

LIFE CYCLE COST DESIGN OF CONCRETE STRUCTURES

HARIKRISHNA NARASIMHAN
(B.Tech. (Civil Engineering), IIT Delhi, India)

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SUMMARY

Concrete in some guise has been used as a construction material for hundreds of years. However, the experience gained in the last few decades has demonstrated that concrete, especially reinforced concrete, degrades with time and is therefore not maintenance free. The durability of concrete has hence been a major area of research for quite some time. Traditionally, the durability design of concrete structures is based on implicit or ‘deem-to-satisfy’ rules for materials, material components and structural dimensions. Examples of such ‘deem-to-satisfy’ rules are the requirements for minimum concrete cover, maximum water/cement ratio, minimum cement content and so on. With such rules, it is not possible to provide an explicit relationship between performance and life of the structure. It is hence necessary to adopt a suitable design approach which provides a clear and consistent basis for the performance evaluation of the structure throughout its lifetime.

A life cycle cost based procedure for the design of reinforced concrete structural elements has been developed in this research. The design procedure attempts to integrate issues of structural performance and durability together with economic cost optimization into the structural design process. The evaluation of structural performance and durability is made on the basis of determination of service life of reinforced concrete. The service life is determined based on the concept of exceedance of defined limit states, a principle commonly used in structural design. Two limit states relevant to corrosion of reinforcement are used – limit state I is based on initiation of corrosion and the limit state II is based on initiation of corrosion and cracking of the concrete cover. The service life hence determined decides the

magnitude and timing of the future costs to be incurred during the design life of the structure. Tradeoffs between initial costs and future costs and the influence of the various design variables and parameters on the life cycle cost are examined and evaluated to determine the optimum design alternative. All these considerations are encapsulated into a computational model that enables the seamless integration of durability and structural performance requirements with the structural design process.

Keywords : concrete durability, service life, life cycle cost, chloride induced corrosion, durability design, performance based design, cost optimization

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Chapter 1

Introduction

1.1 Background

In translating their design concepts into member proportions and structural details, engineers use numerical methods to provide adequate strength, stability and serviceability to the final structure. The skill comes in providing this adequacy at the least cost—usually taken to be the first cost or the cost of construction (Somerville, 1986). The margins and factors of safety are assumed to prevail as soon as the structure is completed as well as during its entire life. Such a traditional approach to structural design tends to focus primarily on the initial cost of structural design and construction. However with time, there is a gradual deterioration in material characteristics and properties and this translates into a decline in the performance and durability of a structure. Such durability and performance related considerations are usually dealt with in structural design through implicit or limiting rules laid out in national standards. A major drawback of this approach is that there is no elaborate consideration given at the structural design stage to the actual future costs that would accrue throughout the life of the structure. Future costs for a building include maintenance and repair costs and can form a substantial part of the total cost to be incurred by the user(s) during the entire lifetime of the structure.

With the ever-increasing paucity of resources in today's world, it has become very essential to achieve their optimum and effective utilization. In view of this, a pragmatic and efficient approach towards structural design would therefore be to a)

develop a framework that provides a joint evaluation of the lifetime performance of a structure and the various components of cost (initial as well as future) incurred during the life and b) incorporate this information in the actual structural design process with the overall objective of achieving overall-cost effective design without compromising on the requirements for structural strength, performance and reliability.

1.2 Conventional Design vis-à-vis Life Cycle Cost based Design

Traditionally concrete structural design has been confined to minimizing the dimension of the structural elements, thereby minimizing the material in use, just sufficient to provide adequate safety against mechanical failure and serviceability related to mechanical loads. The basic aim is hence to attain minimum material and construction cost. In such an approach, issues related to the long term performance and durability of concrete are generally dealt with through ‘deem-to-satisfy’ or implicit rules for materials, material compositions, working conditions and structural dimensions and hence not adequately addressed (Sarja and Vesikari, 1996). Such rules are based on a combination of academic research and practical knowledge accumulated from experience. The application of such general rules means that there is no hence proper insight or appreciation of the service performance of a structure in its uniquely occurring local context. The true economic implications of the costs related to long term performance are therefore not fully understood and accounted for.

The development of procedures for long term performance and durability based design of structures aim to address the above shortcomings. Such design approaches are conceptually based on ensuring that the required performance is maintained throughout the intended life of the structure. However in addition to the performance

stipulations, it is also important to ensure the optimization of the overall costs incurred during the life of the structure. While the requirements related to structural performance can be addressed by defining limit states similar to those used in structural design, the economic implications of the overall lifetime costs can be effectively evaluated by the use of techniques such as life cycle costing.

1.3 Service Life of Concrete Structures

The service life of concrete structures is closely related to the concepts of structural performance, durability and degradation. A formal definition suggested by Masters and Brandt (1987) is as follows: “Period of time after manufacturing or installation during which all essential properties meet or exceed minimum acceptable values, when routinely maintained”. There is a gradual deterioration in properties and performance of reinforced concrete with time. This could be due to corrosion of reinforcement due to chemical processes like chloride ingress and carbonation, chemical attack due to processes like sulphate attack or surface deterioration due to temperature/moisture fluctuations. The time at which this deterioration reaches an unacceptable state is the service life. The determination of the service life is an essential step in any performance/durability based design methodology as it provides a quantifiable basis for the evaluation of stipulated performance benchmarks and also determines the timing and magnitude of costs for any economic analysis.

1.4 Life Cycle Cost Based Design for Concrete Structures

From a structural design point of view, the major costs of significance pertain to the initial costs related to design and construction and the future costs related to

maintenance and repair. Energy and operating costs such as heating and cooling may be significant components in the overall life cycle cost considerations for a structure/building but they generally do not depend on the structural design parameters concerning strength, reliability and serviceability. Hence in a “structural life cycle cost design” process, the primary objective is to achieve an optimum balance between the initial costs of structural design and construction and the future/recurring costs of repair with respect to the various design parameters. The magnitude and timing of these future costs are dependent on the service life of the structure which, in turn, depends on the exposure environment and the level of structural performance expected to be maintained. Hence this design approach involves an integration of service life and the ensuing durability considerations into the structural design process.

1.5 Scope of work

This work is concerned with the development of a life cycle cost based design procedure for design of reinforced concrete structural elements. The timing of the costs incurred during the life of the structure is made through the evaluation of service life for the corrosion of reinforcement due to ingress of chlorides from seawater. The determination of service life is based on the concept of exceedance of defined limit states, a principle commonly used in structural design. The service life determines the magnitude and timing of future costs incurred during the life of the structure. The design approach provides a platform for integration of these lifetime costs with the structural design process to achieve life cycle cost minimization. Since the design process is carried on at an elemental level, the focus is hence on the minimization of life cycle costs for a structural element placed in a specified exposure environment.

1.6 Objectives

The objectives of this research are:

- To determine the service life for reinforced concrete placed in a specified exposure environment based on the principle of exceedance of stipulated performance benchmarks or defined limit states.
- To develop a structural design approach based on life cycle cost considerations that can be adopted for a structural element during its design stage and hence determine the optimum overall cost effective design alternative.
- To analyze and evaluate the influence of the different design input variables parameters on the life cycle cost and structural durability.

Chapter 2

Literature Review

2.1 Service Life

2.1.1 Definition

Service life is the period of time after manufacture or installation during which the prescribed performance requirements are fulfilled (Sarja and Vesikari, 1996). Another formal definition suggested by Masters and Brandt (1987) is as follows: “Period of time after manufacturing or installation during which all essential properties meet or exceed minimum acceptable values, when routinely maintained”. The service life of concrete structures can be treated at different levels. For instance in the case of buildings, at the building level, the end of service life would normally entail complete renovation, reconstruction or rejection of the building. At the structural component or material level, it would mean replacement or major repair of these components or materials.

2.1.2 Types of Service Life

The problem of service life can be approached from three different aspects – technical, functional and economic (Sarja and Vesikari, 1996). Technical requirements related to performance include requirements for the structural integrity of buildings, load bearing capacity of structures and the strength of materials. Functional requirements are set in relation to the normal use of buildings or structures. From the economic point of view a structure, structural component or material is treated as an investment and requirements are set on the basis of profitability.

The aspect of service life problems covered in this research is technical. The technical point of view covers structural performance, serviceability and convenience in use and aesthetics. Among these aspects, the maximum importance is attached to structural performance as it affects the integrity and safety of the structure. The load bearing capacity of structures can be influenced by the degradation of concrete and reinforcement. Structures must be designed so that the required safety is secured during the intended service life despite degradation and ageing of materials. Defects in materials may also lead to poor serviceability or inconvenience in the use of the structure. Aesthetic aspects are included if the aesthetic defects of structures are due to deterioration or ageing of materials (Sarja and Vesikari, 1996).

2.1.3 Prediction of service life for building elements/components

Any service life prediction method involves an understanding of the deterioration pattern or degradation mechanism of the structure. Such prediction methods can be classified into the following approaches (Clifton, 1993):

- 1) estimations based on experience,
- 2) deductions from performance of similar materials,
- 3) accelerated or non-accelerated testing,
- 4) modelling based on deterioration processes,
- 5) application of stochastic concepts

Some examples of service life prediction that are based on the above approaches are reviewed below; the examples involve different building components and are not restricted necessarily to concrete structures.

The first approach consists of a condition appraisal based on an in-situ inspection and expert judgment to predict the future condition profile. For instance, the performance of a concrete structure evaluated at certain time intervals has been extrapolated to the future using this approach (Sayward, 1984). This is a simple and common field method for performance assessment. However it does not allow for a thorough assessment and quantification of the deterioration mechanisms and influencing parameters.

The second approach is based on availability of sufficient information about performance of similar materials/environments. This was used by Purvis et al. (1992) for reinforced concrete bridges to determine progress of deterioration with time. When reference is made to relevant past information deemed sufficient for prediction, this approach is more reliable than the first. However, the deterioration process and influencing parameters are still not comprehensively and quantitatively considered. The uniqueness of every ambient environment and microclimatic condition and extent of similarity between conditions under which the model was developed and conditions where it is applied affect the reliability of this approach. Another method based on this approach was a factorial based method starting with the identification of a “standard service life” from existing databases and adjusting it with coefficients to account for local factors (Architectural Institute of Japan, 1993). However the quantification of relative importance and weightage for each factor is not explicit. Also the method does not provide for a continuous assessment of the deterioration pattern with time.

The third approach uses accelerated or non-accelerated techniques to simulate the deterioration processes. A systematic methodology for service life prediction involving testing procedures was provided by Masters and Brandt (1987). The various stages in the prediction process (problem definition, preparation, pre-testing, testing and interpretation) and the activities to be performed within each stage are described. The methodology is generic and elaborate; its implementation requires a large pool of knowledge of the deterioration processes and extensive testing capability. In a testing approach, the degree of correlation between test results and actual performance is greatly influenced by the extent to which testing conditions simulate actual field conditions. Also the ability of a testing programme to cover several deterioration mechanisms together remains questionable. In a study on the evaluation of paint performance (Roy et al, 1996), the artificial weathering test was found not to provide a good representation of actual paint performance since it monitored deterioration due to chemical weathering only and not that due to mechanical or biological weathering.

Modelling of the deterioration processes based on statistical or simulation techniques are also commonly used for service life prediction. A statistical modelling approach involves data collection concerning the deterioration and influencing parameters and use of suitable statistical methods to determine the deterioration at any point in time. A theoretical modelling approach is based on an analytical understanding of processes involved in the deterioration; parameters relevant to the deterioration are sometimes experimentally determined. Other modelling approaches use techniques like neural networks and expert systems.

Shohet et al (2002) and Shohet and Paciuk (2004) developed a service life prediction method for exterior cladding components based on assessment of actual performance and graphical depiction of deterioration patterns. Evaluation of component performance is made on the basis of a score from physical and visual rating scales. Each value on the scale represents a fixed combination of different defects with specified degrees of severity. This makes it a difficult and inflexible field parameter to measure. Also there is no explicit quantitative relationship between component performance and its influencing factors.

A theoretical model for prediction of concrete deterioration due to corrosion is the modelling of chloride migration, governed mainly by the diffusion mechanism (Tuutti, 1982). A detailed mathematical model can be developed in such cases; however the difficulty encountered in obtaining values for model parameters and incorporating the effect of other contributing mechanisms affects the reliability of this approach. Hjelmstad et al (1996) developed a building materials durability model for cladding on buildings. The serviceability index function used to model the degradation was expressed as a function of temperature, moisture and concentration of aggressive chemicals. The weightage of different defects within this single index value and the conceptual basis for arriving at the model equation was not explicitly provided. Stephenson et al (2002) developed an approach for the prediction of defects on brickwork mortar using expert systems. The approach is based on the ability of the system to capture enough knowledge to predict the likelihood of defects at the pre-construction and construction stages. The method does not provide for evaluation during the lifetime of the building.

A common service life prediction approach based on stochastic methods involves the extension of a theoretically developed model by using statistical distributions rather than single values for model parameters. This approach was used in Siemes et al (1985). The limited use of these methods is due to lack of databases to obtain the required statistical distributions.

2.2 Corrosion of Reinforcement

2.2.1 Introduction

The co-operation of concrete and steel in structures is based partly on the fact that concrete gives the reinforcement both chemical and physical protection against corrosion. The chemical effect of concrete is due to its alkalinity, which causes an oxide layer to form on the steel surface. This phenomenon is called passivation as the oxide layer prevents propagation of corrosion in steel. The concrete also provides the steel with a physical barrier against that promote corrosion such as water, oxygen and chlorides (Tuutti, 1982; Sarja and Vesikari, 1996).

In normal outdoor concrete surfaces, corrosion of reinforcement takes place only if changes occur in the concrete surrounding the steel. The changes may be physical in nature typically including cracking and disintegration of concrete which exposes part of the steel surface to the external environment and leaves it without the physical and chemical protection of concrete. The changes can also be chemical in nature. The most important chemical changes which occur in the concrete surrounding the reinforcement are the carbonation of concrete due to carbon dioxide in air and the penetration of chloride anions into concrete.

Carbonation is the reaction of carbon dioxide in air with hydrated cement minerals in concrete. This phenomenon occurs in all concrete surfaces exposed to air, resulting in lowered pH in the carbonated zone. In carbonated concrete the protective passive film on steel surfaces is destroyed and corrosion is free to proceed. The ingress of chloride anions into concrete also leads to corrosion of reinforcement. The effect of such agents is not based on the decrease in pH as in carbonation but on their ability otherwise to break the passive film.

