A STUDY IN JOINT MAINTENANCE SCHEDULING AND PRODUCTION PLANNING

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Table of Contents

Executive Summary

In practical production planning it is critical to consider reliability/inspection/maintenance parameters. If a production plan fails to take reliability parameters into account, it will be vulnerable to breakdowns and other disruptions due to unreliability of equipment.

Similarly, an optimal maintenance schedule must include production/inventory parameters in practice. A maintenance schedule developed independently from production plan may necessitate a shutdown of equipment to perform preventive maintenance (PM) while, according to the production plan, the equipment cannot be stopped until calculated economic production quantity (EPQ) is achieved.

In each case, shortage, maintenance, and defective costs increase.

The main idea of this thesis is to simultaneously consider these two classes of parameters in a single model to achieve a joint optimal maintenance schedule and production plan.

Production/inventory control models in presence of periodic planned maintenance are selected as the base for development. Joint optimization of buffer stock level and inspection interval in an unreliable production system is studied and an extension is modeled.

Chapter 1 – Introduction

Despite extensive research conducted on maintenance models, those integrating maintenance/inspection schedule with production/inventory control are scarce. Yao *et al.* (2005) found the reason in the fact that most maintenance models rely on reliability measures and ignore production/inventory levels. Similarly, numerous researchers have studied production/inventory control models. However, they have seldom taken possible preventive maintenance actions into account. This is due to modeling the failure processes as two-state (operating-failed) continuous Markov chains which implies the assumption of exponential distribution of lifetimes and a constant failure rate which consequently makes PM unnecessary (Yao *et al.* (2005)).

Research works which consider joint production/PM planning use several general approaches to develop their models. In categorization provided by Cassady and Kutanoglu (2005) they are reactive approach and robustness approach. The former updates production plan when failure occurs, while the latter develops a production plan which is less sensitive to failure. In another categorization, literature is divided into research which studies the effects of machine failures on production plan, and research which develops integrated production/maintenance models (Iravani and Duenyas (2002)). Meller and Kim (1996) classified previous research as concerning either PM policies of a machine operating in isolation or analysis of stochastically failing systems of machines and buffers with no consideration of a PM policy.

As mentioned earlier in Executive Summary, separately derived optimal production/inventory and maintenance policies mostly lead to complications and conflicts in practice between production and maintenance departments of a production environment. On the other hand, true optimal policy cannot be achieved unless parameters of both production and maintenance policies are jointly considered in developing a solution. The studied problem is, therefore, to find joint production/inventory control planning and maintenance/replacement scheduling model.

The motivations behind this study are:

1- To avoid conflicts between production and maintenance departments of a production environment,
2- To find a true joint optimal production/maintenance model parameters,
3- To find mathematically tractable and convenient-to-apply policies,
4- To make the model as general as possible to be theoretically applicable to more cases without compromising its convenience to apply.

Reviewed papers formulate the problem by manipulating five aspects *i.e.*, problem setting (as the way problem is defined), assumptions (e.g. Weibull lifetime distribution with increasing failure rate), objective function (usually minimization of expected cost per unit time), decision variables (e.g. number of maintenance actions, safety stock, and inspection interval length), and optimization procedure (e.g. numerical search methods).

The rest of this thesis is organized as follows. Chapter 2 is dedicated to a review of existing literature on joint maintenance scheduling and production planning. Chapter 3 deals with production/inventory control models in presence of periodic planned maintenance. In Chapter 4 Joint optimization of buffer stock level and inspection interval in an unreliable production system is studied and finally, discussions and conclusions are presented in Chapter 5.

The major contribution of this work is the methodical categorization of literature in the area of joint production planning and maintenance scheduling. Another contribution is an analytical extension of a model presented in chapter 4 along with a sensitivity analysis. The main part of this thesis (Chapter 2) is, therefore, dedicated to literature review and categorization to track the footprint of research through four main research streams, namely production/inventory control models in presence of deterioration and breakdowns, maintenance/replacements models in presence of an inventory control policy, models integrating production and maintenance control, and integrated determination of EPQ and inspection/maintenance schedule.

Chapter 3 provides more details on production/inventory control models in presence of periodic planned maintenance. The purpose of this chapter is to investigate this class of papers which is close to the goal of developing an easy-to-use general model which jointly optimizes production/maintenance control. This chapter provides a basis for Chapter 4.

Chapter 4 studies joint optimization of buffer stock level and inspection interval in an unreliable production system. In this chapter an analytical extension to the model as well as a sensitivity analysis of the results is provided.

Chapter 5 concludes the thesis and discusses its achievements. It also suggests further studies in this area.

Chapter 2 – A literature review on joint maintenance scheduling and production planning

In section 2.1 of this chapter papers reviewing inspection/maintenance models are presented and instances of related optimization models are shown. Section 2.2 deals with production/inventory control models in presence of deterioration and breakdowns. Maintenance/replacement models in presence of an inventory control policy are reviewed in section 2.3. Section 2.4 is dedicated to models which integrate production and maintenance control. In section 2.5, integrated determination of EPQ and inspection/maintenance schedule is discussed. Relevant summarization tables of research development in the area of joint maintenance scheduling and production planning are provided throughout this chapter. For each paper within each of the above-mentioned categories these tables includes problem settings, major assumptions, decision variable(s), objective function, optimization procedure and major achievement(s).

2.1 Review of Inspection/Maintenance models

2.1.1 Papers reviewing maintenance models

Literature on maintenance models and their optimization is abundant. Researchers have classified maintenance models and their optimization procedures in different ways. Wang (2002), for example, classified maintenance models into two basic categories: policies for one-unit systems and policies for multi-unit systems. The former is further classified into age-dependent preventive maintenance (PM) policy, periodic PM policy, failure-limit policy, sequential PM policy, repair-limit policy, and repair number counting and reference time policies.

In failure-limit policy, PM is performed only if a reliability index (usually failure rate) reaches a predetermined level while failures before that time are removed by repair. Sequential PM policy calls for PM action at unequal time intervals which become shorter and shorter as time passes. In repair-limit policy repair is done if its estimated cost is less than a predetermined level, otherwise, the unit is replaced. In repair number counting policy k^{th} failure urges replacement while the first *k-1* failures are corrected by minimal repair. An extension to this policy is that replacement is performed upon k^{th} failure only if it is occurred after a reference time *T*. Maintenance policies for multi-unit systems are classified into group maintenance policy and opportunistic maintenance policies. Wang (2002) focused on policies for single-unit systems.

A classical literature review paper on maintenance models for multi-unit systems is Cho and Parlar (1991). They primarily categorized maintenance models into preventive and preparedness models. Unlike preparedness models, in preventive models the state of units is known. Moreover, they differentiated between discrete-time and continuous-time models. Based on these primary categorizations, their literature review continued in 5 directions: machine interference/repair models, group/block/cannibalization/opportunistic maintenance models, inventory and maintenance models, other maintenance and replacement models, and inspection/maintenance (preparedness maintenance) models. Inventory and maintenance models study optimal maintenance policies when available spare parts are limited. As this provides an opportunity to jointly optimize the inventory control policy and repair/replacement planning, it will be further developed in this review.

With a focus on economic dependence among the units of multi-unit systems, Dekker *et al.* (1997) slightly modified and extended Cho and Parlar (1991) to include papers appeared after 1991. Dekker *et al.* (1991) primarily classified literature into stationary grouping and dynamic grouping models. In the category of stationary grouping, they reviewed grouping corrective maintenance, grouping PM, and opportunistic maintenance. In the category of dynamic grouping models, they reviewed finite time horizon and rolling time horizon.

An interesting area in maintenance models studies is to explore the application of models developed by researchers in real-world situations. Dekker (1996) studied the application of maintenance optimization models and highlighted existing shortcomings. Firstly, these models usually provide no closed-form analytical equation to derive optimal values for decision variables and it is necessary to apply numerical/heuristic approaches to find near-optimal values. Secondly, maintenance optimization models are sensitive to the accuracy and precision of data, and lastly, like any other area, there is a gap between theory and practice.

2.1.2 Instances of inspection/maintenance optimization models

Several papers studying specific inspection/maintenance models and deriving optimal values for their decision variables are reviewed here.

Marquez and Heguedas (2002) explored the trade-off between flexibility and complexity of semi-Markovian probabilistic maintenance models for finite periods of time. By flexibility they meant the attitude of a model to represent a wide range of maintenance situations, while complexity was measured in terms of model data requirements to populate the mathematical formulation and complexity of the model itself. They studied three cases with increasing complexity. In the first case, the only states are operation 100% and corrective maintenance. In the second case, state of operation less than 100% is added. In the last case, the preventive maintenance state is added to the model. They concluded that increasing complexity of model results in added analysis capabilities for maintenance decision maker.

The problem considered in Mercier and Labeau (2004) is to find optimal replacement policy in a series system of n identical and independent components. At the beginning, new-type units replace failed old-type units only. Strategy K dictates that K-1 corrective replacements are performed and when $K^{th} (1 \le K \le n)$ old-type unit fails, this unit together with all the remaining units are replaced with new-type units. Strategy n is a pure corrective approach, while strategy 0 necessitates the replacement of all old-type units with new ones as soon as they appear on the market.

Constant failure rates are assumed for old and new units, replacements are instantaneous, no common cause failure exists, and new units are fully compatible with the system. The objective is to find optimal strategy denoted by strategy K_{opt} as a function of mission time and problem parameters, such that discounted mean total cost with respect to a certain mission time is minimized. Cost components are replacement costs (fixed cost of solicitation of the repair team, corrective replacement and preventive replacement costs) and energy consumption cost. To find the optimal strategy, first, the costs of strategy 0 and 1 are compared, and then the difference between costs of strategies *K* and *K+1* is derived. It is found that for $n \geq 2$ the optimal strategy is either strategy 0, 1, or n; specifically when mission time is short, strategy n is always optimal.

Another research reviewed here is Bartholomew-Biggs *et al.* (2006). They developed a PM scheduling model which minimized a performance function reflecting repair and replacement costs and costs of PM. Decision variables are number of PM actions as well as their optimal timings.

A single k-out-of-N system with deteriorating components is studied in de Smidt-Destombes *et al.* (2006). Each component is either in state 0 (as-good-as new), 1 (degraded), or 2 (failed) with an exponentially distributed sojourn time. As soon as m^{th} component fails maintenance is initiated; however, it actually starts after a fixed lead time *L* and includes replacing all failed and possibly all degraded components by spares. If available spares are not sufficient maintenance period is extended by the time needed to repair the remaining components. Repair time is also exponentially distributed and repairing failed items takes more time on average than that of degraded items. Limiting or steady system availability is defined as in Eq. (2.1).

$$Av_{m,S,c} = \frac{E(T_m) + E(U_m)}{E(T_m) + L + E(D_{m,S,c})} \tag{2.1}$$

where S and c are spare level and repair capacity respectively and $E(T_m)$, $E(U_m)$, and $E(D_{m,S,c})$ are mean time to maintenance initiation, mean uptime during lead time, and mean maintenance duration, respectively. The rest of this study derives the above expressions and considers some extensions.

An example of studies related to inspection scheduling is Cui *et al.* (2004). They studied periodic inspection schemes with emphasis on meeting availability requirement. They studied four optimality criteria (steady-state availability A_{av}, instantaneous availability $A(t)$, long-run inspection rate β, and (through expected number of inspections before time *t*) instantaneous inspection rate $\beta(t)$) for five inspection scheduling models *i.e.,* periodic inspection $PI(\tau)$, single-quantile-based inspection $SQBI(\alpha)$, hybrid inspection $HYBI(\alpha, M)$, multiple-quantile-based inspection $MQBI(\alpha_i, i \in Z^+)$, and time hybrid inspection $THYBI(\alpha, s)$. Failures are non-self-announcing, lifetime of the system follows a known distribution, and inspection and repair/replacement times are negligible. The paper then studies relationships among these inspection schemes and weaknesses and strengths of each scheme.

2.2 Production/inventory control models in presence of deterioration and breakdowns

Production/inventory models in presence of deterioration and breakdowns are also studied extensively in literature. However, no-PM-action assumption prevails. Table 2.1 summarizes the research conducted in this area.

An example in this area is Aka *et al.* (1997). In their study, a component common to n identical parallel machines is prone to failure and has a significant deterministic lead time. When a unit fails an immediate replacement is performed if there is at least one part in inventory; otherwise, it is replaced as soon as a part becomes available.

Time to failure of the component on each machine is exponentially distributed. Implemented inventory policy is such that whenever the inventory level reaches k (renewal point) an order of q units is issued. If, during lead time, fewer than k failures occur no downtime will be faced. If the number of failures is greater than k but is less than q some downtime will occur. More than q failures result in an expedited order in form of an increase in order size to replace failed units and to bring inventory level to k. Expedition cost increases linearly as scheduled delivery time approaches.

Using a direct search procedure over a two-dimensional grid, the paper derived optimal values for q and k for different values of n such that long-run average cost of downtime, expedited ordering, procuring spare parts (contains fixed ordering cost), and holding costs (incorporates shortage cost as well) per unit time was minimized. It is found that an increase in the number of machines, lead time, or cost of downtime increases optimal k and q as well as total cost. An increase in holding cost decreases optimal values for decision variables but increases total cost and an increase in ordering cost increases optimal q and total cost but decreases k.

Table 2.1 Production/inventory control models in presence of deterioration and breakdowns

Research	Problem Setting	Major Assumptions	Decision Variable(s)	Objective Function	Optimizing Procedure	Major Achievement(s)
Aka *et al.* (1997)	One component common to n identical parallel machines	Replacement upon failure, deterministic lead time, time to failure exponentially distributed	Renewal point, order quantity for different values of n	Long-run average cost of downtime, expedited ordering, procuring and holding	Direct search over a two-dimensional grid	Increase in n, lead time, or downtime cost increases optimal decision variables
Iravani and Duenyas (2002)	One system with three state sets	State sets: PI (Produce until an inventory threshold then go idle), PR (Produce until an inventory threshold, then undergo repair), R (undergo repair)	Two inventory thresholds, two state thresholds	Total average costs of holding, lost sales and repair/maintenance	SMDP	A heuristic to reduce any number of states to three
Lin and Gong (2002)	A single deteriorating product, NR continuous review inventory policy	Time to failure and deterioration time exponentially distributed, deterministic and constant demand, perfect repair with fixed duration	Optimal production uptime	Long-run average total cost of setup, repair, inventory, deterioration and lost sales	Derivatives and numerical procedures	NR continuous review inventory policy and a deteriorating product
Yeh and Chen (2006)	A last-K inspection scheme on a production system after having produced a lot L, a minimal repair warranty	Negligible inspection time, higher failure rate for non-conforming items	Lot size L and product inspection scheme parameter K	Expected total cost of production, inspection, restoration, inventory and warranty	Setting EPQ as an upper bound for L, then a search procedure	Better results relative to EPQ model

Iravani and Duenyas (2002) presented a semi-Markov decision process. The objective is to minimize total average costs of inventory holding, lost sales, and repair/maintenance. Optimal policy divides the state of the system to three sets: PI (produce until a certain inventory threshold, then go idle), PR (produce until another certain threshold, then undergo repair), and R (undergo repair). Since optimization is complex, a double-threshold policy is introduced and exact optimal solution is derived for a three-state machine. Implementation of this policy needs two inventory thresholds to determine when to stop production and go idle or undergo repair in set PI and PR respectively as well as two state thresholds to determine the border of sets. The paper suggests a heuristic to reduce any number of states to three.

Another example is Lin and Gong (2006) where a no-resumption (NR) continuous review inventory policy was applied. This means that when failure occurs or a predetermined production time τ is reached, production is stopped until inventory is depleted to zero. For a single deteriorating product, an economic production quantity (EPQ) model was studied in case of random machine breakdowns where both time to failure and product deterioration time were exponentially distributed. Inventory is built during uptime with a rate equal to production rate minus demand rate (*P-D*) offset by product deterioration rate (θ). Renewal epochs are points in time when production starts. If failure happens before τ and downtime T_2 (time to complete depletion of inventory according to NR policy) is greater than repair time R no shortage occurs, otherwise demand is lost.

Demand is deterministic and constant, repair is perfect and its duration is fixed, and no repair or replacement is performed on deteriorated items. Objective, here, is to find optimal production uptime τ^* which minimizes expected long-run average total cost of setup, repair, inventory carrying, deterioration of items and lost sales per unit time. Derivatives and numerical procedures were used to find analytical optimal value for τ while exponential terms were replaced by Taylor series approximation to derive near optimal value for τ. Finally, sensitivity analysis of optimal uptime value with respect to repair time, deterioration rate, and failure rate was conducted and analytical optimal uptime value and its near-optimal approximation for different values of deterioration and failure rate were compared through some numerical examples.

Finally, presence of a warranty scheme is studied by Yeh and Chen (2006). They studied a production system which may shift to out-of-control state (and stays there until the end of a production run) with probability $1-q$. When system is out of control a higher percentage of products will be defective. System is inspected after having produced a lot L and is restored to in-control state if found out of control. All products are sold with a free minimal repair warranty within a period w. A last-K inspection scheme is performed where non-conforming items are reworked and become conforming.

Testing and inspection durations are negligible and a higher failure rate is assumed for non-conforming items. Objective, here, is to find optimal lot size L^* and optimal product inspection scheme parameter K^* which minimize the expected total cost of production cost, system inspection cost, system restoration cost, inventory holding cost, items inspection and rework costs, and post-sale warranty cost per unit time. They found that for a fixed *L*, K^* was either 0, *L*, or an amount between these two which was expressed as a function of *L*. In any case L^* is unique. Suggested algorithm sets classical EPQ as an upper bound for *L*. A search procedure, then, checks each time some specific conditions of problem parameters to find the relevant L^* and hence K^*. Optimal policy is therefore obtained which performs better than traditional EPQ model. It is shown also that as *q* decreases K^* increases and L^* either decreases (when $K^*=0$) or increases (when $K^*>0$).

Before reviewing the literature on joint optimization of production/inventory control policy and inspection/maintenance schedule, some papers which dealt with maintenance/replacement optimization models considering inventory control policy effects are reviewed here.

2.3 Maintenance/replacement models in presence of an inventory control policy

In literature there are papers which derive optimal maintenance/replacement model parameters when a specific inventory control policy is assumed in place. The effect of such a policy on optimality of maintenance/replacement model is therefore studied. In some of these studies optimal values for inventory control parameters are also derived and this is sometimes done jointly with determination of optimal values for maintenance/replacement model parameters. Nevertheless, assuming a specific inventory control policy may hinder the possibility of finding a real optimal joint maintenance-inventory policy. Table 2.2 provides a summary of the research conducted in this area.

In an early study Zohrul Kabir and Al-Olayan (1994) studied a single operating unit with any number of spare units in stock. Inventory policy (s, S) is used which issues an order of $S - s$ when inventory level drops to s. Preventive replacement is scheduled at t_1 if spare is available, otherwise, it is performed as soon as stock arrives. If a failure happens before t_1, the unit is replaced as soon as stock arrives. Time between two successive replacements is a cycle. Order is placed at replacement or at failure if necessary. Unit lifetime and order lead time are randomly distributed (Weibull distribution is adopted in numerical cases). Costs are computed at the end of cycle. Expected total cost of failure and preventive replacement, ordering, inventory holding, and shortage is minimized by finding optimal values for t_1, s, S. Ranges for decision variables, different visualizations of a cycle, and different cost and system parameter sets are used to run simulation. Effects of unit lifetime variability, lead time variability, and various cost parameters are studied through case problems. Jointly optimal (t_1, s, S) policy is more cost-effective than the classical age replacement policy combined with optimal (s, S) inventory policy.

The above paper is extended in Zohrul Kabir and Al-Olayan (1996). They dealt with a continuous review (s, S) type of inventory policy for the case of a single item or a number of identical items. If a failure occurs before t_1 an emergency order is issued and unit is replaced as soon as a spare is available. For a single item a cycle is the time between two successive replacements while for multi-unit case the situation of each unit can be treated separately by recognizing the influence of other units, particularly in relation to spare ordering and replenishment.

Table 2.2 Maintenance/replacement models in presence of an inventory control policy

Research	Problem Setting	Major Assumptions	Decision Variable(s)	Objective Function	Optimizing Procedure	Major Achievement(s)
Zohrul Kabir and Al-Olayan (1994), (1996), and Zohrul Kabir and Farrash (1996)	Single operating unit and later multiple identical units, (s , S) inventory policy, age replacement	Random unit lifetime and order lead time	Age replacement parameter, inventory policy parameters	Expected total cost of replacement, ordering, holding and shortage	Simulation over different cycle visualizations, system parameter sets and ranges for decision variables	Joint policy performs better than classical age replacement combined with (s , S) inventory policy
Van der Duyn Schouten and Vanneste (1995), Kyriakidis and Dimitrikos (2006), and Ribeiro *et al.* (2007)	Deteriorating installation and a non-failing subsequent production system, installation state dependent on its age (working condition and buffer level as well in second paper), partial backlogging of demands (the rest is lost)	Limited buffer inventory capacity, perfect CM and PM with stochastic durations, stochastic time to failure with increasing hazard rate, equidistant monitoring and decision epochs of the installation	Decision to "do nothing", "start PM", or "start CM" at decision epochs. In the third paper planning condition-based maintenance on the first and time-based maintenance on the subsequent machine as well as buffer level	In form of performance measures to evaluate a fixed policy: average lost demand, expected amount of backorders, average buffer content, proportion of time spent on maintenance actions. In the second paper objective function includes operational, maintenance, storage and shortage costs	SMDP, value-iteration algorithm. In the second paper discrete-time MDP is used. Mixed integer programming solved by LINDO software in the third paper	For fixed buffer content, optimal action as a function of age is a control limit rule; this policy performs better than overall optimal policy, no-PM policy and age-replacement no-buffer policy. In the second paper for fixed buffer content and fixed age, the policy is a control-limit rule in terms of working condition

Meller and Kim (1996)	Same as in Van der Duyn Schouten and Vanneste (1995)	Same as in Van der Duyn Schouten and Vanneste (1995) but time to failure and CM duration exponentially distributed, cycle times and PM duration deterministic, PM triggered when buffer capacity limit is reached	Buffer inventory limit but not as a decision variable, rather, several performance measures calculated for different values for it: expected number of unscheduled failures, period length and its variance, percentage of time when the subsequent machine is starving, average inventory	Time-averaged cost function of PM and repair, starving subsequent machine, and holding	no optimization procedure, the model is descriptive rather than prescriptive but	Is an extension to van der Duyn Schouten and Vanneste (1995) and provides a descriptive model rather than prescriptive
Armstrong and Atkins (1996) and Giri *et al.* (2005)	One-component, one-spare system subject to random failure with the possibility of expedited order in the second paper	Constant lead time, perfect replacement with negligible duration, non-decreasing failure rate	preventive replacement time and ordering time (spare inventory time limit and regular ordering time in the second paper)	Expected cost of preventive and corrective replacement, shortage and holding per unit time (inventory, shortage and ordering costs in the second paper)	Karush-Kuhn-Tucker point search (mathematical theorems and lemmas in the second paper)	Larger savings compared to maintenance-only and inventory-only optimal solutions
Das and Sarkar (1999)	Single-product system with (S, s) inventory policy	Markov chain system state, demand arrival as a Poisson process, lost unsatisfied demand, unit production time, TBF, repair and PM times all stochastically distributed	Number of items produced since last repair/maintenance for different values of inventory level	Additional revenue per unit time from increased service level plus savings in repair cost minus maintenance cost per unit time	Gradient search algorithm	Consideration of other performance measures including service level, average level of inventory and productivity of system

Hsu (1999)	Unreliable queue-like production system	Parts arrival as a Poisson process, stochastic processing time, increasing failure rate, stochastic minimal repair, PM and replacement times	Number of parts which triggers PM and number of PM actions which triggers replacement upon failure	Revenue obtained processed pars minus expected minimal repair, PM and replacement cost	Numerical search	Sensitivity analysis which shows that optimal policy is very sensitive to PM effectiveness, cost and life length parameters
Marquez *et al.* (2003)	Production system with a maximum production rate limit and a maximum buffer capacity limit	Random time to failure, constant PM and CM durations, lost unfulfilled demand and variable demand and lead time	Dependent on selected maintenance policy: critical age to perform PM, critical inventory level to perform PM and maximum age to perform PM	In form of a set of performance metrics such as service level and fill rate	System dynamics and a Powell search algorithm	Optimality criteria more important than maintenance policy itself to select optimal maintenance policy parameters

It is assumed that order lead time and unit lifetime are randomly distributed. A simulation procedure is used to find joint optimal values for t_1, s, and S so that expected total cost per unit time is minimized. It consists of preventive and failure replacement, regular and emergency ordering, shortage, and inventory holding costs. Multiple regression analysis shows that holding and shortage costs have the greatest influence on optimal policies while ordering cost has no significant effect independently and failure and preventive replacement costs have considerable influence. ANOVA shows that for Weibull lifetime distribution the shape parameter has significant effect on optimal policy. It was also shown that this policy performed better than classical age replacement policy supported by a (s,S) policy.

