THE INDUSTRIAL ORGANIZATION OF INFORMATION GOODS INDUSTRIES

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SUMMARY

This thesis applies the theories and approaches of industrial organization to study three key issues pertaining to information goods industry.

The first study investigates the incentives of a monopolistic vendor to delay the introduction of a new and improved version of its product in a stationary market with identical consumers. It shows that the vendor's monopoly power is constrained by the mutual cannibalization between the successive generations of products. By deferring sale of the new product, the vendor may charge consumers higher prices for both the old and new products. I characterize the equilibria with delayed introduction, and study their changes with respect to market and product parameters. In particular, I suggest that delayed introduction could occur regardless of whether the seller can offer upgrade discounts to consumers, that instead, it is related to quality improvement brought about by the new product, durabilities, and discount factors.

The second study is about a vendor's strategies to tackle its own installed base when selling a newly improved product. In particular, I investigate the optimal combinations of timing, pricing and product line strategies that the vendor can employ for selling its newly improved product in the presence of an installed base. I characterize the market with either a partly- or fully- covered installed base in terms of consumers' relative willingness to pay for the newly improved version and their relative payoffs from delayed purchase across periods. Different from the conventional proposition of constant consumer reservation price, I propose that if consumers already own an existing (old) version of a durable product, their willingness to purchase the newly improved version indeed increases over time! This effect, interweaving with consumer heterogeneity on valuation of quality and on purchase history may enable perfect intertemporal price discrimination.

Extending the prior research on upgrade pricing to a more general setting, I find that upgrade pricing is not able to segment consumers with different purchase history when consumer heterogeneity is sufficiently high. In that case, instead of upgrade pricing, the vendor would maximize its profit via intertemporal price discrimination, or delayed introduction, or pooling pricing, depending on the characteristics of market structure and technology improvement.

Overcoming the intractability of addressing delayed product introduction in a market with heterogeneous consumers, this study analytically confirms Fishman and Rob's speculation (2000) that the heterogeneity in consumers' valuation of quality may discourage vendor's incentive to launch a new product. I find that two forces may induce the vendor to delay selling the newly improved product: one is the cannibalization of the stock of durable goods in consumers' hand (Hui and Wang 2005); the other is the consumers' anticipation of future price reductions. Particularly, the latter can lead to delayed introduction even when the extent of quality improvement embodied in the new product is high.

The third study is about information security, in particular, the strategic interactions among end-users and between users and hackers. It shows that security efforts by end users are strategic substitutes. This explains the inertia among end-users in taking precautions even in the face of grave potential consequences. Next, by encompassing both direct and indirect effects, this study suggest that reducing user cost of precautions or increasing enforcement against hackers need not enhance overall information security because of the feedback effect through the actions of the other side of the market. Third, the welfare analysis suggests that policy should focus on facilitating user precautions if the users' benefit relative to the cost of precaution and the hackers' expected enjoyment relative to targeting cost are sufficiently high. Finally, we argue for appropriate international authority to make and coordinate policy across borders to resolve international externalities.

These three studies demonstrate that the theories and models developed in traditional industrial organization are effective approaches to study the market-related issues that are enabled by or specific to information goods industry. Further, studying the intriguing relationships between information technology and market structure expands the prior theories and models of industrial organization and opens up avenues of future research.

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CHAPTER 1 INTRODUCTION

1.1 General Background

Industrial organization is the branch of economics that studies the structure of firms and markets and of their interactions (Carlton and Perloff 2005, Pepall, Richards and Norman 2005). This thesis applies the theories and approaches of industrial organization to study three key issues pertaining to the information goods industry. One is about the incentive of a monopolistic vendor to delay the introduction of a new and improved version of its product. The second is about a vendor's strategies to tackle its own installed base when selling a newly improved product. The third is about information security, in particular, the strategic interactions among end-users and between users and hackers.

Industrial organization addresses the imperfect competition using the structure-conduct-performance paradigm as descriptive approach and applying game theory in the analysis of strategic interaction (Carlton, et. al 2005, Pepall et. al 2005). The nature of technology and demand for a product as the basic condition shapes the market structure which in turn influences the conduct of market participants and determines an industry's performance.

An information goods industry is distinct from traditional industries in several aspects. First, production of information goods typically involves high fixed costs and low marginal costs. This is true not just for pure information goods which are immaterial, but even for physical goods such as silicon chips. The specific cost structure leads to significant market power and then monopolistic competition in most information goods markets (Varian 2004). Second, information goods either are made up of "bits" or work in the digital form. The lack of physical constraint facilitates rapid pace of innovation in information goods industry. A new improved successor may render the existing product technologically obsolete well before it is functionally obsolete (Fishman and Rob 2000, Varian 2004). Third, an information goods industry is featured with high interdependency. On the supply side, many functions may be implemented through the connection and cooperation with its complementary products. On the demand side, the benefits that consumers derive from the product may depend on the size of the existing user base. Fourth, information technology provides the convenience for information access and communication. This enables various lucrative business models and improves social efficiency; however, the widespread use of Internet also poses serious threats to security (Whitman 2003).

The above differences determine the unique structure-conduct-performance matrix in an information-goods market. Application of game theory to information goods markets can capture the essential features of the interaction among the market participants, and make clear the underlying structure and the principles governing the market outcome (Carlton, et. al 2005, Pepall et. al 2005). Focusing on different issues, the three essays in this thesis follow this approach and extend the research of industrial organization to information goods industry.

1.2 Delayed Product Introduction

The first essay applies a stylized economic model to investigate the incentives of an information-goods vendor to delay the introduction of a new and improved version of its product.

Facing the rapid pace of information technology (IT) development, IT manufacturers often seem to hesitate in launching new and better products with cutting-edge technologies. We have witnessed the introduction of wideband third-generation (3G) mobile phone services long after the technologies for 3G cellular networks and mobile phones became available several years ago. Similar observations can be found in the markets of DVD video recorders, stereo systems integrating MP3 decoder or home electronic products that adopt MPEG-4 standard, where the vendors seem reluctant to launch new products that incorporate better technologies. Given these intriguing observations, it is interesting to understand the strategic decisions of a vendor who currently sells an existing product, and who can choose whether to sell a new product with better technologies. Specifically, we address the following research questions: Suppose a new technology that improves the quality of an existing product is invented, and applying it to a new product does not involve prohibitively high costs. Would a vendor have incentives to deliberately delay selling the new product? If so, under what circumstances would the vendor adopt such a strategy?

The research about delayed product introduction is closely related to three streams of work in the literature. One of them studies product timing; another focuses on market segmentation and price discrimination; the third examines the monopolist's incentive to plan for product obsolescence. However, the prior research has not formally incorporated the strategic interactions between vendor and consumers when addressing the economic considerations of delayed product introduction. This could be partially due to the intrinsic limitation of the two-period model, which has been widely employed in strategic analysis of new product introduction (Dhebar, 1994, Waldman 1996b, Lee and Lee 1998, Kornish, 2001). As an effective approach to answer the "*yes*/*no*" question about the launching of the newly improved product, the conventional two-period model is unable to explicitly address the important issue of whether to launch it *now* or *later*. In addition, most of the existing studies focus on traditional industrial products, and hence their models cannot capture an important characteristic pertaining to IT-intensive products: they are physically durable; but technologically have much shorter lifespan (Dhebar 1996).

In Chapter 2, we use a three-period model which closely follows the spirit of the classical durable goods monopolist literature to answer the above research questions. It shows that the vendor may prefer to delay introducing a new product, even though the enabling technologies for the product are already available. The underlying motivation is analogous to that found in the durable goods monopolist literature – the vendor suffers from a time inconsistency problem that causes its old and new products to cannibalize each other. Without the ability to remove existing stock of the old product from the market, shorten product durability, or pace research and development (R&D), it may respond by selling the new product later. This study characterizes the equilibria with delayed introduction, and studies their changes with respect to market and product parameters. In particular, it suggests that delayed introduction could occur regardless of whether the vendor can offer upgrade discounts to consumers, that instead, it is related to quality improvement brought about by the new product, durabilities, and discount factors.

1.3 Technology Timing and Pricing in the Presence of an Installed Base

The second essay investigates an IT vendor’s strategies to tackle its own installed base when selling a newly improved product. By simplifying the three-period model into a two-period game, this study extends the first essay to a market with consumers differing in valuation of product quality and purchase history. Other than delayed product introduction, this essay is motivated by the sluggish demand caused by the installed base -- a more general concern in the information goods industry. It considers a much broader set of timing and pricing strategies that the vendor can employ in selling the newly improved product in the presence of an installed base of the old product. Delayed product introduction, together with intertemporal price discrimination, pooling pricing and premium pricing, becomes one of the outcomes of the strategic interaction between vendor and consumers under the impact of technological obsolescence and existing installed base.

Firms in the information technology (IT) industry face a paradox of rapid technological progress: To sustain ongoing industry leadership, a firm should strive to develop the next best technology (Mohr, Sengupta and Slater 2005). However, on one hand, the firm's newly improved technology renders obsolete its older technology, further contributing to competitive volatility. On the other hand, the installed base of its own products adopting the older technology turns to be a formidable competitor to the new technology, particularly when technological progress outpaces users' capacity of fully utilizing technology (Varian 2004). In the personal computer (PC) and mainframe industry, a major concern for vendors is to tackle the reluctance of individual or business users to replace old PCs or mainframes by newer ones (McDonald 2006). Even for computer software, with more than 90% of PCs running some sort of Windows, Microsoft has long considered its main competitor to be the installed base of its own products (Berlind 2005).

Aware of the baulking consumers caused by the rapid technology improvement (Dhebar 1996), prior research in new product introduction suggests that to attract consumers to upgrade, vendor has the incentive to reduce product durability (Bulow 1986, Waldman 1996b). If the vendor is unable to artificially shorten the durability of its products, offering upgrade price discounts to existing consumers may raise its profit and at the same time attain socially optimal outcomes (Waldman 1997, Fishman and Rob 2000).

Compared to the analysis of shortening durability or its variations, studies on timing and pricing strategies to cope with installed base are lacking. Most of the existing research on product line introduction employs a two-period framework, which assumes fixed introduction timing for the new product (launching in the second period or not launching) (e.g., Fudenberg and Tirole 1998). This restricted setting constrains the vendor's wisdom in selling the new product facing a certain installed base of its older product: it can induce consumers' self-selection of whether or not to purchase the new product in the second period, but not *when* to purchase it. This is especially unrealistic considering the fact that vendors often use timing to segment the market. Recurrent model or continuous-time models have also been applied in studying technology innovation and product introduction. However, to make the models tractable, the researchers either assume consumers are homogeneous in valuation of product quality (Fishman and Rob 2000), or just study a single product introduction (Stokey 1979).

Generally, the theoretical limitations of prior research lie in the followings.

- First, previous studies mostly focus on static analysis of the demand in a market consisting of consumers differing in valuation of product quality and purchase history. Little research has addressed the same issue in consideration of the time dimension. The complexity of the demand side lies not only in the heterogeneity among consumers, but also the market as a carrier of history and the future. It is unclear how existing consumers' intention to upgrade changes over time, and how the time trend of intention to upgrade differs among consumer segments.

- Second, given the differing demand for upgrade, upgrade pricing has never been studied along the time dimension. In particular, how should vendors respond to the demand variation by leveraging intertemporal or intra-temporal price discrimination?
- Third, it has long been considered that with upgrade pricing, vendors will adopt socially efficient strategies (Waldman 1997, Lee and Lee 1998, Fishman and Rob 2000). In this study, with a more general setting that encompasses consumer's heterogeneity in valuation of product quality (cf. the high heterogeneity setting in Lee and Lee (1998), or the homogeneous setting in Fishman and Rob (2000)), are there situations where upgrade pricing loses its power to segment the market? If so, can vendors sustain their monopoly power using time as a discrimination instrument?

In Chapter 3, we address the above research questions, and study alternative pricing and product line strategies such as intertemporal price discrimination, delayed product introduction, and upgrade pricing instead of changing durability to alleviate the cannibalization from the installed base, which has often been advocated in the literature. Specifically, we investigate the optimal combinations of timing, pricing and product line strategies that the vendor can employ for selling its newly improved product in the presence of an installed base. We characterize the market with either a partly- or fully- covered installed base in terms of consumers' relative willingness to pay for the newly improved version and their relative payoffs from delayed purchase across periods. Different from the conventional proposition of constant consumer reservation price, we propose that if consumers have already owned an existing (old) version of a durable product, their willingness to purchase the newly improved version indeed increases over time. This effect, interweaving with consumer heterogeneity on valuation of quality and purchase history, may enable perfect intertemporal price discrimination.

Extending the prior research on upgrade pricing to a more general setting, we find that upgrade pricing is not able to segment consumers with different purchase history when consumer heterogeneity is sufficiently high. In that case, instead of upgrade pricing, the vendor would maximize its profit through intertemporal price discrimination, delayed introduction, or pooling pricing, depending on the characteristics of market structure and degree of technology improvement.

Overcoming the intractability of addressing delayed product introduction in a market with heterogeneous consumers, this study analytically confirms Fishman and Rob's speculation (2000) that the heterogeneity in consumers' valuation of quality may discourage vendor's incentive of launching a new product. We find that two forces may induce the vendor to delay selling the newly improved product: one is the cannibalization of the stock of durable goods in consumers' hand (Hui and Wang 2005); the other is the consumers' anticipation of future price reductions. Particularly, the latter can lead to delayed introduction even when the extent of quality improvement embodied in the new product is high.

Without the concern about cost, social welfare directly depends on whether the vendor can sustain its monopoly power facing the mutual cannibalization between the old and new products, and the mutual arbitrage between the heterogeneous consumers[1].

1.4 Information Security: User Precautions and Hacker Targeting

The third essay analyzes the strategic interactions among end-users and between users and hackers. Information security is a critical issue of both national policy and

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[1] Suppose there are two groups of consumers in a market, one group has higher valuation on product quality than the other. The mutual arbitrage between heterogeneous consumers refer to the situations in which either group of consumers have the incentive to accept the price and purchase timing that are originally assigned by firm for the other group.

business operations (Whitman 2003). For instance, in May 2004, Sven Jaschan created the Sasser worm to exploit a vulnerability in the Windows 2000 and XP operating systems. The Sasser worm and its variants caused hundreds of thousands of PCs to crash (ZDnet 2005). In August 2003, the Microsoft Blaster worm exploited a vulnerability in Windows 2000 and XP to infect hundreds of thousands of computers, from which it launched a "denial of service" attack on the Microsoft Windows Update server (Register 2003). During the summer of 2001, the "Code Red" worm and its successor "Code Red II" exploited a vulnerability in the Microsoft Internet Information Server to cause over $2 billion in damage (Moore et al. 2002). The threat of attack and intrusion now extends to mobile phones (Symantec 2005).

Information security depends on user efforts – to fix vulnerabilities, install and update software to detect neutralize viruses and other malicious software, install and configure firewalls, take care with file-sharing programs and email attachments, etc. Security is a critical issue only because of the activities of (unethical) hackers. Industry has systematically tracked hacker behavior: "Attackers continuously look for easy targets, those that will provide them with the maximum return on the time they invest in writing malicious code" (Symantec 2005, page 55). Clearly, hacker activity depends on user behavior.

While there has been some research into the incentives of end-users (Kunreuther and Heal 2003; August and Tunca 2005), and the motivations of hackers (eg, Jordan and Taylor 1998; Van Beveren 2000), there has been little scholarly attention to the strategic interaction between end-users and hackers.

In Chapter 4 of this thesis, we analyze the strategic interactions among end-users and between end-users and hackers. We address several questions in particular. First, it is well known that information security poses grave potential consequences.

Yet, end-users seem quite slow to take precautions (Boss 2005) – to the point that they must be exhorted and goaded by government and vendors (US-CERT 2006). What explains this inertia?

Second, given the strategic interactions, how does information security vary with changes in the user cost of precaution and the rate of enforcement against hackers? This question is not trivial. For instance, a reduction in the user cost of precaution would directly lead users to increase precautions. However, that would make them less attractive targets, and so induce hackers to reduce their targeting, and hence, indirectly lead users to reduce precautions. Accordingly, the net effect depends on the balance between direct and indirect effects.

Third, information security can be and is addressed from two angles – facilitating end-user precautions, and enforcement against hackers. Both policies are costly. Owing to the strategic interaction, facilitation of user precautions will affect hacker behavior, and enforcement against hackers will affect user behavior. From the standpoint of social welfare, what is the right balance between the two classes of policy?

Fourth, viruses and worms do not respect international borders. The Sasser worm illustrated the asymmetric distribution of hackers vis-à-vis users across countries. How should governments address information security when threats cross borders?

This study shows that security efforts by end users are strategic substitutes. This explains the inertia among end-users in taking precautions even in the face of grave potential consequences. Next, we analyze the direct and indirect effects of changes in the user cost of precaution and the rate of enforcement against hackers. For instance, a reduction in the user cost of precaution would directly lead users to increase precautions. However, that would make them less attractive targets, and so induce hackers to reduce their targeting, and hence, indirectly lead users to reduce fixing. Next, we study welfare implications. We show that policy should focus on facilitating user precautions if the users' benefit relative to the cost of precaution and the hackers' expected enjoyment relative to targeting cost are sufficiently high. Finally, we argue for appropriate international authority to make and coordinate policy across borders to resolve international externalities.

1.5 Contribution

1.5.1 Potential Contribution of Delayed Product Introduction

The study about delayed product introduction can provide useful insights for managers of technological products. Popular examples of such products include personal computers, audio-visual equipment, communication tools, and specialized software (e.g., econometrics or statistics software). Vendors of these products often cannot control the schedule of new technologies' arrivals and the obsolescence of old products. Hence, for them, product innovation and introduction are two separate decisions – they might not be able to endogenize the extent of product innovation, but they could always control whether and when to sell new products. Because of this separation of sale from innovation, it is interesting to study whether it is socially optimal for a vendor to defer introducing a new product — an insight that cannot be obtained in prior studies of product introduction (Fishman and Rob 2000; Lee and Lee 1998; Waldman 1996a). This study can also explain why vendors do not deploy new and superior technologies to create new products in some markets. The inclusion of durability as a model parameter allows us to extend our insights directly to products that exhibit different life spans (perhaps due to high dependencies on external parts, technologies, or trends).

This study can contribute to the literature of technology adoption by analyzing both the vendor's and consumers' economic incentives of adopting a new and better technology. Venkatesh and Brown (2001) suggest that rapid technology improvement and the fear for obsolescence have been the major concerns that dissuade consumers from purchasing a product. Based on their insights, this study moves the literature forward by measuring obsolescence through the quality improvement embodied in the new product and the duration over which the product can provide usable services. This setting can capture the nature of the information goods industry where obsolescence does not just result from being superseded by technologically superior successors within the product; it could also happen because of shrinking operational lifespan due to continuously updated external environments, such as peripheral components, communications standards, or hardware and software architectures. By extending the theories developed in the durable-goods monopoly literature [2], we show that delayed product introduction can be a strategic solution for vendors facing consumers who are disconcerted by the fast-paced IT industry[3].

1.5.2 Potential Contribution of Technology Timing and Pricing in the Presence of an Installed Base

This study can provide both theoretical and practical contributes to the literature of new product introduction.

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[2] For an excellent summary of this literature, see Waldman (2003).

[3] As stated by Dhebar (1996, p37), "The rapid introduction of new and improved versions can make a consumer regret a previous purchase, hesitate over any new purchase, and agonize over similar purchase in the future."

- First, it relaxes the conventional assumption that all consumers possess nothing at the beginning of the game (Kornish 2001) and generalizes consumer's utility function by simultaneously incorporating the time dimension, valuation of product quality, and purchase history into the utility function. Hence, it can characterize consumers' purchase patterns over time by considering various scenarios pertaining to consumers' possession of a low-quality product as an initial condition.
- Second, it overcomes the limitation of two-period models and enables the study of flexible product introduction schemes as the combinations of possible timing, pricing, and product line strategies. Other than upgrade pricing, the vendor can choose intertemporal price discrimination, pooling pricing or premium pricing combining with immediate or delayed introduction timing to alleviate the cannibalization due to its own installed base.
- Based on the above approaches, the study of the strategic interaction between vendor and consumers can provide insights on the optimal product introduction scheme for vendor to cope with the existing installed base. This will provide useful guidelines for managers who often have to consider the installed base of their existing products when selling new products. Instead of transforming their business models to control for product durability (as suggested by Bulow 1986, Waldman 1996b), they may adopt flexible timing and pricing strategies. Failing to select the right product line and pricing strategies, they may gravely suffer from the combined dampening effects of the stock of durable goods in consumers' hand, and consumers' anticipation of future price reductions. By contrast, via proper timing, pricing, and product line strategies, the vendor may even be able to practice perfect price discrimination by utilizing the existing consumers' increasing need to upgrade to the new product.

1.5.3 Potential Contribution of Information Security: User Precautions and Hacker Targeting

This study develops a fairly general model of the strategic interaction among end-users in taking security precautions and also the interaction between users and hackers. In a setting with a continuum of user types, this study shows how users' choice of purchase and their effort in fixing depend on hackers' targeting and vice versa. The analysis of the direct and indirect effects of changes in the user cost of precaution and the rate of enforcement against hackers can provide empirical implications as well as recommendations for public policy. While a setting of information security is considered, the analysis can generally apply to any situation in which potential victims take precautions against attack by others.[4]

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[4] See, for instance, Koo and Png (1994) and Kunreuther and Heal (2003).

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CHAPTER 2 DELAYED PRODUCT INTRODUCTION

2.1 Introduction

Technologies for wideband third-generation (3G) cellular networks and mobile phones have been available for several years, yet many telecommunications companies are still reluctant to provide 3G mobile phone services. The use of DVD media for recording, storing and retrieving voluminous data has been popular since the beginning of this century, but it was only recently that we started to see DVD video recorders being actively promoted by hardware vendors. The MP3 compression format for digital music has been well developed since the late 1990s, but it took several more years before we saw vendors of stereo systems include in their products MP3 decoders that read and playback MP3 files on CDs. Similarly, we have yet to see widespread use of the MPEG-4 standard in home electronic products, even though it became an international standard in the year 2000. Why do hardware vendors seem to hesitate in launching new and better products with cutting-edge technologies?

The examples above share a number of characteristics. First, the research and development (R&D) of the new technologies were often pioneered by independent researchers or companies, not by the hardware vendors who apply the technologies to their products. Therefore, the hardware vendors may not control when the new technologies become available. Second, many hardware vendors who consider using the new technologies sell products that incorporate previous generations of similar technologies. For example, most telecommunications companies currently provide mobile phone services on second-generation cellular networks and handsets; many vendors of DVD video recorders are also major vendors of conventional videotape recorders. Third, the markets for products that incorporate these new technologies, or earlier generations of similar technologies, tend to be concentrated – they are often dominated by a small number of large vendors. Fourth, products that use these (new or earlier-generation) technologies are mostly durable. They supply long streams of services to consumers, and the quality of services does not significantly drop over time. Hence, consumers often take into account the durability of such products when making purchase decisions. Finally, the functional values of products that use earlier-generation technologies are not affected by the presence of products with the new technologies. If consumers do not appreciate the new technologies, they can ignore the new products and continue using the products they have previously purchased.

Given these common characteristics and the intriguing observation that some vendors seem reluctant to launch new products that incorporate better technologies, it is interesting to understand the strategic decisions of a vendor who currently sells an existing product, and who can choose whether to sell a new product with better technologies. Specifically, we address the following research questions: Suppose a new technology that improves the quality of an existing product is invented, and applying it to a new product does not involve prohibitively high costs. Would a vendor have incentives to deliberately delay selling the new product? If so, under what circumstances would the vendor adopt such a strategy?

We use a stylized economic model which closely follows the spirit of the classical durable goods monopolist literature[5] to answer the above research questions. Our model consists of three periods, and each period comprises two stages. In the first stage of each period, a monopolistic vendor makes product and pricing decisions. In

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[5] Representative works in this literature include Bulow (1982, 1986), Dhebar (1994), Fishman and Rob (2000), Fudenberg and Tirole (1998), Kornish (2001), Lee and Lee (1998), Levinthal and Purohit (1989), Purohit (1994), Stokey (1981), and Waldman (1993, 1996a, 1996b). For an excellent summary, see Waldman (2003).

the second stage, consumers observe the offers made by the vendor and decide whether to buy the product. The vendor can only sell a low-quality product in the first period. In the second period, owing to external R&D, a new technology arrives that enables him to produce and sell a high-quality product. The vendor has to choose among three courses of action: (a) sell the high-quality product immediately in the second period; (b) sell the high-quality product in the third period; or (c) do not sell the high-quality product in either period. Option (b) corresponds to delayed introduction – the vendor purposely chooses not to sell a better product in an earlier period, even though the product is available and it could feasibly sell it to consumers. We further allow the low- and high-quality products to exhibit various degrees of durability. This facilitates generalizations of our findings to different technological products. The chosen market structure and model features resemble the characteristics of the examples that we have raised (3G mobile phone services, DVD video recorders, etc.).

In our model, we find that under a wide range of conditions, the vendor has incentives not to sell the new (high-quality) product immediately. This is because the existing stock of the old (low-quality) product that has been sold to consumers limits its ability to charge a high price for the new product (i.e., the old product cannibalizes the new product). Further, the expectation that there is going to be a new product in the future may dampen the incentives of consumers to buy the old product. Unless the price of the old product is low, consumers may prefer to wait and buy the new product that promises better quality (i.e., the prospect of a new product in the future cannibalizes the old product). It is these intertemporal cannibalizations between the old and new products that lead the vendor to delay selling the new product.

Specifically, by deferring sale of the new product, the vendor extends the economic life span of the old product, which increases its value to consumers. It also allows the old product to be used for one more period, and hence the old product depreciates more in value; consumers who have bought it in an earlier period would then be willing to pay more to upgrade to the new product in a later period. This allows the vendor to charge a higher price for the new product and earn more profit.[6]

To provide useful strategic insights, we analyze our model in two separate scenarios. In the first scenario, the vendor cannot implement an upgrade policy. Hence, all consumers must pay the same price to acquire the new product. In the second scenario, the vendor can implement an upgrade policy that allows consumers to trade in the old product for the new product at a discounted price. Compared with other consumers, those who own the old product pay less to enjoy the new product (in other words, the vendor can practice price discrimination based on the purchase history of consumers). We find that the vendor chooses different product and pricing strategies in these two scenarios. Generally, it prefers the upgrade policy because it allows him to convince all consumers to purchase the old product as soon as possible. The provision of an upgrade option also leads to socially efficient outcomes.

Regardless of whether an upgrade policy is provided, however, delayed selling of the new product is always optimal for the vendor with some combinations of product and consumer characteristics. Therefore, the provision of an upgrade policy may not be the key determinant in the introduction timing of next-generation technological products (cf. Fishman and Rob 2000). Instead, we find that the vendor's choice of whether to delay selling the new product is related to product durabilities,

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[6] In practice, when deciding whether to upgrade a technological product (e.g., personal computer), consumers often need to assess the remaining service values of their existing products. If the existing products are expected to have short life spans, consumers may be willing to pay more to upgrade to new products.

extent of quality improvement, and the discount factors. These results are robust to changes in the vendor's ability to credibly commit to product strategies.

Our study provides useful insights for managers of technological products, especially those that depend heavily on components developed by external vendors. Popular examples of such products include personal computer, audio-visual equipment, communication tools, and specialized software (e.g., econometrics or statistics software). For vendors of these products, delaying the sales of new products that use better technologies may sometimes be beneficial because of the alleviation of intertemporal cannibalizations. Further, if a vendor is hesitant about when to sell a new product, our model suggests that it should evaluate the durabilities of its products and the improvement in quality that the new product will bring to consumers. Knowledge of whether consumers are patient is also useful in this context.

The remainder of this chapter is organized as follows. Section 2.2 reviews previous research that is related to delayed introduction of new products. Section 2.3 presents our research model. Section 2.4 outlines the analysis and characterizes all equilibria in the studied markets. Section 2.5 relaxes a few assumptions of the model. Section 2.6 considers the case with no commitment. Section 2.7 concludes the chapter.

2.2 Prior Literature

The literature on technology diffusion, adoption and strategic management has suggested that fear of obsolescence may cause consumers to hesitate or refuse to buy current technological products (Cohen et al. 1996; Dhebar 1996; Venkatesh and Brown 2001). The concern of consumers about product obsolescence is particularly noteworthy in high technology markets because the usability of such products is often governed by external technological progress, standards or architectures (Morris and Ferguson 1993; West and Jason 2000). Evidently, hardware products suffer from wear and tear, and sometimes, changes in communications standards with peripheral technologies.[7] Even software products that are supposedly perfectly durable could at times be superseded because of updates in processor instruction sets or operating systems. Therefore, to avoid getting products that could soon be obsolete, consumers may wait for new products and defer making purchases – this is often called leapfrogging in the literature.

In response to the threat of leapfrogging by consumers, firms that sell multiple generations of similar products may wish to slow down the pace of new product introduction. This could serve two purposes. First, delayed introduction may help dissuade consumers from waiting for new products and accumulate potential buyers for future products (Putsis 1993). Second, it may lessen the regret of consumers who have bought old-generation products and persuade them to switch to new products in the future (Dhebar 1996).

Various theoretical studies have responded to the above observations and modeled the timing decisions of firms that sell multiple generations of similar products. Specifically, by studying the decisions of a monopolist in a two-period framework, Dhebar (1994) and Kornish (2001) conclude that a vendor may defer selling new products because if product introduction occurs too frequently, it is difficult to make all products appear attractive to consumers. They have not, however, formally characterized any equilibrium that involves delayed introduction of new products. Chatterjee and Sugita (1990) and Radas and Shugan (1998) show that delayed introduction is optimal when demand is uncertain, or when demand is seasonal

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[7] For example, the growing popularity of the universal serial bus (USB) interface has rendered many computer peripherals that use old communication interfaces, such as parallel or serial ports, obsolete. Similarly, rapid changes in processor and mother board architectures have made old-generation random access memory (RAM) chips incompatible with new PCs.

and expanding. Their results stem from specific assumptions on demand structures and firms' knowledge. By contrast, we show that even if demand is constant and a vendor has perfect knowledge about consumers, delay is still an optimal strategy in many circumstances.

Our study is perhaps closest to that of Fishman and Rob's (2000), which illustrates that in a continuous-time framework with homogeneous consumers, no upgrade policies, and perfectly durable products, a monopolist's rate of product innovation would be too low, which could cause inefficient delays in new product introductions. They show that with an upgrade policy, however, the vendor would not delay new product introductions,[8] which is different from what we report in this study. Our setting differs from that of Fishman and Rob's in three aspects. First, in their setting, product innovation involves R&D costs, and new products are launched as soon as they become available. Hence, their concept of delay is that of innovation, not of introduction per se. In our case, product innovation is exogenous and fixed; we focus on the vendor's strategic choice of when to sell a new product that has just become available – a decision that follows product innovation and does not involve R&D costs. Second, they consider only perfectly durable products. By contrast, we include durability as a model parameter, and that allows us to extend our insights to different technological products, especially those that work closely with base products (as in computer software and hardware) or peripheral components (see footnote 3 and the related discussion above). Third, they do not explicitly demonstrate that the vendor would prefer a strategy with delayed product introduction. By contrast, we illustrate through comprehensive evaluations of possible strategies that under a wide range of

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[8] Lee and Lee (1998) derive similar conclusions in a two-period model with two groups of consumers.

conditions, the vendor would prefer delayed introduction owing to revenue but not cost considerations.

2.3. Basic Setting

Consider a monopolistic vendor who is planning to sell two versions of a product over three periods, $t = 1$, 2 or 3. In period 1, it can only sell a low-quality product, L, with quality q_L. Owing to R&D, a new technology arrives in period 2, which allows him to sell a new product, H, with quality $q_H > q_L$, in either period 2 or 3. For ease of presentation, we normalize q_H to one, and hence $0 < q_L < q_H = 1$. Both the old (low-quality) and new (high-quality) products are of the same durability $n \geq 2$ periods. We shall relax this assumption and allow for different durabilities later in Section 2.5. We further assume zero fixed and marginal costs to focus on the strategic choices of the vendor in response to market demand.[9]

On the demand side, consumers are homogeneous, and we normalize their size and valuation of quality to one. Each consumer demands at most one unit of each version of the product. Within its life span, the product provides a constant stream of service to consumers; once consumers buy it, they enjoy a value that equals its quality in each period of service until it is retired (either because it is replaced by a newer product or because it has exceeded its physical life span). There is no second-hand market; hence, as soon as consumers buy a new product, their old products are retired and provide zero usage or residual values. We use δ to denote a discount factor, which is common to both vendor and consumers. We shall relax this assumption in Section 2.5. Note that $0 \leq \delta \leq 1$. The larger δ is, the smaller the discount in future utilities or prices will be.

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[9] We shall discuss the implications of positive marginal costs in Section 2.5.

Each period in the model further consists of two stages. In the first stage, the vendor makes product and pricing decisions based on its knowledge of consumer profiles (how many people bought the products in previous periods, the utilities they derive from the products, etc.). In the second stage, consumers make purchase decisions, taking into account their valuations for the products and expectations about future products. There is common knowledge on demand, product quality, and technological improvement. Perfect information on history of moves by the vendor and consumers is available. We focus on rational expectations equilibria in which consumers form expectations about the product and pricing decisions of the vendor, and the vendor fulfills such expectations.

We use a tuple $\{\cdot, \cdot, \cdot\}$ to represent the product strategies of the vendor in the three periods. For example, $\{L, \text{-}, H\}$ indicates that the vendor sells the old product in the first period, does not sell any product in the second period, and sells the new product in the third period. Similarly, $\{L, H, \text{-}\}$ denotes a similar strategy except that the new product is sold in the second period instead of the third. Because consumers have identical valuations for products, the vendor would sell only one product in each period. Further, once a product is sold in a period, it would not be sold in subsequent periods.[10]

For each product strategy, the vendor needs to devise a price schedule. An equilibrium is sub-game perfect if the vendor receives optimal profit from its product and price schedules, and if the schedules are consistent with consumer expectations. We use p_t^i to denote the price of product i at time t, where i = L, H and t = 1, 2, or 3,

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[10] In a model where consumers have different valuations for products, or where new consumers enter the market in subsequent periods, it is possible for a vendor to sell both old and new products in the same period, and itmay also sell the same products over time.

and p_t^{Hu} to denote the upgrade price of product *H* at time *t*, where *t* = 2 or 3 (upgrade price is necessary only for the new product, and the new product can only be sold at or after period 2).

Given this setting, there are eight possible product strategies for the vendor, which are listed in the first column of Table 2.1.11 Our analysis proceeds as follows. For each product strategy, we calculate the total utility that consumers can enjoy from using the sequence of products. For example, consider the strategy {*L*, -, -}, where the vendor only sells the old product in the first period. Consumers can enjoy a utility of q_L in the first period, δq_L in the second period, $\delta^2 q_L$ in the third period, and so on, until $\delta^{n-1} q_L$ in the last period of the usable life span of the product. Summing these values, the total utility that a consumer can enjoy,

$$u_{\{L,-,-\}} = \left[\frac{1-\delta^n}{1-\delta}\right] q_L .$$

The utilities that consumers can enjoy from other product strategies are calculated in a similar manner. These are all listed in the second column of Table 2.1.

<Insert Table 2.1 here>

Then, based on the utility that consumers derive from each strategy, we compute an optimal price schedule and the associated profit that the vendor can make by choosing such a strategy. Finally, the profits are compared across the eight product strategies to determine the optimal choices of the vendor. We then characterize a few necessary

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[11] Note that the vendor would consider strategies {*L*, -, -} and {-, *L*, -} only when it can credibly commit to its chosen product strategies. Otherwise, it would defect by subsequently selling the new product, and consumers would adjust their expectations to account for such anticipated defections. In Section 6, we discuss the changes in our results when commitment is infeasible.

and sufficient conditions that lead to equilibria with delayed selling of the new product.

2.4 Analysis

When setting prices, the vendor has to consider consumer expectations and their possessions of products. In particular, consumers would buy or upgrade to a new product if and only if the total surplus that they derive from the purchase or upgrade is positive (participation constraint) and is more than those from any other options (self-selection constraint). Further, consumers expect new products to be available in the future. Hence, before they make a purchase, they would compare its prospect with that of waiting for a new product. It is these considerations about consumer actions that limit the flexibility of pricing for the vendor.

We separate our analysis into two scenarios. In the first scenario, the vendor cannot provide an upgrade option to consumers; hence, all consumers pay the same price for the new product. In the second scenario, the vendor can devise an upgrade policy, which allows consumers to trade in the old product for the new product at a discounted price. In subsequent analysis, we assume the vendor can make credible commitments on product strategies. The case when it cannot make credible commitments is presented in Section 2.6.

2.4.1 With No Upgrade Policies

When the vendor cannot identify consumers who have previously bought its product, or when the administrative cost of trade-in is too high, it is infeasible for him to offer an upgrade discount to consumers. All consumers must then pay the same price for the new product, which means that $p_2^{Hu} = p_2^H$ and $p_3^{Hu} = p_3^H$.

Given the product strategies in Table 2.1, it is easy to calculate the price schedules and profits of the vendor. For illustrative purpose, we present the results with respect to strategies {*L*, -, -} and {*L*, *H*, -} below. The price schedules and profits of other strategies can be computed by following similar procedures.

In strategy {*L*, -, -}, the vendor only sells the old product in period 1. Its problem is:

$$\max_{p_1^L} \pi_{\{L,-,-\}} = p_1^L ,$$

$$\text{s.t.} \left[\frac{1-\delta^n}{1-\delta}\right] q_L - p_1^L \geq 0 . \tag{2.1}$$

The left-hand side of (2.1) is the net surplus that consumers can enjoy by buying the product. Essentially, (2.1) is a participation constraint that ensures that consumers buy the old product in the first period. Solving the above problem, we have:

$$p_1^L = \pi_{\{L,-,-\}} = \left[\frac{1-\delta^n}{1-\delta}\right] q_L .$$

In strategy {*L*, *H*, -}, the vendor sells the old and new products in the first and second periods. It has to set two prices: the price of the old product in period 1, p_1^L, and the price of the new product in period 2, p_2^H. Its profit maximization problem is then:

$$\max_{p_1^L , p_2^H} \pi_{\{L,H,-\}} = p_1^L + \delta p_2^H ,$$

$$\text{s.t.} \left[\frac{1-\delta^n}{1-\delta}\right] q_L - p_1^L + \delta \left\{ \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L - p_2^H \right\} \geq 0 \tag{2.2}$$

$$\left[\frac{1-\delta^n}{1-\delta}\right] q_L - p_1^L + \delta \left\{ \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L - p_2^H \right\} \geq \left[\frac{1-\delta^n}{1-\delta}\right] q_L - p_1^L \tag{2.3}$$

$$\left[\frac{1-\delta^n}{1-\delta}\right]q_L - p_1^L + \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right]q_L - p_2^H\right\} \geq \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - p_2^H\right\} \tag{2.4}$$

The left-hand side of (2.2)-(2.4) is the net surplus that consumers enjoy by buying both products. Similar to (2.1), (2.2) is a participation constraint. Both (2.3) and (2.4) are self-selection constraints; (2.3) prevents consumers from buying only the old product whereas (2.4) prevents them from skipping the old product and buying only the new one. Rearranging the terms, (2.3) and (2.4) can be simplified as:

$$p_2^H \leq \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right]q_L, \tag{2.5}$$

and $p_1^L \leq q_L$. (2.6)

(2.5) imposes a constraint on the feasible price of the new product. Because the existing stock of the old product continues to provide service if consumers do not buy the new product, the vendor could only charge consumers for the incremental value the new product brings. Further, (2.6) says that the price of the old product is bound too because expectations of future switch to the new product reduces the life span of and hence the price that consumers are willing to pay for the old product. Essentially, (2.5) and (2.6) imply that if the vendor wants consumers to buy the new product but cannot offer an upgrade discount to existing customers, then it has to absorb the wastage associated with scrapping the old product prematurely (i.e., before the end of its physical life span).[12]

Solving the vendor's problem subject to (2.2), (2.5) and (2.6), the optimal prices and profits are:

$$p_1^L = q_L,\ p_2^H = \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right]q_L,$$

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[12] Fishman and Rob (2000) make a similar observation in their study of new product introductions.

and $$\pi_{\{L,H,-\}} = q_L + \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right]q_L\right\}.$$

Note that $\pi_{\{L,H,-\}} < u_{\{L,H,-\}}$. That is, if the vendor chooses to sell two products, then it has to leave consumers with positive surplus – the service values of the old product from the second period onwards would have to be a give-away to consumers. This is always true whenever the two products are sold in the planning horizon and the vendor cannot implement an upgrade policy. The price schedules of all product strategies and the associated profits for the vendor are reported in the third and fourth columns of Table 2.1.

