FORMATION AND RECONFIGURATION CONTROL FOR MULTI-ROBOTIC SYSTEMS

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In the name of Allah, Most Gracious, Most Merciful

"Guide us to the straight path."

"Holy Quran"

I present this thesis to my father, mother, sister and brother.

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When I started my study in Singapore, my heart was fulled with hope and concern. I felt Allah very closely and he swept away the concern from my heart. I experienced hard days and sweet days. And thanks to him, I finished this segment of my life. During my master study I learnt a lot from these experiences and I significantly improved both socially and academically.

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Contents

Summary

The cooperative and coordination control of multiple autonomous robots have recently received a significant research interest. This research field is driven by both commercial and military applications. A collection of simple autonomous robots offers greater efficiency and operational freedom, comparing to single complicated robot that performs multiple tasks. We use the term multi-agent system to refer to a group of autonomous robots which work together to achieve the global task. The cooperative control of the multi-agent system has been addressed in number of research papers, workshops, conferences. Also, a huge research funding has dedicated to this subject, but this field is still in its infancy stages and poses significant theoretical and technical challenges.

The key feature of the multi-agent system is that the group behavior of multiple agents is not simply a summation of the individual agent's behavior. The dynamics of each individual and the interaction protocol among agents are very simple; however, as a whole group they can perform complicated tasks and behaviors.

In this thesis, we mainly focus on the cooperative control of multi-agent systems. Specifically, a decentralized cooperative control law for performing a specific formation or coordination among a group of robots is studied and the required conditions for achieving this task is investigated. We develop concrete theoretical foundations, and

also implement the theoretical results in the practice.

This dissertation contributes to cooperative control of multi-agent systems from both theoretical and practical perspectives. Firstly, several essential problems such as controllability, observability and optimality are discussed. Secondly, a formation control among a group of robots is implemented in practice. Specifically, current dissertation provides a graph theoretical interpretation for the controllability property of the multi-agent system. Moreover, a novel consensus observer strategy is proposed, and sufficient and necessary conditions for observability of multi-agent system are driven. Furthermore, a paradigm is introduced which offers a systematic assign the communication weights among a group of robots. Finally, a formation control among a group of three wheeled robots is implemented.

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Chapter 1

Introduction

Multi-robot systems are collection of autonomous robots with a certain degree of capability. Compared to a single multi task robot, these systems provide higher efficiency, robustness and operational capabilities. Multi-robot systems have potential applications in surveillance, combat, distributed sensor network (DSN), autonomous underwater vehicles and unmanned aerial vehicles. Thus, they have recently become so popular [2], [79], [78]. Also, their cooperative control has recently received a significant research interest [59], [55], [82]. In this dissertation, we use the terminology of agent to refer a robot with limited capability. In addition, the expressions multi-agent systems and multi-robot systems are used interchangeably.

Design and analysis of multi-agent system is a complicated task. The dynamics of each individual not only depends on dynamics of its own, but also relays on behavior of its adjacent agents. Moreover, the global behavior of a team is not simply a summation of the individual agent's behavior, but a sophisticated combination of interacting sub modules.

1.1 Motivation

Recent developments of enabling technologies such as communication systems, cheap computation equipment and sensory platforms have greatly enabled the area of multi-agent systems. This area has attracted significant attention worldwide [5], [3], [21], [40], [52], [70]. A group of multi-agent system can perform higher efficiency and operational capabilities, if there exists a kind of simple cooperation among agents.

The cooperative control of multi-agent systems is still in its infancy stages and poses significant theoretical and technical challenges [88], [42]. The cooperative control of such complex networked systems has been highly inspired by biological systems [68]. The research thread in cooperative control has branched into two main venues, homogenous network, where all agents are identical to each other and heterogeneous network, where there exist some agents with superior capabilities. From another point of view, all researches in this field can be categorized into the following branches: sensing, communication, computation and control. This reveals that multi-agent system is a multi-disciplinary area of research including fields such as computer science, engineering, mathematic, biology and control system theory in particular.

There exist so many interesting problems in area of multi-agent system which can be solved using the well-founded control system theory. The interdisciplinary nature of this research has helped the enrichment of control theory. The conjecture of mutual interaction between the multi-agent systems and the control theory has opened new areas such as symbolic control inside the control theory.

Besides to classical control theory, the graph theory has shown to be an effective tool for dealing with coordination control problem. The graph theory encodes the local interaction topology. Moreover, it shades more light on the relation between communication and control i.e. what kind of information topology we need to design an appropriate control law or which kind of control strategy is required for an special communication topology.

1.2 Nature Inspiration

In order to model, analyze and design of a multi-agent system, researchers commenced to explore natural systems, where there exist plenty examples of such systems. These natural systems are quite diverse and range from human society, where each agent is a complex system, to physical particle systems, where each agent has no intelligence [12]. There are several pioneer works [68], [12], [58], [2], [8]. Authors in [68] investigated a flock of birds; they [68], analyzed this phenomenon and validated their results with an animator. [12] proposed a simple model for system of biological particles. In their model, a particle is driven by both a constant term and a term from its neighbors. Based on simulation results, they showed that the model could cause all particles move in the same direction though there is no centralized coordinator. [58] explored the grouping of animal in natural environments. They claimed that they offered a dynamical model for the group size distribution affected by splitting and merging [2].

Several researchers started to make the mathematical justification for natural inspired models. [29], provided a substantial result for convergence of the model similar to [12]. An extended version of the model in [12] is the so called consensus protocol, discussed in [73]. [73] used this model for coordination of first order dynamics agents. It also discusses about the robustness of this algorithm. Inspired by [68], [84] studied the stable flocking motion among a group of agents. They proposed a control paradigm that ensures all agents, will be finally aligned with each other and have the common heading direction. The research focus in this area is on two main streams, homogenous system and heterogeneous system.

1.3 Homogenous network

1.3.1 Consensus Problem

There has been a considerable amount of work which contributed to analyze and design of consensus problem. This problem is also known as agreement , rendezvous and swarming problem in different situations. A group of agents reach consensus, when all of the agents agree on the value. In control language, this agreement means that all state variables asymptotically reach the desired state:

$$\lim_{t\to\infty} x_i(t) = x_d \quad i = 1, \ldots, N. \tag{1.1}$$

The preliminary idea of consensus is to impose the same dynamics on information state of each agent. If continuous communication is allowed among agents or the communication bandwidth is large enough, then state of each agent is updated using differential equation. Otherwise, the discrete model is applicable and states are modified using difference equation. The most common type of agreement law [29],

[66] is given by

$$u_i = -\sum\nolimits_{j \in \mathfrak{N}_i} w_{ij}(x_i - x_j), \tag{1.2}$$

where $\mathfrak{N}_i$ is the neighbor set of the agent i, $w_{ij} \in \mathbf{R}$ is the weight of the edge from agent i to agent j. The weight factor w_{ij} can be evaluated from different angles. If the topology is fixed over the time, weights are set to be constant. However, the topology may evolve over the time [54], [53] and weights could be linear time-variant [46], [65]. [65] considered the consensus among multiple agents with dynamically changing topologies under the confined information exchange. Authors in [46] claimed that general formation can be achieved if convergence to a point is feasible; hence, convergence of the system into the common point is discussed in [46].

The convergence of agreement law (1.2) is highly depend on algebraic topology of the whole system. For instance, [83] proposed a paradigm for flocking motion. They stated that flocking motion can be established, as long as the neighboring graph remains connected. Hence, the connectivity of the whole topology plays a crucial rule in convergence of the algorithm and must be considered in the design of a proper controller. Importance of law convergence and its relation with connectivity are discussed in several research articles. For instance, authors in [30] considered the dynamic changing graph. They proposed an appropriate weights' assignment to the edges in the graphs which guarantees that the connectivity of graph. This problem is further discussed in [91], where authors studied the preserving k-hop connectivity. Based on the k-hop connectivity, agents are allowed to move unless they keep their connection to agents within the k-hop limit. Authors in [92], proposed a hybrid algorithm to preserve the connectivity, while [80] discussed about geometric analysis of connectivity. Moreover, they introduced a function which measures the robustness of local connectedness to variations in position.

While majority of works focused on agents with simple integrator dynamics, recently some researchers have proposed more realistic dynamics for agents. [67] considered the agreement law (1.2) for l-th order system $l > 3$. They showed sufficient and necessary conditions required for convergence of the whole system into the common value. This problem is further discussed in [87], where authors explored the high order dynamics under chain topology. Moreover, the convergence of system was discussed under fixed and dynamic topology.

Under the frame work of homogenous systems, researchers are more concerned about convergence of consensus law. Even though it is important that agents reach the agreement, it could be more interesting to make agents keep a certain formation or reconfigure them between different formations [11], [89], [34], [81].

1.4 Heterogeneous Network

In heterogeneous framework, the majority of agents follow the nearest neighbor law (1.2), but a small group is not confined to this control law. These agents are usually more equipped comparing to other agents. We refer to these advanced agents as leaders and they are able to take the govern of the others. We refer to the rest of agents as followers. This kind of structure, where agents are divided into two sets, is called leader-follower configuration.

A numerous formation control achieved based on leader-follower structure, where either a real agent [14], [15] or a virtual agent [19], [20], [60], [42] takes the lead. For instance, [19] proposes an algorithm for tracking of the desired trajectory.

1.4.1 Flocking, Swarming and Formation Control

Achieving an specific formation and developing a control law that guarantees formation stability is the most important problems in multi-agent systems field [22], [13], [41], [59]. The problem of formation control has been successfully addressed when exploring swarm behaviors, where agents are coordinating based on potential field [64], [23], or some averaging orientation [29], or simply following the leader [83], [84]. Authors, in [84], achieved a stable flocking motion for a group of mobile agents with double integrator dynamics. Moreover, authors in [85], made a relation between the interaction topology to leader-to-formation stability problem. Under this setup, rigidity becomes one of the important issues in formation keeping [71], [18], [4].

A further extension along this direction leads to controllability problem. This problem has become focus of attention recently. Based on the well-developed control theory, as far as system is controllable, it can be driven into any desired state. This elegant result motivated researchers [86], [34], [89] and [49] to investigate the formation and reconfiguration problem of multi-agent problem as controllability problem. Roughly speaking, a multi-agent system is controllable if and only if a whole group of agents can be steered to any desirable configurations under local information from other followers and commands of the leaders.

The controllability problem of multi-agent systems has been investigated in the literature for a while. Tanner proposed this problem in [86] and formulated it as the controllability of a linear system, whose state matrices are induced from the graph Laplacian matrix. Necessary and sufficient algebraic conditions on the state matrices were given based on the well-established linear system theory. Even though we expect that more information leads to better control design, Tanner showed that providing the maximum information violates the controllability of the whole group. Under the same setup, [33] offered a sufficient condition for a system to be controllable. It was shown that the system is controllable if the null space of the leader set is a subset for the null space of follower set. This result is further extended in [34], where authors provided a necessary and sufficient condition. Authors in [34] claimed that a system is controllable if and only if the Laplacian matrix of the follower set and the Laplacian matrix of the whole topology have no common eigenvalues. Even though it is a strong result, but the graphical meaning of these rank conditions related to the Laplacian matrix remains as question. Motivated by this problem, several researchers started exploring the controllability of multi-agent systems from the graph theoretical point of view. For example, [62] proposed a notion of anchored systems showed that symmetry with respect to the anchored vertices makes the system uncontrollable; moreover, the relation of group automorphism and network controllability was discussed in [63]. Authors in [31], introduced a new notation called leader-follower connectedness and characterized some necessary conditions for the controllability problem based on leader-follower connectedness. Most of the available results are focused on continues systems but [48] offered the analysis for controllability of a class of multi-agent systems with discrete-time model. Besides fixed topology, the controllability problem under switching topologies was discussed in [32], [47], [49].

Most of recent results provided just algebraic interpretation, there are few works [89], [49] which offer graphical interpretation of these algebraic conditions. Authors, in [89], consider the weighted graph. They assumed the graph to be weighted and they can be freely assigned. Authors introduced a novel notion of multi-agent systems structural controllability and established a sufficient and necessary condition accordingly.

1.4.2 Centralized Control vs. Decentralized Control

Information interaction among agents is the crucial issue in formation control. In the most cases, the common assumption is that each agent has complete information about the whole group [13], [44], [37]. This is a centralized way of formation control. However, this method suffers from several practical issues such as scalability of group, communication bandwidth and sensors range constraints. As a result, researchers have recently focused on decentralized approach to perform a coordination or maintain a formation among a group of robots. There are plenty of research articles which deal with decentralized control of multi-agent systems. For instance, [5] addressed the problem of coordination control for multiple spacecraft. They proposed the behavioral and virtual-structure approaches to multi-agent systems' coordination problem. Similarly, authors in [19] addressed the coordination control using a virtual vehicle method. Another distributed approach can be seen in [18], where authors introduced systematic method for maintaining rigidity among mobile autonomous vehicles. Authors in [70], studied the decentralized framework for formation stabilization among a group of robots and explored the application of natural potential functions in formation control. Authors in [82] investigated a novel decentralized stability notion so called input-to-state stability. They analyzed the input-to-state stability with help of primitive graphs. A practical of example of decentralized for group of unmanned air vehicles (UAVs) was discussed in [6]. Based on decentralized receding horizon control (RHC) scheme, authors in [6], proposed a decentralized control paradigm which assures the collision avoidance.

Even though the local interaction solves some of the global interaction problems, there are plenty of challenges that need to be solved such as organizing proper communication link, determining of local interaction based on global rule and task scheduling in unknown terrain [35], [36], [38], [39], [10]. In addition, different formations are suitable for different occasions and this decision making mechanism have to be employed in a distributed fashion.

1.4.3 Sensor Capalities

The formation or distributed control is not feasible unless each robot has clear perception from its ambient environment and neighbor robots. Each individual robot can collect data either by peer to peer communication with other robots, or relying on sensor fusion. Since any physical sensor is limited by its range, the required in-

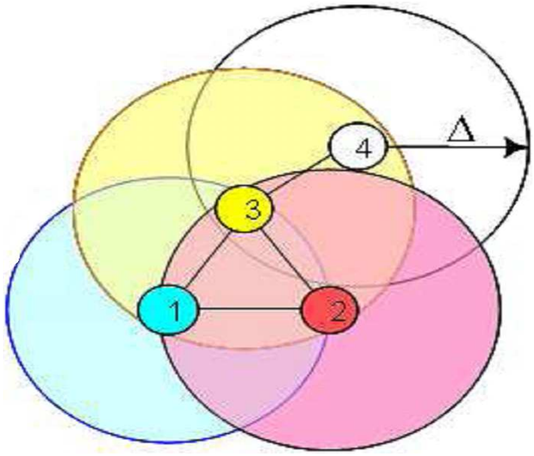


Figure 1.1: Proximity graph

formation must be obtained either by direct observations or state estimation. For instance, in Fig. 1.1, agent four serves as leader, while the rest of agents are followers. It is clearly depicted in Fig. 1.1 that agent four has direct access to states of agent three for design of appropriate control strategy; however, it needs to indirectly observe states of other agents. This problem is closely related to the observer design in the control theory.

Motivated by this problem, several researchers have recently considered the observability problem for multi-agent systems [28], [27], [57]. In [27], the authors used estimator to observe the leader's state. Similarly, the authors in [28], designed the distributed observer for second-order follower-agents to estimate the velocity of leader. Moreover, in [57], the authors studied the observer for the delay systems.

