Meta-heuristics Development Framework:

Design and Applications

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As with other projects, I am greatly indebted to many people, but more so with this than most other works I have undertaken. ***Meta-Heuristics Development Framework (MDF)*** started off in year 2001 with only Professor Lau Hoong Chuin and myself. At that point of time, MDF only has a single meta-heuristic. Realizing the potential of a software tool that could facilitate the ***Meta-heuristics Community*** in rapidly prototyping their imagination into reality, MDF is designed with the intention to condense efforts in research and development and consequently redirect these resources onto the algorithmic aspect. With time, the team expanded with more research engineers and project students, each participating in various roles with invaluable contributions. Many thanks are owed to the persons from this incomplete list:

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TABLE OF CONTENTS

SUMMARY

Recent researches have reported a trend whereby meta-heuristics are successful in solving NP-hard combinatorial optimization problems, many of which surpassed the results obtained by classical search methods. These promising reports naturally captivated the attention of the research communities, especially those in the field of computational logistics. While meta-heuristics are effective in solving large-scale combinatorial optimization problems, in general, they result from an extensively manual trial-and-error algorithmic design tailored to specific problems. This leads to a waste of manpower as well as equipment resources in developing each trial algorithm, which consequently delays the progress in application development. Hence, the demand for a rapid prototyping tool for fast algorithm development became a necessity.

In this thesis, we propose ***Meta-Heuristics Development Framework (MDF)***, a generic meta-heuristics framework that reduces development time through abstract classes and code reuse, and more importantly, aids design through the support of user-defined strategies and hybridization of meta-heuristics. We study two different aspects of MDF. First we examine the *Design Concepts*, which analyze the blueprint of MDF. In this aspect, we will investigate the rationale behind the architecture of MDF such as the interaction between the abstract classes and the meta-heuristic engines. More interestingly, we will examine a novel way of redefining hybridization in MDF through the "request-and-response" metaphor, which form an abstract concept for hybridization. Different hybridization schemes can now be formulated with relative ease, which give the proposed framework its uniqueness. The second aspect of the thesis covers the applications of MDF, in

which we take a more "critic" role by investigating some MDF's applications, and examining their strengths and weaknesses. We begin with the *Traveling Salesman Problem (TSP)* as a "walk-through" in exploring the various facets of MDF, particularly hybridization. As TSP is a single-objective single-constraint problem, the reduced complexity makes it an ideal candidate for a comprehensive illustration. We then extend the problem complexity by augmenting TSP into multiple-objective multiple-constraint problems, with potentially larger search space. The extension results in solving (a) *Vehicle Routing Problem with Time Windows (VRPTW)*, a logistic problem that deals with finding optimal routes for serving a given number of customers; and (b) *Inventory and Routing Problem with Time Windows (IRPTW)*, which adds inventory planning over a defined period to the routing problem. Using the various hybridized schemes supported by MDF, quality solutions can be obtained in good computational time within relatively short developmental cycle, as presented in the experimental results.

LIST OF FIGURES

LIST OF TABLES

CHAPTER 1

INTRODUCTION

[Garey and Johnson, 1979] shows the existence of many non-deterministic polynomial (NP)-hard optimization problems whose solutions are computationally intractable to find. Exact search is no longer a valid option as it is not only operationally infeasible, but also impractical, especially for solving large-scale problems. This motivates the development of intelligent search methods that can achieve good results efficiently. Meta-heuristics have matured rapidly in the recent years and become an excellent substitute for exact methods, due to their algorithmic effectiveness and computational efficiency. Contrary to exact methods however, meta-heuristics do not guarantee global optimality. Rather, they seek to obtain quality solutions within a reasonably time. The fundamental role of meta-heuristics is to "guide" a heuristic (such as greedy) from getting trapped in local optimality and is achieved through their own unique features and strategies.

Meta-heuristic approaches have been shown to achieve promising results for solving NP-hard problems very efficiently, making its industry applications, particularly in the field of logistics, appealing. For two decades, meta-heuristics such as *Tabu Search (TS)*, *Simulated Annealing (SA)*, and *Genetic Algorithms (GA)* have been studied in the literature for obtaining quality results from NP-hard optimization problems. Following the success of these meta-heuristics, there has been an explosive growth of new techniques in line with natural and biological observations, such as *Ant Colony Optimization (ACO*) [Dorigo & Di Caro, 1999], *Squeaky Wheel* [Joslin & Clements, 1999], *Particle Swarm* [Parsopoulos & Vrahatis, 2002] and even mammals like *lab rats* [Yufik and Sheridan, 2002]. This

diffusion, while healthy for seeding new ideas into the community, is met with such numerous and diversity that renders finding the best meta-heuristic intricate.

Till the date of this thesis, there has been no work in the literature that shows one meta-heuristic that could truly dominate the rest for *every* problem. Consequently, this implies the challenge of finding the right meta-heuristic for the right problem. The challenge is further heighten by the observation that the search strategies used within a meta-heuristic have a considerable influence on the effectiveness and efficiency. For example, by determining when to perform exploitation or exploration during an ACO search can yield significant differences in results [Dorigo & Di Caro, 1999]. As such, developers have to face the insurmountable task of trying out different meta-heuristics with varying strategies, and algorithmic parameters, on their problem(s).

Surprisingly, many researchers actually meet this challenge by building meta-heuristics applications from scratch. As such, an enormous amount of resources, in both man and machines, have to be invested for each redevelopment that apparently is uncalled for. Ironically, the process of optimizing problems is not optimized at all! One effective solution is to incorporate a framework that would enable fast development through generic software design. This recycling of design and code conserves the unnecessary wastage of resource, thus allowing researchers to focus on the algorithmic aspects and meaningful experiments rather than mundane implementation issues. However, certain criteria must be imposed to the framework and we list three vital decisive factors.

1. It must be generic.
2. It is able to benchmark fairly on different algorithmic designs.
3. It has an unambiguous object-oriented design.

Genericity has two different meanings in this context. First, the framework must be able to work with most if not all combinatorial optimization problems. Naturally, this is subject to many criticisms as it is not viable to justify the claim. The most convincing "proof" will then be providing illustrations on different applications, which in the scope of this thesis, is restricted to *Routing* related problems. Secondly, genericity also signifies that the framework can support various meta-heuristics as well their strategies. This is especially important, as with the diverse growth of meta-heuristics, we see the potential for advancing the field further if there is provision for algorithm designers to hybridize one technique with another. As expected, each meta-heuristic has its own forte and shortcomings and logically leads to hybrid schemes that could exploit the strengths and cover the weaknesses of one technique with its collaborator(s). Results from the literature have supported the claim that such hybrid methods usually out-perform their predecessors, e.g. [Bent & Hentenryck 2001].

The second point stresses on the role as an unbiased platform for benchmarking, which typically refers to the comparisons of solution quality and computational time. Although effectiveness is likely to be attributed to search strategies, the computational time is more often than not a controversy issue. Aside from algorithmic efficiency, it is obvious that the technical skill of an implementer has a considerable impact on the overall competency. A framework should therefore provide a developmental platform that neglects the impact of programming proficiency. This achieves a more precise comparison on the algorithms' efficiency. Bearing this in mind, the framework should reduce the development efforts by off-loading the routine aspects of meta-heuristics through abstractions and a software library of reusable codes.

Finally, the last point states a software engineering requirement, which may not seem essential but is highly sought-after. *Object-Oriented Programming (OOP)* is adopted because of its clarity in design and ease of integration and extension. As the framework is likely to be a complex tool, each abstract class should be unambiguous and clearly defined for its role. Advantages of a well-designed architecture could give implementers fewer frustrating development hours and is also less prone to programming errors.

By now it is apparent that there is a powerful motivation for a meta-heuristics framework. We propose the *Meta-heuristics Development Framework (MDF)* as an aspirant to compete with other works in the literature. Powered by four different meta-heuristics, MDF provides a platform for both rapid prototyping as well as unbiased benchmarking. The potency of MDF lies in its unique control mechanism, which allows hybridization to be formed effortlessly. In addition, the control mechanism follows the "request-and-response" analogy, which enhances comprehension and easily adopted. The framework also bridges the algorithm designers and the program implementers by having no constraint on the formulation of strategies, thus giving liberty to the designers' imagination and yet easily accommodated by the implementers. In short, MDF is a generic, flexible framework that is constrained only by the developers' mind rather than the restrictions in framework.

The following two sections in this chapter will give a short account on the meta-heuristics' background and some software engineering concepts. For readers who are more concerned with MDF issues, these sections can be skipped without affecting the rest of the thesis. Chapter 2 will be examining the design concepts of MDF, which we term as *fundamental* research and development. In this chapter,

we will be exploring the conceptual design and appreciate the rationale leading to its architecture. Illustrations and pseudo-codes can be found throughout the chapter to enhance its comprehension. Chapter 3 focuses on the *applications* of MDF, particularly to illustrate the flexible design and reuse capability. The chapter will start off with *Traveling Salesman Problem (TSP)*, whose simplicity makes it an excellent illustration on the various formulations of hybridization scheme. We then demonstrated how the *Vehicle Routing Problem with Time Windows (VRPTW)*, using TSP implementations, is solved, followed by the *Inventory Routing Problem with Time Windows (IRPTW)*. Through these applications, we demonstrate how the framework allows reuse, which reduces development time and yet provides excellent results. The experimental results have shown the effectiveness of the proposed framework. Related work in the literature is reviewed in Chapter 4. Finally, Chapter 5 concludes the thesis by reporting the current development and proposing some future extension that is insightful for the growth of MDF.

1.1 Meta-heuristics Background

Meta-heuristics are as flexible as the ingenuity of the algorithm designer, and they can be inspired from physics, biology, nature and any other fields of science. This section provides a brief description on the four meta-heuristics that are incorporated in MDF and they are *Tabu Search (TS)*, *Ant Colony Optimization (ACO)*, *Simulated Annealing (SA)* and *Genetic Algorithm (GA)*. Important concepts are further discussed in ANNEX A-D to enhance the readers' understanding of the strategies discussed in the later chapters of this thesis.

1.1.1 Tabu Search (TS)

In 1986, Fred Glover [Glover, 1986] described TS as *"a meta-heuristic superimposed on another heuristic. The overall approach is to avoid entrapment in cycles by forbidding or penalizing moves that take the solution, in the next iteration, to points in the solution space previously visited (hence 'tabu')"*. TS was inspired from the observation that human behavior appears to operate with a random element that leads to inconsistent behavior given similar circumstances. As a result, the underlying search principle deviates from the conventional charted course: although a poor solution might be regretted as a source of error, it can also prove to be a source of gain. In other words, TS proceeds according to the supposition that a new (poor) solution should be explored if all better paths have already been investigated. This insures new regions of a problems solution space will be investigated in with the goal of avoiding local minima and ultimately finding the desired solution. TS begins by converging to a local minima. To avoid retracing the explored solution, TS stores recent moves in one or more tabu lists. Hence, these tabu lists are historical in nature and they form the TS memory mechanism. Strategies involving TS is usually associated with either diversification or intensification and could change as the algorithm proceeds. For example, at the initialization the goal is make a coarse examination of the solution space (diversification), but as candidate locations are being identified, the search changes to focus on producing improved local optimal in a process of 'intensification'. By alternating between the two opposing techniques, various variations of TS implementation can be formed to optimize a specific problem domain.

1.1.2 Ant Colony Optimization (ACO)

ACO [Dorigo and Di Caro, 1999] can be generalized as a population-based approach in finding a solution to combinatorial optimization problems. The basic concept is to employ a number of simple artificial agents to construct good solutions through an elementary form of communication. While real ants cooperate in their search for food by depositing chemical traces (pheromones) on the paths they traveled, ACO simulates this behavior by using a common memory that is analogous to the deposited pheromone. This artificial pheromone is accumulated at run-time through a learning mechanism and consequently influences the behavior of subsequent search. In short, the artificial ants can be viewed as parallel processes that build solutions using a constructive procedure that is composed of the artificial pheromone and a heuristic function is used to evaluate successive constructive steps. The current trend of using ACO is often associated with the combination of other meta-heuristic, thus giving birth to many hybrid methods.

1.1.3 Simulated Annealing (SA)

SA exploits an analogy between the way in which a metal cools and freezes into a minimum energy crystalline structure (the annealing process). The algorithm is based upon that of [Metropolis et al., 1953], which was originally proposed as a means of finding the equilibrium configuration of a collection of atoms at a given temperature. This technique is subsequently developed by [Kirkpatrick et al., 1983] to form the basis of an optimization technique for combinatorial problems. The major advantage of SA over other meta-heuristics is its ability of avoiding entrapment at local minima. The algorithm employs a random search that not only

accepts changes that improve the objective function, but also some changes that decrease it. The latter are accepted with a probability given by

$$p = \text{exponential}^{-|\Delta x/Ti|} \qquad \text{Eqn 1.1}$$

where Δx is the increase in objective function and T is a control parameter, which is analogous with `temperature' and is irrespective to the objective function.

1.1.4 Genetic Algorithm (GA)

GA was introduced as a computational analogy of adaptive systems that performs *parallelized* stochastic search [Holland, 1992]. It is modeled loosely on the principles of the evolution that evolve the fitness of a population of individuals by undergoing selection processes in the presence of variation-inducing operators such as mutation and recombination (crossover). A fitness function is used to evaluate individuals, and reproductive success varies with fitness. A significant advantage of GA is that it works very well on mixed (continuous *and* discrete), combinatorial problems. In fact GA is less susceptible entrapment in local optima but tends to be more computationally expensive. To order to use GA, the algorithm designer must first represent the solution as a *genome* (or *chromosome*). GA then creates a population of solutions and applies genetic operators such as mutation and crossover to evolve the solutions in order to find the best one(s).

1.2 Software Engineering Concepts

Well-engineered software does not only provide clarity in design, but also gives the ease of integration and extension. While the drawback of obligatory overheads may cause slight degrade in performance, the overall benefits are often

much greater. Among the numerous design standards and practices offered, two useful major concepts are adopted in MDF: *Framework* and *Software library* [Marks Norris et. al., 1999]. The following sections provide brief introductions to these concepts.

1.2.1 Framework

Frameworks [e.g. Microsoft .NET framework (.NET), Java Media Framework (JMF), Apache Struts Web Application framework] are reusable designs of all or part of a software system described by a set of abstract classes and the manner in which instances of those classes collaborate. A good framework can reduce the cost of developing an application by an order of magnitude because it allows the reuse of both designs and codes. They do not require new technology, because they can be implemented with existing object-oriented programming languages. Unfortunately, developing a good framework is time consuming. A framework must be simple enough to be understood yet provides enough features to be used quickly and accommodates for the features that are likely to change. It must embody a theory of the problem domain, and is always the result of domain analysis, whether explicit and formal, or hidden and informal. Therefore, frameworks are developed only when many applications are going to be developed within a specific problem domain, allowing the timesaving from reuse to recoup the time invested in development.

1.2.2 Software Library

Often a framework can be viewed as a top-down approach as it supplies the architectural structure for an implementer to complete by “filling” in the necessary

components (interfaces). As opposed to the concept of frameworks, a software library supplies “ready-codes” to the implementer to speed up the progress of coding. The two software engineering concepts when utilized could form a powerful coalition. For example, the framework could guide the implementer in building his applications through the abstract classes. In addition, it also handles the routines of the underlying algorithm. Such design gives the advantage of clarity in program flows, which in turn prevents coding errors and results in less developing and debugging hours for the implementer. On the other hand, the software library provides the implementer with building blocks to construct the interfaces in the framework. Hence, the tasks of the implementer can be reduced to devising the algorithmic aspects of the problem and coordinating the sequence of events in the framework.

CHAPTER 2

DESIGN CONCEPTS

In this chapter, we discuss the design of MDF. This work has been published in [Lau et. al[1], 2004].

MDF works on a "higher level" than the individual algorithm frameworks in the literature (see Chapter 4 for a more in-depth comparison), and guides the development of both new and existing techniques. In particular, MDF extends the work of TSF++ ([Lau et al [1], 2003]), by working on a higher level where TSF++ serves as a component algorithm. MDF is able to:

a) Act as a development tool to swiftly create solvers for various optimization problems;
b) Benchmark fairly the performance of new algorithm implementations against any existing technique, or other hybridized techniques; and
c) Create hybrid algorithms of any existing technique in the framework, or allow others to adapt their algorithm through reuse;

In short, MDF presents a model to facilitate multi-algorithm inter-operability. MDF uses abstraction and inheritance as the primary mechanism to build adaptable components or interfaces. The architecture of MDF can be categorized into four collections.

1. The *general interfaces* are a collection of generic interfaces that have factored and grouped from the general behavior of meta-heuristics, thus rendering the framework to be robust yet flexible. They include *Solution*, *Move, Constraint*, *Neighborhood Generator, Objective Function*, and *Penalty Function*. These general interfaces do not deal with the actual

algorithm, but provides a common medium in which different algorithms share information and collaborate. We illustrate this concept using the *Move* interface. In TS for example, a move is defined as a translation from current solution to its neighbor. For the case of ACO, a move is defined as a transition while constructing a partial solution to a complete solution. GA treats a move as a solution "mutation" while simulated annealing defines the move as a probabilistic operation to its next state. Although each of these operators exhibits a different behavior, their underlying algorithmic concept is the same. Such realization of common interfaces allows implementation to be easily switched across different meta-heuristics and enables the formation of hybridized models. For example, a common *solution interface* will allow both TS and GA to modify the *solution-inherited* object easily.

2. Extended or *proprietary interfaces* are a collection that built above the general interfaces to support unique behaviors exhibited by each meta-heuristic. In ACO, the proprietary interfaces are the *local heuristic* and *pheromone trail*. In the case of TS, these are *tabu list* and *aspiration criteria* interfaces. SA requires the *annealing schedule* interface and GA has *population* and *recombination* interfaces. Although each proprietary interface is exclusive to its meta-heuristic, the designs and codes can be shared across different problems. For example, the tabu list for TSP can be easily recycled to be applied on VRPTW.
3. The third collection shows the engines that are currently available in MDF; TS, ACO, SA and GA. MDF uses a generic *Engine* interface as a base class for each meta-heuristic to describe the common rudimentary controls.

Some of these controls include *recording of solutions* and specifying the *stopping criteria*. Like engine in reality, a *Switch Box* is incorporated as a container for the tuning parameters, such as *number of iterations* and *tabu tenure*. This centralization design allows fast access and easy modification on the parameters, either manually or through the *Control Mechanism*.

4. The *control mechanism* is the core collection in MDF. It is inspired from the observation that meta-heuristics strategies (including hybridization) can be decomposed into two aspects; first, the point in time when a certain event(s) occur, and second, the action(s) performed on the current search state to bring it to the next state. We define the first aspect as *Requests* and the second aspect as *Responses*. Following this metaphor, the control is devised to bridge requests to their intended responses. This mechanism gives limitless flexibility to the algorithm designers through the many-to-many relationship between requests and responses. Since requests are actually search experiences (events) and responses are the modification made to the search state (handlers), such control implies vast adaptability in search techniques. We will reserve a more in-depth discussion on this mechanism in section 2.4 of this chapter.

In addition, MDF also incorporates an optional built-in *software library* that facilitates developing selected strategies. While these generic strategies are not as powerful as some specific methods that are tailored to a problem type, these components provide a quick and easy means for fast prototyping. In the following sections, we will explain and discuss each of these collections. Figure 2.1 presents an overview of the collections in MDF.

