COMPETITION WITH HORIZONTAL AND VERTICAL DIFFERENTIATION: LOCATION THEORY AND EXPERIMENTS

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For my parents, James and Lucille
In praise and thanksgiving to God

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ABSTRACT

Product differentiation by firms located at the boundary regions of countries or cities is of pertinent significance and interest to various segments of society as a result of its attendant economic benefits and trickle down effects on the rest of the economy. The inside-outside location model presented in this study offers a simple framework for understanding and analysing the price and location decisions of competing duopolists situated on either side of a border, as well as the buying and travel decisions of consumers between the domestic firm and the competing firm beyond their economic precincts.

Formulated in the context of product differentiation analogue to Hotelling's paradigm and drawing on the earlier contributions of Gabszewicz and Thisse (1986; 1992), the inside-outside location model integrates the traditional inside location model and the outside location model. Under horizontal differentiation (inside location), firms offer identical products and compete in price. Consumers will choose the firm that has the lower price, if prices differ. Under vertical differentiation (outside location), products differ in quality. Consumers pay more for products higher up along the quality spectrum.

The inside-outside location model explains firm competition along both horizontal and vertical characteristics. Under parametric firm locations, equilibrium relative prices and market shares are always equal regardless of the nature of transportation costs. When firm location is variable, equilibrium in pure strategies is non-existent under linear transportation costs but exists under non-linear transportation costs. Price and location competition in this model do not necessarily lead to the same results as the traditional location models and possesses stability that is intermediate between the two.

The predictive power of the inside-outside location model is evaluated by means of two experiments. The first experiment corresponds to the short run situation in which firm

location is constant. The second experiment studies the long run situation in which both price and location decisions are made. A simultaneous price-location game is implemented. A total of ten treatments were conducted, half of which institute a 100% increase in transportation costs.

The experimental results accord fairly strong support for the theoretical predictions. Prices and locations under various transportation cost structures generally approached Nash prediction. Under constant location, however, the inside firm players exhibit a strong inclination to price close to levels that monopolise the market. Under variable location when the firms are no longer restricted by competition along a single dimension (i.e., price), the inside firm shows a smaller inclination (or ability) to monopolise the market through low prices. The results show that a reduction in product differentiation under higher transportation costs results in more intensive price competition when location is variable rather than fixed.

Although the inside-outside location model presented here offers solutions in pure competition of price and location, further extensions are feasible with respect to mixed strategies and collusions between firms, especially in instances where a parent company has several outlets on either side of the border. A myriad of other situations present themselves that are worthy of further study by modifying the basic assumptions inherent in the model, e.g., by incorporating price discrimination, production costs and a budget constraint. As such, the situations considered here do not pretend to be either exhaustive or comprehensive in the range of possible applications within this domain.

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CHAPTER 1

INTRODUCTION

Spatial theories of product differentiation have their roots as far back as von Thünen, Launhardt and Weber, long before the seminal contributions of Hotelling and Chamberlin.[1] Theories of product differentiation evolved along two broad themes: the first distinguishes between horizontally and vertically differentiated goods, while the second demarcates goods according to whether they are address or non-address items.[2]

The delineation of product differences along hierarchical lines was first made by Lancaster in the late 1970s.[3] Broadly speaking, two products are said to be *horizontally differentiated* when one contains more of some characteristics but fewer of other characteristics. Consumers exhibiting heterogeneous preferences will choose the product that is closest to their tastes, ceteris paribus. In other words, there will always be positive demand for products offered at the same price. On the other hand, two products are said to be *vertically differentiated* if one contains more of some or all characteristics than the other. All rational consumers will choose the product in which the characteristics are augmented rather than lowered, *ceteris paribus*. Consequently, the product with the augmented characteristics

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[1] The authors are credited as the founding fathers in three areas of location theory: von Thünen for agricultural location (*Der Isolierte Staat* published in 1826), Launhardt for market area analysis (*Mathematische Begrundung der Volkswirtschaftslehre* published in 1885) and Weber for industrial location (*Über den Standort der Industrie* published in 1909). Besides these authors, Christaller and Lösch are known for their contributions to central places theory (major works published in 1933 and 1944 respectively). Others such as Marshall (e.g. *Principles of Economics* first published in 1961) also identified product differentiation but did not cast their work in a spatial context.

[2] Phlips and Thisse (1982) classified theories of product differentiation in location models under categories that distinguished between the pricing mechanism employed, *viz.*, mill pricing versus discriminatory pricing. A sub-category was then introduced for each according to whether the theories differentiated products horizontally or vertically.

[3] Lancaster introduced the concept of differentiated products and consumer tastes in 1966 and subsequently categorised heterogeneity as “vertical” differentiation in 1976 and “outside” in 1979. He explicitly transformed product characteristics into product space à la Hotelling in 1975 in his attempt to find the socially optimal level of product variety. This work was subsequently revised in 1979 (e.g., see Lancaster 1979).

will always capture the whole demand whenever it is offered at the same price as the other product in which the characteristics are lowered.

Horizontal differentiation lies at the heart of Hotelling (1929)'s analysis, while vertical differentiation received a parallel analysis in the same vein as Hotelling only fairly recently by Gabszewicz and Thisse (1986). The authors described horizontal differentiation models as inside location models, and vertical differentiation models as outside location models. In *inside location models,* consumers are located within the same sub-space as firms. In *outside location models*, firms are located outside the residential area of consumers. The product may be homogeneous in all respects except its distance (and hence transportation cost) with respect to consumers. Alternatively, product differentiation may be viewed in terms of brand specification rather than physical location. In terms of product differentiation, the product with lower transportation cost can be viewed as possessing higher quality or brand preference since consumers always prefer to purchase it, *ceteris paribus*.[4] The disutility (if any) arising from consuming the product is then measured by the distance between the product and the consumer.

The alternative method of identifying product differentiation theories is the 'address' versus 'non-address' approach. The *'address'* approach runs along the lines reminiscent of Hotelling. It recognises a product as having spatial characteristics with addresses or coordinates in space, and consumers who similarly possess addresses for their tastes in the same product space. In contrast, the *'non-address'* approach, in the spirit of Chamberlin, assumes that consumer tastes for differentiated goods are defined over a predetermined set of all possible goods (which may be finite or countably infinite) that are purchased by a representative consumer (Eaton and Lipsey 1989). Although the second approach in its original framework is not directly applicable to spatial competition in that it disregards

[4] Cremer and Thisse (1991) showed that horizontal differentiation models are in fact a special case of vertical differentiation models, as long as Shaked and Sutton (1983)'s 'finiteness property' is satisfied, i.e., only a finite number of firms co-exist with positive demand at a price equilibrium where prices exceed marginal cost. This condition is likely to hold in industries where product innovation is accompanied by process innovation, so that marginal cost rises less rapidly than quality increases.

neighbour effects of firms or products, modifications to basic Chamberlinian precepts by authors such as Salop (1979) has made this more tractable.

While the bulk of the existing literature on spatial product differentiation was spawned from either of the two approaches, i.e., horizontal versus vertical, or address versus non-address, relatively fewer attempts have been made to study the co-existence of both attributes within the same spatial framework. Launhardt can be regarded as the pioneer of this third branch of spatial differentiation theories. Generally, such theories attempt to establish a market boundary that segregates markets geographically or through their pricing patterns. In his ‘economic law of market areas’, Fetter (1924) defined a market boundary as a hyperbolic curve separating two geographically competing markets whose position is determined by the relative price and relative freight rate of the two markets. More generally, the market boundary can be described as a family of elliptical curves or hypercircle (e.g. Hyson and Hyson 1950; Hebert 1972). The hyperbolic market curve becomes a straight line when production prices and freight rates are identical.

Spatial models that incorporate the market boundary through geographical market segregation include Salop (1979)’s non-congruent markets along a circle in which a firm in one market sells a homogeneous product while another firm in the other market sells a differentiated product. Cooper (1989) adapted Salop’s model to study indirect competitive effects by having the two markets meet at a single point at which a third firm is located. The two firms located within the markets sell differentiated products in their own market but not outside it, while the straddling firm can sell in both markets. DeGraba (1987) used a similar framework as Salop and Cooper but, instead of circles, the markets are linear with the market boundary at the origin. The two markets are represented by the lines $[L_1, 0]$ and $[0, L_2]$ and contain one firm each which sell only to consumers located inside their own market, while a third firm straddling the two markets at the market boundary sells to consumers in both markets. In a novel approach, Braid (1989) considered location along intersecting roadways to yield an asymmetry in market demand realised by the firm located at the crossroads relative to that obtained by firms located at one of the road segments.

In a modified Hotelling duopoly framework that permits firm location beyond the city boundaries, Tabuchi and Thisse (1995) showed that under quadratic transportation costs, firms locate outside the market at $(-1/4, 5/4)$ if consumers are uniformly distributed, and at $(-\sqrt{6}/9, 5\sqrt{6}/18)$ and $(1-5\sqrt{6}/18, 1+\sqrt{6}/9)$ if the consumer distribution is triangular. The latter of the two asymmetric equilibria has one firm locating outside the market.[5]

Spatial models that define the market boundary through the price structure include Dos Santos Ferreira and Thisse (1996)'s variegated transportation technology model, à la Launhardt. In their framework, firms are located in the same market but encounter different transportation rates in delivering a homogeneous product to consumers within the market. Depending on the distance of the firms from each other, different transportation rates for the product will result in horizontal or vertical product differentiation. On the other hand, Greenhut and Ohta (1975) employed discriminatory pricing to determine the market boundary in their price conjectural variation model. Firms form conjectures about rivals' likely responses and enter these conjectures into their decision-making. In this way, firms select a (delivered) pricing policy to maximise profits subject to a given limit price ceiling at the market boundary.

Although non-exhaustive, the above discussion on spatial product differentiation models with market boundaries shows clearly that such theories are more reflective of the realities of oligopolistic competition. Introducing a market boundary that segregates diverse markets which interact mutually raises the analysis to more realistic levels and hence enhances the practical applicability of the conclusions to be drawn.

With such heuristic intentions in mind, I introduce a new model in Chapter 2 depicting both horizontal and vertical product differentiation characteristics, formulated in the context of product differentiation analogue to Hotelling's paradigm. Drawing on the earlier

[5] See also Lambertini (1997).

contributions of Gabszewicz and Thisse (1986; 1992), an inside-outside location model is proposed which integrates the pure inside location model and the pure outside location model. Two firms, an inside firm and an outside firm, face the same transportation rate and are located in separate linear markets of length $[0,1]$ and $]1,+\infty[$ respectively. The market boundary is located at the point 1. Consumers are located only within one of the markets which constitutes the linear city in this model, *viz.* within $[0,1]$, but may travel to either of the two markets to purchase the product. Firm entry into rival market, however, is closed. It will be shown that the only viable option for the firm outside the city limit is to locate in the vicinity of the market boundary. Intuitively, proximity to the market boundary is crucial for the outside firm to remain in competition with the inside firm (situated in the same area as the consumers) by reducing the transportation costs incurred by the consumers, *ceteris paribus.*

Both horizontal and vertical product differentiation characteristics coexist in this model which segregates the markets geographically. At one extreme, when the inside firm locates at the market boundary at point 1 (i.e., closest to the outside firm), the model reduces to one that mainly exhibits vertical product differentiation characteristics. At the other extreme, when the inside firm locates at 0 (i.e., furthest from the outside firm), horizontal product differentiation characteristics predominate. At locations away from the endpoints of the inside firm, the model naturally displays both horizontal and vertical differentiation attributes.

In this hybrid model, price and location competition do not necessarily lead to the same results as in the pure inside or outside location model. The contrasting findings and all possible equilibria under various types of transportation costs are studied in the ensuing analysis. The proposed inside-outside location model is found to possess stability that is intermediate between the pure location models.

The inside-outside location model constructed in this manner is reflective of many real world situations in which physical entry by firms into rival markets is either too costly or legally prohibitive, but product entry is not. The outside firm either sells the product to consumers by transporting the good to them and charging them the delivered price, or synonymously, consumers travel across the market boundary to purchase the good. In both instances, consumers pay the mill price plus transportation cost. The first situation is reflective of trading nations or cities in which firms produce goods within their own precincts and ship them to neighbouring markets to be sold, while the second is reflective of cross-border shoppers who travel out of their domestic market to shop, and may be adapted to the context of workers who travel to a neighbouring country or city to work and return at the end of each day or year. For example, cross-border shopping is a common phenomenon in the border regions of US and Canada, US and Mexico, several European countries, and Singapore and Malaysia in Southeast Asia (e.g., see Bode *et al*. 1994; Brodowsky and Anderson 2003; Timothy and Butler 1995; Toh 1999). It is worth noting that the IO model is directly applicable to adjoining market areas segmented economically and (or) geographically at the border. It highlights the distinction between an economic boundary and geographical boundary between two regions, which in most of the cases do not necessarily coincide.

The extent to which the IO model has predictive power for the behaviour of duopolistic spatial competition is evaluated in a laboratory setting. Despite the popularity of spatial location theories, experimental tests of such models have been relatively few. Existing experimental studies on spatial firm competition typically draw on the inside location models à la Hotelling (1929) by varying the conditions in which firms compete. There are two broad categories of such studies. The first focuses on firm behaviour with a single strategy, which may be either price or location. The second takes a more realistic approach by studying firm behaviour with dual strategies, i.e., both price and location. The experimental tests of the IO model in this study follow this two-pronged approach. Chapter 3 presents the results of an experiment that assumes constant firm location, while Chapter 4 highlights the experimental study of firm behaviour where both price and location decisions are made.

The study of firm behaviour with a single strategy forms the bulk of existing experimental literature on spatial firm competition. These studies typically observe location decisions by assuming constant price. Brown-Kruse *et al.* (1993) and Brown-Kruse and

Schenk (2000) conducted experiments on location decisions by assuming elastic consumer demand, while Collins and Sherstyuk (2000) and Huck *et al.* (2002) studied location decisions by assuming inelastic consumer demand. On the other hand, Selten and Apesteguia (2004) studied price decisions among varying number of firms with fixed location in a circular market. In all these experiments, buyer decisions are automated.

The experiment presented in Chapter 3 observes price decisions in a short run situation in which firm location is constant. Six treatments are employed, each corresponding to different assumptions of transportation costs. In three treatments, there is a 100% increase in transportation costs. This permits a comparative study of firm decisions under higher transportation costs.

The second approach to the experimental study of spatial firm competition involves both price and location strategic decisions. Among the few studies that adopt this methodology include Barreda *et al.* (2000) and Camacho-Cuena *et al.* (2004). Barreda *et al.* (2000) studied location-then-price decisions in a duopoly faced with horizontal differentiation in a discrete framework. Camacho-Cuena *et al.* (2004) took a novel approach by studying non-automated consumer decisions. Extending Barreda *et al.* (2000)'s study, the authors observed buyer location-purchase decisions in a four-stage game. In the first and second stages, both sellers and buyers make their location decisions. In the third stages, sellers set prices and in the final stage, buyers make purchase decisions.

Chapter 4 highlights an experiment that assumes a long run situation in which firms compete in both price and location. A simultaneous price-location game is implemented. Four treatments are executed in which varying assumptions are made regarding the type of transportation cost structure and its parameters. In two treatments, a 100% increase in transportation costs is assumed.

The conclusions are drawn in Chapter 5. The theoretical and experimental results are summarised, and comparisons are drawn between the main findings of the two experiments.

CHAPTER 2

THE INSIDE-OUTSIDE LOCATION MODEL

2.1 INTRODUCTION

Competition in space arises because market activities occur at dispersed points in space. The study of spatial economic interactions has been well established since Hotelling (1929)'s pioneering work, with notable contributions by Prescott and Visscher (1977), d'Aspremont *et al.* (1979), Gabszewicz and Thisse (1986; 1992), de Palma *et al.* (1985), and Anderson (1988), *inter alia*.

In Hotelling's inside location model, two firms compete in a market along a line segment l (typically normalised to the unit interval $[0,1]$ by subsequent authors) to sell a homogeneous product that is produced at zero cost. The firms have location and price as their decision variables. Consumers are uniformly distributed along the same line segment and encounter transportation costs that increase linearly in distance, i.e., $c(d) = td$, where d is the distance between the firm and the consumer. The firms play a two-stage game in which they decide on location in the first stage and price in the second stage. Under this formulation, Hotelling found that an equilibrium exists which results in firms agglomerating at the market centre, a phenomenon which he termed the *Principle of Minimum Differentiation*.

This solution, however, has been found to be inherently unstable. In a slightly modified version of Hotelling's framework for which a unique price equilibrium in pure strategies exists for any pair of locations (x_1, x_2), D'Aspremont *et al.* (1979) proved that if the transportation costs between firms and consumers increase at a quadratic rate, i.e.,

$c(d) = sd^2$, the *Principle of Maximum Differentiation* holds instead.[1] Rather than clustering at the market centre, firms choose to disperse themselves and locate at opposite ends of the market.

The Principle of Maximum Differentiation has been shown to hold only under certain conditions. Employing a transportation cost function of the form $c(d) = d^{\alpha}$ where $1 \le \alpha \le 2$, Economides (1986) showed that maximum differentiation exists for highly convex transportation cost functions, in particular, for $1.26 \cong \overline{\alpha} < \alpha \le 2$.

Solving a sequential game in which non-uniformly distributed consumers face a quadratic transportation cost function, Neven (1986) showed that the incentive for firms to maximally disperse is reduced with increasing densities of consumers toward the centre while maximum differentiation occurs under uniform consumer distribution.

Prescott and Visscher (1977) obtained maximum firm dispersion as a solution for a foresighted sequential two-firm entry game. No equilibrium exists when there are three firms. A similar result was obtained by Shaked and Sutton (1982) in an extended study involving quality decisions. In a sequential three-stage game, firms make a decision to enter the market in the first stage, followed by a quality choice in the second stage, and a price decision in the third stage. Consumers are identical in tastes but differ in incomes that are uniformly distributed. An equilibrium in pure strategies exists in which only two firms choose to enter the market, produce differentiated products and earn positive profit.

Other authors examined the conditions in which an equilibrium exists under linear transportation costs or a combination of linear and quadratic transportation costs. Osborne and Pitchick (1987) presented a solution under Hotelling's original framework and showed that an equilibrium in pure strategies exists in the location stage and in mixed strategies in the price stage. Gabszewicz and Thisse (1986) studied the case in which transportation costs are

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[1] When transportation costs are linear, a unique price equilibrium exists if and only if $((x_1 + x_2 + 2)/3)^2 \ge 4(2 + x_1 - 2x_2)/3$ and $((4 - x_1 - x_2)/3)^2 \ge 4(1 + 2x_1 - x_2)/3$ when $x_1 + x_2 < 1$, and $p_1^* = p_2^* = 0$ when $x_1 + x_2 = 1$ (see d'Aspremont *et al.* 1979). Note that x_2 is defined from the origin rather than from point 1 in contrast to Hotelling (1929)'s nomenclature.

linear-quadratic, i.e., $c(d) = td + sd^2, t, s > 0$, and showed that no equilibrium in pure strategies exist. Anderson (1988) extended this result by showing the existence of an equilibrium involving pure strategies in the first stage and mixed strategies in the second stage.

Various authors also considered several forms of non-linear markets. For example, Salop (1979) introduced a model in which two firms are located along a circle. A firm in the first market sells a homogeneous product while another firm in the second market sells a differentiated product. If there are n firms, then they locate equidistantly from each other at $1/n$. In terms of prices, three types of equilibria exist: monopoly (segregated or overlapping markets at high prices), competitive (overlapping markets at lower prices) and kinked (the markets just touch). De Frutos *et al.* (2002) showed that under this formulation, the location-then-price game is strategically equivalent regardless of whether transportation costs are convex or concave.

The Principle of Minimum Differentiation is valid under certain assumptions. De Palma *et al.* (1985) showed that firms have a tendency to cluster at the market centre if consumer choices are probabilistic enough, or equivalently, if preferences are sufficiently heterogeneous. Dudey (1990) obtained the same result for a four-stage sequential game involving consumer search. In the first stage, firms choose their location. In the second stage, consumers decide where to shop. In the third stage, firms decide on the quantity to produce. In the final stage, consumers learn the terms-of-trade available from the shopping centre they have decided to visit, and make their purchase at the market clearing price. The authors defined a shopping centre as one in which there are more than one firm at a single location. An equilibrium in pure strategies is obtained in which firms cluster together, i.e., the Principle of Minimum Differentiation. Other variants of this sequence examined by the authors produced the same result, e.g., firms choose quantity and consumers choose shopping location simultaneously, or firms and consumers choose location simultaneously.

While the Principle of Differentiation (whether maximum or minimum) may be an attractive means by which firms attempt to avert rigorous price competition, firms are commonly observed to offer products that possess virtually identical features, e.g., some electronic products (Motorola and Nokia) and automobiles (BMW and Mercedes). Rather than compete among two or more variants of the same product at the same price (*horizontal differentiation*), competition presides over a quality scale in which the product that has a higher quality commands a higher price (*vertical differentiation*).

Gabszewicz and Thisse (1986) presented a vertical differentiation or outside location model in which firms are located along $[1,+\infty[$ outside the residential area of consumers. The product may be homogeneous in all respects except its distance (and hence transportation cost) with respect to consumers. The product with lower transportation cost can be viewed as possessing higher quality since consumers always prefer to purchase it, *ceteris paribus*. An equilibrium in pure strategies always exists for the sequential location-then-price game.

The Hotelling model and its variants have been applied to the study of the impact of brand specification (through product quality, variety, prestige or image) on decisions such as price and brand loyalty. Among the authors in this vein are Grossman and Shapiro (1984), Ben-Akiva *et al*. (1989), Martínez-Giralt (1989), Tremblay and Martins-Filho (2001), Tremblay and Polasky (2002), Wright (2002), Harter (2004), and many others.

In the next section, I present a model that integrates the inside location model and the outside location model. This hybrid model possesses both horizontal and vertical differentiation characteristics. Two firms, an inside firm and an outside firm, produce a homogeneous good. They locate on either side of a market boundary along a line segment of infinite length $[0,+\infty[$ and are prohibited from entering each other's market space. Consumers are located within the same market as the inside firm. They travel to either firm to make their purchase by incurring a transportation cost. This situation is reflective of cross border shoppers who travel beyond their residential area to shop. Synonymously, firms located in adjoining market spaces may deliver the good to consumers who bear the delivery costs. This scenario reflects competition between local and imported goods. The model described in the next section is couched in the first setting *à la* Hotelling in which consumers travel to make their purchase.

2.2 THE INSIDE-OUTSIDE (IO) MODEL

To examine the duopolistic competition between firms selling a homogeneous product in two adjoining markets with entry-barrier to foreign firms, consider an inside-outside location model (hereafter termed IO model) adapted from Hotelling (1929) and Gabszewicz and Thisse (1986; 1992). Figure 2.1 gives a graphic representation of the model.

Two contiguous straight lines represent two markets $i \in \{1,2\}$ that sell a homogeneous product with no storage, distribution or production costs. Market 1 is denoted by the bounded unit interval $[0,1]$ along which firm 1 (the inside firm) and all consumers are located. Market 2 is denoted by the unbounded interval $]1,+\infty[$ along which firm 2 (the outside firm) locates. The two markets meet at the market boundary situated at point 1 and together constitute a continuous straight line of infinite length (although an upper bound is necessary for firm 2 to remain viable, as will be shown in Section 2.3). Consumers are uniformly distributed along $[0,1]$ with density one. Firm 1 is located at distance x_1 from the left endpoint of the line, i.e., $x_1 \in [0,1]$, while firm 2 is located at distance x_2 outside the domestic market with $x_2 \in]1,+\infty[$. The two firms are assumed to have fixed location and compete only in price. This assumption will be relaxed in Sections 2.4 and 2.5. Each consumer buys one unit of the product from the firm charging the lower *full price*, i.e., mill price plus transportation costs. Price ties are resolved in favour of the nearer firm. Consumers are assumed to have control over transport and bear the full burden of the transportation costs. Let $c(d)$ denote the transportation cost function which is continuous, increasing and convex (weakly or strongly) in distance d and presents itself as one of three forms: linear, quadratic and linear-quadratic, with $c(0) = 0$. Let p_1 and p_2 denote the mill price of firm 1 and firm 2

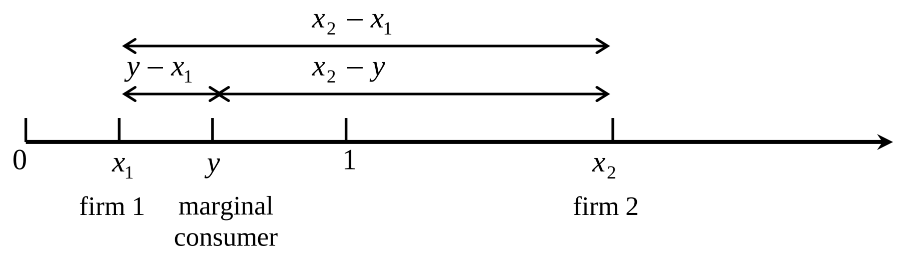

Fig. 2.1 Geographical configuration of the marginal consumer and firms

respectively. Let $m(p_1, p_2)$ be the "marginal consumer" $y \in [0,1]$ who is perfectly indifferent between travelling to firm 1 or firm 2 satisfying

$$p_1 + c(|y - x_1|) = p_2 + c(|x_2 - y|)$$

and is unique whenever he exists.

The market is segmented at $m(p_1, p_2)$: consumers located in $[0, m(p_1, p_2)]$ buy from firm 1 while those in $[m(p_1, p_2), 1]$ buy from firm 2. If $m(p_1, p_2)$ does not exist, then either of the following two conditions holds:

(2.1) $$p_1 + c(|y - x_1|) < p_2 + c(|x_2 - y|) \quad \text{for all } y \in [0,1], \qquad \text{or}$$

(2.2) $$p_1 + c(|y - x_1|) > p_2 + c(|x_2 - y|) \quad \text{for all } y \in [0,1].$$

In the first case, firm 1 serves the whole market at price p_1 while in the second case, the whole market is served by firm 2 at price p_2. The strategies of this two-player game are $p_1 \in [0, +\infty[$ and $p_2 \in [0, +\infty[$ with the payoff function of firm 1 given by

$$\begin{aligned} \Pi_1(p_1, p_2; x_1, x_2) &= p_1 \int_0^{m(p_1, p_2)} f(z)dz && \text{if } m(p_1, p_2) \text{ exists,} \\ &= p_1 && \text{if equation 2.1 holds,} \\ &= 0 && \text{if equation 2.2 holds} \end{aligned}$$

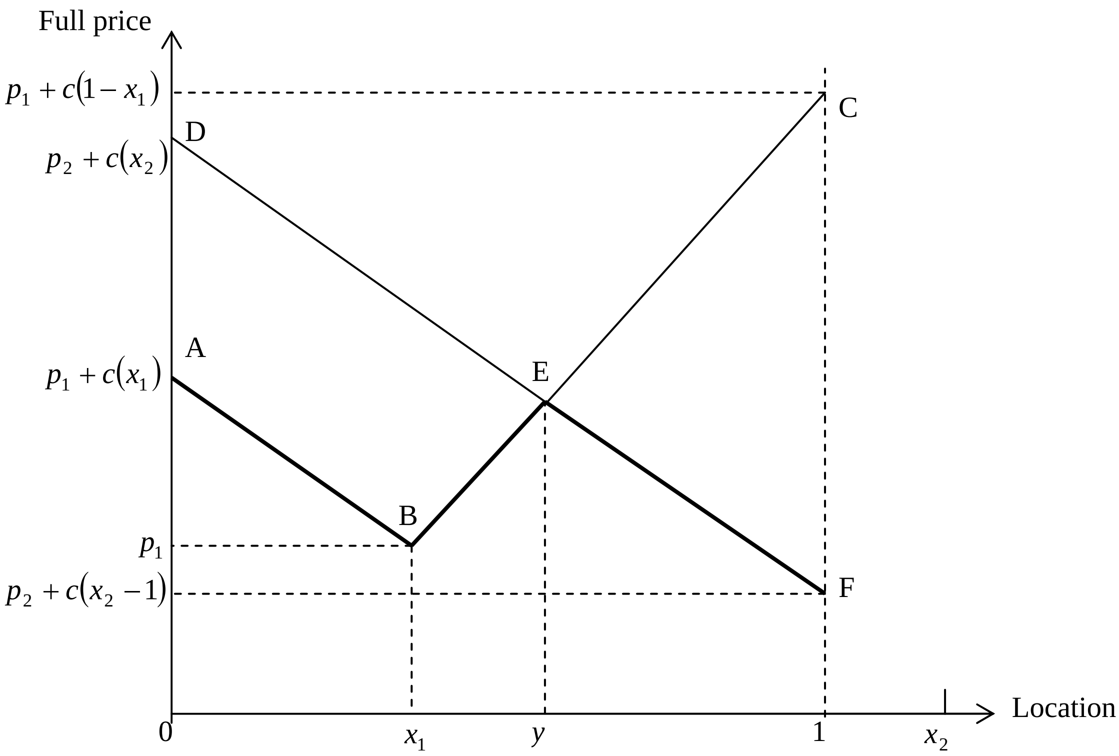

Fig. 2.2 Full price paid by consumers at various locations in $[0,1]$ under linear transportation costs

while the payoff function of firm 2 is defined as

$$\Pi_2(p_1, p_2; x_1, x_2) = p_2 \int_{m(p_1,p_2)}^{1} f(z)dz \qquad \text{if } m(p_1, p_2) \text{ exists,}$$

$$= p_2 \qquad \text{if equation 2.2 holds,}$$

$$= 0 \qquad \text{if equation 2.1 holds.}$$

Assuming linear transportation costs, figure 2.2 illustrates the full price of the good at various locations of the consumer given the cost schedule ABC if he buys from firm 1 and DF if he buys from firm 2. The bold line ABEF depicts the lowest full price at any given location. The intersection of the two cost schedules at y denotes the location of the marginal consumer. It is obvious from the figure that for the marginal consumer to exist, he must locate in $[x_1,1]$.

2.3 EQUILIBRIUM UNDER PARAMETRIC LOCATIONS

Suppose transportation costs are linear-quadratic bearing the form $c(d) = td + sd^2$ where $c(0) = 0$ and $t, s > 0$. If $m(p_1, p_2)$ exists, it must be the solution of the equation

$$p_1 + t(y - x_1) + s(y - x_1)^2 = p_2 + t(x_2 - y) + s(x_2 - y)^2 .$$

Solving, we obtain the demand functions for firm 1 and firm 2, respectively, as

(2.3) $$m_1(p_1, p_2) = \frac{p_2 - p_1}{2[t + s(x_2 - x_1)]} + \frac{(x_1 + x_2)}{2}$$ and

(2.4) $$m_2(p_1, p_2) = \frac{p_1 - p_2}{2[t + s(x_2 - x_1)]} + \frac{(2 - x_1 - x_2)}{2} .$$

with the payoff functions given by $\prod_1(p_1, p_2) = p_1 \cdot m_1(p_1, p_2)$ and $\prod_2(p_1, p_2) = p_2 \cdot m_2(p_1, p_2)$ respectively. Maximising profits on the part of firm 1 and firm 2 by setting $\partial \prod_i (p_1^*, p_2^*) / \partial p_i = 0$ where $i = \{1,2\}$, gives the following best response functions:

(2.5) $$p_1^* = \frac{1}{2}\left[p_2^* + (t + s(x_2 - x_1))(x_1 + x_2)\right]$$ and

(2.6) $$p_2^* = \frac{1}{2}\left[p_1^* + (t + s(x_2 - x_1))(2 - x_1 - x_2)\right].$$

Solving equations 2.5 and 2.6 gives the non-cooperative Bertrand-Nash price equilibrium in pure strategies

(2.7) $$(p_1^*, p_2^*) = \left(\frac{t + s(x_2 - x_1)}{3}(x_1 + x_2 + 2), \frac{t + s(x_2 - x_1)}{3}(4 - x_1 - x_2) \right).$$

For non-zero p_2^*, we assume that $x_1 + x_2 < 4$. In other words, the upper bound on the location of firm 2 for it to remain viable is $x_2 < 4 - x_1$. If $x_1 + x_2 = 4$, an equilibrium exists at

$$(p_1^*, p_2^*) = \left(\frac{t + s(x_2 - x_1)}{2}(x_1 + x_2), 0 \right).$$

Intuitively, this means that when the distance between the two firms becomes too large, firm 1 becomes a monopoly and gains the whole market while firm 2 drops out of the competition. Equation 2.7 shows that the equilibrium prices are dependent on all the parameters of the model, *viz.*, the locations of the two firms as well as the transportation cost. The distribution of market demand between firm 1 and firm 2 at Nash equilibrium is obtained by substituting equation 2.7 into equations 2.3 and 2.4 giving

(2.8) $$\left(m_1^*, m_2^*\right) = \left(\frac{1}{6}\left(x_1 + x_2 + 2\right), \frac{1}{6}\left(4 - x_1 - x_2\right)\right).$$

Equation 2.8 shows that the equilibrium demand is dependent only on the location of the two firms.

A similar exposition can be conducted for the cases in which transportation costs are linear of the form $c(d) = td$ where $c(0) = 0$ and $t > 0$, and quadratic of the form $c(d) = sd^2$ where $c(0) = 0$ and $s > 0$ (See Appendices 1 and 2). In both instances, $p_2^* > 0$ whenever $x_1 + x_2 < 4$. Under linear transportation costs, however, a unique equilibrium exists if and only if $\left(\left(x_1 + x_2 + 2\right)/3\right)^2 \geq 4\left(2 + x_1 - 2x_2\right)/3$ and $\left(\left(4 - x_1 - x_2\right)/3\right)^2 \geq 4\left(1 + 2x_1 - x_2\right)/3$ when $x_1 + x_2 \leq 4$ (see the proof in Appendix 1). The equilibrium price and equilibrium demand are given in Table 2.1 for non-zero p_2^*, along with the contrasting results for the pure inside location and outside location models.

It is obvious from the results that the IO model shares some of the features of the pure inside location model as well as the pure outside location model. The equilibrium price and demand are the same for the IO model and the inside location model for all transportation costs considered, and are identical for the IO model and the outside location model under quadratic transportation costs. Moreover, the equilibrium demand remains the same regardless of the transportation cost structure for both the IO model and the inside location model. The same conclusion, however, cannot be extended to the outside location model where both firms are located beyond the residential area of the consumers. Under duopolistic competition, therefore, it appears that when at least one of the firms is located within the same

Table 2.1
Equilibrium price and demand of the inside, outside and IO models under various transportation cost structures when location is parametric

	Price Equilibrium	Demand Equilibrium
Inside Location Model		
$c(d)=td$	$(p_1^*, p_2^*)=\left(\frac{t}{3}(x_1+x_2+2), \frac{t}{3}(4-x_1-x_2)\right)$	$(m_1^*, m_2^*)=\left(\frac{1}{6}(x_1+x_2+2), \frac{1}{6}(4-x_1-x_2)\right)$
$c(d)=sd^2$	$(p_1^*, p_2^*)=\left(\frac{s}{3}(x_2-x_1)(x_1+x_2+2), \frac{s}{3}(x_2-x_1)(4-x_1-x_2)\right)$	$(m_1^*, m_2^*)=\left(\frac{1}{6}(x_1+x_2+2), \frac{1}{6}(4-x_1-x_2)\right)$
$c(d)=td+sd^2$	$(p_1^*, p_2^*)=\left(\frac{t+s(x_2-x_1)}{3}(x_1+x_2+2), \frac{t+s(x_2-x_1)}{3}(4-x_1-x_2)\right)$	$(m_1^*, m_2^*)=\left(\frac{1}{6}(x_1+x_2+2), \frac{1}{6}(4-x_1-x_2)\right)$
Outside Location Model		
$c(d)=td$	$(p_1^*, p_2^*)=(t(x_2-x_1), 0)$	$(m_1^*, m_2^*)=(1,0)$
$c(d)=sd^2$	$(p_1^*, p_2^*)=\left(\frac{s}{3}(x_2-x_1)(x_1+x_2+2), \frac{s}{3}(x_2-x_1)(4-x_1-x_2)\right)$	$(m_1^*, m_2^*)=\left(\frac{1}{6}(x_1+x_2+2), \frac{1}{6}(4-x_1-x_2)\right)$
$c(d)=td+sd^2$	$(p_1^*, p_2^*)=\left(\frac{(x_2-x_1)}{3}(s(x_1+x_2+2)+t), \frac{(x_2-x_1)}{3}(s(4-x_1-x_2)-t)\right)$	$(m_1^*, m_2^*)=\left(\frac{1}{6}\left(\frac{t}{s}+x_1+x_2+2\right), \frac{1}{6}\left(\frac{t}{s}+4-x_1-x_2\right)\right)$
Inside-Outside Location Model		
$c(d)=td$	$(p_1^*, p_2^*)=\left(\frac{t}{3}(x_1+x_2+2), \frac{t}{3}(4-x_1-x_2)\right)$	$(m_1^*, m_2^*)=\left(\frac{1}{6}(x_1+x_2+2), \frac{1}{6}(4-x_1-x_2)\right)$
$c(d)=sd^2$	$(p_1^*, p_2^*)=\left(\frac{s}{3}(x_2-x_1)(x_1+x_2+2), \frac{s}{3}(x_2-x_1)(4-x_1-x_2)\right)$	$(m_1^*, m_2^*)=\left(\frac{1}{6}(x_1+x_2+2), \frac{1}{6}(4-x_1-x_2)\right)$
$c(d)=td+sd^2$	$(p_1^*, p_2^*)=\left(\frac{t+s(x_2-x_1)}{3}(x_1+x_2+2), \frac{t+s(x_2-x_1)}{3}(4-x_1-x_2)\right)$	$(m_1^*, m_2^*)=\left(\frac{1}{6}(x_1+x_2+2), \frac{1}{6}(4-x_1-x_2)\right)$

Note:

When transportation costs are linear, a unique price equilibrium exists if and only if $((x_1+x_2+2)/3)^2 \geq 4(2+x_1-2x_2)/3$ and $((4-x_1-x_2)/3)^2 \geq 4(1+2x_1-x_2)/3$ when (a) $x_1+x_2<1$, and $p_1^*=p_2^*=0$ when $x_1+x_2=1$ for the inside location model (d'Aspremont *et al.* 1979); and (b) when $x_1+x_2 \leq 4$ for the IO model. A unique price equilibrium exists for all location pairs (x_1, x_2) of the outside location model.

sub-space as the consumers, equilibrium demand depends only on the location of the two firms when location is parametric.

The following propositions encapsulate the results of the IO model whenever a solution exists in pure strategies with non-zero prices (see Appendix 3 for the proofs).

Proposition 1

When firm locations are fixed, the equilibrium relative price p_2^* / p_1^* is independent of the transportation cost structure.

Proposition 2

The equilibrium market demand $\left(m_1^*, m_2^*\right)$ for the good has the following properties:

2.1 It is the same regardless of the transportation cost structure when firm locations are fixed.

2.2 Relative demand is equivalent to relative prices.

Proposition 3

Given a transportation cost structure, the inside firm raises (lowers) its price when faced with higher (lower) transportation costs. The outside firm reacts by raising (lowering) its price but by a smaller amount.

The equilibrium relative price of the good offered by the outside firm to the inside firm is an indication of the "exchange rate" of the good at the two sources. Intuitively, Proposition 1 means that when firm locations are fixed, the outside firm is able to attract the consumer by offering the good at the same price relative to that offered by the inside firm regardless of the nature of the transportation cost structure of the consumer. This result is not surprising since, by Fetter (1924)'s definition, the market boundary is determined by the relative price and the relative transportation costs, and the latter is constant ($t^{'}/t = 1$, $s^{'}/s = 1$) by assumption for this model.

Proposition 2 can be interpreted as follows. The relative market demand, which reflects the market area of the two firms, delineates the market boundary that is determined solely by the relative price in the IO model. This explains the equality of relative market demand and relative price at equilibrium.

Proposition 3 effectively means that under a given transportation cost regime, the inside firm offers the product at a higher price when transportation costs increase. The outside firm reacts by attempting to "compensate" the consumer for the higher transportation costs incurred, resulting in a corresponding but smaller price increase. This result holds similarly when the inside firm is faced with a change in the transportation cost structure from a low cost regime to a high cost regime (see Appendix 3).

It appears that the propositions also apply to the inside location model. Looking at Table 2.1, it is clear that that in the inside location model, the relative price and relative demand under the three transportation cost structures are equivalent. Proposition 3 applies only if $x_1 > x_2$ for an increase in transportation cost to $t^{'} > t$ and $s^{'} > s$ to result in $p_1^{'*} > p_2^{'*}$. The propositions, however, are invalid in the outside location model except for Proposition 2.2 and Proposition 3 under quadratic transportation costs when the results are identical to the IO model.

2.4 THE SIMULTANEOUS PRICE-LOCATION GAME

In the longer run, the locations of firms are not fixed but variable. When firms choose price and location together, we have a simultaneous price-location game. The choice of both price and location in each period of the market game is reflective of situations in which players commit to a price for a period as long as the product lifetime. For example, firms like chain stores publish a catalogue and stick to it for a while. Agency situations may force an employee that work as a seller to commit to the announced price. The ensuing discussion shows that for the IO model, non-existence of equilibrium in pure strategies emerges under linear transportation costs but not under non-linear transportation costs when the game is played simultaneously.

In the simultaneous game, the strategy pairs are (p_1, x_1) for the inside firm 1 and (p_2, x_2) for the outside firm 2, with $p_1, p_2 \in [0,+\infty[$, $x_1 \in [0,1]$ and $x_2 \in]1,+\infty[$. The Nash equilibrium in prices and locations is one in which no firm wishes to change its price and/or location given the price and location it anticipates the other firm will choose. The payoff function for firm 1 at equilibrium satisfies

$$\Pi_1\left(\left(p_1^*, x_1^*\right), \left(p_2^*, x_2^*\right)\right) \geq \Pi_1\left(\left(p_1, x_1\right), \left(p_2^*, x_2^*\right)\right) \tag{2.9}$$

for all $x_1 \in [0,1]$ and $p_1 \geq 0$, while the payoff function for firm 2 satisfies

$$\Pi_2\left(\left(p_1^*, x_1^*\right), \left(p_2^*, x_2^*\right)\right) \geq \Pi_2\left(\left(p_1^*, x_1^*\right), \left(p_2, x_2\right)\right) \tag{2.10}$$

for all $x_2 \in]1,+\infty[$ and $p_2 \geq 0$.

It is readily verified that the only pure strategy equilibrium for the simultaneous game involves $x_1^* \in [0,1]$ and $x_2^* = 1 + \varepsilon$ where $\varepsilon > 0$ is a small constant close to zero representing a physical divide between two countries, e.g., the sea, a mountain, etc.[2] In other words, the dominant location strategy for firm 2 is to locate at $x_2^* = 1 + \varepsilon$. The argument is as follows. Suppose that $x_2 = 1 + \varepsilon$ is not an equilibrium, i.e., we have a candidate equilibrium whereby firm 2 locates at $\tilde{x}_2 > 1 + \varepsilon$ with both firms earning positive market shares. Firm 2 can then increase profit by moving closer to its rival and locating at $x_2^* = 1 + \varepsilon$. Formally,

$$\Pi_2\left(\left(p_1^*, x_1^*\right), \left(p_2^*, \tilde{x}_2\right)\right) < \Pi_2\left(\left(p_1^*, x_1^*\right), \left(p_2^*, x_2^*\right)\right).$$

Hence, $\tilde{x}_2 > 1 + \varepsilon$ cannot be an equilibrium and firm 2 necessarily locates at $x_2^* = 1 + \varepsilon$.

[2] When $\varepsilon = 0$, the market boundary at 1 represents a seamless economic and (or) geographical border between the two markets. In the ensuing discussions, we assume that $\varepsilon > 0$.

2.4.1 Equilibrium Existence

Consider the scenario in which firms 1 and 2 experience linear-quadratic transportation costs of the form $c(d) = td + sd^2$ where $c(0) = 0$ and $t, s > 0$. The profit functions of firm 1 and firm 2 are given by the following equations respectively:

$$\Pi_1((p_1, x_1),(p_2, x_2)) = \frac{p_1 p_2 - p_1^2}{2[t + s(x_2 - x_1)]} + \frac{(x_1 + x_2)}{2} p_1$$

and $$\Pi_2((p_1, x_1),(p_2, x_2)) = \frac{p_1 p_2 - p_2^2}{2[t + s(x_2 - x_1)]} + \frac{(2 - x_1 - x_2)}{2} p_2 .$$

Firm 1 maximises profit by choosing x_1^* with the first order condition given by

$$\frac{\partial \Pi_1((p_1, x_1),(p_2, x_2))}{\partial x_1} = \frac{p_1^*}{2}\left(\frac{s(p_2^* - p_1^*)}{[t + s(x_2^* - x_1^*)]^2} + 1\right) = 0 .$$

Substituting $p_1^* = [(t + s(x_2^* - x_1^*))(x_1^* + x_2^* + 2)]/3$ and $p_2^* = [(t + s(x_2^* - x_1^*))(4 - x_1^* - x_2^*)]/3$ (equation 2.7)) obtained by maximising the respective firm's profit with respect to price into the above gives

$$\frac{s}{18}(x_1^* + x_2^* + 2)\left(2 - 5x_1^* + x_2^* + \frac{3t}{s}\right) = 0 .$$

Since $s > 0$ and $(x_1^* + x_2^* + 2) > 0$, this implies that $(2 - 5x_1^* + x_2^* + 3t/s) = 0$. In other words, the equilibrium location of firm 1 is at

$$x_1^* = \frac{(2 + x_2^*)}{5} + \frac{3t}{5s} \tag{2.11}$$

which gives the response function in location of firm 1.

In the case of firm 2, it maximises profit by choosing x_2^* such that

$$\frac{\partial \Pi_2((p_1, x_1),(p_2, x_2))}{\partial x_2} = -\frac{p_2^*}{2}\left(\frac{s(p_1^* - p_2^*)}{[t + s(x_2^* - x_1^*)]^2} + 1\right) < 0$$

since $p_1^* > p_2^*$ from equation 2.7 for all $x_1 + x_2 < 4$. This implies that firm 2 increases profit by moving towards the market border, i.e., $x_2^* = 1 + \varepsilon$, $\varepsilon > 0$. Solving for x_1^* by substituting $x_2^* = 1 + \varepsilon$ into equation 2.11 gives $x_1^* = 3/5(1 + t/s) + \varepsilon/5$. For a unique equilibrium in location to exist, $x_1^* \le 1$ or $t/s \le 2/3$. The equilibrium prices are obtained by substituting x_1^* and x_2^* into equation 2.7 so that $p_1^* = 2/25[6s + t(7 + t/s)] + 4\varepsilon/25[2t + 7s + 2s\varepsilon]$ and $p_2^* = 2/25[4s + t(3 - t/s)] - 4\varepsilon/25[2t - 3s + 2s\varepsilon]$. The simultaneous price-location equilibrium in pure strategies is, therefore, given by

(2.12) $$\left((p_1^*, x_1^*), (p_2^*, x_2^*)\right) = \left(\left(\frac{2}{25}\left[6s + t\left(7 + \frac{t}{s}\right)\right] + \frac{4\varepsilon}{25}[2t + 7s + 2s\varepsilon],\ \frac{3}{5}\left(1 + \frac{t}{s}\right) + \varepsilon\right), \left(\frac{2}{25}\left[4s + t\left(3 - \frac{t}{s}\right)\right] - \frac{4\varepsilon}{25}[2t - 3s + 2s\varepsilon],\ 1 + \varepsilon\right)\right)$$

where $\varepsilon > 0$.

The simultaneous price-location equilibrium in pure strategies under quadratic transportation costs can be similarly obtained (see Appendix 4).

2.4.2 Equilibrium Non-Existence

We will now turn to the non-existence problem of the simultaneous price-location game when both firms face linear transportation costs. Assume that transportation costs are linear of the form $c(d) = td$ where $c(0) = 0$ and $t > 0$. The profit functions of firm 1 and firm 2 are given by the following equations respectively:

$$\Pi_1((p_1, x_1), (p_2, x_2)) = \frac{p_1 p_2 - p_1^2}{2t} + \frac{(x_1 + x_2)}{2} p_1$$

and $$\Pi_2((p_1, x_1), (p_2, x_2)) = \frac{p_1 p_2 - p_2^2}{2t} + \frac{(2 - x_1 - x_2)}{2} p_2 .$$

Firm 1 chooses the optimal location x_1^* that maximises its profit. Since

$$\frac{\partial \Pi_1((p_1, x_1), (p_2, x_2))}{\partial x_1} = \frac{p_1^*}{2} > 0,$$

firm 1 raises its profit by moving towards firm 2 which gives its equilibrium location as $x_1^* = 1$. At the same time, firm 2’s dominant strategy is to choose $x_2^* = 1 + \varepsilon$. This is obvious from maximising firm 2’s profit with respect to location which gives

$$\frac{\partial \Pi_2((p_1, x_1), (p_2, x_2))}{\partial x_2} = -\frac{p_2^*}{2} < 0 .$$

As a result, firm 2 increases its profit by moving towards the market boundary. The equilibrium location of firm 2 is then given by $x_2^* = 1 + \varepsilon$, $\varepsilon > 0$ is a small constant. Substituting x_1^* and x_2^* into firm 1 and firm 2's response function (equations A3 and A4 in Appendix 1) gives $p_1^* = t/3(4 + \varepsilon)$ and $p_2^* = t/3(2 - \varepsilon)$. This is not possible as the price differential results in a price war between the two firms located next to each other. Their attempt to undercut each other by moving apart naturally generates instability in the location choice of the two firms. The simultaneous price-location equilibrium in pure strategies, therefore, does not exist when transportation costs are linear.

2.4.3 Comparative Analysis

The results of the simultaneous game are summarised in Table 2.2, along with the comparative equilibrium strategies for the pure inside location and outside location models. No simultaneous price-location equilibrium in pure strategies can exist in the inside location model while the simultaneous price-location equilibrium in pure strategies for the outside location model is for the two firms to always locate at $x_1^* = x_2^* = 1$ with prices $p_1^* = p_2^* = 0$ (see Gabszewicz and Thisse 1992).[3] The IO model, with the horizontal differentiation characteristics of the inside location model, has the same instability problem as the inside location model under linear transportation costs. Incorporating the vertical differentiation characteristics of the outside location model, however, has rendered the IO model greater stability than the pure inside location model in that an equilibrium in pure strategies exists when the transportation cost structure is quadratic and linear-quadratic. It can be readily verified that with variable location of firms (as opposed to fixed location) Propositions 1 and 2.1 do not hold in the simultaneous game of the IO model but Propositions 2.2 and 3 remain valid whenever an equilibrium in pure strategies exists (Appendix 5 Propositions 1A to 3A).

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[3] De Palma *et al.* (1985), however, showed that a simultaneous price-location equilibrium exists in the inside location model if the product is heterogeneous enough. Anderson *et al.* (1992) further showed that the only symmetric pure strategy equilibrium occurs when both firms agglomerate at the market centre.

Table 2.2
Simultaneous price-location equilibrium of the inside, outside and IO models under various transportation cost structures

	Location Equilibrium	Price Equilibrium
Inside Location Model		
$c(d)=td$	No equilibrium exists	No equilibrium exists
$c(d)=sd^2$	No equilibrium exists	No equilibrium exists
$c(d)=td+sd^2$	No equilibrium exists	No equilibrium exists
Outside Location Model		
$c(d)=td$	$(x_1^*, x_2^*)=(1,1)$	$(p_1^*, p_2^*)=(0,0)$
$c(d)=sd^2$	$(x_1^*, x_2^*)=(1,1)$	$(p_1^*, p_2^*)=(0,0)$
$c(d)=td+sd^2$	$(x_1^*, x_2^*)=(1,1)$	$(p_1^*, p_2^*)=(0,0)$
Inside-Outside Location Model		
$c(d)=td$	No equilibrium exists	No equilibrium exists
$c(d)=sd^2$	$(x_1^*, x_2^*)=\left(\frac{1}{5}(3+\varepsilon), 1+\varepsilon\right)$	$(p_1^*, p_2^*)=\left(\frac{4s}{25}(3-5\varepsilon-2\varepsilon^2), \frac{4s}{25}(2-5\varepsilon+2\varepsilon^2)\right)$
$c(d)=td+sd^2$	$(x_1^*, x_2^*)=\left(\frac{3}{5}\left(1+\frac{t}{s}\right)+\frac{\varepsilon}{5}, 1+\varepsilon\right)$	$(p_1^*, p_2^*)=\left(\frac{2}{25}\left[6s+t\left(7+\frac{t}{s}\right)\right]+\frac{4\varepsilon}{25}(2t+7s+2s\varepsilon), \frac{2}{25}\left[4s+t\left(3-\frac{t}{s}\right)\right]-\frac{4\varepsilon}{25}(2t-3s+2s\varepsilon)\right)$

Note:
When transportation costs are linear-quadratic, a unique equilibrium in location exists whenever $t/s \le 2/3$ for the IO model.

2.5 THE SEQUENTIAL GAME

When relocation of firms is more costly than price adjustments, a sequential location-then-price game becomes more appropriate. In the sequential game first introduced by Hotelling (1929), there is a two-stage process in which the location strategy is played first in full anticipation of the ensuing price equilibrium, followed by the price strategy in the second stage where prices are decided based on the location choice made in the first stage. The solution to the sequential game is worked out using backward induction. In a subgame consisting of the second stage, a non-cooperative price equilibrium in pure strategies with prices $p_1^*(x_1, x_2)$ and $p_2^*(x_1, x_2)$ are chosen for given locations x_1 and x_2. The pure strategy equilibrium to the first-stage location game is the pair of locations (x_1^*, x_2^*) which maximises the profit function $\prod_i \left(p_1^*(x_1, x_2), p_2^*(x_1, x_2), x_1, x_2\right)$ where $i = \{1,2\}$. This profit function is well defined whenever the price equilibrium exists and is unique. The full (subgame perfect) equilibrium to the game is then given by the quadruple $\left(p_1^*, p_2^*, x_1^*, x_2^*\right)$ where $p_1^* = p_1^*\left(x_1^*, x_2^*\right)$ and $p_2^* = p_2^*\left(x_1^*, x_2^*\right)$. As in the pure inside location model, it will be shown that an equilibrium in pure strategies also fails to exist for the sequential game of the IO model when transportation costs are linear. Unlike the inside location model, however, which possesses an equilibrium for the sequential game when transportation costs are quadratic but not when they are linear or linear-quadratic (d'Aspremont *et al*. 1979; Anderson 1988), an equilibrium always exists for the IO model whenever transportation costs are strictly convex.

2.5.1 Equilibrium Existence

Consider the case in which the transportation cost function is linear-quadratic of the form $c(d) = td + sd^2$ where $c(0) = 0$, $t > 0$ and $s > 0$. When $x_1 + x_2 < 4$, the unique price equilibrium in pure strategies is given by equation 2.7, i.e.,

(2.13) $$\left(p_1^*(x_1, x_2), p_2^*(x_1, x_2)\right) = \left(\frac{t + s(x_2 - x_1)}{3}(x_1 + x_2 + 2), \frac{t + s(x_2 - x_1)}{3}(4 - x_1 - x_2)\right).$$

The profit function of the inside firm 1 is given by

$$\prod_1 \left(p_1(x_1, x_2), p_2(x_1, x_2), x_1, x_2\right) = \frac{p_1 p_2 - p_1^2}{2[t + s(x_2 - x_1)]} + \frac{(x_1 + x_2)}{2} p_1 .$$

Substituting equation 2.13 gives

$$\prod_1 \left(p_1^*(x_1, x_2), p_2^*(x_1, x_2), x_1, x_2\right) = \frac{t + s(x_2 - x_1)}{18}(x_1 + x_2 + 2)^2 .$$

Optimising with respect to x_1 gives

$$\frac{\partial \prod_1 \left(p_1^*(x_1, x_2), p_2^*(x_1, x_2), x_1, x_2\right)}{\partial x_1} = \frac{\left(x_1^* + x_2^* + 2\right)}{18}\left(2t + s\left(x_2^* - 3x_1^* - 2\right)\right).$$

Since $\left(x_1^* + x_2^* + 2\right) > 0$, one possible scenario is that $2t + s\left(x_2^* - 3x_1^* - 2\right) < 0$ or

(2.14) $$\frac{t}{s} < \frac{3x_1^* + x_2^*}{2} + 1 .$$

If equation 2.14 holds, then $\partial \Pi_1\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_1 < 0$ which implies that as x_1 decreases, firm 1's profit increases so that firm 1's optimal location is at the point 0. If the converse of equation 2.14 holds, then two instances can arise: either $\partial \Pi_1\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_1 > 0$ or $\partial \Pi_1\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_1 = 0$. If the former holds, then firm 1's profit is maximised by locating at point 1.

