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CONTRIBUTIONS TO PLANNING AND ANALYSIS OF ACCELERATED TESTING

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Summary

Accelerated Life Testing (*ALT*) and Accelerated Degradation Testing (*ADT*) have become attractive alternatives for reliability assessments as they distinctly save the testing time and testing cost. They are employed when specimens are tested at high stresses to induce early failures or degradations. Through an assumed stress-life or stress-degradation relationship, failure information is extrapolated from the test stress to that at design stress. Although such practice saves time and expense, estimates obtained via extrapolation are inevitably less precise. Hence, a systematic and in-depth study on ALT and ADT data analysis and experiment planning is in demand.

This dissertation involves three parts. The first part addresses the planning of Constant Stress ALT (*CSALT*), in which we propose a method to quantify the departure from the usual optimality criterion. A contour plot is developed to provide the solution space for sample allocations at high and low stress levels in two-stress and three-stress CSADT plans. Based on the output from the contour plot, three related approaches to planning CSALT are then presented. The results show that our plans are: (1) capable of providing sufficient failures at middle stress to detect non-linearity in the stress-life model if it exists; (2) able to serve as follow-up tests during product development; (3) flexible in setting stress levels and sample allocations.

The second part addresses the analysis of Step Stress ADT (*SSADT*) data. We monitor the degradation path with stochastic processes and finally obtain a closed form estimation for unknown parameters. The mean lifetime and its confidence intervals are also derived when failure time follows the Inverse-Gaussian distribution (*IGD*) or

Birnbaum-Saunders distribution (*BSD*). Compared the existing approaches, our method alleviates the difficulty in determining the particular deterministic degradation functions.

The third part deals with the planning of ADT. Motivated by the successful application of stochastic model in ADT data analysis, we present a general formulation to design both CSADT and SSADT by considering the tradeoff between the total experiment cost and the attainable estimate precision level. Decision variables such as the sample size, the test-stopping time or the stress-changing time in a CSDAT or a SSADT are optimized. Influence of the lower stress and inspection time interval on optimal plans is analyzed. Effect of precision parameters on optimal SSADT plans is also studied. The results imply that our formulation is easily coded, and our plans require fewer test samples and less test duration. Hence, testing cost is reduced. Compared with CSADT, SSADT is more powerful in this aspect. Thus implementation of SSADT is highly recommended in real case.

This dissertation also contains numerical examples and simulation studies to demonstrate the validity and efficiency of each approach developed. We highlight the important findings and discuss the comparisons with existing methods. Finally, we point out some possible research directions. Since our current research focuses on single accelerated environment, the planning strategies proposed in this dissertation can be extended to multi-component multi-acceleration environment.

Acronyms

AF	Acceleration Factor
AT	Accelerated Testing
ADT	Accelerated Degradation Testing
ALT	Accelerated Life Testing
BSD	Birnbaum-Saunders Distribution
c.d. f	cumulative density function
CC, PC	Cost Constraint and estimate Precision Constraint
CE	Cumulative Exposure model
CST	Constant Stress Testing
CSADT	Constant Stress ADT
CSALT	Constant Stress ALT
Dev	Deviation
DT	Degradation Testing
DM	Deterministic Model
ED plan	Plans with Equalized Degradation
EL plan	Plans with Equalized Log(degradation)
IGD	Inverse Gaussian Distribution
LED	Light Emitting Diode
LS	Lease Square method
LSE	Lease Square Estimate
ML	Maximum Likelihood method
MLE	Maximum Likelihood Estimate
MMLE	Modified Maximum Likelihood Estimate
ND	Normal Distribution
p.d.f	probability density function
PSADT	Progressive Stress ADT
PSALT	Progressive Stress ADT
PST	Progressive Stress Testing
SM	Stochastic Model
SST	Step Stress Testing
SSADT	Step Stress ADT

SSALT	Step Stress ALT
TC	Total cost
WLSE	Weighted LSE

Notations

A, B, C_1, C_2, a, b, d	unknown parameters
$A_{p\times q}$	a p by q matrix
$A\text{var}(\bullet)$	the asymptotic variance of $(\bullet)$
c, c1, c2, m	reliability bound
C_d	sample cost per unit
C_{de}	total sample cost
C_{me}	total measurement cost
C_{mk}	measurement cost per inspection per unit at X_k, k=0, 1, 2,…
C_{ok}	operation cost per time unit at X_k, k=0, 1, 2,…
C_{op}	total operation cost
$\{D_k(t), t \geq 0\}$	a stochastic process at X_k, k=0, 1, 2,…
$\Delta D_{i,j,k}$	degradation increment, *i*=1, 2, … , n; j=1, 2, …, L; k=0, 1, 2, …
D_c	the threshold of a degradation process
$D_{L\times n}$	a L by n matrix related to degradation increments
Ea	the activation energy (eV)
$E(\bullet)$, $Var(\bullet)$	the expectation and variance of $(\bullet)$
$f(\cdot)$	the p.d.f of a certain distribution
$F(\cdot)$	the c.d.f of a certain distribution
F	the Fisherman Information Matrix
$F_{m,p}^1$	the pth quantile of the F_m^1 distribution
h	$=\left(\frac{\partial\hat{\mu}(X_0)}{\partial a}, \frac{\partial\hat{\mu}(X_0)}{\partial b}, \frac{\partial\hat{\mu}(X_0)}{\partial \sigma}\right)$, the first order partial derivation of $\mu(X_0)$
k	subscript, index of stress level, k=0, 1, 2, L(for low stress), M(for middle stress), H(for high stress), …
K	$=8.623\times10^{-5}\,eV/K$, Boltzmann Constant

L	total number of inspections in an ADT
L_k	number of inspections at X_k
LH, $LH_{i,j,k}$	the likelihood function of $\Delta D_{i,j,k}$
m_{life}	a measure of product lifetime in the Eying model
$Min[A\mathrm{var}(\bullet)]$	the minimum asymptotic variance
n	number of samples
n_k	number of samples allocated at X_k
$n^*(c1)$, $n^*(c_2)$	the optimal sample size when the estimate precision parameter is set at c1 and c2
$n^*(\Delta t)$	the optimal sample size when the inspection time interval is set at Δt
$\bar{n}^*$	the average value of $n^*(\Delta t)$ for different Δt s.
$n^*, \pi_1^*, T_1^*, T_2^*$	optimal n, π_1, T_1, T_2 with correct value of σ/a
$n^0, \pi_1^0, T_1^0, T_2^0$	optimal n, π_1, T_1, T_2 with incorrect value of σ/a
p	a probability bound
p_k	the expected proportion of failures at X_k
p1, p2	the probability related to confidence interval
Pr(*)	probability of (*)
q	the quantile of a distribution
q_k	$= \begin{cases} \frac{T_k}{T} & \text{for CSADT} \\ \frac{n_k}{n} & \text{for SSADT} \end{cases}$
Q	a derivation with respect to asymptotic variance
r	speed of reaction in the Arrhenius model
r_k	$= \begin{cases} \pi_k & for\ CSADT \\ q_k & for\ SSADT \end{cases}$, an indicator in ADT planning

$RowSum(A_{p\times q})$	$=\begin{bmatrix}\sum_{j=1}^{q} a_{1j} \\ \ldots \\ \sum_{j=1}^{q} a_{pj}\end{bmatrix}$
Rn	$=100\cdot\left|\left(n^{*}-n^{0}\right)/n^{*}\right|$
$R\pi_1$, RT_1, RT_2,	related to π_1, T_1 and T_2, definition similar to Rn
$RX_1\|_{c\ or\ p}$	$=\frac{\left|X_1^{*}-\overline{X}_1^{*}\mid \text{given c or p}\right|}{\overline{X}_1^{*}\mid \text{given c or p}}$
S	the applied stress or transformations of the applied stress
$SN_n(\Delta t)$	$=\frac{n^{*}(\Delta t)-\overline{n}^{*}(\Delta t)}{\overline{n}^{*}(\Delta t)}$
t	lifetime
t_f	failure time in the Inverse Power Law model
t_γ	the location parameter in an exponential distribution or a Weibull distribution
t_q	the qth qantile of a failure time distribution
$t_{\gamma Extreme}$	the location parameter in an extreme value distribution
Δt	time interval
Δt_k	the time interval between two continuous inspections in an AT at X_k
T	the termination time of an AT experiment
T_k	testing time at X_k
T_{temp}	testing temperature in Kelvin
$T^*(c_1)$ $T^*(c_2)$	the optimal testing time when the estimate precision parameter is set at c1 and c2
$T^{*}(\Delta t),\overline{T}^{*}$, $SN_T(\Delta t)$	related to the testing time, definition similar to $n^{*}(\Delta t)$, $\overline{n}^{*}, SN_n(\Delta t)$
$U_{i,j,k}$	a transformation of $\Delta D_{i,j,k}$

w_k	$=\begin{cases} q_k & for\ CSADT \\ \pi_k & for\ SSADT \end{cases}$, an indicator in ADT planning
X_k	the standardized testing stress
$X_{L\times 2}$	a L by 2 matrix related to testing stresses
$X^{'} \times X$	the multiplication of $X^{'}_{L\times 2}$ and $X_{L\times 2}$
$X^2_{m,p}$	the pth quantile of the X^2 distribution with m degree of freedom
X^*, $\overline{X}^*$	related to the test stress level, definition similar to n^*, $\overline{n}^*$
y	log of lifetime
Z_p	$=\Phi^{-1}(p)$, inverse of Φ
$(\bullet)_{lcl}, (\bullet)_{ucl}$	the lower and upper confidence limit of $(\bullet)$
$\hat{\bullet}$	the estimate of $\bullet$
*	superscript, index of optimal value
~	distributed as a certain distribution
β	the shape parameter in a Weibull distribution
$\varepsilon_{i,j,k}$	normally distributed variable
η_k	the drift parameter in a stochastic process at X_k
γ_k	the expected number of failures at X_k
λ	the failure rate in an exponential distribution or the dispersion parameter in a stochastic process
Φ	the c.d.f of the standard normal distribution
μ	the mean or the location parameter in a distribution
μ_k	μ at X_k, k=D, L, M, H, 0, 1, 2, 3,…
$\mu_{\ln}$	the log mean in a lognormal distribution
σ	the scale parameter or the dispersion parameter in a distribution
$\sigma_{\ln}$	the log standard deviation in a lognormal distribution
σ_k^2	the diffusion parameter in a stochastic process at X_k
θ	the scale parameter in a Weibull distribution
$\theta_{Extreme}$	the scale parameter in an extreme value distribution

π_k $= \begin{cases} \frac{n_k}{n} & \text{for CSADT} \\ \frac{T_k}{T} & \text{for SSADT} \end{cases}$, the proportion of samples allocated at stress X_k in CST, or proportion of testing time distributed at X_k in SST, $\sum \pi_k = 1$, $\pi_k > 0$

$\hat{v}$ $\equiv \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}$, a matrix of unknown parameters

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Chapter 1

Introduction and Literature Survey

1.1 INTRODUCTION

In manufacturing industry, there is much interest in the lifetime information of products that is traditionally assessed from failure data. However, due to the increasing demand for improved quality and reliability, systems and their individual components are required to have extremely long life span. For example, the lifetime of a Light Emitting Diode (*LED*) can be longer than 10^5 hrs, i.e. 11.5 years. Thus it becomes particularly difficult, if not impossible, to collect enough failure data to estimate the time-to-failure under normal test condition. In order to shorten the testing time and reduce the testing cost, Accelerated Testing (*AT*) is promoted in such circumstances.

AT can be conducted in two ways. One is the Accelerated Life Testing (*ALT*), which is employed at higher than usual stresses to induce early failures. Physical failures are observed during the experiment. Reliability information is estimated under test conditions and then extrapolated to that at use condition through a statistical model. ALT has a high capacity to save testing time and cost once failures are observed.

However, there are still cases in which few data could be obtained even at highly elevated stress levels. Hence the second way is the Accelerated Degradation Testing

(*ADT*). It is imperative in ADT to identify a quantitative parameter (degradation measure) that degrades over time and thus is strongly correlated with product reliability. The degradation path of this parameter is then synonymous to performance loss of the product. Tseng *et al* (1995) defined failures as “soft failures” when the degradation measure of interest passes through a pre-specified threshold. Similar to ALT, degradation data measured at higher stresses are then extrapolated to use condition for prediction of product lifetime.

The key idea to make components degrade or fail faster in an AT is to test the specimens at higher stresses which may involve higher temperature, voltage, acidity, pressure, vibration, load or even combinations of such stress levels. There are mainly three types of stress loading pattern for an ALT or ADT, namely, constant stress, step stress and progressive stress. The former two are the common types of AT in practice.

In Constant Stress Testing (*CST*), test units are assigned to a certain increased stresses. These stresses are held constant throughout the testing until units fail or observations are censored. Figures 1.1-1.2 are demonstrations of Constant-Stress ALT (*CSALT*) and Constant-Stress ADT (*CSADT*) with three test stresses. CST has some advantages. The acceleration models are better developed and can be verified empirically. Besides, because it is simple to maintain the constant stresses once a test is set up, CST is easy to implement and widespread used in industry. However, it is not so easy to select an appropriate level of stress in a CST. If the stress level is too high, specimens under test may fail with a different failure mode from that under use condition. If the stress level is not high enough, many of the tested specimens may not fail within the available testing time frame and thus the collected failure data are not sufficient to get a reliability inference.

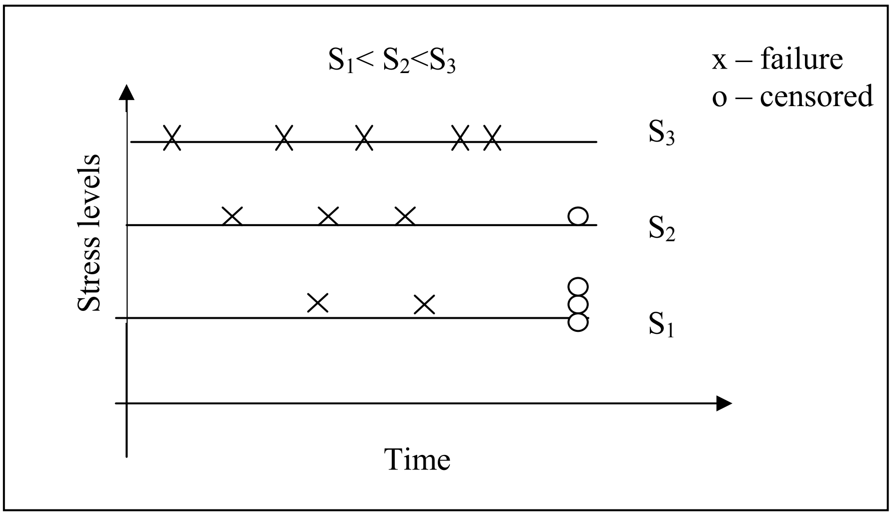

Figure 1.1. An example of the stress-loading pattern in a three-stress CSALT

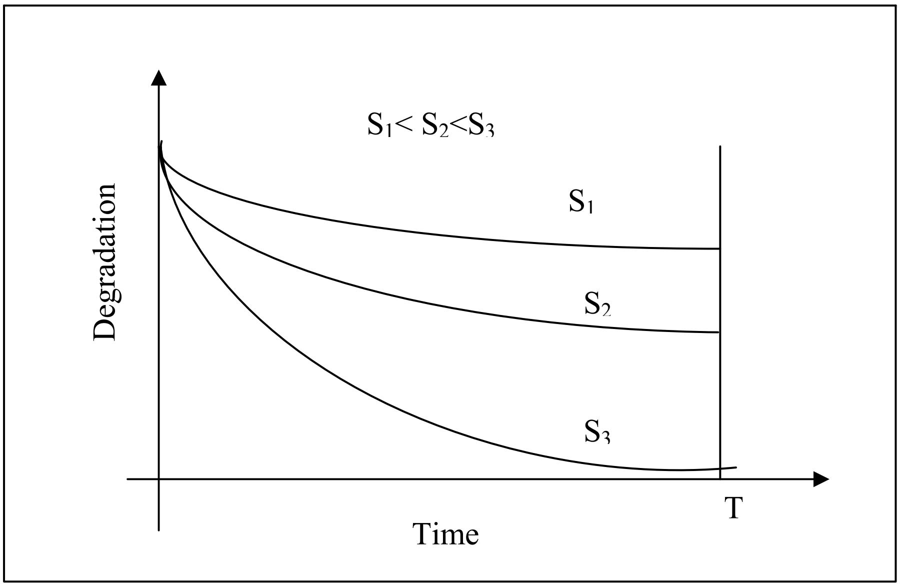

Figure1.2. An example of the stress-loading pattern in a three–stress CSADT

To overcome the problems encountered in CST. Step Stress Testing (*SST*) is adopted. Figures 1.3-1.4 are examples of Step Stress ALT (*SSALT*) and Step Stress ADT

(*SSADT*) plans with three stress levels. Either in SSALT or in SSADT, all units are subjected to the first test stress simultaneously, and the test stress is increased in steps at some pre-specified time points. As a result, each unit runs at each stress for a specific time until it fails or the test is censored. Because of the gradually increased stress level, SST helps to avoid over-stressing of test specimens. The disadvantage of SST is that it is more complex to model the influence of the increasing stress compared with the constant stress in a CST.

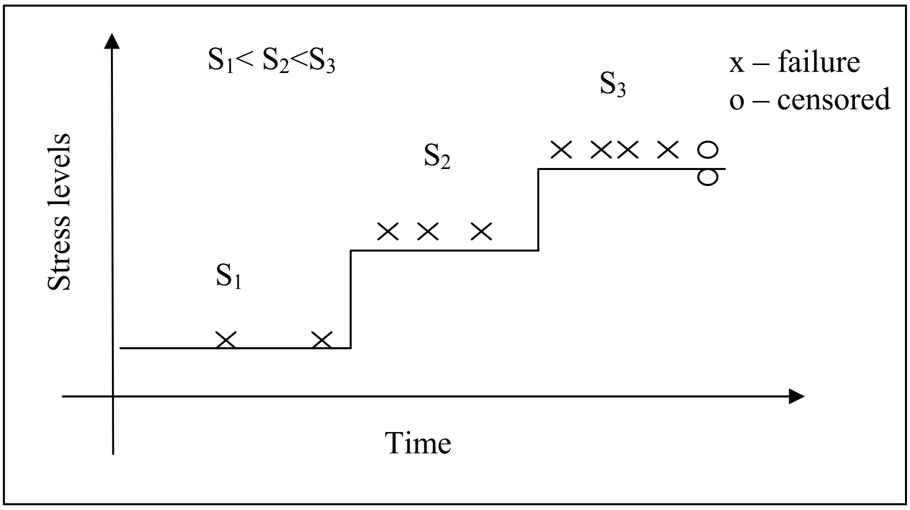

Figure 1.3. An example of the stress-loading pattern in a three-stress SSALT

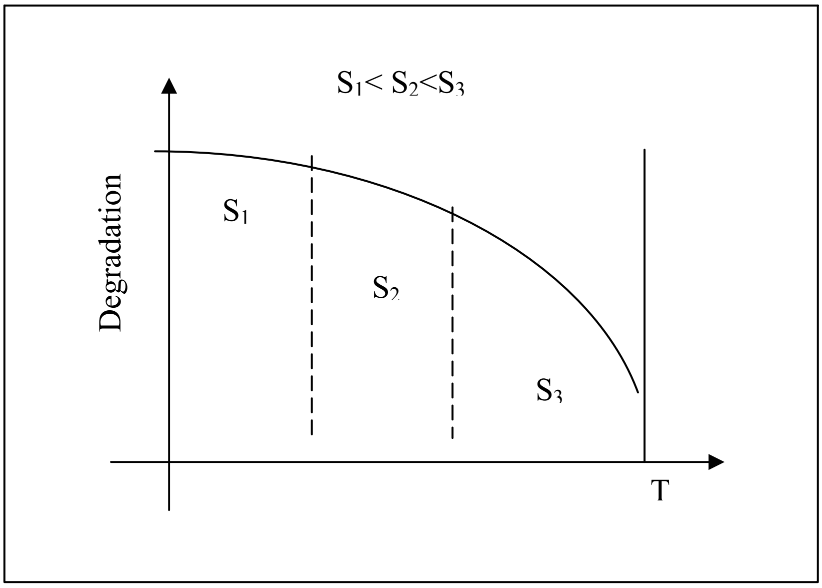

Figure 1.4. An example of the stress-loading pattern in a three-stress SSADT

Progressive Stress Testing (*PST*) is similar to the SST except that the stress applied to the test units is increased continuously. A particular case is called ramp stress test, in which the testing stress is linearly increasing (Tan, 1999). Figure 1.5 is an example of a PST with two different acceleration rates. PST can provide enough failure data within a short time frame, but it is difficult to control the stress changing rate and to model its effect. Thus PST is not commonly adopted in real world. Therefore, in this dissertation, we put our emphasis on data analysis and experiment design of CST and SST. We will not cover details of PST in the following chapters

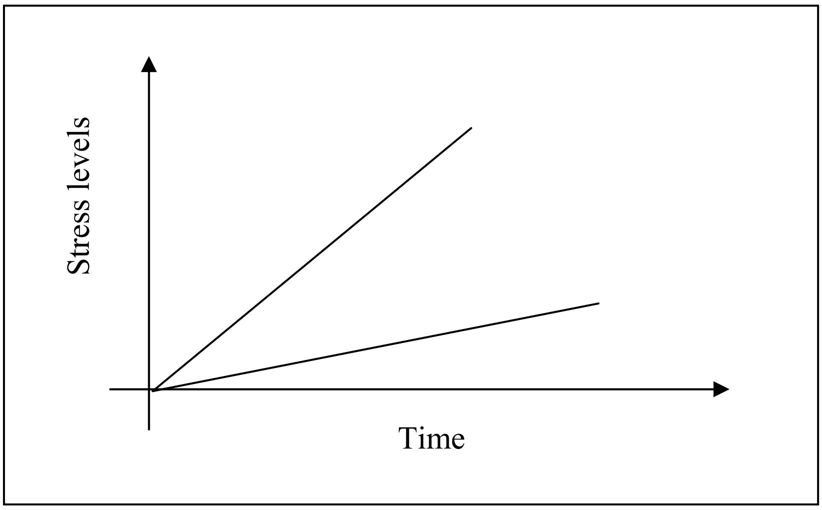

Figure 1.5.An example of the stress-loading pattern in PST with two acceleration rates

Figure 1.6 shows the relationship and differences among these reliability assessment methods.

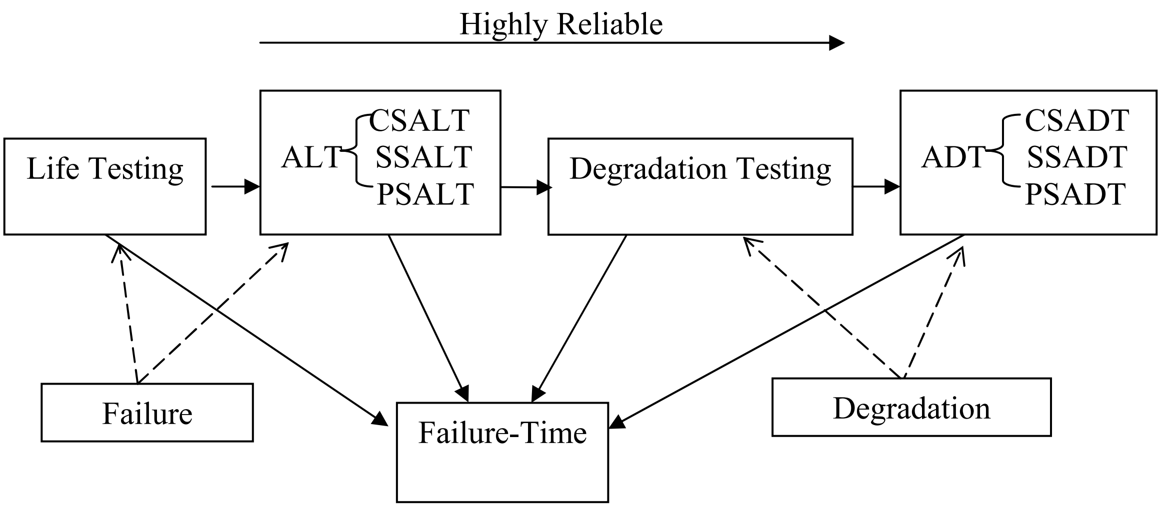

Figure 1.6 Methods to assess reliability information for highly reliable products

ALT and ADT have been studied by many scholars. The distinguished book by Nelson (1990) is a comprehensive resource dealing with their fundamental theories, applications, data analysis methods and experiment planning approaches. Papers about

ALT and ADT also appeared in many journals and proceedings. In the following sections, we first review the basics of AT such as the commonly used lifetime distributions, the commonly used acceleration models, modeling of the degradation paths in an ADT, the parameter estimation methods and failure mechanism validation. After that an extensive literature survey will be given on ALT and ADT data analysis and test planning.

1.2 BASICS OF AT

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1.2.1 The Commonly Used Lifetime Distributions

AT is a quick way to assess reliability inferences on the performance of devices at a lower stress level and at operation time far beyond the length of experiments. These inferences are obtained through extrapolations in two dimensions, i.e. time and stress. Effect of increased stress on failure/degradation can be summarized with three types of models. The first one is Acceleration Factor (AF) model, which means the failure times and different stress level are linked through a deterministic relationship including many different formulations. The lifetime distribution is selected based on past experience, existing engineering knowledge and the underlying failure mechanisms. In this case, the failure at higher stress has the same distribution as that at normal stress but with an altered scale parameter. This type of models are easily understood and widely spread in published research, we will adopt it in our later analysis and give a throughout review in next section. The second type is proportional Hazard model. The hazard function at higher stress is related to hazard function at normal stress through a covariate function involving stress as variables. Meeker et al (2002) has detailed explanation regarding this model. The last one is more general models, where scale and shape parameters change with stress levels. Extrapolation along stresses can be illustrated by acceleration models, which express the lifetime in term of a function of the applied stresses. We have summarized some commonly used life distribution in this section and the acceleration models in next section.

1.2.1.1 The Exponential Distribution

The exponential distribution is the most widely used distribution in mechanical and electronic industry. It owns a constant failure rate. This famous property implies its applications in modeling the long, flat portion of bathtub curve and modeling the failure time of product without significant wear out mechanism. Additionally, because of lack of memory, the exponential distribution is suitable to describe the life of electronic components and electronic systems such as the transistors, resistors, integrated circuits, and capacitors. .

The probability density function (p.d.f) of the exponential distribution is normally expressed as:

$$f(t) = \lambda e^{-\lambda(t-t_\gamma)} \qquad 0 \le \gamma \le t < \propto \tag{1.1}$$

where t is the lifetime; λ represents the failure rate whose reciprocal is the mean time to failure; and t_γ is a location parameter demonstrating the start point of a constant failure rate if the components have been subjected to a burn-in test.

Bartlett's test can be used to check the feasibility of using the exponential distribution as a failure-time model for a given data set (Elsayed, 1996).

1.2.1.2 The Normal Distribution

There are a lot of situations where the normal distribution is applicable. In reliability modeling, the lifetime of mechanical components under cyclic loads or fatigue test is always a normal variable. Degradation increments when a degradation process is modeled with a stochastic process are also normally distributed (Tang & Chang, 1994). Because of its convenient properties, random variables with unknown distributions are often assumed to be normal. Although this can be a dangerous assumption, it is often a good approximation due to the surprising result known as the Central Limit Theory, which states that, the mean of any set of variables with any distribution having a finite mean and variance tends to the normal distribution.

The p.d. f of the normal distribution is:

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(t-\mu)^2}{2\sigma^2}\right] \quad -\infty < t < \infty,\ -\infty < \mu < \infty,\ \sigma > 0 \tag{1.2}$$

where t is the lifetime; μ and σ are respectively the mean and the standard deviation of this distribution. They are also called the location and scale parameters.

1.2.1.3 The Lognormal Distribution

The lognormal distribution is widely used in modeling the failure time of electronic components when they are assigned to high temperature, high electric field, or a combination of both temperature and electric field. It is used for calculating the failure rates due to electromigration in discrete and integrated devices. The lognormal distribution is also powerful to model failures of the fracture of substrate.

The p.d.f of the lognormal distribution is:

$$f(t) = \frac{1}{\sqrt{2\pi}\sigma_{\ln} t} \exp\left[-\frac{(\ln t - \mu_{\ln})^2}{2\sigma_{\ln}^2} \right] \quad -\infty < t < \infty,\ -\infty < \mu_{\ln} < \infty,\ \sigma_{\ln} > 0 \qquad (1.3)$$

where $\mu_{\ln}$ and $\sigma_{\ln}$ are respectively the log mean and log standard deviation.

If a random variable is from a lognormal distribution, the logarithm of this random variable follows a normal distribution.

1.2.1.4 The Weibull Distribution

The Weibull distribution is used across a wide range of applications from electronic components, mechanical components, metal materials, ceramics, to product properties such as strength, elongation and resistance. Because of its flexible ability to include many distributions such as the exponential, the Raleigh, the normal distribution as special cases, it is the recommended model when little knowledge is known about the failure mechanism of products.

The p.d.f of the Weibull distribution can be expressed as:

$$f(t) = \frac{\beta}{\theta}\left(\frac{t - t_\gamma}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t - t_\gamma}{\theta}\right)^{\beta}\right] \quad 0 \le \gamma \le t < \infty,\ \beta > 0,\ \theta > 0 \qquad (1.4)$$

where θ, β and t_γ respectively refer to the scale, shape and location parameter. For $\beta = 1$, this p.d.f reduces to an exponential density; for $\beta = 2$, it describes a Raleigh distribution; for $\beta = 3.44$, it approximately is a normal distribution (Elsayed, 1996). Generally, if $\beta < 1$, failure rate is a decreasing function of t; if $\beta > 1$, failure rate is a increasing function of t.

1.2.1.5 The Extreme Value Distribution

The extreme value distribution is useful in modeling the reliability of components that experience significant wear-out, i.e. highly increasing failure rate.

It is closely related to the Weibull distribution with shape and scale parameters β and θ. Its p.d.f is:

$$f(t) = \frac{1}{\theta_{Extreme}} \exp\left(\frac{t - t_{\gamma_{Extreme}}}{\theta_{Extreme}}\right) \exp\left[-\exp\left(\frac{t - t_{\gamma_{Extreme}}}{\theta_{Extreme}}\right)\right] \quad -\infty < t < \infty \qquad (1.5)$$

where $t_{\gamma_{Extreme}} = \frac{1}{\beta}\log\theta$ and $\theta_{Extreme} = \frac{1}{\beta}$ are the location and scale parameters. The natural logarithm of a Weibull random variable follows an extreme value distribution.

1.2.1.6 The Inverse Gaussian Distribution & The Birnbaum-Saunders Distribution

These two distributions are normally used to model fatigue crack growth in engineering applications. The Inverse Gaussian distribution (*IGD*) has more applications in electrical networks, management sicken, mental health, demography, and environmental science. For its detailed theory and applications, see Chhikara & Folks (1989), Tang & Chang (1994), Gupta & Akman (1996), Iwase & Kanefuji (1996), Seshadri (1998) and Yang (1999). The Birnbaum-Saunders Distribution (*BSD*) was first derived by Birnbaum and Saunders (1969) and later developed by Desmond (1985). Owen & Padgett (1999) investigated the accelerated test models for system strength based on BSD. The confidence interval for the 100*p*th percentile and the point and interval estimates for the critical time of failure rate of the BSD are constructed in Chang & Tang (1993, 1994) and Tang & Chang (1995). For more information about comparisons and contracts of the two distributions, see Bhattacharyya & Fries (1982) and Desmond (1986). We will give the p.d. f of these two distributions later in chapter 3.

Other distributions such as the Raleigh distribution, the Gamma distribution, the Beta distribution and the Half-logistic distribution are also used in modeling the lifetime of products. Their reliability functions and applications have been thoroughly explained in Elsayed (1996).

1.2.2 The Commonly Used Acceleration Models

Statistics-based models, physics-statistics based models, and physics-experimental based models are three kinds of acceleration models. The later two are normally employed to analyze failure time data when the exact relationships between the applied stresses and the failure time of components can be known based on physics or chemistry principles. Specially, the commonly used models are the Arrhenius model, the Inverse Power Law model and the Eyring model.

1.2.2.1 The Arrhenius Model

When only thermal stress is significant, the empirical model, known as the Arrhenius model, has been applied successfully to demonstrate the thermally activated mechanisms such as solid-state diffusion, chemical reactions, semiconductor failure mechanisms, battery life etc (Condra, 2001). The effect equation of temperature on the reaction rate is:

$$r = A\exp(-\frac{Ea}{KT_{temp}}) \tag{1.6}$$

where r is the speed of reaction, A is an unknown constant that needs to be estimated from real data, Ea is the activation energy (eV) that a molecule must own before it can take part in the reaction. Condra (2001) summarized the activation energies for some semiconductor device failure mechanisms. $K = 8.623\times10^{-5}\,eV/K$ is the Boltzmann Constant, and T_{temp} is the testing temperature in Kelvin.

1.2.2.2 The Inverse Power Law Model

The Inverse Power Law model is used when the life of a component is inversely proportional to an applied stress. The main applications of the Inverse Power Law model involve voltage and fatigue due to alternating stress. Failure time under this model can be expressed by the following equation:

$$t_f = \frac{A}{S^B} \tag{1.7}$$

where t_f is the failure time, A and B are constants that relate to the product properties. S is the applied stress.

1.2.2.3 The Eyring Model and the Generalized Eyring Model

The Arrhenius model and the Inverse Power Law model are workable when there is only one stress factor. While, the Eyring model offers a general solution to problems where additional stresses exist. It has the added strength of having a theoretical derivation based on chemical reaction rate theory and quantum mechanics. With temperature as a test stress, the Eyring model has been applied to: (1) accelerated testing of capacitors, with voltage as the second stress; (2) failures caused by electro-migration, with current density as the second stress; (3) epoxy for electronics, with humidity as the additional stress; and (4) rupture of solids with tensile stress as the second stress (Sun, 1995). It is also applicable to describe the dependence of product performance, aging and accelerated stresses in power supply systems (Chang, 1993).

An Eyring model can be expressed as:

$$m_{life} = A\left(\frac{1}{S}\right)B\exp\left(\frac{-E_a}{KT_{emp}}\right) \tag{1.8}$$

where m_{life} is a measure of product life; *A, B* are constants to be estimated from real data; *S* is an applied stress, such as humidity, voltage or their transforms; $\frac{E_a}{KT_{emp}}$ is the Arrhenius exponent.

The Generalized Eyring model allows one or more non-thermal accelerating variables. For one additional non-thermal accelerating variable X, the model can be written as:

$$r = A * \left(T_{temp}\right)^B * \exp\left(C_1 S + \frac{C_2 S}{KT_{temp}}\right)\exp(-\frac{Ea}{KT_{temp}}) \tag{1.9}$$

where *A, B,* C_1, C_2 are characteristics of the particular process. $\frac{E_a}{KT_{emp}}$ is the Arrhenius exponent.

These four models, which can be employed independently or in combinations, are most widely adopted in AT. Some other models, for example the exponential model (Yamakoshi et al 1977, Park & Yum 1997), are also available. Elsayed (1996), Hobbs (2000) and Condra (2001) have discussed how to select the test conditions and how to choose the suitable acceleration models. Based on the mechanical-damage failure mechanism, Guerin et al (2001) also presented the method to analyze and select suitable acceleration models that describe crack propagation of steel components. Considering the acceleration effect of humidity and temperature, Tang & Ong (2003) developed the moisture soak model for surface mounted devices.

1.2.3 Modeling of Degradation Processes

Degradation means gradual loses of characteristic performance. ADT aims to measure the changing process of one or more characteristics of each device under test before an actual failure occurs. Hence, ADT data captures valuable information on the failure mechanisms of the specimens. However, the inferences from ADT are valid only when the underlying degradation model is properly defined. Two types of degradation models, namely deterministic models and stochastic models, have appeared in the published literature.

1.2.3.1 Deterministic Degradation Models

Degradation process can be modeled using a function of time and possibly multidimensional random variables. This kind of models is called deterministic model (Meeker & Escobar 1998, Tseng & Wen 2000, Yang & Yang 2002, Meeker *et a*l 2002).

A deterministic model normally describes the following information: (1) a relationship between degradation measurement and time, i.e. the lifetime distribution over a particular stress; (2) effect of the stress levels on lifetime, i.e. the potential acceleration model; and (3) random effects of individual product characteristics.

There are three types of deterministic models, i.e. linear, convex and concave models. To determine the format of a model, one needs to comprehensively understand the failure mechanisms of the product under test. Historical data, previous testing experience and engineering handbooks will be exactly useful in this aspect.

Deterministic models have several weaknesses. First, the degradation path of one item at a particular stress is determined once the parameters in the pre-assumed model are known, and thus the experimenter only needs to collect a certain number (same as the number of unknown parameters) of degradation points to estimate these parameters. On the contrast, he/she needs more samples to justify the variability of the parameters. Secondly, the error terms in those models are always assumed to be independent and identically distributed. This is not adequate for the correlated process. Moreover, some parameters especially the shape parameters in the assumed life distribution are assumed to be known before testing. This again, sometimes, is not possible in practice. To overcome these problems, stochastic models, which describe the degradation path as a random stochastic process in time, are adopted alternatively.

1.2.3.2 Stochastic Degradation Models

Stochastic models focus on the degradation increments instead of the actual degradation values. Degradation is realized as the additive superposition of a large number of small increments.

The Wiener process is the most widely used stochastic process. Its theory has been thoroughly explained in Park & Beekman (1983) and Dawson et al (1996). Besides, a collection of stochastic processes have been promoted to monitor nondestructive accelerated degradation for power supply units in Tang & Chang (1995). Whitmore & Schenkelberg (1997) demonstrated a degradation process with a time scale transformation. Their model and inference methods have been illustrated using an application case involving self-regulating heating cables.

1.2.4 Parameter Estimation Methods

Parameter estimation plays an important role in reliability assessment. A good estimate should be unbiased, consistent, efficient and sufficient (Elsayed, 1996). Clearly, the accuracy of an estimate depends on the sample size and the method in use for estimating the parameters. In general, two types of approaches, called parametric and nonparametric approaches, are generally employed for parameter estimation.

1.2.4.1 Parametric Methods

The Maximum Likelihood method (*ML*) and the Lease Square method (*LS*) are the two mainly used parametric estimation methods.

Estimate from ML method maximizes the likelihood function, which is a joint probability of an observed sample as a function of the unknown parameters. This method possesses some advantages:

1. It has some desirable mathematical and optimality properties. For example, ML estimate (*MLE*) is unbiased with minimum variance compared with other estimate, and is asymptotically normal for large sample size. As a result, the confidence bounds and hypothesis tests of the reliability interest can easily be obtained.

2. The existing softwares provide excellent algorithms for calculating MLEs for many of the commonly used distributions. This helps saving the computational efforts and mitigating the computational complexity.

However, it also has some disadvantages:

1. The likelihood equations need to be specifically worked out for a given distribution and estimation problem.

2. The numerical estimation is usually non-trivial, particularly if the confidence intervals for the parameters are desired. Except for a few cases where the maximum likelihood formulas are in fact simple, it generally relies on high quality statistical software to obtain MLEs.

LS method assumes that the best estimate of the parameters minimize the sum of the squared deviations, i.e. least square error, from a given set of data. It is normally used for curve fitting. For theory and estimation procedures of MLE and LSE, see Nelson (1990), Elsayed (1996) and Tobias & Trindade (1995).

Other estimate methods such as the graphical method (Nelson 1975, 1990), the Weight Least Square method (Kwon 2000, Wu & Shao 1999), the Moment Estimate approach (Elsayed 1996), the Modified Maximum Likelihood (*MML*) method (Su *et al,* 1999), the Bayes approach (Viertl 1988, Chalone & Larntz 1992, Chaloner & Verdinelli 1995, Dorp 1996, Mazzuchi 1997, Robinson & Crowder 2000) are also available. But, MLE is the most widely adopted method in ALT and ADT analysis. It has the minimum standard deviation for large samples, and the standard deviation is proved comparable to that of other estimates for small samples (Nelson, 1990).

1.2.4.2 Non-parametric Methods

Most AT analysis adopts parametric regression models to estimate the lifetime of products at normal stress. However, when the failure mechanism is unknown, or failure data indicate complicated distributional shapes, semiparametric and nonparametric models can serve as attractive alternatives to relax the difficulty in choosing a distribution function. Among the semiparametric models, multiple regression models and the proportional-hazards model have been highlighted (Millier 1981, Gill 1984, Lawless 1986, Elsayed 1996, Wei 2001). More general models have been introduced in Etezadi-Amoli & Ciampi (1987) and Shyura et al (1999). These models have been successfully used to analyze the survival time of patients in medical applications. Nonparametric approaches for interval estimates of reliability measures have been reported in Tyoskin & Krivolapov (1996) and Shiau & Lin (1999).

Other methods, such as the neural networks method, also appeared in some literature(Chang et al, 1999).

1.2.5 Failure Mechanism Validation

As mentioned early, the basic idea in AT is that we hypothesize that components operating at a well-selected level of elevated stress experience the same failure mechanism as they may experience at normal stress. For example, if corrosion occur at the use temperature and humidity, the same type of failures happens faster in a more moist environment with an increased temperature. If different failure mechanisms are induced, they should be represented by different life distributions and different acceleration models (Tobias & Trindade, 1995). This indicates the essential shortcomings of AT:

1. High level of stresses, sometimes, may induce new failure modes that would not be observed at use condition. For example, instead of simply accelerating a failure-causing chemical process, increased temperature can possibly change the material properties, that leads to a different failure mode. Hence, it is very important to validate the testing stress within a certain range before any testing starts.

   Normally, a new failure mode can be verified by checking the variability of the assumed stress-life relationship. If a new failure mode is recognized, the analysis should be adjusted by treating the failure time caused by the new failure mode as censoring time (Meeker & Escobar, 1998).

2. Multiple operations, sometimes, may cause different failure modes that affect the lifetime of products under test. For example, the filament of an incandescent light bulb goes through a sublimation process and finally fails. There are, however, other operations such as the on-off cycles that induce both thermal & mechanical shocks to shorten the lifetime of the bulb. Correlation

among multiple operations should be clearly understood in order to obtain an accurate estimate in AT analysis.

3. Some failure modes may be ignored when the experiment is focusing on a known failure mechanism. If the unknown modes are the dominating recourse of failures, a wrong estimation is to be obtained. Detailed analysis with respect to mixed failure modes has been published in the early work by Nelson (1975). Jayatilleka & Okogbaa (2001) also presented a method to identify potential failure models in journal bearing.

4. The increase of a certain accelerating variable, sometimes, may cause changes of other accelerating variables. For example, during the testing of electronic products, the increase of temperature may cause the increase of humidity, which fastens the degradation speed too. In additions, the rate of usage for mechanical products is intensively related to corrosion (Meeker & Escobar, 1998). The correlations between acceleration variables should also be counted in AT data analysis and experiment planning.

Hereby, AT experiments should be designed carefully to produce data from only one failure mechanism. Other types of failures should be "censored" out of data analysis. Operations during a test should be well controlled to reduce risks of inducing new failure modes. Despite of the demand to select the highest testing stress as high as possible to minimize the standard deviation of estimate at design stress (Meeker, 1975), the highest stress should validate the same failure mechanism as that at use stress. Normally, the highest stress can be determined based on current product specifications, past experience and engineering handbooks (Yang & Yang, 2002). If there lacks sufficient knowledge about the product, a preliminary test, which is conducted by testing a few samples at a reasonably high stress level over a short time, can be performed to clarify the failure or degradation mode and identify the highest allowable test stress.

1.2.6 Destructive Testing and Non-destructive Testing

In most published literature, AT is assumed to be non-destructive. However, in the situations where the reliability inspection is devastating and the unit cannot fulfill its functional requirements after inspection, destructive testing arises. The time to collect reliability measures in a destructive testing becomes inevitably important because it determines the attainable estimate precision level of the reliability interest. Bergman & Turnbull (1983) presented the optimal inspection time to achieve precise estimates in a destructive life test conducted on animals. Park & Yum (1997, 2004) designed destructive CSADT and SSADT plans by minimizing the asymptotic variance of MLE of mean lifetime at use stress. Sohn (1997) investigated the destructive ALT planning problem with logistic failure-distribution. Yang & Yang (2002) determined the inspection time in a two stress CSADT by considering a tightened degradation critical bound at higher test stress.

Deterministic models are used in all above-mentioned papers. In chapter 7, we will present a planning scheme to determine the inspection time in a two-stress destructive CSADT. Differently, we apply stochastic processes to model the degradation paths.

The optimal testing time as well as the sample size and allocations at each stress are obtained by minimizing the total testing cost under an estimate precision constraint. Comparisons of our proposed plan and the existing plans will also be conducted.

1.3 ANALYSIS OF ALT DATA AND PLANNING OF ALT TEST

An important statistical issue of ALT is how to take advantage of the test data from the severe test conditions to make uncertainty statements about the failure behavior of the items at use stress. Most work related to ALT data analysis and test design is based on the sample theoretic theory (Nelson, 1990).

