ESSAYS IN TECHNOLOGY GAP AND PROCESS SPILLOVERS AT THE FIRM LEVEL

SHRAVAN LUCKRAZ
(B. Soc Sci. (Hons), NUS)

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Summary

This dissertation has attempted to provide a contribution to expanding the literature on both the theory and application of noncooperative R&D by introducing a class of games in which asymmetric spillovers are determined by the level of technology of the players. In particular, we consider the case where the follower is more likely to benefit from such spillovers as compared to the industry leader.

The first essay provides a general framework in which to analyze the relationship between R&D investment and technology catch-up in a differential game and shows that the dynamics of the technology gap play a crucial role in determining whether spillovers necessarily reduce the leader's incentives to invest in R&D. The results provide a sufficient condition for the existence of a steady state in R&D games with spillovers; a finding that is new in the literature.

The second essay presents an application of the theoretical framework by studying the effects of process spillovers on competition in a R&D based endogenous growth model. It finds, firstly, that the innovation strategies of the two firms can be dynamically strategic complements if a large technology gap prevails and, secondly, that there is a case for process reverse engineering as a fall in the level of appropriability may result in higher growth.

The purpose of the third essay is to determine the effects of process R&D spillovers on growth by extending the well-known AHV framework. It demonstrates, without relaxing the assumption of product homogeneity, that competitive behavior can still prevail in a Cournot quantity competition setting. Two main factors drive

competitive behavior in the long-run; firstly, the R&D levels in the neck-and-neck state and, secondly, spillovers occurring due to a lack of appropriability.

The final essay offers a conceptual framework for understanding the role played by spillovers in determining the optimal product and process innovation in a duopoly with a leader-follower configuration. It addresses the question of whether higher spillovers favor more process or more product innovation and contributes to the existing literature by showing that it is always optimal for firms to invest more in product innovations when the rate of spillover falls.

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I. General Introduction

One of the most important applications of the Cournot model can be found in the "R&D" branch of the industrial organization literature. By applying the logic of two stage Cournot games, D'Aspremont and Jacquemin (1988) made a seminal contribution to the analysis of strategic R&D investment in a duopoly with spillovers. While subsequent work by Henriques (1990) and Simpson and Vonortas (1994) highlighted the importance of spillovers in R&D games, Amir, Estignev and Wooders (2003) were the first to endogenize spillovers in the underlying framework. This dissertation introduces an element of asymmetry to the structure of intra-industry spillovers by developing a class of noncooperative R&D games in which the nature of the endogenity of such spillovers turns on the level of technology gap between the two firms. Although some research in the theory of economic growth, such as that by Peretto (1996), has shown that the relationship between R&D investment and technology gap is non-linear, this thesis pioneers the study of the technology gap in strategic R&D games with spillovers. In a series of essays, the dissertation provides both a theoretical framework and some applications of R&D games with asymmetric endogenous spillovers.

The first essay develops a theoretical framework in which a class of dynamic noncooperative R&D games in a duopolistic industry with spillovers and technology gap is considered. In so doing, we examine the extent to which the firm's R&D investment decision is affected by the size of spillovers in the industry. In contrast to previous studies, in which the spillovers are considered to be exogenously given, we allow such externalities to be endogenously determined by the magnitude of the technology gap between the two firms. To this end, we propose a dynamic two stage analysis of a noncooperative game in an asymmetric duopoly. Research efforts, which precede production, are directed to reducing unit cost. While the technological efficiency of the leader firm depends only on its own investment, that of the laggard firm is partly subject to endogenous spillovers. Using a general framework to analyze the relationship between R&D investment and technology catch-up in a differential game, we show that the dynamics of the technology gap play a crucial role in determining whether spillovers necessarily reduce the leader's incentives to invest in R&D, and we derive a sufficient condition for the existence of a steady state in R&D games with spillovers. Our results suggest that in the presence of spillovers the leader will always increase its R&D investment as long as the technology gap does not converge to zero.

In the second essay we provide an application of the theoretical model. Specifically, we develop a non-Schumpeterian endogenous growth model of R&D in which the firm's free-riding behavior, reinforced by a lack of appropriability in its industry, constitutes a major source of growth in the economy. While models analyzing the interaction between either imitation and innovation or spillovers and innovation have already appeared in the literature, we show how imitation via free-riding behavior and spillovers can mutually promote dynamic competition and hence economic growth. The representative industry, which is of duopolistic market structure, comprises a leader who innovates and a laggard who free-rides by exploiting the source of intra-industry spillover. We find firstly that the innovation strategies of the two firms can be dynamically strategic complements if a large technology gap prevails and, secondly, that there is a case for process reverse engineering as a fall in the level of appropriability results in higher growth.

The third essay considers another application of the class of models discussed in the first essay by looking at a more robust equilibrium concept that is, closed-loop equilibrium. The paper extends previous research on the effects of process imitation on economic growth by accounting for stochastic intra-industry spillovers. We employ a non-Schumpeterian growth model to determine the impact of such spillovers on investment in industries where firms are either neck-and-neck or unleveled. Our central finding is that, in an economy where the representative industry is a duopoly, R&D spillovers positively affect growth. While other non-Schumpeterian models assume that the imitation rate of laggard firms is unaffected by the R&D effort of the leader firm, we consider the case where the latter's R&D activity generates some positive externality on its rivals' research. In this construct, the duopolists in each industry play a two-stage game. In the first stage, they invest in R&D which can reduce their costs of production only if they successfully innovate and they compete with each other by using Markovian strategies. In the second stage, they compete in the product market. At any point in time, an industry can either be in the neck-and-neck state or in an unleveled state where the leader is n steps ahead of the follower. At the steady state, the inflow of firms to an industry must be equal to the outflow. By determining the steady state investment levels of each insutry, we demonstrate a positive monotonic relationship between the spillover rate and economic growth.

In the last essay we provide a simple static example of an R&D game when both product and process innovations are possible. The paper proposes a conceptual framework for analyzing how process spillovers can impact on a firm's decision to choose its levels of process and product innovation. In contrast to previous work which considers the interrelation between process and product R&D in a duopoly with no spillovers, we extend the existing literature by introducing process spillovers. A two-stage analysis of a non-cooperative game which entails both demand enhancing product innovation and cost-reducing process innovation in an asymmetric duopoly is developed. While the leader's technological efficiency depends only on its own R&D investment, the follower's productivity depends also on the level of intra-industry spillovers. In the first stage, the duopolists choose their levels of product and process innovations, while in the second stage they compete in the product market. The results obtained confirm the findings highlighted by previous studies that both product and process innovations are strategic substitutes. However, we offer an additional insight in that it is always optimal for the firms to invest more in product innovations when the rate of spillover falls. This new result is important as it portrays the spillover rate as the decisive factor determining the level of product innovation vis-à-vis process innovation.

The four essays, by exploiting the heterogeneity of process spillovers in industries where firms are of different stages of technological development, explain the strategic interaction between firms competing in R&D.

II. Dynamic Noncooperative R&D in Duopoly with Spillovers and Technology Gap

Abstract

In this paper we examine the extent to which the firm's R&D investment decision is affected by the size of spillovers in a duopolistic industry. In contrast to previous studies, in which the spillovers are considered to be exogenously given, we allow such externalities to be endogenously determined by the magnitude of the technology gap between the two firms. To this end, we propose a dynamic two stage analysis of a noncooperative game in an asymmetric duopoly. Research efforts, which precede production, are directed to reducing unit cost. While the technological efficiency of the leader firm depends only on its own investment, that of the laggard firm is partly subject to endogenous spillovers. Using a general framework to analyze the relationship between R&D investment and technology catch-up in a differential game, we show that the dynamics of the technology gap play a crucial role in determining whether spillovers necessarily reduce the leader's incentives to invest in R&D, and we derive a sufficient condition for the existence of a steady state in R&D games with spillovers. Our results suggest that in the presence of spillovers the leader will always increase its R&D investment as long as the technology gap does not converge to zero.

Keywords: process innovation, one-way endogenous spillovers, technology gap, dynamic noncooperative R&D game
JEL Classification Numbers: C7, L1, O3

1. Introduction

It has been well established that when one firm independently develops a cost reducing innovation, the firm's competitors benefit in the sense that they can use the innovation to reduce their own costs. When such spillover effects are significant, noncooperative firms might be expected to research too little from the standpoint of the industry since each firm tends to ignore the positive externality which its research generates on the cost of its rival firm (see D'Aspremont and Jacquemin (1988), Henriques (1990) and Simpson and Vonortas (1994)). However, when spillovers are endogenous it is also observed that the firm's disincentive to engage in R&D activity is partially offset because its own R&D can potentially enhance its capacity to absorb its rival's technology (see Katsoulacos and Ulph (1998), Kultti and Takalo (1998), Kamien and Zang (2000) and Grunfeld (2003)). Moreover, reduced costs of rival firms due to spillovers will lead all firms to compete more intensively in the product market. Empirical findings by Cohen and Levinthal (1989) reinforce the fact that spillovers have two opposing effects on R&D investment in strategic games: firstly, they increase the firm's incentive to raise its own R&D and, secondly, they create a disincentive for the rival firm to invest in R&D as free riding becomes a better strategy. A possible explanation for this behavior is that there exists a threshold level of spillovers beyond which the firm has no incentive to increase its R&D activities.

The purpose of this paper is to show how the dynamics of the technology gap between firms helps demarcate the opposing effects of spillovers on R&D incentives. Our work is motivated by issues originating from the empirical findings of Cameron (1999) who observed that as the technological gap between the leader and the follower narrows, the latter must undertake more formal R&D owing to the exhaustion of imitation possibilities. Also, Peretto (1996) showed that the relationship between R&D investment and technology gap is non-linear; that is, when the gap is large the follower enjoys increasing returns to imitation or reverse engineering[1] and when the gap becomes smaller, there are decreasing returns to such activities. While taking into account such observations, we explore the theoretical link between spillovers as pioneered by D'Aspremont and Jacquemin (1988) (henceforth AJ) and technology gap by allowing the rate of spillovers to depend on the latter.[2] Intuitively, when the follower lags far behind the leader it enjoys larger spillovers and has fewer incentives to conduct its own R&D, but as it moves closer to the frontier[3] it is "forced" to innovate as its free riding possibility set becomes smaller. Thus if there exists a relationship between spillovers and R&D incentives an analogous link must also exist between the latter and the level of technology gap.

In order to demonstrate the relationship between technology gap and R&D incentives, we develop a two stage game of process R&D and output competition for an ex-ante asymmetric duopoly with one-way spillovers.[4] In the model, at the first stage the two firms conduct process R&D and in the second stage, they compete in Cournot fashion in the product market. We go one step further than Katsoulacos and

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[1] For the follower, imitation is a better strategy than innovation as the positive externality created by the leader's research makes learning and reverse engineering easier. However, when all gains from such spillovers have been extracted, the follower might find it more profitable to innovate.

[2] While more recent studies have attempted to endogenize spillovers in an AJ framework (see Amir, Estignev and Wooders (2003)), this is the first attempt to show that the nature of such endogenity turns on the level of technology gap between the two firms.

[3] The frontier is defined as the level of technological efficiency of the leader firm.

[4] In contrast to the traditional AJ framework in which both firms benefit from spillovers, we consider the case in which only the follower can free ride off the leader. Amir and Wooders (2000) also consider one-way spillovers in a two stage game of process R&D.

Ulph (1998) and Kultti and Takalo (1998) who were the first[5] to endogenize spillovers in an AJ framework. Our R&D spillover function does not depend solely on the absorptive capacity effect as in the latter studies since it also takes into account the size of the technology gap between the two firms. We seek to extend existing theoretical framework by incorporating the impact of such endogenous spillovers on the benefits and the costs of R&D.[6]The effect of the spillovers on the cost of undertaking R&D has the following interpretation. Assuming that the spillover rate is endogenous (positively related to the size of the technology gap), then the further away the firm is from the frontier, the less technologically efficient it is, that is; it finds it more costly to undertake R&D when the technology gap is large. Hence, firms operating well within the frontier incur greater costs of doing research since the size of the technology gap (or endogenized spillover) is large.

Given this link between spillovers and technology gap, we consider the dynamic version of a two stage R&D game since we cannot observe changes in the magnitude of the gap over time in a static model. We derive our results based on the steady state values of R&D as well as on their transitional dynamic paths. Finally, we provide a general framework for analyzing dynamic two stage R&D games with endogenous spillovers. We present three different (though non-mutually exclusive) sets of results. First, we present a variant of the static AJ model with one-way endogenous[7] spillovers. We show that the existence of a subgame perfect Nash equilibrium

[5] Subsequent attempts to endogenize spillovers in an AJ framework have been made by Kamien and Zang (2000), Amir, Evstignev and Wooders (2003) and Grunfeld (2003).

[6] In the current literature, while spillovers increase the benefits of the firm's R&D by reducing its costs of production by an amount proportional to its rival's investments, they do not affect the cost of undertaking R&D.

[7] We assume that the marginal cost of production of the follower also depends on the technology gap between the leader and itself. The AJ model will be the special case where the technology gap reduces to zero.

(SPNE) requires that the level of spillovers to be low and the initial marginal cost to be high. We show that the relationship between the free-riding behavior of the laggard and the level of spillovers is non-monotonic. We observe that they are positively related as long as the size of spillovers is small.

Secondly, we develop a dynamic version of the latter model in a differential game setting. It is shown that if each firm in the industry takes into account the dynamic strategic response of its rival, results can be derived by looking at the transitional dynamics of the firms' reaction functions in the neighborhood of the steady state. While in low cost industries[8], we find that there exists no steady state with complete catch-up[9], in high cost industries we observe that there exists a unique and stable steady state with complete catch-up (ex-post symmetry).Lastly, we provide a general framework for analyzing dynamic AJ models with one-way endogenous spillovers. We derive some general conditions that would guarantee the existence of a steady state in a more general class of two stage R&D games with spillovers. In doing so, we also outline the cases when R&D spillovers can act as a deterrent to future research.

Our contribution to the literature might be described as follows. While recent attempts to endogenize spillovers in the AJ paradigm take into account the firm's absorptive capacity only, we show, by introducing the concept of technology gap, that the follower firm's incremental R&D effort does not only enhance its capacity to learn (by reducing its own R&D costs) but also, after some point in time, begins to reduce its marginal benefits too.[10] This is due to decreasing returns to scale to

[8] Here we refer to the marginal cost of production.
[9] By complete catch-up we mean that the technology gap equals zero.
[10] Kamien and Zang (2000) emphasize that the followers themselves must invest in R&D in order to take advantage of the R&D innovations of others (the absorptive capacity effects).

imitation or reverse engineering activities. Thus the follower firm benefits more from the spillovers when the technology gap is large than when it is small; in other words, the laggard's marginal benefits from the spillovers decrease in the level of the technology gap. Also, its cost of R&D falls as the gap becomes smaller due to the absorptive capacity effect. In view of the key role that spillovers play in the current two stage R&D game literature, we believe that it necessary to ascertain whether the existing results remain robust to a more general version of one-way endogenous spillovers. Moreover, our framework will nicely capture the notion that both the R&D benefits and costs of a lagging firm change with its research expenditure when the technology gap is endogenized.

The remainder of the paper is organized as follows. Section 2 discusses some background literature. Section 3 presents a static version of the AJ model with one way spillovers. In Section 4 we study the dynamic version of the AJ model. A general framework is proposed in Section 5. We conclude in Section 6.

2. Related Work

In this section, we provide a brief overview of relevant studies. Our contribution builds on D'Aspremont and Jacquemin's (AJ) (1988) simple model of symmetric duopoly of R&D. The authors compare several equilibrium concepts (the two stage noncooperative solution, the two stage mixed game solution, the two stage fully cooperative solution and the socially optimal solution) in a static two stage game theoretic setting. Two important features of their model are the exogenous nature of spillovers and the range of values of the spillover rate for which the Cournot-Nash equilibrium values of output and R&D are stable. Clearly, assumptions on the level or nature of spillovers can affect results significantly in R&D models with externalities. It is therefore important to treat such spillovers as important determinants of R&D rather than just an exogenously given parameter.

Henriques (1990) shows that for very small spillovers the AJ model's comparison between the pure cooperative and pure noncooperative games does not hold because the noncooperative model would be unstable. This highlighted the importance of setting proper parameter restrictions in accordance to the relevant existence and stability requirements in standard R&D models. Henriques also proposed that this could be achieved by choosing a feasible range of values of the spillover rate for which stability would be guaranteed.[11]Other related studies by Suzumura (1992) and Simpson and Vonortas (1994) have compared the noncoopertive regime with the cooperative one in terms of social efficiency. They found that, while both the noncoopertive and cooperative levels of R&D are suboptimal in the presence of spillovers, the noncoopertive level might overshoot the socially optimum level in the absence of spillovers. Given the important

[11] Henriques found that the stability conditions can be met if and only if the spillovers are not too small.

part played by spillover in the above literature, we propose to conceptualize such spillovers in a somewhat more general approach in order to shed some light on the mechanism by which they affect R&D decisions.[12]

While the above studies model the firm's cost reductions by the sum of its own autonomous R&D and a proportion of the rival's R&D, Kamien , Muller and Zang (1992) measure the spillover effect in terms of R&D dollar expenditure.[13]Our reliance on the technology gap to explain the endogenity of spillovers makes ex-ante asymmetry an important necessary feature of our model; that is, there always exists a leader and a follower configuration at least initially. One way to incorporate such asymmetry in the AJ framework is to consider the case where only the follower can free-ride off the leader. Amir and Wooders (1999) show that it is possible that the standard symmetric two periods R&D model with one-way spillovers leads to an asymmetric equilibrium when there is an endogenous imitator/ innovator configuration. They argue that know-how may only flow from the more R&D intensive firm to its rival but never in the opposite direction. Moreover, in contrast to the existing literature they use a stochastic spillover process and their findings indicate that the extent of the firms' heterogeneity depends on the spillover rate. They also show that an optimal cartel might seek to minimize the spillovers between members. In another study with similar settings, Amir and Wooders (2000) explain the existence of the imitator/ innovator pattern in some industries by using the one-way spillover structure. Furthermore, they demonstrate how the concept of

[12] Although most studies of the current literature compare the cooperative and the noncooperative R&D levels with the socially optimal level, we only look at the noncooperative case as the dynamic version of the cooperative case might require further assumptions.
[13] Kamien , Muller and Zang (KMZ)'s (1992) R&D specification is another way (distinct from AJ) of modeling knowledge externalities. Amir (2000) gives a detailed comparison and a critique of the two frameworks. He also shows the conditions under which equivalence would hold between the two models.

submodularity can be used in the same framework to provide a general analysis of R&D games.

Katsoulacos and Ulph (1998) were the first to endogenize spillovers in the two stage R&D game. In contrast to previous works which considered the spillover rate as purely exogenous when comparing the cooperative case with the nocooperative regime, they focus on the impact of research joint ventures on innovative performance. They argue that "it seems somewhat odd to treat a component of this (innovative performance) – the flow of spillovers from one firm to another – as purely exogenous". They find that either maximal or minimal spillovers will be chosen in a noncooperative setting while partial spillovers are chosen in the cooperative case. The concept of endogenous spillovers is explored further by Kamien and Zang (2000), who argue that the firm cannot capture any spillovers from its rival without engaging in R&D itself. By incorporating absorptive capacity as a strategic variable, they distinguish between two components of spillovers; an exogenous component which represents involuntary spillovers from the firm's R&D activity and an endogenous component that allows the firm to exert control over spillovers. They find that if firms choose identical R&D approaches[14]in the first stage, they would cooperate in the setting of their respective R&D budgets, while if they choose firm specific R&D approaches in the first stage they will not form a research joint venture. Slight changes in the treatment of spillovers can, therefore, alter the results in the two stage R&D game.

More recently, a dynamic feedback game with endogenous absorptive capacity has been developed by Campisi , Mancuso and Nastasi (2001) that derives the existence and uniqueness of Nash equilibrium conditions in a feedback R&D game with spillovers.

[14] R&D approach refers to the firm's choice of the extent of spillovers it allows its rival to enjoy.

However, although they take into account the effects of absorptive capacity, they assume the spillover rate to be constant over time and conclude, not surprisingly, that variations in such externalities hardly affect the firm's R&D investments even if its capacity to exploit such knowledge were to be endogenous. Another study which links learning capacity to spillovers was conducted by Martin (2002). Although his objective is primarily to distinguish between input spillovers (as in KMZ) and imperfect appropriability (as in AJ), his findings, that the firm's value is maximized with complete appropriability, and results which remain robust when the model is extended to allow for endogenous absorptive capacity are helpful to our study.

Grunfeld (2003) shows that contrary to Kamien and Zang's (2000) findings, absorptive capacity effects of the firm's own R&D do not necessarily drive up their investment incentives. Moreover, he argues that learning effects affect the critical rate of spillovers which would determine whether a research joint venture generates more R&D investment than in a noncooperative setting. An important feature of their study is that they highlight the two opposing effects of absorptive capacity created by R&D investment. In a generalized version of R&D games with endogenous spillovers, Amir, Evstignez and Wooders (2003) capture nicely the scope for cooperative behavior by endogenizing the value of the spillover rate and show, by providing a sufficient condition for such an outcome, that firms would always choose extremal spillovers.

Our work is also related to some studies in the area of dynamic games. Ruff (1969) was the first to consider R&D dynamic game in an infinite horizon Cournot economy in which firms choose R&D efforts in the presence of spillovers. He compared the noncooperative solution to the cooperative and the socially optimal ones and his conclusions support the Schumpeterian view that dynamic performance is more important than static efficiency. Reinganum (1982) developed an R&D differential game to derive the dynamic optimal allocation of R&D and found that the availability of perfect information accelerates the development of innovations and that the impact of rivalry on Nash equilibrium investment will depend on the appropriablity level (the spillover rate in our model). Recent developments in Non-Schumpeterian growth models by Vencatachellum (1998) and Traca and Reis (2003) show that dynamic interactions between firms ought to be incorporated into the micro foundations of dynamic general equilibrium models.

By allowing the spillover rate to be endogenously dependent on the level of the technology gap between firms, this paper augments the widely used AJ model in several important ways. It explores further the asymmetry which one-way spillovers can generate while taking seriously the notion of absorptive capacity (as in Katsoulacos and Ulph (1998)). Moreover it introduces a time variant technology gap in a differential game to show under which conditions R&D spillovers crowd out research incentives and when they do not. It is also to be noted that in contrast to the current literature, we do not consider the cooperative and socially optimal cases as we focus only on the noncooperative case.

3. D'Aspremont and Jacquemin (AJ) Revisited – The Static Case

In this section we look at the augmented version of the AJ model. Consider an industry with a duopolistic market structure in which two firms (firm 1 and firm 2) engage in a two stage R&D game. At the first stage, firms 1 and 2 conduct process R&D by choosing their research intensity (the amount by which they reduce their costs of production) X_1 and X_2 respectively. In the second stage, the firms compete in Cournot fashion in the product market. As in AJ we assume that the demand faced by the two rivals is linear with the slope -1.[15] The demand schedule is given by

$P = A - Q$, where $Q = q_i + q_j, i \neq j, i - 1,2$, and q_i is the output of firm i. (1)

We impose an ex-ante asymmetry between the two firms both on the marginal benefit and on the cost of their R&D. In particular, on the marginal benefit (or marginal cost reduction) side of R&D, we assume a one-way spillover structure in which only firm 1, the "follower"[16] can benefit from spillovers from firm 2, the "leader", but not vice-versa. Moreover, the spillover rate depends positively on the technology gap between the two firms. On the cost side of R&D, we assume that the firm benefiting from spillovers (the follower) incurs a higher R&D cost than the leader. Also, the larger the technology gap between the two firms, the higher the R&D cost of the follower. In other words, our assumption states that while free-riding opportunities reduce as the technology gap becomes smaller, so does the R&D cost for a follower firm. The per unit production marginal costs for the follower and the leader are given respectively by the following equations.

$$C_1 = C - X_1 - \beta X_2 \quad (2)$$

[15] This does not lead to a loss of generality.
[16] Note that "leader" and "follower" are not used in the Stackelberg sense here.

$$C_2 = C - X_2 \tag{3}$$

where $X_1, X_2 \in [0, C)$

We shall assume β to be given by[17]

$$\beta_1 \equiv \frac{b(X_2 - X_1)}{1 + bX_2} \tag{4}$$

where $b \in (0,1)$ is the spillover parameter. We also impose the following parameter restriction

$$C > A \tag{5}$$

β is the level of spillovers, due to the leader's (firm 2) R&D, which accrues to the follower (firm 1). A crucial departure from the standard AJ framework is that there exists an asymmetry characterized by one-way spillovers. The leader's marginal cost (3) is decreasing in the level of research that it undertakes and is ex-ante unaffected by the R&D of the follower. The latter's per unit cost (2) is not only reduced by its own R&D effort but also by a proportion of that of the leader's. The spillover parameter b, can account for involuntary leakage or voluntary exchange of technological information. One can also interpret b as a parameter that is inversely related to the degree of patent protection or appropriability. The parameter restriction given by (5) implies the existence of large per unit marginal cost in the industry. It can be easily verified that if the R&D levels for both firms are zero, they will choose not to produce in the second stage. Thus, a minimum R&D level, at least by firm 2, is necessary to ensure a positive output in the product market. While our assumptions restrict our study only to industries where costs are large enough so that (5) is satisfied, they do not rule the possibility in which the

[17] In the next section, we provide a formal way of deriving this expression.

optimal R&D levels for both firms are zero, so that they both do not produce in the second stage. Thus, the infinimum (zero) of the interval containing X_i for i=1, 2 is included in the interval. However, we assume that the initial marginal cost (without R&D) is so large that it is not feasible for firm i to undertake any R&D level X_i such that $X_i = C$. In other words, the supremum (C) of the interval containing X_i for i=1, 2 is not included in the interval. As a corollary, we also have $X_1 + \beta X_2 < C$ for all $\beta \geq 0$ and also that $C_1 > 0$ if $\beta < 0$. Therefore, it is never possible for firm i to reduce its marginal cost to zero in our setting and hence, unlike other AJ models in which the additive marginal benefit functions are unable to exclude the possibility of maximal costs reductions (See Amir, 2000), our assumptions of large costs rule out this possibility. The firms R&D costs functions are assumed to be as follow,

$$C_1^{R\&D} = (1+\beta)C(X_1) + bX_2 \quad \text{for firm 1} \tag{6}$$

$$C_2^{R\&D} = C(X_2) \text{ for firm 2} \tag{7}$$

where $C(\cdot)$ is monotonically increasing in its argument

The R&D cost function of the follower (6) is new and a justification is warranted. The reason for having the term $(1+\beta)$ in the follower's R&D cost function is owing to the fall in its research costs when it approaches the technological frontier. It is not difficult to imagine β as representing the distance between the technology levels of the two firms, in which case a higher (lower) β will imply a larger (smaller) technology gap. Thus, initially, when the follower is a new entrant, β is high and the marginal benefits of free riding are large. However, when its technological efficiency approaches that of the leader, β becomes lower and due to decreasing returns to imitation the marginal benefits begin to fall. Now, when the follower's technology level approaches that of the leader, it must be the case that its R&D costs fall and gradually converge on that of the leader. Hence, while technology catch-up implies reductions in the benefits of free riding, it also lowers research costs. If $\beta = 0$ it means that catch-up is complete as the follower's and the leader's marginal benefits and marginal costs of R&D become identical. If $\beta < 0$, the follower would have leapfrogged the leader and the marginal benefit of free riding would be negative (the last term in the RHS of (2) becomes greater or equal to zero so the total benefit due to spillovers is declining).Also, the cost of R&D of the follower also becomes smaller as β becomes more negative. The last term on the RHS of (6) shows that some learning or reverse engineering costs are incurred by the follower in order to benefit from the leader's research. Thus we assume that the externalities accrued by the follower do not take the form of a pure public good. This assumption also implies that if it is optimal for the follower to free-ride completely on the leader, by choosing $X_1 = 0$, then it also has to bear some costs given by bX_2 . Models which employ the additive marginal benefit function as in the AJ framework are often criticized for they cannot exclude the possibility that a firm can fully free-ride costlessly on its rival (see Amir, 2000). The cost function given by (6) allows our model to survive the so-called Amir's critique. Moreover, for simplicity, we shall assume that $C(X_i)$ for i=1, 2 is linear and is given by $C(X_i) = X_i$.[18] This assumption which does not violate the quasi-concavity of the first

[18] While a strictly convex cost function is one of the necessary ingredients to guarantee the existence of a maximum point in the AJ class of games, the endogenous spillover rate of our model makes this assumption redundant. In other, words, due to (4), usual parameter restrictions can be used to ensure the quasi –concavity of (11) and (12).Moreover, the Hessian matrix of (11) and (12) are found to be negative definite for some region of the parameter space. Poof can be provided upon request.

stage profit function will be relaxed in the next sections. It is also to be noteworthy that the underlying asymmetry (leader/ follower configuration) in our model is only observed through the marginal cost reduction and the R&D cost functions and that we do not impose the condition $X_2 > X_1$. Hence, even when the follower leapfrogs the leader so that $X_2 \leq X_1$, the forms of the benefit and cost functions do not change. Thus, while the follower, being a new entrant with low R&D, can free-ride on the leader's existing technology by incurring some costs, the leader in turn cannot learn or benefit from the laggard's technology. Note that as b approaches 1, the technology gap is still less than 1.