Now consider the profit function for the outside firm 2 which is given by

$$\Pi_2\left(p_1(x_1, x_2), p_2(x_1, x_2), x_1, x_2\right) = \frac{p_1 p_2 - p_2^2}{2\left[t + s(x_2 - x_1)\right]} + \frac{(2 - x_1 - x_2)}{2} p_2 .$$

Substituting equation 2.13 gives

$$\Pi_2\left(p_1^*(x_1, x_2), p_2^*(x_1, x_2), x_1, x_2\right) = \frac{t + s(x_2 - x_1)}{18}(4 - x_1 - x_2)^2 .$$

Optimising with respect to x_2 gives

$$\frac{\partial \Pi_2\left(p_1^*(x_1, x_2), p_2^*(x_1, x_2), x_1, x_2\right)}{\partial x_2} = \frac{\left(4 - x_1^* - x_2^*\right)}{18}\left(s\left(x_1^* - 3x_2^* + 4\right) - 2t\right).$$

Since $x_1^* + x_2^* < 4$, this implies that for $\partial \Pi_2\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_2 = 0$, $s\left(x_1^* - 3x_2^* + 4\right) - 2t = 0$ or

(2.15) $$x_1^* - 3x_2^* + 4 = \frac{2t}{s} .$$

Suppose that equation 2.14 holds, i.e., $\partial \Pi_1\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_1 < 0$ and $x_1^* = 0$. Substituting $x_1^* = 0$ into equation 2.15 and solving gives $x_2^* = 4/3 - 2t/3s$. Since $x_2 \in \left]1, +\infty\right[$, the condition for x_2^* to hold is that $t/s < ½$.

We will now show that the converse of equation 2.14 is never valid. Suppose that $t/s > 1 + \left(3x_1^* + x_2^*\right)/2$ holds so that $\partial \Pi_1\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_1 > 0$ and $x_1^* = 1$. Substituting $x_1^* = 1$ into equation 2.15 and solving gives $x_2^* = 5/3 - 2t/3s$. This solution, however, cannot exist because it contradicts the assumed condition that $t/s > 1 + \left(3x_1^* + x_2^*\right)/2$. Substituting

$x_1^* = 1$ and $x_2^* = 5/3 - 2t/3s$ gives $t/s > 5/2$. This condition, however, suggests that $x_2^* = 5/3 - 2t/3s < 0$ which cannot hold since $x_2 \in]1, +\infty[$.

We will now show that $\partial \prod_1 \left(p_1^*, p_2^*, x_1, x_2\right) / \partial x_1 = 0$ also cannot hold. Suppose on the contrary that $\partial \prod_1 \left(p_1^*, p_2^*, x_1, x_2\right) / \partial x_1 = 0$. In that case, equation 2.14 becomes the equality $3x_1^* - x_2^* + 2 = 2t/s$. Solving this equation together with equation 2.15 gives $x_1^* = t/2s - 1/4$ and $x_2^* = 5/4 - t/2s$. By assumption of the model, we have $x_1 \in [0,1]$ and $x_2 \in]1, +\infty[$. Consequently, $x_1^* = t/2s - 1/4$ implies that $t/s \in [½, \tfrac{5}{2}]$ and $x_2^* = 5/4 - t/2s$ implies that $t/s < ½$, which contradicts $t/s \in [½, \tfrac{5}{2}]$.

The only solution in pure strategies to the first-stage of the sequential game is, therefore, $\left(x_1^*, x_2^*\right) = \left(0, 4/3 - 2t/3s\right)$ for $x_1 + x_2 < 4$ and $t/s < ½$. The second-stage game is then solved by substituting x_1^* and x_2^* into equation 2.13. The equilibrium price pair in pure strategies is given by $\left(p_1^*, p_2^*\right) = \left(2\left(20s + t\left(1 - t/s\right)\right)/27,\ \ 2\left(16s + t\left(8 + t/s\right)\right)/27\right)$. The full (subgame perfect) equilibrium to the sequential game in pure strategies is given by

(2.16) $$\left(p_1^*, p_2^*, x_1^*, x_2^*\right) = \left(\frac{2}{27}\left(20s + t\left(1 - \frac{t}{s}\right)\right), \frac{2}{27}\left(16s + t\left(8 + \frac{t}{s}\right)\right), 0, \frac{4}{3} - \frac{2t}{3s}\right)$$

where $x_1 + x_2 < 4$ and $t/s < ½$.

The equilibrium of the sequential game in pure strategies under quadratic transportation costs can be similarly obtained (see Appendix 6).

2.5.2 Equilibrium Non-Existence

We now turn to the non-existence problem in the sequential game which resurfaces under linear transportation costs. Under linear transportation costs when $x_1 + x_2 < 4$, the unique price equilibrium in pure strategies is given by equation A5, i.e.,

(2.17) $$\left(p_1^*, p_2^*\right) = \left(\frac{t}{3}\left(x_1 + x_2 + 2\right), \frac{t}{3}\left(4 - x_1 - x_2\right)\right).$$

The profit function of firm 1 is given by

$$\Pi_1\left(p_1(x_1,x_2),\, p_2(x_1,x_2), x_1, x_2\right) = \frac{p_1 p_2 - p_1^2}{2t} + \frac{(x_1+x_2)}{2}\, p_1 .$$

Substituting equation 2.17 gives

$$\Pi_1\left(p_1^*(x_1,x_2),\, p_2^*(x_1,x_2), x_1, x_2\right) = \frac{t}{18}(x_1+x_2+2)^2 .$$

Optimising with respect to x_1 gives

$$\frac{\partial \Pi_1\left(p_1^*(x_1,x_2),\, p_2^*(x_1,x_2), x_1, x_2\right)}{\partial x_1} = \frac{t}{9}\left(x_1^* + x_2^* + 2\right) > 0$$

since $t > 0$ and $\left(x_1^* + x_2^* + 2\right) > 0$. Since firm 1's profit increases as x_1 increases, it maximises profit by locating at point 1.

Now consider the profit function for firm 2 which is given by

$$\Pi_2\left(p_1(x_1,x_2),\, p_2(x_1,x_2), x_1, x_2\right) = \frac{p_1 p_2 - p_2^2}{2t} + \frac{(2-x_1-x_2)}{2}\, p_2 .$$

Substituting equation 2.17 gives

$$\Pi_2\left(p_1^*(x_1,x_2),\, p_2^*(x_1,x_2), x_1, x_2\right) = \frac{t}{18}(4-x_1-x_2)^2 .$$

Optimising with respect to x_2 gives

$$\frac{\partial \Pi_2\left(p_1^*(x_1,x_2),\, p_2^*(x_1,x_2), x_1, x_2\right)}{\partial x_2} = -\frac{t}{9}\left(4 - x_1^* - x_2^*\right) < 0$$

since $t > 0$ and $x_1^* + x_2^* < 4$. In other words, firm 2's profit increases as x_2 decreases so that it maximises profit by locating at point $1+\varepsilon$ where ε is a small constant. The solution in pure strategies to the first-stage of the sequential game is, therefore, $\left(x_1^*, x_2^*\right) = (1, 1+\varepsilon)$ for $x_1 + x_2 < 4$.

The second-stage game is then solved by substituting x_1^* and x_2^* into equation 2.17. The equilibrium price pair in pure strategies is then given by $\left(p_1^*, p_2^*\right) = \left((4+\varepsilon)t/3, (2-\varepsilon)t/3\right)$. This is not possible as the price differential creates opportunities for both firms that are situated next to each other to engage in a price war and undercut each other by moving apart, giving rise to instability in the location choice of the two firms. As with the simultaneous game, therefore, an equilibrium of the sequential game in pure strategies does not exist when transportation costs are linear.

2.5.3 Comparative Analysis

The results of the sequential game are summarised in Table 2.3, along with the comparative equilibrium for the pure inside and outside location models. When transportation costs are linear or linear-quadratic, no equilibrium in pure strategies can exist in the inside location model, while the equilibrium in pure strategies for the outside location model always exists (see Gabszewicz and Thisse 1992). The IO model, which possesses the horizontal differentiation characteristics of the inside location model, has the same instability problem as the inside location model under linear transportation costs.

When transportation costs are quadratic, the Principle of Maximum Differentiation is established in the inside location model as well as the IO model. Simply put, the Principle of Maximum Differentiation states that two firms have a tendency to locate in opposite directions towards the end points of the linear city as a result of competition. By locating at $\left(x_1^*, x_2^*\right)=(0,1)$ and $\left(x_1^*, x_2^*\right)=(0, 4/3)$ respectively, firms in the inside and IO model exhibit greater tendency of differentiation under quadratic transportation costs.[4]

When faced with linear-quadratic transportation costs, firms in the outside and IO models make their location decisions based on all the parameters of the model. In the case of the outside location model, some tendency of increasing differentiation is observed although its intensity is lower than that under quadratic transportation costs. Similarly, the IO model reflects a tendency toward increasing differentiation which loses its intensity because of the very nature of the transportation cost function.

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[4] In contrast to Hotelling (1929)'s nomenclature, x_2 in this instance is defined from the origin (see Figure 2.1) instead of from point 1 so that $\left(x_1^*, x_2^*\right)=(0,1)$ rather than $(0,0)$ under quadratic transportation costs for the inside location model.

Table 2.3
Equilibrium in the sequential game of the inside, outside and IO models under various transportation cost structures

	Location Equilibrium	Price Equilibrium
Inside Location Model		
$c(d)=td$	no equilibrium exists	no equilibrium exists
$c(d)=sd^2$	$(x_1^*, x_2^*)=(0,1)$	$(p_1^*, p_2^*)=(s,s)$
$c(d)=td+sd^2$	no equilibrium exists	no equilibrium exists
Outside Location Model		
$c(d)=td$	$(x_1^*, x_2^*)=(1,1)$	$(p_1^*, p_2^*)=(0,0)$
$c(d)=sd^2$	$(x_1^*, x_2^*)=\left(1,\frac{5}{3}\right)$	$(p_1^*, p_2^*)=\left(\frac{28s}{27},\frac{8s}{27}\right)$
$c(d)=td+sd^2$	$(x_1^*, x_2^*)=\left(1,\frac{5}{3}-\frac{t}{3s}\right)$	$(p_1^*, p_2^*)=\left(\frac{2}{27}\left(14s-t\left(5+\frac{t}{s}\right)\right),\frac{2}{27}\left(4s-t\left(4-\frac{t}{s}\right)\right)\right)$
Inside-Outside Location Model		
$c(d)=td$	no equilibrium exists	no equilibrium exists
$c(d)=sd^2$	$(x_1^*, x_2^*)=\left(0,\frac{4}{3}\right)$	$(p_1^*, p_2^*)=\left(\frac{40s}{27},\frac{32s}{27}\right)$
$c(d)=td+sd^2$	$(x_1^*, x_2^*)=\left(0,\frac{4}{3}-\frac{2t}{3s}\right)$	$(p_1^*, p_2^*)=\left(\frac{2}{27}\left(20s+t\left(1-\frac{t}{s}\right)\right),\frac{2}{27}\left(16s+t\left(8+\frac{t}{s}\right)\right)\right)$

Note:
When transportation costs are linear-quadratic, a unique equilibrium in location exists whenever the following conditions hold: (a) outside location model: $t/s \le 2$; and (b) IO model: $t/s < ½$.

As with the simultaneous game, it can be easily determined that under variable firm locations Propositions 1 and 2.1 do not hold in the sequential game of the IO model but Propositions 2.2 and Proposition 3 remain valid whenever an equilibrium in pure strategies exists (see Appendix 7 Propositions 1A to 3A).

2.6 CONCLUSIONS

Product differentiation by firms located at the boundary regions of countries or cities is of pertinent significance and interest to various segments of society as a result of its attendant economic benefits and trickle down effects on the rest of the economy. The IO model presented here offers a simple framework for understanding and analysing the location and pricing decisions of firms situated on either side of the border, as well as the purchase and travel decisions of consumers between the domestic firm and the competing firm beyond their economic precincts. The IO model is readily applicable to analysing cross-border behaviour, whether from the point of view of buying (travel to shop) or selling (travel to work).

When firm locations are fixed, the market boundary is determined solely by the relative price of the two firms. This property satisfies Fetter (1924)'s economic law of market areas. As such, the IO model is directly applicable to situations in which adjoining markets are segmented geographically and (or) economically. It highlights the distinction between an economic boundary and geographical boundary between two regions, which in most cases do not necessarily coincide. Moreover, it appears that under duopolistic competition when at least one firm is located within the same subspace as the consumers, equilibrium prices are dependent on all the parameters of the model while equilibrium demand is determined only by the location of the two firms. The relevance of this property to oligopolistic competition with more than two firms needs to be verified by further study.

Another interesting observation that surfaced under parametric firm locations is that the results of the IO model and the inside location model are identical for all transportation costs considered, and for the IO model and the outside location model when transportation costs are quadratic. On the other hand, under variable firm locations and linear transportation costs, the results of the IO model and the inside location model are identical to a certain extent, i.e., there is a non-existence of equilibrium problem. However, the non-existence problem dissipates in the IO model when price and location decisions are made simultaneously under quadratic and linear-quadratic transportation costs but it persists in the inside location model. Moreover, the IO model does not suffer from the non-existence problem as the inside location model when the game is played sequentially under linear-quadratic transportation costs. This result contrasts with the outside location model where a solution always exists when location is variable. The stability of the IO model can, therefore, be said to be intermediate between the inside location model and the outside location model. This is not surprising since the IO model is an integration of the two models.

Although the IO model focuses on the situation in which there is only one firm on either side of the boundary, the framework presented can be generalised to the case in which there are multiple firms, as well as to a situation in which the neighbouring firm on the other side of the border is capable of locating beyond the boundary. Although this study offers solutions in pure competition of price and location, further extensions are feasible with respect to mixed strategies and collusions between firms, especially in instances where a parent company has several outlets on either side of the border. A myriad of other situations present themselves that are worthy of further study by modifying the basic assumptions inherent in the model, e.g., by incorporating price discrimination, production costs and a budget constraint. As such, the situations considered here do not pretend to be either exhaustive or comprehensive in the range of possible applications within this domain.

The ensuing analyses investigate the extent to which the IO model is validated by actual behaviour in a laboratory setting and by cross-border shopping behaviour between two cities. Two experiments are conducted to examine the IO model: the first studies firm behaviour under constant firm location while the second looks at firm behaviour when location is variable.

CHAPTER 3

EXPERIMENTAL EVIDENCE WITH PARAMETRIC FIRM LOCATION

3.1 INTRODUCTION

A commonly observed phenomenon is the wide dispersion of some firms selling identical products in contrast to the close location of others selling products with similar attributes. For example, convenience stores tend to choose disperse locations close to consumers while fast food restaurants often agglomerate but differentiate their products through advertising. This situation arises from the multitude of dimensions in which firms are able to compete, including price, location and product characteristics.

Theories of oligopolistic competition abound in their attempts to explain competition in its diverse dimensions. A typical vein runs along the spatial product differentiation analogue of Hotelling (1929)'s inside location theory. Two firms select a location along a line segment in which consumers are uniformly distributed. Consumers purchase the good by travelling to the firm that offers the lower *full price,* which is the mill price of the good plus a transportation cost that varies with distance. A large body of work exists in which varying assumptions are made within this spatial framework, such as the nature of transportation costs, the distribution of consumers and the number of firms. These theories have been discussed at length in Chapter 2.

Despite the wide interest generated by theories of spatial firm competition, experimental tests of such models have been relatively few. Efforts in this arena typically focus on the location decision of firms, e.g., Brown-Kruse *et al.* (1993), Brown-Kruse and

Schenk (2000), Collins and Sherstyuk (2000) and Huck *et al.* (2002).[1] These studies occasionally obtain results that are contrary to Hotelling's prediction of minimum product differentiation by varying the conditions in which firms compete.[2]

Brown-Kruse *et al.* (1993) found that in a repeated game of Hotelling's spatial duopoly with elastic demand and probabilistic ending period, minimum product differentiation results when both firms cannot communicate but higher product differentiation emerges when communication is permitted. In particular, firms that engage in non-binding communication or *cheap talk* tend to choose the joint-profit maximising market quartiles at 0.25 and 0.75. Brown-Kruse and Schenk (2000) extended this study by introducing different forms of consumer distribution: uniform, unimodal (consumers concentrated at the market centre) and bimodal (consumers dispersed from the centre). They found that regardless of the form of consumer distribution, communication is a robust facilitator of a higher level of product differentiation than that obtained under non-collusive outcomes.

Collins and Sherstyuk (2000) studied competition among three firms randomly grouped for each period. Consumers are uniformly distributed and have an inelastic demand. The number of periods is fixed but unknown to the players. The authors found that firms generally locate in the central quartiles of the market over $[0.25,0.75]$ in accordance with Shaked (1982)'s prediction. There is, however, a wider dispersion of location choices due to risk averse behaviour and approximate equilibrium behaviour.[3] Since the risk neutral equilibrium at the market centre gives the maximum profit and the highest standard deviation of profit, players who are risk averse are induced to reduce the variance of their profit by locating away from the centre, unless the expected profit at the centre is sufficiently high.

Huck *et al.* (2002) conducted a similar study but with competition among four firms. Although the prediction is for two firms to locate at 0.25 and the other two firms to locate at

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[1] Few experimental studies on spatial firm competition observe price decisions under constant location. One example, discussed in Chapter 4, is Selten and Apesteguia (2004)'s study of price competition of firms located on a circle.
[2] Hotelling argued that both firms should locate at the centre of the market.
[3] An *approximate equilibrium behaviour* results when players encounter relatively small differences in the expected payoff between playing the equilibrium strategy and an alternative strategy, and hence adopt an "almost-optimal" strategy that does not fully optimise.

0.75, only a third of the decisions fall under one of these outcomes while almost 10% congregate at the market centre. None of the six groups of four-firm players converged to the predicted equilibrium. The authors explained this phenomenon by what they termed "myopic" best replies. When one player moved erroneously toward the market centre, best response induced the other players to move likewise, especially when there was no perceivable difference in payoff from a single-deviation outcome vs. prediction. The tendency of location choices at the market extremes, however, is low because players do not follow best response when the distance from equilibrium increases (presumably because the payoff difference between prediction and deviation widens).

This chapter takes the path less trodden by investigating endogenous price strategies of firms with constant location. In the spatial context of product differentiation, this situation is reflective of short run conditions when product redesign is absent. It is also representative of less developed economies where technological innovation or product redesign occurs extremely slowly. For these economies, an entry barrier to large firms is often erected to protect selected domestic industries. This barrier may be economic and (or) geographic. For example, in order to protect small local retailers from hypermarkets, the Malaysian government issued guidelines that permitted hypermarkets to operate only outside a 3.5 km radius of housing estates and town centres, with one hypermarket per 350,000 residents (Malaysia Economic Planning Unit 2003). When consumers within these protected markets travel across the market barrier to purchase the same good at a lower price from rival firms, the situation can be analysed in the context of the IO model developed earlier.

This chapter presents the experimental findings on firm decisions under the assumption of constant location. In an environment that corresponds to the theoretical setup of the IO model, two firms make price decisions on the product they intend to sell. Both firms maintain constant location throughout the experiment, with the inside firm situated at a point along the same linear market as consumers in $[0,1]$ and the outside firm situated at a point beyond the market boundary in $]1,+\infty[$. Consumers are uniformly distributed along

$[0,1]$ and possess inelastic demand for the good. They travel to either firm to purchase the good and bear the full burden of travel costs according to a predetermined transportation cost structure, with price ties resolved in favour of the nearer firm. The theoretical prediction of the game is that regardless of the nature of transportation costs, the market share of both firms remains the same. While the gain in price is higher for the inside firm when transportation costs increase more rapidly, its price relative to that of the outside firm does not change. The constancy of relative demand and relative price in the face of transportation cost changes assuming fixed firm location and convex transportation cost structures was discussed in Chapter 2. Under constant location, two Nash-Bertrand equilibria exist in pure strategies for differing levels of product differentiation. At very high levels of product differentiation, the inside firm monopolises the market. At lower levels of product differentiation, both firms compete for demand, with the outside firm earning a positive price.

In order to investigate the theoretical propositions, an experiment was conducted with six treatments: three treatments characterise different transportation cost structures while the other three treatments implement an increase in transportation costs under each transportation cost structure. In line with theoretical prediction, the experimental results show that market demand and relative price remain the same regardless of transportation costs. Moreover, consumers alleviate higher prices by incurring higher transportation costs. An interesting observation in all treatments is that the inside firm typically attempts to monopolise the market by pricing much lower than predicted at levels supported by higher-than-warranted degrees of product differentiation. In all but one treatment, such low prices could not be sustained as best responses quickly moved prices to prediction.

The next section summarises the theoretical predictions. Section 3.3 presents the experimental procedures. Section 3.4 contains the experimental results. Finally, the concluding remarks are made in Section 3.5.

3.2 THEORETICAL PREDICTIONS

The analyses of this chapter focus on the non-cooperative Nash-Bertrand equilibrium in pure strategies and the propositions of the IO model. This section presents the theoretical framework and a summary of the theoretical predictions for the game under parametric firm location. Consider a differentiated input market in which two firms, an inside firm $i = 1$ and an outside firm $i = 2$, make a decision for an input price p_i in each exchange period. Each firm lies at a point along the segments $[0,1]$ and $]1,+\infty[$ respectively and remains unchanged throughout the experiment. Each firm is assumed to be equally efficient in producing a homogeneous and perfectly divisible good. For simplicity, the firms are assumed to incur no marginal production costs, storage costs or distribution costs other than transportation (or delivery) costs which are borne fully by consumers. The consumers are assumed to be uniformly distributed along the unit interval $[0,1]$ and purchase the good from either firm. Price ties are resolved in favour of the nearer firm. The demand for each firm's good is denoted by m_i. The distance between consumer and firm locations corresponds to the distance d that the consumer travels to purchase the good. The transportation costs incurred by consumers increase according to a predetermined transportation cost schedule which bears one of three functional forms: $c(x) = td$ (linear), $c(x) = sd^2$ (quadratic) and $c(x) = td + sd^2$ (linear-quadratic) where $t > 0$, $s > 0$ and $t/s \leq 2/3$. Each firm earns a profit equivalent to $\prod_i = p_i \cdot m_i - AC$ where AC is the (constant) average cost of production which for simplicity is assumed to be zero.

The non-zero price pair (p_1^*, p_2^*) denotes the non-cooperative Nash equilibrium in pure strategies and is obtained by the intersection of the two best response functions in price, RFp_1 for firm 1 and RFp_2 for firm 2 (see Figure 3.1). For levels of product differentiation $D = x_2 - x_1$ where $x_1 + x_2 < 4$, i.e., $D < 4 - 2x_1$, the equilibrium price is denoted by point *A*. Under very high levels of product differentiation when $D = 4 - 2x_1$, the inside firm captures the whole market and the outside firm earns zero prices. The equilibrium price $(p_1^*, 0)$ for

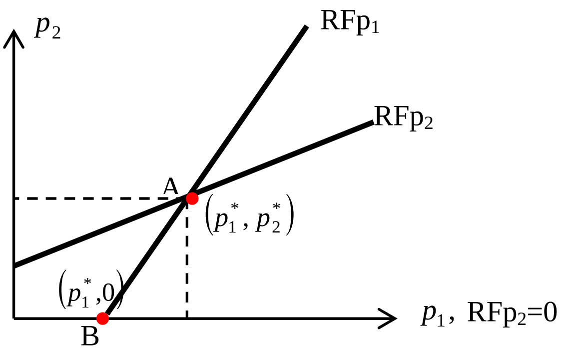

Fig. 3.1 Response functions and price equilibria

$D = 4 - 2x_1$ is denoted by point *B* in the same figure. The best response functions under linear-quadratic transportation costs, derived in Chapter 2, are reproduced below.

(3.1) $$p_1 = \frac{1}{2}\left[p_2 + (t + s(x_2 - x_1))(x_1 + x_2)\right]$$

(3.2) $$p_2 = \frac{1}{2}\left[p_1 + (t + s(x_2 - x_1))(2 - x_1 - x_2)\right]$$

The response functions under quadratic transportation costs and linear transportation costs can be obtained by setting the transportation cost parameters $t = 0$ and $s = 0$ respectively.

Table 3.1 gives the theoretical predictions of the IO model under the three transportation cost structures using the parameter values employed in the experiment. The predicted values for price, demand, relative demand, relative price and profit are shown, along with the predictions for a 100% increase in transportation cost parameters *t* and *s*.

The propositions of the IO model pertaining to the parametric location game for non-zero prices (i.e. for levels of product differentiation $D < 4 - 2x_1$) are recapitulated below.

Table 3.1
Theoretical predictions

Prediction		Linear		Quadratic		Linear-quadratic	
		t = 2.6	t = 5.2	s = 6.5	s = 13	t = 2.6, s = 6.5	t = 5.2, s = 13
Price	(p1, p2)	(3.03, 2.17)	(6.07, 4.33)	(7.58, 5.42)	(15.17, 10.83)	(10.62, 7.58)	(21.23, 15.17)
Demand	(m1, m2)	(0.58, 0.42)	(0.58, 0.42)	(0.58, 0.42)	(0.58, 0.42)	(0.58, 0.42)	(0.58, 0.42)
Relative price	p2/p1	0.71	0.71	0.71	0.71	0.71	0.71
Relative demand	m2/m1	0.71	0.71	0.71	0.71	0.71	0.71
Profit	(r1, r2)	(1.77, 0.90)	(3.54, 1.81)	(4.42, 2.26)	(8.85, 4.51)	(6.19, 3.16)	(12.39, 6.32)

Transportation cost structure spans the Linear, Quadratic and Linear-quadratic columns.

Proposition 1

The equilibrium relative price p_2^* / p_1^* is independent of the transportation cost structure.

Proposition 2

2.1 The equilibrium relative demand m_2^* / m_1^* remains the same regardless of the transportation cost structure.

2.2 The equilibrium relative price is equivalent to equilibrium relative demand.

Proposition 3

For a given transportation cost structure, the inside firm raises (lowers) its price when faced with higher (lower) transportation costs. The outside firm reacts by raising (lowering) its price but by a smaller amount.

Table 3.1 shows that regardless of the transportation cost structure, equilibrium relative price remains constant (Proposition 1) and equals equilibrium relative demand (Proposition 2). When transportation costs increases, e.g., when both *t* and *s* double under linear-quadratic transportation costs, the increase in price by the inside firm is higher than the price increase by the outside firm (Proposition 3).

Table 3.2
Treatments

Treatment no.	Treatment code	Transportation cost structure	Parameter values
1	PL1	Linear	t = 2.6
2	PL2	Linear	t = 5.2
3	PQ1	Quadratic	s = 6.5
4	PQ2	Quadratic	s =13.0
5	PLQ1	Linear-quadratic	t = 2.6, s = 6.5
6	PLQ2	Linear-quadratic	t = 5.2, s = 13.0

3.3 EXPERIMENTAL PROCEDURE

An experimental environment was created that corresponded to the theoretical IO model as closely as possible. Six treatments were organised: three treatments characterise different transportation cost structures, *viz*., linear, quadratic, and linear-quadratic transportation costs. For the other three treatments, there was a 100% increase in transportation cost parameters under each transportation cost structure. The treatments and parameter values are summarised in Table 3.2.

In each treatment, 16 players were seated at computer terminals and given a set of instructions (see Appendix 8). They were randomly assigned the role of inside firm or outside firm, and were informed of their location and the transportation cost structure faced by the consumers. The inside firm was located at 0.25 and the outside firm was located at 1.25. The players' role and location, and the consumers' transportation cost structure all remained unchanged throughout the experiment. In each period, the players made a price decision. They were informed of the functional relationship between price and demand by means of a calculator that computes the demand generated from the price decisions entered. This calculator was made available at all times throughout the experiment. Consumers were located uniformly along the unit interval $[0,1]$ and were automated to purchase one unit of the good from either firm according to the relevant demand function. They travelled a distance *d* to the firm and incurred transportation costs in accordance with a predetermined transportation cost schedule.

In order to avoid collusion, players were randomly paired with each other in such a way as to prevent any two players from meeting more than twice during the whole experiment. There were no interactions among the players in any way, and their decisions were made privately at individual computer terminals. The players were, however, permitted to make clarifications to the facilitator concerning the information provided.

At the end of each period, the price and market share of the player and his rival were displayed. The market share is the percentage demand for player i's good out of total demand, i.e., $m_i/(m_1+m_2)\times 100\% = m_i \times 100\%$ since $m_1+m_2=1$. The payoffs were kept private to each player and were computed as the total profit earned, i.e., $\prod_i = p_i \cdot m_i$. The players were informed of the conversion rate from experimental earnings to actual earnings. They were also told that there were 16 trading periods after an initial trial period. No time restrictions were imposed, and each session lasted an average of one hour.

The experiments were conducted in a computer laboratory at the School of Business, National University of Singapore over a four-day period in February-March 2004. A total of 111 business students were recruited by e-mail. The number of subjects recruited exceeded the number required to run each treatment in order to avoid the problem of no-shows. No subject participated in more than one treatment and almost all had no prior experience in a market experiment. The computerised programmes were developed using ZTree software (Fischbacher 1999).

At the end of the 16 periods, the players were asked to answer a short questionnaire (see Appendix 9) that queried the manner in which pricing decisions were made and the usefulness of the calculator. The responses indicated that all subjects employed the calculator in their price decisions during the initial periods, while two-thirds used the calculator throughout the experiment. After completing the questionnaire, the players received their payment privately in cash. Their earnings averaged S$7.05 including S$4 as show-up fee.

3.4 EXPERIMENTAL RESULTS

Table 3.3 summarises the mean, median, standard deviation in mean, and dispersion of values of all variables under the six treatments. The table shows that the mean and median prices underperform the predicted values, resulting in a lower-than-predicted average profit. The distribution of mean demand is close to the predicted values, with the inside firm gaining slightly higher-than-predicted demand than the outside firm.

The ensuing discussion addresses the theoretical hypotheses and additional issues that emerge from the actual trading behaviour of players. To assess the effect of player experience with the experiment, the data is segregated into the early phase (periods 1-8) and late phase (periods 9-16). The latter is further divided into the late1 phase (periods 9-13) and late2 phase (periods 14-16).

H1: Prices converge to the predicted values.

Figures 3.2 to 3.13 depict the time series of mean or individual price (p_{it}) of each firm, the best responses lagged one period ($RF_{i,t-1}$) and the non-cooperative Nash predictions (p_i^*) for the six treatments. The per period mean price for each firm is obtained by averaging over 8 players with the role of inside or outside firm for each period. The average best response RF_{it} of firm i to the rival firm's price p_{jt} , $i \neq j$, for period t is computed using the relevant response functions given in Chapter 2.

The time series of mean decisions illustrate that in all six treatments, both inside and outside firms typically start the experiment by trading at price levels below prediction. By the end of the 16 periods, the mean price of both firms generally converges to a level below prediction.

Table 3.3
Summary statistics of results

	N	Mean	Median	Prediction	S.D. (mean)	Maximum	Minimum
Treatment 1: PL1							
p1	127	2.47	2.50	3.03	1.04	5.50	0.25
p2	128	1.68	1.60	2.17	0.86	4.00	0.49
m1	127	0.59	0.62	0.58	0.24	1.00	0.00
m2	128	0.40	0.38	0.42	0.25	1.00	0.00
r1	128	1.26	1.17	1.77	0.50	2.62	0.00
r2	128	0.64	0.52	0.90	0.49	2.23	0.00
Treatment 2: PL2							
p1	128	4.95	4.95	6.07	1.29	9.00	2.00
p2	128	3.74	3.50	4.33	1.57	10.00	0.80
m1	128	0.63	0.61	0.58	0.17	1.00	0.19
m2	128	0.37	0.39	0.42	0.17	0.81	0.00
r1	128	2.94	2.89	3.54	0.70	5.99	1.54
r2	128	1.22	1.21	1.81	0.52	2.79	0.00
Treatment 3: PQ1							
p1	127	6.09	6.00	7.58	2.40	12.15	0.25
p2	127	4.25	4.00	5.42	2.37	12.90	0.05
m1	127	0.59	0.60	0.58	0.24	1.00	0.00
m2	127	0.40	0.40	0.42	0.24	1.00	0.00
r1	128	3.13	3.20	4.42	1.20	6.00	0.00
r2	128	1.39	1.38	2.26	0.97	5.06	0.00
Treatment 4: PQ2							
p1	128	10.40	9.89	15.17	4.16	19.90	1.50
p2	128	6.13	6.00	10.83	3.27	15.00	0.01
m1	128	0.59	0.57	0.58	0.19	1.00	0.13
m2	128	0.41	0.43	0.42	0.19	0.87	0.00
r1	128	5.50	5.63	8.85	1.75	9.00	1.04
r2	128	2.21	2.11	4.51	1.31	5.71	0.00
Treatment 5: PLQ1							
p1	128	8.14	8.00	10.62	2.72	15.90	0.55
p2	128	5.45	5.00	7.58	2.34	12.60	0.50
m1	128	0.60	0.60	0.58	0.19	1.00	0.00
m2	128	0.40	0.40	0.42	0.19	1.00	0.00
r1	128	4.50	4.55	6.19	1.29	7.56	0.00
r2	128	1.90	1.91	3.16	0.99	5.72	0.00
Treatment 6: PLQ2							
p1	127	15.95	16.00	21.23	5.15	25.90	5.00
p2	127	12.19	12.10	15.17	4.58	25.00	1.00
m1	127	0.64	0.62	0.58	0.18	1.00	0.20
m2	127	0.35	0.37	0.42	0.18	0.80	0.00
r1	128	9.52	9.57	12.39	2.71	17.45	0.00
r2	128	3.98	3.56	6.32	2.44	11.54	0.00

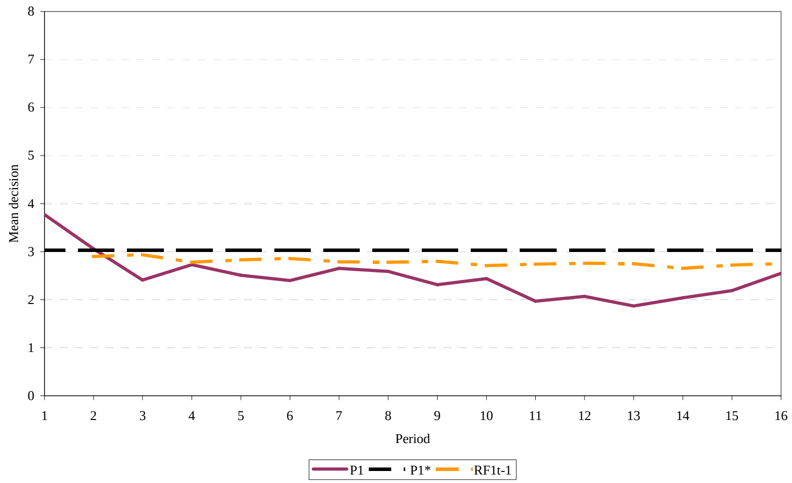

Fig. 3.2(a) Time series of mean prices of inside firm players (PL1)

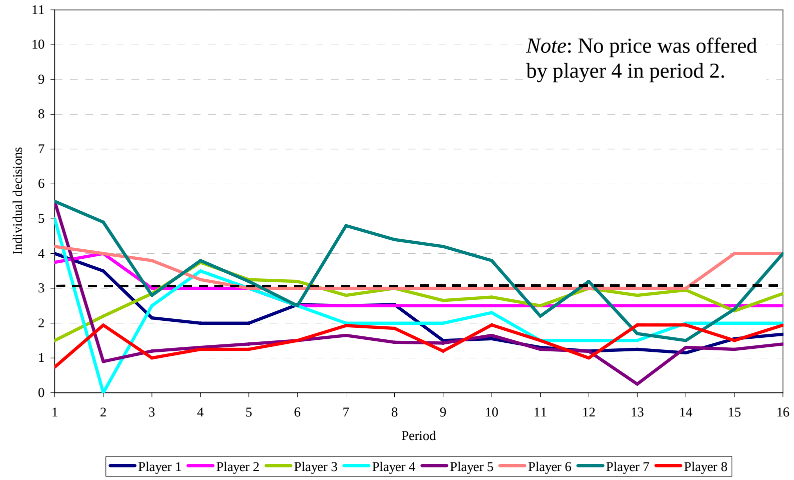

Fig.3.2(b) Time series of individual prices of inside firm players (PL1)

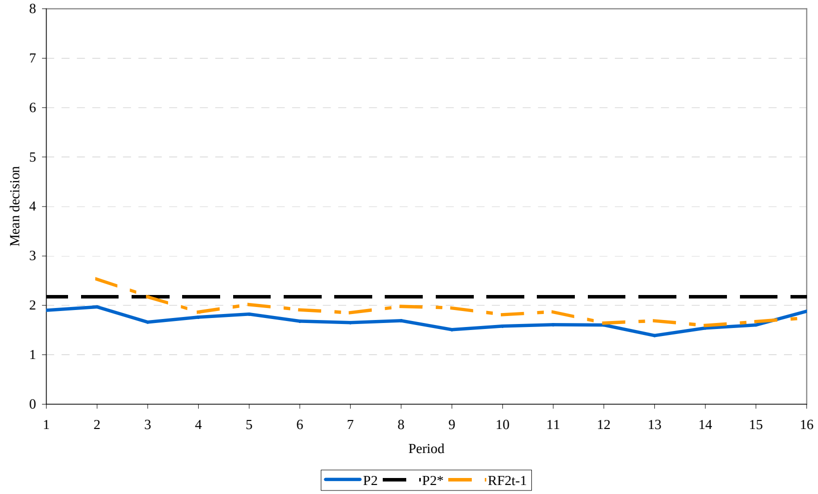

Fig. 3.3(a) Time series of mean prices of outside firm players (PL1)

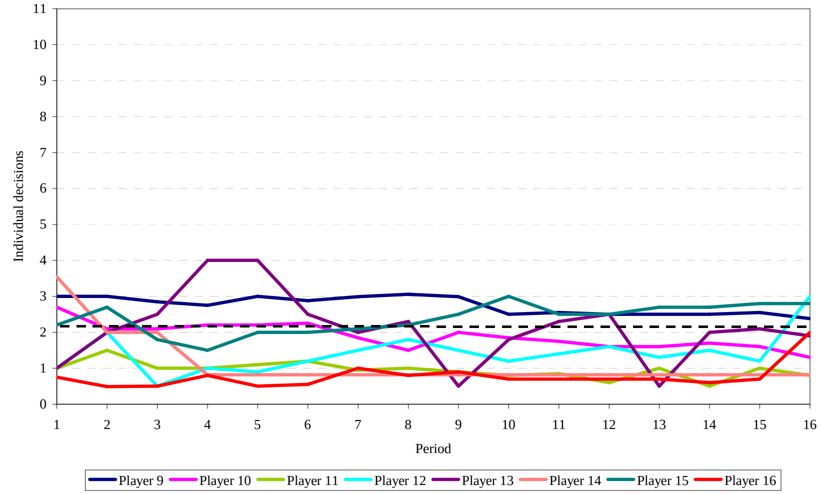

Fig. 3.3(b) Time series of individual prices of outside firm players (PL1)

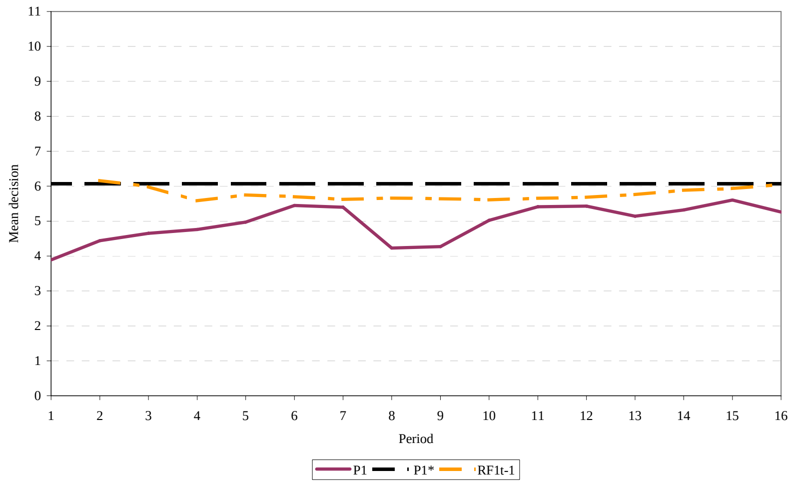

Fig. 3.4(a) Time series of mean prices of inside firm players (PL2)

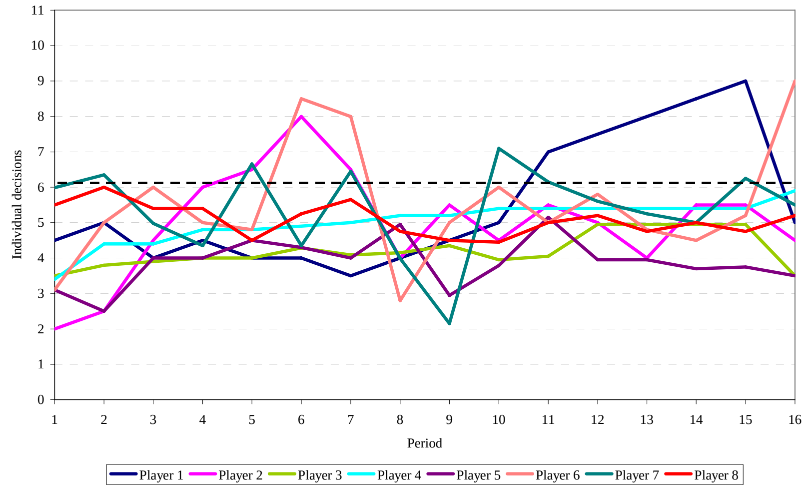

Fig.3.4(b) Time series of individual prices of inside firm players (PL2)

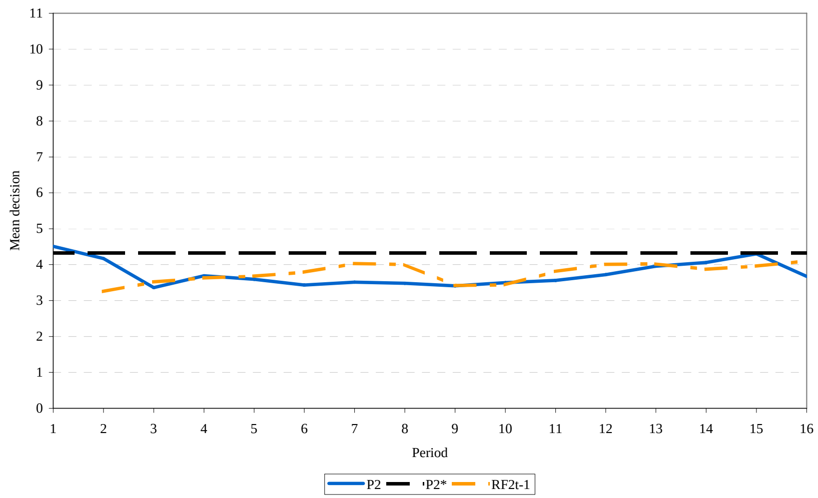

Fig. 3.5(a) Time series of mean prices of outside firm players (PL2)

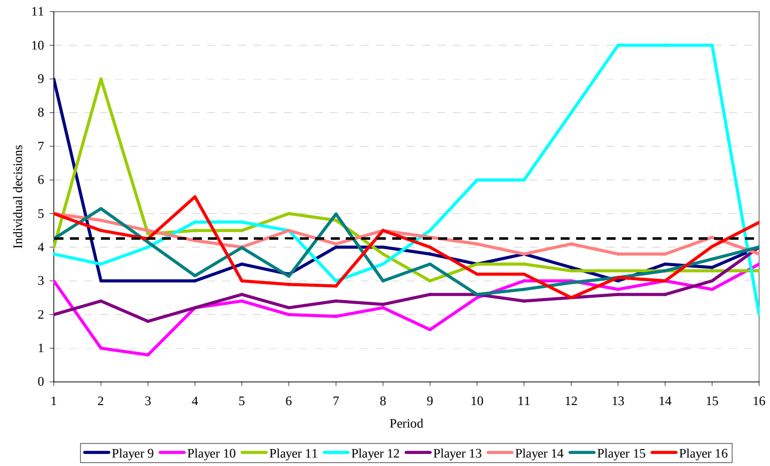

Fig. 3.5(b) Time series of individual prices of outside firm players (PL2)

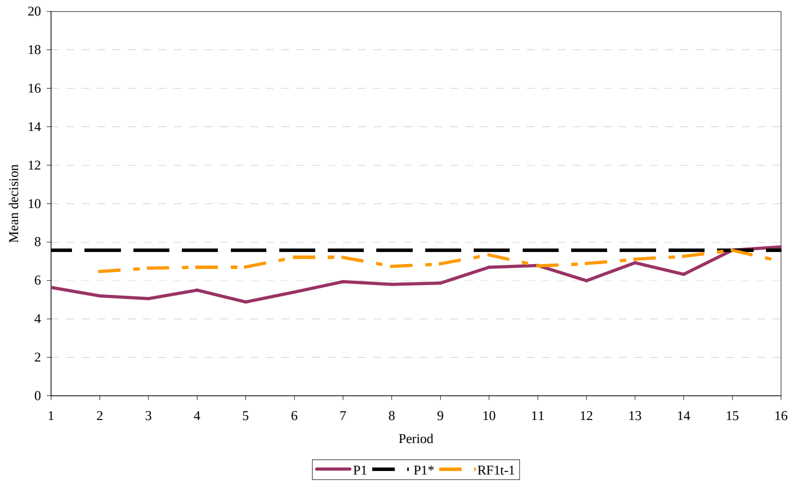

Fig. 3.6(a) Time series of mean prices of inside firm players (PQ1)

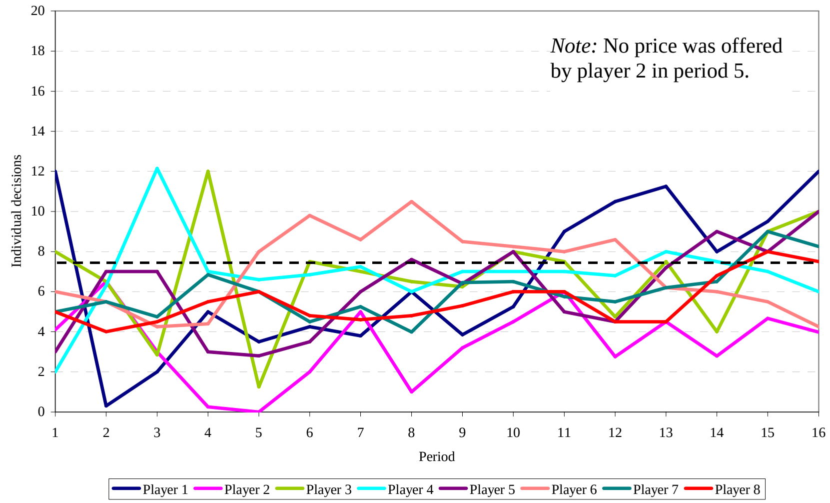

Fig.3.6(b) Time series of individual prices of inside firm players (PQ1)

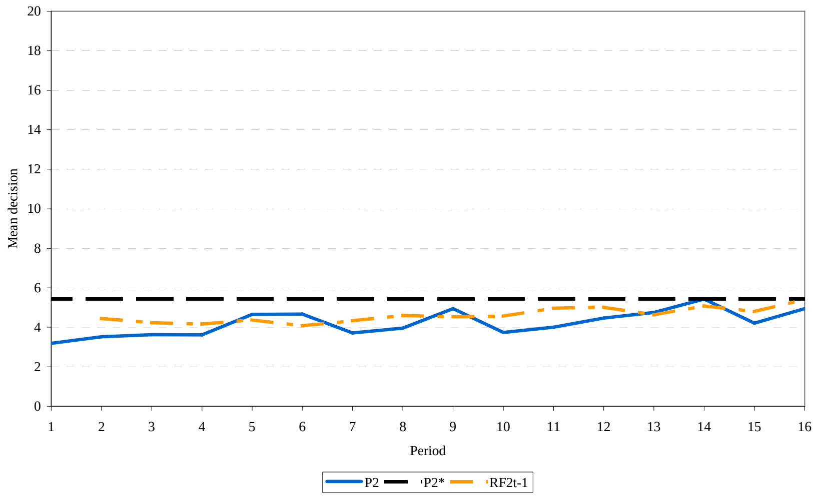

Fig. 3.7(a) Time series of mean prices of outside firm players (PQ1)

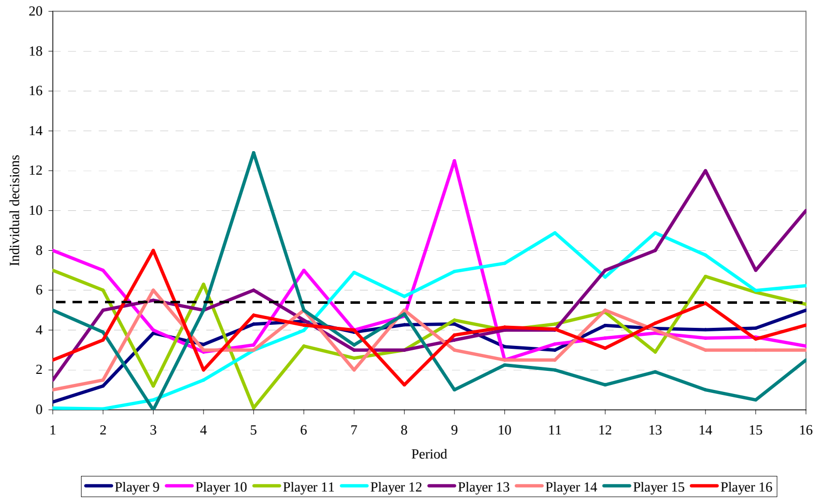

Fig. 3.7(b) Time series of individual prices of outside firm players (PQ1)

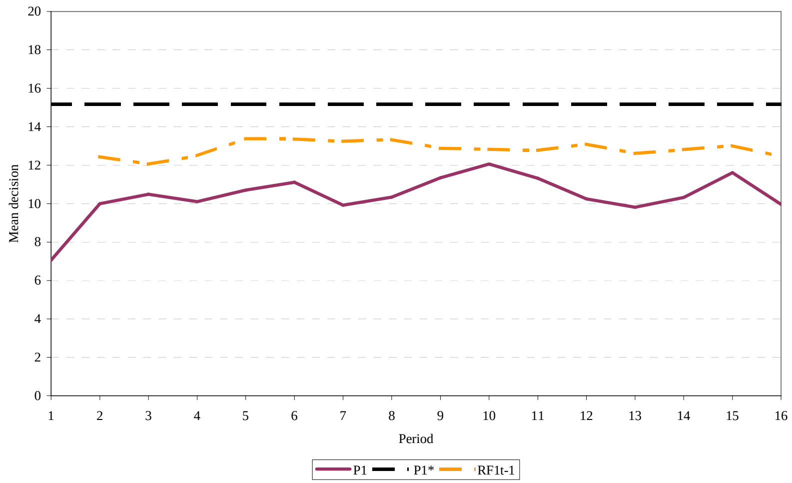

Fig. 3.8(a) Time series of mean prices of inside firm players (PQ2)

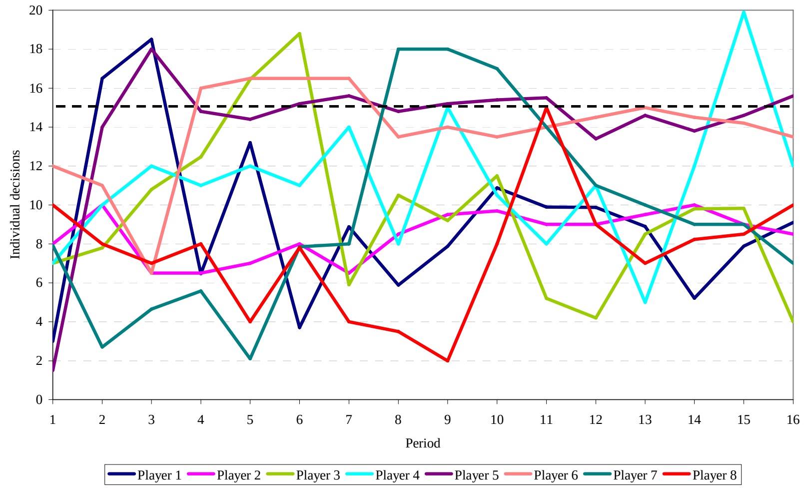

Fig.3.8(b) Time series of individual prices of inside firm players (PQ2)

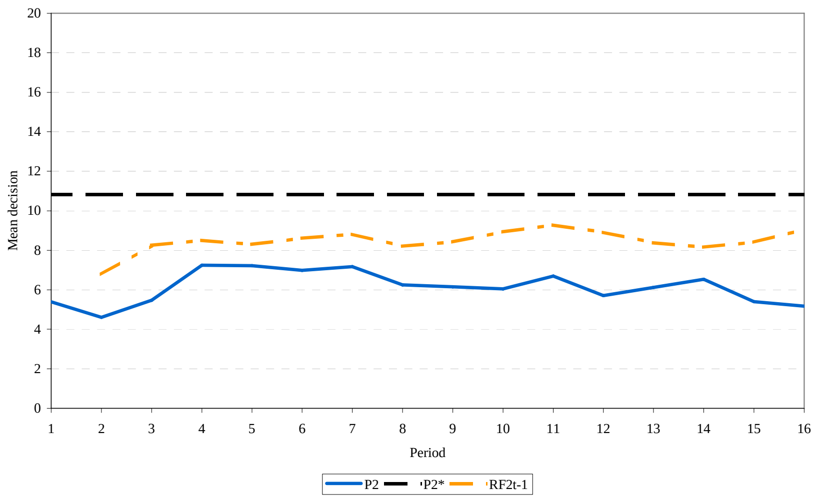

Fig. 3.9(a) Time series of mean prices of outside firm players (PQ2)

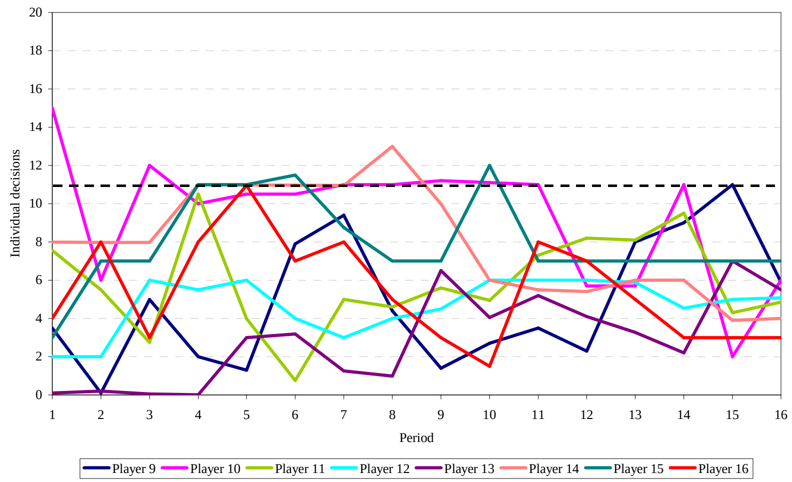

Fig. 3.9(b) Time series of individual prices of outside firm players (PQ2)

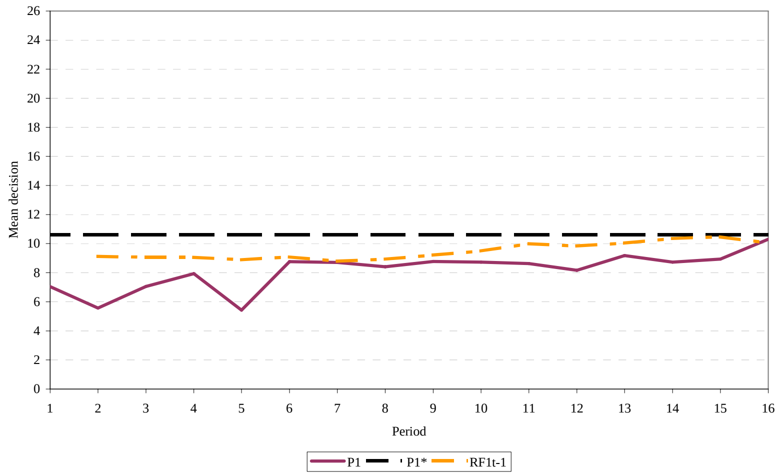

Fig. 3.10(a) Time series of mean prices of inside firm players (PLQ1)

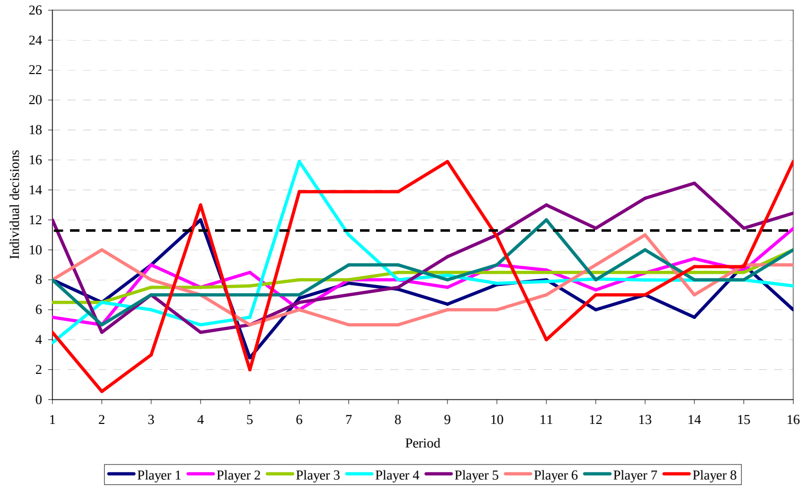

Fig.3.10(b) Time series of individual prices of inside firm players (PLQ1)

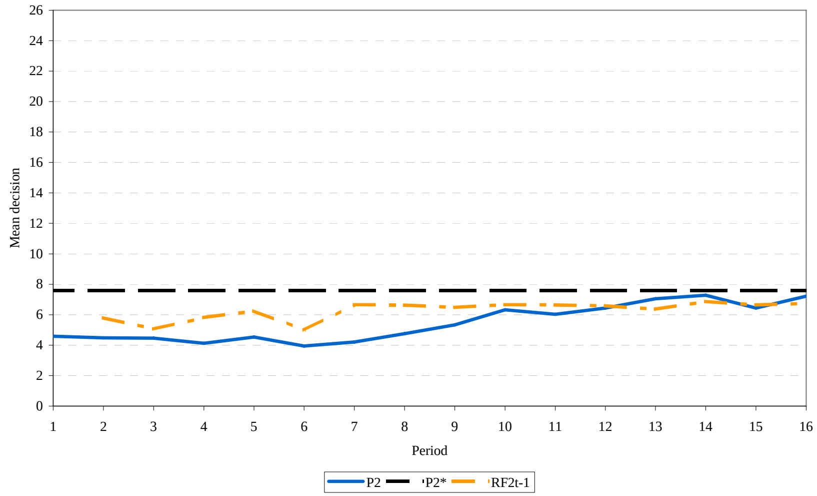

Fig. 3.11(a) Time series of mean prices of outside firm players (PLQ1)

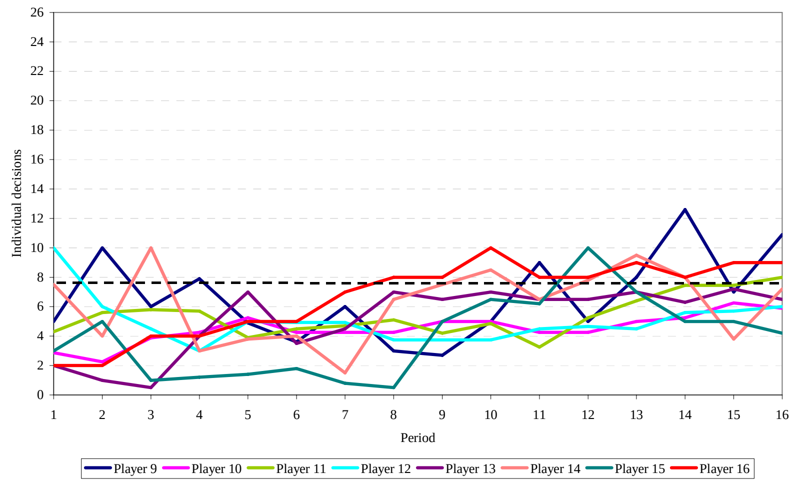

Fig. 3.11(b) Time series of individual prices of outside firm players (PLQ1)

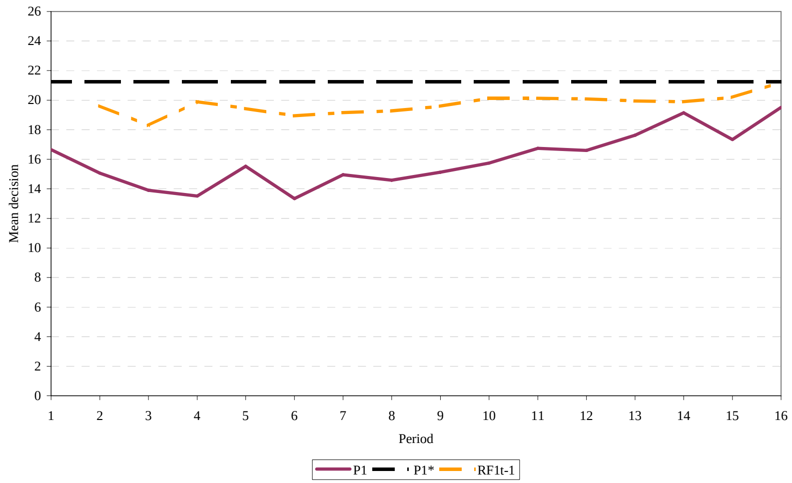

Fig. 3.12(a) Time series of mean prices of inside firm players (PLQ2)

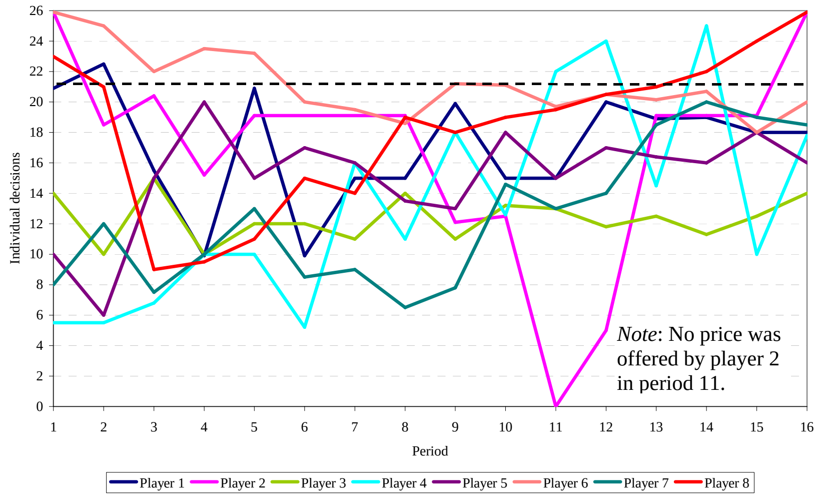

Fig.3.12(b) Time series of individual prices of inside firm players (PLQ2)

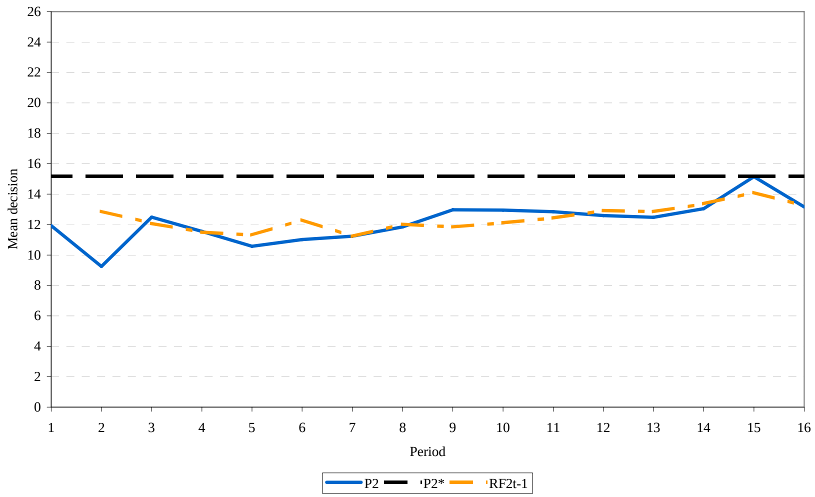

Fig. 3.13(a) Time series of mean prices of outside firm players (PLQ2)

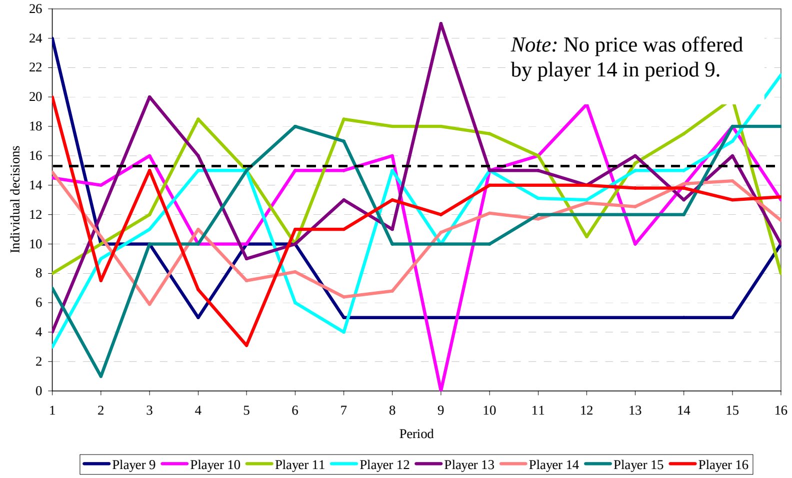

Fig. 3.13(b) Time series of individual prices of outside firm players (PLQ2)

The plots of individual decisions show that in all treatments, four or more players taking on the role of either firm commence trading at prices below prediction. Thereafter, prices rise gradually to a higher level but remain below prediction throughout the experiment. There are a few exceptions to this behaviour: (1) six inside firm players in PL1 start at prices above prediction but only three end at or above prediction. As a result, the mean price of the inside firm in PL1 also converges to a level below prediction; (2) in contrast, six inside firm players in PQ1 commence with low prices but three of these players end with prices above prediction. This results in the mean price of the inside firm in PQ1 converging to a level above prediction.

