CRUDE OIL SCHEDULING IN REFINERY OPERATIONS

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ABSTRACT

Scheduling is one of the tools that facilitates refiner to be proactive for changing scenarios and allows finding solutions that generate enhanced income. Scheduling considerations prevalent with crude oil operations in a petroleum refinery have been addressed in this work. Scheduling of crude oil operations is part of optimization of overall refinery operations and involves unloading crude oil from vessels to storage tanks and charging various mixes of crude oils from tanks to each distillation unit subject to capacity, flow, and composition limitations. Refinery configurations with different unloading facilities such as SBM (Single Buoy Mooring), multiple jetties are considered in this study. Scheduling of crude oil operations is a complex nonlinear problem, especially when tanks hold crude mixes**.** A novel iterative MILP solution approach for optimizing crude oil operations is devised which obviates the need for solving MINLP. This work addresses both discrete and continuous time scheduling models for crude oil scheduling problem and presents a head to head comparison of the two models. The proposed methodology performs much better in comparison to methodologies present in the literature and gives near optimum solutions, thus making them suitable for large-scale operations.

Keywords: scheduling, crude oil, petroleum refinery, Single Buoy Mooring (SBM), multiple jetties, MILP, MINLP.

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SUMMARY

Scheduling considerations prevalent with crude oil operations in a petroleum refinery have been addressed in this work. Scheduling of crude oil operations involves unloading crude oil from vessels to storage tanks and charging various mixes of crude oils from tanks to each distillation unit subject to capacity, flow, and composition limitations. Refinery configurations with different unloading facilities such as SBM (Single Buoy Mooring), multiple jetties are considered in this study. Time consideration in model building is very important. Two different methodologies are adopted namely discrete time, continuous time. Firstly a discrete time MILP (mixed integer linear programming) model is formulated for a refinery configuration with SBM, storage tanks and crude distillation units. The objective is to maximize the gross operating profit. The model is extended to a refinery with multiple jetties alone as crude unloading facility and then to a refinery configuration with jetties along with SBM. A separate MILP model is developed in case of a refinery configuration which includes both SBM, jetties as unloading facility, storage tanks for crude receipt from vessels and crude delivery to distillation units. Mimicking a continuous-time formulation, the developed model allows multiple sequential crude transfers to occur within a time slot. Furthermore, it handles many real-life operational features including brine settling and tank-to-tank transfers.

Scheduling of crude oil operations is a complex nonlinear problem, especially when tanks hold crude mixes**.** Crude mixing generates in bi-linear non-convex equations, which are hard to solve, and the problem is a MINLP. A linearized model poses composition discrepancy in crude charge to CDU. A novel iterative solution approach for optimizing crude oil operations is devised which corrects the concentration discrepancy arising due to crude mixing in tanks without solving any NLP or MINLP.

The size and complexity of MILP in proposed algorithm reduce progressively and provides near-optimal schedules in reasonable time. When the scheduling objective does not involve crude compositions, then proposed algorithm guarantees a globally optimal objective value right in the first MILP, although subsequent iterations are required to correct the composition discrepancy. Both these proposed model and algorithms perform much better in comparison to scheduling methodologies present in the literature and give better solutions for several literature problems, thus making them suitable for large-scale operations. The test examples are solved for a short term scheduling horizon and conclusions are made regarding the system in general.

The second part addresses a continuous time mathematical model for a refinery configuration with SBM. The model is based on synchronized variable length slots on storage tanks. Here an attempt was made to reduce number of binary decisions so that model becomes less compute intensive. For solving the continuous model that provides schedules with no composition discrepancy, we incorporated suitable modifications to the solution algorithm of discrete approach. A direct comparison of discrete and continuous models is carried out and it was concluded that for long horizons continuous time model generates a feasible schedule in reasonable time. For shorter horizons, discrete time models provided a better and quicker solution, thus having an edge over the continuous time models.

Finally the thesis is concluded with a summary of prominent improvements achieved in comparison to previous works and some future directions are proposed.

NOMENCLATURE

ABBREVIATIONS

SBM Single Buoy Mooring.

SPM Single point Mooring.

CDU Crude Distillation Unit.

VLCC Very Large cargo container.

MILP Mixed Integer Linear Program.

MINLP Mixed Integer Non Linear Program.

LP Linear Program.

NLP Non Linear Program.

SYMBOLS

Chapters 2-6 (Part I)

Sets

JP Set of jetty parcels

SP Set of VLCC parcels

PT Set of pairs (parcel p, period t) such that p can connect to SBM line during t

PI Set of pairs (parcel p, tank i) such that i may receive crude from p

IU Set of pairs (tank i, CDU u) such that i can feed crude to CDU u

IC Set of pairs (tank i, crude type c) such that i can hold c

PC Set of pairs (parcel p, crude type c) such that p carries crude c

PV Set of pairs (parcel p, vessel v) such that p is the last parcel of v

II Set of pairs (tank i, tank i') such that transfer between i, i' is allowed

Subscripts

i, i' Storage tanks

c Crude Types

u Crude Distillation Units (CDUs)

v	Vessels
t	Time periods
p	Parcels

Superscripts

U	Upper limit
L	Lower limit

Parameters

ETA_p	Expected time of arrival of parcel *p*
$FPT_{pi}^{L/U}$	Limits on the amount of crude transfer per period from parcel *p* to tank *i*
$FTU_{iu}^{L/U}$	Limits on the amount of crude charge per period from tank *i* to CDU *u*
$FTT_{ii'}^{L/U}$	Limits on the amount of crude transfer per period from tank *i* to *i'*
$FU_{u}^{L/U}$	Limits on the amount of crude processed per period by CDU *u*
$xc_{cu}^{L/U}$	Limits on the composition of crude type *c* in feed to CDU *u*
$xk_{ku}^{L/U}$	Limits on the concentration of key component *k* in feed to CDU *u*
$V_{i}^{L/U}$	Allowable limits on crude inventory in tank *i*
$xt_{ic}^{L/U}$	Limits on the composition of crude type *c* in tank *i*
D	Total crude demand in the scheduling horizon
D_u	Total crude demand per CDU *u* in the scheduling horizon
D_{ut}	Crude demand per CDU *u* in each period *t*
PD_j	Maximum demand for product *j* during scheduling horizon
y_{jcu}	Fractional yield of product *j* from crude *c* in CDU *u*
CP_{cu}	Margin ($/unit volume) for crude *c* in CDU *u*
COC	Cost (k$) per changeover
TTC	Penalty (K$) for occurrence of a tank-to-tank transfer
SSP	Safety stock penalty ($ per unit volume below desired safety stock)
SS	Desired safety stock (kbbl) of crude inventory in any period
SWC_v	Demurrage or Sea waiting cost ($ per period)
ETD_v	Expected time of departure of vessel *v*
ETU_p	Earliest possible unloading period for parcel *p*
PS_p	Size of the parcel *p*
J	Number of identical Jetties

Binary Variables

XP_{pt} 1 if parcel p is connected to SBM/jetty discharge line during period t

XT_{it} 1 if a tank i is connected to SBM/jetty discharge line during period t

Y_{iut} 1 if a tank i feeds CDU u during period t

$Z_{ii't}$ 1 if crude transfers between tanks i and i' during period t

0-1 Continuous Variables

XF_{pt} 1 if a parcel p first connects to the SBM/jetty during period t

XL_{pt} 1 if a parcel p disconnects from the SBM/jetty at time t

X_{pit} 1 if a parcel p and tank i both connect to the SBM line at t

YY_{iut} 1 if a tank i is connected to CDU u during both periods t and (t+1)

CO_{ut} 1 if a CDU u has a changeover during period t

Continuous Variables

TF_p Time at which parcel p first connects to SBM/jetty for unloading

TL_p Time at which parcel p disconnects from SBM/jetty after unloading

FPT_{pit} Amount of crude transferred from parcel p to tank i during period t

FTU_{iut} Amount of crude that tank i feeds to CDU u during period t

FU_{ut} Total amount of crude fed to CDU u during period t

$FCTU_{iuct}$ The amount of crude c delivered by tank i to CDU u during period t

VCT_{ict} Amount of crude c in tank i at the end of period t

V_{it} Crude level in tank i at end of period t

f_{ict} Concentration (volume fraction) of crude c in tank i at the end of period t

DC_v Demurrage cost for vessel v

SC_t Safety stock penalty for period t

ZT_{it} Variable to denote the number of times tank i exchanges crude with another tank in a given period t

$FCTT_{ii'ct}$ The amount of crude c transferred from tank i to tank i' during period t

$FTT_{ii't}$ Total amount of crude transferred between tank i to tank i'during t

$AFTT_{ii't}$ Absolute amount of crude transferred between tank i to tank i' during t

Chapters 7-9 (Part II)

Subscripts

i Storage tanks

c	Crude Types
u	Crude Distillation Units (CDUs)
v	Vessels
t	Time periods
s, s'	Slots
p, p', p''	Parcels

Superscripts

U	Upper limit
L	Lower limit

Sets

PS	Set of pairs (parcel *p*, slot *s*) such that *p* can connect to SBM line during *s*
TS	Set of pairs (period *t*, slot *s*) such that slot *s* is in period *t*
PI	Set of pairs (parcel *p*, tank *i*) such that *i* may receive crude from *p*
IU	Set of pairs (tank *i,* CDU *u*) such that *i* can feed CDU *u*
IC	Set of pairs (tank *i,* crude type *c*) such that *i* can hold *c*
PC	Set of pairs (parcel *p,* crude type *c*) such that *p* has crude *c*
PV	Set of pairs (parcel *p,* vessel *v*) such that *p* is the last parcel of *v*
SS	Set of pairs (slots *s* and *s*′ with $s' > s$) such that a tank receiving crude in *s* may settle brine up to the beginning of *s*′

Parameters

ETA_p	Expected time of arrival of parcel *p*
FPT_{pi}^{U}	Limit on the rate of crude transfer from parcel *p* to tank *i*
$FTU_{iu}^{L/U}$	Limits on the rate of crude charge from tank *i* to CDU *u*
$FU_{u}^{L/U}$	Limits on the crude processing rate of CDU *u*
$xc_{cu}^{L/U}$	Limits on the composition of crude type *c* in feed to CDU *u*
$xk_{ku}^{L/U}$	Limits on the concentration of key component *k* in feed to CDU *u*
$V_{i}^{L/U}$	Allowable limits on crude inventory in tank *i*
$xt_{ic}^{L/U}$	Limits on the composition of crude type *c* in tank *i*
TCD	Total crude demand in the scheduling horizon
DM_u	Total crude demand for CDU *u* in the scheduling horizon
CD_{ut}	Crude demand for CDU *u* in period *t*
CP_{cu}	Marginal profit ($/unit volume) from crude *c* in CDU *u*

COC Cost (k$) per changeover

SSP_t Safety stock penalty ($/ unit volume below desired safety stock) for period t

SS_t Desired safety stock (kbbl) of crude inventory for period t

SWC_v Demurrage or Sea waiting cost ($/unit time)

STD_v Stipulated time of departure as mentioned in the logistics contract of VLCC v

ETU_p Earliest possible unloading period for parcel p

VP_p Size (m^3 or bbl) of parcel p

NP Total number of parcels to unload during the horizon

NS Total Number of slots in the scheduling horizon

D_t Start time of period t

DD_t $= D_t - D_{(t-1)}$ or length of period t

ST Minimum time for crude settling and brine removal

Binary Variables

XP_{ps} 1 if parcel p is connected to the SBM during slot s

XT_{is} 1 if tank i is connected to the SBM during slot s

Y_{ius} 1 if tank i feeds CDU u during slot s

0-1 Continuous Variables

XF_{ps} 1 if parcel p first connects to the SBM during slot s

XL_{ps} 1 if parcel p disconnects from the SBM at the end of slot s

YT_{is} 1 if tank i is delivering crude during slot s

ZT_{is} 1 if tank i is idle or settling during slot s

CO_{us} 1 if CDU u has a changeover at the end of slot s

Continuous Variables

TL_s End-time of slot s

SL_s Length of slot s

RLP_{ps} Time for which p connects to the SBM during s

RLT_{is} Time for which i connects to the SBM during s

RLU_{is} Time for which i feeds crude during s

RLZ_{is} Time for which i is idle or settles brine during s

RU_{ius} Time for which i feeds u during s

RP_{pis} Time for which p unloads crude into i during s

FPT_{pis} Amount of crude transferred from p to i during s

FTU_{ius}	Amount of crude that *i* feeds to *u* during *s*
FU_{us}	Total amount of crude feed to *u* during *s*
$FCTU_{iucs}$	Amount of *c* delivered by *i* to *u* during *s*
VCT_{ics}	Amount of *c* in *i* at the end of *s*
V_{is}	Crude level in *i* at the end of *s*
f_{ics}	Concentration (volume fraction) of *c* in *i* at the end of *s*
DC_v	Demurrage for vessel *v*
SC_t	Safety stock penalty for period *t*

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Chapter 1

INTRODUCTION

1.1 Introduction to Refining industry

Petroleum is the second largest consumable on the planet, second only to water. The modern society is dependent on petroleum products more than any other natural resource for the comforts and convenience. The petroleum business involves many independent operations, beginning with the search for oil and gas and extending to the delivery of finished products, with incredibly complex manufacturing processes in the middle. Each of these processes has unique objectives and demands. Unlike discrete manufacturing, petroleum manufacturing does not start with bill of materials. A bill of materials is a list of all the materials that go into making a particular product. In case of petroleum processes feed stock and intermediates can come from a number of different sources with significantly different qualities, and yet be processed and combined to form precise products, meeting all regulatory specifications as well as shipping times. Petroleum refining is a typical continuous process plant that has a continuous flow of material going in and coming out. Crude oil is basic raw material for producing most of petroleum products. Petroleum refining involves separating crude oil into its constituents, purifying them, and converting them into marketable products. In the refinery, crude enter the crude distillation unit and separated into component streams. Some of these streams are desirable. The undesirable streams are either subject to a series of treatments and undergo specific unit operations and processes in separate units such as crackers, reformers and alkylation units to yield desirable products or blended with desirable streams into finished products.

Undesirable streams may also be sold off or used as low-cost fuels. A general refinery configuration includes crude handling facilities, distillation units, vacuum distillations units, catalytic reforming units, fluid catalytic/Hydro cracking units, hydro treaters, visbraker/delayed coker units and off-site storage/blending facilities to store/process the finished products/intermediate streams. Distillation, vacuum distillation and catalytic reforming units are normally grouped as primary processing units. Fluid catalytic/hydro crackers, hydro treaters, visbraker/delayed coker units are grouped as secondary units where heavier hydrocarbon streams are converted to more useful fuel streams. Quality of the finished petroleum products is very important and has to be strictly complied. Since crude is the basic raw material for producing these products, quality of the crude oil is very crucial. Based on long range forecast of product demand, crude oils are often planned, purchased, and have a delivery schedule set long before they arrive at the refinery. In addition, crudes originated from different sources have different qualities and product patterns, and even some times crudes from the same origin differ in quality. So all the products cannot be produced from every type of crude, and this difference in yield/quality poses different set of constraints on the down stream treatment/processing units resulting in processing bottlenecks, lower throughputs, excessive productions etc. Crude assay provides detailed yields/properties of different cuts based on their boiling range. Impurities present in crudes limit their processing to produce acceptable products. Impurities can be in the form of high sulfur levels, higher metal content, higher levels of basic nitrogen or naphthenic acids etc. Impure crudes are relatively cheaper but fewer refineries are equipped with the required technology that allows processing these crudes. Capital investment in acquiring, updating to the latest technologies is quite high and return on investment is not very attractive. The operating conditions, processing requirements, available technology, and production demands forces refiners to select such crudes which can be processed neat or by blending among the available basket of crudes. Storage or processing segregation of crude types is a common feature in refinery. Segregation is based on key component levels, impurity levels in crudes and processability to yield specific product pattern. Driven by market requirements, refinery operates in different modes, each mode producing a different set of products. Sometimes the streams from different modes are blended to meet product quality specification. Mode switch is changing the feed to distillation unit from one type of crude segregation to other. Every mode switch produces some off-spec production and needs reprocessing and mode switches are inevitable in long run to meet demands. Normally refineries with multi CDU configuration segregate processing and avoid mode switches unless inevitable. Within segregated mode operation, change in feed composition to processing unit results in perturbed operation in the unit resulting some production loss. Given a set of crudes, considerable effort/expertise is required to identify better crude combinations for processing in order to meet customer commitments and to boost gross profit margins. Thus, the refining industry operates under uncertain product demand, and uncertain manufacturing/plant capabilities leading to deviations in actual performance to operating targets set by the monthly refinery plan.

1.2 Need for Scheduling

Every industrial manufacturing business aspires to have maximum profit/return on investment. For being a market leader, the company needs to have a good global coverage, efficient consumer services, lower production costs and reduced inventory levels. Apart from these multiple objectives, the dynamic nature of demands and uncertainties involved in a refinery makes the life of higher management more and more difficult. The data flow into the system, in terms of sale targets/forecasts, material/product inventories, manufacturing costs and deadlines (product delivery dates, crude arrival time, etc), is also enormous. All these factors must be taken into account for planning and scheduling of operations.

With a substantial increase in computational power of modern day computers, there has been renewed interest in finding solutions to these planning/scheduling problems keeping in view as many factors as possible. The next-generation decision support systems, which employ Advanced Planning and Scheduling (APS) techniques, coupled with supply chain management considerations have been in course of development. With the advent of e-business age, when the dynamics of supply chains and collaborations are redefining ways to conduct business, the core of success still relies on seamless flow of information and material across various business ‘nodes’ in ‘networked’ economies. In such a high velocity environment, the goal is to optimize the operations to have maximum gains and planning/scheduling forms the foundation of this process.

In recent years, refining has become an extremely competitive business characterized by fluctuating demands for products, ever-changing raw material prices, and the incessant push towards cleaner fuels. To survive financially, a refinery must operate efficiently. From an operational perspective, a plant would operate the best in a steady state with consistent feed stocks and product requirement and all units operating at full capacity. Any change is undesirable, as it may lead to off-spec products, reduced throughputs, increased equipment wear and tear, uncertainty, and more work. Nevertheless, in the current competitive environment, profit depends on agility, i.e., the ability to exploit short-term opportunities to fill demand at higher profit margins. Processed crude compositions have the greatest influence on refinery margins.

Therefore, refiners tightly control the quality of crude charge and use advanced technologies to plan and schedule crude oil changes.

The increasing reliance on petroleum products is motivation to find better ways to process, and deliver products while maximizing the margins, minimizing the waste, and improving the profitability within the constraints set by the nature of process and environmental regulations. Managing the crude oil operations is vital for better visibility of downstream processes. Since day-to-day operation enhancements are precursors to high throughput and lower operating expenses of a refinery, the focus in this work is on short-term scheduling of crude oil operations. By short term scheduling, it is understood that monthly/weekly targets for the production facilities are known a priori and the objective is to achieve the maximum from the system so as to optimally utilize resources. Before the problem of scheduling is attempted, there is a need for understanding the terminologies involved in refinery business and requirements of a "good schedule".

1.3 Terminology

Crude handling facilities in refinery includes crude unloading facilities, storage facilities and processing facilities. The raw material, crude is commonly transported using sea route in small cargo ship called ‘crude tanker’ or ‘crude vessel’ that can arrive near to the shore for mooring. The mooring station is called ‘Jetty’. Bearing in mind the cost of transportation adding to the cost of raw material and looking at the margins of processing, it is preferable to get different small crude packets normally termed as ‘crude parcel’ in one VLCC (very large crude cargo) ship. VLCC cannot come near to the shore requires special facility mooring called SBM (Single buoy mooring). In both cases a pipeline connects the mooring station to the storage tanks at refinery site. The pipeline connecting the SBM to refinery storage tanks is called

‘SBM line’. Crude is stored in floating roof tanks to avoid the vaporization losses. Crude is normally pumped to a tank and pumped out of tank using different nozzles connected at some height from the bottom of the tank and crude below this nozzle height will be stagnant and is called ‘Heel’ of the tank. This portion facilitates the sediments and brine to settle for periodic removal after every receipt of the crude. The tank operation can be standing gauge or running gauge. Standing gauge operation allows only one operation either receiving or delivering at any one point of time where as running gauge allows both operations. Almost all tank operations in a refinery follow standing gauge operation. ‘Demurrage’ or ‘sea waiting cost’ of a crude cargo ship is the cost associated with delay in unloading crude from crude tanker beyond the accepted time frame as per the logistic contract. A 'graph' , which provides all the information of crude unloading amounts, parcel to tanks allocations, tank to processing unit (CDUs) allocations and the amounts of crude processed that adequately define production is termed as a 'schedule' and problem which yields this solution is the scheduling problem. A schedule thus obtained can be feasible or infeasible. A 'feasible schedule' is one that satisfies all possible constraints - material balances, manufacturing, inventory, demand and user defined constraints. Such a schedule is classified as 'optimum schedule' if it is rated as the best among all feasible schedules based on the objectives - maximization of profits or minimization of production costs. It is worthwhile to now focus on the issues to be addressed in developing optimum schedules for scheduling of crude oil operations.

1.4 Scheduling Considerations

Planning and Scheduling primarily differ in terms of the time frames involved. Planning is generally undertaken for longer time horizon (of the order of months and years) and includes management objectives, policies, etc besides immediate processing requirements. It represents aggregated objectives and usually does not include finer details. Accordingly, the models used are either crude or take simplifying assumptions making them more abstract. If the assumptions overestimate the facility performance giving very little allowances, the resultant plan can become too ambitious and end up as infeasible. On the other hand, if assumptions underestimate the plant's efficiency, the plan thus obtained might be too conservative and lead to under-utilized production capacities. Therefore, for planning operations, one must include the key detailed constraints and their interdependencies in order to get an optimal plan and hence a sound basis for undertaking further scheduling. On the other hand scheduling is the link between the manufacturing process and the customer. The issues addressed by the scheduling vary with the characteristics of the production process and the nature of market served. A formal way of defining the scheduling is the specification of what at each stage of production is supposed to do over short scheduling horizon ranging from several shifts to week. This defines or projects the inputs to and outputs from each production operation. Scheduling is the reality check on the planning process. The objective of scheduling is implementation of the plan, subject to the variability that occurs in the real world. This variability can be in feed stock supplies, quality, the production process, customer requirements or the transport.

Optimization plays an important role in managing the oil refinery. Oil refineries have used optimization techniques for a long time, specifically Linear Programs (LPs) for the planning and scheduling of process operations. While the planning systems provide coordination over several months, the scheduling systems plan the activities over days to weeks. Planning precedes scheduling and uses forecasted product demands. Scheduling includes plant level operations to realize the plan. The refinery planning provides scheduling with volume-based information.

Planning determines the volumes of different feed stocks that will be consumed in different modes. Planning does not provide timing of activities. The function of scheduling group with in refinery is to define charge rates, and the timing of mode/tank changes in response to product requirements and containment problems. A number of refineries decompose the scheduling into crude scheduling, hydraulic scheduling, and product scheduling. Crude schedulers react to the timing of crude arrivals, determine which tank the crude should be placed in, blend crudes as needed to meet targets for yields and qualities off the crude unit, and determine the charging rate to the crude unit. Hydraulic scheduling is concerned with the operations on major units and inventories between the units. The main objective is to have proper control on intermediate inventories. Product scheduling is concerned with defining the timing of blends and the activities required to move the products out of the refinery while ensuring the inventory control.

Crudes are purchased based on the monthly refinery plan. Scheduling subsequently accounts for deviations from the forecast and accounts for changes in demand or plant capability. A priori information of the procured crudes including the types, quantities, and expected times of arrival at the refinery, etc. is used to schedule the short-term activities. These activities include (1) unloading crude oil from vessels to storage tanks and (2) charging various mixes of crude oils to each distillation unit subject to capacity, flow, and composition limitations. Scheduling of crude oil operations defines the timing and the volumes of specific crude mix to be processed to meet the demand while operating continuously with less containment problems.

1.4.1 Planning Objectives

Maximizing the gross refinery margin is the main objective of planning. Some of the factors, which need to be taken into consideration at the highest level in the hierarchy of planning, are:

1. Meeting demand forecast.
2. Efficient utilization of facility's resources (plant capacities, available technology, utilities, manpower, etc).
3. Keeping low Work-In-Process (minimal system hold-up costs).
4. Meeting demands during planned shutdowns of the units.

Many of these affect the planning and in turn affect the daily scheduling. The forethought in these directions becomes imperative when there are highly dynamic demands of products. As discussed earlier, exclusion of these factors might result in schedules, which are good at satisfying local requirements, but can lead to overall inefficiency of the production facility.

1.4.2 Scheduling Objectives

The main objective of scheduling crude oil operations in refinery is to realize the plan considering the changes in plant capabilities, raw material availability and product demand fluctuations. The emphasis in scheduling is the feasibility. The factors that drive crude scheduling are

1. Minimizing the demurrage or sea waiting cost of the unloading tankers
2. Minimizing crude transitions
3. Allocating the better crude combinations for processing
4. Respecting the safe inventory levels.

1.4.3 Processing Constraints

An important and large subset of factors affecting scheduling is called processing constraints. They are essentially divided into three classes:

1. Demand Constraints: They include restrictions on total amount of crude to be processed, amount of specific product to be produced etc.
2. Quality Constraints: They include
   i) Limitations on crude composition to be processed in a distillation unit.
   ii) Limitation on crude composition in a storage tank.
   iii) Limitations of impurities in the feed to distillation units.
3. Quantity and Logic constraints: These consist limitations on the quantity of crude unloading from parcels, crude charging to CDUs and their respective allocations.
4. Inventory Constraints: They consist of crude material balance on storage tanks.

Again, there can be some other miscellaneous or industry specific constraints.

1.4.4 Uncertainties

In spite of the above external and internal limits on the processing facilities, there are instances, beyond anticipation and certainty, when immediate changes need to be brought about in the schedule. They can only be accounted for by suitably taking contingencies in the plan and the corresponding schedules. Some of these uncertainties are as follows:

1. Non availability crude: This can lead to marginal or substantial changes in the current schedule.
2. Spot purchase of crude at lower prices gives an opportunity to increase the profits

3. Specific product demand: There's a fair possibility of getting small to moderately sized specific quality product demand whose production needs to be accommodated within the current schedule.

1.5 Anticipated Benefits

Anticipated benefits vary widely from refinery to refinery, depending on factors such as facility size and complexity; variability of feedstocks; capacity and configuration of tankages; and logistic constraints. Considering the high volumes of crude handled by refineries, the monetary benefits are huge and run into millions of dollars. The following are the generic benefits

1. Enables the scheduler to evaluate the best way to react faster to unexpected changes and maintain optimal refinery operation
2. Optimizes and manages refinery crude storage
3. Allows the scheduler to evaluate the best way to implement the monthly plan, provide guidelines to operations and compare actual operating results to schedules and planning targets.

Specific benefits include:

1. Reduced costs of crude oil processed and chemicals purchased
2. Greater predictability of refinery operations
3. Reduced feed stock and quality issues
4. Better analysis of scheduling options
5. Reduced inventories
6. Superior capacity utilization
7. Increased plant yields
8. Improved visibility of scheduling and inventories across the supply chain

1.6 Research Objective

It can be concluded from the preceding sections that the problem of scheduling can involve enormous considerations for conceiving an optimum schedule taking into account all the objectives. As it appears, in the most general form, the problem is too complicated to formulate mathematically, let alone solving and obtaining an optimum schedule. And even if the problem is formulated, a simplistic approach of enumeration of alternatives sounds preposterous because of the number of possibilities that might exist (combinatorial nature of the problem). A lot of research has been undertaken in this area in the past decade with focus on development of exact and approximate methods to solve short term scheduling problems. Time considerations are important in developing scheduling models. There are two types of time representation in mathematical modeling. One is discrete time modeling, most widely used and studied, where the planning horizon is discretized into pre defined time slots and activities are defined to start and end of the time period. The second is continuous time modeling where the start and end time of each activity is variable and determined by the model solution. Scheduling of crude oil operations is a complex nonlinear problem, especially when tanks hold crude mixes. The resulting problem is a MINLP (mixed integer non linear program) problem that is very compute intensive and difficult to yield an optimal solution. In this work, the focus is on developing MILP (mixed integer linear program) mathematical model (using discrete, continuous time depiction) that represents typical refinery configurations and accommodates most of the real life practices. Since the constraints that define crude mix involve inherent non linearities, a solution algorithm is devised to avoid solving MINLP. Different refinery configurations were considered for this study and illustrated with relevant examples. This work is outlined in the next section.

1.7 Outline Of The Thesis

This thesis is divided into two parts, first part (Chapters 4-6) deals with a discrete time modeling approach and second part deals with continuous time modeling approach to short term scheduling of crude oil operations in a petroleum refinery. Chapter 3 gives a brief introduction to the domain of this system and defines the problem. Along with the problem description it also includes an explanation for the motivation towards crude oil scheduling and presents the drawbacks in the previous work on this problem. An exact mathematical programming formulation based on discrete time modeling approach that account for various crude handling facilities in a refinery is developed in Chapter 4. Subsequently, in Chapter 5, few computational issues pertaining to the exact method are addressed and a solution algorithm is developed and illustrated using motivating example from Chapter 3. In the concluding Chapter of the first part, the model, solution algorithm was evaluated using six different examples with different configurations (SBM, jetties, tank-to-tank transfers), short and long scheduling horizons, and several parcel sizes and arrivals.

In part II (Chapters 7-9), continuous time modeling approach to crude oil scheduling problem is described. An exact mathematical programming formulation based on continuous time modeling approach is developed in Chapter 7 and solution algorithm to avoid the composition discrepancy in the schedule is proposed in Chapter 8. The computational results for such a system are discussed and a comparison with the discrete time approach is presented in detail in Chapter 9. This work is concluded in Chapter 10 with some recommendations for future work in this area.

Chapter 2

Literature Survey

Chemical manufacturing processes can be classified into two types, continuous or batch, based on their modes of operation. A continuous process or unit is the one which produces product in the form of a continuously flowing stream, while a batch unit or process is the one that produces in discrete batches. A semicontinuous unit is a continuous unit that runs intermittently with starts and stops. Batch processes possess the flexibility to produce multiple products and are well suited for producing low-volume, high value products requiring similar processing paths and/or complex synthesis procedures as in the case of specialty chemicals such as pharmaceuticals, cosmetics, polymers, biochemicals, food products, electronic materials etc. In contrast a continuous process, in most cases, is dedicated to produce a fixed product with little or no flexibility to produce another.

In the past years, scheduling problems of various forms have been addressed for the batch/(semi)continuous chemical plants. Extensive reviews in batch processing have been reported in the literature (Reklaitis, 1991, 1992, Rippin, 1993). Many of these problems can be posed as mixed integer optimization programming problems since the corresponding mathematical optimization model involves both discrete and continuous variables that must satisfy a set of equality and inequality constraints. Applequist et al. (1997) and more recently Shah (1998) have given detailed reviews on the perspectives and issues involved in these problems.

Planning and scheduling problems associated with semicontinuous/continuous processing have received less attention are the next addressed area in optimization literature. Kondili et al. (1993) presented a general framework of State Task Network

(STN) that could handle various forms of the scheduling problem arising in multiproduct / multipurpose batch operations. An STN could represent both batch operations (states) and feedstock/products (tasks) explicitly as nodes in a unified directed graph. Processes that share, mix or split raw materials/intermediates were unambiguously represented through this novel technique. The proposed uniform discretized time model though was exhaustive, included too many binary variables and there were some practical considerations required in getting solutions for large scale industrial problems. Many subsequent works by various researchers have extended this model using some additions and deletions (simplifying assumptions) thereby formulating several MILP models. Pantelides (1994) had presented another similar network utilizing the structure of the scheduling problem termed as Resource Task Network (RTN). Zenter et al. (1994) discusses and compares features of uniform discretization models and non uniform continuous models.

Karimi and McDonald (1997) proposed two part models for planning and scheduling of parallel semicontinuous processes. The proposed formulation can handle the problem of single-stage multiproduct facility with parallel semicontinuous processors. Lamba and Karimi (2002) continued the work to include resource constraints.

Cerda et al. (1997) proposed a MILP model for a single-stage batch plant with parallel, non-identical, multiproduct units. Using the concept of job predecessor/ successor to handle sequence-dependent transitions, they allowed restrictions on equipment processing a particular order. They further considered single-product orders, non-preemptive operation and release times for equipment and tasks. Lim May Fong (2002) extended the approach using time slot approach and presented a detailed comparison.

The continuous processing has been the most prevalent and sought-after mode in the chemical processing industry. Few examples of continuous chemical plants are petroleum refineries, petrochemical plants, fertilizer manufacturing plants, polymers and paper production plants etc. Very less attention and much less work has been reported in the scheduling problems of multi product continuous plants despite their practical importance (Sahinidis and Grossmann (1991), Pinto and Grossmann (1994)). Sahinidis and Grossmann (1991) had presented a MINLP model for cyclic scheduling problem for continuous parallel facilities. A key element of the work is the development of the concept of time slots which can be variable in length and within which exactly one product is made. Pinto and Grossmann (1994) extended this work to multistage case.

Quesada and Grossmann (1995) presented a paper that deals with global optimization of networks consisting of splitters, mixers, linear process units that involve multi-component streams and sharp separation networks. During sharp separation of components, non-convexities arise in mass balance equations. The non convex equations involve bilinear terms for flow and composition. They proposed a reformulation linearization technique in order to obtain a relaxed LP formulation. They used some ideas of reformulation linearization technique proposed by Sherali and Alameddine (1992). The proposed techniques are suitable for continuous mixing and for continuous component splitting. They pose limitations in the case where accumulation is inherent in the process. The other main limitation is assumption of linear process units.

Vipul Jain and Grossmann (1998) addressed a problem of scheduling multiple feeds on parallel units with decaying performance over time. They presented a MINLP model using the process information regarding the exponential decay in performance with time to find a cyclic schedule for feed processing. Application of ethylene plant was used to illustrate their methodology.

Alle and Pinto (2002) addressed the problem of the simultaneous scheduling and optimization of the operating conditions of continuous multistage multiproduct plants with intermediate storage. A linearization approach that employs the discretization of non linear variables is presented. A direct comparison of MILP, MINLP models shows that non linear restrictions are more effective than linear discrete restrictions in view of both optimality and computational efforts.

2.1 Planning and Scheduling in Petroleum Refinery

A petroleum refinery is a typical example of multi product, multi unit integrated continuous plant. Refinery planning problems have been studied since the introduction of linear programming in 1950s. Before any computer usage, refinery planning was primarily a volume stock balance over each process and over entire refinery. These plans were essentially linear. The 1950's were a decade of experimentation. The blending of gasoline turned out to be the most popular application of LP techniques. Symonds (1955), Manne (1956) applied linear programming techniques for long term supply and production plan of crude oil and product pooling problems.