The timing of the game is as follows. In the first stage the firms choose their respective R&D levels X_i and in the second stage they choose their respective quantities q_i. As is customary in the analysis of staged games, we work backwards from the last stage to the first. Thus the firm's problem in the second stage is given by

$$\underset{q_i}{Max}\left(A-\left(q_i+q_j\right)\right)q_i - C_i q_i, i=1,2, i \neq j \tag{8}$$

It is well known that the Cournot second stage quantity and profit function for firm i is given by

$$q_i = \frac{A-2C_i+C_j}{3} \tag{9}$$

$$\pi_i = \frac{\left(A-2C_i+C_j\right)^2}{9} \tag{10}$$

(2), (3), (6), (7) and (10) imply that the first stage objective functions for the follower and the leader are given respectively by

$$\underset{X_1}{Max}\frac{\left(A-2\left(C-X_1-\beta X_2\right)+C-X_2\right)^2}{9}-\left(1+\beta\right)C\left(X_1\right) \qquad \text{for firm 1} \tag{11}$$

$$\underset{X_2}{Max}\frac{\left(A-2\left(C-X_2\right)+C-X_1-\beta X_2\right)^2}{9}-C\left(X_2\right) \qquad \text{for firm 2} \qquad (12)$$

where β is given by (4).

The firms' R&D reaction functions are given respectively by[19]

$$-\left(A-C+2X_1-X_2\right)+\frac{9}{4}\left(1+2bX_2-2bX_1\right)=\frac{2bX_2\left(X_2-X_1\right)}{1+bX_2} \quad \text{for firm 1} \qquad (13)$$

$$\frac{2}{9}\left(A-C+\left(2-\frac{b\left(X_2-X_1\right)}{1+bX_2}\right)X_2-X_1\right)\frac{2+2bX_2+bX_1}{\left(1+bX_2\right)^2}=1 \qquad \text{for firm 2} \qquad (14)$$

Proof:

See Appendix

It can be shown that (13) simplifies to

$$\frac{9}{4}-\left(A-C\right)+\left(\left(\frac{27}{4}-\left(A-C\right)\right)b+1\right)X_2-\left(\frac{9}{2}b+2\right)X_1-\frac{9}{2}b^2X_2X_1+\left(\frac{9}{2}b^2-b\right)X_2^2=0 \qquad (15)$$

We can now demonstrate our results via the following propositions.

Proposition 3.1.1

Assume that $C > A$ and b is small, then

(i) *there exists a level of X_2^M beyond which X_1 starts to fall, that is, there exists a level of the leader's investment beyond which the laggard will start to free-ride.*

(ii) *there exists an inverted U-shaped relationship between b and $-\Delta X_2^M$[20], that is, while free-riding increases with the spillover rate initially, it becomes inversely related to the spillover rate as the latter gets larger.*

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[19] The second order conditions are satisfied for small values of b and $C > A$. Proof can be provided upon request.

Proof:

See Appendix

By providing a key relationship between the free-riding and the spillover rate, Proposition 3.1.1 provides some theoretical support to the findings of Aghion et al. (2001) that there exists an inverted U-shaped relationship between imitation and innovation. As shown in the proof, there exists a cut-off value of the spillover rate b beyond which free-riding starts to fall. Thus, as pointed out by Cameron (1999) there are diminishing returns to scale to imitation. Intuitively, this happens when the follower has extracted all possible imitative possibilities from the leader. It is noteworthy that some computer generated simulations of the laggard's reaction function also show the same result. (See Figures 1-4)

Figures 1 – 4 (About Here)

Proposition 3.1.2

Assume that $C > A$ and b is small, then the Nash equilibrium of the game exists and is non-unique.

Proof:

See Appendix.

As shown in the appendix, the derivation of Proposition 3.1.1 is based on the graphs of reaction functions (13) and (14). Since we have already assumed $C > A$ from (5) and we know from the existence proof (see Appendix) that both firms choose some positive X_i, the parameter value of C can be chosen ex-post so that X_i for i=1,2 are

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[20] Note that $-\Delta X_2^M$ (change in the maximum value of firm 2's R&D) captures free-riding.

strictly less than C.[21] We also show that more than one (nonzero) fixed point can exist. Therefore in a leader/follower market configuration both firms will optimally undertake some minimum R&D as long as the spillover rate parameter is not too high and the initial marginal cost is not too low. This is not surprising as excess externalities might lead to a very low level of R&D effort by the follower who would rather free ride on the leader. As a result, the latter's R&D effort might be reduced to zero.

[21] Note that this is possible since the upper bound (C) of X_i is not in the interior of the interval containing X_i.

4. The Dynamic Case

In this section we shall exploit the fact that spillovers are endogenous, together with the dynamic nature of technology gap to extend the existing AJ framework. Subtracting (2) from (3), we can define the gap at time t as

$$G_t = (1-\beta)X_{2t} - X_{1t} \tag{16}$$

We shall assume[22] that the spillover takes the following form

$$\beta_t \equiv bG_t, b \in (0,1) \tag{17}$$

(17) implies a positive relationship between the spillover rate and the technology gap. As mentioned earlier, we emphasize the idea that when the gap is large, the benefits to free riding are greater than when the gap is small.

Since the firms will be playing a differential game, some amendments to the features of the model in the previous section are made without resulting in a loss of generality and without changing the essence of the previous game. First, in the dynamic game, the firms will be choosing their R&D investments I_{it} (rate of change of R&D over time) rather than the R&D level X_{it}. We do not restrict I_{it} to be positive. A negative I_{it} is realized when an asset is sold or disposed. Thus, we have

$$\dot{X}_{it} = I_{it}, i = 1,2 \tag{18}$$

For simplicity it is assumed the depreciation rate of zero. Secondly, the cost of such investments for the follower and the leader are given respectively by;

$$C_{1t}^{R\&D} = \frac{I_{1t}^2(1+\beta_{1t})}{2X_{1t}} \qquad \text{for firm 1} \tag{19}$$

[22] Linearity is assumed for simplicity and to make the solution more tractable.

$$C_{2t}^{R\&D} = \frac{I_{2t}^2}{2X_{2t}} \qquad \text{for firm 2} \tag{20}$$

We observe from (19) that if firm 1 decides to fully free-ride on the leader by choosing a very low X_{1t}, then its cost of R&D would become very large (close to ∞). Such a formulation is robust to the Amir's critique.

The timing of the game is such that there are two stages in each period. Now, since in the second stage of each period the firms choose their output, which are ex-post functions of X_{it} and X_{jt} only, the reduced forms of the firms' first stage objective functions can be derived so that the only choice variables of the dynamic game are R&D investment levels. Thus, firms compete to find their optimal R&D time path. Moreover, we shall use the concept of open-loop Nash equilibrium[23] to solve the differential game. It is also important to note that unlike the case described by Vencatachellum (1998), the open loop Nash equilibrium in our model does not coincide with the myopic strategy whereby the firm does not take into account the R&D of its rival while choosing its optimal path.[24] We shall now use (16) and (17) to endogenize the spillover rate in the model. (17) and (19) can be rewritten as

$$\beta_{1t} \equiv \frac{b(X_{2t} - X_{1t})}{1 + bX_{2t}} \tag{21}$$

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[23] An open-loop Nash equilibrium is found when a competitor takes his rival's reaction function solely as a function of time in his dynamic optimization problem. For more details see Dockner et al (2000). We assume an open-loop equilibrium as in Peretto (1996) since we are unable to find a closed-form solution to analyze the properties of the model for a closed-loop or Markov perfect equilibrium. In principle, if the objective function is of linear quadratic form, the closed-loop equilibrium can be found by setting the Hamilton Jacobi Bellman (HJB) equation.

[24] This special case arises since the Hamiltonian function of his model is linear and separable in its rival's stock of human capital and hence, the latter term vanishes at the first order condition.

$$C_{1t}^{R\&D} = \frac{I_{1t}^2}{2X_{1t}}\left(\frac{1+2bX_{2t}-bX_{1t}}{1+bX_{2t}}\right) \tag{22}$$

Since the quantity and profit functions of the first stage are unchanged, we only find the open loop Nash equilibrium of the first stage.

Definition 4.1.1

The pair (θ_1, θ_2) is called an open-loop Nash equilibrium with function θ_j mapping t $\in [0,T)$ to a real number if for each j = 1, 2 , an optimal control path I_j(.) of the problem below exists and is given by $I_j(t)= \theta_j(t)$.

Using (10), (18), (21) and (22), we can write firm 1 dynamic problem as

$$V_1 = \underset{I_{1t}}{Max}\int_0^\infty e^{-\rho t}\left\{\frac{1}{9}\left(A-2\left(C-X_{1t}-\frac{b(X_{2t}-X_{1t})X_{2t}}{(1+bX_{2t})}\right)+C-X_{2t}\right)^2-C_{1t}^{R\&D}\right\}dt$$ [25]

s.t $\dot{X}_{it} = I_{it}, i=1,2$, $X_{20} > X_{10}$ [26] is given and $\underset{Lim, T\to\infty}{X_{iT}} \geq 0, \rho$ is the interest rate (23)

Using (11), (18), (20) and (21), we can write firm 2's dynamic problem as

$$V_2 = \underset{I_{2t}}{Max}\int_0^\infty e^{-\rho t}\left\{\frac{1}{9}\left(A-2(C-X_{2t})+C-X_{1t}-\frac{b(X_{2t}-X_{1t})X_{2t}}{(1+bX_{2t})}\right)^2-C_{2t}^{R\&D}\right\}dt$$

s.t $\dot{X}_{it} = I_{it}, i=1,2, X_{20} > X_{10}$ is given and $\underset{Lim, T\to\infty}{X_{iT}} \geq 0$ (24)

The current value Hamiltonian function[27] and first order conditions for firm 1 are given by

[25] We assume that the price of investment (P_t) is equal to zero so that $P_t I_{it} + C_{it}^{R\&D} = C_{it}^{R\&D}$
If investment is negative in some period ($I_{it} < 0$), then the firm incurs a loss of $C_{it}^{R\&D}$.

[26] Here we assume that the leader's R&D level is higher, that is, the technology gap is positive at least initially.

$$H_1 = \frac{1}{9}\left(A - 2\left(C - X_{1t} - \frac{b\left(X_{2t} - X_{1t}\right)}{1 + bX_{2t}}\right) + C - X_{2t}\right)^2 - \frac{I_{1t}^2}{2X_{1t}}\left(\frac{1 + 2bX_{2t} - bX_{1t}}{1 + bX_{2t}}\right) + \lambda_{1t}I_{1t} + \lambda_{2t}I_{2t} \tag{25}$$

$$\frac{dH_1}{dI_{1t}} = 0 \Rightarrow \frac{I_{1t}}{X_{1t}}\left(\frac{1 + 2bX_{2t} - bX_{1t}}{1 + bX_{2t}}\right) = \lambda_{1t} \tag{26}$$

$$\dot{\lambda}_{1t} = \rho\lambda_{1t} - \frac{4}{9\left(1 + bX_{2t}\right)}\left(A - C + 2X_{1t} + \left(\frac{2b\left(X_{2t} - X_{1t}\right)}{1 + bX_{2t}} - 1\right)X_{2t}\right) + \frac{I_{2t}^2}{2X_{2t}}\left(\frac{-1 - 2bX_{2t}}{1 + bX_{2t}}\right) \tag{27}$$

$$\dot{\lambda}_{2t} = \rho\lambda_{2t} - \frac{2\left(2bX_{2t} - 2bX_{1t} - 1\right)}{9\left(1 + bX_{2t}\right)^2}\left(A - C + 2X_{1t} + \left(\frac{2b\left(X_{2t} - X_{1t}\right)}{1 + bX_{2t}} - 1\right)X_{2t}\right) + \frac{I_{1t}^2 bX_{1t}}{2X_{1t}^2}\left(\frac{1 + bX_{1t}}{\left(1 + bX_{2t}\right)^2}\right) \tag{28}$$

$$\underset{T \to \infty}{Lim}\, e^{-\rho t}\lambda_{1t}X_{1t} = 0 \tag{29}$$

$$\underset{T \to \infty}{Lim}\, e^{-\rho t}\lambda_{2t}X_{2t} = 0 \tag{30}$$

The current value Hamiltonian function and first order conditions for firm 1 are given by

$$H_2 = \frac{1}{9}\left(A - C + \left(2 - \frac{b\left(X_{2t} - X_{1t}\right)}{1 + bX_{2t}}\right)X_{2t} - X_{1t}\right)^2 - \frac{I_{2t}^2}{2X_{2t}} + \mu_{1t}I_{2t} + \mu_{2t}I_{1t} \tag{31}$$

$$\frac{dH_2}{dI_{2t}} = 0 \Rightarrow \frac{I_{2t}}{X_{2t}} = \mu_{1t} \tag{32}$$

$$\dot{\mu}_{1t} = \rho\mu_{1t} - \frac{2\left(2 + 2bX_{2t} + 2bX_{1t}\right)}{9\left(1 + bX_{2t}\right)^2}\left(A - C + \left(2 - \frac{b\left(X_{2t} - X_{1t}\right)}{1 + bX_{2t}}\right)X_{2t} - X_{1t}\right) - \frac{I_{2t}^2}{X_{2t}^2} \tag{33}$$

[27] We assume that the exogenous variables A, b and C are chosen so that they belong to the subset of real numbers which would satisfy the Mangasarian second order sufficient conditions for the dynamic problems of the two firms.

$$\dot{\mu}_{2t} = \rho\mu_{2t} - \frac{2}{9(1+bX_{2t})}\left(A - C + \left(2 - \frac{b(X_{2t} - X_{1t})}{1+bX_{2t}}\right)X_{2t} - X_{1t}\right) \tag{34}$$

$$\lim_{T\to\infty} e^{-\rho t}\mu_{1t}X_{2t} = 0 \tag{35}$$

$$\lim_{T\to\infty} e^{-\rho t}\mu_{2t}X_{1t} = 0 \tag{36}$$

The above equations enable us to derive the main result of this section. In particular we are now in a position to look at the dynamic equilibrium of the R&D two stage game in which the technology gap evolves over time.

Proposition 4.1.1

Assume that at the steady state $I_{1t} = I_{2t} = 0$*. If the conditions given in the above open loop differential game hold, then*

- *(i)* *there exists a steady state subgame perfect Nash equilibrium given by* $X_{1t}^{*} = X_{2t}^{*} = C - A$ *for all* $C > A$*,*
- *(ii)* *the equilibrium is unique,*
- *(iii)* *there exists a stable path converging to it and*
- *(iv)* *the path is unique as it is the Saddle Path.*
- *(v)* *at the steady state equilibrium of the full game,* $q_{it} \to 0$ *as* $X_{it} \to C - A$ *for i=1,2 and there exists a neighborhood around the steady state in which both* q_{it} *and* X_{it} *are falling*

Proof: See Appendix.

Proposition 4.1.1 states that if the R&D of both firms were to stop growing at a so-called “steady state” then it must be the case that technology catch-up has taken place in the industry as long as the marginal costs are large enough. Therefore, when the basic

AJ model is augmented to allow for an endogenous spillover rate which depends on the dynamics of the technology gap between the two firms, ex-post symmetry occurs. Thus, the follower, benefiting from the externalities, builds up more absorptive capacity and the leader responds to this by reducing its own R&D. As a result, the technology gap shrinks over time.

The intuition behind this proposition is best described in terms of two opposite driving forces affecting R&D incentives. The first one originates from the competitive pressure which the follower puts on the leader so that the latter has no other alternative than to increase its own R&D effort in order to maintain its market share advantage. The second effect arises when the leader reduces its R&D effort in anticipation that the follower might free ride excessively from its research activities. Our findings corroborate those of Grunfeld (2003) who shows that the relative strengths of the two effects are determined by the follower's level of absorptive capacity. Thus when the size of the technology gap is small (high absorptive capacity), there are weaker incentives for R&D efforts. It is also noteworthy that the model predicts a unique and stable Nash equilibrium in contrast to the previous section.[28]

We also find that when the R&D levels of both firms converge to their steady state values, the second period quantity produced converge to zero. We also show in the appendix that this can happen only if the investment levels of both firms are negative in the neighborhood of the steady state. Thus, this is a clear example of creative destruction

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[28] Note that $I_{1t} = I_{2t} = 0$ *is* not the only way of characterizing steady state. $\frac{I_{1t}}{X_{1t}} = \frac{I_{2t}}{X_{2t}}$ is another possible steady state configuration in which all variables grow at a constant rate in equilibrium. Also note that the requirement that $C > A$ would imply that it is possible that the optimal (product) output for one firm is zero in some periods.

or what is also referred to as the Schumpeterian effect at work. Intuitively, the laggard's free-riding behavior causes the leader to invest too little. As a result, inadequate R&D levels fail to reduce the firms' marginal cost of production by an amount which would guarantee a positive quantity of goods produced at all times.

5. A General Model of Dynamic R&D with Endogenous Spillovers

The aim of this section is to provide a more general class of results for dynamic R&D games with uni-directional spillovers. In particular we ask which criteria (if any) are required to determine whether a Schumpeterian effect (as in Section 4) or a Non-Schumpeterian effect will prevail in a dynamic two stage R&D game with endogenous spillovers.

In what follows, we will develop a theory of dynamic optimal investment in an economy where the representative industry is a duopoly with one-way endogenous spillovers and in which firms play a two stage R&D game in each period. Among the few necessary amendments to the model of Section 4 are: An economy in which savers are the shareholders of the firms, a more general specification of the R&D cost function, a generic distance function to characterize the technology gap and some basic assumptions on the smoothness of the profit function. The requirement of a consumer-side economy is necessary not so much for the sake of completing the model in the general equilibrium sense but most importantly to allow for a platform (stock market) on which the firm's R&D investment can be valued.

5.1 The Model

We shall consider an economy as in Aghion, Harris, Howitt and Vickers (2001) in which labor supply is perfectly elastic and each consumer consumes a constant proportion of their income in each period. This setting has two important properties. First, the discount rate is equal to the rate of interest, that is ρ[29] $= r_t$ and secondly, labor supply decisions are exogenously given. Since our main concern is to look at the dynamics of the

[29] Abuse of notation: ρ has been used as the interest rate in the previous section. However, due to the above property they will be treated as equal henceforth.

duopolists' R&D investments, such an extreme simplification of the consumer economy allows us to "transfer" all the dynamics from consumption to production. The production side of the economy consists of n identical duopolistic industries in which the firms play a two stage R&D game. Here one should note that owing to the assumption that the n industries are identical, the growth rate of the whole economy can be extrapolated from the growth rate of the representative industry. Now, since the only variables growing in any industry are the two rival firms' R&D spending, it must be the case that the dynamic two stage R&D game fully characterizes the economy's behavior. Thus, this "reduced form" of the dynamic general equilibrium model of the current section, is similar to the starting points of the R&D games discussed in Sections 3 and 4.

The duopolists will take the demand derived from the consumer problem as given and compete in Cournot fashion in the output market in the second stage of the game. We shall make the following assumptions:

Assumption 5.2.1

The first stage reduced form benefit function is given by

$$\pi_i \equiv \pi_i(X_i, X_j, \beta_t), i = 1,2, \ , i \neq j \tag{37}$$

$$\text{where } \beta \equiv \beta(X_i, X_j, b), \ b \in (0,1) \tag{38}$$

Also $\pi_i(\cdot)$ *is twice continuously differentiable, increasing and strictly concave in* X_i.

Moreover it is decreasing in X_j *and* $\pi_{iX_i}(\cdot)$ *is bounded from above.*

And $\beta_i(\cdot) \in (0,1)$, $\beta_{X_1}(\cdot) \leq 0$, $\beta_{X_2}(\cdot) \geq 0$.

We ought to impose a few more qualifications on (37) and (38). First, the demand schedule derived in the consumer problem and the marginal effective costs of production

(2) and (3) are chosen such that the conditions imposed on the profit function in assumption 5.2.1 are satisfied.[30]

Secondly, given the dependence of (37) on $\beta(X_i, X_j, b)$, one can observe that it is possible that $\pi_{1X_2}(\cdot) \geq 0$ due to the free riding effect. It is, therefore, plausible that at first glance our assumption that $\pi_{1X_2}(\cdot) \leq 0$ may seem contradicting. However, if we recall that the first stage profit function of firm 1 (the follower) is positively related to the effective cost of firm 2 (leader), we would understand that increases in X_2 (apart from the free riding effect) also reduces the leader's cost and hence the follower's profit. Hence the correct interpretation of this negative cross partial derivative would be that the negative effect (due to the leader's own cost reductions) of the leader's incremental research on the follower's profits exceeds the latter's benefits from free riding on the leader's research. Another feature of assumption 5.2.1 is that $\beta(\cdot)$ can be regarded as a distance function that corresponds to the technology gap and it can be shown that in any normed real vector space there exists a general class of functions satisfying the property: $\beta(X_i, X_j, b) = 0$ if and only if $X_i = X_j$.

Assumption 5.2.2

The R&D costs functions of firm 1 is given by

$$C_{1t}^{R\&D} = I_1 + \phi\left(\frac{I_1}{X_1}\right) I_1 \left(1 + \beta(X_1, X_2, b)\right) \quad , \tag{39}$$

[30] It is not difficult to verify that there exist a class of functions for the demand schedule and marginal cost curve that will satisfy those conditions.

$$C^{R\&D}_{1tI_{1t}} = 1 + \left(\phi(\cdot) + \left(\frac{I_1}{X_1}\right)\phi^{'}(\cdot)\right)(1+\beta(\cdot)) \geq 0$$

$$, C^{R\&D}_{1tI_{1t}I_{1t}} = \left(\frac{I_1}{X_1^2}\phi^{"}(\cdot) + \left(\frac{2}{X_1}\right)\phi^{'}(\cdot)\right)(1+\beta(\cdot)) \geq 0 \quad and \; C^{R\&D}_{1tX_{1t}X_{1t}} \geq 0 \qquad (40)$$

The R&D cost functions of firm 2 is given by

$$C^{R\&D}_{2t} = I_2 + \phi\left(\frac{I_2}{X_2}\right)I_2 \quad , \qquad (41)$$

$$C^{R\&D}_{2tI_{2t}} = 1 + \left(\phi(\cdot) + \left(\frac{I_2}{X_2}\right)\phi^{'}(\cdot)\right) \geq 0 \; , \; C^{R\&D}_{2tI_{2t}I_{2t}} = \left(\frac{I_2}{X_2^2}\phi^{"}(\cdot) + \left(\frac{2}{X_2}\right)\phi^{'}(\cdot)\right) \geq 0 \; and$$

$$C^{R\&D}_{2tX_{2t}X_{2t}} \geq 0 \qquad (42)$$

where I_i *is the rate of change of* X_i *over time*[31] *while* $\phi^{'}(\cdot)$ *and* $\phi^{"}(\cdot)$ *are the first and second derivatives of* $\phi(\cdot)$ *with respect to* I_i.

Assumption 5.2.2 simply implies a cost function that is monotonically increasing and convex in investment and is decreasing and convex in R&D stock. (39) and (41) also imply that the price of investment is equal to 1 for both firms. This completes the model.

5.2 Solving the Model

By using the principle of backward induction and bearing in mind that the second stage output level has already been chosen in terms of X_{it} and X_{jt}, we solve for the optimal I_{it} and I_{jt} in the first stage of the game by solving for the subgame perfect open loop Nash equilibrium. Firm 1 objective function is given by

$$\underset{I_{1t}}{Max} \int_0^\infty e^{-\rho t} \{\pi_{1t}(\cdot) - I_{1t} - (1+\beta(\cdot))I_{1t}\phi(\cdot)\} dt$$

s.t $\dot{X}_{it} = I_{it} - \delta X_{it}$, $X_{20} > X_{10}$ is given, and $\underset{Lim, T\to\infty}{X_{iT}} \geq 0, i = 1,2$, δ is the depreciation rate. (43)

[31] Time subscripts have been omitted in the above functions for convenience.

Firm 2 objective function is given by

$$\underset{I_{2t}}{Max}\int_0^{\infty} e^{-\rho t}\{\pi_{2t}(\cdot)-I_{2t}-I_{2t}\phi(\cdot)\}dt$$

$$\text{s.t } \dot{X}_{it}=I_{it}-\delta X_{it}\ ,\ X_{20}>X_{10} \text{ is given and } \underset{Lim,T\to\infty}{X_{iT}}\geq 0, i=1,2\ , \tag{44}$$

The Hamiltonian function and its first order conditions for firm 1 can be given as

$$H_1=\pi_{1t}(\cdot)-I_{1t}-(1+\beta(\cdot))I_{1t}\phi(\cdot)+\sum_{i=1}^{2}\lambda_{it}(I_{it}-\delta X_{it}) \tag{45}$$

$$\frac{dH_1}{dI_{1t}}=0\Rightarrow \lambda_{1t}=1+(1+\beta(\cdot))\left(\phi(\cdot)+\frac{I_{1t}}{X_{1t}}\phi'(\cdot)\right) \tag{46}$$

$$\dot{\lambda}_{1t}=(\rho+\delta)\lambda_{1t}-\left(\pi_{1X_{1t}}(\cdot)+(1+\beta(\cdot))\frac{I_1^2}{X_{1t}^2}\phi'(\cdot)-\phi(\cdot)I_1\beta_{X_{1t}}(\cdot)\right) \tag{47}$$

$$\dot{\lambda}_{2t}=(\rho+\delta)\lambda_{2t}-\left(\pi_{1X_{2t}}(\cdot)-\phi(\cdot)I_1\beta_{X_{t2}}(\cdot)\right) \tag{48}$$

$$\dot{X}_{it}=I_{it}-\delta X_{it}, i=1,2 \tag{49}$$

$$\underset{T\to\infty}{Lim}\, e^{-\rho t}\lambda_{it}X_{it}=0 \tag{50}$$

The Hamiltonian function and its first order conditions for firm 2 can be given as

$$H_2=\pi_{2t}(\cdot)-I_{2t}-I_{2t}\phi(\cdot)+\sum_{i=1}^{2}\mu_{it}(I_{it}-\delta X_{it}) \tag{51}$$

$$\frac{dH_2}{dI_{2t}}=0\Rightarrow \mu_{2t}=1+\left(\phi(\cdot)+\frac{I_{2t}}{X_{2t}}\phi'(\cdot)\right) \tag{52}$$

$$\dot{\mu}_{2t}=(\rho+\delta)\mu_{2t}-\left(\pi_{2X_{2t}}(\cdot)+\frac{I_2^2}{X_{2t}^2}\phi'(\cdot)\right) \tag{53}$$

$$\dot{\mu}_{1t}=(\rho+\delta)\mu_{1t}-\left(\pi_{2X_{1t}}(\cdot)\right) \tag{54}$$

$$\dot{X}_{it} = I_{it} - \delta X_{it}, i = 1,2 \tag{55}$$

$$\underset{T\to\infty}{Lim}\, e^{-\rho t}\mu_{it} X_{it} = 0 \tag{56}$$

Given the above first order conditions[32] we can now derive the main propositions of this paper. We start by deriving an important lemma which constitutes the basis for our propositions.

