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# AN ANALYSIS ON VENDOR HUB

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**A THESIS SUBMITTED**
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The course of writing this dissertation has never been smooth sailing. Days and nights are spent on absorbing the numerous mathematical concepts such as Renewal Theorem, Stochastic Approximation, and in learning Visual C++ programming from the scratch. After all these comes the mammoth task of programming and debugging the Simulator. Finally, comes the tedious process of drafting out the dissertation. Phew … Now that everything is over, I would like to extend special thanks to the following people who have helped me in one way or another.

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**Summary**

Contemporary research in supply-chain management relies on an increasing recognition that the supply chain requires the integration and coordination of different functionalities within a firm. Pioneered by Wal-Mart, Vendor Managed Inventory is an important initiative that aids in the coordination of the supply chain. The study of Vendor Managed Inventory has received much attention from the industry and academia. Though numerous studies have been done on building a theoretical framework for Vendor Managed Inventory, research on developing a model or heuristic for Vendor Managed Inventory is nascent. Current Vendor Managed Inventory literatures on issues such as supplier selection and order splitting are limited. Analysis on industrial polices used in Vendor Managed Inventory was also found to be limited. Comparisons between the popular inventory techniques like Just-In-Time and Vendor Managed Inventory were also seldom made.

This dissertation extends Cetinkaya and Lee's (2000) model to consider constraints like warehouse capacity and lead time. A new performance algorithm is proposed and compared with Cetinkaya and Lee's (2000) model via simulation. In addition, it also seeks to examine the issues of supplier selection and order splitting in Vendor Managed Inventory. In addition, one of the current industrial practices was adapted from our case and analysed. Comparisons were also made between Just-In-Time and Vendor Managed Inventory systems.

Simulation results show this algorithm constantly outperforms Cetinkaya and Lee's (2000) model. The simulation results obtained also point to the importance of strategic supplier selection under Vendor Managed Inventory and show that order- splitting strategies are beneficial. The simulation results also highlighted the rationale of the industrial policy examined. Based on the simulation results, guidelines on choosing the right system is proposed. Guideline on when to use Just-In-Time or Vendor Managed Inventory was proposed using analysis obtained from the simulation results.

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| | |
|---|---|
| **$\lambda$:** | **Average Demand in units** |
| **$\omega$:** | **Warehouse Capacity of Vendor Hub** |
| **$A_R$** | **Order Setup Cost** |
| **C&L** | **Cetinkaya and Lee (2000)** |
| **EDI** | **Electronic Data Interchange** |
| **$A_D$** | **Fixed Delivery Cost to Customer** |
| **g** | **Rental in external warehouse per unit per day** |
| **h** | **Holding cost per unit per day** |
| **MOQ:** | **Minimum Order Quantity** |
| **NPA** | **New Proposed Algorithm** |
| **p** | **Defective Rate** |
| **Q** | **Stock Up To Inventory Level** |
| $\tilde{Q}$ | **Order Quantity** |
| **T** | **Shipment Consolidation time** |
| **VC** | **Variable Delivery Cost to Customer** |
| **VM** | **Variable Dispatch Cost to Vendor** |
| **VMI:** | **Vendor Managed Inventory** |
| **w** | **Waiting Cost per unit per day** |
| **SCM** | **Supply Chain Management** |
| **JIT** | **Just In Time** |
| **L** | **Lead Time** |

# 1 Introduction

Contemporary research in supply-chain management relies on an increasing recognition that the supply chain requires the integration and coordination of different functionalities within a firm. With most industries experiencing intensified cost structures and rising consumer sophistication (Hoover et al., 1996), more emphasis have been placed on supply chain coordination in recent years. In view of this trend, this study will focus on the coordination efforts in integrating inventory and transportation decisions.

Pioneered by Wal-Mart, Vendor Managed Inventory (VMI) is an important initiative that aids in the coordination of the supply chain. In VMI, the vendor takes over the responsibility of inventory management from the retailers by using advanced information tools such as Electronic Data Interchange (EDI). Based on information obtained on the retailers' inventory level, the vendor makes decisions regarding the quantity and timing of shipments. The vendor hub operator usually employs a consolidation *s*hipment strategy where several deliveries are dispatched as a single load to achieve transportation economies. Under a VMI arrangement, the supply chain behaves, as a two-echelon supply chain that will reduce the bullwhip effect existing in the supply chain (Kaminsky and Simichi-Levi, 2000).

## 1.1 Problem Description

The original problem described in Cetinkaya and Lee (2000) is used to develop the model in this paper. In the problem, the vendor observes a sequence of random demands from a group of retailers located in a given geographical region. We consider the case where the vendor uses an (s, S) policy for replenishing inventory, and a time-based, shipment-consolidation policy for delivering customer demands. The vendor also faces the decision of selecting its long-term supplier from a list of potential suppliers.

In addition to the original problem, we consider the model of a real life vendor managed production hub. The vendor managed production hub in our consideration acts as the vendor hub for the raw materials of the customer production line, which produces electronics components and computer products. The production facility is situated near the vendor hub, which effectively eliminates the transportation cost to the customer. The vendor hub is operated by a Third Party Logistics (3PL) service provider. In the vendor hub, inventory is owned by the supplier until an order is triggered by the customer. The inventory policy used in the vendor hub is assumed to be an (s, S) policy unless stated otherwise. As the production plant is just beside the vendor hub, orders are immediately delivered to the production facility without doing any consolidation. The suppliers are supplying different parts /components to the vendor hub and each of them have a different cost structure. All these components are needed in order for the production line to run. A missing component would stall the whole production facility.

## 1.2 Research Motivation

The study of VMI has received much attention from practitioners and academia. Various published accounts and studies have shown that compelling operational benefits are obtained from the implementation of VMI (Achabel et al., 2000; Holmstrom, 1999;

Waller et al., 1999). VMI enables vendors to achieve inventory reduction without sacrificing service level.

Though numerous studies have been done on building a theoretical framework for VMI (James et al., 2000; Achabel et al., 2000; Waller et al., 1999), research on developing a model or heuristic for VMI is limited. In addition, consideration for certain practical constraints such as warehouse capacity of the vendor hub seems to be lacking in these papers.

Single sourcing is one of the primary enablers of an effective VMI system (James et al., 2000). Consequently, supplier selection decisions become important to the vendor hub operator, as a wrong choice of supplier can be fatal to the whole VMI arrangement. Despite the importance of supplier selection in VMI, studies done on this issue is limited.

The current literature on VMI seems to overlook the use of order splitting. Order splitting is a recent proposition made to improve the efficiency of the supply chain. Studies done on order splitting suggest that order splitting is beneficial (Chiang, 2001; Janssen et al., 2000; Chiang and Chiang, 1996). With the potential to achieve cost savings, the feasibility of having an order splitting arrangement in VMI should not be ignored.

The current literature on Just-In-Time (JIT) inventory and VMI inventory is abundant. Much research have been done on examining JIT inventory management system (Schniederjans and Olson, 1999; Schniederjans, 1997; Woodling and Kleiner, 1990;

Jordan, 1988; Schonberger and Schniederjans, 1984). However, little has been done on comparing the performance between JIT and VMI. Given the popularity of these two arrangements, a comparison between these two systems will be helpful to practitioners.

Lastly, we observe that currently modelling/simulation literatures on VMI focuses either on building an optimum policy for vendor hub operators (Disney and Towill, 2002b; Chaouch, 2001; Cetinkaya and Lee, 2000; Ruhul and Khan, 1999) or to provide justifications of implementing VMI (Cheung and Lee, 2002; Aviz, 2002; Dong and Xu, 2002; Disney and Towill, 2002a). Little have been done on analysing current policies that are used by VMI operators in the industry. The insights that could be obtained on analysing industrial practices should not be ignored as they allow the academia to understand VMI inventory systems better.

## 1.3 Research Objectives

The first objective is to develop a feasible heuristic for inventory replenishment and shipment decisions that can be use by VMI practitioners. The second objective is to simulate a VMI supply chain by manipulation of parameters and obtaining insights on supplier selection in a VMI supply chain. The third objective is to determine the performance of JIT and VMI inventory systems under VMI. The last objective is to examine current industrial practices and obtain insights of VMI in the industry

## 1.4 Potential Contributions

This study expands on the VMI model built by Cetinkaya and Lee (2000). Factors such as imperfect quality, Lead Time and Minimum Order Quantity (MOQ), which were overlooked by Cetinkaya and Lee (2000), will be considered in this study. The effect of supplier selection and order splitting under VMI will be examined. This study also looks at the performance between JIT and VMI systems and attempt to propose conditions where one method is preferred over another. Current industry practices will also be examined and analysed. The insights gained from the analysis of the simulation output can help in the understanding of VMI systems.

## 1.5 Chapter Summary and Organisation of Dissertation

This chapter has provided a brief description of the VMI concept. Chapter Two reviews the relevant literature on various studies done on VMI as well as some of the supply chain issues that this study is going to examine. Chapter Three provides the research methodology and describes the steps used to get our results. Chapter Four describes the problem context and present an algorithm to solve the problem. The findings and analysis of the simulation results are presented in Chapter Five. Chapter Six concludes with some key insights and limitations of this study.

## 2. Literature Review

With most industries experiencing intensified cost structure and rising consumer sophistication (Hoover et al., 1996), the effective management of the supply chain has become increasingly important for companies. Advanced information tools like Enterprise Resource Planning (ERP) systems and EDI help to improve information flow within the organisation (Mandal and Gunasekaran, 2002). Coupled with advanced information collection techniques such as radio frequency (RF) data collection systems and bar coding, complexities in managing inventory are reduced. As a result, the responsibility of inventory management is pushed upstream in the supply chain (Inventory Reduction Report, 2000).

Current SCM techniques such as Continuous Replenishment and Quick Response treat inventory as a time-based support. The conventional treatment of inventory as a buffer against delay and disruption is gradually discarded. Trends in inventory management techniques are now pointing toward eliminating or minimising inventory buffers, and the use of inventory to manage the "pull" of material from upstream to facilitate flow (James et al., 2000). VMI is one such technique.

### 2.1. Definition of VMI

Ever since Wal-Mart popularised VMI in the late 1980s, it has attracted attention from researchers from both the marketing and supply chain fields. According to James et al. (2000), VMI is a collaborative strategy whereby the supplier undertakes the responsibility of managing the inventory in an attempt to optimise the availability of products at minimal cost. In the same paper, the environment and primary enablers of an effective VMI system are also established. The environment is identified by six nested subsystems levels, namely capability gap and product characteristics, relative importance from the supplier perspective, ownership and trust issues, framework agreement, primary enablers, and finally objectives and benefits of the VMI system. Information transparency and single sourcing are identified as the primary enablers of an effective VMI system by James et al. (2000). To prove the management theories on VMI, Waller et al. (1999) ran a simulation and found out that compelling operation benefits are derived from VMI systems, even under non-ideal retailing environment. Favourable results obtained from implementing a VMI system on a major apparel manufacturer (Achabal et al., 2000) and a full-scale VMI relationship with a wholesaler (Holmstrom, 1999) proved the practical applicability of VMI to business. Kaipia et al. (2002) analysed the performance of VMI in managing the replenishment process of an entire product range and found that significant savings in inventory and time can be achieved through the implementation of VMI.

VMI can be seen as an example of channel coordination (Achabal et al., 2000). Through effective channel coordination, VMI is able to improve service level and reduce costs for both the suppliers and customers (Waller et al., 1999). The crux of optimising the performance of VMI is to find an optimal inventory decision model that minimises inventory cost without sacrificing the service level. In order to find this optimal inventory decision model, it will require coordination of the vendor hub's replenishment from the supplier and delivery policy to the customer to achieve the best trade-off between inventory costs and service level.

### 2.1.1. Inventory Decision Model

The replenishment policy and delivery policies of the vendor hub face two fundamental decisions: 1. What is the lot size of each order or shipment? 2. When to activate an order or deliver the goods to the customer? These major decisions jointly affect the cost and service level of the whole system. The challenge is to find a replenishment policy for cost minimisation without sacrificing customer service.

#### 2.1.1.1 Lot Sizing Decision

The lot-sizing problem has always received attention from supply chain and decision sciences researchers. The dilemma of the trade-off between inventory costs and other costs components such as transportation have always been the topic for researchers in this field. Higgison and Bookbinder (1994) identified two methods of determining the lot size for consolidation for shipment. They are i) Quantity-Based Consolidation and ii) Time-Based Consolidation.

Quantity-Based policies, such as the Economic Order Quantity (EOQ) and Economic Production Quantity (EPQ), achieve economies of scale in transportation and ordering at the minimal inventory level possible. Using quantity based policies will make sense if demand is a constant (which is one of the assumptions under EOQ models), as all the demands will be fulfilled at a minimal cost. However, in real life, demands are usually driven by stochasticity rather than being a constant. Thus, the quantity-based model might not be optimal in such cases due to the fluctuations of demand. Moreover, stock-outs are now possible as the EOQ might not be able to meet the demand fluctuations. As the theory suggests, quantity-based models will be minimising cost at the expense of service level.

Time-based policies, on another hand, will not have this problem, as the lot size can be dynamic. However, as time-based policies ordering periods are fixed, it is possible for small uneconomical lot sizes to be ordered.

It is observed that quantity-based policies are good in lowering costs in most situations, while time-based systems excel in maximising service level. In the scenarios where consolidation period are short, quantity based consolidation policies constantly outperforms time-based policies. However, when consolidation periods are long, time-based consolidation policies outperform quantity based consolidation policies if the mean arrival rate is relatively high (Higgison and Bookbinder, 1994).

#### 2.1.1.2 Re-Ordering Decisions

Re-ordering decisions are heavily influenced by the lot-sizing decision, and vice versa. This is especially so in quantity-based lot-sizing policies, as re-ordering times are random. In order to determine when to reorder, the required target inventory level and the relevant order lot size will be required. However, the re-ordering period is non-deterministic.

For time-based lot sizing, re-ordering decisions has a completely new meaning. The main objective of the re-ordering decision now is to determine the order cycle time.

#### 2.1.1.3 Inventory Decision Model for VMI

Inventory decision models such as EOQ only deal with a two-party relationship. However, for VMI, the challenge of optimising the inventory decision model has become much complicated. For a VMI vendor to perform, the vendor has to coordinate the replenishment and delivery policy concurrently so that the whole VMI system can be optimised. Both inventory replenishment policies and delivery policies affect the inventory position simultaneously. Optimising the replenishment or delivery policy alone does not guarantee optimality for the VMI vendor, as it does not taken into account the other components in the whole VMI. In order to achieve optimality, both polices have to be considered and solved concurrently as a system.

## 2.2 Research Done on VMI optimisation

In response to this challenge, several studies are done to derive an optimisation model for VMI. Ruhul and Khan (1999) examined the challenge of coordinating between the procurement policy of raw materials and the manufacturing policy of the plant, and derived an optimal batch size for the system operating under periodic delivery policy. Chaouch (2001) attempted to derive an optimal trade off between inventory, transportation and backorder cost in order to increase delivery frequency at the lowest cost. Disney and Towill (2002b) examined the production scheduling problem under a VMI system and presented an optimisation procedure for this problem.

Cetinkaya and Lee (2000) did a related research on the problem of channel coordination faced by a VMI vendor. Their model attempts to find an optimal solution for coordinating inventory and transportation decisions in VMI. In addition, the model considered a Poisson demand pattern. However, the model failed to take into account several important considerations.

### 2.2.1 Imperfect Quality

Firstly, Cetinkaya and Lee's (2000) model failed to consider of the presence of imperfect quality in the products (i.e. defective products or products with a fixed shelf life). Defective products cannot be used to fulfil customer demands and have to be discarded or reworked. Omitting defective product cost may lead to a suboptimal solution.

The problem of imperfect quality has been long researched by academia. Goyal and Giri (2001) had done a review on advances of deteriorating inventory literature since the 1990s and classified them under several categories. Chung and Lin (1998) examined the impact and developed an optimal replenishment model taking into account of the time value of money using the discounted cash-flow approach. Wee (1999) examined the impact of imperfect quality on the inventory decision model by taking into account some real life scenarios like quantity discount. He then developed an optimal deteriorating inventory model taking into consideration quantity discount, pricing and partial back ordering.

So far, the literature cited deals with deteriorating inventory decision models. The impact of defective goods on inventory decision models such as EOQ and EPQ have not been neglected by academia. Schwaller (1988) first examined the problem of imperfect quality in EOQ models. He extended the EOQ model by assuming that a known proportion of defectives must be removed after inspection. He carried on by examining the impact of fixed and variable inspection costs on the EOQ model itself. Dave et al. (1996) examined the interaction of a production lot-sizing model with a uniformly finite replenishment and differential pricing policies. Their model considers the possibility of defective items. In addition to Schwaller's (1988) scenario of rejecting defective items, Dave et al. (1996) considered additional scenarios such as reworking that could be done on the defective product or when defective products reach customers. Salemeh and Jaber (2000) examined the impact of imperfect quality on EPQ and modified the EPQ model to incorporate the effect of imperfect quality to the inventory model. Unlike the treatment of defective items in previous papers, they assumed that defective items have a scrap value and are sold off at a discounted price. Though there are numerous researches done on the problem of imperfect quality in inventory decision models, the literature on the impact of imperfect quality on VMI is scarce.

### 2.2.2 Minimum Order Quantity

Often suppliers specify a MOQ for strategic or physical (e.g. packaging) reasons (Robb and Silver, 1998). Thus, when an inventory decision model recommends an order quantity below MOQ, the vendor has to decide whether to go along with the recommended quantity and pay the penalty charges or order MOQ. Silver and Eng (1998) developed a simple decision criteria for choosing between a manufacturer with MOQ criteria and a wholesaler with no such criteria but higher purchase price. With the introduction of an MOQ requirement, Cetinkaya and Lee's (2000) model might be affected.

### 2.2.3 Order Splitting

Studies done on order splitting suggest that substantial cost savings can be obtained by implementing order splitting in the supply chain. According to Chiang and Chiang (1996), order splitting can yield up to 20% savings by splitting a single order into two equally sized deliveries when the setup-to-holding cost ratio is low or there is a low variability in demand. Jansen et al. (2000) analysed the effects of order splitting on inventory holding cost and shipment cost, and found that lot splitting reduces inventory levels for both customers and manufacturers. Chiang (2001) showed that order splitting could lower cost as long as the dispatch cost of an order is not very small. Though order splitting can generally be cost effective (except in cases where setup-to-holding cost ratio is high), its performance is highly dependent on factors such as the setup cost per dispatch, shipment cost and demand variability. In view of this, we review the use of order splitting in a VMI supply chain.

### 2.2.4 Capacity Constraints of Vendor Hub

Cetinkaya and Lee (2000) have assumed no capacity constraint on the vendor hub. This is quite unrealistic as a vendor hub does have a maximum capacity. Though order quantity rarely exceeds warehouse capacity, this assumption might be breached in cases where the vendor warehouse is small or the cargo handled by the vendor is bulky. Ishii and Nose (1996) examined the problem of inventory control under warehouse capacity constraints. In the paper, excess inventory are stored in a rental warehouse. The rental warehouse charges a higher storage rate than the vendor hub's own holding cost.

### 2.2.5 Lead Time

Lastly, Cetinkaya and Lee's (2000) model fails to take into consideration of lead time. Lead time plays an important role in supply chain management. Lead time affects the level of safety stock in the supply chain. In addition, lead time also amplifies the bull-whip effect that exists in the supply chain (Simchi-Levi et al., 2000). Thus, lead time is usually taken into consideration by the literature dealing with inventory problem (Fujiwara and Sedarage, 1997; Silver and Peterson, 1985; Liu and Yang, 1999). In these works, lead time is viewed either as a prescribed constant or a stochastic variable. Though there are numerous studies done on including lead time in the supply chain, such studies seems to be limited in the VMI context.

## 2.3 Supplier Selection

Supplier selection is one of the fundamental decisions made in Supply Chain Management (SCM). Its importance comes from the fact that suppliers have a direct impact on the cost and service level for the VMI. With the shifting trends in single sourcing, price is no longer the single most important factor in supplier selection. Choi and Harley (1996) found that factors such as quality and delivery consistency have overtaken price as one of the most important factors in supplier selection. This phenomenon is further proved by Swift (1995) who had attempted to determine the differences between supplier selection criteria of single-sourcing and multiple-sourcing firms.

The research by Ghodsypour and O'Brien (2001) is one of the few researches done to examine the effect of supplier selection on cost and performance. They developed a mixed-integer non-linear programming model to solve the problem. The literature on supplier selection in VMI is rare as well. Supplier selection, as one of the fundamental SCM decisions, affects the cost and performance of a VMI system. Hence, the significance of supplier selection in VMI must not be undermined.

## 2.4 Just In Time Inventory Management

Though there were numerous simulations and case studies done on examining VMI, little was done on comparing the VMI with other popular arrangement. One of such arrangement is JIT inventory systems.

A JIT inventory system is build on the following principles: 1) Cut lot sizes and increase frequency of orders, 2) cut buffer inventory, 3) cut purchasing cost, 4)improve material inventory, 5) seek zero inventory and 6) seek reliable suppliers (Woodling and Kleiner, 1990; Schonberger and Schniederjans, 1984; Jordan, 1988; Schniederjans, 1997; Schniederjans and Olson, 1999). JIT inventory systems have received much attention from the academia ever since the pioneering paper by Sugimori et al. (1977) (Fuller, 1995). Most of the research done on JIT management are on rationale of JIT (Burton, 1988), JIT purchasing techniques (Ansari and Mondarres, 1988; Manoochehri, 1984; Freeland, 1991; McDaniel et. al., 1992; Schonberger and Gilbert, 1983), JIT implementation (Ansari and Mondarres, 1986; Ansari and Mondarres, 1987; Ansari and Mondarres, 1988; Schonberger and Ansari, 1984; Raia, 1990), the various prerequisites for successful JIT implementation (Waller, 1991; Ansari and Mondarres, 1988; Schonberger and Ansari, 1984, Macbeth, 1987, Schonberger and Gilbert, 1983,) and the weaknesses associated with JIT inventory management systems (Fuller, 1995). However, works on comparing the performance of the JIT and VMI technique is limited.

## 2.5 Analysis on Industrial Practice

Though current VMI literatures are abundant, we find that studies done on industrial VMI practices are relatively few. The few industry studies that were done on VMI focus mainly on benefits obtained from industrial implementation (Holmstrom, 1998b; Holmstrom, 1998a; Achabal et al., 2000; Kaipia et al., 2002). Studies focusing on investigating the inventory policies used in VMI practitioners are rare.

## 2.6 Issues

Cetinkaya and Lee (2000) developed an optimal model that is able to coordinate transportation and inventory decisions given a Poisson demand. However, the model failed to consider several important factors that a VMI hub operator is likely to face. In view of this, we develop a new model. The possibility of using order splitting under VMI system will be examined. The impact of factors, such as MOQ, has on Cetinkaya and Lee (2000) and the new model will be examined. A comparison will be done between the new model and Cetinkaya and Lee's (2000) model. The issue of supplier selection will be considered in the development of the new model. We will also be doing a comparison on JIT and VMI systems. Lastly, we perform an analysis on the inventory policies current adopted by VMI hub operators and try to understand the rationale behind the policies. From these analyses, we hope to find valuable insights for VMI practitioners to use.

### 2.7 Chapter Summary

This chapter started with the description and definition of VMI. The literature on the various constraints and issues mentioned in Chapter 1 are also reviewed. The chapter ends with a discussion of the research gaps and issues to be tackled in this study. The issues in this study includes building an extension of Cetinkaya and Lee's (2000) model to incorporate constraints such as MOQ and warehouse capacity , a review on issues such as order splitting and supplier selection in VMI, a comparison and analysis of JIT and VMI inventory systems and a analysis on policies currently adopted by VMI hub operators.

# 3 Research Methodology

Given the complexity of a real supply chain system due to its stochastic nature, it is rather difficult and tedious to accurately represent the supply chain under a VMI arrangement using mathematical modelling. In view of the possible analytical difficulties in the modelling of such a system, simulation is usually the preferred solution due to its ease in dealing with the complex supply chain. However, as simulation is an analytical tool rather than an optimization tool (Simchi-Levi et. al, 2000), its does not really suit our purpose here. In view of the various weakness associated with the two common methodologies, we utilise a technique that is found in Hax and Candea (1984) which employs both mathematical optimization and simulation techniques as our research methodology. This chapter presents an overview of the technique of simulation modeling and analytical optimization, followed by the justifications for using the hybrid technique. Following that, we will be touching on the data collecting and experiment procedures used in out sensitivity analysis. We will also be touching on the various aspects of the simulation model and the various configurations used in the simulation in detail. Finally, we will be describing on the algorithms that are used to program the process flow of the simulation model

## 3.1 Overview of Simulation Modelling

Simulation modelling usually involves the development of a computerized model that mimics the behaviour and operation of a real life process of system over time. Usually, the model takes the form of a set of assumptions concerning the operation of the system. These assumptions may take the form of mathematical, logical or symbolic relationships between different components in the system. Once the model is completed and validated, it can be utilized to investigate a wide range of hypothetical scenarios about the real world system and predict the outcome that will be obtained from these situations (Banks et. al., 2000). Through simulation modelling, managers are able to obtain a deeper understanding on the behaviour of the system and be able to make critical decisions on deciding on which configurations to adopt.

The appropriateness and value of simulation modeling as a tool to study system dynamics have discussed by numerous studies (Banks and Gibson, 1997; Banks et al., 2000; Evans and Olson, 2002; Kellner et al., 1999; Pegden et al., 1995; Simichi-Levi et al., 2000). As these studies have already gave a detail discussion on the advantages and disadvantages of simulation modeling, we shall not go through this in detail and will only give a brief summary on the advantages and disadvantages of using simulation modeling.

### 3.1.1 Advantages of Simulation Modeling

The technique of using simulation modeling has become increasingly popular due to several of it distinct strengths. Simulation modeling provides managers and analysts an inexpensive way to evaluate proposed systems or configurations without having to implement them in a real setting. As simulation mimics the system in the real world, results obtained from the simulation technique are usually received with confidence. The simulation model is rather versatile and is able to model any assumptions. This is particularly important when the assumptions are too complex to be modelled by analytical methods. This means that simulation modeling provides an alternative for analysts and managers to look at the problem even conventional management science techniques fails (Evans and Olson, 2002; Banks et al., 2000; Simichi-Levi et al., 2000; Pegden et al., 1995).

### 3.1.2 Disadvantages of Simulation Modeling

Despite the numerous merits of simulation modeling, Simulation modeling is not without its faults. As one of the primary purposes of developing a simulation model is to capture the random nature of the real system, it is not easy to determine whether the results are caused by the change in the system or by the random nature of the inputs. A large amount of time is also required to collect the input data and the development of simulation model and the program. The building and the analysis of simulation models will require the use of skilled professionals, which could be rather expensive (Evans and Olson, 2002; Banks et al., 2000; Simichi-Levi et al., 2000; Pegden et al., 1995). Lastly, though simulation modeling is a great analysis tool, simulation modeling itself is not an optimization tool (Simichi-Levi et al., 2000). Simulation modeling can only be used to evaluate policies. However, it is difficult to generate an optimal or good solution by just utilizing simulation alone.

## 3.2 Overview of Mathematical Modeling

Mathematical modeling belongs to the discipline of Operations Research. It is regarded as the conventional approach to turn the problem into one that is convenient for analysis. Mathematical modeling involves several components such as decision variables, objective functions and constraints. These components represent the assumptions and relationships that are used in the model (Hiller and Lieberman, 1995; Hiller and Lieberman, 1990; Daellenbach et. al., 1983).

### 3.2.1 Advantages of Mathematical Modeling

Mathematical modeling has been used for representations for problems for a very long time due to several strengths it possess. One of its advantages is that a mathematical model is able to describe a problem more concisely as the overall structure of the problem is clearer in a mathematical model. It is also easier to understand the different cause and effect relationships and the interactions between different parameters in a mathematical model. Lastly, mathematical modeling provides a platform for the use of high powered mathematical techniques to analyse and solve the problem (Hiller and Lieberman, 1995; Hiller and Lieberman, 1990; Daellenbach et. al., 1983).

### 3.2.2 Disadvantages of Mathematical Modeling

However, mathematical modeling is not without its flaws. Usually, for a model to be tractable, approximations and simplifying assumptions must be made into the model. Thus, this brings the problem of possible oversimplification or misrepresentation of the problem if these approximations and assumptions are invalid. In complex problems, it may be impossible to represent the behaviour of the system by using mathematical modeling. Though approximations can be used to simplify the problem, one must take extra care that the correct approximation is taken as the wrong approximation will result in a different analysis results being obtained (Hiller and Lieberman, 1995; Hiller and Lieberman, 1990; Daellenbach et. al., 1983).

### 3.3 Hax and Candea Methodology

Due to the various weaknesses found in these methodologies, we are unable to achieve our objective by only applying a single methodology. Hax and Candea (1984) suggested a way to utilize the strengths of both simulation and optimization via mathematical modeling. They suggested that an optimization model to be used first to solve for various scenarios at a macro level. Then, a simulation model can be used to evaluate the solutions generated by optimization in various design alternatives. Variations of this method can be found in later literatures in a different form (Hiller and Lieberman, 1995; Hiller and Lieberman, 1990), where simulation is used for the testing, validation and evaluation of the mathematical model.

### 3.4 Rationale for using Hax and Candea Methodology

There are usually two main approaches in analysing a system: the mathematical modelling/optimisation approach and the simulation approach. As mentioned earlier, both approaches have their own strengths and weakness. In Murty (1995), it is mentioned that simulation modeling fares well in selecting the best policy out of a few configurations. However, when the number of possible configurations is large or infinite, it would be infeasible to use simulation to obtain a good or optimal policy. In such cases, mathematical modeling and optimization would be the better approach. However, due to the various approximations used in mathematical modeling, analysis results obtained might not be received with confidence. Also, approximations and assumptions used in the mathematical model might not be representative of the real system.

Through the use of Hax and Candea's (1984) methodology, it is possible to rectify the weakness of the two approaches. The use of mathematical modeling and optimization in the first step ensure that a good solution is found based on the various approximations and assumptions that are placed within the mathematical model. The next step of using simulation for evaluation and validation ensures the reliability of the results and give the assurance to the users that the solution obtained is indeed a good solution.

### 3.5 Experiment Design

To apply Hax and Candea's methodology, we must first define the problem that we are looking at. After the definition of the problem, the problem is formulated mathematically. From the mathematical model formulated, we will be able to derive a good policy, which will be tested using the simulation model built. Due to the complexities in building the mathematical model, we will be covering it in detail in the next section. Now, we focus on the various aspects and assumptions used in developing the simulation model.

#### 3.5.1 Problem Description

##### 3.5.1.1 Basic Problem : Normal Vendor Distribution Hub (VMI)

The basic problem considered for the simulation model will be used in the first step of our methodology, where we present an algorithm for the parameters of our inventory replenishment and dispatch polices used in the vendor hub. The problem will be similar to Cetinkaya and Lee (2000) paper. The Vendor, V, is facing a group of suppliers/manufacturers ($M_i$) upstream and a group of retailers ($R_{ib}$) the downstream. The inventory policy adopted by the vendor hub will be a (s, S) policy, where s is the cycle stock needed and S =s+Q*. Consolidations are done for a period T* before the goods are dispatched to the retailers. As we will be discussing the detailed assumptions of this model during the mathematical formulation in the next section, we will not go into details into the various assumptions for the basic problem used in the simulation model. The supply chain for the basic problem is depicted in Figure 1 for easy reference.

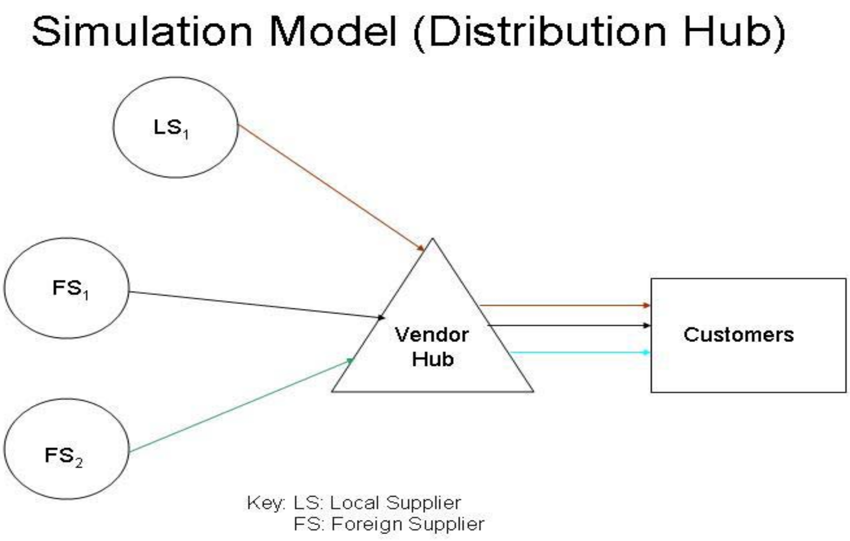


Figure 1: Supply Chain Model for Distribution Hub

### 3.5.1.2 Modified Problem 1: Distribution Hub in a JIT Arrangement

The next problem we will be analysing will be a vendor distribution hub operated using a JIT inventory replenishment system. We will be adopting the inventory policy described in Schniederjans (1999). We assume that the ordering cost and setup cost is negligible in an ideal JIT arrangement (Schniederjans, 1999). However, to let the supplier to implement JIT with the vendor hub operator, it charges a JIT penalty charge per item due to operational reasons. We assume for the JIT system, the retailer facilities are inside the vendor hub itself. Thus, transportation cost to the retailer from the vendor hub is negligible. The inventory policy adopted here would be based on the various assumptions behind the JIT inventory management philosophy found in Schniederjans (1999). We propose to use a (s, s+1) inventory policy, where s is equivalent to the kanban stock needed and the formula as used by Schniederjans (1999). The order up to level is set to be s+1 due to the principle of JIT being reducing the lot size of ordering to a minimum (Schniederjans, 1999). Thus, we set our Q* to be equal to 1 to represent the ideal JIT scenario. The supply chain model will be similar to the one previously depicted in Figure 1.

#### 3.5.1.3 Modified Problem 2: Industry Case Study, A 3PL operated Hub using VMI

In this problem, we replicate a real vendor hub operating in the computer manufacturing industry. Due to confidentiality, we will not be naming the various parties involved in this arrangement. The company in our case employs the services of a 3PL service provider to run its vendor hub operations for it. The 3PL is given a set of guidelines by the company (which will be known as the customer) to run the vendor hub. The vendor hub serves as a material hub for the customer production line. As the customer carries out global sourcing for its components, it is facing with a group of local and foreign suppliers. Unlike traditional VMI arrangement, the inventory stored in the vendor hub belongs to the supplier until the customer activates an order for it. The production facility of the customer is situated beside the vendor hub for ease of transportation. Thus, this effectively eliminates the dispatch cost and the dispatch lead time needed to transfer the components to the production facility. For ease of production, the vendor hub operators are required to assemble various components into kits before sending them to the customer production facility. Due to limited resources in the vendor hub, the kitting can only be done at a deterministic rate. If the vendor hub operator fails to provide the kits in time for the production line, they will be slapped with a penalty charge due to the line down caused by the shortage of kits. For easy referencing, we depict the supply chain model for this problem in Figure 2.

