A STUDY ON GROUP MAINTENANCE POLICIES

KAY THWE HTUN

A THESIS SUBMITTED

FOR THE DEGREE OF MASTER OF ENGINEERING

DEPARTMENT OF INDUSTRIAL & SYSTEMS ENGINEERING

NATIONAL UNIVERSITY OF SINGAPORE

2006

Acknowledgement

I would first like to thank my supervisors, Dr.Ng Kien Ming and Associated Professor Tang Loon Ching, for their invaluable guidance, support and useful suggestions throughout my graduate studies and research process.

I would like to give a special thanks to my parents, U Than Htun and Daw Tin Tin Aye for their unwavering love and support throughout my life. My special gratitude is expressed to all the teachers of the Department of Electronic Engineering, Yangon Technological University, Myanmar. According to their gratitude, I was a qualified student throughout my undergraduate studies and got an opportunity to join NUS.

I also wish to thank the National University of Singapore for offering me a Research Studentship. I would like to express my gratitude to the faculty and staff members of the Department of Industrial and Systems Engineering for providing financial support, and necessary facilities, which are essential in carrying out this research and giving teaching opportunities, throughout my time as a graduate student. I also thank to my colleagues, friends and Lab Officer in Computing Lab for their help, encouragement and sympathetic.

Kay Thwe Htun

July 2006

Table of Contents

Summary

If a system is a multi-components system and there is at least one dependency among the components, the group maintenance policy is the better mode of maintenance. The conventional group maintenance policies are ***T*-based** in which preventive maintenance is performed based on the period where the component has been in operation and ***m*-based** in which preventive maintenance is performed based on the number of failed components within a system. Although these policies are widely used, the shortfalls such as (i) ignoring the status of the component at the time of maintenance and (ii) ignoring the state of the system performance, offer an opportunity for further improvements of the maintenance program.

In this thesis, an effective preventive group maintenance policy is provided as an improvement from the conventional group maintenance policy. The system is inspected during preventive maintenance; the maintenance action for each component is performed depends on its status. Required reliability level, instead of cost, is pre-determined and acted as the primary decision criterion in the cost based model; This deviates from the convention of using lowest cost as the determinant of maintenance action, which can be detrimental to system reliability.

Since maintenance programs differ according to the system types and desired system component integrity condition, two systems with different states of component integrity condition will be considered. The first system has identical components which are linked either in a parallel, parallel-series or k-out-of-n system (k-out-of-n system is a kind of parallel system but at least k components will have to operate to be able to function the whole system). The selection of the applicable preventive group

maintenance policy to this system is then based on the difference in assessing Preventive Maintenance (PM) time, it is then classified into a **reliability-centered *T*-based** and ***m*-based** preventive group maintenance policies.

A required reliability level is pre-defined and the time interval between successive PM is determined to ensure that system reliability does not fall below the defined reliability level. The required optimal parameter in assessing PM time, either T or m, is decided under reliability constraint. A mathematical model has been developed in this thesis for assessing reliability for the given system structures through the minimization of the long-run cost per unit time. Components which have failed during the PM cycle are kept in idle until the next PM time so that unplanned system downtime is reduced. The model developed in this thesis also computes the component downtime cost based on PM time and number of failed components. This assumes that at the time of PM, the whole system is inspected; all failed and non-failed components are either replaced or repaired. Repair times can be assessed in two ways, either as having constant repair times where an exponential distribution with constant repair rate is applied and optimized to provide an optimal maintenance policy. Alternatively, repair times are assumed as monotonically increasing and a Geometric Process (GP) is used in the model. The proposed maintenance policies are applied to the case study and a numerical comparison is then made between the two proposed maintenance policies under different repair time assumptions.

The results reflect that the system's uptime in reliability-centered T-based policy is greater than that in m-based, with a lower per unit maintenance cost and higher system availability. Therefore, reliability-centered T-based maintenance policy is preferable relative to a reliability-centered m-based policy.

The second system, considered in this thesis, is a system in which components are independent but not identical. Since components are different components, their status may not be the same at the time of PM. The problem in determining an optimal group maintenance policy is how maintenance can be performed to attain the required operational conditions. In this thesis, the developed mathematical model can project an optimal preventive group maintenance policy based on the level of maintenance to be carried out, either by repair or replacement, which will be referred to as "maintenance degree". Two cases are classified according to the following different desirable conditions.

1. Maintenance cost of each component at the time of PM is not allowed to be greater than its available maintenance budget and the whole system is renewed at a convenient time so that system reliability is maximized. Available budget percentage of each component is formulated with the basis of its probability of failure and maintenance costs. PM cost, incurred for the maintenance action at the time of PM is described as a function of maintenance degree and number of PM.

2. Reliabilities of the components cannot fall below the acceptable reliability level when PM is performed. The whole system is renewed at a time in order to maintain the minimum maintenance costs.

The required condition is used in each case as a constraint to determine the required maintenance degrees, PM time and number of PM. The determined maintenance degrees are then compared with the historical performance when the same decision was taken. Based on this comparison, suitable maintenance action is then decided. Repair times for both cases are modeled with piecewise exponential distribution function because repeated and identical repair times are not practical.

The results show that the two cases, which are proposed for the purpose of determining an optimal maintenance policy for the non-i.i.d system, are effective and useful. The final program decision is dependent on whether the emphasis is on cost or reliability.

Nomenclature

PM	preventive maintenance
CM	corrective maintenance
iid	independent and identical distributed
GP	Geometric process
rv	random variable
MTBF	mean time between failures
LRI	line replacement item
Scs	subsystem/component
pdf	probability distribution function
cdf	cumulative distribution function
$\Re$	set of real number
$\mathbf{Z}^+$	set of positive integer
n	the total number of components or subsystems
i	the index of PM
j	the index of components or subsystems
m	the index of failed components or subsystems
T	time interval between successive PM
N	number of PM
X_j	random repair time of component j
Y_s	random life time of the whole system
Y_j	random life time of component/system j
Y_{II}	random variable of time interval between successive type II failure

$Y_{m\|\tau}$	m^{th} component failure arrival time, given that each component has age τ
$f(.)$	pdf of Y
$F(t)$	cdf of Y
$F_j(T)$	cdf of Y_j
$R_j(T)$	reliability distribution function of component/system j
R_c	critical reliability (pre-determined reliability level)
R_a	minimal acceptable reliability (pre-determined reliability level, $(R_a < R_c)$
A_o	minimal acceptable system availability
$E[C^*]$	Expected cost per cycle
$E[T^*]$	Expected time to system renewal

List of Figures

List of Tables

Chapter 1

Introduction

Maintenance policies are essential for the proper and efficient functioning of every system. As systems with many components are becoming more complex, maintenance of such systems is becoming one of the major concerns of the system operation.

The goal of a maintenance program is to optimize the system performance through cost reduction and increased availability. This is achieved by reducing the frequency of failures and also the amount of downtime. The system downtime can be reduced by performing group maintenance instead of individual maintenance, whereas the frequency of failures can be reduced by performing maintenance preventively before catastrophic failure. Therefore, the group maintenance policies and preventive maintenance policies which may be the best from the point of view of the system's availability or operation cost, have received more significant attention in the research literatures.

The purpose of this thesis is to improve on conventional preventive group maintenance policy by making "reliability" instead of cost as the primary criterion in selecting a maintenance program. This chapter provides a brief description of multi-component system maintenance, especially in preventive group maintenance policy. The motivation for studying the present work is given in Section 1.1, the scope is described in Section 1.2 and the organization of this thesis is proposed in Section 1.3.

1.1 Introduction to the multi-component system maintenance

A multi-component system is a system comprising of more than one component and all the components work together towards the same objective. A computer system, an electronic power system, a manufacturing system and a computer network system are some examples of the multi-component systems. Normally, the probability of failure increases as a machine or a system or the components ages. It is of great importance to avoid the failure of a system during the actual operation when such an event is costly and/or dangerous. Consequently the study of various preventive maintenance policies to reduce the operating cost as well as the risk of a catastrophic breakdown becomes a vital area of interest in reliability theory.

Preventive maintenance (PM) is a schedule of planned maintenance actions aimed at the prevention of breakdowns and failures. The primary goal of PM is to prevent the failure of equipment before it actually occurs. It is designed to preserve and enhance the system reliability by replacing the worn components before they actually fail. If either economic dependence, failure dependence or structural dependence exists among the components, group maintenance is more economical than individual maintenance. In this thesis, economic dependence means that performing maintenance on several components jointly costs less money and/or time than on each component separately. Failure dependence means that failure distributions of several components are stochastically dependent.

There are basically two practical group maintenance policies for such systems. The first is referred to as a *T*-based policy, in which PM is rescheduled when system is of age *T*. The second policy is referred to as an *m*-based policy in which PM is performed after *m* failures have occurred. Although these two policies are widely applied currently, they have some deficiencies. Firstly, if the whole system is renewed at the time of PM, it may not be economical because non-failed components are replaced together with failed components regardless of their status. This deficiency is removed by adopting a modified *m*-based maintenance policy [10] in which all failed components are replaced together with the non-failed components that were beyond the age of critical threshold at the time of *m*-components failure. According to this policy, all components are not renewed simultaneously, and the age of each component is traced at the time of PM. However, the maintenance program for this policy is tedious to implement.

The second deficiency is concerned with the system performance. Failed components are kept idling for a certain time until the next PM time when all components are repaired or replaced, at the same time, to save time and money. Although failure of one component does not cause the overall failure, the system's performance is reduced. An analogy of this can be made with the failure of one lamp in a chandelier, although this does not cause the failure of the whole chandelier, light output is reduced accordingly. However, to retain the system in an acceptable working condition, taking a proper PM becomes more important during its service.

In this thesis, an effective maintenance policy that can overcome these deficiencies is proposed. This maintenance policy is based upon a "preventive group maintenance policy" where the maintenance is applied to the entire group instead of on a component by component basis. The whole system is inspected at the time of PM. All components, failed or non-failed, are maintained with respective maintenance actions and then next PM is rescheduled. This differs from the group maintenance policies where the whole system is renewed at the prespecified time or when a catastrophic breakdown, whichever occurs first. This maintenance policy can be applied to the systems in which either economic dependence, failure dependence or structural dependence exists among the components. One of the problems, often referred to as the Group Maintenance Problem, is to decide when to carry out the maintenance program for the entire group. This is one of the key areas, this thesis will address with its developed mathematical model.

The conventional group maintenance policies are usually determined by using minimal maintenance cost as the criterion, without consideration of reliability or system performance. If system operators require a particular level of reliability, then this criterion must be formulated as a reliability constraint in the economic model. When reliability becomes the primary decision driver, the group maintenance policies will be changed accordingly, the conventional T-based or m-based determined purely on the lowest cost, are then modified into reliability-centered T-based and m-based preventive group maintenance policies. Current technical research on such reliability-centered approach is still lacking. In addition, when the reliability constraint is taken into account along with the cost factors comprising the component downtime cost, system downtime cost and maintenance costs, a question that requires an answer is whether the T-based or m-based policy is better. The abovementioned facts motivate this study.

Another problem that demands a solution for group maintenance policy is to decide the maintenance action at the time of maintenance. It is reasonable to do the same maintenance action for all independent and identically distributed (iid) components at the time of PM. However, for a system composed of many components that are not identical, the status of each component may differ at the time of PM; some may also be still functional whilst others are not; thus only an appropriate maintenance action should be performed on each component. This involves choosing among the actions such as replacing the whole system, replacing only the failed components and replacing the failed components and repairing the non-failed components.

The cost incurred, remaining life span and reliability of the system are also important factors in choosing a suitable maintenance action. When a system is maintained, the engineers should know the amount of age, reduced by the effect of maintenance. They also should know how much amount should be reduced to retain the system or component in desirable condition in order to choose the proper maintenance action. Although the exact level of required maintenance degree can be evaluated theoretically, it is almost impossible to attain the expected state in practice. This indicates the need to decide a suitable maintenance action for each component with a basis on the reduction amounts that are assessed by both theoretical and practical means. This is a motivation factor to develop an efficient maintenance policy for a system to function properly. It constitutes the second theme for this study. The amount of age reduction is represented by the maintenance degree in this thesis.

1.2 Scope of work

There are two systems considered in this thesis. The first consists of either a parallel, or a combination of parallel and series, or *k*-out-of-*n* operated machines. All of these machines are repairable and subject to stochastic failures from the same distribution. For such systems, reliability-centered T-based and m-based preventive group maintenance policies are analyzed in the first part of this thesis. In these policies, the time interval between successive PM is constrained by a required system's reliability. Therefore, the deficiencies, described in Section 1.1, can be removed. The objective of the research is to formulate the computational mathematical model for T-based and m-based maintenance policies with a reliability constraint. In addition, the corrective and preventive repair costs, component downtime costs and system downtime costs are also included for both to assess which would provide better outcomes for a maintenance program. A comparison of the associated maintenance costs for each would then decide the selection for the mode of maintenance.

The second system considered is for non-identical components that are operated independently. For such system, a model for deciding the suitable maintenance action based on the target and available maintenance degrees is analyzed. Two cases are considered according to the following different conditions desired.

1. Not to incur the maintenance cost beyond the available maintenance budget at the time of PM,
2. Not to allow the system to fall below the pre-determined minimal reliability level.

The mathematical model in this thesis addresses the required maintenance degrees to meet the required condition. After which, the target maintenance degrees are compared with available maintenance degrees, which can be obtained from the historical records to decide the maintenance action to be taken.

1.3 Organization of the thesis

This thesis is organized as follows.

Literatures concerned with the multi-component system maintenance are highlighted in Chapter 2. Model formulations for the reliability-centered T-based and m-based maintenance policies for the first system are presented in Chapter 3. Two cases are classified according to the repair time assumptions. In the first case, repair times are assumed as iid repair times. In the second case, repair times are assumed to be monotonically increasing repair time and Geometric Process (GP) is introduced to model these repair times. Then, the proposed two policies, T-based and m-based, are compared under these assumptions. At the end of Chapter 3, a case study is given as an application of the proposed maintenance policies. A mathematical model of a maintenance policy, proposed for the second system, is proposed in Chapter 4. After determining the required maintenance degrees of the component and after assessment of historical experience on maintenance degrees with different maintenance actions such as replacement or repair, the choice of maintenance action is taken. A detailed description and model formulation are addressed in Chapter 4. A numerical example is given at the end of Chapter 4 in order to demonstrate this work. Summary and conclusions are given in Chapter 5.

Chapter 2

Literature review

This chapter provides an overview of the literature related to the maintenance and reliability of multi-component systems.

2.1 Introduction

In the past several decades, maintenance problems have been extensively discussed in the research literature. Maintenance can be mainly classified as preventive or corrective. Corrective maintenances (CM) are maintenance actions made at the time of system failure. Preventive maintenance (PM) is a schedule of planned maintenance actions aimed at the prevention of breakdowns and failures. Depending on the different assessing PM time, there are various PM policies. A "Periodic Policy" is a preventive maintenance policy which is applied to the system on a fixed time interval. Such a maintenance program is convenient but can be wasteful since a component may have failed and been replaced in the interim, but dictates of the program is for another replacement upon the arrival of the fixed maintenance date. A more economically efficient way would be the "Age-Dependent Policy" which considers the maintenance record of the component, replacing it only after a fixed age. As a system can be expected to require the greater frequencies of maintenance with increase age, the "Sequential policy" shortens the period between maintenance progressively. Other approaches to maintenance include a "Failure Limit Policy" where maintenance is carried out only when the failure rate or other reliable indices of a unit reaches a predetermined level. The another approach of PM policy is "Repair limit Policy" where repair is only undertaken if estimated repair time/repair cost is less than a predetermined limit. The comprehensive descriptions and reviews, concerned with these PM policies are given in [33].

Almost all systems consist of multiple components and sub-components that work together towards a common objective. These components are generally categorized into two groups; repairable and non-repairable. Keeping a system or component in an operational condition calls for a proper maintenance action. For non-repairable components, replacement is the only maintenance policy. Failures of repairable components can be rectified by either repair or overhaul action. Deciding to repair, replace or overhaul depends on the cost and degree of reliability improvement of the components after maintenance. Depending on the maintenance actions and degrees of perfection (improvement), the states of the system after repair or replacement are divided into five groups [6].

1. Perfect repair / replacement: Repaired or replacement action restored the system to as good as new.
2. Minimal repair / replacement: Repaired or replacement action restored the system operationally but system is not improved.
3. Imperfect repair/replacement: Repaired or replacement action restored the system better than before failure but it does not act as a new (original) one.
4. Better than new: Repaired or replacement action, restored the system to better than new with components being replaced by an improved part.
5. Worse than minimal repair/replacement: Repaired or replacement action, resulted in the system getting worse than before failure.

The reality is that the resultant state of the component usually lie somewhere between the perfect and minimal repair state. The repair action has brought about a system that is better than before it failed, but not as good as when it was new. Therefore the choice of imperfect repair has been assumed by many researchers [3, 4, 5 and 21].

Since the foundation of any maintenance model relies on the potential failure behavior of the system after maintenance, it is necessary to know the changes of hazard rates due to the maintenance effect, the mathematical models describing the hazard rates are provided in the following sections for imperfect repair models.

2.1 Imperfect repair models

A model for imperfect repair was first proposed by Brown and Proschan [3]. Failures can be removed by replacement with probability p and rectified the component by minimal repair with probability (1-p). A failure rate PM model is also introduced by Lie and Chun [19]. Although repair action can reset the current failure rate of the component to zero, its failure rate is faster than before repair. Thus, the hazard rate slope is not identical from one PM interval to another. If hazard rate before i^{th} PM is $h_{i-1}(t)$, hazard rate after repair then becomes h_i (t) = $\theta h_{i-1}(t)$, $\theta > 1$ where θ is the adjustment factor.

2.1.1 Age reduction models

Another popular imperfect repair model is the age reduction model proposed by Canfield [4]. In this model the effect of maintenance is measured by a restoration amount "θ". If the effective age before i^{th} PM is t_i, its effective age after repair reduces the restoration interval to θ, this would then become t_i-θ after i^{th} PM. The derived formula for the hazard rate function by applying the age reduction model is as follows:

$$h_k(t) = \begin{cases} h_0(t) & \text{for } 0 \le t \le T \\ \sum_{i=1}^{k} \{h_0(iT-(i-1)\theta) - h_0(i(T-\theta))\} + h_0(t-k\theta) & \text{for } kT \le t \le (k+1)T, k = 1,2,3,... \end{cases}$$

(2.1)

$h_0(t)$ in equation (2.1) refers to the original hazard rate.

By combining the failure rate model and age reduction model, Lin [21] proposed a hybrid model where the hazard rate of failed component reduces to restoration interval and failure rate increases to a value greater than before repair. The hazard rate after k^{th} PM becomes $a_k h_k(b_k t_k + x)$. Although the current age of the system before k^{th} repair (t_k) is reduced to some amount $(b_k t_k)$, it's hazard rate increases because of restoration factor, a_k, is greater than 1. Both restoration factor and restoration interval are assumed as fixed values and these parameters can be estimated by domain experts utilizing real data.

Wu and Croome [34] relaxed this assumption. The restoration factor and restoration interval are assumed as random values. The failure rate of equipment after k^{th} PM is $h_k(t) = \theta^{k-1}h(t)$, and θ represents the restoration factor which is in accordance with an ordinary failure rate model. They assumed that the restoration factor is a random value and it follows a uniform distribution $F(\theta)$. As a result, the ordinary failure rate model becomes:

$$h_k(t) = \left(\int_1^{\infty} \theta \, \mathrm{d}F(\theta) \right)^{k-1} h(t). \tag{2.2}$$

Similar to the failure rate model, the amount of age reduction is assumed as random values and Canfield's age reduction model is changed into:

$$h_k(t) = \begin{cases} h_0(t) & \text{for } 0 \le t \le T \\ \displaystyle\int_0^T \left(\sum_{i=1}^{k} \{ h_0(iT - (i-1)\theta) - h_0(i(T-\theta)) \} + h_0(t - k\theta) \right) \mathrm{d}F(\theta) & \text{for } kT \le t \le (k+1)T, k = 1,2,3,.. \end{cases}$$

(2.3)

2.1.2 Improvement factor models

The effect of a repair action can be expressed in terms of an improvement factor. The concept of improvement factor is used to measure the extent of the restoration for a deteriorating system. If repair action reduces the system age (t) to ($\frac{t}{\theta}$) and the reliability of the system improves from $R(t)$ to $R\left(\frac{t}{\theta}\right)$. A variable "$\theta$" is used to express the improvement of system reliability and it is defined as an improvement factor. Lie and Chun [19] proposed such an improvement factor model in which an improvement factor is a variable that is affected by maintenance cost and system age.

Cheng and Chen [5] extended the improvement factor model proposed by Lie and Chun [19]. They considered the improvement factor as a variable affected by the system's age, cost, and number of maintenance performed. They proposed three different types of improvement factor models to represent three types of restoration effects. Detail description can be seen in [5].

According to the literatures listed above, the imperfect repair models are used mostly to predict the state of the system after repair. Failure rate models, age reduction models, improvement factor models, virtual age models, and the like reflect the imperfect repair effect.

2.3 Multi-component systems maintenance

When there are many components in a system and if there is no dependency among the components, it can be considered as a "single" component. However, if components in a system are not independent and there are dependency relationships among them, maintaining each component separately is not an optimal maintenance policy. For example, if maintenance cost, carried out for components separately, exceed the cost carried out for a group of components, this group of components are said to have economic dependency. For this kind of components, group maintenance is better preferred than individual maintenance. In addition, as failed components are maintained, operable components can also be maintained with marginally additional cost. Therefore, the time of failure of one component can be an opportunity for non-failed components to receive preventive replacement. Two popular maintenance policies for multi-component systems - the group maintenance policy and the opportunistic maintenance policy, are the resultant of these benefits.

Many researchers have modeled and analyzed various policies for multi-component systems over the past decades. Since the systems, discussed in this thesis, are multi-component systems and the main purpose is to give an effective group maintenance policy, these policies are described in the following sections.

2.3.1 Group Maintenance policies

The most popular group maintenance policies are T-based and m-based group maintenance policies. A T-based group maintenance policy is a policy in which all components are replaced at failure before or at pre-specified time T, whichever occurs first. Thus, this policy is named as an age-based group maintenance policy. An m-based group maintenance policy is a policy in which group replacement is made after exactly m components failed. The system renewal time is limited by numbers of failed components. Barlow and Hunter (1960) proposed a periodic group maintenance policy combined with minimal repair. Failures, occurred before pre-specified time T are removed by minimal repair and all are replaced at T. The objective of this model is to find the optimal replacement time T which can minimize the expected cost rate.

Assaf and Shanthikumar [1] considered a group maintenance policy for a set of N machines under continuous and periodic inspection. It is assumed that in the continuous inspection, the number of failed machines is instantly detected. In periodic inspection, the machines are inspected periodically, regardless whether they are in a good state or failed state. Inspection is assumed as perfect inspection. Perfect inspection means that the inspection reveals the true state of the system/component. Inspection costs are given at each inspection time. Later, various group maintenance policies and two-variable maintenance policies are proposed by many researchers.

