fluctuations in the gain of the channel caused by multipath. Flat fading channels are also known as narrowband channels since the bandwidth of the applied signal is narrow as compared to the coherence bandwidth.

On the contrary, if the symbol duration is smaller than the delay spread, or the signal bandwidth is greater than the coherence bandwidth, the channel is a frequency selective one. A frequency selective fading channel possesses a non-constant gain and nonlinear phase over a bandwidth that is smaller than the bandwidth of the transmitted signal. Hence the received signal includes multiple resolvable versions of the transmitted waveform which are attenuated and delayed in time, and thus the received signal is distorted. Frequency selective channels are also known as wideband channels since the signal bandwidth is wider than the bandwidth of the channel impulse response. As time varies, the channel varies in gain and phase across the spectrum of the transmitted signal, thus resulting in time varying distortion in the received signal.

In this thesis, we assume the band of frequency under consideration undergoes frequency selective fading while each sub-carrier undergoes frequency non-selective fading or flat fading. Furthermore, we assume fading between any two sub-carriers is uncorrelated.

#### 2.2 CDMA Systems

CDMA is a multiple access scheme that distinguishes users by assigning unique signature sequences to these users. Although, the users sharing the same frequency spectrum and transmit at the same time, the receiver is able to distinguish each user's information from other users by correlating the received signal with the signature sequences assigned to that particular user. Encoding the user information with its

unique signature sequence usually enlarges the user's signal bandwidth. Hence this technique is also known as spread spectrum (SS) [38].

The signature sequences are designed such that the cross-correlation of signature sequences of any two users is almost zero. This allows multiple users to transmit in the same spectrum without interfering with each other. The cross-correlation between pair of signature sequences is given as

$$C_{ij}(\tau) = \frac{1}{T} \int_{0}^{T} a_{i}(t) \cdot a_{j}(t-\tau) dt.$$
 (2.5)

In CDMA systems, the most common way to spread a signal is Direct Sequence Spread Spectrum (DS-SS). In DS-SS, a narrow band signal is directly multiplied by the DS-CDMA signature sequence. Figure 2.2 shows a block diagram of a DS-CDMA transmitter and receiver.

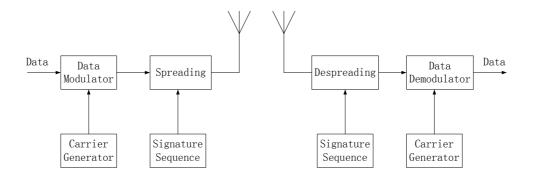


Figure 2.2 Block diagram of DS-CDMA Transmitter and Receiver

The binary data is modulated to a radio frequency carrier. The modulated signal is then spread by the signature sequence. After transmission of the signal, the receiver despreads the SS signal using a locally generated signature sequence. After the demodulation, the original data is recovered. Signature sequences of length equal to the processing gain (PG) can also be used. Such signature sequences are termed as short codes. Assume the CDMA system has K simultaneous users, each is assigned a signature sequence  $a_k(t)$  of duration T, where T is the symbol duration.  $a_k(t)$  is given by

$$a_k(t) = \sum_{g=0}^{G-1} a_k(g) p(t - gT_c), \qquad (2.6)$$

where  $\{a_k(g), g = 0, 1, ..., G - 1\}$  is a short code sequence consisting of G chips that take values  $\{\pm 1\}$  for user  $k \cdot p(t)$  is a pulse shape of duration  $T_c$ , where  $T_c$  is the chip duration. For simplicity, short code will be used for illustration in this thesis, but the algorithm developed can extend to the case when long code is being used.

CDMA has numerous inherent advantages that are derived from the spectral spreading. These advantages include: improved capacity, narrow-band interference rejection and higher privacy, etc [38]. However, CDMA also has some certain disadvantages. First, MAI is the limiting factor for a CDMA system employing the conventional detector [38]. Even if perfect orthogonal signature sequence can be used for short code, multipath will still distort this orthogonality. The conventional detector makes no attempt to overcome MAI and assumes that the aggregate noise plus MAI is white and Gaussian. If the signature sequences are orthogonal (i.e., in (2.5),  $C_{ij}(0) = 0$  for  $i \neq j$ ), the interference from the other users vanishes and the conventional detector is optimum. If multipath is present or the signature sequences are not perfectly orthogonal to the *k*th user's signature sequence, the interference from other users can become excessive if the received power levels of one or more of the

other users are sufficiently larger than the power level of the kth user. This is known as the near-far problem in multi-user communications.

Also, due to the severe ISI as a result of multipath fading, conventional CDMA has been designed only for low- or medium-bit-rate transmission, otherwise many RAKE fingers would have to be used to equalize the received signal. OFDM is the technique proposed to overcome this problem.

## 2.3 Orthogonal Frequency Division Multiplexing

OFDM is a multicarrier modulation scheme [39] [40] [41] [42]. The concept of using parallel data transmission and frequency division multiplexing was published in the mid-1960s [43] [44] and a U.S. patent was filed and issued in 1970 [45]. In OFDM system, multiple data symbols are transmitted in parallel using different sub-carriers. These sub-carriers have overlapping spectrum, but their signal waveforms are specifically chosen to be orthogonal. Mostly, OFDM systems are designed such that each sub-carrier is narrow enough in bandwidth in order to experience frequency flat fading. This also ensures that the sub-carriers remain orthogonal when received over a moderately frequency selective but time-invariant channel.

The orthogonality of the sub-carriers implies that the spectrum of each sub-carrier has a null at the centre frequency of each of the other sub-carriers in the system and therefore guard frequency band is not needed, which makes OFDM is the most efficient FDM scheme [1]. The orthogonality also results in no interference between the sub-carriers. A concept of OFDM signal compared with conventional multicarrier technique signal is shown in Figure 2.3. In practice, inter-channel interference (ICI) due to loss of orthogonality exists due to the presence of Doppler spread and OFDM symbols synchronization error. Some design precautious need to be taken.

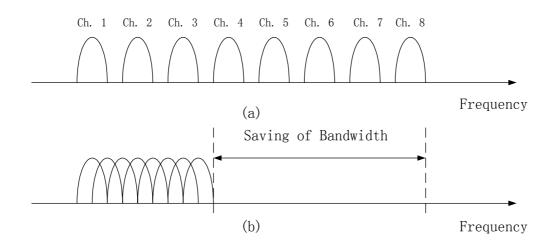


Figure 2.3 Concept of OFDM Signal (a) Conventional multicarrier technique (b) OFDM technique

OFDM is robust against multipath delay spread. In OFDM systems, a data stream of very high rate is converted to multiple parallel data streams of much lower rates using serial-to-parallel converter, and transmitted in parallel over a number of narrow-band sub-carriers. This ensures flat fading for transmission over all the sub-carriers, which will minimize the effect of ISI. The effect of ISI can be reduced further through adding guard periods between the transmitted OFDM symbols. The guard period allows time for multipath signals from the previous OFDM symbol to die away before the information from the current symbol is sent. The most effective guard period to use is a cyclic extension of the symbol. If the last few samples of the OFDM symbol are duplicated at the start of the symbol as the guard period, this effectively extends the length of an OFDM symbol to maintain the orthogonality of the waveform [46].

The parallel transmission of OFDM also can effectively randomize the burst errors caused by the Rayleigh fading [47] because not all sub-carriers will be in deep fading. Instead of a burst of adjacent symbols in a sub-carrier being completely destroyed, parallel transmission will spread the burst of errors over a few error-correction frames. This allows a more reliable recovery of data.

A block diagram of OFDM transceiver is shown in Figure 2.4. Another significant advantage of OFDM is that modulation can be performed by a simple Inverse Discrete Fourier Transform (IDFT) which can be implemented very efficiently as an Inverse Fast Fourier Transform (IFFT) [48] as shown in Figure 2.4. Accordingly in the receiver only an FFT is needed to reverse this operation.

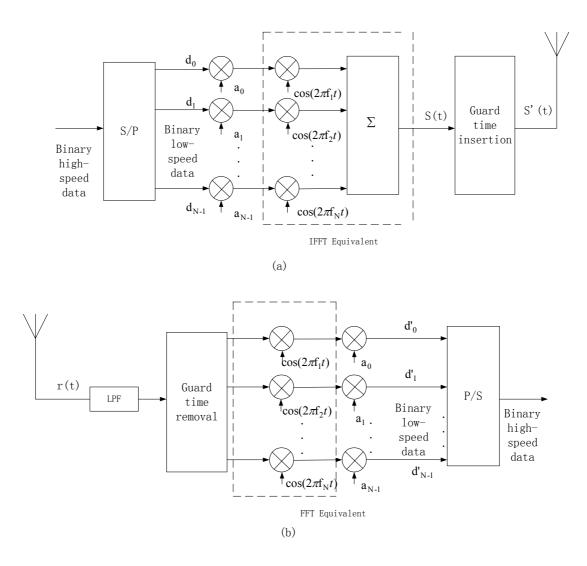


Figure 2.4 OFDM Transmission Systems (a) transmitter (b) receiver

## 2.4 MC-CDMA Systems

In 1993, three types of new multiple access schemes based on a combination of CDMA and OFDM techniques were proposed. That is "MC-CDMA" by N. Yee, J-P. Linnartz, and G. Fettweis [2], K. Fzael and L. Papke [3], and A. Chouly, A. Brajal, and S. Jourdan [4]; "MC-DS-CDMA" by V. DaSilva and E.S. Sousa [5]; and "MT-CDMA" by L. Vandendorpe [6]. These signals are easily transmitted and received using FFT device without increasing the transmitter and receiver

complexities, and have the attractive feature of high spectral efficiency due to minimally densely sub-carrier spacing.

The three MC-CDMA schemes are categorized mainly into two groups, i.e., MCM with spreading in the frequency domain and MCM with spreading in the time domain. There are two schemes corresponding to the second group, MC-DS-CDMA and MT-CDMA, whereas there is only one scheme, known as MC-CDMA, in the first group.

#### 2.4.1 MC-CDMA of Frequency Domain Spreading

#### 2.4.1.1 Transmitter Structure

In this design, the incoming bit stream is copied to N symbols. These N symbols are each modulated onto different orthogonal carrier frequencies. The spreading of the symbol is performed in the frequency domain before modulating to the carrier frequencies. Each carrier is spread with a chip from the signature sequence belonging to the user who sends the data. This is equivalent to performing a N-point S/P conversion after a data stream has been spread by the signature sequence.

Figure 2.5 and 2.6 show MC-CDMA transmitter structure of the *k*th user applying Binary Phase Shift Keying (BPSK) scheme and the power spectrum of the transmitted signal, respectively, where  $G_{MC}$  denotes the PG, N the number of sub-carriers and  $a_k(t) = [a_k(1), a_k(2), \dots, a_k(G_{MC})]$  is the signature sequence of the *k*th user. As shown in Figure 2.5, a single data symbol is replicated into *N* and then modulated to a sub-carrier of  $f_i = [f_1, f_2, ..., f_{G_{MC}}]$ . In the figure, we assume that  $N = G_{MC}$ , however, we do not have to choose  $N = G_{MC}$ , which means long code can also be used. If the original symbol rate causes frequency selective fading in each sub-carrier, multiple bit streams transmission should be used. The data stream needs to be first S/P divided into multiple bit sequence before spreading over the frequency domain. This is because it is crucial for MCM transmission to have flat fading over each sub-carrier. Figure 2.7 and 2.8 show the modification to ensure flat fading in each sub-carrier, where *T* denotes the original symbol duration, and the original data sequence is first converted into *P* parallel sequences, and then each sequence is transmitted over one of the  $G_{MC}$  sub-carriers. ( $N = P \times G_{MC}$ ).

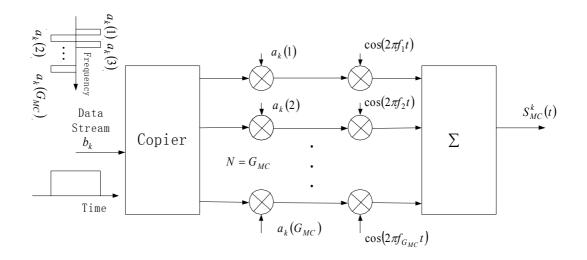


Figure 2.5 Transmitter of MC-CDMA Scheme

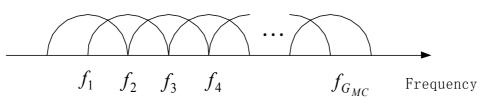


Figure 2.6 Power Spectrum of Transmitted Signal

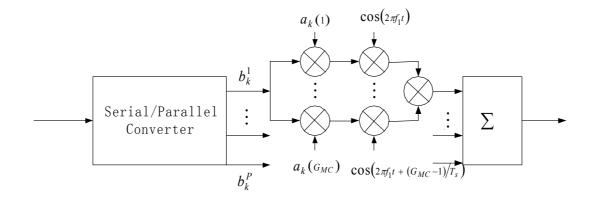


Figure 2.7 Modification of MC-CDMA Scheme, Transmitter

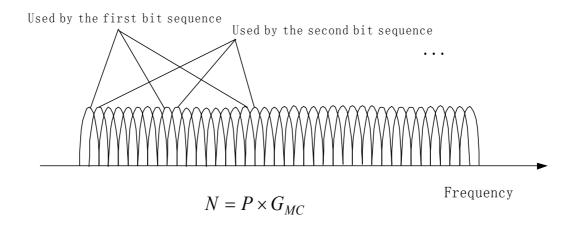


Figure 2.8 Power Spectrum of Transmitted Signal of Modified MC-CDMA Scheme

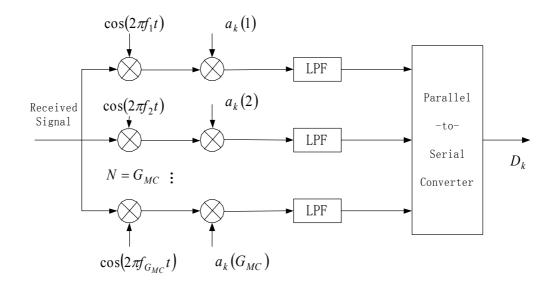


Figure 2.9 Receiver of MC-CDMA Scheme

#### 2.4.1.2 Receiver Structure

Figure 2.9 shows the structure of MC-CDMA receiver of the kth user. The receiver reverses the operation of the transmitter. First, the received signal is demodulated, equivalent to multiplying this signal with N orthogonal carrier frequencies. The demodulated signals are each multiplied with the same signature sequence used at the transmitter and then low pass filtered the resulting signals.

