# A UNIFIED APPROACH FOR THE PERFORMANCE ANALYSIS OF UNITARY SPACE-TIME BLOCK CODES

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# SUMMARY

One effective technique to mitigate the effect of fading is time and frequency diversity. Besides that, in most scattering environments, antenna diversity is a practical, effective, and therefore widely used technique to reduce fading. The classical approach is to employ multiple antennas at the receiver and perform combining or selection and switching in order to improve the quality of the received signal.

Alamouti has proposed a simple transmit diversity scheme which improves the signal quality at the receiver on one side of the link by simple processing across two transmit antennas on the opposite side. The obtained diversity order is equal to that achieved by maximal-ratio receiver combining (MRRC) with two antennas at the receiver.

One assumption Alamouti made in his study is that channel information, in the form of amplitude and phase distortion, is known perfectly to the receiver. In practice, the issue of channel estimation is non-trivial, especially in a fading environment, where the fading gain can change substantially from one bit to the next. One may wish to find out how the performance of Alamouti's scheme will be degraded when channel estimation is imperfect, and closed form expressions for the bit error rate(BER) are also desirable. This thesis presents a new approach, based on quadratic forms of a complex Gaussian random vector, to analytically obtain the performance of various transmit diversity schemes under a variety of conditions. Specifically, we derive closed form expression for the BER under the following cir-

cumstances: perfect and imperfect channel estimation, spatial correlation, temporal correlation, different modulation schemes (e.g. BPSK and QPSK), Number of Tx and Rx antennas. It is also shown that the proposed approach can be used to analyze combinations of the above systems, e.g. QPSK modulated system with spatial correlation. We give one example of the exact performance of a 2 Tx and 2 Rx STBC system, with QPSK modulation, imperfect channel estimation and spatial correlation.

The main result of this project is presented in Chapter 4, "A Unified Approach for the Performance Analysis of Unitary Space-Time Block Codes".

# CHAPTER I

# Introduction

## 1.1 Motivation

The Next-Generation wireless systems are required to have high voice quality and provide high bit rate data services (up to 2 Mbits/s). The fundamental phenomenon which makes reliable wireless transmission difficult is time varying multipath fading. Increasing the quality or reducing the effective error rate in a multipath fading channel is extremely difficult. The improvement in SNR may not be achieved by higher transmit power or additional bandwidth, as it is contrary to the requirements of next generation systems. It is therefore crucial to effectively combat or reduce the effect of fading at both the remote and the base station, without additional power or bandwidth.

One effective technique to mitigate effect of fading is time and frequency diversity. Beside that, in most scattering environments, antenna diversity is a practical, effective, and therefore widely used technique for reducing fading. The classic approach is to install multiple antennas at the receiver and perform combining or selection and switching in order to improve the quality of received signal. Nowadays, however, the remote units are supposed to be small, lightweight, and elegant. It is, therefore, not practical to install multiple antennas on the remote units. As a result, diversity techniques have almost exclusively been applied to base stations to improve their reception quality. It is more economical to add equipment to base stations rather than the remote units. For this reason, transmit diversity schemes are very attractive.

In [1], Alamouti has proposed a simple transmit diversity scheme which improves the signal quality at the receiver on one side of the link by simple processing across two transmit antennas on the opposite side. The obtained diversity order is equal to applying maximal-ratio receiver combining (MRRC) with two antennas at the receiver. The scheme may easily be generalized to two transmit antennas and M receive antennas to provide a diversity order of 2M. This is done without any feedback from the receiver to the transmitter and with small computation complexity. The scheme requires no bandwidth expansion, as redundancy is applied in space across multiple antennas, not in time or frequency.

One assumption Alamouti made in his study is that channel information, in the forms of amplitude and phase distortion, is known perfectly to the receiver. In practice, the issue of channel estimation is non-trivial, especially in a fading environment where the fading gain can change substantially from one bit to the next. One may wish to find out how the performance of Alamouti's scheme will be degraded when channel estimation is imperfect, and closed form expression for BER is also desirable.

## 1.2 Thesis Objectives

The objective of this thesis is to study the impact of imperfect channel estimation on the error performance of the Alamouti's transmission scheme, and to derive closed form BER for various Space-Time Block Code systems. In [2], Buehrer and Kumar has derived a closed form expression for BER of a transmit diversity, block-fading, BPSK modulated STBC system. Much effort, therefore, has been devoted to develop a unified approach to solve more complicated scenario.

## 1.3 Thesis Organization

Chapter 1 of this thesis starts off with an introduction to this project, and then Chapter 2 gives a brief introduction to space time block codes, and Alamouti's scheme is illustrated by a $2 \times 2$ antennas' example, and then two pilot-aided channel estimation strategies for this system are proposed, one based on the decorrelator concept, the other based on the minimum mean square error (MMSE) concept. In both cases, the importance of selecting a proper pilot sequence for channel estimation is illustrated. In Chapter 3 various techniques to analyze effect of imperfect channel estimation are discussed. In Chapter 4, a unified approach for the performance analysis of Unitary Space-Time Block Codes is proposed to solve more complicated problem, and to derive closed form BER of several STBC systems. Chapter 5 concludes the whole project, and discusses several problems left to be solved.

## 1.4 Thesis Contributions

In the project we propose a unified approach to analytically obtain the bit error probability of various transmit diversity schemes under a variety of conditions. we derive closed form expression for the BER under the following circumstances: perfect and imperfect channel estimation, spatial correlation, temporal correlation, different modulation schemes (e.g. BPSK and QPSK), Number of Tx and Rx antennas. It is also shown that the proposed approach can be used to analyze combinations of the above systems, e.g. QPSK modulated system with spatial correlation. We give one example of the exact performance of a 2 Tx and 2 Rx STBC system, with QPSK modulation, imperfect channel estimation and spatial correlation.

# CHAPTER II

# Space Time Block Codes

## 2.1 Introduction

In most situations, the wireless channel suffers attenuation due to destructive addition of multipaths in the propagation media and to interference from other users. Diversity technique provides some less attenuated replica of the transmitted signal to the receiver, which makes it easier for the receiver to reliably determine the correct signal transmitted. Diversity can be provided using temporal, frequency, polarization, and spatial resources. Some interesting approaches for transmit diversity have been suggested by Wittneben [3], [4] for base station simulcasting and later, independently, a similar scheme was suggested by Seshadri and Winters [5] [6]. Later Foschini introduced a multilayered space-time architecture [7]. More recently, space-time trellis coding has been proposed [8] which combines signal processing at the receiver with coding techniques appropriate to multiple transmit antennas.

In [1], Alamouti proposed a simple transmit diversity technique that can provide the same diversity order as maximal-ratio receiver combining (MRRC). One assumption Alamouti made in his study is that channel information, in the forms of amplitude and phase distortion, is known perfectly to the receiver. In practice, the issue of channel estimation is non-trivial, especially in a fading environment where the fading gain can change substantially from one bit to the next. Recently a new class of pilot assisted channel estimation schemes has been proposed in [9], where pilot symbols are superimposed on the data symbols.

The objective of this investigation is to study the impact of imperfect channel estimation on the error performance of the Alamouti's transmission scheme. As in [1], we consider a simple system consisting of two transmit and two receive antennae. For convenience we will refer to this system as the 2 × 2 system. In the first part of this investigation, we propose two pilot-aided channel estimation strategies for this 2 × 2 system, one based on the decorrelator concept, the other based on the minimum mean square error (MMSE) concept. In both cases, we illustrate the importance of selecting a proper pilot sequence for channel estimation. In the second part of this investigation, we outline the approach that we will adopt in relating the performance of the channel estimator to the pairwise error probability of the receiver.

## 2.2 Alamouti's $2 \times 2$ Scheme

Figure 2.1 shows the block diagram of Alamouti's $2 \times 2$ scheme. For comparison, Figure 2.2 shows two branch MRRC diagrams. The $h_i$s, $i = 0, 1, 2, 3$ represent the fading gains in the four physical links. On the other hand, the $n_i$s are the noise terms in these links. All the $h_i$s and $n_i$s are zero mean complex Gaussian random variables.

Let $t$ and $t + T$ be two consecutive transmission instants. The four symbols transmitted by Tx Antenna 0 and Tx Antenna 1 at these two instants are related to each other according to Table 2.1, where * represents complex conjugation. The corresponding received symbols at Rx Antenna 0 and Rx Antenna 1 are shown in Table 2.2. Note that $s_0$ and $s_1$ are binary random variables with a sample space

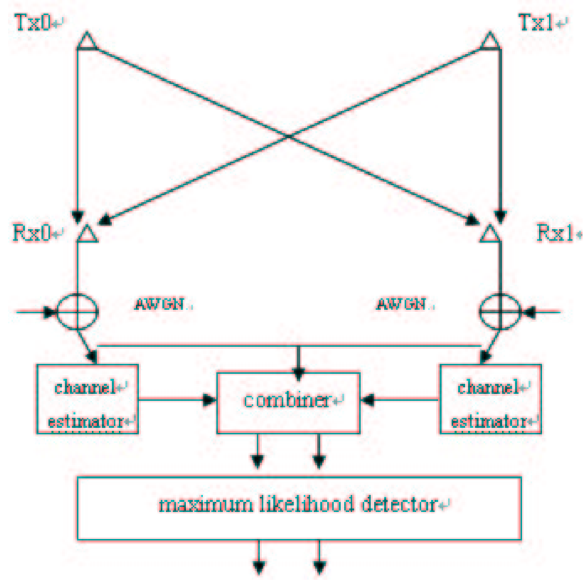


Figure 2.1: Two-branch transmit diversity scheme with two receivers

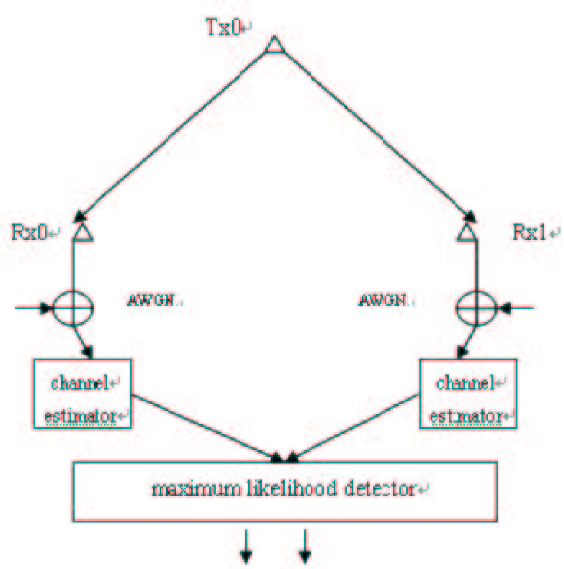


Figure 2.2: Two Branch MRRC

$\{\pm 1\}$. This stems from the fact that we assume binary PSK modulation.

| | tx antenna 0 | tx antenna 1 |
|---|---|---|
| Time $t$ | $s_0$ | $s_1$ |
| Time $t+T$ | $-{s_1}^*$ | ${s_0}^*$ |

Table 2.1: The encoding and transmission sequence for the two-branch transmit diversity scheme

| | rx antenna 0 | rx antenna 1 |
|---|---|---|
| Time $t$ | $r_0$ | $r_2$ |
| Time $t+T$ | $r_1$ | $r_3$ |

Table 2.2: Notations for received signals at two receive antennas

The received symbols in Table 2.2 have the signal structure:

$$r_0 = h_0 s_0 + h_1 s_1 + n_0 \tag{2.1}$$

$$r_1 = -h_0 {s_1}^* + h_1 s_0^* + n_1 \tag{2.2}$$

$$r_2 = h_2 s_0 + h_3 s_1 + n_2 \tag{2.3}$$

$$r_3 = -h_2 {s_1}^* + h_3 s_0^* + n_3 \tag{2.4}$$

If the fading gains $h_i$s, $i = 0, 1, 2, 3$, are known to the receiver, then the received symbols can be combined according to

$$\tilde{s}_0 = h_0^* r_0 + h_1 r_1^* + h_2^* r_2 + h_3 r_3^* \tag{2.5}$$

$$\tilde{s}_1 = h_1^* r_0 - h_0 r_1^* + h_3^* r_2 + h_2 r_3^* \tag{2.6}$$

If $\widetilde{s_0} > 0$, the receiver decides that $s_0 = 1$, else it decides that $s_0 = -1$. Similarly, if $\widetilde{s_1} > 0$, the receiver decides that $s_1 = 1$, else it decides that $s_1 = -1$.