By inspecting the vendor's profits, we see that strategy {*L*, -, -} dominates strategies {-, *L*, -} and {-, -, *L*}; strategy {*L*, *H*, -} dominates strategy {-, *L*, *H*}; and strategy {-, *H*, -} dominates strategy {-, -, *H*}. That is, the vendor would not postpone selling the same sequences of products. The only remaining strategies are {*L*, -, -}, {*L*, *H*, -}, {*L*, -, *H*}, and {-, *H*, -}. Comparing the vendor's profits across these four strategies, our first set of results follows. The proofs of all propositions and corollaries are available in the Appendix.

Proposition 1 [Optimal product strategies with no upgrade policies]:

Suppose that the vendor cannot implement an upgrade policy.[13]

- [*Status quo*]: Strategy {*L*, -, -} is optimal if and only if $q_L > \delta$ and

  $$q_L > \frac{1-\delta^n}{2[1-\delta^{n-2}]}.$$

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[13] For brevity, we assume that the constraints are not binding. If any one of the constraints is binding, then the vendor may choose more than one strategy with equal probability.

- [*Leapfrogging*]: Strategy {-, *H*, -} is optimal if and only if $q_L < \delta$, $q_L < \frac{\delta[1-\delta][1-\delta^n]}{1-2\delta^2+\delta^n}$, and $1-2\delta+\delta^n < 0$.

- [*Immediate sale*]: Strategy {*L*, *H*, -} is optimal if and only if $q_L < \frac{1-\delta^n}{2}$ and $1-2\delta+\delta^n > 0$.

- [*Delayed introduction*]: Strategy {*L*, -, *H*} is optimal if and only if $q_L > \frac{1-\delta^n}{2}$, $q_L > \frac{\delta[1-\delta][1-\delta^n]}{1-2\delta^2+\delta^n}$ and $q_L < \frac{1-\delta^n}{2[1-\delta^{n-2}]}$.

Corollary: As durability, *n*, increases, the necessary and sufficient conditions for delayed introduction become stronger. When $n \to \infty$, the delayed introduction strategy, {*L*, -, *H*}, is always dominated by one of the other three strategies.

Figure 2.1 plots the ranges of parameters for each of the above strategies to be optimal under various durabilities. Generally, three parameters characterize the incentives of the vendor to delay selling the new product: quality of the old product, q_L (since we have normalized q_H to one, q_L also represents the degree of product improvement brought by the new technology; the smaller q_L is, the larger the quality improvement will be), product durability, *n*, and discount factor, δ. As Figure 2.1 clearly shows, the region for delayed introduction gradually shrinks as n increases, and is negligible when n is sufficiently large (say, $n \geq 10$).

<Insert Figure 2.1 here>

By the self-selection constraint (2.3) and hence the price constraint (2.5), it is clear that the existing stock of the old product imposes a negative externality on the new product. Specifically, (2.5) limits the maximum price, which is dependent on the difference in service values provided by the two products at the time of evaluation, that consumers are willing to pay for the new product. In strategy {L, -, H}, the old product is used in both periods 1 and 2. Therefore, its residual value in period 3 is lower than that in period 2 in strategy {L, H, -}. The vendor could then charge a higher price and earn more revenue for the new product. This is confirmed by the fact that $p_3^H > p_2^H$ in strategies {L, -, H} and {L, H, -}.

Further, by (2.4) and hence (2.6), the new product also imposes a negative externality on the old product. This is because consumers expect to buy the new product and scrap the old product in the near future. Hence, they are willing to pay *only* for periods in which the old product is in service (i.e., the new product causes premature obsolescence of the old product). The longer consumers expect to use the old product, the more they are willing to pay for it. Provided that the expected usage of the old product is less than its physical life span, however, the old product's price would fall short of what it should deserve. The vendor then has to subsidize consumers if it wants them to buy the new product before the old product becomes dysfunctional.

Therefore, the vendor has to face mutual cannibalizations between the old and new products if it wants to sell both of them in the planning horizon. To alleviate such cannibalizations, the vendor could either push back the selling of the new product, or remove the old product from the market. The former action could lead to delayed appearance or even shelving of the new product. The latter action could result in leapfrogging by the vendor, and only the new product would be sold in the second period; consumers would not be able to use the old product in the first period. By Proposition 1, all of these outcomes are possible in equilibrium.

If the vendor decides to sell both products but postpones sale of the new product to the last period, then two opposite forces would be in contention. First, it would collect revenue from the new product at a later period, which tends to act against delay. Second, additional depreciation of the old product allows him to charge a higher price for the new product (i.e., the extent of cannibalization is now smaller). Also, because the old product is used for an extended period, the vendor could charge a higher price for it too. These price effects tend to favor delay. Whether delayed introduction is optimal largely depends on the balance of these two opposite forces.

When the discount factor, δ, is large (i.e., the discount of future values is small, or equivalently, the vendor and consumers are patient), the first force above, owing to delayed revenue from the new product, becomes insignificant, and it may suggest that the vendor would prefer the *delayed introduction* strategy; this is indeed consistent with the findings of Kornish (2001) and Radas and Shugan (1998). Interestingly, this is not always the case in our setting because besides the two full product line strategies, $\{L, H, -\}$ and $\{L, -, H\}$, the vendor may also choose to shelf either the old or new product – after all, this is the most effective way to completely resolve the cannibalizations between the products! In general, low quality improvement tends to raise the attractiveness of the *status quo* strategy, $\{L, -, -\}$, and high quality improvement, together with a high discount factor, tends to raise the attractiveness of *leapfrogging*, $\{-, H, -\}$. Delaying sale of the new product within the planning horizon does not necessarily follow a high discount factor.[14]

By contrast, we find that durability, n, and product improvement, q_L, are stronger predictors of delayed introduction. In particular, if both products are durable,

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[14] Kornish (2001) and Radas and Shugan (1998) do not consider the possibility of selling only one product in the planning horizon. Therefore, their conclusion that the choice of delay is tied to the discount factor is not generalizable to other product line strategies. As we shall illustrate below, however, the role of the discount factor in predicting delay is significant when the vendor can implement an upgrade policy.

and if the new product is somewhat better than the old one, then the gain brought by lower cannibalization (the second force above) is minor. The new product prices with and without delay, p_3^H for {L, -, H} and p_2^H for {L, H, -}, are large but similar, however, and hence the loss associated with deferral of revenue from selling the new product is big. The vendor would then prefer to sell the new product as soon as possible, i.e., strategy {L, H, -} becomes optimal.

Proposition 1 characterizes the conditions – low durability and incremental (but not radical) quality improvement – for delayed introduction of new products. These conditions seem to match the features of many high technology products, such as MP3-compatible home stereo systems or DVD video recorders, and hence, may explain why their vendors often launch new products later, even though the enabling advanced technologies are already available.

2.4.2 With an Upgrade Policy

We now consider the case when a vendor can implement an upgrade policy. The provision of the upgrade option is an extra instrument for a vendor to exercise price discrimination based on purchase history (Fudenberg and Tirole 1998; Lee and Lee 1998). If the vendor can identify consumers who have bought the old product, it can charge them for only the incremental utility that they receive by using the new product. For those who do not own the old product, it can charge them for the full utility. In our setting, because all consumers are identical, they exhibit the same behavior in equilibrium. Nevertheless, upgrade is relevant because the vendor can now make a credible threat that if the consumers do not buy the old product in the first period, they will face a very high price for the new product in subsequent periods (cf. those who trade in the old product for the new product). This threat of price discrimination dissuades consumers from leapfrogging and encourages them to buy the old product as soon as it is available for sale.

The analyses with an upgrade policy resemble those that we have presented above; in the five product strategies with single products, the price schedules and profits are identical to the previous case with no upgrade policies. The prices and the vendor's profits in strategies {*L*, *H*, -}, {*L*, -, *H*} and {-, *L*, *H*}, however, differ from those that we have obtained above. For illustrative purpose, we present the results with respect to strategy {*L*, *H*, -} below.

In strategy {*L*, *H*, -}, the vendor needs to set three prices: the price of the old product in period 1, p_1^L, the price of the new product in period 2, p_2^H, and the *upgrade* price of the new product in period 2, p_2^{Hu}.[15] The vendor's profit maximization problem is then:

$$\max_{p_1^L,\, p_2^{Hu}} \pi_{\{L,H,-\}} = p_1^L + \delta p_2^{Hu},$$

$$\text{s.t.} \quad \left[\frac{1-\delta^n}{1-\delta}\right]q_L - p_1^L + \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right]q_L - p_2^{Hu}\right\} \geq 0, \tag{2.7}$$

$$\left[\frac{1-\delta^n}{1-\delta}\right]q_L - p_1^L + \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right]q_L - p_2^{Hu}\right\} \geq \left[\frac{1-\delta^n}{1-\delta}\right]q_L - p_1^L, \tag{2.8}$$

$$\text{and}\left[\frac{1-\delta^n}{1-\delta}\right]q_L - p_1^L + \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right]q_L - p_2^{Hu}\right\} \geq \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - p_2^H\right\}. \tag{2.9}$$

The left-hand side of (2.7)-(2.9) is the net surplus that consumers enjoy by buying the old product in period 1 and upgrading to the new product in period 2. Similar to (2.1),

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[15] The vendor needs to specify the new product price in period 2 to prevent consumers from leapfrogging. As we shall show below, it makes more profits by selling the two products sequentially (offering an upgrade discount to those who own the old product) than by only selling the new product in period 2.

(2.7) is a participation constraint. (2.8) is a self-selection constraint that restricts the upgrade price, and (2.9) is the constraint that ensures consumers do not leapfrog by skipping the old product. Solving the profit maximization problem, the optimal prices and profit are:

$$p_1^L = \left[\frac{1-\delta^n}{1-\delta}\right] q_L \,,\; p_2^{Hu} = \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L \,,\; p_2^H > \left[\frac{1-\delta^n}{1-\delta}\right],$$

and $$\pi_{\{L,H,-\}} = \left[\frac{1-\delta^n}{1-\delta}\right] q_L + \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L\right\} = q_L + \delta\left[\frac{1-\delta^n}{1-\delta}\right].$$

Note that $\pi_{\{L,H,-\}} = u_{\{L,H,-\}}$. That is, the vendor can extract all surplus from consumers. This result holds whenever the vendor can implement an upgrade policy. The price schedules of all product strategies are reported in the fifth column of Table 2.1. With an upgrade policy, the profits of the vendor always equal the consumer utilities in the second column of Table 2.1.

Similar to the case with no upgrade policies (Section 2.4.1), the vendor would not delay selling the same sequences of products. With an upgrade policy, strategy {*L*, *H*, -} further dominates strategies {*L*, -, -} and {-, *H*, -}. The only remaining strategies are the two with full product lines, {*L*, *H*, -} and {*L*, -, *H*}. Our second proposition follows.

Proposition 2 [Optimal product strategies with an upgrade policy]:

Suppose that the vendor can implement an upgrade policy.

- [*Immediate sale*]: Strategy {*L*, *H*, -} is optimal if and only if $q_L < 1-\delta^n$.
- [*Delayed Introduction*]: Strategy {*L*, -, *H*} is optimal if and only if $q_L > 1-\delta^n$.

Corollary: As n increases or δ decreases, the necessary and sufficient condition for *delayed introduction* becomes stronger. When $n \to \infty$ or $\delta \to 0$, *delayed introduction* is not optimal; the vendor would always sell the new product as soon as it is available.

Figure 2.2 depicts the condition in Proposition 2. The implications of Proposition 2 are similar to those of Proposition 1, except that the discount factor now plays a more important role. The interpretations are straightforward – if quality improvement is low, the discount factor is large, or durability is low, then delayed selling of the new product is more likely; otherwise, the vendor would launch the new product immediately. The intuitions of these findings follow directly from those that we have presented in the previous section.

<Insert Figure 2.2 here>

Note the old product's prices in period 1 in Table 2.1 – they are much higher than those in the case when the vendor cannot implement an upgrade policy. Recall that if the vendor sells both products, then the new product may cause premature obsolescence of the old product because the latter is scrapped before the end of its physical life span. The provision of an upgrade policy could internalize such an externality because even though the old product is retired before the end of its physical life span, it allows its owner to buy the new product at a discounted price. Hence, the old product provides a higher value (cf. that in the no upgrade case) to consumers, and the vendor can now charge for its full value. It is also because of this trade-in value of the old product that the vendor does not prefer the single-product strategies – it could always capture all consumer surplus associated with using the products, which is bigger than that in the single-product strategies $\{L, \text{-}, \text{-}\}$ and $\{\text{-}, H, \text{-}\}$.

Finally, with an upgrade policy, the vendor's choices in the equilibria are socially efficient, including those that involve delayed introductions! This is because if the new product is sold too early, the old product, which could still provide useful service to consumers, would be retired prematurely. This means that useful resources are wasted, which causes inefficiency in welfare.[16] Note further that efficient outcomes are not guaranteed when an upgrade policy is not feasible because the vendor's desperate effort to resolve the two-way cannibalizations between its products may cause him to drift away from socially optimal actions.

2.5 Extensions

In this section, we relax a few assumptions that we have made in the above analyses. First, we allow the old and new products to be of different durabilities and study how the vendor's choice of strategies changes when the relative durability of the products varies.

Generally, a more durable product provides a longer stream of services to consumers, who would then be willing to pay more for the product. If the durability of the new product, n_H, is high relative to that of the old product, n_L, then the total utility that consumers enjoy by buying or upgrading to the new product, and hence the prices that the vendor could charge for it, becomes a more important decision factor. If the new product is sold later, the loss in revenue because of the discount factor is significant. The intertemporal cannibalizations between the products are not affected, however, by the durability of the new product. Hence, ceteris paribus, increases in n_H relative to n_L would entice the vendor to sell the new product earlier, or, if originally it

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[16] Although with *delayed introduction* consumers cannot use the new product immediately, they can extend usage of the products into a more distant future. Specifically, they can use the (old and new) products for $n + 2$ periods with strategy $\{L, -, H\}$, but only $n + 1$ periods with strategy $\{L, H, -\}$.

preferred the *status quo* strategy (in the scenario with no upgrade policies), to reconsider introducing the new product within the planning horizon.

By contrast, if n_H decreases relative to n_L, the loss in revenue for selling the new product later is lower because now the new product is worth less to consumers. Cannibalization rises to become a potent threat, and as a result, the vendor would more likely postpone sale of the new product. In the extreme case where the new product has a substantially shorter life span than the old product, the vendor may even decide not to sell the new product in the planning horizon. We summarize these intuitions in the following proposition.

Proposition 3 [Change in durabilities]:

Suppose that n_H increases relative to n_L.

- The necessary and sufficient conditions for the *status quo* and *delayed introduction* strategies, $\{L, \text{-}, \text{-}\}$ and $\{L, \text{-}, H\}$, become stronger. The vendor is less likely to shelf or delay sale of the new product.
- If the vendor originally chose not to sell the new product, it is now possible for him to re-introduce it in the planning horizon and sell it in the last period.

The opposite is true if n_H decreases relative to n_L. If $n_H < n_L - 2$, it is possible for the vendor to shelf the new product (i.e., choose the *status quo* strategy) even though it is able to devise an upgrade policy.

Figure 2.3 shows the changes in the feasible region of each product strategy when the durability of the new product changes relative to that of the old product.

<Insert Figure 2.3 here>

We next consider the case where the vendor and consumers have different discount factors. Let the discount factor of the vendor be θ and that of consumers be δ. Intuitively, one might think that a less patient vendor (i.e., θ is smaller) would sell the new product as soon as possible to capture a higher discounted revenue. This conjecture is indeed correct if the vendor can implement an upgrade policy. Interestingly, if an upgrade policy is not viable, a decrease in θ may induce the vendor to shelf the new product even though it might have originally planned to sell it! This is because the new product reduces the price that it could charge for the old product in the first period, which is now more important owing to a higher discount of future revenue.

Recall that with an upgrade policy, the vendor could internalize the cannibalization caused by the new product, and could always charge the full price for the old product. Hence the change in the discount factor only affects the *incremental* revenue that it receives from those who upgrade to the new product. A lower discount factor (i.e., a higher discount of future revenue) would naturally prompt him to launch the new product earlier.

By contrast, with no upgrade policies, the reverse cannibalization from the new product to the old one is significant. The vendor then has to balance carefully the gain from launching the new product in the future against the loss in first period revenue from the old product. If it is less patient (i.e., when θ decreases), its concern about the old product's price is higher than that about the new product's price. Hence, it has a higher tendency to shelf the new product and a lower tendency to leapfrog. The choice of *delayed introduction* against *immediate sale* and *leapfrogging* is, however, ambiguous.[17] We summarize these results in the following proposition.

Proposition 4 [Change in discount factors]:

Suppose that the vendor's discount factor, θ, decreases relative to that of the consumers, δ.

- With no upgrade policies, the necessary and sufficient conditions for *status quo* become weaker. The vendor has a higher tendency to shelf the new product. Its choice over the other three product strategies is ambiguous.
- With an upgrade policy, the sufficient condition for *delayed introduction* becomes stronger. The vendor has a higher tendency to sell the new product earlier.

Figure 2.4 shows the changes in the feasible region of each product strategy when the discount factor of the vendor decreases relative to that of the consumers.

<Insert Figure 2.4 here>

Finally, we assume zero marginal costs for both the old and new products. If marginal cost is positive and correlates with product quality, its impact on the vendor's product strategies is straightforward: if cost increases with quality, then selling the new product

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[17] The vendor's revenue from selling the new product in *delayed introduction* involves a second-order discount, which reduces its revenue rapidly as θ decreases. Further, in the *delayed introduction* strategy, its revenue from selling the old product is also a function of the consumers' discount factor, δ. The effect of a change in θ relative to δ depends on their relative magnitude. If θ is large, decreases in θ would make the new product substantially less attractive; hence, delay is less appealing to the vendor. By contrast, if θ is small, a further decrease does not make much difference, and delay could be preferred because it increases the old product's price. Therefore, the vendor does not unequivocally prefer or avoid *delayed introduction* (cf. *immediate sale* and *leapfrogging*).

is less attractive to the vendor (cf. the case that we have analyzed in Section 2.4), and it may delay its sale or shelf it. The opposite is true if cost decreases with quality – it may launch it earlier, or in the extreme case, leapfrog and skip the old product. What is more interesting here is that a high marginal cost for the new product may make upgrade less lucrative for the vendor because other than participation and self-selection constraints, there is an additional cost constraint in the vendor's maximization problem that places a lower bound on the upgrade price that is set by the vendor. An immediate conjecture is that the conditions for the vendor to provide upgrade discounts to existing customers would become more stringent, and it would more likely choose product strategies that involve selling only one product in the planning horizon. To prove this result, however, we need a more general model with various marginal costs and endogenous choices of upgrade policies. We leave this problem to future research.

2.6 No Commitment

What happen if the vendor cannot make credible commitment on product strategies? Note that the vendor always has incentives to launch the new product after the old product was sold in an earlier period. When it cannot credibly commit not to sell the new product in the future, consumers would adjust their expectations and refuse to pay a high price for the old product. Therefore, when commitment is incredible, it is not possible for the vendor to obtain the price and profit in the first row of Table 2.1 from the old product. Accordingly, it would always sell the new product, and strategy {*L*, -, -} would not constitute an equilibrium. Proposition One needs to be slightly revised – the *status quo* strategy is no longer stable. In equilibrium, with no upgrade policies, the vendor would choose either *leapfrogging*, *immediate sale*, or *delayed introduction*. Further, when there is no commitment, *delayed introduction* is always optimal under some conditions even if the durability, n, becomes very large (cf. Corollary One, where *delayed introduction* is not optimal when $n \to \infty$).

The results with an upgrade policy (i.e., Proposition 2) are not sensitive to the assumption on commitment, because in equilibrium the vendor would always sell both the old and new products. Hence, regardless of whether the vendor can make credible commitment, consumers always expect to buy both products, and the vendor fulfills such an expectation by setting the corresponding prices. Finally, Propositions Three and Four need to be revised. Again, *status quo* is no longer an equilibrium strategy; the vendor would not shelf the new product.

Note that when there is no commitment the vendor makes less profit in some parameter ranges. This resembles the standard result of durable goods monopolists being worse off because of the inability to commit to restricting output in future periods (e.g., Bulow 1982, Coase 1972). Generally, commitment ameliorates the time inconsistency problem, and this feature persists when we exclusively consider the timing of sale of new products (cf. output choices, which is often studied in the literature).

2.7 Concluding Remarks

Using a three-period model, we have shown that a monopolistic vendor would at times prefer to delay introducing a new product, even if technologies for the product are already developed and it is costless to sell it to consumers. This is because the old and new products affect each other adversely, which is similar to cannibalizations in standard product differentiation studies (e.g., Moorthy 1984; Moorthy and Png 1992; Mussa and Rosen 1978). In the product differentiation literature, the vendor mitigates cannibalizations by dispersing its products; in our setting, it postpones selling the new product. We have characterized a few equilibria with delayed introduction of the new product, and studied their changes with respect to market and product parameters.

The findings of our study are relevant to high-technology products because their quality and life spans are increasingly being determined by external forces, such as developments in peripheral components, communications standards, or hardware and software architectures. In particular, vendors of these products often cannot control when new technologies arrive and when their old products become obsolete. Hence, for them, product innovation and introduction are two separate decisions – they might not be able to endogenize the extent of product innovation, but they could always control whether and when to sell new products. Because of this separation of sale from innovation, we find that in social optimum, a vendor may defer introducing a new product. It also explains why in some markets, vendors do not deploy new and superior technologies to create new products. The inclusion of durability as a model parameter allows us to extend our insights directly to products that exhibit different life spans (perhaps because of high dependencies on external parts, technologies, or trends). Relaxing the conventional assumption of infinite durability, our results suggest that delayed product introduction can be advantageous even when upgrade policy is feasible, an insight that cannot be obtained in prior studies of product introduction with assumption of infinite durability (Fishman and Rob 2000; Lee and Lee 1998; Waldman 1996a)[18].

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[18] For example, in Fishman and Rob's (2000) model, if a vendor can offer an upgrade discount, with infinite durability, the timing decisions of new products are always socially optimal. That is, there is no "delay" in welfare maximizing equilibria.

Our findings provide interesting insights for pricing and policy formulations. First, uniform pricing of new products is not desirable – a vendor should seek to provide upgrade discounts to existing customers whenever feasible. Second, if a vendor cannot control product durability, planned obsolescence that suppresses the value of old products (Bulow 1986; Waldman 1993, 1996b) may not be feasible. Rather than desperately removing old products from the market, the vendor could launch new products later; this enhances the prices of both products and could sometimes result in higher profits. Third, although stipulating minimum levels of durability could lead to slower technological progress (Fishman and Robs 2000), it allows consumers to enjoy new products earlier, provided that the enabling technologies for the new products are available. Hence, it may benefit consumers, if for example, protracted licenses of 3G cellular networks are awarded to telecommunications companies, or long service contracts are extended to pay-TV operators and broadband Internet service providers. Finally, facilitating upgrade or trade-in is especially important in encouraging an impatient vendor to launch better products. Measures that help such a vendor administer trade-in (e.g., reinforcing the credibility of buyback guarantees) or policies that support renting instead of selling a product are useful; those that prohibit the vendor from discriminating against new customers or retaining controls on old products should generally be avoided.

This study contributes to the literature of technology adoption by analyzing both the vendor's and consumers' economic incentives of adopting a new and better technology. Venkatesh and Brown (2001) suggest that rapid technology improvement and the fear for obsolescence have been the major concerns that dissuade consumers from purchasing a product. Based on their insights, we move forward by measuring obsolescence through the quality improvement embodied in the new product and the duration over which the product can provide usable services and analyze the strategic interaction between vendor and consumers. Our findings suggest that the extent of quality improvement and product's usable lifespan are two important factors which positively affect vendor and consumers' incentives to adopt new and better technology.

This study also extends the research on durable goods monopolists. Similar to that literature (e.g., Bulow 1982, 1986; Levinthal and Purohit 1989; Waldman 1993, 1996b), the vendor in our study faces a time inconsistency problem. Departing from that literature, however, it cannot resolve the problem by adjusting output, R&D investment, or durability (all of these are moot in this study). Instead, it can manipulate timing of sale. As advocated by Waldman (2003, p.140), "the problem of time inconsistency is potentially more important for other choices than for output". We have certainly shown that timing of sale per se is yet another such "other choices", and it deserves more attention because vendors have high degrees of freedom in determining when to sell new products (cf. choices of R&D, durability, or planned obsolescence that are often not subject to control by vendors because of external research, or the political and market environments).

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Appendix 2A-1

Proof of Proposition 1.

By comparing the profits of the seller across the four strategies {*L*, -, -}, {*L*, *H*, -}, {*L*, -, *H*}, and {-, *H*, -} in Table 2.1, we have the following set of inequalities:

$\pi_{\{L,-,-\}} > \pi_{\{L,H,-\}}$ if and only if

$$\left[\frac{1-\delta^n}{1-\delta}\right]q_L - q_L - \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right]q_L\right\} > 0\text{, or}$$

$$q_L > \frac{1-\delta^n}{2[1-\delta^{n-1}]}. \tag{2A1}$$

$\pi_{\{L,-,-\}} > \pi_{\{L,-,H\}}$ if and only if

$$\left[\frac{1-\delta^n}{1-\delta}\right]q_L - [1+\delta]\,q_L - \delta^2\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-2}}{1-\delta}\right]q_L\right\} > 0\text{, or}$$

$$q_L > \frac{1-\delta^n}{2[1-\delta^{n-2}]}. \tag{2A2}$$

$\pi_{\{L,-,-\}} > \pi_{\{-,H,-\}}$ if and only if

$$\left[\frac{1-\delta^n}{1-\delta}\right]q_L - \delta\left[\frac{1-\delta^n}{1-\delta}\right] > 0\text{, or}$$

$$q_L > \delta. \tag{2A3}$$

$\pi_{\{L,H,-\}} > \pi_{\{L,-,H\}}$ if and only if

$$q_L + \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right]q_L\right\} - [1+\delta]q_L - \delta^2\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-2}}{1-\delta}\right]q_L\right\} > 0\text{, or}$$

$$q_L < \frac{1-\delta^n}{2}. \tag{2A4}$$

$\pi_{\{L,H,-\}} > \pi_{\{-,H,-\}}$ if and only if

$$q_L + \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right]q_L\right\} - \delta\left[\frac{1-\delta^n}{1-\delta}\right] > 0\text{, or}$$

$$1 - 2\delta + \delta^n > 0. \tag{2A5}$$

$\pi_{\{L,-,H\}} > \pi_{\{-,H,-\}}$ if and only if

$$[1+\delta]q_L + \delta^2\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-2}}{1-\delta}\right]q_L\right\} - \delta\left[\frac{1-\delta^n}{1-\delta}\right] > 0\text{, or}$$

$$q_L > \frac{\delta[1-\delta][1-\delta^n]}{1-2\delta^2+\delta^n}. \tag{2A6}$$

(2A1) to (2A3) together define the necessary and sufficient conditions for q_L, δ, and n such that the seller would prefer the *status quo* strategy. Similarly, (2A3), (2A5) and (2A6) define the conditions for *leapfrogging*; (2A1), (2A4) and (2A5) define the conditions for *immediate sale*; and (2A2), (2A4) and (2A6) define the conditions for *delayed introduction*.

To prove the corollary, observe that:

$$\frac{d}{dn}\left[\frac{1-\delta^n}{2}\right] > 0,\ \frac{d}{dn}\left\{\frac{\delta[1-\delta][1-\delta^n]}{1-2\delta^2+\delta^n}\right\} > 0\text{, and } \frac{d}{dn}\left\{\frac{1-\delta^n}{2[1-\delta^{n-2}]}\right\} < 0.$$

Hence, the feasible region of q_L for *delayed introduction* shrinks when n increases. Finally, for all $\delta > 0$,

$$\lim_{n\to\infty}\frac{1-\delta^n}{2} = \lim_{n\to\infty}\frac{1-\delta^n}{2[1-\delta^{n-2}]} = \frac{1}{2}.$$

This implies that no values of q_L and δ could fulfill all three necessary conditions for *delayed introduction*. In other words, the seller would always choose one of the other three strategies instead of the *delayed introduction* strategy. []

Proof of Proposition 2.

Comparing the seller's profits across the two strategies, we have:

$$\pi_{\{L,H,-\}} - \pi_{\{L,-,H\}} = q_L + \left[\frac{1-\delta^n}{1-\delta}\right]\delta - [1+\delta]q_L + \left[\frac{1-\delta^n}{1-\delta}\right]\delta$$
$$= -\delta\{q_L - [1-\delta^n]\} \quad > 0$$

if and only if $q_L < 1-\delta^n$. For the corollary, it suffices to show that:

$$\frac{d}{dn}[1-\delta^n] = -\delta^n \ln\delta > 0 \text{ and } \frac{d}{d\delta}[1-\delta^n] = -n\delta^{n-1} < 0.$$

Also,

$$\lim_{n\to\infty} -\delta\{q_L - [1-\delta^n]\} = -\delta\{q_L - 1\} > 0,$$

and $-\delta\{q_L - [1-\delta^n]\}$ is always positive when δ is close to but does not equal zero. Therefore, $\pi_{\{L,H,-\}} > \pi_{\{L,-,H\}}$ when $n \to \infty$ or $\delta \to 0$. []

Proof of Proposition 3.

We first consider the case with no upgrade policies. When $n_H \neq n_L$, the profits of the seller with product strategies {*L*, -, -}, {*L*, *H*, -}, {*L*, -, *H*}, and {-, *H*, -} are:

$$\pi_{\{L,-,-\}} = \left[\frac{1-\delta^{n_L}}{1-\delta}\right]q_L, \tag{2A7}$$

$$\pi_{\{L,H,-\}} = q_L + \delta\left\{\left[\frac{1-\delta^{n_H}}{1-\delta}\right] - \left[\frac{1-\delta^{n_L-1}}{1-\delta}\right]q_L\right\}, \tag{2A8}$$

$$\pi_{\{L,-,H\}} = [1+\delta]q_L + \delta^2\left\{\left[\frac{1-\delta^{n_H}}{1-\delta}\right] - \left[\frac{1-\delta^{n_L-2}}{1-\delta}\right]q_L\right\}, \tag{2A9}$$

and $$\pi_{\{-,H,-\}} = \delta\left[\frac{1-\delta^{n_H}}{1-\delta}\right]. \tag{2A10}$$

Comparing these four profit functions, we have the following set of inequalities:

$$\pi_{\{L,-,-\}} > \pi_{\{L,H,-\}} \text{ if and only if } q_L > \frac{1-\delta^{n_H}}{2[1-\delta^{n_L-1}]}, \tag{2A11}$$

$$\pi_{\{L,-,-\}} > \pi_{\{L,-,H\}} \text{ if and only if } q_L > \frac{1-\delta^{n_H}}{2[1-\delta^{n_L-2}]}, \tag{2A12}$$

$$\pi_{\{L,-,-\}} > \pi_{\{-,H,-\}} \text{ if and only if } q_L > \frac{\delta[1-\delta^{n_H}]}{1-\delta^{n_L}}, \tag{2A13}$$

$$\pi_{\{L,H,-\}} > \pi_{\{L,-,H\}} \text{ if and only if } q_L < \frac{1-\delta^{n_H}}{2}, \tag{2A14}$$

$$\pi_{\{L,H,-\}} > \pi_{\{-,H,-\}} \text{ if and only if } 1-2\delta+\delta^{n_L} > 0, \tag{2A15}$$

$$\text{and} \, \pi_{\{L,-,H\}} > \pi_{\{-,H,-\}} \text{ if and only if } q_L > \frac{\delta[1-\delta][1-\delta^{n_H}]}{1-2\delta^2+\delta^{n_L}}. \tag{2A16}$$

Similar to (2A1) to (2A6), (2A11) to (2A16) define a set of necessary and sufficient conditions for each product strategy to be optimal for the seller. In particular, the constraints that define the joint region (see Figure 2.1) for the *status quo* and *delayed introduction* strategies are:

$$q_L > \frac{\delta[1-\delta^{n_H}]}{1-\delta^{n_L}}, \; q_L > \frac{1-\delta^{n_H}}{2}, \text{ and } q_L > \frac{\delta[1-\delta][1-\delta^{n_H}]}{1-2\delta^2+\delta^{n_L}}.$$

Given fixed n_L, the terms on the right-hand side of the three inequalities above increase with n_H, which means that the region for *status quo* and *delayed introduction* contracts with n_H. Further, the boundary between these two strategies is defined by (2A12), which becomes more stringent as n_H increases. That is, the seller may shift from *status quo* to *delayed introduction*.

We next consider the case with an upgrade policy. When $n_H \neq n_L$, three strategies could be optimal for the seller: {*L*, -, -}, {*L*, *H*, -} and {*L*, -, *H*}. The corresponding profits are:

$$\pi_{\{L,-,-\}} = \left[\frac{1-\delta^{n_L}}{1-\delta}\right] q_L, \tag{2A17}$$

$$\pi_{\{L,H,-\}} = q_L + \delta\left[\frac{1-\delta^{n_H}}{1-\delta}\right], \tag{2A18}$$

and $$\pi_{\{L,-,H\}} = [1+\delta]q_L + \delta^2\left[\frac{1-\delta^{n_H}}{1-\delta}\right]. \tag{2A19}$$

As long as $n_H \geq n_L - 2$, strategy {*L*, -, -} is always dominated. The necessary and sufficient condition for *delayed introduction* becomes $\pi_{\{L,-,H\}} > \pi_{\{L,H,-\}}$, or:

$$q_L > 1-\delta^{n_H}. \tag{2A20}$$

Note that n_L does not appear in (2A20). This is because with an upgrade policy, the seller could internalize obsolescence of the old product brought by the new product and capture the full utility that consumers place on the old product. The decision of whether to delay selling the new product then depends on the value that it brings to consumers, which is a function of n_H. As (2A20) clearly shows, the condition for *delayed introduction* is stronger when n_H increases, which means that it is more likely for the seller to launch the new product when its durability or life span increases.

If $n_H < n_L - 2$, then $\pi_{\{L,-,-\}} > \pi_{\{L,-,H\}}$ and $\pi_{\{L,-,-\}} > \pi_{\{L,H,-\}}$ if and only if

$$q_L > \frac{1-\delta^{n_H}}{1-\delta^{n_L-2}}. \tag{2A21}$$

Hence, if the durability of the new product is substantially lower than that of the old product, the *status quo* strategy could emerge as an optimal strategy for the seller (cf. Proposition 2). As in (2A20), if n_H decreases, the chance for the seller to prefer delayed introduction of the new product increases. In the extreme case, when (2A21) is satisfied (perhaps because the old product's quality is similar to that of the new product), he might even shelf the new product even though he is able to offer an upgrade policy. []

Proof of Proposition 4.

We first consider the case with no upgrade policies. When $\theta \neq \delta$, the profits of the seller with product strategies {*L*, -, -}, {*L*, *H*, -}, {*L*, -, *H*}, and {-, *H*, -} are:

$$\pi_{\{L,-,-\}} = \left[\frac{1-\delta^n}{1-\delta}\right] q_L, \tag{2A22}$$

$$\pi_{\{L,H,-\}} = q_L + \theta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L\right\} \tag{2A23}$$

$$\pi_{\{L,-,H\}} = [1+\delta] q_L + \theta^2\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-2}}{1-\delta}\right] q_L\right\}, \tag{2A24}$$

and $$\pi_{\{-,H,-\}} = \theta\left[\frac{1-\delta^n}{1-\delta}\right]. \tag{2A25}$$

Comparing these four profit functions, we have the following set of inequalities:

$$\pi_{\{L,-,-\}} > \pi_{\{L,H,-\}} \text{ if and only if } q_L > \frac{\theta[1-\delta^n]}{[\theta+\delta][1-\delta^{n-1}]}, \tag{2A26}$$

$$\pi_{\{L,-,-\}} > \pi_{\{L,-,H\}} \text{ if and only if } q_L > \frac{\theta^2[1-\delta^n]}{[\theta^2+\delta^2][1-\delta^{n-2}]}, \tag{2A27}$$

$$\pi_{\{L,-,-\}} > \pi_{\{-,H,-\}} \text{ if and only if } q_L > \theta, \tag{2A28}$$

$$\pi_{\{L,H,-\}} > \pi_{\{L,-,H\}} \text{ if and only if } q_L < \frac{\theta[1-\theta][1-\delta^n]}{\delta[1-\delta] + \theta[1-\delta^{n-1}] - \theta^2[1-\delta^{n-2}]}, \tag{2A29}$$

$$\pi_{\{L,H,-\}} > \pi_{\{-,H,-\}} \text{ if and only if } \theta > \frac{1-\delta}{1-\delta^{n-1}}, \tag{2A30}$$

and $$\pi_{\{L,-,H\}} > \pi_{\{-,H,-\}} \text{ if and only if } q_L > \frac{\theta[1-\theta][1-\delta^n]}{[1-\delta^2] - \theta^2[1-\delta^{n-2}]}. \tag{2A31}$$

Comparing against (2A1) to (2A3), it is obvious that when $\theta < \delta$, the necessary and sufficient conditions (2A26) to (2A28) for {*L*, -, -} to be optimal become weaker.

We next consider the case with an upgrade policy. When $\theta \neq \delta$, as in Proposition 2, the seller would only choose either {*L*, *H*, -} or {*L*, -, *H*} in equilibrium. The necessary and sufficient condition for *immediate sale*, {*L*, *H*, -}, is:

$$q_L < \frac{[1-\theta][1-\delta^n]}{[1-\delta^{n-1}]-\theta[1-\delta^{n-2}]}. \tag{2A32}$$

Compared with $[1-\delta^n]$, it is clear that the right-hand side of (2A32) is larger when $\theta < \delta$, i.e., it is more likely for the seller to choose *immediate sale* when he is impatient. []

Appendix 2A-2

Table 2.1 Product Strategies, Prices and Profits

Product strategy	Consumer utility^{+}	Price schedule with no upgrade policies	Monopolist's profit with no upgrade policies	Price schedule with an upgrade policy^{++}
{*L*, -, -}	$u_{\{L,-,-\}} = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$	$p_1^L = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$	$\pi_{\{L,-,-\}} = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$	$p_1^L = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$
{-, *L*, -}	$u_{\{-,L,-\}} = \delta\left[\frac{1-\delta^n}{1-\delta}\right] q_L$	$p_2^L = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$	$\pi_{\{-,L,-\}} = \delta\left[\frac{1-\delta^n}{1-\delta}\right] q_L$	$p_2^L = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$
{-, -, *L*}	$u_{\{-,-,L\}} = \delta^2\left[\frac{1-\delta^n}{1-\delta}\right] q_L$	$p_3^L = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$	$\pi_{\{-,-,L\}} = \delta^2\left[\frac{1-\delta^n}{1-\delta}\right] q_L$	$p_3^L = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$
{*L*, *H*, -}	$u_{\{L,H,-\}} = q_L + \delta\left[\frac{1-\delta^n}{1-\delta}\right]$	$p_1^L = q_L$ $p_2^H = \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L$	$\pi_{\{L,H,-\}} = q_L + \delta\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L\right\}$	$p_1^L = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$ $p_2^{Hu} = \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L$ $p_2^H > \left[\frac{1-\delta^n}{1-\delta}\right]$
{*L*, -, *H*}	$u_{\{L,-,H\}} = [1+\delta] q_L + \delta^2\left[\frac{1-\delta^n}{1-\delta}\right]$	$p_1^L = [1+\delta] q_L$ $p_3^H = \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-2}}{1-\delta}\right] q_L$	$\pi_{\{L,-,H\}} = [1+\delta] q_L$ $+ \delta^2\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-2}}{1-\delta}\right] q_L\right\}$	$p_1^L = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$ $p_3^{Hu} = \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-2}}{1-\delta}\right] q_L$

				$p_3^H > \left[\frac{1-\delta^n}{1-\delta}\right]$
{-, *L*, *H*}	$u_{\{-,L,H\}} = \delta q_L + \delta^2\left[\frac{1-\delta^n}{1-\delta}\right]$	$p_2^L = q_L$ $p_3^H = \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L$	$\pi_{\{-,L,H\}} = \delta q_L + \delta^2\left\{\left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L\right\}$	$p_2^L = \left[\frac{1-\delta^n}{1-\delta}\right] q_L$ $p_3^{Hu} = \left[\frac{1-\delta^n}{1-\delta}\right] - \left[\frac{1-\delta^{n-1}}{1-\delta}\right] q_L$ $p_3^H > \left[\frac{1-\delta^n}{1-\delta}\right]$
{-, *H*, -}	$u_{\{-,H,-\}} = \delta\left[\frac{1-\delta^n}{1-\delta}\right]$	$p_2^H = \left[\frac{1-\delta^n}{1-\delta}\right]$	$\pi_{\{-,H,-\}} = \delta\left[\frac{1-\delta^n}{1-\delta}\right]$	$p_2^H = \left[\frac{1-\delta^n}{1-\delta}\right]$
{-, -, *H*}	$u_{\{-,-,H\}} = \delta^2\left[\frac{1-\delta^n}{1-\delta}\right]$	$p_3^H = \left[\frac{1-\delta^n}{1-\delta}\right]$	$\pi_{\{-,-,H\}} = \delta^2\left[\frac{1-\delta^n}{1-\delta}\right]$	$p_3^H = \left[\frac{1-\delta^n}{1-\delta}\right]$

[+] Since $q_H = 1$, it is omitted from all utility functions.