Zohrul Kabir and Farrash (1996) used a SLAM network program interfaced by TURBO BASIC and Excel spreadsheet to solve the same problem. It was shown that an increase in lead time increased the system cost rate for any given set of system and cost parameters.

Another early study in this area is Van der Duyn Schouten and Vanneste (1995). They studied a deteriorating installation which supplied input to a subsequent production system. A buffer inventory with fixed maximum capacity K can be built up in between. Perfect corrective maintenance is performed after a failure. Installation production rate to build up the buffer is p and when capacity is reached it is reduced to demand rate d.

Time to failure of the installation is a stochastic variable with increasing hazard rate (discretized Weibull is used for comparison of policies) and CM and PM times are stochastic (assumed geometrically distributed for studying structure of optimal policy). PM is perfect and less time consuming than CM. No interruption happens to installation due to lack of input and no failure occurs in production system (hence constant demand rate). Partial backlogging amount ξ may occur but any more demand is lost. State of the installation is $i(0 \le i \le m)$ where m is its maximal age.

System is monitored at discrete equidistant time epochs. Possible actions are "do nothing", "start PM", and "start CM" at decision epochs which are the expiration of a time unit and the end of a maintenance period. Semi-Markov decision process (SMDP) (or Markov decision process (MDP) when PM and CM times are geometrically distributed) is employed to obtain optimal policy. It was found that for fixed buffer content x, optimal action as a function of age was a control limit rule. Optimal policy is found using value-iteration algorithm. The policy calls for PM if the age of installation i and buffer content x satisfy $i \ge N$ and $k \le x \le K$ or $i \ge n$ and $x = K$ for $0 \le n \le N \le m+1$ and $\xi \le k \le K$.

Performance measures are set to evaluate a fixed (n, N, k)-policy. These measures are average lost demand of production unit per unit time, average expected amount of backorders, average buffer content, and proportion of time spent on maintenance actions. Present policy performs very well compared to overall optimal policy, no-PM policy, and age-replacement no-buffer policy.

A similar system was studied in Meller and Kim (1996) where there were two production operations (machines) and a buffer inventory between them. First machine

(M1) which is subject to random failures and random repairs is continuously run at a rate greater than that of second machine (M2) until a failure occurs or when a pre-specified buffer level (b^*) is reached.

It is assumed that no breakdown occurs on M2, no starving happens to M1, and it is very expensive to shut down M2. Operational time between failures and time to repair M1 are exponentially distributed with means MTBF (mean time between failures) and MTTR (mean time to repair) respectively. Cycle times on both machines and PM duration are deterministic. As PM rate increases failure rate decreases on M1. For one cycle including time to perform PM in a PM program, time-averaged cost function of PM and unscheduled repair, starving M2, and inventory holding is derived. Authors did not provide an optimization model; the user, instead, is to derive the cost for different values of b^*, hence, the model is not prescriptive, but rather, descriptive.

For several numerical examples, they showed the impact of different values for b^* on total average cost as well as on some performance measures including expected number of unscheduled failures per period, expected period length, expected percentage of time per period when M2 is starving, the average inventory, and variance of period length.

Infinite-state (age) generalization of the problem in Van der Duyn Schouten and Vanneste (1995) was studied in Kyriakidis and Dimitrikos (2006). The installation is inspected at equidistant time epochs and its working condition is then classified into 0 (new), 1, 2,…, *m+1* (failed). If the installation is found in failed condition, it must undergo corrective maintenance, while a PM action may start if the installation is found to be in condition $i \leq m$. If no PM action is started an operating cost is incurred until the next inspection.

Both PM and CM actions are perfect. Deterioration of the installation depends on its working condition *i* as well as its age *t*. PM and CM times are geometrically distributed. State of the system includes working condition of the installation, its age, and buffer level. The objective is to find the optimal policy at each inspection time epoch among policies 0 (do nothing), 1 (start PM), and 2 (start CM) which minimizes the long-run expected discounted average cost per unit time. Total cost includes operational costs and maintenance costs of the installation, storage and shortage costs. Problem was modeled using discrete-time Markov decision process and was solved using a computationally tractable algorithm. It was also shown that for fixed buffer content and for fixed age of the installation, the policy of starting PM was a control-limit policy in terms of working condition. The same result is obtained for stationary case.

A system comprising of a capacity-constrained resource (CCR) preceded by a non-CCR and a buffer in between was studied in Ribeiro *et al.* (2007). The problem is how to optimally plan condition-based maintenance on CCR, buffer size, and time-based maintenance on the non-CCR. Problem is formulated as a mixed integer linear program and solved by LINDO software.

A one-component and one-spare (in stock or in order) system subject to random failure was considered in Armstrong and Atkins (1996). If a failure occurs before scheduled replacement time t_r the component is replaced immediately or as soon as the spare arrives. If a failure happens before scheduled ordering time t_0 an order is immediately placed. Time between two replacements is one cycle.

Lead time L is constant, replacement is perfect and takes negligible time. System has non-decreasing failure rate (Weibull distribution is used for numerical cases). Objective is to minimize expected cost of preventive replacement, breakage (corrective replacement), shortage and holding the spare in stock per unit time by finding optimal preventive replacement time t_r and optimal ordering time t_0. The paper first found t_r' for maintenance-only problem, then derived t_0' for inventory-only problem, and lastly it developed a joint approach. It was shown that t_r was either $t_0 + L$, infinity, or t_r' for a given t_0. Similarly, t_0 is either zero, $t_r - L$ or t_0' for a given t_r.

Nonlinear programming (specifically Karush-Kuhn-Tucker point search) is employed to find optimal t_0, t_r. Compared to sequential optimal solutions, joint optimization gives large savings especially when all cost coefficients are in balance. If sequential optimization is unavoidable, however, available maintenance information must be used when making subsequent inventory decision.

A similar setting in form of a discrete-time single-unit order-replacement model was studied in Giri *et al.* (2005). There are two decision variables: optimal regular ordering time (n_0^*) and optimal spare inventory time limit (n_1^*). If the unit does not fail before n_0, a spare is regularly ordered at n_0 and is delivered after L_2 time units and is put into operation at $n_0 + L_2$ if a failure has occurred in the interval $[n_0,\ n_0 + L_2]$. Otherwise, delivered spare unit is put into inventory and is put in operation when the original unit fails or when the inventory limit time n_1 is reached after the part's arrival. If the original unit fails before n_0 an expedited order is placed immediately which is more expensive than regular order but its lead time (L_1) is shorter. Optimal values are found such that expected total discounted cost over an infinite planning horizon is minimized. Cost components are expected discounted inventory holding, shortage, and ordering costs. Lead times are constant and deterministic, failed unit is scrapped with no repair, and stocked spare does not deteriorate with time. Using some mathematical theorems and lemmas and conditioning on parameter relationships, the paper found that n_1^* could only be either zero or infinity. In each case and based on some problem parameter relationships, n_0^* could either be finite ($0 < n_0^* < \infty$), zero or infinity.

Das and Sarkar (1999) studied a single-product type production/inventory system with (S, s) inventory control policy meaning that system stopped when inventory level reached S and resumed production when it dropped to s. At the end of production completion epoch of a part if the current inventory level is i and the number of parts produced since last repair/maintenance (product count) is at least N_i maintenance is carried out. System state is a Markov chain and is denoted as (w, i, c) where $w = 0,1,2$ denotes producing, under maintenance, and under repair modes respectively, i is the inventory level, and c is production count.

Demand arrival is a Poisson process, unsatisfied demand is lost, and unit production time, time between failures, repair, and maintenance times all follow general probability distributions (for numerical examples, uniform distribution is assumed for maintenance time and Gamma for the rest). During its vacation, system does not age or fail, and maintained system is as good as repaired system. Objective is to find N_i for $0 \leq i \leq S$ which maximize the average benefit defined as additional revenue per unit time from increased service level plus savings in repair cost minus maintenance cost per unit time. Other performance measures are service level (average percentage of demand satisfied), average level of inventory, and productivity of system (percentage of time when the system is producing). For a set of numerical examples a gradient search algorithm is used. Sensitivity of optimal values to input parameters are higher when repair and maintenance costs and their ratio are high. In this paper, both s and S were assumed fixed and small to facilitate the analysis; however, a brief discussion was presented concerning the alteration of those values concluding that perhaps higher inventory levels constituted a better policy.

An unreliable queue-like production system was studied in Hsu (1999). PM is performed whenever N parts have been processed. If a failure occurs and K PM actions have already been performed, system is replaced, otherwise, a minimal repair is carried out. A production cycle is the time between two successive replacements.

Parts arrive according to a Poisson process, time to process a part is stochastic (it is assumed constant for numerical examples), system has an increasing failure rate (Weibull distribution is assumed for system lifetime in numerical illustration), and effectiveness of a PM action in reducing system's age is a decreasing exponential function of it. Minimal repair, PM, and replacement times are stochastic (their mean values are used). Minimal repair cost is non-decreasing with age of the system (a linear relationship is assumed in numerical illustration), while PM and replacement costs are constant.

For a given K, optimal N is numerically found. This process continues until optimal (K^*, N^*) is obtained which maximizes expected profit (defined as revenue obtained from processed parts minus expected minimal repair cost, PM cost, and replacement cost) per unit time. Sensitivity analysis shows that optimal policy is very sensitive to PM effectiveness, cost parameters and life length parameters.

Marquez *et al.* (2003) dealt with a production system which had a maximum production rate limit and a maximum buffer capacity limit. Decision to start a PM action depends not only on the condition of production unit, but also on buffer inventory level. It is assumed that time to failure is random, PM and CM times are constant (CM takes more time than PM), demand and lead time are variable, and demand which is not fulfilled is lost.

Six performance metrics were provided as follows: service level (percentage of order cycles with no stock-out), fill rate (percentage of demand fulfilled), utilization of production unit, availability of production unit, mean inventory, and maintenance cost. Operations management teams select one or more of these metrics as objective by giving a particular set of weights to them. As an example, improving availability while minimizing maintenance cost can be an objective function.

Elements in the set of decision variables depend on selected maintenance policy. Three policies were studied here: age-based maintenance, age- and buffer-based maintenance, and modified age- and buffer-based maintenance. Decision variables for theses policies are n^* (critical age to perform PM), n^* and k^* (critical inventory level to perform PM), and n^*, k^* and N^* (maximum age to perform PM) respectively. Problem is modeled using system dynamics and a Powell search algorithm is applied to find optimal values for decision variables. Through a numerical case, the paper found that optimality criteria were more important than maintenance policy itself to select optimal maintenance policy parameters.

2.4 Models integrating production and maintenance control

Papers which develop integrated production/maintenance models are reviewed here. Table 2.3 presents a summary of research conducted in this area.

An early work in this area is done by Brandolese *et al.* (1996). They studied a multi-product one-stage production system with parallel flexible machines meaning that several machines could process the same job. Order portfolio is defined by order quantities, release and due dates. Production cost depends on machine and on job to be processed whereas setup cost depends on machine and on job processing sequence. However, processing and setup times are deterministic. Machines have different output rates and any job must be completed on a single machine. Length of maintenance intervention is assumed constant and equal to MTTR of each machine. PM and breakdown costs are known and the latter exceeds the former. Weibull reliability function is assumed for machines lifetime.

Three objectives are investigated: meeting release and due dates, minimizing expected total cost of maintenance, setup, and production, and minimizing total plant utilization time (as a measure of opportunity cost) which consists of machine total job processing time, total setup time, total machine idling time, and total maintenance time. Decision variables for production orders are which order to allocate, which machine to assign the order to, and when to start processing while for maintenance activities they are which intervention to allocate and when to start the intervention. Maintenance is scheduled together with job allocation.

For a specific T (planned interval between two maintenance actions) maintenance cost reaches a minimum. A constraint-based heuristic was applied to find a solution when a value was assigned to each variable that satisfied given constraint (with one-step backtracking). A global priority index is calculated which determines the sequence of allocations satisfying system constraints. Priorities being equal, production orders come before maintenance interventions. System selects the job with the earliest release date. To decide where to allocate a job among all available allocation intervals, system selects the one which implies the lowest total cost.

Table 2.3 Models integrating production and maintenance control

Research	System Setting	Major Assumption(s)	Decision Variables	Objective Function	Optimization Procedure	Major Achievement(s)
Brandolese *et al.* (1996)	Multi-product one-stage production system with parallel flexible machines	Deterministic MTTR and Processing and setup times, Weibull lifetime	Which order to allocate, which machine to assign the order to, when to start processing, which maintenance intervention to allocate, when to start the intervention	Meeting release and due dates, minimizing expected total cost, minimizing total plant utilization	A constraint-based heuristic	A new job and maintenance intervention allocation model with sensitivity analysis on precision and completeness of data provided
Azadivar and Shu (1998)	Four configurations from simple to complex in terms of the number of product states and the number of processes	Five maintenance policies (predictive, reactive, opportunistic, time-based PM and MTBF-based PM)	Type of maintenance policy and size of allowable in-process inventories	Percentage of jobs delivered on time	Computer simulation combined with GA search	GA shows better performance than random search for large systems
Sloan and Shantikumar (2000)	Multiple-product, single-machine, multiple-state production system	Maintenance cost independent of machine condition, state transition independent of product type, machine condition affects different products differently	Probability of making the decision to perform maintenance or to produce one of the items at decision points	Long-run expected average profit	Linear programming	A stationary average reward optimal policy of control limit type exists, substantial gains over sequential approach and FCFS dispatching
Sloan (2004)	Multiple-state single deteriorating machine, single-product	Instantaneous and perfect repair, random demand, binomially distributed yield	Decision to perform repair and how much to input to the production unit	Expected discounted sum of repair, production, backorder and inventory holding cost	MDP	Less cost than sequential approach, control limit policy
Yao *et al.* (2005)	Make-to-stock production system	Stochastic maintenance/repair times, constant	Decision to perform PM and how much to	Total expected discounted PM/CM and inventory costs	Discrete-time MDP	Convenient ontrol-limit PM policy

		demand and integer production rate	produce			
Lee (2005)	Multi-stage multi-component production system	Imperfect production system producing nonconforming components	Investment in inventory and investment in PM	Total investment in inventory, inventory cost, manufacturing cost, backlog cost, stock-out cost, investment in PM and delay cost	Iterative process using sequential quadratic programming method	Investment approach to the problem
Cassady and Kutanoglu (2005) (2003), Sortrakul *et al.* (2005)	Single-machine system	Constant and deterministic processing time, repair and PM durations, minimal repair and perfect PM	Sequential job and PM scheduling then integrating solution using a binary variable as whether or not to schedule a PM before each job	Total expected weighted completion time of jobs (tardiness in the second paper)	Total enumeration (in the third paper a heuristic based on GA is provided)	Relatively simple and convenient to implement
Ji *et al.* (2007), Chen (2006), Liao and Chen (2003)	Multiple non-resumable independent jobs with known processing times and due dates	Deterministic time between maintenance actions and maintenance duration	Job sequence	Total makespan as the maximum/total of completion time of jobs (maximum tardiness in the third paper)	LPT algorithm (a heuristic and a branch-and-bound algorithm in the second paper)	Relatively simple and convenient to implement

To improve the solution, system considers adjacent orders swaps, maintenance shift to a place nearer to optimal T, and stacking the jobs as early as possible to reduce idle time. Numerical experiments to evaluate the performance of the proposed expert system as a pure scheduler and as an integrator were conducted. Sensitivity analysis on precision and completeness of data was provided.

Azadivar and Shu (1998) considered allowable in-process buffer and design parameters of maintenance plan simultaneously. Five maintenance policies (predictive, reactive, opportunistic, time-based PM and MTBF-based PM) were investigated for four configurations ranging from simple to complex in terms of the number of product states and the number of processes used to change the state of part from current to the next. Service level, defined as percentage of jobs delivered on time, was selected as the measure of performance which should be maximized. A methodology combining computer simulation and GA search was used to find the optimal qualitative factors (type of maintenance policy) and quantitative factors (size of allowable in-process inventories). GA showed relatively better performance than random search especially for large systems.

Sloan and Shantikumar (2000) considered a multiple-product, single-machine, and multiple-state production system where state of machine deteriorates over time and equipment condition affected the yield of different product types differently. State of machine in period n is either zero (best condition), or $1,2,...,M$ (worst condition) and is modeled as finite Markov chain.

Objective is to find an optimal production and maintenance policy stated as the decision, at period *n*, to perform maintenance (cleaning specifically which is denoted by $K+1$) or to produce one of the K items to maximize long-run expected average profit. Rewards for actions taken are bounded, cleaning cost is independent of machine condition, and if production is chosen state transition is independent of the choice of product to produce.

It is shown that a stationary average-reward optimal policy exists and is a control limit type and is found using linear programming. More-sensitive products to machine condition are produced when the machine condition is good, then less-sensitive products are produced and when the machine condition reaches a limit cleaning is performed. Decision variable of LP program is x_{ia}^{*} which denotes probability of taking action $a = 1,2,...K, K+1$ when the machine is in state i. Presented method of simultaneous determination of production and maintenance controls and yield-based dispatching showed substantial gains over sequential approach and FCFS dispatching.

A single-stage single-machine single-product case was studied by Sloan (2004). In the beginning of each period, machine state $I_n \in \{0,1,...,M\}$ and inventory level $X_n \in \{...,-2,-1,0,1,2,...\}$ are observed and a decision is made as to whether or not to perform repair (instantaneous and perfect) and how much to input to the production unit. If machine is found in state M a repair is mandatory for leaving this state. Demand (which is random and follows an independent and identical well-behaved distribution for every period) is then experienced and costs are incurred.

Product yield is binomially distributed and depends on equipment condition which deteriorates over time as a result of production according to a Markov chain process with known transition probabilities. Machine deterioration is assumed to be relatively fast as compared to production rate. Yield decreases and it is more likely to go to a worse state than a better state as the machine condition gets worse. Objective is to find optimal equipment maintenance policy ($a_n = 1$ (repair) or $a_n = 0$ (no repair)) and optimal quantity to input (q_n) to the machine at each period such that expected discounted sum of repair, production, backorder, and inventory holding costs is minimized.

A Markov decision process was used to model the problem and a policy improvement algorithm was used to solve it. Through some numerical examples it was found that presented method (simultaneous approach) incurred less cost than sequential approach (maintenance policy optimization, then production optimization). Furthermore, a critical inventory level exists such that input quantity is grater than zero for values equal or smaller than it and a critical machine state exists (and increases as inventory level increases) such that it is necessary to perform repair when the machine state is equal or greater than it.

A joint production/inventory and preventive maintenance optimization was presented in Yao *et al.* (2005). They considered a make-to-stock production system where maintenance/repair times were non-negligible and stochastic. Objective is to minimize total expected discounted preventive/corrective maintenance and inventory costs over an infinite horizon using a discrete-time Markov decision process (MDP) model. Joint policy basically includes (a) whether or not to perform PM, and (b) if not, how much to produce. Optimal policies can have very complex forms. Therefore, some structural properties of optimal solutions under specific assumptions are studied. Specifically, (a) when inventory is below a certain negative level, optimal policy is either to produce at maximum rate or to perform PM; (b) when inventory is above a certain level it is optimal either not to produce at all or to do PM; and finally (c) when inventory goes to infinity or a hedging point policy for production is used or PM setup cost is zero, PM will have a control-limit structure. This structure is attractive due to the ease of its implementation. Demand is considered constant and production rate is assumed to be an integer.

Lee (2005) studied optimal investment in inventory (through optimal service level for component i at stage j ($L(i, j)$)) and optimal investment in PM actions (through optimal proportion of nonconforming components i at stage j ($p(i.j)$)) in an imperfect multi-stage multi-component production system. Total cost to be minimized consists of investment in inventory, inventory cost, manufacturing cost, backlog cost, stock-out cost, investment in PM, and delay cost. An iterative process using sequential quadratic programming method was used to solve this resource allocation problem.

Cassady and Kutanoglu (2005) considered a relatively simple integrated model of a single-machine job scheduling and its preventive maintenance. Objective function is to minimize total expected weighted completion time of jobs. Using WSPT (weighted shortest processing time) they first minimized total weighted completion time of jobs and provided a job sequence. An optimal preventive maintenance interval was then independently obtained by maximizing availability of the machine. Finally, they integrated job scheduling/PM on machine by introducing a binary variable representing whether or not to schedule PM before each job. Processing time of jobs, repair duration, and time required to perform PM were assumed deterministic and constant. Repair was considered minimal and PM was perfect.

In a similar paper, Cassady and Kutanoglu (2003) presented an integrated model of a single-machine job scheduling and preventive maintenance performed on the machine. Objective, this time, was to minimize total weighted tardiness of jobs. Optimal job sequence was found using total enumeration on $n!$ possible sequences of jobs while optimal PM interval was obtained in the same way as Cassady and Kutanoglu (2005). Integrated optimization procedure is slightly different here. The reason is that (a) it is assumed that only one failure can occur during processing of a job, and (b) objective function is now total weighted tardiness of jobs.

Both of the above-mentioned papers used total enumeration as optimization technique, this is, however, feasible only when the number of jobs to be scheduled is low (less than 8). Sortrakul *et al.* (2005) presented a heuristic based on genetic algorithm for the problem formulation of Cassady and Kutanoglu (2005). The focus here was more on optimization procedure rather than efficiency of the model and structural properties of optimal solution.

The case of a single-machine scheduling with periodic maintenance activities was studied in three papers by Ji *et al.* (2007), Chen (2006), and Liao and Chen (2003). In these studies n independent non-resumable jobs $J = \{J_1, J_2, ..., J_n\}$ with known processing times and due dates were scheduled. Maintenance activities which took t time units were performed every T time units.

Objective in Ji *et al.* (2007) was to minimize makespan which was defined as in Eq. (2.2).

$$C_{\max} = \max_{j=1,2,...,n} C_j \tag{2.2}$$

where C_j is the completion time of job J_j. It was shown that LPT (longest processing time) algorithm was the best possible polynomial time approximation since it presented a worst-case ratio of only 2 meaning that relation between minimum makespan produced by LPT (C_{LPT}) and the one produced by an off-line optimal algorithm (C_{OPT}) was $C_{LPT} \le 2C_{OPT}$. According to LPT algorithm jobs are first arranged in descending processing time order and are then scheduled consecutively as early as possible.

Chen (2006) considered minimizing total flow time $\sum_{j \in J} C_j$ as its objective and provided a heuristic and a branch-and-bound algorithm to find near-optimal and optimal schedules respectively. Comparison between the two procedures for a numerical case showed that PED (percentage error deviation) was smaller for large T and small t and it increased slightly as the number of jobs increased. According to computational time it was concluded that the heuristic was applicable to large problems, while branch-and-bound algorithm could only be used for small and medium sized problems.