Figure 2.3 shows BER performance comparison of coherent BPSK with MRRC and two-branch transmit diversity. Note that MRRC (1 Tx, 2 Rx ) and New scheme

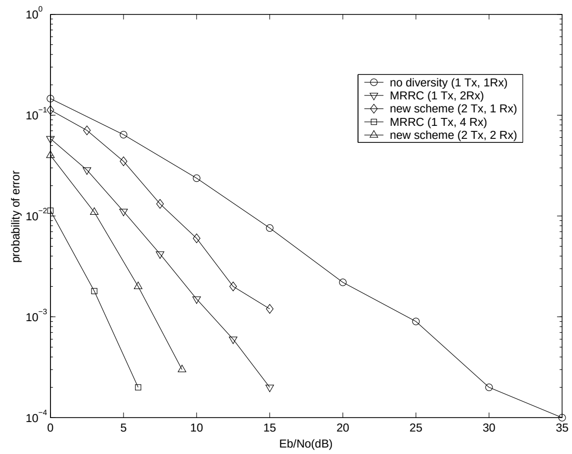


Figure 2.3: The BER performance comparison of coherent BPSK with MRRC and two-branch transmit diversity in Rayleigh fading

(2 Tx, 1 Rx) are equivalent, and MRRC (1 Tx, 4 Rx ) and New scheme (2 Tx, 2 Rx) are equivalent; except a 3 dB gap between, because transmit power is split half on two Tx for the new scheme.

## 2.3 Space-Time Block Codes from Orthogonal Designs

Alamouti [1] has discovered a remarkable scheme for transmission using two transmit antennas. This scheme is much less complex than space-time trellis coding for two transmit antennas but there is a loss in performance compared to space-time trellis codes. Despite this performance penalty, Alamouti's scheme is still appealing in terms of simplicity and performance. The works in [10] then applied the classical mathematical framework of orthogonal designs to construct space-time block codes. It is shown that space-time block codes constructed in this way only exist for few sporadic value of $n$. Subsequently, a generalization of orthogonal designs is shown to provide space-time block codes for both real and complex constellations for any number of transmit antennas. These codes achieve the maximum possible transmission rate for any number of transmit antennas using any arbitrary real constellation such as PAM. For an arbitrary complex constellation such as PSK and QAM, space-time block codes are designed that achieve 1/2 of the maximum possible transmission rate for any number of transmit antennas.The best tradeoff between the decoding delay and the number of transmit antennas is also computed in [10]. Some schemes in [10] will be discussed and analyzed in details in the later part of this thesis.

## 2.4 Two Pilot-aided Channel Estimation Strategies

The received signals in Section 2.2 can be written in matrix form[1] as

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \end{bmatrix} = \begin{bmatrix} s_0 & s_2 & 0 & 0 \\ s_1 & s_3 & 0 & 0 \\ 0 & 0 & s_0 & s_2 \\ 0 & 0 & s_1 & s_3 \end{bmatrix} * \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ n_2 \\ n_3 \end{bmatrix} \tag{2.7}$$

or

$$\mathbf{r} = X * \mathbf{h} + \mathbf{n} \tag{2.8}$$

In a pilot-based channel estimator, the signal matrix $X$ is known and the objective is to estimate the fading gain vector $\mathbf{h}$ as accurately as possible from the receive vector $\mathbf{r}$

We propose two methods to estimate $\mathbf{h}$.

*Decorrelator Approach*: The estimate of $\mathbf{h}$ is

$$\hat{\mathbf{h}} = X^{-1}\mathbf{r} = \mathbf{h} + X^{-1}\mathbf{n} \tag{2.9}$$

and the error is

[1] Here we use capital letters to represent matrices, and bold small letters to represent vectors.

$$\mathbf{e} = \hat{\mathbf{h}} - \mathbf{h} \tag{2.10}$$

Then the mean square estimation error is

$$\varepsilon^2 = \frac{1}{2} trace\{E[\mathbf{ee}^*]\} = \mathbf{N_0} * \mathbf{trace}\{\mathbf{X^{-1}}(\mathbf{X^{-1}})^*\} \tag{2.11}$$

where $N_0$ is noise power.

Since we assume BPSK modulation, there are 16 possibilities for the pilot symbol matrix X. Naturally we would like to use the pilot pattern(s) that minimizes the estimation error. It was found that half of these 16 patterns are non-invertible; therefore they are irrelevant to this method. As for the remaining 8 patterns, they yield the same mean squared estimation error.

*The MMSE Approach*: The decorrelator approach described above can be formulated as $\hat{\mathbf{h}} = T\mathbf{r}$, with $T = X^{-1}$. In principle, we should always choose the transformation matrix $T$ that minimizes the means square estimation error (MMSE). It can be shown the optimal $T$ matrix is

$$T = (\Phi_h X^*)(X \Phi_h X^* + \Phi_n)^{-1} \tag{2.12}$$

where $\Phi_h$ and $\Phi_n$ are the covariance matrices of the fading gains and noise terms respectively. Then the corresponding mean squared estimation error is

$$\varepsilon^2 = \frac{1}{2} trace\{E[\mathbf{ee}^*]\} = \mathbf{trace}((\mathbf{I} - \mathbf{T} * \mathbf{X})\mathbf{\Phi_h}(\mathbf{I} - \mathbf{T} * \mathbf{X})^* + \mathbf{T}\mathbf{\Phi_n}\mathbf{T}^*) \tag{2.13}$$

where $I$ is identity matrix. As in the decorrelator approach, we should use the pilot pattern(s) $X$ that minimize $\varepsilon^2$. This time though, every possible pilot pattern $X$ has a valid $T$ matrix associated with it.

We plot in Figure 2.4 the mean square channel estimation error normalized with total noise power:

$$\varepsilon_0^2 = \frac{\varepsilon^2}{4N_0} \tag{2.14}$$

(a factor of 4 is due to the fact that we add up noise components in all 4 links ) as a function of the signal-to-noise ratio (SNR)

$$\Gamma = \frac{trace\{E[Xh(Xh)^*]\}}{4N_0} = \frac{\varepsilon_s^2}{4N_0} \tag{2.15}$$

( where $\varepsilon_s^2$ is defined as total received signal power ) of the system. In this study, we assume all the links have the same power. For the decorrelator approach, we only present the result for pilot patterns that are invertible. For the MMSE approach, we present two sets of results, one for those pilot patterns that are invertible, and another set for those that are not. It is observed that there is a big difference in performance between the two sets of pilot patterns. If we compare the best results obtained under the MMSE criterion against those obtained under the decorrelator approach, we can conclude that there is not much of a difference between the two at large SNR. There is a noticeable difference at low SNR though.

It is also noted that for both approaches with invertible patterns, normalized estimation error will continuously decrease as SNR increases. In the extreme case where SNR is infinitely large, estimation error will vanish. Therefore both approaches are good estimators.

MMSE approach outperforms decorrelator approach in two aspects: (1) certain patterns of pilot matrix achieve better estimation result, as shown in Figure 2.4. (2) all patterns are usable, unlike decorrelator approach, where only invertible patterns can be used.

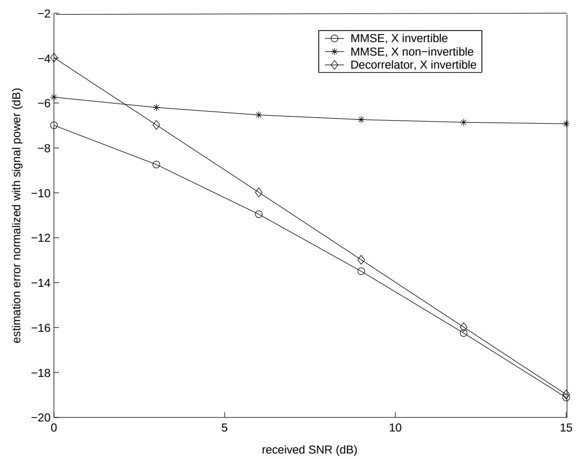


Figure 2.4: Comparison of the decorrelating and MMSE approaches for channel estimation.

On the other hand, decorrelator approach also has two advantages over MMSE approach: (1) it is easy to implement, and we need only to compute $X^{-1}$, while MMSE approach requires greater computational complexity. (2) normalized estimation error is linearly changing with SNR, and this fact makes system design more easily.

Once the receiver obtains channel estimates at the pilot symbol instants, interpolation is used to obtain estimates of the fading gains that affect the data symbols. For brevity, the details are not provided here.

# CHAPTER III

# MIMO Systems with STBC

## 3.1 Introduction

One assumption Alamouti made in his study is that channel information, in the forms of amplitude and phase distortion, is known perfectly to the receiver. In practice, the issue of channel estimation is non-trivial, especially in a fading environment where the fading gain can change substantially from one bit to the next. Given imperfect channel estimates from pilot symbol assisted modulation (PSAM), one may wish to find how estimation error can affect bit error performance of STBC. The author has done a literature survey on how others deal with this problem. The following sections reviews other's findings, as well as the author's attempt to make use of these results to achieve his goal.

## 3.2 Pairwise Error Probability

[11] describes a simple technique for the numerical calculation, within any desired degree of accuracy, of the pairwise error probability (PEP) of space-time codes. This method applies also to the calculation of $E[Q(\sqrt{\xi})]$ for a non-negative random variable whose moment-generating function $\phi_\xi(s) = E[exp(-s\xi)]$ is known.

