CONTROL OF MECHANICAL SYSTEMS WITH BACKLASH PROBLEM

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Abstract

This thesis describes the development of software solutions of backlash problems in mechanical systems. Backlash is common in many components in mechanical and mechatronic systems, such as actuators, sensors and mechanical connections. A typical backlash example is the motion like dead zone due to the gap between gear teeth. This gap leads to degradation of the system's performance. Thus from the early days of classical control theory, the backlash nonlinearity has been recognized as one of the factors which severely limit the performance of feedback systems by causing delays, oscillations and inaccuracy.

Although many control algorithms were developed to overcome the backlash problem, they can not theoretically ensure the system performance criteria such as rise time and overshoot in position control. They have to tune parameters by trial-and-error, which are time-consuming and highly depend on operators' experience. We developed a control approach to satisfy the criteria when backlash exists. The effectiveness of this method was illustrated in simulation results.

We also evaluated two researchers' control algorithms on a real system, a leg of NUS biped, whose motion suffers from backlash in the knee joint. Experiments showed that robust control method was more reliable and had less tracking error.

Present works are dependent on a backlash model which do not resemble backlash in real mechanical connection. Future work would study a reliable control algorithm with a more realistic backlash model in mechanical connections such as gear play.

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Chapter 1

Introduction

Backlash, or backlash-like hysteresis, is a phenomenon that the input and the output are disengaged by imperfect system elements. It is one of the most common non-smooth nonlinearities widespread in mechanical and electrical systems.. For example, if a pair of gears is not precise or well assembled, the driving shaft and the load shaft can be decoupled due to the gap between the teeth of the gears. The driving torque cannot be transferred to the load. Hence, backlash can degrade accurate positioning, lead to chattering, thus severely limit the performance of systems.

Backlash is usually categorized as an imperfection of system components. To solve this problem, there are mainly two classes of approaches: hardware solutions and software solutions.

There are several common hardware solutions to relieve backlash including tightening gear mesh, using precise gears and specific anti-backlash mechanisms. To reduce backlash, engineers may mesh gears tightly. But this inevitably increases friction and even gets gears stuck. Another way is to use precise gears. However, components with high precision are usually expensive, and their maintenance needs specialized personnel. Thus the price of manufactory and maintenance of the systems will be much higher. Sometimes, it is not desired in practical. An alternative to address these difficulties is to apply special anti-backlash mechanisms, as introduced in [10]. These mechanisms are cheaper and can partially compensate for backlash. However, they are cumbersome and unwieldy, and there is still some additional expense. Some of them may introduce other problems such as compliance.

In general, hardware solutions have the following limitations:

- Expensive in assembling, adjusting, maintenance and training.
- Dimension constraints.
- Inconsistent performance due to abrasion.

Due to the above limitations, the request on the application of software solutions arises. The swift advance of computing power technology has already led to new solutions to many stubborn engineering problems in the past. By employing the computational technology we can achieve high accuracy and better performance with imprecise, sound-in-design and inexpensive components. For example, applicability of a noisy sensor can be dramatically broadened by adaptive filtering and other forms of signal conditioning. With a specially designed controller, the “soft” solutions may also be used to remove the harmful effects of backlash in a non-mechanical fashion, without cumbersome and expensive anti-backlash components.

Thus the control of systems with backlash becomes an important area of control system research[6]. An ideal control design for such systems should be able to accommodate system uncertainties. Robust and adaptive methods for the control of systems with partially known or unknown backlash are particularly attractive in many applications. These kinds of techniques are able to provide robust tolerance and adaptation mechanisms for the presence of parametric and system structural uncertainties. However, established robust, adaptive or nonlinear control techniques are for linear systems and some classes of systems with smooth transition nonlinearities. They may not be suitable for backlash which has non-smooth transition. The need for effective control methods to deal with backlash has motivated growing research activities in robust and adaptive control of non-smooth nonlinear systems.

1.1 Objective

In this thesis, we provide a backlash controller in position regulating systems. Many works in literature [6],[11][13][16],[37] concentrated on tracking control, which do not consider the performance criteria like the overshoot and rise time of the system. In this thesis, we worked on position regulation when backlash exists and used overshoot and rise time as the criteria to evaluate the system performance.

This thesis also evaluates several control algorithms on a real test-bed: NUSBIP-I. The purpose is to identify the advantages and limitations of these control algorithms, and formulate more reliable controllers.

1.2 Organization

This thesis is organized as follows. Chapter 2 gives a survey on backlash related research. Chapter 3 presents design of a position controller for systems with backlash. The system can achieve the performance criteria (settling time, over shoot, etc.) when backlash exists. Chapter 4 evaluates five control algorithms on one leg of NUSBIP-I robot. The comparison and discussion are given at the end of this chapter. Chapter 5 concludes this thesis and states the future work.

Chapter 2

Literature Survey

Backlash is a phenomenon which has been a hot research for more than 50 years: from the servo mechanisms in the 1940s to the modern high precision robotic manipulators. The concern for backlash is obvious. For example, in [7], anti-backlash gear boxes were described. Control of servo-lenses for active vision experiments is a more recent illustration. The price of backlash-compensated lenses is much higher than that of those with backlash. Typically the concept of backlash is associated with gear trains and similar mechanical couplings; sometimes it is also used to approximate the delays in drives with elastic cables. In this chapter we will introduce the main research works on the solutions to backlash.

2.1. Backlash Models

In this section, we introduce the common backlash models commonly used. These models are very helpful for the understanding of the characteristic of the backlash. We could also gain some insight to the backlash problem.

However, backlash modeling is itself also an active research topic. These backlash models still have some limitations. It is interesting to note that they all have an important common parameter, which is the backlash gap size. This parameter is important because if the backlash gap size is known, engineers can move the actuators across the backlash gap quickly enough, hence, reducing the harmful effect of backlash.

The typical models in use are static backlash model and sandwiched backlash model. They are also used in Chapter 3 and Chapter 4 respectively in this thesis.

2.1.1. Static Backlash Model

A widely accepted model of backlash [7] is shown in Figure 2.1., where v is the input, u is the output, k is the backlash slope ratio and $c > 0$ is half of the backlash gap. In gear coupling, this gap means the total clearance between the meshing sides of the two gears.

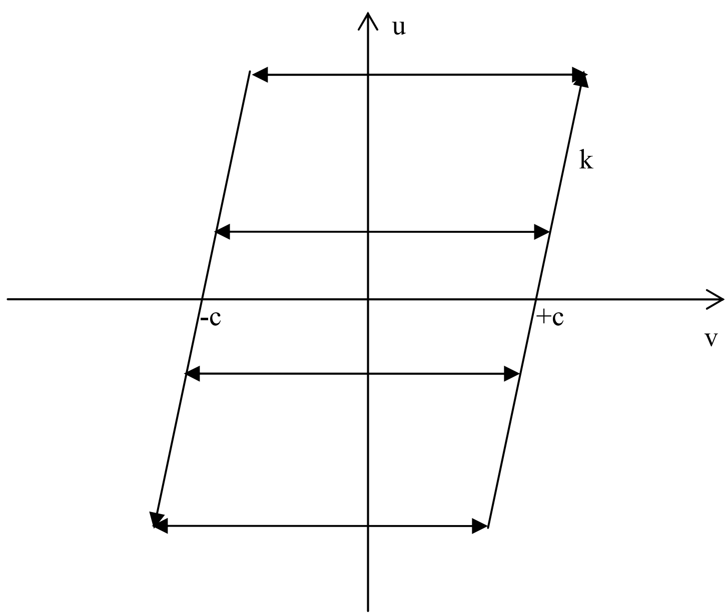

Figure 2.1 Static Backlash Model, v is the input, u is output, c>0 is half gap of the backlash, k>0 is the gear ratio. The double direction arrows mean the input can move into the gap from either slope. The units of u and v usually are mm.

Actually, the model in Figure 2.1 is simplified as it sets the mid-way point as the origin. In reality the gear tooth may not initially be at the mid-way point. A more complicated model is the use of the sum of two parameters $c_r > 0, c_l > 0$ to represent the gap, or set one side of the contact point as the origin and use C as the size of the gap. In this thesis we will use the static backlash model represented by Figure 2.1. So we have a mathematical representation:

$$u = \begin{cases} k(v-c) & \dot{v} > 0, \dot{u} > 0 \\ k(v+c) & \dot{v} < 0, \dot{u} < 0 \\ u^{-} & otherwise \end{cases} \tag{2.1}$$

where u^{-} is the value of last time interval.

A compact description is

$$\dot{u} = \begin{cases} k\dot{v} & \text{if } \dot{v}\text{>0 and u=k(v-c) or} \\ & \text{if } \dot{v}\text{<0 and u=k(v+c)} \\ 0 & \text{otherwise} \end{cases} \tag{2.2}$$

To appreciate the mechanism of this model in Figure 2.1, we will explain it step by step with the help of Figure 2.2, where an L-shaped object is driving a U-shaped object with the contact gap 2c. The input v is the position of the L-shaped object "A" and output u is the position of the U-shaped object "B". Both objects do not have inertia and only their positions are of interest.

Let the starting position in Figure 2.2 be $v = 0, u = c$ and suppose that A begins to move to the right. When v's value reaches $v = +c = u$, contact between A and B is established and B follows A along the upward slope of the characteristic. If at some point A stops and begins to move to the left, B will remain motionless. Hence, the motion of the operating point when B is motionless is represented by the horizontal transition to the left. It is easy to see from Figure 2.2 that the length of the horizontal segment is 2c.

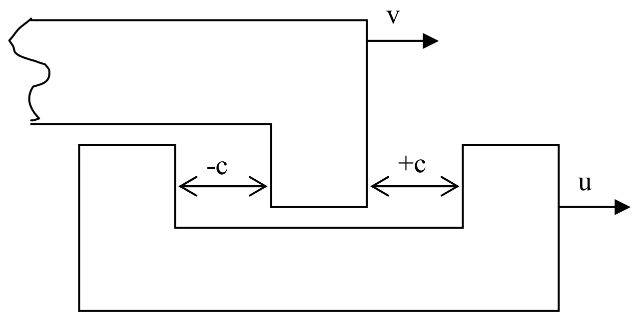

Figure 2.2 Schematic representation of static Backlash model. "-"and "+" represent the negative and positive moving direction respectively.

At the end of this segment, contact is established. Then B begins to move to the left jointly with A, i.e. the operating point moves along the downward slope in Figure 2.1. If at some point A again stops and then moves to the right, B will stop and wait until A traverses the whole segment 2c. The motion is again along a horizontal segment, this time to the right. Surely A can change its direction before it traverses the segment and the next contact may be to the left. Or A can stop before it reaches a new contact, i.e. stay in the horizontal segment.

A typical input-output response of this model is shown in Figure 2.3, where v is a sinusoidal signal with 2 units' amplitude, static backlash has 2 units' gap and u is the output of the backlash. As we can see, the output u does not change until v exceeds it by half of the gap.

This model is very different from some backlash in real systems because it does not consider the inertial of the U shape load and the L shape driver in Figure 2.2. This kind of backlash is only suitable for those components with small inertia. When the load has large inertia, this model is not appropriate any more.

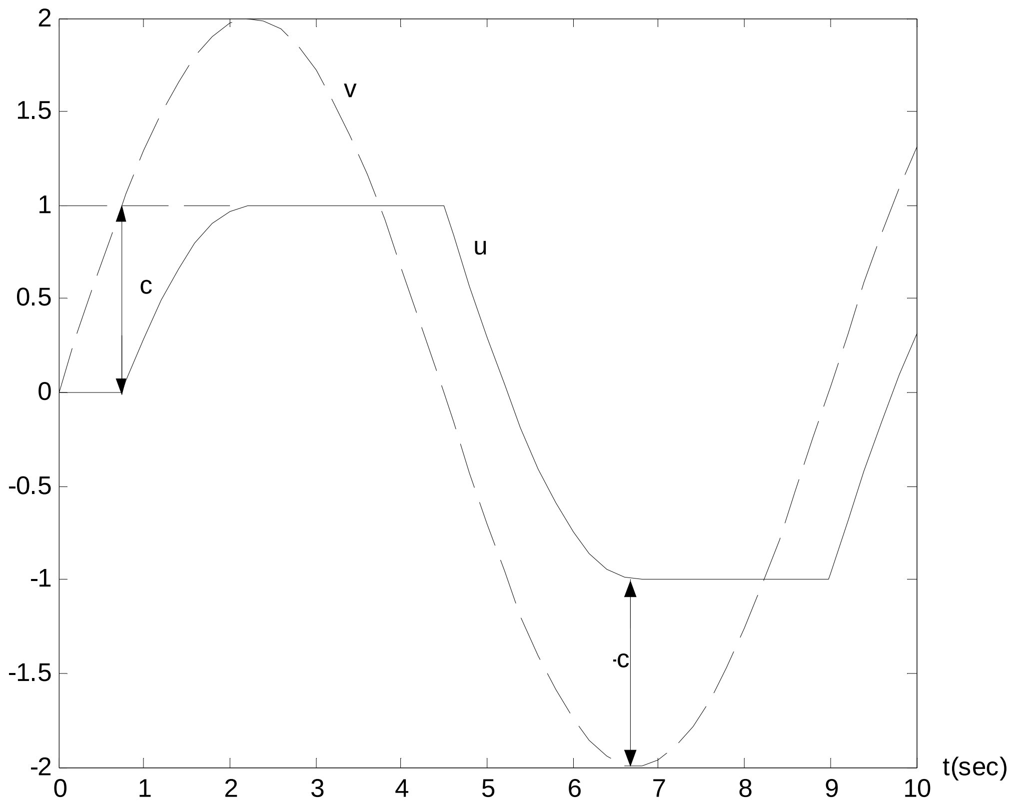

Figure 2.3 Static backlash model responses with 2 units of backlash gap

2.1.2. Sandwiched Backlash Model

When compliance and dynamic effect cannot be neglected, the sandwiched backlash model should be used [8],[9]. This backlash model [8] considers a motor shaft, a load and a backlash. The transferred torque from motor to load is modeled as a spring-damper system. The mathematical description is as Equation 2.3 and 2.4, where the dynamic elements such as inertia are described in the motor and the load's models. The schematic representation of this backlash is shown in Figure 2.4.

$$\tau = k\theta_s + c\dot{\theta}_s,\ \theta_s = DZ(\theta_m - \theta_l) \qquad (2.3)$$

where

$$DZ(\theta_m-\theta_l)=\begin{cases}\theta_m-\theta_l-\alpha & \text{if } \theta_m-\theta_l\geq\alpha\\ 0 & \text{if } -\alpha<\theta_m-\theta_l<\alpha\\ \theta_m-\theta_l+\alpha & \text{if } \theta_m-\theta_l\leq-\alpha\end{cases} \quad (2.4)$$

and τ is the output torque, 2α is the backlash gap, k and c are the elasticity and viscous damping coefficients respectively, θ_m and θ_l are the rotation angles of motor and load respectively.

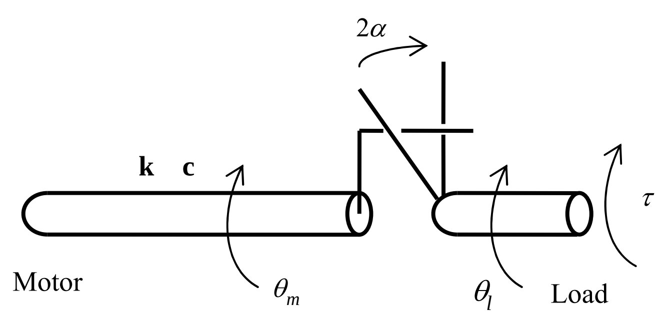

Figure 2.4 The schematic representation of sandwiched backlash model. 2α is the backlash gap, k and c is the elasticity and viscous damping coefficients.