1.3.1 Analysis of ALT Data

The comprehensive sources for analyzing ALT data are Mann *et al* (1974), Cox & Oakes (1984), Viertl (1988) and Nelson (1990).

ALT data are always a mixture of time censoring and failure censoring that cannot be modeled by a normal distribution (Tan, 1999). Usually, we use a Weibull distribution with a constant shape parameter to fit the failure time, a transformed linear function such as the Arrehnius model and the Inverse Power Law model to describe the stress-life relationship, and the parametric method to estimate the unknown parameters (Nelson 1980, Tang 1996, Teng & Yeo 2002). Application of ML in estimation of the linear hazard rate type distribution and exponential step-stress model has been proposed in Shaked (1978) and Khamis & Higgins (1998). Schneider & Weissfeld (1989) derived the confidence intervals of the lifetime in a lognormal distribution for censored ALT. Hirose (1993) and Sohn (1997) discussed the issues of non-constant scale parameters. Hirose (1993) also investigated a threshold below which failures were not likely to occur. Huang & Lin (1994) developed a two stage linear model in case that there were changing points in the assumed Arrhenius model. Bai & Yun (1996) generated a size-effected model in which failure rate was a function of sample size. Wang & Pham (1996) proposed a general statistical definition of accelerated factors and derived the unbiased estimator of the accelerated factor for the Gamma distribution. Tyoskin & Krivolapov (1996) published nonparametric models for SSALT analysis. Mazzuchi et al (1997) presented a Bayesian approach for inference under the Weibull failure model assumption and incorporated prior judgment into the analysis. Kim & Yum (2000) compared exponential life test plans with intermittent inspections. Jayawardhana & Samaranayake (2003) derived the lower prediction bounds for a future observation based on a Weibull lifetime distribution in which the scale parameter was assumed to have inverse power relationship with the stress levels.

1.3.2 Planning of ALT Test

Even the analyzing methods are powerful in obtaining an estimate, the estimates might be significantly different from the true lifetime at use stress. Planning of ALT mitigates this pressure. Usually, the sample size, the test duration and the expected proportion of failures are known due to the availability of samples and testing time. Hereby, decision variables in test planning involves the stress levels, number of units allocated to each stress level in CSALT or holding time assigned at each stress in SSALT. Nelson (1990) summarized the normally used optimization criteria:

1. minimizing the variance of the LSE of reliability interest at use stress (for test plans with complete data); Or
2. minimizing the variance of the MLE of reliability interest at use stress (for tests with censored data); Or
3. minimizing the variance of an estimate over a range of stress; Or
4. minimizing the variance of the estimate of a particular coefficient; Or
5. minimizing the variance of the estimate of the scale parameter; Or
6. be most sensitive to detect non-linearity of the stress-life relationship.

Tian (2002) thoroughly reviewed the models, plans and applications of SSALT. Miller & Nelson (1983) designed a simple two stress SSALT with completed data. Bai *et al* (1993), Tang *et al* (1996) and Khamis & Higgins (1996) presented ideas on three-stress SSALT designs. Tang (1999, 2003) not only optimized stress levels and the holding time, but also achieved a target Acceleration Factor (*AF*) to satisfy the test time constraint with a desirable fraction of failure in SSALT planning, see also Yeo & Tang (1999). They concluded that the statistically optimal way to increase AF in an ALT was to increase the lower stress levels and shorten their holding time. In most of the above-mentioned papers, a certain Weibull distribution has been assumed to model the failure time distribution and the cumulative exposure (*CE*) model has been employed to model the stress-life relationship.

In additions, Xiong & Milliken (1999) investigated the statistical models in SSALT and planned the optimal tests when the stress changing time was an order statistic from an exponential lifetime family. Park & Yum (1998) presented a modified SSALT and CSALT based on an exponential life distribution and a linear acceleration relationship by minimizing the asymptotic variance of the MLE of the mean lifetime at use stress. When the sample size was small, SSALT performs well to capture reliability information (Khamis, 1997).

Statistically optimal CSALT plan (Nelson *et al*, 1978) consists of two stress levels. It yields the most accurate estimate but is unable to validate the assumed linearity of the stress-life relationship. It is not robust to mis-specifications of some pre-estimates either. Three-stress designs have eliminated this disadvantage. The three-stress best standard plan (Nelson & Kielpinski, 1976), which provides poor estimates, sets three equally spaced stresses and equally allocated test units (with allocation ratio 1/3:1/3:1/3) to those stress levels. Three-stress compromised CSALT plan (Meeker 1975, Meeker & Hahn 1985) uses three stresses in which the middle one is the average of the other two, and the allocation ratio for the three stresses is 4/7:2/7:1/7. It is more robust to avoid deviation from the linear stress-life assumption than the statistically optimal plan and more efficient to produce an accurate estimate than the best standard plan. But in this situation, no optimization has been addressed on the middle stress level. It is proved that the statistically optimal plan produces the smallest asymptotic variance. In the above-mentioned plans, the underlying life distribution is assumed to be a Weibull distribution, in which the scale parameter keeps constant for all stresses, and the stress-life relationship is simply linear. Optimality is achieved by minimizing the asymptotic variance of the MLE of a certain percentile of the time–to-failure distribution at the design stress. Under the same assumptions, Yang & Jin (1994) addressed the sample allocations in three-stress CSALT by minimizing the asymptotic variance of the MLE of the mean life at design stress and the total running time. Tan (1999) solved the same problem by constraining the expected failures at design stress not less than a specific amount.

By assuming that the failure time follows a Weibull distribution with non-constant scale parameter at each stress and the allocations at each stress is 4:2:1, Meeter & Meeker (1994) planned the three-stress CSALT by minimizing the asymptotic variance of MLE of the mean life at use stress. While, Tang (1999) gave the optimal stress levels and their allocations by minimizing the variability of scale parameter at each stress, see also Tang et al (2002). Wu et al (2001) addressed the limited failure censored life test plans. Sun (1999) proposed the failure free test plans.
Test plans with other lifetime distributions such as the normal and the exponential distribution are also studied in Yang (1994) and Park & Yum (1996).

Optimality in most of the above-mentioned literature is defined as achieving a minimum asymptotic variance for a certain percentile of reliability interest, that is, to obtain the “best design” that corresponds to “optimization”. In practice, stress levels of the best design may be too harsh to implement and also there may be constraints due to limitation of experiment budget and availability of products and/or test duration. Thus pursuing the best design, sometimes, enhances the difficulty of conducting experiments and increases the experimental expenses. To solve these problems, we consider relaxing the usual optimization criteria to obtain alternate plans. This idea is first initiated by the goal-softening approach.

Ho *et al* (1992) and Ho (1996) introduced the goal-softening approach as to “settle for the good enough solutions with high probability”. Ho *et al* (1992) argued ordinal rather than cardinal optimization concentrating on finding good, better or best designs rather than estimating accurately the performance value of these designs. The interest of goal softening approach is on whether solution A is better than solution B (say, solution A > solution B), not on how much solution A is better than solution B (say, solution A – solution B). Barnhart et al (1994), Ho & Deng (1997) showed examples of this approach for comparison with traditionally cardinal optimization. Lee *et al* (1999) also explained the role of goal softening in ordinal optimization: other than using accurate performance that presumably takes a long time to obtain, one could use the relative order of performance estimate as a basis for comparing and choosing design. Other than picking one single design that is exactly the true optimum in the design space, which is important in the presence of large estimate errors, one could pick a subset in which some good enough designs are contained with high probability. Using the order statistics formulation, they examined the feasibility of this approach to discrete event dynamic systems. It was stated that goal-softening approach was exponentially efficient in terms of matching good designs in a selected group.

We intend to apply goal-softening approach to ALT planning in chapter 2. Instead of finding the unique design solution that corresponds to the smallest asymptotic variance, we are going to solve for the design space in which the asymptotic variances are within a tolerable bound.

1.3.3 Objectives of Our Proposed CSALT Planning Approach

CSALT is wildly used due to several reasons as follows:

- CSALT is easily implemented.

  After the test parameters are determined and the test is set up at the beginning of an experiment, experimenters do not need to adjust the testing equipments during the course of testing.

- CSALT is easily understood.

  Once the test is completed, CSALT data are easier to analyse. The reliability information from each stress is clearer to understand than that in a SSALT.

- CSALT can provide the required reliability information with a certain precision.

Although CSALT has weak points such as the stress levels should be properly determined and the highest stress level should be investigated consciously, a well-designed CSALT can give the required reliability with a certain level of confidence. If the confidence is acceptable, CSALT is regarded superior to SSALT.

Thus in our research, we still focus on design of CSALT. A motivation is to capture the variance departure from the optimal planning when the stress levels and their corresponding allocations vary from the optimal values. In particular, given the highest stress below which a stress-life model is valid, the expected proportion of failure at this stress and that at design stress, our purpose is to develop a procedure that determines near optimal lower stress levels and their respective sample allocations. Here, proximity to optimality is measured by achieving an asymptotic variance for the *p*th quantile of reliability interest not worse than *c* (>1) times that of a statistical optimal plan (In statistics, this is also termed as relative efficiency). Clearly, if *c*=1, the only plan available is the statistically optimum plan. For other *c* values greater than 1, given the desired number of stress levels (>2), we present the solution space for the near optimal stress levels and their corresponding sample allocations. In particular, a contour plot is used to depict this solution space for planning of three-stress ALT.

Briefly, the objectives of planning CSALT in our research are to:

- quantify the departure of actual variance of a test plan from that of the optimally designed plan
- obtain, if exists, the design space of stress levels and their allocations in multiple stress CSALT
- investigate the relationship between the well-known two stress CSALT and multiple stress CSALT
- give solutions to plan multiple CSALTs with allowable variance inflation.

1.3.4 Value of Our Proposed CSALT Planning Approach

The proposed approach is expected to provide experimenters with flexibility in setting stress levels and sample allocations and be able to quantify the tradeoff in terms of how much the variance would be inflated.

Also the approach can be used as a follow-up test during product development when the preliminary inputs are reasonably accurate but the failure mechanisms may have been changed due to modification of design, which results in an unknown stress life model.

1.4 DATA ANALYSIS AND PLANNING OF ADT TEST

ADT is a helpful tool for us to gain insight of the physical mechanisms of a certain degradation process and make inferences on the performance of devices at lower stress levels and at operation time beyond the length of experiments. See Nelson (1990), Meeker & Hamada (1995), Meeker & Escobar (1998b), Chao (1999) and Bagdonavicius & Nikulin (2001) for its theories, models and applications.

Besides reliability prediction, ADT is also useful in phases of product design, development, manufacturing and shipment. With failure analysis and corrective action programs, ADT can drastically reduce the product design and development cycle time, and achieve higher customer satisfaction, lower field failure rate, lower warranty and lower repair field cost. Tseng et al (1994, 1995) applied ADT to experiment design to improve the reliability of fluorescent lamps. Chuai & Hamada (1996b) presented Taguchi's robust design for LED using degradation measurements. Jawaid & Ferguson (2000) applied ADT to design of printed circuit board assembly for a disk drive product. Scibilia et al (2000) applied ADT to improve the reliability of liquid crystal displays. Yu & Chiao (2002) designed the optimal degradation experiment to improve the reliability of LED.

1.4.1 Analysis of ADT Data

One can find resourceful approaches with respect to degradation models, connections and differences between degradation models and failure-time models, statistical methods for data analysis and statistical inferences related to degradation data in books by Nelson (1990), Klinger (1992) and Meeker & Escobar (1998a). These concerns have also been reviewed in papers by Nelson (1981), Meeker & Escobar (1993a, b) and Chao (1999).

Lu & Meeker (1993), Boulanger & Escobar (1994), Tseng et al (1994), Hamada (1995), Chiao & Hamada (1996), and Meeker et al (1998) considered general degradation path models to obtain estimates of the percentile of a failure time distribution. Suzuki et al (1993) used a linear degradation model to study the increase of a resistance measurement over time. Carey & Koenig (1991) used concave degradation models to describe the degradation of electronic components. Meeker and Escobar (1998) used similar models to monitor the growth of failure-causing filaments of clorine-copper compound in printed-circuit boards. Dowling (1993) and Meeker & Escobar (1998) used convex degradation models to study the growth of fatigue cracks. Lu et al (1997) proposed a model with random regression coefficients and standard-deviation function to analyze linear degradation data from semiconductors. Yanagisawa (1997) used degradation models to estimate the degradation of amorphous silicon solar cells. Su et al (1999) considered a random coefficient degradation model with random sample size. A data set from a semiconductor application was used to illustrate their method. Wu & Shao (1999) established the asymptotic properties of the (weighted) least square estimators under the nonlinear mixed-effect model. They used these properties to obtain point estimates and approximate confidence intervals for percentiles of the failure time distribution of metal film resistors and metal fatigue cracks. Chinnam (1999) used finite-duration impulse response multi-layer perception neural network to model degradation measures and self-organizing maps to model degradation variation. This method reduced overall operation cost by facilitating optimal component-replacement and maintenance strategies. Multiple linear regression methodology was established for describing the relationship between random parameters and stresses in Crk (2000). Wu & Tsai (2000) used the optimal fuzzy clustering method. Their procedure could get more accurate estimation results if the patterns of a few degradation paths were different from those of most degradation paths in a test. Tseng & Wen (2000) proposed the CE model to analyze the LED degradation data and then described the life distribution. These papers all focus on estimating the parameters in a degradation model and the percentiles of a failure time distribution.

Parameters in the deterministic models are usually estimated by ML, LS, and other method such as MML (Su *et al,* 1999). However, for most models, it is not easy to obtain the estimates in a simple expression. One usually needs to get the estimates by numerical searching. Pinherio and Bates (2000) presented an estimation scheme that may provide approximate MLEs for the general deterministic models. They also gave a program to achieve the calculation. See also Bates & Watts (1988). Instead of ML and LS method, nonparametric and Bayesian approach could also be used (Shiau & Lin 1999, Robinson & Crowder 2000) to estimate the unknown parameters in a distribution function.

The Wiener diffusion process is the normally used stochastic model to describe degradation paths (Goh et al 1989, Doksum 1991, Ebrahim & Ramallingam 1993, Lawless et al 1995, Whitmore 1995, Doksum & Normand 1995&1996, Whitmore & Schenkelberg 1997, Whitmore et al 1998, Cox 1999, Normand & Doksum 2000). A Wiener process with constant initial damage and constant beginning of damage time has been studied in Kahle &. Lehmann (1998). The failure time as the first passage time followes an IGD. By assuming in each realization of the damage process both the process increments and the failure time are observable, the MLEs of the drift and variance of the damage process, the constant initial damage, the beginning time of damage, and the boundary damage level have been estimated simultaneously. See also Kahle (1994) for the confidence regions of the MLE of these parameters. A limitation of this method is that this model is only applicable at use stress and the testing time is normally too long for the manufactures. An extended model for multiple CSADT analysis has been presented in Doksum & Hoyland (1992). The parameters in the lifetime distribution were estimated by assuming failure time was exactly observable and measurable. But the reliability information in degradation increments was ignored.

Wendet (1998) presented some other stochastic models for damage processes and gave the MLE of unknown parameters for the corresponding lifetime distributions. See also Kahle & Wendt (2000). Cinlar (1980), Singpurwalla & Youngren (1993), Singpurwalla (1997), Bagdonavicius & Nikulin (2000a, b) modeled degradation by a gamma process that includes possibly time-dependent covariates. Other models, for example, the poisson process (Mercer, 1961), the cumulative B-models (Bogdanoff &. Kozin, 1985), the markov and semimarkov process (Cinlar 1984, Kopnov & Kanajev 1994, Kopnov 1999) are also workable.

In most of the above papers, the unknown parameters are estimated by ML method. And the MLEs could not be expressed in a closed form and need to be searched with numerical methods that are quite time-consuming. In chapter 3, we will describe a certain kind of SSADT and present to estimate the unknown parameters by LS method. The estimates are shown with a closed form that reduces labor of calculation.

1.4.2 Planning of ADT Test

The objective of designing degradation experiments is to estimate the amount of degradation at use stress for the commercial lifetime of a system. Several factors such as the stress levels, the sample size, the inspection frequency, and the termination time affect the experimental cost and the precision of estimate significantly.

ADT plans are usually designed to minimize the variance of MLE or LSE of the pth percentile of the lifetime at use condition subject to a cost constraint (Yu & Tseng 1999, Wu & Chang 2002). Marseguerra et al (2003) designed the degradation test in order to achieve accurate estimates of the component reliability characteristics in light of budget limitation. They gave a multi-objective genetic algorithm for tracking the decision variables. Other optimization criteria in ALT planning as mentioned in section 1.3.2 are also applicable.

Boulanger & Escobar (1994) proposed to design CSADT in three stages: 1) optimize stress levels and the corresponding allocations by minimizing the variance of the weighted least squares estimate (*WLSE*) of the mean of the log plateau; 2) determine the measurement frequency by equalizing degradation (*ED Plan*), that is, to make the expected amount of observed degradation between two consecutive measurements a constant, or equalizing Log-spacing *(EL Plan*), that is, the measurement time points are obtained such that they are equidistant in the log-time scale. As a result, it results in more measurements in the lower end of the interval; 3) determine the sample size to meet a pre-specified estimate precision. CSADT plans with only one inspection at the end of experiment, i.e. destructive CSADT, has been presented in Park & Yum (1997) and Yang & Yang (2002). Detailed review on destructive testing can further be found in chapter 7.

Constraining the l-period moving-average to be less than a predetermined value, Tseng & Yu (1997) presented a termination rule for determining an appropriate stopping time for a degradation experiment. This method lacks information on optimizing the sample size. Based on this stopping criterion, Yu, & Tseng (1998) presented an on-line procedure for terminating an ADT with predetermined stress levels and allocations.

It is noted that the degradation paths in all above planning methods are modeled by non-linear mix-effect models, i.e. deterministic models. Stochastic models, which have been proved flexible and acquirable to model degradation paths and widely used in ADT data analysis, so far have not been applied to ADT planning. This motivates us to design ADT in terms of stochastic processes. We will present a general formulation for planning of both CSADT and SSADT in chapter 4. We will also consider a two stress CSADT in nature of destructive inspections, where degradation is modeled by stochastic process in chapter 7. The planning policy is to minimize the total cost of test as well as to achieve a requisite level of estimate precision.

1.4.3 Objectives of Our Proposed ADT Analysis and Planning Approach

Compared with ALT, little information is available on ADT planning because it involves more design variables and is more complex in methodology development. It is also a challenge in industry to justify the goodness of an ADT planning. However, ADT do have advantages over ALT. It requires shorter testing time and reduces the experimental cost greatly. A well-designed ADT gives commercial benefit for industrial practice. Moreover, SSADT is superior to CSADT due to its flexibility in adjusting stress levels and avoiding estimate errors from misused stresses. Because of the disadvantages of deterministic models mentioned in section 1.2.3.1 and the gap of application of stochastic models in ADT planning, we propose to apply stochastic models in ADT data analysis and test designs. Our purposes are to:

- investigate the suitable method for SSADT data analysis, which may mitigate the computation pressure from searching for parameter estimates. More precisely, we use LS method to estimate the unknown parameters. The estimates can be expressed in a closed form. A simple programming algorithm can be coded for analyzing SSADT data.

- present general formulation for planning ADT

  Modeling the degradation path with a stochastic collection, and assuming a linear drift-stress relationship, we develop the general formulation for CSADT and SSADT planning to minimize the total cost of testing as well as to obtain a requisite level of estimate precision. A simple algorithm can be coded to search for optimal plans. CSADT in nature of destructive testing is also considered.

1.4.4 Value of Our Proposed ADT Analysis and Planning Approach

There are two points recommendable in our proposed SSADT analysis approach. First, adopting of stochastic models in degradation path modeling alleviates the difficulty to determine the format of deterministic functions, which is quite useful when less knowledge is known about the tested product. Second, the estimate of unknown parameters can be expressed in a closed form that is easy to solve. It saves the computational efforts significantly.

For planning of ADT, so far there is no literature employing stochastic models. Our proposed approach not only fills up this gap, but also integrates CSADT and SSADT planning problem in a unique formulation. The planning policy, i.e. to minimize the total cost of testing as well as to achieve a certain level of estimate precision, reflects the trade-off between the testing expense and the attainable estimate precision. Because the desire to obtain accurate estimates normally requires more samples and inspections, experimenters need to be aware of the intended reliability of the product without imposing unrealistic precision constraint.

1.5 SCOPE OF THE STUDY

As mentioned earlier, the published methods for planning optimal CSALT are achieved by minimizing the asymptotic variance of the estimated *p*th percentile of the failure time distribution, in which the statistical optimal plan by Nelson *et al* (1978) gives the smallest variance. However, if the variance of estimate is allowed to inflate to some extent, the possible design space of stress levels with their sample allocations is investigated. In ADT data analysis, the unknown parameters are normally estimated by ML method. As a result, it is quite time-consuming to do the numerical search. Even widely used in ADT data analysis, there is no published paper dealing with stochastic models in ADT planning. Therefore, the above problems will be studied in this dissertation. They are divided into three parts: CSALT planning (chapter 2), SSADT analysis (chapter 3), CSADT and SSADT planning (chapter 4, 5, 6, 7).

In chapter 2, the relationship among the stress levels, their allocations and the corresponding variance of reliability estimate in two-stress CSALT plans and three-stress CSALT plans are studied. We then relax the optimality criterion and extend the goal-softening idea to solve for the design spaces that include more than one “good enough” solutions. The design space of stress levels and allocations are illustrated with contour plots. After that, a procedure to design multiple-stress CSALT is generated. As an illustration, three approaches to design three-stress CSALT are presented.

In chapter 3, an approach is presented for SSADT data analysis. Under the assumption that degradation follows a stochastic process, in which the drift-stress relationship is simply linear, LS method that results in the closed form estimates is used for parameter estimation. Based on this methodology, a simple algorithm can be coded to analyze SSADT data.

Based on the assumptions in chapter 3, the general formulation for CSADT and SSADT planning is presented in chapter 4. Considering the trade-off between the testing cost and the attainable estimate precision, we develop the planning policy to minimize the total cost of testing and to achieve a requisite level of estimate precision. Cost functions and the estimate precision constraint are analyzed and generated accordingly.

In chapter 5, optimal CSADT plans are simulated and analyzed. We apply our proposed approach to two-stress plans and compare them with the existing degradation testing plans. The advantages are summarized and its applications are suggested.

In chapter 6, optimal SSADT plans are presented. First, we analyze the two-stress plans, compare them with the existing DT plan and our proposed CSADT plans. The advantages are summarized. After that, three approaches to obtain three-stress SSADT are also presented.

In chapter 7, destructive CSADT is studied. We re-explain the planning policies and their mathematical formulations. After compared with the existing plans, the advantages of this approach are highlighted and its applications are suggested.

Finally, some conclusions and further remarks are provided in Chapter 8.

Chapter 2

Planning of Multiple-Stress CSALT

2.1 INTRODUCTION

For easy implementation, the traditional CSALT plans are conducted with equally spaced test stresses, each with same number of specimens (Nelson, 1990). But these plans are not capable of providing efficient estimates with acceptable accuracy. Problems of planning optimal test plans arise. The motivation is to predict the reliability information at use stress with high or controllable precision using the same number of test specimens and same test duration. Normally, an optimal CSALT plan specifies the test stresses and the sample allocations, i.e. number of test samples assigned to each stress.

Among all published plans, the statistically optimal plan that consists of two stress levels is the simplest one. Optimality in this plan is achieved by minimizing the asymptotic variance of the MLE of the *p*th percentile of the lifetime distribution at design stress. However, It has been pointed out that this plan is not robust against model mis-specification in terms of failure time distribution and value of pre-estimated parameters (Nelson 1990, Meeker & Escobar 1998). More importantly, a two-stress test plan does not allow one to check for the validity of the assumed stress-life relationship. Yang (1994) proposed a three-stress CSALT, in which the optimality is the same as that of statistically optimal plan. However, the questions remains, how to design multiple stress CSALT plans so that the uncertainty involved for some estimates of interest is not worse than that of a statistically optimal plan by a margin determined by the experimenter before the test. This chapter provides a solution to develop such a plan.

Given the maximum possible stress below which a stress-life model is valid, the expected proportion of failure at this stress and that at design stress, we first apply goal-softening approach to determine nearly optimal lower stress levels and their respective sample allocations in a CSALT. Here, proximity to optimality is measured by achieving an asymptotic variance for the *p*th quantile of the lifetime distribution not worse than c= $\exp(m)$ ($m \geq 0$) times that of a statistically optimal plan (In statistics, this is also termed as relative efficiency). Clearly, if m=0, the only plan available is the statistically optimal plan. For other m values greater than 1, given the desired number of stress levels (>2), we present the solution space for the near optimal stress levels and their corresponding sample allocations. In particular, contour plots are used to depict the solution spaces for two and three stress ALT plans. We then discuss the connections of optimal two-stress and three-stress plans. After that, three possible approaches to design three-stress CSALT plans are provided.

2.2 THE EXPERIMENT DESCRIPTION AND MODEL ASSUMPTIONS

The common description of a CSALT states that *n* units available for test are assigned to each stress by:

$$n_k = \pi_k n \qquad \sum \pi_k = 1,\ \pi_k > 0 \tag{2.1}$$

where π_k and n_k are respectively the proportion of sample allocation at X_k and the number of test items at X_k.

Assumptions in our study are as follows:

1. Test stresses are standardized by

$$X_k = \frac{S_k - S_D}{S_H - S_D},\ k = L(\text{for low}),\ M(\text{for middle}),\ H(\text{for high}) \tag{2.2}$$

   in which S_ks are stress factors which may be functions of the applied stress such as temperature, voltage and so on. S_D and S_H represent respectively the use stress and the highest stress under which the same failure mechanism remains. As a result, they are normalized to be 0 and 1. This is to restrict the solution space within the (0,1) interval so that it becomes easier to search for any optimal intermediate stress. The use stress and the highest stress should be specified before planning. They can be selected based on the current product specifications and past experience. If there lacks sufficient knowledge about the product, a preliminary test as mentioned in section 1.2.5 can be performed.

2. Testing at all stress levels is terminated by a common time *T*.

3. The lifetime follows a Weibull distribution of which the natural logarithm of life, $y = Ln(t)$, has a smallest extreme value distribution with the c.d.f as:

$$F(y) = 1 - \exp\left(-\exp\left((y-\mu)/\sigma\right)\right) \quad -\infty < y < +\infty \tag{2.3}$$

where $\mu = \mu(X_k)$ is the location parameter for *y* and σ is the scale parameter.

4. The stress-life relationship is governed by the stress-dependent location parameter $\mu(X_k)$ and σ is assumed to be constant for all stress levels (Yang 1994, Yang & Lin 1994).

$$\mu(X_k) = \mu_D - X_k(\mu_D - \mu_H) \tag{2.4}$$

where μ_D and μ_H, the location parameters at design stress and highest stress, have a relationship with p_D and p_H, the expected proportion of failures at design stress and high stress (Meeker, 1984), that is expressed by:

$$p_k = \Phi((\ln T - \mu_k)/\sigma) \quad k = D, H, \ldots\ldots \tag{2.5}$$

where Φ is the c.d.f of the standard normal distribution.

Meeker (1984) showed that for an accelerated life test with duration *T*, the failure probability at any stress could be uniquely determined by p_D and p_H at:

$$p_k = \Phi\{X_k \Phi^{-1}(p_D) + (1 - X_k)\Phi^{-1}(p_H)\} \tag{2.6}$$

in which Φ^{-1} is the inverse of Φ. Thus, p_D and p_H can be used in place of the parameters of the CSALT model. Meeker and Hahn (1985) adopted this idea.

Once p_k is known, the expected number of failures at X_k can be computed as

$$\gamma_k = n_k p_k \tag{2.7}$$

With this equation, a constraint, that number of failures at a particular stress should not be less than a pre-specified minimum expected amount, can be imposed for planning of multiple-stress CSALT.

2.3 THE GRAPHICAL REPRESENTATION OF NEAR OPTIMAL TWO-STRESS CSALT PLANS

Given n, T, p_H and p_D, the lower stress X_L and its allocation π_L in a statistically optimal plan are normally determined by minimizing the asymptotic variance of the MLE of the *p*th percentile at design stress, say $A\text{var}(\log(t))$. For the derivation of the asymptotic variance of the MLE of a reliability interest, see Nelson (1990), Meeter *et al* (1994) and Yang (1994). Detailed statistically optimal plans have been tabulated in Meeker and Hahn (1985) in terms of π_L and X_L for some percentile of reliability interest. However, it is noted that the optimal low stress levels are typically too high, resulting in too much extrapolation to design condition, and it is recommended that a stress that is lower than the optimal low stress be used. If the asymptotic variance is allowed to inflate to some tolerable extent, may there be any alternate designs to overcome the deficiency of the two-stress CSALT? A possible approach is to soften the optimality criteria to search for near optimal plans.

Instead of minimizing the asymptotic variance, we consider relaxing the optimality criteria to achieve an asymptotic variance not worse than $\exp(m)$ $(m \geq 0)$ times that of a statistically optimal plan, where *m* is a maximum reliability bound tolerable. Clearly, if *m*=0, that is, $\exp(m)$=1, the only plan available is the statistically optimal plan. This idea can be formulated as:

$$Ln\left(\frac{A\operatorname{var}(\log(t_p))}{Min[A\operatorname{var}(\log(t_p))]}\right) \leq m \tag{2.8}$$

where $Min[A\operatorname{var}(\log(t_p))]$ is the minimum asymptotic variance in a statistically optimal plan. Figure 2.1 shows how *m* varies for a two-stress CSALT with respect to X_L and π_L.

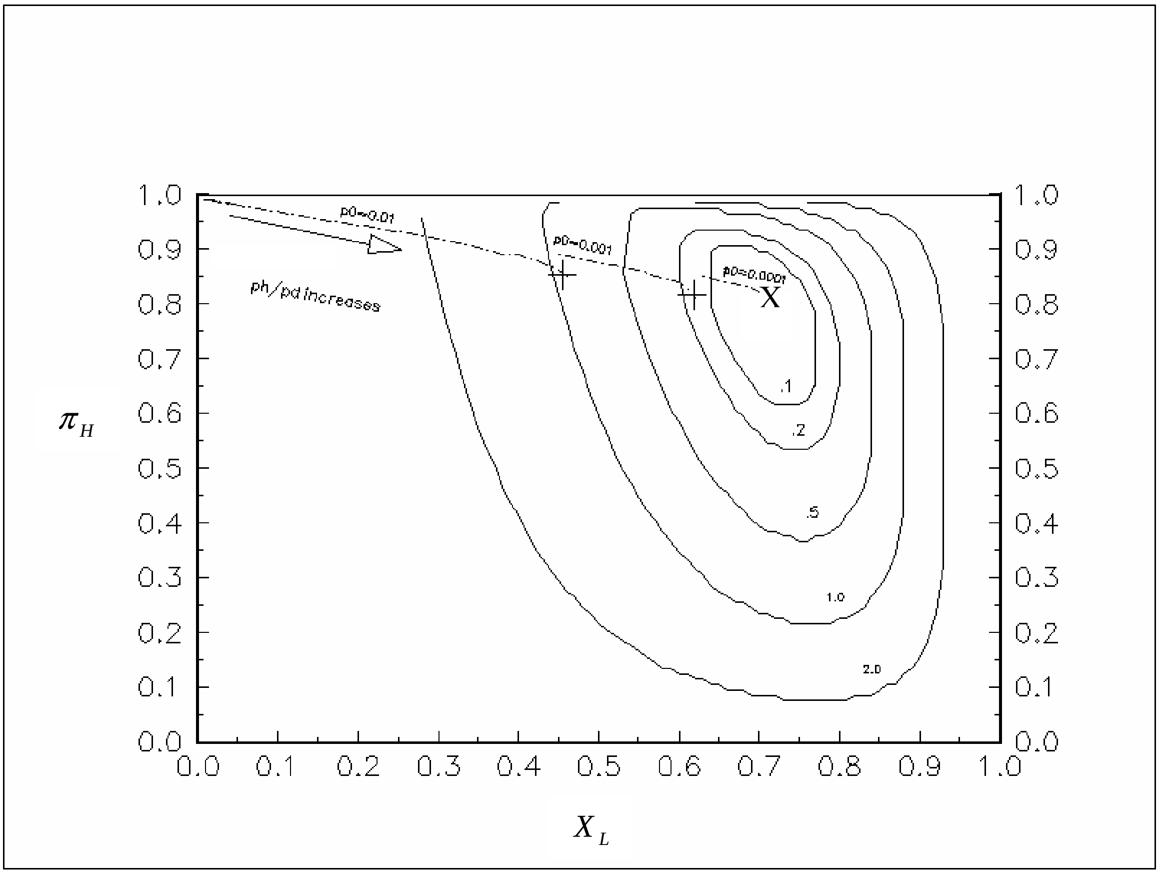

Figure 2.1. An example of the solution space for two-stress CSALT plans

"×" indicating the contour centre for p_D=0.0001, p_H=0.9, T=300, n=300 and σ=1

"+"s indicating the contour centres for other p_Ds and p_Hs

2.4 THE SOLUTION SPACE FOR THREE-STRESS CSALT PLANS

This principle of enlarging the solution space can be generalized to three-stress CSALT plans.

We have numerically plotted $Ln\left(\frac{A\,\mathrm{var}(\log(t_p))}{Min[A\,\mathrm{var}(\log(t_p))]}\right) = m$ for all plans with X$_L \in (0,1)$, X$_M \in (0,1)$, $\pi_D \in (0,1)$, $\pi_D \in (0,1)$ and $\pi_H = 1 - \pi_D - \pi_M$. The results showed that for a fixed percentile of interest and for different combinations of p_D and p_H, the range of π_H as a function of *m* can easily be determined. Figure 2.2 gives an example with various values for *m*.

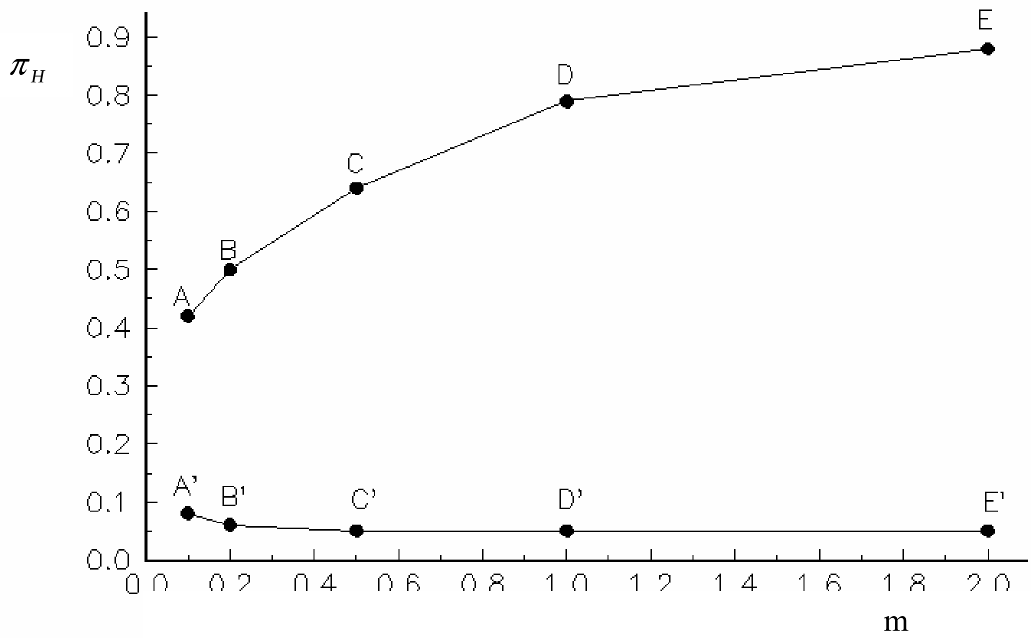

Figure 2.2. The feasible region of π_H for different limits on variances
(P_H=0.9, P_D=0.0001, *n*=300, *T*=300 and σ=1)

Given π_H, the solution space for X_L and X_M can be defined using the optimality criterion (2.8). Note that $X_L < X_M$, the solution is always below a straight line with slope 1. An example is given in Figure 2.3. Out plots also show that solution space for a fixed *m* is approximately a right-angle triangle and those with different *m* share the same slope with gradient 1. As one would usually like to ensure that X_L and X_M are sufficiently far apart, the preferred solution should be at the vertex with the right angle. For different π_H, one could trace the vertices as shown in Figure 2.3 and 2.4. Here we only need to consider π_H within the range that is given Figure 2.2.

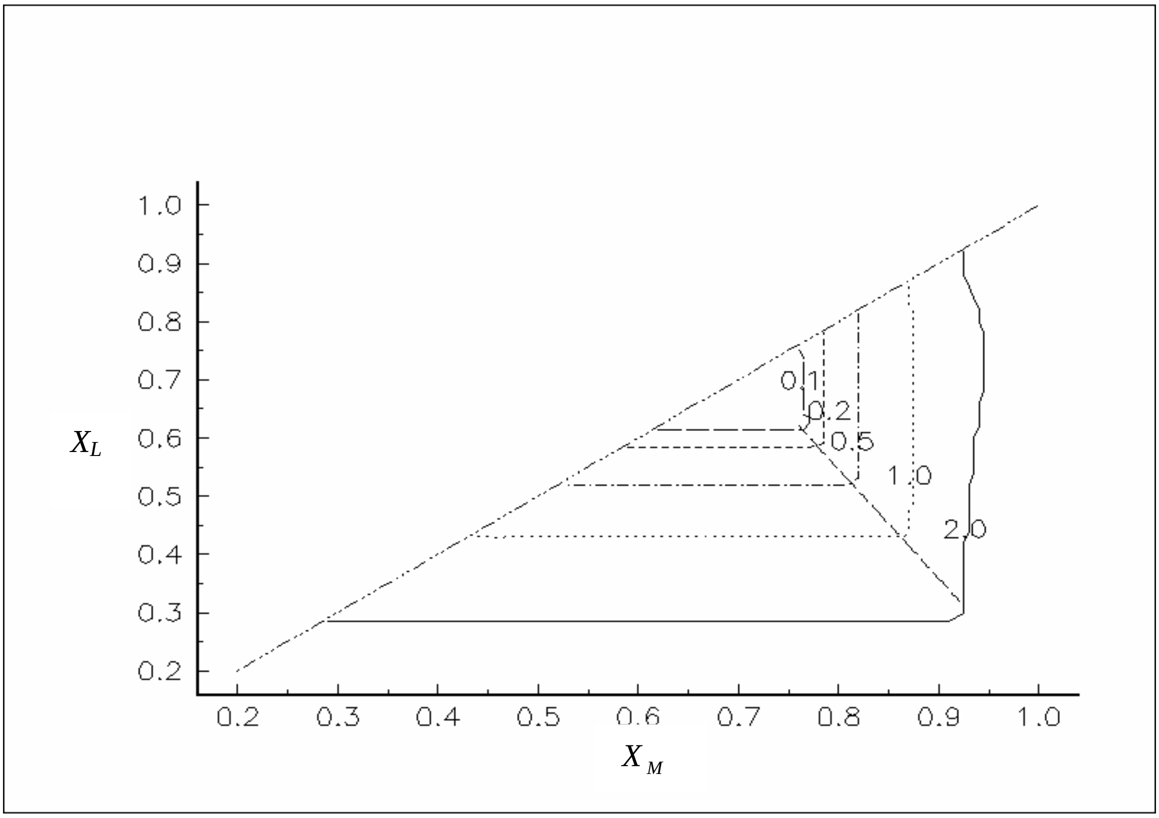

Figure 2.3 The solution space of x_L and x_M
in three-stress CSALT planning ($\pi_H = 0.15$)

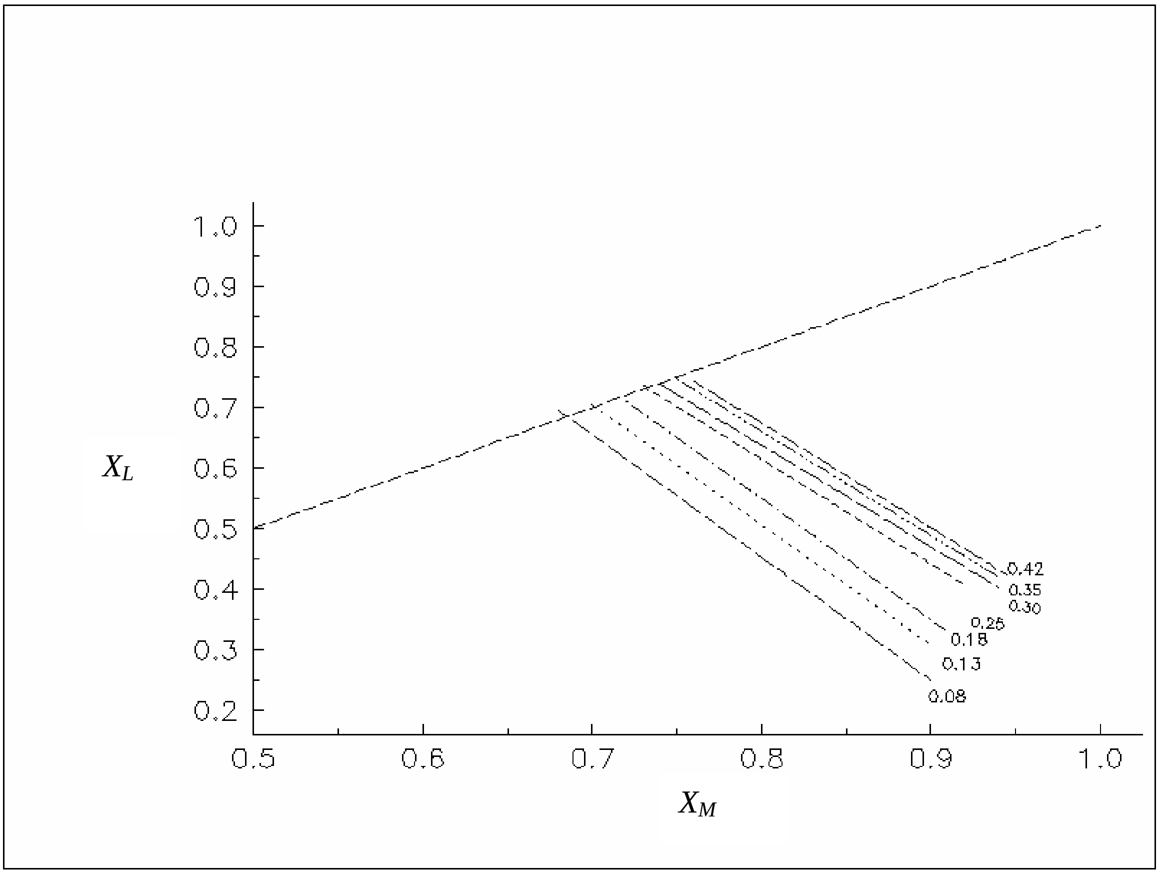

Figure 2.4 Loci of preferred solution with different π_H in three-stress CSALT planning

The above results can be integrated into a single graph using a contour plot shown in Figure 2.5. Along the line of unit gradient, we map the range of π_H with various setting of *m*. The loci of the preferred solutions in the space of X_L and X_M are then superimposed onto the contours of various *m* values.