<u>*Lemma 5.3.1*</u>

If assumptions 5.2.1 and 5.2.2 hold, then λ_{1t} *and* μ_{2t} *are the net marginal values of the follower's and the leader's R&D respectively.*

Proof:

For the follower (firm 1), rewriting (47), yields

$$\dot{\lambda}_{1t} - (\rho + \delta)\lambda_{1t} = -\left(\pi_{1X_{1t}}(\cdot) + (1 + \beta(\cdot))\frac{I_{1t}^2}{X_{1t}^2}\phi'(\cdot) - \phi(\cdot)I_{1t}\beta_{X_{1t}}(\cdot)\right) \tag{57}$$

Multiplying both sides by $e^{-(\rho+\delta)t}$ and taking the integral with respect to time from zero to infinity on both sides and rewriting we have

$$\int_0^\infty \frac{d}{dt}\left(e^{-(\rho+\delta)t}\lambda_{1t}\right)dt = -\int_0^\infty e^{-(\rho+\delta)t}\left(\pi_{1X_{1t}}(\cdot) + (1 + \beta(\cdot))\frac{I_{1t}^2}{X_{1t}^2}\phi'(\cdot) - \phi(\cdot)I_{1t}\beta_{X_{1t}}(\cdot)\right)dt \tag{58}$$

From the First Fundamental Theorem of Calculus we have,

$$\left(e^{-(\rho+\delta)t}\lambda_{1t}\right)\Big|_0^\infty = -\int_0^\infty e^{-(\rho+\delta)t}\left(\pi_{1X_{1t}}(\cdot) + (1 + \beta(\cdot))\frac{I_{1t}^2}{X_{1t}^2}\phi'(\cdot) - \phi(\cdot)I_{1t}\beta_{X_{1t}}(\cdot)\right)dt \tag{59}$$

Using the transversality condition (50) we have

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[32] Note that though $\phi(\cdot)$ has been used interchangeably in both firms' problems, they are not equal in general since while for firm 1 the argument of $\phi(\cdot)$ is $\frac{I_1}{X_1}$, the argument for firm 2 will be $\frac{I_2}{X_2}$.

$$\lambda_{10} = \int_0^{\infty} e^{-(\rho+\delta)t} \left(\pi_{1X_{1t}}(\cdot) + (1+\beta(\cdot)) \frac{I_{1t}^2}{X_{1t}^2} \phi'(\cdot) - \phi(\cdot) I_{1t} \beta_{X_{1t}}(\cdot) \right) dt \tag{60}$$

Thus at any time t we have

$$\lambda_{1t} = \int_t^{\infty} e^{-(\rho+\delta)t} \left(\pi_{1X_{1t}}(\cdot) + (1+\beta(\cdot)) \frac{I_{1t}^2}{X_{1t}^2} \phi'(\cdot) - \phi(\cdot) I_{1t} \beta_{X_{1t}}(\cdot) \right) dt \tag{61}$$

Now from (61), we observe that the bracketed expression consists of three terms. The first one is the marginal increase in profits due to an incremental unit of R&D (X_{1t}). The sum of the second and third terms (both being positive since $\beta_{X_{1t}}(\cdot) \leq 0$) gives us the marginal reduction in the cost of investment due to the incremental unit of R&D (X_{1t}). Thus the RHS of (61) should give us the marginal value of the follower's R&D (X_{1t}) at any time t discounted by the time preference parameter and the depreciation rate. Hence it gives us the net marginal value of the follower's R&D at any point in time. By analogous arguments one can find the net marginal value of the leader's R&D at any point in time and it will be given by the following expression;

$$\mu_{2t} = \int_t^{\infty} e^{-(\rho+\delta)t} \left(\pi_{2X_{1t}}(\cdot) + \frac{I_{2t}^2}{X_{2t}^2} \phi'(\cdot) \right) dt \tag{62}$$

▪

Lemma 5.3.1 gives an important result about the shadow price of the two firms' R&D levels. Savers in this economy would use λ_{1t} and μ_{2t} as indicators[33] for the value of their investments in the stock market.

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[33] Although in practice the shadow price of capital, which is also referred to as Tobin's q in the investment literature, cannot be empirically observed, an equivalent average measure can be used. For more details see Hayashi (1982).

Proposition 5.3.1

If assumptions 5.2.1 and 5.2.2 hold, then $\lambda_{1t} = \mu_{2t}$ *in equilibrium, that is the marginal values of the two firms' R&D should be equal.*

Proof:

From Lemma 5.3.1 we know that $\lambda_{1t} > 0$ and $\mu_{2t} > 0$ are the marginal values of the follower's and the leader's respectively. We claim that $\lambda_{1t} = \mu_{2t}$ in equilibrium. Suppose not, then we have two possible cases (i) $\lambda_{1t} > \mu_{2t}$ or (ii) $\lambda_{1t} < \mu_{2t}$. Case (i) implies that the valuation of firm 1 at time t is greater than that of firm 2. A frictionless stock market would imply that savers would transfer theirs funds from the firm with lower valuation to the firm with the higher valuation such that the marginal value of the latter will start to fall (recall that from assumption 5.2.1 that the firm's benefits from R&D increase at a decreasing rate and from assumption 5.2.2, the reductions in the cost of investment due to an additional unit of R&D also decrease at a decreasing rate) while that of the firm with the lower valuation will start to rise. Thus the no-arbitrage condition would imply that case (i) cannot be equilibrium. By analogous arguments one can show that case (ii) cannot hold. ▪[34]

Proposition 5.3.1 states that if investors (identical savers of this economy) were to allocate their funds in the representative industry comprising of the leader and the follower firms, then the valuations (in terms of the marginal benefits of shareholders) of these two firms on the stock market must equalize in equilibrium. The economic intuition behind this result is given as follows. In a dynamic economy in which agents invest in the two firms of the representative industry, the shadow price of such investments can be

[34] At $\lambda_{1t} = \mu_{2t}$, the consumer is indifferent between investing in firm 1 and investing in firm 2.

observed in terms of the valuation of these firms on the stock market. Now since shareholders are always seeking higher returns they would increase (decrease) their investment in the firm with higher (lower) valuation. But as this process goes on the marginal values of the firms would fall (rise) until they become equal in equilibrium. Proposition 5.3.1 also clearly justifies the importance of having a consumer side in our model. Thus although the R&D activities of the two firms are not a priori comparable, when their respective valuations are translated to the investor's portfolio, we can find a relationship between their marginal values.

Proposition 5.3.2

If assumptions 5.2.1 and 5.2.2 hold, then the two firms' R&D investment strategies are strategic complements; that is, an increase in the leader's investment rate will lead to an increase in the follower's investment rate and vice versa.

Proof:

Lemma 5.3.1 gives us two expressions for λ_{1t} and μ_{2t} respectively. Equating them as per Proposition 5.3.1, and using (46) and (52), we have

$$\left(1+\beta\left(X_{1t},X_{2t},b\right)\right)\left(\phi\left(\frac{I_{1t}}{X_{1t}}\right)+\frac{I_{1t}}{X_{1t}}\phi'\left(\frac{I_{1t}}{X_{1t}}\right)\right)=\left(\phi\left(\frac{I_{2t}}{X_{2t}}\right)+\frac{I_{2t}}{X_{2t}}\phi'\left(\frac{I_{2t}}{X_{2t}}\right)\right) \quad (63)$$

Now treating X_{1t}, X_{2t} and b as exogenous and by taking total derivatives on both sides of (63), we can make use of the convexity assumption (in 5.2.2) to establish a positive relationship between I_{1t}, I_{2t} .▪

Proposition 5.3.2 gives us a very important result in the theory of two stage dynamic R&D games with one-way endogenous spillovers. It states that an increase in the leader's R&D will initially encourage the follower to increase its R&D. Subsequently, the leader will respond to the latter increase to by raising its own R&D level even higher. In this way the technological frontier is always shifting outwards. Intuitively, proposition 5.3.2 can be better described along these lines. If both firms in the duopoly face rising marginal costs of R&D, then an incremental increase in the level of R&D by one firm will increase its marginal cost as well. Now, since in equilibrium the marginal costs of the firms must be equal, the best response of the rival firm must be to increase its level of R&D. The rationale for the result is that when the follower undertakes some research, the leader's technological advantage starts to fall and in order to maintain its lead it has to invest further. Thus the laggard always pushes the leader to do more research and as a result a competitive market structure prevails at all times.

Proposition 5.3.3

If assumptions 5.2.1 and 5.2.2 hold, then

(i) *in the absence of spillovers both technology catch-up and symmetric investment are sufficient conditions for the existence of a steady state in the dynamic two stage R&D game (Schumpeterian economy)*

(ii) *in the presence of spillovers, only technology catch-up is a sufficient condition for the existence of a steady state in the dynamic two stage R&D game (Non-Schumpeterian economy)*

Proof:

Define the steady state as the balanced growth path on which ${}_{g}Q_t = 0$ where ${}_{g}Q_t$ is the growth rate of total output in the representative industry. The latter fully characterizes the behavior of the whole economy. Now, since we know that the only two variables growing in the industry are the two firms R&D stock, we assume that ${}_{g}Q_t = 0$ is equivalent to ${}_g X_{1t} = {}_g X_{2t}$.[35] Thus, at the steady state, we shall have $\frac{I_{1t}}{X_{1t}} = \frac{I_{2t}}{X_{2t}}$ (Note that the steady state $I_{1t} = I_{2t} = 0$ of the previous section can be seen as a special case of this.)

(i) Technology catch-up (sufficiency)

Consider equation (63) for the case where $\beta = 0$, that is there are no spillovers.

$$\left(\phi\left(\frac{I_{1t}}{X_{1t}}\right) + \frac{I_{1t}}{X_{1t}}\phi'\left(\frac{I_{1t}}{X_{1t}}\right)\right) = \left(\phi\left(\frac{I_{2t}}{X_{2t}}\right) + \frac{I_{2t}}{X_{2t}}\phi'\left(\frac{I_{2t}}{X_{2t}}\right)\right) \qquad (64)$$

Now suppose catch up takes place at some time t, then $X_{1t} = X_{2t}$. (64) becomes

$$\left(\phi\left(\frac{I_{1t}}{X_{1t}}\right) + \frac{I_{1t}}{X_{1t}}\phi'\left(\frac{I_{1t}}{X_{1t}}\right)\right) = \left(\phi\left(\frac{I_{2t}}{X_{1t}}\right) + \frac{I_{2t}}{X_{1t}}\phi'\left(\frac{I_{2t}}{X_{1t}}\right)\right) \qquad (65)$$

Since from assumption 5.2.2 we know that $\left(\phi\left(\frac{I_{it}}{X_{1t}}\right) + \frac{I_{it}}{X_{1t}}\phi'\left(\frac{I_{it}}{X_{1t}}\right)\right)$ is monotonically increasing in I_{it}, it must be the case that $I_{1t} = I_{2t}$ (symmetric investment). Thus, since $X_{1t} = X_{2t}$, it must be that $\frac{I_{1t}}{X_{1t}} = \frac{I_{2t}}{X_{2t}}$ and hence technology catch-up implies the existence of a steady state.

Symmetric Investment (sufficiency)

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[35] In general, if total output Q_t can be expressed as a function of the technology gap only , then for the class of distance functions assumed in our framework, ${}_g X_{1t} = {}_g X_{2t}$ if and only if that ${}_g Q_t = 0$. An example of a differential game in which ${}_g X_{1t} = {}_g X_{2t}$ at the steady state can be found in Traca and Reis (2003). The latter prove that ${}_g X_{1t} = {}_g X_{2t}$ should hold at the steady state for a Cobb-Douglas distance function. We also note that the spillover distance function used in the previous section also satisfies this property. More specifically, it can be shown that ${}_g Q_t = 0$ if the growth rate of the spillover distance function converges to zero and this occurs if and only if $\frac{b\dot{X}_{1t}}{1+bX_{1t}} = \frac{b\dot{X}_{2t}}{1+bX_{2t}}$ or ${}_g X_{1t} = {}_g X_{2t}$ for large bX_{1t} and bX_{21t}.

Now suppose that we are at the state where $I_{1t} = I_{2t}$, then (64) becomes

$$\left(\phi\left(\frac{I_{1t}}{X_{1t}}\right)+\frac{I_{1t}}{X_{1t}}\phi'\left(\frac{I_{1t}}{X_{1t}}\right)\right)=\left(\phi\left(\frac{I_{1t}}{X_{2t}}\right)+\frac{I_{1t}}{X_{2t}}\phi'\left(\frac{I_{1t}}{X_{2t}}\right)\right) \tag{66}$$

Since from assumption 5.2.2 we know that $\left(\phi\left(\frac{I_{1t}}{X_{it}}\right)+\frac{I_{1t}}{X_{it}}\phi'\left(\frac{I_{1t}}{X_{it}}\right)\right)$ is monotonically decreasing in X_{it}, it must be the case that $X_{1t} = X_{2t}$. But $X_{1t} = X_{2t}$ implies catch-up if the technology gap belongs to the special class of functions satisfying $T(X_{1t}, X_{2t}) = 0$ if and only if $X_{1t} = X_{2t}$. Again here it must be that $\frac{I_{1t}}{X_{1t}} = \frac{I_{2t}}{X_{2t}}$ since $I_{1t} = I_{2t}$. ▪

(ii) Technology catch-up (sufficiency)

Consider equation (63) again

$$(1+\beta(X_{1t}, X_{2t}, b))\left(\phi\left(\frac{I_{1t}}{X_{1t}}\right)+\frac{I_{1t}}{X_{1t}}\phi'\left(\frac{I_{1t}}{X_{1t}}\right)\right)=\left(\phi\left(\frac{I_{2t}}{X_{2t}}\right)+\frac{I_{2t}}{X_{2t}}\phi'\left(\frac{I_{2t}}{X_{2t}}\right)\right)$$

Now suppose catch up takes place at some time t, then $X_{1t} = X_{2t}$.Given the condition from assumption 5.2.1 that $\beta(X_i, X_j, b) = 0$ if and only if $X_i = X_j$, (65) reduces to

$$\left(\phi\left(\frac{I_{1t}}{X_{1t}}\right)+\frac{I_{1t}}{X_{1t}}\phi'\left(\frac{I_{1t}}{X_{1t}}\right)\right)=\left(\phi\left(\frac{I_{2t}}{X_{1t}}\right)+\frac{I_{2t}}{X_{1t}}\phi'\left(\frac{I_{2t}}{X_{1t}}\right)\right) \tag{67}$$

Since from assumption 5.2.2 we know that $\left(\phi\left(\frac{I_{it}}{X_{1t}}\right)+\frac{I_{it}}{X_{1t}}\phi'\left(\frac{I_{it}}{X_{1t}}\right)\right)$ is monotonically increasing in I_{it}, it must be the case that $I_{1t} = I_{2t}$ and hence $\frac{I_{1t}}{X_{1t}} = \frac{I_{2t}}{X_{2t}}$. This completes the proof. ▪[36]

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[36] We leave it as an exercise to the reader to disprove that symmetric investment is a sufficient condition for the existence of a steady state in the presence of spillovers.

Proposition 5.2.3 gives us an important relationship between the existence of a steady state and the level of technology gap between firms. Before any further description of this result one important general observation between the technology gap and R&D incentives ought to be made. If the technology gap becomes zero at some point in time, then around its neighborhood the leader's R&D level has to be increasing (decreasing) at a lower (higher) rate than that of the follower since $X_{20} > X_{10}$ were assumed to be the initial conditions. Moreover, if symmetric investment always implies technology catch-up, then it must be the case that at some point in time before the catch-up state the leader's investment level rose (fell) at a lower (higher) rate than that of the follower. Intuitively, the follower's incremental investment reduces the leader incentives to undertake high levels of research due to the free riding effect. Hence, this is an example of a Schumpeterian effect due to the possibility of creative destruction. As a corollary to this observation, we have the fact that in a state in which symmetric investment does not imply that the gap shrinks to zero, it is possible that the follower's incremental increase in R&D effort leads to an even larger incremental increase in the leader's effort (excluding of course the case where $I_{1t} = I_{2t} = 0$). Thus in this scenario the leader would always innovate further to maintain its technological lead. This is an example of a Non-Schumpeterian effect whereby the leader always innovates further for fear of losing its technological advantage.

Proposition 5.2.3 states that if there is no technological leakage between the leader and the follower in the dynamic two stage R&D game, then convergence (catch-up or ex-post symmetry) both in R&D stock and at the R&D investment level are sufficient conditions for the existence of a steady state. However, if we introduce endogenous externalities into the model, then it is possible to have symmetric investment with R&D stock asymmetry in which the leader remains the leader and thrives to maintain its lead. While this result primarily helps demarcate between the Schumpeterian and the Non-Schumpeterian effects in a general class of dynamic R&D games, it also leads to an interesting policy implication; with less appropriation (less strict rules against the act of reverse engineering or fewer rules hindering the diffusion of technology) we can always have an economy that enjoys Non-Schumpeterian growth in the sense that competition always takes place in a competitive rather than in a monopolistic environment. This emerges directly from poposition 5.2.3 which postulates that the presence of spillovers can lead to a Non-Schumpeterian economy. Our results, together with the empirical findings of Cohen and Levinthal (1989) provide a strong case in favor of lower appropriations.

While the result shows that technological diffusion in form of imitation/ reverse engineering may not always be a bad thing, its significance varies from industry to industry. For example in manufacturing industries, the innovator is “protected” in two ways. First, the lead-time is long enough to serve the same function as a short-term intellectual property of rights and secondly, complete imitation may be so costly that the follower absorbs only a fraction of the leader’s new technology.

However, in semiconductor industries, chips are vulnerable to rapid and cheap copying which worsens the ability of innovative chip developers such as Intel to recover the high R&D costs. Thus, in the latter case, our results that increasing competition due to imitation may lead to higher industry growth can be used as an argument for reverse engineering.

One possible extension of the above framework is endogenize the spillovers by using the technology gap function and also by making the spillover rate a control variable which the level can choose. While in a static setting, the noncooperative level of spillover will be zero[37], it is harder to predict what may happen in a dynamic setting.

[37] See Kamien and Zang (2000)

6. Summary and Concluding Remarks

We have proposed a representation of an asymmetric two stage R&D game in which technology diffuses from the leader to the follower if and only if the latter undertakes some minimum R&D of its own. Moreover, while such externalities increase the benefits of producing goods, it also increases the cost of doing research. This paper has extended the basic AJ framework by including uni-directional endogenous spillovers and allowing such externalities to depend on the size of the technology gap. In addition, it accommodates these additional features by posting a dynamic two stage game in which the dynamics of the gap can be captured. These two features (endogenous spillovers and R&D costs increasing in the spillover rate) allow us to derive some important relationships between the technology gap and research incentives, as well as the dynamic equilibrium of a differential game.

Our analysis of a static game reveals primarily (in an extended AJ framework) that there exists a non-monotonic relationship between free-riding behavior and the spillover rate. However, the SPNE of the game was not unique. From this observation and the fact that technology gap evolves over time, we formulate an R&D differential game with one-way endogenous spillovers. We found that the technology gap would shrink to zero if there were to exist steady state equilibrium in our dynamic example. Furthermore, the equilibrium would be stable if the industry costs were large enough.

In a more general setting, we found, under some reasonable assumptions, that catch-up and symmetric investment are sufficient conditions for the existence of a steady state in a framework with no spillovers and that steady states with no catch-up exist only in the presence of spillovers. Thus, we infer that the latter can promote welfare (growth) by enhancing dynamic competition in an economy. Promising directions for further investigation include the extension of our analysis to a cooperative setting with research joint ventures (as has been done traditionally for the static and exogenous spillover case). Also, proper characterization of the stability conditions that would guarantee a Saddle Path to the different steady states discussed in Section 5 is an avenue for further inquiry.

7. Appendix

7.1Derivations of (13) and (14)

For firm 1, FOC of (11) (by substituting β by (10) when appropriate)

with respect to X_1 results in

$$\frac{4}{9(1+bX_2)}\left(A-C+2X_1+\left[\frac{2b(X_2-X_1)}{(1+bX_2)}-1\right]X_2\right)=\frac{(1+2bX_2-2bX_1)}{1+bX_2} \qquad \text{(A1)}$$

Simplifying (A1) gives (13).

For firm 2, FOC of (12) with respect to X_2 results in

$$\frac{2(2+2bX_2+bX_1)}{9(1+bX_2)^2}\left(A-C+\left[2-\frac{b(X_2-X_1)}{1+bX_2}\right]X_2-X_1\right)=1 \qquad \text{(A2)}$$

Simplifying (A2) gives (14).

7.2 Proof for Proposition 3.1.1

We shall derive this result from the reaction function of the laggard given by (15). By making X_1 the subject of formula, we find the roots of the graph, that is, we find the values of X_2 when $X_1 = 0$

Thus, we have at $X_1 = 0$

$$X_2 = \frac{-b\left(\frac{27}{4}-f\right)-1\pm\sqrt{\left(\left(\frac{27}{4}-f\right)b+1\right)^2-4\left(\frac{9}{4}-f\right)\left(\frac{9}{2}b^2-b\right)}}{9b^2-2b} \qquad \text{(A3)}$$

where $f = A-C$.

Now, it can be shown that for $f < 0$ and $b < \frac{2}{9}$, there will be one positive root and one negative root which lies to the right of the vertical asymptote. The vertical intercept is given by

$$X_1 = \frac{\frac{9}{4} - f}{\frac{9}{2}b - f} \qquad \text{(A4)}$$

Moreover the part of the graph that is relevant for the parameter space $f < 0$, $b < \frac{2}{9}$ and $X_i. > 0$ is an inverted u-shaped parabola which attains a maximum in the positive orthant. This implies that the laggard's investment will increase up to the maximum point after which it starts to free-ride. The maximum point is where

$$X_2^M = \frac{b\left(\frac{27}{4} + f\right) + 1}{2b - 9b^2} \qquad \text{(A5)}$$

We now try to determine the sign of X_2^M w.r.t b.

Using (A4), we find that the sign is negative if $b < \frac{1}{9}$ and positive if $\frac{1}{9} < b < \frac{2}{9}$. Thus, since $-\Delta X_2^M$ is a proxy to free-riding, we conclude that free-riding increases with b as long as $b < \frac{1}{9}$, and decreases with it for $\frac{1}{9} < b < \frac{2}{9}$. Hence, the relationship between $-\Delta X_2^M$ (free-riding) is non-monotonic and is an inverted U-shaped. ▪

<u>7.3 Proof for Proposition 3.1.2</u>

(A2) divided by (A1) gives

$$\frac{((A-C) + 2X_2 - X_1)(2 + 2bX_2 + bX_1) - \frac{9}{2}(1 + bX_2)^2}{\frac{9}{4}(1 + 2bX_2 - 2bX_1) - [(A-C) + 2X_1 - X_2]} = \frac{(2 + 2bX_2 + bX_1)}{2} \qquad \text{(A6)}$$

(A5) can be simplified as

$$\frac{27}{2}-6(A-C)+\left(\left(\frac{63}{2}-6(A-C)\right)b-6\right)X_2-b\left(\frac{27}{4}+3(A-C)\right)X_1$$
$$-(\frac{9}{2}b^2+3b)X_2X_1+\left(18b^2-6b\right)X_2^2-\frac{9}{2}b^2X_1^2=0 \tag{A7}$$

Now it can be shown that (A7) is the graph of a non-degenerate conic. Thus, it can either a hyperbola or ellipse with some rotation and/or translation from the origin. To prove for existence of a Nash equilibrium we find the X_1 and X_2 intercepts of this conic (which is a reduced-form of the reaction function of the leader firm) and compare their values with the X_1 and X_2 intercepts of the parabola (the reaction function of the laggard firm), the roots of which are given in (A3). From (A7), we find the values of X_2 when $X_1=0$. Thus,

$$X_2=\frac{\left(6-\left[\frac{63}{2}-6f\right]b\right)-1\pm\sqrt{\left(\left(\frac{63}{2}-6f\right)b-6\right)^2-\left(54-24f\right)\left(18b^2-6b\right)}}{2\left(18b^2-6b\right)} \tag{A8}$$

and at $X_2=0$, X_1 will be

$$X_1=\frac{b\left(\frac{27}{4}+3f\right)\pm\sqrt{\left(\left(\frac{27}{4}+3f\right)b\right)^2+\frac{9}{2}b^2\left(54-24f\right)}}{-9b^2} \tag{A9}$$

Now it can be shown that if b is low enough and C is large enough, the X_1 intercept of the conic is always greater than that of the parabola. Moreover, if C is large enough, b is low enough so that terms in b^2 can be ignored, the X_2 intercept of the conic is always less than that of the parabola. If we denote the difference between the parabola and the conic by some function $\Psi\left(X_1X_2\right)$, then as is well-known from the Weierstrass Intermediate Value Theorem that if a continuous function on an interval is sometimes positive and sometimes negative, it must be zero at some point. This proves for existence of Nash equilibrium. However, we cannot guarantee uniqueness since both the parabola and the conic are non-monotonic functions. This completes the proof. ▪

7.4 Proof for Proposition 4.1.1

Assume that at the steady state $I_{1t} = I_{2t} = 0$, then replacing $I_{1t} = 0$ in (26), we have

$$\lambda_{1t} = 0 \tag{A10}$$

Replacing (A10) in (27) and using the fact that (A10) implies $\dot{\lambda}_{1t}$ =0, we have (after re arranging)

$$\left(A - C + 2X_{1t} + \left(\frac{2b(X_{2t} - X_{1t})}{1 + bX_{2t}} - 1 \right) X_{2t} \right) = 0 \tag{A11}$$

Replacing $I_{2t} = 0$ in (32), we have

$$\mu_{1t} = 0 \tag{A12}$$

Replacing (A12) in (33) and using the fact that (A12) implies $\dot{\lambda}_{1t}$ =0, we have (after re arranging)

$$\left(A - C + \left(2 - \frac{b(X_{2t} - X_{1t})}{1 + bX_{2t}} \right) X_{2t} - X_{1t} \right) = 0 \tag{A13}$$

Solving (A11) and (A13) simultaneously gives

$$X_{1t}^* = X_{2t}^* = C - A \ or \frac{-1}{b} \tag{A14}$$

Now since $b \in (0,1)$ and $X_{1t}^*, X_{2t}^* \geq 0$ we choose our parameter values such that $C > A$.[38]

[38] Note that this "high cost industries" assumption might imply that the second stage output for this example is negative in some periods.

The above proves part (i) and (ii) of Proposition 4.1.1. To check the stability we have to linearize the system of differential equations given by (26), (27), (32) and (33) in the neighborhood of the steady state. After some manipulations of these four equations we manage to reduce them the following two equations

$$\left(\frac{1+2bX_{2t}-bX_{1t}}{X_{1t}\left(1+bX_{2t}\right)}\right)\dot{I}_{1t}+\left(\frac{b\left(1+bX_{1t}\right)}{X_{1t}\left(1+bX_{2t}\right)^{2}}\right)I_{1t}I_{2t}-\left(\frac{\left(1+2bX_{2t}\right)}{2X_{1t}^{2}\left(1+bX_{2t}\right)}\right)I_{1t}^{2}-\left(\frac{\rho\left(1+2bX_{2t}-bX_{1t}\right)}{X_{1t}\left(1+bX_{2t}\right)}\right)I_{1t}$$
$$+\left(\frac{4}{9\left(1+bX_{2t}\right)}\right)\left(A-C+2X_{1t}+\left(\frac{2b\left(X_{2t}-X_{1t}\right)}{1+bX_{2t}}-1\right)X_{2t}\right)=0$$

(A15)

$$\frac{\dot{I}_{2t}}{X_{2t}}-\frac{I_{2t}^{2}}{2X_{2t}^{2}}-\frac{\rho I_{2t}}{X_{2t}}+\left(\frac{2\left(2+2bX_{2t}+bX_{1t}\right)}{9\left(1+bX_{2t}\right)^{2}}\right)\left(A-C+\left(2-\frac{b\left(X_{2t}-X_{1t}\right)}{1+bX_{2t}}\right)X_{2t}-X_{1t}\right)=0$$

(A16)

We first compute the Jacobian determinant of the above evaluated at the steady state value $X_{1t}^{*}=X_{2t}^{*}=C-A$. Note that the calculation is made simpler since we know that the steady state values of $I_{1t}^{*},I_{2t}^{*},\dot{I}_{1t}^{*}$ *and* $\dot{I}_{2t}^{*}$ are equal to zero. Bearing this fact in mind and after taking the total derivatives of the above expressions with respect with $X_{1t} and X_{2t}$ we arrive (after some algebraic manipulations) to the following Jacobian matrix:

$$\underset{2\times 2}{J}=\begin{bmatrix}\dfrac{8}{9\left(1-b\left(A-C\right)\right)^{2}} & \dfrac{-4\left(1+b\left(A-C\right)\right)}{9\left(1-b\left(A-C\right)\right)^{2}}\\[2ex] \dfrac{2\left(3b\left(A-C\right)-2\right)}{9\left(1-b\left(A-C\right)\right)^{3}} & \dfrac{2\left(2-3b\left(A-C\right)\right)\left(2-b\left(A-C\right)\right)}{9\left(1-b\left(A-C\right)\right)^{3}}\end{bmatrix} \tag{A17}$$

Now it can be shown that

$$|J| = \frac{8(3b(A-C))}{(b(A-C)-1)^4} \tag{A18}$$

Since $|J| \leq 0$ holds by definition for the parameter restrictions[39] of our model, we conclude that the steady state equilibrium is stable. Hence there exists a stable path converging to the unique steady state of our model. However, it is possible that there is more than one path converging to such steady state. We next prove that this path is unique; that is it is a Saddle Path.