Finally, Liao and Chen (2003) studied the same problem and got the same results considering minimization of maximum tardiness $T_{\max}$ as the objective. If due date of job *j* is d_j, lateness of job *j* is defined as $L_j = C_j - d_j$ and tardiness of job *j* is $T_j = \max\{0, L_j\}$, then $T_{\max} = \max_j \{T_j\}$.

2.4.1 Joint determination of optimal production and preventive maintenance rates

Papers which focus on finding joint optimal preventive maintenance and production rates through analytical and/or simulation-based methods are considered in this part. Reviewed papers mainly use stochastic dynamic programming and/or statistical tools such as ANOVA, regression analysis, and response surface methodology (RSM) to derive optimal production and maintenance rates. Table 2.4 summarizes the research in this area.

An early study in this field is Boukas and Haurie (1990) where a flexible manufacturing system (FMS) which consisted of several machines producing a given item was studied. The case with two machines was analyzed in detail. System state is a hybrid of continuous state equations concerning cumulative surplus of parts and age of machines and discrete state related to operational mode of system which is either operational, under repair, or under maintenance.

Table 2.4 Joint determination of optimal production and preventive maintenance rates

Research	System Setting	Major Assumption(s)	Decision Variables	Objective Function	Optimization Procedure	Major Achievement(s)
Boukas and Haurie (1990)	An FMS of several machines with continuous and discrete state elements	Perfect repair and PM, exponential random repair duration, Poisson process for failures	Production and PM rate of each machine	Total expected discounted cost of part surplus or backlog, repair and PM	Dynamic programming	Hedging surfaces for production rates and age-limit PM policy
Boukas *et al.* (1995)	An FMS of multiple machines and part types subject to breakdown and repair	Finite Markov chain state	Rate of maintenance and rate of production	Supremum of the expected discounted cost of inventory/shortage, production and maintenance	Dynamic programming	Robust objective function is used (minimax)
Boukas and Yang (1996)	A failure-prone single-machine, single-part, two-state manufacturing system	Increasing probability of machine failure, imperfect PM, constant demand, perfect repair with constant duration	Production and maintenance rate	Discounted cost of inventory holding and shortage and PM	Dynamic programming	Optimal control policy for production rate is a hedging surface as a function of machine aging
Kenne and Gharbi (1999) (2000)	One-machine (multiple machines in the second paper), one-product-type, three-state production system with machine-age-dependent downtimes	Constant demand rate and transition rates from PM and repair state to up state	Production and PM rates	Expected discounted cost of inventory holding, backlog, repair and PM	Dynamic programming, simulation, ANOVA, regression analysis, RSM, minimum search	Production and maintenance rates as parameters of a machine-age-dependent hedging point policy
Boukas and liu (2001)	Failure-prone one-machine, one-part-type, four-state manufacturing system	Continuous-time Markov process transitions, constant part deterioration rate and demand	PM, CM, production rates	Discounted expected cost of rejected items, PM, CM, inventory holding, and backlog	Stochastic dynamic programming	Four machine states: good, average, bad, failed. Dependence of part rejection rate on machine state
Kenné and Boukas (2003)	FMS with three-state multiple	Perfect PM and repair	PM and production rates	Expected discounted cost of	Dynamic programming	Optimal PM policy depends on a

	machines and multiple part types			surplus or backlog and PM		machine age at which the machine is sent to PM when certain stock threshold values are achieved
Gharbi and Kenne (2005)	Multiple three-state machines and multiple products	Constant demand	Production and PM rates	Total cost of inventory/backlog, repair and PM	Continuous-time Markov chain process	Production policy is a hedging point structure and PM policy is an age-dependent policy
Kianfar (2005)	FMS	Age-dependent failure rate, time- and advertisement-dependent demand rate	Production, maintenance and advertisement cost rates	Total expected discounted cost of product surplus, repair, maintenance and advertisement minus profits	A proposed numerical method	Inclusion of advertisement
Yuniarto and Labib (2006)	One two-state machine, one-product	Hedging point production policy	Size of hedging point	Total cost of inventory holding and backlog	Fuzzy logic controls	When system is not producing, a PM action is performed from one of five possible strategies

Probability of failure of a component is an increasing function of its age which is itself an increasing function of production rate. Repair and PM are perfect and the former costs much more than the latter. Repair duration is an exponential random variable. Failures occur as a Poisson process. Total expected discounted system cost as a function of part surplus or backlog, repair activity, and maintenance activity is to be minimized by finding optimal production rate of each machine $u_i(t)$ and PM rule $v_i(t)$.

System was modeled by dynamic programming and Hamilton-Jacobi-Bellman (HJB) equations were solved by a specific method. Maintenance rate is determined independently from production rates (u_1 and u_2) which are determined by hedging surfaces. PM is triggered according to an age-limit policy. Hedging surface decreases with an increase in discount rate or a decrease in backlog cost while it increases with an increase in demand rate.

A flexible manufacturing system of *M* machines and *P* part types subject to breakdown and repair was studied in Boukas *et al.* (1995). Machine state $\alpha(t)$ which is either operational (it can produce any part type) or under repair is a finite Markov chain with a generator dependent on aging rate $m(t)$, production rate $u(t)$, and maintenance rate $w(t)$. Inventory/shortage level vector of system and its age vector are denoted by $x(t)$ and $a(t)$ respectively. Aging rate is always nonnegative, linear in *u* and *w*, increasing in *u* and decreasing in *w*, and $m_i(\alpha, u, w) = 0$ when i^{th} machine is not operational. Because of unfavorable fluctuations in demand a robust (minimax) objective function is considered.

Objective is to minimize the supremum of the expected discounted cost of inventory/shortage, production, and maintenance over all demand processes by controlling rate of maintenance and rate of production. Value function satisfies and is the only viscosity solution to associated Hamilton-Jacobi-Isaacs (HJI) equation and has Lipschitz property (it is differentiable almost everywhere). Since it is difficult to derive closed-form solutions, a numerical approach is proposed. The basic idea of this algorithm is an approximation scheme for partial derivatives of value function $v(x,a,\alpha)$ within a finite grid of state vector (x,a) and a finite grid for control vector which transforms the original optimization problem to an auxiliary discounted Markov decision process solved by either successive approximation or policy improvement techniques.

Boukas and Yang (1996) studied a failure prone single-machine manufacturing system producing a single part where $x(t)$ denoted stock surplus and $a(t)$ denoted age of system. Machine has two states, operational ($\xi(t)=1$) and failed ($\xi(t)=2$). Maximal maintenance rate $\overline{v}$ is assumed equal to maximal production rate $\overline{u}$ multiplied by a constant coefficient. Probability of machine failure increases with its age.

PM is imperfect, demand rate is constant, and aging is an increasing function of production rate and a decreasing function of maintenance rate (a linear relationship is assumed). To perform maintenance there is no need to stop the machine. Repair is perfect and its duration is constant. Discounted cost of inventory holding and shortage over an infinite horizon as a function of inventory level and maintenance rate is minimized. PM cost is added in case of its existence. Production and maintenance rates $(u(t),v(t))$ are control variables. When total cost does not include maintenance cost, best maintenance policy is to keep the machine as good as new.

Optimal control policy for production rate is of hedging-level type as a function of machine aging. For nonzero maintenance cost HJB equations must be solved by dynamic programming equation and hedging surface concept is relevant. In this case, if a change in machine age increases cost significantly, machine should be kept as good as new through maintenance, otherwise no PM is optimal. Optimal control policy for production rate is a hedging surface as a function of machine aging.

A one-machine, one-product-type production system with machine-age-dependent downtimes was studied in Kenne and Gharbi (1999). System has three states (up, under repair, and under PM) and is subject to non-homogeneous Markov process. Demand rate and transition rates from PM state and repair state to up state are constants. Condition $\overline{\alpha}U_m > d$ holds where $\overline{\alpha}$ denotes availability of the machine, U_m is the maximum production rate, and d is demand rate. Objective is to find optimal production and PM rates expressed as parameters of a machine-age-dependent hedging point policy which minimize the expected discounted cost of inventory holding, backlog, and cost of keeping the machine in repair or in maintenance states. Value function satisfies certain HJB equations whose analytical solution procedure is difficult. Hence, a combination of analytical control approach and experimental design method based on simulation experiments is used. ANOVA finds significant factors and interactions which makes it possible for regression analysis procedure to find the response surface as a function of those factors. Optimal values for hedging point parameters are found by minimum search.

An extension of the above paper was studied in Gharbi and Kenné (2000). They considered a multiple-identical-machine manufacturing system producing one product and having machine-age-dependent downtimes. System can be in one of operational, under repair, and under PM states. System behavior has a continuous component (inventory/backlog and cumulative age) and a discrete component (number of available machines). Optimal production and maintenance rates are derived through parameters of a modified hedging point policy where *X* is the stock threshold level, *B* is the oldest machine age to send it to PM provided $x(t)$ (stock surplus) is greater than *X*, and *A* is the oldest machine age under which a null stock level is needed and above which a stock of *X* is needed. Therefore, depending on the value of surplus and the age of machines, production rate takes one of these values: αU_m (where α is the number of operational machines and U_m is the maximal production rate of a machine), *d* (the demand rate), and zero.

Objective function is to minimize expected discounted total cost of inventory/backlog, PM, and repair. Since analytical solution of related HJB equations is almost impossible, analytical formalism is combined with simulation-based tools. Proposed method consists of checking if input parameters (*A*, *B*, and *X* derived from analytical part) and their interactions affect the response (using experimental design and ANOVA), estimating the relationship between the cost and significant factors (using RSM), and computing optimal values of significant factors (using a simple numerical grid founding optimal method).

Boukas and Liu (2001) studied a failure prone manufacturing system of one machine and one part type. System is either in good, average, or bad state and part rejection rate depends on machine state. A failed state exists in which no part is produced. State transitions follow continuous-time Markov process, parts deteriorate with a constant rate, and demand is constant. Jump rates from average and bad states to good state (PM rates), from failure state to good state (CM rate), and production rate are control variables whose optimal values are derived such that discounted expected cost of rejected items, PM, CM, inventory holding and backlog is minimized. Stochastic dynamic programming (a nonlinear system with Markov jumping disturbance) is employed. Optimal value function is shown to be the unique viscosity solution to a set of HJB equations.

Production and PM planning control problem for a multi-machine FMS with m machines and n different part types was considered in Kenné and Boukas (2003). Operational mode of system $\zeta_i(t)$ is either 1 (operational), 2 (under repair), or 3 (under PM) for $1 \le i \le m$.

Machines are identical, PM and repair are perfect, and rate of change in machine state is much larger than the rate at which cost is discounted. Their model minimizes expected discounted cost including surplus and backlog costs and PM cost by finding optimal transition rates from operational mode to PM mode $w_i(t)$ for $i = 1,2,...,m$ and production rates $u_j(t)$ for $j = 1,2,...,n$.

Optimal policy is the solution of certain HJB equations. To reduce size of the system, limiting control problem is used. The structure of optimal control policy is as follows. For a machine age greater than A_i, $u_j(\tilde{x})$ is equal to either total maximal production rate of product j $u^j_{\max}$ (if $x_j(t) < X_j$), demand rate d_j (if $x_j(t) = X_j$), or zero (if $x_j(t) > X_j$) where X_j is the threshold stock level of product j. For ages below A_i just replace X_j with zero (no need to stock parts). Optimal machine-age-dependent PM policy depends on a machine age B_i at which machine i is sent to PM when threshold values X_j are achieved.

Gharbi and Kenne (2005) presented a model to find production and PM rates for a multiple-machine, multiple-product system to minimize total cost consisting of inventory/backlog, repair, and PM costs. Each machine has three states: operational, under repair, and under PM. Demand is assumed constant. An interesting result is that the structure of optimal production policy is a hedging point structure and that of PM policy is an age-dependent PM policy. They derive their results using continuous-time Markov chain process. Some statistical experiments on a numerical simple case are conducted to find the impact of inputs on total cost. Finally a sensitivity analysis is performed.

Kianfar (2005) presented simultaneous planning of production and maintenance in a flexible manufacturing system where failure rate is a function of the age of machine and demand is time-dependent and its rate depends on level of advertisement. Objective function is to minimize total expected discounted costs minus profits over an infinite time horizon where cost function consists of product surplus (production over demand) cost as well as repair, maintenance, and advertisement costs. Solution comprises optimal production, maintenance, and advertisement cost rates.

Yuniarto and Labib (2006) considered a case of one machine and one product where production is based on a hedging point policy as follows:

- If $x \prec H$ then $u = \mu\gamma$
- If $x = H$ then $u = d\gamma$
- If $x \succ H$ then $u = 0$

where x is inventory level, H is hedging point as in Eq. (2.3).

$$H = \frac{dT_r}{2} \tag{2.3}$$

where T_r is mean time to repair, μ is maximum production rate, d is demand rate, and γ equals to zero or one when machine is down or operating, respectively. Using fuzzy logic controls, paper finds approximate optimal size of hedging point minimizing total cost of inventory holding and backlog. According to this method when system is not producing, a preventive maintenance action is performed which is one of five possible strategies (from "operation to failure" to "design out maintenance") based on values of mean time to repair and frequency of failures.

2.5 Integrated determination of EPQ and inspection/maintenance schedule

Table 2.5 provides a summary of research conducted in this area.

2.5.1 Joint determination of optimal economic production quantity (EPQ) and inspection/maintenance schedule in a deteriorating production process

In papers reviewed in this part, a production system which may shift to out-of-control state leading to production of some proportion of defective items is considered. Probability distribution function of time spent in control is known. Objective is to find joint optimal economic production quantity (EPQ) (through optimal production run time), number of inspections/PM actions, and their schedule.

A classical work in this field was done by Lee and Rosenblatt (1987). They studied a single production system which produced a single item and might shift to out-of-control state (hence producing α percent defective) after spending an exponentially distributed time in control with mean $1/\mu$. During a production run of T time units, system is inspected at $T_1, T_2, ..., T_n = T$ for $n \geq 1$. System is restored to in-control state once found out of control.

Inspections are error free (type I error is taken into account later in the same paper) and number of defective items produced when the system is in control is negligible. Inspection and restoration times are also negligible (restoration cost as a function of detection delay is briefly discussed). Demand is constant and continuous and no shortage is allowed.

Table 2.5 Integrated determination of EPQ and inspection/maintenance schedule

Research	**System Setting**	**Major Assumptions**	**Decision Variables**	**Objective Function**	**Optimization Procedure**	**Major Achievement(s)**
Lee and Rosenblatt (1987), Lee and Park (1991), Lin *et al.* (2003)	Single-machine, single-product system (multiple raw materials in the third paper)	System may shift to out-of-control state after an exponentially distributed time in control, error-free inspections, constant demand, no shortage allowed	Production run time (hence EPQ), number of inspections, optimal inspection schedule (and order quantities of raw materials in the third paper)	Total cost of setup, inventory holding, inspection, restoration, and defective production (and warranty cost in the second paper and raw materials in the third paper)	Use of derivatives and certain inequalities	Better performance than classical EPQ (Consideration of warranty scheme in the second paper and order quantities of raw materials in the third paper)
Makis and Fung (1995)	A production facility subject to random failures	Same as above, general distribution for time to failure, perfect repair	Lot size, number of inspections during a production run, optimal preventive replacement time	Expected average cost of setup, inventory holding, inspection, restoration, defective production and preventive or failure replacement	Use of derivatives	Stable optimal lot size and number of inspections, more frequent replacements with decreasing time to failure, more inspections with an increase in out-of-control rate
Tseng (1996)	Single-item, single-machine production process	Generally distributed time in control with IFR	Run length, number of maintenance actions, maintenance schedule	Expected total cost of setup, inventory holding, restoration, defective production and maintenance	Use of derivatives	Better performance than the inspection-only policy
Makis (1998)	A production process subject to random deterioration	Imperfect inspections, generally distributed time in control	Inspection schedule, number of inspections and production time	Expected average cost of inventory holding, setup, inspection, restoration, and defective production	Dynamic programming and a one-dimensional search procedure when inspection is only possible at discrete times, nonlinear programming and two-dimensional	Better performance than equal-interval inspection policy and no-inspection policy

					search procedure for continuous case	
Wang and Sheu (2000)	Single-product production system, equal-interval inspections	Perfect PM, deterministic demand and production rates, generally distributed in-control time	Inspection/PM interval length and number of inspections	Total cost of setup, inventory holding, restoration, defective production, inspection and maintenance	Numerical search procedure after having set up upper and lower bounds	Relatively convenient to implement
Kim *et al.* (2001), Wang (2006) (2006)	A deteriorating production process, equal interval inspections	Exponentially distributed time in control (general distribution with non-decreasing failure rate in the second paper)	Run length, number of inspections (run length, number of PM actions and maximum level of backorders in the second paper)	Average cost of setup, inventory holding, inspection, restoration and defective products (and shortage and PM in the second paper)	An algorithm combining derivatives and numerical procedures	Better performance than approximated procedure using McClaurin approximation (consideration of backorders in the second paper)
Alfares *et al.* (2005)	Single deteriorating item in a deteriorating production system, equal interval maintenance inspections	Exponentially distributed in-control time, exponentially decreasing demand	Cycle time, number of inspections	Total average cost of setup, inspection, deterioration of inventory items, inventory holding, defective production, and restoration	Heuristic numerical solution algorithm	Deteriorating product as well as deteriorating equipment
Ben-Daya (2002), Ben-Daya and Noman (2006), Sheu and Chen (2004), Chen (2006)	Single-item production system (a free repair warranty is offered in the second paper)	Generally distributed in-control time with IFR, constant demand, shortage not allowed, reduction in age dependent on PM level	Length of inspection/PM intervals, PM level, optimal number of inspections (optimal run length as well in the second paper)	Total expected cost of setup, inventory holding, PM, inspection, defective production, restoration (revenues from selling items minus costs including manufacturing and warranty costs as well in the second	Modified pattern search technique (a numerical algorithm using Golden section method in the second paper) (stepwise partial enumeration in the fourth paper)	Consideration of PM level as a decision variable (inclusion of warranty scheme in the second paper) (two kinds of out-of-control states are considered in the third paper) (imperfect inspection as well as permitted

				paper) (minimal repair cost as well in the third paper) (shortage and inspection error types incorporated in the fourth paper)		shortage level in the fourth paper)
Rahim and Ben-Daya (1998), Ben-Daya (1999), Ben-Daya and Makhdoum (1998)	Continuous production process, output quality evaluated by $\bar{x}$ -control chart (single-item in the second paper)	Generally distributed in-control time with IFR (constant and continuous demand, no shortage allowed and reduction in age dependent on PM level in the second paper) (three additional variables in the third paper)	Production quantity, inspection schedule (and its number in the second paper), and economic design parameters of control chart (PM schedule parameter and its level in the second paper)	Expected total cost of setup, inventory holding, and using chart	Modified pattern search technique	Consideration of economic design of control charts (and PM level in the second paper) (in the third paper three PM policies are investigated)
Rahim and Ben-Daya (2001)	Deteriorating production process, deteriorating products, output quality monitored by control charts	Arbitrary deterioration distributions, normally distributed quality characteristic	Production run length, inspection schedule, design parameters of control chart	Expected average cost of deteriorated items, setup, using chart, inventory holding	Combined inventory model for deteriorating products and another model for both inventory and quality problems and use of a modified pattern search technique	Better performance than using the two models separately
Lam and Rahim (2002)	A production which can be in or out of control monitored by control charts	Very high replacement costs, generally distributed in-control time with IFR, no shortage allowed	Number of inspection intervals, economic design parameters of control chart, production runtime	Expected total cost of setup, use of control charts, inventory holding, and maintenance	Modified pattern search technique	Extensive sensitivity analysis of the problem parameters and two-factor interactions is conducted

Total cost of setup, inventory holding, inspection, restoration, and production of defective items is minimized by finding optimal production run time T^* (hence economic production quantity Q^*), optimal number of inspections n^*, and optimal inspection schedule $T_1^*, T_2^*, ..., T_n^*$. It is concluded that for given n and T if $s\alpha P/\mu \leq r$ only one maintenance action at the end of production run time is optimal, otherwise, equally spaced maintenance actions are optimal (s is the cost of producing a defective item, P is production rate, and r is restoration cost). Taylor's expansion is applied to replace exponential terms in cost function to facilitate the analysis.

When $n^* = 1$ is optimal, T^* is derived by equating cost function derivative with respect to T to zero and is larger than that of classical EPQ model. In the other case, n^* satisfying a certain inequality is derived and then T^* is found in a similar manner as the case of optimality of one maintenance action. Sensitivity analysis shows that n^* increases with an increase in setup cost and/or failure rate, whereas it decreases when inspection cost and/or inventory holding cost are increased. Also, for small values of restoration cost, n^* tends to be high. Cost penalty for using classical EPQ model is zero when $s\alpha P/\mu = r$ or when $\mu = 0$. This penalty varies directly with the difference between $s\alpha P/\mu$ and r as well as with setup cost K. A large difference between production and demand rates and/or a larger holding cost reduces cost penalty.

Lee and Park (1991) studied a production system that, after an exponentially distributed time in control, might shift to out-of-control state where it produced some proportion of defective items which were either reworked before being shipped or incurred much larger warranty cost. Once found out of control by inspection, system is restored to in-control state.

Inspection and restoration times are negligible. Inspection is perfect and production/inventory follows FIFO policy. Expected annual cost of setup, inventory holding, inspection, restoration, reworking and warranty is minimized by finding optimal values for production run time *T*, number of inspections during each production run *n*, and elapsed time from the beginning of the run until i^{th} inspection T_i where $T_n = T$.

Based on FIFO policy, annual expected cost for cases with and without sold items is studied by conditioning on time of shift to out-of-control state. Policy of equally-spaced inspection intervals is studied in more details and McClaurin approximation for exponential terms is used. For given *n*, relevant derivative of appropriate cost function is set to zero to find $T^*(n)$ and optimal cost $TC^*(n)$. Stepwise partial enumeration procedure is then applied to find *n* which minimizes $TC^*(n)$. Cost penalty of using the same cost for rework before shipping and warranty is shown and sensitivity analysis of using different *n* and *T* from n^* and T^* is conducted.

An EPQ model for a single-product multiple-raw-materials system with imperfect processes under equidistant inspection schedule was studied in Lin *et al.* (2003). After an exponentially distributed time in control process may shift to an out-of-control state where a certain percentage of produced parts are defective. Objective is to minimize total annual cost of before and after sale defectives, setup, inventory holding, inspection and restoration in addition to overall annual cost of all raw materials. McClaurin approximation is used for exponential terms. Optimal production lot size (through optimal production runtime), inspection schedule, and order quantities of raw materials are derived. Model is later extended to consider linear and exponential deterioration, and limited capacity for raw materials as well as the case where deterioration parameters depend on setup cost.

A production facility subject to random failures which, after an exponentially distributed time in control, may shift to out-of-control state and produce some percentage of defective items was studied in Makis and Fung (1995). N equal-interval inspections are performed during production run, the last being at the end of production cycle.

Time to failure follows a general distribution (Weibull distribution is assumed in numerical examples). Perfect repair is done upon failure and preventive replacement is conducted after M production runs. Production cycle starts (renewal point) when inventory is depleted. Inspection, restoration, and replacement take negligible time. Demand is constant and continuous and no shortage is allowed. Objective is to minimize expected average cost of setup, inventory holding, inspection, restoration, producing defective items, and preventive or failure replacement (overhaul) by finding optimal lot size Q^*, optimal number of inspections during a production run N^*, and optimal preventive replacement time (dictated by M^*).

By conditioning on time to machine failure, expected cycle length and expected incurred cost is derived. Optimal lot size and number of inspections are found to be relatively stable. Optimal preventive replacement time decreases (more frequent replacements) with decreasing time to failure. For a given mean time to failure, optimal number of inspections increases with an increase in out-of-control rate.