The tendency of both firms to choose low prices is also shown in Figures 3.14 to 3.19, which depict the distribution of all individual prices, grouped by unit intervals, for the six treatments. The Nash predictions are marked in the figures by a broken vertical line. It is clear that, except for the inside firm in PL1 and PLQ2, the intervals with the highest frequency are invariably below prediction, i.e., to the left of the broken vertical line.

The graphical observations show that a large majority of players in all treatments adopt a low price strategy at the start of the experiment but manage to rise to levels closer to (but still below) prediction by the end of the experiment. The following features of the experiment are evident. First, low price behaviour predominates at the start of the experiment regardless of the role of the players. In fact, two-thirds of all players (67.7%) price below prediction at the start of the experiment, comprising 72.9% of inside firm players and 62.5% of outside firm players. Second, in half of the treatments, the number of low pricers among inside firm players surpasses the number of low pricers among outside firm players. In another two treatments, the number of low pricers among inside firm players and outside firm players is equal.

Why is low price behaviour so prevalent among the players? Why are there more inside firm players who price low compared to outside firm players? If experimental error is an explanation for prices starting below prediction, why are there not more players offering supra-prediction prices instead?

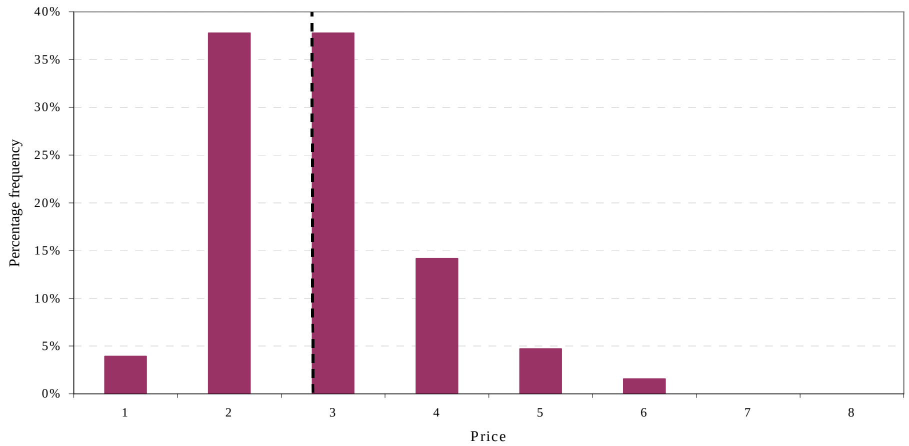

Fig. 3.14(a) Distribution of individual prices of inside firm players (PL1)

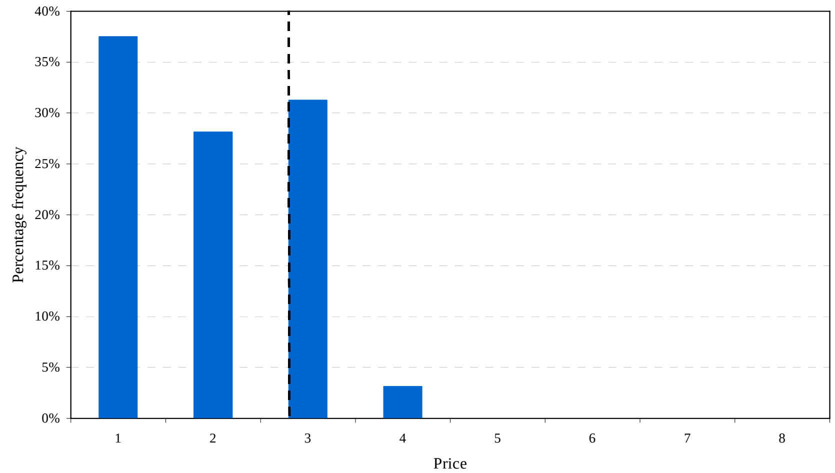

Fig. 3.14(b) Distribution of individual prices of outside firm players (PL1)

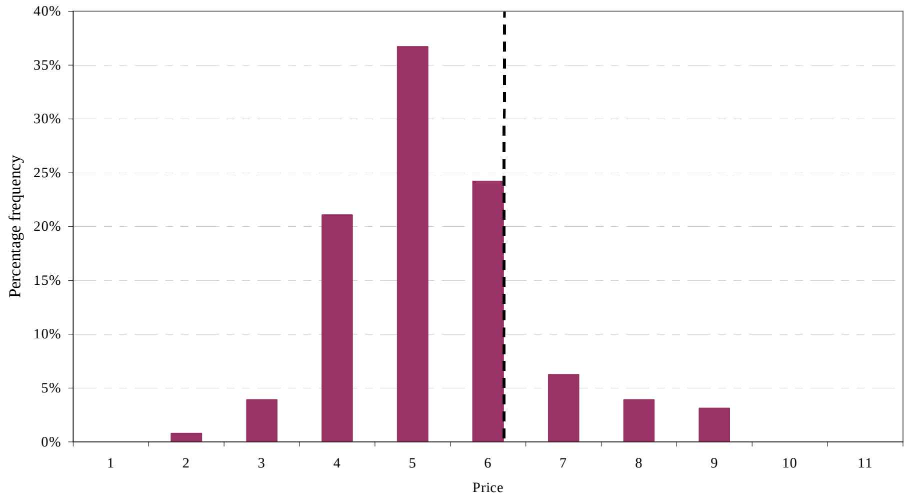

Fig. 3.15(a) Distribution of individual prices of inside firm players (PL2)

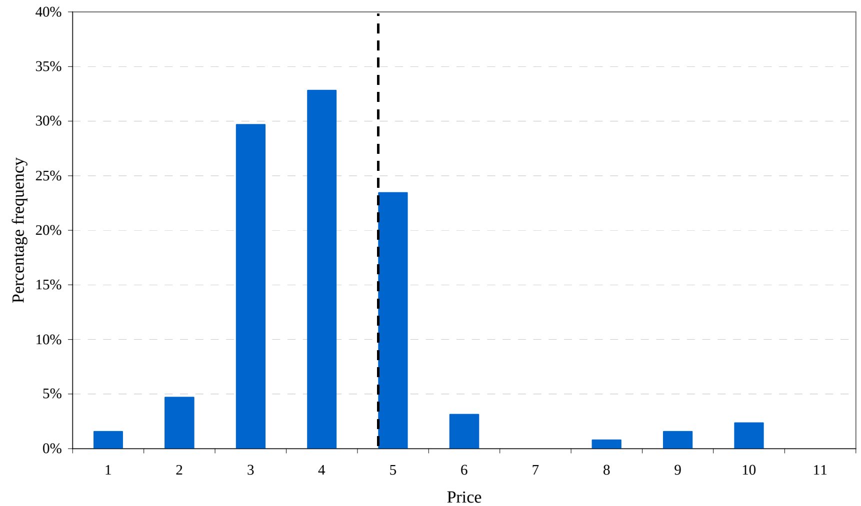

Fig. 3.15(b) Distribution of individual prices of outside firm players (PL2)

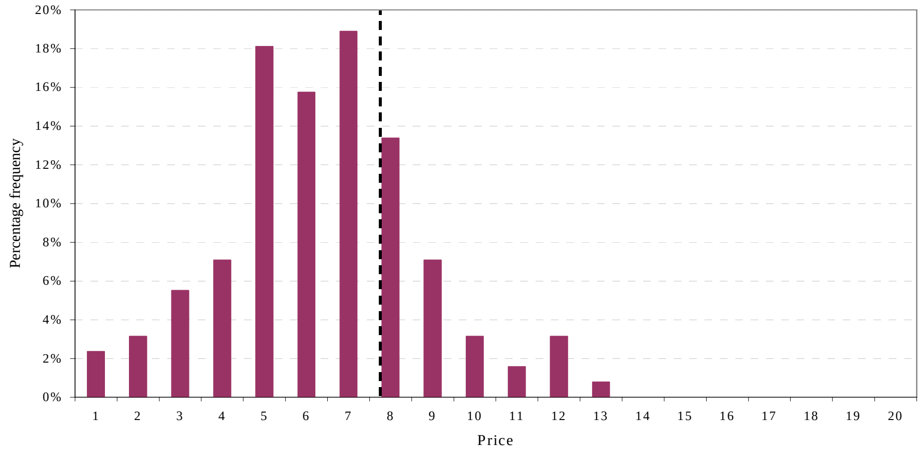

Fig. 3.16(a) Distribution of individual prices of inside firm players (PQ1)

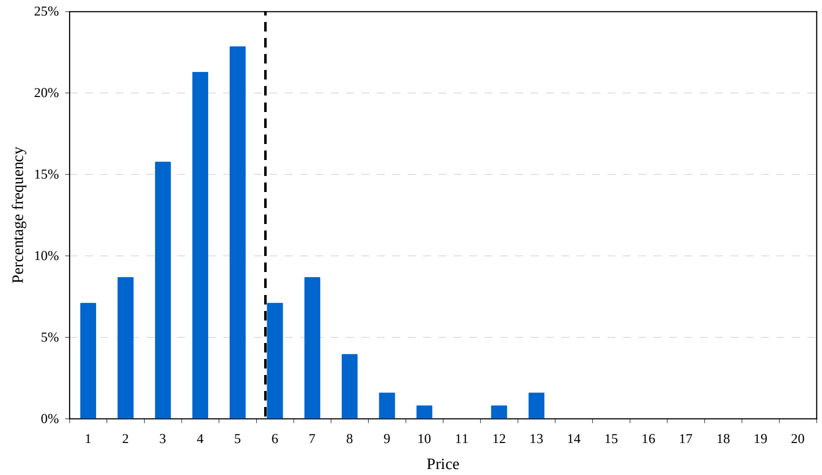

Fig. 3.16(b) Distribution of individual prices of outside firm players (PQ1)

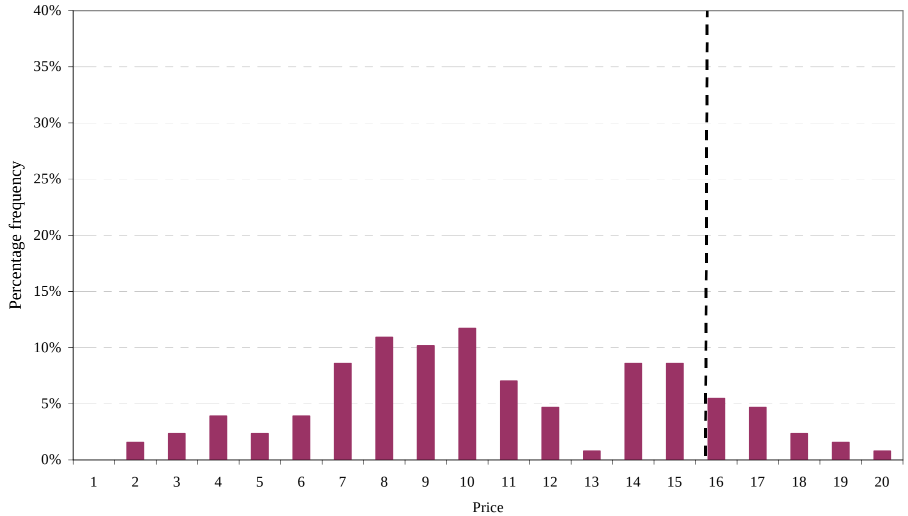

Fig. 3.17(a) Distribution of individual prices of inside firm players (PQ2)

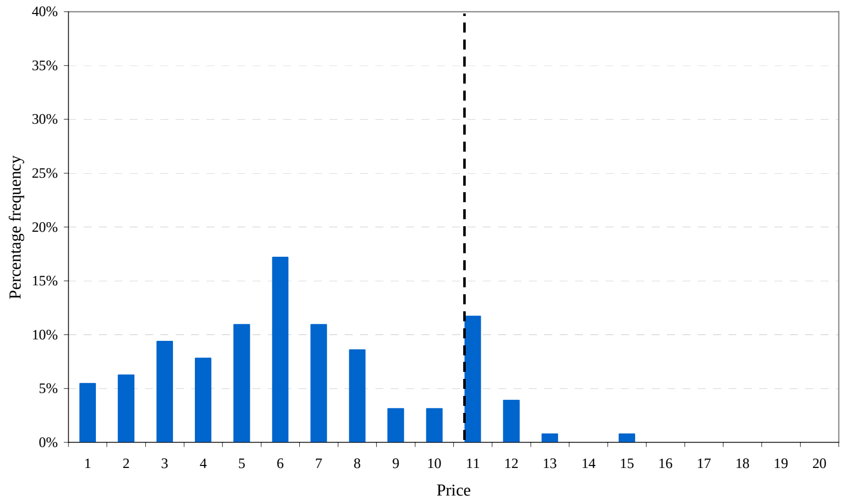

Fig. 3.17(b) Distribution of individual prices of outside firm players (PQ2)

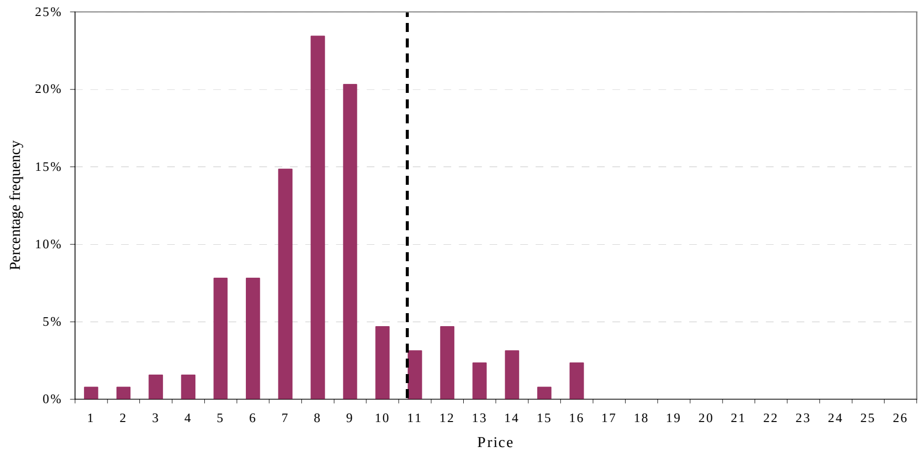

Fig. 3.18(a) Distribution of individual prices of inside firm players (PLQ1)

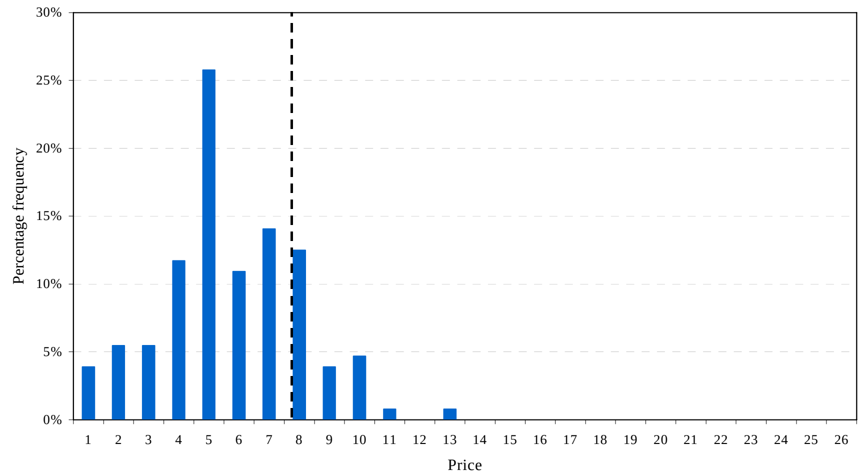

Fig. 3.18(b) Distribution of individual prices of outside firm players (PLQ1)

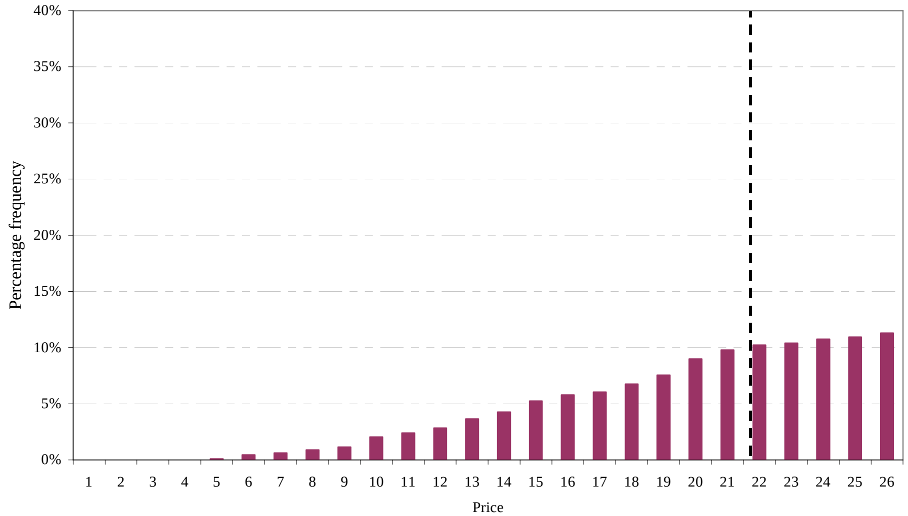

Fig. 3.19(a) Distribution of individual prices of inside firm players (PLQ2)

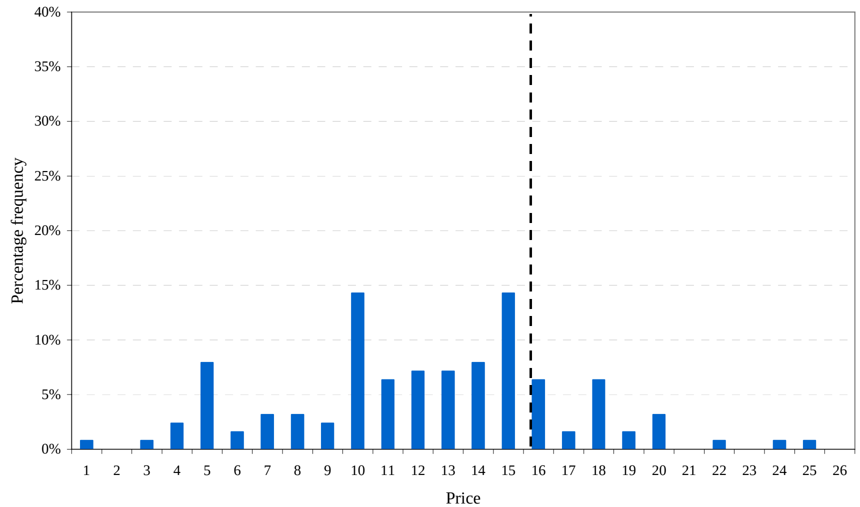

Fig. 3.19(b) Distribution of individual prices of outside firm players (PLQ2)

Table 3.4
Starting price (first three periods), highest monopoly price and predicted price of the inside firm

	p_{1t} , t = 1 to 3		Highest monopoly price	Predicted price
	Mean	Median	(p_2* = 0)	p_1*
Treatment 1: PL1	3.08	3.00	1.95	3.03
Treatment 2: PL2	4.33	4.40	3.90	6.07
Treatment 3: PQ1	5.30	5.00	4.88	7.58
Treatment 4: PQ2	9.18	8.00	9.75	15.17
Treatment 5: PLQ1	6.56	6.50	6.83	10.62
Treatment 6: PLQ2	15.20	15.00	13.65	21.23

Table 3.4 compares the mean and median starting price of the inside firm over the first three periods, its highest monopoly price (which corresponds to point *B* in Figure 3.1 where $p_2^* = 0$) and the predicted price. It is clear that, for four out of six treatments, the mean and median starting prices of the inside firm is close to the highest price at which it can capture the whole market. The two exceptions are PL1 where the starting price of the inside firm nears prediction and PLQ2 where the starting price is between prediction and the highest monopoly price.

At the start of the experiment, therefore, players tend to be over-competitive, with the majority of inside firm players competing to acquire a monopoly. By adopting a low price strategy, both players generally price below prediction, resulting in diminished rent (or producer surplus). Interestingly, as the experiment progresses, both firms realise that they are able to capture higher producer surplus by raising their prices. The discussion under hypothesis H2 shows that, with a few exceptions especially in PQ2, players eventually reach an equilibrium that is closer to prediction as they increasingly play best strategies.

Initial attempts to break out of the low price mould by several players are largely unsuccessful due to the player rotation mechanism.[4] As one outside firm player said, "I aimed to earn more at a mutually higher price level but the opponent refused to collude by raising prices. Instead (he) adopted predatory pricing. The opponent was content with small profits whereas I had wanted both of us to have higher profits by trading at higher price levels. Refer to my period 4 and 5 pricing where I tried to let my opponent see my logic of trading at higher prices but (the) opponent refused and engaged in trading at low prices of 1-2 dollars throughout the rest of the periods."[5]

The following highlights some examples of *price leader behaviour* in which players attempt to initiate an overall movement toward higher prices in order to generate higher profit. Players 1 to 8 are inside firm players while players 9 to16 are outside firm players.

<u>Player 7 in PL1</u> raised prices sharply by 92.0% in period 7 but lowered prices after that when demand fell by a fifth. The rival firm (player 9) raised prices marginally by 2.3% in period 8. When demand dropped with the higher price, the rival quickly reverted to her original price. <u>Player 13 in PLQ2</u> raised prices by 2.3 times in period 9 but immediately reverted to lower prices in the following period when demand dropped to zero. The rival firm (player 5) raised prices by 38.5% in the next period but a similar sharp drop in demand (of 41.2%) brought prices back to the original level.

This scenario is repeated for many other players whose attempts to price lead are made ineffective by the rotation of players in each period and the predominance of low price behaviour among the players.

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[4] The instructions did not reveal that a player rotation mechanism is in place.
[5] Comments by player 13 in PL1 in answer to a question on how his price decisions were made.

To evaluate price convergence to prediction, a Wilcoxon signed rank test is conducted on the null hypothesis that price decisions and prediction are equal. Table 3.5 shows that the null is significant at the 0.01 level for both firms in the late2 phase of PQ1, PLQ1 and PLQ2, and for one firm in PL1 (outside firm) and PL2 (inside firm). In the case of PQ2, the failure of prices to converge to prediction is shown by the insignificant Wilcoxon statistic for all phases of the experiment. Similar conclusions are reached by a Sign test. The tests provide some evidence that, except for PQ2, the firms are pricing near the theoretical prediction towards the end of the experiment.

Alternatively, *t*-tests can be conducted on the null-hypothesis $H_0 : \bar{p}_i = p_i^*$ against the alternative hypothesis $H_a : \bar{p}_i \neq p_i^*$ where $\bar{p}_i$ is the mean price of firm *i* and p_i^* is the Nash prediction for the last three periods. The *t*-statistics require the data to be normally distributed. This is shown to be otherwise for 4 of the 12 price series at the 0.05 level by a Kolmogorov-Smirnov test for normal distribution. *T*-tests on 8 price series that exhibit normal distribution are given in Table 3.6. The results show that H_0 cannot be rejected at the 0.05 level in the late2 phase for PQ1 (inside firm), PLQ1 (outside firm) and PLQ2 (outside firm), and at the 0.01 level for PLQ1 (inside firm). The *t*-statistic is insignificant for PL1 (inside firm), PQ2 (both firms) and PLQ2 (inside firm). For these firms, a difference of means test shows prices falling below prediction, with PQ2 registering the highest mean difference. The results of the *t*-tests are, therefore, similar to that of the Wilcoxon and Sign tests. Since the *t*-tests are not applicable to all data, the Wilcoxon and Sign tests will be the preferred tests henceforth.

Table 3.5
Price convergence to Nash prediction
(probabilities for two-tailed Wilcoxon signed ranks test p_W and Sign test p_S)

Variable	N	pw		ps		Variable	N	pw		ps	
Null hypothesis: $p_{it} = p_i$*											
Treatment: PL1						Treatment: PL2					
All periods						All periods					
p1	128	0.0000		0.0000		p1	128	0.0000		0.0000	
p2	128	0.0000		0.0000		p2	128	0.0000		0.0000	
Early phase						Early phase					
p1	64	0.0180	+	0.0040		p1	64	0.0000		0.0000	
p2	64	0.0008		0.0087		p2	64	0.0001		0.0087	
Late1 phase						Late1 phase					
p1	40	0.0000		0.0000		p1	40	0.0000		0.0000	
p2	40	0.0001		0.0177	+	p2	40	0.0003		0.0000	
Late2 phase						Late2 phase					
p1	24	0.0006		0.0003		p1	24	0.0206	+	0.0015	
p2	24	0.0129	+	0.0639	*	p2	24	0.0055		0.0003	
Treatment: PQ1						Treatment: PQ2					
All periods						All periods	128	0.0000		0.0000	
p1	128	0.0000		0.0000		p1	128	0.0000		0.0000	
p2	128	0.0000		0.0000		p2					
Early phase						Early phase					
p1	64	0.0000		0.0000		p1	64	0.0000		0.0000	
p2	64	0.0000		0.0000		p2	64	0.0000		0.0000	
Late1 phase						Late1 phase					
p1	40	0.0004		0.0027		p1	40	0.0000		0.0000	
p2	40	0.0056		0.0003		p2	40	0.0000		0.0000	
Late2 phase						Late2 phase					
p1	24	0.4748	*	0.8388	*	p1	24	0.0001		0.0000	
p2	24	0.1228	*	0.1516	*	p2	24	0.0000		0.0000	
Treatment: PLQ1						Treatment: PLQ2					
All periods						All periods					
p1	128	0.0000		0.0000		p1	128	0.0000		0.0000	
p2	128	0.0000		0.0000		p2	128	0.0000		0.0000	
Early phase						Early phase					
p1	64	0.0000		0.0000		p1	64	0.0000		0.0000	
p2	64	0.0000		0.0000		p2	64	0.0024		0.0040	
Late1 phase						Late1 phase					
p1	40	0.0000		0.0003		p1	40	0.0000		0.0000	
p2	40	0.0002		0.0072		p2	40	0.0001		0.0003	
Late2 phase						Late2 phase					
p1	24	0.0129	+	0.0066		p1	24	0.0129	+	0.0066	
p2	24	0.0764	*	0.0639	*	p2	24	0.1700	*	0.1516	*

+ indicates significance at 0.01 level
* indicates significance at 0.05 level

Table 3.6
Price convergence to Nash prediction (T-test)

Variable	N	t-statistic	Sig. (2-tailed)		Mean difference from prediction	95% confidence interval of difference lower	upper
Null hypothesis: $p_{it} = p_i^*$							
Treatment: PL1							
p1	24	-4.3850	0.0002		-0.7688	-1.1314	-0.4061
Treatment: PQ1							
p1	24	-0.7804	0.4431	*	-0.3617	-1.3203	0.5970
Treatment: PQ2							
p1	24	-6.1829	0.0000		-4.5388	-6.0573	-3.0202
p2	24	-9.9308	0.0000		-5.1313	-6.2001	-4.0624
Treatment: PLQ1							
p1	24	-2.6609	0.0140	+	-1.3054	-2.3203	-0.2906
p2	24	-1.4744	0.1539	*	-0.6096	-1.4649	0.2457
Treatment: PLQ2							
p1	24	-3.0265	0.0060		-2.5717	-4.3294	-0.8139
p2	24	-1.6085	0.1214	*	-1.3825	-3.1605	0.3955

+ indicates significance at 0.01 level
* indicates significance at 0.05 level

To examine the manner in which individual price decisions are made, the following model is used (additional lags proved insignificant in all treatments):[6]

(3.3) $$p_{it} = \alpha + \beta_1 p_{it-1} + \beta_2 p_{jt-1} + \varepsilon_{it}$$

where p_{it} and p_{jt} are the price of firm i and firm j in period t, $i, j \in \{1,2\}$, $i \neq j$; $t = \{1,...,16\}$. The model hypothesises that the firm makes an adaptive response to its own price as well as a myopic prediction of its rival's price, i.e., it assumes that the rival will post the same price in the current period as it did in the last period.

The regression results are reported in Table 3.7.[7] The following tests are conducted to verify that there is no instability or misspecification of the regressions (significance reported at 0.10 level): (1) Serial correlation in the residuals is shown to be present in almost

[6] A random effects specification is used here rather than a fixed effects specification. Since all diagnostic and specification tests show that the estimates are reliable, in particular, White's heteroskedasticity test, the orthogonality condition (disturbances and regressors are uncorrelated) for a random effects specification is satisfied.
[7] An augmented Dickey-Fuller test rejected the presence of a unit root in the level for all price series at all reported significance levels, indicating that the data are stationary.

all estimates based on significant Q-statistics of the correlogram. This is removed using ARMA estimation and the results are re-examined by the Q-statistics and Breusch-Godfrey serial correlation Lagrange multiplier (LM) test.[8] (2) Heteroskedasticity in the residuals is rejected by White's heteroskedasticity test; (3) For all but 6 estimates, a Jarque-Bera test rejects normal distribution in the residuals. Checks of the residual plot for the estimates that fail the test show 1-3 outlying residual values. As discussed by Brys *et al.* (2004), the Jarque-Bera test fails in the presence of a single outlier, even if the series is normally distributed and the outliers are themselves normally distributed. The failure of the Jarque-Bera test will, therefore, be disregarded in this instance; (4) Ramsey's regression specification error (RESET) test verifies an absence of misspecification of functional form; (5) Chow's breakpoint test at mid-sample ($n = 64$) generally shows an absence of structural change at the breakpoint. For PL2 and PQ2, the failure of Chow's test is due to the location of the outlying residual value at or near the breakpoint.

The regression results show that β_1 is significant at the 0.05 level for both firms in all treatments while β_2 is significant at the 0.05 level for both firms in PLQ1 and at the 0.10 level for the inside firm in PLQ2. In other words, players generally make their price decisions in an adaptive manner to their own price. Only the firms in PLQ1 and PLQ2 take into account the rival's last price in addition to their own last price. For all estimates with significant β 's, $|\beta_1 + \beta_2| < 1$, implying that prices converge to an equilibrium point.

If the firms are making predictions based on their own last price and, in the case of PLQ1 and PLQ2, the last price of their rival as well, are they using best responses to these predictions?

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[8] Since there are lagged dependent variables on the right-hand side of the regression, the Durbin-Watson test for autocorrelation is invalid.

Table 3.7
Regression results for price decisions

Variable	Coefficient	S.E.	t-statistic	
Model:	$p_{it} = \alpha + \beta_1 p_{it-1} + \beta_2 p_{jt-1} + \varepsilon_{it}$			
Treatment: PL1				
p_{1t}				
constant	0.7272	0.2352	3.0918	*
p_{1t-1}	0.7386	0.0759	9.7274	*
p_{2t-1}	-0.0550	0.0737	-0.7459	
AR(1)	-0.2436	0.1095	-2.2256	*
N = 123; Adj R^2 = 0.3588; F = 23.7579 (p = 0.0000); SSE = 81.8871; LM = 0.5748; White = 0.0325; Jarque-Bera = 0.0000; Chow = 0.2788				
p_{2t}				
constant	0.3529	0.1510	2.3376	*
p_{2t-1}	0.8155	0.0571	14.2922	*
p_{1t-1}	-0.0191	0.0429	-0.4465	
AR(1)	-0.2298	0.1031	-2.2287	*
N = 124; Adj R^2 = 0.5493; F = 50.9596 (p = 0.0000); SSE = 38.5263; LM = 0.8830; White =0.0200; Jarque-Bera = 0.0000; Chow = 0.5326				
Treatment: PL2				
p_{1t}				
constant	2.4710	0.4568	5.4088	*
p_{1t-1}	0.4669	0.0793	5.8906	*
p_{2t-1}	0.0464	0.0653	0.7101	
N = 127; Adj R^2 = 0.2129; F = 18.0444 (p = 0.0000); SSE = 163.2839; LM = 0.2092; White = 0.0574; Jarque-Bera = 0.0000; Chow = 0.0024; RESET = 0.3063				
p_{2t}				
constant	1.2513	0.5945	2.1048	*
p_{2t-1}	0.7711	0.0649	11.8729	*
p_{1t-1}	-0.0550	0.0865	-0.6363	
AR(29)	0.4586	0.0832	5.5112	*
MA(15)	0.8592	0.0001	6601.9660	*
N = 98; Adj R^2 = 0.5056; F = 25.7986 (p = 0.0000); SSE = 112.8692; LM = 0.1483; White = 0.4391; Jarque-Bera = 0.0002; Chow = 0.0000				
Treatment: PQ1				
p_{1t}				
constant	3.9949	0.6462	6.1823	*
p_{1t-1}	0.4098	0.0895	4.5784	*
p_{2t-1}	-0.1055	0.0725	-1.4552	
AR(15)	0.1887	0.0777	2.4269	*
N = 106; Adj R^2 = 0.1458; F = 6.9749 (p = 0.0003); SSE = 362.0629; LM = 0.7618; White = 0.7277; Jarque-Bera = 0.5898; Chow = 0.2949				
p_{2t}				
constant	3.2415	0.6202	5.2262	*
p_{2t-1}	0.3604	0.0836	4.3120	*
p_{1t-1}	-0.0862	0.0825	-1.0455	
N = 124; Adj R^2 = 0.1214; F = 9.5007 (p = 0.0001); SSE = 589.4535; LM = 0.0796; White = 0.0311; Jarque-Bera = 0.0000; Chow = 0.9302				

Table 3.7 (contd.)

Variable	Coefficient	S.E.	t-statistic	
Treatment: PQ2				
p_{1t}				
constant	6.0752	1.1277	5.3873	*
p_{1t-1}	0.4892	0.0769	6.3573	*
p_{2t-1}	-0.1146	0.0982	-1.1667	
N = 127; Adj R^2 = 0.2573; F = 22.8311 (p = 0.0000); SSE = 1564.118;				
LM = 0.7167; White = 0.1195; Jarque-Bera = 0.2418; Chow = 0.0075; RESET = 0.0233				
p_{2t}				
constant	1.7717	0.7540	2.3497	*
p_{2t-1}	0.7439	0.0699	10.6414	*
p_{1t-1}	-0.0165	0.0483	-0.3428	
AR(1)	-0.3338	0.0981	-3.4026	*
N = 126; Adj R^2 = 0.3413; F = 22.5865 (p = 0.0000); SSE = 844.5732;				
LM = 0.6598; White = 0.8193; Jarque-Bera = 0.5680; Chow = 0.3405; RESET = 0.6223				
Treatment: PLQ1				
p_{1t}				
constant	4.7610	0.8962	5.3126	*
p_{1t-1}	0.2704	0.0934	2.8967	*
p_{2t-1}	0.2273	0.1033	2.2013	*
AR(5)	-0.2519	0.0900	-2.7997	*
MA(2)	0.1880	0.0957	1.9647	+
MA(7)	-0.2166	0.0947	-2.2869	*
N = 122; Adj R^2 = 0.2306; F = 8.2549 (p = 0.0000); SSE = 655.5547;				
LM = 0.4233; White = 0.0369; Jarque-Bera = 0.0002; Chow = 0.0490; RESET = 0.5748				
p_{2t}				
constant	1.4511	0.6419	2.2605	*
p_{2t-1}	0.4922	0.0770	6.3948	*
p_{1t-1}	0.1649	0.0680	2.4243	*
N = 127; Adj R^2 = 0.2983; F = 27.7859 (p = 0.0000); SSE = 481.6950;				
LM = 0.2546; White = 0.24451; Jarque-Bera = 0.2768; Chow = 0.6556; RESET = 0.1420				
Treatment: PLQ2				
p_{1t}				
constant	8.9896	1.7801	5.0502	*
p_{1t-1}	0.3226	0.0975	3.3095	*
p_{2t-1}	0.1510	0.0849	1.7777	+
AR(2)	0.4132	0.0955	4.3279	*
N = 120; Adj R^2 = 0.3584; F = 23.1564 (p = 0.0000); SSE = 1950.691;				
LM = 0.6880; White = 0.5901; Jarque-Bera = 0.7873; Chow = 0.0495				
p_{2t}				
constant	7.6013	1.4981	5.0741	*
p_{2t-1}	0.4084	0.0797	5.1221	*
p_{1t-1}	-0.0340	0.0719	-0.4730	
N = 124; Adj R^2 = 0.1648; F = 13.1345 (p = 0.0000); SSE = 2002.350;				
LM = 0.3828; White = 0.2049; Jarque-Bera = 0.6107; Chow = 0.4152				

* indicates significance at the 0.05 level. + indicates significance at the 0.10 level.
Probability statistics of the following tests are reported: serial correlation Lagrange multiplier test (LM), White's heteroskedasticity test (White), Jarque-Bera normality test (Jarque-Bera), Chow's breakpoint test (Chow) and Ramsey's RESET test (RESET). White's test includes all cross product terms. RESET test is based on a cubic functional form and is not performed if the sample is discontinuous. Breakpoint for Chow test is at mid-sample (n = 64).

H2: Firms play their best strategies.

To examine the hypothesis that firms play their best strategies, an initial observation is made on the frequency of various types of response behaviour relative to best strategy. The types of response behaviour may be classified as follows:

(1) *Appropriate response behaviour* is one in which a player moves in the same direction as that dictated by best response. The magnitude of the movement may equal or may not equal that called for under best response. If the magnitude of change surpasses best response but is in the correct direction, then it is an *over-increase or over-decrease response behaviour* depending on whether an increase or decrease was expected under best response.

(2) *Inappropriate response behaviour* comprises wrong response behaviour and no response behaviour. *Wrong response behaviour* occurs when a player moves in the opposite direction from that dictated by best response, while *no response* occurs when a player does not make a movement although one is called for under best response.

An over-positive or negative response behaviour comprises both appropriate and inappropriate response behaviour. An *over-positive response behaviour* is defined as comprising *over-increase (appropriate) response behaviour* and *wrong increase inappropriate) response behaviour*. Conversely, an *over-negative response behaviour* comprises both over-decrease response behaviour and wrong decrease response behaviour.

Table 3.8 gives the frequency of various types of response behaviour for both firms in the six treatments. The frequency of appropriate response averages 0.5879 while the frequency of inappropriate response averages 0.4121 for both firms in all treatments. The frequency of appropriate response for the inside firm (0.6025) is about the same as that for the outside firm (0.5866). Both firms, therefore, track their best response equally closely.

Table 3.8
Frequency of appropriate and inappropriate response relative to best strategy

Frequency	Appropriate response	Inappropriate response	Over-positive response			Over-negative response			No response
			Over increase	Wrong increase	Total	Over decrease	Wrong decrease	Total	
All treatments	0.5879	0.4121	0.1038	0.0753	0.1791	0.1052	0.1833	0.2885	0.1536
Inside firm	0.6025	0.3975	0.0983	0.0816	0.1799	0.0900	0.2259	0.3159	0.0900
Outside firm	0.5866	0.4134	0.1127	0.0793	0.1921	0.1148	0.1503	0.2651	0.1837
PL1	0.5523	0.4477	0.0711	0.0669	0.1381	0.0795	0.1339	0.2134	0.2469
Inside firm	0.5630	0.4370	0.0672	0.0168	0.0840	0.0756	0.1597	0.2353	0.2605
Outside firm	0.5417	0.4583	0.0750	0.1167	0.1917	0.0833	0.1083	0.1917	0.2333
PL2	0.6250	0.3750	0.1083	0.0542	0.1625	0.1167	0.1792	0.2958	0.1417
Inside firm	0.6167	0.3833	0.1083	0.0417	0.1500	0.1250	0.2083	0.3333	0.1333
Outside firm	0.6333	0.3667	0.1083	0.0667	0.1750	0.1083	0.1500	0.2583	0.1500
PQ1	0.6513	0.3487	0.1261	0.1134	0.2395	0.1261	0.1933	0.3193	0.0420
Inside firm	0.6639	0.3361	0.1345	0.0924	0.2269	0.1176	0.2101	0.3277	0.0336
Outside firm	0.6387	0.3613	0.1176	0.1345	0.2521	0.1345	0.1765	0.3109	0.0504
PQ2	0.5625	0.4375	0.0833	0.0792	0.1625	0.1042	0.2625	0.3667	0.0958
Inside firm	0.5583	0.4417	0.1000	0.1333	0.2333	0.0833	0.2583	0.3417	0.0500
Outside firm	0.5667	0.4333	0.0667	0.0250	0.0917	0.1250	0.2667	0.3917	0.1417
PLQ1	0.6125	0.3875	0.1208	0.0583	0.1792	0.0875	0.1708	0.2583	0.1583
Inside firm	0.5667	0.4333	0.0917	0.0500	0.1417	0.0583	0.2000	0.2583	0.1833
Outside firm	0.6583	0.3417	0.1500	0.0667	0.2167	0.1167	0.1417	0.2583	0.1333
PLQ2	0.6008	0.3992	0.1134	0.0798	0.1933	0.1176	0.1597	0.2773	0.1597
Inside firm	0.6218	0.3782	0.0672	0.0504	0.1176	0.1008	0.2353	0.3361	0.0924
Outside firm	0.5798	0.4202	0.1597	0.1092	0.2689	0.1345	0.0840	0.2185	0.2269

Inappropriate response frequency is the sum of wrong response frequency and no response frequency. Appropriate response frequency is total response frequency less inappropriate response frequency.

The low price behaviour is reflected by the much higher frequency of over-negative than over-positive response behaviour of both firms in all the treatments. In PQ2 where both firms fared the worst in attaining prediction, the frequency of over-negative behaviour is the highest of all treatments: 0.3917 for the outside firm and 0.3417 for the inside firm.

To study firm experience at playing best strategies, a Wilcoxon signed rank test and a Sign test are conducted on the null hypothesis of price decisions equal one-period lag best responses. The results presented in Table 3.9 show that for four treatments (PL1, PQ1, PLQ1 and PLQ2), the null hypothesis is significant at the 0.01 level by the late2 phase for both firms, suggesting that the firms get better at playing best strategies over the course of the experiment. In PL2, the null is significant only for the outside firm. In PL1, the null is significant for both firms in the early and late2 phases, and only for the outside firm in the late1 phase. This explains the decline in convergence for the inside firm in PL1 during the late1 phase and the subsequent convergence to an equilibrium closer to prediction in the late2 phase. As for PQ2, the inability of prices to attain prediction for both firms is exhibited by the insignificant Wilcoxon statistic throughout the experiment.

Table 3.9
Congruence of price decisions to best response
(probabilities for two-tailed Wilcoxon signed ranks test p_W and Sign test p_S)

Variable	pw		ps		Variable	pw		ps	
Null hypothesis: p_{it} = pbr_{it-1}									
Treatment: PL1					Treatment: PL2				
All periods					All periods				
p1	0.0000		0.0043		p1	0.0000		0.0000	
p2	0.0055		0.1207	*	p2	0.0390	+	0.0552	*
Early phase					Early phase				
p1	0.1253	*	0.6831	*	p1	0.0000		0.0000	
p2	0.0136	+	0.1416	*	p2	0.4264	*	0.3496	*
Late1 phase					Late1 phase				
p1	0.0004		0.0072		p1	0.0008		0.0014	
p2	0.0771	*	0.4292	*	p2	0.1270	*	0.4292	*
Late2 phase					Late2 phase				
p1	0.0101	+	0.0639	*	p1	0.0072		0.0015	
p2	0.8639	*	1.0000	*	p2	0.1064	*	0.1516	*
Treatment: PQ1					Treatment: PQ2				
All periods					All periods				
p1	0.0003		0.0162	+	p1	0.0000		0.0014	
p2	0.0514	*	0.0076		p2	0.0000		0.0000	
Early phase					Early phase				
p1	0.0001		0.0017		p1	0.0005		0.1416	*
p2	0.3096	*	0.4962	*	p2	0.0001		0.0002	
Late1 phase					Late1 phase				
p1	0.1506	*	0.5218	*	p1	0.0032		0.0820	*
p2	0.0513	*	0.0072		p2	0.0000		0.0000	
Late2 phase					Late2 phase				
p1	0.9886	*	0.8388	*	p1	0.0097		0.0227	+
p2	0.4575	*	0.3075	*	p2	0.0001		0.0003	
Treatment: PLQ1					Treatment: PLQ2				
All periods					All periods				
p1	0.0000		0.0000		p1	0.0000		0.0000	
p2	0.0005		0.0034		p2	0.7606	*	1.0000	*
Early phase					Early phase				
p1	0.0001		0.0000		p1	0.0000		0.0000	
p2	0.0000		0.0003		p2	0.2500	*	0.6885	*
Late1 phase					Late1 phase				
p1	0.0006		0.0009		p1	0.0000		0.0007	
p2	0.2912	*	0.4292	*	p2	0.6793	*	1.0000	*
Late2 phase					Late2 phase				
p1	0.0278	+	0.0066		p1	0.0718	*	0.1516	*
p2	0.7380	*	1.0000	*	p2	0.4654	*	0.6776	*

+ indicates significance at the 0.01 level
* indicates significance at the 0.05 level

The following model is estimated to determine if firms play their best strategies and improve their execution of best strategies over time.

(3.4) $$\left| p_{it} - pbr_{it-1} \right| = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_{it}$$

where p_{it} is the price of firm *i* in period *t,* $i \in \{1,2\}$, $t = \{1,...,16\}$; $pbr_{i,t-1}$ is the one-period lag best response in prices; DUM_{1t} is a dummy variable that equals 1 if $t = \{9,...,13\}$ and 0 otherwise; and DUM_{2t} is a dummy variable that equals 1 if $t = \{14,...16\}$ and 0 otherwise. If learning occurs in the late1 phase and late2 phase, then β_1 and β_2 respectively would be negative.

Results from the regression are given in Table 3.10. All diagnostic checks for stationary series, specification and stability are conducted as before. For the inside firm, all the β 's are negative: β_1 is significantly different from zero at the 0.05 level in all treatments except PLQ2 (insignificant), while β_2 is significantly different from zero at the 0.05 level for all treatments except PLQ1 (significant at the 0.10 level) and PL2 (insignificant).

For the outside firm, the β 's are largely insignificant except in two treatments, PL2 and PLQ1: β_1 is significantly different form zero at the 0.10 level in PL2 (positive) and PLQ1 (negative), while β_2 is negative and significantly different from zero at the 0.05 level in PLQ1.

There is evidence, therefore, that learning is occurring for the inside firm but the same cannot be said for the outside firm except in PLQ1. In PL2, prices of the outside firm appear to move away from best response in the late1 phase. In other words, the inside firm in all treatments gets better at using best strategies in making their pricing decisions but this is only true of the outside firm in PLQ1.

Table 3.10
Regression results for price decisions and best strategies

Variable	Coefficient	S.E.	t-statistic	
Model:	$\lvert p_{it} - pbr_{it-1} \rvert = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_{it}$			
Treatment: PL1				
$\lvert p_{1t}$-$pbr_{1t-1} \rvert$				
constant	1.7916	0.3026	5.9202	*
DUM_{1t}	-0.7533	0.2836	-2.6561	*
DUM_{2t}	-1.1023	0.2065	-5.3372	*
AR(1)	0.1761	0.1097	1.6050	
AR(3)	0.4101	0.1146	3.5773	*
AR(18)	0.3350	0.0930	3.6042	*
AR(22)	-0.2481	0.0909	-2.7281	*
N = 74; Adj R^2 = 0.4463; F = 10.8078 (p = 0.0000); SSE = 21.0731; DW = 2.2276; LM = 0.1908; White = 0.2121; Jarque-Bera = 0.3979; Chow = 0.0311				
$\lvert p_{2t}$-$pbr_{2t-1} \rvert$				
constant	0.9178	0.2401	3.8220	*
DUM_{1t}	-0.1128	0.1365	-0.8263	
DUM_{2t}	-0.1862	0.1730	-1.0759	
AR(1)	0.2960	0.1040	2.8461	*
AR(4)	0.2375	0.0938	2.5328	*
AR(18)	0.1920	0.1030	1.8651	+
N = 83; Adj R^2 = 0.1639; F = 4.2156 (p = 0.0019); SSE = 16.9193; DW = 2.0586; LM = 0.6175; White = 0.1048; Jarque-Bera = 0.2183; Chow = 0.2511				
Treatment: PL2				
$\lvert p_{1t}$-$pbr_{1t-1} \rvert$				
constant	1.5508	0.1348	11.5077	*
DUM_{1t}	-0.5568	0.1808	-3.0799	*
DUM_{2t}	-0.4132	0.2615	-1.5805	
AR(8)	0.2072	0.0990	2.0924	*
AR(13)	-0.2073	0.0953	-2.1741	*
N = 92; Adj R^2 = 0.0876; F = 3.1846 (p = 0.0172); SSE = 79.6093; DW = 1.8700; LM = 0.4476; White = 0.6916; Jarque-Bera = 0.0840; Chow = 0.1346				
$\lvert p_{2t}$-$pbr_{2t-1} \rvert$				
constant	0.6926	0.2648	2.6153	*
DUM_{1t}	0.5331	0.2953	1.8052	+
DUM_{2t}	0.0802	0.2752	0.2915	
AR(1)	0.4607	0.1031	4.4674	*
AR(2)	0.4272	0.1258	3.3952	*
AR(4)	-0.3406	0.1330	-2.5619	*
N = 94; Adj R^2 = 0.4388; F = 15.5459 (p = 0.0000); SSE = 65.0985; DW = 2.0838; LM = 0.6870; White = 0.3035; Jarque-Bera = 0.0146; Chow = 0.0000				
Treatment: PQ1				
$\lvert p_{1t}$-$pbr_{1t-1} \rvert$				
constant	2.7757	0.4767	5.8228	*
DUM_{1t}	-0.8557	0.3677	-2.3274	*
DUM_{2t}	-0.9433	0.4028	-2.3418	*
AR(2)	0.2990	0.0997	3.0003	*
AR(3)	0.3143	0.0876	3.5870	*
N = 100; Adj R^2 = 0.2490; F = 9.2074 (p = 0.0000); SSE = 200.1905; DW = 1.7974; LM = 0.9262; White = 0.1071; Jarque-Bera = 0.4973; Chow = 0.2751				
$\lvert p_{2t}$-$pbr_{2t-1} \rvert$				
constant	1.8256	0.2777	6.5731	*
DUM_{1t}	0.0244	0.4538	0.0537	
DUM_{2t}	0.1490	0.4859	0.3065	
AR(8)	-0.2023	0.1032	-1.9608	+
N = 100; Adj R^2 = 0.0176; F = 1.5897 (p = 0.1969); SSE = 277.1170; DW = 1.7768; LM = 0.0780; White = 0.3733; Jarque-Bera = 0.0000; Chow = 0.2192				

Table 3.10 (contd.)