## 2.4.2 MC-CDMA of Time Domain Spreading

This group of scheme includes two types of MC-CDMA schemes, i.e. MC-DS-CDMA and MT-CDMA schemes.

#### 2.4.2.1 Transmitter of MC-DS-CDMA Scheme

Figure 2.10 and 2.11 show the MC-DS-CDMA transmitter of the *k*th user and the power spectrum of the transmitted signal, respectively, where  $G_{MD}$  denotes the PG, N the number of sub-carriers and  $a_k(t) = [a_k(1), a_k(2), ..., a_k(G_{MD})]$  is the signature sequence of the *k*th user.

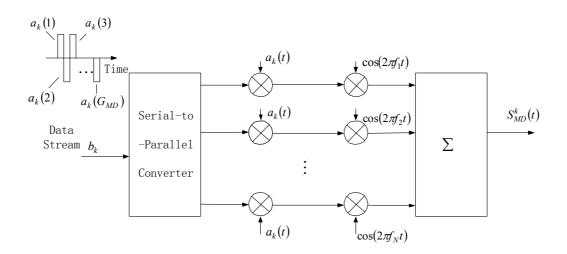


Figure 2.10 Transmitter of MC-DS-CDMA Scheme

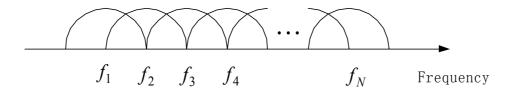


Figure 2.11 Power Spectrum of MC-DS-CDMA Scheme Transmitted Signal

The transmitter spreads the S/P converted data streams using a given signature sequence in the time domain so that the resulting spectrum of each sub-carrier can satisfy the orthogonal condition with the minimum frequency separation, as shown in Figure 2.10.

#### 2.4.2.2 Receiver of MC-DS-CDMA Scheme

Figure 2.12 shows an MC-DS-CDMA receiver structure. The signals are demodulated by the N sub-carriers and despread with the user's signature sequence.

It is important to note that each symbol in the MC-DS-CDMA is spread in time domain by the same signature sequence in one of the sub-carriers while in the MC-CDMA, each symbol is spread by a signature sequence in frequency domain but one chip per sub-carrier.

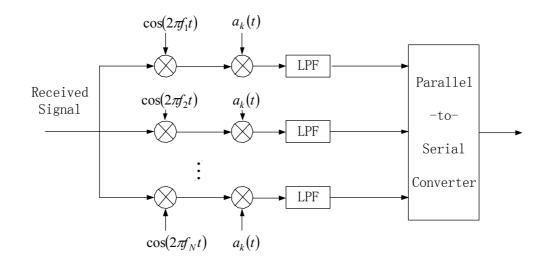


Figure 2.12 Receiver of MC-DS-CDMA Scheme

## 2.4.2.3 Transceiver of MT-CDMA Scheme

The MT-CDMA transmitter spreads the S/P converted data streams using a given signature sequence in the time domain. The spectrum of each sub-carrier of MT-CDMA prior to spreading operation satisfies the orthogonal condition which subsequently loses the orthogonal quality after spreading. Loss of orthogonality after spreading results in inter-sub-carrier interference. In the frequency domain, the bandwidth of each sub-carrier after spreading is larger than the coherence bandwidth

of the channel, therefore, with a high PG, each sub-carrier will experience frequency selective fading. The MT-CDMA scheme uses long codes to spread each sub-carrier signal. Figure 2.13 and 2.14 show the MT-CDMA transmitter of the *k*th user applying BPSK scheme and the power spectrum of the transmitted signal, respectively, where  $G_{MT}$  denotes the PG.

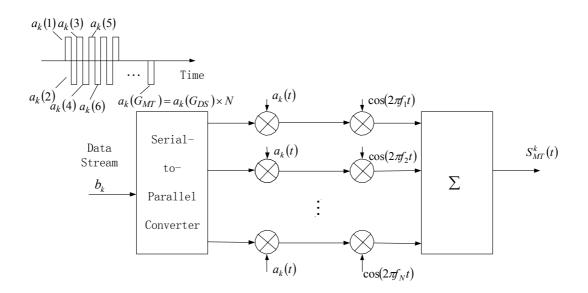


Figure 2.13 Transmitter of MT-CDMA Scheme

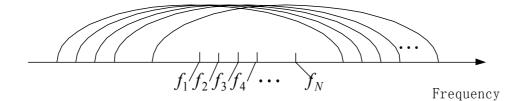


Figure 2.14 Power Spectrum of Transmitted Signal of MT-CDMA Scheme

Figure 2.15 shows an MT-CDMA receiver composed of *N* Rake combiners [15], each of which has the same structure as the DS-CDMA Rake receiver. This is an optimum receiver for an additive white Gaussian Noise (AWGN) channel.

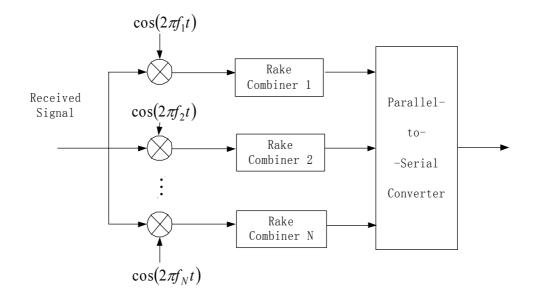


Figure 2.15 Receiver of MT-CDMA Scheme

## 2.5 Comparison of the MC-CDMA Schemes

Based on the description highlighted previously, a comparison on the features among the three schemes is shown in Table 2.1 [52]. The rectangular pulse shape is assumed in all the systems. The required bandwidths of MC-CDMA and MC-DS-CDMA are almost half of that of the DS-CDMA and the bandwidth of MT-CDMA is comparable with that of DS-CDMA scheme.

Table 2.2 briefly compares the advantages and disadvantages of three multicarrier CDMA systems.

	DS-CDMA	MC-CDMA	MC-DS-CDMA	MT-CDMA
Number of sub-carriers	1	N	Ν	Ν
PG	G	G	G	G
Symbol duration at sub-carrier	Т	NT/G	NT	NT
Chip duration	T/G	NT/G	NT/G	T/G
Frequency separation		1/T	G/NT	1/ <i>NT</i>
Required bandwidth	2G/T	(N+1)G/NT	(N+1)G/NT	(2GN+N-1)/NT

Table 2.1 Features of various CDMA Schemes

Table 2.2 Comparison of Advantages and Disadvantages of Three Multicarrier CDMA Schemes

Scheme	Advantages	Disadvantages
MC-CDMA	<ul> <li>Transmits multiple carrier per symbol, therefore diversity combining can be applied</li> </ul>	<ul> <li>Implementation complexity is higher than other MC-CDMA schemes</li> </ul>
MC-DS-CDMA	<ul> <li>Good for uplink transmission because it does not require that the users be synchronized</li> <li>Diversity combining can be applied when the same symbol is repeated on all the sub-carriers</li> <li>It needs fewer carriers and thus allows the PG to be increased</li> <li>Robust to timing errors and</li> </ul>	Performance is not as good as MC- CDMA
MT-CDMA	<ul> <li>frequency offsets</li> <li>Longer spreading codes result in a reduction in self interference and multiple access interference as compared to those experience in conventional CDMA system</li> <li>Detection can be done non- coherently</li> </ul>	ICI

## **Chapter 3**

# System Model and Performance Evaluation of Single User MC-DS-CDMA Systems

## 3.1 MC-DS-CDMA System Model

In this section, we will introduce the MC-DS-CDMA system model used in this thesis. The channel of each sub-carrier is assumed to vary slowly and remains nearly constant within each updating period. Without loss of generality, it is also assumed that the channel undergoes frequency selective Rayleigh fading, but each sub-carrier undergoes frequency non-selective fading and the fading process between sub-carriers is independent of each other. For simplicity, the channel gains of all sub-carriers are independent and identically distributed Rayleigh fading random variables.

#### **3.1.1** Transmitter for the Downlink

Figure 3.1 shows the transmitter at the BS for the downlink. The data stream of the kth user,  $b'_k$ , is spread by the user's signature sequence  $a_k(g)$  {g = 0,1,2,...,G-1}, with PG G in the time domain. The data stream of each user will be split into L parallel sub-streams. Then the S/P converted data sub-streams will pass through a "Downlink Sub-carrier Selector" block. Our sub-carrier allocation scheme to meet the designed objective will be applied in this block. The parallel data stream will be modulated onto the corresponding sub-carriers decided by the "Downlink Sub-carrier Selector" block. Then after summing up, the combined signal is sent out.

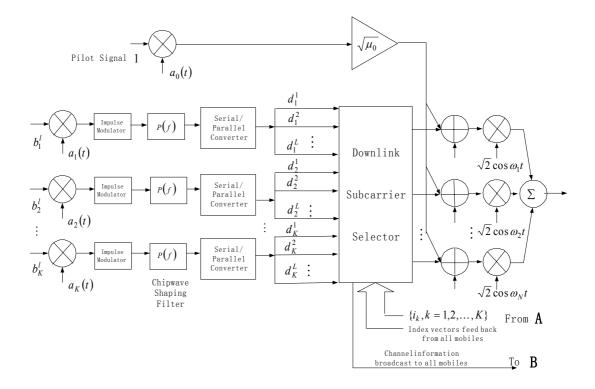


Figure 3.1 System Model Transmitter for the Downlink (at BS)

In Figure 3.1, we should notice that there exists a pilot channel in our system model. The purpose of this pilot channel is to let each user estimate the downlink sub-carriers' channel gain, which will be used in our sub-carrier allocation process. The accurate and in-time estimation of the channel gain is important since the allocation is based on the instantaneous channel condition. The inaccurate estimation of the channel will affect the performance of our sub-carrier allocation algorithm. The data transmitted in the pilot channel is known to the receiver. We assume a sequence of "1" is transmitted.  $a_0(g)$  is the signature sequence allocated to the pilot channel, and  $\mu_0$  is the pilot signal power. The pilot signal is transmitted over all sub-carriers, as shown in Fig 3.1. In this thesis, we assume the channels are perfectly estimated.

The index vector of each user is fed back to the BS, as shown in Figure 3.1, where for "Downlink Sub-carrier Selector" to perform sub-carrier allocation. The channel information will be broadcast to all the mobiles.

#### **3.1.2** Receiver for the Downlink

Figure 3.2 shows the receiver for the downlink. At the receiver, the received signal is coherently demodulated by respective sub-carrier before passing through a chip wave-shaping filter. Coherent detection is used hence the random phase resulting from the channel will not be included in the discussion. Let K denote the number of active users in the system and  $s_{k,n}$  denote the sub-carrier allocation coefficients,

which is obtained from "Downlink Sub-carrier Selector". The received signal at the receiver is given as

$$r(t) = \sqrt{2E_c} \sum_{k=1}^{K} \sum_{l=-\infty}^{\infty} b_k^l a_k (t - lT) \sum_{n=1}^{N} s_{k,n} \alpha_{k,n} \cos(\omega_n t + \phi_{k,n}) + \sqrt{2E_c \mu_0 / N} \sum_{l=-\infty}^{\infty} a_0 (t - lT) \sum_{n=1}^{N} \alpha_{k,n} \cos(\omega_n t + \phi_{k,n}) + w(t), \quad (3.1)$$

where

$$s_{k,n} = \begin{cases} 1 & \text{if } k \text{th user's } n \text{th subcarrier is selected} \\ 0 & \text{if } k \text{th user's } n \text{th subcarrier is not selected} \end{cases}$$
(3.2)

and  $a_k(t)$  has been defined in (2.6). In (3.1),  $E_c$  is the energy per chip, T and  $T_c$ are the symbol and chip period, respectively, with  $T = GT_c$ .  $\alpha_{k,n}$  is the amplitude of the channel gain of the *n*th sub-carrier of the *k*th user.  $\omega_n$  is the carrier angular frequency of the *n*th sub-carrier. The frequency response of the chip waveform p(t)is bandlimited to  $W_s$  and satisfies the Nyquist criterion. The pilot symbol  $b_0^l$  is assumed to be "1" for all l and is sent over all sub-carriers for the user to perform synchronization and sub-carriers' channel estimation. w(t) is the AWGN with zero mean and variance  $\eta_0$ .