A single-item single-machine production process was studied by Tseng (1996). After a generally distributed time having increasing failure rate (IFR) in control (Weibull and extreme-value distributions are assumed in numerical examples), process goes out of control where a percentage of produced items are defective. PM is performed at $t_1, t_2, ..., t_n$. If the process is found in control it is restored to as-good-as-new conditions while if it is found out of control it is restored to in-control state. A maintenance action is always performed at the end of a production cycle.

Objective is to find optimal production run length t^*, optimal number of maintenance actions n^*, and optimal maintenance schedule (interval lengths) $x_1^*, x_2^*, ..., x_n^*$ such that expected total cost of setup, inventory holding, restoration, producing defective items, and maintenance is minimized. For given *n* and *t*, conditions concerning problem parameters for optimality of one maintenance action and equal-interval maintenance actions are explored. For fixed *t* the value of n_t^* is derived by using that part of cost function which depends on $x_1^*, x_2^*, ..., x_n^*$. Finally, t^* is derived using total cost function leading to determination of a policy denoted by $t^*, n_t^*,$ and $(t^* / n_t^*, ..., t^* / n_t^*)$. It is shown that maintenance policy performs better than inspection-only policy unless per-maintenance cost exceeds per-inspection cost by an obtained constant.

Makis (1998) studied a production process subject to random deterioration and monitored by imperfect inspections (type I and II errors exist). A perfect and more expensive inspection is performed at the end of a production run. Time in control follows a general distribution, inspection and restoration times are negligible, and the system may produce defectives both when in control and out of control.

Objective is to obtain optimal inspection schedule $t_1^*, t_2^*, ..., t_n^* = T$, optimal number of inspections n^*, and optimal production time T^* (hence optimal lot size) which minimize long-run expected average cost per unit time consisting of costs of inventory holding, setup, inspection, restoration, and cost of producing defective items.

Two cases are investigated. When inspection is possible only at discrete times, optimal inspection schedule for a given *T* is found by using dynamic programming; a one-dimensional search procedure is then applied to find T^*. In continuous case, assuming exponential lifetime distribution and then a general distribution, the paper uses nonlinear programming to derive optimal inspection schedule for given *T* and *n*; a two-dimensional search procedure is then used to obtain T^* and n^*. In case of exponential lifetime distribution and perfect inspections, *n* is first assumed continuous (denoted by *x*). Optimal policy (T^*, x^*) is obtained by solving two nonlinear equations. Corresponding n^* is derived using a simple inequality for any *T*. Sufficient conditions for optimality of one inspection at the end of cycle is investigated. Comparisons show that suggested model performs better than equal-interval inspection policy and no-inspection policy. Sensitivity analysis confirms that although all cost parameters affect optimal policy and average cost, ratios of lengths of optimal inspection intervals are more affected by risk parameters related to inspection errors.

A single-product production system which may shift to out-of-control state and produce α ($0 \le \alpha \le 1$) proportion of defectives was studied in Wang and Sheu (2000). Inspection/PM is performed at T_i, $i = 1,2,...,n$. Once found out of control, system is restored to in-control state, otherwise a perfect PM is performed. Demand and production rates are deterministic, inspection, restoration, and maintenance times are negligible, optimality of equal-interval inspections holds, and in-control time follows a general distribution (exponential and Weibull distributions are considered in numerical examples). Total cost of setup, inventory holding, restoration (as a linear function of detection delay), production of defective items, and inspection and maintenance is minimized by finding optimal inspection/PM interval length $x^*_{(n)}$ and optimal number of inspections n^*. Upper and lower bound for $x_{(n)}$ and upper bounds for n^* are provided to facilitate numerical search procedure. Value of $x^*_{(n)}$ is found for $n = 1,2,...$ and then n^* is derived based on incurred cost.

Kim *et al.* (2001) considered a deteriorating production process that, after an exponentially distributed in-control time, might shift to out-of-control state where α percent of products would be defective. The system is inspected N times during production runtime. Optimality condition of equal-interval inspection holds, more specifically $s\alpha P/\mu > r$ where s is cost of producing defective items, P is production rate, $1/\mu$ is mean time in control, and r is restoration cost. If machine is found out of control, an instantaneous restoration is performed. An algorithm combining derivatives and numerical procedures is provided to find optimal production run length T^* and optimal number of inspections N^* such that average cost of setup, inventory holding, inspection, restoration, and defective products per unit time is minimized. It is shown that if maximal possible savings earned by inspection and restoration ($s\alpha P/\mu$) is less than sum of inspection and restoration costs ($\nu + r$) only one inspection at the end of cycle is optimal. A numerical experiment shows that this procedure performs better than approximated one using McClaurin approximation of exponential terms in total cost function.

A similar single-item model where in-control time follows a general distribution with non-decreasing failure rate and α percent of items are defective while the system is in out-of-control state was considered in Wang (2006). Restoration cost is a linearly increasing function of detection delay, equal-interval maintenance optimality holds, and inspection, restoration, and PM take negligible time. A multiple-step algorithm based on derivatives and search methods is used to find optimal production run length *T*, optimal number of PM actions *n*, and maximum level of backorders *B* while minimizing total cost per unit time of setup, inspection and PM, restoration, and inventory holding/shortage costs as well as cost of producing defective items. A numerical problem is studied assuming Gamma lifetime distribution.

Wang (2006) was further extended in Wang *et al.* (2006). They studied structural properties of PM problem in case of Weibull shift distribution. It is shown that when expected average savings (EAS) is greater than sum of inspection/PM cost and fixed restoration cost and when n is assumed continuous z, then z^* and T^* are unique variables which minimize cost function. EAS is defined as $(s\alpha P + a)E(X)$ where s is the cost incurred by producing a defective item, P is production rate, a is variable restoration cost per unit time of delay and $E(X)$ is mean lifetime. From z^* it is possible to find integer $n^*(T)$. Moreover, some conditions involving EAS and other problem parameters for optimality of one maintenance action are explored. A multi-step analytical and numerical procedure is used to check different conditions and find n^* and T^*.

Alfares *et al.* (2005) studied a single-item production system where both items and production equipment deteriorated. Deterioration rate per unit time for inventory items is $0 \le \theta \le 1$. Scheduled maintenance inspections are performed at equal intervals. In-control time is exponentially distributed with parameter λ. Fraction of defective items produced during out-of-control state is $0 \le \alpha \le 1$. Demand is exponentially decreasing. Both demand and production rates are time-dependent and production rate is a linear function of demand rate. Shortage is not allowed. Restoration cost (when system is found out of control) is a function of detection delay. Objective here is to minimize total cost per unit time whose components are setup cost, inspection cost, deterioration cost of inventory items, inventory holding cost, quality cost due to producing defective items, and restoration cost while decision variables are cycle time (t_2) and number of inspections during production period (n). A heuristic numerical solution algorithm using an integer search for n and a line search for corresponding production period t_1 (from which t_2 is computable) is used.

2.5.2 Including level of PM in decision variables

For a single-item production system, in Ben-Daya (2002), process is inspected and PM is performed at $t_1, t_2, ..., t_m$. Production cycle ends either when *m* inspection/PM actions are performed or when process is found in out-of-control state in which case the system produces defective items at rate α. System is then restored to as-good-as-new condition and a new cycle starts. In-control time follows a general distribution having an increasing failure rate (IFR). Demand is constant, no shortage is allowed, and inspection/PM time is negligible. PM is assumed to be imperfect after performing which the reduction in age is proportional to the ratio of actual PM level cost to maximum PM level cost. Using a modified pattern search technique, paper finds optimal lengths of inspection/PM intervals $h_1, h_2, ..., h_m$, optimal PM level c_{pm} and optimal number of inspections *m* minimizing total expected cost per unit time which includes setup, inventory holding, PM, inspection, producing defective items, and restoration (as a linear function of detection delay) costs.

An extension to this study was Ben-Daya and Noman (2006) where a free repair warranty was offered within time *W*. Non-conforming items may be produced when process is in control, however, its rate is less than that of when out of control. Hazard rate of non-conforming products is higher than that of conforming items. Items produced when process is out of control are sold at a lower price and inspections are error-free. Objective is to maximize a profit function (revenues from selling items minus costs) per unit produced. Cost function includes manufacturing, inventory holding, setup, inspection and PM, restoration, and warranty costs. Decision variables are optimal production run length t_p, optimal inspection time interval lengths h_j ($j = 1,2,...,n$), and optimal number of PM actions (n). For a numerical example a numerical algorithm using Golden section method and finding h_1^* for each n is used when time in control follows Weibull distribution. A sensitivity analysis of optimal values with respect to mean time to shift to out-of-control state, repair cost, warranty period, PM cost, and investment in PM is conducted.

In a similar study to Ben-Daya (2002), Sheu and Chen (2004) suggested two kinds of out-of-control states *i.e.,* type I and type II with defective product rates α_I and α_{II} respectively. When out of control, system is in type I out-of-control state with probability $1-\theta$ and it is fixed by minimal repair. Cycle ends after m^{th} inspection or when system is found in type II out-of-control state. Cost of minimal repair is incorporated into objective function.

Another extension of this model was studied in Chen (2006) where inspection type-I (α) and type-II (β) errors as well as a maximum permitted shortage level B were taken into account. Paper incorporated shortage cost into cost model of objective function and took care of inspection errors in modeling incurred costs. A stepwise partial enumeration procedure was employed to find optimal values for decision variables (PM level, inspection rate and length of the first inspection interval). It is found that to perform PM activities, maximum PM activities should be selected. Moreover, as inspection errors and maximum permitted shortage increase, total cost increases while integration of minimal repair may result in lower total cost.

2.5.3 Introducing economic design of control charts into problem

Rahim and Ben-Daya (1998) studied a continuous production process which was inspected at $t_1, t_2, ..., t_m$ (or $h_1, h_1 + h_2, ...$) to assess its state where output quality was evaluated by $\bar{x}$ -control chart. Production cycle ends either with a true alarm or at time t_m when a renewal occurs. If process is found out of control, production ceases until accumulated on-hand inventory is depleted to zero. If it is in control, production continues until a predetermined level of inventory is reached.

Inspection for false alarms takes a fixed non-zero time during which the production ceases. In-control time follows a general distribution with IFR (Weibull distribution with shape parameter ν and scale parameter λ is assumed in numerical examples). Sampling and charting take negligible time. Expected total cost per unit time of setup, inventory holding, and maintaining quality using $\bar{x}$ -control chart (expected cost of operating in control/out of control with no alarm, cost of false alarm, repair and sampling costs minus salvage value) is minimized.

Cycle length consists of time for inspection intervals when process is in control and there is no false alarm, time to search in case of false alarm, time to detect the presence of assignable cause, and repair time. Optimal values for production quantity Q (through optimal production run time t_m^*), inspection schedule h_j, $j = 1,...,m$ (through h_1 in case where integrated hazard rate over all intervals is the same), and economic design parameters of $\bar{x}$-control chart (number of inspections m, sample size n, and control limit coefficient k) are found using a modified pattern search technique. Consequences of incorrectly assuming negligible search time for a false alarm are shown. Increasing ν and decreasing λ while maintaining the same mean time to failure result in a design which provides a lower expected cost.

Problem setting in Ben-Daya (1999) was slightly different. Here, a single-item production process was investigated which went out of control after an IFR generally distributed in-control time (Weibull distribution is assumed in numerical examples). Process is inspected at $t_1, t_2, ..., t_m$ by $\bar{x}$-control chart and PM is performed at $t_l, t_{2l}, ...$ where $l = 1,2,...$. If process is found out of control, production ceases until inventory is depleted to zero. Production cycle ends either with a true alarm or after m inspections and the system is restored to as-good-as-new state. Expected production cycle length includes expected time for inspection when process is in control, expected time for detecting the presence of an assignable cause (when no PM is performed time to find the assignable cause is negligible), and repair and PM time.

Demand is constant and continuous, no shortage is allowed, and PM is imperfect meaning that reduction in age of system after PM is a function of the level (cost) of PM performed (linear and nonlinear relationships are investigated). Number of PM levels is assumed finite. As more PM actions are performed their effect degrades. Integrated hazard rate over every interval is the same.

Total expected cost per unit time is minimized. It consists of setup cost, expected quality cost (cost of operation in control, cost of false alarm, cost of operation out of control, repair cost, and cost of sampling minus salvage value), and expected inventory holding cost. Decision variables are *l*, *m*, t_m, cost of actual PM activities C_{pm}, sample size *n*, sampling interval h_j (in Weibull case only h_1 needs to be found), and control limit coefficient *k* of $\bar{x}$ -control chart whose optimal values are found using a modified pattern search technique. It is shown that increasing PM level reduces quality control costs. If maximum PM cost is too high, however, no PM is optimal. Another finding is that as mean time to failure decreases higher PM cost can be justified and when it increases PM is performed less frequently (*l* increases).

Ben-Daya and Makhdoum (1998) studied the same setting and investigate three PM policies. In the first policy PM is performed at $t_l, t_{2l}, \ldots$. The second policy calls for PM at those intervals where failure rate reaches a preset threshold $r_{\max}$. Finally, policy carrying out PM at intervals where two consecutive values of sample means fall in warning zone (with control coefficient w_1 and probability P_w^2) is taken into account. Decision variables added are *l*, $r_{\max}$, and P_w^2 for above-mentioned policies respectively. A different nonlinear relationship, however, is assumed between PM level and reduction in age. In case of low

PM cost optimum scheme could be performing PM at every inspection interval and higher PM levels increase production cycle length.

Rahim and Ben-Daya (2001) dealt with a deteriorating production process (with increasing failure rate) with deteriorating products. System is inspected at $t_1 = h_1, t_2 = h_1 + h_2, \ldots$. Both deterioration times are arbitrarily distributed (Weibull distribution, however, is assumed in numerical examples). Output quality is monitored by an $\bar{x}$-control chart. Quality characteristic is normally distributed. Cycle ends either with a true alarm (out-of-control state is detected) or at t_m. No inspection is carried out at t_m but maintenance is performed. If process is found to be out of control, production ceases until inventory is depleted to zero, otherwise, it continues producing until a predetermined inventory level is accumulated. Time to search the cause in case of a false alarm and time to sample and chart are negligible. Deteriorated items are not replaced, demand and production rates are constant, and no shortage is allowed. A renewal occurs at the end of each cycle.

Objective is to find optimal production run length t_m^* (and hence optimal production quantity Q^*), optimal inspection schedule $h_j, j = 1,2,\ldots,m$ (h_1 only in Weibull lifetime case), and optimal quality control policy parameters (number of inspections *m*, sample size *n*, and control limit coefficient for $\bar{x}$-control chart *k*) such that total expected cost per unit time consisting of setup cost, quality control costs (operation, repair, false alarm, sampling costs minus salvage value), inventory holding cost, and cost of deteriorated items is minimized. Paper combines an inventory model for deteriorating products and another model for both inventory and quality control problems whose performance is shown to be better than that of using those two separately.

A modified pattern search technique is used to find optimal values for decision variables. A sensitivity analysis is also done on parameters of both deterioration distributions. Finally, a comparison is conducted between an economic-statistical design (when a limit on type II inspection error is imposed to the same problem) and a purely economic design. A trade-off between expected cost and desired level of statistical properties can be made.

A production process which could be either in control or out of control was studied in Lam and Rahim (2002). $\bar{x}$-control chart is used to monitor process mean. Inspection and maintenance are performed simultaneously. Production cycle, here, starts when a new component is installed and ends with a repair after detection of a failure (true alarm) or after a specified number of inspection intervals *m*. Process is inspected at $h_1, h_1 + h_2, \ldots$. If process is found out of control, production ceases until on-hand inventory is depleted to zero. Otherwise, production continues until a pre-specified inventory level is reached. Replacement cost is assumed to be very high, in-control process time follows a general probability distribution having increasing failure rate (Weibull for an illustrative numerical example), and no shortage is allowed. Number of inspection intervals *m*, sample size *n*, length of j^{th} inspection/sampling interval h_j, control limit coefficient for the $\bar{x}$-control chart *k*, production runtime w_m (hence production quantity Q) are decision variables.

Objective is to minimize expected total cost for integrated production, inventory, maintenance model, and quality control cycle per unit time. Cost components of objective function are as follows: setup cost, expected quality control costs per cycle (sampling cost, cost of operation in control with no alarm, cost of operation out of control with no alarm, expected cost of a false alarm, and repair cost), inventory holding cost, and maintenance costs (replacement cost, total corrective maintenance costs over a finite horizon, minus salvage value of machine). A modified pattern search technique is used to find optimal values. At the end, an extensive sensitivity analysis of the problem parameters and two-factor interactions is conducted.

2-6 Conclusions

Research on separate maintenance scheduling and production planning is abundant. However, the integration of these two is scarce. In this chapter different types of papers which dealt with this integration are studied. In the first category fall papers whose main purpose is to study the effects of system deterioration and breakdown on optimal production/inventory control model parameters. This class of papers do not take PM action into consideration.

Another group of papers focus on optimal values for maintenance/replacement parameters when a specific inventory control policy is in place. Assumption of a specific type of inventory control policy may hinder the possibility of achieving a true optimal maintenance policy.

There are models presented in papers which find joint optimal production and PM rates. An important advantage of these models is their generality which makes them theoretically applicable to most cases of production/maintenance problems. However, a significant disadvantage is that they are complex and practically difficult to apply to real cases since they often have no closed-form optimal solution.

EPQ along with optimal inspection/maintenance schedule in a deteriorating production process is derived in another category of papers. Extensions to these models are consideration of level of PM and integration of economic design of control charts. This class of papers have the above-mentioned generality advantage yet they are more convenient to apply. However, these models often lead to non-equally-intervalled inspection/maintenance schemes which are again difficult to apply.

Next chapter reviews research on production/inventory control model in presence of periodic planned maintenance in more detail. Periodic planned maintenance is well-adopted by industrial practitioners because of its convenience to apply to real cases.

Chapter 3 – Production/inventory control models in presence of periodic planned maintenance

3.1 Introduction

As mentioned before, the problem of jointly optimizing inventory/production control and inspection/maintenance scheduling policy has just recently received attention in literature despite the fact that these two policies are interrelated. On one hand, when a machine fails or when it is stopped for planned maintenance enough inventory level must be available for subsequent machines in the system. On the other hand, safety stock level must be as low as possible since it incurs miscellaneous costs. Hence, the problem of finding an optimal amount of safety stock/economic production (manufacturing) quantity together with an optimal inspection/maintenance schedule seems to have a high potential to work on. Production/inventory control models in presence of a periodic PM plan are special cases of this problem where PM action is performed in equidistant periods.

In this class of studies an equidistant periodic maintenance policy is in place. Value of maintenance time interval is either given as an input or is obtained together with optimal production quantity, optimal order quantity, or optimal buffer stock level. This model is highly practical because of its simplicity in application in real production environments. However, its limitation is that as maintenance action is assumed periodic and equidistant an opportunity to find better solutions in form of flexible maintenance intervals cannot be sought. Table 3.1 summarizes research in this area.

Table 3.1 Production/inventory control models in presence of periodic planned maintenance

Research	**System Setting**	**Major Assumptions**	**Decision Variables**	**Objective Function**	**Optimization Procedure**	**Major Achievement(s)**
Bresavšček and Hudoklin (2003)	A multi-component system, periodic review inventory policy	IFR	Preventive replacement interval length, maximal inventory level	Average cost of preventive and corrective replacement, ordering, holding, shortage	Numerical iterative procedure	Integration of clock replacement and periodic review inventory policy
Vaughan (2005)	A multi-part system with known PM interval length	Lead time equal to one, binomially distributed number of units replaced, constant failure rate	Order quantity at the beginning of a period	Recursive total cost of holding, ordering, shortage	A twin-bin system through numerical examples	Considers both single-item demand arising from failure replacement and demand generated by PM actions
Cheung and Hausman (1997), Dohi *et al.* (2001)	Single-item, single-machine production environment	Constant demand rate equal to normal production rate, perfect CM and PM, random breakdown with IFR (exponential distribution in the second paper), random CM duration, lost unmet demand, constant PM time	Maintenance period, amount of safety stock	Total expected cost of inventory holding, shortage, PM, and CM (minimal repair cost as well in the second paper)	Derivatives	Condition for optimality of zero safety stock for deterministic repair time is investigated (consideration of minimal repair for removing failures before reaching safety stock in the second paper)
Rezg *et al.* (2004) (2005), Ben Hmida *et al.* (2002), Chelbi and Rezg (2006)	Single- and multiple-machine system, maximum production rate until safety stock level is reached, then production rate equal to demand rate until a maintenance action (a minimum	Critical-age maintenance (CM upon failure and PM after a certain period) (Weibull lifetime and lognormal distribution for CM and PM times in the second paper and Weibull	Critical age of the machine for doing PM, buffer size	Average cost of inventory holding, lost demand and maintenance	Derivatives for single-machine system and a methodology combining simulation and GA algorithms in multiple-machine case	Combining a critical-age maintenance policy and safety stock level control, use of GA for a better choice of inputs to simulation (use of ANOVA, regression analysis and response

	required stationary availability in the fourth paper)	lifetime and exponential PM and CM times in the third paper)				surface in the second paper for single-machine case)
Chelbi and Ait-Kadi (2004)	Repairable production unit similar to the above	Random PM and CM durations with known distribution, no failure before satisfying demand and building buffer stock	PM interval, buffer stock	Sum of PM, CM, inventory holding, and shortage costs	Iterative numerical procedure	Convenient-to-use enumeration method to find optimal values for decision variables
Kenné (2007)	Single-item three-state production process	Perfect CM upon failure and perfect PM at equal intervals both having random durations, unmet demand is lost	Stock level, PM interval	Total cost of inventory holding, lost sales, PM and CM	Numerical case	Inventory threshold value increases with the age according to a staircase structure

3.2 Joint optimization of periodic block replacement in presence of a specific inventory policy

Brezavšček and Hudoklin (2003) studied a system with n components of a given type with increasing failure rate. Failed components are replaced upon failure while all components are preventively replaced at predetermined time intervals of length T which is longer than constant procurement lead time τ. Replacement takes negligible time. Inventory is controlled according to a periodic review (R,S) where $R = T - \tau$ is equidistant reorder interval and S is maximal inventory level. If number of units which are correctively replaced during T is less than inventory level right after planned block replacement ($x_n < S'$) excess of parts will occur, otherwise, shortage will happen. Decision variables are T and S.

x_n is a random variable with probability density function $g(x_n)$ which is approximated by normal probability density function, hence, $E(x_n) = nE(x)$ and $\sqrt{Var(x_n)} = \sqrt{nVar(x)}$ where x is defined as number of components correctively replaced during T when there is only one component in the system. It is shown that $E(x) = H(T)$ where $H(T)$ is component renewal function in time T and is calculated as in Eq. (3.1).

$$H(T) = \sum_{i=1}^{\infty} F_i(T) = \sum_{i=1}^{\infty} \int_0^T f_i(t)dt = \sum_{i=1}^{\infty} \int_0^T \int_0^t f_{i-1}(t-u) f(u) du dt \qquad (3.1)$$

or recursively for practical purposes in Eq. (3.2)

$$H(T) = \sum_{i=0}^{T-1} [1 + H(T-i-1)] \int_i^{i+1} f(t)dt \text{ , } T = 1,2,\ldots; H(0) = 0 \qquad (3.2)$$

where index i specifies cycles. Now assume C_m, C_r, and C_i as expected total, replacement, and inventory cost respectively. Objective is to minimize $C_m = \dfrac{C_r + C_i}{T}$ whose elements are defined in Eqs. (3.3) to (3.8) as follows:

$$C_r = np + cE(x_n) = np + ncH(T) \tag{3.3}$$

$$C_i = C_o + C_h + C_{sh} \tag{3.4}$$

$$C_o = K + s(n + E(x_n)) = K + sn(1 + H(T)) \tag{3.5}$$

$$C_h = hT[\int_0^{S'} (S' - \frac{x_n}{2}) g(x_n) dx_n + \int_{S'}^{\infty} \frac{S'^2}{2x_n} g(x_n) dx_n] \tag{3.6}$$

$$C_{sh} = \begin{cases} 0 & \text{if } x_n \le S' \\ zT\int_{S'}^{\infty} \dfrac{(x_n - S')^2}{2x_n} g(x_n) dx_n & \text{otherwise} \end{cases} \tag{3.7}$$

$$S' = S - n - n(H(T) - H(T - \tau)) = nH(T) \tag{3.8}$$

where p and c are preventive and corrective replacement cost respectively, C_o, C_s, and C_{sh} are respective expected ordering, holding, and shortage cost of inventory and K, s, h, and z are respective fixed ordering cost, purchase cost of items, unit holding cost, and downtime cost due to shortage of spares. Expression $n(H(T) - H(T - \tau))$ is expected number of items correctively replaced during lead time τ. Paper ends with a numerical case study.