In control theory it is well known that if the set of eigenvalues given by the matrix in (A17) contains a positive real part and a negative real part, then the path to the steady state of the system described above is a Saddle path. It can be verified[40] that (A17) has two eigenvalues and given the parameter restrictions of our model, the fact that one is positive and the other is negative holds by definition.

For part (v), we consider four different cases. (1) The investment levels of both firms are negative in the neighborhood of the steady state. (2) The investment level of firm 1 is negative while that of firm 2 is positive. (3) The investment level of firm 1 is positive while that of firm 2 is negative. (4) The investment levels of both firms are positive. For case (1), we let the R&D level of firm 1 and firm 2 be $C-A+\varepsilon_1$ and $C-A+\varepsilon_2$ for small $\varepsilon_1, \varepsilon_2 > 0$ respectively. We then compute $q_{it} = \dfrac{A-2C_i+C_j}{3}$ evaluated at $C-A+\varepsilon_1$ and $C-A+\varepsilon_2$ for i=1,2. We find that $q_{it} > 0$ for i=1,2 as long as $|\varepsilon_1 - \varepsilon_2|$ is not too large. By analogous reasoning, we find

[39] Note we use once more the "high cost" condition that C > A.

[40] The derivations can be provided upon request.

that for each of the cases (2) – (4), at least one firm does not have an incentive to produce a positive output. Thus $q_{it} > 0$ for i=1,2 only for case (1). Hence, we have a positive mass in the neighborhood of the steady state equilibrium of the full game only if the R&D stocks of both firms are falling in its neighborhood. This completes the proof. ▪

FIGURE 1

(b=1/10) Note : X= Leader's R&D , Y= Follower's R&D

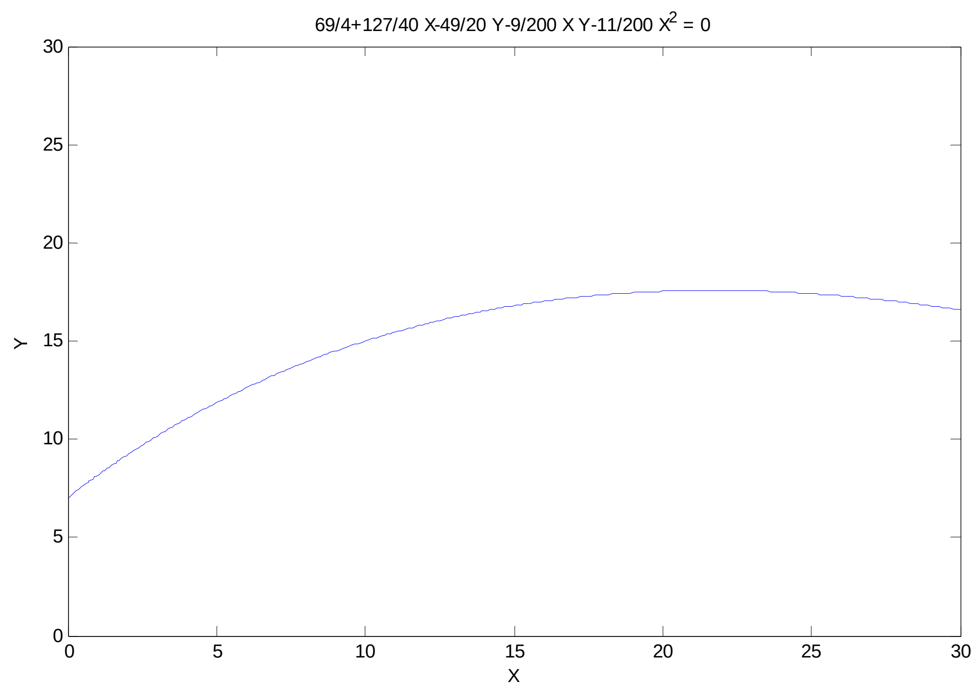


FIGURE 2
(b=1/50)

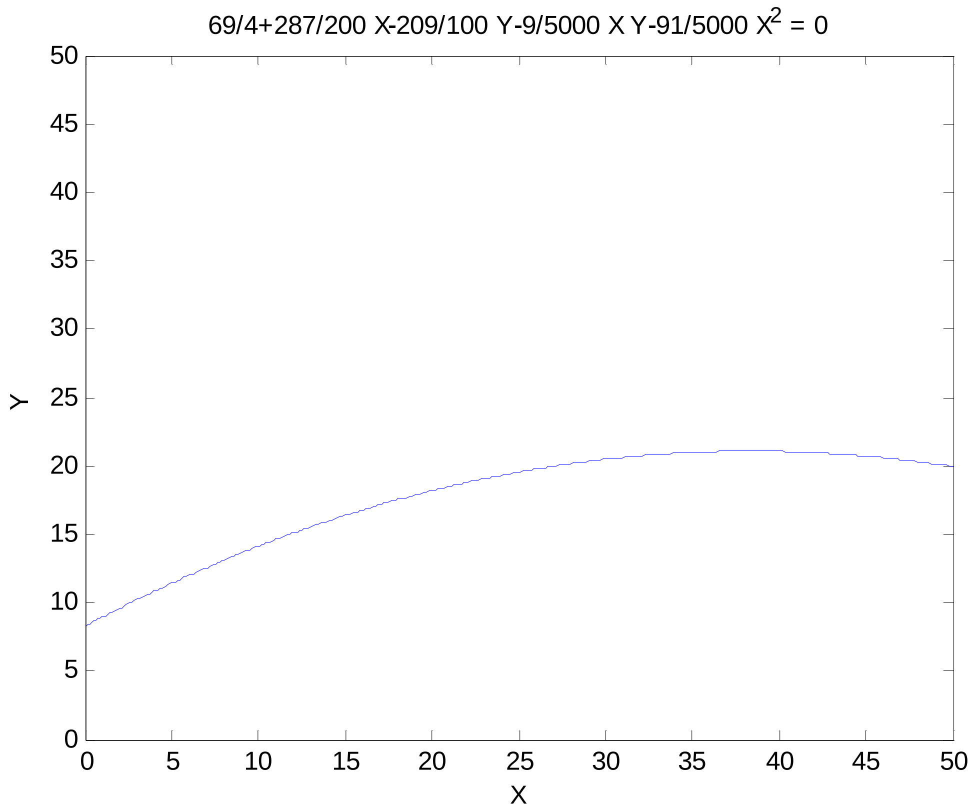


FIGURE 3

(b=1/100)

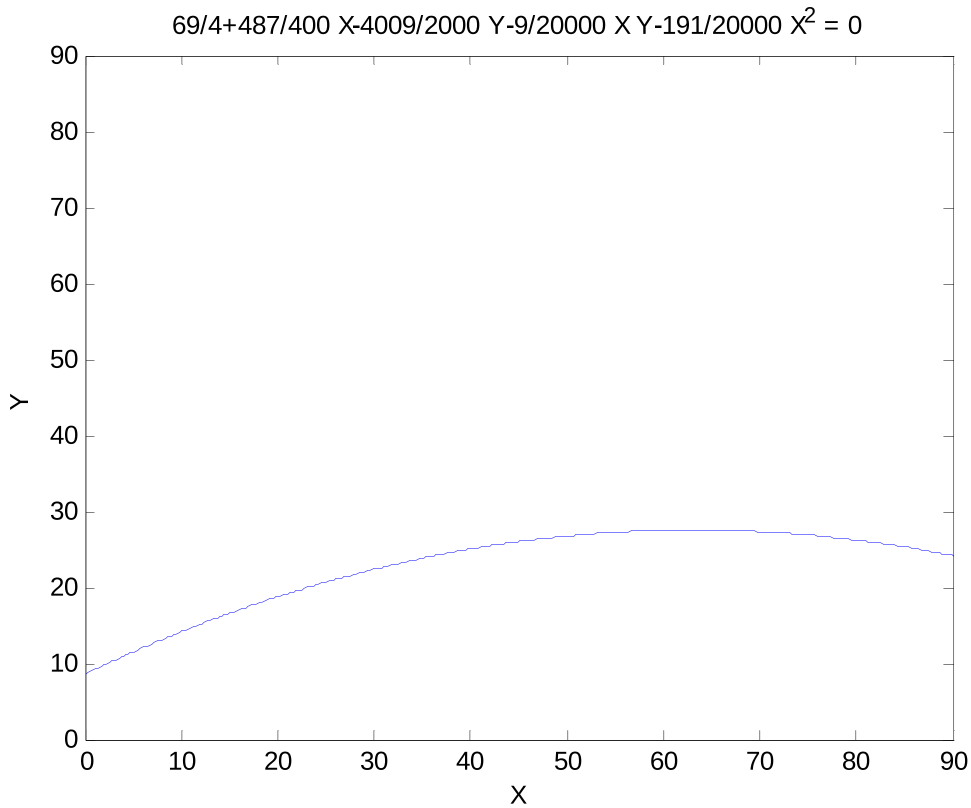
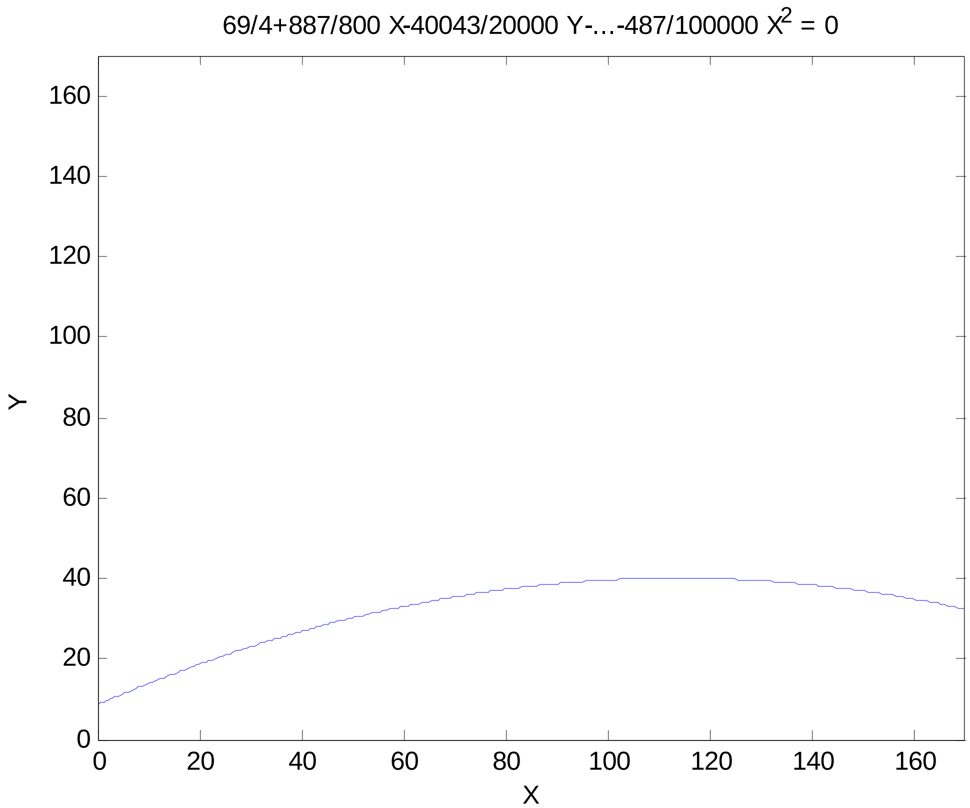


FIGURE 4
(b=1/200)



III. Process Spillovers and Growth.

Abstract

This paper develops a non-Schumpeterian endogenous growth model of R&D in which the firm's free-riding behavior, reinforced by a lack of appropriability in its industry, constitutes a major source of growth in the economy. While models analyzing the interaction between either imitation and innovation or spillovers and innovation have already appeared in the literature, we show how imitation via free-riding behavior and spillovers can mutually promote dynamic competition and hence economic growth. The representative industry, which is of duopolistic market structure, comprises a leader who innovates and a laggard who free-rides by exploiting the source of intra-industry spillover. We find firstly that the innovation strategies of the two firms can be dynamically strategic complements if a large technology gap prevails and, secondly, that there is a case for process reverse engineering as a fall in the level of appropriability results in higher growth.

Keywords: process imitation, innovation, spillovers, technology gap, endogenous growth
JEL Classification Numbers: C7, E0, L1, O3

1. Introduction

One of the prominent features of R&D based endogenous growth models is that the existence of spillovers due to the lack of appropriability and the resulting competition are always detrimental to an economy's growth performance. However, it is difficult to reconcile this prediction with real life facts. According to Kamien and Schwartz (1982), empirical studies on the relationship between market structure and the rate of diffusion of innovations indicate that innovation is positively related to the competitiveness of the industry into which it is introduced. Also, Cohen and Levithal (1989) found that the effect of appropriability on innovative activity is negative and significant, and hence concluded that contrary to traditional results intra-industry spillovers may encourage R&D investments in equilibrium.[41] Clearly, imitation or free-riding behavior driven by the presence of R&D spillovers is a potential source of competitive pressure that deters industry leaders from behaving as monopolists and prompts them to innovate further.

This paper presents a theoretical inspection of the effects of process spillovers on competition in a R&D based endogenous growth model. Our main concern is to characterize the dynamic interaction between innovation and imitation (via free-riding behavior) when spillovers generated by the former activity makes the latter easier. This interaction introduces an element of strategic complementarity between innovative and imitative strategies at the steady state equilibrium. The analytical framework is based on a two-stage noncooperative differential game between two firms; a leader and a follower

[41] They argue that yet another important role of R&D is to enhance the firm's ability to assimilate and exploit existing information. In this paper free-riding via reverse engineering is made possible by spillovers which facilitate the follower's absorption and learning of the leader's technology.

in a representative industry.[42] In this setting, we examine the long-term behavior of each firm given the dynamics of their technology gap. In contrast to previous studies, we show that, owing to the presence of spillovers from innovation, the dynamic best response of the leader facing imitation[43] is to innovate further rather than to dwell on its short-run higher profits. Since the aggregate rate of innovation is given by the sum of the firms' specific innovation or imitation, the economy's growth steady state rate depends on the growth rate of innovation which, in turn, depends on the growth rate of imitation. In the transitional dynamics, the same findings are observed for large technology gaps. Moreover, it is possible that an increase in appropriability reduces innovation and hence growth.

One important assumption of the model is the duopolistic structure of the representative industry. We therefore define the market configuration of the industry in terms of the relative technology gap.[44] While Traca and Reis (2003) compare the market configuration stability between the symmetric and asymmetric cases, we show that along the transitional dynamics the technology gap growth path is stable and we therefore infer that the underlying market configuration in our model is stable.[45]

One noteworthy implication of this paper is that it helps to provide some economic basis for the phenomenon of reverse engineering. The model demonstrates that at least for the case of process reverse engineering, less appropriability is better as it promotes growth. Handa (1995) in a study of legal implications of reverse engineering

[42] Unlike Traca and Reis (2003), in which there is no a priori difference between the leader and the laggard apart from the initial technology gap, we emphasize the fact that the leader only innovates, while the follower, benefiting from a relatively larger spillover free-rides on the leader.
[43] We do not distinguish between imitation and free-riding behavior in this paper.
[44] This implies that a larger gap means that the leader controls a larger market share and hence the market configuration is asymmetric. The symmetric case is when competition is neck and neck.
[45] One possible line of defense in favor of the duopolistic structure is that the barriers to entry or fixed costs ensure that only two firms can thrive in the market at any point in time.

concluded that the Canadian Copyright Act is juridicially underdeveloped and too uncertain to provide solutions. In yet another contribution on legal implications of reverse engineering, Samuelson and Scotchmer (2002) argued that restrictions on reverse engineering ought to be imposed only if they are justified in terms of the specific characteristics of the industry and their economic effects. We see the process of reverse engineering as a key determinant of innovation in the long-run.

The rest of the paper is organized as follows. In section 2 we provide a brief overview of the literature. In section 3 we present a dynamic general equilibrium model featuring innovation, imitation and spillovers. In section 4, we present our results and section 5 contains some brief concluding remarks.

2. Related Work

The main point of departure of our model from the traditional R&D based endogenous growth framework is that it is competitive rather than monopolistic behavior at the R&D level which generates growth. The prevailing paradigm is based on Schumpeter's idea of creative destruction and models within such frameworks are often referred to as Schumpeterian models. Aghion and Howitt (1992) show, in a model of vertical innovations, that the prospect for more future research discourages current research by threatening to destroy the rents created by such research . Similar views are shared by Grossman and Helpman (1991a; 1991b) and Barro and Sala-i-Martin (1995).

Some of these studies highlight the role of ongoing product upgrading and product cycles[46] in characterizing the steady state equilibrium. In particular, the firm holding the state-of-the-art, that is the one with the lowest price adjusted quality, acts as a monopolist in the representative industry.[47] The firms in the latter play a Bertrand game competing on price adjusted quality and such a structure by design leads to a monopolistic market configuration at any point in time.[48] Another consequence of the homogenous Bertrand game assumption is that imitation can be carried out only by relatively lower-cost firms, while successful innovations lead to instantaneous leapfrogging. They show that in general three equilibria exist: the monopolist is a low-cost imitator, the monopolist is a leader who has regained its lead from a low-cost imitator and the monopolist is an innovator who has leapfrogged the leader. Connolly (1997, 1999, 2001) building on the above models, introduces the idea of reverse

[46] This is due to Vernon (1966).
[47] The idea of quality ladder is also pioneered by these authors and a higher step of the ladder is reached only if another firm leapfrogs the current leader.
[48] Though in Segerstom's (1991) model there can be two firms producing the state-of-the-art, the market structure is still monopolistic since those firms would form a coalition.

engineering and learning-to-learn as sources of technological diffusion in North-South trade. Despite her emphasis on the importance of imitation in the transitional dynamics, the concept of creative destruction is still inherent to her analysis.

More recently the question of whether easier imitation of technological leaders is necessarily bad for growth has been given increasing attention. Aghion et al (1997, 2001) have shown that when imitation occurs, leaders tend to innovate further to escape competition or to reestablish their lead. Such models following their spirit have been referred to as non-Schumpeterian models. One of the motivations of this framework is that when the doctrine of creative destruction in R&D based endogenous growth models is applied to real life, it leads to counterfactual predictions. It is therefore possible that there exists some missing link which can explain the empirical failure of Schumpeterian models. In this paper, we show that dynamic interactions between firms in an economy represent a potential candidate for that missing element.

Meanwhile, other studies working in the non-Schumpeterian paradigm, have looked into the relationship between product market competition and growth. Aghion et al (1997, 2001), using a model in which R&D incentives occur only in three possible states, found that innovative incentives are higher in the neck and neck state. However, their models do not incorporate the externalities generated by innovative activities. Our paper is closest to Traca and Reis (2003) who, in a model of duopolistic competition within the endogenous growth paradigm, show that spillovers raise the rate of innovation as they spur a source of competitive pressure on the leader. Although our approach is similar to theirs, our model differs from theirs in non-trivial ways. First, spillovers in our model are heterogeneous as the follower who reverse engineers the leader's innovation benefits from larger externalities than the leader. Such heterogeneity is not addressed in their paper. Secondly, we show that the results remain robust in the transitional dynamics as long as the technology gap is large enough and that the policy maker can control for the nature of the dynamic equilibrium by choosing the level of the industry's appropriability.[49]

[49] Another non-Schumpeterian model with no spillovers is developed by Mukoyama (2003) who shows that subsidizing imitation might increase the economy-wide rate of technological progress and that competition and growth might be positively correlated.

3. The Model

3.1 Overview

We consider a model with n goods, n industries with 2 firms each, and infinitely lived identical consumers. The latter face two optimization problems: temporal and intertemporal utility maximization. Preferences across goods and time are logarithmic. In the intertemporal problem the consumer chooses the optimal labor supply and consumption (or expenditure) for each period. The remaining income is invested in the industries' R&D. To simplify the model we make the following assumptions.[50] Firstly, we normalize expenditures to allow the rate of return to capital (savings of agents) to be constant and equal to the exogenously given discount rate. Moreover, we also assume that the risk of any firm is idiosyncratic and that the stock market values the firm so that its expected rate of return equals the risk free interest rate.[51] Secondly, we assume that labor supply is perfectly elastic.[52]With the optimal amount to be spent in each period chosen, the representative agent can thus derive his demand function for each industry from his temporal optimization problem.

On the production side, the industry demand is derived from the consumer problem and taking such schedule as given, the duopolists in the representative industry compete in Cournot fashion to choose their respective output and research intensities (which are innovation rate for the leader and imitation rate for the laggard). If the productivity of one firm is higher than its competitor, then the former is the leader and the latter is the follower. A further qualification to the structure of our representative industry is the existence of heterogeneous intra-industry spillovers. Being an innovator,

[50] These do not lead to loss of generality in our propositions.
[51] This is similar to Grossman and Helpman (1991).
[52] This follows from Aghion et al (2001).

the leader does not benefit much in terms of externality from the follower[53] , while the follower which practises process reverse engineering benefits more than the leader.

The choices of the firm's variables which are quantity and research intensities are done sequentially. We therefore use backward induction in a two-stage noncooperative game setting to formulate the firms' optimum behavior. In the second stage the firms play a Cournot game to determine their respective quantities. Thus their respective profits as functions of their productivity[54] levels can thus be derived. In the first stage, the leader (follower) plays a differential game to choose their optimal innovation (imitation) time path taking the technology gap (ratio of their productivity levels) dynamics as their state variable. The open-loop Nash equilibrium is then found .[55] Since innovation and imitation are the only variables growing in the economy, the steady state growth rate is determined by the growth rates of those variables. Dynamic reaction functions are used to derive results. Effects of changes in appropriability and growth are analyzed. Finally the path of the technology gap is derived and some stability conditions are imposed.

3.2 Formal Model

Consumers

Let n , C_t , L_t , Q_{it} , R_t and W_t be the number of industries, the consumption of the representative agent, his labor supply,quantity produced in the industry i for i =

---

[53] We assume that the value of the positive externality accruing to the leader is small but non-zero since there might be some heavily located facilities which are inherent to the setting up of a firm and that there might be some interactions among workers.

[54] Productivity is defined in terms of the per unit cost as in Traca and Reis (2003).

[55] An open-loop Nash equilibrium is found when a competitor takes his rival's reaction function solely as a function of time in his dynamic optimization problem. Essentially, there is only one decision node.

1,2...,n, the interest rate and the wage rate respectively at time t. Then the intertemporal preference of the agent can be written as

$$U \equiv \int_0^\infty e^{-\rho t}\left(u(C_t) - L_t\right)dt, \rho > 0 \quad \text{where } \rho \text{ is the discount rate} \tag{1}$$

$$u(C_t) \equiv \ln(C_t) \tag{2}$$

The intertemporal utility maximization problem results in (i) $W_t = 1$ and (ii) $R_t = \rho$ after normalization.[56] The temporal consumer preference is given by

$$u(C_t) \equiv \sum_{i=1}^{n} \ln(Q_{it}) \text{ for all t} \tag{3}$$

The static utility maximization problem results in the industry demand curve[57]

$$Q_{it} = \frac{M}{P_{it}} \qquad \text{where } M = 1/n \tag{4}$$

Producers

---

[56] Let consumer's wealth at time t be A_t , P_t be the price of consumption and $P_t C_t$=1 due to normalization, then we have
$H= \ln(C_t) - L_t + \lambda_t(R_t A_t + W_t L_t - P_t C_t)$
$dH/ dC_t = 0$ implies $1/ P_t C_t = \lambda_t$, but since $P_t C_t =1$, $\lambda_t =1$
$dH/ dL_t = 0$ implies $W_t \lambda_t = 1$ and hence $W_t = 1$
Also $d \lambda_t / dt = \rho \lambda - \lambda R_t$, but since $\lambda_t =1$, $d \lambda_t / dt = 0$ or $\rho - R_t= 0$ and therefore $R_t = \rho$

[57] Static utility maximization leads to

$$\underset{Q_i}{Max} \; u(C_t) \equiv \sum_{i=1}^{n} \ln(Q_{it}) \text{ s.t}$$

$$\sum_{i=1}^{n} P_i Q_i = I$$

and since Income = Expenditure = 1, the constraint becomes

$$\sum_{i=1}^{n} P_i Q_i = 1$$

The logarithmic assumption leads to the following demand curve $Q_{it} = \dfrac{1}{nP_{it}}$

Given the industry demand (4) each firm will choose its respective optimal production q_{ijt} such that[58]

$$\sum_{j=1}^{2} q_{ijt} = Q_{ijt}$$

We assume that firm 1 is the leader and firm 2 is the follower. Each firm's production function is given by

$$q_j = A_j L_j \quad \text{for } j = 1,2 \tag{5}$$

It can easily be inferred from (5) that the per unit cost of each firm is given by W^*/ A_j where W^* is the economy level wage rate. Also, due to our assumption $W_t = 1$, the per unit cost becomes

$$c_j = 1/ A_j \tag{6}$$

The productivity dynamics is assumed to be given by

$$\dot{A}_{jt} = h\Lambda_{jt} L^{R\&D}{}_{jt} \qquad \text{where h is the R\&D productivity} \tag{7}$$

Λ_{jt} is the spillover to firm j and $L^{R\&D}$ is the labor employed in the R&D sector. It is understood from our formulation that each firm operates in two sectors in which it employs labor. Our underlying assumption here is that workers are homogeneous since a constant wage rate ensures that no skill differentials among the workers are observable in the labor market. The term h is the R&D productivity level, which is assumed to be given in the industry. We are therefore left to qualify the spillovers Λ_{jt} which are the underpinnings of our analysis.

[58] In this subsection of the paper, we sometimes omit subscript i for simplicity.

Our definition of spillovers is similar to Cohen and Levinthal (1989) together with some extensions. In particular we define spillovers to include valuable knowledge generated in the research process of the leader and which becomes accessible to the follower if and only if the latter is reverse engineering the innovator's research process. It is important here to note that had the follower not been imitating the leader, the spillover it enjoys would reduce to a small positive number equal to that of the leader (see footnote 13). This also implies that the homogeneous assumption of Traca and Reis (2003) will be a special case of our model. Given this assumption of relatively larger spillovers favoring imitation vis-à-vis innovation, it becomes a better strategy for the follower to imitate by feeding off the leader's innovation at least initially. Thus the follower is necessarily an imitator.