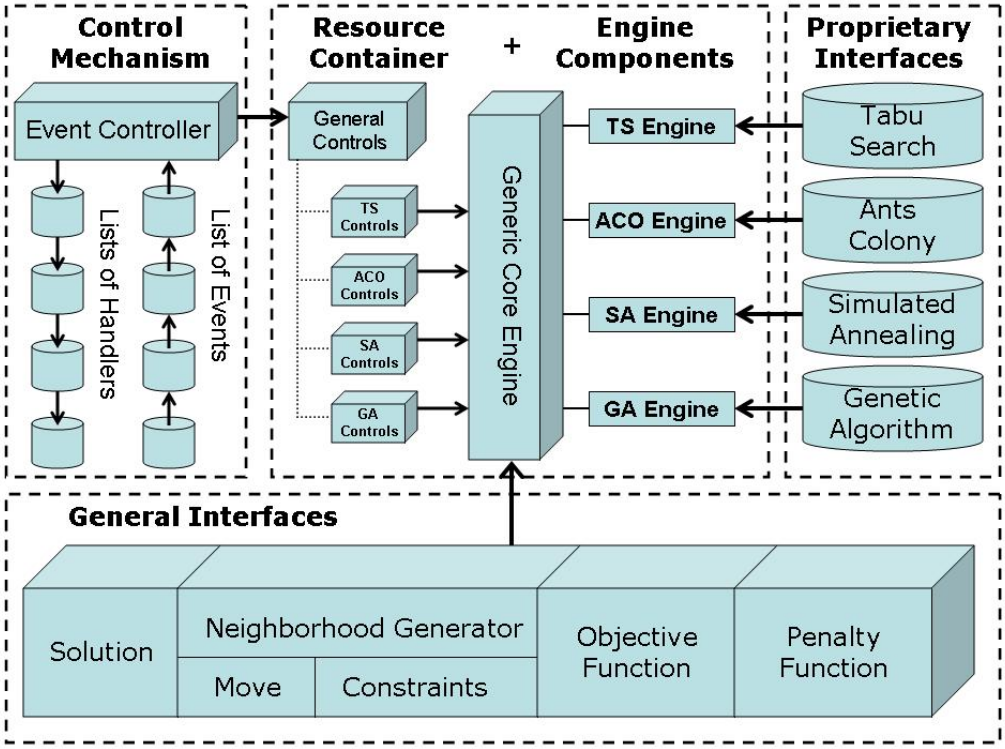


Figure 2.1: The architecture of Meta-heuristics Development Framework

2.1 General Interfaces

The fundamental interfaces are intended to classify the common behaviors of meta-heuristics into distinctive abstract classes. Figure 2.2 illustrates how this common behavior can be formulated into the interfaces. For each interface, we will present the virtual functions that are essential for the objects and a description of their uses.

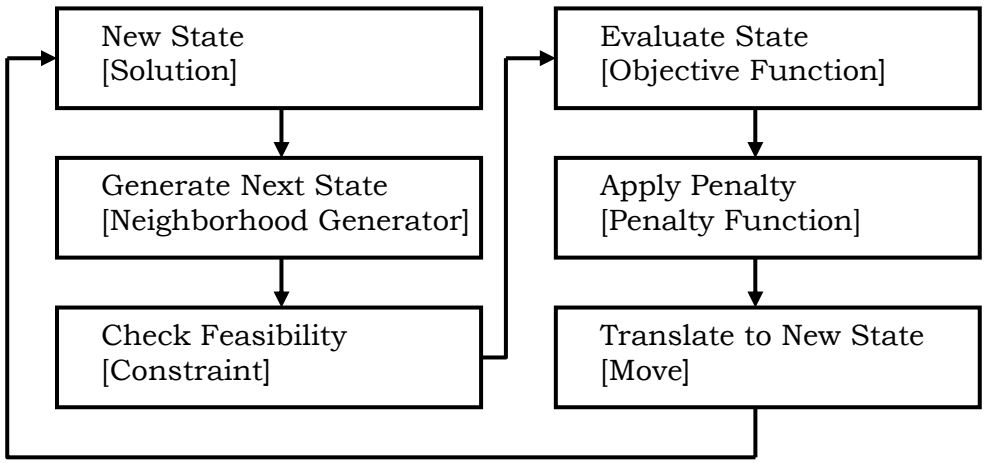


Figure 2.2: The relationship of Meta-heuristics behavior and MDF's fundamental interfaces

2.1.1 Solution Interface

Virtual Function:	
• *Solution* **Clone** (void);*	*Function 1*

Descriptions:

The *Solution* class provides a representation to the result of problem. MDF imposes no restriction on the solution formulation or the type of data structures used because the search engine never manipulates the Solution objects directly. Instead, the engine relies on the *Move* object to translate the Solution, the *Objective Function* object to evaluate the Solution and the Solution itself for cloning. The Solution interface has one virtual function, *Clone (Function 1)*, which returns a cloned instance of the solution object. A pitfall for unaware programmer is the common mistake of using shallow cloning (copy references of the data) instead of deep cloning (copying the data itself) and by doing so, loses valuable results.

2.1.2 Move Interface

Virtual Function:	
• *void **Translate** (Solution* solution);*	*Function 2*

Descriptions:

The *Move* class is used to translate a Solution object from its current state to a new state. However, the definition of a "state" varies across different meta-heuristics. For example in TS, a state refers to the current solution and a new state is defined as a neighbor "adjacent" to the current solution. Hence the move operator delineates the neighborhood around the current solution and translates a current solution to its neighbor. In ACO, a state refers to the paths of the ants. In the beginning, the ant starts from the colony, which corresponds to an empty solution.

When the ant moves from one path (state) to another, the solution is built incrementally. This continues until a complete solution is constructed, which indicate the ant has reached the food source. Hence each move is seen as a transitional phase in which new paths are added into the (partial) solution. In SA, the move operator is a probabilistic operation that generates a random neighbor. This definition of a state is similar to TS except that rather than a neighborhood, only one neighbor in generated in each iteration. Finally in GA, the move operator acts as a mutation to evolve the individuals (solutions). In this way, the current state refers to the current generation and the new state is their offspring.

Surprisingly, there is no rule that prevents one meta-heuristic from using another's move. For example, TS could use ACO incremental move to build up a solution and at the same time, tabu-ing the past constructed solution's components to prevent assembling the same solution (cycling) again. By adopting this view, it becomes probable to assault problems at different angles and even instigate a new technique. In addition, the interface also allows the multiple types of move for a problem through inheritance. In VRPTW for example, both *exchange* and *replace* moves can inherited the same Move interface. Beside moves that perform different operation, it also implies that complex moves such as an *adaptive k-opt* can be implemented to generate *Very Large Scaled Neighborhood (VLSN)*. The *Translate* function (Function 2) modifies the solution in its argument to its next state. Programmer should be aware that the *translate* operation is permanent and cloning should be done to prevent loss of solutions.

2.1.3 Constraint Interface

Virtual Function:

- *int* ***DegreeOfViolation*** *(Solution* solution, Move* move); Function 3*

Descriptions:

The *Constraint* class is usually used to ensure the feasibility of a solution. The *Degree of Violation* function (Function 3) takes in two arguments, a solution and a move objects and return an integer. The return parameter indicates "how much" violation is presented in the candidate neighbor (i.e. neighbor = current solution ⊕ proposed move). A zero value signifies a feasible solution and any integer above zero indicates infeasibility. It is possible to apply some relaxation criteria so that violated solution can be accepted. This is extremely useful in oscillating strategies, in which constraints are sometimes violated to explore previously inaccessible regions and subsequently repaired. However, such tactics often run into the danger of over-violation (solution can no longer to be repaired to feasibility) and a restraint degree of violation can help to confine the risk.

2.1.4 Neighborhood Generator Interface

Virtual Function:

- *Neighborhood* **GenerateMove** (Solution* solution); Function 4*

Descriptions:

The *Neighborhood Generator* class generates the desired next states from the current solution using the *Generate Move* function as shown in Function 4. When the Neighborhood Generator is called, it will use the move objects to generate a list of possible next states. It is possible to control the type of moves that is used to generate the current neighbors. For example, if the search result is stagnant, the

Neighborhood Generator can be adjusted to generate drastic moves. This kind of adaptive selection of moves can be easily programmed using MDF's *control mechanism* and hence guarantees a more controlled search process. After the neighborhood is generated, the constraint objects select the candidates that satisfy their criteria and these chosen candidates are recorded. The resultant neighborhood is sent back for processing. Each meta-heuristic has a different contextual meaning for the Neighborhood Generator. For TS, the neighborhood generator produces a list of desired neighbor with respect to the current solution. In ACO, the Neighborhood Generator determines the possible subsequent paths that can be linked from the partial solution. When no new path is constructed, it implies that the solution has been completely built. In SA, the Neighborhood Generator acts as a generator for generating the random moves and in GA, it performs the selection routine of choosing the individuals for recombination. In short, the functionality of Neighborhood Generator is to generate new candidates so that the meta-heuristics' selection process could continue.

2.1.5 Objective Function Interface

Virtual Function:

- *ObjectiveValueType* ***Evaluate*** *(Solution* solution,*
  Move move);* *Function 5*

- *boolean* ***IsProposedBetterThanOriginal*** *(ObjectiveValueType proposed,*
  ObjectiveValueType original); *Function 6*

Descriptions:

The *Objective Function* evaluates the quality of a solution. It uses a user-defined metric called *ObjectiveValueType* to dictate favorableness of the solution. With

this design, implementer can now define their objective value type to an integer, a double (floating point number) or even a vector of integer or double. This is especially useful for goal programming optimization, in which there are several objectives to be considered and inconvenient to be projected into a single dimension. VRPTW for example has two objectives, which are to minimize the number of vehicle used and the distance traveled. Sometimes, it is impractical to project these two objectives of different dimension together (i.e. how much distance is equivalence to the cost of a vehicle). In MDF, both objective values are stored and compared independently, which allows a case-by-case evaluation.

In order to improve the performance of search, the Objective Function object also supports incremental calculation. Absolute calculation should be done for the initial solution and subsequently switched to incremental calculation for efficiency reasons. An example on absolute and incremental calculation can be illustrated using the Knapsack Problem (KSP). For the initial solution, the objective value is calculated by adding up all the items' values contained in the knapsack. This method is known as the absolute calculation. Subsequent addition and removal can be computed using incremental calculation from the original objective value by adding or subtracting the targeted item value. The *Evaluate* function (Function 5) is designed for this purpose and *Is Proposed Better Than Original* function (Function 6) determines whether a proposed next state is better than current state.

2.1.6 Penalty Function Interface

Virtual Function:

- *ObjectiveValueType* ***ApplyPenalty*** *(Solution* solution, Move* move ObjectiveValueType NeighborObjectValue);* *Function 7*

Descriptions:

The *Penalty Function* gives a temporary penalty to the objective value. This is extremely useful in implementing soft constraints. Typically soft constraints are employed by the algorithm designer to incline the search toward preferred solutions. For example in KSP, bricks and cements are encouraged to be packed together unless the cost is very high and this user-constraint can be easily implemented by applying a "bonus" (negative penalty) to solution value if such arrangement occurred.

2.2 Proprietary Interfaces

This section addresses the interfaces that describe the behaviors exclusive to each meta-heuristic. Interestingly, by formulating these unique behaviors into abstract classes, it gives us valuable insights in forming innovative hybrids. For example, a tabu list can be added to ACO to empower the ants with memory and the annealing schedule can be added to GA as a breeding criterion. In addition, algorithm designers can define their own proprietary interfaces that may mature into a new technique.

Tabu Search

2.2.1 Tabu List Interface

Virtual Function:

- *boolean **IsTabu** (Solution* solution, Move* move);* *Function 8*
- *void **SetTabu** (Solution* solution, Move* move);* *Function 9*

Descriptions:

The *Tabu List* reduces the tendency of solution cycling through the use of memory. The most straightforward implementation is to use a list that stored previously visited solutions for the tenured duration. While this approach looks simplistic, there are a few concerned issues. We consider the case of a solution size of l, a tabu tenure t and running for k iterations and analyze the computational time. In every iteration, each neighbor has to be verified with every element in the tabu list and this requires $O(l * t)$. Suppose there is an average of m neighbors in each iteration, then the total computational time spent in validating the tabu status is $O(l * t * m * k)$. Apparently, the efficiency of the tabu list could be improved if one or more of the four parameters is/are reduced. Since t and k directly affect the algorithm effectiveness, they should be tuned optimality. As for m, it is sometimes possible to reduce the size without sacrificing the quality (such as using a candidate list strategy), but it is generally done heuristically and thus could not be guaranteed. l is the best parameter to cut down as it is usually unnecessary to record the complete solution. A possible approach is to record the hash of the solution rather than the solution itself.

Unfortunately for some problems, it is sometimes impossible or very costly to validate the tabu status even if the solutions are stored. For example in TSP, solution A consisting of a tour of 1-2-3-4 and solution B of a tour of 2-3-4-1 can only be detected as the same solution if rotational comparison is supported. Hence, rather than tabu-ing the solutions, sometimes the move applied can be tabu-ed. Typically, moves only affect some portions of a solution and thus occupy lesser space then the solution. To reduce cycling, subsequent moves are verified to ensure the reverse moves would not be applied. Apparently such technique does not

strictly prevent all forms of solution cycling in its tenure. Nevertheless it is effective and generic to problems.

As oppose to tabu-ing the move, a more restrictive approach is to tabu the objective value. This is based on the assumption that most solutions have an unique objective value and thus tabu-ing the objective value is almost as good as tabu-ing the solution itself. The drawback of this approach is that elite solutions that have the same objective value would be missed.

The *Tabu List* interface supported various kind of tabu techniques (including those that are not mentioned in this thesis) by manipulating the list indirectly through the virtual functions *Is Tabu* (Function 8) and *Set Tabu* (Function 9). The *Is Tabu* verifies if the neighbor is tabu-ed and *Set Tabu* sets accepted neighbors into the list.

2.2.2 Aspiration Criteria Interface

Virtual Function:

- *boolean* ***OverrideTabu*** *(Solution* solution, Move* move,*
  ObjectiveValueType neighborObjectiveValue,
  SwitchBox switchbox);* *Function 10*

Descriptions:

The aspiration criterion is used to override the tabu status of a neighbor if it meets some criteria. For example, when the tabu list is used to tabu the move applied, there is a possibility that good neighbors may be mistaken as tabu-ed solutions. The aspiration criterion could then override the tabu status of a neighbor if its objective value is better than the best-found solution. A virtual function *Override Tabu* (Function 10) is used to perform the exemption.

2.2.3 Pheromone Trail Interface

Virtual Function:

- *double* ***ExtractPheromone*** *(Solution* solution, Move* move)* *Function 11*
- *void* ***UpdateLocalPheromone*** *(Solution* localSolution);* *Function 12*
- *void* ***UpdateGlobalPheromone*** *(Solution* currentSolution, Solution* localSolution);* *Function 13*
- *void* ***PheromoneEvaporation*** *(void);* *Function 14*

Descriptions:

The *Pheromone Trail* object is used to record the pheromone density on the paths. The pheromone trails is one of the two parameters used to determine the transitional probability of ants in choosing their paths. While the local heuristics can be seem as the ant's natural judgment in taking a trail, it is the pheromone density on the trails that influences the ant to change its direction. Each of these factors is assigned a weight, α and β for the pheromone trail and local heuristic respectively. In particular, the probability of moving from node *r* to node *s* is given generally by

$$p_k(r,s) = \begin{cases} \dfrac{[\tau(r,s)]^{\alpha}.[\eta(r,s)]^{\beta}}{\sum_{u \in J_k(r)} [\tau(r,s)]^{\alpha}.[\eta(r,s)]^{\beta}} & \text{if} \quad s \in J_k(r) \\ 0 & \text{otherwise} \end{cases} \qquad \text{Eqn 2.1}$$

where $\tau(r,s)$ = *pheromone for moving from node r to node s*
$\eta(r,s)$ = *local heuristics for moving from node r to node s*

The pheromone trail τ is usually initialized to a fixed value across of the trails prior to being used (Elitism Strategy), and the value it is initialized to, τ_0, is usually given by a generic "baseline" solution to the problem. This solution can be

evaluated using any constructing algorithm likes Greedy Algorithm, or even ACO itself (using a generic pheromone trail initialized to any arbitrary value). τ_0 is a function of this initial solution. The value of $\tau(r,s)$ is retrieved using the *Extract Pheromone* function (Function 11).

After each move is completed, the ant may choose to perform a local pheromone decay or deposit. If no such action is performed, each of the ants in the iteration will be non-collaborative and use only the pheromone trail at the beginning of the iteration. While there are implementations without local pheromone updates with good results, it was generally found that local pheromone update improves solution quality. The logic is that unlike real-ants, the solver of an optimization problem needs to traverse the best path once to record it, and implement other ways to enforce this knowledge (global pheromone update). Meanwhile, it is necessary to search as much of the solution space as possible, and in most cases, it is better to lower the pheromone concentration from a taken trail, so that other ants may try the less trodden paths, which leads to a more aggressive search around the neighborhood as well as to prevent solution cycling. There are many formulas (if implemented) for local pheromone update, but generally,

$$\tau(r,s) \leftarrow (1-\rho_l).\tau(r,s) + \rho_l.\tau_0 \qquad \text{Eqn 2.2}$$

where τ_0 *represents the default pheromone level*
ρ_l represents the local decay factor

Local pheromone update can be performed in two ways. The first, *step-by-step* update is performed as each ant takes a move. The nature of this process makes it more suited for a parallel implementation. The second, online-delayed pheromone update, is performed as each ant completes a solution build, and is more suited for

a serial implementation. This process is updated by the ACO search indirectly through the *Update Local Pheromone* function (Function 12).

While the local pheromone update may be optional, the global pheromone update that occurred at the end of iteration is compulsory. The justification for such an action is by counter-intuition. Suppose there is no pheromone update. Then, each ant will repeatedly find the same probabilities on all the moves. The only variable then is the random choice. While this progresses the solution, it does so very gradually. Furthermore, there tend to be an excessive amount of solution cycling due to the constant nature of the probabilities. This completes the intuition that the pheromone trail should be updated. Global pheromone update can be performed in several ways. Some implementations proposed using the trail from all the ants in the iteration (*AS*, *ASrank*), others advocate using only the best route in the iteration (*MMAS*, *ACO*), and most suggest using the best route found so far. Generally,

$$\tau(r,s) \leftarrow (1-\rho_g).\tau(r,s) + \rho_g.\Delta\tau(r,s) \qquad \text{Eqn 2.3}$$

where ρ_g *represents the global decay factor*

The global update on the pheromone trails is performed via the *Update Global Pheromone* function (Function 13). In synch with global pheromone update is the optional pheromone evaporation, which is updated with the *Pheromone Evaporation* (Function 14). One idea is to use additional reinforcement for unused movements, with equation 2.4, while other approaches perform a simple evaporation on all trails with equation 2.5, for all *i* and *j*:

$$\tau(i,j) \leftarrow \tau(i,j) + \rho_e.\tau_0 \qquad \text{Eqn 2.4}$$

$$\tau(i,j) \leftarrow (1-\rho_e).\tau(i,j) \qquad \text{Eqn 2.5}$$

where ρ_e *represents the evaporation factor*

2.2.4 Local Heuristic Interface

Virtual Function:

- *double* ***ComputeLocalHeuristic*** *(Solution* solution, Move* move)* *Function 15*

Descriptions:

The Local Heuristic interface is used to incorporate the underlying heuristic in solving the problem. Generally a single greedy heuristic is used for its speed and performance. However there are instances of problems, especially those of increased complexity that a single local heuristic does not suffice. For example, there had been implementations of VRPTW with multiple combined local heuristics [Bullnheimer et al., 1997]. In such instances, $\eta(r,s)$ can be formulated as

$$\eta(r,s) = \sum_{j=1}^{n} [\eta_j(r,s)]^{\alpha_j} \qquad \text{Eqn 2.6}$$

where $\alpha_j \geq 0$ *and symbolize the weights of the local heuristics*

The function *Compute Local Heuristics* (Function 15) is used to compute the value of $\eta(r,s)$, which is later used together with $\tau(r,s)$ as shown in Eqn 2,1 to give the transitional probability.

Simulated Annealing

2.2.5 Annealing Schedule Interface

Virtual Function:

- *double* ***RetrieveCoolingTemperature*** *(Solution* solution, ObjectiveValueType neighborObjectiveValue, int currentIteration, int totalIteration)* *Function 16*

Descriptions:

In SA, the probability of transition is a function of the objective values difference between the two states and a global time-dependent parameter called the *temperature.* Suppose δE is the difference in objective values of the current solution and its neighbor, and T is the temperature. If δE is negative (i.e., the new neighbor has a better objective value) then the algorithm moves to the new neighbor with probability 1. If not, it does so with probability $e^{-\delta E/T}$. This rule is deliberately similar to the Maxwell-Boltzmann distribution governing the distribution of molecular energies. It is clear that the behavior of the algorithm is crucially dependent on the T. If T is 0, the algorithm is reduced to greedy, and will always be moving toward a neighbor with a better objective value. If T is ∞, it moves around randomly. In general, the algorithm is sensitive to coarser objective variations for large T and finer variations for small T. This is exploited in designing the annealing schedule, which is the procedure for varying T with time (the number of iterations). At first T is set to infinity, and is gradually decreased to zero ("cooling"). This enables the algorithm to initially get to the general region of the search space containing good solutions, and later hone in on the optimum. The *Annealing Schedule* Object is catered for algorithm designer to devise their cooling function. The *Retrieve Cooling Temperature* function (Function 16) retrieves the time-dependent T, when a non-improving neighbor is encountered.