Ritchken and Wilson [25] combined the advantages of T and m replacement policies as an (m, T) policy. In an (m, T) group maintenance policy, the system is inspected at fixed age, T or the time when exactly m machines have failed, T_m, whichever occurs first. The time interval between successive renewals is random variable T^*, and it is either m^{th} failure arrival time T_m or pre-specified age T. If it is assumed that the system consists of n identical machines and time to failure of each component is independent identically distributed $F(.)$ with finite mean, expected time between successive renewals is:

$$E\{T^*\} = \sum_{i=0}^{m-1} \int_0^T \binom{n}{i} [F(t)]^i [1 - F(t)]^{n-i} \, dt \tag{2.4}$$

In an m-failure group replacement policy, all components are replaced with new ones as soon as exactly m components failed regardless of whether the remaining components are operable or not. This model is modified by Dekker, Meer, Plasmeijer and Wildeman [10] and their model is named as a modified m-failure group maintenance policy. At m^{th} failure arrival time, failed components and non-failed components whose ages are greater than τ are replaced and the remaining components are kept in idle. After repairing, some are new and some are old. So it is evident that the age of each component is needed to be traced. It seems that a renewal theory is not suitable to calculate the expected long-term cost. So an alternate rule which can be effectively used to evaluate such cost is proposed. This policy is known as a renewing modified m-failure group replacement rule. At the time of m failure before threshold age τ, failed components are replaced at that time and all are replaced at next m failure occurrence time. If first m failure occurs beyond threshold age τ, all are replaced at that failure arrival time. In this model, it is assumed that (n-m) is less than m (n-m < m). Let Y be the random life time of a component and it has distribution function $F(.)$. Survival distribution function of a component which has age τ is:

$$\overline{F}(t \mid \tau) = P\{Y > t + \tau \mid Y > \tau\} = \frac{\overline{F}(t+\tau)}{\overline{F}(\tau)} \tag{2.5}$$

If a component has age τ and m components failure arrival time is $Y_{m|\tau}$, survival distribution function of $Y_{m|\tau}$ is

$$\overline{F}_{m|\tau}(t) = P\{Y_{m|\tau} > t\} = \sum_{i=0}^{m-1} \binom{n}{i} (F(t \mid \tau))^{i} \left(\overline{F}(t \mid \tau)\right)^{n-i} \tag{2.6}$$

Expected system renewal cycle is:

$$E[T^*] = \int_0^{\tau} \left\{ x + \int_0^{T-\tau} y dF_{m|\tau}(t) \right\} dF_m(t) + \int_{\tau}^{T} \overline{F}_m(x) dx \tag{2.7}$$

First part of equation (2.7) means that first m failures occurs before threshold age τ and system renewal time is next m failure occurrence time. Second part means that first m failure occurs after threshold age τ and system is renewed at that failure arrival time.

Another type of group maintenance policy is a two-phase group maintenance policy. It is assumed that the components are identical and repairable. Failure of a component is categorized into two such as Type I or Type II. Type I failure is removed by minimal repair and Type II failure is removed by replacement or left idle. Time is divided into two phases. First phase is 0 to T and second phase is T to $T+W$. Group replacement is conducted at the time of k^{th} idle or $T+W$, whichever occurs first. This model is presented by Sheu and Jhan [31]. The objective is to find the optimal T, W and k.

2.3.2 Opportunistic maintenance policies

Another effective group maintenance policy is an opportunistic group maintenance policy. In an opportunistic maintenance policy, the optimal maintenance action for one component depends on the state of the other components. At the time of failure of one component or system breakdown, failed components are performed corrective maintenance and it is an opportunity for non failed components to perform preventive maintenance. In most cases, opportunities cannot be predicted in advance because these are random events. Dekker and Smeitink (1991) considered maintenance opportunities occur randomly and a component has a chance to replace preventively at an opportunity arrival time.

A condition-based opportunistic maintenance policy is proposed by Zheng and Fard [38]. The maintenance action to be performed on failed component is decided on the basis of hazard rate of the component. If a component is failed within its hazard rate 0 and L-U, it is removed by minimal repair. If failure occurs between L-U and L, or it still alive up to L, it is removed by replacement. This replacement is named as an active replacement. Non-failed components whose hazard rates are between L-U and L are replaced at the time of performing active replacement of one component. This replacement is known as a passive replacement. Actually this passive replacement is an opportunistic replacement for non-failed components because replacement of non-failed components depends on both of their hazard rates and condition of the other components.

An age-based opportunistic maintenance policy is proposed by Zheng [36]. This work is based on an ordinary age-based replacement policy in which a component/system is replaced at the time of failure or its age is greater than T. The difference from the ordinary one is that at the time of replacement of one component, the ages of other components are checked; non-failed components whose ages are between (τ and T) are replaced together with failed components. Thus replacement of one component is an opportunity for other components. An opportunity arrives exponentially with rate λ and its distribution function is:

$$\overline{Q}(t) = \exp(-\lambda(t-\tau)) \qquad \tau < \mathrm{t} < \mathrm{T} \tag{2.8}$$

Replacement is classified into three, such as failure replacement, passive replacement (opportunistic) and active replacement. Failure, occurred before τ and within τ and T before the opportunity arrival time, is removed by replacement and it is remarked as a failure replacement. Other two such as active and passive replacements are defined as the same as [36] except that age limits $(\tau \text{ and } T)$ are replaced in place of hazard rates (L and U). Therefore, the renewal time of a component is the time of failure or an opportunity arrival time or T whichever occurs first. The objective being to find an optimal replacement age T and a threshold age τ to get the minimum expected cost.

Failure interaction and minimal repair polices are introduced by many researchers. At the time of failure, this failure may be minor failure with probability $q(t)$ or major failure with $p(t)$. Minor failure is denoted as Type I failure and major failure is denoted as Type II failure. Type I failure is removed by minimal repair and system or component is replaced at the time of Type II failure or at age T whichever occurs first. This is general failure interaction and minimal repair policy. This policy is modified into an opportunistic maintenance policy by Jhang and Sheu [31].

Opportunity arrives according to Poisson process and time between successive opportunities is exponential distribution with rate λ. First opportunity arrival time is Z and its distribution function is

$$\overline{Q}(z) = \exp(-\lambda z) \tag{2.9}$$

The difference from an ordinary one is that system being replaced at Type II failure or at the first opportunity arrival time after age T. If random variable Y_{II} represents the time interval between successive Type II failures and it has life time distribution $F_{II}(t)$, the expect system renewal time T^* is

$$E\left[T^*\right] = \int_0^\infty \int_0^{T+z} \overline{F}_{II}(t)dtdQ(z) \tag{2.10}$$

In the multi-component systems, there exists structural dependence between the components besides failure and economic dependences. In a series system, failure of one component affects the whole system. But in a parallel system, the whole system will fail when all components fail. "*k*-out-of-*n*" system is a kind of parallel system. The main difference from the parallel system is that at least k number of components will have to function so that the whole system is operable. Therefore, the whole system is needed to stop whenever one component failure occurs in the series system, but it is not necessary in the parallel and *k*-out-of-*n* system. If set up costs and system down-time costs are also taken into consideration for the computation of expected cost, there is no doubt that the maintenance cost in series system is greater than that of the parallel system. It is evident that the opportunistic maintenance is more effective in series system.

Neelakanteswara and Bhadury described the evaluation of an opportunistic maintenance on a series system by using simulated approach. Pham and Wang [24] discussed both perfect and imperfect preventive replacement policies for k-out-of-n system. It is assumed that minimal repair action is enough to remove failures occurred in "0 and τ". Failures occurred within "τ and T" are kept in idle until the m^{th} ($m = n-k+1$) failure arrival time. If the m^{th} failure occurs at t ($\tau < t < T$), corrective maintenance and preventive maintenance on failed and non-failed units are preformed at t. Otherwise, all are preventively maintained at time T. In this system, the m^{th} failure occurrence time is an opportunity for other components to perform PM. In other words, an opportunity arrival time, Z, is the m^{th} failure occurrence time, $Y_{m|\tau}$, and its distribution function is

$$Q(t) = P\{Y_{m|\tau} < t\} = 1 - \overline{F}_{m|\tau}(t) \tag{2.11}$$

Similar to this policy, Dekker, Pasmeijer and Wildeman [10] modified m-group replacement policy as an opportunistic group replacement policy in their case study. The modified m-group replacement policy keeps the components idling for a certain time until m components are failed. And then, all the failed components are replaced together with the non-failed components whose age has passed a critical threshold age. If m components fail before threshold age τ, only failed components are replaced and system renewal time is denoted as next m-components failure arrival time. If m components failure occurs over threshold age τ, all are replaced at that time. So in this policy, opportunity arrival rate is the same as m-failure arrival rate. Respective mathematical models for this system are presented in equations (2.5), (2.6) and (2.7).

Optimal maintenance policy concerned with components which can give signals before failure is proposed by [8]. This policy is based on an age-based replacement policy which is the most popular maintenance policy. Some components give prior indications about their health and warn that they are likely to fail. This indication is interpreted as a fault and this fault does not lead to immediate failure. It means there is a time lag between the fault occurrence time and failure time. This time is known as delay time and it follows distribution function $F(.)$. A waiting time *td* is assigned after the occurrence of fault at random time U and if failure arrives before *td*, it is replaced. Otherwise, the whole system is replaced at the end of waiting time *td*. So this policy is known as age replacement during delay time policy (ARDTP). The expected time length for this policy is:

$$E[T^*] = E[U] + \int_0^{td} \overline{F}(t)dt \tag{2.12}$$

If a component or system does not fail during waiting time *td* and an opportunity arrives before failure, this component or system is replaced at an opportunity arrival time. This policy is named as "an opportunistic age replacement during delay time policy" (OARDTP). For this policy, expected system renewal time is an opportunity arrival time or failure occurrence time after waiting time *td* whichever occurs first. Opportunity arrival time *Z* follows $Q(.)$ and expected time length for OARDTP is:

$$E[T^*] = E[U] + \int_0^{\infty} \int_0^{td+z} \overline{F}(t)dtdQ(z) \tag{2.13}$$

When these two policies are compared, cost per unit time of OARDTP is higher than ARDTP if preventive maintenance cost, c_p, and opportunistic maintenance cost, c_o, are the same value. But in real world, c_o is less than c_p. When c_o is only 0.2% c_p, it can be proved that OARDTP is better than ARDTP.

All the models described above are meant to get minimum cost. Howerver, the main purpose of doing maintenance is to reduce failure rate (or) to improve reliability. Thus, an optimal maintenance policy is a policy which can obtain not only minimum cost but also maximum reliability. But it is impossible to satisfy these two requirements simultaneously. It means a maintenance policy which can minimize cost cannot get maximum reliability and vise versa. Therefore, an acceptable (cost or reliability) level is assigned and an optimal maintenance policy is considered as a policy which can maximize (minimize) reliability (cost) and satisfy acceptable cost (reliability) constraint. Various maintenance models concerned with cost and reliability (or) various maintenance models in which reliability is considered as an important factor in the cost models are described in next section.

2.4 Reliability-centered maintenance

The term reliability is defined as the probability that the system will perform its intended function for a specified interval of time under the stated conditions. Determining reliability involves understanding concepts pertaining to failure rate as a function of age. When failure rate is considered as more general hazard rate $h(.)$, general expression of reliability becomes:

$$R(t) = \exp\left(-\int_0^{\infty} h(t)dt\right) \tag{2.14}$$

Mean time between failures (MTBF) is

$$MTBF = \int_0^\infty R(t)dt \tag{2.15}$$

This MTBF is primarily the important factor by which one item can be compared with another. In fact, the above expressions for $R(t)$ and MTBF are the basic mathematical relationships used in reliability prediction. System or component reliability degrades due to usage or age or poor maintenance. In order to improve the degrading reliability and to prevent the failure due to system degradation, PM is performed before failure. At the time of PM, proper maintenance action (repair or replace) is decided by limiting some factors such as cost, time, and reliability, etc. Related papers concerned with repair limit policies and reliability-centered maintenance are listed in the following sections.

2.4.1 Repair limit policies

In all models discussed above, all components have same repair action at each PM time. For example, if maintenance decision is to do imperfect repair at PM time, all components are imperfectly repaired at that time. This assumption is reasonable for iid components. If this assumption is relaxed, performing same maintenance action on all components is not suitable and proper maintenance action should be performed on each component depends on its hazard rate and/or damage level. Basically, at the time of PM, which repair action should be performed is decided based on repair cost, repair time, remaining life time and current age.

In repair cost limit policy, whether failed components are repaired or replaced is decided based on repair cost. When a unit fails, the repair cost is estimated. If this estimated value is less than the predetermined value, repair action is performed and otherwise, replacement is more preferable than repair. Repair time limit policy is same as repair cost limit policy. If repair time is relatively short, failed unit is repaired. Otherwise failed unit is replaced.

Dohi, Kaio and Osaki [11] also discussed the time to stop repairing a unit after it fails. When a unit fails, repair starts immediately. If repair cost is greater than pre-specified cost limit, repair action is stopped. Spare unit is ordered immediately and this unit is installed after lead time L. If repair action is finished before reaching cost limit, repaired unit is installed again. Due to imperfect repair effect, life time of repaired unit is less than that of original one. Optimal repair-cost limit is estimated by using nonparametric method and applying the total time on test (TTT) concept.

Similar to the repair cost limit policy, repair time limit policies are considered by many researchers and related literatures are described in [11]. Dohi, Takeita and Osaki [12] relaxed the assumption of arbitrary repair time and described two models with random repair time. Model 1 is proposed that the failed unit is repaired and if complete repair is finished within limited time, repaired unit is installed. Repair is assumed as perfect repair and expected cycle length is the summation of component life time and repair time. If repair time is exceeded limited time, repaired unit is scrapped and spare unit is ordered and this spare unit is installed after lead time L. Expected cycle length is the summation of component life time, limited repair time and lead time. Mean time to failure is $\frac{1}{\lambda}$ and repair time limit is t_0. If time to complete repair is random value and it follows $G(.)$, and expected cycle length is

$$E[T^*] = \int_0^{t_0} \left(\frac{1}{\lambda} + t \right) dG(t) + \int_{t_0}^{\infty} \left(\frac{1}{\lambda} + t_0 + L \right) dG(t) \tag{2.16}$$

Model 2 is similar to Model 1. But repair time is estimated at failure time. If repair time seems to be greater than repair limit time, spare unit is ordered at failure time and it is installed after lead time *L*. If repair time seems to fall within limited repair time, failed unit is repaired. Expected cycle length for Model 2 is

$$E[T^*] = \frac{1}{\lambda} + \int_0^{t_0} t dG(t) + L\overline{G}(t_0) \tag{2.17}$$

Tsai, Wang and Tsai [32] decided suitable maintenance action by comparing benefit and cost ratios resulting from each type of maintenance. Three maintenance actions are considered at PM such as (1a), (1b) and (2P). (1a) maintenance just only improves the extrinsic state of subsystem or component. (1b) maintenance includes the activities of (1a) maintenance and repairing/replacing for some simple parts. The last one (2P)-maintenance is to replace the subsystem/component. What kind of maintenance action should be performed is decided based on maintenance benefit. The maintenance benefit on the j^{th} PM is:

$$B_{j,k} = \frac{\int_{t_j}^{\infty} R_{i,j+1}(t)dt - \int_{t_j}^{\infty} R_{i,j}(t)dt}{C_{i,k}} \tag{2.18}$$

In this equation, subscripts *k* and *j* refer to the kind of maintenance action and j^{th} PM. If there is no maintenance at time t_j, remaining life time of component *i* is $\int_{t_j}^{\infty} R_{i,j}(t)dt$. If either (1a) or (1b) or (2P) is performed on component *i*, and the remaining life time of component i after j^{th} PM is $\int_{t_j}^{\infty} R_{i,j+1}(t)dt$. Therefore, numerator of (2.18) refers to the effect or benefits of performing maintenane. Denominator is the maintenance cost related to maintenance action. Maintenance action of each component is chosen as the one which can give maximum benefit, $B_{j,k}$.

2.4.2 Reliability-centered maintenance policies

In the maintenance theory, an age-based replacement policy is well known. In this policy, optimal replacement age T is decided based on long run cost. It means age replacement time T is the time that minimizes the long run expected costs. Scarf, Dwight and Musrati [26] decided an optimal replacement time T by setting reliability as a decision criterion and reliability is expressed in various ways. Firstly, reliability is expressed as operational probability. Operational reliability is p and optimal time (T) to get this reliability p can be obtained by equating

$$1 - F(T) = p \tag{2.19}$$

Secondly, reliability is expressed in terms of mean time between failures. Suppose N=n is the number of PM cycle before first operational failure, Y is component life time and it follows distribution function $F(.)$. System renewal time is Y_s= nT+Y, where $Y < T$. Repair action performed at each PM is assumed as perfect repair and expected system renewal time, in other words, mean time between failures is:

$$E[Y_s] = \sum_{n=0}^{\infty} nT\left(\overline{F}(T)\right)^n + \frac{\int_0^T ydF(y)}{F(T)} \tag{2.20}$$

The other way of expression reliability is in terms of some quantile of the distribution of the time between operational failures. The required probability that system renewal time Y_s is greater than nT, is p and it is expressed as follows:

$$\Pr\left(Y_s > nT\right) = p \tag{2.21}$$

Mean time between failures of a system that has periodic maintenance interval T can be described in (2.20). If a system consists of many components or system is highly complex system, there are some difficulties to calculate the system's mean time between failures]. because of integration. So a simple technique for estimating MTTF is proposed by [22]. It is assumed that a system with periodic maintenance has exponentially distributed time between failures with constant rate and its reliability function is expressed in terms of appropriate MTTF (MTTFA).

$$R_A(t) = \exp(-\frac{t}{MTTF_A}) \tag{2.22}$$

If time T and system reliability at time T are known, $MTTF_A$, can be obtained by substituting the reliability at time T in place of $R_A(t)$. The brief description can be studied in [22].

In general, preventive maintenance (PM) policies hold the same time interval for PM actions and are often applied with known failure modes. For a degradation system, hazard rate (reliability) of system increases (decreases) with time t. So the system is assigned to perform imperfect PM at equal PM-time intervals, and as a consequence, the system's reliability will inevitably declined by time. PM opportunities derived from a specified acceptable reliability level are introduced by

Zhao [37]. According to the PM policy of the critical reliability level, there is the same reliability level from one PM cycle to another. From this finding, Zhao [37] deduced a reliability degradation law. For the system with PM policy of critical reliability level, the number of failures in the time intervals of various PM cycles, and degradation ratio of the optimal time intervals and the hazard rates between neighboring PM cycles are the same.

Crocker and Kumar [7] proposed a new approach to RCM. Generally, when machines or components are reconditioned, some may be replaced prematurely. To avoid this case, they defined hard life and soft life for each component. These hard and soft lives are decided under reliability constraints. Failures occurred before soft life or within soft and hard lives before the removal of line replacement unit (LRI), are removed by failure replacement. If LRI removal occurs before failure within hard and soft lives, an opportunistic replacement is performed. If a component does not fail until hard life, a planned replacement is made for this component.

In models described above, optimal PM time or renewal time is limited by critical reliability level and this reliability level is expressed in terms of age or time. It means that time or age which can meet critical reliability level is denoted as PM time or renewal time. But in some cases, maintenance time is decided based on number of failed components. In a parallel system or *k*-out-of-*n* system, the failure of the whole system depends on the number of failed components. In these systems, maintenance time is denoted in terms of number of failed components. So for this system, PM time can be limited in two ways; in terms of age or number of failed components. If failed components are kept in idle until PM time and component downtime cost is taken into account in maintenance cost, the question arisen here is which one is more preferable. In this thesis, optimal PM time is expressed in two ways and two models are presented for the evaluation of optimal PM time, number of PM and give a numerical comparison between them. Detail description and methodology can be seen in Chapter 3.

Generally, the components are assumed as iid components. But in practice, all components are not always iid. For example, there are many different components in complex electronic system, electronic equipment and computer and they have different life spans. For these kind of components, suitable maintenance action is chosen based on remaining life time, maintenance cost, limited repair cost and limited repair time. Related works are described above. In this thesis, a new method to find suitable maintenance action is presented. All components composed in a system are aimed to do imperfect repair or replacement at the time of PM. At each PM, how much current wear amount (failure rate) should be reduced to meet required specification is necessary to know. These required amounts of reduction are termed as target maintenance degrees. However, these amounts are hard to attain exactly in practice. As the reduction amount is at random it is named as an available maintenance degree. Therefore a possible maintenance action is decided by comparing the required and available maintenance degrees. Relevant model formulations for this work are described in Chapter 4.

Chapter 3

Model formulations of reliability-centered *T*-based and *m*-based preventive group maintenance policies

3.1 Introduction

The main purpose of this chapter is to present a model of multi-component systems maintenance.

A system typically consists of many components. In many practical situations, however, a group of units (components) are put in service together. In such a situation, replacing or repairing a group of components instead of individual replacement may result in much cost reduction. This cost saving is known as the economy of scale and comes mainly from the reduction of maintenance set-up cost per unit. For example, car brakes are periodically inspected and replaced by cluster. Since preventive maintenance cost is less than the corrective maintenance cost, preventive maintenance is performed before failure. Here, the main problem is to find the optimal preventive maintenance time. The choice of the optimal replacement age, T, is the main problem in the study of age replacement policies. Deciding optimal number of failed components which is used as a control limit to assign PM time, m, are important in an m-based policy. Mostly, optimal PM time is decided to get minimum cost. But when reliability is taken into consideration in a maintenance problem, it is necessary to keep the system reliability above a minimal acceptable level. In order to do this, PM time must ensure a minimum level of reliability, and this policy is named as a reliability-centered T-based maintenance policy.

In this thesis, this policy is mixed with preventive maintenance policy and a reliability-centered *T*-based preventive group maintenance policy is proposed. A maintenance policy in which the number of failed components is decided under reliability constraint is not expressed in previous papers. Therefore, a reliability-centered *m*-based preventive group maintenance policy in which PM time is limited by number of failed components and these failed components are restricted by reliability, is also provided in this chapter. Then, a comparison is made between the associated maintenance costs of these two policies, *T*-based and *m*-based.

Model formulations for proposed reliability-centered *T*-based and *m*-based policies are described in Sections 3.3 and 3.4. In Section 3.5, a case study along with the discussion is given.

3.1 Statement of the problem

Although failure of one component could not produce a total failure according to the system structure, the system performance will be invariably affected. Suppose that two components are operating in parallel with equal share of workload. If one fails, the remaining one can serve the load. However, life span for such component is less than the one which serves the load by sharing with others. Suppose System A has two components which are connected in parallel and System B operates with only one component. It is assumed that components in System A and B are identical. Probability of System A performing the required function in a given period is greater than that of System B. It means reliability of A is greater than that of B in a given period. This fact points out that the system's reliability depends on the number of operating components in it.

In a system, components can break down from time to time, and system's reliability also decreases with respect to the number of failed components and time. In order to prevent the reduction in system's reliability due to failed components, the failed components must be repaired in time. If the whole system is needed to stop at the time of inspection and repairing, there is an opportunity to do preventive maintenance for non-failed components with an additional cost. This cost is known as preventive maintenance cost. It is always less than failure repaired cost. The loss in production output due to the stoppage of the system during repair time is considered in terms of monetary loss. If a system's down time increases accordingly, and component downtime cost is proportional to inspection time interval while maintenance cost varies with corrective as well as preventive repairs, the whole system needs to be replaced with a new one, after some times, for economic reasons.

Two cases are considered with different definitions of inspection time. In the first case, inspection time is regarded as the system's critical reliability level arrival time and this policy is named as a reliability-centered *T*-based maintenance policy. In the second case, inspection time is regarded as the last item failure arrival time before system reliability is below the critical reliability level. So this policy is named as a reliability-centered *m*-based policy. These two policies will be evaluated in terms of component downtime costs, production loss costs, system's availability and repair time assumptions. The main purpose is to determine the optimal numbers of inspection (*N*) and optimal time interval between successive inspections for each policy. The following notations are used throughout this chapter.