2.2.2 Limit States for Corrosion Of Reinforcement

Two limit states can be identified with regard to service life (Sarja and Vesikari, 1996):

1. The service life ends when the steel is depassivated. Thus the service life is limited to the initiation period of corrosion, that is, the time for the aggressive agent to reach the steel and induce depassivation. The formula for service life used in this case is :

$$T_L = T_0 \quad (2.1)$$

where

T_L = service life

T_0 = initiation time of corrosion

2. The service life includes a certain propagation period of corrosion in addition to initiation period. During propagation of corrosion, the cross-sectional area of steel is progressively decreased, the bond between steel and concrete is reduced and the effective cross-sectional area is diminished due to cracking/spalling of cover. In

this case, the service life is defined as the sum of the initiation time of corrosion and the time for cracking of the concrete cover to a given limit :

$$T_L = T_0 + T_1 \quad (2.2)$$

where

T_L = service life

T_0 = initiation time of corrosion

T_1 = propagation time of corrosion

2.2.3 Modelling of Chloride Ingress into Concrete

The penetration of chlorides into concrete is usually considered as a diffusion process and thus can be described by Fick's second law of diffusion (Crank, 1956). For a general three-dimensional case, the corresponding equation for diffusion can be written as:

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x} \left(D_{CX} \frac{\partial C}{\partial x} \right) + \frac{\partial}{\partial y} \left(D_{CY} \frac{\partial C}{\partial y} \right) + \frac{\partial}{\partial z} \left(D_{CZ} \frac{\partial C}{\partial z} \right) \quad (2.3)$$

where:

C = concentration of chloride ions at any point (x,y,z) in the three-dimensional space at time t

D_{CX} = coefficient of diffusion in the direction x

D_{CY} = coefficient of diffusion in the direction y

D_{CZ} = coefficient of diffusion in the direction z

A common way of modelling the ingress of chlorides into reinforced concrete in one direction is through the assumption of a half-infinite interval for mathematical

simplicity. For such a scenario, if the diffusion coefficient in the concerned direction can be considered to be independent of time and also independent of the spatial coordinates, the diffusion equation in one dimension (say, direction x) can be written as:

$$\frac{\partial C}{\partial t} = D_c \frac{\partial^2 C}{\partial x^2}, x > 0, t > 0 \quad (2.4)$$

If the chloride concentration at the concrete surface is constant, equation 2.4 can be solved to obtain the chloride concentration as:

$$C = C_s \left[1 - \operatorname{erf} \left(\frac{x}{2(D_c t)^{1/2}} \right) \right] \quad (2.5)$$

where

- C = concentration of chloride at depth x at time t
- C_s = the constant chloride concentration at the concrete surface
- x = the depth from the surface
- D_c = diffusion coefficient
- t = time

The mathematical derivation of the solution given in equation 2.5 is presented in Appendix A. Equation 2.5 has been commonly used for modelling of chloride ingress in Liam et al (1992), Englund and Sorensen (1998), Val and Stewart (2003), Khatri and Sirivivatnanon (2004) and several others.

However in marine environments particularly, there is gradual accumulation of chloride predominantly due to salt spray on the concrete surface with time and hence

it is likely that the surface chloride content will increase with the time of exposure. A linear relationship between the surface chloride and the square root of time has been used in Takewaka and Mastumoto (1988), Uji et al (1990), Swamy et al (1994), Stewart and Rosowosky (1998). Hence in the case, the solution of equation 2.4 for a time varying surface chloride concentration is obtained as:

$$C = S\sqrt{t} \left[\exp\left(-\frac{x^2}{4D_c t}\right) - \frac{x\sqrt{\pi}}{2\sqrt{D_c t}} \left\{ 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{D_c t}}\right) \right\} \right] \quad (2.6)$$

where

S = surface chloride content coefficient (in % by weight of cement * sec^{-1/2})

x = depth from the surface (in m)

t = time (in seconds)

D_c = diffusion coefficient (in m²/sec)

erf = error function

C = chloride concentration at depth x at time t (in % by weight of cement)

The mathematical derivation of the solution given in equation 2.6 is presented in Appendix A.

Parameters influencing chloride concentration level – Surface Chloride Concentration

Values for the surface chloride content coefficient published in literature are mostly location/climate/environment specific. The values reported are both constant as well as time varying/accumulating. The range of values for surface chloride levels in a tropical marine structure was reported as 1.3 to 3.1% by weight of cement (Liam et al,

1992). Takewaka and Mastumoto (1988) in a study of marine structures in Japan determined that the surface chloride content was constant for concrete always in contact with seawater at about 0.7 to 1% by weight of concrete; however the surface chloride content in other marine conditions was found to be accumulative and increasing with time at the rate of about 0.01 to 0.1% by weight of concrete per month in a marine splash zone and 0.001 to 0.01% by weight of concrete per month in a marine atmospheric zone. In another study of marine structures in Japan, Uji et al (1990) found the surface chloride content to be proportional to the square root of the time in service; the constant of proportionality was found to vary within a wide range with the maximum in a marine tidal zone followed by the splash and atmospheric zones. Val and Stewart (2003) in an analysis of concrete structures in marine environments used surface chloride values varying with the exposure environment and proximity to seawater. A similar variation of the surface chloride content as that reported in Uji et al (1990) was used by Stewart and Rosowosky (1998) in a study of exposed concrete in temperate climates; the surface chloride content was expressed as a diffusion flux on the concrete surface with a mean value of 7.5×10^{-15} kg/cm²s. In a probabilistic analysis of chloride and corrosion initiation in concrete structures in Denmark, Engelund and Sorensen (1998) considered both temporal as well as spatial variations of the surface chloride content.

It is often not relevant or practical to make use of such values developed in localized situations/environments for other locations. A work of more general nature is published in Swamy et al (1994) where results based on an assessment of data from world wide published laboratory and field tests are provided. When surface chloride

content values are not measured or not available, work of this nature forms a possible basis for use of surface chloride values.

Parameters influencing chloride concentration level – Chloride diffusion coefficient

The chloride diffusion coefficient depends mainly on the properties and specifications of concrete (such as water/cement ratio, composition, degree of hydration and aggregate/cement ratio), environmental conditions (such as temperature and relative humidity) and time. Due to the complexity of the problem, simple empirical and semi empirical models which typically consider the influence of mix proportions and provide mathematical models for computation are usually used. A wide range of chloride diffusivity values are found in the literature. [Tuutti (1982), Takewaka and Mastumoto (1988), Liam et al (1992), Frangopol et al (1997), Stewart and Rosowosky (1998), Vu and Stewart (2000)]. The existence of such a wide range of diffusivity values is because of the vast coverage of a variety of cement/concrete types and exposure conditions, and, in general, is more applicable to marine environments. There is hence no existing computational model in literature for determination of the diffusion coefficient by taking into account all these factors. A typical model for chloride diffusion coefficient proposed by Papadakis et al (1996) is given below; this model is based on the physicochemical processes of chloride penetration and also accounts for the influence of mix proportions such as water/cement ratio and aggregate/cement ratio.

$$D_C = 0.15 \frac{1 + \rho_c \frac{w}{c}}{1 + \rho_c \frac{w}{c} + \frac{\rho_c}{\rho_a} \frac{a}{c}} \left(\frac{\rho_c \frac{w}{c} - 0.85}{1 + \rho_c \frac{w}{c}} \right)^3 D_{Cl^-, H_2O} \quad (2.7)$$

where:

| | | |
|-----------------|---|---|
| D_C | = | diffusion coefficient (in m^2/sec) |
| a/c | = | aggregate/cement ratio |
| w/c | = | water cement ratio |
| ρ_c | = | mass density of cement |
| ρ_a | = | mass densities of aggregate |
| D_{Cl^-,H_2O} | = | diffusion coefficient of Cl^- in an infinite solution (in m^2/sec) |

Parameters influencing chloride concentration level – Cover to reinforcing steel

From a review of past work, the main factors that influence the variability of the concrete cover can be identified as the incorrect placement of reinforcement, mismatch in reinforcement shape or size, complexity of steel fixing, quality control and audit, clashing of services with formwork and reinforcement, formwork erection and movement during concrete casting [Mirza and MacGregor (1979a), Marosszeky and Chew (1990), Clark et al (1997)]. It can be seen that the majority of the factors relate to workmanship and quality control during construction.

Critical Chloride Threshold

The chloride threshold level can be defined as the chloride concentration at the depth of the reinforcing steel which results in a significant corrosion rate leading to corrosion induced deterioration of concrete (Glass and Buenfeld, 1997). Values of the critical chloride threshold ranging from 0.03% to 0.4% chloride by weight of concrete can be found in literature (Tuutti, 1982; Hope and Ip, 1987; Mangat and Molloy, 1994; Glass and Buenfeld, 1997; and others). When the threshold level is defined as a single value of chloride concentration, the time to corrosion activation is determined

as the time which the computed chloride concentration just exceeds the define critical chloride level; this has been commonly used in Liam et al (1992), Stewart and Rosowsky (1998), Anoop et al (2002), and many others. However in a review of chloride threshold levels, Glass and Buenfeld (1997) have stated that chloride threshold is best considered in terms of corrosion risk. Similarly results from the survey of a large number of buildings in Britain published in Everett and Treadway (1980) provide a classification of the corrosion risk in terms of the chloride content. Data for frequency or probability of corrosion as a function of chloride content are also published in Vassie (1984) and Li (2003).

2.3 Life Cycle Costing (LCC)

2.3.1 Introduction

Life cycle costing (LCC) is a method of evaluating the economic performance of investment projects by calculating the total costs of ownership over the life span of the project (Brown and Yanuck, 1985). In this technique, initial costs, all expected costs of significance, disposal value and any other quantifiable benefits to be derived are taken into account. The LCC technique is justified whenever a decision needs to be taken on the acquisition of an asset which would require substantial maintenance costs over its life span.

2.3.2 Relevance of LCC in Design of Concrete Structures

A major cause of concern with the use of reinforced concrete is that it undergoes degradation with time and is hence not maintenance free. The aspect of durability of concrete structures has so far been dealt with in an empirical manner through the specification of guiding and limiting rules concerning materials and properties. The

most common approach to understand the durability problems associated with concrete is through the assessment of service life or the period during which concrete is 'in service' and fulfills all necessary performance requirements. The incorporation of the concept of service life into a design procedure involves an understanding of its economic implications. Life cycle costing provides a tool to quantify the economic implications of service life, thus paving the way for its inclusion into existing design procedures. The adoption of life cycle costing in the design of structures hence enables a thorough understanding of the economic implications of durability on the performance of the structure during its lifetime.

2.3.3 Stepwise Listing of LCC Analysis

The approach to a typical LCC analysis is composed of a number of key steps which are itemized below. (This is extracted from Macedo et al, 1978; Brown and Yanuck, 1985).

Establish Objectives

The first step in LCC analysis is to define requirements and establish basic objectives of what the structure must achieve. These requirements are generally developed from an analysis of the needs of the client or the owner. Also, any special constraints must be identified at this time.

Define Alternatives

A set of alternatives that satisfy the requirements and achieve the basic objectives are selected. It is necessary to identify all practical design approaches for further analysis.

This process of selecting alternatives for further study can be listed as follows:

- Identify feasible design, concept and structural element alternatives.
- Obtain performance requirements for each option.
- Screen alternatives, eliminating those that do not meet defined performance requirements and constraints.
- The remaining alternatives are selected for further study.

Select Life Cycle

This involves deciding upon a finite planning horizon or life cycle applicable to all the alternatives. The selection of a specific number of years for a life cycle establishes the duration of time over which future costs (operating, maintenance etc.) are estimated.

Estimate Costs

All the costs and revenues which are directly relevant to the comparison of alternatives are identified. The initial costs for each alternative are computed first. There are three types of recurring costs : normal operation and maintenance costs incurred on a daily, weekly or monthly basis, the annual costs for utilities and fuels and the recurring costs of repairs, alterations and replacement of structural elements or systems. Estimates of their occurrence and periodicity depend on the estimates of the live cycles derived in the previous step. Also adjustments are made for price escalation.

Compute Present Values or Annual Equivalents

As the various expenditures estimated above take place at different times during the life cycle of the structure, the costs are adjusted to a common time period by converting to present values or annual equivalents. This is done by multiplying these costs by the appropriate discount factors in order to take time value of money into account.

Test sensitivity of results

The results from present value or annual equivalent computations for each alternative establish their ranking. The lowest alternative is the preferred one based on a total life cycle cost approach. However, finally a sensitivity analysis is carried out to assess the influence of the various input parameters on the life cycle cost. Once these sensitivity tests are completed, the resulting lowest life cycle cost alternative is recommended for implementation.

Chapter 3

Development of LCC Design Model

3.1 Basis of Design

The life cycle cost (LCC) based design procedure is developed for 2 limit states related to the corrosion of reinforcement in concrete. The 2 limit states correspond to the following events:

- I) initiation of corrosion
- II) initiation of corrosion and cracking of concrete cover

Figures 3.1 and 3.2 are flow charts listing the stepwise design procedure for limit states I and II respectively. The failure criterion for each limit state is based on the exceedance of a certain maximum allowable probability of failure. These maximum allowable failure probabilities are specified in the form of target reliability indices that are more commonly used in structural design. (The reliability index is the inverse standardized normal distribution function of the probability of failure.) The design procedure for a particular limit state involves the computations of the probability of failure for the corresponding event and then the reliability index at different time points during the intended design life of the structure. As time progresses, there is an increase in the level of deterioration in the condition of the structure as long as no remedial/repair action is undertaken. Hence with time, the probability of failure of the structure based on any of the 2 above defined limit states increases and the reliability index corresponding to this probability of failure decreases.