The output of the wave-shaping filter of the *n*th subcarrier is given by

$$y(t) = \sqrt{E_c} \alpha_{k,n} \sum_{k \in U_n} \sum_{l=-\infty}^{\infty} b_k^l \sum_{g=0}^{G-1} a_k(g) h[t - (lG + g)T_c]$$

+ 
$$\sqrt{E_{c}\mu_{0}/N}\alpha_{k,n}\sum_{l=-\infty}^{\infty}\sum_{g=0}^{G-1}a_{0}(g)h[t-(lG+g)T_{c}]+\widetilde{w}(t),$$
 (3.3)

where  $U_n$  is the set of the active users who share the *n*th sub-carrier among the total K active users, h(t) is the matched filter impulse response and  $\tilde{w}(t)$  is the filtered AWGN. The output signal of the correlator for the *l*th bit can be written as

$$Y_{n} = \sum_{g'=0}^{G-1} a_{k}(g')y[(g'+lG)T_{c}]$$
  
=  $S_{n} + I_{n} + W_{n}$ , (3.4)

and

$$S_n = G\sqrt{E_c}\alpha_{k,n}b_k^l, \qquad (3.5)$$

$$I_{n} = G\sqrt{E_{c}}\alpha_{k,n} \times \left[\sum C_{k,j}(0)b_{j}^{l} + \sqrt{\frac{\mu_{0}}{N}}C_{k,0}(0)\right], \quad (3.6)$$

$$W_n = \sum_{g'=0}^{G-1} a_k(g') \widetilde{w}[(g'+lG)T_c].$$
(3.7)

 $C_{k,j}(0)$  is the cross correlation between the *k*th and *j*th user signature sequences defined by

$$C_{k,j}(0) = \frac{1}{G} \sum_{g=0}^{G-1} a_k(g) a_k(g).$$
(3.8)

Note that  $h[(g'-g)T_c] = 0$  for  $g' \neq g$ , since it satisfies the Nyquist criterion.

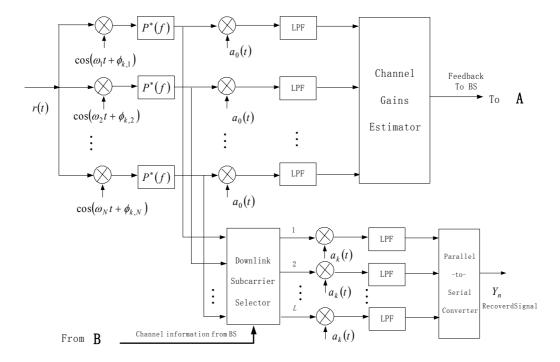


Figure 3.2 System Model Receiver for the Downlink (at Individual Mobile)

#### 3.1.3 Transmitter and Receiver for the Uplink

Figure 3.3 shows the transmitter of the uplink and BPSK is used. Each user will transmit pilot signals to the BS for it to perform channel estimation. Pilot signals are not transmitted on all the sub-carriers. They are transmitted on those un-selected sub-carriers with comparatively lower power. The pilot signals are known to the BS. The transmission of pilot signal is controlled by the "Uplink Sub-carrier Selector", which is shown in Figure 3.3. Figure 3.4 shows the receiver at the BS for the uplink of the *k*th user. The channel estimation is performed on all the sub-carriers using the pilot signals received from mobiles. Compared with the downlink transmission, the uplink transmission uses the pilot signal but not the pilot channel to perform channel

estimation. So the signature sequence for pilot signal is the same as the signature sequence of the respective user for the uplink. "Uplink Sub-carrier Selector" gathers index vectors from all mobiles to perform sub-carrier allocations and broadcasts the results to all mobiles, as shown in Figure 3.4.

In this thesis, we illustrate the sub-carrier allocation algorithms using downlink transmission but this can be easily extended to the uplink.

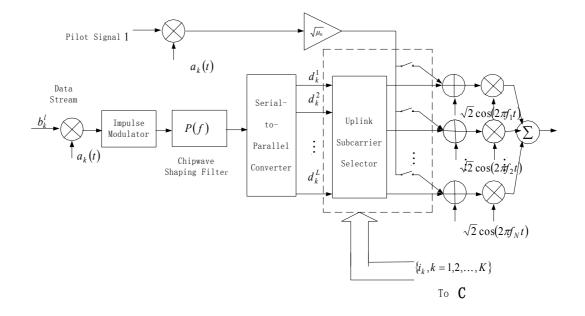


Figure 3.3 System Model Transmitter for the Uplink (at Mobile)

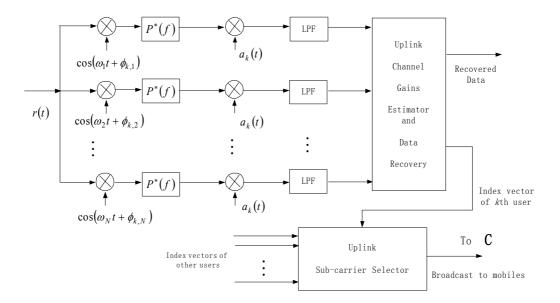


Figure 3.4 System Model Receiver of *k*th user for the Uplink (at BS)

## 3.2 BER Evaluation of Single User MC-DS-CDMA Systems

The BER of a single user employing BPSK modulation scheme in AWGN channel is given by (3.9) [50]

$$P_{be} = Q\left(\sqrt{\frac{2E_b}{\eta_0}}\right),\tag{3.9}$$

where

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{\infty} \exp\left(-\frac{u^2}{2}\right) du .$$
(3.10)

In (3.9),  $E_b/\eta_0$  is defined as the SNR per bit.

Eq. (3.9) can be easily developed to the case where the channel undergoes Rayleigh flat fading. Assume the channel gain is  $\beta = \alpha^2$ , the instantaneous BER can be expressed as

$$P_{be}(\beta) = Q\left(\sqrt{\frac{2E_b\beta}{\eta_0}}\right). \tag{3.11}$$

Since  $\alpha$  is the amplitude of the channel gain and follows Rayleigh distribution.  $\beta$  is exponentially distributed. If we aim to compute the average BER of a given sub-carrier, Eq. (3.11) will be averaged over the probability density function (p.d.f) of  $\beta$ . If a user uses all N sub-carriers to transmit, the performance of a MC-DS-CDMA has no difference from the user using only one sub-carrier to transmit, since all sub-carriers are assumed to be independent and identically distributed.

In some occasions, one user may select only one sub-carrier of a MC-DS-CDMA system to transmit. Assume that the channel gains of all the sub-carriers can be ordered at every updating instant, and the user always selects the sub-carrier having the *p*th largest (p = 1, 2, ..., N) channel gain to transmit its information. In the following, we will show how the BER performance of such user can be evaluated.

Assume there are *N* sub-carriers for user to transmit data. If in the sub-carrier allocation updating occasion, the channel gains of these *N* sub-carriers are given as  $\{\alpha_1^2, \alpha_2^2, ..., \alpha_N^2\}$ . If we make ordering on these channel gains so that  $r_1 = \max(\{\alpha_1^2, \alpha_2^2, ..., \alpha_N^2\})$ ,  $r_2 = \max(\{\alpha_1^2, \alpha_2^2, ..., \alpha_N^2\} - r_1)$  ...  $r_N = \max(\{\alpha_1^2, \alpha_2^2, ..., \alpha_N^2\} - r_1 - r_2 - ... - r_{N-1})$ , we have

$$r_1 \ge r_2 \ge \cdots \ge r_N$$
.

In the following, we derive the statistics of  $r_p$ , (p = 1, 2, ..., N). If  $r_p$  is exponentially distributed with variance  $2\sigma^2$  and  $F_{\alpha^2}(r)$  is the cumulative distribution function (c.d.f) of  $r_p$ , then

$$f_{\alpha^2}(r) = \frac{1}{2\sigma^2} e^{-(r/2\sigma^2)}, \qquad (3.12)$$

$$F_{a^2}(r) = 1 - e^{-(r/2\sigma^2)}.$$
 (3.13)

Since the probability that there are (N - p) sub-carriers (i.e.  $r_{p+1}, ..., r_N$ ) having channel gains less than  $r_p$  is given by  $F_{\alpha^2}(r)$ , and the probability that (p-1)sub-carriers (i.e.  $r_1, r_2, ..., r_{p-1}$ ) have channel gains large than  $r_p$  is given by  $1 - F_{\alpha^2}(r)$ , we can obtain the probability density function of  $r_p$  as [51]

$$f_{p}(r) = \frac{N!}{(N-p)!(p-1)!} F_{\alpha^{2}}(r)^{N-p} (1 - F_{\alpha^{2}}(r))^{p-1} f_{\alpha^{2}}(r).$$
(3.14)

Substituting the expressions of  $f_{\alpha^2}(r)$  and  $F_{\alpha^2}(r)$  into (3.14), we can obtain the probability density function of  $r_p$  given by

$$f_{p}(r) = \frac{N!}{(N-p)!(p-1)!} \frac{1}{2\sigma^{2}} \left(1 - e^{-(r/2\sigma^{2})}\right)^{N-p} \left(e^{-(r/2\sigma^{2})}\right)^{p-1} e^{-(r/2\sigma^{2})}.$$
 (3.15)

Using (3.11) and (3.15), the average BER of the *p*th largest sub-carrier is given by

$$P_{be,p} = \int_{0}^{\infty} Q\left(\sqrt{\frac{2E_{b}\beta}{\eta_{0}}}\right) f_{p}(\beta) d\beta .$$
(3.16)

Substituting (3.10) and (3.15) into (3.16), we will have

$$\begin{split} P_{be,p} &= \int_{0}^{\infty} \int_{\sqrt{\frac{2E_{b}\beta}{\eta_{0}}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^{2}}{2}} du \cdot \frac{N!}{(N-p)!(p-1)!} \Big( 1 - e^{-(\beta/2\sigma^{2})} \Big)^{N-p} \\ &\times \Big( e^{-(\beta/2\sigma^{2})} \Big)^{p-1} \frac{1}{2\sigma^{2}} e^{-(\beta/2\sigma^{2})} d\beta \,. \end{split}$$

By changing the order of the integration, we obtain

$$\begin{split} P_{be,p} &= \int_{0}^{\infty} \int_{0}^{\frac{\eta_{0}u^{2}}{2E_{b}}} \frac{N!}{(N-p)!(p-1)!} \cdot \frac{1}{2\sigma^{2}} \left(1 - e^{-(\beta/2\sigma^{2})}\right)^{N-p} \\ &\times \left(e^{-(\beta/2\sigma^{2})}\right)^{p-1} e^{-(\beta/2\sigma^{2})} d\beta \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{u^{2}}{2}} du \\ &= \int_{0}^{\infty} \int_{0}^{\frac{\eta_{0}u^{2}}{2E_{b}}} \frac{N!}{(N-p)!(p-1)!} \cdot \frac{1}{2\sigma^{2}} \left(1 - e^{-(\beta/2\sigma^{2})}\right)^{N-p} \\ &\times \left(e^{-(\beta/2\sigma^{2})}\right)^{p} d\beta \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{u^{2}}{2}} du \,. \end{split}$$

Since

$$(1 - e^{-(\beta/2\sigma^2)})^{N-p} = \sum_{t=0}^{N-p} {N-p \choose t} (-1)^t e^{-\frac{t}{2\sigma^2}\beta},$$

then we have

$$\begin{split} P_{be,p} &= \int_{0}^{\infty} \int_{0}^{\frac{\eta_{0}u^{2}}{2E_{b}}} \frac{N!}{(N-p)!(p-1)!} \cdot \frac{1}{2\sigma^{2}} \\ &\times \sum_{t=0}^{N-p} \binom{N-p}{t} (-1)^{t} e^{-\frac{(p+t)}{2\sigma^{2}}\beta} d\beta \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{u^{2}}{2}} du \\ &= \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} \binom{N-p}{t} \frac{(-1)^{t}}{t+p} \times \left\{ \frac{1}{2} + \frac{1}{2} \sqrt{\frac{\gamma}{\gamma} + (p+t)} \right\}, \end{split}$$

where  $\overline{\gamma} = E\left\{\alpha_{k,n}^2 E_b / \eta_0\right\}$ . Using the following identity

$$\frac{N!}{(N-p)!(p-1)!}\sum_{t=0}^{N-p} \binom{N-p}{t}\frac{(-1)^t}{t+p} = 1,$$

we obtain

$$P_{be,p} = \frac{1}{2} + \frac{1}{2} \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {\binom{N-p}{t}} \frac{(-1)^t}{t+p} \sqrt{\frac{\overline{\gamma}}{\overline{\gamma} + (p+t)}}.$$
 (3.17)

Eq. (3.17) [9] gives the average BER when a user selects the sub-carrier having the *p*th largest channel gain to transmit, without considering the presence of MAI.

In the next chapter, multiple users will share the same sub-carriers for transmission. We shall modify the analysis to make it suitable for transmission in the presence of MAI. The optimal solution to this sub-carrier allocation for MC-DS-CDMA can be formulated by:

$$\min \sum_{k=1}^{K} \sum_{n=1}^{N} s_{k,n} P_{be,k,n} \qquad , \qquad (3.18)$$

subject to

$$\sum_{n=1}^{N} s_{k,n} = L \qquad \forall k = 1, 2, \dots, K,$$
(3.19)

where

$$\mathbf{S} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1N} \\ s_{21} & s_{22} & \cdots & s_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ s_{K1} & s_{K2} & \cdots & s_{KN} \end{bmatrix},$$
(3.20)

is a  $K \times N$  matrix which means the system has *K* active users and each user with *N* sub-carriers. Its elements  $s_{k,n}$  are the sub-carrier allocation coefficients, as defined in (1.1). BS will update the allocation coefficients at every updating stage so that each user will be allocated with the appropriate sub-carriers. We assume that each user is allocated with the same number of sub-carriers, defined by (3.19), and the value of *L* chosen depends on the data rate to be supported.  $P_{be,k,n}$  is the average BER of the *k*th user in the *n*th sub-carrier. The value can be predicted by (3.17) depending on the ordering index *p* of the sub-carrier. We will modify this expression in the next chapter by taking into consideration the interference of the other users who are also allocated to use the same sub-carrier when evaluating BER. Solving **S** to optimize the optimal objective function (3.18) at every updating stage is not easy, since the search space is very large. In the next chapter, we will formulate a sub-optimal objective function to reduce the complexity of the allocation process.

## **Chapter 4**

# Adaptive Sub-carrier Allocation Scheme in the Presence of Multiple Access Interference

In this chapter, we propose a sub-optimal adaptive sub-carrier allocation scheme in the presence of MAI. The chapter begins with the formulation of the objective function and the constraints. The algorithm to solve the problem iteratively is presented. We then derive the expression to evaluate the average BER that will be used in the algorithm in terms of the unknown allocation coefficients. Simulation results and discussion will be presented.

