VIRTUAL TOPOLOGY DESIGN FOR OPTICAL WDM NETWORKS

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List of Symbols

N: Number of nodes in the network

C: Channel capacity

$G(N_p, E_p)$: Physical topology

$G(N_v, E_v)$: Virtual topology

s: Source node (of a traffic flow)

d: Destination node (of a traffic flow)

i: Starting point (of a virtual link)

j: Terminating point (of a virtual link)

m: Starting point (of a directed physical link)

n: Terminating point (of a directed physical link)

q: Multiplicity variable, integer and $q \geq 1$

Δ^{In}: Ingress degree (limited by the number of receivers at a node)

Δ^{Out}: Egress degree (limited by the number of transmitters at a node)

Δ: In most cases, consider $\Delta^{In} = \Delta^{Out} = \Delta$

$\mathbf{T}$: Traffic matrix (average flows)

T^{sd}: Traffic flows from source s to destination d

T_{ij}: Traffic flows on virtual link i-j

T_{ij}^{sd}: Traffic from s to d routed through virtual link i-j

$\mathbf{b}$: Virtual topology matrix

b_{ij}: Virtual link from node i to node j

$\mathbf{P}$: Physical topology connection matrix

P_{mn}: Physical topology connections between node m and node n

P_{mn}^{ij}: Lightpath routing variables

$\mathbf{H}$: Maximum virtual hop matrix

H_{ij}: Maximum number of hops that a virtual link b_{ij} is permitted to take

M: Maximum number of wavelengths one fiber supports

Abbreviations

ADM: Add/Drop Multiplexer

AON: All Optical Networks

ATD: Average Traffic Delay

EB: Estimated Bound

FDM: Frequency Division Multiplexing

ILP: Integer Linear Programming

LoPL: Lightpaths over a Physical Link

LP: Linear Programming

MAD: Minimum of Average Delay

MILP: Mixed Integer Linear Programming

MTN: Maximum Traffic Net

PD: Physical Degree

PH: Physical Hop

PL: Physical Link

PT: Physical Topology

RLL: Remove Lightpath with Lowest traffic

RWA: Routing and Wavelength Assignment

TCC: Traffic Congestion Control

TDM: Time Division Multiplexing

TL: Total number of Lightpaths

TLoL: Total Length of Lightpath

TW: Traffic Weighted

TI: Traffic Independent

VD: Virtual Degree

VH: Virtual Hop

VT: Virtual Topology

VTD: Virtual Topology Design, or *Logical Topology Design*

VWP: Virtual Wavelength Path

WAN: Wide Area Network

WDM: Wavelength Division Multiplexing

WP: Wavelength Path

Summary

In this thesis, we study a few issues related to the design of virtual topology (VT) of Optical Wavelength-Division-Multiplexing (OWDM) wide area networks. The main focus of virtual topology design (VTD) is on the provision of lightpaths and virtual wavelength paths in the optical layer. VTD has been well-known as a *NP-hard* problem, therefore, to avoid intensive computation when the network size grows, *heuristic algorithms* are widely discussed to achieve effective suboptimal solutions.

First we define some of the basic function blocks related to OWDM networks, including node configurations, optical switches and routers etc. Meanwhile we discuss about the network model used in this thesis. Our study mainly focuses on future OWDM networks, with their nodes having packet switching capability, and hence the traffic flows from a source to a destination can be routed through multiple virtual wavelength lightpaths. When necessary, wavelength converter is assumed to be available although it may not be cost efficient at this stage. The conventional adopted mathematical method is to formulate VTD as a mixed integer linear program with one objective function. The formulation and possible objective functions to be used for optimization are discussed.

Moreover, we examine the existing heuristic algorithms. Performances of some of those algorithms are analyzed, compared and categorized, followed by a discussion on their assumptions and limitations. We also define a new heuristic algorithm named maximum-traffic-net (MTN). MTN-VTD achieves good congestion control by assigning lightpaths to support as much direct high-traffic flows as possible. The algorithm performs VTD by decomposing the problem into four subproblems, each with an ob-

jective function, and to be solved sequentially. It has less computation complexity than MILP but can achieve a congestion level closer to what is predicted using MILP due to the embedding of computation-based optimization in each subproblem. Further it overcomes the disadvantage of existing heuristic algorithms having a similar purpose by eliminating the risk on obtaining a partitioned virtual topology.