Figure 3.1 Design procedure for Limit State I

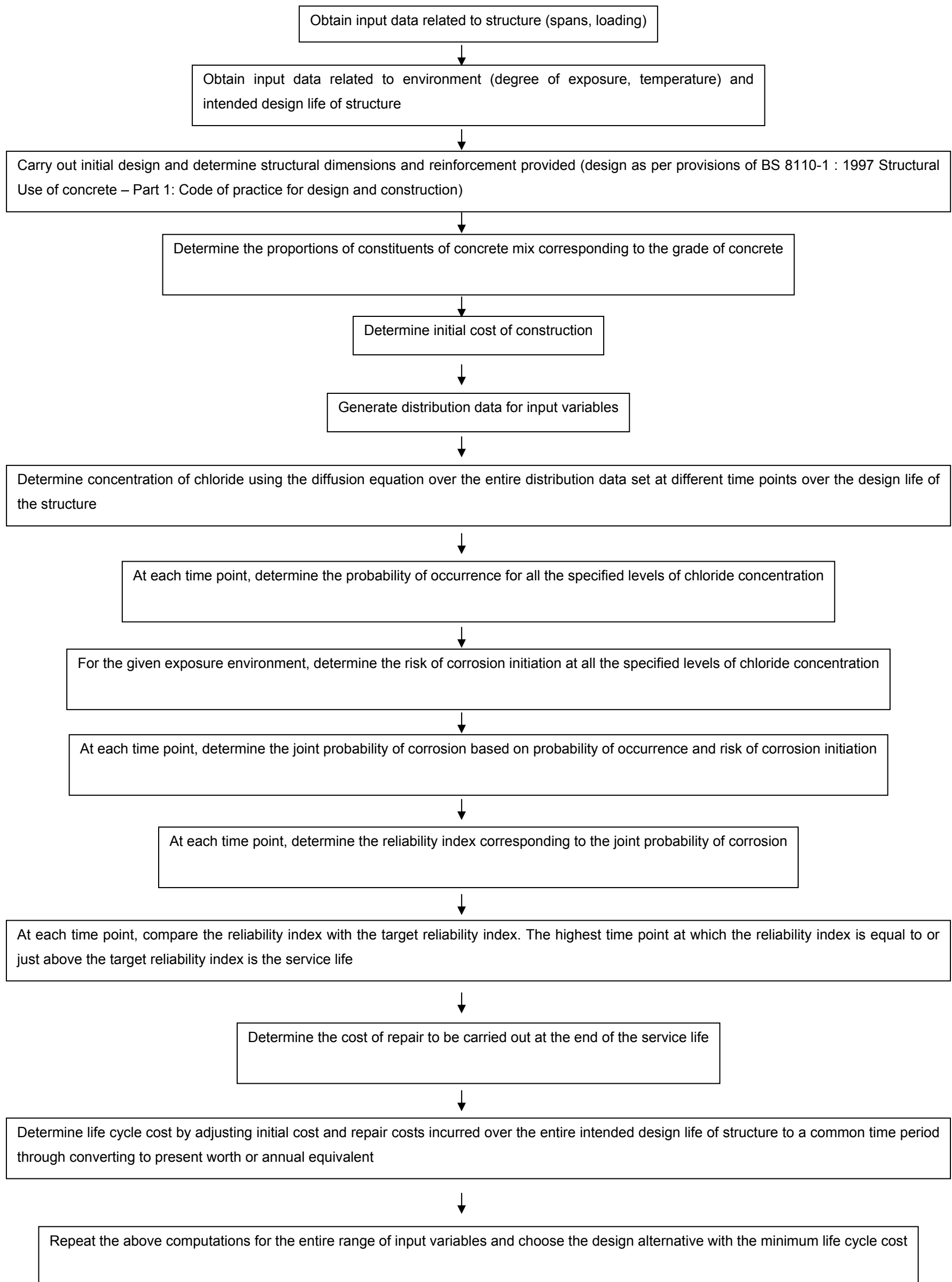
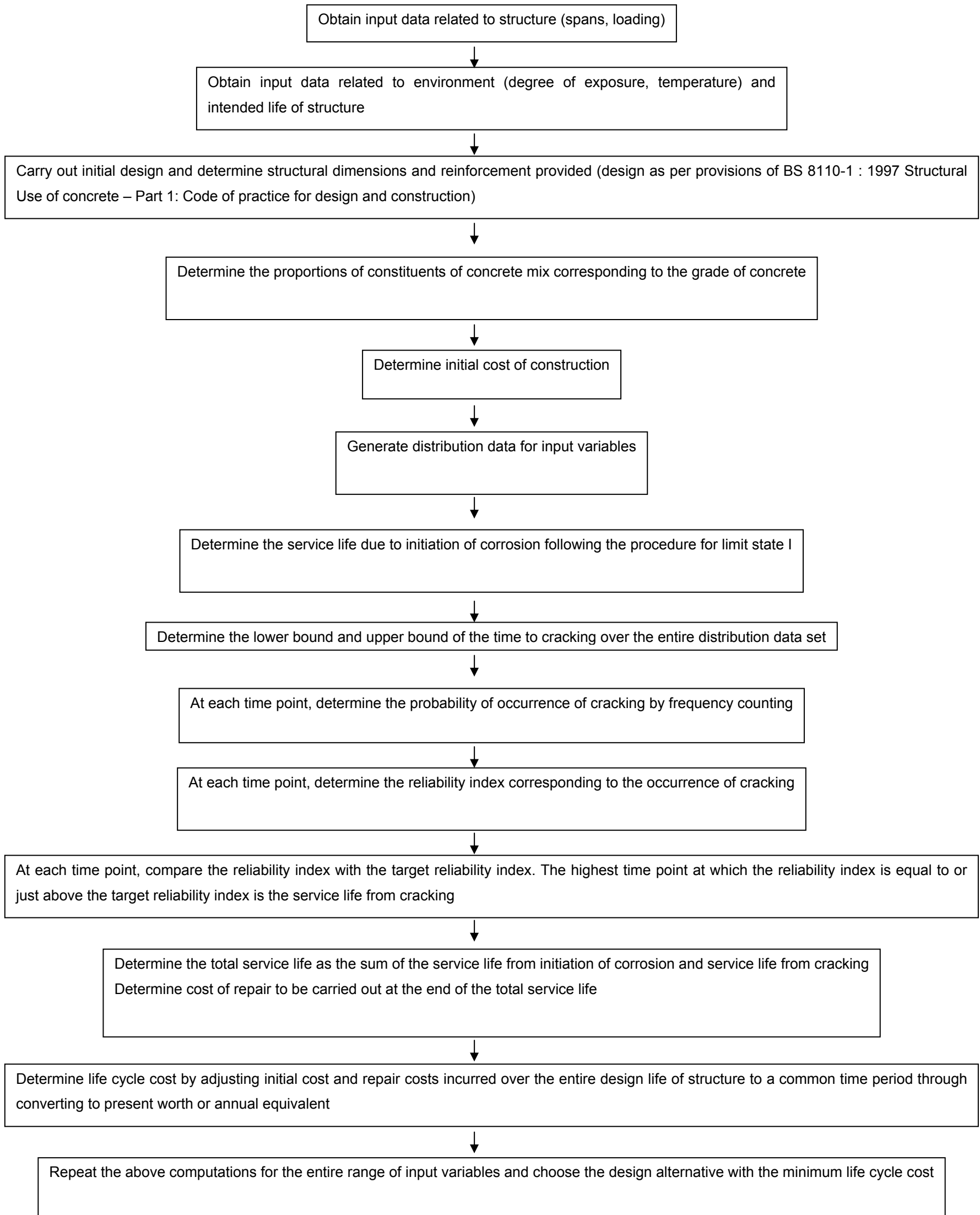


Figure 3.2 Design procedure for Limit State II



The time upto which the reliability index corresponding to the event exceeds the specified target reliability index value is defined as the service life for the structure. In the context of durability design, the service life is the time period at the end of which remedial/repair action is required to bring the structure to an acceptable level of probability of failure/reliability.

The target reliability index values chosen for the 2 limit states are based on guidance given in ISO 2394. These values are based on i) the importance of the structure and ii) the consequence of failure of the structure on account of exceedance of the limit state. In this study, the structures are considered to be of reliability class RC2 as defined in ISO 2394. This is associated with the consequence class CC2 under which failure of the structure has “*medium* consequence for loss of human life with economic, social or environmental consequences *considerable*.”

As we move from limit state I to limit state II, it can be seen that the consequences of failure of the structure increase in their extremity. The more extreme the consequences of failure corresponding to a particular event are, the lower should be its probability of occurrence and consequently the higher should the target reliability index. Keeping this in mind and also based on guidance values given in ISO 2394 and BS EN 1990: 2002, the target reliability index values for limit states I and II are taken as 1.5 and 2.0 respectively.

The target reliability index value for an irreversible serviceability limit state is 1.5 for a structure under reliability class RC2. Failure of the structure defined by initiation of corrosion is considered as a serviceability limit state and hence the value of 1.5 is

chosen. Limit state II involves the cracking of the structure; unlike limit state I, there is visible damage/distress to the structure here though not critical in terms of overall structural stability and integrity. Further some loss in the aesthetic functionality of the structure also occurs. Hence a higher value of 2.0 compared to that for limit state I is chosen for limit state II.

3.2 Categorization of Exposure Environment

Four exposure environments – submerged, tidal/splash, coastal and inland are used; the description of these environments is given in table 3.1. This categorization is derived based on the exposure classes defined in BS 8500-1 : 2000 for category 4 (Corrosion induced by chlorides from seawater) and the classification used in Swamy et al (1994).

Table 3.1 Categorization of exposure environment

| Name | Description | Nearest Matching Exposure Classes from BS 8500 – 1 : 2000 |
|--------------|---|--|
| Submerged | Concrete is below the “Low Water Level” and exposed to seawater always | XS2 Permanently submerged Part of marine structure |
| Tidal/Splash | Concrete is located between “Low Water Level” and “High Water Level” and is exposed to cycles of wet and dry conditions daily due to tidal action Concrete is located just above the “High Water Level” and is exposed to sea water splash | XS3 Tidal, splash and spray zones Part of marine structure |
| Coastal | Concrete is located between Splash and Inland zones. During strong winds and/or high waves, concrete is exposed to sea water splash. | XS3 Tidal, splash and spray zones Part of marine structure XS1 Exposed to airborne salt but not in direct contact with sea water Structures near to or on the coast |
| Inland | Concrete is located about 10m to 20m from sea shore. Concrete is exposed to sea water breeze but not to sea water splash directly | XS1 Exposed to airborne salt but not in direct contact with sea water Structures near to or on the coast |

3.3 Random Variability

The variables in the modelling and design are treated as probabilistic random variables in order to account for their variability. Hence instead of single values or functions, each variable is represented by a distribution type with a certain mean value and standard deviation/coefficient of variation; for computational purposes, the distribution is generated through Monte Carlo random sampling. The choice of the distribution type and parameters is based on existing sources of literature.

3.3.1 Variability in Structural Dimensions and Properties

The statistical parameters for structural dimensions and properties which quantify their variability are listed in table 3.2.

Table 3.2 Statistical parameters for structural dimensions and properties

| VARIABLE | DISTRIBUTION | MEAN | STANDARD DEVIATION | SOURCE |
|--|---------------------|------------------------|--------------------|-----------------------------|
| Structural Dimensions (all in mm) | | | | |
| width | Normal | nominal + 2.3813 | 4.7625 | Mirza and MacGregor (1979a) |
| overall depth | Normal | nominal – 3.175 | 6.35 | Mirza and MacGregor (1979a) |
| top cover | Normal | nominal + 3.175 | 15.875 | Mirza and MacGregor (1979a) |
| bottom cover | Normal | nominal + 1.5875 | 11.1125 | Mirza and MacGregor (1979a) |
| side cover | Normal | nominal + 2.3813 | 13.4938 | Mirza and MacGregor (1979a) |
| Reinforcement Areas (all in mm²) | | | | |
| $\frac{\text{tension area}_{\text{furnished}}}{\text{tension area}_{\text{calculated}}}$ | Modified log-normal | 1.01 | 0.04 | Mirza and MacGregor (1979a) |
| $\frac{\text{compression area}_{\text{furnished}}}{\text{compression area}_{\text{calculated}}}$ | Modified log-normal | 1.01 | 0.04 | Mirza and MacGregor (1979a) |
| Strength (all in N/mm²) | | | | |
| concrete compressive strength | Normal | 0.675*nominal + 7.5862 | 0.175*mean | Mirza et al (1979) |
| steel yield strength | Beta | nominal | 0.1*mean | Mirza and MacGregor (1979b) |

3.4 Limit State I – Initiation of Corrosion

Limit state I is defined by the initiation of corrosion in the reinforcing steel.

3.4.1 Equations used for Modelling

Tidal/Splash and Coastal Environments

In the tidal/splash and coastal environments, there is gradual accumulation of chloride predominantly due to salt spray on the concrete surface with time and hence it is likely that the surface chloride content will increase with the time of exposure. A linear relationship between the surface chloride and the square root of time has been used in Takewaka and Mastumoto (1988), Uji et al (1990), Swamy et al (1994), Stewart and Rosowosky (1998).

In this study, the modelling of surface chloride content for tidal, splash and coastal environments is hence based on a linear relationship with the square root of time. Hence equation 2.6 from chapter 2 which gives the solution of the diffusion equation for a time varying surface chloride concentration is used to determine the chloride concentration at any point of time is used. This equation is reproduced below for reference.

$$C = S\sqrt{t} \left[\exp\left(-\frac{x^2}{4D_c t}\right) - \frac{x\sqrt{\pi}}{2\sqrt{D_c t}} \left\{ 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{D_c t}}\right) \right\} \right] \quad (3.1)$$

where

S = surface chloride content coefficient (in % by weight of cement * sec^{1/2})

x = depth from the surface (in m)

- t = time (in seconds)
 D_c = diffusion coefficient (in m^2/sec)
 erf = error function
 C = chloride concentration at depth x at time t (in % by weight of cement)

The surface chloride coefficient values are derived from results for chloride penetration published in Swamy et al (1994) as these are based on an assessment of data from world wide published laboratory and field tests. The nominal values of the variable 'S' thus obtained are 0.0007716 and 0.00069330 (all with units of % by weight of cement * $s^{-1/2}$) for tidal/splash and coastal environments respectively.

Further the surface chloride content coefficient is modelled as a log-normal distribution with a coefficient of variation of 0.6. Though there is not sufficient relevant literature, the choice is based on the use of the same distribution type and approximately similar coefficient of variation in Stewart and Rosowsky (1998) and Engelund and Faber (2000).

Submerged and Inland Environments

Results published in Swamy et al (1994) show that the level of surface chloride becomes constant after the 2nd year of exposure for submerged exposure conditions and around the 5th year of exposure for inland exposure conditions. Hence equation 2.5 from chapter 2 which gives the solution of the diffusion equation for constant surface chloride concentration is used. This equation is reproduced below for reference.

$$C = C_s \left[1 - \operatorname{erf} \left(\frac{x}{2(D_c t)^{1/2}} \right) \right] \quad (3.2)$$

where

- C = concentration of chloride at depth x at time t
- C_s = the constant chloride concentration at the concrete surface
- x = the depth from the surface
- D_c = diffusion coefficient
- t = time

For submerged environment, the nominal value of C_0 in the above equation is obtained from Swamy et al (1994) as 6 % by weight of cement. C_0 is modelled as a log-normal distribution with a coefficient of variation of 0.5. This choice is based on the estimate of the same distribution type and coefficient of variation by Hoffman and Weyers (1994) from a study of concrete bridge decks in the United States.

For inland environment, the nominal value of C_0 in the above equation is obtained from Swamy et al (1994) as 3.5 % by weight of cement. C_0 is modelled as a log-normal distribution with a coefficient of variation of 0.5. This choice is based on the estimate of the same distribution type and coefficient of variation by McGee (1999) from a study of bridges in atmospheric marine zones in Australia and also used in Vu and Stewart (2000).

Diffusion Coefficient

The expression proposed by Papadakis et al (1996) as given in equation 3.3 is used; this model is based on the physicochemical processes of chloride penetration and also accounts for the influence of mix proportions such as water/cement ratio and aggregate/cement ratio.

$$D_C = 0.15 \frac{1 + \rho_c \frac{w}{c}}{1 + \rho_c \frac{w}{c} + \frac{\rho_c}{\rho_a} \frac{a}{c}} \left(\frac{\rho_c \frac{w}{c} - 0.85}{1 + \rho_c \frac{w}{c}} \right)^3 D_{Cl^-, H_2O} \quad (3.3)$$

where:

D_C = diffusion coefficient (in m²/sec)

a/c = aggregate/cement ratio

w/c = water cement ratio

ρ_c = mass density of cement

ρ_a = mass densities of aggregate

D_{Cl^-, H_2O} = diffusion coefficient of Cl⁻ in an infinite solution (in m²/sec)

The diffusion coefficient is modelled as a log-normal distribution with a coefficient of variation of 0.7. This choice is based on the estimate of the same distribution type and coefficient of variation by Matsushima et al (1998) and Stewart and Rosowsky (1998).

3.4.2 Determination of service life

Using equation 3.1 for tidal/splash and coastal environments and equation 3.2 for submerged and inland environments, the chloride concentration at the level of

reinforcing steel is determined over the entire distribution data set generated using the statistical distributions of the various variables. These chloride concentration computations are carried out for different time points spread over the intended design life of the structure. Hence at each time point, there is one set of chloride concentration output corresponding to the distribution data set. This output set is then discretised into 6 chloride concentration levels from 0.1 to 0.6 % by weight of cement. Thus concentration values between 0 and 0.1% are grouped under 0.1, values between 0.1 and 0.2 % are grouped under 0.2 and so on. Following this, the probability of occurrence for each concentration level is obtained.

This is followed by the determination of the corrosion risk at each of the above defined chloride concentration levels. The corrosion risk, in turn, determines the threshold level for initiation/activation of corrosion of the reinforcing steel. The chloride threshold is considered in terms of corrosion risk as suggested in Glass and Buenfeld (1997).

The determination of the time to activation to corrosion is based on a joint evaluation of the corrosion risk at different levels of chloride concentration rather than being based on comparison with a single critical chloride threshold value. The relationship between corrosion risk and chloride concentration is obtained based on an analysis of data from Vassie (1984) and Li (2003) and is expressed in the form of the following equations:

For tidal/splash and coastal environments,

$$P(\text{corrosion activation}) = -0.868 * C^3 + 12.41 * C^2 - 1.8197 * C + 0.0624 \quad (3.4)$$

For submerged and inland environments,

$$P(\text{corrosion activation}) = 0.0859 * C^2 + 0.2834 * C + 0.1014 \quad (3.5)$$

where:

$P(\text{corrosion activation})$ = probability/risk of corrosion activation

C = chloride concentration (in % by weight of cement)

The Pearson correlation coefficients corresponding to equations 3.4 and 3.5 are obtained as 0.88 and 0.99 respectively.