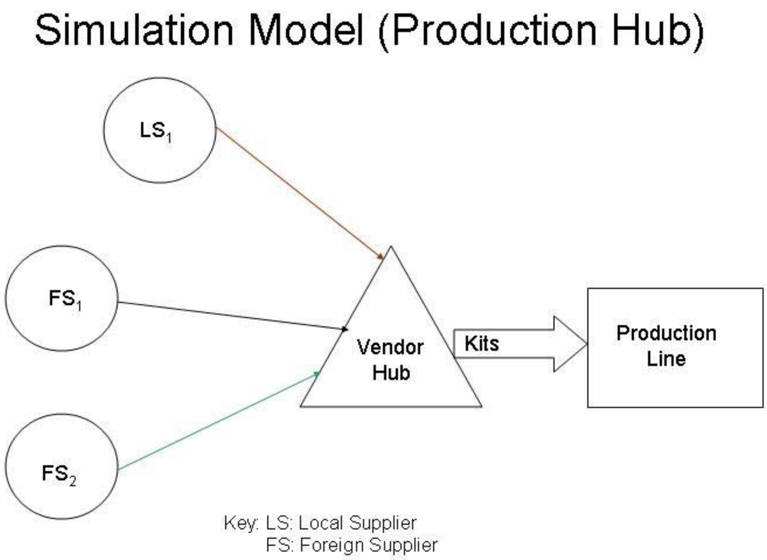


Figure 2: Supply Chain Model for Production Hub

### 3.5.2 Process flow in a vendor hub

The vendor is assumed to adopt a periodic review (s, S) inventory replenishment policy. The inventory position of the vendor hub is reviewed periodically. At every period, the vendor hub will check for orders from the retailers and consolidate the orders into the consolidation pool. The operator will then check whether the consolidation time of the consolidation pool exceeds the pre-determined consolidation period. When the consolidation time exceeds that of the pre-determined consolidation period, the operator will check whether there is enough inventory in the vendor hub to satisfy the demand. If there is enough inventory, the operator will deliver the orders in the consolidation pool to the retailers. In the event when there is not enough inventory at the vendor hub, the operator will issue an order to the supplier. The order size would depend on whether the lot size recommended by the inventory policy is greater that the MOQ of the supplier. If the lot size is lesser than MOQ, then an MOQ amount of goods is ordered. After the consignment reaches the vendor hub after a deterministic period, the operator will inspect the goods for defectives upon receipt. The defectives items are removed and the orders from the consolidation pool are delivered to the retailers. To summarise, a diagram of the replenishment process in the vendor hub is shown below.

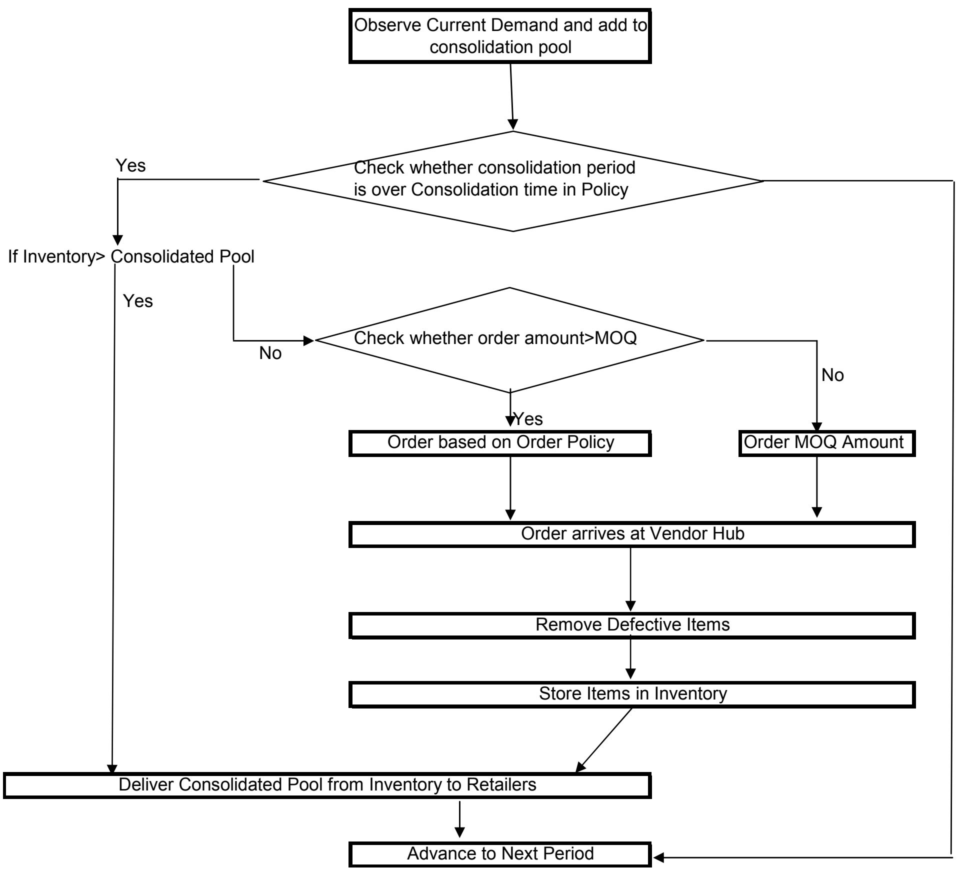


Figure 3: Inventory Replenishment Process Flow in a Vendor Hub

### 3.5.3 Movements of Goods in the Distribution Hub Setting

The distribution hub functions like a typical warehouse. When the suppliers or the vendor hub operator activates an order, goods are immediately sent from the supplier to the vendor hub via the various transportation modes. When the consignment reaches the vendor hub, it is first placed at the receiving area and then processed to be put into the warehouse storage area.

Concurrently, the vendor hub will register the demand from retailers. Orders will be picked and place in the staging area as the demands are triggered by the retailers. After waiting for a time period, T*, the items will then be sent to the customer as a batch. Graphically, the process flow can be depicted by Figure 4.

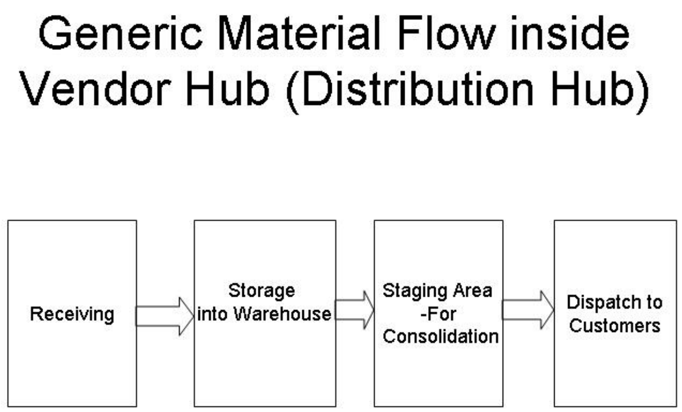


Figure 4: Inventory Flow in a Distribution Hub

### 3.5.4 Production Hub Inventory Process flow

The production hub in our study functions similarly to the distribution hub. The receiving process flow and the ordering process flow are identical to that of the distribution hub. However, in the case of the production hub, inventory ownership is transferred immediately from the supplier to the customer whenever the customer raises an order. As the customer storage place is also in the warehouse itself, thus transfer cost can be considered to be negligible. In the customer storage area, the various components are assembled into kits. The completed kits are then sent directly to the production line. Graphically, the process flow can be depicted by Figure 5.

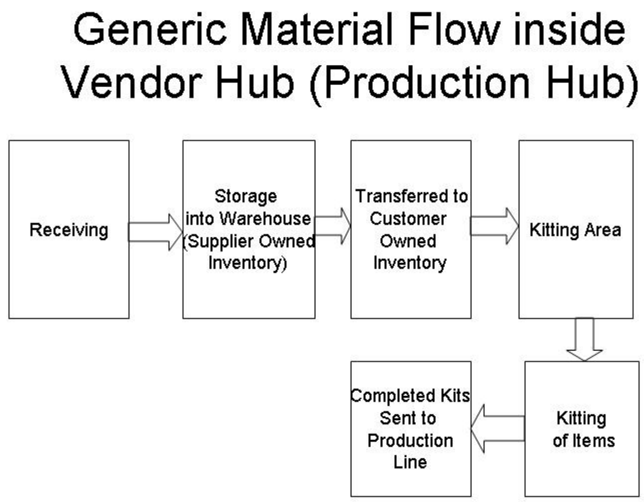


Figure 5: Inventory Flow in a Production Hub

### 3.6 Performance Measure

One practical and credible way of measuring supply chain performance is to consider the average system cost, which is already commonly practised in the industry. As such, we take cost as the unit of measurement (cost is defined as the average total logistical cost incurred by all parties in the supply chain).

Though the average system cost will yield a good measure of the system performance, there are always exceptions to this rule. In such cases, we must analyse deeper into the various components of the total logistical cost. From our case study, we know that a typical VMI arrangement will typically consist of three parties: 1) Suppliers, 2) Vendor Hub Operator and 3) Customer. Thus, for cases where an answer cannot be obtained from the analysis of system cost alone, we will move a step further and analyse the cost of the various players in the arrangement. The exact definition of the various players cost component can be found in the Appendix.

### 3.7 Simulation Model and Validation

Model verification and validation are important steps to be taken in simulation modelling. Model verification is concerned with whether the simulation conceptual model is reflected correctly in the computer program. On the other hand, model validation is the determination of whether the simulation conceptual model is an accurate representation of the real world system (Banks et al. 2000). The simulation model and the program used in this paper are verified in the following ways

- The Computer program was checked by another person who is familiar with Visual C++.
- Step by step tracing is done to ensure the logic of the program codes is accurate.
- The model input and output was examined under a variety of settings to check its face validity.
- The simulator model was given to an industrial practitioner to check the reasonableness of the assumptions and the logic of the various process flows.

### 3.8 Conclusion

A detailed analysis is done on the two incumbent approaches in the analysis the systems: the mathematical / optimization approach and the simulation approach. It is found that no single approach is good enough to fulfil our objectives. It is found that by using the combinational methodology suggested by Hax and Candea (1984), it is possible to remove the flaws from these two approaches. Hax and Candea (1984) approach best fits the objective of our research due to its ability to address the weakness in the two conventional methodology and its various strengths. We have also briefly touched on our research methodology and experiment techniques used in our study. We developed a simulation model that closely resemble the real world operations of a vendor hub, incorporated with all the necessary assumptions and logic that will enable the user to experiment with different configurations to gain insight into the characteristics of the vendor hub. In this way, we can test the proposed heuristics against Cetinkaya and Lee's (2000) solution. In addition, we are able to analyse various inventory polices to gain valuable insights into the world of VMI.

# 4 Mathematical Modelling and Analysis

In this chapter, we build the mathematical model and derive the optimal solution for the model. We will first review the model used in Cetinkaya and Lee (2000). This will be followed by a detailed description of the model characteristics and its underlying assumption. Next, the mathematical formulation of the model will be developed. The model developed will be analysed mathematically, followed by an attempt to obtain an approximated closed-form solution to the problem. This chapter concludes with an algorithm for solving the problem in our paper.

## 4.1 Modification on Cetinkaya and Lee's (2000) Model

The original problem described in Cetinkaya and Lee (2000) is a periodic review inventory system with Poisson demand. Their model assumes negligible lead time and infinite warehouse capacity. Using an approximation, they obtained the optimal solution of Q* (the optimal order quantity) and T* (the optimal consolidation time). This section modifies Cetinkaya and Lee's (2000) approach to provide a better estimate of the optimal values.

## 4.2 Mathematical Model

The model is built on the original problem described in Cetinkaya and Lee (2000). The Vendor, V, faces a group of retailers ($R_i$) in the downstream of the supply chain (See Figure 6). The demand characteristics of each of the retailers can be stable or random. Consolidation of the cargo is done before sending them to the retailers. Unlike the Cetinkaya and Lee (2000) model, the warehouse of the vendor is assumed to have a fixed capacity ω. If the inventory level of the vendor hub is higher than the capacity, the additional goods will be stored at a nearby 3rd party warehouse who will charge an additional charge of $g over the holding cost of the vendor hub.

Using Cetinkaya and Lee (2000) assumptions, delivery lead-time to the retailers is assumed to be negligible. However, the inventory replenishment lead time is assumed to be a constant L, instead of the negligible replenishment lead time assumed in Cetinkaya and Lee (2000). Demands that are not fulfilled immediately are consolidated and shipped in batches. Thus, the vendor will incur customer waiting cost due to the lost of goodwill or relevant penalty charges due to late deliveries. In short, both inventory replenishment and dispatching policies affect the inventory position and total cost faced by the vendor.

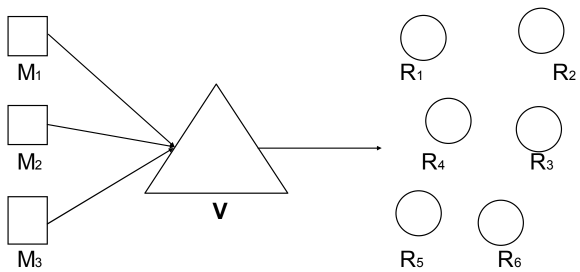

Figure 6: A Graphical Depiction of the problem

## 4.3 Inventory Replenishment Policy

The vendor assumes an (s, S) inventory replenishment policy. In this paper, we assume that reorder point, s, only consists of the cycle stock, which is demand over the lead time. We let the difference between s and S be defined as Q. Thus, the order up to level, S, is equal to Q+s. However, some of the suppliers may impose a MOQ due to strategic considerations. In such cases, the order up to level, S, would be equal to MOQ+s if the Q found is less than MOQ. Manufacturers have a known defective rate $p_i$. Goods from the manufacturers are inspected immediately and the inspection time is assumed to be negligible. Other than the procurement and order charges, delivery charges will be taken into consideration as well. In this paper, we will also consider an incremental discount policy on transportation charges from the supplier to the vendor hub.

### 4.4 Dispatch Policy

Retailer's demands are not fulfilled immediately but consolidated and shipped in batches. The dispatch size depends on the length of order consolidation time. The longer the consolidation time, the larger the batch size that can be consolidated. Dispatch cost to the retailers is assumed to adopt a similar structure as the transportation cost for inventory replenishment. Delivery is assumed to be instantaneous, so retailers will immediately receive the goods once the vendor starts dispatching it to them.

### 4.5 Model Assumptions

The vendor operator faces the problem of selecting a supplier out of a list of potential suppliers. Each of the suppliers has a different cost structure and thus the procurement cost will differ across suppliers. Replenishment costs consist of three main components: fixed cost of replenishing inventory, unit procurement cost and delivery cost. Demand from the retailers is assumed to follow a Poisson distribution and are i.i.d. The lead-time of order replenishment is assumed to be constant.

## 4.6 Model Formulation

The objective of the model is to obtain an optimal target inventory position level, $Q + \lambda L$, and dispatch shipment consolidation period, T, so that expected long-run average cost is minimised. A replenishment cycle is defined as the time interval between two consecutive replenishment decisions. Let C(Q, T) denote the expected long-run average cost. Using the renewal reward theorem, the long run average cost of the vendor is defined by:

$$C(Q,T) = \frac{E[\text{Replenishment Cycle Cost}]}{E[\text{Replenishment Cycle Time}]} \tag{1}$$

The same objective function of model would be:

$$\mathit{Min}\ C\ (Q,\ T)\ \mathit{where}\ Q,\ T \geq 0$$

Let K denotes the number of dispatches in a single order cycle. K is a positive random variable and is defined by

$$K = \inf\left\{k : \sum_{j=1}^{k} N_j(T) > Q\right\}$$

where N(t) is a renewal process that registers the demand consolidated at time t; $N_j(T)$ is defined as demand accumulated at the $j^{th}$ shipment consolidation cycle. It follows that the length of an order cycle (the length of time when an order is made) is

$$\text{E[Order Cycle Length]=E[K]T} \tag{2}$$

However, as lead time is now involved in the model, the actual replenishment lead time for the inventory would be equal to

$$\text{E[Replenishment Cycle Length]=E[K]T + L}$$

However, as L may not be actually divisible by T, the inclusion of the L term may complicate the whole model. To simplify the model, we replace the term L with $\hat{L}T$, where

$$\hat{L} = \left\lceil \frac{L}{T} \right\rceil$$

Thus the replenishment cycle length would be

$$\text{E[Replenishment Cycle Length]=E[K]T} + \hat{L}T$$

Let G(.) be the distribution function of N(T), and $G^{(k)}$(.) denotes the k-fold convolution of G(.). The expected value of K is given by (Cetinkaya and Lee, 2000)

$$E[K] = \sum_{k=1}^{\infty} G^{(k-1)}(Q^*) \tag{3}$$

where Q* is defined as the optimal value for Q

The replenishment cycle cost TC would consist of the following components:

*TC= Inventory Replenishment Cost + Holding Cost + Dispatching cost+ Customer Waiting Cost*

## 4.7 Expected Inventory Replenishment Cost per Replenishment Cycle

Under the cost structure suggested earlier, inventory replenishment cost per cycle would be equal to sum of the fixed ordering cost, $A_R$ and unit procurement cost, $C_R$. Mathematically, its can be expressed as:

*E(Inventory Replenishment Cost per cycle for manufacturer i) =* $A_R + C_R$ *E[Order Quantity]*

Let A(t) be the amount of goods consolidated to meet the outstanding demand.

Thus at time T,

$$N_j(T)=A(jT) \text{ where } j \text{ is the dispatch number}$$

Let $\overline{Q}$ = order quantity. The expected value of $\overline{Q}$ would be

$$E[\overline{Q}] = \mathrm{E}\left[\sum_{j=1`}^{k} N_j(T)\right] = \mathrm{E[K]*E[N(T)], where\ E[N(T)]=}\ \frac{\left[\sum_{j=1`}^{k} N_j(T)\right]}{E[K]}$$

The defective rate, p, lies in [0,1)

$$E[Q] = \frac{E[\overline{Q}]}{1-p},\ \mathrm{p} \neq 1$$

As such, when the defective rate, p, is zero, the equation will simply be transform into

$$E[Q] = E[\overline{Q}]$$

*E(Inventory Replenishment Cost per cycle for manufacturer i)=*$A_i + C_i$ * E[K]*E[N(T)]

## 4.8 Expected Inventory Holding Cost per Replenishment Cycle

By definition,

*E(Inventory Holding Cost per cycle) =Expected Total Inventory Held per cycle X Holding Cost*

Let IP(t) be the inventory position at time t and I(t) be the inventory level at time t. As there is a lead time for the goods to arrive after an order is made, the inventory position would not be the same with the inventory level all the time. The characteristics of the inventory position under consideration imply that

$$IP(t) = \begin{cases} Q + \lambda L & \text{if } 0 \leq t \leq T \\ Q + \lambda L - N_1(T) & \text{if } T < t \leq 2T \\ Q + \lambda L - \sum_{j=1}^{K-1} N_j(T) & \text{if } (K-1)T < t \leq KT \end{cases}$$

In Zipkin (2000) and Axsater (2000), the inventory level for an (s, S) policy at the time period of T+L is given as

$$I(t+\hat{L}T) = IP(t) - D\left(\left(\hat{L}+1\right)T + 1\right) \quad (4)$$

where D( $\left(\hat{L}+1\right)T$ ) is the demand that occurs during the time period of length $\left(\hat{L}+1\right)T$ . As the lead-time for replenishment is assumed to be the constant L, a replenishment order would only be activated if the sum of consolidated batch size A(jT) and the expected demand during $(\hat{L}+1)T$ were greater than the inventory I(jT). Otherwise, no inventory replenishment will be made. If A(jT) ≥ I(jT), a replenishment order quantity Q is placed where

$$E[Q] = \begin{cases} Q + A(t) - I(t), if\,(A(t) + E\left[D((\hat{L}+1)T)\right] \geq I(t)) \\ 0, if\,(A(t) < I(t)) \end{cases}$$

Total inventory held per cycle would be:

$$Total\ Inventory = \int_0^{KT} I(t)\,dt$$

Thus total inventory holding cost would be

$$E(Inventory\ Holding\ Cost\ per\ replenishment\ cycle) = h\,E\left[\int_0^{KT} I(t)\,dt\right]$$

where

\$h= holding cost for inventory stored in the vendor warehouse per unit per unit of time,

KT=replenishment cycle time.

As given by Eqn. (4), the distribution of the I(t+ $\hat{L}T$ ) follows the distribution of IP(t) and D( $(\hat{L}+1)T$ ). As IP(t) is a regenerative process and I(t+ $\hat{L}T$ ) follows the distribution of IP(t), I(t+ $\hat{L}T$ ) should also be a regenerative process. Using the relationship given by Eqn. (3), we have

$$
I(t+\hat{L}T)=\begin{cases} Q+\lambda(\hat{L}+1)T-\sum_{i=1}^{\hat{L}+1}N_i(T) & if\ 0\le t\le T \\ Q+\lambda(\hat{L}+1)T-N_1(T)-\sum_{i=1}^{\hat{L}+1}N_i(T) & if\ T<t\le 2T \\ Q+\lambda(\hat{L}+1)T-\sum_{j=1}^{K-1}N_j(T)-\sum_{i=1}^{\hat{L}+1}N_i(T) & if\ (K-1)T<t\le KT \end{cases}
$$

It can be observed that the structure of I(t) is similar to the inventory level in Cetinkaya and Lee (2000). Thus, using the same concept, we get

$$
H(Q,T)=T\left[(Q+\lambda(\hat{L}+1)T)-E\left[\sum_{i=1}^{\hat{L}+1}N_i(T)\right]\right]+T\sum_{i=0}^{Q}(Q+\lambda(\hat{L}+1)T-i-E\left[\sum_{i=1}^{\hat{L}+1}N_i(T)\right])m_g(i)
$$

where g(.) denotes the probability mass function of A(jT), and $g^{(k)}(.)$ denotes the k-fold convolution of g(.).

$$
m_g(i)=\sum_{k=1}^{\infty}g^{(k)}(i),
$$

where $m_g(.)$ is the renewal density associated with g(.).

We know that the demand arrival N(T) follows a Poisson process with parameter λT.

Under a Poisson distribution, the expected value of $\sum_{i=1}^{\hat{L}+1}N_i(T)$ is simply equal to the expected value of N( $(\hat{L}+1)T$ ), which is

$$
E\left(N\left((\hat{L}+1)T\right)\right)=\lambda(\hat{L}+1)T
$$

By substituting the above value into H(Q, T), we can simplify H(Q, T) as

$$
H(Q,T)=T\left(Q+\lambda(\hat{L}+1)T-\lambda(\hat{L}+1)T\right)+T\sum_{i=0}^{Q}\left\{Q+\lambda(\hat{L}+1)T-i-\lambda(\hat{L}+1)T\right\}m_g(i)
$$

$$
H(Q,T)=TQ+T\sum_{i=0}^{Q}\{Q-i\}m_g(i) \tag{5}
$$

We also know that the vendor hub have a limited warehouse capacity of ω. Thus, if the inventory level is higher than ω, there will be an additional cost of $g per unit per unit time. Let the additional holding cost in storing the goods in an external warehouse be denoted as H' (Q,T) and is defined as

$$HC'(Q,T) = gT(Q-\omega) + gT\sum_{i=\omega}^{Q}\{(Q-i\}m_g(i) \qquad ,Q \geq \omega \tag{6}$$

Thus, the inventory-holding cost per cycle would be denoted by:

E[Inventory Holding Cost Per Cycle]:

$$\begin{aligned} HC(Q,T) &= hH(Q,T) + HC'(Q,T) \\ &= hTQ + hT\sum_{i=0}^{Q}\{(Q-i\}m_g(i) + gT(Q-\omega) + gT\sum_{i=\omega}^{Q}\{Q-i\}m_g(i) \end{aligned} \tag{7}$$

### 4.9 Expected Dispatching Cost per Replenishment Cycle

Let $C_D$ be the variable cost of dispatching to the customer and $A_D$ be the fixed cost of dispatching to the customer. The total dispatching cost would simply be

*E(Delivery cost per Replenishment Cycle)=E[K]*$A_D$ *+E[K]E(N(T))* $C_D$

### 4.10 `Expected Customer Waiting Cost per Replenishment Cycle

Due to the consolidation policy used, demands are not fulfilled immediately. This would leads to backorders. Let $w be the cost of waiting per unit per unit time where

*E[Waiting Cost per Replenishment cycle]=wE[Total Time units waited by Back Orders]*

*E[Total Time units waited by Back Orders]*

*=E[Time units waited during consolidation process]+E[Time units waited when inventory=0]*

*E[Time units waited during the consolidation process]*

*=E[(T-$S_1$)+(T-$S_2$)+ . . . +(T-$S_{N(T)}$)]*

$$=E\left[N(T)T-\sum_{n=1}^{N(T)}S_n\right]$$

Letting

$$\mathrm{W(T)} = E\left[N(T)T-\sum_{n=1}^{N(T)}S_n\right]$$

W(T) is identified by proposition 2 of Cetinkaya and Lee (2000)

$$W(T)=v(T)+\int_0^T v(T-t)dM_F(t)$$

where

$$v(T)=\int_0^T (T-t)dF(t),$$

$$M_F(t)=\sum_{n=1}^{\infty}F^{(n)}(t)=E[N(t)]$$

Let BO(t) denote the shortage amount at time t and is defined as

$$BO(t)=-\left[I(t)\right]_-$$

The characteristics of the number of backorder could be found by taking the negative portion of the I(t) function. Thus,

$$BO(t) = \sum_{j=1}^{K-1} N_j(T) + \sum_{i=1}^{\hat{L}+1} N_i(T) - \left(Q + \lambda\left(\hat{L}+1\right)T\right) \quad if\ (K-1)T < t \leq KT \text{ and I(t)<0.}$$

Using the same principle that is used to find the inventory position, it is easy to simplify BO(t) into

$$BO(Q,T) = T\sum_{i=0}^{Q}\left\{\sum_{e=Q+\lambda(\hat{L}+1)T-i}^{\infty}\left[i + e - \left[Q + \lambda\left(\hat{L}+1\right)T\right]\right]m_b(e)\right\}m_g(i) \tag{8}$$

Where b(.) denote the probability mass function of D( $(\hat{L}+1)T$ ), and $g^{(k)}$(.) denotes the k-fold convolution of b(.).

$$m_b(e) = \sum_{k=1}^{\infty} b^{(k)}(e), \tag{9}$$

where $m_b$(.) is the renewal density associated with b(.)

Thus, the total waiting cost will be

*E[Waiting Cost per Replenishment cycle]*

$$=wE[K]v(T)+wE[K]\int_0^T v(T-t)\,dM_F(t)+w\int_0^{KT} BO(t)\,dt \tag{10}$$

## 4.11 Mathematical Analysis

In order to solve the problem, we have to compute C(Q,T) explicitly. Substituting the various cost equations into the overall cost equations, we have

$$C(Q,T) = \frac{A_R}{E[K]T} + \frac{C_R E[N(T)]}{T} + \frac{HC(Q,T))}{E[K]T} + \frac{A_D}{T} + \frac{C_D E[N(T)]}{T} + \frac{wW(T)}{T} + \frac{BO(T)}{E[K]T}$$

(11)

### 4.11.1 An Explicit Expression of C(Q,T)

As the demand arrivals, N(T), are assumed to follow a Poisson process with parameter λT then N(T) can be said as a Poisson random variable with parameter λT, and G(.) is a Poisson distribution with parameter λT. Then the expected value of N(T) would be

$$E[N(T) = \lambda T \tag{12}$$

Since demand arrivals follow a Poisson process, the interarrival times, Xn, n =1, 2, are by default exponential random variables and thus

$$dF(t) = \lambda e^{-\lambda t} dt \tag{13}$$

The renewal function $M_F(t) = \sum_{n-1}^{\infty} F^{(n)}(t)\lambda dt = E[N(t)]$ is given by λT so that

$$dM_F(t) = \lambda dt \tag{14}$$

Since G(.) is a Poisson distribution with parameter λT, the k-fold convolution of G(.) is simply a Poisson distribution with parameter kλT. Thus

$$g^{(k)}(i) = \frac{(k\lambda T)^i e^{-k\lambda T}}{i!} \tag{15}$$

$$G^{(k)}(Q) = \sum_{i=0}^{Q} \frac{(k\lambda T)^i e^{-k\lambda T}}{i!} \tag{16}$$

We substitute the above equations back to the expression for the renewal density function and the expected value of k function, we will have

$$m_g(i) = \sum_{k=1}^{\infty} \frac{(k\lambda T)^i e^{-k\lambda T}}{i!} \tag{17}$$

$$E[K] = \sum_{k=1}^{\infty} \sum_{i=0}^{Q} \frac{(k\lambda T)^i e^{-k\lambda T}}{i!} \tag{18}$$

The above information was enough to calculate the total cost in Cetinkaya and Lee (2000) paper. However, in our model, this information is insufficient. In order to calculate the total cost in our paper, we need to find the explicit form for a few more variables.

We let B(.) denotes the distribution for the demand that happened during the period $(\hat{L}+1)T$ and is equal to $N[(\hat{L}+1)T]$, which is a Poisson random variable with parameter $\lambda(\hat{L}+1)T$. As B(.) is a Poisson distribution with parameter $\lambda(\hat{L}+1)T$, thus the k fold convolution of B(.) will be a Poisson distribution with parameter $k\lambda(\hat{L}+1)T$. Thus, we have,

$$b^{(k)}(i) = \frac{\left(k\lambda(\hat{L}+1)T\right)^i e^{-k\lambda(\hat{L}+1)T}}{i!} \tag{19}$$

$$B^{(k)}(Q) = \sum_{i=0}^{Q} \frac{\left(k\lambda(\hat{L}+1)T\right)^i e^{-k\lambda(\hat{L}+1)T}}{i!} \tag{20}$$

Though we are able to obtain an explicit form for these variables, these forms do not allow us to get a simple optimal solution directly. In order to obtain such a solution, we have to express these terms in a closed form expression. However, the closed form expressions for these variables are not easily obtainable directly from these equations. To get the closed form expression of these variables, we have to approximate for the term E[K], $m_g(i)$ and $m_b(e)$.

We know that $P(K \geq k+1) = G^{(k)}(Q)$. Thus, $P(K \leq k) = 1 - G^{(k)}(Q)$. Substituting the expression of $G^{(k)}(Q)$, we have

$$P(K \le k) = 1 - \sum_{i=0}^{Q} \frac{(k\lambda T)^i e^{-k\lambda T}}{i!}, k = 1,2,3...$$

The RHS of the above equation behaves like a (Q+1) stage Erlang distribution with parameter λT and mean = (Q+1)/λT. Since the cumulative distribution function of K is equivalent to that of the Q+1 stage Erlang distribution, thus the expected value of K is equivalent to the mean of a Q+1 stage Erlang distribution. That is,

$$E[K] \approx \frac{Q+1}{\lambda T} \tag{21}$$

To approximate for $m_g(i)$, we have to make use of the approximation for E[K]. We know that by definition of the renewal function,

$$m_g(i) = M_G(i) - M_G(i-1) \tag{22}$$

where $M_G(i)$ is the renewal function associated with G(.) and is defined by

$$M_G(i) = \sum_{k=1}^{\infty} G^{(k)}(i)$$

We know that

$$E[K] = \sum_{k=1}^{\infty} G^{(k-1)}(Q)$$

Thus, by relating the two equations above, we have

$$E[K] = M_G(Q) + 1$$

By substituting Eqn. (21) into the above equation, we can solve for $M_G(Q)$, that is,

$$M_G(Q) \approx \frac{Q+1}{\lambda T} - 1 \tag{23}$$

By using Eqn. (22) and (23) will lead to an estimated for $m_g(i)$, that is

$$m_g(i) = \frac{1}{\lambda T} \tag{24}$$

All these equations have been solved in Cetinkaya and Lee (2000) paper. However, in our model, we included a new term $m_b(i)$. To solve for $m_b(i)$, we have to introduce a dummy random variable K' and is defined as

$$K' = \inf\left\{k' : \sum_{j=1}^{k'} N_j((\hat{L}+1)T) > Q'\right\}$$

where Q' is a dummy constant used to compute K' and $N_j\left(\left(\hat{L}+1\right)T\right)$ is the renewal process that registers the demand that are placed during period $\left(\hat{L}+1\right)T$ . B(.) denotes the distribution function of $N\left(\left(\hat{L}+1\right)T\right)$ and $B^{(k')}$ denotes the k' convolution of B(.). Using the same principle used in finding the expected value of K, we find that

$$P(K' \geq k'+1) = B^{(k')}(Q')$$

As K' is also a positive random variable like K, its expected value is given by

$$E[K'] = \sum_{k'=1}^{\infty} B^{(k'-1)}(Q^*)$$

As $P(K' \geq k'+1) = B^{(k)}(Q)$. Thus, $P(K' \leq k') = 1 - B^{(k)}(Q)$. Substituting the expression of $B^{(k)}(Q)$, we have

$$P(K' \leq k') = 1 - \sum_{i=0}^{Q} \frac{\left(k\lambda\left(\hat{L}+1\right)T\right)^i e^{-k\lambda\left(\hat{L}+1\right)T}}{i!}, k' = 1,2,3...$$

Looking at the right side of the above equation, we can observe that it takes the form of the distribution function of a Q+1 stage Erlang distribution with parameter λT and mean = $(Q+1)/\lambda\left(\hat{L}+1\right)T$. Since the distribution function of K' is equivalent to that of a (Q'+1) stage Erlang distribution, the expected value of K equivalent to the mean of a (Q'+10 stage Erlang distribution is

$$E[K'] \approx \frac{Q+1}{\lambda(\hat{L}+1)T} \tag{25}$$

Thus, we can then go on to approximate for $m_b(i)$.