In addition we propose a new objective function named as maximizing virtual hops (MVH), which presents another point of view to partitioned traffic routing and is more suitable for the packet-switched network. Also we present another novel way to formulate VTD problem. We remove the virtual degree (VD) constraints of each individual node and replace them by a single constraint related to the total VD of the entire network. Because we are attempting to optimize the same objective function in a larger search space, a lower optimal value can be achieved than conventional methods but at the expense of increasing computation complexity. However, this new way of modeling can result in the development of a simple heuristic algorithm, named as remove lightpath with lowest traffic (RLL). The solution provides information on whether resources in the network are allocated efficiently, and in case it is not, how re-distribution can be made to achieve better utilization.

Finally, we discuss about the possibility of concurrently considering more than one objective function when designing the virtual topology of an optical WDM network. We formulate VTD problems with two objective functions, and apply Pareto's optimality concept to solve the bi-objective problems. The discussion is demonstrated by using a six-node optical network under various traffic models.

Chapter 1

Introduction

1.1 Background

1.1.1 Evolution of Optical WDM Networks

Wavelength division multiplexing (WDM) [1] [2] [3] is an effective technology utilizing the large bandwidth of an optical fiber by concurrently transmitting multiple data streams over a single optical fiber via different wavelengths. Optical WDM (OWDM) networks [6] are the state-of-art high capacity communication networks, and the potential candidate for the next generation wide-area backbone networks. OWDM networks are very promising because it is easy to upgrade the deployed first generation optical fiber networks to WDM networks. Moreover, OWDM networks, consisting of nodes with switching capability, are able to provide routing, grooming, and restoration at the wavelength level.

Although optical fiber networks exists only for two decades, tremendous progress in photonics technologies has been made. Generally there are four generations of optical networks. The first-generation uses time division multiplexing (TDM) concept and intensity modulated (IM) technology, in which the optical power of the source is modulated by the modulating signal, and recovered by direct detection using a photodetector at the receiver. However, first generation has a major limitation: when

multiplexing traffic from different nodes into a single fiber, TDM requires very high electronic processing speed to achieve high data rate transmission. In addition, first-generation systems generally use optical sources of poor coherency, which prevents the use of more sophisticated modulation techniques.

Thanks to the breakthrough of integrated optic technologies, the use of sophisticated modulation techniques becomes practical in the last decade, which in turn enables the emergence of the second-generation optical systems. This generation is characterized by WDM techniques, in which data are transmitted via different wavelengths over a single fiber, and each wavelength supports a single communication channel operating at whatever rate one desires. Hence, WDM networks, when properly designed, are capable to achieve high system capacity through the use of multiple wavelengths. While the technologies of optical TDM and CDM (Code Division Multiplexing) are still studied in research laboratories due to the bottleneck in electronics processing rate, WDM has become mature and widely used in the industry [6] [9].

The third-generation optical networks use heterodyning receivers, allowing to tune to any wavelength for receiving optical signal rather than the use of optical demultiplexers. This accommodates more wavelengths to be packed into a single fiber, and is known as dense WDM (DWDM) optical systems. In this thesis, WDM is used as a general term referring to the two types of optical WDM networks.

Along with the development of technologies, it is realized recently that optical networks are capable of providing more functions than simple point-to-point transmission. According to the way optical signals are switched and processed, optical networks can be divided into two types. In the usual optical networks, electrical-to-optical (E/O) and optical-to-electrical (O/E) conversions are required at all intermediate nodes, before data reaching their destinations. Hence, the networks are not able to provide protocol-transparency, i.e. the capability of accommodating data comprising different bit rates and formats at the same time. The second type is the all optical networks (AONs), where data can be sent from one node to another entirely in the optical domain, providing complete transparency. In this case, data transported through multiple

links undergo purely all-optical wavelength conversion and multiplexing. Today, AONs are intensively studied in the experiments [1] [7] and are classified as fourth-generation optical networks in the literature [3] [6] [9].

1.1.2 WDM Networks and Optical Devices

Although the success of WDM networks relies heavily upon the available optical components, we are not going to present a comprehensive introduction to all of the optical devices, such as amplifiers, filters and detectors etc. Along with illustrating the WDM network architectures, only the functions of several key photonic components are demonstrated.

Wavelength Add/Drop Multiplexers (WADM)

A *Wavelength Add/Drop Multiplexer* (WADM) enables a small number of contiguous wavelengths to be added and dropped without demultiplexing the entire wavelength bundle.