Notations:

T	time interval between successive inspections
Y_m	rv: m components/subsystems failure arrival time
$Y_{j,i}$	rv: time to failure of component i in subsystem j
X	rv: time to complete repair
X_m	rv: time to complete repair for m failed components
X_c	rv: time to complete corrective repair
X_p	rv: time to complete preventive repair
N_T	rv: number of failed components during inspection time interval T
N_{Y_m}	rv:number of failed components during inspection time interval
$R_s(T)$	system reliability at inspection time T
$R_i(T)$	reliability of component i at inspection time T
$R_{i,j}(T)$	reliability of component i in subsystem j at inspection time T
$F(T)$	life time distribution of iid component
$F_{j,i}(T)$	life time distribution of component i in subsystem j
$P_k(T)$	probability that exactly k components fail at time T
$p_{j,i}$	probability that failure of subsystem j due to component i in subsystem j
n	the total number of components or subsystems
i	the index of PM
j	the index of components or subsystems
m	the index of failed components or subsystems

l	number of components in a subsystem
λ	failure rate
μ_m	corrective repair rate
μ_p	preventive repair rate
c_m	corrective repair cost
c_p	preventive repair cost
c_d	per unit component downtime cost
$c_{m,j,i}$	corrective repair cost of component i in subsystem j
Y_m	random variable of m^{th} component/subsystem failure arrival time
$E[C_R]$	Expected repair cost
$E[C_{R,m}]$	Expected repair cost for m failed components
$E[X_c]$	Expected time to completion of corrective repair
$E[X_p]$	Expected time to completion of preventive repair

3.2 Model formulation of reliability-centered *T*-based maintenance policy for various system structures

Some important definitions and assumptions are given first before presenting the model.

Definitions:

- Inspection cycle: Time interval between two consecutive inspections
- Renewal cycle: Time interval between two consecutive replacements of the whole system
- Reliability: Probability that a system can perform specific function in a specified period
- Critical Reliability: Pre-determined reliability level
- Threshold number of failed components: Maximum acceptable number of failed components during an inspection cycle

Definition 1: Given two random variables, *X* and *Y*, *X* is said to be stochastically greater than *Y*, or *Y* is stochastically less than *X*, if

$$P(X > t) \geq P(Y > t) \quad \text{for } \forall t \in \Re \tag{3.1}$$

It is denoted by $X \geq_{st} Y$ or $Y \leq_{st} X$ (see. Lam, Zhang [17]). Furthermore, stochastic process $\{X_n, n = 1,2,...\}$ is stochastically decreasing if $X_n \geq_{st} X_{n+1}$, and a stochastic process $\{X_n, n = 1,2,...\}$ is stochastically increasing if $X_n \leq_{st} X_{n+1}$ for all *n*=1,2,….

Definition 2: A stochastic process $\{X_n, n = 1,2,...\}$ is called a geometric process (GP), if there exists a real a > 0, such that $\{a^{n-1}X_n, n = 1,2,...\}$ forms a renewal process. The real a is called the ratio of geometric process (see Lam, Zhang [17]).

Obviously, if a >1, then $\{X_n, n = 1,2,...\}$ is stochastically decreasing, i.e.,

$$X_n \geq_{st} X_{n+1} \qquad n=1, 2,\ldots \tag{3.2}$$

If 0 < a <1, then $\{X_n, n = 1,2,...\}$ is stochastically increasing, i.e.

$$X_n \leq_{st} X_{n+1} \qquad n=1, 2,\ldots \tag{3.3}$$

If a=1, then the GP becomes a renewal process.

Definition 3: A replacement cost function exhibits economies of scale if

$$C_G \leq n^* c_m, \qquad \forall n \geq 1 \tag{3.4}$$

Assumptions:

1. Components in a system are independent and identical distributed (iid) components.
2. System is inspected periodically and this inspection is perfect, i.e. they diagnose without any error whether the component is in an operating or failed state.
3. At the time of inspection, the whole system is stopped. After repairing, the whole system returns to perfect condition.

4. After repairing, system cannot operate immediately. It takes time to operate in regular operation.
5. Required service is available at the time of failure.
6. Repair time and repair cost are the same for all components

The first and third assumptions are general assumptions made in group maintenance policy. If these assumptions are relaxed, it is necessary to trace the age (or) failure rate of each component in a system after repair. By adding these assumptions, a maintenance model is easy to implement. Therefore, relaxation on the assumption of iid components is not considered in this chapter. After repairing or replacing the system or component, it is necessary to test whether the system (or) component can run in normal condition or not. Equipment used in wafer fabrication is considered as an example. There are many parts in an equipment. If any part in this equipment fails or is to be preventively maintained, the whole equipment will have to stop and maintenance action is to be performed. After repairing, it cannot operate in normal condition because it takes time to get required air pressure. Assumption 4 is made for this case. Assumption 5 is made in order to highlight that there is no need to wait to get required service at the time of failure. Even if spares are required at the time of failure, there is no need to take lead time to order the required spares and they can be obtained instantly. Under these assumptions, model formulations required for proposed two maintenance policies are described in the following sections.

3.2.1 Reliability formulations for different system structures

Suppose that a system has n iid components which are connected in parallel and each component has life time distribution $F(.)$. If one or more components still alive, the system is also still operable. Probability that a component or system can perform a specific function in a specified period is known as the "reliability". If random life time of the system is represented by Y_s and reliability for a parallel system at inspection time T is given by:

$$P(Y_s > T) = R_s(T) = 1 - \prod_{i=1}^{n}(1 - R_i(T)) \tag{3.5}$$

In this equation, $R_i(T)$ is the reliability distribution function of component i and $R_s(T)$ is the reliability function for the whole system.

Another type of redundant reliability structure besides parallel structure is k-out-of-n redundant system. Its structure is similar to a parallel system. But the difference is that at least k components must function for the whole system to function. If k=1(1-out-of-n), it is the same as a parallel system. All components in the system are iid and reliability for k-out-of-n system is expressed as

$$P(Y_s > T) = R_s(T) = \sum_{i=k}^{n}\binom{n}{i}(R(T))^{i}(F(T))^{n-i} \tag{3.6}$$

If multi-component systems are considered, there are two basic types such as series and parallel systems. Other systems are constructed based on these two types. For example complex reliability structures such as parallel-series and series-parallel systems are built on the combination of these two types. If a system has n

subsystems, connected parallel, and each subsystem has l components in series, then it is called a parallel-series system. If each component has exponential life time distribution $F(.)$, reliability for subsystem j is

$$P(Y_j > T) = R_j = \prod_{i=1}^{l} R_{i,j} \tag{3.7}$$

Reliability for the whole system is

$$R_s = 1 - (\prod_{j=1}^{n}(1 - R_j)) \tag{3.8}$$

Reliability for the whole system during time interval T is

$$P(Y_s > T) = R_s(T) = 1 - \prod_{j=1}^{n}(1 - \prod_{i=1}^{l} R_{i,j}(T)) \tag{3.9}$$

System reliability formulations for respective system structures can be seen in Ebeling Charles [13].

3.3.2 Evaluation of inspection time *T* for different system structures

Suppose that a system is composed of n iid components. If the system is inspected periodically and its reliability is maintained not to fall below the critical reliability R_c, the system will have to be inspected at the time of its reliability arrives at the critical reliability level. Reliability function for each system is equated to R_c and required inspection time T can be obtained by solving it. As an example, inspection time T for a parallel system is considered. R_c is equated with (3.5) and inspection time T for a parallel system whose components have constant failure rate λ is

$$T = \frac{-\ln(1-\sqrt[n]{1-R_c})}{\lambda} \tag{3.10}$$

If components have Weibull distributed life time with parameter (λ,α), (3.10) becomes:

$$T = \frac{\sqrt[\alpha]{-\ln\left(1-\sqrt[n]{1-R_c}\right)}}{\lambda} \tag{3.11}$$

Similar to the parallel system, inspection time T for a parallel-series system is derived by equating (3.9) with critical reliability level, R_c. As shown in (3.6), reliability formulation for k-out-of-n system is more complex compared to other systems. So inspection time T can not be formulated directly as in other systems and this T value for k-out-of-n system can be obtained by using following procedure.

Procedure 1: Compute inspection time T for k-out-of-n system

Step 1: input n, k, $h(t)$, R_c

Step 2: substitute these values in (3.6)

Step 3: apply trial and error method

Step 4: output: T

3.3.3 Component downtime cost consideration

The time interval between successive inspections is known as an inspection cycle. Failed components during an inspection cycle are not repaired immediately and they are kept in idle. Component downtime costs, incurred due to the failure of components before inspection are considered in this section. $\{Y_j, j = 1,2,3,...,n\}$ be random variables which represent j^{th} component failure arrival time. If it is given that *m* components are already failed before *T*, total downtime cost due to the failure of *m* components is

$$E[C_d \mid N_T = m] = c_d * \left[mE[T - Y_m] + \sum_{j=2}^{m} (j-1) E[Y_j - Y_{j-1}] \right] \tag{3.12}$$

From this equation, it is obvious that component downtime costs increase with the number of failed components and length of inspection cycle. Since the system is inspected before failure, the maximum number of failed components before inspection time *T* may be in the range of 0 and *n* for a parallel system.

$$0 \le m < n$$

Possible number of failed components at the time of inspection is a random variable and probability that exactly *m* components fail at the time of inspection is

$$P\{N_T = m\} = P_m(T) = \binom{n}{m} (F(T))^m \left(\overline{F}(T)\right)^{n-m} \tag{3.13}$$

General description for expected component downtime cost due to the failure of *m* components in an inspection cycle is

$$E\left[C_d \mid N_T = m\right] = 0 \qquad \text{if } m = 0$$
$$= c_d * E[T - Y_1] \qquad \text{if } m = 1$$
$$= c_d * \left[\sum_{j=2}^{m} (j-1) * E[Y_j - Y_{j-1}] + m * E[T - Y_m]\right] \qquad \text{if } 2 \le m < n \tag{3.14}$$

Y_m is denoted as random variable of m components failure arrival time and it follows life time distribution $F_m(.)$. Its survival distribution function $\overline{F}_m(.)$ is given by:

$$\overline{F}_m(y) = P\{Y_m > y\} = \sum_{j=0}^{m-1} \binom{n}{j} \left(F(y)\right)^j \left(\overline{F}(y)\right)^{n-j} \tag{3.15}$$

Expected m components failure arrival time within T is

$$E\left[Y_m\right] = \int_0^T \overline{F}_m(y)dy = \int_0^T \sum_{j=0}^{m-1} \binom{n}{j} \left(F(y)\right)^j \left(\overline{F}(y)\right)^{n-j} dy \tag{3.16}$$

By using (3.16), $E[Y_m - Y_{m-1}]$ can be expressed as

$$E[Y_m - Y_{m-1}] = \int_0^T P_{m-1}(y)dy \tag{3.17}$$

By using (3.16) and (3.17), expected component downtime cost expressed in (3.14) can be simplified as:

$$E\left[C_d \mid N_T = m\right] = 0 \qquad \text{if } m = 0$$
$$= c_d * \left(T - \int_0^T P_0(y)dy\right) \qquad \text{if } m = 1$$
$$= c_d * \left[\sum_{j=2}^{m} (j-1) * \int_0^T P_{j-1}(y)dy + m * \left[T - \int_0^T \overline{F}_m(y)dy\right]\right] \text{if } 2 \le m < n$$

$$(3.18)$$

So total expected component downtime cost per inspection cycle is

$$E[C_d] = E\left[C_d \mid N_T = m\right] * P\left\{N_T = m\right\}$$
$$= P_1 * c_d * [T - \int_0^T P_0(y)dy] + \sum_{m=2}^{n-1} P_m * c_d \left[\sum_{j=2}^{m} (j-1) * \int_0^T P_{j-1}(y)dy + m\left[T - \int_0^T \overline{F}_m(y)dy\right]\right]$$

(3.19)

The formulation of component down time cost, described in (3.19) is for a parallel system. If m and c_d are assumed as maximum number of failed subsystems and per unit downtime cost of a subsystem, this equation can be applied for a parallel-series system. For the k-out-of-n system, since failure of the whole system occurs at (n-k+1) components failure arrival time, maximum allowable failed components before inspection, m, is (n-k). When (n-k) is substituted in place of (n-1) in (3.19), expected component downtime cost for k-out-of-n system can be obtained.

3.3.4 Repair time formulation and expected time to system renewal

Generally, repair times are assumed as independent and identical distributed repair times (iid). If X_1 is a random variable which represents the time to complete repair of component 1 and it follows exponential distribution with rate μ, m components failure repair time X_m $\left(\sum_{i=1}^{m} X_i\right)$ has distribution with rate (m, μ). At the time of inspection, failed components are correctively repaired and non-failed components are preventively repaired. Since preventive repair time is less than corrective repair time, repair time (X) is divided into two such as corrective repair time (X_c) and preventive repair time (X_p). If it is given that m components have already failed at the time of inspection, expected repair time for m failed components (X_m) under the assumption of iid repair time is

$$E[X_m \mid N_T = m] = m * E[X_c] + (n-m) * E[X_p] \tag{3.20}$$

$$\begin{aligned} E[X] &= \sum_{m=0}^{n-1} E[X_m] = \sum_{m=0}^{n-1} E[X_m \mid N_T = m] * P(N_T = m) \\ &= \sum_{m=0}^{n-1} \left(m * E[X_c] + (n-m) * E[X_p]\right) P_m(T) = \sum_{m=0}^{n-1} \left(\frac{m}{\mu_c} + \frac{(n-m)}{\mu_p}\right) P_m(T) \end{aligned} \tag{3.21}$$

If the system is renewed at N^{th} inspection, total expected repair time is given as follows.

$$E[X_N] = (N-1) * \left[\sum_{m=0}^{n-1} \left(\frac{m}{\mu_c} + \frac{(n-m)}{\mu_p}\right) P_m(T)\right] \tag{3.22}$$

In practice, general repair procedure includes two steps: inspection for making diagnosis of some questionable parts and repairing or replacing some damaged or failed parts. If an older component fails, failure situations may be complicated and it may take a long time to find the causes of failure and to return to good as new condition. As a result, consecutive repair times increase from one repair time to another. In a deteriorating system, it is assumed that life time is monotonically decreasing while repair time is monotonically increasing. GP is introduced and applied to such maintenance problems. Related papers can be seen in [17]. According to this reason, geometric process (GP) is applied to model monotonically increasing repair times. If $\{X_i, i=1, 2,\ldots, N\}$ represents repair time at i^{th} inspection and it follows GP process with GP ratio a and the value of a is less than 1 because of increasing repair time, $E[X_i]$ is

$$E[X_i] = \frac{E[X_1]}{(a)^{i-1}} \tag{3.23}$$

X_1 and i represent the time to complete repair at first inspection and number of inspection. Expected repair time formulation expressed in (3.21) and (3.22) changes into

$$E[X_i] = \frac{1}{(a)^{i-1}} \sum_{m=0}^{n-1} \left(m * E[X_c] + (n-m) * E[X_p]\right) P_m(T) \tag{3.24}$$

$$E[X_N] = \frac{a\left(1-\left(\frac{1}{a}\right)^{N-1}\right)}{a-1} \left[\sum_{m=0}^{n-1} \left(m * E[X_c] + (n-m) * E[X_p]\right) P_m(T)\right] \tag{3.25}$$

Equations (3.22) and (3.25) are considered for a parallel system. For k-out-of-n system, since failure of the whole system occurs at (n-k+1) components failure arrival time, maximum allowable failure number m before inspection is (n-k). So when (n-k) is substituted in place of (n-1) in (3.22) and (3.25), total expected repair time under iid and increasing repair time assumptions for k-out-of-n system can be obtained.

For a parallel-series system, it is necessary to add up some assumptions.

7. More than one component in the same subsystem does not fail simultaneously.
8. Failure of a subsystem can be removed by repairing a failed component and others in this subsystem are preventively maintained.

Subsystem j is composed of l components connected in series and each component has exponential failure distribution with rate h (.). Probability that failure of subsystem j due to the failure of component i in subsystem j is $p_{j,i}$.

$$p_{j,i} = \frac{h_i(t)}{\sum_{i=1}^{l} h_i(t)} \tag{3.26}$$

This has been proved by [2]. Component i in subsystem j fails and this failure is rectified by corrective repair and remaining components (l-1) is preventively repaired. If $X_{c,i}$ represents the corrective repair time of component i in subsystem j and $X_{p,j'}$ is the preventive repair time of component j' in subsystem j, the time to complete repair of the subsystem j due to the failure of the component i is:

$$E[X_{j,i}] = E[X_{c,i}] + \sum_{j'=1, j'\neq i}^{l-1} E[X_{p,j'}] \tag{3.27}$$

Time to complete repair of the subsystem j is:

$$E[X_j] = E[X_{j,i}] * p_{j,i} = \sum_{i=1}^{l}\left(E[X_{c,i}] + \sum_{j'=1, j'\neq i}^{l-1} E[X_{p,j'}]\right) * p_{j,i} \tag{3.28}$$

According to assumptions (1) and (6), (3.28) can be simplified into

$$E[X_j] = E[X_{j,i}] * p_{j,i} = E[X_c] + (l-1) * E[X_p] \tag{3.29}$$

If a system is composed of n subsystems and m subsystems out of n fail at the time of inspection, total repair time for the whole system is:

$$\begin{aligned} E[X] &= \sum_{m=0}^{n-1} E[X_m] = \sum_{m=0}^{n-1} E[X_m \mid N_T = m] * P\{N_T = m\} = \sum_{m=0}^{n-1} \left[m * E[X_c] + (n*l-m) * E[X_p]\right] P_m(T) \\ &= \sum_{m=0}^{n-1}\left[m * \frac{1}{\mu_c} + (n*l-m) * \frac{1}{\mu_p}\right] P_m(T) \end{aligned} \tag{3.30}$$

If the system is renewed at N^{th} inspection, total expected repair time under iid repair time assumption is given as follows:

$$E[X_N] = (N-1)*\sum_{m=0}^{n-1}\left[m*\frac{1}{\mu_c} + (n*l-m)*\frac{1}{\mu_p}\right]P_m(T) \tag{3.31}$$

Expected total repair time for a parallel-series system under increasing repair time assumption is:

$$E[X_N] = \frac{a\left(1-\left(\frac{1}{a}\right)^{N-1}\right)}{a-1}\sum_{m=0}^{n-1}\left[m*E[X_c] + (n*l-m)*E[X_p]\right]P_m(T) \tag{3.32}$$

After repair, system cannot operate immediately and it has to wait for some time to be able to operate in regular condition (as described in assumption 4). This waiting time is represented by fixed time interval T_w, and if the whole system is replaced at N^{th} inspection and this action takes time T_G, expected time to system renewal is:

$$E[T*] = N*T + E[X_N] + (N-1)T_w + T_G \tag{3.33}$$

3.3.5 Associated maintenance costs consideration

Maintenance costs are related to repair actions performed on each component at inspection time. Failed components are correctly repaired and non-failed components are preventively repaired at the time of inspection and respective fixed costs c_m and c_p are incurred for these actions. Expected repair cost per inspection cycle given that m components have already failed at the time of inspection is:

$$E[C_{R,m} \mid N_T = m] = m*c_m + (n-m)*c_p \tag{3.34}$$

$$E[C_R] = \sum_{m=0}^{n-1}E[C_{R,m}] = \sum_{m=0}^{n-1}E[C_{R,m} \mid N_T = m]*P\{N_T = m\} = \sum_{m=0}^{n-1}\left(m*c_m + (n-m)*c_p\right)P_m(T) \tag{3.35}$$

In Section 3.3.3, component downtime cost incurred due to failed components, kept in idle until inspection time, is described. The remaining cost is system downtime cost (or) production loss cost. This cost is incurred due to the stoppage of the system in order to do maintenance. When repair costs, component downtime costs and production loss costs are taken into account in considering maintenance costs, expected maintenance cost per inspection cycle can be given as:

$$\begin{aligned} E[C] &= E[C_R] + E[C_d] + c_l * (E[X] + T_w) \\ &= \sum_{m=0}^{n-1} (m * c_m + (n-m) * c_p) P_m(T) + E[C_d] + c_l * (E[X] + T_w) \end{aligned} \tag{3.36}$$

c_l in the last term in (3.36) refers to the production loss (or) system down time cost per unit time. If system is renewed at the time of N^{th} inspection, total expected cost for the whole system is:

$$E[C^*] = (N-1)\left[\sum_{m=0}^{n-1} (m * c_m + (n-m) * c_p) P_m(T)\right] + N * E[C_d] \ + c_l * [E[X_N] + (N-1)T_w + T_G] + C_G \tag{3.37}$$

This cost is considered for a parallel system. If (n-k) is replaced in the place of (n-1) in (3.37), the expected cost for k-out-of-n system is obtained. For a parallel-series system, component i in subsystem j is failed and this failure is rectified by corrective repair and remaining components (l-1) is preventively repaired. Repair cost of subsystem j due to the failure of component i is:

$$c_{j,i} = c_{m,i} + \sum_{j'=1, j' \neq i}^{l-1} c_{p,j'} \tag{3.38}$$

Repair cost due to the failure removal of subsystem j is:

$$c_j = \sum_{i=1}^{l} c_{j,i} * p_{j,i} = \sum_{i=1}^{l} \left(c_{m,i} + \sum_{j'=1, j' \neq i}^{l-1} c_{p,j'} \right) * p_{j,i} \tag{3.39}$$

According to the assumptions (1) and (6), it becomes

$$c_j = c_m + (l-1)c_p \tag{3.40}$$

If a system is composed of n subsystems and m subsystems out of n fail at the time of inspection, total repair cost for the whole system is:

$$\begin{aligned} E[C_R] &= \sum_{m=0}^{n-1} E[C_{R,m}] = \sum_{m=0}^{n-1} E[C_{R,m} \mid N_T = m] * P\{N_T = m\} \\ &= \sum_{m=0}^{n-1} [m * c_m + (n * l - m) * c_p] P_m(T) \end{aligned} \tag{3.41}$$

Expected maintenance cost for parallel-series system is obtained by changing the first part of (3.37).

$$E[C^*] = (N-1) * \sum_{m=0}^{n-1} [m * c_m + (n * l - m) * c_p] P_m(T) + N * E[C_d] + c_l * [E[X_N] + (N-1)T_w + T_G] + C_G \tag{3.42}$$

3.3.6 Optimization of the maintenance policy

A reliability-centered T-based preventive maintenance policy is proposed in Section 3.3. In this section, an optimization approach is given for this policy. An optimization aims at discovering the optimal inspection time and the optimal number of inspections while considering at the same time, maintenance cost, system availability and system reliability. The maintenance problem is formulated in the following Section (3.3.6.1).

3.3.6.1 Problem formulation

$$\underset{N}{\text{Minimize}} \frac{E[C^*]}{E[T^*]} = \frac{(N-1)E[C_R] + N * E[C_d] + c_I[E[X_N] + (N-1)T_w + T_G] + C_G}{N * T + E[X_N] + (N-1)T_w + T_G}$$

$$A(N,T) = \frac{N * T}{N * T + E[X_N] + (N-1)T_w + T} \geq A_0$$

$$N \in \mathbf{Z}^+$$

There are two decision variables (T and N) in the problem. Optimal maintenance policy is decided under different repair time assumptions. The first parameter T is decided under pre-determined reliability constraint. Its respective derivations for each system structure are given in Section 3.3.2. So only one variable N is left to be worked out in the problem and this variable is decided as the one which can minimize expected cost under required availability constraint. Optimization approach to find N under iid and increasing repair time assumptions are given in the next sections.

3.3.6.2 Optimization approach for iid repair time assumption

In this section, an optimization approach to find an optimal number of inspections N is given for iid repair time assumption. Followings are the considerations for a parallel system under iid repair time assumption. Inspection time T is decided under pre-determined reliability constraint. If each component has exponential life time distribution with rate λ and system's critical reliability level is R_c, T can be obtained by solving equation (3.10).