Consider the computation of the expectation

$$P \triangleq E[Q(\sqrt{\xi})] \tag{3.1}$$

where $Q(x) = P(\nu > x)$, with real Gaussian random variable with mean zero and unit variance, and $\xi$ is a nonnegative random variable (independent of $\nu$). A simple method is advocated to obtain the value of $P$ based on numerical integration. Assume that the MGF of $\xi$

$$\phi_\xi(s) \triangleq E[exp(-s\xi)] \tag{3.2}$$

is known. In this case, we have

$$E[Q(\sqrt{\xi})] = P(\Delta < 0) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{\phi_\Delta(s)}{s} ds \tag{3.3}$$

where $\Delta = \xi - \nu^2$ . It is straightforward to obtain

$$\begin{aligned} \phi_\Delta(s) &= E[exp(-s\Delta)] \\ &= \phi_\xi(s)\phi_{\nu^2}(-s) \\ &= \phi_\xi(s)(1-2s)^{-1/2} \end{aligned} \tag{3.4}$$

Hence,

$$E[Q(\sqrt{\xi})] = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} \frac{\phi_\xi(s)}{2s}(1-2s)^{-1/2} ds \tag{3.5}$$

Here we assume that $c$ is in the region of convergence (ROC) of $\phi_\Delta(s)$

Finally we use a Gauss-Chebyshev numerical quadrature rule to obtain a numerical result. Then it can be applied to the calculation of PEP error performance analysis of a multiple-antenna fading channel in two cases of fading distribution.

1) Independent fading (IF): we assume that the transmitted symbols in a codeword are affected by independent fading realizations. 2) Block fading (BF): we assume that the transmitted symbols in a codeword are affected by the same fading realization. In both cases, the PEP can be expressed as in (3.1) by suitably defining

the random variable $\xi$. We assume $t$ transmit and $r$ receive antennas, and a code with block length $N$.

A. Independent Fading Channel

The discrete-time low-pass equivalent channel equation can be written as

$$y_i = H_i x_i + z_i, i = 1......N \tag{3.6}$$

where $H_i \in C^{r\times t}$ is the $i$th channel gain matrix, $x_i \in C^{r\times t}$ is the $i$th transmitted symbol vector (each entry transmitted from a different antenna), $y_i \in C^{r\times t}$ is the $i$th received sample vector each entry received from a different antenna), and $z_i \in C^{r\times t}$ is the $i$th received noise sample vector (each entry received from a different antenna). We assume that the channel gain matrices $H_i$ are element-wise independent and independent of each other with $[H_i]_{jk} \sim N_c(0,1)$, i.e., each element is circularly Gaussian distributed with mean zero and variance $E[|[H_i]_{jk}|^2] = 1$. Also, the noise samples are independent with $[z]_i \sim N_c(0, N_0)$.

It is straightforward to obtain the PEP as follows:

$$\begin{aligned} P(X \to \hat{X}) & \\ &= P(\sum_{i=1}^{N}\{||y_i - H_i\hat{x}_i||^2 - ||y_i - H_i x_i||^2\} < 0) \\ &= P(\sum_{i=1}^{N}\{||H_i(x_i - \hat{x}_i) + z_i||^2 - ||z_i||^2\} < 0) \\ &= E\left[Q\left(\sqrt{\frac{1}{2N_0}\sum_{i=1}^{N}||H_i(x_i - \hat{x}_i)||^2}\right)\right] \end{aligned} \tag{3.7}$$

B. Block-Fading Channel

Here we assume that the channel gain matrices $H_i$ coincide and are equal to $H$:

under this assumption, the channel equation can be written compactly as follows:

$$Y = HX + Z \tag{3.8}$$

where $H \in C^{r\times t}$, $X = (x_1 ... x_N) \in C^{t\times N}$, $Y \in C^{r\times N}$, and $Z \in C^{r\times N}$. We assume independent and identically distributed (i.i.d.) entries

$[H]_{ij} \sim N_c(0,1)$ $E[|[H_i]_{jk}|^2] = 1$

and i.i.d. $[Z]_i j \sim N_c(0, N_0)$. We obtain, after straightforward calculations

$$P(X \to \hat{X}) = E\left[Q\left(\frac{||H\Delta||}{\sqrt{2N_0}}\right)\right] \tag{3.9}$$

where $\Delta \triangleq X - \hat{H}$

C. Evaluation of the Moment-Generating Function

Setting

$$\xi \triangleq \begin{cases} \sqrt{\frac{1}{2N_0}\sum_{i=1}^{N} ||H_i(x_i - \hat{x}_i)||^2} & \text{IF channel} \\ \frac{||H\Delta||}{\sqrt{2N_0}} & \text{BF channel} \end{cases} \tag{3.10}$$

Then we can evaluate the PEP by resorting to (3.1).

D. In case of imperfect channel estimation

In (3.7) and (3.9), $H$ need to be replaced by $\hat{H}$, which is the estimated version of $H$.

## 3.3 Imperfect Channel Estimation in Single Antenna System

Consider a single Tx and single Rx system:

$$r = hs + n \tag{3.11}$$

where $h$ is channel gain, $s$ is transmitted signal (assuming BPSK modulation), $n$ is AWGN, and $r$ is the received signal.

Given $\hat{h}$, channel estimate from PSAM, the receiver makes decision by doing the following:

$$z = Re[\hat{h}'r] \tag{3.12}$$

If the transmitted bit is +1, then an error is made if the real part of the decision variable $z$ is negative. Since $\hat{h}$ and $r$ are correlated, zero mean, Gaussian random variables, we can use a standard result for the probability of this event[12]

$$P_b = \frac{1}{2}\left(1 - \sqrt{Re[\rho]^2/(1 - Im[\rho]^2)}\right) \tag{3.13}$$

where $\rho$ is the correlation coefficient between $\hat{h}$ and $r$.

In the case of optimum filter, the correlation coefficient is real, then the error probability becomes:

$$P_b(k) = (1 - \rho)/2 \tag{3.14}$$

A detailed analysis of PSAM can be found in [13]. Then the author wishes to extend this result to MIMO systems with STBC.

## 3.4 Imperfect Channel Estimation in MIMO System with STBC

According to Appendix C.2 in [12], a method has been used in evaluating bit error probability in the general case of diversity reception. This method is as follows: consider a complex Gaussian Random Variable

$$\begin{aligned} z &= \sum_{i=1}^{L} X_k {Y_k}^* \\ &= z_r + j z_i \end{aligned} \tag{3.15}$$

where the random variable $(X_k, Y_k)$ are correlated, complex-correlated, zero-mean, Gaussian, and statistically independent, but identically distributed with any other pair $(X_k, Y_k)$,and phase of $z$ is

$$\Theta = tan^{-1}(\frac{z_i}{z_r}) \tag{3.16}$$

Probability density function of the phase $\Theta$ can be derived according to [24]. Note that for this to be applied, the following conditions must be satisfied:

$$m_{xx} = E(|X_k|^2), \text{identical for all } k \tag{3.17}$$

$$m_{yy} = E(|Y_k|^2), \text{identical for all } k \tag{3.18}$$

$$m_{xy} = E(X_k {Y_k}^*), \text{identical for all } k \tag{3.19}$$

For a PSK modulated system, the phase $\Theta$ is the decision variable. For BPSK, particularly, $z_r$,the real part of $z$, is the decision variable.

It is clear that section 3.3 made use of this result by setting L = 1, i.e., there is only one pair.

It can be shown that this result can also be applied to receiver diversity system. Take a system of 1 Tx and 2 Rx antennas for example, the signals received at two receivers are:

$$r_0 = h_0 s_0 + n_0 \tag{3.20}$$

$$r_1 = h_1 s_0 + n_1 \tag{3.21}$$

Assume perfect estimation, the receiver combining scheme for two branch MRRC is as follows:

$$\begin{aligned} \tilde{s}_0 &= {h_0}^* r_0 + {h_1}^* r_1 \\ &= (|h_0|^2 + |h_1|^2) s_0 + {h_0}^* n_0 + {h_1}^* n_1 \end{aligned} \tag{3.22}$$

Because the two pairs $(h_0, r_0)$, $(h_1, r_1)$ match the criteria mentioned above, we can use that method to determine BER.

Consider Alamouti's scheme of 2 Tx and 1 Rx in [1], the signals received in two intervals are:

$$r_0 = h_0 s_0 + h_1 s_1 + n_0 \tag{3.23}$$

$$r_1 = -h_0 (s_1)^* + h_1 (s_0)^* + n_1 \tag{3.24}$$

Assume perfect estimation, the receiver combining scheme for $s_0$ is,

$$\begin{aligned} \tilde{s}_0 &= {h_0}^* r_0 + h_1 {r_1}^* \\ &= (|h_0|^2 + |h_1|^2) s_0 + {h_0}^* n_0 + h_1 {n_1}^* \end{aligned} \tag{3.25}$$

We find that (3.22) and (3.25) is actually equivalent, which is supported by simulation result. The only difference is a 3-dB gap, because in Alamouti's scheme, transmit power is split half at two transmitters.

Therefore, given perfect channel estimates, Alamouti's transmit diversity scheme can also be analyzed using the above method.

If channel estimation is imperfect in Alamouti's scheme,

$$\tilde{s}_0 = \hat{h}_0^* r_0 + \hat{h}_1 {r_1}^* \tag{3.26}$$

which is no longer equivalent to (3.25). Therefore the above method is no longer applicable and other approaches are needed. Such an approach is presented in the next chapter, which presents a unified approach to evaluating the performance of STBC in the presence of channel estimation error with both spatial and temporal correlation, as well as with different modulation schemes.

# CHAPTER IV

# A Unified Approach for the Performance Analysis of Unitary Space-Time Block Codes

## 4.1 Introduction

Space-time block coding (STBC) has recently emerged as a promising technique to exploit transmit antenna diversity. In [1], Alamouti proposed a simple transmit diversity technique that can provide the same diversity order as maximal-ratio receiver combining (MRRC). Tarokh [10], using the theory of orthogonal designs, generalized these results to an arbitrary number of transmit antennas and constructed codes able to achieve the full diversity promised by multiple transmit and receive antennas. The work in [15] reviews the encoding and decoding algorithms for various codes and provide simulation results demonstrating their performance.

The appeal of STBC is that when the codes satisfy certain orthogonality properties, there exist simple maximum likelihood decoding algorithms based only on linear processing at the receiver. A critical assumption for optimal linear decoding, also made in [1],[8], and[10], is that channel state information (CSI), both amplitude and phase distortion, is known perfectly to the receiver. The main motivation behind this thesis is to evaluate the performance of optimal linear decoding in the presence of imperfect channel state information. In addition, we consider cases where there is spatial correlation (between transmit antennas), temporal correlation (between the fading gains of symbols within one block), different modulation schemes (e.g., BPSK and QPSK), different number of Tx and Rx antennas, and varying code rates.

Recent work on the performance evaluation of STBC includes [11], which computes the exact pairwise error probability (PEP) of STBC with perfect CSI. In practice, the issue of channel estimation is non-trivial, especially in a fading environment where the fading gain can change substantially from one symbol to the next. In [16], an approach based on the quadratic form of a complex Gaussian random vectors (CGRV), is used to compute the pairwise error probability for any coherently demodulated system in arbitrarily correlated Rayleigh fading. The work in [17] also computes the pairwise error probability for space-time codes under coherent and differentially coherent decoding. All of these works assume perfect CSI.