Bodington and Baker (1990) presented a review on the history of Mathematical Programming (MP) in the petroleum industry. Their forecast mentioned that non-linear optimization will gain more wide spread use, especially in the areas of operational planning and process control. Optimization plays an important role in managing the oil refinery. Oil refineries have used optimization techniques for a long time, especially successive linear programming (LP/SLP) for the planning and scheduling of process operations. While the planning systems provide coordination over several months, the scheduling systems plan the activities over days to weeks. Planning precedes scheduling and uses forecasted product demands. Crudes are purchased based on the monthly refinery plan. Scheduling subsequently accounts for deviations from the forecast and accounts for changes in demand or plant capability. The availability of LP-based commercial software for refinery production planning such as RPMS (refinery and petrochemical modeling system- Bonner & Moore Management Science), PIMS (Process Industry Modeling System- Aspentech) has allowed the development of general production plans of the whole refinery, which can be considered as general trends.

Oil refining is one of the most complex chemical industries, which involves many different and complicated processes with various possible connections. Refinery scheduling is an inherent nonlinear problem requiring simultaneous solution to inventory management of raw material crude, processing and converting the raw crude into products and then distribution of the products. It is characterized by discrete decisions and various nonlinear blending relationships. Bodington (1992), while solving the scheduling and planning problem associated with gasoline blending, mentioned that there is a lack of systematic methodology for handling non-linear blending relationships. Ramage, (1998) also refers to nonlinear programming (NLP, MINLP) as a necessary tool for the refineries in 21st century. Pelham and Pharris (1996) pointed out that while planning technology can be considered as well developed, fairly standard, and widely understood, the same couldn't be said for short-term scheduling. There is a need to improve scheduling models to account for the complexity arising from discrete decisions and the various blending relationships.

The lack of rigorous models for refinery scheduling is discussed by Ballintjin (1993), who compared continuous and mixed-integer linear formulations and pointed out the limited applicability of models based only on continuous variables. Coxhead

(1993) identified several applications of planning models for refinery and oil industry, including crude selection, crude allocation to multiple refineries, partnership models for negotiating raw material supply and operations planning.

Many refineries partition scheduling into crude scheduling, hydraulic scheduling, and product scheduling (Bodington, 1995). Crude schedulers react to the crude arrivals, assign destination tank/s for each crude, blend crudes as needed to meet the targets for yields and qualities of the fractionated products from the crude distillation unit (CDU), and determine the charging rate to each crude unit. Hydraulic scheduling involves the operations of major units, and inventories between the units with a view to properly control intermediate inventories. Product scheduling is concerned with the blending and distribution of final products, while ensuring the inventory control. Detailed modeling, effective integration and efficient solution of these scheduling problems is essential for the scheduling of overall refinery operations. It was also mentioned that integration of the main business areas sales, operations, distribution would lead to higher profits. The approach followed in today's processing industry environment for the scheduling problem is to use intuitive graphical user interfaces combined with discrete-event simulators.

Shobrys and White (2002) also supported the argument of integrating the planning, scheduling and process control functions. They specifically pointed out about the economic benefits, which are estimated to be 10 dollars, or more increased margin per ton of product. The review addresses the ground reality that many companies have not achieved integration in spite of multiple initiatives and figure out the reasons for failures.

Magalhães et al (1998) proposed an integrated system for production planning (SIPP). They developed the software using expert system (Gensym G2) and mixed integer optimization models/optimization techniques. They claimed that presently a manual simulation version was developed and development of automated version is in progress. The MIP part of the software is being developed under cooperation between PETROBRAS and Dept of Chem. Eng., Sãopaulo. The approach undertaken was to study individual sections of the plant and then integrate the island modules. They proposed three divisions. The first study deals with the problem of crude management. The second major portion includes the process plants, management of intermediate stocks and the final one deal with product blending.

Pinto et al. (2000) discusses planning and scheduling application for refinery operations. They mentioned that the optimization of the production units does not achieve the global economic optimization of the plant because of conflicting nature of the objectives of individual units and contribute to the suboptimal / many times infeasible over all operation. The main obstacle for achieving this is the lack of computational technology for production scheduling that can integrate with planning and process operations. In their communication, they also proposed a non linear planning model for refinery production that represents a general refinery topology.

2.2 Recent Work

Kelly and Forbes (1998) developed an approach that determines how plant feed stocks should be allocated to storage when there are fewer storage tanks than feed stocks. It is assumed that material from storage tanks is subsequently blended and processed in the down stream units. The allocation of feed stocks to storage tanks is an important issue to be considered for the trouble free, flexible downstream processing. They illustrated the methodology using a crude oil storage case. The limitation of the proposed approach is, it works in making allocation decisions and provides a strategy when incoming feed stocks are to be stored in standing gauge tanks (no simultaneous input to and output from the storage tanks) during the time of allocation process.

Moro et al (1998) proposed a planning model for refinery diesel production. The model represents a general refinery topology and allows implementation of nonlinear process models and blending relationships. A non linear programming (NLP) model was developed for a case of diesel production that considers blending relationships and process equations. Issues of meeting demands, product specification and intermediate product component handling are not clearly mentioned. The property correlations of 350deg C + streams are not well defined in the literature and blending relationships of up to 350deg C boiling range streams is well defined and are additive either on volume (index) or weight basis. They posed the refinery configuration as combination of mixer, splitter and a processing unit that consider linear relationship between input and output streams. The key point of this communication was to use plant operating information to predict the product component property and blending is carried out accordingly.

Pinto and Joly (2000) presented a discrete time MIP model for fuel oil (FO) and asphalt production scheduling problem. First they modeled the problem as MINLP and then used rigorous linearization of viscosity balance constraints to transform the model to MILP. The model is based on assumptions that demands are known *a priori* and there are no dead lines for distribution.

Pinto and Moro (2000) addressed a continuous time MILP model for LPG (Liquefied Petroleum Gas) scheduling. The proposed scheduling model was an extension to their diesel planning problem (Moro et al (1998)).

Zhang and Zhu (2000) proposed a novel decomposition strategy to tackle large scale overall refinery optimization problems. This decomposition approach is derived from the analysis of mathematical structure of a general overall plant model. They divided it into two levels, namely a site level (master) and process (sub) level. The master model determines common issues among processes such as allocation of raw materials and utilities etc. once these common issues are determined, sub models then optimize the processes. They then iteratively feed the sub model solution information to master model for further optimization. The procedure terminates when a convergence criteria is met. However they mentioned that there are some serious doubts about the success of this option in reality. Considering the size and complexity of this problem, not only this will lead to mathematical and computational difficulties that make these kinds of approaches inapplicable and the results generated by this approach may also cause confusion.

Zhang et al.(2001) presented a method for overall refinery optimization through integration of hydrogen network and utility systems with the material processing system. They also used a decomposition approach in which material processing is optimized first using LP techniques to maximize the overall profit and then supporting systems including hydrogen network and utility system are optimized to reduce the operating costs for the fixed process conditions determined from the LP optimization. The discrepancy in hydrogen consumption that is used in traditional LP planning tools was the basic idea on which the model is constructed. Traditional LP models consider a fixed make up hydrogen purity but in reality it varies. This produces the non optimal solution. A Base-Delta formulation is used to integrate the hydrogen model into overall optimization model. The new hydrogen model considers the purity and make up rate changes with the liquid processing at the hydrogen consumer units.

Glismann and Gruhn (2001) proposed an integrated approach to coordinate short-term scheduling of multi-product blending facilities with nonlinear recipe optimization. The developed strategy is based on hierarchical concept consisting of three business levels: long range planning, short-term scheduling and process control. Long range planning is accomplished by solving the nonlinear recipe optimization problem. Resulting blending recipes and production volumes are provided as goals for the scheduling level. The scheduling problem is formulated as a MILP derived from RTN representation. The proposed model is discrete time MILP model. They use iterative methodology and impose new constraints on planning level to avoid bottlenecks during scheduling. Thus the strategy is basically an iterative NLP and MILP combination. However they did not consider any product or component properties in their model.

Xiaoxia Lin et al (2003) addressed a problem of scheduling a fleet of marine vessels for crude oil tanker lightering. Lightering is a shipping industry term that describes the transfer of crude oil from a discharging tanker to smaller vessels to make the tanker "lighter". It is commonly practiced in shallow ports and channels where draft restrictions might prevent some fully loaded tankers from approaching the refinery discharge docks. A continuous time formulation is presented with the objective to minimize the cost associated with the logistics such as demurrage of the mother tanker and transportation cost of the lightering vessels. They did not include any constraints relating to storage allocation of crude oil and its processing.

Jia and Ierapetritou (2003) presented a problem of gasoline blending and distribution scheduling. They modeled the scheduling problem as a continuous time MILP model that is based on the assumption of fixed recipe of the product blend to avoid non linearities. They assumed a constant blending rate and used an artificial variable to determine the deviation in blending rate. The objective used was to minimize the artificial variables which ensure meeting the orders. However the properties of component streams were not considered in the formulation.

Rejowski and Pinto (2003) addressed a scheduling of a multiproduct pipeline system that is used to transfer large quantities of different products to distribution centers to meet the customer orders. Pipe line transportation is the most reliable and economical mode for large amounts of liquid and gaseous products. Two MILP models that are generated from linear disjunctions and rely on discrete time were presented. The first model assumes that the pipe line is divided into packs of equal size, where as second one relaxes this assumption. However, their claim that they had relaxed the assumption is not correct. The reason being, they used different pre defined pack sizes for different segments of pipeline in second model that are still parameters and fixed.

2.3 Crude Oil Scheduling

Most of the research work on supply chain management of petroleum refineries, as mentioned above, agrees to the fact that scheduling of overall refinery operation is a difficult task and decomposition approach is the best suggested methodology to optimize the islands of process operations mainly crude oil operations, operation of intermediate conversion process units and finally operations associated with blending and distribution of the final products. Detailed modeling, efficient solution of these scheduling problems is essential. Effective integration is the final step for the scheduling of overall refinery operations.

Optimizing the crude oil operations, the first step toward optimization of overall refinery operations has received attention very recently as refineries are facing extreme market competition and experiencing lower profit margins. Since crude oil costs account for about 80% of a refinery's turnover, a switch to a cheaper crude oil can have a significant impact on margins. However, some feedstock can lead to processing problems and must be blended with other crudes to maintain plant reliability. Furthermore, most refineries have unsteady supply of crude oil. This creates tremendous pressure on the processing facilities and optimizing crude operations becomes mandatory to squeeze out every pound of product and every penny of profit. Scheduling of crude oil operations is thus a critical component of overall refinery scheduling.

Shah (1996) considers the application of formal, mathematical programming techniques to the problem of scheduling the crude oil supply to the refinery. He reported a discrete-time mixed integer linear programming (MILP) model for crude oil scheduling by decomposing into two sub problems. The upstream problem included portside tanks and offloading and the downstream problem included allocation of charging tanks and CDU operation. The methodology was developed to the problem of optimizing the supply of crude between one port and one refinery using one pipeline. The objective was to minimize the tank heel. This model was a starting point and did not contain most of real life practices.

Almost concurrently, Lee et al. (1996) also reported a MILP model to minimize operating cost arising in crude oil unloading, tank inventory management, and crude charging. They linearized the bilinear terms resulting from crude blending operations using the approximations of Quesada and Grossmann (1995). However, as mentioned above the limitation of reformulation techniques proposed by Quesada and Grossmann (1995) is assumption of linear relationship and flow continuity without accumulation. Since crude tank operation (receive and discharge operation) is disjunctive and accumulation is inherent that lead to composition discrepancy in crude charge to CDU. They used single-crude storage tanks and mixed-crude charging tanks in their configuration, but did not allow splitting of feed to multiple CDUs or multiple tanks charging one CDU. They ensured feed quality by using constraints on the concentration of one key component in charging tanks, but did not consider some real-life operational features such as brine-settling, multiple-parcel vessels, multiple jetties, etc. Furthermore, they processed pre-fixed ranges of crude mixes in charging tanks one after another to meet the total demand.

Recently, Li et al. (2002), recognizing this composition discrepancy, proposed an iterative MILP-NLP combination algorithm to solve the problem. They also attempted to reduce the number of binary decisions by disaggregating the tri-indexed binary variables into bi-indexed ones similar to a methodology proposed by Hui and Gupta (2001), and incorporated new features such as multiple jetties and two tanks feeding a CDU. The solution approach fixes all binary decisions from the MILP solution and solves a NLP for adjusting the continuous flow variables to satisfy the quality, capacity constraints. Fixing the allocation variables reduces the degrees of freedom for NLP optimization problem and may violate quality constraints, thus produce infeasibilities even when a solution exists. Furthermore, their changeover definition leads to double counting and their allocation variables impose undesirable restrictions on charging and unloading options.

Kelly and Mann (2002, 2003a,b) highlight the importance and details of optimizing the scheduling of an oil refinery's crude oil feed stocks from the receipt to the charging of the pipe stills. Though they did not propose any mathematical model in their communication but provided an interesting and enhancing qualitative discussion on related issues of the problem and presented some quantified benefits of better crude oil scheduling. They suggested hierarchical decomposition as a possible strategy for solving this complex problem, in which logistics and quality accounting is done in two separate steps. The logistics sub problem deals with allocation and quantity issues and ignores the quality considerations. The quality problem is solved after the logistics sub problem where the logic variables are fixed and quantity and quality variables are adjusted to respect both the quantity and quality bounds and constraints. However, they correctly anticipated that such a strategy might fail to yield a solution in some cases. In such situations, they mentioned that there be a mechanism to send back some special constraints to the logistics sub problem to force it away from search space that are known to cause infeasibilities. They pointed out that for the immediate future, discrete time formulations have an edge over continuous time formulation. Some of the important features of blending like batch, continuous and within each recipe, specification types are aptly explained. They describe crude scheduling as an application with multi-million dollar benefits and point out the intractability of this problem in general, especially in reasonable time.

Kelly (2002) proposed a chronological decomposition heuristic (CDH) based algorithm which is a simple time-based divide and conquer strategy intended to find integer-feasible solutions to production scheduling optimization problems. This algorithm does not guarantee the global optimum and uses branch and bound technique. CDH makes use of MILP solutions to increase the possibility of finding better solutions and mitigate the possibility of encountering dead-ends or infeasibilities. The simplest form of a dead-end recovery strategy is called “chronological” or “last-in-first-out” backtracking which consists of going back to the most recently instantiated time period node with at least one alternative node left to branch on. They illustrated the methodology on crude oil scheduling problem.

Joly et al. (2002) proposed a continuous-time formulation for the refinery scheduling problem, but their communication did not provide details of their model or objective function. They illustrated their approach using a single column refinery handling 3 different types of crudes. The published results shows many occurrences of tank changeovers to CDU (16 occurrences in 112hr of operation) which is practically not an acceptable solution. They did not talk about the composition discrepancy while illustrating the example.

Jia and Ierapetritou (2003a) also addressed the short-term scheduling of refinery operations based on a continuous-time formulation. They divided refinery operations into three sub problems, the first involving crude oil operations, the second dealing with other refinery processes and intermediate tanks, and the third related to finished products and blending operations. They addressed only the first sub problem in which they used the component balance of Lee et al. (1996), which suffers from the composition discrepancy mentioned earlier. They did not consider the changeover costs arising from crude class or tank changes. Change of crudes and/or tanks is an important operational activity in the refinery, which results in production losses and slop creation. Their proposed model does not allow many operational features such as multiple tanks feeding one crude distillation unit (CDU), single tank feeding multiple CDUs, settling time for brine removal after crude receipt, etc. Besides, their demurrage accounting may be inaccurate, as they seem to compute demurrage for the total time that a vessel spends for unloading crude.

A more elaborated version of the Jia and Ierapetritou (2003a) was presented by Jia et al (2003b) in which they considered transition losses. The definition of occurrence of transitions is not mentioned. They accounted number of transitions as total number of tank beginings to feed CDUs minus the number of tanks. The formulation allows one charging tank to feed a CDU which means the composition constraints on charging tanks drive the production which is similar approach as to Lee et al (1996) and so has the inherent pitfalls of Lee et al(1996). The other draw backs such as demurrage accounting, composition discrepancy in solution remained as it was with Jia and Ierapetritou (2003a).

Magalhães and Shah (2003) also proposed a continuous-time model for crude oil scheduling. While the details of algorithm and model were not reported, they espoused real-world operational rules such as crude segregation, non-simultaneous receipt and delivery of crude by a tank, and settling time for brine removal. They scheduled to achieve the target crude throughput over the scheduling horizon, but did not consider crucial practical aspects such as demurrage and changeover. However, they did acknowledge the importance of these aspects in making the problem realistic.

We now define the objective, scope and extent of this research effort.

2.4 Research Focus

As seen from the above, none of the existing approaches satisfactorily addresses composition discrepancy in crude charge to CDU, transfer lines with non-negligible volumes, and other important operational features such as settling time, tank to tank transfers etc.

This work focuses on the developing of complete mathematical model for crude oil scheduling problem using discrete, continuous time modelling considerations. We address the short-term, deterministic scheduling of crude oil operations in a petroleum refinery, subject to the capacity, quality and demand constraints. In addition to these commonly known constraints, we incorporated several other real-life operational features/rules such as segregated storage/processing, allowing settling time for brine after crude receipt and before crude delivery, single storage tank delivering to multiple CDUs and multiple tanks delivering to single CDU that are very important to build model that resemble the real world operation. Apart from these, we also considered the tank to tank crude transfers feature that is commonly employed in some of refineries and various refinery configurations. We also employ a realistic profit-based objective function that includes marginal crude profits, safety stock penalties, and accurate demurrage accounting. Finally, we devise an iterative, MILP-based solution algorithm that obviates the need for a NLP solution and avoids the pitfall of composition discrepancy.

Firstly, we develop a discrete time mathematical model that incorporates all of the above mentioned features. Most of previous work on discrete models allocated a single receiving task to a fixed time slot, here in this work we attempted to allocate more than one receiving task incase the total time of the slot is not completely utilized. We propose a novel, hybrid (discrete-continuous) formulation that accommodates all industrially important configurational and operational features. We first develop a model for a refinery with SBM as crude unloading facility and then extend the formulation to incorporate other crude oil unloading facility such as multiple jetties. We propose separate model for SBM case alone, several jetties as stand alone unloading facility and combination of both.

In second part of the work, we develop a continuous time mathematical model for the crude scheduling problem using variable length slots as defined by Karimi and McDonald (1997), Lamba and Karimi (2002) and provide the comparison of discrete and continuous time models. In what follows, the system is first described and the scheduling problem is defined.

Chapter 3

PROBLEM DESCRIPTION

3.1 Introduction

Today, petroleum refining is an asset-intensive, operationally complex, low-margin, and extremely competitive industry. The challenge to remain profitable is compounded by ever-changing raw material prices and fluctuating demands for products. Refiners have therefore resorted to advanced decision-making technologies to respond quickly to problems and opportunities and make decisions with a high level of confidence. One of the critical tools that allow the refiners to boost their margins is short term refinery schedule. Crude oil quality is an important aspect of refinery master planning, revamping and daily operations. Its quality and the refinery hardware for processing it determine the refining value of a barrel of crude oil. Crude oil costs account for about 80% of a refinery's turnover; and most refineries have unsteady supply of crude oil. This creates tremendous pressure on the processing facilities and thus optimizing crude operations becomes mandatory for a refiner. Scheduling of crude oil operations is thus a critical component of overall refinery scheduling. The crude schedulers' job in a refinery has become increasingly complex in recent years. They must continuously watch both the crude oil movements and the operational status of the plant and match them to fluctuating demands. In most cases, under intense time pressure and low inventory flexibility, the schedulers rely largely on their experience, and select the first feasible solution found by a spreadsheet model or other tool. However, tremendous opportunity for economic and operability improvement exists in this process. Maximizing productivity, minimizing operating expenses and complying with demand of products are normally the top management priorities in refining business. In many cases, even a marginal gain in overall throughput can mean substantial additional revenues. Crude, manual, time-consuming, and trial and error spreadsheet planning and scheduling approaches that are commonly used are quickly becoming inadequate in the face of intense competition. Thus, there is a need for developing systematic, computer-aided, optimal or nearly optimal methodologies that can solve large-scale scheduling problems.

In this work, we addressed short term scheduling of crude oil operation in a petroleum refinery that considers most of the real-world industry practices. The scheduling objective is to maximize gross operating margin minimizing the demurrage and crude transitions.

3.2 Process Description

Figure 3.1 gives a schematic of the crude oil unloading, storing and processing in a typical refinery. The configuration involves crude offloading facilities such as an SBM (Single Buoy Mooring) or SPM (Single Point Mooring) station and/or one or more jetties, storage facilities such as storage tanks and/or charging tanks and processing facilities such as crude distillation units (CDUs). The operation involves unloading crudes into multiple storage tanks from the ships/tankers arriving at various times and feeding the CDUs from these tanks at various rates over time. Thus, the problem involves both scheduling as well as allocation issues.

Most refineries receive and process several crudes. Crudes arrive in either large multi-parcel tankers or small single-parcel vessels. A very large crude carrier or VLCC has multiple compartments to carry several large parcels of different crudes. However, due to its size, it must dock offshore at a station called SBM or SPM. The SBM connects to the crude tanks in the refinery via one SBM pipeline.

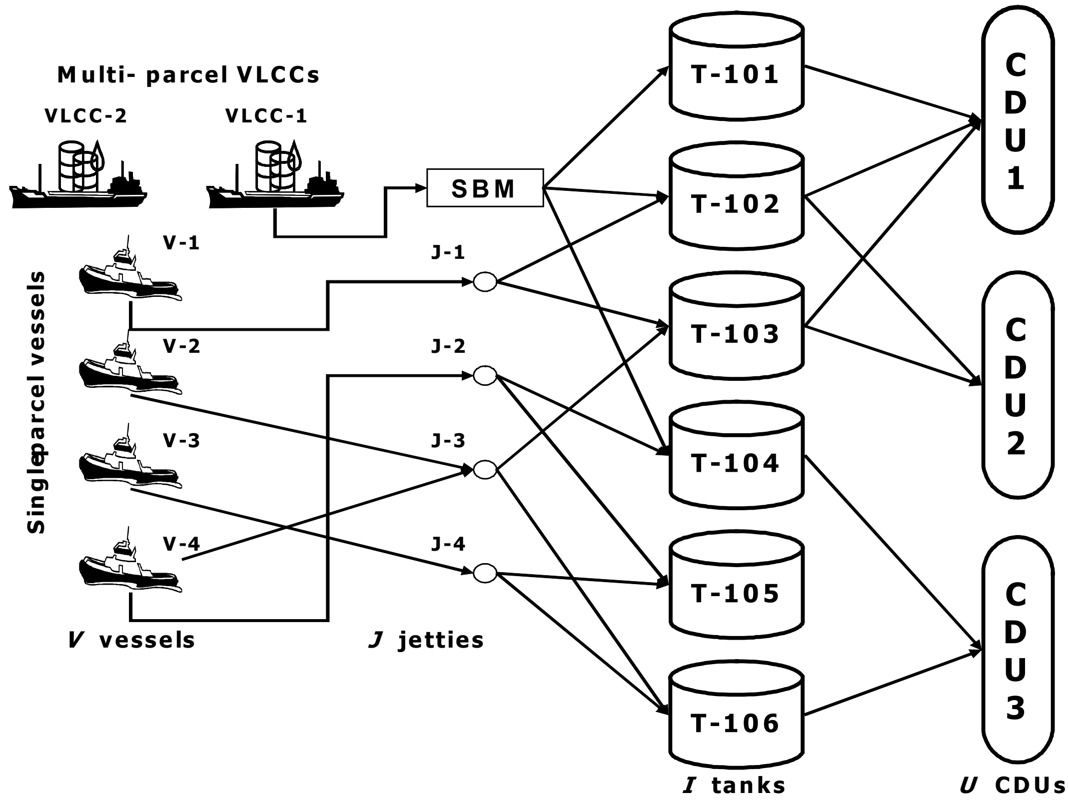


Figure 3.1: Schematic of oil unloading and processing

This SBM line has a significant holdup capacity that cannot be ignored, as the crude type present in the line may not match the crude type of the parcel being unloaded currently from a multi-parcel VLCC. SBMs have become quite important, because transporting large crude parcels reduces unit transportation costs. However, from time to time, a refinery may also receive small parcels of single crudes via small ships that dock at an onshore jetty. A refinery may have multiple such jetties. The characteristics and operations of SBM and jetties are quite different. Usually, there is only one SBM, so VLCCs can dock only one at a time. Similarly, there is only one pipeline, so only one crude parcel can unload at any time. In addition, each parcel must first eject the crude already present in the SBM line. In contrast to the SBM line, the holdup in the pipeline connecting a jetty to a tank is negligible. When there are multiple jetties, multiple ships can dock at the same time and transfer crude parcels simultaneously. In practice, pipelines also transport crude from marine terminals to distant inland refineries or various petroleum products from refineries to far away destinations (Rejowski and Pinto, 2003).

Many types of crude exist in the market, varying widely in properties, processability and product yields. Years of experience have helped the refiners classify crudes based on some key characteristics such as processability, yields of some premium products, impurities or concentrations of some key components that influence the downstream processing. This has led to the common practice of segregating crudes (Kelly and Forbes, 1998) in both storage and processing. Thus, tanks and CDUs usually store or process only specific classes of crudes. With this brief introduction, we now state the crude scheduling problem.

3.2.1 Problem Statement

Given:

1) Arrival times of ships/VLCCs, volumes and crude types of their parcels
2) Configuration details (numbers of CDUs, storage tanks, jetties and their interconnections) of the refinery
3) Holdup in the SBM pipeline and initial crude type
4) Limits on flow rates from the SBM station and jetties to tanks and from tanks to CDUs
5) Limits on CDU processing rates
6) Storage tank capacities, their initial inventory levels and initial volume fractions of crudes in tanks
7) Information about modes of crude segregation in storage and processing

8) Information about key component concentration limits during storage and processing
9) Economic data such as sea waiting costs, pumping costs, crude changeover costs, etc.
10) Production demands during the scheduling horizon. These are normally available from the monthly production plan of the refinery.

Determine:

1) A detailed unloading schedule for each VLCC / vessel
2) Inventory and composition profiles of storage tanks
3) Detailed crude charge profiles for CDUs

3.2.2 Operating Rules

Most refiners use some operating rules. In this work, we assume the following:

A tank receiving crude from another tank, a ship, or a tanker cannot feed a CDU at the same time.

1) Each tank needs some time (8 h) for brine settling and removal after receiving crude.
2) Multiple tanks can feed a single CDU. Most refiners allow at most two tanks to feed a CDU, the operating complexity increases and operation controllability becomes a problem for more than two in absence of individual tank flow control.
3) A tank may feed multiple CDUs. Again, a tank normally does not feed more than two CDUs.

3.2.3 Assumptions

Finally, we make the following assumptions regarding the refinery operation:

1) Only one VLCC can unload at any moment. This is reasonable, as there is only one SBM.

2) The sequence in which a VLCC unloads its parcels is known a priori. A VLCC always unloads parcels in the same sequence that it loads. Refinery planning, which usually takes place weeks before the ship starts sailing towards the refinery and the scheduling activity, predefines the loading sequence of parcels. Thus, the parcel unloading sequence is indeed beyond the purview of crude oil scheduling and this assumption is reasonable.
3) A parcel can unload to only one storage tank at any moment.
4) The SBM pipeline holds only one type of crude at any time and crude flow is plug flow. This is valid, as parcel volumes in a VLCC are much larger than the SBM pipeline holdup.
5) Crude mixing is perfect in each tank and time to changeover tanks between processing units is negligible.
6) For simplicity, only one key component decides the quality of a crude feed to CDU.

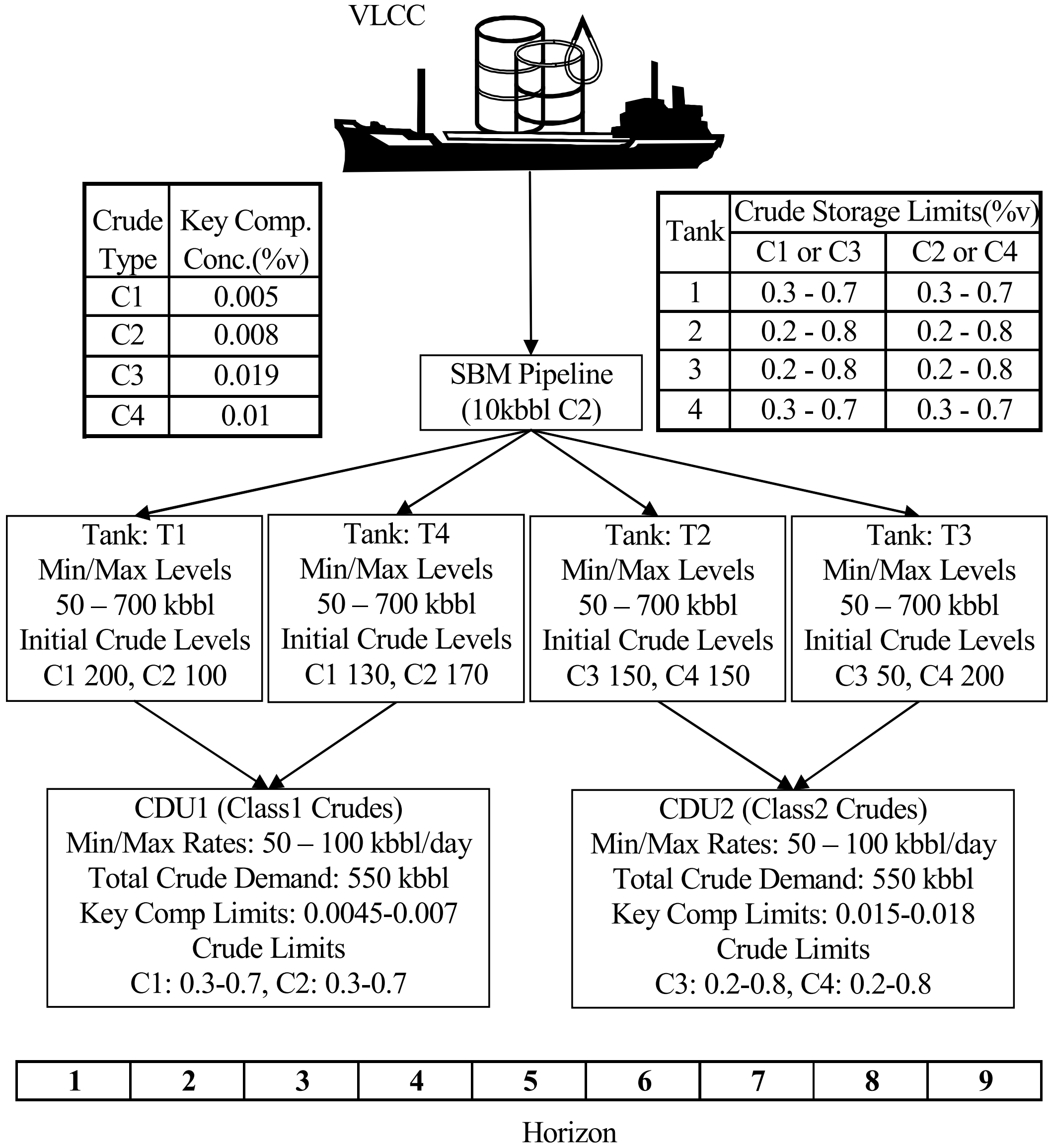
In the following section, we use several motivating examples to highlight the drawbacks of existing approaches and the need for further work.

3.3 Motivating Examples

Three examples are considered to (1) provide some insight into the scheduling problem (2) illustrate its complexity and (3) highlight the issues that previous work has failed to resolve.

A refinery configuration (Figure 3.2) with four storage tanks (T1, T2, T3, and T4), two CDUs (CDU1, CDU2) and one SBM line was considered. The refinery handles four crudes (C1, C2, C3 and C4) and segregates them into two classes (Class1, Class2). As shown in Figure 3.2, C1 and C2 belong to Class1; C3 and C4 belong to Class2; T1, T4 and CDU1 handle Class1 crudes; and T2, T3 and CDU2 handle Class2 crudes.

3.3.1 Example 1



Crude Type	Key Comp. Conc.(%v)
C1	0.005
C2	0.008
C3	0.019
C4	0.01

Tank	Crude Storage Limits(%v)	
	C1 or C3	C2 or C4
1	0.3 - 0.7	0.3 - 0.7
2	0.2 - 0.8	0.2 - 0.8
3	0.2 - 0.8	0.2 - 0.8
4	0.3 - 0.7	0.3 - 0.7

Figure 3.2: Operation schedule for the motivating example

The planning horizon is 9 days and one VLCC carrying three parcels (300 kbbl C1, 300 kbbl C4 and 350 kbbl C3, unloaded in that sequence) arrives at the start of the horizon. The SBM pipeline holds 10 kbbl of C2 initially. Acceptable concentration (% volume) limits of the key component are [0.0045-0.007] for CDU1 and [0.015-0.018] for CDU2. Both CDU1 and CDU2 must process 550 kbbl of crude during the horizon. Figure 3.2 lists the minimum, maximum and initial inventory levels in tanks, and initial levels and acceptable fractions of crudes. It also lists the acceptable limits on crude fractions in the feeds to CDUs. The schedule is in periods of 1 day and the objective is to maximize the gross profit, which is the difference between the marginal profits of crudes and the operating cost. The operating cost includes sea-waiting cost (or demurrage) of VLCC, crude-mix changeover losses and safety stock penalty. Sea waiting cost is \$10K per period (1 day), changeover cost is \$5K per occurrence and penalty is \$0.2K per kbbl per period for depleting the total inventory beyond the minimum stock of 1200 kbbl. The crude margins are \$3K, \$4.5K, \$2K, and \$4K per kbbl of C1, C2, C3 and C4 respectively. We define crude margin as the total value of crude cuts from a crude oil (not the final refinery products) minus the cost for purchasing, transporting and processing the crude.

In many refineries, crude scheduling is a largely manual task with little optimization. It is clear that a manual scheduling approach cannot effectively handle a complex scheduling objective such as the one mentioned above. It is almost impossible for the human scheduler to identify "optimal" crude mixes to process in each CDU. Therefore, such an approach would normally aim to unload the parcels as early as possible and then attempt to maintain constant feed rates to the CDUs, while minimizing crude-mix changeovers. Table 3.1 shows a candidate schedule obtained from such a strategy and the optimal schedule. Table 3.2 compares the profits of the two schedules. In spite of having an extra changeover, the optimal schedule reduces safety stock penalty, but more importantly, increases the profit by using crude mixes in CDU2, which are more profitable. Even for this small example, rigorous optimization increases the gross profit by 3.1%. In the face of narrow margins and intense competition, this can make or break a refinery's bottom line. Besides the above example, previous work (Lee et al., 1996; Li et al., 2002, Kelley and Mann, 2002) has also clearly established the benefits of optimized crude scheduling.