Variable	Coefficient	S.E.	t-statistic	
Treatment: PQ2				
$\lvert p_{1t}-pbr_{1t-1}\rvert$				
constant	5.2220	0.6656	7.8455	*
DUM_{1t}	-1.3703	0.6380	-2.1478	*
DUM_{2t}	-1.7528	0.7884	-2.2232	*
AR(2)	0.1839	0.0964	1.9066	+
AR(4)	0.2131	0.0959	2.2212	*
N = 101; Adj R^2 = 0.1464; F = 5.2869 (p = 0.0007); SSE = 684.0223; DW = 1.6569; LM = 0.5389; White = 0.1292; Jarque-Bera = 0.0406; Chow = 0.4821				
$\lvert p_{2t}-pbr_{2t-1}\rvert$				
constant	3.5160	0.3376	10.4132	*
DUM_{1t}	0.0324	0.4780	0.0678	
DUM_{2t}	0.3723	0.6892	0.5402	
AR(2)	-0.1064	0.1002	-1.0612	
AR(14)	-0.2673	0.1011	-2.6446	*
N = 91; Adj R^2 = 0.0447; F = 2.0520 (p = 0.0942); SSE = 534.6410; DW = 1.2316; LM = 0.3446; White = 0.1571; Jarque-Bera = 0.0515; Chow = 0.4803				
Treatment: PLQ1				
$\lvert p_{1t}-pbr_{1t-1}\rvert$				
constant	2.8190	0.4410	6.3925	*
DUM_{1t}	-0.8322	0.3981	-2.0906	*
DUM_{2t}	-0.8407	0.4510	-1.8641	+
AR(2)	0.3411	0.0874	3.9024	*
AR(3)	0.1748	0.0849	2.0598	*
N = 102; Adj R^2 = 0.1945; F = 7.0962 (p = 0.0000); SSE = 233.1567; DW = 2.0149; LM = 0.7666; White = 0.3996; Jarque-Bera = 0.1081; Chow = 0.1986				
$\lvert p_{2t}-pbr_{2t-1}\rvert$				
constant	2.5203	0.2602	9.6878	*
DUM_{1t}	-0.6747	0.3623	-1.8624	+
DUM_{2t}	-1.0729	0.4582	-2.3416	*
AR(19)	-0.1851	0.0941	-1.9667	+
N = 95; Adj R^2 = 0.0680; F = 3.2853 (p = 0.0243); SSE = 248.0614; DW = 1.6488; LM = 0.8195; White = 0.4180; Jarque-Bera = 0.0000; Chow = 0.0847				
Treatment: PLQ2				
$\lvert p_{1t}-pbr_{1t-1}\rvert$				
constant	6.2989	1.1003	5.7247	*
DUM_{1t}	-1.4104	0.8573	-1.6452	
DUM_{2t}	-1.9898	0.9889	-2.0122	*
AR(2)	0.3570	0.0976	3.6589	*
AR(3)	0.2399	0.0982	2.4434	*
N = 99; Adj R^2 = 0.2421; F = 8.8280 (p = 0.0000); SSE = 1120.511; DW = 1.5453; LM = 0.6480; White = 0.0864; Jarque-Bera = 0.0715; Chow = 0.3094				
$\lvert p_{2t}-pbr_{2t-1}\rvert$				
constant	3.8670	0.7862	4.9185	*
DUM_{1t}	-0.7836	0.8141	-0.9626	
DUM_{2t}	-0.8767	0.9355	-0.9371	
AR(6)	0.2008	0.1113	1.8048	+
AR(18)	0.1951	0.1053	1.8519	+
N = 83; Adj R^2 = 0.0506; F = 2.0917 (p = 0.0898); SSE = 795.5931; DW = 1.3203; LM = 0.9105; White = 0.7438; Jarque-Bera = 0.0010; Chow = 0.7354				

See Table 3.7 for notes. Failure of Chow's test for the outside firm in PL2 is due to an outlying residual value at the breakpoint. Failure of Jarque-Bera test is due to 1-3 outlying values.

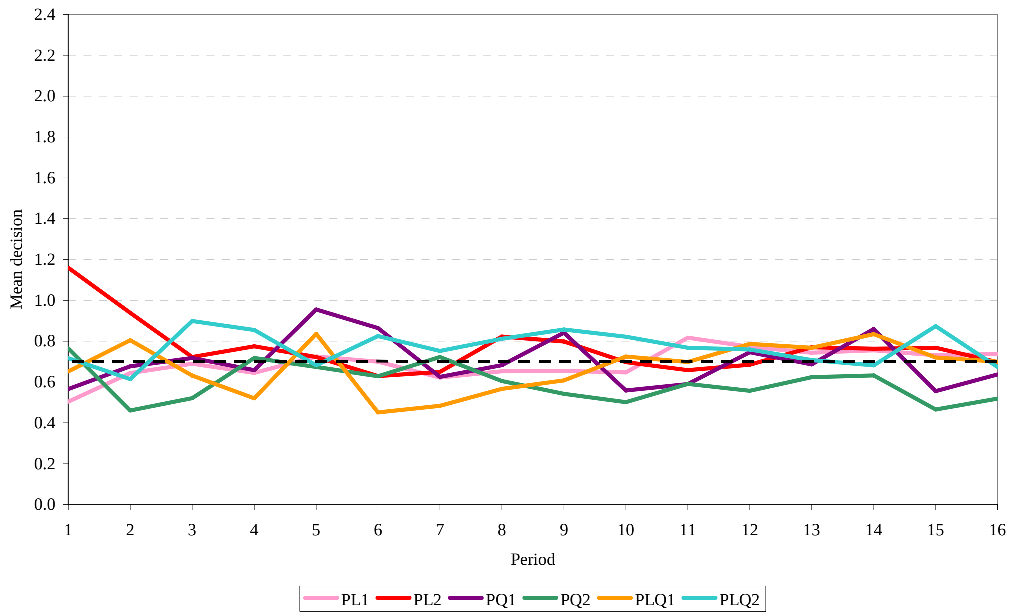

Fig. 3.20 Relative price under different transportation costs

H3: Relative price is the same under different transportation costs (Proposition 1).

Is relative price identical for all transportation cost structures, in accordance with Proposition 1? A plot of the time series of mean relative price (p_2/p_1) in Figure 3.20 shows that the relative price in five treatments is very close to the predicted level of 0.71 (depicted by a broken line) but in PQ2, the relative price is below prediction. This is a direct consequence of the inability of prices in PQ2 to attain prediction.

A Friedman test accords support for the Proposition 1. The null hypothesis is relative price remains the same regardless of transportation costs, i.e., $p_{2jt}/p_{1jt} = p_{2kt}/p_{1kt}$ where p_{ij} and p_{ik} denotes the price of firm i in treatment j and k respectively, $i \in \{1,2\}$ and $j,k \in \{1,...,6\}$, $j \neq k$. The results in Table 3.11 show that relative price is identical for the six treatments at the 0.01 level in the early phase and at the 0.05 level in the late1 and late2 phases.

Table 3.11
Relative price and relative demand are the same under different transportation costs (probabilities for Friedman test p_F)

Variable	N	p_F		N	p_F	
Null hypothesis:	$p_{2jt}/p_{1jt} = p_{2kt}/p_{1kt}$			$m_{2jt}/m_{1jt} = m_{2kt}/m_{1kt}$		
All periods	123	0.0233	+	118	0.0210	+
Early phase	61	0.0331	+	56	0.1481	*
Late1 phase	38	0.2421	*	38	0.0143	+
Late2 phase	24	0.4548	*	24	0.5025	*

+ indicates significance at the 0.01 level
* indicates significance at the 0.05 level
$j \neq k$ represents the treatment number

H4: Relative demand is the same under different transportation costs (Proposition 2.1).

Figure 3.21 shows that the mean relative demand (m_2/m_1) for the six treatments is equivalent under different transportation cost structures and reaches the predicted level of 0.71 (depicted by a broken line) fairly early in the experiment. This observation is verified by a Friedman test on the null hypothesis that the relative demand is the same regardless of transportation costs, i.e., $m_{2jt}/m_{1jt} = m_{2kt}/m_{1kt}$ where m_{ij} and m_{ik} denotes the demand for firm i in treatment j and k respectively, $i \in \{1,2\}$ and $j,k \in \{1,\ldots,6\}$, $j \neq k$. The results in Table 3.11 show that relative demand is identical for the six treatments at the 0.05 level in the early and late2 phases and at the 0.01 level in the late1 phase.

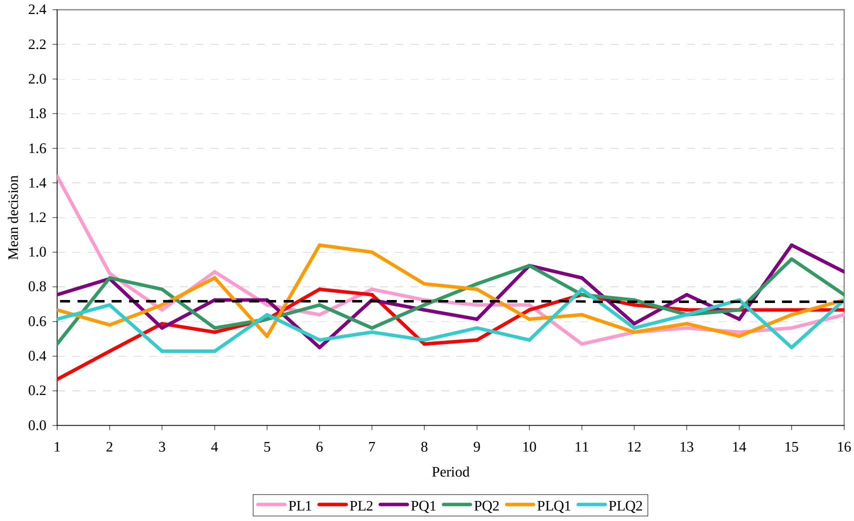

Fig. 3.21 Relative demand under different transportation costs

H5: Relative demand is equivalent to relative price (Proposition 2.2).

It can be expected from the results of hypotheses H3 and H4 that relative demand is equivalent to relative price regardless of the transportation cost structure. Figure 3.22 shows the time series of mean relative demand (m_2/m_1) and mean relative price (p_2/p_1) for each treatment. The predicted equilibrium level is depicted by a broken line. While the two series reach the predicted level within five periods in all six treatments, there appears to be a greater departure of m_2/m_1 from p_2/p_1 in the early phase of PL2 and PLQ2. This observation is verified by a Wilcoxon signed rank test and a Sign test on the null hypothesis that $p_2/p_1 = m_2/m_1$, using individual player data. The results in Table 3.12 indicate that the null is significant at the 0.05 level throughout the experiment, except for PL2 and PLQ2 where the null is insignificant in the early phase at the 0.05 level.[9]

[9] Comparison of the means using standard *t*-tests is not meaningful except for PL2 since a Kolmogorov-Smirnov Test shows normal distribution for only one relative price series (PL2), and three relative demand series (PL2, PQ2, PLQ1) at the 0.01 level and one relative demand series (PLQ2) at the 0.05 level.

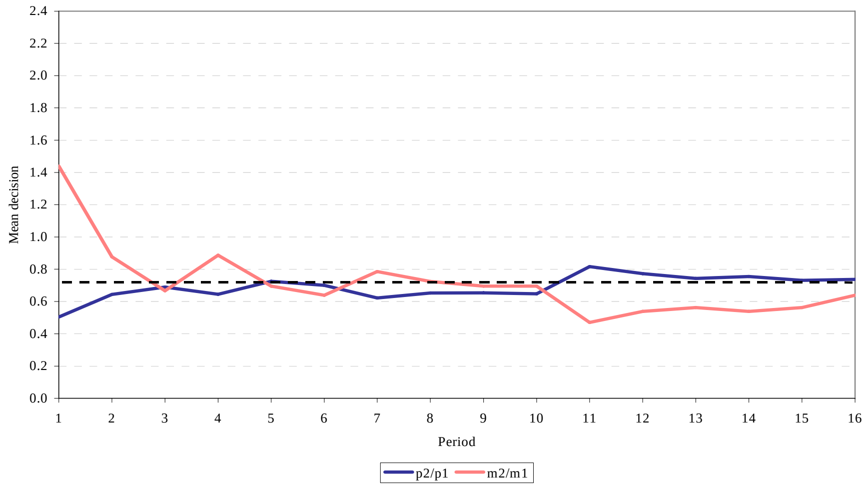

Fig. 3.22(a) Time series of mean relative demand and mean relative price (PL1)

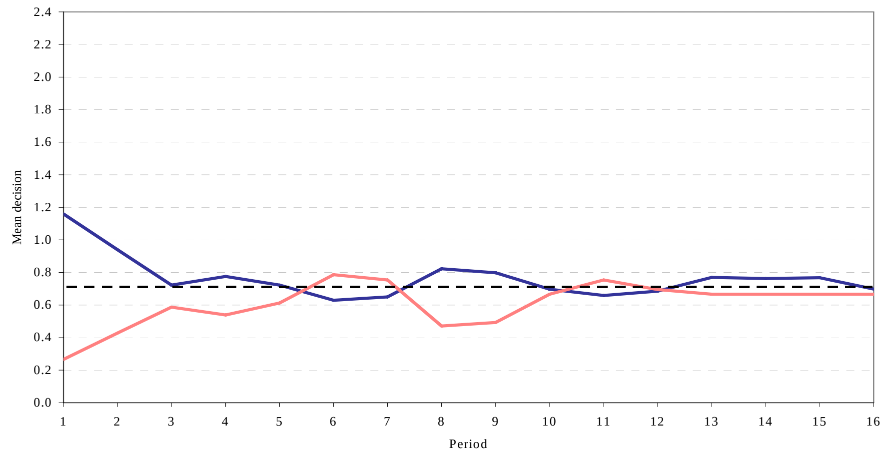

Fig. 3.22(b) Time series of mean relative demand and mean relative price (PL2)

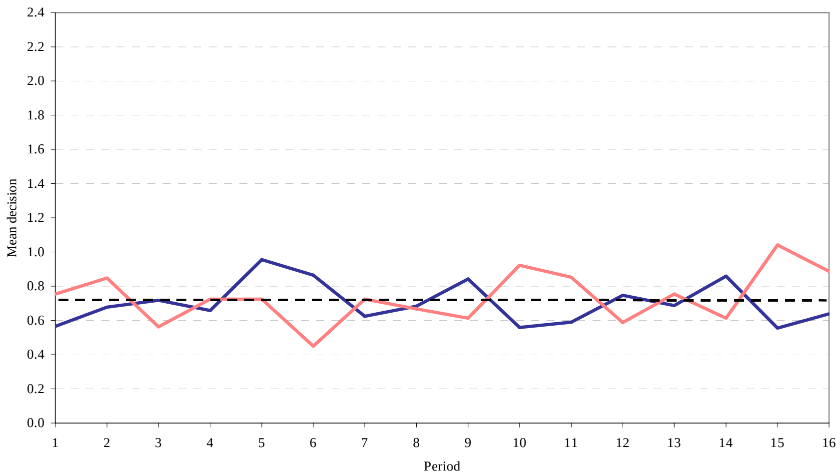

Fig.3.22(c) Time series of mean relative demand and mean relative price (PQ1)

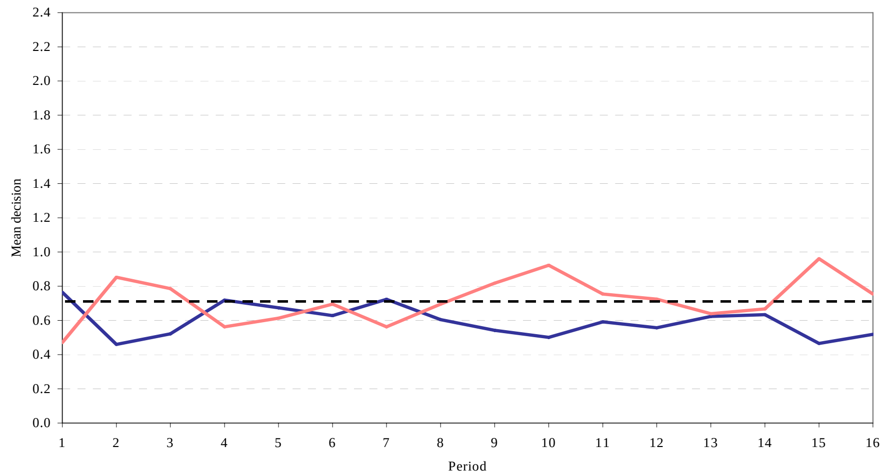

Fig. 3.22(d) Time series of mean relative demand and mean relative price (PQ2)

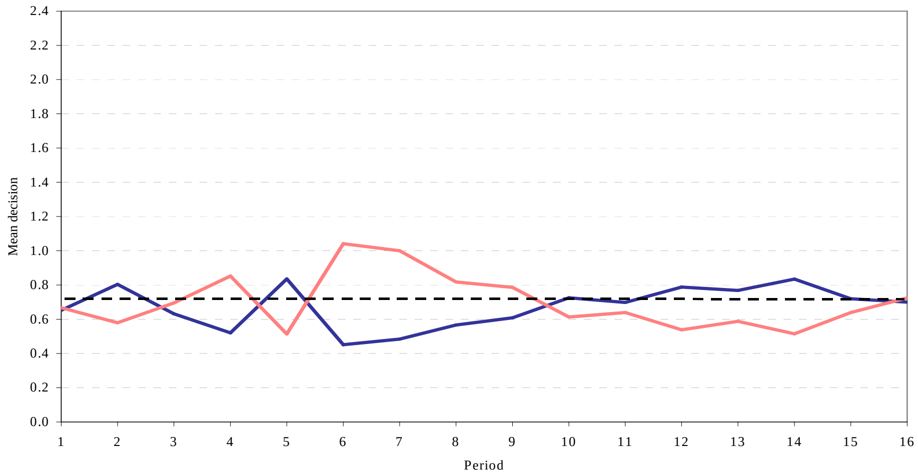

Fig. 3.22(e) Time series of mean relative demand and mean relative price (PLQ1)

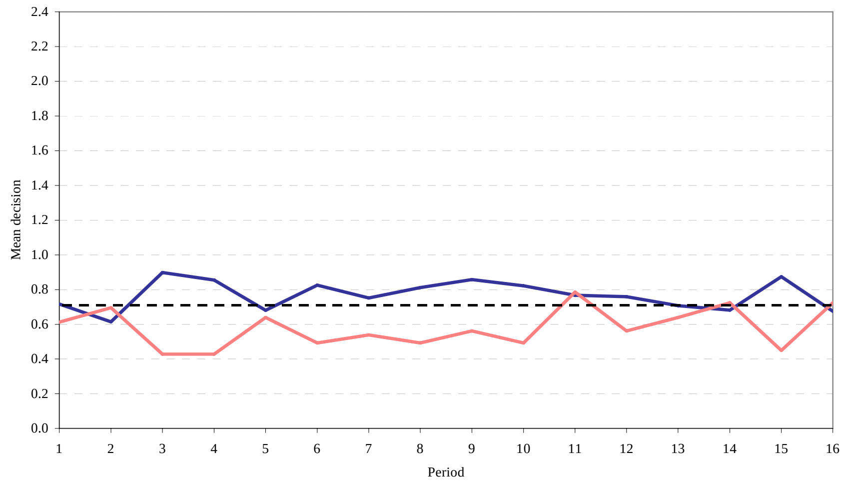

Fig.3.22(f) Time series of mean relative demand and mean relative price (PLQ2)

Table 3.12
Relative demand and relative price
(probabilities for two-tailed Wilcoxon signed ranks test p_W and Sign test p_S)

Treatment	N	pw		ps	
Null hypothesis: $p_2/p_1 = m_2/m_1$					
Treatment: PL1					
All periods	124	0.8850	*	0.7876	*
Early phase	60	0.2959	*	0.2453	*
Late1 phase	40	0.5188	*	0.8744	*
Late2 phase	24	0.5872	*	0.5413	*
Treatment: PL2					
All periods	128	0.0134	+	0.0634	*
Early phase	64	0.0031		0.0040	
Late1 phase	40	0.8193	*	0.8744	*
Late2 phase	24	0.7533	*	1.0000	*
Treatment: PQ1					
All periods	125	0.9617	*	0.5915	*
Early phase	61	0.1602	*	0.6085	*
Late1 phase	40	0.2646	*	0.4292	*
Late2 phase	24	0.4237	*	0.3075	*
Treatment: PQ2					
All periods	128	0.2458	*	0.0931	*
Early phase	64	0.7381	*	0.7077	*
Late1 phase	40	0.0696	*	0.0820	*
Late2 phase	24	0.1888	*	0.5413	*
Treatment: PLQ1					
All periods	127	0.7236	*	0.5944	*
Early phase	63	0.2178	*	0.1306	*
Late1 phase	40	0.5101	*	0.8744	*
Late2 phase	24	0.4405	*	0.5413	*
Treatment: PLQ2					
All periods	126	0.0020		0.0098	
Early phase	64	0.0086		0.0336	+
Late1 phase	38	0.0939	*	0.1443	*
Late2 phase	24	0.6071	*	0.8388	*

+ indicates significance at the 0.01 level
* indicates significance at the 0.05 level

To evaluate the extent of divergence of m_2/m_1 from p_2/p_1, the following equation is estimated for the six treatments:

$$\frac{m_{2t}}{m_{1t}} = \alpha + \beta \frac{p_{2t}}{p_{1t}} + \varepsilon_t \tag{3.5}$$

where p_{it} and m_{it} denote the price and demand respectively of firm *i* in period *t*, $i \in \{1,2\}$.

The regression results are shown in Table 3.13. All diagnostic and stability tests indicate that the estimates are acceptable (normality of the residuals is rejected for all treatments by a Jarque-Bera test but the test is dubious given the presence of one or two outlying values in each instance). The β 's are negative and significant at the 0.05 level in all treatments except PQ1 where it is significant at the 0.10 level. For PQ1, the adjusted R^2 is also very low. In only one treatment, PL1, is $|\beta|$ close to 1. There is, therefore, little evidence from the regressions that relative demand equals relative price. The lack of price convergence to prediction has clearly resulted in an inability of relative price to match relative demand.

H6: The price increase is greater for the inside firm than the outside firm when transportation costs increase for a given transportation cost structure (Proposition 3).

In treatments PL2, PQ2 and PLQ2, the transportation cost parameters are increased by 100% over those in PL1, PQ1 and PLQ1 respectively. The impact of a transportation cost increase on the prices of both firms can, therefore, be observed by comparing prices in PL1 and PL2, PQ1 with PQ2, and PLQ1 with PLQ2.

Figure 3.23 shows that the mean price difference of the inside firm under a 100% increase in transportation cost parameters is generally larger than the mean price difference for the outside firm. There are between one and seven periods in which the converse holds, i.e., the mean price increase of the outside firm surpasses that of the inside firm: (1) linear transportation costs: periods 1, 2 and 8; (2) quadratic transportation costs: period 1; and

Table 3.13
Regression results for relative price and relative demand

Variable	Coefficient	S.E.	t-statistic	
Model:	$\frac{m_{2t}}{m_{1t}} = \alpha + \beta \frac{p_{2t}}{p_{1t}} + \varepsilon_t$			
Treatment: PL1				
m_{2t}/m_{1t}				
constant	1.8719	0.2342	7.9943	*
p_{2t}/p_{1t}	-1.1094	0.2240	-4.9534	*
AR(1)	0.7913	0.1124	7.0388	*
AR(6)	-0.1386	0.0823	-1.6832	+
AR(14)	-0.1884	0.0823	-2.2903	*
N = 99; Adj R^2 = 0.4364; F = 19.9709 (p = 0.0000); SSE = 51.6748; DW = 1.5857; LM = 0.0543; White = 0.1916; Jarque-Bera = 0.0000; Chow = 0.0311				
Treatment: PL2				
m_{2t}/m_{1t}				
constant	1.2182	0.0605	20.1412	*
p_{2t}/p_{1t}	-0.7003	0.0682	-10.2691	*
AR(6)	-0.7988	0.0692	-11.5476	*
MA(6)	0.8000	0.0403	19.8524	*
MA(8)	0.2166	0.0403	5.3773	*
N = 122; Adj R^2 = 0.5335; F = 35.5970 (p = 0.0000); SSE = 8.9981; DW = 1.9307; LM = 0.2305; White = 0.0140; Jarque-Bera = 0.0000; Chow = 0.3240				
Treatment: PQ1				
m_{2t}/m_{1t}				
constant	1.0059	0.1414	7.1164	*
p_{2t}/p_{1t}	-0.1467	0.0853	-1.7193	+
AR(1)	0.1892	0.0910	2.0780	*
N = 122; Adj R^2 = 0.0467; F = 3.9611 (p = 0.0216); SSE = 134.6155; DW = 1.9834; LM = 0.5182; White = 0.7496; Jarque-Bera = 0.0000; Chow = 0.1188				
Treatment: PQ2				
m_{2t}/m_{1t}				
constant	1.0662	0.0828	12.8734	*
p_{2t}/p_{1t}	-0.3350	0.0694	-4.8246	*
AR(6)	0.2342	0.0817	2.8668	*
N = 122; Adj R^2 = 0.2003; F = 16.1545 (p = 0.0000); SSE = 34.2066; DW = 1.8724; LM = 0.3215; White = 0.2550; Jarque-Bera = 0.0000; Chow = 0.1750				
Treatment: PLQ1				
m_{2t}/m_{1t}				
constant	1.1434	0.0678	16.8636	*
p_{2t}/p_{1t}	-0.5201	0.0745	-6.9780	*
N = 127; Adj R^2 = 0.2746; F = 48.6926 (p = 0.0000); SSE = 22.3880; DW = 1.8974; LM = 0.6396; White = 0.0200; Jarque-Bera = 0.0000; Chow = 0.1825				
Treatment: PLQ2				
m_{2t}/m_{1t}				
constant	1.0292	0.0670	15.3531	*
p_{2t}/p_{1t}	-0.4946	0.0684	-7.2281	*
AR(28)	-0.1596	0.0892	-1.7886	+
N = 98; Adj R^2 = 0.3331; F = 25.2285 (p = 0.0000); SSE = 8.9177; DW = 2.2343; LM = 0.4478; White = 0.0266; Jarque-Bera = 0.0000; Chow = 0.9969				

See Table 3.7 for notes. Failure of Jarque-Bera test due to 1-2 outlying values.

(3) linear-quadratic transportation costs: periods 3-4, 6-9 and 15.

To evaluate price increases by the two firms under higher transportation costs over the course of the experiment, the following equation is estimated:

(3.6) $$(p_{12t} - p_{11t}) - (p_{22t} - p_{21t}) = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_t$$

where p_{ij} is the price of firm *i* for treatment of type *j* where $i, j = \{1,2\}$. Type $j = 1$ denotes the treatment before the transportation cost increase while $j = 2$ denotes the treatment with a 100% transportation cost increase. DUM_{1t} is a dummy variable that equals 1 if $t = \{9,...,13\}$ and 0 otherwise, and DUM_{2t} is a dummy variable that equals 1 if $t = \{14,...16\}$ and 0 otherwise.

The regression results are presented in Table 3.14. All diagnostic and stability tests indicate that the estimates are reliable. In all treatments, α is positive and significantly different from zero, while β_1 and β_2 are not significantly different from zero at the 0.05 level. Under quadratic transportation costs, β_2 is positive and significant at the 0.10 level.

This implies that under higher transportation costs, the price increase by the inside firm is consistently higher than the price increase by the outside firm throughout the experiment. Under quadratic transportation costs, the price increase by the inside firm also grows larger during the last three periods of the treatments, widening the price gap between the two firms.

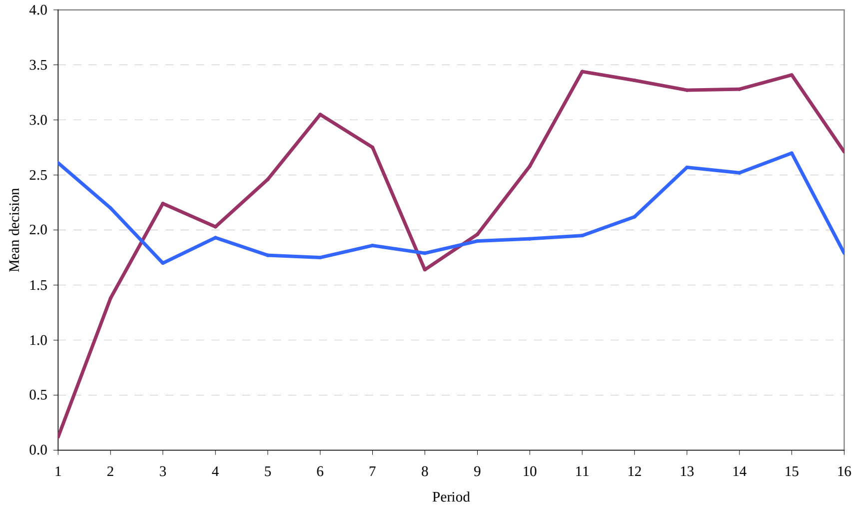

Fig. 3.23 (a) Time series of mean price difference under higher linear transportation costs

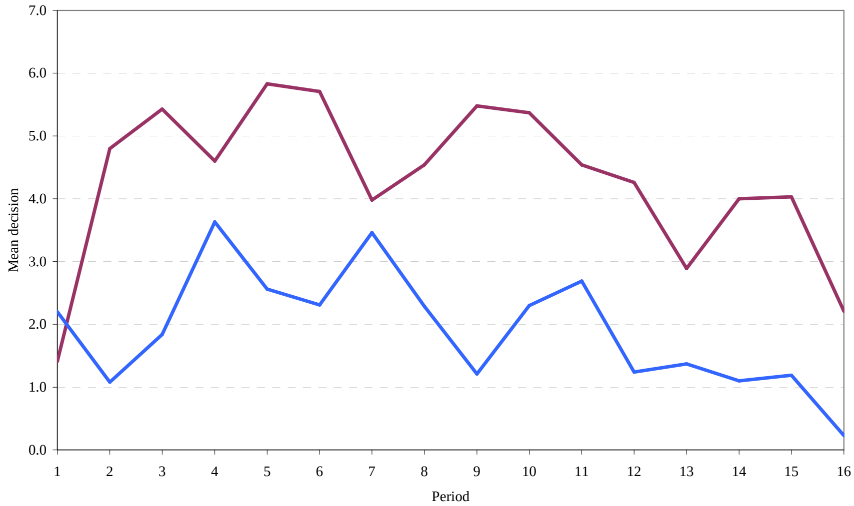

Fig.3.23 (b) Time series of mean price difference under higher quadratic transportation costs

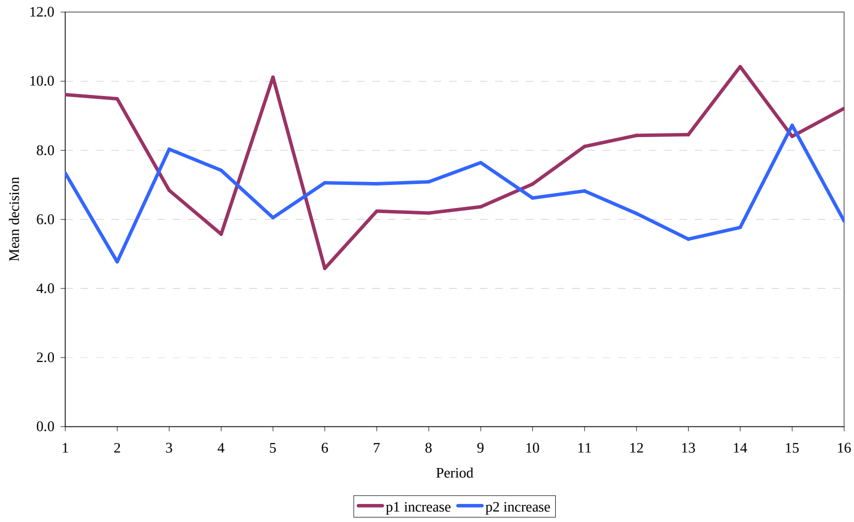

Fig. 3.23 (c) Time series of mean price difference under higher linear-quadratic transportation costs

Table 3.14
Regression results for impact of transportation cost increase on prices

Variable	Coefficient	S.E.	t-statistic	
Model:	$(p_{12t} - p_{11t}) - (p_{22t} - p_{21t}) = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_t$			
Treatments: PL1, PL2				
$(p_{12t}-p_{11t})-(p_{22t}-p_{21t})$				
constant	2.1287	0.3367	6.3231	*
DUM_{1t}	-0.2828	0.3838	-0.7367	
DUM_{2t}	-0.1663	0.4210	-0.3951	
AR(1)	0.5721	0.0746	7.6732	*
N = 125; Adj R^2 = 0.3146; F = 19.9763 (p = 0.0000); SSE = 200.9636; DW = 1.9319; LM = 0.8390; White = 0.9015; Jarque-Bera = 0.0549; Chow = 0.3104				
Treatments: PQ1, PQ2				
$(p_{12t}-p_{11t})-(p_{22t}-p_{21t})$				
constant	4.5131	0.9368	4.8174	*
DUM_{1t}	-0.1218	0.9330	-0.1305	
DUM_{2t}	1.7737	1.0675	1.6615	+
AR(1)	0.4391	0.0843	5.2085	*
AR(4)	0.1419	0.0786	1.8057	*
N = 118; Adj R^2 = 0.2315; F = 9.8134 (p = 0.0000); SSE = 1372.294; DW = 1.9036; LM = 0.5603; White = 0.0245; Jarque-Bera = 0.0550; Chow = 0.0717				
Treatments: PLQ1, PLQ2				
$(p_{12t}-p_{11t})-(p_{22t}-p_{21t})$				
constant	6.7469	0.8495	7.9420	*
DUM_{1t}	-0.1319	1.0901	-0.1210	
DUM_{2t}	-0.2509	1.1970	-0.2096	
AR(1)	0.2463	0.0894	2.7536	*
AR(2)	0.1816	0.0888	2.0458	*
N = 121; Adj R^2 = 0.0910; F = 4.0038 (p = 0.0044); SSE = 1962.361; DW = 1.9219; LM = 0.3391; White = 0.2244; Jarque-Bera = 0.0455; Chow = 0.2172				

See Table 3.7 for notes.

3.5 CONCLUSIONS

The experimental results indicate that the IO model is a behaviourally valid framework for studying competition between two firms situated on either side of a market border. Faced with fixed location, a large majority of players initially price below prediction, resulting in reduced rent. In particular, the inside firm exhibits a strong tendency to adopt low prices that are close to levels that monopolise the market. Nevertheless, best strategies help to increase producer surplus and direct prices closer to (but still below) prediction in all but one treatment.

The results also provide strong support for the propositions that relative price and relative demand remain invariant regardless of the transportation cost structure (Propositions 1 and 2.1). Moreover, an increase in transportation costs is shown to result in higher price increases by the inside firm compared to the outside firm (Proposition 3), i.e., consumers alleviate higher prices by incurring higher transportation costs. Limited evidence, however, is accorded to the equivalence of relative price and relative demand (Proposition 2.2) due to the general convergence of prices to a level below prediction.

In the experiment reported here, constant location appears to foster reduced competition under higher transportation costs, leading to higher prices. This is in accord with Proposition 3. Will the scenario differ under variable location? In the next chapter, we modify the experimental design and examine the situation in which both firms are able to relocate themselves.

CHAPTER 4

EXPERIMENTAL EVIDENCE WITH VARIABLE FIRM LOCATION

4.1 INTRODUCTION

A basic tenet in traditional location literature is that firms should maximise their level of product differentiation to reduce price competition (e.g., Prescott and Visscher 1977; d'Aspremont *et al*. 1979; Shaked and Sutton 1982; Economides 1986). On the other hand, minimum product differentiation is proposed if price competition is absent (e.g., de Palma *et al*. 1985; Dudey 1990; Sorenson 1997), i.e., both firms should locate close together if products are horizontally differentiated (in line with Hotelling (1929)'s arguments).

Despite the prominence accorded to both price and product differentiation strategies in oligopolistic literature, experimental studies on spatial competition typically treat one of these strategies as unchanging. Brown-Kruse *et al.* (1993), Brown-Kruse and Schenk (2000), Collins and Sherstyuk (2000) and Huck *et al.* (2002) assume constant prices, while Selten and Apesteguia (2004) assume constant location.[1] Alternatively, in the context of voting models, the focus is on voter location of candidates with the price of vote held constant. By focusing on the location or price of firms, these studies observe the extent to which either decision conforms to theoretical equilibrium predictions, rather than a dynamic interplay of the two strategies. For example, Brown-Kruse, *et al.* (1993) tested Hotelling's duopoly model in a fixed price environment with elastic (linear, downward sloping) consumer demand and different treatments in which communication among firms is either permitted or absent. Their findings support the theoretical predictions that firms locate at the centre of the market in the absence of communication but congregate towards the collusive equilibrium, i.e., the market

[1] This does not in any way ascribe to the inferiority of these studies as their intent bears a different focus.

quartiles, if communication is allowed. Selten and Apesteguia (2004)'s study of price competition among varying number of firms in a circular market (*à la* Salop (1979) and Beckmann (1989)) showed that when firms have no knowledge of the functional relationship between price and profit, their behaviour is typically influenced by imitation and cooperation tendencies. In such situations, prices converge to outcomes other than Nash equilibrium, *viz*., the imitation equilibrium with sub-Nash prices and the joint profit maximising equilibrium with supra-Nash prices respectively.[2]

Among the few exceptions that study the interaction of price and product differentiation are Barreda *et al.* (2000) and Camacho-Cuena *et al.* (2004). Barreda *et al.* (2000) studied price and product differentiation in a duopoly faced with horizontal differentiation in a discrete framework. Location-then-price decisions are made from finite strategic spaces that coincide with discrete consumer locations. The authors observed that product differentiation tends to be low and firms with marginally fewer consumers tend to price higher than firms with marginally more consumers. Camacho-Cuena *et al.* (2004) extended this study by including buyer location-purchase decisions in a four-stage game. First, sellers make their location decisions, followed by buyers' location decisions. In the ensuing stage, sellers set prices and finally, buyers make purchase decisions. This four-stage game is repeated over a fixed time horizon with two different sub-periods representing different speeds of technological change or product redesign. In the slow-innovation treatment, location decisions are kept constant for five periods during which only price-purchase decisions change. In the rapid-innovation treatment, location decisions are invariant over two periods. The authors found that rapid product redesign is consistent with higher product differentiation and prices. As technology changes more rapidly, buyers attempt to mitigate seller monopolistic power by incurring higher transportation costs.

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[2] Selten and Aspeteguia (2004) reiterated the definition of an *imitation equilibrium*, first introduced in Selten and Ostmann (2001), as one which satisfies the following four stability properties: (1) *finiteness*: all deviation paths are finite, (2) *involvement*: a destination via a deviation path involving a deviator must be the imitation equilibrium, (3) *payoff*: all deviator payoffs other than the one with deviator involvement are lower, and (4) *return*: all return paths are finite and reach the imitation equilibrium.

When firms vary price and location decisions at the same time or with equal rapidity, a simultaneous price-location game becomes more relevant. Numerous products in the real world experience such price and design changes, e.g. computers, mobile phones, computer games and software. Typical of such products is the invariant price of existing models for a short period until a new design enters the market. Such highly rapid innovation situations further shorten Camacho-Cuena *et al.* (2004)'s two-period constant location decision to a single period in which price and location decisions are made at the same time.

This chapter looks at situations in which endogenous price and product differentiation strategies are made simultaneously. Of pertinent interest are situations in which oligopolistic competition occurs with entry barrier of firms into rival markets due to prohibitive legislation or costs. Under these circumstances, firms produce a rapidly innovative product within their own precincts and ship it to neighbouring markets to be sold or alternatively, cross-border shoppers travel out of their domestic market to shop for the product. The IO model proposed earlier presents itself as an appropriate framework for such a study.

In an experimental spatial environment that corresponds to the theoretical setup of the IO model, two firms simultaneously make location and price decisions regarding the product they intend to sell. The *inside firm* locates within the same linear market as consumers along $[0,1]$ while the *outside firm* locates beyond the market boundary along $]1,+\infty[$. Firm entry into rival markets is closed. Consumer demand is generated by the computer and is assumed to be inelastic, with consumer location uniformly distributed along $[0,1]$. Consumers travel to either firm to purchase the product and bear the full burden of travel costs according to a predetermined transportation cost structure. Price ties are resolved in favour of the closer firm. The theoretical prediction of the simultaneous price-location game is that a unique non-cooperative Nash-Bertrand equilibrium in pure strategies exists only under strictly convex transportation costs. The extent of price and product differentiation is predicted to vary with the type of transportation cost structure and rate of transportation cost increase. In addition, relative price and relative demand is expected to be identical under the same transportation cost structure.

Given the significance ascribed to price and product differentiation strategies under varying transportation costs in the IO model, the experiment was conducted with four treatments: two treatments characterise different transportation cost structures while two treatments pertain to an increase in transportation costs under different transportation cost structures. As predicted by the theoretical model, the results show that higher transportation costs entail lower degrees of product differentiation and heightened price competition. Pricing behaviours that do not conform to best response were observed, such as low price behaviour and price leader behaviour. These have a tendency of driving profit away from the non-cooperative Nash-Bertrand equilibrium in pure strategies. Cooperative behaviour was largely ineffective due to the nature of the experimental design: no communication was permitted and players were rotated among themselves with each period of trade. The results accord limited support for the proposition that relative demand and relative prices remain constant within the same transportation cost structure.

This chapter is organised as follows. In Sections 4.2, we present the theoretical predictions. In Section 4.3, we describe the experimental procedures. In Section 4.4, the results are discussed. The final section concludes this chapter.

4.2 THEORETICAL PREDICTIONS

The analyses of this chapter focus on the non-cooperative Nash-Bertrand equilibrium in pure strategies and the propositions of the IO model. This section presents the theoretical framework and a summary of the theoretical predictions for the simultaneous price-location game. Consider a differentiated product market in which two firms, an inside firm $i = 1$ and an outside firm $i = 2$, play a simultaneous price-location game. During each exchange period, the two firms $i \in \{1,2\}$ simultaneously make two decisions: (1) location x_i where $x_1 \in [0,1]$ and $x_2 \in]1, \ell]$, $\ell < +\infty$ corresponds to ℓ available locations to the outside firm, and (2) product price p_i. The strategy pair of each firm is denoted by (p_i, x_i). All locations are uniformly distributed along the segments $[0,1]$ and $]1, \ell]$ and both firms are equally ex ante efficient in producing each firm's good. The good is assumed to be homogeneous and perfectly divisible among consumers. For simplicity, the firms are assumed to incur no marginal production costs and no storage or distribution costs other than transportation (or delivery) costs which are borne fully by consumers. The consumers are assumed to be uniformly distributed along the unit interval $[0,1]$ and purchase the good from either seller. Price ties are resolved in favour of the nearer firm. In making their purchase decisions, each consumer makes a decision on the preferred product specification of the good (price and location) offered by the two firms. The demand for each firm's good is denoted by m_i. The distance between consumer and seller locations corresponds to the distance d that the consumer travels to purchase the good. The transportation costs incurred by consumers increase according to a predetermined transportation cost schedule that bears one of two functional forms: $c(x) = sd^2$ (quadratic) and $c(x) = td + sd^2$ (linear-quadratic) where $t > 0$, $s > 0$ and $t/s \leq 2/3$.[3] Each firm earns a profit equivalent to $\prod_i = p_i \cdot m_i - AC$ where AC is the (constant) average cost of production which for simplicity is assumed to be zero.

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[3] For a unique location equilibrium in pure strategies to exist in the simultaneous game of the IO model, $t/s \leq 2/3$.

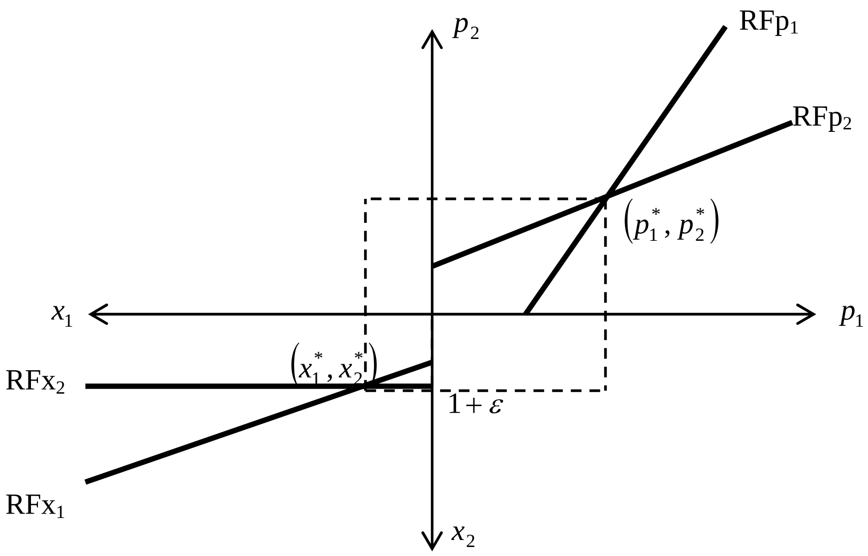

Fig. 4.1 Price-location simultaneous equilibrium

The quadruple $((p_1^*, x_1^*),(p_2^*, x_2^*))$ gives the simultaneous price-location Nash equilibrium in pure strategies and is obtained by the intersection of the four best response functions in price and location of the two firms (Figure 4.1). In the case of linear-quadratic transportation costs, the best response functions in price and location are reproduced in equations 4.1 to 4.4. Regardless of the transportation cost structure, the dominant location strategy for the outside firm is to locate next to the market border at $1+\varepsilon$ where ε is arbitrarily small. The response functions under quadratic transportation costs can be obtained by setting the linear transportation cost parameter $t=0$. The derivation of the response functions was made in Chapter 2.

(4.1) $$p_1 = \frac{1}{2}\left[p_2 + (t + s(x_2 - x_1))(x_1 + x_2)\right]$$

(4.2) $$p_2 = \frac{1}{2}\left[p_1 + (t + s(x_2 - x_1))(2 - x_1 - x_2)\right]$$

(4.3) $$x_1 = \frac{(2 + x_2)}{5} + \frac{3t}{5s}$$

(4.4) $$x_2 = 1 + \varepsilon$$

Table 4.1
Theoretical predictions

Prediction		Transportation cost structure			
		Quadratic		Linear-quadratic	
		s = 6.5	s = 13	t = 2.6, s = 6.5	t = 5.2, s = 13
Location	(x_1, x_2)	(0.61, 1.01)	(0.61, 1.01)	(0.85, 1.01)	(0.85, 1.01)
Price	(p_1, p_2)	(3.14, 2.06)	(6.27, 4.13)	(4.68, 2.60)	(9.37, 5.19)
Demand	(m_1, m_2)	(0.60, 0.40)	(0.60, 0.40)	(0.64, 0.36)	(0.64, 0.36)
Relative price	$rp = p_2/p_1$	0.66	0.66	0.55	0.55
Relative demand	$rm = m_2/m_1$	0.66	0.66	0.55	0.55
Profit	(r_1, r_2)	(1.89, 0.82)	(3.79, 1.64)	(3.01, 0.93)	(6.03, 1.85)

Table 4.1 gives the theoretical predictions of the IO model under the two convex transportation cost structures using the parameter values employed in the experiment. The predicted values for location, price, demand, relative price, relative demand and profit are provided. The predictions before and after a 100% increase in transportation cost parameters *t* and *s* are shown.

Two propositions of the IO model pertaining to the simultaneous price-location game, discussed in Chapter 2, are reproduced below.

Proposition 2.2

For a given transportation cost structure, relative demand is equivalent to relative price at equilibrium.

Proposition 3

For a given transportation cost structure, the inside firm raises (lowers) its price when faced with higher (lower) transportation costs. The outside firm reacts by raising (lowering) its price but by a smaller amount.

Table 4.2
Treatments

Treatment no.	Treatment code	Transportation cost structure	Parameter values
1	VQ1	Quadratic	s = 6.5
2	VQ2	Quadratic	s =13.0
3	VLQ1	Linear-quadratic	t = 2.6, s = 6.5
4	VLQ2	Linear-quadratic	t = 5.2, s = 13.0

Proposition 2.2 is obvious from Table 4.1 which shows the equality of relative demand and relative price under the same transportation cost structure. Proposition 3 is shown by the higher price increase of the inside firm compared to the outside firm when *s* doubles under quadratic transportation costs, and similarly when both *t* and *s* double under linear-quadratic transportation costs.

4.3 EXPERIMENTAL PROCEDURE

The experiment was designed to test the theoretical predictions discussed above. Four treatments were run: one treatment with quadratic transportation cost structure, one treatment with linear-quadratic transportation costs and another two treatments in which there is a 100% increase in transportation parameters under each type of transportation cost structure. The treatments and parameter values are summarised in Table 4.2.

In each treatment, 16 players were randomly assigned the role of inside firm or outside firm at the outset. Their roles remained unchanged throughout the whole experiment. All players were told to make two decisions simultaneously: price and location (up to 2 decimal places) and were informed of the range of feasible locations: $[0,1]$ for the inside firm and $]1,+\infty[$ for the outside firm.[4] In addition, they were informed of the exact nature of transportation costs incurred by consumers, and were aware of the functional relationship between location, price and demand by means of a calculator that computes the demand

[4] Since firms typically price their good up to 2 decimal places in real world situations, players were asked to make their decisions up to 2 decimal places.

generated from the location and price decisions entered. This calculator was made available at all times throughout the experiment. The players were also told that there were 16 trading periods after an initial trial period.

Given this design, the experiment corresponds to the theoretical IO model as closely as possible by minimising uncontrolled or unobservable behaviour arising from player's subjective probabilities on any aspects of the experiment. For example, unlike the experiments by Collins and Sherstyuk (2000) and Brown-Kruse and Schenk (2000), no uncertainty regarding the end period of the experiment was introduced.[5] Attempts at cooperative behaviour were also minimised or made ineffective by rotating players after each trading period.

Consumers were located uniformly along the unit interval $[0,1]$ and were automated to purchase one unit of the good from either seller according to the relevant demand function. They travelled a distance *d* to the seller and incurred transportation costs in accordance with a predetermined transportation cost schedule. At the end of each period, the price, location and market share of the player and his rival were displayed. The market share is the percentage demand for the player i's good out of total demand, i.e., $m_i/(m_1+m_2)\times 100\% = m_i \times 100\%$ since $m_1+m_2=1$. The payoffs, which were kept private to each player, were computed as the total profit earned, i.e., price multiplied by quantity demanded. The players were informed of the conversion rate from experimental earnings to actual earnings.

The experiments were conducted in a computer laboratory at the School of Business, National University of Singapore over a four-day period in March 2004. A total of 98 students of business and economics were recruited by e-mail. None of the subjects participated in more than one treatment or in an earlier experiment on firm behaviour with constant location. Almost all subjects had not taken part in a market experiment before. The number of subjects recruited exceeded the number required to run each treatment in order to avoid the problem of no-shows. The computerised programmes were developed using ZTree

[5] Brown-Kruse and Schenk (2000) randomised the end-period of their experiment by using a bingo cage to determine whether a period would be the last period.

software (Fischbacher 1999). At the start of the experiment, subjects were seated randomly at isolated computer terminals and no communication was allowed between them. Instructions were handed out and clarifications concerning the information provided were answered. No time restrictions were imposed. The treatments lasted an average of 2.5 hours each.

Upon completion of the 16 periods, players were asked to answer a short questionnaire before they received their payment privately in cash. The instructions and questionnaire are provided in Appendices 10 and 9 respectively. The responses from the questionnaire indicated that 92.5% of the players employed the calculator in their decisions and 78.8% used the calculator throughout the experiment. On average, the players received a profit of S$6.83 including S$4 as show-up fee.

4.4 EXPERIMENTAL RESULTS

Table 4.3 summarises the mean and median values of all variables under the four treatments. The standard deviation in mean and dispersion of values are also provided. The results show that the mean prices attained outperform the predicted values, with higher degrees of product differentiation. The distribution of mean demand is very close to the predicted values and the average profit achieved by the players is generally higher than prediction as a result of higher prices.

The following discussion addresses the theoretical hypotheses and highlights additional issues that emerge from the actual trading behaviour of players. Player experience with the experiment is observed by segregating the data into three time intervals: early phase (periods 1-8), late1 phase (periods 9-13) and late2 phase (periods 14-16). The latter two phases together make up the late phase (periods 9-16)

Table 4.3
Summary statistics of results

	N	Mean	Median	Prediction	S.D. (mean)	Maximum	Minimum
Treatment 1: VQ1							
x1	128	0.55	0.60	0.61	0.26	1.00	0.00
x2	127	1.05	1.01	1.01	0.15	2.40	1.01
p1	128	4.19	3.83	3.14	2.61	14.90	0.80
p2	127	3.43	3.00	2.06	2.11	11.00	0.50
m1	128	0.61	0.60	0.60	0.32	1.00	0.00
m2	128	0.39	0.40	0.40	0.32	1.00	0.00
r1	128	2.13	1.91	1.89	1.51	10.45	0.00
r2	128	1.10	0.89	0.82	1.42	11.00	0.00
Treatment 2: VQ2							
x1	127	0.59	0.63	0.61	0.23	1.00	0.00
x2	128	1.04	1.01	1.01	0.11	1.90	1.01
p1	127	8.17	7.50	6.27	3.96	23.90	1.00
p2	128	5.88	5.50	4.13	3.24	20.00	0.42
m1	128	0.58	0.59	0.60	0.32	1.00	0.00
m2	128	0.42	0.41	0.40	0.32	1.00	0.00
r1	128	3.99	3.84	3.79	2.76	14.50	0.00
r2	128	1.94	1.84	1.64	1.73	8.54	0.00
Treatment 3: VLQ1							
x1	128	0.70	0.67	0.85	0.26	1.00	0.00
x2	127	1.03	1.01	1.01	0.06	1.50	1.01
p1	128	5.05	4.50	4.68	2.84	14.00	0.20
p2	127	3.21	2.55	2.60	2.23	10.00	0.00
m1	128	0.63	0.66	0.64	0.31	1.00	0.00
m2	128	0.37	0.34	0.36	0.31	1.00	0.00
r1	128	2.59	2.49	3.01	1.61	10.00	0.00
r2	128	0.89	0.70	0.93	1.05	6.00	0.00
Treatment 4: VLQ2							
x1	128	0.71	0.75	0.85	0.24	1.00	0.00
x2	127	1.02	1.01	1.01	0.02	1.20	1.01
p1	128	10.26	9.90	9.37	3.28	23.90	2.00
p2	128	6.82	6.75	5.19	2.98	18.00	0.00
m1	128	0.66	0.65	0.64	0.24	1.00	0.00
m2	128	0.34	0.36	0.36	0.24	1.00	0.00
r1	128	6.17	6.11	6.03	2.08	13.00	0.00
r2	128	1.98	1.82	1.85	1.44	7.44	0.00

H1: Prices converge to the predicted values.

Figures 4.2 to 4.9 show the time series of mean prices and the time series of all prices for the inside firm players and the outside firm players in Treatments 1 to 4. The figures plot the per-period mean or individual price (p_i) of each firm, their best responses lagged one period ($RF_{i,t-1}$) and the non-cooperative Nash predictions (p_i^*). The mean prices for the two firms are obtained by averaging over 8 players with the role of inside firm and over another 8 players with the role of outside firm. The average best response RF_{it} of firm i to the rival firm's price p_{jt} and location x_{jt}, $i \neq j$, for period t is computed using the relevant response functions given in Chapter 2. The figures show a clear convergence of mean prices to the non-cooperative Nash prediction under quadratic and linear-quadratic transportation costs. Mean prices converge from above to a level at or above prediction for all treatments, except for VQ1 (inside firm) and VLQ1 (both firms) where price convergence occurs at a level below prediction.