Since repair time is assumed as iid, (3.22) is substituted in place of $E[X_N]$. In order to give a simple approach to finding the number of inspection (N), let

$$A = \left[\sum_{m=0}^{n-1} \left(\frac{m}{\mu_c} + \frac{(n-m)}{\mu_p} \right) P_m(T) + T_w \right] \tag{3.43}$$

$$B = \left[\sum_{m=0}^{n-1} \left(m * c_m + (n-m) * c_p \right) P_m(T) \right] + c_l * \left[\left[\sum_{m=0}^{n-1} \left(\frac{m}{\mu_c} + \frac{(n-m)}{\mu_p} \right) P_m(T) \right] + T_w \right] \tag{3.44}$$

$$C = E[C_d] \tag{3.45}$$

$$D = c_l * T_G + C_G \tag{3.46}$$

$$g(N) = A(N,T) \quad \text{and} \tag{3.47}$$

$$f(N) = \frac{E[C^*]}{E[T^*]} \tag{3.48}$$

From constraint $A(N,T) \geq A_0$, it is clear that system's minimal acceptable availability is A_0 and number of inspection (N) to get A_0 is expressed as N_{th} and it can be obtained by changing the constraint as $A(N,T) = A_0$.

Now the problem becomes,

$$\underset{N}{\text{Minimize}}\ f(N) = \frac{(N-1) * B + (N) * C + D}{N * T + (N-1) * A + T_{\text{G}}} \tag{3.49}$$

$$g(N) = \frac{N * T}{N * T + (N-1) * A + T_{\text{G}}} \geq A_0 \tag{3.50}$$

$$N_{th} = \frac{A - T_G}{T + A - \dfrac{T}{A_0}} \tag{3.51}$$

First derivative of *g(N)* with respect to *N* is $\frac{dg(N)}{dN}$

$$\frac{dg(N)}{dN} = \frac{T_G * T - A * T}{(N * T + (N - 1) * A + T_G)^2} \tag{3.52}$$

$$\frac{d^2 g(N)}{dN^2} = \frac{-2 * T * (T_G - A) * (T + A)}{(N * T + (N - 1) * A + T_G)^3} \tag{3.53}$$

First derivative of objective function *f*(*N*) is

$$\frac{df(N)}{dN} = \frac{-A * C + T_G * B + T_G * C + B * T - T * D - A * D}{(N * T + (N - 1) * A + T_G)^2} \tag{3.54}$$

$$\frac{d^2 f(N)}{dN^2} = \frac{-2 * (C * (T_G - A) + B * (T_G + T) - D * (T + A))}{(N * T + (N - 1) * A + T_G)^3} \tag{3.55}$$

From (3.53) and (3.55), we can quote four possible cases to find optimal solution.

Case 1: $T_G > A$ and $B/D \geq \frac{(T + A)}{(T_G + T)}$

In this case $\frac{d^2 g(N)}{dN^2} < 0$ and $\frac{dg(N)}{dN} > 0$, constraint function $g(N)$ is concave and increasing with respect to *N*. Feasible region for number of inspection *N* is

$$N_{th} \leq N < \infty \tag{3.56}$$

To satisfy the constraint $N > 0$ ($N \in \mathbf{Z}^+$), (3.51) becomes

$$N_{th} = \frac{T_G - A}{\max\left(0, {T}/{A_0} - T - A\right)} \tag{3.57}$$

$\frac{d^2 f(N)}{dN^2} < 0$ and $\frac{df(N)}{dN} > 0$, objective function *f(N)* is also concave and increasing with respect to *N*. Range of expected cost per unit time *f(N)* can be expressed as:

$$f(N_{th}) \le f(N) \le f(\infty) \tag{3.58}$$

From (3.56) and (3.58), an optimal number of inspections (*N*) which will meet availability requirements, and cost minimization as well, is N_{th}. Optimal $N = N_{th}$ can be obtained from (3.57) and by substituting this value in (3.49), minimum expected cost per unit time *f(N)* can be obtained. An optimal maintenance policy is that system is inspected at time *T* and the whole system is renewed at $N = N_{th}$.

Case 2: $T_G > A$ and ${B}/{D} < \frac{(T + A)}{(T_G + T)}$

For this case, $\frac{d^2 g(N)}{dN^2} < 0$ and $\frac{dg(N)}{dN} > 0$. Constraint function $g(N)$ is concave and increasing with respect to *N*. Feasible region for number of inspection *N* is:

$$N_{th} \le N < \infty \tag{3.59}$$

For objective function *f(N)*, check

(i) If $C * (T_G - A) > D * (T + A) - B * (T_G + T)$, $\frac{d^2 f(N)}{dN^2} < 0$ and $\frac{df(N)}{dN} > 0$. *f (N)* is concave and increasing with *N*.

So the optimal solution is the same as Case 1 and the optimal number of inspection is N_{th}.

(ii) If $C*(T_G - A) < D*(T + A) - B*(T_G + T)$, $\dfrac{d^2 f(N)}{dN^2} > 0$ and $\dfrac{df(N)}{dN} < 0$.

f(N) is convex and decreasing with *N*. Range of expected cost now becomes

$$f(\infty) < f(N) \le f(N_{th}) \tag{3.60}$$

According to (3.59) and (3.60), there is no need to do the system renewal for this case. Optimal maintenance policy for this case requires that failed parts are correctly repaired and the remaining are preventively repaired at every inspection time.

Case 3: $T_G < A$ and $B/D \le \dfrac{(T + A)}{(T_G + T)}$

$\dfrac{d^2 g(N)}{dN^2} > 0$ and $\dfrac{dg(N)}{dN} < 0$, constraint function $g(N)$ is convex and decreasing with respect to *N*. Feasible region for number of inspection *N* is

$$0 \le N < N_{th} \tag{3.61}$$

$\dfrac{d^2 f(N)}{dN^2} > 0$ and $\dfrac{df(N)}{dN} < 0$, objective function *f(N)* is also convex and decreasing with respect to *N*. Range of expected cost per unit time can be expressed as

$$f(N_{th}) \le f(N) \le f(0) \tag{3.62}$$

So optimal *N* is N_{th} and N_{th} changes into

$$N_{th} = \frac{A - T_G}{\max\left(0, T + A - \frac{T}{A_0}\right)} \tag{3.63}$$

Case 4: $T_G < A$ and $\frac{B}{D} > \frac{(T + A)}{(T_G + T)}$

$\frac{d^2 g(N)}{dN^2} > 0$ and $\frac{dg(N)}{dN} < 0$, constraint function $g(N)$ is convex and decreasing with respect to N. Feasible region for number of inspection N is same as (3.61).

(i) If $C * (T_G - A) > D * (T + A) - B * (T_G + T)$, $\frac{d^2 f(N)}{dN^2} < 0$ and $\frac{df(N)}{dN} > 0$, *f(N)* is concave and increasing with *N*. Range of expected cost now becomes

$$f(0) < f(N) \leq f(N_{th}) \tag{3.64}$$

According to (3.61) and (3.64), optimal number of inspection to get minimum cost is N=0. It means all components, both failed and non-failed components, are renewed at every inspection time.

(ii) If $C * (T_G - A) < D * (T + A) - B * (T_G + T)$, $\frac{d^2 f(N)}{dN^2} > 0$ and $\frac{df(N)}{dN} < 0$, *f(N)* is convex and decreasing with *N*. Range of expected cost now becomes:

$$f(N_{th}) < f(N) \leq f(0) \tag{3.65}$$

From (3.61) and (3.65), it can be interpreted that optimal N is N_{th} (from 3.63) and minimum cost can be obtained by using (3.49), N_{th} and T.

The following procedure is provided in order to find optimal T and N by using the proposed optimization approach.

Procedure 2: Compute the optimal parameter T and N under iid repair time assumption

Step 1: input: n, R_c, $h(t)$, μ_c, μ_p,c_m,c_p, c_l,C_G, T_G, T_w, A_0

Step 2: find T. (Use (3.10) for exponential life time distribution, Use (3.11) for Weibull distribution, Otherwise T is obtained by equating R_c with (3.5))

Step 3: find $P_m(T)$ from m= 0 to n-1 by using (3.13)

Output: $P_0(T)$ to $P_{n-1}(T)$

Step 4: find A as defined by (3.43) by using output from step 3

Step 5: find B as defined by (3.44)

Step 6: find C given in (3.45) by using (3.19)

Step 7: compare T_G *and* A.

If $T_G > A$, check whether Case 1 or Case 2

If $T_G < A$, check whether Case 3 or Case 4

Satisfied Case is followed and decides optimal N

Step 8: find expected cost per unit time $f(N)$ by using (3.49)

Step 9: Outputs: $f(N)$, N and T

3.3.6.3 Optimization approach for increasing repair time assumption

In this section, repair times are assumed as monotonically increasing repair times, and it is modeled with GP and GP ratio, a, is less than 1. Equation (3.25) is substituted in place of $E[X_N]$. A and B, in (3.43) and (3.44) are changed into:

$$A = \left[\sum_{m=0}^{n-1} \left(m * E[X_c] + (n-m) * E[X_p] \right) P_m(T) \right] \tag{3.66}$$

$$B = \left[\sum_{m=0}^{n-1} \left(m * c_m + (n-m) * c_p \right) P_m(T) + c_l * T_w \right] \tag{3.67}$$

Now optimization problem becomes

$$\underset{N}{\text{Minimize}}\, f(N) = \frac{(N-1) * B + N * C + \dfrac{a\left(1-\left(1/a\right)^{N-1}\right)}{a-1} * A * c_l + D}{N * T + \dfrac{a\left(1-\left(1/a\right)^{N-1}\right)}{a-1} * A + (N-1)T_w + T_G} \tag{3.68}$$

$$g\ (N) = \frac{N * T}{N * T + \dfrac{a\left(1-\left(1/a\right)^{N-1}\right)}{a-1} * A + (N-1)T_w + T_G} \geq A_0 \tag{3.69}$$

Equation of N_{th}, described in (3.51), also changes into

$$N_{th} = \frac{T_w - T_G + \left[\dfrac{a\left(1-\left(1/a\right)^{N-1}\right)}{a-1} A \right]}{T - T/A_0 + T_w} \tag{3.70}$$

Objective function *f(N)* and constraint function *g(N)* under increasing repair time assumption are more complex than *f(N)* and *g(N)* under iid assumption. So it is difficult to express whether they are increasing or decreasing with respect to *N* and it is impossible to give an optimization approach as in (3.3.6.2). Feasible region of *N* can not be seen easily as in (3.3.6.2) and this region is searched by substituting *N* values in (3.69). *N* values which can satisfy the constraint (3.69) fall in feasible region and these *N* values are substituted in (3.68). Optimal *N* is decided as the one which can minimize the expected cost. The solution algorithm under the assumption of increasing repair time is given as follows.

Procedure 3: Compute the optimal parameter T and N under increasing repair time assumption

Step 1: input: n, R_c, $h(t)$, μ_c, μ_p, c_l,c_m,c_p,C_G, T_G, T_w, A_0,a

Step 2: find T. (Use (3.10) for exponential life time distribution, Use (3.11) for Weibull distribution, Otherwise T is obtained by equating R_c with (3.5))

Step 3: find $P_m(T)$ from m= 0 to n-1 by using (3.13)

Output: $P_0\ (T)$ to $P_{n-1}(T)$

Step 4: find A as defined by (3.66) by using outputs from step 3

Step 5: find B as defined by (3.67)

Step 6: find C defined in (3.45) by using (3.19) and D

Step 7: Substitute N values starting from 1 in (3.69) and List N values which can satisfy the constraint (3.69)

Output: feasible region of N

Step 8: find expected cost per unit time $f(N)$ by using (3.69) and N values in feasible region. Choose optimal N which can minimize $f(N)$

Step 9: Outputs: $f(N)$, N and T

3.4 Model formulations for a reliability-centered *m*-based maintenance policy for various system structures

In this policy, maximum number of failed components which makes system reliability is greater than or equal to critical level, is denoted as threshold number of failed components. For example, if reliability at m components failure arrival time is greater than critical reliability (R_c) but reliability at (m+1) components failure arrival time is less than R_c, threshold number of failed components is m. Inspection is made at the time of m components failure arrival time.

3.4.1 System reliability formulations

In the first proposed policy (reliability-centered T-based), the time interval between successive inspections is T and reliability is defined as the probability that a system can operate within this time interval. In second policy (reliability-centered m-based), the system is inspected at m component failure arrival time. Random variable Y_m is denoted as m components failure arrival time and it has life time distribution $F_m(.)$. Its survival distribution function is:

$$P\{Y_m > y\} = \overline{F}_m(y) = \sum_{x=0}^{m-1} \binom{n}{x} (F(y))^x \left(\overline{F}(y)\right)^{n-x} \tag{3.71}$$

The time interval between successive inspections is Y_m and probability that the system can perform a specific function during an inspection time window Y_m is defined as reliability. Reliability for a parallel system during an inspection window, given in (3.5), is redefined as:

$$P(Y_s > Y_m) = R_s(y) = 1 - \prod_{i=1}^{n} (1 - \int_0^{\infty} \overline{F}(y) dF_m(y)) \tag{3.72}$$

For a parallel-series system,

$$P(Y_s > Y_m) = R_s(y) = 1 - \prod_{j=1}^{n} (1 - \prod_{i=1}^{l} \int_0^{\infty} \overline{F}_i(y) dF_m(y)) \tag{3.73}$$

For k-out-of-n system, system reliability is:

$$P(Y_s > Y_m) = R_s(y) = \sum_{i=k}^{n} \binom{n}{i} \left(\int_0^\infty \overline{F}(y) dF_m(y) \right)^i \left(\int_0^\infty F(y) dF_m(y) \right)^{n-i} \tag{3.74}$$

3.4.2 Evaluation of threshold number of failed component (*m*)

Maximum allowable failed components during an inspection time interval is defined as threshold number of failed components, m, and m for a parallel system is obtained by setting $R_s(y) = R_c$. Now (3.72) becomes

$$\int_0^\infty \overline{F}(y) dF_m(y) = 1 - \sqrt[n]{1 - R_c} \tag{3.75}$$

Required m value can not be derived directly from this equation. So the following procedure should be used to find the threshold number of failed components m.

Procedure 4: Compute m for a parallel system

Step 1: set m=1

Step 2: find $\overline{F}_m(y)$ by using (3.71) and then find $F_m(y)$ and $\dfrac{dF_m(y)}{dy}$

Step 3: set LHS = $\int_0^\infty \overline{F}(y) dF_m(y)$, RHS = $1 - \sqrt[n]{1 - R_c}$

Step 4: find LHS

If LHS > RHS

m=m+1 and go to step 2

Else if LHS = RHS

$m_{\text{th}} = m$

Else if LHS < RHS

$m_{\text{th}} = m$-1

Step 5: Output: threshold no of failed components m= m_{th}

The abovementioned procedure is for a parallel system. For a parallel-series system, this procedure can be used except that Equation (3.75) is changed into:

$$\int_0^\infty \overline{F}(y)dF_m(y) = \sqrt[l]{1-\sqrt[n]{1-R_c}} \tag{3.76}$$

For k-out-of-n system, (3.74) becomes

$$R_c = \sum_{i=k}^{n} \binom{n}{i} \left(\int_0^\infty \overline{F}(y)dF_m(y) \right)^i \left(\int_0^\infty F(y)dF_m(y) \right)^{n-i} \tag{3.77}$$

Since maximum allowable number of failed components at the time of inspection is (n-k), m is in the range of 0 and (n-k) at the time of inspection.

$$0 \le m \le n - k$$

Procedure 5: Compute m for k-out-of-n system

Step 1: input: n, k, $h(t)$

Step 2: initialize m=1, LHS = R_c

Step 3: find $\overline{F}_m(y)$ from (3.71) and then find $F_m(y)$ and $\dfrac{dF_m(y)}{dy}$.

Step 4: find $\int_0^\infty \overline{F}(y)dF_m(y)$ and $\int_0^\infty F(y)dF_m(y)$=1-$\int_0^\infty \overline{F}(y)dF_m(y)$

Step 5: find RHS of (3.74) by using output from step 4.

Step 6: if LHS > RHS

m=m+1;

go to step 3;

Else If LHS = RHS

$m_{\mathrm{th}}= m$

Else LHS < RHS

$m_{\mathrm{th}}= m$-1

Step 7: output: threshold number of failed component m = m_{th}

System inspection time is m^{th} components failure arrival time Y_m and required m for each system can be calculated by following the respective procedure, described above. Now component downtime costs due to failed components are considered for a reliability-centered m-based policy.

3.4.3 Component downtime cost consideration

In the first policy, number of failed components at the time of inspection is random variable. In this policy, a system is inspected at m components failure arrival time. As a result, probability of exactly m components failing at the time of inspection Y_m is $P\{N_{Y_m} = m\} = P_m(Y_m) = 1$. Total downtime cost due to exactly m components failure is

$$E[C_d] = c_d * \sum_{i=2}^{m}(i-1) * E[Y_i - Y_{i-1}]$$
$$= \sum_{j=0}^{s=m-1}(s-j) * c_d * \int_0^{\infty} P_{(s-j)}(y)dy \qquad (3.78)$$

(3.78) can be used for parallel, parallel-series and k-out-of-n systems.

3.4.4 Repair time formulation and expected time to system renewal

Threshold number of failed components m can be calculated from Section 3.4.2. Difference from the first policy is that m components have already failed at the time of inspection. So total repair time per system renewal cycle under iid and GP (increasing repair time) assumptions is

$$E[X_N] = (N-1)\left[\frac{m}{\mu_c} + \frac{(n-m)}{\mu_p}\right], \qquad 0 < m \le n-1 \qquad (3.79)$$

and

$$E[X_N] = \frac{a\left(1-\left(\frac{1}{a}\right)^{N-1}\right)}{a-1}\left[m * E[X_c] + (n-m) * E[X_p]\right], \qquad 0 < m \le n-1 \qquad (3.80)$$

System is replaced at N^{th} inspection and expected time to system renewal can be written as:

$$E[T^*] = N * \int_0^{\infty} \overline{F}_m(y)dy + E[X_N] + (N-1)T_w + T_G \qquad (3.81)$$

The range of m values expressed in (3.79) and (3.80) are for a parallel system and it is also the same for a parallel-series system. $E[X_N]$ for a parallel-series system can be obtained by setting $P_m(T)$ =1 and then discarding the summation of m $\left[\sum_{m=0}^{n-1}\right]$, in equations (3.31) and (3.32). For k-out-of-n system, however, (n-k+1) is also needed to substitute in place of n in (3.31) and (3.32).

3.4.5 Maintenance cost formulations

Associated maintenance costs, constituted in an inspection cycle, are described in Section 3.3.5. Expected cost per renewal cycle for an m-based policy is directly defined as follows:

$$\mathrm{E}[C^*] = (N-1)*E[C_R] + (N)E[C_d] \; + c_l * \left(E[X_N] + (N-1)T_w + T_G\right) + C_G \tag{3.82}$$

Expected cost per unit time is obtained by dividing equation (3.82) by (3.81).

3.4.6 Optimization of maintenance policy

In this section, an optimization problem for second policy is described. The main purpose of the problem is the same as the first one. But in this policy, since system inspection time is limited by number of failed components, decision variables are threshold number of failed components (m) and optimal number of inspection (N).

3.4.6.1 Problem formulation

$$\underset{N}{\text{Minimize}}\, f(N) = \frac{(N-1)*E[C_R] + N*E[C_d] \; + \mathrm{c}_1 * \left(E[X_N] + (N-1)T_w + T_G\right) + C_G}{N*\int_0^\infty \overline{F}_m(y)dy + E[X_N] + (N-1)T_w + T_G}$$

$$g(N) = \frac{N*\int_0^\infty \overline{F}_m(y)dy}{N*\int_0^\infty \overline{F}_m(y)dy + E[X_N] + (N-1)T_w + T_G} \geq A_0$$

$$N \in \mathbf{Z}^+$$

3.4.6.2 Optimization approach for iid repair time assumption

Threshold numbers of failed components are limited by critical reliability and required *m* value can be evaluated according to the described procedures in Section 3.4.2. So there is only one variable (*N*) in the problem and following parameters are set in order to clarify the problem.

$$Y = \int_{0}^{\infty} \overline{F}_m(y)dy \tag{3.83}$$

$$A = \left[\frac{m}{\mu_c} + \frac{(n-m)}{\mu_p} \right] + T_w \tag{3.84}$$

$$B = \left(m * c_m + (n-m) * c_p \right) + c_l * \left[\frac{m}{\mu_c} + \frac{(n-m)}{\mu_p} + T_w \right] \tag{3.85}$$

$$C = \sum_{j=0}^{s=m-1} (s-j) * c_d * \int_{0}^{\infty} P_{(s-j)}(y)dy \tag{3.86}$$

$$D = c_l * T_G + C_G \tag{3.87}$$

Objective function $f(N)$ and constraint function $g(N)$ for Policy 2 are the same as (3.50) and (3.51). Four possible cases to find the optimal solutions are also the same as a *T*-based maintained policy described in Section 3.3.6.2. Following procedure can be used to find the decision variables *m* and *N* for a reliability-centered *m*-based policy under iid repair time assumption.

Procedure 6: Compute the optimal parameters, m and N, under iid repair time assumption

Step 1: input: n, R_c, $h(t)$, c_m, c_p, T_w, μ_m, μ_p, C_G, T_G, A_0

Step 2: find threshold number of failed components m by using procedure in (3.4.2)

Step 3: find Y by using (3.83) and threshold number of failed components m

Step 4: find A and B defined by (3.84) and (3.85)

Step 5: Find P_{s-j} (y) can be obtained by using (3.13) and random variable y is substituted in place of T and limitation is 0 to infinity, and then find C expressed (3.86)

Step 6: compare T_G and A.

If $T_G > A$, check whether Case 1 or Case 2

If $T_G < A$, check whether Case 3 or Case 4

Follows satisfied Case and decide optimal N. Y is substituted in place of T.

Step 9: find expected cost per unit time $f(N)$ by using (3.50)

Step 10: Outputs: $f(N)$, N and m.

3.4.6.3 Optimization approach for increasing repair time assumption

Repair time E [X_N], expressed in (3.80) is used for the case of increasing repair time assumption and parameter settings A and B expressed in Section 3.4.5 are changed into

$$A = \left(m * E[X_c] + (n-m) * E[X_p]\right) \tag{3.88}$$

$$B = \left(m * c_m + (n-m) * c_p\right) \tag{3.89}$$

Now optimization problem becomes

$$\underset{N}{\text{Minimize}}\ f(N) = \frac{(N\text{-}1)*B + (N)*C + \dfrac{a\left(1-\left(\frac{1}{a}\right)^{N-1}\right)}{a-1}*A + D}{N*\int_0^\infty \overline{F}_m(y)dy + \dfrac{a\left(1-\left(\frac{1}{a}\right)^{N-1}\right)}{a-1}*A + (N\text{-}1)T_w + T_G} \tag{3.90}$$

$$g(N) = \frac{N*\int_0^\infty \overline{F}_m(y)dy}{N*\int_0^\infty \overline{F}_m(y)dy + \dfrac{a\left(1-\left(\frac{1}{a}\right)^{N-1}\right)}{a-1}*A + (N\text{-}1)T_w + T_G} \geq A_0 \tag{3.91}$$

$$N_{th} = \frac{T_w - T_G + \left[\dfrac{a\left(1-\left(\frac{1}{a}\right)^{N-1}\right)}{a-1}A\right]}{T - \frac{T}{A_0} + T_w} \tag{3.92}$$

As described in Section 3.3.6.3, an optimization approach for Policy 2 under increasing repair time is not given. But feasible region of N and optimal N can be obtained by following the solution algorithm, described in Section 3.3.6.3.

3.5 Case Study

Background

In a purified water production system, there are five production lines and they are running in 24 hours. In each line, normally, rinser, filler, capper and packaging machines are included as the major portions of the production system. If all the machines are in good condition, each production line has 120 bpm (bottles per minute) production rate. If one machine fails, the whole line is needed to stop and there is no production for this line until the time to completion of repair. Repair time depends on the types of failure and if a catastrophic failure occurs, it takes about two or three days to be able to run in normal condition.