Table 3.1 Candidate and optimal schedules for the motivating example

Schedule	Tank	Crude Amount [to CDU No.](from Vessel No) in kbbl for period								
		1	2	3	4	5	6	7	8	9
1 (Manual)	1	-50[1]	-50[1]	-14.3[1]	-14.3[1]	-14.3[1]	-14.3[1]	-14.3[1]	-14.3[1]	-14.3[1]
	2	-50[2]	+300(3) +100(4)		-50[2]	-50[2]	-75[2]	-75[2]	-75[2]	-75[2]
	3		-50[2]	-50[2]	+240(4)					
	4	+10(1) +300(2)		-50[1]	-50[1]	-50[1]	-50[1]	-50[1]	-50[1]	-50[1]
2 (Optimal)	1	-50[1]	-50[1]	-8.33[1]	-8.33[1]	-8.33[1]	-8.33[1]	-8.33[1]	-8.33[1]	-100[1]
	2	-13.3[2]	+300(3)		-100[2]	-50[2]	-50[2]	-50[2]	-50[2]	-100[2]
	3	-36.7[2]	-50[2]	-50[2]	+340(4)					
	4	+10(1) +300(2)		-50[1]	-50[1]	-50[1]	-50[1]	-50[1]	-50[1]	

'-' sign represents delivery to [CDU], '+' sign represents receipt from (parcel)

Table 3.2 Profit Comparison for candidate and optimal schedules

Profit component	Manual (k$)	Optimal (k$)
Sea waiting cost	0.00	0.00
Change over cost	**15.00**	20.00
Safety stock penalty	52.14	**40.00**
Marginal profit on CDU1	1905.27	1907.79
Marginal profit on CDU2	1833.08	1936.55
Gross profit	3671.21	3784.33

These benefits increase dramatically for bigger and more complex systems. As mentioned by Lee et al. (1996), in addition to the economic benefits, the automation of operating procedures saves manpower and time by replacing the largely manual approach by the largely automated scheduler. Therefore, there is a clear incentive for developing systematic techniques to handle this scheduling problem. The next two examples show that previous attempts at this problem have not succeeded fully.

3.3.2 Example 2

In this section an example from Lee et al. (1996) was considered to show that composition discrepancy can easily arise in their linearized formulation. Figure 3.3 gives the data for the example.

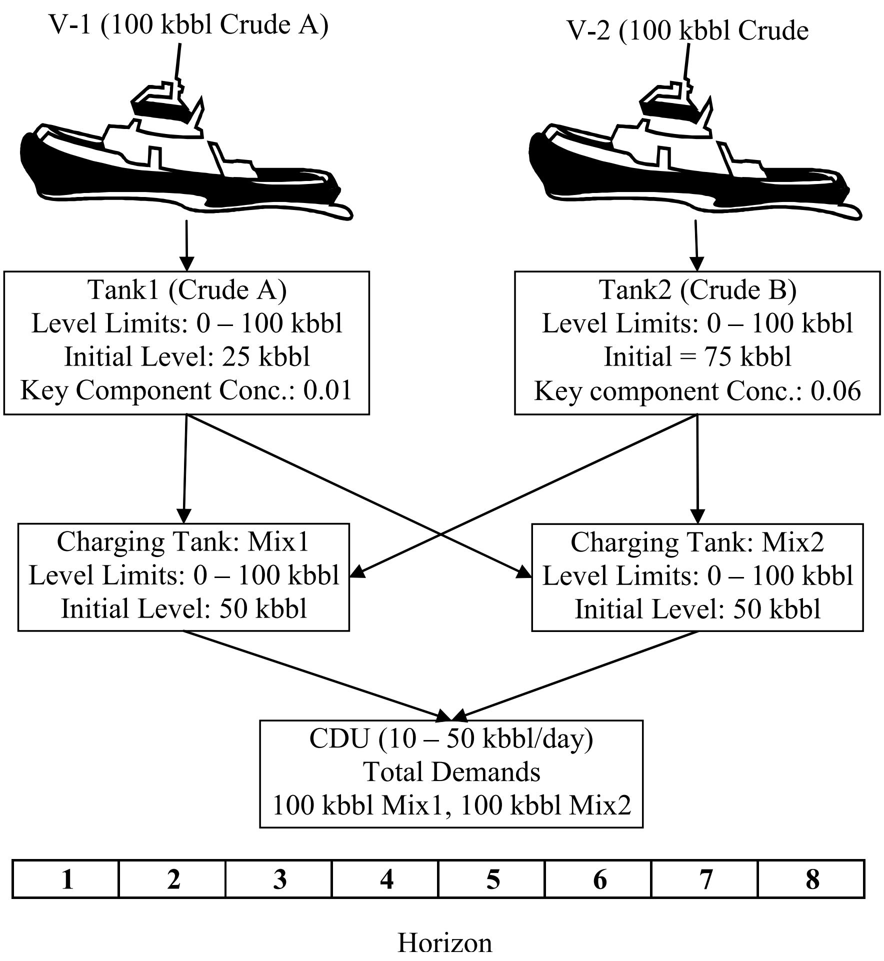


Figure 3.3: Operation schedule for the motivating example of Lee et al. (1996)

Table 3.3 gives the results from the MILP of Lee et al. (1996). During period 3, crude in Mix1 has a key component concentration of 0.032. However, it is clear from Table 3.3; the feed from Mix1 to the CDU has a key component concentration of 0.024 in period 3. The same happens during periods 5 to 8 in case of Mix2. One can also see that a discrepancy also exists in the amounts of individual crude feeds to the CDU. For instance, Table 3.3 shows that the CDU should receive 28 kbbl of crude A and 22 kbbl of crude B in both periods 2 and 3. Instead, it receives 20 kbbl of A and 30 kbbl of B in period 2 and 36 kbbl of A and 14 kbbl of B in period 3. Thus, in the MILP of Lee et al. (1996), the crude composition in a feed tank may not match that in its feed to the CDU. Furthermore, it is clear from Table 3.3 that the key component and crude concentrations in the feed vary from period to period, even when they should remain unchanged. For instance, the key component concentrations are 0.051, 0.052, 0.055 and 0.060 for periods 4 to 7 respectively for Mix2 feed. Similarly, as discussed above, the delivered crude amounts are different in periods 2 and 3 for Mix1 feed. In short, the MILP formulation of Lee et al. (1996) may lead to above two forms of concentration discrepancy. The reason for the discrepancies in Lee et al. solutions is evident. As discussed later in chapter 4, amounts of individual crudes that a charging tank feeds must be in proportion to its composition. When the tank composition is unknown, the constraints that enforce this requirement become bilinear. A discrete-time formulation (e.g., that of Lee et al., 1996) that approximates these bilinear constraints by linear ones will manifest the two discrepancies described above. This is, because when the individual crude amounts (fed to CDU and held in tank) appear in linear constraints only, the optimizer is free to feed individual crudes without regard to their amounts in the tank. And, if there are no constraints that ensure that the tank composition remains constant, when the tank receives no crudes, then the crude amounts vary arbitrarily over periods also, as they are free to do so.

Table 3.3 Key component concentration in the feed to CDU for different allowable concentration limits (Cases 1 & 2) for charging tanks in Lee et al. (1996) motivating example and feed composition discrepancy in Case 2.

Case	Charging tank / Feed	Concentration of Key Component in period								
		t = 0	1	2	3	4	5	6	7	8
1 (Lee et al 1996)	Mix 1 Feed	0.02	0.02			0.025	0.025	0.025	0.025	0.025
	Mix 1	0.02	0.02			0.025	0.025	0.025	0.025	0.025
	Mix 2 Feed	0.05	0.05	0.055	0.055					
	Mix 2	0.05	0.05	0.055	0.055					
2 (Relaxed limits on charging tanks)	Mix 1 Feed	0.02		0.032	0.024					
	Mix 1	0.02		0.032	0.032					
	Mix 2 Feed	0.05	0.05			0.051	0.052	0.055	0.06	0.06
	Mix 2	0.05	0.05			0.051	0.051	0.051	0.051	0.051
	Mix tank . Crude type	Actual(Target) crude flow (kbbl) to CDU in period								
		0	1	2	3	4	5	6	7	8
	Mix1.A			20(28)	36(28)					
	Mix1.B			30(22)	14(22)					
	Mix2.A		10(10)			3(1.8)	3(1.8)	3(1.8)	(1.8)	(1.8)
	Mix2.B		40(40)			7(8.2)	7(8.2)	7(8.2)	10 (8.2)	10 (8.2)

Target represents the crude flow that should be delivered as per tank composition

3.3.3 Example 3

Li et al. (2002) noted the first discrepancy as stated in the previous example and proposed an iterative MILP/NLP approach to correct it. In their algorithm, they first solve a MILP similar to Lee et al. (1996), whose solution may have the concentration discrepancies. Based on that solution, they fix the vessel-to-tank and tank-to-CDU allocation variables and replace the linearized crude blending constraints by the exact nonlinear ones. This results in a NLP model, which they solve to correct the compositions. Then, they use these compositions in the MILP and resolve the MILP with correct linear composition constraints. Thus, they alternately solve MILP and NLP, until they satisfy some termination criteria. The same approach was used to solve the first motivating example (Figure 3.2) discussed earlier. Table 3.4 shows the crude amounts fed to the two CDUs during various periods, as obtained by solving the first MILP. It is clear that the delivered amounts vary from period to period and do not respect the crude compositions in the tanks. Now, to correct this discrepancy, one must solve a NLP by fixing the allocations given by the first MILP and using the correct composition constraints. This means that the assignments of tanks for receipt and delivery are fixed for each period. Surprisingly, it was found that the resulting NLP was infeasible. A closer look revealed that T4 feeds CDU1 exclusively in periods 6-9. Its composition is 29.5% C2 and 70.5% C1, which does not meet the quality requirement (min 30% C2) for CDU1. To satisfy this quality requirement, both T1 and T4 must feed CDU1 in some proportion. However, the NLP of Li et al. (2002) has no freedom to change the tank-to-CDU allocations and hence cannot find a feasible solution. This clearly demonstrates that Li et al.'s algorithm may not find a solution in all cases, and reinforces the assertion of Kelly and Mann (2002) that a decomposition strategy may fail to give a feasible solution, even though one exists.

Table 3.4 Charging schedule of CDUs obtained using Li et al. (2002) approach for the motivating example

Tank	CDU	Crude	Initial Inventory (kbbl)[fraction]	Actual(Target) crude flow (kbbl) to CDU in period or crude{receipt} [fraction] in tank at period end								
				1	2	3	4	5	6	7	8	9
T1	1	C1	200[0.667]	35(33.35)	20(33.35)	35(33.35)	35(33.35)	35(33.35)				
	1	C2	100[0.333]	15(16.65)	30(16.65)	15(16.65)	15(16.65)	15(16.65)				
T2	2	C3	150[0.5]	10(25)	[0.208]		10(10.4)	10(10.4)	10(10.4)	10(10.4)	20(20.8)	20(20.8)
	2	C4	150[0.5]	40(25)	{300} [0.792]		40(39.6)	40(39.6)	40(39.6)	40(39.6)	80(79.2)	80(79.2)
T3	3	C3	50[0.2]		10(10)	10(10)	{340} [0.245]					
	3	C4	200[0.8]		40(40)	40(40)	[0.755]					
T4	4	C1	130[0.433]	{300} [0.705]					35(35.25)	35(35.25)	42(70.5)	70(70.5)
	4	C2	170[0.567]	{10} [0.295]					15(14.75)	15(14.75)	58(29.5)	30(29.5)

Target represents the crude flow that should be delivered as per tank composition

Furthermore, a global optimal solution to NLP cannot be guaranteed in Li et al.(2002) algorithm in all cases.

3.4 Outline

The discussion in the above sections points to the need for an improved methodology for solving the crude scheduling problem. In the subsequent chapter, an MILP formulation is developed using a discrete time representation to obtain an optimal schedule for the abovementioned problem. At first, a complete MILP formulation is developed for scheduling a refinery with one SBM line, *I* storage tanks and *U* crude distillation units (CDUs). Then, the subsequent sections discuss the modifications required to handle *J* jetties instead of one SBM line. The two formulations are merged to obtain a formulation that accommodates one SBM line and *J* jetties. Then, the formulation was modified to extend the model to allow a common practice of tank-to-tank transfers.

Lastly, we present the novel solution algorithm, compare and contrast our model with previous models and demonstrate application of our approach using a range of diverse examples.

Chapter 4

MODEL FORMULATION – Discrete Time (part I)

4.1 Time Representation

To begin with, as in any scheduling problem, the time domain must be appropriately represented. In the chemical scheduling literature, researchers have used primarily two types of time representations - continuous and discrete. Both have their own advantages and disadvantages. Most past attempts at this problem have also used discrete representation of time. Pinto et al. (2000) suggested that although continuous-time models reduce the combinatorial complexity substantially, discrete-time models are still attractive, as they easily handle resource constraints and provide tighter formulations. In this work, a uniform discrete-time representation is used. Let *NT* identical periods (t = 1..*NT*) make up the scheduling horizon. 8 h periods are the best choice, because most refineries operate in 8 h shifts and operators generally prefer to synchronize the starts of critical tasks with the starts of shifts. Shorter periods can give greater accuracy, but would increase the computational burden excessively.

For the sake of simplicity and to improve readability, we first develop a MILP formulation for scheduling a refinery with one SBM line, *I* storage tanks and *U* crude distillation units (CDUs). Then, we discuss the modifications required to handle *J* jetties instead of one SBM line. In the next section, we merge the two to obtain a formulation that accommodates one SBM line and *J* jetties. Lastly, we extend the model to allow a common practice of tank-to-tank transfers.

4.2 Unloading Via SBM

4.2.1 Parcel Creation

As the first step in the formulation, all arriving multi-parcel VLCCs are converted into individual single-crude parcels. Before a VLCC can unload its first parcel, it must first eject the crude residing in the SBM pipeline. This ejected crude will normally not match the crude in the first parcel, hence one must treat it as a distinct single-crude parcel and it must transfer (to a tank) before the first parcel from the VLCC. By the same logic, the last parcel of a VLCC cannot transfer fully, because a portion equal to the SBM line capacity must remain in the SBM line and cannot transfer until the next VLCC starts unloading. As explained above, one must treat that portion as a distinct single-crude parcel; call it an SBM parcel. In other words, unloading of each VLCC results in an extra single-crude SBM parcel from its last parcel. This obviously reduces the size of the last parcel in the VLCC. This gives an ordered list (order of unloading) of all single-crude parcels, those in the VLCCs and their resulting SBM parcels. The first parcel in this list is an SBM parcel that originated from the last VLCC that visited the refinery in the past scheduling horizon. All parcels of the first VLCC to visit the refinery in the scheduling horizon follow next, in the order of their unloading. The last VLCC parcel will have a reduced size, but the SBM parcel that it created will follow next. This continues for all VLCCs. At the end of the current scheduling horizon, the SBM pipeline will hold the SBM parcel originating from the last parcel of the last VLCC to visit the refinery in the current scheduling horizon. The list of parcels for the current scheduling exercise excludes this parcel. To illustrate this parcel creation step, consider a simple example with two VLCCs as in Figure 4.1. VLCC-1 has three parcels: 250 kbbl Oman, 300 kbbl Murban and 110 kbbl Ratawi, which are to unload in that sequence. VLCC-2 also has three parcels: 250 kbbl Escravos, 250 kbbl

Forcados, and 250 kbbl Arabmix, which are to unload in that sequence. At start, the SBM line holds 10 kbbl Kuwait. This is different from Oman in the first parcel of VLCC-1, so 10 kbbl Kuwait in the SBM line be treated as a distinct parcel that must unload first. Thus, the first three parcels in the list become 10 kbbl Kuwait, 250 kbbl Oman and 300 kbbl Murban. The Ratawi parcel of VLCC-1 will be fourth in the list, but with a reduced size of 100 kbbl, because 10 kbbl of Ratawi will remain in the SBM line, until VLCC-2 ejects it. Thus, the remaining parcels (the fourth and later) in the list become 100 kbbl Ratawi, 10 kbbl Ratawi, 250 kbbl Escravos, 250 kbbl Forcados, and 240 kbbl Arabmix. Note that the last parcel (Arabmix of VLCC-2) has a reduced size. The SBM parcel emanating from the Arabmix parcel will be first in the parcel list for the next scheduling horizon. Thus, the list has two extra SBM parcels to model the operation of an SBM line with nonzero holdup effectively.

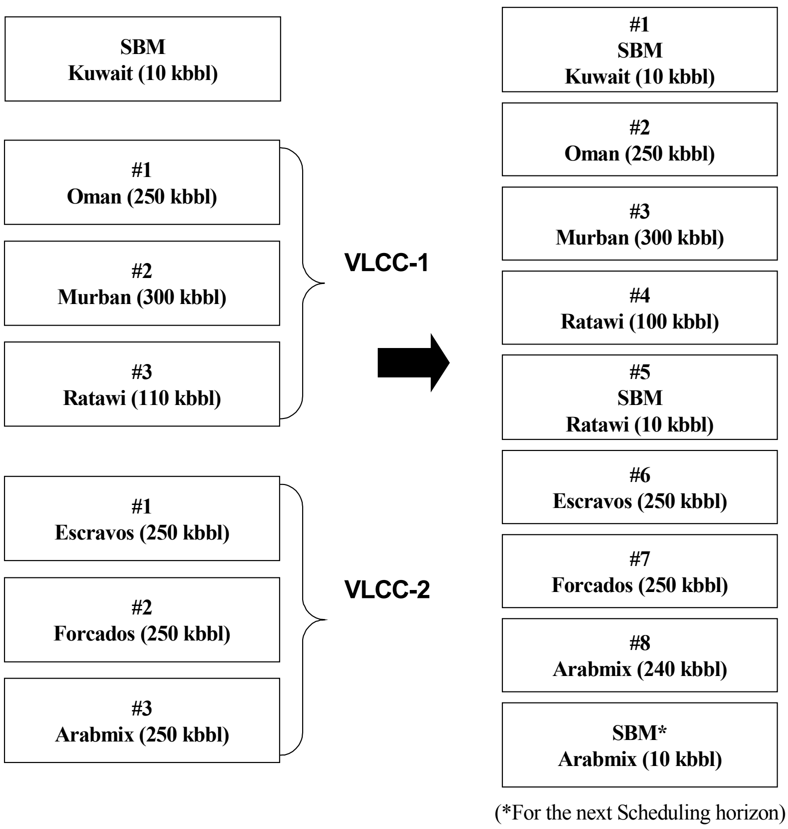


Figure 4.1: Schematic representation of parcel creation

At the end of the parcel creation step, let there be NP parcels (p = 1..NP) in the list, which will unload exactly in the order in which they appear in the list. The assignment of an arrival time ETA_p to parcel p is as follows. ETA_p for a VLCC parcel is the arrival time of its VLCC, while that for an SBM parcel is the arrival time of the next VLCC. Having defined the periods and parcels, now the position is set for developing the constraints in MILP formulation. Let us begin with the parcel unloading operations.

4.2.2 Parcel-to-SBM Connections

The SBM operation demands that each parcel connect to the SBM line to unload and then disconnect after unloading. To model this process of connection/disconnection, we define three binary variables:

$$XP_{pt} = \begin{cases} 1 & \text{if a parcel } p \text{ is connected to the SBM line for unloading during period } t \\ 0 & \text{otherwise} \end{cases}$$

$$XF_{pt} = \begin{cases} 1 & \text{if a parcel } p \text{ first connects to the SBM line at the start of period } t \\ 0 & \text{otherwise} \end{cases}$$

$$XL_{pt} = \begin{cases} 1 & \text{if a parcel } p \text{ disconnects from the SBM line at the end of period } t \\ 0 & \text{otherwise} \end{cases}$$

Based on ETA_p, we can identify the periods in which a parcel p can possibly be connected to the SBM line. Thus, we define XP_{pt}, XF_{pt} and XL_{pt} only for $(p, t) \in \boldsymbol{PT} = \{(p, t) \mid \text{parcel } p \text{ may be connected to the SBM line in } t\}$. The following constraints relate these variables:

$$XP_{pt} = XP_{p(t-1)} + XF_{pt} - XL_{p(t-1)} \qquad (p, t) \in \boldsymbol{PT} \qquad (4.1a)$$

$$XP_{pt} \geq XL_{pt} \qquad (p, t) \in \boldsymbol{PT} \qquad (4.1b)$$

Now, we assume that each parcel connects to and disconnects from the SBM line once and only once, so

$$\sum_t XF_{pt} = \sum_t XL_{pt} = 1 \qquad (p, t) \in \boldsymbol{PT} \qquad (4.2a,b)$$

Eqs. 4.1a,b and 4.2a,b together ensure that XF_{pt} and XL_{pt} will be binary automatically, when the XP_{pt} are so. Therefore, we treat XF_{pt} and XL_{pt} as continuous variables. Using these variables, the time TF_p at which p connects and the time TL_p at which it disconnects are:

$$TF_p = \sum_t (t-1) XF_{pt} \qquad (p, t) \in \boldsymbol{PT} \qquad (4.3a)$$

$$TL_p = \sum_t t XL_{pt} \qquad (p, t) \in \boldsymbol{PT} \qquad (4.3b)$$

As we indicate later in detail, eqs. 4.1- 4.3 represent a novel approach for dealing with parcel unloading, which uses much fewer binary variables than other approaches (Lee et al., 1996; Li et al., 2002) in the literature and gives full flexibility.

Although two parcels cannot connect to the SBM line at a given *instance*, a parcel can disconnect and the next parcel connects during a period t. This would help utilize fully the time available in a period in a discrete-time formulation and embed some continuous-time features in such a formulation. In principle, several small parcels can connect and disconnect in this manner in a given period, but for simplicity, we allow at most two parcels to connect in one period by using,

$$\sum_p XP_{pt} \le 2 \qquad (p, t) \in \boldsymbol{PT} \qquad (4.4)$$

$$TF_{(p+1)} \ge TL_p - 1 \qquad (4.5)$$

Eq. 4.5 ensures that two parcels can connect in one period, only if the first of them disconnects in that period.

Lastly, a parcel can unload only after its arrival time, therefore,

$$TF_p \ge ETA_p \qquad (4.6)$$

4.2.3 SBM-to-Tank Connections

As mentioned earlier, most refineries segregate crudes. To effect this crude segregation, we define a set $\boldsymbol{PI}$ = {(p, i) | tank i may receive crude from parcel p}.

Now, in order to receive crude from a parcel, a tank must also connect to the SBM line. To model this process, we use:

$$XT_{it} = \begin{cases} 1 & \text{if tank } i \text{ is connected to the SBM line during period } t \\ 0 & \text{otherwise} \end{cases}$$

Clearly, a tank *i* cannot receive crude from a parcel during a period, unless the parcel is connected to the SBM line during that period. Therefore, we define a 0-1 continuous variable $X_{pit} = XP_{pt}XT_{it}$, which is one, only when both parcel *p* and tank *i* are connected to the SBM line during *t*. We linearize this nonlinear relation by using:

$$X_{pit} \geq XP_{pt} + XT_{it} - 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (4.7)$$

$$\sum_i X_{pit} \leq 2XP_{pt} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (4.8a)$$

$$\sum_p X_{pit} \leq 2XT_{it} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (4.8b)$$

The above constraints also ensure that X_{pit} will be binary, when XP_{pt} and XT_{it} are so. As we allowed (eq. 4.5) at most two parcels to connect to the SBM line in one period, we do the same for tanks:

$$\sum_i XT_{it} \leq 2 \qquad (4.9)$$

Eqs. 4.5 and 4.9 would admit at most four sequential tank-parcel connections in one period. In order to restrict such connections to exactly two, we further impose,

$$\sum_p \sum_i X_{pit} \leq 2 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (4.10)$$

4.2.4 Tank-to-CDU Connections

For supplying its crude for processing, a tank must connect to one or more CDUs. We model this connection by the following binary variable:

$$Y_{iut} = \begin{cases} 1 & \text{if tank } i \text{ feeds CDU } u \text{ during period } t \\ 0 & \text{otherwise} \end{cases}$$

Operating policies may dictate that a tank may not charge more than some (say two) CDUs simultaneously and vice versa. Thus,

$$\sum_{u} Y_{iut} \leq 2 \qquad (i, u) \in \boldsymbol{IU} \qquad (4.11a)$$

$$\sum_{i} Y_{iut} \leq 2 \qquad (i, u) \in \boldsymbol{IU} \qquad (4.11b)$$

where, $\boldsymbol{IU} = \{(i, u) \mid \text{tank } i \text{ can feed CDU } u\}$. Similarly, most often in practice, a tank cannot feed a CDU, when it is connected to the SBM line or is settling brine after receiving crude. Assuming a brine settling time of 8 h or one period, we use,

$$2XT_{it} + Y_{iut} + Y_{iu(t+1)} \leq 2 \qquad (i, u) \in \boldsymbol{IU} \qquad (4.12)$$

4.2.5 Crude Delivery and Processing

Having modeled the parcel-to-SBM, SBM-to-tank and tank-to-CDU connections, we are ready to transfer crude between tanks and parcels, and tanks and CDUs. We first consider the transfers from parcels to tanks. To this end, we define FPT_{pit} as the amount of crude transferred from parcel p to tank i during period t. This transfer can occur, only when both tank i and parcel p are connected to the SBM line, and its amount must satisfy some lower and upper limits:

$$FPT_{pi}^{L} X_{pit} \leq FPT_{pit} \leq FPT_{pi}^{U} X_{pit} \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (4.13)$$

While the lower limit in the above is the holdup of the SBM line, the upper limit is fixed by the pumping rates of crudes to various tanks. In the event that multiple sequential transfers occur in the same period, the total time required for all transfers must not exceed the period length, therefore,

$$\sum_{p} \sum_{i} \frac{FPT_{pit}}{FPT_{pi}^{U}} \leq 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (4.14)$$

Lastly, each parcel p must unload fully during the scheduling horizon, so if PS_p denotes the size of parcel p, then

$$\sum_{i,t} FPT_{pit} = PS_p \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (4.15)$$

As with tanks, most refineries segregate CDUs too. Therefore, to see if a tank can feed a CDU, we define sets $\boldsymbol{IU} = \{(i, u) \mid \text{tank } i \text{ can feed CDU } u\}$ and $\boldsymbol{IC} = \{(i, c) \mid \text{tank } i$ may have crude c sometime during the horizon$\}$. For delivering crude to CDUs, we define $FCTU_{iuct}$ as the amount of crude c delivered by tank i to CDU u during period t. Then, the total amount FTU_{iut} of crude that tank i feeds to CDU u during t is,

$$FTU_{iut} = \sum_{(i,c)\in \boldsymbol{IC}} FCTU_{iuct} \qquad (i, u) \in \boldsymbol{IU} \qquad (4.16)$$

The above amount can be nonzero, only when tank i is connected to CDU u during t, and it must satisfy some lower and upper limits:

$$Y_{iut} FTU^{L}_{iu} \le FTU_{iut} \le Y_{iut} FTU^{U}_{iu} \qquad (i, u) \in \boldsymbol{IU} \qquad (4.17)$$

Because multiple tanks may feed one CDU, the total feed FU_{ut} to CDU u during t is,

$$FU_{ut} = \sum_{(i,u)\in \boldsymbol{IU}} FTU_{iut} \qquad (4.18)$$

This must be within the processing limits of CDU u, so,

$$FU^{L}_{ut} \le FU_{ut} \le FU^{U}_{ut} \qquad (4.19)$$

In practice, the plant operation may know that its CDUs cannot process crude mixtures with some extreme fractions of crudes. To impose such limitations, we use,

$$FU_{ut} xc^{L}_{cu} \le \sum_{i} FCTU_{iuct} \le FU_{ut} xc^{U}_{cu} \qquad (i, u) \in \boldsymbol{IU}, (i, c) \in \boldsymbol{IC} \qquad (4.20)$$

where, xc^{L}_{cu} and xc^{U}_{cu} are the allowable lower and upper limits on the fraction of crude c in the feed to CDU u. Similarly, the concentration of some key component must also be within certain allowable limits for a CDU. If xk_{kc} is the fraction of key component k in crude c, then we ensure these limits by using,

$$xk^{L}_{ku} FU_{ut} \le \sum_{i}\sum_{c} FCTU_{iuct} xk_{kc} \le xk^{U}_{ku} FU_{ut} \qquad (i, u) \in \boldsymbol{IU}, (i,c) \in \boldsymbol{IC} \qquad (4.21)$$

In real operation, one would want to minimize the upsets caused by changeovers of tanks (thus crudes) feeding to a CDU. To detect such changes, we define a 0-1 continuous variable $YY_{iut} = Y_{iut}Y_{iu(t+1)}$, which is one, if tank i is connected to CDU u during both periods t and $(t+1)$. We linearize YY_{iut} as follows,

$$YY_{iut} \geq Y_{iut} + Y_{iu(t+1)} - 1 \qquad (i, u) \in \boldsymbol{IU} \qquad (4.22a)$$

$$YY_{iut} \leq Y_{iu(t+1)} \qquad (i, u) \in \boldsymbol{IU} \qquad (4.22b)$$

$$YY_{iut} \leq Y_{iut} \qquad (i, u) \in \boldsymbol{IU} \qquad (4.22c)$$

Then, for detecting the presence of a changeover on a CDU, we use,

$$CO_{ut} \geq Y_{iut} + Y_{iu(t+1)} - 2YY_{iut} \qquad (i, u) \in \boldsymbol{IU} \qquad (4.23)$$

When multiple tanks feed a CDU, the composition of feed can change simply due to a change in the feed rates from various tanks. This would upset the CDU operation. However, if only a single tank is feeding a CDU, then a change in its feed rate would not upset the CDU. Therefore, to prohibit a change in composition, when two tanks are feeding a CDU, we force the feed flow rates of individual tanks to remain constant by using the following:

$$M[2 - \sum_i YY_{iut}] + FTU_{iut} \geq FTU_{iu(t+1)} \qquad (i, u) \in \boldsymbol{IU} \qquad (4.24a)$$

$$M[2 - \sum_i YY_{iut}] + FTU_{iu(t+1)} \geq FTU_{iut} \qquad (i, u) \in \boldsymbol{IU} \qquad (4.24b)$$

4.2.6 Crude Inventory

First, to identify the crude in parcel p, we define a set $\boldsymbol{PC} = \{(p, c) \mid \text{parcel } p \text{ carries crude } c\}$. Using VCT_{ict} to denote the amount of crude c in tank i at the end of period t, we get the following from a crude balance on tank i,

$$VCT_{ict} = VCT_{ic(t-1)} + \sum_{(p,c)\in \boldsymbol{PC},(p,t)\in \boldsymbol{PT}} FPT_{pit} - \sum_{(i,u)\in \boldsymbol{IU}} FCTU_{iuct} \qquad (i, c)\in \boldsymbol{IC} \quad (4.25)$$

With this, the total crude level in tank i at the end of period t is given by,

$$V_{it} = \sum_{(i,c)\in \boldsymbol{IC}} VCT_{ict} \qquad (4.26)$$

This must satisfy some upper and lower limits as,

$$V_i^L \le V_{it} \le V_i^U \qquad (4.27)$$

Because of processing and operational constraints, crude fractions in tanks may be kept in some limits as follows,

$$xt_{ic}^L V_{it} \le VCT_{ict} \le xt_{ic}^U V_{it} \qquad (4.28)$$

Crude is normally stored in floating roof tanks to minimize evaporation losses. Such a tank requires a minimum crude level (or heel) to avoid damage to the roof, when the tank goes empty. Due to the presence of heel, crudes usually accumulate in the tank over time. However, a crude type with negligible volume fraction does not affect the overall quality significantly. Thus, to limit the number of crudes in a tank, it is advisable to retain only the crudes with significant volume fractions and normalize their initial fractions in the tank accordingly. Recall that the number of crudes in a tank affects the problem size.

4.2.7 Production Requirements

We can specify them in several ways. One simple way is to specify a crude throughput demand for each CDU in each period as follows:

$$FU_{ut} \ge D_{ut} \qquad (4.29)$$

This obviously requires detailed data that may be difficult to obtain readily. A better way is to specify a throughput demand over the entire horizon for each CDU or groups of CDUs:

$$\sum_t FU_{ut} = D_u \quad \text{or} \quad \sum_u \sum_t FU_{ut} = D \qquad (4.30a,b)$$

In order to integrate the refinery supply chain, we specify demands for the products rather than the crudes. Thus, if PD_j denotes the maximum demand for product *j* during the scheduling horizon, then

$$\sum_i \sum_u \sum_c \sum_t FCTU_{iuct} y_{jcu} \leq PD_j \tag{4.31}$$

where, y_{jcu} is the fractional yield of product *j* from crude *c* processed in CDU *u*.

4.2.8 Scheduling Objective

We use the maximization of total gross profit as the scheduling objective. We define this as the total marginal profit (netback) from crudes minus the operating cost. The former is simply the value of products minus the purchase cost of crude. As mentioned earlier, one aim of short-term crude scheduling is to exploit the benefits of opportunistic crude mixes. We consider gross profit instead of operating cost as the objective, because the former includes the effect of crude compositions and the marginal profits from individual crudes, while the latter does not.

Since the product yields vary with crudes and CDUs, we define CP_{cu} as the marginal profit ($ per unit volume) from crude *c* processed in CDU *u*. Note that the marginal profit does not include any operational costs. We consider these separately in the scheduling objective as follows. First, a change in the feed composition upsets the steady operation of a CDU. This is called a changeover. A changeover lasts a few hours and leads to off-spec products or slops during the transition. In other words, every changeover incurs some cost to the refinery and is undesirable. We let *COC* denote the cost per changeover. Second cost in crude scheduling is the demurrage or sea-waiting cost. The logistics contract with each shipping vessel stipulates an acceptable sea-waiting period. If the vessel harbors beyond this stipulated time, then demurrage (or sea-waiting cost) incurs. We let SWC_v ($ per unit time) denote the demurrage or sea-waiting cost for VLCC *v*. Third, although unloading of crudes does incur costs, we exclude them from our scheduling objective. This is because the amount of crude imported is fixed for the scheduling horizon. Similarly, unlike previous work (Lee et al., 1996; Li et al., 2002), we also do not include the crude inventory cost, because the refiner normally makes the purchasing decisions far in advance and these decisions are beyond the purview of the scheduling activity. Once the refiner purchases crude, it becomes an integral part of the system and incurs inventory cost irrespective of the scheduling. However, one inventory-related decision does fall under the purview of scheduling activity. That is the desire of most refiners to maintain a minimum stock of crude to guard against uncertainty. Let *SS* denote the desired safety stock of crude and *SSP* the penalty ($ per unit volume per period) for under-running the crude safety stock. Based on the above discussion, we obtain the total gross profit as,

$$\text{Profit} = \sum_i \sum_u \sum_c \sum_t FCTU_{iuct} CP_{cu} - \sum_v DC_v - COC \sum_u \sum_t CO_{ut} - \sum_t SC_t \quad (4.32)$$

$$DC_v \geq (TL_p - ETA_p - ETD_v) SWC_v \quad (p, v) \in \boldsymbol{PV} \quad (4.33)$$

$$SC_t \geq SSP(SS - \sum_i V_{it}) \quad (4.34)$$

where, ETD_v is the estimated time of departure of VLCC v as agreed in the logistics contract, $\boldsymbol{PV}$ = {(p, v) | parcel p is the last parcel in VLCC v}, DC_v is the demurrage cost for VLCC v and SC_t is the stock penalty for period t.