It is also implicit from our assumption that it is process imitation rather than product imitation which takes place in our framework. This also means that the conventional definition of reverse engineering as the decompilation of a finished product in order to gain a better understanding of how it was produced as in Handa (1995) does not fit well into our model. Rather we see reverse engineering as the act of extracting know-how or information from the industry leader through channels like the labor market (turnover in R&D personnel, for example) in order to imitate the latter's process (or cost-cutting) innovations.[59] Hence, unlike Schumpeterian models, spillovers are not regarded as a pure public good since some effort (imitation) is involved in acquiring it. We formally let the spillovers for firm 1 and 2 be

$$\Lambda_{1t} = A_{1t}^{1-\sigma_1} A_{2t}^{\sigma_1}$$

[59] Nevertheless, our definition still belongs to a more general class of definitions of reverse engineering.

$$\Lambda_{2t} = A_{2t}^{1-\sigma_2} A_{1t}^{\sigma_2} \tag{8}$$

where σ_1 , σ_2 are less than ½ and $\sigma_2 > \sigma_1$. We let σ_2 be inversely related to the appropriability level of the industry and we see that this expression will also increase the spillovers enjoyed by the follower. One can also think of it as a tool for the policy maker to regulate or protect patents. The second restriction ensures that imitators enjoy larger spillovers than innovators. The technology gap G_t is given by

$$G_t = \frac{A_{1t}}{A_{2t}} > 1 \tag{9}$$

where the inequality shows that firm 1 is the leader. Also, the gap dynamics is given by

$$\dot{G}_t = (\alpha_{1t} - \alpha_{2t})G_t \tag{10}$$

where

$$\alpha_{jt} \equiv \frac{\dot{A}_{jt}}{A_{jt}} \qquad \text{for } j = 1,2$$

α_1 and α_2 are the growth rates of innovation and imitation respectively. This completes the model.

<u>3.3 Solving the Model</u>

We solve the model by backward induction.

Stage 2

Using the inverse demand function (4), firm j's profit maximization problem becomes

$$\underset{q_j}{Max}\ Mq_j/(q_j + q_i) - c_j q_j\ ,\ j = 1,2 \quad \text{and} \quad i \text{ is not equal to } j \tag{11}$$

The Cournot Nash quantity for firm j is given by

$$q_j = (Mc_i)/(c_j + c_i)^2,\ j = 1,2 \quad \text{and} \quad i \text{ is not equal to } j \tag{12}$$

The profit function for firm j is given by

$\Pi_j = (Mc_i^2)/(c_j + c_i)^2,\ j = 1,2$ and i is not equal to j (13)

Proof: See Appendix

So far we have not provided a rationale for why we depart from the prevailing literature which uses a Bertrand differentiated product competition rather than a Cournot competition. We offer two justifications as to why this assumption suits our purpose better here. First, the homogeneous Cournot assumption by design implies that the only way for the leader (follower) to increase his market share is to increase (decrease) the cost differential which is given by the technology gap as shown in the next set of equations. Therefore the possibility of product innovation is ruled out in this setting and hence our model necessarily implies that all the imitation and innovation occur at the process level. Both Aghion et al. (2001) and Traca and Reis (2003) consider only process imitation in the former case and process spillovers in the latter case but yet they use a formulation (Bertand differentiated) in which both product and process innovations are possible. Secondly, our assumption allows the two firms to compete in both the product market and the R&D sector even for the case of product homogeneity unlike the Bertrand homogeneous game. This enables us to compare our model directly with the Schumpeterian paradigm (at least at the micro level) without changing the assumption of product homogeneity.

For the sake of simplifying the remainder of the analysis we rewrite the profit functions of (13) and research costs as functions of the technology gap only G_t .We thus have

$\Pi_{1t} = M/(1 + 1/G_t)^2$ (14)

$\Pi_{2t}= M/(1+ G_t)^2$ (15)

and

R&D cost of firm 1 = $\alpha_{1t}G_t^{\sigma_1}/h$ (16)

R&D cost of firm 2 = $\alpha_{2t}/hG_t^{\sigma_2}$ (17)

Proof: See Appendix.

Stage 1 (The Open-Loop Formulation)

The pair (θ_1, θ_2) is called an open-loop Nash equilibrium with function θ_j mapping t $\in [0,T)$ to a real number if for each j = 1, 2 , an optimal control path $\alpha_j(.)$ of the problem below exists and is given by $\alpha_j(t)= \theta_j(t)$.[60] As shown below the optimal control is performed with this definition as a basis. It is also important to note that, unlike the case described by Vencatachellum (1998), the open-loop Nash equilibrium in our model does not coincide with the myopic strategy whereby the firm does not take into account the productivity of its rival while choosing its optimal path.[61] Firm 1's dynamic optimization problem is given by

$$V_1 = \max_{\alpha_1} \int_0^\infty e^{-\rho t}\left[M\left(1+\frac{1}{G_t}\right)^{-2} - \frac{\alpha_{1t}G_t^{\sigma_1}}{h}\right]dt$$

s.t $\dot{G}_t = (\alpha_1 - \alpha_2)G_t$, G_0 is given, $G_T \geq 0$ as T $\to\infty$ and $\alpha_{1t} \geq 0$ (18)

[60] The general case is formulated by Dockner et al. (2000). We assume an open-loop equilibrium as in Peretto (1996) since we are unable to find a closed-form solution to analyze the properties of the model for a closed-loop or Markov perfect equilibrium. In principle, if the objective function is of linear quadratic form, the closed-loop equilibrium can be found by setting the Hamilton Jacobi Bellman (HJB) equation.

[61] This special case arises since the Hamiltonian of one firm is linear and separable in its rival's stock of human capital and hence, the latter term vanishes at the first order condition.

Firm 2's dynamic optimization problem is given by

$$V_2 = \max_{\alpha_2} \int_0^\infty e^{-\rho t} \left[M(1+G_t)^{-2} - \frac{\alpha_{2t}}{G_t^{\sigma_2} h} \right] dt$$

s.t $\dot{G}_t = (\alpha_1 - \alpha_2) G_t$,G_0 is given, $G_T \geq 0$ as $T \to \infty$ and $\alpha_{2t} \geq 0$ (19)

The Hamiltonian function for firm 1 can be written as

$$H_1 = M\left(1 + \frac{1}{G_t}\right)^{-2} - \alpha_{1t} . \frac{G_t^{\sigma_1}}{h} + \lambda_{1t}(\alpha_{1t} - \alpha_{2t}) G_t \qquad (20)$$

The first order conditions are

$$\frac{dH}{d\alpha_{1t1}} \leq 0, \alpha_{1t} \geq 0$$

$$\lambda_{1t} G_t - \frac{G_t^{\sigma_1}}{h} \leq 0, \alpha_{1t} \geq 0, (\lambda_{1t} G_t - \frac{G_t^{\sigma_1}}{h})\alpha_{1t} = 0 \text{ (Kuhn Tucker conditions)} \qquad (21)$$

$$\dot{\lambda}_{1t} = \rho \lambda_{1t} - \frac{2M}{G_t^2}\left(1 + \frac{1}{G_t}\right)^{-3} + \frac{\alpha_{1t}\sigma_1 G_t^{\sigma_1 - 1}}{h} - \lambda_{1t}(\alpha_{1t} - \alpha_{2t}) \qquad (22)$$

Transversality conditions

$\underset{T \to \infty}{Lim}$ $e^{-\rho t} \lambda_{1T} = 0$ if $G_T > 0$

$\underset{T \to \infty}{Lim}$ $e^{-\rho t} \lambda_{1T} \geq 0$ if $G_T = 0$

Combining and from Kuhn Tucker again we have, $\underset{T \to \infty}{Lim}$ $e^{-\rho t} \lambda_{1T} G_T = 0$ (23)

The Hamiltonian function for firm 2 can be written as

$$H_2 = M(1+G_t)^{-2} - \alpha_{2t} \cdot \frac{1}{h G_t^{\sigma_2}} + \lambda_{2t}(\alpha_{1t} - \alpha_{2t}) G_t \qquad (24)$$

The first order conditions are

$$\frac{dH}{d\alpha_{2t}} \leq 0, \alpha_{2t} \geq 0$$

$$-\lambda_{2t}G_t - \frac{1}{hG_t^{\sigma_2}} \leq 0, \alpha_{2t} \geq 0, (-\lambda_{2t}G_t - \frac{1}{hG_t^{\sigma_2}})\alpha_{2t} = 0 \text{ (Kuhn Tucker conditions)} \quad (25)$$

$$\dot{\lambda}_{2t} = \rho\lambda_{2t} + 2M(1+G_t)^{-3} - \frac{\alpha_{2t}\sigma_2}{hG_t^{\sigma_2+1}} - \lambda_{2t}(\alpha_{1t} - \alpha_{2t}) \quad (26)$$

Transversality conditions

$\lim\limits_{T\to\infty}$ $e^{-\rho t}$ $\lambda_{2T}=0$ if $G_T > 0$

$\lim\limits_{T\to\infty}$ $e^{-\rho t}$ $\lambda_{2T} \geq 0$ if $G_T = 0$

Combining and from Kuhn Tucker again we have, $\lim\limits_{T\to\infty}$ $e^{-\rho t}$ λ_{2T} $G_T = 0$ (27)

4. Results

4.1 Steady State

We now characterize the steady state of the economy by finding the steady state of one industry and assuming that all other industries are operating at their respective steady state levels. As in Traca and Reis (2003), we found that at the steady state the leader's rate of innovation equals the follower's rate of imitation. Formally, the dynamic equilibrium in our model has the following properties at the steady state:

(i) At the steady state $\alpha_1 = \alpha_2$.

(ii) $X_0 > \rho$ is a sufficient condition for both α_1 and α_2 to be positive at time t = 0 where $X_0 \equiv 2Mh(G_0+1)^{-3}$ and G_0 denotes G_t at time t = 0.

(iii) The solution G_s to $\max\{X_s Z_s, X_s Y_s\} = \rho$ is a stagnation steady state with neither innovation nor imitation if $\min\{X_s Z_s, X_s Y_s\} < \rho$

where

$$Z_s \equiv G_s^{\sigma_2+1}$$

and

$$Y_s \equiv G_s^{2-\sigma_1}$$

where G_0 denotes G_s at the steady state.

Proof: See Appendix

We can now formulate the main propositions of this paper.

4.2 Imitation and Appropriability in the transitional dynamics

We saw that rates of innovation and imitation are equal at the steady state. As a corollary, we also have that an increase in imitation by the follower leads to an increase in innovation by the leader and since their constant growth rate is the only variable growing in the representative industry we infer that such an increase would raise the economy's growth rate. Hence by increasing imitation, laggards put pressure on the industry leader to innovate more and it is this interaction which, in turn, drives the economy's engine of growth. Yet another corollary of the previous subsection is that at the steady state the growth rate of the technology gap is zero; thus the market configuration is stable at the steady state. Similar results were also obtained by Traca and Reis (2003). We next show that these results remain robust in the transitional dynamics under some assumptions on the gap.

Proposition 4.2.1(Imitation)

For large technology gaps, imitation and innovation are strategic complements in their transitional dynamics; that is an *increase in imitation by the laggard leads to an increase in innovation by the leader if the technology gap is large enough.*

Proof:

From (A21), we know that

$$\sigma_2\alpha_{1t} = X_t Z_t - \rho \qquad (28)$$

Given X_t and Z_t from (ii) and (iii)

(28) can be rewritten as

$$\sigma_2\alpha_{1t} = \frac{2MhG_t^{\sigma_2+1}}{\left(1+G_t\right)^3} - \rho \qquad (29)$$

We want to find the effect of a change in α_{2t} on α_{1t} .Since from (A24) we know that α_{2t} and G_t are negatively related for all G_t>2 , it suffices to show that α_{2t} and G_t are negatively related. From (29),

$$\frac{d\alpha_{1t}}{dG_t} = \frac{2MhG_t^{\sigma_2}}{\sigma_2(G_t+1)^6}\left[(G^3+3G^2+3G+1)(\sigma_2+1)-(3G^3+6G^2+3G\right] \qquad (30)$$

The above term is negative if and only if

$$\left(2-\sigma_2\right)G_t \quad +\left(3-3\sigma_2\right) > \frac{3\sigma_2}{G_t} + \frac{\sigma_2+1}{G_t^{\,2}} \qquad (31)$$

Given our earlier parameter restrictions on (8) and (9), (31) is true by definition. Hence by chain rule, the impact of α_{2t} on α_{1t} is positive.▪

Proposition 4.2.1 shows that through the establishment of an important long-run relationship between imitation and innovation the steady state result also holds during the transitional dynamics under the assumption of large technology gaps. In this dynamic game between the two firms with imitation and innovation as strategic variables, we observe that at any point in time an increase in imitation rate by the follower will prompt the leader to increase his innovation rate as long as the latter has a significant advantage over the former. Since there are only two variables growing in the representative industry[62] and the dynamic relationship between them is positive, it must be the case that the dynamic interactions between those firms will make the industry competitive at all times. In other words, process imitation creates a source of competitive pressure which deters the leader from maximizing short-run monopoly profits but rather "forces" him to innovate further.

The restriction of large gaps (G_t>2 as shown in the appendix) ensures that no leapfrogging takes place in our model. For small technology gaps, the relationship between imitation and innovation is no longer positive as the leader does not have enough productivity lead and anticipates that the imitator might close the gap by feeding off the

[62] Recall that the economy consists of n such prototypes and hence the economy should be growing at the rate of growth of the representative industry.

intra-industry spillovers. Our assumption of large gaps therefore rules out such possibilities. Moreover, Aghion et.al's (2001) findings that both firms have lower R&D incentives is the special case when the gap equals zero give some insights to our assumption.

The proof for proposition 4.2.1 is instructive since crucial to its construction is the mechanism which explains the above result. This is due to the fact that we derived the effect of imitation on the technology gap first, followed by finding the effect of the latter on innovation and eventually inferred the result by simple chain rule. Hence, imitation first reduces the gap (assuming the gap is not too narrow), and the leader receiving the signal that his technological advantage is shrinking puts in effort to restore his lead. We can also see that without the restriction of large technology gaps, an increase in the technology gap can potentially increase imitation and that the best response of the leader then would be to reduce his innovation to prevent the imitator from benefiting from the positive externalities generated by his activities. Thus in this case the follower is considered as too close to the leader for the latter to allow him to free-ride. Our restriction rules out the occurrence of the above scenario. While proposition 4.2.1 makes a strong case for reverse engineering, it also strengthens the results of most non-Schumpeterian models.

Proposition 4.2.2 (Spillovers)

In the transitional dynamics, due to the existence of a non-Schumpeterian effect, an industry with a relatively lower degree of appropriability does not necessarily grow at a slower rate; that is, an increase in the ease of spillovers or an improvement in the reverse engineering environment does not necessarily lead to a fall in the rate of innovation.

Proof:

From (31), we have

$$\alpha_{1t} = X_t Z_t \left(\frac{1}{\sigma_2}\right) - \rho\left(\frac{1}{\sigma_2}\right) \qquad (32)$$

We observe that there are two components affecting α_{1t}*.Since* ρ *is constant, the effect of a change in* σ_2 *on* α_{1t} *due to the second component of the RHS of (32) is positive. The effect of* σ_2 *on* α_{1t} *due to the first component depends on the effect of* σ_2 *on* $X_t Z_t$ *which in turn depends on the time path of* G_t *.The latter in equilibrium will depend on* σ_2 *,*σ_1 *and t. Since explicit an expression for the time path of* G_t *cannot be found, we consider two cases.*

Case (i) $\dfrac{d}{d\sigma_2}\left(\dfrac{X_t Z_t}{\sigma_2}\right) < 0$

In this case the first component is the usual Schumpeterian effect and the second component, which is unambiguously positive, is our postulated non-Schumpeterian effect.

Case (ii) $\dfrac{d}{d\sigma_2}\left(\dfrac{X_t Z_t}{\sigma_2}\right) \geq 0$

In this case we only have a non-Schumpeterian effect.

But (i) and (ii) imply that the effect of an increase in σ_2 *on* α_{1t} *is at least ambiguous. Therefore we conclude that an increase in* σ_2 *does not necessarily reduce* α_{1t} *.* ▪[63]

Proposition 4.2.2 shows that laws prohibiting process reverse engineering or policies designed to mitigate factors promoting it are not justifiable at least from the economic growth perspective. It demonstrates the impact of a decrease in appropriability (increase in σ_2) on the leader's Nash equilibrium value of innovation. We find that a

[63] Note that for case (i), there will always exist a level of the discount factor which would ensure a non-Schumpeterian effect.

higher patent protection rate in an industry does not increase innovation unambiguously since there exists a non-Schumpeterian effect working in the opposite direction of the Schumpeterian effect. Thus the heterogeneity in spillovers with a higher weight given to the one accrued by the imitator allows us to separate the impact of a general industry-level externality (σ_1) and externalities which enhance imitative behavior (σ_2). This result gives some theoretical insight into Cohen and Levinthal's (1989) empirical studies in which they conclude that the negative incentives effect of spillovers and hence the advantages of policies designed to mitigate them might not be as great as supposed. It also helps shed some light on the recent law debates surrounding the advantages and disadvantages of legalizing the act of reverse engineering.

Proposition 4.2.3 (technology gap)

If the level of appropriability in an industry is bounded from below[64] , then $S_t \leq 0$ is a necessary and sufficient condition for the stability of the dynamic system in our model, where $S_t \equiv \left(\frac{\sigma_2+2}{\sigma_2}\right)G_t^{\sigma_2+\sigma_1-1} + \left(\frac{\sigma_2-1}{\sigma_2}\right)G_t^{\sigma_2+\sigma_1} + G_t - \left(\frac{3-\sigma_1}{\sigma_1}\right)$ *; that is this condition ensures the existence of a Saddle-path to the steady state. Moreover this leads to a stable market configuration.*

Proof: See Appendix

Proposition 4.2.3 shows that as long as the specified condition on the technology gap is satisfied, the latter will always converge and the dynamic system is stable. Although it is a prima facie that this condition is merely to satisfy some technical conditions in control theoretic models, two important corollaries arise from it. First, as described in footnote 5, since the market share of a firm depends on the size of the

[64] This is equivalent to saying that σ_2 is bounded from above.

technology gap, the degree of competition or monopolization will be determined by the dynamics of the latter. Now, since from proposition 4.2.3 we know that the path of technology gap converges and is stable, we can infer that the market structure or configuration is stable in the transitional dynamics under the given condition.[65]

The second interesting corollary to emerge from the above proposition is that the condition given will impose on the gap an upper bound to which its path converges asymptotically. This means that a gap which is very large is not feasible in our model since there will be excessive free-riding from imitation. If we had allowed for the latter to occur by relaxing the condition in proposition 3.2.3, it would have been feasible that the leader might find it optimal not to innovate at all at some point in time. One can also observe that by the definition of G_t the lower bound for technology gap is 1. Now, given the latter and the upper bound restriction of proposition 3.2.3, we conclude that the technology gap and hence the market configuration is bounded in this model. Thus, the symmetric case where competition is neck and neck (when the gap tends to zero) and pure monopolization (when the gap tends to infinity) is never attained in the transitional dynamics. Hence, there is always (at any point in time) a follower who will prompt the leader to innovate further in such a market configuration and this will lead to higher growth. It is also noteworthy that the above phenomenon might be due to increasing returns on the R&D when the gap is large.[66] According to Glass (2000), an important factor in Japan's recent economic slowdown is that they have exhausted all imitation possibilities as they move closer to the world's technology frontier.

Proposition 4.2.4 (Policy Implication)

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[65] Of course the rate of growth of technology gap at the steady state is zero.
[66] See Peretto (1996) for further comment.

If the level of appropriability in an industry and the technology gap are bounded from below, the equilibrium with innovation and imitation as dynamic strategic complements exists, is unique, and is stable; that is, by choosing the spillover parameter, the policy maker can ensure the existence and uniqueness of a steady state with Saddle-path where imitation and innovation are positively related assuming that the gap is not too narrow.

Proof: See Appendix.

Proposition 4.2.4 shows that the policy maker can maneuver the nature of dynamic equilibrium by choosing the level of appropriability (inversely related to σ_2). To see this, let us think of the game in our model as a three stage game with a "pseudo" first stage in which the policy maker chooses $\sigma_2 - \sigma_1$ once and leaves it there.[67] Thus, by the same logic of backward induction described in earlier sections, the latter, acting like a "Stackelberg leader", can ensure the stability of equilibrium with a large technology gap and in which imitation and innovation are dynamic strategic complements. It is therefore possible that the policy maker can promote growth by choosing a lower bound level of appropriability (upper bound to σ_2)

It is also important to note that, as in proposition 4.2.1, this proposition also depends on the assumption of large technology gaps ($G_t>2$); for if it does not hold, the proof shows that the equilibrium with imitation and innovation as dynamic strategic substitutes can be stable. One possible explanation for our result not holding for narrow technology gaps, aside from the one given earlier (that the follower is "too close" to the leader for the latter to allow him to continue free-riding), is that the follower's marginal imitation induces relatively lower change in his market share as compared to when the

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[67] The choice is a one-shot action in this stage as compared to the second stage in which the choices are sequential.

gap is large. In economic terms, there is decreasing returns to scale as the technology gap narrows. This fact is also confirmed by empirical findings in the literature. (see Glass (2000), Peretto (1996)).

5. Concluding Remarks

We have presented an analytical model that deals with process imitation and spillovers in a non-Schumpeterian framework. Our motivation comes mainly from an apparent lacuna in existing non-Schumpeterian models in showing the interrelation between process imitation and spillovers and their impact on growth. Moreover, existing Schumpeterian models, lack adequate empirical evidence to explain growth using the concept of creative destruction. Indeed most of these studies rely heavily on the price undercutting mechanism of the homogeneous Bertrand game. We demonstrate without relaxing the assumption of product homogeneity that competitive behavior can still prevail by using a Cournot quantity competition setting. Two main factors drive competitive behavior in the long-run; firstly, imitation by the follower and, secondly, spillovers occurring due to a lack of appropriability.

The paradigm proposed in this paper can offer a basis for understanding how the dynamic strategic interactions between two firms with a technology gap can determine the economy's growth rate. In particular, imitation acts as a spur by putting pressure on the industry leader to innovate further and this drives the economy's engine of growth. Furthermore, this research can contribute to the literature on "The Law and Economics of Reverse Engineering" (See for example, Samuelson and Scotchmer, 2002) by providing some economic grounds in favor of process reverse engineering. In this regard, it demonstrates the existence of a non-Schumpeterian element in the innovator's best response function. One immediate policy implication of our model is that laws and regulations which hinder process imitation might not always be a good thing in an industry characterized by spillovers.

An obvious extension of our analysis would be to consider alterations to the duopolistic structure. While we measure market configuration (or relative monopolistic structure vis-à-vis competition) only by the technology gap between the two firms, we do not allow for entry (see footnote 6). However, we also believe that more firms entering the industry could only mean more competition and this would provide a case for non-Schumpeterian models. Further research might address the issue of entry in industries with more than two firms, or consider closed-loop games formulation rather than the open-loop case as proposed by Traca and Reis (2003) and in this paper.

6. Appendix

6.1 Derivation of the second stage quantity, profit, and R&D cost functions

In this section we derive (12)-(17).

From (11), firm j's problem is given by

$$\underset{q_j}{Max} \quad Mq_j/(q_j + q_i) - c_j q_j \;, \; j = 1,2 \quad \text{and} \quad i \text{ is not equal to } j \tag{11}$$

FOC for firm j is given by

$$(q_j + q_i)M - q_j M - c_j(q_j + q_i)^2 = 0 \tag{A1}$$

By symmetry we have,

$$(q_i + q_j)M - q_i M - c_i(q_i + q_j)^2 = 0 \tag{A2}$$

Simplifying gives

$$q_i M - c_j(q_j + q_i)^2 = 0 \tag{A3}$$

$$q_j M - c_i(q_i + q_j)^2 = 0 \tag{A4}$$

Solving (A3) and (A4) simultaneously gives (12)

Replacing (12) for both firms in (4) gives P_t

$$P_t - c_j = c_i/(c_j + c_i)^2 \tag{A5}$$

Thus the profit for firm j is given by

$$(P_t - c_j)\, q_j = \Pi_j \tag{A6}$$

(A6) verifies (13)

Using (6) and (13) for firm 1 we have

$$\Pi_1 = M(1/A_2)^2/(1/A_2 + 1/A_1)^2 \tag{A7}$$

$$\Pi_2 = M(1/A_1)^2/(1/A_2 + 1/A_1)^2 \tag{A8}$$

(9), (A7) and (A8) give (14) and (15)

Combining (7) and the identity in (10), we have

$$L_{jt}^{R\&D} = \frac{\alpha_{jt} A_{jt}}{h\Lambda_{jt}} \tag{A9}$$

Now using (A9) , (8) ,(9) combined with the fact that wage rate =1 give (16) and (17)

6.2 Proof for (i) – (iii) of the steady state.

(i) Assuming $\alpha_{1t} > 0$ in the case where the first order condition of the control variable is satisfied with equality and using (21), we have

$$\lambda_{1t} = \frac{G_t^{\sigma_1 - 1}}{h} \tag{A10}$$

Taking the derivative of (A10) w.r.t time we have

$$\dot{\lambda}_{1t} = \left(\frac{\sigma_1 - 1}{h}\right) G_t^{\sigma_1 - 1} \left(\alpha_{1t} - \alpha_{2t}\right) \tag{A11}$$

Combining (22), (A10) ,(A11) by substituting X_t ,Y_t , Z_t where needed and simplifying, we have

$$(\sigma_1 - 1)(\alpha_{1t} - \alpha_{2t}) = \rho + \sigma_1 \alpha_{1t} - (\alpha_{1t} - \alpha_{2t}) - X_t Y_t \tag{A12}$$

We prove (i), that is, that $\alpha_{1t} = \alpha_{2t}$ by contradiction.

Suppose not, then there are two possibilities : (a) $\alpha_{1t} > \alpha_{2t}$,(b) $\alpha_{1t} < \alpha_{2t}$

Case (a) If $\alpha_{1t} > \alpha_{2t} \geq 0$, then $G_t \to \infty$ as $T \to \infty$

Now since $X_t Y_t$ depend on G_t, it can be shown using L'Hopital rule that as $G_t \to \infty$ $X_t Y_t \to 0$[68]

Using the above fact and simplifying (A12) gives

$$\alpha_{2t} = \frac{-\rho}{\sigma_1} \tag{A13}$$

But this is a contradiction since for all $\rho > 0$ and $\sigma_1 > 0$, $\alpha_{2t} < 0$ contradicts $\alpha_{1t} > \alpha_{2t} \geq 0$

[68] Proof can be provided upon request.

Case (b) If $\alpha_{2t} > \alpha_{1t} \geq 0$, then $G_t \rightarrow 0$ as $T \rightarrow \infty$

Now since $X_t Y_t$ depend on G_t, it can be shown using L'Hopital rule that as $G_t \rightarrow 0$, $X_t Y_t \rightarrow \infty$.

Using the above fact and simplifying (A12) gives

$$-\alpha_{2t}\sigma_1 = \rho - X_t Y_t \quad \text{as } X_t Y_t \rightarrow \infty \tag{A14}$$

But this implies that $\alpha_{2t} \rightarrow \infty$ which gives a contradiction to $\alpha_{2t} \in [0, \infty)$ for all t.

Since (a) and (b) are not possible, it must be that $\alpha_{1t} = \alpha_{2t}$. ▪

For consistency sake we show that the proof can also be derived from firm 2's behavior.

Assuming $\alpha_{2t} > 0$ in the case where the first order condition of the control variable is satisfied with equality and using (25), we have

$$\lambda_{2t} = \frac{-1}{hG_t^{\sigma_2+1}} \tag{A15}$$

Taking the derivative of (A15) w.r.t time we have

$$\dot{\lambda}_{2t} = \left(\frac{(\sigma_2 + 1)(\alpha_{1t} - \alpha_{2t})}{hG_t^{\sigma_2+1}} \right) \tag{A16}$$

Combining (26), (A15) ,(A16) by substituting X_t ,Y_t , Z_t where needed and simplifying, we have

$$(\sigma_2 + 1)(\alpha_{1t} - \alpha_{2t}) = -\rho - \sigma_2\alpha_{2t} + (\alpha_{1t} - \alpha_{2t}) + X_t Z_t \tag{A17}$$

We prove (i), that is, that $\alpha_{1t} = \alpha_{2t}$ by contradiction.