Let the probability of occurrence of a certain level of chloride concentration ‘i’ be $P(A_{i,t})$ at time ‘t’. Let the risk of corrosion initiation at this concentration level be $P(B_i)$. Using joint probability and considering all the 6 six defined levels of chloride concentration, the probability of corrosion initiation $P(CI_t)$ at time ‘t’ can be defined as:

$$P(CI_t) = \sum_{i=1}^6 P(A_{i,t}) * P(B_i) \quad (3.6)$$

The probability of corrosion initiation is then converted to a reliability index value using the inverse standardized normal distribution function. Reliability index values are thus obtained at different time points over the intended design life of the structure or the period of analysis. As discussed earlier, the reliability index decreases with time and the time upto which the reliability index remains greater than or equal to the

specified target reliability index value (1.5 in this case) is the service life for initiation of corrosion.

3.5 Limit State II – Initiation of Corrosion and Cracking of Concrete Cover

Limit state II involves initiation of corrosion followed by cracking of the concrete cover. For initiation of corrosion, the design procedure for limit state I as described above is adopted and the time at which initiation of corrosion occurs is first determined.

3.5.1 Equations used for Modelling

For cracking of the concrete cover, the semi-empirical model developed by Liu and Weyers (1998) is used; this model is based on determined of the time required to generate the critical amount of corrosion products that are needed to i) fill the interconnected void spaces around the reinforcing steel and ii) generate sufficient tensile stresses to crack concrete. The equations used to determine the time to corrosion cracking are given below.

$$T_{cr} = \frac{W_{crit}^2}{k_p} \quad (3.7)$$

$$W_{crit} = \frac{\pi \rho_{rust} D \left[\frac{x f_t}{E_{ef}} \left(\frac{b^2 + a^2}{b^2 - a^2} + v_c \right) + d_0 \right]}{1 - \frac{\alpha \rho_{rust}}{\rho_{steel}}} \quad (3.8)$$

$$f_t = 0.56 \sqrt{f_{cu}} \quad (3.9)$$

$$E_{ef} = \frac{E_c}{1 + \psi_{cr}} \quad (3.10)$$

$$a = (D + 2d_0)/2 \quad (3.11)$$

$$b = x + (D + 2d_0)/2 \quad (3.12)$$

$$k_p = \frac{0.000102\pi DI_{corr}}{\alpha} \quad (3.13)$$

$$I_{corr} = 80.7826 * corr \quad (3.14)$$

where:

T_{cr} = time to cracking of concrete cover (in years)

W_{crit} = critical amount of corrosion products

k_p = rate of rust production

ρ_{rust} = density of rust (taken as 3600 kg/m³)

D = diameter of reinforcing steel (in m)

x = concrete cover to reinforcing steel (in m)

f_t = tensile strength of concrete (in N/mm²)

E_{ef} = effective elastic modulus of concrete (in N/mm²)

ν_c = Poisson's ratio of concrete (taken as 0.18)

d_0 = pore band thickness around steel/concrete interface (this is taken as 12.5x10⁻⁶m)

α = ratio of molecular weight of steel to molecular weight of corrosion products (taken as 0.523 for Fe(OH)₃ and 0.622 for Fe(OH)₂)

ρ_{steel} = density of steel (taken as 7860 kg/m³)

f_{cu} = compressive strength of concrete (in N/mm²)

| | | |
|-------------|---|---|
| E_c | = | elastic modulus of concrete (in N/mm ²) |
| Ψ_{cr} | = | creep coefficient (taken as 2.0) |
| I_{corr} | = | corrosion current intensity (in mA/sq ft) |
| $corr$ | = | rate of corrosion (in mm/year) |

From the above equations, the time to cracking is obtained as a function of α (ratio of molecular weight of steel to molecular weight of corrosion products). Two cases are considered – i) when the products are predominantly considered as Fe(OH)₃ for which $\alpha = 0.523$ and ii) when the products are predominantly considered as Fe(OH)₂ for which $\alpha = 0.622$. The values obtained for these 2 cases represent respectively the lower bound and upper bound of the time to cracking. These computations are repeated over the entire distribution input data set to obtain 2 output sets (one for the lower and the other for the upper bound).

Typical ranges of values for corrosion intensity for different exposure conditions based on laboratory specimens as well as on-site structures have been presented in Andrade et al (1990). These are hence used to obtain the rates of corrosion for each of the four exposure environments. The nominal corrosion rates thus obtained are 0.0011, 0.11, 0.011 and 0.0011 (all in mm/year) for submerged, tidal/splash, coastal and inland exposure environments. Further the corrosion rate is modelled as a normally distributed variable with a coefficient of variation of 0.2; this is based on a similar distribution and parameters used in Stewart and Rosowsky (1998).

3.5.2 Determination of service life

For determination of the time to cracking, different time points are considered. The probability of cracking occurring at each time point is determined; this is based on a frequency counting of the number of values in the output data set that are greater than or equal to the concerned time point. The probabilities of cracking thus obtained are converted to reliability index values using the inverse standardized normal distribution function. Reliability index values are thus obtained at different time points over the intended design life of the structure or the period of analysis. The time upto which the reliability index remains greater than or equal to the specified target reliability index value (2.0 in this case) is the service life for cracking of concrete cover. Repeating this for both the lower and upper bound output cases gives the lower bound and upper bound values of the service life respectively.

3.6 Life Cycle Cost Analysis

3.6.1 Range of parameter values

The determination of service life for the 2 limit states is carried out for different combinations of the input variables. These variables are – cover to reinforcing steel, concrete compressive strength, diameter of reinforcing steel, effective depth of beam and effective depth to width ratio. The range of values for each of these variables that are used in the analysis are given in table 3.3

Table 3.3 Range of parameter values used in analysis

| Variable | Range of Values |
|--------------------------------|---|
| cover to reinforcing steel | 20 to 100 mm (in steps of 5 mm) |
| concrete compressive strength | 25 to 50 N/mm ² (in steps of 5 N/mm ²) |
| diameter of reinforcing steel | 16, 20, 25 mm |
| effective depth of beam | (minimum depth) to (minimum depth + 50) mm (in steps of 10 mm) |
| effective depth to width ratio | 1.5, 1.75 and 2 |

3.6.2 Life Cycle Costing and Determination of Optimum Design Alternative

Following the determination of service life, the initial construction costs and the future repair costs are computed based on rates obtained from the schedule of unit rates published by the Building and Construction Authority – the regulatory body for Singapore’s construction industry.

The repair work that needs to be carried out at the end of the service life period for limit state I includes removal of chloride contaminated concrete, surface cleaning of the exposed reinforcement and reinstatement with new concrete. For limit state II, the repair work includes removal of chloride contaminated concrete, surface cleaning of the exposed reinforcement followed by rust removal, addition or lapping with new reinforcement to provide for any corroded reinforcement and reinstatement with new concrete. Since the repair work for limit state II involves additional work compared to that for limit state I, the repair costs for limit state II are higher than that for limit state I.

Since the costs are incurred at different times during the intended design life of the structure, all costs are discounted to present values to provide a uniform basis for comparison. For discounting, a discount rate of 2.02% is used. This is obtained from the following expression: (Fuller and Petersen ,1995)

$$d = \frac{1+D}{1+i} - 1 \quad (3.15)$$

where d is the real discount rate; D the nominal discount rate and i the rate of inflation. The nominal discount rate is obtained as 3.59% based on a seventeen year average between 1988 and 2005 of the Singapore Government Securities (SGS) 5-year bond yield (the choice of the seventeen year period is due to availability of data

obtained from the website of the Monetary Authority of Singapore). The average rate of inflation for the same period is obtained as 1.54% from Consumer Price Index values published by the Department of Statistics Singapore.

The discounted life cycle cost for each design alternative is hence determined and the optimum alternative is identified as the one with the minimum life cycle cost. For limit state II, the service life is defined in terms of the lower and upper bound and hence the timing of the repair activities vary corresponding to the lower and upper bound service life. To use a single timeline for timing of the repair works, the life cycle cost computations are hence carried out for the lower bound.

Chapter 4

Analysis of Model and Discussion

4.1 Illustration of Design Approach

For illustration of the design approach, the design of a simply supported beam is considered. The span of the beam is taken as 6m and the beam is considered to be subjected to a moment of 165 kN m and a shear force of 100 kN. The intended design life of the structure is taken as 100 years. Following the procedure described in the previous chapter, the design is carried out for the two limit states under all the exposure conditions. The results and analysis from the design output are presented in the following sections.

4.2 Optimum Design Solution

The design output for the optimum alternative corresponding to the minimum life cycle cost is presented in tables 4.1 (for limit state I) and 4.2 (for limit state II) for the 4 exposure environments. The important observations that can be noted are:

- ❖ As the severity of exposure environment increases from inland to submerged to coastal to tidal/splash, there is an increase in the specifications for cover to reinforcing steel and concrete compressive strength. This is seen for both the limit states. A change in the cover to reinforcing steel and concrete compressive strength has an influence on both the initial cost and the repair costs and hence both these variables have a significant influence on the life cycle cost. The variation of life cycle cost with cover and concrete strength is examined in detail in sections 4.3 and 4.4 respectively.

- ❖ The diameter of the reinforcing steel bars corresponding to the optimum design solutions is obtained as 25 mm for tidal/splash and coastal environments and 16 mm for submerged and inland environments. For limit state I, the diameter of the reinforcing steel affects the initial cost component of the life cycle cost. However in the case of limit state II, the diameter affects the time to cracking component of the service life and hence influences both the initial cost as well as the repair cost. In this design example, the optimum diameter for limit state II is obtained as a result of a trade-off between initial cost and repair costs. In this case, increasing the diameter from 16mm to 25mm increases the initial cost but also reduces the repair costs. For the submerged and inland exposure environments, the reduction in repair costs is greater than the increase in initial cost and hence the optimum diameter is obtained as 25mm; the reverse is true for tidal/splash and coastal environments and this hence gives the optimum diameter as 16mm. It is important to note that unlike other design variables, the influence of the diameter on the initial cost does not follow any regular pattern; it depends on the difference between the required and actual reinforcement area which varies for different bar diameters from one design situation to another.

- ❖ For both limit states I and II, the optimum section effective depth to width ratio is 2.0 for tidal/splash and coastal environments and 1.5 for submerged and inland environments. Based on this design example, a deeper section may hence seen to be preferred for severe exposure environments. Keeping all other variables constant, the effective depth of the section and the effective depth to width ratio affect only the initial cost; however they play a role albeit minimal in the overall optimization of the life cycle cost for the structure.

❖ The life cycle cost values for limit state II are lower than that for limit state I with the percentage difference between the limit states being 4.6%, 5.6%, 7.7% and 12.4% respectively for submerged, tidal/splash, coastal and inland exposure environments. Limit state I involves repairs to the structure at the end of the corrosion initiation period. The time to initiation indicates the onward point for the onset of corrosion; however there is no corrosion damage or distress to the structure at this point of time. On the other hand, limit state II is based on repairs at the end of the time to cracking over the time to initiation. Due to cracking of the concrete cover and propagation of corrosion of the reinforcement, there is a compromise in the aesthetic as well as protective functionality of the structure. Limit state I can hence be seen to be more conservative than limit state II. The lower life cycle cost values for limit state II compared to limit state I can therefore be seen as a trade-off for accepting a compromise in the aesthetic as well as protective functionality of the structure.

Table 4.1 Design output corresponding to optimum minimum life cycle cost alternative for Limit State I

| Design Parameter | Exposure Environment | | | |
|---|----------------------|----------------|---------|--------|
| | Submerged | Tidal / Splash | Coastal | Inland |
| Concrete Compressive Strength (N/mm ²) | 25 | 30 | 30 | 25 |
| Water to Cement Ratio | 0.57 | 0.52 | 0.52 | 0.57 |
| Cover to Reinforcing Steel (mm) | 80 | 90 | 75 | 60 |
| Effective Section Depth (mm) | 360 | 400 | 400 | 350 |
| Section Width (mm) | 240 | 200 | 200 | 235 |
| Effective Depth to Width Ratio | 1.5 | 2.0 | 2.0 | ~1.5 |
| Diameter of Reinforcing Steel (mm) | 25 | 16 | 16 | 25 |
| Tension Reinforcement Provided (mm ²) | 2454.3 | 2412.7 | 2412.7 | 2945.2 |
| Compression Reinforcement Provided (mm ²) | 981.7 | 201 | 201 | 981.7 |
| Tension Moment of Resistance (kN m) | 168.4 | 179.7 | 179.7 | 196.3 |
| Compression Moment of Resistance (kN m) | 193 | 166.5 | 166.5 | 181.6 |
| Area of Shear Link Reinforcement (mm ²) | 157.0 | 157.0 | 157.0 | 157.0 |

| | | | | |
|--|--------------|--------------|--------------|--------------|
| Spacing of Shear Link Reinforcement (mm) | 385 | 465 | 465 | 395 |
| Shear Resistance (kN) | 158.9 | 146.2 | 146.2 | 161.5 |
| Service Life (years) | 22.3 | 14.5 | 18.1 | 30.2 |
| Life Cycle Cost (\$\$) | 660.6 | 803.5 | 768.9 | 504.4 |

Table 4.2 Design output corresponding to optimum minimum life cycle cost alternative for Limit State II

| Design Parameter | Exposure Environment | | | |
|---|----------------------|----------------|--------------|--------------|
| | Submerged | Tidal / Splash | Coastal | Inland |
| Concrete Compressive Strength (N/mm ²) | 30 | 30 | 30 | 25 |
| Water to Cement Ratio | 0.52 | 0.52 | 0.52 | 0.57 |
| Cover to Reinforcing Steel (mm) | 70 | 95 | 80 | 65 |
| Effective Section Depth (mm) | 370 | 400 | 390 | 350 |
| Section Width (mm) | 250 | 200 | 195 | 235 |
| Effective Depth to Width Ratio | ~1.5 | 2.0 | 2.0 | ~1.5 |
| Diameter of Reinforcing Steel (mm) | 25 | 16 | 16 | 25 |
| Tension Reinforcement Provided (mm ²) | 2454.3 | 2412.7 | 2412.7 | 2945.2 |
| Compression Reinforcement Provided (mm ²) | 490.8 | 201.0 | 402.1 | 981.7 |
| Tension Moment of Resistance (kN m) | 170.2 | 179.7 | 177.0 | 196.3 |
| Compression Moment of Resistance (kN m) | 197.1 | 166.5 | 171.4 | 181.6 |
| Area of Shear Link Reinforcement (mm ²) | 157.0 | 157.0 | 157.0 | 157.0 |
| Spacing of Shear Link Reinforcement (mm) | 370 | 465 | 475 | 395 |
| Shear Resistance (kN) | 166.2 | 146.2 | 141.6 | 161.5 |
| Service Life for Initiation of Corrosion (years) | 25.4 | 16.2 | 19.3 | 36.7 |
| Service Life for Cracking of Cover (years) | 38.8 to 42.3 | 0.6 to 0.8 | 4.8 to 6.5 | 32.3 to 43.5 |
| Total Service Life (years) | 64.2 to 67.7 | 16.8 to 17.0 | 24.1 to 25.8 | 69.0 to 80.2 |
| Life Cycle Cost (\$\$) | 631.6 | 760.8 | 713.7 | 448.6 |