From Figure 2.4, it is easily seen that this model is similar to the static backlash model(Figure 2.2). But from Equation 2.3 this model mathematically considers also compliance and viscous effects. Compared with static model, this model considered velocities of both motor shaft and the load (Equation 2.3). This means during backlash mode[1] the dynamics of the shaft and the load will not affect each other. In this sense, the sandwiched model is more realistic than the static one.

However, many researchers tried to use the above two models to model a backlash with input position and output torque[1],[2][11],[12].

[1] Backlash mode means the driver and the load are not in contact.

2.2 Research on Solutions to Backlash

Recent researches on solutions to backlash include the hardware solutions and the software solutions. Hardware solutions use some dedicated mechanisms to remove backlash. They are usually well applied in the industry. On the other hand, software solutions do not remove or reduce backlash gap physically but utilize control algorithms to reduce backlash effects. In this section, we will briefly describe the works of designing backlash-free mechanisms. Afterwards we will focus on several hot backlash control methods.

2.2.1 Hardware Solutions for Backlash

Hardware solutions to backlash problems refer to specially designed mechanisms, e.g., anti-backlash gear and harmonic drive, to attenuate backlash gap. For example, reference[10] describes eight mechanisms to prevent backlash. These eight mechanisms cleverly utilize springs, bearing and bevels to hold the surfaces in contact; therefore the meshing is “tight”. But such mechanisms, e.g. springs may introduce compliance into the system. Moreover these redundant components are cumbersome, difficult to assembling and maintenance.

In addition, conventional compliant anti-backlash mechanism has some other limitations, i.e., limited motion range, poor kinematic behavior and deformation under multi-axis loading. To address these problems, reference [14] proposes a compliant joint design, in which a split-tube(s) flexure is used to make the shaft. Mechanics analysis shows that this design can result in at least 3 times torsion stiffness compared to conventional flexure shaft. Experiments also prove that the performance is only limited by digital quantization of the trajectory command and sensor noises. However, kinematic and dynamic behaviors are not discussed.

2.2.2 Software Solutions for Backlash

The software solution removes/reduces backlash effects by applying specific control algorithms. This has been extensively studied but the applications are still limited within lab environments. According to the tools used for the software implementations, these solutions can be divided into five categories: using describing function method, adaptive control, robust control, optimization and identification. These five categories are discussed separately in this subsection.

Describing function

Frequency response method is a popular tool to analyze the linear control system. For some nonlinear systems, an extended version of the frequency response method, called the describing function method, can be used to approximately analyze and predict nonlinear behavior.

To control a system containing both non-linear and linear elements, the following procedures are used:

1. Determine the transfer function(s) of the linear elements, G and the describing function(s)[63] of the nonlinear element(s), N. Describing function can be thought as a nonlinear counterpart of transfer function. Therefore the linear controller design methods can be applied in nonlinear system.
2. Adjust the parameters of transfer function G such that G and $-N^{-1}$ do not intersect in their Nyquist plot.

This nonlinear method is used in many research works on solutions to backlash. In [51], a nonlinear compensator circuit is added to the feedback loop to counteract the effect of the backlash. This nonlinear element is tuned based on analysis in Nyquist plot. Simulation proves the efficacy of this nonlinear compensator.

By examining intersection of the Nyquist plot of G and $–N^{-1}$, describing function is also an effective analytical tool to study the occurrence of instabilities[49],[50][52],[54]. Many researchers used this tool to predict stable limit cycles in the presence of backlash. (A limit cycle is a closed periodic trajectory in the phase portrait [63]). [15] studies the performance of variable structure control on system with backlash at the output. In this work, by describing function analysis, Azenha and Machado find that a second order model variable structure controller cannot avoid a limit cycle yet. Although they claim that this controller can improve position accuracy, the results are not shown in simulations.

As a summary, the describing function method transfers backlash mathematical model from time domain to frequency domain. So we can analyze the system stability by control techniques in frequency domain. But this transfer is only an approximation, thus the unstable limit cycle may still exist even when G and $–N^{-1}$ do not intersect.

Adaptive control

By adaptive control methods, it is not necessary to transfer backlash model from time domain to frequency domain, which is not accurate. The adaptive controllers can estimate the backlash gap size and ensure the asymptotical/bounded stability of the system. Most research works fall in this category[21],[22].

A well-known work in this category is the adaptive backlash inverter[33]. The idea is based on the fact that most of the damage caused by backlash comes from the time needed to traverse the inner gap. A backlash inverse having exact backlash gap size makes traversing the inner gap instantaneous and thus canceling the effect of backlash. The exact gap size can be estimated by this adaptive controller. This backlash inverter has been proved as an effective method in [17]. This work has a test bed with a large backlash gap which was modeled as a static one. In [17], evaluation of robustness with overestimated backlash inverter is performed but the results exhibited an oscillatory motion. Underestimated backlash robustness evaluation was not made. The authors also claimed that the inverter would be limited in devices with slow actuators since the "instantaneous jump" action does not exist in physical processes.

Many other papers use similar methods to this backlash inverter([32],[34],[35],[36]). [32] is a discrete time counterpart of [33]. In these aforementioned backlash inverse algorithms, the backlash inverter can traverse the inner segment instantaneously and thus reduce the effect of backlash. However this is not true in real system. Based on this limitation, a new compact continuous model for backlash inverse is presented[37]. A major contribution of [37] is that a parameter is introduced in the backlash inverter. This parameter gives the designer freedom to tune the time for the motor to traverse the backlash gap. This model may be utilized for both backlashes at the input or at the output.

Another interesting adaptive control method is [2]. In this work, Su, C.Y. developed a continuous backlash-like hysteresis. Using this method he designed a robust adaptive controller. We will detail this in Chapter 4.

Other adapting methods like iterative feedback tuning are also found in the literature [16].

Intelligent control

Intelligent control methodologies such as fuzzy logic[42],[44],[45].and neural networks[47] can also be applied to reduce backlash.[41] designed an adaptive fuzzy system to compensate the delays due to backlash nonlinearity. The fuzzy rules can be simply derived from the static backlash model (Equation 2.4). [43] developed a test bed to verify the performance of a fuzzy controller on backlash. The online implementation shows this controller works well in the system having a very small backlash gap (0.1 rad).

Neural network is another intelligent control method which attracts research interests. [46] contributes a neural network controller on a position system. However, the mathematical model of the system, including the motor's model and the backlash model, are not elaborately explained.

[47]provides a backlash dynamic inversion by using an adaptive neural network compensator. Combined with a backstepping controller, the neural network compensator could eliminate the effect of the backlash at the input of the system in the Brunovsky form.[48] is a discrete time counterpart of [47]. Further details on [47]will be provided in Chapter 4.

Robust control

For backlash control problems, the exact backlash gap size is usually unknown. The adaptive control method and the intelligent control method try to estimate it. Unlike these two methods, robust control utilizes the known upper bound and the lower bound of the backlash gap size.

[39] and [40] develop robust control algorithms to solve backlash in actuator devices and generalized to non-smooth nonlinearities in [38]. The proposed controllers can

confine state variables (hence output) inside acceptable bound. In addition, these controllers do not require backlash inverter. This is an advantage because the inversion of a non-smooth nonlinearity is not easy. However, they require actuators which can output discontinuously when the system states changed. Therefore the actuator must be powerful enough for quick crossing of the inner gap.

Optimal control

As we see, most works in backlash control use static backlash model(Section 2.1.2). It is mathematically convenient to study backlash mode and contact mode as a whole with this model. But this model may be a bit simple in some application as we mentioned in Section 2.1. Hence some researchers work on sandwiched backlash model (Section 2.1.3) recently [26],[27][28],[29]. However, the sandwiched backlash model is difficult to be manipulated mathematically for adaptive control, intelligent control and robust control. To get around this problem, backlash mode and contact mode are separately studied. Optimal control is thus used to design an optimal path for the actuator to traverse the backlash segment in backlash mode.

The detail of this control method is provided in[26]. In this work, Tao use the sandwiched backlash model. It treats the compensation of backlash as a optimal control problem. That is, the harmful effect of backlash is reduced by designing an optimal path for the motor to pass the backlash phase. Along this path, the motor reaches the load fast and free of collision.

The drawback of the optimal solution is it is an open-loop control. So a feedback scheme is carried out to improve the performance of the optimal controller. For this optimal control problem, the solution is usually searched by a computer program. The convergence and error analysis of this solution is made in [29].

This optimal method[28] has been evaluated and compared with normal PID controller in[31] and compared with QFT in [30]. However, [30] and [31] only examine the case with exactly known backlash. The robustness of this controller needs further investigation. We will discuss this robustness in Chapter 4.

Identification

Identification is another direction which attracts less research interest([23],[24],[25]). From Optimal Control subsection, an exact backlash gap is known to be very important. By using the identification method, the gap size can be obtained. In [24], three trained neural networks are used to estimate the backlash segment in hydraulic actuators. [25]worked on an identification algorithm of linear system preceded by a nonlinear system. This algorithm is examined in a Hammerstein system which has an unknown backlash clearance.

In the next chapter, we will develop a controller using the describing function approach. In chapter 4, we will select methods from adaptive control, intelligent control, robust control and optimal control. Then these methods will be tested in a test bed to see whether it can be used in the real world.

Chapter 3

Position Controller of Backlash

3.1 Introduction

In this chapter, we will work on the position regulation problem. Position control is broadly used in applications such as transfer lines and laser cutting or automatic welding. More than just placing the end-effecter at the desired position, the controller should also consider certain specifications like overshoot and rise time. However, the classical controller for position regulating could lead to oscillation when backlash exists between the actuator and the load. This oscillation usually leads to an inaccuracy in positioning. This problem is particularly exacerbated when the required position accuracy is high.

Several kinds of hardware have been developed to have no or little backlash. They are harmonic drives, direct drive motors and anti-backlash gears. But hardware solutions usually cost a lot. Therefore designing a good controller to tackle backlash problem has been an alternative solution to this problem. Much efforts on controller design had been made to mitigate the effects of backlash ([54],[55]). However, most works in the literature paid attention to tracking control, which did not consider the overshoot and rise time of the system. In this paper, we work on position regulating when backlash exists and use overshoot and rise time as the criteria to evaluate the performance.

This chapter is organized as follows. In Section 3.2 we formulate the control problem.

In Section 3.3, we present a control approach to regulate the end-effect point position and to fulfill the specifications as backlash exists. In Section 3.4, we illustrate the control method with simulations.

3.2 Problem Statement

In this chapter we consider the following control problem in Figure. 3.1. The input to the actuator is the control signal, for example, the voltage input to DC motors or electro-hydraulic actuators. The output is the position of the actuator. Assume Equation 3.1 is the actuator's transfer function,

$$G(s) = \frac{b_{n-r}s^{n-r} + b_{n-r-1}s^{n-r-1} + \ldots + b_0}{a_n s^n + a_{n-1}s^{n-1} + \ldots + a_0} \tag{3.1}$$

where n > 0 is the order of the characteristic equation of G(s), r is the relative degree of G(s), a_i , b_j are coefficients of G(s), where i =0, 1,…, n and j=0,1…, n-r.

Since the model of backlash shown in Figure 2.1 and the mathematical equation 2.4 is commonly used, we used this model in this chapter.

In the current work, the control objective is to make the step response of the system satisfy overshoot and rise time specification when backlash exists. The following assumptions are also made.

(i) Backlash characteristic is roughly known, that is, c and k have been estimated by experiments.

(ii) The inertia of the load is assumed to be constant and small.

Figure 3.1 Position Control System with Output Backlash

Assumption (i) means that the nominal values of c and k are known. These nominal values may not be exact but this drawback can be overcome by the backlash compensator later on in this chapter.

Based on assumption (ii), the inertia of the load, together with inertia of gears could be ignored and assumed to be zero in this chapter. An example of the output backlash is shown in Figure 3.2. Since the inertia of the camera is trivial comparing to the motor power, we ignore its dynamic behavior.

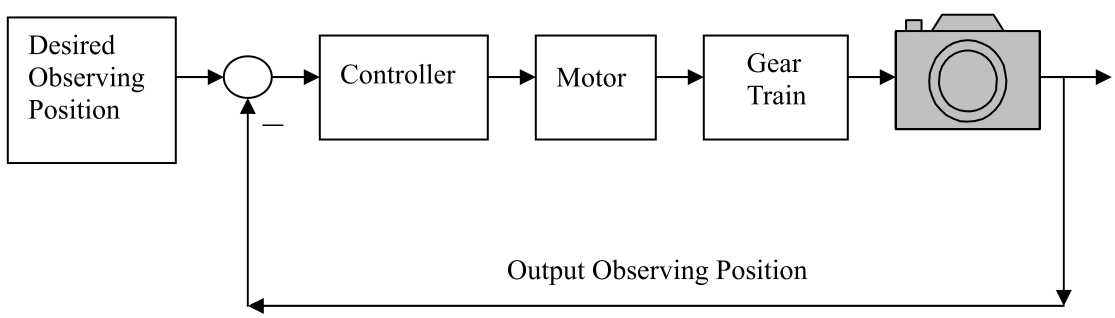

Figure 3.2 A camera Inspection System. Backlash in gear train will lead to great position error if the camera takes pictures from the space.

3.3 Design of Control System with Backlash

Actuator position control is a very classical control topic. The controller design can be found in many papers and text books. However, these controllers' performance may not be retained when backlash has been introduced into the system. In this section, we will show a control idea to retain the controller's performance.

3.3.1. Controller design for nominal plant

To retain the controller's performance, a common method is to pre-compensate the signal before it enters the backlash ([41],[47],[54]). Although these designs may do well in tracking control, they did not consider requirements such as overshoot and rise time which are important in position control. In this chapter, we will also implement a backlash compensator to mitigate the signal distortion due to the backlash. Moreover, the corresponding control loop (Figure 3.3) could help us to design the controller and the compensator separately. In Figure 3.3, C is the controller properly designed with the assumption that backlash does not exist. G is the actuator's nominal transfer function, P is the backlash compensator and N represents the backlash nonlinearity. Thus, the controller design can be separated into two steps. The first step is to design the controller with the classical position control techniques. This step assumes that the backlash does not exist. The second step is to pre-compensate the signal before it passes through backlash unit. Then, connecting these two independent designs, the objective is to achieve a desired step response even if a backlash exists.

In this subsection, we only study the nominal plant of the motor. The robustness will be analyzed in next subsection. In practice, however, a backlash compensator does not exist between the motor and the backlash. To overcome this problem, let us make an addition assumption as follows.

Assumption iii): there exists a solution for compensator P which could make y/z almost equal to one.

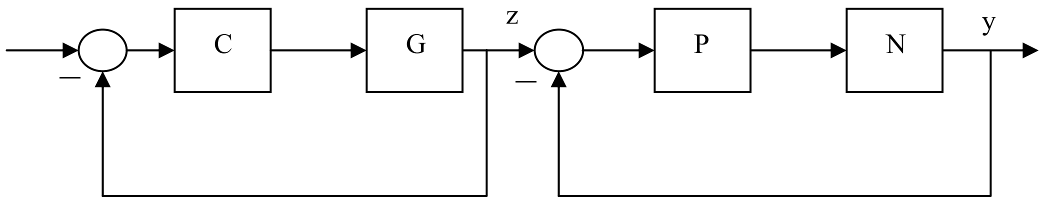

Figure 3.3 System Loop of Controller and Compensator. C is the controller properly designed with the assumption that backlash does not exist. G is the actuator's nominal transfer function, P is the backlash compensator and N represents the backlash nonlinearity.

Here we apply Assumption iii) only for subsection 3.3.1 and subsection 3.3.2. In subsection 3.3.3, the backlash compensator will be designed to relax this assumption. Based on assumption iii), we could have a modified control system loop as Figure 3.4. The transfer function of the system in Figure 3.4 can be denoted as

$$H(s) = \frac{C(s)G(s)P(s)N(A)}{1 + P(s)N(A) + C(s)G(s)P(s)N(A)} \tag{3.2}$$

where N(A) is the describing function of the backlash; A is the amplitude of the sinusoidal signal entering the backlash. For notation simplicity, in the sequel we will write C in short for C(s), G in short for G(s), N in short for N(A) and so on. Equation 3.2 is stable assuming that C(s) and P(s) are chosen so as to make the respective closed loops of Figure 3.3 stable.