This completes our formulation (eqs. 4.1a to 4.34) for a refinery with one SBM pipeline. However, refineries often use jetties with or without an SBM. We need only slight modifications in the above formulation to allow crude unloading via jetties. To this end, we now present a formulation for a refinery with J identical jetties, but no SBM line.

4.3 Unloading via Jetties

Unloading via a jetty is quite analogous to the same via an SBM line except for some differences. We assume that only single-crude vessels berth at jetties, so we can treat a vessel berthing at a jetty as simply a single-parcel VLCC. In contrast to the SBM line, the holdup in a jetty-to-tank transfer line is small and its effect on the composition of the receiving tank is negligible. Thus, we need not consider any new parcels (such as SBM parcels considered earlier) arising from this holdup. The connection / disconnection process of a vessel to a jetty is analogous to that of a parcel to the SBM except that we have J identical berths instead of one SBM station.

Based on the above discussion, it is clear that we can use all the variables in the SBM formulation with their usual meanings to handle jetties. Thus, in the ensuing formulation, we discuss only those constraints that are absent or different from the previous formulation.

Firstly, to allow J vessels to berth and unload simultaneously, we must drop eqs. 4.5 and 4.9 and modify eqs. 4.4 and 4.10 as follows,

$$\sum_{p} XP_{pt} \le J \qquad (p, t) \in \boldsymbol{PT} \qquad (4.4a)$$

$$\sum_{p}\sum_{i} X_{pit} \le 2J \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (4.10a)$$

We continue to use eqs. 4.8a,b to ensure that a parcel can unload to at most two tanks in the same period and a tank can receive from at most two parcels in the same period. This is mainly for simplicity; if facilities permit, we can increase the number as suited. For multiple sequential transfers occurring in the same period, we replace eq. 4.14 by,

$$\sum_{i} \frac{FPT_{pit}}{FPT_{pi}^{U}} \le 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (4.14a)$$

$$\sum_{p} \frac{FPT_{pit}}{FPT_{pi}^{U}} \leq 1 \qquad (p, t) \in \boldsymbol{PT}, (p, i) \in \boldsymbol{PI} \qquad (4.14b)$$

This completes the extension of our formulation to a refinery with *J* jetties and no SBM. Table 4.1 lists its required constraints. Having derived the separate formulations for SBM and jetties, we now combine them into one formulation for a refinery with both one SBM and *J* jetties.

4.4 Unloading Via SBM and Jetties

In this case, both multi-parcel VLCCs and single-parcel vessels will arrive at different times during the scheduling horizon. We treat all of them as vessels. In essence, we have two sets of vessels. One set of vessels unloads via the SBM line, while the other unloads via the jetties. After we create the SBM parcels for the VLCCs as explained in section 4.2.1, we have three types of parcels; namely SBM parcels, VLCC parcels and jetty parcels. For convenience, we include the SBM parcels as the first parcels in the subsequent VLCCs. Thus, we now have only VLCC parcels and jetty parcels, and we use the appropriate constraints developed exclusively for each set. Let ***SP*** denote the set of VLCC parcels and ***JP*** denote the set of jetty parcels.

As discussed earlier, eqs. 4.1a,b, 4.2a,b, 4.3a,b, 4.6, 4.7, and 4.8a,b hold for both parcel sets, eqs. 4.5 and 4.9 hold for ***SP*** only, and eqs. 4.4a and 4.10a hold for ***JP*** only. All other constraints are common to both ***SP*** and ***JP***, except those (eqs. 4.14, and 4.14a,b) governing multiple sequential transfers within a period. Instead of eqs. 4.14 and 4.14a,b, we use eq. 4.14a for both ***SP*** and ***JP*** and add the following constraint,

$$\sum_{p \in \boldsymbol{SP}} \frac{FPT_{pit}}{FPT_{pi}^{U}} + \sum_{p \in \boldsymbol{JP}} \frac{FPT_{pit}}{FPT_{pi}^{U}} \leq 1 \qquad (4.14c)$$

This completes the formulation for a refinery with both one SBM and *J* jetties, and we are now ready to address the practical feature of tank-to-tank transfers.

4.5 Tank-to-Tank Transfers

In practice, one may need to transfer crude from one tank to another to facilitate a quick crude receipt and avoid demurrage. To model these transfers, we use the following binary variable:

$$Z_{ii't} = \begin{cases} 1 & \text{if a crude exchange occurs between tanks } i \text{ and } i' \text{ during period } t \\ 0 & \text{otherwise} \end{cases}$$

$$i' > i,\ (i, i') \in \boldsymbol{II}$$

where, $\boldsymbol{II}$ = {(i, i') | crude transfer between tank i and i' is allowed}. Note that we defined $Z_{ii't}$ only for combinations of i and i' and not permutations. Furthermore, $Z_{ii't}$ defines only the existence of a transfer, not its direction. To identify the number of times a tank i exchanges crude with another tank in a given period, we define,

$$ZT_{it} = \sum_{i'>i,(i,i')\in \boldsymbol{II}} Z_{ii't} + \sum_{i>i',(i,i')\in \boldsymbol{II}} Z_{i'it} \tag{4.35}$$

Tank-to-tank transfers do complicate operations and refiners use them, only when no other option is possible. Therefore, we allow at most one tank-to-tank transfer in a period.

$$ZT_{it} \leq 1 \tag{4.36a}$$

$$\sum_{i} ZT_{it} \leq 2 \tag{4.36b}$$

Note that ZT_{it} will automatically be binary, so we treat it as a continuous variable. Similarly, we restrict the total number of tank-to-tank transfers in the scheduling horizon to a small number m by using,

$$\sum_{t}\sum_{i} ZT_{it} \leq 2m \tag{4.37}$$

As we did earlier (eq. 4.12) with a tank receiving crude, we assume that a tank involved in a tank-to-tank transfer cannot feed a CDU. Therefore,

$$ZT_{it} + Y_{iut} \leq 1 \qquad (i, u) \in \boldsymbol{IU} \tag{4.38}$$

So far, we addressed only the existence of a transfer, but neither its direction nor amount. To model the direction and amount, we define a continuous variable $FCTT_{ii'ct}$ as the amount of crude c transferred from tank i to tank i'. $FCTT_{ii'ct}$ is positive, when the transfer is from i to i', and vice versa. Of the two tanks engaged in a tank-to-tank transfer, one must deliver and the other must receive, therefore,

$$FCTT_{ii'ct} + FCTT_{i'ict} = 0 \qquad (i, i') \in \boldsymbol{II} \qquad (4.39)$$

With this, the total amount of a tank-to-tank transfer from i to i' in period t becomes,

$$FTT_{ii't} = \sum_{c} FCTT_{ii'ct} \qquad (i, i') \in \boldsymbol{II} \qquad (4.40)$$

To obtain the absolute amount $AFTT_{ii't}$ ($i' > i$) of a tank-to-tank crude transfer, we use,

$$AFTT_{ii't} \geq FTT_{ii't} \qquad i' > i, (i, i') \in \boldsymbol{II} \qquad (4.41a)$$

$$AFTT_{ii't} \geq FTT_{i'it} \qquad i' > i, (i, i') \in \boldsymbol{II} \qquad (4.41b)$$

Clearly, $AFTT_{ii't}$ must be zero, when $Z_{ii't}$ is zero, and it must also have an upper limit, so,

$$AFTT_{ii't} \leq FTT_{ii'}^{U}[\delta_{ii'} Z_{ii't} + (1-\delta_{ii'}) Z_{i'it}] \qquad i' > i, (i, i') \in \boldsymbol{II} \qquad (4.42)$$

where, $\delta_{ii'} = 1$ for $i' > i$ and zero otherwise, and $FTT_{i'i}^{U}$ denotes the maximum amount of tank-to-tank transfer possible between i and i' in a period. Having constrained the absolute transfer amount, we can now constrain the individual crude transfer amounts. We do this as follows:

$$-AFTT_{ii't} \max[xt_{ic}, xt_{i'c}] \leq FCTT_{ii'ct} \leq AFTT_{ii't} \max[xt_{ic}, xt_{i'c}]$$

$$i' > i, (i, i') \in \boldsymbol{II} \qquad (4.43)$$

The above transfers modify the individual crude balance (eq. 25) as,

$$VCT_{ict} = VCT_{ic(t-1)} + \sum_{\substack{(p,c)\in \boldsymbol{PC} \\ (p,t)\in \boldsymbol{PT}}} FPT_{pit} - \sum_{(i,u)\in \boldsymbol{IU}} FCTU_{iuct} + \sum_{\substack{i'\neq i \\ (i',c)\in \boldsymbol{IC} \\ (i,i')\in II}} FCTT_{i'ict}$$

$$(i, c) \in \boldsymbol{IC} \qquad (4.25a)$$

Table 4.1 Constraints for different refinery configurations

Refinery Configuration	Constraint equations(#) for							
	Parcel-to-SBM/jetty Connections	SBM/jetty-to-Tank Connections	Tank-to-CDU Connections	Crude Delivery and Processing	Crude Inventory	Product Requirement	Scheduling Objective	Tank-to-Tank Transfers
SBM Only	1a,b, 2a,b, 3a,b, 4-6	7, 8a,b, 9, 10	11a,b, 12	13-21, 22a,b,c 23, 24a,b	25-28	29, 30a,b, 31	32-34, 32a	25a, 35, 36a, 36b, 37-40, 41a,b, 42-44
Jetties Only	1a,b, 2a,b, 3a,b, 4a, 6	7, 8a,b, 10a	11a,b, 12	13, 14a,b, 15-21, 22a,b,c, 23, 24a,b	25-28	29, 30a,b, 31	32-34, 32a	25a, 35, 36a,b, 37-40, 41a,b, 42-44
SBM & Jetties	1a,b, 2a,b, 3a,b, [4, 5], (4a), 6	7, 8a,b, [9,10], (10a)	11a,b, 12	13, 14a,c, 15-21, 22a,b,c, 23, 24a,b	25-28	29, 30a,b, 31	32-34, 32a	25a, 35, 36a,b, 37-40, 41a,b, 42-44

[Equations] are for SBM & VLCC parcels only, while (equations) are for jetty parcels only

#Only Equations numbers are mentioned excluding the chapter number

Lastly, we allow a tank to receive crude from both a parcel and another tank in the same period, as long as the two transfers take place sequentially in the period. Therefore,

$$\sum_{i'\neq i}\frac{AFTT_{ii't}}{FTT_{ii'}^{U}}+\sum_{p}\frac{FPT_{pit}}{FPT_{pi}^{U}}\leq 1 \qquad (i, i') \in \boldsymbol{II} \tag{4.44}$$

In brief, to allow tank-to-tank transfers in our formulation, we replace eq. 4.25 by 4.25a in our SBM/jetty formulation and add eqs. (4.35) to (4.44). Table 4.1 lists the required constraints for various formulations. Tank transfers increase the problem size and complexity drastically due to the additional decisions, variables and constraints, making the problem compute intensive. Since the main goal of tank-to-tank transfers is to avoid demurrage, it may be desirable to allow them only in a few periods. For instance, one may allow them only in the period before the arrival and in periods during the berthing of a vessel. In such a case, we impose eq. 4.25a only for such periods and eq. 4.25 for the remaining periods.

We observed that it was possible to get a better schedule without tank transfers than with because no transfer cost was included in objective function. To avoid such discrepancy, one can include a penalty or cost (*TTC*) in the objective function for each tank transfer operation. With this, the objective function becomes,

$$\text{Profit} = \sum_{i}\sum_{u}\sum_{c}\sum_{t} FCTU_{iuct}CP_{cu} - \sum_{v} DC_{v} - COC\sum_{u}\sum_{t} CO_{ut} - \sum_{t} SC_{t} - \frac{TTC}{2}\sum_{i}\sum_{t} ZT_{it} \tag{4.32a}$$

The crude scheduling problem, as discussed here, is an inherently nonlinear problem due to the blending of crudes in tanks. Our proposed linear formulations, although novel, more general, and improved, as compared to previous work (Lee et al., 1996; Li et al., 2002), are still approximations of the correct nonlinear formulations, and thus suffer from the discrepancies outlined earlier in the motivating examples. This is because the constraints involving $FCTU_{iuct}$ are all linear and the optimizer is free to push arbitrary amounts of individual crudes rather than the correct mixture to CDU. This results in a disproportionate delivery of crudes to CDUs. In the next chapter, we develop a novel iterative strategy to correct the concentration discrepancies without solving any nonlinear problem.

Chapter 5

SOLUTION ALGORITHM – Discrete Time

To avoid concentration discrepancy in crude charge to CDU, one must use the correct nonlinear constraint $FCTU_{iuct} = f_{ict}\, FTU_{iut}$, where f_{ict} is the fraction of crude c in tank i at the end of period t. As discussed earlier in chapter 3, Li et al. (2002) proposed a MILP/NLP iterative approach to solve this problem, but their approach cannot guarantee convergence in all cases. We now develop a novel iterative strategy that simply solves a series of MILPs of reducing size and complexity to obtain a near optimal solution with no discrepancy in composition.

5.1 Description

The main idea behind our algorithm is as follows. First, observe that every tank has blocks of contiguous periods during which its composition does not change. For such a block, if we knew that constant composition (f_{ic}), then the nonlinear equation ($FCTU_{iuct} = f_{ic}\, FTU_{iut}$) becomes linear. Solving such an MILP would yield a solution with no composition discrepancy. Thus, for *each* tank, we can divide all periods into two distinct blocks. One for which we know the tank compositions, and the other for which we do not. For the former, we can use the exact linear constraints ($FCTU_{iuct} = f_{ic}$ FTU_{iut}) and for the latter, we can use the linear approximations as proposed in our formulation. Second, observe that a tank's composition changes, only when it receives crude from a parcel or another tank, otherwise not. During most periods, the tank will receive nothing; hence its composition will be constant. However, we must know the constant composition to make the nonlinear flow constraint linear. To this end, our knowledge of the initial compositions of tanks comes in handy. Because we know the initial composition in each tank, we can identify one initial block of periods for which the composition is constant and known. The length of this block will vary from tank to tank and it could be as short as just one period for some tanks. However, this at least provides a start for our algorithm. As a first try, we can use the exact linear constraints for these first blocks of periods and linear approximations for the remaining periods and solve the MILP. This would give us a solution that has no composition discrepancy at least for the first block of periods on each tank. It will also give us the compositions in all tanks at the end of each period in each block. We now identify the first common block of periods for which we know the compositions in all tanks. We freeze the schedule until the end of that block, and repeat the entire procedure for scheduling the remaining periods. In other words, we now solve another scheduling problem with a reduced horizon. In this manner, we get progressively longer and longer partial schedules, free of composition discrepancies, by solving a series of MILPs, until we have the complete schedule. We now describe the algorithm in full detail.

5.2 Stepwise Methodology

For the time being, we disallow tank-to-tank transfers in describing the algorithm (Figure 5.1). At each iteration of our algorithm, we divide the *NT* periods into two sets. Set 1 includes all periods with $t \le t^*$ for some t^*, and set 2 the rest. The schedule (or all variables Y_{iut}, X_{pit}, FTU_{iut}, $FCTU_{iuct}$, etc. in the MILP) for periods in set 1 is (are) fixed based on previous iterations. However, the MILP variables for periods in set 2 are free. From the schedule for set 1, we know the tank compositions at the end of period t^* or $t = t^*$. Let f_{ic} denote the fraction of crude c in tank i at $t = t^*$.

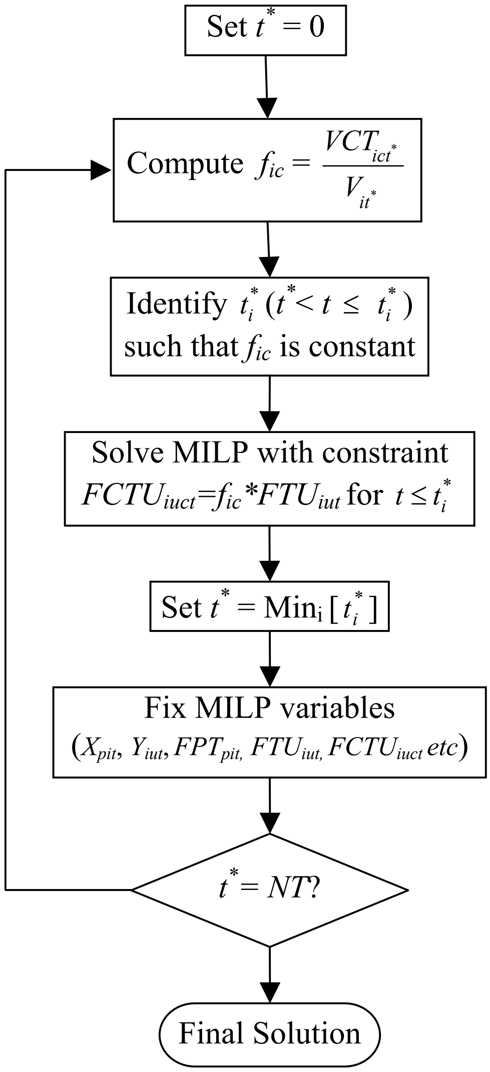


Figure 5.1: Flow chart for the solution algorithm

Then, our iterative algorithm proceeds as follows:

1. Set $t^* = 0$.

2. From the fixed schedule for $t \leq t^*$, compute f_{ic} for each tank i as $f_{ic} = \frac{VCT_{ict*}}{V_{it*}}$ using the known information (VCT_{ict*} and V_{it*}).

3. For each tank i, identify the latest period $t_i^* > t^*$ such that its composition is f_{ic} for all periods $t^* < t \leq t_i^*$. We do this as follows. Let p' denote the last parcel that was unloaded (to any tank) before t^*. Let $\Delta PS_{p'}$ denote the amount of crude remaining in parcel p'. If p' has been unloaded fully before t^*, then $\Delta PS_{p'} = 0$. Now, let p''

denote the earliest ($p'' \geq p'$) parcel that tank i can possibly receive after t^*. If there is no such parcel, then $t_i^* = NT$. If both p' and p'' belong to the same vessel (VLCC or ship), then we get,

$$t_i^* = t^* + \text{ceil}\left(\frac{\Delta PS_{p'}}{\max\limits_{(i,p') \in \boldsymbol{PI}} FPT^U_{p'i}} + \sum_{p=p'+1}^{p''-1} \frac{PS_p}{\max\limits_{(i,p) \in \boldsymbol{PI}} FPT^U_{pi}} \right) \tag{5.1}$$

where, the second term is the minimum time needed to transfer the remainder of parcel p' and all parcels between p' and p''. If p'' belongs to a different vessel, then $t_i^* = ETU_{p''}$.

4. Add the constraint, $FCTU_{iuct} = f_{ic}FTU_{iut}$, for $t^* < t \leq t_i^*$ in the MILP and solve.
5. Fix the MILP variables for periods $t^* < t \leq \min_i[t_i^*]$. Set $t^* = \min_i[t_i^*]$. If $t^* = NT$, then terminate, otherwise go to Step 2.

Note that the size and complexity of MILP in our algorithm reduce progressively by at least one period at each iteration. Although the algorithm does not guarantee an optimal solution, considering the fact that even solving the MILP with linear approximations is a challenging problem, our approach is quite attractive because it does not require solving MINLPs or NLPs and gives near-optimal schedules in reasonable time. In fact, it gives better solutions for several literature problems. When the scheduling objective does not involve crude compositions, then our algorithm guarantees a globally optimal objective value right in the first MILP, although subsequent iterations are required to correct the composition discrepancy. For large and complex problems, solving MILP to zero gap can be compute intensive, hence we may have to use a small relative optimality gap to limit the computation time. However, as iterations proceed in our algorithm, the problem size reduces, which allows us to use decreasing relative gaps to get better schedules. For example, if a problem has NT = 30, then we can use a relative gap of 5% for the first few iterations and 0% for the rest, as the MILP becomes easier to solve.

Tank-to-tank transfers are extremely difficult to handle, because they affect the formulation size and algorithm efficiency immensely. Firstly, we need several additional binary variables and constraints in the formulation to allow tank-to-tank transfers. Because any crude in a given class can be in any tank, the crude transfer and balance constraints must be written for all crudes of the given class. Furthermore, a tank-to-tank transfer can occur in any period; therefore, a constant composition block cannot be longer than one period. In other words, the algorithm must progress only one period in each iteration, until the allowable number of transfers is exhausted. It is also not wise to restrict the number of transfers, because it may be difficult to decide whether to consume the available transfers early or save them for future. All these make the problem extremely difficult.

We now demonstrate the efficacy of our solution algorithm using the motivating example (chapter 3, example 1) and illustrate how our algorithm corrects the discrepancy between the compositions of sent and delivered crudes.

5.3 Illustration

In the motivating example discussed earlier, the VLCC arrives at the start of the scheduling horizon. The first parcel (p = 1) is the SBM parcel with crude C2 of class1. The second is with crude C1 of class1, while the third and the fourth carry class2 crudes. Based on the parcel sizes and maximum possible transfer rates, we set the earliest possible unloading periods for parcels as ETU_1 = 1, ETU_2 = 1, ETU_3 = 2, and ETU_4 = = 2. For the first iteration, t^* = 0, f_{ic} values are as in Table 5.1. Since no parcel has begun unloading, p' is undefined. Parcel 1 is the earliest parcel from which T1 and T4 can receive crude, so p'' = 1 for both and $t_1^* = t_4^* = ETU_1$ = 1. Similarly, parcel 3 is

Table 5.1: Details of individual iterations for the motivating example using the proposed algorithm

Iteration	t^*	Tank	t_i^*	$\text{Min}_i[t_i^*]$	MILP Profit (k$)	Number of Binary Variables	CPU Time (s)	Composition f_{ic} C1 or C3	Composition f_{ic} C2 or C4
0	0	T1	1	1	3879	53	2.719	0.667	0.333
		T2	2					0.500	0.500
		T3	2					0.200	0.800
		T4	1					0.433	0.567
1	1	T1	EOH	2	3879	45	1.235	0.667	0.333
		T2	2					0.500	0.500
		T3	2					0.200	0.800
		T4	EOH					0.705	0.295
2	2	T1	EOH	3	3819.78	35	0.562	0.667	0.333
		T2	3					0.244	0.756
		T3	3					0.200	0.800
		T4	EOH					0.705	0.295
3	3	T1	EOH	4	3815.35	27	0.469	0.667	0.333
		T2	4					0.244	0.756
		T3	4					0.200	0.800
		T4	EOH					0.705	0.295
4	4	T1	EOH	EOH	3784.33	20	0.469	0.667	0.333
		T2	EOH					0.244	0.756
		T3	EOH					0.800	0.200
		T4	EOH					0.705	0.295

EOH: End of Horizon

the earliest parcel from which T2 and T3 can receive crude, so $t_2^* = t_3^* = ETU_3 = 2$. Now, we impose $FCTU_{1uc1} = f_{1c}FTU_{1u1}$, $FCTU_{4uc1} = f_{4c}FTU_{4u1}$, $FCTU_{2uc1} = f_{2c}FTU_{2u1}$, $FCTU_{2uc2} = f_{2c}FTU_{2u2}$, $FCTU_{3uc1} = f_{3c}FTU_{3u1}$, $FCTU_{3uc2} = f_{3c}FTU_{3u2}$ and solve the MILP. MILP solution gives a profit of $3879K, which gives us an upper bound on the globally maximum profit for the exact nonlinear problem.

In the second iteration, $t^* = \min_i[t_i^*] = 1$, so we freeze the schedule for the first period and compute f_{ic} as in Table 6. At $t^* = 1$, we find that both parcels 1 and 2 have unloaded fully, therefore $p' = 2$ for T1 and T4, $\Delta PS_{p'} = 0$ and p'' does not exist. So we get $t_1^* = t_4^* = 9$. For T2 and T3, $p'' = 3$ and using eq. 33, we get $t_2^* = t_3^* = 2$. Then, we compute f_{ic} for all tanks at $t^* = 1$ and impose the linear composition constraints for period 2 for T2 and T3 and for periods 2-9 for T1 and T4. MILP solution gives a profit of \$3879K. Continuing the procedure, we get the detailed results in Table 5.1. Table 5.1 also shows the reduction in problem size with iterations. The algorithm terminates after four iterations with a final profit of \$3784.33K, which is within 2.44% of the upper bound \$3879K. This being a nonconvex MINLP problem, it is difficult to say what the globally best solution is. Considering the fact that the best solution is surely less than \$3879K and we are achieving a solution within 2.44% of that without using any NLP, we can safely consider our solution as near-optimal.

Before we illustrate our methodology on some real-life problems, a few remarks highlighting the salient features of our formulation are in order.

5.4 Remarks

Our proposed formulations and solution approach differ significantly from previous attempts at this problem.

1. The first major difference is that our formulation allows some features of a continuous-time formulation in that tank-to-tank and parcel-to-tank transfers may start at times other than period endpoints. This obviates partially the need for a continuous-time formulation.
2. Unlike Li et al. (2002), our solution approach corrects composition discrepancy without solving a single NLP, although the problem is inherently nonlinear. We have already pointed out earlier that their decomposition strategy can fail to get a solution. However, our algorithm fixes parcel-to-tank and tank-to-CDU allocations based on corrected compositions, thus it cannot produce infeasible results.
3. Although we also model parcel-to-tank connections using bi-index binary variables, our binary variables (XP_{pt} and XT_{it}) are subtly different from those (VT_{vt} and VI_{vi}) of Li et al. (2002). The implications of this are quite subtle, but important.

While a vessel cannot deliver to multiple tanks during the scheduling horizon in Li et al.(2002) formulation, this is possible in our formulation. To prove this point, let us consider the allocation of vessel v to tank i at time t, as given by the variable XW_{vit} in Li et al. (2002). Li et al. (2002) defined VT_{vt} and VI_{vi} as:

$$VT_{vt} = \begin{cases} 1 & \text{if vessel } v \text{ is connected at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$VI_{vi} = \begin{cases} 1 & \text{if vessel } v \text{ is connected to tank } i \\ 0 & \text{otherwise} \end{cases}$$

and fixed XW_{vit} using the constraints: $XW_{vit} \geq VT_{vt} + VI_{vi} - 1$, $XW_{vit} \leq VT_{vt}$ and $XW_{vit} \leq VI_{vi}$. Further, they allowed a vessel v to connect to only one tank i at any time t by imposing $\sum_i XW_{vit} \leq 1$. Now, consider a perfectly possible scenario in which a vessel v delivers to tank T1 in period 1 and T2 in period 2. In this case, $VT_{v1} = VT_{v2} = VI_{v1} = VI_{v2} = 1$. The three constraints for fixing XW_{vit} make $XW_{v11} = XW_{v21} = XW_{v12} = XW_{v22} = 1$ and $\sum_i XW_{vit} = 2$ for the two periods in violation of the constraint $\sum_i XW_{vit} \leq 1$.

4. A similar comment also holds for tank-to-CDU allocation. Li et al. (2002) used bi-index binary variables (IT_{it} and IL_{il}) to define CD_{ilt} by using $CD_{ilt} \geq IL_{il} + IT_{it} - 1$, $CD_{ilt} \leq IL_{il}$, $CD_{ilt} \leq IT_{it}$, and

$$IT_{it} = \begin{cases} 1 & \text{if tank } i \text{ is connected at time } t \\ 0 & \text{otherwise} \end{cases}$$

$$IL_{il} = \begin{cases} 1 & \text{if tank } i \text{ is connected to CDU } l \\ 0 & \text{otherwise} \end{cases}$$

Suppose that tank T1 delivers to CDU1 in period 1 ($CD_{111} = 1$, $CD_{121} = 0$) and to CDU2 ($CD_{112} = 0$, $CD_{122} = 1$) in period 2. Because T1 must connect to CDU1 and CDU2 both, $IL_{11} = IL_{12} = 1$. Similarly T1 must connect in both periods, so $IT_{11} = IT_{12} = 1$. With these, the defining constraints for CD_{ilt} give $CD_{121} = CD_{112} = 1$, which contradicts the assumed values. In other words, once a tank connects to a set of CDU, it cannot connect to any other set of CDU during the scheduling horizon. Even if we were to use bi-index binary variables of the form IT_{it} and LT_{lt} with time dimension in both, it still would not allow all possible charging scenarios. Thus, disaggregating tank-to-CDU binary variables limits the tank-to-CDU connections and does not seem to offer any advantage. Our formulation, on the contrary, allows all viable scenarios.

5. The existing literature (Lee et al., 1996; Li et al., 2002) defines changeover to arise from a change in composition of feed to CDU. However, it ignores the fact that composition may change, even when flows from two tanks feeding to one CDU change. Our eqs. 4.22a-c and 4.23 describe the transitions accurately. When two tanks deliver to one CDU, then change in flow from any tank also causes a transition. To avoid such a transition, refiners keep the flows from both tanks constant. Our eqs. 4.24a,b enforce this industry practice.
6. Unlike our formulation, the changeover constraints of Li et al. (2002) count two changeovers, when a tank stops feeding a CDU and another starts feeding the same. To illustrate this, consider that tank T1 stops feeding and T2 starts feeding to CDU1 from period 3. Thus, the tank-to-CDU allocations are $CD_{112} = CD_{213} = 1$ and $CD_{212} = CD_{113} = 0$. Using Li et al. (2002)'s constraints, $Z_{ilt} \geq CD_{ilt} - CD_{il(t-1)}$ and $Z_{ilt} \geq CD_{il(t-1)} - CD_{ilt}$, we find that $Z_{113} = Z_{213} = 1$. In other words, even though only one changeover occurs on CDU1, Li et al. (2002) count them as two, one for each tank.

We now illustrate our methodology on several examples derived from a local refinery configuration (SRC, Singapore). In order to maintain confidentiality, we used different set of data but kept the nature of problem identical, employed similar operating rules.

Chapter 6

MODEL EVALUATION

6.1 Examples

We use Example-2 of Li et al. (2002) as our first example. For the remaining examples, we take a refinery with 8 tanks (T1-T8), 3 CDUs (CDU1-CDU3) and two classes (Class1 & Class2) of crudes. Tanks T1, T6, T7, T8 and CDU3 store/process Class1 crudes, while the rest store/process Class2 crudes. We selected these examples to illustrate the use of our models for refineries with different configurations (SBM, jetties, tank-to-tank transfers), short and long scheduling horizons, and several parcel sizes and arrivals. Example 1 compares the present approach to that of previous work of Li et al. (2002) on this scheduling problem. Refinery configuration includes 7 storage tanks, four jetties and two CDUs. During 7 day horizon, refinery handles six different types of crudes. The crude arrival data, economic data, and initial crude inventory data for example 1 is given in Table 6.1, Table 6.2, and Table 6.3 respectively. Example 2 describes the refinery configuration with SBM and validates the model. Example 3 uses the same configuration as of example 2 but with long scheduling horizon. Example 4 illustrates the refinery configuration with multiple jetties. Example 5 explains a refinery configuration with SBM and multiple jetties. Lastly, example 6 demonstrates the feature of tank to tank transfers using a refinery configuration with SBM and compares the schedule with that of no tank to tank transfers. For examples 2 to 6, Table 6.4 gives the initial crude levels in tanks and their capacities, Table 6.5 provides the initial crude compositions in tanks, Table 6.6 presents the tanker arrival details, crude demands and key component concentration limits on CDUs, Table 6.7 furnishes the crude types, their marginal profits and key component details and Table 6.8 includes economic data and limits on crude transfer amounts. Wherever possible, we compared our approach with the previous approaches for these examples. We used CPLEX 7.0 solver within GAMS on a Gateway E5250 (Pentium II) machine running Windows NT. Table 6.9 gives the model statistics and histories of model performance for all examples.