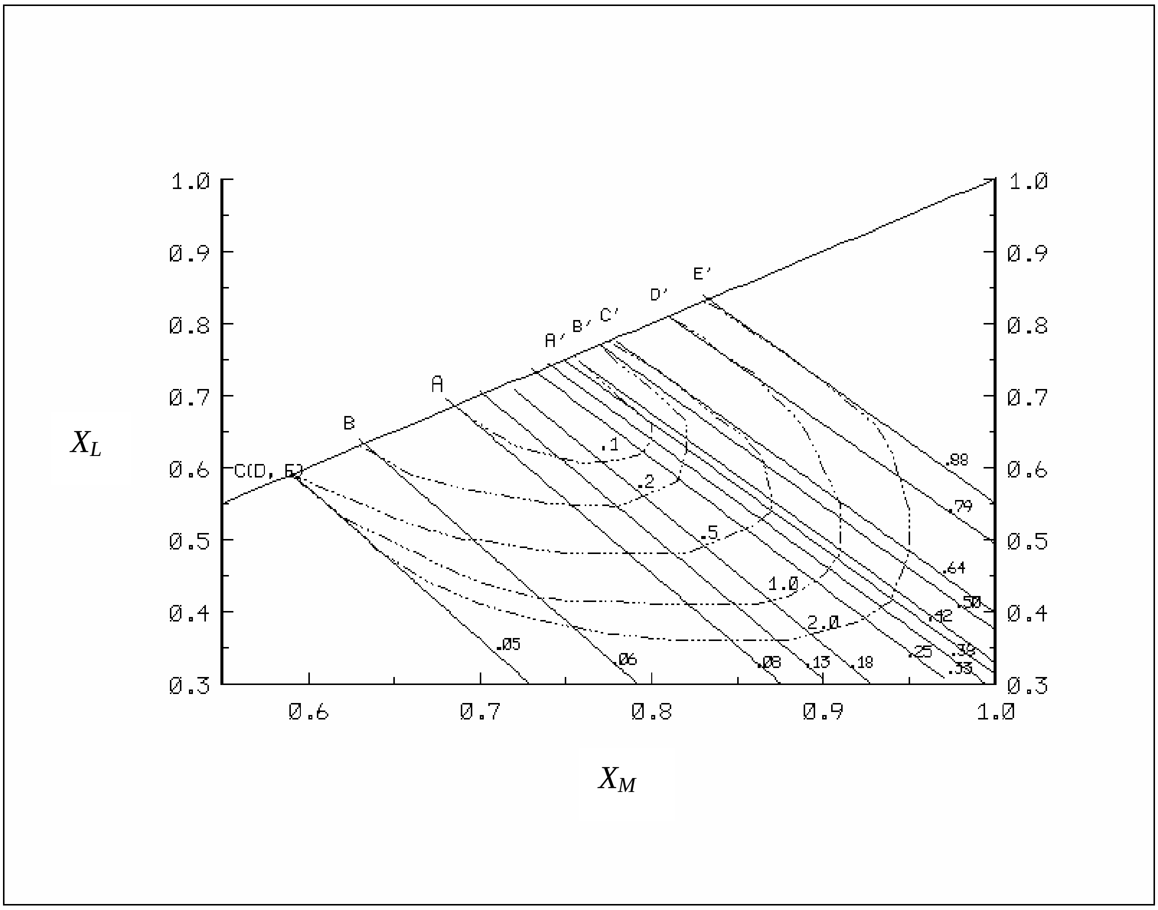

Figure 2.5 The solution space for three-stress CSALT planning

2.5 CONNECTIONS OF TWO-STRESS AND THREE-STRESS CSALT PLANS

It is known that the statistically optimal plan produces the smallest asymptotic variance of the reliability estimate. However, if we set X_L equal to X_M, the three-stress CSALT plan would reduce to a statistically optimal plan. That is, the statistically optimal plan is simply a special case of three-stress plan with $X_L = X_M$. We can dig up the connections of three-stress plans with two-stress plans with the following analysis.

Fixing π_H , we plot $X_M{*}\pi_M$ Vs $X_L{*}\pi_L$ in Figure 2.6. Those whose $A\text{var}(\log(t_p))$ satisfy $Ln\left(\frac{A\text{var}(\log(t_p))}{Min[A\text{var}(\log(t_p))]}\right)=m$ possess a property that $X_M{*}\pi_M+X_L{*}\pi_L$ is a constant.

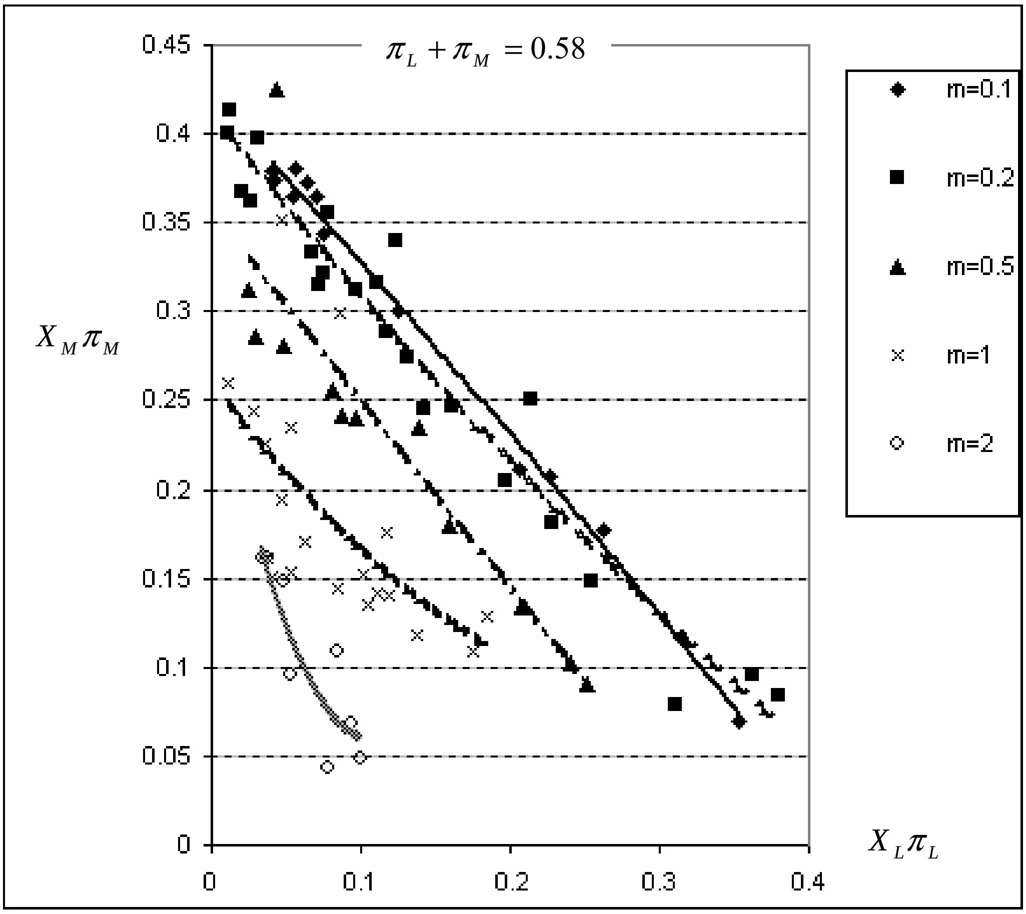

Figure 2.6. $x_M{*}\pi_M$ Vs $x_L{*}\pi_L$ plot for specific $Ln\left(\frac{A\text{var}(\log(t_p))}{Min[A\text{var}(\log(t_p))]}\right)=m$ with fixed π_H in three-stress CSALT planning

This result indicates that the centroid of the lower and middle stress levels, weighted by their respective allocations, in a near optimal three-stress CSALT plan equals to the optimal low stress in the statistically optimal plan (Tang & Yang, 2002). The relationship can be formulated as

$$\frac{X_{3M}\pi_{3M} + X_{3L}\pi_{3L}}{\pi_{3M} + \pi_{3L}} = X^{*}{}_{2L} \tag{2.9}$$

where the subscript number 3 and 2 means the number of stresses at which plans are designed, and the superscript * denotes the optimal values.

As

$$\pi_{3M} + \pi_{3L} = 1 - \pi_{3H} \tag{2.10}$$

Equation (2.9) is rewritten to be

$$\frac{X_{3M}\pi_{3M} + X_{3L}\pi_{3L}}{1 - \pi_{3H}} = X^{*}{}_{2L} \tag{2.11}$$

and

$$\begin{aligned} & X_{3M} * \pi_{3M} + X_{3L} * \pi_{3L} \\ &= X^{*}{}_{2L} * (1 - \pi_{3H}) \\ &= x^{*}_{2L} * (1 - \pi^{*}{}_{2H}) \\ &= x^{*}{}_{2L} * \pi^{*}{}_{2L} \end{aligned} \tag{2.12}$$

That is, the connection of the near optimal three-stress ALT plan with the statistically optimal plan is:

$$X^{*}_{3M} * \pi^{*}_{3M} + X^{*}_{3L} * \pi^{*}_{3L} = X^{*}{}_{2L} * \pi^{*}{}_{2L} \tag{2.13}$$

$$\pi^{*}_{2H} = \pi^{*}_{3H} \tag{2.14}$$

$$\pi^{*}_{3L} + \pi^{*}_{3M} = \pi^{*}_{2L} \tag{2.15}$$

2.6 ALTERNATIVE PROCEDURES FOR THREE-STRESS CSALT PLANNING

From the above results, it can be seen that depending on the preference, past experience and the experimental constraints, there are many possible ways to produce a multiple-stress CSADT. Here, we explain different approaches to determine π_L and π_M under different situations.

2.6.1 Approach 1

Based on the connection with the statistically optimal plan, given p_D, p_H, *n, T,* and σ, a three stress CSALT plan can be determined by the following procedures:

1. Solve for the optimal values of X^*_{2L} and π^*_{2L} (i.e. the optimal point in Figure 2.1).
2. Calculate π_{3H} by

$$\pi_{3H} = 1 - \pi^*_{2L} \tag{2.16}$$

3. Set a value for *m*. And read off the suggested X_{3L} and X_{3M} from Figure 2.5.
4. Calculate π_{3M} using (2.13), noting from (2.15) that

$$\pi_{3L} = \pi^*_{2L} - \pi_{3M} \tag{2.17}$$

This approach can be used as a follow-up test during product development when the preliminary inputs are reasonably accurate but the failure mechanisms may have been changed due to modification of designs, resulting in an unknown stress-life model.

2.6.2 Approach 2

Since the main purpose of having a middle stress is for validating the stress-life model, one may prefer to have sufficient failures for detecting non-linearity if it exists. The minimum allocation at the middle stress can then be determined. This approach is advocated by Meeker & Escobar (1993) and considered by Yang (1994) and Tang (1999):

1. Solve optimal values of X^*_{2L} and π^*_{2L} (i.e. the optimal point in Figure 2.1). And cumulate π_{3H} using (2.16).
2. Choose X_{3L} and X_{3M} from the desirable zone in Figure 2.5.
3. Calculate π_{3M} by

   $$\pi_{3M} = \frac{\gamma_{3M}}{nF_{3M}(T)} \tag{2.18}$$

   where γ_{3M} is the minimum number of failures expected at the middle stress level, and $F_{3M}(T)$ is the probability of failure by end of the test at the middle stress.
4. Calculate π_{3L} using (2.17).

2.6.3 Approach 3

We could also adopt a hybrid of approach 1 and approach 2 as there is no need to fix X_{3L}. This gives rise to:

1. Solve optimal values of X^*_{2L} and π^*_{2L} (i.e. the optimal point in Figure 2.1). And calculate π_{3H} using (2.16).
2. Choose X_{3M} from the desirable zone in Figure 2.5.
3. Calculate π_{3M} by (2.18).
4. Calculate π_{3L} by (2.17)
5. Calculate X_{3L} using (2.14), i.e.

$$X_{3L} = (X^*_{2L} \times \pi^*_{2L} - X_{3M} \times \pi_{3M}) / \pi_{3L} \tag{2.19}$$

The comparisons of these three approaches are illustrated in the following examples.

2.6.4 Numerical Examples

In this section, we give examples to show the procedures for planning a three stress CSALT given that P_D=0.0001, P_H=0.9, n=300, T=300, and σ=1. From (2.5), it is calculated that μ_H=4.8698 and μ_D=14.9148. The statistically optimal plan can be checked from Figure 2.1 as X^*_{2L}=0.71 and π^*_{2L}=0.79. We choose the maximum estimate asymptotic variance bound at m=0.1.

For approach 1, using (2.16), we get

$$\pi_{3H} = 1\text{-}0.79\text{=}0.21.$$

Select X_{3L}=0.62 and X_{3M}=0.78 from Figure 2.5. Using (2.13) and (2.15) we have

$$0.62\pi_{3L} + 0.78\pi_{3M} = 0.79 \times 0.71$$

$$\pi_{3L} + \pi_{3M} = 0.79$$

That is,

$$\pi_L = 0.35\,,\ \pi_M = 0.44.$$

For approach 2, we set X_{3L} =0.62 and X_{3M} =0.78. Using (2.4) and (2.3), we have

$$\mu_{3M} = 14.9148 - 0.78 * (14.9148 - 4.8698) = 7.0795$$

$$F(\ln(300)) = 1 - \exp\left(-\exp\left(\frac{\ln(300) - 7.0795}{1}\right)\right) = 0.223$$

Assuming that γ_{3M} =21, from (2.18) and (2.17), we get

$$\pi_{3M} = \frac{21}{300 * 0.223} = 0.314$$

$$\pi_{3L} = 0.79 - 0.314 = 0.476$$

For approach 3, with π_{3H} = 0.21, X_{3M} =0.78, π_{3M} = 0.314 and π_L =0.477, we have from (2.19):

$$x_{3L} = (0.71 \times 0.79 - 0.78 \times 0.314) / 0.477 = 0.66$$

These results are summarized in Table 2.1. Since the low stress levels for approaches 1 and 2 are determined with m = 0.1, the resulting asymptotic variances should be less than $\exp(m) = 1.1$ times of the best achievable variance. The corresponding relative efficiency for approach 3 is 1.078 (<1.1), which is consistent with our expectation as the low stress (0.66) is larger than 0.62. The sample allocation ratio is approximately 5:3:2. It is quite different from the 4:2:1 ratio recommended by Meeker and Hahn (1985). Despite of a smaller allocation, the expected number of failures is about 8-10 at the lower stress in these plans.

Table 2.1. The proposed three-stress ALT plans
(P_D=0.0001, P_H=0.9, n=300, T=300, σ=1, and m=0.1)

	(X_L, X_M, X_H)	(π_L, π_M, π_H)	(n_L, n_M, n_H)
Approach 1	(0.62,0.78,1)	(0.34,0.45, 0.21)	(102,135,63)
Approach 2	(0.62,0.78,1)	(0.48, 0.31, 0.21)	(143,94,63)
Approach 3	(0.66,0.78,1)	(0.48, 0.31, 0.21)	(143,94,63)

2.7 CONCLUSIONS

By relaxing the optimality criteria in CSALT planning, we obtain the design space for the low/middle stress and their corresponding allocations in two/three stress plans. The proposed approach addresses the quantification of how much the variance of reliability estimate would be inflated if looser optimality criteria are used. It provides experimenters with flexibility in setting stress levels and sample allocations. One can adjust the low/middle stress and their allocations within a pre-specified bound until it is convenient to implement.

Extension of the current principle to design CSALT with four or more stress levels is possible by generalizing (2.11) to one with three or more stress levels in the numerator. This gives additional degrees of freedom in determining the stress levels and the respective allocation. Nevertheless, it should be noted that the middle stress is invariant in the three approaches; which means that we could use it as a reference point to generate more than one lower stress levels. The limiting constraint lies in (2.18), i.e.

are there sufficient samples and/or sufficiently long test time available so that the expected numbers of failures at every stress level are reasonable.

Chapter 3

Analysis of SSADT Data

3.1 INTRODUCTION

ADT, in which the degradation paths of a degradation measure are monitored intermittently, has gained considerable interest in the past years. As shown in section 1.1, it is commonly used to estimate lifetime information of highly reliable products of which failures are rare even under elevated stresses. In ADT, failures typically arise when the amount of degradation reaches a threshold, so that information from degradation paths becomes synonymous to reliability information.

Two essentials are required to analyze degradation data. One is that a model properly describing the underlying degradation mechanism should be selected. From the engineering view, common degradation mechanisms include fatigue, crack, corrosion and oxidation. Various examples can be seen in Nelson (1990) and Meeker & Escobar (1993). So far, deterministic models are widely used to model degradation paths in a large amount of literature. However, in order to determine the particular degradation function, experimenters are required to have sufficient knowledge about the product under test. This adds the difficulty to employ deterministic models. There are also other drawbacks stated in section 1.2.3.1. The alternate choice to model degradation paths is the stochastic model, among which the most widely adopted one is the Wiener process. Detailed survey can be found in section 1.2.3.2.

The other essential to analyze degradation data is the parameter estimation method. ML is normally used whatever the degradation model is. However, when using ML method, the numerical estimation is usually non-trivial. Except for a few cases where the maximum likelihood formulas are quite simple, it generally relies on high quality statistical software to obtain MLEs. LS method has been imposed in ADT data analysis when degradation is modeled with deterministic functions (Tseng & Wen, 2000). Mathematically, it is much easier to obtain the LS estimate. Besides section 1.4.1, the typical papers dealing with ADT analysis are summarized in Table 3.1. There is no evidence of the application of LS in stochastic model estimations yet. This motivates us to do such an investigation.

Table 3.1. A summary of estimation method for ADT analysis

Papers	Degradation Model	Type of Test	Parameter Estimation
Tseng & Wen, 2000	Deterministic model	SSADT	LS method
Meeker & Escobar, 1998		CSADT	ML method
Yang & Yang, 2002			
Wu & Chang, 2002		DT	
Doksum & Hoyland, 1992	Stochastic model	DT	ML method
Kahle & Lehmann, 1998		CSADT	

As discussed in chapter 1, SSADT is superior to CSADT in some aspects. It requires fewer test products and fewer testing chambers, thus it reduces the experimental cost greatly. Hence in this chapter, we focus on SSADT to develop a particular data analysis approach.

In section 3.2, we first present a stochastic process model for SSADT. The emphasis is on a stress-life family such that its unknown parameters can be expressed with a closed form. Section 3.3 discusses the LSEs of the parameters. It provides a simple algorithm for computation. Section 3.4 derives the mean time to failure and its confidence interval if IGD and BSD are used to illustrate the lifetime distribution in different applications. Finally, a numerical example and its comparison with the existing method are presented in section 3.5. Section 6 presents the simulation studies. Section 3.7 gives some remarks of our proposed approach.

3.2 THE EXPERIMENT DESCRIPTION AND MODEL ASSUMPTIONS

In the proposed SSADT experiment, let X_0 be the design stress. n items are put under test from stress X_1 at time T_0=0. The test is changed to increased stresses X_k at time T_{k-1}. The time interval between two continuous inspections is Δt. All items are inspected simultaneously and the inspection time is neglected. Each sample is to be inspected L_k times at stress X_k, thus the total number of inspections on an item is $L = \sum_k L_k$, the inspection time at each stress is $L_k \Delta t,\ k = 1, 2 ...$ and the termination time of the whole test is $L\Delta t$. Figure 3.1 illustrates a two-stress SSADT experiment as described.

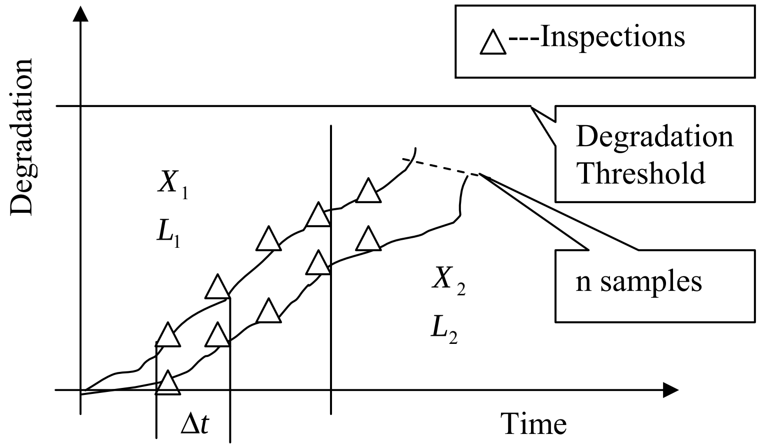

Figure 3.1. An illustration of a two-step-stress ADT experiment

We assume that the degradation follows a stochastic process $\{D_k(t), t \geq 0\}$ with drift $\eta_k > 0$ and diffusion $\sigma_k^2 > 0$ at X_k. It is known that the degradation increments, denoted as $D_k(t+\Delta t) - D_k(t)$, are independently normally distributed random variables (*ND*) with mean $\eta_k \Delta t$ and variance $\sigma_k^2 \Delta t$. If failure is defined as the first time the degradation path crosses a critical value D_c, the failure time of the product under use condition is

$$\mu_0 = \frac{D_c}{\eta_0} \tag{3.1}$$

which follows IGD or BSD depending on the applied failure mechanisms.

Some assumptions in our approach are:

1. Physical failure does not occur, and only increments are observed and recorded during the experiment.

This assumption is acceptable due to the fact that many failure mechanisms in ADT occur gradually. That is, the unit under test experiences structural changes that cause it to degrade gradually over time, rather than to fail catastrophically at a given instant. None of samples would experience "failure" during the time allotted for the test. Viewing from this point, it is necessary for the experimenter to be aware of the product's expected performance in order to define a level of degradation that is equivalent to failure in service. So that degradation of a given property or output of the unit under test can be measured, and can be extrapolated to the "failure" status.

2. Linkage of reliability extrapolation from higher stresses to the use condition can be described with a drift-stress model. We assume it satisfies a log-linear function that can be described as follows:

$$\begin{aligned} \log \eta_k &= a + bx_k \\ \log \sigma_k^2 &= d = \text{Constant} \qquad b > 0 \end{aligned} \tag{3.2}$$

where a, b, d are unknown parameters.

Normally, if variance is large, the paths can indicate with high probability that the process is improving instead of degrading.

3. The test stress is normalized by:

$$X_k = \begin{cases} \dfrac{\log(S_0) - \log(S_k)}{\log(S_0) - \log(S_m)} & \text{for the Power Law model} \\ \dfrac{(1/S_0 - 1/S_k)}{(1/S_0 - 1/S_m)} & \text{for the Arrhenius model} \end{cases} \quad (3.3)$$

where S_m is the highest stress at which the same failure mechanisms as those at use stress are validated. The reason to do such a transformation has been stated in chapter 2. For similar stress standardization, one can refer to Duksum & Hoyland (1992) and Park & Yum (1997). The normalized use stress is always $X_0=0$ and the highest stress is $X_m=1$. Hence (3.2) can be re-written to be $\eta_k = \eta_0 e^{bX_K}$. It is seen that e^{bX_k} is actually the AF reflecting how the failure time has been accelerated at X_k. For instance, if $b = 2$ and at the highest test stress $X_H = 1$, *AF*= $e^{2\times 1}$ =7.389. That is, the lifetime of tested products at the highest test stress is shortened to be 1/7.389 of that at use condition. If we look at the degradation process, it would follow a continuous stochastic process with degradation rate increasing in steps, where the final degradation rate is 7.389 times of that at normal stress. As a result, the total testing time is saved.

These properties are shown in Figure 3.2. n samples are put under test starting from time t=0. The solid lines are degradation paths. At a particular time, degradations of these n specimens are normally distributed as shown by the dotted curves. If failure occurs when degradation exceeds a threshold D_c, then failure time follows an IGD or BSD depends on the real degradation mechanism.

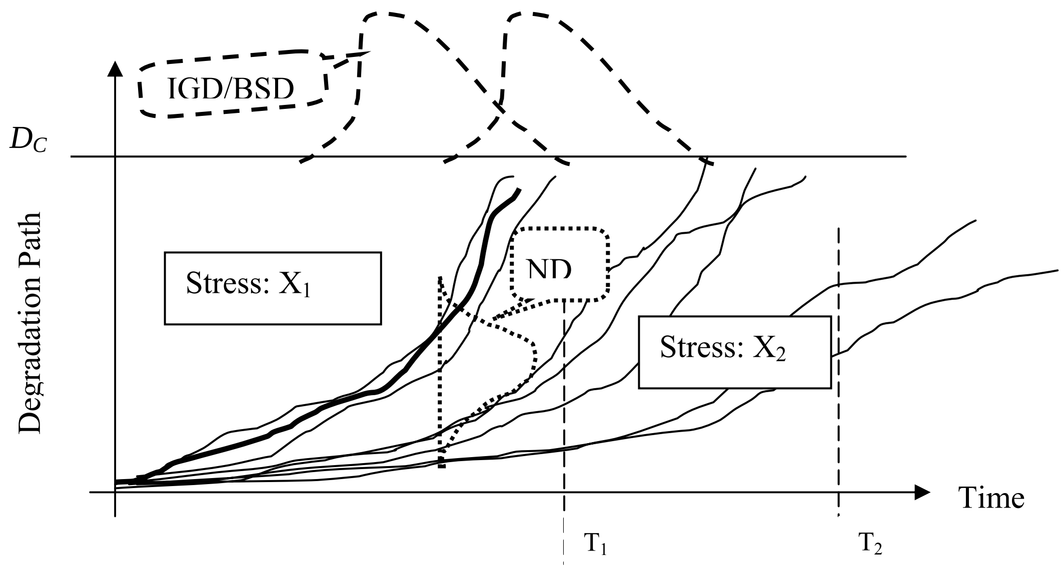

Figure 3.2. An illustration of using a stochastic process to model degradation paths

3.3 PARAMETER ESTIMATION

Analyzing the degradation data generally yields more accurate estimates of reliability related issues than analyzing the failure time data, especially when few failures are observed for highly reliable products. To estimate the products' lifetime governed by a stochastic process, the key point is to estimate η_0, σ_0^2 and b since they are the main parameters to describe the underlying stochastic process. In this section, we present an easily implemented method to estimate these parameters using degradation increments recorded in a SSADT.

3.3.1 Estimation of b and η_0

Denote the degradation increment as $\Delta D_{i,j,k}$, where the subscript *i* indicates the *i*th sample in test, *j* indicates the *j*th inspection, and *k* indicates the *k*th stress level. According to Goh *et al* (1989), each increment is a normally distributed random variable given by

$$\Delta D_{i,j,k} = \Delta t \eta_k + \varepsilon_{i,j,k} \tag{3.4}$$

where $\varepsilon_{i,j,k} \sim N\left(0, \Delta t_{i,j,k} \sigma_k^{\ 2}\right)$ is a normally distributed variable. For large samples, according the Central Limit Theory, the average of $\Delta D_{i,j,k}$ will have a mean value

$$E\left[\Delta D_{i,j,k}\right] = \Delta t \eta_k \tag{3.5}$$

Since the variance at a fixed stress is a constant, by taking the logarithm of (3.4), we get

$$\log \frac{\Delta D_{i,j,k}}{\Delta t} = a + bX_k \tag{3.6}$$

There are totally $n \times L$ such equations. Hence, the LSE of *a*, *b* can be derived from the following equation:

$$\begin{bmatrix} L, & \sum_k L_k X_k \\ \sum_k L_k X_k, & \sum_k L_k X_k^2 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \frac{1}{n} \begin{bmatrix} \sum_{i,j,k} \log \frac{\Delta D_{i,j,k}}{\Delta t} \\ \sum_{i,j,k} \log \frac{\Delta D_{i,j,k}}{\Delta t} X_k \end{bmatrix} \tag{3.7}$$

(Tseng & Yeo 2002).

Hence,

$$\hat{v} = \frac{1}{n} * \left(X'X\right)^{-1} * RowSum(X'*D) \tag{3.8}$$

where

$$\hat{v} \equiv \begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix}$$

$$X_{L\times 2} \equiv \begin{bmatrix} 1, \ ..., 1, & 1, \ ..., \ 1, & ... \\ \underbrace{X_1, \ ..., \ X_1,}_{L_1} & \underbrace{X_2, ..., X_2,}_{L_2} & ... \end{bmatrix}'$$

$$D_{L\times n} \equiv \begin{bmatrix} \log\frac{\Delta D_{1,1,1}}{\Delta t}, ..., \log\frac{\Delta D_{1,L_1,1}}{\Delta t}, & \log\frac{\Delta D_{1,1,2}}{\Delta t}, \ ..., \ \log\frac{\Delta D_{1,L_2,2}}{\Delta t}, & ... \\ \log\frac{\Delta D_{2,1,1}}{\Delta t}, ..., \log\frac{\Delta D_{2,L_1,1}}{\Delta t}, & \log\frac{\Delta D_{2,1,2}}{\Delta t}, \ ..., \ \log\frac{\Delta D_{2,L_2,2}}{\Delta t}, & ... \\ ... \ ... & & \\ \log\frac{\Delta D_{n,1,1}}{\Delta t}, ..., \log\frac{\Delta D_{n,L_1,1}}{\Delta t}, & \log\frac{\Delta D_{n,1,2}}{\Delta t}, \ ..., \ \log\frac{\Delta D_{n,L_2,2}}{\Delta t}, & ... \end{bmatrix}'$$

$$X' \times X \equiv \begin{bmatrix} L, & \sum_k L_k X_k \\ \sum_k L_k X_k, & \sum_k L_k X_k^2 \end{bmatrix}$$

For $A_{p\times q} = \begin{bmatrix} a_{11}, a_{12}, \ ... \ a_{1q} \\ ... \quad ... \\ a_{p1}, a_{p2}, \ ... \ a_{pq} \end{bmatrix}$, we define $RowSum(A_{p\times q}) = \begin{bmatrix} \sum_{j=1}^{q} a_{1j} \\ ... \\ \sum_{j=1}^{q} a_{pj} \end{bmatrix}$

This approach not only achieves closed-form estimates of a and b, but also provides a general algorithm for computation. The detailed Matlab program is given in Appendix A. Compared with the MLE in Kahle &. Lehmann (1998) and Doksum & Hoyland (1992), the approach is much easier for computation (Teng & Yeo, 2002).

Finally, the drift of the stochastic process at use condition X_0 is

$$\hat{\eta}_0 = \exp \hat{a} \tag{3.9}$$

3.3.2 Estimation of σ_0^2

We apply residual analysis to estimate σ_0^2. Note that each increment has the same variance $\sigma_k^2 \Delta t$. σ_0^2 is then estimated as follow:

$$\hat{\sigma}_0^2 = \frac{\sum_{i=1}^{n} \sum_{j=1}^{L_k} \left(\Delta D_{i,j,k} - \eta_k \Delta t \right)^2}{(nL_k - 1)\Delta t} \tag{3.10}$$

3.4 THE MEAN LIFETIME AND ITS CONFIDENCE INTERVAL

The lifetime can be assessed once the drift and diffusion parameters in a stochastic process are estimated. If failure is defined as the first time the degradation process exceeds a threshold D_c. IGD and BSD are two favorable choices to model the failure time distribution.

3.4.1 Modeling the Failure Time with an IGD

The IGD has been investigated as a failure time distribution for various applications from fatigue to stochastic wear out (Bhattacharyya & Fries 1982, Desmond 1986, Duksum & Hoyland 1992, Tang & Chang 1994). It addresses a wider class of lifetime distributions. For example, it is almost an increasing failure rate distribution when it is slightly skewed, thus it is suitable to describe a lifetime distribution that is not dominated by early failures. In addition, the failure rate is a nonzero constant when time approaches infinity. The nearly constant failure rate after a certain time period implies that the occurrence of failure is purely random and is independent of past life, which is similar to the exponential distribution that has been widely used in reliability studies.

A general IGD has two parameters: mean μ and dispersion λ. Its p.d.f is:

$$f(t \mid \mu, \lambda) = \sqrt{\frac{\lambda}{2\pi t^3}} \exp\left(-\frac{\lambda (t-\mu)^2}{2\mu^2 t} \right) \tag{3.11}$$

Other versions of the p.d.f for the IGD are given in Seshadri (1998). The relationship between μ, λ in the IGD and η, σ^2 in the stochastic process is:

$$\mu = \frac{D_c}{\eta} \qquad \lambda = \frac{D_c^{\,2}}{\sigma^2} \tag{3.12}$$

So it is easy to obtain the mean lifetime of a product at use condition and its p% confidence limits as follows:

$$\hat{\mu}_0 = \frac{D_c}{\hat{\eta}_0} \qquad \hat{\lambda}_0 = \frac{D_c^{\,2}}{\hat{\sigma}_0^{\,2}} \tag{3.13}$$

$$
\left(\mu_{0lcl},\mu_{0ucl}\right)=\left(\left[\frac{1}{\hat{\mu}_0}+\sqrt{\frac{F^1_{nL-1,1-p}}{(nL-1)\hat{\mu}_0\hat{\lambda}_0}}\right]^{-1},\left[\frac{1}{\hat{\mu}_0}-\sqrt{\frac{F^1_{nL-1,1-p}}{(nL-1)\hat{\mu}_0\hat{\lambda}_0}}\right]^{-1}\right)
$$
$$
\left(\lambda_{0lcl},\lambda_{0ucl}\right)=\left(\frac{\hat{\lambda}_0 X^2_{nL-1,p/2}}{nL},\frac{\hat{\lambda}_0 X^2_{nL-1,1-p/2}}{nL}\right) \qquad (3.14)
$$

where "lcl" and "ucl" respectively stand for lower confidence limit and upper confidence limit (Tang & Chang,1994).

3.4.2 Modeling the Failure Time with a BSD

Besides the independent property of degradation increments that can be described by an IGD, it is also possible the degradation increment at a particular time point relies on the total degradation accumulated to this time point in realization. In such situations, the BSD can be used. The BSD allows the degradation increment be intrinsically positive. Fixing the inspection time interval large enough to make sure the probability of negative increment is very small, Tang & Chang (1995) applied it to the power supply facilities and derived the failure time confidence bounds and tolerance limits. The c.d.f of the BSD is

$$
F(t;\mu,\lambda)=\phi\left(\sqrt{\lambda}\left(\frac{\sqrt{t}}{\mu}-\frac{1}{\sqrt{t}}\right)\right) \qquad t\geq 0 \qquad (3.15)
$$

The qth percentile of the BSD is:

$$
t_q=\frac{\mu}{4}\left[\sqrt{\frac{\mu}{\lambda}}Z_q+\sqrt{\frac{\mu}{\lambda}Z_q^{\ 2}+4}\right]^2 \qquad (3.16)
$$

The 1-q two sided tolerance limit at (1-p1-p2) confidence level is given by the (1-p1-p2) confidence interval of percentile q:

$$\left(t_{q_{lcl}}, t_{q_{ucl}}\right) = \left(\frac{\mu_{lcl}}{4}\left[\sqrt{\frac{\mu_{lcl}}{\lambda_{ucl}}}Z_q + \sqrt{\frac{\mu_{lcl}}{\lambda_{ucl}}Z_q^{\ 2} + 4}\right]^2, \frac{\mu_{ucl}}{4}\left[\sqrt{\frac{\mu_{ucl}}{\lambda_{lcl}}}Z_q + \sqrt{\frac{\mu_{ucl}}{\lambda_{lcl}}Z_q^{\ 2} + 4}\right]^2 \right)$$

(3.17)

where $Z_p = \phi^{-1}(p)$, (μ_{lcl}, μ_{ucl}) and $(\lambda_{lcl}, \lambda_{ucl})$ are respectively (1-p1) and (1-p2) level confidence interval of μ and λ given in Equation (3.16).

It is noted that when q=0.5, t_q is consistent with the mean lifetime in an IGD.

3.5 A NUMERICAL EXAMPLE

Tseng & Wen (2000) provided an SSADT example of testing LED lamps that are used for scanning manuscripts. The brightness of LED, which determines the quality of facsimile, is chosen as the degradation measure. Temperature is selected as the accelerating variable since it affects the degradation rate intensively. The use condition is 25^0C. 22 lamps are tested at 25^0C, 45^0C, 65^0C, 85^0C and 105^0C. However, according to the diagnostic checking, the valid temperature that would not induce different failure mechanics is less than or equal to 65^0C. Thus we only consider the degradation processes recorded at 25^0C 45^0C and 65^0C in our analysis. Consistent with the Arrhenius model in (3.3), the applied stresses are normalized at:

$$X_1 = \frac{1/(25+273) - 1/(25+273)}{1/(25+273) - 1/(65+273)} = 0$$

$$X_2 = \frac{1/(25+273) - 1/(45+273)}{1/(25+273) - 1/(65+273)} = 0.53$$

$$X_3 = \frac{1/(25+273) - 1/(65+273)}{1/(25+273) - 1/(65+273)} = 1$$

The number of inspections at each stress is $L_1 = 16$, $L_2 = 12$ and $L_3 = 14$. Time interval between two continuous inspections is 48hrs at 25^0C and 168hrs at higher temperatures. The critical degradation value is $D_c = 0.5$.

Due to the negative observations resulted from the low degradation rate, we use IGD as the failure time distribution in our example. Take the observed degradation increments, the stress levels and the corresponding inspection time into equation (3.7), we get $a = -10.748$, $b = 2.03$ and $\hat{\sigma}_0^2 = 5.56\times10^{-4}$. Thus from equations (3.9) and (3.11), the drift at use condition is computed as $\hat{\eta}_0 = 2.148\times10^{-5}$. And the mean lifetime and its variance are computed from equation (3.12) as $\hat{\mu}_0 = 23278hrs$ and $\hat{\lambda}_0 = 449.46$. Substitute values of μ_0 and λ_0 into equation (3.14), their 95% confidence interval are (19497.5hrs, 28877.3hrs) and (408.94hrs^2, 490.84hrs^2). Note that in Tseng and Wen (2002), the lifetime of LED is estimated at 27200hrs. It is within the 95% confidence interval of the mean lifetime in our analysis.

In reliability studies, the choices of failure time distribution are often made on basis of what is understood about the failure mechanism. It is more appropriate to consider the physical characteristics of a failure phenomenon than the goodness of data fit by a distribution to make a choice of a failure mode. When applying deterministic models, it is extremely important to select a reasonable degradation function by prior knowledge of technology (Moura, 1991). Tseng and Wen (2002) assumed an exponentially changing degradation path and applied the lognormal distribution to model the lifetime of LED. However, the failure rate of the lognormal distribution is nearly zero when time approaches infinity. This is not feasible in real life since it indicates that no failure will occur. Our method overcomes this problem. Moreover, when there lacks prior knowledge about the product, our method is generally applicable since it does not contain any pre-specified parameters. Most importantly, our estimations are considered more accurate in terms that the 95% confidence interval of the meantime (17687hrs, 34038hrs) is narrower than that calculated using Tseng and Wen's method (8724hrs, 74795hrs).

3.6 SIMULATIONS

To check the robustness of our proposed approach in analyzing SSADT data, we conducted a simulation study for three-stress level tests. Suppose that the use condition is 25^0C, and the highest stress available to maintain the same failure mechanism is 65^0C. The simulation procedures are employed as follows:

Step1: Pre-guess the mean lifetime, the dispersion parameter of the product and the stress effect factor, i.e. μ, λ and b. This information can be obtained through the historical data. Accordingly, the drift and diffusion parameter can be estimated by (3.1).

As we already have Tseng and Wen's test as reference, we set $\mu_0 = 2.5\times10^4$, $\lambda_0 = 1.5\times10^6$, and $b = 2.0$. The reason here we use a relatively large λ_0 is that we need to control the noise and/or measurement error reasonably small compared with the degradation changes. Because $\eta_0 = \frac{D_c}{\mu_0} = 2\times10^{-5}$, the degradation increment is quite a small number to measure. Here we consider the resolution of measurement instrument is high enough and the measurement errors from operators are controllable. This problem should also be taken into account in real experiment.

Step2: Set the sample size n, inspection time interval Δt and number of inspections L_k at each stress.

Based on Tseng and Wen's test, we set n=1, L_1=15, L_2=L_3=10, $\Delta t_1 = 120$, $\Delta t_2 = 60$, and $\Delta t_3 = 48$. Here the reason we set L_1>L_2=L_3 is that we aim to have more records at the lower stress such that there are more degradation measurements obtained at the lower stress. As a result, the reliability inferences are supposed to be more accurate since the lower stress is closer to the actual used condition. At the same time, we set a larger inspection time interval at the lower stress because the degradation rate at this stress is normally smaller and there won't be obvious degradation changes in a shorter time interval. With a longer inspection time interval that leads to detectable degradation increments, we can also mitigate the effect from external environment disturbances and measurement errors.

Step3: Set the lower stress and middle stress respectively. Here we set the testing temperature at 25^0C and 48^0C. Thus the normalized stresses according to the Arrhenius model in (3.3) are $X_1 = 0$, $X_2 = 0.6$ and $X_3 = 1$.

Step4: Generate the random degradation increments using (3.5). Figure 3.3 is the realizations of degradation paths (1000 runs) based on the above-mentioned parameters.

Step5: Estimate $(\alpha, \beta \text{ and } \delta_k^2)$ and (μ, λ) using (3.2) and (3.13), calculate the 95% confidence interval of $(\hat{\mu}, \hat{\lambda})$ using (3.14) and check whether the true (μ, λ) are contained in the 95% confidence interval.

In 967 of the 1000 runs, the calculated 95% confidence interval of μ_0, λ_0 contains the true value of μ_0, λ_0. Table 3.2 shows the simulation results of 50 runs as an example. We can conclude that this method is robust to estimate the unknown parameters in our approach.

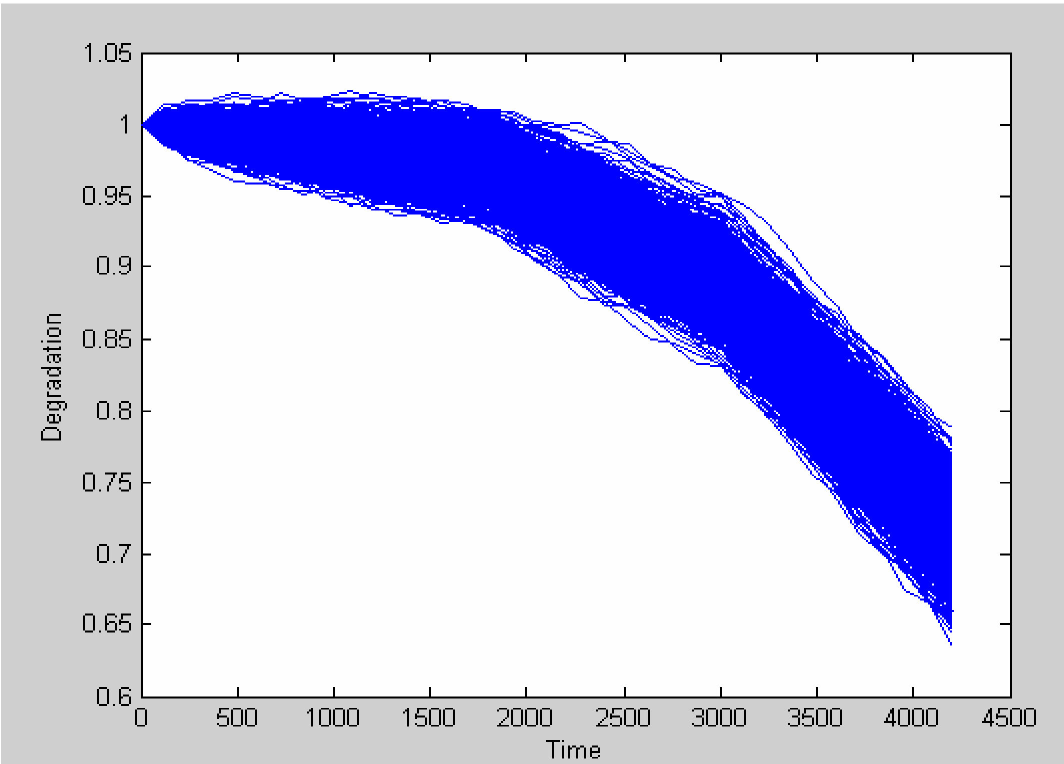

Figure 3.3 Simulation of degradation paths in SSADT

Table 3.2 Simulation results of analysis of three stress SSADT ($\mu_0 = 2.5\times10^4$, $\lambda_0 = 1.5\times10^6$)

μ_{lcl}	μ_{ucl}	λ_{lcl}	λ_{ucl}
24758	26818	929120	1741800
23584	25628	819080	1535500
24404	26595	794270	2082049
23726	25827	792060	2083495
23094	25226	712620	2287349
22870	25071	653070	2068281
24522	26996	641170	1695312
23380	25693	633890	1777710
24274	26730	631440	1961308
24596	27106	630040	1787830
23788	26205	613780	2212885
24484	27020	609510	2027765
23825	26260	607850	2169261
23496	25883	606700	2187966
23916	26370	606280	1894085
22766	25085	584840	2335789
24522	27126	583100	1965658
24384	26967	582770	2124092
24108	26657	577800	2338391
23184	25592	574370	1697237
22932	25303	572940	1796497
23011	25427	558680	1798672
24771	27478	558370	1893594
23940	26513	557000	1804207
22653	25029	551270	2345763
23610	26148	548890	2051499
23534	26066	546380	1724046
23968	26573	545990	1888367
22644	25050	538150	1595803
23446	25984	538040	1697945
24818	27599	534110	2221786
24653	27417	530120	1604833
24391	27115	528540	2376292
23191	25712	527760	1980001
23569	26170	521660	2427364
24085	26786	517940	2077250
24229	26956	517200	1809358
23311	25889	514070	2079963
24881	27737	512660	2424444
23321	25907	511780	1620720
24404	27190	507680	1503123
23664	26324	506810	2037481
23963	26675	506340	1724141
24281	27053	505060	2183117
23158	25736	504390	1775689
23613	26277	502140	2067875
24008	26744	501220	1978019
23003	25565	500570	2231511
23404	26038	499970	1878845

3.7 CONCLUSIONS

We present an approach to analyze SSADT data with stochastic processes in this chapter. It alleviates the difficulty to choose the deterministic degradation path model. The proposed LS method gives closed form estimation for the unknown parameters and provides a general algorithm for computation. Another emphasis of this chapter is on the design of the stress-life model, which contains the Arrhenius model and the Power Law model. It is suitable for various applications whatever the stress refers to temperature or voltage. The IGD and BSD can be chosen to model the lifetime given that failure occurs at the first passage time. Using this method, the mean lifetime and its confidence interval can then be easily predicted. Comparisons with the existing method indicate that our method is more convenient to implement and the estimations are even more accurate.