Physically a compensator does not exist between the plant and the backlash. To avoid this, an equivalent system loop (Figure 3.5) to Figure 3.4 is used, which implements a feed-forward controller, where

$$\begin{aligned} C_1 &= PG^{-1} + CP \\ C_2 &= PG^{-1} \end{aligned} \tag{3.3}$$

By using controllers C_1 and C_2, the compensator does not appear between the motor and the backlash. Thus the backlash pre-compensation can be realized physically.

Remark 1: with the help of Equation 3.3, the control loop in Figure 3.5 enables us to design backlash compensator and the motor controller separately. Hence, the performance specifications can be retained in the system with backlash. And these specifications may be more important in position control, compared to tracking control.

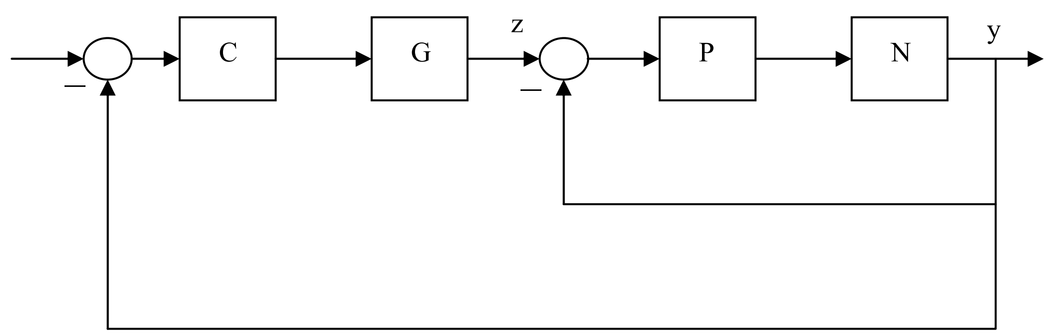

Figure 3.4 Modified System Loop of Controller and Compensator based on Assumption iii

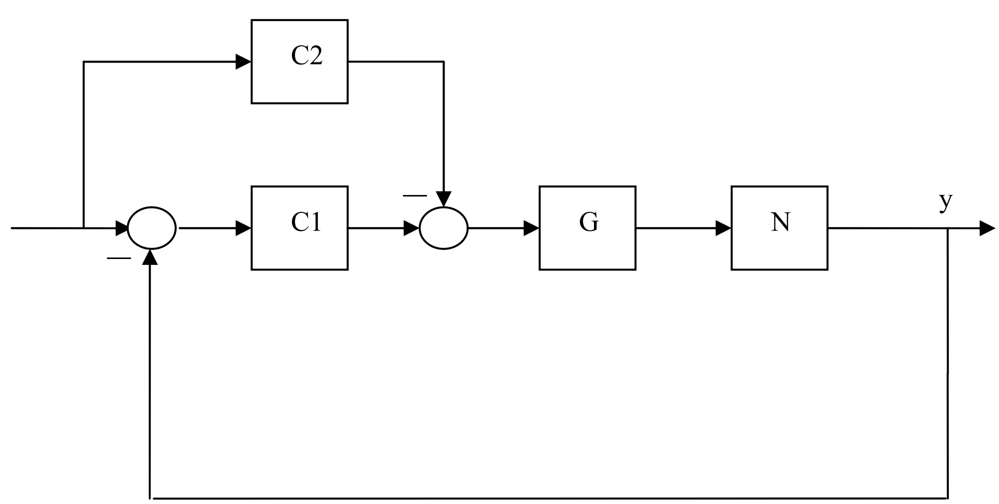

Figure 3.5 Control System Diagram with feedforward controller C2

In fact, feed-forward loop is a common method which is often used in industrial applications. This method could be regarded as a compensation loop such that the system response is improved while feedback controller gain would not be as high as the case that only feedback controller is used. But in most industrial applications, the feed-forward and feedback controller gains are tuned by trial-and-error. In this work, we also use this method to set feed-forward and feedback gains such that backlash in the system could be compensated.

Another problem ensues with design of C_1 and C_2 is that the inverse of the actuator's transfer function are usually improper, that is, the order of the nominator is more than the order of the denominator. Thus the inverse of G(s) is not realizable in the real world. This problem is solved by placing a filter Q(s) in the sequel of C_1 and C_2. A successful design of C_1 and C_2 highly depends on the design of Q(s). Due to its importance, this filter has been extensively studied[58]. And the research results show that Q(s) should be a low-pass filter. A typical kind of Q(s) is Butterworth filter, and the robustness is improved by increasing the order of Q(s)[57], typical forms of Q(s) are:

$$Q(s) = \frac{\sum_{i=1}^{n-r} c_i (\tau s)^i + 1}{\sum_{i=1}^{n} c_i (\tau s)^i + 1} \tag{3.4}$$

where n is the order of the denominator of G(s), r is at least the relative degree of G(s), τ is the cutoff frequency and c_i is the constant coefficient.

3.3.2. Robustness Analysis

We note that the design of C_1 and C_2 needs the inverse of the motor's transfer function. This may lead to instability since G(s) is not the exact motor model. Hence we should make extra effort to deal with the model uncertainty. We used the multiplicative uncertainty in our analysis. The control system loop is shown in Fig. 6, where the motor is modeled as $G(1+\Delta G)$, G is the nominal plant, and $\|\Delta G\| \le \rho$ is the uncertainty multiplier bounded whose norm is bounded by a positive value ρ. Thus, the transfer function of the closed-loop control system is

$$H^*(s) = \frac{(C_1 - C_2)G(1+\Delta G)N}{1 + C_1 G(1+\Delta G)N}$$
$$= \frac{C^* G(1+\Delta G)PN}{1 + (1+\Delta G)PN + C^* G(1+\Delta G)PN} \qquad (3.5)$$

where $C_1 = PG^{-1} + C^* P$. C^* could be designed by using robust controller design techniques. Compared to (3.2) an extra term ΔGPN appears in the denominator of $H^*(s)$. Therefore, transfer function (3.5) may not be stable. To analyze the stability of (3.5), an equivalent system loop of Figure 3.6 is introduced (Figure 3.7). From Figure 3.7, we see that $H^*(s)$ can be thought as a transfer function of the control system loop suffering from measuring noise, where d is a measurement noise such that $d = \Delta G \cdot y$,

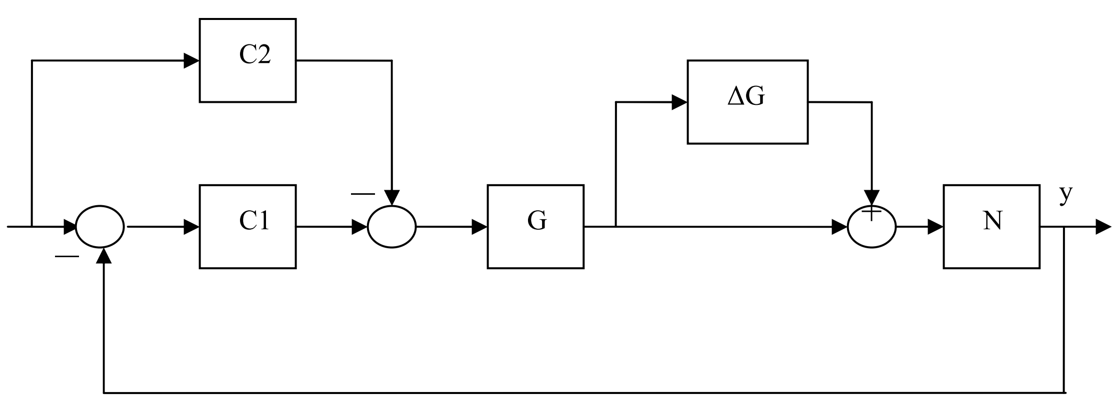

Figure 3.6 System Diagram with Multiplicative Uncertainty of the plant, $\|\Delta G\| \le \rho$

y is the output of the system. Obviously, the bounded measuring noise can be left for the backlash compensator to handle. Therefore, even if there is bounded model uncertainty, the system could be still stable when related controllers are well designed.

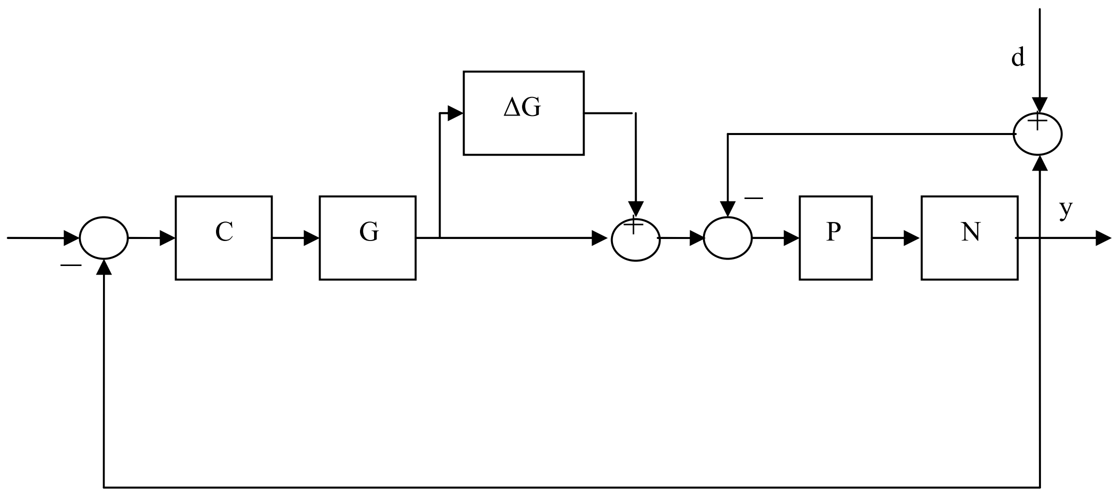

Figure 3.7 System Diagram with Multiplicative Uncertainty and Measuring Noise $d = \Delta G \cdot y$

3.3.3. Design of Backlash Compensator

In Equation 3.3, a virtual backlash compensator is used to formulate C_1 and C_2. The method of designing this compensator is discussed in this subsection.

Some researchers have studied Linear PID backlash compensator ([45]and[59]). Robustness of PID controller is reported in [60]. Reference[52] developed an anti-backlash controller which was equivalent to a P controller. It said that proportional gain should be large enough to make the transfer function of the anti-backlash controller loop equal to one. In [58] backlash was decomposed into a linear part and a disturbance part. The disturbance part was attenuated by a disturbance observer. This inspires us to choose the integral control to mitigate the disturbance effect of backlash nonlinearity. And since backlash is a piece-wise continuous nonlinearity, the mathematical function of backlash is non-differentiable at some points, and using differentiation may lead to great computing error. Hence we set differential control gain to zero. Therefore, we use PI controller as our backlash compensator:

$$P = k_p + \frac{k_i}{s} \tag{3.6}$$

To ensure the limit cycle does not exist in the compensation loop, describing function method is used: suppose the input to backlash is $A\sin(\omega t)$, the describing function of backlash is $N(A) = a + bj$ [45], where

$$a = \frac{Am}{\pi}\left(\frac{\pi}{2} - \sin^{-1}\left(\frac{2c}{A} - 1\right) - \left(\frac{2c}{A} - 1\right)\sqrt{1 - \left(\frac{2c}{A} - 1\right)^2}\right) \tag{3.7a}$$

$$b = \frac{4mc}{\pi A}\left(\frac{2c}{A} - 1\right) \tag{3.7b}$$

Plot of -1/N(A) with c=2 fixed is shown in Figure 3.8. Since Nyquist plot of P is always a straight line crossing the point (kp, 0) and perpendicular to the real axis, we know that the Nyquist curve of P and -1/N(A) will not intersect each other. Thus, the existence of limit cycle is not possible.

3.4 Simulation

To verify the performance of the control scheme derived from the virtual backlash compensator, a simulation is presented in this section. The compensator is designed by using describing function[63]. System's step response specifications are

1) Overshoot M< 20%

2) Rising time ts <1 seconds

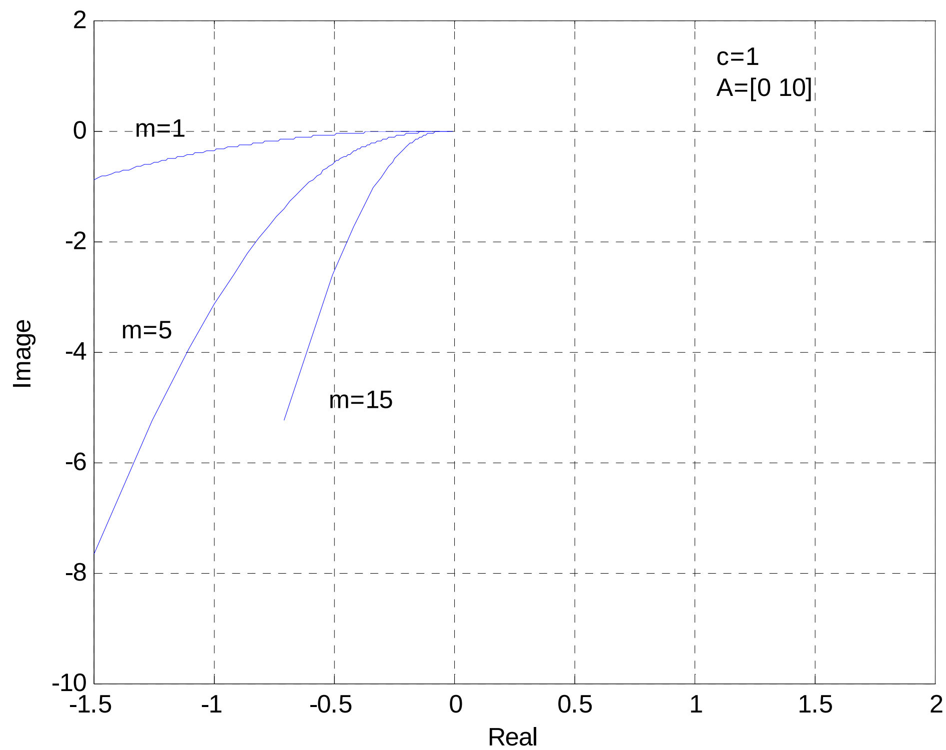

Figure 3.8 Plot of Inverse of Backlash Describing Function, with the gear ratio m=1,5,15, c=1, A ranges from 0 to 10

Remark 2: It should not be considered that the proposed method can only use overshoot and rising time as the specifications. Other specifications such as settling time can also be used. In fact, selection of specifications only depends on the requirements of design.

DC motor is used as the actuator, Transfer function of the DC motor is (3.8):

$$G(s) = \frac{\theta}{V} = \frac{1/K}{s(s\tau_m + 1)}, \tag{3.8}$$

where $\tau_m = RJ/K^2$, J is the moment of the rotor inertia, K is the armature constant, R is the electric resistance, θ is the position of shaft and V is the input voltage.

The values of motor parameters and backlash parameters are shown in Table 3.1.

Table 3.1 DC Motor and Backlash Parameters

J (Nm)	3.2284E-6
K (Nm/A)	2.74E-2
R (ohm)	4
m	1
c (degree)	10

The controller design procedure consists of two steps:

Step 1: We We use PID as the controller. The formulation is as follows,

$$C = K_{pm} + K_{dm}s + K_{im} / s \, , \quad (3.9)$$

to control G(s) assuming that backlash does not exist in the system loop, where K_{pm}=5.5, K_{dm}=1.2, K_{im}=12.