Genetic Algorithm

2.2.6 Recombination Interface

Virtual Function:

- *void* ***Crossover*** *(Solution * parentA, Solution* ParentB Solution* offSpring1, Solution * offSpring2)* *Function 17*

Descriptions:

The *Recombination* object combines the selected individuals to produce their offspring. It incorporates a single tunable variable *probability of crossover (P_c)*, which encodes the probability that two selected individual will actually breed. Generally the value is set between 0.6 and 1.0. For each pair of parent, a random number between 0 and 1 is generated. If the number falls under the crossover threshold, the organisms are reproduced or otherwise, they are propagated into the next generation unchanged. Crossover results in two new child individuals, which are added to the next generation pool. The *Crossover* function shown in Function 17 is dictated for this purpose. During the crossover, the chromosomes of the parents are mixed typically by simply swapping a portion of the underlying data structure, although other more complex merging mechanisms have proved useful for certain types of problems. This process is known as one-point crossover and is repeated with different parent individuals until there are an appropriate number of candidate solutions in the next generation pool.

2.2.7 Population Interface

Virtual Function:	
• *void* ***InitializeFirstGeneration*** *(void)*	*Function 18*
• *void* ***DiscardUnfitIndividuals*** *(void)*	*Function 19*

Descriptions:

GA solution is usually represented as simple strings of data in a manner not unlike instructions for a von Neumann machine, although a wide variety of other data structures for storing chromosomes have also been tested, with varying degrees of

success in different problem domains. The *Population* object is used to keep a collection of such individuals, with each new population (generation) created at the end of every iteration. Initially a *first generation population* is seeded unto the gene pool. This function is implemented in the *Initialize First Generation* function (Function 18) and is used by the Population object to initialize the individuals prior to the start of the algorithm. The first generation can be created randomly or by heuristics such as randomized greedy. However, it is vital that the implementer ensures the diversity of the first generation to prevent rapid convergence of similar individuals. To prevent over-population, GA employs various strategies in selecting the individuals for the next generation (*Fitness Techniques*, *Elitism*, *Linear Probability Curve, Steady Rate Reproduction*). To cater for these strategies, the *Population* object uses the *Discard Unfit Individuals* (Function 19) that mixes parents and their children together and consequently discards some of these individuals in accordance to the user-specific strategies.

2.3 Engine and its Component

Section 2.1 and 2.2 has illustrated the various abstract classes in MDF. In this section, we observe how the MDF search engines put these classes together and then discuss the issues arising from the integration. This section also provides the opportunity to examine the search parameters (contained in the engine switch box) and analyze their effects on the search process.

2.3.1 Engine Interface

Virtual Function:	
• *void* ***StartSolving*** *(void)*	*Function 20*
• *void* ***StopSolving*** *(void)*	*Function 21*
• *Solution** ***GetBestFoundSolution*** *(void)*	*Function 22*

Descriptions:

The *Engine* Interface contains the general operations that are subsequently inherited by the meta-heuristics engines. There are three operations, *Start Solving* (Function 20) that begins the search sequences, *Stop Solving* (Function 21) that terminates the search and *Get Best Found Solution* (Function 22) that returns the best found solution in the search. These virtual functions prevent unnecessary amendment to application codes when an implementer changes the underlying meta-heuristic, thus giving a more generic design.

2.3.2 Switchbox Interface

Parameter:	
• *NumberOfIteration*	*Parameter G1*

Descriptions:

The *Switchbox* interface complement the Engine interface as a container that stores the generic parameter presented in meta-heuristics. The *Number Of Iteration* (parameter G1) indicates the amount of time the meta-heuristics is allowed to run and is often used as a termination criterion. Typically, the quality of solutions improves with the increasing number of iterations, which follows the *law of diminishing returns*. As such, it is important to determine a value that gives

sufficiently well results and yet be computed in a reasonable time. Unfortunately, there are many factors that affects this variable and they include the problem size, the meta-heuristics' parameters (such as tabu tenure, pheromone density), the strategies involved (internsification/diversification) and even instances of the problem. Hence it is a challenge for an algorithm designer to devise an optimization scheme that could produce the best results in the fastest possible time.

2.3.3 TS Engine

TS Engine performs the rudimentary procedures of TS and the pseudo-code is presented in Figure 2.3.

TS Engine

```
procedure
    Initialize a current Solution
    while terminating criteria not reached
        Neighborhood Generator generates a new neighborhood;
        Constraint discards any undesired neighbors;
        Objective Function evaluates selected neighbors;
        Penalty Function applied to neighbors;
        Tabu List and Aspiration Criteria are consulted;
        Move translates current Solution to best neighbor;
        if new Solution is better than best found Solution
            Clones and records new Solution as best found Solution;
        end if
        Tabu List is updated;
    end while
end procedure
```

Figure 2.3: The TS Engine Procedure (pseudo-code)

Prior to the search, TS Engine initializes a solution usually from a problem-specified constructing heuristic. Based on this solution, the *Neighborhood Generator* creates a list of neighbors using the *Move* object(s). The Constraint object(s) then validate each of these created neighbors to select a subgroup of accepted neighbors, which are also known as candidates. These candidates are then evaluated using the *Objective Function*. The *Penalty Function* is applied to the objective value of the candidates and the best non-tabu neighbor is selected after consulting the *Tabu List* and *Aspiration Criteria*. At this point, a new state is selected and the selected *Move* object translates the current solution to the chosen neighbor. The objective value of the new solution is compared against the best-found solution and if the value is better, the new solution will be cloned and then recorded. Finally the *Tabu List* will be updated to prevent reoccurrence of solutions. If the terminating condition is not reached at this time, a new neighborhood will be generated and the iterative search continues.

2.3.4 TS Switchbox

Parameter:	
• *Tabu Tenure*	*Parameter TS1*
• *First Accept*	*Parameter TS2*

Descriptions:

The *Tabu Tenure* (Parameter TS1) determines the tabu-ed duration of visited solution. Apparently a short tenure is ineffective in preventing solution cycling and a long tenure requires a greater validating time. In fact it is almost impossible to find an optimal tenure value even for instances of a same problem. Hence a popular approach is to vary the tenure in accordance to the search events. This

strategy is known as reactive tabu list and is illustrated in *Control Mechanism*. There are several ways to implement the tabu tenure. An effectual implementation is to set up the tabu list is a circular array with size equals to the tenure. In this way, when the list is filled, the first inserted solution will be replaced. Beside the tenure, TS Engine also uses *First Accept* (Parameter TS2). Typically, TS will examine each candidate in the neighborhood to determine the best neighbor to move to. However, when time is crucial (such as real-time optimization), it is possible to speed up the search by accepting the first encountered neighbor with an improving objective value. This strategy is known as first-accept and can be activated by setting the *First Accept* parameter to true.

2.3.5 ACO Engine

ACO Engine performs the rudimentary procedures of ACO and the pseudo-code is presented in Figure 2.4. Using the *Elitism Strategy*, the pheromone trail is first updated with pre-constructed solution(s). ACO Engine then uses a triple nested loop to carry out the ACO routines. The main procedure performs the iterative improvement steps bounded by the number of iteration or user-defined criteria. The colony procedure (second loop) spawns the activity of each ant in the colony and updates the global pheromone trails in accordance to the defined strategies (such as *ASrank*, *MMAS* or *ACO*).

ACO Engine

```
procedure
    Initialize the Pheromone Trail
    while terminating conditions not reached
        while there is still ants in colony and
            while the solution is not completed
                Neighborhood Generator generates a set of new trails;
                Constraint discards any impassible trails;
                Trail chosen by consulting Local Heuristic and Pheromone Trail
                Move translates the Solution with selected trail;
                Local Pheromone Trail Updated
            End while
        end while
        Objective Function evaluates solutions constructed by ants;
        Penalty Function is applied to determine the quality of solutions;
        Global Pheromone Trail is updated;
        If new Solution is better than best found Solution
            Clones and records new Solution as best found Solution;
        end If
        Pheromone Evaporation Occurred;
    end while
end procedure
```

Figure 2.4: The ACO Engine Procedure (pseudo-code)

In addition to the global pheromone update, the colony procedure also executes the pheromone evaporation. The innermost loop describes the ant activity in constructing a solution. The *Neighborhood Generator* generates the ant's trails

with respect to the ant current position (partial solution). The *Constraint* objects then obstruct the trails that would lead to infeasible solution. The transitional probability of each trail is computed by consulting the *Local Heuristic* and *Pheromone Trail*, and the ant randomly chooses a path with this probability. When a trail is selected, the *Move* object adds the new trail into the partial solution. Optional local pheromone update strategies such as ACO, is applied to improve the search and the activity continues until the solution is completely built.

2.3.6 ACO Switchbox

Parameter:

• *Number of Ants*	*Parameter ACO1*
• *α (Pheromone Trail weight)*	*Parameter ACO2*
• *β (Local Heuristic weight)*	*Parameter ACO3*
• *ρ (Decay factors)*	*Parameter ACO4*
• *q0 (Exploitation/Exploration factor)*	*Parameter ACO5*

Descriptions:

There are many arguments on the optimal number of ants and the two most agreed value for this parameter (Parameter ACO1) is a constant value (e.g., 10) or n (problem size) [Bullnheimer et al., 1997; Dorigo et al., 1996]. The impact of a choosing n will increase the computational complexity of the problem by another factor of n. Based on x iterations and n^2 for the probability calculation, choosing a constant number of ants give $O(xn^2)$, whereas n ants gives $O(xn^3)$. However more ants could mean better exploration. Hence, both arguments are valid, and the decision on the value should be up to the implementer. Another two important parameters are the weights value of α (Parameter ACO2) and β (Parameter ACO3). [Dorigo, 1992] found from experimental results that good values of α and β (for

TSP at least) are 1 and 5 respectively. A greater weight is usually placed on the local heuristics (affected by β) to prevent fast convergence to local optimal. Another key parameters are the decay factors ρ (Parameter ACO4). These factors are generally set between 0 and 1, to signify the percentage of decay/evaporation (0 means no decay, 1 means complete decay). Decay factors can be subdivided into three separate parameters (local decay, global decay, and evaporation), although most classic ACO uses the same value for them. Exploration or exploitation factor is another important factor in the ACO algorithm. A complete exploitation reduce the algorithm simply to the power of the local heuristics, in most cases just a greedy approach. Exploration allows an opportunity to search around the best-found solution, a technique that works often in non-linear problems. The decision of exploration or exploitation is defined by the factor q_0 (Parameter ACO5) the exploitation factor, which has a domain of 0 to 1. When q_0 is 0, the ants explore all the time; when q_0 is 1, exploitation occurs all the time.

2.3.7 SA Engine

SA Engine performs the rudimentary procedures of SA and the pseudo-code is presented in Figure 2.5. Similar to TS and ACO, an initial solution is created as the starting point of the search. The *Neighborhood Generator* then generates a random *Move* and this neighbor is evaluated with the *Objective Function*. The *Penalty Function* is the applied to the neighbor's objective value and the adjusted value is then compared against the current solution. If the objective value of the neighbor is better, the *Move* object translates the current solution to its neighbor. Otherwise, the annealing schedule is speculated to see if the non-improving neighbor could be accepted. If the neighbor is accepted, the *Move* object will translate the solution;

otherwise, the current solution will remain unchanged. This procedure is repeated until the any of the terminating criteria is reached.

SA Engine

```
procedure
    Initialize a current Solution;
    while terminating conditions not reached
        Neighborhood Generator generates a random neighbor;
        Constraint validates the feasibility of neighbor;
        Objective Function evaluates solutions;
        Penalty Function temporary adjusts the objective value;
        If new neighbor is better than current Solution
            Move translates Solution to neighbor;
        Else
            Consults the Annealing Schedule;
            If neighbor is accepted
                Move translates Solution to neighbor;
            Else
                Current Solution remains unchanged;
            end if
        end If
        If new Solution is better than best found Solution
            Clones and records new Solution as best found Solution;
        end If
    end while
end procedure
```

Figure 2.5: The SA Engine Procedure (pseudo-code)

2.3.8 SA Switchbox

Parameter:	
• *Temperature*	*Parameter SA1*

Descriptions:

There are various approaches in modeling the temperature and can be classified as either *static* or *dynamic*. In a static annealing schedule, the parameters are fixed and cannot be changed during the execution. In this category, there are two simple static annealing schedules. The first schedule is the *exponential cooling scheme (ECS)*, which has the form of $T_{k+1} = \alpha T_k$, where α is some constant that satisfies $0 < \alpha < 1$, k is the annealing schedule index starting from 0, and T_0 is the initial temperature. This cooling is first proposed by [Kirkpatrick et al., 1983] with $\alpha = 0.95$. Another cooling scheme is the *linear cooling scheme (LCS)* [Randel and Grest, 1986], which has the form of $T_{k+1} = T_k - \Delta T$ (i.e. *T* is reduced for every *L* trials).

2.3.9 GA Engine

GA Engine performs the rudimentary procedures of GA and the pseudo-code is presented in Figure 2.6. Initially, the *Population* object creates the first generation pool. From this gene pool, the *Neighborhood Generator* selects the individuals (*Solutions*) for crossover. The developer specifies the type of crossover and implements in the *Recombination* object. When the crossover is performed, the Constraint objects validate the offspring to ensure their feasibility. The *Move* objects translates/mutates the qualified *Solutions* so as to improve their fitness, which is evaluated by the *Objective Function*. The mutated children are mixed with their parents and the *Penalty Function* is applied to the whole population. The

Population is now over-populated and some of the solutions are discarded based on the modified objective value until the number of solutions is the same as the original population. Subsequently the new population is used to generate the next generation and this procedure is repeated until any of the terminating conditions is reached.

GA Engine

```
procedure
    Initialize the first generation Population;
    while terminating conditions not reached
        Neighborhood Generator selects Solutions for mating;
        Recombination crosses selected Solutions to form new children
        Constraint discards infeasible children;
        Move mutates feasible children;
        Objective Function evaluates children;
        Children are mixed into the parent Population;
        Penalty Function adjusts the objective value of all Solutions in Population;
        Population discards unfit individuals until the population is balanced;
        If any Solution in Population is better than best found Solution
            Clones and records new Solution as best found Solution;
        end If
    end while
end procedure
```

Figure 2.6: The GA Engine Procedure (pseudo-code)

2.3.10 GA Switchbox

Parameter:	
• P_c *(probability of crossover)*	*Parameter GA1*

Descriptions:

P_c (Parameter GA1) is an optional parameter that determines the probability in which two selected individuals are combined. For most GA application, P_c is often set as 1 (i.e. all selected individuals are mated successfully). However, this implies that elite individuals (which have high selection probability) may lose their "good" traits during the crossover or mutation. Hence the primarily function of P_c is to probabilistically preserve some elite parents for each successive generations. It is also possible to model this parameter in accordance to the SA annealing schedule.

2.4 Control Mechanism

The objective of the *Control Mechanism* is to allow the meta-heuristic to adapt itself with the various situations that occurred during a search process. As meta-heuristics suffer from the inability of performing global optimization, it is vital that the local improvement should not depend solely on the underlying meta-heuristics but also on rules or guides that could enhance the search. These rules are better known as search strategies and can be generally categorized as either intensifying or diversifying. It is not difficult to realize that while intensification and diversification work in opposition to each other, they are actions applied to adjust the search trajectory, or more precisely reactions that are executed in response to search situations or events. Based of this observation, we can define ***ALL*** search strategies by two components, *Requests (when is an action necessary)* and their *Responses (what action is needed) (R&R)*. This implies that meta-

heuristic can be viewed as a request-driven simulation, in which the occurrences experienced during the search can be utilized to guide the future exploration.

The above phenomenon can be seen as a feedback path in control engineering, in which information from engines is passed to a centralized control unit that readjusts the control parameters to adapt to the external environment. Figure 2.7 shows an illustration on the feedback control mechanism.

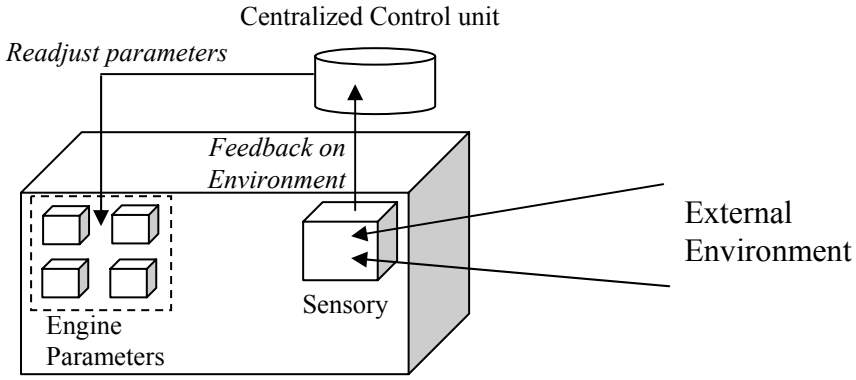


Figure 2.7: Illustration on a feedback control mechanism

Most of the works in the literature present various search strategies and provide detailed explanation on how the they can be performed. Surprisingly, these works seldom describe exactly when these strategies should be performed. For example in the work of [Stutzle and Dorigo, 1999], the importance of exploitation and exploration is illustrated together with a recommended value for *q0*. The authors also proposed how *q0* can be dynamically changed but failed to provide exact details on the factors affecting *q0*. The same predicament surfaced in [Battiti and Tecchiolli, 1994], in which the authors could not present accurate rules that could guide the behavior of the reactive tenure. Obviously, these authors could not be faulted as we realize the considerable efforts involved in coming up with precise rules, especially when some of these rules are problem-specified. However, it would be interesting if we could input different rules into their works and observe

their effects. The control mechanism facilitates this by providing an experimental "playground" that could readily convert user-defined rules into program codes. Rules are turned into search events (*Requests*) and the techniques are converted into handlers (*Responses*) and a "multiple-to-multiple" relationship can be established between these components.

Most interestingly, the *R&R* concept provides a suitable platform for forming hybridized models. Most meta-heuristic hybrids are either loosely coupled such as the two-phase approach (e.g. [Maa and Shanblatt, 1992], [Gehring and Homberger, 2001]) or where one meta-heuristic embedded on another [Stutzle and Dorigo, 1999]. We observe that in these hybrids, each meta-heuristic occupies a certain time phase in the search. These phases can be rotated (loosely coupled hybrids) or interpolated (embedded hybrids). We define an *atomic unit* as the smallest unit time for a meta-heuristic to perform a completed set of routine and assign each phase as an atomic unit. Due to the diverse nature of meta-heuristics, the definition of an atomic unit varies across them. In most cases, an atomic unit is equivalent to one search iteration but in techniques like ACO, an atomic unit means the activity of a single ant. Table 2.1 shows the definition of an atomic unit in TS, ACO, SA and GA. Once the search process is partition into atomic units, each of these units is allocated to a meta-heuristic. The allocation can be adaptive to the previous events and the assigned meta-heuristic is dependent on the rules set by the algorithm designer.

Table 2.1: The definition of an atomic unit in TS, ACO, SA and GA

Meta-heuristics	**Atomic Unit Definition**
Tabu Search	An iteration of the search
Ant Colony Optimization	The activity of an ant
Simulated Annealing	Generating a new random move
Genetic Algorithm	A new generation

In our model, a search state at any atomic time point comprises of the best-found solution, the current solution, the current operating meta-heuristic and the values of search parameters at that point. Prior to the search, the algorithm designer inputs the requests (or rules) that react to event(s) such as *improving solutions*, *non-improving solutions*, *new best solution found* and *end of atomic unit*. A search algorithm begins with an initial search state. As the search proceeds, any occurred event(s) that matches the rules of the requests will be activated, which consequently triggered the desired responses. Suppose we are solving a problem using TS and we implement a response that performs "switch the operating meta-heuristic to SA" and a request that states "execute the switch if 100 non-improving moves are encountered". In this example, each time the search process notices a non-improving solution, the request will be informed. When a hundred non-improving solutions are encountered, the request will trigger the response, which in turn changes the operating meta-heuristic from TS to SA. In the next section, we will examine how *R&R* can be implemented using three mechanisms, *Events*, *Handlers* and *Event Controller*.

2.4.1 Event Interface

Virtual Function:

- *list <EventMessage>* ***TriggerResponse*** *(Engine* currentEngine)* *Function 23*

Descriptions:

The *Event* object implements the user-defined rule(s) in a request. When the operating engine detected an event that matches these rules, the *Trigger Response* function (Function 23) will evoke a list of required action (*Event Message*). Each of these messages is associated with two parameters: the response to be executed and its corresponding priority. There are three priority levels, namely, (a) INSTANT, which is to execute the responses immediately, (b) NORMAL, which is to execute the responses at the end of the atomic unit, and (c) DELAYED, which is only executed after all the responses with priority NORMAL have been performed. The hierarchical nature of the priority queue will allow designers to have additional control over sequence of responses.