[++] The seller's profits that correspond to these price schedules are identical to the consumer utilities reported in the second column.

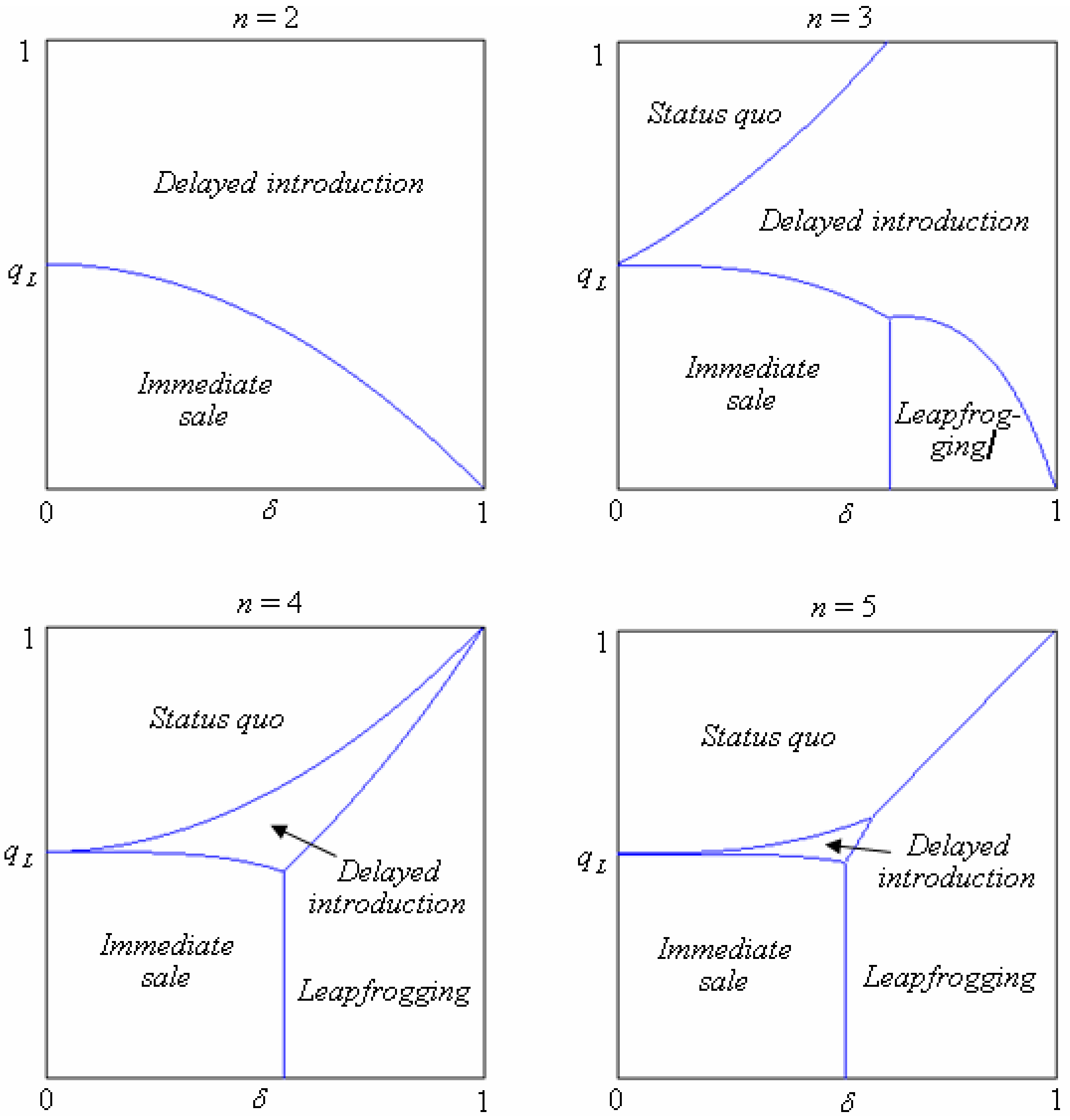


Figure 2.1. Optimal Product Strategies with No Upgrade Policies

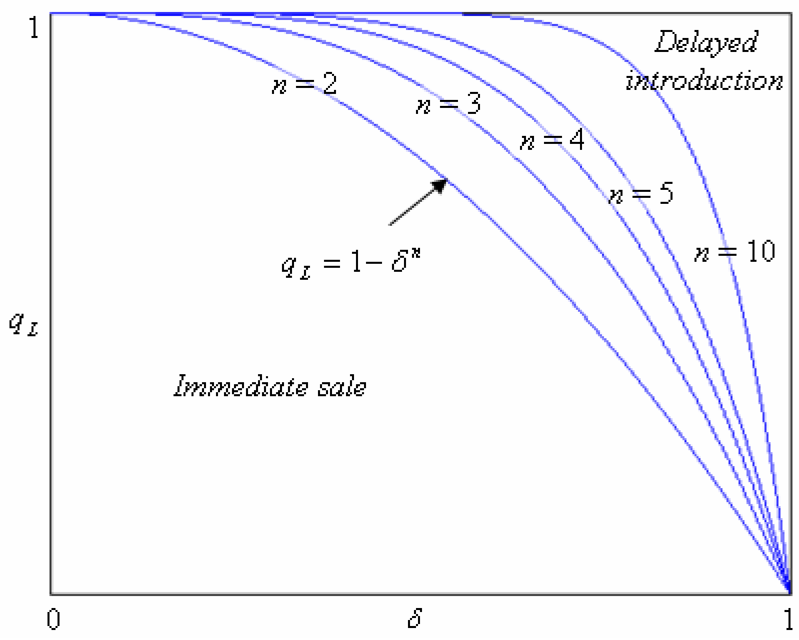
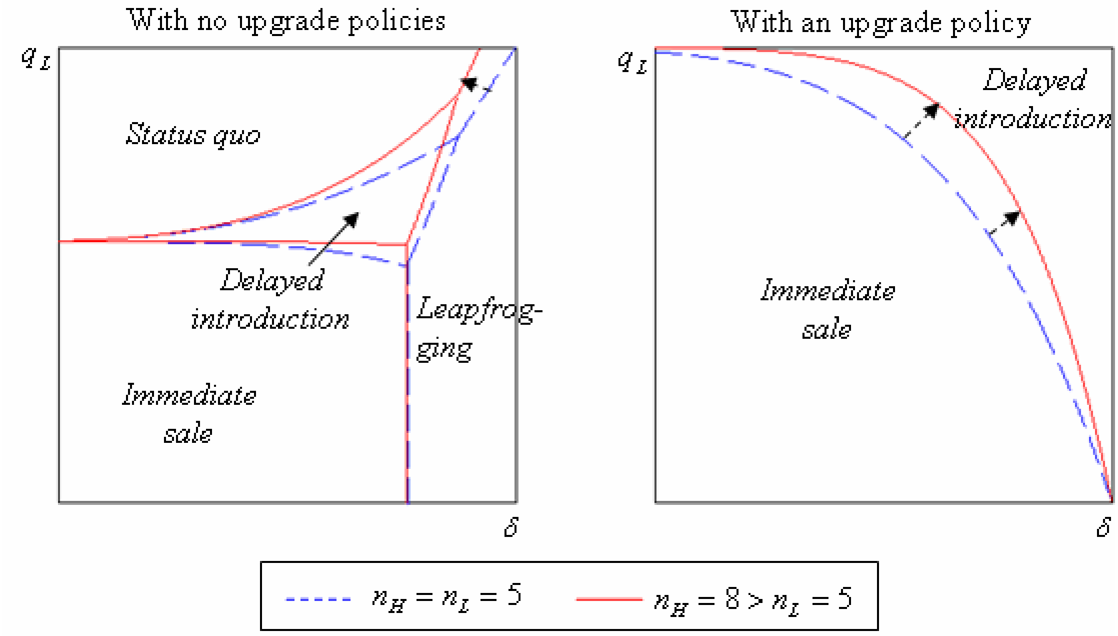


Figure 2.2. Optimal Product Strategies with an Upgrade Policy

(a) When n_H increases relative to n_L



(b) When n_H decreases relative to n_L

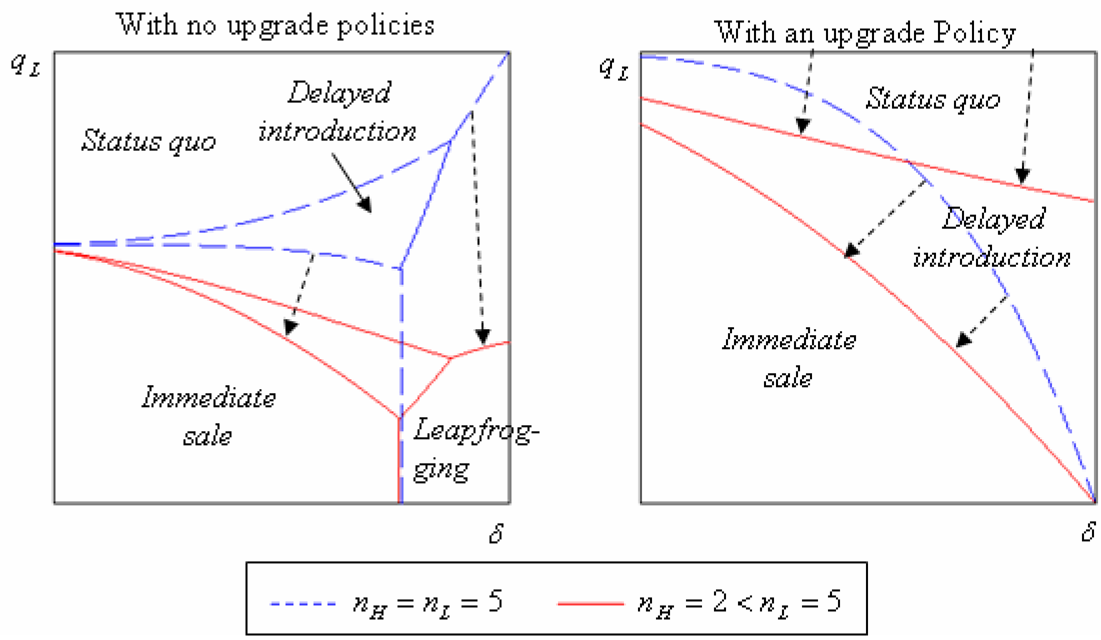


Figure 2.3. Optimal Product Strategies with Different Durabilities

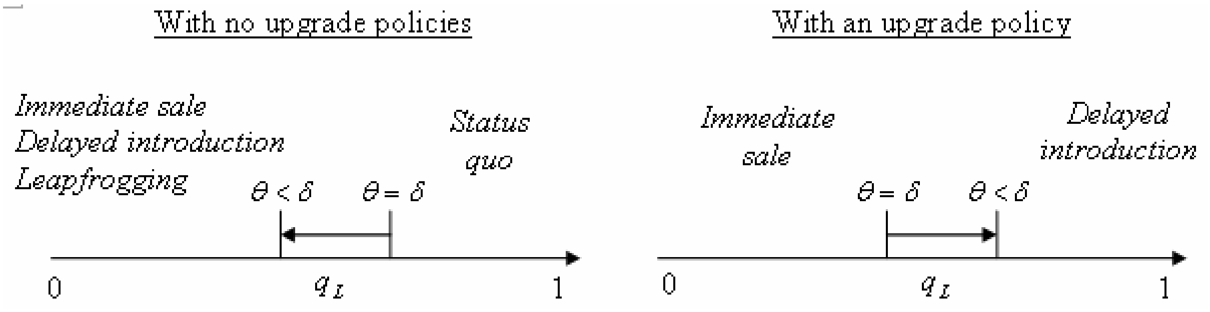


Figure 2.4. Optimal Product Strategies with Different Discount Factors

CHAPTER 3

TECHNOLOGY TIMING AND PRICING IN THE PRESENCE OF AN INSTALLED BASE

3.1 Introduction

Firms in the information technology (IT) industry face a paradox of rapid technological progress: To sustain ongoing industry leadership, a firm should strive to develop the next best technology (Mohr, Sengupta and Slater 2005). However, on one hand, the firm's newly improved technology renders obsolete its older technology, further contributing to competitive volatility. On the other hand, the installed base of its own products adopting the older technology turns to be a formidable competitor to the new technology, particularly when technological progress outpaces users' capacity of fully utilizing technology (Varian 2004). Although both hardware and software vendors make a significant proportion of their revenue by selling users newer versions of what the users already have, many users are increasingly resisting the purchase of upgrades solely for the sake of upgrading (Richter 1995). Thanks to the accumulation of past purchases and the durable nature of many information goods, the number of existing installed units can be an order of magnitude greater than the number sold annually (Wise and Baumgartner 1999). Examples include 3G cellular phone facilities and services versus a large group of existing users who are using their second-generation cellular networks and handsets; DVD video recorders versus the installed base of conventional videotape recorders. In the personal computer (PC) and mainframe computer industry, a major concern for vendors is to tackle the reluctance of individuals or business users to replace old PCs or mainframes by newer ones

(McDonald 2006). Even for computer software, with more than 90% of PCs running some sort of Windows, Microsoft has long considered its main competitor to be the installed base of its own products (Berlind 2005).

Being aware of baulking users caused by rapid technology improvement (Dhebar 1996), prior research in new product introduction suggests that to attract users to upgrade their old products, vendor has the incentivbelow e to reduce product durability (Bulow 1986, Waldman 1996b). If the vendor is unable to artificially shorten the durability of its products, offering upgrade price discounts to existing users can raise its profit and at the same time attain socially optimal outcomes (Waldman 1997, Fishman and Rob 2000).

In information goods industries, new business models are replacing the traditional shipment-based sales model in order to reduce product durability, thus eliminate the negative effect of the installed base (Rappa 2006). Information goods either are made up of "bits" or work in the digital form. Given the lack of physical constraints on production and operation, and being facilitated by the Internet, a few software vendors have managed to control durability by licensing their software or providing periodic services. E.g., Microsoft's software subscription model, Graphsoft's "pay per use", and the more recent "software as a service" (SaaS) model adopted by SalesForce.com. Even hardware vendors have gone downstream along the supply chain towards providing services required to operate and maintain products[19]. Despite their various forms, all of these business models aim at generating perpetual revenue streams by transforming durable products into subscription-based services – a form of 'leasing'.

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[19] Successful examples include IBM global services, Nokia's integrated solution, and Dell's hybrid manufacturing and direct distribution (Wise and Baumgartner 1999).

Nevertheless, transforming business model may not solve the conflict between existing installed base and new products. Users' complaints about "forced upgrades" have never ceased. By paying a recurring fee in exchange for an always up-to-date application, users are likely to spend more in the long run (Kandra 2006). On the vendor's side, many software vendors are unwilling to provide free upgrades even if the users have subscribed to renewable annual services[20]. Further, opposite to the advocates of SaaS, IT consultants have pointed out that SaaS applies best to applications that can be run in isolation, but would not be good for mission critical applications nor make large gains where there is data exchange between service providers and clients (Ferravanti 2004, James 2006). As for the downstream move mentioned above, it makes sense only for some particular combination of market structure and product characteristics[21] (Wise and Baumgartner 1999).

Compared to the analysis of shortening durability or its variations, studies on timing and pricing strategies to cope with installed base are lacking. Most of the existing research on product line introduction employs a two-period framework, which assumes fixed introduction timing for the new product (launching in the second period or not launching) (e.g., Fudenberg and Tirole 1998). This restricted setting constrains the vendor's wisdom in selling the new product facing a certain installed base of its older product: it can induce consumers' self-selection of whether or not to purchase the new product in the second period, but not *when* to purchase it. This is especially

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20 PC World magazine reported the upgrade confusion and pressure to upgrade from the users of Symantec's Norton Antivirus, Intuit's Quicken, Microsoft's Money, Corel's Jasc Photo Album and Mathsoft's Math CAD, etc. http://www.pcworld.com/howto/article/0,aid,123396,00.asp

[21] Wise and Baumgartner (1999) provide a matrix to scope out a downstream opportunity. For instance, a downstream move in VCR market is not attractive considering its low installed-base-to-new-unit ratio and the trivial costs associated with VCR ownership and usage.

unrealistic considering the fact that vendors often use timing to segment the market. Recurrent model or continuous-time models have also been applied in studying technology innovation and product introduction. However, to make the models tractable, the researchers either assume consumers are homogeneous in valuation of product quality (Fishman and Rob 2000), or just study a single product introduction (Stokey 1979).

Generally, the theoretical limitations of prior research lie in the followings.

- First, previous studies mostly focus on static analysis of the demand in a market consisting of consumers differing in valuation of product quality and purchase history. Little research has addressed the same issue in consideration of the time dimension. The complexity of the demand side lies not only in the heterogeneity among consumers, but also the market as a carrier of history and the future. It is unclear how existing consumers' intention to upgrade changes over time, and how the time trend of intention to upgrade differs among consumer segments.
- Second, given the differing demand for upgrade, upgrade pricing has never been studied along the time dimension. In particular, how should vendors respond to the demand variation by leveraging intertemporal or intra-temporal price discrimination?

Third, it has long been considered that with upgrade pricing, vendors will adopt socially efficient strategies (Waldman 1997, Lee and Lee 1998, Fishman and Rob 2000). In this study, with a more general setting that encompasses consumer's heterogeneity in valuation of product quality (cf. the high heterogeneity setting in Lee and Lee (1998), or the homogeneous setting in Fishman and Rob (2000)), are there situations where upgrade pricing loses its power to segment the market? If so, can vendors sustain their monopoly power using time as a discrimination instrument?

This study addresses the above pending questions, and investigates alternative strategies instead of changing durability to alleviate the cannibalization from installed base. Specifically, we investigate the optimal combinations of timing, pricing and product line strategies that the vendor can employ for selling its newly improved product in the presence of an installed base. Two cases: a fully-covered installed base and a partly-covered installed base are studied in a two-period framework with users differing in their valuation of product quality and purchase history, and with a monopolistic vendor selling sequential versions of products with exogenous durability and quality, and zero unit-varied cost. Our contributions are as following:

First, in the full ranges of the extent of user heterogeneity and the extent of quality improvement, we characterize the market with either a partly- or fully- covered installed base in terms of users' relative willingness to pay for the newly improved version and their relative payoffs from delayed purchase across periods.

Second, different from the conventional proposition of constant user reservation price, we propose that if users have already owned an existing (old) version of a durable product, their willingness to purchase the newly improved version indeed increases over time! This effect interweaved with user heterogeneities on valuation of quality and on purchase history can enable a perfect intertemporal price discrimination by which the low valuation users who have not yet purchase any product will purchase the newly improved version earlier than the high valuation users who have owned the previous version. Further, the high valuation users' incentive to postpone purchase may actually make them worse off, which is contrary to popular findings in the price discrimination literature (e.g., Moorthy 1984, Moorthy and Png 1992).

Third, we find that vendor may delay the introduction of the newly improved version because of the combined dampening effects of the installed base and heterogeneous users' anticipation of future price reduction. The equilibria of such delayed introduction exist regardless of the extent of user heterogeneity and the extent of quality improvement between the sequential versions of products.

Fourth, extending the prior research on upgrade pricing to a more general setting, we find that upgrade pricing is not able to segment users with different purchase history when user heterogeneity is sufficiently high. In case of that, instead of upgrade pricing, the vendor would maximize its profit via intertemporal price discrimination, or delayed introduction, or pooling pricing, depending on the characteristics of market structure and technology improvement.

Five, we suggest that without the concern about cost, social welfare directly depends on whether the vendor can sustain its monopoly power facing the mutual cannibalization between the old and new products and the mutual arbitrage between the heterogeneous users.

The remainder of this chapter is organized as follows. Section 3.2 reviews the related literature. Section 3.3 presents our research model. Section 3.4 outlines the analysis and characterizes all equilibria in the case of a fully-covered installed base. Section 3.5 presents the analysis and equilibria in the case of a partly-covered installed base with and without an upgrade policy. Section 3.6 concludes the study.

3.2 Prior Literature

Our research is grounded on the extensive literature of durable goods monopoly, and is related to two streams of works in economics and marketing. One stream of research is about market segmentation and price discrimination spawned by Mussa and Rosen (1978), Stokey (1979) and Moorthy (1984); the other stream is about planned obsolescence triggered by Coasian Dynamics (Coase 1972). The former studies the vendor's strategic choices of timing and quality allocation to extract the surplus of users with heterogeneous valuations for product quality (Moorthy and Png 1992, Padmanabhan, et.al. 1997). The latter studies the vendor's incentive to eliminate the stock of the old durable good and the welfare effects of physical obsolescence (induced by the reduction in durability) (Bulow 1982, 1986, Waldman 1996a) and economic obsolescence (induced by new product introduction) (Waldman 1993, 1996b, Lee and Lee 1998, Fishman, et.al., 1993, Fishman and Rob 2000). Despite their different focuses, these two streams of works are closely related to each other in the context of new product introduction since their studied agents share the common characteristics: a durable-good monopolist seeking to sustain its monopoly power, and a group of users having rational expectations.

The literature on new product introduction is founded on a conventional assumption that all users possess nothing at the beginning of the game (Kornish 2001). Our study relaxes this assumption and generalizes the utility function of users by considering various scenarios pertaining to their possession of a low-quality product at the beginning of the game. This captures the strategic interaction of vendor and users in the presence of an installed base, and at the same time simplifies the analysis of a multi-period game[22].

The stream of research on market segmentation and price discrimination has shown that if the monopolist is unable to identify users with heterogeneous valuation of quality, it could try to induce users' self-selection by offering a menu of qualities

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[22] To study a multi-period game (more than two periods) in a market with heterogeneous consumers can be very complicated because of the exponential increase of the number of combinations of timing, pricing and product line strategies. Hence, prior research with multi-period model settings simplified their models to a market with homogeneous consumers (e.g., Fishman and Rob 2000, Ruize 2003)

simultaneously (Mussa and Rosen 1978), i.e., static price discrimination, or a sequence of prices for the same product over time (Stokey 1979), i.e., intertemporal price discrimination. In the context where the monopolist is able to commit its price and quality trajectories, Salant (1989) addresses the optimality of inducing users' self-selection by synthesizing the works conducted by Stokey (1979), Mussa and Rosen (1978) and Spence (1977, 1980). He proposes that inducing self-selection is suboptimal with linear costs but may be advantageous when users have linear valuations for quality, and when marginal costs are convex in quality. With aspect to static price discrimination, Moorthy (1984) proves that the users with the highest valuation of quality get the socially efficient product quality, while the lower valuation types would be inefficiently served. Moorthy and Png (1992) examine the interaction of quality discrimination and intertemporal price discrimination in a two-period model with the new technology being available at the beginning of the game. They find that employing both time and quality as the instruments to extract users' surplus (sequentially selling two products to the high and low valuation users respectively) is optimal only if cannibalization is a problem and users are relatively more impatient than the vendor. Further, they show that when pre-commitment is infeasible, this sequential selling is less attractive compared with the case with commitment because the vendor's private incentive of maximizing profits in the second period makes itself worse off in the first period (i.e., the vendor faces a time inconsistency problem).

In the above studies, technology innovation is not the main concern. Thus, once any consumer has purchased one unit of the product, she will leave the market forever (Moorthy and Png 1992). This is similar to the first case that we study in this chapter – a fully-covered installed base, where users in the market only differ in their valuation of quality but not their purchase history. The difference is that we assume all users hold the low-quality product at the beginning of the game. We shall investigate the optimality of intertemporal price discrimination in this context.

Prior research considers users' repeat purchase when technology improvement renders the existing technology obsolete. A general setting that has often been studied in this literature is a two-period model in which the vendor can only sell the low-quality (old) product in the first period; at the beginning of the second period, due to technology innovation, the vendor can produce and sell the high-quality (new) product.

The studies on market segmentation and price discrimination are interested in deriving the price sequences that constitute subgame-perfect equilibria (Dhebar 1994, Fudenberg and Tirole 1998, Kornish 2001). Dhebar finds that if users expect the product to improve in present value terms and the monopolist cannot credibly commit to future prices and quality, intertemporal price discrimination may result in disequilibrium regardless of whether upgrade pricing is offered or not. This is due to a demand-side constraint on the rate of product improvement which is imposed for effective intertemporal discrimination. Subsequently, in a similar setting, Kornish complements Dhebar's work and finds that a sub-game perfect equilibrium exists if quality improvement is exogenous and upgrade pricing is not offered. He speculates that a later product introduction may be more profitable in some ranges of the parameters. Both Dhebar's and Kornish's studies imply that the vendor may be better off if it can delay the introduction of the new product in some situations, but they did not explicitly study this phenomenon and characterize the equilibria. Fudenberg and Tirole (1998) incorporate market information as another input variable and analyze the situations where upgrade or buy-back is profitable.

The studies on planned obsolescence concern more about the vendor's incentive to introduce a new product which leads to the economic obsolescence of the old product (Levinthal and Purohit 1989, Purohit 1994, Lee and Lee 1998, Fishman and Rob 2000). Levinthal and Purohit (1989) measure the extent to which the old product is obsolesced by the extent of quality improvement and the degree of substitutability. Lee and Lee (1998) explicitly incorporate price discrimination based on purchase history and valuation of quality. They call the former intra-type price discrimination and the latter inter-type price discrimination. They propose that price discrimination based on purchase history[23] serves to internalize the loss from economic obsolescence for the vendor, thus eliminates the time inconsistency problem induced by new product introduction (although the time inconsistency problem induced by selling to the low valuation users in the subsequent period may still exist.). They suggest that given high heterogeneity in users' valuation of quality, the interaction of intra-type and inter-type price discrimination leads to the reallocation of surplus between the monopolist and the two types of users. The monopolist may only sell the old product rather than the new product to the low valuation users in the second period, if too much consumer surplus has to be given to the high valuation users. Further, if the profit earned from new product introduction cannot balance the innovation cost, the monopolist may prefer to not introduce the new product even though it is socially efficient to introduce it.

Examining the findings of Dhebar (1994), Kornish (2001) and Lee and Lee (1998), we find that the setting of the fixed introduction timing for the new product (launching in the second period or not launching) constrains the monopolist's choice of selling the new product: it can induce users' self-selection of *whether or not* to purchase the new product in the second period, but not *when* to purchase. This is

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[23] Equivalently, upgrade pricing whereby only the old users are offered a discounted price for upgrading to the new product.

unrealistic considering the fact that vendors often use timing to segment the market. Different from Moorthy and Png's setting (1992), here the potential market for the new product consists of not only new users but also the existing users who have owned the old product. Our second case (partly-covered installed base) bridges this gap between the practice and the literature in a tractable setting. Given a partly-covered installed base, we examine the profitability of various combinations of timing and pricing strategies for selling the new product in the presence of existing stock of the old product[24].

Our study particularly contributes to the understanding of vendor's incentive to delay the introduction of new products. Fishman and Rob (2000) suggest that in an infinite-time framework with homogeneous users, no upgrade policies, and perfectly durable products, a monopolist's rate of product innovation would be too low, which can cause inefficient delays in new product introduction. They show that with an upgrade policy, however, the vendor would not delay new product introduction. Our study extends their work into a heterogeneous market and derives various product introduction strategies that are applicable in practice.

3.3 Basic Setting

Consider a model with three periods, t, $t \in \{0,1,2\}$. There exists a monopolist who is planning to sell two versions of a durable product, indexed by x, $x \in \{O,N\}$. Up to period 0, it can only sell a low-quality product, *O*, with quality q_O .Owing to R&D, a new technology arrives in period 1, which allows it to sell a new product, *N*, with quality $q_N > q_O$, in either period 1 or 2. For ease of presentation, we normalize q_N to

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[24] Ruiz (2003) investigates timing and pricing strategies of new product introduction in a three-period model but only with homogeneous users.

one, and $0 < q_O < q_N = 1$. q_O measures the extent of quality improvement embodied in the new product. A smaller q_O represents a higher extent of quality improvement. Both the old (low-quality) and new (high-quality) products are of the same durability of n ($n \geq 2$) periods[25]. n can be considered the relative lifespan of a product compared to the time window of product introduction and technology improvement. The assumption of $n \geq 2$ is to capture the characteristics of IT-intensive products which have a much shorter economic lifespan than its physical lifespan (Fishman and Rob 2000). We further assume zero fixed and marginal costs to focus on the strategic choices of the vendor in response to market demand.

On the demand side, there are two types of consumers, indexed by $e \in \{H, L\}$ and having size d_H and d_L, which differ in their valuation for product quality. Each type-*H* consumer values a unit of the product with quality q_x, $x \in \{O, N\}$, and durability n at $v_H q_x \left[\frac{1-\delta^n}{1-\delta}\right]$; each type-*L* consumer values the same unit of product at $v_L q_x \left[\frac{1-\delta^n}{1-\delta}\right]$. $q_x \left[\frac{1-\delta^n}{1-\delta}\right]$ is the discounted sum (over n periods) of the periodic service value of product x. We normalize $d_H + d_L = 1$. Let $v_H > v_L$, so that type-*H* consumers value quality more highly; *H* is called the high type and *L* the low type. $v_L^H = v_H / v_L$ measures the heterogeneity of users in terms of valuation of quality, larger v_L^H implies a higher user heterogeneity. Each consumer demands at most one unit of each version of the product. Within its life span, the product provides a constant stream of service to users: once users buy it, they enjoy a value that equals its quality in each

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[25] When n goes to infinity, the product is perfectly durable, which is the case studied in Fudenberg and Tirole (1998) and Kornish (2001).

period of service until it is retired (either because it is replaced by a newer product, or because it has exceeded its physical life span). There is no second-hand market; hence, as soon as users buy a new product, their old products are retired and provide zero usage or residual values. We use δ $\left(0 \le \delta \le 1\right)$ to denote a discount factor, which is common to both the vendor and users. The larger δ is, the smaller the discount in future utilities or prices will be. Following Taylor (2002), we consider $d_H v_L^H$ as a measurement of demand elasticity. The larger $d_H v_L^H$, the less elastic is the demand.

In period 0, some users may purchase the old product, which results in an installed base at the beginning of period 1. We study two cases. In the first case (named a fully-covered installed base), both user types have purchased the old product in period 0. In the second case (named a partly-covered installed base), only the high type has purchased the old product in period 0. The case of no purchase in period 0 is not in our interest, since it degenerates to a two-period model similar to the one studied in Moorthy and Png (1992)[26]. We assume that users have separate rationale between their purchase in period 0 and the purchase from period 1 onwards. Thus, we consider the installed base of the old product as an exogenous initial state of the game between the vendor and users.

Periods 1 and 2 further consist of two stages. In the first stage, the vendor makes product and pricing decisions based on its knowledge of consumer profiles (how many people bought the products in the previous periods, the utilities they derive from the products, etc.). In the second stage, users make purchase decisions, taking

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[26] The strategy in period 0 is not in our interest too as we focus on how to launch the new product with an existing installed base but not the old product. Including period 0 into the game would substantially complicate our analysis, and it offers little additional insight. Excluding the strategy in period 0 would not affect our conclusions because our results are derived through a subgame perfect equilibrium path.

into account their valuations for the products and expectations about future products. There is common knowledge on demand, product quality, and technological improvement. Perfect information on history of moves by the vendor and users is available. We focus on rational expectations equilibria in which users form expectations about the product and pricing decisions of the vendor, and the vendor fulfills such expectations.

In the case of a partly-covered installed base, we separate our analysis into two scenarios. In the first scenario, the vendor cannot provide an upgrade option to users; hence, all users pay the same price for the new product. In the second scenario, the vendor can devise an upgrade policy, which allows users to trade in the old product for the new product at a discounted price. In the case of a fully-covered installed base, upgrade is irrelevant since both user types already own the old products. The price of product $x, x \in \{O, N\}$ in period t, $t = 1, 2$ is denoted as p_t^x; the upgrade price of the new product in period t is denoted as p_t^{Nu}.

3.4 Analysis

If users do not possess any product before their purchase, they can derive the full utility that the product can provide in its lifespan. Equation (3.1) represents the *willingness* of a type-e consumer , $e \in \{H, L\}$, to purchase product x, $x \in \{O, N\}$, in period t, $t = 1, 2$:

$$z_{xt}^e = \frac{1-\delta^n}{1-\delta} v_e q_x \tag{3.1}$$

Her corresponding (discounted) utility function can be represented as:

$$Z_{xt}^e = \delta^{t-1} z_{xt}^e \tag{3.2}$$

Alternatively, suppose a consumer has purchased the old product in period t_O, $t_O \in \{0,1,2\}$. Then she only values the *incremental* utility provided by the new product. Equation (3.3) represents this consumer's *willingness* to purchase the new product N in period t, $t = 1,2$ and $t > t_O$:

$$u^e_{t_O t} = \frac{1-\delta^n}{1-\delta} v_e - \frac{1-\delta^{n-[t-t_O]}}{1-\delta} q_O v_e \tag{3.3}$$

Her corresponding utility function can be represented as:

$$U^e_{t_O t} = \delta^{t-1} u^e_{t_O t} \tag{3.4}$$

The first term on the right hand side of equation (3.3) is the full usage value that the consumer can derive from the new product if she does not possess the old product. The second term is the remaining usage value of the old product given that it has been consumed for $[t - t_O]$ periods. Equations (3.1) and (3.3) are equivalent, i.e., $u^e_{t_O t} = z^e_{Nt}$, when the old product does not exist, i.e., $q_O = 0$. The same applies for equations (3.2) and (3.4). Comparing equations (3.1) with (3.2), and (3.3) with (3.4), we find that if the consumer does not own any product, her willingness to purchase remains constant, i.e., $z^e_{x1} = z^e_{x2}$, while she gains higher utility from an earlier purchase, $Z^e_{x1} > Z^e_{x2}$. However, if the consumer bought the old product in period 0, her willingness to purchase *increases* over time, i.e., $u^e_{01} < u^e_{02}$, because the depreciation of the old product raises the incremental benefit she can derive from the new product. Further, the consumer derives higher utility from earlier purchase of the new product, $U^e_{01} > U^e_{02}$, if and only if the extent of quality improvement is sufficiently high, $q_O \le 1 - \delta^n$. Otherwise, if $q_O > 1 - \delta^n$, her utility derived from the new product actually increases over time, $U^e_{01} < U^e_{02}$. Considering the changes of equation (3.2) and (3.4) with respect to t:

$$\frac{\Delta Z_{xt}^{e}}{\Delta t}=\frac{\delta^{\Delta t}-1}{\Delta t}\frac{\delta^{t-1}\left[1-\delta^{n}\right]}{1-\delta}v_{e}q_{x} \tag{3.5}$$

$$\frac{\Delta U_{t_O t}^{e}}{\Delta t}=\frac{\delta^{\Delta t}-1}{\Delta t}\frac{\delta^{t-1}\left[1-\delta^{n}-q_{O}\right]}{1-\delta}v_{e} \tag{3.6}$$

where $\Delta t=1$.

We consider $\frac{\Delta Z_{Nt}^{e}}{\Delta t}$ or $\frac{\Delta U_{t_O t}^{e}}{\Delta t}$ as the margin utility of waiting if type e consumers delay their purchase of the new product for one period.

$$\text{If } q_O \le 1-\delta^n \text{, } \frac{\Delta Z_{Nt}^{e}}{\Delta t}<\frac{\Delta U_{t_O t}^{e}}{\Delta t}\le 0\text{; If } q_O > 1-\delta^n \text{, } \frac{\Delta Z_{Nt}^{e}}{\Delta t}<0<\frac{\Delta U_{t_O t}^{e}}{\Delta t}.$$

Further, the margin utility of waiting increases with q_O, $\frac{\Delta^2 U_{t_O t}^{e}}{\Delta t \Delta q_O}>0$. Proposition 1 summarizes the above findings:

Proposition 1. If a consumer has purchased the old product, because of the depreciation of the old product: (1) her willingness to purchase the new product increases over time; (2) she is better off from delaying the purchase of the new product if and only if $q_O > 1-\delta^n$. Otherwise, if $q_O \le 1-\delta^n$, she would be worse off by delaying the purchase of the new product, but the loss is smaller compared with that of not possessing the old product in period 0. (3). The margin utility of delayed purchase of the new product increases with q_O.

We compare the utility that a consumer can derive from the new product ($U_{t_O t}^{e}$) in three situations and illustrate their time trends in Figure 3.1: (1) no purchase before the game ($q_O=0$, and thus $U_{t_O t}^{e}=Z_{Nt}^{e}$); (2) the consumer owns the old product with quality

$q_O > 1-\delta^n$ one period ahead of the game; (3) the consumer owns the old product with quality $q_O \leq 1-\delta^n$ one period ahead of the game.

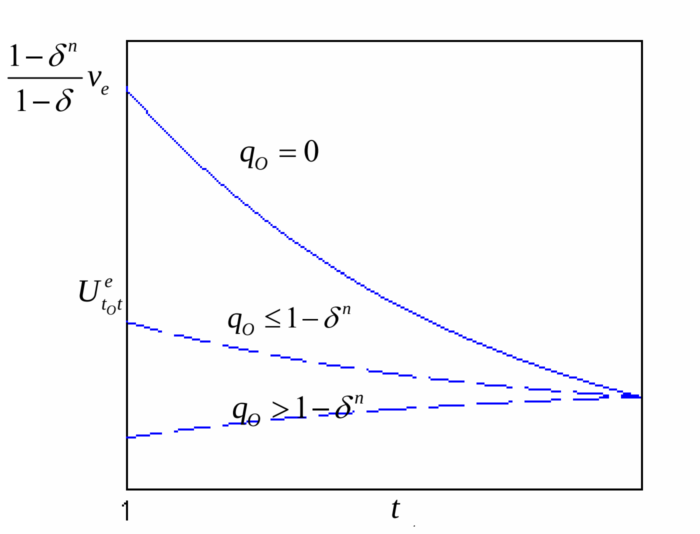


Figure 3.1 Consumer utility from upgrading to the new product

Proposition 1 states our key disposition which is contrary to the prior research. The literature of new product introduction holds a conventional assumption that all users possess nothing at the beginning of the game (i.e., $q_O = 0$) (Kornish 2001). Consequently, consumer's willingness to purchase the new product remains constant and her utility derived from the new product decreases over time. Here, we assume users possess the old product and have already consumed it for one period before the game begins. Thus, the old product in hand depreciates over time which increases the need for consumers to upgrade to the new product. If the quality difference between the old and new products is sufficiently low (i.e., $q_O \geq 1-\delta^n$), over time the depreciation of the old product would increase the value of the new product to consumers. This could outweigh the loss in utility from delaying the consumption of the new product. Hence, the (discounted) net utility that consumers can derive from the new product increases over time as shown in Figure 3.1.

Proposition 1 has important implications for product introduction strategy. One of the main concerns in second-degree price discrimination is the conditions under which a firm can maximize profit by inducing self-selection in quality, quantity or purchasing timing (Mussa and Rosen 1978, Spence 1980, Stokey 1979, Salant 1989). By Proposition 1, having possessed the old product, consumers' marginal utility from delayed purchase of the new product is less negative than (or could even be positive) the case of not possessing the old product. Consequently, compared with the findings in prior research, purchasing earlier may appear less attractive to the high type consumer, and it becomes more difficult for the vendor to induce the high type to purchase ahead of the low type. Hence, we expect that the conditions favoring intertemporal price discrimination on selling the new product will become more stringent in the presence of a fully or partly covered installed base. However, our subsequent analysis shows that this is true in some situations but may lead to adverse results in the others.

3.4.1 A Fully-Covered Installed Base

In a fully-covered installed base, both high and low types bought the old product in period 0 and thus share the same purchase history. The technology to produce the new product is available at the beginning of period 1. The vendor has to solve an old problem that has been extensively studied in the literature of new product introduction (Salant 1989): when should it induce self-selection by selling the new product? The interesting point in this context is that users' willingness to purchase the new product is not constant but rather increases over time, and their discounted incremental utilities derived from the new product is subject to changes as proposed in Proposition 1. The conventional setting of the absence of an installed base can be considered as an extreme case of a fully-covered installed base with $q_O = 0$.