After the gains of PID controller are properly tuned, the requirement specifications are fulfilled (Figure 3.9). However, the performance degraded when the backlash exists (Figure 3.10). In Figure 3.10, we note that backlash nonlinearity introduced a limit cycle into the system loop. This unexpected nonlinear phenomenon caused oscillation when the system is in the steady state. Since the distance from the end-effecter to the rotational center may be large, this angular position oscillation would severely decrease the end-effect position accuracy. To avoid this phenomenon, we have to design a backlash compensator to mitigate the effect of backlash.

Step 2: Based on the analysis of anti-backlash controller in [52], we know that Proportional controller gain should be large. And Integral controller gain should be also large to reject constant disturbance in the system loop. So, we use PI compensator as $P(s) = k_p + k_i / s,$ where k_p=100, k_i=30.

And

$$Q(s) = \frac{1}{(s\tau)^2 + s\tau + 1}, \text{ where } \tau = 0.003 .$$

The performance with proposed control method(Equation 3.3) is shown in Figure 3.11 and 3.12. In Figure 3.11, the dashed line is the step trajectory when nothing has done to deal with backlash. By using the proposed control scheme, the step response is greatly improved. This step response is compared with the response without backlash in Figure 3.12. From Figure 3.12 the step output has been almost restored to the desired response when backlash does not exist.

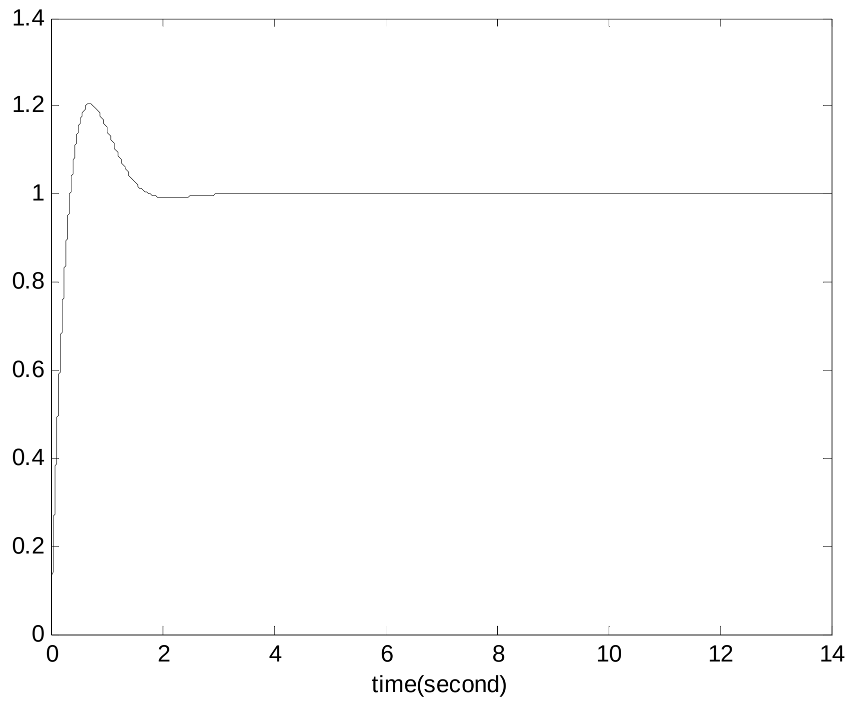

Figure 3.9 Step Response without Backlash at the output

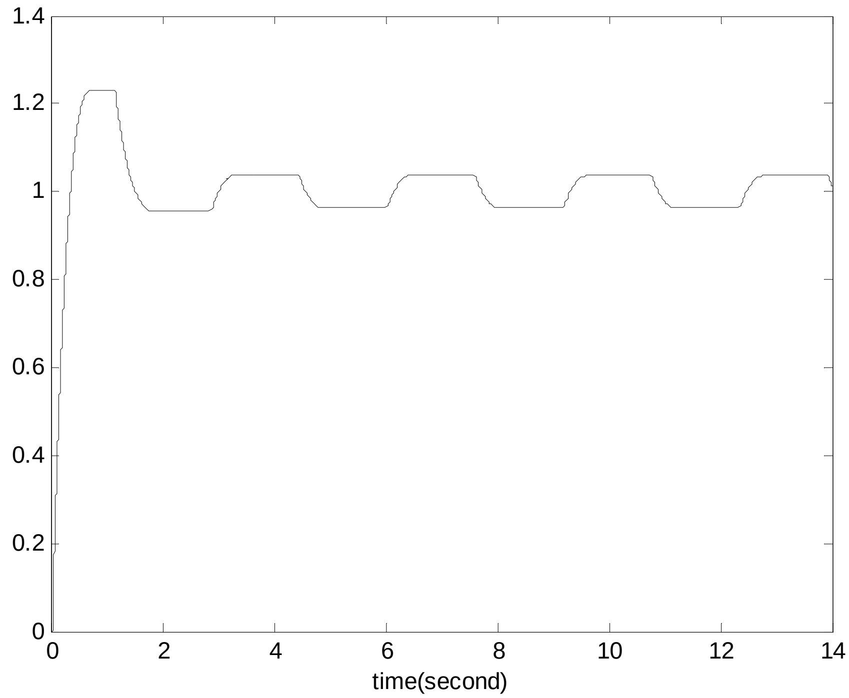

Figure 3.10 Step Response with Backlash at the output

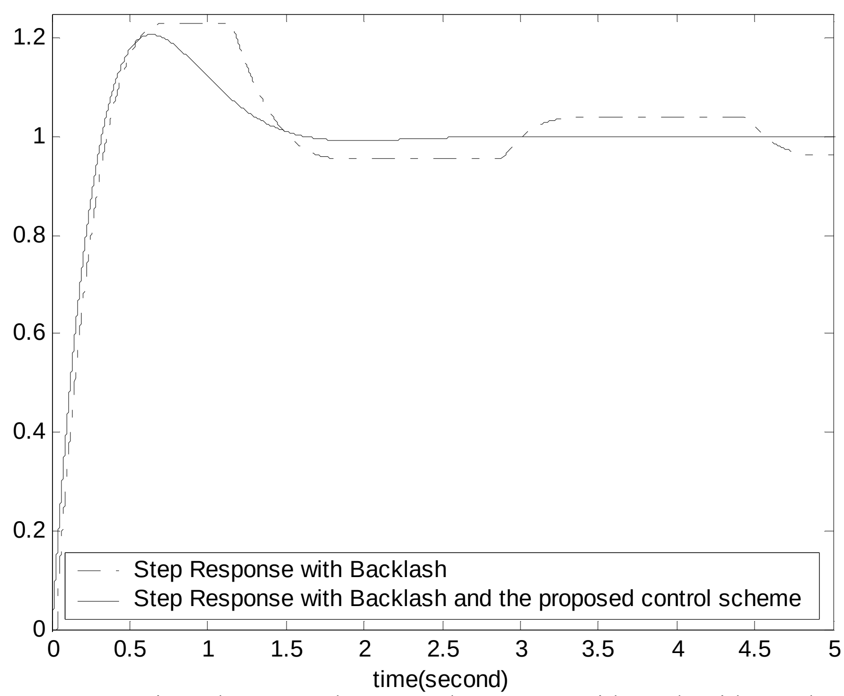

Figure 3.11 Comparison between the control systems with and without the proposed backlash controller.

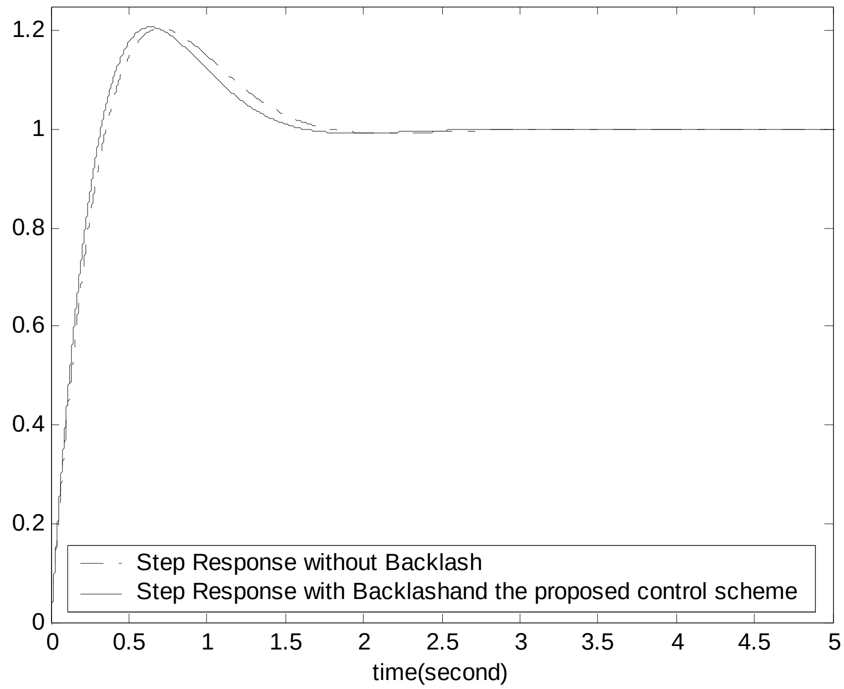

Figure 3.12 Comparison between the step response without backlash and the one with backlash and the proposed controller

3.5 Conclusion

In this chapter, a position control method is designed for systems with backlash by assuming that backlash parameters are known and the inertia of the load is constant. With this method, the design procedure can be divided into two steps. The first step is to design position controller to achieve desired response. This step assumes that backlash does not exist. The second step is to design a backlash pre-compensator to attenuate the backlash nonlinearity. Then using the proposed controller to combine these two, we retained step response as the first step. Robustness of this method, i.e., uncertainty of plant and noise, is also analyzed in this work. The effectiveness of this method is illustrated in simulation results. However, the inertia of the load is practically not always constant. We would work on the case that the load is disturbed by an external step signal in the future.

Chapter 4

Experiment Evaluation of Backlash Controllers

4.1 Introduction

Since high performance is required in certain occasions, control engineers and researchers have to look for more sophisticated solutions to backlash. As we introduce in Chapter 1, a number of approaches to control systems with backlash are reported. However the efficacies of them are mainly proved in simulations; only a few backlash control approaches have been evaluated in real linear systems. Tao's backlash inverter[33] is tested in a linear motor-load position control test bed[17], in which the backlash is as large as 90 degrees. By analyzing the results[17], Dean reports the output error is high when backlash is underestimated. An adaptive fuzzy-neural network controller [61] is evaluated in a motor position control with a constant load. The results show that the oscillation is reduced significantly when backlash is properly compensated. A PID control method [20], a nonlinear observer compensator [22], a fuzzy logic controller [41] and a neural network [46] are tested in real experiments respectively in their papers.

These physically tested approaches are developed for linear systems and tested with constant load. Their performances are nice. Nevertheless the approaches for nonlinear systems[2] (such as in [1] and [2])are not implemented in real system. These nonlinear systems usually have different loads at different system states. For example, at different angle position, a pendulum will have a different torque at its rotating joint. Moreover, control of nonlinear system is more complex than linear system.

In addition, most of these backlash algorithms are derived using a static backlash model. This model is different from the backlash in the gear meshing. So this experiment presented in this chapter will also investigate the efficacy of this backlash model.

The organization of this chapter is as follows. In Section 4.2, we present the details of the experiment system. In Section 4.3 we briefly introduce five types of controllers and select appropriate controllers to be implemented. The experiment results are shown and discussed Section 4.4. Section 4.5 concludes this chapter finally.

4.2 Experiment Hardware

In this section, test platform, motor and computer system for this experiment are described respectively in subsection 4.2.1, 4.2.2 and 4.2.3. In subsection 4.2.4, an analysis of the test platform mechanism is carried out. This gives us an insight to backlash characteristics of this test platform before we move on to the control algorithms.

4.2.1 Test Platform

We will test the backlash control algorithms in one of knee joints in NUSBIP-I (first design) to see whether they can help reduce the effects of the backlash in this robot.

NUSBIP-I is a humanoid robot which has size of an 8-10 years old child. This robot

[2] Here nonlinear systems refer to systems with nonlinear plants, not hard nonlinearities like backlash.

suffered from significant backlash in its joints. Thus the control objective of this experiment is to reduce the backlash effect in one joint. We choose one knee joint because the functions of knee joints are very important in human walking cycle. Compared to the hip joint which has 3 D.O.F., the one D.O.F. knee joint is simple and reduces the complexity of this experiment.

The system plant is the leg presented in Figure 4.1. The femur (upper part of a leg) is fixed to three-ply boards by two screws. The three-ply boards are fixed on the desk by two clamps. The ankle joint is fixed by tapes. This can avoid inertia varying of the part below the knee joint. Thus the system is a one-link pendulum. The mathematical model of the system is Equation 4.1.

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{1}{T}x_2 + mlx_2^2\cos(x_1) - mgl\sin(x_1) + bu \end{aligned} \tag{4.1}$$

where m is the mass of the link, T is the time constant of the motor, l is the length of the link, b is the control coefficient. x_1 and x_2 are the state variables which represents of the position and the velocity.

To get the positions of the motor shaft and the leg, we mounted two encoders, one encoder at the motor; the other encoder at the knee joint.

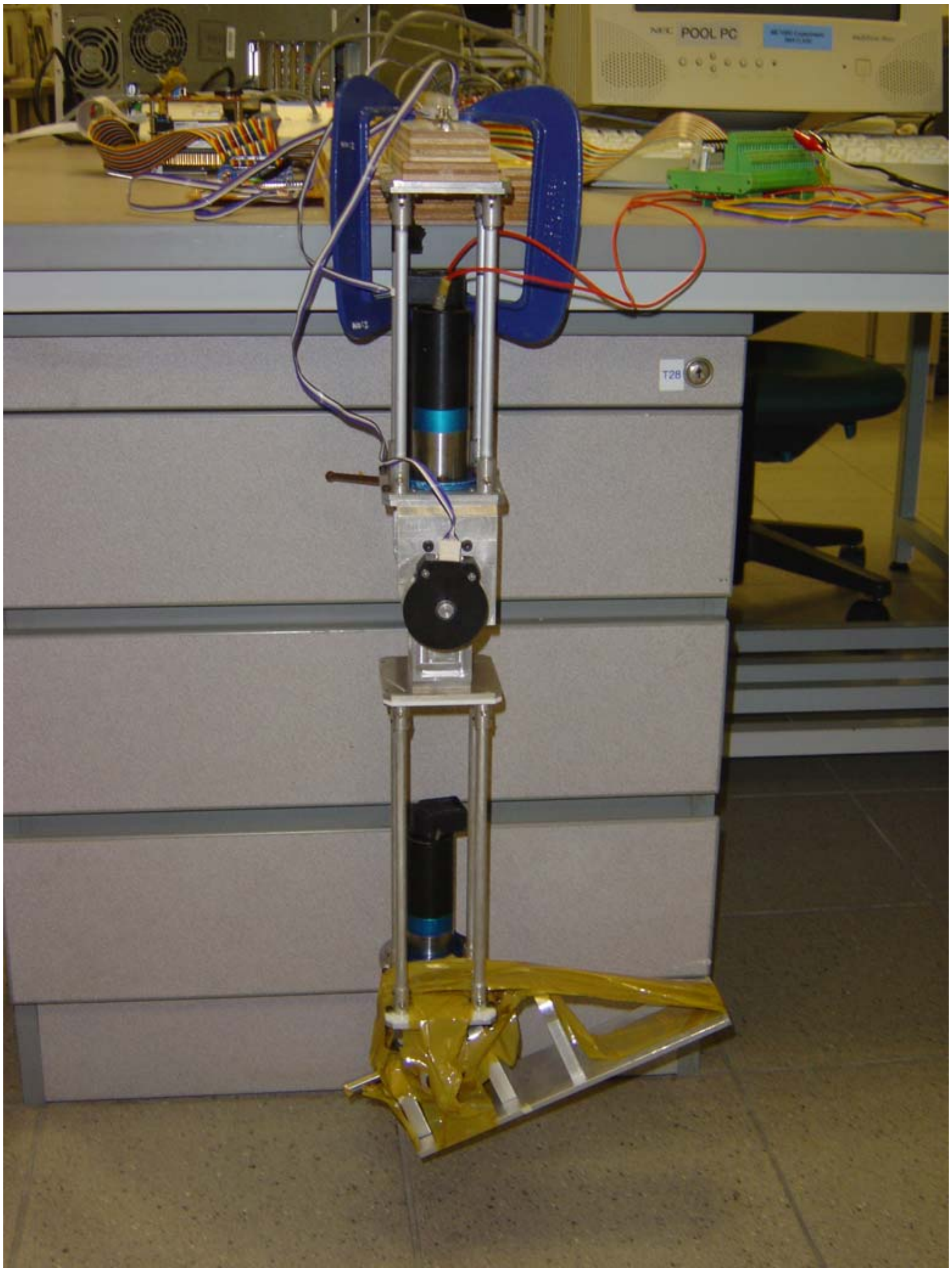

Figure 4.1 One Leg of NUSBIP-I

4.2.2 DC Motor and Servo Amplifier

In this experiment, we used a DC motor (Faulhaber, series 38/1). The selected features are listed in Table 4.1.

