# PRICING AND INVENTORY STRATEGIES IN DUAL-CHANNEL DISTRIBUTION SYSTEM

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**A THESIS SUBMITTED**

**FOR THE DEGREE OF MASTER OF ENGINEERING**

**DEPARTMENT OF INDUSTRIAL AND SYSTEMS ENGINEERING**

**NATIONAL UNIVERSITY OF SINGAPORE**

**2008**

# ACKNOWLEDGEMENTS

I would like to express my deepest gratitude to my supervisors, Associate Professor Poh Kim Leng and Dr Jaruphongsa Wikrom, for their great guidance, suggestions, patience and encouragements throughout my whole research work and life. I have learnt a lot from their knowledge as well as attitude of dealing with work. This dissertation would not have been possible without their help.

I wish to thank the Department of Industrial & Systems Engineering for providing the scholarship and the utilization of its facilities. I would also like to thank other faculty members for the modules I have ever taken: Prof. Goh, Prof. Tang, Prof. Ong, Prof. Tan, and Dr. Ng Kien Ming. Also I would like to thank Ms. Ow Lai Chun and the ISE Computing Lab technician Mr. Cheo for their kind assistance.

I would also like to express my thanks to my friends Qu Huizhong, Wang Yuan, Han Dongling, Cao Yi, Wang Xiaoying, Liu Xiao, Wu Xue, to name a few, for the joy they have brought to me. Specially, I would like to thank my colleagues in the Computing Lab. They are Zhang Haiyun, Yao Zhishuang, Wei Wei, Wong Waipeng, Aldy Gunawan, Jiang Hong, Zhu Zhecheng, Pan Jie, Wu Jun, Xie Yujuan, for the support and help.

Finally, I am grateful for the love and support from my family in China and Mr Long Quan. Their understanding, patience and encouragement have been a great source of motivation for me to pursue my M.Eng.

# CONTENTS

# SUMMARY

Motivated by growing attention to the creation of a manufacturer-owned direct resell channel, the main purpose of this thesis is to study how channel members react to simultaneous horizontal and vertical competition, and if the resulting conflict might overwhelm any potential advantages. Based on a general dual-channel distribution model, where a monopolist manufacturer sells its product through both a traditional retail channel and a direct sell channel, we study the inter-channel competition in two scenarios. First we consider the competition pricing strategy of each channel member under both centralized and decentralized decision making. Our results show that launching a new direct channel raises manufacturer's profit dramatically. Moreover, contrasting to existing studies, retailer's profit decreases even when consumers prefer traditional retailer to direct channel. This suggests that the technique of demand modeling plays an important role in the pricing decision process. In the second scenario, we study the jointly optimal price and inventory decision as a newsvendor problem, considering dynamic customer choice when channel stock out occurs. As the decisions critically depends on the stochastic demand process, we numerically investigate the effect of inter-channel excess demand shifting on the optimal decisions based on linear demand function and multinomial logit model (MNL).

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# CHAPTER 1 **INTRODUCTION**

## 1.1 Background

In the past few decades, we have witnessed a number of significant developments in information technology and major changes in business environment, such as E-commerce, globalization, and intense international cooperation. These forces bring a traditional business concern, distribution channel choice, back to the attention of companies and academic researchers. The expanding role of Internet in consumer and business procurement and the economics of material delivery give birth to direct marketing, where the manufacturer controls the sales and marketing activities. In this business setting, the manufacturer could provide services and information to end customers directly through Internet, reducing operation cost and advertisement cost dramatically. Firms could control dealers and contact small customers in a very cost effective manner, reducing the reliance on dealers. The advantages include extended market coverage, increased sales volume, lower costs, as well as better accommodation of customers' needs and communication.

However, this does not mean that manufacturers could totally abandon intermediate retailers. Indeed, good relationship with retailers leads to high efficiency in making product widely available and large market share in non-brand loyal consumers. Thus, dual-channel supply chains, combining independent retailer channel with direct selling channel, have been widely adopted. For example, Sony has traditional physical retail shops accompanied by an online direct channel. Demand from customers at the retail store is met with the store's on-hand inventory while orders placed through the direct channel are satisfied from the direct channel's warehouse. It is reported in a survey in *The New York Times* (Tedeschi, 2000) that about 42% of top suppliers (e.g. IBM, Pioneer Electronics, Cisco system, Estee Lauder, and Nike) in a variety of industries have begun to sell directly to consumers over the internet, besides their traditional distribution channels.

Although an increasing number of firms now launch direct distribution channel to complement their traditional distribution retail stores, in order to survive and gain advance position in the fierce competing business environment, this vision faces various implementation challenges. Normally, to design a multi-channel distribution system, we must consider whether it is profitable to adopt multiple distribution channels, which channels should be opened, how to coordinate these channels to foster channel confluence and synergy rather than conflict, whether it is possible to reduce the overall distribution cost by a well-designed multiple channels structure. Actually, distribution channel strategy is a real long-term strategy due to the enormous channel build-up cost and significant impact on the company. It is always said that *what you had in the past, the baggage you carry* (Coelho and Easingwood 2004). Thus, a systematic approach for channel decision process is extremely necessary. Rosenbloom (2007) points out that the quality of the channel mix has the most important impact on a firm's customer base. A well-designed distribution channel strategy will be the key to a successful marketing strategy in the future (Fein and Jap, 1999).

## 1.2 Motivation and Research Scope

An obvious and almost inevitable obstacle in a market covered by both intermediated and manufacturer-owned channels is the inter-channel conflict. Specifically, manufacturer could become a competitor to its intermediaries after launching a direct channel. In recent years, this conflict has been quite visible and frequently cited as the most significant barriers to multi-channel strategy, such as in IBM (Nasiretti, 1998), Estee Lauder (Machlis 1998(a)), Avon (Machlis, 1998(b)), etc. The hostility between the manufacturer and its intermediaries can even force manufacturers to abandon a channel entirely. Levi's is an early and famous example of manufacturers which give up the on-line direct channel because of strong protests from key retailers (Davis, 2001).

The primary interests of this thesis are in how channel members react to simultaneously vertical and horizontal competition in terms of their price and inventory decisions, and to what degree this inter-channel conflict would destroy the channel efficiency.

We conduct our study based on a simplified dual-channel system, where a monopolist manufacturer sells its product through both the traditional retail channel and its own direct resale channel (Figure 1.1). Consumers can choose to purchase from either of these two channels, or refer to an outside option where the manufacturer loses this demand. Channel demands are then derived based on consumers' purchase decisions — purchase from which channel. Consumers make their decisions depending on their consumption values and the channel selling prices. Consumption value is the amount of money that the consumer would like to spend on the product or service. Since channels provide various purchase service outputs besides the common product, consumers would have different consumption values for channels according to their purchase service needs.

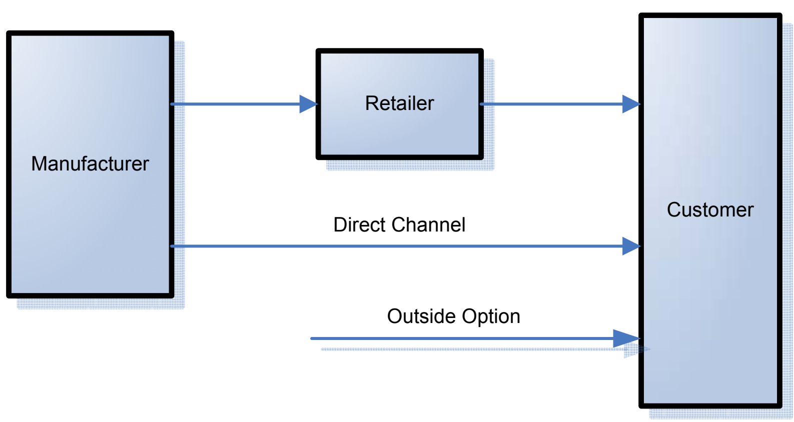



Figure 1.1 General Model Structure

This thesis comprises two scenarios. First we consider the competition pricing strategy in this dual-channel distribution model. Consumer's consumption values are viewed as two independent random variables. As customer's purchasing decision would be affected by the channel resale prices, manufacturer and retailer attempt to maximize their profits through optimal pricing decision. Both of the optimal pricing strategies under centralized decision making and decentralized decision making are studied. In the case of centralized decision making, retail price and direct sell price are determined in coordination to maximize the total profits of both the two channels. In the case of decentralized decision making, manufacturer decides the wholesale price (to retailer) and direct sell price to optimize its profit first; and then retailer decides the resale price to optimize its profit. Compared to single channel distribution system, due to stepping forward to engage in competition for end customer with retailer, manufacturer would make much more profit in dual-channel system, while significantly damaging the retailer, except the cases in which customer's consumption value on direct channel is far less than that on retail channel.

Then, we pay more attention to the effect of inventory on manufacturer and retailer's profits. We study the single period jointly optimal price and inventory decision in dual-channel distribution system. Different from joint price and inventory problem in single channel distribution system, special attention has been put on the effect of inter-channel stock-out demand shifting on manufacturer's pricing and capacity decision. Due to the substitutability between these two channels, customer might shift to the other channel when their first choice channel is out of stock, which would induce manufacturer to over stock. We investigate how customer's shifting behavior would affect the optimal inventory and price decisions as well as the profits.

## 1.3 Summary of Contributions

In this thesis, we mainly focus on the inter-channel competition and coordination in a simplified dual-channel distribution prototype where a monopolist manufacturer sells its product through both the traditional retail channel and its own direct channel.

In Chapter 3, we study the competing pricing strategy in this dual-channel distribution system. Both of the optimal pricing strategies under centralized decision making and decentralized decision making are studied. In the case of decentralized decision making, Stackelberg game model is used to describe manufacturer and retailer's behaviors. Since the demand functions we consider are quadratic functions of $p_r$ and $p_d$, the profit functions become cubic functions, leading to significant increases in computational complexity. Our numerical study shows that manufacturer's profit rises dramatically after launching a new direct channel. On the other hand, contrary to many existing studies, retailer's profit decreases even when consumers prefer traditional retail channel to direct sale channel. This result suggests that the technique for consumer consumption value modeling plays an important role in the pricing decision process.

In Chapter 4, we study the jointly optimal price and inventory decision as a newsvendor problem based on this general dual-channel supply chain model. Special attention has been put on the effect of this inter-channel stock-out demand shifting on manufacturer's pricing and capacity decision. We build up the distribution of the effective direct channel demand conditioning on retailer's order quantity and resell price. Although retailer's problem is a standard newsvendor problem, manufacturer's decision is a combined two-stage price-setting newsvendor problem that no closed form solution could be obtained. To gain management insights, we numerically study this problem based on two demand processes, one being the linear demand function, the other being the multinomial logit model (MNL). It is observed that the effect of inter-channel stock-out demand shifting on manufacturer's capacity is quite sensitive to the retailer's demand variance and the ration that excess demand shift, while that effect on optimal price is not so dramatic.

## 1.4 Organization

This thesis is organized as follows. Literature surveys on related work are presented in Chapter 2, including marketing literature on multi-channel distribution strategy and the operations management literature on pricing and inventory control.

The pricing competition between the traditional retail channel and direct channel is investigated in Chapter 3. Chapter 4 studies the joint price and inventory decisions considering the dynamic customer choice when channel stock out occurs.

Finally, Chapter 5 summarizes all studies in this thesis and gives several directions for future research.

# CHAPTER 2 **LITERATURE REVIEW**

Chapter 2 reviews the previous works related to pricing and inventory strategies in two areas, the marketing literature on multi-channel distribution strategy and the operations management literature on pricing and inventory control.

## 2.1 Multi-channel Distribution Strategy

Multi-channel marketing enables firms to build lasting customer relationships by simultaneously offering their customers prospects information, products, services and supports through synchronized channels. It is noted that multi-channel strategy is becoming increasingly crucial for companies (Easingwood and Storey 1996, and Frazier 1999), but the literature has not matched the practical importance (Bradach and

Eccles 1989, and Coelho and Easingwood 2004). Coelho and Easingwood (2004) presented a comprehensive review of multi-channel systems in services industry.

As our specific interest is in analytic model-based research on strategic behavior of companies when they reach customers through both internal and external distribution channels (e.g., internet, sales force, and value-added resellers), we focus on quantitative managerial models of dual-channel distribution system comprising one direct channel and one indirect channel with intermediary retailers. In this case, manufacturer is not only a channel partner of the retailer, but also a competitor with the retailer over the same market; and therefore conflicts between manufacturer and retailer are incurred. Hence, there are two kinds of competition should be considered: horizontal competition over end-customers and vertical competition between two supply chain entities.

There are two streams of models studying pricing strategies in a hybrid market with respect to methods of demand modeling. One stream investigates the optimal channel strategy and equilibrium prices by assuming a general demand function, usually a linear function with respect to both channel prices. Bell et al. (2002) proposed a linear demand function depending on selling prices at all retailers in the market and their market service level. The authors argue that through forward integration and investment in brand-specific marketing effort, the manufacturer can achieve a form of resale price maintenance.

Tsay and Agrawal (2004) considered the positive impact of direct selling sales effort on demand expansion. They suggested that the addition of a direct channel alongside a reseller channel can benefit both manufacturer and retailer if proper managerial strategies are applied. Corporation strategies are also proposed to address the problem of “channel conflict” and adjust manufacturer-reseller relationship.

In Kumar and Ruan (2006), retailer sells two products with different service levels. Manufacturer can buy high service from retailer with lower wholesale price, adding a direct channel to balance the loss in low wholesale price.

The other stream, focusing on how customer’s channel preference impacts the optimal pricing strategy, uses a demand function induced from a customer channel choice model. This stream of models avoids the difficulty of parameter estimation of general demand functions. Balasubramanian (1998) modeled competition in the multiple-channel environment from a strategic perspective. A circular spatial market model is adopted to capture the relative attractiveness of retail shopping and direct marketer varying across consumers. It is suggested that the level of information disseminated and direct market coverage can be used as tools for competition control.

Hendershott and Zhang (2001) took consumer’s search cost into consideration. They argued that channel conflict is inevitable in dual-channel systems, although the analysis suggested that enough benefit could be generated to fund side payments that would appease the intermediaries.

Chiang et al. (2003) studied a price-competition game between manufacturer and retailer. They assumed a multiplicative relationship between consumer’s reservation prices of retail channel and direct channel with fixed multiplier and random retail channel reservation price. The results show that direct marketing makes the manufacturer more profitable by posing a viable threat to draw customers away from the retailer and alleviate the degree of double marginalization.

Cattani et al. (2006) presented an equal price strategy based on a linear customer utility model where each customer has an independent preference for direct and traditional channels.

Our research in Chapter 3 follows the second stream, deriving demands from customers' choice. The demand functions of the inter-channel pricing competition model we studied are quadratic functions, which greatly increase the complicacy of this problem. We allow unequal channel prices, which is more realistic and generate higher profits than Cattani et al (2006). In Chapter 4, we study the joint price and inventory problem for general linear demand function and Multinomial Logit customer choice model.

## 2.2 Pricing and Inventory in Single Channel System

### 2.2.1 Multiple product inventory models with dynamic customer choice

Dynamic customer choice means that customer would shift to other products when his/her favorite product is out of stock. When this effect is taken into consideration, the optimal inventory policy of each product would depend on the inventory positions and demands of other products.

McGillivray and Silver (1978) started this stream of research. Parlar (1988) studied initial inventory level decisions of two firms carrying similar products facing random demands under competition with simultaneous and sequential moves. In this model, a deterministic fraction of excess demand for each good substitutes to the alternative if the alternative good has excess stock.

Lippman and McCardle (1997) studied an inventory competition game under different rules of initial allocation demands and reallocation of excess demands at any firms. Anupindi and Bassok (1999) investigated the effects of stock-out demand shifting on inventory pooling strategy in a two echelon supply chain with one supplier and two retailers. In this paper, they named the stock-out demand shifting as "market search". Dai et al. (2005) analyzed manufacturer's inventory allocation rule facing the capacity competition of two retailers. This model focuses on the equilibrium of retailer's capacity game, whereas Anupindi and Bassok (1999) ignored this game. Dai et al. (2006) extended this model to consider the capacity allocation game between two manufacturers both of which have traditional and Internet channels.

Netessine and Rudi (2003) studied the optimal inventory policy with *n* substitutable products under both centralized and competitive situation. In this paper, the stock-out demand substitution is named "customer-direct" substitutions. Mahajan and van Ryzin (2001a, 2001b) studied the initial inventory decision assuming that the demand process is a stochastic sequence of heterogeneous customers who choose product dynamically from the available products based on a utility maximization criterion. They proposed a sample path gradient algorithm to find the stationary points of expected profit function.

### 2.2.2 Joint Pricing and Inventory Decision Model

The studies on newsvendor problem considering endogenous price were started in 1950 with the work of Whiting (1955) and Mills (1959). They investigated the optimal pricing and inventory decisions for one retailer selling one product under stochastic demand. Petruzzin and Dada (1999) provided a comprehensive review of this problem. Chen and Simchi-Levi (2002) analyzed a finite horizon, single product, periodic review model in which pricing and production/inventory decisions are made simultaneously. This model is extended to incorporate risk aversion by Chen et al (2007).

Birge et al. (1998) was the first one integrating pricing and capacity decision in a single period problem with two substitutable products. Chen et al. (2004) considered a multiple newsvendors pricing game. Bernstein and Federgruen (2003, 2004, and 2005) addressed coordination of manufacturer and a set of competing retailers facing stochastic demands. All the above models, however, only consider the price-based substitution and ignore stock-out-based substitution. Zhao and Atkins (2008) worked on multi newsvendor simultaneous price and inventory completion considering stock-out demand shifting. They showed the existence and uniqueness of the Nash equilibrium.

## 2.3 Pricing and Inventory in Dual-Channel System

From a supply-chain viewpoint, a great attractiveness of integration of a direct and an indirect channel, besides significant profit gains, is dramatically inventory reduction.