The vendor’s product introduction scheme consists of its decisions on timing, pricing and product line strategies. Since all users possess the old product at the beginning of period 1, the vendor can only market the new product. The vendor may choose to sell the new product only in period 1, only in period 2, or in both periods. To price the new product, the vendor may have three options: (1) premium pricing -- a high price that is only affordable by the high type; (2) pooling pricing -- a low price that is affordable to both consumer types; (3) intertemporal price discrimination -- selling to the high and low types sequentially with different prices. The possible combinations of timing and pricing strategies are listed in Table 3.1a. There are altogether four product introduction schemes denoted as A1, …, A4. Without commitment on its subsequent product introduction schedule, the vendor has the incentive to capture the remaining demand in period 2 by lowering the price. Thus the product introduction scheme of only selling the new product in period 1 with premium pricing cannot form a sub-game perfect equilibrium. Table 3.1b lists the respective equilibrium outcomes of strategies A1 to A4 in terms of users’ purchase sequence. For instance, strategy B2 is related to two sub-columns. The ‘H’ column states that the high type will purchase the new product in period 1; and the ‘L’ column states that the low type will purchase the new product in period 2.

Table 3.1a Feasible Vendor’s Strategies in a Fully-Covered Installed Base

Vendor’s strategies given a fully covered installed base			TIMING		
			Period 1 only	Period 2 only	Both periods
PRICING	Corner solutions	Premium pricing	—	A4	—
		Pooling pricing	A1	A3	—
	Intertemporal price discrimination		—	—	A2

Table 3.1b Feasible Users' Actions in a Fully-Covered Installed Base

Period	Possible Outcomes							
	A1		A2		A3		A4	
	H	L	H	L	H	L	H	L
1	N	N	N	–	–	–	–	–
2	–	–	–	N	N	N	N	–

The prices and the resulting profits for strategies A1, A3 and A4 are straightforward. As for strategy A2, by backward induction, we can derive the price in period 2:

$$p_2^N = u_{02}^L ,$$

which is the low type's reservation price.

The price of the new product in period 1 is subject to self-selection constraints:

$$U_{01}^H - p_1^N \geq U_{02}^H - \delta p_2^N \tag{3.7}$$

$$U_{02}^L - \delta p_2^N \geq U_{01}^L - p_1^N \tag{3.8}$$

Equation (3.7) states that the surplus that the high type obtains from purchasing the new product in period 1 (the left hand side of (3.7)) should not be less than what they would get if they purchase it in period 2 (the right hand side of equation (3.7)). Similarly, equation (3.8) is to ensure that the low type is better off by purchasing in period 2 than in period 1. By equations (3.7) and (3.8), the maximum price that the vendor can charge the high type is:

$$p_1^N = U_{02}^L - \frac{\Delta U_{0t}^H}{\Delta t} \tag{3.9}$$

which is feasible if and only if $\frac{\Delta U_{0t}^H}{\Delta t} \leq 0$, or $q_O \leq \left[1-\delta^n\right]$.

By equations (3.4) and (3.6),

$U_{t_O t}^H > U_{t_O t}^L$, and

$$\left|\frac{\Delta U_{t_O t}^{H}}{\Delta t}\right| > \left|\frac{\Delta U_{t_O t}^{L}}{\Delta t}\right| \text{ or } \begin{cases} 0 < \frac{\Delta U_{t_O t}^{L}}{\Delta t} < \frac{\Delta U_{t_O t}^{H}}{\Delta t}, \text{ if } q_O \geq 1-\delta^n \\ \frac{\Delta U_{t_O t}^{H}}{\Delta t} < \frac{\Delta U_{t_O t}^{L}}{\Delta t} < 0, \text{ if } q_O \leq 1-\delta^n \end{cases} \tag{3.10}$$

Equation (3.10) implies that with the same purchase history, the high type is more sensitive to the change in purchase timing than the low type. If the extent of quality improvement is sufficiently high ($q_O \leq 1-\delta^n$), both types would suffer from delayed purchase and the high type would lose relatively more than the low type. Hence, the high type will always purchase the new product earlier than the low type.

If the extent of quality improvement is not high ($q_O > 1-\delta^n$), both types are better off from delayed purchase, and the high type will gain relatively more than the low type by delaying the purchase. However, the high type will not purchase the new product earlier or later than the low type. That is, in equilibrium, either both types buy simultaneously, or only the high type buy in the last period. Because of the positive marginal utility of waiting, to induce the high type to buy earlier than the low type, the vendor has to provide the high type a price lower than the price charged to the low type in the last period. In that case, instead of waiting until the last period, the low type will purchase the new product in the same period as the high type and earn a positive surplus. On the other hand, the high type would not buy later than the low type. This is because the high type still have higher reservation price than the low type, thus the high type can purchase the new product at any price that is affordable to the low type and get a positive surplus. Proposition 2 summarizes the low and high types' purchase patterns in the presence of a fully-covered installed base.

Proposition 2. In the presence of a fully-covered installed base of the old product, the high type is more sensitive to the change in purchase timing than the low type. The high type will purchase the new product no later than the low type. Specifically, intertemporal price discrimination is feasible only if the extent of quality improvement is sufficiently high ($q_O \le 1-\delta^n$).

We summarize the price sequences and profits of strategies A1-A4 in Table 3.2. Comparing these profits leads to the optimal strategies as presented in Proposition 3.

<Insert Table 3.2>

Proposition 3. In the presence of a fully-covered installed base of the old product, the conditions under which each strategy dominates the others are given in Table 3.3:

Table 3.3 The optimal strategies in the presence of a fully-covered installed base

	The extent of quality improvement	Optimal Strategy		
		No.	Timing	Sales
Elastic demand ($d_H v_L^H < 1$)	Low ($q_O \ge q_2$)	A3	Period 2 only	Both types
	High ($q_O < q_2$)	A1	Period 1 only	Both types
Intermediate case ($1 \le d_H v_L^H < l$)	Low ($q_O \ge q_2$)	A4	Period 2 only	High type only
	High ($q_O < q_2$)	A2	Both periods	First high type followed by low type
Inelastic demand $d_H v_L^H \ge l$	Low ($q_O \ge q_1$)	A4	Period 2 only	High type only
	High ($q_O < q_1$)	A2	Both periods	First high type followed by low type

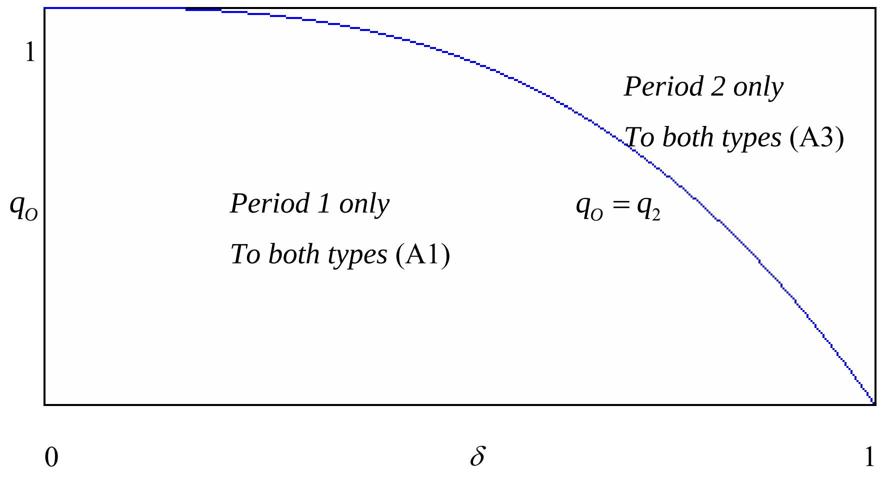
where $q_1 = \dfrac{[1-2\delta][1-\delta^n]}{1-2\delta+\delta^{n-1}}$, $q_2 = 1-\delta^n$, $q_1 < q_2$;

$$l = \frac{\delta\left\{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O\right\}}{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O},\ l > 1 \text{ if } q_O < q_2.$$

Proposition 3 shows that, when users have the same purchase history, whether the vendor would exercise price discrimination is determined by demand elasticity ($d_H v_L^H$). If the demand is elastic ($d_H v_L^H < 1$), penetration pricing can maximize the vendor's profit by capturing the entire demand with a low price. If the demand is inelastic ($d_H v_L^H \geq 1$), the vendor can earn more profit from the high type and at the same time capture the demand of the low type if possible. As for timing, instead of launching the new product in period 1, the vendor may choose to delay the introduction to period 2. When the extent of quality improvement is low, delayed introduction mainly serves to raise users' willingness to purchase the new product. When both the extent of quality improvement and user heterogeneity are high, delayed introduction ensures a non-decreasing price of the new product.

Referring to Figure 3.2a, given an elastic demand (i.e., $d_H v_L^H \leq 1$), the vendor would set a low price to capture the entire demand. The rationale for delayed introduction is similar to the one proposed in Chapter 2 of this thesis: postponing the introduction of the new product allows the old product to be used for one more period, and hence the old product depreciates more in value; users who possess the old product would then be willing to pay more to upgrade to the new product in a later period. This allows the vendor to charge a relatively higher price for the new product and earn more profit. By Proposition 1, in the region $R\left(v_L^H, q_O \geq q_2\right)$, users' payoff from delayed purchase is positive, and so is the vendor's profit.

(a). When demand is elastic ($d_H v_L^H \leq 1$)



(b). When demand is inelastic ($d_H v_L^H > 1$)

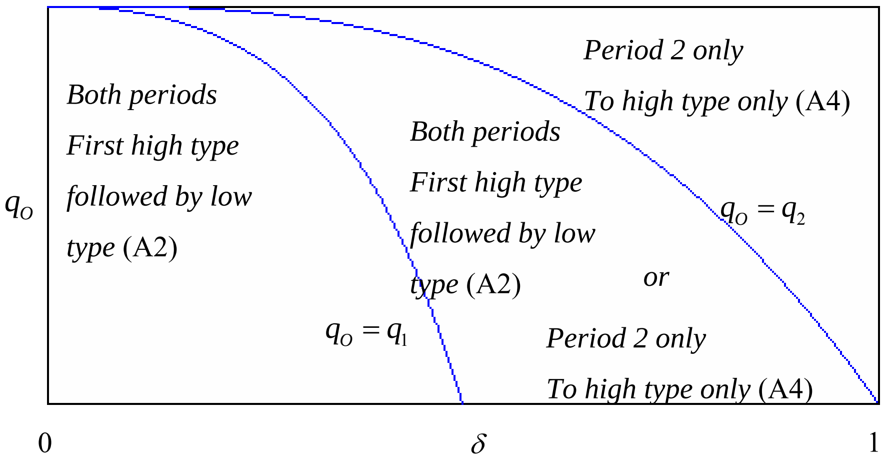


Figure 3.2. Conditions of optimal strategies with a fully-covered installed base (n=3)

Referring to Figure 3.2b, when $d_H v_L^H > 1$, the vendor sets a relatively high price which is unaffordable to the low type in order to capture the premium profit from the high type. More interestingly, compared to the case when demand is elastic and to Proposition 2 in my first essay, even when quality improvement is not low ($q_1 < q_O \leq q_2$), the vendor may prefer delaying the introduction of the new product (A4). In this case, other than the cannibalization from the stock of the old product, it is users' anticipation of price reduction and the vendor's incapability of commitment that further force the vendor to delay selling the new product. On one hand, the high type is capable to pay a relatively high price for the new product. On the other hand, being aware of the vendor's incentive of lowering the price in the subsequent period (to attract the low type), the high type is unwilling to pay a high price in the early period. The lucrative benefit from extracting the full surplus of the high type induces the vendor to delay the launching of the new product. By this, the vendor eliminates its own incentive of lowering the price in the planning horizon, thus enables the equilibrium with premium pricing. However, in some circumstances, the users' rationale about price reduction makes both types being inefficiently served. Similar to Moorthy and Png (1992), the efficient purchase sequence (including purchase timing and bunch) is the one that can maximize consumer's utility given her purchase history and the current technology level. For instance, in the presence of a fully-covered installed base, either the low or high type is efficiently served if:

— they purchase the new product in period 1 when $q_O \le q_2$;

— they purchase the new product in period 2 when $q_O > q_2$.

Consumer's utility in the above two cases is U_{01}^{e} and U_{02}^{e}, $e = H, L$, respectively. Corollary 1 summarizes these findings.

Corollary 1. In the presence of a fully-covered installed base of the old product, when the demand is elastic ($d_H v_L^H \le 1$), both types can get efficient utility. Otherwise, the low type always gets inefficient utility. The high type gets inefficient utility when the demand is very inelastic ($d_H v_L^H \ge l$) and the extent of quality improvement is moderate ($q_1 < q_O < q_2$).

It is useful to compare Proposition 3 with the vendor's optimal strategies in the absence of an installed base ($q_O = 0$). Substituting $q_O = 0$ into equations (3.1) and

(3.2), the consumer's discounted utility from consuming the new product in period t is $Z_{Nt}^{e} = \delta^{t-1}\frac{1-\delta^{n}}{1-\delta}v_{e}$, $t = 1,2$, $e = H, L$. The possible introduction strategies in this case are the same as those listed in Table 3.1a. Obviously, strategy A3 is dominated by strategy A1. Comparing the profits in strategies A1, A2, and A4, we characterize the conditions in which each strategy dominates the others in Table 3.4.

Table 3.4. The optimal strategies in the absence of an installed base

	Discount factor	Optimal Strategy		
		No.	Timing	Sales
Elastic demand ($d_{H}v_{L}^{H} < 1$)		A1	Period 1 only	Both types
Intermediate case ($1 < d_{H}v_{L}^{H} \le l'$)		A2	Both periods	First high type followed by low type
Inelastic demand $d_{H}v_{L}^{H} > l'$	$\delta > \frac{1}{2}$	A4	Period 2 only	High type only
	$\delta \le \frac{1}{2}$	A2	Both periods	First high type followed by low type

Where $l' = \frac{\delta}{2\delta - 1} > l$.

Table 3.4 is a benchmark for examining the effects of an installed base on the optimality of second-degree price discrimination. Comparing the results presented in Tables 3.3 and Table 3.4, we find that the region where A2 is optimal shrinks with a fully-covered installed base. In other words, the presence of a fully-covered installed base makes intertemporal price discrimination less feasible to the vendor. Corollary 2 formally confirms our speculation following Proposition 1.

Corollary 2. In the presence of a fully-covered installed base, the conditions under which intertemporal price discrimination is advantageous becomes more stringent compared to the case with no installed base.

3.4.2 Partly-Covered Installed Base

In this section, we consider the case in which the high type has purchased the old product in period 0 while the low type has not. Other than timing and pricing the new product, the vendor may manage the product line by selling the old product to the low type in period 1 or 2. By Lemma 1 below we can exclude the strategies in which the low type purchases nothing or purchase the old product in period 2.

Lemma 1. In the presence of a partly-covered installed base, suppose there is a sub-game perfect equilibrium in which the vendor sells the new product only to the high type, then it must sell the old product to the low type in period 1 and extract the full surplus from them.

The possible combinations of timing, pricing and product line strategies are summarized in Table 3.5a as strategies B1, …, B8. Table 3.5b lists the respective equilibrium outcomes of these strategies in terms of users' purchase sequence. For instance, in Table 3.5b4b, two sub-columns are listed under strategy B4. The '*H*' column states that the high type will purchase the new product in period 2; the '*L*' column states that the low type will purchase the new product in period 1.

Table 3.5a Feasible Vendor’s Strategies in a Partly-Covered Installed Base

Vendor’s strategies given a partly covered installed base			PRODUCT LINE	TIMING of new product		
				Period 1 only	Period 2 only	Both periods
PRICING of new product	Corner solutions	Premium pricing	both products	—	B6	—
			new product only	—	B8	—
		Penetration pricing	both products	—	B5	—
			new product only	B1	B7	—
	Intertemporal price discrimination	High type first followed by Low type	both products	—	—	B2
			new product only	—	—	B3
		Low type first followed by High type	both products	—	—	—
			new product only	—	—	B4

Table 3.5b Feasible Users’ Actions in a Partly-Covered Installed Base

Period	B1		B2		B3		B4		B5		B6		B7		B8	
	H	*L*	*H*	*L*	*H*	*L*	*H*	*L*	*H*	*L*	*H*	*L*	*H*	*L*	*H*	*L*
1	*N*	*N*	*N*	*O*	*N*	–	–	*N*	–	*O*	–	*O*	–	–	–	–
2	–	–	–	*N*	–	*N*	*N*	–	*N*	*N*	*N*	–	*N*	*N*	–	*N*

In the two-period planning horizon, the vendor may select a product replacement strategy of only selling the new product; alternatively, it can manage a product line consisting of the old and new products and sell the old product to the low type. Both time and product quality can be used as instruments to differentiate users with different valuations for quality. Further, since the low and high types also differ in their purchase history, the vendor may employ an upgrade policy to differentiate users who have purchased or not purchased the old product (Fudenberg and Tirole 1998). We separate our analysis into two scenarios. In the first scenario, the vendor cannot provide an upgrade option to users; hence, all users must pay the same price for the new product. In the second scenario, the vendor can offer an upgrade policy, which allows users to trade in the old product for the new product at a discounted price. In this case, we can examine how price discrimination on purchase history interacts with users' valuation of quality to impact product timing in a heterogeneous market --- a pending question raised by Fishman and Rob (2000). The challenges in marketing to users with different valuations for quality and purchase history lie in the following:

(**Timing and Pricing**) First, selling the new product by intertemporal price discrimination based on valuation of quality and purchase history becomes very tricky even without considering the selling of the old product. This is because the relative net utility of the high and low types derived from the new product ($U_{0t}^{H} - Z_{Nt}^{L}$, $t = 1,2$) and their relative marginal utility of waiting ($\frac{\Delta U_{0t}^{H}}{\Delta t} - \frac{\Delta Z_{Nt}^{L}}{\Delta t}$) are shaped by the extent of quality improvement and user heterogeneity, and they vary with time. By Proposition 1, the high type who owns the old product is better off waiting if and only if $q_O \geq q_2$, while the low type always prefers to purchase the new product in period 1. We characterize consumers' purchase patterns pertaining to a partly-covered installed base in Proposition 4:

Proposition 4. In the presence of a partly-covered installed base, the willingness of the high type to purchase the new product is lower than that of the low type in period t if $v_L^H < \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-t}\right]q_O}$, $t = 1,2$. The low type will lose her utility from delayed purchase of the new product. The high type will derive a lower utility from delayed purchase if and only if $q_O < q_2$, and may even suffer more than the low type if $v_L^H > \frac{1-\delta^n}{1-\delta^n-q_O}$.

Let $v_1 = \dfrac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-2}\right]q_O}$; $v_2 = \dfrac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right]q_O}$; $v_3 = \dfrac{1-\delta^n}{1-\delta^n - q_O}$, then we have $v_1 < v_2 < v_3$. We further illustrate Proposition 4 by Table 3.6 which characterizes the regions $R\left(v_L^H, q_O\right)$ defined by v_L^H and q_O with the relative net utility derived from the new product and the relative marginal utility of waiting. Table 3.6 shows the distinct demand structure in the presence of a partly covered installed base that have not been systematically analyzed in prior research -- Users' relative utility derived from the new product and their relative payoff from delayed purchase vary with user heterogeneity and the extent of quality improvement, and further depend on the purchase timing. This variation in demand results in different pricing functions of the same product introduction strategy depending on the nature of the technology (q_O) and demand (v_L^H). Obviously, intertemporal price discrimination in selling the new product may be feasible in some regions of $R\left(v_L^H, q_O\right)$ but infeasible in others. An upgrade policy, as an exogenous force, further influences the vendor's pricing functions given the same product introduction strategies.

Table 3.6 Properties of the regions defined by (v_L^H, q_O) in the presence of a partly-covered installed base.

v_L^H q_O	$v_L^H \le v_1$	$v_1 < v_L^H \le v_2$	$v_2 < v_L^H \le v_3$	$v_L^H > v_3$
$q_O \ge q_2$	$U_{01}^H < Z_{N1}^L$ $U_{02}^H \le Z_{N2}^L$ $\dfrac{\Delta Z_{Nt}^L}{\Delta t} < 0 \le \dfrac{\Delta U_{0t}^H}{\Delta t}$	$U_{01}^H \le Z_{N1}^L$ $U_{02}^H > Z_{N2}^L$ $\dfrac{\Delta Z_{Nt}^L}{\Delta t} < 0 \le \dfrac{\Delta U_{0t}^H}{\Delta t}$	$U_{01}^H > Z_{N1}^L$ $U_{02}^H > Z_{N2}^L$ $\dfrac{\Delta Z_{Nt}^L}{\Delta t} < 0 \le \dfrac{\Delta U_{0t}^H}{\Delta t}$	
$q_O < q_2$	$U_{01}^H < Z_{N1}^L$ $U_{02}^H \le Z_{N2}^L$ $\dfrac{\Delta Z_{Nt}^L}{\Delta t} \le \dfrac{\Delta U_{0t}^H}{\Delta t} < 0$	$U_{01}^H \le Z_{N1}^L$ $U_{02}^H > Z_{N2}^L$ $\dfrac{\Delta Z_{Nt}^L}{\Delta t} \le \dfrac{\Delta U_{0t}^H}{\Delta t} < 0$	$U_{01}^H > Z_{N1}^L$ $U_{02}^H > Z_{N2}^L$ $\dfrac{\Delta Z_{Nt}^L}{\Delta t} \le \dfrac{\Delta U_{0t}^H}{\Delta t} < 0$	$U_{01}^H > Z_{N1}^L$ $U_{02}^H > Z_{N2}^L$ $\dfrac{\Delta U_{0t}^H}{\Delta t} \le \dfrac{\Delta Z_{Nt}^L}{\Delta t} < 0$

(**Product replacement or Product line**) The strategies targeting at the low type could be very flexible since they could be served either a single product, or a full line of both the old and new products. For instance, if the vendor chooses to sell the new product only to the high type, by Lemma 1, in equilibrium, it can only sell the old product to the low type in period 1, which corresponds to the outcome of strategy B6 in Table 3.5b. If the vendor chooses to sell the new product to the low type in period 2, it has to determine whether or not to also sell them the old product in period 1, which corresponds to strategies B5 and B7 in Table 3.5b. The vendor can earn extra profits by selling the old product, but this extra stock of the old product dampens the price of the new product that the vendor can charge the low and high types. Consequently, the inclusion of the old product further complicates the analysis.

3.4.2.1 With No Upgrade Policies

If the vendor cannot identify users who have previously bought its products, or if the administrative cost of trade-in is too high, it is infeasible for it to offer an upgrade discount to existing users. All users must pay the same price for the new product, which means that $p_1^{Nu} = p_1^N$ and $p_2^{Nu} = p_2^N$. Consider the possible strategies listed in Table 3.5a respectively:

(1) Strategy B1 (selling the new product to both types in period 1) requires a common price that extracts the full surplus from the type of users with a relative lower reservation price for the new product. By Lemma 3, when $v_L^H < v_2$, the willingness of the high type to purchase is lower than that of the low type ($U_{01}^H < Z_{N1}^L$), and thus we have $p_1^N = U_{01}^H$; otherwise, $p_1^N = Z_{N1}^L$.

(2) With strategy B2 or B3, the vendor sells the new product to the high and low types in period 1 and 2 respectively. For strategy B2, the vendor additionally sells the old product to the low type in period 1. Note that without possessing the old product at the beginning of the game, the low type has a higher reservation price for the new product compared to the case in a fully covered installed base ($U_{0t}^{L} < Z_{Nt}^{L}$). Compared to the setting in Corollary 2 of Proposition 3, this exacerbates the difficulty of temporally differentiating the two types of users. Observation 1 states our findings.

Observation 1. Given a partly-covered installed base of the old product, with no upgrade policy, selling the new product in both periods by first serving the high type in period 1 (strategy B2 or B3) is feasible if and only if the extent of quality improvement is sufficiently high, i.e., $q_O \leq q_2$. Further, if the extent of user heterogeneity is not high enough, i.e., $v_L^H \leq v_3$, the vendor has to compensate the low type by selling them the old product with a very low price in period 1 (strategy B2).

Observation 1 results from two forces that suppress the vendor's monopoly power to temporally differentiate the two types of users by first serving the high type:

One is the *mutual arbitrage* between the low and high types. Anticipating future price reductions (i.e., the time inconsistency problem as exhibited in the Coasian Dynamics (Coase 1972)), the high type has the incentive to delay their purchase of the new product to period 2. To attract the high type to purchase in period 1 (Strategy B2 or B3), the vendor has to restrict the price (p_1^N) of the new product in period 1 so as to leave the high type the amount of positive surplus that is no less than what they might gain from waiting to purchase in period 2 (constraint (3A7)). However, if the user heterogeneity is sufficiently low, $v_L^H \leq v_3$, the relatively low price of the new product offered to the high type (p_1^N) may attract the low type to directly purchase the new product in period 1 rather than in period 2. In this situation, the low type would lose even more than the high type from delayed purchase ($\frac{\Delta Z_{Nt}^L}{\Delta t} \leq \frac{\Delta U_{0t}^H}{\Delta t} < 0$ as shown in the region $R(v_L^H \leq v_3, q_O \leq q_2)$ in Table 3.6). Hence, the reservation price of the low type for the new product in period 1 ($Z_{N1}^L = \delta p_2^N - \frac{\Delta Z_{Nt}^L}{\Delta t} = U_{12}^L - \frac{\Delta Z_{Nt}^L}{\Delta t}$) is higher than the maximum price that the vendor can charge the high type in period 1 ($p_1^N = \delta p_2^N - \frac{\Delta U_{0t}^H}{\Delta t} = U_{12}^L - \frac{\Delta U_{0t}^H}{\Delta t}$). To prevent the low type from deviation (i.e., purchasing the new product in period 1), the only measure for the vendor is to further reduce the price of the old product to ensure the low type can be compensated for waiting to the last period (see Strategy B2 and constraint (3A5)).

However, selling the old product in period 1 induces a *mutual cannibalization* between the old and new products. On one hand, anticipating the forthcoming of the new product, the low type may bypass the old product and wait for the newer one; on the other hand, the existing stock of old product limits users' willingness to purchase the new product (see my analysis in Chapter 2). This further constrains the prices (p_1^O , p_2^N) that the vendor can charge the low type for the old and new products (constraints (3A4) and (3A6)).

(3) Contrary to strategy B3, with strategy B4, the vendor sells the new product in two periods by first serving the low type. By Proposition 4 and Table 3.6, the willingness of the low type to purchase the new product is higher than that of the high type in both

periods if $v_L^H \le v_1$, and only in period 1 if $v_1 < v_L^H \le v_2$. We highlight an interesting property of strategy B4 in Observation 2.

Observation 2. Given a partly-covered installed base of the old product, with no upgrade policy, selling the new product in both periods by first serving the low type in period 1 (strategy B4) is feasible if and only if $v_L^H \le v_2$. Specifically, if $v_1 < v_L^H \le v_2$, the vendor achieves perfect intertemporal price discrimination.

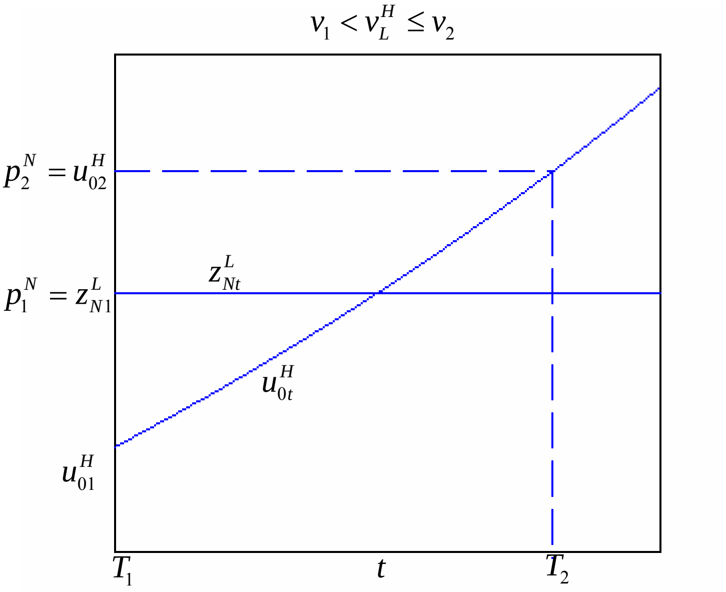


Figure 3.3 Consumers' willingness to purchase the new product

The perfect intertemporal price discrimination by sequentially first selling the new product to the low type followed by the high type (Strategy B4) is a direct result of the existing users' increasing need to upgrade. As shown in Figure 3.3, when heterogeneity in valuation of quality is sufficiently high, i.e., $v_L^H > v_1$, the vendor can extract the full surplus from the low type and leave them with little choice because they have to face an *even higher* price in the subsequent period ($p_2^N = u_{02}^H > z_{N2}^L$) as the willingness of the high type to purchase the new product increases ($\frac{\Delta U_{0t}^{H}}{\Delta t} > 0$). On the other hand, if the heterogeneity in valuation of quality is not too high (e.g. $v_L^H \le v_2$), the high type is unwilling to accept the price in the early period because it is higher than the incremental utility that they can derive from the new product at that time ($p_1^N = z_{N1}^L > u_{01}^H$). Thus they have to wait and purchase the new product after the low type. Because the high type's willingness to purchase increases over time while the low type's remains unchanged (due to the partly-covered installed base), in the region $R\left(v_1 < v_L^H \le v_2, q_O\right)$, the reservation price of the high type for the new product changes from being lower than that of the low type in period 1 to being higher than it in period 2. The increasing need for the new product enables perfect intertemporal price discrimination in which the vendor extracts the full surplus of both consumer types from the new product.

Observations 1 and 2 together exhibit a unique characteristic of intertemporal price discrimination in a market with a partly-covered installed base. Similar to Corollary 2 of Proposition 3, the increasing need for upgrade strengthens the feasible conditions for intertemporal price discrimination. Different from the case of a fully-covered installed base, the low and high types of consumers, being heterogeneous not only in valuation of quality but also in purchase history, exhibit respectively a constant trend and an increasing trend in their willingness to purchase the new product. Thus there exists a certain parameterization of heterogeneity that favors another type of intertemporal price discrimination – one in which the low type with no purchase history consumes the new product earlier.

(4) Strategies B5, B6, B7 and B8 are characterized with delayed introduction of the new product. Both strategies B5 and B6 manage product line by selling the old product to the low type in period 1. Similar to strategy B2, the price sequence of strategy B5 is subject to the mutual cannibalization between the old and new products. Strategy B5 results in a pooling equilibrium in which both types purchase the new product in period 2 with a price equal to the incremental utility of the low type derived from the new product ($p_2^N = u_{12}^L$). By contrast, strategy B6 leads to a separating equilibrium in which only the high type purchases the new product with a price equal to their incremental utility derived from the new product ($p_2^N = u_{12}^H$). In strategy B5, both types are left with positive surplus. In strategy B6, the surplus of both types are fully extracted when the low type cannot afford the price of the new product in period 2 ($v_L^H > v_1$); otherwise, the low type can get a positive surplus. As for strategies B7 and B8, the vendor would not sell any product in period 1. Similar to strategy B1, B7 (selling the new product to both types in period 2) requires a common price that extracts the full surplus from the type of user that has a relatively lower reservation price for the new product in period 2. Strategy B8 sells only the new product to the low type; by Proposition 4, it is only feasible when $v_L^H \le v_1$.

The price sequences and profits for strategies B1-B8 are listed in Table 3.7. Comparing their profits leads to proposition 5.

<Insert Table 3.7>

Proposition 5. With a partly-covered installed base of the old product, and with no upgrade policy, for each region defined by $R\left(v_L^H, q_O\right)$, the optimal strategies given the different demand elasticities are presented below in Table 3.8. Specifically, regardless

of user heterogeneity, strategy B2 (B8) is more profitable than strategy B3 (B5) when the size of the low type is large.

Table 3.8 Optimal Strategy with no upgrade policy in the presence of a partly-covered installed base.[27]

v_L^H \ q_O	$v_L^H \le v_1$	$v_1 < v_L^H \le v_2$	$v_2 < v_L^H \le v_3$	$v_L^H > v_3$
$q_O \ge q_2$	|← B5, B8 →|← B4 → $d_H v_L^H$	**B4**	|← →|← → $d_H v_L^H$	
$q_1 \le q_O < q_2$	|← →|← → $d_H v_L^H$			|← B1 →|← B2, B3 →|← B6 → $d_H v_L^H$
$q_O < q_1$				|← B1 →|← B6 →|← B2, B3 → $d_H v_L^H$

Proposition 5 shows that the conditions under which a specific strategy is optimal are defined by the extent of quality improvement, user heterogeneity, and demand elasticity. For instance, in the region $R\left(v_L^H > v_3, q_1 \le q_O < q_2\right)$, strategy B1 can be optimal when the demand is relatively elastic. As the demand elasticity decreases, strategy B2 or B3 may become optimal; strategy B6 is optimal when the demand is relatively inelastic. It is useful to highlight some interesting observations implied by Proposition 5.

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[27] The detailed analysis of the conditions under which each strategy is optimal is presented in the Appendix 3A-3.

(**Delayed product introduction**) Delayed introduction of the new product is not optimal in the region with moderate heterogeneity, $R\left(v_1 < v_L^H \le v_2, q_O\right)$, or the region with relatively low heterogeneity and high quality improvement, $R\left(v_L^H \le v_1, q_O < q_2\right)$. It can be optimal either in the region $R\left(v_L^H \le v_1, q_O \ge q_2\right)$ or the region $R\left(v_L^H > v_2, q_O\right)$ but with very different reasons.

When both user heterogeneity and the extent of quality improvement are low ($v_L^H \le v_1$, $q_O \ge q_2$), strategy B5 outperforms other strategies because the vendor can profit from selling the old product in period 1 and capturing the entire demand with a common price in period 2. When the low type is dominant (in terms of reservation price and demand size), the vendor may even sell only to the low type in the last period (strategy B8).

When the extent of user heterogeneity is high ($v_L^H > v_2$), strategy B6 can be optimal regardless of the extent of quality improvement. The profit in strategy B6 consists of two parts: one is the sale of the new product to the high type in period 2; the other is the sale of the old product to the low type in period 1. In the region $R\left(v_L^H > v_2, q_O \ge q_1\right)$, strategy B6 outperforms other strategies mainly because of the contribution from selling the new product. The reason behind delayed introduction is similar to the discussion in Section 3.4.1. By postponing the introduction of the new product, the vendor can raise the price of the new product either because of the increasing discounted utility that users derive from the new product ($\frac{\Delta U_{0t}^H}{\Delta t} \ge 0$ in the region $R\left(v_L^H, q_O \ge q_2\right)$), or because of relieving the high type's concern about future price reductions (region $R\left(v_L^H > v_2, q_1 \le q_O < q_2\right)$). By contrast, in the region

$R\left(v_L^H > v_2, q_O < q_1\right)$, strategy B6 cannot outperform strategy B2 and B3 unless the profits from selling the old product to the low type in period 1 is sufficiently large. To make delayed introduction optimal in the region $R\left(v_L^H > v_2, q_O < q_1\right)$, the extent of quality improvement cannot be too high; at the same time the discount factor should be small enough and the size of the low type should be sufficiently large.

Since the competitive advantage of delayed introduction (strategy B6) compared with the other strategies vary with the extent of quality improvement, the optimality of strategy B6 requires a lower demand elasticity (or to say, a relatively large $d_H v_L^H$) than that of strategies B1, B2 and B3 in the region $R\left(v_L^H > v_2, q_O \geq q_1\right)$, whereas a relatively high demand elasticity compared to strategy B2 and B3 is favorable in the region $R\left(v_L^H > v_2, q_O < q_1\right)$.

Delayed introduction with selling the new product to both types and not selling the old product (B7) is never optimal. This is because compared to strategy B7, with relatively low user heterogeneity ($v_L^H \leq v_2$), strategy B4 gives more profit from the low type's early purchase of the new product; with relatively high user heterogeneity ($v_L^H > v_2$), strategy B1 is more profitable by letting both types purchase in period 1.

(**Perfect intertemporal price discrimination**) Selling the new product in both periods by first serving the low type in period 1 (B4) dominates the other strategies regardless of segment sizes in the region $R\left(v_1 < v_L^H \leq v_2, q_O \geq q_2\right)$. By Observation 2, it also achieves perfect price discrimination. Similar to strategy B6, the advantage of strategy B4 relative to the other strategies in the region $R\left(v_L^H \leq v_2, q_O\right)$ varies with the extent of quality improvement. If $q_O \geq q_2$, strategy B4 is optimal when the demand elasticity is relatively low (cf. strategy B5 and B8); if $q_O < q_2$, it is optimal when the demand elasticity is relatively high (cf. strategy B1).

Setting aside the above properties about delayed introduction and perfect intertemporal price discrimination, we next consider the social efficiency of the optimal strategies in Proposition 5. Referring to Proposition 1, the vendor's strategy is socially efficient if it chooses strategy B5 in the region $R\left(v_L^H, q_O > q_2\right)$ and strategy B1 in the region $R\left(v_L^H, q_O \leq q_2\right)$. However, as shown in Table 3.8, for quite a few regions defined by v_L^H and q_O, the outcome is not efficient. Generally, due to the mutual cannibalization between the old and new products and the possible arbitrage between the low and high type users, in most of the parameter spaces, the vendor is unable to extract the full surplus from the users. The relocation of surplus between the vendor, the low type, and the high type induces the vendor to extract more surpluses by choosing inefficient strategies. For instance, to avoid the mutual cannibalization between the old and new products, it may shelve the old product when it is indeed efficient to sell it to the low type (e.g., strategy B4 in the region $R\left(v_L^H \leq v_2, q_O > q_2\right)$); to eliminate the concern of future price reduction, it may delay the introduction of the new product even though it is efficient to launch the new product immediately in period 1 (e.g., strategy B6 in the region $R\left(v_L^H > v_2, q_O \leq q_2\right)$). The latter confirms Fishman and Rob's speculation about the dampening effect of heterogeneity on monopoly profits.

3.4.2.2 With An Upgrade Policy

We now consider the case when a vendor can implement an upgrade policy. The provision of the upgrade option is an extra instrument for a vendor to exercise price discrimination based on purchase history (Fudenberg and Tirole 1998, Lee and Lee 1998). If the vendor can identify users who have bought the old product, it can charge them only the incremental utility that they receive by using the new product. For those who do not own the old product, it can charge them the full utility.

In our setting, only the high type possesses the old product at the beginning of period 1. The implementation of upgrade policy can improve seller's profitability from two perspectives. First, the vendor can now make a credible threat to the low type that if they do not buy the old product in period 1, they will face a very high price of the new product in period 2 (cf. those who trade in the old product for the new product). This threat of price discrimination dissuades the low type from leapfrogging and encourages them to buy the old product as soon as it is available for sale. Consequently, the upgrade policy can eliminate the cannibalization caused by the forthcoming new product on the old product, which in turn improves the profitability of strategies that sell both products to consumers, e.g., strategies B2 and B5.

Second, upgrade policy encourages the high type to reveal their purchase history if their incremental utility from the new product is less than that of the low type. By this means, the vendor can easily differentiate the low and high types and charge the low type a higher price for the new product if they have not possessed the old product. We expect that the presence of the upgrade policy can improve the profitability of product replacement strategies if the high type's incremental utility from the new product is less than that of the low type (e.g., strategies B1 and B4 in the region $R\left(v_L^H \le v_2, q_O\right)$). Compared with Observation 1 in the case with no upgrade policy, Observation 3 summarizes the difference made by an upgrade policy.

Observation 3. Given a partly-covered installed base, with an upgrade policy: (1) the surplus of the low type will always be fully extracted by the seller; (2) selling the new product to the high and low types sequentially (i.e., strategies B2 and B3) is always feasible.

Nevertheless, upgrade policy is not omnipotent. The high type will choose to conceal their purchase history if their incremental utility derived from the new product is still larger than that of the low type. We elaborate this point in Lemma 2.

Lemma 2. Given a partly-covered installed base, upgrade policy is incapable to segment users by their purchase history if user heterogeneity is sufficiently high so that $U_{0t}^{H} > Z_{Nt}^{L}$.

Table 3.9 lists the price sequence and profit for each strategy (B1-B8). The direct effect of Lemma 2 is that some strategies among B1-B8 would not be influenced by the presence of upgrade policy. Moreover, by Observation 2, in the region $v_1 < v_L^H \leq v_2$, strategy B4 achieves perfect intertemporal price discrimination even with no upgrade policy. Comparing Tables 3.7 and 3.9, Lemma 3 lists the strategies where the price sequences remain unchanged in the absence/presence of upgrade policy.