6.1.1 Data Tables

6.1.1.1 Example 1

Table 6.1: Crude arrival information for example 1

Vessel	Capacity (MT)	Arrival time (Day)	Crude type
1	3000	1	C1
2	3000	1	C1
3	3000	2	C1
4	3000	2	C1
5	5000	3	C1
6	5000	3	C1
7	3000	3	C1
8	3000	3	C1
9	3000	4	C1
10	5000	5	C2
11	5000	5	C2
12	3500	5	C2
13	3500	5	C2
14	3000	6	C1
15	3000	6	C1
16	3000	7	C4
17	1500	7	C2
17	1500	7	C6

Table 6.2: Crude types and processing margin for example 1

Crude Type	Crude cost (yuan)	Production cost (yuan)
C1	1130	2130
C2	1574	2574
C3	1418	2418
C4	1150	2150
C5	1150	2150
C6	1671	2671

Table 6.3: Initial crude inventory and storage capacities of storage tanks for example1

Tank	T1	T2	T3	T4	T5		T6		T9
Min capacity		0					0		
Max capacity		25000					40000		
Initial inventory	5000	5000	7000	8000	8000	10000	5000	5000	10000
Crude Type	C1	C2	C3	C1	C1	C2	C4	C6	C3

6.1.1.2 Examples 2-6

Table 6.4: Initial crude levels, capacities of storage tanks for examples 2-6

Tank	Capacity (kbbl)		Heel (kbbl)		Initial Inventory (kbbl)	
	Ex2 to 4, 6	Ex 5	Ex2 to 4, 6	Ex 5	Ex2 to 4,[6]	Ex 5
T1	570	400	60	50	350	250
T2	570	400	60	50	400	200
T3	570	400	60	50	350	300
T4	980	400	110	50	950	350
T5	980	400	110	50	300	250
T6	570	400	60	50	80[240]	100
T7	570	400	60	50	80[120]	100
T8	570	400	60	50	450[550]	250

Data in [] indicate the change in data for Ex6 from Ex2 to 4

Table 6.5: Initial crude composition in storage tanks for examples 2-6

Initial Crude Composition (kbbl)						
Example 2 - 4, [6] #				Example 5 *		
C1or5	C2or6	C3or7	C4or8	C1or5	C2or6	C3or7
50	100	150	50	100	100	50
200	0	50	150	50	100	100
100	100	50	100	100	100	100
200	250	200	300	100	150	100
100	100	50	50	100	75	75
20[30]	20[30]	20[150]	20[30]	25	25	50
20[30]	20[30]	20[50]	20[10]	50	25	25
100[150]	100	100[210]	150[90]	75	75	100

Data in [] indicate the change in data for Ex 6 from Ex 2 to 4
Tanks 1, 6-8 store crudes 1-4; 2-5 store 5-8 for Ex 2-4 & 6
*Tanks 1, 6-8 store crudes 1-3 (Class 1); 2-5 store 4-6 (class 2) for Ex 5

Table 6.6 Tanker arrival details, crude demands and key component concentration limits on CDUs for Examples 2 to 6

Ex	Tanker	Arrival Period	Parcel No: (Crude, Parcel Size kbbl)	Key Comp.Conc. Limits CDU	Lower	Upper	Demand (kbbl)
2	VLCC-1	1	1: (C2, 10), 2: (C3, 250), 3: C4 (300), 4: C5 (100)	CDU1	0.001	0.0130	750
	VLCC-2	14	5: (C5, 10), 6: (C6, 250), 7: (C3, 250), 8:(C8, 240)	CDU2	0.001	0.0125	750
				CDU3	0.001	0.0035	750
3	VLCC-1	1	1: (C2, 10), 2: (C3, 350), 3: (C4, 200), 4: (C5, 300)	CDU1	0.001	0.0135	1000
	VLCC-2	16	5: (C5, 10), 6: (C6, 200), 7: (C8, 250), 8: (C3, 240)	CDU2	0.001	0.0130	1000
	VLCC-3	28	9: (C3,10), 10: (C6,250), 11: (C2,250), 12: (C7,190)	CDU3	0.001	0.0040	1000
4	V1-V2	3	1: (C2, 350), 2: (C3, 350)	CDU1	0.001	0.0130	500
	V3-V5	6	3: (C5, 350), 4: (C1, 300), 5: (C7, 350)	CDU2	0.001	0.0125	500
	V6	9	6: (C8, 250)	CDU3	0.001	0.0035	600
	V7-V8	10	7: (C3, 250), 8: (C6, 300)				
5	VLCC-1	3	1: (C2, 10), 2: (C6, 100), 3: (C1, 100), 4: (C4, 90)	CDU1	0.0125	0.0185	600
	V1-V2	5	5: (C2, 125), 6: (C5, 125)	CDU2	0.0125	0.0175	600
	V3	6	7: (C3, 100)	CDU3	0.004	0.0070	600
6	VLCC-1	2	1: (C2, 10), 2: (C4, 500), 3: (C3, 500), 4: (C5, 440)	CDU1	0.001	0.0140	375
				CDU2	0.001	0.0125	375
				CDU3	0.001	0.0030	400

Table 6.7: Crude types, marginal profits and key component details for examples 2-6

Crude	Key comp. conc. (fr)			Profit ($/bbl)		
	Ex 2-4	Ex 5	Ex 6	Ex 2-4	Ex 5	Ex 6
C1	0.002	0.005	0.0025	1.5	1.5	1.5
C2	0.0025	0.008	0.0025	1.7	1.75	1.7
C3	0.0015	0.004	0.004	1.5	1.85	1.5
C4	0.006	0.015	0.002	1.6	1.25	1.6
C5	0.012	0.01	0.01	1.45	1.45	1.45
C6	0.013	0.02	0.015	1.6	1.65	1.6
C7	0.009		0.014	1.55		1.55
C8	0.015		0.011	1.6		1.6

Table 6.8: Economic data and limits on crude transfer amounts for examples 2-6

Ex	Flow Rate Limits (kbbl / Period)			Demurrage Cost	Changeover Loss	Safe Inventory Penalty
	Parcel-Tk	Tk-CDU	Tk-Tk			
	Min - Max	Min - Max	Min - Max	(k$/period)	(k$/instance)	($/bbl/period)
2 to 4	10 - 400	20-45(40~)		25	10	0.2
5	10 - 250	0 - 50		15	5	0.2
6	10 - 400	0 - 45	0 - 400	50	25	0.2

~ Limit for example 3

6.1.2 Results and discussion

6.1.2.1 Example 1

For the sake of a fair comparison, we used the same definition of changeover, objective function as used by Li et. al (2002) and a relative gap of 10% for MILPs. Our approach yielded a different schedule (Table 6.10) with a 10.3% increase in profit (Table 6.9), as compared to that reported by Li et al. (2002). Our approach solved seven MILPs and took 94 s (Table 6.9), while their approach required four (NLP+MILP)s and took 212 s with a profit of 97902225 Yuan. We also successfully solved this example using a relative gap of 0.01% without much change in profit in 327 s of CPU time. Table 6.11 shows the berth allocation schedule in various periods.

Table 6.9 Model performance and statistics for illustrated examples

Example		No. of Equations	Single Variables	Discrete Variables	Profit (k$)	CPU Time (s)	Number of Iterations	Periods *NT*	Relative MILP Gaps % (Periods)
1	Li et al. (2002)	3247	1285	379	95904657	310	3(NLP+MILP)	7	10%
	Proposed	1996	1022	171	105760000	94	7 (MILP)	7	10% (1-7)
	Proposed	1996	1022	171	105780000	327	7 (MILP)	7	0.01% (1-7)
2		4165	2972	304	3425	1364	7 (MILP)	20	5% (1-5), 3.5% (6-10) 2% (11-15), 0% (16-20)
3		8420	6102	589	4593	11963	10 (MILP)	42	7% (1-17), 4% (18-27) 2% (28-30), 0% (29-42)
4		4705	2383	335	2467	2615	10 (MILP)	15	1% (1-5), 0.5% (6-15)
5		4078	1960	239	2524	1068	5 (MILP)	15	3% (1-3), 2% (4-5), 0.01% (6-15)
6	No tank transfers	3033	1503	162	1720	12835	6 (MILP)	10	0% (1- 10)
	With tank transfers	3579	1831	186	1767.2	41604	6 (MILP)	10	0% (1- 10)

Table 6.10 Operation schedule for Example 2 of Li et al (2002) obtained via our approach

Tank	Crude Amount [to CDU No.](from Vessel No) in ton for period						
	1	2	3	4	5	6	7
1		+3000(3,4)		+3000(9)		+3000(14,15)	
2	-1000[2]	-3000[2]	-1000[2]		+5000(10,11) +3500 (13,14)	-5000[1]	-5000[1]
3	-3000[1]	-4000[1]					
4			+5000(5,6) +3000(7,8)		-5000[1]	-3000[1]	-3000[1]
5	+3000(1,2)		-2000[2]	-3000[2]	-3000[2]	-3000[2]	-3000[2]
6	-5000[1]	-4000[1]	-5000[1]	-3000[1]	-3000[1]		+3000(16,17)
7	-2000[2]		-3000[1]	-5000[1]			

-' sign represents delivery to [CDU], '+' sign represents receipt from (Vessel)

Table 6.11 Jetty allocation Schedule for Example 2 of Li et al (2002) obtained via our approach

Berth	Vessels occupying berths in time periods						
	1	2	3	4	5	6	7
1	V1		V5		V10	V14	
2	V2		V6	V9	V11	V15	
3		V3	V7		V12		V16
4		V4	V8		V13		V17

6.1.2.2 Example 2

This example involves one SBM line, 20 8-h periods and two VLCCs with three parcels each. Table 6.12 shows the operation schedule. Some salient features of the schedule are as follows:

1) T4 with a key component concentration of 0.0126 vol% cannot meet the feed quality for CDU2, so the optimizer combined crudes from T4 and T5 (0.0123 vol %) in periods 1, 2, 10, 11 and 12.
2) Two tanks feed a CDU in several periods. For instance, T6 and T7 feed CDU3 in periods 5-10, T1 and T8 feed CDU3 in periods 11-14, and T4 and T5 feed CDU2 in periods 1, 2 10 and 12. At all times, the optimizer maintained the individual tank feed flows constant to avoid a composition change. As mentioned earlier, many refineries practice this.
3) Sequential, multiple transfers to one or more tanks occur in several periods. For example, in period 1, parcels 1 (SBM) and 2 (1^{st} parcel of VLCC-1) unload to T7. In period 2, parcel 2 and parcel 3 (2^{nd} parcel of VLCC-1) unload to T6. Lastly, in period 3, parcel 3 and parcel 4 unload to T7 and T3 respectively. This is a continuous-time feature in our formulation.
4) T4 delivers to both CDU1 and CDU2 in periods 1, 2, 10, 11 and 12.
5) T2 receives crude in period 14, uses period 15 for brine settling and removal, and starts delivering in only period 16. T7 does the same in periods 4, 5 and 6.
6) Lastly, we see from Figure 6.1 that the key component concentration in feed changes, only when a changeover occurs.

Table 6.13 details profit, CPU time and actual/target relative gaps for iterations.

Table 6.12 Operation schedule for Example 2

Tank	Crude Amount [to CDU No.](from Vessel No) in kbbl for period									
	1 & 11	2 & 12	3 & 13	4 & 14	5 & 15	6 & 16	7 & 17	8 & 18	9 & 19	10 & 20
1										
&	-20[3]	-20[3]	-20[3]	-20[3]	-28.8[3]	-45[3]	-45[3]	-45[3]	-23.1[3]	-23.1[3]
2			-45[2]	-45[2]	-45[2]	-30[2]	-20[2]	-20[2]	-20[2]	
&				+10(5)		-45[1]	-45[1]	-45[1]	-45[1]	-45[1]
				+250(6)						
3			+190(4)							
&			-20[2]	-20[2]	-45[2]	-45[2]	-45[2]	-45[2]	-45[2]	-45[2]
4	-20[1]	-20[1]	-20[1]	-45[1]	-45[1]	-20[1]	-20[1]	-20[1]	-45[1]	-45[1]
	-20[2]	-20[2]								-25[2]
&	-45[1]	-45[1]	-45[1]	-45[1]	-45[1]					
	-25[2]	-25[2]								
5	-20[2]	-20[2]								-20[2]
&	-20[2]	-20[2]				+240(8)				
6		+130(2)			-21.9[3]	-21.9[3]	-21.9[3]	-21.9[3]	-21.9[3]	-21.9[3]
		+270(3)								
&										
7	+10(1)		+30(3)		-20[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]
	+120(2)									
&					+250(7)					
8	-20[3]	-20[3]	-20[3]	-20[3]						
&	-21.1[3]	-21.1[3]	-21.1[3]	-21.1[3]					-21.9[3]	-21.9[3]

'-' sign represents delivery to [CDU], '+' sign represents receipt from (parcel)

Table 6.13: Solution details for Example 2

MILP Iteration	Profit (k$)	CPU Time (s)	Relative Gap (%) Target	Actual
1	3427.60	146.5	5.0	4.6
2	3454.86	292.9	3.5	3.4
3	3495.37	836.3	3.5	2.0
4	3444.32	70	3.5	2.8
5	3433.12	7.8	2.0	1.6
6	3439.00	9.8	0.0	0.0
7	3424.90	0.74	0.0	0.0

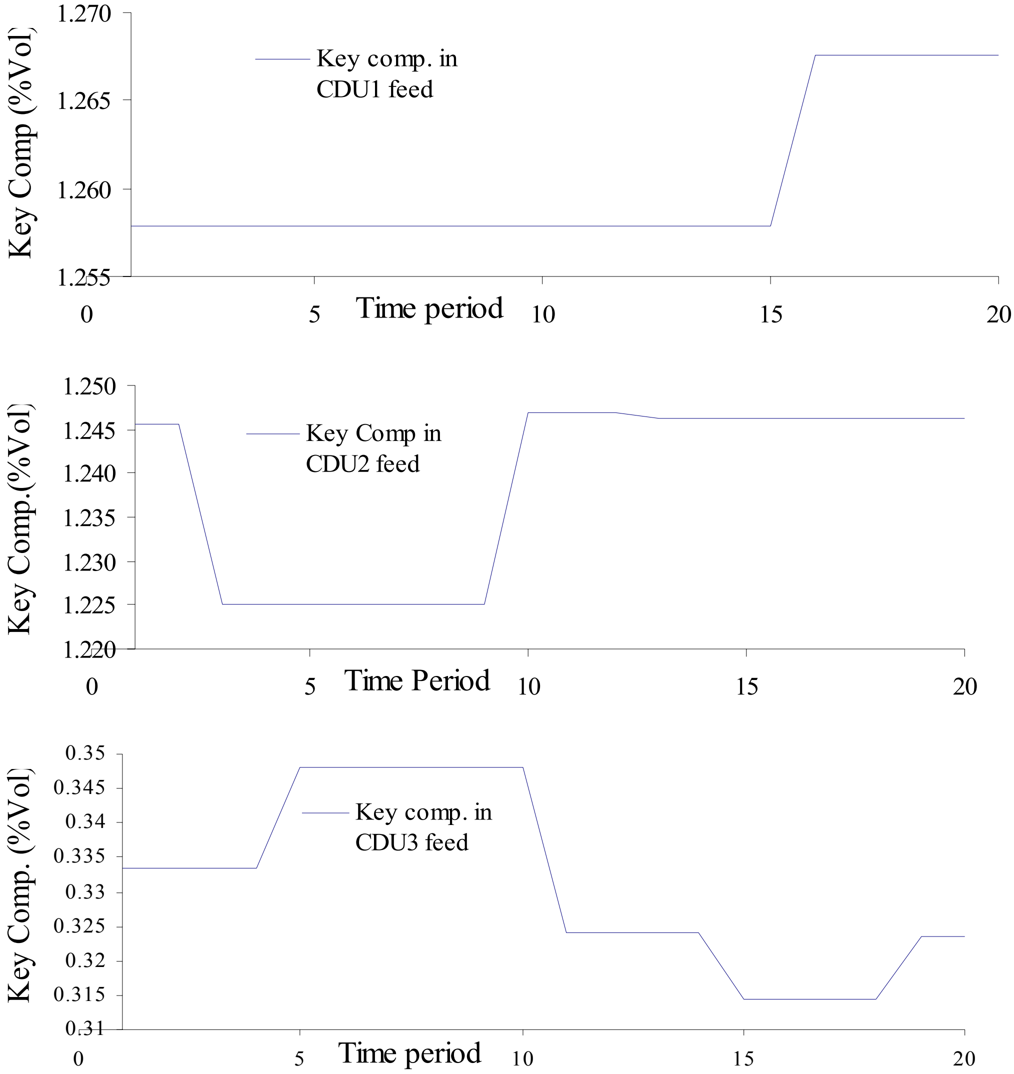


Figure 6.1: Key component concentration in CDU feeds at various time periods for example 2

6.1.2.3 Example 3

To demonstrate our algorithm's ability to solve larger problems, we consider a longer horizon of 42 periods for Example 2, with three VLCCs carrying three parcels each. Table 6.14 shows the operation schedule. The schedule shows all the salient features as explained in the previous discussion for example 2. The model performance (Table 6.9) reveals that longer horizon problems are highly compute-intensive and consume longer CPU time. These problems may require higher relative gaps for the first few MILPs in order to get quicker solutions. Such problems can be solved easily using increased period lengths which in turn reduce total number of decision variables, allowing quicker solution but would approximate the decision making, leading to practical difficulties in its actual realization. Continuous time modeling approach could be helpful in solving longer horizon problems and needs to be explored.

6.1.2.4 Example 4

In this example, we consider three jetties, no SBM, 15 8-h periods and 8 single-parcel vessels (V1 to V8). Table 6.15 shows the operations schedule, while Table 6.16 shows the berth allocations for the arriving vessels. Among some salient features of this schedule, we have two jetties letting two vessels (V3 & V4) unload simultaneously to (T3 & T6) and then to (T5 & T6) in period 6. Also in period 6, V3 unloads to both T3 & T5. In period 11, two vessels (V7 & V8) unload simultaneously to two different tanks (T7 and T2 respectively). From periods 5 to 13, T1 & T8 feed CDU3 and we can see that the composition of feed remains same during this period. The optimizer achieved this by keeping flows from individual tanks constant. Thus, in addition to showing all the features mentioned in Example 2, the schedule shows the simultaneous berth allocations of multiple vessels and simultaneous transfers to multiple tanks in a period.

Table 6.14 Operation schedule for Example 3

Tank	Crude [Charge] (Unload) Amount [CDU No.](Parcel No) in kbbl for period																			
	1	2	3	4	5	6 to 7	7	8 to 15	9	10	11	12	13	14	15	16	17	18	19	20	21
1		-20[3]	-20[3]	-20[3]	-30[3]			-20[3]	-20 [3]	-20 [3]	-20 [3]	-20 [3]	-20 [3]	-20 [3]	-20 [3]	-20[3]	-20[3]	+240(8)			
2	-20[1]		-20[2]	-20[2]	-20[2]	-20[2]	-20 [2]	-20[2]	-20 [2]	-20 [2]	-20 [2]	-20 [2]	-20 [2]	-20 [2]	-20 [2]	-20[2]	-20[2]	-20[2]			
3	-40[2]	-20[2]		-20[2]												+10(5) +190(6)	+10(6), +90(7)				
4		-20[1]	-20[1]	-20[1]	-20[1]	-20[1]	-20 [1]	-20[1]	-20 [1]	-20 [1]	-20 [1]	-20 [1]	-20 [1]	-20 [1]	-20 [1]	-20[1]	-20[1]	-20[1]	-20[1] -40[2]	-20[1] -20[2]	-20[1] -28.7[2]
5			+300(4)															+160(7)			
6	-20[3]		+100(3)																		
7	+10(1) +140(2)	+210(2) +100(3)																			
8						-40[3]	-40 [3]											-20[3]	-20[3]	-20[3]	-20[3]

* '-' sign represent crude feed to [CDU], '+' sign represent receipt of crude from (Parcel)

Table 6.14 contd..

Table 6.14 Operation schedule for Example 3

Tank	Crude [Charge] (Unload) Amount [CDU No.](Parcel No) in kbbl for period																				
	22 to 27	23	24	25	26	27	28	29	30	31 to 34	32	33	34	35	36	37	38	39	40	41	42
1							+10(9)														
2									+190(12)												
3	-20[1]	-20[1]	-20[1]	-20[1]	-20[1]	-20[1]	-20[1]	-20[1]	-20[1]	-40[1]	-40[1]	-40[1]	-40[1]	-40[1]	-30[1]	-20[1]					
4	-20 [2]	-20 [2]	-20 [2]	-20 [2]	-20 [2]	-20 [2]	+240(10)	+10(10)		-40[2]	-40[2]	-40[2]	-40[2]	-20[2]	-20[2]	-20[2]	-40[1] -20[2]	-20[1] -31.3[2]	-20[1] -20[2]	-30[1] -20[2]	-40[1] -20[2]
5							-20[2]	-20[2]	-20[2]												
6								+250(11)		-40[3]	-40[3]	-40[3]	-40[3]	-40[3]	-30[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]
7																					
8	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]												

* '-' sign represent crude feed to [CDU], '+' sign represent receipt of crude from (Parcel)

Table 6.15 Operation schedule for Example 4

Tank	Crude Amount [to CDU No.](from Vessel No) in kbl for period										
	1 to 2	3	4	5	6	7	8	9	10	11	12 to 13	14 to 15
1	-20[3]	-20[3]	-45[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]	-20[3]	
2	-20[2]	-20[2]	-20[2]	-20[2]	-20[2]	-20[2]	-45[2]	-45[2]	+100(6)	+300(8)		
3					+115(3)	+105(5)			-45[2]	-45[2]	-45[2]	-45[2]
4	-20[1]	-20[1]	-20[1]	-20[1]	-20[1]	-20[1]	-45[1]	-45[1]	-45[1]	-45[1]	-45[1]	-45[1]
5					+235(3)	+245(5)		+150(6)				
6		+350(1)			+300(4)							
7			+350(2)							+200(7)		
8				-25[3]	-25[3]	-25[3]	-25[3]	-25[3]	-25[3]	-25[3]	-25[3]	-45[3]

'-' sign represents delivery to [CDU], '+' sign represents receipt from (parcel)

Table 6.16 Berth allocation details for Example 4

Vessel	Jetty allocation	
	Start Period	End Period
V1	3	3
V2	3	4
V3	6	6
V4	6	7
V5	6	7
V6	9	10
V7	10	11
V8	10	11

6.1.2.5 Example 5

In this example, we consider two jetties, one SBM, 15 8-h periods, one VLCC with three parcels and three single-parcel tankers. Table 6.17 shows the schedule. Since the refinery has both SBM and multiple jetties, both should be able to transfer crude simultaneously in any period. This example shows this feature in period 5, when parcel 4 from the VLCC and parcel 6 from V2 unload to T2 and T5. Similarly, during periods 6 & 7, parcels 5 & 7 from V1 and V3 simultaneously unload to T7 & T8. We have a few instances of multiple tanks feeding one CDU: T2 and T4 feed CDU1 during periods 11-15 and T2 and T5 feed CDU2 during the same time.

6.1.2.6 Example 6

In this example, we illustrate the benefits of tank transfer operations. We consider one SBM, 10 8-h periods and one VLCC with three parcels. Table 6.18 gives the schedules for the case with tank-to-tank transfers and for the case without them. For the former, we allowed at most one tank transfer in periods 1-2 only. Note that the last parcel unloads in period 6 for the latter, while the same unloads in period 5 for the former. The optimal schedule for the former shows a transfer of 70 kbbl from T6 to T7 during period 1. This transfer creates the required space and facilitates early unloading of parcel. Thus, the last parcel unloads in period 5 and demurrage is avoided. Furthermore, the profit with transfers is 2.7% greater than that without transfers. Therefore, tank transfers provide additional flexibility to improve profitability. As mentioned in chapter 4 section 4.5, tank-to-tank operations require more CPU resource because it increases the problem size drastically due to additional decisions, variables and constraints. Compared to previous examples, example 6 has less configurational complexity but has more operational complexity and required more CPU resource.

Table 6.17 Operation schedule for Example 5

Tank	Crude Amount [to CDU No.](from Vessel No) in kbbl for period									
	1	2	3	4	5	6	7	8	9	10	11 to 15
1	-25[3]	-20[3]	-20[3]		-20[3]	-20[3]	-46[3]				
2			+100(2)		+76.8(4)					-37.1[2]	-18.4[1]
					+38.1(6)						-37.1[2]
3			-20[1]	-20[1]	-20[1]	-50[1]	-20[1]	-20[1]	-50[1]	-50[1]	
4	-50[1]	-50[1]			-20[2]	-50[2]	-32.1[2]	-20[2]	-20[2]		-31.5[1]
5	-39.5[2]	-39.5[2]	-39.5[2]	-39.5[2]	+13.2(4)					-12.8[2]	-12.8[2]
					+86.5(6)						
6				+100(3)							
7				-49[3]		+87(5)		-7.7[3]	-7.7[3]	-7.7[3]	-7.7[3]
						+10(7)					
8			+10(1)			+38(5)		-42.3[3]	-42.3[3]	-42.3[3]	-42.3[3]
						+90(7)					

'-' sign represents delivery to [CDU], '+' sign represents receipt from (parcel)

Table 6.18 Comparison of schedules without and with tank transfers for Example 6

Case	Tanks	Crude Amount [to CDU No.](from Parcel No) {to/from tank} in kbbl for period									
		1	2	3	4	5	6	7	8	9	10
No Tank Trasfer	1			+220(2)		-19.3[3]	-19.3[3]	-19.3[3]	-19.3[3]	-19.3[3]	-19.3[3]
	2										
	3	-14.6[2]	-14.6[2]	-14.6[2]	-14.6[2]	-14.6[2]	-14.6[2]	-14.6[2]	-14.6[2]	-14.6[2]	-14.6[2]
	4	-20[1] -22.9[2]	-20[1] -22.9[2]	-20[1] -22.9[2]	-45[1] -22.9[2]	-45[1] -22.9[2]	-45[1] -22.9[2]	-45[1] -22.9[2]	-45[1] -22.9[2]	-45[1] -22.9[2]	-45[1] -22.9[2]
	5					+40(4)	+400(4)				
	6		+10(1) +270(2)			-25.7[3]	-25.7[3]	-25.7[3]	-25.7[3]	-25.7[3]	-25.7[3]
	7			+10(2)	+400(3)						
	8	-20[3]	-20[3]	-45[3]	-45[3]	+100(3)					
With Tank Transfer	1	-4.3[3]	-4.3[3]	-4.3[3]	+232.9(3)						
	2				+167.1(4)						
	3	-18.3[1]	-18.3[1]	-18.3[1]	-18.3[1]	-18.3[1]	-18.3[1]	-18.3[1]	-18.3[1]	-18.3[1]	-18.3[1]
	4	-19.1[1] -20[2]	-19.1[1] -20[2]	-19.1[1] -20[2]	-19.1[1] -45[2]	-19.1[1] -45[2]	-19.1[1] -45[2]	-19.1[1] -45[2]	-19.1[1] -45[2]	-19.1[1] -45[2]	-19.1[1] -45[2]
	5					+272.9(4)					
	6	-70{7}		+132.9(2) +267.1(3)							
	7	+70{6}	+10(1) +367.1(2)		-25[3]	-25[3]	-25[3]	-25[3]	-25[3]	-25[3]	-25[3]
	8	-33.7[3]	-33.7[3]	-33.7[3]	-15.8[3]	-15.8[3]	-15.8[3]	-15.8[3]	-15.8[3]	-15.8[3]	-15.8[3]

'-' sign represents delivery, '+' sign represents receipt

6.2 Conclusion

Six examples are used to illustrate the use of our models for refineries with different configurations (SBM, jetties, tank-to-tank transfers), short and long scheduling horizons, and several parcel sizes and arrivals. The solution analysis of these examples shows no discrepancy in composition. In addition to, they explain most of real life operational features such as multiple tanks feeding one CDU, one tank feeding multiple CDUs, SBM pipeline, brine settling, tank-to-tank transfers etc. This work uses fewer binary variables and is different from and superior (both in terms of efficiency and quality of solutions) to those reported in previous work. The proposed approach helps quicker and near-optimal decision making in refinery operations and handles problems with up to 14 days. The solution analysis of example 3, a longer horizon problem, shows the limitation of this approach to yield early solutions. The tank to tank feature (Example 6) is explained here, provides a real benefit but is highly compute intensive. This reveals that the crude scheduling problem is a difficult, nonlinear problem and it needs further work to be able to solve problems with longer scheduling horizons in reasonable time.

Chapter 7

MODEL FORMULATION - Continuous Time (Part-II)

7.1 Introduction

In the previous chapters we developed a hybrid model and an iterative solution algorithm for crude scheduling problem. The hybrid model is based on discrete time approach with some continuous time features. We identified and addressed the inherent problem of nonlinearity associated with crude blending operation and efficiently handled them using the proposed an iterative, hybrid, discrete-time MILP model. Additional real-world refining operational features such as single buoy mooring (SBM) pipeline, jetties, and tank-to-tank transfers were also considered. The number of binary allocation variables was reduced and 0-1 continuous variables introduced to facilitate improved solutions to bigger problems. One attractive feature of the model is that time continuity is approximated by allowing more than one unloading allocation in any time period; thus the entire time period is utilized to the maximum extent, when possible and the need for smaller time intervals is eliminated.

While the above approximation provides an attractive solution to handle time continuity in a discrete formulation, another family of scheduling models uses an inherent continuous-time representation so that the need for predefined, identical slots is obviated. In these models, each activity's start- and end-time are variables that the model solution determines. So far, only discrete-time formulations have stood up to the challenge of this important, nonlinear problem. A continuous-time formulation would portend numerous advantages; however, existing work in this area has just begun to scratch the surface. The continuous-time modeling is particularly suited to crude oil scheduling since refinery activities can range from a few minutes to several hours (Joly et al., 2002). A discrete-time model would necessitate a large number of slots and increase the computational complexity of the problem. Thus, the main advantages (Pinto et al., 2000) of a continuous-time representation are the full utilization of time continuity and the possibly reduced computational complexity due to fewer binary variables.

Joly et al. (2002) proposed a continuous-time formulation for the refinery scheduling problem, but their communication did not provide details of their model or objective function. Jia and Ierapetritou (2003) also addressed the short-term scheduling of refinery operations based on a continuous-time formulation. They divided refinery operations into three sub problems, the first involving crude oil operations, the second dealing with other refinery processes and intermediate tanks, and the third related to finished products and blending operations. They addressed only the first subproblem in which they used the component balance of Lee et al. (1996), which suffers from the composition discrepancy mentioned earlier. They did not consider the changeover costs arising from crude class or tank changes. Change of crudes and/or tanks is an important operational activity in the refinery, which results in production losses and slop creation. Their proposed model does not allow many operational features such as multiple tanks feeding one crude distillation unit (CDU), single tank feeding multiple CDUs, settling time for brine removal after crude receipt, etc. Besides, their demurrage accounting may be inaccurate, as they seem to compute demurrage for the total time that a vessel spends for unloading crude.

Magalhães and Shah (2003) also proposed a continuous-time model for crude oil scheduling. While the details of algorithm and model were not reported, they espoused real-world operational rules such as crude segregation, non-simultaneous receipt and delivery of crude by a tank, and settling time for brine removal. They scheduled to achieve the target crude throughput over the scheduling horizon, but did not consider crucial practical aspects such as demurrage and changeover. However, they did acknowledge the importance of these aspects in making the problem realistic.

As the above discussion suggests, none of the reported continuous-time formulations satisfactorily addresses the composition discrepancy in crude charge to CDU, transfer lines with non-negligible volumes, and other important features such as demurrage, changeovers, brine settling time, etc. This has given a motivation for developing a continuous-time formulation that accommodates some of these industrially important structural and operational features of a refinery.

In this research effort, we present the first complete continuous-time mixed integer linear programming (MILP) formulation for the short-term scheduling of operations in a refinery that receives crude from very large crude carriers via a high-volume SBM (Single buoy mooring) pipeline. This novel formulation accounts for real-world operational practices. We also employ a realistic profit-based objective function that includes marginal crude profits, safety stock penalties, and accurate demurrage accounting. We use an iterative algorithm similar to that developed for discrete approach, to eliminate the crude composition discrepancy that has proven to be the Achilles heel for existing formulations. While it does not guarantee global optimality, the algorithm needs only MILP solutions and obtains excellent maximum-profit schedules for industrial problems with up to seven days of scheduling horizon. We also report the first comparison of discrete-time vs. continuous-time formulations for this complex problem.

7.2 Refinery configuration

In this work we considered a refinery configuration with SBM. Figure 7.1 gives a schematic of the crude oil unloading, storing and processing in a typical refinery. The

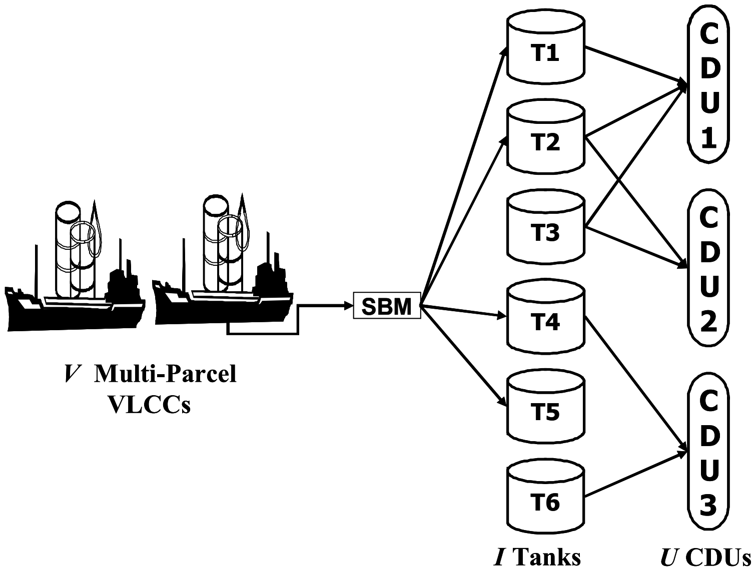


Figure 7.1: Schematic of oil unloading using SBM and processing in CDUs

configuration involves crude offloading facilities such as an SBM (Single Buoy Mooring) or SPM (Single Point Mooring) station, storage facilities such as storage tanks and/or charging tanks and processing facilities such as crude distillation units (CDUs). The operation involves unloading crudes into multiple storage tanks from the ships/tankers arriving at various times and feeding the CDUs from these tanks at various rates over time. Thus, the problem involves both scheduling as well as allocation issues. The problem description is same as explained in chapter 2, here we are attempting to find a solution using continuous time modeling approach.

7.3 MILP Formulation

7.3.1 Time Representation

Time representation is a key to continuous-time formulation. We employ a mix of the strategies used by Karimi and McDonald (1997) and Lamba and Karimi (2002) in that we use synchronized (across tanks) but variable-length time slots within periods defined by known, fixed time events. To this end, we first use the fixed time events to divide the scheduling horizon into several periods with different lengths. In this paper, we use two types of time events, but one could easily use more. These are the scheduled arrival times of VLCCs and their latest *expected* departure times. We take the latest expected departure time to be some hours later than the latest time by which a VLCC must leave to avoid demurrage as stipulated in the VLCC's logistics contract. The first period begins at time zero and ends at the arrival time of the first VLCC. The second period follows the first period and ends at the latest expected departure time of the first VLCC. The next period ends at the arrival time of the second VLCC, and the remaining periods follow likewise. The last period runs from the latest expected departure time of the last VLCC to the end of the scheduling horizon. We use D_t (t = 1 … NT, $D_0 = 0$) to denote the start of period t.

Having defined the periods, we now divide each period into a fixed number of variable-length slots that are synchronized or identical across all storage tanks. The numbers of slots may vary from period to period, but they are the same for all tanks. As in most slot formulations, we must guess the number of slots for each period. During a given slot, there is no activity change in any tank, i.e. the refinery state remains unchanged during the entire slot. However, an activity may continue over two consecutive slots on a given tank.

We consider a refinery with i tanks and u CDUs and one SBM line. If NV VLCCs arrive during the scheduling horizon, then we have at most $2NV+1$ periods. We assign S_t slots to period t to get $NS = S_1 + S_2 + \ldots + S_{NT}$ slots in the entire scheduling horizon. We now derive constraints that govern different aspects of the refinery crude operations.

7.3.2 Parcel Creation

The parcel creation step follows exactly same procedure as the one explained in chapter 4 while describing the discrete time model. At the end of the parcel creation step, let there be NP parcels (p = 1…NP) in the ordered list. We now assign an arrival time ETA_p to each parcel p as follows. ETA_p for a VLCC parcel is the arrival time of its VLCC, while that for an SBM parcel is the arrival time of the next VLCC. Having defined the periods, slots and parcels, we now develop the constraints in our continuous-time MILP formulation. Unless stated otherwise, we write each constraint for all valid values of its defining indices. We begin with the parcel unloading operations.