2.4.2 Handler Interface

Virtual Function:

- *void* ***Execute*** *(EventController* eventController)* *Function 24*

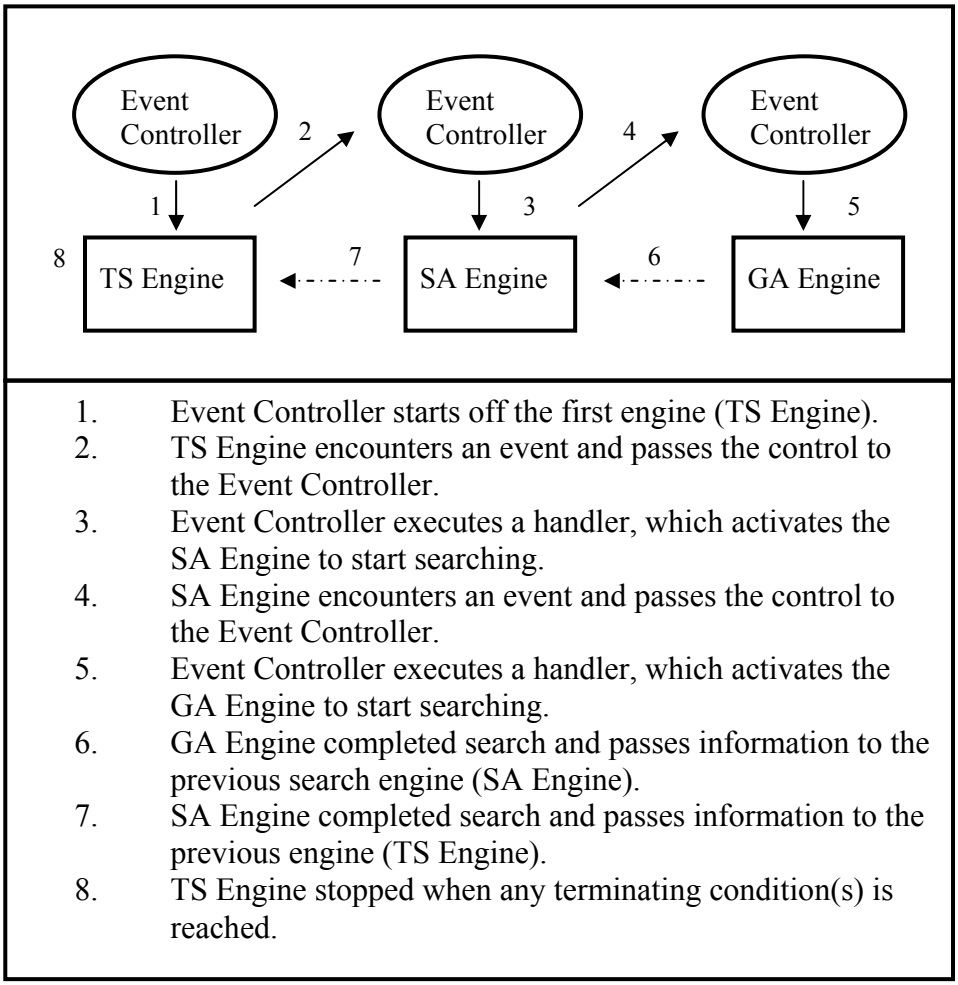
Descriptions:

The *Handler* object implements the responses that readjust the search procedure. Generally these responses can be classified into two categories, parameters-based and techniques-based strategies. Parameters-based strategies such as reactive tabu search and dynamic annealing schedule adapt their search parameters in accordance to events. An example could be a reactive tabu list that shortens the

tabu tenure when an elite solution is encountered and lengthens it when there is solution cycling. For this strategy, two *Handler* objects are required, with one object handling increment and the other decrement. When an event (such as an elite solution) is encountered, the *Handler* will modify the parameter(s) in *Switchbox* via the *Event Controller* using the *Execute* function (Function 24). Techniques-based strategies on the other hand, usually modified parts of the search state. Modifications include changing the current solution, the underlying meta-heuristic and/or the search procedure (e.g. intensification/diversification). These modifications can be evoked using the *Event Controller*, who has control to every aspect of the search state.

2.4.3 Event Controller

The role of the *Event Controller* is to control the search process through the adjustment of search state, which includes the current operating meta-heuristic engine, the search parameters and the current solution. In software term, it acts as a "manager" between the user-defined requests and the meta-heuristics engine and adopts the design of *"Chain of Responsibility"* [Schmidt et. al., 1995]. Initially, the *Event Controller* sets up the search engine. As the search proceeds, events experienced such as the behavior of the solutions' objective values, the structure of the solutions are compared against the user-defined request. If there is a match, the related responses are queued and later triggered according to their priority. If the related response(s) affects the search parameters, the parameters are modified and the search is continued. However, a chain will occur if the response activates another meta-heuristic engine (such as the case of hybridization), which then has the capability to execute yet again another engine as illustrated in Figure 2.8.



1. Event Controller starts off the first engine (TS Engine).
2. TS Engine encounters an event and passes the control to the Event Controller.
3. Event Controller executes a handler, which activates the SA Engine to start searching.
4. SA Engine encounters an event and passes the control to the Event Controller.
5. Event Controller executes a handler, which activates the GA Engine to start searching.
6. GA Engine completed search and passes information to the previous search engine (SA Engine).
7. SA Engine completed search and passes information to the previous engine (TS Engine).
8. TS Engine stopped when any terminating condition(s) is reached.

Figure 2.8: The illustration of the *Chain of Responsibility* pattern adopted by Event Controller.

As illustrated in Figure 2.8, we see that multiple search engine (corresponds to multiple meta-heuristics) can be deployed in a single search. This chain of responsibility ensures that the search is sequential, in which each engine is responsible for their roles as defined by the algorithm designer. In addition, communications between the search engines is possible via the centralized event controller and this enabled them to "share" information experienced in the search. Finally, the *Event Controller* also assures that duplicate responses would not be triggered twice in the same atomic time so as to prevent executing the same handler twice.

2.4.4 Further Illustrations

Our first example illustrates how MDF performs hybridization using the R&R paradigm. The illustrated hybrid scheme is proposed by [Stutzle and Dorigo, 1999], in which ACO and Local Search (LS) were hybridized to solve TSP. The authors' approach was to apply LS to the iteration-best solution before the ants update it into the pheromone trails. This strategy is implemented in MDF as follows; ACO is the operating meta-heuristic and LS is embedded as a *Handler*, which we will denote as the LS handler. We define a request (*End_ACO_Event*) that will be triggered at the end of every ACO search iteration and set it to NORMAL priority. When an iteration is completed, this event will register a match with the request and the Event Controller will execute the LS handler. The LS handler then modifies the search state by applying LS onto the iteration best solution. Subsequently, the enhanced solution will be updated into the pheromone trail for future ants. Figure 2.9 shows the code fragment of the *End_ACO_Event* and *LS handler* as an illustration of this technique-based strategy.

```
class End_ACO_Event : Event
{
    list<EventMessage>TriggeredResponse
            ( Engine* This )
    {
        If ( This->IterationCompleted ( ) )
            list.add ("LS Hander", NORMAL);
            return List;
    }
}
```

```
class LS_hander : Handler
{
   void Execute(EventController* This)
    {
       TSP_Solution* currentBestSoln
           = This->GetCurrentSolution();
        LSEngine->SetInitialSolution(currentBestSoln);
        LSEngine->StartSolving();
        This->SetCurrentSolution
             (LSEngine->GetBestFoundSolution());
     }
}
```

Figure 2.9: An illustration on a technique-based strategy

Our next example is on *reactive tabu search*, in which we illustrate how parameter-based strategies can be implemented using MDF. Reactive tabu search refers to strategies that adaptively adjusting tabu search parameters according to the search trajectory [Battiti and Tecchiolli, 1994]. Many complex heuristics have been proposed with this strategy, each with its own assumptions on the solution space. In fact a popular analogy is to visualize the solution space as a multi-dimensional terrain. The factors include objective value, similarity in the solution structure and time. Based on these factors, the reactive tabu search attempts to navigate along the terrain toward new local optima. In order to simplify our illustration, we only consider two factors, time and objective value and the parameter adjusted is limited to the tabu tenure. Time simply refers to number of iterations performed. Our simplified strategy works as follows. When we encounter a series of non-improving we lengthen our tabu tenure so as to prevent solution cycling. On the other hand, when we encounter a new best solution, we shorten our tenure in order to perform intensification. We implement an event called *Reactive_Event* which trigger two handlers, *Lengthen_Tenure* and *Shorten_Tenure*. The first handler (*Lengthen_Tenure*) will increase the tabu tenure by some *x* amount when the search encounters a series of non-improving moves. On the other hand, when a new best solution is encountered, we will revert back the tenure, discarding any move that have been kept for more than *n* iterations using the *Shorten_Tenure* handler. The code fragment for this implementation is shown in Figure 2.10.

```
class Reactive_Event : Event
{
        Int countBadMove = 0;
        int badMoveLimit = n; // Maximum allowed bad moves
        list<message> TriggeredResponse( Engine* This )
        {
              If ( This->BestSolutionFound( ) )
              {
                  list.add ( “Shorten_Tenure” , NORMAL );
                  countBadMove = 0;
              }
              Else
              {
                   If ( countBadMove = badMoveLimit )
                       list.add ( “Lengthen_Tenure”, NORMAL );
                   Else
                        countBadMove ++;
              }
              return list;
        }
}
class Shorten_Tenure : Handler
{
       void Execute(EventController* This)
       {
             This->SBContainer->TSSwitchBox->TabuList.Tenure = t;
       }
}
class Lengthen_Tenure : Handler
{
       void Execute(EventController* This)
       {
             This->SBContainer->TSSwitchBox->TabuList.Tenure = t + x;
       }
}
```

Figure 2.10: An illustration on a parameter-based strategy

2.5 Software Strategy Library (SSL)

SSL provides a list of tools that facilitates some of the more popular strategies. For example, the two static annealing schedules (*exponential* and *linear*) are incorporated. These tools can be classified as tools for general strategies and for specific meta-heuristics such as the discussed annealing schedules. SSL remains an on-going work due to the numerous strategies (both existing and new) that can and would be included. Sections 2.5.1 and 2.5.2 provides more illustrations on the SSL components.

2.5.1 General Tools Illustration: Elite Recorder

SSL supports this strategy by storing a list of elite solutions during the search in the *Elite Recorder*. Each of these elite solutions can be used as a new initial solution for another meta-heuristic. The rationale is to search these elite solutions more intensively and perhaps differently across various meta-heuristics. The Elite Recorder is embedded as a *Handler* and is triggered when a new best solution is found.

2.5.2 Specific Tools Illustration: Very Large Scaled Neighborhood (VLSN)

VLSN [e.g. Ahuja et al., 2003] works on the principle that by generating a larger neighborhood, it increases the chances of obtaining better solutions. One approach is to repetitively apply the *Move* operator "*k* times" on all the neighbors generated in each move. However, to prevent the neighborhood from expanding exponentially, it is often useful to select only the elite neighbors to narrow down the size. *SSL* provides a *Candidate List* class that inherits from the *Constraint* Interface. It selects the best *n* neighbors from those generated by each k-opt moves.

CHAPTER 3

APPLICATIONS

This chapter reviews some of the MDF applications published. These applications include the *Traveling Salesman Problem (TSP)* [Lau et. al[1], 2004], the *Vehicle Routing Problem with Time Window (VRPTW)* [Lau et. al[1], 2003] and the *Inventory Routing Problem with Time Window (IRPTW)* [Lau et. al[2], 2003].

The choice of TSP, VRPTW, and IRPTW is a generalization of many real-world routing problems, which tend to have multiple objectives and constraints. For instance, the IRPTW considers inventory costs across multiple period of VRPTW, which in turn is the VRP extended with time window, which in turn is extended with optimal fleet (vehicles) size objective from the classic and NP-hard TSP. The extensions of NP-hard problems with more constraints and objectives provide increasing approximate analogy to practical application, increasing the value of solving these problems optimally. As such, these problems are chosen to demonstrate the power of re-use in the framework in solving similar or extended instances of a problem. We believe that MDF framework can be applied in other problems as long as a solution can be formulated for the base problem.

3.1 Traveling Salesman Problem (TSP)

The Traveling Salesman Problem is a classic NP-hard problem, and the mathematical basis related to TSP was treated as early as the 1800s by Irish mathematician Sir William Rowan Hamilton. The development of the general form of TSP, as well as other classic combinatorial optimization problems, is studied by [Schrijver, 1960]. While the problem was well-known, there appears a lack of

reference in the literature to earlier work, and it was not until 1954 that the most popular TSP definition came from [Dantzig et al., 1954]. TSP definitions for general and variant forms of the problems are easily available. In the context of this thesis, TSP is defined in Figure 3.1.

Let

$G = (V,A)$ *be a graph,*

where $V\{ v_1, v_2, \dots , v_n \}$ *be a set of cities* (*vertex set*), *and*

$A = \{ (v_i,v_j) : v_i,v_j \in V, i \neq j \}$ *be the edge set,*

$C(r,s) = C(s,r)$ *be a cost measure associated with edge* (r,s) *w.r.t.* A.

Figure 3.1: Problem definition of the Traveling Salesman Problem

A tour is defined as a *Hamiltonian* circuit passing exactly once through each point in vertices *V*. The TSP objective is to *find a tour of minimum costs/distance*. Interested reader can find the full historical mathematical formulations of TSP at [http://rodin.wustl.edu/~kevin/dissert/node11.html].

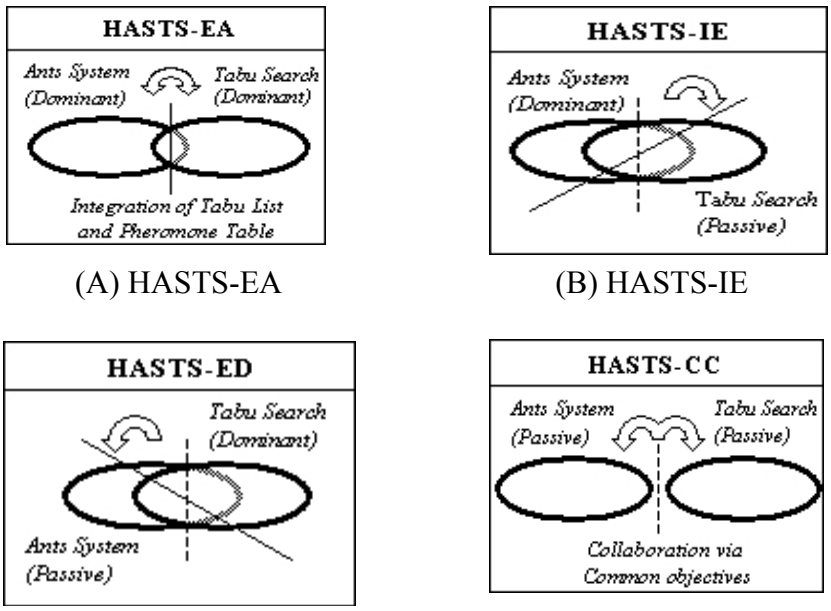
3.1.1 Design Issues

This section illustrates the capability of MDF in supporting different schemes of hybridization. The authors use ACO and TS to exploit on various hybridization schemes in solving the TSP. Their implementation, denoted as *Hybrid Ant System and Tabu Search (HASTS)*, is a flexible hybrid method that spawns *derived models* that utilize the strength of meta-heuristics adept at solving certain problems. Particularly, HASTS takes advantage of the ACO for its nature capability as a constructing heuristic and TS as a local improvement heuristic. By

varying the degree of importance of the inherent algorithms, various derived models are easy formed and formulated with MDF.

The intrinsic flexibility and potential for collaboration allows HASTS to vary the importance of the component meta-heuristics. ACO and TS are argued to be good complements to each other, as ACO works using a preference list, given by the pheromone trail, while TS operates using a forbidden (or tabu) list. The algorithmically opposite techniques offered a high potential that when one algorithm reaches a local optimal, the other algorithm has a higher chance of bring it out and improving the solution henceforth.

HASTS improves results by adjusting the importance level and degree of collaboration of the component meta-heuristics in the hybrid technique, via the framework provided by MDF. Each variant of HASTS has a set of algorithms as the *core* algorithm, while the other algorithm(s) serves as the *aide* algorithm(s). Each of these variant becomes a *derived model* of HASTS. The advantage of the derived models lies in the ability to adapt search to exploit the strength and cover the weakness of the meta-heuristics under the scheme. As such, HASTS is especially suitable for solving complex problems through the use of a divide-and-conquer approach, by first breaking down and identifying the objectives of the sub-problems, and solving them using the best approach optimally. This aptitude will be illustrated in the next two sections, VRPTW and IRPTW. For this section, we focus on illustrating the effects of different hybrids on TSP and observe the efforts required to construct each of them. Figure 3.2 showed four possible derived models of HASTS.



(A) HASTS-EA (B) HASTS-IE

(C) HASTS-ED (D) HASTS-CC

Figure 3.2: The four derived models of HASTS

The four derived models are respectively *Empowered Ants* (HASTS-EA) (Figure 3.2(A)), *Improved Exploitation* (HASTS-IE) (Figure 3.2(B)), *Enhanced Diversification* (HASTS-ED) (Figure 3.2(C)), and *Collaborative Coalition* (HASTS-CC) (Figure 3.2(D)). The framework design ensured that each of these derived models reuses the same implementation for each of the component algorithms. The difference is mainly in where to separate the algorithm, as well as the communication between the algorithms. Hence, for HASTS, MDF guarantees that a generic ACO and TS component engine can be used.

HASTS-EA *(Empowered Ants)*

This derived model arises from the observation that when ACO reaches local optimal solutions, it suffers from a tendency of solution cycling in the near optimum region due to their emphasis on the strong pheromone trails. By empowering the ants with memory, it reduces the chances of reconstructing the

same solution. An analogy can be drawn where each ant becomes more intelligent to find a better trail by *not* following false tracks laid by previous ants. Following this metaphor, ACO optimizes the solution based on its pheromone trails as a "preference" memory, while solution cycling is reduced via the tabu list. Furthermore, TS can be applied to diversify the solutions radically, hence encouraging exploration that helps to escape from local optimality. The tabu list also eliminates the need for local pheromone decay, which reduces one of the parameters. This implementation, however, suffers from a slight increase in computational needs, as well as more computational memory for the additional tabu list. This tradeoff however, is often justified by the increase in performance, especially over large iterations. From an implementation viewpoint, HASTS-EA modifies ACO to include a tabu list, which records the solution made by each ant in a single iteration. Subsequently, each ant in the iteration would check if the next move is tabu-ed. If it is, the move will be dropped and a new move will be generated. The tabu list is reset at the end of the iteration. Figure 3.3 shows the pseudo-code of HASTS-EA.

```
procedure: HASTS – EA ()
        while (termination-criterion-not-satisfied)
                while (Max_Ant_Not_Reached)
                        Ants_generation_and_activity
                        Pheromone_Evaporation
                        Reset_Tabu_List
                        Daemon_actions
                end Schedule_activities
        end while
end procedure
```

```
procedure: Ants_generation_and_activity ()
        while (available_resources)
                Schedule_creation_of_new_ant
                New_Solution = New_active_ant
        update_Tabu_List (New_Solution)
        end while
end procedure

procedure: New_active_ant ()
        Initialize_ant;
        M = read_Pheromone Trail
        T = read_Tabu_List
        while (current_state != target_state)
                A = read_local_ant_routing_table
                P = compute_transitional_probabilities (A, M)
                for Next_state do
                Next_state = apply_ant_decision_policy(P)
                end for
                while (check_Tabu_List (Next_state) == non-tabued)
                Move_to_next_state (next_state)
                if (online_step-by-step_pheromone_update)
                        Deposit pheromone
                        Update M
                end if
        end while
        if (online_delayed_pheromone_update)
                for visited_arc     do
                        Deposit pheromone
                        Update M
                end for
        end if
end procedure
```

Figure 3.3: The pseudo-code of HASTS-EA

In the implementation, the Neighborhood Generator is modified to include a tabu list as an event handler, which records the solution made by each ant in a

single iteration. Subsequent ants in the iteration will trigger an event to check with the handler to prevent them from constructing similar solution structure.

HASTS-IE *(Improved Exploitation)*

In this model, TS is embedded in ACO to conduct *intensification* search on the best solution. A similar design has been employed in [Stutzle and Dorigo, 1999] to produce good solutions for TSP. This model offers two advantages. First, by updating the pheromone trail only after intensifying the best solution, we increase the probability of finding a better solution by subsequent ants. Second, due to the probabilistic guided nature of ants system, this narrows the chances of reaching an optimal solution if it happens to be radically different from local optimum. For example, it is well known that for TSP, the ants system may take a long time before it reaches optimality, due to the presence of "crossings" in the tour, such as those in Figure 3.4. With the help of tabu search, such crossings can be eliminated easily by swap moves such as 2-opt. HASTS-IE, on the other hand, is computational expensive, though it can be extremely effective in situations with many "crossings" in the solution.