In [2],the authors use the quadratic form of the CGRV to analyze the BER of STBC under imperfect channel estimation. This approach, introduced in the next section, deals with block fading, binary modulation and no spatial correlation. In this work, we propose a unifed approach to compute the BER for unitary STBC with imperfect channel estimation, spatial correlation between transmit antennas, temporal correlation between consecutive symbols, different modulation schemes, and different antenna configurations and code rates. We also indicate how the unified approach can be used to analyze space-time-frequency block codes [18] [19]. This demonstrates the well known fact that the quadratic form of the CGRV is a powerful tool in the performance analysis of digital communication systems [20].

For completeness, we mention alternatives when no channel information is available at the receiver. The PEP of decoding of STBC with differentially coherent processing was analyzed in [17] using the quadratic form of a CGRV. The work in

[21] have proposed differential space-time codes (DSTC) based on unitary matrices with a group structure, resulting in a space-time group code. These codes can be decoded without channel knowledge but at a of 3dB cost in signal-to-noise ratio (SNR).

We point out that the tools used in our analysis are well-known and have been applied to similar problems. The primary contribution of this paper is the application of these tools to deal with channel estimation errors, spatial correlation, and temporal correlation in a unified manner.

This chapter is organized as follows. Section 4.2 introduces the system model and our assumptions. Section 4.3 provides an introduction to the quadratic form of a CGRV. Section 4.4 applies this technique to STBC systems with spatial correlation, temporal correlation, BPSK and QPSK modulation, varying number of Tx and Rx antennas, and some of their combinations.

## 4.2 System Model and Assumptions

The STBC system of interest has $t$ transmit antennas and $r$ receive antennas, and a complex symbol alphabet with $n$ symbols. The $i^{th}$ antenna radiates with power $P_i$ per signaling interval and so the total transmit power is $P = \sum_{i=1}^{t} P_i$. The received sampled signal $z_{jn}$ for the $j^{th}$ receive antenna at time $n$ is given by

$$z_{jn} = \sum_{i=1}^{t} \sqrt{P_i} h_{jk} s_{kn} + n_{jn} \tag{4.1}$$

where $n_{jn}$ is a sample of circularly symmetrical Gaussian noise with variance $N_0$, $h_{jn}$ is the complex channel gain from transmit antenna $k$ to receive antenna $j$. The channel between a transmit and a receive antenna is modelled as a frequency non-selective flat Rayleigh-fading process.

Most analysis assume that the channel state information is perfectly known to the receiver, which is generally impractical. If we estimate the channel using either a decorrelating (zero-forcing) or MMSE approach, the channel estimate can be modelled as

$$\hat{h} = h + e \tag{4.2}$$

where $h$ is the actual channel gain and the error term $e$ is a zero mean complex Gaussian variable with variance inversely proportional to estimation power, and $e$ is independent of $h$.

The works in [22] and [21] have proposed differential space-time codes (DSTC), which can be demodulated without channel knowledge, at a loss of 3 dB in SNR. DSTCs are based on unitary matrices with a group structure, forming a space-time group code. [17] has shown that both approaches, differential detection and coherent detection with perfect channel estimation can be analyzed using quadratic form of a CGRV. Later we will show that this approach can even be applied in the case of imperfect channel estimation.

## 4.3 Quadratic Form of a CGRV

In digital communications, we often encounter the evaluation of probability distribution of a generic quadratic form of CGRV. Here, we give the mathematical derivation of the probability distribution of this quadratic form. The approach is described in [20].

The quadratic form of an $N \times 1$ CGRV $x$ is a real valued random variable $z$ given by

$$z = x' M x \tag{4.3}$$

where $x'$ denotes the hermitian (i.e., complex conjugate transpose) of $x$, $M$ is a certain $N \times N$ hermitian matrix, $R$ is the $N \times N$ correlation matrix of $x$, i.e.,

$$R = E[xx'] \tag{4.4}$$

To find the probability distribution function (PDF) of $z$, we first determine the characteristic function of $z$, denoted as $\Phi_z(s)$:

$$\begin{aligned} \Phi_z(s) &= E\{e^{-sz}\} \\ &= \tfrac{1}{\pi^3|R|} \int e^{-sx'Mx} e^{-x'R^{-1}x} dx \\ &= \frac{1}{|sM-R^{-1}||R|} \\ &= \tfrac{1}{|sMR-I|} \end{aligned} \tag{4.5}$$

where $|R|$ denotes the determinant of the matrix $R$. The PDF of $z$ is then

$$f(z) = \begin{cases} \sum_{i=1}^{n} -\frac{k_i}{\lambda_i} e^{-\frac{z}{\lambda_i}}, \ z < 0 \\ \sum_{i=n+p+1}^{N} \frac{k_i}{\lambda_i} e^{-\frac{z}{\lambda_i}}, \ z > 0 \end{cases} \tag{4.6}$$

where $\lambda_i$ for $1 \leq i \leq n$ are the negative eigenvalues of $MR$, $\lambda_i$ for $n+p \leq i \leq N$ are the positive eigenvalues and all other $\lambda_i$ are zero eigenvalues. The coefficients $k_i$ are the residues of $\Phi_z(s)$ evaluated at $\lambda_i$. For distinct eigenvalues, $k_i = \Pi_{k \neq i} \frac{\lambda_i}{\lambda_i - \lambda_k}$. The probability that $z < 0$ is then

$$\begin{aligned} \Pr(z < 0) &= \int_{-\infty}^{0} f(z) dz \\ &= \int_{-\infty}^{0} \sum_{i=1}^{n} -\frac{k_i}{\lambda_i} e^{-\frac{z}{\lambda_i}} dz \\ &= \sum_{i=1}^{n} -\lambda_i (-\frac{k_i}{\lambda_i}) e^{-\frac{z}{\lambda_i}} \Big|_{-\infty}^{0} \\ &= \sum_{i=1}^{n} k_i \end{aligned} \tag{4.7}$$

In the preceding analysis we have assumed that the eigenvalues of MR are distinct. However, when the eigenvalues of MR are not distinct, one can simply go back to (4.5), take the inverse Laplace transform to obtain the PDF of $z$, and then integrate to find the BER theoretically.

## 4.4 Application of Quadratic Forms of a CGRV

Now we use the approach outlined in the previous section to analyze various scenarios in STBC systems by showing that the desired decision statistic can be written in the form of (4.3) by identifying the appropriate CGRV $x$ and the hermitian matrix $M$.

### 4.4.1 Perfect Estimation

The BER performance of a STBC with perfect channel estimation has been studied before. Here we re-derive it, using the quadratic form.

Consider the model of Alamouti [1] where we have 2 Tx and 1 Rx antennas and block fading. The output of the matched filter is

$$z_1 = \sqrt{\frac{P_1}{2}}h_1 s_1 + \sqrt{\frac{P_2}{2}}h_2 s_2 + n_1 \tag{4.8}$$

$$z_2 = \sqrt{\frac{P_1}{2}}h_2 s_1^* - \sqrt{\frac{P_2}{2}}h_1 s_2^* + n_2 \tag{4.9}$$

where $h_1$ and $h_2$ are the channel gains experienced by the signals from antennas 1 and 2 respectively, and $P_1$ and $P_2$ are the transmit power for the first and second transmit antennas respectively.

With perfect channel estimates, the decision statistic for $s_1$ is

$$\begin{aligned}
\hat{s}_1 &= {h_1}^* z_1 + h_2 z_1^* \\
&= {h_1}^* \left( \sqrt{\frac{P_1}{2}}h_1 s_1 + \sqrt{\frac{P_2}{2}}h_2 s_2 + n_1 \right) \\
&\quad + h_2 \left( \sqrt{\frac{P_1}{2}}h_2 s_1^* - \sqrt{\frac{P_2}{2}}h_1 s_2^* + n_2 \right)^* \\
&= \left( \sqrt{\frac{P_1}{2}}|h_1|^2 + \sqrt{\frac{P_1}{2}}|h_2|^2 \right) s_1 + h_1^* n_1 + h_2 n_2^*
\end{aligned} \tag{4.10}$$

Assuming BPSK, it can be shown that the probability of bit error is

$$P_e = \Pr(\text{error}|s_1 \text{ was sent}) = \Pr\{\Re\{\hat{s_1}\} < 0\} = \Pr\{x' M x < 0\} \tag{4.11}$$

where $x$ is given by

$$x = [\ h_1\ \ n_1\ \ h_2\ \ n_2\ ]', \tag{4.12}$$

the corresponding correlation matrix $R$ is

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \sigma_n^2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \sigma_n^2 \end{bmatrix} \tag{4.13}$$

and $M$ can be formed as

$$M = \begin{bmatrix} \frac{\sqrt{P}}{\sqrt{2}} & \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 & 0 \\ 0 & 0 & \frac{\sqrt{P}}{\sqrt{2}} & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} & 0 \end{bmatrix} \tag{4.14}$$

It turns out that the eigenvalues of $MR$ repeat twice, and so we should go back to (4.5), take the inverse Laplace transform to get the PDF of $z$, and integrate to obtain bit error probability. The error probability is again equal to sum of the residues of $\Phi_z(s)$ evaluated at the negative eigenvalues. Fig. 4.1 shows both the simulated and theoretical BER for this STBC system with perfect estimation.

### 4.4.2 Imperfect Channel Estimation

We now deal with imperfect channel estimation using the quadratic form of a CGRV. With imperfect channel estimates $\hat{h}_1$ and $\hat{h}_2$ , we form the decision statistics for $s_1$ as

$$\begin{aligned}\hat{s}_1 &= \hat{h}_1^* z_1 + \hat{h}_2 z_1^* \\ &= (h_1+e_1)^* \left(\sqrt{\frac{P_1}{2}} h_1 s_1 + \sqrt{\frac{P_2}{2}} h_2 s_2 + n_1\right) \\ &\quad + (h_2+e_2)\left(\sqrt{\frac{P_1}{2}} h_2 s_1^* - \sqrt{\frac{P_2}{2}} h_1 s_2^* + n_2\right)^* \\ &= \left(\sqrt{\frac{P_1}{2}} |h_1|^2 + \sqrt{\frac{P_1}{2}} |h_2|^2 + h_1 e_1^* + h_2^* e_2\right) s_1 \\ &\quad + \sqrt{\frac{P_1}{2}} h_2 e_1^* s_2 - \sqrt{\frac{P_1}{2}} h_1^* e_2 s_2 + h_1^* n_1 + h_2 n_2^* + e_1^* n_1 + e_2 n_2^*\end{aligned} \tag{4.15}$$

where the channel estimation error is modelled as $\hat{h}_i = h_i + e_i$ and $e_i$ is a Gaussian random variable with power inversely related to the channel estimation accuracy $\sigma_e^2 = \frac{1}{SNR_{CH}}$.