With reference to the failure and corrective maintenance reports, the main cause of the stoppage of the production line is due to the failure of capper machine. Therefore, the capper machines in the production system are our machines of interest for giving an effective maintenance program. The cap loader, cap sorter and capping mechanisms are important parts of a capper machine. Caps in the storage tank are pushed up to the cap sorter by using the vibrating motor and blower. Filled bottles are capped with the help of cap sorter and capping mechanism. The main causes of failure of capper machine are listed as follows.

(i) Diverting the alignment of the cap sorter, capper head, and star wheel at bottle incoming stage
(ii) Failure of the spring tension at the capper head in the capping mechanism, and
(iii) Failure of the vibrator mechanism at the cap loader

Failures due to the causes, expressed in (i) can be rectified by adjusting the alignment, adding grease and cleaning the routes. Time taken to perform these repair actions, and also the repair costs are not so high. Therefore, these failures are named as minor failures, and the other failures as catastrophic failures (or) major failures. Each machine has Weibull distributed life time with parameters, λ and α. Apart from the machines' life time, the additional factors are to be considered in seeking an effective maintenance program for the capper machines.

(1) There is enough man power to substitute in place of the failed machine. Advantage of using man power is no need to stop the production line. Disadvantages are (i) decreased production rate and (ii) additional labor costs.

(2) Production rate of the other lines can be increased to meet the required customer demands. Even if four machines are down and man power is used instead of these failed machines, the required customer demand can be met by running the remaining machine at full rate. But, all five machines are down; man power will not be enough to keep the required production rate.

(3) There is economic dependency among the machines.

(4) Maintenance engineers do not want to run the machines with full rate in order to control the machine quality as well as the quality of product.

Depending on facts mentioned above, five capper machines in the production system are assumed as the components which are linked as a parallel system. If one component fails, man power is used in the place of failed component and production rates of others will increase to meet the customer demand. Labor costs incurred due to the failed components are considered as the component downtime costs. Required system reliability is pre-determined and preventive maintenance is performed to the system under reliability constraint. The production system is stopped and the required preventive and corrective maintenances are performed at the time of PM. The aim is to provide an effective preventive group maintenance policy for this system. The required parameters are listed in Table 3.1.

Table 3.1 Case parameters

λ	0.3	c_d	50
α	2	c_m	300
μ_c	30	c_p	150
μ_p	60	c_l	800
R_c	0.9	C_G	2500
A_o	1.0	T_w	1/30
a	0.85	T_G	1/15

3.5.1 Results and discussion

The term "Replacement" in this case study does not mean the replacement of the whole capper machine. It means the replacement of one part of capper machine which can cause the catastrophic failure such as the spring in the capper head. Since repair time and costs, which are needed for rectifying the minimal failures are less compared with PM cost and time, minimal repair action is ignored. Negative effects due to the increasing of production rate (e.g. increasing the failure rate, increasing the defects) might also be considered as an extension of this study, but is not taken into account here.

Firstly, an ordinary m-based group maintenance policy is applied to the system and an optimal maintenance plan is decided to minimize the long run cost per unit time. Then, a reliability-centered m-based maintenance policy in which both pre-determined reliability level and cost are taken into account is applied to the system. This policy (Policy 2) is, actually, the modification of an ordinary m-based maintenance policy. So firstly, new results of this modified policy are compared with those of an ordinary one. Secondly, the reliability-centered T-based policy (Policy 1) is also applied and the results of two policies, Policy 1 and Policy 2, are compared in order to give an effective maintenance policy.

Table 3.2 shows the results of the ordinary m-based and the modified reliability-centered m-based policy under iid and increasing repair time assumptions. Decision variables, m and N, are decided to get minimum maintenance cost under availability constraint. In the ordinary m-based policy, maximum number of failed components is decided on the basis of getting minimum maintenance cost. In proposed reliability-centered m-based policy, numbers of failed components are decided with reliability constraint and optimal N is decided as the one which can minimize the long-run cost rate under availability constraint. Thus, while the system is needed to inspect at 3rd components failure arrival time in a reliability-centered m-based policy, it is inspected at 4th component failure arrival time when the ordinary m-based maintenance policy is applied on this system.

Table 3.2 Results for ordinary m-based and reliability-centered m-based policies

	iid		GP (increasing)		
policy	m	N	m	N (A_0=95%)	N (A_0=90%)
m-based	4	_	4	5	12
reliability-centered m-based	3	5	3	3	10

The results for the two proposed policies such as reliability-centered T-based (Policy 1) and reliability-centered m-based policy (Policy 2) are listed in Table 3.3. Under iid repair time assumption, both policies are consistent with Case 3. Both the objective and constraint functions are convex, decreasing with N and system is renewed at N^{th} inspection to get minimum maintenance cost.

Table 3.3 Results for reliability-centered T- based and m-based policies

	reliability-centered T-based		reliability-centered m-based	
repair time	T	N	m	N
iid	3.33	_	3	5
GP (increasing), A_0=95%	3.33	5	3	3
GP (increasing), A_0=90%	3.33	13	3	10

For Policy 1, fixed inspection time T is 3.33 and system availability is greater than the limited availability at every inspection time, as can be seen in Figure 3.1. Since expected cost rate is also decreasing with N (Figure 3.2), optimal maintenance decision for this policy is that there is no need to do system renewal and that the system can be maintained by performing the respective repair actions for failed and non-failed components at every inspection time (T).

For Policy 2, inspection is made at 3rd component failure arrival time. At that time, failed components are corrective repaired and the remaining are preventively repaired. As shown in Figure 3.3, availability is less than A_0 starting from N=5.5. So the system is renewed at 5th inspection time. These results are obtained by assuming that repair times are iid repair times. Under this assumption, at every N value, Policy 1 is more preferable to Policy 2 in terms of cost and system availability.

Under increasing repair time assumption, an algorithm to find an optimal N is described in Section 3.3.6.3. An optimal N which can give minimum maintenance cost for each policy is chosen. Relevant optimal parameters for each policy are listed in Table 3.3 for the purpose of comparing the proposed two policies. Maintenance cost in Policy 2 is 550. Similar to iid repair time assumption, Policy 1 is better than Policy 2 under increasing repair time assumption. Again, the acceptable minimum availability is reduced to (0.9) in order to compare these two policies in long-term. From Table 3.4, it can be seen that Policy1 is better than Policy 2.

Table 3.4. Optimal N and $f(N)$ for both policies under increasing repair time assumption

	A_0=95%		A_0=90%	
policy	N	$f(N)$	N	$f(N)$
reliability-centered T-based	5	474	13	443
reliability-centered m-based	3	550	10	493

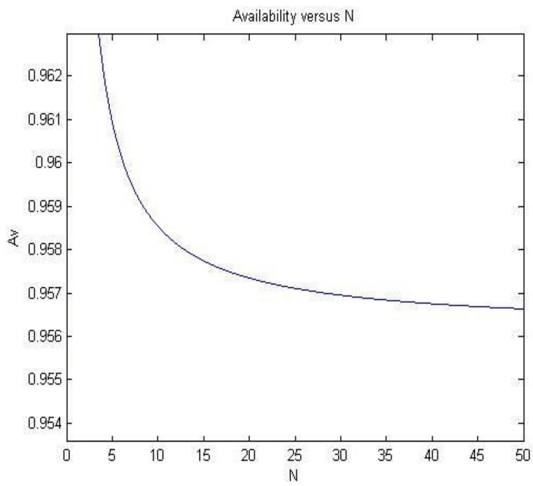

Figure 3.1 Availability versus no of inspection for Policy 1 under iid assumption

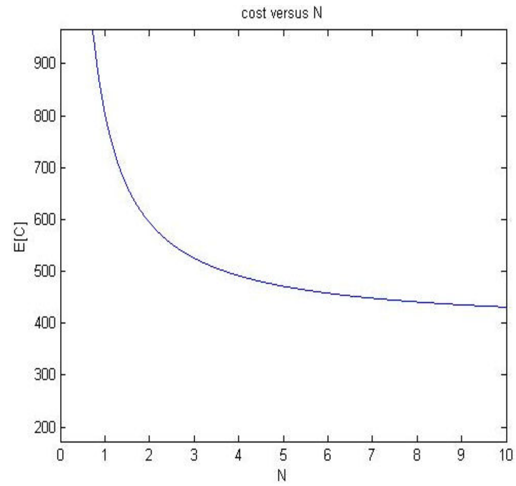

Figure 3.2 Cost versus no of inspection for Policy 1 under iid assumption

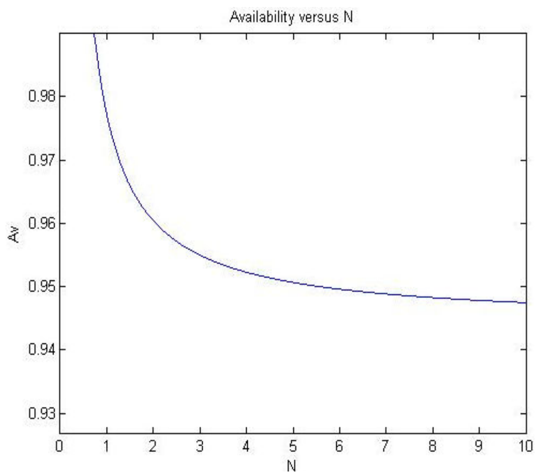

Figure 3.3 Availability versus no. of inspection for Policy 2 under iid assumption

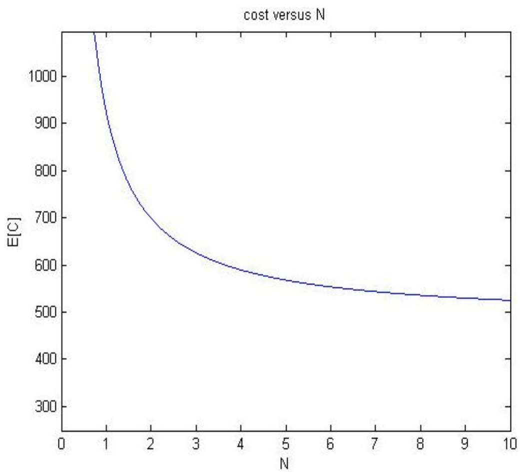

Figure 3.4 Cost versus no. of inspection for Policy 2 under iid assumption

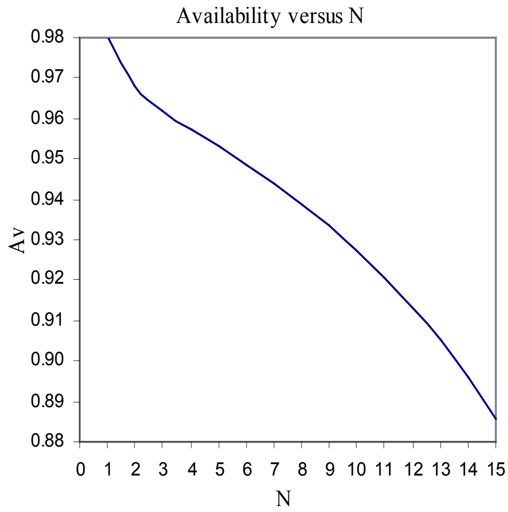

Figure 3.5 Availability versus no of inspection for Policy 1 under increasing repair time assumption

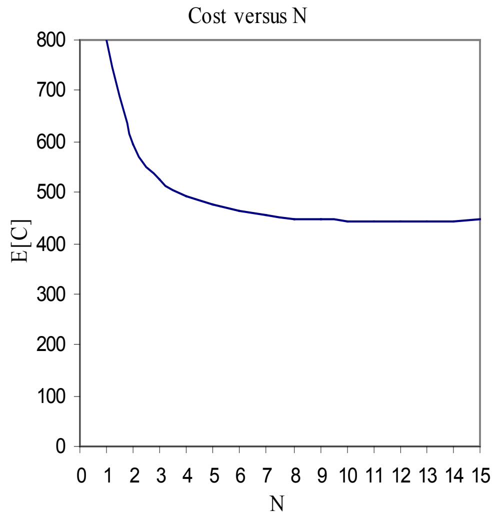

Figure 3.6 Cost versus no of inspection for Policy 1 under increasing repair assumption

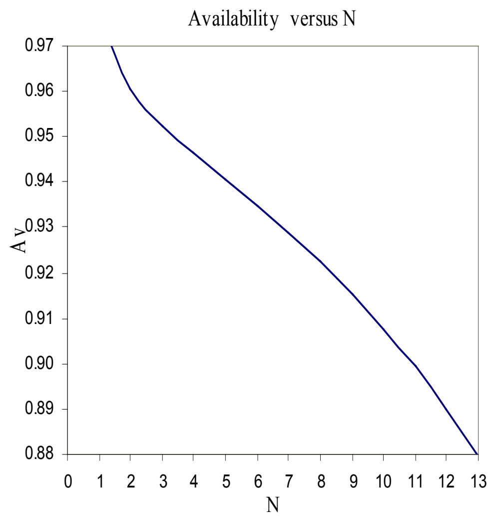

Figure 3.7 Availability versus no of inspection for Policy 2 under increasing repair time assumption

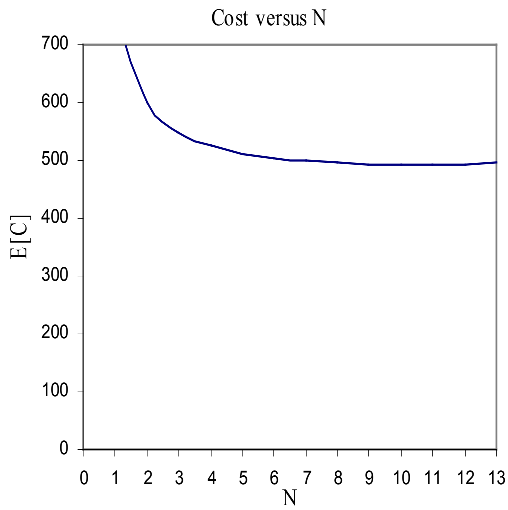

Figure 3.8 Cost versus no of inspection for Policy 2 under increasing repair assumption

Therefore, it can be concluded that reliability-centered T-based maintenance policy is better than m-based policy in both iid and increasing repair time assumptions. The reason is that inspection time in Policy 2 is less than that in Policy 1 and maintenance cost in Policy 2 is greater than Policy 1. The rule of assigning the inspection time in Policy 2 is that if system reliability at m components failure arrival time is greater than the critical level, but system reliability at (m+1) components failure arrival time is lower than R_c, system is inspected at m^{th} component failure arrival time. So, it can be said that PM is taken in advance before the arrival time of critical reliability of Policy 2. Thus, operation time (time interval between consecutive inspections) in Policy 2 is also less than that in Policy 1. As a consequence, total maintenance cost per unit time in Policy 2 is greater than that of Policy 1 under the assumptions of both iid and increasing repair times. Therefore, it can be concluded that under iid repair times or increasing repair times, reliability centered T- based maintenance policy is more preferable than m-based policy in both costs and availability. This is proven by applying both policies to the case study and associated solutions are given in Section 3.5.1. It is evident that if PM time is decided under reliability constraint, there will be no effectiveness in doing PM in advance (before arrival of the assigned critical level) for structural dependence components although degradation costs, production loss costs and maintenance costs are taken into account.

Chapter 4

Deciding suitable maintenance action based on maintenance degrees

4.1 Introduction

According to the literature reviews presented in Chapter 2, it is clear that components in a system are not always iid. When these components fail, suitable corrective measure, on a basis of amount of damages, repair cost, repair time and remaining life time, has to be decided. Related papers are highlighted in Chapter 2. The main purpose of this chapter is to present a model to determine suitable maintenance action with a basis on the target and available maintenance degrees that were under budget and reliability constraints.

4.2 Statement of the problem

For a multi-component system which has economic dependency among the components, preventive maintenance is performed on the entire group of components at the same time which is more economical than individual maintenance. If the components have different hazard rates, there may be some problems at the time of preventive maintenance such as some are still operable and some have already failed and idle. To retain the desirable operating condition and minimize the expected long-run cost rate, it is essential to determine when and how to perform maintenance actions, such as repair or replacement. Dohi, Kaio and Osaki [11] proposed a "repair cost limit" policy in order to decide suitable maintenance action on the basis of repair cost. Similar to this policy, suitable maintenance action, is decided on the basis of repair time, and this "repair time limit" policy is highlighted by Dohi, Takeita, and Osaki [12]. Wang and Tsai [32] suggested three types of maintenance action to be considered at the time of failure. A proper maintenance action is decided by comparing benefit and cost ratios resulting from each type of maintenance. Our purpose is to give the suitable maintenance action for each component at each PM time, considering the target and available maintenance degrees.

The maintenance degree is represented by the amount of age reduction of the system/component after maintenance [36]. In this chapter, maintenance degree is classified into two such as target and available maintenance degrees. Target maintenance degree is the required amount of age reduction which can meet the required target/condition. Available maintenance degree is the amount of age reduction which can be actually obtained in practice when components are actually maintained. Suppose that all components are replaced at time NT and reliability of each component must be greater than or equal to minimal acceptable reliability at that time. This is the required target /condition. If component j has to be repaired with maintenance degree (x_j) at each PM to meet this requirement, maintenance degree x_j is known as the target maintenance degree. But when component j is actually maintained, it is impossible to get the required target amount exactly. The available reduction amount at the time of repair is estimated with a basis on historical experiences and this reduction amount (y_j) is called the available maintenance degree of component j. Suitable maintenance action of component j is decided with a basis on these target and available maintenance degrees.

In this chapter, two cases are considered for different desirable operation conditions. Case 1 is that maintenance cost of each component at each PM time is kept not beyond the available maintenance budget. The objective is to find the optimal preventive maintenance time interval (T), number of PM (N) and suitable maintenance action for each component so that the system's reliability is maximized within the budget frame. Case 2 is to find optimal T, N and suitable maintenance action for each component in order to minimize the maintenance cost under acceptable reliability.

The models, required for proposing two cases are given in Section 4.3. Optimization problems for two cases are expressed in Section 4.4 and a numerical example, results and discussions are added in Section 4.5. The following notations and assumptions are used throughout the whole chapter.

Notations:

X_i	repair time at i^{th} PM
R_c	critical reliability
$h_i(t)$	failure rate after $(i\text{-}1)^{th}$ PM
$h_{i,j}(t)$	failure rate of component j after $(i\text{-}1)^{th}$ PM
x_i	amount of age reduction at i^{th} PM
$x_{i,j}$	amount of age reduction of component j at i^{th} PM
δ, μ	parameters of piecewise exponential distribution
y_j	available reduction amount (or) maintenance degree of component j
$y_{N-1} = \sum_{i=1}^{N-1} x_i$	cumulative amount of age reduction after $(N\text{-}1)^{th}$ PM

$G_j(y)$	cdf of y_j
t_{min}	minimal repair time
T_R	replacement time
T_G	group replacement time
N	number of PM
T	periodic maintenance time interval
$f_j(.)$	pdf of life time distribution of component j with parameters η and β
$E[X_j]$	mean repair time of component j
$E\lfloor X_{D,i,j} \rfloor$	downtime of component j at i^{th} PM
B_G	maintenance budget for all components at each PM
$b_{j,l}$	lower limit of available maintenance budget of component $_j$
$b_{j,u}$	upper limit of available maintenance budget of component j
$l(x_j,i)$	PM cost function varies with maintenance degree x_j at i^{th} PM
C_G	group replacement cost
c_{min}	minimal repair cost
p_j	budget percentage of component j

Assumptions:

1. Components are non-iid and hazard rates of the components are increasing with time t.
2. Group replacement cost is cheaper than cost of replacing units separately.
3. All units are repaired or replaced at the same time. Although repair times are different depending on different maintenance degrees, all units will start work at the same time.
4. Since units are not iid, their hazard rates are not the same. So, the assigned PM costs are not equally distributed on each unit. Available PM budget for each component depends on its hazard rate, repair and replacement costs.
5. Available budget amount at each PM is the same and constant.

4.3 Model formulations

4.3.1 Maintenance cost formulating

Let $h(t)$ be the hazard rate of a component. At each PM, the component is maintained with maintenance degree x and after $(N\text{-}1)^{\text{th}}$ PM, its hazard rate becomes $h_N(t)$.

$$h_N(t) = h(t - y_{N-1}) = h(t - \sum_{i=1}^{N-1} x_i) \tag{4.1}$$

x_i in (4.1) represents the required target reduction amount at i^{th} PM. If it is assumed that target reduction amount at each PM are the same, y_{N-1} becomes $(N-1)*x$. Now (4.1) is changed into:

$$h_N(t) = h(t - (N-1)*x) \tag{4.2}$$

The time interval between successive PMs is denoted as T and failures before T are rectified by minimal repair. If it is assumed that component j is replaced with new one at N^{th} PM, total expected number of minimal repairs up to system renewal time (NT) can be derived as follows.

$$\sum_{i=1}^{N}\int_{0}^{T} h_j((i-1)*T+u-(i-1)*x_j)du \tag{4.3}$$

This is the total number of minimal repairs per system renewal cycle. Fixed cost c_{min} is paid for the removal of failures with minimal repair action and total minimal repair cost is given by

$$c_{\min}\left[\sum_{i=1}^{N}\int_{0}^{T} h_j((i-1)*T+u-(i-1)*x_j)du\right] \tag{4.4}$$

At each PM time before system renewal, all components are maintained so that their hazard rates are reduced by some amounts. Commonly this maintenance action is between the two extremes such as good as new and bad as old. Such PM action is named as imperfect PM. Respective imperfect repair cost is incurred whenever PM action is performed. Imperfect repair cost is named as PM cost and this cost increases with the number of PM and amount of age reduction. Suppose $l(x_j, i)$ represents the imperfect repair cost function (or) PM cost function of component j at i^{th} PM and this cost depends on the number of PM and maintenance degree at each PM. Total PM cost after $(N\text{-}1)^{th}$ PM is:

$$\sum_{i=1}^{N-1} l(x_j, i) \tag{4.5}$$

4.3.2 Consideration of the budget percentage of component *j*

Suppose $B_{G,l}$ and $B_{G,u}$ are the lower and upper limits of the total available maintenance budget for all components at each PM. These amounts are not equally distributed to all components because components are non-iid and their failure rates and maintenance costs are not the same. Now, the available budget percentage of component *j* is considered. Let $p_{c,j}$ be the available budget percentage of component *j*. It is decided solely on the basis of its maintenance cost and thus derived as;

$$p_{c,j} = \frac{c_{\min j} + c_{Rj}}{\sum_{j=1}^{n} \left[c_{\min j} + c_{Rj} \right]} \tag{4.6}$$

It indicates that the budget percentage of component *j* varies directly to its maintenance cost. Thus, a component which has the higher maintenance cost can get a higher maintenance budget. Let $p_{f,j}$ be the available budget percentage of component *j*. If it is decided solely on the basis o f its probability of failure, it becomes

$$p_{f,j} = \frac{F_j(T)}{\sum_{j=1}^{n} \left[F_j(T) \right]} \tag{4.7}$$

If component *j* is a reliable component, it has less failure probability. As a consequence, it's maintenance budget will be lesser compared to others if (4.7) is used for its budget percentage. Suppose component *j* is a reliable component which has high maintenance cost. The available budget percentage of component *j* is high according to (4.6) and is low according to (4.7). Therefore, (4.6) and (4.7) are contradictory to each other for such component. It points out that the budget percentage of a component should not be decided based on only it's maintenance costs (or) probability of failure. Let p_j be the available budget percentage of component *j* and it is decided based on both its maintenance costs and probability. It is decided as the average value of $p_{c,j}$ and $p_{f,j}$. p_j becomes

$$p_j = \frac{\sqrt{p_{c,j} * p_{f,j}}}{\sum_{j=1}^{n} \sqrt{p_{c,j} * p_{f,j}}} \tag{4.8}$$

Now, $B_{G,l}$ and $B_{G,u}$ can be distributed to all components in terms of p_j. The lower limit of available maintenance budget of component *j* in terms of p_j is

$$b_{j,l} = p_j * B_{G,l} \tag{4.9}$$

and the upper limit is

$$b_{j,u} = p_j * B_{G,u}. \tag{4.10}$$

Available total maintenance budget, either $B_{G,l}$ or $B_{G,u}$, is distributed to all components. Thus,

$$\sum_{j=1}^{n} p_j = 1 \quad , \quad \sum_{j=1}^{n} b_{j,l} = B_{G,l} \text{ and } \sum_{j=1}^{n} b_{j,u} = B_{G,u}$$

4.3.3 Reliability consideration

Suppose that Y_j be the random life time of component *j* and it has distribution function $F_j(.)$ and hazard rate $h_j(.)$. Probability that component *j* does not fail within PM time interval *T* is:

$$\begin{aligned} P\{Y_j > T\} &= R_j(T) = \exp(-H_j(T)) \\ &= \exp\left(-\int_0^T h_j(u)du\right) \end{aligned} \tag{4.11}$$

Component j is imperfectly repaired at each PM and current age is reduced to x_j amount after repair. If reduction amounts at each PM time $\{x_1, x_2, ..., x_{N-1}\}$ are assumed to be identical, total reduction amount of component j after $(N$-$1)^{th}$ PM becomes $(N$-$1)x_j$. Survival distribution function of component j within PM time interval T after $(N$-$1)^{th}$ PM is:

$$R_{N-1,j}(T) = P\{Y_j > T \setminus Y_j > (N-1)*(T - x_j)\} = \exp\left(-\int_0^T h_j((N-1)*(T - x_j) + u)du\right) \quad (4.12)$$

4.3.4 Consideration of the suitable maintenance action based on maintenance degrees

Before presenting how to decide the suitable maintenance action based on maintenance degrees, lower and upper limits of available maintenance budget are to be defined first.