Chiang and Monahan (2005) presented a one-to-one inventory control policy in a two-echelon dual-channel supply chain where manufacturer and retailer are competitors and keeping stock independently, considering the inventory positions at these two entities as a Markov stochastic process. They indicated that the dual-channel strategy outperforms retail-only and direct-only channel strategies in most cases. Geng and Mallik (2007) studied a similar problem, but examined the effect of manufacturer's action of cut-off retailer's order when having limited capacity.

Alptekinoglu and Tang (2005) developed a model of a dual-channel distribution network with multiple depots and multiple sales locations, and study the ordering and allocation policies for each depot to minimize the total distribution cost. In their model, the demand pooling effect and how the correlation between demands at sales locations influences the strategic choices are examined. Seifer et al. (2006) also studied the effect of inventory pooling in a dual-channel supply chain. Specifically, they propose a channel coordination strategy of using excess stock at retail stores to fill some online orders, and conclude that this strategy derives significant reduction in channel stock and lost sales.

Dumrongsiri et al. (2006) is the only work on analysis of the pricing and inventory replenishment problem in a hybrid market with intermediary and direct selling. They derived the demand function from consumer's choice model and analyze the effects of demand uncertainty on the equilibrium prices.

The results of the above paper show that inventory cost could impact profits significantly. Thus, differing from existing literature in this area (e.g. Tsay and Agrawal, 2004; Chiang et al., 2003; Cattani et al. 2006), we pay more attention to the inventory part in Chapter 4. Contrasting to the studies of inventory control policy as papers reviewed here, we integrate the joint pricing and inventory decisions in dual-channel system with consideration of inter-channel excess demand shifting.

# CHAPTER 3 **PRICING COMPETITION IN A DUAL-CHANNEL DISTRIBUTION SYSTEM**

Though double marginalization has been widely studied in bilateral distribution channel systems, interactions between independent manufacturer and retailer become more complicated when direct channel is considered, as horizontal and vertical competition exist simultaneously. Here horizontal competition arises when manufacturer's direct channel competes with retailer's shop for the same pool of customers. The vertical competition refers to the buyer-seller's competition which brings forth double-marginalization. In this case, customers' channel preferences and the price competition between these two channels play critical roles in determining their individual sales volumes and profits as well as those of the whole distribution system. If direct channel is targeted at retailer's least profitable customers, manufacturer and retailer could both benefit with appropriate resell prices. Chiang et al. (2003) showed that direct marketing makes the manufacturer more profitable by posing a viable threat to draw customers away from the retailer and alleviate the degree of double marginalization when there is a deterministic relationship between consumer's reservation prices of retail channel and direct channel. Cattani et al. (2006) presented an equal price strategy based on a linear customer utility model where each customer has an independent preference for direct and traditional channels. We extend their works to consider the case that customers have independent random reservation prices and resell prices at these two channels could be unequal.

In this chapter, we study the pure competition pricing strategy in a dual-channel supply chain where a monopolist manufacturer sells its product through a traditional retail channel as well as to consumers directly (Figure 1.1). The manufacturer supplies the exclusive retailer at a wholesale price $w$ and directly sells to consumer at price $p_d$; while the retailer sells its product to consumer at price $p_r$. Merchandising costs per unit $c_d$ and $c_r$, including the cost of selling and logistics, are incurred when product is sold at direct channel and the retail shop, respectively. Without loss of generality, we ignore the manufacturing cost for it is common for products sold in both two channels. A consumer will either purchase from one of the two channels or nothing at all, depending on selling price and his own reservation price.

## 3.1 Channel Demands

In this section, we introduce our distribution system consisting of an exclusive retail channel and a manufacturer own direct selling channel first, and then derive the channel demand functions from a customer choice model.

### 3.1.1 Consumer choice model

Consumer chooses the purchase channel based on price and his own reservation price (alternatively called "willingness to pay" or "consumption value"). This reservation price depends on customer's service preference and the channel service outputs. A direct channel always provides different service outputs from a traditional bricks-and-mortar shop. Take online sale as an example. Online-shop, one of the most extensively used form of direct marketing, usually provides consumers with only virtual description of the product and delayed indoor delivery; while traditional bricks-and-mortar shop can provide consumers with real inspection and immediate possession. On the other hand, different consumers have different needs for the service outputs, and even the same consumer would have different service needs under different situations. Consequently, consumers have different reservation prices of buying product from these two channels. We denote the reservation price for buying from these retailers by $v_r$, and from direct channel by $v_d$.

We assume that consumers are homogenous in the valuation of the product, and for each consumer, $v_r \sim U(0,\alpha_r)$, $v_d \sim U(0,\alpha_d)$, where $\alpha_r$ (or $\alpha_d$) is the highest possible price that customer would like to spend on this product in a retail shop (or in the direct resale channel). This uniform distribution assumption is extensively used in marketing research, for it is usually the situation that marketing managers only know a possible range of consumers' willingness to pay, especially for new product.

Actually, there are two studies related to our research. One is Cattani et al. (2006). They presented an equal price strategy based on a linear customer utility model where each customer has an independent preference for direct and traditional channels. But we study the optimal pricing strategy allowing these two entities to set different prices. This would be more realistic and likely for manufacturer to adopt, because differentiating prices could provide higher profits. The other is Chiang et al. (2003), assuming a multiplicative relationship between $v_r$ and $v_d$, i.e. $v_d = \theta v_r$, where $\theta$ is called consumer channel preference and constant over the whole population. In contrast, $v_r$ and $v_d$ are assumed to be independent random variables in our research to capture the behavior of a more flexible and random consumer population. It is true that there might be certain internal relationship between $v_r$ and $v_d$, as they evaluations for the same product. However, there are too many external factors beyond the product itself, such as methods of product briefing at different channels, customer's locations, manufacturer's brand image, sales force, can influence customer's preference and reservation price. These intangible factors, along with how they would impact customer, are hard to be captured by a simple multiplication with a constant parameter. So, to keep this problem tractable, we assume these two parameters to be independent here. The demand functions are changed from linear to quadratic, which greatly increase the complexity of this problem.

We model an individual customer's utility $u_i$ as a function of both selling price and his reservation price at channel $i$ from which the product is purchased:

$$u_i = v_i - p_i,\ i \in \{r, d\}.$$

In addition, $u_0$ denotes the utility customer could obtain from an outside option. And without loss of generality, we assume that $u_o = 0$. To maximize his/her own utility, customer purchases from the channel providing the highest utility. If $u_o > \max\{u_r, u_d\}$, the customer will not buy this product from either channel and refer to an outside option. Thus, consumer purchases from channel $i$ ($i \in \{r, d, o\}$):

$$i = \arg\max\{u_r, u_d, u_o\}.$$

### 3.1.2 Demands

To determine the demand curves, we first consider the supply chain structure prior to the manufacturer's introduction of a direct selling channel only with traditional retailer. When a product is sold through traditional distribution channel with an outside retail store, consumer will buy this product if $v_r - p_r \geq 0$. Thus, assuming $v_r \sim U(0, \alpha_r)$, the probability that a customer buying from the retail shop is $1 - \frac{p_r}{\alpha_r}$, for $0 \leq p_r \leq \alpha_r$.

With the addition of a direct channel, the probability that a customer buys from the traditional store or direct resale channel will be calculated over appropriate regions of the joint distribution of $v_r$ and $v_d$. Figure 3.1 illustrates the case when $p_r - p_d \leq \alpha_r - \alpha_d$. Consumer buys from traditional store, when

$$u_r \geq \max\{u_d, 0\} = \max\{v_d - p_d, 0\}.$$

Thus, the probability that a customer buys from the traditional store is given by

$$q_r = \Pr\{u_r \geq \max\{u_d, 0\}\}$$

$$= \Pr\{u_r \geq v_d - p_d \mid v_d - p_d \geq 0\} + \Pr\{u_r \geq 0 \mid v_d - p_d \leq 0\}$$

$$= \int_{p_d}^{\alpha_d} \frac{1}{\alpha_d}\left(\int_{v_d - p_d + p_r}^{\alpha_r} \frac{1}{\alpha_r} dv_r\right) dv_d + \int_0^{p_d} \frac{1}{\alpha_d}\left(\int_{p_r}^{\alpha_r} \frac{1}{\alpha_r} dv_r\right) dv_d$$

$$= 1 - \frac{2p_r\alpha_d + (\alpha_d - p_d)^2}{2\alpha_r\alpha_d} \tag{3.1}$$

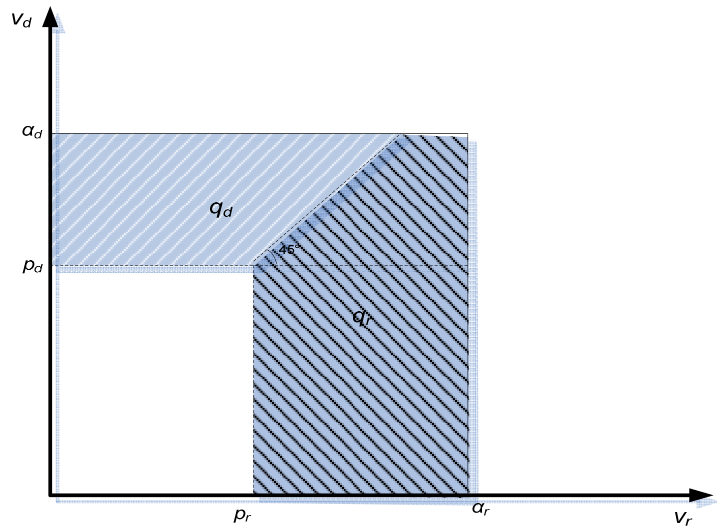



Figure 3.1 Channel demands based on consumers' willingness-to-pay ( $p_r - p_d \le \alpha_r - \alpha_d$ )

Since the probability that consumer buys from either channel is $1 - \frac{p_r p_d}{\alpha_r\alpha_d}$, the probability that a customer buys from the direct resale channel is given by

$$q_d = 1 - \frac{p_r p_d}{\alpha_r\alpha_d} - q_r$$

$$= \frac{(\alpha_d - p_d)^2 + 2p_r\alpha_d - 2p_r p_d}{2\alpha_r\alpha_d} \tag{3.2}$$

Similarly, Figure 3.2 illustrates the case when $p_r - p_d > \alpha_r - \alpha_d$.

$$q_r = \Pr\{u_r \ge \max\{u_d, 0\}\}$$

$$= \int_{p_d}^{\alpha_r - p_r + p_d} \frac{1}{\alpha_d}\left(\int_{v_d - p_d + p_r}^{\alpha_r} \frac{1}{\alpha_r} dv_r\right) dv_d + \int_0^{p_d} \frac{1}{\alpha_d}\left(\int_{p_r}^{\alpha_r} \frac{1}{\alpha_r} dv_r\right) dv_d$$

$$= \frac{2p_d\alpha_r - 2p_r p_d + (\alpha_r - p_r)^2}{2\alpha_r\alpha_d} \tag{3.3}$$

Then,

$$q_d = 1 - \frac{\left(\alpha_r - p_r\right)^2 + 2 p_d \alpha_r}{2\alpha_r \alpha_d} \tag{3.4}$$

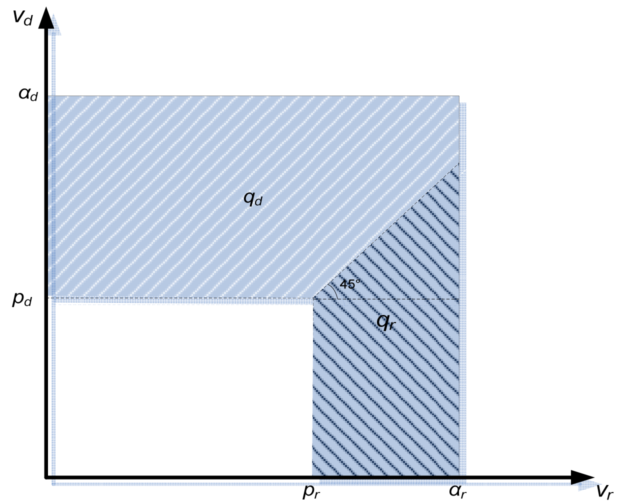



Figure 3.2 Channel demands based on consumers' willingness-to-pay

$(p_r - p_d \geq \alpha_r - \alpha_d)$

**Lemma 3.1** The probability functions that a customer buys from retailer channel $q_r$ and direct channel $q_d$ are continuously differentiable in the region of $R = \left\{\left(p_r, p_d\right) | \, 0 \leq p_r \leq \alpha_r, 0 \leq p_d \leq \alpha_d\right\}$.

*Proof* First we prove that $q_r$ is continuously differentiable. As $q_r$ is an integration of two simple functions, we only need to prove that on the boundary $p_r - p_d = \alpha_r - \alpha_d$ it is differentiable and the derivatives are continuous. Both equations (3.1) and (3.3) give the same value of ${}_r q_r = \frac{\alpha_d^2 - p_d^2}{2\alpha_r \alpha_d}$, which means that $q_r$ is continuous. In addition, the partial derivative of $q_r$ with respect to $p_r$ based on equation (3.1) is

$$\frac{\partial q_r}{\partial p_r} = -\frac{1}{\alpha_r} \quad . \tag{3.5}$$

The partial derivative of $q_r$ with respect to $p_r$ based on equation (3.3) is

$$\frac{\partial q_r}{\partial p_r} = -\frac{p_d + (\alpha_r - p_r)}{\alpha_r \alpha_d}. \tag{3.6}$$

Then at $p_r = \alpha_r - \alpha_d + p_d$, $\left.\frac{\partial q_r}{\partial p_r}\right|_{p_r = \alpha_r - \alpha_d + p_d} = -\frac{1}{\alpha_r}$, which exactly equals to equation (3.5). These mean that $q_r$ is continuous derivable with respect to $p_r$ on $[0,\alpha_r]$.

The partial derivative of $q_r$ with respect to $p_d$ based on equation (3.1) is

$$\frac{\partial q_r}{\partial p_d} = \frac{\alpha_d - p_d}{\alpha_r \alpha_d};$$

and that based on equation (3.3) is

$$\frac{\partial q_r}{\partial p_d} = \frac{\alpha_r - p_r}{\alpha_r \alpha_d}.$$

These two derivatives are also equal on the boundary $p_r - p_d = \alpha_r - \alpha_d$. Thus we show that $q_r$ is a continuously differentiable function in region $R$.

Similarly, we find that $q_d$ is continuously differentiable over the region $R$. Both equations (3.2) and (3.4) give the same value of $q_d$ at $p_r - p_d = \alpha_r - \alpha_d$,

$$q_d = \frac{\alpha_r^2 - p_r^2}{2\alpha_r \alpha_d}, \tag{3.7}$$

and the same value of $\left.\frac{\partial q_d}{\partial p_d}\right|_{p_d = \alpha_d - \alpha_r + p_r}$,

$$\left.\frac{\partial q_d}{\partial p_d}\right|_{p_d = \alpha_d - \alpha_r + p_r} = \frac{1}{\alpha_d}, \tag{3.8}$$

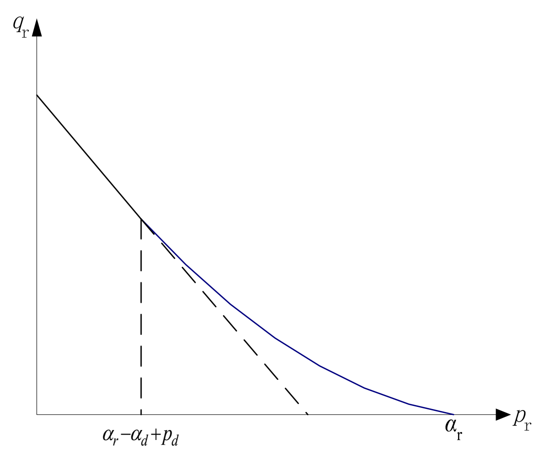
as well as the same value of $\left.\frac{\partial q_d}{\partial p_r}\right|_{p_r = \alpha_r - \alpha_d + p_d}$,

$$\left.\frac{\partial q_d}{\partial p_r}\right|_{p_r=\alpha_r-\alpha_d+p_d} = \frac{\alpha_d - p_d}{\alpha_r \alpha_d} \quad . \tag{3.9}$$

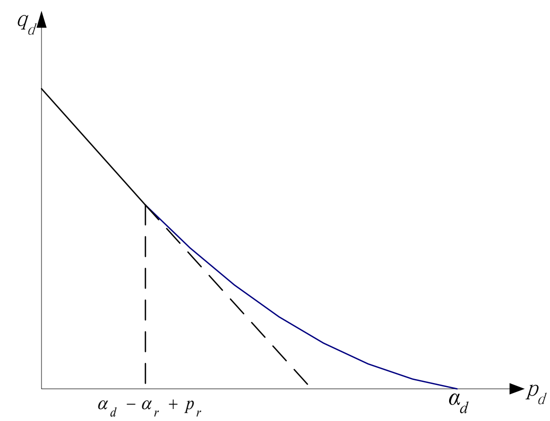
□

Note that $q_r$ and $q_d$ are not twice continuously differentiable. On the boundary $p_r - p_d = \alpha_r - \alpha_d$, $\frac{\partial^2 q_r}{\partial p_r^{\,2}} = 0$ based on equation (3.1) but $\frac{\partial^2 q_r}{\partial p_r^{\,2}} = \frac{1}{\alpha_r \alpha_d}$ per equation (3.3). This inequality could also be found for other second derivatives.

Figure 3.3 plots the probabilities that a customer would buy from each channel as a function of the channel price with given price at the other channel. We can see that, for both channels, the probability of customer choosing channel $i$ is linear when the price at channel $i$ is relatively low compared to that at the other channel, and it becomes quadratic when the channel price exceeds a certain value. Moreover, the line and the quadratic curve are tangent on the boundary $p_r - p_d = \alpha_r - \alpha_d$.