## 4.1 Formulation of the Problem

The optimal solution to the sub-carrier allocation problem given in (3.18) has complexity growing exponentially with the number of users and sub-carriers. In this chapter, we propose a sub-optimal scheme. In our scheme, the objective is to minimize the BER of the respective user, rather than minimizing the overall BER of all users given in (3.18). Since our scheme considers MAI when minimizing each user's BER performance, the total system performance is expected to approach the optimal value. The sub-carrier allocation scheme is for MC-DS-CDMA systems, where random signature sequences are used to identify the users.

Here, let  $s_{k,n}$  denote the sub-carrier allocation coefficient, as defined in (1.1). We can express the allocation process as an objective function

$$\min \sum_{n=1}^{N} s_{k,n} P_{be,k,n} \qquad \forall k = 1, 2, \dots, K,$$
(4.1)

subject to

$$\sum_{n=1}^{N} s_{k,n} = L \qquad \forall k = 1, 2, \dots, K,$$
(4.2)

where

$$\mathbf{S} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1N} \\ s_{21} & s_{22} & \cdots & s_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ s_{K1} & s_{K2} & \cdots & s_{KN} \end{bmatrix}.$$
(4.3)

Note that the single objective function in (3.18) is approximated by *K* objective functions given in (4.1). Hence, this formulation does not guarantee minimum overall BER performance. However, since each user chooses *L* sub-carriers which tries to minimize the BER, the total system BER will approach the minimum also.

By separately considering the BER of respective user, solving of the problem iteratively becomes possible and the algorithm will be described in next section.

## 4.2 Algorithm Description

We now describe the algorithm used to solve (4.1) and (4.2) which is still quite difficult mathematically. Each user estimates the channel gains of all the sub-carriers based on the pilot signal received and generates an index vector according to the order of the sub-carriers channel gains. This index vector of each user is fed back to the BS so that BS will have the order of sub-carriers channel gains of all the users. For example, for user k, the channel gain of each sub-carrier is given in the set  $\{\alpha_{k,1}^2, \alpha_{k,2}^2, \dots, \alpha_{k,N}^2\}$ . If  $\alpha_{k,1}^2$  is the second largest in the set,  $\alpha_{k,2}^2$  is the *N*th largest in the set (i.e. the smallest in the set),  $\ldots$  ,  $\alpha_{k,N}^2$  is the largest in the set, then the index vector of user k  $\{2, N, ..., 1\}$  is fed back to the BS. When calculating BER of  $P_{be,k,n}$ , the index vector fed back from all the users are used. The sub-carrier allocation coefficients of the previous updating stage  $S_{i-1}$  are also used to compute the average BER. We use the objective function together with the constraints defined in (4.1) and (4.2) to obtain the sub-carrier allocation coefficients of the current updating stage  $S_i$ . Similarly, the sub-carrier allocation coefficients  $S_i$ 

obtained will also be used to decide the sub-carrier allocation coefficients of the next updating stage  $S_{i+1}$ .

The basis to the algorithm is that we assume that each of the sub-carrier channel gain is slowly varying, so we can assume the sub-carriers allocated in the previous updating stage to the remaining users remain almost unchanged when computing the BER of a user at the current updating stage. Because the solution to allocate a number of users to each sub-carrier and minimizing the BER of each user are two problems intertwined to each other, which requires to search through a large solution space, this assumption allows us to make use of the previous sub-carrier allocation coefficients, and the solution becomes much easier. The allocation process can begin with  $S_0$  which is obtained by assuming that each user selects its L best sub-carriers regardless of other user's channel condition. Then  $S_0$  and the index vector fed back from users will be used to generate  $S_1$  which is the sub-carrier allocation coefficients of the first updating stage.  $S_1$  and the index vector fed back from users in the second updating stage will be used to generate sub-carrier allocation coefficients  $S_2$ , and so on. The process of generating  $S_i$  from  $S_{i-1}$  is not to be completed in one step, but rather an iteration process. We explain the iteration process with a flow chart as shown in Figure 4.1.

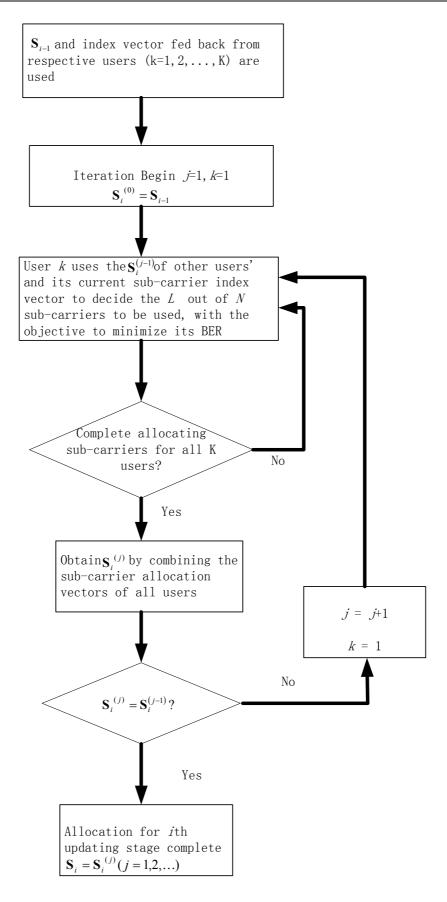


Figure 4.1 Flow Chart of the Iteration

We can summarize the algorithm in five steps:

- 1) Each user orders all the sub-carriers according to the magnitude of its channel gains and generates an index vector. The generated index vectors are fed back to the BS. BS then will have a  $K \times N$  matrix consisting of index information from all the users.
- 2) BS selects sub-carriers for each user separately according to (4.1) and (4.2) iteratively. For each user, *L* sub-carriers which give minimum average BER are selected using the index vector obtained by the user and the sub-carrier allocation coefficients of other users in  $S_{i-1}$ , i.e., when computing the BER of a user in the current updating stage, we assume all the other users still using the previous allocated sub-carriers.
- 3) Repeat step (2) for all users.
- 4) The matrix of the sub-carrier allocation coefficients of the current updating stage  $S_i^{(1)}$  is then obtained. This completes the first iteration. The *j*th iteration can be completed the same way using step (2) and (3).
- 5) To check whether  $\mathbf{S}_{i}^{(1)}$  is the solution, we assume  $\mathbf{S}_{i}^{(0)} = \mathbf{S}_{i-1}$ . If  $\mathbf{S}_{i}^{(1)} = \mathbf{S}_{i}^{(0)}$ , then the sub-carrier allocation coefficients of the *i*th updating stage is given as  $\mathbf{S}_{i} = \mathbf{S}_{i}^{(1)}$  and the process stop. If  $\mathbf{S}_{i}^{(1)} \neq \mathbf{S}_{i}^{(0)}$ , repeat step (2) – (4) to obtain  $\mathbf{S}_{i}^{(2)}$ , and compare whether  $\mathbf{S}_{i}^{(2)} = \mathbf{S}_{i}^{(1)}$ . In general, the *j*th iteration can be checked the

same way by checking whether  $\mathbf{S}_{i}^{(j)} = \mathbf{S}_{i}^{(j-1)}$ . If they are equal, the iteration will stop and  $\mathbf{S}_{i} = \mathbf{S}_{i}^{(j)}$ .

In our simulation, we found that generally only 4-6 iterations are needed for the solution to converge, for a system with four sub-carriers and 16 users. Hence, the algorithm is a promising technique that can be implanted.

## 4.3 Average BER Calculation

Now we show how to obtain the average BER of a sub-carrier which will be used in the objective function (4.1) to allocate sub-carriers. The result obtained in Chapter 3 will be modified here to take into account of MAI.

### 4.3.1 Derivation of Average BER

In Chapter 3, the average BER under a single user case is evaluated. Now we consider a system with K active users. In the system, BPSK is used. We have already obtained the probability of a sub-carrier being the *p*th largest in N sub-carriers, as shown in (3.15). We repeat it here for conveniently reference.

$$f_{p}(r) = \frac{N!}{(N-p)!(p-1)!} \frac{1}{2\sigma^{2}} \left(1 - e^{-(r/2\sigma^{2})}\right)^{N-p} \left(e^{-(r/2\sigma^{2})}\right)^{p-1} e^{-(r/2\sigma^{2})}, \qquad (4.4)$$

where  $2\sigma^2 = E\{\alpha_{k,n}^2\}$  is the variance. The average BER of the *p*th largest sub-carrier for single user case is given by

$$P_{be,p} = \frac{1}{2} + \frac{1}{2} \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {N-p \choose t} \frac{(-1)^t}{t+p} \sqrt{\frac{\gamma}{\gamma+(p+t)}}.$$
 (4.5)

In multiuser case, we will have to take interference into consideration, which is expressed in (3.6). Here, we introduce a new variable

$$\zeta(\kappa_n) = \sum_{j \in U_n^{\prime}} C_{k,j}(0) b_j^l + \sqrt{\frac{\mu_0}{N}} C_{k,0}(0), \qquad (4.6)$$

hence the interference can be written as  $I_n = G\sqrt{E_c} \alpha_{k,n} \zeta(\kappa_n)$ . The set  $U_{n'} = U_n - \{k\}$  and the size of the set  $U_n$  is  $\kappa_n$ , which means there will be  $\kappa_n$  users allocated to the *n*th sub-carrier. When the *l*th transmitted symbol  $b_k^l$  of the *k*th user is assumed to be "-1", we can write the BER conditioned on  $\alpha_{k,n}^2$  and  $\zeta(\kappa_n)$  as

$$P_{be,k,n}[\beta,\zeta(\kappa_n)] = \Pr\{S_n + I_n + W_n \rangle 0 \mid b_k^l = -1, \alpha_{k,n}, \zeta(\kappa_n)\}$$
$$= \Pr\{W_n \rangle G \sqrt{E_c} \alpha_{k,n} (1 - \zeta(\kappa_n)) \mid \alpha_{k,n}, \zeta(\kappa_n)\}$$
$$= \begin{cases} Q\left(\sqrt{\frac{2E_b\beta}{\eta_0}}(1 - \zeta(\kappa_n))\right), & \text{if } \zeta(\kappa_n) < 1\\ 1 - Q\left(\sqrt{\frac{2E_b\beta}{\eta_0}}(1 - \zeta(\kappa_n))\right), & \text{if } \zeta(\kappa_n) \ge 1 \end{cases}.$$
(4.7)

The p.d.f of  $\beta$  is given in (4.4). When  $\beta = r_p$ , which means  $\alpha_{k,n}^2$  is the *p*th largest, we can express the average BER conditioned on  $\zeta(\kappa_n)$  as,

$$P_{be,k,n}[\zeta(\kappa_n)] = \int_0^\infty P_{be,k,n}(\beta,\zeta(\kappa_n))f_p(\beta)d\beta$$

$$= \begin{cases} \frac{1}{2} + \frac{1}{2} \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {N-p \choose t} \frac{(-1)^t}{t+p} \sqrt{\frac{\overline{\gamma}(1-\zeta(\kappa_n))^2}{\overline{\gamma}[1-\zeta(\kappa_n)]^2 + (p+t)}} &, \zeta(\kappa_n) < 1\\ \frac{1}{2} - \frac{1}{2} \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {N-p \choose t} \frac{(-1)^t}{t+p} \sqrt{\frac{\overline{\gamma}(\zeta(\kappa_n)-1)^2}{\overline{\gamma}[1-\zeta(\kappa_n)]^2 + (p+t)}} &, \zeta(\kappa_n) < 1 \end{cases}$$

where  $\overline{\gamma} = E\left\{\alpha_{k,n}^2 E_b / \eta_0\right\}$ . The derivation of (4.8) is quite similar to the case of single user environment given in Section 3.2 and will be presented shortly in Section 4.2 [9]. The difference is that (4.8) consists of multiple users access interference terms other than fading.

If the data symbols  $b_k^l$  of all the users and the chips of random signature sequences are equally distributed, then the mean of the interference term  $\zeta(\kappa_n)$  is zero and its variance is given by

$$\sigma_{\zeta(\kappa_n)}^2 = E\left\{\zeta(\kappa_n)^2\right\} = \sum_{j \in U_n^+} E\left\{C_{k,j}^2(0)\right\} + \frac{\mu_0}{N} E\left\{C_{k,0}^2(0)\right\}$$
$$= \frac{\kappa_n - 1}{G} + \frac{\mu_0}{GN}.$$
(4.9)

Here we assume that  $E\{C_{k,j}^2(0)\} = E\{C_{k,0}^2(0)\} = 1/G, \zeta(\kappa_n)$  can be assumed to be Gaussian if G is large enough by central limit theorem (CLT). We then can obtain the BER averaged over  $\zeta(\kappa_n)$  conditioned on  $\kappa_n$ , which is given by [9]

$$P_{be,k,n|\kappa_{n}} = \int_{-\infty}^{\infty} P_{be,k,n}(\zeta(\kappa_{n})) f_{\zeta(\kappa_{n})}(x) dx$$
  
$$= \frac{1}{2} + \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {N-p \choose t} \frac{(-1)^{t+1}}{t+p} \frac{1}{\sqrt{2\pi\sigma_{\zeta(\kappa_{n})}^{2}}}$$
  
$$\times \int_{0}^{\infty} \sqrt{\frac{x^{2}}{x^{2} + (t+p)/\overline{\gamma}}} \exp\left(-\frac{x^{2}+1}{2\sigma_{\zeta(\kappa_{n})}^{2}}\right) \sinh\left(\frac{x}{\sigma_{\zeta(\kappa_{n})}^{2}}\right) dx.$$
(4.10)

If the sub-carriers for different users fade independently, the probability of choosing one sub-carrier among N sub-carriers will then be equal in the long run. Therefore, the average BER can be formulated using

$$P_{be,k,n} = \sum_{\kappa_n=1}^{K} \binom{K}{\kappa_n} \left(\frac{1}{N}\right)^{\kappa_n} \left(1 - \frac{1}{N}\right)^{K-\kappa_n} P_{be,k,n|\kappa_n}, \qquad (4.11)$$

Eq. (4.11) is used by the objective function (4.1) to select sub-carriers to transmit data.