Chapter 4

A General Formulation for Planning of ADT

4.1 INTRODUCTION

Planning of ADT is a necessary step to obtain precise reliability inferences from ADT data. Armed with a certain degradation model, a test plan for CSADT specifies the stress levels, sample size, sample allocations, inspection frequencies and number of inspections. Typically, a CSADT plan is designed so that a good estimate of a particular reliability measure, such as one with minimum bias or variance, can be obtained (Boulanger & Escobar 1994, Yu & Tseng 1999&2004, Wu & Chang 2002, Yang & Yang 2002). An SSADT plan specifies the stress levels, sample size, number of inspections and holding time at each stress. Compared with CSADT, SSADT is more complicated in terms of design, implementation and analysis as the test stress is increased step by step from a lower one to a higher one. However, as a result of the mild stress gradient, SSADT helps to prevent over-stressing specimens, which is significant in maintaining the same failure modes during the whole test. Detailed survey about ADT planning can be found in section 1.4.2.

Similar to SSADT data analysis that we have discussed in chapter 3, planning of ADT also requires specifying the underlying degradation model. So far in all the published work, all scholars employed deterministic functions to model the degradation paths. These published methods are useful in conditions where the planner masters sufficient knowledge about the product under test, thus s/he can determine the particular degradation function confidently. However difficulties arise if the product is newly developed and no historical information is available to understand the possible degradation process. Motivated by the successful application of stochastic models in ADT data analysis in chapter 3, we propose to design ADT experiments using stochastic models.

Moreover, it is traced that the existing planning methods either concentrate on CSADT or on SSADT, there is no literature presenting both of the two types test planning in one formulation. Here we investigate such an approach, with which, a single function to design CSADT and SSADT is obtained. And as a result, a general program can be coded to search for optimal solutions for both CSADT and SSADT experiments.

In this chapter, the stochastic process similar to that in chapter 3 is used. The unique formulation applicable to both CSADT and SSADT planning are then developed. The layout of this chapter is arranged as follows: in section 4.2, we give the description and the assumptions of our proposed method. Reasons to validate these assumptions are also stated. In section 4.3, we begin by comprehensively analyzing the trade-off between the cost budget and the attainable estimation accuracy in ADT designs. We then propose our planning policy and formulise the objective function as well as the constraints. In section 4.4, we present an example to demonstrate the planning procedure. And in section 4.5, we integrate the experiment plans and data analysis with some simulation studies. Based on the optimal CSADT and SSADT plans generated in section 4.4, the mean lifetime of the product under test is estimated by simulating the degradation processes. Finally in section 4.5, we state some concluding remarks. This chapter only covers the overall view of ADT planning, detailed optimal CSADT plans and SSADT plans will be analyzed in chapter 5 - 7.

4.2 THE EXPERIMENT DESCRIPTION AND MODEL ASSUMPTIONS

Without loss of generality, we consider a CSADT or a SSADT with two stress levels. The experiment description and our assumptions are summarized as follows:

1. Test stress X_k is normalized by $X_k = \frac{S_k - S_0}{S_2 - S_0}$, $k = 0,1,2$, in which S_ks are functions of the applied stresses. For instance, when the acceleration variable is temperature, to be consistent with the Arrhenius model, S is suggested to take a form of $\frac{1}{Temp}$. With such a transformation, $X_0 = 0 < X_1 < X_2 = 1$. This again is for easy searching for the intermediate stress within a (0,1) plane. The design stress indexed with subscript 0 and the maximum allowable stress indexed with subscript 2 should be specified before planning. They can be selected based on the current product specifications and/or past experience. If there lacks such information, a preliminary test can then be performed by forcing the degradation of the device over a short time to clarify the degradation modes and to identify the highest stress. That is, in order to determine the stress range, a pre-test is needed by testing a few samples at a reasonably high stress level.

Once the corresponding degradation data are collected, method in Meeker & Escobar (1998a) can then be used to justify whether there are different failure modes from the ones at use stress.

2. n units are subjected to test, in which n_k are put under X_k. The relationship between n and n_k can be expressed by:

$$n = \begin{cases} \sum_{k=1}^{2} n_k & \text{for CSADT} \\ n_k & \text{for SSADT} \end{cases} \tag{4.1}$$

3. The test duration at stress X_k is T_k, and the stopping time of the whole test is T. The relationship between T and T_k is:

$$T = \begin{cases} \text{maximum}(T_1, T_2) & \text{for CSADT} \\ \sum_{k=1}^{2} T_k & \text{for SSADT} \end{cases} \tag{4.2}$$

4. All samples are inspected simultaneously and continuously with a time interval Δt until the stress changing time T_k at X_k. Unit *i* is inspected L_k times at X_k and the degradation values observed at time $t_{i,1}, t_{i,2}, \ldots t_{i,l_i}$, $t_{i,0} = 0 < t_{i,1} < t_{i,2} < \ldots < t_{i,l_k} \le T_k$ are denoted as $D_{i,1}, D_{i,2}, \ldots D_{i,l_k}$.

5. Only degradation increments are measured throughout the test. This assumption is mild since the products in ADT are always highly reliable and no physical failures would occur in the testing time frame (Kahle & Lehmann, 1998). Further explanation can be seen in section 3.2.

6. The degradation is governed by a stochastic process $\{D_k(t),\, t \ge 0\}$ with drift $\eta_k > 0$ and diffusion $\sigma_k^{\ 2} > 0$ at X_k, in which drift is stress-dependent by:

$$\eta_k = a + bX_k \tag{4.3}$$

and diffusion remains constant for all stresses:

$$\sigma_k^2 = \sigma^2 \tag{4.4}$$

where a, b, σ^2 are unknown parameters that need to be pre-estimated either from engineering handbooks or other ways before experiment planning (Yang & Yang, 2002). Most product managers should have available at least some data from prior performance of similar products, earlier tests, component suppliers or other sources. These data are usually the least expensive data available, and should be used as extensively as possible to pre-estimate the unknown parameters. In cases, however, these sources are not sufficient, experimental data should be collected through a preliminary test (Condra, 2001). η and σ are normally very small for a highly reliable product. In this sense, the probability of a failure occurring during the testing duration is very small and negligible.

Since X_0=0, X_2=1 and drift at use condition is $\eta_0 = a$. (4.3) can be rewritten as $\eta_k = a + bX_k = \eta_0\left(1 + \dfrac{b}{\eta_0} X_k\right) = \eta_0 o(X_k)$, $o(X_k) \ge 1$ indicates a function of X_k. This relationship is well recommended in Duksum & Hoylan (1992). Generally, our approach is applicable to those that can be "linearized" as in (4.3). For example, (4.3) may be the result after taking logarithm of the actual sliding wear versus the logarithm of the applied loading; this helps to stabilize the variance. In the event that drift is an exponential function of the stress factor, taking logarithm on both sides of the equation results in a linear function between the Log(Drift), which is η_k, versus the stress factor, X_k.

The advantage of using (4.3) and (4.4) also lies in its simplicity in that the stress factor is zero under use stress. As a result, only coefficient "*a*" has a bearing on the precision constraint.

4.3 A GENERAL FORMULATION FOR PLANNING OF CSADT AND SSADT

The objective of planning accelerated testing is to obtain precise estimate of the reliability interest. So that the optimization criteria in many existing methods are to minimize the variance of a particular estimate as summarized in section 1.3.2. Obviously, the larger the number of samples and inspections is, the more accurate the statistical inferences will be. However, increasing number of testing samples and testing time also increase the testing cost. There is a trade-off between the attainable precision of the estimate and the total testing cost. Neglecting the cost issues, Park & Yum (1997) and Yu & Chiao (2002) designed to fulfill a precision constraint for optimal planning. While Boulanger & Escobar (1994), Yu & Tseng (1999) and Yu & Chiao (2002) generated cost functions according to their test disciplines. Considering this trade-off, we propose a precision constraint and a cost function based on our assumptions. Basically, the optimal ADT plan is obtained such that the total test expense is minimized while the probability that the estimated mean lifetime at use stress locates within a pre-described range of its true value should not be less than a precision level *p*. We discuss the cost functions and the precision constraint in the following subsections.

Define

$$\pi_k = \begin{cases} \dfrac{n_k}{n} & \text{for CSADT} \\ \dfrac{T_k}{T} & \text{for SSADT} \end{cases} \tag{4.5}$$

$$q_k = \begin{cases} \dfrac{T_k}{T} & \text{for CSADT} \\ \dfrac{n_k}{n} & \text{for SSADT} \end{cases} \tag{4.6}$$

thus

$$0 < \pi_k < 1 \ , \ \sum \pi_k = 1 .$$

And for CSADT,

$$q_1 = 1, \ 0 < q_2 \le 1 ;$$

for SSADT,

$$q_k = 1 .$$

With this definition, the proportions of the sample allocation in CSADT and the holding time in SSADT are comparable. Since the samples are assigned to two stresses in CSADT and the total test time is distributed to two stresses in SSADT, the consistency of physical meaning of π_k is achieved. Another merit of this definition is that it is easier to code a program for optimization computation, as the sum of π_k always equals 1.

We have investigated that in a CSADT, the optimal testing time at stress X_2 is always shorter than that of X_1 with some preliminary study, i.e. $T_2 < T_1$, so that q_2 is less than 1.

4.3.1 The Cost Functions

The total cost of testing normally consists of three parts: the sample cost, the measurement cost, and the manpower cost.

1. Sample cost C_{de}

   This is the cost due to consuming of test units. Let C_d be the cost per test unit, for both CSADT and SSADT, the total sample cost is

$$C_{de} = C_d \cdot n \tag{4.7}$$

2. Measurement cost C_{me}

   This cost is induced by using inspection equipments and materials. It depends on the number of units and the number of inspections. Let C_{mk} denote the cost per inspection per unit at X_k. For both CSADT and SSADT, the number of inspections at X_k is $n_k L_k = \frac{n \cdot T}{\Delta t} \pi_k \cdot q_k$, thus the total inspection cost is:

$$\begin{aligned} C_{me} &= \frac{n \cdot T}{\Delta t} \sum C_{mk} \cdot \pi_k \cdot q_k \\ 0 < \pi_i < 1,\ & \sum \pi_k = 1,\ \begin{cases} q_1 = 1,\ 0 < q_1 \le 1 & \text{for CSADT} \\ q_1 = q_2 = 1 & \text{for SSADT} \end{cases} \end{aligned} \tag{4.8}$$

3. Manpower cost C_{op}

   This kind of cost comprises of the salary of operators and depreciation of test equipments. It is proportional to the experimental time. Let C_{ok} be the operation cost per time unit at stress X_k, then:

   for CSADT, the total operation cost is: $\sum C_{ok} \cdot T_k = T \sum C_{ok} \cdot q_k$;

   for SSADT, the total operation cost is: $\sum C_{ok} \cdot T_k = T \sum C_{ok} \cdot \pi_k$.

   To be standardized, the operation cost can be described as follow:

$$C_{op} = T \sum C_{ok} \cdot w_k \qquad \begin{cases} w_k = q_k & \textit{for CSADT} \\ w_k = \pi_k & \textit{for SSADT} \end{cases} \tag{4.9}$$

In general, the total cost of testing can be expressed by:

$$TC = C_d \cdot n + \frac{n \cdot T}{\Delta t} \sum C_{mk} \cdot \pi_k \cdot q_k + T \sum C_{ok} \cdot w_k$$
$$0 < \pi_k < 1, \sum \pi_k = 1, \begin{cases} w_k = q_k,\ q_1 = 1,\ 0 < q_2 \le 1 & \text{for CSADT} \\ w_k = \pi_k,\ q_k = 1 & \text{for SSADT} \end{cases} \tag{4.10}$$

4.3.2 The Precision Constraint

Suppose that the mean lifetime at use condition, $\mu(X_0)$, is of interest in our planning. To obtain an estimate close to its true value with a certain level of confidence, we impose a precision constraint by limiting the sampling risk in estimating $\mu(X_0)$ with its MLE, i.e. $\hat{\mu}(X_0)$, to be reasonably small. Mathematically, the above proposal can be expressed as:

$$\Pr\left(\frac{\mu(X_0)}{c} \le \hat{\mu}(X_0) \le c \cdot \mu(X_0)\right) \ge p \tag{4.11}$$

where c>1 and 0<p<1 are given constants. The asymptotic variance of $\hat{\mu}(X_0)$ is needed for further explanation of (4.11).

It is well known that the degradation increment $D_k(t+\Delta t) - D_k(t)$ in a stochastic process follows a normal distribution with mean $\eta_k \Delta t$ and variance $\sigma_k^2 \Delta t$, i.e. $\Delta D_{i,j,k} \sim N(\eta_k \Delta t_{i,j,k}, \sigma^2 \Delta t_{i,j,k})$ with the p.d.f

$$f(\Delta D_{i,j,k}) = \frac{1}{\sqrt{2\pi}\left(\sqrt{\Delta t \sigma^2}\right)} \exp\left(-\frac{(\Delta D_{i,j,k} - \Delta t \eta_k)^2}{2\Delta t \sigma^2}\right) \tag{4.12}$$

We can write the log-likelihood of an individual degradation increment $D_{i,j,k}$ as

$$\ln LH_{i,j,k} = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\Delta t) - \ln \sigma - \frac{U_{i,j,k}^2}{2} \tag{4.13}$$

where

$$U_{i,j,k} = \frac{(\Delta D_{i,j,k} - \Delta t \eta_k)}{\sqrt{\Delta t}\sigma}, \begin{cases} k = 1 & \text{if} \quad j \le L_1 \\ k = 2 & \text{otherwise} \end{cases} \tag{4.14}$$

Hence, the log-likelihood function for all degradation increments of *n* items is given by:

$$\ln LH = \sum_{i=1}^{n} \sum_{j=1}^{L} \ln LH_{i,j,k} \tag{4.15}$$

Given the degradation critical value D_c, $\mu(X_0)$ is given by the ratio of this threshold over the drift at use condition, i.e.

$$\mu(X_0) = Dc / \eta_0 = Dc / a \tag{4.16}$$

Let $\{\hat{a}, \hat{b}, \hat{\sigma}\}$ be the MLE of $\{a, b, \sigma\}$, then, by the invariant property, the MLE of $\mu(X_0)$ is given by:

$$\hat{\mu}(X_0) = Dc / \hat{\eta}_0 = Dc / \hat{a} \tag{4.17}$$

Then the asymptotic variance of $\hat{\mu}(X_0)$ can be obtained using:

$$A\,\mathrm{var}(\hat{\mu}(X_0)) = \hat{h}' F^{-1} \hat{h} \tag{4.18}$$

where $h = \left(\frac{\partial \hat{\mu}(X_0)}{\partial a}, \frac{\partial \hat{\mu}(X_0)}{\partial b}, \frac{\partial \hat{\mu}(X_0)}{\partial \sigma} \right)'$, and F is a Fisher Information Matrix displayed as follows, in which we take use of $E(U_{i,j}) = 0$ and $Var(U_{i,j}) = 1$. The caret ^ indicates that the derivative is evaluated at $\{a, b, \sigma\} = \{\hat{a}, \hat{b}, \hat{\sigma}\}$. The first and second order partial differential can be seen in Appendix B1.

$$F=\begin{bmatrix} E\left(-\frac{\partial^2 \ln LH}{\partial a^2}\right), & E\left(-\frac{\partial^2 \ln LH}{\partial a \partial b}\right) & E\left(-\frac{\partial^2 \ln LH}{\partial a \partial \sigma}\right) \\ & E\left(-\frac{\partial^2 \ln LH}{\partial b^2}\right) & E\left(-\frac{\partial^2 \ln LH}{\partial b \partial \sigma}\right) \\ & symmetric & E\left(-\frac{\partial^2 \ln LH}{\partial \sigma^2}\right) \end{bmatrix}$$

$$=\begin{cases} \frac{n}{\sigma^2}\begin{bmatrix} \sum_k \pi_k T_k, & \sum_k X_k \pi_k T_k & 0 \\ & \sum_k X_k^2 \pi_k T_k & 0 \\ symmetric & & 2\sum_k \pi_k L_k \end{bmatrix} & \text{for CSADT} \\ \frac{n}{\sigma^2}\begin{bmatrix} L\Delta t, & \Delta t \sum_{k=1}^{2} X_k L_k & 0 \\ & \Delta t \sum_{k=1}^{2} X_k^2 L_k & 0 \\ symmetric & & 2L \end{bmatrix} & \text{for SSADT} \end{cases}$$

(4.19)

Thus we have

$$\text{Avar}(\hat{\mu}(X_0)) = \frac{\hat{\sigma}^2}{n} \cdot \frac{D_c^2}{\hat{a}^4} \cdot Q . \tag{4.20}$$

where Q is generated from the derivation of the asymptotic variance of MLE as:

$$Q=\begin{cases} \dfrac{\sum_{k=1}^{2} X_k^2 \pi_k T_k}{\left(\sum_{k=1}^{2} X_k^2 \pi_k T_k\right)\left(\sum_{k=1}^{2} \pi_k T_k\right)-\left(\sum_{k=1}^{2} X_k \pi_k T_k\right)^2} & \text{for CSADT} \\ \dfrac{\sum_{k=1}^{2} X_k^2 L_k}{\sum_{k=1}^{2} L * \Delta t * \sum_{k=1}^{2} X_k^2 L_k - \Delta t * \left(\sum_{k=1}^{2} X_k L_k\right)^2} & \text{for SSADT} \end{cases} \tag{4.21}$$

With definitions (4.5) and (4.6), a universal format of Q can be obtained as:

$$Q = \frac{\sum_{k=1}^{2} X_k^2 \pi_k q_k}{T\left(\left(\sum_{k=1}^{2} r_k q_k\right)\left(\sum_{k=1}^{2} X_k^2 \pi_k q_k\right) - \left(\sum_{k=1}^{2} X_k \pi_k q_k\right)^2\right)} \tag{4.22}$$

$$r_k = \begin{cases} \pi_k & for\ CSADT \\ q_k & for\ SSADT \end{cases}$$

Because the MLE is asymptotically normal and consistent, for large n, approximately we have:

$$\hat{\mu}(X_0) \sim N(\mu(X_0), A\mathrm{var}(\hat{\mu}(X_0))) \tag{4.23}$$

which can be rewritten as

$$\frac{\hat{\mu}(X_0)}{\mu(X_0)} \sim N\left(1, \frac{\hat{\sigma}^2}{n\hat{a}^2} \cdot Q\right) \tag{4.24}$$

From (4.11), we have

$$\Pr\left(\frac{1}{c} \le \frac{\hat{\mu}(X_0)}{\mu(X_0)} \le c\right) \ge p \tag{4.25}$$

This translates to the following precision constraint

$$\Phi\left(\frac{(c-1)\cdot\sqrt{n}}{\frac{\hat{\sigma}\cdot}{\hat{a}}\cdot\sqrt{Q}}\right) - \Phi\left(\frac{(\frac{1}{c}-1)\cdot\sqrt{n}}{\frac{\hat{\sigma}\cdot}{\hat{a}}\cdot\sqrt{Q}}\right) \ge p \tag{4.26}$$

$$c > 1,\ 0 < p < 1$$

where $\Phi(\cdot)$ is the c.d.f of the standard normal distribution.

Finally, planning of ADT can be generally formulated as follows:

Minimizing:

$$C = C_d \cdot n + \frac{n \cdot T}{\Delta t} \sum C_{mk} \cdot \pi_k \cdot q_k + T \sum C_{ok} \cdot w_k$$

Subject to:

$$\Phi\left(\frac{(c-1)\cdot\sqrt{n}}{\frac{\sigma \cdot}{a} \cdot \sqrt{Q}}\right) - \Phi\left(\frac{(\frac{1}{c}-1)\cdot\sqrt{n}}{\frac{\sigma \cdot}{a} \cdot \sqrt{Q}}\right) \geq p$$

$$c > 1, \quad p > 0$$

$$Q = \frac{\sum_{k=1}^{2} X_k^2 \pi_k q_k}{T\left(\left(\sum_{k=1}^{2} r_k q_k\right)\left(\sum_{k=1}^{2} X_k^2 \pi_k q_k\right) - \left(\sum_{k=1}^{2} X_k \pi_k q_k\right)^2\right)}$$

$$0 < \pi_k < 1, \sum \pi_k = 1, \begin{cases} w_k = q_k,\ r_k = \pi_k, q_1 = 1,\ 0 < q_2 \leq 1, & \text{for CSADT} \\ w_k = \pi_k, r_k = q_k \quad q_k = 1 & \text{for SSADT} \end{cases} \tag{4.27}$$

Decision variables in an ADT are explored in Table 4.1.

Table 4.1 Variables in a two-stress ADT

	Decision variables				Other variables		
	n	T_k	π_1	q_2	π_2	q_1	X_1
CSADT	✓	✓	✓	✓	1-π_1	1	Given
SSADT	✓	✓	✓	1			

This formulation has been coded in a VBA program to search for the optimal values of decision variables. We use it to analyze the CSADT plans and SSADT plans respectively in chapter 5 and chapter 6. In addition, an interactive user form can be developed. We show it in the following examples.

4.4 NUMERICAL EXAMPLES

Yu & Tseng (1999) presented a degradation testing (*DT*) experiment for dot-matrix display unit conducted at use stress. The lifetime of a dot-matrix display unit is technically defined as the time when the standardized light intensity of LED lamp degrades below the critical value D_C=0.5. Optimization criteria in their paper are to minimize the variance of the LSE of the 100*p*th percentile lifetime at use stress under the condition that the test cost does not exceed a cost budget. They used a lognormal function to model the degradation path. With Co=12.25/48=0.255$/hr*time, C_m=3.65$/unit*time, C_d=86$/unit and some other pre-estimated parameters, they obtained the optimal sample size n=30, the optimal inspection frequency Δt =240hrs and number of inspections L=21. That is, the stopping time is T=21*240=5040hrs. The total cost in their planned experiment is:86*30+0.255*240*21+3.65*21*30=6165.75$.

To compare our method with that in Yu & Tseng (1999), we use the same or slightly changed cost coefficients. Individual sample cost remains the same at C_d=86$/unit. Operation and inspections at high stress usually cost more, thus we set the operation and measurement coefficients at C_{o1}=0.3$/hr*time, C_{o2}=0.4$/hr*time, C_{m1}=4$/unit*time and C_{m2}=4.5$/unit*time, which are greater than those in Yu & Tseng (1999).

It is noted that our planning is based on a pre-estimation of σ / a, the ratio of the degradation process dispersion relative to its drift at use condition. So it is necessary to do a pilot test under use condition if σ / a is currently unknown or unpredictable in the product design stage. A preliminary test has been conducted in Yu & Tseng (1999). The degradation drift at use condition is estimated as $\hat{a} = 10^{-5}$. Information on variability is usually difficult to obtain, here we set a rough value of $\hat{\sigma} = 10^{-3}$ to compute the optimal plans. Additionally, we set $X_1 = 0.3$, c=5 and p=0.9.

The optimal SSADT and CSADT plans are computed by the interactive VBA program attached in Appendix B2. Optimal values for the decision variables are obtained by numerical search within the whole design space. After inputting the cost coefficients, the selected stress level X_1, the precision parameters c and p, the inspection time interval Δt, and the type of ADT, the final plan is shown in the lower half of the dialog window. Commercially, it is helpful for the experimenter to get the optimization result quickly and easily. Figure 4.1 and 4.2 are respectively the final CSADT and SSADT results.

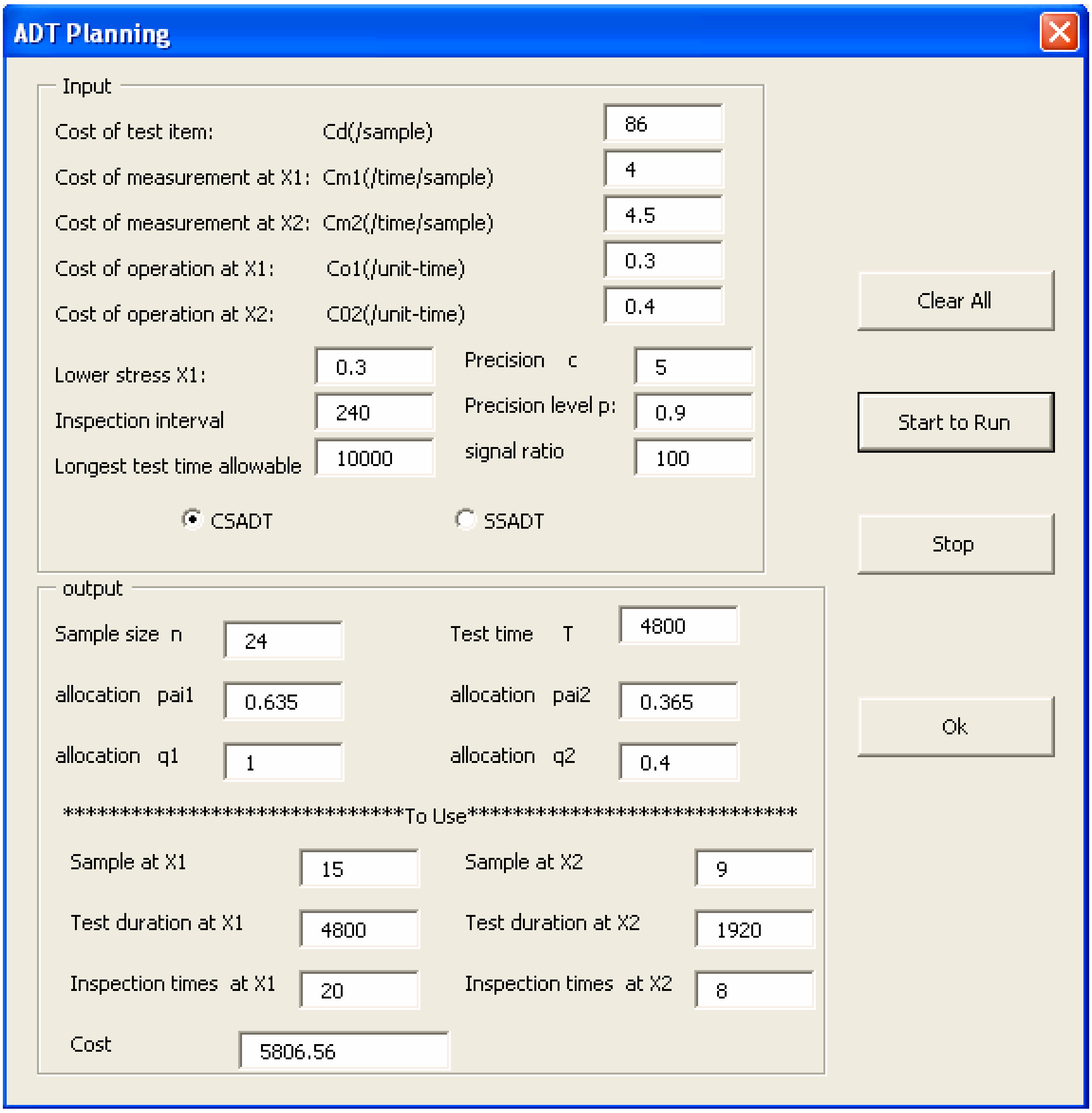

Figure 4.1 A user-interactive window for CSADT planning

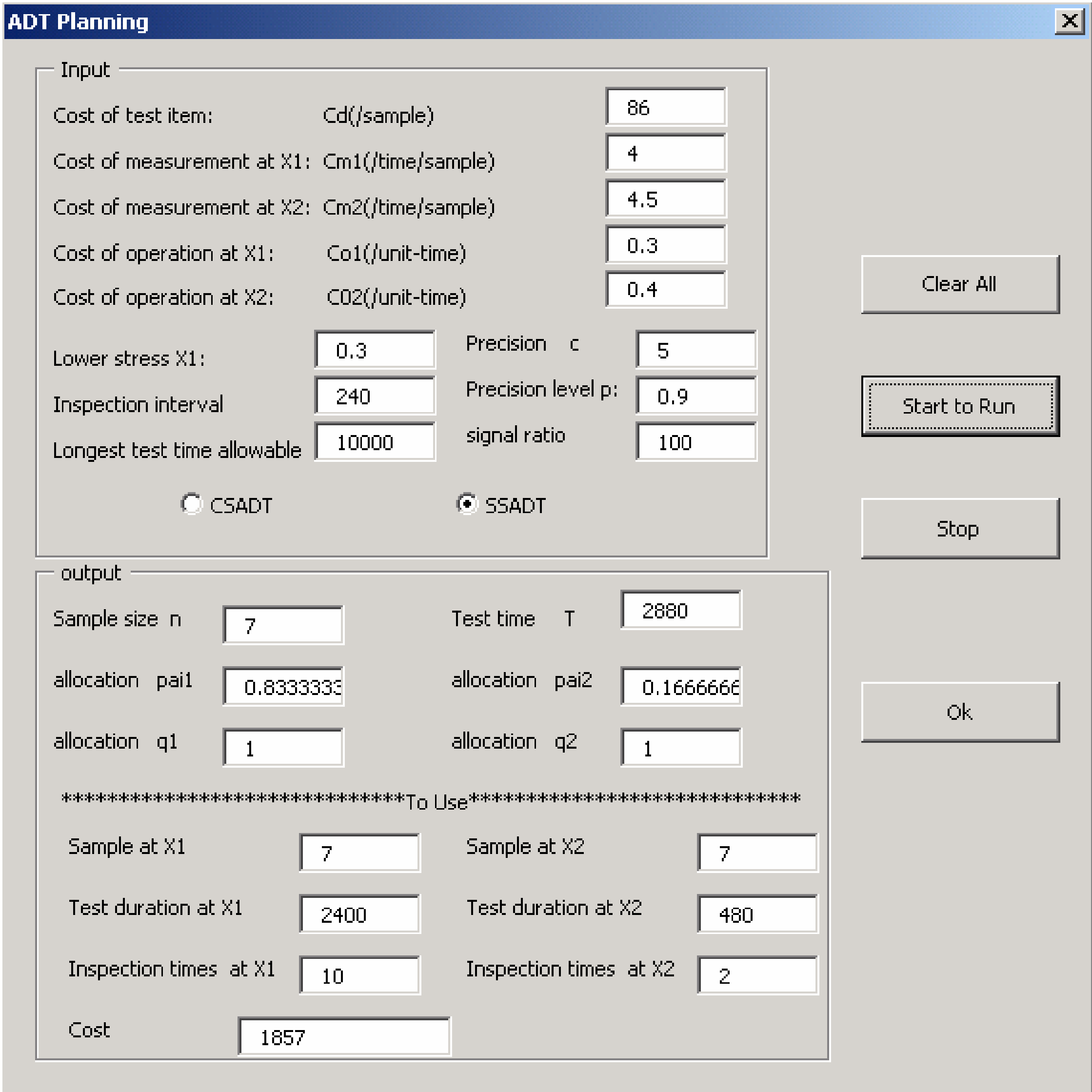

Figure 4.2 A user-interactive window for SSADT planning

Optimal plans are summarized in Table 4.2

<table>
<tr><th colspan="9">Table 4. 2 Comparisons of our proposed ADT with the existing plan</th></tr>
<tr><th colspan="2">Approach</th><th>Stress</th><th colspan="2">Sample size</th><th colspan="2">Test duration</th><th>Inspection times</th><th>Cost</th></tr>
<tr><td colspan="2">Yu & Tseng’s plan</td><td>$X_0=0$</td><td colspan="2">30</td><td colspan="2">5040</td><td>21</td><td>6167.8</td></tr>
<tr><td rowspan="4">Our proposed planning</td><td rowspan="2">CSADT</td><td>$X_1=0.3$</td><td rowspan="2">24</td><td>15</td><td rowspan="2">4800</td><td>4800</td><td>20</td><td rowspan="2">5806.6</td></tr>
<tr><td>$X_2=1$</td><td>9</td><td>1920</td><td>8</td></tr>
<tr><td rowspan="2">SSADT</td><td>$X_1=0.3$</td><td colspan="2" rowspan="2">7</td><td rowspan="2">2880</td><td>2400</td><td>10</td><td rowspan="2">1857</td></tr>
<tr><td>$X_2=1$</td><td>480</td><td>2</td></tr>
</table>

Basically, Yu and Tseng aimed to obtain the absolute optimized plan. However, if the testing time is too long or the available samples are limited, an optimal plan will be difficult for implementation in real industry. To overcome this problem, instead of minimizing the variance of an estimate, we propose to soften the planning criteria to obtain an estimate precision dominated by parameters c and p. Provided that the estimate precision constraint with c=5 and p=0.9 is acceptable, our proposed plan requires fewer samples and less testing time compared with Yu & Tseng (1999). This consequently reduces the total cost of testing. It would be favorable when the individual sample is extremely expensive or the product is newly developed such that number of available samples is limited. Our plan is also more efficient than the existing DT plan since it allows for reliability assessment within shorter test duration.

It is preferred in situations when manufacturers need to know the reliability result urgently (Tang et al, 2004).

4.5 SIMULATIONS

In order to verify the efficiency of our method in planning CSADT and SSADT experiments, here we show some simulation studies based on the results generated from the above example. The simulation procedures are illustrated as follows:

4.5.1 Simulation Study of the Optimal CSADT Plan

Step1. With reference to Yu & Tseng (1999), suppose the degradation drift and dispersion at use condition are respectively $\eta_0 = a = 10^{-5}$ and $\sigma = 10^{-3}$. Moreover, $b = 9\times10^{-5}$, $D_c = 0.5$. That is, the lifetime of the tested product at normal stress is $\mu_0 = Dc/\eta_0 = 5\times10^4\, hrs$.

Step 2: According to the optimal CSADT plan in section 4.4, set $X_1 = 0.3$, c=5, p=0.9, n_1=15, n_2=9, L_1=20 and L_2=8. Generate degradation increments from (4.3) (4.4) and (4.12). Detailed values of the degradation paths and their realizations are shown in Table 4.3 and Figure 4.3.

Step 3: Using (4.13), (4.14) and (4.15), calculate the MLE of $(\hat{a}, \bar{b}, \hat{\sigma})$.

Here given initial values of $(10^{-5}, 9\times10^{-5}, 10^{-3})$, the MLE of $(\hat{a},\vec{b},\hat{\sigma})$ are estimated at $\hat{a}=1.147\times10^{-5}$, $\hat{b}=7.651\times10^{-5}$ and $\hat{\sigma}=1.04\times10^{-3}$ by invoking the "Solver" function in Excel.

Step 4: Using (4.17), (4.20) and (4.21),

$$\hat{\mu}_0 = 43574.8hrs$$

and $$\text{Avar}(\hat{\mu}(X_0)) = 47505229$$

Step 5: Substitute c=5, $\hat{\mu}_0$ and $\text{Avar}(\hat{\mu}(X_0))$ into (4.23), (4.24) and (4.25), compute the actual estimate precision level and compare it with desired estimate precision level p=0.9. The actual estimate precision level is:

$$\Pr\left(\frac{1}{5} \le \frac{\hat{\mu}(X_0)}{\mu(X_0)} \le 5\right) = 0.99 > p = 0.9\,.$$

The result indicates that our method is efficient to estimate the reliability interest as we have proposed.

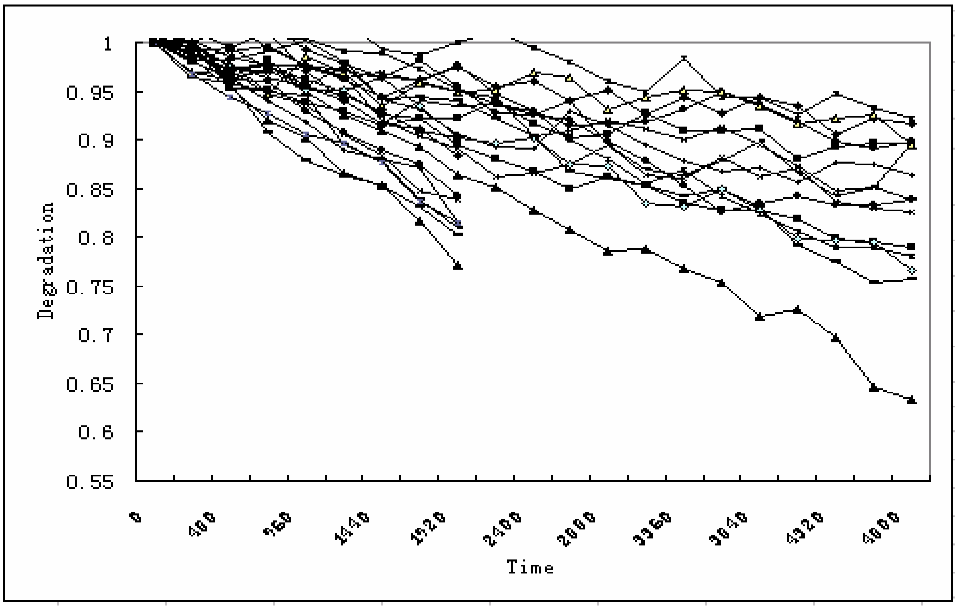

Figure 4.3 Realizations of the simulated CSADT plan

Table 4.3.1 Simulation of degradation paths in a CSADT experiment (X_1=0.3)

Time(hr)	Samples														
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
240	1.0174	0.9902	0.9697	0.9839	0.9999	1.0048	0.9963	1.0016	0.9822	1.0001	0.9843	0.9691	0.9899	1.0174	0.9877
480	1.0419	0.9953	0.9613	0.9904	1.0170	1.0191	0.9767	0.9863	0.9815	0.9750	0.9537	0.9701	0.9729	1.0290	0.9710
720	1.0192	0.9750	0.9723	0.9963	1.0120	0.9953	0.9633	0.9767	0.9917	0.9759	0.9506	0.9471	0.9699	1.0055	0.9771
960	0.9930	0.9618	0.9485	0.9758	1.0029	0.9761	0.9538	0.9551	1.0023	0.9510	0.9385	0.9850	0.9737	1.0030	0.9767
1200	0.9774	0.9490	0.9251	0.9647	1.0230	0.9620	0.9761	0.9403	0.9796	0.9506	0.9300	0.9685	0.9632	0.9909	0.9709
1440	0.9639	0.9257	0.9090	0.9198	0.9931	0.9442	0.9411	0.9165	0.9417	0.9640	0.9140	0.9351	0.9692	0.9894	0.9634
1680	0.9610	0.9226	0.8927	0.9037	0.9874	0.9598	0.9304	0.9395	0.9436	0.9347	0.9220	0.9590	0.9609	0.9722	0.9813
1920	0.9751	0.8942	0.8634	0.8907	1.0001	0.9769	0.9063	0.9342	0.9392	0.9016	0.9227	0.9483	0.9493	0.9516	0.9544
2160	0.9539	0.8804	0.8514	0.8626	1.0101	0.9478	0.8922	0.9360	0.9224	0.8959	0.9395	0.9517	0.9442	0.9363	0.9283
2400	0.9601	0.8677	0.8280	0.8656	0.9949	0.9291	0.8913	0.9258	0.9037	0.9012	0.9172	0.9690	0.9034	0.9287	0.9254
2640	0.9400	0.8499	0.8071	0.8740	0.9799	0.9152	0.9296	0.9406	0.8670	0.8755	0.9016	0.9631	0.9088	0.9001	0.9203
2880	0.9503	0.8613	0.7862	0.8939	0.9608	0.9166	0.9148	0.8942	0.8610	0.8729	0.9062	0.9316	0.9193	0.8800	0.8982
3120	0.9258	0.8521	0.7875	0.8637	0.9499	0.9189	0.8956	0.8702	0.8529	0.8353	0.9278	0.9437	0.9120	0.8552	0.8783
3360	0.9446	0.8340	0.7673	0.8609	0.9833	0.9303	0.8784	0.8636	0.8413	0.8310	0.9096	0.9507	0.9007	0.8690	0.8532
3600	0.9270	0.8280	0.7529	0.8824	0.9439	0.9499	0.8671	0.8786	0.8499	0.8492	0.9098	0.9490	0.9136	0.8421	0.8266
3840	0.9430	0.8273	0.7179	0.8628	0.9430	0.9365	0.8707	0.8986	0.8318	0.8280	0.9111	0.9345	0.8946	0.8249	0.8352
4080	0.9343	0.8194	0.7264	0.8720	0.9264	0.9178	0.8563	0.8664	0.7910	0.7982	0.8800	0.9174	0.8736	0.8050	0.8425
4320	0.9050	0.7994	0.6974	0.8361	0.9478	0.8980	0.8757	0.8428	0.7746	0.7970	0.8924	0.9227	0.8477	0.7893	0.8340
4560	0.9212	0.7955	0.6466	0.8303	0.9325	0.8915	0.8745	0.8504	0.7536	0.7952	0.8969	0.9257	0.8520	0.7903	0.8334
4800	0.9170	0.7903	0.6337	0.8262	0.9221	0.8985	0.8639	0.8383	0.7572	0.7653	0.8968	0.8949	0.8962	0.7799	0.8376

Table 4.3.2 Simulation of degradation paths in a CSADT experiment (X_2=1)

Time(hr)	Samples								
	1	2	3	4	5	6	7	8	9
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
240	0.9996	0.9959	0.9633	1.0126	0.9803	0.9939	0.9882	0.9668	0.9959
480	0.9643	0.9579	0.9591	0.9949	0.9641	0.9540	0.9796	0.9441	0.9622
720	0.9395	0.9487	0.9078	1.0111	0.9817	0.9206	0.9467	0.9282	0.9597
960	0.9189	0.9457	0.8779	0.9718	0.9555	0.9013	0.9370	0.9050	0.9301
1,200	0.8900	0.8947	0.8642	0.9407	0.9462	0.8663	0.9051	0.8958	0.9070
1,440	0.8806	0.8782	0.8529	0.9166	0.9301	0.8531	0.8847	0.8762	0.8899
1,680	0.8706	0.8389	0.8314	0.9107	0.9094	0.8163	0.8471	0.8360	0.8743
1,920	0.8147	0.8087	0.8015	0.8847	0.9025	0.7722	0.8390	0.8148	0.8427

4.5.2 Simulation Study of the Optimal SSADT Plan

In the previous section, the simulation of CSADT experiment shows the feasibility of our method in planning of CSADT plans. In the section, simulation and analysis of SSADT experiments will be conducted. The procedures are as follows:

Step1. With reference to Yu & Tseng (1999), suppose the degradation drift and dispersion at use condition are respectively $\eta_0 = a = 10^{-5}$ and $\sigma = 10^{-3}$. That is, the lifetime of the tested product at normal stress is $\mu_0 = Dc/\eta_0 = 5\times10^4\,hrs$ .Moreover, we set $b = 9\times10^{-5}$ and $D_c = 0.5$.

Step 2: According to the optimal SSADT plan in section 4.4, set $X_1 = 0.3$, c=5, p=0.9, n_1=n_2=7, L_1=10 and L_2=2. Generate degradation increments from (4.3) (4.4) and (4.12). Detailed values of the degradation paths and their realizations are shown in Table 4.4 and Figure 4.4.

Step 3: Using (4.13), (4.14) and (4.15), calculate the MLE of $(\hat{a}, \vec{b}, \hat{\sigma})$.

Here given initial values of $(10^{-5}, 9\times10^{-5}, 10^{-3})$, the MLE of $(\hat{a}, \vec{b}, \hat{\sigma})$ are estimated at $\hat{a} = 1.598\times10^{-5}$, $\hat{b} = 7.511\times10^{-5}$ and $\hat{\sigma} = 9.58\times10^{-4}$ by invoking the "Solver" function in Excel.

Step 4: Using (4.17), (4.20) and (4.21),

$$\hat{\mu}_0 = 31297.8hrs$$

and

$$\text{Avar}(\hat{\mu}(X_0)) = 40023043$$

Step 5: Substitute c=5, $\hat{\mu}_0$ and Avar$(\hat{\mu}(X_0))$ into (4.23), (4.24) and (4.25), compute the actual estimate precision level and compare it with desired estimate precision level p=0.9. The actual estimate precision level is:

$$\Pr\left(\frac{1}{5} \leq \frac{\hat{\mu}(X_0)}{\mu(X_0)} \leq 5\right) = 0.99 > p = 0.9 .$$

Again, the result indicates that our method is efficient to estimate the reliability interest as we have proposed.