Figures 4.10 to 4.13 show the distributions of all individual prices, grouped by unit intervals, for the four treatments. The Nash predictions are marked in the figures by a broken vertical line. It can be seen immediately that the interval with the highest frequency in VQ1, VLQ1 (2 intervals in the case of the inside firm) and VLQ2 coincide with Nash prediction. In the case of VQ2, there is a broader range of intervals in which the highest frequencies coincide with Nash prediction: 4 intervals for the inside firm and 3 intervals for the outside firm.

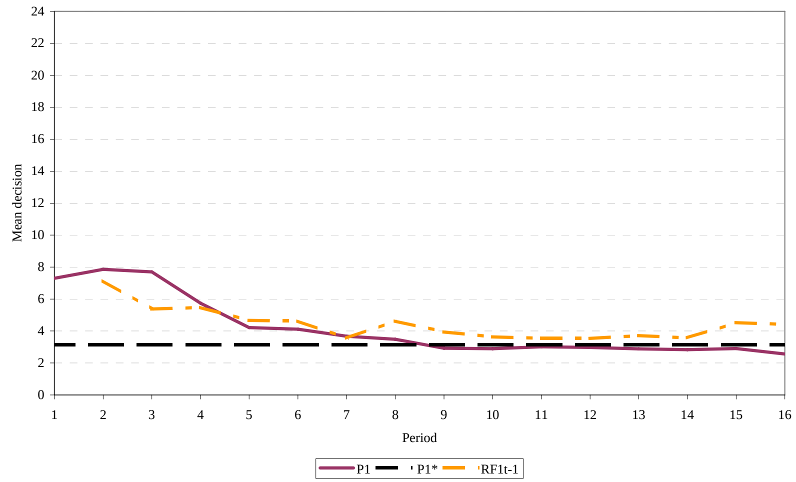

Fig. 4.2(a) Time series of mean prices of inside firm players (VQ1)

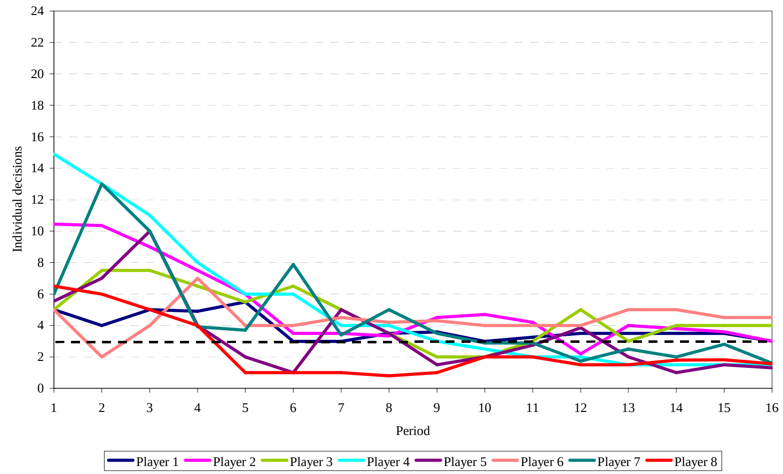

Fig. 4.2(b) Time series of individual prices of inside firm players (VQ1)

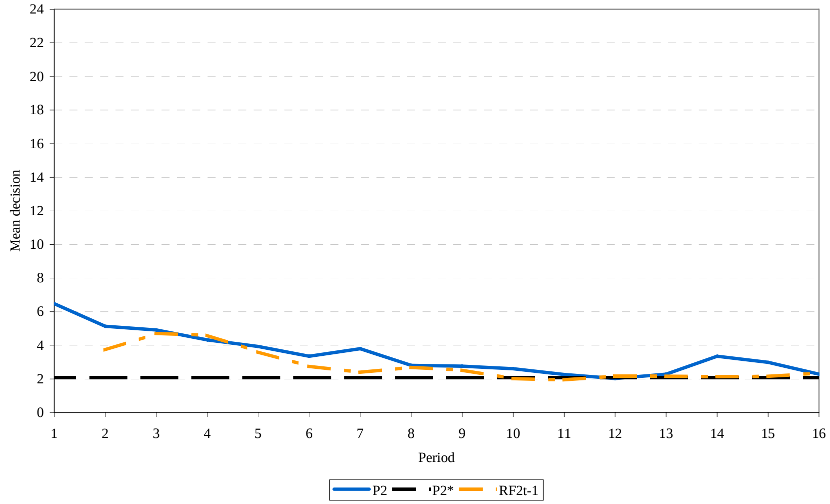

Fig. 4.3(a) Time series of mean prices of outside firm players (VQ1)

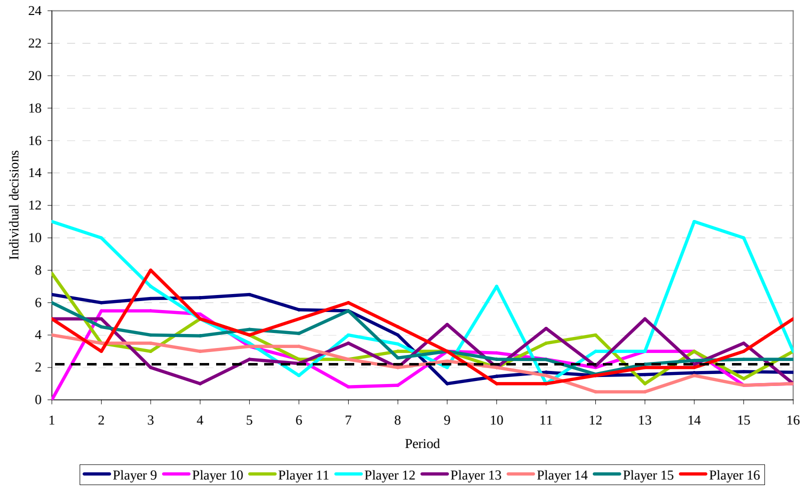

Fig. 4.3(b) Time series of individual prices of outside firm players (VQ1)

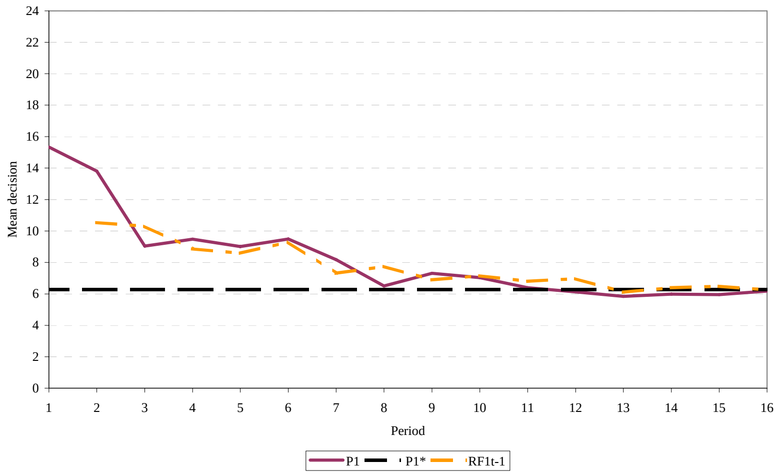

Fig. 4.4(a) Time series of mean prices of inside firm players (VQ2)

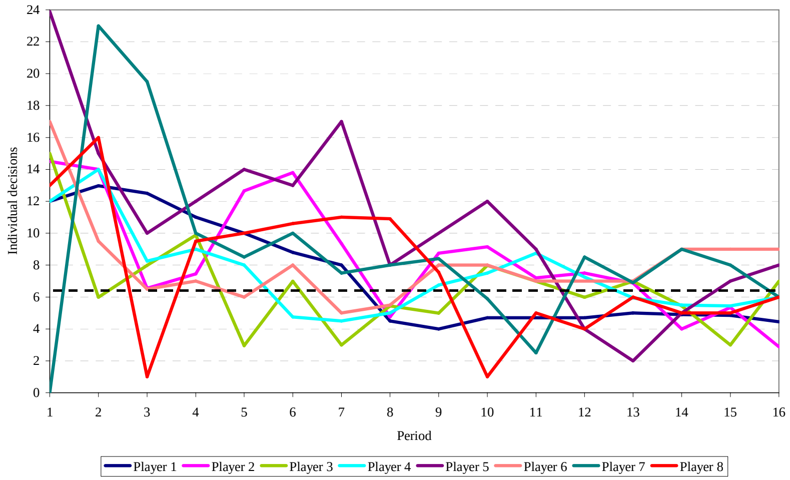

Fig. 4.4(b) Time series of individual prices of inside firm players (VQ2)

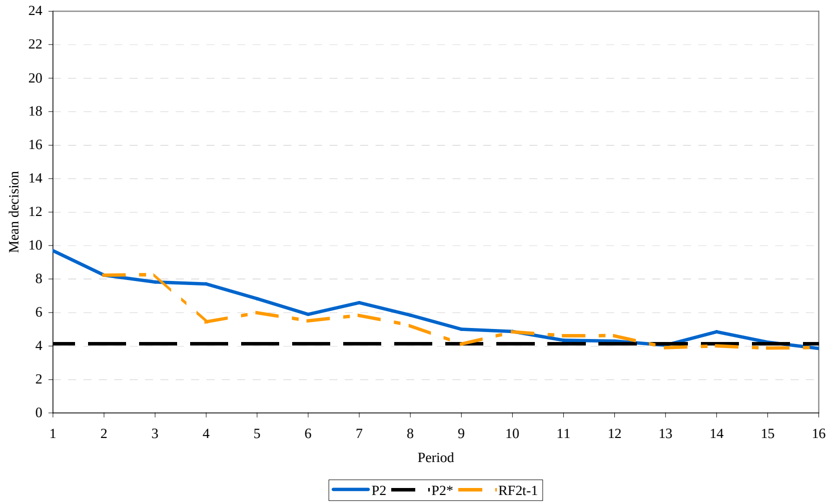

Fig. 4.5(a) Time series of mean prices of outside firm players (VQ2)

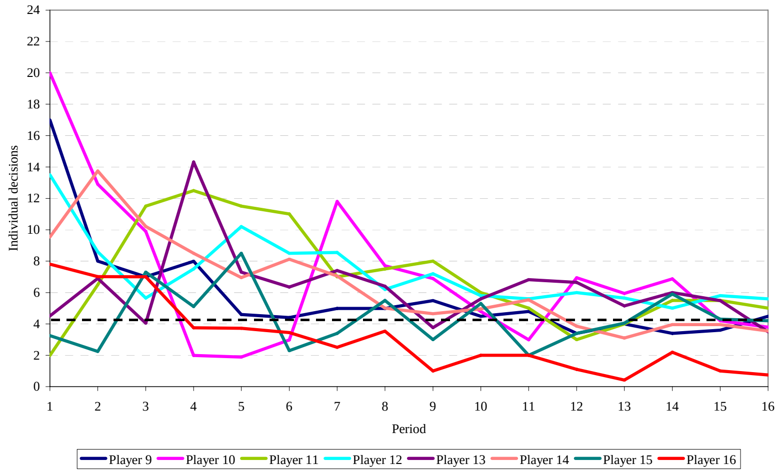

Fig. 4.5(b) Time series of individual prices of outside firm players (VQ2)

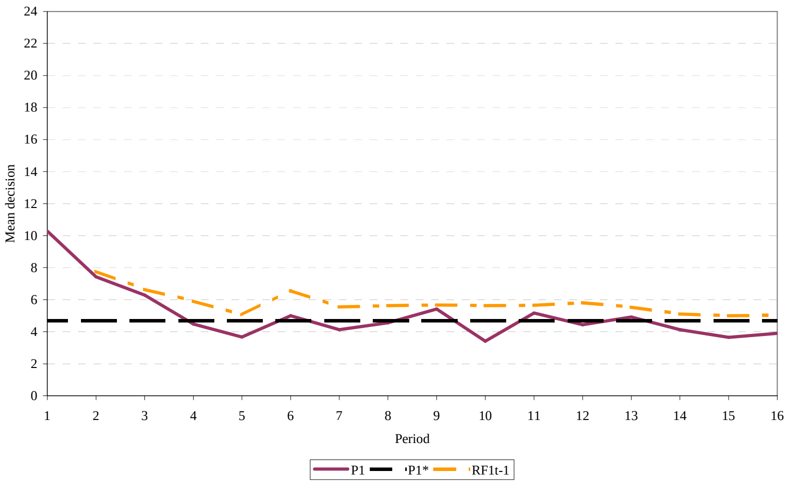

Fig. 4.6(a) Time series of mean prices of inside firm players (VLQ1)

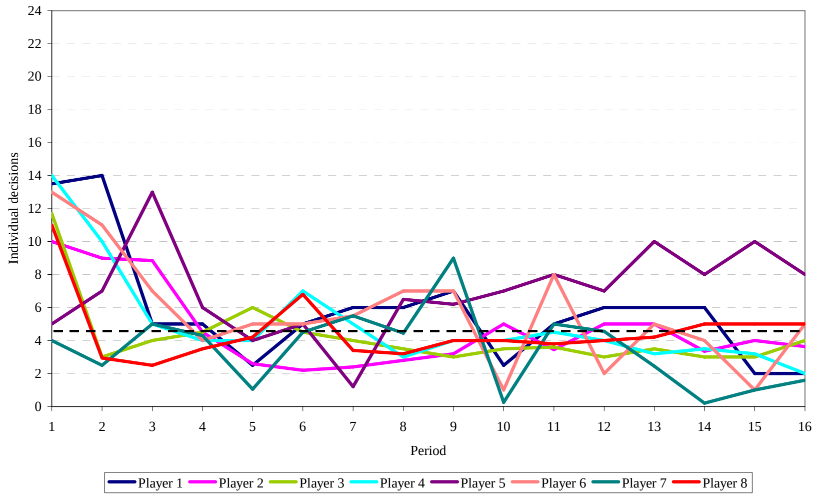

Fig. 4.6(b) Time series of individual prices of inside firm players (VLQ1)

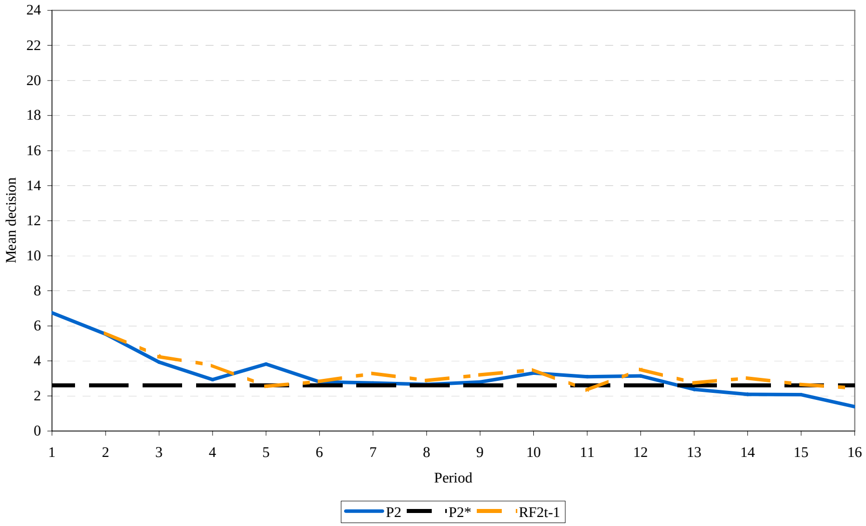

Fig. 4.7(a) Time series of mean prices of outside firm players (VLQ1)

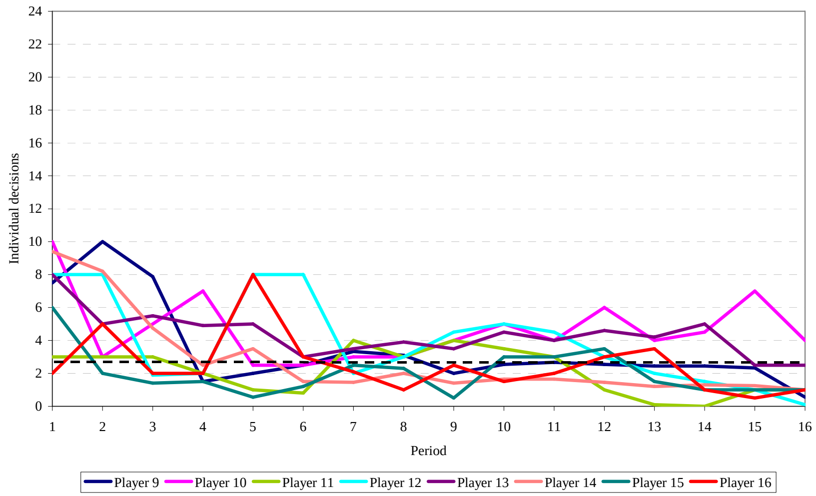

Fig. 4.7(b) Time series of individual prices of outside firm players (VLQ1)

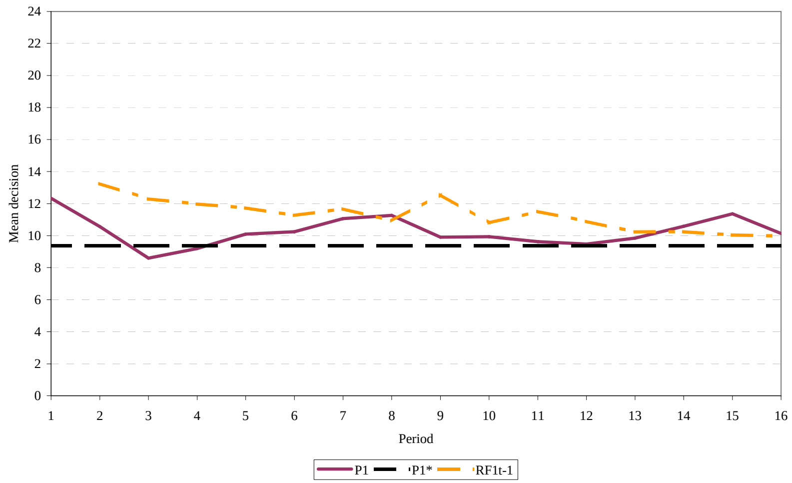

Fig. 4.8(a) Time series of mean prices of inside firm players (VLQ2)

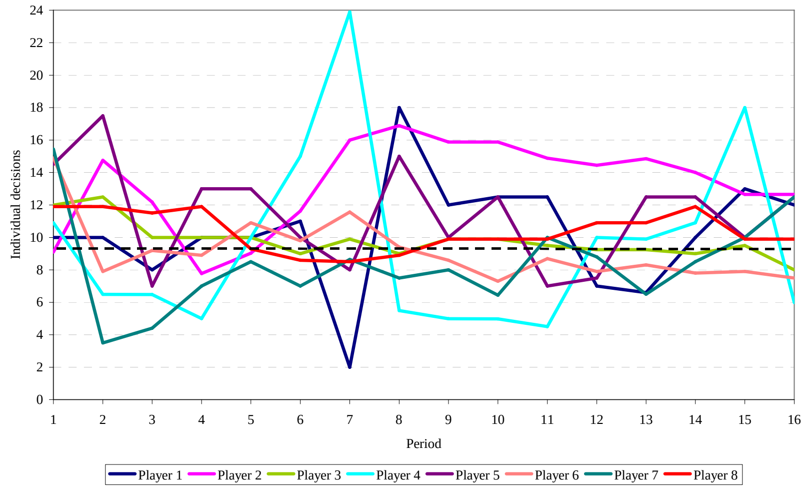

Fig. 4.8(b) Time series of individual prices of inside firm players (VLQ2)

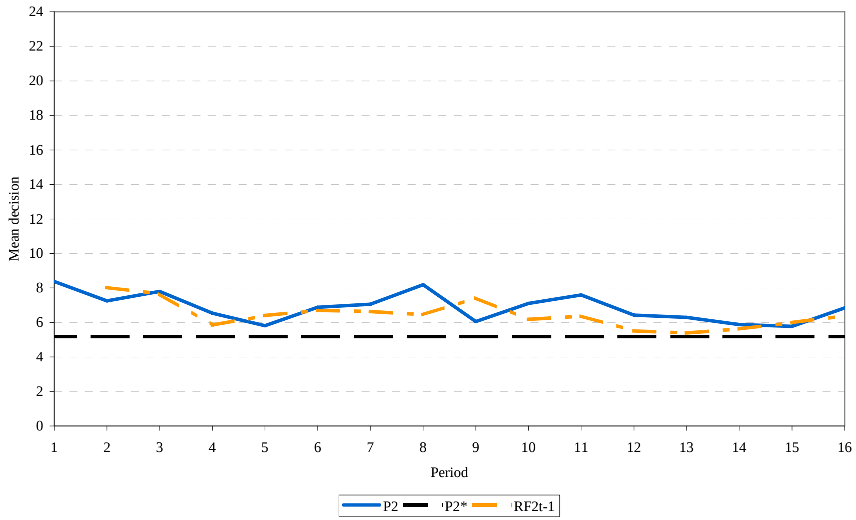

Fig. 4.9(a) Time series of mean prices of outside firm players (VLQ2)

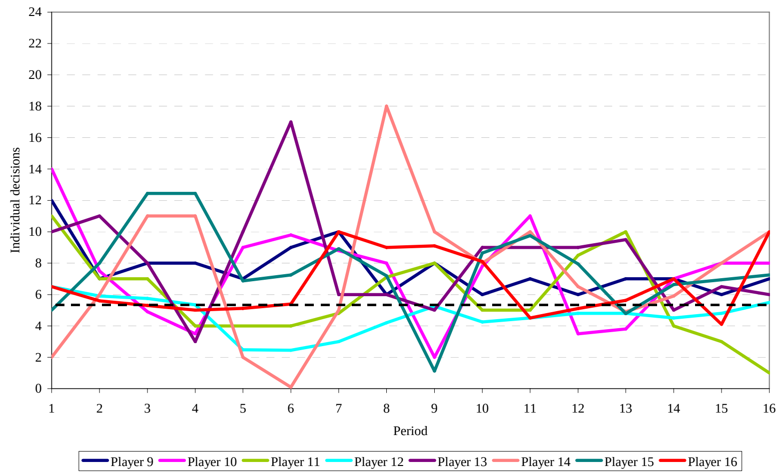

Fig. 4.9(b) Time series of individual prices of outside firm players (VLQ2)

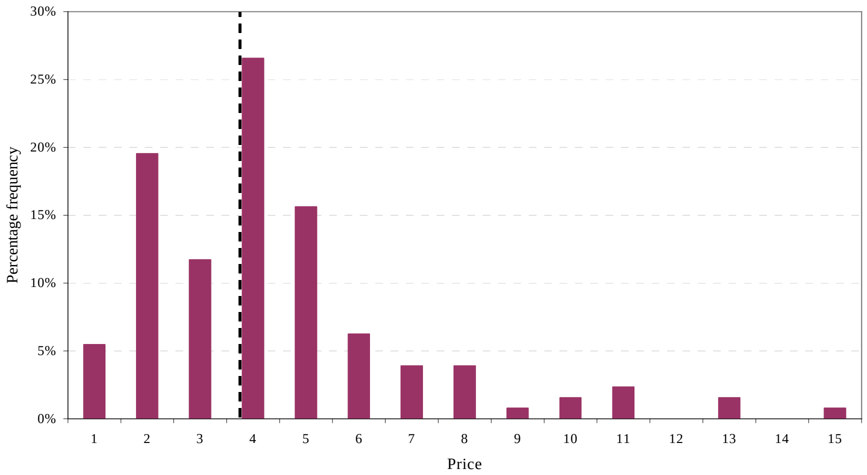

Fig. 4.10(a) Distribution of individual prices of inside firm players (VQ1)

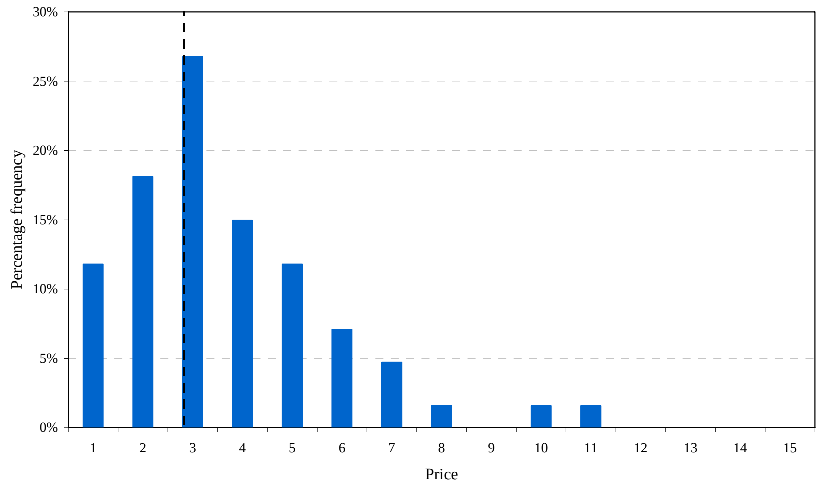

Fig. 4.10(b) Distribution of individual prices of outside firm players (VQ1)

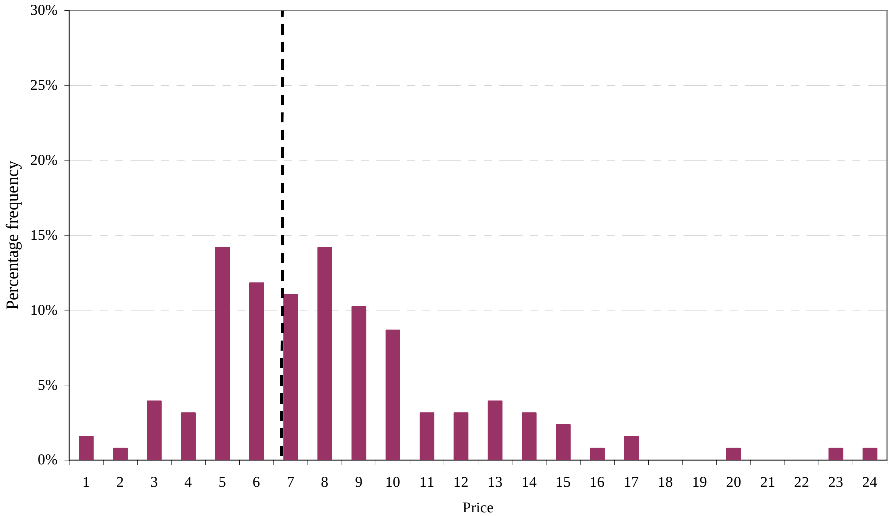

Fig. 4.11(a) Distribution of individual prices of inside firm players (VQ2)

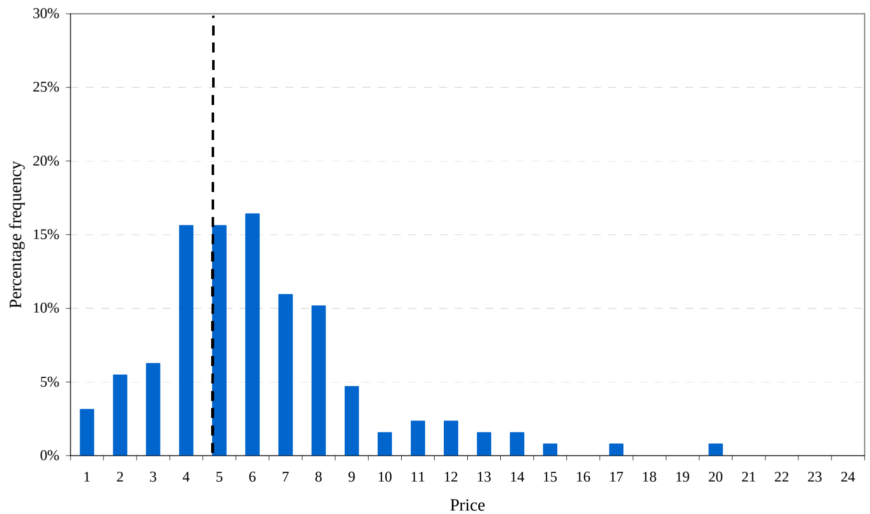

Fig. 4.11(b) Distribution of individual prices of outside firm players (VQ2)

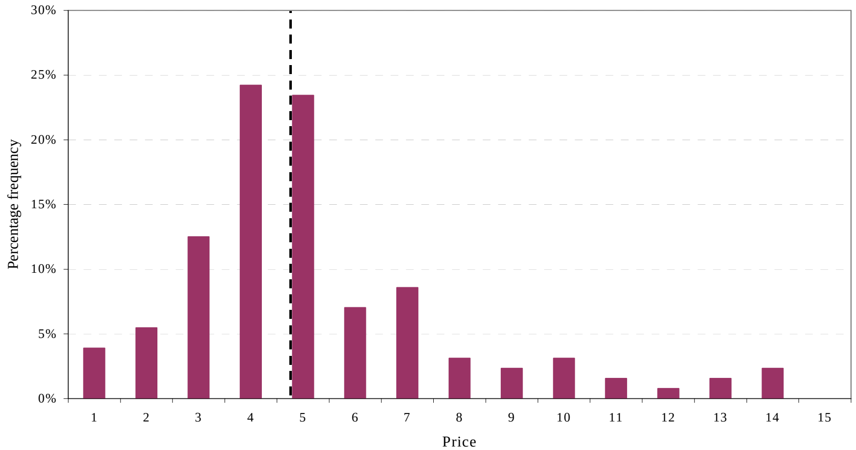

Fig. 4.12(a) Distribution of individual prices of inside firm players (VLQ1)

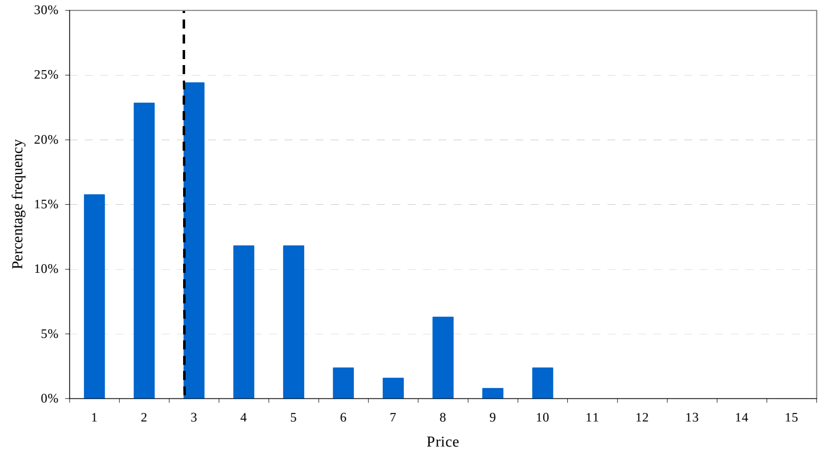

Fig. 4.12(b) Distribution of individual prices of outside firm players (VLQ1)

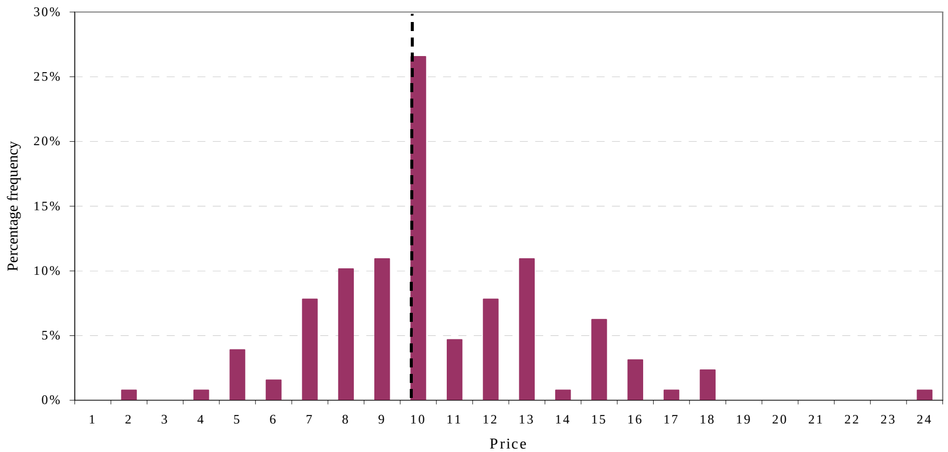

Fig. 4.13(a) Distribution of individual prices of inside firm players (VLQ2)

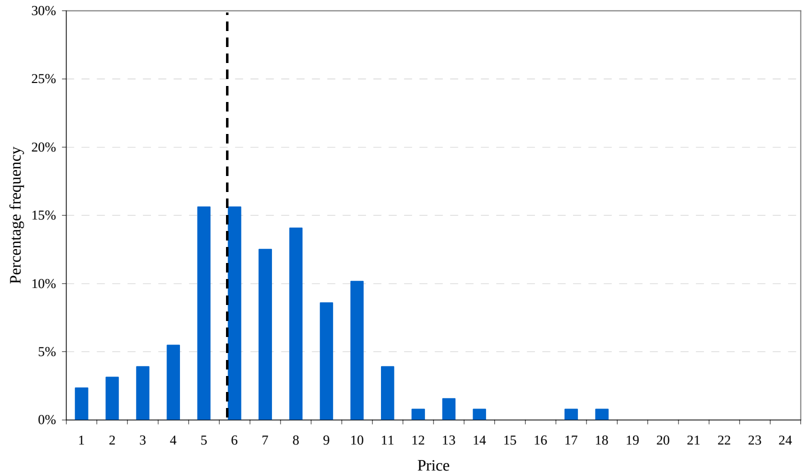

Fig. 4.13(b) Distribution of individual prices of outside firm players (VLQ2)

Table 4.4
Inadequate and inappropriate price response

Frequency	Over-positive response behaviour			Over-negative response behaviour			No response	Total
	Over increase	Wrong increase	Total	Over decrease	Wrong decrease	Total		
All treatments	0.0670	0.0565	0.1236	0.1047	0.2880	0.3927	0.1099	0.6710
Inside firm	0.0962	0.1004	0.1967	0.1569	0.1569	0.3138	0.1318	0.7188
Outside firm	0.0377	0.0126	0.0503	0.0524	0.4193	0.4717	0.0881	0.6229
VQ1	0.0586	0.0502	0.1088	0.1088	0.2469	0.3556	0.1548	0.5103
Inside firm	0.0833	0.0750	0.1583	0.1167	0.1500	0.2667	0.2083	0.4917
Outside firm	0.0336	0.0252	0.0588	0.1008	0.3445	0.4454	0.1008	0.5292
VQ2	0.0667	0.0875	0.1542	0.1500	0.2708	0.4208	0.0458	0.6926
Inside firm	0.0917	0.1500	0.2417	0.2167	0.0833	0.3000	0.0667	0.6593
Outside firm	0.0417	0.0250	0.0667	0.0833	0.4583	0.5417	0.0250	0.7260
VLQ1	0.0714	0.0420	0.1134	0.0756	0.3361	0.4118	0.1176	0.6471
Inside firm	0.1092	0.0840	0.1933	0.1429	0.2017	0.3445	0.1176	0.6723
Outside firm	0.0336	0.0000	0.0336	0.0084	0.4706	0.4790	0.1176	0.6218
VLQ2	0.0714	0.0462	0.1176	0.0840	0.2983	0.3824	0.1218	0.5000
Inside firm	0.1008	0.0924	0.1933	0.1513	0.1933	0.3445	0.1345	0.5378
Outside firm	0.0420	0.0000	0.0420	0.0168	0.4034	0.4202	0.1092	0.4622

While prices generally follow the best response correspondence (this will be discussed under hypothesis H3 below), there exist certain forms of pricing behaviour that contribute to decreased proximity of price convergence to Nash prediction. As a reflection of real world behaviour, *low price behaviour* appears to be a common market strategy among the players in which low prices are offered to capture market demand at the expense of profit. The ensuing intense price war results in an occasional diminution of profit to near-zero levels. Less frequently observed is *price leader behaviour* in which players attempt to initiate an overall movement toward higher prices in order to generate higher profit.[6] If either behaviour predominates and successfully results in imitation, a decline in price convergence to Nash prediction results.

An indication of both behaviour types may be obtained by the frequency of *over-positive (negative) response behaviour* which is defined in Chapter 3 as comprising the frequency of (1) *over-increase (decrease) response behaviour* in which players increase (decrease) prices above (below) the response price in the preceding period and (2) *wrong*

[6] The instructions did not reveal that a rotation mechanism is in place.

increase (decrease) response behaviour in which players increase (decrease) prices when the opposite direction or no change is called for. An over-increase (decrease) in price corresponds to an excessive price increase (decrease) in relation to best response while a wrong price increase (decrease) corresponds to a price movement that is opposite in direction to best response.

Table 4.4 shows the frequencies of over-positive and over-negative response behaviour. The table also shows the frequency of “no response” which reflects a failure on the part of players to make a price adjustment when one is called for as best response. The total over-response and no response frequencies reflect the overall departure from best response. The table clearly shows that over-negative response behaviour surpasses over-positive response behaviour as well as no response. Over-negative behaviour is more prevalent among outside firm players than inside firm players for all treatments considered. This is not surprising since the outside firm is more limited in varying product differentiation (given a dominant location strategy) and effectively has only one strategy (price) to vary in the face of stiff competition.

The following highlights some examples of low price behaviour and price leader behaviour observed among the players. Players 1 to 8 are inside firm players while players 9 to 16 are outside firm players.

<u>Low price behaviour</u> was observed in player 4 of VQ1 who adopted a twin strategy of increasing product differentiation and reducing price. On reaching maximum differentiation at point 0 in period 6, the player maintained high differentiation and lowered prices with each drop in demand for the remaining periods. Another player, player 5, made deep price cuts of 50% or more in his attempt to attract demand. These price cuts did not necessarily coincide with a drop in demand in the preceding period (e.g., periods 5, 6 and 9). A third player, player 8, encountered a low-pricing rival (player 13) in period 4 which immediately ignited an aggressive price war. This player adhered to a low-price regime not exceeding 2.0 for the rest of the experiment, allowing him to capture the whole market for 8 out of the remaining 12 periods.

In VQ2, player 16 priced at very low levels from period 9 onwards after suffering a sharp loss of 57.2% in demand of 0.43 in period 8 compared to an average demand of 0.65 in the periods before that. The player's low price behaviour, however, results in his near zero profit by the end of the experiment.

In VLQ1, player 11 offered deep price discounts of 50% in period 5 and 90% in period 13 (the latter despite experiencing higher demand in the preceding period) as well as a zero price offer in period 14. Player 7 and player 16 also exhibited low price behaviour and priced below Nash at the start of the experiment. The combined effect of these low pricers and over-negative response by the other players was strong enough to result in a convergence below prediction for this treatment.

<u>Price leader behaviour</u> was observed in player 12 of VQ1. Starting the experiment at a price higher than any of his counterparts, the player attempted to price lead in periods 10, 14 and 15 by setting high prices of $p_2 = 7,$ 11 and 10 respectively. The ineffectiveness of this price leader behaviour is obvious as no upward imitation attempts were made by the corresponding rival players in the ensuing period (player 3, player 7 and player 6 respectively) resulting in a quick return by this player to lower prices and eventual convergence toward the Nash prediction.

To evaluate price convergence to prediction, a Wilcoxon signed rank test is conducted on the null hypothesis that price decisions and prediction are equal. Table 4.5 shows that the null is significant at the 0.05 level for both firms in all treatments in the late2 phase (at the 0.01 level for the outside firm in VLQ1 and the inside firm in VLQ2). Similar results are obtained by a Sign test. The alternative *t*-test on the null-hypothesis $H_0 : \overline{p}_i = p_i^*$ (where $\overline{p}_i$ is the mean price of firm *i* and p_i^* is the prediction) can be performed on only 3 of the 8 price series shown to be normally distributed by a Kolmogorov-Smirnov test. *T*-tests on these price series in the late2 phase accept H_0 at the 0.05 level for VLQ2 (outside firm) but not VQ2 (both firms) (Table 4.6). Given the non-normal distribution of several price series, the Wilcoxon and Sign tests will be employed henceforth where appropriate.

Table 4.5
Price convergence to Nash prediction
(probabilities for two-tailed Wilcoxon signed ranks test p_W and Sign test p_S)

Variable	N	pw		ps		Variable	N	pw		ps	
Null hypothesis: $p_{it} = p_i$*											
Treatment: VQ1						Treatment: VQ2					
All periods						All periods					
p1	128	0.0002		0.0035		p1	127	0.0000		0.0014	
p2	127	0.0000		0.0000		p2	128	0.0000		0.0001	
Early phase						Early phase					
p1	64	0.0000		0.0000		p1	63	0.0000		0.0000	
p2	63	0.0000		0.0000		p2	64	0.0000		0.0000	
Late1 phase						Late1 phase					
p1	40	0.2107	*	0.2684	*	p1	40	0.3101	*	0.2684	*
p2	40	0.4591	*	1.0000	*	p2	40	0.1621	*	0.4292	*
Late2 phase						Late2 phase					
p1	24	0.1611	*	0.5413	*	p1	24	0.4232	*	0.1516	*
p2	24	0.4398	*	0.8388	*	p2	24	0.3992	*	0.5413	*
Treatment: VLQ1						Treatment: VLQ2					
All periods						All periods					
p1	128	0.9053	*	0.4263	*	p1	128	0.0070		0.0421	+
p2	127	0.1121	*	1.0000	*	p2	128	0.0000		0.0001	
Early phase						Early phase					
p1	64	0.1540	*	0.7077	*	p1	64	0.0470	+	0.2606	*
p2	64	0.0087		0.2606	*	p2	64	0.0000		0.0007	
Late1 phase						Late1 phase					
p1	40	0.6474	*	0.4292	*	p1	40	0.5540	*	0.6353	*
p2	39	0.1755	*	0.5218	*	p2	40	0.0018		0.1547	*
Late2 phase						Late2 phase					
p1	24	0.0553	*	0.1516	*	p1	24	0.0320	+	0.0639	*
p2	24	0.0231	+	0.0015		p2	24	0.0716	*	0.1516	*

+ indicates significance at 0.01 level
* indicates significance at 0.05 level

Table 4.6
Price convergence to Nash prediction (T-test)

Variable	N	t-statistic	Sig. (2-tailed)		Mean difference from prediction	95% confidence interval of difference lower	upper
Null hypothesis: $p_{it} = p_i$*							
Treatment: VQ2							
p1	24	-5.6844	0.0000		-0.2362	-1.0137	0.5412
p2	24	-5.0918	0.0000		0.1842	-0.4520	0.8203
Treatment: VLQ2							
p2	24	-1.8276	0.0806	*	0.7342	-0.2798	1.7482

* indicates significance at 0.05 level

How do firms make their price decisions? Are their decisions based purely on their own last period decisions? Or do they also take into account the price and location decisions of their rival? The following model is used to examine these questions:

(4.5) $$p_{it} = \alpha + \beta_1 p_{it-1} + \beta_2 p_{it-2} + \beta_3 p_{jt-1} + \beta_4 p_{jt-2} + \beta_5 x_{it} + \beta_6 x_{it-1} + \beta_7 x_{it-2} + \beta_8 x_{jt-1} + \beta_9 x_{jt-2} + \varepsilon_{it}$$

where p_{it} and p_{jt} are the price of firms *i* and *j* respectively in period t, $i, j \in \{1,2\}$, $i \neq j$, $t = \{1,...,16\}$, x_{it} and x_{jt} are the location of firms *i* and *j* respectively.

The regression was run using ordinary least squares based on price data pooled from 8 inside firm players and 8 outside firm players for all periods in each treatment. Serial correlation was removed using ARMA estimation. The results are presented in Tables 4.7. [7] Tests for stability and misspecification show that the estimates are reliable at the 0.01 level (the tests follow that described in Chapter 3). In all instances where the Jarque-Bera test for normality of residuals failed, the residual plots show the presence of one to three outlying residual values. Since the Jarque-Bera test fails in the presence of a single outlier, even if the series is normally distributed and the outliers are themselves normally distributed (Brys *et al.* 2004), the failure of the test will be disregarded. The failure of the Chow test in three instances (p_1 in VQ1, p_2 in VQ2 and p_1 in VQL2) is attributable to the location of an outlying residual near the breakpoint at $n = 64$.

The adjusted R^2 for the outside firm in all treatments is very low. One-period lag price coefficients of the outside firm are statistically insignificant except for VQ2 where β_1 is significant at the 0.05 level. Two period lag price coefficients are significantly different from zero for β_2 in VLQ1 (at 0.10 level) and in VLQ2 (at 0.05 level), and β_4 in VLQ2 (at 0.10 level). In contrast, all one-period lag price coefficients (β_1 and β_3) of the inside firm are statistically significant at the 0.05 level in all treatments although two-period lag price

[7] An augmented Dickey-Fuller test rejects the presence of a unit root in the level for all price and location series at all reported significance levels, indicating that the data are stationary. Note that the Durbin-Watson test for autocorrelation is not valid here due to the presence of lagged dependent variables on the right-hand side of the regression. The Breusch-Godfrey serial correlation LM Test is reported instead.

coefficients (β_2 and β_4) are all insignificant. In other words, the inside firm makes its price decisions by adapting to last period prices of itself and its rival, while the outside firm generally does not take into account last period prices (except in VQ2) but more of two-period ago prices of itself (in VLQ1) and its rival as well (in VLQ2).

With regards to price convergence, evidence is shown that prices converge to an equilibrium for the inside firm in all treatments and for the outside firm in VLQ1 and VLQ2 since $|\beta_1 + \beta_2| < 1$ for these treatments.

Looking at the relative importance of own and rival prices in determining price decisions, it can be seen that for the inside firm, prices adjust equally according to own price and rival price in VQ2, since β_1 and β_3 are statistically significant and approximately equal at the 0.05 level. In the other three treatments (VQ1, VLQ1 and VLQ2), $\beta_1 > \beta_3$, indicating that the inside firm adapts more to its own last price than that of its rival. In the case of the outside firm, no evidence is found that price decisions are made by taking into account rival prices, except in VLQ2 where $|\beta_2| > |\beta_4|$, indicating that the outside firm adapts more to its own two-period ago price than that of its rival.

There is some evidence that the two firms make their price decisions according to their own location and that of their rival. One or more location coefficients of both firms in all treatments are found to be statistically different from zero at the 0.05 level or 0.10 level. The only exception is the outside firm in VQ2 which shows no evidence of making its price decisions based on location decisions of itself and that of its rival ($\beta_5, \beta_6, \beta_7, \beta_8$ and β_9 are not significantly different from zero at the 0.05 level). In contrast to the inside firm, none of the outside firm in the four treatments makes its price decisions based on rival location (β_8 and β_9 are all insignificant at the 0.05 level).

Table 4.7
Regression results for price decisions

Variable	Coefficient	S.E.	t-statistic	
Model: $p_{it} = \alpha + \beta_1 p_{it-1} + \beta_2 p_{it-2} + \beta_3 p_{jt-1} + \beta_4 p_{jt-2} + \beta_5 x_{it} + \beta_6 x_{it-1} + \beta_7 x_{it-2} + \beta_8 x_{jt-1} + \beta_9 x_{jt-2} + \varepsilon_{it}$				
Treatment: VQ1				
p_{1t}				
constant	-4.6669	1.7496	-2.6674	*
p_{1t-1}	0.5865	0.1022	5.7389	*
p_{1t-2}	-0.0216	0.0905	-0.2385	
p_{2t-1}	0.1958	0.0897	2.1833	*
p_{2t-2}	0.0635	0.0881	0.7211	
x_{1t}	0.7286	0.9489	0.7679	
x_{1t-1}	1.1369	1.0079	1.1281	
x_{1t-2}	-2.0750	0.9026	-2.2990	*
x_{2t-1}	4.7096	1.1088	4.2476	*
x_{2t-2}	0.6017	1.2516	0.4807	
AR(16)	0.2569	0.0993	2.5875	*
N = 107; Adj R^2 = 0.5397; F = 13.4270 (p = 0.0000); SSE = 303.7832; LM = 0.4211; White = 0.9751; Jarque-Bera = 0.0000; Chow = 0.0028				
p_{2t}				
constant	0.6195	2.4020	0.2579	
p_{2t-1}	-0.0004	0.1010	-0.0040	
p_{2t-2}	-0.0669	0.1040	-0.6437	
p_{1t-1}	0.0571	0.1137	0.5020	
p_{1t-2}	0.1627	0.1048	1.5521	
x_{2t}	2.4418	1.3046	1.8717	+
x_{2t-1}	-0.3506	1.2638	-0.2774	
x_{2t-2}	-0.4388	1.3803	-0.3179	
x_{1t-1}	0.8265	1.0341	0.7993	
x_{1t-2}	-0.3395	1.0035	-0.3383	
AR(6)	-0.2794	0.0979	-2.8525	*
N = 114; Adj R^2 = 0.0690; F = 1.8377 (p = 0.0630); SSE = 432.0067; LM = 0.5339; White = 0.7977; Jarque-Bera = 0.0000; Chow = 0.1614				
Treatment: VQ2				
p_{1t}				
constant	4.6226	3.7271	1.2403	
p_{1t-1}	0.4802	0.0927	5.1825	*
p_{1t-2}	0.0322	0.0818	0.3939	
p_{2t-1}	0.4978	0.0894	5.5672	*
p_{2t-2}	-0.1121	0.0972	-1.1534	
x_{1t}	1.1431	1.8189	0.6285	
x_{1t-1}	1.2253	2.0944	0.5851	
x_{1t-2}	-2.6624	1.7090	-1.5579	
x_{2t-1}	2.6419	2.3799	1.1101	
x_{2t-2}	-5.3725	2.0631	-2.6041	*
AR(16)	0.5431	0.0887	6.1256	*
N = 104; Adj R^2 = 0.3936; F = 7.6862 (p = 0.0000); SSE = 674.5523; LM = 0.0582; White = 0.8806; Jarque-Bera = 0.0000; Chow = 0.3054				

Table 4.7 (contd.)

Variable	Coefficient	S.E.	t-statistic	
Treatment: VQ2				
p_{2t}				
constant	5.1166	4.3055	1.1884	
p_{2t-1}	0.1826	0.0927	1.9701	*
p_{2t-2}	-0.0183	0.0924	-0.1982	
p_{1t-1}	0.1103	0.0782	1.4113	
p_{1t-2}	0.0168	0.0724	0.2327	
x_{2t}	-1.8523	2.6973	-0.6867	
x_{2t-1}	-1.1102	2.4053	-0.4616	
x_{2t-2}	0.9035	2.3595	0.3829	
x_{1t-1}	0.7354	1.7165	0.4284	
x_{1t-2}	0.3647	1.7755	0.2054	
AR(15)	0.2730	0.0925	2.9529	*
N = 107; Adj R^2 = 0.1249; F = 2.5125 (p = 0.01000); SSE = 667.3489; LM = 0.3686; White = 0.1267; Jarque-Bera = 0.0101; Chow = 0.0002				
Treatment: VLQ1				
p_{1t}				
constant	2.5180	11.9387	0.2109	
p_{1t-1}	0.3129	0.0987	3.1709	*
p_{1t-2}	0.0244	0.0951	0.2565	
p_{2t-1}	0.2798	0.1070	2.6137	*
p_{2t-2}	-0.0975	0.1103	-0.8838	
x_{1t}	-2.1900	1.0421	-2.1015	*
x_{1t-1}	1.9841	1.0369	1.9135	+
x_{1t-2}	-0.4766	1.0445	-0.4563	
x_{2t-1}	-3.4586	7.5770	-0.4565	
x_{2t-2}	4.1093	7.3870	0.5563	
AR(5)	0.2143	0.0940	2.2784	*
N = 117; Adj R^2 = 0.1808; F = 3.5597 (p = 0.0004); SSE = 636.5835; LM = 0.9601; White = 0.8342; Jarque-Bera = 0.0000; Chow = 0.5514				
p_{2t}				
constant	12.0114	13.9797	0.8592	
p_{2t-1}	0.0635	0.1003	0.6326	
p_{2t-2}	-0.1786	0.1041	-1.7161	+
p_{1t-1}	0.0192	0.0864	0.2220	
p_{1t-2}	-0.1124	0.0864	-1.3011	
x_{2t}	-13.3433	6.8328	-1.9528	+
x_{2t-1}	0.8296	7.0604	0.1175	
x_{2t-2}	5.0077	6.5607	0.7633	
x_{1t-1}	-0.8574	0.9800	-0.8749	
x_{1t-2}	0.4825	0.9464	0.5098	
AR(11)	-0.3346	0.0984	-3.3989	*
N = 109; Adj R^2 = 0.0832; F = 1.9798 (p = 0.0436); SSE = 488.5638; LM = 0.3181; White = 0.3359; Jarque-Bera = 0.0004; Chow = 0.9056				

Table 4.7 (contd.)

Variable	Coefficient	S.E.	t-statistic	
Treatment: VLQ2				
p_{1t}				
constant	5.1883	20.1881	0.2570	
p_{1t-1}	0.3440	0.1070	3.2148	*
p_{1t-2}	0.1008	0.1007	1.0016	
p_{2t-1}	0.2477	0.1133	2.1858	*
p_{2t-2}	0.0763	0.1091	0.6993	
x_{1t}	-5.3785	1.7222	-3.1231	*
x_{1t-1}	4.1317	2.1776	1.8974	+
x_{1t-2}	0.3056	1.7415	0.1755	
x_{2t-1}	23.5787	14.0886	1.6736	+
x_{2t-2}	-24.6041	13.6749	-1.7992	+
AR(21)	-0.1970	0.0988	-1.9938	*
N = 103; Adj R^2 = 0.1982; F = 3.5213 (p = 0.0006); SSE = 805.0833; LM = 0.7423; White = 0.1421; Jarque-Bera = 0.0000; Chow = 0.0002				
p_{2t}				
constant	-54.5467	19.8560	-2.7471	*
p_{2t-1}	-0.0220	0.0859	-0.2559	
p_{2t-2}	-0.1816	0.0884	-2.0553	*
p_{1t-1}	-0.0270	0.0880	-0.3065	
p_{1t-2}	-0.1541	0.0857	-1.7989	+
x_{2t}	29.7729	11.0167	2.7025	*
x_{2t-1}	-8.0383	11.2873	-0.7122	
x_{2t-2}	42.7015	11.2941	3.7809	*
x_{1t-1}	-0.8153	1.3851	-0.5886	
x_{1t-2}	-0.4271	1.3803	-0.3094	
N = 123; Adj R^2 = 0.1275; F = 2.9803 (p = 0.0032); SSE = 803.7951; LM = 0.9401; White = 1.0000; Jarque-Bera = 0.0001; Chow = 0.8384				

* indicates significance at the 0.05 level. + indicates significance at the 0.10 level.
Probability statistics of the following tests are reported: serial correlation Lagrange multiplier test (LM), White's heteroskedasticity test (White), Jarque-Bera normality test (Jarque-Bera) and Chow's breakpoint test (Chow). White's test includes all cross product terms. Breakpoint for Chow test is at mid-sample (n = 64). Failure of Jarque-Bera test due to 1-3 outlying values. Failure of Chow's test for p_1 in VQ1, p_2 in VQ2 and p_1 in VLQ2 due to outlying residual value near breakpoint.

H2: Locations converge to the predicted values.

Figures 4.14 to 4.21 show the time series of mean locations and the time series of all locations for the inside firm players and the outside firm players in Treatments 1 to 4. As with the time series plots for mean prices, three lines are shown on the graphs of mean locations: mean location of each firm (x_i), the best responses lagged one period ($RF_{i,t-1}$) and the non-cooperative Nash prediction (x_i^*). The mean locations for the two firms in each treatment are obtained by averaging over 8 inside firm players and 8 outside firm players. The best response RF_{it} of firm i to the rival firm's location x_{jt} , $i \neq j$, for period t is computed using the relevant response functions given in Chapter 2. The graphs of individual locations show the locations over time of each player (x_i) and the best response lagged one period ($RF_{i,t-1}$).

A cursory observation of the graphs reveal that while mean locations for both players clearly converge to Nash prediction by the second half of the experiment, there is a general failure on the part of several inside firm players in VQ1 and VQ2 to reach prediction values. In the case of the outside firm players, the dominant strategy of locating next to the market border is generally adopted within the first two periods. The exceptions are four players in VQ2 (location equilibrium reached after 4-6 periods) and one player in VQ1 (failure of attaining location equilibrium).

The distribution of individual locations, grouped by intervals of 0.1, for the four treatments (Figures 4.22 to 4.25), shows that the interval with the highest frequency invariably coincides with Nash prediction (depicted by a broken vertical line) for the outside firm but this is true only for the inside firm in VQ1 and VQ2. Moreover, there is a wider distribution of location choice by the inside firm under quadratic transportation costs (VQ1 and VQ2) than under linear-quadratic transportation costs (VLQ1 and VLQ2). This suggests that when transportation costs increase at a rapid rate, the level of product differentiation tends to decrease. This point will be explored further under hypothesis H4.