A certain percentage of group maintenance cost, qC_G, is defined as the lower limit of available maintenance budget, $B_{G,l}$. The value of q is assumed as the known and fixed value.

Group maintenance cost, C_G, is defined as the upper limit of available maintenance budget, $B_{G,u}$.

In Case 1, available maintenance budgets, $B_{G,l}$ and $B_{G,u}$, which will have to use at the time of PM, are pre-determined. These amounts are distributed to all components according to (4.9) and (4.10). The summation of minimal repair costs between successive PMs and preventive maintenance cost at PM is named as the maintenance cost. If the maintenance cost is greater than the lower limit of maintenance budget, the decision to make group replacement is more worthwhile than repair. Thus, if maintenance cost of component *j*, obtained by the summation of minimal repair costs between $(N\text{-}1)^{th}$ and $(N)^{th}$ PMs and PM cost at N^{th} PM, is greater than $b_{j,l}$, component *j* is renewed at N^{th} PM. Although total maintenance cost at N^{th} PM can be greater than $b_{j,l}$, it does not allow for overcoming the upper limit of maintenance budget, $b_{j,u}$. Therefore, the maintenance cost of component *j* at N^{th} PM is limited as follows.

$$b_{j,l} < c_{\min j}\int_0^T h_j((N-1)*\left(T-x_j\right)+u)du + l(x_j, N) \le b_{j,u} \tag{4.13}$$

If the maintenance cost of component *j* at N^{th} PM satisfies the constraint, given in (4.13), maintenance decision of component *j* is to repair at $(N\text{-}i)^{th}$ PM, where i=1,2,…,N-1, and to replace with new one at N^{th} PM. x_j in (4.13) refers to the required amount of age reduction(maintenance degree) of component *j* at the time of repair. The inequality signs in (4.13) are changed into equal sign and x_j is also changed into $x_{j,l}$ and $x_{j,u}$ respectively. (4.13) becomes

$$c_{\min j}\int_0^T h_j((N-1)*\left(T-x_{j,l}\right)+u)du + l(x_{j,l}, N) = b_{j,l} \tag{4.14}$$

$$c_{\min j}\int_0^T h_j((N-1)*\left(T-x_{j,u}\right)+u)du + l(x_{j,u}, N) = b_{j,u} \tag{4.15}$$

$x_{j,l}$ which can satisfy the described equation (4.14) at a given *N* and *T*, is the lower limit of target maintenance degree of component *j* at a given *N* and *T*. However, this amount is not allowed to be negative value. So it can be expressed as:

$$x'_{j,l} = \max(0, x_{j,l}) \tag{4.16}$$

$x_{j,u}$ which can satisfy the described equation (4.15) at a given N and T, can be defined as the upper limit of target maintenance degree of component j at a given N, T pair. This upper limit is not allowed to be greater than T because of imperfect repair action. Thus, this value can be redefined as:

$$x'_{j,u} = \min(T, x_{j,u}) \tag{4.17}$$

Traditionally, lower limit is always less than the upper limit. Therefore,

$$x'_{j,l} < x'_{j,u} \tag{4.18}$$

Which maintenance action should be performed for each component at each PM is decided by comparing the target and available maintenance degrees. If available maintenance degree is between the upper and lower limits of target maintenance degrees, equation (4.13) will be satisfied and maintenance decision is to repair at $(N\text{-}i)^{th}$ (i=1,2,…,N-1) PM and to replace with new one at N^{th} PM. If available maintenance degree is greater than the upper limit, the maintenance cost due to repair action is greater than the available maintenance budget. The possible maintenance decision for such case which can satisfy (4.31), might be "do nothing". However, "do nothing" case is not considered in this chapter. So, the case in which $y_j > x'_{j,u}$ is discarded. If available maintenance degree is less than the lower limit, repair cost is less than the lower limit of available budget. The possible maintenance decision to meet the required condition (4.13) is to replace with new one.

Let $I(N,T,y_j)$ be an indicator function, which is a function of *N*, *T* and available reduction amount of component j, y_j. $I(N,T,y_j)=1$ represents that maintenance decision of component *j* is imperfectly repaired. $I(N,T,y_j)=0$ is for the replacement decision. A mathematical expression of an indicator function is

$$I(N,T,y_j)=\begin{cases}1\ , & \text{if } x'_{j,l} \le y_j \le x'_{j,u}\\ 0\ , & y_j < x'_{j,l}\end{cases} \tag{4.19}$$

Case 2 is aimed to maintain the components' reliability. If $R_{N-1,j}(T)$ of a component, defined in Section 4.3.3, is less than or equal to critical reliability R_c, this component is replaced at time *NT*. Although $R_{N-1,j}(T)$ can be less than R_c, this probability should not be zero. Therefore, minimum acceptable reliability is assigned. If $R_{N-1,j}(T)$ is between the critical and acceptable reliability levels, the next PM time, *NT*, is denoted as the renewal time of component *j*. Thus, the renewal time of component *j* is limited by giving the following reliability constraint.

$$R_a < R_{N-1,j}(T) \le R_c \tag{4.20}$$

$$R_a < \exp\left(-\int_0^T h_j((N-1)*(T-x_j)+u)du\right) \le R_c \tag{4.21}$$

In Case 2, the value of x_j which can get minimal acceptable reliability, R_a , at a given *N* and *T* is the lower limit of target maintenance degree of component *j*, $x_{j,l}$. The value of x_j which can get critical reliability, R_c , at a given *N* and *T* is the upper limit of target maintenance degree of component *j*, $x_{j,u}$. The definition of indicator function, expressed in (4.19) can be used for Case 2.

4.3.5 Expected cost per renewal cycle

In Section 4.3.4, the maintenance decision, either repair or replace is to be made. If maintenance decision of component *j* is to repair, it is repaired at the time of PM. Its actual maintenance degree at the time of repair is y_j, not $x'_{j,l}$ nor $x'_{j,u}$ because y_j is the available maintenance degree at the time of repair and $x'_{j,l}$ and $x'_{j,u}$ are the required maintenance degrees. Therefore, the repair time and repair cost of component *j* is concerned with only the available maintenance degree y_j. If maintenance decision is to repair, the repair cost $l(y_j, i)$ is incurred for this action and replacement cost $c_{R,j}$ is paid for the replacement action. PM cost of component *j* at *i*th PM is

$$I(N,T,y_j)*l(y_j,i)+\left(1\text{-}I(N,T,y_j)\right)*c_{R,j} \tag{4.22}$$

System is composed of *n* components and PM cost of the whole system is given by

$$\sum_{i=1}^{N-1}\left\{\sum_{j=1}^{n}\left[I(N,T,y_j)*l(y_j,i)+\left(1\text{-}I(N,T,y_j)\right)*c_{R,j}\ \right]\right\} \tag{4.23}$$

The time interval between successive group replacements is defined as the system renewal cycle and maintenance costs included in a renewal cycle are total minimal repair costs, total PM costs up to system renewal time and group replacement cost. $E[C^*]$ is denoted as the total expected cost per renewal cycle and it can be expressed as:

$$E[C^*]\ =\sum_{j=1}^{n}c_{\min j}\sum_{i=1}^{N}\int_0^T h_j\left((i-1)*(T-y_j)+u\right)du+\sum_{i=1}^{N-1}\left\{\sum_{j=1}^{n}\left[I(N,T,y_j)*l(y_j,i)+\left(1\text{-}I(N,T,y_j)\right)*c_{R,j}\right]+C_G\right\} \tag{4.24}$$

4.3.6 Reliability at the time of system renewal

Suppose the maintenance decision of component j is to repair at $(N\text{-}i)^{\text{th}}$ PM and to renew at N^{th} PM. Although component j is renewed at NT, its actual age is $(NT\text{-}(N\text{-}1)^{*}y_j)$ due to imperfect repair action (the reason, why y_j is used has explained in Section 4.3.5). So reliability within a renewal cycle or reliability at the time of NT is

$$\mathrm{P}\{Y_j > (NT-(N-1)y_j\} = R_j(NT) = \exp\left(- \int_0^{NT-(N-1)y_j} h_j(u)du \right) \tag{4.25}$$

$R_s(NT)$ is denoted as the total reliability of all components at the time of system renewal and expressed in terms of indicator function as follows.

$$R_s(NT) = \prod_{j=1}^{n} I(N,T,y_j)R_j(NT) + (1-I(N,T,y_j))R_j(T) \tag{4.26}$$

4.3.7 Repair time formulation and expected system renewal time

Mostly, repair times are assumed as iid repair times that follow with exponential distribution. But in the real systems, constant or repeated repairs are impossible. Repair times are independent but not identical. So the assumption of iid repair times is relaxed. If $\{X_1, X_2, ..., X_n\}$ represents i^{th} repair time and they are independent but not identical, piecewise exponential distribution is applied to model such repair times and expected repair time for per unit reduction at i^{th} PM, X_i, is exponential with a mean of

$$E[\ X_{\mathrm{i}}\ ] = \frac{1}{\mu}\delta(i)^{\delta-1} \tag{4.27}$$

When $\delta=1$, repair times have the iid exponential case because the expectation of X_i is equal to μ which is independent of *i*. When $\delta > 1$, then $E[X_i]$ is an increasing function of *i*; this corresponds to increasing repair time from one PM to another. Expected repair time of component *j* for y_j unit reduction is

$$E[X_{i,j}] = \frac{1}{\mu}(y_j)\delta(i)^{\delta-1} \tag{4.28}$$

In this equation, $\delta > 1$ and repair time increases with *i* and reduction amount *x* from one PM time to another. If component *j* is repaired at i^{th} PM, then (4.28) is used as the repair time of component *j*. However, if component *j* is needed to replace with new one, it takes $T_{R,j}$ for the replacement of component *j*. In this case, it is assumed that required spares are available at the time of failure and there is no need to take lead time to order these spares. Obviously, maintenance time also depends on maintenance decision. If replacement time of component *j* is $T_{R,j}$ and imperfect repair down time of component *j* at i^{th} PM is $E[X_{i,j}]$, and downtime of component *j* at i^{th} PM can be written as:

$$E\lfloor X_{D,i,j} \rfloor = I(N,T,y_j) * E[X_{i,j}] + (1 - I(N,T,y_j)) * T_{R,j} \quad ,j=1, 2,\ldots, n \tag{4.29}$$

As described in assumption (3), all components are preventively maintained at the same time in order to reduce the system downtime and to restart the whole system after repair soon. If down time of component *j* is the longest of *n* components; all components will operate again after repairing of component *j*. Downtime of the system at i^{th} PM is

$$\text{Max}\,(E\lfloor X_{D,i,j} \rfloor), \qquad j=1, 2, 3,..,n \tag{4.30}$$

All are replaced at N^{th} PM and it takes $T_{G.}$ Expected system renewal time can be defined as follows

$$E\,[T^*] = NT + \sum_{i=1}^{N-1} \text{Max}\,(E\left[X_{D,i,j}\right]) + T_G \tag{4.31}$$

Per unit expected cost is $\dfrac{E[C^*]}{E\,[T^*]}$.

After formulating the required functions, we are ready to present the maintenance problem. The next section describes the maintenance problems and the respective optimization approaches under budget cost/ reliability constraint.

4.4 Optimization of the maintenance policy

4.4.1 Problem formulation under maintenance budget cost constraints

From equation (4.13), it can be seen that maintenance cost increases with T, N and maintenance degree x. When available maintenance budget is of fixed value and either T or N increases, maintenance degree decreases to cover the available maintenance budget. So if the time interval between successive PMs is short, the reduction amount to meet the required target maintenance budget is large (approaches to T). Lower and upper limits of target maintenance degrees are close to T. As a result, the chance of component replacement is greater than that of repair because the chance of catching the available maintenance degree within the range of target maintenance degrees is less. This result indicates that the shorter the time interval between successive PMs, the greater the chance of replacement is. Even for replacements, PM time interval T or system renewal time should be long enough in order to do so. For this reason, system availability is used as a constraint in order to control T and N. System availability is

$$Av(N,T)=\frac{NT-\sum_{j=1}^{n} t_{\min j}\int_{0}^{T} h_j\left((N-1)*\left(T-y_j\right)+u\right)du}{NT+\sum_{i=1}^{N-1}\text{Max }\left(E\left[X_{D,i,j}\right]\right)+T_G} \tag{4.32}$$

Optimal PM time interval and numbers of PM for each component with suitable maintenance action are determined under limited maintenance budget with the objective of achieving maximum reliability. Since the "do nothing" approach is not considered here, the case in which available maintenance degree is greater than the upper limit of target maintenance degree ($y_j > x'_{j,u}$) is discarded. In other words, N and T pairs which can give available maintenance degree is greater than the upper limit of target maintenance degrees, are not considered in deciding optimal values. Maintenance problem under budget cost constraint is

Objective: $\underset{N,T}{\max imize}\ \ R_s(NT)$ (4.33)

Constraints: $x'_{j,l} < x'_{j,u}$ (4.34)

$$b_{j,l} < c_{\min j}\int_{0}^{T} h_j((N-1)*\left(T-x_j\right)+u)du+l(x_j,N)\leq b_{j,u} \tag{4.35}$$

$$Av\left(N,T\right)\geq A_0 \tag{4.36}$$

Second constraint is used to decide replacement time, numbers of PM and respective target reduction amounts. Pairs of N, T and target reduction amounts obtained from second constraint are filtered by the first constraint. A suitable maintenance decision is decided by using (4.19) and N,T pairs are filtered again by using availability constraint (4.36). Respective reliability for each component is evaluated and optimal solution (N, T) is obtained by choosing one with maximum reliability. A solution algorithm is described in detail as follows;

4.4.1.1 Solution algorithm

In this section, a solution algorithm is given for the purpose of finding T, N and suitable maintenance action which can keep the system in the desirable operation condition. Before presenting the detailed approach, the sets which are necessary for finding optimal solutions are defined first.

- $\boldsymbol{S_j}(N,\ T,\ x'_{j,l}, x'_{j,u})$ is the set of N, T and associated lower and upper limits of reduction amount of component j .
- $\boldsymbol{S}(N,\ T,\ x'_{1,l}, x'_{1,u}, \ldots,\ x'_{n,l}, x'_{n,u})$ is the set of organizing the associated lower and upper limits of target reduction amounts of all components under the same N, T pairs.
- $\boldsymbol{S_d}$ (N, T) is the set of N, T and suitable maintenance action for each component.
- $\boldsymbol{S_{Av}}$ (N, T) is the set of N, T and suitable maintenance action for each component. All the elements in $\boldsymbol{S_{Av}}$ satisfy the constraint (4.36).

Thus, the detailed expression of an optimization algorithm is as follows;

Step 1: input: $h_1(t), h_2(t), \ldots, h_n(t), c_{min,1}, c_{min,2}, \ldots, c_{min,n}, B_{G,L}, B_{G,U}$

Step 2: find $p_j (j=1, \ldots, n)$

When p_j values are evaluated in **Step 2**, there is unknown variable T in p_j equation. T is the time interval between successive PMs and also a decision variable. Thus, it is impossible to insert exact T value when p_j values are calculated. In order to find p_j values, desired PM period is determined first. It means if it is desired to do PM within one month, PM time T will be within this duration ($0 < T \leq 1$). p_j values are decided by varying T values within these limitations. The following procedure is proposed in order to find p_j values.

Procedure 1: Calculating p_j values

Step 1: Set desired PM period, T_{start}, T_{end} and T_{int}

Step 2: Set $T = T_{\text{start}}$ and i=1

Step 3: Find p_j by using (4.8)

If $\sum_{j=1}^{n} p_j = 1$, $\boldsymbol{Pi}$ ={p_j, j=1,…,n}, i=i+1

Else i=i

Step 4: T=T+T_{int}

if T<=T_{end}, go to Step 3

Step 4: Output: $\boldsymbol{Pi'}$, (i' =1,..,i)

Step 3: set i' =1

Step 4: find $b_{j,L}$ and $b_{j,U}$ (j=1,…,n) by using $p_j \in Pi'$, equations (4.9) and (4.10)

Step 5: find $\boldsymbol{S_j}$

The following **Procedure 2** is aimed to find $\boldsymbol{S_j}$

Procedure 2: Forming a set, $\boldsymbol{S_j}$

Step 1: set N=1

Step 2: set $T= T_{\text{start}}$

Step 3: find $x_{j,l}$ which are satisfied (4.14)

Step 4: find $x'_{j,l}$ by using (4.16)

Step 5: find $x_{j,u}$ which are satisfied (4.15)

Step 6: find $x'_{j,u}$ by using (4.17)

Step 7: If (4.18) is satisfied, Output: N,T and $x'_{j,l}$ and $x'_{j,u}$, list in $\boldsymbol{S_j}$

Else $T=T+T_{\text{int}}$

if $T<= T_{\text{end}}$

Go to Step 3 (Under **Procedure 2**)

Else

$N=N+1$, Go to Step 2 (Under **Procedure 2**)

Step 8: Output $\boldsymbol{S_j}$

Step 6: find $\boldsymbol{S}$

The required set, $\boldsymbol{S}$, can be obtained by using the outputs of **Step 5 ($\boldsymbol{S_j}$**, j=1,…,n). The definition of the set, $\boldsymbol{S}$, is already described and the following procedure can be used to create the set, $\boldsymbol{S}$**,** easily.

Procedure 3: Forming a set, $\boldsymbol{S}$

Step 1: Select N and T pairs so that $\boldsymbol{S}(N,T)$ = $\boldsymbol{S_1}(N,T) \cap ... \cap \boldsymbol{S_n}(N,T)$

Step 2: Organize $x'_{j,l}$ and $x'_{j,u}$, which are associated with each N, T pair in $\boldsymbol{S}(N,T)$

Step 7: Make maintenance decision for each component according to the expression described in (4.19).

Output: $\boldsymbol{S_d}$

Step 8: find $\boldsymbol{S_{Av}}$ by using the set of $\boldsymbol{S_d}$ and constraint (4.36)

Step 9: Evaluate reliability for each component according to N, T and associated maintenance action listed in $\boldsymbol{S_{Av}}$ by using (4.11) and (4.25) .

Step 10: choose optimal N and T pair which can maximize reliability.

Step 11: $i' = i' + 1$

if $i' \neq i$, Go to **Step 4**

else

Go to **Step 12**

Step 12: Choose the optimal N, T and $\boldsymbol{Pi'}$ ($i' \in \{1,...,i\}$), which can give maximum reliability

Output: N, T, p_j and suitable maintenance action for each component

4.4.2 Problem formulation under reliability constraints

Under reliability constraints, if the time interval between successive PMs is long, the reduction amount to meet the required target reliability is large (approaches to T), and the chance of replacement is greater. However, the maximum reduction amount should not be greater than PM time interval (T). If time interval T is short, required maintenance degrees become small (approach to zero) and repair action is more suitable than replacement. But minimum reduction amount must not be less than zero. Constraints of maintenance degree, expressed in (4.16),(4.17) and (4.18) can be used in this case. Maintenance problem under reliability constraint is formulated as follows.

Objective: $\underset{N,T}{\min imize} \dfrac{E[C^*]}{E[T^*]}$ (4.37)

Constraints: $x'_{j,l} < x'_{j,u}$ (4.38)

$R_a < R_{N-1,j}(T) \le R_c$ (4.39)

$Av(N,T) \ge A_0$ (4.40)

4.4.2.1 Solution algorithm

Reliability constraint described in (4.39) is reconstituted as follows to find the upper and lower reduction amounts for each component.

$$R_{N-1,j}(T) = R_a \tag{4.41}$$

$$R_{N-1,j}(T) = R_c \tag{4.42}$$

The following algorithm can be used to find optimal T, N and suitable maintenance action for each component under reliability constraints.

Step 1: input: $h_1(t)$, $h_2(t)$,…, $h_n(t)$, $c_{min,1}$, $c_{min,2}$,…,$c_{min,n}$, R_c, R_a

Step 2: find $\boldsymbol{S_j}$

Procedure 2, expressed in Section 4.4.1.1, can be used. However, (4.14) in Step 4 is changed into (4.41) and (4.15) in Step 6 is changed into (4.42).

Step 3: find $\boldsymbol{S}$

Procedure 3, expressed in Section 4.4.1.1, can be used to find $\boldsymbol{S}$.

Step 4: find $\boldsymbol{S_d}$

Step 5: find $\boldsymbol{S}_{A\nu}$

Step 6: Evaluate maintenance cost rate for each component according to N, T and associated maintenance action listed in $\boldsymbol{S}_{A\nu}$ by using (4.37), (4.24) and (4.31).

Step 7: choose optimal N and T pair which can minimize cost.

4.5 Numerical example

In this section, a numerical example, proposed by [32], is presented in order to demonstrate the proposed model. Since the aim of giving a numerical example is just to demonstrate the proposed maintenance policy and not to compare with [32], only the relevant parameters are used.

Example:

This example involves a mechatronic system which consist of five SCs (subsystem or component) (1) control, (2) power, (3) transmission, (4) sensing, and (5) tool. The reliabilities of the SCs are formulated by using Weibull function because the most useful probability distributions in reliability are Weibull. Required parameters including Weibull parameters for each component are listed in Table 4.1. Available reduction amount of component j (y_j) is uniformly distributed within 0 and successive PM time interval T. Its cdf is

$$G_j(y) = \begin{cases} \dfrac{y}{T}, & \text{if } 0 < y < T \\ 0, & \text{otherwise} \end{cases}$$

Table 4.1 Case parameters

	unit 1	unit 2	unit 3	unit 4	unit 5
θ	2000	2400	2600	3400	2000
β	2.5	2.5	3.2	2.8	3.1
c_{min}	44	72	95	76	78
c_R	150	240	400	320	260
t_{min}	1	1	1	1	1
$1/\mu$	0.12	0.12	0.12	0.12	0.12
T_R	90	90	90	90	90
δ			1.2		
B			0.25		
q			0.8		
T_G			120		
$l(x_j,i)$			$c_{minj} + B*x_j*(i)^{(\delta-1)}$		
R_c, R_a			*0.9,0.8*		
A_0			*0.85*		

4.5.1 Results and discussion

The first case is to determine optimal maintenance plan under budget cost constraint. According to the proposed algorithm, available budget percentage of component *j* is considered first by assuming that T_{start}=300, T_{end}=1000 and T_{int}=100. The following table is obtained as a result of **Procedure 2**, proposed in Section 4.4.1.1.