Suppose not, then there are two possibilities : (a) $\alpha_{1t} > \alpha_{2t}$,(b) $\alpha_{1t} < \alpha_{2t}$

Case (a) If $\alpha_{1t} > \alpha_{2t} \geq 0$, then $G_t \rightarrow \infty$ as $T \rightarrow \infty$

Now since $X_t Z_t$ depend on G_t, it can be shown using L'Hopital rule that as $G_t \rightarrow \infty$ $X_t Z_t \rightarrow 0$

Using the above fact and simplifying (A12) gives

$$\alpha_{1t} = \frac{-\rho}{\sigma_2} \tag{A18}$$

But this is a contradiction since for all $\rho>0$ and $\sigma_1>0$, $\alpha_{1t}<0$ contradicts $\alpha_{1t} > \alpha_{2t} \geq 0$

Case (b) If $\alpha_{2t} > \alpha_{1t} \geq 0$, then $G_t \rightarrow 0$ as $T \rightarrow \infty$

Now since X_tZ_t depend on G_t , it can be shown using L'Hopital rule that as $G_t \rightarrow 0$ $X_tZ_t \rightarrow \infty$.

Using the above fact and simplifying (A12) gives

$$-\alpha_{1t}\sigma_2 = \rho - X_t Z_t \quad \text{as } X_tZ_t \rightarrow \infty \tag{A19}$$

But this implies that $\alpha_{1t} \rightarrow \infty$ which yields a contradiction to $\alpha_{2t} > \alpha_{1t}$ and $\alpha_{2t} \in [0, \infty)$ for all t.

Since (a) and (b) are not possible, it must be that $\alpha_{1t} = \alpha_{2t}$. ▪

(ii) We now show that as assumed by (i), α_1 and α_2 are indeed positive at the steady state.

(A14) can be rewritten as

$$\alpha_{2t}\sigma_1 = X_t Y_t - \rho \tag{A20}$$

By visual inspection of (A20), we see that $X_0 Y_0 > \rho$ as initial condition at t = 0 is a sufficient condition for α_2 to be positive in the initial state.

Also, (A19) can be rewritten as

$$\alpha_{1t}\sigma_2 = X_t Z_t - \rho \tag{A21}$$

By visual inspection of (A21), we see that $X_0 Z_0 > \rho$ as initial condition at t = 0 is a sufficient condition for α_2 to be positive in the initial state.

But since both Y_0 and Z_0 are > 1 by definition, it must be that

$X_0 > \rho$ is a sufficient condition for both α_1 and α_2 to be positive at time t = 0.▪

(iii) Those conditions can easily be inferred from (A20) and (A21).

6.3 Proof for negative relationship between α_{2t} and G_t for large G_t.

Rewriting (A20), we have

$$\sigma_1 \alpha_{2t} = \frac{2MhG_t^{2-\sigma_1}}{(1+G_t)^3} - \rho \quad \text{(A22)}$$

$$\frac{d\alpha_{2t}}{dG_t} = \frac{2MhG_t^{1-\sigma_1}}{\sigma_1 (G_t+1)^6}\left[(G^3+3G^2+3G+1)(2-\sigma_1)-(3G^3+6G^2+3G\right] \quad \text{(A23)}$$

The above term is negative if and only if

$$(1+\sigma_1)G_t \ + 3\sigma_1 > \frac{3-3\sigma_1}{G_t} + \frac{2-\sigma_1}{G_t^{\ 2}} \quad \text{(A24)}$$

Given our earlier parameter restrictions on (8) and (9), (A24) is true for all G_t>2.▪

6.4 Proof of Proposition 4.2.3

Using (A12) and (10) by letting $\dot{G}_t \ / G_t = g_t$, we have

$$\sigma_1 g_t = \rho + \sigma_1 \alpha_{1t} - X_t Y_t \quad \text{(A25)}$$

Using (A17) and (10) by letting $\dot{G}_t \ / G_t = g_t$, we have

$$-\sigma_2 g_t = \rho + \sigma_2 \alpha_{2t} - X_t Z_t \quad \text{(A26)}$$

From (A25), we have

$$\alpha_{1t} = (-\rho + \sigma_1 g_t + X_t Y_t)/\sigma_1 \quad \text{(A27)}$$

From (A26), we have

$$\alpha_{2t} = (-\rho - \sigma_1 g_t + X_t Z_t)/\sigma_2 \quad \text{(A28)}$$

Solving (A27), (A28) and (10) by letting $\dot{G}_t / G_t = g_t$, we have

$$g_t = \frac{X_t Z_t}{\sigma_2} - \frac{X_t Y_t}{\sigma_1} + \left(\frac{\sigma_2 - \sigma_1}{\sigma_2 \sigma_1}\right)\rho \tag{A29}$$

$$\frac{d\dot{G}_t}{dG_t} = \frac{2MhG_t^{2-\sigma_1}}{(1+G_t)^4}\left\{(1+G_t)\left[\left(\frac{\sigma_2+2}{\sigma_2}\right)G_t^{\sigma_2+\sigma_1-1} - \left(\frac{3-\sigma_1}{\sigma_1}\right)\right] - 3\left[\frac{G_t^{\sigma_2+\sigma_1}}{\sigma_2} - \frac{G_t}{\sigma_1}\right]\right\} + \frac{(\sigma_2-\sigma_1)\rho}{\sigma_2\sigma_1} \tag{A30}$$

For $\sigma_2 - \sigma_1 \rightarrow 0$ (which can hold only if σ_2 is bounded from above), the RHS of (A30) is negative if and only if

$$S_t \equiv \left(\frac{\sigma_2+2}{\sigma_2}\right)G_t^{\sigma_2+\sigma_1-1} - \left(\frac{1-\sigma_2}{\sigma_2}\right)G_t^{\sigma_2+\sigma_1} + G_t - \left(\frac{3-\sigma_1}{\sigma_1}\right) \leq 0 \tag{A31}$$

This completes the proof.▪

<u>6.5 Proof of Proposition 4.2.4</u>

Using (A29) and the fact that at the steady state $g_t = 0$, we have

$$\rho(\sigma_2 - \sigma_1) = X_t Y_t \sigma_2 - X_t Z_t \sigma_1 \tag{A32}$$

Re-arranging by substituting the expressions for X_t, Y_t, and Z_t,we have

$$\frac{(1+G_t)^3 \rho}{2Mh} = \frac{G_t^{2-\sigma_1}\sigma_2 - G_t^{\sigma_2+1}\sigma_1}{\sigma_2 - \sigma_1} \tag{A33}$$

Since $G_t^{2-\sigma_1}\sigma_2 > G_t^{\sigma_2+1}\sigma_1$ (given our early parameter restrictions σ_2 ,σ_1<1/2 , $\sigma_2 > \sigma_1$ and G_t>1), we observe from (A33) that as $\sigma_2 - \sigma_1 \rightarrow 0$ (assuming that if σ_2 is bounded from above), there exists some $\underline{G_t}$ such that the RHS of (A33) > LHS of (A33). Hence, we have $\underline{G_t}$ where $1 < \underline{G_t} < \infty$ such that RHS> LHS. Now it can also be shown that both the RHS and LHS of (A33) are monotonically increasing and convex for our early parameter restrictions σ_2 ,σ_1<1/2 , σ_2 >σ_1 and G_t>1[69]. Thus by visual inspection of (A33), we see that as $G_t \rightarrow\infty$, the LHS of (A33) > RHS of (A33) since the power of the terms in G_t of the LHS are always higher than that of the RHS. Therefore, given the monotonocity of the LHS and the RHS, we infer that there exists some $\overline{G_t}$ such that the LHS of (A33) > RHS of (A33). Hence we have $\overline{G_t}$ where $1 < \underline{G_t} < \overline{G_t} < \infty$ such that LHS> RHS. Using (A33), we define a function F_t given by

$$F_t(G_t) = \frac{(1+G_t)^3 \rho}{2Mh} - \frac{G_t^{2-\sigma_1}\sigma_2 - G_t^{\sigma_2+1}\sigma_1}{\sigma_2 - \sigma_1} \tag{A34}$$

As noted above, for some G_t , RHS> LHS and for some G_t, LHS> RHS. Thus for some G_t , F_t is positive and for some G_t , F_t is negative. It well-known from the Weierstrass Intermediate Value Theorem that if a continuous function on an interval is sometimes positive and sometimes negative, it must be zero at some point. Let this point be G_t^* . This proves the existence of a fixed point such that $1 < \underline{G_t} < G_t^* < \overline{G_t} < \infty$. The proof for uniqueness follows from the monotonocity of both sides off (A33).

We now prove for Saddle path stability assuming $\sigma_2 - \sigma_1 \rightarrow 0$ (this can hold if σ_2 is bounded from above).

We observe that for earlier parameter restrictions σ_2 ,σ_1<1/2 , σ_2 >σ_1 and G_t>1, the fourth term of (A31) is larger than its first term. A sufficient condition for stability is therefore that the second term is larger than the third term. This is true if and only if

[69] Proof can be provided upon request.

$$\left(\frac{1-\sigma_2}{\sigma_2}\right)G_t^{\sigma_2+\sigma_1} > G_t \qquad \text{(A35)}$$ or

$$G_t > \left[\frac{1-\sigma_2}{\sigma_2}\right]^{\frac{1}{1-\sigma_2-\sigma_1}} \qquad \text{(A36)}$$ This

establishes a lower bound for the technology gap which will ensure stability. Hence if the gap is large enough ($G_t > \left[\frac{1-\sigma_2}{\sigma_2}\right]^{\frac{1}{1-\sigma_2-\sigma_1}}$) and the level of appropriability is bounded from below (σ_2 is bounded from above and thus, $\sigma_2 - \sigma_1 \rightarrow 0$), the system is stable. We now show that the equilibrium with innovation and imitation as dynamic strategic complements is stable.

Recall from Proposition 4.2.1 that if $G_t>2$, innovation and imitation are strategic complements in their transitional dynamics. In other words, we need the lower bound on the technology gap to be larger than 2, that is

$$G_t > \left[\frac{1-\sigma_2}{\sigma_2}\right]^{\frac{1}{1-\sigma_2-\sigma_1}} > 2 \text{ or} \qquad \text{(A37)}$$

$$\frac{1-\sigma_2}{\sigma_2} > 2^{1-\sigma_2-\sigma_1} \qquad \text{(A38)}$$

But (A38) holds by definition given our parameter restrictions. Hence, the path on which innovation and imitation are strategic complements is a Saddle path. This completes the proof. ▪

IV. Economic Growth and Process Spillovers with Step-by-Step Innovation

Abstract

This paper extends previous research on the effects of process imitation on economic growth by accounting for stochastic intra-industry spillovers. We employ a non-Schumpeterian growth model to determine the impact of such spillovers on investment in industries where firms are either neck-and-neck or unleveled. Our central finding is that, in an economy where the representative industry is a duopoly, R&D spillovers positively affect economic growth. While other non-Schumpeterian models assume that the imitation rate of laggard firms is unaffected by the R&D effort of the leader firm, we consider the case where the latter's R&D activity generates some positive externality on its rivals' research. In this construct, the duopolists in each industry play a two-stage game. In the first stage, they invest in R&D which can reduce their costs of production only if they successfully innovate and they compete with each other by using Markovian strategies. In the second stage, they compete in the product market. At any point in time, an industry can either be in the neck-and-neck state or in an unleveled state where the leader is n steps ahead of the follower. At the steady state, the inflow of firms to an industry must be equal to the outflow. By determining the steady state investment levels of each industry, we demonstrate a positive monotonic relationship between the spillover rate and economic growth.

Keywords: Step-by-Step Innovation, non-Schumpeterian Growth, Process Spillovers, Imitation.

JEL Classification Numbers: C7, E0, L1, O3

1. Introduction

Endogenous growth theorists have investigated extensively the impact of low appropriability on the growth rate of an economy. While Schumpeterian models[70] posit a negative relationship between them, more recently developed non-Schumpeterian models have shown that the relationship is non-monotonic and that a strict negative relationship only holds whenever the level of appropriability is very low.[71] Aghion et al. (2001) made a seminal contribution to the non-Schumpeterian branch of endogenous growth by showing that static monopoly is not always a necessary evil for long-run efficiency in a step-by-step innovation growth model (referred to as the AHV model). Their result is consistent with the empirical findings of Blundell, Griffith and Reenen (1995), Nickell et al. (1996) and the theoretical predictions of D'Aspremont et al. (2002).[72] The AHV model, together with contributions noted above, however, downplay the role of externalities in strategic interactions among firms. In contrast, an important strand in the industrial organization (IO) literature argues that process spillovers play a key role in two stage non-cooperative R&D games[73] because they capture the diffusion of technology between leaders and laggards. Since such externalities depend on the level of appropriability in an industry, the effect of lower or higher appropriability on growth can be observed by the impact of spillovers on the latter. The purpose of this study is to determine the effects of process R&D spillovers on growth by extending the AHV framework.

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[70] See Gossman and Helpman (1991) and Segerstrom (1991).

[71] In particular, they find an inverted-U shaped relationship between innovation and a parameter which promotes competition in their models.

[72] Blundell et al. (1995) found that the firm's market power and R&D competition are not necessarily negatively related, Nickell (1996) provides the empirical evidence that a larger number of firms is usually associated with a higher level of productivity.

[73] D'Aspremont and Jacquemin (1989) pioneered this framework.

The relationship between spillovers and R&D incentives has two aspects. First, in many industries, firms undertake R&D investments in order to develop new products or processes. One feature of R&D investment that distinguishes itself from other forms of investment is that firms which do the investing are often not able to exclude others from freely benefiting from their investments. Thus, the benefits from R&D investments spill over to other firms in the economy.[74] Now since the laggards can improve their own technology by free-riding on the leader's research, technologically more advanced firms might have a disincentive to undertake more research since their productivity lead might be significantly reduced in the presence of such spillovers. Hence, the first characteristic of R&D spillovers is that it can potentially reduce research incentives. The second aspect of spillovers is related to the concept of "escape competition".[75] When the laggard firms benefit from process R&D spillovers, they improve their own technology and thereby reduce the technology gap[76] between the leader and themselves. As a result, there will be competitive pressure on the leader to innovate further to maintain its lead. Those two opposing forces of spillovers on R&D are observed in the empirical findings of Cohen and Levinthal (1989).[77] Given the importance of spillovers in the strategic interactions between firms investing in R&D, we allow them to play a major role in our model.

The purpose of this paper is to explore the interdependence of spillovers, appropriability and growth in a framework where strategic interactions between firms are taken into account. Our work is primarily motivated by the empirical study of Zachariadis

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[74] Griliches (1979) emphasized the significance of spillovers in modeling and estimating the effects of R&D investments.
[75] "Escape competition" refers to the motive of innovating in order to escape competition; that is, firms in the neck-and-neck state will innovate to obtain a productivity lead over their rivals.
[76] Cameron (1999) found that there were more free-riding or imitation possibilities when the technology gap is large than when it was low.
[77] They found, contrary to previous studies, that intra-industry spillovers may encourage rather than deter R&D investment.

(2003) who finds strong support for technological spillovers from aggregate research intensity to industry-level innovation success.[78] We develop a dynamic general equilibrium model which is distinct from the AHV model in two major ways. First, we consider the case for homogeneous Cournot competition rather than differentiated Bertrand competition.[79] Secondly, we assume the hazard rate of imitation to be dependent on the spillovers induced by the leader's R&D. Thus, owing to the presence of externalities, the probability of the laggard making a successful innovation is a fraction of the leader's probability of doing so. We therefore highlight the role of spillovers in an economy in which firms play a differential R&D game. We consider a one-sector endogenous growth model whereby in the second stage, duopolists in the representative industry sell a homogenous good to consumers who spend a fixed proportion of their income in each period.[80] In the first stage, while the industry leader innovates and moves one step up the technology ladder with some probability, the follower imitates and catches up with the leader with some hazard rate. Thus, at any point in time, we can have industries in different states with the technology gap ranging from 0 to n.[81] Stationarity implies that for some state n, the inflow of industries in that state should be equal to the outflow. By computing the growth rate at the steady state, we derive two sets of results.

First, as in the AHV model, we look at the case where the innovation lead of the leader is so high that it has no incentive to increase its lead by one step. This simplification allows us to reduce the number of states to only state 0 and state 1. By

[78] They used US manufacturing data to estimate a system of three equations implied by a model of R&D induced growth at the steady state.
[79] Although, in another paper, Aghion et al. (1997) compared the Bertrand and the Cournot cases, they did not allow the industry leader to extend its lead by more than one step in their model.
[80] This simplification, also found in Grossman and Helpman (1991), Segerstrom (1991) and AHV (2001), "transfers" the dynamics from the consumer side to the producer side of the economy.
[81] Note that state 0 is also known as the neck-and neck-state.

computing the fraction of industries in those states and by deriving the optimal neck-and-neck and unleveled innovation rates[82], we can derive the steady state growth rate of the economy by using the stationarity condition that the fraction of industries in every state is constant in long-run equilibrium. We then use comparative statics to find the impact of the spillover parameter on the growth rate and consider whether lack of appropriablity necessarily reduces the growth rate for the case of a large innovation lead.

Secondly, we consider the case where the innovation lead can be small and hence, in the case at hand, the leader has incentives to extend its lead further than one step. While AHV (2001) use a method of asymptotic expansion to derive results for the "small innovation lead" case, our results are derived directly from the Bellman's equations since our assumption of Cournot competition allows our profit functions to be independent of the competition parameter as opposed to the differentiated Bertrand case. Thus, we also solve for the optimum R&D effort of the leader who wants to move more than one step ahead of its follower and derive the fraction of industries which might be in that state in equilibrium.[83] We shall then have three optimal levels of R&D effort as well as three steady state fractions of industries in the respective states. Results are derived in a similar fashion as in the previous case with "large innovation lead" and we can therefore analyze how the policymaker, by varying the appropriability rate of the industry, can affect the research incentives in the neck-and-neck state, as well as the unleveled states. Some policy implications on whether larger appropriability promotes growth are then drawn.

Our results show that the growth rate is unaffected by the spillover rate for the "large innovation lead" case. Thus, in contrast to the traditional Schumpeterian argument,

[82] As in AHV, the innovation rate refers to the probability of success of R&D.
[83] For simplicity, we shall consider only states 0,1,2.

we find that lower appropriability does not necessarily reduce growth, when the rate of imitation depends on the R&D spillovers. Moreover, we find as in the AHV model that the level of R&D effort is greatest at the neck-and-neck state and that this constitutes a major component of the economy's growth rate. For the case of "small innovation lead", our findings indicate that process spillovers affect growth positively and that imitation and innovation can be strategic substitutes.[84] We also note that the fraction of industries in the state in which the leader is more than one step ahead is positively related to the spillover rate and to the R&D effort in the neck-and-neck state. Clearly, it follows from our results that the trade-off between short-run monopoly and long-run efficiency is not observable in a framework where both strategic interactions between firms and diffusion of technology are taken into account. Hence, an immediate policy implication is that greater appropriability is not always good for the growth rate of the economy.

On the normative side, one interpretation of the main result is that the more technologically advanced firm will innovate even further in the face of process spillovers in order to maintain its productivity lead. It is noteworthy that unlike the AHV result, ours is not heavily dependent on the product differentiability parameter. Thus, the model identifies the "pure" effect of process spillovers, which enhance imitation, on welfare. Specifically, imitation and unintended technological diffusion can promote growth. As a consequence, we shed some light on the ongoing debate as to whether or not restricting the act of reverse engineering is justifiable on economic grounds.[85] We believe that in an industry where reverse engineering can hasten the diffusion of technology via process spillovers, the strategic interaction between rival firms will guarantee that a competitive

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[84] While R&D effort in the neck-and-neck state is negatively related to the imitation rate of the laggard, it is positively related to the innovation rate of the leader.

[85] See Scotchmer and Samuelson (2002) and Handa (1995).

environment always prevails. Furthermore, the innovator does not have an incentive to lay back as a monopolist as its technological lead might fall. Hence, growth is always enhanced by more competition.

The remainder of this paper is organized as follows. In Section 2, we review the relevant literature. Section 3 presents the formal model, derives the steady state growth rate and provides some comparative static results. Section 4 offers some concluding remarks.

2. Related Work

In this section we briefly review some related studies. It was recognized in the early work of Ruff (1969) that it is very difficult to decide whether the Schumpeterian argument that the static inefficiency of less competitive firms was more than offset by superior dynamic performance. Ruff analyzed the symmetric equilibrium in a multi-firm economy[86] and concluded that technological progress will be optimal in an economy if research opportunities, population and the initial technological level are large enough and the discount rate is small enough. Also, he found that dynamic efficiency can be achieved in an economy where firms form a cooperative research lab. Recent work has criticized Ruff's assumption of non-Markovian strategies in a noncooperative differential game setting since members are not allowed to respond to feedback during the game. However, in spite of this limitation, his assumption of a dynamic Cournot economy best describes industries in which firms undertake process R&D. In our model, we consider a dynamic Cournot economy where firms play Markovian strategies while choosing their R&D levels.

The prevailing paradigm stems from Schumpeter's idea of creative destruction.[87] In a model of vertical innovations, Aghion and Howitt (1992) showed that the prospect for more future research discourages current research by threatening to destroy rents created by the latter. This finding was substantiated by Grossman and Helpman (1991a; 1991b), Segerstrom (1991) and Barro and Sala-i-Martin (1995). Some of these studies point out the role of ongoing product-upgrading and product cycles in characterizing the

[86] This has also been referred to as the Cournot economy.
[87] See Schumpeter (1934) for more details.

steady state equilibrium. In particular, the firm holding the state-of-the-art, that is the one with the lowest price adjusted quality acts as monopolist in the representative industry. Moreover, due to the homogenous Bertrand assumption in such models, imitation can be carried out only by the relatively lower cost firms, while successful innovation leads to instantaneous leapfrogging. These models show that, in general, three equilibria can exist: The monopolist is a low-cost innovator; the monopolist is a leader who has regained its lead from a low-cost imitator and the monopolist is an innovator who has leapfrogged the leader. In a related study, the idea of reverse engineering and learning to learn was introduced by Connolly (1997) in North-South trade models.

The non-Schumpeterian framework of endogenous growth was pioneered by Aghion et al. (1997, 2001). Our paper is closest to theirs as we also share their view that in an economy where imitation of the technological leader is made easier, growth is not negatively affected. The main argument of non-Schumpeterian models is that the leaders would have an incentive to innovate further in order to reestablish their lead. AHV (2001), in a model of growth with step-by-step innovation considered the relationship between product market competition and growth in which the laggard's imitation rate is enhanced by its R&D investment but does not contribute to the economy's stock of knowledge. While firms compete in the product market in the second stage, in the first stage there are three states in which R&D can take place; the neck-and-neck state, an unleveled state in which the leader is one step ahead and an unleveled state in which the leader can be more than one step ahead of its rival. They showed that the R&D level is higher at the neck-and-neck state and concluded that the latter state promotes growth as it is the only state where two firms (instead of one) are trying to advance the industry's technological frontier. Aghion et al. (1997) give an example of the above general model by assuming that the leader has no incentive to extend its lead by more than one step.

More recently, non-Schumpeterian models have supported the argument that imitation and more product competition can enhance economic growth. Traca and Reis (2003) developed an endogenous growth model of duopolistic competition in which there are knowledge spillovers induced by the firms' R&D and such spillovers raise the level of innovation as they spur a source of competitive pressure on the leader. Spillovers increase innovation since they reduce the laggard's innovation costs and this signals the leader to innovate further, lest it will forfeit its competitive advantage. Hence, the leader's incentive to innovate increases as it anticipates that it might lose its market share. Thus spillovers expand the R&D of both firms. Our model differs from that of Traca and Reis (2003) in the following ways. First, we consider the case where the leader innovates and the follower imitates. Secondly, we assume a one-way spillover structure in our model to highlight the leader-follower configuration. Such asymmetry is not addressed in their paper. Another non-Schumpeterian model with no spillovers is developed by Mukoyama (2003), who showed that subsidizing imitation might increase the economy-wide rate of technological progress and that competition and growth might be positively correlated.

The main point of departure of our work from the existing literature is our simplistic assumption of Cournot competition. The latter, in contrast to Bertrand competition, implies that the only way for the leader (follower) to increase its market share is to increase (decrease) the cost differential which is given by the technology gap, as will become clear in the next section. We therefore rule out the possibility of product innovations as our Cournot assumption necessarily implies that all imitation and innovation occur at the process level. It is noteworthy that both AHV (2001) and Traca and Reis (2003) consider only process imitation/innovation but used the differentiated Bertrand assumption in which both product and process innovations are possible. In addition, our assumption not only allows the two firms in the representative industry to compete in both the product market and the R&D sector for the homogenous product case (in contrast to the homogenous Bertrand case), but it also helps us to derive results without having to depend on the product differentiability parameter as in AHV.[88] We therefore contribute to the literature by introducing a framework of step-by-step innovation with spillovers in a leader-follower configuration.

[88] AHV (2001) used a transformation of product differentiability as a proxy for competition in their model.

3. The Model

3.1 Overview

We consider a model with d goods, d industries with 2 firms each, and infinitely lived identical consumers. The latter face two optimization problems: temporal and intertemporal utility maximization. Preferences across goods and time are logarithmic. In the intertemporal problem the consumer chooses the optimal labor supply and consumption (or expenditure) for each period. The remaining income is invested in the industries' R&D. To simplify the model we make the following assumptions.[89] Firstly, we normalize expenditures to allow the rate on return to capital (savings of agents) to be constant and equal to the exogenously given discount rate. Moreover, we also assume that the risk of any firm is idiosyncratic and that the stock market values the firm so that its expected rate of return equals the risk free interest rate.[90] Secondly, we assume that labor supply is perfectly elastic.[91]With the optimal amount to be spent in each period chosen, the representative agent can thus derive his demand function for each industry from his temporal optimization problem.

On the production side, the industry demand is derived from the consumer problem and taking the demand schedule as given, the duopolists in the representative industry compete in Cournot fashion to choose their respective output and research intensities (which are innovation rate for the leader and imitation rate for the laggard). If the productivity of one firm is higher than its competitor, then the former is the leader and the latter is the follower. Moreover, the leader moves up the technology ladder with a Poisson hazard rate by employing some units of labor, while the follower catches up with

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[89] These do not lead to loss of generality in our propositions.
[90] This is similar to Grossman and Helpman (1991).
[91] This follows from Aghion et al (2001).

some Poisson hazard rate which consists of two components (one of which is proportional to the leader's success rate). The leader does not benefit in terms of externality from the follower, while the latter which practices process reverse engineering benefits from the leader.

The choices of the firm's variables which are quantity and research intensities are found sequentially. We therefore use backward induction in a two-stage non cooperative game setting to formulate the firms' optimum behavior. In the second stage the firms play a Cournot game to determine their respective quantities. Their respective profits as functions of their productivity[92] levels can thus be derived. In the first stage, the leader (follower) plays a differential game to choose its optimal innovation (imitation) in each possible state taking the technology gap as given. The Markovian Nash equilibrium is then found. The steady state growth rate is determined by the optimal values of imitation and innovation and is used to derive results. The effects of changes in appropriability and growth are subsequently analyzed.

3.2 Formal Model

Consumers

Let d , C_t , L_t , Q_{it} , R_t and W_t be the number of industries, the consumption of the representative agent, his labor supply, quantity produced in the industry i for i = 1,2...,d, the interest rate and the wage rate respectively at time t. Then the intertemporal preference of the agent can be written as

$$U \equiv \int_0^\infty e^{-\rho t}\left(u(C_t) - L_t\right)dt, \rho > 0 \quad \text{where } \rho \text{ is the discount rate} \tag{1}$$

$$u(C_t) \equiv \ln(C_t) \tag{2}$$

[92] Productivity is defined in terms of the per unit cost as in Traca and Reis (2003).