Table 4.1 Selected features of the DC Motor

Backlash at no-load	$\le 1^\circ$
Reduction ratio	134:1
Shaft Load, max	Both $\le 300N$ in radial and axial
Recommended Max Current Input	3.99 A
Max Output Torque(Continuous)	10 Nm
Max Output Torque(Intermittent)	15 Nm

The length from the center of rotation to the center of the gravity of the shank is 0.4 m, its weight is 2.115 kg (counting in the motors mounted at the ankle joints). Because robot walking does not need a very fast rotation of the shank, the power of this motor is enough for this experiment.

This motor does not have a current sensor. Thus the output torque can not be easily controlled because of lack of current feedback. The only way to read the current data is through one pin of the servo amplifier (Copley Controls Corp, Model 413ce Since this current is controlled by the voltage command input to the servo amplifier, an identification of voltage-to-current ratio is carried out. With input voltage range 0.2~1.3 V, this ratio is 2.8 in our experiments. Hence the current in our experiments is calculated from the command voltage multiplied by this ratio.

4.2.3 Central Process Unit

Many control systems use DSP or PC as their processor to generate command voltage. But DSP is not fit for the present experiments because designers have to spend much time on the instruction periods and peripheral devices like keyboard and display. Average PCs are easier to use. But considering this research result may be used in a real robot and the limited space in the robot, PC104 is selected as an alternate. PC104 is small and powerful so it is convenient for such an experiment. Moreover, DAQ card (Diamond-MM-32-AT in this experiment) and motion controller cards can be easily stacked up through ISA bus connector (Figure 4.2).

The system architecture is shown in system diagram (Figure 4.3). The experiment setting is shown in Figure 4.4.

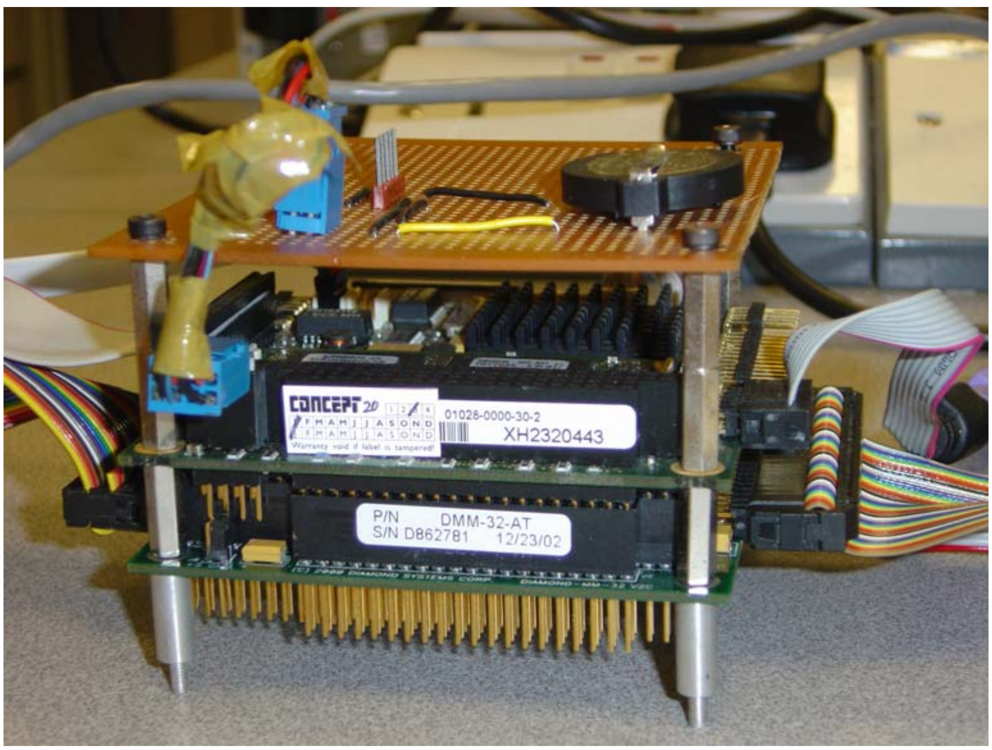

Figure 4.2 Computer System. Power supply circuit is at the top level; PC104 is at the middle level; Dmm-32-AT DAQ card is at the bottom level; more DAQ cards can be stacked at the bottom)

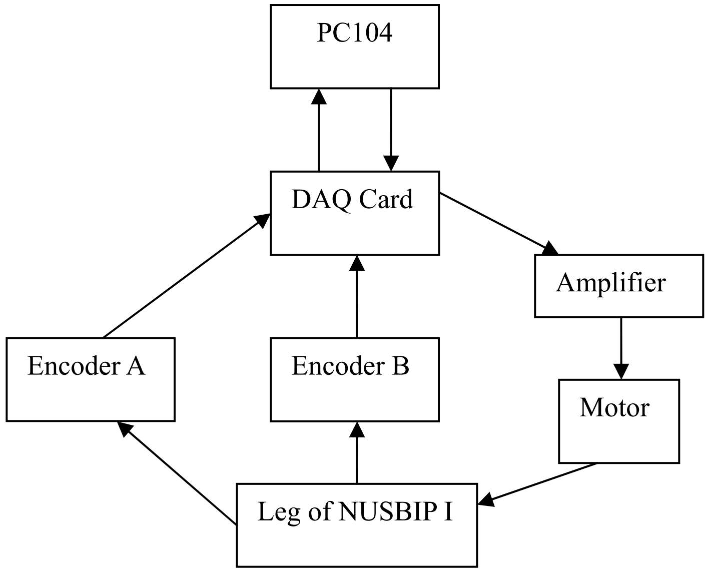

Figure 4.3 Diagram of overall control system architecture. Encoder A and B are mounted at the end of the motor and the knee joint respectively. The timing circuit between encoders and the DAQ Card is not shown.

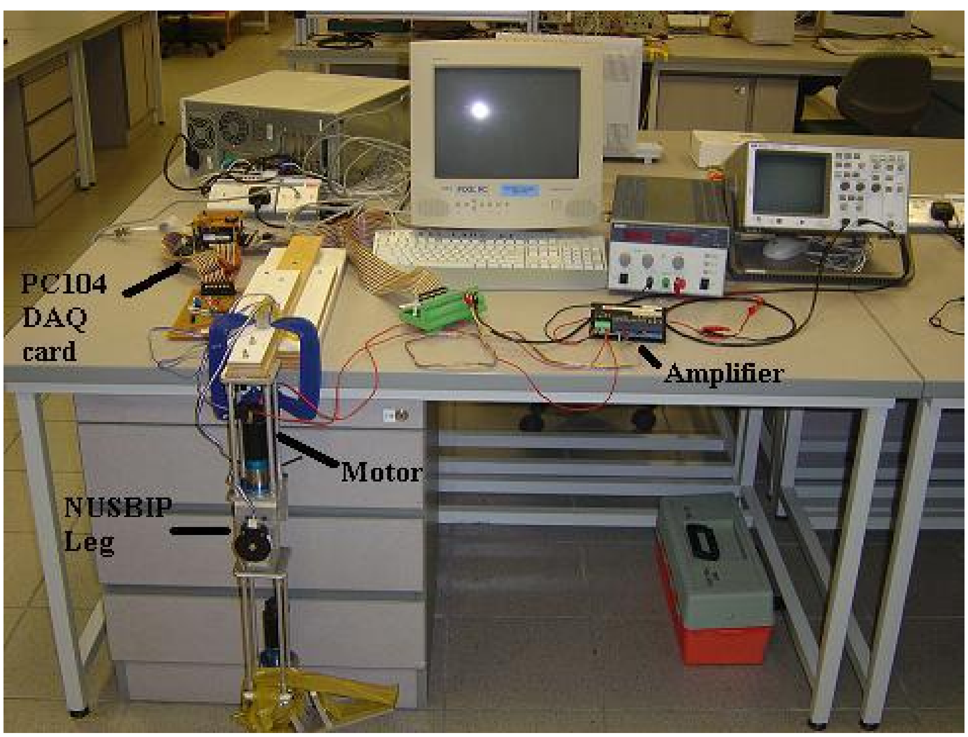

Figure 4.4 the Real Control System

4.2.4 Analysis of Mechanisms

In this subsection we will analyze the mechanism of the knee joint. This can help us locate where the backlash exists and understand the characteristics of the backlash. Such knowledge will help us appreciate the controllers' performance described in Section 4.3.

The mechanism of NUSBIP-I knee joint is shown in Figure 4.5. A shaft which connects the motor shaft and the gear is fixed in a ball bearing. The ball bearing is, in turn, fixed to a plate. This plate is fixed on the frame by four screws (two on each side.) The seam between the frame and the plate is around 1-2 millimeters and can be adjusted by the four screws. This adjustment would affect the gear's vertical axis position and the inter tooth space, hence the gap size of the backlash. Moreover the screws could be loosed when the leg is rotating and the inter tooth space will change. Thus the backlash is not a constant value. Tests show this mechanism has a backlash of 1.5 to 3 degrees in the gear meshing and the size of the backlash gap may change from time to time.

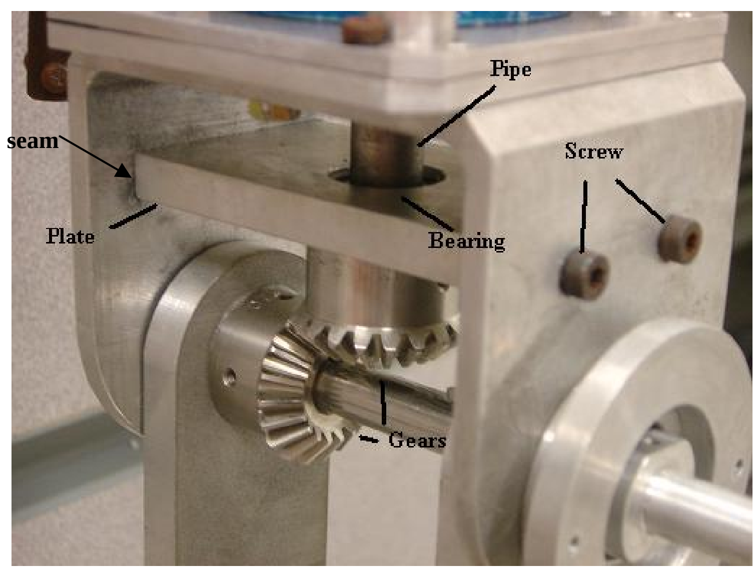

Figure 4.5 Mechanism of NUSBIP-I knee joint

The biggest backlash happens at the motor mount, i.e., in the coupling of the motor shaft and the pipe which connects the shaft and the gear. The inner hole of this shaft should have clutched tightly the shaft. But due to the oversize radius and abrasion of the inner hole, a sliding happens between them. Hence backlash here is about 23 degrees. The phenomena are shown in Figure 4.6. In this figure, we see there are screw holes on the pipe. It is supposed to put screws here to prohibit the pipe sliding on the motor shaft by the friction. However, this works only if the load is small enough and the system does not oscillate. But we will see in next chapter these two conditions are not satisfied in this experiment.

Another backlash is inside the motor, but it is reported less than one degree (see Table 4.1).

As a summary, this knee joint suffers from a gradually changed small backlash, a less-than-1-degree backlash a friction force and a great backlash about 23 degrees. For simplicity, in this experiment 23-degree backlash at the motor mount is assumed not to increase due to abrasion.

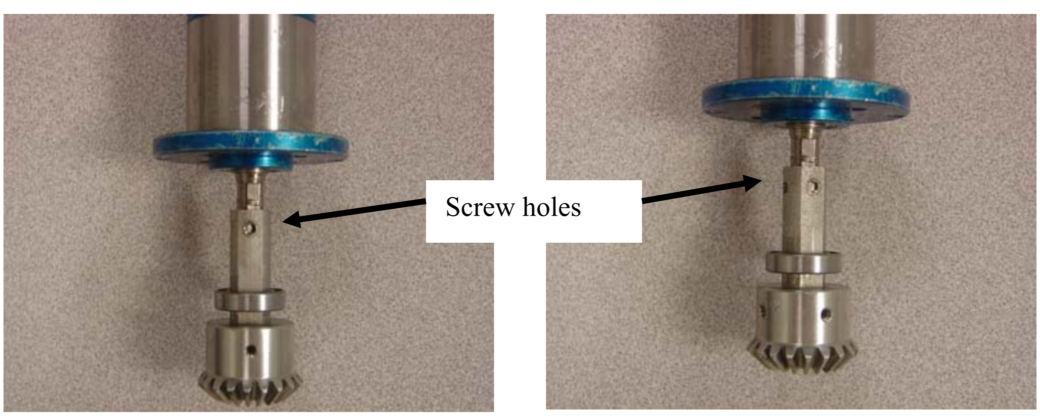

Figure 4.6 Backlash at the motor mount. This picture is to show qualitatively the backlash size at the motor mount.

4.3 Control Algorithms

In this section we select and describe briefly five types of control algorithms for nonlinear systems. These types are PID, robust control[1], adaptive control[2], neural network control[47] and optimal control[28]. Except PID, these algorithms are selected according to the categories of backlash control methods we introduced in Section 2.2.

4.3.1 PID Control

In industry nowadays, classical linear control techniques such as PID control are very mature and they are common solutions to many control problems. It has been used in many fields in technology, from mechanics and pneumatics to microprocessors and integrated circuits. Despite a linear controller, it can control some nonlinear system[64]. In the literature[19], backlash effects can also be reduced by this controller. To do this, PID controller is usually tuned slow enough such that the motor shaft does not collide to the load severely[31].

In various textbooks, PID control algorithm is usually described as

$$u(t) = K_p e(t) + K_i \int_0^t e(\tau) d\tau + K_d \frac{de(t)}{dt} \qquad (4.2)$$

where e(t)=r(t)-y(t), r(t) is the reference signal, y(t) is the output of the system. The control signal is thus a sum of three terms: the P(proportional) term which is proportional to the error e(t), the I(integral) term which is proportional to the integral of the error e(t), and the D(derivative) term which is proportional to the derivative of the error e(t). The controller parameters are proportional gain *Kp*, integral gain *Ki*, and derivative gain *Kd*. The proportional, integral, and derivative parts can be interpreted as control actions based on the past, the present and the future information. The effects of increasing these parameters are listed in Table 4.2. Usually a control engineer begins with tuning of proportional gain, then derivative gain, and integral gain at last.