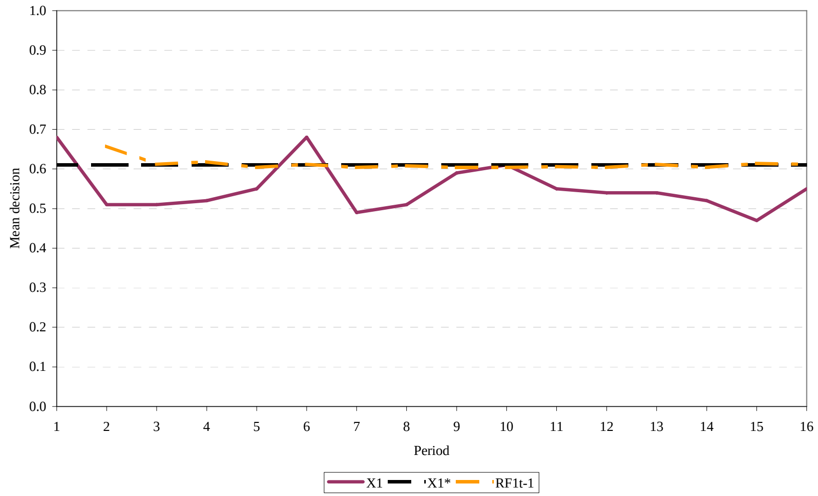

Fig. 4.14(a) Time series of mean locations of inside firm players (VQ1)

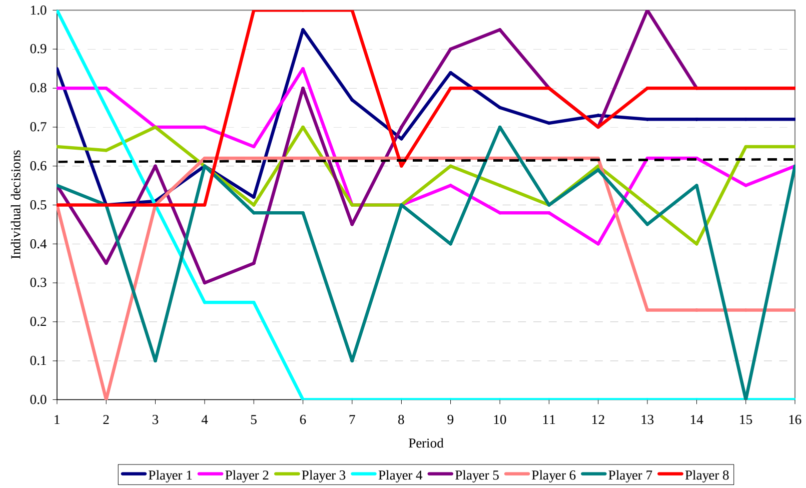

Fig. 4.14(b) Time series of individual locations of inside firm players (VQ1)

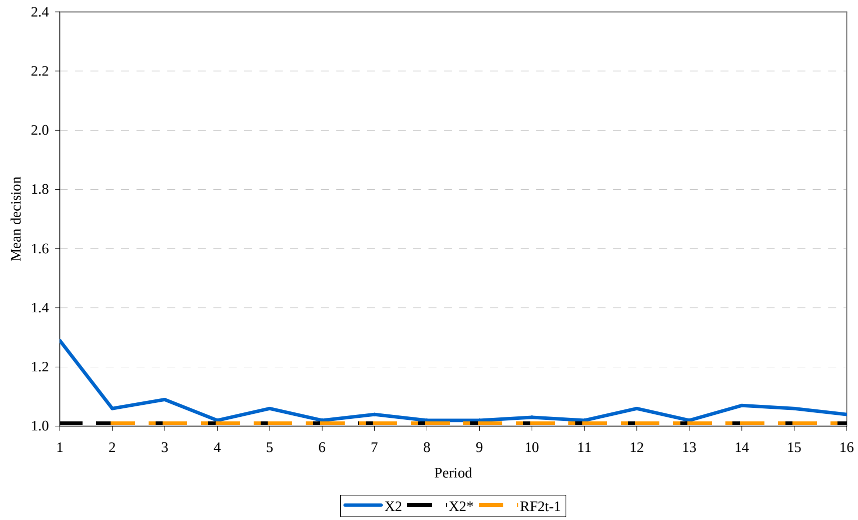

Fig. 4.15(a) Time series of mean locations of outside firm players (VQ1)

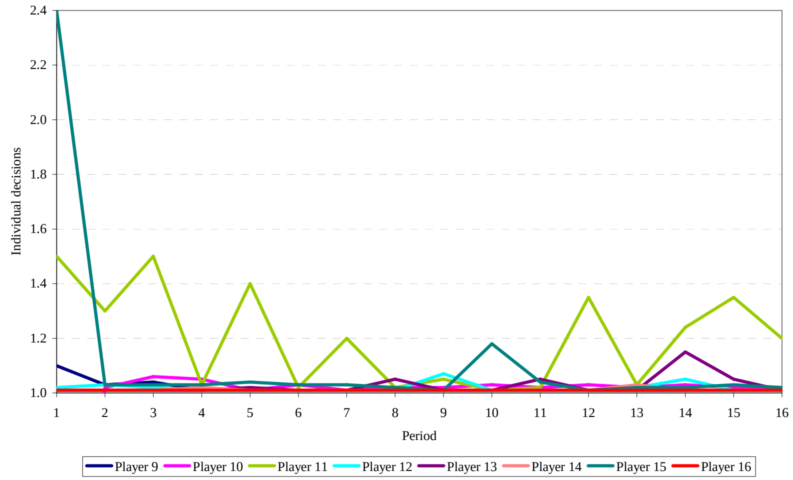

Fig. 4.15(b) Time series of individual locations of outside firm players (VQ1)

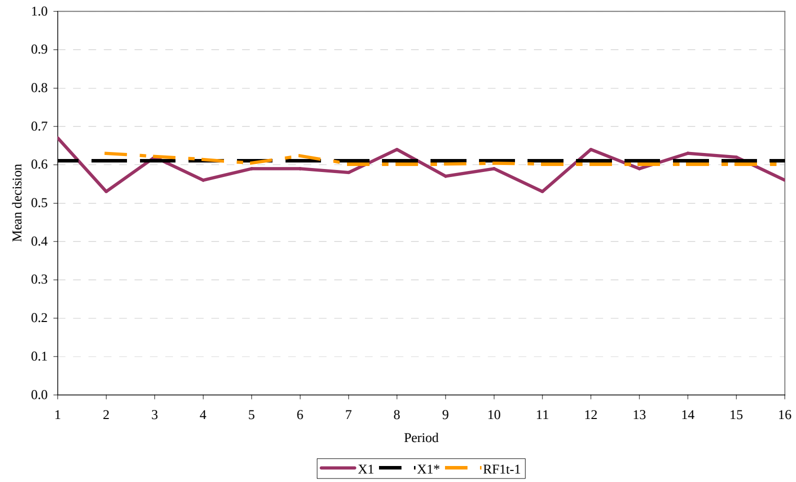

Fig. 4.16(a) Time series of mean locations of inside firm players (VQ2)

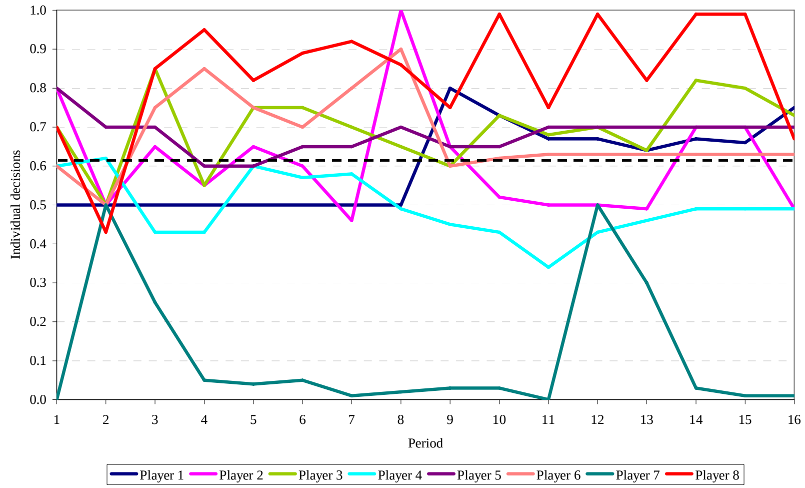

Fig. 4.16(b) Time series of individual locations of inside firm players (VQ2)

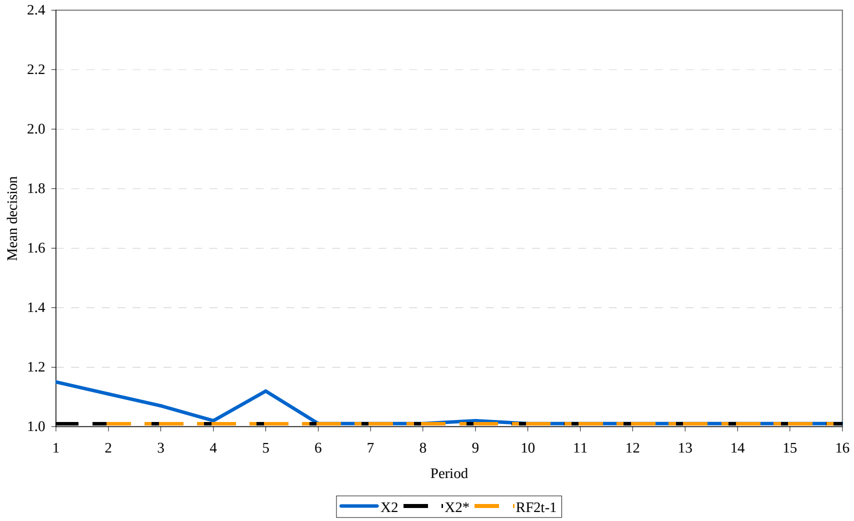

Fig. 4.17(a) Time series of mean locations of outside firm players (VQ2)

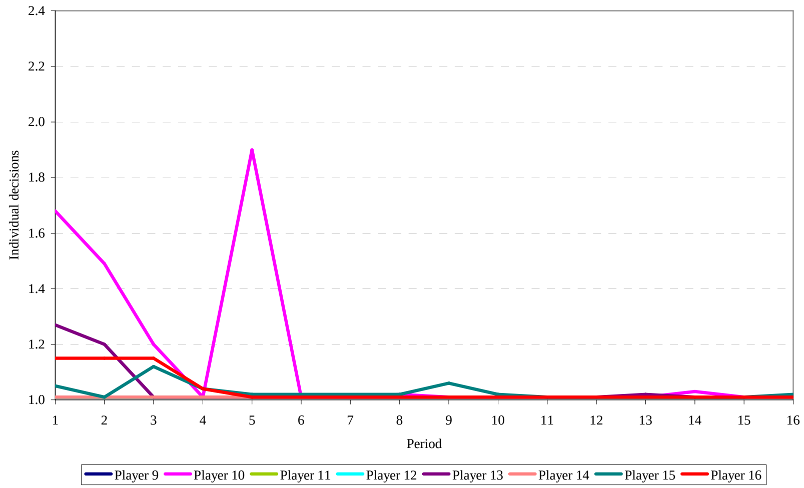

Fig. 4.17(b) Time series of individual locations of outside firm players (VQ2)

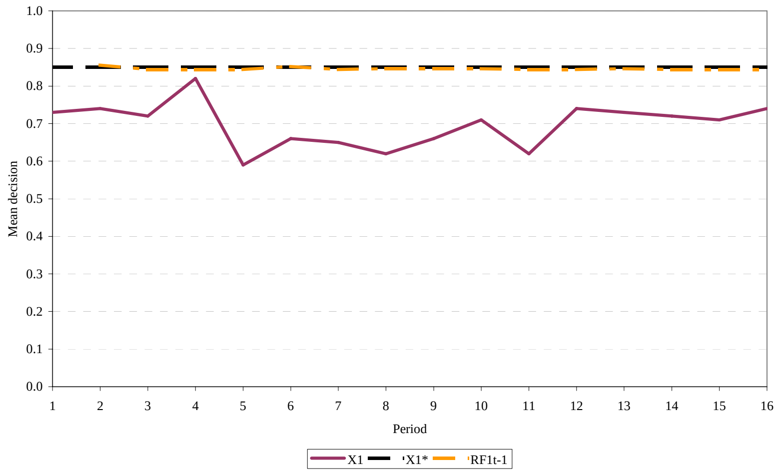

Fig. 4.18(a) Time series of mean locations of inside firm players (VLQ1)

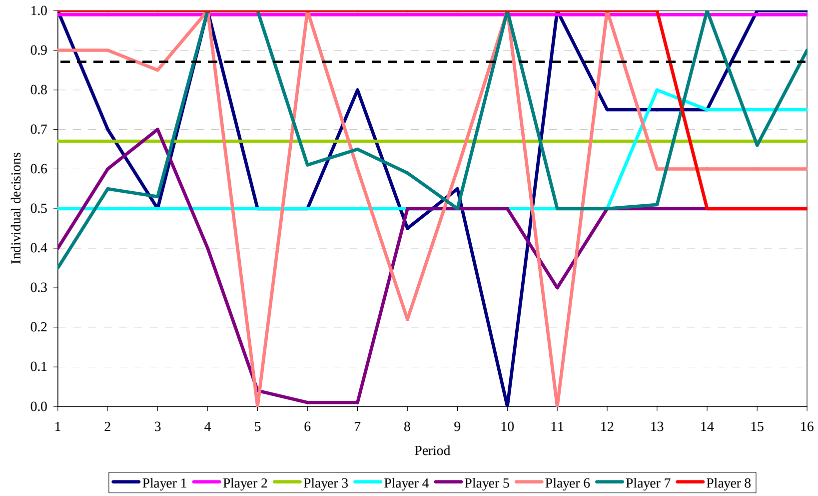

Fig. 4.18(b) Time series of individual locations of inside firm players (VLQ1)

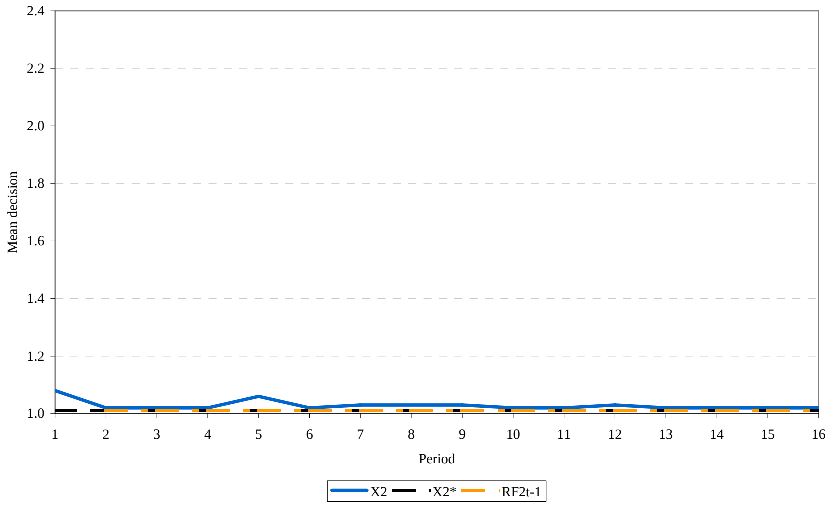

Fig. 4.19(a) Time series of mean locations of outside firm players (VLQ1)

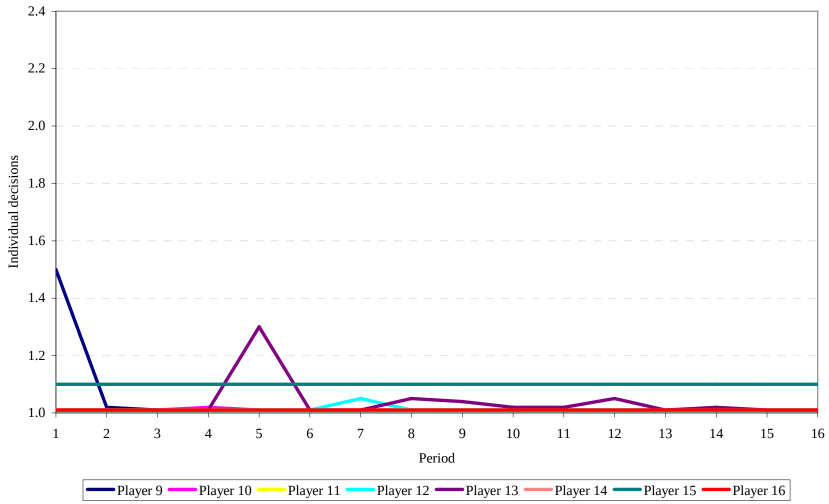

Fig. 4.19(b) Time series of individual locations of outside firm players (VLQ1)

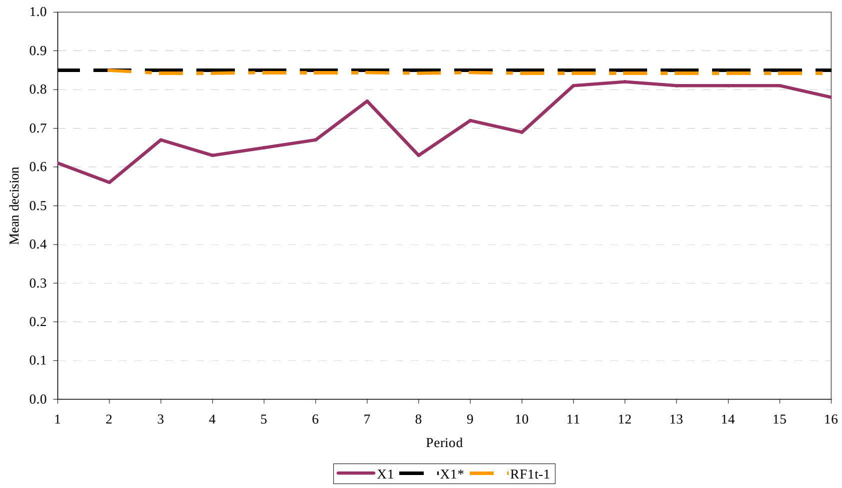

Fig. 4.20(a) Time series of mean locations of inside firm players (VLQ2)

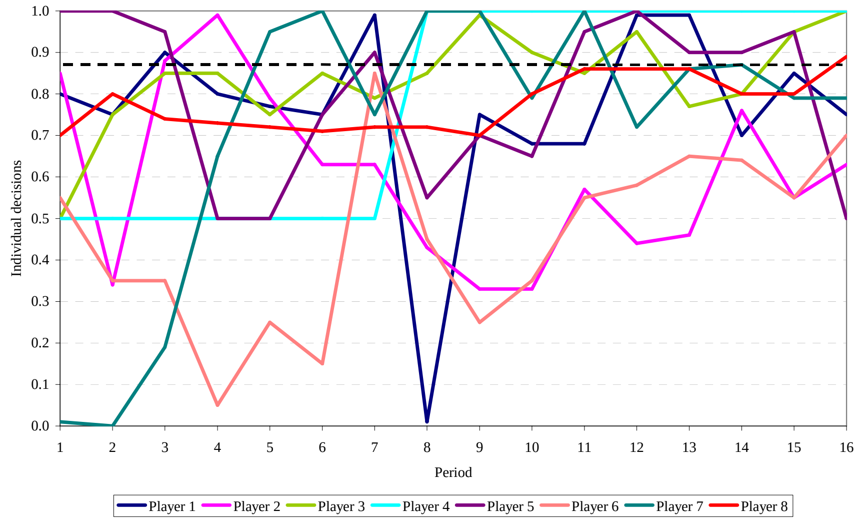

Fig. 4.20(b) Time series of individual locations of inside firm players (VLQ2)

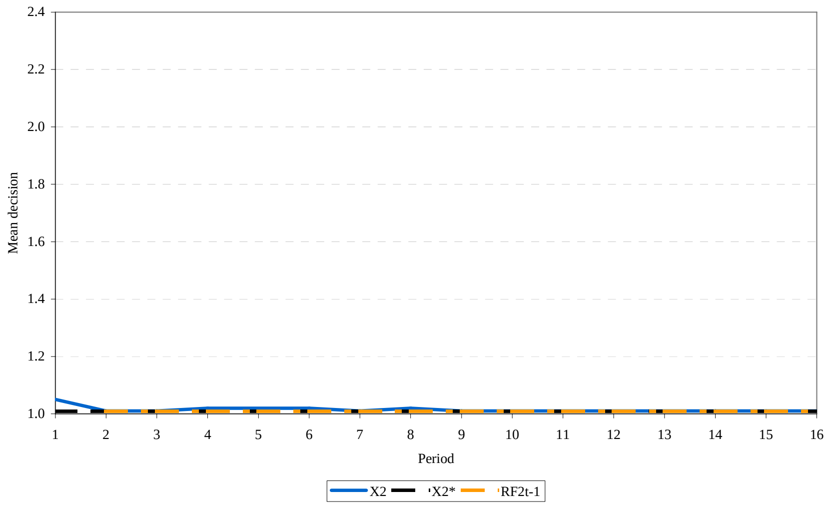

Fig. 4.21(a) Time series of mean locations of outside firm players (VLQ2)

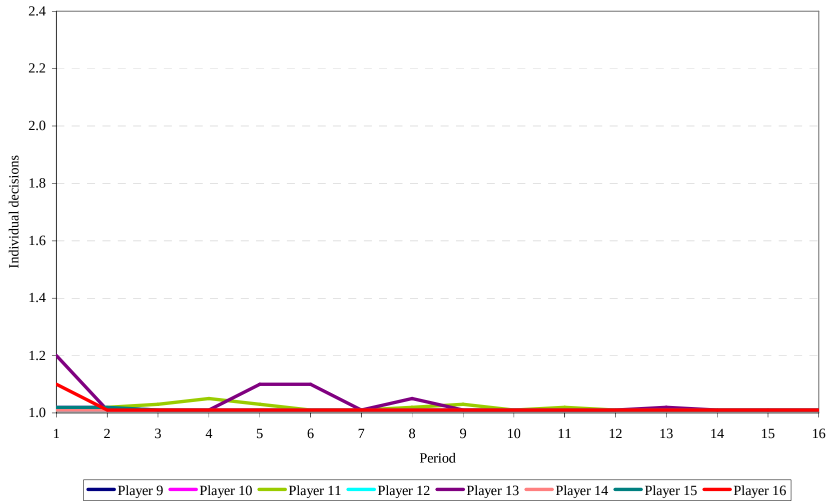

Fig. 4.21(b) Time series of individual locations of outside firm players (VLQ2)

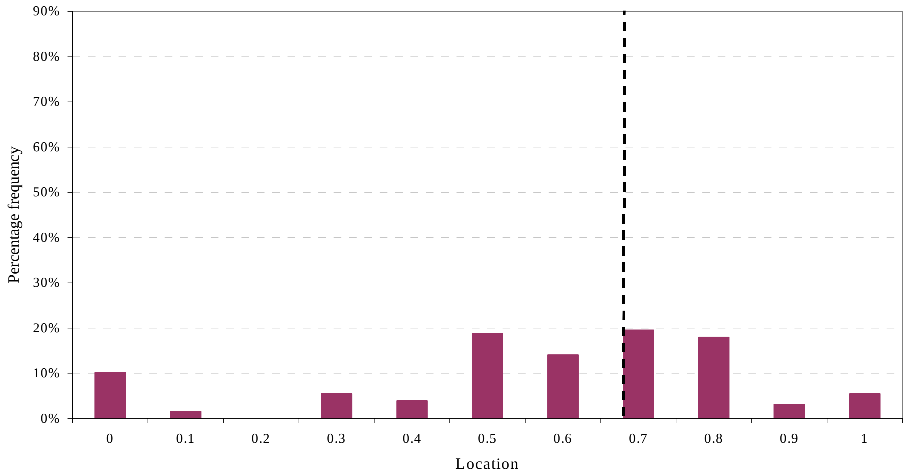

Fig. 4.22(a) Distribution of individual locations of inside firm players (VQ1)

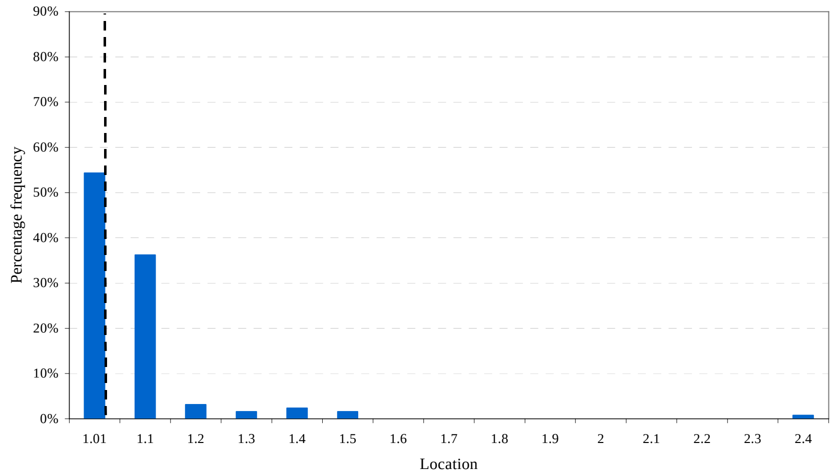

Fig. 4.22(b) Distribution of individual locations of outside firm players (VQ1)

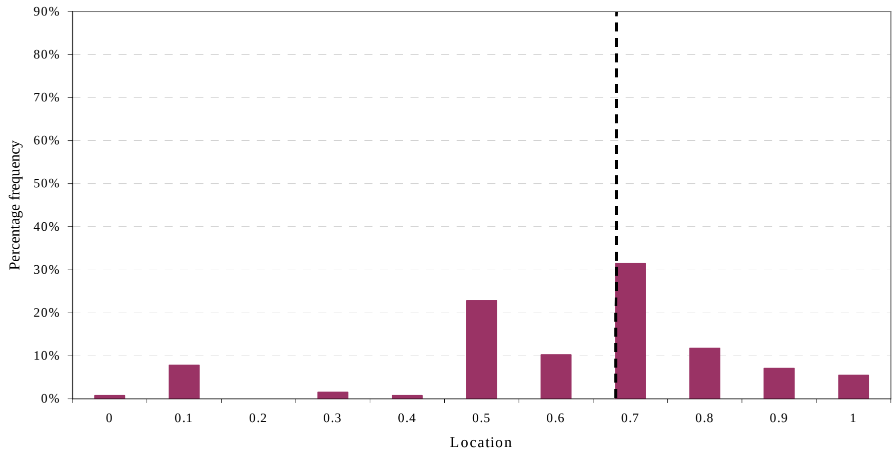

Fig. 4.23(a) Distribution of individual locations of inside firm players (VQ2)

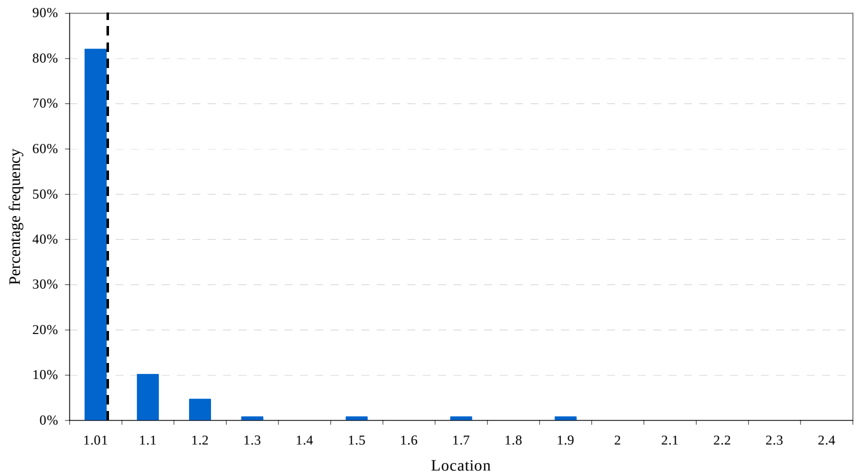

Fig. 4.23(b) Distribution of individual locations of outside firm players (VQ2)

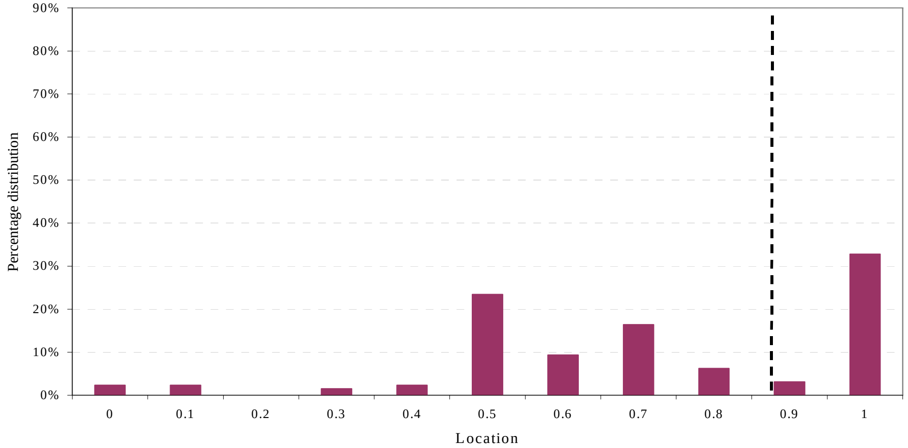

Fig. 4.24(a) Distribution of individual locations of inside firm players (VLQ1)

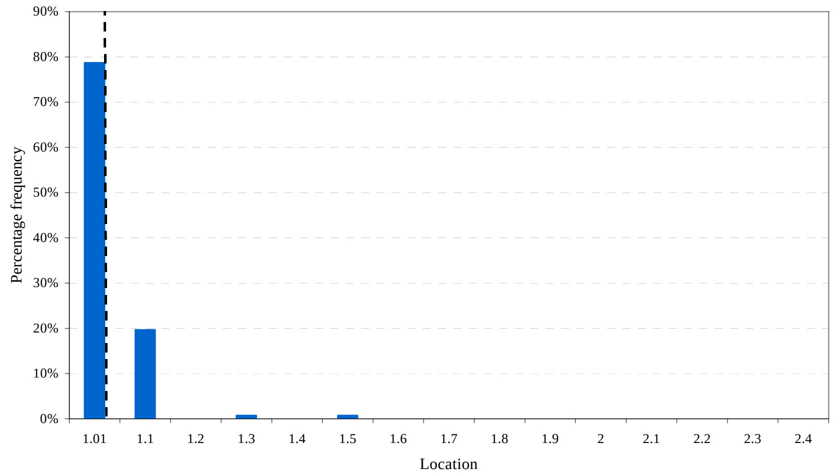

Fig. 4.24(b) Distribution of individual locations of outside firm players (VLQ1)

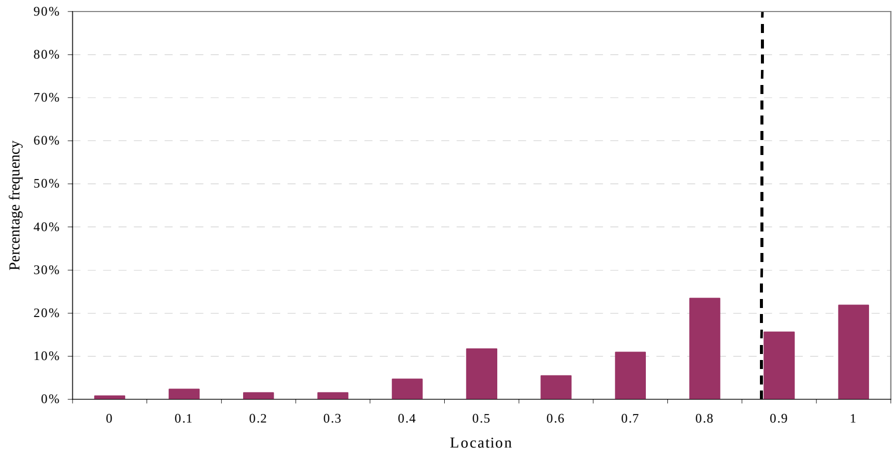

Fig. 4.25(a) Distribution of individual locations of inside firm players (VLQ2)

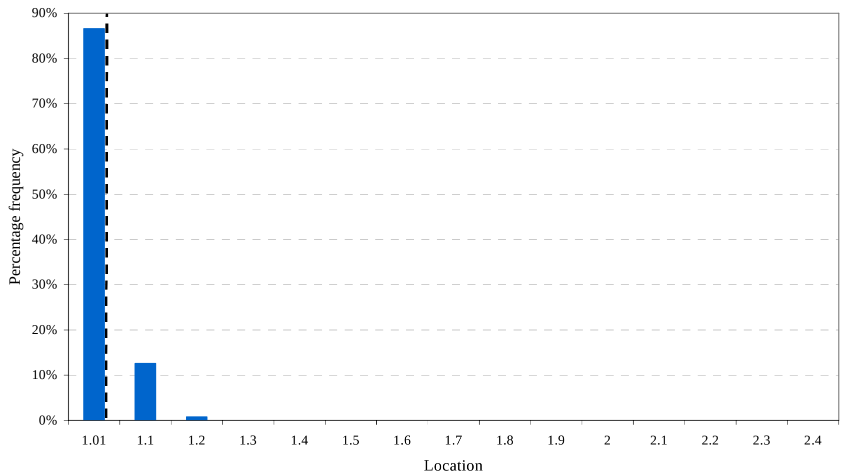

Fig. 4.25(b) Distribution of individual locations of outside firm players (VLQ2)

Table 4.8
Inadequate and inappropriate location response

Frequency	Over-positive response behaviour			Over-negative response behaviour			No response	Total
	Over increase	Wrong increase	Total	Over decrease	Wrong decrease	Total		
All	0.0723	0.0409	0.1132	0.0786	0.2872	0.3658	0.2317	0.7107
Inside firm	0.1069	0.0692	0.1761	0.1048	0.1551	0.2600	0.3753	0.8113
Outside firm	0.0000	0.0453	0.0453	0.0000	0.0000	0.0000	0.0948	0.1401
VQ1	0.0546	0.0462	0.1008	0.0882	0.2647	0.3529	0.2395	0.6933
Inside firm	0.0756	0.0672	0.1429	0.0756	0.1849	0.2605	0.3782	0.7815
Outside firm	0.0000	0.1333	0.1333	0.0000	0.0000	0.0000	0.0500	0.1833
VQ2	0.0750	0.0792	0.1542	0.0958	0.3042	0.4000	0.1417	0.6958
Inside firm	0.1083	0.1333	0.2417	0.1083	0.1500	0.2583	0.2583	0.7583
Outside firm	0.0000	0.0083	0.0083	0.0000	0.0000	0.0000	0.0417	0.0500
VLQ1	0.0588	0.0042	0.0630	0.0504	0.2857	0.3361	0.3697	0.7689
Inside firm	0.0840	0.0084	0.0924	0.0924	0.1008	0.1933	0.6218	0.9076
Outside firm	0.0000	0.0084	0.0084	0.0000	0.0000	0.0000	0.1345	0.1429
VLQ2	0.1008	0.0336	0.1345	0.0798	0.2941	0.3739	0.1765	0.6849
Inside firm	0.1597	0.0672	0.2269	0.1429	0.1849	0.3277	0.2437	0.7983
Outside firm	0.0000	0.0286	0.0286	0.0000	0.0000	0.0000	0.1619	0.1905

While the outside firm players recognise their optimal market strategy fairly quickly, i.e., to locate as close to the consumers as possible so as to reduce the transportation costs borne by them, the effectively wider range of location decisions available to inside firm players naturally results in a higher distribution of location choices. Under linear-quadratic transportation costs, transportation costs increase much more rapidly with distance than under quadratic transportation costs. Consequently, fewer inside firm players would choose to locate away from those consumers who reside close to the rival firm. This explains the lower frequencies for intervals in the vicinity of point 0 in VLQ1 and VLQ2.

Table 4.8 shows the relative frequencies of inadequate and inappropriate location movements *vis-à-vis* best response behaviour. An over-increase (decrease) in location corresponds an excessive location movement toward (away from) the market endpoint at 1.0 in relation to best response, while a wrong increase (decrease) corresponds to a location movement that is opposite in direction to best response. "No response" represents an absence of location movement when one is called for as best response. The total over-response and no response frequencies reflect the overall departure from best response.

The table clearly shows a higher frequency of inappropriate and inadequate location movements by the inside firm players with respect to best response: 0.8113 compared to 0.1401 by outside firm players. For all treatments, inappropriate movements by the inside firm away from the market border exceeds inappropriate movements toward it. This is reflected by a higher frequency of over-negative response compared to over-positive response. There is, therefore, a higher degree of product differentiation than is called for under best response.

We will now look at a few examples of players who failed to reach Nash prediction in location. The earlier discussion on low price behaviour has highlighted the strategy by player 4 in VQ1 of lowering price and locating further from his rival with each decline in demand. On reaching point 0 (corresponding to maximum product differentiation) in period 6, the player maintained this position and lowered price with each fall in demand. Similarly, player 7 in VQ2 reached point 0.01 in period 7 in the same manner, i.e., lowering price and moving further from his rival with each drop in demand. Maintaining this position for the next 5 periods, the player decided to test the profitability of an alternative location with higher price. A failure to gain higher demand with this move gave the player the erroneous belief that the earlier location was more optimal, resulting in a return to point 0 with continual dives in demand and profit.

In VQ1, there is an outside firm player (player 11) who, contrary to her best response, did not locate next to the market border but made somewhat erratic location decisions based purely on a one-period lag location of her rival. A decrease in proximity of the rival would mean a movement towards the border while the converse holds for an increase in proximity. In response to a question on the reasons for her decisions, the player stated that she made location decisions “depending on the location of the previous seller” and price “as low as possible to compete with the other seller”. Such an erratic strategy contributed to an inability by this player to reach any price or location equilibrium.

Table 4.9 shows that under a Wilcoxon signed rank test, the null hypothesis that location equals prediction is significant at the 0.05 level in the late2 phase for all but the inside firm in VQ2 and the outside firm in VQ1. For these two exceptions, the former shows significance at the 0.05 level under a Sign test but the latter remained insignificant at the 0.01 and 0.05 levels.[8]

The table highlights the universal convergence in location by late1 period of all treatments, with the abovementioned exceptions. The low or decreased location convergence under quadratic transportation costs compared to linear-quadratic transportation costs can be explained by a higher incidence of location movements contrary to best response. Table 4.8 shows that for VQ1 and VQ2, there is a higher frequency of wrong increase contrary to the direction dictated by best response and a higher frequency of over-decrease (greater movement toward point 0 than called for under best response) compared to the other treatments. In addition, VQ2 has a higher frequency of wrong decrease compared to all other treatments.

The higher location convergence of the inside firm under linear-quadratic transportation costs than under quadratic transportation costs concurs with the earlier observation of a smaller distribution of location frequencies of the inside firm under linear-quadratic transportation costs. Under a faster rate of transportation cost increase, fewer inside firm players choose to locate away from those consumers who reside close to the rival firm.

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[8] A *t*-test on the hypothesis that the mean price equals prediction cannot be performed because none of the location series are shown to be normally distributed by a Kolmogorov-Smirnov test for normal distribution: seven location series failed the test at the 0.05 level and one location series failed the test at the 0.01 level.

Table 4.9
Location convergence to Nash prediction
(probabilities for two-tailed Wilcoxon signed ranks test p_W and Sign test p_S)

Variable	N	pw		ps		Variable	N	pw		ps	
Null hypothesis: $x_{it} = x_i$*											
Treatment: VQ1						Treatment: VQ2					
All periods						All periods					
x1	128	0.0979	*	0.4263	*	x1	127	0.7351	*	0.2141	*
x2	127	0.0000		0.0000		x2	128	0.0000		0.0000	
Early phase						Early phase					
x1	64	0.0678	*	0.1691	*	x1	63	0.9372	*	0.8011	*
x2	63	0.0000		0.0000		x2	64	0.0002		0.0000	
Late1 phase						Late1 phase					
x1	40	0.7723	*	0.8744	*	x1	40	0.0004		0.0820	*
x2	40	0.0004		0.0000		x2	39	0.0066		0.0039	
Late2 phase						Late2 phase					
x1	24	0.3743	*	1.0000	*	x1	24	0.0050		0.0639	*
x2	24	0.0021		0.0005		x2	24	0.0588	*	0.1250	*
Treatment: VLQ1						Treatment: VLQ2					
All periods						All periods					
x1	128	0.0000		0.0014		x1	128	0.0000		0.0004	
x2	127	0.0000		0.0000		x2	127	0.0002		0.0000	
Early phase						Early phase					
x1	64	0.0000		0.0778	*	x1	64	0.0000		0.0000	
x2	64	0.0008		0.0001		x2	64	0.0009		0.0001	
Late1 phase						Late1 phase					
x1	40	0.9785	*	0.2684	*	x1	40	0.0712	*	1.0000	*
x2	40	0.1025	*	0.2500	*	x2	40	0.1025	*	0.2500	*
Late2 phase						Late2 phase					
x1	24	0.4740	*	0.0639	*	x1	24	0.1560	*	0.4049	*
x2	24	0.1797	*	0.5000	*	x2	24	1.0000	*	1.0000	*

* indicates significance at 0.05 level

To examine the manner in which firms make their location decisions, the following model is applied to pooled location data for 8 inside firm players and 8 outside firm players for each treatment:

(4.6) $$x_{it} = \alpha + \beta_1 x_{it-1} + \beta_2 x_{it-2} + \beta_3 x_{jt-1} + \beta_4 x_{jt-2} + \beta_5 p_{it} + \beta_6 p_{it-1} + \beta_7 p_{it-2} + \beta_8 p_{jt-1} + \beta_9 p_{jt-2} + \varepsilon_{it}$$

where p_{it} and p_{jt} are the price of firm *i* and firm *j* respectively in period t, $i, j \in \{1,2\}$, $i \neq j$; $t = \{1,...,16\}$, and x_{it} and x_{jt} are the location of firm *i* and firm *j* respectively.

The regression results are given in Table 4.10.[9] All diagnostic and stability checks are made to ensure that the estimates are reliable. Normality of most of the residuals, however, is rejected by a Jarque-Bera test at the 0.05 level. A graphical observation and residual statistics show that for these estimates, there is an extremely low variance of residuals around the mean zero with a small number of outliers.[10] We will, therefore, relegate the problem of a non-normal distribution of residuals as one of limited importance in these instances. The failure of Chow's breakpoint test for the outside firm in VLQ1 and VLQ2 is the result of the location of an outlying residual value near the breakpoint.

The results show that for the inside firm in all treatments, the coefficients for its own last location (β_1) and two-period lag own location (β_2) are positive and significantly different from zero at the 0.05 level, with the exception of a negative two-period lag own location in VLQ2. Since $|\beta_1 + \beta_2| < 1$, there is convergence in location of the inside firm in all treatments. Moreover, $|\beta_2| < |\beta_1| < 1$ in VQ1, VQ2 and VLQ2, indicating that the inside firm makes its location decisions by adapting more to its last location than its two-period ago location. The reverse is true for VLQ1 where $|\beta_1| < |\beta_2| < 1$. In the case of the outside firm, all β_1's and β_2's are not significantly different from zero at the 0.05 level in all treatments. This is the result of the outside firm predominantly playing its dominant strategy throughout the experiment so that generally no adjustments to earlier locations are necessary.

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[9] An augmented Dickey-Fuller test indicates that all location data series are stationary: the null hypothesis of a unit root is rejected in the level at the 0.05 and 0.10 levels for all series and at the 0.01 level as well for 5 of the 8 series.

[10] For these residuals, the standard deviation from the mean of zero ranges from 0.0070 to 0.2002.

Given the largely unvarying location of the outside firm, only the inside firm in VQ1 and VLQ2 have significant coefficients for two-period lag rival location (β_4) at the 0.05 level. All other coefficients for last rival location (β_3) and two-period lag rival location (β_4) in the other treatments are not significantly different from zero at the 0.05 level. Moreover, only the outside firm in VLQ1 and VLQ2 show significant coefficients for last rival location (β_3) and only in VLQ2 is the coefficient for two-period lag rival location (β_4) significant.

There is also some evidence that the two firms make their location decisions according to their own price and that of their rival. One or more price coefficients ($\beta_5, \beta_6, \beta_7, \beta_8$ and β_9) in all four treatments are found to be statistically different from zero at the 0.05 level or 0.10 level. In contrast to the price decisions of the outside firm which generally do not seem to take into account rival locations (see discussion under hypothesis H1), location decisions by the outside firm generally take into consideration rival prices (β_8 and β_9 are significantly different from zero at the 0.05 level or 0.10 level except in VQ2).

The results from the regression of equations 4.5 and 4.6 show that when firms make their price and location decisions simultaneously, price decisions play an important role in location decisions, and vice versa.

Table 4.10
Regression results for location decisions

Variable	Coefficient	S.E.	t-statistic	
Model: $x_{it} = \alpha + \beta_1 x_{it-1} + \beta_2 x_{it-2} + \beta_3 x_{jt-1} + \beta_4 x_{jt-2} + \beta_5 p_{it} + \beta_6 p_{it-1} + \beta_7 p_{it-2} + \beta_8 p_{jt-1} + \beta_9 p_{jt-2} + \varepsilon_{it}$				
Treatment: VQ1				
x_{1t}				
constant	0.4548	0.1824	2.4931	*
x_{1t-1}	0.5696	0.1117	5.1007	*
x_{1t-2}	0.2870	0.1077	2.6662	*
x_{2t-1}	-0.0242	0.1309	-0.1850	
x_{2t-2}	-0.3447	0.1302	-2.6469	*
p_{1t}	0.0115	0.0096	1.1978	
p_{1t-1}	-0.0207	0.0114	-1.8213	+
p_{1t-2}	0.0046	0.0093	0.4996	
p_{2t-1}	-0.0005	0.0100	-0.0520	
p_{2t-2}	0.0074	0.0099	0.7491	
AR(46)	-0.4654	0.1366	-3.4082	*
N = 78; Adj R^2 = 0.6119; F = 13.1393 (p = 0.0000); SSE = 2.3408; LM = 0.5037; White = 0.1395; Jarque-Bera = 0.1771; Chow = 0.0832				
x_{2t}				
constant	1.2055	0.5804	2.0771	*
x_{2t-1}	0.0326	0.1134	0.2876	
x_{2t-2}	0.0164	0.1088	0.1504	
x_{1t-1}	0.0196	0.0681	0.2879	
x_{1t-2}	-0.0988	0.0700	-1.4122	
p_{2t}	0.0084	0.0065	1.3033	
p_{2t-1}	-0.0050	0.0065	-0.7672	
p_{2t-2}	0.0166	0.0066	2.4985	*
p_{1t-1}	-0.0240	0.0068	-3.5174	*
p_{1t-2}	0.0147	0.0075	1.9603	+
AR(47)	0.9408	0.1415	6.6479	*
N = 76; Adj R^2 = 0.3450; F = 4.9503 (p = 0.0000); SSE = 1.2911; LM = 0.4455; White = 0.0548; Jarque-Bera = 0.0000; Chow = 0.4350				
Treatment: VQ2				
x_{1t}				
constant	0.0637	0.1708	0.3727	
x_{1t-1}	0.5680	0.0929	6.1139	*
x_{1t-2}	0.3344	0.0968	3.4554	*
x_{2t-1}	-0.1049	0.1183	-0.8867	
x_{2t-2}	0.1451	0.1180	1.2295	
p_{1t}	0.0043	0.0039	1.1266	
p_{1t-1}	-0.0059	0.0041	-1.4378	
p_{1t-2}	0.0055	0.0034	1.6200	
p_{2t-1}	-0.0078	0.0044	-1.7528	+
p_{2t-2}	-0.0051	0.0044	-1.1637	
AR(11)	0.2371	0.1002	2.3672	*
N = 109; Adj R^2 = 0.6607; F = 22.0343 (p = 0.0000); SSE = 1.7972; LM = 0.2335; White = 0.0418; Jarque-Bera = 0.0000; Chow = 0.5255				

Table 4.10 (contd.)

Variable	Coefficient	S.E.	t-statistic	
Treatment: VQ2				
x_{2t}				
constant	0.8417	0.1472	5.7189	*
x_{2t-1}	0.0173	0.1029	0.1680	
x_{2t-2}	0.1169	0.1031	1.1341	
x_{1t-1}	0.0510	0.0627	0.8137	
x_{1t-2}	-0.0170	0.0620	-0.2749	
p_{2t}	-0.0022	0.0037	-0.6097	
p_{2t-1}	0.0006	0.0037	0.1630	
p_{2t-2}	0.0044	0.0039	1.1297	
p_{1t-1}	0.0005	0.0029	0.1615	
p_{1t-2}	0.0014	0.0026	0.5294	
AR(20)	0.1882	0.0847	2.2207	*
N = 102; Adj R^2 = -0.0116; F = 0.8840 (p = 0.5511); SSE = 0.8611; LM = 0.9754; White = 0.3227; Jarque-Bera = 0.0000; Chow = 0.9027				
Treatment: VLQ1				
x_{1t}				
constant	1.5155	0.7054	2.1483	*
x_{1t-1}	0.2803	0.0873	3.2104	*
x_{1t-2}	0.3275	0.0872	3.7538	*
x_{2t-1}	-0.8370	0.5257	-1.5921	
x_{2t-2}	-0.3458	0.3692	-0.9369	
p_{1t}	-0.0146	0.0084	-1.7327	+
p_{1t-1}	0.0020	0.0088	0.2304	
p_{1t-2}	-0.0070	0.0077	-0.9050	
p_{2t-1}	0.0085	0.0098	0.8665	
p_{2t-2}	0.0126	0.0099	1.2735	
N = 124; Adj R^2 = 0.2871; F = 6.5046 (p = 0.0000); SSE = 5.7376; LM = 0.1717; White = 0.1438; Jarque-Bera = 0.0002; Chow = 0.2403				
x_{2t}				
constant	1.1846	0.1636	7.2429	*
x_{2t-1}	-0.0815	0.1178	-0.6920	
x_{2t-2}	-0.0838	0.0820	-1.0210	
x_{1t-1}	0.0176	0.0088	1.9951	*
x_{1t-2}	-0.0125	0.0092	-1.3636	
p_{2t}	-0.0012	0.0010	-1.1398	
p_{2t-1}	-0.0013	0.0011	-1.2305	
p_{2t-2}	-0.0009	0.0011	-0.8597	
p_{1t-1}	0.0018	0.0009	1.9512	+
p_{1t-2}	-0.0020	0.0008	-2.4091	*
AR(8)	0.2189	0.0802	2.7290	*
AR(17)	0.2185	0.0822	2.6582	*
AR(30)	0.3927	0.0640	6.1331	*
N = 84; Adj R^2 = 0.4903; F = 7.6523 (p = 0.0000); SSE = 0.0319; LM = 0.2015; White = 0.0558; Jarque-Bera = 0.0000; Chow = 0.0001				

Table 4.10 (contd.)

Variable	Coefficient	S.E.	t-statistic	
Treatment: VLQ2				
x_{1t}				
constant	0.8925	0.9288	0.9609	
x_{1t-1}	1.1804	0.1358	8.6910	*
x_{1t-2}	-0.3872	0.1311	-2.9540	*
x_{2t-1}	0.7651	0.7799	0.9810	
x_{2t-2}	-1.5769	0.7817	-2.0173	*
p_{1t}	-0.0158	0.0059	-2.6965	*
p_{1t-1}	0.0179	0.0068	2.6092	*
p_{1t-2}	-0.0055	0.0058	-0.9400	
p_{2t-1}	0.0223	0.0059	3.7839	*
p_{2t-2}	-0.0057	0.0070	-0.8169	
AR(1)	-0.4234	0.1329	-3.1853	*
AR(30)	-0.1632	0.0737	-2.2134	*
N = 94; Adj R^2 = 0.5597; F = 111.7481 (p = 0.0000); SSE = 2.2420; LM = 0.7446; White = 0.0142; Jarque-Bera = 0.0130; Chow = 0.4926				
x_{2t}				
constant	1.0432	0.0613	17.0309	*
x_{2t-1}	-0.0269	0.0411	-0.6549	
x_{2t-2}	-0.0111	0.0434	-0.2552	
x_{1t-1}	0.0108	0.0055	1.9550	+
x_{1t-2}	-0.0114	0.0053	-2.1576	*
p_{2t}	-0.0001	0.0003	-0.3531	
p_{2t-1}	-0.0002	0.0003	-0.4864	
p_{2t-2}	-0.0001	0.0004	-0.3751	
p_{1t-1}	0.0013	0.0003	4.1050	*
p_{1t-2}	-0.0003	0.0003	-0.9768	
AR(48)	0.2961	0.0336	8.8010	*
N = 75; Adj R^2 = 0.5577; F = 10.3310 (p = 0.0000); SSE = 0.0038; LM = 0.6147; White = 0.9110; Jarque-Bera = 0.0000; Chow = 0.0000				

See Table 4.7 for notes. Failure of Jarque-Bera test due to 1-2 outlying values. Failure of Chow's test for x_2 in VLQ1 and VLQ2 due to outlying residual value near breakpoint.

Table 4.11
Frequency of appropriate and inappropriate response relative to best strategy

Frequency	Location		Price	
	Appropriate response	Inappropriate response	Appropriate response	Inappropriate response
All treatments	0.4403	0.5597	0.5455	0.4545
Inside firm	0.4004	0.5996	0.6109	0.3891
Outside firm	0.8599	0.1401	0.4801	0.5199
VQ1	0.4496	0.5504	0.5481	0.4519
Inside firm	0.3697	0.6303	0.5667	0.4333
Outside firm	0.8167	0.1833	0.5294	0.4706
VQ2	0.4750	0.5250	0.5958	0.4042
Inside firm	0.4583	0.5417	0.7000	0.3000
Outside firm	0.9500	0.0500	0.4917	0.5083
VLQ1	0.3403	0.6597	0.5042	0.4958
Inside firm	0.2689	0.7311	0.5966	0.4034
Outside firm	0.8571	0.1429	0.4118	0.5882
VLQ2	0.4958	0.5042	0.5336	0.4664
Inside firm	0.5042	0.4958	0.5798	0.4202
Outside firm	0.8095	0.1905	0.4874	0.5126

Inappropriate response frequency is the sum of wrong response frequency and no response frequency. Appropriate response frequency is total response frequency less inappropriate response frequency.

H3: Firms play their best strategies.

How closely do the firms follow their best response? Do they get better at playing their best strategies over time? An indication of the proximity of firm behaviour to best response can be obtained by looking at the frequency of appropriate response behaviour relative to best strategy. An *appropriate response* was defined in Chapter 3 as one in which a player moves in the same direction as that dictated by best response. Table 4.11 shows that the frequency of appropriate price response for both firms averages 0.5455 while the frequency of appropriate location response averages 0.4403 for all treatments. The frequency of appropriate location response for the outside firm (0.8599) is higher than that for the inside firm (0.4004). On the other hand, the frequency of appropriate price response for the inside firm (0.6109) is higher than that for the outside firm (0.4801). In other words, the inside firm tracks its best response for price more closely while the outside firm tracks its best response for location for more closely. Among the four treatments, VLQ1 has the lowest frequency of appropriate location response (by the inside firm) and the lowest frequency of appropriate price response (by the outside firm).

In terms of firm experience at playing best strategies, a Wilcoxon signed rank test and a Sign test show that the price and location decisions of both firms match best response by the late1 phase in most treatments. Table 4.12 shows that the null hypothesis of an equivalence of location decisions and best response (lagged one period) is predominantly insignificant in the early phase but is significant in the late2 phase at the 0.05 level for all treatments except the outside firm in VQ1 and the inside firm in VLQ1. On the other hand, the null hypothesis of an equivalence of price decisions and best response (lagged one period) is significant throughout the experiment for both firms at the 0.05 level and 0.01 level in all treatments. Exceptions are the price decisions of the inside firm in VLQ1 and VLQ2 which do not exhibit any significant parity with best response in the early phase at the 0.05 level.

To evaluate the extent to which firms play their best strategies and improve their execution of best strategies over time, the following equations are estimated:

$$(4.7) \qquad \left|p_{it} - pbr_{it-1}\right| = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_{it}$$

$$(4.8) \qquad \left|x_{it} - xbr_{it-1}\right| = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_{it}$$

where p_{it} and x_{it} are the price and location respectively of firm *i* in period *t*, $i \in \{1,2\}$, $t = \{1,...,16\}$; $pbr_{i,t-1}$ is the one-period lag best response in price; $xbr_{i,t-1}$ is the one-period lag best response in location; DUM_{1t} is a dummy variable that equals 1 if $t = \{9,...,13\}$ and 0 otherwise; and DUM_{2t} is a dummy variable that equals 1 if $t = \{14,...16\}$ and 0 otherwise. If learning occurs in the late1 phase and late2 phase, then β_1 and β_2 respectively would be negative.

The results of the regression can be found in Tables 4.13 and 4.14. All diagnostic checks for stationary series, specification and stability are conducted as before.[11]

[11] An augmented Dickey-Fuller test shows that all the best response data series in the level are stationary at all reported levels (except VLQ1pbr$_2$ which rejects the null hypothesis of a unit root at the 0.01 level). The ADF test is not performed for the best response of x_2 in all treatments because of a near singular matrix.