Fig. 1.1 shows a WADM consisting of a wavelength Demultiplexer (DEMUX), a set of 2×2 switches and a wavelength Multiplexer (MUX). The 2×2 switches provided here are simple 2×2 crosspoint elements. A 2×2 crosspoint element routes optical signals from two input ports to two output ports and has two states: cross state and bar state as shown in Fig. 1.1. In the cross state, the signal from the upper input port is switched to the lower output port, and the signal from the lower input port is switched to the upper output port. In the bar state, the signal from the upper input port is switched to the upper output port, and the signal from the lower input port is switched to the lower output port.

With the exception of λ_i, wavelengths λ_1 to λ_ω flow through the 2×2 switches in the 'bar' state. However, the 2×2 switch for wavelength λ_i is configured in the 'cross' state. The signal on the wavelength λ_i is dropped locally, and a new signal can be added on to the same wavelength accordingly.

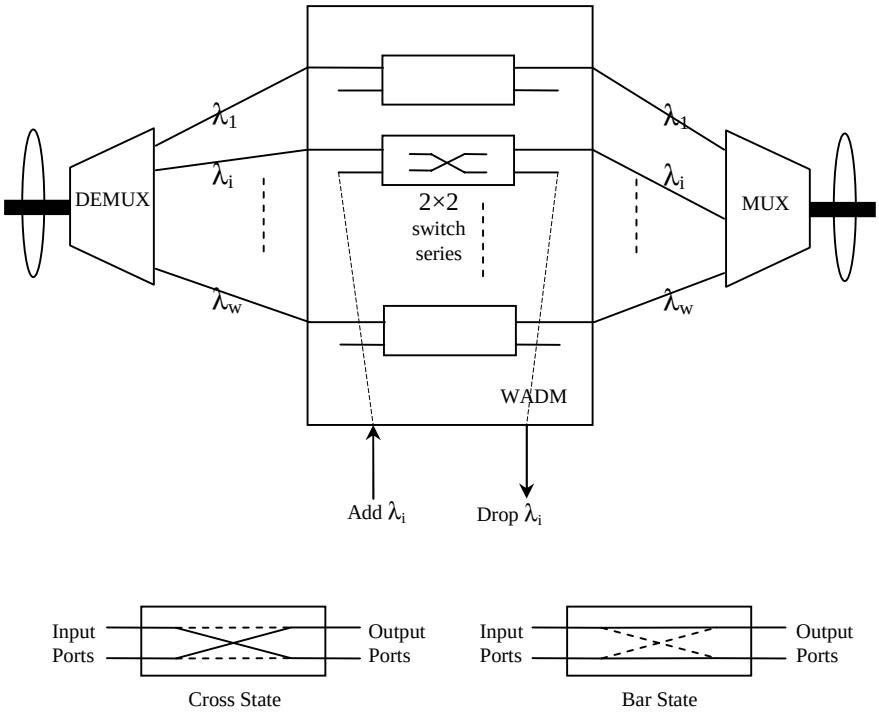


Figure 1.1: Wavelength Add/Drop Multiplexer

Wavelength Converter

When a *lightpath* is established from the source to the destination, usually a particular wavelength should be used along all the links, this is known as *Wavelength Continuity Constraint*. However, wavelength converters allow the conversion of data on any input wavelength to any other wavelength at the intermediate nodes in the optical domain, thereby eliminating the wavelength continuity constraint and improving the utilization of the network bandwidth in high-speed networks.

Normally wavelength converters are located in the switches (or crossconnectors). However, such a wavelength-convertible switch is not cost-efficient since not all of the wavelength converters will be used all the time. Effective methods to cut cost, such as sharing and sparse-implementation are discussed in [7] [33].

Although wavelength converters are used in AONs, this does not mean that wavelength continuity cannot be removed in the usual WDM networks. Wavelength conversion for WDM networks also can be made if the nodes have the O/E and E/O capability.