Table 4.2 Effects of PID control parameters

Close-loop Response	Rise Time	Overshoot	Settling Time	Steady State Error
Kp	Decrease	Increase	Small Change	Small Decrease
Ki	Decrease	Increase	Increase	Great Decrease
Kd	Small Change	Decrease	Decrease	Small Change

According to [31], Kp, Ki and Kd should be tuned so that the motor speed is slow enough to avoid severe collision but swift enough to traverse the backlash gap. This is not simple. For one thing, the desired motor speed should be related with the gap size. However, PID controller does not tell us the relationship. So the nature of implementing such a controller is a trial-and-error tuning. In this experiment, we have tuned these parameters many times. Although the parameters may not be optimal, they work well in that (i) no severe oscillation happens; (ii) the output is acceptable.

4.3.2 Robust Control

In [1], a robust controller is designed so the backlash mode is avoided. Its superiority to PID is that the information on the bound of uncertain parameters can be utilized. The general description of plant considered in [1] is

$$\dot{X} = h(X) + \Delta h(X) + g(X)u + d(X) \tag{4.3}$$

where

$X \in R^n$	state vector;
$u \in R$	plant input;
$g(X) \in R^n \to R^n$	smooth state-input map;
$h(X) \in R^n \to R^n$	smooth function describing the intrinsic plant nonlinearity.

The uncertain terms $\Delta h(X)$ and $d(X)$ account for parameter variations and external disturbances, respectively. In this experiment, $d(X)=0; \Delta h(X)$ lumps the uncertainty of the pendulum length, i.e. $m\Delta l x_2^2 \cos(x_1) - mg\Delta l \sin(x_1)$; the uncertainty of the backlash gap size is lumped in the controller (see [1]for details); the gear ratio m is known exactly.

Since the controller in this paper is for position regulation, here we modified it to make it suitable for tracking control. The modified parts are only as follows, described in Equation 4.3.

$$w(X) = \frac{\partial S}{\partial e} h(X)$$

$$r(X) = \frac{\partial S}{\partial e} g(X)$$

$$\delta(X) = \frac{\partial S}{\partial e} (\Delta h(X) + d(X) - \dot{X}_d)$$

where e=X-Xd is the error vector. Xd is a reference signals vector for the plant to follow. The other parts of the controller are left intact. The proof of the modified controller is the same as the proof procedure of [1].

4.3.3 Adaptive Control

Unlike robust control, an adaptive controller is developed in [2] which can estimate the unknown parameters. However, this work does not estimate the backlash gap size. It only uses a disturbance to integrate the backlash effects. To understand how this integration is done, [2] presents a continuous static backlash model. This backlash model is described as Equation 4.4.

$$\frac{du}{dt} = \alpha \left| \frac{dv}{dt} \right| (mv - u) + B_1 \frac{dv}{dt} \tag{4.4}$$

where v is the backlash input, u is the backlash output, α >0and B_1>0 are model parameters. The simulation of this model is shown in Figure 4.7. Equation 4.4 is solved explicitly for v piecewise monotone. The solution is Equation 4.5

$$\begin{aligned} u(t) &= mv(t) + d(v) \\ d(v) &= [u_0 - mv_0]e^{-\alpha(v-v_0)\operatorname{sgn}\dot{v}} + e^{-\alpha v \operatorname{sgn}\dot{v}} \int_{v_0}^{v} [B_1 - m]e^{-\alpha\varsigma\operatorname{sgn}\dot{v}} d\varsigma \end{aligned} \tag{4.5}$$

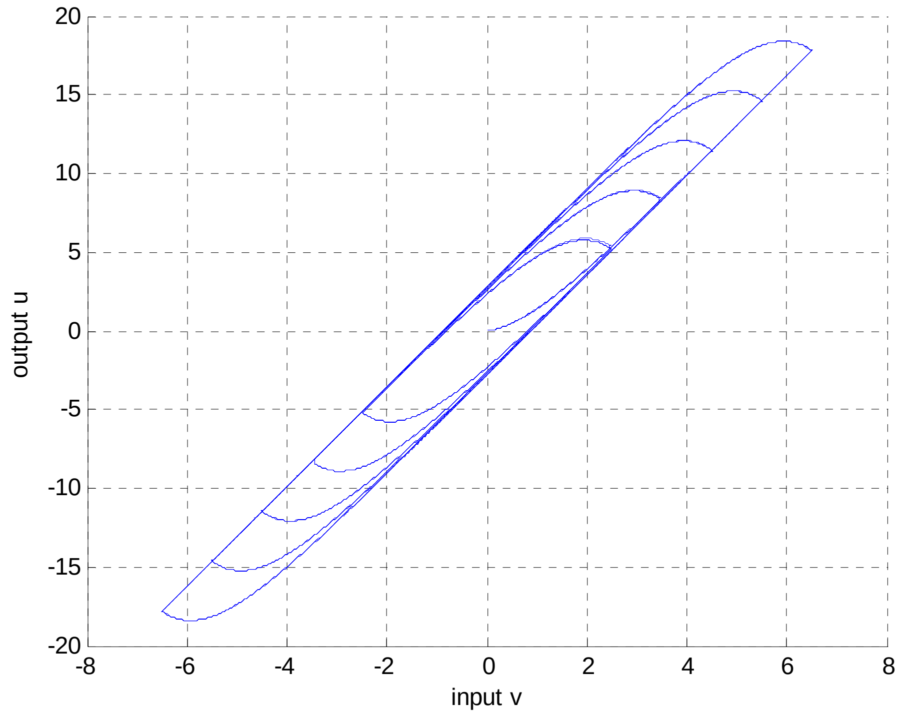

Figure 4.7 Su, C.Y.'s backlash model with 2 units' backlash gap

where u_0, v_0 are initial value of backlash output and input. Therefore, the static backlash model becomes a linear equation adding a lumped disturbance. By assuming $\| d(v) \| \le \rho$ is known, the control problem is much simplified to a system with an input adding a bounded disturbance. A general system is described in Equation 4.6

$$x^{(n)}(t) + \sum_{i=1}^{r} a_i Y_i(x(t), \dot{x}(t) \ldots x^{(n-1)}(t)) = bmv(t) + bd(v(t)) \tag{4.6}$$

where n is the order of the system, a_i is system coefficient and b is the control coefficient. Afterwards an adaptive sliding mode controller ([63]) has been designed for this system.

4.3.4 Intelligent Control

Many approaches have been well developed for intelligent control such as Artificial Neural Networks, Fuzzy, Reinforcement Learning and Expert systems. Here, we introduced an artificial neural networks control method for backlash reported in [47]. In this subsection, we will briefly introduce why neural networks can be used in controller design first; then we will describe the scheme of the control system in [47].

One kind of neural networks may be written in terms of vectors as Equation. 4.7

$$y = W^T \sigma(V^T x) \tag{4.7}$$

where σ could be any continuous sigmoid function.[62]; W and V are weight vectors; x is the input vector.

Theoretically, any sufficiently smooth function can be approximated arbitrarily closely on a compact set using a two-layer neural network with appropriate number of neurons [62]. This is often called universal approximation ability of neural network.

Because of this ability, a neural network is used in the backlash control in [47]. The control system is shown in Figure 4.8.

In this diagram, the parts outside the dotted square are to calculate the desired torque τ_{des}. These parts are well established for nonlinear system controller design[63]. The input v_1 is a robust term to compensate the estimation error of the nonlinear system; Λ is a coefficient vector; x_d is a vector which consists of zero order derivative to n-1 order derivative of the desired output; n is the relative degree of the nonlinear system. $y_d^{(n)}$ is n order derivative of the desired output.

The parts inside the dotted square compose the neural network pre-compensator. The compact static backlash model can be described as

$$\begin{aligned}\dot{\tau} &= B(\tau,\hat{v},\dot{\hat{v}}) = \hat{B}(\tau,\hat{v},\dot{\hat{v}}) + \tilde{B}(\tau,\hat{v},\dot{\hat{v}}) \\ &= \hat{\phi} + \tilde{B}(\tau,\hat{v},\dot{\hat{v}})\end{aligned}.$$

Where $\hat{B}(\tau,\hat{v},\dot{\hat{v}})$ is the backlash slope line (Figure 2.1); $\tilde{B}(\tau,\hat{v},\dot{\hat{v}})$ is the backlash output error due to the inner gap.

By using neural network to calculate y_{nn} equal to backlash error $\tilde{B}(\tau,\hat{v},\dot{\hat{v}})$ and subtracting it in $\hat{\phi}$, we can have desired torque τ_{des} at the input to the nonlinear system.

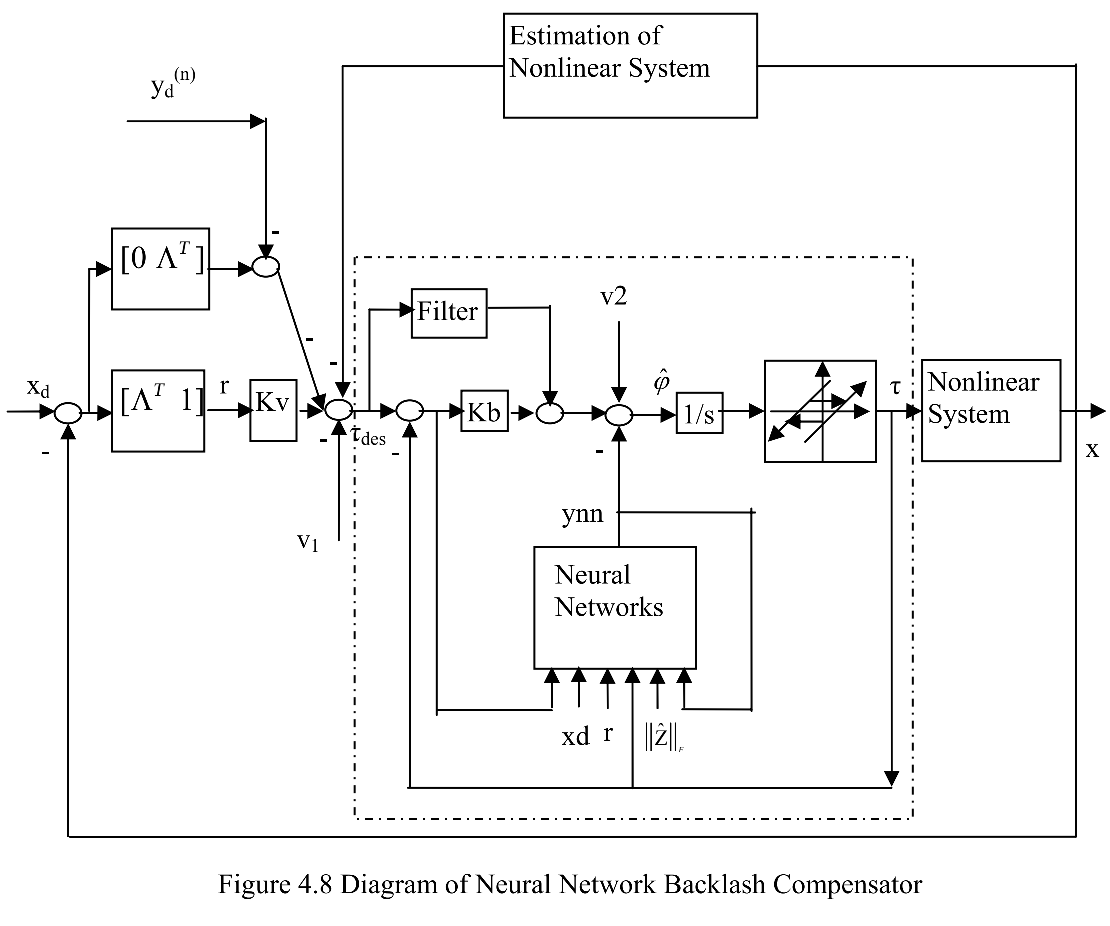

Figure 4.8 Diagram of Neural Network Backlash Compensator

Although the stability of this neural network controller is proved in [47], we still have to be aware that backlash is not a smooth function. So due to universal approximation ability of smooth functions, the neural network may not work well here.

4.3.5 Optimal Control

Backlash control algorithms are doing one thing: to traverse the backlash gap swiftly and to keep the system stable at the same time. A problem naturally arises: is it desirable for the system to leave the backlash gap as fast as possible? Obviously this can make the actuator in contact with load for more time. However, it is not practical as actuators in practice have limited power and response time. Even when an actuator is powerful enough, this solution would lead to a severe collision or oscillation when the actuator and the load get contact. This is similar to the result with high proportional gain in PID controller.

So [28] developed a controller which used sandwiched backlash model(Figure 2.5) and optimizes the motion of the system when it is in the backlash mode. The optimization constraints are

a) To make contact. The actuator and the load must be kept in contact.

b) To have soft contact. When the actuator and the load get contact, their speed must be the same.

The objective function is

$$J = \int_{t_0}^{t_f} \{1 + \rho u^T(t)u(t)\} dt$$

which optimizes the control signal power. t_0 is the time when the system enters backlash modes; t_f is the time when the system leaves backlash modes.

However, this method is not practical because it requires the knowledge of the exact value of the backlash gap. In fact the control system is quite sensitive to the estimation error of the backlash gap. “Sensitive” here means a small estimation error of gap size leads to a large error or instability in the output of the control system. A simulation below illustrates this.

In the simulation, the system is modeled as a link with motor dynamics (Equation. 4.1). The parameters are described in Table 3.1 and Table 4.3. Let c be the real half gap, C be the estimated half gap. In this simulation, c is 1. When C is equal to c, the output is following the reference signal (Figure 4.9); when C is very close to c, the controller can still work with a big spike (Figure 4.10). When C is a bit higher, the system is unstable (Figure 4.11). Because of its sensitivity on estimated gap size, we will not evaluate this optimal controller in our real test bed.

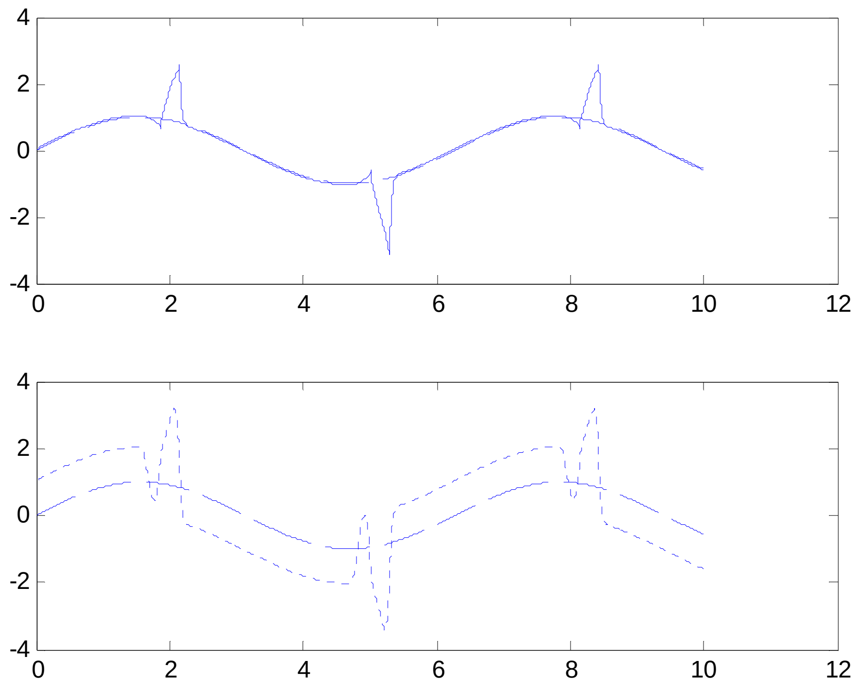

Figure 4.9 The optimal control system output with both the actual c and the estimated C equal to one. The solid line is the position of the link, the dotted line is the motor's position, and the dashed line is the reference signal.