We know that by definition of the renewal function,

$$m_b(i) = M_B(i) - M_B(i-1) \tag{26}$$

where $M_B(i)$ is the renewal function associated with B(.) and is defined by

$$M_B(i) = \sum_{k=1}^{\infty} B^{(k)}(i)$$

We know that

$$E[K'] = \sum_{k=1}^{\infty} B^{(k-1)}(Q')$$

Thus, by relating the two equations above, we have

$$E[K'] = M_B(Q')+1$$

If we use the estimate for E[K'] and solve for $M_B(Q')$, then

$$M_B(Q') \approx \frac{Q'+1}{\lambda(\hat{L}+1)T} - 1 \tag{27}$$

By using Eqn. (26) and (27) will lead to an estimated for $m_b(i)$, that is

$$m_b(i) = \frac{1}{\lambda(\hat{L}+1)T} \tag{28}$$

With these approximations, we would be able to obtain the closed form expressions of the various components. By substituting Eqns. (24) and (28) into Eqn. (7), we would be able to get the closed form expression of HC(Q,T) through a few simple algebraic manipulations.

$$HC(Q,T) = hTQ + \frac{h}{2\lambda}(Q+1)Q + gT(Q-\omega) + gT\frac{(Q-\omega)(Q-\omega+1)}{2}\frac{1}{\lambda T}$$

$$HC(Q,T) = hTQ + gT(Q-\omega) + \frac{h}{2\lambda}(Q+1)Q + \frac{g\left(Q^2-(2\omega-1)Q+\omega^2-\omega\right)}{2\lambda} \qquad (29)$$

However, for BO(Q, T), we would not be able to directly compute the closed form expression by substituting the various approximations into Eqn. (8). This is due to the term

$$\sum_{e=Q+\lambda(\hat{L}+1)T}^{\infty}\left[i+e-\left[Q+\lambda(\hat{L}+1)T\right]\right]m_b(e)$$

which is extremely difficult to compute. We have to find an approximate for this term. In order to do this, let's take a look at the explicit form of $m_b(e)$ in Eqn. (9).

$$m_b(e) = \sum_{k=1}^{\infty} b^{(k)}(e),$$

We know b(.) is the probability mass function of $N((\hat{L}+1)T)$ and it is a Poisson process with the following mass function

$$b(i) = \frac{\left(\lambda(\hat{L}+1)T\right)^i e^{-\lambda(\hat{L}+1)T}}{i!}, \text{ i=1, 2, 3....}$$

By the definition of a Poisson process, we also know when i> $\lambda(\hat{L}+1)T$ ,

$$\lim_{i\to\infty}\left[\frac{\left(\lambda(\hat{L}+1)T\right)^i e^{-\lambda(\hat{L}+1)T}}{i!}\right] \to 0, \text{ i> } \lambda(\hat{L}+1)T$$

This means as i larger, b(i) will decrease and will eventually reach 0. This would mean that $m_b(e)$ will tend to zero at some big e as $m_b(e)$ is a function of b(i). However, we are unable to directly determine the point whereby b(i) will be insignificant from the expression itself. Fortunately we are able to get a good estimate of the number from the properties of a Poisson distribution. We know that the Poisson distribution can be approximated by a Normal distribution with parameters (λ,λ), where λ is the mean of the Poisson distribution. By the characteristics of a normal distribution, we know that

$$P(X<x)\rightarrow 1,\ x \geq \mu+3\sigma$$

This implies that

$$P(X>x \mid x \geq \mu+3\sigma)\rightarrow 0,$$

From the above, we can infer that b(e) →0 when $e \geq \lambda\left(\hat{L}+1\right)T + 3\sqrt{\lambda\left(\hat{L}+1\right)T}$ . However, to make our calculation simpler, we relax the upper bound restriction and let the upper bound restriction to be $\lambda\left(\hat{L}+1\right)T + 3\sqrt{\lambda\left(\hat{L}+1\right)T}$ . Thus, we now change the upper bound restriction from ∞ to $\lambda\left(\hat{L}+1\right)T + 3\sqrt{\lambda\left(\hat{L}+1\right)T}$ . Eqn. (8) becomes

$$BO(Q,T) = T\sum_{i=0}^{Q}\left\{\sum_{e=Q+\lambda\left(\hat{L}+1\right)T-i}^{Q+\lambda\left(\hat{L}+1\right)T+3\sqrt{\left(\lambda\left(\hat{L}+1\right)T\right)}-i}\left[i+e-\left[Q+\lambda\left(\hat{L}+1\right)T\right]\right]m_b(e)\right\}m_g(i) \tag{30}$$

Eqn. (30) can be simplified by substituting Eqn. (24) and (28) into Eqn. (30). After a series of mathematical manipulation, we will have

$$
\begin{aligned}
BO(Q,T) &= T\sum_{i=0}^{Q}\left\{\sum_{e=Q+\lambda(\hat{L}+1)T-i}^{Q+\lambda(\hat{L}+1)T+3\sqrt{(\lambda(\hat{L}+1)T)}-i}\left[i+e-\left[Q+\lambda(\hat{L})T\right]\right]m_b(e)\right\}m_g(i) \\
&= T\sum_{i=0}^{Q}\left\{\frac{1}{\lambda(\hat{L}+1)T}\sum_{e=Q+\lambda(\hat{L}+1)T-i}^{Q+\lambda(\hat{L}+1)T+3\sqrt{(\lambda(\hat{L}+1)T)}-i}\left[i+e-\left[Q+\lambda(\hat{L}+1)T\right]\right]\right\}m_g(i) \\
&= T\sum_{i=0}^{Q}\left\{\frac{1}{\lambda(\hat{L}+1)T}\left[\frac{\left(9\lambda(\hat{L}+1)T\right)+\left(3\sqrt{\lambda(\hat{L}+1)T}\right)}{2}\right]\right\}m_g(i) \\
&= \frac{T}{2\lambda(\hat{L}+1)T(\lambda T)}\sum_{i=0}^{Q}\left\{\left[\left(9\lambda(\hat{L}+1)T\right)+\left(3\sqrt{\lambda(\hat{L}+1)T}\right)\right]\right\} \\
&= \frac{1}{2\lambda^2(\hat{L}+1)T}\sum_{i=0}^{Q}\left\{\left[9\left(\lambda(\hat{L}+1)T\right)\right]+\left(3\sqrt{\lambda(\hat{L}+1)T}\right)\right\} \\
&= \frac{1}{2\lambda^2(\hat{L}+1)T}\left[(Q+1)9\left(\lambda(\hat{L}+1)T\right)+(Q+1)\left(3\sqrt{\lambda(\hat{L}+1)T}\right)\right] \\
&= \frac{3(Q+1)}{2\lambda^2(\hat{L}+1)T}\left[3\left(\lambda(\hat{L}+1)T\right)+\sqrt{\lambda(\hat{L}+1)T}\right]
\end{aligned}
$$

$$
BO(Q,T)=\frac{3(Q+1)}{2\lambda^2(\hat{L}+1)T}\left[3\left(\lambda(\hat{L}+1)T\right)+\sqrt{\lambda(\hat{L}+1)T}\right] \tag{31}
$$

From Cetinkaya and Lee (2000), we know that

$$
W(T)=\frac{\lambda T^2}{2} \tag{32}
$$

To get the complete closed form expression for the average cost, we substitute Eqns. (12), (25), (29), (31) and (32) into Eqn. (11), we will obtain

$$
\begin{aligned}
C(Q,T) &= \frac{A_R\lambda}{(Q+1)} + C_R\lambda + \frac{A_D}{T} + C_D\lambda + \frac{w\lambda T}{2} + \frac{w\lambda T}{(Q+1)T}\frac{3(Q+1)}{2\lambda^2(\hat{L}+1)T}\left[3\left(\lambda(\hat{L}+1)T\right) + \sqrt{\lambda(\hat{L}+1)T}\right] \\
&+ \frac{\lambda}{(Q+1)}\left\{hTQ + \frac{h}{2\lambda}(Q+1)Q + gT(Q-\omega) + \frac{g\left(Q^2-(2\omega-1)Q+\omega^2-\omega\right)}{2\lambda}\right\} \\
&= \frac{A_R\lambda}{(Q+1)} + C_R\lambda + \frac{A_D}{T} + C_D\lambda + \frac{w\lambda T}{2} + \frac{3w}{2\lambda(\hat{L}+1)T}\left[3\left(\lambda(\hat{L}+1)T\right) + \sqrt{\lambda(\hat{L}+1)T}\right] + \frac{\lambda hTQ}{(Q+1)} + \frac{hQ}{2} \\
&+ \frac{\lambda gT(Q-\omega)}{Q+1} + \frac{gQ^2}{2(Q+1)} - \frac{gQ(2\omega-1)}{2(Q+1)} + \frac{g(\omega^2-\omega)}{2(Q+1)} \qquad (33)
\end{aligned}
$$

Observing Eqn. (33), we note that presence of the term $\sqrt{\lambda(\hat{L}+1)T}$ will complicate the whole expression when we are solving for T*. To simplify the equation, we need to introduce a term that will simplify the whole equation. Let us take a look at the term $\frac{1}{2}\lambda(\hat{L}+1)T$.

For the range of $\lambda(\hat{L}+1)T < 4$, it can be proven that the difference between the term $\frac{1}{2}\lambda(\hat{L}+1)T$ and $\sqrt{\lambda(\hat{L}+1)T}$ is small in the range of [0, 4). Thus, it could be inferred that $\frac{1}{2}\lambda(\hat{L}+1)T$ can be used to approximate $\sqrt{\lambda(\hat{L}+1)T}$ in the range of [0, 4).

Without the lost of generality, we replace $\sqrt{\lambda(\hat{L}+1)T}$ with this relaxed approximation $\frac{1}{2}\lambda(\hat{L}+1)T$ to simplify the equation, then Eqn. (33) becomes

$$
\begin{aligned}
C(Q,T) &= \frac{A_R\lambda}{(Q+1)} + C_R\lambda + \frac{A_D}{T} + C_D\lambda + \frac{w\lambda T}{2} + \frac{3w}{2\lambda(\hat{L}+1)T}\left[3\left(\lambda(\hat{L}+1)T\right) + \frac{1}{2}\left(\lambda(\hat{L}+1)T\right)\right] + \frac{\lambda hTQ}{(Q+1)} + \frac{hQ}{2} \\
&+ \frac{\lambda gT(Q-\omega)}{Q+1} + \frac{gQ^2}{2(Q+1)} - \frac{gQ(2\omega-1)}{2(Q+1)} + \frac{g(\omega^2-\omega)}{2(Q+1)} \qquad (33a)
\end{aligned}
$$

To simplify the computation, we let $\hat{Q} = Q+1$ and substitute these into Eqn. (33a).

$$C(\hat{Q},T)=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+\frac{A_D}{T}+C_D\lambda+\frac{w\lambda T}{2}+\frac{30w}{4}+\frac{\lambda hT(\hat{Q}-1)}{\hat{Q}}+\frac{h(\hat{Q}-1)}{2}$$

$$+\frac{\lambda gT(\hat{Q}-1-\omega)}{\hat{Q}}+\frac{g(\hat{Q}-1)^2}{2\hat{Q}}-\frac{g(\hat{Q}-1)(2\omega-1)}{2\hat{Q}}+\frac{g(\omega^2-\omega)}{2\hat{Q}}$$

$$=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+\frac{A_D}{T}+C_D\lambda+\frac{w\lambda T}{2}+\frac{30w}{4}+\frac{\lambda hT(\hat{Q}-1)}{\hat{Q}}+\frac{h(\hat{Q}-1)}{2}$$

$$+\frac{\lambda gT(\hat{Q}-1-\omega)}{\hat{Q}}+\frac{g\hat{Q}}{2}-g+\frac{g}{2\hat{Q}}-\frac{g(2\omega-1)}{2}+\frac{g(2\omega-1)}{2\hat{Q}}+\frac{g(\omega^2-\omega)}{2\hat{Q}}$$

$$C(\hat{Q},T)=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+\frac{A_D}{T}+C_D\lambda+\frac{w\lambda T}{2}+\frac{30w}{4}+\frac{\lambda hT(\hat{Q}-1)}{\hat{Q}}+\frac{h(\hat{Q}-1)}{2}$$

$$+\frac{\lambda gT(\hat{Q}-1-\omega)}{\hat{Q}}+\frac{g\hat{Q}}{2}-g+\frac{g}{2\hat{Q}}-\frac{g(2\omega-1)}{2}+\frac{g(2\omega-1)}{2\hat{Q}}+\frac{g(\omega^2-\omega)}{2\hat{Q}} \qquad (34)$$

The solution to our problem will be

$$\text{Min } C(\hat{Q},T)$$

$$\text{s.to } \begin{array}{l}\hat{Q}\ge 1\\ T\ge 0\end{array}$$

To check the convexity of the function, we compute

$$\frac{dC(\hat{Q},T)}{d\hat{Q}}=-\frac{A_R\lambda}{\hat{Q}^2}+\frac{\lambda hT}{\hat{Q}^2}+\frac{h}{2}+\frac{\lambda gT}{\hat{Q}^2}+\frac{\lambda gT(\omega)}{\hat{Q}^2}+\frac{g}{2}-\frac{g}{2\hat{Q}^2}-\frac{g(2\omega-1)}{2\hat{Q}^2}-\frac{g(\omega^2-\omega)}{2\hat{Q}^2} \qquad (35)$$

$$\frac{d^2C(\hat{Q},T)}{d\hat{Q}^2}=\frac{2A_R\lambda}{\hat{Q}^3}-\frac{2\lambda hT}{\hat{Q}^3}-\frac{2\lambda gT}{\hat{Q}^3}-\frac{2\lambda gT(\omega)}{\hat{Q}^3}+\frac{g}{\hat{Q}^3}+\frac{g(2\omega-1)}{\hat{Q}^3}+\frac{g(\omega^2-\omega)}{2\hat{Q}^3} \qquad (36)$$

$$\frac{dC(\hat{Q},T)}{dT}=-\frac{A_D}{T^2}+\frac{w\lambda}{2}+\frac{\lambda h(\hat{Q}-1)}{\hat{Q}}+\frac{\lambda g(\hat{Q}-1-\omega)}{\hat{Q}} \qquad (37)$$

$$\frac{d^2C(\hat{Q},T)}{dT^2}=\frac{2A_D}{T^3} \tag{38}$$

$$\frac{d^2C(\hat{Q},T)}{d\hat{Q}dT}=\frac{\lambda h}{\hat{Q}^2}+\frac{\lambda g}{\hat{Q}^2}+\frac{\lambda g\omega}{\hat{Q}^2} \tag{39}$$

From the various derivatives, it can be seen that $C(\hat{Q},T)$ must be convex in T for all positive T values. However, $C(\hat{Q},T)$ may not be necessary convex in $\hat{Q}$ for all positive $\hat{Q}$ values. The complication is due to the term $\frac{h\lambda T(\hat{Q}-1)}{\hat{Q}}+\frac{\lambda gT(\hat{Q}-1-\omega)}{\hat{Q}}$ in Eqn. (34).

Let (Q*, T*) denote the solution to Eqn. (34). Since we let $\hat{Q}=Q+1$, thus the solution of the problem by solving Eqn. (34) would be (Q*-1,T*). The necessary conditions for the optimal solution from Eqn (34) are

$$\hat{Q}^*=\sqrt{\frac{2\lambda(A_R-hT-gT-gT\omega)+g\omega+g\omega^2}{h+g}} \tag{40}$$

$$T^*=\sqrt{\frac{2A_D\hat{Q}}{\lambda(\hat{Q}w-2h+2\hat{Q}h-2g+2gQ-2g\omega)}} \tag{41}$$

From the above equations, we can see that it is difficult to compute Q* and T* directly due to the recursive nature of the equation. Thus, the optimal solution obtained here might not be unique. To solve this problem, we have the following analysis.

If we substitute Eqn. (41) into (34), the function C($\hat{Q}$,T) reduces to C($\hat{Q}$). After several simple algebraic manipulations, we will get

$$C\left(\hat{Q},T\right)=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+C_D\lambda+\frac{g\hat{Q}}{2}-g+\frac{g}{2\hat{Q}}-w\frac{g(2\omega-1)}{2}+\frac{g(2\omega-1)}{2\hat{Q}}+\frac{g\left(\omega^2-\omega\right)}{2\hat{Q}}+\frac{h\left(\hat{Q}-1\right)}{2}+\frac{3w}{2}$$
$$+\frac{2\hat{Q}A_D}{2\hat{Q}T}+\frac{\hat{Q}w\lambda T^2}{2\hat{Q}T}+\frac{2\hat{Q}T(\lambda hT)}{2\hat{Q}T}-\frac{2\lambda hT^2}{2\hat{Q}T}+\frac{2\lambda gT^2\left(\hat{Q}-1-\omega\right)}{2\hat{Q}T}$$
$$=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+C_D\lambda+\frac{g\hat{Q}}{2}-g+\frac{g}{2\hat{Q}}-w\frac{g(2\omega-1)}{2}+\frac{g(2\omega-1)}{2\hat{Q}}+\frac{g\left(\omega^2-\omega\right)}{2\hat{Q}}+\frac{h\left(\hat{Q}-1\right)}{2}+\frac{3w}{2}$$
$$+\frac{2\hat{Q}A_D+\hat{Q}w\lambda T^2+2\lambda(g+h)\hat{Q}T^2-2\lambda(h+g)T^2-2\lambda g\omega T^2}{2\hat{Q}T}$$
$$=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+C_D\lambda+\frac{g\hat{Q}}{2}-g+\frac{g}{2\hat{Q}}-w\frac{g(2\omega-1)}{2}+\frac{g(2\omega-1)}{2\hat{Q}}+\frac{g\left(\omega^2-\omega\right)}{2\hat{Q}}+\frac{h\left(\hat{Q}-1\right)}{2}+\frac{3w}{2}$$
$$+\frac{2\hat{Q}A_D}{2\hat{Q}T}+\frac{\left(\hat{Q}w\lambda+2\lambda(g+h)\hat{Q}-2\lambda(h+g)-2\lambda g\omega\right)T}{2\hat{Q}}$$
$$C(Q)=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+C_D\lambda+\frac{g\hat{Q}}{2}-g+\frac{g}{2\hat{Q}}-w\frac{g(2\omega-1)}{2}+\frac{g(2\omega-1)}{2\hat{Q}}+\frac{g\left(\omega^2-\omega\right)}{2\hat{Q}}+\frac{h\left(\hat{Q}-1\right)}{2}+\frac{3w}{2}$$
$$+\sqrt{\frac{A_D\lambda\left(\hat{Q}w-2h+2\hat{Q}h-2g+2gQ-2g\omega\right)}{2\hat{Q}}}+\sqrt{\frac{A_D\left(\hat{Q}w\lambda+2\lambda(g+h)\hat{Q}-2\lambda(h+g)-2\lambda g\omega\right)}{2\hat{Q}}}$$
$$=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+C_D\lambda+\frac{g\hat{Q}}{2}-g+\frac{g}{2\hat{Q}}-w\frac{g(2\omega-1)}{2}+\frac{g(2\omega-1)}{2\hat{Q}}+\frac{g\left(\omega^2-\omega\right)}{2\hat{Q}}+\frac{h\left(\hat{Q}-1\right)}{2}+\frac{3w}{2}$$
$$+\sqrt{\frac{2A_D\lambda\left(\hat{Q}w-2h+2\hat{Q}h-2g+2gQ-2g\omega\right)}{2\hat{Q}}}$$

$$C\left(\hat{Q}\right)=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+C_D\lambda+\frac{g\hat{Q}}{2}-g+\frac{g}{2\hat{Q}}-w\frac{g(2\omega-1)}{2}+\frac{g(2\omega-1)}{2\hat{Q}}+\frac{g\left(\omega^2-\omega\right)}{2\hat{Q}}+\frac{h\left(\hat{Q}-1\right)}{2}+\frac{3w}{2}$$
$$+\sqrt{\frac{2A_D\lambda\left(\hat{Q}w-2h+2\hat{Q}h-2g+2gQ-2g\omega\right)}{2\hat{Q}}} \tag{42}$$

Let us define

$$C_1\left(\hat{Q}\right)=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+C_D\lambda+\frac{g\hat{Q}}{2}-g+\frac{g}{2\hat{Q}}-w\frac{g(2\omega-1)}{2}+\frac{g(2\omega-1)}{2\hat{Q}}+\frac{g\left(\omega^2-\omega\right)}{2\hat{Q}}+\frac{h\left(\hat{Q}-1\right)}{2}+\frac{3w}{2} \tag{43}$$

$$C_2\left(\hat{Q}\right)=\sqrt{\frac{2A_D\lambda\left(\hat{Q}w-2h+2\hat{Q}h-2g+2gQ-2g\omega\right)}{2\hat{Q}}} \quad (44)$$

We also let $C'(\hat{Q}), C_1^{'}(\hat{Q})$ and $C_2^{'}(\hat{Q})$ denotes the first derivative of $C(\hat{Q}), C_1(\hat{Q})$ and $C_2(\hat{Q})$ respectively. Then

$$C'(\hat{Q}) = C_1^{'}(\hat{Q}) + C_2^{'}(\hat{Q}) \quad (45)$$

and Q* is a solution of

$$C_1^{'}(\hat{Q}) + C_2^{'}(\hat{Q}) = 0 \quad (46)$$

$$C'_1\left(\hat{Q}\right)=-\frac{A_R\lambda}{\hat{Q}^2}+\frac{g}{2}-\frac{g}{2\hat{Q}^2}-\frac{g(2\omega-1)}{2\hat{Q}^2}-\frac{g\left(\omega^2-\omega\right)}{2\hat{Q}^2}+\frac{h}{2}$$

$$C'_1\left(\hat{Q}\right)=-\frac{2A_R\lambda+g\omega+g\omega^2}{2\hat{Q}^2}+\frac{g+h}{2} \quad (47)$$

$$C_2\left(\hat{Q}\right)=\sqrt{\frac{2A_D\lambda(w)}{2}-\frac{2A_D\lambda(h)}{\hat{Q}}+\frac{2A_D\lambda(2h)}{2}+\frac{2A_D\lambda\left(9w\bar{L}\right)}{2}-\frac{2A_D\lambda(g)}{\hat{Q}}+2A_D\lambda(g)-\frac{2A_D\lambda(g\omega)}{\hat{Q}}}$$
$$=\sqrt{2A_D\lambda}\sqrt{\frac{(w)}{2}+(g+h)-\frac{(g\omega+h+g)}{\hat{Q}}}$$

$$C'_2\left(\hat{Q}\right)=\frac{\sqrt{A_D\lambda}(g\omega+h+g)}{2\hat{Q}^2\sqrt{\frac{(w)}{2}+(g+h)-\frac{(g\omega+h+g)}{Q}}} \quad (48)$$

For Eqn. (45) to hold, $-C_1^{'}(\hat{Q}) = C_2^{'}(\hat{Q})$ must be true. Upon analysing Eqn. (46), it can be easily seen that $C_1^{'}(\hat{Q})$ is increasing over the range [1,+∞). This implies that $-C_1^{'}(\hat{Q})$ is decreasing over the same range too. Analysing Eqn. (47), it is seen that $C_2^{'}(\hat{Q})$ is decreasing over [1,+∞). At large $\hat{Q}$, we observe that - $C_1^{'}(\hat{Q}) \rightarrow -\frac{(g+h)}{2}$ and

$C_2^{'}(\hat{Q}) \to 0$. This suggest that the gradient of $-C_1^{'}(\hat{Q})$ is steeper than the gradient of $C_2^{'}(\hat{Q})$. It can be inferred that $-C_1^{'}(\hat{Q})$ and $C_2^{'}(\hat{Q})$ will intersect at most once. In addition,

$$-C_1^{'}(1) \geq C_2^{'}(1)$$

At $\hat{Q}$=1, we have

$$C'_1(1) = -A_R\lambda - \frac{g\omega^2}{2} + \frac{h}{2} + \frac{g}{2} \tag{49}$$

$$C'_2(1) = \frac{\sqrt{2A_D\lambda}(g\omega + h + g)}{2\sqrt{\frac{w}{2} - (g\omega)}} \tag{50}$$

Substituting the Eqns (49) and (50) into Eqn. (45), we have

$$C'(1) = -A_R\lambda - \frac{g\omega^2}{2} + \frac{h}{2} + \frac{g}{2} + \frac{\sqrt{2A_D\lambda}(g\omega + h + g)}{2\sqrt{\frac{w}{2} - (g\omega)}} \tag{51}$$

Analysing Eqn. (51), we observe that when

$$2A_R\lambda + g\omega^2 - h - g > \frac{\sqrt{2A_D\lambda}(g\omega + h + g)}{\sqrt{\frac{w}{2} - (g\omega)}} \tag{52}$$

then $-C_1'(1) > C_2'(1)$, i.e. $-C_1'(\hat{Q})$ and $C_2'(\hat{Q})$ do not intercept at the range of $[1,\infty)$. If Eqn. (52) holds, it also means that $C'(1)>0$ (i.e. $C(\hat{Q})$ is increasing in the range [0,1). From Eqn. (42), we can see that $C(\hat{Q})$ is increasing as $\hat{Q}$ goes to infinity. This in turn implies that if Eqn. (52) is true, it is implied that the global minimum will be $\hat{Q}^*=1$ and $T^* = \sqrt{\frac{2A_D}{\lambda(w - 2g\omega)}}$. (T* is obtained by substituting $\hat{Q}$=1 into equation (41). Recall that

$\hat{Q}$=Q-1. Thus when Eqn. (52) holds, the optimal inventory level would be zero and the optimal consolidation cycle time would be $\sqrt{\frac{2A_D}{\lambda(w-2g\omega)}}$ time units.

If Eqn. (52) does not hold, the optimal solution will be given by ($\hat{Q}$*-1, T*). Looking at the optimal solution for $\hat{Q}$* and T*, it can be seen that the optimal solutions of $\hat{Q}$* and T* (i.e. Eqn. (40) and (41)) is dependent on the values obtained for the other dependent variable. This suggests that the solution have to be obtained iteratively. However, this process is a tedious process, especially if the initial estimates used for $\hat{Q}$* and T* are far away from the optimal values. To simplify this process, we suggest a reasonably fast and good approximation algorithm to obtain the values

We note that for large $\hat{Q}$,

$$\frac{h\lambda T(\hat{Q}-1)}{\hat{Q}} \to h\lambda T \tag{53}$$

However, the approximation of

$$\frac{g\lambda T(\hat{Q}-1-\omega)}{\hat{Q}} \to g\lambda T$$

will only be true if $\hat{Q}$>>ω, which may not be the case. Thus, this approximation cannot be done. In order for us to deal with this term, let us denote

$$z = \frac{(\hat{Q}-1-\omega)}{\hat{Q}} \qquad z \in )[0,1) \tag{54}$$

By substituting the approximation (53) and Eqn. (54) into Eqn. (34), we have

$$C\left(\hat{Q},T\right)=\frac{A_R\lambda}{\hat{Q}}+C_R\lambda+\frac{A_D}{T}+C_D\lambda+\frac{w\lambda T}{2}+\frac{30w}{4}+\lambda hT+\frac{h\left(\hat{Q}-1\right)}{2}$$
$$+z\lambda gT+\frac{g\hat{Q}}{2}-g+\frac{g}{2\hat{Q}}-\frac{g(2\omega-1)}{2}+\frac{g(2\omega-1)}{2\hat{Q}}+\frac{g\left(\omega^2-\omega\right)}{2\hat{Q}} \qquad (55)$$

Let us look at the global minimum of T for the equation (55)

$$\frac{dC\left(\hat{Q},T\right)}{dT}=-\frac{A_D}{T^2}+\frac{w\lambda}{2}+\lambda h+z\lambda g$$

$$T^*=\sqrt{\frac{2A_D}{(w\lambda+2\lambda h+2z\lambda g)}} \qquad (56)$$

We know that z $\in$ [0,1]. Thus, we are able to get the bounds of T* by simply substituting the bounds of z into Eqn. (56)

$$\sqrt{\frac{2A_D}{(w\lambda+2\lambda h+2\lambda g)}}<T^*\leq\sqrt{\frac{2A_D}{(w\lambda+2\lambda h)}} \qquad (57)$$

From Eqn. (56), we know that value of T* depends on z and

$$\frac{dT}{dz}=\frac{1}{2}\left(\frac{2A_D}{(w\lambda+2\lambda h+2z\lambda g)}\right)^{-\frac{1}{2}}\left(\frac{0-(2A_D)(2\lambda g)}{(w\lambda+2\lambda h+2z\lambda g)^2}\right)$$
$$=-\left(\frac{(\lambda g)\sqrt{(2A_D)}}{(w\lambda+2\lambda h+2z\lambda g)^{\frac{3}{2}}}\right).$$

To get the percentage change in T if z is changed by 1 unit, we let

$$\frac{\frac{dT}{dz}}{T}=\frac{-\left(\frac{(\lambda g)\sqrt{(2A_D)}}{(w\lambda+2\lambda h+2z\lambda g)^{\frac{3}{2}}}\right)}{\sqrt{\frac{2A_D}{(w\lambda+2\lambda h+2z\lambda g)}}}$$

$$=-\left(\frac{(\lambda g)\sqrt{(2A_D)}}{(w\lambda+2\lambda h+2z\lambda g)^{\frac{3}{2}}}\right)\sqrt{\frac{(w\lambda+2\lambda h+2z\lambda g)}{2A_D}}$$

$$=-\left(\frac{(g)}{(w+2\lambda h+2zg)}\right)$$

Thus the above equation, it can be seen that T is relative insensitive to any change in the variable z as the numerator term is usually much smaller than the denominator term. This means that the choice of the value for the term z would not result in the estimate of T being deviated from the optimal value of T* too much. Let us set the initial value of z to be at its upper bound. Thus, this would mean that T* would adopt its initial value at its lower bound

$$T=\sqrt{\frac{2A_D}{(w\lambda+2\lambda h+2\lambda g)}} \tag{58}$$

Thus, using this estimate of T, we substitute Eqn. (58) into Eqn. (40) to get the 1st estimate of $\hat{Q}$. We then use this estimate of $\hat{Q}$ to get the approximation of T* by substituting the estimate for $\hat{Q}$ into Eqn. (41). Lastly, we will use this approximate of T* to get the approximate for $\hat{Q}$ * by substituting it into Eqn. (40).

We know when C($\hat{Q}$,T) is convex, the minimum is given by Q*. Even when C($\hat{Q}$, T) is not a convex function, we have proven that it is an increasing function after 1. Thus, when an MOQ is applied and the MOQ is higher than Q*, then it make sense to set MOQ as Q*. Then we substitute MOQ into Eqn. (41) to get the optimal T*.

### 4.11.2 Algorithm for finding Optimal Q* and T*

We shall now summarise the steps in finding our approximate Q* and T*

1) Obtain $T_1$, an initial estimate for T* using Eqn. (58)
2) Substitute the estimate $T_1$ into Eqn (40) to obtain $Q_1$, which is an initial estimate of Q*.
3) Substitute $Q_1$ into Eqn (41) to get a final estimate of T*. Then we substitute T* again into Eqn (40) to get final estimate for Q*. If we are unable to compute T* or/and Q*, retain the initial estimates as the final Q* and T*
4) Check for any MOQ criteria. If there is an MOQ, check if Q* is lower that MOQ. If Q* is lower than MOQ, then go to (5). Else stop.
5) Set Q* to be equal to MOQ. Substitute MOQ into Eqn (41) to get an estimate of T*.

# 5 Results and Analysis

This chapter begins with a brief description of the VMI simulation program that is used to develop the simulator model in Chapter 3. This is followed by a sensitivity analysis of the model. We compare the simulated average total cost of Cetinkaya and Lee's (2000) model with the algorithm proposed in Chapter 4 to determine the performance gap. Insights are obtained on supplier selection in VMI. Using the proposed algorithm as the control policy, we test its performance with various other policies.

## 5.1 VMI Simulator

The VMI Simulator[1] acts as a simple yet effective decision toolkit to help understand the impact of various parameters, such as holding cost, target inventory level and consolidation time, on the average total cost. The VMI Simulator helps the user to compute $\hat{Q}^*$ and T* based on Cetinkaya and Lee's (2000) model and NPA. In addition, the VMI Simulator also allows the user to perform a "What if" analysis. The description for the simulation model used in the VMI Simulator is described in Chapter 3. For portability, the VMI simulator is coded in Microsoft Visual C++ 6.0 and SQL in Microsoft Access 2000 database, which can be run on a Microsoft Windows 98/2000/XP platform. To obtain convergence, we run the simulation for 20 iterations of 3000 days each (Waller et al., 1999). For this discrete event simulation, the values of T* are rounded up to the nearest hour (Dim [T] =days and we take 1 day=24 hours).

[1] The VMI Simulator is specially built to model the problem described in this study.

## 5.2 Base Case Scenario

For the purpose of the sensitivity analysis, a base case scenario (S1), with no constraints imposed, is used to act as a reference for the other scenarios. We borrow the values in Cetinkaya and Lee (2000) for S1, namely $A_R$=\$125 per replenishment, h=\$7 per unit per day, $A_D$=\$50 per delivery, w=\$10 per unit per day and $\lambda$=10 units per day. The numerical solution is obtained by computing Eqn. (32) with $\hat{Q}*$ and the average cost function C(Q, T) rounded up to the second decimal place.

| Heuristic used | $\hat{Q}$ | T | Simulated Average Cost (\$) | Average Cost (\$) (Eqn. 33) |
|---|---|---|---|---|
| C&L[2] | 18.89 | 0.645 | 239.27 | 281.32 |
| NPA | 18.89 | 0.645 | 238.28 | 281.32 |

Table 1: Results for base case scenario S1

Table 1 shows that Cetinkaya and Lee's (2000) model and the NPA produce identical results in both the numerical computation and simulation of the average cost. This validates the NPA for the base case.

### 5.2.1 Sensitivity Analysis

As Cetinkaya and Lee (2000) and the New Proposed Algorithm are identical when there are no constraints, only the results obtained from Cetinkaya and Lee (2000) model will be shown in this section to illustrate the model sensitivity and response to the various model parameters.

In order to understand the effect of demand, $\lambda$, on VMI system employing the model, a set of scenarios with different demand will be used to illustrate the impact of demand on

[2] C & L: Cetinakaya and Lee's (2000) model.

the average cost incurred by a VMI vendor. The results obtained are tabulated in table 2 (Note that the average cost calculated is based on Eqn. (33)).

| Heuristic used | λ | $\widehat{Q}$ | T | Simulated Average Cost ($) | Average Cost ($) (Eqn. 33) | Approximate Average Cost($) |
|---|---|---|---|---|---|---|
| C & L | 20 | 26.73 | 0.456 | 339.39 | 400.28 | 402.67 |
| C & L | 50 | 42.26 | 0.2887 | 520.21 | 636.32 | 638.71 |
| C & L | 100 | 59.76 | 0.2041 | 707.35 | 902.34 | 904.73 |
| C & L | 200 | 84.52 | 0.1443 | 986.79 | 1278.54 | 1280.93 |

Table 2: Impact of Demand on Average Cost

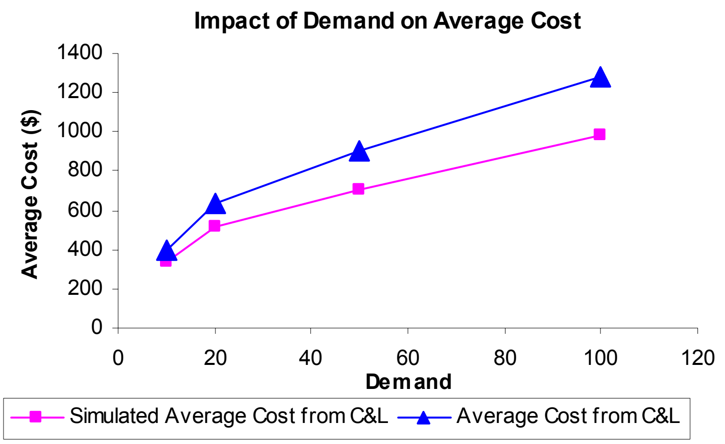


Figure 7: Impact on Demand on Average Cost

Table 4 shows that as the λ increases, the resulting $\widehat{Q}*$ and average cost values increases while the corresponding T value decreases. As shown in figure 7, the impact of demand on average cost is rather constant. With every 100% increase in λ, the average cost value increases approximately 42 % with $\widehat{Q}*$ increases approximately by 41 % and T decreases by approximately 29%.

Another set of scenarios examined the impact of the fixed inventory replenishment cost, $A_R$, have on the average cost incurred. Different $A_R$ values will use in each scenario. The results obtained are tabulated in Table 3 and the trend is shown in Figure 8.

| Heuristic used | $A_R$ | $\hat{Q}$ | T | Simulated Average Cost ($) | Average Cost ($) (Eqn. 33) | Approximate Average Cost($) |
|---|---|---|---|---|---|---|
| C & L | 100 | 16.90 | 0.645 | 213.20 | 267.07 | 269.74 |
| C & L | 200 | 23.90 | 0.645 | 262.07 | 316.86 | 318.75 |
| C & L | 250 | 26.73 | 0.645 | 297.66 | 336.81 | 338.5 |
| C & L | 500 | 37.80 | 0.645 | 360.09 | 414.80 | 416 |

Table 3: Impact of Fixed Replenishment Cost on Average Cost

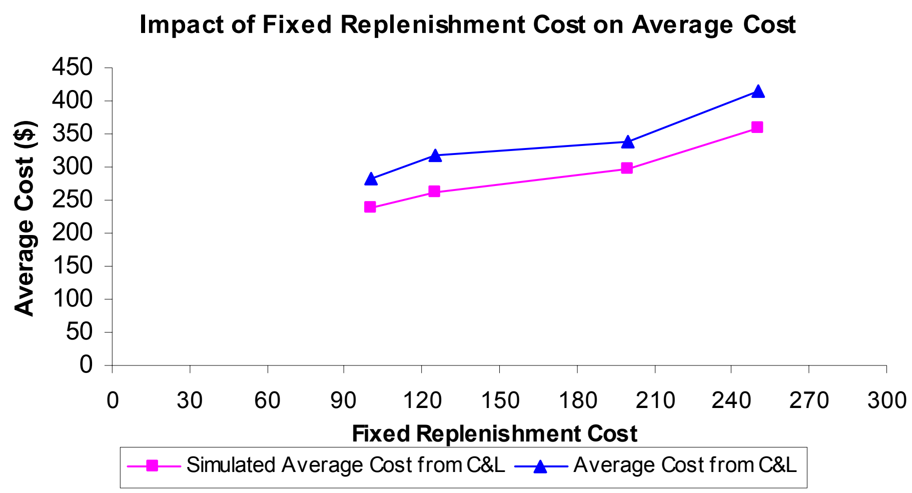


Figure 8: Impact of Fixed Replenishment Cost on Average Cost

Table 3 and Figure 8 shows that as the fixed inventory replenishment cost increases, the resulting $\hat{Q}*$ and average cost values increases while the corresponding T value remains unchanged. This is because the computation of the optimal T does not take into account of $A_R$. The average cost increases approximately by 20% with a 100% increase in $A_R$. It can be seen from figure 8 and table 3 that the rate of change of average cost increases with $A_R$. $\hat{Q}*$ increases approximately by 41% with a corresponding 100% increase in $A_i$.