All the existing work focused on the estimation of the leaders' state, while another interesting question is whether or under what condition we can reconstruct the followers' state based on readings from the leaders. The motivation for this observability problem comes from the study of controller synthesis for leaders to herd all agents to a desirable configuration. To design control signals for leaders, the operators need to know all agents states. However, due to communication constraints the leader cannot measures all agents states directly and it requires to estimates the states of the agents just based on the readings from the leader. By saying a multi-agent systems is observable from the leader, we mean that one can reconstruct all agents' states just based on the output reading from the leader. We consider in [90] the classical notion of observability for a group of autonomous agents interconnected through the nearest neighbor law. In addition, the sufficient and necessary conditions are presented from both algebraic and graph theoretic perspectives. Similar problems were considered in [50], where the authors specifically focused on the controllability and observability of the two configurations, the cyclic topology and the chain topology, and their interconnections.

1.5 Graph Theory

The graph theory has proved to be a useful tool for handling the control theory problems [45], [16], [26] and multi-agent systems problems [75], [76], [56], [65], [46], [21], [24].

For instance, while [21] made a connection between control theory and graph theory to analyze the formation stabilization. Authors in [24] showed that rank of graph Laplacian relates to connectivity . Similar results have been shown while studying the convergence of agreement law [75]. Authors in [75], proposed a convergence analysis for agreement control based on properties of balanced graph. This idea is further extended in [76], where a connection between performance of the nearest neighbor law and the Fiedler eigenvalue of the graph Laplacian was established. Hence, the graph topology not only determines the convergence, but also determines the performance of the system. Within the same line, [73] considered a spatial adjacency matrix for obtaining the formation among a group of agents which are equipped with sensors of limited range. [65], discussed the dynamic topology and claimed that the systems asymptotically converge to common value if union of interaction topologies over some time intervals has a spanning tree. Moreover, [71] set a graph theoretic framework which relates the uniqueness of graph realization to stability of formations. The connectivity of graph has shown to be an important issue in multi-agent systems [56], [91], [30]. For example. [56] introduces a paradigm for topology characterization based on the connectivity graphs.

The application of the graph theory is not confined to this; it turns out that some of the well-known control theory problem can be better expressed under graph theoretic framework. Early effort in this area can be seen in [45] which offered more general definition for controllability problem. Comparing to algebraic conditions, graph theoretic conditions offers better insight into the problem. For instance, effort of [45] has further continued by [49] which offers a neat graph theoretic result for multi-agent systems controllability. This result has true privilege over other similar existing result. It not only leads us to design of communication link, but also shades more light on controllability of switching systems which is an open problem in hybrid control area. Due to the importance of graph theory in our discussion, in this part some of the basic concepts of graph theory are presented.

1.5.1 Some Basic Notations in the Graph Theory

A *weighted graph* is an appropriate representation for the communication or sensing links among agents because it can represent both existence and strength of each link. The weighted graph $\mathcal{G}$ with N vertices consists of a vertex set $\mathcal{V}$ =$\{v_1, v_2, \ldots, v_N\}$ and an edge set $\mathcal{I}$ =$\{e_1, e_2, \ldots, e_N\}$, which is the interconnection links among the vertices. Each edge in the weighted graph represents a bidirectional communication or sensing media. The *order* of the weighted graph is denoted to be the cardinality of its vertex set. Similarly, the cardinality of the edge set is defined as its *degree.* Two vertices i and j are known to be *neighbors* if $(i, j) \in e$, and the number of neighbors for each vertex is its *valency.* A graph is so called *regular* if all vertices have the same degree. If all vertices of graph $\mathcal{G}$ are pairwise neighbor, then $\mathcal{G}$ is *complete.* A N order complete graph is denoted by K_N. An alternating sequence of distinct vertices and edges in the weighted graph is called a *path.* The weighted graph is said to be *connected* if there exists at least one path between any distinct vertices. A number of edges of a path is its *length.*

The *incidence matrix* ***In*** of $\mathcal{G}$ is a $|\mathcal{V}| \times |\mathcal{I}|$ which is defined as

$$\boldsymbol{In}_{kl} = \begin{cases} k_{ij} & \text{if node k is the head of edge l,} \\ -k_{ij} & \text{if node k is the tail of edge l.} \end{cases}$$

The *adjacency matrix*, $\boldsymbol{A}_{ij}$, is defined as

$$\boldsymbol{A}_{ij} = \begin{cases} \beta_{ij} & (i,j) \in e, \\ 0 & \text{otherwise,} \end{cases}$$

where $\beta_{ij} \neq 0$ stands for the weight of edge (i,j). Here, the adjacency matrix $\boldsymbol{A}$ is $|\mathcal{V}| \times |\mathcal{V}|$ and $|.|$ is the cardinality of a set.

Define another $|\mathcal{V}| \times |\mathcal{V}|$ matrix, $\boldsymbol{D}$, called *degree matrix*, as a diagonal matrix which consists of the degree numbers of all vertices.

The *Laplacian matrix* of a graph $\mathcal{G}$, denoted as $\boldsymbol{L}(\mathcal{G}) \in \mathcal{R}^{|\mathcal{V}|\times|\mathcal{V}|}$ or $\boldsymbol{L}$ for simplicity, is defined as

$$\boldsymbol{L}_{ij} = \begin{cases} \sum_{i \neq j} w_{ij} & i = j, \\ -w_{ij} & (i,j) \in e, \\ 0 & \text{otherwise.} \end{cases}$$

The Laplacian matrix $\boldsymbol{L}$ can be expressed as

$$\boldsymbol{L} = \boldsymbol{D} - \boldsymbol{A}.$$

It turns out that Laplacian matrix is a key to solve control agreement problem [75], [24].

For example, if all weights are set to unity, the adjacency matrix and the Laplacian matrix of a graph shown in Fig. 1.2 can be written as :

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{bmatrix} \quad \mathbf{L} = \begin{bmatrix} 2 & 0 & 0 & -1 & -1 \\ 0 & 2 & -1 & 0 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ -1 & 0 & 0 & 2 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}$$

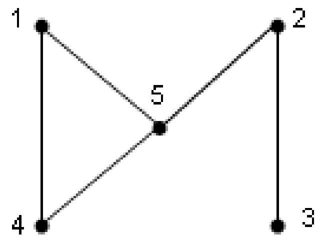


Figure 1.2: A graph on $\mathcal{V}$=$\{1, 2, 3, 4, 5\}$ and edge set $\mathcal{I}$ =$\{(1, 4), (1, 5), (4, 5), (5, 2), (2, 3)\}$

It can be easily verified that the Laplacian matrix has several interesting properties:

1. It is positive semi-definite matrix and its spectrum has following order

$$\lambda_N \geq \lambda_{N-1} \geq ... \geq \lambda_2 \geq \lambda_1 = 0$$

   where λ_i is the i-th ordered eigenvalue of the graph. The multiplicity of zero eigenvalue of a graph equals its connected components

2. The laplacian of a graph does not depend on its orientation

3. The laplacian is not only non-negative but also symmetric.

4. The topology is connected if and only if $\lambda_2 > 0$

5. If the topology $\mathcal{G}$ is connected, then the null space of $\mathbf{L}$ is span$\{\mathbf{1}\}$, where $\mathbf{1}$ denotes a vector with all unit entries.

6. For a graph $\mathcal{G}$ with N vertices

$$\sum_i \lambda_i < N$$

   if and only if $\mathcal{G}$ has no isolated vertices.

7. if $\lambda_i = 0$ and $\lambda_{i+1} \neq 0$ then $\mathcal{G}$ has excatly $i + 1$ connected components.

1.6 Contributions

This thesis has several important contributions to area of multi-agent systems cooperative control. It contributes to this area from both theory and practice. Several fundamental issues related to multi-agent systems are discussed in this dissertation which helps us in analysis and design of multi-agent systems. We focus on two profound properties of multi-agent systems controllability and observability. In contrast to the existing literatures on this topic, we investigate the problem from graph theory point of view and establish a connection between graph theory and these fundamental properties. Some sufficient and necessary conditions for observability and controllability of multi-agents are obtained which shade light on design of communication link.

Despite existing literatures, we study the multi-agent systems under a weighted graph topology. Under this setup, a novel notion of multi-agent systems structural controllability is proposed. It is clearly shown that for multi-agent systems are structurally controllable if and only if the communication topology remains connected. Hence, as far as there exists a connected communication link among agents, multi-agent systems can be configured into any desired configuration.

Due to the sensors' constraint, information collection from agents may not be feasible all the time. However, availability of states is a necessary fact for design of proper control law. Motivated by this problem, we focused on the estimation problem of multi-agent systems. A novel notion of multi-agent systems observability is proposed as an extension to the well-known observability notion. The observability problem for multi-agent systems is investigated from algebraic point of view and observability property of some well-known topology such as the path graph or the complete graph is discussed. Besides algebraic point of view, the problem is also discussed from graph theory prespective. A novel notion of structural observability is proposed and a required sufficient and necessary condition is obtained. It turns out that the connectivity of communication topology is both necessary and sufficient for a system to be observable.

It is clear that controllability and observability notion purely depend on the topology of communication link. Hence, an optimal solution for configuring the topology is proposed. Our algorithm determines a set of the best weight among a plenty of possible weight meanwhile it guarantees the final desired states.

This dissertation is not just confined to theoretical results. The structural controllability of multi-agent systems is implemented on a group of wheeled robots with a leader and the experiment results are reported.

1.7 Organization

This dissertation consists of two parts. First, the problem of formation control is studied from theoretical point of view. The formation control problem is stated as controllability problem for multi-agent systems. Several interesting problems are discussed under this part. In Chapter 2, we introduce a problem of structural controllability for multi-agent systems. Consequently, a sufficient and necessary condition for structural controllability of multi-agent systems are proposed. The problem is studied from graph theoretic perspective which is quite novel. A controllable system can be steered into any desired configuration, but design of an appropriate control law requires availability of state variables. However, due to the communication limitation, availability of a state variable is not always feasible. Motivated by this problem, an observability problem for multi-agent systems is studied in Chapter 3. The proper controller is proposed to drive all the agents into the favorite destination. In Chapter 4, a method is proposed for the design of connection weights among the agents. This method not only guarantees the reachability of the final destination, but also tries to keep the control effort given to whole system, at the minimum possible level.

In last part, we mainly focus on practical implementation of result obtained in first part. A group of three e-puck robots is used as test bench. The leader-follower approach is obtained, where one of agents serves as the leader and the rest two are followers. Each robot is equipped with limited computation and sensing capabilities this makes the test bench suitable for exploring the swarm configuration.

Chapter 2

Structural Controllability of Multi-Agent Systems

2.1 Introduction

In this chapter, the controllability problem for a group of multi-agent system is investigated. In particular, the case of a single leader under a fixed topology is considered. Moreover, the graph is assumed to be weighted and one may freely assign the weights. Under this setup, the system is controllable if one may find a set of weights so as to satisfy the classical controllability rank condition. It turns out that this controllability notation purely depends on the topology of the communication scheme, and the multi-agent system is controllable if and only if the graph is connected. Furthermore, we propose an optimal control based control scheme to steer the followers to desired configurations. Finally, some simulation results and numerical examples are presented to illustrate the approach.

The rest of the chapter is structured as follows. In the next section, a new notation, structural controllability for multi-agent systems is proposed, and the problem studied in this chapter is formulated. In Section 2.3, a necessary and sufficient condition for the structural controllability problem is given. In Section 2.4, an optimal control based control law is designed for the leader to steer the followers into the desired configurations. Section 2.5 presents some numerical examples to illustrate the derived theoretical results and design methods. Finally, the chapter concludes with comments and plans for our further work.

2.2 Problem Formulation

Our objective in this chapter is to control N agents based on the leader-follower framework. We specifically will consider the case of a single leader and fixed topology. Without loss of generality, assume the N-th agent serves as the leader and take commands and controls from outside operators directly, while the rest $N-1$ agents are followers and take controls as the nearest neighbor law.

Mathematically, each agent's dynamics can be seen as a point mass and follows

$$\dot{x}_i = u_i. \tag{2.1}$$

The control strategy for driving all follower is

$$u_i = -\sum\nolimits_{j \in \mathfrak{N}_i} w_{ij}(x_i - x_j), \tag{2.2}$$

where $\mathfrak{N}_i$ is the neighbor set of the agent i, and w_{ij} is weight of the edge from agent i to agent j. On the other hand, the leader's control signal is not influenced by the followers and need to be designed, which can be represented as

$$\dot{x}_N = u_N.$$

In other words, the leader affects its nearby agents, but it does not get directly affected from the followers since it only accepts the control input from an outside operator. For simplicity, we will use z to stand for x_N in the sequel.

According to the algebraic graph theory [9], it is known that the whole system can be written in a compact form

$$\begin{bmatrix} \dot{x} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} A_{aq} & B_{aq} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ z \end{bmatrix} + \begin{bmatrix} 0 \\ u_N \end{bmatrix} . \tag{2.3}$$

Or, equivalently

$$\begin{cases} \dot{x} &= A_{aq}x + B_{aq}z \\ \dot{z} &= u_N \end{cases} \tag{2.4}$$

where $A_{aq} \in \mathbf{R}^{(N-1)\times(N-1)}$ and $B_{aq} \in \mathbf{R}^{(N-1)\times 1}$ are both sub-matrices of the corresponding graph Laplacian matrix $\boldsymbol{L}$. The matrix A_{aq} reflects the interconnection among followers, and the column vector B_{aq} represents the relation between followers and the leader.

The problem is whether we can find a weighting scheme, i.e., set values for w_{ij}, such that it is possible to drive these agents to any configuration or formation (if the states stand for the positions of agents) by properly designed control signals u_N for the leader. This is related to the controllability of the system (2.4). Once the weights w_{ij} are all selected and fixed, the system (2.4) is reduced to a LTI system and its controllability can be directly answered by the well-developed linear system theory, see e.g. [1]. Actually, a special case when all weights $w_{ij} = 1$ (an unweighed graph) has been investigated in the past literature, e.g., [86]. However, Tanner in [86] showed that the complete graph is uncontrollable as illustrates in the following example.

Example 1 *Consider a multi-agent system with six agents, whose communication*

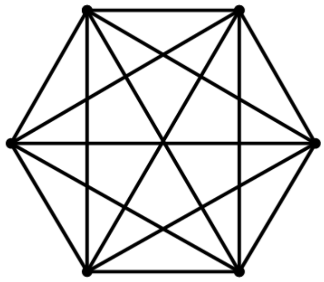


Figure 2.1: A complete graph with 6 vertices.

topology is a complete graph with six vertices as shown in Fig. 2.1. Following the formulation in [86] that the matrices A_{aq} *and* B_{aq} *in (2.4) can be written as*

$$A_{aq} = \begin{bmatrix} 5 & -1 & -1 & -1 & -1 \\ -1 & 5 & -1 & -1 & -1 \\ -1 & -1 & 5 & -1 & -1 \\ -1 & -1 & -1 & 5 & -1 \\ -1 & -1 & -1 & -1 & 5 \end{bmatrix}, \ B_{aq} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}. \tag{2.5}$$

It is not difficult to see that this pair is uncontrollable. This is quite counter intuitive, since the complete graph is an ideal case which provides the maximum information for the control purpose. It should be the case that more information exchanges among agents imply better control performances. The problem seems to be how we use this information. To treat all available information in an equal way seems not be a good choice. One should use the information in a selective way. This motivates us to impose different weights according to the information resources.

