STUDIES ON CHAIN SAMPLING SCHEMES IN QUALITY AND RELIABILITY ENGINEERING

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Summary

Chain Sampling scheme is the first topic covered in this thesis. The interest in chain sampling plans is sparked by an industry project, in which a suitable sample scheme is required to conduct destructive test on fire-retard door and fire-retard cable. Some features of this testing are: (I) this testing is destructive, so it is favorable to take as few samples as possible, and (II) testing units are selected from the same continuous process and it is reasonable to expect a certain kind of relationship between the ordered samples. For example, units after good units (conformities) are more likely to be good, and bad units (non-conformities) are more likely to happen after bad units. In our research, we proposed a chain-sampling plan for Markovian process to address these problems. The chain sampling has it unique strength in dealing with scarce information and a two stage Markov chain model is demonstrated to be able to model such process adequately.

Another important assumption for chain sampling plan is the error-free inspection assumption, which assumes that inspection procedures are completely flawless. In reality, however, inspection tasks are seldom error free. While inspection errors incurred during acceptance sampling for attributes are often unintentional and in most cases neglected, they nevertheless can severely distort the quality objective of a sampling system design. This motivated our study of the effect of inspection errors on chain sampling schemes to be part of our chain sampling studies.

The error study of chain sampling plans is done through three phases: 1. the effect of constant inspection errors; 2. the effect of variable inspection errors; and 3. the design of chain sampling plan under inspection error. The first two stages is the basis of the inspection error study and the final stage, design of chain sampling plan, completes

this study on inspection errors. The ultimate goal of this series of error study is to devise a procedure to design chain-sampling plan under error inspection. This includes the binomial model, the proposed design approach and its series of tables etc. After complete the correlation and error effect of chain sampling, we find that the chain inspection actually can have a much broader application in such areas as reliability acceptance test and the high yield process etc. An outline of its application in reliability test is given and demonstrated.

Some additional work has been done during the course of my research stint in NUS, which have their unique contributions in terms of researching. However, it is not very consist with the above-mentioned topics and not easy to be incorporated in a cohesive structure. Rather than simply drop them off, we decide to document them in the appendix for future reference. These include the mathematical deviation of ratio of two normal in the multivariate process control and the SWOT (Strengths, Weaknesses, Opportunities and Threats) analysis to Six Sigma Strategy.

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Nomenclature

N	Lot size
D	Number of nonconforming items in a lot
n	Sample size
y	Number of nonconforming items in a sample
z	Number of observed nonconforming items in a sample
z_m	Number of items in a sample classified as nonconforming in the mth preceding lot, m ranging from 0 to $k-1$ with 0 denoting the current lot
z_{total}	Total number of items classified as nonconforming from the current as well as the previous $k-1$ lots
z_{pre}	$z_{total} - z_0$
c	Acceptance number of a sampling plan
c_1	Acceptance number for the first stage
c_2	Acceptance number for the cumulative stage
r	Rejection number
k	Total number of lots (inclusive of the current lot) used in ChSP
e_1	Probability that a conforming item is observed as nonconforming
e_2	Probability that a nonconforming item is observed as conforming
w	Number of nonconforming items classified as nonconforming
w'	Number of conforming items classified as nonconforming
p	True fraction of nonconforming items in a lot
q	$q = 1 - p$
π	Apparent (observed) fraction of nonconforming items in a lot
P_a	Probability of acceptance

P_s	Probability of acceptance for a single sampling plan
P_{ch}	Probability of acceptance for a chain-sampling plan
AOQ	Average Outgoing Quality
ATI	Average Total Inspection

1. Introduction

Quality and reliability engineering has gained its overwhelming application in industries as people become aware of its critical role in producing quality product and/or service for quite a long time, especially since the beginning of last century. It has been developed into a variety of areas of research and application and is continuously growing due to the steadily increasing demand.

Acceptance sampling is one field of Statistical Quality Control (SQC) with longest history. Dodge and Romig popularized it when U.S. military had strong need to test its bullets during World War Two. If 100 percent inspection were executed in advance, no bullets would be left to ship. If, on the other hand, none were tested, malfunctions might occur in the field of battle, which may result in potential disastrous result. Dodge proposed a "middle way" reasoning that a sample should be selected randomly from a lot, and on the basis of sampling information, a decision should be made regarding the disposition of the lot. In general, the decision is either to accept or reject this lot. This process is called *Lot Acceptance Sampling* or just *Acceptance Sampling.*

Single sampling plans and double sampling plans are the most basic and widely applied testing plans when simple testing is needed. Multiple sampling plans and sequential sampling plans provide marginally better disposition decision at the expense of more complicated operating procedures. Other plans such as the continuous sampling plan, bulk-sampling plan, and Tighten-normal-tighten plan etc., are well developed and frequently used in their respective working condition.

Among these, chain-sampling plans have received great attention because of their unique strength in dealing with destructive or costly inspection, which the sample size is kept as low as possible to minimize the total inspection cost without compromising the protection to suppliers and consumers. Some characteristics of these situations are (I) the testing is destructive, so it is favorable to take as few samples as possible, and/or (II) physical or resource constraint makes mass inspection an insurmountable task.

The original chain sampling plan-1 (ChSP-1) was devised by Dodge (1977) to overcome the inefficiency and less discriminatory power of the single sampling plan when the acceptance number is equal to zero. Two basic assumptions embedded with the design of chain sampling plans are independent process and perfect inspection, which means all the product inspected are not correlated and the inspection activity itself is error free. These assumptions make the model easy to manage and apply, though they are challenged as manufacturing technology advances.

The interest of studying chain-sampling plans was driven by a real industrial project, where appropriate sampling plans were required to test fire-retard door and fire-retard cable.

Some features of this testing are: (I) this testing is destructive, so it is favorable to take as few samples as possible, and (II) testing units are selected from the same continuous process and it is reasonable to expect a certain kind of relationship between the ordered samples. For example, units after good units (conformities) are more likely to be good, and bad units (non-conformities) are more likely to happen after bad units.

For the first problem, suitable sampling schemes are needed and chain-sampling plan stands up to be a perfect candidate because of its power in making use of the limited information. As for the second question, a suitable way needs to be found to capture the dependency between testing units. This becomes the starting point of our research on the chain sampling schemes. The problem actually addresses one of the underlying assumptions for the chain sampling plan---uncorrelated process. In the original ChSP-1, all products inspected are assumed to come from the same process and follow an identical independent distribution (i.i.d.). This strict assumption has to be relaxed in our project and a Markovian model is proposed later on to model this kind of correlation.

Another important assumption for chain sampling plan is the error-free inspection assumption, which assumes that inspection procedures are completely flawless. In reality, however, inspection tasks are seldom error free. On the contrary, they may even be error prone. A variety of causes may contribute to these error commitments. In manual inspection, errors may result from factors such as the complexity and difficulty of the inspection task, inherent variation in the inspection procedure, subjective judgment required by human inspectors, mental fatigue and inaccuracy or problem of gages or measurement instruments used in the inspection procedures. Automated inspection system has been introduced to reduce the inspection time as well as to eliminate errors incurred as a result of human fatigue. However, inspection errors may still be present due to factors such as complexity and difficulty of the inspection task, resolution of the inspection sensor, equipment malfunctions and "bugs" in the computer program controlling the inspection procedure etc. In short, any activities related to human being are subject to mistake as "To err is human".

There are two types of errors present in inspection schemes, namely, Type I and Type II inspection errors, where Type I inspection error refers to the situation in which a conforming item is incorrectly classified as nonconforming and Type II error occurs when a nonconforming unit is erroneously classified as conforming.

While inspection errors incurred during acceptance sampling for attributes are often unintentional and in most cases neglected, they nevertheless can severely distort the quality objective of a sampling system design. This motivated our study of the effect of inspection errors on chain sampling schemes to be part of our chain sampling studies. This research has been completed phase by phase in three stages, the effect of constant inspection errors, the effect of variable inspection errors and the design of chain sampling plan under inspection errors.

The final part of this thesis goes to the reliability engineering, while the previous two topics fall in the category of quality engineering. In this part, the chain sampling is extended to reliability acceptance test and (a new approach to design chain sampling plans for reliability acceptance test is proposed) proposes our approaches to design chain-sampling plans for reliability acceptance test. Its mathematical models are relatively straightforward, but results are useful in application.

Some additional work has been done during the course of my research stint in NUS, which have their unique contributions in terms of researching. However, it is not very consist with the above-mentioned topics and not easy to be incorporated in a cohesive structure. Rather than simply drop them off, we decide to document them in the appendix for future reference. These include the mathematical deviation of ratio of two normal in the multivariate process control and the SWOT (Strengths, Weaknesses, Opportunities and Threats) analysis to Six Sigma Strategy.

A detail review of related topics will be presented in the next chapter, which includes the historical development of acceptance sampling, the review of chain sampling plan and the study of correlated production, the effect of inspection errors on the acceptance sampling, specifically, the error effect on chain sampling plan, and the chain sampling plan for production reliability acceptance test.

In chapter three, the effect of correlation on chain sampling plan will be studied. This study can be served as an abstract and extension of an industrial project. A new model named as Chain Sampling Plan with Markov Property is developed, and the numerical analysis is conducted. Some parameter study is also included.

Chapter four starts the study of the effect of inspection errors on chain sampling plan, in which inspection errors are assumed constant throughout inspection, i.e. the constant error model. In this chapter, the inspection error is considered in chain sampling schemes and a mathematical model is constructed to investigate the performance of chain sampling schemes when inspection errors are taken into consideration. Expressions of performance measures are derived, such as the operating characteristic function, average total inspection and average outgoing quality to aid the analysis of a general chain sampling scheme, ChSP-4A (c1, c2) r, developed by Frishman (1960).

Chapter five is a counterpart of chapter four with the underlying assumption changed from constant inspection error to variable inspection error The variable error is in fact very complicated, so Biegel (1974) linear model is adopted to simplify the problem. The similar study is conducted in chapter four and five so as to highlight the difference between two models.

Chapter six is the most important part of the inspection error effect study. Procedures of designing chain-sampling plans are proposed when constant inspection errors are taken into consideration. Two approaches to design chain-sampling plans for imperfect inspection are proposed with the comparison and examples included for reference.

Chapter seven focuses on the application of chain sampling plan in Reliability Acceptance Testing (RAT) or Product Reliability Acceptance Testing (PRAT), in which this chain sampling scheme for reliability acceptance test is proposed to complement the existing commonly used two schemes: single sampling plan and sequential sampling plan. In addition to the mathematical description, tables for the selection of sampling parameter, and Excel templates are also provided to facilitate designing and flexible usage. Examples are included to illustrate the application of proposed methods.

A summarization of results and conclusions is presented in chapter eight, from which a quick understanding of this study on chain sampling schemes can be found. Reference is listed after chapter eight and the appendix part can be found after reference.

2. Literature Review

2.1 Historical Development of Acceptance Sampling

The development of the statistical science of acceptance sampling has a long history that can be traced back to the formation of the Inspection Engineering Department of Western Electric's Bell Telephone Laboratories in 1924. The department made lots of contributions in this area and some members of the department became gurus in this area later such as H.F. Dodge, who is considered by some to be the father of acceptance sampling. Other pioneers were W.A. Shewhart, Juran and H.G. Romig.

In 1924, Shewhart from this department presented the first control chart, the symbolic start of the era of statistical quality control (SQC). Meanwhile, many, if not most, of the acceptance sampling terminologies was coined by this department between 1925 to 1926 such as single sampling plan, double sampling plan, consumer's risk, producer's risk, probability of acceptance, OC curves, ATI etc. In 1941, H.F. Dodge and H.G. Romig published the famous Dodge-Romig table "Single Sampling and Double Sampling Inspection Tables", which provided plans based on fixed consumer risk (LTPD protection) and also plans for rectification (AOQL protection), which guaranteed stated protection after 100 percent inspection of the rejected lots.

The Second World War witnessed a great development of quality control and particularly acceptance sampling. This included the development, by the Army's Office of the Chief of Ordnance (1942), of "Standard Inspection Procedures" of which the Ordnance sampling tables, using a sampling system based on a designated acceptable quality level, were a part. Also in this period, H.F. Dodge (1943) developed a sampling plan for continuous production indexed by AOQL and A. Wald (1943), a member of the Statistical Research Group in Columbia University, put forward his new theory of sequential sampling which was the ultimate extension of multiple sampling plans, where items were selected from a lot one at a time and after inspection of each item a decision was made to accept or reject the lot or select another unit.

The Statistical Research Group of Columbia University (1945) made outstanding contributions during the Second World War. Their output consisted of advancements in variables and attributes sampling in addition to sequential analysis. Some of these were documented in the Statistical Research Group (1947) "Techniques of Statistical Analysis." They were active in theoretical developments in process quality control, design of experiments, and other areas of industrial and applied statistics as well. Out of the work of the Statistical Research Group came a manual on sampling inspection prepared for the U.S navy, office of Procurement and Material. Like the Army Ordnance Tables, it was a sampling system based on specification of an acceptable quality level (AQL) and was later published by the Statistical Research Group (1948) under the title "Sampling Inspection". In 1949 the manual became the basis for the Defense Department's non-mandatory Joint Army-Navy Standard JAN-105. And later, a committee of military quality control specialists was formed to reach a compromise between JAN-105 and the ASF tables, which resulted in MIL-STD-105A issued in 1950 and subsequently revised as 105B, 105C and 105D, which was still a handbook for current inspection practitioners in industries.

The research of acceptance sampling became less active after 1970s and 1980s as more and more research were streamed into statistical process control and design for quality. There is clear indication that acceptance sampling is playing a lesser role in research, which can be easily identified by its decreasing proportion in the Statistical Quality Control textbooks. However, research paper and works still appear sometime focusing on the development or improvement of specified acceptance techniques.

2.2 Chain Sampling Plan

The principle of a continuous sampling plan (CSP-1), which was originally applied to a steady stream of individual items from the process and required sampling of a specified fraction, f, of the items in order of production, with 100 percent inspection of the flow at specified times, could be extended to apply to a continuing series of lots or batches of material rather than to individual product units. This led Dodge (1955) to propose the skip-lot sampling plan (SkSP). Its underlying principle was almost the same as that of the CSP and the only difference lied in that the SkSP plans dealt with series of lots or batches while the CSP plans handled with series of units. The application of these plans and ideas was formulized by Dodge and Perry (1971), Perry (1970, 1973a, 1973b) and later documented by ANSI/ASQC Standard S1-1987 (1987).

Both the continuous sampling plan and skip-lot sampling plan were members of, so-called, cumulative results plans, which made decision not only based on the current lot, but also made use of the cumulative lots information. Another member of this cumulative results plans is the chain sampling plan (ChSP) introduced by Dodge (1955), which made use of previous lots results, combining with the current lot information, to achieve a reduction of sample size while maintaining or even extending protection. The ChSP plans were first conceived to overcome the problem of lack of discrimination of the single sampling plan when acceptance number c=0, and had been received wide application in industries where the test is either costly or destructive. Its operating procedure is illustrated in Figure2.1.

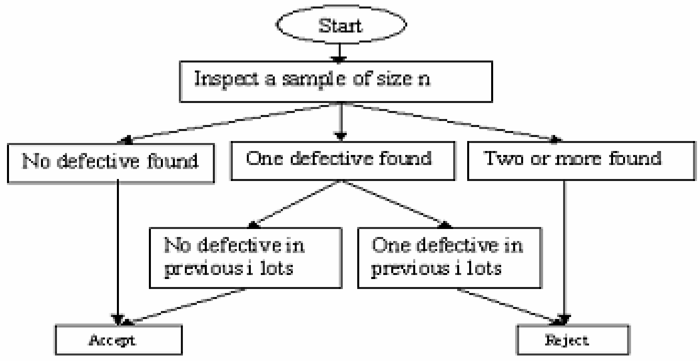

Figure 2. 1Dodge Chain Sampling Plan

Zwickl (1963) and Soundarajan (1978a and 1978b) had carried out further evaluations of ChSP-1 type sampling plans. Since the invention of ChSP-1, numerous works had been done on the extensions to chain sampling plans. These included plans designated ChSP-2 and ChSP-3, which was done by Dodge (1958) but kept unpublished, partly due to the complexities of its operating procedures. Frishman (1960) presented extended chain sampling plans designated ChsSP-4 and ChSP-4A (perhaps contemplating publication of designations 2 and 3 by Dodge). His plans were developed from an application in the sampling inspection of torpedoes for Naval Ordnance as a check on the control of the production process and test equipment (including 100% inspection). Features of these plans included a basic acceptance number greater than zero, an option for forward or backward accumulation of results for an acceptance-rejection decision on the current lot, and provision for rejecting a lot on the basis of the results of a single sample (ChSP-4A). Its operating procedure is illustrated in Figure 2.2.

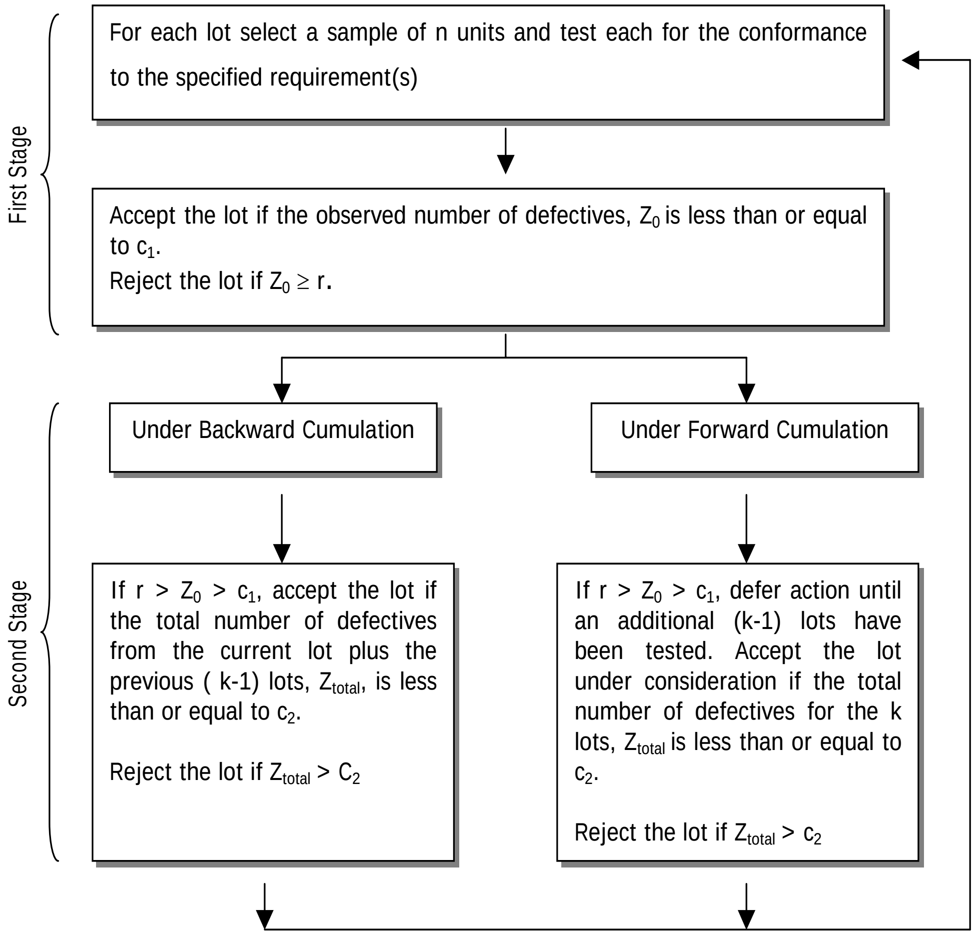

Figure 2. 2Chain Sampling Plan (4A)

Some variations of chain sampling for which cumulative results were used in the sentencing of lots had also been developed by Anscomber, Godwin, and Plackett (1947); Page (1955); Hill, Horsnell, and Warner (1959); Ewan and Kemp (1960); Kemp (1962); Beattle (1962); Cone and Dodge (1964); Wortham and Moog (1970), and Soundarajan (1978a and 1978b). Further extensions to a general family of chain sampling inspection plans had been developed by Dodge and Stephens and published in numerous technical reports, conference papers, and journal articles.

Raju (1996a, 1996b, 1991,1995, 1997) did extensive research work on chain sampling plan both cooperatively and independently. His contribution included extending idea of ChSP-1 and devising tables based on the Poisson model for the construction of two-stage chain sampling plans ChSP (0,2) and ChSP (1,2) under difference sets of criteria, outlining the structure of a generalized family of three- stage chain sampling plans, which extended the concept of two-stage chain sampling plans of Dodge and Stephens (1966). He also authored a series of 5 papers, which presented procedures and tables for the construction, and selection of chain sampling plans ChSP-4A (c1, c2). Govindaraju (1998) extended the idea of chain sampling plans to variable inspection and examined the related properties and listed the desired table.

2.3 Correlated Production

All the abovementioned research works were done based on the assumption of independent life distributions and perfect inspection. In the other direction of research, some researchers had questioned the unrealistic assumption of i.i.d. (identical independent distribution).

Lieberman (1953) presented an analysis of CSP-1 under the assumption that the probability of a defective unit was not constant for each unit. He found that the worst situation would be the one where only defective units were produced under fractional sampling and non-defective unites were produced under 100 percent inspection. In practice, it was unlikely that automated mass production would follow such a case. Sackrowitz (1975) studied the unrestricted AOQL and remarked: “What happened apparently is that, the assumption of statistical control was recognized as being too restrictive and unrealistic and so was relaxed completely. However, assuming that the production process could always do anything may be too unrealistic.”

Broadbent (1958) described a production process where a mold continuously produced glass bottles in an automatic manufacturing process. He reported that non-defective and defective bottles occurred in runs and suggested, therefore, a Markov model with non-defective (0) and defective (1) as two states. He introduced a Markovian character because of the fact that a defect was likely to occur in a succession of bottles from a single mold until the cause of the defect was corrected.

Preston (1971), while discussing a two-state Markov chain model of a production process, pointed out that if the serial correlation coefficient of the Markov chain was positive, long strings of non-defectives and defective were more likely; whereas if the serial correlation coefficient was negative, alternating sequences of non-defectives and defectives were more likely.

Rajarshi and Kumar (1983), Kumar, and Rajarshi (1987), studied the behavior of three continuous sampling CSP-1, CSP-2 and MLP2 under the assumption of a continuous production process follows a two-state time-homogeneous Markov chain. The AOQL formula of these plans were also derived and presented. The study shows that if the serial correlation coefficient of the Markov chain was positive (negative), the AOQL in increase (decreased) as compared to the case when the successive units in the production process followed a Bernoulli pattern.

McShane and Turnbull (1991) investigated the performance of CSP-1 when the production run lengths were short or moderate or when the input process was not i.i.d. Bernoulli. They considered both rectifying and non-rectifying inspections and compared the AOQL for the i.i.d. case and the Markov case and the unrestricted AOQL values. They concluded that great care should be taken in interpreting the AOQ and AOQL, which were the usual measures of the effectiveness of CSP-1 plans. Even if the input process was in statistical control, these long-run average measures could be very deceiving for finite production runs because the AOQ and AOQL may differ from their finite run counterparts and they didn’t take any measure of variability into account.

Kumar and Vasantha (1995) presented their studies of the continuous inspection of Markov processes with a clearance interval. A common conclusion from these studies showed that it's more reasonable to expect the production unit from the same process to exhibit a Markov property than the identical independent distribution. Chen and Wang (1999) derived the minimum AFI for CSP-1 plan under the Markov processes, which could be seen as a comparable work with Resnifoff (1960) and Ghosh (1988), which addressed the problem of constructing a minimum average fraction inspected (AFI) for a CSP-1 plan when the production process was under control. These work mainly dealt with the dependency existing between the product units from the same production process.

Another direction of research in the area of CSP went to the study of the effects of the inspection errors. Up to now, few researchers were involved with this as all assumed the inspection is perfect. Johnson and Kotz (1980, 1981, 1982a, 1982b, 1984), however, contributed in this area and studied the effects of the inspection error on the performance of acceptance sampling plans. Kotz and Johnson (1984) also considered the economic impact of the sampling plans and proposed a simple model to simulate them.

2.4 Effect of Inspection Errors

There are two types of errors present in inspection schemes, namely, Type I and Type II inspection errors, where Type I inspection error refers to the situation in which a conforming item is incorrectly classified as nonconforming and Type II error occurs when a nonconforming unit is erroneously classified as conforming.

Effects of inspection error on the statistical quality control objectives are well documented in literatures. In a series of four papers devoted to the effects of inaccuracies of inspection sampling for attributes, Johnson and Kotz had derived the hyper geometric probability distributions for several types of inspection schemes namely, single stage acceptance sampling schemes [1], double stage, link and partial link acceptance sampling schemes [2], Dorfman screening procedures [3] and modified Dorfman screening procedures [4]. While, in reality, all inspection procedures are governed by the hyper geometric distribution (as sampling is done without replacement from a finite lot), the mathematical models derived by Johnson and Kotz are often complex and computationally intensive. As such, a number of quality control analysts (Maghsoodloo and Bush (1985) for instance) have employed the binomial distribution to evaluate error prone sampling procedures instead. Such approximation is satisfactory in situations where lot size is more than ten times the sample size.

Dorris and Foote (1978) had given a literature review of the research works being done pertaining to the effect of inspection errors. Most recent work can be found in Beainy and Case (1981), Kotz and Johnson (1984), Shin and Lingayat (1992), Fard & Kim (1993), Tang (1987), Ferrell and Chhoker (2002).

In order to examine the effects of inspection errors on statistical quality control procedures, it is necessary to have a model of the process generating the errors. One particular model for errors in the inspection of items on the basis of attributes assumes constant error probabilities. That is the probability of committing inspection errors does not change thorough out the inspection. This assumption, though simple and mathematical appealing, does not provide a good representative of the real case. Actually there are number of argument that inspection errors are fluctuating and different model (Biegel (1974) for example) has been proposed to model this fluctuation.

2.5 Reliability Acceptance Test

Reliability acceptance sampling Reliability Acceptance Testing (RAT) or Product Reliability Acceptance Testing (PRAT) is used to sentence a lot according to some reliability requirements. This test may be conducted either by the supplier or the customer or both based on agreed sampling plans and acceptance rules.

It is probably the oldest reliability testing techniques and also almost the least explored topic in current reliability study, which due partly to the commonly existed misconception that it is too simple to deserve further study. In the 1950s and 1960s, life test had been the subject of extensive research and some concrete results had been produced and became the basis of the later reliability acceptance test techniques. In a series of papers devoted to life test (Epstein & Sobel 1953, Epstein 1954, Epstein & Sobel 1955), Epstein and Sobel presented their results of life test based on exponential distribution. In 1961, Gupta and Groll carried out a similar study of life test sampling plans based on gamma distribution.

Similar research about Weibull distribution was deferred until 1980, when Fertig and Mann published their paper “Life-test sampling plans for two parameter Weibull populations”. One major reason for this deference lied in the difficulty and complexity of deriving the parameter estimate and its distribution as well as finding its feasible approximation.

Besides the above-mentioned one-stage life test plans, two-stage life test, which offers a better risk control and an average less sampling cost, were also appear in literature. Bulgren and Hewett (1973) considered a two-stage test of exponentially distributed lifetime with failure censoring at each state. Fairbanks (1988) presented his two-stage life test for exponential parameter with a hybrid censoring at each stage.

A thorough survey of two-stage methods, as well as examples of experiments, was provided by Hewett and Spurrier (1983).

3. Chain Sampling Plan for Correlated Production

3.1 Introduction

Acceptance sampling is one of major areas of statistical quality control in quality and reliability engineering. It began to take root during the era of industrial revolution in the early nineteenth century and flourished during the Second World War. It continued to prosper in the second half of the last century, during which period various sampling plans had been formulated to cater for various testing situations and quality requirements.

Single sampling plan and double sampling plan are the most basic and widely applicable testing plans when simple testing is needed. Multiple sampling plans and sequential sampling plans help make marginally better disposition decision at the expense of more complicated operating procedures. Other plans such as continuous sampling plans, bulk-sampling plans, and Tighten-normal-tighten plans etc., are well developed and frequently used in their respective working conditions. Among these, chain sampling plans have received great attention from industries because of its unique strength in dealing with destructive or expensive inspections, where the number of sample size is kept at as low as possible to minimize the total inspection cost. This feature supports the application of chain sampling plans to the testing of products such as the fire-retard door and the fire-retard cable.

The characteristics of these testings are: (I) testing is destructive, so it is favorable to take as few samples as possible, and (II) testing units (or their components) are cut from the same process and it is reasonable to expect a certain kind of relationship between ordered samples. For example, units after good units (conformities) are more likely to be good, and units after bad units (nonconformities) are more likely to be bad.

The objective of this chapter is thus to extend chain sampling plans to plans that could capture the dependency between test units within a sample.

In this chapter, the starting point is the Dodge Chain Sampling Plan (ChSP-1), first introduced by Dodge (1977). Its original intention was to overcome the problem of the lack of discrimination of a single sampling plan when the acceptance number $c=0$. Today, this plan and its extensions (Ewan and Kemp (1960), Frishman (1960), Govindaraju & Kuralmani (1991), Jothikumar & Raju (1996), Raju (1991), Raju & Jothikumar (1997), Raju & Murthy (1995 & 1996), Soundarajan (1978) etc.) have become the most frequently used plans in destructive or expensive inspections. Its operating procedure was illustrated in Figure 2.1.

Theoretical calculations of ChSP-1 plan are made on assumptions that:

I. Inspection is perfect;

II. The production process is in "statistical control";

III. The quality characteristic of interest follows an independent identical distribution (i.i.d.).

Above-mentioned assumptions are obviously too restrictive, especially for products under continuous production and/or for samples collected in some pre-determined order, for example, fire-retard cables and fiber optics, etc. For obvious economic reasons, samples are taken at the beginning or at the end of each reel. As a result, it seems more reasonable to expect some kind of dependency in the quality characteristics within a sample.

Broadbent (1958) studied various models for quality characteristics of this type of production processes, and among them, a two-state Markov chain model is a simple and yet versatile choice. It usually offers a satisfactory fit for correlated production processes and the dependency can be characterized using model parameters, which, in turn, can be estimated from the data or assumed known a priori. Some literatures have presented models of various sampling plans with Markovian property. For example, Kumar & Rajarshi (1987) presented their Markov chain model for continuous sampling plans and Bhat et al (1990) showed their studies on a sequential inspection plans for Markov dependent production process. Related works have also been carried out by Kumar and Vasantha (1995), McShane and Turnbull (1991), Chen and Wang (1999), and Rajarshi & Kumar (1983) etc. However, to the best of author's knowledge, such an extension to the chain-sampling plan has yet to appear in literature.

A correlation study is conducted to bridge this gap, with the aim of capturing the correlation between testing units. Assume that a Markov chain can model the dependency of product units within a sample and there is no dependency between samples. In the next section, an extension to the Dodge chain-sampling plan is proposed and the related characteristic functions are derived. This is followed by results and discussions; and finally, a conclusion is given in the last section.

3.2 Chain Sampling Plan for Markov Dependent Process

In this section, an extension to the Dodge chain sampling, called as chain sampling plan for Markov dependent process is described, in which the correlation of quality characteristics of testing units within a sample is assumed be a Markov chain. For mathematical tractability, assume these characteristics are independent among different samples. Here no distinction is made between the number of samples and the number of previous lots, as only one sample (with sample size equal to n) is taken from each lot. Therefore, the number of samples and the number of lots are identical in this context. Some basic assumptions are as follows:

1. The quality characteristic of interest follows a 2-state Markov chain within each sample (subgroup);
2. The quality characteristics of interest are independent between different samples;
3. All samples (subgroups) come from the same process.

Following above assumptions, define the sequence of random variables $\{X_n, n \geq 0\}$ by

$X_n = 1$ if the *n*th unit is defective

$X_n = 0$ if the *n*th unit is non-defective (3-1)

Suppose that $\{X_n, n \geq 0\}$ follows a 0-1-valued time-homogeneous Markov-chain, initial distribution and transition probability matrix, respectively, are given by

$$P[X_0 = 1] = \pi_0 \qquad 0 \leq \pi_0 \leq 1, \tag{3-2}$$

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{cc} 0 & \quad 1 \end{array} \\ \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} \end{array} \qquad 0 < a, b < 1 \tag{3-3}$$

Where: a = Pr {(*i+1*)th unit is defective| the *i*th unit is non defective}

b = Pr {(*i+1*)th unit is non-defective| the *i*th unit is defective}.

For the convenience of derivation, introduce the following new parameters:

$$p = a(a+b)^{-1}, \delta = (a+b), \lambda = 1-\delta, \ q = (1-p) \tag{3-4}$$

So that: $a = p\delta$, $b = q\delta$

Thus obtain: $\max[0, 1-\delta^{-1}] < p < \min[\delta^{-1}, 1]$.

The physical interpretation of above parameters is listed as follows:

p --- is a long-run proportion of defective units.

λ , δ --- can be viewed as dependency parameters of the process, i.e. a serial correlation coefficient between X_n and X_{n+1} provided (that) the stationary distribution is taken to be the initial distribution. Particularly, $\lambda = 0$ gives Bernoulli model.

The k -step transition matrix is given by:

$$P^k = \begin{bmatrix} q & p \\ q & p \end{bmatrix} + (1-\delta)^k \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} \tag{3-5}$$

It will be proved in the following that the Markov model transitional probability matrix is as described in equation (3-5). The physical interpretation of the parameter will also be explained:

As defined in equation (3-3):

$$P = \begin{array}{c} \\ 0 \\ 1 \end{array} \begin{array}{c} \begin{array}{cc} 0 & 1 \end{array} \\ \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix} \end{array} \qquad 0 < a, b < 1$$

For k =1

$$\begin{aligned} P^1 = P &= \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix} = \begin{bmatrix} 1-p\delta & p\delta \\ q\delta & 1-q\delta \end{bmatrix} = \begin{bmatrix} p+q-p\delta & p-p+p\delta \\ q-q+q\delta & p+q-q\delta \end{bmatrix} \\ &= \begin{bmatrix} q+p(1-\delta) & p-p(1-\delta) \\ q-q(1-\delta) & p+q(1-\delta) \end{bmatrix} = \begin{bmatrix} q & p \\ q & p \end{bmatrix} + \begin{bmatrix} p(1-\delta) & -p(1-\delta) \\ -q(1-\delta) & q(1-\delta) \end{bmatrix} \\ &= \begin{bmatrix} q & p \\ q & p \end{bmatrix} + (1-\delta) \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} \end{aligned} \tag{3-6}$$

For k =2

$$P^2 = P*P = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix} * \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix} = \begin{bmatrix} 1-p\delta & p\delta \\ q\delta & 1-q\delta \end{bmatrix} * \begin{bmatrix} 1-p\delta & p\delta \\ q\delta & 1-q\delta \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \delta \begin{bmatrix} -p & p \\ q & -q \end{bmatrix} \right\} * \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \delta \begin{bmatrix} -p & p \\ q & -q \end{bmatrix} \right\} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} * \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \delta \begin{bmatrix} -p & p \\ q & -q \end{bmatrix} \right\}$$

$$+ \delta \begin{bmatrix} -p & p \\ q & -q \end{bmatrix} * \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \delta \begin{bmatrix} -p & p \\ q & -q \end{bmatrix} \right\}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \delta \begin{bmatrix} -p & p \\ q & -q \end{bmatrix} + \delta \begin{bmatrix} -p & p \\ q & -q \end{bmatrix} + \delta^2 \begin{bmatrix} p^2+pq & -p^2-pq \\ -pq-q^2 & pq+q^2 \end{bmatrix}$$

$$= \begin{bmatrix} 1-2p\delta+p\delta^2 & 2p\delta-p\delta^2 \\ 2q\delta-q\delta^2 & 1-2q\delta+q\delta^2 \end{bmatrix} = \begin{bmatrix} q+p(1-2\delta+\delta^2) & p-p(1+2\delta-\delta^2) \\ q-q(1+2\delta-\delta^2) & p+q(1-2\delta+\delta^2) \end{bmatrix}$$

$$= \begin{bmatrix} q+p(1-\delta)^2 & p-p(1-\delta)^2 \\ q-q(1-\delta)^2 & p+q(1-\delta)^2 \end{bmatrix} = \begin{bmatrix} q & p \\ q & p \end{bmatrix} + \begin{bmatrix} p(1-\delta)^2 & -p(1-\delta)^2 \\ -q(1-\delta)^2 & q(1-\delta)^2 \end{bmatrix}$$

$$= \begin{bmatrix} q & p \\ q & p \end{bmatrix} + (1-\delta)^2 \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} \tag{3-7}$$

Suppose the formula for $k=n$ is correct. That is:

$$P^n = \begin{bmatrix} q & p \\ q & p \end{bmatrix} + (1-\delta)^n \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} \tag{3-8}$$

Then, when $k=n+1$,

$$P^{n+1} = P^n * P = \left(\begin{bmatrix} q & p \\ q & p \end{bmatrix} + (1-\delta)^n \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} \right) \left(\begin{bmatrix} q & p \\ q & p \end{bmatrix} + (1-\delta) \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} \right)$$

$$= \left(\begin{bmatrix} q & p \\ q & p \end{bmatrix} \begin{bmatrix} q & p \\ q & p \end{bmatrix} + (1-\delta)^n \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} \begin{bmatrix} q & p \\ q & p \end{bmatrix} \right)$$

$$+ \left(\begin{bmatrix} q & p \\ q & p \end{bmatrix} (1-\delta) \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} + (1-\delta)^n \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} (1-\delta) \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} \right)$$

$$= \begin{bmatrix} q^2+pq & pq+p^2 \\ q^2+pq & pq+p^2 \end{bmatrix} + (1-\delta)^n \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + (1-\delta) \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + (1-\delta)^{n+1} \begin{bmatrix} p^2+pq & -p^2-pq \\ -pq-q^2 & pq+q^2 \end{bmatrix}$$

$$= \begin{bmatrix} q^2+pq & pq+p^2 \\ q^2+pq & pq+p^2 \end{bmatrix} + (1-\delta)^{n+1} \begin{bmatrix} p^2+pq & -p^2-pq \\ -pq-q^2 & pq+q^2 \end{bmatrix}$$

$$= \begin{bmatrix} q(q+p) & p(q+p) \\ q(q+p) & p(q+p) \end{bmatrix} + (1-\delta)^{n+1} \begin{bmatrix} p(q+p) & -p(p+q) \\ -q(p+q) & q(p+q) \end{bmatrix}$$

$$= \begin{bmatrix} q & p \\ q & p \end{bmatrix} + (1-\delta)^{n+1} \begin{bmatrix} p & -p \\ -q & q \end{bmatrix} \tag{3-9}$$

Therefore, the transitional probability for the Markov model is:

$$P^k = \begin{bmatrix} q & p \\ q & p \end{bmatrix} + (1-\delta)^k \begin{bmatrix} p & -p \\ -q & q \end{bmatrix}$$

The physical interpretation of parameter p and δ:

Suppose that D=defective, and G=good (non defective); from the definition, obtain $a = P(D \mid G)$ and $b = P(G \mid D)$.

$$p = \frac{a}{a+b} = \frac{P(D \mid G)}{P(D \mid G) + P(G \mid d)} = \frac{\frac{P(DG)}{P(G)}}{\frac{P(DG)}{P(G)} + \frac{P(DG)}{P(D)}}$$
$$= \frac{P(DG)}{P(G)} * \frac{1}{\frac{P(DG)}{P(G)} + \frac{P(DG)}{P(D)}} = \frac{P(DG)}{P(G)} * \frac{1}{\frac{(P(G)+P(D))P(DG)}{P(G)P(D)}} \qquad (3\text{-}10)$$
$$= \frac{P(DG)}{P(G)} * \frac{P(G)P(D)}{P(DG)} = P(D) = P(defective)$$

Therefore, p is the long- term proportion of defects.

Similarly:

$$\delta = a + b = \frac{P(DG)}{P(G)} + \frac{P(DG)}{P(D)} = \frac{(P(G)+P(D))P(DG)}{P(G)P(D)} = \frac{P(DG)}{P(G)P(D)} = \frac{P(D \mid G)}{P(D)} = \frac{P(G \mid D)}{P(G)} \qquad (3\text{-}11)$$

This can be viewed as the dependency parameter as it is the ratio of the conditional probability and the probability of the respective event.

So for the initial state: $(1-\pi_0, \pi_0)$, compute the state of the *k*th unit within each sample using k-step transition probability P^k.

$$\left(\Pr(X_k = 0) \quad \Pr ob(X_k = 1)\right) = \left(1-\pi_0 \quad \pi_0\right) \times P^k$$
$$= \left[q + (1-\delta)^k (p - \pi_0) \quad p + (1-\delta)^k (\pi_0 - p)\right] \qquad (3\text{-}12)$$

Therefore, the probability that the *k*th unit within each sample is in state 0 and state 1 respectively is given by:

$$\Pr(X_k = 0) = q + (1-\delta)^k (p - \pi_0) \qquad (3\text{-}13)$$

$$\Pr(X_k = 1) = p + (1-\delta)^k (\pi_0 - p) \qquad (3\text{-}14)$$

The probability of finding a non-defective in a sample is:

$$
\begin{aligned}
P(0) &= \Pr(X_1 = 0, X_2 = 0, \ldots, X_n = 0) \\
&= \Pr(X_1 = 0) * \Pr(X_2 = 0, \ldots, X_n = 0 \mid X_1 = 0) \\
&= \Pr(X_1 = 0) * \Pr(X_2 = 0 \mid X_1 = 0) * \ldots * \Pr(X_n = 0 \mid X_1 = 0, \ldots, X_{n-1} = 0) \\
&= \Pr(X_1 = 0) * \Pr(X_2 = 0 \mid X_1 = 0) * \ldots * \Pr(X_n = 0 \mid X_{n-1} = 0) \\
&= (1-\pi_0)(1-a)^{n-1} = (1-\pi_0)(1-p\delta)^{n-1}
\end{aligned}
\tag{3-15}
$$

Similarly, the probability of finding one defective in a sample is:

$$
\begin{aligned}
P(1) &= \Pr(X_1 = 1, X_2 = 0, \ldots, X_n = 0) \\
&\quad + \Pr(X_1 = 0, X_2 = 1, X_3 = 0, \ldots, X_n = 0) \\
&\quad + \Pr(X_1 = 0, X_2 = 0, X_3 = 1, X_4 = 0, \ldots, X_n = 0) \\
&\quad \vdots \\
&\quad + \Pr(X_1 = 0, \cdots, X_{n-2} = 0, X_{n-1} = 1, X_n = 0) \\
&\quad + \Pr(X_1 = 0, \cdots, X_{n-1} = 0, X_n = 1) \\
&= \pi_0 b(1-a)^{n-2} + (1-\pi_0)ab(1-a)^{n-3} + \cdots + (1-\pi_0)(1-a)^{n-3}ab + (1-\pi_0)(1-a)^{n-2}a \\
&= \pi_0 b(1-a)^{n-2} + (n-2)(1-\pi_0)ab(1-a)^{n-3} + (1-\pi_0)(1-a)^{n-2}a \\
&= (1-a)^{n-3}\left[\pi_0 b(1-a) + (n-2)(1-\pi_0)ab + (1-\pi_0)(1-a)a\right] \\
&= (1-a)^{n-3}\left[\pi_0 b - \pi_0 ba + (n-2)(ab - \pi_0 ab) + (1-\pi_0)(a-a^2)\right] \\
&= (1-a)^{n-3}\left[\pi_0 b + (n-2)ab - (n-1)\pi_0 ab + a - a^2 - a\pi_0 + a^2\pi_0\right] \\
&= (1-a)^{n-3}\left[\pi_0\left(b - (n-1)ab - a + a^2\right) + (n-2)ab + a - a^2\right] \\
&= (1-p\delta)^{n-3}\left[\pi_0\left(q\delta - (n-1)pq\delta^2 - p\delta + p^2\delta^2\right) + (n-2)pq\delta^2 + p\delta - p^2\delta^2\right] \\
&= (1-p\delta)^{n-3}\left[\pi_0\left(\delta - (n-1)p(1-p)\delta^2 - 2p\delta + p^2\delta^2\right) + (n-2)p(1-p)\delta^2 + p\delta - p^2\delta^2\right] \\
&\qquad \text{for } n \ge 2
\end{aligned}
\tag{3-16}
$$

Secondly, treat all samples independently and follow rules of ChSP-1 that the whole batch will be accepted either when there is no defective found in the current sample or when one defective found in the current sample but no defectives found in the previous i samples.

Therefore, the probability of acceptance of a batch is given by:

$$P_a = P(0) + P(1) \times (P(0))^i \tag{3-17}$$

In this case:

$$P_a = (1-\pi_0)(1-p\delta)^{n-1}$$
$$+(1-p\delta)^{n-3}\left[\pi_0\left(\delta-(n-1)p(1-p)\delta^2-2p\delta+p^2\delta^2\right)+(n-2)p(1-p)\delta^2+p\delta-p^2\delta^2\right]$$
$$*\left((1-\pi_0)(1-p\delta)^{n-1}\right)^i$$

(3-18)

and:

$$AOQ = p * P_a$$
$$= p(1-\pi_0)(1-p\delta)^{n-1}$$
$$+ p(1-p\delta)^{n-3}\left[\pi_0\left(\delta-(n-1)p(1-p)\delta^2-2p\delta+p^2\delta^2\right)+(n-2)p(1-p)\delta^2+p\delta-p^2\delta^2\right]$$
$$*\left((1-\pi_0)(1-p\delta)^{n-1}\right)^i$$

(3-19)

where i is the number of previous samples (or number of previous lots).

3.3 Results and Discussion

An Excel Visual Basic Application program is developed to carry out the numerical study of the new model, particularly for generating the *OC* curve and *AOQ* curve. The results are illustrated below.

1. A comparison of the *OC* curve of the proposed model with that of the former Dodge plan is illustrated in Figure 3.1. The correlation parameter δ is changed from 0.2 to 1.8. For δ =1, the corresponding *OC* curve is identical to that of the Dodge ChSP-1 plan. For δ <1, units within a sample are positively correlated and; for δ >1, the correlation is negative.

   When δ >1, the new model reveals that for a given " p ", the probability of acceptance is smaller than Dodge ChSP-1, when the negative correlation is taken into consideration. In other words, the proposed plan is more discriminating than the Dodge ChSP-1 and the discriminating power increases as δ increases. The converse is true. When the correlation coefficient is positive, the corresponding probability of acceptance is larger for a given " p " when $\delta < 1$ and thus the discrimination power is less than that of Dodge plan. The implication in practice is that when the Dodge ChSP-1 plan is applied to samples with positive correlation, the resulting probability of acceptance is smaller than what it is supposed to be and will lead to a more conservative decision. On the other hand, when there is a negative correlation, Dodge ChSP-1 plan must be used with caution as its probability of acceptance and average outgoing quality are larger than actual values given in this plan.

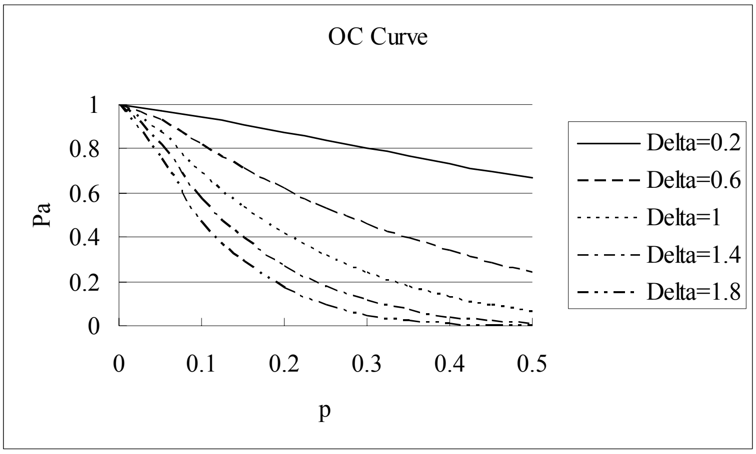

Figure 3. 1OC curve of new model (i=5, n=5)

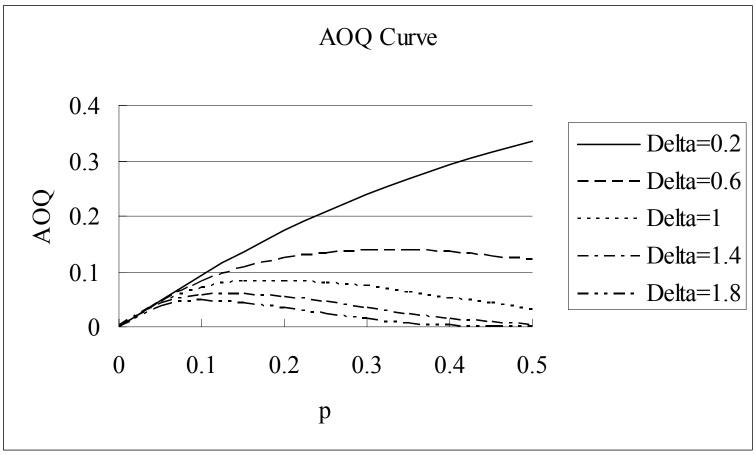

Figure 3. 2AOQ curve of new model (i=5, n=5)

Figure 3.2 shows the effect of correlation on AOQ of this model for different δ ranging from 0.2 to 1.8. For δ=1, the corresponding AOQ curve is identical to that of the Dodge plan. It can be seen that AOQ becomes smaller when the correlation pattern changes from positive to negative for a given incoming lot quality. Moreover, the $AOQL$ also decreases for a larger δ. This is consistent with the earlier observation that the proposed plan is more discriminating under the negative correlated production.

2. The effect of sample size on the performance of OC curves is illustrated in Figure 3.3, 3.4, and 3.5. Here, the sample size n is changed for a fixed value of δ and lots number. In Figure 3.3, the correlation parameter δ is fixed at 0.4 and the previous lots number i is fixed at five. It represents an example of positively correlated scenario (δ<1). In Figure 3.4, δ is set to one and is actually the plot of Dodge ChSP-1 as there is no correlation between testing units (δ=1). Figure 3.5 is an example of negatively correlated cases, in which the correlation coefficient δ is fixed at 1.4 (δ>1). These three graphs exhibit the same trend when the sample size n is changed. For a given "p", the probability of acceptance decreases with the increase of sample size, which means an increase in the discriminating power.