The next set of scenarios examined the effect of holding cost, $h_i$, on average cost. A set of scenarios with varying h will be used. The scenarios will be using different h values. The results obtained are tabulated in Table 4 and the trend is shown in Figure 9.

| Heuristic used | H | $\hat{Q}$ | T | Simulated Average Cost ($) | Average Cost ($) (Eqn. 33) | Approximate Average Cost($) |
|---|---|---|---|---|---|---|
| C & L | 14 | 13.36 | 0.513 | 284.60 | 369.64 | 375.02 |
| C & L | 28 | 9.449 | 0.3892 | 420.51 | 495.95 | 507.48 |
| C & L | 125 | 4.472 | 0.1961 | 738.82 | 951.61 | 1006.42 |
| C & L | 250 | 3.162 | 0.14 | 986.80 | 1269.02 | 1379.71 |

Table 4: Impact of unit holding cost on Average Cost

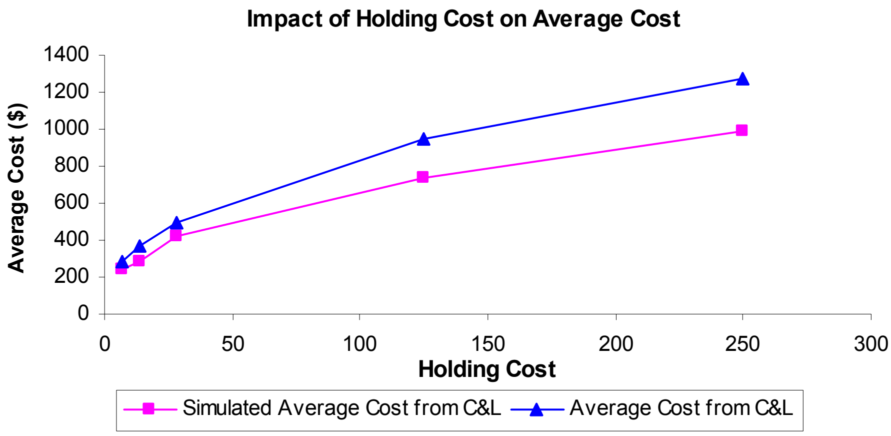


Figure 9: Impact of holding Cost on Average Cost

As shown in Table 4 and Figure 9, as the unit holding cost, *h*, increases, the resulting average cost values increases while the $\hat{Q}*$ and T values decrease. The average cost increases approximately by 33% with a 100% increase in h. On the other hand, the recommended $\hat{Q}*$ and T values decreases approximately by 29% and 24% respectively. It is noted that the rate of decrease for T increases with *h*.

The next set of scenarios examined the effect of waiting cost, w, on average cost. The scenarios will use different waiting cost values.

| Heuristic used | W | $\hat{Q}$ | T | Simulated Average Cost ($) | Average Cost ($) (Eqn. 33) | Approximate Average Cost($) |
|---|---|---|---|---|---|---|
| C & L | 5 | 18.898 | 0.725 | 204.83 | 263.94 | 266.63 |
| C & L | 20 | 18.898 | 0.542 | 277.87 | 311.17 | 313.17 |
| C & L | 125 | 18.898 | 0.268 | 462.36 | 500.62 | 501.61 |
| C & L | 250 | 18.898 | 0.194 | 590.78 | 641.88 | 642.60 |
| C & L | 1250 | 18.898 | 0.088 | 1209.61 | 1252.80 | 1253.13 |

Table 5: Impact of waiting cost on Average Cost

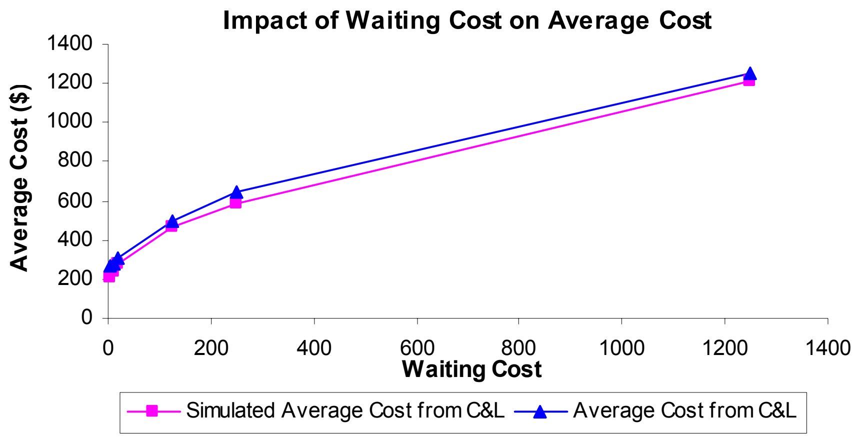


Figure 10: Impact of waiting cost on average cost

As shown in Table 5 and Figure 10, as the waiting cost, *w*, increases, the resulting average cost values increases while T value decreases. The resulting $\hat{Q}*$ remains unaffected by the change in *w*. This is because the computation of the optimal $\hat{Q}*$ does not include the parameter *w*. It is noted that the rate of increase of the resulting average cost value increases with *w* while the rate of decrease of T increases with *w*.

The next set of scenarios examined the effect of fixed outbound transportation cost, $A_D$, on average cost. The scenarios will use different $A_D$ values.

| Heuristic used | $A_D$ | $\hat{Q}$ | T | Simulated Average Cost ($) | Average Cost ($) (Eqn. 33) | Approximate Average Cost($) |
|---|---|---|---|---|---|---|
| C & L | 25 | 18.898 | 0.456 | 207.93 | 236.64 | 238.33 |
| C & L | 75 | 18.898 | 0.791 | 249.03 | 315.59 | 318.52 |
| C & L | 100 | 18.898 | 0.913 | 300.45 | 344.49 | 347.88 |
| C & L | 200 | 18.898 | 1.29 | 340.146 | 433.84 | 438.63 |
| C & L | 1000 | 18.898 | 2.886 | 651.28 | 810.92 | 821.61 |

Table 6: Impact of outbound transportation cost on average cost

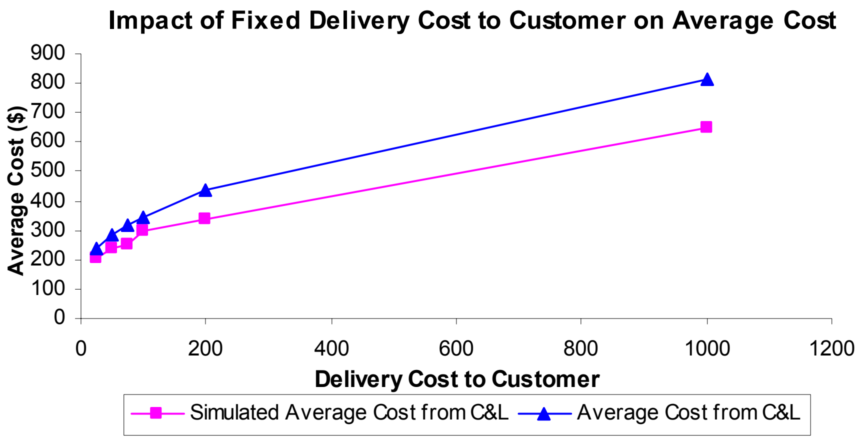


Figure11: Impact on Fixed Delivery Cost to Customer on Average Cost

Table 6 shows that as the outbound transportation cost, $A_D$, increases, the resulting average cost and recommended T values increases. The resulting $\hat{Q}*$ remains unaffected by the change in $A_D$. This is because the computation of the optimal $\hat{Q}*$ does not include the parameter $A_D$. It is noted that the rate of increase of the resulting average cost value increases with $A_D$. T increases approximately by 41% with a 100% increase in $A_D$.

### 5.2.2 Price and Quality

In order to examine the impact of quality on the model as a whole, the unit cost of the product is needed as the cost of defective rate is affected by the unit cost indirectly.

In order to compare the impact of price and quality on average cost, a new base case scenario S2 is set up. The base values for the various model parameters would be similar to base case scenario S1. The base value for unit cost C is set at $10 and the defective rate p is set at 0%. Using these values, the following simulation results were obtained.

| Heuristic used | $\hat{Q}$ | T | Simulated Average Cost ($) |
|---|---|---|---|
| Simulated C&L | 18.89 | 0.645 | 338.08 |

Table 7: Base case with Unit cost=10 (Base Case Scenario S2)

To examine the impact of price on average cost, a set of scenarios with different unit prices are used. The simulated results obtained are as shown in table 8.

| Heuristic used | C | $\hat{Q}$ | T | Simulated Average Cost ($) |
|---|---|---|---|---|
| C&L | 10.1 | 18.89 | 0.645 | 339.07 |
| C&L | 10.2 | 18.89 | 0.645 | 340.04 |
| C&L | 10.5 | 18.89 | 0.645 | 343.19 |
| C&L | 11 | 18.89 | 0.645 | 348.15 |
| C&L | 11.5 | 18.89 | 0.645 | 353.119 |

Table 8: Impact of Price on Average Cost

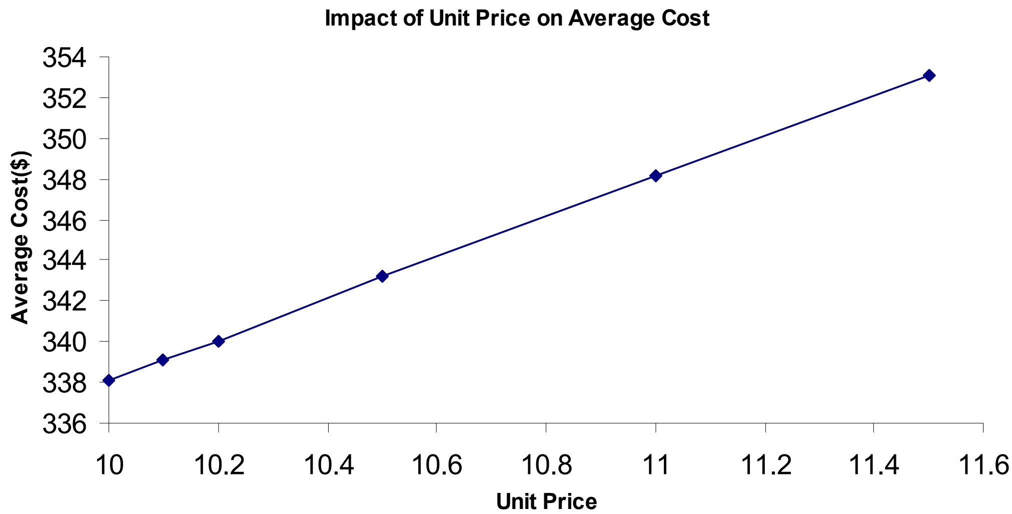


Figure 12 Impact of Unit Price on Average Cost

Similarly, to examine the impact of quality on average cost, a set of scenarios with different defective rates are used. The simulated results obtained are as shown in Table 9.

| Heuristic used | p(%) | $\hat{Q}$ | T | Simulated Average Cost ($) |
|---|---|---|---|---|
| C&L | 1 | 18.89 | 0.645 | 339.17 |
| C&L | 2 | 18.89 | 0.645 | 340.20 |
| C&L | 5 | 18.89 | 0.645 | 343.40 |
| C&L | 10 | 18.89 | 0.645 | 349.27 |
| C&L | 15 | 18.89 | 0.645 | 355.73 |

Table 9: Impact of Defective Rate on Average Cost

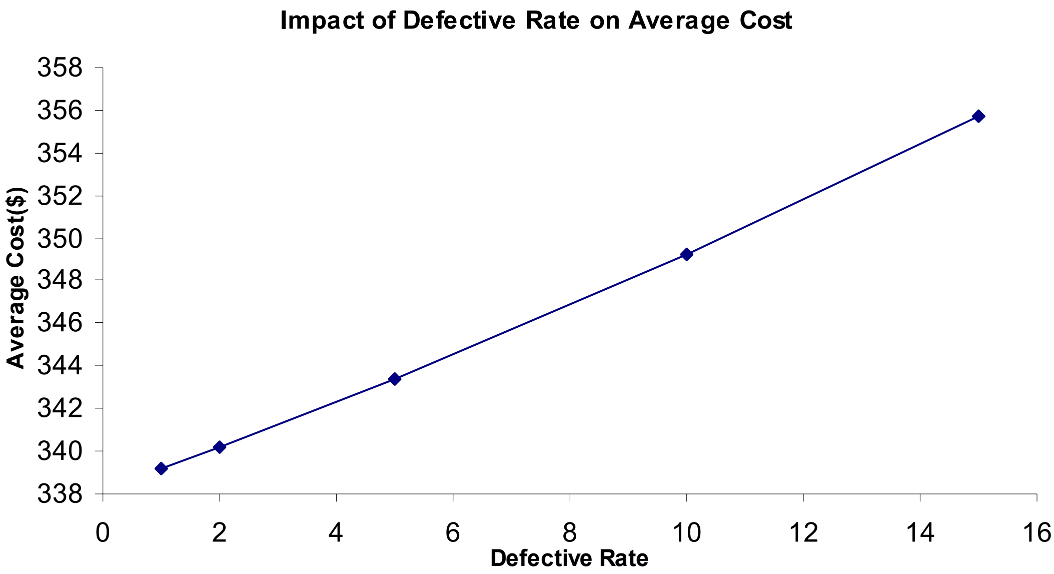


Figure 13: Impact of Defective Rate on Simulated Average Cost

Figures 12 and 13 show that unit price and defective rate have a linear relationship with average cost. Upon deeper analysis on the simulated results in Tables 8 and 9, it can be seen that defective rate have a larger impact on average cost than price.

### 5.3 Comparison of Performance

When various constraints are imposed, the $\hat{Q}*$ and T* computed by the New Proposed Algorithm (NPA) will be different from that obtained from Cetinkaya and Lee (2000) solution. To examine the performance of the new proposed algorithm against Cetinkaya and Lee (2000) solution, we have to include the various constraints considered in this paper in our simulation. A sensitivity analysis would be done to determine whether the proposed algorithm in this paper is better than Cetinkaya and Lee (2000) solution. [We round off Q to the nearest integer]

### 5.3.1 Base Scenario for Comparison (Scenario S2)

We will be borrowing values from the original base scenario in 6.2. For the parameters used in our comparison. For parameters not defined in the original base case scenario, we set them as follows: External Warehousing Cost, g: $10 per day; Warehouse Capacity, ω: 10 units; Lead Time, L: 1 day. The average cost and the simulated values of using the two different polices are shown below. Note that the superior policy is highlighted in bold.

| Heuristic used | $\hat{Q}$ | T | Average Cost ($) | Simulated Average Cost ($) |
|---|---|---|---|---|
| C&L | 18 | 0.645 | 342.87 | 265.73 |
| **NPA** | **14** | **0.6349** | **339.27** | **257.38** |

Table 10: Comparison of Performance in S2

As we can observe from Table 10, the NPA outperforms the Cetinkaya and Lee (2000) model. However, we are unable to conclude that the NPA is better than Cetinkaya and Lee (2000) solution based only this result only. More test need to be done to affirm this hypothesis that the performance of NPA in our paper. For this purpose, we will be performing sensitivity analysis on the solutions provided by the NPA and the Cetinkaya and Lee (2000) model to verify whether NPA will outperform Cetinkaya and Lee (2000) solution in all situations

#### 5.3.1.1 Sensitivity Analysis/Performance Comparison of the 2 models

To prove the superiority of the NPA, we conducted a series of sensitivity analysis to determine the performance gap. By varying the various parameters, we simulated the performance of the system. For simplicity, we tabulate the results in Appendix A. From Appendix A, it can be seen that the New Proposed Algorithm generally outperforms Cetinkaya and Lee (2000). It can also be observed that the New Proposed Algorithm relative performance to Cetinkaya and Lee's (2000) solution is better when warehouse capacity is low or/and external warehouse storage rate is high. This is because the New Proposed Algorithm is designed to obtain a better solution when the warehousing constraint problem is serious (i.e. when the vendor warehouse is small and alternative storage rates are high). In cases where the penalty cost to holding cost ratio is low (g/h) and high warehouse capacity, the solution found using NPA is found to be as good as Cetinkaya and Lee (2000) solution. Thus, from the results of the sensitivity analysis and simulation study, we can infer that our proposed algorithm in our paper is a more comprehensive and better solution than Cetinkaya and Lee (2000).

## 5.4 Comparison of VMI and JIT policies

After obtaining a good policy for our VMI problem, we shall now move on to compare the performance of JIT and VMI inventory systems. As we have already proven that our Proposed Algorithm, the NPA, is a good solution for VMI inventory system under the circumstances described in our problem, we will then use the NPA to derive the policies parameters for our VMI system in this comparison.

### 5.4.1 Base Case Scenario S3

We will be borrowing values from the original base scenario S2 for the parameters used in our comparison. The only parameter not defined in that scenario is the additional cost charged for implementing JIT, which is set at \$5 per unit. The average cost and the simulated values of using the two different polices are shown below. Note that the superior policy is highlighted in bold.

| Policies used | Simulated Average Cost ($) |
|---|---|
| **JIT** | **68** |
| VMI | 257.74 |

Table 11: Comparison of Performance in S3

As we can observe from Table 11, the simulated cost from using JIT inventory systems is much lower than that of the VMI system in the base scenario. This result is not surprising as ordering cost were virtually eliminated in the ideal JIT inventory system. Together with the low holding cost typical in a JIT system, average cost is kept to a minimal. However, we are unable to conclude that JIT systems are better than VMI systems just by one single result alone. More test need to be done for us to reach a conclusion on the performance of JIT systems and VMI systems. For this purpose, we will be performing sensitivity analysis on the simulated cost obtained from both VMI and JIT inventory systems.

#### 5.4.1.1 Sensitivity Analysis/Performance Comparison of the 2 polices

To compare the performance between the two policies, we conducted a series of sensitivity analysis to determine the performance gap. The first of the parameter to be tested is $A_R$.

| Policies used | $A_R$ | Simulated Average Cost ($) |
|---|---|---|
| **JIT** | **125** | **68** |
| VMI | 125 | 257.54 |
| **JIT** | 100 | **68** |
| VMI | 100 | 243.2 |
| **JIT** | 75 | **68** |
| VMI | 75 | 227 |
| **JIT** | 50 | **68** |
| VMI | 50 | 210.14 |
| **JIT** | 25 | **68** |
| VMI | 25 | 191.77 |
| **JIT** | 10 | **68** |
| VMI | 10 | 167.2 |

Table 12: Impact of Inventory Replenishment Cost on JIT/VMI performance

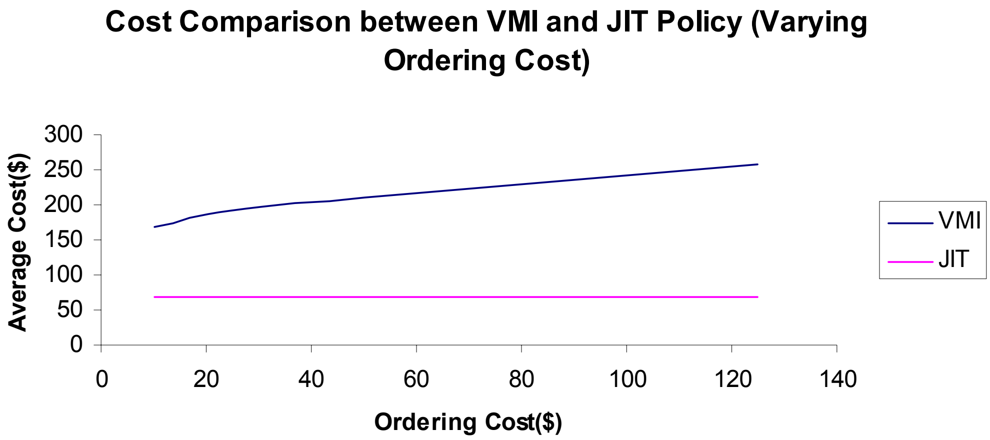


Figure 14: Cost Comparison between VMI and JIT Policy (Vary $A_R$)

From Table 12 and Figure 14, it can be seen that JIT outperforms VMI inventory system in all scenarios. To complete the sensitivity analysis, we perform numerous simulations by varying one parameter at a time while keeping others at the base rates.

| Policies used | $A_D$ | Simulated Average Cost ($) |
|---|---|---|
| **JIT** | **50** | **68** |
| VMI | 50 | 210.14 |
| **JIT** | 40 | **68** |
| VMI | 40 | 193.6 |
| **JIT** | 30 | **68** |
| VMI | 30 | 177.21 |
| **JIT** | 20 | **68** |
| VMI | 20 | 164.77 |
| **JIT** | 10 | **68** |
| VMI | 10 | 138.29 |

Table 13: Impact of Fixed Dispatch Cost on JIT/VMI performance

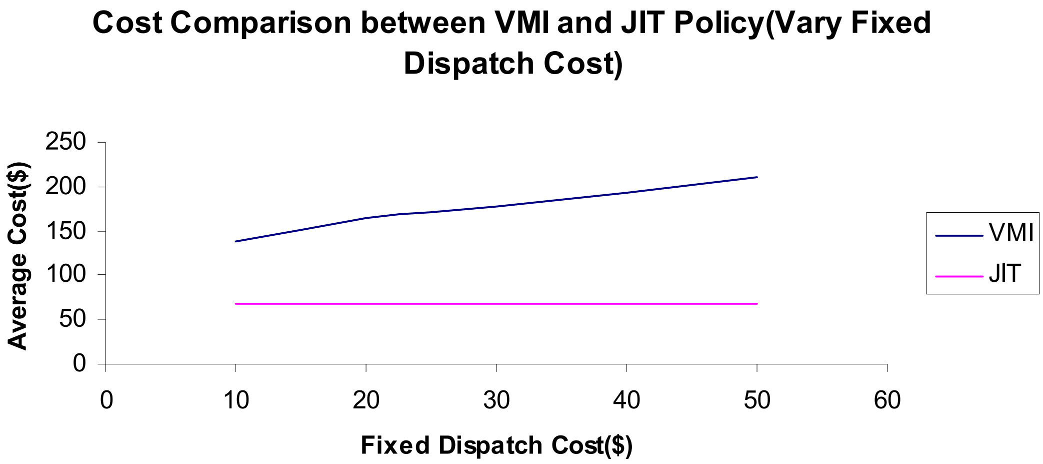


Figure 15: Cost Comparison between VMI and JIT Policy (Vary $A_D$)

| Policies used | JIT Penalty | Simulated Average Cost ($) |
|---|---|---|
| **JIT** | **5** | **68** |
| VMI | 5 | 210.14 |
| **JIT** | **10** | **114.29** |
| VMI | 10 | 210.14 |
| JIT | 20 | 207.75 |
| VMI | 20 | 210.14 |
| JIT | 50 | 478.89 |
| **VMI** | **50** | **210.14** |
| JIT | 100 | 949.22 |
| **VMI** | **100** | **210.14** |

Table 14: Impact of JIT Penalty Cost on JIT/VMI performance

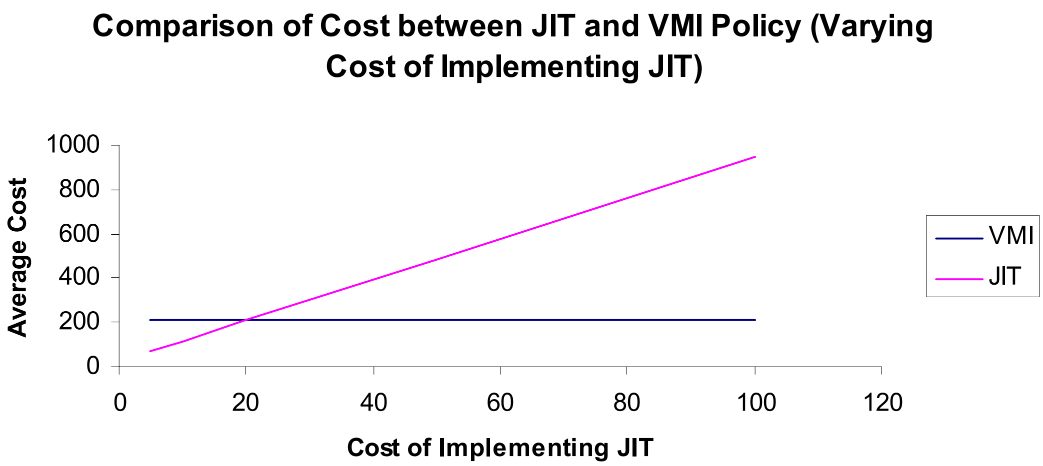


Figure 16: Cost Comparison between VMI and JIT Policy (Vary JIT Penalty)

| Policies used | λ | Simulated Average Cost ($) |
|---|---|---|
| **JIT** | **10** | **68** |
| VMI | 10 | 210.14 |
| **JIT** | **20** | **125.31** |
| VMI | 20 | 311.98 |
| **JIT** | **50** | **292.59** |
| VMI | 50 | 495.54 |
| **JIT** | **100** | **563.25** |
| VMI | 100 | 698.4 |
| JIT | 200 | 1100.61 |
| **VMI** | **200** | **978.81** |
| JIT | 500 | 2694.25 |
| **VMI** | **500** | **1577.42** |
| JIT | 1000 | 5325.53 |
| **VMI** | **1000** | **2396.38** |

Table 15: Impact of demand on JIT/VMI performance

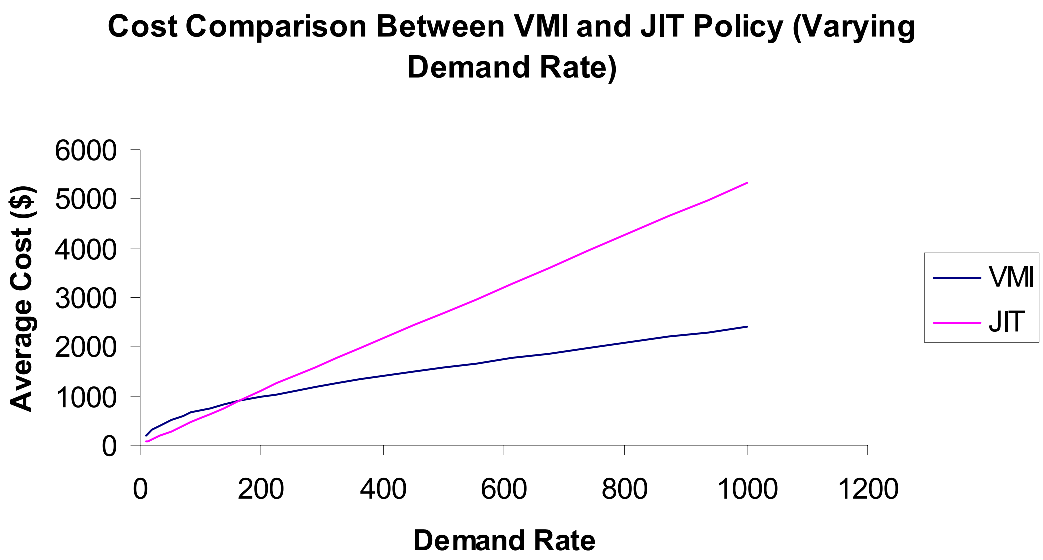


Figure 17: Cost Comparison between VMI and JIT Policy (Vary Demand)

| Policies used | w | Simulated Average Cost ($) |
|---|---|---|
| **JIT** | **1** | **64.27** |
| VMI | 1 | 175.29 |
| **JIT** | **5** | **66.24** |
| VMI | 5 | 191.22 |
| **JIT** | **10** | **68** |
| VMI | 10 | 210.14 |
| **JIT** | **20** | **73.14** |
| VMI | 20 | 237.48 |
| **JIT** | **50** | **90.18** |
| VMI | 50 | 316.83 |
| **JIT** | **100** | **111.3** |
| VMI | 100 | 387.48 |

Table 16: Impact of Waiting Cost on JIT/VMI performance

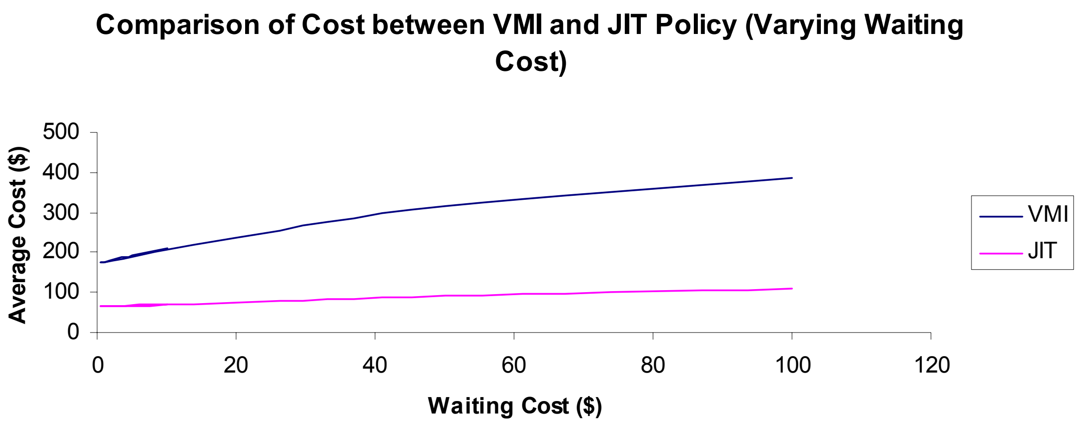


Figure 18: Cost Comparison between VMI and JIT Policy (Vary Waiting Cost)

| Policies used | L | Simulated Average Cost ($) |
|---|---|---|
| **JIT** | **1** | **68** |
| VMI | 1 | 210.14 |
| **JIT** | **2** | **77.39** |
| VMI | 2 | 212.46 |
| **JIT** | **5** | **101.82** |
| VMI | 5 | 243.47 |
| **JIT** | **10** | **135.43** |
| VMI | 10 | 274.18 |
| **JIT** | **50** | **489.13** |
| VMI | 50 | 635.51 |
| **JIT** | **100** | **1208.16** |
| VMI | 100 | 1352.86 |

Table 17: Impact of Lead Time on JIT/VMI performance

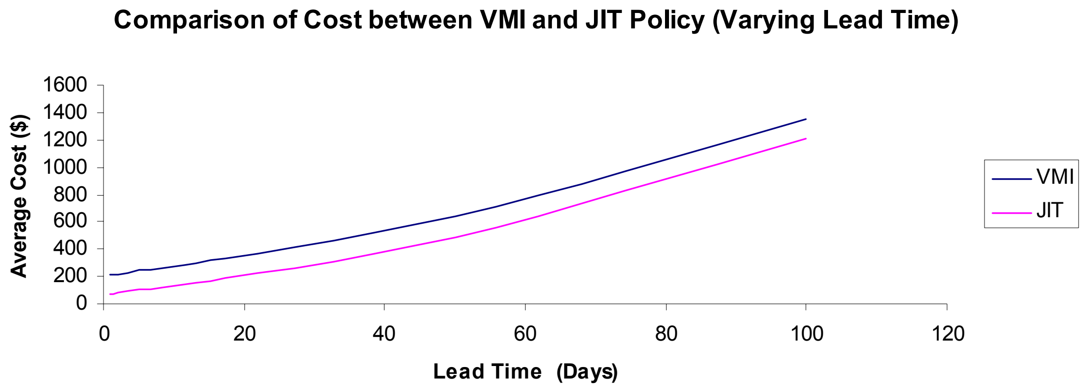


Figure 19: Cost Comparison between VMI and JIT Policy (Vary Lead Time)

| Policies used | h | Simulated Average Cost ($) |
|---|---|---|
| **JIT** | **3** | **58.52** |
| VMI | 3 | 174.34 |
| **JIT** | **7** | **68** |
| VMI | 7 | 210.14 |
| **JIT** | **10** | **76.29** |
| VMI | 10 | 232.86 |
| **JIT** | **30** | **130.07** |
| VMI | 30 | 379.61 |
| **JIT** | **50** | **177.04** |
| VMI | 50 | 473.65 |
| **JIT** | **100** | **305.59** |
| VMI | 100 | 689.39 |
| **JIT** | **200** | **536.66** |
| VMI | 200 | 1020.97 |

Table:18: Impact of holding cost on JIT/VMI performance

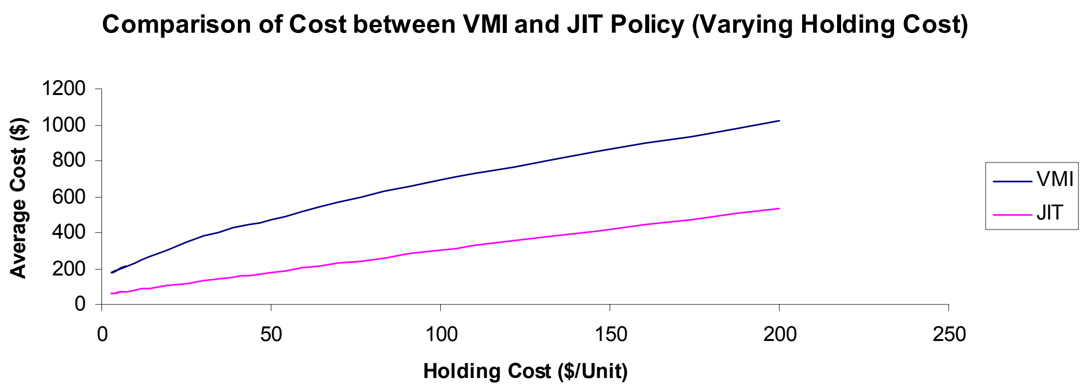


Figure 20: Cost Comparison between VMI and JIT Policy (Vary Holding Cost)

| Policies used | g | Simulated Average Cost ($) |
|---|---|---|
| **JIT** | **3** | **68** |
| VMI | 3 | 210.14 |
| **JIT** | **7** | **68.95** |
| VMI | 7 | 217.23 |
| **JIT** | **10** | **68.8** |
| VMI | 10 | 233.59 |
| **JIT** | **20** | **68.65** |
| VMI | 20 | 246.05 |
| **JIT** | **50** | **68.62** |
| VMI | 50 | 298.86 |
| **JIT** | **100** | **68.26** |
| VMI | 100 | 334.7 |

Table 19: Impact of External Warehouse Penalty on JIT/VMI performance

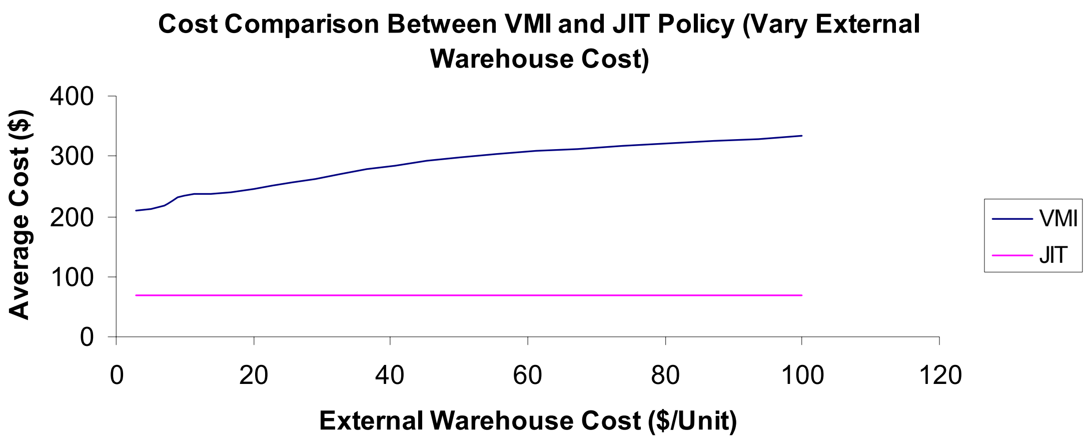


Figure 21: Cost Comparison between VMI and JIT Policy (Vary External Warehouse Penalty)

Tables 13 to 19 and Figure 15 to 21 have illustrated the sensitivity of the 2 different policies relative to changes in various model parameters. It can be seen that JIT inventory systems generally outperforms VMI inventory systems in most of the scenarios considered. However, from Table 14 and Figure 16, we observe that VMI outperform JIT in scenarios where JIT implementation cost. This result is concur with the proposal in Schniederjans (1999) that JIT should only be implemented if the cost of implementing JIT is smaller than the savings of switching from other inventory policy to JIT. We also observe from Table 15 and Figure 17 that VMI inventory system outperforms JIT inventory system when $\lambda$ is large. Unlike the JIT implementation cost, the inferiority of

JIT inventory system at high λ cannot be explained by Schniederjans (1999) alone as λ also represent the variance of the demand distribution (Property of Poisson Distribution). Thus, the inferiority of JIT inventory system could also be due to the variance of the demand distribution. To fully understand the reasons behind JIT inferiority to VMI, we have to do a more detailed study on the effect on variance on JIT performance.