Vaughan (2005) studied a system of n identical and non-repairable parts with exponential lifetime with parameter λ. Upon failure a single-unit demand is generated and at given regularly scheduled PM intervals of length T an (n,p)-binomially distributed number of parts are replaced. A stochastic dynamic programming is applied as follows.

$D(t)$ is the number of units required during period t when period 0 is a PM period and backward numbering is used such that period 1 comes immediately prior to PM period and so on. Hence, $0,T,2T,...$ are PM periods and any t where $t \bmod T \neq 0$ is an operating period where random failure process generates Poisson-distributed demand. We have Eq. (3.9) which is:

$$P_t(d) = P(D(t)=d) = \begin{cases} e^{-\lambda n}(\lambda n)^d / d!, d=0,1,2,...,t=1,2,3,...,t \bmod T \neq 0 \\ \binom{n}{d} p^d (1-p)^{n-d}, d=0,1,2,...,n, t=0,T,2T,... \end{cases} \tag{3.9}$$

Therefore, the problem is to find $TC_t(x(t))$ from Eq. (3.10) as follows:

$$\begin{aligned} &TC_t(x(t)) = \min_{Q(t)}\{C_O \delta(Q(t)) + \\ &\sum_{d=0}^{\infty}[B(t)[d-x(t)]^+ + C_H[x(t)-d+Q(t)]^+ + TC_{t-1}([x(t)-d+Q(t)]^+)]P_t(d)\} \end{aligned} \tag{3.10}$$

for $t=1,2,3,...$ where $TC_t(x(t))$ is minimum total expected cost of inventory system over $t, t-1, t-2,...,0$ if $x(t)$ is on-hand inventory at the beginning of period t, $Q(t)$ is the number of units ordered at the beginning of period t (and received at the end of t) in response to $x(t)$, C_O is ordering cost, $\delta(q) = \begin{cases} 0 & if\ q=0 \\ 1 & if\ q>0 \end{cases}$,

$B(t) = \begin{cases} B_1 & t=1,2,3,...,t \bmod T \neq 0 \\ B_2 & t=0,2T,3T,... \end{cases}$ is shortage penalty, C_H is unit carrying cost,

and $TC_0(x(0)) = \sum_{d=x(0)+1}^{n} B(0)(d-x(0))P_0(d)$.

Since analytical evaluation of this model is difficult, a numerical characterization is conducted which shows once the analysis escapes end-of-horizon effects, policy becomes cyclical-stationary which means that it depends on t only through $t \bmod T$. We have $(s(k_t), S(k_t))$ policy where $k_t = t \bmod T$ and $Q^*(t) = \begin{cases} 0 & if \ x(t) > s(k_t) \\ S(k_t) - x(t) & if \ x(t) \le s(k_t) \end{cases}$.

As k_t becomes large $(s(k_t), S(k_t))$ converges to (s, S) policy. The rest of this paper studies different realizations of this policy for different parameter set values.

3.3 Joint determination of PM interval length and safety stocks in an unreliable production environment

For a single item and single process, Cheung and Hausmann (1997) defined $U(m)$ and $V(m)$ as expected time to PM and expected time to machine breakdown respectively where PM is performed every m time periods as in Eqs. (3.11) and (3.12):

$$U(m) = m[1 - F(m)] + \int_0^m [t + E(R) + U(m)] f(t) dt = \\ \{\int_0^m [1 - F(t)] dt + E(R) F(m)\} / [1 - F(m)] \tag{3.11}$$

$$V(m) = [1 - F(m)][m + M + V(m)] + \int_0^m t f(t) dt = \\ \{\int_0^m [1 - F(t) dt + M[1 - F(m)]\} / F(m) \tag{3.12}$$

where

$F(.)$ is cumulative distribution function of time to machine breakdown T,

R is time to perform corrective maintenance with probability density function $g(r)$, cumulative distribution function $G(r)$, and mean $E(R)$,

$f(.)$ is probability density function of T, and

M is constant time to perform PM. $V(m)$ is increasing in m when process has decreasing failure rate (DFR) and decreasing in m when process has increasing failure rate (IFR) and in any case $U(m)$ is increasing in m. Total cost per unit time $TC(s,m)$ is shown in Eq. (3.13):

$$TC(s,m) = hs + [c_R + L(s)]/[V(m) + E(R)] + c_M /[U(m) + M] \tag{3.13}$$

where

h is inventory holding cost per unit time,

s is safety stock level and long-run average stocking level (assuming PM time and safety stock building time are small relative to mean time between failures (MTBF)),

c_R is cost of corrective maintenance,

$L(s)$ is expected penalty cost for stock-out demand during downtime due to machine breakdown given s, and

c_M is PM cost.

$L(S)$ is derived as in Eq. (3.14) which follows:

$$L(s) = \int_{s/p}^{\infty} (pr - s)c_0 g(r)dr = c_0\{p[E(R) + \int_0^{s/p} G(r)dr] - s\} \tag{3.14}$$

where c_0 is penalty cost for each unit of lost demand and p is constant demand and normal production rate (compared to $p_0 > p$ which is the maximum production rate to build up safety stock). TC is decreasing in m when process failure rate $r(t) = f(t)/[1 - F(t)]$ is non-increasing. In this case no PM action is optimal ($m^* = \infty$).

In case of increasing failure rate PM action must be taken into account in addition to safety stock. s^* is derived as follows

$dTC/ds = 0$

so s^* should satisfy $G(s^*/p) = 1-(h/c_0)[V(m)+E(R)]$. It is shown that $s^*(m) > 0$ if $h < c_0/[V(m)+E(R)]$ and $s^*(m) = 0$ otherwise.

To get a mathematically tractable solution two cases are considered: (a) deterministic repair time R' and (b) exponential repair time with parameter w. In case (a) TC is reduced to Eq. (3.15):

$$TC = hs + [c_R + (pR' - s)c_0]/[V(m)+R')] + c_M/[U(m)+M] \tag{3.15}$$

which is either non-increasing or non-decreasing . $s^*(m)$ is hence either equal to zero or demand during repair time (pR'). In any case m^* must satisfy $dTC/dm = 0$ which for the second case is reduced to Eq. (3.16):

$$\frac{f(m)\int_0^m [1-F(t)]dt}{[1-F(m)]} + \frac{c_R M - c_M R'}{c_R - c_M}\frac{f(m)}{1-F(m)} - F(m) = \frac{c_M}{c_R - c_M} \tag{3.16}$$

If $[c_R M - c_M R']/(c_R - c_M)$ is nonnegative, then there is at most one solution, otherwise, no PM is optimal.

In case (b)

$G(s^*/p) = 1 - e^{-ws^*/p} = 1-(h/c_0)[V(m)+(1/w)]$

$s^* = s^*(m) = -(p/w)\ln\{(h/c_0)[V(m)+(1/w)]\}$

m^* should satisfy $dTC(s^*(m), m)/dm = 0$

In numerical examples, Cheung and Hausman (1997) considered Weibull and Gamma distributions for time to machine breakdown and exponential distribution for repair time.

According to Dohi *et al.* (2001) this model was mathematically incomplete and conclusions were not justified. They believed that shortcomings were due to implicit assumption of no failure during safety stock accumulation and the fact that this model did not consider time to accumulate safety stock s in its definition of cycle time. They used a more precise stock average level in their calculation of expected inventory costs. Dohi *et al.* (2001), therefore assumed that failures before reaching s are removed by minimal repair which costs c_m and takes negligible time. Machine lifetime is exponentially distributed with parameter μ. Total expected cost per cycle $V(s,m)$ is equal to sum of following elements

$V_M(s,m) = c_M(1 - F(m))$ which is expected PM cost in one cycle,

$V_R(s,m) = c_R F(m)$ as expected corrective maintenance cost per cycle,

$V_O(s,m) = c_0 F(m)\int_{s/p}^{\infty}(px - s)dG(x)$ as expected shortage cost during a cycle,

$V_m(s,m) = c_m F(m)\{\frac{\mu s}{p_0 - p}[1 - G(s/p)] + \int_0^{s/p}\frac{\mu px}{p_0 - p}dG(x)\} + c_m[1 - F(m)]\frac{\mu pM}{p_0 - p}$ which is expected minimal repair cost per cycle, and

$V_h(s,m)$ as expected inventory holding cost in a cycle.

Expected cost per unit time $TC(s,m)$ equals to total cost per cycle divided by length of a cycle $T(s,m)$. Solution procedure has three main parts: (i) for a fixed m find $S^* = s^* / p$ which satisfies $\partial TC(pS^*, m) / \partial S = 0$. If repair time distribution function $G(t)$ is IHR then $TC(pS, m)$ is strictly convex. Based on some conditions on problem parameters s^* can be either zero, a finite value ($pM < s^* < 0$) or ∞. (ii) For a fixed s find m^* which satisfies $\partial TC(s, m^*) / \partial m = 0$. Based on some other conditions on problem parameters, $TC(s,m)$ is shown to be monotonically increasing or decreasing in m for any s, hence m^* is either zero or ∞ respectively. (iii) Joint optimization where, based on certain problem parameters and for a certain bounded S_0^* and IFR repair time, optimal pair (s^*, m^*) is one of (pM,0), (pS_0^*, ∞), (pM, ∞), and (∞, ∞). A numerical example where repair time follows Weibull distribution with IFR shows that when failure rate is high policy (0, 0) is optimal.

Rezg and his colleagues studied the issue of jointly optimizing buffer stock level and maintenance interval length as follows. They divided an operation cycle into three phases, (I) building safety stock h at rate $U_{\max} - d$ where $U_{\max}$ is the maximum production rate and d is constant demand rate, (II) keeping inventory level $s(t)$ at h and reducing production rate to d, and (III) interruptions due to failures or planned PM where stock level drops at rate d. The first two phases in cycle k are working time W_k and the third is down time D_k which together comprise an operation cycle time T_k. Maintenance is of critical-age type which means corrective maintenance (CM) is performed upon failure and PM after T time units.

Rezg *et al.* (2004) first provided analysis for PM policy as follows. Total maintenance cost over $(0,t)$ is shown in Eq. (3.17):

$$\Phi(t) = M_C N_C(t) + M_P N_P(t) \tag{3.17}$$

where M_C and M_P are respective CM and PM costs and $N_C(t)$ and $N_P(t)$ are number of CM and PM actions in $(0,t)$ respectively. Average long-term cost rate is reduced to Eq. (3.18):

$$\varphi(T) = \lim_{t\to\infty}\{\frac{E(\Phi(t))}{t}\} = \frac{M_C F(T) + M_P R(T)}{\int_0^T R(u)du + \mu_P R(T) + \mu_C F(T)} \tag{3.18}$$

where $F(T)$ is failure distribution function and $R(T) = 1 - F(T)$ and μ_P and μ_C are mean time to perform PM and CM respectively. It is shown that if machine failure distribution has IFR and $M_C > M_P$ and $M_C / \mu_C \geq M_P / \mu_P$, then T^* is the unique value which minimizes $\varphi(T)$ and satisfies

$d\varphi(T)/dT = 0$ which is reduced to Eq. (3.19):

$$\lambda(T)\int_0^T R(u)du + \lambda(T)\frac{\mu_P M_C - \mu_C M_P}{M_C - M_P} - F(T) = \frac{M_P}{M_C - M_P} \tag{3.19}$$

Next, inventory control policy is considered assuming no failure during phase (I). Two cases may occur, (i) when $D_k \leq h/d$ no loss is faced and cost of inventory control equals to Eq. (3.20):

$$\Gamma_{NL}(h) = C_S(h(D_k + W_k) - \frac{dU_{\max} D_k^2}{2(U_{\max} - d)}) \tag{3.20}$$

and (ii) when $D_k > h/d$ there is loss and cost of inventory control is as in Eq. (3.21):

$$\Gamma_{WL}(h) = C_S(h^2/2d + h^2/2(U_{\max} - d) + h(W_k - \frac{h}{U_{\max} - d})) + C_P d(D_k - h/d) \tag{3.21}$$

where C_S and C_P are unit storage cost and unit loss cost respectively. Total inventory cost is then as in Eq. (3.22):

$$\Gamma(h) = E[\Gamma_{NL}(h)]P_{NL} + E[\Gamma_{WL}(h)]P_{WL} \tag{3.22}$$

where P_{NL} and P_{WL} are probability of having a cycle without loss or with loss respectively based on the length of down time D_k. Cost rate $\delta_c(h)$ equals to total inventory cost divided by average cycle length $E(T_k)$. Hence h^* is the value which satisfies $d(\delta_c(h))/dh = 0$.

Finally joint optimization of stock level and PM interval is taken into account. Average total cost of inventory control and maintenance actions per unit of time is shown in Eq. (3.23):

$$\Pi(h,T) = \frac{\delta_{c/p}(h,T) + M_P R(T) + M_C F(T)}{E(T_k)} \tag{3.23}$$

where $\delta_{c/p}(h,T)$ is basically the same as inventory control policy case without PM with $W_k = \min\{X_k, T\}$ where X_k is time to failure and D_k is a mixture of time to perform PM and time to perform CM depending on whether W_k equals to T or X_k respectively. In case where there is only CM and no PM is scheduled ($T = \infty$), total cost is reduced to $\Pi(h,\infty) = \dfrac{\Gamma(h) + M_C}{E(T_k)}$. h^* and T^* satisfy

$$\partial\Pi(h,T)/\partial h = 0$$
$$\partial\Pi(h,T)/\partial T = 0$$

To take care of the possibility of failure during phase (I) and also to study the case of more than one machine, Rezg *et al.* (2004) proposed GA for a better choice of h and T as inputs to simulation and statistical analysis. Numerical examples for Weibull lifetimes and exponential time to PM and CM are conducted.

Rezg *et al.* (2005) studied the same setting for a single machine and conduct numerical examples for Weibull lifetime and lognormal distribution for CM and PM times. h and T are input to simulation model, ANOVA is applied to evaluate their impact as well as impact of their interactions on total cost. Regression analysis is then utilized to find the relationship between significant parameters and total cost. Optimal values for decision variables are derived using regression equation (response surface).

With a similar approach to Rezg *et al.* (2005), Ben Hmida *et al.* (2002) used a simple simulation algorithm assuming Weibull lifetime and exponential PM and CM times.

Same problem subject to a minimum required stationary availability was studied in Chelbi and Rezg (2006). Stationary availability as a function of maintenance interval T is defined as in Eq. (3.24):

$$SA(T) = \frac{\int_0^T R(u)du}{\int_0^T R(u)du + \mu_P R(T) + \mu_C F(T)} \tag{3.24}$$

Considering the fact that for systems with IFR there is a unique T^* which maximizes availability, solution procedure first finds $[T_1, T_2]$ for which the availability constraint is satisfied. Then it finds T within this interval which minimizes $\Pi(h,T)$. Numerical examples assume Weibull lifetime and lognormal CM and PM times.

System studied in Chelbi and Ait-Kadi (2004) undergoes PM at $T, 2T, 3T, ...$ and failures within each cycle are perfectly repaired. Machine lifetime, PM duration, and repair time are all random variables with respective probability density functions of $g(.)$, $f(.)$, and $h(.)$. At the beginning of each cycle a safety stock S is built with rate $\alpha+\beta$, then production rate is reduced to demand rate β of subsequent assembly line. It is assumed that no failure occurs before reaching S and that $T >> S/\alpha$. Average consumption during repair periods following breakdowns within T is $M(T).\beta.MTTR$ where $M(T)$ is the average number of operation-repair cycles within T and $MTTR = \int_0^\infty t dH(t)$. We have Eq. (3.25):

$$M(T) = \sum_{n=1}^{\infty} L^n(T) \tag{3.25}$$

where $L(t) = \int_0^t G(t-x) dH(x)$ is convolution product of lifetime and repair time distributions and $L^n(T) = \int_0^t L^{n-1}(t-x) dL(x)$ which is the major difficulty in this model.

Total expected cost C_{tot} is the sum of total average maintenance, shortage, and inventory holding costs respectively shown as cm, cpt, and cst as follows.

$cm = C_1 M(T) + C_2$ where C_1 and C_2 are CM and PM costs respectively and $C_1 > C_2$,

$cpt = C_p \beta \int_{\frac{(S-R)}{\beta}}^{\infty} (t - \frac{S-R}{\beta}) dF(t)$ where C_p is unit shortage cost and

$R = S - (M(T).MTTR.\beta)$, and

cst is the area under inventory-time curve multiplied by unit holding cost.

Therefore, expected cost rate is $CT = \dfrac{C_{tot}}{E(T+T_{pm})}$ where $E(T+T_{pm}) = T + \int_0^\infty t dF(t)$ is average duration of a PM cycle. Problem becomes

$$\begin{cases} Min\ Z = CT(S,T) \\ s.t. \quad T > 0, [M(T).MTTR.\beta] \le S < \alpha T \end{cases}$$

A numerical procedure based on a simple enumeration is used to find the optimal values for decision variables S and T which yield optimal value for CT.

Kenné *et al.* (2007) studied a single-item production process which undergoes PM every T time units and CM upon random breakdowns both with random durations. It has a production capacity of $u_{\max}$ and a constant demand rate d where $d < u_{\max}$. System state is denoted by (α, x) where $\alpha(t) \in \{1,2,3\}$ is mode of operation (operational, under CM, and under PM respectively) and x is surplus level. When at mode 1 production rate is given by an extended version of hedging point policy as

$$u(x) = \begin{cases} u_{\max} & if\ x < S(a) \\ d & if\ x = S(a) \\ 0 & otherwise \end{cases}$$

where $S(a)$ is stock threshold level when age of system is a. Total incurred cost of inventory holding and lost sale in period $[0,T]$ is shown in Eq. (3.26):

$$L(S,T) = c^+ .S(t).\int_t^T R(t)dt + \int_0^T (c^+ .\frac{S(t)^2}{2d} + c^- .\int_{S/d}^{\infty} (x - \frac{S(t)}{d}).g(x)dx)dF(t) \tag{3.26}$$

where c^+ and c^- are unit holding cost and unit loss cost respectively, $R(t)$ is reliability function of the machine, and $f(t)$ and $g(t)$ are probability density functions of machine lifetime and CM duration respectively. To find optimal S and T we have $\frac{\partial L(.)}{\partial S} = 0$ and $\frac{\partial L(.)}{\partial T} = 0$. The former leads to Eq. (3.27):

$$S^* .\frac{c_{mp}}{c^+}.R_r(S/d) = -d.\frac{\int_t^{t+\Delta t} R(t)dt}{\Delta F(t)} \tag{3.27}$$

but the latter is complex. Hence, first order sufficient condition for optimality of S is solved, and then a numerical search over a given computational grid is applied to derive the minimal cost and relevant optimal control parameters (S,T). A numerical case is studied where time to breakdown follows a Rayleigh distribution (special case of Weibull distribution where $\eta = 2$) and PM and CM times follow Lognormal distribution. Inventory value increases with the age according to a staircase structure such that we have S_i^* for each age partition $dA_i, i = 1,...,k$ where k and interval of each range dA_i depend on T and values of input costs. Modified hedging point production policy becomes

$$u_i(x, age) = \begin{cases} u_{\max} & if\ x < S_i\ and\ age \in dA_i \\ d & if\ x = S_i\ and\ age \in dA_i, \quad j = 1,...,k. \\ 0 & if\ x > S_i\ and\ age \in dA_i \end{cases}$$

Sensitivity analysis shows that an increase in unit holding cost leads to an increase in T^* and in length of the first interval and a reduction of overall cost while a rise in unit loss cost increases S^* and overall cost and decreases the length of the first interval. Moreover, an increase in CM cost reduces PM period and a rise in PM cost increases it while both have little influence on production policy and its staircase structure.

3-4 Conclusions

This chapter studies production/inventory control models in presence of periodic planned maintenance in more details where the derivation of equations are given as well as the problem setting, assumptions, decision variables, optimization procedure, and major achievements. This chapter is organized in this way because it is necessary for the reader to know in more details about the way these models are derived to be able to go through the next chapter which studies joint optimization of buffer stock level and inspection interval in an unreliable production system.

The first category in this class of models is the research on joint optimization of periodic block replacement in presence of a specific inventory policy. In this class a block replacement is performed at equidistant intervals the length of which is either given or found as a decision variable. A specific inventory policy (a twin-bin policy) is in place as the background. These models are convenient to apply to real cases. However, the assumption of a specific inventory policy as well as the adoption of block replacement policy may not be the optimal choice in target production environment.

Another category is joint determination of PM interval length and safety stocks in an unreliable production environment. This category provides more easy-to-implement procedures while keeping versatility. The limitation is that adopting this model may hinder the application of more optimal procedures. A special case of this category is presented in the next chapter and an extension is provided.

The next chapter deals with joint optimization of buffer stock level and inspection interval in an unreliable production system. This setting makes it possible for the models to simultaneously take the convenience advantage of equally-distanced PM policy and the advantage of general maintenance and inventory policy. Hence, these models lead to true optimal solution to the presented cases yet they are convenient to apply.

Chapter 4 – Joint optimization of buffer stock level and inspection interval in an unreliable production system

4.1 Introduction

Among categories of papers reviewed to develop the present thesis, those studying a production/inventory control model in presence of a periodic PM plan are selected as the basis for further extension. More specifically, model developed by Salameh and Ghattas (2001) and its extension in Zequeira *et al.* (2004) is extended in this part. We study an unreliable production system which may shift to out-of-control state where it produces a certain percentage of defective items. System is inspected at equidistant intervals. If it is in control PM is performed, otherwise CM is carried out. CM is more costly that PM and on average takes more time. Objective is to minimize average total cost of stock holding, shortage, maintenance, and defective production. Decision variables are buffer stock level and/or maintenance interval length. Derivatives and search methods are used to find optimal values for decision variables.

As described in Chapter 2 of this thesis, research conducted on joint maintenance scheduling/production control optimization is categorized as (1) production/inventory models in presence of deterioration and breakdowns where no-PM-action assumption is prevailing (e.g., Yeh and Chen (2006), Lin and Gong (2006), Iravani and Duenyas (2002), etc.), (2) maintenance/replacement policy when a specific inventory control policy is assumed in place (e.g., Ribeiro *et al.* (2007), Kyriakidis and Dimitrikos (2006), Van der

Duyn Schouten and Vanneste (1995), etc.), (3) production/inventory control policy in presence of a PM plan (e.g., Brezavšček and Hudoklin (2003), Dohi *et al.* (2001), Cheung and Hausman (1997), etc.), (4) joint determination of optimal production and preventive maintenance rates (e.g., Gharbi and Kenné (2005), Kenné and Boukas (2003), Boukas and Liu (2001), etc.), and (5) joint determination of optimal economic production quantity and inspection/maintenance schedule in a deteriorating production process (e.g., Wang *et al.* (2006), Kim *et al.* (2001), and Makis (1998), etc.).

4.2 Problem Description and Solution Procedures

A single-unit single-product process which undergoes equally distanced periodic preventive maintenance is studied here. Buffer S is accumulated at rate α before the end of operational time taking S/α unit time and is consumed at rate β during maintenance action. Since the process is a Just-In-Time (JIT) setting, its normal production rate is equal to its consumption/demand rate β; hence, no inventory is kept during normal operational time. Maintenance action takes random time with known stochastic distribution which can result in shortage of items. During normal operational time defective items are produced once the system shifts to out-of-control state. Incurred cost function, therefore, include shortage, holding, maintenance and defective items costs. Decision variables are buffer size S and operational time T whose values which minimize cost function are sought. Major assumptions are as follows:

1. Maintenance action is perfect and restores the system to a good-as-new condition.

2. Operational time T is much longer than maintenance action duration. Therefore, buffer accumulation always starts from a zero level.