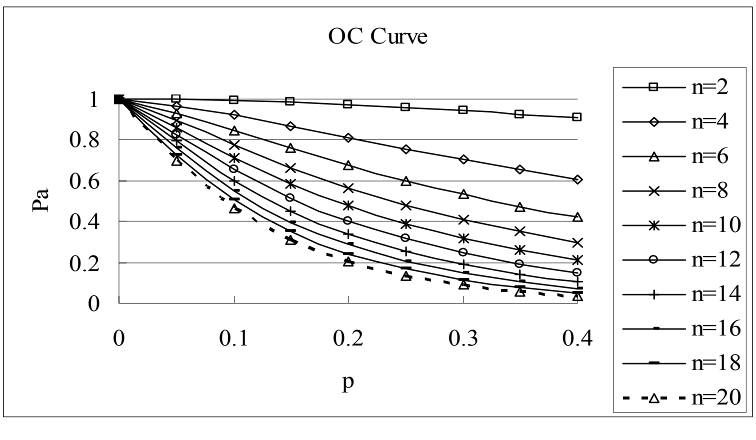

Figure 3. 3 OC curve comparison of sample size (*i*=5, δ =0.4)

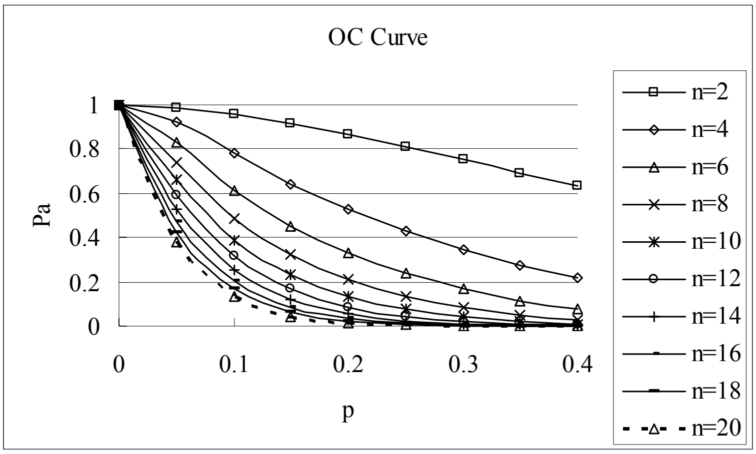

Figure 3. 4 OC curve comparison of sample size (*i*=5, δ =1)

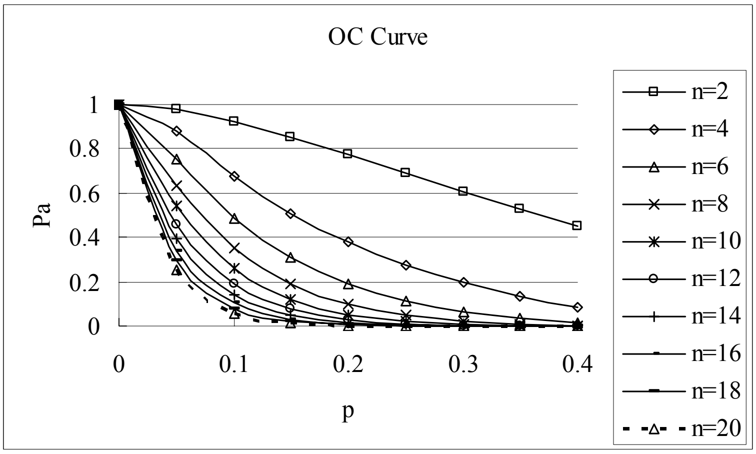

Figure 3. 5 OC curve comparison of sample size (*i*=5, δ =1.4)

Corresponding AOQ curves are illustrated in Figures 3.6, 3.7, and 3.8 respectively. These three figures are used to study the effect of sample size on AOQ curves when there is a positive correlation (δ <1), no correlation (δ =1) and negative correlation (δ >1) respectively. The results are similar to those of OC curves. For a given “ p ”, the $AOQL$ decreases with the increase of sample size.

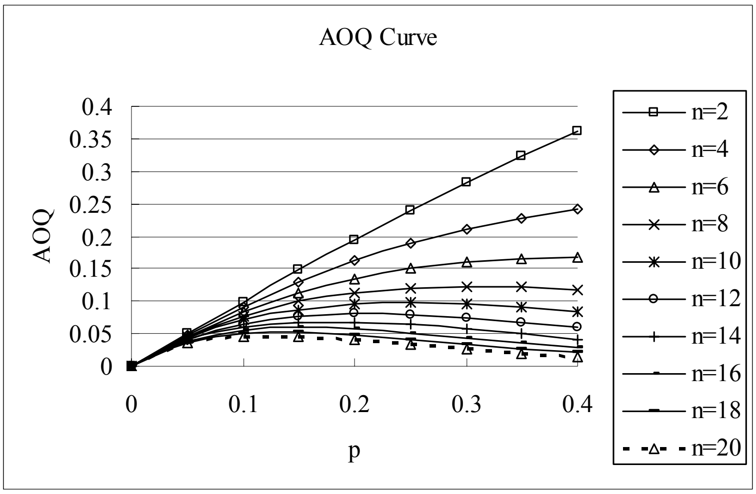

Figure 3. 6 AOQ comparison of sample size (*i*=5, δ =0.4)

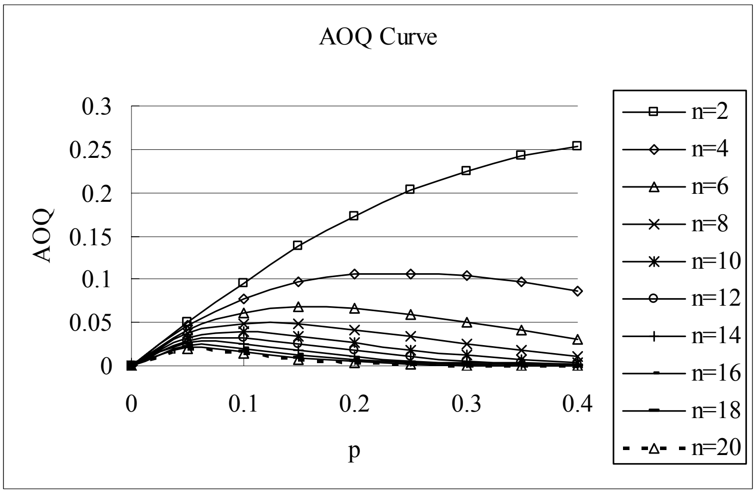

Figure 3. 7 AOQ comparison of sample size (*i*=5, δ =1)

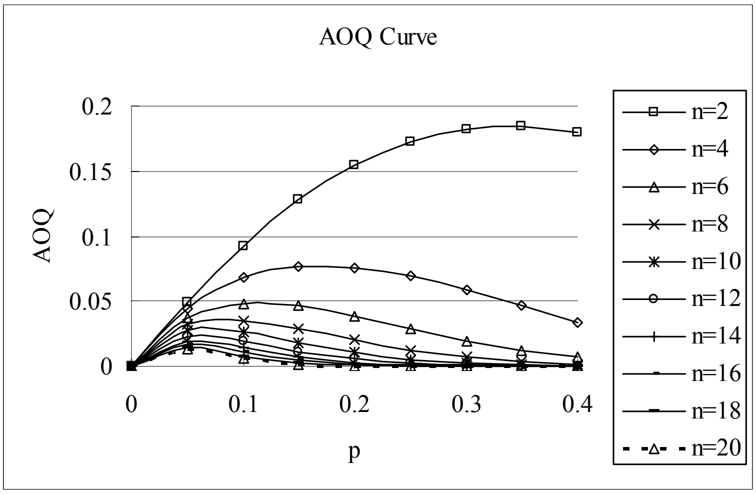

Figure 3. 8 AOQ comparison of sample size (*i*=5, δ =1.4)

3. The effect of the cumulative number of previous lots on the performance of *OC* curves is illustrated in Figures 3.9, 3.10, and 3.11 for $\delta<1$, $\delta=1$ and $\delta>1$ respectively. The number of previous lots i differs from 1 to 5 for fixed values of δ and sample size. The trends revealed in these three graphs are similar. For a given " p ", the probability of acceptance decreases with the increase of the number of previous lots, which implies an increase of the discriminating power.

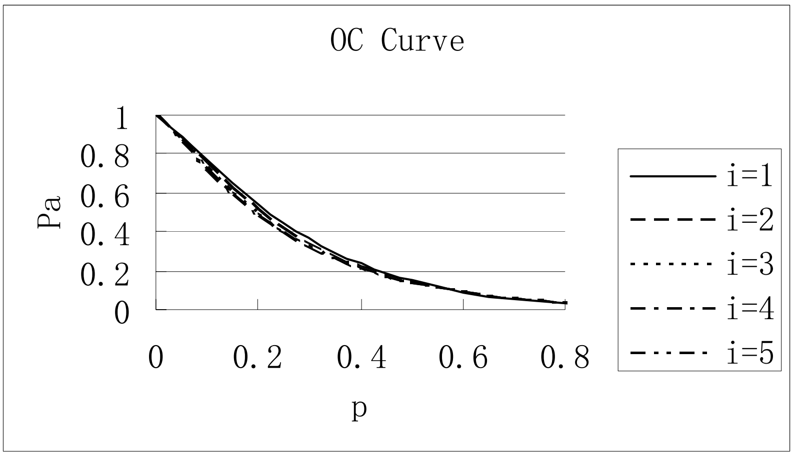

Figure 3. 9 OC curve comparison of lots no. (*n*=10, δ=0.4)

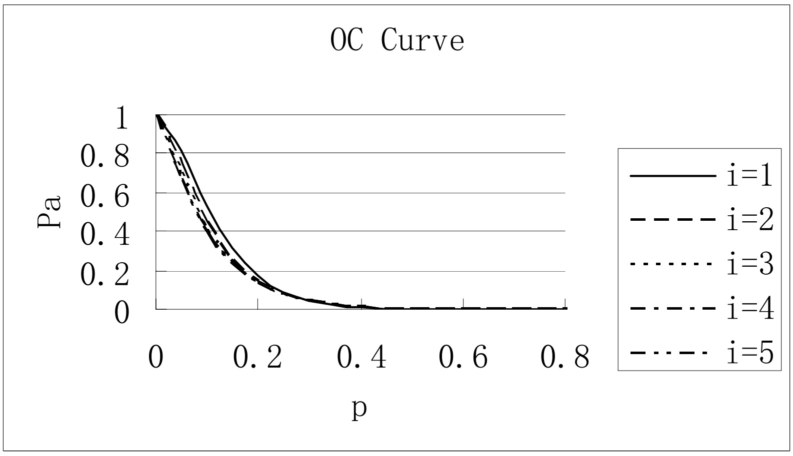

Figure 3. 10 OC curve comparison of lots no. (*n*=10, δ=1.0)

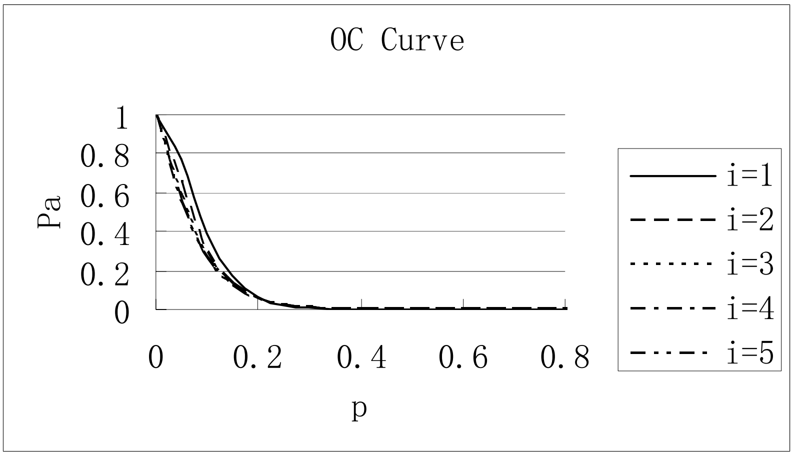

Figure 3. 11 OC curve comparison of lots no. (*n*=10, δ=1.4)

Their corresponding *AOQ* curves are illustrated in Figure 3.12, 3.13, and 3.14 respectively, in which δ is set to 0.4, 1, and 1.4 respectively. Results are similar to those of their *OC* curve counterparts. For a given " p ", *AOQL* decreases with the increase of the number of previous lots. In other words, the discriminating power increases when the number of previous lots increases and vice versa.

Another important finding revealed by this study is that both *OC* curve and *AOQ* curve fluctuate sharply to the change of the number of previous lots when it is small. However, such fluctuation becomes much more moderate when this number becomes large. For example, when the number of previous lots i is greater than 3, changes in *OC* curve and *AOQ* curve turn to be minor. It is therefore recommended to select a number of previous lots of 3 to maintain a relatively robust performance while not increasing the inspection cost by including a large number of previous lots.

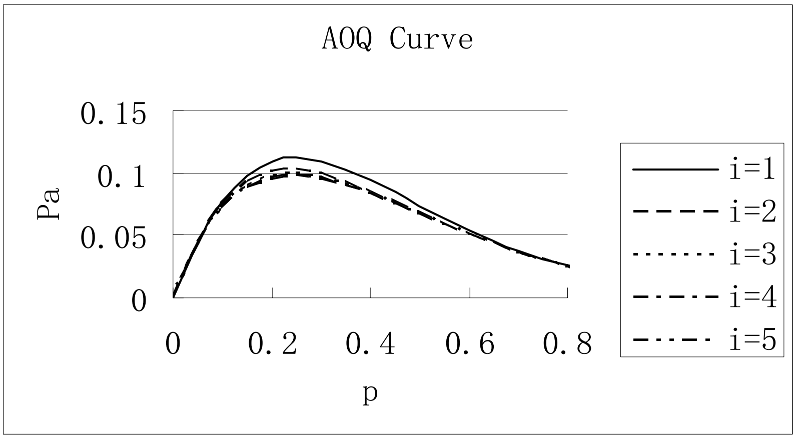

Figure 3. 12 AOQ curve comparison of lots no. (*n*=10, δ=0.4)

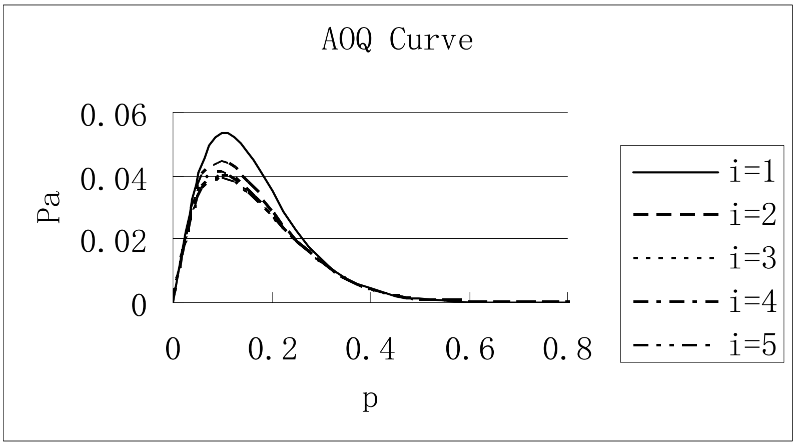

Figure 3. 13 AOQ curve comparison of lots no. (*n*=10, δ=1.0)

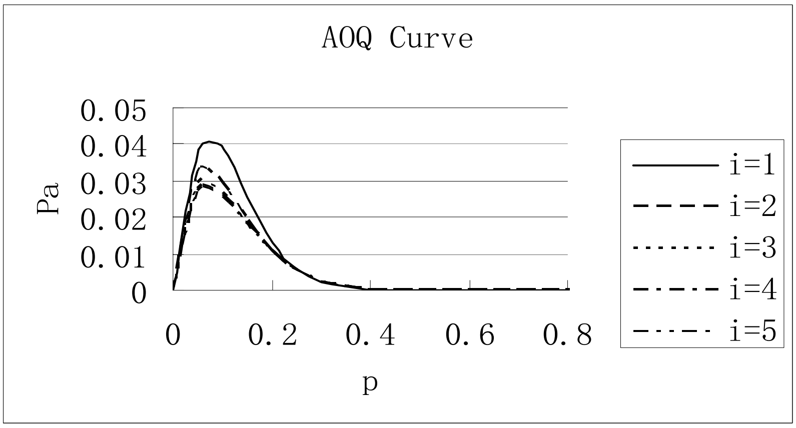

Figure 3. 14 AOQ comparison of lots no. (*n*=10, δ=1.4)

3.4 Conclusion

The study presented in this chapter is mainly motivated by the intention to model the correlation between testing units in chain sampling plans since the former Dodge ChSP-1 did not take this into consideration. Numerical results reveal that for a given " p ", the probability of acceptance is smaller when a negative correlation is taken into consideration. In other words, when the correlation is negative, the new extension, Chain Sampling Plan with Markov Property (ChSP-MP), is more discriminating than the Dodge ChSP-1 and the discriminating power increases as the correlation parameter δ increases. The reverse is true when the correlation coefficient is positive. The corresponding probability of acceptance is larger for a given " p " and thus the discrimination power is less than that of the Dodge plan.

The implication in practice is that when the Dodge ChSP-1 plan is applied to samples with positive correlation, the resulting probability of acceptance is smaller than what it is supposed to be and will lead to a more conservative decision. On the other hand, when there is a negative correlation, the Dodge ChSP-1 plan must be used with caution as its probability of acceptance and average outgoing quality are larger than actual values given in this plan.

Numerical results show that it is advisable to use three previous lots as a choice of an important design parameter. The reason is simple. As the OC curve and AOQ curve indicate that any lots number less than three will compromise robustness and any lots number more than three will incur additional cost.

4. Chain Sampling Scheme under Inspection Errors (I: For Constant Errors)

4.1 Introduction

Acceptance sampling by attributes is a fundamental tool in statistical quality control. It deals with procedures by which an accept/reject decision to a production lot is made based on results of inspection of samples. Sampling schemes, rather than 100% inspection of a production lot, are widely employed in industries to achieve a more economical and efficient use of company resources. Also, sampling schemes are applied to cases where it is impossible to carry out destructive inspection procedure on an entire production lot.

Embedded within the design of acceptance sampling plans for attributes is an implicit assumption that inspection procedures are completely flawless. In reality, however, inspection tasks are seldom error free. On the contrary, they may even be error prone. A variety of channels may contribute to these error commitments. In manual inspection, errors may result from factors such as complexity and difficulty of the inspection task, inherent variation in the inspection procedure, subjective judgment required by human inspectors, mental fatigue and inaccuracy or problem of gages or measurement instruments used in inspection procedures. Automated inspection system has been introduced to reduce the inspection time as well as to eliminate errors incurred as a result of human fatigue. However, inspection errors may still present due to factors such as complexity and difficulty of the inspection task, resolution of the inspection sensor, equipment malfunctions and “bugs” in the computer program controlling the inspection procedure etc.

There are two types of errors present in inspection schemes, namely, type I and type II inspection errors, where type I inspection error refers to the situation in which a conforming item is incorrectly classified as nonconforming and type II error occurs when a nonconforming unit is erroneously classified as conforming.

While inspection errors incurred during acceptance sampling for attributes are often unintentional and in most cases neglected, they nevertheless can severely distort quality objectives of a system design. This has motivated the study of the effect of inspection errors on different sampling schemes. In a series of four papers devoted to the effect of inaccuracies of inspection sampling for attributes, using the hyper-geometric distribution, Johnson and Kotz (1985, 1986, 1988 & 1990) analyzed several types of inspection schemes namely, single stage acceptance sampling schemes (1985), double stage, link and partial link acceptance sampling schemes (1986), Dorfman screening procedures (1988) and modified Dorfman screening procedures (1990). While, in reality, all inspection procedures are governed by the hyper-geometric distribution (as sampling is done without replacement from a finite lot size), mathematical models derived by Johnson and Kotz are often complex and computationally intensive. As such, a number of quality control analysts (Maghsoodloo and Bush (1985) for instance) had employed the binomial distribution to evaluate error prone sampling procedures instead. Such approximation is satisfactory in situations where lot size is more than ten times of the sample size.

While the above work considered inspection error in other sampling plans, thus far, no literature is found to deal with the study of chain sampling plans with inspection errors. The motivation to study chain sampling plans was driven by a real industrial project, where appropriate sampling plans were required to test fire-retard doors and fire-retard cables. One impeding feature of these tests is that they are destructive and very costly, so it is favorable to take as few samples as possible. Chain sampling plan stands up to be the best choice for this scenario among various collections of sampling plans because of its unique strength in dealing with destructive and costly testing by tracking previous information.

The primary aim of this paper is to extend the inspection error consideration to chain sampling schemes so as to better understand its implications and to quantify the related sampling risk. Expressions of performance measures such as operating characteristic function, average total inspection and average outgoing quality will be derived to aid the analysis of a general chain sampling scheme, ChSP-4A (c1, c2) r, developed by Frishman (1960). Effects of all sampling parameters on the performance of inspection schemes will be investigated to serve as a foundation for future plan designing purpose. A detailed model description will be introduced in Section two and followed the analysis and discussion in Section three. Section four gives conclusions.

4.2 Mathematical Model

4.2.1 Single sampling plan with inspection errors

Single stage sampling plans form the theoretical framework of chain sampling plans as chain-sampling plans rely on the result of single stage sampling plans to make acceptance/rejection decisions. Therefore, in order to develop a mathematical expression of chain sampling plans, it is essential to first develop a mathematical model for single stage sampling plans in the presence of inspection errors. Johnson et al (1985) derived the mathematical expression for single sampling based on hyper-geometric distribution. The probability distribution is given by:

$$\Pr[Z = z \mid n; D, N; \rho, \rho'] = \sum_y \frac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}} \bullet \sum_w \binom{y}{w}\binom{n-y}{z-w} \rho^w (1-\rho)^{y-w} p'^{z-w} (1-\rho')^{n-y-z+w}$$

(4-1)

where the author used ρ' and $1-\rho$ to stand for the type I and type II inspection error respectively; other notations are agreeable with ours in the nomenclature table.

To facilitate the subsequent derivation, an outline of the derivation is given as follows. Let T (rue) and A (pparent) represent the true state and the observed state of inspected items respectively. Define:

$T = 0$ when the inspected item is truly conforming,

$T = 1$ when the inspected item is truly nonconforming; and

$A = 0$ when the inspected item is observed (or classified) as conforming,

$A = 1$ when the inspected item is observed (or classified) as nonconforming.

The combination of the relationship between these two variables is illustrated in Table 4.1 where letter e_1 stands for type I inspection error and e_2 stands for type II inspection error.

Table 4. 1Types of inspection errors

A \ T	$T = 0$	$T = 1$
$A = 0$	No inspection error	Type II error (e_2)
$A = 1$	Type I error (e_1)	No inspection error

So:

$$e_1 = \Pr(A = 1 \mid T = 0) = \Pr(\frac{A = 1 \,\&\, T = 0}{T = 0}) \qquad (4\text{-}2)$$

$$e_2 = \Pr(A = 0 \mid T = 1) = \Pr(\frac{A = 0 \,\&\, T = 1}{T = 1}) \qquad (4\text{-}3)$$

The number of nonconforming items in a sample follows a hyper-geometric distribution with parameters $(n; D, N)$ i.e.

$$Y \sim Hypg(n; D, N) \qquad (4\text{-}4)$$

and its probability is given by:

$$\Pr(Y = y \mid n; D, N) = \binom{D}{y}\binom{N-D}{n-y} \Big/ \binom{N}{n} \qquad (4\text{-}5)$$

where,

$$\max(0, n - N + D) \le y \le \min(n, D)$$

Given Y, Z is the sum of two mutually independent variables, W and W', with each following binomial distribution:

$$\begin{aligned} W \mid Y &\sim Bin(Y, (1-e_2)) \\ W' \mid Y &\sim Bin(n-Y, e_1) \end{aligned} \qquad (4\text{-}6)$$

and

$$Z = W + W' \qquad (4\text{-}7)$$

Then, $Z \mid Y \sim Bin(Y, (1-e_2)) * Bin(n-Y, e_1)$, where $*$ stands for convolution.

The probability distribution of the number of observed nonconformings in a sample is given by

$$\Pr(Z = z \mid Y; n; D, N) = \sum_w \left(\binom{Y}{w} (1-e_2)^w e_2^{Y-w} \binom{n-Y}{z-w} e_1^{z-w} (1-e_1)^{n-Y-z+w} \right)$$

(4-8)

Where w takes a value from $\max(0, z-n+Y)$ to $\min(Y, z)$ inclusive.

Finally, the overall distribution of Z is given by:

$$\Pr[Z = z \mid n; D, N, e_1, e_2] = \sum_y \frac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}} \left(\sum_w \left(\binom{Y}{w} (1-e_2)^w e_2^{Y-w} \binom{n-Y}{z-w} e_1^{z-w} (1-e_1)^{n-Y-z+w} \right) \right)$$

(4-9)

Formula (4-9) is essentially identical to the initial equation (4-1)

For a single stage sampling plan the decision rule is "If the number of apparent defective items in a sample size n exceeds c, the acceptance number, reject the lot; otherwise accept it"; or mathematically expressed as:

"Reject if $Z > c$; accept if $Z \le c$"

Therefore the probability of acceptance for a single stage acceptance sampling is given by:

$$P_s = \sum_{z=0}^{c} \left(\Pr\left(Z = z \mid n, D, N, e_1, e_2 \right) \right) \tag{4-10}$$

$$P_s\left(c, n, D, N, e_1, e_2\right) = \sum_{z=0}^{c} \left(\sum_{y} \frac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}} \left(\sum_{w} \left(\binom{Y}{w}\binom{n-Y}{z-w} e_1^{z-w} e_2^{Y-w} \left(1-e_1\right)^{n-Y-z+w} \left(1-e_2\right)^{w} \right) \right) \right) \tag{4-11}$$

4.2.2 Mathematical Model for Chain Sampling Plans, ChSP (c_1, c_2) r

Unlike double, multiple and sequential sampling plans, where the probability of acceptance for a production lot of high quality is enhanced by taking extra sample(s) from the same production lot, chain sampling inspection schemes do not require additional samples from a lot to increase the chance of acceptance. In fact, in chain sampling, each production lot undergoes a simple single stage acceptance sampling and the verification of quality of any production lot in doubt hinges on the cumulative result of the immediate $(k-1)$ preceding lots. The operating procedure and probability tree for chain sampling is given in Figure 4.1.

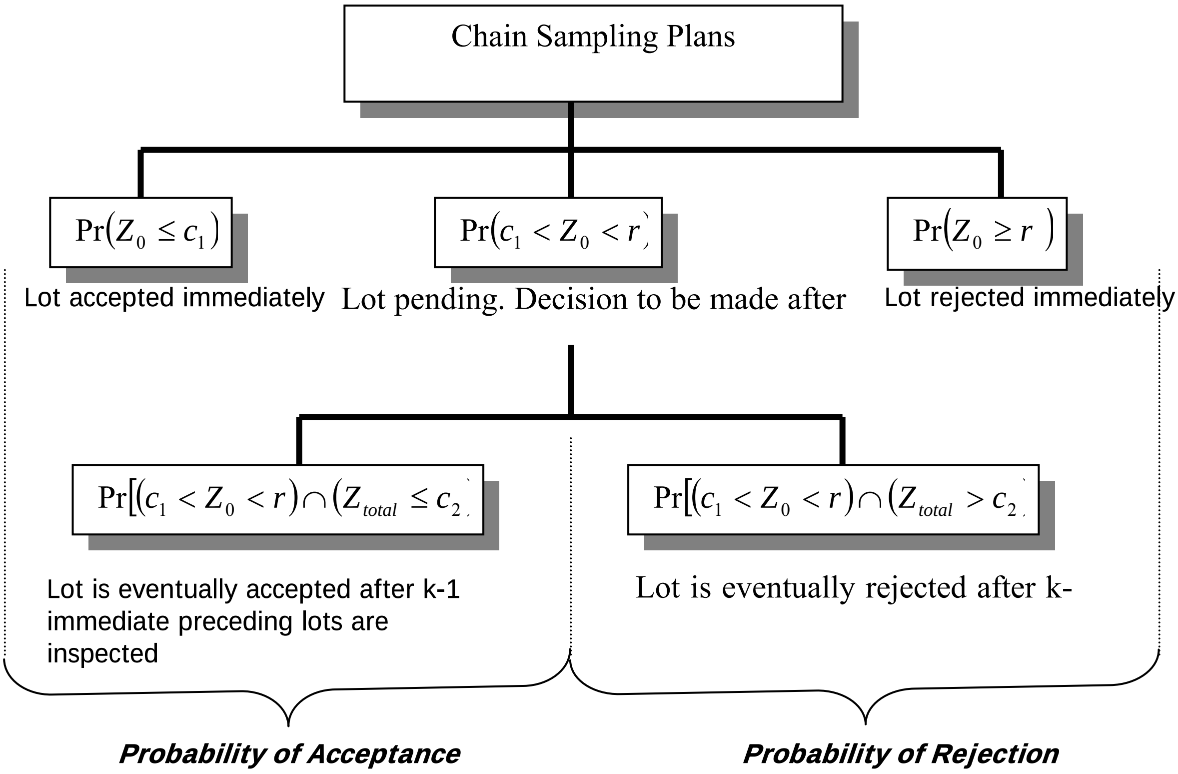

Figure 4. 1Probability tree for chain sampling plans

Here assumptions are:

1. The process should be in a state of statistical control and all lots follow i.i.d.
2. No switching rules will be adopted
3. Inspection errors will remain constant throughout inspection activities. (Variable inspection errors will be addressed in chapter 5)

Based on above mentioned assumptions and procedures, the general expression of the probability of acceptance for chain sampling plans in the presence of constant inspection errors, P_{ch} is given by:

$$P_{ch} = \Pr(Z_o \le c_1 \mid n; D, N, e_1, e_2) + \Pr(Z_{total} \le c_2 \mid c_1 < Z_0 < r; k, n, D, N, e_1, e_2) \quad (4\text{-}12)$$

$$
\begin{aligned}
P_{ch} &= \Pr\left(Z_o \le c_1 \mid n; D, N, e_1, e_2\right) + \Pr\left(Z_{total} \le c_2 \mid c_1 < Z_0 < r; k, n, D, N, e_1, e_2\right) \\
&= P_s\left(c_1, n, D, N, e_1, e_2\right) \\
&\quad + \sum_{z_0=c_1+1}^{r-1} \left(\left(\Pr\left(Z = z_0 \mid n; D, N, e_1, e_2\right)\right) \bullet \sum_{Z_{total}=z_0}^{c_2} \left(\Pr\left(Z = z_{total} \mid kn; kD, kN, e_1, e_2\right)\right) \right) \\
&= \sum_{z=0}^{c_1} \left(\sum_y \frac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}} \left(\sum_w \left(\binom{Y}{w}\binom{n-Y}{z-w} e_1^{\,z-w} e_2^{Y-w} \left(1-e_1\right)^{n-Y-z+w} \left(1-e_2\right)^{w} \right)\right)\right) \\
&+ \sum_{z_0=c_1+1}^{r-1} \left(\begin{array}{l}
\displaystyle \sum_y \frac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}} \left(\sum_w \left(\binom{Y}{w}\binom{n-Y}{z-w} e_1^{\,z-w} e_2^{Y-w} \left(1-e_1\right)^{n-Y-z+w} \left(1-e_2\right)^{w} \right)\right) \\
\displaystyle \bullet \sum_{z_{pre}=0}^{c_2-z_0} \left(\begin{array}{l}
\displaystyle \sum_i \frac{\binom{(k-1)D}{i}\binom{(k-1)(N-D)}{(k-1)n-i}}{\binom{(k-1)N}{(k-1)n}} \\
\displaystyle \bullet \left(\sum_{w_2} \binom{i}{w_2}\binom{(k-1)n-i}{z_{pre}-w_2} \left(1-e_2\right)^{w_2} e_2^{\,i-w_2} e_1^{\,z_{pre}-w_2} \left(1-e_1\right)^{(k-1)n-i-z_{pre}+w_2} \right)
\end{array} \right)
\end{array} \right)
\end{aligned}
\tag{4-13}
$$

where:

$$\max\left(0, n - N + D\right) \le y \le \min\left(n, D\right)$$

$$\max\left(0, z_0 - n + y\right) \le w \le \min\left(z_0, y\right)$$

$$\max\left(0, (k-1)(n - N + D)\right) \le i \le \min\left((k-1)n, (k-1)D\right)$$

$$\max\left(0, z_{pre} - (k-1)n + i\right) \le w_2 \le \min\left(z_{pre}, i\right)$$

$$c_2 - c_1 \le r$$

$$z_{pre} = z_{total} - z_0$$

Calculating acceptance probabilities for different values of true fraction nonconforming $\frac{\mathbf{D}}{\mathbf{N}}$ using Equation (4-13) will yield the operating characteristic (OC) curve of an inspection scheme. This curve displays the discriminating power of the inspection scheme. More precisely, it shows the probability that the lot submitted with a certain fraction of nonconforming items will be accepted.

4.2.3 Average Outgoing Quality

Average Outgoing Quality (AOQ) defines the quality of a production lot that leaves the inspection and can be mathematically expressed as the ratio of total number of outgoing nonconforming items to the total number of outgoing items. This ratio highly depends on different samples and lot disposition policies. In this paper, all apparent nonconforming items in a sample will be replaced and any rejected lot will undergo 100% screening with all apparent nonconforming items replaced.

In order to construct a formula for AOQ, two important expressions, namely the apparent fraction nonconforming and the conditional probability that an apparent (observed) conforming item is actually a nonconforming item, must be established.

Apparent Fraction Nonconforming

Let **p** be the true fraction nonconforming, and **π** be the apparent (observed) fraction nonconforming. The relationship between two variables is given by:

$$\pi = (1-p)e_1 + p(1-e_2) = p - p(e_1 + e_2) + e_1 \qquad (4\text{-}14)$$

Conditional Probability

To compute the conditional probability that an apparent (observed) conforming item is actually a nonconforming item, define following events:

Accept be the event that an item is classified as conforming,

Reject be the event that an item is classified as nonconforming,

Good be the event that an item selected is conforming, and

Bad be the event that an item selected is nonconforming.

Then:

$$\Pr(Bad) = p, \qquad \Pr(Good) = 1 - p, \qquad\qquad (4\text{-}15)$$
$$\Pr(\mathrm{Re}\,ject) = \pi, \qquad \Pr(Accept) = 1 - \pi$$

and

$$\Pr(\mathrm{Re}\,ject \mid Bad) = 1 - e_2, \qquad \Pr(Accept \mid Bab) = e_2 \qquad\qquad (4\text{-}16)$$
$$\Pr(\mathrm{Re}\,ject \mid Good) = e_1, \qquad \Pr(Accept \mid Good) = 1 - e_1$$

With a simple manipulation, obtain the following probability:

$$\Pr(Bad \mid Accept) = \frac{\Pr(Bad \ \&\ Accept)}{\Pr(Accept)} = \frac{\Pr(Accept \mid Bad)\Pr(Bad)}{\Pr(Accept)}$$
$$= \frac{(1 - e_2)p}{1 - \pi} = \frac{pe_2}{1 - p + p(e_1 + e_2) - e_1}$$

(4-17)

The expected total nonconforming items sending out came from five different sources for this particular sample / rest of lot disposition policy, namely:

1. Number of nonconforming items in an unscreened portion of an accepted production lot, which is given by:

   $$(N - n)p$$

   and the probability of such occurrence is P_{ch}.

2. Number of nonconforming items in a screened portion that are misclassified as conforming, which is given by:

   $$(N - n)pe_2$$

   and the probability of such occurrence is $1 - P_{ch}$.

3. Number of nonconforming items misclassified as conforming in a sample, which is given by:

   $$npe_2$$

   and the probability of such occurrence is **1**.

4. Number of nonconforming items that are used to replenish the rejected fraction of a screened portion, is given by:

$$(N-n)(p-p(e_1+e_2)+e_1)\frac{pe_2}{1-p+p(e_1+e_2)-e_1}$$

and the probability of such occurrence is $1-P_{ch}$.

5. Number of nonconforming items that are used to replenish rejected fraction of a sample, which is given by:

$$n(p-p(e_1+e_2)+e_1)\frac{pe_2}{1-p+p(e_1+e_2)-e_1}$$

and the probability of such occurrence is **1**.

Therefore, AOQ is defined as:

$$AOQ=\frac{(N-n)pP_{ch}+(N-n)pe_2(1-P_{ch})+npe_2}{N}$$

$$+\frac{\left((N-n)(p-p(e_1+e_2)+e_1)\frac{pe_2}{1-p+p(e_1+e_2)-e_1}\right)(1-P_{ch})}{N}$$

$$+\frac{n(p-p(e_1+e_2)+e_1)\frac{pe_2}{1-p+p(e_1+e_2)-e_1}}{N}$$

$$=\frac{De_2}{(1-e_1)(N-D)+De_2}+\frac{(N-n)}{N}\frac{D}{N}\left(\frac{(N-D)(1-e_1-e_2)}{(1-e_1)(N-D)+De_2}\right)P_{ch}$$

(4-18)

The average outgoing quality of chain sampling plans can be obtained by using equation (4-13) to substitute the P_{ch} in equation (4-18).

4.2.4 Average Total Inspection

ATI (Average Total Inspection) is essentially the expected amount of items inspected per lot in a long run. Like AOQ, the computation of average total inspection depends on the sample /lot disposition policy. Adhering to the same policy as described in the previous section (all apparent nonconforming items in a sample will be replaced and any rejected lot will undergo 100% screening with all apparent nonconforming items replaced), the average total inspection under inspection errors is given by:

$$ATI = P_{ch}\left(n + \frac{n\pi}{1-\pi}\right) + \left(1 - P_{ch}\right)\left(N + \frac{N\pi}{1-\pi}\right) \\ = \frac{N^2 - N(N-n)P_{ch}}{(1-e_1)(N-D) + De_2} \quad (4\text{-}19)$$

4.3 Analysis and Discussion

A series of Microsoft Excel Visual Basic Application routines have been developed to compute the complex cumulative hyper-geometric equation of the probability of acceptance for chain sampling plans as well as its OC curve, AOQ curve and ATI curve, which are three major measurements of any sampling schemes. Brief description of the programs is included in this section and they are available from the author upon request.

4.3.1 Effects of Inspection Errors

There are two types of inspection errors, namely type I and type II errors. The probability of incurring type I error is defined as e_1 and the probability of the occurrence of type II error is defined as e_2, where both e_1 and e_2 range from 0 to 1. Note that values of e_1 and e_2 used in Figure 4.2 do not cover the entire range from 0 to 1 for both e_1 and e_2. Figure 4.2 is a 3D plot of the ChSP (1, 3) 4 (Lot size = 1000, Number of defectives =10, Sample Size = 5, and $(k-1)$ =2). Rather, we select the respective range corresponding to situations most likely to be encountered in current high yield industry to highlight its typical behavior. Nevertheless, the entire range for each e_1 and e_2 is plotted in Figure 4.2a and Figure 4.2b respectively to obtain a more conclusive result in the following analysis.

For all chain sampling plans, the result for any production lot of a small fraction of nonconforming items displays two prominent trends as illustrated in Figures 4.2, 4.2a, and 4.2b: a) as type I inspection error, e_1 increases, the probability of acceptance decreases; and b) as type II inspection error, e_2 increases, the probability of acceptance increases. The increase, however, is almost negligible as compared to that of the change in probability of acceptance when e_1 decreases while lot size is large and there is only a small fraction of defectives.

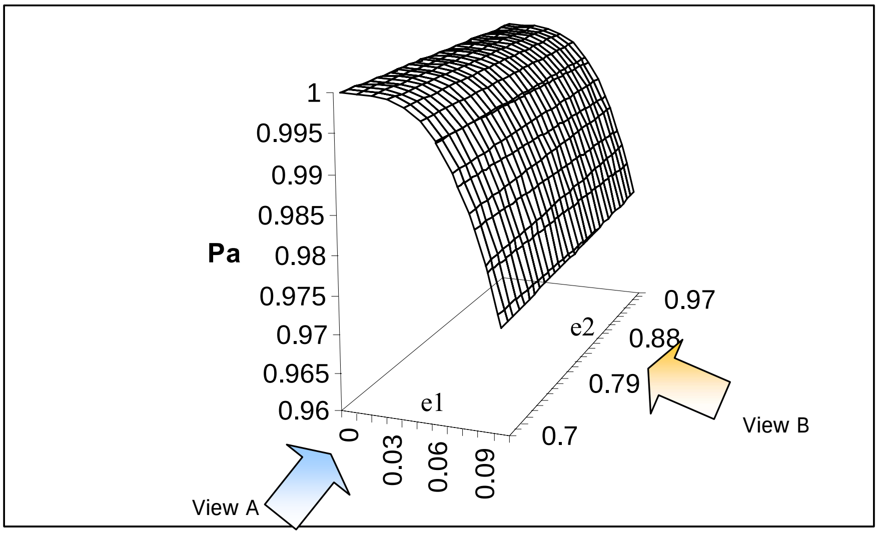

Figure 4. 2 3D plot of effects of inspection errors

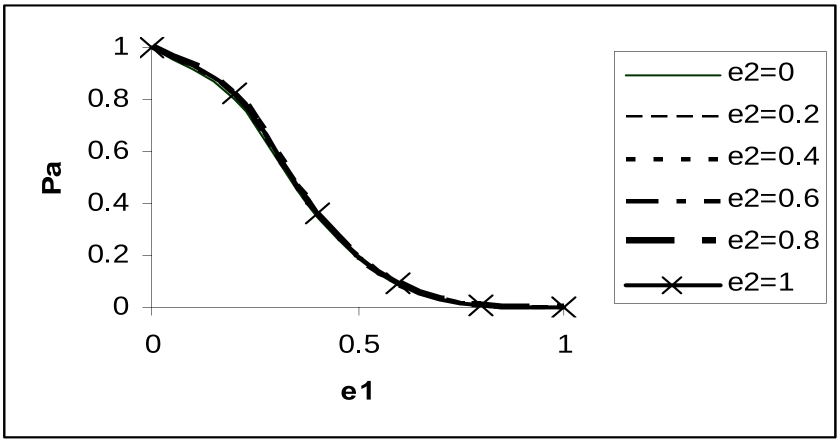

Figure 4.2a View A

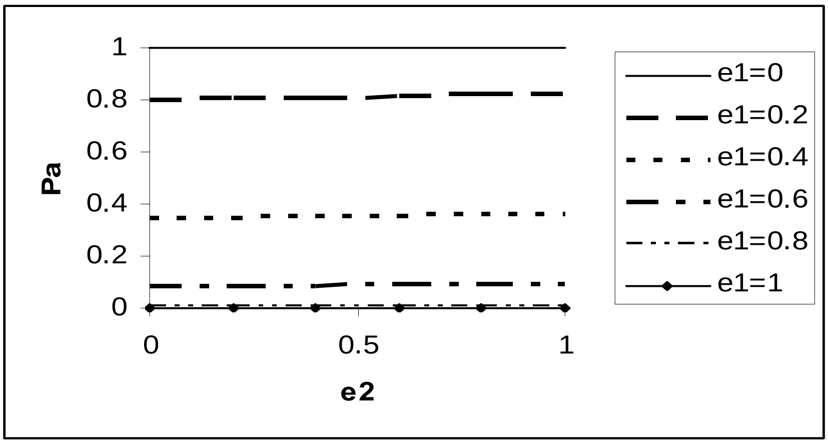

Figure 4.2b View B

The observation is consistent with results obtained in all literatures pertaining to inspection error models. The reason for the occurrence of such phenomenon is that given that there is only a small fraction of nonconforming items, while the probability of misclassifying them (type II error) may be high, the expected number of misclassified nonconforming items in a sample remains small; and hence, the increase in probability of acceptance incurred by type II error is almost negligible.

On the other hand, a good production lot with a small fraction of nonconforming items implies a higher probability of selecting a large fraction of conforming items for sampling. Therefore, a slight increase in the probability of committing a type I error will significantly increase the expected number of nonconforming items through misclassification of a large pool of conforming items. Thus, it can be concluded that the probability of acceptance is insensitive to e_2 as small as 0.05 but is sensitive to e_1 as small as 0.01 when the lot size is very large with few nonconforming items presence.

A Visual Basic Application program in Excel is designed to compute the acceptance probability and to draw the OC curve automatically. Readers can request the program and reproduce results by following the interface provided in this chapter. The program input interface for Figure 4.2 is shown in Figure 4.3.

Chain Sampling Under Inspection Error

1. Effect of Inspection Error

Sampling Parameters	
1000	N Lot Size
10	D Lot defective number
5	n Sample Size
1	c1,acceptance number for first stage
3	c2,acceptance number for cumulative lots
4	r, rejection number for single lot
3	k, number of total lots
Error range	
0	Mininum type I inpsection error
0.1	Maximium type I inspection error
0.01	increment of type I inspection error
0.7	Mininum type II inpsection error
1	Maximium type II inspection error
0.01	increment of type II inspection error

Calculate Probability of Acceptance for ChSP under Inspection Error

Figure 4. 3 Screen snapshot of the program input interface

Its input screen snapshot for Figure 4.2a and 4.2b is shown in Figure 4.4 below.

Sampling Parameters	
1000	N Lot Size
10	D Lot defective number
5	n Sample Size
1	c1,acceptance number for first stage
3	c2,acceptance number for cumulative lots
4	r, rejection number for single lot
3	k, number of total lots
Error range	
0	Mininum type I inpsection error
1	Maximium type I inspection error
0.1	increment of type I inspection error
0	Mininum type II inpsection error
1	Maximium type II inspection error
0.1	increment of type II inspection error

Calculate Probability of Acceptance for ChSP under Inspection Error

Figure 4. 4 Screen snapshot of inspection error rang

4.3.2 Effect on OC Curve

Next, the OC curve under the effect of inspection errors will be presented. Assume that only type I inspection error is present (that is, $e_2 = 0$). As e_1 increases, the probability of acceptance decreases more significantly as compared to that of a faultless inspection. The decrease is more prominent in the region where the true fraction of nonconforming is small as shown in Figure 4.5 below.

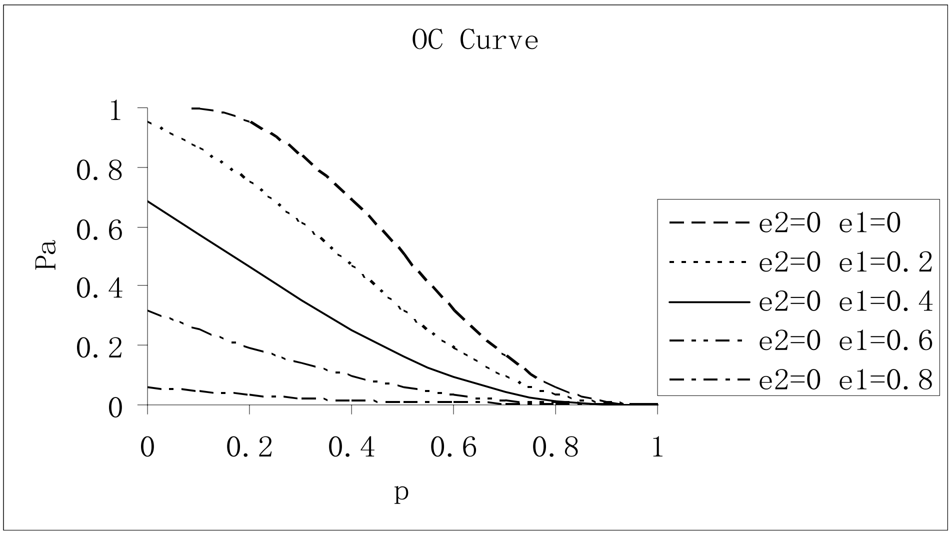

Figure 4. 5 OC curves for ChSP (2, 5) 5, n =5, (k-1) =5 with type I inspection errors

When only type II inspection error is present (i.e. $e_1 = 0$), the acceptance probability increases as type II inspection error (e_2) increases. The increase in the acceptance probability is relatively greater in the region of large true fraction of nonconforming items. (Figure 4.6)

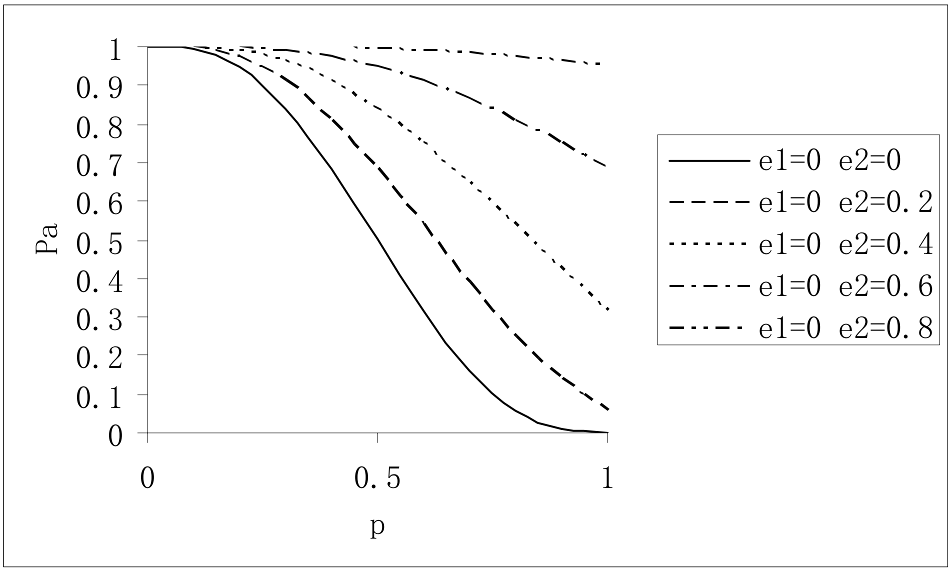

Figure 4. 6 OC curves for ChSP (2, 5) 5, n =5, (k-1) =5 with type II inspection errors

Very often, inspection tasks are subjected to two types of inspection errors simultaneously. Except for rare cases where inspection errors cancel out each other, operating characteristics curve will be distorted by the presence of inspection errors. Figure 4.7 below shows effects of different combinations of type I and type II inspection errors on the behavior of OC Curve.