#### 5.4.1.2 Sensitivity Analysis on Variance

To determine the source of the poor performance under JIT inventory systems at high λ, we conduct a sensitivity analysis on the standard deviation of the demand distribution. We set our λ to be 1000 in this case. However, as we are doing a sensitivity analysis on the standard deviation of the demand distribution, we assume the demand random variable follows the normal distribution with mean λ and its standard deviation be defined in the various scenarios used in the sensitivity analysis. We tested the two inventory systems in various scenarios with different standard deviation and tabulate the results in Table 20 and Figure 22.

| Policies used | Demand Deviation | Simulated Average Cost ($) |
|---|---|---|
| **JIT** | **3** | **1131.67** |
| VMI | 3 | 1908.08 |
| **JIT** | **10** | **1133.5** |
| VMI | 10 | 1900.15 |
| **JIT** | **20** | **1125.63** |
| VMI | 20 | 1813.85 |
| **JIT** | **50** | **1312.33** |
| VMI | 50 | 1818.19 |
| JIT | 100 | 1813.71 |
| **VMI** | **100** | **1780.32** |
| JIT | 200 | 2836.74 |
| **VMI** | **200** | **2288.12** |
| JIT | 500 | 5831.51 |
| **VMI** | **500** | **5244.78** |
| JIT | 1000 | 10961.84 |
| **VMI** | **1000** | **10088.29** |

Table 20: Impact of Standard Deviation of Demand on JIT/VMI performance

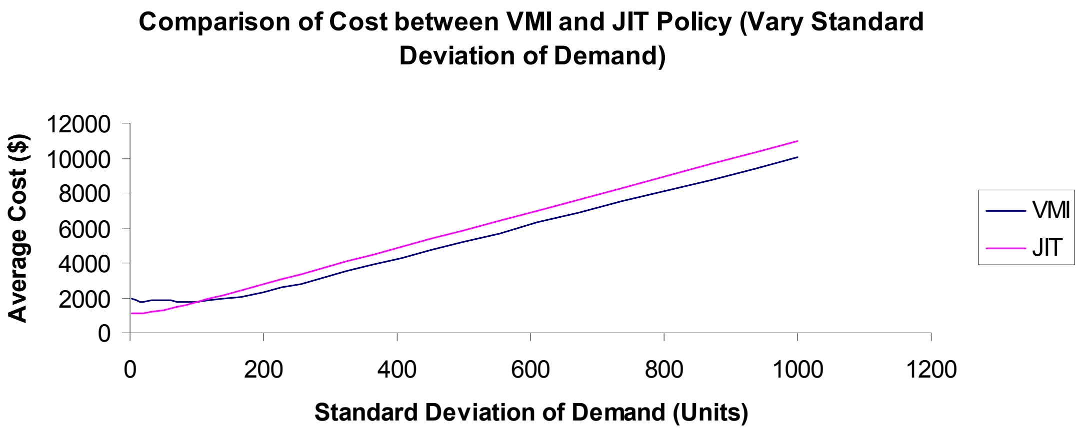


Figure 22: Cost Comparison between VMI and JIT Policy (Vary Standard Deviation of Demand)

From Table 20 and Figure 22, we observe that at low level of standard deviation, JIT inventory system still performs better than VMI inventory system. However, at high level of standard deviation, we can clearly see that VMI inventory system outperforms JIT inventory system. This seems to imply that the source of the poor perform of JIT at high $\lambda$ comes from the variance of the demand distribution itself. The mean plays a relatively minor part in this finding. To confirm this observation, we let the deterministic parameter, Lead Time, to be a random variable that follows the normal distribution curve. We let the mean of the lead time distribution to be set at 14 days and vary its standard deviation in our sensitivity analysis. The results obtained from the sensitivity analysis are shown in Table 21 and Figure 23.

| Policies used | Lead Time Deviation | Simulated Average Cost ($) |
|---|---|---|
| JIT | 1 | 2968.51 |
| VMI | 1 | 3700.26 |
| JIT | 2 | 2777.69 |
| VMI | 2 | 3468.80 |
| JIT | 5 | 2316.85 |
| VMI | 5 | 2981.69 |
| JIT | 8 | 2374.86 |
| VMI | 8 | 2892.30 |
| JIT | 10 | 2646.84 |
| VMI | 10 | 2964.87 |
| JIT | 12 | 2871.79 |
| VMI | 12 | 3078.05 |
| JIT | 20 | 4770.93 |
| VMI | 20 | 4494.89 |
| JIT | 25 | 6150.81 |
| VMI | 25 | 5642.37 |
| JIT | 50 | 7374.24 |
| VMI | 50 | 6864.17 |

Table 21: Impact of Standard Deviation of Lead Time on JIT/VMI performance

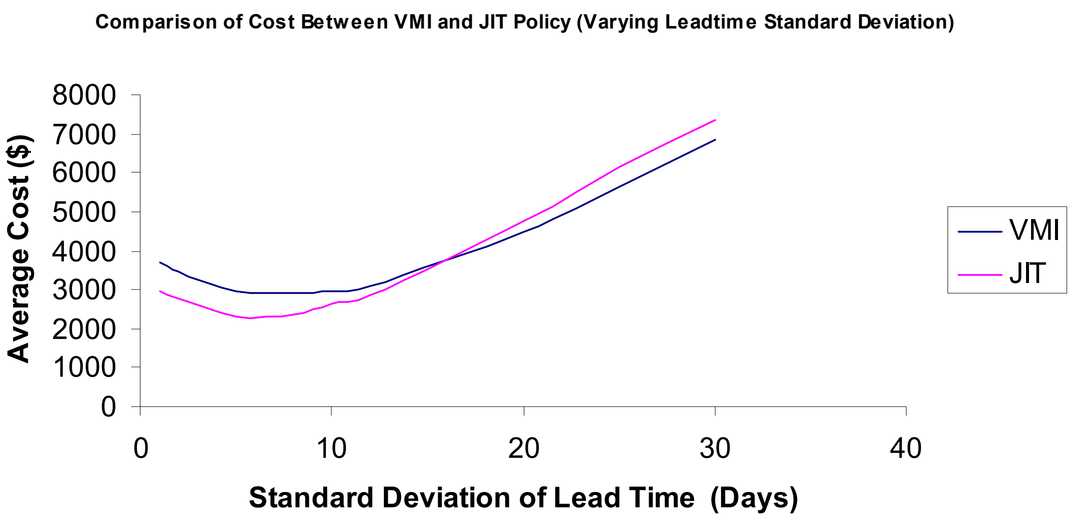


Figure 23: Cost Comparison between VMI and JIT Policy (Vary Standard Deviation of Lead Time)

From Figure 23 and Table 21, we can clearly see that VMI is the preferred inventory system when the standard deviation of the Lead Time distribution is high. Like the sensitivity analysis done for the standard deviation of demand, it is found that JIT is the better inventory system under low standard deviation and VMI being the preferred inventory system under high standard deviation. However, from the figures above, we are only able to understand the behaviour of the model with respect to uncertainty in one parameter. To get a more detailed understanding of JIT and VMI system reacts towards uncertainty, we do a sensitivity analysis across uncertainties in lead time and demand.

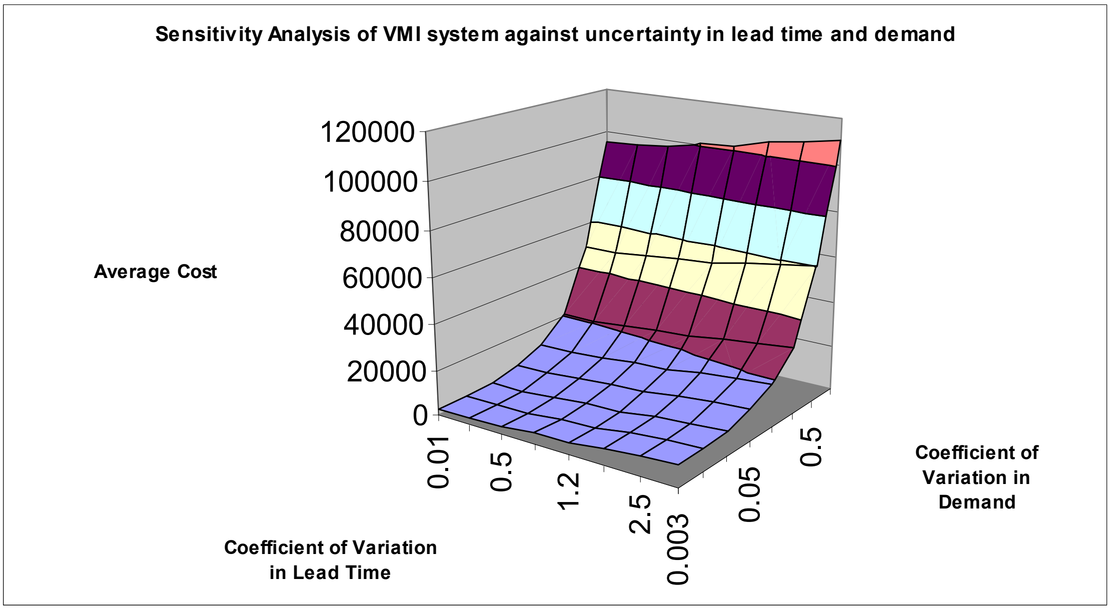


Figure 24: Sensitivity Analysis of VMI System on Uncertainty in demand and lead time

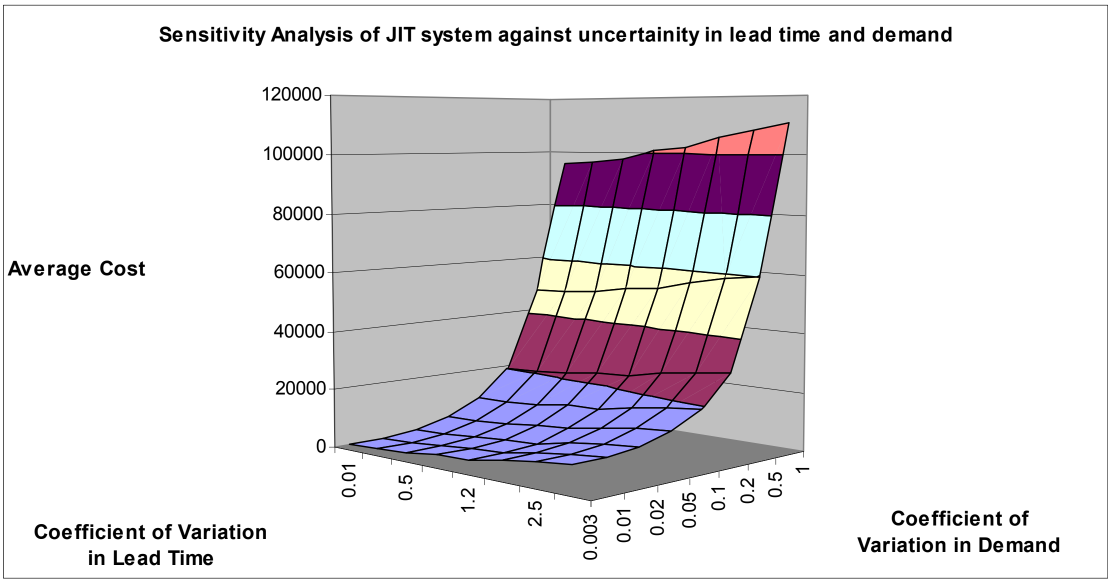


Figure 25: Sensitivity Analysis of JIT System on Uncertainty in demand and lead time

| COV of Leadtime<br><br>COV of Demand | 0.1 | 0.2 | 0.5 | 1 | 1.2 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|---|
| **0.003** | JIT | JIT | JIT | JIT | JIT | JIT | JIT | VMI |
| **0.01** | JIT | JIT | JIT | JIT | JIT | VMI | VMI | VMI |
| **0.02** | JIT | JIT | JIT | JIT | VMI | VMI | VMI | VMI |
| **0.05** | JIT | JIT | JIT | JIT | VMI | VMI | VMI | VMI |
| **0.1** | JIT | JIT | JIT | JIT | VMI | VMI | VMI | VMI |
| **0.2** | JIT | JIT | JIT | JIT | VMI | VMI | VMI | VMI |
| **0.5** | JIT | JIT | JIT | JIT | VMI | VMI | VMI | VMI |
| **1** | VMI | VMI | VMI | VMI | VMI | VMI | VMI | VMI |

Table 22: Optimal strategy for different scenarios

Where COV= Coefficient of Variation= $\dfrac{Standard\ Deviation}{Mean}$

Figure 24 and 25 depicts the sensitiveness of the average cost of VMI and JIT inventory systems against the uncertainty in demand and lead time. As we can see from the figures, the average cost is more sensitive towards uncertainty of demand than the uncertainty of the lead time. In Table 22, the optimal strategy is displayed for the various combinations. As we can see, VMI is the optimal strategy for high uncertainty in lead time and demand. To confirm the results, we vary the mean for the lead time and demand to determine the impact of uncertainty on the average cost incurred on both systems.

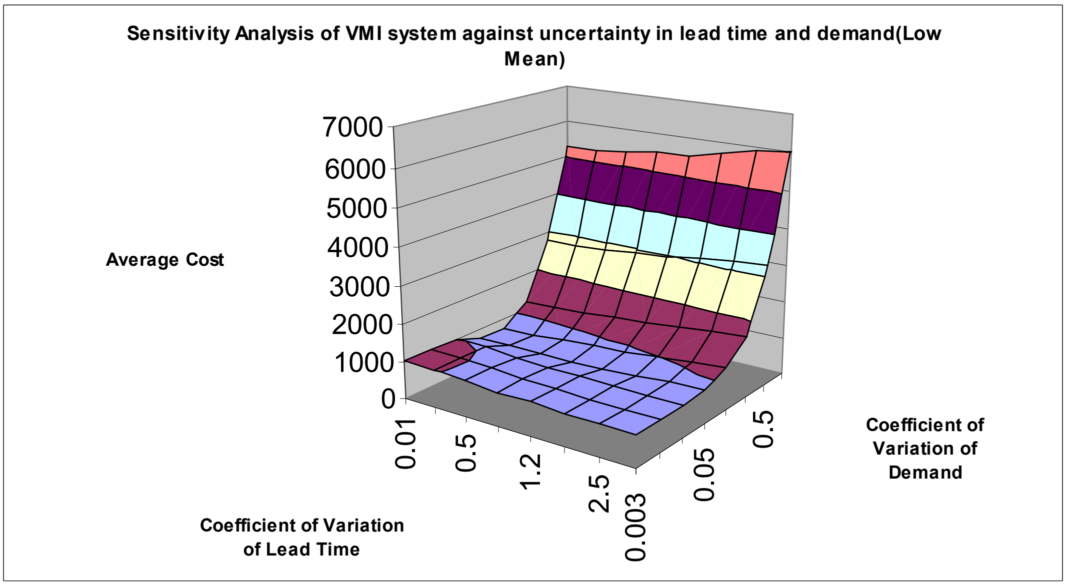


Figure 26: Sensitivity Analysis of VMI System on Uncertainty in demand and lead time (low mean)

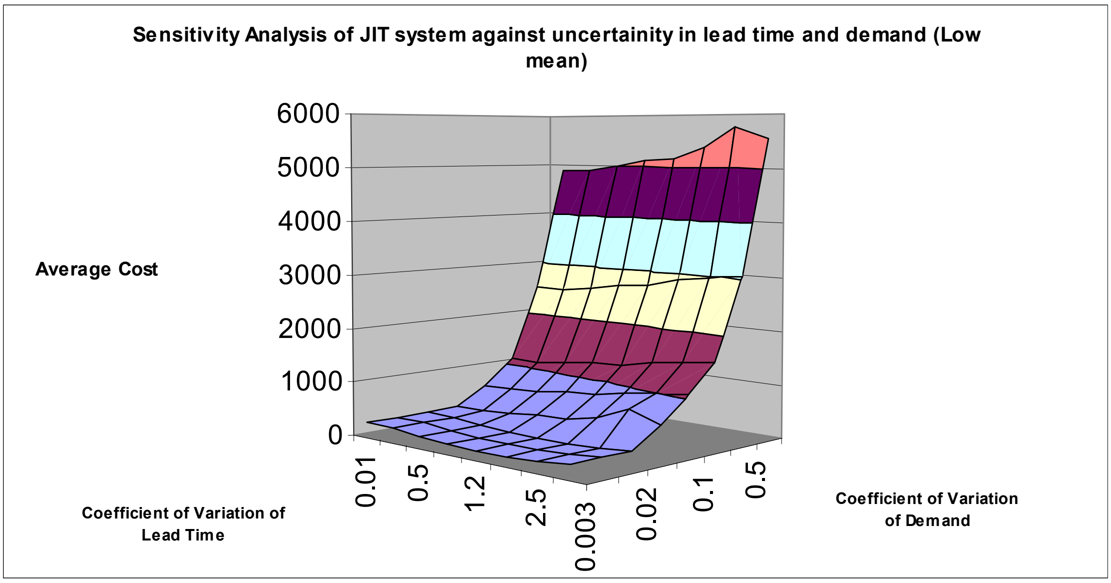


Figure 27: Sensitivity Analysis of JIT System on Uncertainty in demand and lead time (low mean)

| **COV of Leadtime**<br><br>**COV of Demand** | **0.1** | **0.2** | **0.5** | **1** | **1.2** | **2** | **2.5** | **3** |
|---|---|---|---|---|---|---|---|---|
| **0.003** | JIT | JIT | JIT | JIT | JIT | JIT | JIT | JIT |
| **0.01** | JIT | JIT | JIT | JIT | JIT | JIT | JIT | JIT |
| **0.02** | JIT | JIT | JIT | JIT | JIT | JIT | JIT | JIT |
| **0.05** | JIT | JIT | JIT | JIT | JIT | JIT | JIT | JIT |
| **0.1** | JIT | JIT | JIT | JIT | JIT | JIT | JIT | JIT |
| **0.2** | JIT | JIT | JIT | JIT | JIT | JIT | JIT | JIT |
| **0.5** | JIT | JIT | JIT | JIT | JIT | JIT | JIT | JIT |
| **1** | JIT | JIT | JIT | JIT | JIT | JIT | JIT | JIT |

Table 23: Optimal Strategy for different scenarios (low mean)

Interestingly, we find that the sensitivities of the average cost towards the uncertainty in demand and lead time rather similar in general. The average cost is still relatively more sensitive towards uncertainty in demand than uncertainty in lead time. JIT is also found to be more sensitive towards uncertainty compared to VMI. However, we find that JIT manage to outperform VMI in all scenarios with low mean in demand and lead time. From this analysis we can infer a few conclusions.

1) Supply Chains in general are more sensitive to fluctuations in demand than supply.

2) The impact of uncertainty is large on JIT inventory management systems than VMI inventory management systems.

At high level of uncertainty, VMI will be preferred if the impact of uncertainty is higher than the savings obtained from JIT Implementation

## 5.5 Order Splitting Feasibility

Order splitting is one of the new propositions that could help obtain substantial inventory savings. The effects of order splitting will be examined here due to its potential to harness substantial inventory savings in a VMI arrangement. The effects of order splitting policy would be examined by reducing the recommended stock up to level by half. Let the ratio

$r = \frac{A_D}{A_R}$ and $v = \frac{A_R}{h}$.

The basic sub model M1 would be used to illustrate to impact of order splitting. By using a set of scenarios different fixed vendor delivery rates, the following values are computed.

| Heuristic used(Policy) | $A_D$ ($) | $\hat{Q}$ | T | Simulated Average Cost ($) |
|---|---|---|---|---|
| C&L (No Order Splitting) | 35 | 21.38 | 0.645 | 261.99 |
| **C&L (Order Splitting)** | **35** | **11.19** | **0.645** | **219.29** |
| C&L (No Order Splitting) | 50 | 22.36 | 0.645 | 257.84 |
| **C&L (Order Splitting)** | **50** | **11.68** | **0.645** | **233.30** |
| C&L (No Order Splitting) | 75 | 23.90 | 0.645 | 261.90 |
| **C&L (Order Splitting)** | **75** | **12.45** | **0.645** | **256.36** |
| C&L (No Order Splitting) | 100 | 25.35 | 0.645 | 279.59 |
| **C&L (Order Splitting)** | **100** | **13.17** | **0.645** | **278.56** |
| C&L (No Order Splitting) | 125 | 26.73 | 0.645 | 297.74 |
| **C&L (Order Splitting)** | **125** | **13.80** | **0.645** | **297.06** |
| C&L (No Order Splitting) | 150 | 28.03 | 0.645 | 309.21 |
| **C&L (Order Splitting)** | **150** | **14.51** | **0.645** | **302.58** |

Table 24: Impact of Ratio r on Average Cost

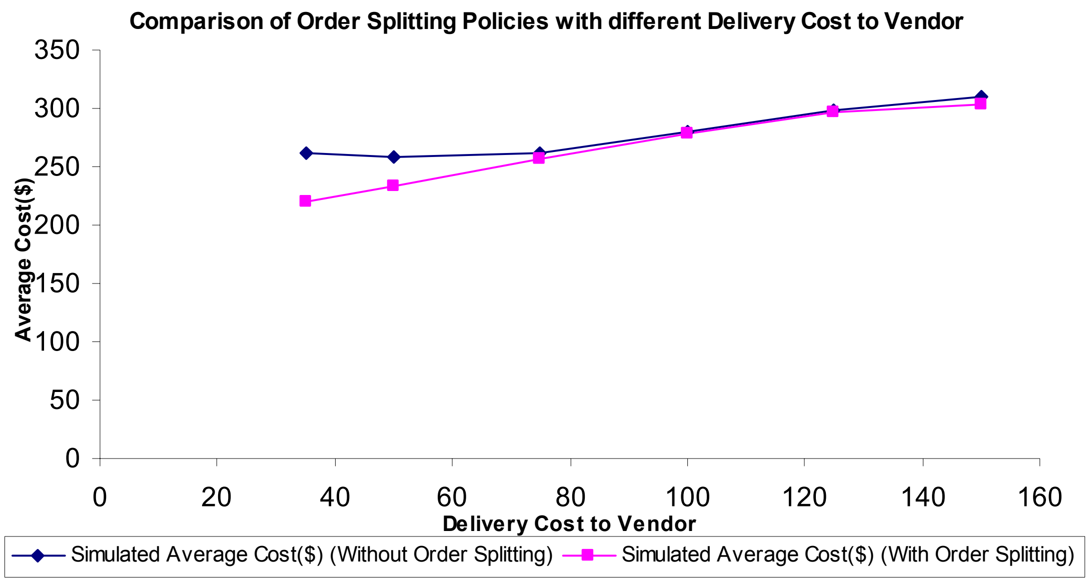


Figure 28: Comparison of Order Splitting policies with different Delivery cost to Vendor

By using another set of scenarios with different holding cost (Note that the base value for delivery cost, $A_D$, is \$75), the following values are computed

| Heuristic used(Policy) | H(\$/unit) | $\hat{Q}$ | T | Simulated Average Cost (\$) |
|---|---|---|---|---|
| C&L (No Order Splitting) | 14 | 16.90 | 0.513 | 365.49 |
| **C&L (Order Splitting)** | **14** | **8.95** | **0.513** | **301.28** |
| C&L (No Order Splitting) | 25 | 12.65 | 0.408 | 452.24 |
| **C&L (Order Splitting)** | **25** | **6.82** | **0.408** | **368.77** |
| C&L (No Order Splitting) | 50 | 8.94 | 0.301 | 567.12 |
| **C&L (Order Splitting)** | **50** | **4.97** | **0.301** | **484.27** |
| C&L (No Order Splitting) | 100 | 6.32 | 0.218 | 863.00 |
| **C&L (Order Splitting)** | **100** | **3.66** | **0.218** | **787.58** |
| C&L (No Order Splitting) | 125 | 5.66 | 0.196 | 954.22 |
| **C&L (Order Splitting)** | **125** | **3.32** | **0.196** | **866.90** |

Table 25: Comparison of Order Splitting policies with different holding cost

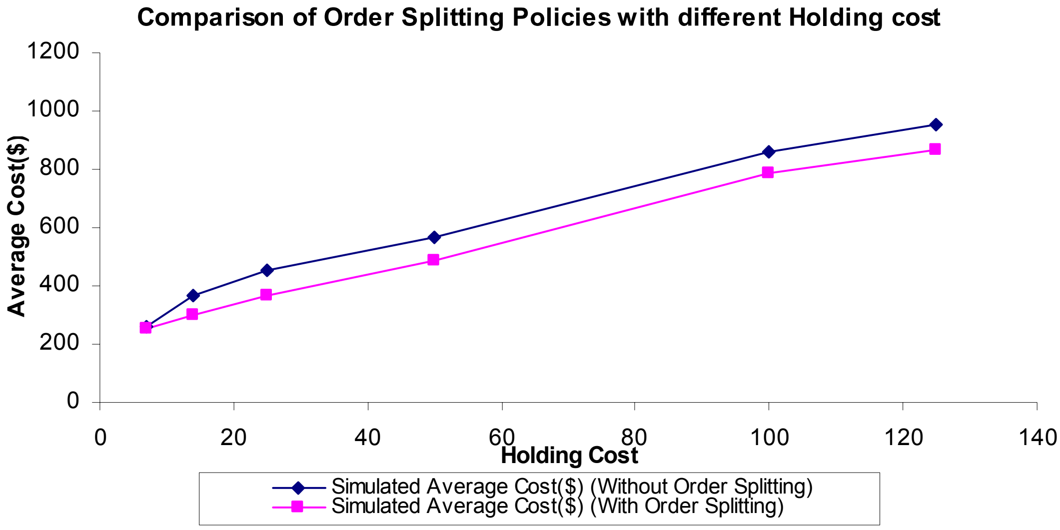


Figure 29: Comparison of Order Splitting policies with different holding cost

From the above tables and figures, it can be seen that scenarios that considers order splitting generally experience a lower cost than scenarios that only have a single delivery per order. The cost savings that were obtained from order splitting policy range from 2% to 18.45%. However, it can be observed from Figure 28 and 29 that as the ratio r and v increases, order splitting tends to be less beneficial. This is because if the fixed vendor delivery cost is relatively larger than the order setup cost, the cost savings that results from a lower inventory may be nullified by the increases in delivery cost to the vendors. On the other hand, if the ratio is low, order splitting becomes more attractive as the increase in delivery cost will be lower than the cost savings derived from holding a lower inventory. It is also observed that as the holding cost increases, order splitting becomes more desirable. This is because as holding cost increases, savings derived from inventory savings would increase and thus enhance the benefits of an order splitting policy. The findings are in coherence with Chiang and Chiang (1996) and Chiang (2001) where order splitting policy is found to be most attractive in scenarios where the ratio of setup cost to holding cost, v, is low or/and order dispatching cost is not low. However, in addition to

Chiang (2001) conclusion that the dispatching cost of an order must not be small in order to let order splitting lower cost, it is found that the ratio r must be low too so that savings from order splitting can be reaped.

### 5.6 Evaluation of Inventory policy used in the Industry

Our last objective of this paper is to look into policies currently adopted by the industry. During our data collection phase in the vendor hub, we found that VMI hub operators are now implementing a Uniform Minimum inventory policy across all suppliers, regardless of whether the supplier is a local or foreign supplier. This policy puzzle us we know that a Uniform Minimum inventory policy will definitely incur a higher system cost that setting a different Minimum Maximum inventory level for each of the suppliers. To understand the rationale of this policy, we will simulate the inventory systems under different policy in a VMI Production Hub environment (Please refer to the Chapter 3 for a detailed description on the VMI Production Hub environment used in this paper). We assume a Vendor Hub is currently having a local supplier and a foreign supplier from the different components currently used by their customer. The local supplier is assumed to have a lead time of 1 day and the foreign supplier is assumed to have a lead time of 14 days. For the vendor hub to satisfy the customer, it must assemble the kits, which consist of one component each from the local and foreign supplier, before it can send to its customer.

From the results obtained from the simulation, we hope to be able to insights on the rational behind of this policy. In the process, we will also attempt to find a better inventory policy that will meet industry rationale of using the Uniform Minimum inventory policy.

### 5.6.1 Comparison of Performance between Uniform and Non Uniform Minimum Policy

To compare the performance between the two policies, we conducted a series of sensitivity analysis to determine the performance gap. The first parameter to be tested is the inventory replenishment cost, $A_R$. The results are shown in Figure 37

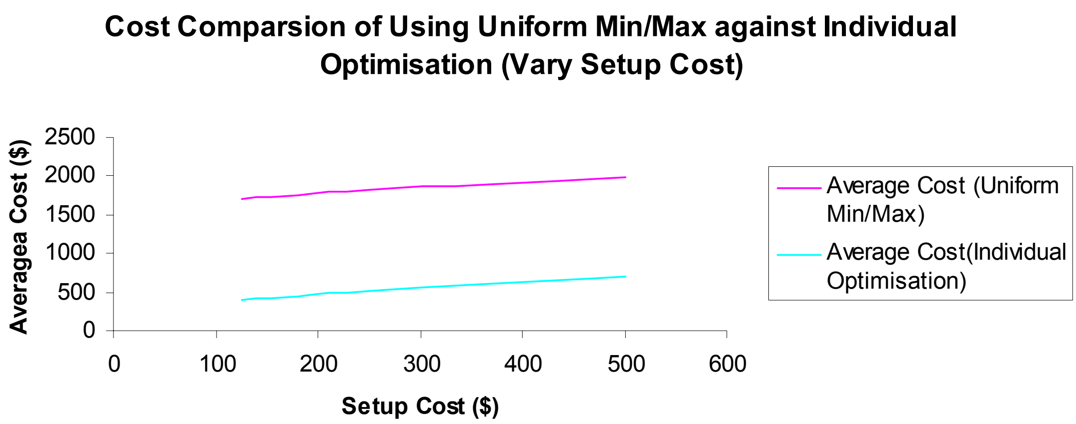


Figure 30: Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary $A_R$)

As seen in Figure 30, we can see that the average cost of the Uniform Minimum inventory system is much higher than that of the using the NPA. Analysing the policies based on system cost, there seems to be no reason for Vendor Hub operators to implement a Uniform Minimum inventory policy. However, since this policy is quite popular with vendor hub operators, there must be rationale behind this. A deeper analysis on the simulated results has let us discover an interesting phenomenon in the customer proportion of the average cost. Referring at Figure 38, we observe that the Uniform Minimum inventory policy outperforms the Non Uniform Minimum inventory policy

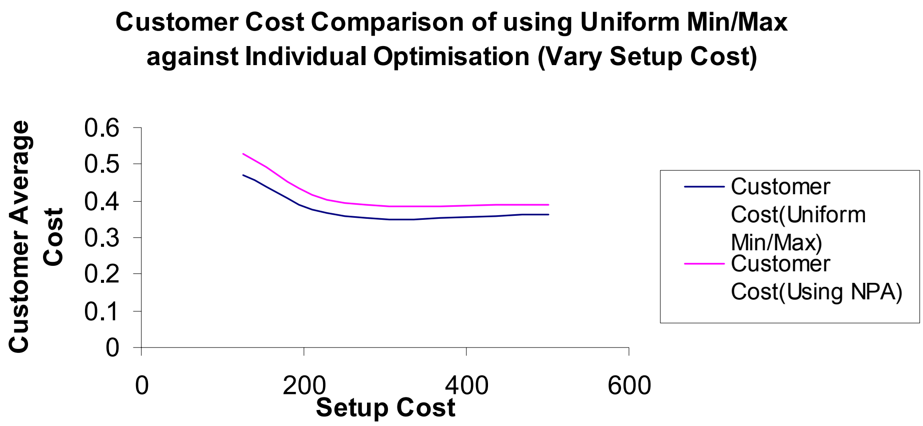


Figure 31: Customer's Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary $A_R$)

To ascertain this observation, we conduct a sensitivity analysis on various parameters used in the model. The simulation results is tabulated in Appendix B

From Figures in Appendix B, we can see that our initial hypothesis of customer cost being lower in a Uniform Minimum inventory policy is true. In all cases, we can see that though the total system costs are higher in a Uniform Minimum inventory policy, the customers incur less cost in this policy too. Thus, we can infer that the popularity of this policy is due to the low customer cost. As customers are usually the one with the bigger bargaining power in a VMI relationship, thus it is of no surprise that the customer would want to implement a policy that is beneficial to them. However, as we can see in most cases, the system cost of using a Uniform Minimum-Maximum policy is much higher than using the Non Uniform Minimum-Maximum Policy. Thus, the optimal policy for the customer is detrimental to the other players in the VMI supply Chain.

### 5.6.2 Alternative Policies for the VMI Supply Chain

As mentioned in the previous section, we have found that by using the Uniform Minimum-Maximum Inventory policy, we are able to lower customer's cost but we increased the cost incurred by other players tremendously. To solve this problem, we try out several alternative configurations in an attempt to solve this problem**.**

The first configuration that we are considering will be our base case configuration, the Optimised VMI configuration. This configuration assumes that both the foreign and local supplier adopts the algorithm developed in this paper for their inventory replenishments decisions. As for the second configuration, we adopt a hybrid inventory system, the JIT/VMI configuration. This configuration assumes that the vendor hub operator lets the local supplier to run on a JIT inventory policy while the foreign supplier supplies the vendor hub using the VMI inventory policy that is derived from our paper. The remaining configurations tested would be based on various manipulations on the s, S parameter in the s, S policy considered in the problem. A summary of the various configurations and their characteristics are listed below.

| Config No. | Configuration Type | Local Supplier Policy | Foreign Supplier Policy |
|---|---|---|---|
| 1 | Optimised VMI | ($s^{\#}$,S) policy, S = s+Q* | (s,S) policy, S = s+Q* |
| 2 | JIT/VMI | JIT | VMI |
| 3 | Full-Max | (s,S) policy S = s+λL | (s,S) policy, S = s+λL |
| 4 | Local Half-Max | (s,S) policy, S = s+$^{1}/_{2}$ λL | (s,S) policy, S = s+Q* |
| 5 | Local Full-Max | (s,S) policy, S = s+ λL | (s,S) policy, S = s+Q* |
| 6 | Local Half Min-Max | (s,S) policy, where s is the cycle stock+$^{1}/_{2}$ λL, S = s+Q* | (s,S) policy, S = s+Q* |
| 7 | Local Full Min-Max | (s,S) policy, where s is the cycle stock + λL, S = s+Q* | (s,S) policy, S = s+Q* |
| 8 | Total Half Min, Maintain Max | (s,S) policy, where s is the cycle stock +Min($^{1}/_{2}$λL , $^{1}/_{2}$Q*), S = λL +Q* | (s,S) policy, where s is the cycle stock +Min($^{1}/_{2}$λL , $^{1}/_{2}$Q*), S = λL +Q* |
| 9 | Total Full Min-Max | (s,S) policy, where s is the cycle stock +Min(λL , Q*), S = λL +Q* | (s,S) policy, where s is the cycle stock +Min(λL , Q*), S = λL +Q* |

Table 26: List of Configurations

#we define the default s to be equal to the cycle stock, where s= λL

To determine the performance of the various policies, a sensitivity analysis is needed to examine the performance of the different configurations under different conditions.

#### 5.6.2.1 Comparison of Performance between JIT/VMI hybrid system and pure VMI Inventory systems

The first comparison to be conducted would be the JIT/VMI hybrid system against the pure VMI inventory system. We would first conduct the sensitivity analysis on the parameter inventory replenishment cost, $A_R$, for the two different policies. The results are shown in Figure 32 to 36.