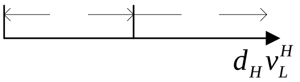
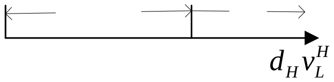
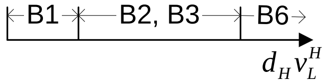
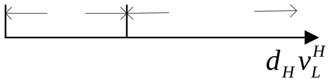
<Insert Table 3.9>

Lemma 3. The price sequences of the following strategies would not be affected by the presence of upgrade policy: (1) Strategy B1 in the region $v_L^H > v_2$; (2) Strategy B3 in the region $v_L^H > v_3$ and $q_O \leq q_2$; (3) Strategy B4 in the region $v_1 < v_L^H \leq v_2$ and $q_O > q_2$; (4) Strategy B6 in the region $v_L^H > v_2$ and $q_O > q_1$.

Comparing their profits listed in Table 3.9 leads to Proposition 6.

Proposition 6. With a partly-covered installed base of the old product, and with an upgrade policy, for each region defined by $R\left(v_L^H, q_O\right)$, the optimal strategies given the different demand elasticities are presented below. Specifically, regardless of the extent of user heterogeneity, strategy B2 (B5) is more profitable then strategy B3 (B1) when the segment size of the low type is large.

Table 3.10 Optimal Strategy with an upgrade policy in the presence of a partly-covered installed base.

v_L^H q_O	$v_L^H \le v_1$	$v_1 < v_L^H \le v_2$	$v_2 < v_L^H \le v_3$	$v_L^H > v_3$
$q_O \ge q_2$	 $d_H v_L^H$		 $d_H v_L^H$	
$q_1 \le q_O < q_2$	**B1**		 B1 B2, B3 B6 $d_H v_L^H$	
$q_O < q_1$			 $d_H v_L^H$	

As expected, the upgrade policy improves the profitability of strategies that sell the entire product line, i.e., B2 and B5, and the profitability of product replacement strategies in the region with relatively low user heterogeneity, i.e., B1 and B4. Referring to Table 3.10, this is reflected by the expansion of the regions in which these strategies are optimal, as compared with Table 3.8. As a result, some strategies are squeezed out of the original regions, e.g., strategy B8 disappears in the region $R\left(v_L^H \le v_2, q_O \ge q_2\right)$ and B6 becomes suboptimal in the region $R\left(v_L^H > v_3, q_O < q_1\right)$. We highlight some useful points in the following.

(**Delayed product introduction**) It is not optimal to delay the introduction of the new product when no product is sold in period 1. Compared with Table 3.8, the region in which the delayed introduction strategy selling both the old and new products to the low type (B5) is optimal extends to the region with relatively high user heterogeneity ($v_L^H > v_2$). The region where the delayed introduction strategy selling only the old product to the low type (B6) is optimal is squeezed by strategies B2 and B5 towards relatively high user heterogeneity and low quality improvement, and thus is not optimal in the region $R\left(v_L^H > v_2, q_O < q_1\right)$. It shows that as strategy B2 becomes increasingly profitable, selling the old product in period 1 is never a competitive advantage for strategy B6.

(**Perfect price discrimination based on purchase history**) The strategy selling the new product to both types in period 1 (B1) achieves perfect price discrimination in the region $R\left(v_L^H \leq v_2, q_O \leq q_2\right)$ and is also socially optimal. With an upgrade policy, in this region, the vendor simultaneously offers a discounted upgrade price to the high type and a relatively high price to the low type.

(**Perfect intertemporal price discrimination**) Selling the new product in both periods by first serving the low type in period 1 (B4) achieves perfect price discrimination in the region $R\left(v_L^H \leq v_2, q_O\right)$. However, the region where strategy B4 is optimal shrinks towards relatively high user heterogeneity and low quality improvement due to the increasing profitability of strategies B1 and B5.

Quite a few studies in various settings advocate the benefit of the upgrade policy in maximizing the vendor's profit and promoting socially efficient production (Waldman 1997, Lee and Lee 1998, Fishman and Rob 2000). By Table 3.10, we notice that the strategies listed in Lemma 3 can be optimal within certain ranges of demand elasticity. This suggests that, in a more general setting with differing user heterogeneity (cf. the high heterogeneity setting in Lee and Lee (1998), or the homogeneous setting in Fishman and Rob (2000)), the upgrade policy may not be a necessary condition to maximize profit. Referring to Table 3.10, denote $d_H v_L^H\left(B_i, v_L^H, q_O\right)$, $i = 1..8$, as the range of demand elasticity in which strategy B_i is optimal given the parameterization $\left(v_L^H, q_O\right)$, Proposition 7 summarizes the conditions in which the upgrade policy will not be used.

Proposition 7. With a partly-covered installed base of the old product, suppose that the vendor is able to offer an upgrade policy. It will not implement the upgrade policy in any of the following conditions:

(1) $v_1 < v_L^H \le v_2$, $q_O > q_2$, and $d_H v_L^H \left(B_4, v_L^H, q_O\right)$;

(2) $v_L^H > v_2$ and $d_H v_L^H \left(B_1, v_L^H, q_O\right)$;

(3) $v_L^H > v_3$, $q_O \le q_2$, $d_H v_L^H \left(B_3, v_L^H, q_O\right)$, and $d_L < \delta \frac{1-\delta^{n-1}}{1-\delta}$;

(4) $v_L^H > v_2$, $q_O > q_1$, and $d_H v_L^H \left(B_6, v_L^H, q_O\right)$.

The first condition in Proposition 7 implies that in certain parameterizations, the *perfect intertemporal price discrimination* strategy can outperform the upgrade policy to maximize the vendor's profit. The condition is defined by (i) the region of $R\left(v_1 < v_L^H \le v_2, q_O > q_2\right)$ in which by employing strategy B4, the vendor can practice perfect intertemporal price discrimination without the assistance of an upgrade policy and (ii) the range of demand elasticity ($d_H v_L^H$) in which strategy B4 is optimal in the specific region $R\left(v_1 < v_L^H \le v_2, q_O > q_2\right)$. Similarly, the second, third and fourth conditions suggest that given some parameterizations of v_L^H, q_O, and $d_H v_L^H$, instead of the upgrade policy, *a penetration pricing strategy* (B1), an *intertemporal price* *discrimination* strategy (B3), or a *delayed introduction strategy* (B6), can maximize the vendor's profit.

Proposition 7 also implies that with its incentive to differentiate users, the vendor may not choose socially efficient strategies even if an upgrade policy is feasible. The inability of upgrade policy to segment users by their purchase history as proposed in Lemma 2 may lead to both types of users be inefficiently served, e.g., strategy B6 in the region $R\left(v_L^H, q_O \leq q_2\right)$. Referring to Tables 3.8 and 3.7, the welfare of users depends on how much the vendor can profit from its production. If the vendor can sustain its monopoly power, e.g., in the region $R\left(v_L^H \leq v_2, q_O \leq q_2\right)$, via an upgrade policy, both types of users can get efficient quality. Otherwise, whether and which type can be efficiently served is determined by demand elasticity. Generally, the high type tends to get efficient quality when the demand is inelastic, whereas both types of users may be efficiently served when the demand is elastic.

3.5 Concluding Remarks

It is useful to highlight our results in light of the existing literature. First, in their study of recurrent innovation, Fishman and Rob (2000) speculate that in the setting with heterogeneous users, the monopolist may slow down the pace of innovation because of the combined (dampening) effects of the stock of durable goods in use, and the users' anticipation of future price reductions. Nevertheless, they did not characterize the profit-maximizing strategies with heterogeneous users. Their speculation has not been analytically validated due to the intractable setting.

Our study confirms Fishman and Rob's speculation about the dampening effect of user heterogeneity on monopoly profits. We find that there is a wide class of equilibria in which the vendor may forgo the chance of selling the new product earlier, and instead delay selling the new product (even though it is costless to sell it earlier) to a later period Such delayed introduction strategies can be optimal regardless of the extent of user heterogeneity and quality improvement.

Second, prior research in various settings advocates the benefit of upgrade pricing in maximizing the vendor's profits and promoting socially efficient productions (Waldman 1997, Lee and Lee 1998, Fishman and Rob 2000). However, in a more general setting with differing user heterogeneity (cf. the homogeneous setting in Fishman and Rob (2000)), we characterize the situations in which upgrade pricing may lose its ability to segment the market. We suggest that if the heterogeneity on valuation of quality is sufficiently high, instead of upgrade pricing, the vendor would maximize its profit via intertemporal price discrimination, delayed introduction, or penetration pricing, depending on market structure and the extent of technology improvement.

Third, studies on product timing and pricing have used both quality and time as segmentation variables (Stokey 1979, Moorthy 1984). In this literature, the reservation price of users for a new product decreases over time because of discounted future utility. To optimize profit, a vendor would first sell a new product to high valuation users, and then inefficiently serve low valuation users by distorting product quality (Moorthy and Png 1992); high valuation users would always buy a new product before low valuation users do, and they can get the efficient quality that they want.

However, our key disposition is that if some users have already owned the existing (old) version of a durable product, their willingness to purchase the newly improved version could indeed increase over time! This is because the old product depreciates over time and this increases the need for users to upgrade to the new product. Further, if a market consists of users with different purchase histories (i.e., some existing high-valuation users who own the old product and new low-valuation users who do not), their reservation prices for the new product may change with time in adverse directions. We show that these inconsistent trends among consumers could enable a perfect intertemporal price discrimination strategy with which a vendor sells the new product sequentially by first serving the low valuation users who have not purchased anything. When heterogeneity in user valuation is sufficiently high, the vendor might extract the full surplus from the low valuation users and leave them with little choice because they may face an even *higher* price in the subsequent period, because the willingness of the high valuation users to purchase the new product would increase over time. On the other hand, if user heterogeneity is not too high, the high valuation users are unwilling to accept the price in the early period because it is higher than the incremental utility that they can derive from the new product at that time. Thus, they may want to wait and purchase the new product after the low valuation users. This incentive to postpone purchase may actually make them worse off, which is contrary to popular findings in the price discrimination literature (e.g., Moorthy 1984, Moorthy and Png 1992).

Fourth, prior research assumes user heterogeneity on valuation of quality to be sufficiently high so as to exclude the cases where the willingness of the low valuation users to purchase an advanced version of the product is higher than that of the high valuation users who have possessed the old product (Waldman 1993, Lee and Lee 1998). This assumption significantly simplifies the analysis, but it raises a question which may be common in reality: in a moderately heterogeneous market, if the high valuation users have purchased the old product, how should the vendor schedule the launching of the new product? In this study, we characterize the properties of this market with respect to users' relative utility derived from the new product and their relative payoff from delayed purchase. We suggest that if the extent of quality improvement is high, an upgrade policy can help the vendor extract the full surplus from both types of users and at the same time provide socially optimal outcome. Otherwise, depending on the demand elasticity, the vendor may delay the launch of the new product or first sell to the low valuation users.

Our findings provide useful guidelines for product managers who often have to consider the installed base of their existing products when selling new products. Instead of transforming their business models to manipulate product durability (as suggested by Bulow 1986 and Waldman 1996b), they may adopt flexible timing and pricing strategies to cope with the existing installed base. Our setting is particularly applicable to IT products, especially those that depend heavily on components developed by external vendors. Sellers of these products often cannot control when new technologies are invented and when old products would become obsolete. Hence, for them, product innovation and introduction are two separate decisions – they might not be able to endogenize the extent of product innovation, but they could always control the timing and pricing of new products. Because IT vendors often face a huge installed base of existing customers using their previous products, they need to exercise extreme care when launching and pricing a new product that incorporates advanced technologies. With suboptimal strategies, they may greatly suffer from the combined dampening effects of the stock of durable goods in users' hand, and the users' anticipation of future price reductions. However, via proper timing, pricing and product line strategies, the seller could even practise perfect price discrimination by exploiting the existing users' increasing need to upgrade to the new product.

The immediate direction for further work is to extend our research in a competitive market. Further, it would also be interesting to consider settings where the demand is uncertain, or where there is asymmetric information.

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Appendix 3A-1

Table 3.2 Prices and profits with a fully-covered installed base

Strategy		Price sequence	Profit
A1		$p_1^N = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right) v_L$	$\pi_{A1} = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_L \right) v_L$
A2	$q_O < \left(1-\delta^n\right)$	$p_1^N = \delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_L + \left(1-\delta^n - q_O\right) v_H$ $p_2^N = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_L$	$\pi_{A2} = \left[\delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_L + \left(1-\delta^n - q_O\right) v_H \right] d_H + \delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_L d_L$
A3		$p_2^N = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_L$	$\pi_{A3} = \delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_L$
A4		$p_2^N = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_H$	$\pi_{A4} = \delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_H d_H$

Table 3.7 Prices and profits in a partly-covered installed base with no upgrade policy

Strategy		Price sequence	Profit
B1	$v_L^H \le \frac{1-\delta^n}{1-\delta^n - \left(1-\delta^{n-1}\right) q_O}$	$p_1^N = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right) v_H$	$\pi_{B1} = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right) v_H$
	$v_L^H > \frac{1-\delta^n}{1-\delta^n - \left(1-\delta^{n-1}\right) q_O}$	$p_1^N = \frac{1-\delta^n}{1-\delta} v_L$	$\pi_{B1} = \frac{1-\delta^n}{1-\delta} v_L$

B2	$q_O < 1-\delta^n$ and $v_L^H \le \frac{1-\delta^n}{1-\delta^n - q_O}$	$p_1^O = \left(1-\delta^n - q_O\right)v_H - \left(1-\delta^n - q_O\right)v_L$ $p_1^N = \delta\left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta}q_O\right)v_L + \left(1-\delta^n - q_O\right)v_H$, $p_2^N = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta}q_O\right)v_L$	$\pi_{B2} = \delta\left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta}q_O\right)v_L$ $+\left(1-\delta^n - q_O\right)v_H - \left(1-\delta^n - q_O\right)v_L d_L$
	$q_O < 1-\delta^n$ and $v_L^H > \frac{1-\delta^n}{1-\delta^n - q_O}$	$p_1^O = q_O v_L$, $p_1^N = \delta\left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta}q_O\right)v_L + \left(1-\delta^n - q_O\right)v_H$, $p_2^N = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta}q_O\right)v_L$	$\pi_{B2} = \delta\left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta}q_O\right)v_L$ $+\left(1-\delta^n - q_O\right)v_H d_H + q_O v_L d_L$
B3	$q_O < 1-\delta^n$ and $v_L^H > \frac{1-\delta^n}{1-\delta^n - q_O}$	$p_1^N = \delta\frac{1-\delta^n}{1-\delta}v_L + \left(1-\delta^n - q_O\right)v_H$, $p_2^N = \frac{1-\delta^n}{1-\delta}v_L$	$\pi_{B3} = \delta\frac{1-\delta^n}{1-\delta}v_L + \left(1-\delta^n - q_O\right)v_H d_H$
B4	$v_L^H \le \frac{1-\delta^n}{1-\delta^n - \left(1-\delta^{n-2}\right)q_O}$	$p_1^N = \delta\left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H + \left(1-\delta^n\right)v_L$, $p_2^N = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H$	$\pi_{B4} = \delta\left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H + \left(1-\delta^n\right)v_L d_L$
	$v_L^H > \frac{1-\delta^n}{1-\delta^n - \left(1-\delta^{n-2}\right)q_O}$ and $v_L^H \le \frac{1-\delta^n}{1-\delta^n - \left(1-\delta^{n-1}\right)q_O}$	$p_1^N = \frac{1-\delta^n}{1-\delta}v_L$, $p_2^N = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H$	$\pi_{B4} = \delta\left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H d_H + \frac{1-\delta^n}{1-\delta}v_L d_L$
B5		$p_1^O = q_O v_L$, $p_2^N = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta}q_O\right)v_L$	$\pi_{B5} = \delta\left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta}q_O\right)v_L + v_L q_O d_L$

B6	$v_L^H \le \frac{1-\delta^n}{1-\delta^n-\left(1-\delta^{n-2}\right)q_O}$	$p_1^O = \delta\left(\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H - \frac{1-\delta^n}{1-\delta}\left(\delta-q_O\right)v_L$, $p_2^N = \left(\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H$	$\pi_{B6} = \delta\left(\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H - \frac{1-\delta^n}{1-\delta}\left(\delta-q_O\right)v_L d_L$
	$v_L^H > \frac{1-\delta^n}{1-\delta^n-\left(1-\delta^{n-2}\right)q_O}$	$p_1^O = \frac{1-\delta^n}{1-\delta}v_L q_O$, $p_2^N = \left(\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H$	$\pi_{B6} = \delta\left(\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H d_H + \frac{\left(1-\delta^n\right)}{1-\delta}v_L q_O d_L$
B7	$v_L^H \le \frac{1-\delta^n}{1-\delta^n-\left(1-\delta^{n-2}\right)q_O}$	$p_2^N = \left(\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H$	$\pi_{B7} = \delta\left(\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right)v_H$
	$v_L^H > \frac{1-\delta^n}{1-\delta^n-\left(1-\delta^{n-2}\right)q_O}$	$p_2^N = \frac{1-\delta^n}{1-\delta}v_L$	$\pi_{B7} = \frac{\delta\left(1-\delta^n\right)}{1-\delta}v_L$
B8	$v_L^H < \frac{1-\delta^n}{1-\delta^n-\left(1-\delta^{n-2}\right)q_O}$	$p_2^N = \frac{1-\delta^n}{1-\delta}v_L$	$\pi_{B8} = \delta\frac{1-\delta^n}{1-\delta}v_L d_L$

Table 3.9 Prices and profits in a partly-covered installed base with an upgrade policy

Strategy		**Price sequence**	**Profit**
B1	$v_L^H \le \frac{1-\delta^n}{1-\delta^n-\left(1-\delta^{n-1}\right)q_O}$	$p_1^{Nu} = \left(\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right)v_H$, $p_1^N = \frac{1-\delta^n}{1-\delta}v_L$	$\pi_{B1}^u = \left(\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right)v_H d_H + \frac{1-\delta^n}{1-\delta}v_L d_L$
	$v_L^H > \frac{1-\delta^n}{1-\delta^n-\left(1-\delta^{n-1}\right)q_O}$	$p_1^{Nu} = p_1^N = \frac{1-\delta^n}{1-\delta}v_L$	$\pi_{B1}^u = \frac{1-\delta^n}{1-\delta}v_L$

B2		$p_1^O = \frac{1-\delta^n}{1-\delta} q_O v_L$, $p_2^{Nu} = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right) v_L$, $p_1^{Nu} = \delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right) v_L + \left(\left(1-\delta^n\right) - q_O \right) v_H$	$\pi_{B2}^u = \delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right) v_L$ $+ \left(1-\delta^n - q_O\right) v_H d_H + \frac{1-\delta^n}{1-\delta} q_O v_L d_L$
B3	$v_L^H \le \frac{\left(1-\delta^n\right)}{\left(1-\delta^n\right) - \left(1-\delta^{n-2}\right) q_O}$	$p_1^{Nu} = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right) v_H$, $p_2^N = \frac{1-\delta^n}{1-\delta} v_L$	$\pi_{B3}^u = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right) v_H d_H + \frac{\delta\left(1-\delta^n\right)}{1-\delta} v_L d_L$
	$v_L^H > \frac{\left(1-\delta^n\right)}{\left(1-\delta^n\right) - \left(1-\delta^{n-2}\right) q_O}$	$p_1^{Nu} = \frac{\delta\left(1-\delta^n\right)}{1-\delta} v_L + \left(\left(1-\delta^n\right) - q_O \right) v_H$, $p_2^N = \frac{1-\delta^n}{1-\delta} v_L$	$\pi_{B3}^u = \frac{\delta\left(1-\delta^n\right)}{1-\delta} v_L + \left(\left(1-\delta^n\right) - q_O \right) v_H d_H$
B4	$v_L^H \le \frac{1-\delta^n}{1-\delta^n - \left(1-\delta^{n-1}\right) q_O}$	$p_1^N = \frac{1-\delta^n}{1-\delta} v_L$; $p_2^{Nu} = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_H$	$\pi_{B4}^u = \delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_H d_H + \frac{\left(1-\delta^n\right)}{1-\delta} v_L d_L$
B5		$p_1^O = \frac{1-\delta^n}{1-\delta} q_O v_L$, $p_2^{Nu} = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right) v_L$	$\pi_{B5}^u = \delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right) v_L + \frac{\left(1-\delta^n\right)}{1-\delta} v_L q_O d_L$
B6		$p_1^O = \frac{1-\delta^n}{1-\delta} v_L q_O$, $p_2^{Nu} = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_H$	$\pi_{B6}^u = \delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_H d_H + \frac{\left(1-\delta^n\right)}{1-\delta} v_L q_O d_L$
B7	$v_L^H \le \frac{1-\delta^n}{1-\delta^n - \left(1-\delta^{n-2}\right) q_O}$	$p_2^{Nu} = \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_H$, $p_2^N = \frac{1-\delta^n}{1-\delta} v_L$	$\pi_{B7}^u = \delta \left(\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right) v_H d_H + \frac{\delta\left(1-\delta^n\right)}{1-\delta} v_L d_L$
	$v_L^H > \frac{1-\delta^n}{1-\delta^n - \left(1-\delta^{n-2}\right) q_O}$	$p_2^{Nu} = p_2^N = \frac{1-\delta^n}{1-\delta} v_L$	$\pi_{B7}^u = \frac{\delta\left(1-\delta^n\right)}{1-\delta} v_L$
B8	$v_L^H < \frac{1-\delta^n}{1-\delta^n - \left(1-\delta^{n-2}\right) q_O}$	$p_2^N = \frac{1-\delta^n}{1-\delta} v_L$	$\pi_{B8}^u = \delta \frac{1-\delta^n}{1-\delta} v_L d_L$

Appendix 3A-2

Proof of Lemma 1. Suppose in equilibrium the vendor wants to sell the new product only to the high type, it must sell it in period 2. Otherwise, the vendor can always capture the remaining demand by selling the new product with any positive price in period 2. In such an equilibrium, the low type would purchase the old product in period 1 because the price of the old product will not decrease and the price of the new product in period 2 will only be affordable to the high type. []

Proof of Proposition 4. In a partly-covered installed base, the utilities that the high and low types derive from purchasing the new product in period t, $t = 1,2$, are represented as U_{0t}^{H} and Z_{Nt}^{L} respectively. By equations (3.2), (3.4), (3.5) and (3.6), we have:

(i). $U_{0t}^{H} > Z_{Nt}^{L}$ if and only if $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-t}\right]q_O}$; otherwise $U_{0t}^{H} \leq Z_{Nt}^{L}$.

(ii).When $q_O \geq q_2$, $\frac{\Delta Z_{Nt}^{L}}{\Delta t} < 0 \leq \frac{\Delta U_{0t}^{H}}{\Delta t}$.

(iii).When $q_O < q_2$, $\frac{\Delta U_{0t}^{H}}{\Delta t} < \frac{\Delta Z_{Nt}^{L}}{\Delta t} < 0$ if and only if $v_L^H > \frac{1-\delta^n}{1-\delta^n-q_O}$; otherwise, $\frac{\Delta Z_{Nt}^{L}}{\Delta t} \leq \frac{\Delta U_{0t}^{H}}{\Delta t} < 0$. []

Proof of Observation 1. Referring to Table 4b, with strategy B3, the vendor adopts product replacement strategy by only selling the new product in periods 1 and 2. The vendor will extract the full surplus from the low type in period 2 by charging the price $p_2^N = z_{O1}^L$. Moving backwards, the price of the new product in period 1 is subject to self-selection constraints, such that both types would not deviate from the assigned purchase sequences:

$$U^H_{01} - p^N_1 \geq U^H_{02} - \delta p^N_2 \tag{3A1}$$

$$Z^L_{N2} - \delta p^N_2 \geq Z^L_{N1} - p^N_1 \tag{3A2}$$

Substituting (3A1) and (3A2) with $p^N_2 = z^L_{O1}$ results in the constraint for the price of the new product in period 1:

$$Z^L_{O1} - \frac{\Delta Z^L_{Nt}}{\Delta t} \leq p^N_1 \leq Z^L_{O1} - \frac{\Delta U^H_{0t}}{\Delta t} \tag{3A3}$$

which is feasible if and only if $\frac{\Delta U^H_{0t}}{\Delta t} \leq \frac{\Delta Z^L_{Nt}}{\Delta t} < 0$. Referring to Table 3.6, only the region $R\left(v^H_L > v_3, q_O \leq q_2\right)$ can satisfy this constraint. The maximum price that the vendor can charge the high type in period 1 is: $p^N_1 = Z^L_{O1} - \frac{\Delta U^H_{0t}}{\Delta t}$.

As for strategy B2, similarly, we get $p^N_2 = u^L_{12}$. In equilibrium, both types of users will follow the purchase sequences as outlined in Table 3.5b if they are unable to benefit from the other alternative choices:

$$Z^L_{O1} + U^L_{12} - p^O_1 - \delta p^N_2 \geq Z^L_{O1} - p^O_1 \tag{3A4}$$

$$Z^L_{O1} + U^L_{12} - p^O_1 - \delta p^N_2 \geq Z^L_{N1} - p^N_1 \tag{3A5}$$

$$Z^L_{O1} + U^L_{12} - p^O_1 - \delta p^N_2 \geq Z^L_{N2} - \delta p^N_2 \tag{3A6}$$

$$U^H_{01} - p^N_1 \geq U^H_{02} - \delta p^N_2 \tag{3A7}$$

Substituting $p^N_2 = u^L_{12}$ into (3A7) gives the maximum price that the vendor can charge the high type in period 1:

$$p^N_1 = U^H_{01} - \left(U^H_{02} - U^L_{12}\right) = U^L_{12} - \frac{\Delta U^H_{0t}}{\Delta t} \ ,$$

which is lower than the high type's incremental utility derived from the new product (U_{01}^{H}), and equals the discounted price charged to the low type in period 2 (U_{12}^{L}) plus the relative benefit the high type gains from the early purchase ($-\frac{\Delta U_{0t}^{H}}{\Delta t}$). Thus, p_1^N is feasible if and only if $\frac{\Delta U_{0t}^{H}}{\Delta t} \leq 0$, which corresponds to the region $R\left(v_L^H, q_O \leq q_2\right)$ in Table 3.6. Similarly, substituting $p_2^N = u_{12}^{L}$ into (3A4)-(3A6) ,

If $v_L^H \leq v_3$, $p_1^O = U_{12}^L - U_{02}^H + U_{01}^H + Z_{O1}^L - Z_{N1}^L$; otherwise $p_1^O = U_{12}^L - Z_{N2}^L + Z_{O1}^L$. []

Proof of Observation 2. For strategy B4, the price of the new product in period 2 is $p_2^N = u_{02}^H$. The price (p_1^N) in period 1 is subject to self-selection constraints such that neither type is willing to deviate from the assigned purchase path:

$$Z_{N1}^L - p_1^N \geq Z_{N2}^L - \delta p_2^N \tag{3A8}$$

$$U_{02}^H - \delta p_2^N \geq U_{01}^H - p_1^N \tag{3A9}$$

Meanwhile, p_1^N is also subject to a participation constraint so that the low type's surplus from purchasing in period 1 is nonnegative:

$$Z_{N1}^L - p_1^N \geq 0 \tag{3A10}$$

Substituting $p_2^N = u_{02}^H$ into (3A6)-(3A8) gives the constraint for the price of the new product in period 1:

$$U_{01}^H \leq p_1^N \leq \min\left\{U_{02}^H - \frac{\Delta Z_{Nt}^L}{\Delta t}, Z_{N1}^L\right\} \tag{3A11}$$

which is feasible only in the region $R\left(v_L^H \leq v_2, q_O\right)$ as shown in Table 3.6. Further,

$$\text{if } v_1 < v_L^H \leq v_2,\ p_1^N = Z_{N1}^L;\text{ otherwise } p_1^N = U_{02}^H - \frac{\Delta Z_{Nt}^L}{\Delta t}.$$

According to the above analysis, in the region $R\left(v_1 < v_L^H \leq v_2, q_O\right)$, the vendor can extract the full utility of the low and high types. The following proof shows that the low type, being aware of their zero surplus from purchasing in period 1, is unwilling to deviate.

Given $v_1 < v_L^H \leq v_2$, suppose there are a small group of low type who delay their purchase of the new product to period 2. Denote the size of the deviating users as γ. Since $p_2^N = u_{12}^H$ is larger than z_{N2}^L, to capture the extra demand of the deviating users, the vendor has to reduce the price to be no higher than z_{N2}^L. Setting $p_2^N = z_{N2}^L$, the vendor's profit in the last period is $\pi_{A4}^2 = z_{N2}^L \times \left(d_H + \gamma\right)$ which is less than $u_{12}^H \times d_H$ when $\gamma \to 0$ with $v_L^H > v_1$. Consequently, given an infinite small group of the low type who rejects the offer in period 1, the vendor would not capture the deviating users by lowering the price to their affordable level. Anticipating that they would not benefit from delayed purchase, no low type users would reject the offer in period 1. []

Proof of Observation 3. By Lemma 1, the surplus of the low type will be fully extracted if they only purchase the old product (i.e., strategy B6).

If the low type purchases the old and new products in periods 1 and 2 respectively, as discussed above, the forthcoming new product will not cannibalize the sales of the old product with the presence of an upgrade policy. The vendor can charge the low type the full service value of the old product even though they anticipate the new product to be available in the next period (see my analysis in Chapter 2). In addition, the low type who has purchased the old product in period 1 has a lower reservation price for the new product than that of the high type ($u_{02}^{H} > u_{12}^{L}$). Thus, the vendor can always extract the full surplus from the low type.

As for those strategies where the low type only purchases the new product in period 1 or 2 (i.e., B1, B3, B4, B7, B8), if the low type has a higher reservation price for the new product than the high type (e.g., when the user heterogeneity is relatively low), as discussed above, the high type is willing to reveal their purchase history and obtain a discounted price for the new product. And the low type will be offered a higher price that equals their full utility derived from the new product. If the low type has a lower reservation price for the new product, of course, they will be left with zero surplus.

Thus, in any of the above cases, the surplus of the low type will be fully extracted.

Since the vendor can identify users who have bought the old product, it can offer a relatively low price for the high type and simultaneously charge a high price to the low type, which is infeasible in the case with no upgrade policy. Hence, different from Observation 1, in the region where the low type has a higher reservation price for the new product (e.g., $R\left(v_{L}^{H} \le v_{2}, q_{O}\right)$) or the region where the high type is better off from delayed purchase (e.g., $R\left(v_{L}^{H}, q_{O} > q_{2}\right)$), the vendor can still sell the new product to the high type in period 1 and to the low type in period 2 by offering an upgrade price.[]

Appendix 3A-3

Case A: A fully-covered installed base

--- Both type *H* and type *L* have purchased the old product q_O in period 1

Period	Possible Outcomes							
	A1		*A2*		*A3*		*A4*	
	H	*L*	*H*	*L*	*H*	*L*	*H*	*L*
1	*N*	*N*	*N*	–	–	–	–	–
2	–	–	–	*N*	*N*	*N*	*N*	–

Case B: A partly-covered installed base

--- Only type H have purchased the old product q_O in period 1

Period	Possible Outcomes															
	B1		*B2*		*B3*		*B4*		*B5*		*B6*		*B7*		*B8*	
	H	*L*	*H*	*L*	*H*	*L*	*H*	*L*	*H*	*L*	*H*	*L*	*H*	*L*	*H*	*L*
1	*N*	*N*	*N*	*O*	*N*	–	–	*N*	–	*O*	–	*O*	–	–	–	–
2	–	–	–	*N*	–	*N*	*N*	–	*N*	*N*	*N*	–	*N*	*N*	–	*N*

3A.1 Case A: Price Sequences and Profits of Strategies A1-A4

We denote the profit generated by strategy s as π_s.

A1. $p_1^N = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L$; $\pi_{A1} = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L$

A2. $p_2^N = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_L$

$$\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_H - p_1^N \geq \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_H - \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_L$$

$$p_1^N > \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L ;\ p_1^N \leq \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_H$$

$$p_1^N = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_L + \left[1-\delta^n - q_O\right] v_H$$

A2 is feasible only when $q_O < 1-\delta^n$

$$\pi_{A2} = \left\{ \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_L + \left[1-\delta^n - q_O\right] v_H \right\} d_H + \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_L d_L$$

A3. $p_2^N = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_L$; $\pi_{A3} = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_L$

A4. $p_2^N = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_H$; $\pi_{A4} = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_H d_H$

The Analysis of A1:

$\pi_{A1} > \pi_{A2}$ if and only if $\left[1-\delta^n - q_O\right] > \left[1-\delta^n - q_O\right] v_L^H d_H$;

$\pi_{A1} > \pi_{A3}$ if and only if $1-\delta^n > q_O$

$\pi_{A1} > \pi_{A4}$ if and only if $d_H v_L^H < \dfrac{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_O\right\}}$

A1 is optimal if and only if: $q_O < 1-\delta^n$, $d_H v_L^H < 1$.

The Analysis of A2:

$\pi_{A2} > \pi_{A1}$ if and only if $\left[1-\delta^n - q_O\right] < \left[1-\delta^n - q_O\right] d_H v_L^H$;

$\pi_{A2} > \pi_{A3}$ if and only if $q_O < 1-\delta^n$;

$\pi_{A2} > \pi_{A4}$ if and only if

$$\left\{[2\delta-1]\left[1-\delta^n\right] - \left[2\delta-1-\delta^{n-1}\right] q_O\right\} d_H v_L^H < \delta\left\{\left[1-\delta^n\right] - \left[1-\delta^{n-2}\right] q_O\right\}$$

A2 is optimal if and only if: $q_O < 1-\delta^n$, $d_H v_L^H > 1$, and either of the following:

(1). $\left[2\delta-1-\delta^{n-1}\right] q_O > [2\delta-1]\left[1-\delta^n\right]$; or

(2). $\left[2\delta-1-\delta^{n-1}\right] q_O < [2\delta-1]\left[1-\delta^n\right]$ and $d_H v_L^H < \dfrac{\delta\left\{\left[1-\delta^n\right] - \left[1-\delta^{n-2}\right] q_O\right\}}{[2\delta-1]\left[1-\delta^n\right] - \left[2\delta-1-\delta^{n-1}\right] q_O}$

The Analysis of A3:

$\pi_{A3} > \pi_{A1}$ if and only if $q_O > 1-\delta^n$;

$\pi_{A3} > \pi_{A2}$ if and only if $q_O > 1-\delta^n$;

$\pi_{A3} > \pi_{A4}$ if and only if $d_H v_L^H < 1$;

A3 is optimal if and only if: $q_O > 1-\delta^n$, $d_H v_L^H < 1$

The Analysis of A4:

$\pi_{A4} > \pi_{A1}$ if and only if $d_H v_L^H > \dfrac{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_O\right\}}$;

$\pi_{A4} > \pi_{A2}$ if and only if $d_H v_L^H > \dfrac{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_O\right\}}{\left[1-2\delta+\delta^{n-1}\right] q_O - \left[1-2\delta\right]\left[1-\delta^n\right]}$ and

$\left[2\delta-1-\delta^{n-1}\right] q_O < \left[2\delta-1\right]\left[1-\delta^n\right]$;

$\pi_{A4} > \pi_{A3}$ if and only if $d_H v_L^H > 1$.

Note that:

$$\frac{\left[1-\delta^n\right] - \left[1-\delta^{n-1}\right] q_O}{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_O\right\}} < \frac{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_O\right\}}{\left[1-2\delta+\delta^{n-1}\right] q_O - \left[1-2\delta\right]\left[1-\delta^n\right]}\text{; and}$$

when $q_O > 1-\delta^n$, $\dfrac{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_O\right\}}{\left[1-2\delta+\delta^{n-1}\right] q_O - \left[1-2\delta\right]\left[1-\delta^n\right]} < 1$;

when $q_L \le 1-\delta^n$, $\dfrac{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_L\right\}}{\left[1-2\delta+\delta^{n-1}\right] q_O - \left[1-2\delta\right]\left[1-\delta^n\right]} > 1$

A4 is optimal if and only if:

when $q_O > 1-\delta^n$, $d_H v_L^H > 1$;

when $q_L \le 1-\delta^n$, $d_H v_L^H > \dfrac{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}{\left[1-2\delta+\delta^{n-1}\right]q_O-\left[1-2\delta\right]\left[1-\delta^n\right]}$,

$\left[2\delta-1-\delta^{n-1}\right]q_O<\left[2\delta-1\right]\left[1-\delta^n\right]$.

3A.2 Case B: Price Sequences and Profits of Strategies B1-B8

3A.2.1 No Upgrade Policy

B1.

If $v_L^H \le \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, then $p_1^N=\left[\dfrac{1-\delta^n}{1-\delta}-\dfrac{1-\delta^{n-1}}{1-\delta}q_O\right]v_H$,

$$\pi_{B1}=\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_H$$

If $v_L^H > \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, then $p_1^N=\dfrac{1-\delta^n}{1-\delta}v_L$, $\pi_{B1}=\dfrac{1-\delta^n}{1-\delta}v_L$

B2.

$$p_2^N=\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L$$

$$\frac{1-\delta^n}{1-\delta}q_Ov_L+\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L-p_1^O-\delta p_2^N \ge \frac{1-\delta^n}{1-\delta}v_L-p_1^N$$

$$\frac{1-\delta^n}{1-\delta}q_Ov_L+\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L-p_1^O-\delta p_2^N \ge \delta\frac{1-\delta^n}{1-\delta}v_L-\delta p_2^N$$

$$\frac{1-\delta^n}{1-\delta}q_Ov_L+\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L-p_1^O-\delta p_2^N \ge \frac{1-\delta^n}{1-\delta}q_Ov_L-p_1^O$$

$$\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_H-p_1^N \ge \delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H-\delta p_2^N$$

$$p_1^O \le p_1^N + \frac{1-\delta^n}{1-\delta} q_O v_L - \frac{1-\delta^n}{1-\delta} v_L$$

$$p_1^O \le q_O v_L$$

$$p_1^N \le \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L + \left[1-\delta^n - q_O\right] v_H$$

$$p_1^N = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L + \left[1-\delta^n - q_O\right] v_H$$

$p_1^O \le q_O v_L$; $p_1^O \le \left[1-\delta^n - q_O\right] v_H - \left[1-\delta^n - q_O\right] v_L$ which implies that $q_O < 1-\delta^n$

If $q_O < 1-\delta^n$ and $v_L^H \ge \frac{1-\delta^n}{1-\delta^n - q_O}$, then

$$p_1^N = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L + \left[1-\delta^n - q_O\right] v_H,$$

$$p_1^O = q_O v_L, p_2^N = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L$$

$$\pi_{B2} = \left\{\delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L + \left[1-\delta^n - q_O\right] v_H\right\} d_H + \left\{q_O v_L + \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L\right\} d_L$$

If $q_O < 1-\delta^n$ and $v_L^H < \frac{1-\delta^n}{1-\delta^n - q_O}$, then

$$p_1^N = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L + \left[1-\delta^n - q_O\right] v_H,$$

$$p_1^O = \left[1-\delta^n - q_O\right] v_H - \left[1-\delta^n - q_O\right] v_L, \; p_2^N = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_L$$

$$\pi_{B2}=\left\{\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L+\left[1-\delta^n-q_O\right]v_H\right\}d_H$$

$$+\left\{\left[1-\delta^n-q_O\right]v_H-\left[1-\delta^n-q_O\right]v_L+\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L\right\}d_L$$

B3.

$$p_2^N=\frac{1-\delta^n}{1-\delta}v_L$$

$$\delta\frac{1-\delta^n}{1-\delta}v_L-\delta p_2^N\geq\frac{1-\delta^n}{1-\delta}v_L-p_1^N$$

$$\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_H-p_1^N\geq\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H-\delta p_2^N$$

$$p_1^N\geq\frac{1-\delta^n}{1-\delta}v_L$$

$p_1^N=\delta\frac{1-\delta^n}{1-\delta}v_L+\left[1-\delta^n-q_O\right]v_H$ and this requires $\left[1-\delta^n-q_O\right]v_H\geq\left[1-\delta^n\right]v_L$

In summary: $q_O<1-\delta^n$ and $v_L^H\geq\frac{1-\delta^n}{1-\delta^n-q_O}$

$$\pi_{B3}=\left\{\delta\frac{1-\delta^n}{1-\delta}v_L+\left[1-\delta^n-q_O\right]v_H\right\}d_H+\delta\frac{1-\delta^n}{1-\delta}v_Ld_L$$

B4.

$$p_2^N=\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H$$

$$\frac{1-\delta^n}{1-\delta}v_L-p_1^N\geq\delta\frac{1-\delta^n}{1-\delta}v_L-\delta p_2^N$$

$$\frac{1-\delta^n}{1-\delta}v_L-p_1^N\geq 0$$

$$\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H-\delta p_2^N \geq \left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_H-p_1^N$$

$$p_1^N \leq \delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H+\left[1-\delta^n\right]v_L$$

$$p_1^N \leq \frac{1-\delta^n}{1-\delta}v_L$$

$$p_1^N \geq \left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_H$$

If $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, then $p_1^N=\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H+\left[1-\delta^n\right]v_L$

$$\pi_{B4}=\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_Hd_H+\left\{\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H+\left[1-\delta^n\right]v_L\right\}d_L$$

If $\frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}<v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, then $p_1^N=\frac{1-\delta^n}{1-\delta}v_L$

$$\pi_{B4}=\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_Hd_H+\frac{1-\delta^n}{1-\delta}v_Ld_L$$

B5.

$$p_2^N=\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L$$

$$\frac{1-\delta^n}{1-\delta}q_Ov_L+\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L-p_1^O-\delta p_2^N \geq \delta\frac{1-\delta^n}{1-\delta}v_L-\delta p_2^N$$

$p_1^O=q_Ov_L$; $p_2^N=\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L$

$$\pi_{B5} = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L + v_L q_O d_L$$

B6.

$$p_2^N = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H$$

$$\frac{1-\delta^n}{1-\delta}v_L q_O - p_1^O \geq \delta\frac{1-\delta^n}{1-\delta}v_L - \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H$$

$$\frac{1-\delta^n}{1-\delta}v_L q_O - p_1^O \geq 0$$

$$p_1^O \leq \frac{1-\delta^n}{1-\delta}v_L q_O$$

$$p_1^O \leq \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H - \left[\delta\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^n}{1-\delta}q_O\right]v_L$$

If $v_L^H > \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-2}\right]q_O}$, then $p_1^O = \frac{1-\delta^n}{1-\delta}v_L q_O$

$$\pi_{B6} = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H d_H + \frac{\left[1-\delta^n\right]}{1-\delta}v_L q_O d_L$$

If $v_L^H \leq \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-2}\right]q_O}$, then

$$p_1^O = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H - \left[\delta\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^n}{1-\delta}q_O\right]v_L$$

$$\pi_{B6} = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H d_H + \left\{\delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H - \left[\delta\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^n}{1-\delta}q_O\right]v_L\right\}d_L$$

B7.