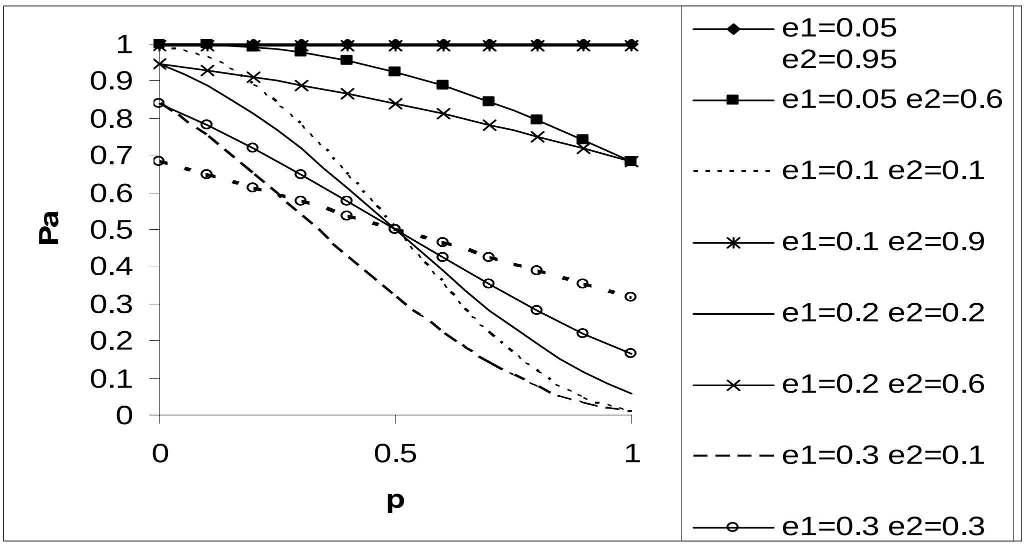

Figure 4. 7 OC curve for combined inspection errors (ChSP (2, 5) 5, n =5, (k-1) =5)

A screen snapshot of the program, which is used to conduct the analysis of the effect of inspection error on OC curves, is shown in Figure 4.8. Users can follow the figure to key (in) the parameter input and reproduce the result.

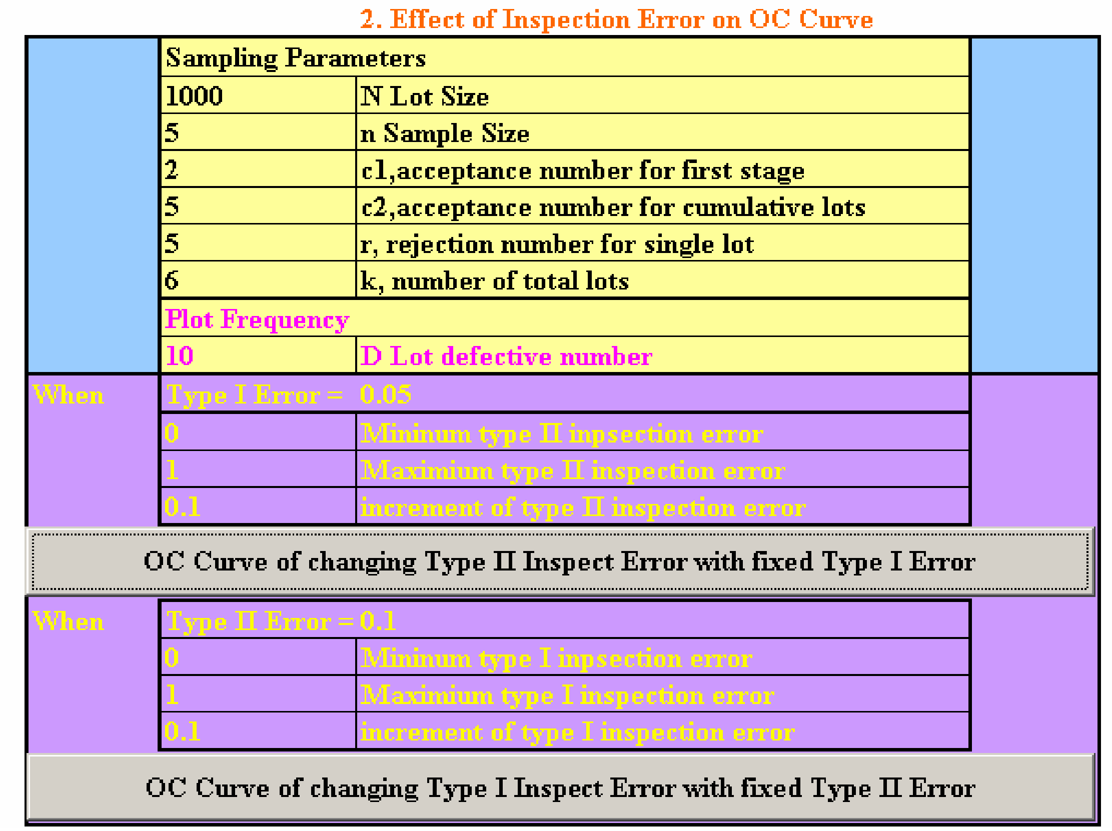

Figure 4. 8 Program input of the OC curve analysis

The effect of inspection errors on the OC curve is actually the exhibition of the effect of inspection errors on the observed number of nonconforming items. When there are no inspection errors, the observed number of nonconforming items is the same as the true number of nonconforming items from a lot. Whenever the inspection error is present, the observed number of nonconforming items is a “false” representative of the true number of nonconforming with a certain degree of distortion. Its relationship can be mathematically expressed as:

$$z = ye_2 + (n - y)e_1 \qquad (4\text{-}20)$$

The observed fraction of nonconforming π, for a sample is therefore given by:

$$\pi = \frac{z}{n} = \frac{ye_2 + (n-y)e_1}{n} = \frac{y}{n}e_2 + \left(1 - \frac{y}{n}\right)e_1 \qquad (4\text{-}21)$$

Suppose sampling is done randomly and the sample is a true representative of the lot quality, so:

$$\frac{y}{n} = \frac{D}{N} = p \qquad (4\text{-}22)$$

Combining equation (4-21) and (4-22), we obtain:

$$\pi = (1 - e_1 - e_2)p + e_1 \qquad (4\text{-}23)$$

Assume that two types of inspection errors, e_1 and e_2, remain constant throughout the inspection. It will, without exception, observe π fraction of nonconforming items from a lot size of N with p fraction of nonconforming items. This implies that the probability of acceptance for a lot with p fraction of nonconforming items subjected to inspection errors of e_1 and e_2 respectively, is equivalent to the probability of acceptance of the same lot with π fraction of nonconforming items undergoing perfect inspections.

Hence, it implies that the OC curve for any inspection scheme subjected to inspection errors is essentially the OC curve for a similar inspection scheme that undergoes following two transformations:

- Stretch by a factor of $\frac{1}{e_2 - e_1}$ in the direction of **p**;
- The resulting curve is then shifted by a factor of $-e_1$ in the direction of **p**.

Therefore, the probability of acceptance for a single stage-sampling plan (4-9) can be rewritten as:

$$\Pr(Z = z) = \frac{\binom{D^*}{z}\binom{N - D^*}{n - z}}{\binom{N}{n}} \tag{4-24}$$

Where D^* is the equivalent number of nonconforming items from a perfect inspection to that of D nonconforming items when the inspection is subjected to type I and type II errors simultaneously, and D^* is given by:

$$D^* = D(1 - e_2) + (N - D)e_1 \tag{4-25}$$

Similarly, the complicated chain sampling acceptance probability formula (4-12) and (4-13) can be reduced to the following:

$$P_{a,chain} = \Pr(Z_o \le c_1 \mid n; D^*) + \Pr(Z_{total} \le c_2 \mid c_1 < Z_0 < r; k, n; D^*, N)$$

$$= \sum_{z_0=0}^{c_1} \frac{\binom{D^*}{z_0}\binom{N - D^*}{n - z_0}}{\binom{N}{n}} + \sum_{z_0=c_1+1}^{r-1} \left(\frac{\binom{D^*}{z_0}\binom{N - D^*}{n - z_0}}{\binom{N}{n}} \sum_{z_{pre}=0}^{c_2 - z_0} \frac{\binom{(k-1)D^*}{z_{pre}}\binom{(k-1)(N - D^*)}{(k-1)n - z_{pre}}}{\binom{(k-1)N}{(k-1)n}} \right)$$

(4-26)

Equation (4-26) can be used to approximate the complicated equation (4-13). Slight departure from the original OC curve may occur due to the rounding up error (as D^* must be an integer to compute the hyper-geometric probability function). Nevertheless, it serves as a more comprehensible model and simpler approximation to the true distribution. A sensitivity analysis of the rounding up error is illustrated in Figure 4.9, and it is clear that those three OC curves are almost identical; which means that the rounding up error is insignificant in most cases.

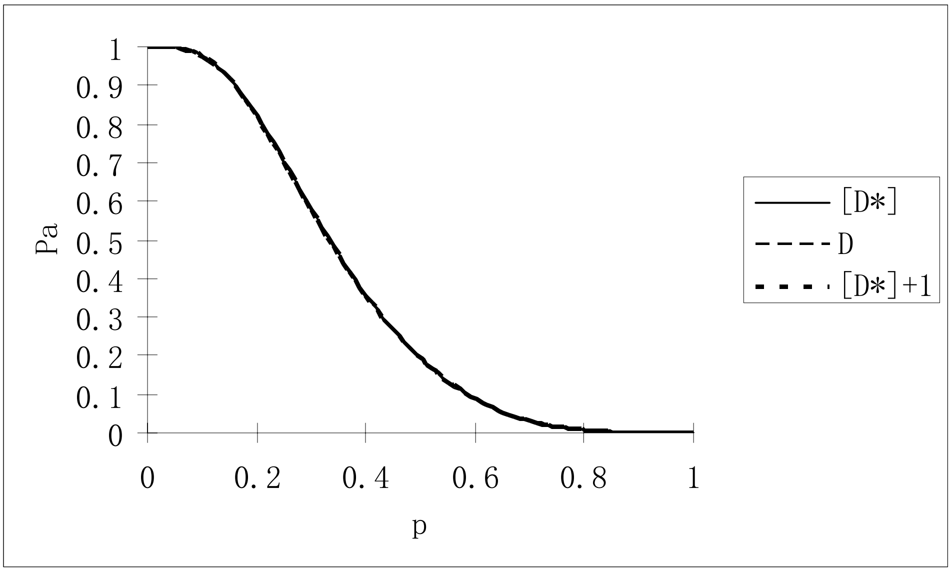

Figure 4. 9 Effect of roundup error

4.3.3 Effects on AOQ and ATI

Besides OC curves, ATI (Average Total Inspection) and AOQ (Average Outgoing Quality) are other two performance measures for assessing sampling schemes subjected to inspection errors. It is important to point out that while the OC curve of a sampling scheme subjected to inspection errors can be approximated by the OC curve of a prefect sampling scheme undergoing two transformations, the same approximation cannot be applied to ATI and AOQ as the computation for two performance measures are highly dependent on the type of error and the disposition policy. The study of the two performance measures is critical as both of them have direct impact on the economic aspect of sampling procedures.

An introduction of the program used for AOQ and ATI analysis will be given first. Since the input for AOQ curve and ATI curve are the same, they are incorporated into one input interface to make it more concise. To run the simulation, just key in required sampling parameters and press the left button for AOQ computation and the right button for ATI computation. The input interface for this program is illustrated in Figure 4.10 below.

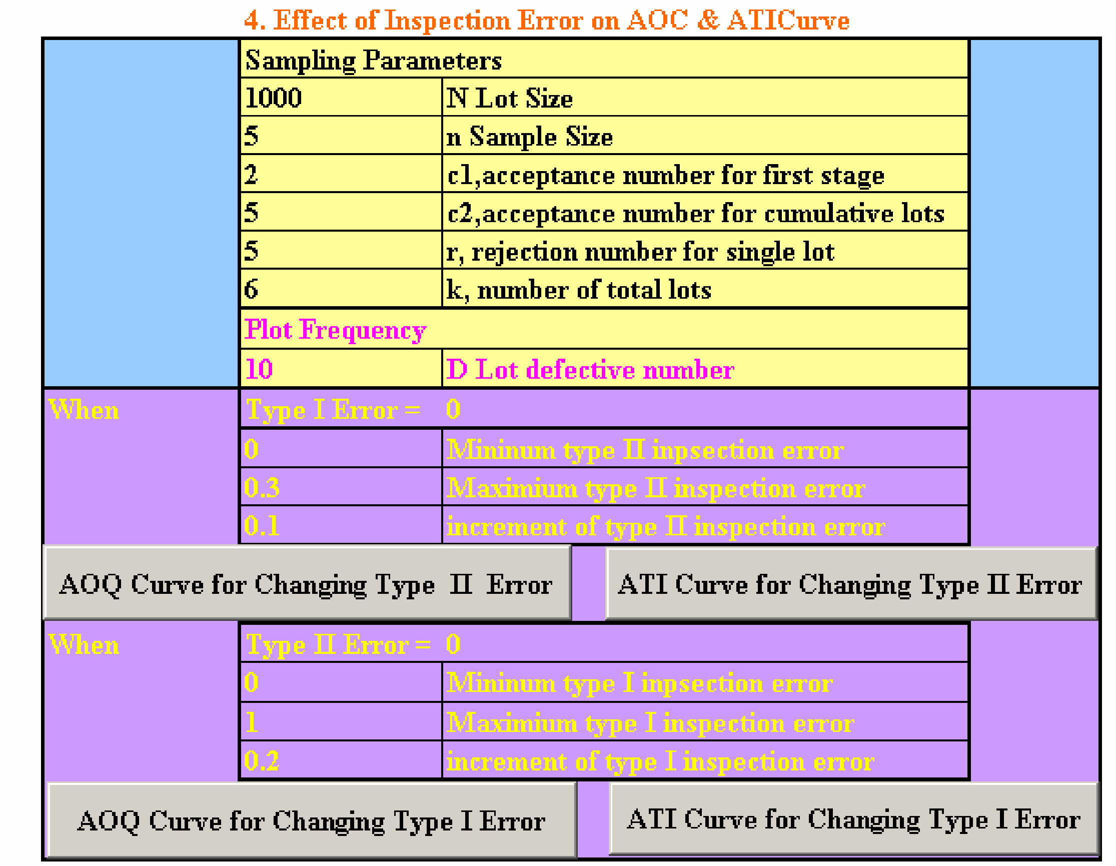

Figure 4. 10 Program input interface for AOQ and ATI analysis

The same sampling settings are used in order to provide a simpler illustration, i.e. the ChSP (2, 5) 5, $k = 6$ and $n = 5$, throughout this section for comparison.

In Figure 4.11, the type I inspection error is fixed at zero. It is obvious that as the type II inspection error increases, the AOQ will increase accordingly. This observation is intuitively clear as the larger the type II inspection error, the more nonconforming items escape from inspection. The average outgoing quality is therefore worsened. This trend holds for constant type I inspection errors. Figure 4.12 shows the AOQ curve changes when type I inspection error is equal to 0.2.

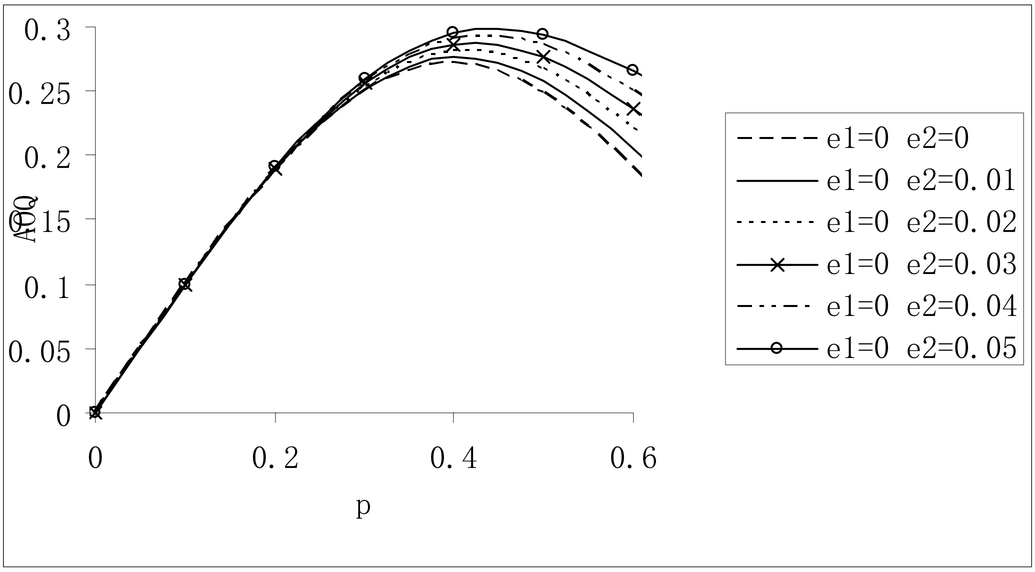

Figure 4. 11 AOQ curve of type II inspection error (e_1=0)

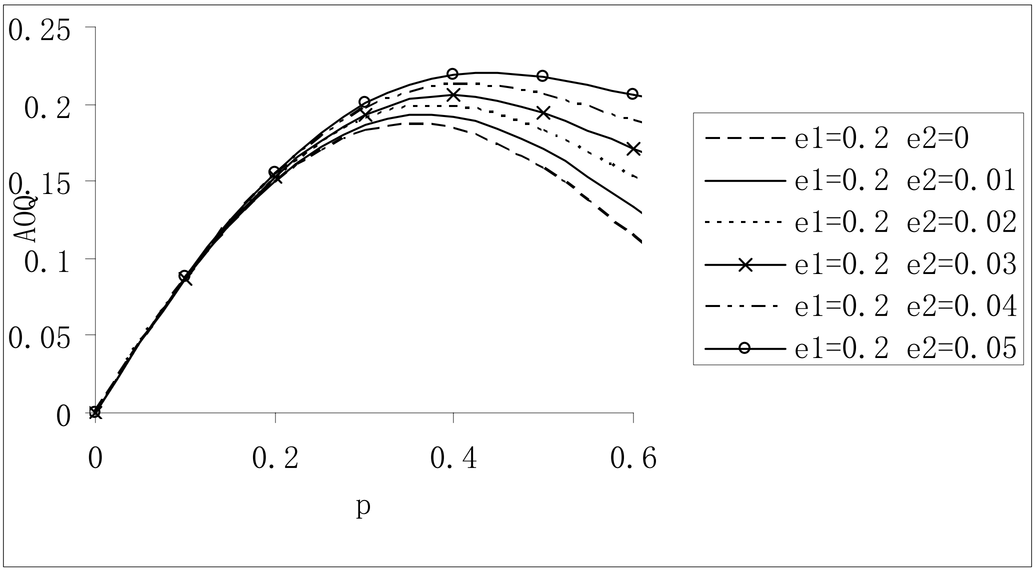

Figure 4. 12 AOQ curve of type II inspection error (e_1=0.2)

On the other hand, Figure 4.13 & 4.14 illustrate the effect of different type I inspection error with type II inspection error set to zero and 0.1 respectively. The trend is reverse of that of type II inspection error. When type I inspection error becomes larger, the corresponding average outgoing quality becomes smaller. This is because more conforming items are mis-classified as nonconforming, which means that the actual number of nonconforming items is fewer, and thus better AOQ.

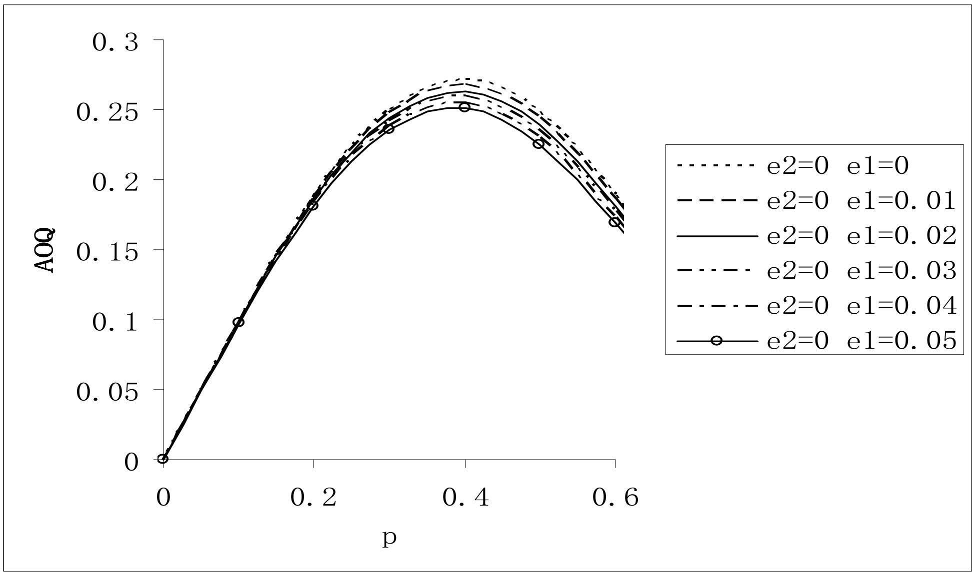

Figure 4. 13 AOQ curve of type I inspection error (e_2=0)

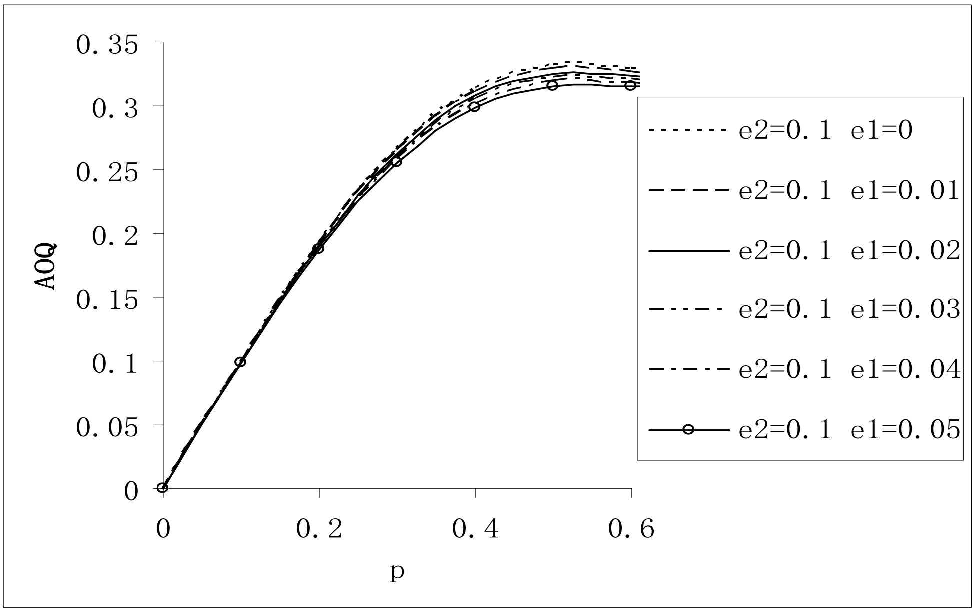

Figure 4. 14 AOQ curve of type II inspection error (e_2=0.1)

It should be noted that the effect of type II inspection error is more prominent on the AOQL (average outgoing quality limits). Figure 4.15 depicts the AQO curve for different type I inspection errors with type II inspection error equal to zero. Figure 4.16 is a counterpart of Figure 4.15 where the type II inspection error is set at 0.01. There is a strong indication that even a very small type II inspection error will lead the final AOQL to almost 1.

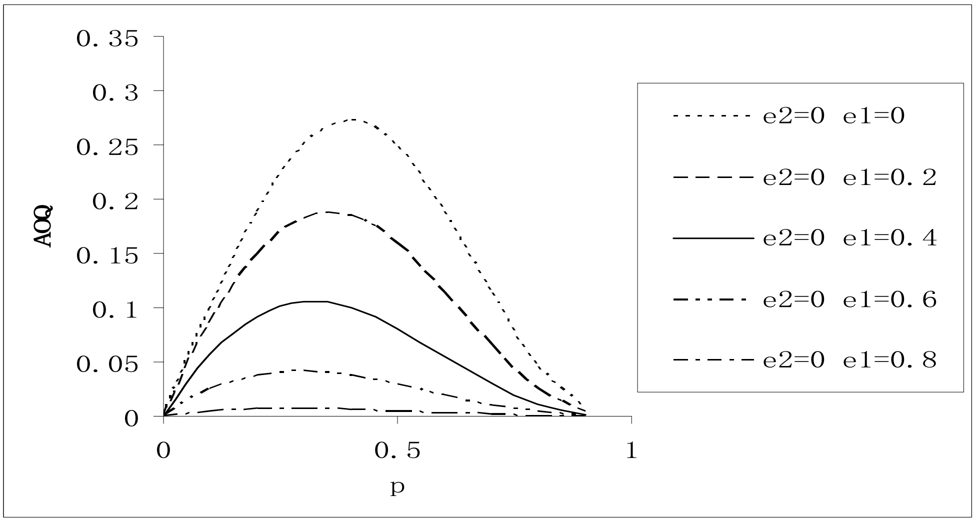

Figure 4. 15 AOQ curve of increased type I inspection error (e_2=0)

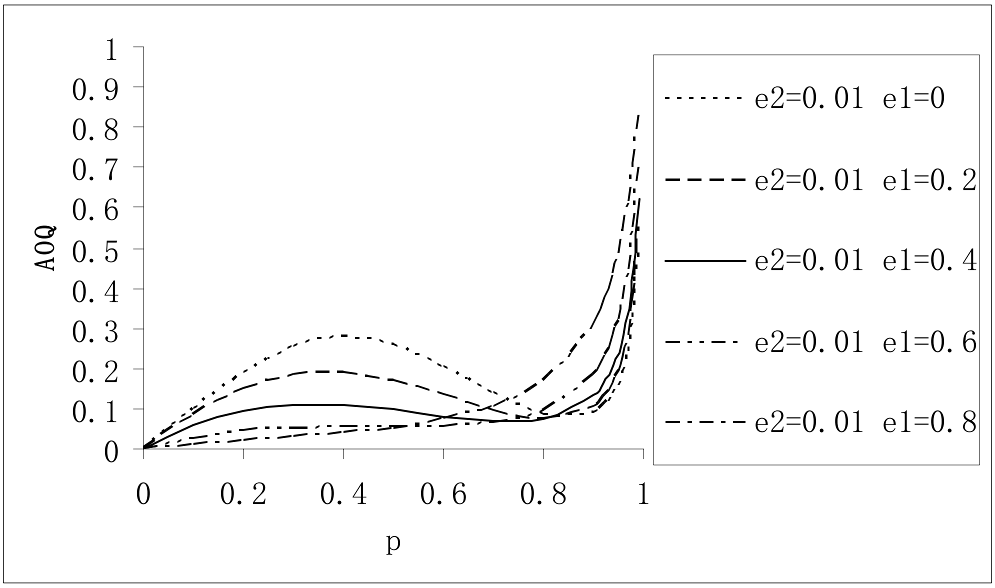

Figure 4. 16 AOQ curve of increased type I inspection error (e_2=0.01)

It is also necessary to point out that the ranges for two types of inspection errors should be within reasonable ranges, namely 0～0.01 for type I inspection error and 0～0.05

for type II inspection error as larger inspection errors will incapacitate the effectiveness of any sampling plan.

From the plot of ATI (Figure 4.17), it is shown that for values of type II inspection error less than one, the average total inspection is less than that of a perfect inspection. As type II inspection error increases, the ATI decreases. This is because as e_2 increases, the probability of nonconforming items that is correctly classified decreases, implying that the probability of nonconforming items classified as conforming increases. Suppose that e_1 is zero (that is to say, there is no misclassification of conforming items during the sampling process), the overall probability of having items classified as conforming increases. Hence, the probability of rejecting a lot decreases. Since the probability of rejection decreases, the chance of carrying out 100% screening of the rejected lot will decrease.

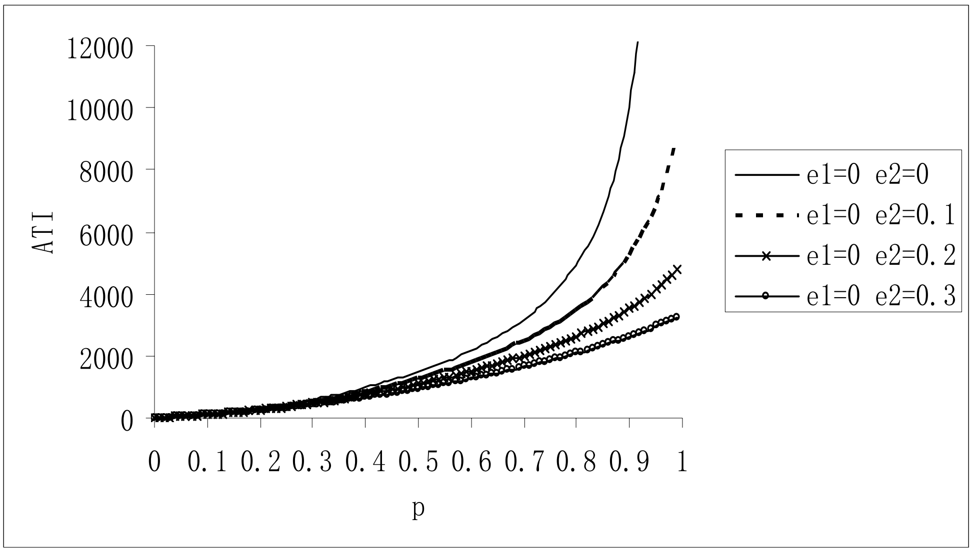

Figure 4. 17 ATI curve of increased type II inspection error (e_1=0)

Similarly, as depicted in Figure 4.18, when the probability of type I inspection error increases, ATI increases. This is because as e_1 increases, the probability of erroneously classifying conforming items as nonconforming increases, implying higher probability of rejecting a lot; and consequently, a higher probability of carrying out 100% screening on the lot.

Inference drawn on the ATI of chain sampling plans in this section is similar to those in other inspection procedures such as single and double stage sampling plans. It is important to note that while the presence of type II inspection error reduces total inspection efforts required, readers should not be misled into thinking that the type II inspection error is favorable. Combined with the previous discussion of AOQ curve and OC curve, it is obvious that two types of inspection errors should be minimized.

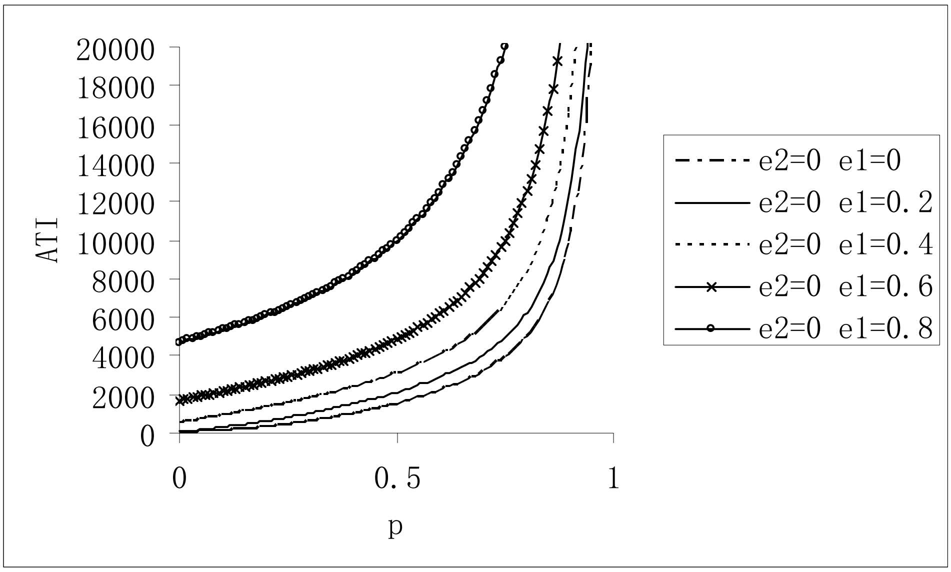

Figure 4. 18 ATI curve of increased type I inspection error (e_2=0)

4.3.4 Effects of other sampling parameters

The study of the effect of other sampling parameters of chain sampling scheme such as lot size, lot defect number, sample size, acceptance number(s), etc. is essential to better understand the behavior of chain sampling plan when inspection errors cannot be ignored. In subsection 4.3.2, it is shown that the presence of inspection errors can be treated equivalently to that of flawless inspection after substituting the true number of non conforming items, D, with its equivalent D^*. This implies that the effect of sampling parameters on the acceptance probability under inspection errors would behave similar to that of one perfect inspection. Therefore, the study of the effect of sampling parameters under imperfect inspection can be reduced to the study of the perfect chain sampling parameter without loss of its generality. The following investigation in this subsection will focus on sampling plans under perfect inspection only, but the conclusion can be applied to those of imperfect inspection.

Lot Size N

Figure 4.19 shows the relationship between the acceptance probability and lot size N. For a fixed process, where the long-term process quality is assumed to be $p = \frac{D}{N} = 0.1$, four different chain-sampling plans are compared. It is observed that as N increases, the probability of acceptance P_a becomes increasingly insensitive to N. That is, after a certain point, P_a exhibits little change as N increases. OC curve of ChSP (1, 2) 2 is actually the same as that of single sample plan with acceptance number equal to one. It is essentially the case of using binomial distribution to approximate hyper-geometric distribution.

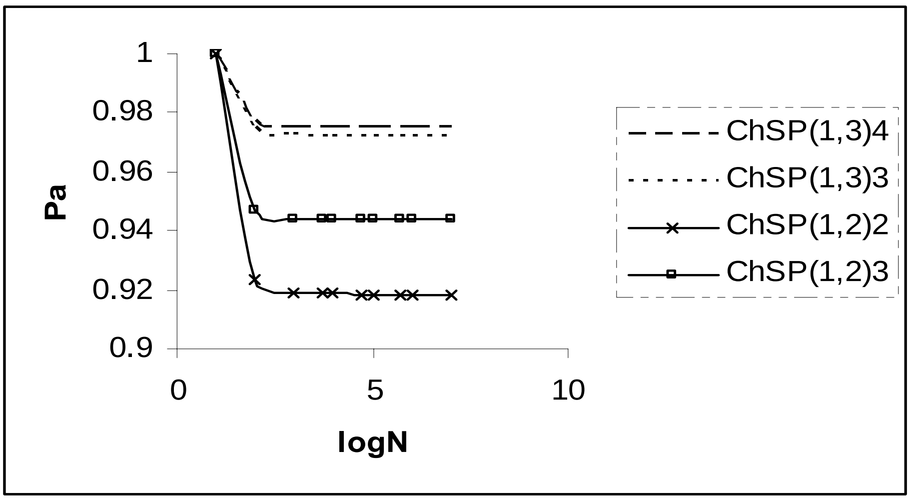

Figure 4. 19 Effects of lot size (1)

From the plot, it can be shown that as N increases, the probability of acceptance increases. As it continues to increase to infinity, the probability of acceptance approaches an asymptotic value. The asymptotic value depends on the process quality and also on the value of inspection errors. Generally speaking, if the presence of inspection error is significant, the convergence occurs at a larger N. The operating characteristic curve for a particular inspection scheme (ChSP (2, 4) 4, $k-1$=5) where N =100, 1000, 10000,100000 and 1000000 respectively is given in Figure 4.20. The graph shows that these curves superimpose on each other, implying that for a large N, probabilities of acceptance are almost equivalent.

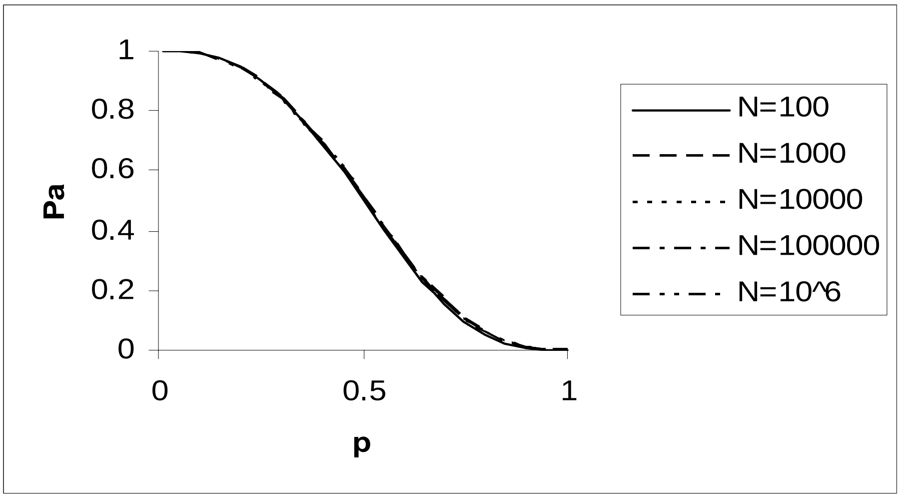

Figure 4. 20 Effects of lot size (2)

Sample Size n

Figure 4.21 succinctly illustrates the influence of sample size on the operating characteristic function. In this figure, the lot size of the sampling plan is set to 1000. As the sample size n, increases, the slope of the OC curve becomes steeper and will approach a vertical line when 100% inspection is carried out. This implies that implementing inspection schemes with a large sample size can increase the discriminatory power of acceptance sampling. However, having a larger sample size implies a larger ATI. That is, more items have to be inspected per lot. It is a time consuming and uneconomical approach. Therefore, an optimum sampling plan should consist of a small sample size that has the required discriminatory power.

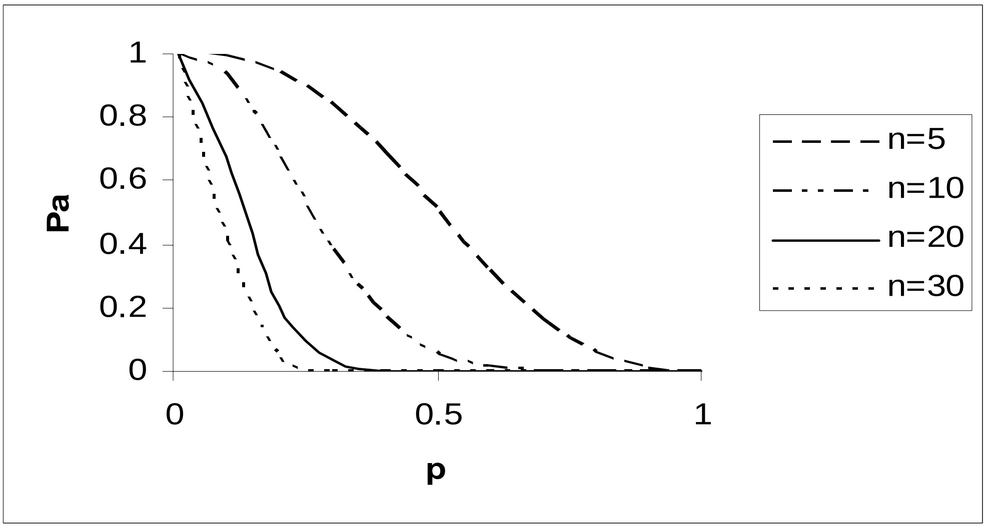

Figure 4. 21 Effects of sample size

Effects of acceptance number for first stage, c_1

A reduction in the acceptance number for the first stage, c_1, has the effect of compressing the shape of the OC curve, causing the probability of acceptance to decrease. In other words, the discriminatory power of the inspection scheme increases as c_1 decreases (as illustrated in Figure 4.22). The setting for this plan is that lot size equals to 1000 and sample size is 5.

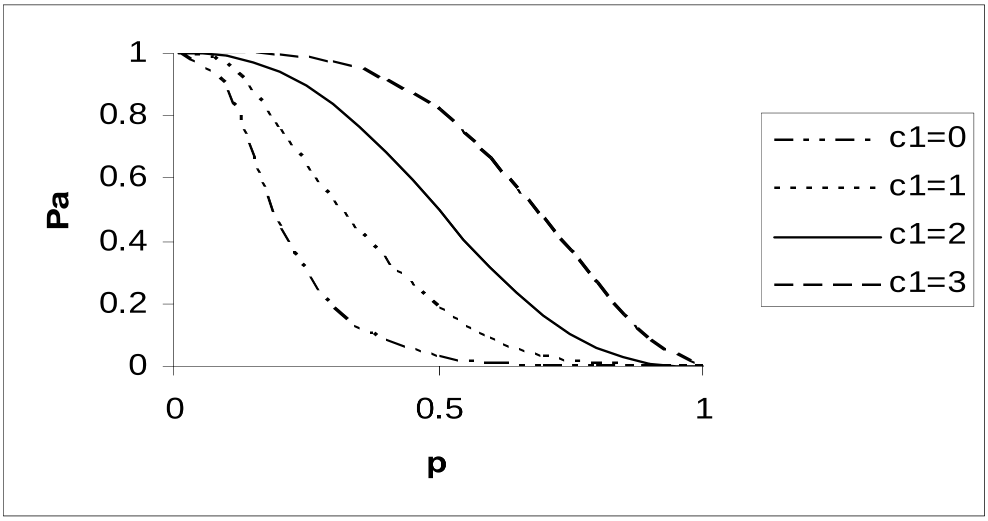

Figure 4. 22 Effect of c_1

Number of proceeding lots used for the cumulative criterion (k-1)

One of the most important characteristics of chain sampling plans is the number of preceding lots $(k-1)$ to be used for the cumulative criterion. The effect of cumulative criterion leads to more discriminating plans for smaller true fraction of nonconforming. Figure 4.23 shows that the probability of acceptance at the region of smaller true fraction of nonconforming will be greatly enhanced by utilizing fewer preceding lots especially, when $(k-1)$ equals to 1 or 2, for the verification of a production lot in doubt. However, such practice tends to increase consumer's risk, as utilizing only one or two of the preceding sampling result is not sufficient to reflect the process consistency. In practice, the value of $(k-1)$ varies from 3 to 5 as larger $(k-1)$ value will adversely lengthen the time to reach a decision (if a forward cumulative approach is used) and will increase the administration cost. Another important observation to be noted is that as k increases, the OC curve virtually converges with that of a single sampling plan.

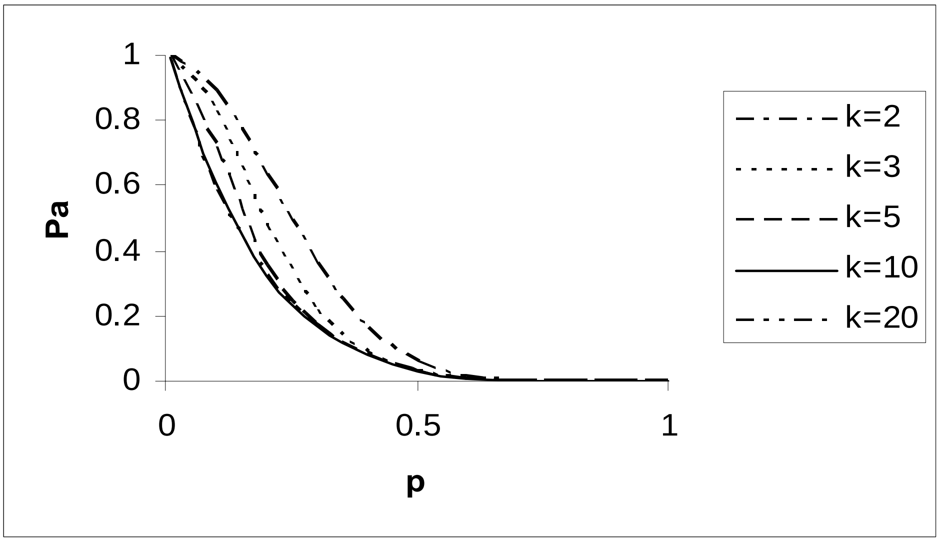

Figure 4. 23 Effects of k, number of lots

Rejection number, r

The rejection number also plays a pivotal role in the cumulative criterion, as it defines the quality of a production lot, which is in doubt and the range at which the cumulative criterion can take place. As the rejection number increases, the probability of acceptance increases (as shown in Figure 4.24). While increasing r, chances of acceptance will be enhanced; it also inevitably increases consumers' risk. Therefore, it is often undesirable to define a large value of r. On the contrary, reducing r has the same effect of increasing k, which means reducing r will cause the OC curve approaching that of single sampling. It is therefore desirable to select a moderate value of r.

Acceptance number for second stage, c_2

The last but equally important component of the cumulative criterion is the acceptance number c_2 for the second stage. From Figure 4.25, the increase of acceptance number for the second stage will similarly alter the shape of the OC curve in the region of principal interest, in which the fraction of nonconforming is small. Any attempt to decrease producers' risk by defining a chain sampling inspection scheme with a large c_2, would similarly incur an objectionable increment in the consumers' risk. In practice, c_2 takes only two values, $r-1$ and r.

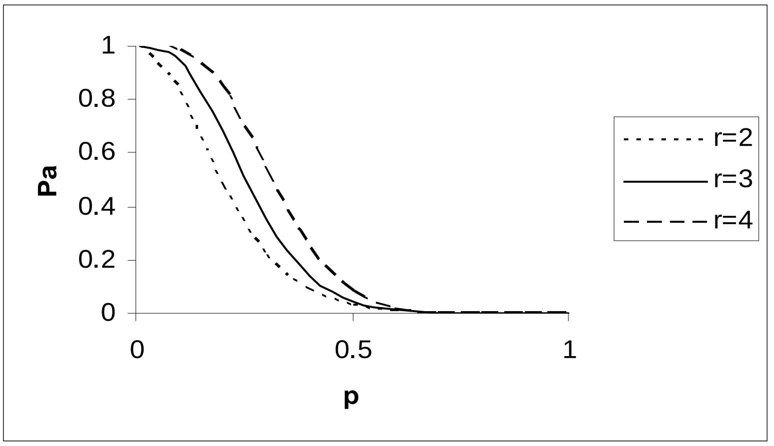

Figure 4. 24 Effects of rejection no. r

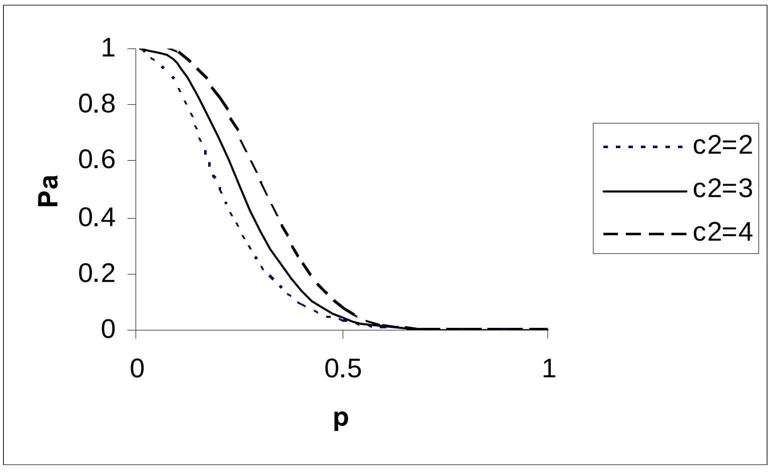

Figure 4. 25 Effects of c_2

4.4 Conclusion and Remark

Acceptance sampling, rather than 100% inspection of a production lot, is widely employed in industries to achieve a more economical and efficient use of company resources. An implicit assumption in the design of acceptance sampling plans for attributes is that inspection procedures are completely flawless. In reality, however, inspection tasks are seldom error free. On the contrary, they may even be error prone.

While inspection errors incurred during an acceptance sampling for attributes are often unintentional and in most cases neglected, they nevertheless can severely distort the quality objective of a system design.

In this chapter, the inspection error is considered in chain sampling schemes and a mathematical model is developed to investigate the performance of chain sampling schemes when inspection errors are taken into consideration. Expressions of performance measures are derived to aid the analysis of a general chain sampling scheme, ChSP-4A $(c_1, c_2)r$, developed by Frishman (1960), such as the operating characteristic function, average total inspection and average outgoing quality.

The study reveals that as type I inspection error increases, the acceptance probability will decrease while the increment of type II inspection error will increase the acceptance probability. The effect of type II error on the sampling acceptance probability is very marginal as compared to that of type I error especially when the true fraction of nonconforming is small.

An important conclusion from this study is that the effect of inspection errors can be "eliminated" by transforming to its equivalent perfect inspection counterpart, thus greatly reducing the complexity of the analysis.

Effects of inspection errors on the AOQ curve and ATI curve are also complicated. As the type I inspection error increases, the corresponding AOQ value will decrease and its ATI will increase. The effect of the type II inspection is on the reverse, i.e. when the type II inspection error becomes bigger, the AOQ will become larger and its ATI will become smaller accordingly. These confounding effects deserve careful consideration before any decision can be reached. One guideline is that the type I inspection error usually plays a prominent role in the small fraction of defectives while the type II inspection error has more weight on the large fraction of nonconforming product. Accordingly, the type II inspection error plays a dominant role in determining the final average outgoing quality limit (AOQL). Simulation shows that even a small type II inspection error will lead the final AOQL to almost one.

Analysis of the AOQ and ATI also shows that the effectiveness of sampling plans can only be maintained when two types of inspection errors are relatively small. If an inspection error is large, either type I or type II, sampling schemes will not be effective any more. The final outgoing quality after inspection will be barely improved, which implies that there is no point to implement sampling plan when the inspection error is large.

The study of how other sampling parameters such as lot size, sample size, number of previous lot, etc. behave helps identify guidelines in setting such parameters. Some suggestions include choosing a lot size at least one hundred, the number of previous lot ranging from three to five, setting the acceptance number for second stage, c_2 equal to the rejection number, r, or $r-1$. Sample size is usually determined by resource and cost constraints.

Future complementary research directions include the study of the effect of fluctuating inspection errors and a general procedure for designing chain-sampling plan under general inspection error. Next chapter will focus on variable inspection errors, which means the stringent constant error assumption in this chapter will be relaxed. The result in the next chapter will provide a more realistic picture of how sampling plans behave under inspection errors.

Error effect is not the final goal, though it serves a good foundation to investigate chain-sampling schemes. Once this foundation is built, a way will be found as to how to design a chain-sampling plan with the presence of inspection errors. This will be the subject of another chapter. In this chapter, the hype-geometrical model will be modified to the most commonly used binomial model propose our own library of tables and algorithms to make such a design easy and agreeable.

5. Chain Sampling Scheme under Inspection Errors (II: For Varying Errors)

5.1 Introduction

In the previous chapter, the effect of inspection error on chain sampling plan is studied by assuming that inspection errors are constant throughout the inspection. In this chapter, this strict assumption will be relaxed and the effect of fluctuating inspection errors on chain sampling plan will be investigated.