7.3.3 Parcel-to-SBM Connections

The SBM operation demands that each parcel connection to the SBM line to unload and then disconnection after unloading. To model this process of connection/disconnection, we define three binary variables:

$$XP_{ps} = \begin{cases} 1 & \text{if parcel } p \text{ is connected to the SBM line for unloading during slot } s \\ 0 & \text{otherwise} \end{cases}$$

$$XF_{ps} = \begin{cases} 1 & \text{if parcel } p \text{ first connects to the SBM line at the start of slot } s \\ 0 & \text{otherwise} \end{cases}$$

$$XL_{ps} = \begin{cases} 1 & \text{if parcel } p \text{ disconnects from the SBM line at the end of slot } s \\ 0 & \text{otherwise} \end{cases}$$

Because each VLCC arrival coincides with the start of a period, we can identify the slots in which each parcel p may connect to the SBM line. We define XP_{ps}, XF_{ps} and XL_{ps} only for such slots, which we define by $(p, s) \in \boldsymbol{PS} = \{(p, s) \mid \text{parcel } p \text{ may connect to the SBM line in slot } s\}$. The following constraints relate these variables:

$$XP_{ps} = XP_{p(s-1)} + XF_{ps} - XL_{p(s-1)} \qquad (p, s) \in \boldsymbol{PS} \qquad (7.1a)$$

$$XP_{ps} \geq XL_{ps} \qquad (p, s) \in \boldsymbol{PS} \qquad (7.1b)$$

Assuming that each parcel connects to and disconnects from the SBM line once and only once, we get,

$$\sum_s XF_{ps} = \sum_s XL_{ps} = 1 \qquad (p, s) \in \boldsymbol{PS} \qquad (7.2a,b)$$

Eqs. 7.1a-b and 7.2a-b together ensure that XF_{ps} and XL_{ps} are binary, when XP_{ps} are so. Therefore, we treat XF_{ps} and XL_{ps} as 0-1 continuous variables. In order to unload parcels one at a time in the given sequence, we make a parcel start unloading only after the previous one has unloaded completely and has disconnected. The following constraints ensure this.

$$\sum_p XP_{ps} \leq 1 \qquad (p, s) \in \boldsymbol{PS} \qquad (7.3)$$

$$XF_{ps} \leq \sum_{s'<s} XL_{(p-1)s'} \qquad (p, s) \in \boldsymbol{PS} \qquad (7.4)$$

7.3.4 SBM-to-Tank Connections

For a tank to receive crude from a parcel, it must connect to the SBM line while the parcel is connected. To model this SBM-to-tank connection, we define:

$$XT_{is} = \begin{cases} 1 & \text{if tank } i \text{ is connected to the SBM line during slot } s \\ 0 & \text{otherwise} \end{cases}$$

We allow only one tank to receive crude from an unloading parcel at a time, so only one tank can connect to the SBM line at a time. In addition, the tank cannot connect if the parcel is not connected. We enforce both these restrictions by using,

$$\sum_i XT_{is} \leq \sum_p XP_{ps} \qquad (p, s) \in \boldsymbol{PS}, (p, i) \in \boldsymbol{PI} \qquad (7.5)$$

where, $\boldsymbol{PI} = \{(p, i) \mid \text{tank } i \text{ may receive crude from parcel } p\}$.

7.3.5 Tank-to-CDU Connections

To supply its crude for processing, a tank must connect to one or more CDUs. We model this connection by the following binary variable:

$$Y_{ius} = \begin{cases} 1 & \text{if tank } i \text{ feeds CDU } u \text{ during slot } s \\ 0 & \text{otherwise} \end{cases}$$

Operating policies may dictate that a tank may not charge more than some (say two) CDUs simultaneously and vice versa. Thus,

$$\sum_{u} Y_{ius} \leq 2 \qquad (i, u) \in \boldsymbol{IU} \qquad (7.6a)$$

$$\sum_{i} Y_{iut} \leq 2 \qquad (i, u) \in \boldsymbol{IU} \qquad (7.6b)$$

where, $\boldsymbol{IU} = \{(i, u) \mid \text{tank } i \text{ can feed CDU } u\}$.

7.3.6 Tank Activity

A tank may do one of three activities at any given time. First, it may receive crude from a parcel. Second, it may feed crude to one or more CDUs. Third, it may idle or settle brine. XT_{is} modeled the first activity. For the remaining two, we define:

$$YT_{is} = \begin{cases} 1 & \text{if tank } i \text{ is delivering crude during slot } s \\ 0 & \text{otherwise} \end{cases}$$

$$ZT_{is} = \begin{cases} 1 & \text{if tank } i \text{ is idle or brine-settling during slot } s \\ 0 & \text{otherwise} \end{cases}$$

Because a tank must do exactly one of the above three activities in a given slot, we get,

$$XT_{is} + YT_{is} + ZT_{is} = 1 \qquad (7.7)$$

A tank must be in the state of delivery in a slot s, if it is feeding a CDU i and vice versa. In other words,

$$YT_{is} \geq Y_{ius} \qquad (i, u) \in \boldsymbol{IU} \qquad (7.8a)$$

$$YT_{is} \leq \sum_{u} Y_{ius} \qquad (i, u) \in \boldsymbol{IU} \qquad (7.8b)$$

The above constraints make YT_{is} binary, when Y_{ius} are so. Thus, we treat YT_{is} as 0-1 continuous variables. As we treat XT_{is} as binary and YT_{is} are automatically binary, eq. 7.7 forces ZT_{is} to be binary. Thus, we treat ZT_{is} also as 0-1 continuous variables.

7.3.7 Activity Durations

As stated earlier, we synchronize slots across all tanks, so we assign a unique slot length SL_s to each slot s. Thus, if TL_s denotes the time at which a slot s ends, then

$$TL_s = TL_{(s-1)} + SL_s \qquad TL_0 = 0 \tag{7.9}$$

Because each period consists of slots, we have,

$$\sum_s SL_s = DD_t \qquad D_0 = 0,\ (t, s) \in \boldsymbol{TS} \tag{7.10}$$

where, $\boldsymbol{TS} = \{(t, s) \mid \text{slot } s \text{ is in period } t\}$ and $DD_t = D_t - D_{(t-1)}$ is the length of period t.

Recall that we trigger new slots, only when there is a change in activity somewhere in the refinery. Therefore, if an activity occurs during a slot s, then it must span the entire slot duration. To ensure this, we define the following continuous variables for the durations of various activities during slot s:

$RLP_{ps} = XP_{ps}SL_s$ = Time for which parcel p connects to the SBM line (7.11a)

$RLT_{is} = XT_{is}SL_s$ = Time for which tank i connects to the SBM line (7.11b)

$RLU_{is} = YT_{is}SL_s$ = Time for which tank i feeds crude (7.11c)

$RLZ_{is} = ZT_{is}SL_s$ = Time for which tank i idles or settles brine (7.11d)

$RU_{ius} = Y_{ius}SL_s$ = Time for which tank i feeds CDU u (7.11e)

$RP_{pis} = XT_{is}XP_{ps}SL_s$ = Time for which parcel p unloads into tank i (7.11f)

Because eqs. 7.11a-f are nonlinear, we develop linear approximations for them.

First, we multiply eq. 7.8 by SL_s and use eqs. 7.11b-d to get,

$$SL_s = RLT_{is} + RLU_{is} + RLZ_{is} \tag{7.12}$$

Now, we know that RLP_{ps}, RLT_{is}, RLU_{is}, RLZ_{is}, RU_{ius} and RP_{pis} must be zero, if their respective binary variables XP_{ps}, XT_{is}, YT_{is}, ZT_{is} and Y_{ius} are zero, so we use,

$$RLP_{ps} \le DD_t XP_{ps} \qquad (t, s) \in \boldsymbol{TS},\ (p,s) \in \boldsymbol{PS} \tag{7.13a}$$

$$RLT_{is} \le DD_t XT_{is} \qquad (t, s) \in \boldsymbol{TS} \tag{7.13b}$$

$$RLU_{is} \le DD_t YT_{is} \qquad (t, s) \in \boldsymbol{TS} \qquad (7.13c)$$

$$RLZ_{is} \le DD_t ZT_{is} \qquad (t, s) \in \boldsymbol{TS} \qquad (7.13d)$$

$$RU_{ius} \le DD_t\, Y_{ius} \qquad (i, u) \in \boldsymbol{IU}, (t, s) \in \boldsymbol{TS} \qquad (7.13e)$$

$$\sum_p RP_{pis} \le RLT_{is} \qquad (p, s) \in \boldsymbol{PS}, (p, i) \in \boldsymbol{PI} \qquad (7.14a)$$

$$\sum_i RP_{pis} \le RLP_{ps} \qquad (p, s) \in \boldsymbol{PS}, (p, i) \in \boldsymbol{PI} \qquad (7.14b)$$

Then, we multiply eqs. 7.4, 7.5 and 7.8a-b by SL_s to get,

$$\sum_p RLP_{ps} \le SL_s \qquad (p, s) \in \boldsymbol{PS} \qquad (7.15)$$

$$\sum_i RLT_{is} \le \sum_p RLP_{ps} \qquad (p, s) \in \boldsymbol{PS}, (p, i) \in \boldsymbol{PI} \qquad (7.16)$$

$$RLU_{is} \ge RU_{ius} \qquad (i, u) \in \boldsymbol{IU} \qquad (7.17a)$$

$$\sum_u RU_{ius} \ge RLU_{is} \qquad (i, u) \in \boldsymbol{IU} \qquad (7.17b)$$

Lastly, to make RP_{pis} and RU_{ius} equal slot length, when their respective activities take place, we use,

$$RU_{ius} \ge RLU_{is} - DD_t(1 - Y_{ius}) \qquad (i, u) \in \boldsymbol{IU}, (t, s) \in \boldsymbol{TS} \qquad (7.18)$$

$$RP_{pis} \ge RLT_{is} + RLP_{ps} - SL_s \qquad (p, \mathrm{s}) \in \boldsymbol{PS}, (p, i) \in \boldsymbol{PI} \qquad (7.19)$$

We can derive eqs. 7.14a-b and 7.19 as follows:

Define a 0-1 continuous variable X_{pis} as:

$$X_{pis} = \begin{cases} 1 & \text{if tank } i \text{ is receiving crude from parcel } p \text{ during slot } s \\ 0 & \text{otherwise} \end{cases}$$

Since a tank i can receive crude from parcel p, only if both the tank and the parcel are connected to the SBM line (i.e., $XT_{is} = 1$ and $XP_{ps} = 1$). In other words, $X_{pis} = XP_{ps}\, XT_{is}$. A linear approximation for this is given by,

$$\sum_p X_{pis} \le XT_{is} \qquad (p, s) \in \boldsymbol{PS}, (p, i) \in \boldsymbol{PI} \qquad (i)$$

$$\sum_i X_{pis} \le XP_{ps} \qquad (p, s) \in \boldsymbol{PS}, (p, i) \in \boldsymbol{PI} \qquad (ii)$$

$$X_{pis} \geq XP_{ps} + XT_{is} - 1 \qquad (p, s) \in \boldsymbol{PS}, (p, i) \in \boldsymbol{PI} \qquad \text{(iii)}$$

The first equation states that a tank cannot receive crude from any parcel, if it is not connected to the SBM line. Similarly, the second equation states that no tank can receive crude from a parcel, if that parcel is not connected to the SBM line. The third equation states that crude can transfer if both parcel and tank are connected to the SBM line. Multiplying the above three equations by SL_s, we get eqs. 7.14a-b and 7.19. By using eqs. 7.14a-b and 7.19, we have avoided the variable X_{pis} and associated constraints. Note that most of the available literature (Lee et al., 1996; Jia and Ierapetritou, 2003) uses even this variable as a tri-index binary.

7.3.8 Crude Unloading

Having defined the parcel-to-SBM and SBM-to-tank connections, we now compute the amount of crude unloaded in slot s. Let FPT_{pis} be the crude volume that parcel p unloads to tank i during slot s. If we take the pumping rate FPI_{ps}^{U} from parcel p to tank i as constant, then

$$FPT_{pis} = FPI_{p}^{U} RP_{pis} \qquad (p, s) \in \boldsymbol{PS}, (p, i) \in \boldsymbol{PI} \qquad (7.20)$$

To make each parcel p unload fully during the scheduling horizon, we use,

$$\sum_{i}\sum_{s} FPT_{pis} = VP_{p} \qquad (p, s) \in \boldsymbol{PS}, (p, i) \in \boldsymbol{PI} \qquad (7.21)$$

where, VP_p denotes the size of parcel p.

7.3.9 Crude Processing

Let FTU_{ius} denote the crude volume that tank i feeds CDU u during slot s. If FTU_{iu}^{L} and FTU_{iu}^{U} are the minimum and maximum feed rates from tank i to CDU u, then we have,

$$FTU_{iu}^{L} RU_{ius} \leq FTU_{ius} \leq FTU_{iu}^{U} RU_{ius} \qquad (i, u) \in \boldsymbol{IU} \qquad (7.22)$$

For individual crudes fed to CDUs, we define $FCTU_{iucs}$ as the amount of crude c fed by tank i to CDU u during slot s. Then, the total crude amount FTU_{ius} that tank i feeds CDU u during s is,

$$FTU_{ius} = \sum_{c} FCTU_{iucs} \qquad (i, u) \in \boldsymbol{IU}, (i, c) \in \boldsymbol{IC} \qquad (7.23)$$

where, $\boldsymbol{IC}$ = {(i, c) | tank i may hold crude c}. Because multiple tanks may feed a CDU, the total feed FU_{us} to CDU u during slot s is,

$$FU_{us} = \sum_{i} FTU_{ius} \qquad (i, u) \in \boldsymbol{IU} \qquad (7.24)$$

This must be within the processing limits of CDU u:

$$FU_{u}^{L} SL_{s} \le FU_{us} \le FU_{u}^{U} SL_{s} \qquad (7.25)$$

where, FU_{u}^{L} and FU_{u}^{U} are the lower and upper limits on the processing rate of CDU u.

In practice, the plant operation may know that a CDU cannot process a feed with some extreme fractions of crudes. To impose such limitations, we use,

$$FU_{us} xc_{cu}^{L} \le \sum_{i} FCTU_{iucs} \le FU_{us} xc_{cu}^{U} \qquad (i, u) \in \boldsymbol{IU}, (i, c) \in \boldsymbol{IC} \qquad (7.26)$$

Similarly, the concentration of a key component may also be critical to the operation of a CDU and must be within certain allowable limits. If xk_{kc} is the fraction of a key component k in crude c, then we respect these limits by using,

$$xk_{ku}^{L} FU_{us} \le \sum_{i} \sum_{c} FCTU_{iucs} xk_{kc} \le xk_{ku}^{U} FU_{us}, \quad (i, u) \in \boldsymbol{IU}, (i, c) \in \boldsymbol{IC} \qquad (7.27)$$

7.3.10 Crude Balance

First, to identify the crude in parcel p, we define a set $\boldsymbol{PC}$ = {(p, c) | parcel p carries crude c}. Using VCT_{ics} to denote the amount of crude c in tank i at the end of slot s, we write a crude balance on tank i as,

$$VCT_{ics} = VCT_{ic(s-1)} + \sum_{(p,c) \in \boldsymbol{PC}, (p,i) \in \boldsymbol{PI}} FPT_{pis} - \sum_{(i,u) \in \boldsymbol{IU}} FCTU_{iucs} \qquad (i, c) \in \boldsymbol{IC} \qquad (7.28)$$

With this, the total crude level in tank i at the end of slot s:

$$V_{is} = \sum_{c} VCT_{ics} \qquad (i, c) \in \boldsymbol{IC} \qquad (7.29)$$

Now, this must satisfy some upper and lower limits as,

$$V_i^L \le V_{is} \le V_i^U \qquad (7.30)$$

where, V_i^L and V_i^L are the lower and upper limits on the crude volume that tank i may hold.

Because of processing and operational constraints, the refinery may also keep crude fractions in tanks within some limits as follows,

$$xt_{ic}^L V_{is} \le VCT_{ics} \le xt_{ic}^U V_{is} \qquad (7.31)$$

where, xt_{ic}^L and xt_{ic}^U are the allowable lower and upper limits on the fraction of crude c in tank i.

7.3.11 Brine Settling

After it receives crude, a tank must idle for some time to settle and remove brine. Thus, if it receives crude in slot s, then cannot deliver in slot (s+1), i.e.,

$$XT_{is} + YT_{i(s+1)} \le 1 \qquad (7.32)$$

It is possible that the length of slot (s+1) is insufficient for the required settling time ST. In that case, the tank must continue to settle in slot (s+2) as well. In general, we need to allocate certain slots after slot s for possible settling. Let $\boldsymbol{SS}_s$ = {s' | slot $s' > s$ such that a tank receiving crude in slot s may require time up to beginning of slot s' for settling}. Then, to ensure minimum settling time, we need,

$$ST\,(YT_{is'} + XT_{is} - 1) \le TL_{(s'-1)} - TL_s \quad s' \in \boldsymbol{SS}_s \qquad (7.33)$$

7.3.12 Changeovers

In real operation, it is desirable to minimize the upsets caused by changeovers of tanks feeding to a CDU. To detect such changes, we define a 0-1 continuous variable CO_{us} as:

$$CO_{us} = \begin{cases} 1 & \text{if there is a tank switch on CDU } u \text{ at the end of slot } s \\ 0 & \text{otherwise} \end{cases}$$

$$CO_{us} \geq Y_{ius} - Y_{iu(s+1)} \qquad (i, u) \in \boldsymbol{IU} \qquad (7.34a)$$

$$CO_{us} \geq Y_{iu(s+1)} - Y_{ius} \qquad (i, u) \in \boldsymbol{IU} \qquad (7.34b)$$

7.3.13 Crude Demand

We can specify the desired throughput of crude in several ways. One simple way is to specify it for each CDU in each period as follows:

$$\sum_i \sum_u \sum_s FTU_{ius} = CD_t \qquad (i, u) \in \boldsymbol{IU}, (t, s) \in \boldsymbol{TS} \qquad (7.35a)$$

This obviously requires detailed data that may be difficult to obtain readily. A better way is to specify a throughput over the entire horizon for each CDU or groups of CDUs:

Crude demand per CDU: $$\sum_i \sum_s FTU_{ius} = DM_u \qquad (i, u) \in \boldsymbol{IU} \qquad (7.35b)$$

Total crude demand in the horizon: $$\sum_i \sum_u \sum_s FTU_{ius} = TCD \qquad (i, u) \in \boldsymbol{IU} \qquad (7.35c)$$

7.3.14 Demurrage

A key operating cost in crude scheduling is the demurrage or sea-waiting cost. The logistics contract with each VLCC stipulates an acceptable sea-waiting period. If the VLCC harbors beyond this stipulated period, then the demurrage (or sea-waiting cost) is incurred. Let STD_v be the stipulated time of departure in the logistics contract of VLCC v. The demurrage incurs if the last parcel of VLCC v remains connected to the

SBM line beyond STD_v. If VLCC v arrives at the start of period t and its demurrage or sea-waiting cost is SWC_v per unit time, then the demurrage incurred is:

$$DC_v \geq SWC_v[TL_s - STD_v XP_{ps} - D_t(1-XP_s)] \qquad (t, s) \in \boldsymbol{TS}, (p, v) \in \boldsymbol{PV} \qquad (7.36)$$

where, $\boldsymbol{PV} = \{(p, v) \mid \text{parcel } p \text{ is the last parcel in VLCC } v\}$.

7.3.15 Objective

We use total gross profit as the scheduling objective. We define this as the total marginal profit (netback) from crudes minus the operating cost. The former is simply the value of products minus the purchase cost of crude. Since the product yields vary with crudes and CDUs, we define CP_{cu} as the marginal profit ($ per unit volume) for crude c processed in CDU u. Note that the marginal profit does not include any operating costs.

The operating costs include the changeover costs, the demurrage and the penalty for under-running the crude safety stock. As noted earlier, a change in the feed composition (tanks) to a CDU is called a changeover. A changeover lasts a few hours and leads to off-spec products or slops during the transition. In other words, every changeover incurs some cost to the refinery and is undesirable. Let COC be the cost per changeover. The refinery may wish to keep a minimum crude inventory SS_t at the end of period t. We call this the desired safety stock of crude. To prevent the crude inventory from dropping below this level, we assign a penalty SSP_t ($ per unit volume) for period t for under-running the crude safety stock. Based on the above discussion, we compute the operating costs and obtain the total gross profit as,

$$\text{Profit} = \sum_i \sum_u \sum_c \sum_s FCTU_{iucs} CP_{cu} - \sum_v DC_v - COC \sum_u \sum_s CO_{us} - \sum_t SC_t \qquad (7.37)$$

$$SC_t \geq SSP_t(SS_t - \sum_i V_{is}) \qquad (t, s) \in \boldsymbol{LAST} \qquad (7.37a)$$

where, $\boldsymbol{LAST} = \{(t, s) \mid \text{slot } s \text{ is the last slot in period } t\}$.

This completes our continuous-time formulation (eqs. 7.1-7.37a) for a refinery with one SBM line. A coastal refinery may use a single jetty to receive small ships carrying single crude parcels. Since the pipeline connecting the jetty to the refinery tank farm usually has negligible volume, we can ignore its crude holdup. This makes the SBM parcels disappear from our formulation and we can use the proposed formulation as is to handle a single jetty by treating the individual marine vessels as single-parcel VLCCs.

As discussed in chapter 5, the discrete time MILP model experiences the non linearity issue inherent to crude mixing in the storage tanks. The continuous time model too suffers from this problem and leads to a solution with composition discrepancy. In the next chapter we develop a solution algorithm for continuous time model using an approach identical to that developed for discrete model in chapter 5.

Chapter 8

SOLUTION ALGORITHM – Continuous Time

8.1 Introduction

The blending of crudes in tanks makes the crude scheduling problem inherently nonlinear. The proposed linear formulation is thus an approximation. The linear crude balance constraints (Eqs. 7.22-7.27) allow the optimizer to push arbitrary amounts of individual crudes to a CDU rather than in the proportions dictated by the tank composition. This results in a disproportionate delivery of crudes to CDUs. For a discrete-time formulation, chapter 5 presented a novel iterative strategy to eliminate this problem. In the next section, we modify this strategy to suit the proposed continuous-time approach without solving any nonlinear problem.

8.2 Description

To avoid disproportionate delivery, we must use the correct nonlinear constraint $FCTU_{iucs} = f_{ics}\, FTU_{ius}$, where f_{ics} is the fraction of crude c in tank i at the end of slot s. We now devise an iterative strategy that solves a series of MILPs of reducing size and complexity to obtain a near-optimal solution with no discrepancy in the crude composition delivered to a CDU.

Observe that every tank has blocks of contiguous slots during which its composition does not change, because it receives no crude. For such a block, if we knew that constant composition (f_{ic}), then the nonlinear equation ($FCTU_{iucs} = f_{ic}$ FTU_{ius}) becomes linear. Solving an MILP with these constraints would yield a solution with no composition discrepancy. Thus, for *each* tank, we divide all slots into two distinct blocks: one for which we know the tank compositions, and the other for which we do not. For the former, we use the exact linear constraints ($FCTU_{iucs} = f_{ic}\, FTU_{ius}$) and for the latter, we use the linear approximations as in our formulation. Furthermore, note that a tank's composition changes, only when it receives crude from a parcel or another tank, otherwise not. During most slots, the tank will receive nothing; hence its composition will be constant. However, we must know that composition to make the nonlinear flow constraints linear. To this end, our knowledge of the initial compositions of tanks proves useful. Because we know the initial composition in each tank, we can identify one initial block of slots for which the composition is constant and known. The length of this block will vary from tank to tank and it could be as short as just one slot for some tanks. However, this at least provides a start for our algorithm. As a first try, we use the exact linear constraints for these first blocks of slots and linear approximations for the remaining slots and solve the MILP. This gives us a solution that has no composition discrepancy at least for the first block of slots on each tank. It also gives us the compositions in all tanks at the end of each slot in each block. We now identify the first common block of slots for which we know the compositions in all tanks. We freeze the schedule until the end of that block, and repeat the entire procedure for scheduling the remaining slots. In other words, we now solve another scheduling problem with a reduced horizon. In this manner, we get progressively longer and longer partial schedules, free of composition discrepancies, by solving a series of MILPs, until we have the complete schedule. We now describe the algorithm in full detail.

8.3 Step-wise Procedure

At each iteration of our algorithm, we divide the *NS* slots into two sets. Set 1 includes all slots with $s \leq s^*$ for some s^*, and set 2 the rest. The schedule (or all variables Y_{ius}, XP_{ps}, XT_{is} ,FTU_{ius}, $FCTU_{iucs}$, etc. in the MILP) for slots in set 1 is (are) fixed based on previous iterations. However, the MILP variables for slots in set 2 are free. From the schedule for set 1, we know the tank compositions at the end of slot s^*. Let f_{ic} denote the fraction of crude c in tank i at $s = s^*$. Then, our iterative algorithm proceeds as follows:

1. Set $s^* = 0$.
2. From the fixed schedule for $s \le s^*$, compute f_{ic} for each tank i as $f_{ic} = VCT_{ics^*} / V_{is^*}$ using the known information (VCT_{ics^*} and V_{is^*}).
3. For each tank i, identify the latest slot $s_i^* > s^*$ such that its composition is f_{ic} for all slots $s^* < s \le s_i^*$. We do this as follows. Let p' denote the last parcel that was unloaded (to any tank) before s^* and p'' denote the earliest parcel ($p'' \ge p'$) that tank i can possibly receive after s^*. Two possibilities exist, namely that p' has finished unloading or p' has partially unloaded.

   First, consider that p' has fully unloaded, i.e. $XL_{p's^*} = 1$. If parcel p'' does not exist, then $s_i^* = NS$. If both p' and p'' belong to the same VLCC, then $s_i^* = s^* + (p'' - p')$. This is because we need at least one slot to unload a parcel. If p'' belongs to a different vessel, then $s_i^* = ETU_{p''}$, which is the slot in which p'' can possibly start unloading. Next, consider that p' has partially unloaded. Then, $s_i^* = s^* + (p'' - p') + 1$ for an i that cannot receive crude from p' and $s_i^* = s^* + 1$ for an i that can do so.
4. Add the constraint, $FCTU_{iucs} = f_{ic} FTU_{ius}$, for $s^* < s \le s_i^*$ in the MILP and solve.
5. Fix the MILP variables for $s^* < s \le \min_i[s_i^*]$. Set $s^* = \min_i[s_i^*]$. If $s^* = NS$, then terminate, otherwise go to Step 2.

8.4 Discussion

Note that the size and complexity of the MILP in our algorithm reduce progressively by at least one slot at each iteration. However, for periods in which no VLCC arrives, the size and complexity reduce by one full period. Because our approach fixes MILP variables based on a solution from linear approximation, it cannot guarantee an optimal solution. However, considering the fact that even solving the approximate MILP is a challenging problem, our approach is quite attractive because it does not require solving MINLPs or NLPs and gives near-optimal schedules in reasonable time. When the scheduling objective does not involve crude compositions (minimizing operating cost as objective instead of profit), then our algorithm guarantees an optimal objective value right in the first MILP, although further iterations are required to correct the composition discrepancy. For large and complex problems, solving even the MILP to a zero gap is compute-intensive; hence we may have to target a small optimality gap for the first few iterations. As iterations proceed in our algorithm, the problem size reduces. This reduces the computation time drastically and allows us to target even smaller relative gaps in progressive iterations. For example, if a problem has NS = 30, then we can use a relative gap of 5% for the first few iterations and 0% for the rest, when the MILP becomes easier to solve.

In the next chapter, we illustrate our methodology on several examples derived from a local refinery (SRC Singapore). In order to maintain confidentiality, we used different set of data but kept the nature of problem identical, employed similar operating rules

Chapter 9

COMPUTATIONAL RESULTS

9.1 EXAMPLES

We illustrate our methodology using three examples. We take a refinery with 8 tanks (T1-T8), 3 CDUs (CDU1-CDU3) and two classes (Class1 & Class2) of crudes. Tanks T1, T6, T7, T8 and CDU3 handle Class1 crudes, while the rest handle Class2 crudes. For all Examples 1 to 3, Table 8.1 gives the tanker arrival details, crude demands and key component concentration limits on CDUs. Table 8.2 gives the initial crude levels in tanks and tank capacity details. Table 8.3a gives the economic data and limits on crude transfer amounts. Table 8.3b provides information about crudes, their marginal profits and key component concentrations. As mentioned earlier, detailed model information and problem data for the literature (Joly et al., 2002; Jia and Ierapetritou, 2003; Magalhães and Shah, 2003) on continuous formulations are not available, so we compare continuous time approach with the discrete-time approach developed and explained earlier. It is important to do this, because the issue of which approach (continuous-time or discrete-time) is better for this problem is yet to be resolved conclusively. However, there is some difficulty in comparing the schedules and profits from the two approaches on the same footing because of the following reasons.

1) In a discrete-time model, demurrage counts for multiples of the uniform slot, while in a continuous-time model, its counting is on a per unit time basis and thus accurate.
2) Violation of minimum safety stock is checked and penalized at every slot in a discrete-time model, while it is done at event points (periods) only in a continuous-

time model. This is because an inventory penalty depends on time as well and variable-length slots would result in a nonlinear formulation.

3) A discrete-time model allows amount transferred to a tank in a period to vary between some lower and upper limits. This makes us the transfer rate flexible. In a continuous-time model, however, the transfer rate is fixed and the amount depends on the slot length. It would be desirable to use the maximum possible transfer rate to minimize demurrage, so a fixed transfer rate is closer to real practice, but a variable one may prove prudent in some situations.
4) In the discrete time approach, we considered even a change in the tank-to-CDU flow as a changeover, when two tanks are feeding to one CDU, as such a change alters feed composition. A discrete model has the advantage of avoiding such changeovers by forcing the flows from respective tanks to be constant in contiguous periods. In the proposed continuous model, a comparison of flow rates in consecutive slots introduces a nonlinearity that forces the algorithm to adjust the flow by generating MILP at every slot and making the problem compute intense. In fact, it is easier to adjust the flow rates to avoid such changeovers, after we get the final schedule. Therefore, we avoid the flow matching constraints in both models and use the changeover definition mentioned in this paper for a fair comparison of both models.

For the three examples, we used CPLEX 7.0 solver within GAMS on a SUN enterprise 250 server using SUN OS 5.7, Single Ultra SPARC II 400 MHz Processor and 2 GB RAM. Tables 9.4 and 9.5 give the model statistics and performances respectively for all examples. We see (Table 9.5) that the continuous-time formulation uses more variables and constraints, but fewer binary variables.

Table 9.1: Tanker arrival details, crude demands and key component concentration limits on CDUs

Ex	Horizon (h)	Tanker	Arrival times (h) Conti. / Discrete	Parcel No: [Parcel Size (kbbl) crude]	Key Comp. Limits (vol%) CDU	Lower	Upper	Demand (kbbl)
1	72	VLCC-1	15 / 8	1: [10 C2], 2: [250 C3], 3: [300 C4], 4: [190 C5]	CDU1	0.10	1.40	300
					CDU2	0.10	1.30	300
					CDU3	0.10	0.40	300
2	80	VLCC-1	8 / 8	1: [10 C2], 2: [500 C4], 3: [500 C3], 4: [440 C5]	CDU1	0.10	1.35	375
					CDU2	0.10	1.20	375
					CDU3	0.10	0.35	400
3	160	VLCC-1	15 / 8	1: [10 C2], 2: [250 C3], 3: [300 C4], 4: [190 C5]	CDU1	0.10	1.30	700
		VLCC-2	100 / 104	5: [10 C5], 6: [250 C6], 7: [250 C3], 8: [240 C8]	CDU2	0.10	1.25	700
					CDU3	0.10	0.35	700

Table 9.2: Storage tank capacities and initial inventory of crude stock

Tank	Capacity (kbbl)	Heel (kbbl)	Initial Inventory (kbbl)		Initial Crude Composition (kbbl)							
					Examples 1 & 3				Example 2			
	Exs 1-3	Exs 1-3	Exs 1&3	Ex 2	C1 or C5	C2 or C6	C3 or C7	C4 or C8	C1 or C5	C2 or C6	C3 or C7	C4 or C8
T1	570	60	350	350	50	100	150	50	50	100	150	50
T2	570	60	400	400	200	0	50	150	200	0	50	150
T3	570	60	350	350	100	100	50	100	100	100	50	100
T4	980	110	950	950	200	250	200	300	200	250	200	300
T5	980	110	300	300	100	100	50	50	100	100	50	50
T6	570	60	80	240	20	20	20	20	30	30	150	30
T7	570	60	80	120	20	20	20	20	30	30	50	10
T8	570	60	450	550	100	100	100	150	150	100	210	90

Tanks 1, 6-8 store crudes 1-4 and 2-5 store 5-8 for all Examples

Table 9.3a: Economic data and limits on crude transfer amounts

Ex	Flow Rate Limits (kbbl / 8-h) Parcel-to-Tank Min - Max	Flow Rate Limits (kbbl / 8-h) Tank-to-CDU Min - Max	Demurrage Cost (k$/h)	Change-over loss (k$)	Safe Inventory Penalty ($/bbl) for model: Discrete (per slot)	Safe Inventory Penalty ($/bbl) for model: Continuous (per period 1, 2, 3, 4, 5)
1	10 - 400	16 - 48	3.125	5	0.2	0.375, 0.5, 0.925, -, -
2	10 - 400	20 - 45	6.25	25	0.2	0.2, 1.0, 0.8, -, -
3	10 - 400	16 - 48	3.125	5	0.2	0.375, 0.625, 1.5, 0.625, 0.875

Table 9.3b: Crude types, key component concentrations and marginal profits

Crude Type	Key Component Concentrations (vol%) Exs 1 & 3	Key Component Concentrations (vol%) Ex 2	Marginal Profit ($/bbl) Exs 1-3
C1	0.20	0.25	1.50
C2	0.25	0.25	1.70
C3	0.15	0.40	1.50
C4	0.60	0.20	1.60
C5	1.20	1.00	1.45
C6	1.30	1.50	1.60
C7	0.90	1.40	1.55
C8	1.50	1.10	1.60

Table 9.4: Performances of the continuous and discrete models on the examples

Iteration	Discrete model (Case1)			Discrete model (Case2)			Continuous Model		
	CPU time (s)	Profit (k$)	%Relative Gap (Actual/Target)	CPU time (s)	Profit (k$)	%Relative Gap (Actual/Target)	CPU time (s)	Profit (k$)	%Relative Gap (Actual/Target)
Example 1 [Total CPU times 105 s, 49.2 s & 68.4 s respectively]									
1	102	1450.6	0/0	45.9	1450.0	0/0	60.9	1449.9	0/0
2	1.9	1449.2	0/0	3.0	1449.2	0/0	4.2	1449.1	0/0
3	0.4	1426.9	0/0	0.3	1409.3	0/0	2.4	1448.0	0/0
4	0.2	1409.3	0/0				0.8	1427.2	0/0
5							0.2	1409.3	0/0
Example 2 [Total CPU times 17591 s, 556 s & 815 s respectively]									
1	17093	1773.5	0.5/0.5	531	1809.1	0.5/0.5	665	1828.1	0.5/0.5
2	491.4	1767.0	0/0	16.4	1802.5	0/0	63.6	1826.4	0/0
3	3.5	1758.4	0/0	7.6	1799.1	0/0	55.8	1821.6	0/0
4	2.5	1755.3	0/0	0.4	1786.9	0/0	25.7	1814.8	0/0
5	0.8	1736.7	0/0	0.2	1771.7	0/0	3.6	1813.5	0/0
6	0.2	1723.4	0/0				0.4	1789.5	0/0
7							0.3	1766.4	0/0
Example 3 [Total CPU times 37909 s (4663 s), 3389 s & 3242 s respectively]									
1	33469 (223)	3306.8	1.0 (1.1) / 1.0 (2.0)	1663	3315.4	0.8/1.0	754	3316.2	0.68/1.0
2	948	3319.0	0.2/0.5	905	3323.4	0.38/0.5	789	3326.7	0.15/0.5
3	3164	3312.6	0.35/0.5	815	3299.3	0.21/0.5	373	3316.0	0.48/0.5
4	319	3294.0	0.25/0.5	4.2	3283.3	0.48/0.5	735	3324.7	0.05/0.5
5	4.8	3285.1	0.14/0.5	1.0	3255.3	0.25/0.5	577	3304.4	0.18/0.5
6	2.8	3265.6	0.48/0.5				4.6	3290.7	0.3/0.5
7	1.1	3229.3	0.39/0.5				5.2	3292.1	0.01/0.5
8							3.8	3257.0	0.28/0.5
9							0.8	3252.9	0.15/0.5

Case1 allows two parcel transfers per slot, while Case2 allows three. Statistics in () indicate the performance, if the 1st MILP is solved with a relative gap of 2%.