If $v_L^H \le \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, then $p_2^N = \left[\dfrac{1-\delta^n}{1-\delta} - \dfrac{1-\delta^{n-2}}{1-\delta} q_O\right] v_H$

$$\pi_{B7} = \delta \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_H$$

If $v_L^H > \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, then $p_2^N = \dfrac{1-\delta^n}{1-\delta} v_L$, $\pi_{B7} = \dfrac{\delta\left[1-\delta^n\right]}{1-\delta} v_L$

In the region $v_L^H \le \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, B7 is dominated by B4;

In the region $v_L^H > \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, B7 is dominated by B1.

B8.

There must be $\dfrac{1-\delta^n}{1-\delta} v_L > \left[\dfrac{1-\delta^n}{1-\delta} - \dfrac{1-\delta^{n-2}}{1-\delta} q_O\right] v_H$ or $v_L^H < \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$.

$$p_2^N = \frac{1-\delta^n}{1-\delta} v_L$$

B8 is feasible only $v_L^H < \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$; $\pi_{B8} = \delta \dfrac{1-\delta^n}{1-\delta} v_L d_L$.

The Analysis of B1:

First consider B1 in the region $v_L^H \le \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$:

- In the region $q_O < 1-\delta^n$ and $v_L^H < \dfrac{1-\delta^n}{1-\delta^n - q_O}$, $\pi_{B1} > \pi_{B2}$ if and only if $v_L^H > \dfrac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} - \dfrac{\left[1-\delta^n-q_O\right]d_L}{\delta\left[\dfrac{1-\delta^n}{1-\delta} - \dfrac{1-\delta^{n-2}}{1-\delta} q_O\right]} < 1$. Thus B2 is dominated by B1 here.

- In the region $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, $\pi_{B1} > \pi_{B4}$ if and only if $q_O < 1-\delta^n$ and $v_L^H > \frac{\left[1-\delta^n\right]d_L}{1-\delta^n-q_O}$ which is equivalent to $d_L < \frac{1-\delta^n-q_O}{1-\delta^n}v_L^H$.
- In the region $\frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} < v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $\pi_{B1} > \pi_{B4}$ if and only if

$$v_L^H > \frac{\left[1-\delta^n\right]d_L}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O-\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}d_H} \text{ and}$$

$$d_H < \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$$

Alternatively, in this region,

$$\pi_{B1} > \pi_{B4} \text{ if and only if } \begin{array}{l} \left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L d_L\left\{v_L^H-\left[\frac{1-\delta^n}{1-\delta^n-\left(1-\delta^{n-1}\right)q_O}\right]\right\} \\ >\delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_O\right]v_H d_H-\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_H d_H \end{array}$$

If $q_O \geq 1-\delta^n$, the left side of the above inequality is negative; the right side is positive, thus B1 is dominated by B4. If $q_O < 1-\delta^n$, $\pi_{B1} > \pi_{B4}$ if and only if

$$d_L < \frac{[1-\delta]\left[1-\delta^n-q_O\right]v_L^H}{\left[1-\delta^n\right]-\delta\left\{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O\right\}v_L^H}$$

- In the region $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, $\pi_{B1} > \pi_{B5}$ if and only if $v_L^H > \delta + \frac{q_O d_L}{\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]}$. According to the analysis of B5 in the subsequent section, we know that B5 cannot be optimal if $q_O < 1-\delta^n$. Since in the region $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, B1 is optimal only if $q_O < 1-\delta^n$, we do not need to consider B5 in this case.
- According to the analysis of B6 in the subsequent section, B6 is not optimal in this region.

- In the region $v_L^H < \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, $\pi_{B1} > \pi_{B10}$ if and only if

  $v_L^H > \frac{\delta\left[1-\delta^n\right]d_L}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$.

Then, consider B1 in the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$:

- In the region $q_O < 1-\delta^n$ and $v_L^H \geq \frac{1-\delta^n}{1-\delta^n-q_O}$,

$\pi_{B1} > \pi_{B2}$ if and only if $d_H v_L^H < \frac{1-\delta^n+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O - q_O d_L}{1-\delta^n-q_O}$ which is equivalent to

$$d_L > \frac{v_L^H\left[1-\delta^n-q_O\right]-\left[1-\delta^n\right]-\delta\frac{1-\delta^{n-1}}{1-\delta}q_O}{v_L^H\left[1-\delta^n-q_O\right]-q_O}.$$

- In the region $q_O < 1-\delta^n$ and $v_L^H < \frac{1-\delta^n}{1-\delta^n-q_O}$,

$\pi_{B1} > \pi_{B2}$ if and only if $v_L^H < \frac{1-\delta^n+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O}{1-\delta^n-q_O}+d_L > \frac{1-\delta^n}{1-\delta^n-q_O}$, thus B2 is dominated by B1 here.

- In the region $q_O < 1-\delta^n$ and $v_L^H \geq \frac{1-\delta^n}{1-\delta^n-q_O}$, $\pi_{B1} > \pi_{B3}$ if and only if

  $d_H v_L^H < \frac{1-\delta^n}{1-\delta^n-q_O}$ which is equivalent to $d_L > 1-\frac{1-\delta^n}{v_L^H\left[1-\delta^n-q_O\right]}$.

- In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, B5 is not optimal. $\pi_{B1} > \pi_{B6}$ if and only if

  $d_H v_L^H < \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$.

B1 is optimal if and only if:

In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left(1-\delta^{n-1}\right)q_O}$, $d_H v_L^H < \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$;

if $q_O < 1-\delta^n$ and $v_L^H \ge \frac{1-\delta^n}{1-\delta^n-q_O}$, then it further requires

$$d_L > \frac{v_L^H\left[1-\delta^n-q_O\right]-\left[1-\delta^n\right]-\delta\frac{1-\delta^{n-1}}{1-\delta}q_O}{v_L^H\left[1-\delta^n-q_O\right]-q_O} \text{ and } d_L > 1-\frac{1-\delta^n}{v_L^H\left[1-\delta^n-q_O\right]}.$$

In the region $\frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} < v_L^H \le \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$,

$$q_O < 1-\delta^n \text{ and } d_L < \frac{\left[1-\delta\right]\left[1-\delta^n-q_O\right]v_L^H}{\left[1-\delta^n\right]-\delta\left\{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O\right\}v_L^H}.$$

In the region $v_L^H \le \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, $q_O < 1-\delta^n$ and $d_L < \frac{1-\delta^n-q_O}{1-\delta^n}v_L^H$.

The Analysis of B2:

In the region $q_O < 1-\delta^n$ and $v_L^H < \frac{1-\delta^n}{1-\delta^n-q_O}$, B2 is dominated by B1. So we only consider the region $q_O < 1-\delta^n$ and $v_L^H \ge \frac{1-\delta^n}{1-\delta^n-q_O}$. In this region, B4, B5 and B10 are not optimal.

- In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $\pi_{B2} > \pi_{B1}$ if and only if

  $$d_H v_L^H > \frac{1-\delta^n+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O-q_O d_L}{1-\delta^n-q_O}$$ which is equivalent to

  $$d_L < \frac{v_L^H\left[1-\delta^n-q_O\right]-\left[1-\delta^n\right]-\delta\frac{1-\delta^{n-1}}{1-\delta}q_O}{v_L^H\left[1-\delta^n-q_O\right]-q_O}.$$

- $\pi_{B2} > \pi_{B3}$ if and only if $d_L > \delta\frac{1-\delta^{n-1}}{1-\delta}$.

In the region $v_L^H > \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $\pi_{B2} > \pi_{B6}$ if and only if

$$\left\{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}v_L^H d_H < \delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O\left[1+d_L\right]$$

B2 is optimal if and only if: $q_O < 1-\delta^n$, $v_L^H \geq \dfrac{1-\delta^n}{1-\delta^n-q_O}$,

$d_L < \dfrac{v_L^H\left[1-\delta^n-q_O\right]-\left[1-\delta^n\right]-\delta\dfrac{1-\delta^{n-1}}{1-\delta}q_O}{v_L^H\left[1-\delta^n-q_O\right]-q_O}$, $d_L > \delta\dfrac{1-\delta^{n-1}}{1-\delta}$, and

$$\left\{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}v_L^H d_H < \delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O\left[1+d_L\right]$$

The Analysis of B3:

- In the region $v_L^H > \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $\pi_{B3} > \pi_{B1}$ if and only if

  $d_H v_L^H > \dfrac{1-\delta^n}{1-\delta^n-q_O}$ which is equivalent to $d_L < 1-\dfrac{1-\delta^n}{v_L^H\left[1-\delta^n-q_O\right]}$.
- $\pi_{B3} > \pi_{B2}$ if and only if $d_L < \delta\dfrac{1-\delta^{n-1}}{1-\delta}$.
- In the region $v_L^H > \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $\pi_{B3} > \pi_{B6}$ if and only if

  $\left\{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}d_H v_L^H < \dfrac{1-\delta^n}{1-\delta}\left[\delta-q_O d_L\right]$.

B3 is optimal if and only if: $q_O < 1-\delta^n$, $v_L^H \geq \dfrac{1-\delta^n}{1-\delta^n-q_O}$,

$d_L < 1-\dfrac{1-\delta^n}{v_L^H\left[1-\delta^n-q_O\right]}$, $d_L < \delta\dfrac{1-\delta^{n-1}}{1-\delta}$ and

$$\left\{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}d_H v_L^H < \frac{1-\delta^n}{1-\delta}\left[\delta-q_O d_L\right]$$

The Analysis of B4:

According to the analysis of B1-B3 and B5-B6, we know that B2, B3 and B6 are not optimal in the region where B4 is feasible.

- In the region $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, $\pi_{B4} > \pi_{B1}$ if and only if $\left[1-\delta^n - q_O\right]v_L^H < \left[1-\delta^n\right]d_L$. Thus, if $q_L > 1-\delta^n$, B4 dominates B1.
- In the region $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, $\pi_{B4} > \pi_{B5}$ if and only if $v_L^H > \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} - \frac{\left[1-\delta\right]\left\{\left[1-\delta^n\right]-q_O\right\}d_L}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$ which is equivalent to $d_L < \left\{v_L^H - \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}\right\}\frac{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}{(1-\delta)\left\{q_O-\left[1-\delta^n\right]\right\}}$.
- In the region $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, $\pi_{B4} > \pi_{B8}$ if and only if $v_L^H > \frac{\left[2\delta-1\right]\left[1-\delta^n\right]d_L}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$.
- In the region $\frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} < v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$,

$\pi_{B4} > \pi_{B1}$ if and only if

$$\left\{\left\{1-\delta^n-\left[1-\delta^{n-1}\right]q_O\right\}-\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}d_H\right\}v_L^H < \left[1-\delta^n\right]d_L$$

B4 is optimal if and only if:

In the regions $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$,

then $v_L^H > \frac{\left[2\delta-1\right]\left[1-\delta^n\right]d_L}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$,

if $q_O \geq 1-\delta^n$, then $d_L < \left[v_L^H - \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}\right]\frac{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}{\left[1-\delta\right]\left\{q_O-\left[1-\delta^n\right]\right\}}$;

if $q_O < 1-\delta^n$, then $d_L > \left[\dfrac{1-\delta^n - q_O}{1-\delta^n}\right] v_L^H$.

In the region $\dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} < v_L^H \le \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$,

if $q_O \ge 1-\delta^n$, B4 always dominates other strategies;

if $q_O < 1-\delta^n$, then $d_L > \dfrac{v_L^H\left[1-\delta\right]\left\{\left[1-\delta^n\right]-q_O\right\}}{\left\{1-\delta^n - v_L^H\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}\right\}}$.

The Analysis of B5:

- In the region $v_L^H \le \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_L}$, according to the analysis of B1, B1 is not optimal if $q_O > 1-\delta^n$. Thus, in the region $q_O < 1-\delta^n$, $\pi_{B5} > \pi_{B1}$ if and only if $v_L^H < \delta + \dfrac{q_O d_L\left[1-\delta\right]}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, which implies $q_O > \dfrac{1-\delta^n}{\left[1-\delta^{n-1}\right]+d_L}$ and $d_L > \delta^{n-1}\left[1-\delta\right]$.
- In the region $v_L^H > \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $\pi_{B5} > \pi_{B1}$ if and only if

$\delta\left[1-\delta^{n-1}\right]q_O < q_O d_L\left[1-\delta\right]-\left[1-\delta^n\right]\left[1-\delta\right]$, which implies $d_L > \dfrac{\delta\left[1-\delta^{n-1}\right]}{1-\delta}$ and

$q_O > \dfrac{\left[1-\delta\right]\left[1-\delta^n\right]}{\left[1-\delta\right]d_L-\delta\left[1-\delta^{n-1}\right]}$.

- In the region $q_O < 1-\delta^n$ and $v_L^H \ge \dfrac{1-\delta^n}{1-\delta^n-q_O}$, B5 is dominated by B2.
- In the region $q_L < 1-\delta^n$ and $v_L^H < \dfrac{1-\delta^n}{1-\delta^n-q_O}$, $\pi_{B5} > \pi_{B2}$ if and only if $v_L^H < \dfrac{\left[1-\delta^n\right]d_L}{1-\delta^n-q_O}$.
- In the region $v_L^H \le \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_L}$, $\pi_{B5} > \pi_{B4}$ if and only if

$v_L^H < \frac{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O} + \frac{\left[1-\delta\right]\left\{q_O - \left[1-\delta^n\right]\right\} d_L}{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_O\right\}}$, which is equivalent to

$d_L > \left\{ v_L^H - \frac{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O} \right\} \frac{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_O\right\}}{\left[1-\delta\right]\left\{q_O - \left[1-\delta^n\right]\right\}}$, and which implies

$\frac{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O} + \frac{\left[1-\delta\right]\left\{q_O - \left[1-\delta^n\right]\right\} d_L}{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_O\right\}} > 1$, which requires

$q_O > 1-\delta^n$ and $d_L > \frac{\delta^{n-1} q_O}{q_O - \left[1-\delta^n\right]}$.

- In the region $\frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O} < v_L^H \leq \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}$,

$\pi_{B5} > \pi_{B4}$ if and only if $d_H v_L^H < \frac{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O} + \frac{\left\{q_O\left[1-\delta\right] - \left[1-\delta^n\right]\right\} d_L}{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right] q_O\right\}}$, which

implies $d_L < \frac{\delta\left[1-\delta^n\right] - \delta\left[1-\delta^{n-1}\right] q_O}{\left[1-\delta^n\right] - \left[1-\delta\right] q_O}$.

- In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}$, $\pi_{B5} > \pi_{B6}$ if and only if $d_H v_L^H < \frac{1-\delta^n - \left[1-\delta^{n-1}\right]\left[1+d_L\right] q_O}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O}$. In the region $v_L^H \leq \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}$, B6 is dominated by B4.
- In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O}$, $\pi_{B5} > \pi_{B7}$ if and only if $d_L > \delta \frac{1-\delta^{n-1}}{1-\delta}$.

This conflicts with the comparison between B5 and B4 that requires

$d_L < \frac{\delta\left[1-\delta^n\right] - \delta\left[1-\delta^{n-1}\right] q_O}{\left[1-\delta^n\right] - \left[1-\delta\right] q_O}$. Thus, in the region:

$\frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O} < v_L^H \leq \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}$, B5 cannot be optimal.

In addition, consider the conditions which make both B6 and B7 dominated by B5 in the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$: $d_H v_L^H < \frac{1-\delta^n-\left[1-\delta^{n-1}\right]\left[1+d_L\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$ and $d_L > \delta\frac{1-\delta^{n-1}}{1-\delta}$, they imply that $q_O < \frac{1-\delta}{1-\delta^{n-1}}$. Then, consider the condition making B1 dominated by B5 in the same region: $q_O > \frac{\left[1-\delta\right]\left[1-\delta^n\right]}{\left[1-\delta\right]d_L-\delta\left[1-\delta^{n-1}\right]}$. To satisfy both implied conditions, we must have $d_L > \frac{\left[1-\delta^{n-1}\right]\left[1+\delta-\delta^n\right]}{1-\delta} > 1$, which is impossible.

Hence, B5 cannot be optimal in the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$.

In the region $v_L^H \le \frac{1-\delta^n}{1-\delta^n-\left(1-\delta^{n-2}\right)q_L}$, $\pi_{B5} > \pi_{B8}$ if and only if

$$\left[\delta\frac{1-\delta^n}{1-\delta}-q_O\right]d_L < \delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]$$

B5 is optimal if and only if: $q_O > 1-\delta^n$, $v_L^H \le \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$,

$d_L > \left\{v_L^H - \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}\right\}\frac{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}{\left[1-\delta\right]\left\{q_O-\left[1-\delta^n\right]\right\}}$ and

$\left[\delta\frac{1-\delta^n}{1-\delta}-q_O\right]d_L < \delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]$ or $q_O > \delta\frac{1-\delta^n}{1-\delta}$

The Analysis of B6:

In the region $v_L^H \le \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, B6 is dominated by B4.

In the region $v_L^H > \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$:

$\pi_{B6} > \pi_{B1}$ if and only if $d_H v_L^H > \dfrac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-2}\right]q_O}$ which is equivalent to

$v_L^H > \dfrac{1-\delta^n}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$ and $d_L < \dfrac{v_L^H\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}-\left[1-\delta^n\right]}{v_L^H\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}-\left[1-\delta^n\right]q_O}$.

$\pi_{B6} > \pi_{B2}$ if and only if: $q_O < 1-\delta^n$ and $v_L^H \geq \dfrac{1-\delta^n}{1-\delta^n-q_O}$

$$\left\{\left[2\delta-1\right]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}v_L^H d_H > \delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O\left[1+d_L\right]$$

$\pi_{B6} > \pi_{B3}$ if and only if $q_O < 1-\delta^n$ and $v_L^H \geq \dfrac{1-\delta^n}{1-\delta^n-q_O}$

$$\left\{\left[2\delta-1\right]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}v_L^H d_H > \left[1-\delta^n\right]\left[\delta-q_O d_L\right]$$

Consequently,

B6 is optimal if and only if:

$v_L^H > \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $v_L^H > \dfrac{1-\delta^n}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$ and

$$d_L < \frac{v_L^H\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}-\left[1-\delta^n\right]}{v_L^H\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}-\left[1-\delta^n\right]q_O}$$

Further, if $q_O < 1-\delta^n$ and $v_L^H \geq \dfrac{1-\delta^n}{1-\delta^n-q_O}$, then:

$$\left\{\left[2\delta-1\right]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}v_L^H d_H > \delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O\left[1+d_L\right]$$

$$\left\{\left[2\delta-1\right]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}v_L^H d_H > \left[1-\delta^n\right]\left[\delta-q_O d_L\right]$$

The Analysis of B8:

$\pi_{B4} < \pi_{B8}$ if and only if $v_L^H < \dfrac{[2\delta-1]\left[1-\delta^n\right]d_L}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$ which implies $\delta > \dfrac{1}{2}$

$\pi_{B5} < \pi_{B8}$ if and only if $d_L > \dfrac{\delta\left[\dfrac{1-\delta^n}{1-\delta}-\dfrac{1-\delta^{n-1}}{1-\delta}q_O\right]}{\delta\dfrac{1-\delta^n}{1-\delta}-q_O}$ which implies $1-2\delta+\delta^n < 0$.

Consequently,

B8 is optimal if and only if: $v_L^H < \dfrac{[2\delta-1]\left[1-\delta^n\right]d_L}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$ and

$$d_L > \frac{\delta\left\{1-\delta^n-\left[1-\delta^{n-1}\right]q_O\right\}}{\delta\left[1-\delta^n\right]-q_O[1-\delta]}.$$

Summary of the optimal strategies with aspects of v_L^H, q_O, and $d_H v_L^H$

In the region $v_L^H \le \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, $q_O \ge 1-\delta^n$,

Optimal strategies: B4, B5 and B8

B4: $v_L^H > \dfrac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}-\dfrac{[1-\delta]\left\{\left[1-\delta^n\right]-q_O\right\}d_L}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$,

$$v_L^H > \frac{[2\delta-1]\left[1-\delta^n\right]d_L}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$$

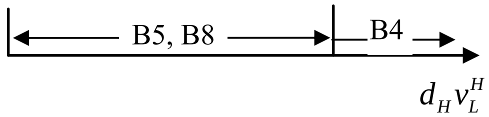
B5: $v_L^H < \dfrac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}-\dfrac{[1-\delta]\left\{\left[1-\delta^n\right]-q_O\right\}d_L}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$,

$$\left[\delta\frac{1-\delta^n}{1-\delta}-q_O\right]d_L < \delta\left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]$$

B8: $v_L^H < \frac{[2\delta-1]\left[1-\delta^n\right]d_L}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$, $d_L > \frac{\delta\left\{1-\delta^n-\left[1-\delta^{n-1}\right]q_O\right\}}{\delta\left[1-\delta^n\right]-q_O[1-\delta]}$, $\delta > \frac{1}{2}$,

$q_O < \delta\frac{1-\delta^n}{1-\delta}$

The demand elasticity for each strategy to be optimal:



In the region $\frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} < v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $q_O \geq 1-\delta^n$,

Optimal strategy: **B4**

In the region $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $q_O < 1-\delta^n$,

Optimal strategies: **B1 and B4**

B1: $d_L < \frac{1-\delta^n-q_O}{1-\delta^n}v_L^H$, $d_L < \frac{[1-\delta]\left[1-\delta^n-q_O\right]v_L^H}{\left[1-\delta^n\right]-\delta\left\{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O\right\}v_L^H}$

B4: $d_L > \left[\frac{1-\delta^n-q_O}{1-\delta^n}\right]v_L^H$, $d_L > \frac{v_L^H[1-\delta]\left\{\left[1-\delta^n\right]-q_O\right\}}{\left\{1-\delta^n-v_L^H\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}\right\}}$

The demand elasticity for each strategy to be optimal:

In the region

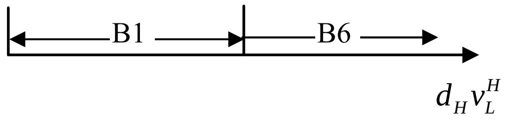
$$\frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O} < v_L^H \leq \frac{1-\delta^n}{1-\delta^n-q_O} \text{ or } v_L^H > \frac{1-\delta^n}{1-\delta^n-q_O} \,\&\, q_O \geq 1-\delta^n,$$

Optimal strategies: **B1** and **B6**

B1: $d_H v_L^H < \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$

B6: $d_H v_L^H > \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-2}\right]q_O}$

The demand elasticity for each strategy to be optimal:



In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-q_O}$, $q_O < 1-\delta^n$,

Optimal strategies: **B1**, **B2**, **B3** and **B6**

B1: $d_H v_L^H < \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$, $d_H v_L^H < \frac{1-\delta^n+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O-q_O d_L}{1-\delta^n-q_O}$, and

$d_H v_L^H < \frac{1-\delta^n}{1-\delta^n-q_O}$

B2: $d_H v_L^H > \frac{1-\delta^n+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O-q_O d_L}{1-\delta^n-q_O}$, $d_L > \delta\frac{1-\delta^{n-1}}{1-\delta}$, and

$\left\{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}v_L^H d_H < \delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O\left[1+d_L\right]$

B3: $d_H v_L^H > \frac{1-\delta^n}{1-\delta^n-q_O}$, $d_L < \delta\frac{1-\delta^{n-1}}{1-\delta}$, and

$\left\{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}d_H v_L^H < \frac{1-\delta^n}{1-\delta}\left[\delta-q_O d_L\right]$

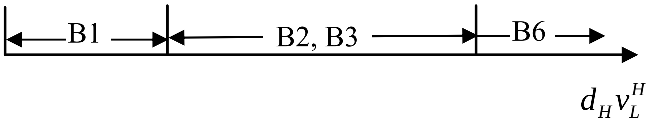
B6: $d_H v_L^H > \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-2}\right]q_O}$,

$\left\{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}v_L^H d_H > \delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O\left[1+d_L\right]$, and

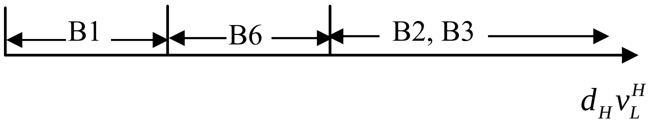
$$\left\{[2\delta-1][1-\delta^n]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}v_L^H d_H > \left[1-\delta^n\right][\delta-q_O d_L]$$

The demand elasticity for each strategy to be optimal:

If $q_O > \dfrac{[1-2\delta]\left[1-\delta^n\right]}{1-2\delta+\delta^{n-1}}$:



If $q_O < \dfrac{[1-2\delta]\left[1-\delta^n\right]}{1-2\delta+\delta^{n-1}}$:



3A.2.2 Upgrade Policy

In this case, strategy B8 is not optimal because the seller can earn more by selling the new product to the high type with a discounted price.

B1.

If $v_L^H \le \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $p_1^{Nu} = \left[\dfrac{1-\delta^n}{1-\delta}-\dfrac{1-\delta^{n-1}}{1-\delta}q_O\right]v_H$; $p_1^N = \dfrac{1-\delta^n}{1-\delta}v_L$

$$\pi_{B1}^u = \left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_H d_H + \frac{1-\delta^n}{1-\delta}v_L d_L$$

If $v_L^H > \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $p_1^{Nu} = p_1^N = \dfrac{1-\delta^n}{1-\delta}v_L$; $\pi_{B1}^u = \dfrac{1-\delta^n}{1-\delta}v_L$

B2.

$$p_1^O = \frac{1-\delta^n}{1-\delta}q_O v_L \text{ ; } p_2^{Nu} = \left[\frac{1-\delta^n}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_O\right]v_L$$

$$\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_{O}\right]v_{H}-p_{1}^{Nu}\geq\delta\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_{O}\right]v_{H}-\delta p_{2}^{Nu}$$

$$p_{1}^{Nu}\leq\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_{O}\right]v_{H}\text{ and }p_{1}^{Nu}\leq\delta\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_{O}\right]v_{L}+\left\{\left[1-\delta^{n}\right]-q_{O}\right\}v_{H}$$

$$\because\frac{1-\delta^{n}-\left[1-\delta^{n-1}\right]q_{O}}{1-\delta^{n}-\left[1-\delta^{n-2}\right]q_{O}}<1,\therefore v_{L}^{H}>\frac{1-\delta^{n}-\left[1-\delta^{n-1}\right]q_{O}}{1-\delta^{n}-\left[1-\delta^{n-2}\right]q_{O}}$$

$$p_{1}^{Nu}=\delta\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_{O}\right]v_{L}+\left\{\left[1-\delta^{n}\right]-q_{O}\right\}v_{H}$$

$$\pi_{B2}^{u}=\left\{\delta\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_{O}\right]v_{L}+\left\{\left[1-\delta^{n}\right]-q_{O}\right\}v_{H}\right\}d_{H}+\left\{\frac{1-\delta^{n}}{1-\delta}q_{O}v_{L}+\delta\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_{O}\right]v_{L}\right\}d_{L}$$
$$=\delta\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_{O}\right]v_{L}+\left\{\left[1-\delta^{n}\right]-q_{O}\right\}v_{H}d_{H}+\frac{1-\delta^{n}}{1-\delta}q_{O}v_{L}d_{L}$$

In this region: B2 dominates B5 if and only if $q_{L}<1-\delta^{n}$.

B3.

$$p_{2}^{N}=\frac{1-\delta^{n}}{1-\delta}v_{L}$$

$$\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_{O}\right]v_{H}-p_{1}^{Nu}\geq\delta\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-2}}{1-\delta}q_{O}\right]v_{H}-\delta p_{2}^{N}$$

$$p_{1}^{Nu}\leq\left[\frac{1-\delta^{n}}{1-\delta}-\frac{1-\delta^{n-1}}{1-\delta}q_{O}\right]v_{H}\text{ and }p_{1}^{Nu}\leq\frac{\delta\left[1-\delta^{n}\right]}{1-\delta}v_{L}+\left\{\left[1-\delta^{n}\right]-q_{O}\right\}v_{H}$$

If $v_{L}^{H}\geq\frac{\left[1-\delta^{n}\right]}{\left[1-\delta^{n}\right]-\left[1-\delta^{n-2}\right]q_{O}}$, $p_{1}^{Nu}=\frac{\delta\left[1-\delta^{n}\right]}{1-\delta}v_{L}+\left\{\left[1-\delta^{n}\right]-q_{O}\right\}v_{H}$

If $v_L^H < \frac{\left[1-\delta^n\right]}{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O}$, $p_1^{Nu} = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_H$

If $v_L^H \geq \frac{\left[1-\delta^n\right]}{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O}$,

$$\pi_{B3}^u = \left\{ \frac{\delta\left(1-\delta^n\right)}{1-\delta} v_L + \left\{\left[1-\delta^n\right]-q_O\right\} v_H \right\} d_H + \frac{\delta\left[1-\delta^n\right]}{1-\delta} v_L d_L$$

If $v_L^H < \frac{\left[1-\delta^n\right]}{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O}$, $\pi_{B3}^u = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_H d_H + \frac{\delta\left[1-\delta^n\right]}{1-\delta} v_L d_L$

We can see that in the region $v_L^H < \frac{\left[1-\delta^n\right]}{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O}$, B1 dominates B3.

B4.

$$p_2^{Nu} = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_H$$

$$\delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_H - \delta p_2^N \geq \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_H - p_1^N;\ p_1^N \leq \frac{1-\delta^n}{1-\delta} v_L$$

$$\Rightarrow \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O\right] v_H \leq p_1^N \leq \frac{1-\delta^n}{1-\delta} v_L$$

Thus, B4 is feasible only when $v_L^H \leq \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right]q_O}$.

$$p_1^N = \frac{1-\delta^n}{1-\delta} v_L;\ p_2^{Nu} = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_H,$$

$$\pi_{B4}^u = \delta\left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O\right] v_H d_H + \frac{\left[1-\delta^n\right]}{1-\delta} v_L d_L$$

B5.

$$p_1^O = \frac{1-\delta^n}{1-\delta} q_O v_L \,;\, p_2^{Nu} = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right] v_L$$

$$\pi_{B5}^u = \delta \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-1}}{1-\delta} q_O \right] v_L + \frac{\left[1-\delta^n\right]}{1-\delta} v_L q_O d_L$$

B6.

$$p_1^O = \frac{1-\delta^n}{1-\delta} q_O v_L \,;\, p_2^{Nu} = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right] v_H$$

$$\pi_{B6}^u = \delta \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right] v_H d_H + \frac{\left[1-\delta^n\right]}{1-\delta} v_L q_O d_L$$

$\pi_{B4}^u > \pi_{B6}^u$ if and only if $v_L^H \le \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}$.

B7.

If $v_L^H \le \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O}$, $p_2^{Nu} = \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right] v_H \,;\, p_2^N = \frac{1-\delta^n}{1-\delta} v_L$

$$\pi_{B7}^u = \delta \left[\frac{1-\delta^n}{1-\delta} - \frac{1-\delta^{n-2}}{1-\delta} q_O \right] v_H d_H + \frac{\delta\left[1-\delta^n\right]}{1-\delta} v_L d_L$$

In this region, $\pi_{B4}^u > \pi_{B7}^u$.

If $v_L^H > \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O}$, $p_2^{Nu} = p_2^N = \frac{1-\delta^n}{1-\delta} v_L \,;\, \pi_{B7}^u = \frac{\delta\left[1-\delta^n\right]}{1-\delta} v_L$

In this region, $\pi_{B1}^u > \pi_{B7}^u$.

The Analysis of B1:

First consider the region $v_L^H \le \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}$:

- In the region $q_O < 1-\delta^n$, $\pi^u_{B1} > \pi^u_{B2}$ if and only if $v_L^H - \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} > d_L\left\{v_L^H - \frac{\left[1-\delta^n\right]\left[1-q_O\right]}{\delta\left\{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O\right\}}\right\}$ in which $v_L^H - \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} > v_L^H - \frac{\left[1-\delta^n\right]\left[1-q_O\right]}{\delta\left\{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O\right\}}$ if $q_O < 1-\delta^n$. Thus, B1 dominates B2.
- In the region $v_L^H \geq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$, $\pi^u_{B1} > \pi^u_{B3}$ if and only if $v_L^H\delta\left\{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O\right\}-\delta\left[1-\delta^n\right] > \left\{v_L^H\delta\left\{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O\right\}-\left[1-\delta^n\right]\right\}d_L$ in which $v_L^H\delta\left\{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O\right\}-\delta\left[1-\delta^n\right] > v_L^H\delta\left\{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O\right\}-\left[1-\delta^n\right]$. Thus B1 dominates B3.

- $\pi^u_{B1} > \pi^u_{B4}$ if and only if $q_O < 1-\delta^n$.
- $\pi^u_{B1} > \pi^u_{B5}$ if and only if $d_L\left\{v_L^H\left\{1-\delta^n-\left[1-\delta^{n-1}\right]q_O\right\}-\left[1-\delta^n\right]\left[1-q_O\right]\right\} < \left[v_L^H-\delta\right]\left\{1-\delta^n-\left[1-\delta^{n-1}\right]q_O\right\}$ in which $v_L^H\left\{1-\delta^n-\left[1-\delta^{n-1}\right]q_O\right\}-\left[1-\delta^n\right]\left[1-q_O\right] < \left[v_L^H-\delta\right]\left\{1-\delta^n-\left[1-\delta^{n-1}\right]q_O\right\}$ if $q_O < 1-\delta^n$.Thus if $q_O < 1-\delta^n$, B1 dominates B5.
- $\pi^u_{B1} > \pi^u_{B6}$ if and only if $\left[1-\delta^n-q_O\right]v_H d_H + \frac{1-\delta^n}{1-\delta}v_L d_L\left[1-q_O\right] > 0$ which is valid if $q_O < 1-\delta^n$.

Then, consider the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$:

In the region $q_O < 1-\delta^n$, $\pi^u_{B1} > \pi^u_{B2}$ if and only if

$$d_H v_L^H < \frac{\left\{\left[1-\delta^n\right]+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O\right\}-\frac{1-\delta^n}{1-\delta}q_O d_L}{\left[1-\delta^n\right]-q_O}.$$

In the region $v_L^H \geq \frac{1-\delta^n}{\left[1-\delta^n\right]-\left[1-\delta^{n-2}\right]q_O}$, $\pi^u_{B1} > \pi^u_{B3}$ if and only if

$\left[1-\delta^n-q_O\right]v_L^H d_H < 1-\delta^n$. Thus, if $q_O \geq 1-\delta^n$, B1 dominates B3.

$\pi_{B1}^{u} > \pi_{B5}^{u}$ if and only if $d_L < \frac{1-\delta}{q_O} + \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$. Thus, if $q_O < 1-\delta^n$, B1 dominates B5.

$\pi_{B1}^{u} > \pi_{B6}^{u}$ if and only if $d_H v_L^H < \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$.

B1 is optimal if and only if:

In the region $v_L^H \le \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $q_O < 1-\delta^n$;

In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $d_H v_L^H < \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$,

Further, if $q_O \ge 1-\delta^n$, then $d_L < \frac{1-\delta}{q_O} + \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$;

if $q_O < 1-\delta^n$, then

$$d_H v_L^H < \frac{\left\{\left[1-\delta^n\right]+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O\right\}-\frac{1-\delta^n}{1-\delta}q_O d_L}{\left[1-\delta^n\right]-q_O} \text{ and } d_H v_L^H < \frac{1-\delta^n}{1-\delta^n-q_O}.$$

The Analysis of B2:

B2 is dominated by B5 if $q_O \ge 1-\delta^n$; B2 is dominated by B1 if $q_O < 1-\delta^n$ and $v_L^H < \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$. Thus we only consider the case $q_O < 1-\delta^n$ and $v_L^H \ge \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$.

$\pi^u_{B2} > \pi^u_{B1}$ if and only if $d_H v_L^H > \dfrac{\left[1-\delta^n+\delta\dfrac{1-\delta^{n-1}}{1-\delta}q_O\right]-\dfrac{1-\delta^n}{1-\delta}q_O d_L}{\left[1-\delta^n\right]-q_O}$ which is equivalent to

$$v_L^H > \frac{\left[1-\delta^n\right]+\delta\dfrac{1-\delta^{n-1}}{1-\delta}q_O}{\left[1-\delta^n\right]-q_O} \text{ and } d_L < \frac{v_L^H\left[1-\delta^n-q_O\right]-\left[1-\delta^n\right]-\delta\dfrac{1-\delta^{n-1}}{1-\delta}q_O}{v_L^H\left[1-\delta^n-q_O\right]-\dfrac{1-\delta^n}{1-\delta}q_O}.$$

$\pi^u_{B2} > \pi^u_{B3}$ if and only if $d_L > \dfrac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$;

$\pi^u_{B2} > \pi^u_{B6}$ if and only if

$$\left\{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}d_H v_L^H < \delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O$$

B2 is optimal if and only if:

$v_L^H \geq \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $q_O < 1-\delta^n$, $d_L > \dfrac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$,

$d_H v_L^H > \dfrac{\left[1-\delta^n+\delta\dfrac{1-\delta^{n-1}}{1-\delta}q_O\right]-\dfrac{1-\delta^n}{1-\delta}q_O d_L}{\left[1-\delta^n\right]-q_O}$ and

$$\left\{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}d_H v_L^H < \delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O$$

The Analysis of B3:

We only need to consider the region $v_L^H \geq \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$:

- In the region $v_L^H < \dfrac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$ and $q_O < 1-\delta^n$, B3 is dominated by B1.

- In the region $v_L^H \geq \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}$, $\pi_{B3}^u > \pi_{B1}^u$ if and only if

$d_H v_L^H > \frac{1-\delta^n}{1-\delta^n - q_O}$ and $q_O < 1-\delta^n$.

- $\pi_{B3}^u > \pi_{B2}^u$ if and only if $d_L \leq \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$.
- In the region $\frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-2}\right] q_O} < v_L^H \leq \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}$ and $q_O \geq 1-\delta^n$, B3 is dominated by B4.
- In the region $q_O < 1-\delta^n$, B5 is dominated by B1.

- $\pi_{B3}^u > \pi_{B6}^u$ if and only if

$\left\{[2\delta-1]\left[1-\delta^n\right] - \left[2\delta-1-\delta^{n-1}\right] q_O\right\} d_H v_L^H < \left[1-\delta^n\right]\left[\delta - q_O d_L\right]$.