Assuming BPSK, the probability of bit error is

$$P_e = \Pr\{\Re\{\hat{s_1}\} < 0\} = \Pr\{x' M x < 0\} \tag{4.16}$$

where the CGRV $x$ given by

$$x = [\ h_1\ \ n_1\ \ e_1\ \ h_2\ \ n_2\ \ e_2\ ]' \tag{4.17}$$

has correlation matrix $R$ given by

$$R = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_n^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_e^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_n^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_e^2 \end{bmatrix} \tag{4.18}$$

and $M$ is given by

$$M = \begin{bmatrix} \frac{\sqrt{P}}{\sqrt{2}} & \frac{1}{2} & \frac{\sqrt{P}}{\sqrt{8}} & 0 & 0 & \frac{\sqrt{P}}{\sqrt{8}} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{\sqrt{P}}{\sqrt{8}} & \frac{1}{2} & 0 & \frac{\sqrt{P}}{\sqrt{8}} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{P}}{\sqrt{8}} & \frac{\sqrt{P}}{\sqrt{2}} & \frac{1}{2} & \frac{\sqrt{P}}{\sqrt{8}} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{\sqrt{P}}{\sqrt{8}} & 0 & 0 & \frac{\sqrt{P}}{\sqrt{8}} & \frac{1}{2} & 0 \end{bmatrix} \tag{4.19}$$

Here we have assumed that the two TX antennas are placed far enough apart to ensure no spatial correlation, the channel has unit power, and the two Tx antennas have equal transmit power, i.e., $P_1 = P_2 = P$.

Note that some entries of the $R$ and $M$ given here are different from those in Buehrer and Kumar's paper [2]. If the values for $R$ and $M$ in [2] are used, the theoretical value for final BER does not seem to match with simulation. The reason is, the first entry on the diagonal of $R$ in [2] should be $\sqrt{P_1}$ instead of $P_1$, and other entries should also be changed accordingly.

The analysis above is verified by simulating a STBC system with 2 Tx and 1 Rx (equal transmit power on both Tx antennas), and with imperfect channel estimation (estimation SNR = 10 dB). Fig. 4.1 shows that both the theoretical and simulated BER match quite well. The BER with perfect channel estimation is also shown to contrast the effect of channel estimation error.

Fig. 4.2 shows the impact of channel estimation error on BER. For each received SNR, BER performance of perfect estimation is also plotted for comparison. It shows that when estimation SNR is above 20 dB, the performance gap is negligible. It is also noted that as received SNR gets lower, channel estimation error matters less.

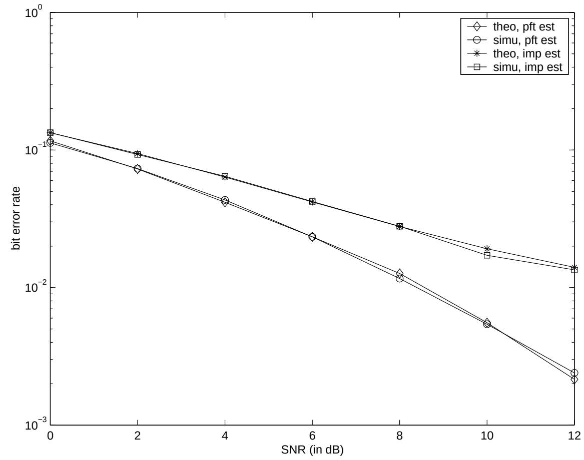


Figure 4.1: Effect of imperfect channel estimation on 2 × 1 STBC (channel estSNR = 10 dB)

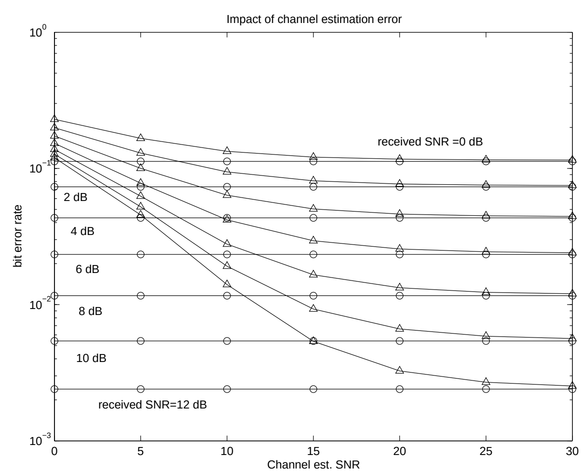


Figure 4.2: Impact of imperfect channel estimation(circle for perfect estimation and triangle for imperfect estimation)

### 4.4.3 Spatial Correlation and Imperfect Channel Estimation

If the two TX antennas are placed close to each other, there will be spatial correlation, i.e. $h_1$ and $h_2$ are no longer uncorrelated. We model this as follows: generate a Gaussian random variable $h_1$, and then generate $h_2$ as

$$h_2 \;=\; \rho h_1 + \sqrt{1-\rho^2}g \qquad (4.20)$$

where $g$ is another complex Gaussian random variable independent of $h_1$ but with the same mean and variance, and $\rho$ is correlation between $h_1$ and $h_2$ and $0 < \rho < 1$.

Here instead of (4.17), we let $x$ be

$$x = [\; h_1 \;\; n_1 \;\; e_1 \;\; g \;\; n_2 \;\; e_2 \;]' \qquad (4.21)$$

where $e_1$ and $e_2$ are estimation errors for $h_1$ and $h_2$ respectively, referring to Section 4.2.

Then $R$ is same as in (4.18) and $M$ is now

$$M = \begin{bmatrix} \frac{\sqrt{P}}{\sqrt{2}}(1+\rho^2) & \frac{1}{2} & \frac{\sqrt{P}}{\sqrt{8}}(1+\rho) & \frac{\sqrt{P}}{\sqrt{2}}\rho q & \frac{\rho}{2} & \frac{\sqrt{P}}{\sqrt{8}}(\rho-1) \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{\sqrt{P}}{\sqrt{8}}(1+\rho) & \frac{1}{2} & 0 & \frac{\sqrt{P}}{\sqrt{8}}q & 0 & 0 \\ \frac{\sqrt{P}}{\sqrt{2}}\rho q & 0 & \frac{\sqrt{P}}{\sqrt{8}}q & \frac{\sqrt{P}}{\sqrt{2}}q^2 & \frac{q}{2} & \frac{\sqrt{P}}{\sqrt{8}}q \\ \frac{\rho}{2} & 0 & 0 & \frac{q}{2} & 0 & \frac{1}{2} \\ \frac{\sqrt{P}}{\sqrt{8}}(\rho-1) & 0 & 0 & \frac{\sqrt{P}}{\sqrt{8}}q & \frac{1}{2} & 0 \end{bmatrix} \qquad (4.22)$$

where $\rho$ is spatial correlation and $q = \sqrt{(1-\rho^2)}$.

Using the quadratic form approach, Fig. 4.3 shows both the simulated and theoretical BER when $\rho = 0.8$. Fig. 4.4 shows that effect on the BER as the spatial correlation $\rho$ increases.

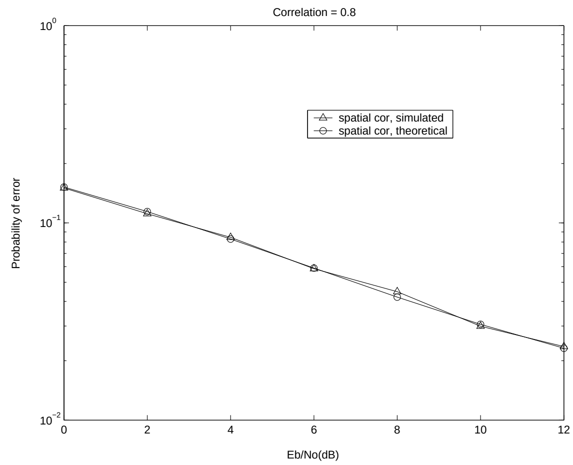


Figure 4.3: Simulated and theoretical BER of $2 \times 1$ STBC with spatial correlation ($\rho = 0.8$) and imperfect channel estimation (estSNR=10dB)

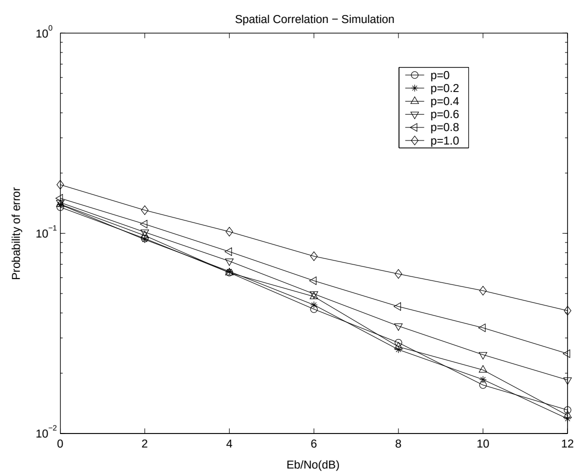


Figure 4.4: Theoretical BER when spatial correlation increases

### 4.4.4 Temporal Correlation

If the fading is not constant during the whole data blocks, (4.8) and (4.9) should be modified as

$$z_1 = \sqrt{\frac{P_1}{2}}h_{10}s_1 + \sqrt{\frac{P_2}{2}}h_{20}s_2 + n_1 \tag{4.23}$$

$$z_2 = \sqrt{\frac{P_1}{2}}h_{21}s_1^* - \sqrt{\frac{P_2}{2}}h_{11}s_2^* + n_2 \tag{4.24}$$

where $h_{10}$ and $h_{20}$ ($h_{11}$ and $h_{21}$)are fading gains in the first (second) time interval from Tx 1 and Tx 2 respectively to the receive antenna. The temporal correlation model, similar to the spatial correlation model used previously, is

$$\begin{aligned} h_{11} &= \rho_1 h_{10} + g_1\sqrt{1-\rho_1^2} \\ h_{21} &= \rho_2 h_{20} + g_2\sqrt{1-\rho_2^2} \end{aligned} \tag{4.25}$$

where $g_1$ ($g_2$) is another CGRV independent of $h_{10}$ ($h_{20}$) but with the same mean and variance, and $\rho_1$ ($\rho_2$) is correlation between $h_{11}$ and $h_{10}$ ($h_{21}$ and $h_{20}$) and $0 < \rho_1, \rho_2 < 1$.

To find out the effect of temporal correlation, we still use the Linear Maximum Likelihood (LML) decoder as in static fading. In temporal correlation case, we have 4 observed fading gains (assuming perfect), two in each time interval. To apply LML, we arbitrarily choose fading gains in one time interval, and assumes those in the other symbol interval are the same. For example, we choose fading gains in the first time interval, then LML decoder works as:

$$\hat{s}_1 = {h_{10}}^* z_1 + h_{20} z_2^* \tag{4.26}$$

$$\hat{s}_2 = h_{20}^* z_1 - h_{10} z_2^* \tag{4.27}$$

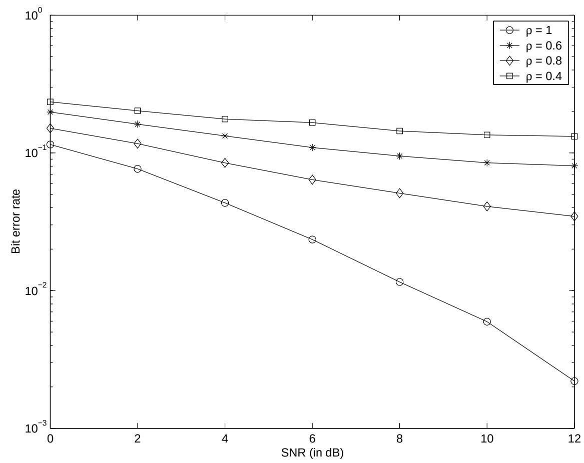


Figure 4.5: Effect of temporal correlation on LML (by simulation)

Fig. 4.5 shows performance of LML decoder when temporal correlation $\rho = 1$ (static fading), 0.8, 0.6, 0.4. It is clear that the error performance becomes very bad as $\rho$ decreases. We can conclude that LML decoder is not a good choice in a fast fading environment. Various decoders for temporal correlation are discussed in the next section.