The intertemporal utility maximization problem results in (i) $W_t = 1$ and (ii) $R_t = \rho$ after normalization.[93] The temporal consumer preference is given by

$$u(C_t) \equiv \sum_{i=1}^{d} \ln(Q_{it}) \text{ for all t} \tag{3}$$

The static utility maximization problem results in the industry demand curve[94]

$$Q_{it} = \frac{M}{P_{it}} \qquad \text{where M = 1/d} \tag{4}$$

Producers

Given the industry demand (4) each firm will choose its respective optimal production q_{ijt} such that [95]

$$\sum_{j=1}^{2} q_{ijt} = Q_{ijt}$$

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[93] Let consumer's wealth at time t be A_t , P_t be the price of consumption and $P_t C_t = 1$ due to normalization, then we have

$H = \ln(C_t) - L_t + \lambda_t(R_t A_t + W_t L_t - P_t C_t)$

$dH/ dC_t = 0$ implies $1/ P_t C_t = \lambda_t$, but since $P_t C_t = 1$, $\lambda_t = 1$

$dH/ dL_t = 0$ implies $W_t \lambda_t = 1$ and hence $W_t = 1$

Also $d \lambda_t / dt = \rho \lambda - \lambda R_t$, but since $\lambda_t = 1$, $d \lambda_t / dt = 0$ or $\rho - R_t = 0$ and therefore $R_t = \rho$

[94] Static utility maximization leads to

$$\text{Max} (Q_i) \; u(C_t) \equiv \sum_{i=1}^{d} \ln(Q_{it}) \text{ s.t}$$

$$\sum_{i=1}^{d} P_i Q_i = I$$

and since Income = Expenditure = 1, the constraint becomes

$$\sum_{i=1}^{d} P_i Q_i = 1$$

The logarithmic assumption leads to the following demand curve $Q_{it} = \dfrac{1}{dP_{it}}$

[95] In this subsection of the paper, we sometimes omit subscript i for simplicity.

We assume that firm 1 is the leader and firm 2 is the follower. Each firm's production function is given by

$q_j = A_j L_j$ for j =1,2 (5)

It can easily be inferred from (5) that the per unit cost of each firm is given by W^*/ A_j where W^* is the economy level wage rate. Also, due to our assumption $W_t = 1$, the per unit cost becomes

$$c_j = 1/ A_j \tag{6}$$

We denote the productivity lead of leader n steps ahead as follows

$$\left(\frac{c_2}{c_1}\right)^n \equiv \left(\frac{A_1}{A_2}\right)^n \qquad \text{for } n = 1,2,... \tag{7}$$

We also denote the size of the lead by

$$\left(\frac{A_1}{A_2}\right) \equiv \gamma \tag{8}$$

Thus an increase in γ and/or n will increase (decrease) the leader's (follower's) profit. We assume innovative and imitative activities to be randomly determined. Specifically, we assume (as in AHV) that the leader or a neck-and-neck firm in state n, by employing $\psi(x)$ units of labor in R&D moves one step ahead with a Poisson hazard rate[96] of x_n for n = 0,1,2…, while the follower catches up with its rival with a Poisson hazard rate of $\bar{x}_n + h_n$[97], where the R&D cost function $\psi(x)$ is an increasing and convex function of the

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[96] Formally, we let $H(n) = x_i u(n)$ be the hazard rate in state n where $u(n)$ is the hazard function. Using the exponential distribution which has been widely used in the literature, $u(n) = 1$ and hence $H(n) = x_i$ for firm i.

[97] In any state n, x_n is the leader's success rate while $\bar{x}_n$ is the follower's catching up probability.

R&D effort and h_n is the ease of imitation or R&D spillovers parameter. We define such spillovers as follows

$$h_n = bx_n \tag{9}$$

Our definition of spillovers is similar to Cohen and Levinthal (1989) although we pursue some extensions. In particular, we define spillovers to include valuable knowledge generated in the research process of the leader, which becomes accessible to the follower if and only if the latter is reverse engineering the innovator's research process.[98] Given that spillovers favor imitation, it becomes a better strategy for the follower to imitate by feeding off the leader's innovation at least initially. Thus the follower is necessarily an imitator.

It is also implicit from our assumption that it is process imitation rather than product imitation which takes place in our framework. This also means that the conventional definition of reverse engineering as the decompilation of a finished product in order to gain a better understanding of how it was produced as in Handa (1995) does not fit well into our model. Rather we see reverse engineering as the act of extracting know-how or information from the industry leader through channels like the labor market (turnover in R&D personnel, for example) in order to imitate the latter's process (or cost-cutting) innovations.[99] It is also to be noted that while AHV make use of an "ease to imitate" parameter and a "competition parameter" to proxy the absence of institutional, legal and regulatory impediments connected with patent laws and regulations, in our model b in (9) includes all of these factors.

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[98] We assume, however, the follower incurs a fixed cost of undertaking reverse engineering. Such costs do not affect R&D decisions since they vanish when the first order conditions are found.

[99] Nevertheless, our definition still belongs to a more general class of definitions of reverse engineering.

We shall consider the stationary closed-loop Nash equilibrium in Markovian strategies in which each firm's R&D effort depends on its current[100] state as well as its current R&D level and not on the industry to which the firm belongs or the time. We assume without loss of generality that the R&D cost function is given by

$$\psi(x) = \frac{\beta x^2}{2} \qquad \text{for } \beta > 0 \tag{10}$$

This completes the model.

3.3 Solving the Model

We solve the model by backward induction.

Stage 2

Using the inverse demand function (4), firm j's profit maximization problem becomes

$\text{Max}(q_j)\quad Mq_j/(q_j + q_i) - c_j q_j$, j =1,2 and i is not equal to j (11)

The Cournot Nash quantity for firm j is given by

$q_j = (Mc_i)/(c_j + c_i)^2$, j =1,2 and i is not equal to j (12)

The profit function for firm j is given by

$\Pi_j = (Mc_i^2)/(c_j + c_i)^2$, j =1,2 and i is not equal to j (13)

Proof: See Appendix

Remark 3.3.1

For all $\gamma \geq 1$, *(i)* $\overline{\pi}_n = \dfrac{M}{\left(1+\gamma^n\right)^2}$ *is strictly decreasing in* γ *and* $\pi_n = \dfrac{M}{\left(1+\gamma^{-n}\right)^2}$ *is strictly increasing in* γ *; (ii)* $\dfrac{M}{\left(1+\gamma^n\right)^2} = \dfrac{M}{\left(1+\gamma^{-n}\right)^2}$ = *¼M if* γ *=1 and* $\dfrac{M}{\left(1+\gamma^n\right)^2} + \dfrac{M}{\left(1+\gamma^{-n}\right)^2}$ >

[100] Note that the word "current" is used across states rather than over time.

½M other wise where π_n *and* $\overline{\pi}_n$ *are the profits of the leader and the follower respectively in state n.*

Proof: See Appendix

The first part of Remark 3.3.1 states that a higher (lower) relative cost, that is, the larger (lower) the technology gap in favor of the leader (follower) is always strictly advantageous to its profit. The second part of Remark 3.3.1 states firstly that when the firms are in the neck-and-neck state, they have equal profits and, secondly, that the sum of the firm's profit in an asymmetric duopoly is larger than the sum of profits when firms are symmetric. Thus, when there is more than the minimal degree of competition, total profits are lower if firms are neck-and-neck; with identical costs than if one has a relative cost advantage. This fact, which is also consistent the AHV Bertrand differentiated product case, is important for the derivation of our results.

Stage 1 (The Closed-Loop Formulation)

The N-tuple $(\phi_1, \phi_2, ..., \phi_N)$ *of functions* $\phi_i : X \times \{1,2...N\} \mapsto R^{m^i}$ *, state* $i \in \{1,2..N\}$*, is called a Markovian or closed loop Nash equilibrium if, for each* $i \in \{1,2..N\}$*, a rule* $u_i(.)$ *of the problem below exists for each player and is given by* $u_i(.) = \phi_i\ (x(i), i)$*.*[101]

Let V_0, V_n and $\overline{V_n}$ denote the expected present value of the profits of the neck-and-neck firm, the leader and the follower respectively. Given that the equilibrium interest rate equals the rate of time preference, we derive V_0, V_n and $\overline{V_n}$ heuristically from the Bellman equations as follow:

[101] The general case is formulated by Dockner et al. (2000).

$$V_n = \underset{x_n}{Max}\left\{\left(\pi_n - \frac{\beta x_n^2}{2}\right)dt + e^{-rdt}\left[x_n dt V_{n+1} + \left(\overline{x}_n + bx_n\right)dt V_0 + \left(1 - x_n dt - \left(\overline{x}_n + bx_n\right)dt\right)V_n\right]\right\} \tag{14}$$

$$\overline{V}_n = \underset{\overline{x}_n}{Max}\left\{\left(\overline{\pi}_n - \frac{\beta \overline{x}_n^2}{2}\right)dt + e^{-rdt}\left[x_n dt \overline{V}_{n+1} + \left(\overline{x}_n + bx_n\right)dt V_0 + \left(1 - x_n dt - \left(\overline{x}_n + bx_n\right)dt\right)\overline{V}_n\right]\right\} \tag{15}$$

$$V_0 = \underset{x_0}{Max}\left\{\left(\pi_0 - \frac{\beta x_0^2}{2}\right)dt + e^{-rdt}\left[x_0 dt \overline{V}_1 + x_0 dt V_1 + \left(1 - x_0 dt - x_0 dt\right)V_0\right]\right\} \tag{16}$$

As in AHV (14) can be interpreted as follows: the value of currently being a leader n steps ahead at time t equals the discounted value at time (t +dt), plus the current profit flow $\pi_n dt$, minus the current R&D cost $(\frac{\beta x^2}{2})dt,$ plus the expected discounted capital gain from innovation, thereby moving one step ahead of the follower, minus the discounted expected capital “loss” from having a follower catch up. Similar interpretations can be made for (15) and (16). For dt small, $e^{-rdt} \cong 1 - rdt$ and the second order terms in (dt) can be ignored. Then (14)-(16) can be rewritten as follow:

$$rV_n = \pi_n - \frac{\beta x_n^2}{2} + x_n\left[V_{n+1} - V_n\right] + \left(\overline{x}_n + bx_n\right)\left[V_0 - V_n\right] \tag{17}$$

$$r\overline{V}_n = \overline{\pi}_n - \frac{\beta \overline{x}_n^2}{2} + x_n\left[\overline{V}_{n+1} - \overline{V}_n\right] + \left(\overline{x}_n + bx_n\right)\left[V_0 - \overline{V}_n\right] \tag{18}$$

$$rV_0 = \pi_0 - \frac{\beta x_0^2}{2} + x_0\left[\overline{V}_1 - V_0\right] + \overbrace{x_0}^{R\&D\ effort\ of\ the\ rival}\left[V_1 - V_0\right] \tag{19}$$

Maximizing the RHS of (17)-(19), we have

$$x_n = \frac{(V_{n+1} - V_n)}{\beta} + \frac{(V_0 - V_n)b}{\beta} \tag{20}$$

$$\bar{x}_n = \frac{(V_0 - \bar{V}_n)}{\beta} \tag{21}$$

$$x_0 = \frac{(V_1 - V_0)}{\beta} \tag{22}$$

We can now use equations (17)-(22) to solve recursively for the sequence $\{x_n, \bar{x}_{n+1}, V_n, V_{n+1}\}_{n \geq 0}$.

It can be shown[102] that after some recursions, the system above reduces to the following three equations.

$$rV_1 = \pi_1 + \frac{\beta x_1^2}{2} - \beta \bar{x}_1 x_0 \tag{23}$$

$$rV_0 = \pi_0 + \frac{\beta x_0^2}{2} - \beta \bar{x}_1 x_0 \tag{24}$$

$$r\bar{V}_1 = \bar{\pi}_1 + \frac{\beta \bar{x}_1^2}{2} - \beta b \bar{x}_1 x_1 \tag{25}$$

In addition, the two equations below solve the above system

$$\frac{x_0^2}{2} + rx_0 = \frac{\Gamma_0}{\beta} + \frac{x_1^2}{2} \tag{26}$$

$$\frac{\bar{x}_1^2}{2} + (r + [x_0 + bx_1])\bar{x}_1 = \frac{\Gamma_{-1}}{\beta} + \frac{x_0^2}{2} \tag{27}$$

where

[102] Proof can be provided upon request.

Γ_0 and Γ_{-1} are given by $\pi_1 - \pi_0$ *and* $\pi_0 - \overline{\pi}_1$ respectively. A corollary of Remark 3.3.1 is that $\Gamma_0 > \Gamma_{-1}$ and this implies there is more incentive for the leader to do research in order to escape competition when it is in the neck-and-neck state.

3.4 Steady state

We now characterize the steady state of the economy by finding the steady state of one industry and assuming that all other industries are operating at their respective steady state levels. Let μ_n denote the steady state fraction of industries with technological gap $n \geq 0$ so that we have

$$\sum_{n\geq 0} \mu_n = 1 \tag{28}$$

As mentioned earlier, and as in AHV, stationarity will imply that for any state n, the flow of industries into it should be equal to the flow out. For example, during time interval dt, in $\mu_n(\overline{x}_n + bx_n)dt$ industries with technological gap $n \geq 1$ the follower catches up with the leader and thus the total flow of industries into state 0 is

$$\sum_{n\geq 1} \mu_n(\overline{x}_n + bx_n)dt \tag{29}$$

Also, in $\mu_0(2x_0)dt$ neck-and-neck industries one firm secures a lead, and the total flow of industries out of state 0 is $2\mu_0(x_0)dt$. We thus have

$$2\mu_0 x_0 = \sum_{n\geq 1} \mu_n(\overline{x}_n + bx_n) \tag{30}$$

For state 1 and then for states $n \geq 2$, we have:

$$\mu_1(x_1 + \overline{x}_1 + bx_1) = 2\mu_0 x_0 \qquad \text{and} \tag{31}$$

$$\mu_n(x_n + \overline{x}_n + bx_n) = \mu_{n-1}x_{n-1}\,, \qquad n \geq 2 \tag{32}$$

The asymptotic growth rate of the representative industry is given by

$$g = \lim_{\Delta t \to \infty} \frac{\Delta \ln Q_i}{\Delta t} \tag{33}$$

As in AHV we say that an industry i is said to go through a (p+1) cycle if the technological gap n goes through the sequence $\{0,1,..., p-1, p,0\}$. Since the value of $\ln Q_i$ rises by $\ln \gamma^p = p \ln \gamma$, $\Delta \ln Q_i$ can be approximated by $\Delta \ln Q_i \cong \sum_{p \geq 1} \#_p \left(p \ln \gamma\right)$ where $\#_p$ is the number of (p+1) cycles the industry has gone through over the interval. Thus we rewrite (33) as

$$g = \lim_{\Delta t \to \infty} \sum_{p \geq 1} \left(\lim_{\Delta t \to \infty} \frac{\#_p}{\Delta t} \right) (p \ln \gamma) \tag{34}$$

where $\lim_{\Delta t \to \infty} \frac{\#_p}{\Delta t}$ is the asymptotic frequency of (p+1) cycles. While the latter equals the steady state flow of industries from state p to state 0, it is also equal to the fraction of industries in state p given by μ_p times the probability that the follower catches up with the leader in such an industry. Thus, again as in AHV we have

$$g = \sum_{p \geq 1} \mu_p \left(\bar{x}_p + b x_p\right)(p \ln \gamma) \tag{35}$$

(30)-(32) and (35) imply that

$$g = (2\mu_0 x_0 + \sum_{k \geq 1} \mu_k x_k)(p \ln \gamma) \tag{36}$$

Proof : See Appendix.

It is clear from (36) that the largest component of growth comes from the neck-and-neck state. Intuitively, this happens since there are two firms trying to advance the technology frontier in that state compared to only one in any other state. Hence, technology would advance twice as fast on average in a neck-and-neck state if all efforts were the same.

3.5 Very Large Innovative lead

In this section, we consider the case where a one step lead is so large that the leader has no incentives to increase its lead by more than one step. In other words, we consider the case where $\gamma \to \infty$. Thus in this section the maximum permissible lead is one step. Consequently, $x_1 = 0$.

Proposition 3.4.1

Assume that the conditions in the above game hold and that the productivity lead of the leader is large, then

(i) an industry with a relatively lower degree of appropriability does not necessarily grow at a slower rate; that is, an increase in the ease of spillovers or an improvement in the reverse engineering environment does not necessarily lead to a fall in the rate of innovation

(ii) and the level of R&D effort is higher in the neck-and-neck state than in the unleveled state.

Proof:

(i) Since the maximum permissible lead is one step, we have $x_1 = 0$. Thus (26) and (27) can be rewritten as

$$\frac{x_0^2}{2} + rx_0 = \frac{\Gamma_0}{\beta} \tag{37}$$

$$\frac{\bar{x}_1^2}{2} + (r + x_0)\bar{x}_1 = \frac{\Gamma_{-1}}{\beta} + \frac{x_0^2}{2} \tag{38}$$

Using the fact that there are only 2 states in this case, we have, using (28),

$$\mu_1 = 1 - \mu_0 \tag{39}$$

Replacing (39) in (31), we have

$$\mu_0 = \frac{\bar{x}_1}{2x_0 + \bar{x}_1} \tag{40}$$

Using the fact that $x_k = 0$ for all $k \geq 1$ in (36) and (40), we have the growth rate

$$g = \frac{2x_0\bar{x}_1}{2x_0 + \bar{x}_1} \tag{41}$$

Now since $x_0 *$ and $\bar{x}_1 *$ can be found by solving (37) and (38) simultaneously, we can derive g* only in terms of the exogenous parameters. Visual inspection of (37), (38) and (41) show that g* is independent of the spillover parameter b.

(ii) Using the fact that $\Gamma_0 > \Gamma_{-1}$ and after some algebraic manipulation of (37) and (38), we can establish that $x_0 > \bar{x}_1$.▪

Part (i) of Proposition 3.4.1 states that the spillover parameter b does not affect growth whenever the lead of one step is large enough. Part (ii) of Proposition 3.4.1 states that when firms are in the leveled state they have more incentive to undertake innovation than in any other states. Thus the usual Schumpeterian effect of more intense competition in the neck and neck state is outweighed by the increased incentive for firms to innovate in order to escape competition. Moreover, unlike the AHV model our result does not depend on the product differentiability parameter. Hence, we find that a competitive environment can stimulate R&D by increasing the incremental profit from innovating, that is, by strengthening the motive for neck-and-neck rivals to innovate so as to become the leader. Intuitively, since externalities are present only in the unleveled state and that the R&D level of the leader is zero in that state, the spillover rate in the model (for the case of large innovative lead) becomes zero too. Since growth is driven by the innovation rate in the neck-and-neck state and the imitation rate of the unleveled state, it is independent of the spillover rate. Therefore, this proposition reinforces the case put forward by AHV by showing that R&D incentives are higher at the neck-and-neck state and that greater appropriability does not necessarily increase growth even when there are externalities to the leader's R&D.

3.6 Very Small Innovative lead

In this section we look at the extreme opposite case of the previous section, that is the case in which the one step lead is small and hence, the leader does have an incentive to increase its lead by more than one step. Therefore, we look into eh case where $x_1 \neq 0$. Thus, we consider the case where $\gamma \to 0$. For simplicity we assume only three states; state 0 which is the neck-and neck-state, state 1 where the leader has a one step lead and state 2 where the leader has a two step lead. Similar results can be derived for more than two states as will be shown in the proof Proposition 3.5.1.

Proposition 3.5.1

Assume that the conditions in the above game hold and that the productivity lead of the leader is small, then growth rate and process spillovers are positively related; that is, an increase in the spillover rate unambiguously leads to an increase in the growth rate.

Proof: See Appendix

Proposition 3.5.1 states the main result of the paper. It gives us an important long-run relationship between imitation rate and growth at the steady state without making the assumption of large technology gap (large innovative lead). In this Markovian game between the two duopolists, with imitation and innovation as strategic variables, we observe that, at any point in time, an increase in imitation rate will always prompt the leader to increase its innovation rate in equilibrium. Thus process imitation creates a source of competitive pressure which deters the leader from maximizing short run monopoly profit but rather “forces” him to innovate further. The mechanism driving this result can be observed from the construction of the proof. For a small productivity lead, we show that the relationship between the R&D effort of the follower and the spillover rate is negative since the laggard has less incentive to innovate when it can feed off the leader’s effort. Since, we also show that the R&D effort of the two rivals in the unleveled state are inversely related at the steady state, it must be the case that the leader’s innovation is positively related to the level of externalities.

Thus, an increase in the spillover rate reduces the effort of the follower as it can free ride on the leader who, by receiving the signal that his technological advantage is shrinking, puts in effort to restore its lead. Proposition 3.5.1 implies that the policymaker can enhance the economy’s growth rate by choosing a lower level of appropriability. Hence, there is always (at any point in time) a follower who will prompt the leader to innovate further in such a market configuration, and this will lead to higher growth. It is also noteworthy that the above phenomenon might be due to increasing returns on the R&D when the gap is large.[103] According to Glass (2000), an important factor in Japan’s recent economic slowdown has been the exhaustion of all imitation possibilities as they move closer to the world’s technology frontier.

[103] See Peretto (1996) for further comments.

4. Conclusion

We have presented an analytical model that deals with process imitation and spillovers in a non-Schumpeterian framework. Our motivation stems mainly from the fact that the existing non-Schumpeterian models depend heavily on the level of competition in showing the interrelation between process imitation and spillovers and their impact on growth. Moreover, existing Schumpeterian models lack adequate empirical evidence to explain growth using the concept of creative destruction. Indeed, most of these studies rely heavily on the price undercutting mechanism of the homogeneous Bertrand game. We demonstrate, without relaxing the assumption of product homogeneity, that competitive behavior can still prevail by using a Cournot quantity competition setting. Two main factors drive competitive behavior in the long-run; firstly, the R&D level in the neck-and-neck state and, secondly, spillovers occurring due to a lack of appropriability.

Moreover, this paper can offer a basis for understanding how the dynamic strategic interactions between two firms with a technology gap can determine the economy's growth rate when there is uncertainty. In particular, imitation acts as a spur by putting pressure on the industry leader to innovate further and this drives the economy's engine of growth. Furthermore, this research can contribute to the literature on "The Law and Economics of Reverse Engineering" (see, for example, Samuelson and Scotchmer, 2002) by providing some economic grounds in favor of process reverse engineering. In this regard, it demonstrates the existence of a non-Schumpeterian element in the innovator's best response function. One immediate policy implication of our model is that relaxing laws and regulations which hinder process imitation might not always be a good thing in an industry characterized by spillovers.

5. Appendix

5.1 Derivation of the second stage quantity and profit functions

In this section we derive (12) , (13) and Remark 3.3.1

From (11), firm j's problem is given by

$\text{Max}(q_j)\quad Mq_j/(q_j + q_i) - c_j q_j$, $j = 1,2$ and i is not equal to j (11)

FOC for firm j is given by

$(q_j + q_i)M - q_jM - c_j(q_j + q_i)^2 = 0$ (A1)

By symmetry we have,

$(q_i + q_j)M - q_iM - c_i(q_i + q_j)^2 = 0$ (A2)

Simplifying gives

$q_iM - c_j(q_j + q_i)^2 = 0$ (A3)

$q_jM - c_i(q_i + q_j)^2 = 0$ (A4)

Solving (A3) and (A4) simultaneously gives (12)

Replacing (12) for both firms in (4) gives P_t

$P_t - c_j = c_i/(c_j + c_i)^2$ (A5)

Thus the profit for firm j is given by

$(P_t - c_j)\, q_j = \Pi_j$ (A6)

(A6) verifies (13)

Using (6) and (13) for firm 1 we have

$\Pi_1 = M(1/A_2)^2/(1/A_2 + 1/A_1)^2$ (A7)

$\Pi_2 = M(1/A_1)^2/(1/A_2 + 1/A_1)^2$ (A8)

Now using (7), (8) on (A7) and (A8) and differentiating the result w.r.t γ establishes the first part of Remark 3.3.1. Part (ii) of Remark 3.3.1 is obtained by replacing γ =1 in (7), (8) and hence on (A7) and (A8). Part (iii) of Remark 3.3.1 holds since

$$\frac{1+\left(\gamma^{n}\right)^{2}}{\left(1+\gamma^{n}\right)^{2}} > 1/2 \tag{A9}$$

5.2 Derivation of the Steady State Growth rate (36)

Note that (35) can be rewritten as

$$g = \ln\gamma \sum_{p\geq 1} \mu_p\left(\bar{x}_p + bx_p\right)p \tag{A10}$$

Moreover,

$$\sum_{p\geq 1} \mu_p\left(\bar{x}_p + bx_p\right)p = \sum_{p\geq 1} \mu_p\left(\bar{x}_p + bx_p\right) + \sum_{p\geq 2} \mu_p\left(\bar{x}_p + bx_p\right) + \sum_{p\geq 3} \mu_p\left(\bar{x}_p + bx_p\right) + \ldots \tag{A11}$$

The first term of the RHS of (A11) is $2\mu_0 x_0$ from (30). Now taking the summation on both sides of (32) and re-arranging we have,

$$\sum_{p\geq k} \mu_p\left(\bar{x}_p + bx_p\right) = \mu_{k-1}x_{k-1} \tag{A12}$$

Replacing (A12) in (A11) and then in (A10),we have (36).

5.3 Proof of Proposition 3.5.1

Assume that n = 0,1,2.

We first derive the fraction of industries in state 2, μ_2

We know that

$$\mu_1 + \mu_2 + \mu_3 = 1, \text{ thus } \mu_0 = 1 - \mu_1 - \mu_2 \tag{A13}$$

Using (31) and (32) we have the stationarity condition

$$\mu_1\left(x_1+\bar{x}_1+bx_1\right)=2\mu_0x_0 \qquad \text{and} \tag{A14}$$

$$\mu_2\left(x_2+\bar{x}_2+bx_2\right)=\mu_1x_1\ , \tag{A15}$$

Solving (A13)-(A15) simultaneously gives

$$\mu_2{*}=\frac{2x_0x_1}{2x_0x_1+\left[(1+b)x_1+\bar{x}_1+2x_0\right]\left[(1+b)x_2+\bar{x}_2\right]} \tag{A16}$$

Thus,

$$\mu_1=1-\mu_0-\mu_2{*} \tag{A17}$$

Now by solving for μ_0 as we did for Proposition 3.4.1, we derive the growth rate also in similar fashion and it is given by[104]

$$g=\frac{2x_0\left((1+b)x_1+\bar{x}_1\right)\left(1-\mu_2{*}\right)}{2x_0+\bar{x}_1+(1+b)x_1} \tag{A18}$$

Now it can be shown[105] that the partial derivative of g in (A18) w.r.t to b, x_0, x_1 and $\bar{x}_1$ are all positive as long as the partial derivative of $\mu_2{*}$ w.r.t to b, x_0, x_1 and $\bar{x}_1$ are small enough. We next solve (26) and (27) to find $\bar{x}_1{*}$ in terms of b, x_0 and other exogenous parameters only. We thus have

$$\bar{x}_1^{*}=\frac{\sqrt{\Omega^2+4\Lambda}-\Omega}{2} \tag{A19}$$

where $\Omega=2(r+x_0+b\sqrt{x_0^2+2+x_0-\frac{2\Gamma_0}{\beta}}\ )$ and $\Lambda=\Gamma_{-1}+x_0^2$

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[104] For the case where the number of states is greater than 2, we replace $\mu_2{*}$ in (A18) by $\sum_{n=2}^{N}\mu_n$ .Thus, the proof can be extended for the case of more than one state.

[105] Proof can be provided upon request.

It can be shown[106] that the partial derivatives of $\bar{x}_1$* w.r.t b, x_0 and x_1 are negative[107] and thus, by chain rule the partial derivatives of x_0* and x_1* w.r.t b must be positive. We can now find the total change in the growth rate (g) as a result of a change in b.

$$\operatorname{sgn}\left(\frac{dg}{db}\right) = \operatorname{sgn}\left\{\frac{dg}{dx_0}\times\frac{dx_o}{db}+\frac{dg}{dx_1}\times\frac{dx_1}{db}+\frac{dg}{d\bar{x}_1}\times\frac{d\bar{x}_1}{db}+\frac{dg}{db}\right\} \quad \text{(A20)}$$

It can be shown[108] that the RHS of (A20) is always positive as long as $x_1 \geq \frac{d\bar{x}_1}{db}$ which we can reasonably impose as a restriction. ▪

[106] Proof can be provided upon request.
[107] Note that although x_1 is not present in (A19), we can still deduce this relationship since we know from (26) that x_1 and x_0 are positively related.
[108] Proof can be provided upon request.