#### 4.3.2 Derivation for (4.8) and (4.10)

Here we show the derivation of (4.8) and (4.10). We first show how (4.8) is derived when  $\zeta(\kappa_n) < 1$ . The (4.8) is given as

$$P_{be,k,n}[\zeta(\kappa_n)] = \int_0^\infty P_{be,k,n}(\beta,\zeta(\kappa_n))f_p(\beta)d\beta, \qquad (4.12a)$$

when  $\zeta(\kappa_n) < 1$  from (4.7),  $P_{be,k,n}(\beta, \zeta(\kappa_n)) = Q\left(\sqrt{\frac{2E_b\beta}{\eta_0}}(1-\zeta(\kappa_n))\right)$ , substituting

(4.4) into (4.12a), we can obtain

$$P_{be,k,n}[\zeta(\kappa_{n})] = \int_{0}^{\infty} \int_{\sqrt{\frac{2E_{b}\beta}{\eta_{0}}}(1-\zeta(\kappa_{n}))}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^{2}}{2}} du \cdot \frac{N!}{(N-p)!(p-1)!} \left(1-e^{-(\beta/2\sigma^{2})}\right)^{N-p} \times \left(e^{-(\beta/2\sigma^{2})}\right)^{p-1} \frac{1}{2\sigma^{2}} e^{-(\beta/2\sigma^{2})} d\beta .$$
(4.12b)

By changing the order of two random variables when perform integration, we have

$$P_{be,k,n}[\zeta(\kappa_{n})] = \int_{0}^{\infty} \int_{0}^{\frac{\eta_{0}u^{2}}{2E_{b}[1-\zeta(\kappa_{n})]^{2}}} \frac{N!}{(N-p)!(p-1)!} \cdot \frac{1}{2\sigma^{2}} \left(1-e^{-(\beta/2\sigma^{2})}\right)^{N-p} \\ \times \left(e^{-(\beta/2\sigma^{2})}\right)^{p-1} e^{-(\beta/2\sigma^{2})} d\beta \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{u^{2}}{2}} du \\ = \int_{0}^{\infty} \int_{0}^{\frac{\eta_{0}u^{2}}{2E_{b}[1-\zeta(\kappa_{n})]^{2}}} \frac{N!}{(N-p)!(p-1)!} \cdot \frac{1}{2\sigma^{2}} \left(1-e^{-(\beta/2\sigma^{2})}\right)^{N-p} \\ \times \left(e^{-(\beta/2\sigma^{2})}\right)^{p} d\beta \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{u^{2}}{2}} du , \qquad (4.13)$$

Using the relationship

$$\left(1 - e^{-\left(\beta/2\sigma^{2}\right)}\right)^{N-p} = \sum_{t=0}^{N-p} \binom{N-p}{t} (-1)^{t} e^{-\frac{t}{2\sigma^{2}}\beta}, \qquad (4.14)$$

and substituting (4.14) into (4.13), we have

$$\begin{split} P_{be,k,n}[\zeta(\kappa_{n})] &= \int_{0}^{\infty} \int_{0}^{\frac{\eta_{h}u^{2}}{2E_{h}[1-\zeta(\kappa_{n})]^{2}}} \frac{N!}{(N-p)!(p-1)!} \cdot \frac{1}{2\sigma^{2}} \\ &\times \sum_{t=0}^{N-p} {\binom{N-p}{t}} (-1)^{t} e^{-\frac{(p+t)}{2\sigma^{2}}\beta} d\beta \cdot \frac{1}{\sqrt{2\pi}} e^{-\frac{u^{2}}{2}} du \\ &= \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {\binom{N-p}{t}} \frac{(-1)^{t}}{(1-\zeta(\kappa_{n}))^{2}} \frac{u^{2}}{2} \end{bmatrix} du \\ &\times \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{u^{2}}{2}} \left\{ 1 - \exp\left[ -\frac{(p+t)}{\overline{\gamma}(1-\zeta(\kappa_{n}))^{2}} \frac{u^{2}}{2} \right] \right\} du \\ &= \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {\binom{N-p}{t}} \frac{(-1)^{t}}{t+p} \\ &\times \left\{ \frac{1}{2} + \int_{0}^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left\{ -\left[ \frac{\overline{\gamma}(1-\zeta(\kappa_{n}))^{2} + (p+t)}{\overline{\gamma}(1-\zeta(\kappa_{n}))^{2}} \right] \frac{u^{2}}{2} \right\} du \right\} \\ &= \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {\binom{N-p}{t}} \frac{(-1)^{t}}{\overline{\gamma}(1-\zeta(\kappa_{n}))^{2}} \end{bmatrix} (du) \\ &= \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {\binom{N-p}{t}} \frac{(-1)^{t}}{\overline{\gamma}(1-\zeta(\kappa_{n}))^{2}} \end{bmatrix} (du) \\ &= \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {\binom{N-p}{t}} \frac{(-1)^{t}}{(N-p)!(p+t)!} \\ &\times \left\{ \frac{1}{2} + \frac{1}{2} \sqrt{\frac{\overline{\gamma}(1-\zeta(\kappa_{n}))^{2}}{\overline{\gamma}[1-\zeta(\kappa_{n})]^{2} + (p+t)!}} \right\}, \end{split}$$
(4.15)

where we note that

$$\frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {\binom{N-p}{t}} \frac{(-1)^t}{t+p} = 1.$$
(4.16)

By substituting (4.16) into (4.15), we have

$$P_{be,k,n}[\zeta(\kappa_n)] = \frac{1}{2} + \frac{1}{2} \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} {N-p \choose t} \frac{(-1)^t}{t+p} \sqrt{\frac{\overline{\gamma}(1-\zeta(\kappa_n))^2}{\overline{\gamma}[1-\zeta(\kappa_n)]^2 + (p+t)}},$$

which gives the first expression of (4.8).

Similarly, when  $\zeta(\kappa_n) > 1$ , the second expression of (4.8) can be obtained.

We next show the derivation of (4.10). For a given  $\kappa_n$ , the distribution of  $\zeta(\kappa_n)$  is assumed to be Gaussian with variance given by (4.9). Conditioned on a given  $\kappa_n$ , the average BER is given by

$$P_{be,k,n|\kappa_n} = \int_{-\infty}^{\infty} P_{be,k,n}(\zeta(\kappa_n)) f_{\zeta(\kappa_n)}(x) dx$$
  

$$= \frac{1}{2} + \frac{1}{2} \int_{-\infty}^{1} \frac{N!}{(N-p)!(p-1)!}$$
  

$$\times \sum_{t=0}^{N-p} \binom{N-p}{t} (-1)^t \frac{1}{p+t} \left( \sqrt{\frac{\overline{\gamma}(1-x)^2}{\overline{\gamma}(1-x)^2 + (p+t)}} \right) \times \frac{\exp(-x^2/2\sigma_{\zeta(\kappa_n)}^2)}{\sqrt{2\pi\sigma_{\zeta(\kappa_n)}^2}} dx$$
  

$$- \frac{1}{2} \int_{1}^{\infty} \frac{N!}{(N-p)!(p-1)!}$$
  

$$\times \sum_{t=0}^{N-p} \binom{N-p}{t} (-1)^t \frac{1}{p+t} \left( \sqrt{\frac{\overline{\gamma}(1-x)^2}{\overline{\gamma}(1-x)^2 + (p+t)}} \right) \times \frac{\exp(-x^2/2\sigma_{\zeta(\kappa_n)}^2)}{\sqrt{2\pi\sigma_{\zeta(\kappa_n)}^2}} dx . (4.17)$$

By substituting 1 - x = u when  $x \le 1$  and 1 - x = v when x > 1, respectively, into (4.17), we can obtain

$$P_{be,k,n|\kappa_n} = \frac{1}{2} + \frac{1}{2} \int_{\infty}^{0} \left[ \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} \binom{N-p}{t} (-1)^t \frac{1}{p+t} \right]$$

$$\times \left(\sqrt{\frac{\overline{\gamma}u^2}{\overline{\gamma}u^2 + (p+t)}}\right) \times \frac{\exp\left(-(1-u)^2/2\sigma_{\zeta(\kappa_n)}^2\right)}{\sqrt{2\pi\sigma_{\zeta(\kappa_n)}^2}}d(-u)$$
$$-\frac{1}{2}\int_0^{-\infty} \frac{N!}{(N-p)!(p-1)!} \sum_{t=0}^{N-p} \binom{N-p}{t}(-1)^t \frac{1}{p+t}$$
$$\times \left(\sqrt{\frac{\overline{\gamma}v^2}{\overline{\gamma}v^2 + (p+t)}}\right) \times \frac{\exp\left(-(1-v)^2/2\sigma_{\zeta(\kappa_n)}^2\right)}{\sqrt{2\pi\sigma_{\zeta(\kappa_n)}^2}}d(-v).$$
(4.18)

By substituting u = -v into the second integral of (4.18), we can obtain (4.10).

### 4.4 Simulation Results and Discussions

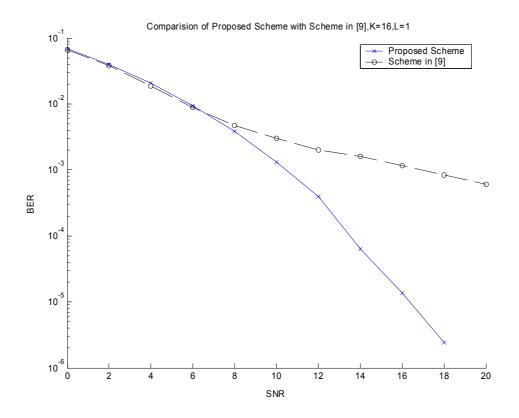
In this section, the performance of the proposed scheme is evaluated using computer simulation under Rayleigh fading channel [53]. We compare the performance achieved by our scheme with that of the sub-carrier allocation scheme in [9] where a simple sub-carrier allocation scheme is proposed, and that of the system without sub-carrier allocation scheme in [8].

The channel is assumed to experience slow fading. A Doppler frequency of 30Hz is used in our simulation when we generate channel gains of the sub-carriers. The updating period is 1.0 ms so that the channels only made reasonable change over each updating period. For a transmission rate of 1M symbols/s per sub-carrier, this is equivalent to transmitting 1000 symbols/s per sub-carrier. It was found from our simulations that the choice for the initial sub-carrier allocation can be arbitrarily assigned (such as select the best L without considering MAI) and only a few iterations is needed for the algorithm to converge.

#### 4.4.1 **Performance Comparison with Scheme in [9]**

We first compare our scheme with the scheme in [9]. The comparison is carried out given the same number of users and total sub-carriers. Also each user has the same number of sub-carriers allocated to transmit data.

Figure 4.2 and 4.3 show the comparison of our scheme with [9] where the number of users is 16 and 30, respectively. The total number of sub-carriers for each user is 4 and each user select one sub-carrier to transmit data. From these figures, we can see that when SNR is low, our allocation scheme has almost the same performance as the scheme in [9]. However, with the increase of the signal-to-noise ratio (SNR), the BER performance improves more significantly for our scheme.





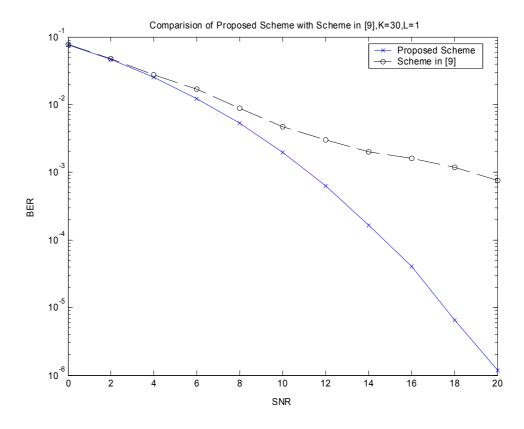


Figure 4.3 Comparison of Proposed Scheme with Scheme in [9] user=30,L=1

To further compare our scheme with the scheme in [9], we simulate our scheme when each user selects two sub-carriers to transmit data. For a fair comparison, we modify the scheme in [9] where each user also selects two sub-carriers with largest channel gains to transmit data. Figure 4.4 and 4.5 show the simulation results where the number of user is equal to 16 and 30, respectively. The total number of sub-carriers of each user is 4 and each user is allocated 2 sub-carriers to transmit data.

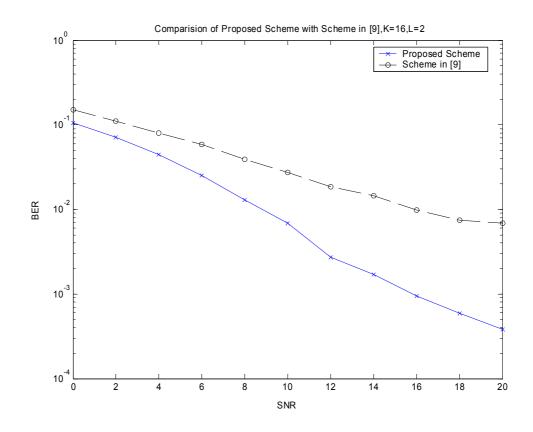


Figure 4.4 Comparison of Proposed Scheme with Scheme in [9], user=16,L=2

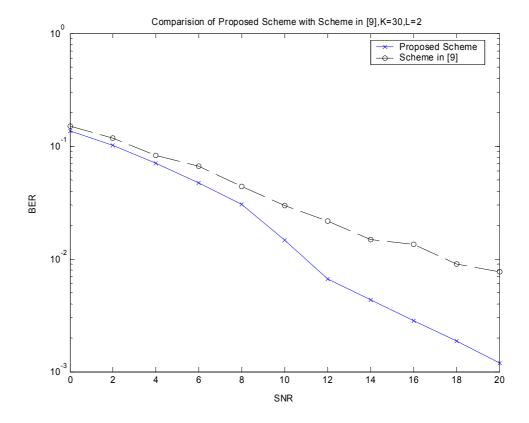


Figure 4.5 Comparison of Proposed Scheme with Scheme in [9], user=30,L=2

From these figures, we can see that our scheme has better BER performance although SNR is low. With the increase of the SNR, the BER performance improves accordingly.