### 4.4.5 Various Decoders for Temporal Correlation

The work in [15] deals with decoding of STBC on time-varying Rayleigh-fading channels. Several detectors, such as the ML detector, decision-feedback detector, and zero-forcing linear detector, are discussed in [15]. The ML detector has the best BER performance over all, but it is time consuming and theoretical analysis is difficult.

A linear detector attempts to make decision about $s_1$ and $s_2$ separately. In other words we wish to choose $C$ so that

$$y = C * z \tag{4.28}$$

decouples $s_1$ and $s_2$, where $z = [z_1, z_2^*]^T$ is received vector.

From [15], for Zero Forcing detector, $C$ is chosen as,

$$C = \frac{|h_{10}h_{11}^* + h_{20}h_{21}^*|}{h_{10}h_{11}^* + h_{20}h_{21}^*} \begin{bmatrix} (|h_{11}|^2 + |h_{20}|^2)^{-1/2} & 0 \\ 0 & (|h_{10}|^2 + |h_{21}|^2)^{-1/2} \end{bmatrix} \begin{bmatrix} h_{11}^* & h_{20} \\ h_{21}^* & -h_{10} \end{bmatrix} \tag{4.29}$$

Substituting (4.29),(4.23)and (4.24) into (4.28) yields,

$$y = |h_{10}h_{11}^* + h_{20}h_{21}^*| \begin{bmatrix} (|h_{11}|^2 + |h_{20}|^2)^{-1/2} & 0 \\ 0 & (|h_{10}|^2 + |h_{21}|^2)^{-1/2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \hat{n} \tag{4.30}$$

where $\hat{n}$ is the product term of $C$ and original AWGN. It is clear that decision statistics for $s_1$ and $s_2$ are indeed decoupled. It should also be noted that ZF detector cannot completely decouple $s_1$ and $s_2$ in the presence of channel estimation noise.

Theoretical results for zero-forcing detector with perfect channel estimation are provided in [15]; but it is no longer applicable in the presence of channel estimation error. As mentioned before, [16] also proposes a quadratic form approach to solve for the pairwise error probability for any coherent demodulated systems in arbitrarily correlated Rayleigh fading, but channel estimation error is not covered.

We wish to find out a new detector for temporal correlation, which should satisfy the following conditions: 1) able to carry out theoretical analysis for imperfect channel estimation; 2) has comparable performance as ZF detector. Inspired by LML detector and matched filter , and with the aim of increasing received SNR, we introduce a new detector, namely Decoupling Zero Forcing (DZF), which has better performance than the ZF detector at low SNR but worse at higher SNR.

For DZF, given imperfect channel estimates, we form decision statistics for $s_1$ and $s_2$ as

$$\hat{s}_1 = \hat{h}_{10}^* z_1 + \hat{h}_{21} z_2^* \tag{4.31}$$

$$\hat{s}_2 = \hat{h}_{20}^* z_1 - \hat{h}_{11} z_2^* \tag{4.32}$$

The quadratic form approach can be used to derive the theoretical BER for this new detector under channel estimation errors, while it does not seem to be possible for the ZF and ML detectors.

To analyze BER of $s_1$, letting the CGRV $x$ be given by

$$x = [\ h_{10}\ \ e_{10}\ \ n_1\ \ h_{20}\ \ e_{20}\ \ n_2\ \ h_{11}\ \ e_{11}\ \ h_{21}\ \ e_{21}\ ]' \tag{4.33}$$

where $e_{ij}$ represent estimation error for channel $h_{ij}$.

the correlation matrix $R$ is given by

$$R = diag\left(\left[\ 1\ \ \sigma_e^2\ \ \sigma_n^2\ \ 1\ \ \sigma_e^2\ \ \sigma_n^2\ \ 1\ \ \sigma_e^2\ \ 1\ \ \sigma_e^2\ \right]\right) \tag{4.34}$$

and the matrix $M$ is given by

$$M = \begin{bmatrix} \frac{\sqrt{P}}{\sqrt{2}} & \frac{\sqrt{P}}{\sqrt{8}} & \frac{1}{2} & \frac{\sqrt{P}}{\sqrt{8}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{P}}{\sqrt{8}} & 0 & \frac{1}{2} & \frac{\sqrt{P}}{\sqrt{8}} & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{1}{2} & \frac{1}{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{\sqrt{P}}{\sqrt{8}} & \frac{\sqrt{P}}{\sqrt{8}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{\sqrt{P}}{\sqrt{8}} & -\frac{\sqrt{P}}{\sqrt{8}} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{\sqrt{P}}{\sqrt{8}} & 0 & \frac{\sqrt{P}}{\sqrt{2}} & \frac{\sqrt{P}}{\sqrt{8}} \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} & -\frac{\sqrt{P}}{\sqrt{8}} & 0 & \frac{\sqrt{P}}{\sqrt{8}} & 0 \end{bmatrix} \tag{4.35}$$

We test the DZF with 2 cases, 1). $\rho_1 = \rho_2 = 0$ (so that the fading in the two symbol intervals is uncorrelated); 2). $\rho_1 = \rho_2 = 0.5$ (general case). Fig. 4.6 shows that both the theoretical and simulation performance of the proposed detector match quite well for the above two cases.

Fig.4.7 shows that BER will go up as temporal correlation decreases, for both ZF and DZF. It is also interesting to note that the cross-over point of DZF and ZF move to the right as temporal correlation decreases, which means that DZF will outperform ZF in a fast fading environment.

Fig.4.8 compares the performance of DZF and ZF, with both perfect and imperfect channel estimates. It shows that, with perfect estimation, DZF has lower BER than ZF at low SNR, but there is a cross-over point (around 6 dB), when the ZF starts to do better than DZF. With imperfect estimation, this cross-over point moves to the higher end (around 9 dB). This shows that DZF is more robust against estimation error, compared to the ZF detector.

The explanation for their performance is as follows: ZF decoder tries to reduce inter-symbol interference (ISI), while DZF attempts to increase signal to noise ratio. At lower SNR, additive Gaussian noise dominates, therefore DZF outperforms ZF. At high SNR, however, ISI dominates, as a result, ZF performs better than DZF after a intersection.

To conclude, the proposed DZF detector performs better than traditional ZF in fast fading channel, and is more robust against estimation error. Moreover, performance of DZF can be precisely analyzed by our unified approach. The weakness of DZF is that, it shows bad performance at high SNR, with some error floor. Therefore, given certain conditions(received SNR, channel estimation error, and temporal correlation, etc), one can switch between DZF and ZF to get better BER performance.

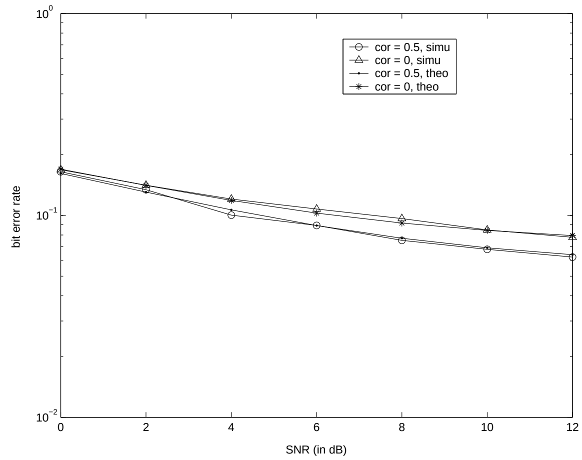


Figure 4.6: Theoretical and simulation performance for DZF with temporal correlation (i.i.d fading) and imperfect channel estimation (estSNR=10dB)

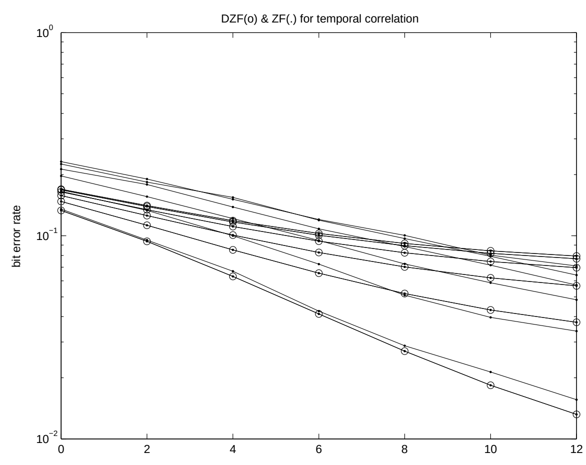


Figure 4.7: Theoretical and simulation performance for DZF and ZF with temporal correlation (i.i.d fading) and imperfect channel estimation (estSNR=10dB), and $\rho$ decreases from 1.0 to 0.0, from bottom to top

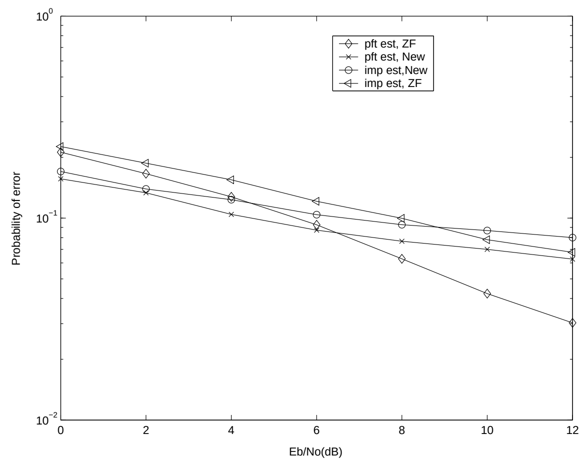


Figure 4.8: Performance comparison for DZF (by analysis)and ZF (by simulation) detectors with perfect and imperfect estimation

### 4.4.6 QPSK and Imperfect Channel Estimation

A QPSK symbol is basically two orthogonal BPSK symbols and (4.15) gives the decision statistics for both BPSK and QPSK. We can compare the two situations to get some hints on how to solve the QPSK case.

For BPSK, assume $s_1 = 1$ and $s_2 = 1$ are sent, then (4.15) gives

$$\begin{aligned}\hat{s}_1 = &\left(\sqrt{\frac{P_1}{2}}|h_1|^2 + \sqrt{\frac{P_1}{2}}|h_2|^2\right) + \\ &h_1 e_1^* + h_2^* e_2 + \sqrt{\frac{P_1}{2}} h_2 e_1^* - \sqrt{\frac{P_1}{2}} h_1^* e_2 + \\ &h_1^* n_1 + h_2 n_2^* + e_1^* n_1 + e_2 n_2^*\end{aligned} \tag{4.36}$$

An error is made if $\mathrm{Re}[\hat{s}_1] < 0$, and that is the BER for BPSK system. For QPSK, assume $s_1 = 1+\mathrm{j}$ and $s_2 = 1 + \mathrm{j}$ are sent (with the same bit energy as BPSK). Then

(4.15) gives

$$\begin{aligned}\hat{s}_1 = &\left( (\sqrt{\frac{P_1}{2}}|h_1|^2 + \sqrt{\frac{P_1}{2}}|h_2|^2 \right)(1+j) + \\ &\left( h_1 e_1^* + h_2^* e_2 + \sqrt{\frac{P_1}{2}} h_2 e_1^* - \sqrt{\frac{P_1}{2}} h_1^* e_2 \right)(1+j) + \\ &h_1^* n_1 + h_2 n_2^* + e_1^* n_1 + e_2 n_2^* \\ = &Z_1 + Z_2 * j\end{aligned} \tag{4.37}$$

where $Z_1$ and $Z_2$ are real and imaginary part of $\hat{s}_1$ respectively.