3. At maintenance points, inspection takes negligible time.

Main notations are as follows. Other notations are introduced in context.

S buffer stock level

T operational time (time between the end of one maintenance action and the start of the next one)

α buffer replenishment rate

β consumption/demand rate (equal to normal production rate)

h unit buffer holding cost per unit time

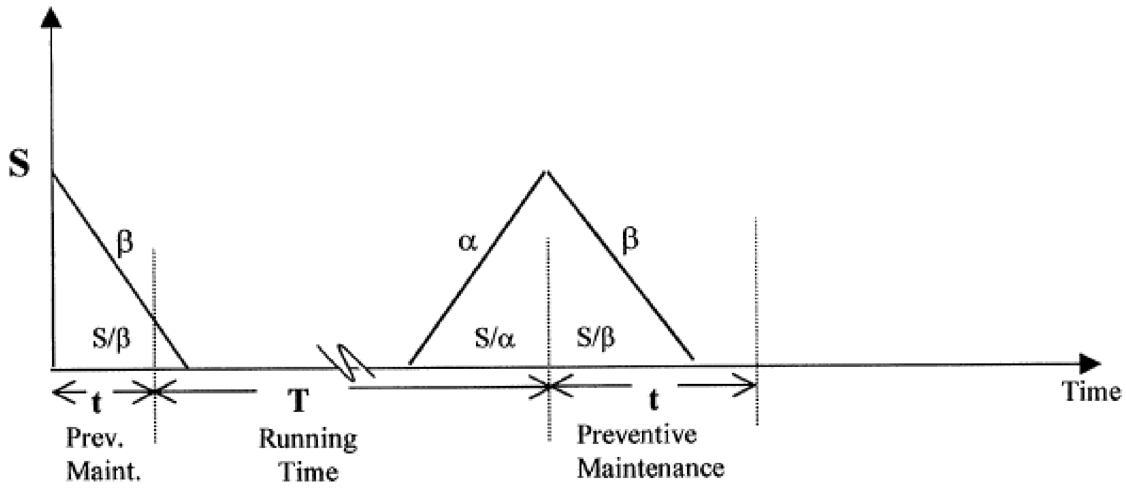
ρ unit shortage cost

This section provides an analytical extension to studies carried out on this problem setting. Before dealing with the extension, the model itself is explained.

Salameh and Ghattas (2001) studied this system but ruled out the possibility of defective production during given operational time T; hence only PM action is performed which takes random duration of t time units. Total cost rate function is comprised of average holding and average shortage costs. Optimal value for decision variable S which minimizes total cost rate function is found using differentiation. When h is unit buffer holding cost per unit time and ρ is unit shortage cost, average inventory cost is $\frac{hS^2(\alpha+\beta)}{2\alpha\beta}E(\frac{1}{t+T})$ and average shortage cost equals $\rho\frac{\beta}{T+E(t)}\int_{S/\beta}^{\infty}(t-\frac{S}{\beta})f(t)dt$ and total cost rate $TCU(S)$ is the sum of these two costs. These expressions are derived as follows:

Fig. 4.1 depicts an inventory cycle of the assumed setting.

Fig. 4.1 depiction of an inventory cycle of the assumed setting (Salameh and Ghattas (2001))



Average inventory level during $t+T$ is shown by $I(t)$ (Eq. 4.1):

$$I(t)=[\frac{S^2}{2\beta}+\frac{S^2}{2\alpha}][\frac{1}{t+T}] \tag{4.1}$$

Expected average inventory level per unit time is $E(I(t))$ (Eq. 4.2):

$$E(I(t))=[\frac{\alpha+\beta}{\alpha\beta}]\frac{S^2}{2}E(\frac{1}{t+T})=[\frac{S^2}{2\beta}+\frac{S^2}{2\alpha}]\int_0^{\infty}\frac{f(t)}{t+T}dt \tag{4.2}$$

Average inventory cost is therefore $\frac{hS^2(\alpha+\beta)}{2\alpha\beta}E(\frac{1}{t+T})$.

Shortage occurs when maintenance duration exceeds buffer supply time which is S/β.

Stock-out time is $(t-S/\beta)$ and the number of units short is $\omega(t)=\beta(t-S/\beta)=\beta t-S$.

The expected number of units short is $E(\omega(t)) = \beta \int_{S/\beta}^{\infty} (t - \frac{S}{\beta}) f(t) dt$ and the expected number of units short per unit time is given in Eq. (4.3):

$$\frac{E(\omega(t))}{E(t+T)} = \frac{\beta}{T+E(t)} \int_{S/\beta}^{\infty} (t - S/\beta) f(t) dt \tag{4.3}$$

Average shortage cost is therefore $\rho \frac{\beta}{T+E(t)} \int_{S/\beta}^{\infty} (t - \frac{S}{\beta}) f(t) dt$.

We will have then

$$TCU(S) = \frac{hS^2(\alpha+\beta)}{2\alpha\beta} E(\frac{1}{t+T}) + \rho \frac{\beta}{T+E(t)} \int_{S/\beta}^{\infty} (t - \frac{S}{\beta}) f(t) dt$$

Optimal value for decision variable S^* satisfies $dTCU(S)/dS = 0$ which is reduced to Eq. (4.4):

$$\int_{S^*/\beta}^{\infty} f(t) dt = \frac{hS^*(\alpha+\beta)(T+E(t))}{\alpha\beta\rho} E(\frac{1}{T+t}) \tag{4.4}$$

A numerical case for exponentially distributed PM duration is provided here. Table 4.1 presents numerical values of parameters used in this case study. PM duration is assumed to be exponentially distributed with $\lambda = 1/2$ (PM takes two days on average) as shown in Fig. 4.2. Optimal value for S is found to be 146 units which incurs a minimum total cost of 1875. The result obtained from Mathematica v5.0 is presented in Appendix A.

Fig. 4.2 Probability density function of PM duration

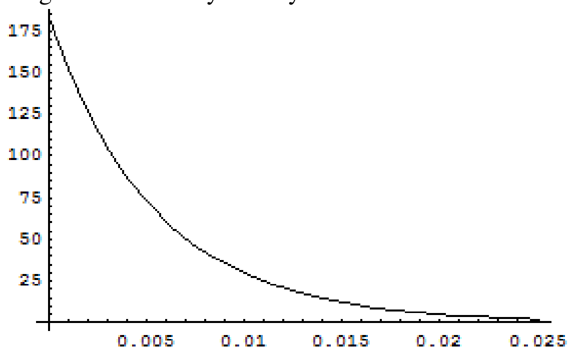


Table 4.1 – Values applied in numerical study of optimal buffer stock level when PM duration is exponentially distributed with parameter λ

Parameter	***Description***	***Numerical Value***	***Units***
α	Buffer replenishment rate	15000	Items per year
β	Demand and normal production rate	30000	Items per year
ρ	Unit shortage cost	1	Monetary unit per shortage
λ	Parameter of PM exponential distribution	½	Per day
H	Unit holding cost	28	Monetary unit per item per year
T	Maintenance interval	30	Days

Zequeira *et al.* (2004) extended this study by considering the possibility of imperfect production during T which is now another decision variable and dependency of duration and cost of maintenance action on the state of production facility at maintenance points. At random time τ (with a distribution function $F(\tau)$) process shifts to out-of-control state. If $\tau < T$ items produced from τ to T are defective with probability q. At T if process is found in control PM action is taken which costs c_1 and takes random time Y_1 whose distribution function is $G_1(z)$. If the process is found out of control, it undergoes corrective maintenance (CM) which costs c_2 and takes random duration of Y_2 with distribution function $G_2(z)$. Moreover, it is assumed that $E(Y_2) > E(Y_1)$ and $c_2 > c_1$. Total cost per cycle $TC(S,T)$ equals the sum of following cost components:

$L_d(T) = dq\beta\int_0^T F(y)dy$ is defective production cost in a cycle (d is defective cost per item),

$L_h(S) = h\frac{S^2}{2}[\frac{\alpha+\beta}{\alpha\beta}]$ is inventory holding cost per cycle,

$L_s(S,T) = \rho\beta(\overline{F}(T)\int_{S/\beta}^{\infty} \overline{G}_1(y)dy + F(T)\int_{S/\beta}^{\infty} \overline{G}_2(y)dy)$ is shortage cost per cycle, and

$c_1\overline{F}(T) + c_2 F(T)$ is expected maintenance cost per cycle.

Cost rate $TCR(S,T)$ equals to $TC(S,T)$ divided by expected length of a cycle $EC(T)$ where

$$EC(T) = T + E(Y) = T + \overline{F}(T)E(Y_1) + F(T)E(Y_2)$$

Solution procedure is as follows: firstly an initial value T_1 is assigned to T, then value for S which satisfies Eq. (4.5) is found,

$$\frac{\partial}{\partial S}TCR(S,T) = \frac{1}{T+E(Y)}\left\{hs(\frac{\alpha+\beta}{\alpha\beta}) - \rho[\overline{F}(T)\overline{G}_1(S/\beta) + F(T)\overline{G}_2(S/\beta)]\right\} = 0 \quad (4.5)$$

obtained value for S is put into Eq. (4.6) and the value for T which satisfies it is found,

$$\frac{\partial}{\partial T}TCR(S,T) = \frac{1}{(T+E(Y))^2}[(T+E(Y))\frac{\partial}{\partial T}TC(S,T) - TC(S,T)\frac{\partial}{\partial T}(T+E(Y))] = 0 \quad (4.6)$$

where

$$\frac{\partial}{\partial T}TC(S,T) = dq\beta F(T) + f(T)[(c_2 + \rho\beta\int_{S/\beta}^{\infty} \overline{G}_2(y)dy) - (c_1 + \rho\beta\int_{S/\beta}^{\infty} \overline{G}_1(y)dy)]$$

and

$$\frac{\partial}{\partial T}(T+E(Y)) = 1 + f(T)(E(Y_2) - E(Y_1))$$

New value for T is again used to generate a new value for S. This procedure continues until no further change is observed in T. Obtained S and T incur minimal total cost rate.

Appendix B presents a numerical case solved using Mathematica v5.0. Numerical values of parameters to this case study are presented in Table 4.2. Time to shift is exponentially distributed with parameter $\lambda = 1$ which yields an average lifetime of one year. PM duration follows uniform distribution with minimum 0.5 and maximum 1 day. CM duration also has a uniform distribution but with a minimum 2 and a maximum 5 days. Optimal values for decision variables are found to be $S^* = 232$ items and $T^* = 50$ days starting the solution procedure from an initial value for S rather than T. Minimum total cost rate is $TCR(S^*, T^*) = 4630$.

Table 4.2 – Values applied in numerical study of optimal buffer stock level and maintenance interval when PM and CM duration are uniformly distributed and time to shift to out-of-control state follows exponential distribution with parameter λ

Parameter	***Description***	***Value***	***Units***
α	Buffer replenishment rate	6000	Items per year
β	Demand and normal production rate	30000	Items per year
ρ	Unit shortage cost	10	Monetary unit per shortage
d	Defective cost	10	Monetary unit per defective item
h	Unit holding cost	20	Monetary unit per item per year
c_1	PM cost	150	Monetary unit per PM action
c_2	CM cost	450	Monetary unit per CM action
λ	Parameter of time-to-shift exponential distribution	1	Per year
q	Percentage of defectives when out of control	%10	-
a_{PM}	Minimum PM duration	0.5	Day
b_{PM}	Maximum PM duration	1	Day
a_{CM}	Minimum CM duration	2	Days
b_{CM}	Maximum CM duration	5	Days

An extension to this study is presented in the next section.

4.3 Shift to out-of-control state as a discrete random variable

One way to extend a model is to make it easier to understand and implement. In this section a possible such an extension to this model is presented which consists of considering shift to out-of-control state as a function of number of produced items, hence a discrete random variable. In this case there will only be a single decision variable in the model which is the number of produced items (S) which triggers maintenance action. To prevent misunderstanding of the presented model and its results we call S economic production quantity instead of buffer stock level. When S items are produced (when production quantity reaches S) the system is inspected. If the system is in control PM is performed, while CM is carried out if system is found out of control. A certain percentage of produced items are defective once the system goes out of control. CM is more costly than PM and takes more time on average than PM. Low values for S incur high shortage cost and high values for S result in increased holding, defective, and maintenance costs. Therefore, there is an optimal value for this decision variable which minimizes cost rate per produced item.

Cost function comprises of following elements,

Defective items cost if system goes out of control before producing S^{th} item (i.e. upon producing x^{th} item where $x \prec S$), a certain percentage of produced items from $(x+1)^{\text{th}}$ to S^{th} item will be defective. This percentage is shown by q. It is assumed that defective items are identified through a perfect test (neither type I nor type II errors exists) with negligible time. This test is carried out after production of each item, therefore, the item upon producing which the system has gone out of control is known. Each defective item costs d to be identified and reworked. Therefore, total defective items cost per cycle is shown in Eq. (4.7),

$$L_d(T) = dq\beta \sum_{x=1}^{S} P(X = x) \tag{4.7}$$

where β is production rate and $P(X = x)$ is the probability that system goes out of control upon producing x^{th} item. The summation $\sum_{x=1}^{S} P(X = x)$ represents the case where system goes out of control before producing S^{th} item.

Holding cost this element is similar to Salameh and Ghattas (2001) and is derived from Fig. 4.1 and Eqs. (4.1) and (4.2) which results in total holding cost per cycle as in Eq. (4.8),

$$L_h(S) = h\frac{S^2}{2}[\frac{\alpha+\beta}{\alpha\beta}] \tag{4.8}$$

where h is unit holding cost and α is the added production rate to accumulate S.

Shortage cost shortage occurs when maintenance action (whether preventive or corrective) takes longer than the time to consume S which equals to S/β. Shortage cost includes two elements related to two cases as follows: (1) the case where system is found in control upon producing S^{th} item and preventive maintenance action takes longer than S/β. (2) the case where system is found out of control upon producing S^{th} item and corrective maintenance takes longer than S/β. This is represented in Eq. (4.9):

$$L_s(S) = \rho\beta(\sum_{x=S}^{\infty} P(X=x)\int_{S/\beta}^{\infty} G_1(y)dy + \sum_{x=1}^{S} P(X=x)\int_{S/\beta}^{\infty} G_2(y)dy) \tag{4.9}$$

where ρ is unit shortage cost and $G_1(y)$ and $G_2(y)$ are probability distribution function for PM and CM durations respectively. The summations $\sum_{x=S}^{\infty} P(X=x)$ and $\sum_{x=1}^{S} P(X=x)$ represent the cases where system goes out of control after and before S^{th} item respectively. The integrations $\int_{S/\beta}^{\infty} G_1(y)dy$ and $\int_{S/\beta}^{\infty} G_2(y)dy$, respectively, represent the cases where PM and CM take longer than the time to consume economic production quantity which is S/β.

In our numerical case where PM and CM durations are assumed to be uniformly distributed Eq. (4.9) is reduced to Eq. (4.10):

$$L_s(S) = \rho\beta(\sum_{x=S}^{\infty} P(X=x)\int_{S/\beta}^{b_{PM}} G_1(y)dy + \sum_{x=1}^{S} P(X=x)\int_{S/\beta}^{b_{CM}} G_2(y)dy) \tag{4.10}$$

where b_{PM} and b_{CM} are maximum PM and CM durations respectively when uniformly distributed PM and CM durations are assumed.

Maintenance cost this cost also includes two elements related to two cases as follows: (1) the case where system is found in control upon producing S^{th} item, and (2) the case where system is found out of control upon producing S^{th} item. Maintenance cost per cycle is shown in Eq. (4.11):

$$c_1\sum_{x=S}^{\infty} P(X=x) + c_2\sum_{x=1}^{S} P(X=x) \tag{4.11}$$

where c_1 and c_2 are PM and CM costs, respectively.

Total cost per cycle $TC(S)$ is the sum of above components and total cost rate per item, $TCR(S)$, is $\frac{TC(S)}{S}$ which is to be minimized through numerical search for optimal S. A numerical case is presented (values in Table 3) and solved here using base model numerical values and assuming a geometric distribution for shifting to out-of-control state as in Eq. (4.7):

$$P(X = x) = p(1-p)^{x-1} \tag{4.7}$$

Parameter p of the distribution is assumed 0.01 which is the probability of going out of control after producing the first item. Shifting to out-of-control state is illustrated in Fig. 4.3 for numerical values assumed here. It is assumed that the process is in burn-in phase when the probability of going out of control decreases as the process produces more parts. Table 4.3 shows numerical values for parameters of this case. When $S^* = 3610$ the cost per produced item reaches its minimum at $TCR(S^*) = 24.4417$.

Fig. 4.3 Probability of going out of control as a function of number of produced items

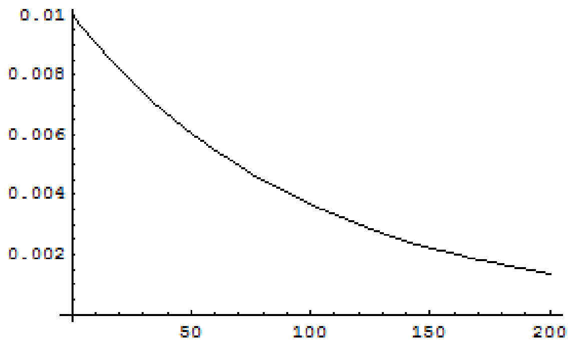


Table 4.3 – Values applied in numerical study of optimal buffer stock level which triggers maintenance action when PM and CM duration are uniformly distributed and shift to out-of-control state is a discrete random variable as a function of number of units produced to stock

Parameter	***Description***	***Value***	***Units***
α	Buffer replenishment rate	6000	Items per year
β	Demand and normal production rate	30000	Items per year
ρ	Unit shortage cost	10	Monetary unit per shortage
d	Defective cost	10	Monetary unit per defective item
h	Unit holding cost	20	Monetary unit per item per year
c_1	PM cost	150	Monetary unit per PM action
c_2	CM cost	450	Monetary unit per CM action
P	Probability of going out of control after producing an item in geometrical distribution	0.01	-
q	Percentage of defectives when out of control	%10	-
a_{PM}	Minimum PM duration	0.5	Day
b_{PM}	Maximum PM duration	1	Day
a_{CM}	Minimum CM duration	2	Days
b_{CM}	Maximum CM duration	5	Days

Numerical case solved by Mathematica v5.0 is shown in appendix C.

4.4 Sensitivity analysis

A sensitivity analysis is conducted for different values of probability of going out of control after producing the first item, p in geometrical distribution and the relevant values of average cost per item and economic production quantity. The results of this sensitivity analysis are presented in Table 4.4 and Fig. 4.4 and Fig. 4.5.

Table 4.4 Sensitivity analysis for different values of p and relevant cost function elements, total and average cost

p	S*	Defective items cost	Holding cost	Shortage cost	Maintenance cost	Total cost	Average cost
0.003	3623	29909.4	26252.3	32130.3	449.094	88741.1	24.4938
0.005	3620	29850	26208.8	32106.8	448.5	88614.1	24.4790
0.01	**3610**	**29700**	**26064.2**	**32023.3**	**447**	**88234.5**	**24.4417**
0.02	3592	29400	25804.9	31876.2	444	87525.1	24.3667
0.03	3573	29100	25532.7	31719	441	86792.7	24.2913
0.1	3438	27000	23639.7	30599.2	420	81658.9	23.7519

Fig. 4.4 Diagram of sensitivity analysis for different values of p and their relevant average cost for optimal S values

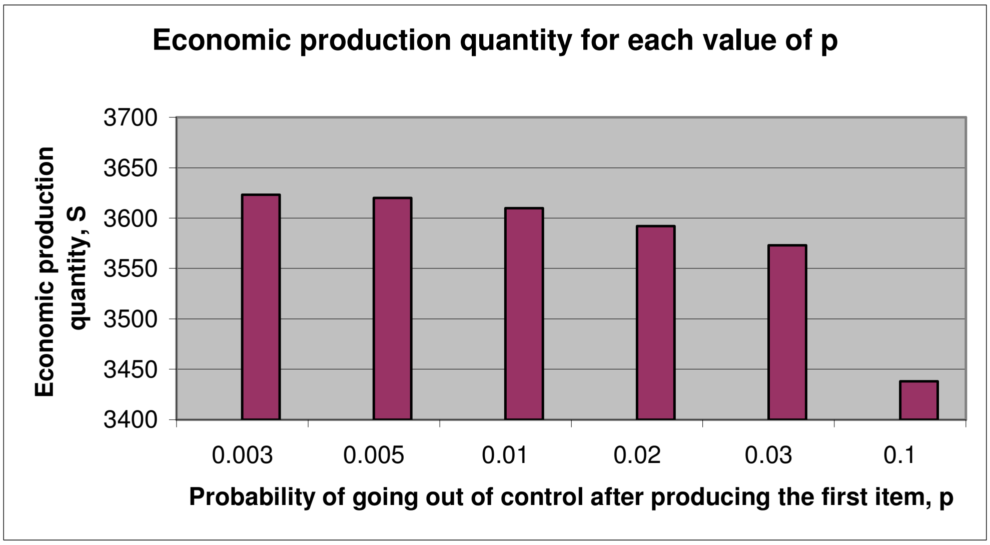
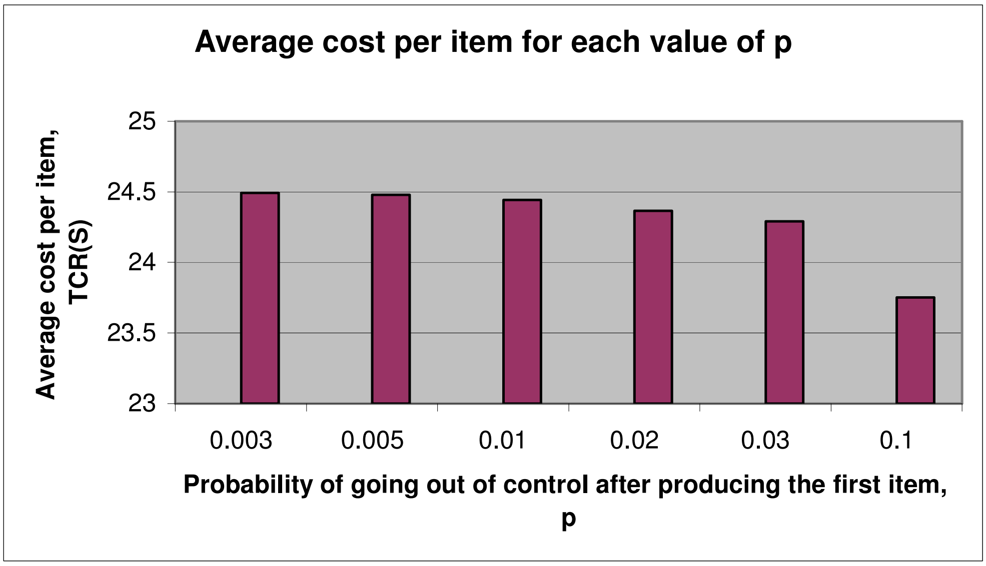


Fig 4.5 Diagram of sensitivity analysis for different values of p and their relevant economic production quantity, S.