(a)




(b)
Figure 3.3 Channel demands (a) Retailer (b) Direct Channel

## 3.2 Pricing Strategy

### 3.2.1 Centralized SC

Under centralized decision making, a vertically integrated firm controls all three decisions: manufacturing, traditional retailing, and the direct marketing. As customer could access either the retail shop or the direct channel, the firm should make appropriate pricing decision for these two channels according to customer's consumption value distribution so that it could achieve profit-maximized customer segmentation. Given the customer's purchasing probability functions in equations from (3.1) to (3.4), if the vertically integrated firm sets retail price $p_r$ and direct market price $p_d$, then the expected profit per customer it would earn is

$$\pi_I(p_r, p_d) = (p_r - c_r)q_r + (p_d - c_d)q_d \tag{3.10}$$

where $c_r$ and $c_d$ are marginal costs incurred by the manufacturer for the product sold through the retailer and direct market, respectively.

Since $\pi_I(p_r, p_d)$ is a continuous function on the bounded area $R = \{(p_r, p_d) \mid 0 \le p_r \le \alpha_r, 0 \le p_d \le \alpha_d\}$, there exists pricing strategy $(p_r, p_d)$ in $R$

giving the maximum value of $\pi_I$. If either channel's price was set to the upper bound of customer's consumption value ($\alpha_r$ or $\alpha_d$), the probability that customer chooses that channel becomes zero, which is equivalent to the single channel situation and out of our research scope in this section. Also, the firm cannot price either channel to zero, or customer could definitely purchase from that channel resulting in a negative system profit. Because all the points on the boundary of $R$ do not result in positive value of profit, we know the optimal pricing strategy $\left(p_r^*, p_d^*\right)$must be a stationary point of $\pi_I(p_r, p_d)$ located in the inner set of $R$, i.e. $\left(p_r^*, p_d^*\right)$ should satisfy $\frac{\partial \pi_I}{\partial p_r} = 0$ and $\frac{\partial \pi_I}{\partial p_d} = 0$.

As the functions of the probability that customer chooses each channel have different forms with the relationship between $p_r - p_d$ and $\alpha_r - \alpha_d$, we will figure out all the stationary points in R in the following two cases, $p_r - p_d \le \alpha_r - \alpha_d$ and $p_r - p_d > \alpha_r - \alpha_d$, respectively (Figure 3.4). The optimal price strategy $\left(p_r^*, p_d^*\right)$ should be the stationary point which provides the highest profit.

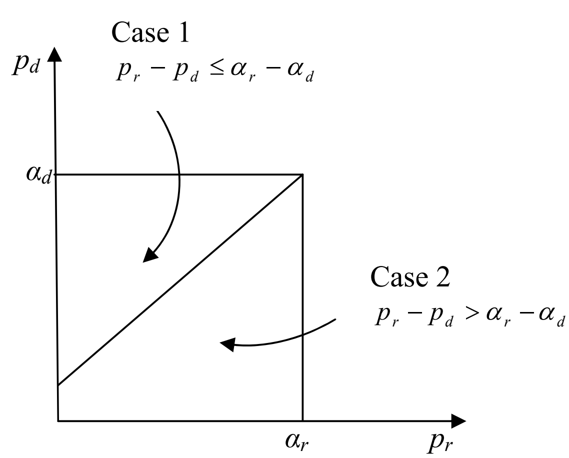



Figure 3.4 Illustration of regions ($\alpha_r < \alpha_d$)

**Case 1** $p_r - p_d \le \alpha_r - \alpha_d$

The stationary points in this range should satisfy $\dfrac{\partial \pi_I}{\partial p_r} = 0$ and $\dfrac{\partial \pi_I}{\partial p_d} = 0$ .

By setting $\dfrac{\partial \pi_I}{\partial p_r} = 0$ and $\dfrac{\partial \pi_I}{\partial p_d} = 0$ , we have

$$q_r + (p_r - c_r)\left(-\frac{1}{\alpha_r}\right) + (p_d - c_d)\frac{\alpha_d - p_d}{\alpha_r \alpha_d} = 0 \qquad (3.11)$$

and

$$(p_r - c_r)\frac{\alpha_d - p_d}{\alpha_r \alpha_d} + q_d + (p_d - c_d)\frac{p_d - p_r - \alpha_d}{\alpha_r \alpha_d} = 0 \ , \qquad (3.12)$$

which result in ( $p_{r1}^*$ , $p_{d1}^*$ ) where

$$p_{r1}^* = \frac{-3p_d^{\ 2} + 2(2\alpha_d + c_d)p_d + 2c_r\alpha_d - 2c_d\alpha_d + 2\alpha_r\alpha_d - \alpha_d^{\ 2}}{4\alpha_d} \qquad (3.13)$$

and

$$p_{d1}^* = \frac{1}{3}\left(3p_r + 2\alpha_d + c_d - c_r - \sqrt{9p_r^{\ 2} - 6c_r p_r + c_r^{\ 2} + c_d^{\ 2} + \alpha_d^{\ 2} - 2c_r c_d + 2\alpha_d c_r - 2c_d\alpha_d}\right). \qquad (3.14)$$

The solutions of equation (3.13) and (3.14) $\left(p_{r1}^*, p_{d1}^*\right)$ are candidates of the optimal pricing strategy if it satisfies the condition $p_r - p_d \le \alpha_r - \alpha_d$ . Actually we have another solution of $p_d$,

$$\frac{1}{3}\left(3p_r + 2\alpha_d + c_d - c_r + \sqrt{9p_r^{\ 2} - 6c_r p_r + c_r^{\ 2} + c_d^{\ 2} + \alpha_d^{\ 2} - 2c_r c_d + 2\alpha_d c_r - 2c_d\alpha_d}\right) \qquad (3.15)$$

by equation (3.12), but when substituting this value of $p_d$ and equation (3.13) into $\dfrac{\partial^2 \pi_I}{\partial p_d^{\ 2}}$, we find that

$$\frac{\partial^2 \pi_I}{\partial p_d^{\ 2}} = \frac{-c_d + c_r + 3p_d - 3p_r - 2\alpha_d}{\alpha_r \alpha_d}$$
$$= \frac{\sqrt{9p_r^2 - 6c_r p_r + c_r^2 + c_d^2 + \alpha_d^2 - 2c_r c_d + 2\alpha_d c_r - 2c_d \alpha_d}}{\alpha_r \alpha_d}$$
$$= \frac{\sqrt{(3p_r - c_r)^2 + (\alpha_d - c_d)^2 + 2c_r(\alpha_d - c_d)}}{\alpha_r \alpha_d} > 0.$$

As equation (3.15) results in a positive value of $\frac{\partial^2 \pi_I}{\partial p_d^{\ 2}}$, it could not be a local maximum point. Thus in the region of $p_r - p_d < \alpha_r - \alpha_d$ the optimal prices can only come out from equation (3.13) and (3.14). It is possible that there are more than one the solution of equation (3.13) and (3.14). In this case, we only need to keep the pairs satisfying $p_r - p_d \le \alpha_r - \alpha_d$ and providing the highest value of $\pi_I(p_r, p_d)$.

If none of the roots of equation (3.13) and (3.14) falls into the range $p_r - p_d \le \alpha_r - \alpha_d$, it indicates that profit function $\pi_I(p_r, p_d)$ keeps increasing till the bound $p_r - p_d = \alpha_r - \alpha_d$ and the optimal $(p_r^*, p_d^*)$ does not exist in this range.

**Case 2** $p_r - p_d > \alpha_r - \alpha_d$

Similarly, we could find the stationary points in Case 2 by solving $\frac{\partial \pi_I}{\partial p_r} = 0$ and $\frac{\partial \pi_I}{\partial p_d} = 0$ with $q_r$ in equation (3.3) and $q_d$ in equation (3.4). If no stationary point is found in this area, it means that the profit decreases monotonously and the optimal pair of prices $(p_r^*, p_d^*)$ would occur in Case 1.

By setting $\frac{\partial \pi_I}{\partial p_r} = 0$ and $\frac{\partial \pi_I}{\partial p_d} = 0$ we have the stationary point ( $p_{r2}^*$, $p_{d2}^*$ ) should satisfy the following two equations

$$q_r + (p_r - c_r)\frac{p_r - \alpha_r - p_d}{\alpha_r \alpha_d} + (p_d - c_d)\frac{\alpha_r - p_r}{\alpha_r \alpha_d} = 0 \tag{3.16}$$

and

$$(p_r - c_r)\frac{\alpha_r - p_r}{\alpha_r \alpha_d} + q_d + (p_d - c_d)\left(-\frac{1}{\alpha_d}\right) = 0\,. \tag{3.17}$$

Then we could obtain

$$p_{r2}^* = \frac{1}{3}\left(3p_d + 2\alpha_r + c_r - c_d - \sqrt{9p_d^{\,2} - 6c_d p_d + c_d^{\,2} + c_r^{\,2} + \alpha_r^{\,2} - 2c_r c_d + 2\alpha_r c_d - 2c_r \alpha_r}\right) \tag{3.18}$$

and

$$p_{d2}^* = \frac{-3p_r^{\,2} + 2(2\alpha_r + c_r)p_r + 2c_d\alpha_r - 2c_r\alpha_r + 2\alpha_d\alpha_r - \alpha_r^{\,2}}{4\alpha_r}\,. \tag{3.19}$$

Similarly, we could also show that the other root of $p_r$ from equation (3.16),

$$\frac{1}{3}\left(3p_d + 2\alpha_r + c_r - c_d + \sqrt{9p_d^{\,2} - 6c_d p_d + c_d^{\,2} + c_r^{\,2} + \alpha_r^{\,2} - 2c_r c_d + 2\alpha_r c_d - 2c_r \alpha_r}\right),$$

would give a positive value of $\dfrac{\partial^2 \pi_I}{\partial p_r^{\,2}}$. Thus the candidates of the optimal price pair in Case 2 can only come out from $\left(p_{r2}^{\;*}, p_{d2}^{\;*}\right)$.

If either $\left(p_{r1}^*, p_{d1}^*\right)$ or $\left(p_{r2}^*, p_{d2}^*\right)$ falls into its feasible region, the feasible one is our optimal solution. If both $\left(p_{r1}^*, p_{d1}^*\right)$ and $\left(p_{r2}^*, p_{d2}^*\right)$ are feasible solutions, we choose the one providing higher total profit $\pi_I$.

Next we look at the special case of Case 1 that $p_r - p_d = \alpha_r - \alpha_d$. In this case, the optimal price pair $\left(p_{r3}^{\;*}, p_{d3}^{\;*}\right)$ is

$$p_{r3}^* = \frac{1}{6}\left(c_d + c_r - 2\alpha_d + 2\alpha_r + \sqrt{\left(c_d + c_r - 2\alpha_d + 2\alpha_r\right)^2 + 12\left(\alpha_r\alpha_d + c_d\alpha_d - c_d\alpha_r\right)}\right), \tag{3.20}$$

$$p_{d3}^* = \frac{1}{6}\left(c_d + c_r + 4\alpha_d - 4\alpha_r + \sqrt{\left(c_d + c_r - 2\alpha_d + 2\alpha_r\right)^2 + 12\left(\alpha_r\alpha_d + c_d\alpha_d - c_d\alpha_r\right)}\right). \tag{3.21}$$

Specifically, we can see that when $c_r = c_d$ and $\alpha_r = \alpha_d$,

$$p_r^* = p_d^* = \frac{1}{3}\left(c_r + \sqrt{c_r^{\,2} + 3\alpha_r^{\,2}}\right). \tag{3.22}$$

However, in the situation that there is only retail shop distribution channel, system profit

$$\pi_I^S = \left(p_r - c_r\right)q_r = \left(p_r - c_r\right)\left(1 - \frac{p_r}{\alpha_r}\right) \tag{3.23}$$

and

$$\frac{\partial \pi_I^S}{\partial p_r} = \left(p_r - c_r\right)\left(-\frac{1}{\alpha_r}\right) + \left(1 - \frac{p_r}{\alpha_r}\right). \tag{3.24}$$

Thus, the optimal resell price $p_r^{S^*} = \dfrac{\alpha_r + c_r}{2}$ which is less than or equal to the optimal price shown in equation (3.22) in dual-channel distribution system when $c_r = c_d$ and $\alpha_r = \alpha_d$. This is because

$$\begin{aligned}
p_r^* = p_d^* &= \frac{1}{3}\left(c_r + \sqrt{c_r^{\,2} + 3\alpha_r^{\,2}}\right) \\
&= \frac{1}{3}\left(cr + \sqrt{\left(\frac{c_r}{2} + \frac{3\alpha_r}{2}\right)^2 + \frac{3}{4}\left(c_r - \alpha_r\right)^2}\right) \\
&\geq \frac{1}{3}\left(cr + \frac{c_r}{2} + \frac{3\alpha_r}{2}\right) \\
&= \frac{1}{2}\left(cr + \alpha_r\right).
\end{aligned}$$

The equality of $p_r^{S^*} = p_r^*$ occurs only when $c_r = c_d = \alpha_r = \alpha_d$. One reason to this phenomenon is that in dual-channel distribution system, customer with low consumption value for retail channel might have high consumption value for direct channel. In this case, the lost demand at retail channel whose consumption value for direct channel is higher than direct selling price could be captured by that channel. Therefore, manufacturer is able to adjust the price in dual channel system a little higher than that in single channel system, as long as the possibility that customer buys the product from either of these two channels holds large enough to generate higher system profit.

### 3.2.2 Decentralized SC competition and coordination

In this subsection, we analyze the pricing strategy of manufacturer and traditional retailer in a decentralized supply chain where each of the two entities looks at his own profit when setting the price.

We incorporate the direct channel into the Stackelberg game model to examine the interaction between the manufacturer and the retailer. A Stackelberg game is a one-round game introduced by the German economist Heinrich Freiherr von Stackelberg (Gibbons, 1992). There are two players in the game: the *leader* moves first, then the *follower* moves, and then the game is over. The follower optimizes its own objective function, knowing the leader's move. The leader, sometimes referred to as the *market leader,* has to optimize its own objective function by anticipating the optimal response of the follower.

In our case, manufacturer is the game leader who is able to predict retailer's response to his wholesale price and direct price decisions. In the first stage, manufacturer decides whether to engage in direct sales, and sets the wholesale price $w$ as well as direct selling price $p_d$ (if the direct channel is opened). To keep the retailer from buying through the direct channel or other arbitrators with a lower price, we require $w \le p_d$. Manufacturer maximizes its expected profit per customer, $\pi_m = wq_r + \left(p_d - c_d\right)q_d$, taking the retailer's expected decision with given $w$ and $p_d$ into account, where $q_r$ and $q_d$ are the probabilities that a customer buys from each channel. In the second stage, the retailer, as a follower, chooses the retail price $p_r$ to maximize its profit $\pi_r = (p_r - w - c_r)q_r$. In the remainder of this subsection we solve this problem backwards, first retailer's optimal decisions upon any possible manufacturer's behavior, and then manufacturer's decision by anticipating retailer's reaction.

#### 3.2.2.1 Retailer's pricing reaction

Given manufacturer's pricing decisions, retailer chooses the optimal $p_r$ to maximize its profit $\pi_r(p_r \mid p_d, w) = (p_r - w - c_r)\cdot q_r(p_r \mid p_d)$. As shown in Figure 3.3(a), $q_r(p_r \mid p_d)$ is a piecewise function with a line segment and a curve segment. According to Lemma 3.1, $q_r(p_r \mid p_d)$ is continuously differentiable over [0, $\alpha_r$]. So, retailer's profit function with given $p_d$ and $w$ is also continuously differentiable over [0, $\alpha_r$]. Since $p_r$ = 0 or $\alpha_r$ leads to non-positive profit, the optimal price of retail channel should be a stationary point of $\pi_r\left(p_r \mid p_d, w\right)$.

As $q_r(p_r \mid p_d)$ has different expressions in the range of $\left[0, \alpha_r - \alpha_d + p_d\right]$ and $\left(\alpha_r - \alpha_d + p_d, \alpha_r\right]$, we first find out the stationary points of $\pi_r\left(p_r \mid p_d, w\right)$ with $q_r$ in equation (3.1) and (3.3) separately. The optimal $p_r$ is the feasible one among these stationary points that generates the highest profit.

**Case 1** $p_r \le \alpha_r - \alpha_d + p_d$

In this case, $q_r$ follows the form in equation (3.1), which is a linear decreasing function, $\pi_r\left(p_r \mid p_d, w\right)$ is a concave function with respect to $p_r$ for given $p_d$ and $w$. Then, by setting $\frac{\partial \pi_r}{\partial p_r} = 0$, i.e.,

$$\left(1 - \frac{2p_r\alpha_d + (\alpha_d - p_d)^2}{2\alpha_r\alpha_d}\right) + (p_r - w - c_r)\left(-\frac{1}{\alpha_r}\right) = 0, \tag{3.25}$$

we have one stationary point

$$x_1 = \frac{c_r + w + \alpha_r}{2} - \frac{(\alpha_d - p_d)^2}{4\alpha_d}. \tag{3.26}$$

Note that the feasibility of $x_1$, despite that $x_1 \le \alpha_r - \alpha_d + p_d$ , requires that $c_r + w < x_1 < \alpha_r$ . $x_1 < \alpha_r$ is guaranteed under the natural constraint of $c_r + w < \alpha_r$ which stands for the profitability of retail channel. And $x_1 > c_r + w$ is satisfied when

$$\frac{\alpha_r - (c_r + w)}{2} - \frac{(\alpha_d - p_d)^2}{4\alpha_d} > 0,$$

that is,

$$2\alpha_d(\alpha_r - c_r - w) > (\alpha_d - p_d)^2. \tag{3.27}$$

**Case 2** $p_r > \alpha_r - \alpha_d + p_d$

As $q_r$ follows the form in equation (3.3), which is a quadratic function of $p_r$, $\pi_r\left(p_r \mid p_d, w\right)$ becomes a cubic function of $p_r$. By setting $\frac{\partial \pi_r}{\partial p_r} = 0$, i.e.,

$$\left(\frac{2p_d\alpha_r - 2p_r p_d + (\alpha_r - p_r)^2}{2\alpha_r\alpha_d}\right) + (p_r - w - c_r)\left(-\frac{\alpha_r - p_r + p_d}{\alpha_r\alpha_d}\right) = 0, \qquad (3.28)$$

we obtain two stationary points

$$x_2 = \frac{1}{3}\left(2p_d + w + c_r + 2\alpha_r - \sqrt{(\alpha_r - w - c_r + p_d)^2 + 3{p_d}^2}\right), \qquad (3.29)$$

$$x_3 = \frac{1}{3}\left(2p_d + w + c_r + 2\alpha_r + \sqrt{(\alpha_r - w - c_r + p_d)^2 + 3{p_d}^2}\right). \qquad (3.30)$$

But as $\left.\frac{\partial^2 \pi_r}{\partial p_r^2}\right|_{p_r = x_2} < 0$ and $\left.\frac{\partial^2 \pi_r}{\partial p_r^2}\right|_{p_r = x_3} > 0$, only $x_2$ is a local maximum point. In addition, noting that $c_r + w < \alpha_r$ gives rise to

$$\alpha_r < (\alpha_r - w - c_r + p_d)^2 + 3{p_d}^2 < 4(\alpha_r - w - c_r + p_d)^2,$$

we know that $c_r + w < x_2 < \alpha_r$ and $x_3 > \alpha_r$, which means that if $x_2 > \alpha_r - \alpha_d + p_d$ then it is a feasible solution to retailer's pricing problem.