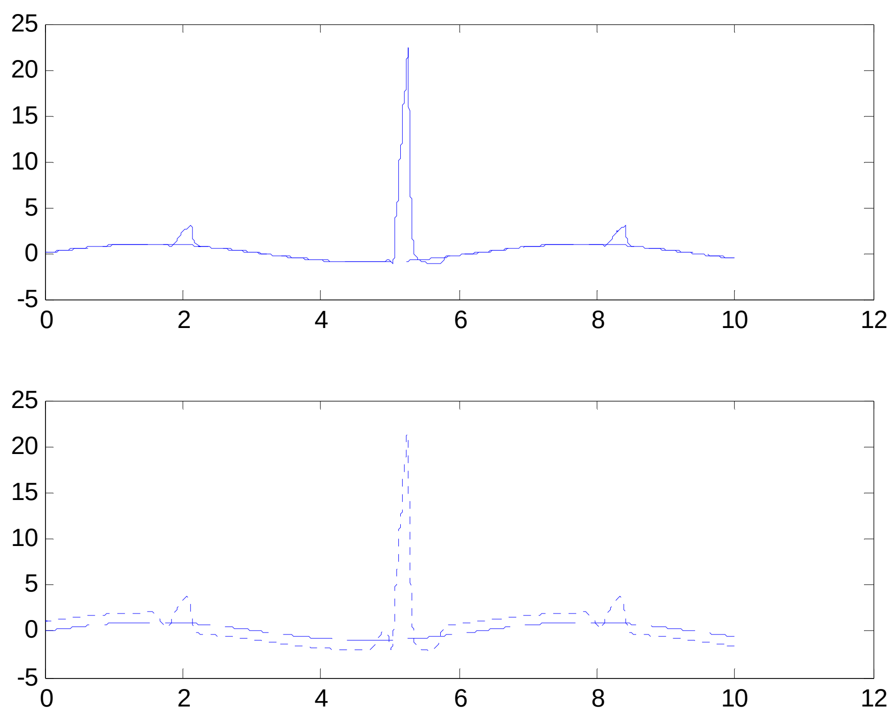

Figure 4.10 The optimal control system output with the actual c =1 and the estimated C =1.05. The solid line is the position of the link, the dotted line is the motor's position, and the dashed line is the reference signal.

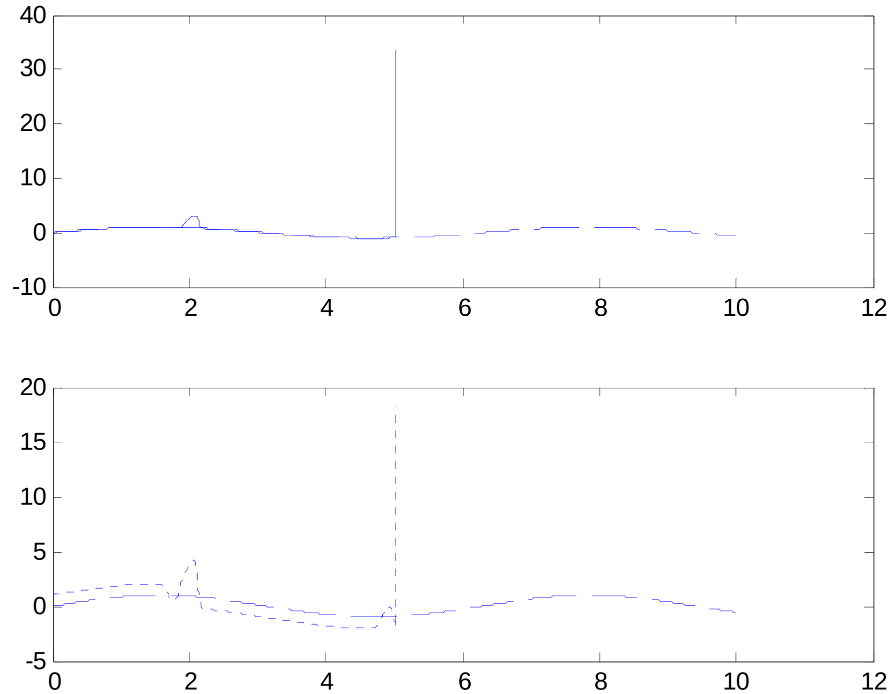

Figure 4.11 The optimal control system output with both the actual c =1 and the estimated C =1.1. The solid line is the position of the link, the dotted line is the motor's position, and the dashed line is the reference signal.

4.4 Experiment Results and Discussion

4.4.1 Results

Since we do not know the exact backlash gap size, the optimal control method cannot work. We only evaluated the other four methods on the real test platform: PID, the robust control, the adaptive control and the neural network control.

The experiment parameters are shown in Table 4.3

Table 4.3 The experiment parameters

Mass of the shank	M=2.115 kg;
Gravity acceleration	g=9.8 N/s^2;
Length of the leg	L=0.4m;
Uncertain range of leg length	ΔL=0.05m
Time Constant of the motor	T=0.2s;

Sampling frequency	1kHz

To protect the motor from high current input, we limit controller's output between -1volt and 1 volt. The shank starts moving to the perpendicular position. We also assume the backlash gap size does not change in this experiment. The controllers' parameters and results are listed in Table 4.4.

Table 4.4 Controller's parameters

Controller type	Parameters value	Results
PID	Kp=35, Ki=10, Kd=5.	Figure 4.12
Robust Control	$S=\dot{e}+\lambda e$, $\rho_m=0$, $m=1$, $c_r=0.2$, $c_l=-0.4$, $\rho_{c_l}=0.05$, $\rho_{c_r}=0.01, \lambda=1, \varepsilon=0.37$	Figure 4.13
Adaptive control	$S=\dot{e}+\lambda e$, ε=0.2; λ=1.0; Kd=0.3; K=0.5; r=0.1; η=0.1; c_{min}=0.8; c_{max}=1.2	Figure 4.14
Neural Network	λ=0.1; Kv=0.1; Kb=0.1; Kz1=5; Kz2=2; Kz3=5; K=0.001; Zm=10; S and T are initialized to be unit matrices. Number of hidden neurons: 10;	N/A

By using the neural network method we found that the motor only jittered. This is because the motor has limited response time. When the control signal change too fast and abruptly, the motor fails to work. Since the neural network control result is very bad, we neglected it here.

4.4.2 Discussion

From Figure 4.12a, we can see if the controller's parameters are well adjusted, the output error is acceptably small. But this does not mean PID control is able to do a good job all the time. PID has its limitations. First, the contradiction of PID is if the gain is high, saturation and oscillation may spoil its performance; if the gain is low, performance is not satisfactory. This means we have to do a trade-off. Secondly the backlash effect can be mitigated theoretically by increasing PID gains. To decrease the error further, we increased gains of PID. However, the result did not get better: it

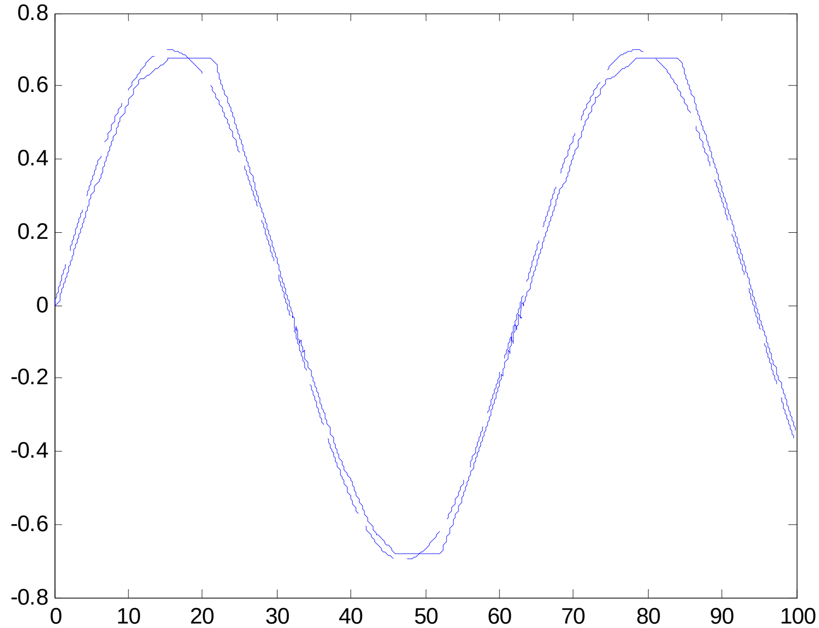

Figure 4.12a Results of PID control. Dashed line is the reference signal; solid line is the leg's angular position

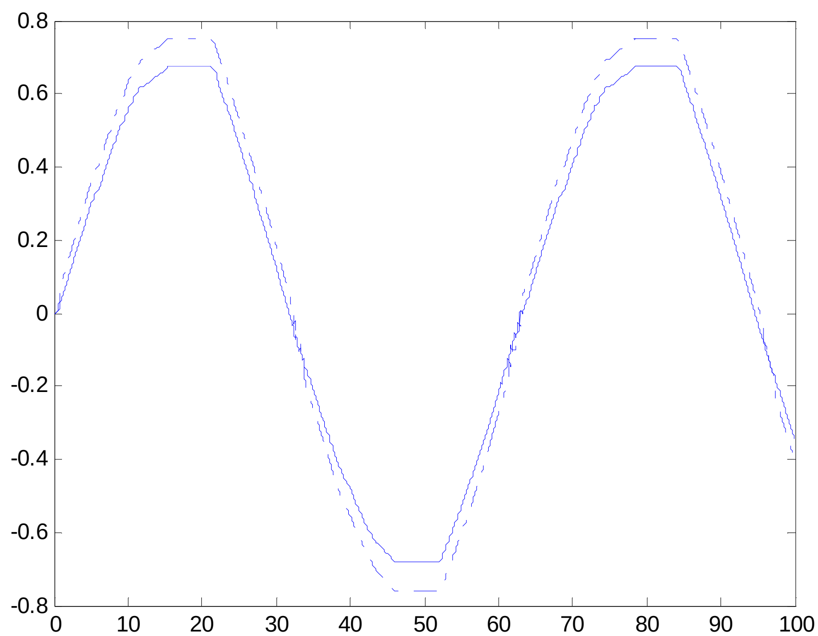

Figure 4.12b Results of PID Control. Dashed dotted line is the motor output position; solid line is the leg's angular position.

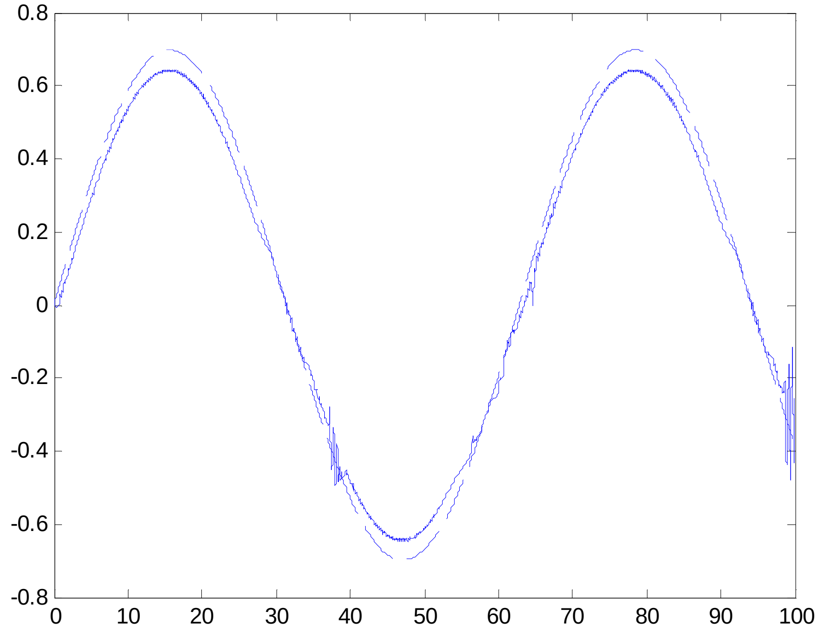

Figure 4.13a Results of Robust control. Dashed line is the reference signal; solid line is the leg's angular position

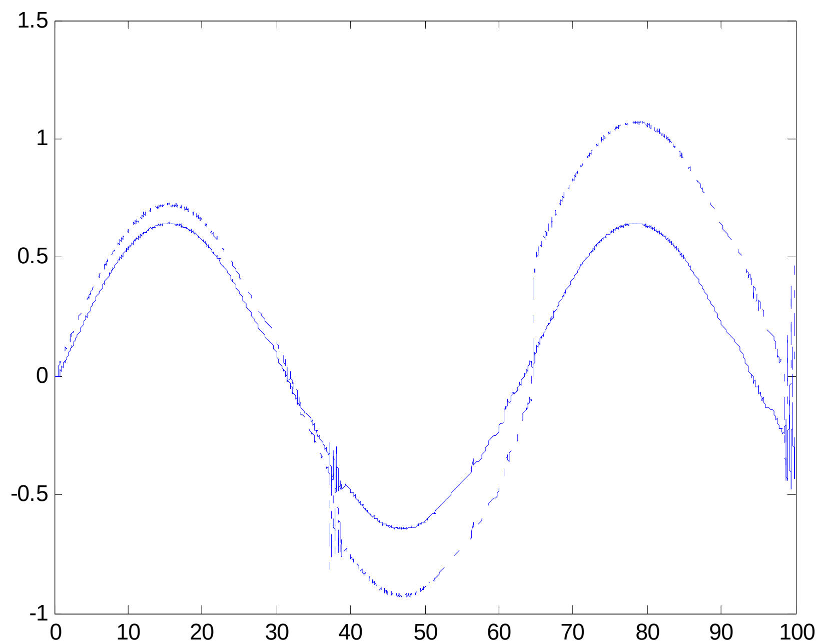

Figure 4.13b Results of Robust control. Dashed dotted line is the motor's output position; solid line is the leg's angular position

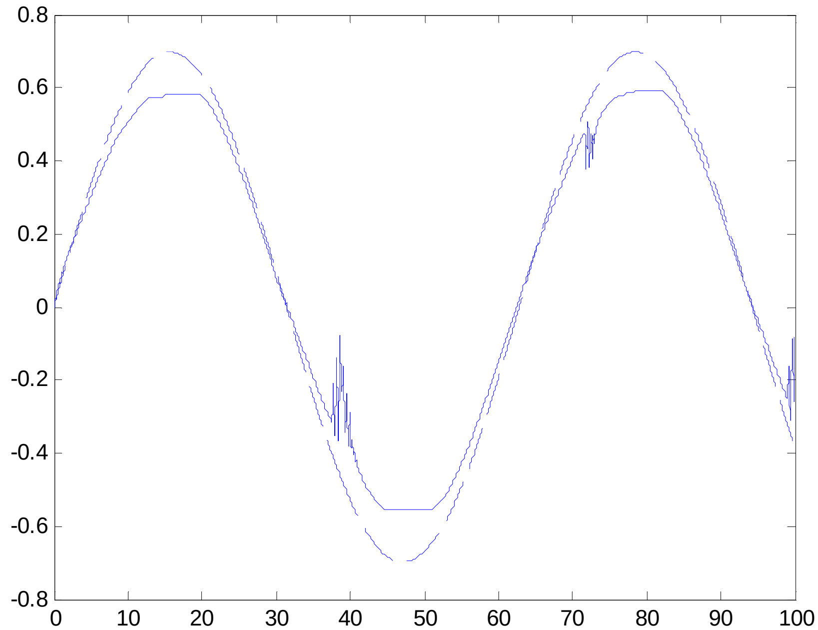

Figure 4.14a Results of Adaptive control. Dashed line is the reference signal; solid line is the leg's angular position

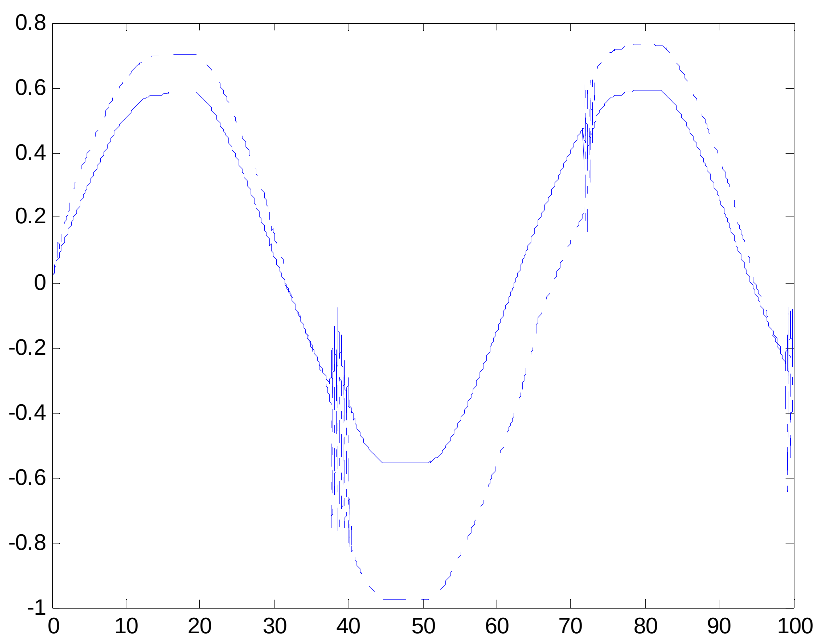

Figure 4.14b Results of Adaptive control. Dashed dotted line is the motor output position; solid line is the leg's angular position

oscillated. This oscillation is due to non-rigidity of the biped shank and the elastic impact between motor shaft and the shank. Thirdly, PID controller does not need a very high gain if there is enough friction: backlash effects are reduced by the friction in this joint because the friction makes the motor shaft and the load coupled again. Finally, other elements also affect the PID performance such as frequency of reference signal, sampling time, etc. But these two elements are not dominant in this experiment.