However, our scheme works well only when either a large number of users are present or when each user uses more sub-carriers to transmit data. Figure 4.6 shows the BER performance comparison of the proposed scheme with the scheme in [9]. The number of sub-carriers of each user is equal to 4 and we have 8 active users. From the 4 sub-carriers, we choose 1 sub-carrier to transmit data. From Figure 4.6, we can see our proposed system has worse performance than the scheme in [9]. This is because our scheme considers the interference from other users using the same sub-carrier. When the number of users is small, the interference is not large enough to see the advantage. On the other hand, the decision based on previous sub-carrier allocation will generate inaccuracy to the allocation process. Under such circumstances, simply using the method described in [9] to select sub-carriers to transmit is sufficient.

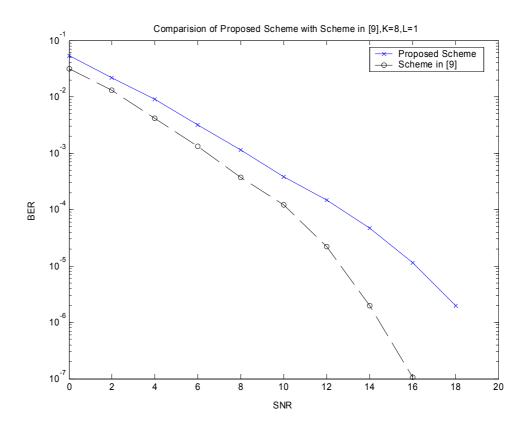


Figure 4.6 Comparison of Proposed Scheme with Scheme in [9], user=8,L=1

Figure 4.7 shows the BER performance when the number of users is also 8 but we select two sub-carriers to transmit data. From the figure, we can see our scheme has better BER performance compared with the scheme in [9]. This is because when we select more sub-carriers to transmit data, the level of interference from other users and the chance that one sub-carrier accommodates more users to transmit data is increased, compared with the case where each user only selects one sub-carrier. So compared with the scheme in [9], we can see that when the system has reasonable large number of users or when each user demands many sub-carriers for transmission, our scheme shows a significant BER improvement.

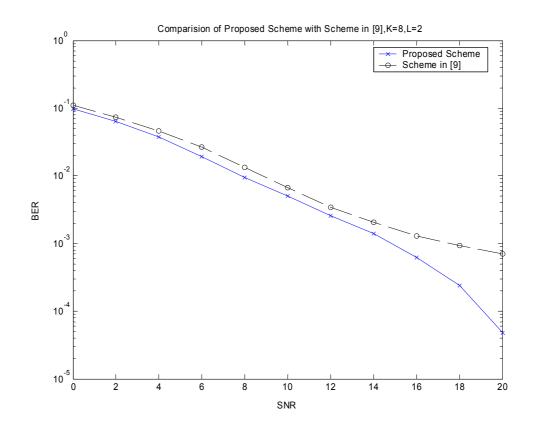


Figure 4.7 Comparison of Proposed Scheme with Scheme in [9], user=8,L=2

## 4.4.2 Performance Comparison with Scheme without Sub-carrier Allocation Scheme

In this section, we will present the simulation results and compare our scheme with MC-DS-CDMA systems without adaptive sub-carrier allocation. We will show that our scheme will also have BER improvement.

Figure 4.8 to Figure 4.10 show the BER performance comparisons between our scheme and MC-DS-CDMA systems with fixed sub-carriers being allocated to each user.

From Figure 4.8 to Figure 4.10, we can see that with different number of users in the system, our scheme shows better BER performance compared with MC-DS-CDMA systems without any adaptive sub-carrier allocation schemes.

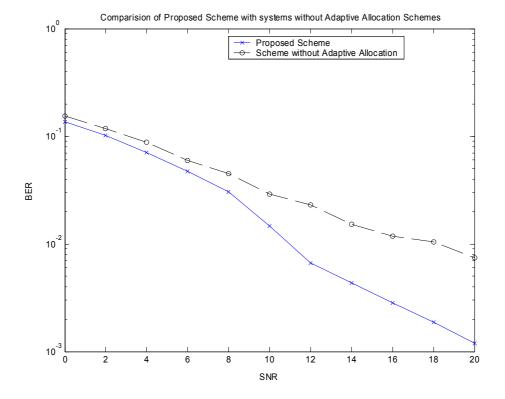


Figure 4.8 Comparison of Proposed Scheme with MC-DS-CDMA Systems without Adaptive Allocation Schemes, user=30

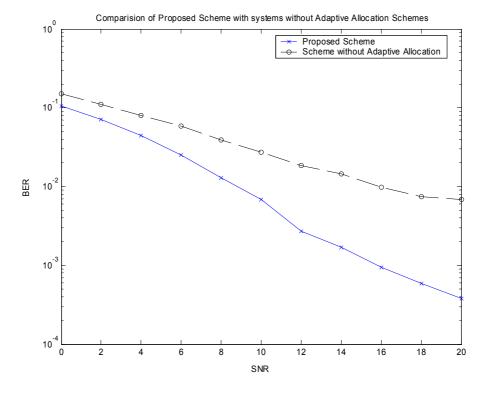


Figure 4.9 Comparison of Proposed Scheme with MC-DS-CDMA Systems without Adaptive Allocation Schemes, user=16

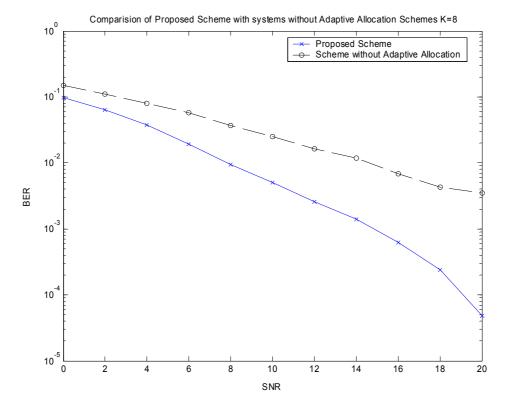


Figure 4.10 Comparison of Proposed Scheme with MC-DS-CDMA Systems without Adaptive Allocation Schemes, user=8

From the simulation results, we show that in general there is an improvement in the BER performance of our scheme when compared to scheme [9] and systems without any adaptive sub-carrier allocation schemes. The main reason is that we jointly consider the effect of MAI when we allocate sub-carrier to users. When the number of users or sub-carriers is large, our scheme shows larger improvement.

# Chapter 5

# Sub-carrier Allocation Scheme Using Quadra--tic Programming

In the previous chapter, we propose a sub-optimal sub-carrier allocation scheme for MC-DS-CDMA systems. The scheme considers the effect of other user's interference when allocating the sub-carriers. This scheme shows good BER performance improvement and is a promising solution to the practical system. This is because, as we have mentioned in Chapter 4, the amount of interference power will increase accordingly with the increase of the number of the active users or the number of sub-carriers allocated to each user. Our scheme is capable of optimizing the BER performance in the allocation process since the MAI is considered when allocating the sub-carriers to users. From the simulation results, we can conclude that the performance improves when the SNR or the number of active users increases. In that scheme, we use the previous allocation coefficients to decide the current allocation coefficients, since calculating the number of users in each sub-carrier and computing the BER performance are intertwined with each other. The use of the previous allocation coefficients can reduce the complexity in this aspect and is reasonable if the channel is slow time-varying. However, when the amount of MAI is small, this method shows discrepancy. In this chapter, another method will be proposed.

We formulate the objective function by allocating sub-carriers according to the actual values of channel gains while considering the MAI rather than using the previous allocation coefficients, i.e., we will try to solve the single optimal objective function similar to (3.18) instead of minimizing each user BER as in Chapter 4. Such scheme is not as simple as our scheme in Chapter 4 in terms of complexity; however it improves the system's performance more significantly. In this new method, we use quadratic programming (QP) to formulate the objective function. The reason to use QP is that it can include the interference terms and the allocation coefficients in one objective function. The way we formulate the problem will be explained in Section 5.2. From the mathematical point of view, the QP problem is solvable although it belongs to non-linear programming (NLP).

This chapter is organized as follows. In Section 5.1, we will first give background knowledge of NLP problems. Generally, solvable QP problem must meet some conditions, such as the Hessian matrix must be positive definite and symmetric [57]. The solution to QP problem needs to follow some given procedures, which will also

be introduced in this section. In Section 5.2, we will explain how we formulate our objective functions. We show how we can modify the formulated objective functions and transform them into the general forms of the QP problems. To make our objective functions solvable, we convert our objective functions to linear objective functions employing some mathematical methods, i.e., Kuhn-Tucker conditions. Finally, we present our computer simulation results and compare them with conventional scheme in [9] and our scheme in Chapter 4.

#### 5.1 Introduction to Non-Linear Programming

#### 5.1.1 Non-linear Programming

Non-linear programming (NLP) deals with the problem of optimizing an objective function in the presence of equality and inequality constraints. If not all of these functions are linear, we call the problem a NLP problem.

The development of efficient and robust algorithms and software for linear programming (LP) has made LP an important tool for solving problems in diverse fields. However, many realistic problems cannot be adequately represented or approximated as a LP due to the nature of the nonlinearity of the objective function and/or the nonlinearity of any of the constraints [57]. Much effort has been spent to solve such NLP problems efficiently, and this has resulted in rapid progress during the past few decades. But generally, NLP problems are far more complicated than LP problems. They are hard to solve and do not have a general algorithm like simplex algorithm in LP. Actually, there is no available algorithm to solve all the NLP problems yet. Each algorithm has its own advantages and limitations [58] [59].

We give a general form of NLP optimization problem for convenience:

Minimize 
$$f(x_1, x_2, ..., x_i, ..., x_I),$$
 (5.1)

Subject to 
$$h_p(x_1, x_2, ..., x_i, ..., x_I) = 0$$
,  $p = 1, 2, ..., P$ , (5.2)

$$g_q(x_1, x_2, \dots, x_i, \dots, x_I) \le 0, \quad q = 1, 2, \dots, Q,$$
 (5.3)

$$b_{l,i} \le x_i \le b_{u,i}$$
,  $i = 1, 2, \dots, I$ . (5.4)

Eq. (5.1) is called objective function, while (5.2) and (5.3) are called equality and inequality constraints, respectively. Eq. (5.4) is defined as side constraints where  $b_{l,i}$  represents lower bounds and  $b_{u,i}$  represents upper bounds where  $x_i$  can take. If not all the functions in (5.1) – (5.3) are linear, then this problem is called a NLP problem. If a NLP problem does not have (5.4), then it is called unconstrained NLP, otherwise it is a constrained NLP problem.

In a general NLP problem given in (5.1)-(5.4), we can actually combine the constraints (5.2) with (5.3) by adding slack variables  $y_j (j = 1, 2, ..., P + Q)$ , i.e., (5.2) and (5.3) can be rewritten into one general form as:

$$w_j(x_1, x_2, \dots, x_i, \dots, x_I) + y_j = 0, \quad j = 1, 2, \dots P + Q, \quad y_j \ge 0,$$
 (5.5)

 $y_j$  is equal to zero for the equality constraints in (5.2) and  $y_j$  is positive or zero for the inequality constraints in (5.3). So a general NLP problem can be rewritten as:

Minimize 
$$f(x_1, x_2, ..., x_i, ..., x_I),$$
 (5.6)

Subject to 
$$w_j(x_1, x_2, \dots, x_i, \dots, x_i) + y_j = 0, \quad j = 1, 2, \dots P + Q, \quad (5.7)$$

$$b_{l,i} \le x_i \le b_{u,i}, \qquad y_i \ge 0, \quad i = 1, 2, \dots, I.$$
 (5.8)

#### 5.1.2 Kuhn-Tucker Condition for NLP

Kuhn-Tucker (KT) conditions were derived by Kuhn and Tucker in 1951, which is one of the most important contributions in solving NLP problem. It is a necessary condition to decide whether a feasible solution x is an optimal one. If the feasible solution x satisfies KT conditions, it must be the optimal solution. It can also be used to solve the QP, which is one specific type of the NLP problems.