Table 4.12
Congruence of price and location decisions to best response
(probabilities for two-tailed Wilcoxon signed ranks test p_W and Sign test p_S)

Variable	N	pw		ps		Variable	N	pw		ps	
Null hypothesis: $x_{it} = xbr_{it-1}$; $p_{it} = pbr_{it-1}$											
Treatment: VQ1						Treatment: VQ2					
All periods						All periods					
x1	119	0.0410	+	0.1994	*	x1	120	0.9916	*	0.3153	*
x2	120	0.0000		0.0000		x2	120	0.0001		0.0000	
p1	119	0.0377	+	0.2713	*	p1	119	0.9218	*	0.3593	*
p2	119	0.2433	*	0.1956	*	p2	119	0.5867	*	0.4633	*
Early phase						Early phase					
x1	55	0.0127	+	0.1272	*	x1	56	0.7694	*	0.6831	*
x2	56	0.0000		0.0000		x2	56	0.0009		0.0001	
p1	55	0.7951	*	1.0000	*	p1	55	0.8048	*	1.0000	*
p2	55	0.1480	*	0.1344	*	p2	55	0.4334	*	0.7874	*
Late1 phase						Late1 phase					
x1	40	0.8846	*	0.8711	*	x1	40	0.9653	*	0.1443	*
x2	40	0.0004		0.0000		x2	40	0.1025	*	0.2500	*
p1	40	0.0513	*	0.6353	*	p1	40	0.9357	*	0.6353	*
p2	40	0.9722	*	1.0000	*	p2	40	0.7318	*	0.8744	*
Late2 phase						Late2 phase					
x1	24	0.3458	*	0.8318	*	x1	24	0.4398	*	0.0639	*
x2	24	0.0021		0.0005		x2	24	0.1797	*	0.5000	*
p1	24	0.0093		0.1516	*	p1	24	0.7861	*	0.3075	*
p2	24	0.6891	*	0.5413	*	p2	24	0.5111	*	0.5413	*
Treatment: VLQ1						Treatment: VLQ2					
All periods						All periods					
x1	119	0.0000		0.0000		x1	119	0.0000		0.0172	+
x2	119	0.0000		0.0000		x2	119	0.0013		0.0002	
p1	119	0.0000		0.0034		p1	119	0.0024		0.0103	+
p2	118	0.0265	+	0.0342	+	p2	119	0.9672	*	0.3593	*
Early phase						Early phase					
x1	56	0.0000		0.0000		x1	56	0.0000		0.0021	
x2	56	0.0017		0.0005		x2	56	0.0047		0.0020	
p1	56	0.0012		0.0050		p1	56	0.0035		0.0111	+
p2	56	0.2211	*	0.0824	*	p2	56	0.6655	*	0.8937	*
Late1 phase						Late1 phase					
x1	40	0.0000		0.0000		x1	40	0.1595	*	1.0000	*
x2	39	0.0066		0.0039		x2	40	0.1025	*	0.2500	*
p1	39	0.0707	*	0.5218	*	p1	40	0.0326	+	0.0820	*
p2	38	0.4464	*	1.0000	*	p2	40	0.4721	*	0.2684	*
Late2 phase						Late2 phase					
x1	23	0.0000		0.0000		x1	23	0.3609	*	0.6776	*
x2	24	0.0588	*	0.1250	*	x2	23	1.0000	*	1.0000	*
p1	24	0.0397	+	0.3075	*	p1	23	0.5034	*	0.6776	*
p2	24	0.0520	*	0.0639	*	p2	23	0.7610	*	0.4049	*

+ indicates significance at the 0.01 level
* indicates significance at the 0.05 level

Table 4.13
Regression results for price decisions and best strategies

Variable	Coefficient	S.E.	t-statistic	
Model:	$\lvert p_{it} - pbr_{it-1} \rvert = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_{it}$			
Treatment: VQ1				
$\lvert p_{1t}-pbr_{1t-1} \rvert$				
constant	1.9570	0.2453	7.9791	*
DUM_{1t}	-0.6510	0.3760	-1.7315	+
DUM_{2t}	-0.4529	0.4422	-1.0242	
N = 118; Adj R^2 = 0.0100; F = 1.5901 (p = 0.2084); SSE = 373.5757; DW = 1.5024; LM = 0.9148; White = 0.1110; Jarque-Bera = 0.0000; Chow = 0.9522				
$\lvert p_{2t}-pbr_{2t-1} \rvert$				
constant	1.8881	0.2243	8.4195	*
DUM_{1t}	-0.4756	0.3419	-1.3909	
DUM_{2t}	-0.1319	0.4017	-0.3283	
N = 117; Adj R^2 = -0.0002; F = 0.9862 (p = 0.3761); SSE = 303.8549; DW = 1.6452; LM = 0.3783; White = 0.1891; Jarque-Bera = 0.0000; Chow = 0.4054				
Treatment: VQ2				
$\lvert p_{1t}-pbr_{1t-1} \rvert$				
constant	4.3601	0.6243	6.9841	*
DUM_{1t}	-2.4938	0.9377	-2.6596	*
DUM_{2t}	-2.4281	1.0979	-2.2117	*
AR(16)	0.2885	0.0961	3.0020	*
N = 100; Adj R^2 = 0.1852; F = 8.4985 (p = 0.0000); SSE = 832.1263; DW = 1.5994; LM = 0.9129; White = 0.0324; Jarque-Bera = 0.0000; Chow = 0.3506				
$\lvert p_{2t}-pbr_{2t-1} \rvert$				
constant	3.3861	0.3420	9.9012	*
DUM_{1t}	-0.4726	0.5243	-0.9015	
DUM_{2t}	-1.4478	0.6165	-2.3483	*
N = 118; Adj R^2 = 0.0292; F = 2.7587 (p = 0.0676); SSE = 726.3093; DW = 1.6300; LM = 0.6171; White = 0.2434; Jarque-Bera = 0.0000; Chow = 0.3185				

Table 4.13 (contd.)

Variable	Coefficient	S.E.	t-statistic	
Treatment: VLQ1				
$\lvert p_{1t}\text{-}pbr_{1t-1}\rvert$				
constant	2.5916	0.2405	10.7760	*
DUM_{1t}	-0.7998	0.3734	-2.1422	*
DUM_{2t}	-0.4687	0.4363	-1.0742	
N = 118; Adj R^2 = 0.0226; F = 2.3507 (p = 0.0999); SSE = 365.8456; DW = 1.6211; LM = 0.7768; White = 0.0968; Jarque-Bera = 0.0000; Chow = 0.4686				
$\lvert p_{2t}\text{-}pbr_{2t-1}\rvert$				
constant	2.8436	0.2519	11.2874	*
DUM_{1t}	-0.9986	0.4112	-2.4284	*
DUM_{2t}	-0.6889	0.4605	-1.4959	
AR(6)	-0.1819	0.0966	-1.8833	+
N = 103; Adj R^2 = 0.0735; F = 3.6969 (p = 0.0143); SSE = 267.8592; DW = 1.7133; LM = 0.9703; White = 0.0358; Jarque-Bera = 0.0001; Chow = 0.1325				
Treatment: VLQ2				
$\lvert p_{1t}\text{-}pbr_{1t-1}\rvert$				
constant	3.4001	0.4362	7.7951	*
DUM_{1t}	-0.6364	0.6169	-1.0316	
DUM_{2t}	-0.9283	0.7060	-1.3150	
AR(1)	0.1881	0.0945	1.9911	*
N = 110; Adj R^2 = 0.0331; F = 2.2427 (p = 0.0876); SSE = 654.0084; DW = 1.9908; LM = 0.9251; White = 0.0377; Jarque-Bera = 0.0000; Chow = 0.9299				
$\lvert p_{2t}\text{-}pbr_{2t-1}\rvert$				
constant	3.4233	0.3450	9.9215	*
DUM_{1t}	0.3354	0.5289	0.6341	
DUM_{2t}	-0.8792	0.6220	-1.4134	
N = 118; Adj R^2 = 0.0127; F = 1.7538 (p = 0.1777); SSE = 739.3184; DW = 1.9798; LM = 0.8822; White = 0.2750; Jarque-Bera = 0.0000; Chow = 0.1943				

See Table 4.7 for notes. Failure of Jarque-Bera test due to 1-3 outlying values.

Table 4.14
Regression results for location decisions and best strategies

Variable	Coefficient	S.E.	t-statistic	
Model:	$\left\|x_{it} - xbr_{it-1}\right\| = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_{it}$			
Treatment: VQ1				
$\|x_{1t}-xbr_{1t-1}\|$				
constant	0.1829	0.0597	3.0650	*
DUM_{1t}	-0.0031	0.0404	-0.0765	
DUM_{2t}	0.0601	0.0459	1.3093	
AR(1)	0.2716	0.1060	2.5620	*
AR(4)	0.2993	0.1079	2.7748	*
AR(6)	0.2989	0.1150	2.5988	*
AR(7)	-0.1959	0.1018	-1.9242	+
N = 81; Adj R^2 = 0.3792; F = 9.1435 (p = 0.0000); SSE = 1.6147; DW = 1.7260; LM = 0.1356; White = 0.1388; Jarque-Bera = 0.1192; Chow = 0.0001				
$\|x_{2t}-xbr_{2t-1}\|$				
constant	0.0647	0.0263	2.4595	*
DUM_{1t}	-0.0440	0.0335	-1.3147	
DUM_{2t}	-0.0167	0.0390	-0.4273	
AR(4)	0.1700	0.0957	1.7768	+
N = 106; Adj R^2 = 0.0190; F = 1.6786 (p = 0.1763); SSE = 2.4275; DW = 1.1506; LM = 0.2919; White = 0.4146; Jarque-Bera = 0.0000; Chow = 0.5491				
Treatment: VQ2				
$\|x_{1t}-xbr_{1t-1}\|$				
constant	0.3241	0.1757	1.8449	+
DUM_{1t}	-0.1496	0.0519	-2.8826	*
DUM_{2t}	-0.0770	0.0718	-1.0722	
AR(1)	0.7699	0.0695	11.0730	*
AR(15)	0.1376	0.0697	1.9751	+
N = 90; Adj R^2 = 0.5746; F = 31.0547 (p = 0.0000); SSE = 1.0857; DW = 2.2368; LM = 0.5896; White = 0.5686; Jarque-Bera = 0.0403; Chow = 0.2944				
$\|x_{2t}-xbr_{2t-1}\|$				
constant	0.0464	0.0142	3.2673	*
DUM_{1t}	-0.0446	0.0219	-2.0401	*
DUM_{2t}	-0.0451	0.0257	-1.7523	+
N = 119; Adj R^2 = 0.0277; F = 2.6805 (p = 0.0728); SSE = 1.2847; DW = 1.7077; LM = 0.9557; White = 0.2296; Jarque-Bera = 0.0000; Chow = 0.8391				

Table 4.14 (contd.)

Variable	Coefficient	S.E.	t-statistic
Treatment: VLQ1			
$\lvert x_{1t}\text{-}xbr_{1t-1}\rvert$			
constant	9.5176	0.1990	47.8328 *
DUM_{1t}	0.2233	0.3404	0.6559
DUM_{2t}	-0.4085	0.4350	-0.9393
AR(28)	-0.1989	0.1022	-1.9468 +
N = 85; Adj R^2 = 0.0417; F = 2.2199 (p = 0.0921); SSE = 157.4149; DW = 2.1506; LM = 0.8058; White = 0.2457; Jarque-Bera = 0.0020; Chow = 0.4509			
$\lvert x_{2t}\text{-}xbr_{2t-1}\rvert$			
constant	0.0101	0.0057	1.7713 *
DUM_{1t}	0.0049	0.0071	0.7005
DUM_{2t}	-0.0067	0.0090	-0.7435
AR(30)	0.4249	0.0620	6.8513 *
N = 82; Adj R^2 = 0.3616; F = 16.2938 (p = 0.0000); SSE = 0.0482; DW = 2.0264; LM = 0.2520; White = 0.6489; Jarque-Bera = 0.0000; Chow = 0.0001			
Treatment: VLQ2			
$\lvert x_{1t}\text{-}xbr_{1t-1}\rvert$			
constant	0.2251	0.0386	5.8357 *
DUM_{1t}	-0.0107	0.0442	-0.2426
DUM_{2t}	-0.0974	0.0478	-2.0389 *
AR(1)	0.3553	0.1272	2.7936 *
AR(2)	0.2672	0.1176	2.2722 *
AR(19)	-0.1450	0.0790	-1.8347 +
N = 83; Adj R^2 = 0.3425; F = 9.5425 (p = 0.0000); SSE = 1.3039; DW = 1.8175; LM = 0.9088; White = 0.1357; Jarque-Bera = 0.0000; Chow = 0.8461			
$\lvert x_{2t}\text{-}xbr_{2t-1}\rvert$			
constant	0.0211	0.0072	2.9431 *
DUM_{1t}	-0.0182	0.0074	-2.4670 *
DUM_{2t}	-0.0294	0.0079	-3.7351 *
AR(17)	0.6082	0.1397	4.3550 *
N = 96; Adj R^2 = 0.1299; F = 5.7264 (p = 0.0012); SSE = 0.0453; DW = 1.2138; LM = 0.8149; White = 0.2181; Jarque-Bera = 0.0000; Chow = 0.0003			

See Table 4.7 for notes. Failure of Jarque-Bera test due to 1-2 outlying values. Failure of Chow's test for $\lvert x_{1t}\text{-}xbr_{1t-1}\rvert$ in VQ1 and $\lvert x_{2t}\text{-}xbr_{2t-1}\rvert$ in VLQ1 and VLQ2 due to outlying residual value near breakpoint.

Looking at the results for price decisions and best strategies (Table 4.13), it is clear that one or more β's for the inside firm are negative and significant in all but one treatment (VLQ2). β_1 is negative and significantly different from zero at the 0.10 level in VQ1 and at the 0.05 level in VQ2 and VLQ1. β_2 is negative and significantly different from zero at the 0.05 level in VQ2 and is insignificant in all the other treatments. For the outside firm, the β's are mostly insignificant except for β_1 and β_2 in VLQ1 and VQ2 respectively which are negative and significantly different from zero at the 0.05 level.

There is evidence, therefore, that learning is occurring for the inside firm in three treatments during the late1 phase but in only one treatment is learning occurring during the late2 phase as well. As for the outside firm, there is evidence that learning is occurring in the late1 and late2 phases in only two treatments. In VLQ2, neither firm shows signs of learning as the experiment progresses.

Turning to location decisions and best responses (Table 4.14), there is little evidence of learning with respect to location, with β_1 negative and significantly different from zero only for both firms in VQ2 (at 0.05 level), and β_2 negative and significantly different from zero only for the outside firm in VQ2 (at 0.10 level) and for both firms in VLQ2 (at 0.05 level). The largely insignificant β's for the outside firm is a natural phenomenon given that the outside firm nearly always plays its best response (i.e., the dominant strategy of locating next to the market border) throughout the experiment.

The greater proximity of location decisions than price decisions to best strategies for both firms is reflected in the smaller α's for location decisions (significant at the 0.05 level in all treatments for both price and location, except at 0.10 level for the inside firm in VQ2). One exception is the inside firm in VLQ1 where α is higher for location decisions.

H4: The level of product differentiation decreases with transportation costs.

Figure 4.26 shows the distribution of individual product differentiation decisions, grouped by intervals of 0.1, for the four treatments. A product differentiation decision is obtained by taking the difference between the location decision of each outside firm player and the counterpart inside firm player for each period. The broken vertical line corresponds to the theoretical prediction. The graphs show that the interval with the highest frequency in VQ2 coincides with prediction while the interval with the highest frequency is below prediction in VLQ1 and above prediction in VQ1 and VLQ2.

Figure 4.27 shows the time series of mean product differentiation decisions for Treatments 1 to 4. It is evident that the level of product differentiation is higher under quadratic transportation costs (VQ1 and VQ2) than under linear-quadratic transportation costs (VLQ1 and VLQ2). When transportation costs increase at a more rapid rate with distance, product differentiation decreases as fewer inside firm players choose to locate away from consumers who reside close to the rival firm.

To test the hypothesis that the level of product differentiation decreases when transportation costs increase more rapidly, the following equation is estimated:

$$\left|\Delta D_{jt} - \Delta D^{*}\right| = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_{jt} \tag{4.9}$$

where ΔD_{jt} is the difference between product differentiation decisions under transportation costs of type *j* in period *t*, $j \in \{1,2\}$ where $j = 1$ denotes the ex ante treatment before a transportation cost increase and $j = 2$ denotes the ex post treatment after transportation cost increase. ΔD_1 is the difference between product differentiation decisions under VQ1 and VLQ1 while ΔD_2 is the difference between product differentiation decisions under VQ2 and VLQ2.[12] $\Delta D^{*} = 0.24$ is the predicted equilibrium value. DUM_{1t} is a dummy variable that equals 1 if $t = \{9,...,13\}$ and 0 otherwise; and DUM_{2t} is a dummy variable that equals 1 if $t = \{14,...16\}$ and 0 otherwise. If the change in product differentiation

[12] Note that product differentiation under the same transportation cost structure remains unchanged under a uniform 100% increase in transportation cost parameters.

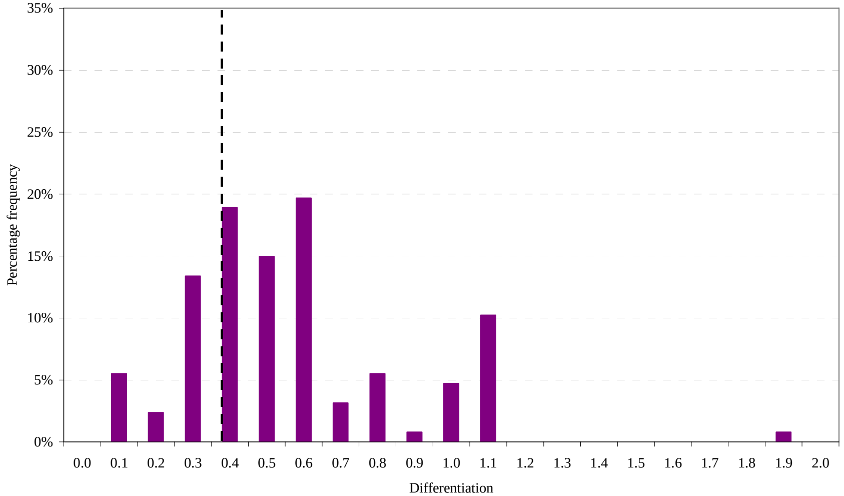

Fig. 4.26(a) Distribution of individual product differentiation decisions (VQ1)

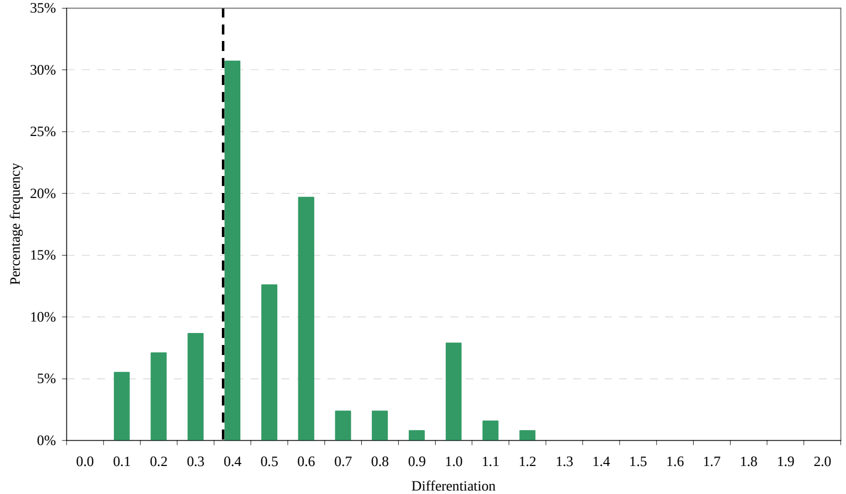

Fig. 4.26(b) Distribution of individual product differentiation decisions (VQ2)

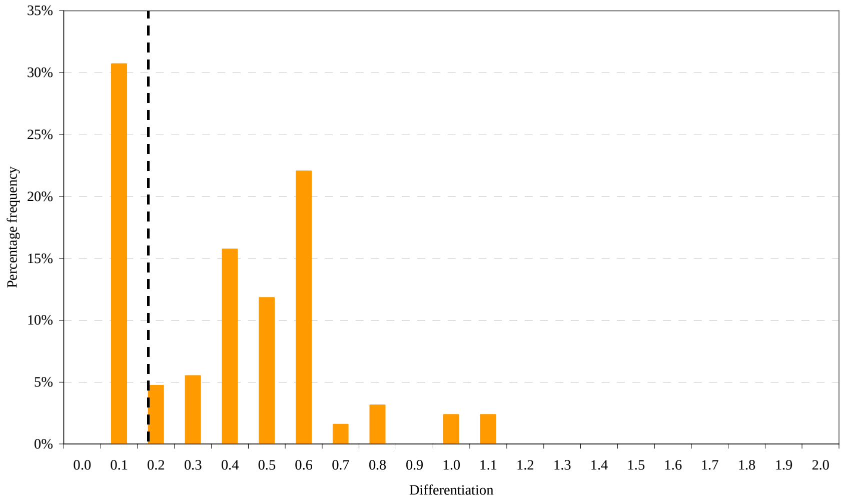

Fig. 4.26(c) Distribution of individual product differentiation decisions (VLQ1)

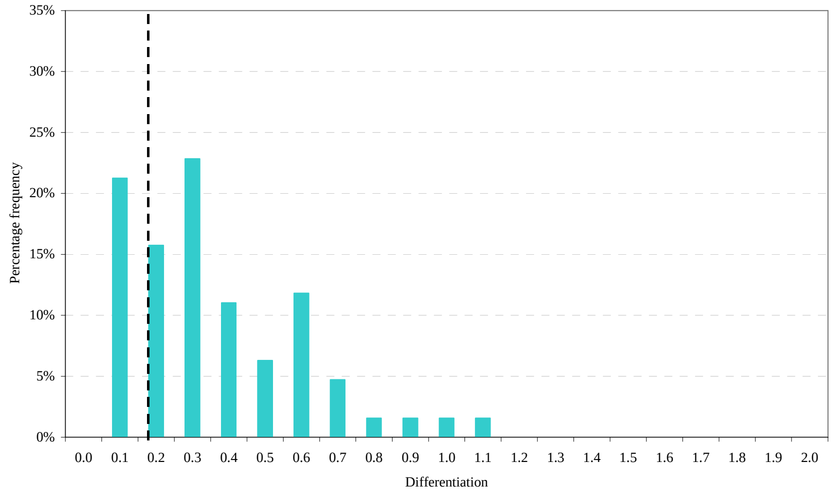

Fig. 4.26(d) Distribution of individual product differentiation decisions (VLQ2)

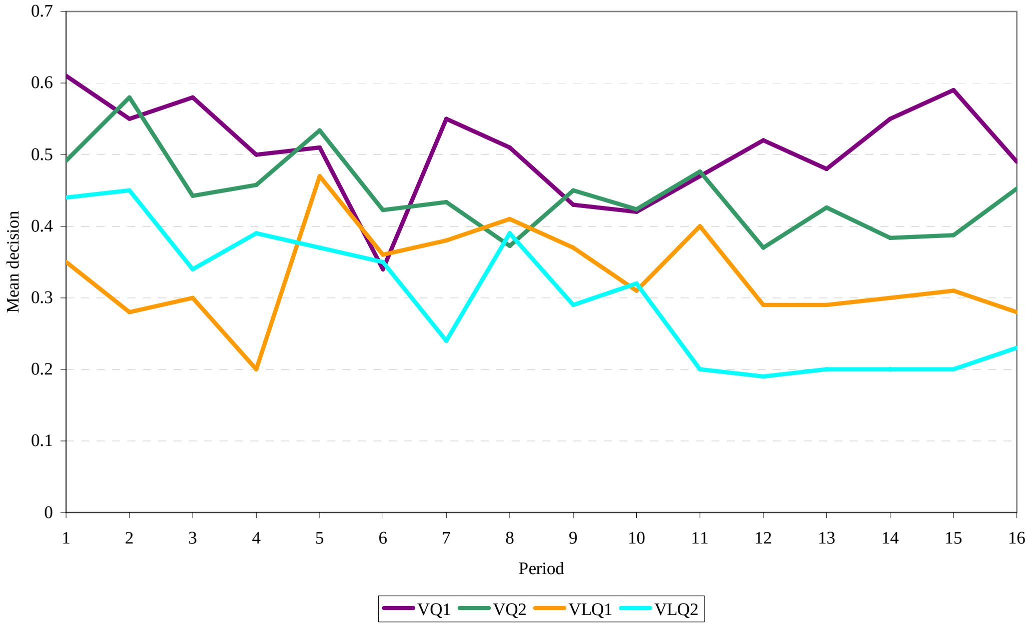

Fig. 4.27 Time series of mean product differentiation decisions

under higher transportation costs grows closer to prediction in the late1 phase and late2 phase, then β_1 and β_2 respectively would be negative.

The regressions results are reported in Table 4.15. For $\left|\Delta D_1 - \Delta D^*\right|$, $\alpha = 0.0879$ is close to zero and significant at the 0.05 level. For $\left|\Delta D_2 - \Delta D^*\right|$, α is not significantly different from zero at the 0.05 level. In other words, ΔD_1 and ΔD_2 are very close to the predicted value of $\Delta D^* = 0.24$. This provides strong evidence of a decrease in the level of product differentiation when transportation costs grow at a more rapid rate under a higher transportation cost structure.

With regards to increasing proximity of ΔD_1 and ΔD_2 to prediction over the course of the experiment, only β_2 is negative and significantly different from zero at the 0.05 level for ΔD_2. There is, therefore, growing parity of ΔD_2 to prediction in the late2 phase but there is no evidence that ΔD_1 grows closer to prediction over time.

Table 4.15
Regression results for product differentiation under higher transportation costs

Variable	Coefficient	S.E.	t-statistic
Model: $\left\|\Delta D_{jt} - \Delta D^*\right\| = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_{jt}$			
$\left\|\Delta D_{1t} - \Delta D^*\right\|$			
constant	0.0879	0.0391	2.2506 *
DUM_{1t}	-0.0146	0.0581	-0.2506
DUM_{2t}	-0.0026	0.0654	-0.0396
AR(1)	0.3044	0.0874	3.4844 *
N = 123; Adj R^2 = 0.0692; F = 4.0253 (p = 0.0091); SSE = 5.9643; DW = 2.1215; LM = 0.0879; White = 0.2723; Jarque-Bera = 0.0585; Chow = 0.0589			
$\left\|\Delta D_{2t} - \Delta D^*\right\|$			
constant	0.0081	0.0441	0.1830
DUM_{1t}	-0.0366	0.0547	-0.6699
DUM_{2t}	-0.1421	0.0606	-2.3464 *
AR(1)	0.5201	0.0986	5.2765 *
AR(2)	0.3610	0.0990	3.6444 *
AR(6)	-0.1766	0.0760	-2.3221 *
AR(29)	-0.2306	0.0902	-2.5562 *
N = 93; Adj R^2 = 0.5582; F = 20.3756 (p = 0.0000); SSE = 2.6467; DW = 1.8509; LM = 0.4423; White = 0.4835; Jarque-Bera = 0.0163; Chow = 0.2817			

See Table 4.7 for notes.

H5: Higher product differentiation yields higher prices.

It has been highlighted at the start of this chapter that location literature typically proposes that firms should maximise their level of product differentiation to reduce price competition. In other words, higher product differentiation would lead firms away from fierce price competition and the resulting lower prices. To test the positive correlation between the level of product differentiation and price, the following equation is estimated for the late phase of the four treatments: [13]

(4.10) $$p_{it} = \alpha + \beta D_t + \varepsilon_{it}$$

where p_i is the price of firm $i \in \{1,2\}$ and D_t is the level of product differentiation.

[13] One-period lag and two-period lags of D_t in equation 4.10 are all insignificant at the 0.05 level.

The regression results in Table 4.16 show clearly that β is positive and significant at the 0.05 level for the inside firm in all treatments except VQ1. For the outside firm, the β's are not significantly different from zero at the 0.05 level in all treatments. The poor fit of equation 4.10 for the outside firm is also evident from the negative adjusted R^2. These findings are in accord with the results from a Spearman rank-order correlation test and a Kendall rank-order correlation test (Table 4.17). While higher product differentiation clearly results in reduced price competition (and hence higher prices) for the inside firm, the impact on the outside firm is not obvious.

H6: Relative demand is equivalent to relative price (Proposition 2.2).

Is relative demand equivalent to relative price under a given transportation cost structure, as suggested by Proposition 2.2 of the theoretical model? The time series of mean relative demand (m_2/m_1) and mean relative price (p_2/p_1) in Figure 4.28 shows that the two series track each other very closely and approach the predicted value (represented by the broken horizontal line) for all treatments. In VQ1, mean relative demand appears to track mean relative price less closely than the other treatments. This observation is borne out by a Wilcoxon signed rank test and a Sign test on relative demand and relative price based on individual player data in all treatments. The results given in Table 4.18 indicate that the two series are equivalent at the 0.05 level throughout the experiment, with VQ1 having the lowest probability of acceptance in the late2 phase among all treatments.

Table 4.16
Regression results for relationship between product differentiation and price

Variable	Coefficient	S.E.	t-statistic
Model: $p_{it} = \alpha + \beta D_{it} + \varepsilon_{it}$			
Treatment: VQ1			
p_{1t}			
constant	2.5639	0.4268	6.0078 *
D_t	0.4311	0.4764	0.9049
AR(1)	0.7079	0.0927	7.6362 *
N = 63; Adj R^2 = 0.4821; F = 29.8544 (p = 0.0000); SSE = 40.6792; DW = 2.0335; LM =0.8073; White = 0.7698; Jarque-Bera = 0.0249; Chow = 0.0001			
p_{2t}			
constant	2.5480	0.4789	5.3210 *
D_t	0.0385	0.8436	0.0456
N = 64; Adj R^2 =-0.0161; F = 0.0021 (p = 0.9638); SSE = 220.6081; DW = 2.2655; LM =0.2575; White = 0.3167; Jarque-Bera = 0.0000; Chow = 0.4362			
Treatment: VQ2			
p_{1t}			
constant	5.5965	0.4525	12.3673 *
D_t	2.3202	0.9671	2.3992 *
AR(3)	-0.2621	0.1385	-1.8925
AR(17)	-0.3055	0.1509	-2.0250 *
N = 47; Adj R^2 =0.1979; F = 4.7830 (p = 0.0058); SSE = 154.9954; DW = 1.5156; LM =0.6461; White = 0.6667; Jarque-Bera = 0.9725; Chow = 0.2958			
p_{2t}			
constant	4.8566	0.5327	9.1164 *
D_t	-1.0181	0.8312	-1.2249
AR(14)	0.3944	0.1332	2.9615 *
N = 50; Adj R^2 =0.1361; F = 4.8603 (p = 0.01206); SSE = 123.9945; DW = 1.8286; LM =0.5193; White = 0.6540; Jarque-Bera = 0.9165; Chow = 0.9859			
Treatment: VLQ1			
p_{1t}			
constant	2.1910	0.4657	4.7051 *
D_t	6.5206	0.9618	6.7793 *
AR(1)	0.4972	0.1269	3.9172 *
AR(16)	-0.2706	0.1163	-2.3259 *
N = 48; Adj R^2 =0.6227; F = 25.2101 (p = 0.0000); SSE = 90.5326; DW = 2.1434; LM =0.3239; White = 0.9336; Jarque-Bera = 0.6275; Chow = 0.2206			
p_{2t}			
constant	2.4743	0.3335	7.4194 *
D_t	0.1539	0.8384	0.1835
N = 64; Adj R^2 =-0.0158; F = 0.0337 (p = 0.8550); SSE = 156.5205; DW = 2.2340; LM =0.28194; White = 0.5060; Jarque-Bera = 0.2101; Chow = 0.1177			
Treatment: VLQ2			
p_{1t}			
constant	8.7601	0.7248	12.0870 *
D_t	5.1076	1.9416	2.6306 *
AR(1)	0.4789	0.1147	4.1757 *
N = 63; Adj R^2 =0.3303; F = 15.7933 (p = 0.0000); SSE = 314.3429; DW = 2.1523; LM =0.2319; White = 0.0157; Jarque-Bera = 0.0004; Chow = 0.0007			
p_{2t}			
constant	6.6827	0.4477	14.9265 *
D_t	-0.7550	1.4688	-0.5140
N = 64; Adj R^2 =-0.0120; F = 0.2642 (p = 0.6091); SSE = 322.6108; DW = 1.9379; LM =0.8546; White = 0.1576; Jarque-Bera = 0.64015; Chow = 0.2293			

See Table 4.7 for notes. Chow's breakpoint test is at n = 32. Failure of Jarque-Bera test due to 1-3 outlying values. Failure of Chow's test in VLQ2 due to an outlying residual value near breakpoint at n = 31.

Table 4.17
Product differentiation and prices
(one-tailed Spearman and Kendall rank-order correlation tests)

Variables	N	Spearman correlation	p_{SP} (1-tailed)		Kendall correlation	p_K (1-tailed)	
Treatment: VQ1							
p_1, D	64	0.1633	0.0987		0.1377	0.0606	
p_2, D	64	-0.0419	0.3712		-0.0307	0.3653	
Treatment: VQ2							
p_1, D	64	0.3714	0.0013	*	0.2582	0.0017	*
p_2, D	64	-0.1354	0.1430		-0.1014	0.1238	
Treatment: VLQ1							
p_1, D	63	0.4959	0.0000	*	0.3610	0.0000	*
p_2, D	63	-0.0117	0.4637		-0.0148	0.4357	
Treatment: VLQ2							
p_1, D	63	0.3693	0.0014	*	0.2870	0.0007	*
p_2, D	63	-0.0890	0.2439		-0.0599	0.2519	

* indicates significance at the 0.05 level.

Table 4.18
Relative demand and relative price
(probabilities for two-tailed Wilcoxon signed ranks test p_W and Sign test p_S)

Treatment	N	pw		ps	
Null hypothesis: $p_2/p_1 = m_2/m_1$					
Treatment: VQ1					
All periods	117	0.0953	*	0.0777	*
Early phase	55	0.1856	*	0.1775	*
Late1 phase	38	0.7115	*	0.8711	*
Late2 phase	24	0.1909	*	0.2100	*
Treatment: VQ2					
All periods	116	0.9961	*	0.6239	*
Early phase	55	0.3303	*	0.1447	*
Late1 phase	37	0.5784	*	0.6069	*
Late2 phase	23	0.6148	*	1.0000	*
Treatment: VLQ1					
All periods	120	0.5063	*	0.4074	*
Early phase	58	0.2055	*	0.3580	*
Late1 phase	39	0.8327	*	0.7423	*
Late2 phase	23	1.0000	*	1.0000	*
Treatment: VLQ2					
All periods	126	0.2203	*	0.1268	*
Early phase	62	0.3475	*	0.2530	*
Late1 phase	40	0.2617	*	0.6353	*
Late2 phase	24	0.8967	*	0.5235	*

* indicates significance at the 0.05 level

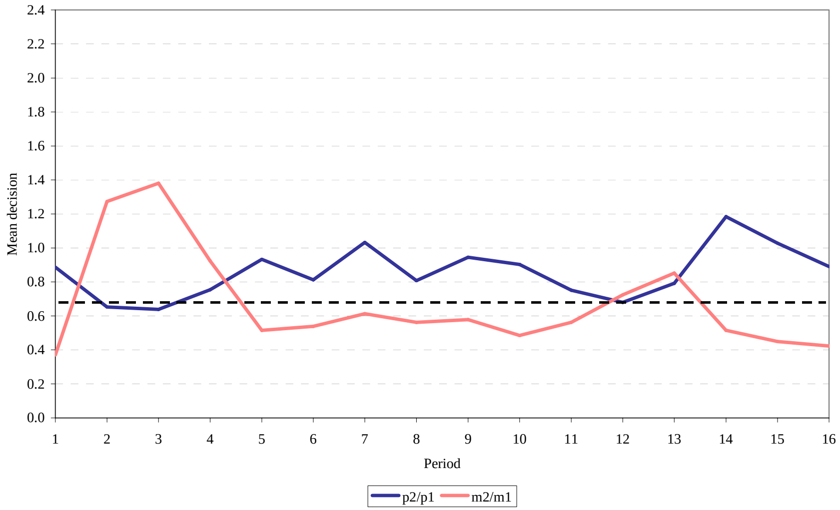

Fig. 4.28(a) Time series of mean relative demand and mean relative price (VQ1)

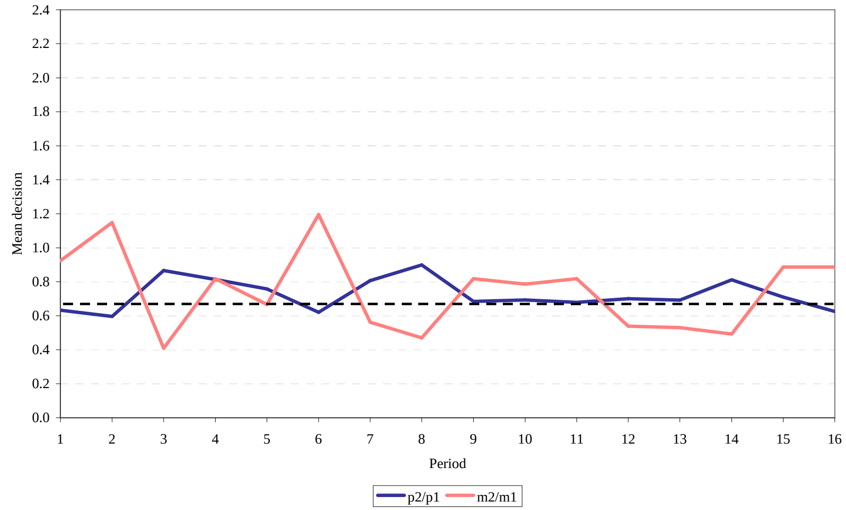

Fig. 4.28(b) Time series of mean relative demand and mean relative price (VQ2)

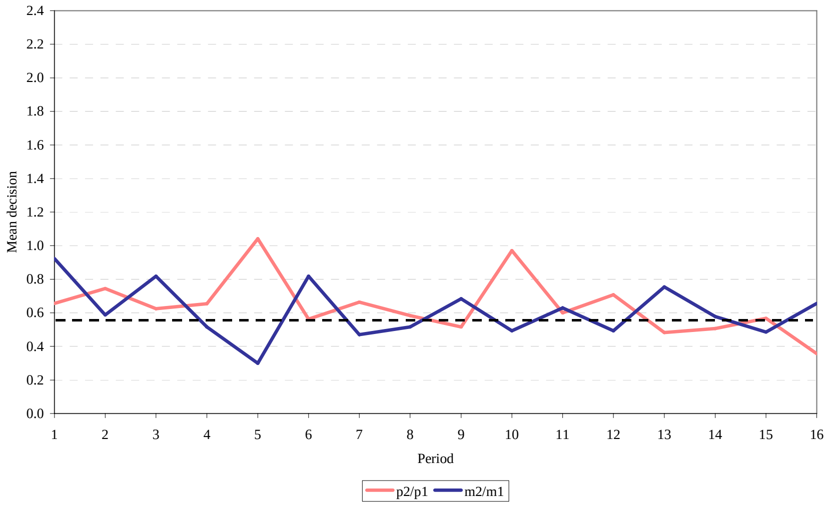

Fig.4.28(c) Time series of mean relative demand and mean relative price (VLQ1)

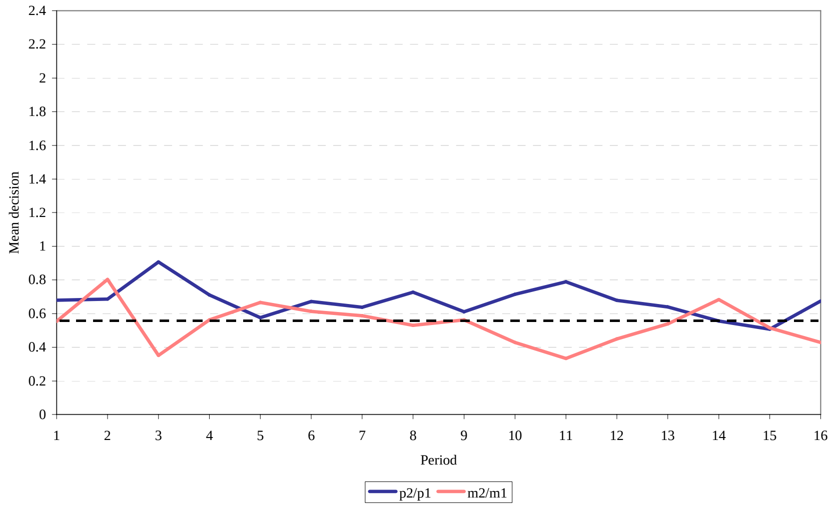

Fig. 4.28(d) Time series of mean relative demand and mean relative price (VLQ2)

To examine the relationship between relative demand and relative price, the following equation is estimated:

$$(4.11) \qquad \left|\frac{m_{jt}}{m_{it}} - \frac{p_{jt}}{p_{it}}\right| = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_{ijt}$$

where p_{it} and p_{jt} denote the price of firm 1 and firm 2 respectively in period *t*; and m_{it} and m_{jt} denote the demand of firm 1 and firm 2 respectively in period *t*. DUM_{1t} is a dummy variable that equals 1 if $t = \{9,...,13\}$ and 0 otherwise; and DUM_{2t} is a dummy variable that equals 1 if $t = \{14,...16\}$ and 0 otherwise. If relative demand grows closer to relative price in the late1 phase and late2 phase, then β_1 and β_2 respectively would be negative.

The regression results are shown in Table 4.19. All diagnostic and stability tests indicate that the estimates are acceptable (except the Jarque-Bera test which fails because of one to three outlying residual values).[14] The results do not support the hypothesis that relative demand equals relative price. Since α is significantly different from zero in all treatments at the 0.05 level, relative demand falls short of or surpasses relative price. In VLQ1 and VLQ2, the β's are insignificant at the 0.05 level. In VQ2, proximity between relative demand and relative price grows in the late1 phase (β_1 is negative and significant at the 0.05 level) while in VQ1, relative demand grows closer to relative price in the late2 phase (β_2 is negative and significant at the 0.10 level). It appears that decreased price convergence (VLQ1, VLQ2) and the lack of location convergence (inside firm of VQ2) to prediction during the late2 phase has manifested itself in the inability of relative price to match relative demand (see Tables 4.5 and 4.9). This explains the absence of a significant β_2 in the late2 phase of treatments other than VQ1.

[14] All relative price and relative demand data are stationary in the level at all reported significance levels based on an augmented Dickey-Fuller test.

Table 4.19
Regression results for relative price and relative demand

Variable	Coefficient	S.E.	t-statistic
Model: $\left\lvert \frac{m_{jt}}{m_{it}} - \frac{p_{jt}}{p_{it}} \right\rvert = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_{ijt}$			
Treatment: VQ1			
$\left\lvert \frac{m_{jt}}{m_{it}} - \frac{p_{jt}}{p_{it}} \right\rvert$			
constant	1.3542	0.1572	8.6131 *
DUM_{1t}	0.1374	0.2460	0.5586
DUM_{2t}	-0.4916	0.2895	-1.6978 +
N = 116; Adj R^2 = 0.0203; F = 2.1904 (p = 0.1166); SSE = 153.6303; DW = 1.7882; LM = 0.4714; White = 0.6687; Jarque-Bera = 0.0000; Chow = 0.0615			
Treatment: VQ2			
$\left\lvert \frac{m_{jt}}{m_{it}} - \frac{p_{jt}}{p_{it}} \right\rvert$			
constant	2.3119	0.5003	4.6211 *
DUM_{1t}	-0.9216	0.4515	-2.0412 *
DUM_{2t}	-0.5643	0.5393	-1.0465
AR(24)	0.3988	0.1257	3.1735 *
N = 83; Adj R^2 = 0.0902; F = 3.7108 (p = 0.0149); SSE = 422.0608; DW = 1.8189; LM = 0.8149; White = 0.1218; Jarque-Bera = 0.0000; Chow = 0.3310			
Treatment: VLQ1			
$\left\lvert \frac{m_{jt}}{m_{it}} - \frac{p_{jt}}{p_{it}} \right\rvert$			
constant	1.7279	0.4485	3.8529 *
DUM_{1t}	0.0816	0.5752	0.1419
DUM_{2t}	0.1213	0.6828	0.1776
AR(28)	0.2519	0.1154	2.1824 *
N = 88; Adj R^2 = 0.0229; F = 1.6809 (p = 0.1773); SSE = 516.1871; DW = 1.9597; LM = 0.7943; White = 0.8545; Jarque-Bera = 0.0000; Chow = 0.3906			
Treatment: VLQ2			
$\left\lvert \frac{m_{jt}}{m_{it}} - \frac{p_{jt}}{p_{it}} \right\rvert$			
constant	0.9606	0.1733	5.5422 *
DUM_{1t}	-0.1111	0.2768	-0.4015
DUM_{2t}	0.0469	0.3281	0.1428
N = 126; Adj R^2 = -0.0142; F = 0.1233 (p = 0.8841); SSE = 229.1188; DW = 1.9171; LM =0.5466; White = 0.1497; Jarque-Bera = 0.0000; Chow = 0.1348			

See Table 4.7 for notes. Failure of Jarque-Bera test due to 1-3 outlying values.

H7: The price increase is greater for the inside firm than the outside firm when transportation costs increase under a given transportation cost structure (Proposition 3).

In treatments VQ2 and VLQ2, the transportation cost parameters are increased by 100% over those in VQ1 and VLQ1 respectively. The impact of a transportation cost increase under a given transportation cost structure on the prices of both firms can be observed by comparing their prices in VQ1 with VQ2, and VLQ1 with VLQ2. Figure 4.29 shows that the mean price difference of the inside firm under a 100% increase in transportation cost parameters is larger than the mean price difference for the outside firm (except for two periods).

To validate the above observation, the following equation is estimated using ordinary least squares:

(4.12) $$(p_{12t} - p_{11t}) - (p_{22t} - p_{21t}) = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_t$$

where p_{ij} is the price of firm *i* for treatment of type *j* where $i, j = \{1,2\}$. Type $j = 1$ denotes the ex ante treatment before a transportation cost increase while $j = 2$ denotes the ex post treatment after a transportation cost increase. DUM_{1t} is a dummy variable that equals 1 if $t = \{9,...,13\}$ and 0 otherwise, and DUM_{2t} is a dummy variable that equals 1 if $t = \{14,...16\}$ and 0 otherwise.

The regression results are presented in Table 4.20. All diagnostic and stability checks indicate that the estimates are reliable at the 0.05 level (except the Jarque-Bera test which fails due to the presence of two outlying residual values in each instance). In all treatments, α is positive and significantly different from zero at the 0.05 level. β_1 and β_2 are negative and significantly different from zero at the 0.05 level for both transportation cost structures (β_2 is negative and significant at the 0.10 level under linear-quadratic transportation costs). This implies that, throughout the experiment, the inside firm invariably raises its price by a larger amount than the outside firm when faced with higher transportation costs (which are borne by

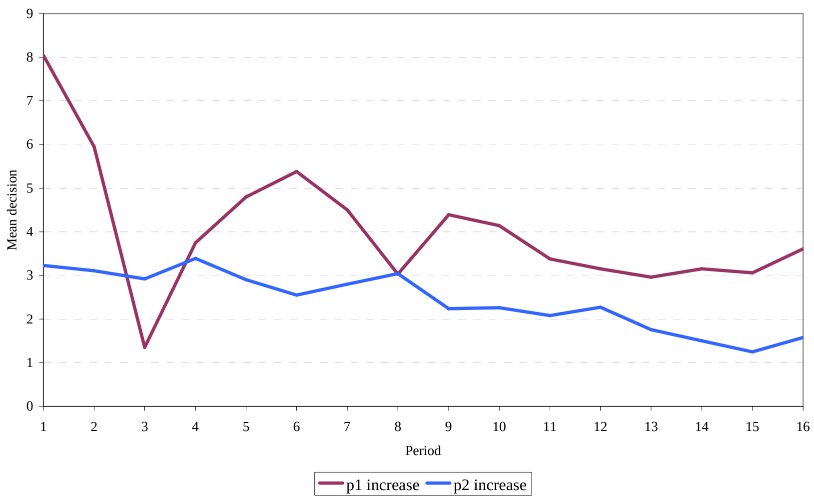

Fig. 4.29 (a) Time series of mean price difference under higher quadratic transportation costs

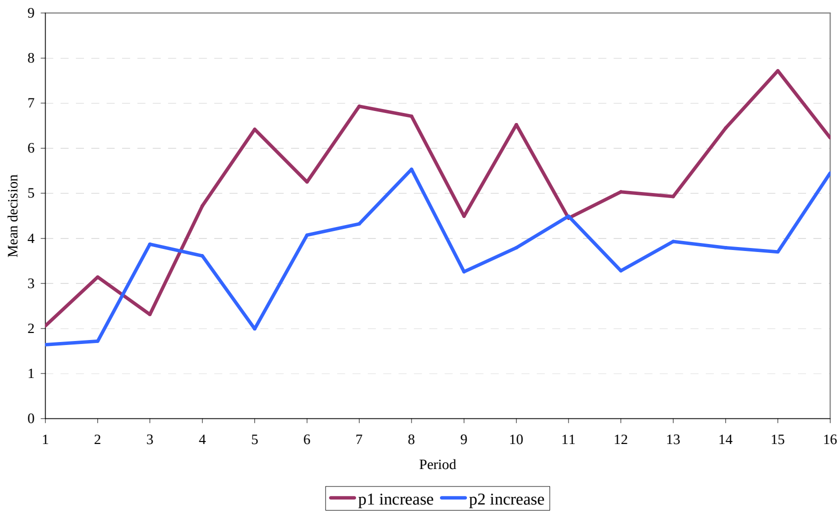

Fig. 4.29 (b) Time series of mean price difference under higher linear-quadratic transportation costs

Table 4.20
Regression results for impact of transportation cost increase on prices

Variable	Coefficient	S.E.	t-statistic	
Model:	$(p_{12t} - p_{11t}) - (p_{22t} - p_{21t}) = \alpha + \beta_1 DUM_{1t} + \beta_2 DUM_{2t} + \varepsilon_t$			
Treatments: VQ1, VQ2				
$(p_{12t}-p_{11t})-(p_{22t}-p_{21t})$				
constant	5.6896	1.7060	3.3349	*
DUM_{1t}	-1.8911	0.6032	-3.1350	*
DUM_{2t}	-2.2104	0.7388	-2.9920	*
AR(4)	0.1881	0.0999	1.8826	+
AR(5)	0.1853	0.0995	1.8615	+
AR(6)	0.2028	0.1016	1.9962	*
AR(26)	0.1924	0.1072	1.7953	+
N = 96; Adj R^2 = 0.1477; F = 3.7439 (p = 0.0023); SSE = 1051.444; DW = 1.8372; LM = 0.8880; White = 0.1507; Jarque-Bera = 0.0000; Chow = 0.5827				
Treatments: VLQ1, VLQ2				
$(p_{12t}-p_{11t})-(p_{22t}-p_{21t})$				
constant	4.4534	0.8482	5.2501	*
DUM_{1t}	-3.2229	0.7813	-4.1250	*
DUM_{2t}	-1.5364	0.9090	-1.6902	+
AR(3)	-0.2077	0.0995	-2.0885	*
AR(8)	0.2496	0.0868	2.8751	*
AR(15)	0.2370	0.0856	2.7695	*
AR(34)	0.2167	0.0850	2.5495	*
N = 89; Adj R^2 = 0.2694; F = 6.4095 (p = 0.0044); SSE = 734.6055; DW = 2.3510; LM = 0.7666; White = 0.0122; Jarque-Bera = 0.0020; Chow = 0.8816				

See Table 4.7 for notes. Failure of Jarque-Bera test due to 2 outlying values.

consumers). There is also evidence that the increase in price by the inside firm declines over the late1 phase and late2 phase given the negative β's.

4.5 CONCLUSIONS

Under variable firm location, the experimental results show that an increase in the level of product differentiation results in reduced price competition and higher prices for the inside firm, but the effect on the firm located outside the market space is not obvious. Low price behaviour heightens price competition, regardless of the level of product differentiation, to the extent that prices are occasionally driven to very low levels.

Under higher transportation costs, the level of product differentiation decreases. At first sight, the transitive effect of lower product differentiation (due to higher transportation costs) in reducing prices (due to intensified price competition) does not appear to hold. Both firms gain higher prices for the good, with the inside firm earning higher prices *vis-à-vis* the outside firm. This is in accord with Proposition 3. In other words, consumers mitigate seller power in raising prices by incurring positive transportation costs. A similar finding was made by Camacho-Cuena *et al*. (2004).

A closer look reveals that lower product differentiation arising from higher transportation costs does have a price reduction effect through heightened price competition. This is obvious when one contrasts the situation in which firms are unable to relocate themselves in the face of higher transportation costs, i.e., the level of product differentiation remains invariant at a higher level ($D = 1.00$ of the experiment with constant location vs. $D = 0.16$ and $D = 0.40$ for linear-quadratic and quadratic transportation costs respectively of the experiment with variable location). Under this situation, price competition is smaller as both firms are able to reap higher prices and profits (compare the mean prices of the results in Tables 3.3 and 4.3). In other words, when consumers have no choice on the level of product differentiation, they incur even higher transportation costs in their attempts to reduce seller power.

The results in this chapter accord support for the IO model's equilibrium predictions and propositions. No evidence, however, is obtained for Proposition 2.2 and Fetter's law of market areas, i.e., relative demand is equivalent to relative price under a given transportation cost structure as a result of a decline in price and location convergence to prediction in the final three periods of the experiment.

CHAPTER 5

CONCLUSIONS

5.1 THEORY: SUMMARY AND IMPLICATIONS

The mainstream of economic theories explain spatial firm competition in terms of horizontal or vertical product differentiation (alternatively address or non-address goods). Following the pioneering steps of Launhardt, this study introduces a model that possesses both horizontal and vertical product characteristics within a single framework. The IO model presented in Chapter 2 integrates Hotelling (1929)'s inside location model and Gabszewicz and Thisse (1986, 1992)'s outside location model, thereby inheriting the properties of both models. At the same time, the IO model has properties that are unique to itself.

An interesting result arising under fixed firm location is that the market boundary is determined solely by the relative price of the two firms. This property satisfies Fetter (1924)'s economic law of market areas, i.e., the market boundary is determined by the relative price and relative transportation costs. The latter is assumed to be constant ($t^{'}/t = 1$, $s^{'}/s = 1$) for the IO model. Another result is that, regardless of whether firm location is fixed or variable, relative price is equivalent to relative demand at equilibrium.

Moreover, when firm location is variable, no equilibrium in pure strategies exists under linear transportation costs but a unique Nash equilibrium in pure strategies exists under quadratic and linear-quadratic transportation costs. This is true whether the location-price game is played simultaneously or sequentially. In contrast, for the inside location model, an equilibrium in pure strategies exists only when the sequential location-then-price game is played under quadratic transportation costs. In the case of the outside location model, an equilibrium in pure strategies always exists. The stability of the IO model is, therefore, intermediate between the inside location model and the outside location model. This is not surprising since the IO model is an integration of the two models.

The IO model is directly applicable to situations in which adjoining markets are segmented geographically and (or) economically. It highlights the distinction between an economic boundary and geographical boundary between two regions, which in most cases do not necessarily coincide. The model is a useful framework to analyse firm competition on a broader dimension than is permissible under either the inside location model or the outside location model. It explains why many firms compete in both horizontal and vertical product differentiation characteristics rather than along a single spectrum. This scenario is reflective of firms that compete in both price and quality of a product, e.g., the Filière Qualité brand of Carrefour, Charles Shaw wines, etc.

5.2 EXPERIMENTS: SUMMARY AND IMPLICATIONS

The experimental results accord fairly strong support for the predictions of the IO model. Prices and locations under various transportation cost structures generally approached Nash prediction. There are, however, instances in which players' behaviour (e.g., their preference for low price strategy rather than best response) resulted in a failure to reach prediction. This is especially true of the experiment in Chapter 3 that assumes fixed firm location.

Two contrasting forms of behaviour emerge under constant location (parametric firm location game in Chapter 3) and variable location (simultaneous location-price game in Chapter 4). While equilibrium prices under constant location generally fall below prediction, the opposite is true under variable location. Under constant location, the inside firm players exhibited a strong inclination to price close to levels that monopolise the market. By adopting low price behaviour, the inside firm players attempt to price their rivals out of the market and capture full market demand. Under variable location when the firms are no longer restricted by competition along a single dimension (i.e., price), the inside firm shows a smaller inclination (or ability) to monopolise the market through low prices. While there exist a number of inside firm players who adopted low price behaviour under variable location, these are relatively few compared to the large majority engaged in low price behaviour under constant location.

For both experiments, players adhere to best responses although not absolutely. Regression of prices with one-period lag best responses provide evidence that the inside firm generally gets better at playing best strategies over time but the same cannot always be said of the outside firm. The greater proximity of prices to best strategies under variable location (as shown by the generally lower α values) result in prices reaching an equilibrium that is closer to prediction than was obtained under constant location.

As a result of the disparity of equilibrium prices from prediction, there is an inability of the findings to provide full support to the theoretical proposition that relative price and relative demand are equal at equilibrium (Proposition 2.2). Unequivocal support, however, is accorded to the other propositions of the IO model: equilibrium relative price (Proposition 1) and equilibrium relative demand (Proposition 2.1) are the same regardless of transportation cost structure, and the inside firm raises its price by a greater amount than the outside firm under higher transportation costs (Proposition 3).

The results show that when transportation costs increase as consumers are faced with a higher transportation cost structure (e.g., from quadratic transportation costs to linear-quadratic transportation costs), the level of product differentiation decreases. The price increase under variable location, however, is smaller than under constant location with relatively higher product differentiation. A reduction in product differentiation under higher transportation costs, therefore, results in more intensive price competition in an environment faced with variable location than when location is constant.