Table 4.4 Simulation of degradation paths in a SSADT experiment (X_1=0.3 and X_2=1)

Stress	Time (hr)	Samples						
		1	2	3	4	5	6	7
X_1=0.3	0	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
	240	0.987882	1.012572	1.004518	0.9949	1.000499	0.980991	1.001686
	480	0.971093	1.026891	1.010027	0.996637	0.954895	0.990494	1.036393
	720	0.96772	1.017109	0.9958	0.98293	0.912616	0.95464	1.036887
	960	0.945132	1.003177	0.984459	0.995638	0.902479	0.955056	0.992599
	1200	0.933198	1.007135	0.972326	0.962309	0.900699	0.951109	0.969865
	1440	0.933777	1.017404	0.970086	0.94025	0.864381	0.907864	0.96033
	1680	0.943347	0.972359	0.933037	0.911272	0.862001	0.880161	0.959319
	1920	0.92904	0.983817	0.93277	0.928677	0.837779	0.870673	0.967596
	2160	0.910868	0.973345	0.920436	0.897862	0.853405	0.867113	0.955244
	2400	0.91514	0.957783	0.916177	0.898705	0.847295	0.866632	0.95061
X_2=1	2640	0.908173	0.930981	0.899866	0.883259	0.84164	0.826224	0.930932
	2880	0.870833	0.905679	0.870372	0.862475	0.826422	0.817435	0.892879

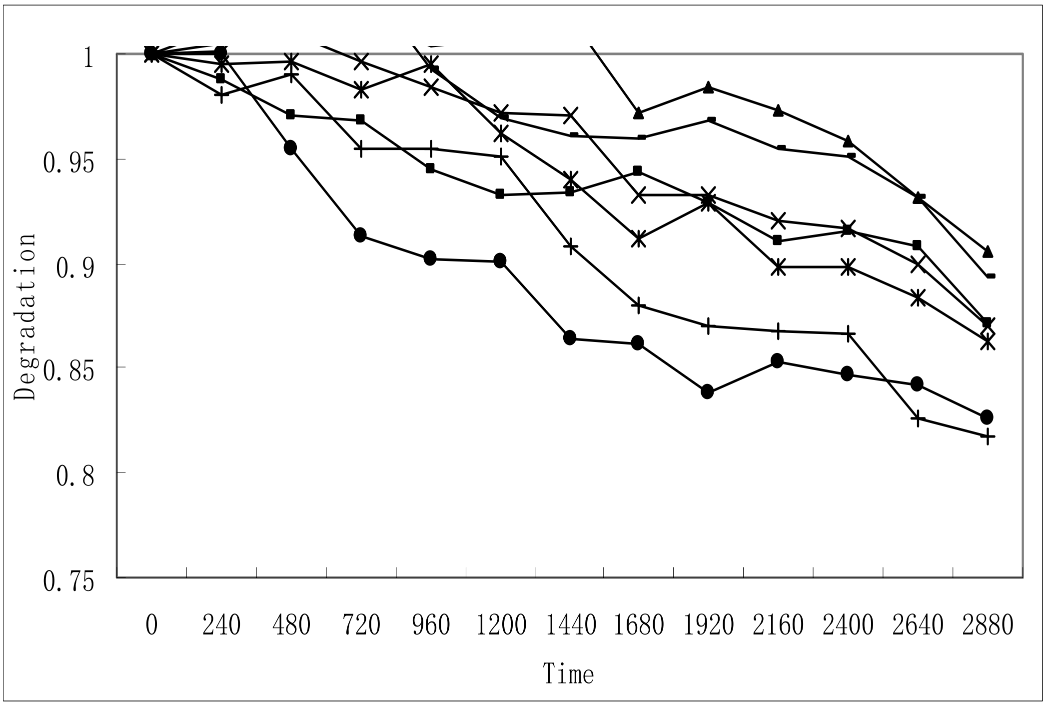

Figure 4.4 Realizations of the simulated SSADT plan

It is seen that the estimated mean lifetime in a SSADT is less accurate compared with that in a CSADT. It is explainable since SSADT saves quite a number of testing units and a plenty of time. Therefore, it inevitably captures less degradation information for reliability estimation.

4.5 CONCLUSIONS

We present to design the optimal ADT plans in a general formulation in this chapter. Different from all the existing methods, we adopt stochastic processes to model the degradation paths. If the lower stress and the inspection time interval are known, the optimal design is determined by minimizing the total testing cost under the condition that the probability the estimated mean lifetime at use stress within a range of its true value is not less than a pre-specified precision level. We combine SSADT and CSADT planning in a general formulation and further code it into a program for computation. In CSADT, we obtain the optimal sample size allocated at each stress, the stopping time (or number of inspections) at each stress. And in SSADT, we obtain the optimal sample size, the stress changing time and the stopping time of the whole test. We also provide a user-interactive dialog window for easy implementation.

Compared with the existing DT plan in Yu & Tseng (1999), some concluding remarks can be drawn as follows:

1. With a properly chosen lower stress, the proposed plans need less test samples. The test duration in our proposed ADT is much shorter, and with a well-set inspection interval, the test cost is also reduced.

2. The proposed planning gives more freedom for the experimenter to choose the optimal plan according to the setting of the lower stress and inspection time interval, so that they can conduct the test at the convenience of practical circumstance.

3. Compared with CSADT, SSADT saves time and cost significantly. Implementation of SSADT in real industry is highly recommended.

Simulation study integrating the experiment planning and data analysis are also provided. The results imply that our proposed method is efficient in planning both CSADT and SSADT experiments. However, because of its strength in saving testing time and testing cost, SSADT is inevitably less powerful to offer an accurate result as compared with CSADT.

Chapter 5

Optimal CSADT Plans

5.1 INTRODUCTION

The general formulation for planning of ADTs has been presented in chapter 4. In this chapter, we focus on optimal CSADT plans and further discuss their properties and some of our findings. CSADT have been widely studied in Boulanger & Escobar (1994), Park & Yum (1997), Yu & Tseng (1999), Wu & Chang (2002), Yang & Yang (2002) and Yu & Chiao (2002). Normally, an ideal optimal CSADT plan needs to specify the stress levels, sample size, inspection time interval, sample allocations, and testing time at each stress. However, in some experiments, due to the limitation of real test conditions, some variables are not adjustable and should be fixed before testing. For example, the temperature in a test oven may only be adjusted within a certain range or even be fixed at a particular value. Besides the survey in section 1.4.5, Table 5.1 is briefly a summary of these published papers on DT and CSADT plans.

It is seen that in Table 5.1, the later three papers are related to multiple stress ADTs. However, only Boulanger & Escobar (1994) considered multiple-inspection test and addressed number of measurements under each test condition. They determined the design variables with three steps. But the testing time or number of inspections at each stress is not genuinely optimized with ED and EL constraints. Based on the formulation in chapter 4, we provide two-stress CSADT plans with multiple inspections in this chapter. Compared with Boulanger & Escobar (1994), our approach is simpler since the decision variables can be obtained simultaneously.

Table 5.1 A summary of the existing DT and CSADT plans

	levels	X_k	n	π_k	Δt	L_k	Planning Criteria
Wu & Tseng, 2002	1	/	√	/	√	√	Minimizing Avar of LSE(), s.t. CC
Yu & Chiao, 2002	1	/	√	/	√	√	Minimizing Avar of LSE() s.t. PC
Yu & Tseng, 1999	1	/	√	/	√	√	Minimizing Avar of LSE()
Yang & Yang 2002	2	√	×	√	/	1	Minimizing Avar of MLE()
Park & Yum, 1997	3	√	√	√	/	1	Minimizing Avar of MLE()
Boulanger & Escobar, 1994	3	√	√	√	√	×	Minimizing Avar of WLSE() s.t. ED or EL, and PC

Remarks:
√ --- Optimized; /---not included; ×--- not addressed
Avar---asymptotic variance;
CC, PC---cost constraint or estimate precision constraint
ED, EL --- equalized degradation or equalized log-spaced degradation
LSE(), WLSE(), MLE()---LSE, Weighted LSE or MLE of the mean lifetime at use stress

We first give the optimal CSADT plans for various values of X_1 and Δt in section 5.2. We then analyze the influence of X_1 and Δt on the decision variables in an optimal plans. Because these two variables are supposed to be known before planning, the findings provide useful tips to determine proper values of them. Sensitivity study of optimal plans to mis-specified unknown parameters is involved in section 5.3. And finally, conclusions are drawn in section 5.4.

5.2 OPTIMAL TWO-STRESS CSADT PLANS

Continuing with examples in Section 4.4, we set the cost coefficients at C_d=86$/unit, C_{m1} = 4$/unit*time, C_{m2} = 4.5$/unit*time, C_{o1} = 0.3$/hr*time, and C_{o2} = 0.4$/hr*time, the precision parameter c=5 and p=0.9, the stochastic process parameters σ / a =100. The two-stress CSADT plans for Δt =48, 48*2, 48*3, 48*4, 48*5, 48*6 have been computed as shown in Table 5.2. Because the total cost of testing is extremely high (>30,000$) when X_1>0.7, we only show the results for X_1<0.7.

From Table 5.2, it suggests that:

1. The influence of Δt on the optimal number of sample size *n* (or number of samples allocated at each stress n_k) is not significant. For a particular X_1, this can be justified by investigating the ratio of sample deviation relative to the average number of samples resulted from different Δt s.

   Define

   $$SN_n(\Delta t) = \frac{n^*(\Delta t) - \bar{n}^*(\Delta t)}{\bar{n}^*(\Delta t)} \tag{5.1}$$

   where $n^*(\Delta t)$ is the optimal sample size when inspection time interval is set at Δt . $\bar{n}^*$ is the average value of $n^*(\Delta t)$ for different Δt s.

   $SN_n(\Delta t)$ has the same nature as the signal to noise ratio that represents how widely the needed samples would disperse when Δt varies. A small value of SN_n implies that the optimal sample size is stable regardless how Δt changes.

As shown in Table 5.3, $SN_n(\Delta t) < 0.05$ except for $X_1<0.1$. So that we can conclude that n is not sensitive to Δt.

Table 5.2 Optimal two-stress CSADT plans
(c=5, p=0.9)

X_1	Δt	n^*	n_1^*	n_2^*	$T_1^*(T^*)$	T_2^*	L_1^*	L_2^*	Cost
0.05	48	11	9	2	3360	528	70	11	4784.2
	96	12	10	2	3072	480	32	5	3470.6
	144	11	9	2	3456	432	24	3	3046.6
	192	11	9	2	3456	384	18	2	2802.4
	240	10	8	2	3840	480	16	2	2734
	288	10	9	1	3456	864	12	3	2687.9
0.1	48	14	11	3	3216	960	67	20	5770.8
	96	14	11	3	3264	864	34	9	4146.3
	144	14	11	3	3312	864	23	6	3636.2
	192	15	12	3	3072	768	16	4	3340.8
	240	12	10	2	3840	960	16	4	3244
	288	12	9	3	4032	864	14	3	3131.7
0.15	48	15	11	4	3840	1248	80	26	6929.2
	96	16	12	4	3552	1152	37	12	4894.4
	144	16	11	5	3888	864	27	6	4211
	192	15	11	4	4032	960	21	5	3897.6
	240	15	11	4	4080	960	17	4	3718
	288	15	10	5	4320	864	15	3	3599.1
0.2	48	18	12	6	4080	1440	85	30	8238
	96	17	12	5	4224	1536	44	16	5815.6
	144	17	12	5	4320	1440	30	10	4999
	192	18	12	6	4224	1344	22	7	4597.8
	240	18	13	5	3840	1680	16	7	4361.5
	288	19	13	6	4032	1152	14	4	4140.4
0.25	48	20	13	7	4560	1728	95	36	9853.2
	96	20	13	7	4512	1824	47	19	6845.7
	144	20	13	7	4464	1872	31	13	5829.5
	192	21	13	8	4608	1536	24	8	5338.8
	240	19	13	6	4560	2160	19	9	5097
	288	20	14	6	4608	1728	16	6	4851.6
0.3	48	23	14	9	4992	2064	104	43	11866.7
	96	23	15	8	4704	2304	49	24	8114.8
	144	23	15	8	4896	2016	34	14	6797.2
	192	24	16	8	4608	2112	24	11	6223.2
	240	23	15	8	4800	2160	20	9	5806
	288	23	15	8	4896	2016	17	7	5525.2
0.35	48	27	17	10	4992	2592	104	54	14358.4
	96	27	17	10	4992	2592	52	27	9607.4
	144	28	18	10	4752	2592	33	18	8056.4
	192	27	17	10	4992	2688	26	14	7292.8
	240	27	17	10	4800	2880	20	12	6814
	288	29	18	11	4608	2592	16	9	6510.7

To be continued

Table 5.2 Optimal two-stress CSADT plans
c=5, p=0.9
(Continued)

X_1	Δt	n^*	n_1^*	n_2^*	$T_1^*(T^*)$	T_2^*	L_1^*	L_2^*	Cost
0.4	48	33	21	12	4896	3072	102	64	17559.6
	96	32	20	12	4992	3264	52	34	11551.2
	144	33	21	12	4896	3024	34	21	9506.4
	192	31	20	11	4992	3648	26	19	8643.3
	240	35	23	12	4560	2880	19	12	7926
	288	35	21	14	4896	2592	17	9	7510.6
0.45	48	40	26	14	4944	3600	103	75	21800.2
	96	41	26	15	4896	3456	51	36	14111.2
	144	42	26	16	4896	3168	34	22	11468
	192	40	25	15	4992	3648	26	19	10279.3
	240	40	26	14	4800	3840	20	16	9504
	288	42	26	16	4896	3168	17	11	8908
0.5	48	50	32	18	4992	3984	104	83	27426.2
	96	48	31	17	4992	4512	52	47	17473.9
	144	49	32	17	4896	4464	34	31	14191.9
	192	50	32	18	4992	4032	26	21	12439.4
	240	49	32	17	4800	4560	20	19	11491.5
	288	49	32	17	4896	4320	17	15	10734.3
0.55	48	61	39	22	4992	4992	104	104	35260.4
	96	61	39	22	4992	4992	52	52	22000.4
	144	64	41	23	4896	4608	34	32	17704
	192	61	39	22	4992	4992	26	26	15370.4
	240	64	43	21	4800	4800	20	20	14194
	288	65	42	23	4896	4320	17	15	13195.3
0.6	48	83	52	31	4992	4896	104	102	46455
	96	83	52	31	4992	4896	52	51	28524.5
	144	85	56	29	4896	4896	34	34	22790.2
	192	83	54	29	4992	4992	26	26	19641.4
	240	86	55	31	4800	4800	20	20	17946
	288	86	56	30	4896	4608	17	16	16676
0.65	48	115	72	43	4992	4944	104	103	63247.7
	96	115	74	41	4992	4992	52	52	38370.4
	144	117	71	46	4896	4896	34	34	30183.2
	192	115	74	41	4992	4992	26	26	25877.4
	240	119	74	45	4800	4800	20	20	23564
	288	119	75	44	4896	4608	17	16	21814
0.7	48	166	103	63	4992	4992	104	104	90102.4
	96	166	103	63	4992	4992	52	52	53936.4
	144	170	105	65	4896	4896	34	34	42272.2
	192	166	103	63	4992	4992	26	26	35853.4
	240	172	104	68	4800	4800	20	20	32592
	288	172	105	67	4896	4608	17	16	30068

2. Δt has less effect on the optimal stopping time T_k. Taking T=T$_1$ as an example, similarly we define

$$SN_T(\Delta t) = \frac{T^*(\Delta t) - \overline{T}^*(\Delta t)}{\overline{T}^*(\Delta t)} \tag{5.2}$$

where $T^*(\Delta t)$ is the optimal testing time when inspection time interval is set at Δt. $\overline{T}^*$ is the average value of $T^*(\Delta t)$ for different Δt s.

It is seen that $SN_T(\Delta t) < 0.05$ for $X_1 > 0.15$ from Table 5.3. So that the influence of Δt on the optimal stopping time T is not obvious. Actually, for a given X_1, the optimal testing time for different Δt are almost the same. But the plan with larger Δt, i.e. longer inspection time interval, costs less. In practice, we suggest the experimenters select a longer measurement interval based on the practical test circumstance.

Table 5.3 Influence of Δt on n and T in optimal two-stress CSADT plans

X_1	$SN_n(\Delta t)$	$SN_T(\Delta t)$
0.05	0.069487	0.071513
0.1	0.090722	0.111458
0.15	0.033678	0.065482
0.2	0.042212	0.041953
0.25	0.031623	0.012327
0.3	0.017622	0.029341
0.35	0.030424	0.033326
0.4	0.048304	0.032824
0.45	0.024078	0.01301
0.5	0.015311	0.015906
0.55	0.029711	0.015906
0.6	0.017852	0.015906
0.65	0.016855	0.015906
0.7	0.053755	0.015906

3. However, both the optimal n and T are sensitive to X_1. Hence, experimenters need to pay more attention on selection of X_1.

Actually, X_1 can be determined by considering a cost budget. For example, according to Yu & Tseng (1999), their cost budget is 7500$ and their actual testing cost is 6175$. Let us set Δt =240hrs, then to meet the cost budget at 7500$, we should choose plans with $X_1<0.4$; and to meet the cost budget at 6175, we should choose plans with $X_1<0.3$.

There is one thing to point out. The testing time in our plan is always shorter than that of Yu & Tseng's. It means that with a properly chosen X_1, our test plan not only saves testing expense but also saves testing time.

5.3 SENSITIVITY ANALYSIS

It is noted that the optimal plans are obtained based on guessed values of unknown parameters in the degradation path models. However, due to intrinsic difference of the materials, and the inevitable non-homogeneity of the experimental circumstances, the above assumptions may be violated. Thus, a simulation study is needed to investigate the robustness of the proposed method with respect to those parameters. Here in our proposed planning strategy, σ/a is the only parameter that needs to be pre-estimated. Suppose the true value of σ/a is 100, in the following subsection, we analyze the sensitivity of optimal n, π_1, T_1, T_2 to guessed values of σ/a at 80, 90, 110 and 120 respectively. The corresponding optimal plans are attached in Appendix C.

Denote *Dev*. be the deviation (in percentage) of σ/a from its true value.

Let $n^*, \pi_1^*, T_1^*, T_2^*$ be optimal n, π_1, T_1, T_2 with correct value of σ/a;

$n^0, \pi_1^0, T_1^0, T_2^0$ be optimal n, π_1, T_1, T_2 with incorrect value of σ/a;

Rn= $100 \cdot \left|\left(n^* - n^0\right)/n^*\right|$ be the relative ratio of increase of n in percentage;

$R\pi_1$, RT_1, RT_2, similar to Rn, be the relative ratio of increase of π_1, T_1 and T_2 in percentage.

Figures 5.4 to 5.7 are sensitivity analysis of n, π_1, T_1 and T_2 to the mis-specified σ/a for p=0.9 and c from 2 to 5.

Table 5. 4 Sensitivity of Rn to σ/a in two-stress CSADT plans

X_1	Dev	c					Dev	C			
	σ/a	2	3	4	5	X_1	σ/a	2	3	4	5
0.05	-20	25.00	21.43	16.67	20.00	0.4	-20	35.90	31.82	27.03	28.57
	-10	20.00	14.29	8.33	10.00		-10	19.23	13.64	18.92	22.86
	10	25.00	0.00	0.00	20.00		10	20.51	15.91	16.22	5.71
	20	40.00	21.43	25.00	40.00		20	43.59	38.64	37.84	25.71
0.1	-20	28.00	13.33	20.00	8.33	0.45	-20	36.63	31.48	23.26	27.50
	-10	16.00	6.67	13.33	0.00		-10	18.81	16.67	13.95	15.00
	10	20.00	13.33	6.67	33.33		10	18.81	22.22	20.93	12.50
	20	36.00	33.33	6.67	25.00		20	41.58	44.44	41.86	35.00
0.15	-20	33.33	33.33	25.00	20.00	0.5	-20	35.66	34.29	30.91	28.57
	-10	13.33	19.05	12.50	0.00		-10	19.38	18.57	16.36	20.41
	10	20.00	0.00	6.25	6.67		10	20.93	20.00	21.82	20.41
	20	40.00	19.05	25.00	40.00		20	43.41	42.86	43.64	42.86
0.2	-20	32.43	19.05	22.22	27.78	0.55	-20	35.88	35.87	36.00	35.94
	-10	18.92	14.29	5.56	11.11		-10	19.41	16.30	21.33	18.75
	10	13.51	23.81	22.22	5.56		10	14.12	20.65	18.67	20.31
	20	35.14	33.33	33.33	22.22		20	8.82	43.48	38.67	43.75
0.25	-20	30.23	26.92	18.18	10.53	0.6	-20	0.00	36.29	33.67	36.05
	-10	16.28	19.23	13.64	0.00		-10		19.35	19.39	18.60
	10	18.60	15.38	13.64	21.05		10		20.97	20.41	20.93
	20	37.21	30.77	27.27	36.84		20		43.55	43.88	44.19
0.3	-20	30.77	31.25	29.63	21.74	0.65	-20		0.00	36.03	34.45
	-10	19.23	18.75	18.52	4.35		-10			19.12	18.49
	10	17.31	12.50	7.41	17.39		10			20.59	21.01
	20	36.54	31.25	22.22	39.13		20			43.38	44.54
0.35	-20	32.26	27.78	31.03	25.93						
	-10	16.13	11.11	10.34	14.81						
	10	20.97	16.67	24.14	14.81						
	20	43.55	38.89	41.38	33.33						

Table 5. 5 Sensitivity of $R\pi_1$ to σ/a in two-stress CSADT plans

X_1	Dev σ/a	c				X_1	Dev σ/a	C			
		2	3	4	5			2	3	4	5
0.05	-20	1.96	4.55	8.00	9.38	0.4	-20	1.26	2.22	2.93	14.78
	-10	2.94	2.78	1.82	13.64		-10	2.26	3.51	2.78	1.45
	10	3.53	0.00	0.00	4.17		10	2.46	6.41	0.39	6.93
	20	0.84	3.92	4.00	7.14		20	0.19	8.20	5.80	3.75
0.1	-20	2.78	5.77	6.25	1.82	0.45	-20	2.24	11.66	10.14	4.51
	-10	4.76	1.79	5.77	0.00		-10	0.62	2.11	7.83	4.98
	10	4.17	2.94	1.56	2.50		10	0.42	0.96	0.20	5.98
	20	6.62	0.00	1.56	4.00		20	0.47	2.02	2.09	8.26
0.15	-20	2.17	6.25	0.00	2.27	0.5	-20	0.16	6.83	10.03	8.13
	-10	0.33	0.37	4.76	0.00		-10	2.15	0.25	3.06	2.08
	10	1.45	0.00	1.96	2.27		10	0.77	4.76	4.28	6.41
	20	2.48	0.25	0.00	3.90		20	2.38	2.86	0.27	7.19
0.2	-20	1.33	1.18	1.10	4.14	0.55	-20	0.36	2.25	0.27	1.99
	-10	0.49	1.11	2.26	4.81		-10	3.97	4.02	8.19	0.18
	10	7.67	2.31	5.59	5.26		10	6.85	0.55	2.20	3.35
	20	6.89	10.00	1.92	0.70		20	5.71	0.55	4.34	0.30
0.25	-20	5.94	8.77	2.22	5.43	0.6	-20		3.11	5.77	0.50
	-10	2.95	3.17	7.37	7.69		-10		2.02	3.09	2.75
	10	2.76	1.11	0.27	1.67		10		0.02	1.38	2.24
	20	4.77	6.21	4.76	1.18		20		1.10	0.09	3.40
0.3	-20	4.97	7.44	2.63	2.22	0.65	-20			2.97	1.04
	-10	1.00	4.90	4.55	4.55		-10			0.52	1.13
	10	3.19	1.01	3.45	2.22		10			2.78	0.61
	20	6.00	3.90	9.09	5.42		20			3.00	1.91
0.35	-20	8.92	5.85	5.75	4.71						
	-10	6.14	1.00	5.19	3.58						
	10	0.26	2.86	3.33	7.59						
	20	0.69	3.68	0.98	10.29						

Table 5.6 Sensitivity of RT to σ/a in two-stress CSADT plans

X_1	Dev σ/a	c					Dev	C			
		2	3	4	5	X_1	σ/a	2	3	4	5
0.05	-20	15.00	18.75	26.67	25.00	0.4	-20	0.00	0.00	5.00	0.00
	-10	0.00	6.25	13.33	31.25		-10	0.00	0.00	0.00	5.26
	10	5.00	18.75	20.00	0.00		10	0.00	0.00	0.00	5.26
	20	0.00	18.75	13.33	0.00		20	0.00	0.00	0.00	5.26
0.1	-20	5.00	31.58	20.00	31.25	0.45	-20	0.00	0.00	5.00	5.00
	-10	0.00	10.53	13.33	18.75		-10	0.00	0.00	0.00	0.00
	10	0.00	0.00	6.67	6.25		10	0.00	0.00	0.00	0.00
	20	0.00	5.26	26.67	18.75		20	0.00	0.00	0.00	0.00
0.15	-20	0.00	6.25	16.67	17.65	0.5	-20	0.00	0.00	0.00	0.00
	-10	0.00	0.00	5.56	23.53		-10	0.00	0.00	0.00	0.00
	10	0.00	25.00	5.56	11.76		10	0.00	0.00	0.00	0.00
	20	0.00	25.00	11.11	5.88		20	0.00	0.00	0.00	0.00
0.2	-20	0.00	15.00	21.05	6.25	0.55	-20	0.00	0.00	0.00	0.00
	-10	0.00	5.00	10.53	0.00		-10	0.00	0.00	0.00	0.00
	10	0.00	5.00	0.00	25.00		10	0.00	0.00	0.00	0.00
	20	0.00	0.00	5.26	18.75		20	0.00	0.00	0.00	0.00
0.25	-20	0.00	5.00	25.00	26.32	0.6	-20		0.00	0.00	0.00
	-10	0.00	0.00	5.00	15.79		-10		0.00	0.00	0.00
	10	0.00	0.00	0.00	0.00		10		0.00	0.00	0.00
	20	0.00	0.00	0.00	5.26		20		0.00	0.00	0.00
0.3	-20	0.00	0.00	10.00	20.00	0.65	-20			0.00	0.00
	-10	0.00	0.00	0.00	20.00		-10			0.00	0.00
	10	0.00	0.00	0.00	0.00		10			0.00	0.00
	20	0.00	0.00	0.00	0.00		20			0.00	0.00
0.35	-20	0.00	5.00	0.00	5.00						
	-10	0.00	5.00	5.00	0.00						
	10	0.00	0.00	0.00	0.00						
	20	0.00	0.00	0.00	0.00						

Table 5.7 Sensitivity of RT_2 to σ/a in two-stress CSADT plans

X_1	Dev σ/a	c 2	3	4	5	X_1	Dev σ/a	C 2	3	4	5
0.05	-20	25.00	0.00	0.00	0.00	0.4	-20	0.00	17.65	28.57	16.67
	-10	0.00	0.00	0.00	0.00		-10	0.00	17.65	0.00	8.33
	10	0.00	50.00	0.00	0.00		10	0.00	17.65	14.29	41.67
	20	25.00	50.00	0.00	0.00		20	0.00	17.65	14.29	41.67
0.1	-20	33.33	0.00	0.00	25.00	0.45	-20	5.26	20.00	36.84	25.00
	-10	16.67	25.00	33.33	25.00		-10	0.00	10.00	15.79	12.50
	10	0.00	50.00	66.67	25.00		10	5.26	5.00	0.00	25.00
	20	33.33	25.00	100.0	0.00		20	5.26	0.00	5.26	25.00
0.15	-20	22.22	33.33	0.00	25.00	0.5	-20	0.00	10.00	20.00	26.32
	-10	33.33	0.00	0.00	0.00		-10	0.00	0.00	10.00	5.26
	10	0.00	0.00	75.00	25.00		10	0.00	0.00	0.00	5.26
	20	11.11	0.00	50.00	50.00		20	0.00	0.00	0.00	5.26
0.2	-20	20.00	37.50	14.29	28.57	0.55	-20	0.00	0.00	0.00	0.00
	-10	0.00	0.00	28.57	28.57		-10	0.00	10.00	11.11	0.00
	10	30.00	12.50	0.00	14.29		10	0.00	0.00	5.56	0.00
	20	30.00	37.50	14.29	14.29		20	0.00	0.00	11.11	0.00
0.25	-20	28.57	22.22	14.29	33.33	0.6	-20		0.00	10.00	0.00
	-10	14.29	0.00	0.00	22.22		-10		0.00	0.00	0.00
	10	7.14	22.22	28.57	0.00		10		0.00	0.00	0.00
	20	21.43	44.44	57.14	0.00		20		0.00	0.00	0.00
0.3	-20	25.00	20.00	0.00	11.11	0.65	-20			0.00	5.00
	-10	0.00	0.00	0.00	0.00		-10			0.00	0.00
	10	12.50	30.00	50.00	11.11		10			0.00	0.00
	20	25.00	40.00	75.00	11.11		20			0.00	0.00
0.35	-20	20.00	26.67	21.43	25.00						
	-10	15.00	20.00	21.43	16.67						
	10	0.00	13.33	7.14	16.67						
	20	0.00	13.33	7.14	25.00						

From Table 5.4, it is seen that the optimal n is sensitive to specification of σ/a, which can also be proved from Figure 5.1. Because deviation of n is normally larger than that of σ/a, the influence of σ/a on Rn is obvious. Compared with overestimating, underestimating of σ/a would have less effect on Rn. Thus experimenters should have accurate information of the underlying degradation process before they conduct test

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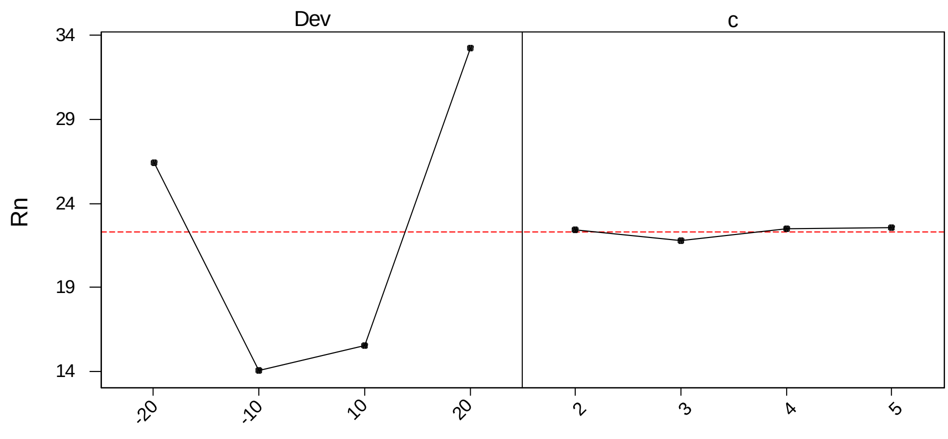

Figure 5.1 Main effect plot of sensitivity of n to mis-specified σ/a in two-stress CSADT plans

From Table 5.6, the stopping time at X_2 is moderately sensitive to the underestimated σ/a. And from Table 5.4 and 5.5, the other variables are not sensitive to mis-specified σ/a.

5.4 CONCLUSIONS

The optimal CSADT plans have been studied in this chapter. Even the inspection time interval and the lower test stress are assumed to be known in our planning, we analyzed the effect of these two variables on the optimal plans. For different settings of inspection time interval, the simulated plans are almost the same. Hence, experimenters can set the inspection time interval freely at their convenience. However, they should bear in mind that the lower stress level affects the optimal plans distinctly. Generally, a plan with a small X_1 saves cost. Thus in order to determine a proper X_1, a cost budget should be taken into account.

Sensitivity study of optimal plans due to misspecifications of σ/a has also been carried out in this chapter. It is concluded that optimal sample size is sensitive to changes of σ/a. The stopping time is sensitive to the underestimated σ/a. While other variables are not sensitive to σ/a. The result suggests experimenters have a thorough understanding of the tested product before they start to design any test plan. If they are not confident of the accuracy of σ/a, a larger value would be helpful to obtain a better plan that is close to the truly optimal one.

Chapter 6

Optimal SSADT Plans

6.1 INTRODUCTION

For SSADT, an optimal plan specifies the sample size, stress levels, the time interval between two continuous inspections, holding time or number of inspections at each stress. Despite of the superiority of a SSADT to a CSADT as we have discussed in section 1.1 and section 3.1, there is no literature addressing the problem of planning SSADT yet. Based on our proposed formulation in chapter 4, we would like to further present the two-stress and three-stress SSADT plans in this chapter.

We first analyze the two-stress plans in section 6.2. After discussing the optimal plans, we give the guidance on how to determine values of the lower test stress, the inspection time interval and the precision parameters. To investigate the influence of mis-specified pre-estimated parameters on the decision variables, sensitivity study is conducted. Sequentially, we present to plan three-stress plans with additional constraints in section 6.3. Three options are illustrated with examples. Finally, concluding remarks are given in section 6.4

6.2 OPTIMAL TWO-STRESS SSADT PLANS

To compare with the plan in Tseng & Yu (1999), again we set the cost coefficients C_d=86\$/unit, C_{m1} = 4\$/unit*time, C_{m2} = 4.5\$/unit*time, C_{o1} = 0.3\$/hr*time, and C_{o2} = 0.4\$/hr*time, the precision parameter c=5 and p=0.9, the stochastic process parameters σ / a =100. We have the optimal two-stress SSADT plans for Δt =48, 48*2, 48*3, 48*4, 48*5, 48*6 in Table 6.1.

From Table 6.1, it is seen that the optimal sample size n^* and the stopping time T^* are not sensitive to the lower stress X_1 and the inspection time interval Δt. As shown in Figure 6.1 and 6.2, the optimal n is between 6 and 10. Compared with the optimal sample size 30 in Yu & Tseng's plan, our value is smaller, which suggests our SSADT plan is superior in cases where the product is newly developed such that number of available samples is limited or the price of a single item is expensive. The optimal stopping time T is around 2888hrs in our plan, which is shorter compared with Yu & Tseng's plan, i.e. 5040hrs. Our shorter testing time implies that experimenters can save nearly half of the time to get the reliability information. As a result, the test expense is greatly reduced from 6000$+ to 3300$-.

Table 6.1 Optimal SSADT plans
(c=5, p=0.9)

Δt (hrs)	X_1	n^*	T^*	π_1^*	L^*	L_1^*	L_2^*	Cost
48	0.05	7	2016	0.952381	42	40	2	2399.4
	0.1	7	2208	0.934783	46	43	3	2577.3
	0.15	7	2400	0.92	50	46	4	2755.2
	0.2	7	2544	0.849057	53	45	8	2915.6
	0.25	7	2736	0.877193	57	50	7	3076.9
	0.3	8	2496	0.807692	52	42	10	3188.8
	0.35	8	2640	0.836364	55	46	9	3319.2
	0.4	8	2736	0.807018	57	46	11	3429.6
	0.45	8	2832	0.79661	59	47	12	3531.2
	0.5	8	2928	0.819672	61	50	11	3615.2
	0.55	9	2688	0.857143	56	48	8	3670.8
	0.6	9	2736	0.859649	57	49	8	3721.2
	0.65	9	2832	0.966102	59	57	2	3766.2
	0.7	9	2832	0.949153	59	56	3	3775.5
	0.75	8	3216	0.985075	67	66	1	3805.6
	0.8	8	3216	0.985075	67	66	1	3805.6
	0.85	8	3216	0.985075	67	66	1	3805.6
	0.9	8	3216	0.985075	67	66	1	3805.6
	0.95	8	3216	0.985075	67	66	1	3805.6
96	0.05	7	2016	0.952381	21	20	1	1807.9
	0.1	7	2208	0.913043	23	21	2	1934.6
	0.15	7	2400	0.92	25	23	2	2048.2
	0.2	7	2592	0.888889	27	24	3	2174.9
	0.25	8	2400	0.88	25	22	3	2248.8
	0.3	8	2496	0.807692	26	21	5	2336.8
	0.35	8	2688	0.857143	28	24	4	2444.8
	0.4	8	2784	0.862069	29	25	4	2505.6
	0.45	9	2496	0.730769	26	19	7	2557.5
	0.5	9	2592	0.777778	27	21	6	2608.2
	0.55	9	2688	0.857143	28	24	4	2644.8
	0.6	9	2784	0.931034	29	27	2	2681.4
	0.65	8	3168	0.939394	33	31	2	2721.6
	0.7	8	3168	0.909091	33	30	3	2735.2
	0.75	9	2880	0.966667	30	29	1	2732.1
	0.8	9	2880	0.966667	30	29	1	2732.1
	0.85	9	2880	0.966667	30	29	1	2732.1
	0.9	9	2880	0.966667	30	29	1	2732.1
	0.95	9	2880	0.966667	30	29	1	2732.1

To be continued

Table 6.1 Optimal SSADT plans c=5, p=0.9 (continued)								
Δt (hrs)	X_1	n^*	T^*	π_1^*	L^*	L_1^*	L_2^*	Cost
144	0.05	6	2448	0.941176	17	16	1	1675.8
	0.1	6	2592	0.944444	18	17	1	1743.0
	0.15	7	2448	0.941176	17	16	1	1830.3
	0.2	7	2592	0.888889	18	16	2	1919.4
	0.25	7	2736	0.842105	19	16	3	2008.5
	0.3	7	2880	0.85	20	17	3	2079.7
	0.35	7	3024	0.809524	21	17	4	2168.8
	0.4	8	2736	0.789474	19	15	4	2190.4
	0.45	8	2880	0.85	20	17	3	2247.2
	0.5	9	2592	0.777778	18	14	4	2275.2
	0.55	8	3024	0.857143	21	18	3	2322.4
	0.6	9	2736	0.842105	19	16	3	2335.5
	0.65	8	3168	0.909091	22	20	2	2379.2
	0.7	8	3168	0.909091	22	20	2	2379.2
	0.75	9	2880	0.95	20	19	1	2376.9
	0.8	9	2880	0.95	20	19	1	2376.9
	0.85	9	2880	0.95	20	19	1	2376.9
	0.9	9	2880	0.95	20	19	1	2376.9
	0.95	9	2880	0.95	20	19	1	2376.9
192	0.05	7	2112	0.909091	11	10	1	1566.3
	0.1	7	2304	0.916667	12	11	1	1651.9
	0.15	8	2112	0.909091	11	10	1	1696.8
	0.2	8	2304	0.916667	12	11	1	1786.4
	0.25	9	2112	0.818182	11	9	2	1851.0
	0.3	8	2496	0.769231	13	10	3	1922.4
	0.35	8	2688	0.857143	14	12	2	1988.8
	0.4	9	2496	0.846154	13	11	2	2038.2
	0.45	9	2496	0.692308	13	9	4	2085.6
	0.5	8	3072	0.9375	16	15	1	2144.8
	0.55	9	2688	0.857143	14	12	2	2131.8
	0.6	8	3072	0.8125	16	13	3	2191.2
	0.65	10	2496	0.846154	13	11	2	2177.2
	0.7	9	2880	0.933333	15	14	1	2201.7
	0.75	9	2880	0.933333	15	14	1	2201.7
	0.8	9	2880	0.933333	15	14	1	2201.7
	0.85	9	2880	0.933333	15	14	1	2201.7
	0.9	9	2880	0.933333	15	14	1	2201.7
	0.95	9	2880	0.933333	15	14	1	2201.7
To be continued								

Table 6.1 Optimal SSADT plans
c=5, *p*=0.9 (continued)

Δt (hrs)	X_1	n^*	T^*	π_1^*	L^*	L_1^*	L_2^*	Cost
240	0.05	6	2400	0.9	10	9	1	1503.0
	0.1	6	2640	0.909091	11	10	1	1599.0
	0.15	7	2400	0.9	10	9	1	1629.5
	0.2	7	2640	0.909091	11	10	1	1729.5
	0.25	8	2400	0.8	10	8	2	1784.0
	0.3	7	2880	0.833333	12	10	2	1857.0
	0.35	8	2640	0.818182	11	9	2	1888.0
	0.4	7	3120	0.769231	13	10	3	1984.5
	0.45	8	2880	0.833333	12	10	2	1992.0
	0.5	9	2640	0.818182	11	9	2	2019.0
	0.55	9	2640	0.727273	11	8	3	2047.5
	0.6	8	3120	0.846154	13	11	2	2096.0
	0.65	8	3120	0.846154	13	11	2	2096.0
	0.7	9	2880	0.916667	12	11	1	2098.5
	0.75	9	2880	0.916667	12	11	1	2098.5
	0.8	9	2880	0.916667	12	11	1	2098.5
	0.85	9	2880	0.916667	12	11	1	2098.5
	0.9	9	2880	0.916667	12	11	1	2098.5
	0.95	9	2880	0.916667	12	11	1	2098.5
288	0.05	5	2880	0.9	10	9	1	1525.3
	0.1	6	2592	0.888889	9	8	1	1541.4
	0.15	6	2880	0.9	10	9	1	1651.8
	0.2	7	2592	0.888889	9	8	1	1663.9
	0.25	6	3168	0.818182	11	9	2	1794.0
	0.3	8	2592	0.888889	9	8	1	1786.4
	0.35	7	3168	0.909091	11	10	1	1892.7
	0.4	7	3168	0.818182	11	9	2	1925.0
	0.45	9	2592	0.888889	9	8	1	1908.9
	0.5	9	2592	0.777778	9	7	2	1942.2
	0.55	7	3456	0.833333	12	10	2	2039.4
	0.6	8	3168	0.909091	11	10	1	2023.2
	0.65	8	3168	0.909091	11	10	1	2023.2
	0.7	8	3168	0.909091	11	10	1	2023.2
	0.75	9	2880	0.9	10	9	1	2031.3
	0.8	9	2880	0.9	10	9	1	2031.3
	0.85	9	2880	0.9	10	9	1	2031.3
	0.9	9	2880	0.9	10	9	1	2031.3
	0.95	9	2880	0.9	10	9	1	2031.3

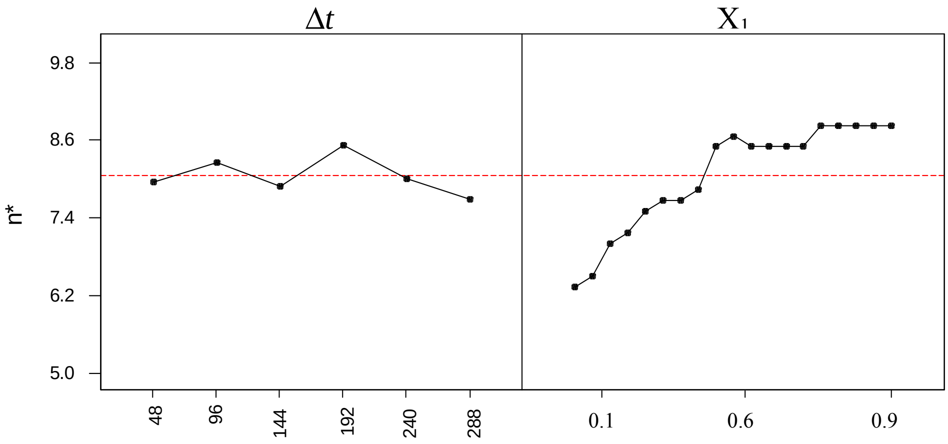

Figure 6.1 Main effect plot of optimal sample size n in SSADT planning

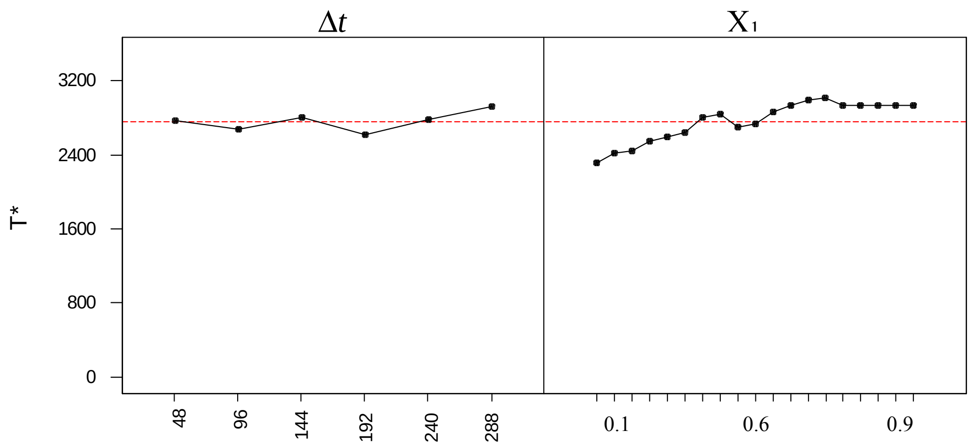

Figure 6.2 Main effect plot of optimal stopping time T in SSADT planning

6.2.1 Determination of the Lower Stress X_1 and the Inspection Time Interval Δt

There are also some findings by investigating the plot of L_2/L_1 Vs X_1. These findings are especially useful to determine the optimal lower stress X_1 and inspection time interval Δt.

In Figure 6.3, it is seen that for a given c and p, the polynomial trend of L_2/L_1 for various X_1 shows contour curves. L_2/L_1 always reaches its maximum value at around $X_1 \in (0.4, 0.5)$. Also, the plots show contour patterns for various Δt s. L_2/L_1 increases when Δt increases from 48hrs to 240hrs and then deceases when Δt is greater than 240hrs. That is, L_2/L_1 is maximized at Δt =240hrs, which is also the optimal inspection time interval in Yu & Tseng' plan. We suggest the optimal Δt and the optimal X_1 be determined by referring to the highest curve in the countour, and the apex of this curve. Mathmatically, X_1 can be caculated by setting the first order difference of the polynomial function equal to 0. For example, in Figure 6.3, it is read the optimal Δt =240hrs and the optimal X_1=0.45.

Kielpinski & Nelson (1975) suggested the tester run more experiments at a lower stress rather than at a higher stress such that less extrapolation is made and the estimate would be more accurate. However, the consequence is that the test time becomes longer. The reason here we set our optimization criterion to achieve the highest value of L_2/L_1, i.e. T_2/T_1 is that we aim to assign more proportion of time to the higher stress, with which we can shorten the testing time but still grant to meet the estimate precision constraint.