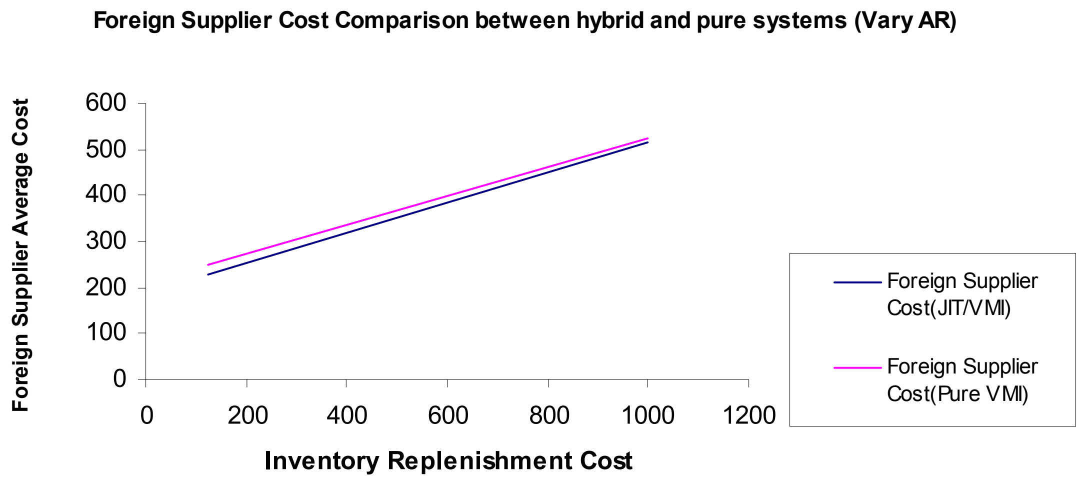


Figure 32: Foreign Supplier Cost Comparison between Hybrid and Pure system (Vary $A_R$)

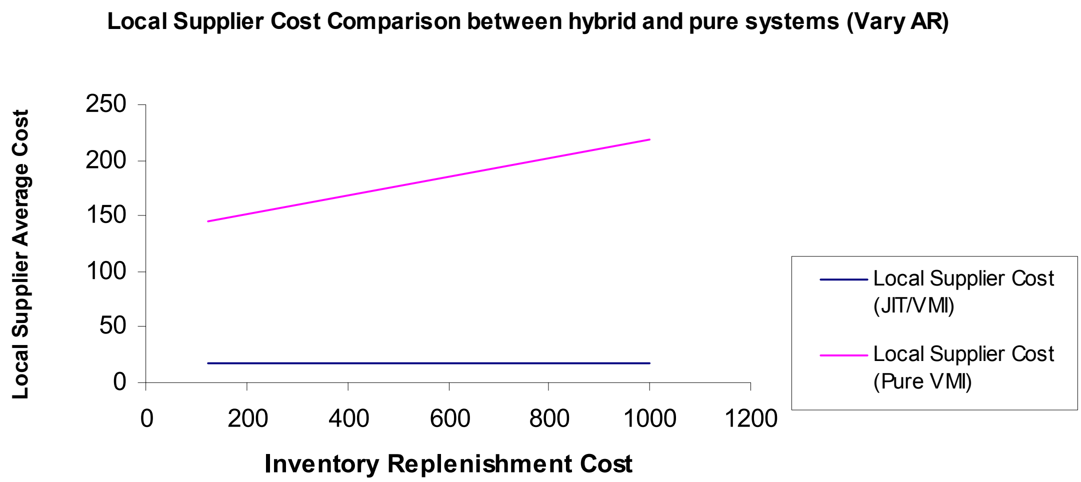


Figure 33: Local Supplier Cost Comparison between Hybrid and Pure system (Vary $A_R$)

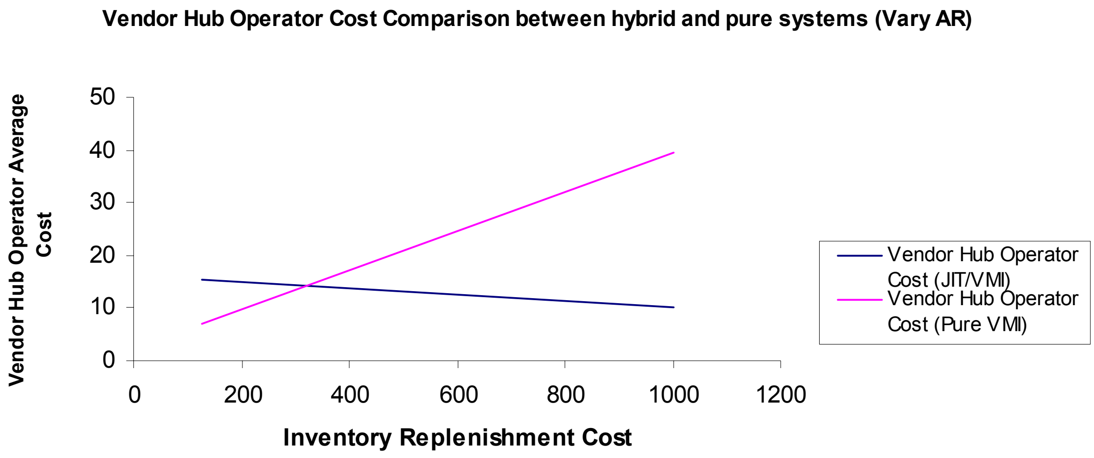


Figure 34: Vendor Hub Operator Cost Comparison between Hybrid and Pure system (Vary $A_R$)

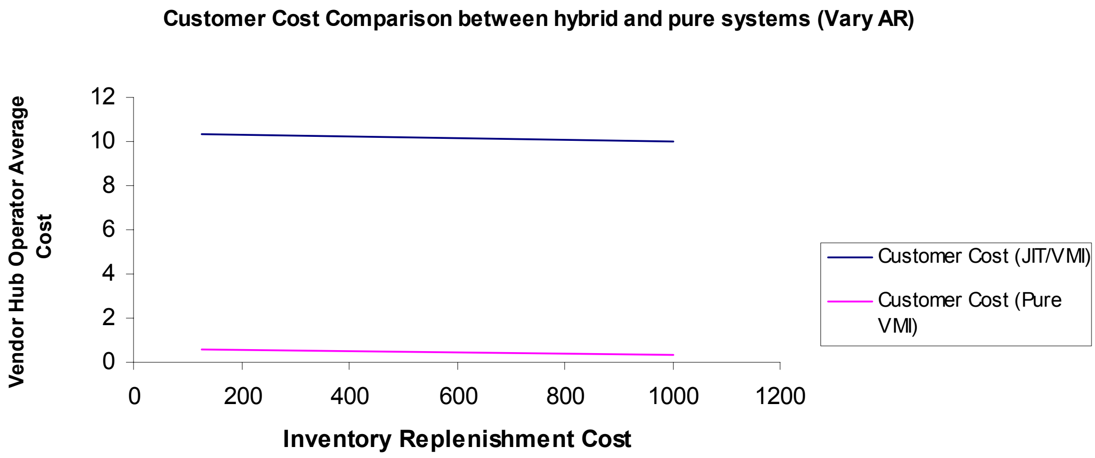


Figure 35: Customer Cost Comparison between Hybrid and Pure system (Vary $A_R$)

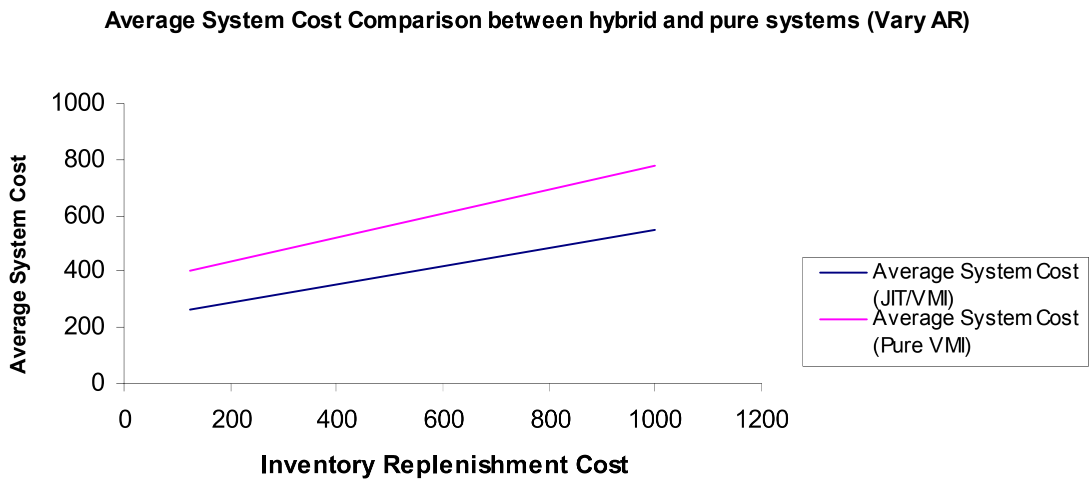


Figure 36: Average System Cost Comparison between Hybrid and Pure system (Vary $A_R$)

From Figure 32 to 36, we can see that the hybrid system outperforms the pure VMI system in the Foreign Local Supplier Cost, the Vendor Hub Operator Cost and the Average System Cost. However, in terms of customer cost, the hybrid JIT/VMI system is inferior compare to the pure VMI system. To get a conclusive analysis on the performance of hybrid system with pure VMI systems, we conduct the sensitivity analysis for the remaining parameters. From the sensitivity analysis conducted, we observe that the various cost components generally reacts similarly to the two policies. However, there are cases where the results generated are different from what we got from the sensitivity analysis on the inventory replenishment cost. For simplicity, we will highlight the cases that are different and leave out the results for those cases where the cost behaviour similarly.

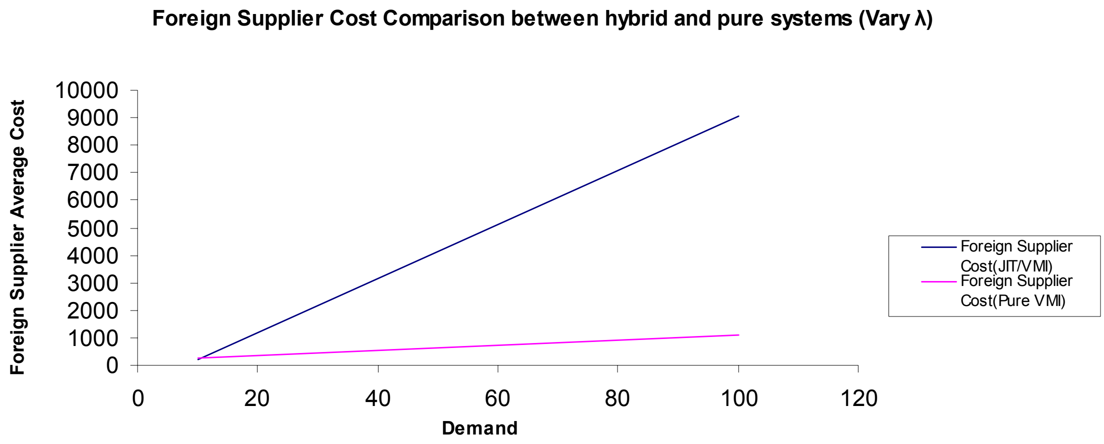


Figure 37: Foreign Supplier Cost Comparison between Hybrid and Pure system (Vary λ)

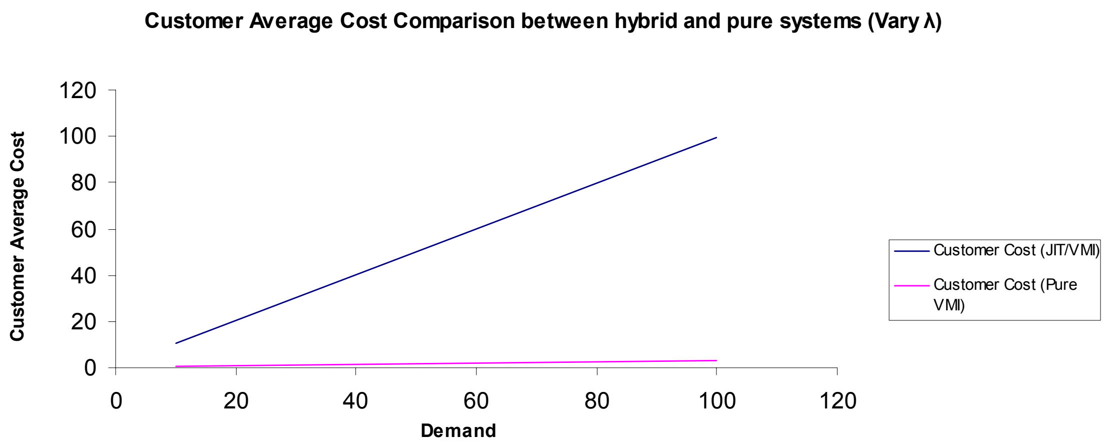


Figure 38: Customer Average Cost Comparison between Hybrid and Pure system (Vary λ)

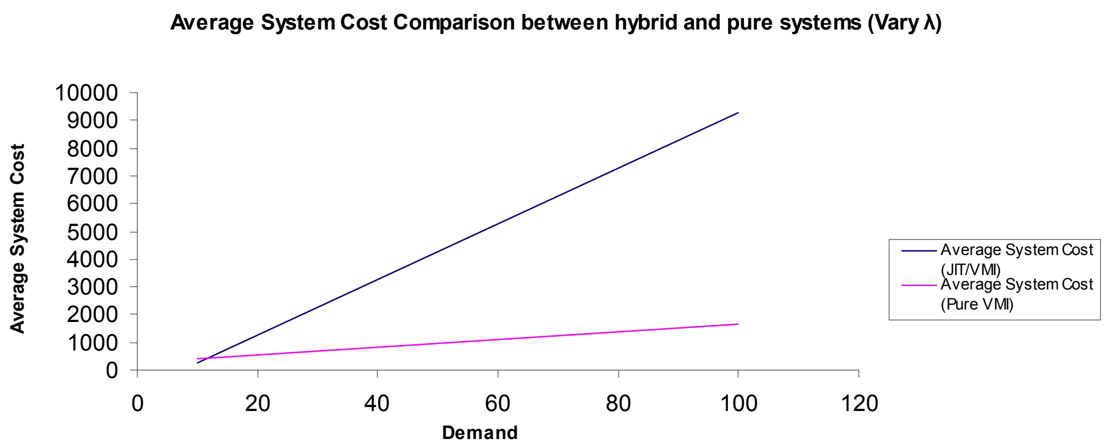


Figure 39: Average System Cost Comparison between Hybrid and Pure system (Vary λ)

From our sensitivity analysis, we discovered that in cases of high demand, JIT/VMI hybrid systems tend to fail in comparison with pure VMI systems. This is coherent with our findings that JIT system will tend to fail in cases of high lambda. This poor performance of the JIT system result the hybrid system performing poorly at such scenarios.

### 5.6.2.2 Comparison of Performance between by increasing minimum levels for local suppliers.

The next analysis to be conducted would be manipulating the minimum level s while maintaining the Q* level for the local supplier. We would be conducting a similar procedure to the previous comparison by conducting a sensitivity analysis on the parameter inventory replenishment cost, $A_R$, for the few policies. The results are shown in Figures 40 to 44.

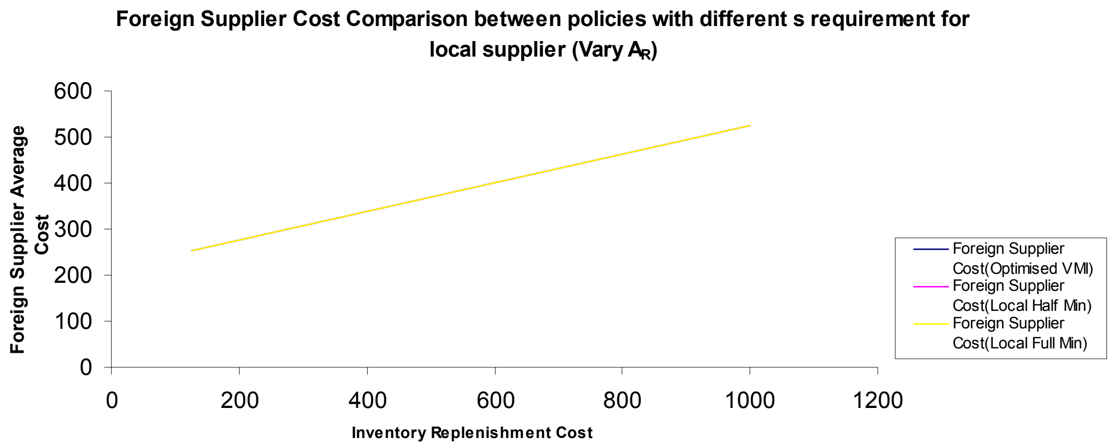


Figure 40: Foreign Supplier Cost Comparison between policies with different s requirement for local supplier (Vary $A_R$)

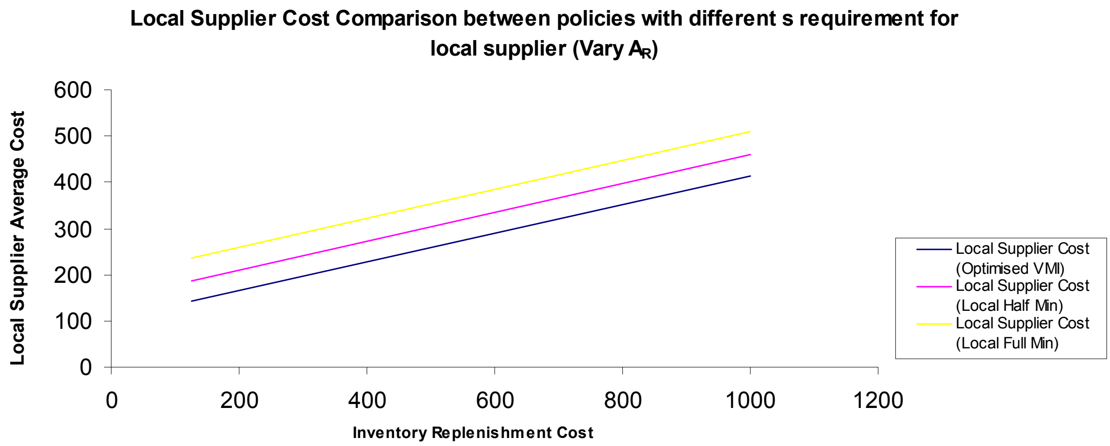


Figure 41: Local Supplier Cost Comparison between policies with different s requirement for local supplier (Vary $A_R$)

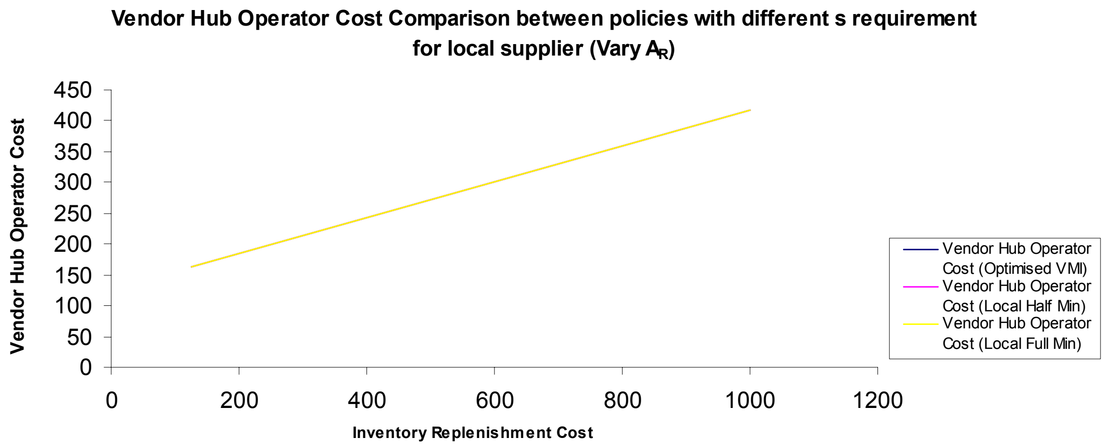


Figure 42: Vendor Hub Operator Cost Comparison between policies with different s requirement for local supplier (Vary $A_R$)

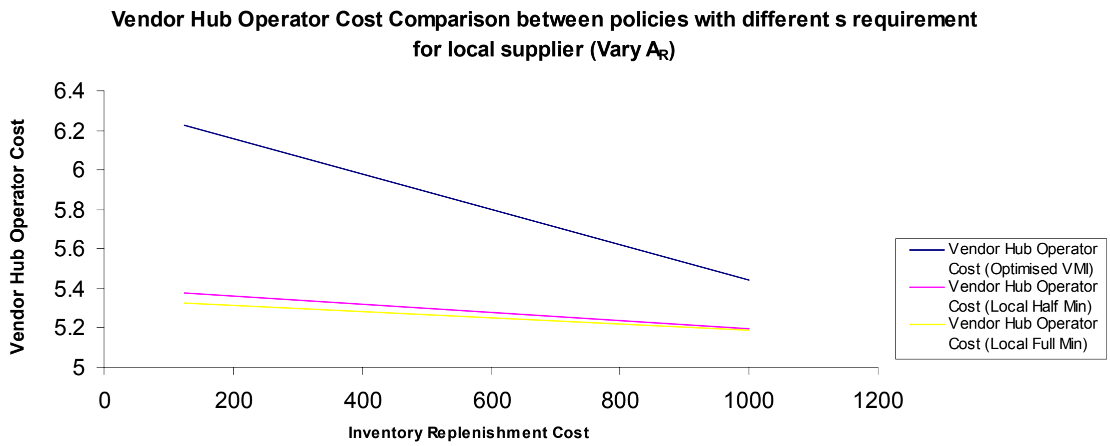


Figure 43: Customer Cost Comparison between policies with different s requirement for local supplier (Vary $A_R$)

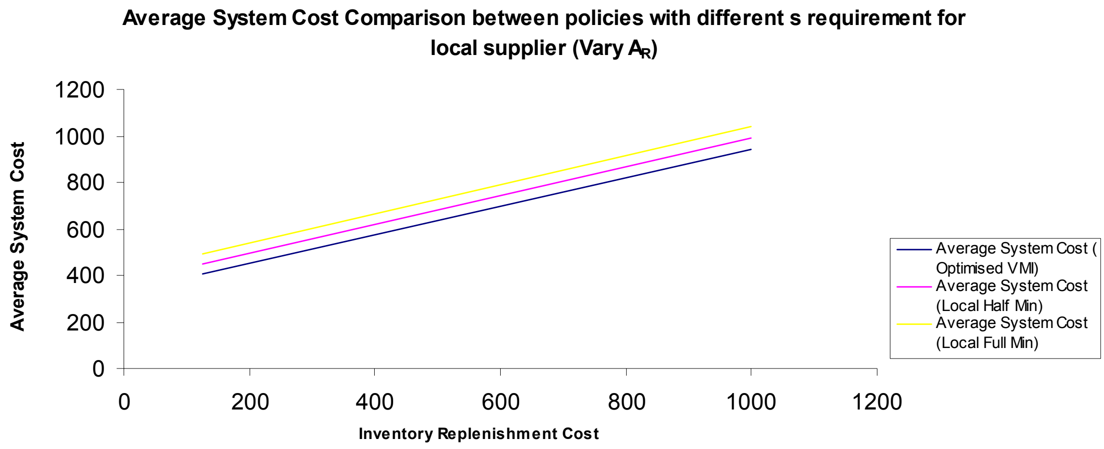


Figure 44: Average System Cost Comparison between policies with different s requirement for local supplier (Vary $A_R$)

Looking at Figures 40 to 44, we observe that the foreign supplier cost and the vendor hub operator seems unaffected by the change in policies. This result is expected as the foreign supplier cost is not affected by the different configuration in the local supplier. We observe that when we increase the minimum level required for local supplier, we decrease the customer cost while increasing the local supplier cost and the average system cost in the process. For us to get the complete picture of the impact of increasing s while maintaining Q*, we will do a complete sensitivity analysis of these policies with regards to other parameters. For simplicity, we will only show figures that exhibit a different behaviour from the sensitivity analysis done on the inventory replenishment cost.

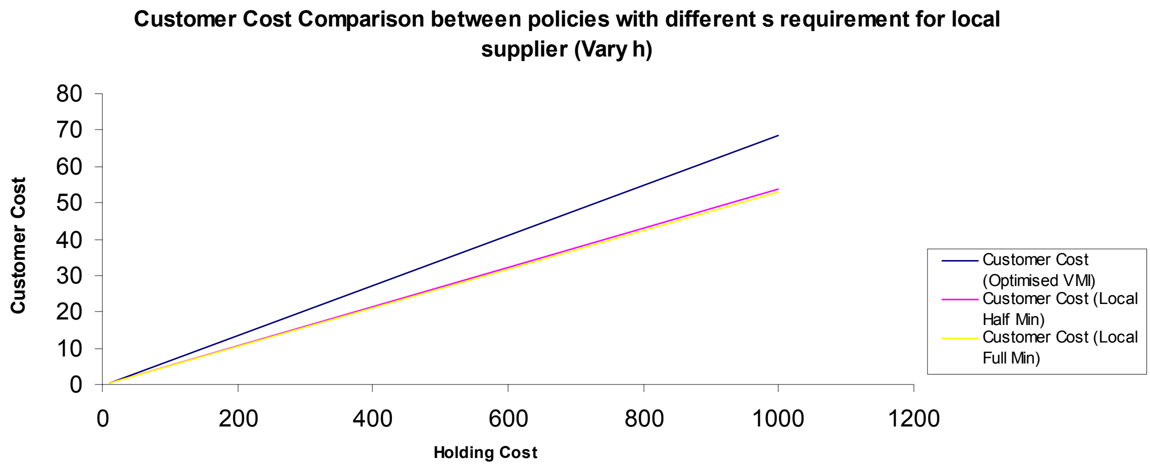


Figure 45: Customer Cost Comparison between policies with different s requirement for local supplier (Vary h)

From the sensitivity analysis conducted, we found out that the observation that we made during the sensitivity analysis for the parameter, $A_R$, still holds. However, we highlight an interesting result that we obtained from the sensitivity analysis conducted. We found that the reduction in customer cost by increasing the s is minimal when the increment passes the ½ λL mark. Using Figure 45 as an example, we can clearly see that the customer cost reduction is almost negligible when we increase the s level from ½ λL to λL.

#### 5.6.2.3 Comparison of Performance between by increasing Q* levels for local suppliers

The third analysis to be conducted would be manipulating the maximum level S while maintaining the minimum level s for the local supplier. We would be conducting a similar procedure to the previous comparison by conducting a sensitivity analysis on the parameter inventory replenishment cost, $A_R$, for the few policies. The results are shown in Figures 46 to 50.

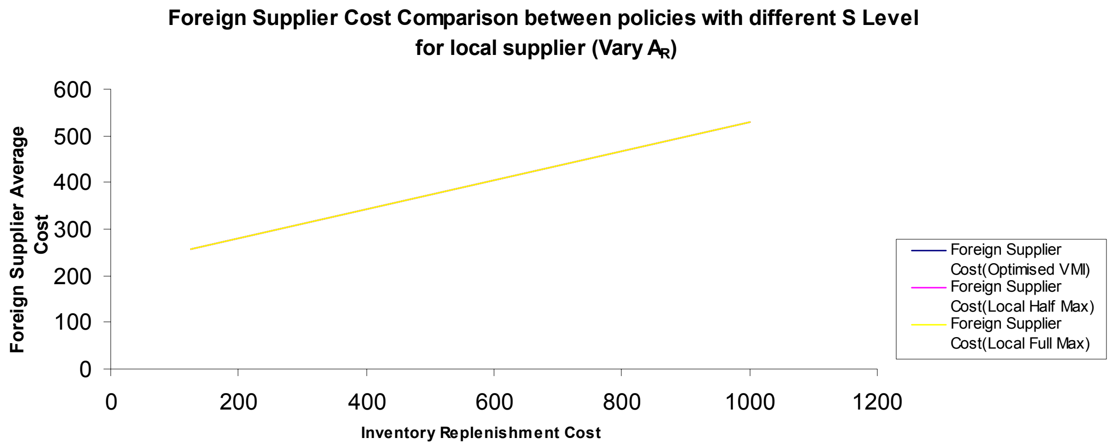


Figure 46: Foreign Supplier Cost Comparison between policies with different S Level for local supplier (Vary $A_R$)

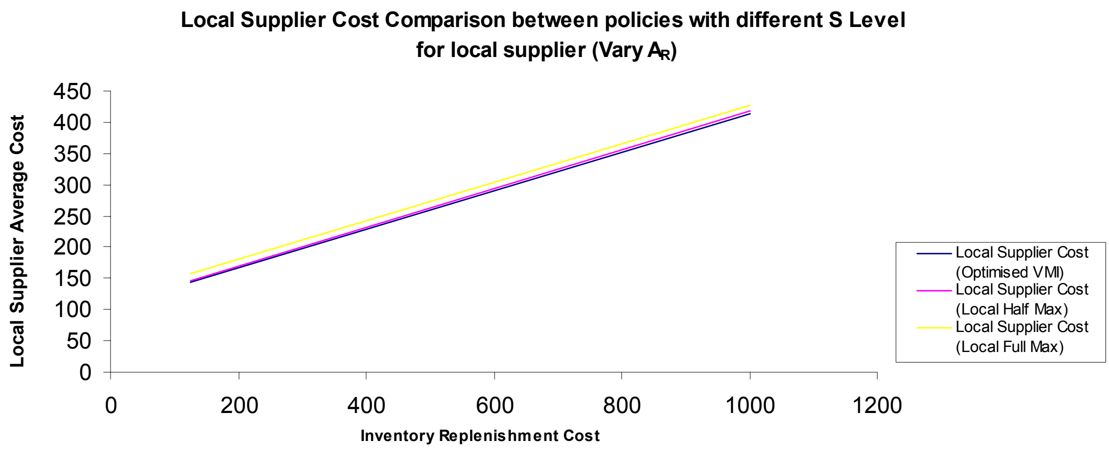


Figure 47: Local Supplier Cost Comparison between policies with different S Level for local supplier (Vary $A_R$)

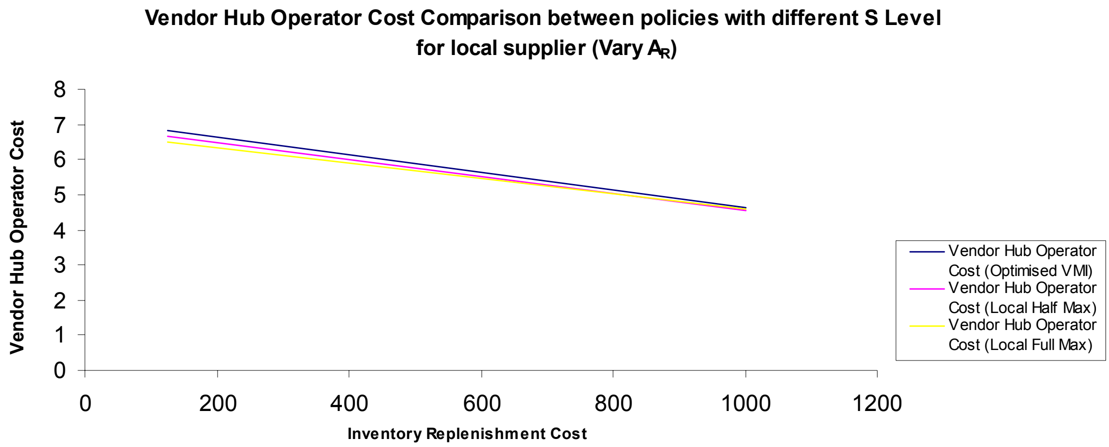


Figure 48: Vendor Hub Operator Cost Comparison between policies with different S Level for local supplier (Vary $A_R$)

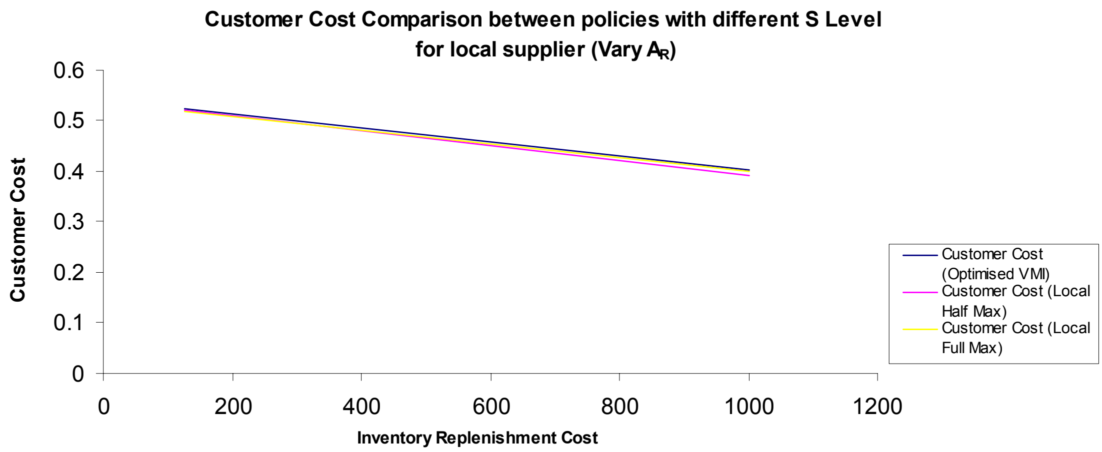


Figure 49: Customer Cost Comparison between policies with different S Level for local supplier (Vary $A_R$)

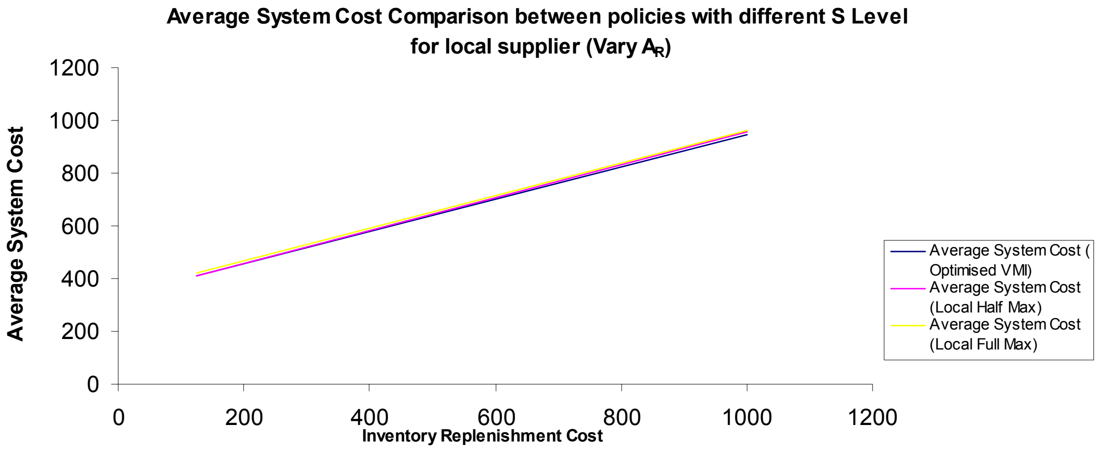


Figure 50: Average System Cost Comparison between policies with different S Level for local supplier (Vary $A_R$)

Looking at Figures 46 to 50, we observe that the foreign supplier cost is unaffected by the change in policies. This result is expected as the foreign supplier cost is not affected by the different configuration in the local supplier. We observe that when we increase the maximum level required for local supplier, we decrease the customer and the vendor hub operator cost while increasing the local supplier cost and the average system cost in the process. However, the degree of change for increasing the maximum level is not as large as the configuration of changing the minimum. For us to get the complete picture of the impact of increasing Q*, we will do a complete sensitivity analysis of these policies with regards to other parameters. For simplicity, we will only show figures that exhibit a different behaviour from the sensitivity analysis done on the inventory replenishment cost.

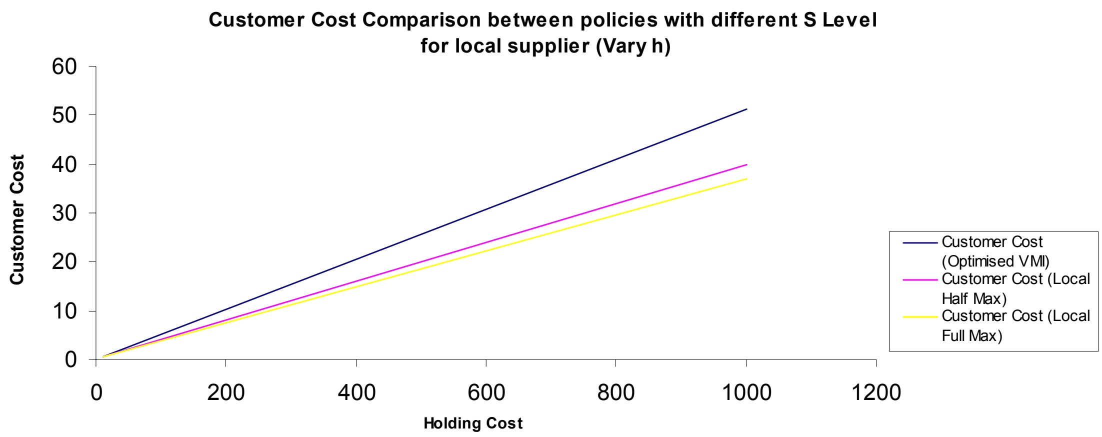


Figure 51: Average System Cost Comparison between policies with different S Level for local supplier (varying h)

From the sensitivity analysis conducted, we found out that the observation that we made during the sensitivity analysis for the parameter, $A_R$, still holds. When we increase the maximum level required for the local supplier, the customer and vendor hub operator cost are decreased while increasing the local supplier cost and the average system cost. The decrease in cost for the customers by increasing the maximum level is relatively small compared to the decrease given by increasing the minimum requirement. In addition, the increase in cost for local suppliers and average system cost is lower than that of the minimum increase requirement policy. However, looking at Figure 51, we find that in scenarios where holding cost are high, increasing the maximum levels yields bigger savings for the customer compared to increasing the minimum requirement levels. In addition, the increase in local supplier cost and average system cost by increasing the maximum is still lower in increasing the minimum requirement levels.