With the set-up in (2.4), a set of weight can be assigned such that the controlla-

bility rank is satisfied; for instance, the pair (A_{aq}, B_{aq}) can be written as

$$A_{aq} = \begin{bmatrix} 7 & -2 & -2 & -2 & -1 \\ -2 & 9 & -3 & -2 & -2 \\ -2 & -3 & 13 & -5 & -3 \\ -2 & -2 & -5 & 11 & -2 \\ -1 & -2 & -3 & -2 & 8 \end{bmatrix}, \; B_{aq} = \begin{bmatrix} -1 \\ -2 \\ -5 \\ -3 \\ -1 \end{bmatrix}. \tag{2.6}$$

One can check that this (A_{aq}, B_{aq}) pair is controllable.

This example motivates us to give a more general definition for controllability of multi-agent systems as follows.

Definition 1 *The linear system Σ in (2.4) is said to be structurally controllable if and only if there exists $w_{ij} \neq 0$ which can make the system (2.4) controllable.*

Here, we are especially interested in a necessary and sufficient condition on the graphical topology of a multi-agent system to make it structurally controllable. That is, under exactly what condition of the graph that we can always find a weighting scheme w_{ij} so as to make the multi-agent system (2.4) controllable.

2.3 Structural Controllability

First, a lemma on controllability of (2.4) when weights are fixed is due.

Lemma 1 *For the system (2.4) with a fixed weighting w_{ij}, the following statements are equivalent:*

i) The system (2.4) is controllable.

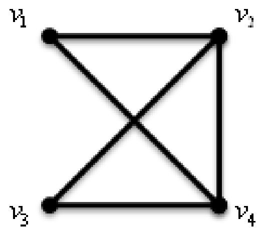


Figure 2.2: Topology $\mathcal{G}$

ii) The controllability matrix

$$\mathcal{U} = \left[\begin{array}{cccc} B_{aq} & A_{aq}B_{aq} & \ldots & A_{aq}^{N-1}B_{aq} \end{array} \right].$$

is of full row rank.

iii) The controllability Gramian matrix

$$W(t_0, t_f) = \int_{t_0}^{t_f} e^{A_{aq}\tau} B_{aq} B_{aq}^T e^{A_{aq}^T \tau} d\tau$$

is nonsingular for all $t > 0$.

iv) The matrix $\begin{bmatrix} A_{aq} - \lambda I & B_{aq} \end{bmatrix}$ *has full row rank for all eigenvalues* λ *of* A_{aq}.

The above lemma is a direct consequence of the well-known linear systems theory, see e.g., [1], due to the fact that the system (2.4) is reduced to a LTI system once weighting is fixed; however, for the structural controllability of multi-agent system we need the following definitions from [45].

Definition 2 *The pair* (A_{aq}, B_{aq}) *in (2.4) is said to be reducible if they can be written* *into the form below;*

$$A_{aq} = \begin{bmatrix} A_{aq_{11}} & 0 \\ A_{aq_{21}} & A_{aq_{22}} \end{bmatrix}, \quad B_{aq} = \begin{bmatrix} 0 \\ B_{aq_{22}} \end{bmatrix}, \tag{2.7}$$

where $A_{aq_{11}} \in \boldsymbol{R}^{p\times p}$, $A_{aq_{21}} \in \boldsymbol{R}^{(N-1-p)\times p}$, $A_{aq_{22}} \in \boldsymbol{R}^{(N-1-p)\times(N-1-p)}$ *and* $B_{aq_{22}} \in \boldsymbol{R}^{(N-1-p)}$.

It was shown in [45] that the controllability matrix for this structure cannot be of the full row rank no matter how one chooses the weighting w_{ij}. Hence, the system (2.4) is not structurally controllable under this situation.

Another obviously uncontrollable scenario is captured as follows.

Lemma 2 *[45] The system (2.4) is not structurally controllable if the matrix* $[A_{aq}, B_{aq}]$*, which is* $N-1\times N$ *matrix, can be written as*

$$Q = \begin{pmatrix} Q_{11} \\ Q_{22} \end{pmatrix}, \tag{2.8}$$

where Q_{22} *is of* $(N-1-p)\times N$ *and* Q_{11} *is of* $p\times N$ *with at most* $p-1$ *nonzero entries and the rest of columns are all zero.*

Interestingly, except these two obviously uncontrollable scenarios, the system (2.4) will be structurally controllable as the following lemma states.

Lemma 3 *[45] The pair* (A_{aq}, B_{aq}) *is structurally controllable if and only if it is neither reducible nor writable into the form of (2.8) in Lemma 2.*

Our next task is to interpret the above results in a graph theory point of view. It has been shown in [9] that the relation of a pair (A_{aq}, B_{aq}) can be depicted in a pictorial representation and the notion of flow structure plays an important role here. Hence, we introduce some necessary notations which we need for further discussions in this chapter.

Definition 3 *The pair* (A_{aq}, B_{aq}) *matrix can be represented by a digraph, defined as a flow structure,* $\mathfrak{F}$*, with vertex set* $\mathcal{V}' = \{v_1', v_2', ..., v_N'\}$*. There exists an edge from* v_i' *to* v_j' *in the flow structure if and only if* $A_{aq}(j,i) \neq 0$ *and an edge from* v_N' *to* v_i' *if and only if* $B_{aq}(i) \neq 0$.

Remark 1 *Directions of links in flow structure has no dependence on the sign of their corresponding entries in matrix* A_{aq}.

For example, the flow structure for the graph shown in Fig. 2.2 is depicted in Fig. 2.3. There are some well known flow structure that have interesting controllability properties, such as the flow structure of an ordered vertex set $\mathcal{V}'$ =$\{v_1', v_2', ..., v_n'\}$ with a sequence of edges, where terminal vertex of each edge is initial point for the following edge. This is known as a stem [45], as depicted in Fig. 2.4. The corresponding state matrices for a stem, denoted as (A^*_{aq}, B^*_{aq}), can be written as

$$A^*_{aq} = \begin{bmatrix} 0 & * & 0 & \dots & 0 \\ & & & & \\ \vdots & & \ddots & \ddots & \\ & & & 0 & * \\ 0 & & \cdots & & 0 \end{bmatrix}, \quad B^*_{aq} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ * \end{bmatrix},$$

where the symbol $*$ is used to represent the unknown but nonzero elements that depends on the weighting for edges. This falls into the controllable canonical form,

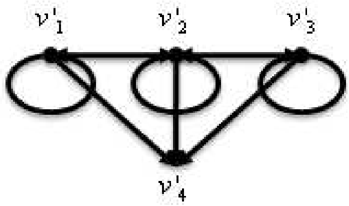


Figure 2.3: Flow graph

Figure 2.4: Stem

so the controllability is obvious for a stem structure.

Another interesting structure grows from a stem. If the vertex v'_n of a stem structure coincides with v'_2, the structure is called a bud [45] and its corresponding flow structure is shown in Fig. 2.5. For a bud, the corresponding pair (A^*_{aq}, B^*_{aq}) can be written as

$$A^*_{aq} = \begin{bmatrix} 0 & * & 0 & \dots & 0 \\ \vdots & & \ddots & \ddots & \\ & & & 0 & * \\ * & & \cdots & & 0 \end{bmatrix} \quad B^*_{aq} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ * \end{bmatrix}.$$

A union of a stem $\mathfrak{S}$ and buds $\mathfrak{B}_i$, $1 \leq i \leq d$, is called a cactus if none of the buds $\mathfrak{B}_i$ share a common initial vertex in $\mathfrak{S}$. A set of mutually disjoint cactus is called a cacti, as illustrated in Fig. 2.6.

Based on these notations, we have the following sufficient condition to characterize the structural controllability of the multi-agent system (2.4).

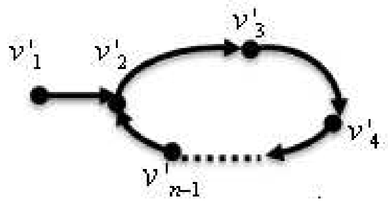


Figure 2.5: Bud

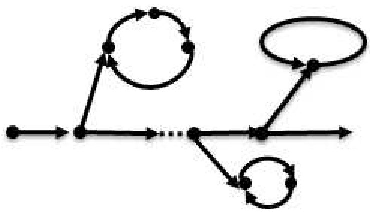


Figure 2.6: Cacti

Proposition 1 *The multi-agent system (2.4) is structurally controllable if its corresponding flow structure can be spanned by a cacti.*

Proof: Suppose that the graph can be spanned by a union of mutually disjoint cactus $\mathfrak{C}_i$, $1 \leq \ i \leq p$. Under this scenario all edges equal to zero except those pertaining with one cacti. With the help of the permutation matrix, A^*_{aq} can be written in form I as

$$A^*_{aq} = \left[\begin{array}{c|c|c|c}
\begin{bmatrix} 0 & * & & \\ & & \ddots & \\ & & \ddots & * \\ 0 & & & 0 \end{bmatrix} & 0 & & \cdots \\ \hline
\begin{bmatrix} 0 & \cdots & 0 \\ & & \\ & \ddots & \\ & * & 0 \end{bmatrix} & \begin{bmatrix} 0 & * & \cdots & 0 \\ & & \ddots & \\ & & \ddots & * \\ * & & & 0 \end{bmatrix} & 0 & \\ \hline
\begin{bmatrix} 0 & \cdots & 0 \\ & & \\ & \ddots & \\ & 0 & * \end{bmatrix} & & \begin{bmatrix} 0 & * & \cdots & 0 \\ & & \ddots & \\ & & \ddots & * \\ * & & & 0 \end{bmatrix} & \\ \hline
 & \begin{bmatrix} 0 & \cdots & 0 \\ & \ddots & \\ & \ddots & \\ * & & 0 \end{bmatrix} & & \begin{bmatrix} 0 & * & \cdots & 0 \\ & & \ddots & \\ & & \ddots & * \\ * & & & 0 \end{bmatrix}
\end{array}\right]$$

(Form I)

while B_{aq}^* has the structure in the form of ;

$$\begin{bmatrix} 0 \\ 0 \\ \vdots \\ * \end{bmatrix}.$$

Hence, the matrix $\begin{bmatrix} A_{aq}^* - \lambda_i I & B_{aq}^* \end{bmatrix}$ has generic full row rank for all λ_i, $1 \leq i \leq N$, which implies the structural controllability. ∎

The above result is a direct application of some known structural controllability results for linear systems in [45] through the introduction of the flow structure. What does this imply in the original graph? The following theorem answers this and provides a nice graphical interpretation.

Theorem 1 *The multi-agent system (2.4) under the communication topology $\mathcal{G}$ is structurally controllable if and only if $\mathcal{G}$ is connected.*

Proof: *Necessity:* Assume that the graph $\mathcal{G}$ is disconnected. For simplicity, we will prove by contradiction for the case that there exists only one disconnected agent. There are two possibilities: First, this isolated agent is the leader. Then, B_{aq} is a null vector in this case , and the system is uncontrollable no matter what the weights are. Secondly, the isolated agent is one follower. For this case, (A_{aq}^*, B_{aq}^*) is reducible, which implies uncontrollability. Both cases end with a contradiction, so the necessity holds. The proof can be straightforwardly extended to more general cases with more than one disconnected agents.

Sufficiency: For the sufficiency part, we show that a connected graph cannot be written either in a reducible form or in the form of (2.8). Note that $w_{ij} \neq 0$ if and only if $w_{ji} \neq 0$. Then, (A^*_{aq}, B^*_{aq}) is in a reducible form if and only if A^*_{aq} is of a block diagonal matrix, this implies that the graph is disconnected. This contradicts with our assumption on the graph connectivity. On the other hand, the graph contains isolated vertex if and only if $\mathcal{D}$ matrix contains zero diagonal elements. So, (A_{aq}, B_{aq}) pair can be written in the form of (2.8) in Lemma 2 if and only if it has a group of isolated agents. Therefore, according to Lemma 3, the graph is structurally controllable. ∎

Example 2 *A star graph is shown in Fig. 2.7. It is assumed that the central agent which is denoted with bold point in Fig. 2.7 serves as the leader and reset are just followers. This structure can be steered to any desired configuration because leader has direct access to all followers. Under the notion of structural controllability one can find a set of weight to make controllability rank condition satisfied; for example, the pair can be written as*

$$A_{aq} = \begin{bmatrix} 1 & & & \\ & 5 & & \\ & & 3 & \\ & & & 2 \end{bmatrix} \quad B_{aq} = \begin{bmatrix} -1 \\ -5 \\ -3 \\ -2 \end{bmatrix}$$

Another interesting phenomenon is demonstrated in the following example.

Example 3 *The graph shown in Fig. 2.8. The middle agent, depicted with bold dot is the leader. It is claimed in [53] that symmetry with respect to the sufficient condition for a system to be uncontrollable. However, under the setup in (2.4), pair*

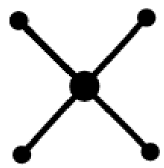


Figure 2.7: Star graph

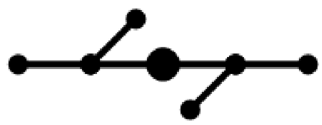


Figure 2.8: Symmetrical structure

(A_{aq}, B_{aq}) can be written as the following form:

$$A_{aq} = \begin{bmatrix} 3 & -2 & 0 & 0 & 0 & 0 \\ -2 & 7 & -4 & 0 & 0 & 0 \\ 0 & -4 & 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & -2 & -3 \\ 0 & 0 & 0 & -2 & 2 & 0 \\ 0 & 0 & 0 & -3 & 0 & 3 \end{bmatrix} \quad B_{aq} = \begin{bmatrix} 0 \\ -1 \\ 0 \\ -1 \\ 0 \\ 0 \end{bmatrix}$$

2.4 Optimal Control Law

In this section, we present an optimal control scheme to drive the system into its desired position.