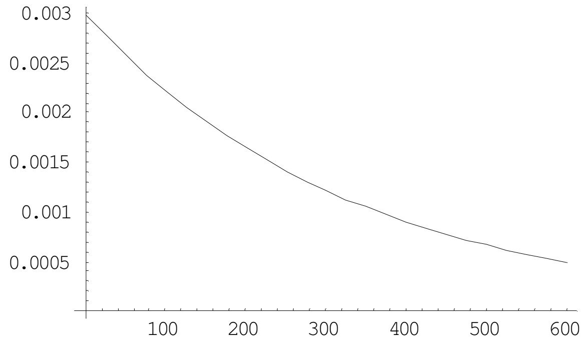
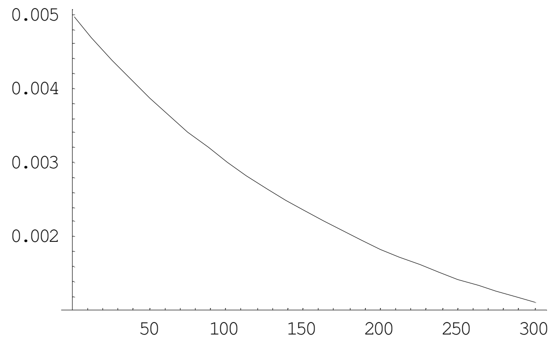
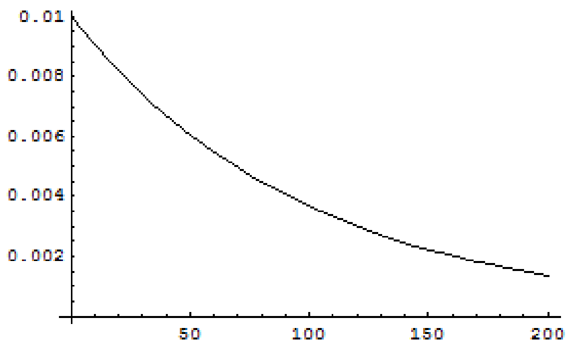
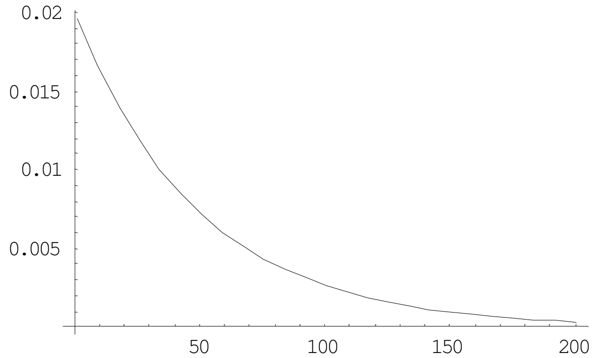
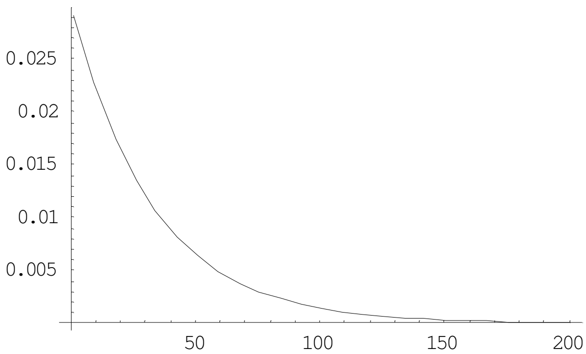
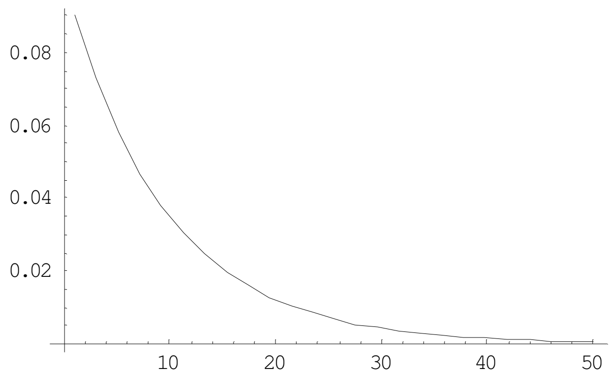


As it can be seen from Table 4.4 and the diagram in Fig. 4.4, an increase in p results in a decrease in the economic production size. This might be counterintuitive at the first sight. However, reader's attention is drawn to the point that it is assumed that the system is in its break-in period. This means that the probability of going out of control decreases as more items are produced. On the other hand, it can be observed in Table 4.5 that as p increases, the reduction in probability of going out of control accelerates. This means that when p increases, the break-in period finishes earlier. In other words, higher p results in earlier stabilization of the system, hence, a more reliable system with smaller economic production size.

Defective items cost, shortage cost, and maintenance cost decrease accordingly for higher p. Also, holding cost is less for higher p values since their corresponding S is less. Total and average costs also decrease for higher p.

Again, Note that it is assumed that the production facility is in burn-in phase and the probability of going out of control is a decreasing function of the number of produced items.

Table 4.5 Reduction in probability of going out of control in break-in phase and its rate for different values of p. Horizontal axis shows the number of produced items.

$p = 0.003$ 	$p = 0.005$ 
$p = 0.01$ 	$p = 0.02$ 
$p = 0.03$ 	$p = 0.1$ 

Overall conclusions and discussions as well as suggested further studies are presented in the next chapter.

Chapter 5 – Discussions and conclusions

In practical production planning it is critical to consider reliability/inspection/maintenance parameters. Lifetime of equipment can usually be assumed to follow a probability distribution function; hence breakdowns and/or shift to out-of-control state are likely to happen. If a production plan has not taken reliability parameters into account, it is vulnerable to breakdowns and other disruptions due to unreliability of equipment.

Similarly, an optimal maintenance schedule must include production/inventory parameters in practice. A maintenance schedule developed independently from production plan may necessitate a shutdown of equipment to perform PM while according to the production plan the equipment cannot be stopped until calculated EPQ is achieved. In both cases, shortage, maintenance, and defective costs increase.

The main idea of this thesis is to simultaneously consider these two classes of parameters in a single model to achieve a joint optimal maintenance schedule and production plan.

In present report we first reviewed and classified existing literature on different models with various problem settings, assumption sets, objective functions, decision variables, and optimization procedures. Production/inventory control models in presence of periodic planned maintenance were selected as the main base for further development. In the final chapter joint optimization of buffer stock level and inspection interval in an unreliable production system was studied and an extension was modeled.

In presented extension to the existing model shift to out-of-control state is a function of number of produced items. This assumption reduces the number of decision variables to only one which is the optimal number of items that should be produced before performing maintenance action. This extension makes the model easier to understand and implement. The system is assumed to be in its break-in period; that is, the probability of going out of control decreases after having produced each item. The model gives an optimal joint maintenance schedule and production plan with no conflict between production and maintenance disciplines in practice since no relevant parameter is ignored.

More extensions can be suggested as follows:

- *Imperfect diagnosis (possibility of type I and II errors in inspection).*

  The inclusion of type I and II errors in inspection can be an extension to the model. So far it is assumed that the diagnosis at the maintenance point is perfectly able to determine whether the system is in control or out of control. A possible extension to the model is to include the case when the system is actually in control but is diagnosed as being out of control (type I error) and vice versa (type II error). In each case extra cost components are added to total cost function. These components will be the cost of unnecessary CM and the cost of insufficient PM due to type I and II errors in diagnosis, respectively.

- *Non-negligible inspection time.*

Inspection is assumed to have negligible duration. An extension can be to relax this assumption and modify the model accordingly.

- *Imperfect production during in-control time.*

In the model presented in Chapter 4, it is assumed that system is perfect when it is in control. In other words, when system is in control it produces no defective item. This assumption can be relaxed and defective items cost component can be modified to include imperfection in the system when it is in control.

- *Imperfect production rate as a function of time or number of produced items.*

A possible extension is to assume degrading production rate as a function of time or number of produced items. This means that as the number of produced items increases and/or as time passes the production rate decreases due to aging equipments.

- *Random demand (consumption rate).*

Random consumption rate is another suggested extension to the model. It makes the model more realistic but more complex and less convenient to apply.

- *Minimal repair.*

A possibility in a production system is to perform minimal repair once it goes out of control to prevent producing defective items. This reduces maintenance cost at maintenance points but takes time that could otherwise be consecrated to production (even with a certain percentage of defective items). A trade-off can be found between (1) doing minimal repair and avoiding CM at maintenance point; and (2) producing some defective items and performing CM at maintenance point. Parameters related to cost and time of minimal repair and defective production determine trade-off point.

- *Production rate as a decision variable*.

It might be insightful to include production rate as a decision variable. This extension, however, necessitates flexible capacity of facilities which may not be available in practice.

- *Deteriorating buffer stock.*

Buffer stock may deteriorate over time and incur an additional cost of rework to get it back in appropriate conditions. Including this in the model is a way to extend it.

References:

[1] M. Aka, S. M. Gilbert, P. Ritchken, Joint inventory/replacement policies for parallel machines, IIE Transactions 29 (1997) 441-449

[2] H. K. Alfares, S. N. Khursheed, S. M. Noman, Integrating quality and maintenance decisions in a production-inventory model for deteriorating items, International Journal of Production Research 43 (2005) 899-911

[3] M. J. Armstrong, D. R. Atkins, Joint optimization of maintenance and inventory policies for a simple system, IIE Transactions 28 (1996) 415-424

[4] F. Azadivar, J. V. Shu, Use of simulation in optimization of maintenance policies, Proceedings of the 1998 Winter Simulation Conference 1061-1067

[5] M. Bartholomew-Biggs, B. Christianson, M. Zuo, Optimizing preventive maintenance models, Computational Optimization and Applications (2006)

[6] J. Ben Hmida, A. Chelbi, N. Rezg, Optimization of flows and preventive maintenance in a manufacturing unit, Proceedings of 2002 IEEE International Conference on Systems, Men, and Cybernetics, Vol. 3

[7] M. Ben-Daya, Integrated production maintenance and quality model for imperfect processes, IIE Transactions 31 (1999) 491-501

[8] M. Ben-Daya, The economic production lot-sizing problem with imperfect prodcution processes and imperfect maintenance, International Journal of Production Economics 76 (2002) 257-264

[9] M. Ben-Daya, M. Makhdoum, Integrated production and quality model under various preventive maintenance policies, Journal of the Operational Research Society 49 (1998) 840-853

[10] M. Ben-Daya, S. A. Noman, Lot sizing, preventive maintenance, and warranty decisions for imperfect production systems, Journal of Quality in Maintenance Engineering 12 (2006) 68-80

[11] E. –K. Boukas, A. Haurie, Manufacturing flow control and preventive maintenance: a stochastic control approach, IEEE Transactions on Automatic Control 35 (1990) 1024-1031

[12] E. K. Boukas, Z. K. Liu, Production and maintenance control for manufacturing systems, IEEE Transactions on Automatic Control 46 (2001) 1455-1460

[13] E. K. Boukas, H. Yang, Optimal control of manufacturing flow and preventive maintenance, IEEE Transactions on Automatic Control 41 (1996) 881-885

[14] E. K. Boukas, Q. Zhang, G. Yin, Robust production and maintenance planning in stochastic manufacturing systems, IEEE Transactions on Automatic Control 40 (1995) 1098-1102

[15] M. Brandolese, M. Franci, A. Pozzetti, Production and maintenance integrated planning, International Journal of Production Research 34 (1996) 2059-2075

[16] A. Brezavšček, A. Hudoklin, Joint optimization of block-replacement and periodic-review spare-provisioning policy, IEEE Transactions on Reliability 52 (2003) 112-117

[17] C. R. Cassady, E. Kutanoglu, Integrating preventive maintenance planning and production scheduling for a single machine, IEEE Transactions on Reliability 54 (2005) 304-309

[18] C. R. Cassady, E. Kutanoglu, Minimizing job tardiness using integrated preventive maintenance planning and production scheduling, IIE Transactions 35 (2003) 503-513

[19] A. Chelbi, D. Ait-Kadi, Analysis of a production/inventory system with randomly failing production unit submitted to regular preventive maintenance, European Journal of Operational Research 156 (2004) 712-718

[20] A. Chelbi, N. Rezg, Analysis of a production/inventory system with randomly failing production unit subjected to a minimum required availability level, International Journal of Production Economics 99 (2006) 131-143

[21] W. J. Chen, Minimizing total flow time in the single-machine scheduling problem with periodic maintenance, Journal of Operational Research Society 57 (2006) 410-415

[22] Y. –C. Chen, Optimal inspection and economical production quantity strategy for an imperfect production process, International Journal of Systems Science 37 (2006) 295-302

[23] K. L. Cheung, W. H. Hausman, Joint determination of preventive maintenance and safety stocks in an unreliable production environment, Naval Research Logistics 44 (1997) 257-272

[24] D. I. Cho, M. Parlar, A survey of maintenance models for multi-unit systems, European Journal of Operational Research 51 (1991) 1-23

[25] L. Cui, M. Xie, H. –T, Loh, Inspection schemes for general systems, IIE Transactions 36 (2004) 817-825

[26] T. K. Das, S. Sarkar, Optimal preventive maintenance in a production inventory system, IIE Transactions 31 (1999) 537-551

[27] K. S. de Smidt-Destombes, M. C. van der Heijden, A. van Harten, On the interaction between maintenance, spare part inventories and repair capacity for a *k*-out-of-*N* system with wear-out, European Journal of Operational Research 174 (2006) 182-200

[28] R. Dekker, R. E. Wildeman, F. A. van der Duyn Schouten, A review of multi-component maintenance models with economic dependence, Mathemetical Methods of Operations Research 45 (1997) 411-435

[29] R. Dekker, Applications of maintenance optimization models: a review and analysis, Reliability Engineering and System Safety 51 (1996) 229-240

[30] T. Dohi, H. Okamura, S. Osaki, Optimal control of preventive maintenance schedule and safety stocks in an unreliable manufacturing environment, International Journal of Production Economics 74 (2001) 147-155

[31] A. Gharbi, J. P. Kenne, Production and preventive maintenance rates control for a manufacturing system: An experimental design approach, International Journal of Production Economics 65 (2000) 275-287

[32] A. Gharbi, J. -P. Kenné, Maintenance scheduling and production control of multiple-machine manufacturing systems, Computers and Industrial Engineering 48 (2005) 693-707

[33] B. C. Giri, T. Dohi, N. Kaio, A discrete-time order-replacement model with time discounting and spare part provisioning, Journal of Quality in Maintenance Engineering 11 (2005) 190-205

[34] L. –F. Hsu, Simultaneous determination of preventive maintenance and replacement policies in a queue-like production system with minimal repair, Reliability Engineering and Systems Safety 63 (1999) 161-167

[35] S. M. R. Iravani, I. Duenyas, Integrated maintenance and production control of a deteriorating production system, IIE Transactions 34 (2002) 423-435

[36] M. Ji, Y. He, T. C. E. Cheng, Single-machine scheduling with periodic maintenance to minimize makespan, Computers and Operations Research 34 (2007) 1764-1770

[37] J. P. Kenné, E. K. Boukas, Hierarchical control of production and maintenance rates in manufacturing systems, Journal of Quality in Maintenance Engineering 9 (2003) 66-82

[38] J. P. Kenne, A. Gharbi, Experimental design in production and maintenance control problem of a single machine, single product manufacturing system, International Journal of Production Research 37 (1999) 624-637

[39] J. –P. Kenné, A. Gharbi, M. Beit, Age-dependent production planning and maintenance strategies in unreliable manufacturing systems with lost sale, European Journal of Operational Research 178 (2007) 408-420

[40] F. Kianfar, A numerical method to approximate optimal production and maintenance plan in a flexible manufacturing system, Applied Mathematics and Computation 170 (2005) 924-940

[41] C. H. Kim, Y. Hong, S. Y. Chang, Optimal production run length and inspection schedules in a deteriorating production process, IIE Transactions 33 (2001) 421-426

[42] E. G. Kyriakidis, T. D. Dimitrikos, Optimal preventive maintenance of a production system with an intermediate buffer, European Journal of Operational Research 168 (2006) 86-99

[43] K. K. Lam, M. A. Rahim, A sensitivity analysis of an integrated model for joint determination of economic design of $\bar{x}$-control charts, economic production quantity and production run length for a deteriorating production system, Quality and Reliability Engineering International 18 (2002) 305-320

[44] H. –H. Lee, A cost/benefit model for investments in inventory and preventive maintenance in an imperfect production system, Computers and Industrial Engineering 48 (2005) 55-68

[45] J. S. Lee, K. S. Park, Joint determination of production cycle and inspection intervals in a deteriorating production system, Journal of Operational Research Society 42 (1991) 775-783

[46] H. L. Lee, M. J. Rosenblatt, Simultaneous determination of production cycle and inspection schedules in a production system, Management Science 33 (1987) 1125-1136

[47] C. J. Liao, W. J. Chen, Single-machine scheduling with periodic maintenance and nonresumable jobs, Computers and Operations Research 30 (2003) 1335-1347

[48] C. –S. Lin, C. –H. Chen, D. E. Kroll, Integrated production-inventory models for imperfect production processes under inspection schedules, Computers and Industrial Engineering 44 (2003) 633-650

[49] G. C. Lin, D. –C. Gong, On a production-inventory system of deteriorating items subject to random machine breakdowns with a fixed repair time, Mathematical and Computer Modelling 43 (2006) 920-932

[50] V. Makis, Optimal lot sizing and inspection policy for an EMQ model with imperfect inspections, Naval Research Logistics 45 (1998) 165-186

[51] V. Makis, J. Fung, Optimal preventive replacement, lot sizing and inspection policy for a deteriorating production system, Journal of Quality in Maintenance Engineering 1 (1995) 41-55

[52] A. C. Marquez, J. N. D. Gupta, A. S. Heguedas, Maintenance policies for a production system with constrained production rate and buffer capacity, International Journal of Production Research 41 (2003) 1909-1926

[53] A. C. Marquez, A. S. Heguedas, Models for maintenance optimization: a study for repairable systems and finite time periods, Reliability Engineering and System Safety 75 (2002) 367-377

[54] R. D. Meller, D. S. Kim, The impact of preventive maintenance on system cost and buffer size, European journal of operational research 95 (1996) 577-591

[55] S. Mercier, P. –E. Labeau, Optimal replacement policy for a series system with obsolescence, Applied Stochastic Models in Business and Industry 20 (2004) 73-91

[56] M. A. Rahim, M. Ben-Daya, A generalized economic model for joint determination of production run, inspection schedule and control chart design, International Journal of Production Research 36 (1998) 277-289

[57] M. A. Rahim, M. Ben-Daya, Joint determination of production quantity, inspection schedule, and quality control for an imperfect process with deteriorating products, Journal of the Operational Research Society 52 (2001) 1370-1378

[58] N. Rezg, X. Xie, Y. Mati, Joint optimization of preventive maintenance and inventory control in a production line using simulation, International Journal of Production Research 42 (2004) 2029-2046

[59] N. Rezg, A. Chelbi, X. Xie, Modeling and optimizing a joint inventory control and preventive maintenance strategy for a randomly failing production unit: analytical and simulation approaches, International Journal of Computer Integrated Manufacturing 18 (2005) 225-235

[60] M. A. Ribeiro, J. L. Silveira, R. Y. Qassim, Joint optimisation of maintenance and buffer size in a manufacturing system, European Journal of Operational Research 176 (2007) 405-413

[61] M. K. Salameh, R. E. Ghattas, Optimal just-in-time buffer inventory for regular preventive maintenance, International Journal of Production Economics 74 (2001) 157-161

[62] S. -H. Sheu, J. -A. Chen, Optimal lot-sizing problem with imperfect maintenance and imperfect production, International Journal of Systems Science 35 (2004) 69-77

[63] T. W. Sloan, A periodic review production and maintenance model with random demand, deteriorating equipment, and binomial yield, Journal of the Operational Research Society 55 (2004) 647-656

[64] T. W. Sloan, J. G. Shanthikumar, Combined production and maintenance scheduling for a multiple-product, single-machine production system, Production and Operations Management 9 (2000) 379-399

[65] N. Sortrakul, H. L. Nachtmann, C.R. Cassady, Genetic algorithms for integrated preventive maintenance planning and production scheduling for a single machine, Computers in Industry 56 (2005) 161-168

[66] S. –T. Tseng, Optimal preventive maintenance policy for deteriorating production systems, IEE Transactions 28 (1996) 687-694

[67] F. A. Van der Duyn Schouten, S. G. Vanneste, Maintenance optimization of a production system with buffer capacity, European Journal of Operational Research 82 (1995) 323-338

[68] T. S. Vaughan, Failure replacement and preventive maintenance spare parts ordering policy, European Journal of Operational Research 161 (2005) 183-190

[69] C. –H. Wang, Optimal production and maintenance policy for imperfect production systems, Naval Research Logistics 53 (2006) 151-156

[70] C. –H. Wang, S. –H. Sheu, Fast approach to the optimal production/PM policy, Computers and Mathematics with Applications 40 (2000) 1297-1314

[71] C. –H. Wang, R. H. Yeh, P. Wu, Optimal production time and number of maintenance actions for an imperfect production system under equal-interval maintenance policy, Journal of the Operational Research Society 57 (2006) 262-270

[72] H. Wang, A survey of maintenance policies of deteriorating systems, European Journal of Operational Research 139 (2002) 469-489

[73] X. Yao, X. Xie, M.C. Fu, S.I. Marcus, Optimal joint preventive maintenance and production policies, Naval Research Logistics 52 (2005) 668-681

[74] R. H. Yeh, T. –H. Chen, Optimal lot size and inspection policy for products sold with warranty, European Journal of Operational Research 174 (2006) 766-776

[75] M. N. Yuniarto, A. W. Labib, Fuzzy adaptive preventive maintenance in a manufacturing control system: a step towards self-maintenance, International Journal of Production Research 44 (2006) 159-180

[76] R. I. Zequeira, B. Prida, J. E. Valdés, Optimal buffer inventory and preventive maintenance for an imperfect production process, International Journal of Production Research 42 (2004) 959-974

[77] A. B. M. Zohrul Kabir, A. S. Al-Olayan, Joint optimization of age replacement and continuous review spare provisioning policy, International Journal of Operations and Production Management 14 (1994) 53-69

[78] A. B. M. Zohrul Kabir, A. S. Al-Olayan, A stocking policy of spare part provisioning under age based preventive replacement, European Journal of Operational Research 90 (1996) 171-181

[79] A. B. M. Zohrul Kabir, S. H. A. Farrash, Simulation of an integrated age replacement and spare provisioning policy using SLAM, Reliability Engineering and System Safety 52 (1996) 129-138

Appendix A

In this appendix the case of an exponentially distributed PM duration with numerical values provided in Table 1 is solved using Mathematica v5.0 according to the flowchart presented in figure A1. The parameters' numerical values are from Salameh and Ghattas (2001), however, they assumed PM duration to follow uniform distribution instead.