In order to examine the effect of inspection errors on statistical quality control procedures, it is necessary to have a model of the process generating errors. One particular model assumes constant error probability. That is, the probability of committing inspection errors does not change throughout the inspection. This assumption, though simple and mathematical appealing, does not provide a good representative of the real case. Actually there are arguments that inspection errors are fluctuating and different models (Biegel (1974) for example) have been proposed to model this fluctuation.

In this chapter, we adopt the Biegel (1974) linear model to assume that the error probability is a linear function of the process quality. This is the most reasonable and useful model available so far.

In the next section, a chain sampling under varying inspection error model will be outlined first and the detailed derivation will be given subsequently. After that the analysis and discussion section will follow. Conclusion and remarks are summarized at the end of this chapter.

5.2 Mathematical Model

5.2.1 Chain sampling plan for linearly varying inspection error.

Recall from chapter four that there are two types of inspection errors, whose relationship was illustrated in Table 4.1. Where T stands for T(rue) state of an inspected item and A stands for A(pparent) or classified state of an inspected item. Letter e_1 and e_2 stand for type I and type II inspection error respectively.

In the constant error model, inspection errors are assumed unchanged throughout the inspection. That is to say that both e_1 and e_2 are constants in the model and are given by the following equation:

$$e_1 = P(A=1 \mid T=0) = P(\frac{A=1 \,\&\, T=0}{T=0}) \qquad (5\text{-}1)$$

$$e_2 = P(A=0 \mid T=1) = P(\frac{A=0 \,\&\, T=1}{T=1}) \qquad (5\text{-}2)$$

There are two types of varying error models. One is to assume that inspection errors are changing when the process quality is changing. The other is to assume that errors are fluctuating between inspection items from a fixed process quality while the process quality is also changing. The second model is obviously a more realistic representative of the real scenario. However, it is too mathematically intractable to be easily incorporated here. The first type of model will be used to study inspection errors fluctuating with process quality with not loss too much of generality.

Through experimental studies, Biegel (1974) found that error rate is related to the process quality or process fraction of defectives p . From the viewpoint of an inspector, this means that the likelihood of "catching" a defective item is dependent upon the frequency, with which a defective happens. Hence, he proposed his linear model:

$$\begin{aligned} e_1(p) &= a_1 + b_1 p \\ e_2(p) &= a_2 + b_2 p \end{aligned} \qquad (5\text{-}3)$$

where p is the process fraction of defectives and ranging from zero to one.

Following a similar derivation procedure of that in chapter four, mathematical expression of the probability of acceptance, under this linear model, can be obtained for the single stage sampling plan and the chain-sampling plan. They are given by:

$$P_s\left(c,n,D,N,e_1(p),e_2(p)\right)$$
$$=\sum_{z=0}^{c}\left(\begin{array}{l}\sum_y \dfrac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}}\\ *\left(\sum_w\left(\binom{Y}{w}(1-a_2-b_2p)^w(a_2+b_2p)^{Y-w}\binom{n-Y}{z-w}(a_1+b_1p)^{z-w}(1-a_1-b_1p)^{n-Y-z+w}\right)\right)\end{array}\right)$$

(5-4)

$$P_{ch}=\sum_{z=0}^{c_1}\left(\begin{array}{l}\sum_y \dfrac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}}\\ *\left(\sum_w\left(\binom{Y}{w}\left(\dfrac{N-Na_2-Db_2}{N}\right)^w\left(\dfrac{Na_2+Db_2}{N}\right)^{Y-w}\binom{n-Y}{z-w}\left(\dfrac{Na_1+Db_1}{N}\right)^{z-w}\left(\dfrac{N-Na_1-Db_1}{N}\right)^{n-Y-z+w}\right)\right)\end{array}\right)$$
$$+\sum_{z_0=c_1+1}^{r-1}\left(\begin{array}{l}\sum_y \dfrac{\binom{D}{y}\binom{N-D}{n-y}}{\binom{N}{n}}\\ *\left(\sum_w\left(\binom{Y}{w}\left(\dfrac{N-Na_2-Db_2}{N}\right)^w\left(\dfrac{Na_2+Db_2}{N}\right)^{Y-w}\binom{n-Y}{z-w}\left(\dfrac{Na_1+Db_1}{N}\right)^{z-w}\left(\dfrac{N-Na_1-Db}{N}\right)^{n-Y-z+w}\right)\right)\\ *\sum_{z_{pre}=0}^{c_2-z_0}\left(\sum_i \dfrac{\binom{(k-1)D}{i}\binom{(k-1)(N-D)}{(k-1)n-i}}{\binom{(k-1)N}{(k-1)n}}\left(\sum_{w_2}\begin{array}{l}\binom{i}{w_2}\binom{(k-1)n-i}{z_{pre}-w_2}\left(\dfrac{N-Na_2-Db_2}{N}\right)^{w_2}\left(\dfrac{Na_2+Db_2}{N}\right)^{i-w_2}\\ *\left(\dfrac{Na_1+Db_1}{N}\right)^{z_{pre}-w_2}\left(\dfrac{N-Na_1-Db}{N}\right)^{(k-1)n-i-z_{pre}+w_2}\end{array}\right)\right)\end{array}\right)$$

(5-5)

where:

$$\max(0,n-N+D)\le y\le\min(n,D)$$

$$\max(0,z_0-n+y)\le w\le\min(z_0,y)$$

$$\max\left(0,(k-1)(n-N+D)\right) \le i \le \min\left((k-1)n,(k-1)D\right)$$

$$\max\left(0, z_{pre} - (k-1)n + i\right) \le w_2 \le \min\left(z_{pre}, i\right)$$

$$c_2 - c_1 \le r$$

$$z_{pre} = z_{total} - z_0$$

Plotting acceptance probabilities P_{ch} against different values of the true fraction of nonconforming p ($p = \frac{D}{N}$) will yield the operating characteristic (OC) curve of the chain-sampling scheme under varying inspection error.

5.2.2 AOQ and ATI

Based on the same disposition policy as that of chapter four, i.e. all apparent nonconforming items in a sample will be replaced and any rejected lot will undergo 100% screening with all apparent nonconforming items replaced, obtain expressions of AOQ and ATI for linearly varying inspection errors as follows:

$$AOQ = \frac{(N-n)pP_{ch} + (N-n)pe_2(p)(1-P_{ch}) + npe_2(p)}{N}$$
$$+\frac{\left((N-n)(p-p(e_1(p)+e_2(p))+e_1(p))\dfrac{pe_2(p)}{1-p+p(e_1(p)+e_2(p))-e_1(p)}\right)(1-P_{ch})}{N}$$
$$+\frac{n(p-p(e_1(p)+e_2(p))+e_1(p))\dfrac{pe_2(p)}{1-p+p(e_1(p)+e_2(p))-e_1(p)}}{N}$$
$$=\frac{De_2(p)}{(1-e_1(p))(N-D)+De_2(p)}+\frac{(N-n)}{N}\frac{D}{N}\left(\frac{(N-D)(1-e_1(p)-e_2(p))}{(1-e_1(p))(N-D)+De_2(p)}\right)P_{ch}$$
$$=\frac{D(a_2+b_2p)}{(1-a_1-b_1p)(N-D)+D(a_2+b_2p)}$$
$$+\frac{(N-n)}{N}\frac{D}{N}\left(\frac{(N-D)(1-a_1-b_1p-a_2-b_2p)}{(1-a_1-b_1p)(N-D)+D(a_2+b_2p)}\right)P_{ch}$$
$$=\frac{D(Na_2+Db_2)}{(N-Na_1-Db_1)(N-D)+D(Na_2+Db_2)}$$
$$+\frac{(N-n)}{N}\frac{D}{N}\left(\frac{(N-D)((1-a_1-a_2)N-(b_1+b_2)D)}{(N-Na_1-Db_1)(N-D)+D(Na_2+Db_2)}\right)P_{ch} \tag{5-6}$$

The average outgoing quality of chain sampling plans can be obtained by using equation (5-6) to substitute the P_{ch} in equation (5-14).

$$ATI = P_{ch}\left(n+\frac{n\pi}{1-\pi}\right)+(1-P_{ch})\left(N+\frac{N\pi}{1-\pi}\right)$$
$$=\frac{N^3-N^2(N-n)P_{ch}}{(N-Na_1-Db_1p)(N-D)+D(Na_2+Db_2)} \tag{5-7}$$

Similarly, substituting P_{ch} in equation (5-5) using equation (5-7) will give the exact formula to calculate ATI value.

5.2.3 Parameter Estimation

In adopting Biegel's linear model, it is important to outline the way to estimate model parameters. In this chapter, Biegel's approach will be used again, i.e. using linear regression to estimate parameters. For example, suppose the maximum type I and type II inspection error are 0.01 and 0.05 respectively, then:

$$\begin{aligned} e_1(0) &= a_1 + b_1 * 0 = 0.01 \\ e_1(1) &= a_1 + b_1 * 1 = 0 \end{aligned} \Rightarrow \begin{aligned} a_1 &= 0.01 \\ b_1 &= -0.01 \end{aligned} \Rightarrow e_1(p) = 0.01 - 0.01p$$

and

$$\begin{aligned} e_2(0) &= a_2 + b_2 * 0 = 0 \\ e_2(1) &= a_2 + b_2 * 1 = 0.05 \end{aligned} \Rightarrow \begin{aligned} a_2 &= 0 \\ b_2 &= 0.05 \end{aligned} \Rightarrow e_2(p) = 0.05p$$

There is no reference or application data available in determining the maximum value of type I inspection and type II inspection error. In this chapter, results from the previous study in chapter four will be adopted, i.e. the probability of acceptance is insensitive to e_2 as small as 0.05 but is sensitive to e_1 as small as 0.01 when the lot size is very large with very little nonconforming items presence. This guideline will be used for further reference.

5.3 Analysis and Discussion

A series of Microsoft Excel Visual Basic Application routines has been developed to compute the complex cumulative hyper-geometric equation of the probability of acceptance for the chain sampling plan as well as its OC curve, AOQ curve and ATI curve, which are the three major measurements of any sampling schemes. Brief description of the program will be included in the relevant section.

5.3.1 Effects of Inspection Errors

There are two types of inspection errors, namely type I and type II errors. The probability of incurring type I error is e_1 and the probability of the occurrence of type II error is e_2, where both e_1 and e_2 range from 0 to 1. The effect of inspection error on the probability of acceptance is illustrated in Figure 5.2, which is a 3D plot of the ChSP (1, 3) 4 (Lot size = 1000, Number of defectives =10, Sample Size = 5, and $k-1$=2). It should be noted that Figure 5.2 is similar to that of Figure 4.2. This is due to the fixed setting of chain sampling plain. When the lot size and lot defectives (here are 1000 and 10 respectively), the process quality is fixed ($p = \frac{D}{N}$). The value of type I inspection error and type II inspection error is therefore constant because of the definition of the linear model.

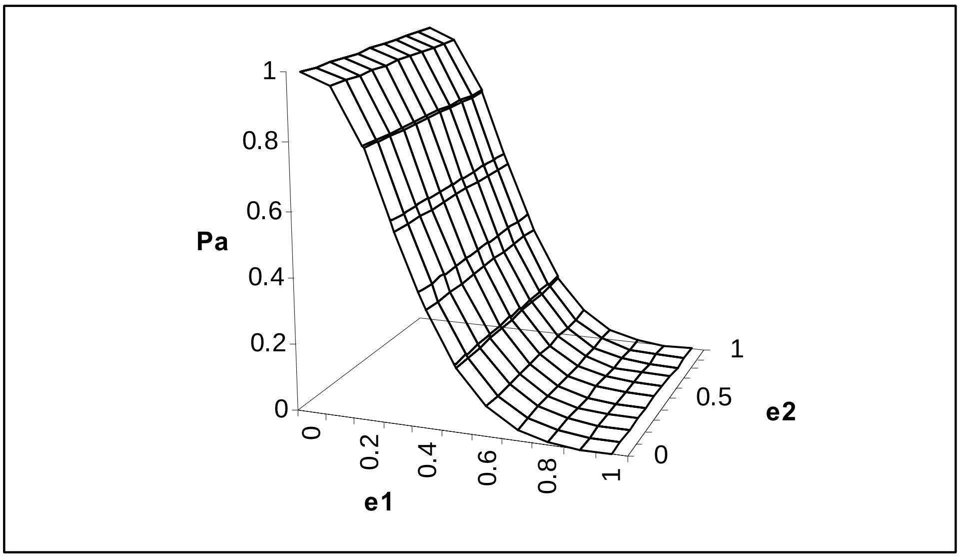

Figure 5. 1 3D Plot of effects of varying inspection errors

It is not surprising to find from the resultant figure (figure 5.1) that the effect of inspection error on probability of acceptance displays two prominent trends:

- As the maximum value of e_1 increases, the probability of acceptance decreases. That is, as the probability of type I inspection error increases, the probability of acceptance decreases.
- As the probability of maximum type II inspection error e_2 **,** increases, the probability of acceptance increases. The increase, however, is almost negligible as compared to that of the change in the probability of acceptance when e_1 decreases while the lot size is large with only a small fraction of defectives.

The observation here is almost identical to that of chapter four for constant inspection error except that in this chapter the maximum inspection error is introduced and in chapter four inspection errors are assumed constant.

Similar to chapter four, a brief illustration of the Excel VBA program is given first to compute the acceptance probability and to draw the OC curve automatically. Readers can request the program and reproduce results by following the interface provided in this chapter. The program input interface for Figure 5.1 is shown in Figure 5.2.

Chain Sampling Under Inspection Error

1. Effect of Inspection Error

Sampling Parameters	
1000	N Lot Size
10	D Lot defective number
5	n Sample Size
1	c1,acceptance number for first stage
3	c2,acceptance number for cumulative lots
4	r, rejection number for single lot
3	k, number of total lots
Error range	
0	Smallest maximum type I inpsection error
1	Largest maximium type I inspection error
0.1	Increment of type I inspection error
0	Smallest maximum type II inpsection error
1	Largest maximium type II inspection error
0.1	Increment of type II inspection error

Calculate Probability of Acceptance for ChSP under Inspection Error

Figure 5. 2 Screen snapshot of the program input interface of Figure 5.1

5.3.2 Effect on OC Curve

The statistical effect of inspection errors on the operating characteristic (OC) curve is the basic measure of any sampling plans and will be evaluated in this section for linear inspection error model.

To make figure captions more concise, all figures, unless stated otherwise, in this subsection will follow the same chain sampling setting (ChSP (2, 5) 5, n=5, $k-1$=5) and sampling parameters will not appear in the caption.

A screen snapshot of the program, which is used to conduct the analysis of the effect of linearly varying inspection errors on OC curves, is shown in Figure 5.3. Readers can follow the figure to key in the parameter input and reproduce the result.

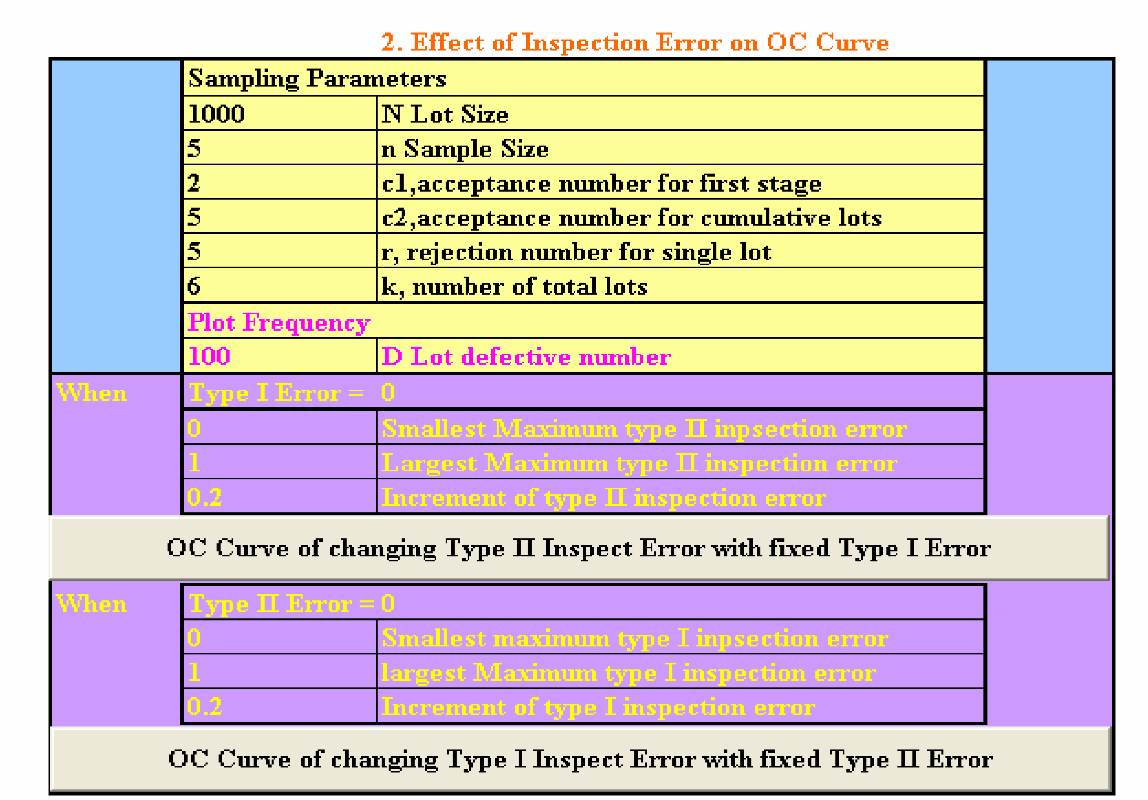

Figure 5. 3 Program input of the OC curve analysis for linear error model

Under this linear model, the behavior of OC under the effect of type I inspection error only will be studied first. It is illustrated in Figure 5.4.

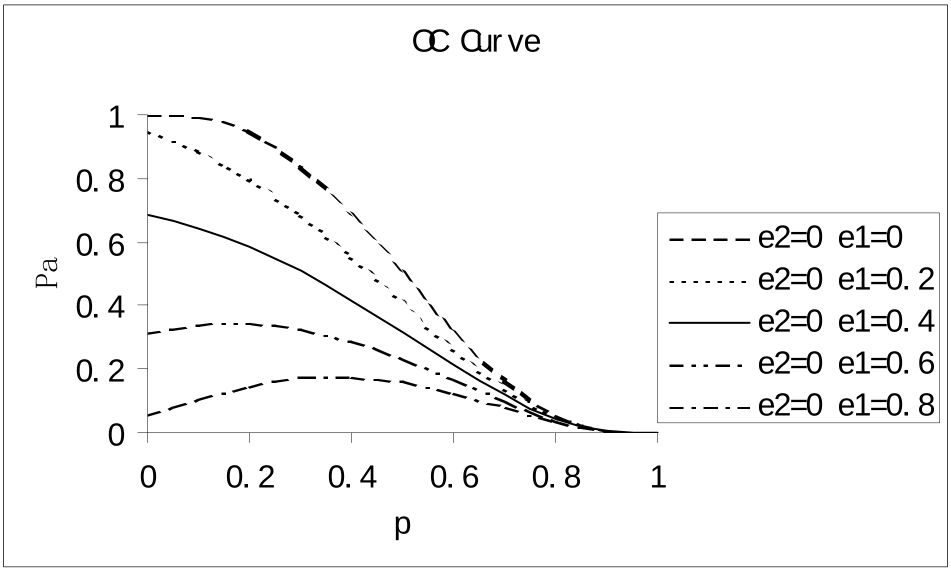

Figure 5. 4 OC curves for type I inspection errors (e_2=0)

Seen from the figure, when only type I inspection error exits, the overall probability of acceptance decreases with the increase of the type I inspection error. This decrease is more prominent in the area of the small fraction of nonconforming. It is reasonable to have this observation because as the type I inspection error increases, the probability of misclassifying conforming items as nonconforming items increases. Hence the process quality "deteriorates", and the probability of acceptance therefore decreases. This trend is agreeable to that of chapter four and is valid for any type II inspection error. Figure 5.5 have the same setting as Figure 5.4 expect that the maximum type II inspection error is set to 0.2.

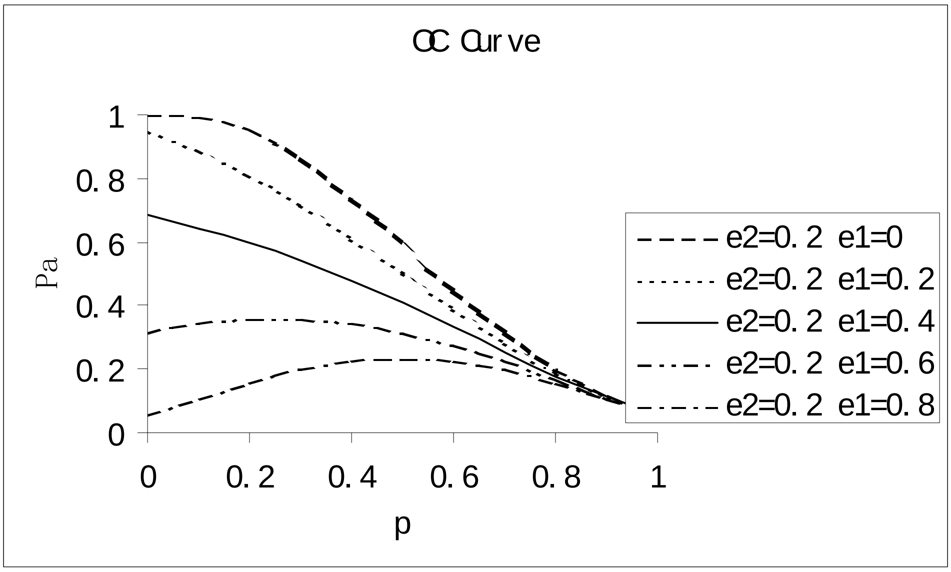

Figure 5. 5 OC curves for type I inspection errors (e_2=0.2)

It is clear that Figure 5.5 and 5.4 exhibit litter difference and the trend is exactly the same.

The most important observation lies in the difference between constant error model and linear error model. Figure 5.6, 5.7, and 5.8 show these differences when the average type I inspection error in the linear model is equal to that of the constant error model.

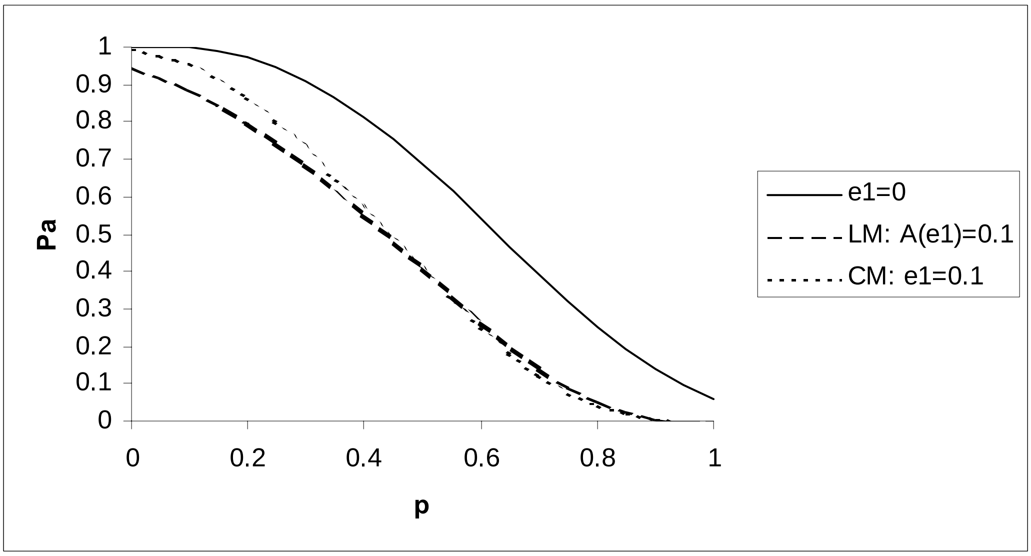

Figure 5. 6 Comparison of linear model and constant model (e2=0, e1=0.1)

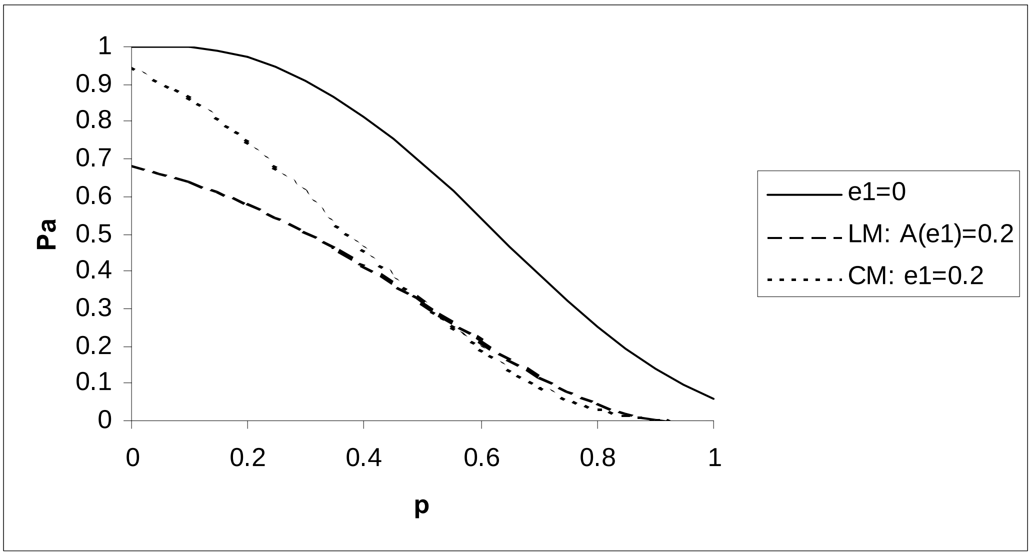

Figure 5. 7 Comparison of linear model and constant model (e2=0, e1=0.2)

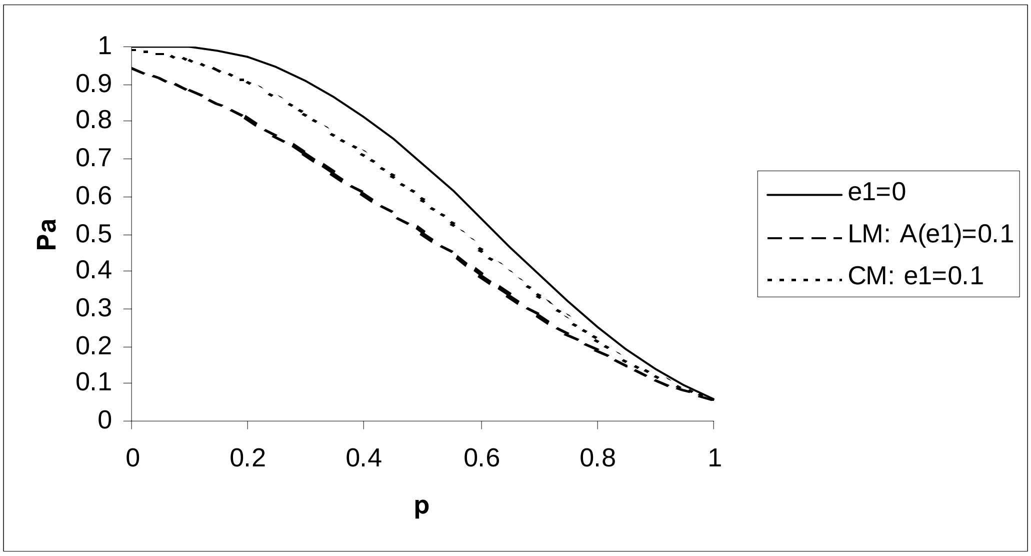

Figure 5. 8 Comparison of linear model and constant model (e2=0.2, e1=0.1)

In Figure 5.6, the type II inspection error is fixed at zero, i.e. there is no type II inspection error. Its Constant Model (LM) curve is obtained by setting the type I inspection error to 0.1, and the Linear Model (LM) is obtained by setting the average type I inspection error equal to zero. Since the error in the linear model will start from zero, the average inspection error is half of the maximum value that the inspection error takes. Similarly, the setting for Figure 5.7 is that the type II inspection error equals to zero and the type I inspection error (either average for LM or constant for CM) equals to 0.2. Settings for Figure 5.8 is the same as that of Figure 5.6 expect that its type II inspection error is set to 0.2 to show the combined effect of both errors.

The predominant trend in these three figures is that the difference between the linear error model and the constant error model is more obvious in the region of small fraction of nonconforming. When there is only type I inspection error, two OC curves (LM and CM) will intersect each other at the region of middle quality values. The difference is the biggest at the origin point ($p = 0$), where the probability of acceptance for the linear model is bigger than that of the constant model. As p increases, this difference becomes smaller until two curves intersect each other at some point in the middle. The trend is then reversed after this point, i.e. the probability of acceptance for the linear model will become less than that of the constant error model when p continue to increase till one. This trend can be summarized in short that the OC curve for the constant error model is more discriminating than that of the linear model when only type I inspection error exists.

When the type II inspection error is not zero (Figure 5.8), the probability of acceptance for the constant error model is always greater than that of the linear error model.

On the contrary, when the value of the type II inspection error is changed with a fixed type I inspection error, the effect is reverse to that of the type I inspection error as discussed before. Figure 5.9, 5.10 & 5.11 display the comparison of the linear error model and the constant error model for different type II inspection errors.

In Figure 5.9 and 5.10, where the type II inspection error is set to zero and the type I inspection error is set to 0.1 and 0.2 respectively, it is shown that two OC curves are intersecting each other and the trend is converse to that of type I inspection error. In the region of the small fraction of nonconforming, the probability of acceptance for the constant error model is greater than that of the linear error model and as p increases this difference decreases. After intersection, this trend is reverse. That is, after intersection, the probability of acceptance for the constant error model is less than that of the linear model. As p increases, this difference increases and reaches its maximum value when p reach one.

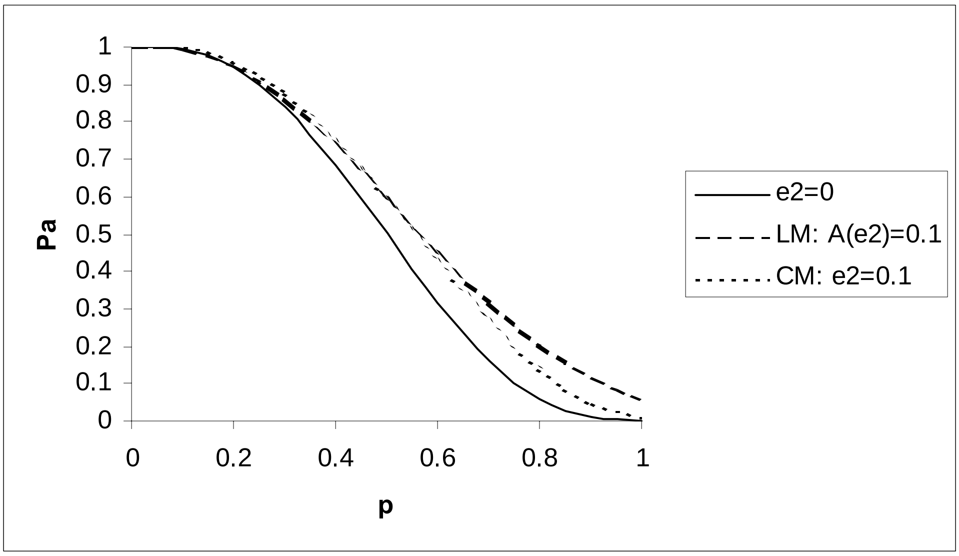

Figure 5. 9 Comparison of linear model and constant model (e_1=0, e_2=0.1)

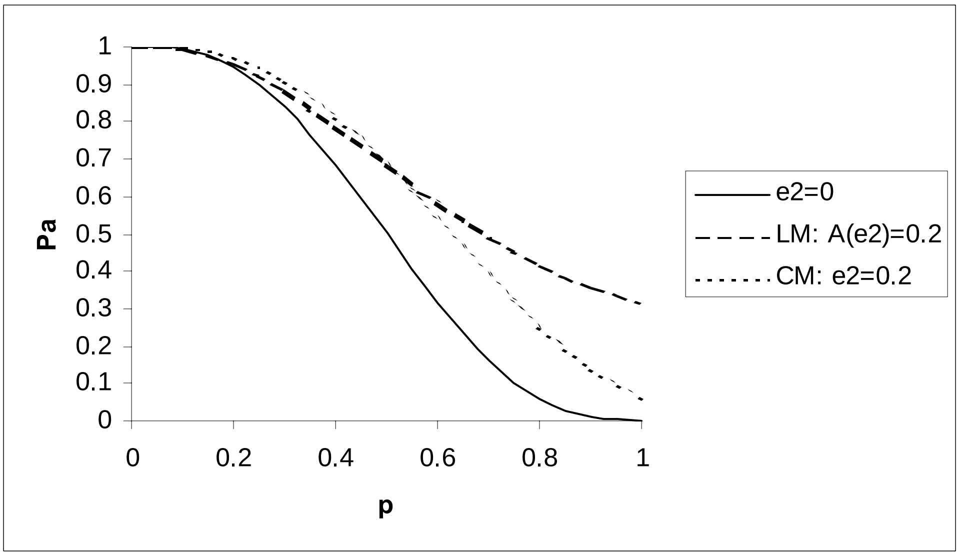

Figure 5. 10 Comparison of linear model and constant model (e_1=0, e_2=0.1)

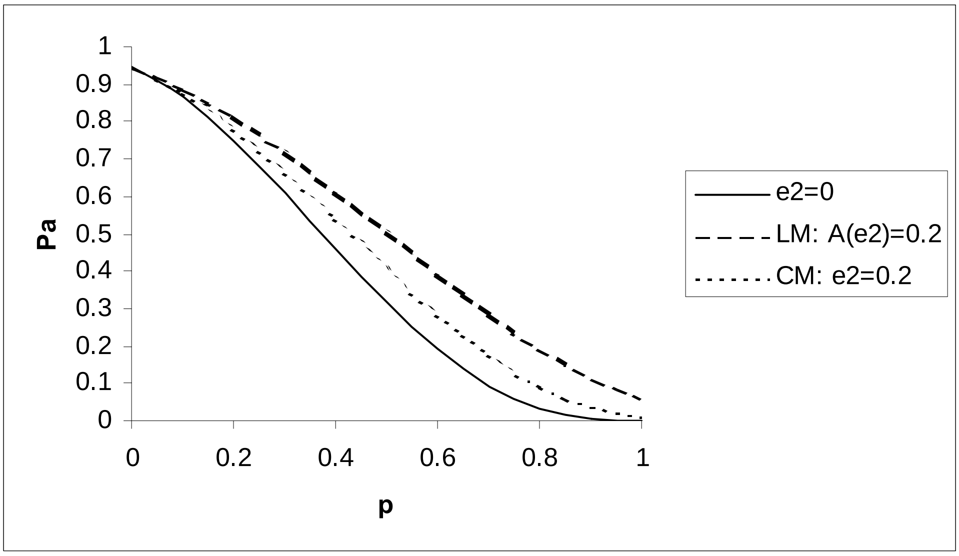

Figure 5. 11 Comparison of linear model and constant model (e_1=0.1, e_2=0.1)

Figure 5.11 shows the combined effect of two types of inspection errors. In this case, the probability of acceptance for the linear error model is always greater than that of the constant error model.

In all of these figures (Figure 5.6~5.11), the ideal OC curve is also plotted. The observation is agreeable to that of chapter four, in which it is shown that the type I inspection error will decrease the probability of acceptance while the type II inspection error will increase the acceptance probability.

The comparison of the linear error model and the constant error model in this section provides a guideline in analyzing these differences. However, in practice, there is no too much difference between two models because the value of two types of inspection errors is assumed to take moderately small value. Larger inspection error will invalidate the inspection process and there is no point to carry out further. Figure 5.12 illustrates the difference between two models when two types of inspection errors are taking reasonable small values (here both equal to 0.01).

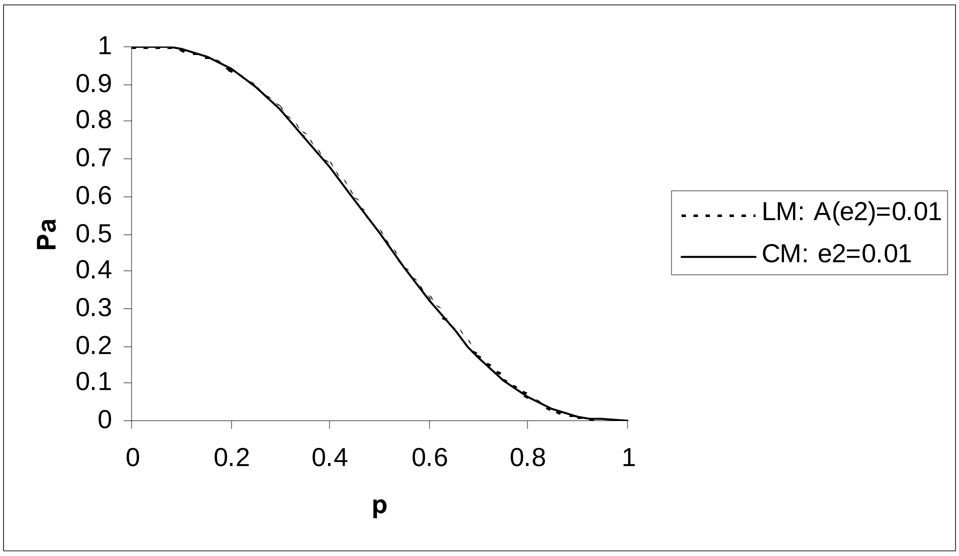

Figure 5. 12 Comparison of linear model and constant model (e_1=0.01, e_2=0.01)

There is a clear indication that when both types of inspection error are small, the differences between two models are smaller enough to be negligible. People may argue that 0.01 is a very small value. However, it is a very loose requirement in real application, especially in current high yield production. One out of one hundred chances is actually a very high probability of error commitment. True value should be far less that 0.01. It is therefore very safe for application purposes to assume constant error model rather than the complicated linear error model without loss of too much accuracy.

This result is very important in that it provides a guideline that the constant error model, rather than the complicated linear model, can be used at the design stage without loss of accuracy. In the next chapter (chapter six), procedures to design the chain sampling plan with inspection errors will be proposed. Two types of inspection errors are assumed constant throughout the inspection activity in that chapter. The justification for this assumption lies in the important result above-mentioned in this section.

5.3.3 Effects on AOQ and ATI

In this section, the behavior of AOQ (Average Outgoing Quality) curve and ATI (Average Total Inspection) curve will be studied when inspection errors are linearly associated with process quality p. Both curves are important performance measures that can be employed to assess sampling schemes subjected to inspection errors besides the famous OC curve. It is important to note that the study of the two performance measures is critical as both of them have direct impact on the economic aspect of the sampling procedure. As pointed out in chapter four, the computation of both AOQ and ATI are highly dependent on the sample/lot disposition policy. In this section, the disposition policy remains the same as that of chapter four, i.e. all apparent nonconforming items in a sample be replaced and any rejected lot undergoes 100% screening with all apparent nonconforming items replaced. That is the basis of this analysis.

Similar to the previous section, to make captions concise, sampling parameters in captions of all figures in this section will not be specified. All figures, unless stated otherwise, are from the same sampling setting of ChSP (2, 5) 5, $k = 6$ and $n = 5$.

Before proceed to the analysis, a short description of the program used for AOQ and ATI analysis will be given first. Since the input for AOQ curve and ATI curve are same, incorporate them in one input interface to make it more concise. To run the simulation, just key in required sampling parameters and press the left button for AOQ computation and the right button for ATI computation. The input interface for this program is illustrated in Figure 5.13 below.

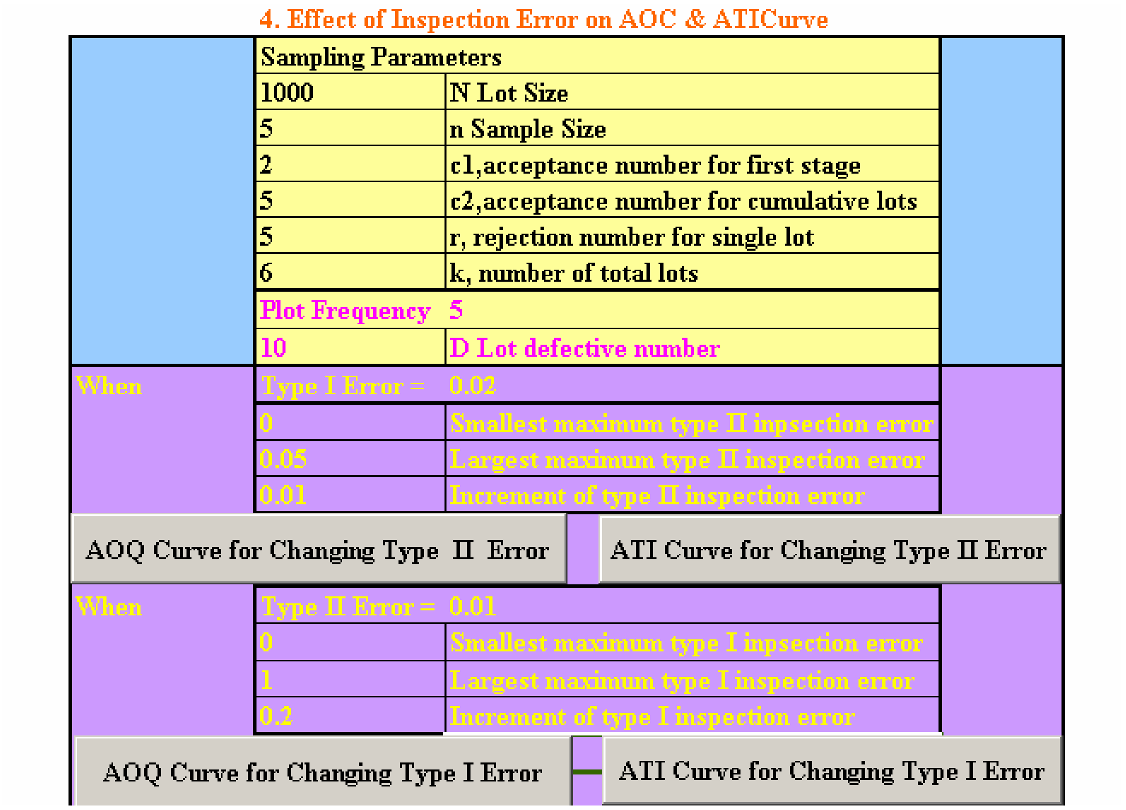

Figure 5. 13 Program input interface for AOQ and ATI analysis (LM)

Figure 5.14 shows AOQ curves for changing type II inspection error while the type I inspection error is fixed to zero, i.e. no type I inspection error. In this section, all values of inspection errors are the maximum value they can assume. The trend is similar to that of the constant error model. That is, under the linear error model, when the type II inspection error increases, the AOQ will increase accordingly. The same trend holds for other settings of the type I inspection error. These observations conform to intuitive understanding because the bigger the type II inspection error, the more nonconforming items escape from inspection. The resultant average outgoing quality therefore increases. This justification is valid for both the constant error and the linear error models. In Figure 5.15 the value of the type I inspection error is set at 0.2 and the type II inspection error is changed. Two figures suggest that for a fixed type I inspection error, regardless of its value, the AOQ will increase with the increase of the type II inspection error. The trend is much more prominent in the region of large fraction of defectives while the difference is very small and almost negligible for small fraction of defectives. This observation agrees with that of previous literatures.

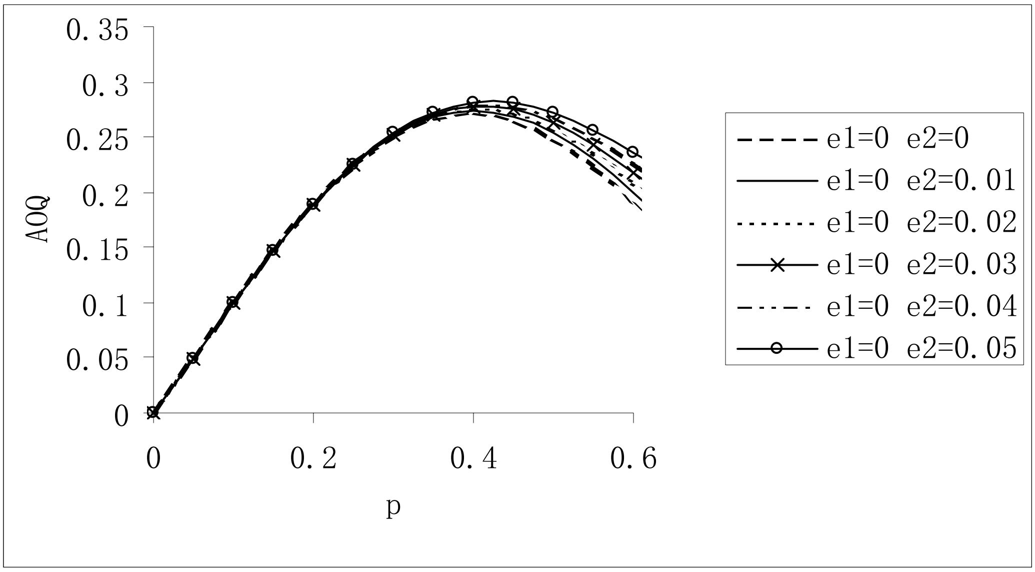

Figure 5. 14 AOQ curve of different type II inspection error (e_1=0)

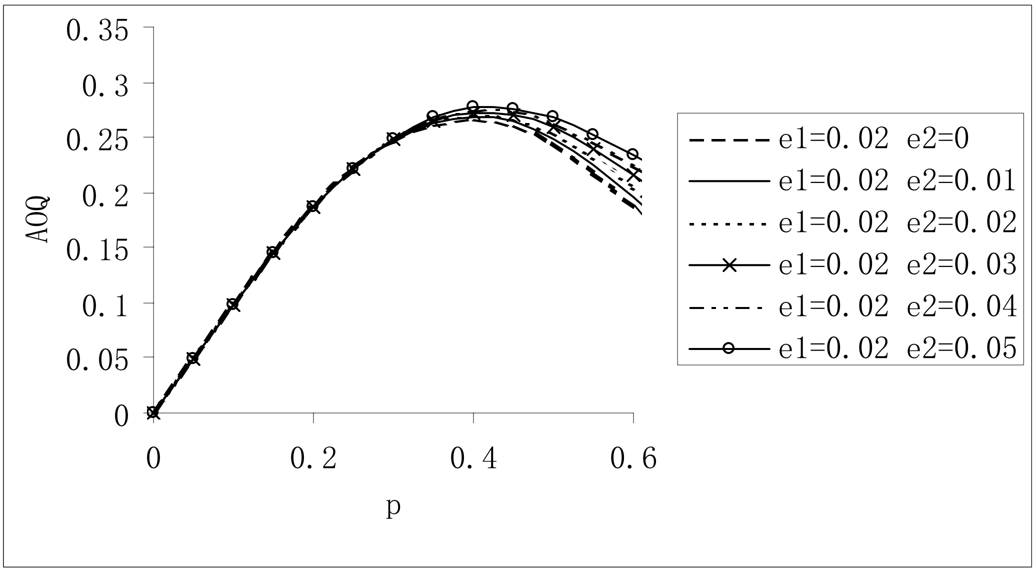

Figure 5. 15 AOQ curve of different type II inspection error (e_1=0.2)

It should be pointed out that the difference between AOQ curves for the constant error model and the linear error model is very small, even smaller than that of the OC curve difference. Figure 5.16~5.18 illustrate these differences for difference inspection error values.