Based on the above discussion, the optimal 2-stress CSADT plan is summarized in Table 6.2.

Table 6.2 Optimal two-stress SSADT plan1

X_1	Δt (hrs)	n	T(hrs)	T_1(hrs)	T_2(hrs)	L	L_1	L_2	Cost ($)
0.45	240	7	3120	2400	720	13	10	3	1789.5

However, if the operator prefers more inspections at the high stress level, i.e. X_2, s/he can choose Δt =48, i.e.

Table 6.3 Optimal two-stress SSADT plan 2

	Δt (hrs)	n	T(hrs)	T_1(hrs)	T_2(hrs)	L	L_1	L_2	Cost ($)
0.45	48	8	2832	2256	576	59	47	12	3248.2

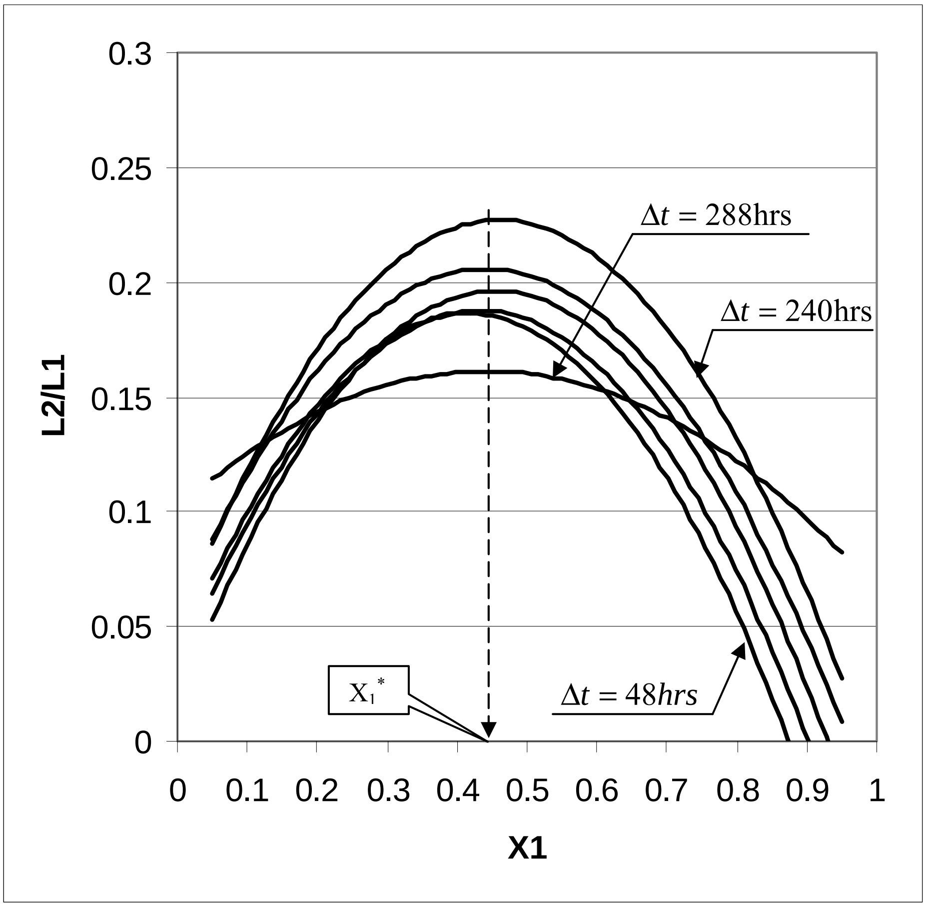

Figure 6.3. Plot of L_2/L_1 Vs X_1 in two-stress SSADT plans

6.2.2 Determination of the Precision Parameters c and p

The contour pattern does not exist for all values of {c, p}. We have plotted L_2/L_1 vs X_1 for c∈ (1, 5) and p∈ (1, 5). The results showed that the contour pattern mentioned in Figure 6.3 would only happen when p is greater than a particular value for a specified c. There is no obvious contour display for some values of c coped with a relatively smaller p. That is, the estimate precision constraint looses its function when c and p are set improperly. For a fixed c, p should be greater than a critical value such that the above-mentioned contour curves exist for different Δt s**.** The boundaries of p is plotted in Figure 6.4. c and p should be chosen above the bold line. A small value of c and a relatively larger p are recommended if the operator wants a tighten precision constraint, vice versa.

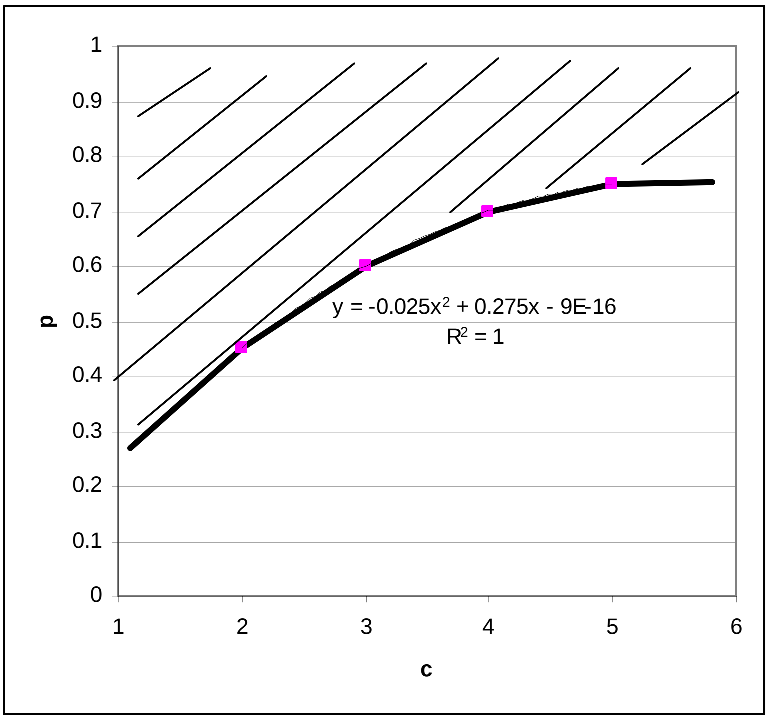

Figure 6.4 Boundaries of {c, p}, the precision constraint in SSADT

We next investigate the influence of (c, p) on the optimal X_1 and Δt. For various (c, p)'s, the optimal X_1 and Δt are summarzed in Table 6.4 and 6.5.

Define $RX_1|_{c\,or\,p} = \frac{\left|X_1^* - \overline{X}_1^* \mid \text{given c or p}\right|}{\overline{X}_1^* \mid \text{given c or p}}$. Shown in Table 6.4 and 6.5, with a fixed c or p, $RX_1|_{c\text{ or }p}$ is normally no greater than 10%. It indicates that optimal X_1 is not sensitive to c and p.

Table 6.4 Optimal X_1 and Δt given {c, p} in two-stress SSADT planning

c	p	X_1*	Δt *	RX_1 (%)	c	p	X_1*	Δt *	RX_1 (%)
2	0.5	0.5	192	12.2	4	0.5'	0.466	48	2.1
	0.55	0.467	240	4.8		0.55'	0.434	96	4.8
	0.6	0.512	240	14.9		0.6'	0.43	144	5.7
	0.65	0.424	240	4.78		0.65'	0.472	192	3.5
	0.7	0.442	240	0.74		0.7	0.455	144	0.2
	0.75	0.459	240	3.08		0.75	0.464	192	1.7
	0.8	0.411	144	7.70		0.8	0.478	288	4.8
	0.85	0.414	192	7.03		0.85	0.461	240	1.0
	0.9	0.395	240	11.30		0.9	0.461	240	1.10
	0.95	0.429	192	3.66		0.95	0.439	240	3.73
3	0.5	0.481	48'	4.75	5	0.5'	0.505	144	12.30
	0.55	0.485	48'	5.62		0.55'	0.367	144	18.39
	0.6	0.461	240	0.39		0.6'	0.5	192	11.19
	0.65	0.427	240	7.01		0.65'	0.351	288	21.95
	0.7	0.495	240	7.79		0.7'	0.471	144	4.74
	0.75	0.485	240	5.62		0.75	0.459	96	2.07
	0.8	0.491	288	6.93		0.8	0.47	240	4.51
	0.85	0.433	192	5.71		0.85	0.476	192	5.85
	0.9	0.439	240	4.40		0.9	0.461	240	2.51
	0.95	0.395	240	13.98		0.95	0.437	192	2.82

'indicates the contour pattern does not exist for the corresponding {c, p}.

Table 6.5 Optimal X_1 and Δt given {p, c}in two-stress SSADT planning

p	c	x1	Δt		RX_1
0.5	2	0.5	192		2.46
	3	0.481	48	’	1.43
	4	0.466	48	’	4.51
	5	0.505	144	’	3.48
0.55	2	0.467	240		6.56
	3	0.485	48	’	10.67
	4	0.434	96	’	0.97
	5	0.367	144	’	16.26
0.6	2	0.512	240		7.62
	3	0.461	240		3.10
	4	0.43	144	’	9.62
	5	0.5	192	’	5.10
0.65	2	0.424	240		1.31
	3	0.427	240		2.03
	4	0.472	192	’	12.78
	5	0.351	288	’	16.13
0.7	2	0.442	240		5.10
	3	0.495	240		6.28
	4	0.455	144		2.31
	5	0.471	144	’	1.13
0.75	2	0.459	240		1.66
	3	0.485	240		3.91
	4	0.464	192		0.59
	5	0.459	96		1.66
0.8	2	0.411	144		11.14
	3	0.491	288		6.16
	4	0.478	288		3.35
	5	0.47	240		1.62
0.85	2	0.414	192		7.17
	3	0.433	192		2.91
	4	0.461	240		3.36
	5	0.476	192		6.73
0.9	2	0.395	240		10.02
	3	0.439	240		0.00
	4	0.461	240		5.01
	5	0.461	240		5.01
0.95	2	0.429	192		0.94
	3	0.395	240		7.06
	4	0.439	240		3.29
	5	0.437	192		2.82

’indicates the contour pattern does not hold for the corresponding {c, p}.

The study also shows Δt =240hrs is the most frequently occurring optimal inspection time interval for various (c, p)'s as in Table 6.6. what is more, if p=0.9, whatever c is, the optimal Δt^*=240hrs.

Table 6.6 Frequency of the optimal Δt in two-stress SSADT plans

Δt	No. of Occurrences	Percentage
240	17	58.6%
192	7	24.1%
288	3	10.3%
144	2	7%
Total	29	100%

6.2.3 Sensitivity Analysis

Again, we need to conduct sensitivity analysis on optimal SSADT plans to mis-specified pre-set parameters. Suppose the true value of σ/a is 100, we analyze the sensitivity of optimal n, π_1, T_1, T_2 to guessed values of σ/a at 80, 90, 110 and 120 respectively. The detailed data are given in Appendix D.

Tables 6.7 to 6.10 are sensitivity analysis of n, T, T_1 and T_2 to the mis-specified σ/a.

Table 6. 7 Sensitivity of Rn to σ/a in two-stress SSADT plans

X_1	Dev	c				X_1	Dev	c			
	σ/a	2	3	4	5		σ/a	2	3	4	5
0.05	-20	20.00	12.50	28.57	16.67	0.55	-20	28.57	27.27	11.11	33.33
	-10	0.00	12.50	0.00	16.67		-10	7.14	9.09	11.11	11.11
	10	20.00	0.00	0.00	0.00		10	7.14	9.09	11.11	11.11
	20	20.00	12.50	28.57	33.33		20	21.43	18.18	22.22	22.22
0.1	-20	9.09	25.00	14.29	0.00	0.6	-20	21.43	20.00	22.22	25.00
	-10	9.09	12.50	14.29	0.00		-10	7.14	0.00	11.11	12.50
	10	18.18	0.00	28.57	33.33		10	14.29	20.00	11.11	12.50
	20	18.18	12.50	28.57	33.33		20	14.29	20.00	33.33	25.00
0.15	-20	9.09	20.00	25.00	28.57	0.65	-20	14.29	20.00	30.00	25.00
	-10	9.09	10.00	0.00	14.29		-10	14.29	0.00	20.00	12.50
	10	27.27	10.00	0.00	0.00		10	14.29	20.00	10.00	12.50
	20	27.27	10.00	12.50	42.86		20	14.29	30.00	0.00	25.00
0.2	-20	9.09	22.22	12.50	14.29	0.7	-20	20.00	18.18	30.00	22.22
	-10	9.09	11.11	12.50	0.00		-10	20.00	18.18	10.00	11.11
	10	36.36	11.11	0.00	28.57		10	13.33	0.00	10.00	11.11
	20	36.36	22.22	12.50	28.57		20	13.33	18.18	10.00	22.22
0.25	-20	35.71	22.22	33.33	25.00	0.75	-20	13.33	18.18	36.36	
	-10	28.57	11.11	22.22	25.00		-10	26.67	18.18	18.18	
	10	7.14	11.11	11.11	0.00		10	0.00	0.00	9.09	
	20	14.29	22.22	11.11	12.50		20	13.33	18.18	0.00	
0.3	-20	15.38	30.00	12.50	14.29	0.8	-20	13.33	18.18	27.27	
	-10	7.69	10.00	12.50	0.00		-10	26.67	18.18	18.18	
	10	15.38	0.00	12.50	14.29		10	0.00	0.00	9.09	
	20	23.08	10.00	25.00	42.86		20	20.00	18.18	0.00	
0.35	-20	23.08	0.00	11.11	12.50	0.85	-20	13.33	18.18	11.11	
	-10	23.08	11.11	0.00	0.00		-10	26.67	18.18	0.00	
	10	15.38	22.22	11.11	12.50		10	0.00	0.00	11.11	
	20	23.08	44.44	22.22	12.50		20	20.00	27.27	22.22	
0.4	-20	23.08	22.22	25.00	14.29	0.9	-20	26.67	18.18	11.11	
	-10	15.38	0.00	12.50	0.00		-10	26.67	18.18	0.00	
	10	7.69	11.11	12.50	14.29		10	0.00	0.00	11.11	
	20	23.08	33.33	25.00	57.14		20	20.00	27.27	22.22	
0.45	-20	28.57	11.11	22.22	12.50	0.95	-20	26.67	25.00	11.11	
	-10	7.14	11.11	11.11	12.50		-10	26.67	25.00	0.00	
	10	14.29	22.22	11.11	12.50		10	0.00	8.33	11.11	
	20	28.57	22.22	22.22	12.50		20	20.00	16.67	22.22	
0.5	-20	14.29	10.00	20.00	22.22						
	-10	0.00	10.00	10.00	11.11						
	10	7.14	20.00	10.00	11.11						
	20	28.57	20.00	0.00	11.11						

Table 6. 8 Sensitivity of RT to σ/a in two-stress SSADT plans

X_1	Dev σ/a	c				X_1	Dev σ/a	c			
		2	3	4	5			2	3	4	5
0.05	-20	18.75	27.27	10.00	20.00	0.55	-20	10.53	7.69	30.77	0.00
	-10	18.75	9.09	20.00	0.00		-10	10.53	7.69	7.69	9.09
	10	0.00	18.18	20.00	20.00		10	15.79	15.38	7.69	9.09
	20	18.75	27.27	10.00	10.00		20	21.05	23.08	15.38	18.18
0.1	-20	31.25	16.67	27.27	36.36	0.6	-20	20.00	20.00	15.38	15.38
	-10	12.50	8.33	9.09	18.18		-10	15.00	20.00	7.69	7.69
	10	0.00	16.67	9.09	9.09		10	5.00	0.00	7.69	7.69
	20	18.75	25.00	9.09	9.09		20	25.00	20.00	7.69	15.38
0.15	-20	29.41	20.00	10.00	10.00	0.65	-20	25.00	20.00	8.33	15.38
	-10	11.76	10.00	20.00	0.00		-10	5.00	20.00	0.00	7.69
	10	5.88	40.00	20.00	20.00		10	5.00	0.00	8.33	7.69
	20	11.76	60.00	30.00	0.00		20	25.00	13.33	41.67	15.38
0.2	-20	27.78	16.67	27.27	27.27	0.7	-20	21.05	21.43	8.33	16.67
	-10	11.11	8.33	9.09	18.18		-10	0.00	0.00	8.33	8.33
	10	11.11	8.33	18.18	9.09		10	5.26	21.43	33.33	8.33
	20	5.56	16.67	27.27	9.09		20	26.32	21.43	33.33	16.67
0.25	-20	0.00	15.38	0.00	10.00	0.75	-20	26.32	21.43	0.00	
	-10	13.33	7.69	10.00	10.00		-10	10.53	0.00	0.00	
	10	13.33	7.69	10.00	20.00		10	21.05	21.43	36.36	
	20	26.67	15.38	30.00	30.00		20	26.32	21.43	45.45	
0.3	-20	23.53	8.33	25.00	25.00	0.8	-20	26.32	21.43	9.09	
	-10	11.76	8.33	8.33	16.67		-10	10.53	0.00	0.00	
	10	5.88	25.00	8.33	8.33		10	21.05	21.43	36.36	
	20	17.65	33.33	16.67	0.00		20	21.05	21.43	45.45	
0.35	-20	16.67	35.71	27.27	27.27	0.85	-20	26.32	21.43	28.57	
	-10	5.56	7.14	18.18	18.18		-10	10.53	0.00	21.43	
	10	5.56	0.00	9.09	9.09		10	21.05	21.43	7.14	
	20	16.67	0.00	18.18	27.27		20	21.05	14.29	14.29	
0.4	-20	15.79	20.00	15.38	23.08	0.9	-20	10.53	21.43	28.57	
	-10	5.26	20.00	7.69	15.38		-10	10.53	0.00	21.43	
	10	10.53	6.67	7.69	7.69		10	21.05	21.43	7.14	
	20	15.79	6.67	15.38	7.69		20	21.05	14.29	14.29	
0.45	-20	11.11	26.67	16.67	25.00	0.95	-20	10.53	15.38	28.57	
	-10	11.11	6.67	8.33	8.33		-10	10.53	7.69	21.43	
	10	5.56	0.00	8.33	8.33		10	21.05	30.77	7.14	
	20	11.11	20.00	16.67	25.00		20	21.05	23.08	14.29	
0.5	-20	26.32	28.57	18.18	18.18						
	-10	21.05	7.14	9.09	9.09						
	10	10.53	0.00	36.36	9.09						
	20	10.53	21.43	45.45	27.27						

Table 6. 9 Sensitivity of RT_1 to σ/a in two-stress SSADT plans

X_1	Dev. σ/a	c					Dev. σ/a	c			
		2	3	4	5	X_1		2	3	4	5
0.05	-20	20.00	30.00	11.11	22.22	0.55	-20	13.33	10.00	50.00	25.00
	-10	20.00	10.00	22.22	0.00		-10	0.00	10.00	8.33	12.50
	10	0.00	20.00	22.22	22.22		10	33.33	40.00	0.00	12.50
	20	20.00	30.00	11.11	11.11		20	33.33	30.00	0.00	25.00
0.1	-20	33.33	18.18	30.00	40.00	0.6	-20	26.32	15.38	9.09	18.18
	-10	13.33	9.09	10.00	20.00		-10	26.32	23.08	0.00	9.09
	10	0.00	18.18	10.00	10.00		10	0.00	0.00	0.00	9.09
	20	13.33	27.27	10.00	10.00		20	21.05	23.08	9.09	18.18
0.15	-20	26.67	22.22	11.11	11.11	0.65	-20	22.22	23.08	9.09	27.27
	-10	13.33	11.11	22.22	0.00		-10	0.00	30.77	9.09	9.09
	10	6.67	44.44	11.11	11.11		10	0.00	7.69	0.00	9.09
	20	13.33	55.56	33.33	0.00		20	22.22	23.08	27.27	18.18
0.2	-20	26.67	10.00	30.00	30.00	0.7	-20	27.78	23.08	0.00	18.18
	-10	13.33	10.00	10.00	20.00		-10	5.56	0.00	0.00	9.09
	10	6.67	10.00	10.00	20.00		10	0.00	23.08	30.00	9.09
	20	6.67	20.00	30.00	0.00		20	27.78	23.08	50.00	18.18
0.25	-20	0.00	9.09	12.50	0.00	0.75	-20	27.78	23.08	10.00	
	-10	16.67	0.00	25.00	25.00		-10	11.11	0.00	0.00	
	10	16.67	9.09	12.50	25.00		10	22.22	23.08	40.00	
	20	33.33	18.18	37.50	37.50		20	22.22	23.08	50.00	
0.3	-20	15.38	0.00	20.00	30.00	0.8	-20	27.78	23.08	0.00	
	-10	7.69	0.00	10.00	10.00		-10	11.11	0.00	11.11	
	10	15.38	44.44	10.00	10.00		10	22.22	23.08	55.56	
	20	23.08	55.56	20.00	10.00		20	22.22	15.38	66.67	
0.35	-20	20.00	36.36	25.00	33.33	0.85	-20	27.78	23.08	30.77	
	-10	6.67	0.00	12.50	22.22		-10	11.11	0.00	23.08	
	10	6.67	0.00	12.50	11.11		10	22.22	23.08	7.69	
	20	13.33	0.00	25.00	22.22		20	22.22	15.38	15.38	
0.4	-20	12.50	30.77	20.00	20.00	0.9	-20	11.11	23.08	30.77	
	-10	6.25	23.08	10.00	0.00		-10	11.11	0.00	23.08	
	10	6.25	0.00	10.00	20.00		10	22.22	23.08	7.69	
	20	12.50	0.00	20.00	0.00		20	22.22	15.38	15.38	
0.45	-20	14.29	18.18	11.11	20.00	0.95	-20	11.11	16.67	30.77	
	-10	0.00	9.09	0.00	10.00		-10	11.11	8.33	23.08	
	10	7.14	9.09	11.11	10.00		10	22.22	33.33	7.69	
	20	7.14	36.36	11.11	10.00		20	22.22	25.00	15.38	
0.5	-20	29.41	27.27	12.50	22.22						
	-10	29.41	0.00	0.00	11.11						
	10	0.00	9.09	50.00	11.11						
	20	5.88	27.27	62.50	22.22						

Table 6. 10 Sensitivity of RT_2 to σ/a in two-stress SSADT plans

X_1	Dev. σ/a	c				X_1	Dev. σ/a	c			
		2	3	4	5			2	3	4	5
0.05	-20	0.0	0.0	0.0	0.0	0.55	-20	0.0	66.7	200.0	66.7
	-10	0.0	0.0	0.0	0.0		-10	50.0	66.7	0.0	0.0
	10	0.0	0.0	0.0	0.0		10	50.0	66.7	100.0	0.0
	20	0.0	0.0	0.0	0.0		20	25.0	0.0	200.0	0.0
0.1	-20	0.0	0.0	0.0	0.0	0.6	-20	100.0	50.0	50.0	0.0
	-10	0.0	0.0	0.0	0.0		-10	200.0	0.0	50.0	0.0
	10	0.0	0.0	0.0	0.0		10	100.0	0.0	50.0	0.0
	20	100.0	0.0	0.0	0.0		20	100.0	0.0	0.0	0.0
0.15	-20	50.0	0.0	0.0	0.0	0.65	-20	50.0	0.0	0.0	50.0
	-10	0.0	0.0	0.0	0.0		-10	50.0	50.0	100.0	0.0
	10	0.0	0.0	100.0	100.0		10	50.0	50.0	100.0	0.0
	20	0.0	100.0	0.0	0.0		20	50.0	50.0	200.0	0.0
0.2	-20	33.3	50.0	0.0	0.0	0.7	-20	100.0	0.0	50.0	
	-10	0.0	0.0	0.0	0.0		-10	100.0	0.0	50.0	
	10	33.3	0.0	100.0	100.0		10	100.0	0.0	50.0	
	20	0.0	0.0	0.0	100.0		20	0.0	0.0	50.0	
0.25	-20	0.0	50.0	50.0	50.0	0.75	-20	0.0	0.0	100.0	
	-10	0.0	50.0	50.0	50.0		-10	0.0	0.0	0.0	
	10	0.0	0.0	0.0	0.0		10	0.0	0.0	0.0	
	20	0.0	0.0	0.0	0.0		20	100.0	0.0	0.0	
0.3	-20	50.0	33.3	50.0	0.0	0.8	-20	0.0	0.0	50.0	
	-10	25.0	33.3	0.0	50.0		-10	0.0	0.0	50.0	
	10	25.0	33.3	0.0	0.0		10	0.0	0.0	50.0	
	20	0.0	33.3	0.0	50.0		20	0.0	100.0	50.0	
0.35	-20	0.0	33.3	33.3	0.0	0.85	-20	0.0	0.0	0.0	
	-10	0.0	33.3	33.3	0.0		-10	0.0	0.0	0.0	
	10	0.0	0.0	0.0	0.0		10	0.0	0.0	0.0	
	20	33.3	0.0	0.0	50.0		20	0.0	0.0	0.0	
0.4	-20	33.3	50.0	0.0	33.3	0.9	-20	0.0	0.0	0.0	
	-10	0.0	0.0	0.0	66.7		-10	0.0	0.0	0.0	
	10	33.3	50.0	0.0	33.3		10	0.0	0.0	0.0	
	20	33.3	50.0	0.0	33.3		20	0.0	0.0	0.0	
0.45	-20	0.0	50.0	33.3	50.0	0.95	-20	0.0	0.0	0.0	
	-10	50.0	50.0	33.3	0.0		-10	0.0	0.0	0.0	
	10	0.0	25.0	0.0	0.0		10	0.0	0.0	0.0	
	20	25.0	25.0	33.3	100.0		20	0.0	0.0	0.0	
0.5	-20	0.0	33.3	33.3	0.0						
	-10	50.0	33.3	33.3	0.0						
	10	100.0	33.3	0.0	0.0						
	20	50.0	0.0	0.0	50.0						

The results show that the optimal n, T, T_1 and T_2 are moderately sensitive to the deviation of the pre-estimate of σ/a. Overestimating and underestimating σ/a have nearly the same effects on these decision variables. Thus it is important to estimate the degradation process parameters as accurate as one can before the test planning.

6.3 OPTIMAL THREE-STRESS SSADT PLANS

6.3.1 Introduction

Compared with ALT, ADT is more efficient to provide precise reliability estimates especially for highly reliable products since it captures degradation information to obtain reliability reference rather than captures physical failures. Compared with CSADT, it is proved in chapter 4 that SSADT is useful to shorten testing time and save testing samples, therefore reduce the total experiment cost. We have studied two-stress SSADT plans in the previous sections. However, two-stress SSADT plans have some inefficiency:

1. Experimenters need to specify a highest stress before planning. While, this high stress may cause different failure modes from that at design stress with a consequence that data collected at high stress are less informative. We have mentioned in section 4.2, experimenters need to know from historical information or to do a pre-test to roughly identify the highest allowable stress before test starts. Actually, an intermediate stress is needed to check whether the failure modes remain the same through out the stress range.

2. We have assumed a linear stress-drift relationship in our two-stress planning. However, this linear relationship is possibly inadequate. Normally, there should be at least three stress levels to check the curvature of the assumed relationship.

3. It is possible that the degradation rate at the lower stress is very small such that the degradation increment is difficult to detect or the measurement error is relatively large. So, an intermediate stress, which is hopefully greater than the optimal lower stress in a two-stress SSADT, is needed for better estimation.

Therefore, we consider planning SSADT with three test stresses. The test method and assumptions remains as we did in the two-stress ADT design. Except that X_k is normalized as:

$$X_k = \frac{S - S_0}{S_3 - S_0}. \quad X_0=0<X_1<X_2< X_3=1 \tag{6.1}$$

S_0, the design stress, and S_3, the highest stress that validates the same failure mechanism, should be specified before test planning. Decision variables include sample size n, stress levels X_1, X_2, and number of inspections at each stress L_1, L_2, and L_3.

Using the same planning criteria stated in chapter 4, planning of three-stress SSADT can be formulated as follows:

Minimizing:

$$\begin{aligned} &TC(n, L_k) = C_d \cdot n + n \cdot \sum_{k=1}^{3} C_{mk} \cdot L_k + \Delta t \cdot \sum_{k=1}^{3} C_{ok} \cdot L_k \\ &C_d > 0,\ C_{mk} > 0,\ C_{ok} > 0 \end{aligned} \tag{6.2}$$

Subject to:

$$\Phi\left(\frac{(c-1)\cdot\sqrt{n}}{\frac{\hat{\sigma}}{\hat{a}}\cdot\sqrt{Q}}\right)-\Phi\left(\frac{(\frac{1}{c}-1)\cdot\sqrt{n}}{\frac{\hat{\sigma}}{\hat{a}}\cdot\sqrt{Q}}\right)\geq p \qquad c>1,\ p>0$$

$$Q=\frac{\sum_{k=1}^{3}X_k^2L_k}{L*\Delta t*\sum_{k=1}^{3}X_k^2L_k-\Delta t*\left(\sum_{k=1}^{3}X_kL_k\right)^2} \tag{6.3}$$

where C_d, C_{mk}, C_{ok}, c, p are the same as defined in chapter 4

6.3.2 Three-stress SSADT Plans

The optimal three-stress plan, for example, for $\Delta t = 240$hrs, c=2, p=0.9, $\frac{\hat{\sigma}}{\hat{a}}$=100, is:

Table 6.11 Optimal three-stress SSADT plan 1

Plans	Cost coefficients	n	X_1	L_1	X_2	L_2	L_3	L	Cost
Three-stress SSADT	C_d=86, C_{m1} = 3.2, C_{m2} = 4, C_{m3} = 4.5, Co1 = 0.2, C_{o2} = 0.25, C_{o3} = 0.3	8	0	11	0	1	1	12	1697.6
Two-stress SSADT	C_d=86, C_{m1} = 3.65, C_{m3} = 4.5, C_{o1} = 0.26, C_{o3} = 0.3	13	0.395	16	/	/	3	19	3281.5

It is seen that the employment of an intermediate stress not only overcomes the disadvantages of two-stress SSADT, but also further saves the test samples, time, and cost. However, the result is not adoptable since X_1 and X_2 are not optimal. One more constraint is needed to properly enhance them. It can be done through two ways.

6.3.2.1 Approach 1

By disassembling the optimal low stress in a two-stress SSADT to be the two lower stresses in a three-stress SSADT, we add in an additional constraint as:

$$X_1 \cdot L_1 + X_2 \cdot L_2 = X_L^* \cdot L_L^* \\ L_3 = L_H^* \tag{6.4}$$

where X_L^*, L_L^* and L_H^* are respectively the optimal lower stress, inspections at low stress and inspections at high stress in a two-stress SSADT. The illustration is shown in Figure 6.5.

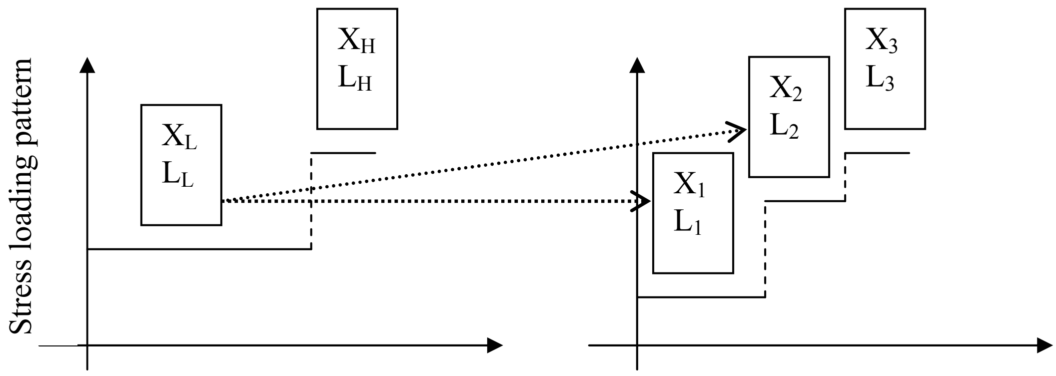

Figure 6.5. An illustration of 3-stress SSADT planning (extended from 2-stress SSADT plans)

For example, with Δt = 240hrs, c=2, p=0.9, $\frac{\hat{\sigma}}{\hat{a}}$=100, the optimal plan satisfying this additional rule is listed in Table 6.12:

Table 6.12 Optimal three-stress SSADT plan 2

Plans	Cost coefficients	n	X_1	L_1	X_2	L_2	L_3	L	Cost
Three-stress SSADT	C_d=86, C_{m1} = 3.2, C_{m2} = 4, C_{m3} = 4.5, Co1 = 0.2, C_{o2} = 0.25, C_{o3} = 0.3	7	0.37	9	0.99	3	3	15	1810.1

It is observed in the three-stress SSADT, the lower stress is slightly smaller than that in the two-stress one. And the intermediate stress approaches to the high stress. Additionally, the sample size, total number of inspections, and total cost of test are reduced.

6.3.2.2 Approach 2

Since X_1 and X_2 are close to the design stress in Table 6.11, let the test starts from $X_1=X_0=0$. We add in one more constraint that the intermediate stress X_2 should be greater than X_L^*, i.e.

$$X_1=0, \quad X_2>X_L^* \tag{6.5}$$

X_2, L_1, L_2, L_3 are then to be determined by fulfilling (6.2), (6.3) and (6.5).

For example, with $\Delta t = 240$hrs, c=2, p=0.9, $\frac{\hat{\sigma}}{\hat{a}}$ =100, the optimal plan is:

Table 6.13 Optimal three-stress SSADT plan 3

Plans	Cost coefficients	n	X_1	L_1	X_2	L_2	L_3	L	Cost
Three-stress SSADT	C_d=86, C_{m1} = 3.2, C_{m2} = 4, C_{m3} = 4.5, Co1 = 0.2, C_{o2} = 0.25, C_{o3} = 0.3	7	0	9	0.75	3	3	15	1810.1

With approach 2, a proper intermediate stress that fastens the degradation rate for better measurement is obtained. Compared with the two-stress SSADT, the test samples, inspections and the test cost are saved.

6.4 CONCLUSIONS

The optimal SSADT plans are studied in this chapter. Besides the optimal sample size and holding time at each stress, we present the guidance on how to determine the lower test stress, the inspection time interval and the precision parameters. Three-stress SSADT plans are also studied in this chapter. To compare with each other, a summary of the proposed plans is given in Table 6.14.

Table 6.14 Comparisons of optimal SSADT plans
($\Delta t = 240$hrs, $\frac{\hat{\sigma}}{\hat{a}}$ =100, c=2 and p=0.9)

Plans	Cost coefficients	n	X_1	L_1	X_2	L_2	L_3	L	Cost	Planning criteria
Two-stress SSADT	C_d=86, C_{m1} = 3.65, C_{m3} = 4.5, C_{o1} = 0.26, C_{o3} = 0.3	13	0.395	16	/	/	3	19	3281.5	(4.10) (4.25)
Three-stress SSADT	C_d=86, C_{m1} = 3.2, C_{m2} = 4, C_{m3} = 4.5, Co1 = 0.2, C_{o2} = 0.25, C_{o3} = 0.3	8	0	11	0	1	1	12	1697.6	(6.2) (6.3)
		7	0.37	9	0.99	3	3	15	1810.1	(6.2) (6.3) (6.4)
		7	0	9	0.75	3	3	15	1810.1	(6.2) (6.3) (6.5)

The conclusions are:

1. A three-stress SSADT requires fewer samples and inspections to achieve the same level of estimation precision. And consequently, saves the total cost of test.

2. As suggested by solution 1, the best plan that minimizes the test cost and provides an acceptable level of sampling risk is to test at design stress and the highest stress. It requires the least cost and time. But, the three-stress ADT is

reduced to a two-stress one. To involve an intermediate stress, solution 2 and solution 3 can be adopted.

3. Solution 3 is recommended since the test starts from the design stress, which reflects the prototype of degradation process we are interested in and is helpful to provide better reliability reference. The intermediate stress is adequate to justify the model assumptions, and to fasten the degradation speed for easier measurement.

Chapter 7

Planning of Destructive CSADT

7.1 INTRODUCTION

In engineering testing, there are cases where the degradation measurement is devastating, that is, items under test are destroyed and cannot fulfill their functional requirements after inspection. In such a situation, each unit can only be inspected one time in the degradation experiment. This kind of testing is called "destructive testing" (Park & Yum, 1997). In a destructive ADT, the time at which to collect the degradation data becomes more important because it significantly affects the estimate precision.

Park & Yum (1997) presented a method to design destructive CSADT, which involves two, or three test stresses. Given the total number of samples available and the longest test time allowable, they determined the stress levels, test time and the sample allocations at each stress by assuming that the degradation process follows a lognormal distribution and the degradation rate is simply constant at a particular stress. Considering the influence of tightened degradation critical value on the lifetime of tested products, Yang & Yang (2002) proposed a two-stress CSADT planning method in which samples are inspected at the end of the whole testing. Park et al (2004) developed to optimize destructive SSADT plans by assuming a simple constant-rate relationship and a cumulative exposure effect between the stress and the performance of a unit. The optimality in the three papers is defined to minimize the MLE of the mean lifetime at use stress.

Motivated by the application of stochastic processes in CSADT and SSADT planning, in this chapter, we model the degradation process by a stochastic process with drift η_k and dispersion σ_k^2 at stress X_k, and assume that the drift-stress relationship is simple linear while dispersion keeps the same for all the stress levels. For simplicity, we only consider two-stress level CSADT, where the highest stress is known and normalized to be 1. The planning policy is the same as we present through chapter 4 to chapter 6, i.e. to minimize the total cost of testing as well as to achieve a requisite level of estimate precision.

First, we present the optimal plans with known lower stresses. Decision variables include the samples size, sample allocations and the test time at each stress. A numerical example compared with the existing plan shows the advantages of our proposed plan. Subsequently, we present to determine the lower test stress with/without test time and sample size constraints. Finally, sensitivity analysis is conducted to show the influence of mis-specified pre-estimated parameters on the decision variables. This sensitivity study also suggests how the pre-set parameters be determined and how the results be influenced if the pre-set parameter is not well estimated.

A comparison of Park & Yum (1997)'s plan and our proposed plan is given below in Table 7.1.

Table 7.1 Comparisons of our proposed plan with the existing destructive CSADT plan

	Park & Yum, 1997	The proposed method
Type of ADT	CSADT	
Number of Stress	2 or 3	2
Inputs	Sample size	Stress levels
Decision variables	Stress levels	Sample size
	Test time at each stress Sample allocations at each stress	
Planning policy	To minimize: the MLE of mean life under use condition	To minimize: the total cost of testing S. t. a requisite level of estimate precision

7.2 PLANNING OF THE DESTRUCTIVE CSADT

7.2.1 Experiment Description & Model Assumptions

In this destructive CSADT, the basic description and assumptions are the same as those in chapter 4. That is, n samples in total are put into test, among which $n\pi_k$ are under X_k, where $\sum_{k=1}^{2}\pi_k = 1$. Testing time at each stress is respectively T_1 and T_2. The only difference lies that there is no need to specify the inspection time interval. At stress X_k, the degradation changes follow a stochastic process with drift η_k and dispersion σ_k^2 that satisfy a relationship described in equation (4.3) and (4.4). Degradation increments of unit i follow a normal distribution, i.e. $\Delta D_i \sim N\left(\eta_k T_k, \sigma^2 T_k\right)$ with the p.d.f

$$f(\Delta D_i) = \frac{1}{\sqrt{2\pi}\left(\sqrt{(\sigma^2 T_k}\right)} \exp\left(-\frac{\left(\Delta D_i - T_k\eta_k\right)^2}{2T_k\sigma^2}\right) \tag{7.1}$$

7.2.2 Planning Policy

Same as the planning policy stated before, we aim to minimize the cost under the condition that the estimate precision satisfies a requisite level.

Because there is only one inspection on each unit, the measurement cost reduces to:

$$C_{me} = n \cdot \sum_{k=1}^{2} C_{mk} \cdot \pi_k \tag{7.2}$$

Sample cost and manpower cost remains the same as:

$$C_{de} = C_d \cdot n \tag{7.3}$$

and

$$C_{op} = \sum_{k=1}^{2} C_{ok} \cdot T_k \tag{7.4}$$

The cost function becomes:

$$TC(n, T_1, T_2) = C_d \cdot n + n\sum_{k=1}^{2} \pi_k C_{mk} + \sum_{k=1}^{2} C_{ok} \cdot T_k$$
$$C_d > 0,\ \ C_{mk} > 0,\ \ C_{ok} > 0 \tag{7.5}$$

where C_d, C_{mk}, C_{ok} are the cost coefficients defined in chapter 4.

We are interested in the estimate of the mean lifetime at use condition, i.e. $\mu(X_0)$. By limiting the sampling risk in estimating $\mu(X_0)$ to be reasonably small, i.e. (4.11), the precision constraint mathematically reduces to:

$$\Phi\left(\frac{(c-1)\cdot\sqrt{n}}{\frac{\hat{\sigma}\cdot}{\hat{a}}\cdot\sqrt{Q}}\right) - \Phi\left(\frac{(\frac{1}{c}-1)\cdot\sqrt{n}}{\frac{\hat{\sigma}\cdot}{\hat{a}}\cdot\sqrt{Q}}\right) \geq p$$
$$c > 1 \qquad 0 < p < 1 \tag{7.6}$$
$$Q = \frac{\pi_1 T_1 X_1^2 + (1-\pi_1) T_2 X_2^2}{\pi_1 (1-\pi_1) T_1 T_2 (X_1 - X_2)^2}$$

See Appendix E1 for the detailed derivations.

With inspection times at each stress level equal to 1, the optimization criteria in our proposed method is a a special case of models in Chapter 4 that is expressed as:

Minimizing:

$$TC(n, T_1, T_2) = C_d \cdot n + n\sum_{k=1}^{2} \pi_k C_{mk} + \sum_{k=1}^{2} C_{ok} \cdot T_k$$
$$C_d > 0,\ \ C_{mk} > 0,\ \ C_{ok} > 0$$

Subject to:

$$\Phi\left(\frac{(c-1)\cdot\sqrt{n}}{\frac{\hat{\sigma}\cdot}{\hat{a}}\cdot\sqrt{Q}}\right) - \Phi\left(\frac{(\frac{1}{c}-1)\cdot\sqrt{n}}{\frac{\hat{\sigma}\cdot}{\hat{a}}\cdot\sqrt{Q}}\right) \geq p$$
$$c > 1 \qquad 0 < p < 1$$

7.3 OPTIMAL DESTRUCTIVE CSADT PLANS

7.3.1 Simulations

We have run simulations to obtain the optimal plans given C_d=86$/unit, C_{m1} = 4$/unit*time, C_{m2} = 4.5$/unit*time, C_{o1} = 0.3$/hr*time, C_{o2} = 0.4$/hr*time and $\hat{\sigma}/\hat{a} = 100$. The results are shown in Table 7.2.

Table 7.2 Optimal two-stress destructive CSADT plans

	X_1	n^*	π_1^*	n_1*	n_2*	T_1*(hrs)	T_2*(hrs)
c=5 p=0.9	0.1	15	0.8	12	3	3312	528
	0.15	16	0.75	12	4	3792	912
	0.2	19	0.736842	14	5	3984	1152
	0.25	22	0.681818	15	7	4368	1344
	0.3	24	0.666667	16	8	4848	1824
	0.35	28	0.642857	18	10	5280	2064
	0.4	32	0.625	20	12	5760	2496
	0.45	36	0.611111	22	14	6240	3216
	0.5	42	0.595238	25	17	7008	3648
	0.55	48	0.583333	28	20	7872	4512
	0.6	56	0.571429	32	24	9072	5376
	0.65	67	0.567164	38	29	9984	6816
	0.7	89	0.573034	51	38	9960	8392
	0.75	127	0.574803	73	54	9960	9832
	0.8	209	0.555024	116	93	9960	9928
	0.85	393	0.557252	219	174	9960	9928
	0.9→1	>500					
c=4 p=0.9	0.1	16	0.8125	13	3	3504	576
	0.15	17	0.764706	13	4	4080	960
	0.2	20	0.75	15	5	4176	1344
	0.25	23	0.695652	16	7	4656	1536
	0.3	26	0.692308	18	8	5088	1920
	0.35	29	0.655172	19	10	5568	2448
	0.4	34	0.617647	21	13	6144	2688
	0.45	38	0.605263	23	15	6864	3360
	0.5	44	0.590909	26	18	7584	3984
	0.55	50	0.58	29	21	8640	4896
	0.6	59	0.559322	33	26	9792	5808
	0.65	74	0.567568	42	32	9984	7296
	0.7	98	0.581633	57	41	9960	9064
	0.75	144	0.583333	84	60	9960	9928
c=4 p=0.9	0.8	238	0.567227	135	103	9960	9928
	0.85	447	0.55481	248	199	9960	9928
	0.9→1	>500					

Table 7.2 Optimal two-stress destructive CSADT plans (Continued)

	X_1	n^*	π_1^*	n_1^*	n_2^*	T_1^*(hrs)	T_2^*(hrs)
C=3 P=0.9	0.1	17	0.823529	14	3	4080	768
	0.15	20	0.75	15	5	4368	1056
	0.2	22	0.727273	16	6	4992	1392
	0.25	26	0.692308	18	8	5088	1824
	0.3	29	0.655172	19	10	5856	2112
	0.35	33	0.6363	21	12	6432	2544
	0.4	38	0.631579	24	14	6864	3120
	0.45	43	0.604651	26	17	7632	3792
	0.5	49	0.591837	29	20	8640	4512
	0.55	57	0.578947	33	24	9552	5472
	0.6	69	0.57971	40	29	9984	6816
	0.65	90	0.577778	52	38	9984	8016
	0.7	120	0.6	72	48	9960	9832
	0.75	183	0.579235	106	77	9960	9832
	0.8	302	0.566225	171	131	9960	9880
	0.85→1	>500					
C=2 P=0.9	0.1	23	0.826087	19	4	5568	1056
	0.15	26	0.769231	20	6	6288	1440
	0.2	30	0.733333	22	8	6672	1968
	0.25	34	0.705882	24	10	7344	2448
	0.3	39	0.666667	26	13	8016	2928
	0.35	45	0.644444	29	16	8640	3504
	0.4	51	0.627451	32	19	9552	4224
	0.45	60	0.616667	37	23	9984	5136
	0.5	72	0.625	45	27	9984	6480
	0.55	90	0.611111	55	35	9984	7584
	0.6	111	0.621622	69	42	9984	9648
	0.65	153	0.620915	95	58	9960	9928
	0.7	221	0.60181	133	88	9960	9928
	0.75	337	0.581602	196	141	9960	9928
	0.8→1	>500					

Some findings can be seen as follows:

1. Once the experimenter specifies the sampling risk level and the lower stress, i.e. c, p and X_1, the optimal plan implies that number of samples allocated at the lower stress is larger than that at higher stress; and the testing time required at

the lower stress is longer than that at higher stress. That is, $n_1^*>n_2^*$, $T_1^*>T_2^*$ given c, p and X_1.