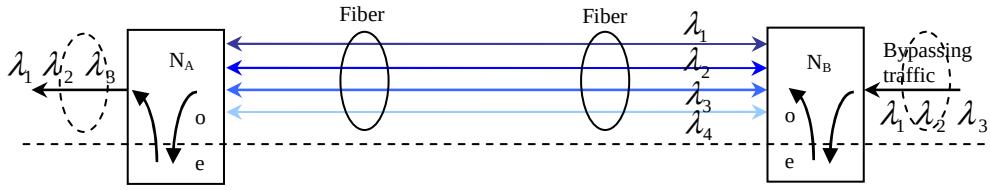


Figure 1.2: Point-to-point WDM communication

The recovered electrical signal can be re-modulated into another wavelength through controlling the emitting frequency of the laser sources.

Point-to-Point WDM Systems

The first and also the simplest implementation of WDM techniques is used for point-to-point transportation [1] [28] [9]. Fig. 1.2 given on this page shows a typical four-channel point-to-point WDM transmission system. The transfer of information between N_A and N_B can be set up using one of the four wavelengths (λ_4 in this example) available in the connecting fiber. Traffic routed via the link from node N_B to node N_A have to undergo O/E and E/O conversions at the two intermediate nodes. Generally these systems implement only the minimum number of optical devices. They simply achieve high-speed transportation through parallel bit streams having different wavelengths, and they do not have any routing capability.

Broadcast-and-Select Optical WDM Networks

Generally broadcast and select networks are only useful for Local Area Networks (LANs) and Metropolitan Area Networks (MANs) [1] [28] [9]. The simplest broadcast and select star network is shown in Fig. 1.3. Network nodes are connected to a passive star via two-way fibers. Different nodes or hosts connected to the nodes transmit using different pre-defined wavelengths. Their signals are optically combined by the star coupler and broadcasted to all the nodes. Each node uses a tunable optical filter to select the desired wavelength, as shown in Fig. 1.3. The passive star network can also support multicast services because any of the receiving nodes can simultaneously

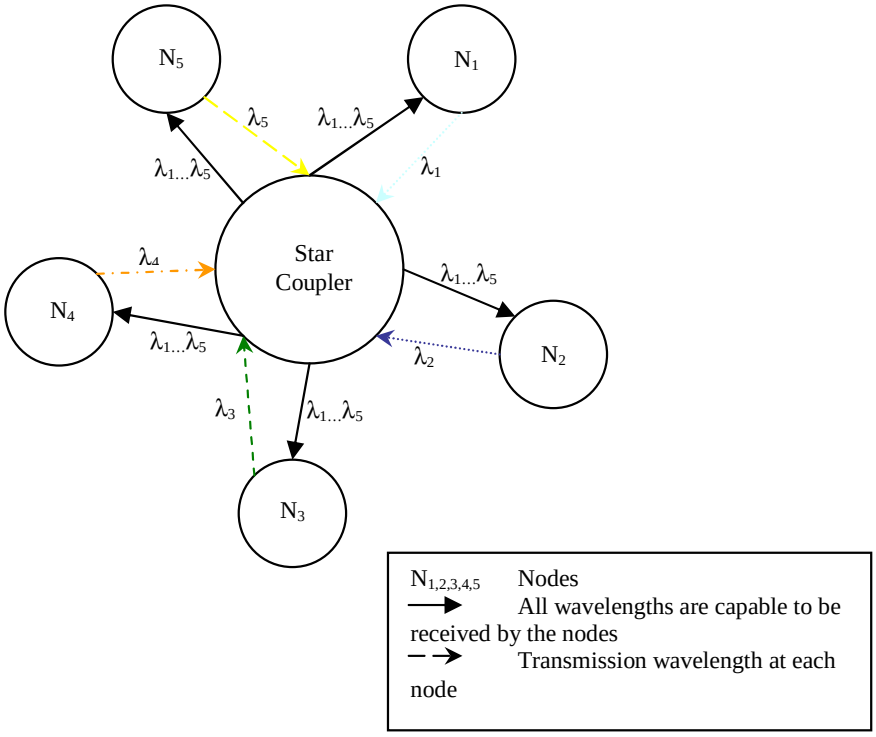


Figure 1.3: Broadcast and select WDM networks

be tuned to the same particular wavelength. Since the end nodes perform the switching functions in terms of wavelength tunability, the broadcast-and-select networks can still function if one node is down.

Configuration of a passive star coupler is presented in Fig. 1.4. The passive star broadcasts information from every input fiber to every output fiber — light powers from all input fibers are each split equally to all the output fibers. The input and output ports of the passive star copuler are connected to the transmitter and receiver of each node as shown in Fig. 1.3.