Therefore, each of these two cases has one local maximum, and retailer's optimal price should be either one of these two points. To show the existence and uniqueness of the optimal solution, we first have the following lemma regarding the unimodality of $\pi_r(p_r \mid p_d, w)$ on $[0, \alpha_r]$.

**Lemma 3.2** Given $p_d$ and $w$, exactly one of the following three formulas is satisfied:

i) $x_1 < \alpha_r - \alpha_d + p_d$;

ii) $x_2 > \alpha_r - \alpha_d + p_d$;

iii) $x_1 = x_2 = \alpha_r - \alpha_d + p_d$.

*Proof* Let $f_1(p_d, w) = x_1 - (\alpha_r - \alpha_d + p_d)$ , $f_2(p_d, w) = x_2 - (\alpha_r - \alpha_d + p_d)$ . The Lemma 3.2 is equivalent to $f_1 \cdot f_2 \geq 0$ and the equality can be attained only when $f_1 = f_2 = 0$.

Firstly, we deduce the sufficient and necessary conditions of $f_1 = 0$ and $f_2 = 0$. When $f_1 = 0$, we have

$$\frac{c_r + w + \alpha_r}{2} - \frac{(\alpha_d - p_d)^2}{4\alpha_d} = \alpha_r - \alpha_d + p_d, \tag{3.31}$$

simplified to

$$2\alpha_d(c_r + w - \alpha_r) = p_d^{\ 2} - 3\alpha_d^{\ 2} + 2\alpha_d p_d. \tag{3.32}$$

So equation (3.32) is equivalent to $x_1 = \alpha_r - \alpha_d + p_d$. And

On the other hand, from

$$\begin{aligned} f_2 &= x_2 - (\alpha_r - \alpha_d + p_d) \\ &= \frac{1}{3}\left(-p_d + w + c_r - \alpha_r + 3\alpha_d - \sqrt{(\alpha_r - w - c_r + p_d)^2 + 3p_d^{\ 2}}\right) = 0, \end{aligned} \tag{3.33}$$

we know

$$3\alpha_d - (\alpha_r - w - c_r + p_d) = \sqrt{(\alpha_r - w - c_r + p_d)^2 + 3p_d^{\ 2}}. \tag{3.34}$$

Letting $A \equiv 3\alpha_d - (\alpha_r - w - c_r + p_d)$, $B \equiv \sqrt{(\alpha_r - w - c_r + p_d)^2 + 3p_d^{\ 2}}$,

$$\begin{aligned} A^2 - B^2 &= (-p_d + w + c_r - \alpha_r + 3\alpha)^2 - \left((\alpha_r - w - c_r + p_d)^2 + 3p_d^{\ 2}\right) \\ &= 9\alpha_d^{\ 2} - 3p_d^{\ 2} - 6\alpha_d p_d + 6\alpha_d(w + c_r - \alpha_r) \\ &= 3\left(3\alpha_d^{\ 2} - p_d^{\ 2} - 2\alpha_d p_d + 2\alpha_d(c_r + w - \alpha_r)\right). \end{aligned} \tag{3.35}$$

Equation (3.34) implies that A=B and $A^2 - B^2 = 0$. So we obtain the necessary condition of $f_2 = 0$ is $2\alpha_d(c_r + w - \alpha_r) = p_d^{\ 2} - 3\alpha_d^{\ 2} + 2\alpha_d p_d$ which is exactly the same as equation (3.32). In addition, under this condition,

$3\alpha_d - (\alpha_r - w - c_r + p_d) = 3\alpha_d - p_d + \frac{1}{2\alpha_d}\left(p_d^{\ 2} - 3\alpha_d^{\ 2} + 2\alpha_d p_d\right) = \frac{3}{2}\alpha_d + \frac{p_d^{\ 2}}{2\alpha_d} > 0$.

Thus equation (3.22) is the necessary and sufficient condition of $f_2 = 0$. Hence, we proved $f_1 \cdot f_2 = 0$ can be attained only when $f_1 = f_2 = 0$, i.e., $x_1 = x_2 = \alpha_r - \alpha_d + p_d$.

Next, we show that $f_1 \cdot f_2 > 0$ when $f_1$ and $f_2$ are not equal to zero. Suppose $f_1 < 0$ and $f_2 > 0$, i.e., $x_1 < \alpha_r - \alpha_d + p_d$ and $x_2 > \alpha_r - \alpha_d + p_d$. From

$$x_1 = \frac{c_r + w + \alpha_r}{2} - \frac{(\alpha_d - p_d)^2}{4\alpha_d} < \alpha_r - \alpha_d + p_d$$

we obtain

$$2\alpha_d(c_r + w - \alpha_r) < p_d^{\ 2} - 3\alpha_d^{\ 2} + 2\alpha_d p_d. \qquad (3.36)$$

Then from $x_2 > \alpha_r - \alpha_d + p_d$ we know

$x_2 - (\alpha_r - \alpha_d + p_d) =$

$$\frac{1}{3}\left(-p_d + w + c_r - \alpha_r + 3\alpha_d - \sqrt{(\alpha_r - w - c_r + p_d)^2 + 3p_d^{\ 2}}\right) > 0. \qquad (3.37)$$

When $A > 0$ and $B > 0$, $A^2 - B^2$ is positive because it has the same sign as that of equation (3.37). So equation (3.35) is larger than zero, which implies $2\alpha_d(c_r + w - \alpha_r) > p_d^{\ 2} - 3\alpha_d^{\ 2} + 2\alpha_d p_d$ . This contradicts equation (3.36). Thus, $f_1 < 0$ and $f_2 > 0$ cannot held simultaneously.

Similarly, we show that $f_1 > 0$ and $f_2 < 0$ , i.e., $x_1 > \alpha_r - \alpha_d + p_d$ and $x_2 < \alpha_r - \alpha_d + p_d$, also do not hold simultaneously. As $f_1 > 0$,

$$2\alpha_d(c_r + w - \alpha_r) > p_d^{\ 2} - 3\alpha_d^{\ 2} + 2\alpha_d p_d. \qquad (3.38)$$

Then we have

$$A = -p_d + 3\alpha_d + w + c_r - \alpha_r$$

$$> -p_d + 3\alpha_d + p_d + \frac{p_d^{\ 2}}{2\alpha_d} - \frac{3}{2}\alpha_d = \frac{3}{2}\alpha_d + \frac{p_d^{\ 2}}{2\alpha_d} > 0 .$$

But from equation (3.35) we know

$$A^2 - B^2 = 3\left(3\alpha_d^{\ 2} - p_d^{\ 2} - 2\alpha_d p_d + 2\alpha_d\left(c_r + w - \alpha_r\right)\right)$$

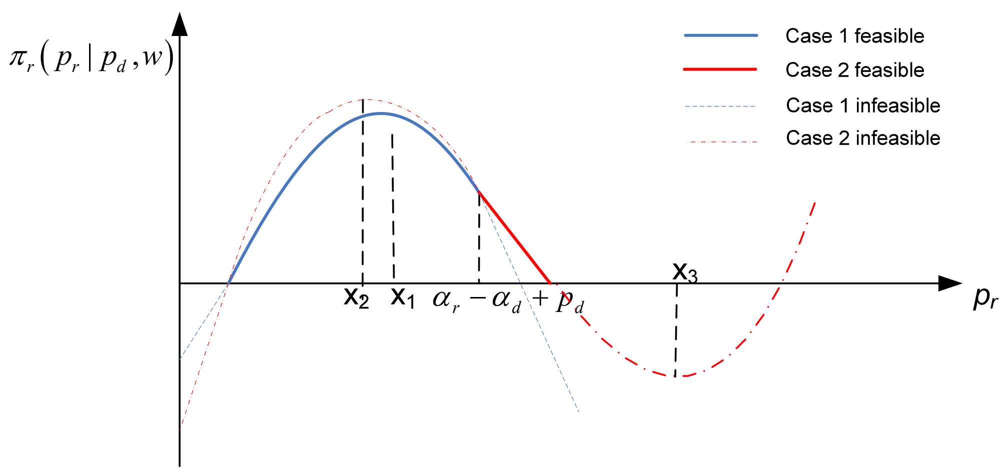
$$> 3\left(3\alpha_d^{\ 2} - p_d^{\ 2} - 2\alpha_d p_d + p_d^{\ 2} - 3\alpha_d^{\ 2} + 2\alpha_d p_d\right) = 0 .$$

This indicates $A > B$; or equivalently $f_2 > 0$. So it is not possible that $f_1 > 0$ and $f_2 < 0$ hold simultaneously.

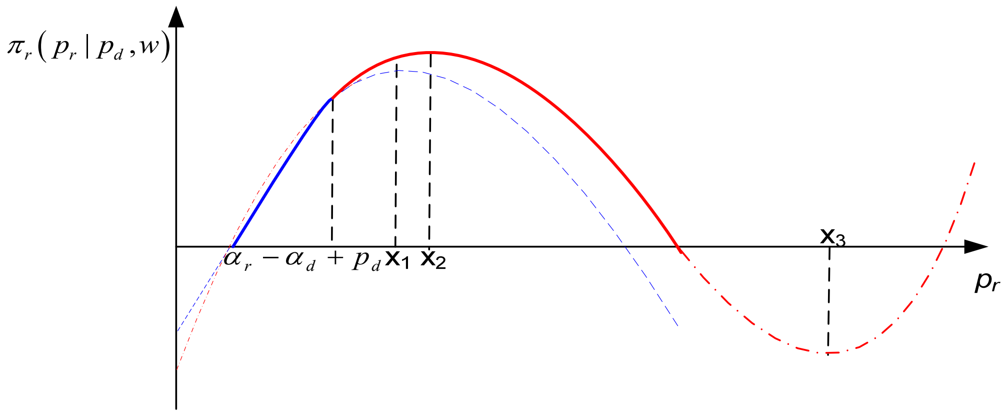
Hence, we proof $f_1 \cdot f_2 > 0$. □

Remark. We can also explain Lemma 3.2 from the perspective of derivative of $\pi_r\left(p_r \mid p_d, w\right)$. As $x_1$ is a local maximum, $\left.\frac{\partial \pi_r}{\partial p_r}\right|_{p_r = x_1} = 0$ and $\left.\frac{\partial^2 \pi_r}{\partial p_r^2}\right|_{p_r = x_1} < 0$. Since when $p_r < \alpha_r - \alpha_d + p_d$ only one stationary point $x_1$ exists, we know that $\left.\frac{\partial \pi_r}{\partial p_r}\right|_{p_r = \alpha_r - \alpha_d + p_d} < 0$ if $x_1 < \alpha_r - \alpha_d + p_d$. Similarly, the fact that $x_2$ is a local maximum shows that $\left.\frac{\partial \pi_r}{\partial p_r}\right|_{p_r = x_2} = 0$ and $\left.\frac{\partial^2 \pi_r}{\partial p_r^2}\right|_{p_r = x_2} < 0$. Because the other stationary point $x_3$ is larger than $x_2$, which means that $\left.\frac{\partial \pi_r}{\partial p_r}\right|_{p_r < x_2} > 0$ and $\left.\frac{\partial \pi_r}{\partial p_r}\right|_{x_2 < p_r < x_3} < 0$. As $\pi_r\left(p_r \mid p_d, w\right)$ is continuously differentiable over [0, $\alpha_r$], we know that $x_2 < \alpha_r - \alpha_d + p_d$. If $x_1 = \alpha_r - \alpha_d + p_d$, then $\left.\frac{\partial \pi_r}{\partial p_r}\right|_{p_r = \alpha_r - \alpha_d + p_d} = 0$. $\pi_r\left(p_r \mid p_d, w\right)$ with $q_r$ in equation (3.3) has only two stationary points; and the local minimum point $x_3$ is larger than the local maximum point $x_2$. So from the continuous differentiability of $\pi_r(p_r \mid p_d, w)$ over [0, $\alpha_r$], we know that $x_2 = \alpha_r - \alpha_d + p_d$.

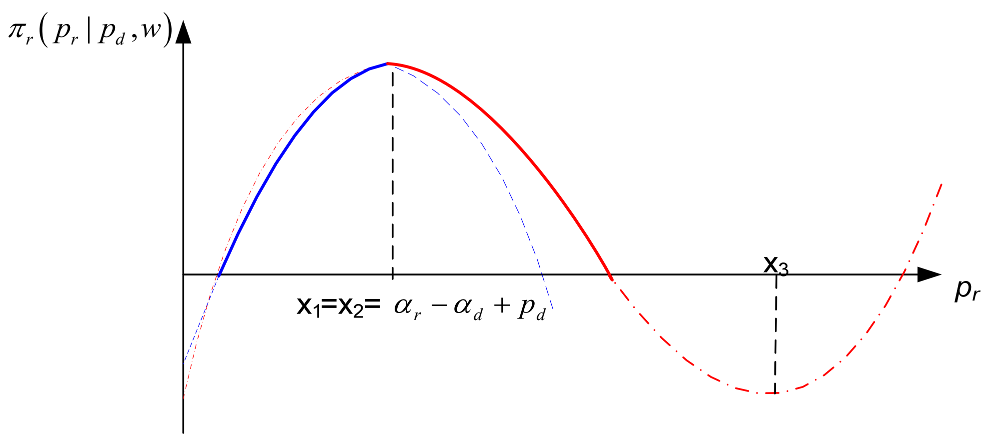
Figure 3.5 provides a visual illustration of Lemma 3.2. The plot in Figure 3.5 (a) shows the case that $x_1 < \alpha_r - \alpha_d + p_d$ and $x_2 < \alpha_r - \alpha_d + p_d$. We can see that $x_1$ is the only one local maximum of $\pi_r(p_r \mid p_d, w)$ when $p$ is in the range of [0, $\alpha_r$]. The plot in Figure 3.5 (b) shows the case that both $x_1$ and $x_2$ are larger than $\alpha_r - \alpha_d + p_d$, and in which $x_2$ becomes the only one local maximum of $\pi_r(p_r \mid p_d, w)$ on [0, $\alpha_r$]. Figure 3.5 (c) shows the case that $x_1 = x_2 = \alpha_r - \alpha_d + p_d$. So the only local maximum is $\alpha_r - \alpha_d + p_d$.




(a)




(b)




(c)

Figure 3.5 Illustration of Lemma 3.2

Lemma 3.2 suggests that $\pi_r(p_r \mid p_d, w)$ has only one local maximum point over [0, $\alpha_r$], either $x_1$, or $x_2$, or $x_1 = x_2 = \alpha_r - \alpha_d + p_d$. Thus we obtain the following theorem on retailer's optimal price $p_r^*$.

**Theorem 3.1** i) When $p_d$ and $w$ fall into the region $R_1 = \{(p_d, w) | 2\alpha_d(c_r + w - \alpha_r) \le p_d^2 - 3\alpha_d^2 + 2\alpha_d p_d\}$, the optimal resell price

$$p_r^* = \frac{c_r + w + \alpha_r}{2} - \frac{(\alpha_d - p_d)^2}{4\alpha_d}; \tag{3.39}$$

especially, when $2\alpha_d(c_r + w - \alpha_r) = p_d^2 - 3\alpha_d^2 + 2\alpha_d p_d$, $p_r^* = \alpha_r - \alpha_d + p_d$;

ii) When $p_d$ and $w$ fall into the region $R_2 = \{(p_d, w) | 2\alpha_d(c_r + w - \alpha_r) > p_d^2 - 3\alpha_d^2 + 2\alpha_d p_d\}$, the optimal resell price

$$p_r^* = \frac{1}{3}\left(2p_d + w + c_r + 2\alpha_r - \sqrt{(\alpha_r - w - c_r + p_d)^2 + 3p_d^2}\right). \tag{3.40}$$

*Proof* When $p_d$ and $w$ fall in $R_1$, equation (3.32) shows that $x_1 < \alpha_r - \alpha_d + p_d$, and Lemma 3.2 suggests that $x_2 < \alpha_r - \alpha_d + p_d$. This is just the case shown in Figure 3.5 (a) where $x_1$ is the only possible optimal solution. Noting that $3\alpha_d^2 - p_d^2 - 2\alpha_d p_d > (\alpha_d - p_d)^2$, we know $x_1$ is larger than $c_r + w$ per equation (3.27) and so is feasible. Therefore, when $(p_d, w) \in R_1$, $p_r^* = x_1$. The special case in $2\alpha_d(c_r + w - \alpha_r) = p_d^2 - 3\alpha_d^2 + 2\alpha_d p_d$ is the plot shown in Figure 3.5 (c). Similarly, we can show that when $(p_d, w) \in R_2$ both $x_1$ and $x_2$ are larger than $\alpha_r - \alpha_d + p_d$ (Figure 3.5 (b)). And as $c_r + w < x_2 < \alpha_r$, we have $p_r^* = x_2$. □

#### 3.2.2.2 Manufacturer's pricing problem

With the anticipation of retailer's reaction functions, the manufacturer's problem is to maximize its own profit by choosing wholesale price $w$ and direct price $p_d$ subject to $w \le p_d$. The manufacturer cannot do price discrimination by charging a higher wholesale price than direct sale price, because the retailer would switch its purchase to the direct channel and refuse to pay higher wholesale price. As we can see, manufacturer becomes much more powerful than retailer after the addition of direct channel.