The control law given to the leader minimizes the following performance index

$$J = \frac{1}{2}(x(t_f) - x_d)^T P(x(t_f) - x_d) + \frac{1}{2}\int_{t_0}^{t_f} [(x_d - x)^T Q(x_d - x) + u_N^T R u_N]dt,$$

where x_d stands for the desired final position at the final time t_f, and $Q > 0$, $R > 0$ and $P > 0$ are specification matrices. It can be shown that solution is in the form of

$$z = -\Xi, \tag{2.9}$$

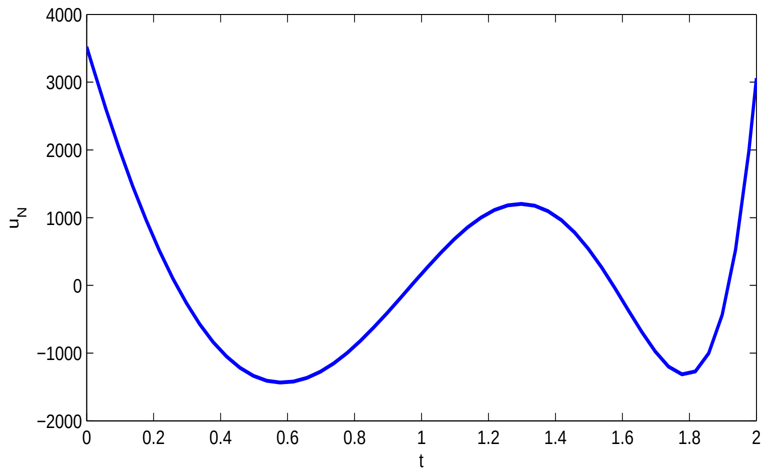
where Ξ is gained by solving the following equations

$$\begin{aligned}
&-\dot{H} = A_{aq}^T H + HA - HB_{aq}R^{-1}B_{aq}^T H + Q, \qquad H(t_f) = P \\
&M(t) = R^{-1}B^T H \\
&-\dot{S} = (A_{aq} - B_{aq}M)^T S + Q, \qquad S(t_f) = Px_d(t_f) \\
&\Xi = -Mx + R^{-1}B_{aq}^T S,
\end{aligned} \tag{2.10}$$

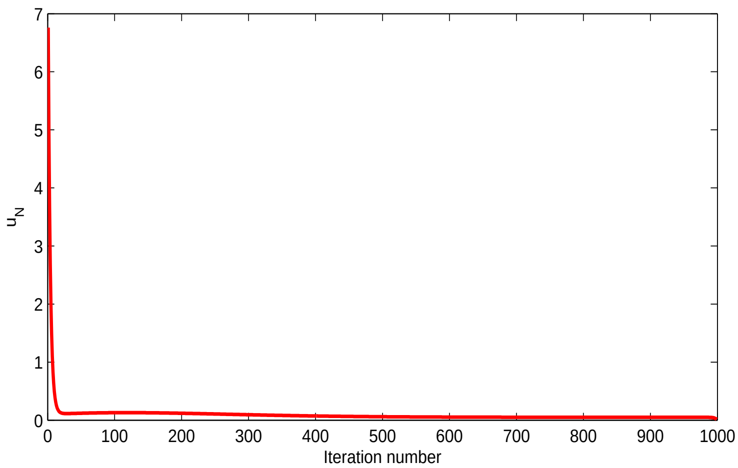
where $x = [x_1, x_2, ..., x_{N-1}]$.

Next, the proposed optimal control law with and the well-known Gramian integral control paradigm are compared. And, their corresponding performances are further investigated.

Example 4 *The control law in (2.9) is deployed for the x position convergence of a group of five agents shown in Fig. 2.1 and use the weights in Example 1. The control effort shown in Fig. 2.9(b) is quite negligible comparing to Fig. 2.9(a). The Fig. 2.9(a) depicts control input just based on Gramian integral. Moreover, the x positions trajectory for both Gramian integral and (2.9) are depicted in Fig. 2.10 and Fig. 2.11, respectively. Since the displacement in Fig. 2.11 is reasonable, each agent*



(a) Control effort from Gramian intergral



(b) Control effort from control law (2.9) with total time of five second

Figure 2.9: Control effort from two control strategy

under control (2.9) will behave more efficiently to reach its desired position. However, the draw back for (2.9) is that equation (2.10) which need to be calculated offline.

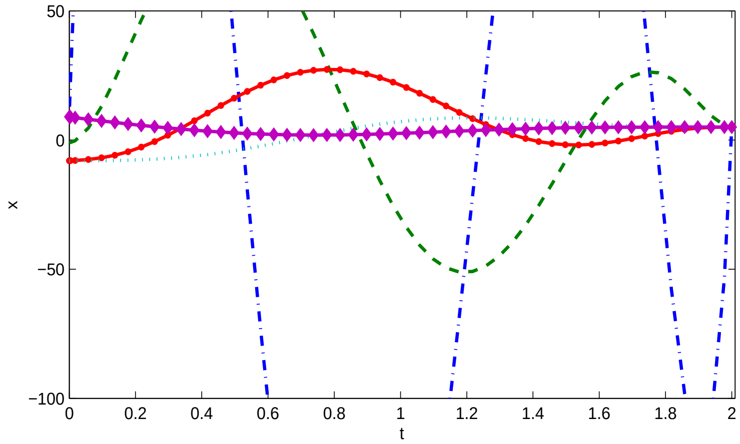


Figure 2.10: The x position trajectory based on the Gramian integral input

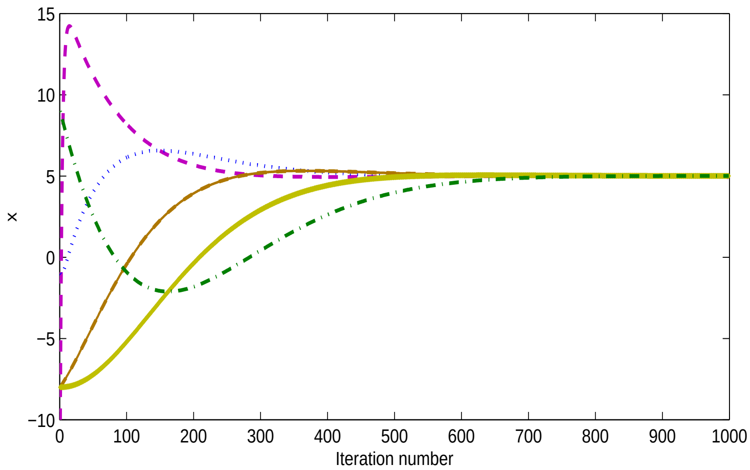


Figure 2.11: The x position trajectory based on the optimal law (2.9)

2.5 Numerical Examples

In this section, we give some numerical examples to illustrate the theoretical results demonstrated in the earlier sections. In section 2.3 we just mentioned the controllability for one dimensional case. However, all the results can be readily extended to higher dimensions by Kronecker product, as argued in [86]. In this section, we will consider the formation control among a group of agents on the plane, while each agent's state is of three dimensions, the x, y positions and its heading angle. Assume that interconnected topology is as depicted in Fig. 2.2, where the vertex v_1 is selected to be the leader and the remaining three are followers. Thus, the corresponding (A_{aq}, B_{aq}) with proper weighting selections is

$$A_{aq} = \begin{bmatrix} 5 & -1 & -2 \\ -1 & 4 & -2 \\ -2 & -2 & 4 \end{bmatrix}, B_{aq} = \begin{bmatrix} -1 \\ -3 \\ 0 \end{bmatrix}.$$

Some desired formation such as horizontal line, vertical line and triangular shape

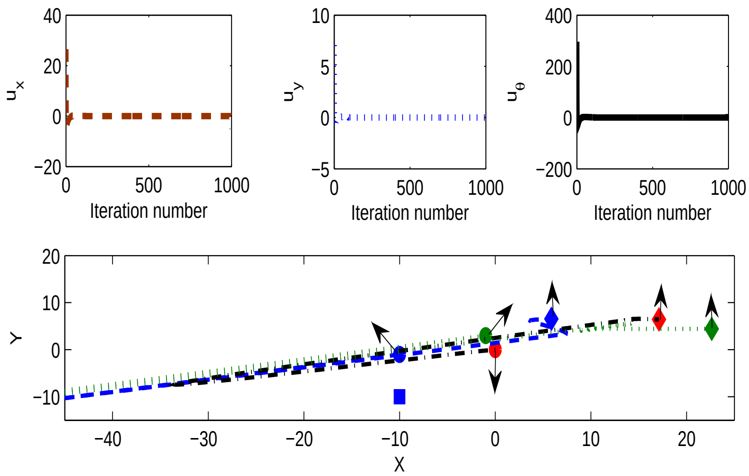


Figure 2.12: Horizontal line formation. Heading control effort (solid line), X position control effort (dashed line), Y position control effort (dotted line). Initial position (circle), final position (diamond), the leader (square)

are applied to this topology. The initial position and final position are denoted with circle and diamond, respectively. The leader is denoted by a square, and is deployed

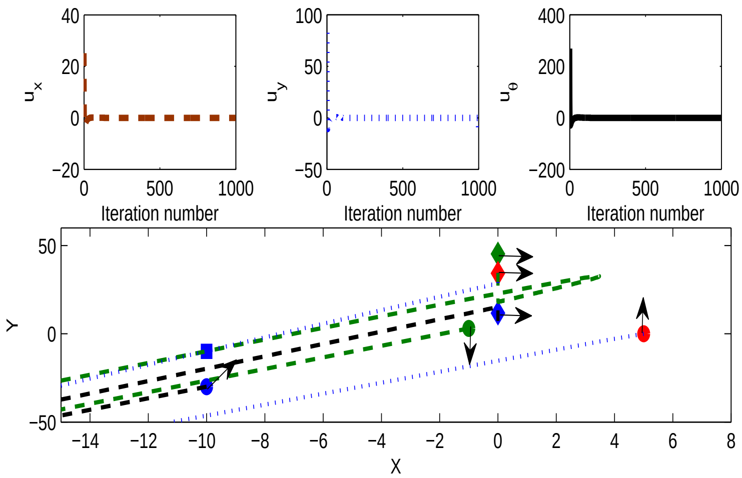


Figure 2.13: Vertical line formation. Heading control effort (solid line), X position control effort (dashed line), Y position control effort (dotted line). Initial position (circle), final position (diamond), the leader (square)

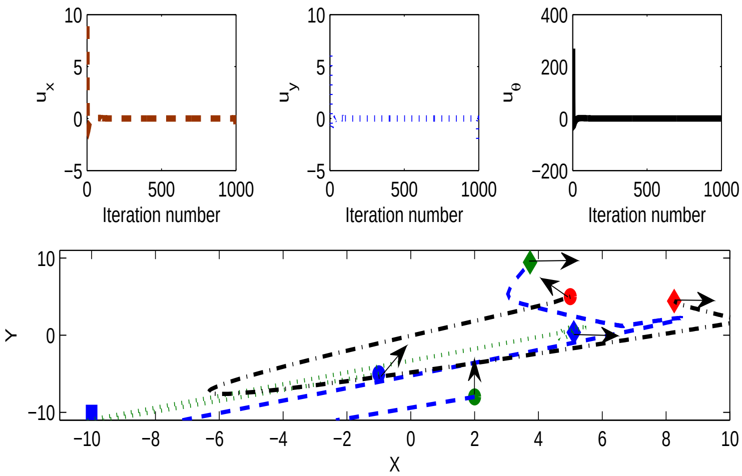


Figure 2.14: Triangular shape formation. Heading control effort (solid line), X position control effort (dashed line), Y position control effort (dotted line). Initial position (circle), final position (diamond), the leader (square)

with the control strategy (2.9) with

$$P = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, Q = \begin{bmatrix} 0.1 & 0 & 0 \\ 0 & 0.1 & 0 \\ 0 & 0 & 0.1 \end{bmatrix}, R = 1.$$

Based on this parameters, numerical methods can be used to solve equation (2.10). And, it is the same for all dimensions, but their initial and final condition may be different.

The optimal control input for the x position, y position and heading are depicted with the dashed line, the dotted line and the solid line, correspondingly. The magnitude of control effort needed for the heading is relatively large comparing to the positioning control efforts. Also, the final positions have slightly drifted from the desired one because the optimal control input is bounded within a reasonable region. On the other hand, exact positioning can be achieved by the Gramian integral input. However, Example 2 showed that the Gramian integral method produces a considerably large control effort. And, this is the drawback for this method.

2.6 Conclusion

In this chapter, the controllability problem for multi-agent systems interconnected via a fixed weighted topology was investigated. A novel notion of multi-agent structural controllability was proposed, and a necessary and sufficient condition was derived accordingly. It was shown that the connectivity is not only necessary, but also sufficient for structural controllability of interconnected systems. The simulation results seem promising and underscores their theoretical counterparts.

Assuming more than one leader in a group and high order dynamics realizations for each agent are our future challenges.

Chapter 3

Observability of Multi-Agent Systems

3.1 Introduction

The cooperative control of multi-agent systems has recently become a hot research topic. This area has largely been inspired by natural swarms such as fish schooling, bees flocking and ant colonies, see e.g., [58]. Design of a convenient strategies for local interaction among agents is the most important challenges in this area. The most popular distributed control laws is nearest neighbor law. The neighbor law for each agent is established on average between agent's information and that of its neighbors [83]. This interesting control strategy has recently been discussed in literature as consensus problem.

Consensus is more considered about stability problem, but although it is important to keep rigid formation, it could be more important to make multi-agent systems reconfigurable between different formations. The main question is that whether agents can be steered to any desired configuration or not. Several researchers have made connections between latter problem and well-established control theory. [86], [89], [34], [31]. They modified the consensus law's structure and recomposed it as a controllability problem; subsequently, some sufficient and necessary algebraic conditions was introduced.

Compared to the controllability problem, there exists very few literatures that discussed about the observability problem and observer design issue for multi-agent system. In [27], the authors used estimator to observe the immeasurable and time-varying states of leader. It was shown that the agents can follow the leader if the acceleration of the leader is known. This work is further extended in [28], where authors designed a distributed observer for estimation of leader's velocity under switching topology. Moreover, in [57], authors studied an observer-based multi-agent system with communication time delay. The model discussed in [57] is a general state space model, and the consensus problem is extended from state feedback to the output feedback case. In [25], the authors proposed a reduced-order estimator for observer-based control of multi-agent system. Even though the algorithm in [25] is decentralized, but there is no information exchange among agents.

In this chapter, we will focus on the observability issues of multi-agent systems. We assume that the multi-agent system is under the leader-follower framework while all interactions between agents and outside operators are though the leaders. We further assume that the underneath communication topologies are time-invariant and only there exists a single leader for the whole group. First, the classical notion of observability for dynamical systems is considered, and an algebraic necessary and sufficient condition for the observability is presented. However, some counter intuitive examples under this setup are found, which motivate us to propose a new observability definition, called structural observability. The multi-agent system is said to be structurally observable if there exists a set of weights which can make the system observable. It is shown that the proposed structural observability presents a generalization of the traditional observability, and is more suitable for multi-agent systems since it has a clear graphical interpretation. It turns out that the multi-agent system is structurally observable if and only if the communication topology forms a connected graph.

The outline of the chapter is as follows. In Section 3.2, the problem studied in this chapter is formulated; moreover, new notions of multi-agent observability and multi-agent structural observability are proposed and sufficient and necessary conditions are provided for each. The controller for steering the system into its desired configuration is designed in Section 3.3. Section 3.4 provides a numerical example to illustrate the derived theoretical results. Finally, Section 3.5 closes the chapter with comments and pointing into our future works.

3.2 Multi-Agent Observability

Objective in this chapter is to observe the followers' states under the leader-follower framework. A case of fixed topology and single leader is specifically investigated. We assume that there exists an agent which serves as the leader, while the rest of agents are followers and take controls from the nearest neighbor law.

Consider N point mass agents with first order dynamics

$$\dot{x}_i = u_i, \qquad i = 1, ..., N, \tag{3.1}$$

where x_i is denoted to be state of each agent and can have arbitrary dimension but all agents are required to have same dimension. Although the analysis that follows is valid for any dimension, for sake of simplicity we will present the one-dimensional case. All expressions can be readily generalized to any dimension case via Kronecker product.

Without loss of generality, we assume that the N-th agent serves as the leader and takes commands and controls from outside operators directly,

$$\dot{x}_N = u_N, \tag{3.2}$$

while other $N-1$ agents are followers and take controls as the nearest neighbor law:

$$u_i = -\sum\nolimits_{j\in\mathfrak{N}_i} w_{ij}(x_i - x_j), \tag{3.3}$$

where $\mathfrak{N}_i$ is the neighbor set of the agent i, $w_{ij} \in \mathbf{R}$ is weight of the edge from agent i to agent j.