Table 9.5: Statistics of the continuous and discrete models for the examples

Ex	Discrete model			Continuous model		
	Single Variables	Binary Variables	Constraints	Single Variables	Binary Variables	Constraints
1	1235	136	2323	1358	115	2668
2	1395	162	2601	1778	160	3471
3	2759	301	5041	2893	242	5510

9.2 Example 1

This example illustrates a case with 72 h of scheduling horizon (nine 8-h slots in the discrete-time model) and one VLCC with three parcels. The VLCC arrives at 15 h into the horizon and the stipulated time for unloading without demurrage is 16 h in the logistics contract. For the continuous-time model, we allow 4 h extra for unloading and hence define three periods of lengths 15 h, 35 h and 32 h. We assign seven slots to period 1, five slots to period 2 and one slot to period 3. We use a target relative gap of 0% for all MILPs. Table 9.6 shows the operation schedule obtained from the proposed continuous-time model. The schedule has the following noteworthy features.

1) Two tanks feed a CDU in several slots, e.g. T1 and T8 feed CDU3 in slots 1-9. Similarly, one tank feeds two CDUs in several slots, as T4 feeds CDUs 1 and 2 in slots 1-9.
2) Though the key component concentrations and of feeds from T1 (0.00314) and T8 (0.0033) independently meet the feed quality requirement (max 0.004) for CDU3, the optimizer feeds a blend from T1 and T8 in slots 1-9 to achieve a profit higher than what T1 or T8 alone can achieve. Note that the margin for the crude in T1 is 1.586, that for T8 is 1.566 and that for the blend feed is 1.581 k$ per kbbl. Thus, T1 has a higher margin than the blend, but it has only 290 kbbl of crude supply. Exclusive use of T1 would require a changeover costing 5 k$ during the horizon,

because the asking rate for crude throughput is 300 kbbl. The optimizer makes up the shortfall of 10 kbbl by using some crude from T8 and avoids the changeover to achieve an increase in profit of 3.85 k$. This aptly illustrates (a) the trade-off between margin and changeover and (b) the judicious use of available resources to minimize changeovers.

In this example, sufficient ullage for crude receipt and adequate supply for crude are available. Thus, no changeover takes place and other features of our model such as the brine-settling time, though inherently taken care of, are absent in the schedule, as tanks that receive crude never deliver.

To compare our continuous model with the discrete model, we obtain two solutions using the latter. In Case1, we allow two parcel-to-tank transfers in one slot, while three in Case2. Table 9.6 also gives the schedule from the discrete model for Case2. Apart from the timing differences that arise due to the different arrival times (Table 9.1) in the two models, the schedules from the continuous and discrete models are essentially the same. As expected, Case1 assigns 24 h for crude unloading, while Case2 assigns 16 h. Thus, in terms of time utilization, Case2 produces a schedule closer to the one from the continuous model. All schedules have the same profit, because the receiving tanks are not required to feed CDUs. However, the discrete model for Case2 seems 28% faster than the continuous model (Table 9.6). Note (Table 9.4) that Case1 solves significantly slower than Case2. In addition, the continuous model needs five iterations as compared to three for Case2. However, this does not affect the profit, because the tanks whose compositions change do not feed CDUs.

Table 9.6: Operation schedules for Example 1 obtained from the continuous and discrete (Case2) models

Tank	Amounts [to CDU No.](from Vessel No) in kbbl for slots with end-times							
	Continuous model							
Tank	15 h	15.2 h	20.2 h	26.2 h	30 h	35 h	72 h	-
T1	-30[3]	-0.4[3]	-10[3]	-12[3]	-15.2[3]	-10[3]	-78.4[3]	-
T3					+190(4)			-
T4	-30[1]	-0.4[1]	-10[1]	-12[1]	-7.6[1]	-30[1]	-210[1]	-
	-30[2]	-0.4[2]	-10[2]	-12[2]	-7.6[2]	-30[2]	-210[2]	
T6		+10(1)	+250(2)					-
T7				+300(3)				-
T8	-30[3]	-0.4[3]	-10[3]	-12[3]	-7.6[3]	-10[3]	-74[3]	-
	Discrete model (Case2)							
Tank	8 h	16 h	24 h	32,40 h	48 h	56 h	64 h	72 h
T1	-16[3]	-16[3]	-16[3]	-16[3]	-16[3]	-16[3]	-16[3]	-28[3]
T3			+190(4)					
T4	-16[1]	-16[1]	-16[1]	-48[1]	-44[1]	-16[1]	-48[1]	-48[1]
	-16[2]	-16[2]	-16[2]	-48[2]	-44[2]	-16[2]	-48[2]	-48[2]
T6		+10(1)						
		+250(2)						
		+140(3)						
T7			+160(3)					
T8	-16[3]	-16[3]	-16[3]	-16[3]	-16[3]	-16[3]	-16[3]	-16[3]

'-' sign represents delivery to [CDU], '+' sign represents receipt from (parcel)

9.3 Example 2

In this example, we use a horizon of 80 h (ten 8-h slots in the discrete model) and one VLCC with three parcels. In this example, we consider a situation where the ullage is sparsely distributed among the tanks, parcel sizes are bigger and key component concentration limits on feeds to CDUs are tighter. The VLCC arrives at 8 h and the plant has 32 h to unload its parcels without incurring demurrage. We allow eight hours more for unloading in our continuous model and use three periods of lengths 8 h, 40 h and 32 h. We assign one slot to period 1, seven to period 2 and one to period 3. We target a relative gap of 0.5% for the first MILP and 0% for all others. Table 9.7 shows the operation schedule generated by the continuous model with the following salient features:

1) The model ensures the required time for brine settling/removal between crude receipt and delivery. For instance, T1 receives crude from the SBM parcel and the first parcel of the VLCC during slots 2 and 3 and then idles for brine removal during slots 4 and 5. Slots 4 and 5 have a combined length of 14.6 h, which exceeds the required settling time of 8 h. Similarly, T6, after receiving crude from parcel 2 in slot 4, allows 8 h of settling in slot 5 before feeding CDU3.
2) Although T2 with a key component concentration of 0.0108 can alone feed CDU2 (max 0.012), the optimizer feeds a blend of T2 and T4. This is because T4 has a higher margin, but T4 with a key component concentration of 0.01257 alone cannot meet the feed quality requirement of 0.012. By using a blend, the optimizer also avoids a changeover.
3) The schedule also exhibits other features mentioned earlier in Example 1 such as single tank (T4) feeding multiple CDUs (1 and 2), and two tanks (T1 and T6) feeding one CDU (CDU3).

We again compare the continuous-time solution with two solutions from the discrete model. As before, Case1 allows two parcel-to-tank transfers in one slot, while Case2 allows three. Table 9.7 describes the schedule for Case2. Again, Case2 schedule is closer to the continuous-time schedule, but it gives a greater profit (1771.7 vs. 1766.4 in Table 9.4). Case1 uses five slots for unloading, while Case2 uses only four. Case2 also solves 31.8% faster than the continuous model. Reasons for the latter's faster performance are multiple parcel transfers in a single slot. They help reduce the MILP iterations. By allowing three transfers per slot, the Case2 model has longer constant composition zones that allow the optimizer to select better blends and minimize changeovers. On the other hand, the continuous model uses shorter blocks of constant composition, which limits the blend adjustments and can increase changeovers. However, longer horizons mean more binary variables in the discrete model, making the problems more difficult, while the continuous model should require fewer slots. That should make the continuous model faster for the bigger problems, as the slots are of variable lengths. To demonstrate this, we use the next example.

9.4 Example 3

We use 160 h of horizon (Twenty 8-h slots in the discrete model) and two VLCCs with three parcels each. VLCC-1 arrives at 15 h and VLCC-2 at 100 h. The stipulated berthing time for both VLCCs is 20 h. Allowing 5 h extra for unloading, we define five periods with lengths 15 h, 25 h, 60 h, 25 h and 35 h. We assign one, five, two, five and two slots to periods 1 through 5 respectively. We use a target relative gap of 1% for the first MILP and 0.5% for the rest. Table 8 shows the operation schedule from the continuous model. Note that:

1) Though we used 15 slots, the optimizer needed only 14. Slot 8 in period 3 proved extra and its length is zero. Slots 7 and 8 have identical activities on tanks.

Table 9.7: Operation schedules for Example 2 obtained from the continuous and discrete (Case2) models

	Amounts [to CDU No.](from Vessel No) in kbbl for slots with end-times								
	Continuous model								
Tank	8 h	8.2 h	11.6 h	18.2 h	26.2 h	28.2 h	37 h	48 h	80 h
T1		+10(1)	+170(2)			-4[3]	-31.9[3]	-39.9[3]	-116[3]
T2	-16[2]	-0.4[2]	-6.8[2]	-13.2[2]	-16[2]	-4.0[2]	-17.6[2]	-22[2]	-64[2]
T4	-20[1]	-0.5[1]	-8.5[1]	-26.5[1]	-20[1]	-8.1[1]	-49.5[1]	-61.9[1]	-180[1]
	-16[2]	-0.4[2]	-6.8[2]	-13.2[2]	-16[2]	4.0[2]	-17.6[2]	-39.9[2]	-101.1[2]
T5							+440(4)		
T6				+330(2)		-7.25[3]	-17.6[3]	-22[3]	-64[3]
T7					+400(3)				
T8	-20[3]	-0.5[3]	-8.5[3]	-23.3[3]	-45[3]	+100(3)			
	Discrete model (Case2)								
Tank	8 h	16 h	24 h	32 h	40 h	48 h	56 h	64 h	72,80 h
T1				+220(3)					
T2					+10(4)				
T4	-20[1]	-20[1]	-20[1]	-45[1]	-45[1]	-45[1]	-45[1]	-45[1]	-45[1]
	-20[2]	-20[2]	-20[2]	-45[2]	-45[2]	-45[2]	-45[2]	-45[2]	-45[2]
T5				+50(4)	+380(4)				
T6		+10(1)		-45[3]	-45[3]	-45[3]	-45[3]	-45[3]	-45[3]
		+320(2)							
T7			+180(2)						
			+220(3)						
T8	-20[3]	-20[3]	-45[3]	+60[3]					

'-' sign represents delivery to [CDU], '+' sign represents receipt from (parcel)

Table 9.8: Operation schedules for Example 3 obtained from the continuous and discrete (Case2) models

Amounts [to CDU No.](from Vessel No) in kbbl for slots with end-times for the continuous model

Tank	15 h	15.2 h	20.2 h	26.2 h	30 h	40 h	100 h	100.2 h	105.2 h	110.2 h	115 h	125 h	154.8 h	160 h
T1	-30[3]	-0.4[3]	-10[3]	-12[3]	-7.6[3]	-20[3]					+240(8)			
T2	-30[2]	-0.4[2]	-10[2]	-12[2]	-7.6[2]	-20[2]	-200.4[2]	+10(5)	+250(6)	+250(7)			-59.6[2]	
T3					+190(4)			-0.4[2]	-10[2]	-20[2]	-9.6[2]	-20[2]		
T4	-30[1]	-0.4[1]	-10[1]	-12[1]	-7.6[1]	-20[1]	-260.8[1]	-0.4[1]	-30[1]	-30[1]	-28.8[1]	-60[1]	-178.8[1] -119.2[2]	-31.2[1] -20.8[2]
T5	-30[2]	-0.4[2]	-10[2]	-12[2]	-7.6[2]			-0.4[2]	-20[2]	-10[2]	-19.2[2]	-40[2]		-10.4[2]
T6		+10(1)		+300(3)			-162.5[3]	-0.57[3]						
T7			+250(2)				-178.3[3]	-0.62[3]						
T8									-10[3]	-10[3]	-9.6[3]	-60[3]	-178[3]	-10.4[3]

Amounts [to CDU No.](from Vessel No) in kbbl for slots with end-times for the discrete (Case2) model

Tank	8 h	16 h	24 h	32,40 h	48 h	56 h	64,72 h	80,88 h	96 h	104 h	112 h	120 h	128 h	136 h	144-160 h
T1								-16[3]	-16[3]	-16[3]	-16[3]	-16[3]	-29.7[3]	-32[3]	-32[3]
T2	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-32[2]	-16[2]
T3	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]	-16[2]		+160(7)			
T4	-16[1]	-16[1]	-16[1]	-48[1]	-48[1]	-44[1]	-16[1]	-16[1]	-48[1]	-48[1]	-16[1] -16[2]	-48[1] -16[2]	-48[1] -16[2]	-48[1]	-48[1] -28[2]
T5			+190(4)								+10(5) +250(6) +90(7)			-16[2]	
T6		+10(1) +250(2) +90(3)	+140(3)									-32[3]	-16[3]	-16[3]	-16[3]
T7			+70(3)									+240(8)			
T8	-16[3]	-16[3]	-16[3]	-48[3]	-46.3[3]	-16[3]	-16[3]	-16[3]	-32[3]	-16[3]	-16[3]				

'-' sign represents delivery to [CDU], '+' sign represents receipt from (parcel)

2. A tank may receive over several slots. For instance, T2 receives crude from parcels 5-7 during slots 9, 10 and 11. It then settles and removes brine during slots 12 and 13, before feeding CDU2 in slot 14.
3. The schedule also supports other features illustrated in the previous examples.

We use the same two cases for the discrete model as in the previous examples. Table 9.8 also presents the schedule for Case2. Case2 gives a slightly higher profit than the continuous model, but as expected, the latter is slightly faster than the former.

From Table 9.4, we see that the solution times decrease drastically for the discrete models after the first iteration for all three examples. However, the same is not true for the continuous model. In addition, the solution time decreases, as we allow more parcels to transfer in a slot in the discrete model. Another point to note is the huge difference in the solution times for the first iterations between the discrete and continuous models.

9.5 CONCLUSION

This work presents the first complete, continuous-time MILP model for short-term scheduling of crude operations of a refinery with an SBM pipeline. We used three examples to show the efficacy of model and solution algorithm to solve the difficult problem in reasonable time. In addition to accounting for real practices such as multiple tanks feeding one CDU, one tank feeding multiple CDUs, unloading via SBM pipeline, brine settling, etc., the proposed algorithm gives near-optimal schedules without solving a single NLP or MINLP as illustrated through three industrial problems. A direct head-to-head comparison between the two competing formulations (discrete, continuous) reveals that the proposed continuous-time approach should out do the discrete-time one for problems with longer horizons.

Chapter 10

CONCLUSIONS AND RECOMMENDATIONS

10.1 Conclusions

In part I of this work, a discrete-time MILP model that allows some tasks to begin even at intermediate points in a period and a novel solution algorithm that shows no composition discrepancy were developed for the scheduling of crude oil operations in a refinery. In addition to including several real features such as multiple tanks feeding one CDU, one tank feeding multiple CDUs, SBM pipeline, brine settling, tank-to-tank transfers etc., the proposed model uses fewer binary variables and is different from and superior (both in terms of efficiency and quality of solutions) to those reported in previous work. The proposed model considers configurationally different crude oil unloading facilities and uses different set of constraints for each facility. The main feature of our algorithm is that it solves the oil quality, transfer quantity, tank allocation and oil blending issues simultaneously without solving a single NLP or MINLP. The proposed approach helps quicker and near-optimal decision making in refinery operations and handles problems with up to 14 days. Longer horizon problems increase the number of binary decisions and problem size, making it highly compute intensive. Though some of continuous time features were incorporated in this model, we had to use increased optimality gap to get a quicker solution. Alternative methodology was to consider an inherent continuous time model to find a solution to such problems. Despite being the Holy Grail, continuous-time formulations are far behind their discrete counterparts in solving the complex problem of scheduling short-term crude operations in an oil refinery. In fact, none of the reported continuous-time formulations has succeeded in addressing the problem satisfactorily.

Part II of this work, presents the first complete, continuous-time MILP model for short-term scheduling of crude operations of a refinery with an SBM pipeline. Our novel model uses fewer binary variables than the existing ones and our solution algorithm eliminates composition discrepancy. In addition to accounting for real practices such as multiple tanks feeding one CDU, one tank feeding multiple CDUs, unloading via SBM pipeline, brine settling, etc., the proposed algorithm gives near-optimal schedules without solving a single NLP or MINLP as illustrated through three industrial size problems. A direct head-to-head comparison between the two competing formulations reveals that the proposed continuous-time approach should outdo the discrete-time one for problems with longer horizons. Although the latter currently appears to be better for smaller and more complex problems, we expect that, with future improvements, the superiority of the former would be clearly established. Other exact methods based on mathematical programming prove to be computationally very expensive even for small problems. From a comprehensive scheduling perspective, the new methodology developed shows promising results both in terms of efficiency and quality of solutions

Further work is also needed to solve the challenges of composition discrepancy, longer scheduling horizons, and more complex operational aspects, all with full optimality in reasonable time. To this end, we believe that this work is a significant step in the right direction.

10.2 Recommendations

The developed model and algorithm allow us to solve the issues of crude feed quality; crude transfer quantity and the tank allocations in one go but provide a near optimal solution for the crude scheduling problem. One needs to look into a different approach than that is presented here for finding a better solution or optimal solution. The developed approach imitates a roll forward approach and always corrects the schedule for any composition discrepancy and proceeds further. At present, we had concluded that the present approach produces a feasible solution if one exists based on evaluation of our approach on various examples. We did not thoroughly examine all the possible infeasibilities associated with our approach. Thus there is further scope and need to explore the infeasibilities (if any) associate with the present approach. Another area of improvement for this problem can be use of some better heuristics. Application of heuristics reduces the complexity of the problem and may facilitate a quicker and good feasible solution to the problem.

The developed model can be easily extended to incorporate the product properties, cut properties that are nonlinear in relationship but respective property indexes are linear in blending. The present formulation assumes that key component limits on crude feed determine the product pattern and quality. Using index relationships, we can easily extend the present formulation to adjust the crude feed based on the product qualities and quantities. Crude oil scheduling is first part of overall refinery operations scheduling. Developing the other two parts namely hydraulic scheduling, product blending & distribution scheduling, integrating all three parts and providing a user interface should be the last stage of extension to this problem.

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APPENDIX A

A.1: GAMS file - case-1 discrete time model for example-1 of part II

$TITLE Example 1 of Part2 discrete time modeling approach

OPTION solprint = off;

OPTION MIP = cplex;

OPTION limrow = 0;

OPTION limcol = 0;

OPTION sysout = off;

* Sets definition.

SETS

C	different types of crudes	/1*8/
I	crude oil storage tank	/1*8/
K	key component of crude oil	/1/
U	crude distillation units	/1*3/
T	time interval	/1*9/
V	crude vessel	/1/
P	compartments	/1*4/
RCC(i,c)	RCC crude types	/1.(1*4), (6*8).(1*4)/
NRCC(i,c)	NRCC crude types	/(2*5).(5*8)/
FI(i,t)	tank and period combination for which composition is constant and known	/1.1*2, 2.1*3, 3.1*3,4.1*3, 5.1*3, 6.1*2, 7.1*2, 8.1*2/
PT(p,t)	feasible unloading time for vessel v	/1.(2), 2.(2*3), 3.(3), 4.(3*5)/
PIT(p,i,t)	feasible time period for vessel v to unload to storage tank i this is combination of sets PI,PT	

/1.1.2, 1.6*8.2, 2.1.2*3, 2.6*8.2*3, 3.1.3, 3.6*8.3, 4.(2*5).(3*5) /

VP(v,p)	oil types that vessel carries	/1.(1*4)/
FP(v,p)	set of first containers of crude vessel v	/1.1/
LP(v,p)	set of last containers of crude vessel v	/1.4/
RCP(v,p)	last RCC parcel and vessel combination	/1.3/
NRP(v,p)	last NRCC parcel and vessel combination	/1.4/
IC(i,c)	oil types in storage tanks	/1.1*4, 2.5*8, 3.5*8, 4.5*8, 5.5*8, 6.1*4, 7.1*4, 8.1*4/
IU(i,u)	probable Storage tanks that can be given to CDUs	/1.3, 2.1*2, 3.1*2, 4.1*2, 5.(1*2), 6.3, 7.3, 8.3/
UC(u,c)	crudes that CDU u can process	/1.(5*8), 2.(5*8), 3.(1*4)/
PC(p,c)	crudes that are being carried by containers	/1.2, 2.3, 3.4, 4.5/
PI(p,i)	feasible storage tank i for vessel v	/1.(1),1.(6*8), 2.(1), 2.(6*8), 3.1, 3.(6*8), 4.(2*5)/

Alias(t,tt);

Alias (i,ii);

* Parameters definition

PARAMETERS

C_prof(c)	profit margin on crude processing k$per kbbl	/1 1.5, 2 1.7, 3 1.5, 4 1.6, 5 1.45,6 1.6, 7 1.55, 8 1.6/
C_SEA	sea waiting cost k$of vessel v per unit time interval	/25/
C_SET	change over cost for crude mix transition from storage tank i to CDU l	/5/
PEN	penalty k$ per kbbl for safety stock per period	/0.2/
DM(u)	demand of crude mix j by cdus during the schedule horizon	/1 300, 2 300, 3 300/
DMMIN(u)	minimum demand of crude oil by cdu l each day	/1 16, 2 16, 3 16/

DMMAX(u) maximum demand of crude oil by cdu l each day /1 48, 2 48, 3 48/

EV(p,c) concentration of crude oil c in vessel v /1.2 1, 2.3 1, 3.4 1, 4.5 1/

EC(c, k) /1.1 0.002, 2.1 0.0025, 3.1 0.0015, 4.1 0.006, 5.1 0.012, 6.1 0.013, 7.1 0.009, 8.1 0.015/

ELMIN(u,k) lower limit of key component on processing units /1.1 0.001, 2.1 0.001, 3.1 0.001/

ELMAX(u,k) upper limit of key component on processing units /1.1 0.014, 2.1 0.0130, 3.1 0.004/

ESMIN(i,c) minimum concentration of crude oil c in storage tank i

/ 1.1 0, 1.2 0, 1.3 0, 1.4 0, 2.5 0.0, 2.6 0, 2.7 0, 2.8 0, 3.5 0, 3.6 0, 3.7 0, 3.8 0,

4.5 0, 4.6 0, 4.7 0, 4.8 0, 5.5 0, 5.6 0, 5.7 0, 5.8 0, 6.1 0, 6.2 0, 6.3 0, 6.4 0,

7.1 0, 7.2 0, 7.3 0, 7.4 0, 8.1 0, 8.2 0, 8.3 0, 8.4 0/

ESMAX(i,c) maximum concentration of crude oil c in storage tank i

/1.1 1, 1.2 1, 1.3 1, 1.4 1, 2.5 1, 2.6 1, 2.7 1, 2.8 1, 3.5 1, 3.6 1, 3.7 1, 3.8 1,

4.5 1, 4.6 1, 4.7 1, 4.8 1, 5.5 1, 5.6 1, 5.7 1, 5.8 1, 6.1 1, 6.2 1, 6.3 1, 6.4 1,

7.1 1, 7.2 1, 7.3 1, 7.4 1, 8.1 1, 8.2 1, 8.3 1, 8.4 1/

FPTMIN minimum crude oil transfer rate from parcel p to storage tank i /10.0/

FPTMAX minimum crude oil transfer rate from parcel p to storage tank i /400/

FTUMIN minimum crude oil transfer rate from tank i to CDU u /16/

FTUMAX maximum crude oil transfer rate from tank i to CDU u /48/

TARR(V) the arrival time of vessel v /1 1/

VV0(p) initial volume of crude oil in crude vessel v /1 10, 2 250.0, 3 300.0, 4 190.0/

SS safety stock penalty in any period /1500/

VSMIN(i) minimum crude oil volume of storage tank i /1 60.0, 2 60.0, 3 60.0, 4 110.0, 5 110.0, 6 60.0,7 60.0, 8 60.0/

VSMAX(i) maximum crude oil volume of storage tank i /1 570.0, 2 570.0, 3 570.0, 4 980.0, 5 980.0, 6 570.00, 7 570.00, 8 570.00/

VS0(i) initial crude oil volume in storage tank i /1 350, 2 400.0, 3 350.000, 4 950.0, 5 300.0, 6 80.0, 7 80, 8 450.0/

VSC0(i,c) initial volume of crude oil c in storage tank I
/1.1 50, 1.2 100, 1.3100, 1.4 100, 2.5 100, 2.6 100, 2.7 100, 2.8 100, 3.5 100, 3.6 100,

3.7 50, 3.8 100, 4.5 200, 4.6 250, 4.7 200, 4.8 300, 5.5 100, 5.6 100, 5.7 50, 5.8 50,

6.1 20, 6.2 20, 6.3 20, 6.4 20,7.1 20, 7.2 20, 7.3 20, 7.4 20,8.1 100, 8.2 100, 8.3 100,

8.4 150/

TMIN(v) Expected departure time of vessel v /1 3/;

Table ESCMIN(c,u) details what amount in percentage crude c type can be processed in a CDU u

	1	2	3
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0
7	0	0	0
8	0	0	0

;

Table ESCMAX(c,u) details what amount in percentage crude c type can be processed in a CDU u

	1	2	3
1	0	0	1

2	0	0	1
3	0	0	1
4	0	0	1
5	1	1	0
6	1	1	0
7	1	1	0
8	1	1	0

;

* Variable definition

Variables

XF	XF(p t) A binary variable to denote if vessel v starts unloading at time t
XL	XL(p t) a binary variable to denote if vessel v completes unloading at t
XI	XI(i t) a binary variable used for tank connections for unloading at t
XP	XP(p t) a binary variable used for parcel connections at t
CD	CD(i u t) crude oil mix in storage tank i charges CDU u at time t
FTU	FTU(i u t) volumetric flow rate of crude oil from storage tank i to CDU u at time t
FCTU	FCTU(i u c t) volumetric flow rate of crude oil c from storage tank i to CDU u at time t
FPT	FPT(p i t)volumetric flow rate of crude oil from parcel p to storage tank i at time t
PROFIT	total profit of the refinery during the scheduling horizon
TF	TF(v) vessel v unloading initiation time
TL	TL(v) vessel v unloading completion time
TFF	TFF(p) container p start unloading time
TLL	TLL(p) parcel p unloading completion time

FKTU	FKTU(i u k t) component k transfer rate from tank i to cdu u at t
VS	VS(i t) volume of crude oil in storage tank i at time t
VCT	VCT(i c t) volume of crude oil c in storage tank i at time t
Z	Z(i u t) 0-1 a continuous variable to denote transition of storage tanks that charge CDU l at time t
XPI	XPI(p i t) 0-1 continuous variable to denote unloading from container p to storage tank i at time t
CW	waiting period

Binary variables

XP

XI

CD;

Positive variables

XF, XL, FTU, FCTU, FPT, FU, TF, TFF, TL, TLL, FKTU, VS, VCT, SSP, Z, XPI,

CW;

Equations

unl1(p)

unl221(p,t)

unl2(p)

unl4(p)

unl5(p)

unl6(p)

unl61(P)

modl2(p,t)

mod3(t)

mod4(t)

unp1(p,i,t)

unp23(i,u,t)

unp21(t)

extra1(p,t)

extra2(i,t)

meb21(p,i,t)

meb22(p,i,t)

meb221(t)

meb3(p)

meb7(i,u,t)

dem11(u,T)

dem1(u)

com1(i,c,t)

com11(i,c,t)

com2(u,c,t)

com3(u,c,t)

com33(i,u,t)

com34(i,u,t)

com4(i,c,t)

com5(i,c,t)

com6(i,t)

com9(i,u,c,t)

comk2(u,k,t)

```
comk3(u,k,t)

char1(i,t)

char2(u,t)

chg21(i,u,T)

chg22(i,u,T)

extra4(t)

extra5(v,p)

OBJ2          ;

*Constraints

unl1(p)..sum(t$PT(p,t),XF(p,t))=e=1;

unl221(p,t)$PT(p,t)..XP(p,t)=e=XP(p,t-1)+XF(p,t)-XL(p,t-1);

unl2(p)..sum(t$PT(p,t),XL(p,t))=e=1;

unl4(p)..TFF(p)=e=sum(t$PT(p,t),(ord(t))*XF(p,t))-1;

unl5(p)..TLL(p)=e=sum(t$PT(p,t),ord(t)*XL(p,t));

unl6(p)$(ord(p) lt card(p))..TFF(p+1)=g=TLL(p)-1;

unl61(P)..TLL(p)=g=TFF(p)+1;

modl2(p,t)$PT(p,t)..XP(p,t)=g=XL(p,t);

mod3(t)..sum(p$PT(p,t),XP(p,t))=l=2;

mod4(t)..sum(i,XI(i,t))=l=2;

extra1(p,t)$PT(p,t)..sum(i$PIT(p,i,t),XPI(p,i,t))=l=2*XP(p,t);

extra2(i,t)..sum(p$PIT(p,i,t),XPI(p,i,t))=l=2*XI(i,t);

unp1(p,i,t)$PIT(p,i,t)..XPI(p,i,t)=g=XP(p,t)+XI(i,t)-1;

unp21(t)..sum((p,i)$PIT(p,i,t),XPI(p,i,t))=l=2;

unp23(i,u,t)$IU(I,u)..2*XI(i,t)+CD(i,u,t)+CD(i,u,t+1)=l=2;
```

```
meb21(p,i,t)$(PIT(p,i,t))..FPTMIN*XPI(p,i,t)=l=FPT(p,i,t);

meb22(p,i,t)$(PIT(p,i,t))..FPT(p,i,t)=l=FPTMAX*XPI(p,i,t);

meb221(t)..sum((p,i)$PIT(p,i,t),FPT(p,i,t))/FPTMAX =l=1;

meb3(p)..sum((i,t)$(PIT(p,i,t)),FPT(p,i,t))=e=VV0(p);

meb7(i,u,t)$(IU(i,u))..FTU(i,u,t)=e=sum(c$(IC(i,c)and
             UC(u,c)),FCTU(i,u,c,t));

dem11(u,t)..FU(u,t)=e=sum(i$IU(i,u),FTU(i,u,t));

dem1(u)..sum(t,FU(u,t))=e=DM(u);

com1(i,c,t)$(IC(i,c) and (ord(t) gt 1))..VCT(i,c,t)=E=VCT(i,c,t-1)+SUM(p$(PC(p,c)
             and PIT(p,i,t)),FPT(p,i,t)*EV(p,c))-SUM(u$IU(i,u),FCTU(i,u,c,t));

com11(i,c,t)$(IC(i,c) and (ord(t) eq 1))..VCT(i,c,t)=e=VSC0(i,c)+SUM(p$(PC(p,c)
             and PIT(p,i,t)),FPT(p,i,t)*EV(p,c))-SUM(u$IU(i,u),FCTU(i,u,c,t));

com2(u,c,t)..FU(u,t)*ESCMIN(c,u)=l=sum(i$(IU(i,u) and IC(i,c)),FCTU(i,u,c,t));

com3(u,c,t)..sum(i$(IU(i,u) and IC(i,c)),FCTU(i,u,c,t))=l=FU(u,t)*ESCMAX(c,u);

com33(i,u,t)$(IU(i,u))..FTU(i,u,t)=L=FTUMAX*CD(i,u,t);

com34(i,u,t)$(IU(i,u))..FTUMIN*CD(i,u,t)=L=FTU(i,u,t);

com4(i,c,t)$(IC(i,c))..VS(i,t)*ESMIN(i,c)=l=VCT(i,c,t);

com5(i,c,t)$(IC(i,c))..VCT(i,c,t)=l=VS(i,t)*ESMAX(i,c);

com6(i,t)..sum(c$IC(i,c), VCT(i,c,t))=e=VS(i,t);

com9(i,u,c,t)$(IU(i,u) and IC(i,c)and
             FI(i,t))..FCTU(i,u,c,t)*VS0(i)=e=VSC0(i,c)*FTU(i,u,t);

comk2(u,k,t)..ELMIN(u,k)*sum(i$IU(i,u),FTU(i,u,t))=l=sum((i,c)$(IU(i,u) and IC(i,c)
             and UC(u,c)),FCTU(i,u,c,t)*EC(c,k));

comk3(u,k,t)..sum((i,c)$(IU(i,u) and IC(i,c) andUC(u,c)),FCTU(i,u,c,t)*EC(c,k))
             =l=ELMAX(u,k)*SUM(i$IU(i,u),FTU(i,u,t));

char1(i,t)..sum(u$IU(i,u),CD(i,u,t))=l=2;

char2(u,t)..sum(i$IU(i,u),CD(i,u,t))=l=2;
```

```
chg22(i,u,t)$(IU(i,u)and (ord(t) lt card(t)))..Z(u,t)=g=CD(i,u,t+1)-CD(i,u,t);

chg21(i,u,t)$(IU(i,u)and (ord(t) lt card(t)))..Z(u,t)=g=CD(i,u,t)-CD(i,u,t+1);

extra4(t)..SSP(t)=g=SS-sum(i,VS(i,t));

extra5(v,p)$LP(v,p)..CW(v)=g=(TLL(p)-TARR(v)-TMIN(v))*C_sea;

OBJ2..PROFIT=e=SUM((i,u,c,t)$(IC(i,c)and IU(i,u)),C_PROf(c)*FCTU(i,u,c,t))-
          SUM(v,CW(v))-SUM(t,PEN*SSP(t))-SUM((u,t)$(ord(t) lt card(t)),
          C_SET*Z(u,T));

*Bounds that can be incorporated

SSP.lo(t)=0;

SSP.up(t)=SS;

XPI.lo(p,i,t)=0;

XPI.up(p,i,t)=1;

VS.lo(i,t)=VSmin(i);

VS.up(i,t)=VSmax(i);

FU.lo(u,t)=DMMIN(u);

FU.up(u,t)=DMMAX(u);

Z.lo(u,t)=0;

Z.up(u,t)=1;

* Variable values that are knows and are fixed

VS.fx(i,'0')=VS0(i);

VCT.fx(i,c,'0')=VSC0(i,c);

XP.fx(p,'0')=0;

XP.fx('1','1')=0;

XP.fx('2','1')=0;

XP.fx('3','2')=0;
```

```
XP.fx('4','2')=0;
XI.fx(i,'6')=0;
XI.fx(i,'7')=0;
XI.fx(i,'8')=0;
XI.fx(i,'9')=0;
XI.fx(i,'10')=0;
XI.fx(i,'1')=0;
XI.fx('1','4')=0;
XI.fx('6','4')=0;
XI.fx('7','4')=0;
XI.fx('1','5')=0;
XI.fx('6','5')=0;
XI.fx('7','5')=0;
XI.fx('8','5')=0;
XI.fx('2','2')=0;
XI.fx('3','2')=0;
XI.fx('4','2')=0;
XI.fx('5','2')=0;
* Model definition
Model ResearchProblem /all/;
ResearchProblem.optca=0.01;
ResearchProblem.optcr=0.00;
ResearchProblem.iterlim = 10000000;
ResearchProblem.reslim = 1000000;
```

```
Solve ResearchProblem maximizing profit using MIP;
```