4.3 Variation of Reliability Index with Time

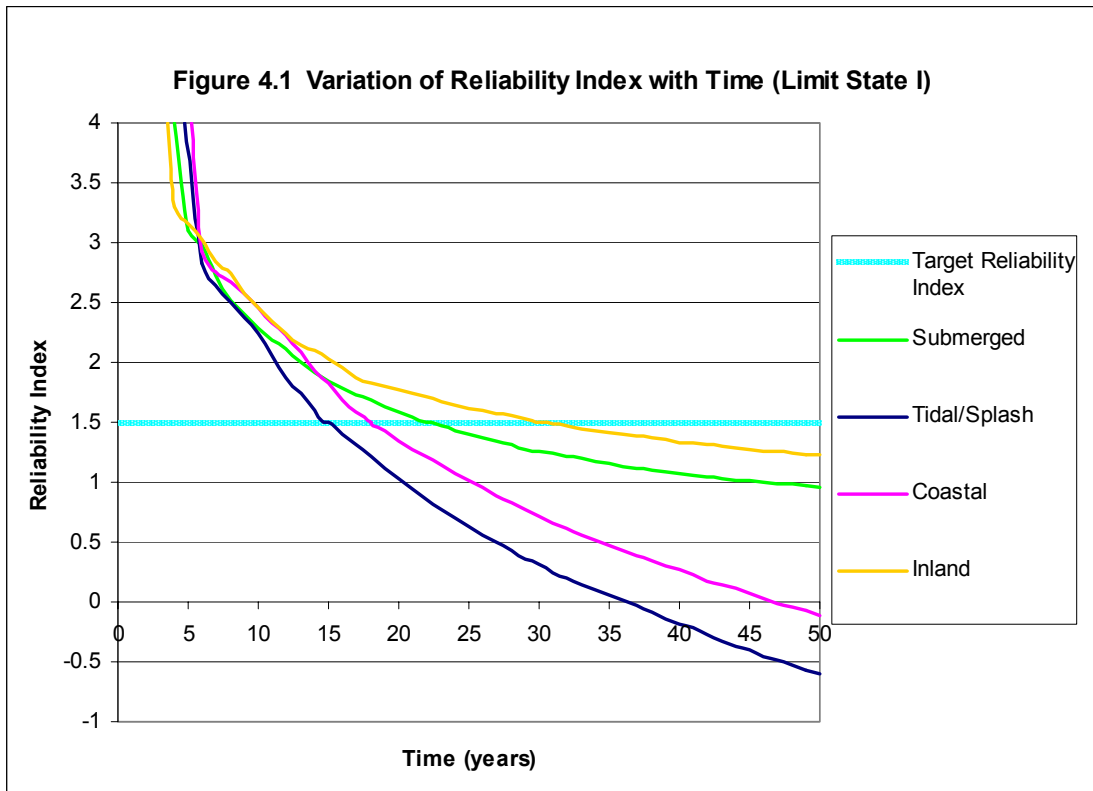
Figure 4.1 shows the variation in reliability index with time for limit state I and the optimum design solutions presented in table 4.1; these values are plotted over a period of 50 years. For limit state II, the variation in reliability index with time after initiation of corrosion are shown in figures 4.2 and 4.3 for the lower and upper bounds of the service life respectively; due to wide variation in values between the various

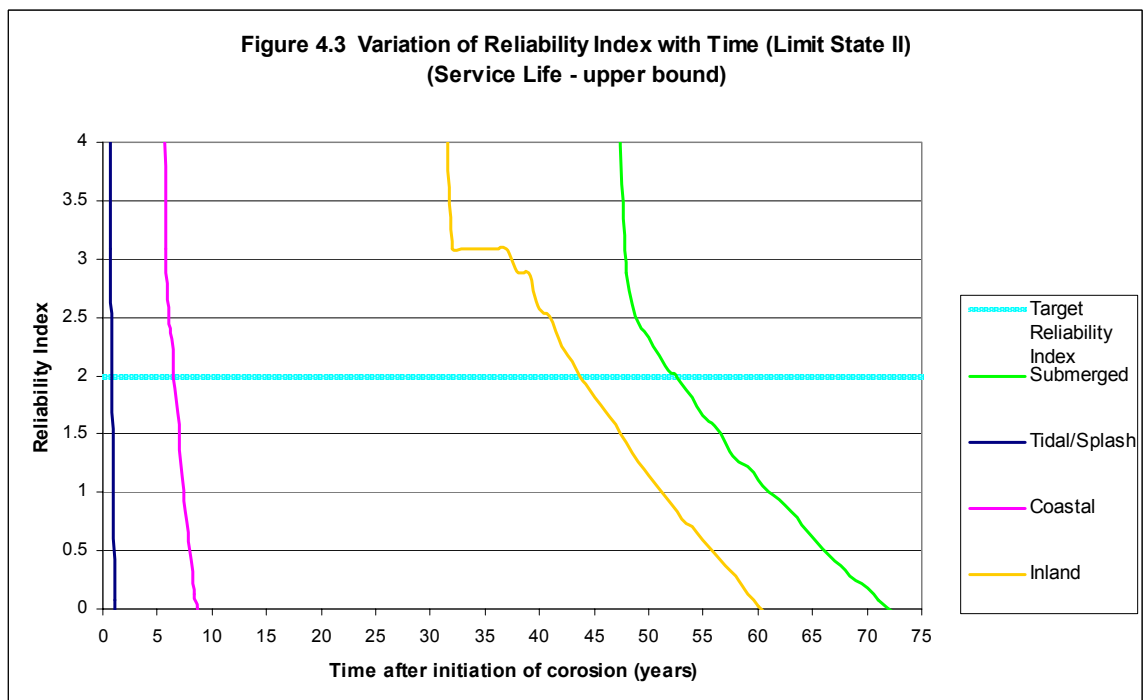
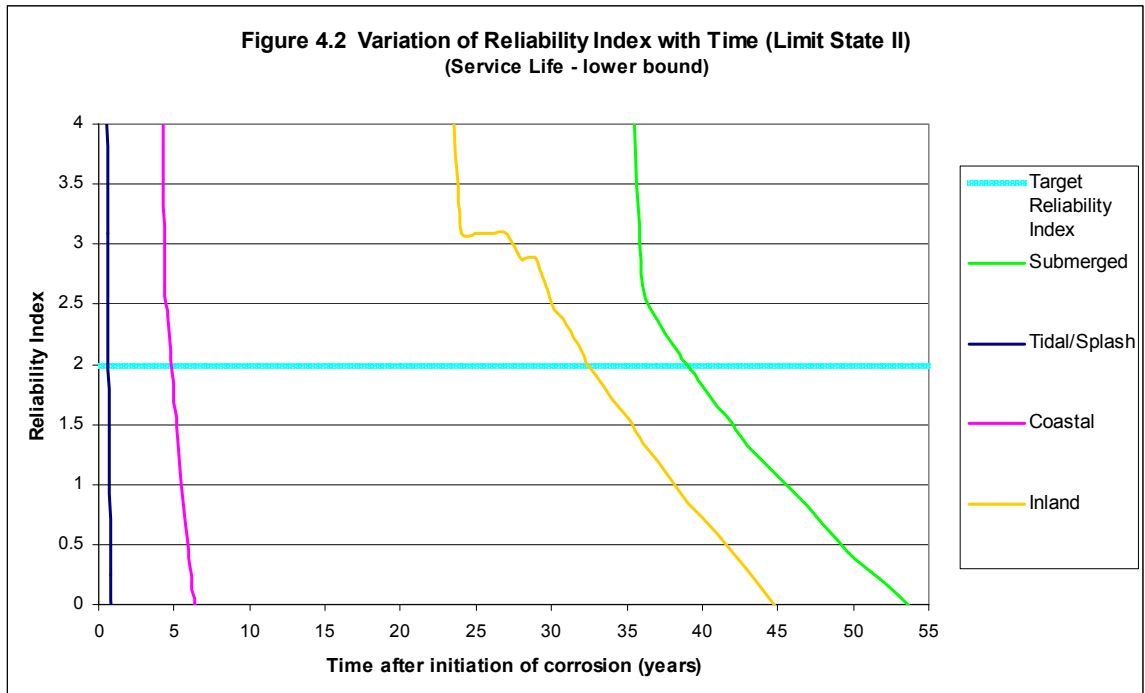
exposure environments, the values in this case are plotted till reliability index reaches zero. Initially when the probability of initiation of corrosion is zero, the reliability index value is infinity and the curves hence start from infinity. As time progresses, the probability of failure (defined by the event of initiation of corrosion) increases and hence the associated reliability index decreases. As explained in the previous chapter, the 2 limit states are defined by the exceedance of their respective target reliability index values (1.5 for limit state I and 2.0 for limit state II). The time upto which the reliability index remains greater than or equal to the target reliability index value is obtained as the service life; the service life values corresponding to the optimum design solutions are listed in tables 4.1 and 4.2 respectively for limit states I and II and can also be read off from the corresponding figures 4.1 and 4.2.

For limit state I, the rate of decrease of reliability index with time after the 5th and later is seen to be much higher for tidal/splash and coastal environments compared to the submerged and inland exposure environments. Further it is seen that the reliability index drops to 0 (which corresponds to a value of 0.5 for the probability of initiation of corrosion) around the 35th and 46th year respectively for tidal/splash and coastal environments.

For limit state II, the high difference in corrosion rates between the different exposure environments translates to a corresponding difference for the time to cracking. The service life due to cracking is understandably lower for the submerged environment as the lack of sufficient oxygen at level of the reinforcing steel results in very low corrosion rates. Figures 4.2 and 4.3 show the decrease in the reliability index with time till the reliability index drops to zero. This happens between 53 (lower bound) &

72 (upper bound) years, 0.8 & 1.1 years, 6.4 & 8.6 years and 44 & 60 years respectively for submerged, tidal/splash, coastal and inland exposure environments.





4.4 Sensitivity Analysis

In order to determine the influence of the various input variables – cover to reinforcing steel, concrete compressive strength, diameter of reinforcing steel,

effective depth of section and depth to width ratio, a sensitivity analysis is conducted and the percentage change in life cycle cost for a unit percentage change in input variable is determined. The absolute values of the minimum, average and maximum sensitivities obtained through this analysis for the different variables are given in tables 4.3 and 4.4 for limit states I and II respectively.

Table 4.3 Results from sensitivity analysis of life cycle cost for limit state I

| Variable | Exposure Environment | Sensitivity | | |
|-------------------------------|------------------------|----------------|----------------|----------------|
| | | Minimum | Average | Maximum |
| Cover to Reinforcing Steel | Submerged | 0.01044 | 0.36311 | 0.98296 |
| | Tidal/Splash | 0.00104 | 0.32355 | 1.01202 |
| | Coastal | 0.02091 | 0.33153 | 1.03797 |
| | Inland | 0.01013 | 0.34429 | 1.28712 |
| | OVERALL AVERAGE | 0.01063 | 0.34062 | 1.08002 |
| Concrete Compressive Strength | Submerged | 0.00632 | 0.19718 | 0.67657 |
| | Tidal/Splash | 0.00080 | 0.18314 | 0.52096 |
| | Coastal | 0.00045 | 0.16388 | 0.52541 |
| | Inland | 0.00145 | 0.18482 | 0.63375 |
| | OVERALL AVERAGE | 0.00225 | 0.18225 | 0.58917 |
| Diameter of Reinforcing Steel | Submerged | 0.00049 | 0.02401 | 0.04958 |
| | Tidal/Splash | 0.00036 | 0.02218 | 0.04831 |
| | Coastal | 0.00042 | 0.02437 | 0.04627 |
| | Inland | 0.00009 | 0.02118 | 0.04789 |
| | OVERALL AVERAGE | 0.00034 | 0.02294 | 0.04801 |
| Effective Section Depth | Submerged | 0.00074 | 0.01753 | 0.03532 |
| | Tidal/Splash | 0.00056 | 0.01868 | 0.03481 |
| | Coastal | 0.00070 | 0.01645 | 0.03450 |
| | Inland | 0.00067 | 0.01846 | 0.03597 |
| | OVERALL AVERAGE | 0.00067 | 0.01778 | 0.03515 |
| Depth to Width Ratio | Submerged | 0.00040 | 0.01318 | 0.03022 |
| | Tidal/Splash | 0.00050 | 0.01495 | 0.02960 |
| | Coastal | 0.00055 | 0.01506 | 0.03076 |
| | Inland | 0.00048 | 0.01398 | 0.03294 |
| | OVERALL AVERAGE | 0.00048 | 0.01429 | 0.03088 |

Table 4.4 Results from sensitivity analysis of life cycle cost for limit state II

| Variable | Exposure Environment | Sensitivity | | |
|-------------------------------|------------------------|----------------|----------------|----------------|
| | | Minimum | Average | Maximum |
| Cover to Reinforcing Steel | Submerged | 0.00914 | 0.34441 | 0.90020 |
| | Tidal/Splash | 0.01091 | 0.36940 | 0.99055 |
| | Coastal | 0.00884 | 0.37564 | 0.95531 |
| | Inland | 0.00780 | 0.35214 | 1.26109 |
| | OVERALL AVERAGE | 0.00917 | 0.36040 | 1.02679 |
| Concrete Compressive Strength | Submerged | 0.00083 | 0.18312 | 0.52096 |
| | Tidal/Splash | 0.00045 | 0.16409 | 0.52541 |
| | Coastal | 0.00065 | 0.16878 | 0.53083 |
| | Inland | 0.00155 | 0.20056 | 0.64526 |
| | OVERALL AVERAGE | 0.00087 | 0.17914 | 0.55561 |
| Diameter of Reinforcing Steel | Submerged | 0.00160 | 0.03528 | 0.09304 |
| | Tidal/Splash | 0.00174 | 0.03832 | 0.09906 |
| | Coastal | 0.00169 | 0.03967 | 0.09973 |
| | Inland | 0.00156 | 0.03656 | 0.09039 |
| | OVERALL AVERAGE | 0.00165 | 0.03746 | 0.09555 |
| Effective Section Depth | Submerged | 0.00049 | 0.01821 | 0.03158 |
| | Tidal/Splash | 0.00041 | 0.01562 | 0.03305 |
| | Coastal | 0.00073 | 0.01578 | 0.03165 |
| | Inland | 0.00052 | 0.01758 | 0.03651 |
| | OVERALL AVERAGE | 0.00054 | 0.01680 | 0.03320 |
| Depth to Width Ratio | Submerged | 0.00058 | 0.01480 | 0.03004 |
| | Tidal/Splash | 0.00032 | 0.01597 | 0.03103 |
| | Coastal | 0.00044 | 0.01521 | 0.03238 |
| | Inland | 0.00066 | 0.01521 | 0.03147 |
| | OVERALL AVERAGE | 0.00050 | 0.01530 | 0.03123 |

Tables 4.3 and 4.4 show that the cover to reinforcing steel followed by concrete compressive strength have the greatest influence on the life cycle cost; the two variables influence both the initial cost as well as the repair costs. For a unit percentage change in the value of cover, there is an average change of 0.3406% (for limit state I) and 0.3604% (for limit state II) in the life cycle cost whereas a unit percentage change in the value of concrete compressive strength leads to an average change of 0.1823% (for limit state I) and 0.1723% (for limit state II) in the life cycle cost. The higher sensitivity for the cover compared to concrete strength can be linked

to its direct and hence greater influence on the chloride concentration level obtained from the diffusion equation. On the other hand, the influence of concrete strength on chloride concentration level is indirect – the strength affects the water cement ratio and permeability of the concrete and this, in turn, influences the diffusion coefficient of the concrete and the chloride concentration level.