B3 is optimal if and only if: $v_L^H \geq \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}$, $q_O < 1-\delta^n$,

$d_H v_L^H > \frac{1-\delta^n}{1-\delta^n - q_O}$, $d_L \leq \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$ and

$\left\{[2\delta-1]\left[1-\delta^n\right] - \left[2\delta-1-\delta^{n-1}\right] q_O\right\} d_H v_L^H < \left[1-\delta^n\right]\left[\delta - q_O d_L\right]$.

The Analysis of B4:

We only need to consider the region $v_L^H \leq \frac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right] q_O}$:

According to the above analysis,

$\pi_{B4}^u > \pi_{B1}^u$ if and only if $q_O > 1-\delta^n$.

In the region $q_O \geq 1-\delta^n$, $\pi_{B4}^u > \pi_{B5}^u$ if and only if

$$d_H v_L^H > \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} - \frac{\left[1-\delta^n\right]\left[1-q_O\right]d_L}{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-2}\right]q_O} \text{ which is equivalent to}$$

$$d_L < v_L^H - \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} \Bigg/ v_L^H - \frac{\left[1-\delta^n\right]\left[1-q_O\right]}{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-2}\right]q_O}.$$

B6 is dominated by B4.

B4 is optimal if and only if: $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $q_O > 1-\delta^n$, and

$$d_L < v_L^H - \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} \Bigg/ v_L^H - \frac{\left[1-\delta^n\right]\left[1-q_O\right]}{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-2}\right]q_O}.$$

The Analysis of B5:

In the region $q_O < 1-\delta^n$, B1 dominates B5. Thus, we only consider the region

$q_O \geq 1-\delta^n$ in which B2 and B3 are not optimal.

- In the region $v_L^H \leq \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, B6 is dominated by B4; $\pi_{B5}^u > \pi_{B4}^u$ if and only if $d_H v_L^H < \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} - \frac{\left[1-\delta^n\right]\left[1-q_O\right]d_L}{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-2}\right]q_O}$, which is equivalent to $d_L > v_L^H - \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} \Big/ v_L^H - \frac{\left[1-\delta^n\right]\left[1-q_O\right]}{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-2}\right]q_O}$ and implies that $d_L < \frac{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O}{\left[1-\delta^n\right]\left[1-q_L\right]} > \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$.

  - In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $\pi_{B5}^u > \pi_{B1}^u$ if and only if $q_O > \frac{\left[1-\delta\right]\left[1-\delta^n\right]}{\left[1-\delta^n\right]d_L-\delta\left[1-\delta^{n-1}\right]} > 1-\delta^n$, which implies $d_L > \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$.

$\pi^u_{B5} > \pi^u_{B6}$ if and only if $d_H v^H_L < \dfrac{1-\delta^n - \left[1-\delta^{n-1}\right]q_O}{1-\delta^n - \left[1-\delta^{n-2}\right]q_O}$ which is equivalent to

$$d_L > 1 - \frac{1-\delta^n - \left[1-\delta^{n-1}\right]q_O}{v^H_L\left\{1-\delta^n - \left[1-\delta^{n-2}\right]q_O\right\}}.$$

B5 is optimal if and only if: $q_O \geq 1-\delta^n$, and

(1) In the region $v^H_L \leq \dfrac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right]q_O}$, $d_L > \dfrac{v^H_L - \dfrac{1-\delta^n - \left[1-\delta^{n-1}\right]q_O}{1-\delta^n - \left[1-\delta^{n-2}\right]q_O}}{v^H_L - \dfrac{\left[1-\delta^n\right]\left[1-q_O\right]}{\delta\left[1-\delta^n\right] - \delta\left[1-\delta^{n-2}\right]q_O}}$;

(2) In the region $v^H_L > \dfrac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right]q_O}$, $d_L > 1 - \dfrac{1-\delta^n - \left[1-\delta^{n-1}\right]q_O}{v^H_L\left\{1-\delta^n - \left[1-\delta^{n-2}\right]q_O\right\}}$,

$d_L > \dfrac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$, and $q_O > \dfrac{\left[1-\delta\right]\left[1-\delta^n\right]}{\left[1-\delta^n\right]d_L - \delta\left[1-\delta^{n-1}\right]}$

The Analysis of B6:

In the region $v^H_L \leq \dfrac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right]q_O}$, B6 is dominated by B4. Thus we only consider the region $v^H_L > \dfrac{1-\delta^n}{1-\delta^n - \left[1-\delta^{n-1}\right]q_O}$.

- $\pi^u_{B6} > \pi^u_{B1}$ if and only if $d_H v^H_L > \dfrac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n - \left[1-\delta^{n-2}\right]q_O\right\}}$;

In the region $q_O < 1-\delta^n$:

- $\pi^u_{B6} > \pi^u_{B2}$ if and only if $d_H v^H_L > \dfrac{\delta\left[1-\delta^n\right] - \delta\left[1-\delta^{n-1}\right]q_O}{\left[1-2\delta+\delta^{n-1}\right]q_O - \left[1-2\delta\right]\left[1-\delta^n\right]}$ and

$\left[2\delta - 1 - \delta^{n-1}\right]q_O < \left[2\delta-1\right]\left[1-\delta^n\right]$;

- $\pi_{B6}^{u} > \pi_{B3}^{u}$ if and only if
$\left\{[2\delta-1]\left[1-\delta^{n}\right]-\left[2\delta-1-\delta^{n-1}\right]q_O\right\}d_H v_L^H > \left[1-\delta^{n}\right][\delta-q_O d_L]$.
- $\pi_{B6}^{u} > \pi_{B5}^{u}$ if and only if $d_H v_L^H > \dfrac{1-\delta^{n}-\left[1-\delta^{n-1}\right]q_O}{1-\delta^{n}-\left[1-\delta^{n-2}\right]q_O}$ which implies that

$d_L < 1-\dfrac{1-\delta^{n}-\left[1-\delta^{n-1}\right]q_O}{v_L^H\left\{1-\delta^{n}-\left[1-\delta^{n-2}\right]q_O\right\}}$.

B6 is optimal if and only if:

$v_L^H > \dfrac{1-\delta^{n}}{1-\delta^{n}-\left[1-\delta^{n-1}\right]q_O}$, $d_H v_L^H > \dfrac{\left[1-\delta^{n}\right][1-q_O d_L]}{\delta\left\{1-\delta^{n}-\left[1-\delta^{n-2}\right]q_O\right\}}$, and

(1) If $q_O > 1-\delta^{n}$, $d_H v_L^H > \dfrac{1-\delta^{n}-\left[1-\delta^{n-1}\right]q_O}{1-\delta^{n}-\left[1-\delta^{n-2}\right]q_O}$;

(2) If $q_O < 1-\delta^{n}$,

$\left[2\delta-1-\delta^{n-1}\right]q_O < [2\delta-1]\left[1-\delta^{n}\right]$, which is equivalent to $\delta \geq \dfrac{1}{2}$, or

$\delta < \dfrac{1}{2}$ & $q_O > \dfrac{[2\delta-1]\left[1-\delta^{n}\right]}{\left[2\delta-1-\delta^{n-1}\right]}$,

$d_H v_L^H > \dfrac{\delta\left[1-\delta^{n}\right]-\delta\left[1-\delta^{n-1}\right]q_O}{\left[1-2\delta+\delta^{n-1}\right]q_O-[1-2\delta]\left[1-\delta^{n}\right]}$, and

$d_H v_L^H > \dfrac{\left[1-\delta^{n}\right][\delta-q_O d_L]}{\left[1-2\delta+\delta^{n-1}\right]q_O-[1-2\delta]\left[1-\delta^{n}\right]}$

Summary of the optimal strategies with aspects of v_L^H, q_O, and $d_H v_L^H$

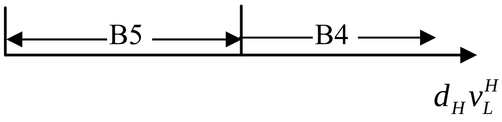
In the region $v_L^H \leq \dfrac{1-\delta^{n}}{1-\delta^{n}-\left[1-\delta^{n-1}\right]q_O}$, $q_O \geq 1-\delta^{n}$,

Optimal strategies: **B4 and B5**

B4: $d_H v_L^H > \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} - \frac{\left[1-\delta^n\right]\left[1-q_O\right]d_L}{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-2}\right]q_O}$

B5: $d_H v_L^H < \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O} - \frac{\left[1-\delta^n\right]\left[1-q_O\right]d_L}{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-2}\right]q_O}$

The demand elasticity for each strategy to be optimal:



In the region $v_L^H \le \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $q_O < 1-\delta^n$,

Optimal strategy: **B1**

In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $q_O \ge 1-\delta^n$,

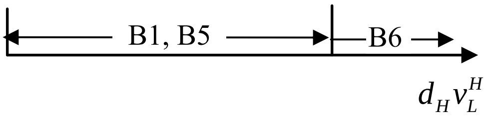
Optimal strategies: **B1, B5 and B6**

B1: $d_L < \frac{1-\delta}{q_O} + \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$, $d_H v_L^H < \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$

B5: $d_L > \frac{1-\delta}{q_O} + \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$, $d_H v_L^H < \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$

B6: $d_H v_L^H > \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$, $d_H v_L^H > \frac{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}{1-\delta^n-\left[1-\delta^{n-2}\right]q_O}$

The demand elasticity for each strategy to be optimal:



In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $\frac{[1-2\delta]\left[1-\delta^n\right]}{1-2\delta+\delta^{n-1}} \le q_O < 1-\delta^n$,

Optimal strategies: **B1, B2, B3 and B6**

B1:, $d_H v_L^H < \frac{\left[1-\delta^n+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O\right]-\frac{1-\delta^n}{1-\delta}q_O d_L}{\left[1-\delta^n\right]-q_O}$, $d_H v_L^H < \frac{1-\delta^n}{1-\delta^n-q_O}$, and

$$d_H v_L^H < \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$$

B2: $d_H v_L^H > \frac{\left[1-\delta^n+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O\right]-\frac{1-\delta^n}{1-\delta}q_O d_L}{\left[1-\delta^n\right]-q_O}$, $d_L > \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$, and

$$d_H v_L^H < \frac{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O}{\left[1-2\delta+\delta^{n-1}\right]q_O-[1-2\delta]\left[1-\delta^n\right]}$$

B3: $d_H v_L^H > \frac{1-\delta^n}{1-\delta^n-q_O}$, $d_L < \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$, and

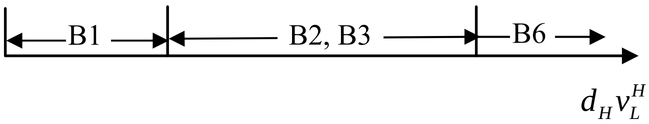
$$d_H v_L^H < \frac{\left[1-\delta^n\right]\left[\delta-q_O d_L\right]}{\left[1-2\delta+\delta^{n-1}\right]q_O-[1-2\delta]\left[1-\delta^n\right]}$$

B6: $d_H v_L^H > \frac{\delta\left[1-\delta^n\right]-\delta\left[1-\delta^{n-1}\right]q_O}{\left[1-2\delta+\delta^{n-1}\right]q_O-[1-2\delta]\left[1-\delta^n\right]}$,

$$d_H v_L^H > \frac{\left[1-\delta^n\right]\left[\delta-q_O d_L\right]}{[2\delta-1]\left[1-\delta^n\right]-\left[2\delta-1-\delta^{n-1}\right]q_O},\text{ and}$$

$$d_H v_L^H > \frac{\left[1-\delta^n\right]\left[1-q_O d_L\right]}{\delta\left\{1-\delta^n-\left[1-\delta^{n-2}\right]q_O\right\}}$$

The demand elasticity for each strategy to be optimal:



In the region $v_L^H > \frac{1-\delta^n}{1-\delta^n-\left[1-\delta^{n-1}\right]q_O}$, $q_O < \frac{\left[1-2\delta\right]\left[1-\delta^n\right]}{1-2\delta+\delta^{n-1}}$,

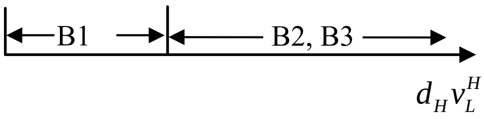
Optimal strategies: **B1, B2 and B3**

B1:, $d_H v_L^H < \frac{\left[1-\delta^n+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O\right]-\frac{1-\delta^n}{1-\delta}q_O d_L}{\left[1-\delta^n\right]-q_O}$, $d_H v_L^H < \frac{1-\delta^n}{1-\delta^n-q_O}$,

B2: $d_H v_L^H > \frac{\left[1-\delta^n+\delta\frac{1-\delta^{n-1}}{1-\delta}q_O\right]-\frac{1-\delta^n}{1-\delta}q_O d_L}{\left[1-\delta^n\right]-q_O}$, $d_L > \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$

B3: $d_H v_L^H > \frac{1-\delta^n}{1-\delta^n-q_O}$, $d_L < \frac{\delta\left[1-\delta^{n-1}\right]}{1-\delta^n}$

The demand elasticity for each strategy to be optimal:



CHAPTER 4

INFORMATION SECURITY: USERS PRECAUTIONS AND HACKER TARGETING

4.1 Introduction

Information security is a critical issue of both national policy and business operations (Whitman 2003). For instance, in May 2004, Sven Jaschan created the Sasser worm to exploit a vulnerability in the Windows 2000 and XP operating systems. The Sasser worm and its variants caused hundreds of thousands of PCs to crash (*BBC News* 2005).

Referring to Figure 4.1, in the second half of 2005, Symantec (2006) observed 10,992 *new* Win32 viruses and worms, and blocked 8.242 million phishing messages per day (an increase of 45% over the number in the first half of 2005).

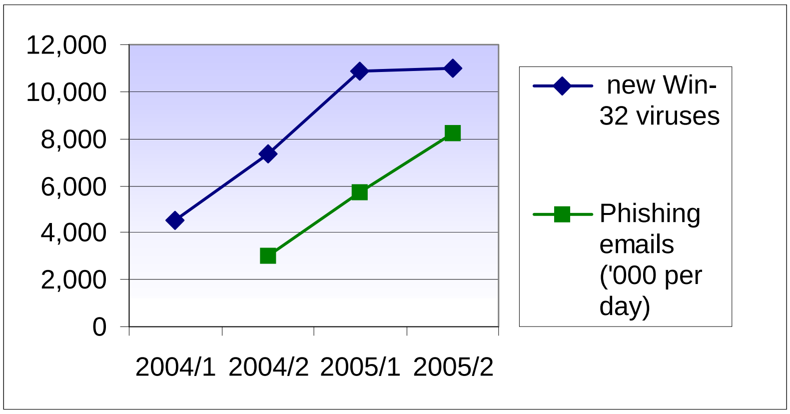


Figure 4.1 Security attacks

Information security depends on user efforts – to fix vulnerabilities, install and update software to detect neutralize viruses and other malicious software, install and configure firewalls, take care with file-sharing programs and email attachments, etc.

Security is a critical issue only because of the activities of (unethical) hackers.[28] Industry has systematically tracked hacker behavior: "Attackers continuously look for easy targets, those that will provide them with the maximum return on the time they invest in writing malicious code" (Symantec 2005, page 55). Clearly, hacker activity depends on user behavior.

While there has been some research into the incentives of end-users (Kunreuther and Heal 2003; August and Tunca 2005), and the motivations of hackers (eg, Jordan and Taylor 1998; Van Beveren 2000), there has been little scholarly attention to the *strategic* interaction between end-users and hackers.

In this study, we analyze the strategic interactions among end-users and between end-users and hackers. We address several questions in particular. First, it is well known that information security poses grave potential consequences. Yet, end-users seem quite slow to take precautions (Boss 2005) – to the point that they must be exhorted and goaded by government and vendors (US-CERT 2006). What explains this inertia?

Second, given the strategic interactions, how does information security vary with changes in the user cost of precaution and the rate of enforcement against hackers? This question is not trivial. For instance, a reduction in the user cost of precaution would directly lead users to increase precautions. However, that would make them less attractive targets, and so induce hackers to reduce their targeting, and hence, indirectly lead users to *reduce* precautions. Accordingly, the net effect depends on the balance between direct and indirect effects.

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[28] We will focus on *unethical* hackers, and, for brevity, simply refer to them as "hackers".

Third, information security can be and are addressed from two angles – facilitating end-user precautions, and enforcement against hackers. Both policies are costly. Owing to the strategic interaction, facilitation of user precautions will affect hacker behavior, and enforcement against hackers will affect user behavior. From the standpoint of social welfare, what is the right balance between the two classes of policy?

4.2 Prior Literature

Information security is a major concern for governments and businesses worldwide (Whitman 2003). Generally, it involves four groups of persons – users, hackers, software vendors, and security specialists such as CERT/CC. Further, it is now recognized to be as much as an issue of economic incentives as a technological problem (Anderson 2001).

Most economic analysis has focused on the policies of software vendors, CERT/CC and other security specialists to disclose security flaws and provide the appropriate patches (see, for instance, Cavusoglu, Cavusoglu, and Raghunathan 2004; Choi, Fershtman, and Gandal 2004; Nizovtsev and Thursby 2005; Arora, Caulkins, and Telang 2005; Jaisingh and Li 2005). Other analyses have considered users' incentives to share information (Gal-Or and Ghose 2005) and implementation of detection systems (Cavusoglu, Mishra, and Raghunathan 2005).

August and Tunca (2005) consider the behavior of users, and specifically, their incentive to patch security flaws. In a finding that is reminiscent of the public-health literature on infectious diseases, they show that mandatory patching is not optimal (Brito et. al. 1991; Philipson 2001). With commercial software, the optimal policy is a subsidy on patching when security risk and patching cost are high, and no policy otherwise. However, with open-source software, the optimal policy is a subsidy on patching when both security risk and patching costs are low, and a tax on software usage otherwise.

August and Tunca (2005) assume that users consider the risk of attack when deciding whether to fix their software. This assumption is consistent with empirical analyses of crime and victim precautions. For instance, in a study of migration from urban areas, Cullen and Levitt (1999) found that each additional reported crime was associated with a decline in urban population by about one person. In particular, the migration of highly educated households and those with children was relatively more sensitive to crime.

However, in the specific context of software security, the risk of attack did not have a significant effect on experimental subjects' intention to take precautions (Boss 2005). Further, in a recent survey of residential Internet users, 78% of respondents felt "somewhat safe", "not very safe", or "not at all safe" from online threats, but only 67% protected themselves with a firewall (National Cyber Security Alliance, 2005). Apparently, users are still slow to expend effort in information security. The question is why?

Kunreuther and Heal (2003) study a positive network externality among users in taking precautions against attack. Specifically, they assumed that each user makes an all-or-nothing choice between taking precautions or not taking precautions, and that, the expected loss to any user decreases with others' precautions. They show that, for a wide range of cost and risk parameters, there are two equilibria – either all users invest in precautions or no one does. Kunreuther and Heal (2003), however, did not analyze why the expected loss to any user decreased with others' precautions, and specifically, the role of hackers.[29]

Previous research into hacker behavior has focused on their motivation. External factors encourage hackers – the perception that hacking is seldom punished and peer approval from other hackers (Van Beveren 2000). However, greed, power, and revenge are superseding curiosity and other benign motivations (Jordan and Taylor 1998). Symantec (2005) also observed that the motivation of hackers has shifted towards making money. This trend portends greater losses as hackers aim "to create more malicious code and that will become stealthier and more selective" (Symantec 2005, page 9).

The work closest to ours is by Choi, Fershtman, and Gandal (2005), who analyze the interaction of a software vendor, users, and hackers. There are two types of user – the high type gets more benefit from the software and incurs a loss from attack while the low type receives gets less benefit and incurs no loss from attack. Choi et al. focus on the vendor's investment in software quality: While the vendor's profit-maximizing investment depends on the impact on the marginal user, the socially optimal investment depends on the effect on the average user.

By contrast with the previous research, we focus on the strategic interaction among end-users and between users and hackers in a setting with a continuum of user types. Our analysis shows how users' effort in fixing depends on hackers' targeting and vice versa. Accordingly, we can show how changes in policy toward hackers will affect user behavior, and, also how policy changes toward users will influence hacker behavior.

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[29] See also Varian (2004).

4.3 Basic Setting

Consider the market for some service, which is provided by a monopoly at a uniform price p. (We assume a simple market structure, in order to focus on the interaction between end-users and hackers.) We use the term “service” generically to encompass software and systems as well.

End-users derive benefit, v, from use of the service. The vendor would set price such that $v > p$, else there would be no demand. End-users differ in naivete, n, which is distributed according to the cumulative distribution function, $\Phi(n)$ between [0, 1], with 0 representing the least naïve (most sophisticated) user and 1 representing the most naïve user. All users are risk-neutral.

A user suffers an attack with probability $\alpha(f)\chi$, where $\alpha(f)$ is a probability that depends on the user’s effort, f, in precautions such as installing patches and scanning suspicious emails, and where χ is a probability that measures the effectiveness of hackers’ targeting. The properties of $\alpha(f)$ are

$$\alpha(0) = 1, \lim_{f\to\infty} a(f) = 0, \frac{d\alpha}{df} < 0, \frac{d^2\alpha}{df^2} > 0. \tag{4.1}$$

If the user suffers an attack, she will not derive any benefit, and in addition, will suffer some harm, h.[30] The user’s cost of precautions is ncf. Each potential user decides whether to buy and, if so, chooses precautions to maximize her expected net benefit.

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[30] This set-up is similar to that in the literature on enforcement against copyright piracy (see, for instance, Chen and Png (2003)).

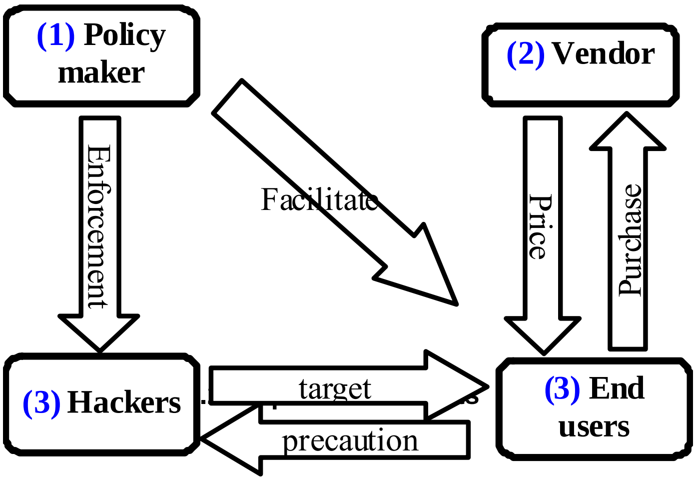
There are Z identical hackers $i = 1,...,Z$, each of whom chooses targeting k_i. The total targeting, $K = k_1 + ... + k_Z$, determines the effectiveness probability $\chi = \chi(K)$, where

$$\chi(0) = 0, \lim_{K \to \infty} \chi(K) = 1, \frac{d\chi}{dK} > 0, \frac{d^2\chi}{dK^2} < 0 . \tag{4.2}$$

Each hacker derives enjoyment, e from an attack on a user, provided that he is not discovered. With enforcement probability, η, the authorities discover the hacker and then impose a penalty with monetary value, s, and prevent his enjoyment. Further, the cost of targeting is $c_K(k)$, which is convex in k.

The hacker chooses targeting to maximize his expected net benefit. This modeling assumption is consistent with Symantec's (2005, page 55) observation that hackers direct efforts against targets that provide the maximum return.

The vendor sets price p to maximize profit. The sequence of events is as follows:



4.4 User-Hacker Equilibrium

Consider the end-user with type (naivete), n. If she buys the item, her expected net benefit, given hackers' targeting K, would be

$$B(n\,|\,K) = [1-\alpha(f)\chi]v - \alpha(f)\chi h - p - ncf \ . \tag{4.3}$$

Maximizing with respect to f ,

$$-[v+h]\chi\frac{d\alpha}{df} = nc\text{, or } \frac{d\alpha}{df} = \frac{nc}{-[v+h]\chi} \tag{4.4}$$

which defines the net-benefit maximizing precaution, $f(n)$, as a function of the user type. By inspection of (4.4), we have[31]

Observation 1. User precaution, f , is continuous and decreasing in naivete, n, and the cost of precaution, c, independent of price, p, and continuous and increasing in hacker targeting, K, such that, if $K = 0$, then $f(n) = 0$, and there exists $f_\infty(n) > 0$ such that $\lim_{K\to\infty} f(n) = f_\infty(n)$.

We next characterize the demand for the service. By (4.3), every user for whom $B(n) \geq 0$ will buy the item. It is relatively straightforward to prove that $B(n)$ is decreasing in n. Accordingly, we have

Observation 2. Either all users buy or there exists a marginal user, $\hat{n}$, defined by

$$B(\hat{n}) = v - [v+h]\alpha(f(\hat{n}))\chi - p - \hat{n}cf(\hat{n}) = 0\,, \tag{4.5}$$

and such that only users with $n \leq \hat{n}$ buy.

The demand for the item arises from the users with $n \leq \hat{n}$, hence the quantity demanded (equal to the vendor's sales) is

$$\int_0^{\hat{n}} d\Phi(n)\,. \tag{4.6}$$

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[31] We prove all results in the Appendix 4A.

The following result shows how the demand for service depends on hackers' targeting and the vendor's price.

Observation 3. The marginal user type, $\hat{n}$, is continuous and decreasing in the user cost of precaution, c, and the price, p, and continuous, decreasing and convex in hackers' targeting, K. In addition, $\lim_{K\to 0}\hat{n}=1$ and $\lim_{K\to\infty}\hat{n}=\hat{n}_0$, where $0\le\hat{n}_0\le 1$.

Having analyzed user behavior (choices of whether to buy the item and, if so, precaution) as a function of hackers' targeting, we now consider the hackers' targeting as a function of user behavior. Hacker i chooses k_i to maximize expected net benefit,

$$H_i(k_i\,|\,\hat{n}, f(n), k_1,...,k_{i-1},k_{i+1},...,k_Z)=[1-\eta]\int_0^{\hat{n}}\alpha(f(n))d\Phi(n)\cdot\chi(k_i+\sum_{\substack{j=1\\ j\neq i}}^{Z}k_j)e-\eta s-c_K(k_i)$$

$$. \qquad (4.7)$$

By (4.2) and since $c_K(k_i)$ is convex, the function H_i is concave in k_i. Maximizing H_i with respect to k_i, the first-order condition is

$$e[1-\eta]\frac{d\chi}{dk_i}\int_0^{\hat{n}}\alpha(f(n))d\Phi(n)=\frac{dc_K}{dk_i}. \qquad (4.8)$$

Since hackers are identical, they all choose the same targeting, as characterized by (4.8).

Observation 4. Hacker targeting, k_i, is continuous and decreasing in the enforcement rate, η. Further, hacker targeting, k_i, is continuous and increasing in the marginal user type, $\hat{n}$, and, if $\hat{n}=0$, then $k_i(\hat{n})=0$, and there exists some $\widetilde{k}>0$ such that if $\hat{n}=1$, then $k_i(\hat{n})=\widetilde{k}$. In addition, hacker targeting, k_i, is continuous and decreasing in user precaution, f, and there exists some $k_0>0$ such that if $f=0$, then $k_i=k_0$, and there exists some $k_\infty\ge 0$ such that if $f\to\infty$, then $k_i=k_\infty$.

As hackers are identical, we focus on a symmetric equilibrium, in which $k_i^* = k^*$, say. For the analysis to be meaningful, we must show that there exists a non-trivial equilibrium. To prove existence, it is useful to consider the rate at which end-users are subject to security attack, conditional on hacker effectiveness, $\chi(K)$, i.e, the *conditional vulnerability*,

$$A(K) = \int_0^{\hat{n}(K)} \alpha(f(n) \mid K) d\Phi(n) . \tag{4.9}$$

Accordingly, the function $A(K)\chi(K)$ is the rate at which users actually suffer security attack, i.e, the *effective vulnerability of users*.

Lemma 1 proves the existence of equilibrium by considering the relation between $A(K)$ and targeting, k. The effective vulnerability is a continuous, decreasing function of hacker targeting, and similarly, hacker targeting is a continuous, increasing function of the effective vulnerability of users. Figure 4.3 illustrates the result.

Lemma 1. There exists a non-trivial equilibrium between end-users and hackers, k^*, $\hat{n}(k^*)$ and $f(\hat{n} \mid k^*)$.

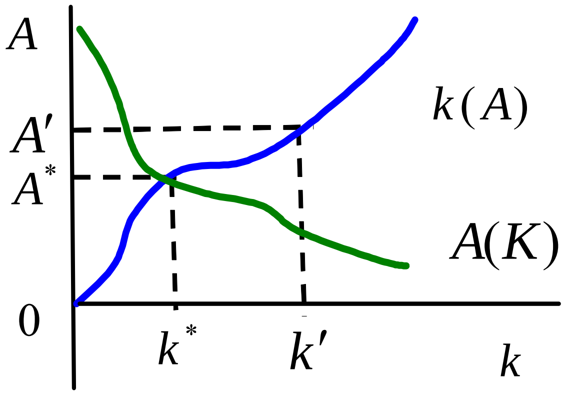

Figure 4.3 User-hacker equilibrium

Our first substantive result focuses on the strategic interaction among end-users. Proposition 1 shows that, given hacker behavior, users' precautions are *strategic substitutes* (Bulow et al. 1985).

Proposition 1. Given hacker behavior, user precautions are strategic substitutes: The higher the precautions of others, the lower the precaution of any particular individual.

By contrast with our Proposition 1, Kunreuther and Heal (2003) suggest that user efforts in security are *strategic complements*, and that the equilibrium outcome is of an all-or-nothing nature – either all users or none take precautions. By contrast, our analysis implies that user efforts are strategic substitutes, hence the equilibrium outcome could involve an intermediate level of precaution.

Empirical evidence appears to lend stronger support to our analysis than that of Kunreuther and Heal (2003). Table 4.1 reports data from an annual survey of U.S. residential computer users. It suggests that users take an intermediate level of precautions, rather than follow an all-or-nothing approach.

Table 4.1 User security measures

Security measure	*2004*	*2005*
Equipped with anti-virus software	85%	83%
Equipped with properly configured firewall	28%	56%
With active or open file sharing program	23%	11%
Source: National Cyber Security Alliance, 2004 and 2005.		

Proposition 1 implies that a free-rider problem exists in user security. If other users raise their precautions, they will reduce the expected harm to any particular user, and she will rationally respond by reducing her precaution. The analysis of August and Tunca (2005) also points to a similar free-rider problem in user security. This free-rider problem is reminiscent of that arising from concealed precautions, such as Lojack, by potential crime victims (Koo and Png 1994; Ayres and Levitt 1998).[32]

4.5 Empirical Implications

Responding to public concern, software vendors have invested heavily to facilitate user precautions. For instance, in August 2004, Microsoft released Service Pack 2, to enhance user security. Service Pack 2 enabled automatic update of Microsoft security patches and included a firewall. Other vendors followed Microsoft to provide automatic updating of patches. How would automatic updating affect overall vulnerability?

In the preceding sections, we considered only the direct effects of changes in vendor strategy and government policy, and ignored the indirect (feedback) effects through the actions of the other side of the market. We now consider the effects of changes in vendor strategy and government policy on the equilibrium between users and hackers. Accordingly, this analysis encompasses both direct and indirect effects.

Our analysis points to some unintended effects: specifically, actions to reduce user cost of precautions such as automatic updating of patches need not enhance overall information security. As reported in Proposition 2 below, actions to reduce user cost of precautions would *enhance* overall security (as measured by effective user

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[32] Lojack is a concealed device that allows police to track a stolen vehicle. Ayres and Levitt (1998) measured a significant positive externality from Lojack installation: an owner who installed Lojack significantly reduced the likelihood of theft of other vehicles.

vulnerability) if the cost of precaution is sufficiently high, but *reduce* overall security if the cost of precaution is sufficiently low.

Next, consider an increase in the rate of enforcement against hackers, η. This directly leads hackers to reduce their targeting. However, there is also an indirect effect: users would respond to the reduced hacking by reducing their precautions, and the demand for the item would increase. As reported in Proposition 2 below, the net impact on security (as measured by the effective user vulnerability), which balances the direct and indirect effects, is ambiguous.

Similarly, an increase in the price, *p*, would have direct and indirect effects on the demand for the service. The price increase would directly reduce $\hat{n}$, i.e., cause some users to stop buying the item. However, the price increase would also have an indirect (consequential) effect through hackers' response to user choices. With fewer users of the item, hackers would reduce targeting, which reduces the probability of attack and therefore raises users' expected net benefit. Thus, the indirect effect from hackers tends to offset the direct effect of the price increase. Accordingly, the demand for the item is *less elastic* than it would appear from studying the direct effect alone.

Proposition 2. Effective user vulnerability, χA, is decreasing in the price of the service, p; increasing in the user cost of precaution, c, if it is sufficiently high, but decreasing if it is sufficiently low; and ambiguous in the rate of enforcement against hackers and the hackers' targeting cost.

Table 4.2 summarizes the net effect on users' precautions, hackers' targeting and demand for the service with regard to changes in the price, p, enforcement rate, η, hacking cost, c_K, and the user cost of precaution, c.

Table 4.2. Empirical Implications

Change		Users' precaution f	Hackers' targeting k	Demand $\hat{n}$	Conditional user vulnerability A	Effective user vulnerability χA
Price, p		decreasing	decreasing	decreasing	decreasing	decreasing
Cost of precaution, c	$\partial A/\partial c \geq 0$	decreasing	increasing	decreasing	increasing	increasing
	$\partial A/\partial c < 0$	decreasing	decreasing	decreasing	decreasing	decreasing
Enforcement rate, η		decreasing	decreasing	increasing	increasing	ambiguous
Targeting cost, c_K		decreasing	decreasing	increasing	increasing	ambiguous

4.6 Welfare

Information security can be and is addressed from two angles – facilitating end-user precautions, and enforcement against hackers. How much should society spend on these policies?

In general, welfare could possibly include the net benefits of users, the vendor, and hackers. However, following Trumbull (1990), we exclude hackers' benefits and costs from the measure of welfare. Accordingly, social welfare is

$$\begin{aligned} W &= v\int_0^{\hat{n}} d\Phi(n) - [v+h]\int_0^{\hat{n}} \alpha(f(n))\chi(K)d\Phi(n) - \int_0^{\hat{n}} ncf(n)d\Phi(n) \\ &= v\int_0^{\hat{n}} d\Phi(n) - [v+h]\chi(K)A(K) - \int_0^{\hat{n}} ncf(n)d\Phi(n) \end{aligned} \tag{4.10}$$

Consider enforcement against hackers. Differentiating (4.10) with respect to η,

$$\begin{aligned} \frac{dW}{d\eta} &= v\frac{d\hat{n}}{d\eta}\frac{d\Phi(\hat{n})}{dn} - [v+h]\left\{\frac{dA}{d\eta}\chi(K) + \frac{d\chi(K)}{dK}\frac{dK}{d\eta}A\right\} \\ &\quad - \left\{\frac{d\hat{n}}{d\eta}\frac{d\Phi(\hat{n})}{dn}\hat{n}cf(\hat{n}) + c\int_0^{\hat{n}} n\frac{df(n)}{d\eta}d\Phi(n)\right\}. \end{aligned} \tag{4.11}$$

Substituting from (4.4), (4.5) and (4.8), this simplifies to

$$\frac{dW}{d\eta}=p\frac{d\hat{n}}{d\eta}\frac{d\Phi(\hat{n})}{dn}-\left[v+h\right]\frac{d\chi(K)}{dK}\frac{dK}{d\eta}A\,. \tag{4.12}$$

By Table 4.2, $d\hat{n}/d\eta>0$ and $dK/d\eta<0$, hence, $dW/d\eta>0$, i.e, social welfare unambiguously increases with enforcement.

Society should invest in enforcement up to the level that the marginal increase in welfare from additional enforcement, $dW/d\eta$, just balances the marginal cost of enforcement. By (4.12), the marginal increase in welfare from additional enforcement and hence the optimal enforcement rate is increasing in the benefit, *v*, harm, *h*, and price, *p*.

Next, consider the facilitation of end user precautions. Substituting from (4.9) in (4.10) and then differentiating with respect to c, we obtain

$$\begin{aligned}\frac{dW}{dc}&=v\frac{d\hat{n}}{dc}\frac{d\Phi(\hat{n})}{dn}-\left[v+h\right]\left\{\frac{dA}{dc}\chi(K)+\frac{d\chi(K)}{dK}\frac{dK}{dc}A\left(K\right)\right\}\\&-\left\{\frac{d\hat{n}}{dc}\frac{d\Phi(\hat{n})}{dn}\hat{n}cf(\hat{n})+\int_0^{\hat{n}}n\left[f(n)+c\frac{df(n)}{dc}\right]d\Phi(n)\right\}.\end{aligned} \tag{4.13}$$

Further, substituting from (4.4), (4.5) and (4.8), this simplifies to

$$\frac{dW}{dc}=p\frac{d\hat{n}}{dc}\frac{d\Phi(\hat{n})}{dn}-[v+h]\frac{d\chi(K)}{dK}\frac{dK}{dc}A-\int_0^{\hat{n}}nf(n)d\Phi(n)\,. \tag{4.14}$$

By Table 4.2, $d\hat{n}/dc<0$, hence the first term on the right-hand side of (4.14) is negative. Also, the third term on the right-hand side of (4.14) is unambiguously negative. With regard to the second term, by Table 4.2, the sign of dK/dc depends on the sign of $\partial A/\partial c$. Accordingly, the effect of changes in the user cost of precaution on welfare is *a priori* ambiguous.

It might seem surprising that the conditional vulnerability could be decreasing in the user cost of precaution, $\partial A / \partial c < 0$. An increase in the cost of precaution would lead all users to reduce precautions, thus *increasing* the conditional vulnerability. However, it would also cause the marginal user to drop out of the market, hence *reducing* the conditional vulnerability. The net effect on the conditional vulnerability depends on the balance of these two effects.

If $\partial A / \partial c < 0$, then, by Table 4.2, $dK / dc < 0$, and if dK / dc is sufficiently negative, then, by (4.14), $dW / dc > 0$, i.e., welfare is *increasing* with the user cost of precaution. We consider this to be a theoretical curiosity, and not realistic. Accordingly, from this point onward, we suppose that $\partial A / \partial c > 0$ and hence that $dK / dc > 0$.

Suppose that the investment in facilitating user precautions is *I*. Then, society should invest in facilitation up to the level that the marginal increase in welfare from additional facilitation equals the marginal cost of investment, i.e.,

$$\frac{dW}{dI} = \frac{dW}{dc}\frac{dc}{dI} - 1 = 0. \qquad (4.15)$$

By (4.14), the marginal increase in welfare from additional precaution and hence the optimal level of facilitation is *increasing* in the benefit, *v*, harm, *h*, and price, *p*.

Realistically, the government may have a limited budget, and may not be able to fully optimize enforcement and facilitation. In such circumstances, it is important to understand how best to allocate a fixed amount of resources. For instance, if the government has an additional dollar to spend, should it spend that money on enforcement or facilitating user precautions? The following result addresses this question.

Proposition 3. Policy should focus on facilitating user precautions rather enforcement against hackers if users' benefit relative to the cost of precaution, $[v - p]/c$, and hackers' expected enjoyment relative to targeting cost,

$$\frac{e[1-\eta]}{\frac{dc_K}{dk}}, \tag{4.16}$$

are sufficiently high.

Proposition 3 is quite intuitive. The higher is end-users' benefit relative to the cost of precaution, the greater would be the users' incentive to take precautions, and hence the more effective would be any policy to facilitate precaution. Further, the higher is hackers' expected enjoyment relative to targeting cost, the greater would be hackers' incentive to attack, and hence the less effective would be enforcement against hackers. Accordingly, under these conditions, it makes sense to focus policy on facilitating end user precaution.

According to Proposition 3, intuitively, if users' benefit relative to the cost of precaution and hackers' expected enjoyment relative to targeting cost are low, then it may be optimal to focus on enforcement against hackers.

4.7 Limitations and Future Research

We have developed a fairly general model of the strategic interaction among end-users in taking security precautions and also the interaction between users and hackers. The analysis has provided empirical implications as well as recommendations for public policy. While we considered a setting of information security, the analysis is fairly general and would apply to any situation in which potential victims take precautions against attack by others.[33]

The key direction for future research is to test the various empirical implications. There are two major challenges to empirical research. One is to identify changes in the security environment (user cost of precaution, enforcement rate, and penalties). The other challenge is to acquire sufficient data on user behavior. The best source of information on user behavior that we know of is the AOL/NCSA Online Safety Study. However, that has been conducted only twice and only on an annual basis – in 2004 and 2005, which limits the available time series of data on user behavior.