Following retailer's expected response with given manufacturer's decision, we need to examine manufacturer's decisions in two regions $R_1$ and $R_2$ in Theorem 3.1 respectively, and then fix the region of ($p_d$, $w$) which provides the higher profit.

First, consider region

$$\mathrm{R}_1 = \left\{(p_d, w) \middle| 2\alpha_d(c_r + w - \alpha_r) \le p_d^2 - 3\alpha_d^2 + 2\alpha_d p_d\right\},$$

where retailer's expected price $p_r$ follows equation (3.39), $q_r$ follows equation (3.1) and $q_d$ follows equation (3.2). Manufacturer's problem is

$$\text{DOP1:} \quad \max_{w, p_d} \pi_m = wq_r + (p_d - c_d)q_d$$

$$\text{s.t.} \quad 2\alpha_d(c_r + w - \alpha_r) \le p_d^2 - 3\alpha_d^2 + 2\alpha_d p_d$$

$$p_d \ge w$$

$$0 \le p_d \le \alpha_d$$

$$0 \le w \le \alpha_r - c_r$$

Similarly, we can model manufacturer's pricing problem in $R_2$ as DOP 2, where retailer's expected price $p_r$ follows equation (3.40), $q_r$ follows equation (3.3) and $q_d$ follows equation (3.4).

$$\text{DOP2:} \quad \max_{w, p_d} \pi_m = wq_r + (p_d - c_d)q_d$$

$$\text{s.t.} \quad 2\alpha_d(c_r + w - \alpha_r) > p_d^2 - 3\alpha_d^2 + 2\alpha_d p_d$$

$$p_d \ge w$$

$$0 \le p_d \le \alpha_d$$

$$0 \le w \le \alpha_r - c_r$$

### 3.2.2.3 Algorithm

We will apply searching method to solve manufacturer's problem. Regarding DOP1, for each possible $p_d$, as

$$\frac{\partial \pi_m}{\partial w} = q_r + \frac{1}{2}\left[ w\left( -\frac{1}{\alpha_r} \right) + (p_d - c_d)\frac{\alpha_d - p_d}{\alpha_r \alpha_d} \right]$$

$$= \frac{1}{2\alpha_r \alpha_d}\left[ -2\alpha_d w + \alpha_d(\alpha_r - c_r) - \frac{1}{2}(\alpha_d - p_d)^2 + (p_d - c_d)(\alpha_d - p_d) \right],$$

we could obtain the root of $\frac{\partial \pi_m}{\partial w} = 0$,

$$w_0 = \frac{1}{2\alpha_d}\left[ \alpha_d(\alpha_r - c_r) - \frac{1}{2}(\alpha_d - p_d)^2 + (p_d - c_d)(\alpha_d - p_d) \right]. \tag{3.42}$$

If $w_0 \le \min(p_d, \alpha_r - c_r)$, the optimal wholesale price $w^* = w_0$; or, $w^* = \min(p_d, \alpha_r - c_r)$. Regarding DOP2, due to the complexity of derivatives of $p_r$ in equation (3.40) with respect to $p_d$ and $w$, exhaustive method is used to find out the optimal pair of ($p_d$, $w$).

Since DOP1 and DOP2 have non-overlapping feasible regions, any pair of ($p_d$, $w$) could be feasible solution for only one of these two. For each possible value of $p_d$, as $0 < w \le p_d$, it could only be feasible solution to DOP1 with any possible value of $w$ when $p_d^2 \ge 3\alpha_d^2 - 2\alpha_d(\alpha_r - c_r)$ and only to DOP2 when $p_d^2 + 2\alpha_d p_d < 3\alpha_d^2 - 2\alpha_d(\alpha_r - c_r)$. When

$$p_d^2 < 3\alpha_d^2 - 2\alpha_d(\alpha_r - c_r) \le p_d^2 + 2\alpha_d p_d,$$

that is,

$$\sqrt{2\alpha_d(\alpha_d - \alpha_r + c_r)} - \alpha_d \le p_d < \sqrt{3\alpha_d^2 - 2\alpha_d(\alpha_r - c_r)},$$

it depends on the value of $w$ to determine which problem we should look into. When

$w \le \frac{1}{2\alpha_d}\left(p_d^{\,2} + 2\alpha_d p_d - 3\alpha_d^{\,2}\right) + \left(\alpha_r - c_r\right)$, ($p_d$, $w$) falls in $R_1$ and becomes a feasible solution of DOP1; otherwise, it falls in feasible region of DOP2.

Let X= $\sqrt{2\alpha_d\left(\alpha_d - \alpha_r + c_r\right)}$ , Y= $\sqrt{3\alpha_d^{\,2} - 2\alpha_d\left(\alpha_r - c_r\right)}$ , and Z= $\frac{1}{2\alpha_d}\left(p_d^{\,2} + 2\alpha_d p_d - 3\alpha_d^{\,2}\right) + \left(\alpha_r - c_r\right)$. The solution approach is as follows:

(1) Exhaustively search each $c_d \le p_d \le X$ and $0 \le w \le \min(p_d, \alpha_r - c_r)$ for DOP2.

a. Discrete the range of $\left[c_d, X\right]$ with step size $\eta_1$ into series $\left\{p_d^0, p_d^1, \ldots, p_d^n\right\}$, and $p_d^0 = c_d$, $p_d^n = X$.

b. Discrete the range $\left[0, \min(p_d, \alpha_r - c_r)\right]$ with step size $\eta_2$ into series $\left\{w^0, w^1, \ldots, w^m\right\}$, and $w^0 = 0$, $w^m = \min\left(p_d, \alpha_r - c_r\right)$.

c. For each $p_d^i$ ($i$ = 0, …,$n$), calculate the value of objective function for each $w^j$ ($j$ = 0, …,$m$), and record the pair providing maximum objective value.

(2) Exhaustively search each $X < p_d < Y$ and $Z \le w \le \min(p_d, \alpha_r - c_r)$ for DOP2; the detailed method is the same as step (1).

(3) For each $X < p_d < Y$, the optimal $w$ of DOP1 is $w_1$=min ($w_0$, Z).

(4) For each $Y \le p_d \le \alpha_d$, the optimal $w$ of DOP1 is $w_2$=min ($w_0$, $p_d$).

(5) Check all the above optimums and choose the pair that provides the highest profit.

## 3.3 Numerical Studies

### 3.3.1 An illustrative example

In this subsection we present a numerical example to illustrate the solution to pricing optimization problem of a dual-channel distribution system. The related input parameters are given as $\alpha_r = 10$, $\alpha_d = 8$, $c_r = c_d = 2$. The results are shown in Table 3.1.

Table 3.1 Numerical Example Results

| | Vertically Integrated Firm | | Independent Firms | |
|---|---|---|---|---|
| | Single channel | Dual-channel | Single channel | Dual-channel |
| Retail Price | 6 | 6.35456 | 8 | 7.90 |
| Wholesale Price | n/a | n/a | 4 | 4.31 |
| Direct Price | n/a | 5.44 | n/a | 5.14 |
| Retailer Demands | 0.4 | 0.324 | 0.2 | 0.16 |
| Direct Channel Demands | n/a | 0.244 | n/a | 0.33 |
| Retailer Profits | n/a | n/a | 0.4 | 0.25 |
| Manufacturer Profits | n/a | n/a | 0.8 | 1.73 |
| Total Profits | 1.6 | 2.25 | 1.2 | 1.98 |

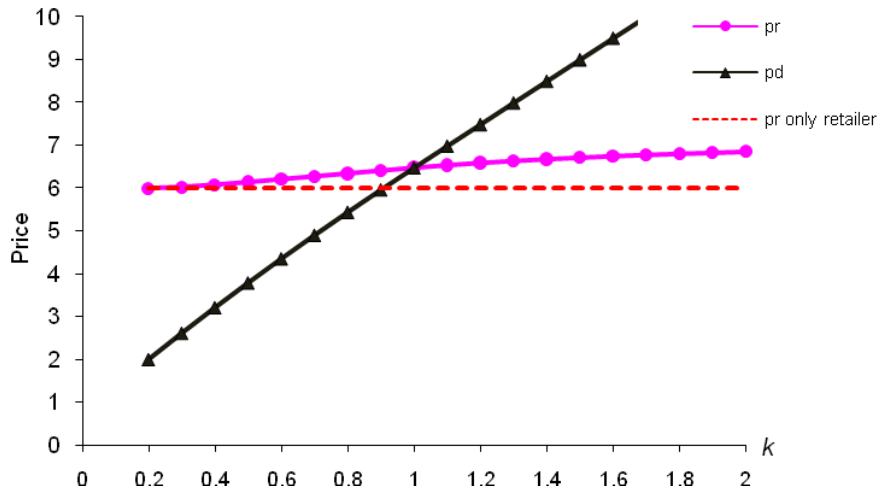
The first two columns of Table 3.1 show the pricing strategy in a vertically integrated firm that optimizes the total expected profit of the direct channel and the traditional channel jointly over $p_r$ and $p_d$ for single channel system and dual-channel system. We can see that adding a direct channel leads to a higher price at the traditional retailer and a lower price at the direct channel. This is because the consumers' consumption value at the retailer is a bit higher than that at the direct channel. Total demand at these two channels is increased and so is the total profit. The last two columns show the pricing strategies when the manufacturer and retailer act as independent firms and compete with each other. It is shown that retailer's profit decreases 37.5% while manufacturer's profit increases 116.25%. Furthermore, we can see from Table 3.1 that the total profit under centralized decision making is much higher than that under decentralized system. Hence, the opportunity that manufacturer and retailer both could achieve higher profits exists. Profit sharing strategies would be worth for studying.

### 3.3.2 Parametric analysis

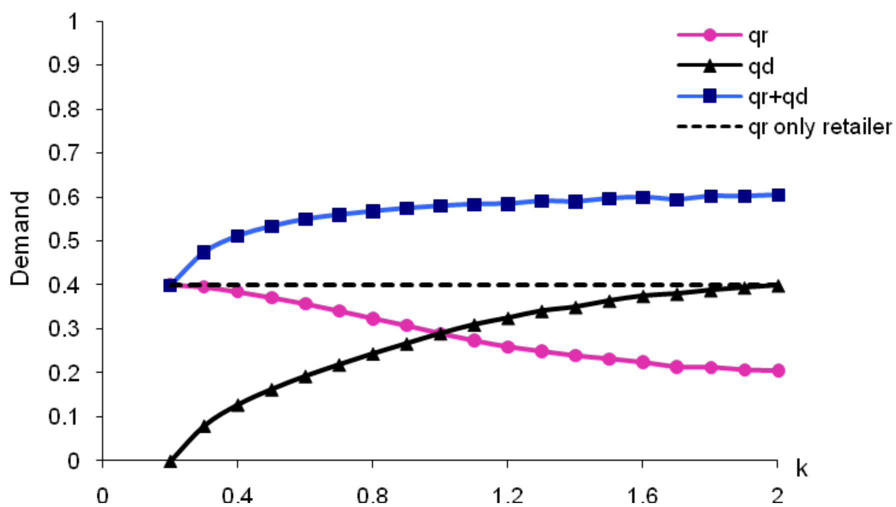
To gain managerial insights of the dual-channel distribution system, we investigate impact of consumer's preference of the direct channel and direct selling cost $c_d$ on our pricing strategy using the models described in this chapter.

Similar to Cattani et al. (2006), we define $k = \dfrac{\alpha_d}{\alpha_r}$ to represent consumer's relative channel preference. When $k < 1$, consumer has lower average consumption value of buying from direct channel versus the retail shop; otherwise, consumption value is higher at direct channel.

First we study the vertical integrated supply chain model. Figure 3.6 shows the impact of $k$ on pricing strategy and channel demands. We note that in Figure 3.6 (a) the prices of both channels rise as $k$ increases. And retail price in a vertically integrated dual-channel system is a little higher than that in a vertically integrated single channel system. In addition, Figure 3.6 (b) indicates the total demand also increases because the direct demand grows at a faster rate than retailer demand falls down at as the retail price increases. As $\alpha_d$ increases, which means that customer's reservation price for direct channel stochastically increases, manufacturer can raise the direct channel price to make more margin. Meanwhile, the possibility that customer would go to direct channel grows up, resulting in decrease of retail demand. Then the manufacturer has to raise the price at retail channel to sustain profitability at the retail channel. These findings suggest that coordinated dual-channel distribution system could expand sales and increase profits.




(a)




(b)

Figure 3.6 Impact of channel preference $k$ on pricing strategy ($\alpha_r = 10$, $c_r = c_d = 2$) under centralized decision making: (a) optimal prices, (b) demands

In the case of $c_r = c_d$, we can see from Figure 3.6(a) that $p_r > p_d$ when $\alpha_r > \alpha_d$ and $p_r < p_d$ when $\alpha_r < \alpha_d$. We plot the difference between $p_r$ and $p_d$ in Figure 3.7. It is shown here that these two differences are positively related, which follows our intuition that greater difference in acceptance between these two channels leads to larger difference between two prices. In addition, we can see that $p_r - p_d < \alpha_r - \alpha_d$ when $k < 1$ (i.e., $\alpha_r > \alpha_d$) and $p_r - p_d > \alpha_r - \alpha_d$ when $k > 1$ (i.e., $\alpha_r < \alpha_d$). This observation would simplify the price determination process stated in Section 3.2.1. If $\alpha_r \geq \alpha_d$, the optimal price pair falls into the area of Case 1 in Figure 3.4 and could be determined by equations (3.13) and (3.14); otherwise, it is as Case 2 and could be determined by equations (3.18) and (3.19). Indeed, we can see from Figure 3.7 that these two cases are central symmetric to the point of $\alpha_r = \alpha_d$, which means that these two channels are symmetrically priced according to $\alpha_r$ and $\alpha_d$ in the case of equal unit cost.

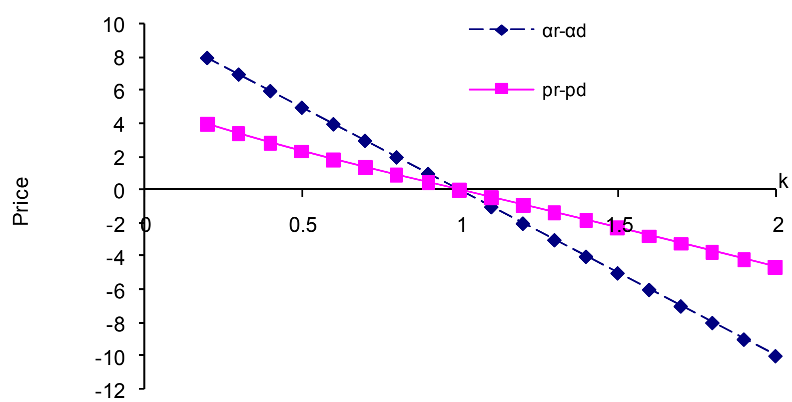
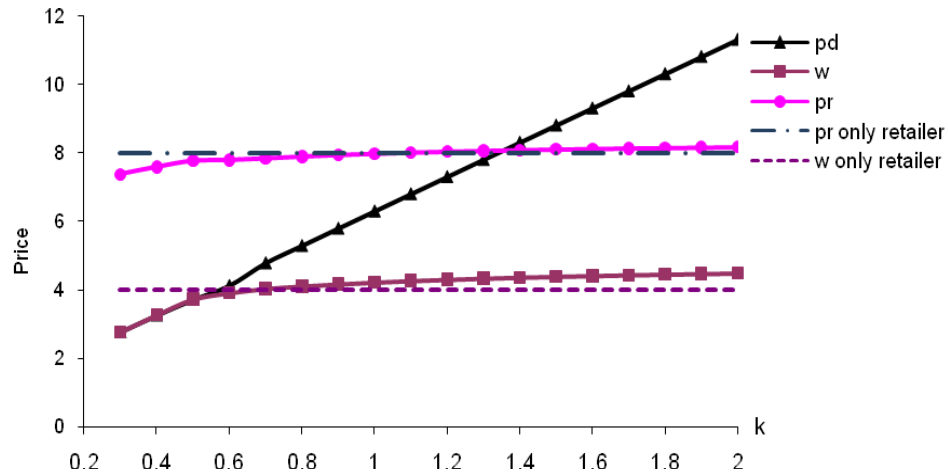


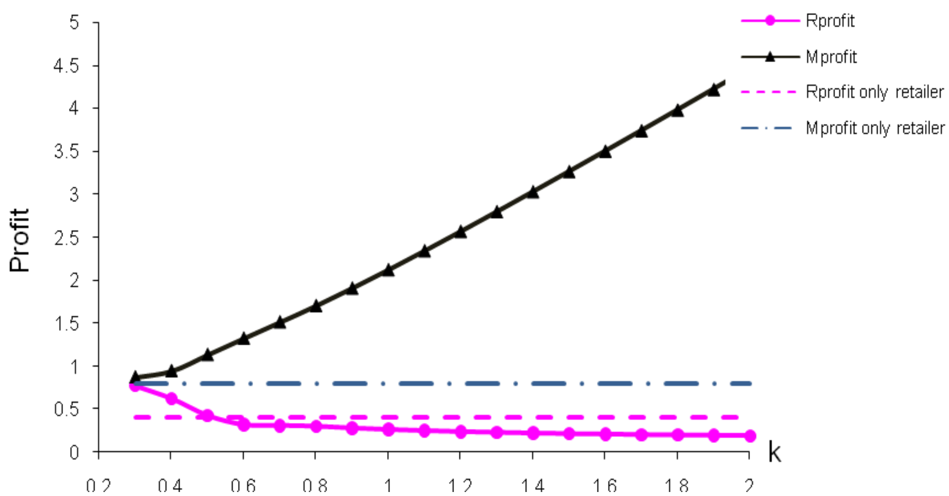

Figure 3.7 Difference between $p_r$ and $p_d$

Now we consider the pricing strategy of independent competing manufacturer and retailer. Figure 3.8 shows that, though retailer price and wholesale price increase as $k$

increases, the increments are small so that they are almost the same as in the case of single channel. When direct channel is much less preferred than the retail shop ($k$ < 0.5), direct price is very low, which restricts the wholesale price since it is required that $w \leq p_d$ to prevent retailer buying from direct channel. The lower wholesale price leads to lower retailer price and higher retailer profits. However, retailer's profit shrinks sharply as $\alpha_d$ approaches $\alpha_r$ since its market power drops quickly. From the graph, we can see that the retailer's profit decreases, but it does not seem to approach zero. To explain this phenomenon, we let $\alpha_d$ approach infinity, and then $p_d$ would also approach infinity with a slower rate. This is because that $p_d$ is monotonically increasing as $\alpha_d$. As $\alpha_d$ increases, which means that customer's reservation price for direct channel stochastically increases, manufacturer can raise the direct channel price to make more margin. The reason the increasing rate of $p_d$ is lower than that of $\alpha_d$ is that manufacturer need to restrict the rate to boost the demand as well as margin to achieve the highest profitability. But $p_r$ stays at a finite value leading to a positive demand at the retailer and giving rise to a positive profit. Thus, a direct channel is highly detrimental to retailer in most cases, except when direct channel is far less preferred to retailer shop; it is consistent with what we have seen in reality where retailer is always opposed to this action.