Wavelength-Routed Networks

A simple model for wavelength-routed optical network is shown in Fig. 1.5. Such a network is highly scalable and suitable for wide area networks (WANs) [1] [28] [9]. A wavelength-routed network (WRN) consists of wavelength routers and the fiber links that interconnect them. Wavelength routers are capable of routing the light signal of a

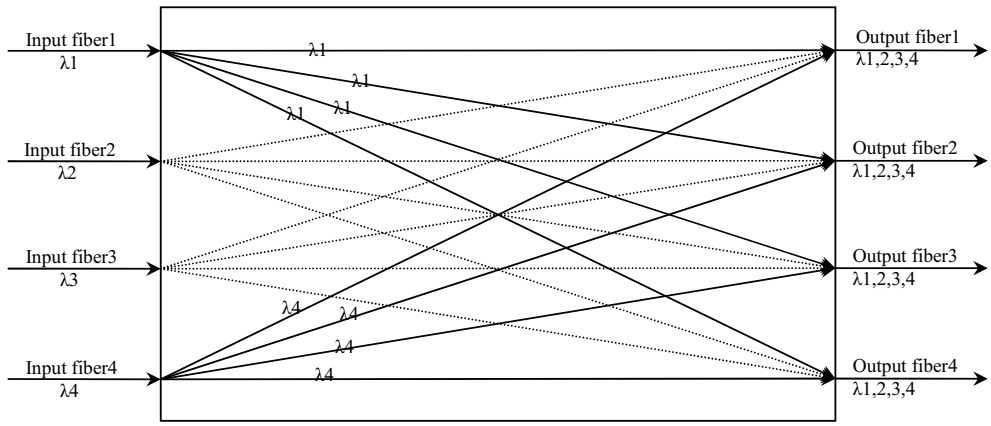


Figure 1.4: Functions of a node as a passive star coupler.

given wavelength from any input to any output port so that WRN can establish end-to-end lightpaths, i.e. direct optical connections without any intermediate electronics. The same wavelength may be spatially reused to carry multiple connections. A lightpath uses the same wavelength on every link in its path since there is no wavelength conversion capability at each node. This requirement is referred as the wavelength continuity constraint. For example in Fig. 1.5, a signal travels from $Host_1$ to $Host_2$ using wavelength λ_2. If a signal originates at $Host_4$ via wavelength λ_2 and reaches the middle wavelength router R_2, it cannot continue towards router R_1 because the wavelength λ_2 is already taken by the signal from $Host_2$. The signal is not allowed to switch to wavelength λ_1 even though λ_1 is not in use.

Several devices can be used in the nodes of WRNs, such as passive routers, active routers or wavelength-routing switches (WRS). The differences on their re-configurable capabilities are discussed below.

When signal reaches a passive router, traffic flows from a certain ingress fiber using certain wavelength are routed to a 'pre-defined' egress fiber, as shown in Fig. 1.6, where traffic routing cannot be flexibly controlled. For example, as shown in Fig. 1.6, traffic flows coming through input fiber 2 using wavelength λ_4 are all routed to output fiber 3 using the same wavelength.

As shown in Fig. 1.7, an active switch can support $N \times N$ simultaneous connections,