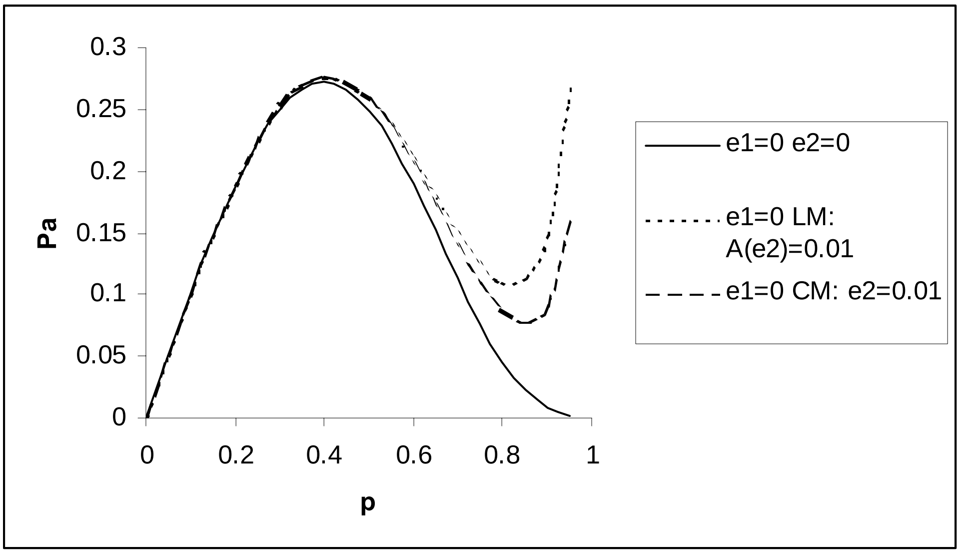

Figure 5. 16 AOQ curve for LM and CM (e_1=0, e_2=0.01)

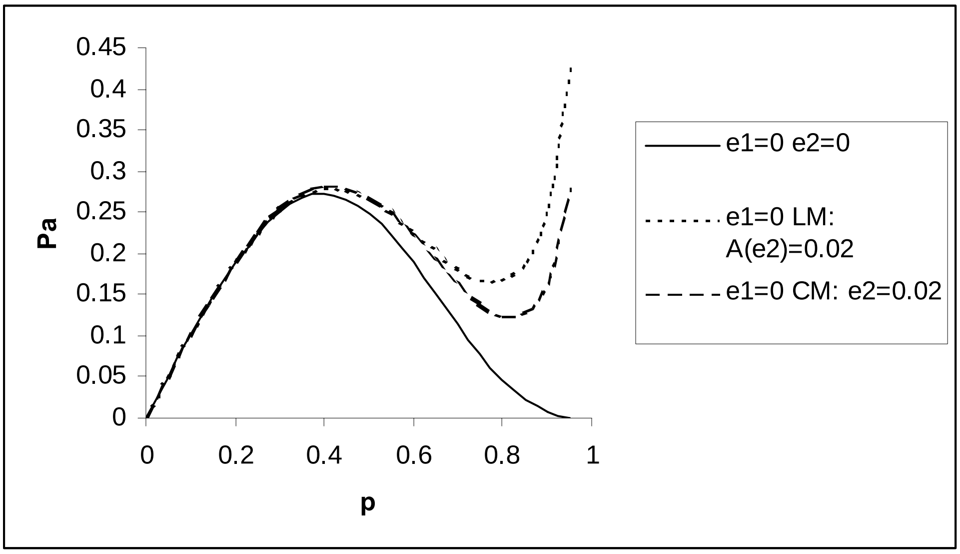

Figure 5. 17 AOQ curve for LM and CM (e_1=0, e_2=0.02)

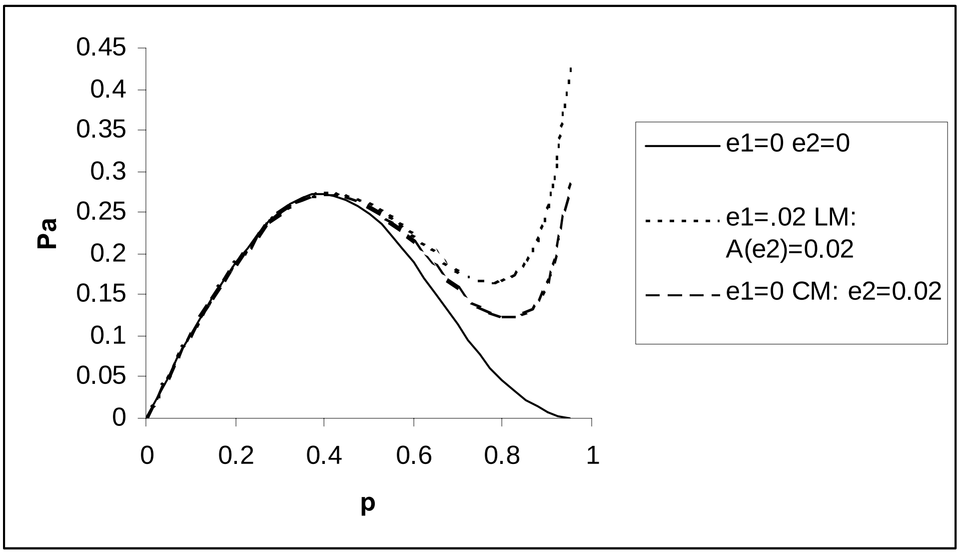

Figure 5. 18 AOQ curve for LM and CM (e_1=0.02, e_2=0.02)

There is an interesting finding that the AOQ curve displays little difference for both error models and also displays little difference with that of the perfect inspection when process quality, p is less than 0.5. However, the difference of AOQL (Average Outgoing Quality Limit) for both models is much more obvious and it is far from satisfactory when compared with that of the perfect inspection. These effects on AOQL are greatly "discounted" by the fact that most, if not all, of the application lies in the small region of process quality p .

On the other hand, Figure 5.19 & 5.20 illustrate the scenario of the effect of different type I inspection errors while the type II inspection error is fixed to zero (no type I inspection error) and 0.1 respectively based on the linear error model. The trend is reverse to that of the type II inspection error. When the type I inspection error becomes larger, the corresponding average outgoing quality becomes smaller. This conforms to intuition that as the portion of conforming items classified as nonconforming increases, the actual portion of nonconforming items in the passing lot is smaller than observation. Therefore, the actual AOQ value will decrease. This serves as adopting a more “stringent” acceptance policy and the AOQ value will increase equivalently.

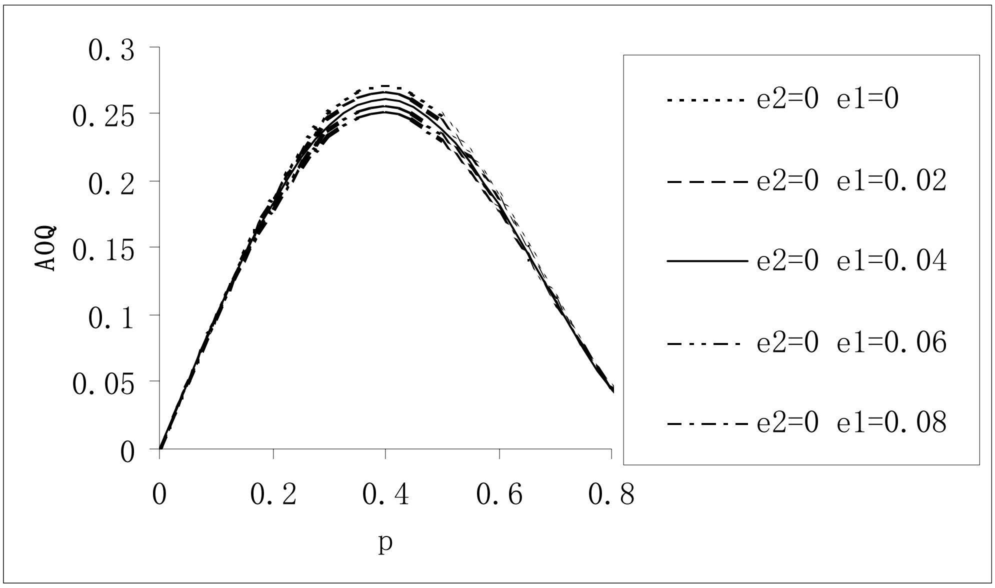

Figure 5. 19 AOQ curve of increased type I inspection error (e_2=0)

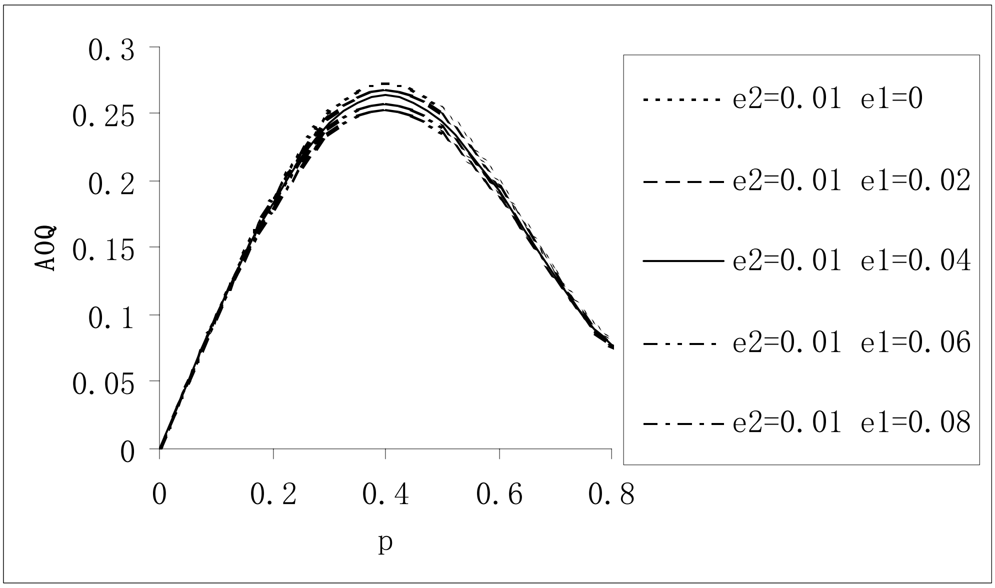

Figure 5. 20 AOQ curve of different type I inspection error (e_2=0.01)

The comparison between two models in terms of the AQO curve for difference type I inspection values is displayed in Figure 5.21~5.23. The observation is similar to that of Figure 5.16 ~5.18.

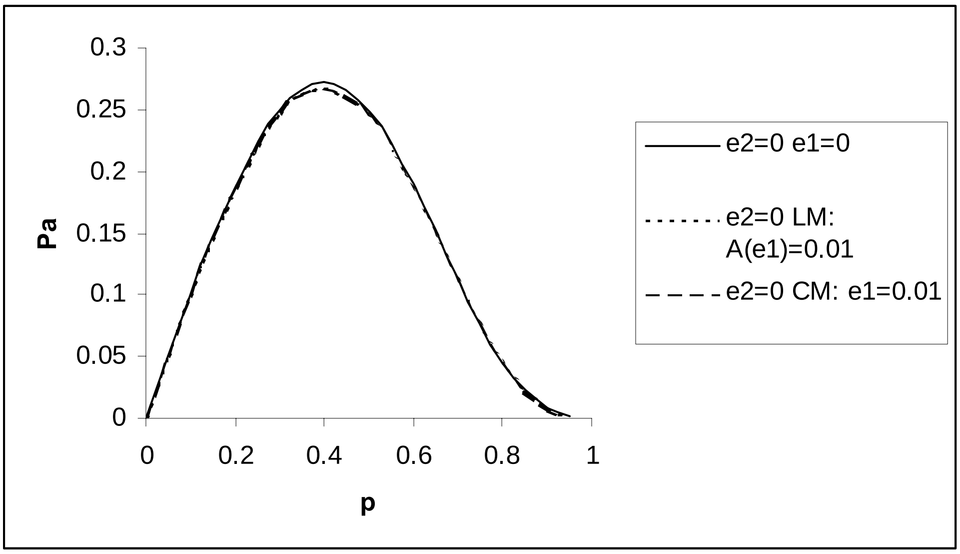

Figure 5. 21 AOQ curve for LM and CM (e_2=0, e_1=0.01)

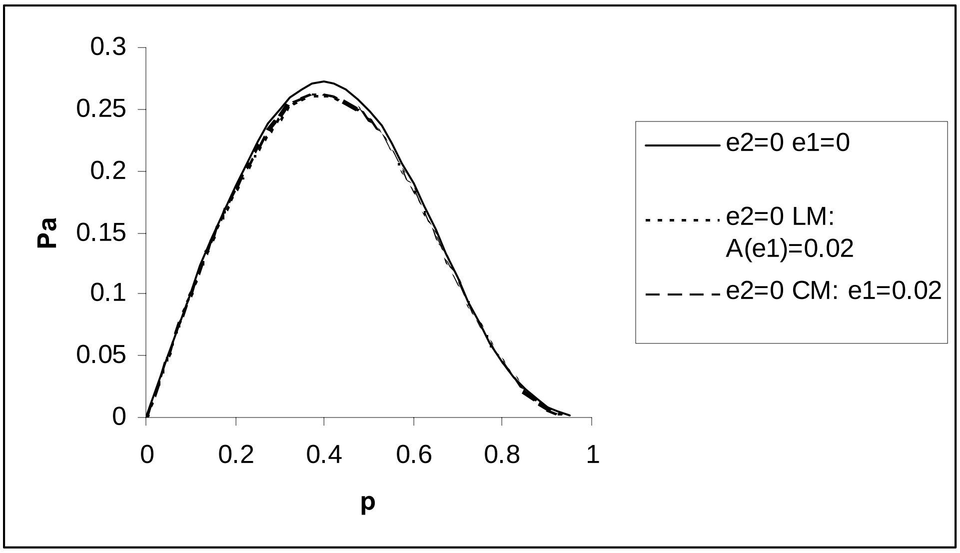

Figure 5. 22 AOQ curve for LM and CM (e_2=0, e_1=0.02)

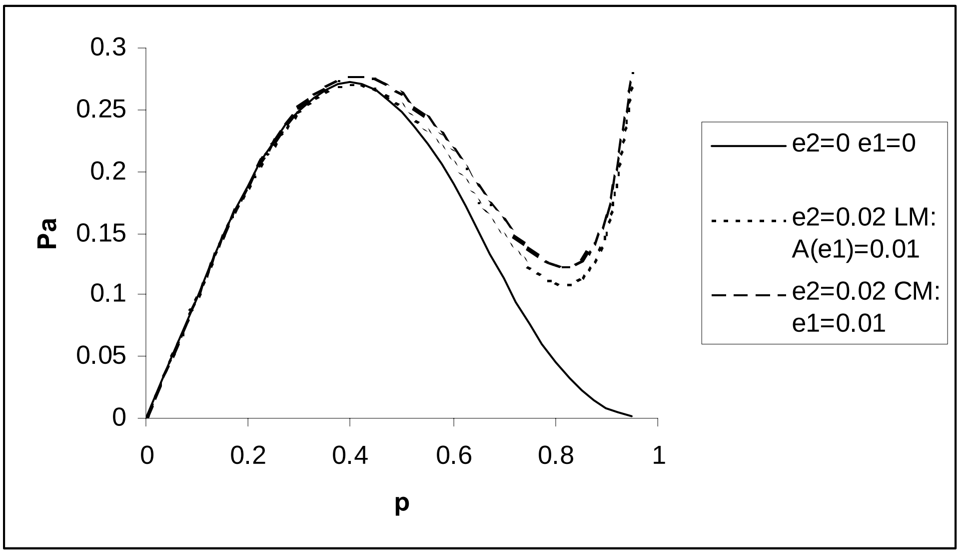

Figure 5. 23 AOQ curve for LM and CM (e_2=0.02, e_1=0.01)

One important result from this study in this section is that the AOQ difference between two models (the constant error model and the linear error model) is very small and it can be regarded as negligible in the real application.

The behavior of the ATI curve for the linearly proportioned inspection error is illustrated in Figure 5.24 and 5.25. The observation is consistent with that of the constant error model and the elaboration is omitted here.

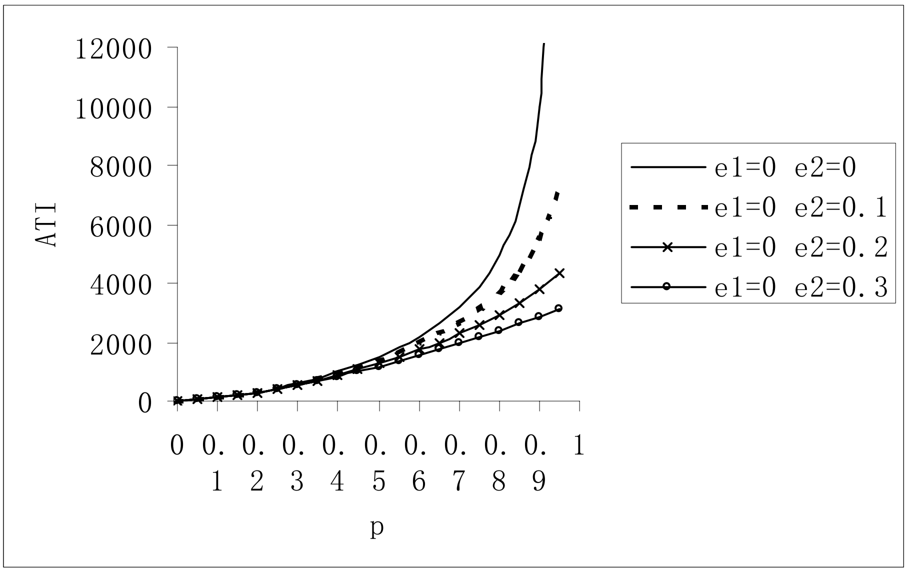

Figure 5. 24 ATI for LM model (e_1=0)

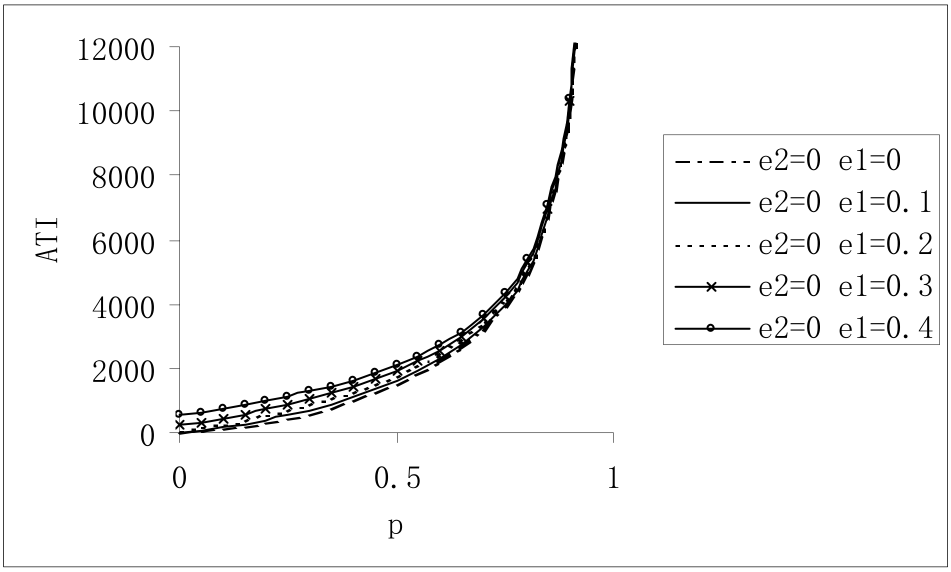

Figure 5. 25 ATI for LM model (e_2=0)

5.4 Conclusion and Remark

Sampling schemes, rather than complete inspection of a production lot, are widely employed in industries to achieve a more economical and efficient use of company resources. Embedded within the design of acceptance sampling plans for attributes is an implicit assumption that the inspection procedure is completely flawless. In reality, however, inspection tasks are seldom error free. On the contrary, they may even be error prone. While inspection errors incurred during the acceptance sampling for attributes are often unintentional and in most cases neglected, they nevertheless can severely distort quality objectives of a system design.

In the previous chapter, the effect of inspection error on chain sampling plan studied by assuming that inspection errors are unchanged throughout the inspection. In this chapter, this strict assumption is relaxed and the effect of inspection errors on chain sampling plan with fluctuating inspection errors is investigated.

In order to examine the effect of inspection errors on statistical quality control procedures, it is necessary to have a model of the process generating the errors. One particular model for errors in the inspection of items on the basis of attributes assumes constant error probability. That is, the probability of committing inspection errors does not change throughout the inspection, which is the basic assumption for the previous chapter. This assumption, though simple and mathematical appealing, does not provide a good representative of the real case.

In this chapter the Biegel (1974) linear model adopted to assume that the error probability is a linear function of the process quality. This is the most reasonable and useful model available so far.

The primary aim of this chapter is to study the behavior of chain sampling schemes when inspection errors are linearly associated with process quality p. Mathematical model and expressions of performance measures such as operating characteristic function, average total inspection and average outgoing quality are derived to aid the analysis of a general chain sampling scheme, ChSP-4A $(c_1, c_2)\, r$, developed by Frishman (1960).

The study further confirms that as the type I inspection error increases, the acceptance probability will decrease while the increment of the type II inspection error will increase the acceptance probability. When both types of inspection errors are small, the difference between two models is small enough to be negligible. It is therefore very safe for application purposes to assume constant error model rather than the complicated linear error model without loss of too much accuracy.

This result is very important in that it provide a justification that the constant error model, rather than the complicated linear model, can be used at the design stage without loss of accuracy. This can greatly reduce the complexity and difficulty in the design stage.

The Study of AOQ curve and ATI curve produces similar result to that of chapter four. That is when the type I inspection error increases, the corresponding AOQ value will decrease, and its ATI will increase. The effect of the type II inspection is on the reverse, i.e. when the type II inspection error gets bigger and bigger, the AOQ will become larger and larger and its ATI will become smaller accordingly. More importantly, the difference of AOQ curve and ATI curve between the constant error model and the linear error model is relatively small and can be neglected in most applications.

The above-mentioned conclusion, together with that of chapter four, forms a good foundation for the further study of chain sampling schemes. In the next chapter, procedures of design chain sampling plan under inspection errors will be proposed, which is the last part, also the most important part, of this work to the error effect on chain sampling schemes.

6. Design of Chain Sampling Plan for Inspection Errors

6.1 Introduction

In the previous two chapters (chapter four and chapter five), the effect of inspection errors on chain sampling plans is studied for the constant error model and the linear error model respectively. The ultimate goal of this study is to propose procedures of designing chain-sampling plans with inspection errors, which is the subject of this chapter.

In this chapter, procedures to design chain-sampling plans when inspection errors are taken into account will be proposed. The proposal is mainly based on the constant error assumption, which means the inspection error remains unchanged throughout inspection activities. As shown in the previous chapter that the difference of the acceptance probability between the constant error model and the linear error model is small enough to be neglected, therefore the constant error model, rather than the linear error model, is used in the design stage to avoid the mathematical complexity and difficulties incurred by the varying error model.

In the next section, the common approach in the design of sampling plans will be presented and a method in the design of chain sampling plans under inspection errors will be proposed. Economical consideration is also touched and is presented in section three. Section four concludes this chapter.

6.2 Binomial model and tables

The primary aim of studying the effect of inspection errors and the influence of other sampling parameters is to design an optimum inspection scheme in the presence of inspection errors. The common approach to design a sampling plan is to fix the OC curve in accordance with the desired degree of discrimination – the OC curve is fixed by suitably choosing parameters such as considering two points on it, usually (AQL,

$1-\alpha$) and (LTPD, β). AQL (the Acceptable Quality Level) reflects a customer's willingness to accept a lot with a small proportion of defectives. It essentially defines the worst quality level for the process, which would be considered acceptable as an overall process average. The probability of rejecting a production lot with such acceptable quality is referred to as the producer's risk, and is designated by $(1-\alpha)$. On the other hand, LTPD (Lot Tolerance Percent Defective) defines the worst quality level that would be considered acceptable as an individual lot. The probability of accepting such a lot is referred to as the consumer's risk, and is designated by the Greek symbol, β. The optimum sampling parameter for chain sampling plans under inspection errors based on the hyper-geometric model can be found by satisfying following inequalities and minimizing the sample size, n:

$$P_{ch}\left(AQL \mid n, N, D, c_1, c_2, r, k, e_1, e_2\right) \geq 1-\alpha \qquad (6\text{-}1)$$

$$P_{ch}\left(LTPD \mid n, N, D, c_1, c_2, r, k, e_1, e_2\right) \leq \beta \qquad (6\text{-}2)$$

Stemming from the discussion in the chapter four that the effect of inspection errors can be "eliminated" by transforming to its equivalent perfect inspection, equation (6-1) and (6-2) can be reduced to the following:

$$P_{ch}\left(AQL^* \mid n, N, D, c_1, c_2, r, k\right) \geq 1-\alpha \qquad (6\text{-}3)$$

$$P_{ch}\left(LTPD^* \mid n, N, D, c_1, c_2, r, k\right) \leq \beta \qquad (6\text{-}4)$$

where,

$$AQL^* = \rho * AQL + \rho'(1-AQL) \qquad (6\text{-}5)$$

$$LTPC^* = \rho * LTPC + \rho'(1-LTPC) \qquad (6\text{-}6)$$

The simplest method to solve non-linear equations (6-3) and (6-4), is to use a constrained optimization software routine such as Solver in Microsoft's Excel.

However, the fact that all parameters must be integer makes the problem too intractable for Solver to handle.

Another difficulty of the hyper-geometric model lies in the lot size (N) and lot defectives (D). In order to use this model to design the sampling plan, the lot size and lot defectives must be specified in advance. This, however, entails great difficulty or inconvenience in application because the common language in industry is the process quality, or the fraction of nonconforming items p. It is thus desirable to use p rather than N and D to be design parameter.

In mathematical arena, binomial distribution is often used to approximate the hyper-geometric distribution when the lot size is large. Binomial model, rather than the hyper-geometric model will be used in the subsequent design because the binomial model meets the above-mentioned requirement in that it employs p rather than N and D as its parameter(s).

The derivation of chain sampling plan based on the binomial model is relatively straight forward, and an outline of this derivation is given here:

The binomial model for the single stage-sampling plan is given by:

$$P_s(p \mid n,c) = \sum_{z=0}^{c} \binom{n}{z} p^z (1-p)^{n-z} \qquad (6\text{-}7)$$

where p is the fraction of nonconforming.

When inspection errors are taken into account, the above formula (6-7) becomes:

$$P_s(\pi \mid n,c) = \sum_{z=0}^{c} \binom{n}{z} \pi^z (1-\pi)^{n-z} \qquad (6\text{-}8)$$

where π is the apparent (observed) fraction of nonconforming, and is given by:

$$\pi = p(1-e_2) + (1-p)e_1 = p - p(e_1+e_2) + e_1 \qquad (6\text{-}9)$$

where e_1 and e_2 are the type I and type II inspection error respectively.

Therefore, the binomial model for the single sampling plan under inspection errors becomes:

$$P_s\left(p \mid n, c, e_1, e_2\right) = \sum_{z=0}^{c} \binom{n}{c} \pi^z \left(1-\pi\right)^{n-z}$$
$$= \sum_{z=0}^{c} \binom{n}{c} \left(p - p\left(e_1 + e_2\right) + e_1\right)^z \left(1 - p + p\left(e_1 + e_2\right) - e_1\right)^{n-z} \tag{6-10}$$

A chain-sampling plan with inspection errors based on the binomial model is therefore given by:

$$P_{ch}\left(p \mid n, c_1, c_2, r, k, e_1, e_2\right)$$
$$= P_s\left(p \mid c_1, n, e_1, e_2\right) + \sum_{z_0=c_1+1}^{r-1} \left(\left(\Pr\left(Z = z_0 \mid n; p, e_1, e_2\right)\right) \bullet \sum_{Z_{total}=z_0}^{c_2} \left(\Pr\left(Z = z_{total} \mid kn; p, e_1, e_2\right)\right) \right)$$
$$= \sum_{z_0=0}^{c_1} \binom{n}{z_0} \left(p - p\left(e_1 + e_2\right) + e_1\right)^{z_0} \left(1 - p + p\left(e_1 + e_2\right) - e_1\right)^{n-z_0}$$
$$+ \sum_{z_0=c_1+1}^{r-1} \left(\begin{array}{l} \binom{n}{z_0} \left(p - p\left(e_1 + e_2\right) + e_1\right)^{z_0} \left(1 - p + p\left(e_1 + e_2\right) - e_1\right)^{n-z_0} \\ \bullet \sum_{z_{pre}=0}^{c_2 - z_0} \left(\binom{kn}{z_{pre}} \left(p - p\left(e_1 + e_2\right) + e_1\right)^{z_{pre}} \left(1 - p + p\left(e_1 + e_2\right) - e_1\right)^{kn - z_{pre}} \right) \end{array} \right)$$

(6-11)

where:

$$c_2 - c_1 \le r$$

$$z_{pre} = z_{total} - z_0$$

One solution to solving difficult non-linear equations (6-3) and (6-4) is to use existing tables indexed by AQL and LTPD constructed under the assumption that the inspection activity is error free for specific values of α and β. One such table is that from Raju and Jothikumar (1997). Here a new library of tables for chain sampling plans will be developed under perfect inspection, and the following example is used to illustrate the application of such tables.

Example 6.1

In the situation in which management wishes to find the optimum chain sampling plan with following characteristics: AQL = 0.1%, α = 0.05, LTPD = 8% and β = 0.05 and its inspection activity is described by parameters, $e_1 = 0.01$ and $e_2 = 0.02$.

AQL^* and $LTPD^*$ can be determined by the following:

$$AQL^* = (1 - 0.1/100) * 0.01 + 0.1/100 * (1 - 0.02) = 0.01097 = 1.097\%$$

$$LTPD^* = (1 - 8/100) * 0.01 + 8/100 * (1 - 0.02) = 0.0876 = 8.76\%$$

Since the table does not provide exact values of AQL^* and $LTPD^*$ calculated. The value of AQL^* has to be rounded **up** to the nearest division (that is 1.1) and $LTPD^*$ rounded **down** to the nearest division (that is, 8.5). Note that the rounding up and down rules are essential to ensure the sampling plan selected would be able to satisfy inequalities (6-3) and (6-4). By using the table, the optimum sampling parameter is found to be ChSP (0,3) 4. The number of preceding lots to be used is 4 and the sample size is 31.

Table 6. 1 Table for chain sampling plans

Table for chain sampling plans in the form of n;ChSP(c1,c2)r for
α = 0.05
β = 0.05
k-1 = 3

LTPD(%)	AQL(%) 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
1.5	199 ;ChSP(0,2)3	204 ;ChSP(0,4)5										
2.0	149 ;ChSP(0,2)3	150 ;ChSP(0,3)4	153 ;ChSP(0,4)5									
2.5	119 ;ChSP(0,1)2	119 ;ChSP(0,2)3	120 ;ChSP(0,3)4	122 ;ChSP(0,4)5								
3.0	99 ;ChSP(0,1)2	99 ;ChSP(0,2)3	100 ;ChSP(0,3)4	102 ;ChSP(0,4)5	102 ;ChSP(0,4)5							
3.5	85 ;ChSP(0,1)2	85 ;ChSP(0,2)3	85 ;ChSP(0,3)4	85 ;ChSP(0,3)4	87 ;ChSP(0,4)5	87 ;ChSP(0,4)5						
4.0	74 ;ChSP(0,1)2	74 ;ChSP(0,2)3	74 ;ChSP(0,2)3	74 ;ChSP(0,3)4	74 ;ChSP(0,3)4	76 ;ChSP(0,4)5	76 ;ChSP(0,4)5					
4.5	66 ;ChSP(0,1)2	66 ;ChSP(0,1)2	66 ;ChSP(0,2)3	66 ;ChSP(0,3)4	66 ;ChSP(0,3)4	67 ;ChSP(0,4)5	67 ;ChSP(0,4)5	67 ;ChSP(0,4)5				
5.0	59 ;ChSP(0,1)2	59 ;ChSP(0,1)2	59 ;ChSP(0,2)3	59 ;ChSP(0,2)3	59 ;ChSP(0,3)4	59 ;ChSP(0,3)4	61 ;ChSP(0,4)5	61 ;ChSP(0,4)5	60 ;ChSP(0,5)6			
5.5	53 ;ChSP(0,1)2	53 ;ChSP(0,1)2	54 ;ChSP(0,2)3	54 ;ChSP(0,2)3	54 ;ChSP(0,3)4	54 ;ChSP(0,3)4	54 ;ChSP(0,3)4	54 ;ChSP(0,4)5	55 ;ChSP(0,4)5	55 ;ChSP(0,4)5		
6.0		49 ;ChSP(0,1)2	49 ;ChSP(0,2)3	49 ;ChSP(0,2)3	49 ;ChSP(0,2)3	49 ;ChSP(0,3)4	49 ;ChSP(0,3)4	50 ;ChSP(0,4)5	50 ;ChSP(0,4)5	50 ;ChSP(0,4)5	50 ;ChSP(0,5)6	
6.5		45 ;ChSP(0,1)2	45 ;ChSP(0,1)2	45 ;ChSP(0,2)3	45 ;ChSP(0,2)3	45 ;ChSP(0,3)4	45 ;ChSP(0,3)4	45 ;ChSP(0,3)4	46 ;ChSP(0,4)5	46 ;ChSP(0,4)5	46 ;ChSP(0,4)5	46 ;ChSP(0,5)6
7.0		42 ;ChSP(0,1)2	42 ;ChSP(0,1)2	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	42 ;ChSP(0,3)4	42 ;ChSP(0,3)4	42 ;ChSP(0,4)5	43 ;ChSP(0,4)5	43 ;ChSP(0,4)5	43 ;ChSP(0,4)5
7.5		39 ;ChSP(0,1)2	39 ;ChSP(0,1)2	39 ;ChSP(0,2)3	39 ;ChSP(0,2)3	39 ;ChSP(0,2)3	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	40 ;ChSP(0,4)5	40 ;ChSP(0,4)5	40 ;ChSP(0,4)5
8.0		36 ;ChSP(0,1)2	36 ;ChSP(0,1)2	36 ;ChSP(0,2)3	36 ;ChSP(0,2)3	36 ;ChSP(0,2)3	36 ;ChSP(0,3)4	37 ;ChSP(0,3)4	37 ;ChSP(0,3)4	37 ;ChSP(0,3)4	37 ;ChSP(0,4)5	37 ;ChSP(0,4)5
8.5		34 ;ChSP(0,1)2	34 ;ChSP(0,1)2	34 ;ChSP(0,1)2	34 ;ChSP(0,2)3	34 ;ChSP(0,2)3	34 ;ChSP(0,2)3	35 ;ChSP(0,3)4	35 ;ChSP(0,3)4	35 ;ChSP(0,3)4	35 ;ChSP(0,3)4	35 ;ChSP(0,4)5
9.0		32 ;ChSP(0,1)2	32 ;ChSP(0,1)2	32 ;ChSP(0,1)2	32 ;ChSP(0,2)3	32 ;ChSP(0,2)3	32 ;ChSP(0,2)3	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4
9.5		31 ;ChSP(0,1)2	31 ;ChSP(0,1)2	31 ;ChSP(0,1)2	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4
10.0		29 ;ChSP(0,1)2	29 ;ChSP(0,1)2	29 ;ChSP(0,1)2	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4

A check on the validity of the suggested plan at two stipulated points of OC curve yields the following:

$$P_{ch}\left(AQL^{*}\right)=0.9505>(1-\alpha)$$

$$P_{ch}\left(LTPD^{*}\right)=0.0411<\beta$$

The above-mentioned example can succinctly demonstrate that tables developed under the assumption of perfect inspection can be used to find the most suitable inspection schemes subjected to inspection errors. However, the drawback of employing such a methodology to obtain a sampling plan is that statisticians seldom include high AQL values in the table – the value of AQL is generally limited to 2%. If e_1 is relatively large, for example 0.05, AQL^{*} would normally exceed 5% (since $AQL^{*}=\rho * AQL+\rho'(1-AQL)$). In such a situation, the management will not be able to search for a suitable plan. Another inherent shortcoming of the table is that it does not offer specific plan for intermediate AQL or LTPD values (that is, if LTPD = 8.8, user has to use a plan for LTPD = 8.5) as illustrated above in Example 6.1. This will lead to inefficiency in sampling procedures – two conditions (6-3) and (6-4) cannot be satisfied by using a smaller sample size. In the long run, the time and economic loss as a result of using a less optimum inspection plan will be quite significant.

6.3 Solution Algorithm

To overcome difficulties that arise from using existing library of tables, a solution algorithm is written to solve for an optimum inspection plan. In a nutshell, the solver tries to find suitable sampling parameters iteratively by hinging on the assumed convergence point ($LTPD$, β) for the single stage and chain sampling plans with the same sample size and acceptance number of the first stage, c_1. Figure 6.1 shows the algorithm for the computer routine developed to find the optimum chain-sampling plan.

STEP 1:

Findings in chapter 4 reveal that chain-sampling plans converge with single sampling plans as the true fraction of nonconforming increases. Therefore, the first step in this analytical approach is to find the smallest sample size for single stage sampling plans that is able to satisfy the following condition:

Minimize n

Subject to: $P_s\left(LTPD^* \mid n,c\right) \leq \beta$

The initial value of acceptance number c is set as 0 so that n can be minimized. For c=0, the smallest sample size, n that is able to satisfy the given condition can be found by the following equation:

$$n = \frac{\ln(\beta)}{Ln\left(1 - LTPD^*\right)} \qquad \text{(6-12)}$$

The value of n found should be rounded up to the nearest integer. It is important to note that the solution obtained will be the initial guess of the sample size, n and the acceptance number for the first stage c_1.

STEP 2:

The rejection number is then increased from 2 till it satisfies the condition:

Minimize: r

Subject to: $P_{ch}\left(AQL^* \mid n, c_1, c_2, r, k,\right) \geq \left(1 - \alpha\right)$

The value of c_2 is initiated to be equivalent to r and $k-1$ is set to be 3. While the number of preceding lots $k-1$ to be used in chain sampling plans can take a value from 1 onwards, the choice of $k-1$ in the algorithm in the view that it is the smallest value that is able to reflect production process consistency (refer to chapter 4), and at the same time, ensure a higher chance of convergence with the single sampling plan below ($LTPD$, β). This choice of k can be altered to meet management needs.

STEP 3:

The acceptance number for the second stage c_2 is decreased from the value of r till the two conditions are satisfied

Minimize: c_2

Subject to: $P_{ch}\left(AQL^* \mid n, c_1, c_2, r, k\right) \geq (1-\alpha)$

The rejection number is then adjusted to the value of c_2 + 1 if c_2 is less than the initial guess of r .

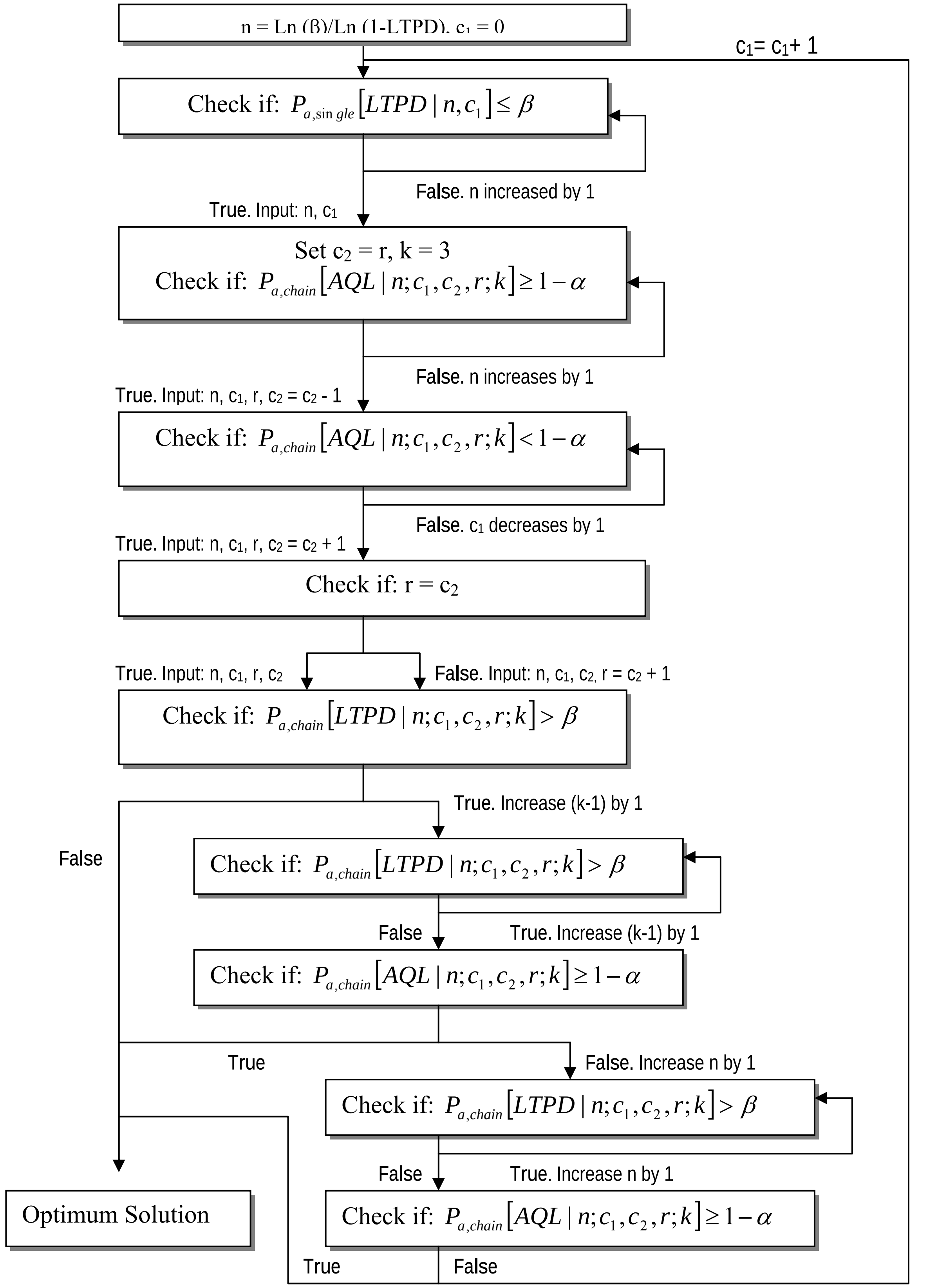

Figure 6. 1 Solution algorithm to design chain sampling plans

STEP 4:

If $P_{ch}\left(LTPD^* \mid n, c_1, c_2, r, k\right) \leq \beta$, the optimum solution is obtained. Otherwise k is increased gradually to satisfy two conditions (6-3) and (6-4) simultaneously. However, increasing k may cause the $P_{ch}\left(AQL^*\right)$ to fall below (1- α). When such situation occurs, the sample size n is increased gradually to satisfy two conditions with k reset to 3. Increasing the sample size has a similar effect on P_{ch} as k, but to a lesser extent. If similar situation occurs, such that $P_{ch}\left(AQL^*\right)$ falls below (1- α), the acceptance number of the first stage c_1 will be increased by 1 and STEPS 1, 2, 3, 4 will be repeated. It is important to note that selection of k and n in the following sequence is motivated by the assumption that the increment in k will incur a smaller cost (than that by) as compared to the increase of sample size n.

Example 6.2

The same condition as example 6.1 is used where management wishes to find the optimum chain sampling plan with following characteristics: AQL = 0.1%, α = 0.05, LTPD = 8% and β = 0.05 and its inspection activity is described by parameters, p = 0.98 and p' = 0.01.

The constructed computer routine based on the iterative methodology is used to find that the optimum sampling plan is ChSP (0, 3) 4, the preceding lot results to be used is 3 and the sample size is 33.

A check on the validity of using the suggested plan at two stipulated points of the OC curve yields the following.

$$P_{ch}\left(AQL^*\right) = 0.95838039 > (1-\alpha)$$

$$P_{ch}\left(LTPD^{*}\right) = 0.04982369 < \beta$$

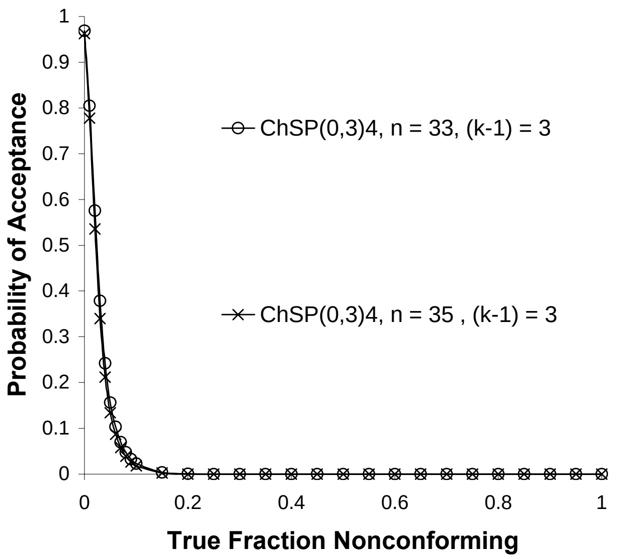

Figure 6. 2 OC curves for both sampling schemes

Figure 6.2 clearly illustrates that the discriminatory power for both inspection schemes is similar to each other since two OC curves superimpose on each other. This implies that there is no unique plan for a set of specified AQL and $LTPD$ values. In situations where two plans are able to satisfy sampling requirements, the deciding factor for the selection of an inspection scheme is the cost of implementing the sampling procedure.

6.5 Conclusion

This chapter is a continuation of chapter four and chapter five, in which the effect of inspection errors on chain sampling plans is studied for the constant error model and the linear error model respectively. In this chapter, procedures of designing chain sampling plans are proposed with inspection errors, which is the most important and useful part of studies of chain sampling plans.

The proposal is mainly based on the constant error assumption, which means inspection errors remain unchanged throughout inspection activities. The justification for this constant error assumption lies in the result from previous chapters, where the difference in the probability of acceptance between the constant error model and the linear error model is proven to be small enough to be neglected. Therefore the constant error model, rather than the linear error model is used in the design stage to avoid the mathematical complexity and difficulties incurred by the varying error model.

The study reveals that two approaches can be adopted to design chain-sampling plans for imperfect inspection. One is to use the existing perfect inspection tables with the adjusted AQL and $LTPD$ value, and another is to use the proposed solution algorithm to search the optimal sampling plan. The first approach is easy to implement but with possible limitation of unavailable tables. The second one is more versatile in terms of the value of AQL and $LTPD$, but at the expense of more complicated and difficult operating procedure. Users can determine their choice based on their available resources.

In all, while plans can be designed to accommodate predetermined level of inspection errors, inspection schemes suggested are generally time consuming and expensive since they all involve a larger sample size. This is especially serious in the presence of the type I error (e_1). In order to minimize such a loss, one solution is to reduce inspection errors through better training and providing a more conducive environment for inspection activity to be carried out. However, the selection of chain plans with consideration of inspection errors still has to be employed, as inspection errors will never be fully eradicated.

7. Chain Sampling Plan for Reliability Acceptance Test

7.1 Introduction

Reliability Acceptance Testing (RAT) or Product Reliability Acceptance Testing (PRAT) is used to sentence a lot according to some reliability requirements. This test may be conducted either by supplier or by customer or both based on agreed sampling plans and acceptance rules.

It is probably the oldest reliability testing technique and also almost the least explored topic in current reliability study, which due partly to the commonly existing misconception that it is too simple to deserve further study. In the 1950s and 1960s, life test had been the subject of extensive research and some concrete results had been produced and become the basis of later reliability acceptance test techniques. In a series of papers devoted to the life test (Epstein & Sobel 1953, Epstein 1954, Epstein & Sobel 1955), Epstein and Sobel presented their results of the life test based on an exponential distribution. In 1961, Gupta and Groll carried out a similar study of life test sampling plans based on a gamma distribution. Similar research about Weibull distribution was deferred until 1980, when Fertig and Mann published their paper "Life-test sampling plans for two parameter Weibull populations". One major reason for this deference lied in the difficulty and complexity of deriving the parameter estimate and its distribution as well as finding its feasible approximation.

Besides the above-mentioned one-stage life test plans, two-stage life test, which offers a better risk control and on average less sampling cost, also appears in literature. Bulgren and Hewett (1973) considered a two-stage test of exponentially distributed lifetime with failure censoring at each state. Fairbanks (1988) presented his two-stage life test for an exponential parameter with a hybrid censoring at each stage. Hewett and Spurrier (1983) gave a thorough survey of two-stage methods, as well as examples of experiments. It is a rather thick and thorough paper consisting of more than one hundred pages.

The focus of the previous approach was on two major sampling schemes: single sampling and sequential sampling. These are two extreme cases of sampling plans, where single sampling plan is the easiest one in terms of operating complexity at the expense of less accurate disposition decision while sequential sampling has the most complicated operation procedure but provides higher degree of disposition accuracy. A possible improvement over these two schemes is to leverage on the advent of information system, which makes the tracking of the previous lot results an easy task. The natural candidate is the Chain sampling plan, a member of cumulative sampling plans, with the feature of incorporating past information. In this chapter, the chain sampling idea will be applied to reliability acceptance testing and design examples based on exponential distribution will be provided.

In the next section, a general procedure to conduct chain sampling reliability acceptance test will be outlined. A detailed discussion based on exponential distribution will be presented in section three, in which examples are also included. Conclusion and remarks are in section four.

7.2 Chain Sampling Plan for Reliability Acceptance Test

In this section, a new reliability acceptance test scheme namely chain-sampling plan will be described for reliability acceptance test. The basic assumption is that the quality characteristic of the test item, life time, follows an identical independent distribution.

The former Dodge (1945) chain sampling plan, though simple in operation, has the advantage of taking previous information into account to achieve a better risk protection at the same or lower sampling cost. Recall its operating procedures from the previous chapter in Figure 2.1.

To adopt the chain idea into reliability acceptance test, propose the following test scheme:

a. Place n items on test until time t,

b. Observe the number of failures, d, occurred in the test

c. If $d \le c$, accept the lot

$d \ge c+2$, reject the lot

$d = c+1$, trace back the information of previous i lots and accept the lot if each of the previous lot has a number of failures less than c, reject the lot otherwise.

The parameter design of the scheme is relatively straightforward:

1. For any test item, fit a suitable distribution to model its lifetime, e.g. $t \sim f(t)$, where t is the time to fail, $f(t)$ is the probability density function of the life model.

2. Obtain c, the probability of failure within time t, through $p = \int_0^t f(t)dt = F(t)$ where, and $F(t)$ is the cumulative density function of the life model.

3. Use binomial theory, and find P_d, the probability of observing d failures within time t for a test sample of sample size n, where $P_d = \binom{n}{d} p^d (1-p)^{n-d}$.

4. The probability of acceptance for this chain sampling reliability acceptance test is therefore given by: $P_{ch} = \sum_{d=0}^{c} P_d + P_{c+1} * P_0^i$

The biggest advantage of this scheme lies in its mathematical simplicity without loss of rigidity, which enables it to incorporate other life model easily. Generally speaking, it applies to all life models so long as the life distribution can be obtained. In the next section this scheme will be illustrated by using exponential distribution, one of the most basic life model in use.

7.3 Exponential Examples

Suppose the lifetime of testing items follows an exponential distribution:

$$x \sim f(x;\theta) = \frac{1}{\theta}\exp(-\frac{x}{\theta}), \qquad (7\text{-}1)$$

where θ is the mean time to failure. The probability of failure within time t is:

$$p = \int_0^t f(t)dt = \int_0^t \frac{1}{\theta}\exp(-\frac{x}{\theta})dx = 1-\exp(-\frac{t}{\theta}). \qquad (7\text{-}2)$$

The probability of observing d failures from a sample of size n within time t is:

$$P(d) = \binom{n}{d} p^d (1-p)^{n-d} = \binom{n}{d}\left(1-\exp(-\frac{t}{\theta})\right)^d \left(\exp(-\frac{t}{\theta})\right)^{n-d},$$

(7-3)

The probability of acceptance based on chain sampling schemes is:

$$\begin{aligned}
P_{ch} &= \sum_{d=0}^{c} P_d + P_{c+1} * P_0^i \\
&= \sum_{d=0}^{c} \binom{n}{d}\left(1-\exp(-\frac{t}{\theta})\right)^d \left(\exp(-\frac{t}{\theta})\right)^{n-d} \\
&\quad + \left[\binom{n}{c+1}\left(1-\exp(-\frac{t}{\theta})\right)^{c+1}\left(\exp(-\frac{t}{\theta})\right)^{n-c-1}\right] * \left[\binom{n}{0}\left(1-\exp(-\frac{t}{\theta})\right)^{0}\left(\exp(-\frac{t}{\theta})\right)^{n-0}\right]^i \\
&= \sum_{d=0}^{c} \binom{n}{d}\left(1-\exp(-\frac{t}{\theta})\right)^d \left(\exp(-\frac{t}{\theta})\right)^{n-d} + \binom{n}{c+1}\binom{n}{c}^i * \left(1-\exp(-\frac{t}{\theta})\right)^{c+1}\left(\exp(-\frac{t}{\theta})\right)^{n(i+1)-c-1}
\end{aligned}$$

(7-4)

To design a chain sampling reliability acceptance test, five parameters are required in advance: testing time t, acceptable mean life θ_0, unacceptable mean life θ_1, producer's risk α, consumer's risk β and the number of previous lots i. Usually, the testing time t and the unacceptable mean life θ_1 are expressed in the ratio of the acceptable mean life θ_0 such as $\frac{t}{\theta_0}$ and $\frac{\theta_1}{\theta_0}$.