The average sensitivity values for the other variables – diameter of reinforcing steel, effective section depth and depth to width ratio are 0.0229%, 0.0178% & 0.0143% (for limit state I) and 0.0375%, 0.0168% & 0.01530% (for limit state II) respectively. While the effective section and the depth to width ratio only influence the initial cost for both the limit states, the choice of diameter of the reinforcing steel influences the initial cost for limit state I and the initial cost as well as repair costs for limit state II; this has been explained in section 4.2. The sensitivities for these 3 variables are seen to be minimal compared to the cover and compressive strength. Hence the influence of the cover and concrete compressive strength on the life cycle cost is explored further through a detailed analysis in the following sections.

The sensitivity values for all variables across the four exposure environments are seen to be quite close to one another without much variation. Further the sensitivity values for the two limit states are seen to be quite close to one another for all the variables except the diameter of reinforcing steel. For the diameter, the 63% increase in the sensitivity value for limit state II over limit state I is because the choice of diameter of the reinforcing steel influences only the initial cost for limit state I whereas it affects both the initial cost as well as repair costs for limit state II.

4.5 Variation of Life Cycle Cost with Cover

The variation of life cycle cost with the cover to reinforcing steel and concrete compressive strength is shown in figures 4.4 to 4.7 for limit state I and figures 4.8 to 4.11 for limit state II. For a fixed value of concrete compressive strength, it is seen that the life cycle cost decreases with the cover to reach a minimum value beyond which it starts increasing. Till this minimum value is reached, providing a higher cover increases the initial cost but also reduces the lifetime repair costs. The use of higher cover increases the depth of penetration for chloride attack; this increases the service life of the structure and leads to a reduction in the number of repair activities during the intended design life of the structure. However beyond a certain value of the cover, it remains no longer economical to prolong the service life by increasing the initial cost. From this point onward, an increase in the cover only leads to an increase in the life cycle cost as the increase in the initial cost now becomes greater than the savings offered by the reduced overall repair cost.

This same trend is obtained for all the 4 exposure environment conditions. For a fixed compressive strength, the optimum cover corresponding to the minimum life cycle cost increases with an increase in severity of the exposure environment from inland to tidal / splash exposure conditions. On the other hand, for a fixed exposure condition, the optimum cover corresponding to the minimum life cycle cost decreases with an increase in compressive strength of the concrete.

For fixed values of concrete compressive strength, the optimum cover values corresponding to the minimum life cycle cost are tabulated in tables 4.5 and 4.6 for the 2 limit states. The knowledge of these values is particularly useful in design

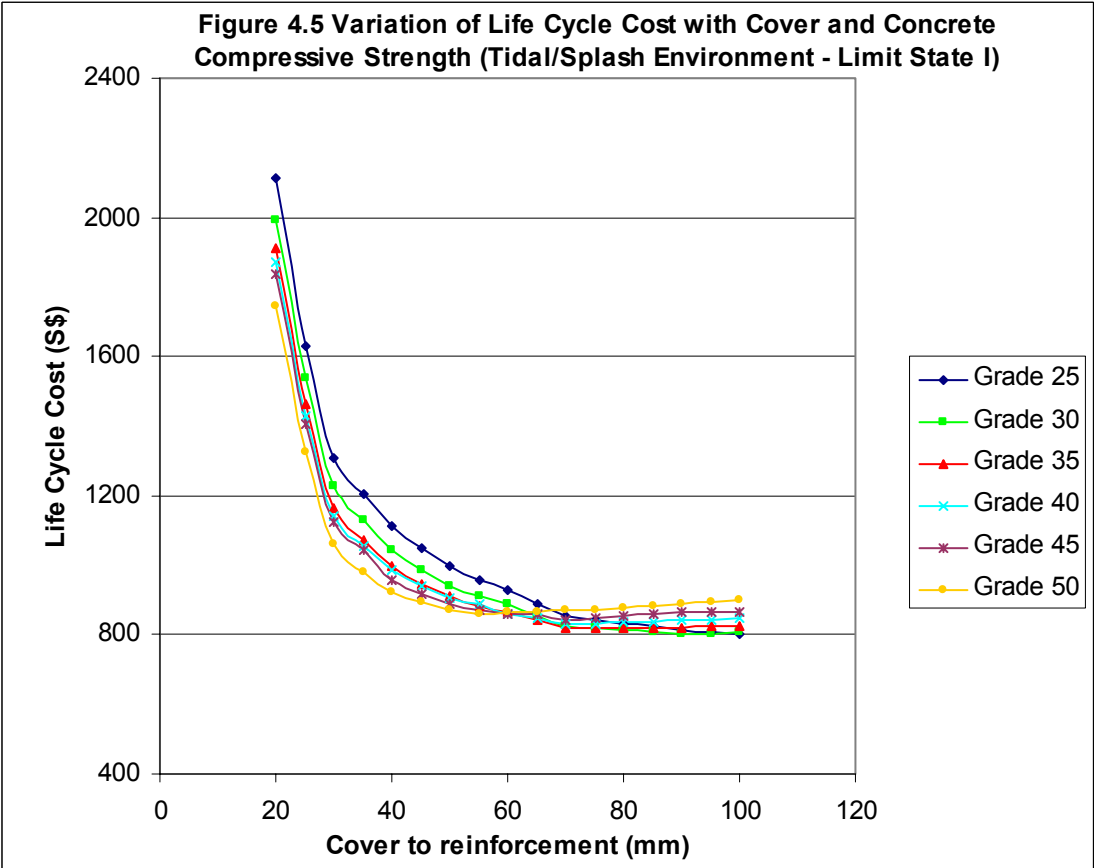
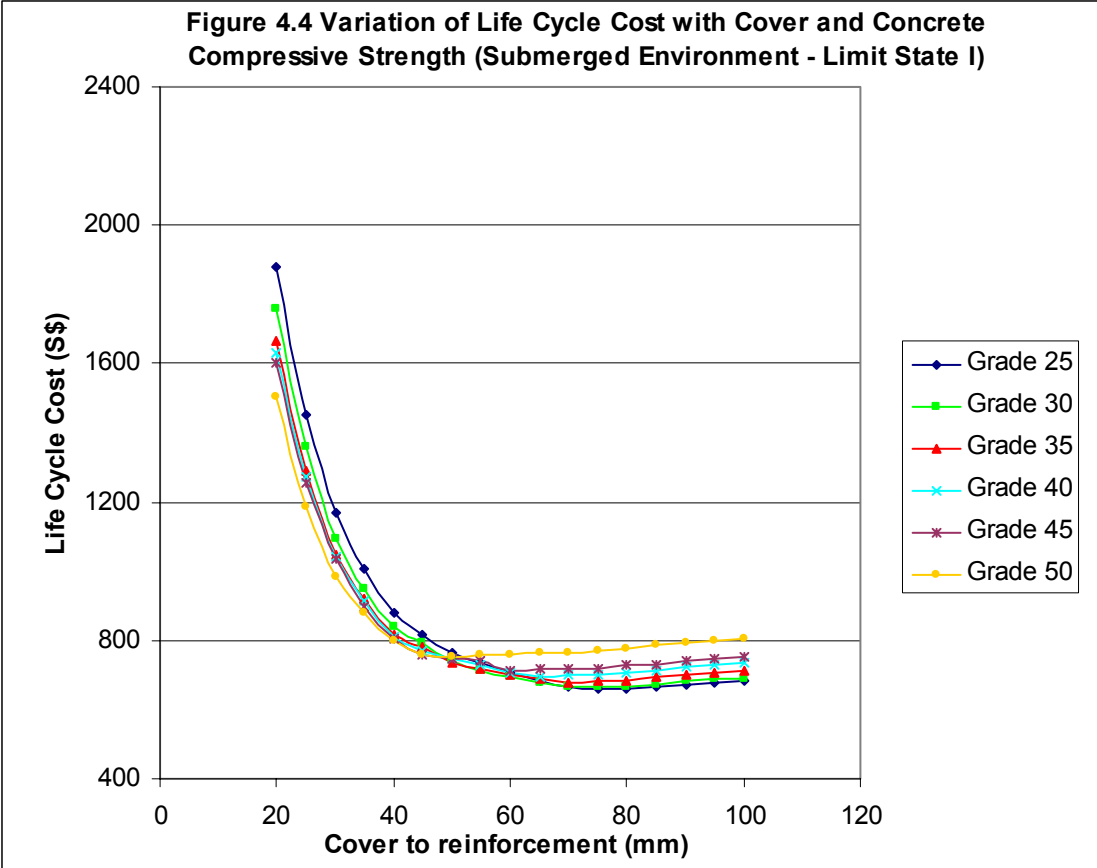
situations when there are constraints in the form of the use of concrete of only certain compressive strengths and/or certain cover values.

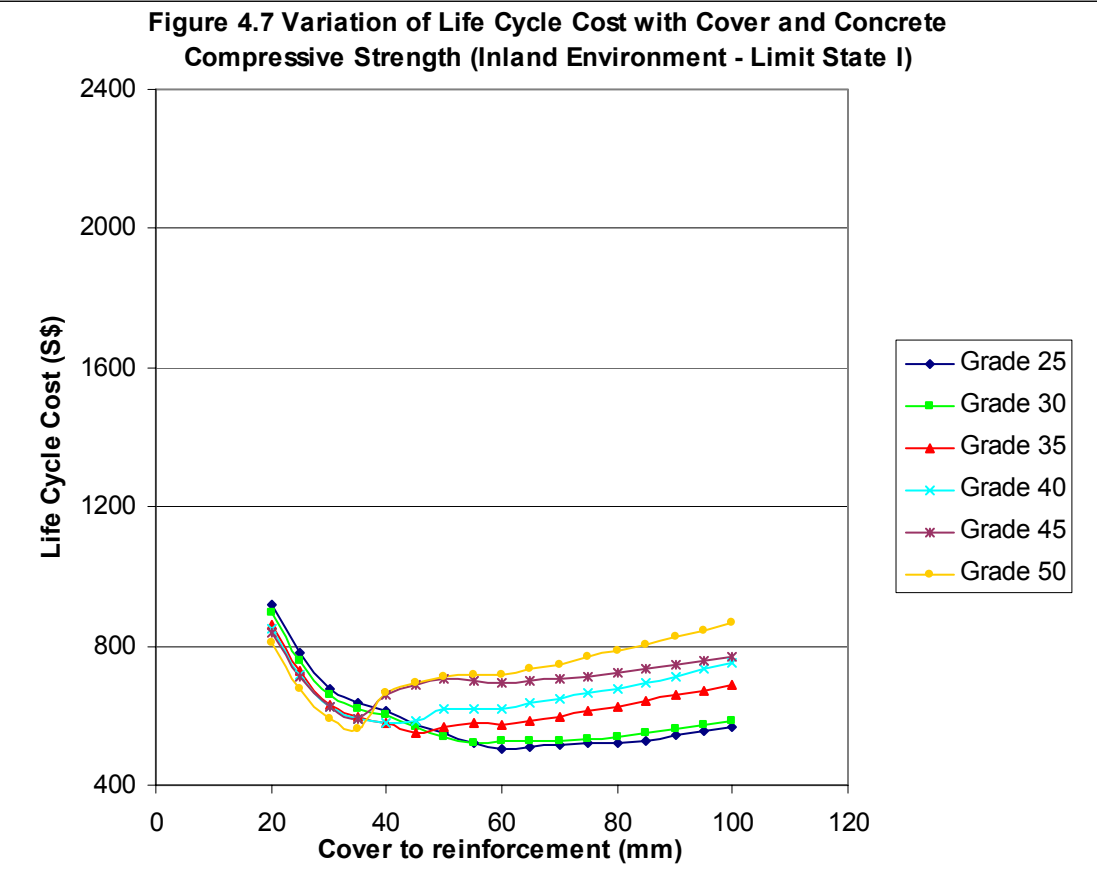
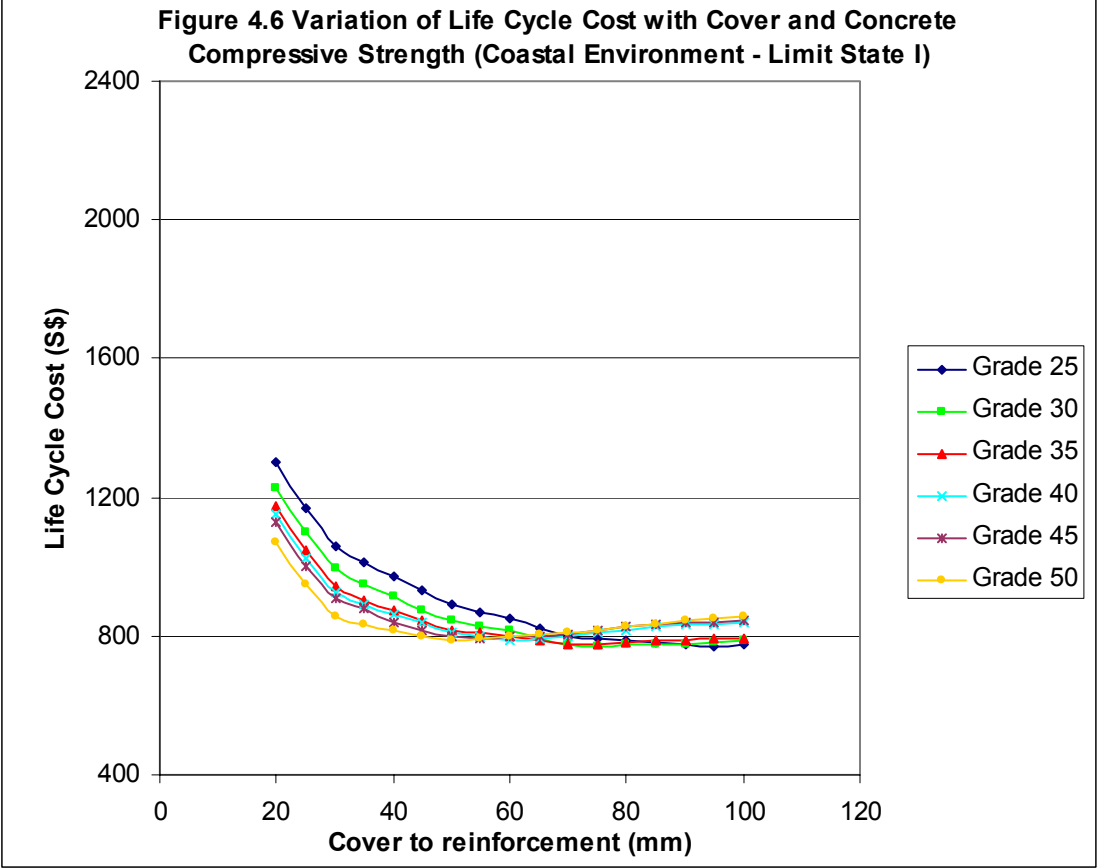
Table 4.5 Optimum cover for a given concrete compressive strength – Limit State I