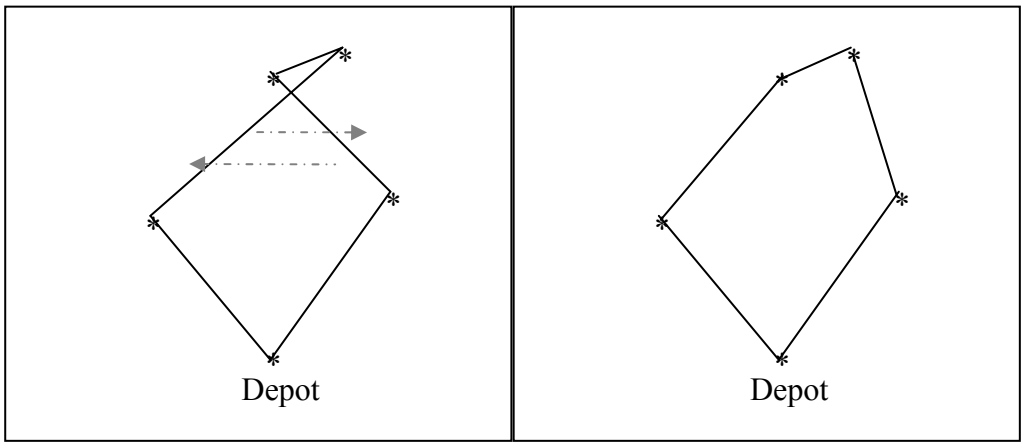


Figure 3.4: Crossings and Crossing resolved by a swap operation

In the implementation, TS is applied adaptively by adjusting the terminating criterion with respect to the number of non-improving moves. An event is set to detect the time when an iteration best solution is found. Before the solution is updated into the pheromone trail, a handler will apply TS to optimize the solution until it reaches 100 non-improving moves.

HASTS-ED ***(Enhanced Diversification)***

In this model, ACO is proposed as a diversifier for tabu search. As TS suffers from local optimality, a diversification strategy is to apply another meta-heuristic as a diversifier [Li and Lim, 2001]. HASTS-ED uses ACO as the TS diversifier with the following rationales. First, the probabilistic nature of the ants system gives a higher chance of successfully diversifying from the local optimum. Second, the diversifier should make a radical move from the current solution so as to explore new regions. Although a random restart is a good strategy, the new starting solution is often poor. Ants system provides a remedy to this by reconstructing quality solutions. However, appropriate parameters for the ACO diversifier should be set, such as a low q_0 that is unusually in most other effective ACO implementation.

In the implementation, a counter event is used to adaptively apply ants to diversify as a non-linear function of non-improving moves. A recommended function is to cumulatively increment the number of non-improved move tolerated for every diversification applied. The diversification technique is embedded into the handler, which reconstructs the part of best-found solution in TS using ACO.

HASTS-CC ***(Collaborative Coalition)***

HASTS-CC proposes a collaborative coalition between the ACO and TS. This model offers the least coupling between the two meta-heuristics but allows great flexibility in the formulation of the problem. One configuration of HASTS-CC is to espouse the two-phase approach as advocated by [Schulze and Fahle, 1997]. This approach consists of a construction phase follow by an optimization phase. ACO work extremely well for the construction phase as it could be used independently to obtain quality solutions. Being an optimization heuristic, tabu search fit naturally into the second phase of the approach. Such collaboration exploits the natural heritage of each meta-heuristic.

For the implementation, an event is set to switch from ACO to TS when ACO has completed its intended iterations.

Hyper-hybrid models

In addition to the four hybrid schemes, [Lau et al.[1], 2004] also illustrates the ability of MDF in combining hybrid to hyper-hybrid. The authors introduce two hyper-hybrid schemes, HASTS-CCED and HASTS-IEEA. HASTS-CCED replaces the TS in HASTS-CC to HASTS-ED. This aims to enhance the optimizing phase. For HASTS-IEEA, it fuses the tabu list strategy in HASTS-EA to HASTS-IE, thus allowing HASTS-IE to develop a more aggressive diversifying capability. HASTS-CCED and HASTS-IEEA are simple illustrations of how hyper-hybrids can be easily formed from previously constructed hybrids when MDF is applied. Initial experimentation of these hyper-hybrids has shown promising results with low additional development cost.

3.1.2 Experimental Observations and Discussion

We demonstrate experimentally the cost-effectiveness of MDF in hybridization. The TSP test problems are obtained from TSPLIB [Reinelt, 1991].

Development Cost of Hybrids

The most obvious and necessary incentive for using a framework is cost-savings in development time. However, it is difficult to measure accurately the amount of resources required as it is subject to numerous factors. The metric used in [Lau et al.[1], 2004] is to record the lines of code, which reveals partially the programming efforts. Unfortunately, the number of lines of code alone is often inadequate to reflect exact development time, as some programmers are known to write condensed codes. In addition, this metric only considers the implementation time and not the validation time. Intuitionally, if each hybrid scheme were developed independently, they would have to be validated separately. An implicit benefit of MDF is the reduction in validation cost. Usually the time required to validate an application increases non-linearly with the amount of code. Hence the savings could be considerable especially in complex applications such as meta-heuristic hybridization. Figure 3.5 approximates the metric for each model. From the comparison, it is apparent that developing strict TS and ACO requires less effort than building from scratch (which typically requires around 1500 lines of code). The large amount of code in MDF and the relatively smaller additional code to formulate MDF to solve TSP, strongly suggests that the MDF has provided the bulk of the implementation. Consequently, this implies that MDF has a strong software reuse capability that could greatly save development time, satisfying the primary objective of the framework.

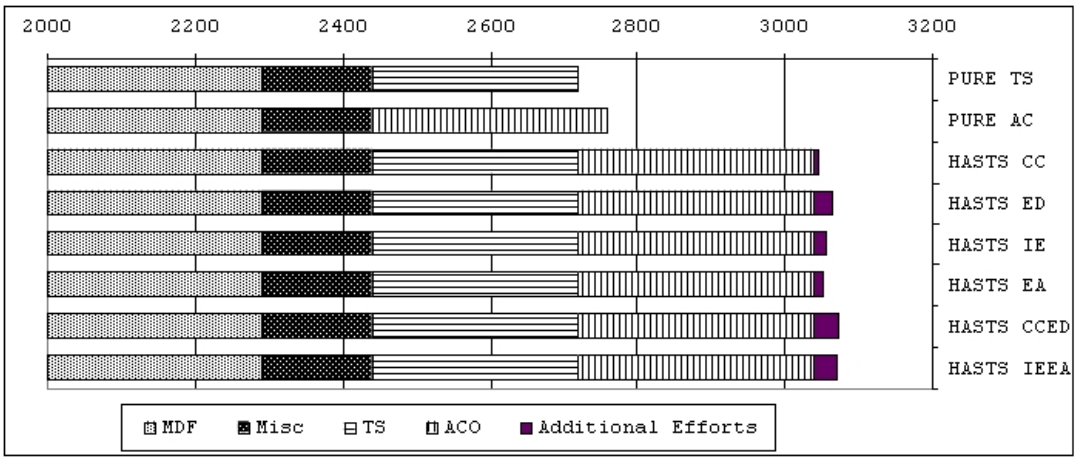
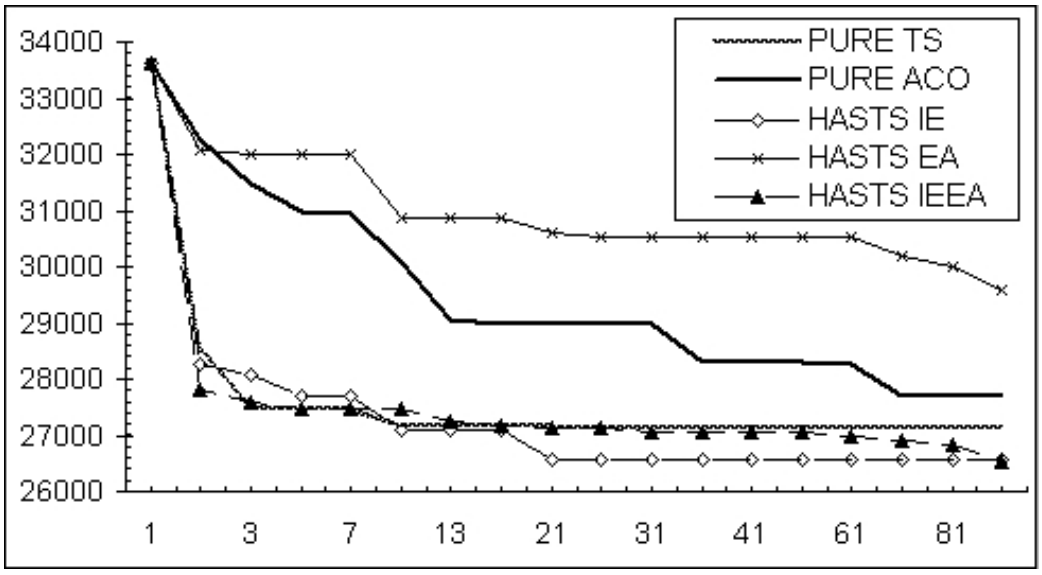


Figure 3.5: Approximation of development time

Cost Effectiveness Comparison of Hybrids

The authors compare the effectiveness of the various hybrid schemes on an Athlon XP 3200+ processor with 512MB of memory, and the results are taken after 90 seconds regardless of the instance size. For each scheme, a greedy heuristic based on the nearest neighbor is used to construct the initial solution. Two test cases, KROA150 (Figure 3.6) and LIN318 (Figure 3.7) are analyzed in the following.



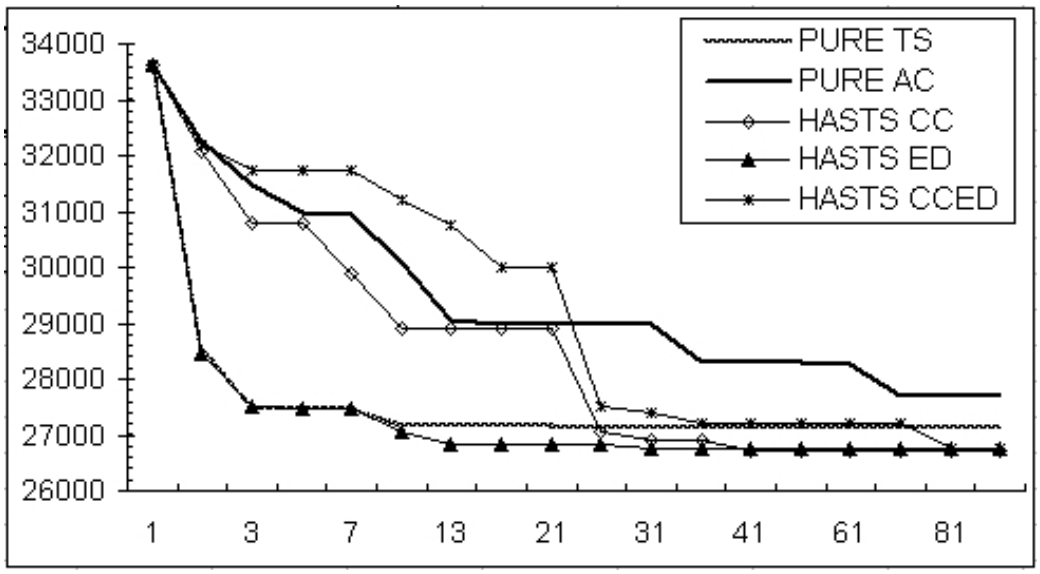


Figure 3.6: Result of test case KROA150

In test case KROA150, we observed that Pure TS converged faster then Pure AC. However the solution quality of TS stops improving at around 10 seconds while Pure AC continued to improve on its solution. HASTS-CC, HASTS-ED and HASTS-CCED produced the same result at 90 seconds although HASTS-ED converged the fastest. Although HASTS-CCED appeared to be slowest to reach the local optimum, we observe a rapid improvement from 22nd seconds to the 26th seconds. The winner of this instance is HASTS-IEEA where the local optimum is reached at 88 seconds. HASTS-EA has the weakest result showing the unsuitability of the scheme in this instance.

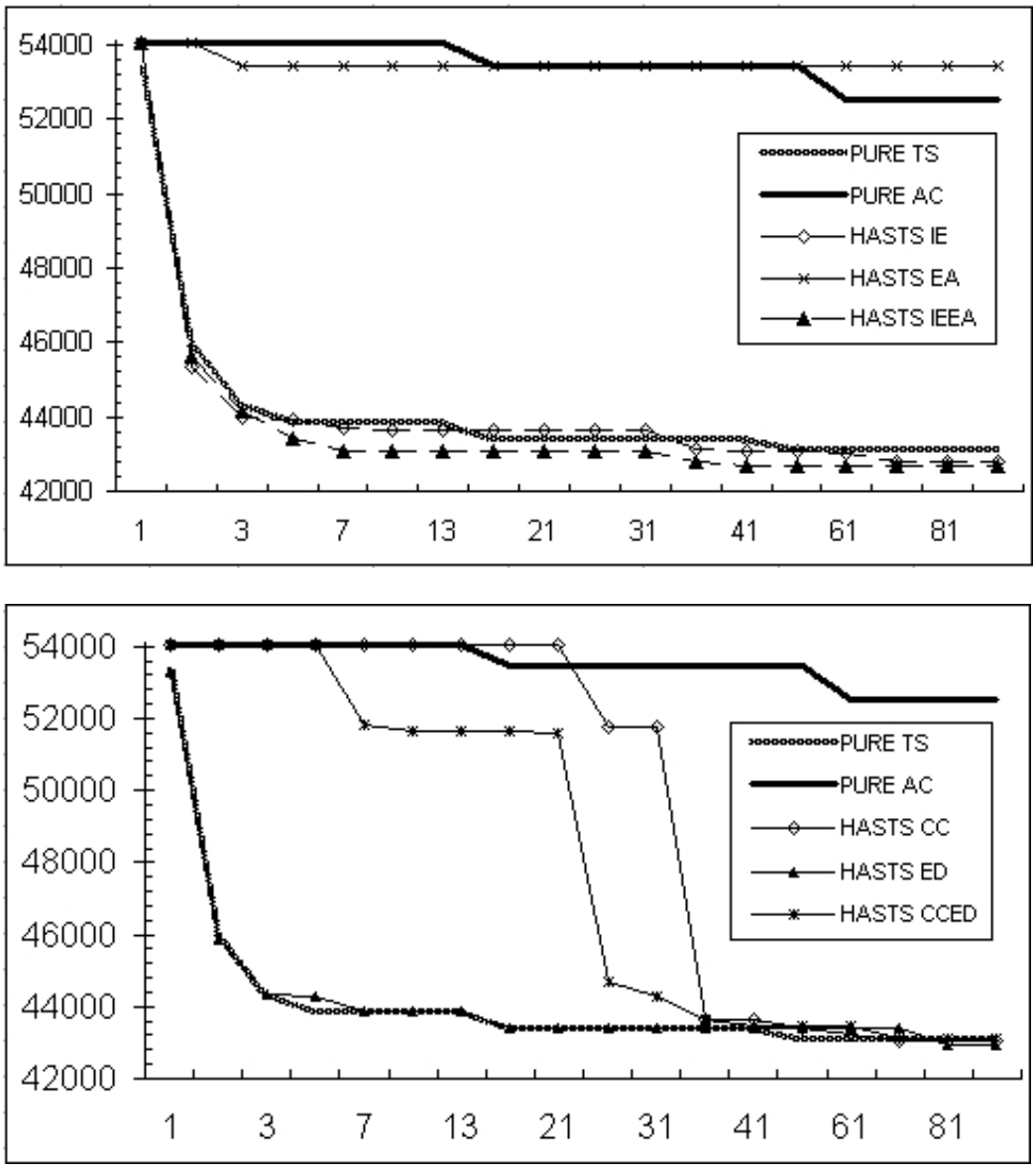


Figure 3.7: Result of test case LIN318.

In test case LIN318, Pure AC could not improve significantly on the initial solution. This phenomenon has been observed by the authors in [Stützle and Dorigo, 1999], which comments that Pure AC does not solve TSP well for large instances. Due to the weakness of Pure AC, HASTS-CC and HASTS-CCED are rendered ineffective. Fortunately, the TS component compensates the weakness to produce results that are comparable with Pure TS. HASTS-ED also produces result close to Pure TS due to the ineffectiveness of the diversifier. The HASTS-IEEA

emerged as the winner although HASTS-IE is only a step behind. Again, we see that HASTS-EA has little improvement as it was greatly affected by the limitation of Pure AC.

In addition, another 13 test cases from the TSPLIB are recorded in Table 3.1. The "Bound" column shows the best-published results to date. Each column gives the objective value and the percentage gap when compared with best-published results. In summary, the table shows that HASTS-IEEA produces the best results and has the best standard deviation. Although it is not conclusive, we have a strong belief that hybrids usually out-perform their parents. Hence, with MDF, complex hybridized schemes are now possible to be developed in much less development time, allowing hybridization to become a practical solution for algorithm improvement.

Table 3.1: Results for TSP from TSPLIB test cases

Name	**Bound**	**Pure TS**		**Pure ACO**		**HASTS-EA**		**HASTS-IE**	
Att48	10628	10755	1.19	10847	2.06	10860	2.18	**10628**	**0.00**
ei151	426	427	0.23	430	0.94	430	0.94	427	0.23
Pr76	108159	109186	0.95	111994	3.55	111435	3.03	**108159**	**0.00**
kroA100	21282	21296	0.07	21559	1.30	22092	3.81	**21282**	**0.00**
kroB100	22141	22235	0.42	23145	4.53	22936	3.59	22220	0.36
Wil101	629	**629**	**0.00**	649	3.18	638	1.43	**629**	**0.00**
Ch130	6110	6196	1.41	6492	6.25	6492	6.25	6124	0.23
kroA150	26524	27125	2.27	27682	4.37	27621	4.14	**26550**	**0.10**
kroB150	26130	26178	0.18	27909	6.81	28499	9.07	26132	0.01
d198	16780	15909	0.82	17397	10.25	17213	9.08	**15780**	**0.00**
kroA200	29368	29487	0.41	34087	16.07	35859	22.10	29565	0.67
kroB200	29437	30121	2.32	36980	25.62	36980	25.62	29813	1.28
a280	2579	2669	3.49	3157	22.41	3157	22.41	2598	0.74
Lin318	42029	43123	2.60	52156	24.10	50053	19.09	42777	1.78
pcb442	50778	52025	2.46	61979	22.06	61979	22.06	51873	2.16
STD Deviation			1.11		9.15		9.13		0.70

Name	Bound	HASTS-ED		HASTS-CC		HASTS-CCED		HASTS-IEEA	
Att48	10628	**10628**	**0.00**	10653	0.24	**10628**	**0.00**	**10628**	**0.00**
ei151	426	**426**	**0.00**	**426**	**0.00**	**426**	**0.00**	**426**	**0.00**
Pr76	108159	**108159**	**0.00**	**108159**	**0.00**	**108159**	**0.00**	**108159**	**0.00**
kroA100	21282	**21282**	**0.00**	**21282**	**0.00**	21292	0.05	**21282**	**0.00**
kroB100	22141	22210	0.31	22200	0.27	22271	0.59	**22141**	**0.00**
wil101	629	**629**	**0.00**	**629**	**0.00**	**629**	**0.00**	**629**	**0.00**
ch130	6110	6128	0.29	6150	0.65	**6113**	**0.05**	**6113**	**0.05**
kroA150	26524	26767	0.92	26727	0.77	26762	0.90	**26525**	**0.00**
kroB150	26130	26152	0.08	26860	2.79	26391	1.00	**26130**	**0.00**
d198	16780	16876	0.61	15796	0.10	15799	0.12	15781	0.01
kroA200	29368	29668	1.02	29487	0.41	29603	0.80	**29479**	**0.38**
kroB200	29437	30121	2.32	30121	2.32	30121	2.32	**29543**	**0.36**
a280	2579	2658	3.06	2669	3.49	2654	2.91	**2579**	**0.00**
lin318	42029	42938	2.16	43123	2.60	43083	2.51	**42665**	**1.51**
pcb442	50778	51860	2.13	52025	2.46	51955	2.32	**51654**	**1.73**
STD Deviation			1.05		1.26		1.07		**0.56**

3.2 Vehicle Routing Problem with Time Windows (VRPTW)

The Vehicle Routing Problem [Toth and Vigo, 2002] is a generic class of complex combinatorial optimization problems extended from the TSP and the Bin Packing Problem (BPP), and was first formulated by [Dantzig and Ramser, 1959]. The VRP is a generalization of the TSP, with additional *m* constraints, the *m*-TSP, inductively making VRP NP-hard. Inversely, the TSP is the VRP with one un-capacitated vehicle (which is the elementary version of VRP, the Capacitated Vehicle Routing Problem – CVRPT), no depot, and customers with no demand. Such observation inspired some approach to solving VRP using a divide and conquer method to break VRP into several Multiple TSP (MTSP, a TSP with *m* identical duplicated origin and *m* salesman) (e.g., [Bullnheimer et al., 1997]). VRP and its variations had been well examined and solved using various techniques from exact methods (e.g., [Baldacci et al., 1999], [Balinski and Quandt, 1964], [Christofides and Eilon, 1969], [Christofides et al., 1981], [Cook and Rich, 1999], [Cullen et al., 1981], [Fisher, 1988], [Fisher and Jaikumar, 1981], and [Foster and

Ryan, 1976]), to heuristics and meta-heuristics (e.g., [Braysy, 2001], [Chiang and Russell, 1997], [Cordeau et al., 2000], [Gillet and Miller, 1974], and [Rousseau et al., 1999]).