A library of tables is provided for different occasions based on different design parameters.

Example 7.1

Find a life test plan, which will be stopped at the occurrence of the fifth failure and will accept a lot having acceptable mean life of 1000 hours with probability 0.95.

Solution:

In this case, $\alpha = 1 - 0.95 = 0.05$, $\theta_0 = 1000$, $r = 5$ (letter r will be used instead of c in the following to denote the number of failures observed);

Suppose we have resources of previous three lots, which is obtained from historical record (the necessary condition for the use of chain sampling plan)

First select a sample of size 20.

From table 7.1 obtain that for $k = 3, r = 5, \alpha = 0.05$, and $n = 20$, the minimum required testing time is $\frac{T}{\theta_0} = 0.1502$.

Therefore: $T = 0.1502 \times \theta_0 = 0.1502 \times 1000 = 150.2$ hours.

The desired sampling plan is thus obtained as follows:

1. Place 20 items in a test for a period of 150.2 hours, and observe the number of failures, d.

2. If the number of observed failures, d is less than or equal to 5 (the specified r), or if 6 failures from the current lot are observed, but no failure is found in the previous three lots, then accept the current lot.
3. Otherwise, reject the current lot.

Table 7. 1Test time for chain sampling reliability acceptance test (T/θ_0)

r	α=0.05 k=3													
	n=													
	2r	3r	4r	5r	6r	7r	8r	9r	10r	11r	12r	13r	14r	15r
1	0.2837	0.1663	0.1184	0.0921	0.0754	0.0638	0.0553	0.0489	0.0437	0.0396	0.0362	0.0333	0.0308	0.0287
2	0.2891	0.1691	0.1201	0.0932	0.0763	0.0645	0.0559	0.0493	0.0441	0.0399	0.0365	0.0335	0.0311	0.0289
3	0.3167	0.1851	0.1313	0.1019	0.0832	0.0704	0.0610	0.0538	0.0481	0.0435	0.0398	0.0366	0.0339	0.0315
4	0.3413	0.1996	0.1417	0.1099	0.0898	0.0759	0.0658	0.0580	0.0519	0.0469	0.0429	0.0394	0.0365	0.0340
5	0.3616	0.2117	0.1502	0.1165	0.0952	0.0805	0.0698	0.0615	0.0550	0.0498	0.0455	0.0418	0.0387	0.0360
6	0.3785	0.2217	0.1574	0.1221	0.0998	0.0844	0.0731	0.0645	0.0577	0.0522	0.0476	0.0438	0.0406	0.0378
7	0.3929	0.2303	0.1635	0.1268	0.1036	0.0876	0.0759	0.0670	0.0599	0.0542	0.0495	0.0455	0.0421	0.0392
8	0.4054	0.2376	0.1687	0.1309	0.1069	0.0904	0.0783	0.0691	0.0618	0.0559	0.0511	0.0470	0.0435	0.0405
9	0.4163	0.2441	0.1733	0.1344	0.1098	0.0929	0.0805	0.0710	0.0635	0.0574	0.0524	0.0482	0.0447	0.0416
10	0.4259	0.2497	0.1773	0.1376	0.1124	0.0950	0.0823	0.0726	0.0650	0.0588	0.0537	0.0494	0.0457	0.0426
11	0.4346	0.2548	0.1809	0.1404	0.1147	0.0970	0.0840	0.0741	0.0663	0.0600	0.0548	0.0504	0.0466	0.0434
12	0.4424	0.2594	0.1842	0.1429	0.1168	0.0987	0.0855	0.0755	0.0675	0.0611	0.0557	0.0513	0.0475	0.0442
13	0.4494	0.2636	0.1871	0.1452	0.1186	0.1003	0.0869	0.0767	0.0686	0.0620	0.0566	0.0521	0.0482	0.0449
14	0.4559	0.2674	0.1898	0.1473	0.1203	0.1018	0.0882	0.0778	0.0696	0.0629	0.0575	0.0529	0.0489	0.0456
15	0.4618	0.2708	0.1923	0.1492	0.1219	0.1031	0.0893	0.0788	0.0705	0.0638	0.0582	0.0535	0.0496	0.0462
16	0.4673	0.2741	0.1946	0.1510	0.1234	0.1043	0.0904	0.0797	0.0713	0.0645	0.0589	0.0542	0.0502	0.0467
17	0.4724	0.2771	0.1967	0.1526	0.1247	0.1054	0.0913	0.0806	0.0721	0.0652	0.0595	0.0548	0.0507	0.0472
18	0.4771	0.2798	0.1987	0.1541	0.1260	0.1065	0.0923	0.0814	0.0728	0.0659	0.0601	0.0553	0.0512	0.0477
19	0.4816	0.2824	0.2005	0.1556	0.1271	0.1075	0.0931	0.0821	0.0735	0.0665	0.0607	0.0558	0.0517	0.0481
20	0.4857	0.2848	0.2022	0.1569	0.1282	0.1084	0.0939	0.0829	0.0741	0.0670	0.0612	0.0563	0.0521	0.0485
21	0.4896	0.2871	0.2039	0.1582	0.1292	0.1093	0.0947	0.0835	0.0747	0.0676	0.0617	0.0568	0.0526	0.0489
22	0.4933	0.2893	0.2054	0.1594	0.1302	0.1101	0.0954	0.0841	0.0753	0.0681	0.0622	0.0572	0.0529	0.0493
23	0.4967	0.2913	0.2068	0.1605	0.1311	0.1109	0.0960	0.0847	0.0758	0.0686	0.0626	0.0576	0.0533	0.0496
24	0.5000	0.2932	0.2082	0.1615	0.1320	0.1116	0.0967	0.0853	0.0763	0.0690	0.0630	0.0580	0.0537	0.0500
25	0.5031	0.2950	0.2095	0.1625	0.1328	0.1123	0.0973	0.0858	0.0768	0.0694	0.0634	0.0583	0.0540	0.0503

Example 7.2

Find a life test plan, which will be stopped at the occurrence of the fifth failure and will reject a lot having acceptable mean life less than 500 hours with probability 0.95.

Solution:

In this case, $\beta = 0.05$, $\theta_1 = 500$, $r = 5$;

First, choose a sample of size 20.

From table 7.2 obtain that for $k = 3$, $r = 5$, $\beta = 0.05$, and $n = 20$, the minimum required testing time is $\frac{T}{\theta_1} = 0.6078$.

Therefore: $T = 0.6078 \times \theta_1 = 0.6078 \times 500 = 303.9$ hours.

Table 7. 2 Test time for chain sampling reliability acceptance test (T / θ_1)

r	β=0.05 k=3													
	n=													
	2r	3r	4r	5r	6r	7r	8r	9r	10r	11r	12r	13r	14r	15r
1	3.6741	1.9988	1.3912	1.0707	0.8714	0.7351	0.6358	0.5603	0.5009	0.4529	0.4133	0.3801	0.3519	0.3275
2	2.3257	1.3038	0.9151	0.7067	0.5762	0.4866	0.4212	0.3713	0.3320	0.3003	0.2741	0.2521	0.2334	0.2172
3	1.8755	1.0640	0.7491	0.5792	0.4725	0.3992	0.3456	0.3047	0.2725	0.2465	0.2250	0.2069	0.1916	0.1783
4	1.6450	0.9391	0.6622	0.5124	0.4181	0.3533	0.3059	0.2697	0.2412	0.2182	0.1992	0.1832	0.1696	0.1579
5	1.5026	0.8611	0.6078	0.4705	0.3840	0.3245	0.2810	0.2478	0.2216	0.2005	0.1830	0.1683	0.1558	0.1451
6	1.4048	0.8073	0.5702	0.4415	0.3604	0.3046	0.2637	0.2326	0.2080	0.1882	0.1718	0.1580	0.1463	0.1362
7	1.3330	0.7675	0.5424	0.4200	0.3429	0.2898	0.2510	0.2213	0.1980	0.1791	0.1635	0.1504	0.1392	0.1296
8	1.2776	0.7367	0.5208	0.4034	0.3294	0.2784	0.2411	0.2126	0.1902	0.1720	0.1570	0.1444	0.1337	0.1245
9	1.2334	0.7121	0.5036	0.3901	0.3185	0.2692	0.2331	0.2056	0.1839	0.1663	0.1519	0.1397	0.1293	0.1204
10	1.1972	0.6918	0.4894	0.3791	0.3096	0.2617	0.2266	0.1998	0.1788	0.1617	0.1476	0.1358	0.1257	0.1170
11	1.1668	0.6749	0.4775	0.3699	0.3021	0.2553	0.2211	0.1950	0.1744	0.1578	0.1440	0.1325	0.1227	0.1142
12	1.1410	0.6604	0.4673	0.3621	0.2957	0.2499	0.2164	0.1909	0.1707	0.1544	0.1410	0.1297	0.1201	0.1118
13	1.1186	0.6478	0.4585	0.3553	0.2901	0.2452	0.2124	0.1873	0.1675	0.1516	0.1383	0.1273	0.1178	0.1097
14	1.0991	0.6368	0.4507	0.3493	0.2853	0.2411	0.2088	0.1842	0.1647	0.1490	0.1360	0.1251	0.1158	0.1078
15	1.0818	0.6271	0.4439	0.3440	0.2810	0.2375	0.2057	0.1814	0.1623	0.1468	0.1340	0.1232	0.1141	0.1062
16	1.0665	0.6184	0.4378	0.3393	0.2771	0.2342	0.2029	0.1789	0.1600	0.1448	0.1322	0.1216	0.1126	0.1048
17	1.0526	0.6106	0.4324	0.3351	0.2737	0.2313	0.2004	0.1767	0.1581	0.1430	0.1305	0.1201	0.1112	0.1035
18	1.0402	0.6036	0.4274	0.3312	0.2705	0.2287	0.1981	0.1747	0.1563	0.1413	0.1290	0.1187	0.1099	0.1023
19	1.0288	0.5972	0.4229	0.3278	0.2677	0.2263	0.1960	0.1729	0.1546	0.1399	0.1277	0.1174	0.1087	0.1012
20	1.0184	0.5913	0.4188	0.3246	0.2651	0.2241	0.1941	0.1712	0.1531	0.1385	0.1264	0.1163	0.1077	0.1002
21	1.0089	0.5859	0.4150	0.3216	0.2627	0.2221	0.1923	0.1696	0.1517	0.1373	0.1253	0.1153	0.1067	0.0993
22	1.0001	0.5809	0.4115	0.3189	0.2605	0.2202	0.1907	0.1682	0.1505	0.1361	0.1242	0.1143	0.1058	0.0985
23	0.9920	0.5763	0.4082	0.3164	0.2584	0.2185	0.1892	0.1669	0.1493	0.1350	0.1233	0.1134	0.1050	0.0977
24	0.9844	0.5721	0.4052	0.3141	0.2565	0.2169	0.1878	0.1657	0.1482	0.1340	0.1224	0.1126	0.1042	0.0970
25	0.9773	0.5681	0.4024	0.3119	0.2548	0.2154	0.1865	0.1645	0.1472	0.1331	0.1215	0.1118	0.1035	0.0964

The desired sampling plan is thus obtained as follows:

1. Place 20 items in a test for a period of 303.9 hours, and observe the number of failures d.
2. If the number of observed failures, d is less than or equal to 5 (the specified r), or if 6 failures from the current lot are observed, but no failure is found in the previous three lots, then accept the current lot.
3. Otherwise, reject the current lot.

Example 7.3

Find a life test plan, which will accept a lot having acceptable mean life of 1000 hours with probability 0.95 and reject a lot having acceptable mean life less than 500 hours with probability 0.95.

Table 7. 3 Value of θ_1/θ_0 for chain sampling reliability acceptance test

r	α=0.05 β=0.05 k=3													
	n=													
	2r	3r	4r	5r	6r	7r	8r	9r	10r	11r	12r	13r	14r	15r
1	0.0832	0.0851	0.0860	0.0865	0.0868	0.0870	0.0872	0.0873	0.0874	0.0875	0.0875	0.0876	0.0876	0.0877
2	0.1297	0.1312	0.1319	0.1323	0.1326	0.1327	0.1328	0.1329	0.1330	0.1330	0.1331	0.1331	0.1332	0.1332
3	0.1739	0.1753	0.1759	0.1762	0.1763	0.1765	0.1765	0.1766	0.1767	0.1767	0.1767	0.1768	0.1768	0.1768
4	0.2126	0.2139	0.2145	0.2147	0.2149	0.2150	0.2151	0.2151	0.2151	0.2152	0.2152	0.2152	0.2152	0.2152
5	0.2458	0.2472	0.2477	0.2480	0.2481	0.2482	0.2483	0.2484	0.2484	0.2484	0.2485	0.2485	0.2484	0.2485
6	0.2747	0.2760	0.2766	0.2768	0.2770	0.2771	0.2772	0.2772	0.2773	0.2772	0.2773	0.2773	0.2773	0.2774
7	0.3000	0.3014	0.3019	0.3022	0.3023	0.3024	0.3025	0.3026	0.3026	0.3026	0.3026	0.3027	0.3026	0.3027
8	0.3225	0.3239	0.3244	0.3247	0.3249	0.3249	0.3250	0.3251	0.3251	0.3251	0.3251	0.3252	0.3252	0.3252
9	0.3427	0.3441	0.3446	0.3449	0.3450	0.3451	0.3452	0.3452	0.3453	0.3453	0.3454	0.3453	0.3454	0.3453
10	0.3610	0.3623	0.3628	0.3631	0.3633	0.3634	0.3634	0.3635	0.3635	0.3636	0.3636	0.3636	0.3636	0.3636
11	0.3776	0.3789	0.3795	0.3797	0.3799	0.3800	0.3800	0.3801	0.3801	0.3801	0.3802	0.3802	0.3802	0.3803
12	0.3928	0.3941	0.3947	0.3949	0.3951	0.3952	0.3953	0.3953	0.3954	0.3954	0.3954	0.3954	0.3954	0.3955
13	0.4068	0.4081	0.4087	0.4089	0.4091	0.4092	0.4093	0.4093	0.4094	0.4094	0.4094	0.4094	0.4094	0.4095
14	0.4198	0.4211	0.4217	0.4219	0.4220	0.4222	0.4222	0.4223	0.4223	0.4223	0.4224	0.4224	0.4224	0.4224
15	0.4319	0.4332	0.4337	0.4340	0.4341	0.4342	0.4343	0.4343	0.4344	0.4344	0.4344	0.4344	0.4344	0.4344
16	0.4431	0.4444	0.4449	0.4452	0.4454	0.4455	0.4455	0.4456	0.4456	0.4456	0.4457	0.4456	0.4457	0.4456
17	0.4537	0.4550	0.4555	0.4557	0.4559	0.4560	0.4560	0.4561	0.4561	0.4561	0.4562	0.4562	0.4562	0.4562
18	0.4636	0.4648	0.4653	0.4656	0.4658	0.4659	0.4659	0.4660	0.4660	0.4660	0.4661	0.4660	0.4661	0.4660
19	0.4729	0.4742	0.4746	0.4749	0.4750	0.4752	0.4752	0.4753	0.4753	0.4753	0.4754	0.4754	0.4754	0.4754
20	0.4817	0.4829	0.4834	0.4837	0.4838	0.4839	0.4840	0.4840	0.4840	0.4841	0.4841	0.4841	0.4841	0.4841
21	0.4900	0.4913	0.4917	0.4920	0.4922	0.4922	0.4923	0.4924	0.4924	0.4924	0.4925	0.4924	0.4924	0.4925
22	0.4979	0.4991	0.4996	0.4999	0.5000	0.5001	0.5001	0.5002	0.5003	0.5003	0.5003	0.5003	0.5003	0.5004
23	0.5054	0.5066	0.5071	0.5074	0.5075	0.5076	0.5076	0.5077	0.5077	0.5077	0.5078	0.5078	0.5078	0.5078
24	0.5126	0.5138	0.5142	0.5145	0.5146	0.5147	0.5148	0.5148	0.5148	0.5149	0.5149	0.5149	0.5149	0.5150
25	0.5194	0.5206	0.5210	0.5213	0.5215	0.5215	0.5216	0.5216	0.5216	0.5217	0.5217	0.5217	0.5217	0.5218

Solution:

In this case, $\alpha = \beta = 0.05$, $\theta_0 = 1000$, $\theta_1 = 500$; $\theta_1/\theta_0 = 500/1000 = 0.5$

Suppose we have resources of previous three lots, which is obtained from historical record (the necessary condition for the use of chain sampling plan)

First, select a suitable combination of acceptance number and the sample size. From Table 7.3, when $r = 22$ and $n = 6r = 132$, the value of θ_1/θ_0 is exactly 0.5, therefore choose the acceptance number as 22 and the sample size as 132.

Next, repeat the procedure of Example 7.1.

From Table 7.1 obtain that for $k = 3$, $r = 22$, $\alpha = 0.05$, and $n = 132$, the minimum required testing time is $\frac{T}{\theta_0} = 0.1302$.

Therefore: $T = 0.1302 \times \theta_0 = 0.1302 \times 1000 = 130.2$ hours.

The desired sampling plan is thus obtained as follows:

1. Place 132 items in a test for a period of 130.2 hours, and observe the number of failures, d .
2. If the number of observed failures, d is less than or equal to 22 (the specified r), or if 23 failures from the current lot are observed, but no failure is found in the previous three lots, then accept the current lot.
3. Otherwise, reject the current lot.

In Example 7.3, after the acceptance number and the sample size are fixed, Table 7.2 can also be used to find the required testing time. In this case, obtain $\frac{T}{\theta_1} = 0.2605$ and $T = 0.2605 \times \theta_1 = 130.3$ hours. The result is the same as that obtained from Table 7.1.

A library of tables is provided in the appendix for more values of α , β , and k . Users can follow the similar procedure of above examples to design their required chain sampling reliability acceptance test plans.

Similar to that of chapter 6, the use of indexed tables will be cumbersome in some cases, and it is also possible to find that the desired table that is not available as the library of tables provided is covering the most frequently used value only. To overcome these difficulties, an Excel routine template provided to facilitate the design of the sampling plan. They are explained in the following figure:

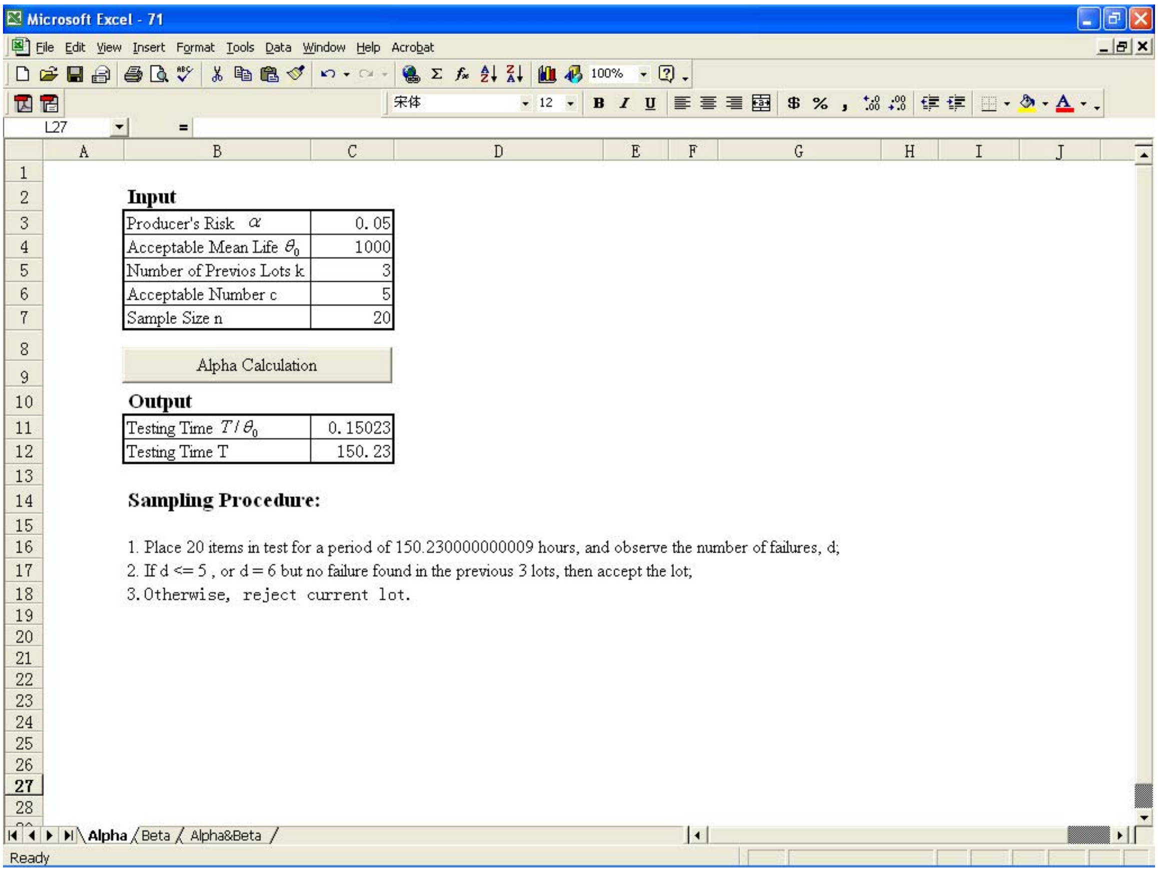

Figure 7. 1Excel template for example 7.1

The usage of this template is very straightforward. Key in the required parameters and press the "Alpha Calculation" button, results will appear within seconds. Users can simply follow the provided sampling procedures to conduct their reliability acceptance test. The result in this template is more accurate than those obtained from tables.

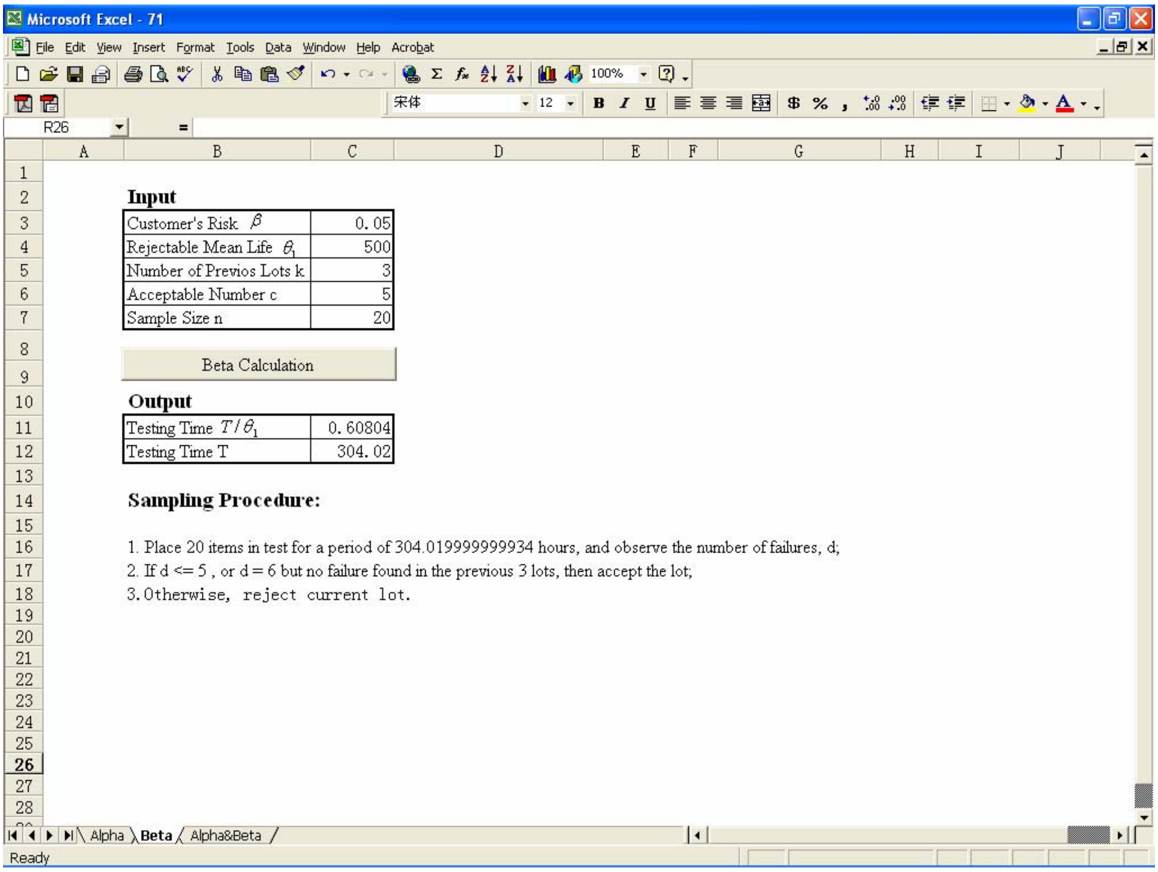

Figure 7. 2 Excel template for example 7.2

Figure 7.2 and 7.3 are the Excel template for example 7.2 and 7.3 respectively. They are easy to operate and users have the flexibility of choosing their desired parameters rather rigidly following the table. Careful comparison of the result from these templates and those from the table reveals a little difference, and users can choose either the existing tables or these templates at their convenience. One inconvenience lies in Figure 7.4, which requires users to wait for a much longer time in order to obtain the result. Efforts are required to revise the algorithm to make the computation faster.

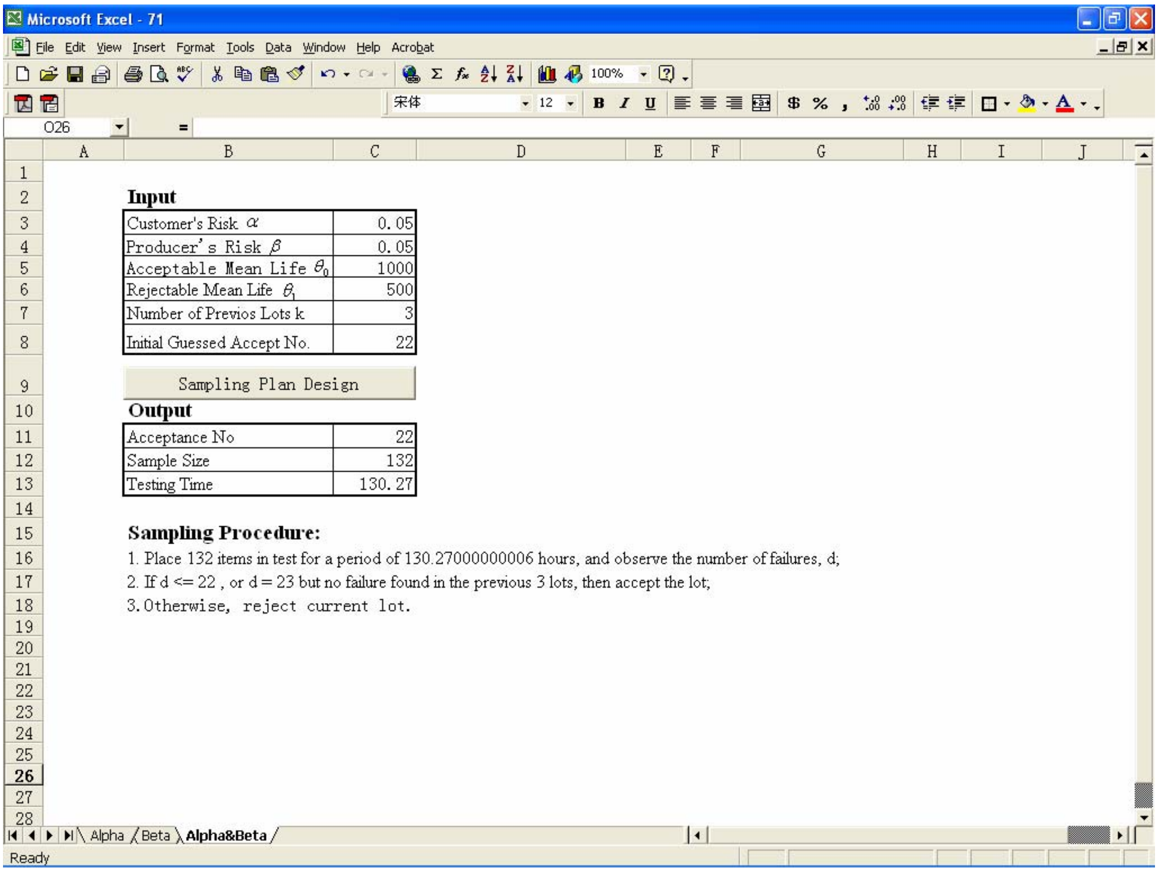

Figure 7. 3 Excel template for example 7.3

7.4 Conclusion and Remark

Reliability Acceptance Testing (RAT) or Product Reliability Acceptance Testing (PRAT) is probably the oldest reliability testing technique, which is used to sentence a lot according to some reliability requirements, which is conducted either by suppliers or by customers or both based on agreed sampling plans and acceptance rules.

It is a hot subject with extensive research in the 1950s and 1960s, after which it became silent because of the misconception that it is too simple to deserve further study. It is therefore not surprising to find that most of the techniques developed that period are still serving industries now.

In this chapter, chain sampling schemes for reliability acceptance test are proposed to complement the existing commonly used two schemes: single sampling plan and sequential sampling plan. Besides the mathematical description, tables for the selection of sampling parameters and Excel templates are provided to facilitate the design procedure and to give users more flexibility in application.

This is a rather interesting and useful topic, and the content presented is this chapter is only the beginning of this research. Further extensive research is required for other lifetime distribution models. Its massive application is to be addressed as well.

8. Conclusions and Remarks

This thesis focuses on Chain Sampling Schemes, a versatile sampling plan that is found useful in the costly or destructive testing. Several issues related to this sampling scheme are addressed. The first concern is the effect of the correlation on the performance of chain sampling plan, which is driven by an industrial project where the correlated production are subjected to testing for disposition decision.

It is where our interest in chain sampling plan is developed from, and serves as an introduction and catalyst to spark our interest to further explore it. We then proceed to study the effect of inspection errors on the sampling plan, specifically, its effect on chain sampling plan. It is a difficult task and cost most of my time in my research. We tackle this problem phase by phase and gradually we manage to come up the final result. The three stages or phases of this project are effect of constant inspection errors, effect of variable inspection errors, and the design of chain sampling plan. The first two stages is the foundation of our inspection error study and the final stage, design of chain sampling plan, complete our study on the inspection error.

The final part of this thesis goes to the reliability engineering, while the previous two topics fall in the category of quality engineering. In this part, we extend the chain sampling to reliability acceptance test and propose our approaches to design chain-sampling plans for reliability acceptance test. Its mathematical models are relatively straightforward, but the results are useful in application.

In chapter three, we present our study of the effect of correlation on chain sampling plan, which is actually an abstract and extension of our industrial project. We develop our model, Chain Sampling Plan with Markov Property, and conduct the numerical analysis. Our analysis reveals that for a given " p ", the probability of acceptance is smaller when a negative correlation is taken into consideration, i.e. when the correlation is negative, the proposed model is more discriminating than the Dodge ChSP-1. The discriminating power increases as δ , the correlation parameter, increases. The reverse is true when the correlation coefficient is positive. The corresponding probability of acceptance is larger for a given " p " and thus the discrimination power is less than that of the Dodge plan. The implication in practice is that when the Dodge ChSP-1 plan is applied to samples with positive correlation, the resulting probability of acceptance is smaller than what it is supposed to be and will lead to a more conservative decision. On the other hand, when there is a negative correlation, the Dodge ChSP-1 plan must be used with caution as its probability of acceptance and average outgoing quality are larger than the actual values given in our plan.

Another finding from chapter three is that it is advisable to use a number of previous lots of three as the choice of an important design parameter. The reason is simple. As the OC curve and AOQ curve indicate that any lots number less than three will compromise robustness and a larger lots number than three will incur additional cost.

Chapter 4 starts the study of the effect of inspection errors on chain sampling plan, in which inspection errors are assumed to remain constant thorough out the inspection, i.e. the constant error model. In this chapter, we extend the inspection error consideration to chain sampling schemes and develop a mathematical model to investigate the performance of chain sampling schemes when inspection errors are taken into consideration. We also derive expressions of performance measures such as the operating characteristic function, average total inspection and average outgoing quality to aid the analysis of a general chain sampling scheme, ChSP-4A (c_1 , c_2) r , developed by Frishman (1960).

Our study reveals that as type I inspection error increases, the acceptance probability will decrease while the increment of type II inspection error will increase the acceptance probability. The effect of type II error on the sampling acceptance probability is very marginal as compared to that of type I error especially when the true fraction of nonconforming is small. An important conclusion from this chapter is that the effect of inspection errors can be "eliminated" by transforming to its equivalent perfect inspection counterpart, thus greatly reduces the complexity of the analysis.

Effects of inspection errors on the AOQ curve and ATI curve are complicated. As type I inspection error increases, the corresponding AOQ value will decrease and its ATI will increase. The effect of type II inspection is on the reverse, i.e. when type II inspection error gets bigger, the AOQ will become larger and its ATI will become smaller accordingly. These confounding effects deserve careful consideration before any decision can be reached. One guideline is that type I inspection error usually plays a prominent role in small fraction defectives while type II inspection error has more weight on large fraction of nonconforming product. Accordingly, type II inspection error plays a dominant role in determining the final average outgoing quality limit (AOQL). Simulation shows that even a small type II inspection error will lead the final AOQL to almost 1.

Analysis of the AOQ and ATI also tells us that the effectiveness of sampling plans can only be maintained when both types of inspection error are relatively small. If the inspection error is large, either type I or type II, sampling schemes will not be effective any more. The final outgoing quality after inspection will be barely improved, which implies that there is no point to implement sampling plan when the inspection error is large.

Chapter five is a counterpart of chapter four with the underlying assumption changed from constant error model to variable error model. The variable error is in fact very complicated and we adopt the Biegel (1974) linear model to simplify the problem. We go through the similar study to that of chapter 4 focusing the difference between these two models. Our study further confirms that as type I inspection error increase, the acceptance probability will decrease while the increment of type II inspection error will increase the acceptance probability. When both types of inspection error are small, the differences between both models are smaller enough to be negligible. It is therefore very safe for application purpose to assume constant error model rather than the complicated linear error model without loss of too much accuracy.

This result is very important in that it provide us justifications that when come to the design stage we can use the constant error model rather than the complicated linear model without loss of accuracy. This can greatly reduce the complexity and difficulties in the design stage.

Chapter 6 is the final and the most important part of our inspection error effect study. We propose our procedures of designing chain-sampling plans when inspection errors are taken into consideration; we propose two approaches to design the chain sampling plans for imperfect inspection. One is to use the existing perfect inspection tables with adjusted AQL and LTPD values, and the other is to use our solution algorithm to search the optimal sampling plans. The first approach is easy to implement but with possible limitation of unavailable tables. The second one is more versatile in terms of values of *AQL* and LTPD, but at the expense of a more complicated and difficult operating procedures. To conclude our error effect study, it should be noted that while plans can be designed to accommodate predetermined level of inspection errors, the inspection schemes suggested are generally more time consuming and costly since they all involve a larger sample size. This is especially serious in the presence of type I error (e_1). In order to minimize such losses, the solution would be to reduce inspection errors through better training and providing a more conducive environment for inspection activity to be carried out. However, the selection of chain plans with consideration of inspection errors will still have to be employed as inspection errors will never be fully eradicated.

Chapter 7 focuses on the application of chain sampling plan in Reliability Acceptance Testing (RAT) or Product Reliability Acceptance Testing (PRAT), which is probably the oldest reliability testing technique. Its purpose is to supply suitable sampling procedures and based on which a lot is sentenced according to some reliability requirements. It is conducted either by the supplier or the customer or both based on agreed sampling plans and acceptance rules.

This was a very hot topic and received extensive study in the 1950s and 1960s, after which it became silent because of the misconception that it is too simple to deserve further study. It is therefore no surprised to find that most of the techniques developed at that period are still serving our industries now.

In this chapter, we proposed our chain sampling schemes for reliability acceptance test to complement the existing commonly used two schemes: single sampling plan and sequential sampling plan. Besides the mathematical description we provide our tables for the selection of sampling parameter, and Excel templates are also provided to facilitate the design and provide more flexibility for the usage. Examples are included to illustrate the use of proposed methods. It is a rather interesting and useful topic, and the content presented is this chapter is only the beginning of this research. Further extensive research is required for other lifetime distribution models. Its massive application is to be addressed as well.

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Appendix A Tables for Chain Sampling Plan

Table A. 1Tables for Selection of Chain Sampling Plans indexed by α and β

Tables of sampling plans available for different values of α and β	Page
α = 0.01 , β = 0.01	152
α = 0.01 , β = 0.05	153
α = 0.01 , β = 0.10	154
α = 0.05 , β = 0.01	155
α = 0.05 , β = 0.05	156
α = 0.05 , β = 0.10	157
α = 0.10 , β = 0.01	158
α = 0.10 , β = 0.05	159
α = 0.10 , β = 0.10	160

Table for chain sampling plans in the form of n;ChSP(c1,c2)r for

$\alpha =$ 0.01
$\beta =$ 0.01
k-1 = 3

LTPD(%)	AQL(%) 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
1.5	305 ;ChSP(0,4)5	315 ;ChSP(0,7)8										
2.0	228 ;ChSP(0,3)4	229 ;ChSP(0,5)6	236 ;ChSP(0,7)8									
2.5	182 ;ChSP(0,3)4	183 ;ChSP(0,5)6	185 ;ChSP(0,6)7	186 ;ChSP(0,7)8								
3.0	152 ;ChSP(0,3)4	152 ;ChSP(0,4)5	152 ;ChSP(0,5)6	157 ;ChSP(0,7)8	161 ;ChSP(0,8)9							
3.5	130 ;ChSP(0,2)3	130 ;ChSP(0,4)5	130 ;ChSP(0,5)6	131 ;ChSP(0,6)7	134 ;ChSP(0,7)8	138 ;ChSP(0,8)9						
4.0	13 ;ChSP(0,2)3	113 ;ChSP(0,3)4	113 ;ChSP(0,4)5	114 ;ChSP(0,5)6	115 ;ChSP(0,6)7	117 ;ChSP(0,7)8	121 ;ChSP(0,8)9					
4.5	101 ;ChSP(0,2)3	101 ;ChSP(0,3)4	101 ;ChSP(0,4)5	101 ;ChSP(0,5)6	102 ;ChSP(0,6)7	104 ;ChSP(0,7)8	104 ;ChSP(0,7)8	107 ;ChSP(0,8)9				
5.0	90 ;ChSP(0,2)3	90 ;ChSP(0,3)4	90 ;ChSP(0,4)5	91 ;ChSP(0,5)6	91 ;ChSP(0,5)6	91 ;ChSP(0,6)7	93 ;ChSP(0,7)8	93 ;ChSP(0,7)8	96 ;ChSP(0,8)9			
5.5	82 ;ChSP(0,2)3	82 ;ChSP(0,3)4	82 ;ChSP(0,4)5	82 ;ChSP(0,4)5	82 ;ChSP(0,5)6	83 ;ChSP(0,6)7	83 ;ChSP(0,6)7	85 ;ChSP(0,7)8	83 ;ChSP(0,7)8	87 ;ChSP(0,8)9		
6.0	75 ;ChSP(0,2)3	75 ;ChSP(0,3)4	75 ;ChSP(0,3)4	75 ;ChSP(0,4)5	75 ;ChSP(0,5)6	75 ;ChSP(0,5)6	76 ;ChSP(0,6)7	76 ;ChSP(0,6)7	77 ;ChSP(0,7)8	80 ;ChSP(0,8)9	80 ;ChSP(0,8)9	
6.5	69 ;ChSP(0,2)3	69 ;ChSP(0,3)4	69 ;ChSP(0,3)4	69 ;ChSP(0,4)5	69 ;ChSP(0,5)6	69 ;ChSP(0,5)6	70 ;ChSP(0,6)7	70 ;ChSP(0,6)7	71 ;ChSP(0,7)8	71 ;ChSP(0,7)8	74 ;ChSP(0,8)9	74 ;ChSP(0,8)9
7.0	64 ;ChSP(0,2)3	64 ;ChSP(0,2)3	64 ;ChSP(0,3)4	64 ;ChSP(0,4)5	64 ;ChSP(0,4)5	64 ;ChSP(0,5)6	64 ;ChSP(0,6)7	65 ;ChSP(0,6)7	65 ;ChSP(0,6)7	66 ;ChSP(0,7)8	66 ;ChSP(0,7)8	66 ;ChSP(0,7)8
7.5	60 ;ChSP(0,2)3	60 ;ChSP(0,2)3	60 ;ChSP(0,3)4	60 ;ChSP(0,4)5	60 ;ChSP(0,4)5	60 ;ChSP(0,4)5	60 ;ChSP(0,6)7	60 ;ChSP(0,6)7	60 ;ChSP(0,6)7	60 ;ChSP(0,6)7	62 ;ChSP(0,7)8	62 ;ChSP(0,7)8
8.0	58 ;ChSP(0,1)2	56 ;ChSP(0,2)3	56 ;ChSP(0,3)4	56 ;ChSP(0,3)4	56 ;ChSP(0,4)5	56 ;ChSP(0,4)5	56 ;ChSP(0,5)6	56 ;ChSP(0,5)6	57 ;ChSP(0,6)7	57 ;ChSP(0,6)7	58 ;ChSP(0,7)8	58 ;ChSP(0,7)8
8.5	52 ;ChSP(0,1)2	52 ;ChSP(0,2)3	52 ;ChSP(0,3)4	52 ;ChSP(0,3)4	52 ;ChSP(0,4)5	52 ;ChSP(0,4)5	53 ;ChSP(0,5)6	53 ;ChSP(0,5)6	53 ;ChSP(0,6)7	53 ;ChSP(0,6)7	53 ;ChSP(0,6)7	54 ;ChSP(0,7)8
9.0	49 ;ChSP(0,1)2	49 ;ChSP(0,2)3	49 ;ChSP(0,3)4	49 ;ChSP(0,3)4	49 ;ChSP(0,4)5	49 ;ChSP(0,4)5	50 ;ChSP(0,5)6	50 ;ChSP(0,5)6	50 ;ChSP(0,6)7	50 ;ChSP(0,6)7	50 ;ChSP(0,6)7	50 ;ChSP(0,6)7
9.5	47 ;ChSP(0,1)2	47 ;ChSP(0,2)3	47 ;ChSP(0,3)4	47 ;ChSP(0,3)4	47 ;ChSP(0,4)5	47 ;ChSP(0,4)5	47 ;ChSP(0,5)6	47 ;ChSP(0,5)6	47 ;ChSP(0,5)6	47 ;ChSP(0,6)7	47 ;ChSP(0,6)7	47 ;ChSP(0,6)7
10.0	44 ;ChSP(0,1)2	44 ;ChSP(0,2)3	44 ;ChSP(0,2)3	44 ;ChSP(0,3)4	44 ;ChSP(0,3)4	44 ;ChSP(0,4)5	44 ;ChSP(0,5)6	44 ;ChSP(0,5)6	44 ;ChSP(0,5)6	44 ;ChSP(0,5)6	45 ;ChSP(0,6)7	45 ;ChSP(0,6)7

Table for chain sampling plans in the form of n;ChSP(c1,c2)r for

$\alpha =$ 0.01
$\beta =$ 0.05
k-1 = 3

LTPD(%)	AQL(%)											
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
1.5	200 ;ChSP(0,3)4	212 ;ChSP(0,5)6										
2.0	150 ;ChSP(0,3)4	153 ;ChSP(0,4)5	156 ;ChSP(0,5)6									
2.5	119 ;ChSP(0,2)3	122 ;ChSP(0,4)5	120 ;ChSP(0,5)6	120 ;ChSP(0,6)7								
3.0	99 ;ChSP(0,2)3	100 ;ChSP(0,3)4	102 ;ChSP(0,4)5	106 ;ChSP(0,5)6	111 ;ChSP(0,6)7							
3.5	85 ;ChSP(0,2)3	85 ;ChSP(0,3)4	86 ;ChSP(0,4)5	86 ;ChSP(0,4)5	90 ;ChSP(0,5)6	95 ;ChSP(0,6)7						
4.0	74 ;ChSP(0,2)3	74 ;ChSP(0,3)4	74 ;ChSP(0,3)4	76 ;ChSP(0,4)5	75 ;ChSP(0,5)6	78 ;ChSP(0,5)6	83 ;ChSP(0,6)7					
4.5	66 ;ChSP(0,2)3	66 ;ChSP(0,2)3	66 ;ChSP(0,3)4	67 ;ChSP(0,4)5	67 ;ChSP(0,4)5	67 ;ChSP(0,5)6	67 ;ChSP(0,5)6	74 ;ChSP(0,6)7				
5.0	59 ;ChSP(0,2)3	59 ;ChSP(0,2)3	59 ;ChSP(0,3)4	61 ;ChSP(0,4)5	61 ;ChSP(0,4)5	61 ;ChSP(0,5)6	61 ;ChSP(0,5)6	61 ;ChSP(0,6)7	67 ;ChSP(0,6)7			
5.5	53 ;ChSP(0,2)3	54 ;ChSP(0,2)3	54 ;ChSP(0,3)4	54 ;ChSP(0,3)4	54 ;ChSP(0,4)5	55 ;ChSP(0,4)5	54 ;ChSP(0,5)6	55 ;ChSP(0,5)6	55 ;ChSP(0,6)7	60 ;ChSP(0,6)7		
6.0	49 ;ChSP(0,2)3	49 ;ChSP(0,2)3	49 ;ChSP(0,3)4	49 ;ChSP(0,3)4	50 ;ChSP(0,4)5	50 ;ChSP(0,4)5	50 ;ChSP(0,5)6	52 ;ChSP(0,5)6	52 ;ChSP(0,5)6	55 ;ChSP(0,6)7	55 ;ChSP(0,6)7	
6.5	45 ;ChSP(0,1)2	45 ;ChSP(0,2)3	45 ;ChSP(0,3)4	45 ;ChSP(0,3)4	45 ;ChSP(0,3)4	46 ;ChSP(0,4)5	46 ;ChSP(0,4)5	46 ;ChSP(0,5)6	47 ;ChSP(0,5)6	47 ;ChSP(0,5)6	51 ;ChSP(0,6)7	51 ;ChSP(0,6)7
7.0	42 ;ChSP(0,1)2	42 ;ChSP(0,2)3	42 ;ChSP(0,3)4	42 ;ChSP(0,3)4	42 ;ChSP(0,3)4	43 ;ChSP(0,4)5	43 ;ChSP(0,4)5	43 ;ChSP(0,4)5	45 ;ChSP(0,5)6	45 ;ChSP(0,5)6	45 ;ChSP(0,5)6	47 ;ChSP(0,6)7
7.5	39 ;ChSP(0,1)2	39 ;ChSP(0,2)3	39 ;ChSP(0,2)3	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	40 ;ChSP(0,4)5	40 ;ChSP(0,4)5	40 ;ChSP(0,4)5	42 ;ChSP(0,5)6	42 ;ChSP(0,5)6	42 ;ChSP(0,5)6	40 ;ChSP(0,5)6
8.0	36 ;ChSP(0,1)2	36 ;ChSP(0,2)3	36 ;ChSP(0,2)3	37 ;ChSP(0,3)4	37 ;ChSP(0,3)4	37 ;ChSP(0,3)4	37 ;ChSP(0,4)5	38 ;ChSP(0,4)5	38 ;ChSP(0,4)5	37 ;ChSP(0,5)6	39 ;ChSP(0,5)6	39 ;ChSP(0,5)6
8.5	34 ;ChSP(0,1)2	34 ;ChSP(0,2)3	34 ;ChSP(0,2)3	35 ;ChSP(0,3)4	35 ;ChSP(0,3)4	35 ;ChSP(0,3)4	35 ;ChSP(0,4)5	35 ;ChSP(0,4)5	35 ;ChSP(0,4)5	36 ;ChSP(0,4)5	35 ;ChSP(0,5)6	37 ;ChSP(0,5)6
9.0	32 ;ChSP(0,1)2	32 ;ChSP(0,2)3	32 ;ChSP(0,2)3	32 ;ChSP(0,2)3	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,4)5	33 ;ChSP(0,4)5	33 ;ChSP(0,4)5	33 ;ChSP(0,5)6	35 ;ChSP(0,5)6
9.5	31 ;ChSP(0,1)2	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4	31 ;ChSP(0,4)5	31 ;ChSP(0,4)5	31 ;ChSP(0,4)5	31 ;ChSP(0,4)5	33 ;ChSP(0,5)6
10.0	29 ;ChSP(0,1)2	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4	30 ;ChSP(0,4)5	30 ;ChSP(0,4)5	30 ;ChSP(0,4)5	30 ;ChSP(0,4)5	31 ;ChSP(0,5)6