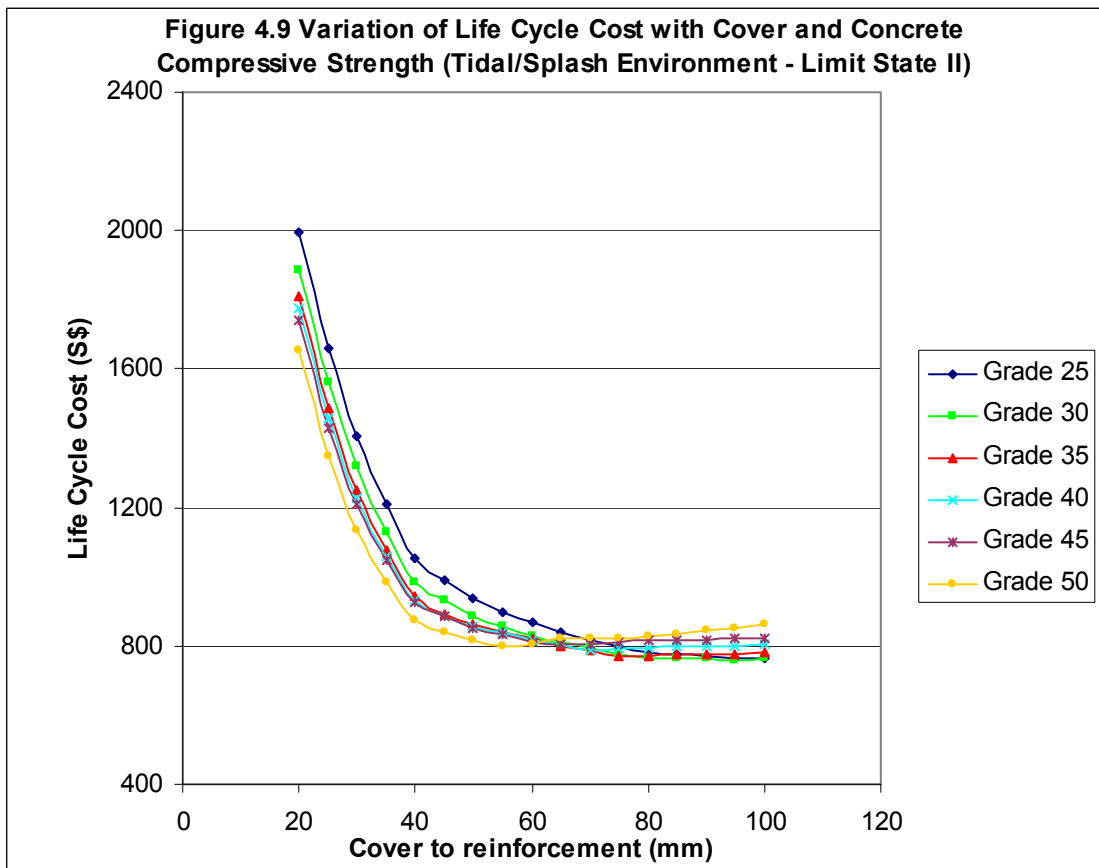
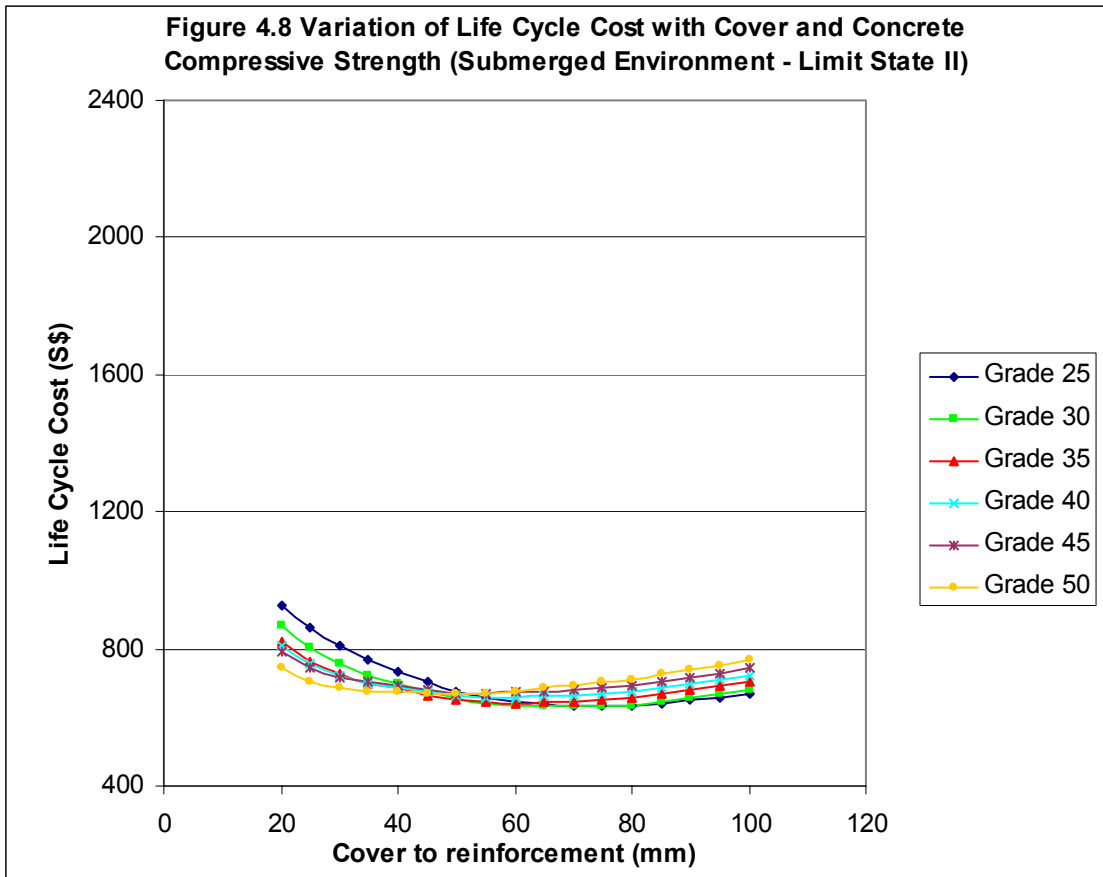
| Concrete Compressive Strength (N/mm ²) | Optimum Cover (mm) | | | |
|--|--------------------|--------------|---------|--------|
| | Submerged | Tidal/Splash | Coastal | Inland |
| 25 | 80 | 100 | 95 | 60 |
| 30 | 75 | 90 | 75 | 55 |
| 35 | 70 | 75 | 70 | 45 |
| 40 | 65 | 70 | 60 | 40 |
| 45 | 60 | 60 | 55 | 35 |
| 50 | 50 | 55 | 50 | 35 |

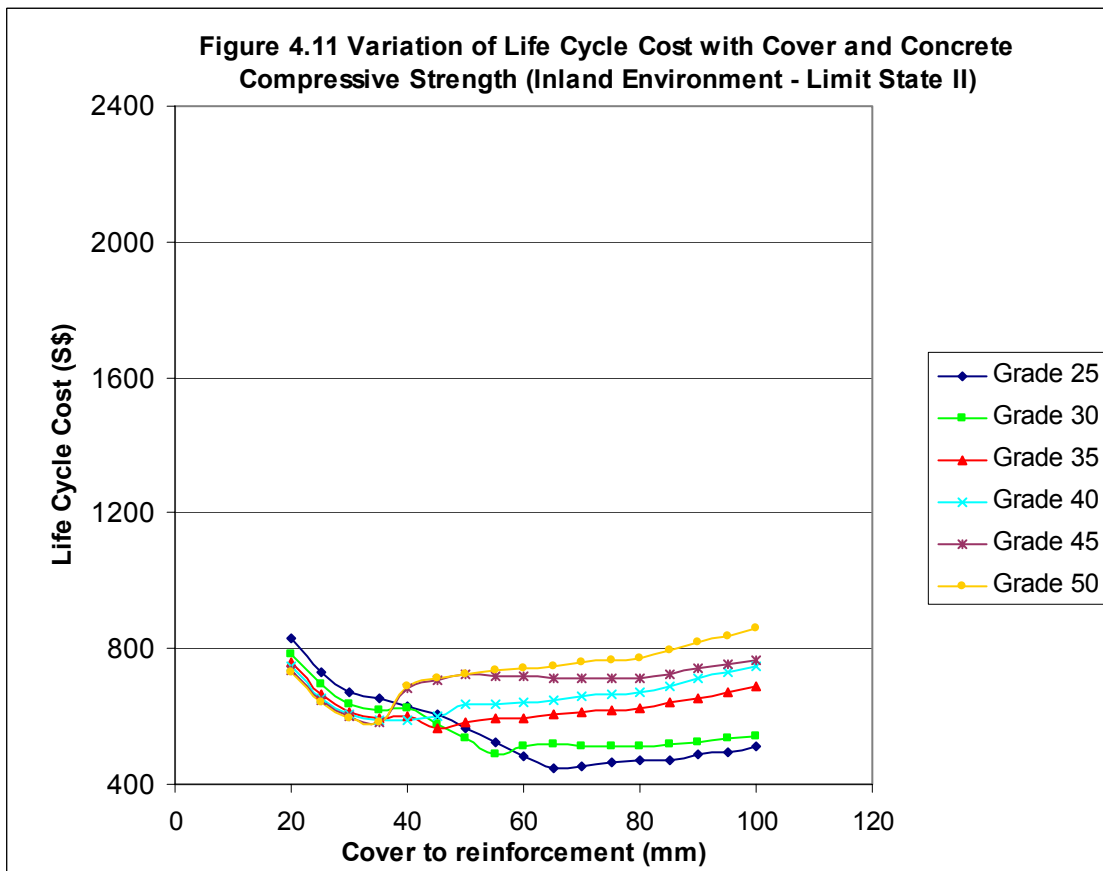
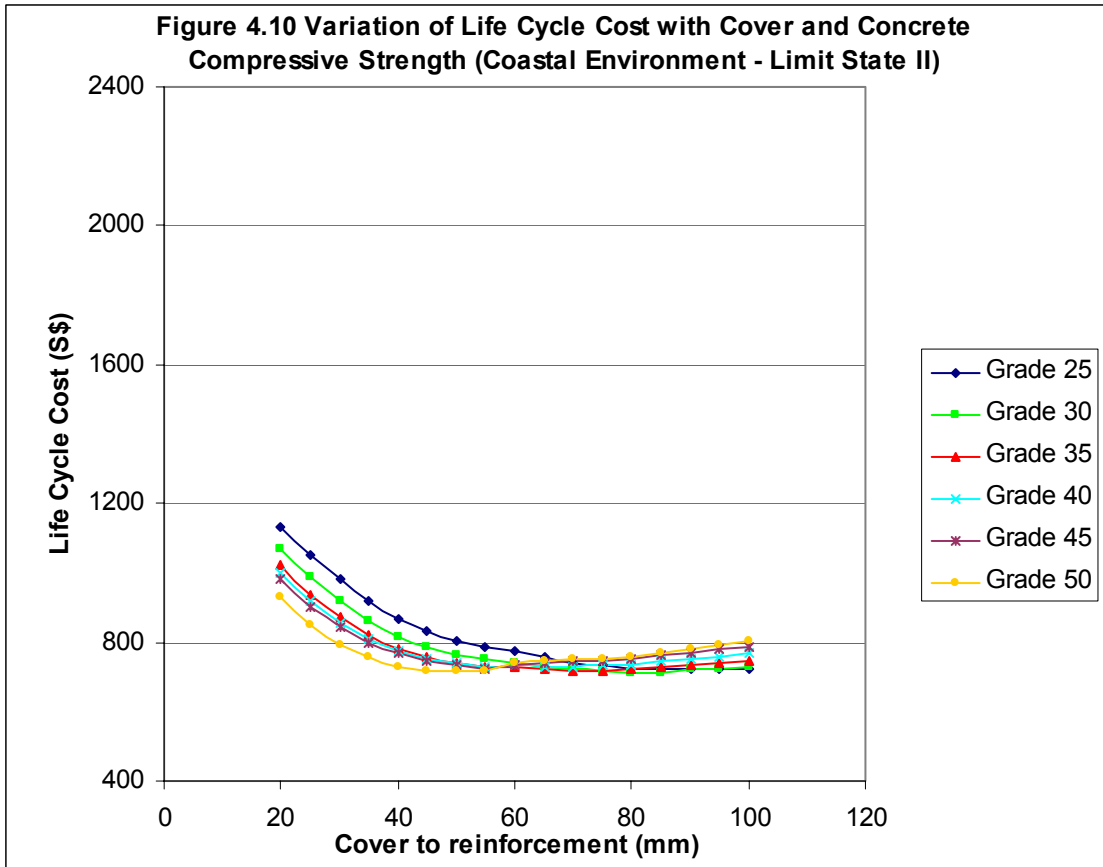
Table 4.6 Optimum cover for a given concrete compressive strength – Limit State II

| Concrete Compressive Strength (N/mm ²) | Optimum Cover (mm) | | | |
|--|--------------------|--------------|---------|--------|
| | Submerged | Tidal/Splash | Coastal | Inland |
| 25 | 80 | 100 | 95 | 65 |
| 30 | 70 | 95 | 80 | 55 |
| 35 | 60 | 75 | 75 | 45 |
| 40 | 55 | 70 | 65 | 40 |
| 45 | 50 | 65 | 55 | 35 |
| 50 | 45 | 55 | 50 | 35 |









4.6 Variation of Life Cycle Cost with Concrete Compressive Strength

The variation of life cycle cost with the cover to reinforcing steel and concrete compressive strength is shown in figures 4.4 to 4.7 for limit state I and figures 4.8 to 4.11 for limit state II. It is seen that the variation of life cycle cost with concrete compressive strength depends on the cover value. For lower cover values, there is generally a decrease in life cycle cost with an increase in concrete compressive strength. In such cases, the use of higher strength concrete leads to a reduction in the water-to-cement ratio of the concrete and hence reduces the permeability and the diffusion coefficient of the concrete. This, in turn, increases the service life of the concrete and provides monetary savings through reduced repair costs. However for higher values of the cover, the life cycle cost decreases with increasing concrete compressive strength to reach a certain minimum value beyond which it starts increasing. The strength at which this minimum value is attained generally decreases as the cover value increases. At higher cover values, the use of concrete of relatively lower strength hence becomes more economical. The use of the higher cover value in such cases itself gives a longer service life and provides the necessary savings in repair costs and life cycle costs and hence it becomes uneconomical to use high strength concrete.

This same trend is obtained for all the 4 exposure environment conditions. For a fixed cover, the optimum concrete strength corresponding to the minimum life cycle cost generally increases (or at least remains same) with an increase in severity of the exposure environment from inland to tidal / splash exposure conditions. For fixed values of cover, the optimum concrete compressive strength values corresponding to the minimum life cycle cost are tabulated in tables 4.7 and 4.8 for the 2 limit states.

The knowledge of these values is particularly useful in design situations when there are constraints in the form of the use of concrete of only certain compressive strengths and/or certain cover values.