Now we describe the KT conditions. Consider a general NLP optimization problem as in (5.6)-(5.8). Let  $\mathbf{x}_0$  be a feasible solution. Suppose the objective function f(x) and the constraints  $w_j(x)$  are differentiable at  $\mathbf{x}_0$ . Then there exist scalars  $\mathbf{u} = (u_1, u_2, \dots, u_j, \dots, u_{P+Q})^T$  such that

$$\nabla f(\mathbf{x}_0) + \sum_{j=1}^{P+Q} u_j \nabla w_j(\mathbf{x}_0) = 0, \qquad (5.9)$$

$$u_j w_j (\mathbf{x}_0) = 0, \qquad j = 1, 2, \dots P + Q, \qquad (5.10)$$

$$b_{l,i} \le x_i \le b_{u,i}, \qquad i = 1, 2, \dots, I,$$
 (5.11)

$$u_j \ge 0. \tag{5.12}$$

The vector  $\mathbf{u} = (u_1, u_2, \dots, u_j, \dots, u_{p+Q})^T$  is defined as Lagrangian multiplier. Eq. (5.9)-(5.12) are called KT conditions. We should note that in (5.10), for each j, the multiplier  $u_j$  or the corresponding constraint  $w_j$  should be zero. [59] [60]

# 5.1.3 Quadratic Programming and Using KT Conditions to Convert Quadratic Programming

QP represents a special class of NLP in which the objective function is quadratic and the constraints are linear. A QP problem can be formulated as:

Minimize 
$$f(\mathbf{x}) = \sum_{i=1}^{I} c_i x_i + \frac{1}{2} \sum_{i=1}^{I} \sum_{k=1}^{I} H_{i,k} x_i x_k$$
, (5.13)

Subject to 
$$\sum_{i=1}^{l} m_{p,i} x_i = n_p$$
,  $p = 1, 2, ..., P$ , (5.14)

$$\sum_{i=1}^{I} l_{q,i} x_i \le r_q, \qquad q = 1, 2, \dots, Q, \qquad (5.15)$$

$$0 \le x_i \le 1, x_i \in \{0,1\}, i = 1,2..., I, k = 1,2,..., I, H_{i,k} = H_{k,i},$$
 (5.16)

where  $\mathbf{x} = (x_1, x_2, \dots, x_I)^T$ ,  $c_i$  is the element of  $\mathbf{c} = (c_1, c_2, \dots, c_I)^T$  having a dimension of I,  $H_{i,k}$  is the element of a symmetric  $I \times I$  matrix **H**, which is positive definite.  $m_{p,i}$  and  $l_{q,i}$  are the elements of equality and inequality constraints, respectively, which is given by

$$\mathbf{A}_{1} = \begin{bmatrix} m_{1,1} & m_{1,2} & \dots & m_{1,I} \\ m_{2,1} & m_{2,2} & \dots & m_{2,I} \\ \vdots & \vdots & \ddots & \vdots \\ m_{P,1} & m_{P,2} & \dots & m_{P,I} \end{bmatrix}, \qquad \mathbf{A}_{2} = \begin{bmatrix} l_{1,1} & l_{1,2} & \dots & l_{1,I} \\ l_{2,1} & l_{2,2} & \dots & l_{2,I} \\ \vdots & \vdots & \ddots & \vdots \\ l_{Q,1} & l_{Q,2} & \dots & l_{Q,I} \end{bmatrix}.$$

 $n_p$  and  $r_q$  are the elements of vector  $\mathbf{n} = (n_1, n_2, \dots, n_p)^T$  and  $\mathbf{r} = (r_1, r_2, \dots, r_Q)^T$ , where  $\mathbf{A}_1 \mathbf{x} = \mathbf{n}, \mathbf{A}_2 \mathbf{x} \le \mathbf{r}$ .

The constraints (5.14) and (5.15) can also be combined in the similar way as we have combined (5.2) and (5.3) in the general NLP problem. We add slack variables  $\mathbf{y} = (y_1, y_2, \dots, y_j, \dots, y_{P+Q})^T$  to (5.14) and (5.15) so we can rewrite (5.13)-(5.16) as:

Minimize 
$$f(\mathbf{x}) = \sum_{i=1}^{I} c_i x_i + \frac{1}{2} \sum_{i=1}^{I} \sum_{k=1}^{I} H_{i,k} x_i x_k$$
, (5.17)

Subject to 
$$\sum_{i=1}^{I} a_{j,i} x_i + y_j = b_j$$
,  $j = 1, 2, ..., P + Q$ , (5.18)

$$0 \le x_i \le 1$$
,  $i = 1, 2..., I$ ,  $x_i \in \{0,1\}$ ,  $H_{i,k} = H_{k,i}$ , (5.19)

where  $a_{j,i} = m_{j,i}$  for j = 1,2,...,P,  $a_{j,i} = l_{j-P,i}$  for j = P+1, P+2,...,P+Q,  $y_j$  is equal to zero for j = 1,2,...,P and  $y_j \ge 0$  for j = P+1, P+2,...,P+Q.  $b_j = n_j$  for j = 1,2,...,P and  $b_j = r_{j-P}$  for j = P+1, P+2,...,P+Q. As we have mentioned in Section 5.1.2, a NLP problem can be solved using KT condition defined in (5.9)-(5.12). Since a QP problem is a kind of NLP problem, so the use of KT conditions is available. We use KT conditions defined in (5.9)-(5.12) to convert a general QP problem defined in (5.17)-(5.19). If we take (5.19) as a constraint also, we can convert the lower and upper bounds of  $x_i$  (5.19) to two constraints of  $x_i$ given in (5.19a) and (5.19b), respectively

$$x_i - e_i = 0$$
,  $i = 1, 2..., I$  (5.19a)

$$x_i + e_{I+i} = 0$$
,  $i = 1, 2..., I$ . (5.19b)

If the Lagrangian multiplier vectors of (5.18) and (5.19a), (5.19b) are  $\mathbf{u} = (u_1, u_2, \dots, u_j, \dots, u_{P+Q})^T$ ,  $\mathbf{v}_1 = (v_1, v_2, \dots, v_i, \dots, v_I)^T$ ,  $\mathbf{v}_2 = (v_{I+1}, v_{I+2}, \dots, v_{I+i}, \dots, v_{2I})^T$ ,

respectively. The KT conditions can be written as [57] [58]:

$$-\sum_{k=1}^{I} H_{i,k} x_k - \sum_{j=1}^{P+Q} u_j a_{j,i} + v_i + v_{I+i} = c_i, \qquad i = 1, 2..., I, \quad (5.20)$$

$$x_i v_i = 0$$
,  $i = 1, 2..., I$ , (5.21)

$$x_i v_{I+i} = 0$$
,  $i = 1, 2..., I$ , (5.22)

$$u_j y_j = 0,$$
  $j = 1, 2..., P + Q,$  (5.23)

$$x_i, v_i, v_{I+i}, u_j, y_j \ge 0, H_{i,k} = H_{k,i}.$$
 (5.24)

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The solution to (5.20)-(5.24) will be the optimal solution to problem (5.17)-(5.19), which is obtained by solving the equations in (5.20)-(5.24). We can modify (5.20) to convert (5.20)-(5.24) to a LP problem which has the same solution as the original defined problem in (5.17)-(5.19). We introduce an artificial scalar  $\mathbf{z} = (z_1, z_2, \dots z_L)^T$ , and define

$$\begin{cases} \operatorname{sgn}(c_i) = 1 & \text{if } c_i \ge 0\\ \operatorname{sgn}(c_i) = -1 & \text{if } c_i < 0 \end{cases}$$
(5.25)

Considering the following LP problem

Minimize 
$$\varphi(z) = \sum_{i=1}^{I} z_i$$
, (5.26)

Subject to 
$$-\sum_{k=1}^{I} H_{i,k} x_k - \sum_{j=1}^{P+Q} u_j a_{j,i} + v_i + v_{I+i} + \operatorname{sgn}(c_i) z_i = c_i$$
, (5.27)

$$x_i v_i = 0$$
,  $i = 1, 2..., I$ , (5.28a)

$$x_i v_{I+i} = 0$$
,  $i = 1, 2..., I$ , (5.28b)

$$u_j y_j = 0, \qquad j = 1, 2..., P + Q, \qquad (5.29)$$

$$x_i, v_i, v_{I+i}, u_j, y_j, z_i \ge 0, \quad H_{i,k} = H_{k,i},$$
 (5.30)

The solution to (5.26) - (5.30) will be the same to the original QP problem (5.17) - (5.19) when  $z_i = 0, (i = 1, 2, ..., I)$  since it fulfills the KT conditions. We should note that (5.26) is a LP problem and hence the QP problem in the form of (5.17)-(5.19) are converted to a LP in the form of (5.26)-(5.30).

### 5.2 Formulation of the Problem

#### 5.2.1 Conversion of our Problem to a QP Problem

Suppose the allocation results we want to obtain is still the allocation coefficients **S** as defined in (4.3), we rewrite it here for convenience.

$$\mathbf{S} = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1N} \\ s_{21} & s_{22} & \cdots & s_{2N} \\ \vdots & \vdots & \vdots & \vdots \\ s_{K1} & s_{K2} & \cdots & s_{KN} \end{bmatrix},$$
(5.31)

where

$$s_{k,n} = \begin{cases} 1 & \text{if } k\text{th user's } n\text{th subcarrier is selected} \\ 0 & \text{if } k\text{th user's } n\text{th subcarrier is not selected} \end{cases}$$
(5.32)

As we have mentioned above, we want to further improve the system's performance through allocating sub-carriers according to the actual values of the channel gains of current updating stage rather than using the allocation coefficients of the previous stage. The objective is still to minimize the BER of the system. The objective function we formulate is

$$\max f(s_{k,n}) = \max \sum_{k=1}^{K} \sum_{n=1}^{N} \left[ F_{k,n} s_{k,n} - s_{k,n} \sum_{g=1}^{K} \left[ C_{k,g}(0) F_{g,n} s_{g,n} \right] \right], \quad g \neq k \quad (5.33)$$

subject to

$$\sum_{n=1}^{N} s_{k,n} = L \qquad \forall k = 1, 2, \dots, K,$$
(5.34)

$$0 \le s_{k,n} \le 1, \ s_{k,n} \in \{0,1\}$$
(5.35)

where  $f(s_{k,n})$  is the objective function with variables  $s_{k,n}$ . The constraint defined in (5.34) is the same as (4.2) which means each user is allocated with the same number of sub-carriers. The value of *L* depends on the data rate to be supported. **F** is a  $K \times N$  matrix consists of all the sub-carriers channel gains and **C** is a  $K \times K$ matrix consists of all the cross correlation between signature sequences. Their elements are denoted by  $F_{k,n}$  and  $C_{k,g}(0)$ , respectively.  $F_{k,n}$  gives the *k*th user's *n*th sub-carrier's channel gain in the sampling time.  $C_{k,g}(0)$  gives the cross correlation between the *k*th and gth user signature sequences defined by (2.5).

The interference is illustrated by the second item in (5.33). The coefficient of the quadratic term is given by the multiplication of the channel gain and the cross correlation between the desired user k and the "interference" user g ( $g \neq k$ ). The reason we formulate the problem this way is that, under the assumption that all the users are allocated identical power on each sub-carrier, if the channel gain of the gth user ( $g \neq k$ ) is large, which means the gth user's channel is in good condition, it may cause greater interference to other users who are using the same sub-carrier. We can find that many publications define the amount of interference using the channel gain [54] [55] [56]. The same reason applies to the cross

correlation of the desired user k with the interference users. If the signature sequences are more correlated, then interference power will be greater and we expect a poorer performance to user k.

#### 5.2.2 Conversion of the Formulated Problem Using KT Conditions

For the convenience of making use of KT conditions, we have to make changes to the indices of the variables that appear in (5.33) - (5.35). This is because the variables in a QP problem should take the form of a one-dimension vector. However, **S** is a matrix and its elements  $s_{k,n}$  have double indices (i.e. k and n). Hence we have to modify it to a one-dimensional index. The modified index will arrange the  $K \times N$ variables the  $S_{kn}$ in form as  $\vec{t} = \{s_{1,1}, s_{1,2}, \dots, s_{1,N}, s_{2,1}, s_{2,2}, \dots, s_{2,N}, \dots, s_{K \times N}\} = \{t_1, t_2, \dots, t_{\alpha}, \dots, t_{K \times N}\}$ . Suppose the one-dimensional index is given by  $\alpha$ . We define  $\alpha_c = \operatorname{ceil}(\alpha/N)$  is the nearest integer towards infinity of  $\alpha/N$  and  $\alpha_m = \mod(\alpha/N)$  is the modulus of  $\alpha/N$ , i.e., we have  $\alpha = (\alpha_c - 1) \times N + \alpha_m$ . We should notice that if we substitute  $k = \alpha_c$ ,  $n = \alpha_m$  into (5.33) - (5.35), we have the following form

$$\max f(t_{\alpha}) = \max \sum_{\alpha=1}^{K \times N} \left[ t_{\alpha} F_{\alpha_{c},\alpha_{m}} - t_{\alpha} \sum_{g=1}^{K} \left[ C_{\alpha_{c},g}(0) F_{g,\alpha_{m}} t_{[(g-1) \times N + \alpha_{m}]} \right] \right]$$
$$= \max \sum_{\alpha=1}^{K \times N} t_{\alpha} F_{\alpha_{c},\alpha_{m}} - \sum_{\alpha=1}^{K \times N} \sum_{g=1}^{K} \left[ C_{\alpha_{c}g}(0) F_{g,\alpha_{m}} \cdot t_{\alpha} \cdot t_{[(g-1) \times N + \alpha_{m}]} \right], \quad g \neq \alpha_{c} , (5.36)$$

subject to

$$\sum_{\alpha_c=j} t_{\alpha} = L \qquad \forall j = 1, 2, \dots, K, \alpha = 1, 2, \dots, K \times N, \qquad (5.37)$$

$$0 \le t_{\alpha} \le 1, \ t_{\alpha} \in \{0,1\}$$
 (5.38)

In the above, we change the two-dimensional indices k and n to a one-dimensional index  $\alpha \{ \alpha = 1, 2, ..., K \times N \}$ . After the change, we can find (5.36)-(5.38) are of the same form with (5.17)-(5.19), after replacing  $c_i$  by  $F_{\alpha_c,\alpha_m}$ ,  $x_i$  by  $t_\alpha$ ,  $b_j$  by L,  $H_{\alpha,(g-1)\times N+\alpha_m}$  by  $C_{\alpha_c,k}(0)F_{k,\alpha_m}$  and  $K \times N = I$ , we then obtain the KT conditions of (5.36) – (5.38) using (5.26) – (5.30). The double summation part of (5.36) has a total  $K \times K \times N$  terms, but (5.13) has  $K^2 \times N^2$  terms. Those absent terms will be replaced by 0. Finally, in order to obtain the symmetry property of  $H_{\alpha,(g-1)\times N+\alpha_m}$  given in (5.19), the coefficients of  $t_\alpha \cdot t_{[(g-1)\times N+\alpha_m]}$  and  $t_{[(g-1)\times N+\alpha_m]} \cdot t_\alpha$  each will take  $\frac{1}{2} [C_{\alpha_c,g}(0) \cdot F_{g,\alpha_m} + C_{g,\alpha_c}(0) \cdot F_{\alpha_c,\alpha_m}].$ 