Parameter

```
f(i,c,t)

r

NE

NE1

NET

NET1

Next

Next1   ;

NE=0;

NE1=0;

NET=0;

NET1=0;

Next=0;

Next1=0;

r=3;
```

Equation

```
compiter(i,u,c,t)

compiter2(i,u,c,t)

compiter3(i,u,c,t)  ;

compiter(i,u,c,t)$(IU(i,u) and IC(i,c) and (ord(t)le r)) .. FCTU(i,u,c,t) =e=
            f(i,c,t)*FTU(i,u,t);

compiter2(i,u,c,t)$(IU(i,u) and RCC(i,c) and  (ord(t)ge NE) and (ord(t)le NE1))..
            FCTU(i,u,c,t)=e=f(i,c,t)*FTU(i,u,t);
```

```
compiter3(i,u,c,t)$(IU(i,u) and NRCC(i,c) and  (ord(t)ge NET) and (ord(t)le NET1))..
                FCTU(i,u,c,t)=e=f(i,c,t)*FTU(i,u,t);

Model sai /
unl1, unl2, unl4, unl5, unl6, unl61, modl2, mod3, mod4, extra1, extra2, unl221, unp1,

unp21, unp23, meb21, meb22, meb221, meb3, dem11, dem1, com1, com11, com2,

com3, com33, com34, meb7, com4, com5, com6, com9, comk2, comk3, char1, char2,

chg22, chg21, extra4, extra5, OBJ2, compiter, compiter2, compiter3 /;

Sai.optca=0.00;

Sai.optcr=0.000;

Sai.iterlim = 10000000;

Sai.reslim= 1000000;

* Iterations for correcting the composition

while ((r le card(t)),

loop(i,

loop(c$IC(i,c),

loop(t$(FI(i,t)),

f(i,c,t)=VSC0(i,c)/VS0(i););););

loop(v$(ord(v) eq card(v)),

loop(p$RCP(v,p),

loop(t$(ord(t) eq r-1),

if((XL.l(p,t) eq 1),

NE=r;

NE1=card(t););););););

loop(i,

loop(c$RCC(i,c),
```

```
loop(tt$(ord(tt) eq NE),

loop(t$((ord(t) ge NE) and (ord(t) le NE1)),

f(i,c,t)=VCT.l(i,c,tt-1)/VS.l(i,tt-1); ););););

loop(v$(ord(v) eq card(v)),

loop(p$NRP(v,p),

loop(t$(ord(t) eq r-1),

if((XL.l(p,t) eq 1),

NET=r;

NET1=card(t);););););

loop(i,

loop(c$NRCC(i,c),

loop(tt$(ord(tt) eq Net),

loop(t$((ord(t) ge NET) and (ord(t) le NET1)),

f(i,c,t)=vct.l(i,c,tt-1)/vs.l(i,tt-1);););););

loop(i,

loop(c$IC(i,c),

loop(t$((ord(t)le r)and (ord(t) gt 1)),

f(i,c,t)=VCT.l(i,c,t-1)/VS.l(i,t-1););););

loop(v$(ord(v) eq card(v)),

loop(p$LP(v,p),

loop(t$(ord(t)eq r-1),

if((xl.l(p,t) eq 1),

Next=r;

Next1=card(t);););););
```

```
loop(u,

loop(i$(IU(i,u)),

loop(c$IC(i,c),

loop(t$(ord(t)le r-1),

FTU.fx(i,u,t)=FTU.l(i,u,t);

CD.fx(i,u,t)=CD.l(i,u,t);

VS.fx(i,t)=VS.l(i,t);

FCTU.fx(i,u,c,t)=FCTU.l(i,u,c,t);

VCT.fx(i,c,t)=VCT.l(i,c,t); ););););)

loop(p,

loop(i$PI(p,i),

loop(t$(ord(t)le r-1),

XPI.fx(p,i,t)=XPI.l(p,i,t);

XP.fx(p,t)=XP.l(p,t);

XI.fx(i,t)=XI.l(i,t);

XL.fx(p,t)=XL.l(p,t);

XF.fx(p,t)=XF.l(p,t);

FPT.fx(p,i,t)=FPT.l(p,i,t););););

* Adjusting the optimality gap as the problem size decreases

if(r gt 2,

Sai.optcr=0.0;);

Solve Sai maximizing profit using MIP;

Display XF.l, XL.l, TFF.l, TLL.l, XP.l, XI.l, XPI.L, CD.l, Z.l, FPT.l, FTU.l, FCTU.l,
        VS.l, VCT.l, PROFIT.L, NE, NE1, NET, NET1;

* Incase if r is reached a criteria to end of horizon then assign it as end else increment
```

```
if((r eq Next),

  r=Next1 ;);

  r=r+1; );

Display XF.L, XL.L, XP.l, XI.l, XPI.L, CD.l, Z.l, FPT.l, FTU.l, FCTU.l, VS.l, VCT.l,
        PROFIT.L, NE, NE1, NET, NET1;
```

A.2: GAMS file – continuous time model for example-1 of part II

```
$TITLE Example 1 of Part2 continuous time modeling approach

OPTION solprint    =off;
OPTION MIP         =cplex;
OPTION limrow      = 0;
OPTION limcol      = 0;
OPTION sysout      =off;

* Set definitions.

Sets
C           different types of crudes                                /1*8/
I           crude oil storage tank                                   /1*8/
K           key component of crude oil                               /1/
U           crude distillation units                                 /1*3/
T           time interval                                            /1*3/
V           crude vessel                                             /1/
P           compartments                                             /1*4/
S           slots                                                    /1*7/
RCC(i,c)    RCC crude types                        /1.(1*4), (6*8).(1*4)/
NRCC(i,c)   NRCC crude types                                /(2*5).(5*8)/
CST(t,s)    combination of slots and time periods   /1.(1), 2.(2*6), 3.(7)/
CPS(p,s)    combination of slots and parcels in which the parcel can be
            possibly get unloaded          /1.(2), 2.(3*4),3.(4*5), 4.(5*6)/
VP(v,p)     oil types that vessel carries                            /1.(1*4)/
```

FP(v,p)	set of first containers of crude vessel v	/1.1/
LP(v,p)	set of last containers of crude vessel v	/1.4/
RCP(v,p)	last RCC parcel and vessel combination	/1.3/
NRP(v,p)	last NRCC parcel and vessel combination	/1.4/
RP(v,p)	RCC parcels and vessel combinations	/1.(1*3)/
NP(v,p)	NRCC parcels and vessel combinations	/1.(4)/
RFP(v,p)	First RCC parcel and vessel combinations	/1.1/
NFP(v,p)	First NRCC parcels and vessel combinations	/1.4/
IC(i,c)	oil types in storage tanks	/1.1*4, 2.5*8, 3.5*8, 4.5*8, 5.5*8, 6.1*4, 7.1*4, 8.1*4/
IU(i,u)	probable Storage tanks that can be given to CDUs	/1.3, 2.1*2, 3.1*2, 4.1*2, 5.(1*2), 6.3, 7.3, 8.3/
UC(u,c)	crudes that CDU u can process	/1.(5*8), 2.(5*8), 3.(1*4)/
PC(p,c)	crudes that are being carried by containers	/1.2, 2.3, 3.4, 4.5/
PI(p,i)	feasible storage tank i for vessel v	/1.(1),1.(6*8), 2.(1), 2.(6*8), 3.1, 3.(6*8), 4.(2*5)/
Last(t,s)	last slot in period t	/1.1, 2.6, 3.7/
VLP(p)	vessel last parcel	/4/
FI(i,s)	constant composition slots	/1.(1*2), (2*5).(1*5), (6*8).(1*2)/

Alias(s,ss);

Alias(s,sss);

Alias(i,ii);

Alias(p,pp);

CSS(s,ss)	slot combinations for settling time	/2.(3*4), 3.(4*5), 4.(5*6), 5.(6*7)/
RP1(v,pp)	RCC parcels and vessel combinations	/1.(1*3)/

NP1(v,pp)	NRCC parcels and vessel combinations	/1.(4)/
RFP1(v,pp)	First RCC parcel and vessel combinations	/1.1/
NFP1(v,pp)	First NRCC parcels and vessel combinations	/1.4/
IU1(ii,u)	Probable Storage tanks that can be given to CDUs	/1.3, 2.1*2, 3.1*2, 4.1*2, 5.(1*2), 6.3, 7.3, 8.3/
IC1(ii,c)	oil types in storage tanks	/1.1*4, 2.5*8, 3.5*8, 4.5*8, 5.5*8, 6.1*4, 7.1*4, 8.1*4/
CST1(t,ss)	combination of slots and time periods	/1.(1), 2.(2*6), 3.(7)/
CST2(t,sss)	combination of slots and time periods	/1.(1), 2.(2*6), 3.(7)/
CPS2(pp,ss)	combination of slots and parcels in which the parcel can be possibly get unloaded	/1.(2), 2.(3*4),3.(4*5), 4.(5*6)/

Parameters

C_prof(c)	margin profits of crude c	/1 1.5, 2 1.7, 3 1.5, 4 1.6, 5 1.45, 6 1.6, 7 1.55, 8 1.6/
C_SEA	sea waiting cost of vessel v per unit time interval	/3.125/
C_SET	change over cost for crude mix transition toCDU u	/5/
PEN(t)	penalty for safety stock per period	/1 0.375, 2 0.5, 3 0.925/
DM(u)	demand of crude mix j by cdus during the schedule horizon	/1 300, 2 300, 3 300/
DMMIN(u)	minimum demand of crude oil by CDU u each hr	/1 2, 2 2, 3 2/
DMMAX(u)	maximum demand of crude oil by CDU u each hr	/1 6, 2 6, 3 6/
EV(p,c)	concentration of crude oil c in vessel v	/1.2 1, 2.3 1, 3.4 1, 4.5 1/
EC(c,k)	key component in crude	/1.1 0.002, 2.1 0.0025, 3.1 0.0015, 4.1 0.006, 5.1 0.012, 6.1 0.013, 7.1 0.009, 8.1 0.015/
ELMIN(u,k)	Limits on key components in CDU u feed	/1.1 0.001, 2.1 0.001, 3.1 0.001/
ELMAX(u,K)		/1.1 0.014, 2.1 0.0130, 3.1 0.004/

ESMIN(I,C) minimum concentration of crude oil c in storage tank i

/ 1.1 0, 1.2 0, 1.3 0, 1.4 0, 2.5 0.0, 2.6 0, 2.7 0, 2.8 0, 3.5 0, 3.6 0, 3.7 0, 3.8 0,

4.5 0, 4.6 0, 4.7 0, 4.8 0, 5.5 0, 5.6 0, 5.7 0, 5.8 0, 6.1 0, 6.2 0, 6.3 0, 6.4 0,

7.1 0, 7.2 0, 7.3 0, 7.4 0, 8.1 0, 8.2 0, 8.3 0, 8.4 0/

ESMAX(I,C) maximum concentration of crude oil c in storage tank i

/1.1 1, 1.2 1, 1.3 1, 1.4 1, 2.5 1, 2.6 1, 2.7 1, 2.8 1, 3.5 1, 3.6 1, 3.7 1, 3.8 1,

4.5 1, 4.6 1, 4.7 1, 4.8 1, 5.5 1, 5.6 1, 5.7 1, 5.8 1, 6.1 1, 6.2 1, 6.3 1, 6.4 1,

7.1 1, 7.2 1, 7.3 1, 7.4 1, 8.1 1, 8.2 1, 8.3 1, 8.4 1/

FPTMAX Transfer rate of crude from parcel /50/

FTUMIN minimum crude oil transfer rate from tank i to CDU u /2/

FTUMAX maximum crude oil transfer rate from tank i to CDU u /6/

TARR(V) the arrival time hr of vessel v /1 15/

ETA(p) parcel arrival time /1 15, 2 15, 3 15, 4 15/

D(t) event points /1 15,2 35, 3 72/

DD(t) length of time period /1 15, 2 20, 3 37/

VV0(p) initial volume of crude oil in crude vessel v /1 10, 2 250.0, 3 300.0, 4 190.0/

VSMIN(I) minimum crude oil volume of storage tank i /1 60, 2 60, 3 60, 4 110, 5 110, 6 60, 7 60, 8 60/

VSMAX(I) maximum crude oil volume of storage tank i /1 570, 2 570, 3 570, 4 980, 5 980, 6 570, 7 570, 8 570/

VS0(I) initial crude oil volume in storage tank i /1 350, 2 400.0, 3 350, 4 950.0, 5 300.0, 6 80.0, 7 80, 8 450.0/

SAS(t) safety stock end of period /1 1500, 2 1500, 3 1500/

VSC0(I,C) initial volume of crude oil c in storage tank i

/1.1 50, 1.2 100, 1.3 100, 1.4 100, 2.5 100, 2.6 100, 2.7 100, 2.8 100, 3.5 100, 3.6 100,

3.7 50, 3.8 100, 4.5 200, 4.6 250, 4.7 200, 4.8 300, 5.5 100, 5.6 100, 5.7 50, 5.8 50,

6.1 20, 6.2 20, 6.3 20, 6.4 20, 7.1 20, 7.2 20, 7.3 20, 7.4 20,8.1 100, 8.2 100, 8.3 100, 8.4 150/

TMIN(v) allowable demurrage /1 18/

Tset settling time /8/

f(i,c,s) crude c composition of tank I at the end of slot s;

f(i,c,'1')=VSC0(i,c)/VS0(i);

Table ESCMIN(c,u)

	1	2	3
1	0	0	0
2	0	0	0
3	0	0	0
4	0	0	0
5	0	0	0
6	0	0	0
7	0	0	0
8	0	0	0

Table ESCMAX(c,u)

	1	2	3
1	0	0	1
2	0	0	1
3	0	0	1
4	0	0	1
5	1	1	0

6	1	1	0
7	1	1	0
8	1	1	0;

Variables

YT	YT(i s)	tank i delivering in slot s
XT	XT(i s)	tank i connected to SBM for receipt
XP	XP(p s)	parcel p is connected to SBM line for unloading
ZT	ZT(i s)	tank is either idle or settling in slot s
Y	Y(i u s)	tank i is delivering to CDU u in slot s
RLP	RLP(p s)	time length for which parcel p is connected to SBM line in slot s
RLT	RLT(i s)	time length for which tank i is connected to SBM line in slot s
RLU	RLU(i s)	time length for which tank i is connected to CDU in slot s
RLZ	RLZ(i s)	time length for which tank i is idle settling in slot s
RRP	RRP(p i s)	time required to transfer crude from parcel p to tank i in slot s
RU	RU(i u s)	time required to transfer crude from tank i to CDU u in slot s
XF	XF(p s)	parcel p first connected to SBM in slot s for unloading
XL	XL(p s)	parcel p disconnected to SBM in slot s after unloading
ETU	ETU(p)	end time of parcel p unloading
TF	TF(s)	start time of slot s
TL	TL(s)	end time of slot s
SL	SL(s)	slot length
CO	CO(u s)	change over occurrence on CDU u in slot s
DC	DC(p)	variable to calculate demurrage charge
SC	SC(s)	variable that calculates penalty for not meeting safety inventory

FTU FTU(i u s) volumetric flow rate of crude oil from storage tank i to CDU u at time s

FU FU(u s) feed to CDU u in slot s

FCTU FCTU(i u c s) volumetric flow rate of crude oil c from storage tank i to CDU u at time s

FPT FPT(p i s) volumetric flow rate of crude oil from parcel p to storage tank i at time s

PROFIT total profit of the refinery during the scheduling horizon

VS VS(i,s) volume of crude oil in storage tank i at time s

VCT VCT(i c s) volume of crude oil c in storage tank i at time s;

Binary variables

Y, XP, XT;

Positive variables

YT, ZT, RLP, RLT, RLU, RLZ, RRP, RU, XF, XL, ETU, TF, TL, SL, CO, DC, SC, FTU, FU, FCTU, FPT, VS, VCT;

Equations

unl1(p,s)

unl4(p,s)

unl5(p)

unl6(p)

unl7(p,pp,s)

unl8(s)

*SBM to Tank connection

unl9(s)

*Tank to CDU connection

unl10(i,s)

unl11(u,s)

*Activity assignment

act1(i,u,s)

act2(i,s)

act3(i,s)

*Activity time accounting

act4(s)

act5(s)

act6(p,s,t)

act7(s)

act8(i,s,t)

act9(s)

act10(s)

act11(i,s,t)

act12(i,s,t)

act13(i,s)

* Crude transfer from parcel to tank

ulp3(p,i,s)

ulp4(i,s)

ulp5(p,s)

ulp6(t)

ulp7(p,i,s)

ulp8(p)

*Crude transfer from tank to CDUs

chr1(i,u,s,t)

chr2(i,u,s,t)

chr3(i,u,s)

chr4(i,u,s)

chr5(i,u,s)

chr6(i,u,s)

chr7(u,s)

chr8(u,s)

chr9(u,s)

chr10(u)

chr11(i,u,c,s)

*Crude balance

bal1(i,c,s)

bal2(i,c,s)

bal3(i,s)

*Component balance

com1(u,c,s)

com12(u,c,s)

com2(i,c,s)

com22(i,c,s)

com3(u,k,s)

com32(u,k,s)

*Change over calculation

co1(i,u,s)

co2(i,u,s)

*Settling time

set1(i,s)

set2(i,s,ss)

*Demurrage accounting,safety inventory penalty accounting

extra4(t,s)

extra5(v,p,s,t)

* Objective function

OBJ2 ;

***equations**

*parcel to SBM connections

unl1(p,s)$CPS(p,s)..XP(p,s+1)=e=XP(p,s)+XF(p,s+1)-XL(p,s);

unl4(p,s)$CPS(p,s)..XL(p,s)=l=XP(p,s);

unl5(p)..sum(s$CPS(p,s),XF(p,s))=e=1;

unl6(p)..sum(s$CPS(p,s),XL(p,s))=e=1;

unl7(p,pp,s)$(CPS(p,s)and (ord(pp) eq (ord(p)-1)))..XF(p,s)=l=sum(ss$((ord(ss) lt ord(s)) and CPS2(pp,ss)),XL(pp,ss));

unl8(s)..sum(p$CPS(p,s), XP(p,s))=l=1;

*SBM to Tank connection

unl9(s)..sum(i,XT(i,s))=l=sum(p$CPS(p,s),XP(p,s));

*Tank to CDU connection

unl10(i,s)..sum(u$IU(i,u),Y(i,u,s))=l=2;

unl11(u,s)..sum(i$IU(i,u),Y(i,u,s))=l=2;

*Activity assignment

```
act1(i,u,s)$IU(i,u)..YT(i,s)=g=Y(i,u,s);

act2(i,s)..sum(u$IU(i,u),Y(i,u,s))=g=YT(i,s);

act3(i,s)..XT(i,s)+YT(i,s)+ZT(i,s)=e=1;

*Activity time accounting

act4(s)$(ord(s) gt 1)..TL(s)=e=TL(s-1)+SL(s);

act5(s)$(ord(s) eq 1)..TL(s)=e=TL('0')+SL(s);

act6(p,s,t)$(CST(t,s)and CPS(p,s))..RLP(p,s)=l=DD(t)*XP(p,s);

act7(s)..sum(p$CPS(p,s),RLP(p,s))=l=SL(s);

act8(i,s,t)$CST(t,s)..RLT(i,s)=l=DD(t)*XT(i,s);

act9(s)..sum(i,RLT(i,s))=l=SL(s);

act10(s)..sum(i,RLT(i,s))=l=sum(p$CPS(p,s),RLP(p,s));

act11(i,s,t)$CST(t,s)..RLU(i,s)=l=DD(t)*YT(i,s);

act12(i,s,t)$CST(t,s)..RLZ(i,s)=l=DD(t)*ZT(i,s);

act13(i,s)..RLT(i,s)+RLU(i,s)+RLZ(i,s)=e=SL(s);

* Crude transfer from parcel to tank

ulp3(p,i,s)$(PI(p,i) and CPS(p,s))..RRP(p,i,s)=g=RLT(i,s)+RLP(p,s)-SL(s);

ulp4(i,s)..sum(p$(PI(p,i)and CPS(p,s)),RRP(p,i,s))=l=RLT(i,s);

ulp5(p,s)$(CPS(p,s))..sum(i$PI(p,i),RRP(p,i,s))=l=RLP(p,s);

ulp6(t)..sum(s$cst(t,s),SL(s))=e=DD(t);

ulp7(p,i,s)$(PI(p,i) and CPS(p,s))..FPT(p,i,s)=e=FPTmax*RRP(p,i,s);

ulp8(p)..sum((i,s)$(PI(p,i) and CPS(p,s)),FPT(p,i,s))=e=VV0(p);

*Crude transfer from tank to CDUs

chr1(i,u,s,t)$(IU(i,u) and CST(t,s))..RU(i,u,s)=l=DD(t)*Y(i,u,s);

chr 2(i,u,s,t)$(IU(i,u) and CST(t,s))..RU(i,u,s)=g=RLU(i,s)-DD(t)*(1-Y(i,u,s));
```

```
chr 3(i,u,s)$IU(i,u)..RU(i,u,s)=l=RLU(i,s);

chr 4(i,u,s)$IU(i,u)..FTUmin*RU(i,u,s)=l=FTU(i,u,s);

chr 5(i,u,s)$IU(i,u)..FTUmax*RU(i,u,s)=g=FTU(i,u,s);

chr 6(i,u,s)$IU(i,u)..FTU(i,u,s)=e=sum(c$IC(i,c),FCTU(i,u,c,s));

chr 7(u,s)..FU(u,s)=e=sum(i$IU(i,u),FTU(i,u,s));

chr 8(u,s)..DMmin(u)*SL(s)=l=FU(u,s);

chr 9(u,s)..FU(u,s)=l=DMmax(u)*SL(s);

chr 10(u)..sum(s,FU(u,s))=e=DM(u);

chr 11(i,u,c,s)$(IU(i,u) and IC(i,c)and FI(i,s))..FCTU(i,u,c,s)=e=f(i,c,'1')*FTU(i,u,s);

*Crude balance

bal1(i,c,s)$(IC(i,c)and (ord(s) gt 1))..VCT(i,c,s)=e=VCT(i,c,s-1)+sum(p$(PI(p,i) and
                          PC(p,c)and CPS(p,s)),FPT(p,i,s))-Sum(u$IU(i,u),FCTU(i,u,c,s));

bal 2(i,c,s)$(IC(i,c)and (ord(s) eq 1))..VCT(i,c,s)=e=VSC0(i,c)+sum(p$(PI(p,i) and
                          PC(p,c)and CPS(p,s)),FPT(p,i,s))-Sum(u$IU(i,u),FCTU(i,u,c,s));

bal 3(i,s)..VS(i,s)=e=Sum(c$IC(i,c),VCT(i,c,s));

*component balance

com1(u,c,s)..FU(u,s)*ESCmin(c,u)=l=sum(i$(IC(i,c) and IU(i,u)),FCTU(i,u,c,s));

com 12(u,c,s)..sum(i$(IC(i,c) and IU(i,u)),FCTU(i,u,c,s))=l=FU(u,s)*ESCmax(c,u);

com 2(i,c,s)$(IC(i,c))..VS(i,s)*ESmin(i,c)=l=VCT(i,c,s);

com 22(i,c,s)$(IC(i,c))..VCT(i,c,s)=l=VS(i,s)*ESmax(i,c);

com 3(u,k,s)..FU(u,s)*ELmin(u,k)=l=sum((i,c)$(IC(i,c) and
                                              IU(i,u)),FCTU(i,u,c,s)*EC(c,k));

com 32(u,k,s)..sum((i,c)$(IC(i,c) and
                   IU(i,u)),FCTU(i,u,c,s)*EC(c,k))=l=FU(u,s)*ELmax(u,k);

*change over calculation

co1(i,u,s)$(IU(i,u)and (ord(s) lt card(s)))..CO(u,s)=g=Y(i,u,s)-Y(i,u,s+1);
```

```
co2(i,u,s)$(IU(i,u)and (ord(s) lt card(s)))..CO(u,s)=g=Y(i,u,s+1)-Y(i,u,s);

*settling time

set1(i,s)..XT(i,s)+YT(i,s+1)=l=1;

set2(i,s,ss)$CSS(s,ss)..Tset*(YT(i,ss)+XT(i,s)-1)=l=TL(ss-1)-TL(s);

*Demurrage accounting,safety inventory penalty accounting

extra4(t,s)$(last(t,s))..SC(t)=g=pen(t)*(sAs(t)-sum(i,VS(i,s)));

extra5(v,p,s,t)$(LP(v,p)and cps(p,s)and cst(t,s)) ..DC(v)=g=(TL(s)-
                              (ETA(p)+Tmin(v))*XP(p,s)-D(t)*(1-XP(p,s)))*C_sea;

* Objective function

OBJ2..profit=e=sum((i,u,c,s)$(IU(i,u) and IC(i,c)),FCTU(i,u,c,s)*C_prof(c))-
                              sum(v,DC(v))-C_set*sum((u,s),CO(u,s))-Sum(t,SC(t));

*Bounds that CAN BE INCORPORATED

CO.lo(u,s)=0;

CO.up(u,s)=1;

YT.lo(i,s)=0;

YT.up(i,s)=1;

ZT.lo(i,s)=0;

ZT.up(i,s)=1;

VS.lo(i,s)=VSmin(i);

VS.up(i,s)=VSmax(i);

XF.lo(p,s)=0;

XF.up(p,s)=1;

XL.lo(p,s)=0;

XL.up(p,s)=1;

TL.fx('1')=D('1');
```

```
TL.fx('6')=D('2');
TL.fx('7')=D('3');
TL.fx('0')=0;
VS.fx(i,'0')=VS0(i);
VCT.fx(i,c,'0')=VSC0(i,c);
XF.fx('1','1')=0;
XL.fx('1','3')=0;
XF.fx('2','2')=0;
XL.fx('2','5')=0;
XF.fx('3','3')=0;
XL.fx('3','6')=0;
XF.fx('4','4')=0;
XL.fx('4','7')=0;
XP.fx(p,'1')=0;
XP.fx(p,'7')=0;
XP.fx('1','3')=0;
XP.fx('1','4')=0;
XP.fx('1','5')=0;
XP.fx('1','6')=0;
XP.fx('2','2')=0;
XP.fx('2','5')=0;
XP.fx('2','6')=0;
XP.fx('3','2')=0;
XP.fx('3','3')=0;
```

```
XP.fx('3','6')=0;
XP.fx('4','2')=0;
XP.fx('4','3')=0;
XP.fx('4','4')=0;
XT.fx(i,'1')=0;
XT.fx(i,'7')=0;
XT.fx('1','6')=0;
XT.fx('6','6')=0;
XT.fx('7','6')=0;
XT.fx('8','6')=0;
XT.fx('2','2')=0;
XT.fx('2','3')=0;
XT.fx('2','4')=0;
XT.fx('3','2')=0;
XT.fx('3','3')=0;
XT.fx('3','4')=0;
XT.fx('4','2')=0;
XT.fx('4','3')=0;
XT.fx('4','4')=0;
XT.fx('5','2')=0;
XT.fx('5','3')=0;
XT.fx('5','4')=0;
Model ResearchProblem /all/;
ResearchProblem.optca=0.00;
```

ResearchProblem.optcr=0.000;

ResearchProblem.iterlim = 10000000;

ResearchProblem.reslim= 1000000;

Solve ResearchProblem maximizing profit using MIP;

Display XF.l, XL.l, TL.L, SL.l, RLP.l, RLT.l, RLZ.l, RLU.l, RRP.l, RU.l, XP.l, XT.l, YT.l, ZT.l, Y.l, CO.l, FPT.l, FTU.l, FCTU.l, VS.l, VCT.l, PROFIT.L;

Parameter

f(i,c,s)

r

Next

Next1

NE

NE1

NET

NET1;

r=3;

Next=0;

Next1=0;

NE=0;

NE1=0;

NET=0;

NET1=0;

Equation

cor1(i,u,c,s)

```
cor5(i,u,c,s)

cor6(i,u,c,s)

cor7(i,u,c,s)

;

cor1(i,u,c,s)$(IU(i,u) and IC(i,c) and (ord(s)le r))..FCTU(i,u,c,s)=e=f(i,c,s)*FTU(i,u,s);

cor5(i,u,c,s)$(IU(i,u) and RCC(i,c) and  (ord(s)ge NE) and (ord(s)le
                                              NE1))..FCTU(i,u,c,s)=e=f(i,c,s)*FTU(i,u,s);

cor6(i,u,c,s)$(IU(i,u) and NRCC(i,c) and  (ord(s)ge NET) and (ord(s)le
                                              NET1))..FCTU(i,u,c,s)=e=f(i,c,s)*FTU(i,u,s);

cor7(i,u,c,s)$(IU(i,u) and IC(i,c) and  (ord(s)ge Next) and (ord(s)le
                                              next1))..FCTU(i,u,c,s)=e=f(i,c,s)*FTU(i,u,s);

Model Sai /

unl1, unl4, unl5, unl6, unl7, unl8, unl9, unl10, unl11, act1, act2, act3, act4, act5, act6,
act7, act8, act9, act10, act11, act12, act13, ulp3, ulp4, ulp5, ulp6, ulp7, ulp8, chr1,
chr2, chr3, chr4, chr5, chr6, chr7, chr8, chr9, chr10, chr11, bal1, bal2, bal3, com1, com
12, com 2, com 22, com 3, com 32, co1, co2, set1, set2, extra4, extra5, cor1, cor5,
cor6, cor7, OBJ2 /;

Sai.optca=0.00;

Sai.optcr=0.000;

Sai.iterlim = 10000000;

Sai.reslim= 1000000;

While ((r le card(s)),

loop(i,

loop(c$IC(i,c),

loop(s$((ord(s)le r)and (ord(s) gt 1)),

f(i,c,s)=VCT.l(i,c,s-1)/VS.l(i,s-1););););

loop(v$(ord(v) eq card(v)),
```

```
loop(p$RCP(v,p),

loop(s$(ord(s) eq r-1),

if((XL.L(p,s) eq 1),

NE=r;

NE1=card(s););););););

loop(i,

loop(c$RCC(i,c),

loop(ss$(ord(ss) eq NE),

loop(s$((ord(s) ge NE) and (ord(s) le NE1)),

f(i,c,s)=VCT.l(i,c,ss-1)/VS.l(i,ss-1););););););

loop(v$(ord(v) eq card(v)),

loop(p$NRP(v,p),

loop(s$(ord(s) eq r-1),

if((XL.L(p,s) eq 1),

NET=r;

NET1=card(s););););););

loop(i,

loop(c$NRCC(i,c),

loop(ss$(ord(ss) eq NET),

loop(s$((ord(s) ge NET) and (ord(s) le NET1)),

f(i,c,s)=VCT.l(i,c,ss-1)/VS.l(i,ss-1););););););

loop(v$(ord(v) eq card(v)),

loop(p$LP(v,p),

loop(s$(ord(s)eq r-1),
```

```
if((XLl.l(p,s) eq 1),
Next=r;
Next1=card(s););););
loop(u,
loop(i$(IU(i,u)),
loop(c$IC(i,c),
loop(s$(ord(s)le r-1),
FTU.fx(i,u,s)=FTU.l(i,u,s);
Y.fx(i,u,s)=Y.l(i,u,s);
VS.fx(i,s)=VS.l(i,s);
FCTU.fx(i,u,c,s)=FCTU.l(i,u,c,s);
VCT.fx(i,c,s)=VCT.l(i,c,s);
SL.fx(s)=SL.l(s);
RLU.fx(i,s)=RLU.l(i,s);
RLT.fx(i,s)=RLT.l(i,s);
RLZ.fx(i,s)=RLZ.l(i,s);
YT.fx(i,s)=YT.l(i,s);
ZT.fx(i,s)=ZT.l(i,s);
XT.fx(i,s)=XT.l(i,s);
RU.fx(i,u,s)=RU.l(i,u,s););););)
loop(p,
loop(i$PI(p,i),
loop(s$(ord(s)le r-1),
XP.fx(p,s)=XP.l(p,s);
```

```
XL.fx(p,s)=XL.l(p,s);

XF.fx(p,s)=XF.l(p,s);

FPT.fx(p,i,s)=FPT.l(p,i,s);

RLP.fx(p,s)=RLP.l(p,s);

RRP.fx(p,i,s)=RRP.l(p,i,s););););

Solve Sai maximizing profit using MIP;

Display XF.l, XL.l, TL.L, SL.l, RLP.l, RLT.l, RLZ.l, RLU.l, RRP.l, RU.l, XP.l, XT.l,
YT.l, ZT.l, Y.l, CO.l, FPT.l, FTU.l, FCTU.l, VS.l, VCT.l, PROFIT.L, Next, Next1,
NE, NE1, NET, NET1;

If((r eq Next),

r=Next1;);

r=r+1;

Next=0;

Next1=0;);

Display XF.l, XL.l, TL.L, SL.l, RLP.l, RLT.l, RLZ.l, RLU.l, RRP.l, RU.l, XP.l, XT.l,
YT.l, ZT.l, Y.l, CO.l, FPT.l, FTU.l, FCTU.l, VS.l, VCT.l, PROFIT.L, Next, Next1,
NE, NE1, NET, NET1;
```