Then, the bit error probability for QPSK is,

$$\begin{aligned}BER &= \frac{1}{2}\left(\Pr[Z_1 > 0\&Z_2 < 0] + 2 * \Pr[Z_1 < 0\&Z_2 < 0] + \Pr[Z_1 < 0\&Z_2 > 0]\right) \\ &= \frac{1}{2}((\Pr[Z_1 > 0\&Z_2 < 0] + \Pr[Z_1 < 0\&Z_2 < 0]) + \\ &(\Pr[Z_1 < 0\&Z_2 < 0] + \Pr[Z_1 < 0\&Z_2 > 0]) \\ &= \frac{1}{2}(\Pr[Z_1 < 0] + \Pr[Z_2 < 0])\end{aligned} \tag{4.38}$$

Considering the symmetrical structure between $Z_1$ and $Z_2$, we can conclude that, $\Pr[Z_1 < 0]$=$\Pr[Z_2 < 0]$. Therefore (4.38) becomes

$$BER = \Pr[Z_1 < 0] \tag{4.39}$$

From (4.37),

$$\begin{aligned}Z_1 = &\left( \sqrt{\frac{P_1}{2}}|h_1|^2 + \sqrt{\frac{P_1}{2}}|h_2|^2 \right) + \\ &\Re\left[ \left( h_1 e_1^* + h_2^* e_2 + \sqrt{\frac{P_1}{2}} h_2 e_1^* - \sqrt{\frac{P_1}{2}} h_1^* e_2 \right)(1+j) \right] \\ &+ \Re[h_1^* n_1 + h_2 n_2^* + e_1^* n_1 + e_2 n_2^*]\end{aligned} \tag{4.40}$$

Comparing $Z_1$ of QPSK with Re$[\hat{(s)}_1]$ of BPSK in (4.21), we find that only difference is that, $Z_1$ has a multiple of (1+j) in the second summation term. (1+j) has a

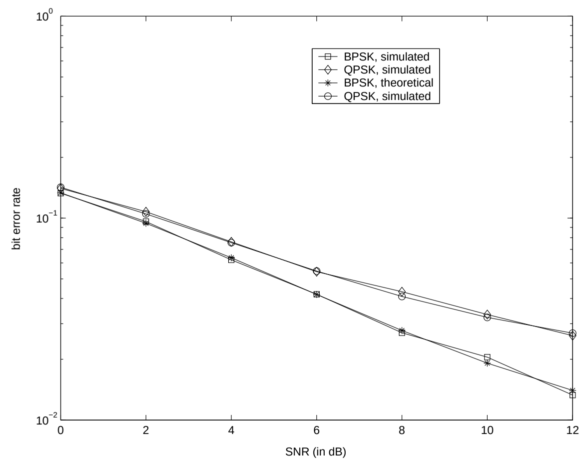


Figure 4.9: BPSK and QPSK modulation, with imperfect estimation

magnitude of $\sqrt{2}$, therefore we replace (4.19) as

$$M = \begin{bmatrix} \frac{\sqrt{P}}{\sqrt{2}} & \frac{1}{2} & \frac{\sqrt{P}}{\sqrt{8}}\sqrt{2} & 0 & 0 & \frac{\sqrt{P}}{\sqrt{8}}\sqrt{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{\sqrt{P}}{\sqrt{8}}\sqrt{2} & \frac{1}{2} & 0 & \frac{\sqrt{P}}{\sqrt{8}}\sqrt{2} & 0 & 0 \\ 0 & 0 & \frac{\sqrt{P}}{\sqrt{8}}\sqrt{2} & \frac{\sqrt{P}}{\sqrt{2}} & \frac{1}{2} & \frac{\sqrt{P}}{\sqrt{8}}\sqrt{2} \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{\sqrt{P}}{\sqrt{8}}\sqrt{2} & 0 & 0 & \frac{\sqrt{P}}{\sqrt{8}}\sqrt{2} & \frac{1}{2} & 0 \end{bmatrix} \tag{4.41}$$

Fig. 4.9 shows both the theoretical and simulated BER for the $2 \times 1$ system with QPSK modulation and imperfect channel estimation. The BPSK system is also plotted for comparison (with the bit energy kept constant for fair comparison). For MPSK modulation, our unified approach will lead to pair-wise symbol error probability and therefore give bounds on the BER.

### 4.4.7 Number of TX and RX Antennas

To achieve diversity, we can use 2 Tx antennas and $M$ Rx antennas where M is any positive integer. Starting from the original $2 \times 1$ STBC system, whenever we add one more Rx antenna, the receiver receives one more copy of the transmitted signals, and we can then construct $M$ and $R$ accordingly. Therefore, this $2 \times M$ system can also be analyzed using the quadratic form approach discussed above. Examples are given in this subsection and in a later part of the paper.

From [10], as far as square matrices are concerned (i.e. number of Tx and Rx antennas are equal), real orthogonal designs only exist of sizes $2 \times 2$, $4 \times 4$, and $8 \times 8$, while complex orthogonal design only exist for size $2 \times 2$. As long as we have an orthogonal design, we can always write the decision statics in the form of (4.3), and the quadratic form approach can be applied.

### 4.4.8 Rate 3/4 Code from Complex Orthogonal Designs

The work in [10] presented some generalized complex linear processing orthogonal designs for transmission for general number of transmit and receive antennas. [23] gives examples of some complex space-time block codes for four Tx Antennas. We now take the $n = 3$ example from [10], and show that this rate 3/4 complex orthogonal code can also be analyzed using our unified approach. The rate 3/4 code is given by:

$$X = \begin{bmatrix} x_1 & x_2 & x_3/\sqrt{2} \\ -{x_2}^* & {x_1}^* & x_3/\sqrt{2} \\ {x_3}^*/\sqrt{2} & {x_3}^*/\sqrt{2} & (-x_1 - {x_1}^* + x_2 - {x_2}^*)/2 \\ {x_3}^*/\sqrt{2} & -{x_3}^*/\sqrt{2} & (x_2 + {x_2}^* + x_1 - {x_1}^*)/2 \end{bmatrix} \tag{4.42}$$

Assuming perfect CSI, to decode $x_1$, we form the decision statistic as

$$\begin{aligned} \hat{s}_1 &= {h_1}^* z_1 + h_2 z_1^* - \frac{1}{2}(h_3 {z_3}^* + {h_3}^* z_3) + \frac{1}{2}({h_3}^* z_4 - h_3 {z_4}^*) \\ &= \left(|h_1|^2 + |h_2|^2 + |h_3|^2\right) s_1 + W_1 \end{aligned} \tag{4.43}$$

where $z_1, z_2, z_3, and z_4$ are the received signals at the 4 symbol intervals and $W_1$ represents the sum of noise product terms.

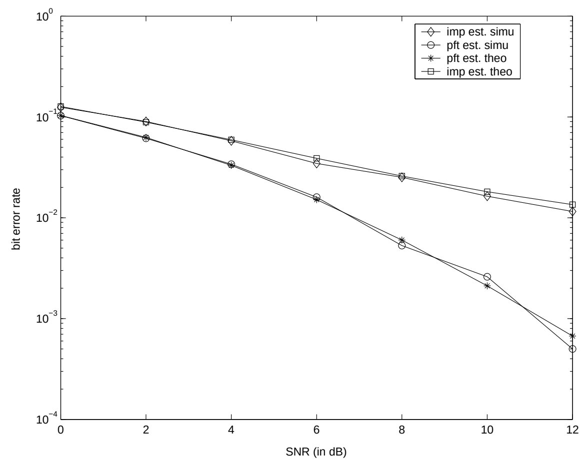


Figure 4.10: Rate 3/4 code, QPSK modulation, with perfect and imperfect estimation

Note that the transmitted signals are decoupled. The decision statistics for $s_2$ and $s_3$ are similar. Comparing (4.10) and (4.43), we see that they are both sum of Gaussian products, where quadratic form method is applicable. The techniques in the previous sections indicate how to build the CGRV $x$ and the $M$ matrix when there is imperfect channel estimation and different modulation schemes.

Fig. 4.10 shows both simulation and theoretical BER for this rate 3/4 code (QPSK modulation), with and without channel estimation error.

### 4.4.9 Space-Time-Frequency Block codes

Space-Frequency Block codes (SFBC) [18] are very similar to STBC, with the main difference being that,in SFBC, the encoding technique is carried out across space and frequency.

Consider a 2 Tx 1 Rx antenna SFBC system, where the data symbol on each of the two carrier frequencies $f_0$ and $f_1$ are transmitted in two symbol periods. Then the received signals on carrier frequencies $f_0$, $f_1$ at time $t$ is exactly same as (4.8) and (4.9). Therefore SFBC systems can also be analyzed with our unified approach, in the exactly same way as STBC. The benefit of SFBC is apparent in fast, frequency non-selective fading.

Space-Time Frequency block codes (STFBC) [19] combine advantages of SFBC and STBC, and can be used in fast, frequency selective fading. With appropriate modifications, our unified approach may also be used to analyze this case.

### 4.4.10 Application to Combination of Situations

The previous sections discuss several scenarios where the quadratic form approach can be utilized to obtain closed form expressions for the BER of unitary transmit diversity systems. We can combine these techniques to solve more complicated problems. Consider a $2 \times 2$, QPSK modulated STBC system, with imperfect channel estimation and a spatial correlation of 0.8. Since the $2 \times 2$ system is just a receiver diversity version of a $2 \times 1$ system, we can consider the $2 \times 1$ system first, and then add another branch. Making use of the methods discussed in previous sections, we form $M$ for $2 \times 1$ system as,

$$M_{21} = \begin{bmatrix} \frac{\sqrt{P}}{\sqrt{2}}(1+\rho^2) & \frac{1}{2} & \frac{\sqrt{P}}{\sqrt{8}}(1+\rho)\sqrt{2} & \frac{\sqrt{P}}{\sqrt{2}}\rho q & \frac{\rho}{2} & \frac{\sqrt{P}}{\sqrt{8}}(\rho-1)\sqrt{2} \\ \frac{1}{2} & 0 & \frac{1}{2} & 0 & 0 & 0 \\ \frac{\sqrt{P}}{\sqrt{8}}(1+\rho)\sqrt{2} & \frac{1}{2} & 0 & \frac{\sqrt{P}}{\sqrt{8}}q\sqrt{2} & 0 & 0 \\ \frac{\sqrt{P}}{\sqrt{2}}\rho q & 0 & \frac{\sqrt{P}}{\sqrt{8}}q\sqrt{2} & \frac{\sqrt{P}}{\sqrt{2}}q^2 & \frac{q}{2} & \frac{\sqrt{P}}{\sqrt{8}}q\sqrt{2} \\ \frac{\rho}{2} & 0 & 0 & \frac{q}{2} & 0 & \frac{1}{2} \\ \frac{\sqrt{P}}{\sqrt{8}}(\rho-1)\sqrt{2} & 0 & 0 & \frac{\sqrt{P}}{\sqrt{8}}q\sqrt{2} & \frac{1}{2} & 0 \end{bmatrix} \tag{4.44}$$

Defining $M_0$ as the all zero $6 \times 6$ matrix, the $M$ matrix for the $2 \times 2$ system is

$$M_{22} = \begin{bmatrix} M_{21} & M_0 \\ M_0 & M_{21} \end{bmatrix} \tag{4.45}$$

Fig. 4.11 shows that the analytical results for the $2 \times 2$ system resulting from the combination of issues discussed above match well with simulation results.