There are several obvious directions for further analytical research. First, while we assumed that the enforcement rate was exogenous, it may realistically vary with hacker's targeting. So, it would be useful to explore the policy implications if the enforcement rate increases with hacker's targeting.

Second, we considered a setting with just one product. Realistically, users might have a choice of service. Then, the product with a larger user base would attract more hacking. As mentioned above, Mozilla Firefox has been recommended over Internet Explorer, in part because its smaller user base has attracted less hacking (cio.com 2005). In this context, any increase in demand would attract more targeting by hackers and hence diminish users' expected net benefit. Accordingly, there would be a negative network effect in demand. Another direction for future research is to consider the implications of such negative network effects on public policy.

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[33] See, for instance, Koo and Png (1994) and Kunreuther and Heal (2003).

Third, as mentioned above, there is an important interaction between software piracy and hacker targeting. Piracy increases the user base and hence attracts hacking. This context presents challenging issues for vendors and policy-makers. For instance, should vendors facilitate precautions by users of pirated software? On the one hand, this would raise the benefit to users of pirated software and so, cannibalize the legitimate demand. On the other hand, this would discourage hackers and so benefit legitimate users. Accordingly, a third direction for further analysis is to consider the implications of piracy with hacking for vendor strategy and public policy.[34]

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[34] One approach would be to suppose that users vary on two dimensions – benefit from use and cost of precautions. Users of pirated items would be subject to enforcement and loss of enjoyment of use (Chen and Png 2003). In equilibrium, users would divide into three segments – those with relatively higher benefit and lower cost of precautions would buy the legitimate item, those with relatively lower benefit and lower cost of precautions would pirate, while those with relatively lower benefit and higher cost of precautions would not use the item.

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Appendix 4A

Proof of Observation 1. Differentiating (4.4) with respect to *c*,

$$-[v+h]\chi\frac{d^2\alpha}{df^2}\frac{\partial f}{\partial c}=n\,,$$

and hence, by (4.1),

$$\frac{\partial f}{\partial c}=\frac{n}{-[v+h]\chi\dfrac{d^2\alpha}{df^2}}<0\,. \qquad (4A1)$$

Differentiating (4.4) with respect to n,

$$-[v+h]\chi\frac{d^2\alpha}{df^2}\frac{\partial f}{dn}=c\,,$$

and hence, by (4.1),

$$\frac{\partial f}{\partial n}=-\frac{c}{[v+h]\chi\dfrac{d^2\alpha}{df^2}}<0\,. \qquad (4A2)$$

Differentiating (4.4) with respect to χ,

$$-[v+h]\frac{d\alpha}{df}-[v+h]\chi\frac{d^2\alpha}{df^2}\frac{\partial f}{\partial\chi}=0\,,$$

and hence, by (4.1),

$$\frac{\partial f}{\partial\chi}=-\frac{d\alpha/df}{\chi\dfrac{d^2\alpha}{df^2}}>0\,. \qquad (4A3)$$

By (4.2), $d\chi/dK>0$, hence $\partial f/\partial K>0$. If $K=0$, then $\chi(K)=0$, hence by (4.4), $f(n\,|\,K=0)=0$, $\forall n\in[0,1]$. Further, if $K\to\infty$, $\chi(K)\to 1$, hence, by (4.4), $\lim_{K\to\infty} f(n)=f_\infty(n)$. [ ]

Proof of Observation 2. We first prove that $B(n)$ is monotone decreasing in n. Consider n_1 and n_2 such that $n_1 < n_2$. Let user n_1 choose the precautions, $f(n_2)$, associated with user n_2. Since $n_1 < n_2$, her expected net benefit would be

$$\begin{aligned} & v-[v+h]\alpha(f(n_2))\chi - p - n_1 cf(n_2) \\ & \qquad > v-[v+h]\alpha(f(n_2))\chi - p - n_2 cf(n_2) \equiv B(n_2 \mid K), \end{aligned} \tag{4A4}$$

By (4.3), the precautions $f(n_1)$ must provide user n_1 with the maximum expected net benefit, and, in particular,

$$\begin{aligned} B(n_1 \mid K) &= v-[v+h]\alpha(f(n_1))\chi - p - n_1 cf(n_1) \\ & \geq v-[v+h]\alpha(f(n_2))\chi - p - n_1 cf(n_2). \end{aligned} \tag{4A5}$$

Hence, by (4A4) and (4A5), $B(n_1|K) > B(n_2 \mid K)$, which is the result.

Since $B(n)$ is monotone decreasing in n, the demand of the software is characterized as follows. Consider the most sophisticated user, $n = 0$. By (4.3), her cost of precaution is zero and therefore she will choose the highest precaution, i.e., $f(0) \to \infty$. Under the assumption that $v > p$ and by (4.1), the most sophisticated user would buy since $B(0) = v - p > 0$. Consider the most naïve user, $n = 1$. If $B(1) \geq 0$, then, $B(n) > 0$ for all $n < 1$ and all other users buy the software. However, if $B(1) < 0$, the most naïve user does not buy the software, and there exists some critical level as claimed. [ ]

Proof of Observation 3. Differentiating (4.5) with respect to c,

$$-[v+h]\chi \frac{d\alpha(f(\hat{n}))}{df}\left[\frac{\partial f(\hat{n})}{\partial c} + \frac{\partial f(\hat{n})}{\partial n}\frac{\partial \hat{n}}{dc}\right] - \hat{n}f(\hat{n}) - cf(\hat{n})\frac{\partial \hat{n}}{dc} - c\hat{n}\left[\frac{\partial f(\hat{n})}{\partial c} + \frac{\partial f(\hat{n})}{\partial n}\frac{\partial \hat{n}}{dc}\right] = 0$$

hence, using (4.4),

$$\frac{\partial \hat{n}}{\partial c} = -\frac{\hat{n}}{c} < 0, \tag{4A6}$$

i.e., the marginal user, $\hat{n}$, is decreasing in *c*.

Differentiating (4.5) with respect to *p*,

$$-[v+h]\chi\frac{d\alpha(f(\hat{n}))}{df}\left[\frac{\partial f(\hat{n})}{\partial p}+\frac{\partial f(\hat{n})}{\partial n}\frac{\partial \hat{n}}{\partial p}\right]-1-cf(\hat{n})\frac{\partial \hat{n}}{\partial p}-\hat{n}c\left[\frac{\partial f(\hat{n})}{\partial p}+\frac{\partial f(\hat{n})}{\partial n}\frac{\partial \hat{n}}{\partial p}\right]=0,$$

hence, using (4.4),

$$\frac{\partial \hat{n}}{\partial p}=-\frac{1}{cf(\hat{n})}<0 \qquad (4A7)$$

Differentiating (4.5) with respect to K,

$$-[v+h]\chi\frac{d\alpha}{df}\left[\frac{\partial f}{\partial K}+\frac{\partial f}{\partial n}\frac{\partial \hat{n}}{\partial K}\right]-[v+h]\alpha(f(\hat{n}))\frac{d\chi}{dK}-\frac{\partial \hat{n}}{\partial K}cf(\hat{n})-c\hat{n}\left[\frac{\partial f}{\partial K}+\frac{\partial f}{\partial n}\frac{\partial \hat{n}}{\partial K}\right]=0 \quad ,$$

hence, using (4.4),

$$\frac{\partial \hat{n}}{\partial K}=-\frac{[v+h]\alpha(f(\hat{n}))}{cf(\hat{n})}\frac{d\chi(K)}{dK}<0 \qquad (4A8)$$

Further differentiating (4A8) with respect to *K*,

$$\frac{\partial^2 \hat{n}}{\partial K^2}=\frac{-[v+h]\alpha(f(\hat{n}))}{cf(\hat{n})}\frac{d^2\chi}{dK^2}-\frac{[v+h]}{cf^2}\frac{d\chi}{dK}\left[\frac{\partial f}{\partial K}+\frac{\partial f}{\partial n}\frac{\partial \hat{n}}{\partial K}\right]\left[f\frac{d\alpha}{df}-\alpha\right]>0, \qquad (4A9)$$

which follows from (4.1), (4.2), (4A2), (4A3), and (4A8). By (4A8) and (4A9), $\hat{n}$ is decreasing and convex in *K*.

If $K\to 0$, then $\chi(K)\to 0$. Hence by (4.3), users' expected net benefit, $B(n)\to v-p-ncf(n)$, which is maximized with $f(n)=0$. Thus $B(n)\to v-p$, for all *n*. Since $v>p$, all users buy the software. Accordingly, if $K\to 0$, then $\hat{n}\to 1$.

Now, if $K\to\infty$, then $\chi(K)\to 1$, hence, by (4.3), users' expected net benefit, $B(n)\to v-[v+h]\alpha(f(n))-p-ncf(n)$. As proved by Observation 2, the most sophisticated user would buy the software, i.e., $B(0 \mid K \to \infty) > 0$. Consider the user with $n = 1$. If her expected net benefit, $B(1) \to v - [v + h]\alpha(f(1)) - p - cf(1) \geq 0$, then by Observation 2, $B(n) > 0$ for all *n*. Hence all users will buy the software. Otherwise, if $B(1) < 0$, then there exists some $\hat{n}_0$ such that

$$B(\hat{n}_0) \to v - [v + h]\alpha(f(\hat{n}_0)) - p - \hat{n}_0\, cf(\hat{n}_0) = 0,$$

which completes the proof.

Proof of Observation 4. To simplify notation, define

$$A(K) = \int_0^{\hat{n}(K)} \alpha(f(n) \mid K) d\Phi(n). \tag{4A10}$$

Since $d\alpha/df < 0$, then $\partial A/\partial f < 0$. Further $\partial \mathrm{A}/\partial \hat{n} > 0$. Substituting (4A10) in (4.8), and then differentiating with respect to η,

$$eA\left[-\frac{d\chi}{dk_i} + [1-\eta]\frac{d^2\chi}{dk_i^2}\frac{\partial k_i}{\partial \eta}\right] = \frac{d^2 c_K}{dk_i^2}\frac{\partial k_i}{\partial \eta},$$

which simplifies to

$$\frac{\partial k_i}{\partial \eta} = \frac{e\dfrac{d\chi}{dk_i}A}{e[1-\eta]A\dfrac{d^2\chi}{dk_i^2} - \dfrac{d^2 c_K}{dk_i^2}} < 0. \tag{4A11}$$

Similarly, differentiating (4.8) with respect to $\hat{n}$,

$$e[1-\eta]\left[\frac{\partial A}{\partial \hat{n}}\frac{d\chi}{dk_i} + A(\hat{n})\frac{d^2\chi}{dk_i^2}\frac{\partial k_i}{\partial \hat{n}}\right] = \frac{d^2 c_K}{dk_i^2}\frac{\partial k_i}{\partial \hat{n}},$$

which simplifies to

$$\frac{\partial k_i}{\partial \hat{n}} = \frac{e[1-\eta]\frac{\partial \mathrm{A}}{\partial \hat{n}}\frac{d\chi}{dk_i}}{\frac{d^2 c_K}{dk_i^2} - e[1-\eta]\mathrm{A}(\hat{n})\frac{d^2\chi}{dk_i^2}} > 0\,. \tag{4A12}$$

When $\hat{n} = 0$, no one buys the software, it doesn't pay for the hackers to attack the software, hence $K = 0$. When $\hat{n} = 1$, all users buy the software. Since the hacker's expected net benefit, (4.7), is concave in k_i, there exists $\tilde{k} > 0$ that satisfies the first order condition, (4.8), and maximizes the expected net benefit.

Similarly, we can show that

$$\frac{\partial k_i}{\partial f} = \frac{e[1-\eta]\frac{\partial \mathrm{A}}{\partial f}\frac{d\chi}{dk_i}}{\frac{d^2 c_K}{dk_i^2} - e[1-\eta]\mathrm{A}(\hat{n})\frac{d^2\chi}{dk_i^2}} < 0\,, \tag{4A13}$$

and that there exists some $k_0 > 0$ such that if $f = 0$, then $k_i = k_0$, and there exists some $k_\infty > 0$ such that if $f \to \infty$, then $k_i = k_\infty$.

Proof of Lemma 1. By Observations 1 and 3 respectively, f is increasing in K and $\hat{n}$ is decreasing in K. Accordingly, $A(K)$ is monotonically decreasing in K, regardless of the user distribution $\Phi(n)$. Further, if $K = 0$, then by (4.2), $\chi = 0$, hence all users would choose $f(n) = 0$ and, by (4.3), get $B(n \mid K) = v - p$. By assumption, $v - p > 0$, hence, if all $k_i = 0$, then $K = 0$, and $\hat{n} = 1$, and so, $A > 0$.

With regard to hacker targeting, by Observation 4, k_i is monotonically increasing in A. Further, if $A = 0$ (because either $\hat{n} = 0$ or $\alpha(f(n)) = 0$, for all n), then hackers will not target the software, $k_i = 0$.

Figure 4.3 depicts $k_i(A)$ and $\mathrm{A}(k)$, which describe the best response functions of the hackers and users, respectively. Since the functions are continuous, they have a non-trivial intersection, say (k_i^*, A^*).

Given hacker targeting $k_1^*,...,k_Z^*$, let $K^* \equiv k_1^* + ... + k_Z^*$, and further, let $\hat{n}(K^*)$ and $f(n \mid K^*)$ be the marginal user and user precautions respectively. Then, by (4.9), the conditional vulnerability

$$A' = \int_0^{\hat{n}(K^*)} \alpha(f(n) \mid K^*) d\Phi(n).$$

Now, we claim that $A^* = A'$, and prove the claim by contradiction as follows.

(i) Suppose otherwise that $A' > A^*$. Then, referring to Figure 4.3, the function $k_i(A)$ gives the hacker's best-response k_i'. Since $k_i(A)$ is monotonically increasing in A, we have $k_i' > k_i^*$, and so, $K' \equiv k_1^* + ... + k_i' + ... + k_Z^* > k_1^* + ... + k_i^* + ... + k_Z^* = K^*$. Since $\hat{n}$ is decreasing in K and $f(.)$ is increasing in K, it follows that $\hat{n}(K') < \hat{n}(K^*)$ and $f(n \mid K') > f(n \mid K^*)$, which implies that $A' < A^*$, which contradicts the original assumption.

(ii) Suppose otherwise that $A' < A^*$. Then, referring to Figure 4.3, the function $k(\mathrm{A})$ gives the hacker's best-response k'. Since $k(\mathrm{A})$ is monotonically increasing in A, we have $k_i' < k_i^*$, and so, $K' \equiv k_1^* + ... + k_i' + ... + k_Z^* < k_1^* + ... + k_i^* + ... + k_Z^* = K^*$. Since $\hat{n}$ is decreasing in K and $f(.)$ is increasing in K, it follows that $\hat{n}(K') > \hat{n}(K^*)$ and

$f(n \mid K') < f(n \mid K^*)$, which implies that $A' > A^*$, which contradicts the original assumption.

Therefore, we must have $A^* = A'$. In symmetric equilibrium, $k_i^* = k^*$, $i = 1,...,Z$. Hence, there exists a non-trivial equilibrium comprising k^*, $\hat{n}(k^*)$ and $f(\hat{n} \mid k^*)$. [ ]

Proof of Proposition 1. Expand (4.8) to distinguish between the precaution of end-user n' denoted $f(n')$ and the precautions of all other users, f,

$$e[1-\eta]\frac{d\chi}{dk_i}\left[\int_{[0,n')}\alpha(f(n))d\Phi(n)+\alpha(f(n'))d\Phi(n')+\int_{(n',\hat{n}]}\alpha(f(n))d\Phi(n)\right]=\frac{dc_K}{dk_i}.\text{(4A14)}$$

By (4A14), an increase in precautions, f, by all other users except n' would reduce the term in brackets, and hence induce all hackers to reduce targeting, $\Delta k_i < 0$, all i. This would imply $\Delta\chi < 0$, which in (4.4), shifts down the left-hand side. Therefore, user n' would reduce $f(n')$. [ ]

Proof of Proposition 2. This follows directly from the proof of Table 4.2, by noting that (4A16) will hold, and hence $\partial A / \partial c \geq 0$, if c is sufficiently high, and not hold if c is sufficiently low.

Proof of Table 4.2

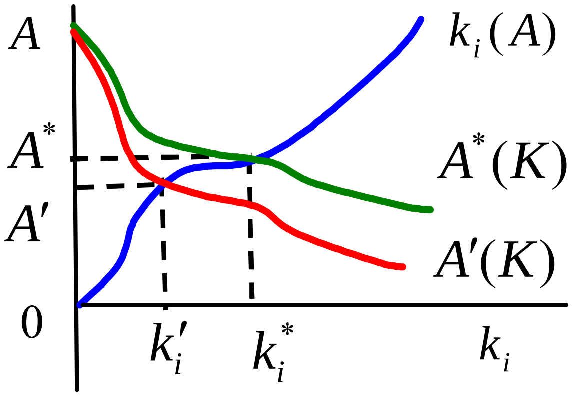

Figure 4A Increase in price, p

User cost of precaution, c

By Observations 1 and 4, an increase in the user cost of precaution, c, directly leads to reduced user precautions, f, and software demand, $\hat{n}$. By (4.9), these have mixed effects on the users' best-response function, $A(K)$. By (4.8), the increase in the user cost of precaution has no direct effect on $k(A)$. Accordingly, the net effect on targeting, k, and conditional vulnerability, A, depends on the sign of $\partial A/\partial c$, which is calculated as follows,

$$\frac{\partial A(K)}{\partial c} = \alpha(f(\hat{n}))\frac{\partial \hat{n}}{\partial c}\frac{d\Phi(\hat{n})}{dn} + \int_0^{\hat{n}} \frac{d\alpha}{df}\frac{\partial f(n)}{\partial c}d\Phi(n) \tag{4A15}$$

Substituting from (4.4) and (4A1), it follows that $\partial A/\partial c \geq 0$ if and only if

$$-\frac{1}{[v+h]\chi}\int_0^{\hat{n}} \frac{n \cdot d\alpha/df}{d^2\alpha/df^2}d\Phi(n) \geq \frac{\hat{n}\alpha(f(\hat{n}))}{c}\frac{d\Phi(\hat{n})}{dn}$$

or

$$c \geq \frac{[v+h]\chi\alpha(f(\hat{n}))\hat{n}}{-\int_0^{\hat{n}} \frac{n \cdot d\alpha/df}{d^2\alpha/df^2}d\Phi(n)}\frac{d\Phi(\hat{n})}{dn}. \tag{4A16}$$

We analyze two cases below.

(i) $\partial A / \partial c \geq 0$. Referring to Figure 4A, an increase in c would lead to a new equilibrium, with higher targeting, $k_i' \leq k_i^*$, higher conditional vulnerability, $A' \leq A^*$, and hence higher effective vulnerability, $\chi(K')A' \leq \chi(K^*)A^*$, where $K' = k_1' + ... + k_Z'$ and $K^* = k_1' + ...k_{i-1}' + k_i^* + k_{i+1}' + ... + k_Z'$. In sum, when $\partial A / \partial c \geq 0$, we must have $dk_i / dc \geq 0$, all *i*, and $dA / dc \geq 0$.

With regard to the marginal user, i.e., software demand,

$$\frac{d\hat{n}}{dc} = \frac{\partial \hat{n}}{\partial c} + \frac{\partial \hat{n}}{\partial K}\frac{dK}{dc} = \frac{\partial \hat{n}}{\partial c} + \frac{\partial \hat{n}}{\partial K}\left[\frac{dk_1}{dc} + ... + \frac{dk_Z}{dc}\right]. \tag{4A17}$$

By Observation 3, $\partial \hat{n} / \partial c < 0$ and $\partial \hat{n} / \partial K < 0$, while from above, $dk_i / dc \geq 0$, for all *i*. Hence, substituting in (4A17), we have $d\hat{n} / dc < 0$.

Regarding the precautions, from above, $A' \leq A^*$, hence by (4.9).

$$\frac{dA}{dc} = \alpha(f(\hat{n}))\frac{d\hat{n}}{dc}\frac{d\Phi(\hat{n})}{dn} + \int_0^{\hat{n}} \frac{d\alpha}{df}\frac{df(n)}{dc}d\Phi(n) \geq 0. \tag{4A18}$$

Now, $d\hat{n} / dc < 0$, hence, substituting in (4A18), it follows that $df / dc < 0$.

(ii) $\partial A / \partial c < 0$. Referring to Figure 4A, an increase in c would lead to a new equilibrium, with lower targeting, $k_i' > k_i^*$, lower conditional vulnerability, $A' > A^*$, and hence lower effective vulnerability, $\chi(K')A' > \chi(K^*)A^*$, where $K' = k_1' + ... + k_Z'$ and $K^* = k_1' + ...k_{i-1}' + k_i^* + k_{i+1}' + ... + k_Z'$. In sum, when $\partial A / \partial c < 0$, we must have $dk_i / dc < 0$, all *i*, and $dA / dc < 0$.

With regard to user precautions,

$$\frac{df}{dc} = \frac{\partial f}{\partial c} + \frac{\partial f}{\partial K}\frac{dK}{dc} = \frac{\partial f}{\partial c} + \frac{\partial f}{\partial K}\left[\frac{dk_1}{dc} + ... + \frac{dk_Z}{dc}\right]. \tag{4A19}$$

By Observation 1, $\partial f / \partial c < 0$ and $\partial f / \partial K > 0$, while from above, $dk_i / dc < 0$, for all *i*. Hence, substituting in (4A19), we have $df / dc < 0$.

Regarding the marginal user, from above, $A' > A^*$, hence by (4.9),

$$\frac{dA}{dc} = \alpha(f(\hat{n}))\frac{d\hat{n}}{dc}\frac{d\Phi(\hat{n})}{dn} + \int_0^{\hat{n}} \frac{d\alpha}{df}\frac{df(n)}{dc}d\Phi(n) < 0. \qquad (4A20)$$

Now, $df / dc < 0$, hence, substituting in (4A20), it follows that $d\hat{n}/dc < 0$.

Enforcement rate, η, and hacking cost, $c_K(.)$

First, consider the effect of an increase in enforcement, η. By Observations 1 and 3, the increase in enforcement has no direct effect on users' precautions or demand $\hat{n}$. Hence, by (4.9), the best-response function $A(k)$ remains unchanged. By Observation 4, the enforcement increase directly leads hackers to reduce targeting, hence their best-response function, $k_i(A)$, shifts to the left. Accordingly, in the new equilibrium, targeting is lower, $k_i' > k_i^*$, and the conditional vulnerability is higher, $A' < A^*$.

Since the increase in enforcement results in lower targeting, k_i, hence lower hacker effectiveness, $\chi(K)$, but higher conditional vulnerability, A, the impact on the effective user vulnerability, χA, depends on the balance of the effects on hackers and users.

With regard to user precautions,

$$\frac{df}{d\eta} = \frac{\partial f}{\partial \eta} + \frac{\partial f}{\partial K}\frac{dK}{d\eta}. \qquad (4A21)$$

By (4.4), $\partial f / \partial \eta = 0$, by Observation 1, $\partial f / \partial K > 0$, while from above, $dK / d\eta < 0$. Hence, substituting in (4A21), we have $df / d\eta < 0$.

Similarly, with regard to the marginal user, i.e., software demand,

$$\frac{d\hat{n}}{d\eta}=\frac{\partial \hat{n}}{\partial \eta}+\frac{\partial \hat{n}}{\partial K}\frac{dK}{d\eta}. \tag{4A22}$$

By (4.5), $\partial \hat{n}/\partial \eta = 0$, by Observation 3, $\partial \hat{n}/\partial K < 0$, while from above, $dK/d\eta < 0$. Hence, substituting in (4A22), we have $d\hat{n}/d\eta > 0$, which completes the proof.

The effect of an increase in the hacking cost is similar. For brevity, we omit the proof.

Price, $\underline{p}$

By Observation 1, a price increase has no direct effect on user precautions, while, by Observation 3, the price increase directly reduces the demand, $\hat{n}$. Accordingly, by (4.9), for $k_i > 0$, the best-response function $A(k)$ shifts downward, while, by (4.9), for $k_i = 0$, $A(0)$ does not change with p. By (4.8), the price increase has no direct effect on $k_i(A)$.

Figure 4A depicts the new equilibrium: the users' best-response function shifts from $A^*(K)$ downward to $A'(K)$, while the hackers' best-response function remains unchanged. In the new equilibrium, targeting is lower, $k_i^* > k_i'$, and the conditional vulnerability is lower, $A^* > A'$.

Given that the increase in price, p, leads to lower targeting, k, it would, by (4.2) result in lower hacker effectiveness, χ. Thus, the effective user vulnerability, χA, decreases with price, p.

With regard to user precautions,

$$\frac{df}{dp}=\frac{\partial f}{\partial p}+\frac{\partial f}{\partial K}\frac{dK}{dp}. \tag{4A23}$$

By (4.4) $\partial f / \partial p = 0$, by Observation 1, $\partial f / \partial K > 0$, while from above, $dK / dp < 0$. Hence, substituting in (4A23), we have $df / dp < 0$.

Regarding the marginal user, from above, $A^* > A'$, hence, by (4.9),

$$\frac{dA}{dp} = \alpha(f(\hat{n}))\frac{d\hat{n}}{dp}\frac{d\Phi(\hat{n})}{dn} + \int_0^{\hat{n}} \frac{d\alpha}{df}\frac{df(n)}{dp}d\Phi(n) < 0 . \qquad (4A24)$$

From above, $df / dp < 0$, hence substituting in (4A24), it follows that $d\hat{n} / dp < 0$, which completes the proof. [ ]

Proof of Proposition 3. By assumption, $\partial A / \partial c > 0$, hence $dK / dc > 0$ and $dW / dc < 0$. By (4.12) and (4.14), $|dW / dc| > |dW / d\eta|$ if and only if

$$\int_0^{\hat{n}} nf(n)d\Phi(n) > p\frac{d\Phi(\hat{n})}{dn}\left[\frac{d\hat{n}}{d\eta} + \frac{d\hat{n}}{dc}\right] - [v+h]\frac{d\chi(K)}{dK}\left[\frac{dK}{d\eta} + \frac{dK}{dc}\right]A ,$$

(4A25)

where

$$\frac{d\hat{n}}{d\eta} = \frac{\partial\hat{n}}{\partial K}\frac{dK}{d\eta},$$

(4A26)

$$\frac{d\hat{n}}{dc} = \frac{\partial\hat{n}}{\partial c} + \frac{\partial\hat{n}}{\partial K}\frac{dK}{dc},$$

(4A27)

$$\frac{dK}{d\eta} = \frac{\partial K}{\partial\eta} + \frac{\partial K}{\partial A}\frac{dA}{d\eta},$$

(4A28)

$$\frac{dA}{d\eta} = \frac{\partial A}{\partial\eta} + \frac{\partial A}{\partial K}\frac{dK}{d\eta} = \frac{\partial A}{\partial K}\frac{dK}{d\eta}.$$

(4A29)

Substituting from (4A6) and (4A8) in (4A27),

$$\frac{d\hat{n}}{dc}=-\frac{\hat{n}}{c}-\frac{[v+h]\alpha(f(\hat{n}))}{cf(\hat{n})}\frac{d\chi(K)}{dK}\frac{dK}{dc}.$$

(4A30)

Further, by differentiating (4.8) with respect to c,

$$\frac{dk_i}{dc}=\frac{e[1-\eta]\frac{d\chi}{dk_i}\frac{dA}{dc}}{\frac{d^2c_K}{dk_i^2}-e[1-\eta]A\frac{d^2\chi}{dk_i^2}}.$$

(4A31)

In symmetric equilibrium, $k_i^*=k$, $i=1,...,Z$, thus, by (4A31)

$$\frac{dK}{dc}=\frac{dk_1}{dc}+...+\frac{dk_Z}{dc}=Z\frac{dk_i}{dc}=\frac{eZ[1-\eta]\frac{d\chi}{dk}\frac{dA}{dc}}{\frac{d^2c_K}{dk^2}-e[1-\eta]A\frac{d^2\chi}{dk^2}}.$$

(4A32)

Similarly, in symmetric equilibrium, by (4A11),

$$\frac{\partial K}{\partial\eta}=\frac{\partial k_1}{\partial\eta}+...+\frac{\partial k_Z}{\partial\eta}=Z\frac{\partial k_i}{\partial\eta}=\frac{-eZ\frac{d\chi}{dk}A}{\frac{d^2c_K}{dk^2}-e[1-\eta]A\frac{d^2\chi}{dk^2}}.$$

(4A33)

Substituting from (4A29) in (4A28), and then substituting from (4A33), we have

$$\frac{dK}{d\eta}=\frac{\frac{\partial K}{\partial\eta}}{1-\frac{\partial K}{\partial A}\frac{\partial A}{\partial K}}=\frac{\frac{eZ\frac{d\chi}{dk}A}{e[1-\eta]A\frac{d^2\chi}{dk^2}-\frac{d^2c_K}{dk^2}}}{1-\frac{\partial K}{\partial A}\frac{\partial A}{\partial K}}.$$

(4A34)

Substituting from (4A26), (4A8) and (4A30) in (4A25),

$$p\frac{d\Phi(\hat{n})}{dn}\frac{\hat{n}}{c}+\int_0^{\hat{n}} nf(n)d\Phi(n) > -[v+h]\frac{d\chi(K)}{dK}\left[p\frac{d\Phi(\hat{n})}{dn}\frac{\alpha(f(\hat{n}))}{cf(\hat{n})}+A\right]\left[\frac{dK}{d\eta}+\frac{dK}{dc}\right].$$

(4A35)

Further substituting from (4A32) and (4A34) in (4A35), and then simplifying, we have

$$c\left\{\frac{p\dfrac{d\Phi(\hat{n})}{dn}\dfrac{\hat{n}}{c}+\displaystyle\int_0^{\hat{n}} nf(n)d\Phi(n)}{p\dfrac{d\Phi(\hat{n})}{dn}\dfrac{\alpha(f(\hat{n}))}{cf(\hat{n})}+A}\right\}\left\{\frac{\dfrac{d^2c_K}{dk^2}-e[1-\eta]A\dfrac{d^2\chi}{dk^2}}{eZA[v+h]\left[\dfrac{d\chi}{dK}\right]^2}\right\} > \frac{c}{1-\dfrac{\partial K}{\partial A}\dfrac{\partial A}{\partial K}}-[1-\eta]\frac{c}{A}\frac{dA}{dc}.$$

(4A36)

Now,

$$\frac{dA}{dc}=\frac{\partial A}{\partial c}+\frac{\partial A}{\partial K}\frac{dK}{dc}=\frac{\partial A}{\partial c}+\frac{\partial A}{\partial K}\frac{dK}{dA}\frac{dA}{dc},$$

which implies

$$\frac{dA}{dc}=\frac{\dfrac{\partial A}{\partial c}}{1-\dfrac{\partial K}{\partial A}\dfrac{\partial A}{\partial K}}. \tag{4A37}$$

Substituting from (4.5) and (4A37) in (4A36), then multiplying both sides by $1/[1-\eta]$, and then substituting from (4.8), and simplifying, we have

$$c\left\{\frac{\dfrac{p\dfrac{d\Phi(\hat{n})}{dn}}{cf(\hat{n})}\left[\dfrac{v-p}{c}-\dfrac{[v+h]\alpha(f(\hat{n}))\chi}{c}\right]+\displaystyle\int_0^{\hat{n}} nf(n)d\Phi(n)}{\dfrac{p\dfrac{d\Phi(\hat{n})}{dn}}{cf(\hat{n})}\alpha(f(\hat{n}))+A}\right\}\left\{\frac{\dfrac{d^2c_K}{dk^2}}{\dfrac{dc_K}{dk}}+\frac{e[1-\eta]A}{\dfrac{dc_K}{dk}}\left[-\frac{d^2\chi}{dk^2}\right]\right\}$$

$$> \frac{\left[\dfrac{c}{1-\eta}-\dfrac{c}{A}\dfrac{\partial A}{\partial c}\right]}{1-\dfrac{\partial K}{\partial A}\dfrac{\partial A}{\partial K}}Z[v+h]\frac{d\chi}{dK}. \tag{4A38}$$

Condition (4A38) will be satisfied if the users' benefit relative to the cost of precaution, $\left[v-p\right]/c$, and the hackers' expected enjoyment relative to targeting cost, $e\left[1-\eta\right]\Big/\frac{dc_K}{dk}$, are sufficiently large. []

CHAPTER 5 CONCLUSION AND FUTURE WORK

In this chapter, I will briefly review the results of these three essays, and propose a few possible directions for future research.

5.1 Delayed Product Introduction

The study of delayed product introduction investigated a monopolistic vendor's incentive to delay the introduction of its newly improved product in a stationary market with identical consumers. In this three-period game, the vendor can only sell a low-quality (old) product in the first period. In the second period, due to external technology improvement, the vendor is able to produce a high-quality product incorporating the advanced technology. The vendor has to choose whether and when to sell the high-quality product. We find that the vendor's monopoly power is constrained by the mutual cannibalization between the successive generations of products: The remaining stock of the old product that has already been sold to consumers limits their willingness to purchase the new product. On the other hand, anticipating the forthcoming of the new product, consumers may bypass the current product, choosing instead to wait and observe prospective technology.

To alleviate the mutual cannibalization, the vendor may delay selling the new product, which allows the vendor to charge consumers higher prices for both the old and new products. By deferring sale of the new product, the vendor extends the economic life span of the old product, which increases its value to consumers. It also allows the old product to be used for one more period, and hence the old product depreciates more in value. Since consumers who possess the old product are willing to pay only the incremental utility that they can derive from the new product, the further deterioration of the old product raises their reservation price for upgrading to the new product in a later period.

With an upgrade policy, the vendor can internalize the cannibalization caused by the new product but not the cannibalization from the old product. The above analysis explains why delayed product introduction can be advantageous even when an upgrade policy is possible -- an insight that prior studies of product introduction with assumption of infinite durability (Fishman and Rob 2000; Lee and Lee 1998; Waldman 1996) cannot show.

This study contributes to the literature on technology adoption by analyzing both the vendor's and consumers' economic incentives of adopting a new and better technology.

This study opens up several avenues of future research. First, we could allow for dynamic demands with new consumers entering the market in each period, or incorporate heterogeneity in taste for quality. Second, it would be interesting to see if competition dilutes the incentives to delay new product introductions. Third, we have assumed that the new product does not affect the quality of the old product, but this might not be the case for products that exhibit network externalities or require compatible standards (Padmanabhan et al. 1997). Finally, it may be worthwhile to study the interplay of delayed introduction and preannouncement (e.g., Bayus et al. 2001; Hendricks and Singhal 1997). Preannouncement is commonly practiced for software, information technology and electronics products. It is instructive to investigate if preannouncement raises consumer expectations of new products, and whether delay in such a context serves the same function as in this paper.

Despite these future extensions, it is clear that the incentives of durable goods sellers to deploy advanced technologies in new products must be closely monitored, or else consumers may simply not see the light of better products.

5.2 Technology Timing and Pricing in the Presence of an Installed Base

This study investigated a monopolistic vendor's timing, pricing and product line strategies for selling a new improved product in the presence of an installed base of its old product. The market consists of two groups of consumers with either low or high valuation of product quality. In the scenario of a fully-covered installed base, both types already possess the old product at the beginning of the game. In the scenario of a partly-covered installed base, only the high type owns the old product, thus they are distinct from the low segment not only in valuation of quality but also in purchase history. By generalizing users' utility function incorporating their valuation of quality, purchase history and upgrade timing, we characterized the purchase pattern of users who already own an existing (old) version of a durable product: (1) their willingness to purchase the new product increases over time; (2) their (discounted) utility derived from the new product is less sensitive to the upgrade timing compared to not possessing the old product and may even increase over time if the extent of quality improvement is sufficiently low; (3) with the same purchase history, the high type is more sensitive to the change in purchase timing than that of the low type.

Users' purchase pattern (as elaborated above) and the nature of the product (as defined by the extent of quality improvement and durability) together determine the vendor's optimal strategies. We were particularly interested in the strategies incorporating intertemporal price discrimination, delayed product introduction or upgrade pricing since they exhibit distinct features compared to the existing research in the absence of an installed base:

- The presence of an installed base strengthens the conditions under which intertemporal price discrimination by first serving the high type is advantageous. However, if the market is only partly covered with the old product, a moderate heterogeneity in valuation of quality may favor another type of intertemporal price discrimination – in which the low type with no purchase history consumes the new product earlier.
- Because of the combined dampening effects of the installed base and heterogeneous users' anticipation of future price reduction, delaying the introduction of the newly improved version can be advantageous regardless of the extent of user heterogeneity and the extent of quality improvement between the sequential versions of products. This is different from the finding in a market with identical users, which suggests that delayed product introduction only occurs with relatively low extent of quality improvement (Hui and Wang 2005). Further, the competitive advantages of delayed product introduction vary with the parameterization of the extent of quality improvement and the extent of user heterogeneity. When the extent of user heterogeneity is relatively low, by postponing selling the new product, the vendor can capture the whole demand in the last period. When the extent of user heterogeneity is relatively high, via delayed introduction, the vendor can earn more profit from selling the old product to the low type with a relatively low extent of quality improvement or from selling the new product only to the high type with a relatively high extent of quality improvement.
- Unlike prior studies which advocate the benefits of the upgrade pricing in maximizing the vendor's profits and promoting socially efficient production (Waldman

1997, Lee and Lee 1998, Fishman and Rob 2000), this study suggested that upgrade policy cannot segment users by their purchase history if user heterogeneity is sufficiently high. In this case, the vendor would maximize its profit via intertemporal price discrimination, or delayed introduction, or pooling pricing, depending on the characteristics of market structure and technology improvement.

Based on the economic theories of consumer segmentation and price discrimination, this study provides an effective framework to address product timing and pricing strategies for tackling the installed base. Interesting economic factors can be included into the model for further research. For instance, demand-side positive network effects can be incorporated into consumer utility function. With backward compatibility, users of the newly improved version can enjoy the positive network externality from the installed base of the old version; while the benefit to users of the old product decreases as users upgrade to the new version. It is interesting to examine the impact of the various combinations of timing, pricing and product line strategies in the presence of network effects.

Another direction is to test various empirical implications. Most of the propositions derived in the first and second essays focus on the strategic decisions of information goods vendor on launching newly improved product, in particular, the effects of the extent of quality improvement, durability, installed base and consumer heterogeneity on vendor’s timing, pricing and product line strategies. The empirical research requires sufficient data on vendor behavior and demand structure. Vendor behavior can be acquired based on the time series of regional sales and the installed base of certain product categories; while demand structure can be captured through demographic information and surveying business users. With the availability of data, the findings in the above two essays can be validated.

5.3 Information security: User Precautions and Hacker Targeting

Via a fairly general model, the study of information security analyzed the strategic interactions among end-users and between end-users and hackers and investigated the impacts of changes in the user cost of precaution and the rate of enforcement against hackers on information security and social welfare. First, we found that since hacker maximizes his expected net benefit based on the overall vulnerability of the user base rather than specific user precautions, there is inertia among end-users in taking precautions even facing grave potential consequences. Second, by encompassing both direct and indirect effects, we showed that reducing user cost of precautions or increasing enforcement against hackers need not enhance overall information security because of the feedback effect through the actions of the other side of the market. Third, our welfare analysis suggested that policy should focus on facilitating user precautions if the users' benefit relative to the cost of precaution and the hackers' expected enjoyment relative to targeting cost are sufficiently high. Finally, we argue for appropriate international authority to make and coordinate policy across borders to resolve international externalities.

Other than the future research directions mentioned in Section 4.9, potential research opportunity also lies in the cross-boundary study between information security and product introduction or information privacy. First, will the threat of hacking affect the vendor's decision on launching newly improved product? If yes, what are the impacts on product design, timing and pricing? Next, enhancing information security has conflicting effects on end-users' privacy. On the one hand, it can protect end-users' privacy from hacking; on the other hand, implementation of security safeguards as such often require access to end-users' systems and end-users' personal information. What is the appropriate trade-off between end-users' privacy and information security measures?

5.4 Conclusion

According to Varian (2004), "high-technology industries are subject to the same market forces as every other industry……. but forces that were relatively minor in the industrial economy turn out to be critical in the information economy". This thesis extended the research of industrial organization into information goods industry, specifically in the areas of new product introduction and information security. The three studies included in this thesis demonstrate that the theories and models developed in traditional industrial organization are effective approaches to study the economic relationships existing in the emerging business models or market activities which are enabled by or specific to information goods industry. Further, studying the intriguing relationship between information technology and market structure extends the prior theories of industrial organization and opens up important avenues for future research.

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