A popular and important variant to the VRP, the Vehicle Routing Problem with Time Windows (VRPTW), introduce additional constraints to the original definition, specifying that each costumer must be served within a specific time window. Other variants of the problem are multi-depot, fixed routes, fixed areas, etc. Such variants are formulated as they better approximate practical scenarios. This thesis in particular looks at VRPTW, which is defined in Figure 3.8.

Let

$G = (V, A)$ *be a graph,*

where $V = \{v_0, v_1, \ldots, v_n\}$ *is the vertex set, and*

$A = \{(v_i, v_j) \mid v_i, v_j \in V, i \neq j\}$ *is the edge set.*

Figure 3.8: Problem definition of the Vehicle Routing Problem with Time Windows

This definition is similar to the TSP definition. The difference is in the additional constraints. The depot vertex v_0, has m identical vehicles, each with a maximum load capacity Q and a maximum route duration D. The remaining vertex $v_i \in V$ represent customers to be serviced, each with a non-negative demand q_i, a service time s_i, and a service time window comprised of a ready time r_i and a due time l_i. A waiting time w_i is incurred if customer i is serviced before its ready time. Each edge (v_i, v_j) has an associated non-negative $cost_{ij}$, interpreted as the travel time t_{ij} between location i and j. A complete tour is defined by the order in which the n customers are serviced by m vehicles, and the objective of VRPTW is *to determine a complete tours starting and ending at the depot, such that each*

customer is visited exactly once within its time window, the total demand of any vehicle route does not exceed Q, the duration of any vehicle route does not exceed D and the total cost of all routes is minimized.

Due to the number of constraints in the problem, there are many definitions on the problem optimality. A widely debated factor is whether to consider distance or number of vehicles as the primal optimality factor, with more researchers focusing on the latter as the primary factor with the former as the secondary factor, due in part to the challenge among the community in solving [Solomon, 1987] benchmark test cases. ([Larsen, 1999], [Mester, 2002], and [Mester and Braysy, 2002]) provides further references on the VRPTW.

3.2.1 Design Issues

The problem being solved in this instance, the VRPTW, is an NP-hard multi-objective optimization problem. Traditional approach in solving VRPTW involves projecting all objectives into a single dimension. However, the correlation between these various objectives are usually weak and difficult to express using a common aspect. In addition, during the search, the optimizer has no insight to which objective it is improving. This resulted in redundancy spent in optimizing the secondary objectives while the primary objective is being optimized. To resolve this, an approach is to optimize the problem by independently considering each of its objectives, allowing precise strategies to be employed. In solving this problem, a decision can be made to decompose the problem into the following objectives:

Objective 1: *Minimize the number of vehicles given a set number of customers. The dual problem is to maximize the total number of customers given a set of vehicles.*

Objective 2: *Minimizes the total distance traveled given a fixed set of vehicles.*

This divide-and-conquer formulation suggests the suitability of using HASTS. As had been mentioned earlier, each derived model of HASTS share the same implementation for the component algorithm. It is also seen that VRPTW is an extension of the TSP. Hence, in the implementation, HASTS utilizes a generic ACO and TS implementation for TSP, and reuse this implementation with modifications to handle the additional constraints in VRPTW, to provide a solver for VRPTW. This solver is then extended by each derived model, and modified according to the specifications of the sub-problem it is assigned to solve. Figure 3.9 shows the evolution of the MDF implementation in solving VRPTW using HASTS.

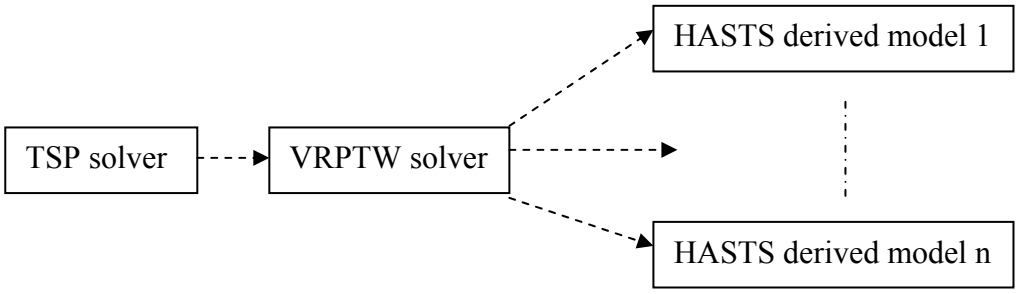


Figure 3.9: Codes reuse for MDF implementation

For this problem, HASTS requires only two derived models, HASTS-IE and HASTS-ED described earlier. The first objective can be reformulated to its dual model and writing it as maximizing the customers served in given a set of vehicles, and reduce the required vehicles each time a solution that serves all the

customers is found with the lesser fleet size. The *HASTS-EA* derived model is appropriate for this sub-problem. ACO is a good meta-heuristic for this objective as it optimizes the solution quality through reconstruction. TS, although possible, is not a suitable candidate as it tries to 'pull' the solution to feasibility through optimizing the customers' sequence in the tour, which is a slow process. Instead, tabu search is used to empower the ant system by intelligently rupturing the pheromone trails left by the ants, and in doing so, helped the ants from being ensnared in a local optimum. Initially *m* vehicles are obtained by applying a greedy heuristic to serve all customers. The algorithm then reduces the value of *m* by 1 and seeks to construct a feasible solution that services all the customers. Once a feasible solution is found, the number of vehicles is reduced to the best-found number of vehicles and the process is repeated for a new feasible solution. This sub-problem requires the search to find a configuration where the customers can fit into the pre-set vehicles. HASTS-EA performs well since the tabu list assists each ant in an iteration to construct a radically different solution. Although other derived models can also be used, they lack of the intensified exploration that HASTS-EA provides.

Objective 2 is attempted after Objective 1 had been optimized, and as a result, this sub-problem will consist of a tighter solution space. In spite of the success by HASTS-EA in optimizing the number of vehicles, this derived model is not very effective for this objective because of the difficulties involved in constructing different feasible solutions on an allowed number of vehicles due to the nature of ACO. Instead, another derived model, *HASTS-ED*, is employed to minimize the total distance on a fixed set of vehicles. HASTS-ED uses tabu search as the core heuristic with ants system acting as the diversifier. Tabu search is

effective in solving this sub-problem as it optimizes the route distance rather than reconstructs the solutions. However, tabu search still faces the danger of being entrapped in a local optimum during its search. To address this issue, when tabu search encounters a local optimum, it randomly selects some of the routes to be reconstructed by ACO, which assists tabu search by radically re-configuring the selected partial routes. Details on this objective rely mainly on the operations of Tabu Search and are examined in further detail in [Lau et. al.[2], 2003].

3.2.2 Experimental Observations and Discussion

VRPTW, as mentioned, as extended from the TSP. The classical and most common comparison for VRPTW solvers in the literature is with the Solomon's VRPTW benchmark [Solomon, 1987], consisting of a total of 56 test cases covering different scenarios. These test cases included a set of problems consisting of *Clustered* nodes (C101-C109, and C201-208), which generally is best solved by assigning vehicles to service the same or nearby clusters in the problem; a set of problems consisting of *Random* nodes (R101-R112, and R201-R211), which has nodes randomly assigned, and solving it optimally will be problem specific; and a set of problems consisting of a combination of *Random and Clustered* nodes (RC101-108, and RC201-208). Table 3.2 tabulates the results obtained.

Table 3.2: Results for VRPTW from the Solomon's original test cases (n=100)

Test cases	TS	ACO	HASTS
C101	10/828.94	10/855.07	10/828.94
C102	10/852.97	10/1072.24	10/845.61
C103	10/858.62	10/1435.26	10/840.88
C104	10/856.87	10/1182.64	10/857.57
C105	10/828.94	10/936.47	10/828.94
C106	10/828.94	10/958.91	10/828.94
C107	10/828.94	10/877.99	10/828.94
C108	10/828.94	10/1033.81	10/828.94

C109	10/828.94	10/1900.94	10/828.94
R101	19/1686.24	19/1929.05	19/1686.24
R102	18/1518.93	18/1886.77	18/1493.31
R103	14/1301.64	14/1679.71	14/1301.64
R104	11/1072.04	10/1198.69	10/1025.38
R105	14/1459.84	14/1651.43	14/1458.60
R106	13/1324.38	12/1564.99	12/1314.69
R107	11/1165.87	10/1144.72	10/1140.27
R108	10/1002.56	10/1117.25	10/ 994.66
R109	12/1287.62	12/1502.57	12/1207.58
R110	11/1218.33	11/1348.78	11/1166.65
R111	11/1104.93	11/1239.53	11/1172.66
R112	10/1039.55	10/1242.24	10/1041.36
RC101	15/1742.29	15/1899.97	15/1698.50
RC102	13/1605.30	13/1780.98	13/1551.32
RC103	11/1337.04	11/1567.12	11/1371.40
RC104	11/1249.13	10/1353.87	10/1187.97
RC105	15/1633.39	14/1899.54	14/1618.01
RC106	12/1428.88	12/1620.67	12/1434.33
RC107	12/1312.84	11/1468.59	11/1266.92
RC108	11/1258.40	10/1326.94	10/1273.12
C201	3/591.56	3/ 591.56	3/ 591.56
C202	3/591.56	3/ 993.62	3/ 591.56
C203	3/617.32	3/1065.81	3/ 605.23
C204	3/673.46	3/1046.87	3/ 594.80
C205	3/604.67	3/ 913.03	3/ 588.88
C206	3/632.35	3/ 647.29	3/ 588.49
C207	3/621.02	3/ 646.69	3/ 588.49
C208	3/588.88	3/ 646.72	3/ 588.49
R201	4/1308.84	4/2048.31	4/1366.34
R202	4/1123.34	3/1755.11	3/1239.22
R203	3/1013.59	3/1625.26	3/1000.29
R204	3/817.60	3/1159.14	3/ 781.86
R205	4/1022.02	3/1678.53	3/1063.29
R206	4/963.94	3/1525.34	3/ 955.34
R207	3/863.60	3/1258.12	3/ 866.35
R208	3/761.94	2/1016.07	2/1016.07
R209	4/934.45	3/1551.01	3/ 979.30
R210	3/1000.53	3/1659.90	3/ 968.32
R211	3/816.33	3/1143.96	3/ 865.51
RC201	4/1704.92	4/2226.23	4/1445.00
RC202	4/1265.78	4/1878.00	4/1204.45
RC203	3/1118.19	3/1706.48	3/1091.71
RC204	3/884.70	3/1342.81	3/ 826.27
RC205	4/1435.06	4/2271.26	4/1469.25
RC206	4/1162.96	3/1717.62	3/1259.12
RC207	4/1178.01	3/1733.47	3/1127.19
RC208	3/931.76	3/1422.07	3/ 937.78

Table 3.2 is read as follows: *TS* refers to the results obtained using a standard Tabu Search implementation on MDF-TSF. *ACO* refers to the results after passing the data through derived model HASTS-EA, a predominantly ACO technique implemented with MDF-ACF that focus on solving the first objective (minimizing the fleet size of vehicles). Finally, the *HASTS* column tabulates the results obtained after the entire HASTS process mentioned earlier – in effect after a combination of HASTS-EA and HASTS-ED.

Note the effectiveness of the hybrid HASTS compared against TS and ACO, which adequately showed the effectiveness of MDF and a divide and conquer hybrid approach. Also, the results from TS are generally better than ACO in this instance due to the different objectives of the approach. TS has an objective of minimizing distance, and perform it so well that for some instances, such as R202, it performs better in terms of distance, but is worse off by the problem definition specifying the fleet size as primary priority, while the ACO results focuses mainly on reducing the fleet size of vehicles. It should also be further noted that the development of the TS implementation takes about 3 months man-hours, while the ACO implementation takes a lesser amount of time at about 2 months, due to its simpler nature. Meanwhile, with the availability of MDF, HASTS requires only less than a week man-hours to develop.

3.3 Inventory Routing Problem with Time Window (IRPTW)

The Inventory Routing Problem with Time Window (IRPTW) follows as a natural extension from the VRPTW, with the additional constraint over multiple time-periods, which better reflect practical scenarios of a known future period planning. Despite the complexity, literature survey showed that IRPTW can be

solved optimally if major restrictions are imposed. [Carter et al., 1996] proposed a Lagrangean heuristic to solve a single-supplier, single-warehouse instance of the problem, but it is sensitive to the values of several parameters where there are no good heuristics for setting them, and is unable to guarantee feasibility. [Chan et al., 1998] modeled a *single-item*, *constant demand* distribution system and presented worst case as well as probabilistic bounds. However, it is doubtful that any of the asymptotically optimal heuristic proposed will perform well for realistic problems with time-varying demand due to the unrealistic assumption on demand. [Campbell et al., 1998] proposed a computationally intensive integer programming approach to a similar problem. [Lau et al., 2000; Lau et al., 2002] proposed a divide and conquer approach of decomposing IRPTW into two sub-problems, then defined an interface to allow the two corresponding algorithms to collaborate in a *master-slave* fashion and provided a proof of convergence. This approach is unable to guarantee feasibility, when the output of the first module is infeasible for the second; and the quality of solution is necessarily low, since there is no provision for an iterative improvement heuristics. IRPTW is defined as in Figure 3.10.

Given

S:	*set of suppliers*
R:	*set of retailers*
J:	*set of items*
T:	*consecutive days in the planning period {1,2,…,n}*
D_{ijt}:	*demand of retailer I for item j on day t*
Q_v:	*vehicle capacity*
Q_w:	*warehouse storage capacity*
Q_i:	*storage capacity of retailer i*

W_i: *time window of retailer i*

C_j: *inventory holding cost per unit item j per day at the warehouse*

C_{ij}: *inventory holding cost per unit item j per day at retailer i*

B_{ij}: *backlog cost per unit item j per day at retailer i*

T_{ik}: *transportation cost incurred by visiting retailer i followed by k on the same route*

and

$G = (V,A,T)$ *is a multi-period graph*

where $V = (v_1,v_2,\ldots,v_i,\ldots,v_m)$ *is the vertex set, and*

$A = \{(v_i, v_j) \mid v_i, v_j \in V, i \neq j\}$ *is the edge set, and*

T : as defined above

Output the following:

[1] The distribution plan denoted by

x_{sjt}: integral flow of amount of item *j* from supplier *s* to warehouse on day *t*, and

x_{ijt}: integral flow amount of item *j* from the warehouse to retailer *i* on day *t*

[2] The set of daily transportation routes Φ, which carry the flow amounts in (1) from the warehouse to the retailers such that the sum of the following linear costs is minimized:

(a) inventory cost at the warehouse (C_j)

(b) inventory cost at the retailer (C_{ij})

(c) backlog cost (B_{ij})

(d) transportation cost from the warehouse to the retailers (T_{ik})

Figure 3.10: Problem Definition for the Inventory Routing Problem with Time Windows

3.3.1 Design Issues

The design of the IRPTW solver is presented in [Lau et al.[2], 2003]. Following the similar concept proposed in [Lau et al., 2000], the algorithm works by decomposing this complex problem into the relatively simpler VRPTW and DLP. Since VRPTW can be further broken down using its separate objectives as described in the previous sub-section, IRPTW then can be formulated to the following three sub-objectives:

Objective 1: *Minimize the number of vehicle used subject to customer time windows of the given set of customers.*

Objective 2: *Minimize the total distance traveled, subject to customer time windows and the given fleet of vehicles.*

Objective 3: *Minimize the inventory holding and backlog costs, subject to the vehicle capacity and retailer holding capacity constraints.*

It can be seen that objectives 1 and 2 forms the VRPTW part of the problem, while objective 3 specifies the DLP sub-problem. Having previously used HASTS to solve VRPTW, it become logical to reuse this implementation to solve IRPTW once it was apparent IRPTW can be broken down into the VRPTW and DLP.

In order to reduce inventory or backlog, more frequent deliveries have to be made, hence increasing the transportation cost. Hence, the goal for objective 3 is to minimize the number of retailers (or customers) served each day without increasing the total cost. That is, the objective is to delete retailers from routes in a manner that does not incur additional costs. Many techniques are available to handle this objective, but in line with reusing HASTS, which is already used to solve the problem involving the other two objectives, it is a straightforward matter

to reuse the same ACO and TS engines by employing another derived model catered to the problem, *HASTS-IE*, such as in Figure 6.6. HASTS-IE uses ACO to construct different solutions. It then uses tabu search to improve its exploitation to reduce missing elite solutions. The tabu search uses the standard "add", "remove" and "swap" moves that attempt to improve the solution quality found by the ACO. The output is a distribution plan that induces the set of customers to be served for objective 1, to facilitate iterative improvement.

3.3.2 Experimental Observations and Discussion

The results for IRPTW are obtained from an implementation that reuses the implementation for the VRPTW. There are no well-known sets of test cases for the IRPTW, but there are implementations in the literature that extends the set of test cases from the Solomon's benchmark for the VRPTW with additional constraints. As such, the MDF application was experimented on the set of problems in [Lau et al., 2002], which provided a good set of test cases for IRPTW.

Specifically, the planning period is 10 days. The vehicle capacity, locations and time-windows of the customers and depot are as specified in the corresponding Solomon instances. The demand d_{it} of customer i for day t ($t=1,...,10$) is equal to the demand d_i of the Solomon instance, by partitioning the value $10*d_i$ into 10 parts, i.e. $d_{i1}, d_{i2},...,d_{i,10}$ randomly such that d_{it} is within the range $[0.5*d_i, 1.5*d_j]$. The capacities of consumers and warehouse are the vehicle capacity and infinity respectively. As for cost coefficients, the inventory cost and backlog cost for each customer are *1* and *2* respectively, symbolizing a preference to holding a unit of inventory over a day than suffer a lost of customer trust on a backlog of a corresponding unit of inventory. The transportation cost of each route is 10 times

its total distance. Table 3.3 shows the results of the test cases. The table also presented the addition test cases from the RC2 series that was not in the original benchmark problems. The columns *VRPTW*, *ILS+VRP* and *TS+VRP* denote the results obtained, where VRPTW is the approach taken from adopting a standard two-phase heuristics; ILS+VRP is the results obtained using Iterated Local Search ([Gu, 1992] and [Johnson 1990]) and TS+VRP employs a Tabu Search technique. The column HASTS presents the results obtained using our proposed hybrid algorithm implemented from the MDF (ACF+TSF).

Table 3.3: Results for IRPTW extended from Solomon's original test cases

Test Cases	**VRPTW**	**ILS+VRP**	**TS+VRP**	**HASTS**
C201	178650	113263	112821	54905
C202	192818	117483	124312	53404
C203	200615	131920	122055	53620
C204	216447	136384	142300	54778
C205	175378	116147	109248	51907
C206	177331	123978	127876	50507
C207	177447	122204	117735	51453
C208	175268	124110	125667	52501
R201	304779	111330	116893	85014
R202	291492	116982	114717	70533
R203	247122	110215	115070	68865
R204	227381	114118	114118	61944
R205	284759	122333	123009	73455
R206	260760	120928	123251	64652
R207	223527	115438	115438	63697
R208	338033	120011	117255	59285
R209	249036	116840	120725	69200
R210	-	-	-	69545
R211	-	-	-	61816
RC201	-	-	-	97417
RC202	-	-	-	87245
RC203	-	-	-	80114
RC204	-	-	-	71795
RC205	-	-	-	92560
RC206	-	-	-	86144
RC207	-	-	-	83326
RC208	-	-	-	71740

Results for the VRPTW, ILS+VRP, and TS+VRP columns are obtained on a Pentium 666MHz machine, while the results from the HASTS column is obtained on a Pentium 1.13GHz machine, which is estimated to perform at twice the power. As such, for comparison, the HASTS implementation is obtained under 90 seconds, to compensate for the 180 seconds upper bound used for the other implementations.