### 5.6.2.4 Comparison of Performance between by increasing (s,S) levels

The previous configuration manipulations were done purely on the local supplier as the Uniform Minimum Maximum Policy usually affects only the local supplier. We will conduct manipulation to both suppliers inventory policy to determine whether imposing changes on both suppliers works better than imposing changes on one supplier. The next analysis to be conducted would be manipulating the Minimum and Maximum level for both suppliers. We would be conducting a similar procedure to the previous comparison by conducting a sensitivity analysis on the parameter inventory replenishment cost, $A_R$, for the few policies. The results are shown in Figures 52 to 56.

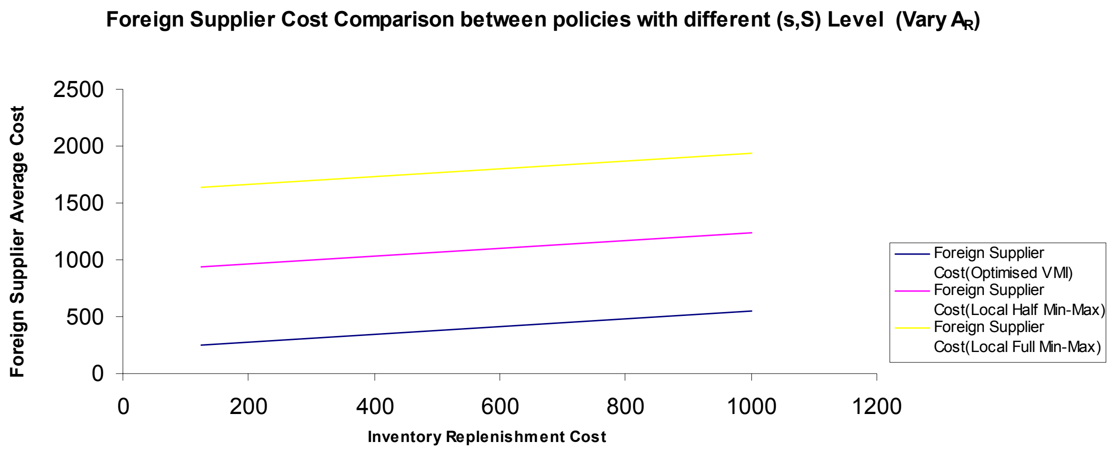


Figure 52: Foreign Supplier Cost Comparison between policies with different (s, S) Level (Vary $A_R$)

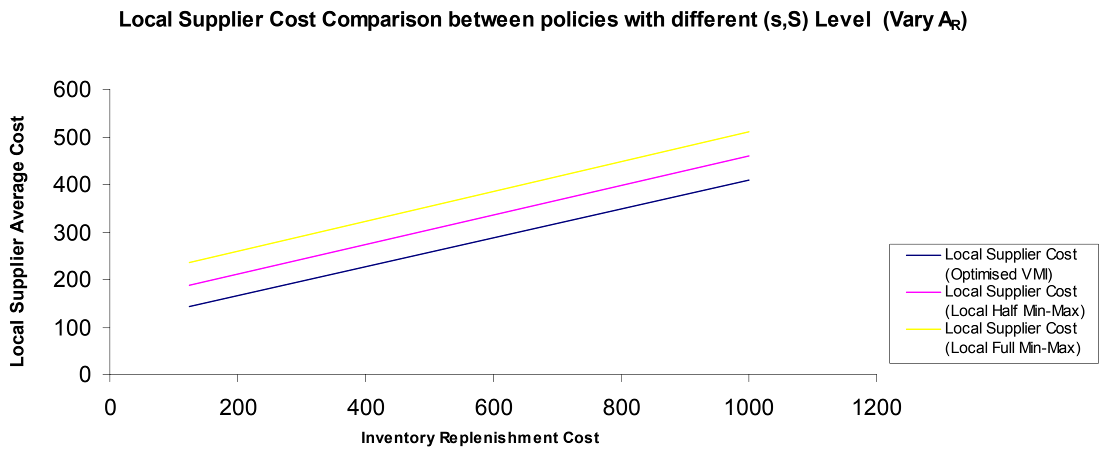


Figure 53: Local Supplier Cost Comparison between policies with different (s, S) Level (Vary $A_R$)

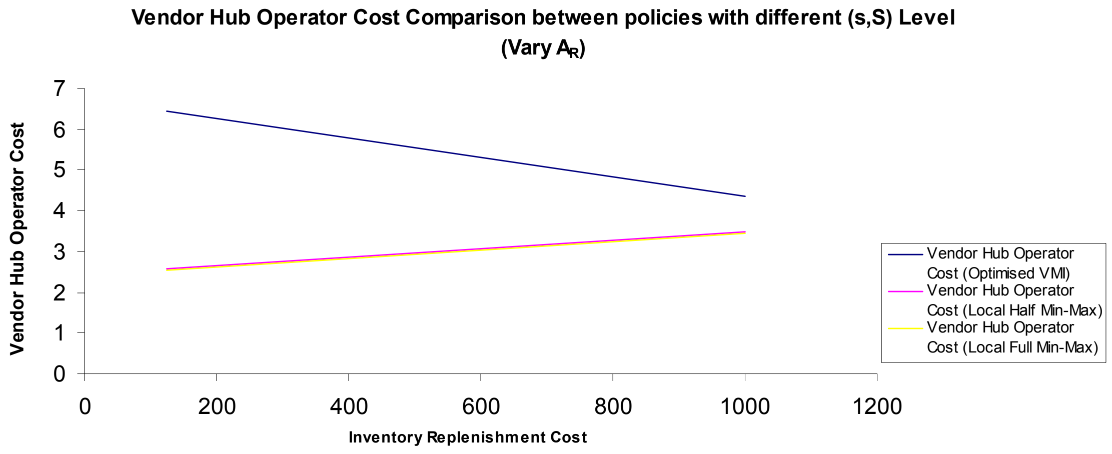


Figure 54: Vendor Hub Operator Cost Comparison between policies with different (s, S) Level (Vary $A_R$)

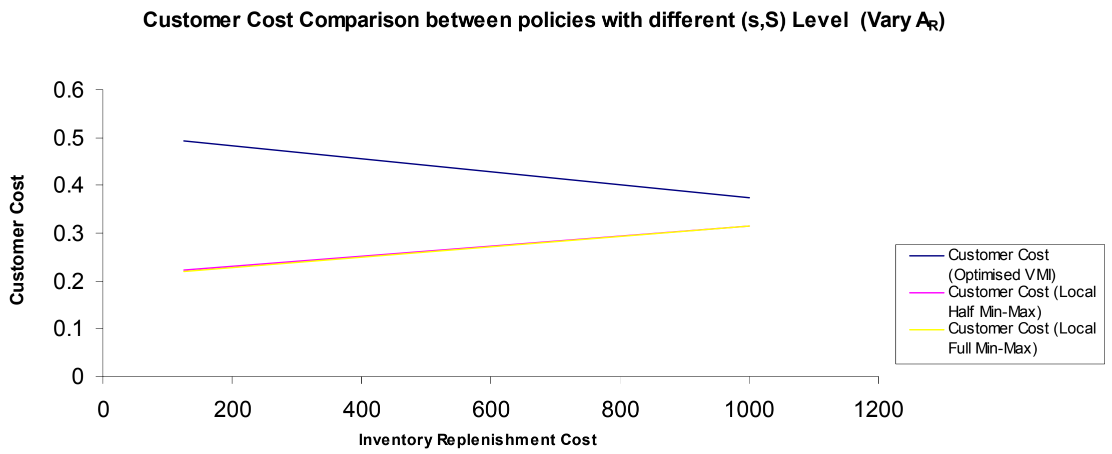


Figure 55: Customer Cost Comparison between policies with different (s, S) Level (Vary $A_R$)

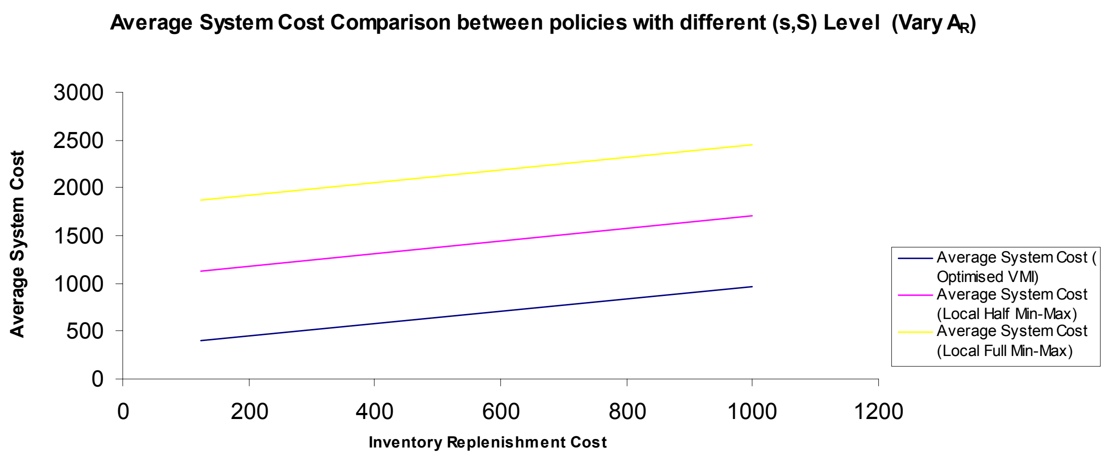


Figure 56: Average System Cost Comparison between policies with different (s, S) Level (Vary $A_R$)

From Figures 52 to 56, we observe that the vendor hub operator cost is relatively unaffected by the change in policies. The other cost components are affected by the choice of the inventory policy. We observe that when we increase the parameters for the (s, S) policy for both suppliers, we decrease the customer cost while increasing the suppliers' cost and the average system cost in the process. However, like the case of increasing the minimum level of the single supplier, the reduction of cost for the customer seems to be quite minute minimal when the increment passes over the ½ $\lambda L$ marks. For us to get a definite conclusion, we conducted a complete sensitivity analysis of these policies with regards to other parameters. From the sensitivity analysis conducted, we found out that the observation that we made during the sensitivity analysis for the parameter, $A_R$, still holds. However, we do observe that the magnitude of the change is much higher than the other policies. Customer Cost decreased by a larger portion from an increase in (s, S) for both suppliers. Concurrently, the supplier's cost and the average system cost increased by a bigger proportion too.

#### 5.6.2.5 Comparison of Performance between by increasing s level while maintaining S level

The last manipulation that we will be conducting to both suppliers inventory policy is to increase the Minimum level, s, for both suppliers without increasing the Maximum level, S. We would be conducting a sensitivity analysis on the parameter inventory replenishment cost, $A_R$, for the few policies. The results are shown in Figure 74 to 78.

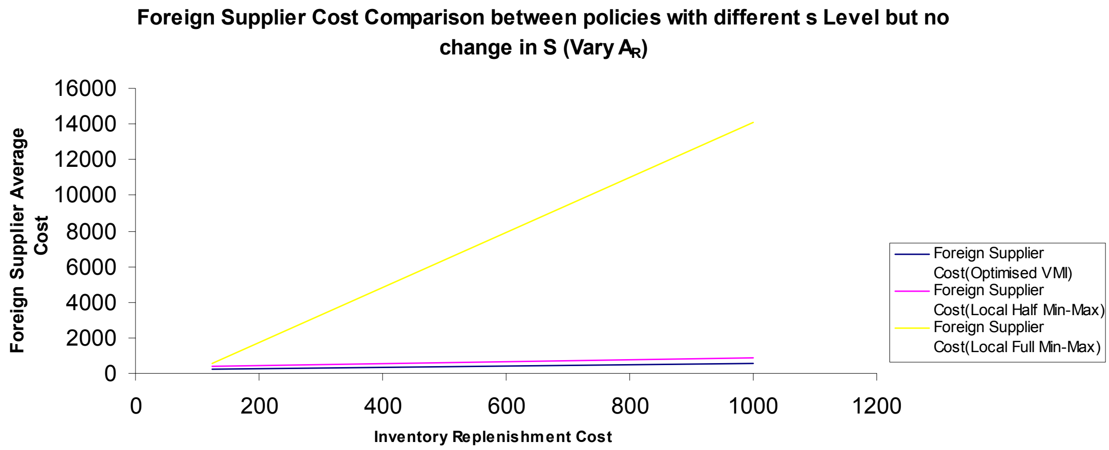


Figure 57: Foreign Supplier Cost Comparison between policies with different s but same S Level (Vary $A_R$)

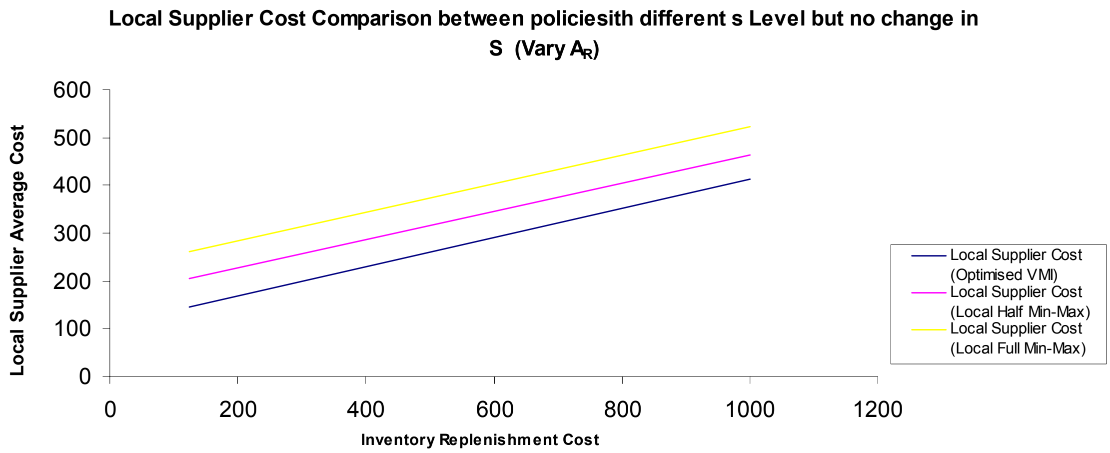


Figure 58: Local Supplier Cost Comparison between policies with different s but same S Level (Vary $A_R$)

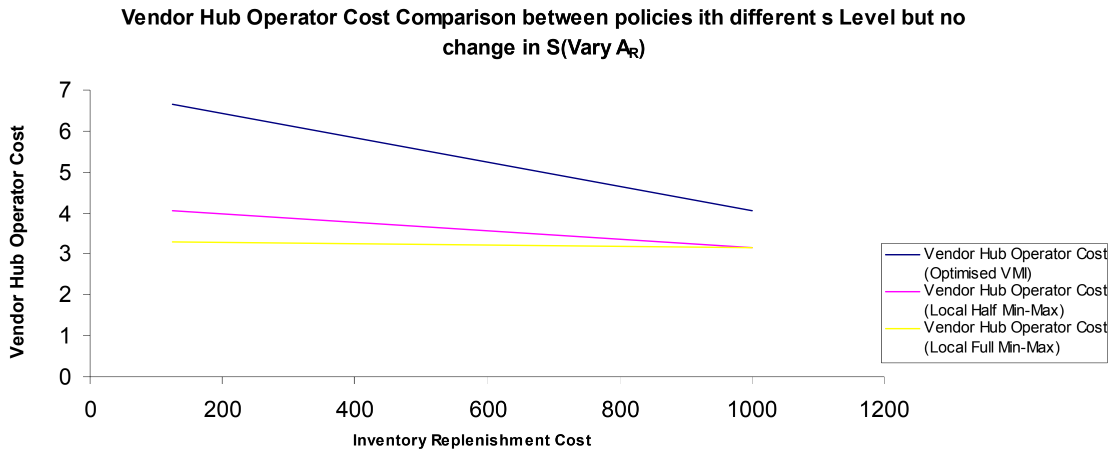


Figure 59: Vendor Hub Operator Cost Comparison between policies with different s but same S Level (Vary $A_R$)

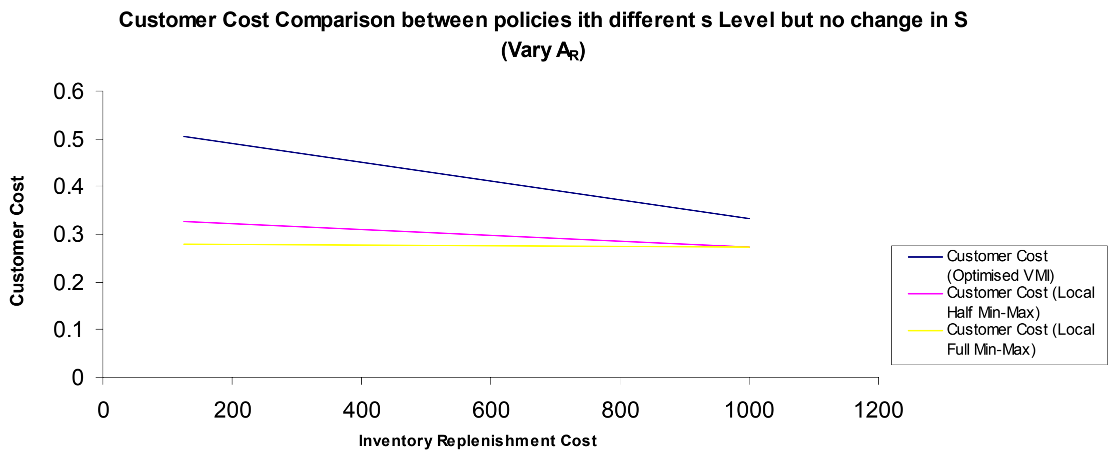


Figure 60: Customer Cost Comparison between policies with different s but same S Level (Vary $A_R$)

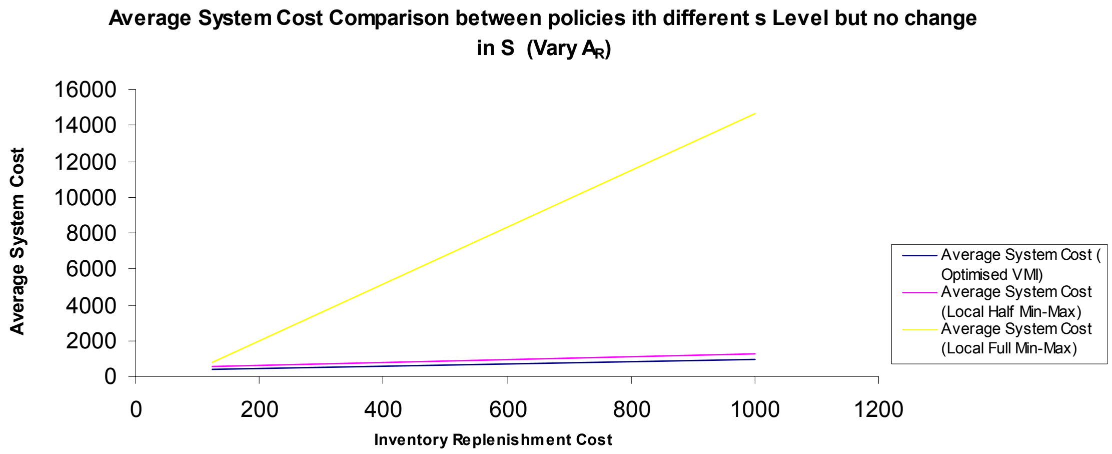


Figure 61: Average System Cost Comparison between policies with different s but same S Level (Vary $A_R$)

From Figures 57 to 61, we observe that when the parameters for the minimum level for both suppliers is increased without changing the maximum level, S, we decrease the customer and the vendor hub operator costs while increasing the suppliers' and the average system cost in the process. A complete sensitivity analysis of these policies is conducted with regards to other parameters to analyse the results. From the sensitivity analysis conducted, we found out that the observation that we made during the sensitivity analysis for the parameter, $A_R$, still holds. However, we do observe that the magnitude of the change is not as high as the change in (s, S) for both suppliers though it is higher than the other policies.

## 5.7 Discussion of Results

After obtaining all the results, we are now going to consolidate the results and attempt to analyse them. Our analysis can be broken into 3 parts: Supplier Selection Issues, VMI and JIT comparison and Industrial Practice.

### 5.7.1 Supplier Selection Issues

Supplier selection is one of the most fundamental decisions made in a supply chain. Selecting the right suppliers significantly reduces costs and improves corporate competitiveness (Dobler et al., 1990). As found in James et al. (2000), single sourcing is considered as one of the primary enablers in a VMI arrangement. Thus, only the issues of supplier selection in a single sourcing environment will be analysed due to its applicability in a VMI arrangement.

Upon analysing the various results in the sensitivity analysis, it can be seen that defective rate and price have the largest impact on average cost, followed by other model parameters like MOQ, demand, holding cost and warehouse capacity. This is in contrary to the traditional belief of selecting suppliers based on price alone. As seen, other supplier specific parameters, such as MOQ, has also have a significant impact on total logistics cost. This observation is coherent with Moore and Fearon (1973) proposition that price, quality and delivery are important criteria for supplier selection.

In addition to this observation, another observation can be drawn from the analysis. Supplier specific parameters are not the only factors that affect the total logistical cost of a vendor hub. Vendor hub specific parameters that , such as demand, holding cost in the vendor hub and warehouse capacity, also have a significant impact on the logistics cost incurred by the vendor hub. This is due to the interaction that the vendor hub specific parameters have with supplier specific parameters. For example, an increase in demand will increase the impact of price and defective rate on average cost. On another hand, an increase in holding cost/alternative storage cost and/or a decrease in warehouse capacity will increase the impact of MOQ on average cost. All these interactions may change the importance of the various supplier specific parameters used in the selection process.

From all the above observation and analysis, it can be concluded that supplier selection should not be based on supplier specific parameters alone. The vendor must also consider its current capabilities and resources (the vendor hub specific parameters) and matches these factors with the supplier considered.

### 5.7.2 Comparison of JIT and VMI

In our sensitivity analysis, we found that JIT, if operated at the ideal scenario, is usually the better policy to adopt compared to VMI. However, even at the ideal environment where all the basic principles of JIT inventory management is adhered to, JIT still fails in scenarios where the demand or lead time deviation is high. This implies that JIT do not work well in situations where the demand is relatively unknown or in scenarios where the supplier is unreliable (which is represented by the high deviation in the lead time). This result is coherent to Fuller (1995) findings from the analysis on JIT literatures, stating that dependable deliveries are vital to JIT inventory systems. In addition, we find that if the implementation cost for suppliers to switch to JIT is large, JIT become much more expensive to adopt compared to VMI systems. We also have to take the note that the tests are conducted under perfect JIT conditions. In reality, it is quite difficult to achieve zero setup cost. In such non-ideal, situations, JIT may not outperform VMI even when implementation cost and/or deviation of demand and lead time parameters are low.

### 5.7.3 Analysis on Industry Practice

Upon analysing the consolidated results from the sensitivity analysis of different configurations, we manage to obtain some interesting finding. Firstly, through the test results, we have obtained insights on the reason behind the popularity behind the Uniform Minimum Policy in vendor hub operators. By using the Uniform Minimum policy rather than the NPA algorithm that we are suggesting, the customer/buyer are able to achieve a lower cost comparative to the inventory policy generated by the NPA. On the other hand, suppliers find their cost is much higher in using such policy rather than a policy generated by the NPA Algorithm. This would translate in a higher system cost for the Uniform Minimum Policy compared to the NPA policy. Thus, we can see that only customers benefits from the Uniform Minimum Policy. This must mean that the customer bargaining power must be much bigger than those of the supplier in order for them to have the power to force the suppliers to implement such a policy. This finding concurs with Ramsay (1994) and Stannack (1996) where they find that when the purchasing power or the supply chain power of the buyer is high, the buyers are able to force the sellers to act in an unfavourable manner. In our case, the customers are able to force the suppliers to implement policies that are favourable to the customers and detrimental to the suppliers. This proposition is further proven by the fact that the inventory policy in the vendor hub used in our study is determined by the customer alone.

#### 5.7.3.1 Alternative Configurations

We now take a look at the simulations results of the different configurations. The impact of each manipulation will be examined in detail.

We find the hybrid JIT/VMI inventory system is the best system in terms of system cost in general. The hybrid system suffer the same problem as the pure JIT inventory system policy in failing to outperform the VMI inventory management system in the scenarios of high standard deviation of demand and lead time. Other than this apparent weakness, we also find that the customer cost in the hybrid system is higher than the NPA VMI systems. Increasing the minimum level for the local supplier while maintaining Q* produces a similar effect to the uniform Minimum Policy. By increasing s, the system cost is increased but customer cost is reduced. We find that the decrease in customer cost come to a stand till at $^1/_2$ λL. Any further increase will have little impact on customer cost. This implies that the Uniform Minimum policy might be too conservative if the suppliers' lead time is much lesser than the Uniform Minimum requirement.

Increasing the maximum level for the local supplier while maintaining the s level also produces a similar effect to uniform Minimum Policy. Like increase s, the increase in S increases the system cost but reduces customer cost. However, the extent of the change is not as high as increasing the minimum. In addition, we have to remember that increasing S may mean a corresponding increase in warehouse space needed. This may not be feasible if an external warehouse is not readily available.

Increasing the minimum level for both suppliers and maintaining Q* also produce a similar effect to the Uniform Minimum Policy. By increasing the minimum level for both suppliers, we increase the system cost but reduces the customer cost. We find that the customer cost is reduced much more than the Uniform Minimum Policy but the increase in system cost is also much larger.

By increasing the minimum level without changing S also increase system cost and reduces customer cost. We find that the customer cost decrease is similar to that of increasing the minimum of the suppliers. However, we find increase in system cost is much higher than most other policies. This is due to the huge increase in the fixed replenishment cost component of supplier as frequency of replenishment increase.

In the various sensitivity analysis to find a replacement policy for the Uniform Minimum Policy, we find that when in policies where customer costs are low than the policy that is focus of optimising system cost, the supplier cost are usually much higher than what it would have been in the system optimised policy. This means that if we want to reduce the customer cost by manipulating the s, S parameters in the replenishment policy, ceteris paribus, the supplier cost would be increased. In addition, the magnitude of the decrease in customer cost is found to be proportional to the increase in supplier cost. In other words, this mean that the greater the reduction in customer cost a policy gives; the greater the cost increase is for the supplier.

For easy reference, we summarise our analysis in the table below. We ranked the magnitude of the impact of various polices on cost of the various parties in the supply chain. The JIT/VMI hybrid system is not ranked as the impact is reverse of that of the manipulation of the s, S parameters.

| Manipulation Type | Impact on System Cost | Impact on Supplier Cost | Impact of Customer Cost | Impact on Vendor Hub Operator Cost | Comments |
|---|---|---|---|---|---|
| JIT/VMI Hybrid | Decrease | Decrease | Increase | Increase | Best Performance in terms of system cost, but cost of customer and vendor hub operator is much higher than other policies |
| Increase both s, maintain Q | Increase(2) | Increase(2) | Decrease(1) | Decrease(1) | Effective if increase is less than $^1/_2\ \lambda L$ |
| Increase both s, maintain S | Increase(1) | Increase(1) | Decrease(2) | Decrease(2) | Increase Supplier cost tremendously. |
| Increase s, maintain Q | Increase(3) | Increase(3) | Decrease(3) | Decrease(3) | Effective if increase is less than $^1/_2\ \lambda L$ |
| Increase both S, maintain s | Increase(4) | Increase(4) | Decrease(4) | Decrease(4) | Will pose a problem if there are space constraints. |

Table 28: Analysis on manipulations of various parameters in a vendor hub

## 5.8 Conclusion

By applying the simulation technique, we have proven that our New Proposed Algorithm is indeed better than Cetinkaya and Lee (2000) solution. Using our New Proposed Algorithm, we compare the performance between JIT and VMI inventory systems and have reached a conclusion on the performance between the two systems. In addition, we have also analysed current practices adopted by the industry. We have proposed several configurations to replace the current practice and have compared the performance between these systems.

# 6 Conclusion

In this chapter, the results presented in chapter 6 are summarised. This is followed by a discussion of the strategic implications drawn from the study. The limitations of this study and suggestions for further research will also be presented.

## 6.1 Research Contribution

Several researches were done on developing an optimal model for VMI or supply chain of a similar nature. Ruhul and Khan (1999) and Cetinkaya and Lee (2000) examined the problem and developed an optimal solution for various decisions that exists in a VMI system. However, certain real life supply chain constraints such as Minimum Order Quantity (Robbs and Silver, 1998), warehouse capacity (Ishii and Nose, 1996) and imperfect quality (Schwaller, 1998) were omitted from Cetinkaya and Lee (2000) study. Issues such as supplier selection and order splitting in a VMI supply chain were also not examined in detailed by past literature. In addition, past VMI literatures also failed to make any comparison between VMI and JIT systems. We also find that past literatures failed to analyse present industry practices used. Thus, this study fills the gaps that exist in the literature and attempt to derive a new algorithm that will surpass Cetinkaya and Lee (2000) model under the various constraints mentioned.

## 6.2 Summary of Results

The New Proposed Algorithm is found to be a better heuristic in determining the order up to level and consolidation period when the vendor hub capacity is limited. Thus, we are able to conclude that our recommended algorithm is a better solution compared to Cetinkaya and Lee (2000) solution.

In examining the supplier selection issues in VMI, it is found that other than price, other supplier specific parameters such as MOQ and defective rate of the supplier is as important as the price of the product itself as they have significant impact on cost. The vendor hub operator in a VMI supply chain should also take note of its own resources and capability as their interaction with the supplier specific parameters to would affect the magnitude of the impact caused by supplier specific parameters.

We have also compared the performance between JIT and VMI inventory systems. We found that in general, JIT systems fare better than VMI systems. However, JIT systems are inferior compared to VMI systems in cases where the variance of the demand/lead time parameters or the cost of JIT implementation is high.

Lastly, we examined the industrial practice of using a Uniform Minimum level policy for all suppliers, regardless of their different replenishment lead times. We find that the policy main objective is to reduce customer cost at the expense of increasing supplier system cost. Thus, this policy can only be implemented when customer have considerable purchasing power compared to supplier. Thus, the popularity of such an industrial policy infers that customer in a VMI relationship usually have a higher bargaining power compared to the supplier. However, this policy increases the system cost drastically to achieve the aim of reducing customer cost, which make it undesirable for suppliers in a

VMI relationship. In view of this problem, we propose various configurations to find a good alternative for the Uniform Minimum Policy.

## 6.3 Strategic Implications

In drawing the managerial implications from this study, it is important to emphasise that they relate to the practical decisions which are involved in VMI. In this aspect, the implications should concern with three main groups of people namely, (1) Vendor Hub Operators, (2) Suppliers adopting VMI and (3) Customers who are implementing VMI. The implications of each of the following will be discussed in the following sections.

### 6.3.1 Vendor Hub Operators

Vendor hub operators own and operate the vendor hub, which is the nerve centre in a VMI supply chain. Their priority is to ensure that the whole VMI supply chains operate efficiently and push the whole logistics cost incurred in the supply chain to the lowest.

Vendor hub operators must make various decisions that will in turn affect the cost and efficiency of the supply chain. On inventory decisions, other than obtaining the optimal stock up to level and shipment consolidation period, the vendor hub operator should also examined the feasibility of having an order splitting arrangement with the supplier so that its inventory cost can be kept low.

Other than inventory decisions, supplier selection decisions can also impact the overall cost and effectiveness of the VMI supply chain. Thus vendor hub operators must excise real caution in selection of suppliers. The vendor hub operators should not only consider the various supplier specific parameters alone as the basis of selecting supplier. They should instead strive to find a strategic match of their capabilities and resources with the various suppliers. In doing so, then the vendor hub operators will be assured that the best and right supplier is chosen, which in turn will lower the total logistical cost of the vendor hub operator

### 6.3.2 Suppliers

The suppliers in VMI strive to lower their cost and obtain the supplier contracts from the vendor hub operator. In a VMI arrangement, the replenishment decisions are made by the vendor hub operators. Thus, the supplier ability to reduce cost tends to be very limited. However, the suppliers can introduce certain policies that will attract vendor hub operators to order at quantities that are beneficial to them. One of such policies would be an order splitting arrangement. Order splitting would entice vendor hub operators to order at higher quantity, as the recommended quantity would be increased in an order splitting arrangement (Chiang, 1996). Order splitting would also make the suppliers more attractive to vendor hub operators as an order splitting arrangement is generally able to reduce the total logistical cost incurred by the vendor.

In making quotation and proposal to vendor hub operators for contract purpose, suppliers should note that other than price, quality and MOQ criteria are also equally important. In additional, suppliers should not fall into the thinking that supplier selection is based on supplier specific parameters only. If possible, suppliers should do some research into the vendor hub operators and attempt to offer the best terms based on the vendor hub own capabilities and resources.

Suppliers should also be made aware of the bargaining power of the customers when entering into a VMI relationship. Given the high bargaining power of the customers, the suppliers may be forced to enter a VMI arrangement that is unfavourable to them. Thus, suppliers may find it to their advantage if they could give some concessions to the suppliers so that the customers may implement policies that are more favourable to them. Alternatively, if they are able to propose policies that would reduce costs for themselves and yet without increasing cost for the customers, the customers would be more ready to accept the alternative kind of arrangement.

### 6.3.3 Customers

Customers are usually the initiators of a VMI arrangement. Thus, they are able to determine the policy parameters to start off with. Due to their bargaining power, they usually set the policy to their advantage. However, in the process, the suppliers are forced to adapt unfavourable polices that increase their cost greatly. For a VMI relationship to be successful, mutual trust between the suppliers and the customers is very important James et al. (2000). If the customer exploit its bargaining power when implementing VMI, then trust between the suppliers and customers would be very hard to be established. This might lead to the VMI arrangement to fail. Thus, customers should instead also take into account the supplier cost when they are drafting the basic guidelines for inventory polices and contracts in a VMI relationship. They should choose a beneficial inventory policy to both suppliers and customers.

Customers should also take extra caution when they are planning to implement any arrangement such as JIT or VMI with the suppliers. They should do a detailed analysis on the characteristics of the products before deciding which inventory management system to adopt as both of these inventory management systems have their own strength and weaknesses. When the right policy is chosen, they will be able to unlock all the potentials benefits of the policy into their logistical network.

From the results that we obtained, we propose a general guideline for customers to follow when they are considering using VMI or JIT inventory management systems. As a general rule, if the customer products are already in the maturity phase of the product life cycle (i.e. demand is stable), JIT should be chosen as the inventory management system to maximise profits. If the customer's products are in the introductory or growing stage in the product life cycle, VMI inventory management systems should be adopted due to their robustness and ability to adjust to any fluctuations in demand. If the customer is venturing into a new market and they are using new suppliers where the reliability of the suppliers is unknown, it will be more prudent for customers to adapt VMI inventory management systems. As a general rule, if the demand follows a distribution that have a very high standard deviation relative to its demand (High C.O.V of demand), VMI should be adopted. If the C.O.V of lead time is high, one should take a look at the C.O.V. of demand to determine whether VMI should be use instead of JIT. For easy referencing, we summarise our proposed guidelines into the Figures 62 and 63 grouped based on Product Life Cycle and various characteristics of the product.