Table for chain sampling plans in the form of n;ChSP(c1,c2)r for

α = 0.01
β = 0.1
k-1 = 3

LTPD(%)	AQL(%)											
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
1.5	154 ;ChSP(0,3)4	166 ;ChSP(0,4)5										
2.0	115 ;ChSP(0,2)3	115 ;ChSP(0,3)4	121 ;ChSP(0,5)6									
2.5	92 ;ChSP(0,2)3	92 ;ChSP(0,3)4	94 ;ChSP(0,4)5	96 ;ChSP(0,5)6								
3.0	77 ;ChSP(0,2)3	77 ;ChSP(0,3)4	77 ;ChSP(0,3)4	83 ;ChSP(0,4)5	89 ;ChSP(0,5)6							
3.5	66 ;ChSP(0,2)3	66 ;ChSP(0,2)3	67 ;ChSP(0,3)4	67 ;ChSP(0,4)5	69 ;ChSP(0,4)5	76 ;ChSP(0,5)6						
4.0	57 ;ChSP(0,2)3	57 ;ChSP(0,2)3	59 ;ChSP(0,3)4	59 ;ChSP(0,3)4	62 ;ChSP(0,4)5	62 ;ChSP(0,4)5	67 ;ChSP(0,5)6					
4.5	51 ;ChSP(0,1)2	51 ;ChSP(0,2)3	52 ;ChSP(0,3)4	52 ;ChSP(0,3)4	52 ;ChSP(0,4)5	55 ;ChSP(0,4)5	53 ;ChSP(0,5)6	59 ;ChSP(0,5)6				
5.0	45 ;ChSP(0,1)2	46 ;ChSP(0,2)3	46 ;ChSP(0,3)4	46 ;ChSP(0,3)4	47 ;ChSP(0,3)4	48 ;ChSP(0,4)5	49 ;ChSP(0,4)5	49 ;ChSP(0,4)5	52 ;ChSP(0,5)6			
5.5	41 ;ChSP(0,1)2	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	43 ;ChSP(0,3)4	43 ;ChSP(0,3)4	42 ;ChSP(0,4)5	45 ;ChSP(0,4)5	45 ;ChSP(0,4)5	45 ;ChSP(0,5)6	47 ;ChSP(0,5)6		
6.0	38 ;ChSP(0,1)2	38 ;ChSP(0,2)3	38 ;ChSP(0,2)3	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	39 ;ChSP(0,4)5	41 ;ChSP(0,4)5	42 ;ChSP(0,5)6	43 ;ChSP(0,5)6	43 ;ChSP(0,5)6	
6.5	35 ;ChSP(0,1)2	36 ;ChSP(0,2)3	35 ;ChSP(0,2)3	36 ;ChSP(0,3)4	36 ;ChSP(0,3)4	36 ;ChSP(0,3)4	36 ;ChSP(0,4)5	36 ;ChSP(0,4)5	38 ;ChSP(0,4)5	37 ;ChSP(0,5)6	41 ;ChSP(0,5)6	41 ;ChSP(0,5)6
7.0	32 ;ChSP(0,1)2	33 ;ChSP(0,2)3	33 ;ChSP(0,2)3	33 ;ChSP(0,2)3	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,4)5	35 ;ChSP(0,4)5	35 ;ChSP(0,4)5	34 ;ChSP(0,5)6	38 ;ChSP(0,5)6
7.5	30 ;ChSP(0,1)2	30 ;ChSP(0,2)3	30 ;ChSP(0,2)3	30 ;ChSP(0,2)3	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4	31 ;ChSP(0,4)5	31 ;ChSP(0,4)5	33 ;ChSP(0,4)5	33 ;ChSP(0,4)5	32 ;ChSP(0,5)6
8.0	28 ;ChSP(0,1)2	28 ;ChSP(0,1)2	28 ;ChSP(0,2)3	28 ;ChSP(0,2)3	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4	29 ;ChSP(0,4)5	29 ;ChSP(0,4)5	31 ;ChSP(0,4)5	29 ;ChSP(0,5)6
8.5	26 ;ChSP(0,1)2	26 ;ChSP(0,1)2	27 ;ChSP(0,2)3	27 ;ChSP(0,2)3	27 ;ChSP(0,2)3	27 ;ChSP(0,3)4	27 ;ChSP(0,3)4	27 ;ChSP(0,3)4	27 ;ChSP(0,4)5	27 ;ChSP(0,4)5	29 ;ChSP(0,4)5	29 ;ChSP(0,4)5
9.0	25 ;ChSP(0,1)2	25 ;ChSP(0,1)2	25 ;ChSP(0,2)3	25 ;ChSP(0,2)3	25 ;ChSP(0,2)3	26 ;ChSP(0,3)4	26 ;ChSP(0,3)4	26 ;ChSP(0,3)4	26 ;ChSP(0,3)4	26 ;ChSP(0,4)5	26 ;ChSP(0,4)5	27 ;ChSP(0,4)5
9.5	24 ;ChSP(0,1)2	24 ;ChSP(0,1)2	24 ;ChSP(0,2)3	24 ;ChSP(0,2)3	24 ;ChSP(0,2)3	24 ;ChSP(0,3)4	24 ;ChSP(0,3)4	24 ;ChSP(0,3)4	24 ;ChSP(0,3)4	26 ;ChSP(0,4)5	26 ;ChSP(0,4)5	26 ;ChSP(0,4)5
10.0	22 ;ChSP(0,1)2	22 ;ChSP(0,1)2	23 ;ChSP(0,2)3	23 ;ChSP(0,2)3	23 ;ChSP(0,2)3	23 ;ChSP(0,2)3	23 ;ChSP(0,3)4	23 ;ChSP(0,3)4	23 ;ChSP(0,3)4	23 ;ChSP(0,4)5	23 ;ChSP(0,4)5	23 ;ChSP(0,4)5

Table for chain sampling plans in the form of n;ChSP(c1,c2)r for

α = 0.05
β = 0.01
k-1 = 3

LTPD(%)	AQL(%) 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
1.5	305 ;ChSP(0,3)4	306 ;ChSP(0,5)6										
2.0	228 ;ChSP(0,2)3	229 ;ChSP(0,4)5	229 ;ChSP(0,6)7									
2.5	182 ;ChSP(0,2)3	182 ;ChSP(0,3)4	183 ;ChSP(0,5)6	185 ;ChSP(0,6)7								
3.0	152 ;ChSP(0,2)3	152 ;ChSP(0,3)4	152 ;ChSP(0,4)5	152 ;ChSP(0,5)6	154 ;ChSP(0,6)7							
3.5	130 ;ChSP(0,1)2	130 ;ChSP(0,3)4	130 ;ChSP(0,4)5	130 ;ChSP(0,4)5	130 ;ChSP(0,5)6	131 ;ChSP(0,6)7						
4.0	113 ;ChSP(0,1)2	113 ;ChSP(0,2)3	113 ;ChSP(0,3)4	113 ;ChSP(0,4)5	114 ;ChSP(0,5)6	114 ;ChSP(0,5)6	115 ;ChSP(0,6)7					
4.5	101 ;ChSP(0,1)2	101 ;ChSP(0,2)3	101 ;ChSP(0,3)4	101 ;ChSP(0,4)5	101 ;ChSP(0,4)5	101 ;ChSP(0,5)6	102 ;ChSP(0,6)7	102 ;ChSP(0,6)7				
5.0	90 ;ChSP(0,1)2	90 ;ChSP(0,2)3	90 ;ChSP(0,3)4	90 ;ChSP(0,3)4	90 ;ChSP(0,4)5	91 ;ChSP(0,5)6	91 ;ChSP(0,5)6	91 ;ChSP(0,6)7	91 ;ChSP(0,6)7			
5.5	82 ;ChSP(0,1)2	82 ;ChSP(0,2)3	82 ;ChSP(0,2)3	82 ;ChSP(0,3)4	82 ;ChSP(0,4)5	82 ;ChSP(0,4)5	82 ;ChSP(0,5)6	82 ;ChSP(0,5)6	83 ;ChSP(0,6)7	83 ;ChSP(0,6)7		
6.0	75 ;ChSP(0,1)2	75 ;ChSP(0,2)3	75 ;ChSP(0,2)3	75 ;ChSP(0,3)4	75 ;ChSP(0,3)4	75 ;ChSP(0,4)5	75 ;ChSP(0,4)5	75 ;ChSP(0,5)6	75 ;ChSP(0,5)6	76 ;ChSP(0,6)7	76 ;ChSP(0,6)7	
6.5	69 ;ChSP(0,1)2	69 ;ChSP(0,1)2	69 ;ChSP(0,2)3	69 ;ChSP(0,3)4	69 ;ChSP(0,3)4	69 ;ChSP(0,4)5	69 ;ChSP(0,4)5	69 ;ChSP(0,5)6	69 ;ChSP(0,5)6	70 ;ChSP(0,6)7	70 ;ChSP(0,6)7	70 ;ChSP(0,6)7
7.0	64 ;ChSP(0,1)2	64 ;ChSP(0,1)2	64 ;ChSP(0,2)3	64 ;ChSP(0,3)4	64 ;ChSP(0,3)4	64 ;ChSP(0,3)4	64 ;ChSP(0,4)5	64 ;ChSP(0,4)5	64 ;ChSP(0,5)6	64 ;ChSP(0,5)6	65 ;ChSP(0,6)7	65 ;ChSP(0,6)7
7.5	60 ;ChSP(0,1)2	60 ;ChSP(0,1)2	60 ;ChSP(0,2)3	60 ;ChSP(0,2)3	60 ;ChSP(0,3)4	60 ;ChSP(0,3)4	60 ;ChSP(0,4)5	60 ;ChSP(0,4)5	60 ;ChSP(0,5)6	60 ;ChSP(0,5)6	60 ;ChSP(0,5)6	60 ;ChSP(0,6)7
8.0		56 ;ChSP(0,1)2	56 ;ChSP(0,1)2	56 ;ChSP(0,2)3	56 ;ChSP(0,3)4	56 ;ChSP(0,3)4	56 ;ChSP(0,3)4	56 ;ChSP(0,4)5	56 ;ChSP(0,4)5	56 ;ChSP(0,5)6	56 ;ChSP(0,5)6	56 ;ChSP(0,5)6
8.5		52 ;ChSP(0,1)2	52 ;ChSP(0,1)2	52 ;ChSP(0,2)3	52 ;ChSP(0,3)4	52 ;ChSP(0,3)4	52 ;ChSP(0,3)4	52 ;ChSP(0,4)5	52 ;ChSP(0,4)5	52 ;ChSP(0,4)5	53 ;ChSP(0,5)6	53 ;ChSP(0,5)6
9.0		49 ;ChSP(0,1)2	49 ;ChSP(0,1)2	49 ;ChSP(0,2)3	49 ;ChSP(0,2)3	49 ;ChSP(0,3)4	49 ;ChSP(0,3)4	49 ;ChSP(0,4)5	49 ;ChSP(0,4)5	49 ;ChSP(0,4)5	50 ;ChSP(0,5)6	50 ;ChSP(0,5)6
9.5		47 ;ChSP(0,1)2	47 ;ChSP(0,1)2	47 ;ChSP(0,2)3	47 ;ChSP(0,2)3	47 ;ChSP(0,3)4	47 ;ChSP(0,3)4	47 ;ChSP(0,3)4	47 ;ChSP(0,4)5	47 ;ChSP(0,4)5	47 ;ChSP(0,4)5	47 ;ChSP(0,5)6
10.0		44 ;ChSP(0,1)2	44 ;ChSP(0,1)2	44 ;ChSP(0,2)3	44 ;ChSP(0,2)3	44 ;ChSP(0,3)4	44 ;ChSP(0,3)4	44 ;ChSP(0,3)4	44 ;ChSP(0,4)5	44 ;ChSP(0,4)5	44 ;ChSP(0,4)5	44 ;ChSP(0,4)5

Table for chain sampling plans in the form of n;ChSP(c1,c2)r for
α = 0.05
β = 0.05
k-1 = 3

LTPD(%)	AQL(%) 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
1.5	199 ;ChSP(0,2)3	204 ;ChSP(0,4)5										
2.0	149 ;ChSP(0,2)3	150 ;ChSP(0,3)4	153 ;ChSP(0,4)5									
2.5	119 ;ChSP(0,1)2	119 ;ChSP(0,2)3	120 ;ChSP(0,3)4	122 ;ChSP(0,4)5								
3.0	99 ;ChSP(0,1)2	99 ;ChSP(0,2)3	100 ;ChSP(0,3)4	102 ;ChSP(0,4)5	102 ;ChSP(0,4)5							
3.5	85 ;ChSP(0,1)2	85 ;ChSP(0,2)3	85 ;ChSP(0,3)4	85 ;ChSP(0,3)4	87 ;ChSP(0,4)5	87 ;ChSP(0,4)5						
4.0	74 ;ChSP(0,1)2	74 ;ChSP(0,2)3	74 ;ChSP(0,2)3	74 ;ChSP(0,3)4	74 ;ChSP(0,3)4	76 ;ChSP(0,4)5	76 ;ChSP(0,4)5					
4.5	66 ;ChSP(0,1)2	66 ;ChSP(0,1)2	66 ;ChSP(0,2)3	66 ;ChSP(0,3)4	66 ;ChSP(0,3)4	67 ;ChSP(0,4)5	67 ;ChSP(0,4)5	67 ;ChSP(0,4)5				
5.0	59 ;ChSP(0,1)2	59 ;ChSP(0,1)2	59 ;ChSP(0,2)3	59 ;ChSP(0,2)3	59 ;ChSP(0,3)4	59 ;ChSP(0,3)4	61 ;ChSP(0,4)5	61 ;ChSP(0,4)5	80 ;ChSP(0,5)6			
5.5	53 ;ChSP(0,1)2	53 ;ChSP(0,1)2	54 ;ChSP(0,2)3	54 ;ChSP(0,2)3	54 ;ChSP(0,3)4	54 ;ChSP(0,3)4	54 ;ChSP(0,3)4	54 ;ChSP(0,4)5	55 ;ChSP(0,4)5	55 ;ChSP(0,4)5		
6.0		49 ;ChSP(0,1)2	49 ;ChSP(0,2)3	49 ;ChSP(0,2)3	49 ;ChSP(0,2)3	49 ;ChSP(0,3)4	49 ;ChSP(0,3)4	50 ;ChSP(0,4)5	50 ;ChSP(0,4)5	50 ;ChSP(0,4)5	50 ;ChSP(0,5)6	
6.5		45 ;ChSP(0,1)2	45 ;ChSP(0,1)2	45 ;ChSP(0,2)3	45 ;ChSP(0,2)3	45 ;ChSP(0,3)4	45 ;ChSP(0,3)4	45 ;ChSP(0,3)4	46 ;ChSP(0,4)5	46 ;ChSP(0,4)5	46 ;ChSP(0,4)5	46 ;ChSP(0,5)6
7.0		42 ;ChSP(0,1)2	42 ;ChSP(0,1)2	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	42 ;ChSP(0,3)4	42 ;ChSP(0,3)4	42 ;ChSP(0,4)5	43 ;ChSP(0,4)5	43 ;ChSP(0,4)5	43 ;ChSP(0,4)5
7.5		39 ;ChSP(0,1)2	39 ;ChSP(0,1)2	39 ;ChSP(0,2)3	39 ;ChSP(0,2)3	39 ;ChSP(0,2)3	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	40 ;ChSP(0,4)5	40 ;ChSP(0,4)5	40 ;ChSP(0,4)5
8.0		36 ;ChSP(0,1)2	36 ;ChSP(0,1)2	36 ;ChSP(0,2)3	36 ;ChSP(0,2)3	36 ;ChSP(0,2)3	36 ;ChSP(0,3)4	37 ;ChSP(0,3)4	37 ;ChSP(0,3)4	37 ;ChSP(0,3)4	37 ;ChSP(0,4)5	37 ;ChSP(0,4)5
8.5		34 ;ChSP(0,1)2	34 ;ChSP(0,1)2	34 ;ChSP(0,1)2	34 ;ChSP(0,2)3	34 ;ChSP(0,2)3	34 ;ChSP(0,2)3	35 ;ChSP(0,3)4	35 ;ChSP(0,3)4	35 ;ChSP(0,3)4	35 ;ChSP(0,3)4	35 ;ChSP(0,4)5
9.0		32 ;ChSP(0,1)2	32 ;ChSP(0,1)2	32 ;ChSP(0,1)2	32 ;ChSP(0,2)3	32 ;ChSP(0,2)3	32 ;ChSP(0,2)3	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4
9.5		31 ;ChSP(0,1)2	31 ;ChSP(0,1)2	31 ;ChSP(0,1)2	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4
10.0		29 ;ChSP(0,1)2	29 ;ChSP(0,1)2	29 ;ChSP(0,1)2	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4

Table for chain sampling plans in the form of n;ChSP(c1,c2)r for

α = 0.05
β = 0.1
k-1 = 3

LTPD(%)	AQL(%) 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
1.5	154 ;ChSP(0,2)3	154 ;ChSP(0,3)4										
2.0	115 ;ChSP(0,1)2	115 ;ChSP(0,2)3	119 ;ChSP(0,3)4									
2.5	92 ;ChSP(0,1)2	92 ;ChSP(0,2)3	92 ;ChSP(0,3)4	95 ;ChSP(0,3)4								
3.0	76 ;ChSP(0,1)2	77 ;ChSP(0,2)3	77 ;ChSP(0,2)3	77 ;ChSP(0,3)4	77 ;ChSP(0,3)4							
3.5	65 ;ChSP(0,1)2	65 ;ChSP(0,1)2	66 ;ChSP(0,2)3	66 ;ChSP(0,3)4	67 ;ChSP(0,3)4	67 ;ChSP(0,4)5						
4.0	57 ;ChSP(0,1)2	57 ;ChSP(0,1)2	57 ;ChSP(0,2)3	57 ;ChSP(0,3)4	59 ;ChSP(0,3)4	59 ;ChSP(0,3)4	58 ;ChSP(0,4)5					
4.5		51 ;ChSP(0,1)2	51 ;ChSP(0,2)3	51 ;ChSP(0,2)3	52 ;ChSP(0,3)4	52 ;ChSP(0,3)4	52 ;ChSP(0,3)4	52 ;ChSP(0,4)5				
5.0		45 ;ChSP(0,1)2	45 ;ChSP(0,1)2	46 ;ChSP(0,2)3	46 ;ChSP(0,3)4	46 ;ChSP(0,3)4	46 ;ChSP(0,3)4	47 ;ChSP(0,3)4	46 ;ChSP(0,4)5			
5.5		41 ;ChSP(0,1)2	41 ;ChSP(0,1)2	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	43 ;ChSP(0,3)4	43 ;ChSP(0,3)4	43 ;ChSP(0,3)4	42 ;ChSP(0,4)5		
6.0		38 ;ChSP(0,1)2	38 ;ChSP(0,1)2	38 ;ChSP(0,2)3	38 ;ChSP(0,2)3	38 ;ChSP(0,2)3	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	39 ;ChSP(0,4)5	
6.5		35 ;ChSP(0,1)2	35 ;ChSP(0,1)2	35 ;ChSP(0,2)3	35 ;ChSP(0,2)3	35 ;ChSP(0,2)3	35 ;ChSP(0,2)3	36 ;ChSP(0,3)4	36 ;ChSP(0,3)4	36 ;ChSP(0,3)4	36 ;ChSP(0,3)4	36 ;ChSP(0,4)5
7.0		32 ;ChSP(0,1)2	32 ;ChSP(0,1)2	32 ;ChSP(0,1)2	33 ;ChSP(0,2)3	33 ;ChSP(0,2)3	33 ;ChSP(0,2)3	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4
7.5		30 ;ChSP(0,1)2	30 ;ChSP(0,1)2	30 ;ChSP(0,1)2	30 ;ChSP(0,2)3	30 ;ChSP(0,2)3	30 ;ChSP(0,2)3	30 ;ChSP(0,2)3	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4	31 ;ChSP(0,3)4
8.0		28 ;ChSP(0,1)2	28 ;ChSP(0,1)2	28 ;ChSP(0,1)2	28 ;ChSP(0,2)3	28 ;ChSP(0,2)3	28 ;ChSP(0,2)3	28 ;ChSP(0,2)3	28 ;ChSP(0,2)3	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4	29 ;ChSP(0,3)4
8.5		26 ;ChSP(0,1)2	26 ;ChSP(0,1)2	26 ;ChSP(0,1)2	26 ;ChSP(0,1)2	27 ;ChSP(0,2)3	27 ;ChSP(0,2)3	27 ;ChSP(0,2)3	27 ;ChSP(0,2)3	27 ;ChSP(0,3)4	27 ;ChSP(0,3)4	27 ;ChSP(0,3)4
9.0			25 ;ChSP(0,1)2	25 ;ChSP(0,1)2	25 ;ChSP(0,1)2	25 ;ChSP(0,2)3	25 ;ChSP(0,2)3	25 ;ChSP(0,2)3	25 ;ChSP(0,2)3	25 ;ChSP(0,2)3	23 ;ChSP(0,3)4	26 ;ChSP(0,3)4
9.5			24 ;ChSP(0,1)2	24 ;ChSP(0,1)2	24 ;ChSP(0,1)2	24 ;ChSP(0,2)3	24 ;ChSP(0,2)3	24 ;ChSP(0,2)3	24 ;ChSP(0,2)3	24 ;ChSP(0,2)3	24 ;ChSP(0,3)4	24 ;ChSP(0,3)4
10.0			22 ;ChSP(0,1)2	22 ;ChSP(0,1)2	22 ;ChSP(0,1)2	22 ;ChSP(0,1)2	23 ;ChSP(0,2)3	23 ;ChSP(0,2)3	23 ;ChSP(0,2)3	23 ;ChSP(0,2)3	23 ;ChSP(0,2)3	23 ;ChSP(0,3)4

Table for chain sampling plans in the form of n;ChSP(c1,c2)r for

α = 0.1
β = 0.01
k-1 = 3

LTPD(%)	AQL(%) 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
1.5	305 ;ChSP(0,2)3	305 ;ChSP(0,4)5										
2.0	228 ;ChSP(0,2)3	228 ;ChSP(0,3)4	229 ;ChSP(0,5)6									
2.5	182 ;ChSP(0,1)2	182 ;ChSP(0,3)4	183 ;ChSP(0,4)5	183 ;ChSP(0,4)5								
3.0	152 ;ChSP(0,1)2	152 ;ChSP(0,2)3	152 ;ChSP(0,3)4	152 ;ChSP(0,4)5	152 ;ChSP(0,5)6							
3.5	130 ;ChSP(0,1)2	130 ;ChSP(0,2)3	130 ;ChSP(0,3)4	130 ;ChSP(0,4)5	130 ;ChSP(0,4)5	130 ;ChSP(0,5)6						
4.0	113 ;ChSP(0,1)2	113 ;ChSP(0,2)3	113 ;ChSP(0,2)3	113 ;ChSP(0,3)4	113 ;ChSP(0,4)5	114 ;ChSP(0,5)6	114 ;ChSP(0,5)6					
4.5		101 ;ChSP(0,1)2	101 ;ChSP(0,2)3	101 ;ChSP(0,3)4	101 ;ChSP(0,4)5	101 ;ChSP(0,4)5	101 ;ChSP(0,5)6	101 ;ChSP(0,5)6				
5.0		90 ;ChSP(0,1)2	90 ;ChSP(0,2)3	90 ;ChSP(0,3)4	90 ;ChSP(0,3)4	90 ;ChSP(0,4)5	90 ;ChSP(0,4)5	91 ;ChSP(0,5)6	91 ;ChSP(0,5)6			
5.5		82 ;ChSP(0,1)2	82 ;ChSP(0,2)3	82 ;ChSP(0,2)3	82 ;ChSP(0,3)4	82 ;ChSP(0,3)4	82 ;ChSP(0,4)5	82 ;ChSP(0,4)5	82 ;ChSP(0,5)6	82 ;ChSP(0,5)6		
6.0		75 ;ChSP(0,1)2	75 ;ChSP(0,2)3	75 ;ChSP(0,2)3	75 ;ChSP(0,3)4	75 ;ChSP(0,3)4	75 ;ChSP(0,4)5	75 ;ChSP(0,4)5	75 ;ChSP(0,5)6	75 ;ChSP(0,5)6	75 ;ChSP(0,5)6	
6.5		69 ;ChSP(0,1)2	69 ;ChSP(0,1)2	69 ;ChSP(0,2)3	69 ;ChSP(0,3)4	69 ;ChSP(0,3)4	69 ;ChSP(0,3)4	69 ;ChSP(0,4)5	69 ;ChSP(0,4)5	69 ;ChSP(0,5)6	69 ;ChSP(0,5)6	69 ;ChSP(0,5)6
7.0		64 ;ChSP(0,1)2	64 ;ChSP(0,1)2	64 ;ChSP(0,2)3	64 ;ChSP(0,2)3	64 ;ChSP(0,3)4	64 ;ChSP(0,3)4	64 ;ChSP(0,4)5	64 ;ChSP(0,4)5	64 ;ChSP(0,4)5	64 ;ChSP(0,5)6	64 ;ChSP(0,5)6
7.5		60 ;ChSP(0,1)2	60 ;ChSP(0,1)2	60 ;ChSP(0,2)3	60 ;ChSP(0,2)3	60 ;ChSP(0,3)4	60 ;ChSP(0,3)4	60 ;ChSP(0,3)4	60 ;ChSP(0,4)5	60 ;ChSP(0,4)5	60 ;ChSP(0,5)6	60 ;ChSP(0,5)6
8.0		56 ;ChSP(0,1)2	56 ;ChSP(0,1)2	56 ;ChSP(0,2)3	56 ;ChSP(0,2)3	56 ;ChSP(0,2)3	56 ;ChSP(0,3)4	56 ;ChSP(0,3)4	56 ;ChSP(0,4)5	56 ;ChSP(0,4)5	56 ;ChSP(0,4)5	56 ;ChSP(0,5)6
8.5			52 ;ChSP(0,1)2	52 ;ChSP(0,1)2	52 ;ChSP(0,2)3	52 ;ChSP(0,2)3	52 ;ChSP(0,3)4	52 ;ChSP(0,3)4	52 ;ChSP(0,3)4	52 ;ChSP(0,4)5	52 ;ChSP(0,4)5	52 ;ChSP(0,4)5
9.0			49 ;ChSP(0,1)2	49 ;ChSP(0,1)2	49 ;ChSP(0,2)3	49 ;ChSP(0,2)3	49 ;ChSP(0,2)3	49 ;ChSP(0,3)4	49 ;ChSP(0,3)4	49 ;ChSP(0,3)4	49 ;ChSP(0,4)5	49 ;ChSP(0,4)5
9.5			47 ;ChSP(0,1)2	47 ;ChSP(0,1)2	47 ;ChSP(0,2)3	47 ;ChSP(0,2)3	47 ;ChSP(0,2)3	47 ;ChSP(0,3)4	47 ;ChSP(0,3)4	47 ;ChSP(0,3)4	47 ;ChSP(0,4)5	47 ;ChSP(0,4)5
10.0			44 ;ChSP(0,1)2	44 ;ChSP(0,1)2	44 ;ChSP(0,2)3	44 ;ChSP(0,2)3	44 ;ChSP(0,2)3	44 ;ChSP(0,3)4	44 ;ChSP(0,3)4	44 ;ChSP(0,3)4	44 ;ChSP(0,3)4	44 ;ChSP(0,4)5

Table for chain sampling plans in the form of n;ChSP(c1,c2)r for

α = 0.1
β = 0.05
k-1 = 3

LTPD(%)	AQL(%) 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
1.5	199 ;ChSP(0,1)2	200 ;ChSP(0,3)4										
2.0	149 ;ChSP(0,1)2	149 ;ChSP(0,2)3	150 ;ChSP(0,3)4									
2.5	119 ;ChSP(0,1)2	119 ;ChSP(0,2)3	120 ;ChSP(0,3)4	120 ;ChSP(0,3)4								
3.0		99 ;ChSP(0,1)2	99 ;ChSP(0,2)3	100 ;ChSP(0,3)4	102 ;ChSP(0,4)5							
3.5		85 ;ChSP(0,1)2	85 ;ChSP(0,2)3	85 ;ChSP(0,2)3	85 ;ChSP(0,3)4	87 ;ChSP(0,4)5						
4.0		74 ;ChSP(0,1)2	74 ;ChSP(0,2)3	74 ;ChSP(0,2)3	74 ;ChSP(0,3)4	74 ;ChSP(0,3)4	76 ;ChSP(0,4)5					
4.5		66 ;ChSP(0,1)2	66 ;ChSP(0,1)2	66 ;ChSP(0,2)3	66 ;ChSP(0,2)3	66 ;ChSP(0,3)4	66 ;ChSP(0,3)4	67 ;ChSP(0,4)5				
5.0		59 ;ChSP(0,1)2	59 ;ChSP(0,1)2	59 ;ChSP(0,2)3	59 ;ChSP(0,2)3	59 ;ChSP(0,3)4	59 ;ChSP(0,3)4	59 ;ChSP(0,3)4	61 ;ChSP(0,4)5			
5.5		53 ;ChSP(0,1)2	53 ;ChSP(0,1)2	54 ;ChSP(0,2)3	54 ;ChSP(0,2)3	54 ;ChSP(0,2)3	54 ;ChSP(0,3)4	54 ;ChSP(0,3)4	54 ;ChSP(0,3)4	54 ;ChSP(0,4)5		
6.0			49 ;ChSP(0,1)2	49 ;ChSP(0,1)2	49 ;ChSP(0,2)3	49 ;ChSP(0,2)3	49 ;ChSP(0,2)3	49 ;ChSP(0,3)4	49 ;ChSP(0,3)4	49 ;ChSP(0,3)4	50 ;ChSP(0,4)5	
6.5			45 ;ChSP(0,1)2	45 ;ChSP(0,1)2	45 ;ChSP(0,2)3	45 ;ChSP(0,2)3	45 ;ChSP(0,2)3	45 ;ChSP(0,3)4	45 ;ChSP(0,3)4	45 ;ChSP(0,3)4	46 ;ChSP(0,4)5	46 ;ChSP(0,4)5
7.0			42 ;ChSP(0,1)2	42 ;ChSP(0,1)2	42 ;ChSP(0,1)2	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	42 ;ChSP(0,3)4	42 ;ChSP(0,3)4	42 ;ChSP(0,3)4	43 ;ChSP(0,4)5
7.5			39 ;ChSP(0,1)2	39 ;ChSP(0,1)2	39 ;ChSP(0,1)2	39 ;ChSP(0,2)3	39 ;ChSP(0,2)3	39 ;ChSP(0,2)3	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4
8.0			36 ;ChSP(0,1)2	36 ;ChSP(0,1)2	36 ;ChSP(0,1)2	36 ;ChSP(0,2)3	36 ;ChSP(0,2)3	36 ;ChSP(0,2)3	36 ;ChSP(0,2)3	37 ;ChSP(0,3)4	37 ;ChSP(0,3)4	37 ;ChSP(0,3)4
8.5				34 ;ChSP(0,1)2	34 ;ChSP(0,1)2	34 ;ChSP(0,1)2	34 ;ChSP(0,2)3	34 ;ChSP(0,2)3	34 ;ChSP(0,2)3	34 ;ChSP(0,2)3	35 ;ChSP(0,3)4	35 ;ChSP(0,3)4
9.0				32 ;ChSP(0,1)2	32 ;ChSP(0,1)2	32 ;ChSP(0,1)2	32 ;ChSP(0,2)3	32 ;ChSP(0,2)3	32 ;ChSP(0,2)3	32 ;ChSP(0,2)3	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4
9.5				31 ;ChSP(0,1)2	31 ;ChSP(0,1)2	31 ;ChSP(0,1)2	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,2)3	31 ;ChSP(0,3)4
10.0				29 ;ChSP(0,1)2	29 ;ChSP(0,1)2	29 ;ChSP(0,1)2	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,2)3	29 ;ChSP(0,3)4

Table for chain sampling plans in the form of n;ChSP(c1,c2)r for

α = 0.1
β = 0.1
k-1 = 3

LTPD(%)	AQL(%) 0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0	1.1	1.2
1.5	153 ;ChSP(0,1)2	154 ;ChSP(0,2)3										
2.0	115 ;ChSP(0,1)2	115 ;ChSP(0,2)3	115 ;ChSP(0,2)3									
2.5		92 ;ChSP(0,1)2	92 ;ChSP(0,2)3	92 ;ChSP(0,3)4								
3.0		78 ;ChSP(0,1)2	77 ;ChSP(0,2)3	77 ;ChSP(0,2)3	77 ;ChSP(0,3)4							
3.5		65 ;ChSP(0,1)2	65 ;ChSP(0,1)2	65 ;ChSP(0,2)3	66 ;ChSP(0,2)3	66 ;ChSP(0,3)4						
4.0		57 ;ChSP(0,1)2	57 ;ChSP(0,1)2	57 ;ChSP(0,2)3	57 ;ChSP(0,2)3	57 ;ChSP(0,2)3	59 ;ChSP(0,3)4					
4.5			51 ;ChSP(0,1)2	51 ;ChSP(0,1)2	51 ;ChSP(0,2)3	51 ;ChSP(0,2)3	52 ;ChSP(0,3)4	52 ;ChSP(0,3)4				
5.0			45 ;ChSP(0,1)2	45 ;ChSP(0,1)2	46 ;ChSP(0,2)3	46 ;ChSP(0,2)3	46 ;ChSP(0,2)3	46 ;ChSP(0,3)4	46 ;ChSP(0,3)4			
5.5			41 ;ChSP(0,1)2	41 ;ChSP(0,1)2	41 ;ChSP(0,1)2	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	42 ;ChSP(0,2)3	43 ;ChSP(0,3)4	43 ;ChSP(0,3)4		
6.0			38 ;ChSP(0,1)2	38 ;ChSP(0,1)2	38 ;ChSP(0,1)2	38 ;ChSP(0,2)3	38 ;ChSP(0,2)3	38 ;ChSP(0,2)3	38 ;ChSP(0,2)3	39 ;ChSP(0,3)4	39 ;ChSP(0,3)4	
6.5				35 ;ChSP(0,1)2	35 ;ChSP(0,1)2	35 ;ChSP(0,1)2	35 ;ChSP(0,2)3	35 ;ChSP(0,2)3	35 ;ChSP(0,2)3	36 ;ChSP(0,3)4	36 ;ChSP(0,3)4	36 ;ChSP(0,3)4
7.0				32 ;ChSP(0,1)2	32 ;ChSP(0,1)2	32 ;ChSP(0,1)2	33 ;ChSP(0,2)3	33 ;ChSP(0,2)3	33 ;ChSP(0,2)3	33 ;ChSP(0,2)3	33 ;ChSP(0,3)4	33 ;ChSP(0,3)4
7.5				30 ;ChSP(0,1)2	30 ;ChSP(0,1)2	30 ;ChSP(0,1)2	30 ;ChSP(0,1)2	30 ;ChSP(0,2)3	30 ;ChSP(0,2)3	30 ;ChSP(0,2)3	30 ;ChSP(0,2)3	31 ;ChSP(0,3)4
8.0				28 ;ChSP(0,1)2	28 ;ChSP(0,1)2	28 ;ChSP(0,1)2	28 ;ChSP(0,1)2	28 ;ChSP(0,2)3	28 ;ChSP(0,2)3	28 ;ChSP(0,2)3	28 ;ChSP(0,2)3	28 ;ChSP(0,2)3
8.5					26 ;ChSP(0,1)2	26 ;ChSP(0,1)2	26 ;ChSP(0,1)2	26 ;ChSP(0,1)2	27 ;ChSP(0,2)3	27 ;ChSP(0,2)3	27 ;ChSP(0,2)3	27 ;ChSP(0,2)3
9.0					25 ;ChSP(0,1)2	25 ;ChSP(0,1)2	25 ;ChSP(0,1)2	25 ;ChSP(0,1)2	25 ;ChSP(0,2)3	25 ;ChSP(0,2)3	25 ;ChSP(0,2)3	25 ;ChSP(0,2)3
9.5					24 ;ChSP(0,1)2	24 ;ChSP(0,1)2	24 ;ChSP(0,1)2	24 ;ChSP(0,1)2	24 ;ChSP(0,2)3	24 ;ChSP(0,2)3	24 ;ChSP(0,2)3	24 ;ChSP(0,2)3
10.0					22 ;ChSP(0,1)2	22 ;ChSP(0,1)2	22 ;ChSP(0,1)2	22 ;ChSP(0,1)2	22 ;ChSP(0,1)2	23 ;ChSP(0,2)3	23 ;ChSP(0,2)3	23 ;ChSP(0,2)3

Appendix B The Use of a Ratio Test in Multi-Variate SPC

Hotelling T^2 is a common statistics used in multivariate process control and was shown effective in detecting out of control signal. However, once an out-of-control signal was detected, T^2 performed poorly in identifying which variable or set of variables is the source of signal. It also fails to address the correlation effect.

Many researchers have shown interested in this topic and a lot of research has been devoted to address the abovementioned problems. We are no exception. In our research, we propose a percentage decomposition method to solve the problem. The model is outlined below:

1. For multivariate process data, obtain its μ and Σ
2. Use Hotelling T^2 method to do multivariate process control
3. When an out-of-control signal is detected, plot the percentage decomposition chart.
4. Examine the percentage chart and identify source of out-of-control variable(s).

The percentage decomposition chart is illustrated below:

1. Normalize the ith observation

$$\overset{X_i}{\begin{bmatrix} x_{i1} \\ x_{i2} \\ \vdots \\ x_{ip} \end{bmatrix}} \Rightarrow \overset{Z_i}{\begin{bmatrix} z_{i1} \\ z_{i2} \\ \vdots \\ z_{ip} \end{bmatrix}} \text{ where } z_{ij} = \frac{(x_{ij} - u_{.j})}{\sigma_{.j}},\ j = 1,2,\dots p$$

2. Calculate the percentage of each variable in the observation

$$\overset{Z_i}{\begin{bmatrix} z_{i1} \\ z_{i2} \\ \vdots \\ z_{ip} \end{bmatrix}} \Rightarrow \overset{P_i}{\begin{bmatrix} p_{i1} \\ p_{i2} \\ \vdots \\ p_{ipn} \end{bmatrix}} \text{ where } p_{ij} = \frac{z_{ij}}{\sum_{i=1}^{r} z_{ij}}$$

3. Plotting p_{ij} based on it three sigma limits, and we can tell which variable is the source of out-of-control.

In our research, we proceed to derive the mathematical distribution for the ratio of two normal, as when the multivariate data follows a multi-normal distribution its summation is still normal. However, our proposed method does not provide satisfactory result based on the model derivation. The research was therefore aborted and our effort on this topic was no longer continued.

However, the mathematical derivation of the ratio two normal itself has its unique contribution to literature. It is wastage if we throw all of them. We decided to document our derivation process in the appendix for future reference.

The following is the derivation of ratio of two normal distributions:

Notations:

X_i, Y_i, Z_i: Capital letters with one subscript is used to denote vectors and observations

X_{ij}, Y_{ij}, Z_{ij}: Capital letters with two subscripts is used to denote individual observation readings.

x, y, z: Small letters stand for variables

Greek letters are used to stand for population statistics and English letter for sample statistics.

1. What is the distribution of $Z = \frac{X}{Y}$, here X and Y are independent.

$$F(Z) = P(Z \le z) = P\left(\frac{X}{Y} \le z\right)$$

$$= \int_{-\infty}^{o} P(X \ge az) f_y(a) da + \int_{0}^{+\infty} P(X \le az) f_y(a) da$$

$$= \int_{-\infty}^{o} [1 - P(X \le az)] f_y(a) da + \int_{0}^{+\infty} P(X \le az) f_y(a) da$$

$$= \int_{-\infty}^{o} [1 - F_X(az)] f_y(a) da + \int_{0}^{+\infty} F_X(az) f_y(a) da$$

$$= \int_{-\infty}^{o} f_y(a) da - \int_{-\infty}^{o} F_X(az) f_y(a) da + \int_{0}^{+\infty} F_X(az) f_y(a) da$$

$$= \Phi\left(\frac{-\mu_y}{\sigma_y}\right) - \int_{-\infty}^{o} F_X(az) f_y(a) da + \int_{0}^{+\infty} F_X(az) f_y(a) da$$

$$f(z) = \frac{\partial F(z)}{\partial z} = \frac{\partial \left\{ \Phi\left(\frac{-\mu_y}{\sigma_y}\right) - \int_{-\infty}^{o} F_X(az) f_y(a) da + \int_{0}^{+\infty} F_X(az) f_y(a) da \right\}}{\partial z}$$

$$= \frac{\partial \left\{ - \int_{-\infty}^{o} F_X(az) f_y(a) da + \int_{0}^{+\infty} F_X(az) f_y(a) da \right\}}{\partial z}$$

$$= -\int_{-\infty}^{o} \frac{\partial (F_X(az) f_y(a))}{\partial z} da + \int_{0}^{+\infty} \frac{\partial (F_X(az) f_y(a))}{\partial z} da$$

$$= -\int_{-\infty}^{0} f_x(az) * f_y(a) * ada + \int_{0}^{+\infty} f_x(az) * f_y(a) * ada$$

$$f(z) = -\int_{-\infty}^{0} f_x(az)*f_y(a)*ada + \int_{0}^{+\infty} f_x(az)*f_y(a)*ada$$

$$= -\int_{-\infty}^{0} \frac{1}{\sqrt{2\pi}\sigma_x}\exp\left(\frac{-(az-\mu_x)^2}{2\sigma_x^2}\right)\frac{1}{\sqrt{2\pi}\sigma_y}\exp\left(\frac{-(a-\mu_y)^2}{2\sigma_y^2}\right)ada$$

$$+ \int_{0}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma_x}\exp\left(\frac{-(az-\mu_x)^2}{2\sigma_x^2}\right)\frac{1}{\sqrt{2\pi}\sigma_y}\exp\left(\frac{-(a-\mu_y)^2}{2\sigma_y^2}\right)ada$$

$$= -\frac{1}{2\pi\sigma_x\sigma_y}\int_{-\infty}^{0}\exp\left(\frac{-(az-\mu_x)^2}{2\sigma_x^2} + \frac{-(a-\mu_y)^2}{2\sigma_y^2}\right)ada$$

$$+\frac{1}{2\pi\sigma_x\sigma_y}\int_{0}^{+\infty}\exp\left(\frac{-(az-\mu_x)^2}{2\sigma_x^2} + \frac{-(a-\mu_y)^2}{2\sigma_y^2}\right)ada$$

$$= -\frac{1}{2\pi\sigma_x\sigma_y}\int_{-\infty}^{0}\left(\exp\left(\frac{-(az-\mu_x)^2}{2\sigma_x^2} + \frac{-(a-\mu_y)^2}{2\sigma_y^2}\right)\right)ada$$

$$+\frac{1}{2\pi\sigma_x\sigma_y}\int_{0}^{+\infty}\left(\exp\left(\frac{-(az-\mu_x)^2}{2\sigma_x^2} + \frac{-(a-\mu_y)^2}{2\sigma_y^2}\right)\right)ada$$

$$= -\frac{1}{2\pi\sigma_x\sigma_y}\int_{-\infty}^{0}\left(\exp\left(\frac{-\sigma_y^2(az-\mu_x)^2 - \sigma_x^2(a-\mu_y)^2}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$+\frac{1}{2\pi\sigma_x\sigma_y}\int_{0}^{+\infty}\left(\exp\left(\frac{-\sigma_y^2(az-\mu_x)^2 - \sigma_x^2(a-\mu_y)^2}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$= -\frac{1}{2\pi\sigma_x\sigma_y}\int_{-\infty}^{0}\left(\exp\left(\frac{-\sigma_y^2(a^2z^2 - 2az\mu_x + \mu_x^2) - \sigma_x^2(a^2 - 2a\mu_y + \mu_y^2)}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$+\frac{1}{2\pi\sigma_x\sigma_y}\int_{0}^{+\infty}\left(\exp\left(\frac{-\sigma_y^2(a^2z^2 - 2az\mu_x + \mu_x^2) - \sigma_x^2(a^2 - 2a\mu_y + \mu_y^2)}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$= -\frac{1}{2\pi\sigma_x\sigma_y}\int_{-\infty}^{0}\left(\exp\left(\frac{-(\sigma_y^2a^2z^2 - \sigma_y^2 2az\mu_x + \sigma_y^2\mu_x^2) - (\sigma_x^2a^2 - \sigma_x^2 2a\mu_y + \sigma_x^2\mu_y^2)}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$+\frac{1}{2\pi\sigma_x\sigma_y}\int_{0}^{+\infty}\left(\exp\left(\frac{-(\sigma_y^2a^2z^2 - \sigma_y^2 2az\mu_x + \sigma_y^2\mu_x^2) - (\sigma_x^2a^2 - \sigma_x^2 2a\mu_y + \sigma_x^2\mu_y^2)}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$= \frac{1}{2\pi\sigma_x\sigma_y}\left(\begin{array}{l} -\int_{-\infty}^{0}\left(\exp\left(\frac{-(\sigma_y^2a^2z^2 - \sigma_y^2 2az\mu_x + \sigma_y^2\mu_x^2) - (\sigma_x^2a^2 - \sigma_x^2 2a\mu_y + \sigma_x^2\mu_y^2)}{2\sigma_x^2\sigma_y^2}\right)\right)ada \\ +\int_{0}^{+\infty}\left(\exp\left(\frac{-(\sigma_y^2a^2z^2 - \sigma_y^2 2az\mu_x + \sigma_y^2\mu_x^2) - (\sigma_x^2a^2 - \sigma_x^2 2a\mu_y + \sigma_x^2\mu_y^2)}{2\sigma_x^2\sigma_y^2}\right)\right)ada \end{array}\right)$$

The first integration:

$$\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(\sigma_y^2a^2z^2-\sigma_y^2 2az\mu_x+\sigma_y^2\mu_x^2\right)-\left(\sigma_x^2a^2-\sigma_x^2 2a\mu_y+\sigma_x^2\mu_y^2\right)}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$=\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(\left(\sigma_y^2z^2+\sigma_x^2\right)a^2-2\left(\sigma_y^2z\mu_x+\sigma_x^2\mu_y\right)a+\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2\right)}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$=\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(a^2-2\left(\frac{\sigma_y^2z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2z^2+\sigma_x^2}\right)a+\frac{\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2}{\sigma_y^2z^2+\sigma_x^2}\right)}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2z^2+\sigma_x^2}}\right)\right)ada$$

$$=\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(a^2-2\left(\frac{\sigma_y^2z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2z^2+\sigma_x^2}\right)a+\left(\frac{\sigma_y^2z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2z^2+\sigma_x^2}\right)^2\right)+\left(\frac{\sigma_y^2z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2z^2+\sigma_x^2}\right)^2-\frac{\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2}{\sigma_y^2z^2+\sigma_x^2}}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2z^2+\sigma_x^2}}\right)\right)ada$$

$$=\exp\left(\frac{\left(\frac{\sigma_y^2z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2z^2+\sigma_x^2}\right)^2-\frac{\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2}{\sigma_y^2z^2+\sigma_x^2}}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2z^2+\sigma_x^2}}\right)\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2z^2+\sigma_x^2}}\right)\right)ada$$

the first part:

$$\exp\left(\frac{\left(\frac{\sigma_y^2 z\mu_x + \sigma_x^2\mu_y}{\sigma_y^2 z^2 + \sigma_x^2}\right)^2 - \frac{\sigma_y^2\mu_x^2 + \sigma_x^2\mu_y^2}{\sigma_y^2 z^2 + \sigma_x^2}}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2 + \sigma_x^2}}\right)$$

$$= \exp\left(\frac{\left(\sigma_y^2 z\mu_x + \sigma_x^2\mu_y\right)^2 - \left(\sigma_y^2 z^2 + \sigma_x^2\right)\left(\sigma_y^2\mu_x^2 + \sigma_x^2\mu_y^2\right)}{\left(\sigma_y^2 z^2 + \sigma_x^2\right)^2} * \frac{\sigma_y^2 z^2 + \sigma_x^2}{2\sigma_x^2\sigma_y^2}\right)$$

$$= \exp\left(\frac{\left(\sigma_y^2 z\mu_x\right)^2 + 2\sigma_y^2 z\mu_x\sigma_x^2\mu_y + \left(\sigma_x^2\mu_y\right)^2 - \sigma_y^2 z^2\sigma_y^2\mu_x^2 - \sigma_y^2 z^2\sigma_x^2\mu_y^2 - \sigma_x^2\sigma_y^2\mu_x^2 - \sigma_x^2\sigma_x^2\mu_y^2}{2\sigma_x^2\sigma_y^2\left(\sigma_y^2 z^2 + \sigma_x^2\right)}\right)$$

$$= \exp\left(\frac{\sigma_y^4 z^2\mu_x^2 + 2\sigma_x^2\sigma_y^2 z\mu_x\mu_y + \sigma_x^4\mu_y^2 - \sigma_y^4 z^2\mu_x^2 - \sigma_x^2\sigma_y^2 z^2\mu_y^2 - \sigma_x^2\sigma_y^2\mu_x^2 - \sigma_x^4\mu_y^2}{2\sigma_x^2\sigma_y^2\left(\sigma_y^2 z^2 + \sigma_x^2\right)}\right)$$

$$= \exp\left(\frac{2\sigma_x^2\sigma_y^2 z\mu_x\mu_y - \sigma_x^2\sigma_y^2 z^2\mu_y^2 - \sigma_x^2\sigma_y^2\mu_x^2}{2\sigma_x^2\sigma_y^2\left(\sigma_y^2 z^2 + \sigma_x^2\right)}\right) = \exp\left(\frac{2z\mu_x\mu_y - z^2\mu_y^2 - \mu_x^2}{2\left(\sigma_y^2 z^2 + \sigma_x^2\right)}\right)$$

$$= \exp\left(-\frac{\left(\mu_x - z\mu_y\right)^2}{2\left(\sigma_y^2 z^2 + \sigma_x^2\right)}\right)$$

$$\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)a\,da=\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}+\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)da$$

$$=\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)da+\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)da$$

$$=-\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)d\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)$$

$$+\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)\sqrt{2\pi\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\int_{-\infty}^{0}\left(\frac{1}{\sqrt{2\pi\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}}\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)da$$

$$=-\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\Bigg|_{-\infty}^{0}+\frac{\sqrt{2\pi}\sigma_x\sigma_y\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\Phi\left(\frac{-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}}{\sqrt{\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}}\right)$$

$$=-\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}\exp\left(\frac{-\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)+\frac{\sqrt{2\pi}\sigma_x\sigma_y\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\Phi\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\sigma_x\sigma_y\sqrt{\left(\sigma_y^2 z^2+\sigma_x^2\right)}}\right)$$

$$=-\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}\exp\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)^2}{2\sigma_x^2\sigma_y^2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)+\frac{\sqrt{2\pi}\sigma_x\sigma_y\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\Phi\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\sigma_x\sigma_y\sqrt{\left(\sigma_y^2 z^2+\sigma_x^2\right)}}\right)$$

Therefore, the first integration is:

$$\int_{-\infty}^{0}\left(\exp\left(\frac{-\left(\sigma_y^2 a^2 z^2-\sigma_y^2 2az\mu_x+\sigma_y^2\mu_x^2\right)-\left(\sigma_x^2 a^2-\sigma_x^2 2a\mu_y+\sigma_x^2\mu_y^2\right)}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$=\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)*\left(\begin{array}{l}-\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}\exp\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)^2}{2\sigma_x^2\sigma_y^2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)\\+\frac{\sqrt{2\pi}\sigma_x\sigma_y\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\Phi\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\sigma_x\sigma_y\sqrt{\left(\sigma_y^2 z^2+\sigma_x^2\right)}}\right)\end{array}\right)$$