With the objective being to minimize the cost, Table 3.3 amply showed that HASTS offers a set of superior results compared to previous works. While this could be due in part to the originality of IRPTW in the literature, and hence not well studied as yet, it can still be claimed that the effectiveness when solving VRPTW is not lost when reused to solve IRPTW. Furthermore, it can be seen that the framework provided generality and flexibility for reuse, which enabled development to take minimal effort and implementation to be achieved in less than 2 weeks man-hours.

Beside the applications illustrated in this chapter, other publications of MDF include the *Multi-Periods Multi-dimension Knapsack problems* (MPMKP) [Lau et al.[2], 2004] and *Quadratic Assignment Problems* [Lau et al.[1], 2003].

CHAPTER 4

RELATED WORKS

This chapter examines some of the software frameworks in the literature that share similar design goals with MDF, and yet differ in their structurally designs. These frameworks include *OpenTS* [Harder, 2003], *Localizer* ++ [Michel and Van Hentenryck, 2001], *EasyLocal* ++ [Gaspero and Schaerf, 2001], and *HotFrame* [Fink and Voß, 2002], and will be introduced respectively in the following sections.

4.1 OPENTS

OpenTS is one of the project initialized by *Computational Infrastructure for Operations Research (COIN-OR)* to spur the development of open-source software for the operations research community. It is a java-based tabu search framework that has a well-defined, object-oriented design. The generic aspect of the framework is achieved through inheritance, using well-structured interfaces, which includes *Solution*, *Move*, *Move Manager*, *Objective Function*, *Tabu List* and *Aspiration Criteria*. This unambiguous decomposition defined clearly the collaborative role of each interface in the algorithm. In addition, the author presumes that most TS applications adopt the "tabu-ing the move" strategy and hence provides "helper" classes such as *SimpleTabuList*, *ComplexMove* and *ComplexTabuList* classes to assist the implementation.

OpenTS also supports the implementation of TS strategies through the use of the *EventListener* objects. These objects can be embedded into any of the interface-inherited objects so as adjust their parameters. However OpenTS only

support a static set of search events and does not cater for user-defined events such as the presence (or absence) of certain component(s) in solutions. This causes difficulties in implementing strategies that are based on the solution structures (such as recency and frequency based strategies). The absence of a centralized control mechanism also poses a limitation to the framework capability. For example, when two EventListeners are triggered in the same iteration, their order of execution follows a First-In-First-Out (FIFO) sequence, thus giving no control to the algorithm designer. It is also probable for two conflicting EventListener objects (such as intensification and diversification) to be performed together.

4.2 LOCALIZER ++

The literature presented another framework known as the Localizer ++ that incorporates *Constraint Local Search (CLS)* in C++. The framework is structured into a two-level architecture, which composes of *Declarative* and *Search* components. The Declarative components are the core of the architecture and are used to maintain the complex data structure in local search. In addition, it also incorporates a *Constraint Library* that provides a set of frequently used constraints, such as the *alldiff* which verifies that every element in the data structure has a different value. The Search component on the other hand, operates around the Declarative component and is procedural in nature. Generally, this component implements the general procedure of local search and thus could be used to implement any meta-heuristics that follow to this general behavior (i.e. such as iterative local search and tabu search).

Before Localizer ++ can be deployed, it requires the algorithm designer to formulate the problem into its mathematical equivalence form in order for the

framework to recognize and subsequent manage the variables. Algorithm designers are required to implement the routines of the local search such as the local moves and the selection criteria, and together with the Constraint Library, to construct the solver. Due to the numerous possible types of constraint, it is improbable for the Constraint Library to provide all forms of constraint and thus Localizer ++ copes with this limitation by supporting the extension to the library through the addition of invariants. The framework also supports user-defined search strategies that are triggered at static points of the search (such as at the start or the end of the search) rather than dynamically in response to search events. New search procedures can be extended from Localizer ++ through inheritance.

4.3 EASYLOCAL ++

EasyLocal ++ is another object-oriented framework that can be used as a general tool for the development of local search algorithms in C++. EasyLocal ++ relies on programming techniques such as the "Template Method" that specifies and implements the invariant parts of various search algorithms, and the "Strategy Method" for the communication between the main solver and its component classes, in order to achieve the generic aspect. The classes in EasyLocal ++ can be classified into four categories, *Basic Data*, *Helpers*, *Runners* and *Solvers*. The Basic Data is a group of data structure with their managers and is used to maintain the states of the search space, the moves, and the input/output data. The Basic Data classes are supplied to the other classes of the framework by means of template instantiation. The local search problem is embodied in the Helpers classes, which perform actions that are related to some specific aspects of the search, such as maintaining the states or exploring the neighborhood of a solution. The Runners

represent the algorithmic core of the framework and are responsible for performing the routine of the meta-heuristic. Currently, EasyLocal ++ supports several common meta-heuristics such as Hill Climbing heuristic, SA and TS.

EasyLocal ++ can be easily deployed by first defining the data classes and the derived helper classes, which encode the specific problem description. These classes are then "linked" with the required Runners and Solvers and the application is ready to run. EasyLocal ++ also supports diversification techniques through the *Kickers* classes. The Kickers objects are incorporated into the Solver and triggered at specific iteration of the search. Hence, this mechanism relies on the knowledge of the algorithm designer to determine the best moment to trigger the diversification. While this may be achievable for most experience designer, it may be a bit demanding for unfamiliar implementer coping in a new problem. Hybridization is also very restricted as the framework only supports three meta-heuristics. In addition, TS can be seen as a Hill-Climbing heuristic with memory and hence the most probable candidates for hybridization are TS and SA. Hence very limited hybridized schemes can formed with Easy Local++.

4.4 HOTFRAME

HotFrame is a meta-heuristics framework implemented in C++, which provides adaptable components to incorporate different meta-heuristics and common problem-specific complements. Currently HotFrame includes meta-heuristics such as basic and iterated local search, SA and their variations, different variants of tabu search, evolutionary methods, variable depth neighborhood search, candidate list approaches and some hybrid methods. HotFrame provides several reusable data structure classes to incorporate common solution spaces such as

binary vectors, permutations, combined assignment and sequencing and also some standard neighborhood operations like bit-flip, shift, or swap moves. These classes can be deployed immediately or be used as base classes for subsequent customized derived classes. This design encourages software reuse especially for problems that can be formulated with the components that are already presence in the framework.

Meta-heuristics strategies can be implemented in HotFrame through the templates design, which incorporates a set of type parameters that can be extended to support both problem-specific and generic strategies. A benefit of this design is that it gives HotFrame a concise and declarative system specification, which would decrease the conceptual gap between program codes and domain concepts. HotFrame adopts a hierarchical configuration for the formulation of the search techniques in order to separate problem-specific with the generic meta-heuristic concepts. Generic meta-heuristic components are pre-defined in the configuration as a higher-level control while the problem-specific definitions are incorporated inside these meta-heuristic components to form a two level architecture (i.e. each problem-specific strategy will be embedded to a meta-heuristic scheme).

4.5 Frameworks Comparison

Apparently each of these frameworks has its own forte and drawbacks and we conclude that there is no one universal model that truly dominates the rest. Hence we proposed six different facets that we consider as important criteria in benchmarking these frameworks. Table 4.1 presents a summarized tabulation on the performance of the frameworks.

Table 4.1: A comparison of MDF and the four reviewed frameworks

	MDF	***OpenTS***	***Localizer++***	***EasyLocal++***	***HotFrame***
Programming Language?	C++	Java	C++	C++	C++
Number of meta-heuristics supported?	04	01	02	03	04
Support for Hybridization?	Supported	None	None	Limited	Supported
Adaptive control?	Yes	Yes	No	No	No
Usage Friendliness?	Easy	Easy	Moderate	Easy	Moderate
Extended library included?	Yes	No	Yes	No	Yes

The first aspect compares the programming language platform among these frameworks. OpenTS appears to be the only framework that is implemented in Java, which is well-known for its "across platform" capability. However, both MDF and EasyLocal ++ can be used with *Windows*, *Linux* and *Solaris,* and although not explicitly mentioned, we believe Localizer ++ and HotFrame could be deployed in these platforms as well. We consider C++ to be a better candidate for writing framework due to its efficiency (since C++ is a native language, it occurs a smaller overheads than Java, which requires additional interpretation from the virtual machine) and also the supports of templates design (static polymorphism). Implicitly, the evidence of more number of frameworks written in C++ also implies that the committees are more inclined toward C++.

The second aspect considers the number of meta-heuristics supported, which implicitly measures the extensiveness of the framework. We consider only the fundamental meta-heuristics (excluding any variations). MDF and HotFrame both support four different core algorithms, EasyLocal ++ three algorithms, Localizer ++ two algorithms and OpenTS supporting only TS. Hence in this aspect, MDF and HotFrame offer more varieties to the algorithm designer.

The third aspect is very much related to the second as it examines the supporting mechanism provided for hybridization, which is deemed as a vital consideration for modern meta-heuristics framework. Having only a single meta-heuristic, OpenTS does not perform very well in this criterion. The EventListener provides an awkward means for hybridization and added to the absence of reusable codes, the merging of multiple meta-heuristics could be an inconvenient if not intricate task. Similarly Localizer ++ suffers from this aspect as it provides no mechanism for hybridization and. Only limited hybridization can be achieved through overriding some of the abstract classes. EasyLocal++ offers a Kickers classes as the mechanism to support hybridization (i.e. TS as the core algorithm and SA as the diversifier). However, this provides very limited hybridized schemes and is generally not considered as very flexible. HotFrame offers a greater amount of flexibility and convenience by providing deployable codes as that can be readily inherited to form hybrids. Unfortunately, we observe that the framework does not facilitate the formation of hyper-hybrids (hybridizing hybrids) nor encourages the recycling of derived hybrids codes (i.e. once the hybrid is formed from the base class, the derived hybrid's codes cannot be easily recycled onto another hybridized scheme). These two issues are easily resolved in MDF. Firstly the Event Controller provides a centralized scheme that facilitates the merging of hybrids (see Section 3.1.1). In addition, every hybridized scheme is formulated into Event and Handler objects that could be easily recycled from one scheme to another, which solve the second issue. Hence MDF is more prominent when hybridization is concerned.

The fourth aspect regards the adaptability of the frameworks to the search events. Localizer ++, EasyLocal ++ and HotFrame have no mechanism to support adaptive controls and usually the task for reactively adjusting the search trajectory

is fallen onto the algorithm designers' shoulder. MDF and OpenTS incorporates a feedback mechanism that links the search engine with a decision unit. This forms a learning environment for their applications. Algorithm designer can generalize their decisions on when to perform a strategy into rules rather than specifying a time to trigger the strategy. This results in more dynamism in directing the search.

The fifth aspect looks at the user friendliness in deploying the framework. Localizer ++ requires implementer to formulate the problem into its mathematical forms and this may cause difficulties to unfamiliar implementer. EasyLocal ++ and HotFrame relies on templates to offload the routine behaviors of meta-heuristics from the implementer. In addition EasyLocal ++ also provides a *Testers* class that facilitate implementers on their debugging process. MDF and OpenTS on the other hand, relies more on inheritance for its generic aspect and handle the derived objects indirectly through the routine in the search engine(s). This provides the additional advantage of design clarity as the interfaces specify clearly the role of the inherited classes, which is often less confusing than using template classes.

The final aspect complements the fifth by observing if the framework provides any additional tools to facilitate development. These tools are often grouped together to form a software library. MDF provides tools that facilitate general and meta-heuristics strategies implementation (Section 2.5). HotFrame has a more matured library that contains general data storage classes and standard neighborhood operators. Localizer ++ offers a constraints library that provides general constraints to the implementers. There is no report of a library in OpenTS and EasyLocal ++.

CHAPTER 5

CONCLUSION

In this thesis, we presented the designs and architecture of MDF, which is a generic framework capable of integrating any number of separate heuristics to aid algorithmic collaboration and performance comparisons. The primary objectives of the thesis are to demonstrate MDF as a versatile platform for strategy development, particularly hybridization, as well as to exemplify the potential of reuse, which can decrease developmental resources and increase productivity. These capabilities are illustrated with implemented examples, which include TSP and the extended VRPTW and IRPTW. The TSP and VRPTW implementations obtained good results, even when compared against state-of-the-art techniques in the literature, and when reused for IRPTW, the excellent results achieved clearly show the value of software reuse in this instance. By induction, it is logical to state that as long as a good implementation is found for a base problem, it is simple to reuse that implementation for similar or extended scenarios of that base problem. Unfortunately, the versatility of MDF is not without a price. In order for MDF to achieve the generic aspect, control codes are required to sequence the order of events, which inevitably induces overheads to the framework. However, we consider this nearly negligible outlay of efficiency a small price to pay with respect to the advantages that have been illustrated throughout the thesis. In the next few sections, we list the contributions of this thesis and report the current development as well as future goal of MDF.

5.1 Thesis Contributions

This thesis is a contribution to the application of meta-heuristics. It describes a new meta-heuristics framework that is a paradigm of a software solution for combinatorial optimization problems. The following are our main contributions:

1. It presents a wide discussion on the current state-of-art in meta-heuristics and their techniques.
2. It proposes a novel approach of characterizing different meta-heuristics into common behavior, which consequently enables codes reuse across different meta-heuristics.
3. It describes the design and realization on how meta-heuristics can adopts a *Request and Response (R&R)* scheme that facilitates the formation hybridized schemes and related strategies

These results are also reported in [Lau et.al., 2004], [Lau et.al.[1], 2003] and [Lau et.al.[2], 2003].

5.2 Current Development

MDF is currently undergoing an enhancement phase. Areas of interest in which development is in progress include those listed in section 5.2.1, 5.2.2 and 5.2.3.

5.2.1 Parallel Computing

Works in literature [e.g. Perry, 1990] have shown parallel computing does not only reduced computation time but produced better solutions for several

problems. However, an obstacle in this approach is the difficulty in implementing the multi-processes application. This is further hindered by the fact that each new application usually demands re-implementation, which is both tedious and prone to errors. MDF is viewed as a potential candidate to reduce this hazard through extending the framework to support parallel computing.

5.2.2 Human-guided Visualization

Manual or adaptive tuning of search parameters could be a demanding task for either the algorithm designer or machine. A logical solution is then to bridge human instinct with artificial intelligence. A collaborative venture can be formed with a visualizer that provides information to the algorithm designer, which in turn devises new rules to guide the machine. This technique is currently investigated as a possible extension to MDF.

5.2.3 Solving problems with scholastic demands

Meta-heuristics are often applied on deterministic problems even if their underlying techniques are scholastic. This is due to the difficulty in computing the objective value of a scholastic problem analytically. Simulation on the other hand, excels at handling scholastic problems but usually has no means of optimization. Apparently there is no rule that prevents their collaboration and this initiates another extension that could be made to the MDF architecture.

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Annex A

Tabu Search (TS)

History

The roots of TS can be traced back to the 1970's and was first formally introduced in its present form by [Glover, 1986]. Incidentally, the basic ideas had also been sketched in the works of [Hansen, 1986]. Additional efforts of formalization are later reported in [Glover, 1989], [de Werra & Hertz, 1989], [Glover, 1990]. Many computational experiments have shown TS to be competitive against most known techniques and through its flexibility, could out-perform many classical procedures. Surprisingly till today, there is yet a formal explanation of this good behavior. Theoretical aspects of TS have been investigated in the works of ([Faigle & Kern, 1992], [Fox, 1993]). A didactic presentation of tabu search and a series of applications have also been collected in a book [Glover, Taillard, Laguna & de Werra, 1993]. Its interest lies in the fact that success with tabu search often implies that a serious effort of modeling was done from the beginning. The applications in [Glover and Laguna, 1997] provide many such examples together with a collection of references.

Basic Concept

Formally let us consider an optimization problem in the following way: Given a set S of feasible solutions and a function $f : S \rightarrow \Re$, find some solution i^* in S such that $f(i^*)$ is acceptable subjected to some constraints. Generally the acceptability for a solution i^* is to have $f(i^*) \leq f(i)$ for every i in S. In such a situation TS would be an exact minimization algorithm provided the exploration process can guarantee

that after a finite number of steps such an i^* would be reached. However in most situation, there is no guarantee on an i^* and therefore TS could simply be viewed as an extremely general heuristic procedure. The general procedure of TS is presented in Figure A.1.

Tabu Search

Step 1. Choose an initial solution i in S. Set $i^* = i$ and $k = 0$.

Step 2. Set $k = k+1$ and generate a subset V^* of solution in $N(i,k)$ such that either one of the tabu conditions is violated or at least one of the aspiration conditions holds.

Step 3. Choose a best j = i Å m in V* (with respect to f) and set $i = j$.

Step 4. If $f(i) < f(i^*)$ then set $i^* = i$.

Step 5. Update tabu and aspiration conditions.

Step 6. If a stopping condition is met, then stop. Else go to Step 2.

Notations	
S:	Available Search Space
i:	Current Solution
i*:	Best Found Solution
k:	Current iteration
N(i,k):	Neighborhood

Figure A.1: The Tabu Search (TS) Procedure

Generally $V^* = N(i)$, which indicates the complete neighborhood generated from the current solution i. However this neighborhood is often large and it may be too time-consuming to search each individual. Hence an appropriate size of V^* would be a substantial improvement. The iterative exploration process (local search) should accept non-improving moves from i to j in V^* (i.e. $f(j) > f(i)$) if one would like to escape from a local minimum. However, as soon as non-improving moves are possible, the risk of re-visiting a solution (cycling) becomes a serious concern. TS reduces this likelihood through the use of memory, which forbids moves that might lead to recently visited solutions. If such a memory is introduced, the structure of $N(i)$ will depend upon the iteration k and so the neighborhood becomes $N(i,k)$ instead of $N(i)$. It is important to realize that the definition of $N(i, k)$ at each

iteration k and the choice of V^* are crucial. The definition of $N(i, k)$ implies that some recently visited solutions are removed from $N(i)$. These removed solutions are known as "tabu-ed" solutions and should be avoided in some future iterations. Such usage of recency-based memory will prevent cycling for the length of "tabu-ed" duration (tabu tenure). For instance, keeping at iteration k a tabu list of the last T solutions visited will prevent cycles of size at most T. However, keeping a tabu list of the last T solutions is sometimes cumbersome and it is often simplified to keep track of the last T moves associated with the translation of i to j ($j = i \oplus m$). It is clear that this restriction has a loss of information and hence will have no guarantee that there is no cycling for a length of T. The drawback of the simplification (replacement of solutions by moves) could result in giving a "tabu-ed" status to solutions, which may be unvisited so far. As such, it is compelled to have a relaxation on the tabu status when the tabu-ed solutions will look attractive. This relaxation is known as *aspiration criterion*. For example, a tabu-ed move m applied to a current solution i may appear attractive because it results in a solution that is better than the best found so far. Finally the stopping conditions also assert certain influence on the search procedure and some immediate stopping conditions could be the following:

- $N(i, k+1) = \text{NULL}$
- k is larger than the maximum number of iterations allowed
- The number of iterations since the last improvement of i^* is larger than a specified number
- Evidence can be given than an optimum solution has been obtained.

Strategies

Most of the TS strategies are associated the memory. So far the described usage of memory is an essential part of TS and is considered as a short-term memory that prevents cycling to some extent. On the other hand, long-term memory often involves collecting information from the search and applied strategies in response to these information. Among these strategies, there are three distinctive tactics, variable (reactive) tabu list size, intensification and diversification.

Reactive tabu list

The basic role of the tabu list is to prevent cycling. Ideally, the tabu tenure should be small as a lengthy list affects both the search efficiency as well as the memory consumption. However, if the length of the list is too small, the role might not be achieved. Given an optimization problem it is often difficult or even impossible to determine a value that prevents cycling and does not excessively restrict the search for all instances of the problem of a given problem size. An effective way for circumventing this difficulty is to use a tabu list with variable size. The size of the list would response to the search information based on the instance it is solving and changes accordingly. To prevent extreme sizes being used, it is often bounded by given maximal and minimal values.