This property is explainable since degradation information collected at the lower stress is more preferable because there would be less extrapolation from the lower stress to the use condition than that from the higher stress to use condition. More samples set at the lower stress are helpful to achieve the desired estimate precision with a smaller sample size.

2. Given c and p, the required sample size and testing time increase if the lower test stress increases. That is, $n^*(X_1^1)<n^*(X_1^2)$, $T^*(X_1^1)<T^*(X_1^2)$ if $X_1^1<X_1^2$.

   Since a higher test stress level means larger extrapolation, this finding can be explained in the way that to achieve the same precision, more samples are requested to compensate the larger extrapolation.

3. For the same X_1, a tightened test plan requires more samples and longer testing time. That is, given X_1 and p, $n^*(c_1)>n^*(c_2)$, $T^*(c_1)>T^*(c_2)$, if c1<c2.

   With a specified sampling risk p, it is reasonable that if the experimenter wants the MLE of the mean lifetime at use condition drops within a smaller range ($c_1<c_2$) of its true value, it needs more samples and longer test time.

7.3.2 A Numerical Example

In Park & Yum (1997), an example, in which temperature is the accelerated stressor, was provided to illustrate their approach. The operation temperature is 120^0C. And the highest temperature validating the same failure mode is 270^0C. Their two-stress optimal CSADT plan is: lower test temperature 181^0C and sample size 193.

Park & Yum (2004) also presented a two stress destructive SSADT plan where the highest testing temperature and use temperature are respectively 2750C, 1500C. The longest allowable testing time is 600hr. By optimizing the asymptotic variance of the MLE of the 100pth quantile of the lifetime distribution at the use condition, they obtained the optimal sample size as: n_0=104, n_1=343, n_2=74, T_1=486hrs and T_2=600hrs.

To compare with Park and Yum's plans, we set the low temperature and the high temperature at 181^0C and 270^0C respectively. Thus the lower stress is normalized as:

$$X_1 = \frac{1/(181+273) - 1/(120+273)}{1/(270+273) - 1/(120+273)} = 0.4863$$

In the first plan, we set sample size n=193, and determine the sample allocations and testing time. In the second plan, we optimally determine the sample size, sample allocations, and test time at each stress. The results are summarized in Table 7.3.

Table 7.3. Numerical comparisons of the proposed plans with the existing plans

	Stress		Sample size		Test time (hrs)	Total Cost
Park & Yum 1997	CSADT	181^0C	193	164	1200	18482.5
		270^0C		29	1200	
Park & Yum 2004	SSADT	150^0C	601	184	0	54508.8
		245^0C		343	486	
		275^0C		74	600	
The proposed plans (Two options)	CSADT Plan 1	181^0C	193	130	528	17675.1
		270^0C		63	288	
	CSADT Plan 2	181^0C	24	14	4224	4315.4
		270^0C		10	2208	

Remarks:
1. Temperature in use condition is 120^0C, and the highest allowable temperature is 270^0C.
2. Precision parameters are set at c=1.5 and p=0.9.
3. Cost coefficients are respectively: $C_d = 86$, $C_{m1} = 4$, $C_{m2} = 4.5$, $C_{o1} = 0.3$, $C_{o2} = 0.4$

The comparisons show that:

1. The proposed optimal plan requires fewer samples but longer testing time than that of the existing plans.

2. With the same sample size, the proposed method requires less testing time to obtain the same estimate precision than the existing plans.

3. To achieve the same estimate precision, it is seen that the required sample size and the testing time are compensated by each other, which results in different plans. In this situation, the total cost of testing becomes a concern to determine the optimal plan as we have proposed in this dissertation. If the individual cost of sample is relatively high, the best plan always indicates a smaller sample size and longer test time. However, if the test result is expected to be obtained

in a short time frame, a larger sample size is needed and the experimenter must endure a higher test expense.

4. Among all the above plans, our plan is most economical in terms of saving testing cost and testing time, especially when the unit sample is expensive. The existing SSADT can be adopted if no more than one test equipment can be employed or if the operation cost is relatively high.

7. 4 DETERMINATION OF THE LOWER STESS X_1

In section 7.3, we have assumed that the lower stress is known. However, the optimal X_1 or the proper X_1 under some constraints, such as the limitation of test duration and availability of test samples, can be analyzed from the above simulation results.

7.4.1 Determination of the Optimal Lower Stress X_1 without Constraints

Plots of n_2/n_1 Vs X_1 for various c (given p=0.9) show that the ratio of allocations goes up till a plateau and then goes up again. What's more, for different c, the plateau approximately locates at $X_1=0.6\pm0.05$. A suggestion is that the optimal stress level be drawn within (0.55, 0.65).

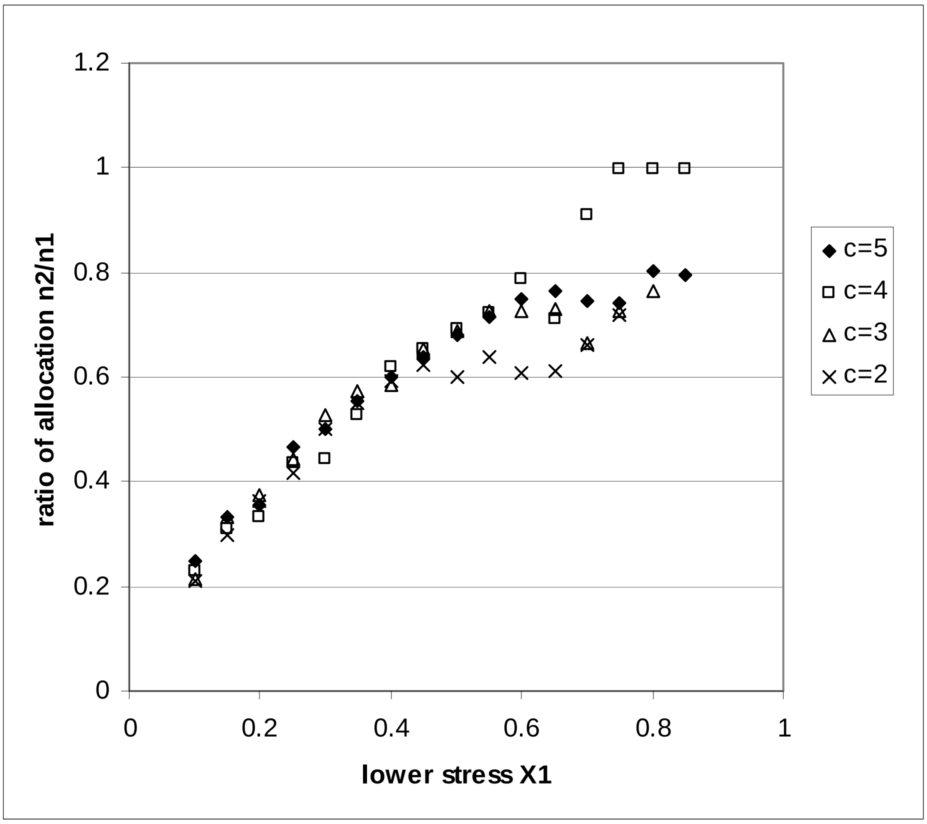

Figure 7.1. Plot of n_2/n_1 Vs X_1 for various c in destructive CSADT plans

7.4.2 Determination of the Optimal Lower Stress X_1 with the Test Time Constraint

It is observed that the optimal testing time at X_1 is longer than that at X_2, i.e $T_1 > T_2$. And to satisfy the precision constraint, T_1 is normally longer than 9000hrs such that the test is a little bit time-consuming. In the cases where manufacturers need to know the test result within a certain time span, a test time constraint can be added to determine a proper X_1.

Figure 7.2 demonstrates the influence of X_1 on optimal test time T_1 and T_2. For example, if the allowable testing time is 5000hrs, and the estimate precision requirement is represented as p=0.9 and c=3, a proper lower test stress is approximately read off from Figure 7.2 at X_1=0.204. It also can be computed by linear extrapolations. Since $T_1^*{}_{(X1=0.2)}$=4992hrs and $T_1^*{}_{(X1=0.25)}$=5088hrs, to get X_1 where the corresponding T_1 is 5000hrs, we take a linear extrapolation in between X_1=0.2 and X_1=0.25, the calculation is as follows:

$$X_1=0.2 +(0.25-0.2)*(5000-4992)/(5088-4992)=0.204$$

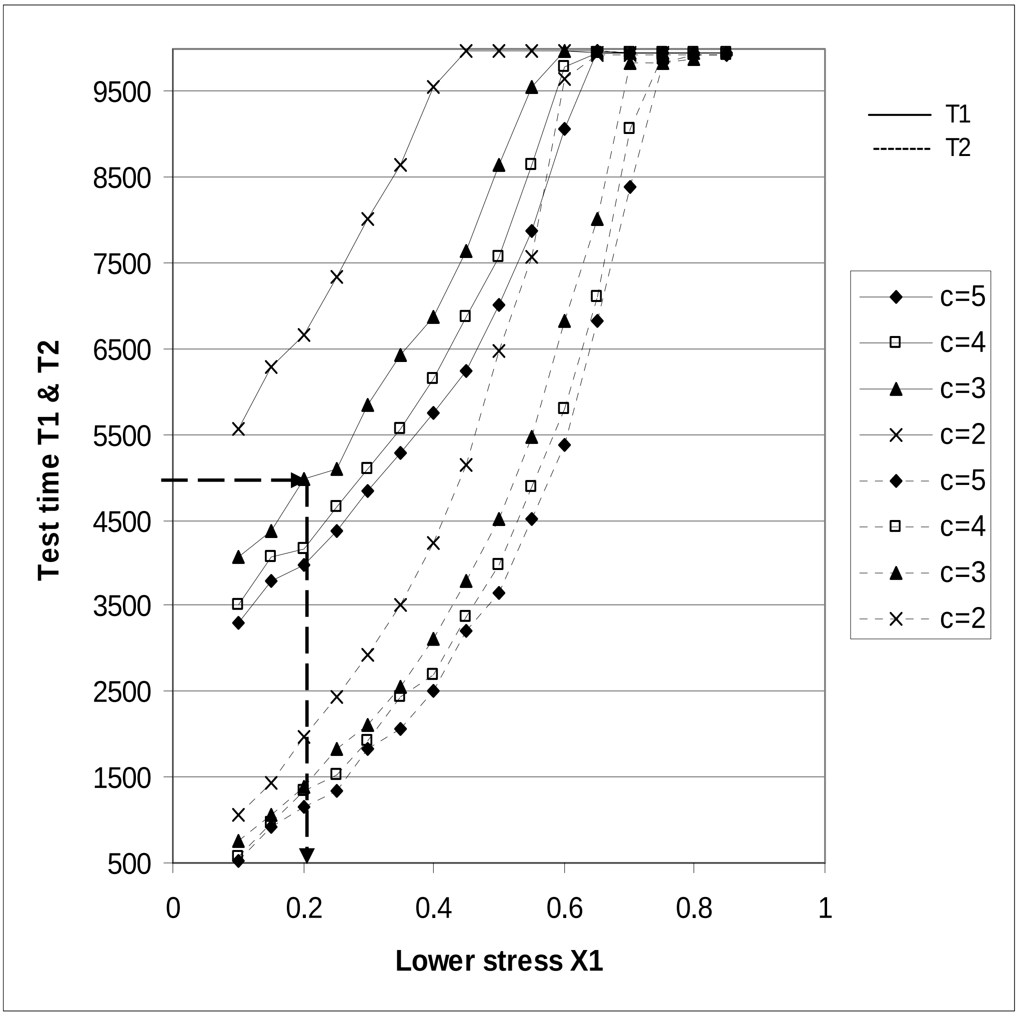

Figure 7.2 Plot of optimal test time (T_1 & T_2) Vs X_1 for various c in destructive CSADT plans

7.4.3 Determination of the Optimal Lower Stress X_1 with the Sample Size Constraint

Similarly, since the test requires a lot of samples (n>50) when tested at the optimal X_1, there may be a sample size constraint especially when the products are newly developed and a limited number of samples are available for test. A proper X_1 can be determined from an n Vs X1 plot as shown in Figure 7.3.

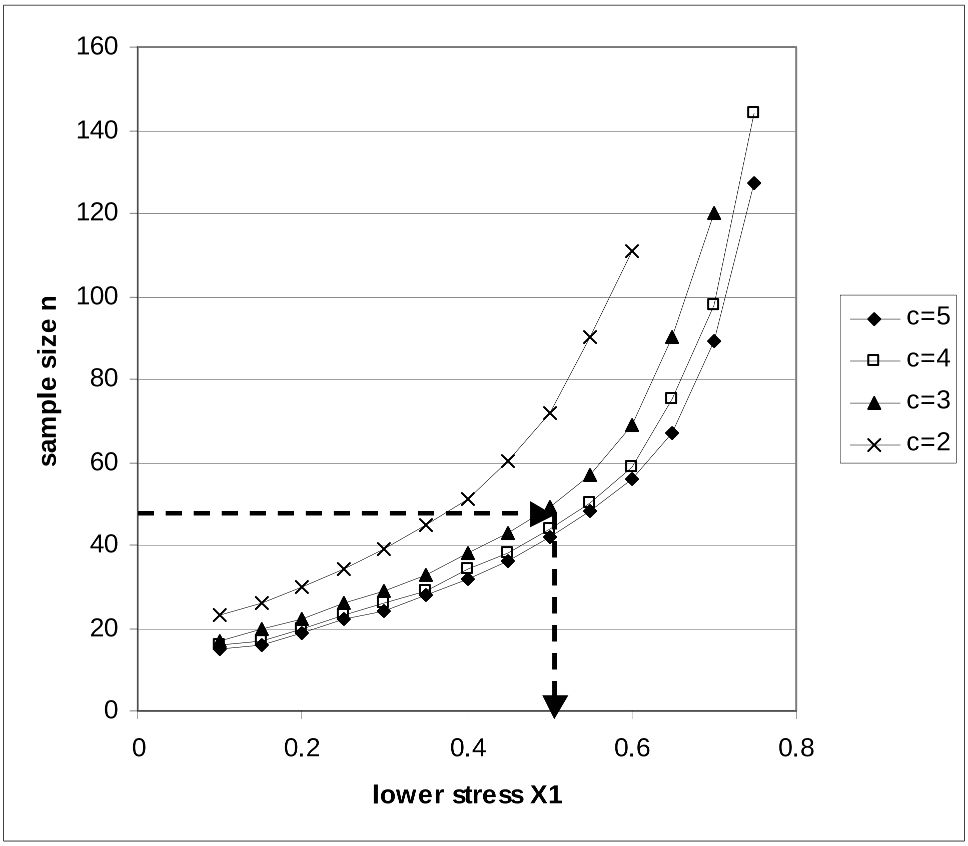

Figure 7.3. Plot of n Vs X_1 for various c in destructive CSADT plans

For example, if the number of available samples is 50 and the estimate precision requirement is defined as p=0.9 and c=3. The proper X_1 is approximately determined from Figure 3 or it can be calculated by taken a linear extrapolation in between X_1= 0.5 and X_1=0.55 as:

$$X_1=0.5 +(0.55-0.5)*(50-49)/(57-49)=0.506$$

7.4.4 Determination of the Optimal Lower Stress X_1 with Both Test Time and Sample Size Constraints

In the condition both the test time and the sample size are limited, a proper X_1 should be chosen to satisfy the two constraints. Mathematically, it is represented as follow:

X_1= Minimum (X_1|under the test time constraint, X_1|under the sample size constraint)

For example, if the test time is limited to 5000hrs, the number of available samples is 50, and the experimenter defines the estimate precision at p=0.9 and c=3, the lower test stress should be determined as below:

X_1= Minimum (X_1|under the test time constraint, X_1|under the sample size constraint)

= Minimum (0.202, 0.506)

=0.202

7.5 ROBUSTNESS ANALYSIS

Since the optimal plans are designed based on a guessed value of σ/a, we conduct the sensitivity analysis of design variables to the mis-specification of σ/a. For consistency, we assume the true value of σ/a is 100. Simulations of the optimal plans have been run for σ/a at 80, 90, 110 and 120 respectively. The detailed data set can be found in Appendix E2.

7.5.1 Sensitivity of n to σ/a

Table 7.4 summarizes the values of Rn for various deviations of σ/a from its true value 100. The results show that the optimal planning is moderately sensitive to the deviation of the pre-estimate of σ/a. Overestimating and underestimating σ/a have nearly the same effect on Rn. But when X_1 is selected greater than 0.5, the sensitivity is evident since Rn >deviation of σ/a. Hereby, we suggest $X_1<0.5$ be used in practice.

Table 7.4 Sensitivity of n to σ/a

X_1	σ/a dev. (%) \ Rn (%)	C = 2	3	4	5
0.1	-20	17.4	17.6	25	26.7
	-10	8.7	11.8	12.5	13.3
	10	8.7	11.8	6.3	6.7
	20	17.4	23.5	18.8	13.3
0.2	-20	20	18.2	20	21.1
	-10	10	4.5	10	10.5
	10	13.3	13.6	10	10.5
	20	23.3	22.7	20	15.8
0.3	-20	20.5	17.2	19.2	20.8
	-10	7.7	10.3	7.7	8.3
	10	12.8	13.8	11.5	12.5
	20	23.1	20.7	19.2	20.8
0.4	-20	19.6	21.1	20.6	21.9
	-10	9.8	10.5	11.8	9.4
	10	13.7	7.9	8.8	6.3
	20	27.5	18.4	17.6	18.8
0.5	-20	25	20.4	20.5	21.4
	-10	13.9	10.2	11.4	11.9
	10	18.1	10.2	9.1	7.1
	20	33.3	24.5	18.2	16.7
0.6	-20	27.9	23.2	20.3	21.4
	-10	13.5	13	8.5	10.7
	10	19.8	17.4	15.3	10.7
	20	42.3	31.9	32.2	25
0.7	-20	35.7	30	29.6	29.2
	-10	19	17.5	14.3	15.7
	10	21.3	20	16.3	14.6
	20	43.9	43.3	38.8	33.7

7.5.2 Sensitivity of π_1 to σ/a

Table 7.5 is a summary of values of $R\pi_1$ for various deviations of σ/a from its true value. The results show that the plans are robust against the departure of guessed σ/a from its true value. Overestimating and underestimating σ/a have nearly the same effect on $R\pi_1$.

Table 7.5 Sensitivity of π_1 to σ/a

X_1	σ/a dev. (%) \ $R\pi_1$ (%)	C = 2	3	4	5
0.1	-20	4.4	4.6	2.6	2.3
	-10	2	2.9	3.3	3.8
	10	3.2	4.1	1.4	1.6
	20	1.4	1.7	2.8	2.9
0.2	-20	3.4	0.7	0	0.5
	-10	1	1.8	3.7	4.2
	10	0.3	1	3	3.1
	20	0.5	1.9	5.6	1.3
0.3	-20	1.6	1.8	3.7	2.6
	-10	0	0.2	3.7	2.3
	10	2.3	1.8	5.4	0
	20	0	4.7	2.2	1.7
0.4	-20	1.1	0.3	1.9	2.4
	-10	0.5	2.2	2.5	0.7
	10	1.7	0.4	5	1.2
	20	3	1.5	1.2	1.1
0.5	-20	5.2	0.4	1.5	1.8
	-10	4.5	0.2	0.2	0.1
	10	1.6	0.1	2.2	0.8
	20	3.3	0.3	0.9	0.6
0.6	-20	5.5	2.4	2.7	0.6
	-10	2.8	2.2	0.7	2
	10	2.8	4.4	2.5	1.2
	20	2.8	4.3	5.4	0
0.7	-20	0.5	6.7	5.3	5.8
	-10	0.3	2.4	1.8	2.3
	10	0.4	0.5	2.6	2.7
	20	3.5	0.2	3.7	5.6

7.5.3 Sensitivity of T_1 and T_2 to σ/a

Values of RT_1 and RT_2 for various deviation of σ/a are shown in Table 7.6 and Table 7.7. The results indicate T_1 is moderately sensitive to departure of σ/a when X_1 <0.5, and is robust to the departure when X_1 > 0.5. While T_2 is moderately sensitive to mis-specified σ/a. Because overestimate of σ/a request longer testing time both at the lower stress and the highest stress. It satisfies the precision constraint conservatively.

Table 7.6 Sensitivity of T_1 to σ/a

X_1	σ/a dev. (%) \ RT_1 (%)	C = 2	C = 3	C = 4	C = 5
0.1	-20	19	20	19.2	17.4
	-10	12.1	5.9	6.8	7.2
	10	13.8	11.8	8.2	10.1
	20	23.3	18.8	20.5	23.2
0.2	-20	17.3	22.1	20.7	18.1
	-10	10.1	15.4	6.9	8.4
	10	6.5	7.7	13.8	9.6
	20	16.5	16.3	25.3	25.3
0.3	-20	21	23.8	19.8	20.8
	-10	12.6	9.8	12.3	10.9
	10	6	4.9	11.3	5.9
	20	15	15.6	19.8	20.8
0.4	-20	21.6	21	20.3	21.7
	-10	12.1	7.7	11.7	13.3
	10	3	11.2	8.6	14.2
	20	4.5	21.7	21.1	19.2
0.5	-20	7.7	20.6	19.6	21.2
	-10	1	10.6	9.5	8.9
	10	0	10	10.1	11
	20	0	15.6	20.3	23.3
0.6	-20	0	12	20.1	19
	-10	0	2.9	10.8	9.5
	10	0	0	2	9.5
	20	0	0	2	10.1
0.7	-20	0	0.2	0.2	0.2
	-10	0	0.2	0.2	0.2
	10	0	0.2	0.2	0.2
	20	0.2	0	0.2	0.2

Table 7.7 Sensitivity of T_2 to σ/a

X_1	σ/a dev. (%) \ RT_2 (%)	C = 2	C = 3	C = 4	C = 5
0.1	-20	36.4	31.3	8.3	9.1
	-10	9.1	18.8	8.3	0
	10	0	6.3	41.7	27.3
	20	18.2	6.3	25	45.5
0.2	-20	26.8	20.7	17.9	20.8
	-10	9.8	13.8	17.9	12.5
	10	7.3	3.4	0	8.3
	20	17.1	20.7	7.1	20.8
0.3	-20	16.4	20.5	22.5	15.8
	-10	11.5	9.1	12.5	13.2
	10	9.8	9.1	2.5	10.5
	20	21.3	27.3	22.5	15.8
0.4	-20	18.2	15.4	17.9	11.5
	-10	6.8	12.3	1.8	5.8
	10	12.5	13.8	16.1	13.5
	20	29.5	21.5	25	25
0.5	-20	24.4	18.1	19.3	14.5
	-10	13.3	8.5	7.2	6.6
	10	6.7	9.6	12	15.8
	20	23	16	24.1	23.7
0.6	-20	25.9	22.5	19	17.9
	-10	15.4	12	12.4	8.9
	10	3	7.7	9.1	8.9
	20	3.5	23.9	19	22.3
0.7	-20	1	19.4	19.5	20.5
	-10	0	4.8	12.6	8.5
	10	0.5	1.5	9.6	13.3
	20	0.1	1	9.1	19

7.6 CONCLUSIONS

Planning of CSADT in nature of destructive testing is studied in this chapter. The optimal sample size in total, the allocations and the testing time at each stress are determined to minimize the total cost of experiment and to achieve a requisites level of estimate precision. Additionally, the consideration of properly setting the lower stress is presented in case that the testing time or sample size is limited. Sensitivity analysis indicates that π_1 is robust to the mis-estimated σ/a, while n, T_1, T_2 are moderately sensitive to σ/a. X_1 <0.5 is suggested for a test plan to save the total cost of testing if individual sample is expensive.

Chapter 8

Conclusions and Future Research

This dissertation has surveyed the literature of main aspects related to AT, such as the commonly used lifetime distributions, the acceleration models, modeling of degradation process in ADT, parameter estimation methods, failure mechanism validation, non-destructive and destructive testing, analysis of AT data, planning of ALT/ADT and so on. After that, ALT and ADT have been studied in three aspects, namely, planning of multiple-stress CSALT (chapter 2), analysis of SSADT data (chapter 3) and planning of CSADT, SSADT and destructive CSADT (chapter 4 to 7).

Planning of multiple-stress CSALT has been presented in chapter 2. Different from the existing CSALT plans that minimize the asymptotic variance of the estimate of a particular reliability interest, we quantified the influence of variance inflation by relaxing the optimization criteria to the test plans. Assume that the failure time of product follows a Weibull distribution and the stress-life model is simply linear, we developed the design space for the low/middle stress and their corresponding allocation(s) in two/three stress plans. The result implies that the centroid of the lower and middle stress levels, weighted by their respective allocation, in our near optimal three-stress CSALT plan equals to the optimal low stress in the two-stress statistically optimal plan, which owns the smallest asymptotic variance. Based on the design space and the connection of three-stress plan with two-stress plans, three approaches were further imposed to plan three-stress tests. Our proposed plans allow the experimenter to validate the stress-life model by minimizing allocation to the middle stress such that there are sufficient failures for detecting non-linearity if it exists. Our plans can also serve as follow-up tests during product development when the failure mechanisms are possibly changed due to modification of designs. Furthermore, our plans are friendly in practical use. They provide flexibility for experimenters to set stress levels and sample allocations. Experimenters can adjust the lower/middle stress and their allocations within a range until it is convenient to implement.

A new way to analyze SSADT data has been studied in chapter 3. With a general stochastic model and a linear function to monitor the degradation process and the drift-stress relationship, we achieved the closed from estimation of unknown parameters. This method not only alleviates the difficulty to determine the particular deterministic degradation functions, but also provides an analytical solution for various applications when the acceleration variables are temperature, voltage and so on. From the expression of the reliability estimate, it is clear, as expected, that the unknown parameters can be solved easily and efficiently.

Planning of ADT has been studied in chapter 4 to chapter 7. In chapter 4, we presented to design both CSADT and SSADT in one general formulation. Motivated by the successful applications of stochastic models in ADT data analysis, we used stochastic processes to monitor the degradation paths in our ADT planning. Considering the tradeoff between the total cost of testing and the attainable estimate precision, we optimized the ADT plans by minimizing the total testing cost under the condition that the probability the estimated mean lifetime at use stress within a range of its true value is not less than a pre-specified precision level. Given the lower stress and the inspection time interval, we obtained the optimal sample size allocated at each stress, the stopping time (or number of inspections) at each stress in CSADT, and the optimal sample size, the stress changing time and the stopping time of the whole test in SSADT

It is seen that the general formulation is easily coded. Compared with the existing DT plan, our proposed plans require fewer test samples and less test duration if the lower stress and the inspection time interval are properly selected. As a result, the test cost can be greatly reduced. Compared with CSADT, SSADT saves time and cost significantly. Hence implementation of SSADT is highly recommended in real industry.

Based on the formulation in chapter 4, the optimal CSADT plans have been simulated and analyzed in chapter 5. We studied the influence of the lower test stress and the inspection time interval on optimal plans. It is shown that the inspection time interval has less effect on the optimal results, while the lower stress affects the optimal results intensively. Additionally, the proper X_1 can be determined by taking a cost budget into account. Sensitivity analysis of optimal plans to misspecifications of degradation parameters, σ/a, has also been carried out. It is seen that the optimal sample size and the stopping time are sensitive to σ/a when $X_1>0.5$. The other variables are not sensitive to σ/a. It is suggested regardless what planning method the experimenters are going to use, they should utilize all available information to get an accurate σ/a

before test planning. If it is not possible, an overestimate of σ/a would be helpful to lead to a conservative plan, which is more close to the optimal one.

The optimal SSADT plans have been studied in chapter 6. Besides the optimal sample size and holding time at each stress, we presented the guidance on how to determine the lower test stress, the inspection time interval and the precision constraint parameters. We recommended the lower stress and the inpection time interval be deteremined by the apex in the L_2/L_1 Vs X_1 convex contour plots. However, the contour pattern only exists for some combinations of c and p, hence, we further analyzed the scope of p copped with c. As a result, experimenters can tighten or loosen a plan by adjusting the values of c and p.

To overcome the inefficiency of two-stress plans, we also designed three-stress SSADT plans with additional planning rules in chapter 6. The results indicate that three-stress SSADT requires less samples and inspections to achieve the same level of estimate precision, and consequently, saves the total cost of test. Moreover, existence of intermediate stress is adequate to verify the stress-drift relationship, and to fasten the degradation speed.

Destructive CSADT plans have been addressed in chapter 7. The optimal sample size, the allocations and the testing time at each stress are determined. How to select a lower stress is also discussed in cases where the total testing time or sample size is limited. Sensitivity analysis indicates that ratio of optimal testing time at different stress levels is robust to the mis-estimated σ/a, while the optimal sample size and the testing time are moderately sensitive to σ/a. We recommend experimenters choose a lower stress less than 0.5 for a destructive CSADT because such a plan saves the total testing cost if an individual sample is expensive.

AT data analysis and experiment planning involves a wide range of problems. This dissertation mainly discussed SSADT data analysis, multiple-stress CSALT planning, two-stress CSADT planning (with or without destructive inspections), and two/three-stress SSADT planning. The planning strategy presented in this dissertation can be applied to design SSALTs and multiple-stress CSADTs for future research. However, it is not applicable to destructive SSADT planning because the Fisher Information Matrix, which plays an important role in deriving the asymptotic variance of MLE of the lifetime at use stress, does not exist. Thus, the estimate precision constraint based on ML estimates cannot be generated.

As in ALT planning, we have assumed the stress-life relationship is simply linear, current research can be extended to discuss the nonlinearity models. Some digamous analysis (Tseng & Wen, 2000) can be used to check whether the failure times or scale parameters at different stresses are linear or not. Additionally, one can future discuss the possible statistic tests that can be employed to test the null hypothesis that a relationship is linear versus an alternate hypothesis that it is not.

Another limitation of current research is that in three-stress ADT planning, the decision variables are six-dimensional, i.e. lower stress, middle stress, samples allocated at each stress or holding time at each stress, inspection times and inspection intervals. But the discussion has been restricted to a lower dimension by assuming lower stress and inspection interval are known. Even further details have been explained on how to determine these two variables, a more sophisticated method that can optimize all the six factors simultaneously and can achieve a global optimum are expected.

The ADT models allow lot of score for new research in terms of modeling the degradations, inspection points and the use of data in a dynamic manner to obtain estimates, Kalman filtering (Singpurwalla and Meinhold, 1986) offers scope for further research.

In addition, the emphasis of current work is to analyze or test individual components in a single acceleration environment. Future research can be directed to test two/three different components in multiple environments. The early paper by Zelen (1959) presented a factorial exponential method to analyze failure data collected from different components treated in different environments. The recent work by Singpurwalla (1986) and Kvam and Samaniego (1993, 1997) extended Zelen's model. Integrating the planning strategy presented in this dissertation and the analytical ideas mentioned in the above papers, future research of planning ALT and ADT in random environments can be promoted to improve the applicability of AT in practice.

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Appendix A: A Matlab program for analysing SSADT data

```
Function [V_final]=SSADT_analysis (CriD, n, Xk, Lk,DI, DeltT,  percentage, q)

% n is the number of samples in test
% xk is vector of test stress
% Lk is a verctor of inspection frequencies
% LogDIoverDeltT is an L by n matrix that records ln(DI/deltT)
%% deltT is the inspection time interval

%% the assumption is that the dispersion drift at different stress level follows log-linear function:
%% log(D/delatT)=alpha+beta*Xk

% the output itaU, deltaUsqure are the drift and diffusion parameters at use condition
% the output beta is the parameter in the log-linear function. It is the accelerated factor due to changing of stress level

% number of total inspection per item L
L=sum(Lk);
% Number of stress levels
TotNumS=sum(size(Xk))-1;

%X is the L by 2 matrix
X=ones(L,2);
p=1;
for i=1:TotNumS
    X(p:(p+Lk(i)-1),2)=Xk(i);
    p=p+Lk(i);
end
i=1;
j=1;
for i=1:L
  for j=1:n
    if DI(i,j)==0
      DI(i,j)=10^(-15);
   end
 end
end

LogDIoverDeltT=log(abs(DI./(DeltT*ones(1,n))));
% XD is X' Times D(L*n), a 2 by n matrix
XD=X'*LogDIoverDeltT;
% FinalsumXD is the r.h.s of the equations in report. It is the summarizations of all the increments, and the sum of the increment*stress
FinalsumXD=(sum(XD'))';

V=1/n*inv(X'*X)*FinalsumXD;

alpha=V(1,1);
```

```
beta=V(2,1);

%split the D matrix into parts, Dk---the Degradation increments at Xk
%calculate itak
p=1;
DeltUSqure=0;
for i=1:TotNumS
Ita(i)=exp(alpha+beta*Xk(i));
itakDeltT=(Ita(i)*DeltT)*ones(1,n);
PDeltUSqure=(sum((sum((DI(p:(p+Lk(i)-1),:)-itakDeltT(p:(p+Lk(i)-
1),:)).*(DI(p:(p+Lk(i)-1),:)-itakDeltT(p:(p+Lk(i)-1),:))))'))/(n*Lk(i)-1);
DeltUSqure=DeltUSqure+PDeltUSqure;
p=p+Lk(i);
end

ExpBetaXk=exp(beta*Xk);
SumExpBetaXk=sum(ExpBetaXk');

ItaU=exp(alpha);
DeltUSqure=DeltUSqure/SumExpBetaXk;
MeanU=CriD/ItaU;
LamdaU=CriD^2/DeltUSqure;

MeanU_lower=…
(1/MeanU+sqrt(FCDF((1-percentage), 1, (n*L-1))/(MeanU*LamdaU*(n*L-1))))^(-1);
MeanU_upper=…
1/(1/MeanU-sqrt(FCDF((1-percentage), 1, (n*L-1))/(MeanU*LamdaU*(n*L-1))));

LamdaU_lower=LamdaU*chi2inv(percentage/2, n*L-1)/(n*L);
LamdaU_upper=LamdaU*chi2inv(1-percentage/2, n*L-1)/(n*L);

Tq=MeanU/4*(sqrt(MeanU/LamdaU)*norminv(q, 0, 1)+…
sqrt(MeanU/LamdaU*norminv(q, 0, 1)^2+4))^2;

Tq_lower=MeanU_lower/4*(sqrt(MeanU_lower/LamdaU_upper)*norminv(q, 0, 1)+…
sqrt(MeanU_lower/LamdaU_upper*norminv(q, 0, 1)^2+4))^2;
Tq_upper=MeanU_upper/4*(sqrt(MeanU_upper/LamdaU_lower)*norminv(q, 0, 1)+…
sqrt(MeanU_upper/LamdaU_lower*norminv(q, 0, 1)^2+4))^2;

V_final=[ItaU, DeltUSqure, beta, MeanU, LamdaU, MeanU_upper, MeanU_lower,
LamdaU_upper, LamdaU_lower,Tq, Tq_lower];
```

Appendix B1: First and second order partial derivations of $LnL_{i,j,k}$

From equation (4.12)

$$\ln LH_{i,j,k} = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln(\Delta t) - \ln\sigma - \frac{U_{i,j,k}^2}{2}$$

where $$U_{i,j,k} = \frac{(\Delta D_{i,j,k} - \Delta t\eta_k)}{\sqrt{\Delta t}\sigma}, \begin{cases} k = 1 & \text{if} \quad j \le L_1 \\ k = 2 & \text{otherwise} \end{cases}$$

and equation (4.14) $$\ln LH = \sum_{i=1}^{n}\sum_{j=1}^{L} \ln LH_{i,j,k}$$

The first and second order partial differential of the $LnL_{i,j,k}$ can be derived as follows:

$$\frac{\partial LnL_{i,j}}{\partial a} = \frac{U_{i,j}\Delta t}{\sigma\sqrt{\Delta t}} \qquad \frac{\partial LnL_{i,j}}{\partial b} = \frac{U_{i,j}X_k\Delta t}{\sigma\sqrt{\Delta t}} \qquad \frac{\partial LnL_{i,j}}{\partial \sigma} = -\frac{1}{\sigma} + \frac{U_{i,j}{}^2}{\sigma}$$

$$\frac{\partial Ln^2 L_{i,j}}{\partial a^2} = -\frac{\Delta t}{\sigma^2} \qquad \frac{\partial Ln^2 L_{i,j}}{\partial b^2} = -\frac{X_k^2\Delta t}{\sigma^2} \qquad \frac{\partial Ln^2 L_{i,j}}{\partial \sigma^2} = \frac{1-3U_{i,j}{}^2}{\sigma^2}$$

$$\frac{\partial Ln^2 L_{i,j}}{\partial a\partial b} = -\frac{X_k\Delta t}{\sigma^2} \qquad \frac{\partial Ln^2 L_{i,j}}{\partial a\partial \sigma} = -\frac{2U_{i,j}\sqrt{\Delta t}}{\sigma^2} \qquad \frac{\partial Ln^2 L_{i,j}}{\partial b\partial \sigma} = -\frac{2U_{i,j}X_k\sqrt{\Delta t}}{\sigma^2}$$

Appendix B2: A VBA program to optimise CSADT and SSADT plans with a interactive dialog window

```
Dim st As Boolean

Private Sub CommandButton1_Click()
TextBoxcd.Value = Null
TextBoxcm1.Value = Null
TextBoxcm2.Value = Null
TextBoxco1.Value = Null
TextBoxco2.Value = Null

TextBoxsignalratio.Value = Null
TextBoxc.Value = Null
TextBoxp.Value = Null

TextBoxX1.Value = Null
TextBoxTcrita.Value = Null
TextBoxdeltT.Value = Null
End Sub

Private Sub CommandButton2_Click()
save_output
Hide
End Sub

Private Sub CommandButton3_Click()
st = True
End Sub

Private Sub CommandButtonstarrun_Click()

Dim Cd, Cm1, Cm2, Co1, Co2, signalratio, c, p, X1, deltT, Tcritical As Double

clear_output
Repaint
st = False

If TextBoxcd.Value = "" Then
MsgBox ("Please enter a value of individual sample cost Cd.")
Exit Sub
End If

If TextBoxcm1.Value = "" Then
MsgBox ("Please enter a value of measurement cost at X1 Cm1.")
Exit Sub
End If
```

```
If TextBoxcm2.Value = "" Then
MsgBox ("Please enter a value of measurement cost at X2 Cm2.")
Exit Sub
End If

If TextBoxco1.Value = "" Then
MsgBox ("Please enter a value of operation cost at X1 Co1.")
Exit Sub
End If

If TextBoxco2.Value = "" Then
MsgBox ("Please enter a value of operation cost at X2 Co2.")
Exit Sub
End If

If TextBoxX1.Value = "" Then
MsgBox ("Please enter a value of lower stress, X1.")
Exit Sub
End If

If TextBoxTcrita.Value = "" Then
MsgBox ("Please enter a value of longest allowable test time, Tcritical.")
Exit Sub
End If

If TextBoxdeltT.Value = "" Then
MsgBox ("Please enter a value of time interval between two inspection, deltT.")
Exit Sub
End If

If TextBoxc.Value = "" Then
MsgBox ("Please enter a value of precision requirement, c.")
Exit Sub
End If

If TextBoxp.Value = "" Then
MsgBox ("Please enter a value of precision level, p.")
Exit Sub
End If

If TextBoxsignalratio = "" Then
MsgBox ("Please enter a value of signalratio.")
Exit Sub
End If

If CSADT = False And SSADT = False Then
MsgBox ("Please seclet a the type of ADT")
Exit Sub
End If
```

```
Cd = TextBoxcd.Value
Cm1 = TextBoxcm1.Value
Cm2 = TextBoxcm2.Value
Co1 = TextBoxco1.Value
Co2 = TextBoxco2.Value

signalratio = TextBoxsignalratio.Value
c = TextBoxc.Value
p = TextBoxp.Value

X2 = 1
X1 = TextBoxX1.Value
Tcritical = TextBoxTcrita.Value
deltT = TextBoxdeltT.Value

Dim n As Integer
Dim T, T1, T2, pai1, pai2, q1, q2, r1, r2, w1, w2 As Double
Dim fai As Double
Dim cost As Double

Dim Ln As Integer
Dim LT, Lp, Lq As Double
Dim Costsave, Tsave, pai1save, q2save As Double
Dim nsave, rol As Integer

Ln = 1
q1 = 1
Costsave = 10000000

If CSADT.Value = True Then

For n = 1 To 200 Step Ln
Lp = 100 / n
For T = deltT To Tcritical Step deltT
Lq = 100 / (T / deltT)
For pai11 = 1 To 99 Step Lp
pai1 = pai11 / 100
pai2 = 1 - pai1
'for CSADT, wk=qk, rk=paik, q1=1, 0<q2<1
For q22 = Lq To 100 Step Lq
q2 = q22 / 100
w1 = q1
w2 = q2
r1 = pai1
r2 = pai2
Q = -(X1 * X1 * pai1 * q1 + X2 * X2 * pai2 * q2) / (T * ((X1 * pai1 * q1 + X2 * pai2
* q2) ^ 2 - (X1 * X1 * pai1 * q1 + X2 * X2 * pai2 * q2) * (r1 * q1 + r2 * q2)))
fai = Application.WorksheetFunction.NormSDist((c - 1) * n ^ 0.5 / (signalratio * Q ^
0.5)) - Application.WorksheetFunction.NormSDist((1 / c - 1) * n ^ 0.5 / (signalratio *
Q ^ 0.5))
```

```
If fai >= p Then
cost = Cd * n + n * T / deltT * (Cm1 * pai1 * q1 + Cm2 * pai2 * q2) + T * (Co1 * w1
+ Co2 * w2)
If Costsave >= cost Then
Costsave = cost
nsave = n
Tsave = T
pai1save = pai1
q2save = q2
End If
End If

Next
Next
Next
Next
TextBoxn.Value = nsave
TextBoxT.Value = Tsave
TextBoxpai1.Value = pai1save
TextBoxpai2.Value = 1 - pai1save
TextBoxq1.Value = q1
TextBoxq2.Value = q2save
TextBoxcost.Value = Costsave

TextBoxn1.Value = Round(nsave * pai1save)
TextBoxn2.Value = nsave - Round(nsave * pai1save)
TextBoxT1.Value = Tsave
TextBoxT2.Value = Tsave * q2save
TextBoxL1.Value = Tsave / deltT
TextBoxL2.Value = Tsave * q2save / deltT
End If


If SSADT.Value = True Then
q1 = 1
q2 = 1
r1 = q1
r2 = q2

For n = 1 To 200 Step Ln
For T = deltT To Tcritical Step deltT
Lp = 100 / (T / deltT)
For pai11 = 1 To 99 Step Lp
pai1 = (pai11 - 1) / 100
pai2 = 1 - pai1
w1 = pai1
w2 = pai2

If st = True Then
Exit Sub
```

```
End If

Q = -(X1 * X1 * pai1 * q1 + X2 * X2 * pai2 * q2) / (T * ((X1 * pai1 * q1 + X2 * pai2
* q2) ^ 2 - (X1 * X1 * pai1 * q1 + X2 * X2 * pai2 * q2) * (r1 * q1 + r2 * q2)))
fai = Application.WorksheetFunction.NormSDist((c - 1) * n ^ 0.5 / (signalratio * Q ^
0.5)) - Application.WorksheetFunction.NormSDist((1 / c - 1) * n ^ 0.5 / (signalratio *
Q ^ 0.5))
If fai >= p Then
cost = Cd * n + n * T / deltT * (Cm1 * pai1 * q1 + Cm2 * pai2 * q2) + T * (Co1 * w1
+ Co2 * w2)
If Costsave >= cost Then
Costsave = cost
nsave = n
Tsave = T
pai1save = pai1
End If
End If

Next
Next
Next
TextBoxn.Value = nsave
TextBoxT.Value = Tsave
TextBoxpai1.Value = pai1save
TextBoxpai2.Value = 1 - pai1save
TextBoxq1.Value = q1
TextBoxq2.Value = q2
TextBoxcost.Value = Costsave
TextBoxn1.Value = nsave
TextBoxn2.Value = nsave
TextBoxT1.Value = Tsave * pai1save
TextBoxT2.Value = Tsave - Tsave * pai1save
TextBoxL1.Value = TextBoxT1.Value / deltT
TextBoxL2.Value = TextBoxT2.Value / deltT
End If

End Sub


Private Sub CSADT_Click()
CSADT.Enabled = True
End Sub

Private Sub Label2_Click()
MsgBox ("this is the measurment cost")
End Sub

Private Sub SSADT_Click()
SSADT.Enabled = True
End Sub
```

```
Private Sub save_output()
Dim myRange As Range
Set myRange = Worksheets("user_interactive").Range("A1:D5")
'myRange.Cells.Item(1, 1) = TextBoxn.Value
TextBoxn.Value = nsave
TextBoxT.Value = Tsave
TextBoxpai1.Value = pai1save
TextBoxpai2.Value = 1 - pai1save
TextBoxq1.Value = q1
TextBoxq2.Value = q2
TextBoxcost.Value = Costsave
TextBoxn1.Value = nsave
TextBoxn2.Value = nsave
TextBoxT1.Value = Tsave * pai1save
TextBoxT2.Value = Tsave * (1 - pai1save)
End Sub

Private Sub clear_output()
TextBoxn.Value = Null
TextBoxT.Value = Null
TextBoxpai1.Value = Null
TextBoxpai2.Value = Null
TextBoxq1.Value = Null
TextBoxq2.Value = Null
TextBoxcost.Value = Null
TextBoxn1.Value = Null
TextBoxn2.Value = Null
TextBoxT1.Value = Null
TextBoxT2.Value = Null
TextBoxL1.Value = Null
TextBoxL2.Value = Null
End Sub
```