The second integration in f (z) is

$$\int_{0}^{+\infty}\left(\exp\left(\frac{-\left(\sigma_y^2 a^2 z^2-\sigma_y^2 2az\mu_x+\sigma_y^2\mu_x^2\right)-\left(\sigma_x^2 a^2-\sigma_x^2 2a\mu_y+\sigma_x^2\mu_y^2\right)}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$=\int_{0}^{+\infty}\left(\exp\left(\frac{-\left(\left(\sigma_y^2 z^2+\sigma_x^2\right)a^2-2\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)a+\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2\right)}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$=\int_{0}^{+\infty}\left(\exp\left(\frac{-\left(a^2-2\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)a+\frac{\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2}{\sigma_y^2 z^2+\sigma_x^2}\right)}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)ada$$

$$=\int_{0}^{+\infty}\left(\exp\left(\frac{-\left(a^2-2\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)a+\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2\right)+\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2-\frac{\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2}{\sigma_y^2 z^2+\sigma_x^2}}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)ada$$

$$=\exp\left(\frac{\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2-\frac{\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2}{\sigma_y^2 z^2+\sigma_x^2}}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\int_{0}^{+\infty}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)ada$$

Similarly, the first part is equal to:

$$\exp\left(\frac{\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2-\frac{\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2}{\sigma_y^2 z^2+\sigma_x^2}}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)=\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)$$

$$\int_0^{+\infty}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)ada=\int_0^{+\infty}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}+\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)da$$

$$=\int_0^{+\infty}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)da+\int_0^{+\infty}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)da$$

$$=-\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}\int_0^{+\infty}\left(\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)d\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)$$

$$+\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)\sqrt{2\pi\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\int_0^{+\infty}\left(\frac{1}{\sqrt{2\pi\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}}\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\right)da$$

$$=-\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}\exp\left(\frac{-\left(a-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)\Bigg|_0^{+\infty}+\frac{\sqrt{2\pi}\sigma_x\sigma_y\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\left(1-\Phi\left(\frac{-\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}}{\sqrt{\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}}\right)\right)$$

$$=\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}\exp\left(\frac{-\left(\frac{\sigma_y^2 z\mu_x+\sigma_x^2\mu_y}{\sigma_y^2 z^2+\sigma_x^2}\right)^2}{2\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}}\right)+\frac{\sqrt{2\pi}\sigma_x\sigma_y\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\left(1-\Phi\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\sigma_x\sigma_y\sqrt{\left(\sigma_y^2 z^2+\sigma_x^2\right)}}\right)\right)$$

$$=\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}\exp\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)^2}{2\sigma_x^2\sigma_y^2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)+\frac{\sqrt{2\pi}\sigma_x\sigma_y\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\left(1-\Phi\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\sigma_x\sigma_y\sqrt{\left(\sigma_y^2 z^2+\sigma_x^2\right)}}\right)\right)$$

Therefore, the second integration is:

$$\int_0^{+\infty}\left(\exp\left(\frac{-\left(\sigma_y^2 a^2 z^2-\sigma_y^2 2az\mu_x+\sigma_y^2\mu_x^2\right)-\left(\sigma_x^2 a^2-\sigma_x^2 2a\mu_y+\sigma_x^2\mu_y^2\right)}{2\sigma_x^2\sigma_y^2}\right)\right)ada$$

$$=\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)*\left(\begin{array}{l}\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2 z^2+\sigma_x^2}\exp\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)^2}{2\sigma_x^2\sigma_y^2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)\\+\frac{\sqrt{2\pi}\sigma_x\sigma_y\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\left(1-\Phi\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\sigma_x\sigma_y\sqrt{\left(\sigma_y^2 z^2+\sigma_x^2\right)}}\right)\right)\end{array}\right)$$

the Final express for f (z) is:

$$f(z)=\frac{1}{2\pi\sigma_x\sigma_y}*$$

$$\left(\begin{aligned}&-\exp\left(-\frac{(\mu_x-z\mu_y)^2}{2(\sigma_y^2z^2+\sigma_x^2)}\right)*\left(-\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2z^2+\sigma_x^2}\exp\left(\frac{-(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)^2}{2\sigma_x^2\sigma_y^2(\sigma_y^2z^2+\sigma_x^2)}\right)+\frac{\sqrt{2\pi}\sigma_x\sigma_y(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)}{(\sigma_y^2z^2+\sigma_x^2)^{3/2}}\Phi\left(\frac{-(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)}{\sigma_x\sigma_y\sqrt{(\sigma_y^2z^2+\sigma_x^2)}}\right)\right)\\&+\exp\left(-\frac{(\mu_x-z\mu_y)^2}{2(\sigma_y^2z^2+\sigma_x^2)}\right)*\left(\frac{\sigma_x^2\sigma_y^2}{\sigma_y^2z^2+\sigma_x^2}\exp\left(\frac{-(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)^2}{2\sigma_x^2\sigma_y^2(\sigma_y^2z^2+\sigma_x^2)}\right)+\frac{\sqrt{2\pi}\sigma_x\sigma_y(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)}{(\sigma_y^2z^2+\sigma_x^2)^{3/2}}\left(1-\Phi\left(\frac{-(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)}{\sigma_x\sigma_y\sqrt{(\sigma_y^2z^2+\sigma_x^2)}}\right)\right)\right)\end{aligned}\right)$$

$$=\frac{1}{2\pi\sigma_x\sigma_y}\frac{2\sigma_x^2\sigma_y^2}{\sigma_y^2z^2+\sigma_x^2}\exp\left(\frac{-(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)^2}{2\sigma_x^2\sigma_y^2(\sigma_y^2z^2+\sigma_x^2)}\right)*\exp\left(-\frac{(\mu_x-z\mu_y)^2}{2(\sigma_y^2z^2+\sigma_x^2)}\right)$$

$$+\frac{1}{2\pi\sigma_x\sigma_y}\frac{\sqrt{2\pi}\sigma_x\sigma_y(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)}{(\sigma_y^2z^2+\sigma_x^2)^{3/2}}\exp\left(-\frac{(\mu_x-z\mu_y)^2}{2(\sigma_y^2z^2+\sigma_x^2)}\right)*\left(1-2\Phi\left(\frac{-(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)}{\sigma_x\sigma_y\sqrt{(\sigma_y^2z^2+\sigma_x^2)}}\right)\right)$$

$$=\frac{\sigma_x\sigma_y}{\pi(\sigma_y^2z^2+\sigma_x^2)}\exp\left(\frac{-(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)^2}{2\sigma_x^2\sigma_y^2(\sigma_y^2z^2+\sigma_x^2)}-\frac{(\mu_x-z\mu_y)^2}{2(\sigma_y^2z^2+\sigma_x^2)}\right)$$

$$+\frac{(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)}{\sqrt{2\pi}(\sigma_y^2z^2+\sigma_x^2)^{3/2}}\exp\left(-\frac{(\mu_x-z\mu_y)^2}{2(\sigma_y^2z^2+\sigma_x^2)}\right)*\left(1-2\Phi\left(\frac{-(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)}{\sigma_x\sigma_y\sqrt{(\sigma_y^2z^2+\sigma_x^2)}}\right)\right)$$

The first part is:

$$\frac{\sigma_x\sigma_y}{\pi(\sigma_y^2z^2+\sigma_x^2)}\exp\left(\frac{-(\sigma_y^2z\mu_x+\sigma_x^2\mu_y)^2-\sigma_x^2\sigma_y^2(\mu_x-z\mu_y)^2}{2\sigma_x^2\sigma_y^2(\sigma_y^2z^2+\sigma_x^2)}\right)$$

$$=\frac{\sigma_x\sigma_y}{\pi(\sigma_y^2z^2+\sigma_x^2)}\exp\left(\frac{-\left((\sigma_y^2z\mu_x)^2+2\sigma_y^2z\mu_x\sigma_x^2\mu_y+(\sigma_x^2\mu_y)^2\right)-\sigma_x^2\sigma_y^2(\mu_x^2-2\mu_xz\mu_y+z^2\mu_y^2)}{2\sigma_x^2\sigma_y^2(\sigma_y^2z^2+\sigma_x^2)}\right)$$

$$=\frac{\sigma_x\sigma_y}{\pi(\sigma_y^2z^2+\sigma_x^2)}\exp\left(\frac{-(\sigma_y^4z^2\mu_x^2+2\sigma_x^2\sigma_y^2z\mu_x\mu_y+\sigma_x^4\mu_y^2+\sigma_x^2\sigma_y^2\mu_x^2-2\sigma_x^2\sigma_y^2\mu_xz\mu_y+\sigma_x^2\sigma_y^2z^2\mu_y^2)}{2\sigma_x^2\sigma_y^2(\sigma_y^2z^2+\sigma_x^2)}\right)$$

$$=\frac{\sigma_x\sigma_y}{\pi(\sigma_y^2z^2+\sigma_x^2)}\exp\left(\frac{-(\sigma_y^4z^2\mu_x^2+\sigma_x^4\mu_y^2+\sigma_x^2\sigma_y^2\mu_x^2+\sigma_x^2\sigma_y^2z^2\mu_y^2)}{2\sigma_x^2\sigma_y^2(\sigma_y^2z^2+\sigma_x^2)}\right)$$

$$=\frac{\sigma_x\sigma_y}{\pi(\sigma_y^2z^2+\sigma_x^2)}\exp\left(\frac{-\left(\sigma_y^2z^2(\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2)+\sigma_x^2(\sigma_x^2\mu_y^2+\sigma_y^2\mu_x^2)\right)}{2\sigma_x^2\sigma_y^2(\sigma_y^2z^2+\sigma_x^2)}\right)$$

$$=\frac{\sigma_x\sigma_y}{\pi(\sigma_y^2z^2+\sigma_x^2)}\exp\left(\frac{-(\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2)(\sigma_y^2z^2+\sigma_x^2)}{2\sigma_x^2\sigma_y^2(\sigma_y^2z^2+\sigma_x^2)}\right)$$

$$=\frac{\sigma_x\sigma_y}{\pi(\sigma_y^2z^2+\sigma_x^2)}\exp\left(\frac{-(\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2)}{2\sigma_x^2\sigma_y^2}\right)$$

Therefore, the final expression is:

$$f(z)=\frac{\sigma_x\sigma_y}{\pi\left(\sigma_y^2 z^2+\sigma_x^2\right)}\exp\left(\frac{-\left(\sigma_y^2\mu_x^2+\sigma_x^2\mu_y^2\right)}{2\sigma_x^2\sigma_y^2}\right)$$
$$+\frac{\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\sqrt{2\pi}\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)*\left(1-2\Phi\left(\frac{-\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\sigma_x\sigma_y\sqrt{\left(\sigma_y^2 z^2+\sigma_x^2\right)}}\right)\right)$$

2. Ratio of two normal and its mean (correlated):

Suppose $X\sim N(\mu,\Sigma)$, then $x_i\sim N\left(\mu_i,\sigma_i^2\right)$

Let $y=x_1+x_2+\ldots+x_p$

Then, $y\sim N\left(\sum_{i=1}^{p}\mu_i, C\Sigma C'\right)$ and where matrix C is $1\times p$ matrix with all the elements equal to one.

What is the distribution of $z=\frac{x_i}{y}$, here x and y is correlated, not independent.

$$F(Z)=P(Z\le z)=P\left(\frac{X_i}{Y}\le z\right)$$
$$=\int_{-\infty}^{o}P\left(X_i\ge az\right)f_y(a)da+\int_{0}^{+\infty}P\left(X_i\le az\right)f_y(a)da$$
$$=\int_{-\infty}^{o}\left[1-P\left(X_i\le az\right)\right]f_y(a)da+\int_{0}^{+\infty}P\left(X_i\le az\right)f_y(a)da$$
$$=\int_{-\infty}^{o}\left[1-F_{X_i}(az)\right]f_y(a)da+\int_{0}^{+\infty}F_{X_i}(az)f_y(a)da$$
$$=\int_{-\infty}^{o}f_y(a)da-\int_{-\infty}^{o}F_{X_i}(az)f_y(a)da+\int_{0}^{+\infty}F_{X_i}(az)f_y(a)da$$
$$=\Phi\left(\frac{-\mu_y}{\sigma_y}\right)-\int_{-\infty}^{o}F_{X_i}(az)f_y(a)da+\int_{0}^{+\infty}F_{X_i}(az)f_y(a)da$$

Applying the same derivation process we get:

$$f(z)=\frac{\sigma_{x_i}\sigma_y}{\pi\left(\sigma_y^2 z^2+\sigma_{x_i}^2\right)}\exp\left(\frac{-\left(\sigma_y^2\mu_{x_i}^2+\sigma_{x_i}^2\mu_y^2\right)}{2\sigma_{x_i}^2\sigma_y^2}\right)$$
$$+\frac{\left(\sigma_y^2 z\mu_{x_i}+\sigma_{x_i}^2\mu_y\right)}{\sqrt{2\pi}\left(\sigma_y^2 z^2+\sigma_{x_i}^2\right)^{3/2}}\exp\left(-\frac{\left(\mu_{x_i}-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_{x_i}^2\right)}\right)*\left(1-2\Phi\left(\frac{-\left(\sigma_y^2 z\mu_{x_i}+\sigma_{x_i}^2\mu_y\right)}{\sigma_{x_i}\sigma_y\sqrt{\left(\sigma_y^2 z^2+\sigma_{x_i}^2\right)}}\right)\right)$$

Next step will focus on the derivation of the respective cumulative density function.

$$F(t)=\int_{-\infty}^{t} f(z)dz=\int_{-\infty}^{t}\frac{\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\sqrt{2\pi}\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)dz$$

$$\frac{d\left(\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)\right)}{dz}$$

$$=\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)\left(-\frac{1}{2}\frac{\left(\sigma_y^2 z^2+\sigma_x^2\right)*2\left(\mu_x-z\mu_y\right)\left(-\mu_y\right)-\left(\mu_x-z\mu_y\right)^2*2\sigma_y^2 z}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^2}\right)$$

$$=\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)\left(\frac{\left(\sigma_y^2 z^2+\sigma_x^2\right)*\left(\mu_x-z\mu_y\right)\mu_y+\left(\mu_x-z\mu_y\right)^2*\sigma_y^2 z}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^2}\right)$$

$$=\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)\left(\frac{\left(\mu_x-z\mu_y\right)\left(\left(\sigma_y^2 z^2+\sigma_x^2\right)\mu_y+\left(\mu_x-z\mu_y\right)\sigma_y^2 z\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^2}\right)$$

$$=\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)\left(\frac{\left(\mu_x-z\mu_y\right)\left(\sigma_y^2 z^2\mu_y+\sigma_x^2\mu_y+\sigma_y^2 z\mu_x-\sigma_y^2 z^2\mu_y\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^2}\right)$$

$$=\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)\left(\frac{\left(\mu_x-z\mu_y\right)\left(\sigma_x^2\mu_y+\sigma_y^2 z\mu_x\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^2}\right)$$

$$F(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t}\frac{\left(\sigma_y^2 z\mu_x+\sigma_x^2\mu_y\right)}{\left(\sigma_y^2 z^2+\sigma_x^2\right)^{3/2}}\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)dz$$

$$=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{t}\frac{\sqrt{\left(\sigma_y^2 z^2+\sigma_x^2\right)}}{\left(\mu_x-z\mu_y\right)}d\left(\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)\right)$$

$$=\frac{1}{\sqrt{2\pi}}\left(\left.\frac{\sqrt{\left(\sigma_y^2 z^2+\sigma_x^2\right)}}{\left(\mu_x-z\mu_y\right)}*\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)\right|_{-\infty}^{t}-\int_{-\infty}^{t}\exp\left(-\frac{\left(\mu_x-z\mu_y\right)^2}{2\left(\sigma_y^2 z^2+\sigma_x^2\right)}\right)d\left(\frac{\sqrt{\left(\sigma_y^2 z^2+\sigma_x^2\right)}}{\left(\mu_x-z\mu_y\right)}\right)\right)$$

the latter part:

$$\int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) d\left(\frac{\sqrt{(\sigma_y^2 z^2 + \sigma_x^2)}}{(\mu_x - z\mu_y)}\right)$$

$$= \int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) \frac{(\mu_x - z\mu_y)\frac{2\sigma_y^2 z}{2\sqrt{(\sigma_y^2 z^2 + \sigma_x^2)}} - \sqrt{(\sigma_y^2 z^2 + \sigma_x^2)} * (-\mu_y)}{(\mu_x - z\mu_y)^2} dz$$

$$= \int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) \frac{\sigma_y^2 z(\mu_x - z\mu_y) + (\sigma_y^2 z^2 + \sigma_x^2)\mu_y}{\sqrt{(\sigma_y^2 z^2 + \sigma_x^2)}(\mu_x - z\mu_y)^2} dz$$

$$= \int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) \frac{(\sigma_y^2 z\mu_x - \sigma_y^2 z^2 \mu_y + \sigma_y^2 z^2 \mu_y + \sigma_x^2 \mu_y)}{\sqrt{(\sigma_y^2 z^2 + \sigma_x^2)}(\mu_x - z\mu_y)^2} dz$$

$$= \int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) \frac{(\sigma_y^2 z\mu_x + \sigma_x^2 \mu_y)}{\sqrt{(\sigma_y^2 z^2 + \sigma_x^2)}(\mu_x - z\mu_y)^2} dz$$

$$= \int_{-\infty}^{t} \left(\frac{(\mu_x - z\mu_y)(\sigma_x^2 \mu_y + \sigma_y^2 z\mu_x)}{(\sigma_y^2 z^2 + \sigma_x^2)^2}\right) \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) \frac{(\sigma_y^2 z^2 + \sigma_x^2)^{3/2}}{(\mu_x - z\mu_y)^3} dz$$

$$= \int_{-\infty}^{t} \frac{(\sigma_y^2 z^2 + \sigma_x^2)^{3/2}}{(\mu_x - z\mu_y)^3} d\left(\exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right)\right)$$

$$= \frac{(\sigma_y^2 z^2 + \sigma_x^2)^{3/2}}{(\mu_x - z\mu_y)^3} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right)\Bigg|_{-\infty}^{t} - \int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) d\left(\frac{(\sigma_y^2 z^2 + \sigma_x^2)^{3/2}}{(\mu_x - z\mu_y)^3}\right)$$

$$= \frac{(\sigma_y^2 z^2 + \sigma_x^2)^{3/2}}{(\mu_x - z\mu_y)^3} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right)\Bigg|_{-\infty}^{t}$$

$$- \int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) \frac{(\mu_x - z\mu_y)^3 \frac{3}{2}(\sigma_y^2 z^2 + \sigma_x^2)^{1/2} 2\sigma_y^2 z - (\sigma_y^2 z^2 + \sigma_x^2)^{3/2} 3(\mu_x - z\mu_y)^2(-\mu_y)}{(\mu_x - z\mu_y)^6} dz$$

$$\int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) \frac{(\mu_x - z\mu_y)^3 \frac{3}{2}(\sigma_y^2 z^2 + \sigma_x^2)^{1/2} 2\sigma_y^2 z - (\sigma_y^2 z^2 + \sigma_x^2)^{3/2} 3(\mu_x - z\mu_y)^2(-\mu_y)}{(\mu_x - z\mu_y)^6} dz$$

$$= 3\int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) \frac{(\mu_x - z\mu_y)(\sigma_y^2 z^2 + \sigma_x^2)^{1/2} \sigma_y^2 z + (\sigma_y^2 z^2 + \sigma_x^2)^{3/2} \mu_y}{(\mu_x - z\mu_y)^4} dz$$

$$= 3\int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) \frac{(\sigma_y^2 z^2 + \sigma_x^2)^{1/2}(\sigma_y^2 z\mu_x - \sigma_y^2 zz\mu_y + \sigma_y^2 z^2 \mu_y + \sigma_x^2 \mu_y)}{(\mu_x - z\mu_y)^4} dz$$

$$= 3\int_{-\infty}^{t} \exp\left(-\frac{(\mu_x - z\mu_y)^2}{2(\sigma_y^2 z^2 + \sigma_x^2)}\right) \frac{(\sigma_y^2 z^2 + \sigma_x^2)^{1/2}(\sigma_y^2 z\mu_x + \sigma_x^2 \mu_y)}{(\mu_x - z\mu_y)^4} dz$$

$$F(z) = \Phi\left(\frac{-\mu_y}{\sigma_y}\right) - \int_{-\infty}^{o} F_X(az) f_y(a) da + \int_{0}^{+\infty} F_X(az) f_y(a) da$$

$$= \Phi\left(\frac{-\mu_y}{\sigma_y}\right) - \int_{-\infty}^{o}\int_{-\infty}^{az} \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(\frac{-(x-\mu_x)^2}{2\sigma_x^2}\right) dx \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(\frac{-(a-\mu_y)^2}{2\sigma_y^2}\right) da$$

$$+ \int_{0}^{+\infty}\int_{-\infty}^{az} \frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(\frac{-(x-\mu_x)^2}{2\sigma_x^2}\right) dx \frac{1}{\sqrt{2\pi}\sigma_y} \exp\left(\frac{-(a-\mu_y)^2}{2\sigma_y^2}\right) da$$

Appendix C SWOT Analysis of Six Sigma Strategy

Introduction

Nowadays, Six Sigma has become the "fashion" and has gained much popularity worldwide. Not only manufacturing companies, but also service industries, such as financial and educational institutions, are starting to embrace this "all-around" strategy. In certain areas it has become synonymous with business excellence. The success stories of Motorola, General Electric, Seagate Technologies and Allied Signal have enticed organizations to adopt Six Sigma as their ultimate tool towards perfection and customer satisfaction (Harry and Schroeder 2000). However, twenty years after its introduction, Six Sigma has never stopped drawing debates and criticisms. Some criticisms include the huge investment costs it entails and the uncertainty of successful implementation it poses. As a matter of fact, the debates over its pros and cons never cease but have instead increased in intensity over the past years as this program gains in popularity in many industries. Whether Six Sigma should be implemented is a dilemma faced by all organizations. This is an important decision in all forward looking companies because if they chose otherwise or other less effective quality programs, they may be left behind from competition and other undesirable consequences. The need to be competitive is also fueled by the threats of the highly competitive global environment today. On the other hand, implementing Six Sigma is not an easy task and does not guarantee corporate survival in these hostile market conditions. While some giant corporations are vocally praising the Six Sigma (Harry and Schroeder 2000), stories of failed efforts and remarks of dissatisfaction are also heard (www.isixsigma.com). Furthermore, organization must be financially prepared since the program requires huge cash outflow, especially in training, before significant results can be seen. In consideration of these elements, the organization must quantify the cost of doing nothing and the cost of implementing Six Sigma or going to other programs.

In view of these dilemmas, it is very important to examine carefully the nature of Six Sigma and how it can help the organization. In this paper, we perform the SWOT analysis to identify strengths, weaknesses, opportunities and threats to Six Sigma strategy, and based on these identifications, we give our views regarding this dilemma and also outline the possible improvement strategies for this program.

In section two, we give a brief description of the SWOT analysis method and section three goes to the introduction of Six Sigma strategy. Our detailed analysis will be presented in section four and conclusions are to be included in section five.

SWOT Analysis

What is it?

SWOT analysis stands for strengths, weaknesses, opportunities and threats analysis. It is a popular way and an effective tool of analyzing a company or an organization by identifying its inherent strengths and weaknesses, and examining the external opportunities and threats, which may affect the organization. This method provides a formal framework for summarizing and integrating the analyses of an organization's external environment and its internal resource and capabilities. The aim of SWOT analysis is to match likely external environment changes with its internal capabilities, to test these out and challenge how an organization can capitalize on new opportunities, or defend itself against future threats. The exercise seeks to challenge the robustness of an organization's current strategy and highlight areas that might need to change in order to sustain or develop its competitive position.

Who performs it?

To carry out a SWOT Analysis effectively, a team consisting of members from various departments of an organization is usually required for the initial brainstorming session. The need for a team approach is largely due to the need for an effective integration of diverse domain knowledge from each department of the organization. In addition, a team approach facilitates the convergence of ideas and directions amongst the different specialist departments within an organization. In most case, the final summarization and integration may be done individually, usually in the hands of the managers.

How to use it?

SWOT analysis should be conducted as objectively as possible based on a thorough investigation of all possible influencing factors in each category. To carry out a SWOT analysis, the following questions in each category may be used as a guide to obtain the necessary information.

Strengths:

What are the advantages you hold in the market?

What do you do well?

What makes you difference from your competitors?

Weaknesses:

What do you do badly?

What should you avoid?

What could you improve on?

What are the causes of your problems and complaints?

Opportunities:

Where are the good opportunities facing you?

What are the interesting trends you are aware of?

Some guidelines for introducing more information are:

Changes in technology and markets on both broad and narrow scale

Changes in government policy related to your field

Changes in social patterns, population profiles, lifestyle, etc.

Local events

Threats:

What are your obstacles?

What are your competitors good at?

Is the changing technology threatening your position?

Do you have bad debt or cash-flow problems?

After identifying all the possible factors, all these information can be consolidated into a SWOT table as illustrated below.

	Positive Factors	Negative Factors
Internal	Strengths	Weaknesses
External	Opportunities	Threats

The consolidated information will then be used as the foundation of further decision and action.

Brief introduction to Six Sigma

In 1988, Motorola, Inc. developed and actively pursued a quality management program called Six Sigma. Since then, it attributed much of its quality improvement to this program. Motorola's World Website (www.motorola.com) states their reason for establishing the Six Sigma process, "In order to achieve the goal of doing it right the first time, we established and communicated the process that we termed Six Sigma."

Six Sigma is a way to measure the probability that companies can produce any given unit of a product (or service) with only 3.4 defects per million units or operations. This measurement standard essentially stems from the need to combat variations in mass manufacturing environments in efforts to improve the quality of the products. Essentially the Six Sigma program aims to identify, measure, reduce and control variations found in mass manufacturing environment. The Six Sigma crusade that began at Motorola has since spread to other companies, such as General Electric (GE), Allied Signal, Seagate Technologies, and so on.

There have been many common interpretations of what is Six Sigma. The following are some common understanding of Six Sigma in the industry:

- A set of complex statistical techniques applied by engineers or statistician to improve the business process.
- Techniques used to achieve the performance target of operating 3.4 defects per million opportunities.
- A 'sweeping' cultural change of an organization to steer toward greater customer satisfaction, profitability, and competitiveness.

These descriptions of Six Sigma serve only to partially define it. Harry and Schroeder (2000) defined it as "A business process that allows companies to drastically improve their bottom line by designing and monitoring everyday business activities in ways that minimize waste and resources while increasing customer satisfaction." Despite these numerous definitions, we prefer the definition given by Pande et al (2000):

"A comprehensive and flexible system for achieving, sustaining and maximizing business success, Six Sigma is uniquely driven by close understanding of customer needs, disciplined use of facts, data and statistical analysis, and diligent attention to managing, improving, and reinventing business process."

In general, the concepts underlying Six Sigma deal with the fact that process and product variation is known to be a strong factor affecting manufacturing lead times, product and process costs, process yields, product quality, and, ultimately, customer satisfaction. A crucial part of Six Sigma work is to define and measure variation with the intent of discovering its causes and to develop efficient operational means to control and reduce the variation. The expected outcomes of Six Sigma efforts are faster and more robust product development, more efficient and capable manufacturing processes, and more confident overall business performance.

Given the tools and techniques used, one might conclude that Six Sigma is nothing new. It uses statistical methods that focus on defect reduction, which results in quality improvement through a project-by-project improvement basis. Although many of these tools of Six Sigma are not new, the approach and its deployment are unique and are the source of its success.

Six Sigma has both management and technical components. On the management side, it focuses on getting the right process metrics and goals, the right projects and right people to work on the projects, and the use of management systems to complete the projects successfully and sustain the gains over time. On the technical side, the focus is on enhancing process performance by improving the average level of performance and reducing variation using process data, statistical thinking and methods.

The traditional Six Sigma process improvement framework is based on a disciplined and focused process-improvement methodology, which has four key stages: Measure, Analyze, Improve, and Control, with an up-front stage (Define) sometimes added (DMAIC). These key stages are defined as follows:

Define (D): Define the problem to be solved, including customer impact and potential benefits.

Measure (M): Identify the critical-to-quality characteristics (CTQs) of the product or service. Verify measurement capability. Baseline the current defect rate and set goals for improvement.

Analyse (A) : Understand root causes of why defects occur; identify key process input variables (KPIVs) that cause defects.

Improve (I) : Quantify influences of key process input variables on the CTQs, identify acceptable limits of these variables, and modify the process to stay within these limits, thereby reducing defect levels in the CTQs.

Control (C) : Ensure that the modified process now keeps the key process output variables (KPOVs) within acceptable limits, in order to maintain the gains in the long term.

Successful Six Sigma implementation in any organization is a top-down initiative carried out by a hierarchy of trained personnel designated as Champions, Master Black Belts, Black Belts and Green Belts. Each designation reflects the level of competence with respect to DMAIC knowledge, practice, and experience. Six Sigma is based on factual data, hard techniques, and purposeful changes. It is an improvement initiative enforced top-down and never meant to be bottom-up phenomenon. It is not conducted via past quality management practices such as slogans, pep talks, will power, accreditation, audit, or certification.

In Six Sigma, strong emphasis is placed on personnel training and deployment. The conscious use of formal statistical tools makes it possible to base decisions on facts rather than arbitrary opinions or preferences. Six Sigma is deployed on a project-by-project basis, each with clear objectives, time frame and results, with the results commonly expressed in financial terms.

Implementing Six Sigma in manufacturing means more than delivering products without defects, it also means eliminating almost all defects, rework, and scrap. It includes operating processes under statistical control, controlling input variables, rather than inspecting for defects at the end of the process, and it means maximizing equipment uptime and optimizing cycle time.

SWOT Analysis to Six Sigma

The consolidated table of SWOT analysis is presented in Table1. Detailed explanation of each factor would be presented subsequently.

Table 1 Consolidated Matrix of SWOT Analysis on Six Sigma Strategy

	Positive Factors	Negative Factors
Internal	Customer focus Data-driven and statistical approach to problem solving Top-down support and corporate-wide involved culture Well-structured project team Clear problem solving framework (DMAIC) Project-based approach Systematic HR development Project tied to bottom line	Heavy investment Highly dependent on corporate culture (receptiveness to change) No uniformly accepted standards Inability to measure and improve intangibles such as innovation and creativity
External	Highly competitive market and demanding customer Fast development of IT and data mining technology Growing research interest in quality and reliability engineering	Resistance to change Highly competitive job market. Cyclical economic conditions

	Previous implementation of quality programs has laid foundation for the easy adoption of Six Sigma.	

Strengths:

Customer focus:

Customer focus is addressed in many quality systems such as TQM and Taguchi methods. It's the core of the quality and the ultimate goal of any successful process. Similarly, customer focus is heavily stressed and is implicitly the top priority in any Six Sigma implementation. Apart from the traditional Six Sigma program, the systematic framework of the Design for Six Sigma methodology for the design phase of any Six Sigma product always begins with a thorough study of customers' requirements. This conforms to the philosophy that any Six Sigma product should stem from a consideration of customers' requirements. In the traditional Six Sigma program for process improvements, the aim is to build what the customers want and its improvements are defined by their impact on customer satisfaction through the proper control of the process to achieve the specifications of the Critical to Quality (CTQ) factors. These CTQs would have been transmitted downwards from the initial design phases of these products. Hence, Six Sigma implementation serves to accurately define customer requirements and measure performance against them. This would enable new development initiatives to be clearly defined with strong customer focus.

Data-driven and statistical approach to problem solving:

A strong focus on technically sound quantitative approaches rather than qualitative approaches is the most important feature of Six Sigma program. The once-fashionable quality program, TQM, seemed to be no different with Six Sigma program in view of many quality practitioners as they found both systems share many in common (Pyzdek, 2001). However, Six Sigma adopted a systematic quantitative approach that overcomes the difficulties incurred by the general and abstract guidelines in TQM. These guidelines could hardly be turned into a successful deployment strategy (Pyzdek, 2001).

Six Sigma is well rooted in mathematics and statistics. Statistical tools are used systematically to measure, collect, analyze and interpret the data and hence to identify the working directions and areas for process improvement. It is a data-driven approach or information-driven approach. Montgomery (2001) observed that Six Sigma could work very well because it is based on sound statistical science and contains in it an effective problem identification and solution framework. This quantitative approach makes quality an attractive, agreeable and manageable task.

Top-down support and corporate-wide involved culture:

Six Sigma requires a top-down management approach. The initiative must come from the top management to drive through every level of the organization. The top management cannot just approve the Six Sigma implementation by just approving the budget for it without any involvement. If this is not the case, these Six Sigma implementations are doomed to failure from the start (Howell, 2001). With this top-down approach, it facilitates the way in acquiring resources for sustaining the activities. This creates a sense of 'urgency' to members of Six Sigma project to devote 100% of their time to these projects. GE is a good illustrative example, where its former CEO, Jack Welch, started its Six Sigma program and drove down through the whole organization, which brought 2 billion dollars returns to GE in 1999 (Goh, 2001). He once told his employees that if they want to be promoted, they'd better be Black Belts.

The huge financial returns incurred by this program make GE almost the model of every Six Sigma practitioner and entices many other companies to join.

Project-based approach:

Unlike other quality system such as TQM and Taguchi methods, Six Sigma is usually carried out on a project basis. The spirit or the essence is still the same—continuous improvement, but the manifestation is different. Continuous improvement may have seemed to be a good slogan and brand name to have, but it is too intangible to be handled with. Adopting a project-based approach forms a cycle of a Six Sigma program and can be easily identified and managed. A typical Six Sigma project is usually selected by the Master Black Belts and the typical project team is composed of Black Belts and Green Belts. The associated team players may be within or cross department. Theoretically all staff should be liable to the project when necessary (Henderson & Evans, 2000). A clear target must be specified in advance and examined to see whether it would be feasible for implementation. Such projects usually last between four and six months and the performance is usually measured in term of monetary saving returns.

Well-structured project team:

Associated with the project-based approach, Six Sigma has a well-designed project team structure. A Six Sigma project team consists of Executive Champion, Deployment Champions, Master Black Belts, Black Belts, and Green Belts. The CEO adopts Six Sigma publicly through a company wide training effort and assigns someone from top management to be the 'Executive Champion' (Henderson & Evans, 2000). The Executive Champion assigns Deployment Champions and Master Black

Belts (also called Project Champions) from the next highest levels of management. The Master Black Belts oversee Six Sigma Projects. Master Black Belts also act as internal Six Sigma consultants for new initiatives. They pick up the projects and people, and teach, coach, and monitor them. Black Belts are the core and the fulltime carrier of a typical Six Sigma project. They are the heart and soul of the Six Sigma quality initiative. Their main purpose is to lead quality projects and work full time until they are complete. Black Belts can typically complete four to six projects per year with savings of approximately $230,000 per project (**www.isixsigma.com**). They also hold the responsibility of coaching Green Belts on their projects. Green Belts are employees trained in Six Sigma who spend a portion of their time completing projects while maintaining their regular work role and responsibilities. Master Black Belts assign the Black Belts and Green Belts to help lead and contribute to the projects. This clear and comprehensive team structure makes the program tangible and manageable.

Clear problem solving framework (DMAIC):

Six Sigma provides a clear systematic problem-solving framework, DMAIC, as the core of its technological base. Statistical tools such as DOE, SPC and Monte Carlo simulations and structured decision support tools such as QFD and FMEA, etc are integrated together under this framework to be explored with their fullest potential. Statistical jargons are no longer barrier to the practitioners, but are integrated for better understanding and ease of use. The Define-Measure-Analyze-Improve-Control approach is applicable to both the manufacturing and service sector (Goh, 2001). It begins by defining (D) who are the customers and what are their priorities, and proceeds to measure (M) the process, i.e. identifies the key internal processes that influence CTQs and measures the defects currently generated relative to those processes. The project team then goes the analyze (A) stage to analyze what are the most important causes of defects and how to improve (I) these defects by removing the causes of defects. The final stage is to control (C)—how can we maintain the improvements (Henderson & Evans, 2000). The DMAIC approach mainly focuses on combating the variations, the biggest enemy of quality. In addition, the DFSS framework offers a systematic means to address quality problems from the design phase of any product. All these provide clear, unambiguous, continuous frameworks for the practitioners to follow and implement.

Systematic HR development:

Six Sigma emphasizes on human resource development and invests heavily in staff training. Practitioners of Six Sigma hold different titles such as Green Belts, Black Belts, Master Black Belts and Champions, which are related to the level of personal competency and roles in carrying out the projects. Practitioners usually start from the more basic and applied Green Belt training from which they will gain the necessary experience and desire to learn more. Then they will proceed on to the next higher level of training to be a Black Belt, which would deal more in depth with the different tools used. Subsequently, their technical competencies would be elevated to that of a Master Black Belt when they would have gained the necessary technical and management experience for them to progress and effectively act as internal consultants to any Six Sigma programs.

In addition, associated with the project-based approach is the reward system of the Six Sigma program. With a project-based approach, the intangible aspects of any “continuous improvement” objective of other quality programs can be more effectively managed by instituting tangible end results to be achieved thereby motivating efforts for quality improvements. Every project will commence with a specified target in mind and finish with a thorough check of the achievement of these targets. Every favorable result will be tied to the bottom-line with strong customer focus. In order to motivate the practitioners, rewards that are tied to bottom-line savings would be instituted. The incentive mechanism fit the human nature well and greatly summons people's interest in quality performance. It ensures that everyone on the track is having well-defined performance indicators, hence, consequently, a fulfilling career.

Project tied to bottom line:

Six Sigma implementations are conducted on a project basis. Once the key business processes are identified, every project will have a deadline and they are all tied to the dollar savings in the bottom-line. There is usually an accountant from the finance department to audit the newly improved way of operating the business process and work out the potential saving as compared to that of the old ways. Therefore, it helps the company to assess the effectiveness of each project through the dollar savings these projects can achieve. Once these savings are verified, it is easier to convince the management to embark on further Six Sigma projects.

Weakness:

Huge Investments:

Large amount of investment is required to train employees to be certified Green Belts, Black Belts, Master black Belts, etc. Training a Black Belt by Singapore Quality Institute require S$24,995.00. In a table given in page 192 of Harry and Schroeder (2000), an average of one Black Belt is required per 100 employees. A 10,000-employee organization needs 100 Black Belts and spends about S$2.5 million for training, exclusive of green belt training fees.

Furthermore, for any Six Sigma project to be effective, the returns are usually not realized in the short term. In contradiction, there may be a possibility of negative returns. Hence, companies who wish to embark of Six Sigma projects would have to adopt such an expectation to maintain commitment in the project. This is usually not easy to justify without concrete results.

Highly Dependent on Corporate Culture:

The success of any Six Sigma implementation is very much dependent on the flexibility of the organization in being able to adapt its already established functions and processes to the structured and disciplined Six Sigma approach. The Six Sigma program is not just a technically sound program with a strong emphasis on statistical tools and techniques, but it also requires the establishment of a strong management framework.

In comparison with the common TQM models, Six Sigma places more emphasis on successful management elements. As such, to have a successful implementation, a shift in the corporate culture within the organization is usually a necessity. This entails a shift in the internalized values and beliefs of the organization, which ultimately leads to some change in the behaviors, and practices of the organization. This implies that if the company contains an established and strong traditional approach in its practices, the change in management perspective would be more difficult.

Furthermore, the necessary statistical tools would need to be relearned by the engineers and managers who may not as yet be fluent in their usage. As such, there may be added difficulties in trying to establish these new skills. These techniques, if not properly taught and applied, will easily undermine the confidence in Six Sigma.

No Uniformly Accepted Standards:

There is yet to be any governing body for the certification of Six Sigma though there are many diverse organizations issuing Six Sigma certificate. No unified standards and procedures are set up and accepted so far. Every organization can claim itself to be a Six Sigma Company with their interpretation of Six Sigma but would not be able achieve the level of quality expected of a Six Sigma company. This does not augur well for the reputation of Six Sigma to the public as companies may utilize such label to improve both customers and investors relations in the market.

For companies who consider building up a core Six Sigma expertise, the lack of standardized body of knowledge and a governing body to administer them may result in a varying level of competency amongst so-called "certified" Six Sigma practitioners. Every training organization uses its own set of course content for training. Many of these training courses may be unbalanced in their focus or lack some critical elements that would be necessary to ensure success.

The lack of a governing body for Six Sigma certification coupled with the tendency of the industry to place higher value on these certifications rather than proper academic qualifications from accredited institutions may result in the loss of confidence in such quality programs in the future.

Inability to measure and improve intangibles

In a globally competitive environment, the ability for a company to innovate and delight customers has become a necessity to stay ahead of cutthroat competition. Due to the fact that Six Sigma strategy focuses on combating variations measured by "sigma" levels, it is still as yet unable to measure and improve intangibles such as creativity and innovation.

In addition, in consideration of the competitive global marketplace, issues such as customization and synergism in product design would have to be dealt with seriously. These may not be easily captured and improved through a Six Sigma framework. The DMAIC framework, which is effective for combating variations in a mass manufacturing environment, has not yet been synergistically integrated with efforts to streamline manufacturing and distribution operations for highly customizable product over diversified geographical markets.

Opportunities

Highly competitive market and demanding customer.

The current globalization and free trade agreements make the competition for market share more hostile and open. Manufacturers are not competing locally or regionally, but globally. To gain or maintain one's market share requires much more efforts and endeavor than ever before. Higher quality and reliability is no longer a conscious choice of the organization but a requirement of the market. For any organization to be successful, quality and reliability in the products that they offer have become one of the essential competing margin and those without them are bound to lose. As Kano theory indicates, customer requirements are growing gradually as time advances. An air conditioner equipped in a car would greatly delight the customer twenty years ago, but now it has become an essential feature. No customer would be excited by an air conditioner in a car nowadays, but will be quite disappointed without it. All these indicate the same phenomenon. That is the demand for high quality is growing with time. This opens a great opportunity for Six Sigma because the essence of Six Sigma is to achieve higher quality continuously and systematically. The more competitive the market is, and the more demanding customers are, the more opportunity would be for Six Sigma to flourish.

Fast development of IT and data mining technology:

The technological aspect of Six Sigma deals heavily with data. Its measurement, collection, analysis, summarization and interpretation form the foundation of Six Sigma technology. Without data, Six Sigma will become meaningless. Accordingly, data manipulation and analysis techniques play an important role in Six Sigma. Advanced IT technology and data mining techniques greatly enhance the applicability of Six Sigma because modern technologies make data analysis no longer a complicated, tedious job, but an easy task. Simply pressing a few buttons or several clicks on advanced software package would produce all the results one wants. This certainly is a good opportunity for the application of Six Sigma because it gets rid of technological hurdle of Six Sigma.

Growing research interest in quality and reliability engineering:

The growing interest in quality and reliability engineering research opens another opportunity for Six Sigma because these researches would contribute to the further development or improvement of Six Sigma methodology. For example, research in robust design combined with Six Sigma produce an important improvement to Six Sigma—DFSS (Design for Six Sigma). While the traditional DMAIC approach mainly deals with the existing process, the new DFSS addresses issues mainly in the design stage and introduces the idea of designing a process with Six Sigma capability instead of transforming an existing process to Six Sigma capability. Interest in quality and reliability engineering research is growing and the potential for the improvement of Six Sigma is far from its limits.

Previous implementation of quality programs has laid foundation for the easy adoption of Six Sigma:

Modern quality awareness started about 80 years ago. During this period, various quality programs have been developed and adopted in practice. These programs did a very good preparation for the adoption of Six Sigma. For example, TQM, the once fashioned quality program, shares some similarities with Six Sigma such as customer satisfaction and continuous improvement. That's why some people argue that Six Sigma and TQM are the same. Six Sigma requires a top-down management approach and corporate-wide culture change. However, cultural change usually happened gradually, not suddenly. Companies took part in TQM were, more or less, already experiencing this change. This have been justified by the phenomenon that companies which implemented other quality programs before actually experienced less difficulties in adopting Six Sigma than those which are new to any quality program. The wide spread quality awareness during the last century served as good "warm-up exercises" and have gotten us ready for this new quality breakthrough.

Threats

Resistance to Change:

The success of Six Sigma requires culture change within the organization (Hendriks & Kelbaugh, 1998; Jerome, 1999). Six Sigma should be embraced in the organization as a philosophy rather than merely a quality initiative. Six Sigma revolutionized the way an organization should work by introducing a new set of paradigm in doing things. The organization may need to give up some old traditions in order to accept certain new elements in this paradigm. Although Six Sigma tools are not difficult to learn, the managers and the rest of the workforce who have been with the organization for a long time often view these as additional load that are impractical. These managers rely on mainly their experience in dealing with problems and are confident enough to use their intuition rather than resort to statistical tools deriving information from available data. Such attitude may be harmful to the success of Six Sigma. The middle managers and supervisors who have experienced many other quality initiatives may regard Six Sigma as any other previously known quality initiatives, which will soon pass away.

When people are placed in a comfort zone for long, these people are unwilling to move out of the comfort and face the challenge of an uncertain environment. Furthermore, it may be rather difficult for experienced people to accept the fact that their usual ways of doing things may need to be improved, especially if the advice was to come from a Six Sigma practitioner who may be less experienced then himself. Hence, the implementation of changes to processes that may impact process owners would have to be undertaken with tact and sensitivity.

Highly Competitive Job Market:

Few companies practice life-long employment strategy in today's competitive job market. This is even more prevalent given the rapidly changing economic, social and technological environment. People tend to more frequently change jobs in pursuit of "better prospects".

When Six Sigma practitioners "job hop", they bring with then the valuable skill set that the company may have invested in them for them to effectively contribute to the company's process development initiatives. Hence, companies may lose confidence in potential of success that Six Sigma initiatives can achieve. The impact of the frequent job-changing phenomenon is further worsened by the fact that appreciable benefits from serious Six Sigma work can only be visible few years after the project was initiated.

Corporate leadership plays a vital role in the successful implementation of Six Sigma. The implementation structure of Six Sigma demands strong support from the Champions, or the executive management (e.g. Henderson and Evans, 2000). Any changes in the executive management will have adverse effects to the implementation. With the hostile market conditions, corporate leadership has become relatively more volatile. CEO's are changed frequently or changes may be brought about through the mergers and acquisitions between organizations. When higher-level management is changed frequently, it may be difficult to maintain the same level of top-down commitment to Six Sigma initiatives in the company.

It is well known that the success of Six Sigma is dependent on how soon it can be successfully implemented in a company (Clifford 2000). From experience, companies would realize the full benefit of Six Sigma only after the fourth year of implementation. The first three years are considered learning or transition phases during which financial results are not significant. If during this period, changes in corporate leadership occur, the implementation of Six Sigma would be seriously compromised. The risk of phasing out this methodology in favor of other management strategy has thus been enhanced.

Cyclical Economic Conditions:

Economic trends are usually cyclical. In times of good economic situations, companies may be more willing to spend additional income on process improvement efforts. This tendency may be reversed during situations of economic downturn as companies struggle to keep afloat. Such practices may be unhealthy for Six Sigma implementation in consideration of the much longer training and transition phase that is required before significant financial gains can be seen. As discussed in Section 4.2 and from Figure 1, negative returns may be encountered in the initial phases of projects implementation.

This situation may be compounded by the widely held misconception that quality improvement efforts result in additional cost but not profit or customer satisfaction. This could be due to the myopic viewpoints held by companies, which may not be true, as good quality does not imply higher costs [www.industryweek.com, September 2001]. Six Sigma has explicitly dealt with this misconception by tying in quality improvement efforts with the Voice of the Customers (VOC) and the company's bottom-line for each project undertaken.

Conclusions

Six Sigma strategies has been somewhat at the forefront of the quality movement in recent years. However, due to its popularity, it has encountered its fair share of criticisms or negative comments. Six Sigma is a natural product of the long term quality march that has involved many other quality management philosophies. Amongst these, it has presented itself as an excellent systematic integration of the qualitative and quantitative approaches to quality improvement. Its emphasis on customer focus and continuous improvement is the continuation of the former TQM methodology and its quantitative techniques are well rooted in mathematics and statistics. The original motivation was to combat variations, the natural enemy of quality. This was eventually developed into a systematic and methodical framework, which is both philosophically and technically sound.

Six Sigma is a unique strategy, which would be able to address many issues that past quality programs have neglected. It will continue to play an important role in the quality arena because the current and future environment is advantageous to its proliferation and full exploitation. However, due to its integrated nature with techniques deeply rooted in sound statistical thinking, it is suggested that companies go for a full Six Sigma after a deeper understanding and proper deployment strategy is reached. The implementation and deployment of Six Sigma should be conducted in a systematic toll-gated manner that would ensure useful organizational learning throughout with regards to the sound statistical thinking and effective management techniques within the organization.

The understanding of Six Sigma strategy varies from organizations to organizations. Some regard it as a management philosophy and some take it as a well-designed statistical package. However, the correct interpretation in order to exploit its full potential is to view it as both. The key elements of its success involve the commitment from the top management and the corporate culture. If the top management is highly committed and the corporate culture is dynamic and receptive to change, Six Sigma can be used as a strategic guideline that will guarantee both financial returns and business excellence. However, if the top management is not keen in this regard and the corporate culture is repulsive to change, it would be better to stay away from it and wait until the top management or the corporate culture is mature enough to harvest its fruits.

A “middle” way is also possible as some companies are currently practicing. This school of thought view Six Sigma as a package of tools that will enhance the implementation of many quality management philosophies that has successfully worked its way into some organizations (Kaizen, TQM, Lean, etc). While keeping their operations and corporate culture unchanged, these organizations pick up Six Sigma projects whenever they deem suitable and make use of the advantages of these tool. The usefulness of such a strategy is still debatable in the ability to achieve synergy with other methodologies rather than just co-exist with them. Used in this way, they reduce the risk of implementing Six Sigma but are not exploring the full potential of this program.

A more healthy view of Six Sigma is that it is a great tool to most problems, but not an answer to all. It will achieve its full potential only when the corporate culture is ready for it. It should also be noted that Six Sigma strategy is not static but constantly evolving. Research in quality and reliability engineering and advanced IT technology will provide many opportunities for its improvement.