We then use KT conditions to convert (5.36) - (5.38) into the form of (5.26)-(5.30). When differentiating the objective function (5.36), we will have

 $\nabla f(t_{\alpha})$ 

$$= F_{\alpha_{c},\alpha_{m}} - \sum_{g=1}^{K} C_{\alpha_{c},g}(0) \cdot F_{g,\alpha_{m}} \cdot t_{[(g-1) \times N + \alpha_{m}]} - \sum_{g=1}^{K} C_{g,\alpha_{c}}(0) \cdot F_{\alpha_{c},\alpha_{m}} \cdot t_{[(g-1) \times N + \alpha_{m}]}$$
$$= F_{\alpha_{c},\alpha_{m}} - \sum_{g=1}^{K} \left[ C_{\alpha_{c},g}(0) \cdot F_{g,\alpha_{m}} + C_{g,\alpha_{c}}(0) \cdot F_{\alpha_{c},\alpha_{m}} \right] \cdot t_{[(g-1) \times N + \alpha_{m}]},$$
(5.39)

and

$$\nabla \left[\sum_{\alpha_c=j} t_{\alpha} - L\right] = 1 \qquad \forall j = 1, 2, \dots, K, \alpha = 1, 2, \dots, K \times N.$$
(5.40)

Substituting (5.39) and (5.40) into (5.27) and assuming Langrangian multiplier given by  $\mathbf{u} = (u_1, u_2, \dots, u_j, \dots, u_K)^T$ ,  $\mathbf{v}_1 = (v_1, v_2, \dots, v_\alpha, \dots, v_{K \times N})^T$  and  $\mathbf{v}_2 = (v_{1+K \times N}, v_{2+K \times N}, \dots, v_{\alpha+K \times N}, \dots, v_{2(K \times N)})^T$ , respectively, we can get

$$\sum_{g=1}^{K} \left[ C_{\alpha_c,g}(0) \cdot F_{g,\alpha_m} + C_{g,\alpha_c}(0) \cdot F_{\alpha_c,\alpha_m} \right] \cdot t_{\left[ (g-1) \times N + \alpha_m \right]}$$
$$+ u_j + v_\alpha + v_{K \times N + \alpha} + \operatorname{sgn}(F_{\alpha_c,\alpha_m}) \cdot z_\alpha = F_{\alpha_c,\alpha_m}.$$
(5.41)

Using KT conditions (5.28) and (5.29), from (5.41), we will have the following

Minimize 
$$\varphi(z) = \sum_{\alpha=1}^{K \times N} z_{\alpha}$$
, (5.42)

$$\sum_{g=1}^{K} \left[ C_{\alpha_{c},g}(0) \cdot F_{g,\alpha_{m}} + C_{g,\alpha_{c}}(0) \cdot F_{\alpha_{c},\alpha_{m}} \right] \cdot s_{\left[ (g-1) \times N + \alpha_{m} \right]}$$
$$+ u_{j} + v_{\alpha} + v_{K \times N + \alpha} + \operatorname{sgn} \left( F_{\alpha_{c},\alpha_{m}} \right) \cdot z_{\alpha} = F_{\alpha_{c},\alpha_{m}}, \qquad (5.43)$$

$$\sum_{\alpha_c=j} t_{\alpha} = L \qquad \forall j = 1, 2, \dots, K, \alpha = 1, 2, \dots, K \times N, \qquad (5.44)$$

$$t_{\alpha} \cdot v_{\alpha} = 0 , \qquad (5.45)$$

$$t_{\alpha} \cdot v_{K \times N + \alpha} = 0, \qquad (5.46)$$

$$u_j \ge 0, \ v_{\alpha} \ge 0, \ v_{K \times N + \alpha} \ge 0, z_{\alpha} \ge 0,$$
  
 $\alpha = 1, 2, \dots, K \times N, \ j = 1, 2, \dots, K.$  (5.47)

Eq. (5.42) - (5.47) is a solvable LP problem. The solution to this problem will be the same with the original formulated QP problem (5.33) - (5.35). Since the slack variables for (5.37) are all zeros, so KT conditions (5.29) will not appear in (5.42) - (5.47).

From the derivation of the (5.42) - (5.47), we can see that the objective function allocates sub-carriers according to the actual values of the channel gains, taking the MAI into consideration. The allocation process does not use the allocation coefficients of the previous updating stage. This will avoid any discrepancy from propagating through different stages. Our simulation in the next section also shows that this scheme can improve the system's performance even more significantly. Since the sub-carrier allocation coefficient of the previous stage  $S_{i-1}$  is not needed, the algorithm is expected to work in channel having larger Doppler spread. The original formulation is a QP problem, but we convert the problem to a solvable LP problem. Many efficient and robust algorithms and software have been developed for LP [58] [59] [60].

### 5.3 Simulation Results and Discussion

In this Section, we compare the scheme we propose in this chapter with the conventional scheme in [9] as well as the scheme we proposed in the previous chapter through simulation. We call the scheme we propose in previous chapter "Proposed Scheme (Linear)", and the scheme we proposed in this chapter "Proposed Scheme (Quadratic)".

In the simulations, the channel use Jakes Model and the Doppler shift is set to 30Hz. The updating period of allocation is 1ms. Given a transmitting data rate of 1M symbols per second per sub-carrier, this means 1000 symbols will be transmitted in each updating period.

Figure 5.1 shows the simulation results of the system of 8 active users with each user selects 1 sub-carrier to transmit data. From the figure, we can see that this new scheme has BER improvement compared to both the Proposed Scheme (linear) and the conventional scheme in [9]. As we have mentioned in the previous chapter, the performance of Proposed Scheme (linear) is worse than the conventional scheme in [9] in this case. The reason is that the number of users in the system is not so large so that MAI is small. Based on previous allocation coefficient however causes non-optimal allocation to propagate resulting in larger error. However, our Proposed Scheme (linear) performs well in the case when interference affects the allocation

coefficients much and the performance improves with the increase of interference correspondingly. Our proposed scheme (Quadratic) performs the best in these three schemes.

Figure 5.2 is the simulation results with 8 active users and each user selects two sub-carriers to transmit data. From the figure, we can see that our Proposed Scheme (linear) and Proposed Scheme (quadratic) both improve BER performance compared to the conventional scheme in [9]. Also, our Proposed Scheme (quadratic) has some BER performance improvement to the Proposed Scheme (linear). This is because our Proposed Scheme (quadratic) does not use the previous allocation coefficients to decide the current allocation coefficients, so this scheme is more accurate than our Proposed Scheme (linear).

Figure 5.3 and 5.4 show the simulation results of 16 users, and each user selects one and two sub-carriers to transmit data, respectively. The conclusion is similar to what we can draw from Figure 5.2.

From simulation results, our Proposed Scheme (linear) and Proposed Scheme (Quadratic) both show BER improvement compared to conventional scheme in [9]. The reason is that we jointly consider the effects of MAI when allocate sub-carriers

to users. Our Proposed Scheme (Quadratic) shows even better performance. This is because the scheme uses the instantaneous channel information without using the previous allocation coefficients. This can improve the accuracy of the scheme.

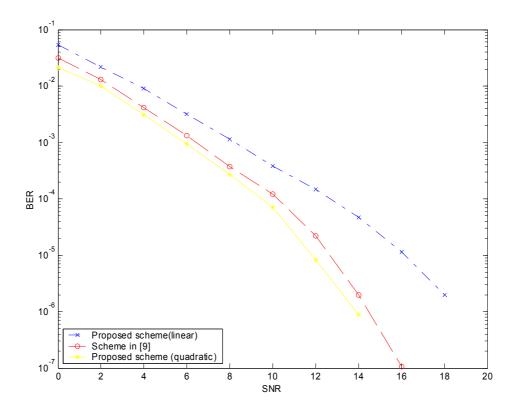


Figure 5.1 Simulation results with 8 users, 4 sub-carriers, each user selects 1 to transmit data

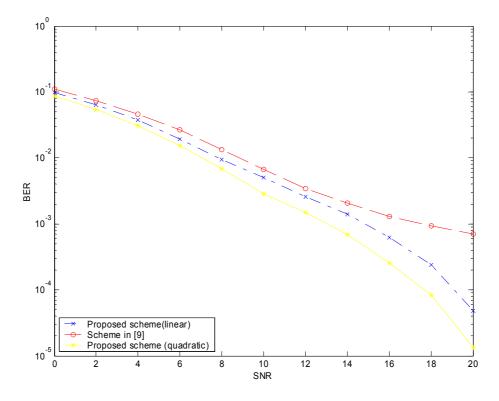


Figure 5.2 Simulation results with 8 users, 4 sub-carriers, each user selects 2 to transmit data

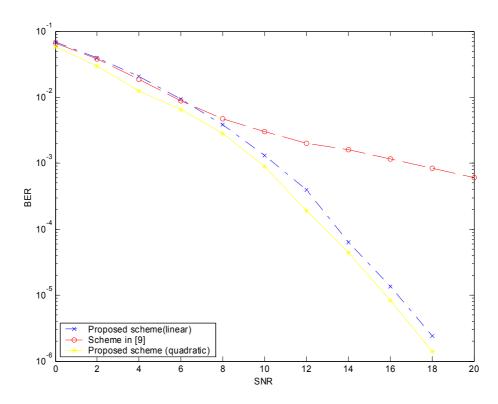


Figure 5.3 Simulation results with 16 users, 4 sub-carriers, each user selects 1 to transmit

10<sup>0</sup> 딾 10<sup>-2</sup> 10<sup>-3</sup> Proposed scheme(linear) Scheme in [9] Proposed scheme (quadratic) SNR 

Figure 5.4 Simulation results with 16 users, 4 sub-carriers, each user selects 2 to transmit data

# **Chapter 6**

### **Conclusions and Future Research**

### 6.1 Conclusions

Multi-path induced ISI is a severe problem in wireless communications. In order to overcome this problem, OFDM is introduced to combine with CDMA, which is the MC-CDMA systems. As a result, the problem of the sub-carrier allocation in MC-CDMA systems arises. We have conducted a review of the previous work on sub-carrier allocation schemes for OFDM and MC-CDMA systems. From the review, we find that those schemes do not consider MAI when allocating sub-carriers to users, so the existing schemes may degrade when MAI is dominant. In this thesis, we aim to design adaptive sub-carrier allocation schemes in the presence of MAI for MC-DS-CDMA systems.

In this thesis, we proposed two sub-carrier allocation schemes for MC-DS-CDMA systems in the presence of Multiple Access Interference. Both schemes show BER

performance improvement compared with conventional sub-carrier allocation schemes and those without sub-carrier allocation schemes.

The first algorithm is a sub-optimal one. We formulate objective functions aimed to minimize each user's BER performance considering MAI. In order to avoid the intertwine of the unknown allocation coefficient and computation of MAI, we use the allocation coefficient of last stage  $S_{i-1}$  and the ordering index of user's channel gain of the current stage to jointly decide the allocation coefficients of the current stage  $S_i$ . This process involves solving for the coefficients iteratively and the algorithm has been described in detail in Chapter 4. From the simulation results, our first method generally shows BER performance improvement compared to conventional method in [9] and those without adaptive sub-carrier allocation method. The performance improvement increases with the increase of the SNR. When SNR is low (usually below 8dB), our method is quite similar to the scheme in [9], when SNR increases, the SNR improvement can be 4-10 dB with different number of users or sub-carriers selected. However, our method performs well when either a large number of users are present or when each user uses more sub-carriers to transmit data. The decision based on previous sub-carrier allocation may generate discrepancy to the allocation process if the effect due to MAI is small.

In order to overcome the limitation of the first method, we define a new objective function using the instantaneous channel gains and cross-correlation coefficients, which appears to be Quadratic Programming (QP) problem. Such scheme is not as simple as the first one, but this scheme can improve the system performance even

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more significantly. This is because the method allocates sub-carriers according to the actual values of channel gains while considering the MAI without using the previous allocation coefficients. We use Kuhn-Tucker conditions to convert our objective function to a solvable linear problem without changing the solution to the original problem. From the simulation results, this method outperforms all the schemes mentioned in this thesis and shows more accuracy compared with our first method.

### 6.2 Suggestions for Future Research

From the derivation of average BER in Chapter 4, we calculate average BER using the index vector feedback from the users. These indexes give the order of the sub-carriers channel gain. However, the BER considered should relate to the actual value of the instantaneous channel gain. We use average BER may have constraints and limitations when we allocate sub-carriers. Further study can be carried out to solve this problem.

Also, the objective function we formulated in Chapter 5 is an 0-1 integer programming problem. Now, many researchers are working on efficient and simple algorithms to solve the 0-1 programming problem, a famous algorithm is cutting-edge algorithm, we should look into developing more efficient cutting-edge algorithm to solve our specific problem.

## **List of Publication**

- Hai LONG, Yong Huat CHEW, "An Adaptive Subcarrier Allocation Scheme for MC-DS-CDMA Systems in the Presence of Multiple Access Interference", IEEE Proc. International Conference on Communications, Vol. 5, pp. 2894-2898, 20-24 June 2004
- Hai LONG, Yong Huat CHEW, "Two Sub-optimal Subcarrier Allocation Schemes for MC-CDMA System", under preparation, to be submitted to IEEE Trans. Vehicular Technology.

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