It is interesting to notice that motor output in Figure 4.12b is different from the ones in Figure 4.13b and Figure 4.14b. In Figure 4.13b and Figure 4.14b, motor's output position increases or decreases abruptly at about the 40th second and the 70th second. This means that the motor shaft quickly traverses the gap and pushes the shank to the other direction. But this phenomenon does not happen in Figure 4.12b. Although a gap between the motor shaft and the shank is clear in Figure 4.12b, the motor does not jerk to pass the gap while the shank still follows the motor. This is contradictory to what we predict according to backlash models. Obviously backlash does not significantly affect the shank motion in Figure 4.12b. A reasonable explanation is friction interferes in the process. The socket and the motor shaft (Figure 4.6) do not contact very tightly but they contacts. As the shank rotates, it presses the socket against the shaft, which increases the friction. This friction exists when PID controller is applied. But it disappears when we use robust control and adaptive control. This is because these two control methods introduce oscillations which can almost remove the friction. The first half period In Figure 4.13b and 4.14b also verifies this argument. At the beginning, the motor and the shank are well contact. This makes control signal smooth since it does not need a jerk to pass the gap which excites oscillations. So the friction has not been reduced too much and it will help move the shank. Thus, the gap measured in the first half period is less than the actual value.

But where does the oscillation come from? The two controllers' architecture may have introduced the oscillation. It is probably because of the non-rigidity of the shank and the elastic impact between the socket and the shaft. Apart from the controller architecture and compliance, another explanation is these controllers are derived based on the static backlash model while they were tested in a real sandwiched backlash. A simulation can prove this. In this simulation, we only simulated the sandwich backlash and omit the friction and compliance, i.e. the shaft of the motor will keep contact with the shank once they impact. This omission is reasonable because it simplifies the simulation and the results of the simulation can still predict their performance on the system with friction and compliance. The simulation results are shown in Figure 4.15 and Figure 4.16.

From Figure 4.15a and Figure 4.16a, we see vague oscillations when the output crosses the zero line. If compliance exists in the system, it can amplify the vague oscillation. Hence the adaptive controller is more possible to cause a devastating oscillation. Though oscillations in both Figures are obscure, it is still clear that the oscillation in Figure 4.15a is less severe than the one in Figure 4.16a. In Figure 4.15a, the output oscillates only when it goes downward and crosses the time axis. In Figure 4.16a, the output oscillates whenever it crosses the time axis. From Figure 4.15b and Figure 4.16b, we see that robust control signal oscillates mostly in the negative part while adaptive control signal oscillates in the whole period. This explains why oscillations happen in different places for robust control and adaptive control (Figure 4.15a and Figure 4.16a).

A difference between Figure 2.3 and Figure 4.12b~4.14b should also be noticed. In Figure 2.3, when the input's direction reverses, the input is in the gap and the backlash output will keep unchanged until the input passes the gap. But when the motion of the motor shaft changes direction in Figure 4.12b~4.14b, the shank still follows. This is because of the dynamics of the shank. When the motor shaft changes its moving direction, the shank will swing to the perpendicular position, too. So the static backlash model is not accurate for this time interval. It is almost accurate only when the shank goes through the perpendicular position.

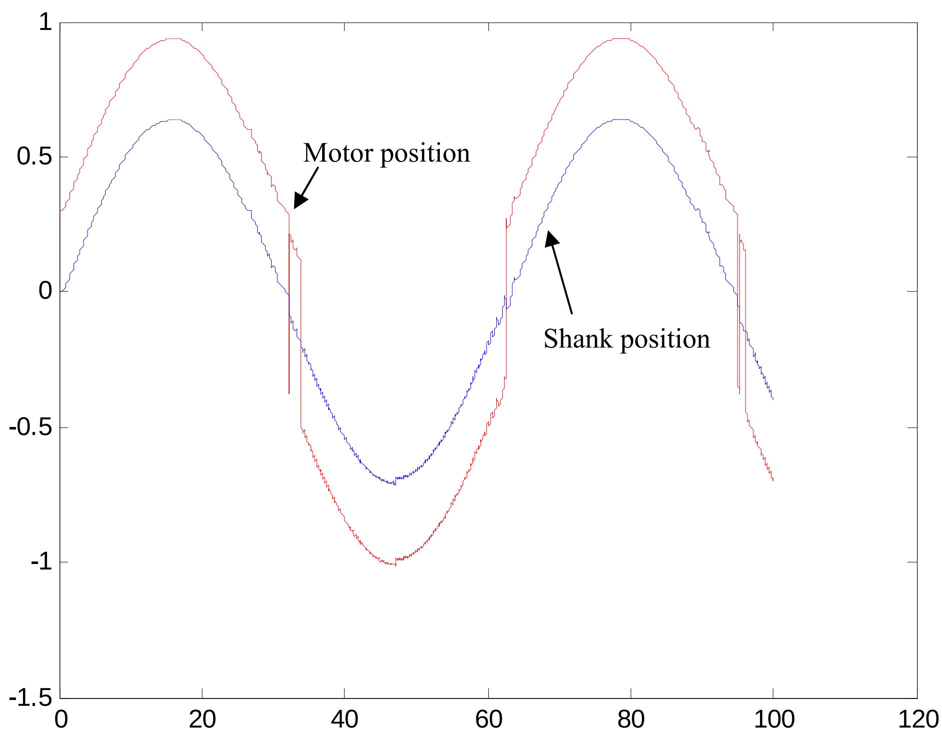

Figure 4.15a motor and system output of robust control simulation.

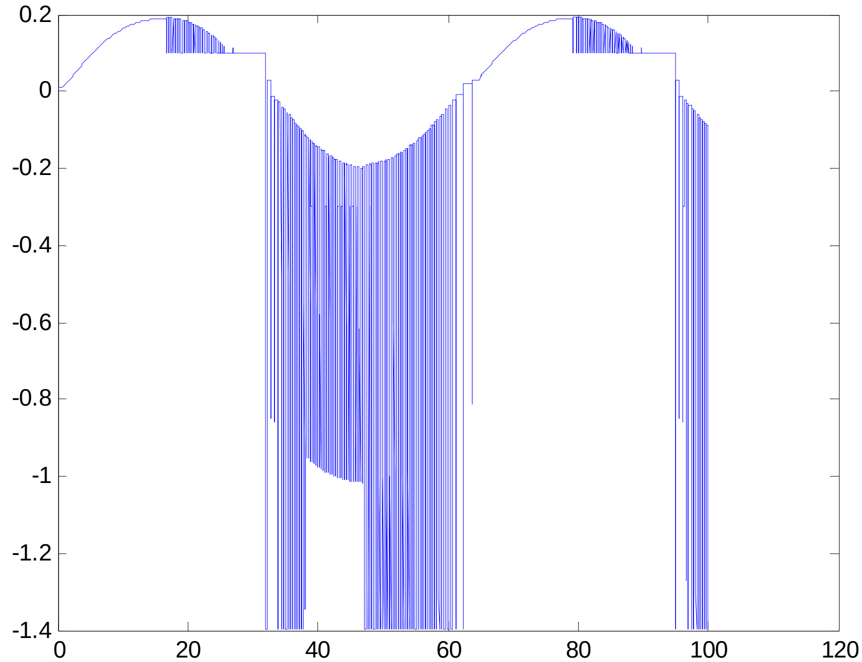

Figure 4.15b controller output of robust control simulation.

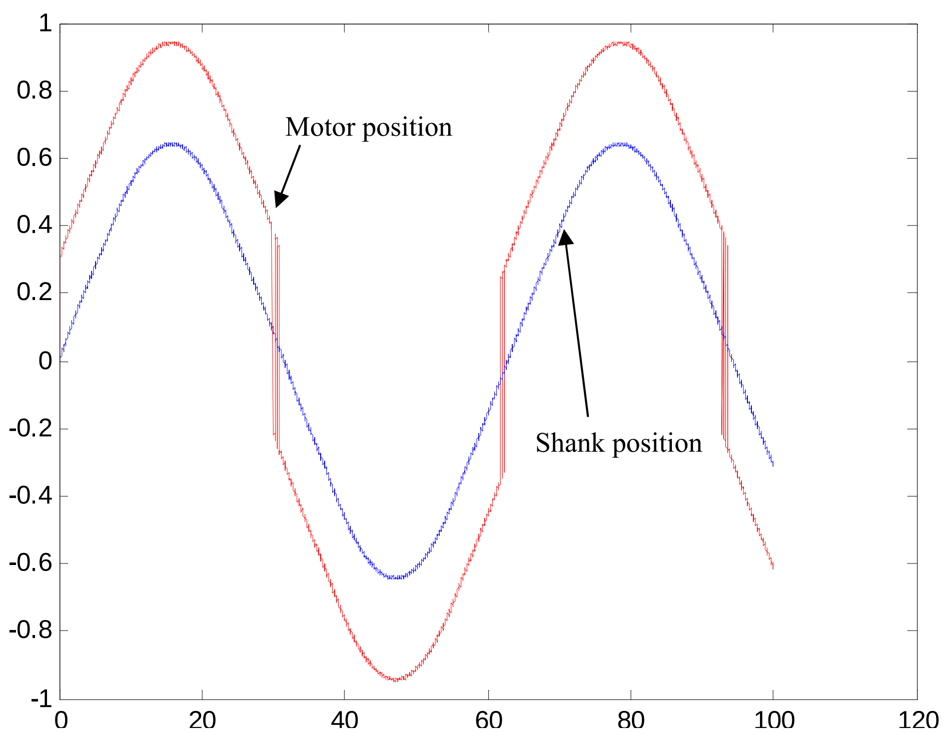

Figure 4.16a motor and system output of adaptive control simulation.

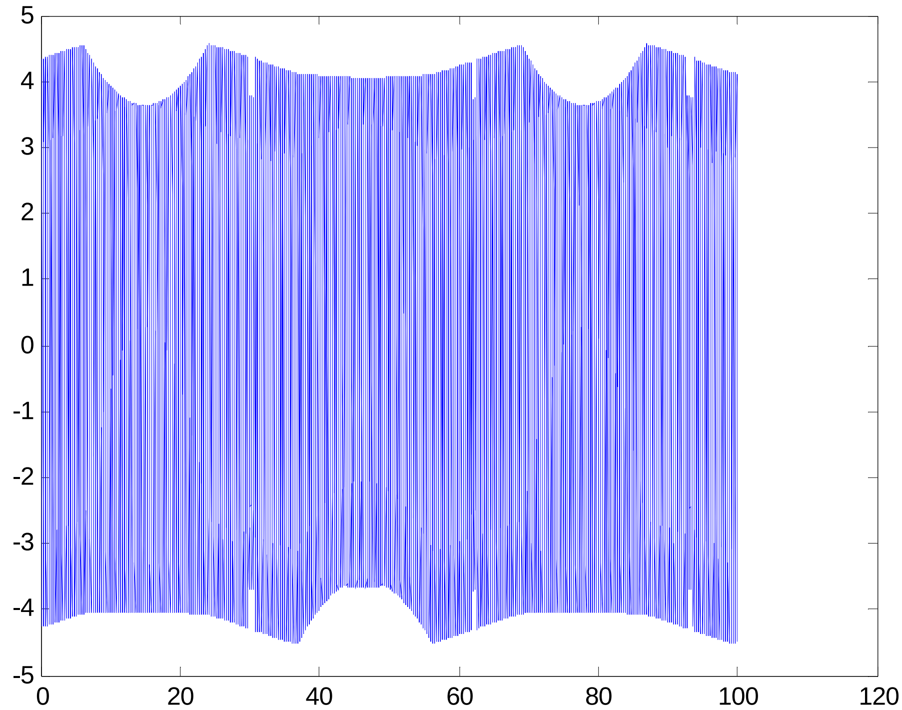

Figure 4.16b controller output of adaptive control simulation.

To summarize, neural network and optimal control is not fit for real application. The robust control is better than the adaptive control since it needs controller output is less than the adaptive one. But robust control may not outperform PID controller. If enough friction exists or has a powerful actuator, PID is preferred; if the system has less friction and permits a bit oscillation, robust control should be used. If neither of the PID controller and the Robust controller meets the requirement, a mechanical redesign should be considered.

4.5 Conclusion

In this chapter, we have analyzed the mechanism of the test platform which leads to backlash and chose 5 types backlash controller and tested 4 of them on our test bed. The experiment results show that neural networks controller and optimal controller do not work very well as shown in those papers. The robust control shows superiority to the adaptive control. Grading PID control and robust control is difficult. So where they can be used is given. A detailed discussion explains the reasons with helps of simulations.

Chapter 5

Conclusion

Backlash is widespread in mechanical and electrical systems and has been in engineers' minds for more than half a century. It is an undesired nonlinearity, which can affects systems' regulating and tracking performance. To solve it, control methods such as describing function method, adaptive control, intelligent control, robust control and optimal control have been used.

In Chapter 3 of this thesis, we focus on position regulating for systems with backlash at the output. The developed controller is based on describing function analysis. This position controller can restore the desired position regulation specifications whereas the conventional controllers only consider the stability of the system. With the developed method, controlling a system with backlash is simplified to two steps: first design a controller for the plant and design a backlash compensator for the backlash at the output. By using classical frequency analysis method, this controller can achieve the desired specifications such as overshoot, rise time, etc; second, using developed formula to design a forward loop and a feedback loop. Second step can retain the system performance achieved by the first step. The robustness of this method is analyzed, i.e., the uncertainty of the backlash gap and system modeling is analyzed and modeled as a disturbance.

There are many backlash control methods developed for tracking problem, too (actually most methods are for this problem). However, these methods are still limited to simulation or strict experiment protocol, e.g. small or exactly known backlash gap. To evaluate these methods for practical applications, in Chapter 4 we implement several well-known backlash controllers in a real test platform and compare them with PID controller. Two of these controllers can work on our platform.

For future work, we will consider practical issues in our backlash controller design. These issues are compliance, backlash mixed with friction, hardware constraints and so forth.

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