Table 4.2 Lists of ***Pi'***

	unit 1	unit 2	unit 3	unit 4	unit 5
P1	25	20	15	15	25
P2	25	20	20	15	20
P3	25	25	15	15	20
P4	20	20	20	15	25
P5	20	25	20	15	20
P6	20	25	15	15	25
P7	25	25	20	10	20
P8	25	25	15	10	25
P9	25	20	20	10	25
P10	20	25	20	10	25

The next step is to find ***Sj*** by using ***Pi'***. Only ***P4*** and ***P5*** can give the required set, ***Sj***. Then, ***S***, $\boldsymbol{S_d}$ **and** $\boldsymbol{S_{Av}}$ are searched according to the proposed solution algorithm. Then, reliability for each component for each (N, T) pair in $\boldsymbol{S_{Av}}$ is evaluated. Results are listed in Table 4.3. Optimal N, T pairs which can give maximum reliability are chosen for each pair of budget percentage. According to the results listed in Table 4.3, both ***P4*** and ***P5*** give the same answer. Maximum reliability is obtained by performing PM at time interval T, 550 hrs, and all are replaced at 5th PM.

Table 4.3 Lists of reliability and cost for each N,T pair under budget cost constraints

	T	N	1	2	3	4	5	Availability	$R_s(NT)$	cost
P4	500	6	R	R	Im	Im	R	0.84	0.4779	4.4704
	550	5	R	R	Im	Im	R	0.85	0.5379	3.4610
P5	550	5	R	R	Im	Im	R	0.85	0.5379	3.4610

Now the optimal maintenance plan for the second case is decided under pre-determined reliability levels, R_c and R_a. ***S***, $\boldsymbol{S_d}$ and $\boldsymbol{S_{Av}}$ under reliability constraints are evaluated first according to the described solution algorithm in Section 4.4.2.1. And then, maintenance cost for each N,T pairs, listed in $\boldsymbol{S_{Av}}$ is calculated. The optimal cost is the minimum cost. From Table 4.4, it can be seen that minimum maintenance cost is obtained at PM time interval 600 and the number of PM is 5. Maintenance action for component 3 and 4 is to repair and others are to be replaced at each PM and the whole system is renewed at 5th PM.

Table 4.4 Lists of reliability and cost for each N,T pair under reliability constraints

T	N	1	2	3	4	5	Availability	$R_s(NT)$	cost
450	8	R	R	Im	Im	R	0.85	0.3095	1.8724
500	7	R	R	Im	Im	R	0.85	0.3275	1.7420
550	6	R	R	Im	Im	R	0.86	0.3732	1.6301
600	5	R	R	Im	Im	R	0.86	0.4468	1.5330

Table 4.5 Lists of reliability and cost for optimal N,T pair under budget and reliability constraints

	T	N	1	2	3	4	5	Availability	$R_s(NT)$	cost
Cost constraint	550	5	R	R	Im	Im	R	0.85	0.5379	3.4610
Reliability constraint	600	5	R	R	Im	Im	R	0.86	0.4468	1.5330

The results for both cases (under budget cost constraints and reliability constraints) are listed in Table 4.5. When the results from both cases are compared, reliability in Case 1 is greater than that of Case 2. But Case 2 is more preferable to Case 1 when maintenance costs in both cases are compared. Therefore, Case 2 should be emphasized for cost reduction, whereas Case 1 should be emphasized for better reliability.

.

Chapter 5

Summary and Conclusions

This thesis addresses the preventive group maintenance policies for two different systems. First system is a parallel or parallel-series or k-out-of-n system in which components have the same life time distribution. Reliability-centered T-based and m-based preventive group maintenance policies are proposed for this system. The second system is a system whose components are independent but not identical. A preventive group maintenance policy in which suitable maintenance action for each component is decided on the basis of maintenance degrees is presented for the second system.

The model formulations and solution algorithms for the proposed maintenance policies are described in Chapters 3 and 4. A case study and relevant numerical example, results and discussions are also provided. In this chapter, a brief description of our studies, concluding remarks, the main contributions of these studies and possible future research areas are presented.

5.1 Summary and conclusions

A comprehensive review on the maintenance of multi-component systems is presented. Group maintenance policies, opportunistic maintenance policies, reliability-centered maintenance policies and repair limit policies are also reviewed in Chapter 2. Although these maintenance policies have different names and/or procedures, all of them invariably aimed at performing group maintenance in order to save time and money and to maintain the system's reliability.

In this thesis, preventive group maintenance policies for two different systems are proposed. First system is a parallel or parallel-series or a k-out-of-n system in which components are identical components. The main problem in proposing an effective maintenance policy for such system is to decide preventive maintenance (PM) time. Required reliability is pre-determined and it is used as a primary decision criterion in assessing PM time. This is the main difference from the conventional group maintenance policies in which PM time is decided mostly on the basis of attaining minimum maintenance cost. Two policies are classified according to different assessing PM time.

The first policy is reliability-centered T-based preventive group maintenance policy (Policy 1) in which PM time is determined by critical reliability, expressed in terms of time unit. The second policy is reliability-centered m-based preventive group maintenance policy (Policy 2) in which PM time is assessed by critical reliability, expressed in terms of the number of failed units. The model formulations for these two policies are described in Chapter 3. Establishing an effective maintenance policy for five capper machines, operated in the purified water production system is given as a case study. Proposed two policies are applied to this case study. Then, a numerical comparison is made between Policy 1 and Policy 2. The results show that

(i) system's uptime (operation time) in Policy 1 is greater than Policy 2.

(ii) per unit maintenance cost in Policy 1 is less than Policy 2

Therefore, when Policy 1 and Policy 2 are compared on the basis of system availability and cost, Policy 1 is more preferable to Policy 2. The reason is due to the differences between these two policies, expressed as follows.

(i) numbers of failed components at the time of inspection are random variable in Policy 1 (i.e. $0 < P_m(T) < 1$, where $m=0,1,2\ldots,n$) while exactly m components are failed at the time of inspection in Policy 2 ($P_m(Y_m) = 1$), and

(ii) system is inspected at the time of $R_s(T) = R_c$ in Policy 1 while $R_s(Y_m) \geq R_c$ in Policy2.

Finally, it can be concluded that if system inspection time (or) PM time is limited by reliability, there is no effectiveness in performing PM in advance before arriving pre-specified critical reliability level although degradation costs, production loss costs and maintenance costs are taken into account.

The second system is a system whose components are independent but not identical. Since components are non-iid, the status of the components may not be the same at the same time. Proper maintenance action is decided based on maintenance degrees. Again these maintenance degrees are considered to get the desirable operational conditions such as maintenance cost and system reliability. Two cases are considered according to different desired operation conditions.

The desirable condition in first case is not to incur the cost beyond the available maintenance budget at the time of PM. Lower and upper limits of available maintenance budget at the time of PM, $B_{G,l}$ and $B_{G,u}$ are pre-determined and these amounts are distributed to all components. Since components are non-iid components, it is impossible to distribute uniformly to all components. Therefore, available budget percentage of each component p_j is formulated with the basis on the component's maintenance cost and failure probability. $B_{G,l}$ and $B_{G,u}$ are distributed to all components with the help of p_j. By this way, the required lower and upper limits of maintenance budget for each component, $b_{j,l}$ and $b_{j,u}$, are obtained and these amounts are used as constraints in determining the target maintenance degrees $x_{j,l}$ and $x_{j,u}$.

The desirable condition of second case is not to allow the system to fall below the pre-determined reliability level. In this case, the lower and upper reliability levels are pre-determined as minimal and critical reliability levels, R_a and R_c. These values are used as constraints and $x_{j,l}$ and $x_{j,u}$ are determined under these constraints.

Since the exact amount of maintenance degrees cannot be obtained in practice, the required target amounts and available amount are compared and suitable maintenance action is decided on the basis of this comparison. Model formulations as to how to decide proper maintenance actions based on maintenance degrees are presented in Chapter 4. Relevant solution algorithms for each case are also provided to be more convenient in finding optimal parameters. A mechatronic system, proposed by [32] is used as a numerical example in order to demonstrate the proposed maintenance policy and to compare these two cases.

The results prove that both of them are effective and useful. The choice between these two depends on the emphasis of cost or reliability. Moreover, in the proposed model, the available maintenance degrees are not put into a fixed model/distribution function. Therefore, the available maintenance degrees can be fixed reduction amounts (or) random values. In whichever way, the proposed model can be applied.

5.2 Main Contribution

Group maintenance policies are proposed by many researchers [1,25,33]. However, the maintenance cost is used as a decision criterion. Later, reliability becomes an important factor in maintenance policy and reliability is also used as a decision criterion. In the first part of this thesis, reliability-centered T-based and m-based preventive group maintenance policies are presented. Although reliability-centered T-based maintenance policies have been proposed by many researchers [32, 38], component downtime costs are added into consideration here. Moreover, reliability-centered m-based policy is presented as a modification of an m-based group maintenance policy. Without lost of generality, repair times are modeled as iid repair times and mathematical models of these two policies under iid repair times are given. To be more realistic, repair times are assumed as monotonically increasing repair times and model formulations for both the reliability-centered T-based and m-based policies for various system structures are provided in Chapter 3. An algorithm for iid repair time assumption that ensures the optimal parameters, is developed. The advantage of this algorithm is that the required optimal parameter N can be obtained by choosing one out of four possible Cases. For increasing repair time assumption, although an algorithm as in iid assumption is not given, the way of choosing optimal N that can minimize cost is presented. Another contribution is that a numerical comparison is made between the reliability-centered T-based and m-based policies through the cost reduction and increasing availability.

In the second part of this thesis, a preventive group maintenance policy is presented with the aim of providing an answer for the question "how to decide the suitable maintenance action for non-iid components". In "Repair limit policies", proposed by [11] and [12], proper maintenance action is decided based on repair time and repair cost. Tsai and Wang [32] proposed a maintenance policy in which either repair or replacement action is decided based on the remaining life time after maintenance and maintenance cost. In this thesis, proper maintenance action is decided with the basis of maintenance degrees, obtained by the theoretical and practical means.

5.3 Future research

Several extensions are possible to the proposed model in Chapter 4. In this model, (i) only two maintenance actions, such as repair or replace, are considered and "do nothing" case ($y_j > x'_{j,u}$) is discarded, (ii) in a given T and N, if $x'_{j,l} \le y_j \le x'_{j,u}$, component j is repaired at every PM and replaced with new one at N^{th} PM. If $y_j < x_{j,l}$, component j cannot be retained in the required operation conditions at N^{th} PM by making repair action on it at every PM. Since the proposed maintenance policy is that all components are either repaired or replaced at PM and all are renewed at the same time, the maintenance decision for component j is to replace with new one at every PM. The status of component j at $(N\text{-}1)^{th}$ PM and $(N+1)^{th}$ PM are not taken into account here. If the statuses at $(N\text{-}1)^{th}$ and $(N+1)^{th}$ are taken into consideration in deciding suitable maintenance action, all the components in the same system may or may not be in the same group. It means there may be more than one group in the same system and optimal maintenance policies are needed to adopt for each group and also for the whole system. This field may be an extension of the proposed work.

The other point of consideration is business policy. In the proposed policies, business environment is assumed as constant. Therefore, optimal PM time, T, and number of PM, N, are considered under infinite time horizon in a given environment. For high-tech equipments, fast changing business environment and technologies necessitates rapid change of process specifications too. Thus, equipment usage (or) life cycle cannot be considered with the basis only in terms of life time of a component. It is needed to anticipate the arrival of a new technology and a planned route map in deciding an optimal maintenance policy. Thus, proposing a maintenance policy which can give suitable maintenance action for high-tech equipments might be of material for the future work.

References

[1] Assaf D. and George Shanthikumar, J. Optimal group maintenance policies with continuous and periodic inspections, Management Science, vol.33, No 11, 1987.

[2] Bai, J. and Pham, H. Discounted warranty cost of minimally repaired series systems, IEEE Trans Reliab., vol. 53, pp. 37-42, 2004.

[3] Brown, M., Proschan, F. "Imperfect repair", J. Applied Probability, vol 22. pp. 370-385, 1982.

[4] Canfield , R. V. Cost optimization of periodic preventive maintenance, IEEE Trans Reliab., vol. 35, pp. 78-81, 1986.

[5] Cheng, C.Y. and Chen, M.C. The periodic preventive maintenance policy for deteriorating systems by using improvement factor model, International Journal of Applied Science and Engineering, vol. 2, pp.114-122,2003.

[6] Chukova, S., Arnold, R., Wang, Dong Q. Warranty analysis: An approach to modeling imperfect repairs, Int. J. Production Economics, vol. 89, pp. 57-68, 2004.

[7] Crocker,J., Kumar, U. D. Age-related maintenance versus reliability centered maintenance: a case study on aero-engines, Reliability Eng. And System Safety, vol. 67, pp.113-118, 2000.

[8] Das, A. N. and Acharya, D. Age replacement of components during IFR delay time, IEEE Trans Reliab., vol. 53, pp.306-312, 2004.

[9] Dekker, R., Ingrid Roelvink, F. K. Marginal cost criteria for preventive replacement of a group of components, European J. Operation Research, vol. 84, pp. 465-480, 1995.

[10] Dekker, R., Meer, R.V., Plasmeijer, R. Ph., Wildeman, R. E., Bruin, J.J. de Maintenance of light standards, a case-study, 1996.

[11] Dohi, T., Kaio, N., Osaki, S. A graphical method to repair-cost limit replacement policies with imperfect repair, Mathematical and Computer Modeling, vol.31, pp.99-106, 2000.

[12] Dohi, T., Takeita, K., Osaki, S. Graphical methods for determining/estimating optimal repair-limit replacement policies, Int. J. Reliab. Qual. Safety Eng, vol.7, pp.43-60, 2000.

[13] Ebeling.Charles, E. An Introduction to Reliability and Maintainability Engineering

[14] Ebrahimi, N. Two new replacement policies, IEEE Trans Reliab., vol. 42, pp. 141-147, 1993.

[15] Jhang,J. P., Sheu, S. H. Opportunity-based age replacement policy with minimal repair, Reliability Eng. And System Safety, vol. 64, pp.339-344, 1999.

[16] Klutke, G. A. and Yang, Y. The availability of inspected systems subject to shocks and graceful degradation, IEEE Trans Reliab., vol. 51, pp.371-374, 2002.

[17] Lam,Y. and Zhang, Y. L. A geometric-process maintenance model for a deteriorating system under a random environment, IEEE Trans Reliab., vol. 52, pp. 83-89, 2003.

[18] Lam, Y., Zhang, Y. L. A shock model for the maintenance problem of a repairable system, Computers & Operations Research, vol.31, pp. 1807-1820, 2004.

[19] Lie, C. H. and Chun, Y. H. An algorithm for preventive maintenance policy, IEEE Trans Reliab., vol. 1, R-35, pp.71-75, 1986.

[20] Lim, J. H., Park, D. H. Evaluation of average maintenance cost for imperfect-repair model, IEEE Trans Reliab., vol. 48, pp. 199-204, 1999.

[21] Lin, D., Zuo, M. J., Yam, R. C. M. General sequential imperfect preventive maintenance models, Int. J. Reliab. Qual. Safety Eng, vol.7, pp.253-266, 2000.

[22] Mitchell, J. M. Approximation of mean time between failures when a system has periodic maintenance, IEEE Trans Reliab., vol. 51, pp. 166-167, 2002.

[23] Pham,H., Wang, H. Imperfect maintenance, European J. Operation Research, vol. 94, pp.425-438, 1996.

[24] Pham, H., Wang, H. Optimal (τ, T) opportunistic maintenance of a k-out-of-n :G system with imperfect pm and partial failure, Dept of Industrial Engineering, Rutgers University, Piscataway, New Jersey 08854-0909.

[25] Ritchken, P., Wilson, J.G. (m,T) group maintenance policies, Management Science, vol.36, pp.632-639, 1990.

[26] Scarf, P. A., Dwight, R., A1-Musrati, A. On reliability criteria and implied cost of failure for a maintained component, Reliability Eng. And System Safety, vol.89, pp.199-207, 2005.

[27] Sheu, S. H., A generalized age and block replacement of a system subject to shocks, European J. Operation Research, vol. 108, pp. 345-362, 1998.

[28] Sheu, S.H., Griffith, W.S. Optimal age-replacement policy with age-dependent minimal-repair and random-lead-time, IEEE Trans Reliab., vol.50, pp.302-309, 2001.

[29] Sheu, S. H., Griffith, W. S. Extended block replacement policy with shock models and used items, European J. Operation Research, vol. 140, pp.50-60, 2002.

[30] Sheu, S.H., Griffith, W.S., Nakagawa, T. Extended optimal replacement model with random minimal repair costs, European J. Operation Research, vol. 85, pp. 636-649, 1995.

[31] Sheu, S., Jhang, J. A generalized group maintenance policy, European J. Operation Research, vol. 96, pp. 232-247, 1997.

[32] Tsai, Y. T., Wang, K. S., Tsai, L.C. A study of availability-centered preventive maintenance for multi-component systems, Reliability Eng. And System Safety, vol.84, pp.261-270, 2003.

[33] Wang, H. A survey of maintenance policies of deteriorating systems, European J. Operation Research, vol. 139, pp.469-489, 2002.

[34] Wu, S., Croome, D. C. Preventive maintenance models with random maintenance quality, Reliability Eng. And System Safety, vol.90, pp.99-105, 2005.

[35] Vaurio, J. K. Availability and cost functions for periodically inspected preventively maintained units, Reliability Eng. And System Safety, vol.63, pp.133-140, 1999.

[36] Zheng, X. All opportunity-triggered replacement policy for multiple units system, IEEE Trans Reilab., vol. 44, pp.648-652, 1995.

[37] Zhao, Y. X. On preventive maintenance policy of a critical reliability level for system subject to degradation, Reliability Eng. And System Safety, vol.79, pp.301-308, 2003.

[38] Zheng, X., Fard, N. A maintenance policy for repairable systems based on opportunistic failure-rate tolerance, IEEE Trans Reilab., vol. 40, pp. 237-244, 1991.

[39] Zhang, F. and Jardine, A. K. S. Optimal maintenance models with minimal repair, periodic overhaul and complete renewal, *IIE* Transactions, vol. 30, pp. 1109-1119, 1998.

Appendix

1. Source code

1. Under iid assumption

a. Excel spread sheet for reliability-centered *T*-based maintenance policy

This spread sheet is used to calculate the required parameters (*A*, *B* and *C*) which are needed to decide suitable case (either case 1 or 2 or 3 or 4) in order to find optimal *N*.

m	P_m	downtime	$\int_0^T P_m(y)dy$	$\int_0^T \overline{F}_m(y)dy$	$E[C_d]$	first part of B
0	0.0068	0.0006	1.319			5.1
1	0.0585	0.0059	0.7552	1.319	5.8766	52.65
2	0.2001	0.0233	0.5981	2.0742	32.6463	210.105
3	0.3421	0.0456	0.4156	2.6723	67.0311	410.52
4	0.2924	0.0439	0.1961	3.0879	60.8046	394.7400
					1st part of B	1073.1150
					2nd part of B	122.0547

A	B	C
0.1526	1195.1697	166.3586

b. Matlab program for reliability-centered *m*-based maintenance policy

The main purpose of this program is used to calculate required parameters (*A*, *B* and *C*) but also to provide the graphs of availability versus *N* and cost versus *N* under iid assumption.

Main Program

```
syms m t u i N y  j l b

%define I =1 for T based policy
I  =1;
alpha   =  2;
lambra =  0.3;
valmu  =  30;
cl    =  800;
cd   =  50;
cp   =  150;
cr    =  300;
cG  =  2500;

tpm =  1/60;
tcr   =  1/30;
tG    =  2/valmu;
tw    =  1/valmu;

a     =  0.85;
Rc   =  0.9;
valn =  5;
Av0 =  0.95;
 totrc  =  0;
downtime  =  0;
downtimecost  =  0;
totalcomdtc     =  0;
totnof  =  0;
dtt      =  0;

%inspection time
squrc   =  (-logm(1-(1-Rc)^(1/valn)))^(1/alpha);
```

```
valT   = double(squrc/lambra);
FT     = 1-exp(-(lambra*valT)^alpha);
FbT    = 1-FT;

%call function of repair cost and repair time
valm   = 0;
[totreptime,totrepcost]  =   reptimecost(valm,valn,alpha,lambra,cr,cp,tpm,tcr,valT,I)

%call comdtcost fun
totalcomdtc   =  comdtcost(valm,valn,alpha,lambra,cd,valT,Rc,I);

%expected repair time and cost
intotreptime  =  0;

    for l    =  1:60
     valN  =  l;

     if valN  ==  1
        explen  =  valT+tG;
        availa   =  double(valT/explen);
        expcost =  cG+totalcomdtc+cl*(tG);
        expcostpercy  =  double(expcost/explen);
        fprintf  ('availability(%d) =%d\n', valN,availa);
%           fprintf ('expcost(%d) = %d\n', valN,expcostpercy);

     else
        intotreptime  =  0;

        for b  =  1:valN-1
           incfaca  =  1/((a)^(b-1));
           increptime   =  incfaca*totreptime;
           intotreptime =  intotreptime+increptime;
      end
```

```
            explen     = double(valN*valT+intotreptime+(valN-1)*tw+tG);
            uptime     = valN*valT;
            expavaila  = double(uptime/explen);
            fprintf ('availability(%d)  =%d\n', valN, expavaila);
            expcost  = (valN-1)*totrepcost+valN*totalcomdtc+cl*(intotreptime+(valN-
1)*tw+tG)+cG;
            expcostpercy  = double(expcost/explen);
%           fprintf ('expcost(%d)  =%d\n', valN,expcostpercy);
      end
 end

%end of program
```

2. Under increasing repair time assumption

The main purposes of following programs (a and b) are the same as programs described in (1) except that the repair time assumption is different.