5.3 CONCLUDING REMARKS

The experimental results indicate that the IO model has some predictive power for the analysis of spatial competition between two firms located along two contiguous line segments separated by a market border. In accordance with theory, players in the simultaneous price-location experiment adopted higher product differentiation under transportation costs that increase less rapidly (quadratic costs) than more rapidly (linear-quadratic costs). Players' behaviour, however, do not correspond exactly to Nash prediction under constant location. At the start of the experiment, players tend to be over-competitive and price below prediction, with the majority of inside firm players competing to acquire a monopoly. Mutual best responses gradually managed to bring prices closer to theoretical prediction, enabling both firms to capture higher producer surplus.

When firms experience higher transportation costs, the full price of their product naturally increase since it is the sum of the mill price of the product and transportation costs. Since the mill price of the product is assumed to be the same for both firms and is constant throughout the experiment, the full price of the product is directly dependent on the nature of transportation cost increase. Consequently, when transportation costs increase more rapidly (e.g., under linear-quadratic transportation costs), firms are able to increase their price by a greater extent.

The amount of price increase also depends on the ability of firms to vary location (or brand specification). Under constant location, firms are able to increase prices by a larger amount than when location is variable. This is not surprising since a product that enjoys a short period before a new design enters the market will not be able to command as large a price increase as one that has a longer shelf life. This provides some anecdotal support for products that typically offer a new brand in a short interval of time to be launched at a much higher price level than similar products that adhere to a single brand specification.

Under horizontal differentiation, firms offer identical products and compete in price. Well-informed consumers will choose the firm that has the lower price, if prices differ. Under vertical differentiation, products differ in quality. Consumers pay more for products higher up along the quality spectrum. The IO model explains firm competition along both horizontal and vertical characteristics. Firms choose the optimal mix of horizontal and vertical differentiation for a product. This is more reflective of many real world situations.

The IO model presents itself as a useful framework to analyse consumer travel and firm competition on either side of a market boundary. It can be adapted to the study of other forms of travel decisions between two entities in adjoining markets, e.g., worker migration between two cities, or leisure choice between neighbouring destinations.

The applicability of the IO model to the study of actual firm and consumer behaviour, however, is rather limited given its present framework and its restrictive assumptions. Further modifications of the model, therefore, may be useful in enhancing the applicability of the IO model to the study of real world situations. By modifying the basic assumptions inherent in the model, a myriad of other situations present themselves that are worthy of further study, both in theoretical work and in experimental laboratories. For example, future studies may examine the effect of non-uniform consumer distributions, concave transportation costs, incorporation of a budget constraint (both temporal and monetary), and the effect of interactive communication among firms. What happens if consumers are able to select their own location besides deciding which firm to buy the good from, or if they can withhold consumption of the good? Will the results differ markedly if production costs are variable rather than constant in either or both market? What if there are three or more firms, or when there is sequential firm entry by foreign firms into the domestic market? Can firms choose transportation rates that will deter the entry of new firms? These questions remain open to further examination. Any research in these areas would certainly help to bring new insights into firm competition with horizontal and vertical differentiation.

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APPENDICES

APPENDIX 1

PARAMETRIC LOCATIONS WITH LINEAR TRANSPORTATION COSTS

Assume that the transportation cost function is linear of the form $cd = td$, where $c(0) = 0$ and $t > 0$. If $m(p_1, p_2)$ exists, it must be the solution of the equation

$$p_1 + t(y - x_1) = p_2 + t(x_2 - y).$$

Solving, we obtain the demand functions for firm 1 and firm 2, respectively, as

(A1) $$m_1(p_1, p_2) = \frac{p_2 - p_1}{2t} + \frac{(x_1 + x_2)}{2}$$ and

(A2) $$m_2(p_1, p_2) = \frac{p_1 - p_2}{2t} + \frac{(2 - x_1 - x_2)}{2}.$$

The payoff functions are given by $\Pi_1(p_1, p_2) = p_1 \cdot m_1(p_1, p_2)$ and $\Pi_2(p_1, p_2) = p_2 \cdot m_2(p_1, p_2)$ respectively. Maximising profits on the part of firm 1 and firm 2 gives the following response functions:

(A3) $$p_1^* = \frac{1}{2}\left[p_2^* + t(x_1 + x_2)\right]$$ and

(A4) $$p_2^* = \frac{1}{2}\left[p_1^* + t(2 - x_1 - x_2)\right].$$

Solving equations A3 and A4 gives the non-cooperative Bertrand-Nash price equilibrium in pure strategies, i.e.,

(A5) $$\left(p_1^*, p_2^*\right) = \left(\frac{t}{3}(x_1 + x_2 + 2), \frac{t}{3}(4 - x_1 - x_2)\right).$$

For non-zero p_2^*, we assume that $x_1 + x_2 < 4$. If $x_1 + x_2 = 4$, an equilibrium occurs at $\left(p_1^*, p_2^*\right) = \left(t(x_1 + x_2)/2, 0\right)$.

The distribution of market demand between firm 1 and firm 2 at Nash equilibrium is given by substituting A5 into A1 and A2:

(A6) $$\left(m_1^*, m_2^*\right)=\left(\frac{1}{6}\left(x_1+x_2+2\right), \frac{1}{6}\left(4-x_1-x_2\right)\right).$$

Equilibrium Existence under Linear Transportation Costs

The following result states the necessary and sufficient conditions for the existence of an equilibrium under linear transportation costs.

Lemma 1

For $x_1+x_2<4$, there is an equilibrium in prices if and only if

1. $$\left(\frac{x_1+x_2+2}{3}\right)^2 \geq \frac{4}{3}\left(2+x_1-2x_2\right)$$

2. $$\left(\frac{4-x_1-x_2}{3}\right)^2 \geq \frac{4}{3}\left(1+2x_1-x_2\right)$$

and whenever it exists, the equilibrium is uniquely determined by

3. $$p_1^*=t\left(\frac{x_1+x_2+2}{3}\right)$$

4. $$p_2^*=t\left(\frac{4-x_1-x_2}{3}\right)$$

For $x_1+x_2=4$, $p_1^*=\dfrac{t\left(x_1+x_2\right)}{2}$ and $p_2^*=0$.

Proof

Let $x_1+x_2 \leq 4$. We will show that any equilibrium $\left(p_1^*, p_2^*\right)$ must satisfy the condition $\left|p_1^*-p_2^*\right|<t\left(x_2-x_1\right)$.

Suppose on the contrary $\left(p_1^*, p_2^*\right)$ is an equilibrium but $\left|p_1^*-p_2^*\right|>t\left(x_2-x_1\right)$. Then the firm that charges the higher (strictly positive) price and with zero profit can gain by charging a price equal to the price of the other firm, which contradicts the assumption that $\left(p_1^*, p_2^*\right)$ is an equilibrium.

Suppose that $\left|p_1^* - p_2^*\right| = t\left(x_2 - x_1\right)$. If $p_1^* = 0$ and firm 1 earns zero profit, then firm 1 can gain by charging a positive price below $p_2^* + t\left(x_2 - x_1\right)$. If $p_1^* > 0$, then either (1) firm 1 captures the whole market and firm 2, which charges a positive price, can increase profit by decreasing price, or (2) firm 1 has a fraction of the market, in which case it can capture the whole market and increase profit by decreasing its price. In both instances, the assumption that $\left(p_1^*, p_2^*\right)$ is an equilibrium is contradicted.

Any equilibrium $\left(p_1^*, p_2^*\right)$ must, therefore, satisfy $\left|p_1^* - p_2^*\right| < t\left(x_2 - x_1\right)$. A consequence of this condition is that for any $\left(p_1^*, p_2^*\right)$, p_1^* must maximise $\Pi_1 = \left(\left(p_1 p_2 - p_1^2\right)/2t\right) + p_1\left(x_1 + x_2\right)/2$ in the open interval $\left]p_2^* - t\left(x_2 - x_1\right), p_2^* + t\left(x_2 - x_1\right)\right[$ while p_2^* must maximise $\Pi_2 = \left(\left(p_1 p_2 - p_2^2/2t\right)\right) + p_2\left(2 - x_1 - x_2\right)/2$ in the open interval $\left]p_1^* - t\left(x_2 - x_1\right), p_1^* + t\left(x_2 - x_1\right)\right[$. Taking first order conditions gives equations 3 and 4. For the case when $x_1 + x_2 = 4$, the first order conditions give equation $p_1^* = \frac{t\left(x_1 + x_2\right)}{2}$ and $p_2^* = 0$.

For $\left(p_1^*, p_2^*\right)$ to be an equilibrium strategy given any $\left(x_1, x_2\right)$, we must have for any $\varepsilon > 0$ where ε is arbitrarily small

$$\Pi_1\left(p_1^*, p_2^*\right) = \frac{t}{18}\left(x_1 + x_2 + 2\right)^2 \geq p_2^* - t\left(x_2 - x_1\right) - \varepsilon$$

The right hand side of the above inequality is the profit of firm 1 should it charge a price slightly lower than p_2^*. We will now show that the above inequality can be rewritten as equation 1. Substituting equation 4 gives

$$\frac{t}{18}\left(x_1 + x_2 + 2\right)^2 \geq \frac{t}{3}\left(4 - x_1 - x_2\right) - t\left(x_2 - x_1\right)$$

or $$\left(\frac{x_1 + x_2 + 2}{3}\right)^2 \geq \frac{4}{3}(2 + x_1 - 2x_2)$$

which is equivalent to equation 1.

Similarly, for (p_1^*, p_2^*) to be an equilibrium strategy given any (x_1, x_2), we must have for any $\varepsilon > 0$

$$\Pi_2(p_1^*, p_2^*) = \frac{t}{18}(4 - x_1 - x_2)^2 \geq p_1^* - t(x_2 - x_1) - \varepsilon .$$

The right hand side of the above inequality is the profit of firm 2 should it charge a price slightly lower than p_1^*. The following shows that the above inequality can be rewritten as equation 2. Substituting equation 3 gives

$$\frac{t}{18}(4 - x_1 - x_2)^2 \geq \frac{t}{3}(x_1 + x_2 + 2) - t(x_2 - x_1)$$

or $$\left(\frac{4 - x_1 - x_2}{3}\right)^2 \geq \frac{4}{3}(1 + 2x_1 - x_2)$$

which is equivalent to equation 2.

To show that conditions 1 and 2 are also sufficient for (p_1^*, p_2^*) to be an equilibrium, it is easily verifiable that they imply that $\left|p_1^* - p_2^*\right| < t(x_2 - x_1)$.

QED

APPENDIX 2

PARAMETRIC LOCATIONS WITH QUADRATIC TRANSPORTATION COSTS

Assume that the transportation cost function is quadratic of the form $c(d) = sd^2$, where $c(0) = 0$ and $s > 0$. If $m(p_1, p_2)$ exists, it must be the solution of the equation

$$p_1 + s(y - x_1)^2 = p_2 + s(x_2 - y)^2$$

Solving, we obtain the demand functions for firm 1 and firm 2, respectively, as

(A7) $$m_1(p_1, p_2) = \frac{p_2 - p_1}{2s(x_2 - x_1)} + \frac{(x_1 + x_2)}{2} \qquad \text{and}$$

(A8) $$m_2(p_1, p_2) = \frac{p_1 - p_2}{2s(x_2 - x_1)} + \frac{(2 - x_1 - x_2)}{2}.$$

The payoff functions are given by $\Pi_1(p_1, p_2) = p_1 \cdot m_1(p_1, p_2)$ and $\Pi_2(p_1, p_2) = p_2 \cdot m_2(p_1, p_2)$ respectively. Maximising profits on the part of firm 1 and firm 2 gives the following response functions:

(A9) $$p_1^* = \frac{1}{2}\left[p_2^* + s\left(b^2 - a^2\right)\right] \qquad \text{and}$$

(A10) $$p_2^* = \frac{1}{2}\left[p_1^* + s(x_2 - x_1)(2 - x_1 - x_2)\right]$$

Solving equations A9 and A10 gives the non-cooperative Bertrand-Nash price equilibrium in pure strategies

(A11) $$\left(p_1^*, p_2^*\right) = \left(\frac{s}{3}(x_2 - x_1)(x_1 + x_2 + 2), \frac{s}{3}(x_2 - x_1)(4 - x_1 - x_2)\right).$$

For non-zero p_2^*, we assume that $x_1 + x_2 < 4$. If $x_1 + x_2 = 4$, an equilibrium exists at $\left(p_1^*, p_2^*\right) = \left(s\left(x_2^2 - x_1^2\right)/2, 0\right)$. The distribution of market demand between firm 1 and firm 2 at Nash equilibrium is given by substituting A11 into A7 and A8:

(A12) $$\left(m_1^*, m_2^*\right) = \left(\frac{1}{6}(x_1 + x_2 + 2), \frac{1}{6}(4 - x_1 - x_2)\right).$$

APPENDIX 3

PROOF OF PROPOSITIONS 1, 2 AND 3

<u>Proposition 1</u>

When the transportation cost structure is linear, i.e., when $c(d)=td$, $t>0$, the unique price equilibrium in pure strategies exists at the pair of prices

$$\left(p_1^*, p_2^*\right)=\left(\frac{t}{3}\left(x_1+x_2+2\right), \frac{t}{3}\left(4-x_1-x_2\right)\right)$$

The equilibrium relative price of the good is given by

(A13) $$\frac{p_2^*}{p_1^*}=\frac{t\left(4-x_1-x_2\right)/3}{t\left(x_1+x_2+2\right)/3}=\frac{4-x_1-x_2}{x_1+x_2+2}$$

When transportation costs are quadratic, i.e., when $c(d)=sd^2$, $s>0$, the unique equilibrium in pure strategies exists at the pair of prices

$$\left(p_1^*, p_2^*\right)=\left(\frac{s}{3}\left(x_2-x_1\right)\left(x_1+x_2+2\right), \frac{s}{3}\left(x_2-x_1\right)\left(4-x_1-x_2\right)\right).$$

The equilibrium relative price of the good is given by

(A14) $$\frac{p_2^*}{p_1^*}=\frac{s\left(x_2-x_1\right)\left(4-x_1-x_2\right)/3}{s\left(x_2-x_1\right)\left(x_1+x_2+2\right)/3}=\frac{4-x_1-x_2}{x_1+x_2+2}.$$

In the case of linear-quadratic transportation costs, i.e., when $c(d)=td+sd^2$, $t>0$ and $s>0$, the unique equilibrium in pure strategies exists at the pair of prices

$$\left(p_1^*, p_2^*\right)=\left(\frac{t+s\left(x_2-x_1\right)}{3}\cdot\left(x_1+x_2+2\right), \frac{t+s\left(x_2-x_1\right)}{3}\cdot\left(4-x_1-x_2\right)\right).$$

The equilibrium relative price of the good is given by

(A15) $$\frac{p_2^*}{p_1^*}=\frac{\left[t+s\left(x_2-x_1\right)\right]\left(4-x_1-x_2\right)/3}{\left[t+s\left(x_2-x_1\right)\right]\left(x_1+x_2+2\right)/3}=\frac{4-x_1-x_2}{x_1+x_2+2}.$$

Equations A13, A14 and A15 are all equivalent.

QED

Proposition 2

Part 1 of the proposition is obvious from the equivalence of equations 8, A6 and A12. Part 2 of the proposition is proven as follows. Since the equilibrium demand is the same for all transportation costs and is given by

$$\left(m_1^*, m_2^*\right) = \left(\frac{1}{6}\left(x_1 + x_2 + 2\right), \frac{1}{6}\left(4 - x_1 - x_2\right)\right),$$

we have for all transportation cost functions, the equilibrium relative demand as

(A16) $$\frac{m_2^*}{m_1^*} = \frac{\frac{1}{6}\left(4 - x_1 - x_2\right)}{\frac{1}{6}\left(x_1 + x_2 + 2\right)} = \frac{4 - x_1 - x_2}{x_1 + x_2 + 2}$$

Since A16 is equivalent to A15, we have $m_2^* / m_1^* = p_2^* / p_1^*$ for all transportation cost functions.

QED

Proposition 3

Increase in Transportation Costs Within the Same Transportation Cost Structure

Linear Transportation Costs

When the transportation cost structure is linear, i.e., when $c(d) = td$, $t > 0$, the unique price equilibrium in pure strategies exists at the pair of prices

$$\left(p_1^*, p_2^*\right) = \left(\frac{t}{3}\left(x_1 + x_2 + 2\right), \frac{t}{3}\left(4 - x_1 - x_2\right)\right)$$

Firm 2 offers a lower price than firm 1 since $4 - x_1 - x_2 < x_1 + x_2 + 2$, i.e., $x_1 + x_2 > 1$ (always true given the firms' configuration). When t increases to $t^{'}$, therefore, firm 1 offers a higher price at $t^{'}\left(x_1 + x_2 + 2\right)/3$, while firm 2 correspondingly raises its price but by a smaller amount to $t^{'}\left(4 - x_1 - x_2\right)/3$.

Quadratic Transportation Costs

When transportation costs are quadratic, i.e., when $c(d)=sd^2$, $s>0$, the unique equilibrium in pure strategies exists at the pair of prices

$$\left(p_1^*, p_2^*\right)=\left(\frac{s}{3}(x_2-x_1)(x_1+x_2+2), \frac{s}{3}(x_2-x_1)(4-x_1-x_2)\right)$$

for $x_1+x_2<4$. Firm 2 offers a lower price than firm 1 since $4-x_1-x_2<x_1+x_2+2$, i.e., $x_1+x_2>1$. When s increases to s', therefore, firm 1 offers a higher price at $s'(x_2-x_1)(x_1+x_2+2)/3$, while firm 2 correspondingly raises its price but by a smaller amount to $s'(x_2-x_1)(4-x_1-x_2)/3$.

Linear-Quadratic Transportation Costs

In the case of linear-quadratic transportation costs, i.e., when $c(d)=td+sd^2$, $t>0$ and $s>0$, the unique equilibrium in pure strategies exists at the pair of prices

$$\left(p_1^*, p_2^*\right)=\left(\frac{t+s(x_2-x_1)}{3}\cdot(x_1+x_2+2), \frac{t+s(x_2-x_1)}{3}\cdot(4-x_1-x_2)\right)$$

for $x_1+x_2<4$. Firm 2 offers a lower price than firm 1 since $4-x_1-x_2<x_1+x_2+2$, i.e., $x_1+x_2>1$. When t and s increases to t' and s' respectively, firm 1 offers a higher price at $\left[t'+s'(x_2-x_1)\right](x_1+x_2+2)/3$, while firm 2 correspondingly raises its price but by a smaller amount to $\left[t'+s'(x_2-x_1)\right](4-x_1-x_2)/3$.

QED

Increase in Transportation Costs From Lower to Higher Transportation Cost Structure

Linear to Quadratic Transportation Costs

The difference in equilibrium price offered by firm 1 under quadratic and linear transportation costs is given by

$$\left(p_1^*\right)_Q - \left(p_1^*\right)_L = \frac{\left[s\left(x_2 - x_1\right) - t\right]\left(x_1 + x_2 + 2\right)}{3}$$

which is >0 if and only if $t/s < x_2 - x_1$. Therefore, under the higher quadratic transportation cost structure, firm 1 offers the good at a higher price than when transportation costs are linear if $t/s < x_2 - x_1$.

The difference in equilibrium price offered by firm 2 under quadratic and linear transportation costs is given by

$$\left(p_2^*\right)_Q - \left(p_2^*\right)_L = \frac{\left[s\left(x_2 - x_1\right) - t\right]\left(4 - x_1 - x_2\right)}{3}$$

which is >0 if and only if $t/s < x_2 - x_1$. For $p_2^* > 0$, $x_1 + x_2 < 4$. Therefore, under the higher quadratic transportation cost structure, firm 2 offers the good at a higher price than when transportation costs are linear if $t/s < x_2 - x_1$ and $x_1 + x_2 < 4$.

Since $4 - x_1 - x_2 < x_1 + x_2 + 2$ or $x_1 + x_2 > 1$, firm 1 offers a higher price than firm 2.

Linear to Linear-Quadratic Transportation Costs

The difference in equilibrium price offered by firm 1 under linear and linear-quadratic transportation costs is given by

$$\left(p_1^*\right)_{LQ} - \left(p_1^*\right)_L = \frac{s\left(x_2 - x_1\right)\left(x_1 + x_2 + 2\right)}{3}$$

which is >0 since $s > 0$ and $x_2 > x_1$. Therefore, under the higher linear-quadratic transportation cost structure, firm 1 offers the good at a higher price than when transportation costs are linear.

The difference in equilibrium price offered by firm 2 under linear and linear-quadratic transportation costs is given by

$$\left(p_2^*\right)_{LQ} - \left(p_2^*\right)_L = \frac{s(x_2 - x_1)(4 - x_1 - x_2)}{3}$$

which is > 0 since $s > 0$ and $x_2 > x_1$. For $p_2^* > 0$, $x_1 + x_2 < 4$. Therefore, under the higher linear-quadratic transportation cost structure, firm 2 offers the good at a higher price than when transportation costs are linear.

Since $4 - x_1 - x_2 < x_1 + x_2 + 2$ or $x_1 + x_2 > 1$, firm 1 offers a higher price than firm 2.

Quadratic to Linear-Quadratic Transportation Costs

The difference in equilibrium price offered by firm 1 under quadratic and linear-quadratic transportation costs is given by

$$\left(p_1^*\right)_{LQ} - \left(p_1^*\right)_Q = \frac{t(x_1 + x_2 + 2)}{3}$$

which is > 0 since $t > 0$. Therefore, under the higher linear-quadratic transportation cost structure, firm 1 offers the good at a higher price than when transportation costs are quadratic.

The difference in equilibrium price offered by firm 2 under quadratic and linear-quadratic transportation costs is given by

$$\left(p_2^*\right)_{LQ} - \left(p_2^*\right)_Q = \frac{t(4 - x_1 - x_2)}{3}$$

which is > 0 since $t > 0$. For $p_2^* > 0$, $x_1 + x_2 < 4$. Therefore, under the higher linear-quadratic transportation cost structure, firm 2 offers the good at a higher price than when transportation costs are linear.

Since $4 - x_1 - x_2 < x_1 + x_2 + 2$ or $x_1 + x_2 > 1$, firm 1 offers a higher price than firm 2.

QED

APPENDIX 4

SIMULTANEOUS PRICE-LOCATION GAME WITH QUADRATIC TRANSPORTATION COSTS

Assume that the transportation cost function is of the form $c(d) = sd^2$ where $c(0) = 0$ and $s > 0$. The profit functions of firm 1 and firm 2 are given by the following respective equations:

$$\Pi_1((p_1, x_1), (p_2, x_2)) = \frac{p_1 p_2 - p_1^2}{2s(x_2 - x_1)} + \frac{(x_1 + x_2)}{2} p_1$$

and
$$\Pi_2((p_1, x_1), (p_2, x_2)) = \frac{p_1 p_2 - p_2^2}{2s(x_2 - x_1)} + \frac{(2 - x_1 - x_2)}{2} p_2 .$$

Firm 1 maximises profit by choosing x_1^* with the first order condition given by

$$\frac{\partial \Pi_1((p_1, x_1), (p_2, x_2))}{\partial x_1} = \frac{p_1^*}{2}\left(\frac{p_2^* - p_1^*}{s(x_2^* - x_1^*)^2} + 1\right) = 0 .$$

Substituting $p_1^* = s(x_2^* - x_1^*)(x_1^* + x_2^* + 2)/3$ and $p_2^* = s(x_2^* - x_1^*)(4 - x_1^* - x_2^*)/3$ (equation A11) obtained by maximising the respective firm's profit with respect to price into the above gives

$$\frac{s}{9}(x_1^* + x_2^* + 2)(2 - 5x_1^* + x_2^*) = 0 .$$

Since $s > 0$ and $(x_1^* + x_2^* + 2) > 0$, this implies that $(2 - 5x_1^* + x_2^*) = 0$. In other words, the equilibrium location of firm 1 is at

(A17) $$x_1^* = \frac{2 + x_2^*}{5}$$

which gives the response function in location of firm 1.

In the case of firm 2, it maximises profit by choosing x_2^* such that

$$\frac{\partial \Pi_2((p_1, x_1), (p_2, x_2))}{\partial x_2} = -\frac{p_2^*}{2}\left(\frac{p_1^* - p_2^*}{s(x_2^* - x_1^*)^2} + 1\right) < 0$$

since $p_1^* > p_2^*$ from equation A11 for all $x_1 + x_2 < 4$. This implies that firm 2 increases profit by moving toward the market border, i.e., $x_2^* = 1 + \varepsilon$ with $\varepsilon > 0$ arbitrarily small. Solving for x_1^* by substituting $x_2^* = 1 + \varepsilon$ into equation A17 gives $x_1^* = (3 + \varepsilon)/5$. Finally, the equilibrium prices are obtained by substituting x_1^* and x_2^* into equation A11 which gives $p_1^* = 4s\left(3 - 5\varepsilon - 2\varepsilon^2\right)/25$ and $p_2^* = 4s\left(2 - 5\varepsilon + 2\varepsilon^2\right)/25$. The simultaneous price-location equilibrium in pure strategies is, therefore, given by

(A18) $$\left(\left(p_1^*, x_1^*\right), \left(p_2^*, x_2^*\right)\right) = \left(\left(\frac{4s}{25}\left(3 - 5\varepsilon - 2\varepsilon^2\right), \frac{(3+\varepsilon)}{5}\right), \left(\frac{4s}{25}\left(2 - 5\varepsilon + 2\varepsilon^2\right), 1 + \varepsilon\right)\right)$$

where $\varepsilon > 0$ is a small constant.

APPENDIX 5

RELEVANCE OF PROPOSITIONS 1, 2 AND 3 TO THE SIMULTANEOUS PRICE-LOCATION GAME UNDER VARIABLE LOCATIONS

The following proves that Propositions 2.2 and 3 hold for the simultaneous price-location game of the IO model whenever an equilibrium in pure strategies exists, but Propositions 1 and 2.1 do not hold.

Proposition 1A

When firm locations are variable, the equilibrium relative price p_2^* / p_1^* varies with the transportation cost structure.

Proof

Under quadratic transportation costs, the equilibrium relative price is

$$\frac{p_2^*}{p_1^*} = \frac{4s}{25}\left(2 - 5\varepsilon + 2\varepsilon^2\right) \Big/ \frac{4s}{25}\left(3 - 5\varepsilon - 2\varepsilon^2\right)$$

(A19) $$\frac{p_2^*}{p_1^*} = \left(2 - 5\varepsilon + 2\varepsilon^2\right) / \left(3 - 5\varepsilon - 2\varepsilon^2\right) \cong \frac{2}{3} \text{ as } \varepsilon \to 0 .$$

Under linear-quadratic transportation costs, the equilibrium relative price is

$$\frac{p_2^*}{p_1^*} = \frac{\frac{2}{25}\left[4s + t\left(3 - \frac{t}{s}\right) - \frac{4\varepsilon}{25}\left(2t - 3s + 2s\varepsilon\right)\right]}{\frac{2}{25}\left[6s + t\left(7 + \frac{t}{s}\right) + \frac{4\varepsilon}{25}\left(2t + 7s + 2s\varepsilon\right)\right]}$$

(A20) $$\frac{p_2^*}{p_1^*} = \frac{4s + t\left(3 - \frac{t}{s}\right) - \frac{4\varepsilon}{25}\left(2t - 3s + 2s\varepsilon\right)}{6s + t\left(7 + \frac{t}{s}\right) + \frac{4\varepsilon}{25}\left(2t + 7s + 2s\varepsilon\right)} \cong \frac{4s + t\left(3 - \frac{t}{s}\right)}{6s + t\left(7 + \frac{t}{s}\right)} \text{ as } \varepsilon \to 0 .$$

Since equations A19 and A20 are not equal, Proposition 1 does not hold.

QED

Proposition 2A

Under variable firm locations, the equilibrium market demand $\left(m_1^*, m_2^*\right)$ for the good has the following properties:

2.1A It varies with the transportation cost structure

2.2A Relative demand is equivalent to relative prices.

<u>Proof</u>

Under quadratic transportation costs, the equilibrium demand is obtained by substituting $\left(\left(p_1^*, x_1^*\right), \left(p_2^*, x_2^*\right)\right)$ into equations A7 and A8 which gives

$$\left(m_1^*, m_2^*\right) = \left(\frac{3+11\varepsilon+10\varepsilon^2}{5(1+2\varepsilon)}, \frac{2-\varepsilon-10\varepsilon^2}{5(1+2\varepsilon)}\right).$$

The relative demand is given by

(A21) $$\frac{m_2^*}{m_1^*} = \frac{\dfrac{2-\varepsilon-10\varepsilon^2}{5(1+2\varepsilon)}}{\dfrac{3+11\varepsilon+10\varepsilon^2}{5(1+2\varepsilon)}} = \frac{2-\varepsilon-10\varepsilon^2}{3+11\varepsilon+10\varepsilon^2} \cong \frac{2}{3} \text{ as } \varepsilon \to 0.$$

which is equivalent to equation A19 since ε is a small constant close to 0.

Under linear-quadratic transportation costs, the equilibrium demand is obtained by substituting $\left(\left(p_1^*, x_1^*\right), \left(p_2^*, x_2^*\right)\right)$ into equations 3 and 4 which gives

$$\left(m_1^*, m_2^*\right) = \left(\frac{1}{10(t+s+2s\varepsilon)}\left(7t+6s+\frac{t^2}{s}+4t\varepsilon+14s\varepsilon+4s\varepsilon^2\right), \frac{1}{10(t+s+2s\varepsilon)}\left(3t+4s-\frac{t^2}{s}-4t\varepsilon+6s\varepsilon-4s\varepsilon^2\right)\right)$$
.

The relative demand is given by

(A22) $$\frac{m_2^*}{m_1^*} = \frac{4s+3t-\dfrac{t^2}{s}-4t\varepsilon+6s\varepsilon-4s\varepsilon^2}{6s+7t+\dfrac{t^2}{s}+4t\varepsilon+14s\varepsilon+4s\varepsilon^2} \cong \frac{4s+3t-\dfrac{t^2}{s}}{6s+7t+\dfrac{t^2}{s}} \text{ as } \varepsilon \to 0.$$

which is equivalent to A20 as $\varepsilon \to 0$. Since equations A21 and A22 are not identical, Proposition 2.1 does not hold. Since $m_2^*/m_1^* = p_2^*/p_1^*$ for ε close to 0, Proposition 2.2 holds for the simultaneous game of the IO model whenever an equilibrium in pure strategies exists.

QED

Proposition 3A

Given a transportation cost structure, the inside firm raises (lowers) its price when faced with higher (lower) transportation costs. The outside firm reacts by raising (lowering) its price but by a smaller amount.

Proof

When the transportation costs are quadratic, the unique price equilibrium in pure strategies exists at the pair of prices

$$p_1^* = \frac{4s}{25}\left(3 - 5\varepsilon - 2\varepsilon^2\right) \quad \text{and} \quad p_2^* = \frac{4s}{25}\left(2 - 5\varepsilon + 2\varepsilon^2\right).$$

Firm 2 offers a lower price than firm 1 since $8s/25 < 12s/25$ as long as $\varepsilon < 1/2$. When s increases to s', firm 1 offers a higher price at $\frac{4s'}{25}\left(3 - 5\varepsilon - 2\varepsilon^2\right)$, while firm 2 correspondingly raises its price but by a smaller amount to $\frac{4s'}{25}\left(2 - 5\varepsilon + 2\varepsilon^2\right)$, $\varepsilon < 1/2$.

When the transportation costs are linear-quadratic, the unique price equilibrium in pure strategies exists at the pair of prices

$$p_1^* = \frac{2}{25}\left[6s + t\left(7 + \frac{t}{s}\right) + 2\varepsilon(2t + 7s + 2s\varepsilon)\right] \quad \text{and} \quad p_2^* = \frac{2}{25}\left[4s + t\left(3 - \frac{t}{s}\right) - 2\varepsilon(2t - 3s + 2s\varepsilon)\right]$$

Firm 2 offers a lower price than firm 1 since

$$\frac{2}{25}\left[4s + t\left(3 - \frac{t}{s}\right) - 2\varepsilon(2t - 3s + 2s\varepsilon)\right] < \frac{2}{25}\left[6s + t\left(7 + \frac{t}{s}\right) + 2\varepsilon(2t + 7s + 2s\varepsilon)\right]$$

When t increases to t' and s increases to s', firm 1 offers a higher price at $2\left[6s' + t'\left(7 + \frac{t'}{s'}\right) + 2\varepsilon\left(2t' + 7s' + 2s'\varepsilon\right)\right]\Big/25$, while firm 2 correspondingly raises its price but by a smaller amount to $2\left[4s' + t'\left(3 - \frac{t'}{s'}\right) - 2\varepsilon\left(2t' - 3s' + 2s'\varepsilon\right)\right]\Big/25$.

Proposition 3, therefore, holds for the simultaneous price-location game of the IO model whenever an equilibrium in pure strategies exists.

QED

APPENDIX 6

SEQUENTIAL GAME WITH QUADRATIC TRANSPORTATION COSTS

Under quadratic transportation costs when $x_1 + x_2 < 4$, the unique price equilibrium in pure strategies is given by equation A11, i.e.,

$$\left(p_1^*, p_2^*\right) = \left(\frac{s}{3}(x_2 - x_1)(x_1 + x_2 + 2), \frac{s}{3}(x_2 - x_1)(4 - x_1 - x_2)\right).$$

The profit function of firm 1 is given by

$$\Pi_1\left(p_1(x_1, x_2), p_2(x_1, x_2), x_1, x_2\right) = \frac{p_1 p_2 - p_1^2}{2s(x_2 - x_1)} + \frac{(x_1 + x_2)}{2} p_1.$$

Substituting equation A11 gives

$$\Pi_1\left(p_1^*(x_1, x_2), p_2^*(x_1, x_2), x_1, x_2\right) = \frac{s(x_2 - x_1)}{18}(x_1 + x_2 + 2)^2.$$

Optimising with respect to x_1 gives

$$\frac{\partial \Pi_1\left(p_1^*(x_1, x_2), p_2^*(x_1, x_2), x_1, x_2\right)}{\partial x_1} = -\frac{s}{18}\left(x_1^* + x_2^* + 2\right)\left(3x_1^* - x_2^* + 2\right)$$

since $s > 0$ and $\left(x_1^* + x_2^* + 2\right) > 0$. There are two possible scenarios. If $3x_1^* - x_2^* + 2 > 0$, or

(A23) $\quad x_2^* < 3x_1^* + 2$

then $\partial \Pi_1\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_1 < 0$ and firm 1 increases profit by moving away from firm 2. The equilibrium location for firm 1 would then be $x_1^* = 0$. Otherwise, if the converse of equation A23 holds, then either $\partial \Pi_1\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_1 > 0$ when $x_2^* > 3x_1^* + 2$ so that firm 1 will locate at $x_1^* = 1$, or $\partial \Pi_1\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_1 = 0$ when $x_2^* = 3x_1^* + 2$.

Now consider the profit function for firm 2 which is given by

$$\Pi_2\left(p_1(x_1, x_2), p_2(x_1, x_2), x_1, x_2\right) = \frac{p_1 p_2 - p_2^2}{2s(x_2 - x_1)} + \frac{(2 - x_1 - x_2)}{2} p_2.$$

Substituting equation A11 gives

$$\Pi_2\left(p_1^*(x_1,x_2),\, p_2^*(x_1,x_2), x_1, x_2\right)=\frac{s(x_2-x_1)}{18}(4-x_1-x_2)^2 .$$

Optimising with respect to x_2 gives

$$\frac{\partial \Pi_2\left(p_1^*(x_1,x_2),\, p_2^*(x_1,x_2), x_1, x_2\right)}{\partial x_2}=\frac{s}{18}\left(4-x_1^*-x_2^*\right)\left(4+x_1^*-3x_2^*\right).$$

Since $s>0$ and $4-x_1^*-x_2^*>0$, we have, for $\partial\Pi_2\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_2=0$,

(A24) $\qquad 4+x_1^*-3x_2^*=0$

Suppose that equation A23 holds so that $x_1^*=0$. Substitution of $x_1^*=0$ into equation A24 then gives $x_2^*=4/3$.

We will now show that the converse of equation A23 is not valid. Suppose instead that $x_2^*>3x_1^*+2$ which implies that $x_1^*=1$ since $\partial\Pi_1\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_1>0$. The condition then gives $x_2^*>5$. Substituting $x_1^*=1$ into equation A24 gives $x_2^*=5/3$ which contradicts $x_2^*>5$. This implies that $x_2^*>3x_1^*+2$ cannot hold.

Next, suppose that $\partial\Pi_1\left(p_1^*, p_2^*, x_1, x_2\right)/\partial x_1=0$ because $x_2^*=3x_1^*+2$. Solving this equality together with equation A23 gives $x_1^*=1/4$ and $x_2^*=5/4$. This solution, however, contradicts the equality condition assumed at the outset, since substitution of $x_1^*=1/4$ gives $x_2^*=11/4$.

We have now established that the only solution in pure strategies to the first stage game if $x_1+x_2<4$ is $\left(x_1^*, x_2^*\right)=(0,4/3)$. The second-stage game is then solved by substituting x_1^* and x_2^* into equation A11. The equilibrium price pair in pure strategies, therefore, is given by $\left(p_1^*, p_2^*\right)=(40s/27, 32s/27)$. The full (subgame perfect) equilibrium to the sequential game in pure strategies is then given by

(A25) $\qquad \left(p_1^*, p_2^*, x_1^*, x_2^*\right)=\left(\frac{40s}{27},\frac{32s}{27},0,\frac{4}{3}\right)$

where $x_1+x_2<4$.

APPENDIX 7

RELEVANCE OF PROPOSITIONS 1, 2 AND 3 TO THE SEQUENTIAL GAME UNDER VARIABLE LOCATIONS

The following proves that Propositions 2.2 and 3 hold for the sequential game of the IO model whenever an equilibrium in pure strategies exists, but Propositions 1 and 2.1 do not hold.

Proof of Proposition 1A

Under quadratic transportation costs, the equilibrium relative price is

(A26) $$\frac{p_2^*}{p_1^*} = \left(\frac{32s}{27}\right) \Big/ \left(\frac{40s}{27}\right) = \frac{4}{5}$$

Under linear-quadratic transportation costs, the equilibrium relative price is

(A27) $$\frac{p_2^*}{p_1^*} = \frac{16s + t\left(8 + \frac{t}{s}\right)}{20s + t\left(1 - \frac{t}{s}\right)}$$

Since equations A26 and A27 are not equal, Proposition 1 does not hold.

QED

Proof of Proposition 2A

Under quadratic transportation costs, the equilibrium demand is obtained by substituting $\left(p_1^*, p_2^*, x_1^*, x_2^*\right)$ into equations A7 and A8 which gives

$$\left(m_1^*, m_2^*\right) = \left(\frac{5}{9}, \frac{4}{9}\right).$$

The relative demand is given by

(A28) $$\frac{m_2^*}{m_1^*} = \frac{\frac{4}{9}}{\frac{5}{9}} = \frac{4}{5}$$

which is equivalent to equation A26.

Under linear-quadratic transportation costs, the equilibrium demand is obtained by substituting $\left(p_1^*, p_2^*, x_1^*, x_2^*\right)$ into equations 3 and 4 which gives

$$\left(m_1^*, m_2^*\right) = \left(\frac{20s + t\left(1 - \frac{t}{s}\right)}{9(4s+t)}, \frac{16s + t\left(8 + \frac{t}{s}\right)}{9(4s+t)}\right).$$

The relative demand is given by

$$\text{(A29)} \qquad \frac{m_2^*}{m_1^*} = \frac{16s + t\left(8 + \frac{t}{s}\right)}{20s + t\left(1 - \frac{t}{s}\right)}$$

which is equivalent to A27.

Since equations A28 and A29 are not identical, Proposition 2.1 does not hold. Since $m_2^* / m_1^* = p_2^* / p_1^*$, Proposition 2.2 holds whenever an equilibrium in pure strategies exists.

QED

Proof of Proposition 3A

When the transportation costs are quadratic, the unique price equilibrium in pure strategies exists at the pair of prices

$$p_1^* = \frac{40s}{27} \quad \text{and} \quad p_2^* = \frac{32s}{27}.$$

Firm 2 offers a lower price than firm 1 since $32s/27 < 40s/27$. When s increases to s', firm 1 offers a higher price at $40s/27$, while firm 2 correspondingly raises its price but by a smaller amount to $32s/27$.

When the transportation costs are linear-quadratic, the unique price equilibrium in pure strategies exists at the pair of prices

$$p_1^* = \frac{2}{27}\left[20s + t\left(1 - \frac{t}{s}\right)\right] \quad \text{and} \quad p_2^* = \frac{2}{27}\left[16s + t\left(8 + \frac{t}{s}\right)\right].$$

Firm 2 offers a lower price than firm 1 since $16s + t(8 + t/s) < 20s + t(1 - t/s)$. When t increases to $t^{'}$ and s increases to $s^{'}$, firm 1 offers a higher price at $\left(2\left[20s^{'} + t^{'}\left(1 - t^{'}/s^{'}\right)\right]/27\right)$, while firm 2 correspondingly raises its price but by a smaller amount to $\left(2\left[16s^{'} + t^{'}\left(8 + t^{'}/s^{'}\right)\right]/27\right)$.

Proposition 3, therefore, holds for the sequential game of the IO model whenever an equilibrium in pure strategies exists.

QED

APPENDIX 8

INSTRUCTIONS FOR EXPERIMENT WITH PARAMETRIC FIRM LOCATION

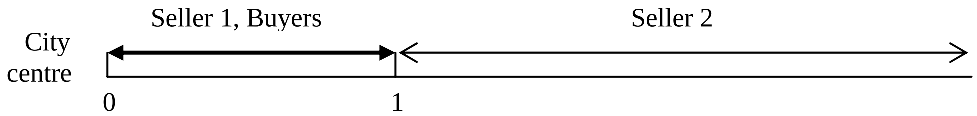
Welcome to the experiment! Please read these instructions and follow them carefully. **Do not talk to any person other than the facilitator until the end of the experiment.** If you have any questions, you may ask the facilitator after reading the instructions.

In this experiment, we are going to set up a market in which buyers and sellers trade a single commodity. Trading will commence with one practice period, followed by a sequence of 16 actual periods.

The prices that you negotiate in each trading period will determine your earnings in experimental dollars. At the end of the experiment, your earnings will be paid to you after conversion to Singapore dollars. The exchange rate is set at 2 experimental dollars to 1 Singapore dollar.[1]

<u>Instructions</u>

In this experiment, you will function as a seller. In your market, there is one other seller and many buyers. The buyers and sellers are located at different distances from the city centre along the same main road. The distances are measured in ED (experimental distance) units.



One seller is located between 0 and 1 ED unit while the other seller is located at distances beyond 1 ED unit. The buyers are evenly located along the main road from 0 to 1 ED unit.

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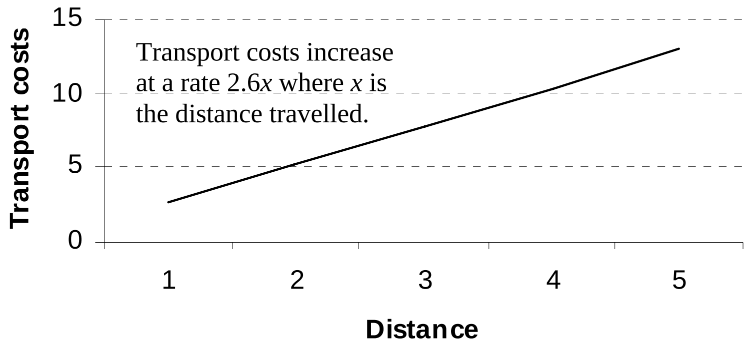
[1] The exchange rate varies for each treatment.

When you start trading, the computer will inform you of your location and the location of the other seller. The locations of all participants remain unchanged throughout the whole of the experiment.

Each buyer incurs a travel cost to arrive at either seller to purchase the commodity. If *x* is the distance a buyer travels to the seller, the buyer pays a transport cost of 2.6*x*.[2] Therefore, if the buyer travels *x* = 1 ED unit, he incurs a transport cost of 2.6, while if *x* = 2 ED units, he incurs a transport cost of 5.2.



The buyers choose the seller who offers the lower offer price plus transport cost. They do not consider other factors such as the inconvenience of buying one unit from one seller compared to the other seller, or the time spent on travelling. If there is a tie in offer price plus transport cost, the buyers purchase from the seller closer to them.

In this experiment, you make a decision on the price to sell your commodity. You must choose a price from zero upwards.

[2] The transportation cost structure and its parameters vary for each treatment.

To help you with your decision, you are provided with a Market Share Calculator which you can use at any time. The Market Share Calculator determines the percentage of buyers out of the total number of buyers in the market that you may capture at the price you have chosen. To access the Market Share Calculator, press Alt-Tab to reveal the Excel spreadsheet (see Figure below). You can enter alternative offer price pairs for yourself and the other seller and see the resulting market shares. Once you have decided on your offer price, press Alt-Tab again to return to the experiment screen.

MARKET SHARE CALCULATOR

Please enter your price in the box below:

0.00

Please enter a price that you think the other seller may offer in the box below:

0.00

The market share (percentage of buyers) for you and the other seller at these prices are:

Your market share	75.00	%
The other seller's market share	25.00	%

You may continue to enter different offer prices in the blue boxes above to compute the market share you may get.
When you are done, press Alt-Tab to return to the previous screen.

Enter your offer price in the experiment screen (see Figure below). Then click on the button "Offer". The number of buyers who accept the commodity at the price you have chosen will be shown to you. You will also be shown the price and market share of the other seller.

Period 1 Remaining time{sec}: 272

You are seller number 1

You are located at (in ED units) 0.25 The other seller is at (in ED units) 1.25

Please enter your offer price in the box below:

No offer Offer

INSTRUCTIONS

You must decide on a price to sell your commodity. Choose a price between 0 and 7.9.
Enter your choice in the box above. Then click the "Offer" button.
If you do not wish to offer any price, click the "No Offer" button. To help you with your decision, you may use the Market Share Calculator which can be accessed by pressing Alt-Tab. To return to this screen, press Alt-Tab again.

Your earnings are equal to your market share multiplied by the price you charge. This profit is then added to any profits you may earn in the earlier periods to determine your total profits in each period.

Period 1

Seller Number	Price	Percentage market share
1	5	35.8
2	4	64.2

Your earnings this period are 2.79
Your total earnings are 2.79

If you have no questions, we will proceed with one trial trading period, followed by the actual trading periods. After you have completed the experiment, you will be asked to complete a short questionnaire. We will then privately pay your earnings after conversion to Singapore dollars, including a show-up fee of S$4.

APPENDIX 9

QUESTIONNAIRE FOR EXPERIMENT

Personal Data

Name

Age

Gender

Nationality

Telephone

E-mail

What is your Faculty?

What are your subjects of study? Please state your major subject first.

Which year of study are you? ❘ First − Second ❘ Third • Honours • Masters └ PhD

Have you participated in a market experiment before? └ Yes ┐ No

Have you participated in a market experiment the same as the one you just did? └ Yes ┐ No

Would you like to participate in other experiments? └ Yes ┐ No

Questions on the experiment

How did you arrive at your price decisions?

Did you find the Market Share Calculator useful?

Did you use the Market Share Calculator to arrive at an optimal price target that you have set for yourself?

Please write down any other comments you have about this experiment.

Additional questions for experiment on variable firm location

How did you arrive at your location decisions?

Did you use the Market Share Calculator to arrive at an optimal location target that you have set for yourself?

APPENDIX 10

INSTRUCTIONS FOR EXPERIMENT WITH VARIABLE FIRM LOCATION

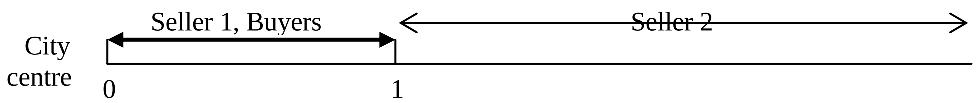
Welcome to the experiment! Please read these instructions and follow them carefully. **Do not talk to any person other than the facilitator** until the end of the experiment. If you have any questions, you may ask the facilitator after reading the instructions.

In this experiment, we are going to set up a market in which buyers and sellers trade a single commodity. Trading will commence with one practice period, followed by a sequence of 16 actual periods.

The prices that you negotiate in each trading period will determine your earnings in experimental dollars. At the end of the experiment, your earnings will be paid to you after conversion to Singapore dollars. The exchange rate is set at 21 experimental dollars to 1 Singapore dollar.[3]

<u>Instructions</u>

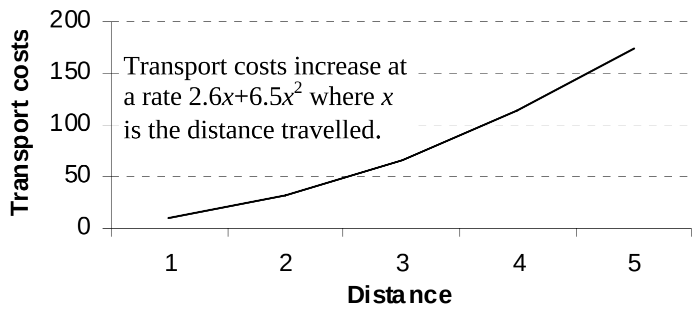
In this experiment, you will function as a seller. In your market, there is one other seller and many buyers. The buyers and sellers are located at different distances from the city centre along the same main road. The distances are measured in ED (experimental distance) units.



[3] The exchange rate varies for each treatment.

One seller is located between 0 and 1 ED unit while the other seller is located at distances beyond 1 ED unit. When you start trading, the computer will inform you which of the two sellers you are. The buyers are evenly located along the main road from 0 to 1 ED unit.

Each buyer incurs a travel cost to arrive at either seller to purchase the commodity. If x is the distance a buyer travels to the seller, the buyer pays a transport cost of $2.6x + 6.5x^2$.[4] Therefore, if the buyer travels $x = 1$ ED unit, he incurs a transport cost of 9.1, while if $x = 2$ ED units, he incurs a transport cost of 31.2.



The buyers choose the seller who offers the lower offer price plus transport cost. They do not consider other factors such as the inconvenience of buying one unit from one seller compared to the other seller, or the time spent on travelling. If there is a tie in offer price plus transport cost, the buyers purchase from the seller closer to them.

[4] The transportation cost structure and its parameters vary for each treatment.

The numerical examples below illustrate how the total cost (price plus transport cost) of 5 buyers change with the sellers' location and price.

Buyer located at	**Seller 1 located at 0**			**Seller 2 located at 1.5**		
	Transport cost	Price	Total Cost	Transport cost	Price	Total Cost
0.00	0.00	14.40	14.40	18.53	8.23	26.76
0.25	1.06	14.40	15.46	13.41	8.23	21.64
0.50	2.93	14.40	17.33	9.10	8.23	17.33
0.75	5.61	14.40	20.01	5.61	8.23	13.84
1.00	9.10	14.40	23.50	2.93	8.23	11.16
Market share	**50.02%**			**49.98%**		

Buyer located at	**Seller 1 located at 0**			**Seller 2 located at 1.01**		
	Transport cost	Price	Total Cost	Transport cost	Price	Total Cost
0.00	0.00	14.40	14.40	9.26	14.31	23.57
0.25	1.06	14.40	15.46	5.73	14.31	20.04
0.50	2.93	14.40	17.33	3.02	14.31	17.33
0.75	5.61	14.40	20.01	1.12	14.31	15.43
1.00	9.10	14.40	23.50	0.03	14.31	14.34
Market share	**50.01%**			**49.99%**		

Buyer located at	**Seller 1 located at 1**			**Seller 2 located at 1.01**		
	Transport cost	Price	Total Cost	**Transport cost**	**Price**	**Total Cost**
0.00	9.10	13.00	22.10	9.26	10.36	19.62
0.25	5.61	13.00	18.61	5.73	10.36	16.09
0.50	2.93	13.00	15.93	3.02	10.36	13.38
0.75	1.06	13.00	14.06	1.12	10.36	11.48
1.00	0.00	13.00	13.00	0.03	10.36	10.39
Market share	**50.97%**			**49.03%**		

In this experiment, you make two decisions: (1) Deciding where to locate your shop to sell your commodity, and (2) Deciding what price to sell your commodity.

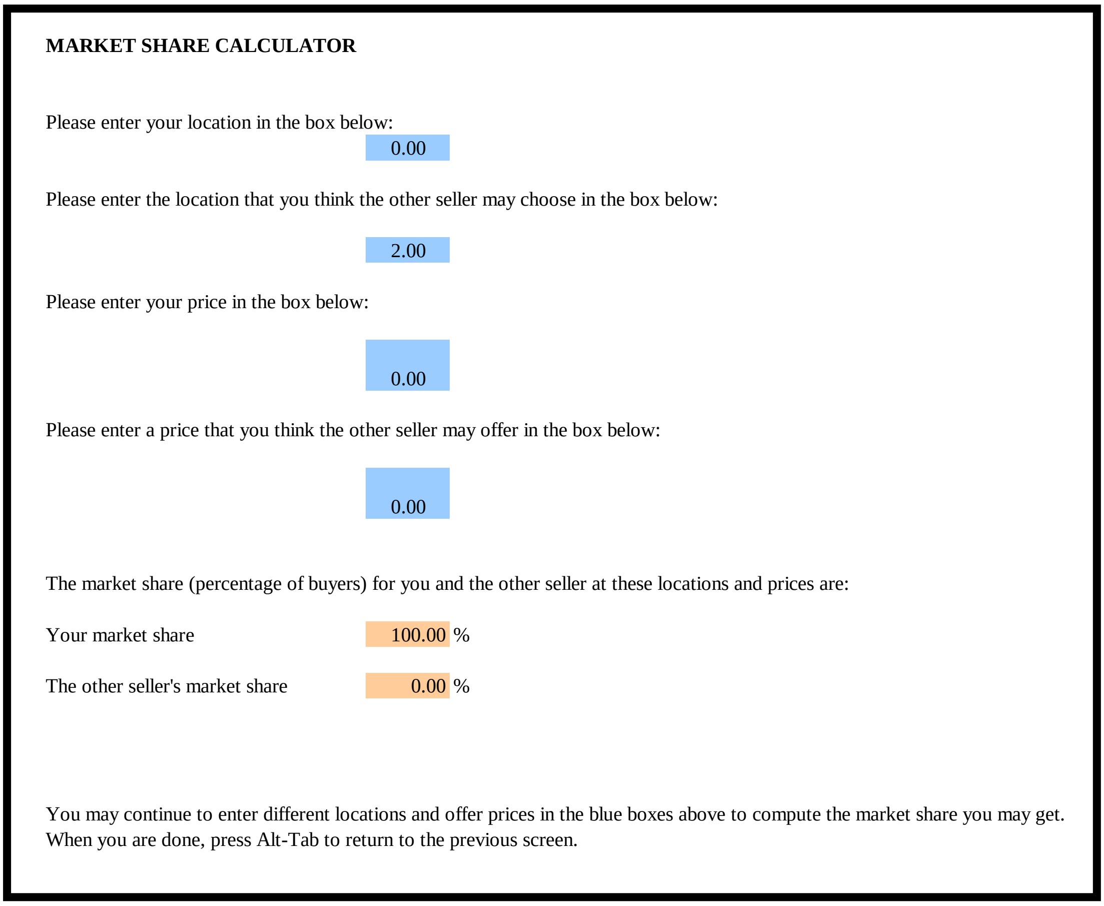
(1) Deciding Your Location

At the start of each trading period, you will be asked to decide on a location for your shop. The computer will inform you whether you are Seller 1 who must locate within 0 to 1 ED unit, or Seller 2 who must locate at distances beyond 1 ED unit. You must then make your location decisions within the relevant boundaries. Your role as Seller 1 or Seller 2 will not change throughout the experiment.

(2) Deciding on Your Price

Next, you must decide on a price to sell the commodity. You must choose a price from zero upwards.

To help you with your decisions, you are provided with a Market Share Calculator which you can use at any time. The Market Share Calculator determines the percentage of buyers out of the total number of buyers in the market that you may capture at the location and price you have chosen. To access the Market Share Calculator, press Alt-Tab to reveal the Excel spreadsheet (see Figure below). You can enter alternative offer price pairs for yourself and the other seller and see the resulting market shares. Once you have decided on your offer price, press Alt-Tab again to return to the experiment screen.



Enter your location and offer price in the experiment screen (see Figure below). Then click on the button "Offer". The number of buyers who accept the commodity at the location and price you have chosen will be shown to you. You will also be shown the location, price and market share of the other seller.

Period 1 Remaining time{sec}: 272

You are seller number 1

Enter your location choice in the box below: Enter your offer price in the box below:

No offer Offer

Instructions

You must decide on a location for your shop and a price at which to sell your commodity.
First, choose a location from 0 to 1. Next, choose a price between 0 and 14.9.
Enter your choices in the relevant boxes above. Then press the "Offer" button.
If you do not wish to make a price or location decision, press the "No Offer" button.
To help you with your decisions, you may use the Market Share Calculator which can be accessed by pressing Alt-Tab. To return to this screen, press Alt-Tab again.

Your earnings are equal to your market share multiplied by the price you charge. This profit is then added to any profits you may earn in the earlier periods to determine your total profits in each period.

Period 1

Seller Number	Location	Price	Percentage market share
1	0.25	7	55.77
2	1.25	6	44.23

Your earnings this period are 2.65
Your total earnings are 2.65 OK

If you have no questions, we will proceed with one trial trading period, followed by the actual trading periods. After you have completed the experiment, you will be asked to complete a short questionnaire. We will then privately pay your earnings after conversion to Singapore dollars, including a show-up fee of S$4.