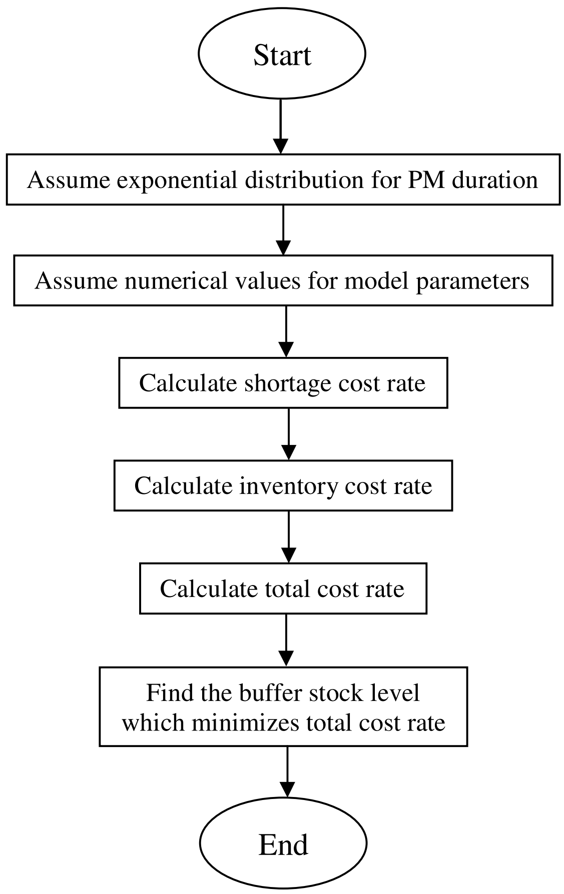


Figure A1 – Flowchart of a numerical solution case for the model solved using Mathematica v5.0

In[151]:=

```
Clear["Global`*"]
```

In[152]:=

```
<< Statistics`ContinuousDistributions`
```

In[153]:=

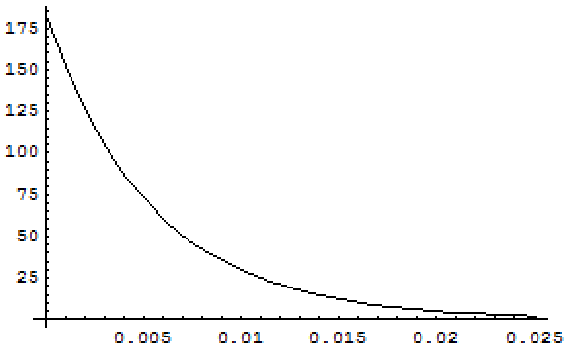
```
expdist = ExponentialDistribution[λ]; exp = PDF[expdist, t];
(*Exponential distribution for PM duration*)
```

In[154]:=

```
α = 15000; (*Buffer accumulation rate (per year)*)
β = 30000; (*Buffer consumption rate (per year)*)
λ = (1/2) * 365; (*parameter of exponential distribution for PM duration*)
ρ = 1; (*Shortage cost per unit*)
h = 28; (*Inventory holding cost per unit per year*)
T = 30 / 365; (*Interval between two consecutive PM actions (year)*)
```

In[160]:=

```
Plot[exp, {t, 0, .025}]
```



Out[160]=

- Graphics -

In[161]:=

$$\text{eshortage[S\_]} := \frac{\rho * \beta * \int_{S/\beta}^{\infty} \left(t - \frac{S}{\beta}\right) * \text{exp}\, dt}{T + \text{Mean[expdist]}} \quad \text{(*Shortage cost rate*)}$$

In[162]:=

$$\text{einventory[S\_]} := \frac{h * S\hat{}2 * (\alpha + \beta) * \text{ExpectedValue}\left[\frac{1}{T+t}, \text{expdist}, t\right]}{2 * \alpha * \beta}$$

(*Inventory cost rate*)

In[163]:=

```
eTCU[S_] := eshortage[S] + einventory[S] (*Total cost rate*)
```

In[164]:=

```
FindMinimum[eTCU[S], {S, 0}](*Finds where minimum total cost occurs*)
```

Out[164]=

```
{1112.98, {S → 146.22}}
```

In[165]:=

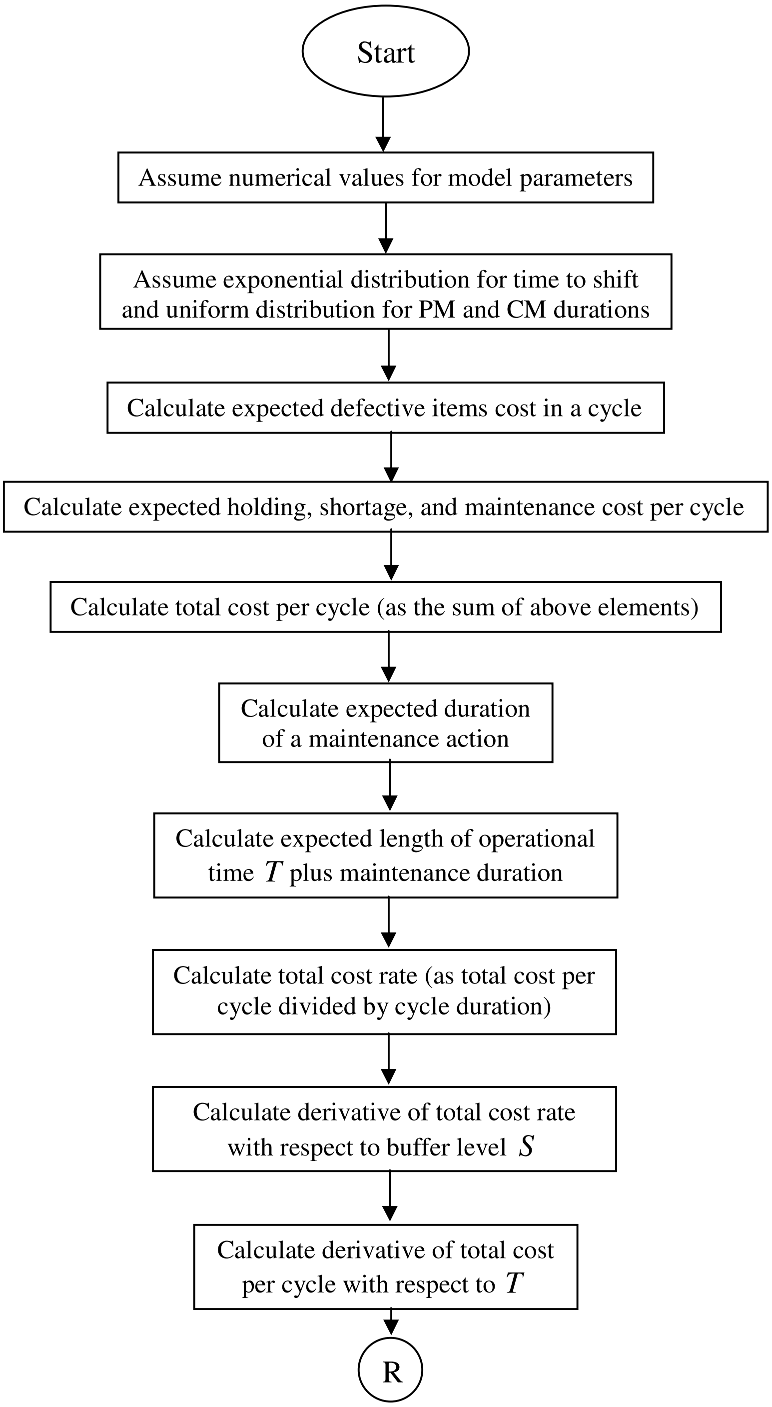
```
eTCU[S = 0](*Total cost rate where no inventory is kept*)
```

Out[165]=

```
1875
```

Appendix B

This appendix presents a numerical case based on model presented in Zequeira *et al.* (2004) using Mathematica v5.0. Numerical values are from Table 2 and the flowchart is presented in Figure B1.



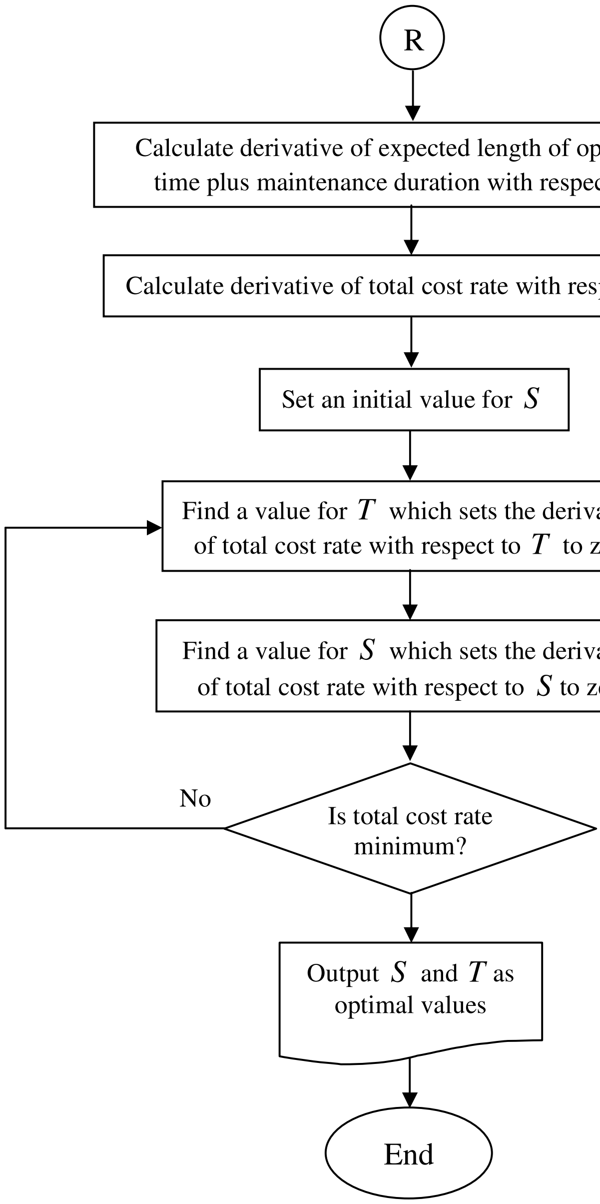


Figure B1 – Flowchart of a numerical solution case for the extended model solved using Mathematica v5.0

In[61]:=

```
Clear["Global`*"]
```

In[62]:=

```
<< Statistics`ContinuousDistributions`
```

In[64]:=

```
α = 6000; (*Buffer replenishment rate*)
 β = 30000; (*Consumption rate and normal production rate*)
 ρ = 10; (*Unit shortage cost*)
 d = 10; (*Unit defective item cost*)
 q = .1; (*Probability of producing defectives while out of control*)
 c1 = 150;
 (*Maintenance cost when the process is in control at maintenance
   interval (PM cost)*)
 c2 = 450;
 (*Maintenance cost when the process is out of control at maintenance
   interval (CM cost)*)
 h = 20; (*Unit holding cost*)
 λ = 1; (*Constant failure rate when time to shift is distributed exponentially*)
 apm = .5 / 365; (*Minimum PM duration when it is uniformly distributed*)
 bpm = 1 / 365; (*Maximum PM duration when it is uniformly distributed*)
 acm = 2 / 365; (*Minimum CM duration when it is uniformly distributed*)
 bcm = 5 / 365; (*Maximum CM duration when it is uniformly distributed*)
```

In[77]:=

```
Off[General::Spell1]
```

In[78]:=

```
expdist = ExponentialDistribution[λ]; unidistpm = UniformDistribution[apm, bpm];
 unidistcm = UniformDistribution[acm, bcm];
 exppdf = PDF[expdist, t]; expcdf = CDF[expdist, t];
 cmpdf = PDF[unidistcm, y]; cmcdf = CDF[unidistcm, y];
 pmpdf = PDF[unidistpm, y]; pmcdf = CDF[unidistpm, y];
 (*Uniform distribution for PM and CM and exponential distribution for
  time to shift*)
```

In[82]:=

$$\text{ld}[T\_] := d\, q\, \beta * \int_0^T \text{expcdf}\, dt \ \text{(*Expected defective items cost in a cycle*)}$$

In[83]:=

$$\text{lh}[S\_] := \frac{h * (S\verb|^|2) * (\alpha + \beta)}{2 * \alpha * \beta} \ \text{(*Expected holding cost per cycle*)}$$

In[84]:=

$$ls[S\_, T\_] := \rho * \beta * \left(\left(1 - \int_0^T exppdf\, dt\right) * \int_{S/\beta}^{\infty} (1 - pmcdf)\, dy + \left(\int_0^T exppdf\, dt\right) * \int_{S/\beta}^{\infty} (1 - cmcdf)\, dy\right)$$

(*Expected shortage cost per cycle*)

In[85]:=

$$lm[T\_] := c1 * \left(1 - \int_0^T exppdf\, dt\right) + c2 * \left(\int_0^T exppdf\, dt\right)$$

(*Expected maintenance cost per cycle*)

In[86]:=

TC[S_, T_] := ld[T] + lh[S] + ls[S, T] + lm[T] (*Total cost per cycle*)

In[87]:=

$$ey[T\_] := \left(1 - \int_0^T exppdf\, dt\right) * Mean[unidistpm] + \left(\int_0^T exppdf\, dt\right) * Mean[unidistcm]$$

(*Expected duration of a maintenance action*)

In[88]:=

ec[T_] := T + ey[T] (*Expected length of operational time plus maintenance duration*)

In[89]:=

$$TCR[S\_, T\_] := \frac{TC[S, T]}{ec[T]}$$

(*Total cost rate*)

In[90]:=

$$dTCRS[S\_] := \frac{1}{T + ey[T]} \left(h * S * \left(\frac{\alpha + \beta}{\alpha * \beta}\right) - \rho * \left(\left(1 - \int_0^T exppdf\, dt\right) * \left(1 - \int_0^{S/\beta} pmpdf\, dy\right) + \left(\int_0^T exppdf\, dt\right) * \left(1 - \int_0^{S/\beta} cmpdf\, dy\right)\right)\right)$$

(*Derivative of total cost rate with respect to buffer level S*)

In[91]:=

$$dTCT[T\_] := d * q * \beta * \int_0^T exppdf\, dt + \lambda * e^{-\lambda * T} * \left(c2 + \rho * \beta * \int_{S/\beta}^{\infty} (1 - cmcdf)\, dy - c1 - \rho * \beta * \int_{S/\beta}^{\infty} (1 - pmcdf)\, dy\right)$$

(*Derivative of total cost per cycle with respect to operational time T*)

In[92]:=

```
decT[T_] :=
 1 + λ * e^(-λ*T) (Mean[unidistcm] - Mean[unidistpm])
   (*Derivative of expected length of operational time plus maintenance
     duration with respect to T*)
```

In[93]:=

```
dTCRT[T_] :=
 1/((T + ey[T]) ^ 2) ((T + ey[T]) * dTCT[T] - TC[S, T] * decT[T])
  (*Derivative of total cost rate with respect to T*)
```

In[94]:=

```
Clear[S, T]
```

In[95]:=

```
S = 213; (*Initial value of S from Zequeira (2004)*)
```

In[96]:=

```
FindRoot[dTCRT[T] == 0, {T, 0}]
```

Out[96]=

```
{T → 0.13206}
```

In[97]:=

```
T = 0.13206;
```

In[98]:=

```
Clear[S]
```

In[99]:=

```
dTCRS[S = 229]
```

Out[99]=

```
0.0227865
```

In[100]:=

```
TCR[S, T]
```

Out[100]=

```
4631.76
```

In[101]:=

```
Clear[T]
```

In[102]:=

```
FindRoot[dTCRT[T], {T, 0}]
```

Out[102]=

```
{T → 0.13593}
```

In[103]:=

```
T = 0.13593;
```

In[104]:=

```
Clear[S]
```

In[105]:=

```
dTCRS[S = 231]
```

Out[105]=

```
-0.0258447
```

In[106]:=

```
TCR[S, T]
```

Out[106]=

```
4630.17
```

In[107]:=

```
Clear[T]
```

In[108]:=

```
FindRoot[dTCRT[T], {T, 0}]
```

Out[108]=

```
{T → 0.136427}
```

In[109]:=

```
T = 0.136427;
```

In[110]:=

```
Clear[S]
```

In[111]:=

```
dTCRS[S = 232]
```

Out[111]=

```
0.0173237
```

In[112]:=

```
TCR[S, T]
```

Out[112]=

```
4630.13
```

In[113]:=

```
Clear[T]
```

In[114]:=

```
FindRoot[dTCRT[T], {T, 0}]
```

Out[114]=

```
{T → 0.136676}
```

In[115]:=

```
T = 0.136676;
```

In[116]:=

```
Clear[S]
```

In[117]:=

```
dTCRS[S = 232]
```

Out[117]=

```
0.00600702
```

In[118]:=

```
TCR[S, T](*Minimum total cost rate attainable*)
```

Out[118]=

```
4630.13
```

In[119]:=

```
S = 232;(*Optimal value for buffer level*)
```

In[120]:=

```
T = 0.136676;(*Optimal operational time (year)*)
```

Appendix C

This appendix presents a numerical case using Mathematica v5.0 according to the flowchart presented in Figure C1 for the case where shift to out-of-control state is a discrete random variable as a function of number of produced items before going out of control.

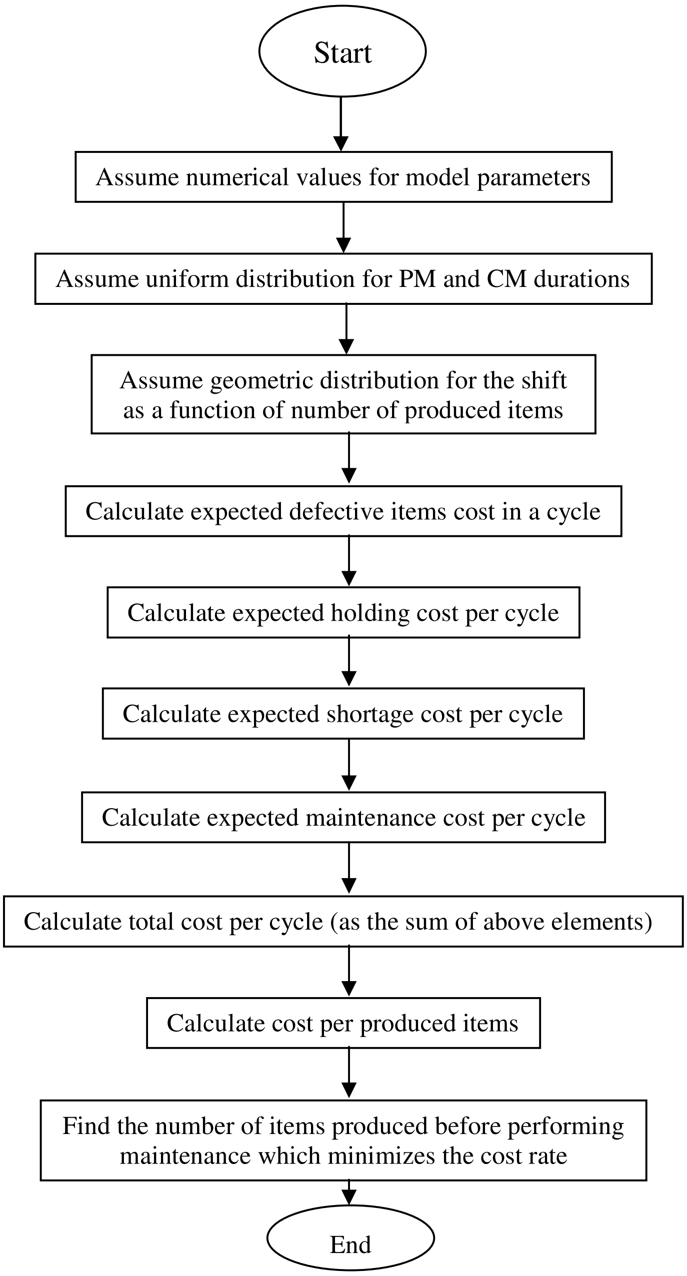


Figure C1 – Flowchart of a numerical solution case for the suggested extension

```
Clear["GLobal`*"]

<< Statistics`ContinuousDistributions`

<< Statistics`DiscreteDistributions`

α = 6000; (*Buffer replenishment rate*)
 β = 30000; (*Consumption rate and normal production rate*)
 ρ = 10; (*Unit shortage cost*)
 d = 10; (*Unit defective item cost*)
 q = .1; (*Probability of producing defectives while out of control*)
 c1 = 150;
 (*Maintenance cost when the process is in control at maintenance interval (PM cost)*)
 c2 = 450;
 (*Maintenance cost when the process is out of control at maintenance interval
   (CM cost)*)
 h = 20; (*Unit holding cost*)
 apm = .5 / 365; (*Minimum PM duration when it is uniformly distributed*)
 bpm = 1 / 365; (*Maximum PM duration when it is uniformly distributed*)
 acm = 2 / 365; (*Minimum CM duration when it is uniformly distributed*)
 bcm = 5 / 365; (*Maximum CM duration when it is uniformly distributed*)
 p = .01; (*Geometric distribution parameter as the probablity of going out
  of control after producing each item*)

Off[General::Spell1]

unidistpm = UniformDistribution[apm, bpm];
unidistcm = UniformDistribution[acm, bcm];

pmpdf = PDF[unidistpm, y]; pmcdf = CDF[unidistpm, y];
 cmpdf = PDF[unidistcm, y]; cmcdf = CDF[unidistcm, y];
 (*Uniform distribution for PM and CM durations*)

geodistout = GeometricDistribution[p];
 outpdf = PDF[geodistout, x];
 (*Geometric distribution with parameter p=
  0.01 for the shift to out-of-control state as a function of the number of
    produced items*)
```

$$\mathrm{ld}[S\_] := d\, q\, \beta * \sum_{x=1}^{S} (\mathrm{outpdf})\ \text{(*Expected defective items cost in a cycle*)}$$

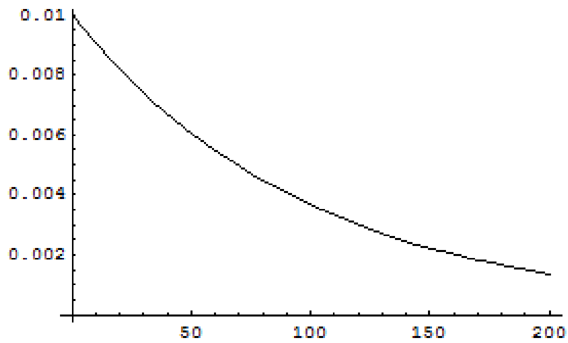
$$\mathrm{lh}[S\_] := \frac{h * (S \char`^ 2) * (\alpha + \beta)}{2 * \alpha * \beta}\ \text{(*Expected holding cost per cycle*)}$$

$$\mathrm{ls}[S\_] := \rho * \beta * \mathrm{Abs}\left[\left(\left(1 - \sum_{x=1}^{S} \mathrm{outpdf}\right) * \int_{S/\beta}^{\mathrm{bpm}} (\mathrm{pmcdf})\, dy + \left(\sum_{x=1}^{S} \mathrm{outpdf}\right) * \int_{S/\beta}^{\mathrm{bcm}} (\mathrm{cmcdf})\, dy\right)\right]$$

```
  (*Expected shortage cost per cycle*)
```

$$\mathrm{lm}[S\_] := c1 * \left(1 - \sum_{x=1}^{S} \mathrm{outpdf}\right) + c2 * \left(\sum_{x=1}^{S} \mathrm{outpdf}\right)\ \text{(*Expected maintenance cost per cycle*)}$$

```
TC[S_] := ld[S] + lh[S] + ls[S] + lm[S] (*Total cost per cycle*)
TCR[S_] := TC[S] / S(*Cost per produced item*)
Clear[S]
Plot [outpdf, {x, 1, 200}]
```




```
- Graphics -
S = 3610;
(*Optimal value for S, decision variable representing the number of items to
  produce before performing maintenance action*)
TCR[S]
24.4417
NumberForm[%, 10]
24.44168634
ld[S]
29700.
lh[S] // N
26064.2
ls[S] // N
32023.3
lm[S]
447.
TC[S]
88234.5
24.44168634`(*Minimum cost per item achieved when S=3610*)
```

Appendix D – List of Notations

ANOVA – ANalysis Of VAriance

CCR – Capacity-Constrained Resource

CM – Continuous Maintenance

DFR – Decreasing Failure Rate

EAS – Expected Average Savings

EPQ – Economic Production Quantity

FCFS – First Come, First Serve

FIFO – First In, First Out

FMS – Flexible Manufacturing System

GA – Genetic Algorithm

HJB – Hamilton-Jacobi-Bellman

HJI – Hamilton-Jacobi-Isaacs

IFR – Increasing Failure Rate

JIT – Just-In-Time

LP – Linear Programming

MDP – Markov Decision Process

MTBF – Mean Time Between Failures

MTTR – Mean Time To Repair

NR – No-Resumption

PED – Percentage Error Deviation

PM – Preventive Maintenance

RSM – Response Surface Methology

SMDP – Semi-Markov Decision Process

WSPT – Weighted Shortest Processing Time

Appendix E – List of Tables

Appendix F – List of Figures