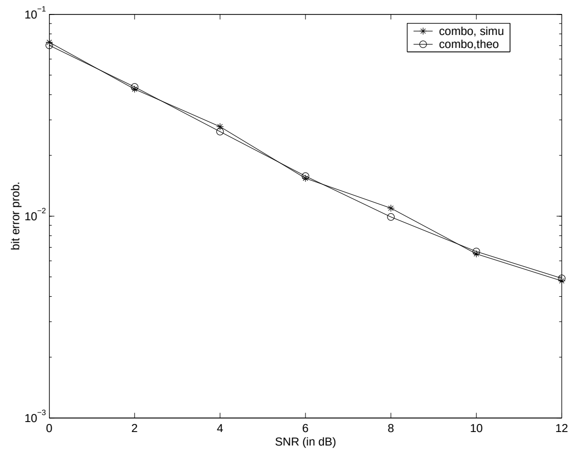


Figure 4.11: $2 \times 2$ QPSK modulated system with spatial correlation (estSNR=10dB)

## 4.5 Conclusion

In this chapter, we propose a unified approach to analytically obtain the bit error probability of various transmit diversity schemes under a variety of conditions. Closed form expressions for the BER are derived under a variety of circumstances including imperfect channel estimation, spatial correlation, temporal correlation, different modulation schemes (e.g. BPSK and QPSK), different number of Tx and Rx antennas and their combinations. We give one example of the exact performance with 2 Tx and 2 Rx antenna STBC system, with QPSK modulation, imperfect channel estimation and spatial correlation.

# CHAPTER V

# Conclusion

## 5.1 Conclusion

Diversity techniques provide a less attenuated replica of the transmitted signal to the receiver, which makes it easier for the receiver to reliably determine the correct signal transmitted. Diversity can be provided using temporal, frequency, polarization, and spatial resources.

Alamouti proposed a simple transmit diversity technique that can provide the same diversity order as maximal-ratio receiver combining (MRRC). One assumption Alamouti made in his study is that channel information, in the forms of amplitude and phase distortion, is known perfectly to the receiver. In practice, the issue of channel estimation is non-trivial, especially in a fading environment where the fading gain can change substantially from one bit to the next.

The objective of this thesis is therefore to study the impact of imperfect channel estimation on the error performance of the Alamouti's transmission scheme.

In the first phase of this project, we consider two pilot-aided channel estimation strategies for this $2 \times 2$ system, one based on the decorrelator concept, the other based on the minimum mean square error (MMSE) concept. In both cases, we illustrate the importance of selecting a proper pilot sequence for channel estimation.

Buehrer and Kumar used an approach, based on Hermitian quadratic forms, to find the closed form expression for BER of STBC, given imperfect channel estimation. Buehrer and Kumar's approach, however, only deals with block fading, with BPSK modulation and without spatial correlation. The second phase of this project is to extend their method to solve more complicated scenarios.

Several issues are considered: perfect estimation, spatial correlation, temporal correlation, different modulation schemes (e.g. BPSK and QPSK), Number of Tx and Rx antennas. It is shown that the extended quadratic form approach can be used to obtain closed form expression for BER of the above systems. Furthermore, quadratic form can also solve some combinations of the above situations.

There are two cases that the quadratic form approach can not solve, namely, decorrelator detector and MMSE detector in time varying fading channel. It is found that these two problems can be modelled as product of two quadratic forms, where the two random vector are correlated. If probability density function of this product term can be obtained, then bit error probability of the above two detector can be expressed in closed form.

## 5.2 Future Works

As described in Chapter 4, quadratic form has been extended to solve various scenario for STBC systems. However, there are some other problems still left to be solved. This chapter is devoted to the two unsolved problem.

### 5.2.1 MMSE in Time Varying Fading Channel

In [24], three methods for detecting an Alamouti Space time block code over Time-Varying Rayleigh fading channels were proposed.

Take a system of 2 Tx and 1 Rx Antenna for example. The receiver observations

$r_1$ and $r_2$ corresponding to the two symbol periods are given by:

$$\begin{bmatrix} r_1 \\ {r_2}^* \end{bmatrix} = \begin{bmatrix} h_1 & h_2 \\ \tilde{h}_2^* & -\tilde{h}_1^* \end{bmatrix} * \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} n_1 \\ {n_2}^* \end{bmatrix} \tag{5.1}$$

or with obvious notations:

$$r = Hx + n \tag{5.2}$$

A. The Maximum Likelihood (ML) Detector Because of the white Gaussian noise, the joint ML detector chooses the pair of symbols $x$ to minimize:

$$||r - Hx||^2 \tag{5.3}$$

B. The Decision-Feedback Detector The decision-feedback detector uses a decision about $x_1$ to help make a decision about $x_2$.

C. The zero Forcing linear Detector

A linear detector computes:

$$y = Cr \tag{5.4}$$

then makes a decision about $x_i$ based solely on $y_i$, for $i$ = 1,2, where $C$ is set as:

$$C = \frac{|h_1\tilde{h}_1^* + h_2\tilde{h}_2^*|}{h_1\tilde{h}_1^* + h_2\tilde{h}_2^*} \begin{bmatrix} (|\tilde{h}_1|^2 + |h_2|^2)^{-1/2} & 0 \\ 0 & (|\tilde{h}_2|^2 + |h_1|^2)^{-1/2} \end{bmatrix} \begin{bmatrix} \tilde{h}_1^* & h_2 \\ \tilde{h}_2^* & -h_1 \end{bmatrix} \tag{5.5}$$

Substituting (5.5) to (5.2) yields:

$$y = |h_1\tilde{h}_1^* + h_2\tilde{h}_2^*| \begin{bmatrix} (|\tilde{h}_1|^2 + |h_2|^2)^{-1/2} & 0 \\ 0 & (|\tilde{h}_2|^2 + |h_1|^2)^{-1/2} \end{bmatrix} x + \tilde{n} \tag{5.6}$$

Then [24] gives analytical results of BER for ZF detector and lower bound BER for DF detector. Note that [24] assumes perfect channel estimation.

The author proposes another detector, names Minimum Mean Square Error (MMSE) detector, which has better performance than ZF detector.

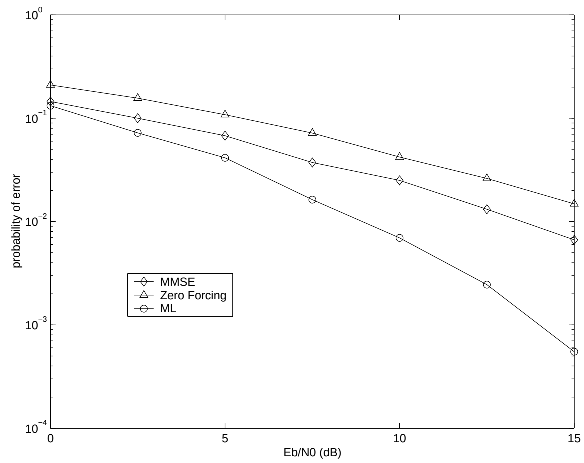


Figure 5.1: Performance comparison for various detectors in time-varying fading

For MMSE, the basic concept is to choose T, such that mean square error is minimized. Define:

$$H = \begin{bmatrix} h_1 & h_2 \\ \tilde{h_2}^* & -\tilde{h_1}^* \end{bmatrix} \tag{5.7}$$

and $E_b$ as bit energy,$E_N$ as noise power, then

$$T = E_b H^* (E_b H H^* + E_N I)^{-1} \tag{5.8}$$

where I is identity matrix and $(.)^*$ denotes conjugate transpose of a matrix.

Figure 5.1 plots the BER performance of ML, ZF and MMSE, assuming the channel is completely uncorrelated between two symbol intervals, which means that, there is no correlation between $h_1$ and $\tilde{h_1}$, $h_2$ and $\tilde{h_2}$. The scenario is similar when the correlation is between 0 and 1. When correlation is 1, i.e. the fading is static, the three detectors have same performance.

Though MMSE detector outperforms ZF detector, it is difficult to analyze its BER performance. the problem is even more complicated when channel estimation is imperfect. The same problem exists for Zero-Forcing detector, which will be discussed in the next section.

### 5.2.2 Imperfect Channel Estimates for Zero-Forcing Detector

[24] discusses how to analyze BER performance of Zero-Forcing detector, given perfect estimation. The method is no longer applicable if channel estimation is imperfect. Here we try to analyst this problem.

We form decision statistics for $x_1$ as:

$$
\begin{aligned}
\hat{x}_1 &= \frac{1}{\hat{h}_{10}\hat{h}_{11}^* + \hat{h}_{20}\hat{h}_{21}^*}(\hat{h}_{11}^* r_1 + \hat{h}_{20} r_2^*) \\
&= \frac{1}{|\hat{h}_{10}\hat{h}_{11}^* + \hat{h}_{20}\hat{h}_{21}^*|^2}(\hat{h}_{10}^*\hat{h}_{11} + \hat{h}_{20}^*\hat{h}_{21})(\hat{h}_{11}^* r_1 + \hat{h}_{20} r_2^*) \\
&= \frac{1}{|\hat{h}_{10}\hat{h}_{11}^* + \hat{h}_{20}\hat{h}_{21}^*|^2} Z_1 Z_2 \qquad (5.9)
\end{aligned}
$$

where $\hat{h}_{ij}$ is the estimated version of $h_{ij}$, and $Z_1 = \hat{h}_{10}^*\hat{h}_{11} + \hat{h}_{20}^*\hat{h}_{21}$, $Z_2 = \hat{h}_{11}^* r_1 + \hat{h}_{20} r_2^*$

It is clear that $Z_1$ and $Z_2$ are of quadratic form. So the problem now is, given the product of two quadratic form random variable, what is its probability density function? It is easily shown that decision in section (5.2) can be simplified to the similar form.

If the two random variable vector in $Z_1$ and $Z_2$ (refer to Chapter 4 )are independent of each other, then we can derive PDF of $Z_1 Z_1$ from that of $Z_1$ and $Z_2$. But here the two vectors are obvious correlated, therefore it seems a difficult problem in mathematics.

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