Intensification

Intensifying strategies are based on the assumption that better solutions can be found by exploring the search space around elite solutions. In order to intensify the search in promising regions, a preliminary search is performed to collect a list of elite solutions (mostly local optimal). Each elite solution is then "examined"

closely by decreasing the size of the tabu list for a small number of iterations. In some cases, more elaborate techniques may be used. Another strategy inspired from the classic divide-and-conquer paradigm consists of partitioning an optimization problem into sub-problems, solving them (optimally) and finally combining the partial solutions. A post-optimization phase may sometimes be performed on the combined solution. Obviously, the difficulty lies in finding a good partitioning technique. Other ways for intensifying the search are the use of more elaborate heuristics or even exact methods, or the enlargement of the neighborhood around elite solutions. It is also possible to perform an intensification based on long-term memory. As each solution or move can be characterized by certain components for their "goodness", these components are memorized for future selection of neighbors. This usage of long-term memory can be viewed as a kind of learning process.

Diversification

As oppose to intensification, diversifying strategies focus on searching the unexplored regions. While intensification attempts to improve on the solution quality, it is not necessary for a solution to diversify to a better neighbor. The underlying notion is to “jump” away from the current solution structure. The simplest diversification is to perform random restarts. A different approach, which ensures the exploration of unvisited regions is to penalize frequently, performed moves or certain component(s) presence in the neighbors. Some diversifications involve oscillating between feasible and infeasible solutions. This is achieved by relaxing the constraints for a small number of iteration before repairing the

feasibility. However, there are times in which the solution is beyond repair and it is then necessary to “backtrack” to the original solution or to restart the search again.

Annex B

Ants Colony Optimization (ACO)

History

Ant Colony Optimization (ACO) is a recently proposed meta-heuristic approach, which is inspired from the foraging behavior of real ants using pheromones as a communication medium. In analogy to the biological example, ACO is based on the indirect communication of a colony of simple agents, called (artificial) ants, mediated by (artificial) pheromone trails. The pheromone trails in ACO serve as distributed, numerical information, in which individual ants use to probabilistically construct solutions. The ants adapt by "depositing" different amount of pheromone to reflect their search experience. The first ACO algorithm proposed was *Ant System (AS)* [Dorigo et al., 1991]. At the early stage, AS was applied to some rather small instances of the TSP with the problem size of up to 75 cities. Experimental results show that it was more than a match in performance compared to other meta-heuristics such as evolutionary computation ([Dorigo, 1992], [Dorigo et al., 1996]). Despite the initial encouraging results, AS loses its edges for large instances in TSP. Since then, a substantial amount of research has been invested on ACO algorithms. The more recent algorithms are direct extensions of AS with added advanced features, and have established their creditability in obtaining good results ([Dorigo and Gambardella, 1997], [Dorigo and Di Caro, 1999]). Ironically, while these features improve on the effectiveness, they also render the behaviors of ACO to draft away from the resemblance of its biological counter-parts.

Basic Concept

While TS is considered as an enhancement to the local search technique, ACO can be interpreted as an extension of traditional construction heuristics. Informally, the ACO algorithm can be summarized as follows: A colony of ants is concurrently and asynchronously moving through adjacent states of the problem, which incrementally build up a solution to the optimization problem. Each "chosen" state depends on a stochastic local decision policy that uses a combination of pheromone trails and heuristic information. During the construction of a solution, the ant evaluates the (partial) solution and deposits pheromone trails on the components or connections it used. This pheromone information is used later to direct the search of the future ants. Beside the ants' activity, there are two other concurrent events, *pheromone trail evaporation* and *daemon actions*. *Pheromone evaporation* is the process in which the pheromone trail intensity on the components decreases over time. This phenomenon is necessary to avoid a rapid convergence towards a sub-optimal region. Analogically, it can be seen as "forgetting" the previously favored paths and begins the exploration of new areas of the search space. *Daemon actions* are used to implement centralized actions that cannot be performed by a single ant. For example, a daemon action can be the collection of global information that can be used to decide whether it is useful to deposit additional pheromone to guide the search process away from local optimum. A pseudo code of ACO is presented in Figure B.1.

Ants Colony Optimization

```
procedure ACO
    ScheduleActivities
        ManageAntsActivity()
        EvaporatePheromone()
        DaemonActions()
    end ScheduleActivities
end ACO
```

Figure B.1: The pseudo code of Ants Colony Optimization (ACO)

As discussed, the three components of ACO algorithms: (i) *ManageAntsActivity*, (ii) *EvaporatePheromone*, and (iii) *DaemonActions* are encapsulated under *ScheduleActivities*. These three activities need not be performed in any particular order. Rather, they can be executed in a completely parallel and independent way, or with some kind of synchronization among them when necessary. There are two technical issues concerned with managing the ants' activities. First is the definition of stochastic local decision policy. [Dorigo, 1992] proposed an equation for computing the probability of acceptance for each (partial) solution states and is given as:

$$p_{ij} = \left[\tau_{ij}\right]^{\alpha}\left[\eta_{ij}\right]^{\beta} \Big/ \sum_{l \in N_i^k}\left[\tau_{il}\right]^{\alpha}\left[\eta_{il}\right]^{\beta} \qquad if\ j \in N_i^k \qquad \text{Eqn B.1}$$

where η_{ij} is a priori available heuristic information, τ_{il} is the relative strength of pheromone trails, α and β are two parameters that determine the relative influence of pheromone trail and heuristic information and N_j^k is the feasible neighborhood of ant k. If $\alpha = 0$, the selection probabilities are proportional to $[\eta_{ij}]^{\beta}$ and the states

with the best heuristic value will more likely be selected. In this case, ACO behaves like a classical stochastic greedy algorithm. If β = 0, only pheromone amplification is at work and would lead to the rapid emergence of a *stagnation* solution (ie all the ants converge to a same solution usually sub-optimal). The second issue arises from updating the pheromone trails. Equation B.1 was recommended by Dorigo as a formula for update and is shown below.

$$\tau_{ij} = (1-\rho).\tau_{ij} + \sum_{k=1}^{m} \Delta\tau_{ij}^{k} \qquad (\forall i, j)$$

Eqn B.1

where $0 < \rho \leq 1$ is the pheromone trail evaporation rate and m is the number of ants. The parameter ρ is used to avoid unlimited accumulation of the pheromone trails and enables the algorithm to "forget" previous (bad) decisions. Hence, on paths that are not chosen by the ants, the associated pheromone strength will decrease exponentially with the number of iterations.

Strategies

As mentioned earlier, naive AS approach was not competitive with most other meta-heuristics in large-scale instances. As such, the algorithm is extended with additional features to improve its search. These enhancements include *Elitist Strategy*, *Rank-Based version of Ant System (ASrank)*, *MAX – MIN Ant System (MMAS)*, and *Ant Colony System (ACS)*.

Elitist Strategy

The Elitist Strategy was introduced in ([Dorigo, 1992], [Dorigo et al., 1996]). Prior to the start of the search, a good (elite) solution is acquired through means such as greedy heuristics or iterative local searches. Pheromone is then deposited onto the

"path" contained in the elite solution. When the search begins, the additional pheromone will render the ants to favor taking the "good" paths. Hence, this strategy can be also viewed as intensifying the ants to search around the elite solution.

Rank-Based Ants System (ASrank)

Following the same concept of intensification, ASrank [Bullnheimer et. al., 1999] can be seen as an extension of the Elitist Strategy. For each round of optimization (iteration), the solutions constructed by the ants are sorted according to their quality. The selected best w solution is then updated into the pheromone trails. In addition, the strength of the updated pheromone depends on the quality of the solution. For example, the r best ant will be updated with $(w - r)$ amount of pheromone onto its trail. An advantage of this strategy is that it removes the false trails left by poorly constructed solutions, and hence reduces the probability of constructing poor solutions.

MAX –MIN Ant System (MMAS)

In *MMAS* ([Stutzle et al., 1997], [Stutzle, 1999], [Stutzle et al., 2000]), upper and lower bounds are enforced to the values of the pheromone trails, as well as a different initialization of their values. This helps to avoid sudden convergence to stagnation solution and promote a higher degree of exploration. For each round of optimization, MMAS only update the best ants' trail (the global-best or the iteration-best ant). Similar to the ASrank, the idea is to prevent deposition of pheromone in false trails. Computational results have shown that best results are

obtained when pheromone updates are performed using the global-best solution with increasing frequency during the algorithm execution.

Ants Colony System (ACS)

ACS ([Gambardella and Dorigo, 1996], [Dorigo and Gambardella, 1997]) focuses more on the exploitation of information collected by previous ants than the exploration of the search space. There are three mechanisms involved. Firstly, a *pseudo-random proportional* rule [Dorigo and Gambardella, 1997] is used to guide the ants in choosing their "paths". This rule uses a parameter q_0 to determine whether an ant is performing exploitation or exploration. In exploitation, the ants are stimulated to intensify their search on paths with stronger pheromone whereas in exploration, the ants are encouraged to diversify their search on unexplored ground. When the value q_0 is set to a value close to 1, the ants will favor exploitation over exploration. Conversely, when q_0 is set to 0, the probabilistic decision rule becomes the same as in AS. Secondly, ACS follows the concepts of MMAS by only updating the trails of the best ants with pheromone. The best ants could be the global-best or the iteration-best ants. Thirdly, to counter the effect of over-exploitation, the last mechanism (known as the local evaporation), is used to lessen the pheromone on a trail whenever an ant moves through it. The local evaporation can be imagined as ants "absorbing" some of the pheromone as they move along the trails. The effect is to encourage subsequent ants to explore new regions rather than to follow previous ants' paths. In addition to the three mechanisms, some ACS algorithms also incorporate local search to enhancement their results.

Annex C

Simulated Annealing (SA)

History

In 1983 three IBM researchers [Kirkpatrick et al., 1983] published a paper in *Science* magazine called *Optimization by Simulated Annealing*. They described a computational intensive algorithm for finding solutions to general optimization problems. Their method is based on the way nature performs an "optimization of energy" of a crystalline solid when it is annealed to remove defects in the atomic arrangement. As an analogy to this physical process, *Simulated Annealing (SA)* uses the objective function of an optimization problem instead of the energy level of a real material. The simulated thermal fluctuations are changes in the adjustable parameters of the problem rather than atomic positions. If the annealing schedule achieves effective thermal equilibrium at each temperature (i.e., enough accepted random moves), then the objective function reaches its global minimum when the simulated temperature reaches the vicinity of zero.

Basic Concept

SA is a global optimization method that distinguishes between different local optimal. Starting from an initial point, the algorithm generates a random neighbor and the objective function is evaluated on the neighbor. Any improving move is accepted and the process repeats from this new point. However, a non-improving move may be accepted in order to allow the search to escape from local optimal. This "anti-greedy" decision is made by the *Metropolis* criteria [Metropolis et al.

1953]. Generally, as the optimization process proceeds, the probability of acceptance declines. The complete pseudo code is presented in Figure C.1.

Simulated Annealing

Choose an initial state i at random

While termination-condition is not satisfied, do

Pick at random, a neighbor j of the current state

Let Δx be the improvement in $\Delta x = f(j) - f(i)$

If $\Delta x > 0$ then

Set current state to the selected neighbor, j = i

Else

Calculate probability p = exponential$^{-|\Delta x/Ti|}$

Set the current state j = i with probability p

Notations	
Δx:	Difference in objective value between current new state
i:	Current State
j:	New State
Ti:	Temperature, dependent on time (iteration)

Figure C.1: The pseudo code of Simulated Annealing (SA)

One technical issue of the algorithm is the formulation the acceptance probability. Generally, there are two factors to be considered when deciding the probability. The first is the variable Δx, which measures the desirability of the random neighbors. Following the same rationale as the *hill climbing heuristic*, a neighbor with a smaller regression is more favored. The second consideration is annealing schedule, which is time-dependent. The basic idea is that the algorithm is more likely to accept a "bad" neighbor at the start of the search. As search time gets shorter, the algorithm would "insist" on better solutions and hence the acceptance probability decreases. A general acceptance probability is given in equation C.1.

$$p = \text{exponential}^{-|\Delta x/Ti|} \qquad \text{Eqn C.1}$$

The literature has also proposed many variations of the annealing schedule such as the *Boltzmann Annealing* [Metropolis et al. 1953], which was essentially

introduced as a Monte Carlo importance-sampling technique for doing large-dimensional path integrals arising in statistical physics problems. This method was later generalized to apply on non-convex cost-functions arising from a variety of optimization problems. *Fast Annealing* [Szu and Hartley, 1987] was later extended from the Boltzmann Annealing, by replacing the Boltzmann forms with the Cauchy distribution.

Strategies

In most optimization, SA is rarely used alone. This is because of the lengthy computational time involved before the algorithm could obtain quality results. On the other hand, SA excellent capability in escaping from local optimal made it too valuable to be ignored. As such, modern techniques often hybridize SA (or its variations) as a mechanism to escape local entrapment. For example, a simple hybrid scheme can be formed with the hill-climbing heuristic. The hill-climbing heuristic is an iterative improvement technique that adopted a greedy approach to increase the solution quality. When the heuristic is ensnared in local optimal, SA could then be applied as a "kick" to diversify the search to a new region. In such strategies, SA acts as a probabilistic diversifier and has been known to obtain good results when hybridize in similar fashion with many other meta-heuristics.

Annex D

Genetic Algorithm (GA)

History

GA originated from the studies of cellular automata, conducted by Holland [Holland, 1992], and his colleagues at the University of Michigan. Holland's book that was published in 1975 is generally acknowledged as the beginning of the research of GA. Until the early 1980s, the research in genetic algorithms was mainly theoretical [Davidor, 1991], with few real applications. From the early 1980s the community of genetic algorithms has experienced an abundance of applications that spread across a large range of disciplines. Each and every additional application gave a new perspective to the theory. Furthermore, in the process of improving performance, new and important findings regarding the generality, robustness and applicability of genetic algorithms became available. Following the last decades of rapid development, GA, in various guises has been successfully applied to various optimization problems.

Basic Concept

Genetic algorithm is a model of machine learning that derives its behavior from a metaphor of the processes of evolution in nature. A population of individuals can be represented by their chromosomes. Nature compels each individual to go through a process of evolution which, according to [Darwin, 1979], is made up of the principles of selection and mutation. The selection process allows only the "fittest" to survive and consequently passed down their genes to their offspring. Natural mutation on the other hand, "alters" the individuals' chromosomes, usually

to improve survivability. Optimization can be formulated as an evolutionary process. For example, a solution can be represented as a set of characters or byte/bit strings, which corresponds to the chromosomes. The selection criterion then becomes the objective function. Table D.1 gives a list of GA components with its evolutionary counterparts. With these components in place, the pseudo-code of GA is presented in Figure D.1.

Table D.1: Allegory of GA components and their evolutionary counterparts

Natural	**Genetic Algorithm**
Individual	Solution
Chromosome	String Representation
Gene	Feature, character or detector
Allele	Feature value
Locus	String position
Genotype	Structure, or population
Phenotype	Parameter set, alternative solution, a decode structure
Fitness	Objective Function
Reproduction	Recombination Function
Mutation	Local Improvement Function

Genetic Algorithm

```
Initialize and evaluate population P (0);
While not last generation, do
        P'(t) := Select_Parents P(t);
        Recombine P'(t);
        Mutate P'(t);
        Evaluate P(t);
        P(t + 1) := survive P(t), P'(t);
end while
```

Figure D.1: The pseudo code of Genetic Algorithm (GA)

GA starts off with a population of strings (original parents) that is used to generate successive populations (generations). The initialization randomly constructs some individuals for the first generation. These individuals are evaluated on their fitness, which in turn determine their probability of selection. In the selection process, a fitter individual has a higher likelihood to be selected (several times) for reproduction (or recombination). The recombination process consists of a crossover operator that extracts certain traits (structures) from both parent and then recombines them to form a new offspring. Each offspring then undergoes a mutation process, in which some fast heuristics are used to improve on its fitness. Sometimes, these new offspring are evaluated and mixed with their parents. Finally a new generation is obtained through sampling of the combined population to remove away the individuals who are considered as "unfit". The algorithm is then repeated for a pre-determined number of generations. It is essential for the solution to be formulated as characters or byte strings before GA can be applied. This restriction demands some ingenuity from the algorithm designers when they devise their approaches. In addition, the modeling of GA does not take into account the possibility of infeasible solutions. In GA, infeasible solutions are often treated as "unfit" individual and eventually discarded. However, there is no mechanism that prevents producing infeasible individual and thus renders the algorithm to be less suitable for problems with tight constraints.

Strategies

Aside from hybridization (which will be discussed further in Chapter 2), there are several strategies that improve the effectiveness and efficiency of GA search. Usually these strategies involve one or more GA components collaborating

together. Among these strategies, we introduce *Fitness Techniques*, *Elitism*, *Linear Probability Curve* and *Steady Rate Reproduction*.

Fitness Techniques

At the start of GA search, it is common to have a few elite individuals in a population of mediocre contemporaries. If left to the normal selection rule of the simple GA, the elite individuals would soon take over a significant proportion of the finite population in a single generation and this leads to an undesirable cause of *premature convergence*. In the later part of the search, the population average fitness may come close to the population best fitness. If this situation is left alone, the average individuals and best individuals will have nearly the same structure in future generations and the survival of the fittest necessary for improvement becomes a random walk among the mediocre. There are three proposed solutions in the literature and they are *linear scaling*, *windowing* and *linear normalization*. *Linear scaling* requires a linear relationship between the original raw fitness f and the scaled fitness f' as shown in equation 1.4.

$$f' = a * f + b \qquad \text{Eqn 1.4}$$

The coefficients *a* and *b* may be calculated from f_{min}, f_{max} and f_{avg} in as follows.

$$a = \frac{C_{mult} - 1}{\delta} * f_{avg} \qquad b = f_{avg} * \frac{f_{max} - f'_{max}}{\delta} \qquad \text{Eqn 1.5}$$

with $f'_{max} = C_{mult} * f_{avg}$, and $\delta = f_{max} - f_{avg}$.

In this way, the number of offspring given to each population member with maximum raw fitness is controlled by the parameter C_{mult} (the number of expected selections desired for the best population member). *Windowing* is a technique for assigning "vitamins" to a population of chromosomes to boost the fitness of the weaker members, in order to prevent their elimination. The technique works by first determining a threshold for the minimum fitness in the population. Each chromosome below this minimum is assigned a small random amount so that it exceeds this minimum. This creates a guard against the lowest chromosomes to have no chance of reproduction. The last technique is known as *Linear Normalization*, which takes the fairness inherent in windowing to an extreme by first normalizing the fitness for all chromosomes in the population.

Elitism

The Elitism strategy is inspired from the observation that for every new generation, there is a chance that elite parents may be lost through the algorithm's probabilistic selection. This could result in an unstable algorithm and a slower convergence. The Elitism strategy is proposed to overcome this problem by retaining some of the best parents of each generation into the succeeding generations. Although this may heighten the risk of domination by a superior individual, but on balance it appears to improve the performance.

Linear Probability Curves

The Linear Probability is another technique for giving the better individuals a higher survival rate. This could be achieved by assigning a "survival probability" to each individual in the population using a linear probability curve [Barberio,

1996]. For example, the best individual could be assigned to a probability of 0.9, and the worst individual to a probability of 0.1. In this way, not all the least fit individuals would necessarily perish, and not all the fittest individuals would survive and subsequently reproduce. If an individual is assigned to a probability of 1, then the strategy behaves similarly to the Elitism Strategy.

Steady State Reproduction

When the simple GA reproduces, it replaces its entire set of parents by their children. This technique has some potential drawbacks and even with an Elitism Strategy, there is no guarantee that the best individuals would reproduce and hence their genes may be lost. It is also possible that mutation or crossover may alter the best chromosomes' genes such that their "good" traits are lost. The *steady-state* reproduction can be used to resolve this problem. The strategy work as follows: As pairs of solutions are produced, they replace the two worst individual in the population. This is repeated until the number of new offspring added to the population since the last generation is equal to the original number of individuals in the population [Parker, 1992]. The *steady-state without duplicates* [Davis, 1991] improves this strategy by discarding the children that are the duplicates of current chromosomes in the population.

Other Advanced Techniques

In addition to the discussed GA strategies, some strategies improve on the GA components. For example, the works of [Davis, 1991, Goldberg, 1989, Starkweather et al., 1991] showed that advanced recombination methods such as *two-point crossover*, *uniform crossover*, *partially mixed crossover* and *uniform*

order-based crossover have several advantages over the original *one-point crossover*. One apparent drawback of the one-point crossover is that it cannot merge certain combinations of features encoded on chromosomes and hence schemata with a large defining length are easily disrupted. Beside the recombination methods, the works of [Davis, 1991, Grant, 1995] have also shown some advanced improvements made for the mutation operator.