Table 4.7 Optimum concrete compressive strength for a given cover – Limit State I

| Cover (mm) | Optimum Concrete Compressive Strength (N/mm ²) | | | |
|------------|--|--------------|---------|--------|
| | Submerged | Tidal/Splash | Coastal | Inland |
| 20 | 50 | 50 | 50 | 50 |
| 25 | 50 | 50 | 50 | 50 |
| 30 | 50 | 50 | 50 | 50 |
| 35 | 50 | 50 | 50 | 50 |
| 40 | 50 | 50 | 50 | 40 |
| 45 | 45 | 50 | 50 | 35 |
| 50 | 35 | 50 | 50 | 30 |
| 55 | 30 | 50 | 50 | 25 |
| 60 | 30 | 45 | 40 | 25 |
| 65 | 30 | 35 | 35 | 25 |
| 70 | 30 | 35 | 35 | 25 |
| 75 | 25 | 35 | 30 | 25 |
| 80 | 25 | 30 | 30 | 25 |
| 85 | 25 | 30 | 30 | 25 |
| 90 | 25 | 30 | 30 | 25 |
| 95 | 25 | 30 | 25 | 25 |
| 100 | 25 | 25 | 25 | 25 |

Table 4.8 Optimum concrete compressive strength for a given cover – Limit State II

| Cover (mm) | Optimum Concrete Compressive Strength (N/mm ²) | | | |
|------------|--|--------------|---------|--------|
| | Submerged | Tidal/Splash | Coastal | Inland |
| 20 | 50 | 50 | 50 | 50 |
| 25 | 50 | 50 | 50 | 50 |
| 30 | 50 | 50 | 50 | 50 |
| 35 | 50 | 50 | 50 | 50 |
| 40 | 50 | 50 | 50 | 40 |
| 45 | 35 | 50 | 50 | 35 |
| 50 | 35 | 50 | 50 | 30 |
| 55 | 30 | 50 | 50 | 30 |
| 60 | 30 | 50 | 35 | 25 |
| 65 | 30 | 35 | 35 | 25 |
| 70 | 30 | 35 | 35 | 25 |
| 75 | 25 | 35 | 30 | 25 |
| 80 | 25 | 30 | 30 | 25 |
| 85 | 25 | 30 | 30 | 25 |
| 90 | 25 | 30 | 30 | 25 |
| 95 | 25 | 30 | 25 | 25 |
| 100 | 25 | 30 | 25 | 25 |

4.7 Comparison with Codal Specifications

BS 8500-1:2002 provides limiting values for composition and properties of concrete for different exposure classes related to environmental conditions. In particular, exposure category XS deals with corrosion of reinforcement induced by chlorides from sea water. As discussed in the previous chapter, the exposure environment categorization used in this study is derived partly based on the different exposure classes defined under this category XS.

In BS 8500-1:2002, the design objective in specifying the properties of concrete is to ensure that the concrete remains in service during its “intended working life”. The limiting values in BS 8500-1:2002 for exposure class XS are provided for an “intended working life” of at least 50 years. The “intended working life” is considered to be the same as “design working life” which is defined in BS EN 1990 as the “assumed period for which a structure or part of it is to be used for its intended purpose with anticipated maintenance but without major repair being necessary”. Although “major repair” in this definition is not elaborated further, the repair of deteriorated reinforced concrete involving removal of chloride contaminated concrete and/or cleaning and addition of reinforcement can be reasonably inferred to fall under this category. Hence the design basis in this case for achieving the design objective is to specify concrete properties (mainly concrete strength and cover) that i) prolong the service life to an extent where repairs of the kind mentioned above can be avoided altogether during the design working life and ii) require only minor routine maintenance (such as surface cleaning, patching surface cracks, etc.) to be carried out.

The objective of the life cycle cost based design developed in this research remains the same as above – to ensure that the concrete remains in service during the intended design life/intended working life. However unlike BS 8500-1:2002, the design basis for achieving these objectives is by specifying the properties of concrete to achieve a minimum life cycle cost. This is done by i) estimating the service life of concrete based on exceedance of a defined limit state and ii) carrying out repair of the structure during the intended design life at time points equal to this known service life in order to restore the structure to an acceptable level of reliability for the specified limit state.

Table 4.9 Concrete cover and strength specifications from LCC Design and BS 8500

| Exposure Environment in LCC Design | Optimum Values from LCC Design for Limit State I | | Nearest BS 8500-1:2002 Exposure Class | Limiting Values in BS 8500-1:2002 | |
|------------------------------------|--|---------------------------------|---------------------------------------|--|---------------------------------|
| | Concrete Compressive Strength (N/mm ²) | Cover to Reinforcing Steel (mm) | | Concrete Compressive Strength (N/mm ²) | Cover to Reinforcing Steel (mm) |
| Submerged | 25 | 80 | XS2 | 35 40 50 | 65 60 55 |
| Tidal/Splash | 30 | 90 | XS3 | 50 | 75 |
| Coastal | 30 | 75 | XS3 XS1 | 50 45 50 | 75 65 60 |
| Inland | 25 | 60 | XS1 | 45 50 | 65 60 |

Table 4.10 Percentage difference in life cycle cost between LCC Design and BS 8500

| Exposure Environment in LCC Design | Nearest BS 8500-1:2002 Exposure Class | Percentage increase in life cycle cost for providing BS 8500-1:2002 specification over life cycle cost of optimum LCC design alternative for Limit State I |
|------------------------------------|---------------------------------------|--|
| Submerged | XS2 | 4.1 6.5 14.7 |
| Tidal/Splash | XS3 | 8.6 |
| Coastal | XS3 XS1 | 6.2 3.8 3.9 |
| Inland | XS1 | 38.5 42.7 |

The specifications for concrete strength and cover as obtained from the LCC design and BS 8500-1:2002 respectively are given in table 4.9. The nominal cover values given in BS 8500-1:2002 comprise of a specified minimum cover plus a tolerance to accommodate fixing precision; the typical range of tolerance values is suggested as 5mm to 15mm. Further it is suggested in BS 8500-1:2002 to increase the nominal cover values by 15 mm in order to use the values to achieve a working life of at least 100 years. Hence the nominal cover values listed are the minimum cover values plus 25mm (10mm as the average tolerance and 15mm for achieving a working life of at least 100 years).

Although the design basis for the two specification approaches are different as explained earlier, the same design objective for the two approaches provides some basis for a comparison. A comparison of the values given in table 4.9 shows that the optimum cover obtained from the LCC design is higher or at least the same as that specified in BS 8500-1:2002. On the other hand, the optimum concrete compressive strength from the LCC design is seen to be much lower than that specified in BS 8500-1:2002. Based on the LCC design, using a relatively low strength concrete and providing a higher cover is seen to be more economical than using concrete of higher strength and providing a relatively lower cover. The design basis of the BS 8500-1:2002 specifications essentially require that the service life (or the time at the end of which repairs are required to restore the structure to an acceptable level of reliability) is sufficiently close to the design working life so as to avoid the structural repairs. This is achieved by specifying concrete of much higher strengths in addition to the use of moderately high cover values. Clearly the BS 8500-1:2002 specification approach is more conservative in nature.

Table 4.10 lists the costs increase resulting from providing the BS 8500-1:2002 specifications compared to the optimum LCC design specifications; the values range from 3.8% to 14.1% for the submerged, tidal/splash and coastal environments. This difference can be considered as an “excess durability cost” that needs to be incurred to achieve the specifications of the more conservative BS 8500-1:2002 approach over the LCC design approach developed in this research. The much higher percentage difference in life cycle cost values for inland environment is due to an absence of a perfect matching between the exposure environments for the LCC design and the exposure classes in BS 8500-1:2002 with the result that exposure class XS1 is greater in severity for the inland environment.

Chapter 5

CONCLUSION

A life cycle cost based design procedure for the design of reinforced concrete structural elements has been developed in this research. The design procedure attempts to integrate issues of structural performance and durability together with economic cost optimization into the structural design process. The evaluation of structural performance and durability is made on the basis of determination of the service life of reinforced concrete. The service life is determined based on the concept of exceedance of defined limit states that is commonly used in structural design. Two limit states relevant to corrosion of reinforcement are used – limit state I is based on initiation of corrosion and the limit state II is based on initiation of corrosion and cracking of the concrete cover. The service life hence determined decides the magnitude and timing of the future costs to be incurred during the design life of the structure. The life cycle cost is then determined based on discounting of the initial construction cost and the future repair costs to present values to ensure a time-consistent comparison of costs. Repeating the life cycle cost computations for a range of input variables and parameters leads to the determination of the optimum design alternative with the least overall cost or life cycle cost.

A detailed analysis of a design example using this procedure was carried out. The main inferences that can be drawn from this analysis are:

- ❖ The cover to reinforcing steel followed by concrete compressive strength are seen to have the greatest influence on the life cycle cost. The two variables influence both the initial cost as well as the repair costs. The greater influence

for the cover compared to concrete strength can be linked to its direct and hence greater influence on the chloride concentration level obtained from the diffusion equation. On the other hand, the influence of concrete strength on chloride concentration level is only indirect – the strength affects the water cement ratio and permeability of the concrete and this, in turn, influences the diffusion coefficient of the concrete and the chloride concentration level.

- ❖ For a fixed value of concrete compressive strength, it is seen that the life cycle cost decreases with the cover to reach a minimum optimum value beyond which it starts increasing. This optimum cover corresponding to the minimum life cycle cost increases with an increase in severity of the exposure environment. For a fixed exposure condition, the optimum cover corresponding to the minimum life cycle cost decreases with an increase in compressive strength of the concrete.
- ❖ For lower cover values, there is generally a decrease in life cycle cost with an increase in concrete compressive strength. However for higher values of the cover, the life cycle cost decreases with an increase in the concrete compressive strength to reach a certain minimum value beyond which it starts increasing. The strength at which this minimum value is attained generally decreases as the cover value increases. For a fixed cover, the optimum concrete strength corresponding to the minimum life cycle cost generally increases (or at least remains same) with an increase in severity of the exposure environment.
- ❖ For limit state I (based on initiation of corrosion), the diameter of the reinforcing

steel only affects the initial cost component of the life cycle cost. However in the case of limit state II (based on initiation of corrosion and cracking of the concrete cover), the diameter affects the time to cracking component of the service life and hence influences both the initial cost as well as the repair cost. The optimum diameter for limit state II is hence obtained as a result of a trade-off between reduction in repair costs and increase in initial cost.

- ❖ The effective depth of the section and the effective depth to width ratio are seen to affect only the initial cost and hence their influence on the life cycle is minimal; however they play a role in the overall optimization of the life cycle cost for the structure.
- ❖ The life cycle cost values for limit state II (based on initiation of corrosion and cracking of the concrete cover) are lower than that for limit state I (based on initiation of corrosion) by 4% to 13% depending on the exposure environment. Limit state I is more conservative than limit state II. The lower life cycle cost values for limit state II compared to limit state I are hence obtained as a trade-off for accepting a compromise in the aesthetic as well as protective functionality of the structure.
- ❖ There is a 3.8% to 14.1% increase in life cycle costs for providing the concrete specifications specified in the BS 8500-1:2002 (the British Standard for concrete specifications) compared to the optimum specifications from the developed LCC design approach. This difference can be considered as an “excess durability cost” that needs to be incurred to provide the specifications of the more

conservative BS 8500-1:2002 approach over the LCC design approach developed in this research.

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Appendix A

Derivation of the Solutions for the Diffusion Equation

A.1 Constant Surface Chloride Concentration

As discussed in section 2.2.3 of chapter 2, the equation for modelling the ingress of chlorides into reinforced concrete in one direction for a constant diffusion coefficient can be written as:

$$\frac{\partial C}{\partial t} = D_c \frac{\partial^2 C}{\partial x^2}, x > 0, t > 0 \quad (\text{A.1})$$

where

C = concentration of chloride at depth x and time t

x = the depth from the surface

D_c = the diffusion coefficient

t = time

For the case where there is a constant surface chloride concentration, the boundary condition is:

$$C = C_s; x = 0, t > 0 \quad (\text{A.2})$$

The method of Laplace transformation is used to solve the differential equation (A.1).

In general, the Laplace transform $\bar{f}(p)$ of a function $f(t)$ can be written as:

$$\bar{f}(p) = \int_0^{\infty} e^{-pt} f(t) dt \quad (\text{A.3})$$

The Laplace transform of equation (A.1) can hence be obtained by multiplying both sides of the equation by e^{-pt} and integrating with respect to t from 0 to ∞ which gives:

$$\int_0^{\infty} e^{-pt} \frac{\partial^2 C}{\partial x^2} dt - \frac{1}{D_c} \int_0^{\infty} e^{-pt} \frac{\partial C}{\partial t} dt = 0 \quad (\text{A.4})$$

Assuming that the orders of differentiation and integration can be interchanged (Crank, 1956), the first term on the left hand side of equation (A.4) can be written as:

$$\int_0^{\infty} e^{-pt} \frac{\partial^2 C}{\partial x^2} dt = \frac{\partial^2}{\partial x^2} \int_0^{\infty} C e^{-pt} dt = \frac{\partial^2 \bar{C}}{\partial x^2} \quad (\text{A.5})$$

Integrating by parts the second term on the left hand side of equation (A.4),

$$\int_0^{\infty} e^{-pt} \frac{\partial C}{\partial t} dt = [C e^{-pt}]_0^{\infty} + p \int_0^{\infty} C e^{-pt} dt = p \bar{C} \quad (\text{A.6})$$

Hence from equations (A.5) and (A.6), equation (A.4) can be re-written as:

$$D_c \frac{\partial^2 \bar{C}}{\partial x^2} = p \bar{C} \quad (\text{A.7})$$

By treating the boundary condition in the same manner, equation (A.2) can be obtained as:

$$\bar{C} = \int_0^{\infty} C_s e^{-pt} dt = \frac{C_s}{p}; x = 0 \quad (\text{A.8})$$

Hence the application of the Laplace transformation reduces the partial differential equation (A.1) to an ordinary differential equation (A.7). The solution of equation (A.7) satisfying the transformed boundary condition (A.8) and for which \bar{C} remains finite as x approaches infinity (Crank, 1956) is:

$$\bar{C} = \frac{C_S}{p} e^{-\sqrt{\frac{p}{D_C}}x} \quad (\text{A.9})$$

The inverse Laplace transformation is now applied to transform \bar{C} to C in order to obtain the final solution of the differential equation (A.1) satisfying the boundary condition (A.2). The function whose Laplace transform is given by equation (A.9) can be obtained from Carslaw and Jaeger (1947) as:

$$C = C_S \left[1 - \operatorname{erf} \left(\frac{x}{2(D_C t)^{1/2}} \right) \right] \quad (\text{A.10})$$

This gives the solution of the diffusion equation (A.1) for a constant surface chloride concentration.

A.2 Time Varying Surface Chloride Concentration

In the case of a time varying surface chloride concentration, the equation for modelling the ingress of chlorides remains the same as given in equation (A.1).

However there is a change in the boundary condition which now becomes:

$$C = S\sqrt{t}; x = 0, t > 0 \quad (\text{A.11})$$

where

S = surface chloride content coefficient

Since there is no change in the differential equation, the Laplace transform of equation (A.1) also remains the same as given by equation (A.7).

The Laplace transform of the boundary condition defined in equation (A.11) is now obtained as:

$$\bar{C} = \int_0^{\infty} S\sqrt{t}e^{-pt} dt; x = 0 \quad (\text{A.12})$$

The solution of the integral in equation (A.12) is obtained from Carslaw and Jaeger (1947) as:

$$\bar{C} = \frac{S\sqrt{\pi}}{2p^{3/2}}; x = 0 \quad (\text{A.13})$$

As seen earlier, the application of the Laplace transformation reduces the partial differential equation (A.1) to an ordinary differential equation (A.7). The solution of equation (A.7) satisfying the transformed boundary condition (A.13) and for which \bar{C} remains finite as x approaches infinity (Carslaw and Jaeger, 1947) is:

$$\bar{C} = \frac{S\sqrt{\pi}}{2p^{3/2}} e^{-\sqrt{\frac{p}{D_c}}x} \quad (\text{A.14})$$

As before, the inverse Laplace transformation is now applied to transform \bar{C} to C in order to obtain the final solution of the differential equation (A.1) satisfying the boundary condition (A.11).

The function whose Laplace transform is given by equation (A.14) can be obtained from Carslaw and Jaeger (1947) as:

$$C = S\sqrt{t} \left[\exp\left(-\frac{x^2}{4D_c t}\right) - \frac{x\sqrt{\pi}}{2\sqrt{D_c t}} \left\{ 1 - \operatorname{erf}\left(\frac{x}{2\sqrt{D_c t}}\right) \right\} \right] \quad (\text{A.15})$$

This gives the solution of the diffusion equation (A.1) for a time varying surface chloride concentration as defined in equation (A.11).