The N-th agent has a freedom to choose arbitrary control input and an operator can deploy different control strategies to the leader. Based on the well-known linear system theory [1], the leader needs all followers' states for design of an appropriate control strategy. However, not all agents can establish at least a communication link to the leader. Thus, there exist some followers which are not able to communicate with the leader directly. This phenomenon can be captured as

$$y_i = \lambda_{iN} q_{iN} x_i, \tag{3.4}$$

where q_{iN} is weight of edge from agent i to the leader. The leader just has access to y_i; hence, it needs to estimate followers' states. Based on observed states, the leader design an appropriate control law consequently. It can be clearly seen from Fig. 3.1 that the leader requires a topology map for design of control law, meanwhile it accepts commands from the operator.

The algebraic graph theory [24] helps us to rewrite the system dynamics (3.1), (3.2), (3.3), (3.4) into the matrix form :

$$\begin{cases} \dot{x} &= A_{aq}x + B_{aq}z \\ \dot{z} &= u_N \\ y &= C_{aq}x \end{cases}, \tag{3.5}$$

where $z = x_N$, $A_{aq} \in \mathbf{R}^{(N-1)\times(N-1)}$ and $B_{aq} \in \mathbf{R}^{(N-1)}$. The matrix A_{aq} reflects followers' interconnection and vectors B_{aq} and C_{aq} represent the relation between followers and the leader.

Definition 4 *The linear system Σ in (3.5) is observable if and only if following observability matrix is full column rank.*

$$\mathcal{O} = \begin{bmatrix} -C_{aq} \\ C_{aq}A_{aq} \\ -C_{aq}A_{aq}^2 \\ \vdots \\ (-1)^n C_{aq}A_{aq}^{n-1} \end{bmatrix}.$$

In the following, the observability problem is solved from both algebraic and graph theoretic point of views. Firstly, algebraic tools are used to explore the problem; sec-

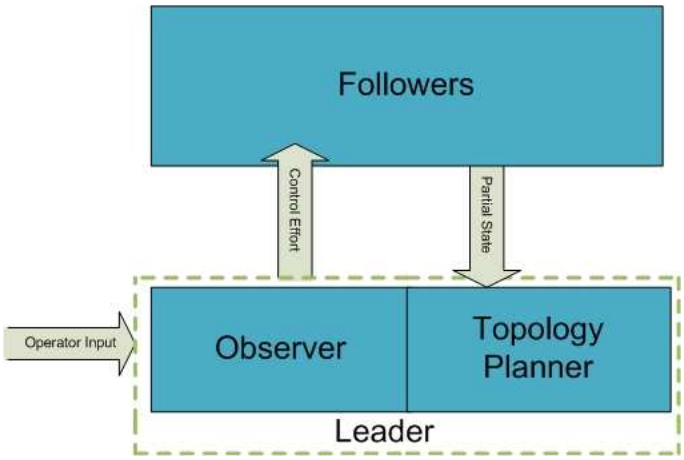


Figure 3.1: The leader based observer

ondly, a new notion of multi-agent systems structural observability is introduced and multi-agent systems observability problem is investigated from graph theory perspective.

3.2.1 Algebraic Condition

Theorem 2 *A class of multi-agent systems is observable if and only if the following holds:*

1. *The eigenvalues of A_{aq} are all distinct*
2. *The eigenvectors of A_{aq} are not reciprocal to C_{aq}*

Proof: The matrix A_{aq} is symmetric; thus, it can be expressed as $A_{aq} = SJS^T$, where S is orthonormal matrix and J is diagonal matrix consisting of the eigenvalues of A_{aq}. The observability matrix is

$$\mathcal{O} = \begin{bmatrix} -C_{aq} \\ C_{aq}A_{aq} \\ -C_{aq}A_{aq}^2 \\ \vdots \\ (-1)^n C_{aq}A_{aq}^{n-1} \end{bmatrix}. \tag{3.6}$$

O can be rewritten as:

$$\mathbf{O} = \begin{bmatrix} -C_{aq} \\ C_{aq}SJS^T \\ -C_{aq}(SJS^T)^2 \\ \vdots \\ (-1)^n C_{aq}(SJS^T)^{n-1} \end{bmatrix}, \tag{3.7}$$

this can be reduced into

$$\mathbf{O} = \begin{bmatrix} -C_{aq}S \\ C_{aq}SJ \\ -C_{aq}SJ^2 \\ \vdots \\ (-1)^n C_{aq}S(J)^{n-1} \end{bmatrix} S^T. \tag{3.8}$$

Since S is rank efficient, it does not affect the rank of right side of (3.8)and we only discuss about the rank matrix $\mathbf{O}$.

$$\mathbf{O} = \begin{bmatrix} -C_{aq}S \\ C_{aq}SJ \\ -C_{aq}SJ^2 \\ \vdots \\ (-1)^n C_{aq}S(J)^{n-1} \end{bmatrix}. \tag{3.9}$$

Since matrix J is a diagonal and nonsingular, the multiplication of J with a vector will be scaling along its dimensions. In order to maintain (3.9) full rank, each element of $C_{aq}S$ should be nonzero. Moreover, the distinctiveness of the eigenvalues of A_{aq} guaranties the observability of system (3.5). ∎

It can be induced from Theorem 2 that observability property of system (3.5) is related to the topology of interaction graph. This motivates us to investigate the observability properties of some well-known graphs.

Proposition 2 *The system (3.5) is unobservable if there is an isolated agent among followers.*

Proof: Without loss of generality, we assume that the $(N-1)$-th is isolated, then

$$C_{aq} = \begin{bmatrix} * & \dots & * & 0 \end{bmatrix}$$

$$A_{aq} = \begin{bmatrix} & & & 0 \\ & * & & \vdots \\ & & & 0 \\ 0 & \dots & 0 & 1 \end{bmatrix}$$

.

It is quite obvious that the last column of **O** contains all zeros. Thus, (3.5) is unobservable. ∎

Proposition 3 *A path P_N is observable.*

Proof: For simplicity, we prove this for a case which $w_{ij} = w_{ji} = q_{iN} = 1$. Thus, for a path graph P_N the pair (A_{aq}, C_{aq}) can be written as

$$C_{aq} = \begin{bmatrix} 0 & \dots & 0 & 1 \end{bmatrix}$$

$$A_{aq} = \begin{bmatrix} 1 & -1 & 0 & \dots & 0 \\ -1 & 2 & -1 & \dots & \\ & & \ddots & & \\ 0 & \dots & -1 & 2 & -1 \\ 0 & 0 & \dots & -1 & 2 \end{bmatrix}$$

The rank of $\mathbf{O}$ can readily be obtained from simple computation,

$$\mathrm{rank}(\mathbf{O}) = \mathrm{rank}\left\{\begin{array}{c}\begin{bmatrix} 0 & \dots & 0 & 1 \end{bmatrix} \\ \begin{bmatrix} 0 & \dots & 1 & * \end{bmatrix} \\ \vdots \\ \begin{bmatrix} 1 & \dots & * & * \end{bmatrix}\end{array}\right\} = \mathrm{N}$$

Hence, a path is observable. ∎

Example 5 *Consider a multi-agent system with six agents, whose communication topology is a complete graph with six vertices as shown in Fig. 2.1. The leader has access to all followers and it just does summation among followers data to observe the followers' states. The following pair of* (A_{aq}, C_{aq}) *fails to satisfy the condition in Theorem 1; hence, a complete graph is not controllable under unity weights' assignment.*

$$A_{aq} = \begin{bmatrix} 5 & -1 & -1 & -1 & -1 \\ -1 & 5 & -1 & -1 & -1 \\ -1 & -1 & 5 & -1 & -1 \\ -1 & -1 & -1 & 5 & -1 \\ -1 & -1 & -1 & -1 & 5 \end{bmatrix}, \quad C_{aq} = \begin{bmatrix} -1 \\ -1 \\ -1 \\ -1 \\ -1 \end{bmatrix}'. \tag{3.10}$$

Although the condition in Theorem 1 is strong and easy to check, it does not provide any insight into the problem from graph theory point of view. It is crucial that this problem be explored from graph topology perspective because graph theoretic approach is not only more intuitive, but also provides some required condition for establishment of communication topology. Hence, apart from algebraic conditions, we investigate required condition from graph topology perspective.

3.2.2 Structural Observability

Our result in this section is inspired by results in area of descriptor systems [16].

Definition 5 *The linear system Σ in (3.5) is said to be structurally observable if and only if there exists a set of $w_{ij} \neq 0$ which can make the system (3.5) observable.*

Here, we are interested to observe that under which topology we can always find a set of weights to make the system observable.

Definition 6 *The pair (A_{aq}, C_{aq}) in (3.5) is said to be reducible if they can be written into the form below;*

$$C_{aq} = \begin{bmatrix} 0 & C_{aq_{22}} \end{bmatrix},$$

$$A_{aq} = \begin{bmatrix} A_{aq_{11}} & A_{aq_{12}} \\ 0 & A_{aq_{22}} \end{bmatrix}, \tag{3.11}$$

where $A_{aq_{11}} \in \boldsymbol{R}^{p\times p}$, $A_{aq_{12}} \in \boldsymbol{R}^{p\times(N-1-p)}$, $A_{aq_{22}} \in \boldsymbol{R}^{(N-1-p)\times(N-1-p)}$ and $C_{aq_{22}} \in \boldsymbol{R}^{(N-1-p)}$.

One can verify that regardless of the choice of weights, the observability matrix for this structure cannot be full column rank. Thus, system (3.5) is not structurally observable.

Another famous unobservable structure can be expressed as the following lemma.

Lemma 4 *The system (3.5) is not structurally observable if the matrix* $[C_{aq}, A_{aq}]$*, which is* $N-1 \times N$ *matrix, can be written as*

$$[C_{aq}, A_{aq}] = \begin{bmatrix} P_{11} & P_{22} \end{bmatrix}, \tag{3.12}$$

where P_{22} *is of* $N \times (N-1-p)$ *and* P_{11} *is* $N \times p$ *with at most* $p-1$ *nonzero entries and the rest of rows are all zero.*

Apart from these structures, the well-known linear system [16] guarantees that there exists at least a set of weight which can make the system (3.5) observable.

Lemma 5 *The pair* (C_{aq}, A_{aq}) *is structurally observable if and only if it is neither reducible nor writable into the form (3.12).*

The theory of linear structural system helps us to a give general answer for structural observability of networked systems but we need to establish a linkage between these lemmas from linear descriptor systems and graph theory. The following theorem establishes this linkage and interprets these lemmas into graph theory language.

Theorem 3 *The multi-agent system (3.5) with the communication topology* $\mathcal{G}$ *is structurally observable if and only if* $\mathcal{G}$ *is connected.*

Proof: *Necessity:* Assume that the graph $\mathcal{G}$ is disconnected. For simplicity, we will prove by contradiction for the case that there exists only one disconnected agent. There are two possibilities: First, this isolated agent is the leader. Then, C_{aq} is a null vector in this case , and the system is uncontrollable no matter what the weights are. Secondly, the isolated agent is one follower. For this case, (C_{aq}, A_{aq}) is reducible, which implies uncontrollability. Both cases end with a contradiction, so the necessity holds. The proof can be straightforwardly extended to more general cases with more than one disconnected agents.

Sufficiency: For the sufficiency part, we show that a connected graph cannot be written either in a reducible form or in the form of (3.12). Note that $w_{ij} \neq 0$ if and only if $w_{ji} \neq 0$. Then, (C_{aq}, A_{aq}) is in a reducible form if and only if A_{aq} is of a block diagonal matrix, which implies that the graph is disconnected. This contradicts with our assumption on the graph connectivity. On the other hand, the graph contains isolated vertex if and only if $\mathcal{D}$ matrix contains zero diagonal element. So, (C_{aq}, A_{aq}) pair can be written in the form of (3.12) in Lemma 4 if and only if it has a group of isolated agents. Therefore, according to Lemma 5, the graph is structurally observable. ∎

Remark 2 *The notion of multi-agent system structural observability offers the possibility of weights' assignment and it gives more degree of freedom in setup of communication topology among multi-agent systems. Thus, it is the more general definition compared to multi-agent system observability notion. For instance, there are cases in which a system fails to satisfy condition in Theorem 1. However, one can design weights to make the system observable. This is further illustrated in the following example.*

Example 6 *Consider a multi-agent system with four agents, whose communication topology is shown in Fig. 3.2. If all the edges' weight are assigned to unity, the matrices A_{aq} and C_{aq} can be written as*

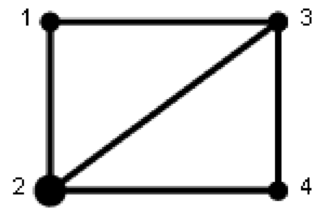


Figure 3.2: A multi-agent system with four agents, where bold agent serves as the leader .

$$A_{aq} = \begin{bmatrix} 3 & -1 & 0 \\ -1 & 4 & -1 \\ 0 & -1 & 3 \end{bmatrix}, \; C_{aq} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}'. \tag{3.13}$$

It can be easily determined that this pair is unobservable. However, under structural observability setup, a set of weights can be assigned such that the observability condition is satisfied; for instance, the pair (A_{aq}, C_{aq}) can be chosen as the following observable pair:

$$A_{aq} = \begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & -3 \\ 0 & -3 & 4 \end{bmatrix}, \; C_{aq} = \begin{bmatrix} -1 \\ -2 \\ -5 \end{bmatrix}'. \tag{3.14}$$

3.3 Output Feedback Controller for Multi-Agent systems

An output feedback control strategy is used to steer followers into their final destinations in finite time. The well-known Kalman filter based observer is chosen to estimate the states. The observer has the following dynamics:

$$\dot{\hat{x}} = A_{aq}\hat{x} + B_{aq}u_N + K(y - \hat{y}),$$
$$\hat{y} = C_{aq}\hat{x},$$

where $\mathbf{K}$ is Kalman filter gain, calculated as:

$$\mathbf{K} = PC^T R^{-1},$$

where P is positive definite solution of the following Riccati equation,

$$PA_{aq}^T + A_{aq}P - PC_{aq}^T R^{-1} C_{aq} P + Q = 0,$$

where Q is positive definite matrix

Based on observed states, a state feedback controller can be designed as:

$$z = -\Lambda x. \tag{3.15}$$

Above equation tends to minimize the following cost function

$$J(x, u, Z, L) = \int_0^\infty (x^T Z x + z^T L_e z) dt,$$

where Z and L_e are semi positive definite and positive definite matrices, respectively. The parameter Λ can be readily obtained from the Riccati equation :

$$P_e A_{aq} + A_{aq}^T P_e - P_e B_{aq} L_e^{-1} B_{aq}^T P_e + Z = 0.$$

And,

$$\Lambda = L_e^{-1} B^T P_e.$$

3.4 Numerical Example

In this section we present a numerical example on how the output feedback controller can be used in order to control a group of an interconnected system into its defined destination. Simulation results shows that how an interconnected system can perform a specific formation when the leader has partial access to followers' positions. A group of multi-agent systems consists of ten agents is depicted in Fig. 3.3 and the agent number ten is selected to be leader. An operator must regulate motion of the leader such that the interconnected system can be herded to the desired position. Our objective is to steer followers from $y = 0$ to $y = 5$ just based on partial information.