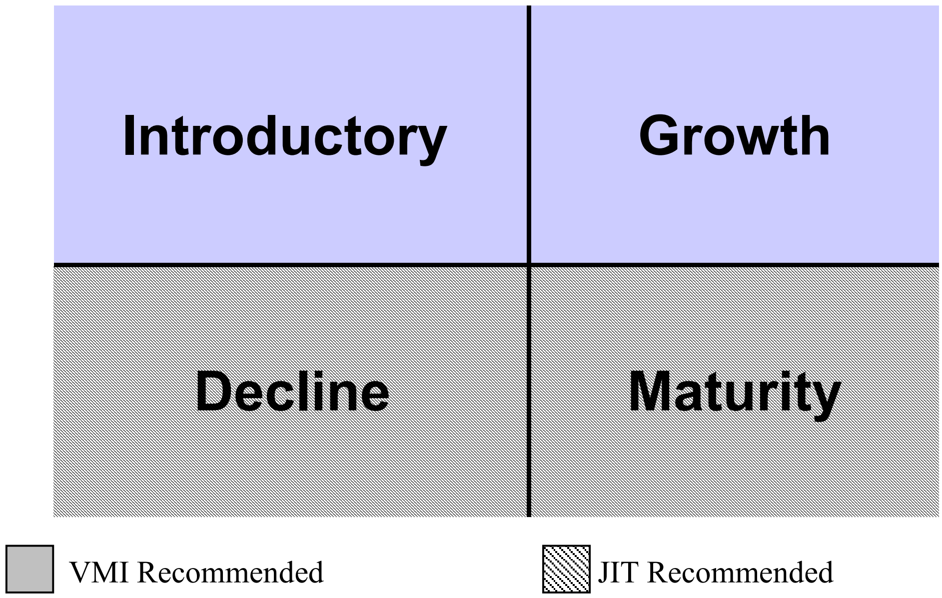


Figure 62: Proposed Guideline for Selecting VMI /JIT according to Product Life Cycle

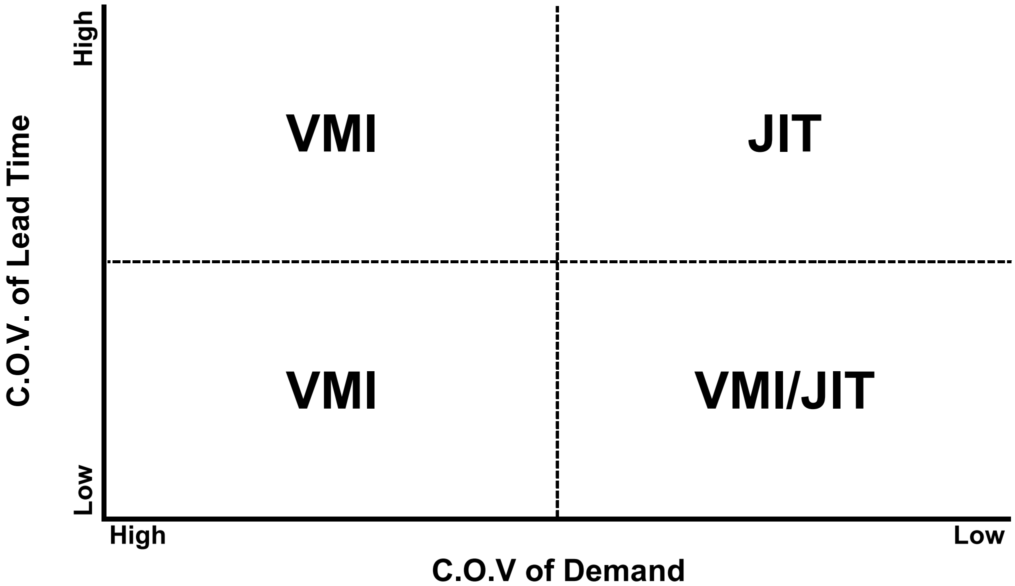


Figure 63: Proposed Guideline of Selecting JIT/VMI according to supply chain characteristics

## 6.4 Limitations of Study

This study has proposed an algorithm to obtain the optimum stock up to level and consolidation time for a VMI operator. Although efforts are made to ensure the validity of the algorithm proposed, there are some limitations that should be noted when analysing conclusion from this study.

Firstly, this study assumes that the lead time for replenishment is deterministic. In assuming deterministic lead time, the model has ignored the impact of unreliable suppliers on cost. Although we have examined the problem of stochastic lead time in our simulation model, we have not included this aspect in our mathematical model.

Secondly, we failed to examine the impact of obsolete cost of raw materials in our problem. Obsolete cost is a very large cost in supply chain involving with high tech products. Ignoring this part of the cost can mean a very big change in our solution

Thirdly, the mathematical model used in this study is based on a single item VMI operation. Thus, the solution derived from the mathematical model might not prove to be a good solution for the production hub scenario as multiple items are not considered in the model.

## 6.5 Recommendations for Future Research

On future research, one potential area to include is the lead time for delivery. With the inclusion of lead time, the heuristics provided for supplier selection would be more accurate and useful for VMI practitioners to adopt.

Another potential extension for this study is the development of an algorithm to minimise cost while keeping customer cost at a minimal. Though we have identified minimising customer cost being one of the main concerns of practitioners in the industry, we had not developed any solution to reduce supplier cost while minimising customer cost.

Lastly, we could include the impact of obsolescence of components to the supply chain. The impact of obsolescence could not be underestimated, especially with the shortening of the product life cycle of various products. Thus, the inclusion of obsolescence cost will provide valuable insights to both the academia and practitioners.

### 6.6 Conclusion

This study has attempted to provide an easy to use algorithm for VMI practitioners to use to optimise their inventory replenishment and delivery consolidation decision. It has extended the theoretical framework of Cetinkaya and Lee (2000) to enable it to incorporate several real life supply chain constraints and problems. The proposition of an order splitting strategy is also examined in this paper. In additional, various factors for supplier selections are also examined and interesting conclusions have been made on supplier selection criteria. Through this study, strategic implications are drawn for the various players involved in a VMI supply chain, in particular the vendor hub operators and the suppliers of a VMI supply chain.

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## Appendix A: Results Comparison for NPA and C&L

| Heuristic used | $A_R$ | $\bar{Q}$ | T | Average Cost ($) | Simulated Average Cost ($) |
|---|---|---|---|---|---|
| C&L | 50 | 11 | 0.645 | 280.96 | 214.53 |
| **NPA** | **50** | **8** | **0.6833** | **276.99** | **209.04** |
| C&L | 125 | 18 | 0.645 | 342.87 | 265.73 |
| **NPA** | **125** | **14** | **0.635** | **339.27** | **257.38** |
| C&L | 250 | 26 | 0.645 | 410.85 | 326.9 |
| **NPA** | **250** | **21** | **0.618** | **406.44** | **317.85** |
| C&L | 500 | 37 | 0.645 | 505.97 | 414.19 |
| **NPA** | **500** | **30** | **0.6** | **500.135** | **403.21** |
| C&L | 1000 | 52 | 0.645 | 639.77 | 542.4 |
| **NPA** | **1000** | **44** | **0.585** | **631.84** | **528.51** |

Table A1: Impact of Fixed Replenishment Cost on Average Cost

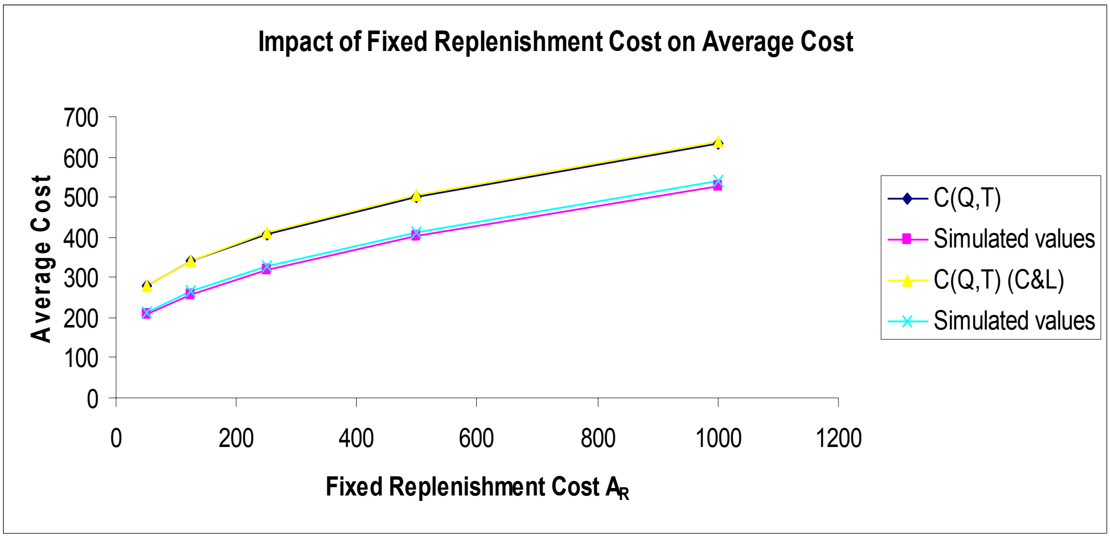


Figure A1: Impact of Fixed Replenishment Cost on Average Cost

| Heuristic used | $A_D$ | $\bar{Q}$ | T | Average Cost ($) | Simulated Average Cost ($) |
|---|---|---|---|---|---|
| C&L | 50 | 18 | 0.645 | 342.87 | 265.73 |
| NPA | 50 | 14 | 0.635 | 339.27 | 257.38 |
| C&L | 100 | 18 | 0.913 | 408.99 | 334.44 |
| NPA | 100 | 14 | 0.90 | 403.86 | 327.59 |
| C&L | 200 | 18 | 1.29 | 502.68 | 392.48 |
| NPA | 200 | 12 | 1.285 | 494.71 | 374.09 |
| C&L | 500 | 18 | 2.04 | 688.86 | 567.11 |
| NPA | 500 | 10 | 2.07 | 671.91 | 539.23 |
| C&L | 1000 | 18 | 2.86 | 898.91 | 772.19 |
| NPA | 1000 | 5 | 3.09 | 857.96 | 754.65 |

Table A2: Impact of Fixed Dispatch Cost on Average Cost

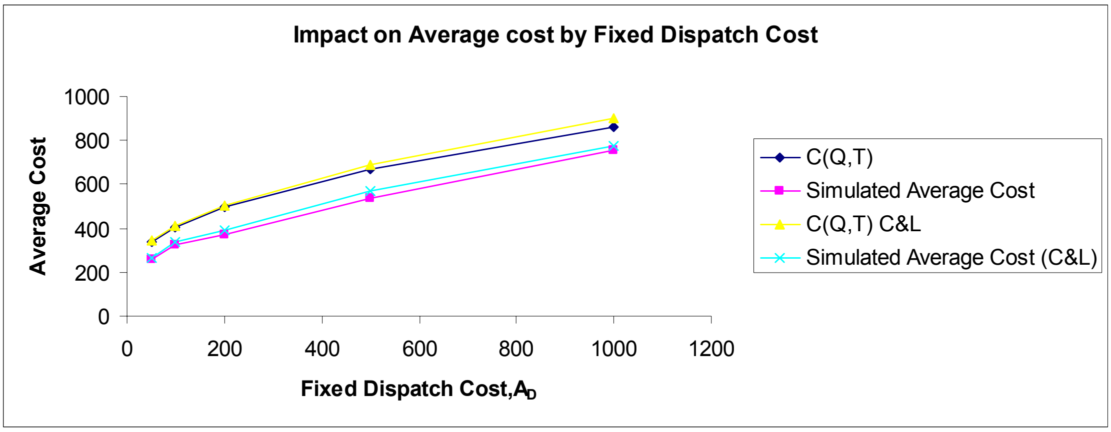


Figure A2: Impact of Fixed Dispatch Cost on Average Cost

| Heuristic used | h | $\hat{Q}$ | T | Average Cost ($) | Simulated Average Cost ($) |
|---|---|---|---|---|---|
| C&L | 7 | 18 | 0.645 | 342.87 | 265.73 |
| **NPA** | **7** | **14** | **0.635** | **339.27** | **257.38** |
| C&L | 14 | 12 | 0.513 | 421.4 | 331.95 |
| **NPA** | **14** | **11** | **0.526** | **419.64** | **328.2** |
| C&L | 28 | 8 | 0.389 | 542.57 | 453.81 |
| NPA | 28 | 8 | 0.415 | 541.13 | 453.81 |
| C&L | 50 | 6 | 0.302 | 684.96 | 595.53 |
| NPA | 50 | 6 | 0.331 | 682.76 | 595.33 |
| C&L | 100 | 4 | 0.218 | 914.65 | 866.89 |
| NPA | 100 | 4 | 0.251 | 909.51 | 866.89 |

Table A3: Impact of Holding Cost on Average Cost

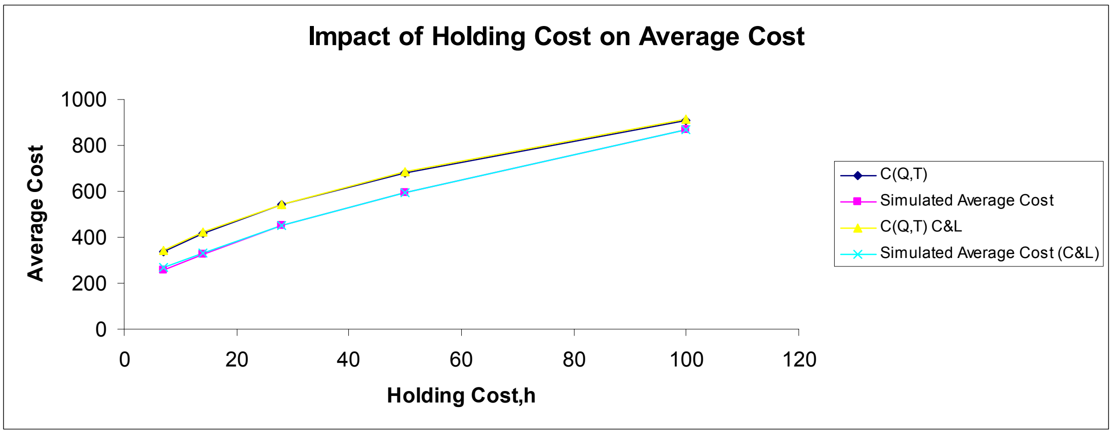


Figure A3: Impact of Holding Cost on Average Cost

| Heuristic used | g | $\hat{Q}$ | T | Average Cost ($) | Simulated Average Cost ($) |
|---|---|---|---|---|---|
| C&L | 3 | 18 | 0.645 | 342.87 | 265.73 |
| **NPA** | **3** | **14** | **0.635** | **339.27** | **257.38** |
| C&L | 7 | 18 | 0.645 | 361.09 | 286.62 |
| **NPA** | **7** | **12** | **0.63** | **345.94** | **267.47** |
| C&L | 14 | 18 | 0.645 | 392.99 | 324.5 |
| **NPA** | **14** | **9** | **0.656** | **345.33** | **280.01** |
| C&L | 28 | 18 | 0.645 | 456.8 | 395.8 |
| **NPA** | **28** | **3** | **0.799** | **270.17** | **315.37** |
| C&L | 50 | 18 | 0.645 | 557.06 | 509.8 |
| **NPA** | **50** | **8** | **0.283** | **397.38** | **370.9** |
| C&L | 100 | 18 | 0.645 | 782.04 | 773.9 |
| **NPA** | **100** | **8** | **0.211** | **436.36** | **412.77** |

Table A4: Impact of Penalty Cost on Average Cost

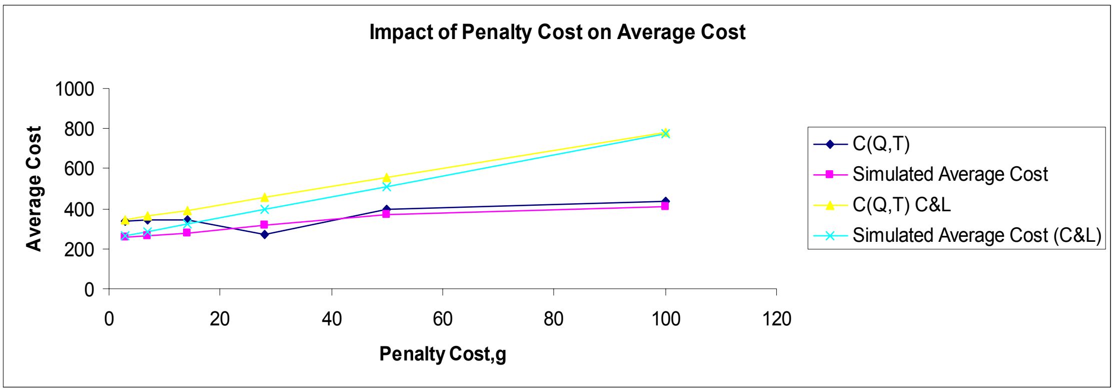


Figure A4: Impact of Penalty Cost on Average Cost

| Heuristic used | w | $\hat{Q}$ | T | Average Cost ($) | Simulated Average Cost ($) |
|---|---|---|---|---|---|
| C&L | 10 | 18 | 0.645 | 342.87 | 265.73 |
| **NPA** | **10** | **14** | **0.635** | **339.27** | **257.38** |
| C&L | 20 | 18 | 0.542 | 419.70 | 293.57 |
| **NPA** | **20** | **14** | **0.535** | **416.62** | **287.19** |
| C&L | 50 | 18 | 0.395 | 632.84 | 364.26 |
| **NPA** | **50** | **15** | **0.392** | **630.42** | **360.26** |
| C&L | 100 | 18 | 0.296 | 961.22 | 437.63 |
| **NPA** | **100** | **15** | **0.294** | **959.18** | **433.21** |
| C&L | 200 | 18 | 0.216 | 1576.88 | 548.89 |
| **NPA** | **200** | **15** | **0.215** | **1575.131** | **546.91** |

Table A5: Impact of Waiting Cost on Average Cost

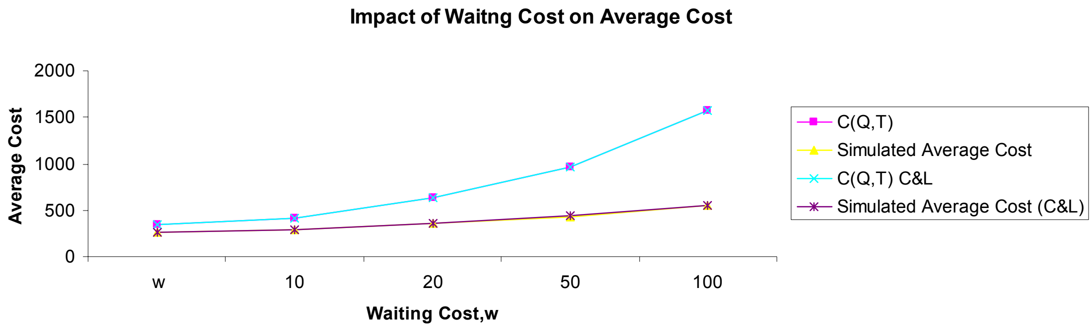


Figure A5: Impact of Waiting Cost on Average Cost

| Heuristic used | Ω | $\hat{Q}$ | T | Average Cost ($) | Simulated Average Cost ($) |
|---|---|---|---|---|---|
| C&L | 10 | 18 | 0.645 | 342.87 | 265.73 |
| **NPA** | **10** | **14** | **0.635** | **339.27** | **257.38** |
| C&L | 12 | 18 | 0.645 | 338.47 | 262.75 |
| **NPA** | **12** | **14** | **0.644** | **335.25** | **254.29** |
| C&L | 13 | 18 | 0.645 | 336.51 | 259.83 |
| **NPA** | **13** | **15** | **0.649** | **333.51** | **254.73** |
| C&L | 14 | 18 | 0.645 | 334.71 | 258.88 |
| **NPA** | **14** | **15** | **0.652** | **331.94** | **252.87** |
| C&L | 15 | 18 | 0.645 | 333.06 | 257.76 |
| **NPA** | **15** | **15** | **0.657** | **330.54** | **252.42** |

Table A6: Impact of Warehouse capacity on Average Cost

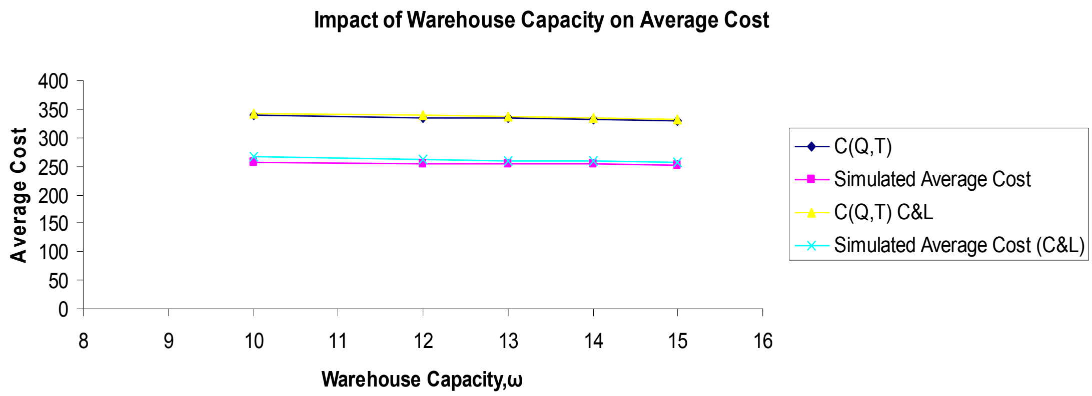


Figure A6: Impact of Warehouse Capacity on Average Cost

| Heuristic used | L | $\hat{Q}$ | T | Average Cost ($) | Simulated Average Cost ($) |
|---|---|---|---|---|---|
| C&L | 1 | 18 | 0.645 | 342.87 | 265.73 |
| **NPA** | **1** | **14** | **0.635** | **339.27** | **257.38** |
| C&L | 2 | 18 | 0.645 | 342.52 | 271.58 |
| **NPA** | **2** | **14** | **0.635** | **338.92** | **263.05** |
| C&L | 5 | 18 | 0.645 | 341.86 | 300.48 |
| **NPA** | **5** | **14** | **0.635** | **338.25** | **291.45** |
| C&L | 8 | 18 | 0.645 | 341.53 | 323.91 |
| **NPA** | **8** | **14** | **0.635** | **337.925** | **314.27** |
| C&L | 10 | 18 | 0.645 | 341.39 | 343.65 |
| **NPA** | **10** | **14** | **0.635** | **337.78** | **335.24** |

Table A7: Impact of Lead Time on Average Cost

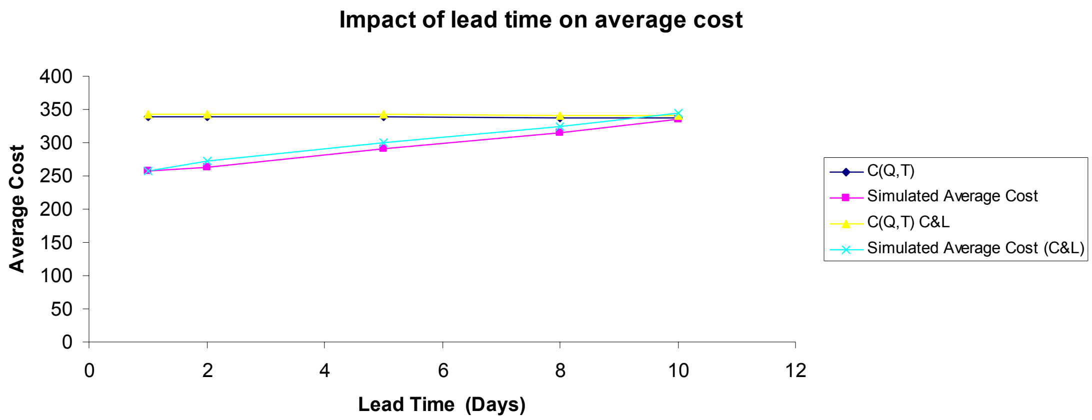


Figure A7: Impact of Lead Time on Average Cost

Appendix B: Simulated Results for Uniform Min/Max vs. NPA

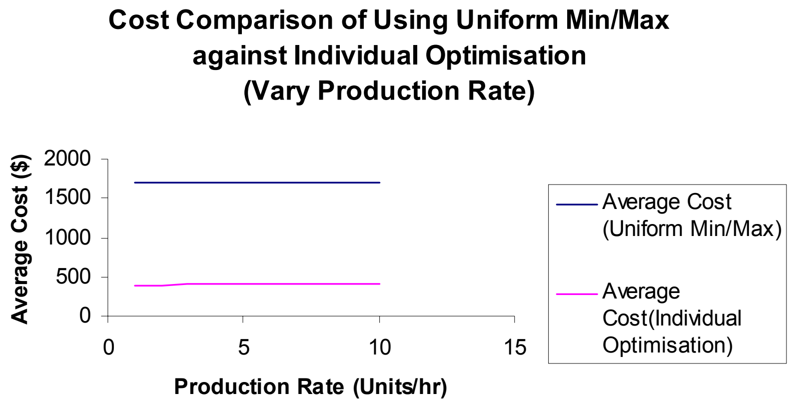


Figure B1: Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary Production Rate)

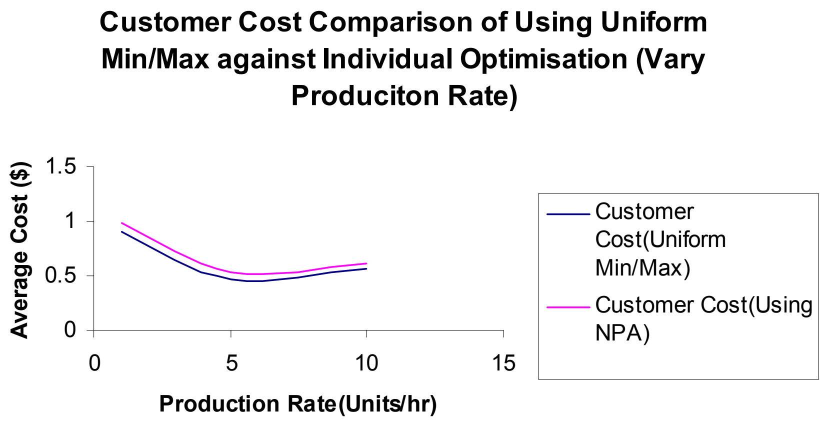


Figure B2: Customer Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary Production Rate)

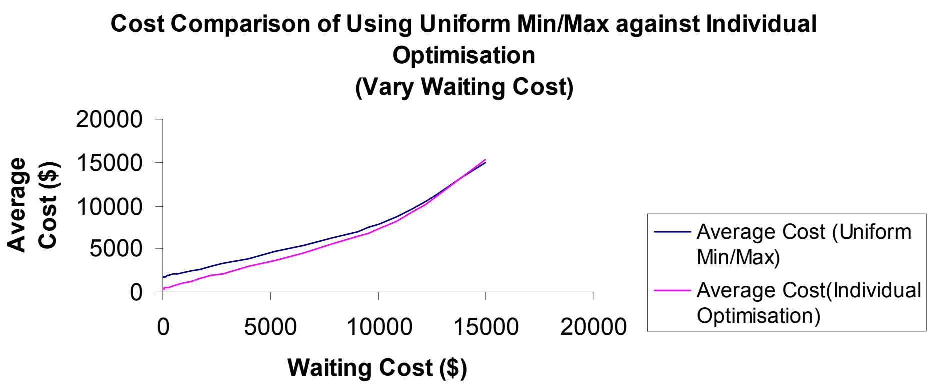


Figure B3: Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary Waiting Cost)

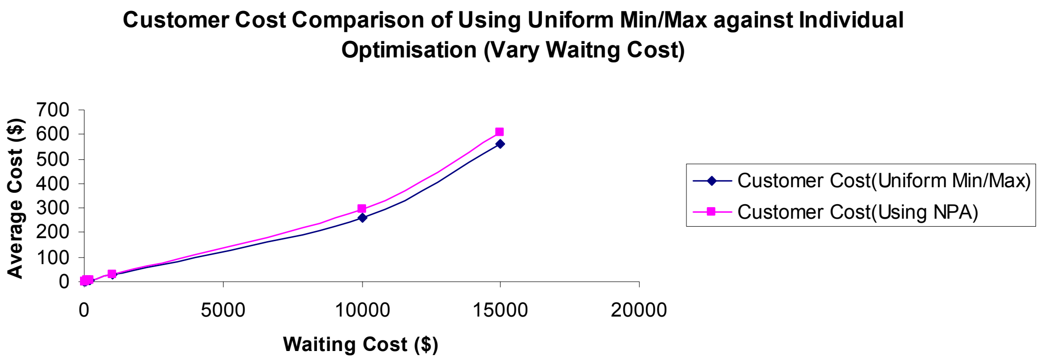


Figure B4: Customer Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary Waiting Cost)

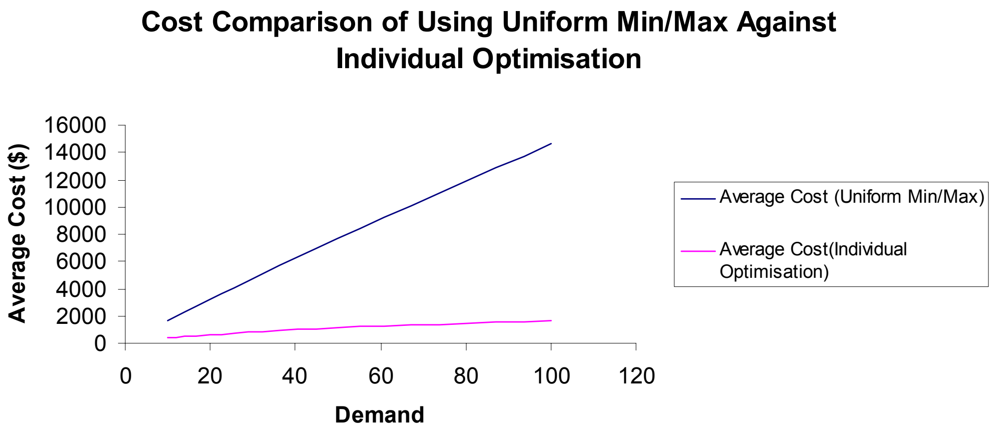


Figure B5: Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary Demand)

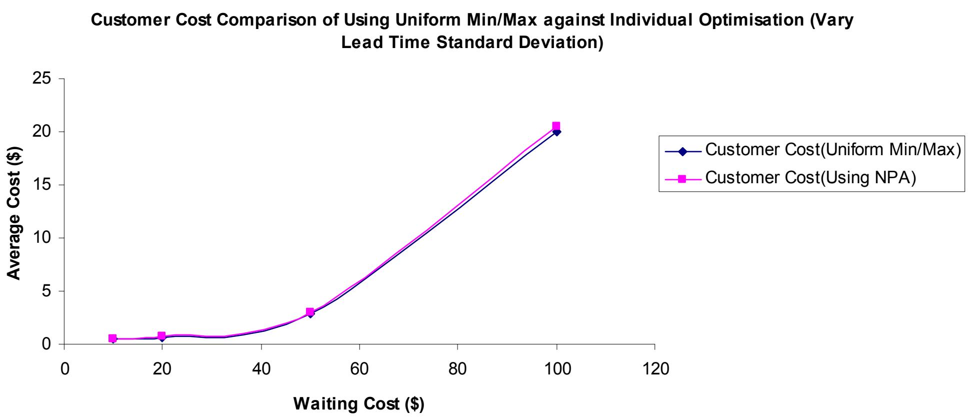


Figure B6: Customer Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary Demand)

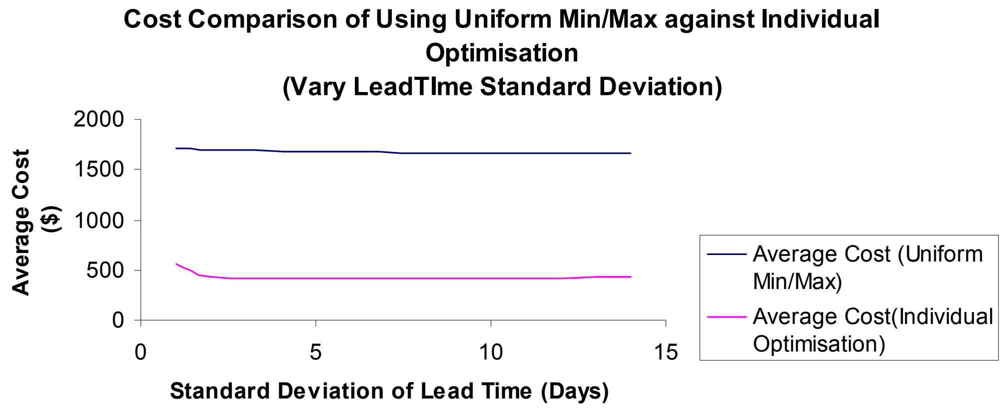


Figure B7: Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary S.D. for Lead Time)

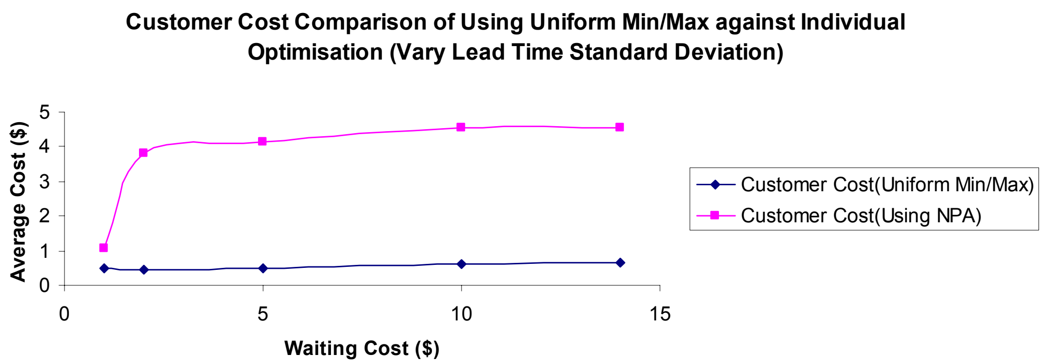


Figure B8: Customer Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary S.D. for Lead Time)

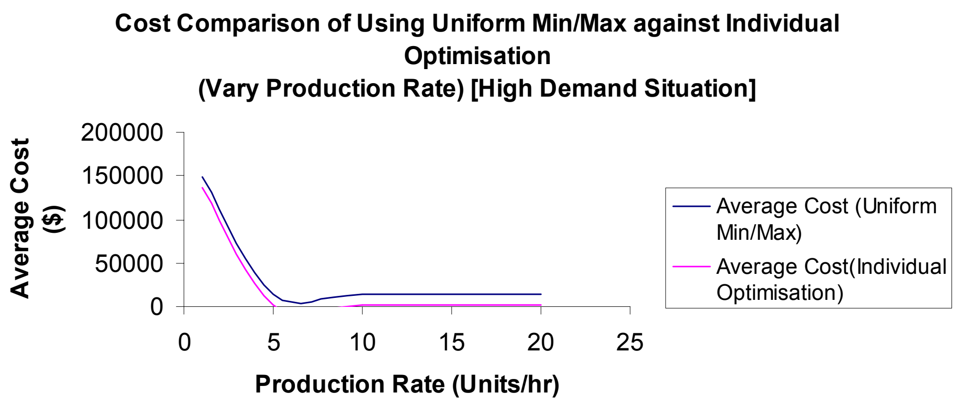


Figure B9: Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary Production Rate, High Lambda)

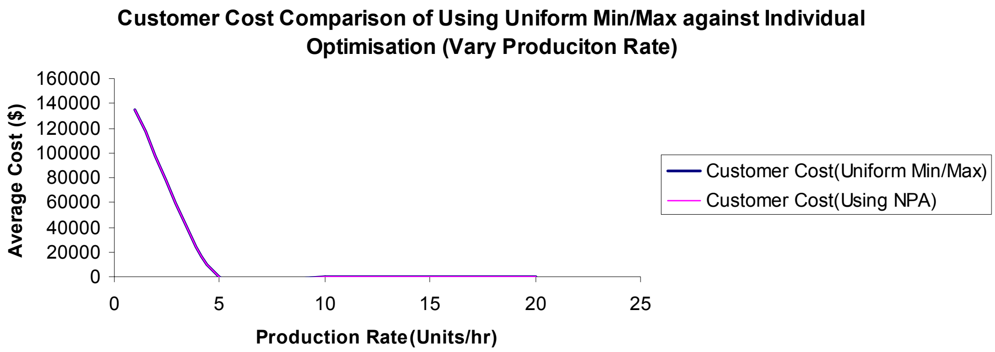


Figure B10: Customer Cost Comparison between Uniform and Non Uniform Inventory Policy (Vary Production Rate, High Lambda)