a. Matlab program for reliability-centered *T*-based maintenance policy

Main program

```
syms m t u i N y  j l b

%define I=1 for T based policy
I  =  1;
alpha    =  2;
lambra  =  0.3;
valmu   =  30;
cl   =  800;
cd  =  50;
cp  =  150;
```

```
cr   =  300;
cG  =  2500;

tpm  =  1/60;
tcr    =  1/30;
tG    =  2/valmu;
tw    =  1/valmu;

a     =  0.85;
Rc   =  0.9;
valn =  5;
Av0 =  0.95;

totrc  =  0;
downtime   =  0;
downtimecost  =  0;
totalcomdtc  =  0;
totnof  =  0;
dtt       =  0;

 %inspection time
squrc  =  (-logm(1-(1-Rc)^(1/valn)))^(1/alpha);
valT    =  double(squrc/lambra);
FT       =  1-exp(-(lambra*valT)^alpha);
FbT     =  1-FT;

%call function of repair cost and repair time
valm  =  0;
[totreptime,totrepcost]  =   reptimecost(valm,valn,alpha,lambra,cr,cp,tpm,tcr,valT,I)

%call comdtcost fun
totalcomdtc  =  comdtcost(valm,valn,alpha,lambra,cd,valT,Rc,I);
```

```
%expected repair time and cost
intotreptime = 0;

   for l = 1:60
    valN = l;

    if valN == 1
      explen = valT+tG;
      availa  = double(valT/explen);
      expcost = cG+totalcomdtc+cl*(tG);
      expcostpercy = double(expcost/explen);
      fprintf ('availability(%d) =%d\n',valN,availa);
%        fprintf('expcost(%d)=%d\n',valN,expcostpercy);

    else
        intotreptime = 0;

      for b = 1:valN-1
        incfaca      = 1/((a)^(b-1));
        increptime  = incfaca*totreptime;
        intotreptime = intotreptime+increptime;
      end

        explen = double(valN*valT+intotreptime+(valN-1)*tw+tG);
        uptime = valN*valT;
        expavaila = double(uptime/explen);
        fprintf ('availability(%d) =%d\n',valN, expavaila);
        expcost = (valN-1)*totrepcost+valN*totalcomdtc+cl*(intotreptime+(valN-
1)*tw+tG)+cG;
        expcostpercy = double(expcost/explen);
%          fprintf('expcost(%d)=%d\n',valN,expcostpercy);
    end
end
%end of program
```

b. Matlab program for reliability-centered *m*-based maintenance policy

Main Program

```
syms m t u i N y  j l b

%define I=0 for m-based policy
I  =  0;
alpha   =  2;
lambra =  0.3;
valmu  =  30;

cl   =  800;
cd  =  50;
cp  =  150;
cr   =  300;
cG  =  2500;

tpm  =  1/60;
tcr    =  1/30;
tG     =  2/valmu;
tw     =  1/valmu;

a  =  0.85;
valrc  =  0.9;
valn   =  5;

Av0  =  0.95;
lhs   =  0;
limr =  1-((1-valrc)^(1/valn));
rhs   =  limr;
Fy   =  1-(exp(-(lambra*y)^alpha));
Fby =  1-Fy;
Fbm=  0;
```

```
limr  =  1-((1-valrc)^(1/valn));
rhs   =  limr;
lhs   =  0;
Fbm =  0;

%call subfunction to find threshold number of failed component m
maxnof   =   theshnof(valn,lambra,alpha,valrc);
valm       =  4;

%calculating component downtime cost
valT  =  0;
totalcomdtc  =  comdtcost(valm,valn,alpha,lambra,cd,valT,Rc,I);

%call function for repair time and repair cost
[totreptime,totrepcost]  =   reptimecost(valm,valn,alpha,lambra,cr,cp,tpm,tcr,valT,I);

%expected inspection time
Fym  =  0;

for k = 0:valm-1
   valk  =  k;
   fack  =  0;
   fack  =  factorial(valn)/(factorial(valk)*factorial(valn-valk));
   Fyk  =  fack*(Fy)^(valk)*(Fby)^(valn-valk);
   Fym =  Fym+Fyk;
end
 Elm  =  int(Fym,y,0,inf);

%expected cost and time
intotreptime  =  0;

     for l  = 1:60
       valN  =  l;
```

```
    if valN == 1;

      explen = Elm+tG;
      availa = double(Elm/explen);
      expcost = cG+totalcomdtc+cl*(tG);
      expcostpercy = double(expcost/explen);
      fprintf ('availability(%d) =%d\n',valN,availa);

%         fprintf('expcost(%d)=%d\n',valN,expcostpercy);

    else
      intotreptime = 0;

      for b = 1:valN-1
        incfaca = 1/((a)^(b-1));
        increptime = incfaca*totreptime;
        intotreptime = intotreptime+increptime;
      end

        explen = double(valN*Elm+intotreptime+(valN-1)*tw+tG);
        uptime = valN*Elm;
        expavaila = double(uptime/explen);
        fprintf ('availability(%d) =%d\n',valN, expavaila);
        expcost = (valN-1)*totrepcost+valN*totalcomdtc+cl*(intotreptime+(valN-
1)*tw+tG)+cG;
        expcostpercy = double(expcost/explen);
%          fprintf ('expcost(%d) =%d\n',valN,expcostpercy);
    end
   end
```

Subroutine 1: Compute threshold numbers of failed components

```
function maxnof = theshnof(valn,lambra,alpha,valrc)
```

```
syms y i
vallam  =  lambra;
valalph = alpha;

Fy    =  1-exp(-(vallam*y)^valalph);
Fby  =  1-Fy;
Fbm =  0;
lhs   =  0;
limr  =  1-((1-valrc)^(1/valn));
rhs    =  limr;

for i  =  1:valn
   vlm  =  i;
   vali  =  i-1;
   fac   =  factorial(valn)/(factorial(vali)*factorial(valn-vali));
   Fbmi=  fac*(Fy)^(vali)*(Fby)^(valn-vali);
   Fbm=  Fbm+Fbmi;
   Fm  =  1-Fbm;
   dFm=  diff(Fm,y);
   lhs   =  double(int(Fby*dFm,y,0,inf));

   if lhs == rhs
      maxnof  =vlm;
      fprintf ('m =%d\n',valm);
      fprintf ('lhs =%d\n',lhs);
      break;

   elseif lhs < rhs
      maxnof  =vlm-1;
      valm  =  vlm-1;
      fprintf ('m =%d\n',valm);
      fprintf ('lhs =%d\n',lhs);
      break;
   else
```

```
    end
end
```

Subroutine 2: Compute total repair time and repair cost

```
function [totreptime,totrepcost] = reptimecost (valm,valn,alpha,lambra,…
                                    cr,cp,tpm,tcr,valT,I)

syms y m

FT  = 1-exp(-(lambra*valT)^alpha);
FbT= 1-FT;
totreptime = 0;
totrepcost = 0;

if I ==1

   for m = 0:valn-1
        valm = m;
        facm = factorial(valn)/(factorial(valm)*factorial(valn-valm));
        pm   = (facm*(FT)^(valm)*(FbT)^(valn-valm));
        reptime    = pm*(m*tcr+(valn-m)*tpm);
        totreptime = totreptime+reptime;
        repcost    = pm*(m*cr+(valn-m)*cp);
        totrepcost = totrepcost+repcost;
   end

elseif I == 0
   correptime = valm*tcr;
   prereptime = (valn-valm)*tpm;
   correpcost = valm*cr;
   prerepcost = (valn-valm)*cp;
   totreptime = correptime+prereptime;
   totrepcost = correpcost+prerepcost;
```

```
    else
      fprintf('pls select 0 or 1\n:I=1 if T-based\n, I=0 if m-based\n');
end
```

Subroutine 3: Compute total component downtime cost

```
function totalcomdtc = comdtcost(valm,valn,alpha,lambra,cd,valT,Rc,I)

syms y j

totalcomdtc = 0;
Fy  = 1-exp(-(lambra*y)^alpha);
Fby = 1-Fy;
comdtc = 0;

FT  = subs(Fy,y,valT);
FbT = 1-FT;
comdtc = 0;
Fbjy = 0;

%component downtime cost
if I == 1
for j  = 1:valn-1
   valj = j-1;
   facj = factorial(valn)/(factorial(valj)*factorial(valn-valj));
   Fbj  = facj*(Fy)^(valj)*(Fby)^(valn-valj);
   Fbjy = Fbjy+Fbj;
   EYj  = double(int(Fbjy,y,0,valT));

   if j == 1
     valj = j;
     EY1= EYj;
      dt1 = valT-EY1;
```

```
      p1f =  (factorial(valn)/(factorial(valj)*factorial(valn-
valj)))*(FT)^(valj)*(FbT)^(valn-valj)
     comdtc =  double(cd*dt1*p1f);

   else if j = =  2
      valj   =  j;
      Ey2    =  EYj;
      dt2    =  double(Ey2-EY1);
      p2f    =  (factorial(valn)/(factorial(valj)*factorial(valn-
valj)))*(FT)^valj*(FbT)^(valn-valj);
     comdtc =  double((cd*dt2+2*cd*(valT-Ey2))*p2f);

   else if j  = =  3
      valj  =  j;
      Ey3   =  EYj;
      dt3   =  (Ey3-Ey2);
      p3f   =  (factorial(valn)/(factorial(valj)*factorial(valn-
valj)))*(FT)^(valj)*(FbT)^(valn-valj);
    comdtc =  double((cd*dt2+2*cd*dt3+3*cd*(valT-Ey3))*p3f);
    else if j = = valn-1
      valj  =  j;
      Ey4 =  EYj;
      dt4  =  (Ey4-Ey3);
      p4f  =  (factorial(valn)/(factorial(valj)*factorial(valn-
valj)))*(FT)^(valj)*(FbT)^(valn-valj);
     comdtc  =  double((cd*dt2+2*cd*dt3+3*cd*dt4+4*cd*(valT-Ey4))*p4f);

        else
           fprintf('wrong loop=%d\n',j);
        end
      end
   end
   end
        totalcomdtc  =  double(totalcomdtc+comdtc);
```

```
end

elseif I  = = 0

   for j  =  1:valm
   valx  =  valm-j;
   facx  =  factorial(valn)/(factorial(valx)*factorial(valn-valx));
   pj      =   facx*(Fy)^(valx)*(Fby)^(valn-valx);
   lenj   =  int(pj,y,0,inf);
   comdtc  =  cd*valx*lenj;
   totalcomdtc  =  totalcomdtc+comdtc;
   end

else
   fprintf('pls select 0 or 1\n:I=1 if T-based\n, I=0 if m-based\n');

end
```

3. *N*, *T* and respective target maintenance degrees under budget constraint

Target maintenance degree for a given *N*,*T* pairs for each component under budget cost constraint are calculated by using the following program. Reduction amounts obtained from the following program are lower limit of target reduction amounts because lower limit of budget amounts are used for each component. If upper limits are required, RHS of lines (27,28 and 29) are exchanged with upper limit of maintenance budget.

```
%N,T and target maintenance degree for each component

%define a=l for lower limit
%define a=h for high limit
```

```
syms t u j i y1 y2 y3 y4 y5 T x1 x2 x3 z n a

a = char('l');                    %for lower limit of y
valT = 400;
totn = 5;
error = 1;

while valT <= 400

  theta1 = 2000;
   beta1 = 2.5;
   theta2 = 2400;
   beta2 = 2.5;
   theta3 = 2600;
   beta3 = 3.2;
   theta4 = 3400;
   beta4 = 2.8;
   theta5 = 2000;
   beta5 = 3.1;

   cR1 = 150;
   cR2 = 240;
   cR3 = 400;
   cR4 = 320;
   cR5 = 260;

   cmin1 = 44;
   cmin2 = 72;
   cmin3 = 95;
   cmin4 = 76;
   cmin5 = 78;

   cg = double(0.9*(cR1+cR2+cR3+cR4+cR5));
```

```
if a == 'h'
cgst1 = 0.25*(cg);
cgst2 = 0.2*(cg);
cgst3 = 0.2*(cg);
cgst4 = 0.15*(cg);
cgst5 = 0.2*(cg);

elseif a == 'l'
cgst1   = 0.25*(0.8*cg);
cgst2   = 0.2*(0.8*cg);
cgst3   = 0.2*(0.8*cg);
cgst4   = 0.15*(0.8*cg);
cgst5   = 0.2*(0.8*cg);

else
   fprintf ('check a again');

end

valdel = 1.2;
valn   = 5;
valB   = 0.25;

det  = 0;
dtt  = 0;
dt   = 0;
dtt  = 0;
totb = 0;

for i = 4:10
vali = i;
det  = (vali)^(valdel-1);
y1   = 100;
y2   = 100;
```

```
y3   = 100;
y4   = 100;
y5   = 100;

for n = 1:valn
   num = n;

   if num == 1
   hu = (beta1/theta1)*(u/theta1)^(beta1-1);
   y   = y1;
   beta  = beta1;
   bdp  = ((cmin1)/((theta1)^(beta1)));
   cmin = cmin1;
   cgst  = cgst1;

   elseif num == 2
      hu = (beta2/theta2)*(u/theta2)^(beta2-1);
      y   = y2;
      beta = beta2;
      cmin = cmin2;
      cgst = cgst2;
      bdp = ((cmin2)/((theta2)^(beta2)));

   elseif num == 3
      hu = (beta3/theta3)*(u/theta3)^(beta3-1);
      y   = y3;
      cmin = cmin3;
      beta  = beta3;
      cgst  = cgst3;
      bdp  = ((cmin3)/((theta3)^(beta3)));

   elseif num == 4
      hu   = (beta4/theta4)*(u/theta4)^(beta4-1);
      y    = y4;
```

```
        beta  =  beta4;
        cmin =  cmin4;
        cgst  =  cgst4;
        bdp  =  ((cmin4)/((theta4)^(beta4)));

    elseif num  ==  valn
        hu  =  (beta5/theta5)*(u/theta5)^(beta5-1);
        y    =  y5;
        beta  =  beta5;
        cmin =  cmin5;
        cgst  =  cgst5;
        bdp  =  ((cmin5)/((theta5)^(beta5)));
    else
       fprintf  ('check tot num of comp and n loop\n');

    end
% reduction amount loop(y loop) for component 1
  for j  =  1:10000
      lhsfp  =  bdp*(vali*(valT-y)+y)^(beta);
      lhssp  =  bdp*((vali-1)*(valT-y))^(beta);
      lhstp  =  valB*det*y;
      lhs     =  lhsfp-lhssp+lhstp;
      rhs     =  cgst-cmin;

      er  =  rhs-lhs;

       if er  <   0
          err =  er*(-1);

        else
          err  =  er;

       end
          if err  <=  error
```

```
            if y  <=  valT && y >0
                x  =  y;
                 fprintf ('(%d) =%d\n',num,x);
                 fprintf ('pm T =%d\n',valT);
                 fprintf ('no of pm =%d\n',vali);
                 break;

            elseif  y  <  0
               x  =  max(0,y);
               fprintf  ('(%d)  =%d\n',valn,x);
               fprintf ('pm T=%d\n',valT);
               fprintf  ('no of pm =%d\n',vali);
               break;

              else
          end
        end

      y  =  y+0.1;
      end
  end
 end
    valT  =  valT+50;
  end
```

4. *N*, *T* and respective target maintenance degrees under reliability constraint

In this program, required target maintenance degrees for a given N,T pairs are evaluated under reliability constraint.

```
%N,T and target maintenance degrees
syms t u j i y1 y2 y3 y4 y5 T x1 x2 x3 x4 x5 z n a
```

```
%define a=l for lower limit
%define a=h for high limit
a = 'h';
valT = 400;
totn  = 5;

if a == 'h'
    valrc = 0.9;
elseif a == 'l'
   valrc = 0.8;
 else
      fprintf ('check a again');
end

while valT <= 400

   theta1 = 2000;
   beta1  = 2.5;
   theta2 = 2400;
   beta2  = 2.5;
   theta3 = 2600;
   beta3  = 3.2;
   theta4  = 3400;
   beta4  = 2.8;
   theta5 = 2000;
   beta5  = 3.1;

   valn = 5;

% N loop

for i  = 4:20
  vali = i;
  y1  = 200;
```

```
y2  = 147;
y3  = 100;
y4  = 100;
y5  = 100;
for n  =  1:valn
    num  =  n;

    if num  = =  1
    hu  =  (beta1/theta1)*(u/theta1)^(beta1-1);
    y   =  y1;

    elseif num= =2
      hu=(beta2/theta2)*(u/theta2)^(beta2-1);
       y=y2;

    elseif num  = =  3
        hu  =  (beta3/theta3)*(u/theta3)^(beta3-1);
        y    =  y3;

    elseif num  = =  4
        hu  =  (beta4/theta4)*(u/theta4)^(beta4-1);
        y    =  y4;

    elseif num  = =  valn
        hu = (beta5/theta5)*(u/theta5)^(beta5-1);
        y   = y5;

    else
       fprintf  ('check tot num of comp and n loop\n');
    end

% reduction amount loop(y loop) for component 1
  for j   =  1:10000
     hus =  subs(hu,u,((vali-1)*(valT-y))+u);
```

```
        HNT  =  int(hus,u,0,valT);
        RNT  =  double(exp(-HNT));

     if RNT   >= valrc

          if y >= valT
             x   =  min(valT,y);
          elseif y < 0
             x  =  max(0,y);
          else
             x  =  y;
          end

        fprintf ('reduction amount(%d) =%d\n',num,x);
        fprintf  ('pm T =%d\n',valT);
        fprintf  ('no of pm  =%d\n',vali);
        break;

    end  %(end of if loop)
    y  =  y+0.1;

    end %(end of j loop)

 end
end
       valT  =  valT+50;
end
```

5. Calculate reliability, cost and repair time for each pair of *N*, *T*

Main Program

```
syms t u j i y d1 d2 d3 k n
```

```
%define d=1 for imperfect repair
%define d=0 for replacement
%input: d1,d2,d3,d4,d5 T and N
d1 = 0;
d2 = 0;
d3 = 0;
d4 = 1;
d5 = 0;

   valT = 450;
   theta1 = 2000;
   beta1 = 2.5;
   theta2 = 2400;
   beta2 = 2.5;
   theta3 = 2600;
   beta3 = 3.2;
   theta4 = 3400;
   beta4 = 2.8;
   theta5 = 2000;
   beta5 = 3.1;
   cmin1 = 44;
   cmin2 = 72;
   cmin3 = 95;
   cmin4 = 76;
   cmin5 = 78;
   tmin1 = 1;
   tmin2 = 1;
   tmin3 = 1;
   tmin4 = 1;
   tmin5 = 1;

   TG = 120;
   cg = double(0.9*(cR1+cR2+cR3+cR4+cR5));
```

```
valdel = 1.2;
valB  = 0.25;

hu1 = (beta1/theta1)*(u/theta1)^(beta1-1);
hu2 = (beta2/theta2)*(u/theta2)^(beta2-1);
hu3 = (beta3/theta3)*(u/theta3)^(beta3-1);
hu4 = (beta4/theta4)*(u/theta4)^(beta4-1);
hu5 = (beta5/theta5)*(u/theta5)^(beta5-1);

%min repair cost
totaldtresult = 0;
totminrept   = 0;
totmaincost = 0;

for i  = 4:4
valN = i;

for n  =  1:5
   valn =  n;
   if valn = =  1
      hu  =  hu1;
      tmin  =  tmin1;
      cmin  = cmin1;
      cR  =  cR1;

      if d1  = = 1
         d = 1;
      else
         d = 0;
      end

   elseif valn  = = 2
      hu     =  hu2;
      tmin  =  tmin2;
```

```
    cmin =  cmin2;
    cR    = cR2;

    if d2  = =  1
       d   =  1;
    else
       d  =  0;
    end

    elseif valn  = =  3
    hu  =  hu3;
    tmin  =  tmin3;
    cmin =  cmin3;
    cR    =  cR3;

    if d3  = =  1
       d   =  1;
    else
       d  =  0;
    end

    elseif valn  = =  4
    hu    = hu4;
    tmin =  tmin4;
    cmin=  cmin4;
    cR   =  cR4;

    if d4  = =  1
       d   =  1;
    else
       d  =  0;
    end

 else
```

```
        hu    = hu5;
        tmin =  tmin5;
        cmin=  cmin5;
        cR   =  cR5;

        if d5  = =  1
           d   =  1;
        else
           d  =  0;
        end
  end

     R  =  reliability (hu,valT,valN,d);
     [maincost,totmindt]  =  costandtmin (hu,valT,valN,valdel,valB,d,cmin,tmin,…
                                    cR);
     if valn  = =  1
        fprintf ('R(%d) =%d\n',valn,R);
      elseif valn  = = 2
        fprintf ('R(%d) =%d\n',valn,R);
      elseif valn  = =  3
        fprintf  ('R(%d) =%d\n',valn,R);
      elseif valn  = =  4
        fprintf ('R(%d) = %d\n',valn,R);
      else
        fprintf ('R(%d) = %d\n',valn,R);
      end
     totminrept   =  double(totmindt+totminrept);
     totmaincost =  double(maincost+totmaincost);
  end

totmaindowntime  =  maintenancedownt (valN,valdel,valT,d1,d2,d3,d4,d5);
EC  =  (totmaincost+cg)/(valN*valT+totmaindowntime+TG);
Av  =  double((valN*valT-totminrept)/(valN*valT+totmaindowntime+TG));
fprintf ('total cost = %d\n',EC);
```

```
fprintf ('Av = %d\n\n',Av);
end
```

Subroutine 1: Compute reliability

```
function R  =  reliability (hu,valT,valN,d)

syms u y

Gy  =  y/valT;
dGy =  diff(Gy,y);
if d  = = 0
       HT = int(hu,u,0,valT);
       R   =  double(exp(-HT));

  else     %(imperfect repair)

       RTN  = 1;
       HT     = int(hu,u,0,(valN*valT-(valN-1)*y));
       HdT    =  int(HT*dGy,y,0,valT);
       R       =  double(exp(-HdT));
end
```

Subroutine 2: Compute maintenance cost and time

```
function [maincost, totmindt]  =  costandtmin(hu,valT,valN, valdel, valB,…
                                               d,cmin,tmin,cR)
syms u y

Gy   = y/valT;
dGy = diff(Gy,y);
totmindt  =  0;
maincost =  0;
```

```
 if d = =  0 %(replacement)
       HT          =  int(hu,u,0,valT);
       mincost  =  valN*cmin*HT;
       maincost =  mincost+cR*(valN-1);
       totmindt  =  valN*tmin*HT;

else     %(imperfect repair)

%for cost
       minr   =  0;
       mindt =  0;
       maindtime  =  0;
       totdt           =  0;
       maincost     =  0;

   for i     =  1:valN
       vali = i;
       h1u = subs(hu,u,(vali-1)*(valT-y)+u);
       H1u = double(int(int(h1u*dGy,y,0,valT),u,0,valT));
       minr = minr+H1u;
   end

mincost  =  cmin*minr;
totmindt  = tmin*minr;

% maintenace cost
   maincost=0;

   for k  =  1:valN-1
       valk =  k;
       det   =  (valk)^(valdel-1);
       main =  cmin+double(int(valB*y*det*dGy,y,0,valT));
       maincost =  maincost+main;
   end
```

```
   maincost   =  mincost+maincost;
end
```

Subroutine 3: Compute maintenance downtime

```
%maintainance downtime
function totmaindowntime  =  maintenancedownt(valN,valdel,valT,d1,d2,d3,d4,d5)

syms y n k

   Gy   =  y/valT;
   dGy  =  diff(Gy,y);

   timp1 =  0.12;
   trep1  =  90;
   timp2 =  0.12;
   trep2  =  90;
   timp3 =  0.12;
   trep3  =  90;
   timp4 =  0.12;
   trep4  =  90;
   timp5 =  0.12;
   trep5 =  90;
  totmaindowntime  =  0;

  for k  = 1:valN-1
   downtimenew  =  0;
   maindowntime =  0;

   if  d1= =0 && d2= =0 && d3= =0 && d4= =0 && d5= =0

          max12   =  max(trep1,trep2);
          max123  =  max(max12,trep3);
          max1234 = max(max123,trep4);
```

```
        maindowntime = max(max1234,trep5);

  elseif (d1= =0 |d2= =0 |d3= =0|d4= =0 |d5= =0) && (d1= =1 |d2= =1 |d3=
=1|d4==1 |d5= =1)

     for n = 1:5
        if n = = 1
           d = d1;
           trep = trep1;
           timp = timp1;

        elseif n = = 2
           d = d2;
           trep = trep2;
           timp = timp2;

           elseif n = = 3
           d = d3;
           trep = trep3;
           timp = timp3;

            elseif n = = 4
            d = d4;
            trep = trep4;
            timp = timp4;

        else
           d = d5;
           trep = trep5;
           timp = timp5;
        end
      if d = = 0
        downtime = trep;
```

```
        else
          valk = k;
           dot = (valk)^(valdel-1);
           maindtrep = double(int((valdel*timp)*y*dGy* dot,y,0,valT));
           downtime = maindtrep;
        end
      downtimenew  = downtime;
       maxdowntime = max(downtime,downtimenew);
end

  maindowntime   =  maxdowntime;

 else
          tmax12   =  max(timp1,timp2);
          tmax123 =  max(tmax12,timp3);
          tmax1234  =  max(tmax123,timp4);
          tmax12345=  max(tmax1234,timp5);
          valk  =  k;
          dot   =  (valk)^(valdel-1);
          maindowntime  =  double(int((valdel*tmax12345)*y*dGy* dot,y,0,valT));
end
  totmaindowntime  =  totmaindowntime+maindowntime;

  end
```