The system (3.5) can be expressed as

$$A_{aq} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 5 & -1 & 0 & 0 & 0 & -1 & 0 & -1 \\ 0 & -1 & 2 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & -1 & -1 \\ 0 & 0 & -1 & 0 & 0 & 2 & 0 & 0 & -1 \\ 0 & -1 & 0 & 0 & 0 & 0 & 2 & 0 & -1 \\ -1 & 0 & 0 & 0 & -1 & 0 & 0 & 4 & 0 \\ 0 & -1 & 0 & 0 & -1 & -1 & -1 & 0 & 5 \end{bmatrix},$$

$$B_{aq} = \begin{bmatrix} 0 & -2 & 0 & -3 & 0 & 0 & 0 & -2 & -1 \end{bmatrix}^T,$$

$$C_{aq} = \begin{bmatrix} 0 & -1 & 0 & -1 & 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

The Kalman estimator is used to observe the y position of the group of robots shown in Fig. 3.3. The design parameters of R and Q are set to 1 and 2, respectively.

The actual position and observer result are depicted in Fig. 3.4. Comparing Fig. 3.4(a) and Fig. 3.4(b) reveals that the estimated parameters are quite close to their actual counterpart. Then, based on estimated states of leader, operator can design the appropriate controller to steer system into the desired state. Optimal state feedback is one of the best possible solutions. Design parameters Z, L_e are set to the following values:

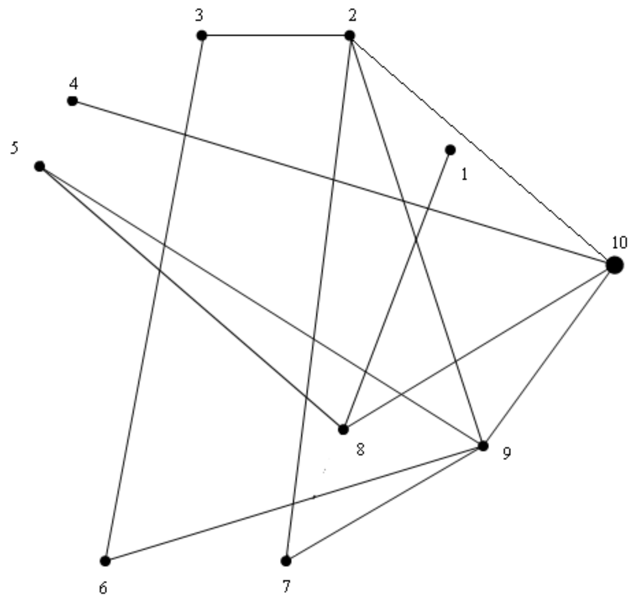
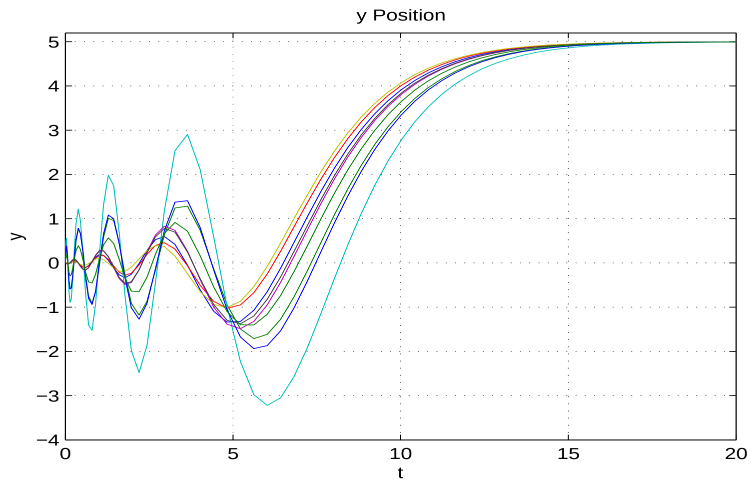


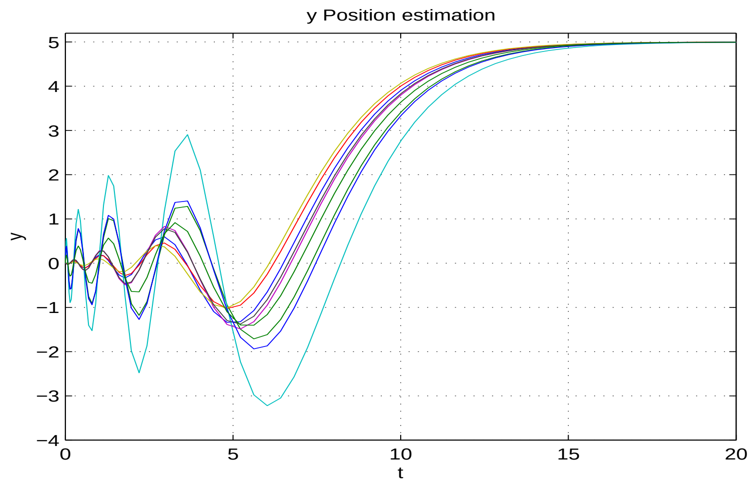
Figure 3.3: The observable structure consisting of ten vertices and vertex ten is the leader.

$$L_e = 1, \quad Z = \begin{bmatrix} 1 & & & 0 \\ & 1 & & \\ & & \ddots & \\ 0 & & & 1 \end{bmatrix}_{9\times 9}.$$

Followers initial positions are shown in Fig. 3.6. The leader of group shown in Fig. 3.3 deploys the optimal control law (3.15) to its followers. This control signal is shown in Fig. 3.5. At $t = 16$, the system reached the desired position as shown in Fig. 3.7. It can be seen from Fig. 3.7 that the system has successfully performed the required task, while the control effort given to the system, Fig. 3.5, is finite and implementable.



(a) Actual y position trajectory of group of ten agents in Fig. 3.3



(b) The observer output trajectory of group of ten agents in Fig. 3.3

Figure 3.4: Observer output trajectory versus actual trajectory

3.5 Conclusion

In this chapter, we investigated the observability problem for multi-agent systems under a leader. In addition, the interconnection topology assumed to be weighted. It was demonstrated interconnected topology affects the observability of overall system. Some new notions for observability of multi-agent systems were provided and sufficient

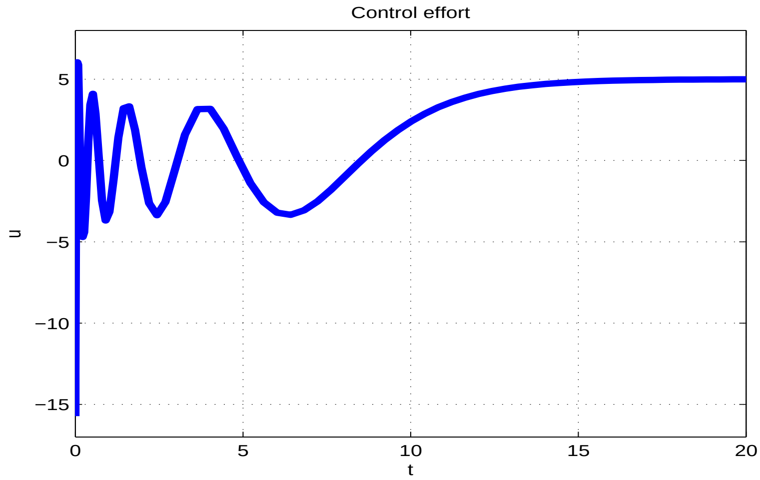


Figure 3.5: Optimal control effort deployed to the leader

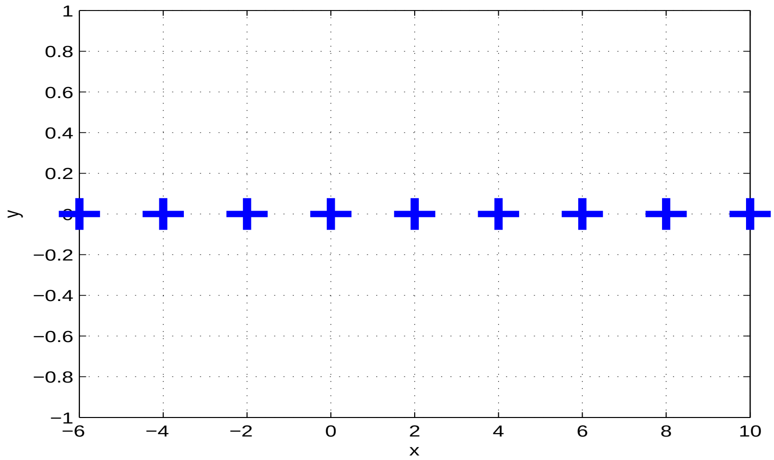


Figure 3.6: The initial position for the followers (t=0)

and necessary conditions were driven, consequently. Under the novel notion of multi-agent system structural observability, it was shown that the connectivity of graph is not only necessary, but also sufficient for observability. The simulation results seems promising and underscore the theoretical part.

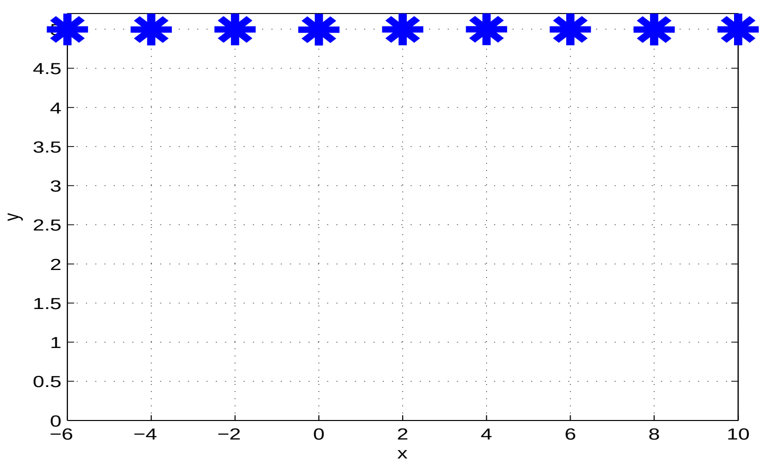


Figure 3.7: The final position for the followers (t=16)

Chapter 4

Weights' Assignments Among a Group of Multi-Agent Systems

4.1 Introduction

The main objective of this chapter is to design an optimal solution for weights' assignment in formation and reconfiguration control among a group of robots. The optimal solution must keep the control effort given to the whole system at its minimum possible level and guarantees that the desired configuration can be reached. In particular, the case of a single leader under a time-varying topology is considered. In contrast to the existing literature on this topic, we assume that the graph is weighted and time-varying; moreover, weights can be assigned freely. Under this setup, there are plenty of possible weights. However, determining a set of the best weights remains as an open problem because the control effort given to the agents must be minimized meanwhile the final desired states must be assured. In this chapter, the problem of weights' assignment is discussed and formulated using the optimal control theory. The optimal control strategy is designed based on minimization of an index function and a solution is found using Hamilton-Jacobi-Bellman equations. Finally, some simulation results are presented to illustrate the approach.

4.2 Main Result

In this chapter, we use the same problem formulation as Chapter 3. It is assumed that each agent has first order dynamics; moreover, there exists an agent which could accept commands from outside operator. The leader also collects the information from followers for sake of proper control law design. Following the problem formulation in Chapter 3, on can get the whole systems dynamics as the following matrix form :

$$\left\{ \begin{array}{rcl} \dot{x} & = & A_{aq}x + B_{aq}x_N \\ \dot{x}_N & = & u_N \\ y & = & C_{aq}x \end{array} \right. , \tag{4.1}$$

where $A_{aq} \in \mathbf{R}^{(N-1)\times(N-1)}$ and $B_{aq} \in \mathbf{R}^{(N-1)\times 1}$ are both sub-matrices of the corresponding graph Laplacian matrix $\mathbf{L}$. The matrix A_{aq} reflects the interconnection among followers, and the column vector B_{aq}, C_{aq} represents the relation between followers and the leader.

Our objective is to design a paradigm that can minimize the control effort given to the whole group. In section next the optimal control approach is used for solving this problem. In sequel, we assume that the communication topology remains connected during the whole maneuver. This assumption guarantees that the system is both observable and controllable; moreover, it assures the existence of the solution.

4.2.1 Cost Function Definition

Let Σ represent the system in (4.1).

Definition 7 *The linear time-invariant system Σ is said to be structurally controllable if and only if there exists a set of fixed w_{ij} which can make the system Σ controllable.*

Definition 8 *The linear time-invariant system Σ is said to be structurally observable if and only if there exists a set of fixed w_{ij} which can make the system Σ observable.*

Lemma 6 *The multi-agent system Σ under the fixed communication topology $\mathcal{G}'$ is structurally controllable if and only if $\mathcal{G}'$ is connected.*

Proof: See the proof in Chapter 2. ∎

Lemma 7 *The multi-agent system Σ under the communication fixed topology $\mathcal{G}'$ is structurally observable if and only if $\mathcal{G}'$ is connected.*

Proof: See the proof in Chapter 3. ∎

The next step is to design the global optimal control strategy which can put into account whole dynamics. Moreover, it should be able to minimize the control effort given to each agent.

Let us define the following index function for overall system:

$$J = \int_0^T \left[(A_{aq}x)^T Q(A_{aq}x) + x^T Sx + u_N R u_N \right] dt + (x(T) - x_f)^T E\,(x(T) - x_f)\,, \quad (4.2)$$

where x_f stands for the desired final position at the final time T, and $Q > 0$, $S > 0$ and $R > 0$ are specification matrices.

Remark 3 *The cost function introduced in (4.2) is in a quadratic form. It is chosen such that it minimizes the control effort given to the whole system. It not only minimizes the leader's control effort, but also penalizes followers' control signals.*

4.2.2 Hamiltton-Jacobi-Bellman(HJB) Equations

The problem of finding the minimum value for the general cost function, can be solved by help of HJB set of equations. This method is applicable to the general finite horizon case [17]. Assume a system with the following dynamics

$$\dot{X} = f(t, X, u), \tag{4.3}$$

The objective is to minimize the the following cost function

$$J = \int_0^T g(t, X, u)dt + \lambda(X(T)). \tag{4.4}$$

A set of HJB equations can be written for solving the optimal problem in (4.3) and (4.4):

$$-\frac{\partial W}{\partial t}(t, X) = \min_{u \in U} \Xi(t, X, u)$$

$$W(T, X) = \lambda(X(T))$$

$$u^* = \arg\min_{u \in U} \{\Xi(t, X, u)\} \tag{4.5}$$

where W is so called value function.

$$\Xi(t, X, u) = g(t, X, u) + \frac{\partial W}{\partial X}(t, X)f(t, X, u).$$

The solvability of the above minimization problem is depend on whether the PDE can be solved or not. In another word, one needs to find the value function W such that it satisfies the PDE.

4.2.3 Optimal Control Problem for Multi-Agent Systems

The HJB equations can be rewritten for the system given in (4.1):

$$-\frac{\partial W}{\partial t}(t, X) = \min u \in U \Xi(t, X, u)$$

$$W(T, X) = x(T)^T E x(T) + \phi(T) \tag{4.6}$$

$$\Xi(t, X, u) = (A_{aq}x)^T Q (A_{aq}x) + x^T S x + u_N R u_N + \frac{\partial W}{\partial x}(A_{aq}x + B_{aq}u). \tag{4.7}$$

Existence of a solution : The existence of solution to the above minimization problem can be guaranteed if certain controllability and observability conditions are satisfied [43]. Moreover, it was just shown that as long as the topology graph $\mathcal{G}'$ remains connected, the controllability and observability requirement are both realized. Thus, the existence of solution for this minimization problem is guaranteed.