V. A Strategic Analysis of Product and Process Innovation with Spillovers

Abstract

In this paper we propose a conceptual framework for analyzing how process spillovers can impact on a firm's decision to choose its levels of process and product innovation. In contrast to previous works which consider the interrelation between process and product R&D in a duopoly with no spillovers, we introduce process spillovers into the framework. A two-stage analysis of a noncooperative game which entails both demand enhancing product innovation and cost-reducing process innovation in an asymmetric duopoly is developed. While the leader's technological efficiency depends only on its own R&D investment, the follower's productivity depends also on the level of intra-industry spillovers. In the first stage the duopolists choose their levels of product and process innovations, while in the second stage they compete in the product market. The results obtained confirm the findings highlighted by previous studies that both product and process innovations are strategic substitutes. A new result is that it is always optimal for the firms to invest more in product innovations when the rate of spillover falls.

Keywords: Product and Process Innovation, R&D, Process Spillovers, Imitation.

JEL Classification Numbers: C7, L1, O3

1. Introduction

It has been established that both product and process innovations play important roles in determining the competitiveness and performance of firms in various industries. Indeed, studies by Athey and Schmutzler (1995) and Yin and Zuscovitch (1998) highlight the importance of distinguishing between process and product innovations in two-stage non-cooperative R&D games.[109] Increasing attention has also been given to the potential for R&D spillovers impacting on the technology of firms. Cohen and Levinthal (1989) found that, in contrast to the traditional result, intra-industry spillovers may encourage equilibrium industry R&D investment.[110] Nadiri (1993) and Mohnen (1996) have considered a more general form of such externalities. Nevertheless, the extent to which R&D spillovers affect product and process innovations in a framework where strategic interaction is present remains a subject of relatively less attention for which there has not been formal theoretical modeling. Our study is primarily motivated by this apparent absence in the existing literature. Few empirical studies highlight the distinct role of R&D spillovers in influencing product and process innovation decisions. For example, Ornaghi (2002) proposed a new empirical approach to assess the impact of knowledge spillovers on product and process innovation.[111]

It has been observed that the relationship between spillovers and R&D incentives has two aspects. First, in many industries, firms undertake R&D investments in order to develop new products or processes. One feature of R&D investment that distinguishes

[109] While Yin and Zuscovitch (1998) study the relationship between the size of firms and their decisions about product and process R&D, Athey and Schmutzler (1995) look at the relationship between the firms' short-run decision variables and their long run decisions about product and process flexibility.

[110] Cohen and Levinthal formally study the two opposing effects of spillovers on the R&D incentives of firms.

[111] Ornaghi (2002) found that the technological diffusion of product innovation is larger than the one driven by process innovation.

itself from other forms of investment is that firms which do the investing are often not able to exclude others from freely benefiting from their investments. Thus the benefits from R&D investments spill over to other firms in the economy.[112] Since the laggards can improve their own technology by free-riding on the leader's research, technologically more advanced firms might have a disincentive to undertake further research since their productivity lead might be significantly reduced in the presence of such spillovers. Hence, the first characteristic of R&D spillovers is that it can potentially reduce research incentives. The second aspect of spillovers is related to the concept of "escape competition".[113] When the laggard firms benefit from process R&D spillovers, they improve their own technology and thereby reduce the technology gap[114] between the leader and themselves. As a result, there will emerge competitive pressure on the leader to innovate further to maintain its lead. These two opposing forces of spillovers on R&D are observed in the empirical findings of Cohen and Levinthal (1989).[115] Given the importance of spillovers in the strategic interactions between firms investing in process and product R&D, we allow both of them to play a major role in our model.

However, if process spillovers are incorporated into the existing theoretical framework, the interrelation between product and process R&D is rendered inherently more complex.[116] While the literature on R&D with product and/or process innovations have pointed out how a two-stage non-cooperative game can help to explore the R&D

[112] Griliches (1979) emphasized the significance of spillovers in modeling and estimating the effects of R&D investments.
[113] "Escape competition' refers to the motive of innovating in order to escape competition.
[114] Cameron (1999) found that there are more free-riding or imitation possibilities when the technology gap is large than when it is low.
[115] Cohen and Levinthal found, contrary to previous studies, that intra-industry spillovers may encourage rather than deter R&D investment.
[116] Yin and Zuscovitch (1998) recognized this difficulty in characterizing an implicit form Nash equilibrium in the R&D game in which there are both process and product innovations.

incentives of a firm deciding to undertake either product or process innovations, they downplay the role of process spillovers in the firm's investment decisions. This suggests potential theoretical limitations in the study of R&D investment and its determinants. In addition, the existing empirical analysis by Ornaghi (2002) shows clearly that there exists some evidence that spillovers affect product and process innovavtion at least in manufacturing industries. Therefore the relationship between product and process innovation, in a two-stage non-cooperative R&D game setting, when externalities are present ought to be determined. Also, the results of Yin and Zuscovitch (1998) ignored the existence of such spillovers. Thus, some new findings on the interrelation between product and process innovation and spillovers can be found in this new setting.

The objective of this paper is to offer a conceptual framework for understanding the role played by spillovers in determining the optimal product and process innovation in a duopoly with a leader-follower configuration. A central concern is to address the question of whether higher spillovers favor more process or more product innovations. We develop a two-stage non-cooperative R&D game of process and product innovation in a duopoly model which is distinct from Yin and Zuscovitch (1998) in the following way. Unlike the latter, we allow for process spillovers from which only the follower benefits in the model so that the follower's marginal cost of production is reduced not only by its own process innovation but also by a fraction of the leader's process investment.[117] At the first stage of the game, the duopolists (the leader and the follower) will engage in product and process R&D. While product R&D is stochastic (in the sense that it realizes with a probability) and leads to the instantaneous discovery of a new product which leads to an outward shift of the firm's demand schedule, process R&D

[117] This follows from D'Aspremont and Jacquemin (1989).

reduces the marginal cost of production with certainty.[118] The two firms compete in the product market in the second stage. As in Yin and Zuscovitch (1998), results are derived by assuming that in the first stage the firm chooses product innovation taking process innovation as given and vice versa and finally the impact of spillovers on product and process strategies is found.

Our results show that the spillover rate plays a critical role in analyzing the interplay between process and product innovations. Specifically we find that when spillovers are high, the firms will invest less in product innovation. Thus a negative relationship prevails between product innovation and spillovers. One natural interpretation that emerges from this result is that when the spillover rate falls, there are fewer imitation possibilities available to the follower. Thus the extent to which the laggard is able to free ride on the leader is reduced and it becomes optimal for the former to change its strategy by increasing its product innovation. Our results also show that Yin and Zuscovitch's (1998) findings, that the product/process innovation rates of the two firms are strategic substitutes, remain robust when spillovers are introduced into the model. Hence, while the results remain unchanged in our new setting, we are also able to give some insights as to when it is optimal for a firm to change its strategy from process to product innovation.

One immediate policy implication is that greater appropriability in an industry might not always be a "good" thing if higher innovation rates improve social welfare. In particular, if an increase in the spillover rate can lead to an increase in the process innovation rates (due to relatively less product innovation) of both firms, then the overall

[118] The asymmetry is observed by the difference in the marginal cost schedules of the two firms and by the process spillovers from which only the follower benefits.

industry's level of R&D will also rise. Thus our model, by helping us to identify the effects of spillovers or the ease of imitation on the innovation rate, also addresses some issues in the area of "The Law and Economics of Reverse Engineering". Handa (1995) and Samuelson and Scotchmer (2002) recognized that it is difficult to find an economically sound argument to justify the restriction or the legalizing of the act of reverse engineering.[119]Our analysis sheds some light on the ongoing debate as to whether or not restricting the act of reverse engineering is justifiable on economic grounds. We believe that in an industry where reverse engineering can speed up the diffusion of technology via process spillovers, the strategic interaction between rival firms will guarantee that a competitive environment always prevails. Furthermore, the leader does not have an incentive to sit back as a monopolist as its technological lead might dissipate in this setting. Hence, is welfare is always enhanced by more competition.

Our model is closest to Yin and Zuscovitch (1998) who show that different innovation incentives might cause the larger firm to invest more in process innovations and the smaller firm to allocate more resources to product innovations. They use a two-stage game model in a duopoly setting, in which product innovation is stochastic and instantaneous while process innovation is incremental. Their results, which are also consistent with empirical findings, show that the large firm will be a leader in process innovation while the small firm will be a leader in product innovation. They therefore conclude that the structure of R&D expenditure should be taken into consideration together with the conventional R&D investment level. We extend their model by allowing the laggard to benefit from process spillovers by free riding on the leader's

[119] See Samuelson and Scotchmer (2002) for more details.

process innovation and we derive a new result in which both firms find it optimal to invest in product innovations in an industry where the spillover rate is falling.

The rest of this paper is organized as follows. Section 2 presents an overview of the model, as well as our results. Section 3 gives some concluding remarks.

2. Model

2.1 Model Overview

An asymmetric duopoly model of technological competition in cost-reducing process and demand enhancing product innovation is proposed, where reference will be made to a leader (with a lower marginal cost of production) and a follower. Thus there are two forms of R&D activity – product and process. A firm is said to undertake process innovations if it employs some resources to reduce its marginal cost of production. While the leader's cost is reduced only by its own investment level, the laggard's cost is reduced by a positive fraction[120] of the leader's investment level in addition to its own investment level. A firm is said to undertake product innovations if it successfully introduces a new product which instantaneously increases its demand schedule (given that it continues to sell the original product). Product innovation is uncertain and episodic. This proposed analysis of product and process R&D in duopolistic competition entails sequential decisions which can be treated as two distinct stages in a noncooperative game; product and process R&D decisions by competing duopolists are made at the first stage and product market competition at the second stage. We use the standard methodology of backward induction to solve the game. Results are derived from the first stage process and product reaction functions of the two firms by assuming that each firm can undertake only one strategy (either process or product) at any point in time.

2.2 Formal Model

[120] We shall formally define this fraction as the spillover rate.

As in Yin and Zuscovitch (1998) we consider an asymmetric duopoly in which initially both firms produce a homogeneous product a with constant marginal costs c . Each firm face the following linear demand schedule

$$p^{a} = l - m\left(q^{a1} + q^{a2}\right) \tag{1}$$

where p^{a} and q^{ai} are the price and quantities respectively and l and m are positive constants. Each firm spend $f\left(s^{i}\right)$ dollars to search for a new product b with a probability of success $s^{i} \in [0,1]$ and spend $g\left(r^{i}\right)$ to reduce their unit cost c by r^{i}. Thus cost-reducing innovation is non-stochastic and incremental while product innovation is uncertain and instantaneous. In particular as in Yin and Zuscovitch (1998), we assume that as soon as the new product is introduced into the market, the inverse demand schedule for both commodities become

$$p^{i} = l - m\left(q^{i1} + q^{i2}\right) - n\left(q^{j1} + q^{j2}\right) \quad , i, j = a, b; i \neq j \text{ [121]} \tag{2}$$

The effective marginal costs of firms 1 and 2 after process cost-reductions are given respectively by

$$C^{1} = c - r^{1} \tag{3}$$

$$C^{2} = c - r^{2} - \beta r^{1} \tag{4}$$

where $\beta \in (0,1)$

[121] Naturally, $m > n > 0$ implies that the two goods are substitutes and that the price elasticity of demand for each good is greater than their cross elasticity of demand.

Assume $C^1 < C^2$, then firm 1 is the leader and firm 2 is the follower. We formally define the technology gap between the two firms by

$$X \equiv C^2 - C^1 \quad (5)$$ [122]

β represents the spillover rate which the follower benefits from the leader. (3) and (4) characterize the one-way spillover structure of the model. Our definition of spillovers is similar to Cohen and Levinthal (1989) together with some extensions. In particular, we define spillovers to include valuable knowledge generated in the research process of the leader and which becomes accessible to the follower if and only if the latter is reverse engineering the innovator's research process.[123] Given that spillovers favor imitation, it becomes a better strategy for the follower to imitate by feeding off the leader's innovation at least initially. Thus, the follower is necessarily an imitator.

We assume without loss of generality that the R&D cost functions are given by

$$f(s^i) = \frac{s^{i2}}{2} \quad (6)$$

$$f(r^i) = \frac{r^{i2}}{2} \quad (7)$$

In the first stage, each firm simultaneously determines their product and process innovation strategies s^i and r^i respectively. They then engage in Cournot competition in the product markets in the second stage of the game. We solve the game by backward induction. The equilibrium concept is the standard subgame-perfect Nash equilibrium.

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[122] It can be shown easily that $X = (1-\beta)r^1 - r^2$.

[123] We, however, assume the follower incurs a fixed cost when undertaking reverse engineering. Such costs do not affect R&D decisions since they vanish when the first order conditions are found.

2.3 Second Stage

As in Yin and Zuscovitch's (1998) model, the total profit function for firm i in the second stage subgame is

$$\pi^i\left(\vec{q},C^i\right)=\left(p^a-C^i\right)q^{ai}+\left(p^b-c\right)q^{bi} \qquad \text{for i=1,2} \tag{8}$$

where $\vec{q}=\left(q^{a1},q^{a2},q^{b1},q^{b2}\right)$ is the output vector and C^i is given by (3) and (4). Since product innovation is stochastic, it is possible that $s^i=0$. Hence, there are four possible outcomes in the first stage subgame. (i) Both firms succeed in introducing the new product; (ii) firm i succeeds but its rival fails; (iii) firm i fails but its rival succeeds; (iv) both firms fail. We also let $\vec{q}_k=(k=1,...4)$, the equilibrium output of the four above cases as in Yin and Zuscovitch (1998). It can be shown that the equilibrium output and prices are:[124]

$$q_1^{ai}=\max\left\{\left[2m\left(l-C^i\right)-m\left(l-C^j\right)-n(l-c)\right]/3\left(m^2-n^2\right),0\right\} \tag{9}$$

$$q_1^{bi}=\max\left\{\left[n\left(l-C^j\right)-2n\left(l-C^i\right)-m(l-c)\right]/3\left(m^2-n^2\right),0\right\} \tag{10}$$

$$p_1^a=\frac{\left(l+C^i+C^j\right)}{3} \quad ; \quad p_1^b=\frac{(l+2c)}{3} \tag{11}$$

$$q_2^{ai}=\max\left\{\left[\left(4m^2-n^2\right)\left(l-C^i\right)-3mn(l-c)\right]/\left(6m\left(m^2-n^2\right)\right)\ -\left(1-C^j\right)/3m,0\right\} \tag{12}$$

$$q_2^{bi}=\max\left\{\left[m(l-c)-n\left(l-C^i\right)\right]/2\left(m^2-n^2\right),0\right\} \tag{13}$$

$$p_2^a=\frac{\left(l+C^i+C^j\right)}{3} \quad ; \quad p_2^b=\frac{n\left(2C^j-C^i-l\right)}{6m}+\frac{(l+c)}{2} \tag{14}$$

$$q_3^{ai}=\frac{\left[2\left[l-C^i\right]-\left[l-C^j\right]\right]}{3m} \quad ; \quad q_3^{bi}=0 \tag{15}$$

[124] Moreover, they are the same as Yin and Zuscovitch (1998).

$$p_3^a = \frac{\left(l + C^i + C^j\right)}{3} \quad ; \quad p_3^b = \frac{n\left(2C^i - C^j - l\right)}{6m} + \frac{\left(l + c\right)}{2} \tag{16}$$

$$q_4^{ai} = \frac{\left[l - 2C^i + C^j\right]}{3m} \quad ; \quad q_4^{bi} = 0 \tag{17}$$

$$p_4^a = \frac{\left(l + C^i + C^j\right)}{3} \tag{18}$$

We shall consider only interior solutions. The total (sum of the profits made for each product) profit function for firm i for each of the four cases are given as follows[125].

$$\pi_1^i = \frac{4m^2\left(A^i + X\right)^2 - 8mA\left[A^i + X\right] + 4m^2A^2}{\left[36m\left(m^2 - n^2\right)\right]} \tag{19}$$

$$\pi_2^i = \frac{\left[A^i + X\right]\left[2\left(4m^2 - n^2\right)A^i - 6mnA - 4A^j\left(m^2 - n^2\right)\right] + \left(mA - nA^i\right)\left[3n\left(X - A^j\right) + 9Am\right]}{\left[36m\left(m^2 - n^2\right)\right]}$$

(20)

$$\pi_3^i = \frac{\left(2A^i - A^j\right)\left(A^i + X\right)}{9m} \tag{21}$$

$$\pi_4^i = \frac{\left(A^i + X\right)^2}{9m} \tag{22}$$

where $A^i = l - C^i$ for i,j , $A = l - c$ and $X = C^j - C^i$

2.4 First Stage

The first stage payoff for firm i is

$$V^i\left(s^i, s^j, r^i, r^j\right) = s^i\left[s^j\pi_1^i + \left(1 - s^j\right)\pi_2^i\right] + \left(1 - s^i\right)\left[s^j\pi_3^i + \left(1 - s^j\right)\pi_4^i\right] - f\left(s_i\right) - g\left(r_i\right) \tag{23}$$

We analyze the R&D choice by looking at the first stage reaction functions of the firms. As in Yin and Zuscovitch's model we shall consider product or process innovation,

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[125] Derivations can be provided upon request.

assuming the other R&D strategies are exogenously given, that is; each firm can only choose the level of one strategy at any point in time.[126]

Proposition 2.4.1

Assume that the conditions of the above game hold, then the product innovations of the two firms are strategic substitutes; that is, are an increase in one firm's investment in product R&D reduces its rival's investment in product R&D.

Proof:

We rewrite (23) as

$$\underset{s^i}{Max}\frac{1}{\theta}\{\ s^i\left[s^j\theta\pi_1^i+\left(1-s^j\right)\theta\pi_2^i\right]+\left(1-s^i\right)\left[s^j\theta\pi_3^i+\left(1-s^j\right)\theta\pi_4^i\right]\ \}-f\left(s_i\right)\ -g\left(r_i\right) \qquad (24)$$

Using (6), (7), taking the first derivative of (23) w.r.t s^i *and re-arranging, we have the following reaction function*

$$s^j=\frac{s^i+\theta\pi^i{}_4-\theta\pi^i{}_2}{\theta\pi^i{}_1-\theta\pi^i{}_2} \qquad where\ \theta=36m\left(m^2-n^2\right) \qquad (25)$$

But since $\pi_2^i>\pi_1^i$*, a negative relationship between* s^i *and* s^j *holds.*▪

Proposition 2.4.2

Assume that the conditions of the above game hold, then the process innovations of the two firms are strategic substitutes; that is, an increase in one firm's investment in process R&D reduces its rival's investment in process R&D.

Proof:

Owing to the asymmetry in the cost structure of the two firms, we derive their reaction functions separately.

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[126] Unlike Yin and Zuscovitch (1998), we do not emphasize the existence and stability of the Nash equilibrium in this game as we only use the reaction functions to derive results. Moreover, we do not compare the strategic behavior of a large firm with that of a small firm.

We rewrite (23) as

$$\underset{r^i}{Max}\frac{1}{\theta}\{\ s^i\left[s^j\theta\pi_1^i+\left(1-s^j\right)\theta\pi_2^i\right]+\left(1-s^i\right)\left[s^j\theta\pi_3^i+\left(1-s^j\right)\theta\pi_4^i\right]\ \}-f\left(s_i\right)\ -g\left(r_i\right) \qquad (26)$$

Using (6), (7), (19)-(22) and taking the first derivative of (26) w.r.t r^i *for i=1, 2, we have*

$$r^i\theta=\frac{d}{dr^i}\left[\begin{array}{l} s^is^j\left(\left(2-4n\right)mAZ+4m^2A^2\right)+s^i\left(s^j-1\right)\left(2n^2ZY-\left(3nY+9Am\right)\left(mA-nA^i\right)\right) \\ +s^i\left(4n^2m^2-6AmZ\right)+4Z^2\left(m^2-n^2\right) \end{array}\right] \qquad (27)$$

and

$$r^j\theta=\frac{d}{dr^j}\left[\begin{array}{l} s^is^j\left(\left(-2+4n\right)mAY+4m^2A^2\right)+s^j\left(s^i+1\right)\left(-2n^2ZY-9A\left(mA-nA^i\right)\right) \\ +s^j\left(s^i-1\right)3nZ\left(mA-nA^j\right)+s^j\left(4y^2n^2=6mAY\right)+4Y^2\left(m^2-n^2\right) \end{array}\right] \qquad (28)$$

where $Z = A^i + X$ *and* $Y = X - A^j$[127]

Now, after simplifying and re-arranging (27), we compare the coefficients of r^i *and* r^j *in the reduced form of the reaction function. Since the coefficients are of opposite sign, a negative relationship between* r^i *and* r^j *holds. Analogous methods are used on (28) and again we find that a negative relationship between* r^i *and* r^j *holds.* ▪

Proposition 2.4.1 and Proposition 2.4.2 show that the two R&D activities (process and product) are strategic substitutes. They show that the results of Yin and Zuscovitch (1998) remain robust in a framework with externalities. (See Yin and Zuscovitch (1998) for the economic rationale for the above propositions.)

Proposition 2.4.3

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[127] Derivatives of these two terms with respect to r^i are also found. A detailed proof can be provided upon request.

Assume that the conditions of the above game hold. If $\left(\frac{1-2\beta}{2}\right)r^i < r^j < (1-\beta)r^i$ *and* $m^2 - n^2$ *is small, then* s^i *and* s^j *are negatively related to* β*; that is, if the follower's process R&D is bounded and the two products are close substitutes, then the product innovations of both the leader and the follower decrease with the spillover rate.*

Proof:

Owing to the asymmetry in the cost structure of the two firms, we have to consider (25) for i=1,2.

Thus we have

$$\frac{ds^j}{d\beta} = \frac{\left(\theta\pi_1^i - \theta\pi_2^i\right)\left(\frac{ds^i}{d\beta} + \frac{d}{d\beta}(\theta\pi^i{}_4 - \theta\pi^i{}_2)\right) - \frac{d}{d\beta}\left(\theta\pi_1^i - \theta\pi_2^i\right)\left(s^i + \left(\theta\pi_4^i - \theta\pi_2^i\right)\right)}{[\cdot]^2}$$

(29)

A similar expression is derived for $\frac{ds^i}{d\beta}$*. It can be shown that if* $m^2 - n^2$ *is small,*

$\frac{d}{d\beta}(\theta\pi_k^i - \theta\pi_l^i) > 1$ *and* $(\theta\pi_k^i - \theta\pi_l^i) > 1$ *for all k,l = 1,..4 and i=1,2*

Hence, $\frac{d}{d\beta}(\theta\pi_k^i - \theta\pi_l^i) > \frac{ds^i}{d\beta}$ *and* $(\theta\pi_k^i - \theta\pi_l^i) > s^i$ (30)

We next derive $\frac{d\pi_k^i}{d\beta}$ *for all k and I using (19) –(22).*[128]

Now it can be shown that $\left(\frac{1-2\beta}{2}\right)r^i < r^j < (1-\beta)r^i$ *is a sufficient condition for both* $\frac{ds^i}{d\beta} \leq 0$ *for i=1,2.*[129] *This completes the proof.* ▪

[128] Derivations can be provided upon request.

Proposition 2.4.3 gives us an important relationship between product innovation and the spillover rate. It tells us that a fall in the spillover rate might imply that firms might switch from process to product innovations.[130] Intuitively, the spillover rate starts to fall when the follower has exhausted all possible benefits from free-riding off the leader's process innovations. As a result, the laggard, who is now left with less free-riding opportunities, has no other alternatives than to change its strategy by undertaking more product innovations. The leader would then respond to the laggard's move by also increasing its product innovation so that it can maintain its market share lead. Hence, a decrease in the spillover rate raises both the leader's and the follower's levels of product innovation and this leads to an increase in the industry's level of product innovations. One policy implication which emerges from this result is that greater appropriability and laws which prohibit reverse engineering by restricting technological diffusion might not always improve social welfare since process innovations fall although product innovations increase.

On the normative side, another possible interpretation of our result is that firms might switch from process to product innovation when the technology gap becomes small. Cameron (1999) found that there are more free-riding or imitation possibilities when the technology gap is large than when it is small. Thus there might be decreasing returns to scale to imitation. It is therefore possible that owing to such decreasing marginal benefits, the follower might find it optimal to switch from process to product innovation, with the leader responding to it to maintain its lead. Hence, if the technology

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[129] Full details of the proof can be provided upon request.

[130] Note that "switch" should be interpreted as the decision of the firm to choose more of one strategy and less of the other rather than reducing one strategy to zero.

gap dynamics of an industry can be observed, one can determine when an industry's innovations will shift from product to process.

The results of Bonanno and Haworth (1998) that Cournot competition favors cost-reducing innovations is likely to corroborate our findings that process spillovers do not hinder process innovations. However, the reader is cautioned that the framework of vertical differentiation described in their paper may not be directly comparable to ours; they have a high and a low quality product unlike the case at hand.

3. Conclusion

One possible limitation of the existing literature on the interrelation between product and process innovations in two-stage non-cooperative R&D games is the assumption that technological diffusion does not take place between the leader and the follower of the industry. Indeed, the previous work by Yin and Zuscovitch (1998) considers the case where process innovations have no externalities. We augment the latter framework by incorporating process spillovers. We consider the case of one-way spillovers whereby only the follower can benefit from the leader and not vice-versa. The central contribution of our work is to offer a conceptual model for determining the impact of spillovers on the industry's innovation level and also for understanding the factors which might cause a firm to change its strategy from process to product when the spillover rate becomes small. Our results demonstrate that there exists a negative relationship between the spillover parameter and the product innovations of both the leader and follower. This suggests that we may observe switching behavior in an industry when the spillover rate becomes small.

The model proposed in this paper can offer a basis for determining whether policy makers should always aim at increasing the level of appropriability in industries as has been done conventionally. In particular, we offer some economic arguments against the restriction of the act of reverse engineering. Promising directions for further investigations include the extension of our model to incorporate product spillovers as well, and the endogenizing of the spillover rate by allowing it to depend on the technology gap between the firms.

VI. General Conclusions

This dissertation has attempted to provide a contribution to expanding the literature on both the theory and application of noncooperative R&D by introducing a class of games in which asymmetric spillovers are determined by the level of technology of the players. In particular, we consider the case where the follower is more likely to benefit from such spillovers as compared to the industry leader.

The first essay provides a general framework in which to analyze the relationship between R&D investment and technology catch-up in a differential game and shows that the dynamics of the technology gap play a crucial role in determining whether spillovers necessarily reduce the leader's incentives to invest in R&D. The results provide a sufficient condition for the existence of a steady state in R&D games with spillovers; a finding that is new in the literature.

The second essay presents an application of the theoretical framework by studying the effects of process spillovers on competition in a R&D based endogenous growth model. It finds, firstly, that the innovation strategies of the two firms can be dynamically strategic complements if a large technology gap prevails and, secondly, that there is a case for process reverse engineering as a fall in the level of appropriability may result in higher growth.

The purpose of the third essay is to determine the effects of process R&D spillovers on growth by extending the well-known AHV framework. It demonstrates, without relaxing the assumption of product homogeneity, that competitive behavior can still prevail in a Cournot quantity competition setting. Two main factors drive competitive behavior in the long-run; firstly, the R&D levels in the neck-and-neck state and, secondly, spillovers occurring due to a lack of appropriability.

The final essay offers a conceptual framework for understanding the role played by spillovers in determining the optimal product and process innovation in a duopoly with a leader-follower configuration. It addresses the question of whether higher spillovers favor more process or more product innovation and contributes to the existing literature by showing that it is always optimal for firms to invest more in product innovations when the rate of spillover falls.

This dissertation can contribute to the literature on “The Law and Economics of Reverse Engineering”. By providing some economic grounds in favor of process reverse engineering, this dissertation has extended the existing literature on “Law and Economics of Reverse Engineering” by demonstrating the existence of a non-Schumpeterian element in the innovator’s best response function. One immediate policy implication of the result is that laws and regulations which hinder process imitation might not always be a good thing in an industry characterized by spillovers since they might lead to lower economic growth.

From a theoretical perspective, some directions for further investigation include the extension of our analysis to a cooperative setting with research joint ventures (as has been done traditionally for the static and exogenous spillover case). Also, proper characterization of the stability conditions that would guarantee a Saddle Path in our general class of R&D game models is an avenue for further inquiry. A closed-looped analysis of the general model would be another robustness check of our theorems.

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