(a)




(b)

Figure 3.8 Impact of channel preference $k$ on pricing strategy ($\alpha_r$ =10, $c_r$ = $c_d$ =2) under decentralized decision making: (a) optimal prices (b) demands

From manufacturer's point of view, introduction of a direct channel raises its market power by weakening that of retailer. Figure 3.9 shows a set of manufacturer's profit curves with different values of $c_d$. Even with a significant disadvantage of the direct channel ($c_d$ = 4 vs. $c_r$ = 2), manufacturer has strong incentive to open a direct channel. Considering retailer's strong opposition to the direct channel, the most serious problem for a manufacturer to open a direct channel would be to persuade retailer to accept this marketing strategy. Profits-sharing strategy would be studied further. Another point that needs to be highlighted in this graph is that higher valuation of direct channel brings more profit increment for the overall supply chain. It is obviously a strong incentive for manufacturer to improve the service quality at the direct channel.

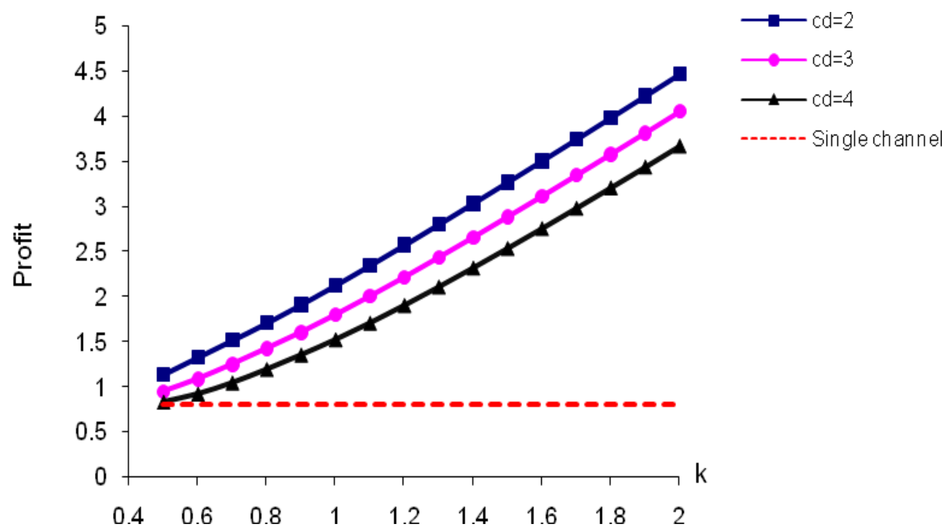



Figure 3.9 Impact of $c_d$ on manufacturer profit

## 3.4 Conclusion

In this chapter, we focus on the competing pricing strategy in a dual-channel distribution system. With the probabilities that customer would purchase from each channel deriving from consumer choice model, we obtain the optimal prices under centralized decision making as a benchmark. Then, under decentralized decision making, we built up a two-stage Stackelberg game model to study the simultaneously horizontal and vertical competition between a monopolistic manufacturer and retailer.

Our research shows that adding one more channel could significantly increase total profit in both centralized and decentralized supply chains, but the effects of horizontal and vertical competitions reduce this increase in decentralized one. Unlike a traditional distribution structure in which double-marginalization and substitutability tend to counteract, channel inefficiencies are always present and enlarge as customer's consumption value on direct channel grows up. We also examine the impact of variable cost at direct channel on manufacturer's profit. Even with a significant disadvantage of direct channel, manufacturer has strong incentive to open direct channel. However, due to dramatic damage to retailer's profit, how to persuade retailer to accept and cooperate should be a substantial topic to work on.

Although we only investigate in this model manufacturer and retailer's pricing decisions with constant unit merchandising cost, inventory decision as one of the most important ordinary operations is worth studying in dual-channel distribution system. In the following chapter, we will do research on this problem.

# CHAPTER 4 JOINT PRICE AND INVENTORY DECISION WITH DYNAMIC CUSTOMER CHOICE

There are a few common characteristics among the manufacturers adopting dual-channel distribution system. Firstly, the products in dual-channel supply chains usually have short life cycles. For instance, the life cycle of fashion products, like Nike, is just one season about 3 to 6 months (McIntyre and Perlman, 2000). Consumer electronics, due to rapid development of high technologies, usually refresh every 12 to 18 months.

Facing long production lead times and short product life cycles, manufacturers need to acquire precise demand information and to broaden market coverage, which is exactly the reason for popularity of dual-channel supply chains.

Secondly, these manufacturers usually have certain market power and a population of brand loyal customers, so that they could control the market prices to avoid throat-cutting price competition and sustain a stable brand reputation when facing fierce inter-channel substitution. According to various marketing literature, it is not advisable for small companies or non-famous manufacturers to open direct channel, because uncontrollable inter-channel conflicts would severely destroy the relationship with existing retailers, and result in disastrous damage to the manufacturer itself.

We focus on the problem of stock levels balance between these two channels and the optimal price of product, combining independent retail distribution channel with a direct channel. Due to the long production lead times and short product life cycles, retailers have to place orders to manufacturers firstly, and manufacturers have to decide production capacity and finish the production before the beginning of the selling season. Facing a dual-channel supply chain, manufacturers' inventories cover both retailers' orders and customer demands at direct channel. Since the inventory for retailer orders is make-to-order, while that for demands at direct channel is make-to-stock, manufacturers have to forecast the direct channel demands. As independent retailers and manufacturer direct channel hold inventory separately, it is observed that, when stock-out occurs at retail channel, unmet customer demands at the retailer would shift to direct channel. In contrast, when stock-out occurs at direct channel, customers usually will not visit a local retailer, but they may shift to another brand with similar product, resulting in lost sales (Dai et al., 2006). Thus, different from retailer's ordering decision, manufacturers have to consider the part of unfulfilled demand at retail channel that would probably shift to direct channel.

However, this kind of demand shifting has been widely ignored among literatures on dual-channel supply chain, except Geng and Mallik (2007) who consider a capacity allocation game with heterogeneous market price. Actually, there are scarce studies on both price and inventory decisions in dual-channel supply chains. The objective of this study is to examine how these operational decisions would be impacted by the inter-channel shifting of excess demands and also the optimal profits.

In this chapter, we study the jointly optimal price and inventory decision in a single period general dual-channel supply chain model where a monopolist manufacturer sells its product to customers through a traditional retailer and also its own direct channel. Manufacturer sets identical resell price for both two channels to avoid inter-channel price competition. Independent retailer announces its order quantity to manufacturer, and then manufacturer decides the production capacity. Special attention has been put on the effect of this inter-channel stock-out demand shifting on manufacturer's pricing and capacity decision. We build up the distribution of the effective direct channel demand conditioning on retailer's order quantity and resell price. Although retailer's problem is a standard newsvendor problem, manufacturer's decision is a combined two-stage price-setting newsvendor problem that no closed form solution could be obtained. As the optimal behaviors depend critically on the type of stochastic demand functions, we numerically study this problem based on two of the most frequently used classes of demand processes in the marketing and industrial organization literature, one the linear demand function, the other the multinomial logit model (MNL). It is observed that the effect of inter-channel stock-out demand shifting on manufacturer's capacity is quite sensitive to the retailer's demand variance and the rate that spillover demand shifts, while that effect on optimal price is not so dramatic.

## 4.1 The Model

This chapter studies a single period joint pricing and inventory decision problem in a general dual-channel supply chain where a monopolist manufacturer sells its product to customers through a traditional retailer as well as its own direct channel (Figure 1.1). Several assumptions are made in this model.

1) The manufacturer is a monopoly player in the market, and produces a single period/seasonal product.

2) Different from the model in Chapter 3, manufacturer implements a control price policy and sets the same selling price $p$ across retail channel and direct channel.

To name a real instance of our model, we can look at the fresh launching period of iPhone. Though mobile phone market is quite mature, iPhone, with its smooth, almost button-free contours, elegance appearance and diverse ingenious features, attracted a large population of customers who disregarded other products in the market and were eager to experience this epoch-making phone. Since the development cycle of such a product would take at least 6-12 months, there was no competitor in the first half year and iPhone acts as a monopoly in the market. Customers can buy it through retailer as well as online channels. Its strong market power allows Apple to maintain the same price at these two channels. But as time went by, various product experience reviews, rapid development of competing products, along with news of prompt launching of second generation iPhone, faded away customers desire for this product and changed the market significantly. iPhone lost its monopoly power and the value of this product shrieked. So for the first six months after launching, iPhone can be viewed as a single period monopoly product with same price across two distribution channels.

3) A constant unit wholesale price contract between manufacturer and retailer is adopted here, and wholesale price $w = kp$, where $k$ is a representative of firms' bargaining power and assumed to be fixed among various products that the manufacturer sells to the retailer. This simple contract can be used when two firms have long-term co-operation in a supply chain keeping a stable value of $k$ for various products.

Then with given selling price and corresponding wholesale price, retailer determines its order quantity $q_r$. Manufacturer at last builds up its capacity $C = q_r + q_d$, where $q_d$ is manufacturer's order quantity for the direct channel. Both the manufacturer and retailer have only one replenishment chance before the selling season. Manufacturer's unit production cost is $c_p$ and direct channel unit selling cost is $c_d$. Retailer's unit selling cost is $c_r$. At the end of selling season, all leftover inventories could be salvaged at value of $v$.

The demand faced by each channel $i$ ($i \in \{r, d\}$, where $r$ = traditional retail channel and $d$ = direct channel), $D_i$, has a distribution that may depend on the resell price. We assume that the demand variables are of the additive form, i.e.,

$$D_i = y_i(p) + \varepsilon_i, \tag{4.1}$$

where $\varepsilon_i$ is a normal distributed random variable and independent of resale price $p$. The additive form implies that the variances of the channel demands are exogenous constants and also independent of resale price $p$. Without loss of generality, we normalize $E(\varepsilon_i) = 0, i \in \{r, d\}$, so that $E(D_i) = y_i(p)$. In other words, the functions $y_i(p)$ may be viewed as representing the expected channel sales volumes. Let $\mathrm{Var}(\varepsilon_i) = \sigma_i^2$, and then $\varepsilon_i = u_i \sigma_i$ where $u_i$ follows standard normal distribution whose cumulative distribution function is $\Phi(\cdot)$ and probability density function is $\phi(\cdot)$. For analytical tractability, we assume that $\varepsilon_r$ and $\varepsilon_d$ are independent, as well as $D_r$ and $D_d$. At this stage, we only make a general assumption regarding the shape of the mean sales function $\{y_i(p)\}$ that

$$\frac{dy_i(p)}{dp} \le 0, \; i \in \{r, d\}. \tag{4.2}$$

which means that $y_i(p)$ is a non-increasing function of $p$.

The decision process is that manufacturer sets $p$ first, and then retailer decides $q_r$, and lastly manufacturer decides $q_d$. Observe retailer and manufacturer's order quantities impact only their own profits but not that of the other channel without consideration of demand shift when stock-out occurs. Both of these two channels' capacity decisions could be modeled as independent standard newsvendor problems with given $p$. However, when taking inter-channel excess demand shifting into account, manufacturer's profit would be influenced by retailer's order quantity, because excess demand at retailer may shift to direct channel. Low inventory level at the retailer may induce a large amount of demand loss, part of which would switch to direct channel, and then increase the demand at the direct channel. Since it is assumed that there is no excess demand shifting to retailer from direct channel, retailer's profit would still only depend on resale price $p$ and order quantity $q_r$.

Similar as Chapter 3, this decision process is modeled as a Stackelberg game where manufacturer plays as a game leader and retailer as a follower. We solve their problems backwards, first retailer's inventory decision with given resell price, and then manufacturer's inventory and price decision.

### 4.1.1 Retailer's problem

In this section, we investigate retailer's optimal order quantity with given resale price $p$ in both situations that inter-channel excess demand shifting is considered or not. As retailer's profit is only influenced by $p$ and $q_r$ no matter whether this shifting is taken into account, its order quantity decision would only depend on $p$ in these two scenarios.

Retailer's profit function is

$$\pi_r(q_r \mid p) = (p - w - c_r)q_r - (p - v)E[q_r - D_r]^+ , \tag{4.2}$$

Let $q_r = y_r(p) + z_r\sigma_r$, and then $z_r$ could be viewed as the safety stock factor in most inventory models. As $w = kp$, $\pi_r$ could be rewritten as

$$\pi_r(z_r \mid p) = [(1-k)p - c_r]y_r(p) - L(z_r \mid p), \tag{4.3}$$

where

$$L(z_r \mid p) = \sigma_r \left\{[(1-k)p - c_r]E[u - z_r]^+ + (kp + c_r - v)E[z_r - u]^+\right\}. \tag{4.4}$$

Indeed, $L(z_r \mid p)$ is the loss function of uncertainty of demand. With given $p$, retailer's order quantity problem is a standard newsvendor problem. $\pi_r$ is concave with respect to $z_r$, and by solving $\frac{\partial \pi_r}{\partial z} = 0$, we obtain

$$z_r^* = \Phi^{-1}\left(\frac{(1-k)p - c_r}{p-v}\right) \tag{4.5}$$

and

$$q_r^*(p) = \sigma_r \Phi^{-1}\left(\frac{(1-k)p - c_r}{p-v}\right) + y_r(p). \tag{4.6}$$

From the solution of standard newsvendor problem, we know $F(z_r) = \dfrac{\text{underage cost}}{\text{underage cost} + \text{overage cost}}$. In this case, the underage cost is $(1-k)p - c_r$, and the overage cost is $kp - v + c_r$. The following proposition shows how the optimal safety stock factor $z_r^*$ would change with resale price $p$.

**Proposition 4.1** From equation (4.5) we have

i) If $k > \dfrac{v-c_r}{v}$, $\dfrac{dz_r^*}{dp} > 0$;

ii) if $k = \dfrac{v-c_r}{v}$, $\dfrac{dz_r^*}{dp} = 0$ and $z_r^* = \Phi^{-1}\left(\dfrac{c_r}{v}\right)$;

iii) if $k < \dfrac{v-c_r}{v}$, $\dfrac{dz_r^*}{dp} < 0$;

*Proof* . Note $\dfrac{dz_r^*}{dp} = \dfrac{c_r - (1-k)v}{\phi(z_r^*)(p-v)^2}$. As $\phi(z_r^*) > 0$, the sign of $\dfrac{dz_r^*}{dp}$ depends on that of $c_r - (1-k)v$. □

Remark. As $k \in [0,1]$, if $v < c_r$, $z_r$ always increases with $p$, which means that higher resell price would always induce retailer to set a high service level.

### 4.1.2 Manufacturer's inventory problem

Manufacturer's problem consists of determining the optimal order quantity for direct channel $q_d$ and the resale price $p$. We start with manufacturer's order quantity decision.