The above minimization problem has the optimal control law in the form of:

$$u^* = -\frac{1}{2} R^{-1} (\frac{\partial W}{\partial x})^T, \tag{4.8}$$

and one of the possible choice for W can be expressed as

$$W = -\frac{1}{2} x^T K(t) x + \phi(t). \tag{4.9}$$

The following Lemma shows how parameter K can be calculated such that the PDEs in (4.6) and (4.7) have solutions.

Theorem 4 *Assume that the group of agent has first order dynamics, and are connected through the nearest neighbor law. The following control law would minimize the cost function (4.2).*

$$u^* = -\frac{1}{2} R^{-1} (\frac{\partial W}{\partial x})^T, \tag{4.10}$$

where $K(t)$ satisfies the following equation:

$$-\dot{K} = 2(S + A_{aq}^T Q A_{aq}) + \frac{K^T R^{-1} K}{2} \quad K(T) = 2E. \tag{4.11}$$

Proof: Equation (4.7) can be written as:

$$\Xi(t, X, u^*) = (A_{aq}x)^T Q(A_{aq}x) + x^T Sx + u_N^* R u_N^* + \frac{\partial W}{\partial x}(A_{aq}x + B_{aq}u) = -\frac{\partial W}{\partial t}(t, x). \tag{4.12}$$

By substituting W and u_N^* from (4.9) and (4.10; one gets

$$-x^T \frac{\dot{K}}{2} x - \dot{\phi} = X(A_{aq}x)^T Q(A_{aq}x) + x^T Sx + (-\frac{1}{2}R^{-1}(\frac{\partial W}{\partial x})^T)^T R(-\frac{1}{2}R^{-1}(\frac{\partial W}{\partial x})^T) + A_{aq}, \tag{4.13}$$

this can be written into a compact form:

$$-x^T \frac{\dot{K}}{2} x - \dot{\phi} = (A_{aq}x)^T Q(A_{aq}x) + x^T Sx + \frac{x^T K^T R^{-1} K x}{4} + A_{aq}. \tag{4.14}$$

By comparing the corresponding terms in $x^T x$, we get

$$-\dot{K} = 2(S + A_{aq}^T Q A_{aq}) + \frac{K^T R^{-1} K}{2}. \tag{4.15}$$

On the other hand, final condition can be verified as follows

Again by comparing corresponding term in $x(T)^T x(T)$,

$$K(T) = 2E. \tag{4.16}$$

This completes the proof. ∎

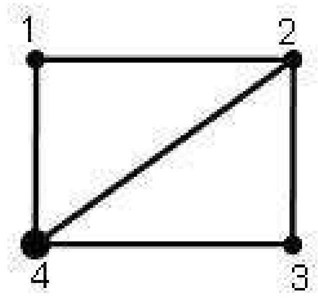


Figure 4.1: A system consists of four agent and agent four serves as the leader

Remark 4 *The problem of weights' assignment can be solved by help of Theorem 1. The optimal law (4.10) can be replaced in (4.1); hence, entries of matrix* A_{aq} *are updated, or in another word weights among the agents are modified such that not only the leader's control effort becomes optimum, but also the control effort given to the whole system is optimized.*

The result in Remark 2 is further illustrated in the next section

4.3 Numerical Example

In this section, we give a numerical examples to illustrate the theoretical results demonstrated in the earlier sections. Assume topology as shown in Fig. 4.1 which consists of three followers and a leader. We assume that x represents each agent's position. The dynamics of whole system can be written as the following:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0.25 & -0.25 & 0 \\ -0.25 & 0.5 & -0.25 \\ 0 & -0.25 & 0.25 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} u. \tag{4.17}$$

Design parameters are set as below:

$$Q = \begin{bmatrix} 1 & & \\ & 1 & \\ & & 1 \end{bmatrix}, \; E = \begin{bmatrix} 0.1 & & \\ & 0.1 & \\ & & 0.1 \end{bmatrix},$$

$$R = 1, \; S = \begin{bmatrix} 0.875 & 0.1875 & -0.0625 \\ 0.1875 & 0.6250 & 0.1875 \\ -0.0625 & 0.1875 & 0.8750 \end{bmatrix}$$

One can write (4.11) for above setup as

$$-\dot{K} = 2I + \frac{K^2}{2} I \quad K(T) = 20I, \tag{4.18}$$

where I is the identity matrix. Above problem can be easily solved as

$$K = -2I \tan(2t - c),$$

where c can be obtained from the boundary condition. Henceforth, the feedback law can be written as

$$W = -\frac{1}{2}(x_1^2 + x_2^2 + x_3^2)k + \phi(t)$$

$$\frac{\partial W}{\partial x} = -\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} k$$

$$u^* = -\frac{1}{2} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} k, \tag{4.19}$$

Where k is a diagonal element of the matrix K.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0.25 & -0.25 & 0 \\ -0.25 & 0.5 & -0.25 \\ 0 & -0.25 & 0.25 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0.5 \\ 0.5 \\ 0.5 \end{bmatrix} \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} k. \tag{4.20}$$

Consequently, we get

$$\dot{X} = \begin{bmatrix} 0.25 + 0.5k & -0.25 + 0.5k & 0.5k \\ -0.25 + 0.5k & 0.5 + 0.5k & -0.25 + 0.5k \\ 0.5k & -0.25 + 0.5k & 0.25 + 0.5k \end{bmatrix} X,$$

where $X = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T$. The above equation illustrates how the optimal solution in Theorem 1 can assign weights among a group of connected agents. The system (4.17) initiates from random initial conditions in $2D$ space and under the control law (4.19), all the followers are forced to converge into the origin within the finite time $t = 2$.

The system in (4.17) is exposed to the optimal control law (4.19). The states trajectory is depicted in Fig. 4.3. It is clearly shown in Fig. 4.4 that how followers are moving in 2D space till they reach the desired point. In Fig. 4.4 initial positions are marked by plus sign and the destination is illustrated by star sign. It can be seen from Fig. 4.4 that the proposed control strategy is capable of driving the system into its desired position. Furthermore, the control effort given to the system is shown in

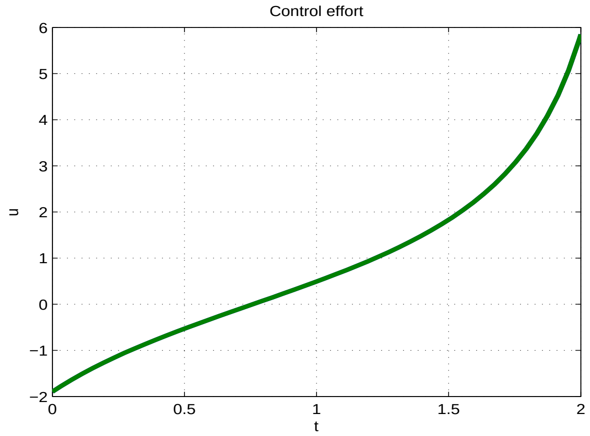


Figure 4.2: The optimal control effort (4.19) given to the system (4.17).

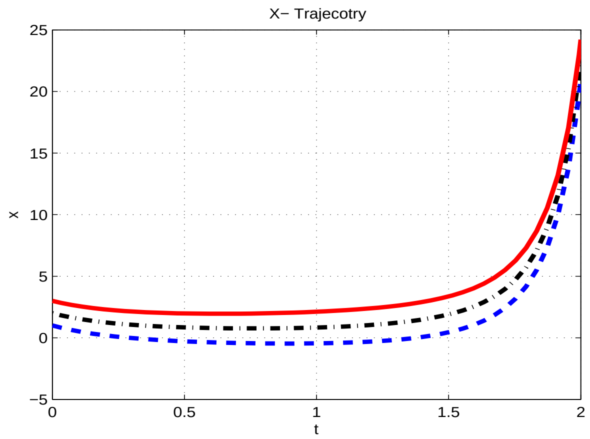


Figure 4.3: X position trajectory of the system (4.17) this is driven by the optimal law (4.19) and design parameters (4.3).

Fig. 4.2. Investigating Figures 4.2 and 4.4 reveals that not only the desired formation is obtained, but also control efforts given to the system is quite negligible. This supports that the optimal law in (4.19) has modified the weights such that control effort given to the system is optimized.

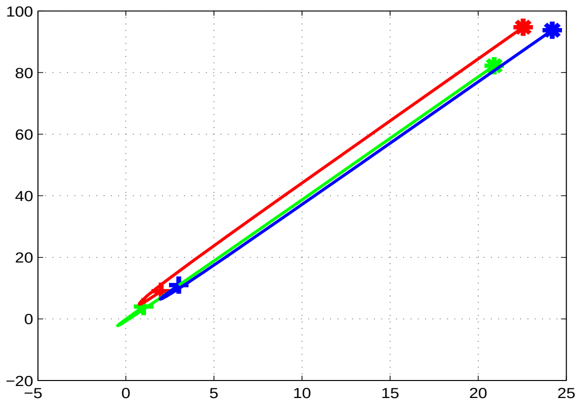


Figure 4.4: The X-Y position trajectory of the system (4.17). Followers' initial positions (plus), final positions (star).

4.4 Conclusion

In this chapter, the optimal control paradigm was proposed for weights' assignment among multi-agent systems. It was assumed that system is under a leader. The problem of weights' assignment was written as an optimal control problem; henceforth, index function was minimized with the help of HJB equations. Finally, simulation results were introduced which underscore their theoretical counterpart. Our approach guarantees that after the weights' assignment the whole group can reach the final destination. However, since the resultant system becomes time-variant, the controllability property of a new system must be further investigated. In addition, the model assumed for each agent can be modified to the general state space dynamics. We optimized the system with respect to the weights. Another interesting research question could be optimization of the system with respect to the topology.

Chapter 5

Implementation

5.1 Introduction

This chapter describes the software and hardware development for performing a formation among a group of the e-puck robots. The coordination control problem introduced in early chapters is implemented in practice and experimental results are reported. The test bench consists of the three e-puck robots where one of them serves as the leader and the rest are followers. Our experimental results are carried out entirely on physical robot without human interference. The leader herds the group into its desired destination. The control of autonomous robot has been addressed in several literatures [6], [7]; however, in our current work we adapted the classical concept from linear system theory and based on this we have implemented a formation control among group of robot in a systematic way.

The outline of this chapter is as following. Firstly, the hardware structure of an e-puck robot is introduced. Secondly, the software preparation for implementing a program on the e-puck robot is given. Finally, in the last section, those theoretical results from early chapter are implemented in real world application. This chapters concludes with conclusion and our further research directions.

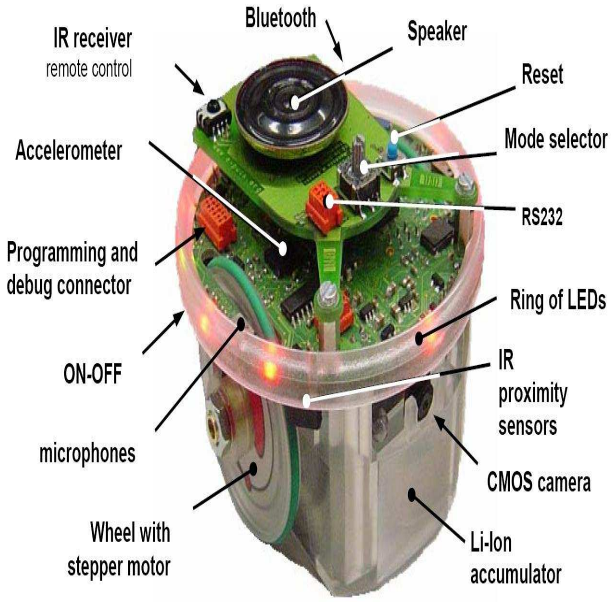


Figure 5.1: E-puck

5.2 Hardware

The hardware of the e-puck robot is discussed in this section. And, different parts of this robot are presented. It is clearly demonstrated that how available sensors on e-puck robots can be exploited to perform a localization method.

The e-puck robot was originally designed by Michael Bonani and Francesco Mondada at the ASL laboratory of Prof. Roland Siegwart at EPFL (Lausanne, Switzerland). It is an open source product in both software and hardware. There are several companies active in production of this product. The e-puck robot and its different parts are clearly depicted in Fig. 5.1. The e-puck beneficiates from a neat and flexible design; moreover, there are several noncommercial software dedicated to the e-puck robot.

The e-puck robot uses dsPIC as its processor core. This series of microcontrollers

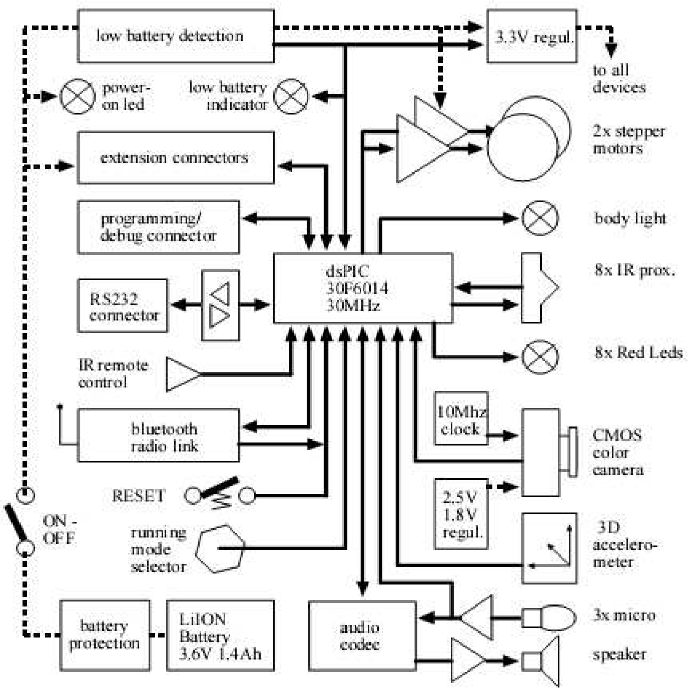


Figure 5.2: E-puck block diagram

are produced by the Microchip company. There are several softwares offered by Microchip to facilitate use of dsPIC processor (www.mirochip.com). The e-puck robot also features a large number of sensors and actuators, described in Table. 5.1. The interconnection among different parts of robot is shown in Fig. 5.2.

The e-puck robot is equipped by two high precision stepper motors. These motors are driven using differential steering system. With help of this steering system, the robot can be easily localized in a terrain.