Appendix C: Optimal CSADT plans with mis-specified σ/a

σ/a	c	X_1^o	n^o	T_1^o	T_2^o	T^o	n_1^o	n_2^o	L_1^0	L_2^o
80	2	0.05	15	4080	720	4080	13	2	17	3
		0.1	18	4560	960	4560	14	4	19	4
		0.15	20	4800	1680	4800	15	5	20	7
		0.2	25	4800	1920	4800	18	7	20	8
		0.25	30	4800	2400	4800	21	9	20	10
		0.3	36	4800	2880	4800	25	11	20	12
		0.35	42	4800	3840	4800	29	13	20	16
		0.4	50	4800	4800	4800	37	13	20	20
		0.45	64	4800	4800	4800	46	18	20	20
		0.5	83	4800	4800	4800	58	25	20	20
		0.55	109	4800	4800	4800	74	35	20	20
		0.6	146	4800	4800	4800	92	54	20	20
	3	0.05	11	3120	480	3120	9	2	13	2
		0.1	13	3120	960	3120	11	2	13	4
		0.15	14	4080	960	4080	10	4	17	4
		0.2	17	4080	1200	4080	12	5	17	5
		0.25	19	4560	1680	4560	12	7	19	7
		0.3	22	4800	1920	4800	14	8	20	8
		0.35	26	4560	2640	4560	17	9	19	11
		0.4	30	4800	3360	4800	20	10	20	14
		0.45	37	4800	3840	4800	23	14	20	16
		0.5	46	4800	4320	4800	30	16	20	18
		0.55	59	4800	4800	4800	40	19	20	20
		0.6	79	4800	4800	4800	50	29	20	20
	4	0.05	10	2640	480	2640	9	1	11	2
		0.1	12	2880	720	2880	9	3	12	3
		0.15	12	3600	960	3600	9	3	15	4
		0.2	14	3600	1440	3600	10	4	15	6
		0.25	18	3600	1440	3600	12	6	15	6
		0.3	19	4320	1920	4320	13	6	18	8
		0.35	20	4800	2640	4800	13	7	20	11
		0.4	27	4560	2400	4560	17	10	19	10
		0.45	33	4560	2880	4560	20	13	19	12
		0.5	38	4800	3840	4800	23	15	20	16
		0.55	48	4800	4320	4800	30	18	20	18
		0.6	65	4800	4320	4800	40	25	20	18
		0.65	87	4800	4800	4800	54	33	20	20
To be continued										

Continued

σ/a	c	X_1^o	n^o	T_1^o	T_2^o	T^o	n_1^o	n_2^o	L_1^0	L_2^o
90	2	0.05	16	4800	960	4800	14	2	20	4
		0.1	21	4800	1200	4800	16	5	20	5
		0.15	26	4800	1440	4800	20	6	20	6
		0.2	30	4800	2400	4800	22	8	20	10
		0.25	36	4800	2880	4800	26	10	20	12
		0.3	42	4800	3840	4800	31	11	20	16
		0.35	52	4800	4080	4800	37	15	20	17
		0.4	63	4800	4800	4800	45	18	20	20
		0.45	82	4800	4560	4800	58	24	20	19
		0.5	104	4800	4800	4800	71	33	20	20
		0.55	137	4800	4800	4800	89	48	20	20
		0.6	185	4800	4800	4800	118	67	20	20
	3	0.05	12	3600	480	3600	10	2	15	2
		0.1	14	4080	720	4080	11	3	17	3
		0.15	17	3840	1440	3840	13	4	16	6
		0.2	18	4560	1920	4560	13	5	19	8
		0.25	21	4800	2160	4800	15	6	20	9
		0.3	26	4800	2400	4800	17	9	20	10
		0.35	32	4560	2880	4560	22	10	19	12
		0.4	38	4800	3360	4800	25	13	20	14
		0.45	45	4800	4320	4800	31	14	20	18
		0.5	57	4800	4800	4800	40	17	20	20
		0.55	77	4800	4320	4800	49	28	20	18
		0.6	100	4800	4800	4800	64	36	20	20
	4	0.05	11	3120	480	3120	9	2	13	2
		0.1	13	3120	960	3120	11	2	13	4
		0.15	14	4080	960	4080	10	4	17	4
		0.2	17	4080	1200	4080	12	5	17	5
		0.25	19	4560	1680	4560	12	7	19	7
		0.3	22	4800	1920	4800	14	8	20	8
		0.35	26	4560	2640	4560	17	9	19	11
		0.4	30	4800	3360	4800	20	10	20	14
		0.45	37	4800	3840	4800	23	14	20	16
		0.5	46	4800	4320	4800	30	16	20	18
		0.55	59	4800	4800	4800	40	19	20	20
		0.6	79	4800	4800	4800	50	29	20	20
		0.65	110	4800	4800	4800	70	40	20	20

To be continued

Continued										
σ/a	c	X_1^o	n^o	T_1^o	T_2^o	T^o	n_1^o	n_2^o	L_1^0	L_2^o
110	2	0.05	25	4560	960	4560	22	3	19	4
		0.1	30	4800	1440	4800	25	5	20	6
		0.15	36	4800	2160	4800	28	8	20	9
		0.2	42	4800	3120	4800	33	9	20	13
		0.25	51	4800	3600	4800	39	12	20	15
		0.3	61	4800	4320	4800	46	15	20	18
		0.35	75	4800	4800	4800	57	18	20	20
		0.4	94	4800	4800	4800	67	27	20	20
		0.45	120	4800	4800	4800	84	36	20	20
		0.5	156	4800	4800	4800	108	48	20	20
	3	0.05	14	4560	720	4560	12	2	19	3
		0.1	17	4560	1440	4560	14	3	19	6
		0.15	21	4800	1440	4800	16	5	20	6
		0.2	26	4560	2160	4560	19	7	19	9
		0.25	30	4800	2640	4800	21	9	20	11
		0.3	36	4800	3120	4800	25	11	20	13
		0.35	42	4800	4080	4800	30	12	20	17
		0.4	51	4800	4800	4800	37	14	20	20
		0.45	66	4800	4560	4800	46	20	20	19
		0.5	84	4800	4800	4800	56	28	20	20
		0.55	111	4800	4800	4800	74	37	20	20
		0.6	150	4800	4800	4800	98	52	20	20
	4	0.05	12	4320	480	4320	10	2	18	2
		0.1	16	3840	1200	3840	13	3	16	5
		0.15	17	4560	1680	4560	13	4	19	7
		0.2	22	4560	1680	4560	15	7	19	7
		0.25	25	4800	2160	4800	17	8	20	9
		0.3	29	4800	2880	4800	20	9	20	12
		0.35	36	4800	3120	4800	24	12	20	13
		0.4	43	4800	3840	4800	28	15	20	16
		0.45	52	4800	4560	4800	35	17	20	19
		0.5	67	4800	4800	4800	47	20	20	20
		0.55	89	4800	4560	4800	57	32	20	19
		0.6	118	4800	4800	4800	76	42	20	20
		0.65	164	4800	4800	4800	102	62	20	20
To be Continued										

Continued										
σ/a		X_1^o	n^o	T_1^o	T_2^o	T^o	n_1^o	n_2^o	L_1^0	L_2^o
120	2	0.05	28	4800	1200	4800	24	4	20	5
		0.1	34	4800	1920	4800	29	5	20	8
		0.15	42	4800	2400	4800	33	9	20	10
		0.2	50	4800	3120	4800	39	11	20	13
		0.25	59	4800	4080	4800	46	13	20	17
		0.3	71	4800	4800	4800	55	16	20	20
		0.35	89	4800	4800	4800	67	22	20	20
		0.4	112	4800	4800	4800	82	30	20	20
		0.45	143	4800	4800	4800	101	42	20	20
		0.5	185	4800	4800	4800	126	59	20	20
	3	0.05	17	4560	720	4560	14	3	19	3
		0.1	20	4800	1200	4800	16	4	20	5
		0.15	25	4800	1440	4800	19	6	20	6
		0.2	28	4800	2640	4800	22	6	20	11
		0.25	34	4800	3120	4800	25	9	20	13
		0.3	42	4800	3360	4800	30	12	20	14
		0.35	50	4800	4080	4800	36	14	20	17
		0.4	61	4800	4800	4800	45	16	20	20
		0.45	78	4800	4800	4800	56	22	20	20
		0.5	100	4800	4800	4800	68	32	20	20
		0.55	132	4800	4800	4800	88	44	20	20
		0.6	178	4800	4800	4800	115	63	20	20
	4	0.05	15	4080	480	4080	13	2	17	2
		0.1	16	4560	1440	4560	13	3	19	6
		0.15	20	4800	1440	4800	15	5	20	6
		0.2	24	4800	1920	4800	17	7	20	8
		0.25	28	4800	2640	4800	20	8	20	11
		0.3	33	4800	3360	4800	24	9	20	14
		0.35	41	4800	3600	4800	28	13	20	15
		0.4	51	4800	3840	4800	35	16	20	16
		0.45	61	4800	4800	4800	42	19	20	20
		0.5	79	4800	4800	4800	53	26	20	20
		0.55	104	4800	4800	4800	68	36	20	20
		0.6	141	4800	4800	4800	92	49	20	20
		0.65	195	4800	4800	4800	121	74	20	20

Appendix D: Optimal SSADT plans with mis-specified σ/a

σ / a	c	X_1	n	T_1	T_2	T	L	L_1	L_2
		0.05	8	2880	240	3120	13	12	1
		0.1	10	2400	240	2640	11	10	1
		0.15	10	2640	240	2880	12	11	1
		0.2	10	2640	480	3120	13	11	2
		0.25	9	2880	720	3600	15	12	3
		0.3	11	2640	480	3120	13	11	2
		0.35	10	2880	720	3600	15	12	3
		0.4	10	3360	480	3840	16	14	2
		0.45	10	2880	960	3840	16	12	4
		0.5	12	2880	480	3360	14	12	2
		0.55	10	3120	960	4080	17	13	4
		0.6	11	3360	480	3840	16	14	2
		0.65	12	3360	240	3600	15	14	1
		0.7	12	3120	480	3600	15	13	2
	2	>=0.75	13	3120	240	3360	14	13	1
		0.05	7	1680	240	1920	8	7	1
		0.1	6	2160	240	2400	10	9	1
		0.15	8	1680	240	1920	8	7	1
		0.2	7	2160	240	2400	10	9	1
		0.25	7	2400	240	2640	11	10	1
		0.3	7	2160	480	2640	11	9	2
		0.35	9	1680	480	2160	9	7	2
		0.4	7	2160	720	2880	12	9	3
		0.45	8	2160	480	2640	11	9	2
		0.5	9	1920	480	2400	10	8	2
		0.55	8	2640	240	2880	12	11	1
		0.6	8	2640	240	2880	12	11	1
		0.65	8	2400	480	2880	12	10	2
	3	>=0.7	9	2400	240	2640	11	10	1
		0.05	5	1920	240	2160	9	8	1
		0.1	6	1680	240	1920	8	7	1
		0.15	6	1920	240	2160	9	8	1
		0.2	7	1680	240	1920	8	7	1
		0.25	6	2160	240	2400	10	9	1
		0.3	7	1920	240	2160	9	8	1
		0.35	8	1440	480	1920	8	6	2
		0.4	6	1920	720	2640	11	8	3
		0.45	7	1920	480	2400	10	8	2
		0.5	8	1680	480	2160	9	7	2
		0.55	8	1440	720	2160	9	6	3
		0.6	7	2400	240	2640	11	10	1
		0.65	7	2400	240	2640	11	10	1
		0.7	7	2400	240	2640	11	10	1
		0.75	7	2160	480	2640	11	9	2
	4	>=0.8	8	2160	240	2400	10	9	1
		0.05	5	1680	240	1920	8	7	1
80	5	0.1	6	1440	240	1680	7	6	1
To be continued									

Continued									
σ / a	c	X_1	n	T_1	T_2	T	L	L_1	L_2
		0.15	5	1920	240	2160	9	8	1
		0.2	6	1680	240	1920	8	7	1
		0.25	6	1920	240	2160	9	8	1
		0.3	6	1680	480	2160	9	7	2
		0.35	7	1440	480	1920	8	6	2
		0.4	6	1920	480	2400	10	8	2
		0.45	7	1920	240	2160	9	8	1
		0.5	7	1680	480	2160	9	7	2
		0.55	6	2400	240	2640	11	10	1
		0.6	6	2160	480	2640	11	9	2
		0.65	6	1920	720	2640	11	8	3
80	5	>=0.7	7	2160	240	2400	10	9	1
		0.05	10	2880	240	3120	13	12	1
		0.1	10	3120	240	3360	14	13	1
		0.15	10	3120	480	3600	15	13	2
		0.2	10	3120	720	3840	16	13	3
		0.25	10	3360	720	4080	17	14	3
		0.3	12	2880	720	3600	15	12	3
		0.35	10	3840	720	4560	19	16	3
		0.4	11	3600	720	4320	18	15	3
		0.45	13	3360	480	3840	16	14	2
		0.5	14	2880	720	3600	15	12	3
		0.55	13	3600	480	4080	17	15	2
		0.6	13	3360	720	4080	17	14	3
		0.65	12	4320	240	4560	19	18	1
		0.7	12	4080	480	4560	19	17	2
	2	>=0.75	11	4800	240	5040	21	20	1
		0.05	7	2160	240	2400	10	9	1
		0.1	7	2400	240	2640	11	10	1
		0.15	9	1920	240	2160	9	8	1
		0.2	8	2160	480	2640	11	9	2
		0.25	8	2640	240	2880	12	11	1
		0.3	9	2160	480	2640	11	9	2
		0.35	8	2640	480	3120	13	11	2
		0.4	9	2400	480	2880	12	10	2
		0.45	8	2880	480	3360	14	12	2
		0.5	9	2640	480	3120	13	11	2
		0.55	10	2640	240	2880	12	11	1
		0.6	10	2400	480	2880	12	10	2
		0.65	10	2160	720	2880	12	9	3
	3	>=0.7	9	3120	240	3360	14	13	1
		0.05	7	1680	240	1920	8	7	1
		0.1	6	2160	240	2400	10	9	1
		0.15	8	1680	240	1920	8	7	1
		0.2	7	2160	240	2400	10	9	1
		0.25	7	2400	240	2640	11	10	1
		0.3	7	2160	480	2640	11	9	2
		0.35	9	1680	480	2160	9	7	2
		0.4	7	2160	720	2880	12	9	3
		0.45	8	2160	480	2640	11	9	2
90	4	0.5	9	1920	480	2400	10	8	2
To be continued									

Continued

σ / a	c	X_1	n	T_1	T_2	T	L	L_1	L_2
		0.55	8	2640	240	2880	12	11	1
		0.6	8	2640	240	2880	12	11	1
		0.65	8	2400	480	2880	12	10	2
	4	>=0.7	9	2400	240	2640	11	10	1
		0.05	5	2160	240	2400	10	9	1
		0.1	6	1920	240	2160	9	8	1
		0.15	6	2160	240	2400	10	9	1
		0.2	7	1920	240	2160	9	8	1
		0.25	6	2400	240	2640	11	10	1
		0.3	7	2160	240	2400	10	9	1
		0.35	8	1680	480	2160	9	7	2
		0.4	7	2400	240	2640	11	10	1
		0.45	7	2160	480	2640	11	9	2
		0.5	8	1920	480	2400	10	8	2
		0.55	8	1680	720	2400	10	7	3
		0.6	7	2400	480	2880	12	10	2
		0.65	7	2400	480	2880	12	10	2
90	5	>=0.7	8	2400	240	2640	11	10	1
		0.05	12	3600	240	3840	16	15	1
		0.1	13	3600	240	3840	16	15	1
		0.15	14	3360	480	3840	16	14	2
		0.2	15	3360	480	3840	16	14	2
		0.25	15	3360	720	4080	17	14	3
		0.3	15	3600	720	4320	18	15	3
		0.35	15	3840	720	4560	19	16	3
		0.4	14	4080	960	5040	21	17	4
		0.45	16	3600	960	4560	19	15	4
		0.5	15	4080	960	5040	21	17	4
		0.55	15	4800	480	5280	22	20	2
		0.6	16	4560	480	5040	21	19	2
		0.65	16	4320	720	5040	21	18	3
		0.7	17	4320	480	4800	20	18	2
	2	>=0.75	15	5280	240	5520	23	22	1
		0.05	8	2880	240	3120	13	12	1
		0.1	8	3120	240	3360	14	13	1
		0.15	9	3120	240	3360	14	13	1
		0.2	10	2640	480	3120	13	11	2
		0.25	10	2880	480	3360	14	12	2
		0.3	10	3120	480	3600	15	13	2
		0.35	11	2640	720	3360	14	11	3
		0.4	10	3120	720	3840	16	13	3
		0.45	11	2880	720	3600	15	12	3
		0.5	12	2400	960	3360	14	10	4
		0.55	12	3360	240	3600	15	14	1
		0.6	12	3120	480	3600	15	13	2
		0.65	12	2880	720	3600	15	12	3
	3	>=0.7	11	3840	240	4080	17	16	1
		0.05	7	2640	240	2880	12	11	1
		0.1	9	2160	240	2400	10	9	1
		0.15	8	2400	480	2880	12	10	2
110	4	0.2	8	2640	480	3120	13	11	2

To be continued

Continued

σ / a	c	X_1	n	T_1	T_2	T	L	L_1	L_2
		0.25	10	2160	480	2640	11	9	2
		0.3	9	2640	480	3120	13	11	2
		0.35	10	2160	720	2880	12	9	3
		0.4	9	2640	720	3360	14	11	3
		0.45	10	2400	720	3120	13	10	3
		0.5	9	2880	720	3600	15	12	3
		0.55	10	2880	480	3360	14	12	2
		0.6	10	2640	720	3360	14	11	3
		0.65	11	2640	480	3120	13	11	2
		0.7	9	3120	720	3840	16	13	3
	4	>=0.75	10	3360	240	3600	15	14	1
		0.05	6	2640	240	2880	12	11	1
		0.1	8	2160	240	2400	10	9	1
		0.15	7	2400	480	2880	12	10	2
		0.2	9	1920	480	2400	10	8	2
		0.25	8	2400	480	2880	12	10	2
		0.3	8	2640	480	3120	13	11	2
		0.35	9	2400	480	2880	12	10	2
		0.4	8	2880	480	3360	14	12	2
		0.45	9	2640	480	3120	13	11	2
		0.5	10	2400	480	2880	12	10	2
		0.55	10	2160	720	2880	12	9	3
		0.6	9	2880	480	3360	14	12	2
		0.65	9	2880	480	3360	14	12	2
110	5	>=0.7	10	2880	240	3120	13	12	1
		0.05	12	4320	240	4560	19	18	1
		0.1	13	4080	480	4560	19	17	2
		0.15	14	4080	480	4560	19	17	2
		0.2	15	3840	720	4560	19	16	3
		0.25	16	3840	720	4560	19	16	3
		0.3	16	3840	960	4800	20	16	4
		0.35	16	4080	960	5040	21	17	4
		0.4	16	4320	960	5280	22	18	4
		0.45	18	3600	1200	4800	20	15	5
		0.5	18	4320	720	5040	21	18	3
		0.55	17	4800	720	5520	23	20	3
		0.6	16	5520	480	6000	25	23	2
		0.65	16	5280	720	6000	25	22	3
		0.7	17	5520	240	5760	24	23	1
		0.75	17	5280	480	5760	24	22	2
	2	>=0.8	18	5280	240	5520	23	22	1
		0.05	9	3120	240	3360	14	13	1
		0.1	9	3360	240	3600	15	14	1
		0.15	9	3360	480	3840	16	14	2
		0.2	11	2880	480	3360	14	12	2
		0.25	11	3120	480	3600	15	13	2
		0.3	11	3360	480	3840	16	14	2
		0.35	13	2640	720	3360	14	11	3
		0.4	12	3120	720	3840	16	13	3
		0.45	11	3600	720	4320	18	15	3
120	3	0.5	12	3360	720	4080	17	14	3

To be continued

Continued									
σ / a	c	X_1	n	T_1	T_2	T	L	L_1	L_2
		0.55	13	3120	720	3840	16	13	3
		0.6	12	3840	480	4320	18	16	2
		0.65	13	3840	240	4080	17	16	1
		0.7	13	3840	240	4080	17	16	1
		0.75	13	3840	240	4080	17	16	1
		0.8	13	3600	480	4080	17	15	2
	3	>=0.85	14	3600	240	3840	16	15	1
		0.05	9	2400	240	2640	11	10	1
		0.1	9	2640	240	2880	12	11	1
		0.15	9	2880	240	3120	13	12	1
		0.2	9	3120	240	3360	14	13	1
		0.25	10	2640	480	3120	13	11	2
		0.3	10	2880	480	3360	14	12	2
		0.35	11	2400	720	3120	13	10	3
		0.4	10	2880	720	3600	15	12	3
		0.45	11	2400	960	3360	14	10	4
		0.5	10	3120	720	3840	16	13	3
		0.55	11	2880	720	3600	15	12	3
		0.6	12	2880	480	3360	14	12	2
		0.65	10	3360	720	4080	17	14	3
	4	>=0.7	11	3600	240	3840	16	15	1
		0.05	8	2400	240	2640	11	10	1
		0.1	8	2640	240	2880	12	11	1
		0.15	10	2160	240	2400	10	9	1
		0.2	9	2400	480	2880	12	10	2
		0.25	9	2640	480	3120	13	11	2
		0.3	10	2160	720	2880	12	9	3
		0.35	9	2640	720	3360	14	11	3
		0.4	11	2400	480	2880	12	10	2
		0.45	9	2640	960	3600	15	11	4
		0.5	10	2640	720	3360	14	11	3
		0.55	11	2400	720	3120	13	10	3
		0.6	10	3120	480	3600	15	13	2
		0.65	10	3120	480	3600	15	13	2
120	5	>=0.7	11	3120	240	3360	14	13	1

Appendix E1: Derivation of estimate precision constraint for destructive CSADT planning

From equation (7.1), the log-likelihood of each increment is

$$\ln L_i = -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln\left(\sum_{k=1}^{2} T_k\right) - \ln\sigma - \frac{U_i^2}{2}$$

$$\text{where } U_i = \frac{\left(\Delta D_i - T_k\left(a + bX_k\right)\right)}{\sqrt{T_k}\,\sigma}$$

The first partial derivates of U_i and LnL are respectively:

$$\frac{\partial U_i}{\partial a} = -\frac{T_k}{\sigma\sqrt{T_k}}$$

$$\frac{\partial U_i}{\partial b} = -\frac{T_k X_k}{\sigma T_k}$$

$$\frac{\partial U_i}{\partial \sigma} = -\frac{U_i}{\sigma}$$

$$\frac{\partial LnL_i}{\partial a} = (-U_i)\cdot\left(-\frac{T_k}{\sigma\sqrt{T_k}}\right) = \frac{U_i T_k}{\sigma\sqrt{T_k}}$$

$$\frac{\partial LnL_i}{\partial b} = -U_i * \left(-\frac{T_k X_k}{\sigma\sqrt{T_k}}\right) = U_i \frac{T_k X_k}{\sigma\sqrt{T_k}}$$

$$\frac{\partial LnL_i}{\partial \sigma} = -\frac{1}{\sigma} - U_i\cdot\left(-\frac{U_i}{\sigma}\right) = -\frac{1}{\sigma} + \frac{U_i^2}{\sigma}$$

The second order partial derivates of LnL are:

$$\frac{\partial Ln^2 L_i}{\partial a^2} = -\left(\frac{\sum_{k=1}^{2} T_k}{\sigma\sqrt{\sum_{k=1}^{2} T_k}}\right)^2 = -\frac{T_k}{\sigma^2}$$

$$\frac{\partial Ln^2 L_i}{\partial b^2} = -\frac{\left(T_k X_k\right)^2}{\sigma^2 T_k} = -\frac{T_k X_k^2}{\sigma^2}$$

$$\frac{\partial Ln^2 L_i}{\partial \sigma^2} = \frac{1}{\sigma^2} + \frac{2U_i}{\sigma}\left(-\frac{U_i}{\sigma}\right) - \frac{U_i^2}{\sigma^2} = \frac{1-3U_i^2}{\sigma^2}$$

$$\frac{\partial Ln^2 L_i}{\partial a \partial b} = \frac{T_k}{\sigma\sqrt{T_k}}\cdot\left(-\frac{T_k X_k}{\sigma\sqrt{T_k}}\right) = -\frac{T_k X_k}{\sigma^2}$$

$$\frac{\partial Ln^2 L_i}{\partial a \partial \sigma} = \frac{T_k}{\sigma\sqrt{T_k}}\cdot\left(-\frac{U_i}{\sigma}\right)\cdot 2 = -\frac{2U_i T_k}{\sigma^2\sqrt{T_k}}$$

$$\frac{\partial LnL_i}{\partial b \partial \sigma} = -\frac{2U_i T_k X_k}{\sigma^2\sqrt{T_k}}$$

Hence, the fisher information matrix is derived as:

$$F = \begin{bmatrix} E\left(-\sum_{i=1}^{n}\frac{\partial^2 \ln L_i}{\partial a^2}\right), & E\left(-\sum_{i=1}^{n}\frac{\partial^2 \ln L_i}{\partial a \partial b}\right) & E\left(-\sum_{i=1}^{n}\frac{\partial^2 \ln L_i}{\partial a \partial \sigma}\right) \\ & E\left(-\sum_{i=1}^{n}\frac{\partial^2 \ln L_i}{\partial b^2}\right) & E\left(-\sum_{i=1}^{n}\frac{\partial^2 \ln L_i}{\partial b \partial \sigma}\right) \\ symmetric & & E\left(-\sum_{i=1}^{n}\frac{\partial^2 \ln L_i}{\partial b^2}\right) \end{bmatrix}$$

$$= \frac{n}{\sigma^2}\begin{bmatrix} \pi_1 T_1 + (1-\pi_1)T_2, & \pi_1 T_1 X_1 + (1-\pi_1)T_2 X_2, & 0 \\ & \pi_1 T_1 X_1^2 + (1-\pi_1)T_2 X_2^2, & 0 \\ & symmetric & 2 \end{bmatrix}$$

and

$$|F| = 2\left[(\pi_1 T_1 + (1-\pi_1)T_2)(\pi_1 T_1 X_1^2 + (1-\pi_1)T_2 X_2^2) - (\pi_1 T_1 X_1 + (1-\pi_1)T_2 X_2)^2\right]$$

Let

$$Q = F_{11} = \frac{\sum_{k=1}^{2}\pi_k T_k X_k^2}{\left(\sum_{k=1}^{2}\pi_k T_k\right)\left(\sum_{k=1}^{2}\pi_k T_k X_k^2\right) - \left(\sum_{k=1}^{2}\pi_k T_k X_k\right)^2}$$

$$= \frac{\pi_1 T_1 X_1^2 + (1-\pi_1)T_2 X_2^2}{(\pi_1 T_1 + (1-\pi_1)T_2)(\pi_1 T_1 X_1^2 + (1-\pi_1)T_2 X_2^2) - (\pi_1 T_1 X_1 + (1-\pi_1)T_2 X_2)^2}$$

$$= \frac{\pi_1 T_1 X_1^2 + (1-\pi_1)T_2 X_2^2}{\pi_1(1-\pi_1)T_1 T_2 (X_1 - X_2)^2}$$

Then the asymptotic variance of MLE of $\mu(X_0)$ is calculated as:

$$A\operatorname{var}(\hat{\mu}(X_0)) = \left(\frac{\partial\hat{\mu}(X_0)}{\partial a}, \frac{\partial\hat{\mu}(X_0)}{\partial b}, \frac{\partial\hat{\mu}(X_0)}{\partial \sigma}\right)' F^{-1}\left(\frac{\partial\hat{\mu}(X_0)}{\partial a}, \frac{\partial\hat{\mu}(X_0)}{\partial b}, \frac{\partial\hat{\mu}(X_0)}{\partial \sigma}\right)$$

$$= \frac{\hat{\sigma}^2}{n}\cdot\frac{D_c^2}{\hat{a}^4}\cdot Q$$

Appendix E2: Destructive CSADT plans

σ / a	c	X_1	n	π_1	T_1	T_2
80	2	0.1	19	0.789474	4512	672
		0.15	21	0.761905	4848	1248
		0.2	24	0.708333	5520	1440
		0.25	28	0.714286	5760	1872
		0.3	31	0.677419	6336	2448
		0.35	36	0.638889	6816	2880
		0.4	41	0.634146	7488	3456
		0.45	47	0.617021	8304	4080
		0.5	54	0.592593	9216	4896
		0.55	63	0.587302	9888	6144
		0.6	80	0.5875	9984	7152
		0.65	101	0.594059	9984	9120
		0.7	142	0.598592	9960	9832
		0.75	216	0.592593	9960	9928
		0.8	357	0.57423	9960	9928
		>0.85	>500			
	3	0.1	14	0.785714	3264	528
		0.15	16	0.75	3600	768
		0.2	18	0.722222	3888	1104
		0.25	20	0.7	4320	1440
		0.3	24	0.666667	4464	1680
		0.35	26	0.653846	5040	2208
		0.4	30	0.633333	5424	2640
		0.45	34	0.617647	6240	3024
		0.5	39	0.589744	6864	3696
		0.55	46	0.565217	7728	4224
		0.6	53	0.566038	8784	5280
		0.65	64	0.546875	9936	6336
		0.7	84	0.559524	9984	7920
		0.75	117	0.581197	9984	9840
		0.8	192	0.557292	9984	9984
		0.85	362	0.546961	9960	9928
		>0.9	>500			
	4	0.1	12	0.833333	2832	624
		0.15	14	0.785714	3168	768
		0.2	16	0.75	3312	1104
		0.25	19	0.684211	3600	1200
		0.3	21	0.666667	4080	1488
		0.35	23	0.652174	4560	1920
		0.4	27	0.62963	4896	2208
		0.45	31	0.612903	5376	2640
		0.5	35	0.6	6096	3216
		0.55	41	0.585366	6720	3840
		0.6	47	0.574468	7824	4704
		0.65	56	0.553571	8976	5712
		0.7	69	0.550725	9936	7296
		0.75	95	0.568421	9984	9216
		0.8	153	0.568627	9960	9832
		0.85	286	0.552448	9960	9928
		>0.9	>500			

To be continued

Continued						
σ / a	c	X_1	n	π_1	T_1	T_2
80	5	0.1	11	0.818182	2736	576
		0.15	13	0.769231	2928	768
		0.2	15	0.733333	3264	912
		0.25	17	0.705882	3504	1200
		0.3	19	0.684211	3840	1536
		0.35	23	0.652174	4080	1632
		0.4	25	0.64	4512	2208
		0.45	28	0.607143	5328	2496
		0.5	33	0.606061	5520	3120
		0.55	38	0.578947	6432	3600
		0.6	44	0.568182	7344	4416
		0.65	53	0.54717	8304	5328
		0.7	63	0.539683	9984	6672
		0.75	87	0.551724	9984	8400
		0.8	134	0.567164	9984	9888
		0.85	252	0.551587	9960	9880
		>0.9	>500			
90	2	0.1	21	0.809524	4896	960
		0.15	25	0.76	5376	1152
		0.2	27	0.740741	6000	1776
		0.25	32	0.6875	6384	2064
		0.3	36	0.666667	7008	2592
		0.35	40	0.65	7776	3264
		0.4	46	0.630435	8400	3936
		0.45	53	0.603774	9216	4656
		0.5	62	0.596774	9888	5616
		0.55	75	0.613333	9984	7056
		0.6	96	0.604167	9984	8160
		0.65	124	0.604839	9984	9840
		0.7	179	0.603352	9960	9928
		0.75	273	0.582418	9960	9928
		0.8	452	0.577434	9960	9928
		>0.85	>500			
	3	0.1	15	0.8	3840	624
		0.15	18	0.777778	3984	912
		0.2	21	0.714286	4224	1200
		0.25	23	0.695652	4704	1632
		0.3	26	0.653846	5280	1920
		0.35	30	0.633333	5616	2352
		0.4	34	0.617647	6336	2736
		0.45	38	0.605263	6960	3504
		0.5	44	0.590909	7728	4128
		0.55	52	0.576923	8496	4848
		0.6	60	0.566667	9696	6000
		0.65	77	0.571429	9936	7104
		0.7	99	0.585859	9984	9360
		0.75	147	0.571429	9984	9984
		0.8	243	0.559671	9984	9984
		0.85	459	0.562092	9960	9928
		>0.9	>500			
	4	0.1	14	0.785714	3264	528
		0.15	16	0.75	3600	768
		0.2	18	0.722222	3888	1104
To be continued						

Continued

σ / a	c	X_1	n	π_1	T_1	T_2
90	4	0.25	20	0.7	4320	1440
		0.3	24	0.666667	4464	1680
		0.35	26	0.653846	5088	2160
		0.4	30	0.633333	5424	2640
		0.45	34	0.617647	6048	3168
		0.5	39	0.589744	6864	3696
		0.55	45	0.577778	7824	4368
		0.6	54	0.555556	8736	5088
		0.65	64	0.546875	9984	6288
		0.7	84	0.571429	9984	7920
		0.75	117	0.589744	9984	9840
		0.8	193	0.57513	9960	9928
		0.85	362	0.552486	9960	9928
		>0.9	>500			
	5	0.1	13	0.769231	3072	528
		0.15	15	0.8	3264	816
		0.2	17	0.705882	3648	1008
		0.25	19	0.684211	3984	1344
		0.3	22	0.681818	4320	1584
		0.35	25	0.64	4608	2016
		0.4	29	0.62069	4992	2352
		0.45	33	0.606061	5712	2688
		0.5	37	0.594595	6384	3408
		0.55	43	0.581395	7056	4128
		0.6	50	0.56	8208	4896
		0.65	60	0.55	9408	5856
		0.7	75	0.56	9936	7680
		0.75	104	0.567308	9984	9552
		0.8	170	0.576471	9960	9880
		0.85	318	0.54717	9960	9928
		>0.9	>500			
100	2	0.1	23	0.826087	5568	1056
		0.15	26	0.769231	6288	1440
		0.2	30	0.733333	6672	1968
		0.25	34	0.705882	7344	2448
		0.3	39	0.666667	8016	2928
		0.35	45	0.644444	8640	3504
		0.4	51	0.627451	9552	4224
		0.45	60	0.616667	9984	5136
		0.5	72	0.625	9984	6480
		0.55	90	0.611111	9984	7584
		0.6	111	0.621622	9984	9648
		0.65	153	0.620915	9960	9928
		0.7	221	0.60181	9960	9928
		0.75	337	0.581602	9960	9928
		>0.8	>500	0.581602	9960	9928
	3	0.1	17	0.823529	4080	768
		0.15	20	0.75	4368	1056
		0.2	22	0.727273	4992	1392
		0.25	26	0.692308	5088	1824
		0.3	29	0.655172	5856	2112
		0.35	33	0.6363	6432	2544
		0.4	38	0.631579	6864	3120

To be continued

Continued						
σ / a	c	X_1	n	π_1	T_1	T_2
100	3	0.45	43	0.604651	7632	3792
		0.5	49	0.591837	8640	4512
		0.55	57	0.578947	9552	5472
		0.6	69	0.57971	9984	6816
		0.65	90	0.577778	9984	8016
		0.7	120	0.6	9960	9832
		0.75	183	0.579235	9960	9832
		0.8	302	0.566225	9960	9880
		>0.85	>500			
	4	0.1	16	0.8125	3504	576
		0.15	17	0.764706	4080	960
		0.2	20	0.75	4176	1344
		0.25	23	0.695652	4656	1536
		0.3	26	0.692308	5088	1920
		0.35	29	0.655172	5568	2448
		0.4	34	0.617647	6144	2688
		0.45	38	0.605263	6864	3360
		0.5	44	0.590909	7584	3984
		0.55	50	0.58	8640	4896
		0.6	59	0.559322	9792	5808
		0.65	75	0.56	9960	7096
		0.7	98	0.581633	9960	9064
		0.75	144	0.583333	9960	9928
		0.8	238	0.567227	9960	9928
		0.85	447	0.55481	9960	9928
		>0.9	>500			
	5	0.1	15	0.8	3312	528
		0.15	16	0.75	3792	912
		0.2	19	0.736842	3984	1152
		0.25	22	0.681818	4368	1344
		0.3	24	0.666667	4848	1824
		0.35	28	0.642857	5280	2064
		0.4	32	0.625	5760	2496
		0.45	36	0.611111	6240	3216
		0.5	42	0.595238	7008	3648
		0.55	48	0.583333	7872	4512
		0.6	56	0.571429	9072	5376
		0.65	67	0.567164	9984	6816
		0.7	89	0.573034	9960	8392
		0.75	127	0.574803	9960	9832
		0.8	209	0.555024	9960	9928
		0.85	393	0.557252	9960	9928
		>0.9	>500			
110	2	0.1	25	0.8	6336	1056
		0.15	30	0.766667	6432	1632
		0.2	34	0.735294	7104	2112
		0.25	39	0.692308	7680	2640
		0.3	44	0.681818	8496	3216
		0.35	50	0.64	9312	3888
		0.4	58	0.637931	9840	4752
		0.45	70	0.642857	9984	5664
		0.5	85	0.635294	9984	6912
		0.55	103	0.631068	9984	8784
To be continued						

To be continued						
σ / a	c	X_1	n	π_1	T_1	T_2
110	2	0.6	133	0.639098	9984	9936
		0.65	185	0.616216	9960	9928
		0.7	268	0.604478	9960	9880
		0.75	408	0.585784	9960	9928
		>0.8	>500			
	3	0.1	19	0.789474	4560	720
		0.15	22	0.772727	4752	1200
		0.2	25	0.72	5376	1440
		0.25	29	0.689655	5712	1824
		0.3	33	0.666667	6144	2304
		0.35	37	0.648649	6816	2832
		0.4	41	0.634146	7632	3552
		0.45	47	0.617021	8544	4128
		0.5	54	0.592593	9504	4944
		0.55	64	0.59375	9888	6240
		0.6	81	0.604938	9984	7344
		0.65	102	0.598039	9984	9408
		0.7	144	0.597222	9984	9984
		0.75	221	0.579186	9960	9880
		0.8	365	0.575342	9960	9928
		>0.85	>500			
	4	0.1	17	0.823529	3792	816
		0.15	20	0.75	4176	1008
		0.2	22	0.727273	4752	1344
		0.25	26	0.692308	4992	1632
		0.3	29	0.655172	5664	1968
		0.35	32	0.65625	6096	2688
		0.4	37	0.648649	6672	3120
		0.45	42	0.619048	7488	3696
		0.5	48	0.604167	8352	4464
		0.55	56	0.571429	9408	5232
		0.6	68	0.573529	9984	6336
		0.65	86	0.581395	9984	8016
		0.7	114	0.596491	9984	9936
		0.75	175	0.582857	9960	9832
		0.8	288	0.569444	9960	9928
		>0.85	>500			
	5	0.1	16	0.8125	3648	672
		0.15	18	0.777778	3984	1056
		0.2	21	0.714286	4368	1248
		0.25	23	0.695652	4944	1632
		0.3	27	0.666667	5136	2016
		0.35	30	0.633333	6000	2304
		0.4	34	0.617647	6576	2832
		0.45	40	0.6	6912	3408
		0.5	45	0.6	7776	4224
		0.55	53	0.584906	8688	4896
		0.6	62	0.564516	9936	5856
		0.65	77	0.584416	9984	7680
		0.7	102	0.588235	9984	9504
		0.75	153	0.575163	9960	9928
		0.8	253	0.565217	9960	9928
		0.85	475	0.545263	9960	9928
To be continued						

Continued						
σ / a	c	X_1	n	π_1	T_1	T_2
110	5	>0.9	>500			
120	2	0.1	27	0.814815	6864	1248
		0.15	33	0.757576	7008	1728
		0.2	37	0.72973	7776	2304
		0.25	42	0.690476	8592	2832
		0.3	48	0.666667	9216	3552
		0.35	55	0.654545	9936	4320
		0.4	65	0.646154	9984	5472
		0.45	79	0.64557	9984	6528
		0.5	96	0.645833	9984	7968
		0.55	120	0.641667	9984	9312
		0.6	158	0.639241	9984	9984
		0.7	318	0.597484	9960	9928
		0.75	486	0.592593	9960	9928
		0.8	>500			
	3	0.1	21	0.809524	4848	816
		0.15	24	0.75	5328	1200
		0.2	27	0.740741	5808	1680
		0.25	30	0.7	6480	2160
		0.3	35	0.685714	6768	2688
		0.35	40	0.65	7440	3168
		0.4	45	0.622222	8352	3792
		0.45	51	0.607843	9264	4608
		0.5	61	0.590164	9984	5232
		0.55	74	0.608108	9984	6624
		0.6	91	0.604396	9984	8448
		0.65	119	0.605042	9984	9888
		0.7	318	0.597484	9960	9928
		0.75	263	0.581749	9960	9880
		0.8	434	0.569124	9960	9928
		>0.85	>500			
	4	0.1	19	0.789474	4224	720
		0.15	21	0.761905	4656	1200
		0.2	24	0.708333	5232	1440
		0.25	27	0.703704	5664	1920
		0.3	31	0.677419	6096	2352
		0.35	35	0.657143	6816	2784
		0.4	40	0.625	7440	3360
		0.45	46	0.608696	8112	4032
		0.5	52	0.596154	9120	4944
		0.55	62	0.580645	9936	5760
		0.6	78	0.589744	9984	6912
		0.65	99	0.59596	9984	8688
		0.7	136	0.602941	9984	9888
		0.75	208	0.591346	9960	9880
		0.8	343	0.574344	9960	9928
		>0.85	>500			
	5	0.1	17	0.823529	4080	768
		0.15	20	0.75	4368	1056
		0.2	22	0.727273	4992	1392
		0.25	26	0.692308	5136	1776
		0.3	29	0.655172	5856	2112
To be continued						

Continued						
σ / a	c	X_1	n	π_1	T_1	T_2
120	5	0.35	34	0.647059	6192	2496
		0.4	38	0.631579	6864	3120
		0.45	43	0.604651	7632	3792
		0.5	49	0.591837	8640	4512
		0.55	56	0.571429	9840	5472
		0.6	70	0.571429	9984	6576
		0.65	87	0.586207	9984	8688
		0.7	119	0.605042	9984	9984
		0.75	183	0.584699	9960	9832
		0.8	301	0.55814	9960	9928
		0.85	566	0.558304	9960	9928
		>0.9	>500			

Publications

Tang LC, Yang GY & Xie M. Planning step-stress accelerated degradation tests, Proceedings of the Annual Reliability and Maintainability Symposium, pp287-292, 2004, (Stan Ofsthun Award).

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