As part of the loss demand at the retailer would shift to direct channel, the true demand facing direct channel, $R_d$, includes the "first-choice" customers as well as any spill-over customers from the retail channel. The shifting rate, denoted by λ, describes the percentage of unfulfilled retail demand transfer ring to direct channel. It is assumed to be deterministic in this study. Then

$$R_d = D_d + \lambda(D_r - q_r)^+ = y_d(p) + \varepsilon_d + \lambda(D_r - q_r)^+$$

$$= y_d(p) + \widetilde{R}_d, \tag{4.7}$$

where

$$\widetilde{R}_d = \varepsilon_d + \lambda(D_r - q_r)^+ = \begin{cases} \varepsilon_d + \lambda(\varepsilon_r - z_r\sigma_r), & \varepsilon_r \geq z_r\sigma_r \\ \varepsilon_d, & \varepsilon_r < z_r\sigma_r \end{cases}. \tag{4.8}$$

In the case that there is no demand loss at the retail channel, $\widetilde{R}_d$ denotes the random part of the "first-choice" demand faced by the direct channel $\varepsilon_d$. When excess demand exists at the retail channel, $\widetilde{R}_d$ contains $\varepsilon_d$ and the shifting spill-over demand $\lambda(\varepsilon_r - z_r\sigma_r)$. Let $\overline{R}_d = \widetilde{R}_d / \sigma_d$. The cumulative distribution function of $\overline{R}_d$:

$$H(t \mid p, z_r) \equiv \Pr(\overline{R}_d \leq t \mid p, z_r)$$

$$= \Pr(\varepsilon_d \leq t, \varepsilon_r < z_r\sigma_r) + \Pr\left(\frac{\varepsilon_d + \lambda(\varepsilon_r - z_r\sigma_r)}{\sigma_d} \leq t, \varepsilon_r \geq z_r\sigma_r\right)$$

$$= \Pr(u_d \leq t, u_r \leq z_r) + \Pr\left(u_d + \frac{\sigma_r}{\sigma_d}\lambda(u_r - z_r) \leq t, u_r \geq z_r\right)$$

$$= \int_{-\infty}^{t} \phi(u_d) \int_{-\infty}^{z_r} \phi(u_r) du_r du_d + \int_{z_r}^{+\infty} \phi(u_r) \int_{-\infty}^{t-\frac{\sigma_r}{\sigma_d}\lambda(u_r - z_r)} \phi(u_d) du_d du_r$$

$$= \int_{-\infty}^{t} \phi(u) \Phi\left( z_r + \frac{(t-u)\sigma_d}{\lambda \sigma_r} \right) du . \quad (4.9)$$

and probability density function

$$h(t \mid p, z_r) \equiv \frac{\partial}{\partial t} \int_{-\infty}^{t} \phi(u) \Phi\left( z_r + \frac{(t-u)\sigma_d}{\lambda \sigma_r} \right) du$$

$$= \Phi(z_r)\phi(t) + \Phi(t)\phi\left( z_r + \frac{\sigma_d}{\lambda \sigma_r} \right). \quad (4.10)$$

As $E[D_r - q_r]^+ = \sigma_r \int_{z_r}^{+\infty} (1 - \Phi(t)) dt$, we know $E[R_d] = y_d(p) + \lambda \sigma_r \int_{z_r}^{+\infty} (1 - \Phi(t)) dt$. Since the later addition of $E[R_d]$ is always positive, the expected effective demand at the direct channel is positive proportional to the shifting rate and variation of retailer's demand. Moreover, lower retailer safety factor $z_r$ would also increase $E[R_d]$. In the next section, we are going to examine the effects of these three parameters.

The expected profit from the direct channel is

$$\pi_d(q_d \mid p) = (p - c_d - c_p) q_d - (p - v) E[q_d - R_d]^+ , \quad (4.11)$$

With given $p$, the derivative of $\pi_d(q_d \mid p)$ with respect to $q_d$,

$$\frac{d}{dq_d} \pi_d(q_d \mid p) = (p - c_d - c_p) - (p - v) \Pr(R_d \le q_d), \quad (4.12)$$

is a monotone decreasing function and so $\pi_d(q_d \mid p)$ is concave with respect to $q_d$ for given $p$. The optimal capacity allocation for direct channel $q_d$ could be obtained by solving $\frac{d}{dq_d}\pi_d(q_d \mid p) = 0$ which holds if $\Pr(R_d \le q_d) = \frac{(p - c_d - c_p)}{(p - v)}$. Assuming $q_d(p) = y_d(p) + z_d\sigma_d$, we have with equation (4.7) that

$$\Pr\left(\widetilde{R}_d \le z_d\sigma_d\right) = \frac{(p - c_d - c_p)}{(p - v)}, \tag{4.13}$$

which gives rise to

$$z_d^{\ *} = H^{-1}\left(\frac{p - c_d - c_p}{p - v}\right). \tag{4.14}$$

Note that the case of no inter-channel excess demand shifting considered can be represented by simply setting $\lambda = 0$. Then we have $H(t \mid p, z_r) = \Phi(t)$, and the optimal safety factor is $\Phi^{-1}\left(\frac{p - c_d - c_p}{p - v}\right)$. Comparing the equation (4.9) with cumulative distribution function of standard normal distribution, multiplying with $\Phi\left(z_r + \frac{(t-u)\sigma_d}{\lambda\sigma_r}\right)$ that is always less than 1 makes the integrand of $H(t \mid p, z_r)$ less than $\phi(u)$. Thus we could expect a higher order quantity when considering inter-channel excess demand shifting, which is consistent with our intuitive.

Finally, we solve manufacturer's pricing problem, to figure out the optimal price in the range of $[\underline{p}, \overline{p}]$ to maximize manufacturer's profit

$$\pi_m(p) = (kp - c_p)q_r^*(p) + \pi_d(q_d^*(p), p). \tag{4.15}$$

Here, $\underline{p}$ ($\overline{p}$) denotes the lowest (highest) possible price. The optimal price should not only maximize manufacturer's profit, but also keep retailer's expected profit positive. Joint pricing and inventory decision is, in general, a very difficult task, which is further complicated in our model due to the dependency of the distribution of $\widetilde{R}_d$ on the stocking levels. Since the concavity of $\pi_m(p)$ does not always hold, it is hard to efficiently obtain the optimal price. Similar to Petruzzi and Dada (1999), we could find the optimal $p$ through a one-dimensional search.

## 4.2 Numerical Studies

As the characteristics of retailer and manufacturer's optimal behaviors depend critically on the type of stochastic demand functions, we in this section try to gain some management insights from numerical studies with two specific demand models, linear demand model and MNL model. Due to its simplicity, linear demand function focusing on the statistical behavior of a large population of customers is the most widely used model in researches studying the pricing strategies (Bell et al 2002, Tsay and Agrawal 2004, Kumar and Ruan 2006). On the other hand, MNL model, deducted from the customer choice model, is prominent on investigating how customer's channel preference impacts the pricing strategy.

### 4.2.1 General linear demand function

We now consider the case where the expected demand functions $y(p)$ are linear in price. This linear structure was considered in many papers, such as Bell et al. (2002), Tsay and Agrawal (2004), and Kumar and Ruan (2006). Let

$$y_r(p) = a_1 - \beta_1 p,$$

$$y_d(p) = a_2 - \beta_2 p,$$

with $a_1$, $a_2$, $\beta_1$, $\beta_2$ being positive constants, to ensure that demands are decreasing with price.

To investigate the properties of manufacturer's profit function under linear demand structure, we first study manufacturer's optimal pricing problem in a special case where merchandise cost $c_r$ and $c_d$, as well as salvage value of leftover stocks $v$ are ignored. From equation (4.5), $z_r$ equals to $\Phi^{-1}(1-k)$ which is independent of $p$. From equation (4.14), we know $z_d^{\ *} = H^{-1}\left(\frac{p-c_p}{p}\right)$. Then equation (4.15) could be written as the following

$$\pi_m(p) = (kp - c_p)q_r + (p - c_p)y_d + p\int_{-\infty}^{z_d\sigma_d} th(t)dt\,. \tag{4.16}$$

Taking the derivative with respect to $p$,

$$\frac{d}{dp}\pi_m(p) = kq_r + (kp - c_p)y_r' + y_d + (p - c_p)y_d' + \left[\int_{-\infty}^{z_d\sigma_d} th(t)dt + \frac{c_p}{p}z_d\sigma_d\right] \tag{4.17}$$

Note that under the linear demand assumption

$$\frac{d}{dp^2}\pi_m^2(p) = -2k\beta_1 - 2\beta_2 + \frac{c_p^2}{p^3}\frac{1}{h(z_d\sigma_d)}$$

$$< -2k\beta_1 - 2\beta_2 + \frac{1}{p}\frac{1}{h(z_d\sigma_d)}$$

where the last inequality is because $c_p < p$. Let $\underline{z_d} = H^{-1}\left(\frac{p-c_p}{p} \mid p = \underline{p}\right)$. As $\frac{1}{h(z_d\sigma_d)}$ >0 and $-2k\beta_1 - 2\beta_2 + \frac{1}{p}\frac{1}{h(z_d\sigma_d)}$ is decreasing with $p$, $\pi_m(p)$ is quasi-concave with respect to $p$ on $[\underline{p}, \overline{p}]$ if $-2k\beta_1 - 2\beta_2 + \frac{1}{\underline{p}^3}\frac{1}{h(\underline{z_d}\sigma_d)} < 0$. Thus, we know that the optimal price should be a root of equation (4.17). For general cases with merchandising costs and salvage value of leftover inventory, we observed that $\pi_m(p)$ is also quasi-concave with respect to $p$ on $[\underline{p}, \overline{p}]$ in various numerical examples, though we could not prove it theoretically.

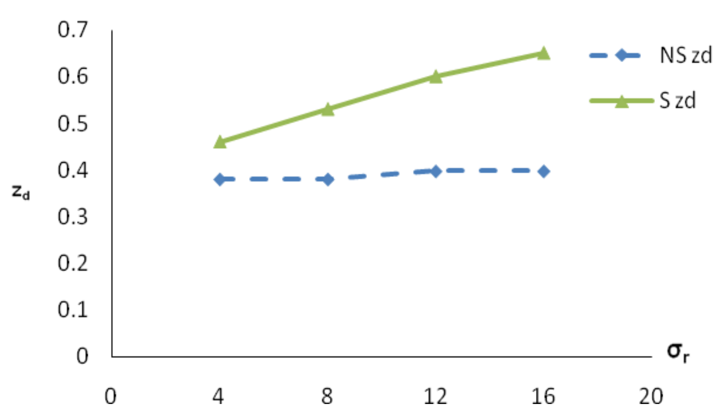
Next we do sensitivity analysis for demand variance at the retail channel $\sigma_r$ and shifting rate $\alpha$ to explore the effects of inter-channel excess demand shifting on manufacturer and retailer's optimal decisions. In Table 4.1, we assign all the basic value of parameters.

Table 4.1 Parameter setup for linear demand model

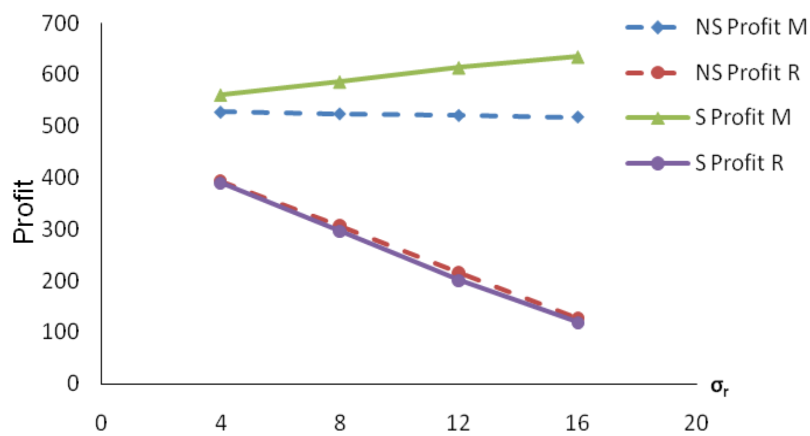
| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $a_1$ | 50 | $c_p$ | 15 |
| $\beta_1$ | 0.5 | $c_r$ | 12 |
| $\sigma_r$ | 8 | $c_d$ | 12 |
| $a_2$ | 50 | $v$ | 8 |
| $\beta_2$ | 0.5 | $\lambda$ | 0.5 |
| $\sigma_d$ | 8 | $k$ | 0.4 |

Here we use exhaustive searching method to locate the optimal price. Regarding the distribution of effective demand at the direct channel $R_d$, we are going to apply Monte Carlo simulation method to approximate this distribution, instead of using equation (4.9) which is too time consuming to calculate the integral for each $t$ from -3 to 3 ($\pm 3\sigma$ where $\sigma$=1 for standard normal distribution). With a step size of 0.1, for each value of price $p$, we calculate $z_r$ and then generate the distribution of $R_d$ which is used to find out $z_d$. The sample size of Monte Carlo simulation is 500,000.

In the first place, we estimate the effect of retailer's demand variance on manufacturer's stock level and profit. Higher demand variation induces larger expected loss demand at the retailer, which in turn augments the expected demand at the direct channel and stock level. Figure 4.1(a) shows how the retail demand variance affects the optimal safety factor $z_d$, where the label "S" ("NS") denotes the situation with (without) considering inter-channel demand shifting. Unsurprisingly, $z_d$ increases obviously from 0.45 to 0.65 as $\sigma_r$ rises from 4 to 16. From Figure 4.1(b), we see that manufacturer's profit when taking inter-channel excess demand into consideration, compared to ignoring this effect, increases 3% to 15 % without hurting retailer's profit. This increment has two contributors. One is the increase of retailer's order quantity $q_r$ according to equation (4.6) with $\sigma_r$ as the optimal price stays around 65 when $\sigma_r$ changes and so does retailer's optimal safety stock factor $z_r$. The other is that the expected demand at the direct channel grows up.




(a)



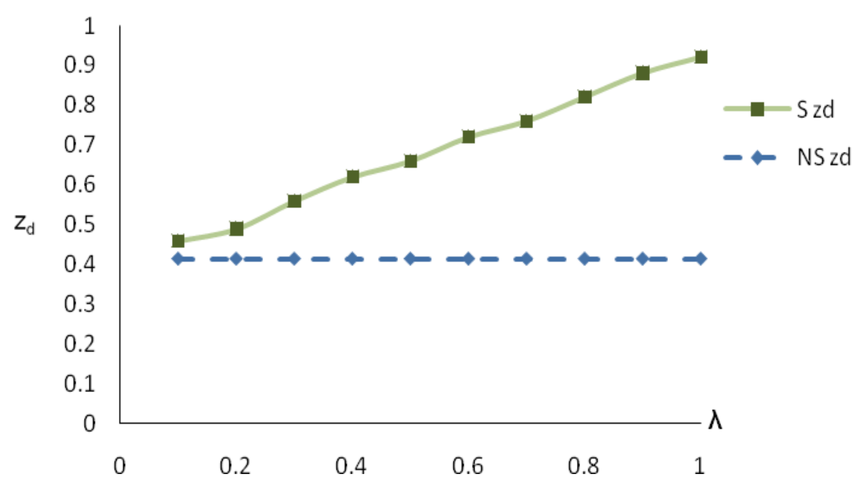

(b)

Figure 4.1 Effects of $\sigma_r$ on (a) $z_d$ (b) profit

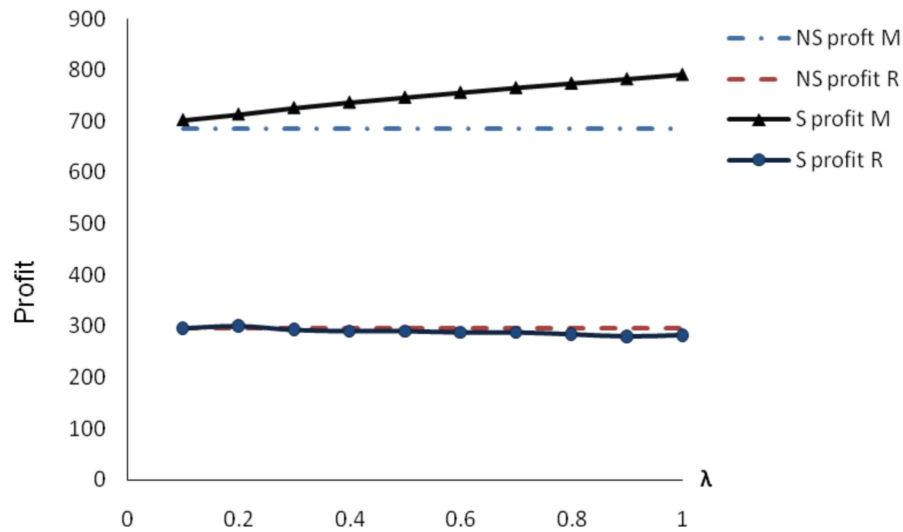
Next, we examine the effects of shifting rate α. Intuitively, substitutability of direct channel to traditional retail channel also induces high stocking at the direct channel because it results in larger amount of demand loss transmitting. As we can see from Figure 4.2(a), $z_d$ rises up dramatically from 0.41 to 0.91 as shifting rate increases from 0.1 to 1. Further, from Figure 4.2(b) we could see that manufacturer's profit also increases from 2% to 13.3% compared to the case without demand shifting being considered. At the same time, we find that retailer's profit scarcely changes, which shows that all the increase of manufacturer's profit comes from the capture of unmet demands at the retailer. The unsmooth of the line of $z_d$ when considering demand shifting is due to the randomness brought about by the approximation of the distribution of $R_d$.

Indeed, both increases on $\sigma_r$ and $\alpha$ would lead to more demands at the retailer shifting to direct channel, resulting in the rise of expected demand at the direct channel and corresponding manufacturer profit. Furthermore, we observe from our numerical experiments that the optimal price is rather insensitive to demand variance and shifting rate. When $\sigma_r$ changes from 4 to 16 or $\alpha$ changes from 0.1 to 1, the optimal price stays around 65 and the fluctuation keeps within 1. Thus, it is the increasing satisfaction of customer demands that accounts for the growth of profit.

Though we only study the special case $a_1$=$a_2$, $\beta_1$=$\beta_2$ here, above results can be extended to general cases in which these four parameters can have different values. From equation (4.17), the difference between optimal price solving from taking the inter channel demand shift into consideration and that from not considering this effect depends on $\lambda$ and $\sigma_r$. Meanwhile, $z_d$ also depends on these two parameters but not on $a_j$ ($j$ = 1, 2) and $\beta_j$ ($j$ = 1, 2).