5.2.1 Localization

The well-known method of odometry can localize the e-puck robot in smooth terrain using data from actuators for localization. In our case stepper motors' pulses can be

Size, weight	70 mm diameter,55 mm height, 150 g
Battery autonomy	5Wh LiION rechargeable battery providing about3 hours autonomy
Processor	dsPIC 30F6014A @ 60 Mhz (15 MIPS) 16 bit DSP microcontroller
Memory	RAM: 8 KB; FLASH: 144 KB
Motors	2 stepper motors with a 50:1 reduction gear, resolution: 0.13 mm
Speed	Max: 15 cm/s
Mechanical structure	Transparent plastic body supporting PCBs, battery and motors
IR sensors	8 infra-red sensors measuring ambient light
Camera	VGA color camera with resolution of 480x640 (typical use: 52x39)
Microphones	3 omni-directional microphones for sound localization
Accelerometer	3D accelerometer along the X, Y and Z axis
LEDs	8 independent red LEDs on the ring, green LEDs in the body
Speaker	On-board speaker capable of WAV and tone sound playback
Switch	16 position rotating switch on the top of the robot
PC connection	Standard serial port up to 115 kbps
Wireless	Bluetooth for robot-computer and robot-robot wireless communication
Remote control	Infra-red receiver for standard remote control commands

Table 5.1: Features of the e-puck robots

used to estimate the position over time. Based on odometry, position of each robot can be estimated relative to starting location. It is clearly known that the odometry method is sensitive to errors. Hence, the system's error will be accumulated if the terrain is not well designed or equipments are not calibrated. The geometry of robot

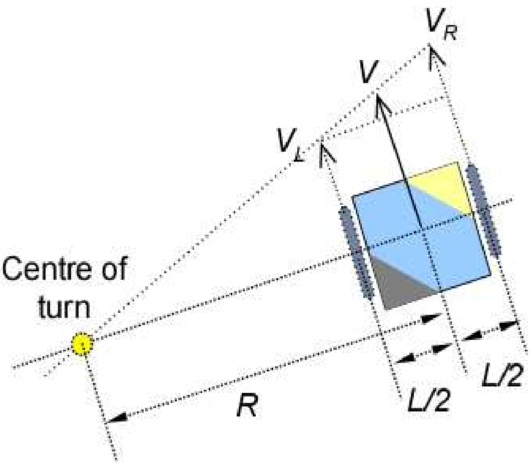


Figure 5.3: Geometry of the e-puck robot

is presented in Fig. 5.3. Based on the robot geometry, localizing the e-puck robot becomes a trivial robotic question [61]. After a discussion about the e-puck robot's dynamics, next section introduces programming of the e-puck robot.

5.3 Software

This section discusses about programming of e-puck robots. The Microchip company has developed a MPLAB IDE, development package, which can handle programming a large series of PIC microcontrollers. The C language is the most efficient language for low level programming. Hence, this language is chosen for our implementation purpose. Several setups need to be done for programming of the e-puck robot. There are three steps for programming of the e-puck robot:

1. Make a project in MPLAB IDE.
2. Compile the code.

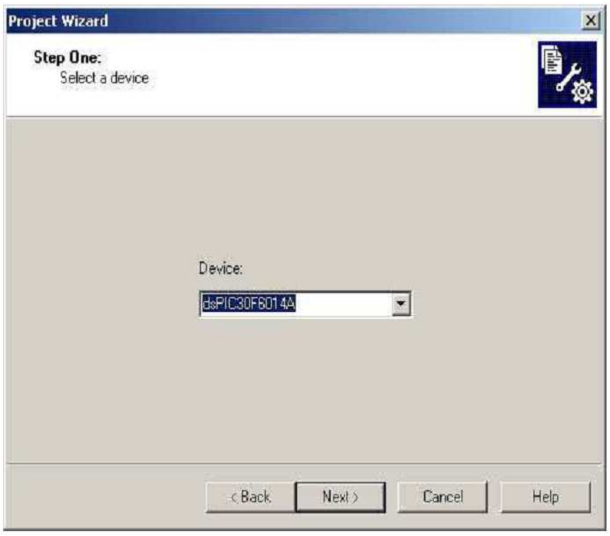


Figure 5.4: Project wizard, step 1, select device

3. Program via Bluetooth.

5.3.1 Creating a Project

At first a project and workspace must be created in MPLAB IDE. There must be only one project in workspace at a time. Each project contains several files such as source code, linker script files and etc. MPLAB IDE project wizard helps to create a new project.

1. Create a new project (Project>Project Wizard).

2. Select a device as dspic30F6014 (see Fig. 5.4).

   The MPLAB IDE needs a C compiler to produce desired output file. Thus, the software C30 should be installed and patched to the MPLAB IDE. The Toolsuite includes the required files that will be used.

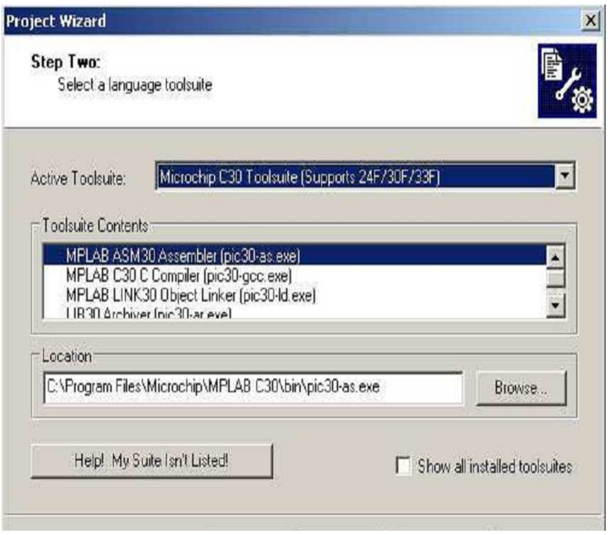


Figure 5.5: Project wizard, step 2, select language Toolsuite

3. From the activate Toolsuite pull-down menu, select Microchip C30 Toolsuite (see Fig. 5.5).

4. Name the project.

5. Add files to the project.

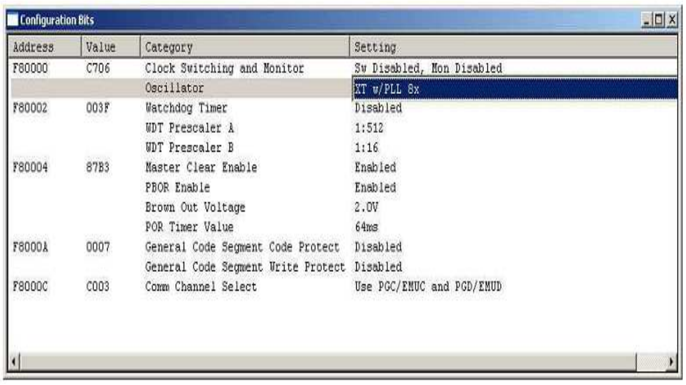
   After the project wizard completes, the main C file must be added to the project. In addition to software setup, device configurations must be modified. Following configuration parameters need to be set as:

   - Oscillator: XT w/PLL 8x
   - Watchdog Timer: Disabled

   This setup is clearly depicted in Fig. 5.6.

6. Build the project(Make>Project)

The program is ready to download when "BUILD SUCCEEDED" display.



Configuration Bits

Address	Value	Category	Setting
F80000	C706	Clock Switching and Monitor	Sw Disabled, Mon Disabled
		Oscillator	XT w/PLL 8x
F80002	003F	Watchdog Timer	Disabled
		WDT Prescaler A	1:512
		WDT Prescaler B	1:16
F80004	87B3	Master Clear Enable	Enabled
		PBOR Enable	Enabled
		Brown Out Voltage	2.0V
		POR Timer Value	64ms
F8000A	0007	General Code Segment Code Protect	Disabled
		General Code Segment Write Protect	Disabled
F8000C	C003	Comm Channel Select	Use PGC/EMUC and PGD/EMUD

Figure 5.6: Configuration bits

5.3.2 Programming of the E-puck Robot

The e-puck robot can be programmed through Bluetooth communication link. To do so, the e-puck robot and computer should be firstly paired. Each e-puck robot has a specific Bluetooth ID for communication. The e-puck robot establishes Bluetooth communication to PC with this ID. The ID is printed on each individual robot, as shown in Fig. 5.7. Once the communication link has been established between a computer and the e-puck robot, the e-puck robot is recognized as serial communication. The PC can communicates with the e-puck via this dummy serial communication. The "Tiny Bootloader" is an application program which helps to download a program into the e-puck robot through a dummy serial port. The user should browse the corresponding .hex file and burn the flash memory. The main screen of Tiny Bootloader is shown in Fig. 5.8.

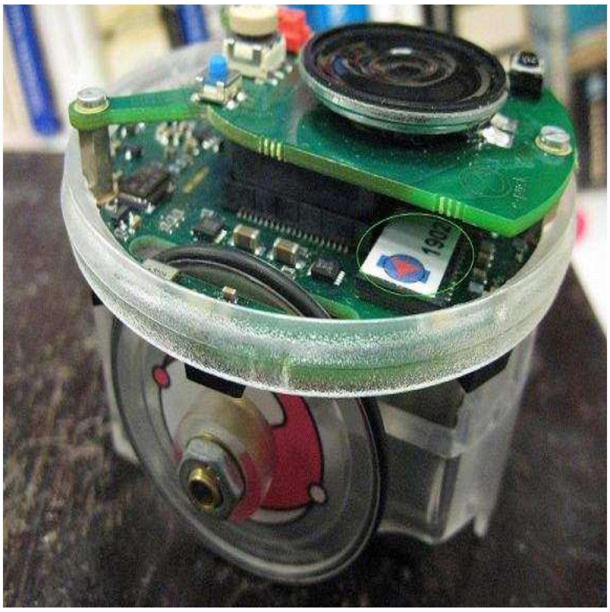


Figure 5.7: Bluetooth ID

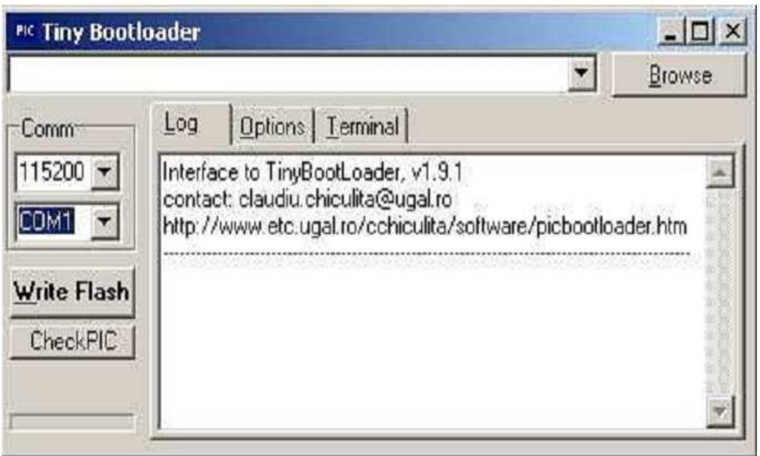


Figure 5.8: Tiny Bootloader main page

5.4 Implementation Results

In previous sections a tutorial about programming of the e-puck robot is given. This section discusses about implementation of theoretical results introduced early chapters of this dissertation.

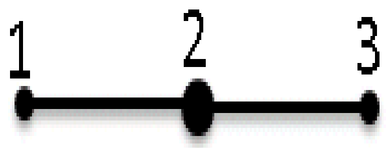


Figure 5.9: Communication topology among the e-puck robtos

Figure 5.10: Initial position of e-puck robots

A group of three e-puck robots is chosen as the test bench. These robots are selected to obtain a certain formation. The communication topology among robot is depicted in Fig. 5.9. An agent number two serves as leader and sends the required commands to followers. The mission is to perform a line formation among robots. All the robots are aligned in vertical line as shown in Fig. 5.10. The mission is to form a horizontal line at the end of maneuver. Moreover, robots must have a same heading at the end of the maneuver. It is assumed that the initial position of each robot is known. Hence, the position of each robot can be easily measured with help of odometry. The trajectory of followers are demonstrated in Fig. 5.11, where initial

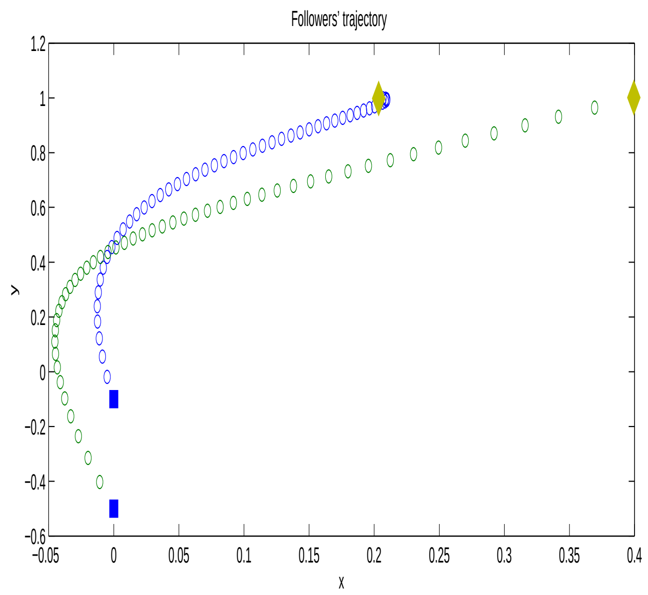


Figure 5.11: Followers' trajectory in implementation

Figure 5.12: Final position of e-puck robots

and final positions are demonstrated by rectangular and diamond, respectively. The determination of the leader's position become a trivial task since it can be controlled directly. Based on our problem formulation, followers' position is our main focus. At the end of maneuver, our group of robots successful obtains the desired formation, as demonstrated in Fig. 5.12. The team of robots achieves its defined task and fulfills the formation requirement.

5.5 Conclusion

In this chapter, the implementation of controllability of multi-agent system theory on real world application was discussed. It was shown that with help of a simple control law a formation control among robots can be obtained in real application. Moreover, a comprehensive introductory to the e-puck robot and its programming were given.

Even though our results was successfully applied in practice, but there are several interesting problems still unsolved. The formation control algorithm will be more robust if there exist several leaders among agents; moreover, a high order dynamics could replace a mass point dynamics for each agent. In addition, even though the cooperation among robots was achieved for a group of three, the results need to be tested on larger group of robots to test the scalability of our algorithm. The optimality of solution also need be tested in practice.

Chapter 6

Conclusions

Several challenges involved with multi-agent systems cooperative control were addressed in this dissertation. This thesis contributed to the area of multi-agent system by solving some fundamental challenges such as controllability and observability. The multi-agent systems were investigated from both theory and practice. Moreover, the systematic paradigm for configuration of communication topology was proposed. And, the practical implementation of results was reported.

Firstly, the controllability problem for multi-agent systems for a fixed weighted topology was studied. We introduced the novel notion of multi-agent system structural controllability and derived a necessary and sufficient condition accordingly. The connectivity of topology is both necessary and sufficient for structural controllability of interconnected systems.

Secondly, the observability of multi-agent system for a single leader case was investigated. We showed that the interaction topology directly affects the observability. The classical notion of observability was extended to multi-agent systems observability and a sufficient and necessary condition was obtained. However, some counter intuitive examples showed the need for the general definition of observability. Thus, We proposed a new observability definition, called structural observability. It was illustrated that the connectivity of graph is not only necessary, but also sufficient for structural observability.

Thirdly, we focused on design of a systematic paradigm for weights' assignment in multi-agent systems. This problem was formulated as an optimal control problem. In order to solve the optimal control problem, the general cost function was written. Then, the cost function was minimized with help of HJB functions.

Finally, the idea of controllability for multi-agent systems was implemented in practice. We used the group of e-puck robot to emulate our idea. In our experiment, there was only a leader which led the group and two followers. The implantation results were promising and offered opportunity for more research.

As the direction for future work, there are several interesting opportunities. Due to the complexity of analysis, we just focused on the agents with simple integrator dynamics; however, it is more interesting to discuss observability and controllability problem for a group of agents with the general state space dynamics. Moreover, a group of robots is more robust, when there exist more than one leader in the group. Our results can be further extended for multi-leader case. From optimization point of view, the optimization of the multi-agent system with respect to the topology is a significant research question.

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- H.Nejati, Z. Azimifar, M. Zamani, "Crop Field Weed Detection using Fast Fourier Transform," *Proc. of IEEE system mans and cybernetics conference*, 2008.