(a)




(b)

Figure 4.2 Effect of shifting rate on (a) $z_d$ (b) profits

### 4.2.2 MNL

In this subsection we use the standard multinomial logit model (MNL), a commonly used random utility model in economics and marketing literature (see Anderson et al 1992), to describe the demand process. In MNL, customer utility $u_j$ takes the form of

$$u_i = s_i - p + \xi_i\,,\ \ i = r,\, d,$$

where $s_j$ is customer's consumption value for channel $i$ and $\xi_i$ is an noise term that allows for unobservable heterogeneity in taste. The noise term, $\xi_i$, follows an extreme value distribution with mean of 0 and variance of $\frac{\pi\mu^2}{6}$. Let $u_o$ denote the utility that customer could obtain from outside option. Then similar as the principle of utility maximization in the choice model in Chapter 3, customer would choose to purchase from the channel providing the highest utility. If $u_o$ is the largest of these three, customer would not buy this product. The probability of a customer choosing channel $i$

$$r_i = P\left(u_i = \max_{j=r,d,o} u_j\right) = \frac{e^{\frac{s_i}{\mu}}}{e^{\frac{s_r}{\mu}} + e^{\frac{s_d}{\mu}} + e^{\frac{u_o}{\mu}}}. \tag{4.18}$$

Assuming the number of customers coming to buy this product follows a Poisson distribution with mean of $T$, the number of customers choosing channel $i$ is assumed to be a thinned Poisson process with mean of $r_iT$. Using Normal distribution to approximate this demand process, the expected demand

$$y_i(p) = T\frac{e^{\frac{s_i-p}{\mu}}}{e^{\frac{s_r-p}{\mu}} + e^{\frac{s_d-p}{\mu}} + e^{\frac{u_o}{\mu}}} \tag{4.19}$$

and the random term

$$\varepsilon_i \sim N\left(0, \sqrt{T\frac{e^{\frac{s_i-p}{\mu}}}{e^{\frac{s_r-p}{\mu}} + e^{\frac{s_d-p}{\mu}} + e^{\frac{u_o}{\mu}}}}\right). \tag{4.20}$$

Though the shifting rate λ can be a given constant just as in general linear model, we can calculate it out using the given parameters in MNL, which saves extra information for estimation. The customer, who chooses retail channel and would like to shift to direct channel when retailer is out of stock, should have $u_d \geq u_o$. Then the probability of customer demand shifting, i.e. the demand shifting rate, is

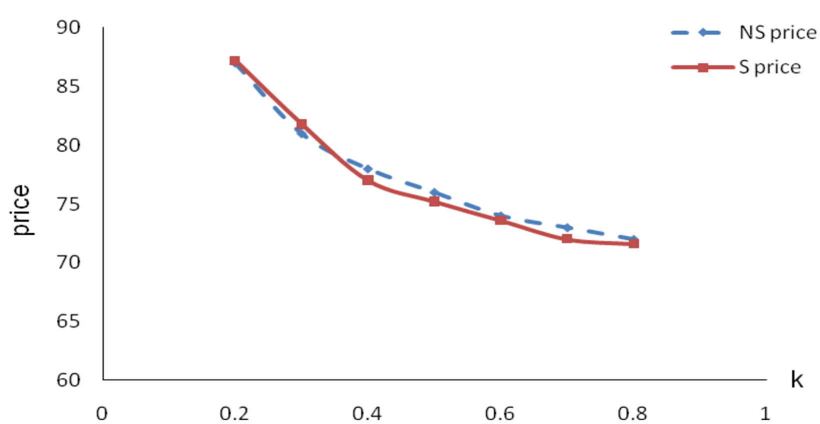
$$\lambda = \Pr(u_d \geq u_o) = \frac{e^{\frac{s_d-p}{\mu}}}{e^{\frac{s_d-p}{\mu}} + e^{\frac{u_o}{\mu}}}. \tag{4.21}$$

As both the variation of demand at retail channel and inter-channel excess demand shifting rate in the MNL model depend on the value of $\mu$, the monotonicity of inventory level at direct channel does not hold. In the remainder of this subsection, we explore the effect of wholesale price $k$ on manufacturer's optimal behavior. Table 4.2 shows the basic values of experimental factors considered.

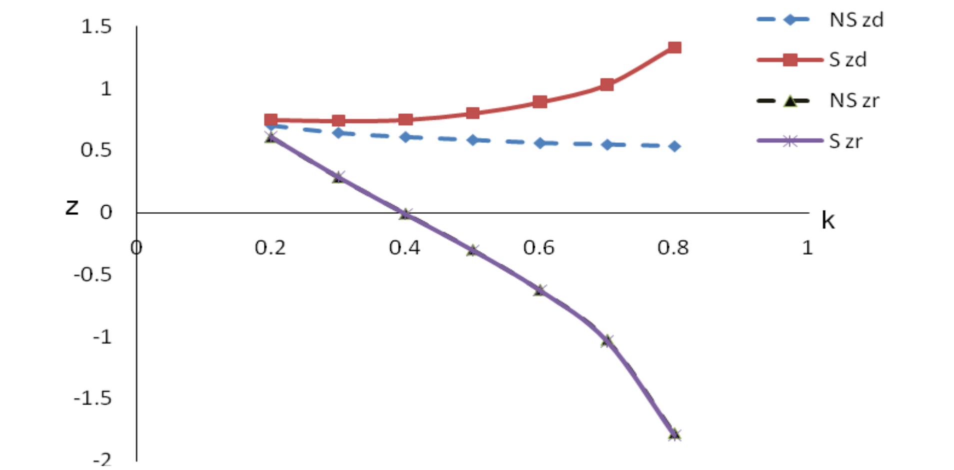
Table 4.2 Parameter setup for MNL model

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $s_0$ | 0 | $c_p$ | 15 |
| $s_r$ | 85 | $c_r$ | 12 |
| $s_d$ | 85 | $c_d$ | 12 |
| $k$ | 0.4 | $v$ | 8 |
| $\mu$ | 15 | | |

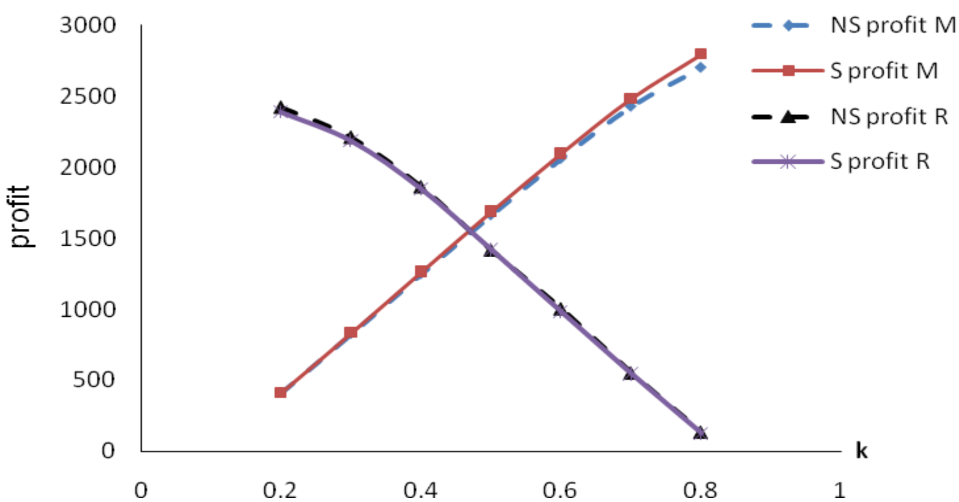
As discussed earlier, the contraction parameter $k$ is actually a representative of the two companies' bargaining power. It is also a reflection of double-marginalization arousing in vertical competition, which induces high resale price and over-stocking. In the case that manufacturer has great bargaining power, it would be set high wholesale price to gain large revenue and then aggravate the effect of double marginalization. On the other hand, retailer also tends to price very high to cover the wholesale price and keep its own margin.




(a)




(b)




(c)

Figure 4.3 Effects of bargaining power $k$ on (a) price (b) $z_d$ (c) profit

However, as in our model the resale price is controlled by manufacturer, the results are much different. We can see from Figure 4.3 (a) that manufacturer tends to set a very high retail price to gain enough margins from retailer to cover its production cost when retailer has great bargaining power. As $k$ increases, retailer's margin shrinks rapidly, whereas manufacturer's profit rockets. Large margin also stimulates manufacturer to set a high level of safety factor at the direct channel (Figure 4.3 (b)).

On the other hand, we find from Figure 4.3 (c) that the effect of capturing transmitting excess demand on manufacturer's profit is not significant even though customer valuation varies greatly (when $k$ is 0.7, the profit improvement is only 3%). This is probably due to one of the major assumptions lying in the customer choice model that all customers are considered to be homogeneous. It would be easy to predict the demands. The coefficient of variance of demands in channel $j$, $COV_j = \frac{1}{\sqrt{r_j T}}$, is converging to zero with $T$. Thus, it leads to very small demand variety and then few lost sales at the retailer. Consequently, manufacturer could not capture much lost sales from the retailer to improve its profit.

## 4.3 Conclusion

In this chapter, we study the joint optimal price and inventory decision in a single period general dual-channel supply chain model where a monopolist manufacturer sells its product to customers through a traditional retailer and also its own direct channel. We propose a model to capture the major features of such supply chains and use this model to investigate the effect of inter-channel excess demand shifting on manufacturer's pricing and capacity decision. We try to gain some basic idea of manufacturer's behavior from the generated distribution of the effective direct channel demand conditioning on retailer's order quantity and resell price. Following this, we obtain the optimal price and capacity decisions.

Within our model, as the results significantly depend on the stochastic demand process, we numerically estimate the impact of inter-channel excess demand shifting based on two general used demand processes, one being the linear demand function, the other being the multinomial logit model (MNL).

In linear demand model, our results show that considering this excess demand shifting effect would dramatically increase manufacturer's capacity reserved for direct channel and also the optimal profit, even when demand variance is low and demand shift rate is moderate. Yet the optimal price is insensitive to the demand variance and demand shift rate. This suggests that it is necessary for manufacturer to take this demand shift into account while making capacity decisions when two channels face a pool of customers.

In MNL model, due to the underlying assumption of homogeneous customers, demand variance is extremely low as number of customers increases, which results in scarce excess demand at the retailer and also low benefit from considering the excess demand shift effects. But as manufacturer's market power increases, retailer's profit margin shrinks and sets low service level, which gives large excess demands and stimulates the increase of direct channel demand.

Thus, whether it is worth to consider this inter-channel excess demand shifting when making inventory decision critically depends on the demand variance and shifting rate; but it mainly depends on the demand process when setting the resell price.

Our model also has a few limitations. We only study the decisions made in decentralized situation. Comparing our results with the decisions made in centralized supply chain, we could know how this kind of demand shifting would affect the behavior of over-stocking. In addition, the assumption that $D_r$ and $D_d$ are independent is not realistic in many cases and needs to be relaxed. It would be interesting to investigate how the correlation of these two first choice demands would change their decisions. Another restriction of this model is that the price is controlled by manufacturer. Consequently, retailer does not have much power to react to manufacturer's direct competition with end customers. If retailer could decide its own selling price, it could raise the price very high to reach a large margin to compensate the low demand, but manufacturer might lose money due to the low demand at retailer.

# CHAPTER 5 **CONCLUSIONS AND FUTURE WORK**

Motivated by growing attention of both business community and academic society to the creation of a manufacturer-owned direct resell channel, the main purpose of this thesis is to study how channel members react to simultaneous horizontal and vertical competition, and if the resulting conflict might overwhelm any potential advantages. This chapter summarizes the research findings and contributions in this thesis, and discusses their implications and limitations. Several directions for future research will also be presented.

## 5.1 Main Findings

In this thesis, we mainly focus on the inter-channel competition and coordination in a simplified dual-channel distribution prototype. We consider that a monopolist manufacturer sells its product through both the traditional retail channel and its own direct channel (Figure 1.1). Consumers have different consumption values for purchasing from these two channels, and then they make their purchase decisions of which channel to purchase from depending on their consumption values and the channel selling prices according to utility maximization principle.

In Chapter 3, we consider the competing pricing strategy in a dual-channel distribution system. Consumer's consumption values, $v_r$ and $v_d$, are viewed as two independent random variables. The demand functions of these two channels are derived based on a linear utility consumer choice model. Both of the optimal pricing strategies under centralized decision making and decentralized decision making are studied. In the case of centralized decision making, retail price and direct sell price are determined in coordination to maximize the total profits of both the two channels. In the case of decentralized decision making, manufacturer acts as a pricing Stackelberg game leader, who decides the wholesale price (to retailer) and direct sell price to optimize his profit first. And then, retailer acts as the game follower, in deciding the resale price to optimize his own profit with known wholesale price and direct sale price. Retailer's response for given wholesale price and direct sale price is obtained. With retailer's response function, we provide an algorithm for manufacturer to find its optimal pricing strategy. Since the demand functions we consider are quadratic functions of $p_r$ and $p_d$, the profit functions become cubic functions, leading to significant increases in computational complexity. $p_r$ and $p_d$ are treated as independent arguments, which follows the real characteristic of a competing environment and provides higher system-wide profits. Our numerical study shows that manufacturer's profit rises dramatically after launching a new direct channel. On the other hand, contrary to many existing studies, retailer's profit decreases even when consumers prefer traditional retail channel to direct sale channel. This result suggests that the technique for consumer consumption value modeling plays an important role in the pricing decision process.

In Chapter 4, we study the jointly optimal price and inventory decision as a newsvendor problem based on this general dual-channel supply chain model. Manufacturer sets identical resell price for both the two channels to avoid inter-channel price competition. Independent retailer announces its order quantity to manufacturer, and then manufacturer decides the production capacity. Special attention has been put on the effect of this inter-channel stock-out demand shifting on manufacturer's pricing and capacity decision. We build up the distribution of the effective direct channel demand conditioning on retailer's order quantity and resell price. Although retailer's problem is a standard newsvendor problem, manufacturer's decision is a combined two-stage price-setting newsvendor problem that no closed form solution could be obtained. To gain management insights, we numerically study this problem based on two demand processes, one being the linear demand function, the other being the multinomial logit model (MNL). It is observed that the effect of inter-channel stock-out demand shifting on manufacturer's capacity is quite sensitive to the retailer's demand variance and the rate that spillover demand shifts, while that effect on optimal price is not so dramatic.

## 5.2 Suggestions for Future Work

### 5.2.1 Channel coordination under decentralized decision making

From our study, we can see that inter-channel competition under decentralized decision making induces significantly channel conflict and losses of total system profits. The problem of channel conflict is also the biggest obstacle of multi-channel marketing. The channel conflict between channels can be modeled by the derivative of channel profit with respect to the prices at these two channels. A channel coordination strategy and supply chain contract under decentralized decision making are worth studying to minimize the channel conflicts as well as maximize the channel profits.

### 5.2.2 Multi-channel design with multiple products

Based on the existing works in this thesis, we can further consider the channel design problem for a manufacturer producing multiple products. As we have shown, the profitability of multi-channel distribution system depends much on the demand characteristics of products, and designing the channels that products are distributed through should consider both the characteristics of product and that of channel. In addition, as differing resell assortments at channels, suggested by various business reports, is a useful way to diversify targeted customer segments and alleviate channel conflict, it could be studied together with channel coordination, serving as a coordination instrument.

### 5.2.3 Equilibrium channel structure with price and service competition

In most literature on multiple-channel distribution strategy, it is pointed out that manufacturer's direct marketing would raise channel conflict and damage the relationship with existing independent retailers. This relationship would be interpreted as the sales effort that the retailer would like to spend on this brand to promote its demand, like the salesmen's explanation, or the position where the product is put in the store. Beyond using only price as the competing instrument in this thesis, it would be interesting to integrate market service level into investigation of, in a scenario with two manufacturers and one common retailer, how the direct marketing strategy would impact this relationship, how manufacturer's competitive position would be changed, and what the equilibrium channel structure is when competitive manufacturers exist.

A similar two-stage game model could be employed. Manufacturers need to set the optimal resell prices and wholesale prices given the channel structure in stage one. In the second stage, given wholesale prices and resell prices, the common retailer decides its sales effort to be dedicated to each brand. Three kinds of competition could be studied at the same time, manufacturer-level horizontal competition, manufacturer-retailer vertical competition (double-marginalization), and manufacturer-retailer horizontal competition.

### 5.2.4 Replenishment policy

Consider the situation that manufacturer and retailer had replenishment opportunities during the selling season. Before the start of the selling season, retailer announces his forecast and order quantity to the manufacturer, while manufacturer forecasts the demand at the direct channel and installs his production capacity. During the season, as time goes by, manufacturer and retailer could replenishment inventory based on their upgraded demand forecasting for the remaining selling time and on-hand inventory level. Manufacturer has to decide whether to replenish, when to replenish and how much to replenish (with or without retailer's replenishment decision).

Manufacturer's replenishment lead time might depend on his order size, while the replenishment cost depends on the lead time and order size. If retailer foresees a stock-out and finds that it would be profitable to increase its inventory level, it would place a replenishment order to the manufacturer. This replenishment would incur a cost higher than the original wholesale price.

There are two possibilities. One is that manufacturer replenishes its inventory before the retailer proposes its $2^{nd}$ order, and the other is the reverse. Without the information whether retailer would place the $2^{nd}$ order, manufacturer decide to start his $2^{nd}$ production when he anticipates a shortage in direct channel during the remaining selling season. If the manufacturer is short sighted, he would only consider the demand at the direct channel and ignore the possibility that the retailer might place replenishing order. Otherwise, he should take the possibility of retailer's replenishment into account when making decisions.

When retailer announces his $2^{nd}$ order, another problem is that whether manufacturer should accept this order. If manufacturer accepts this order and decides to satisfy it using his on-hand inventory, it could be delivered immediately. Manufacturer would also postpone the delivery if he is waiting for his replenishment and does not want to satisfy retailer's order using his now on-hand inventory because this would lead to his out-of-stock situation. However, this delay would impact retailer's order quantity because some demand might already be lost. Moreover, manufacturer would reject retailer's order if he finds that replenishing retailer is not profitable.

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