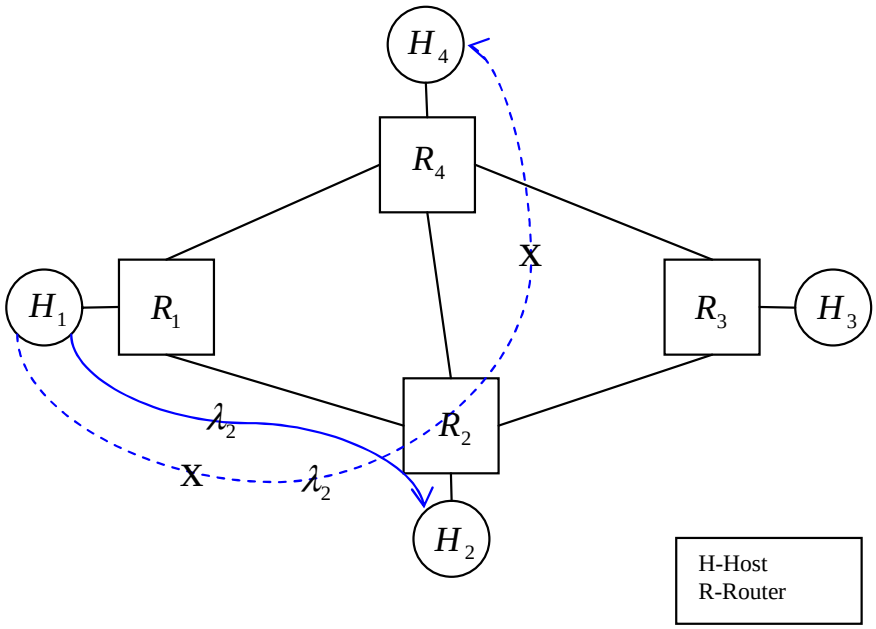


Figure 1.5: A simple model for Wavelength-Routed Networks

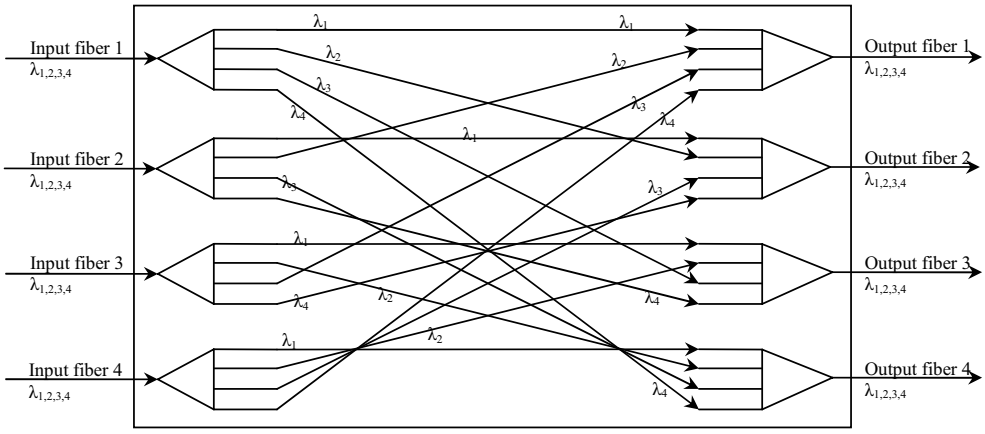


Figure 1.6: Functions of a node as a passive router.

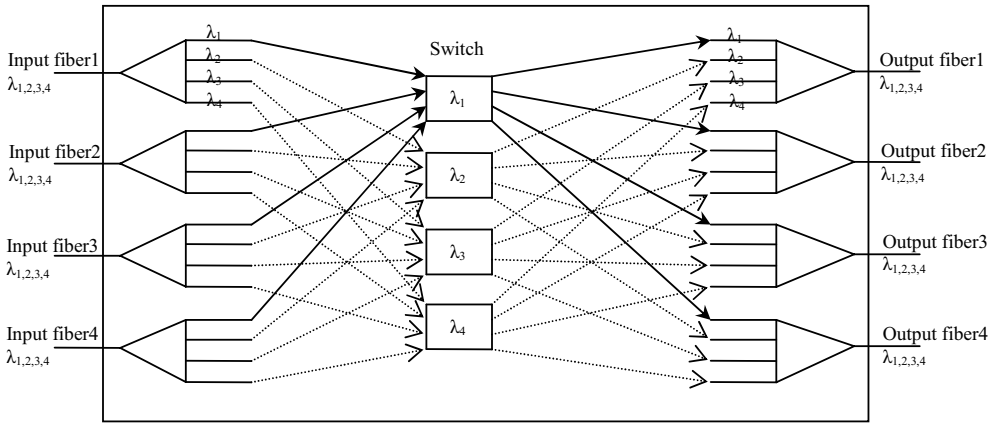


Figure 1.7: Functions of a node as an active switch

moreover it allows wavelength reuse. An active star has a further enhancement over the passive router, because connections between its input ports and output ports can be reconfigured on demand. For example, traffic flows coming through input fiber 2 using wavelength λ_2 can be re-routed to any of the output fibers through controlling switch λ_2. Active switches need to be powered up and are not so fault-tolerant as the passive stars, whilst the passive routers do not need to be powered up. The active switch can be WRS, WSXC (wavelength selective crossconnector), or just crossconnector for short.

1.2 Related Work

1.2.1 Wavelength Routing

A basic VTD issue in OWDM networks is the routing and wavelength assignment for dealing with all the transmission requests in the network. To begin, we first differentiate the two common terms used in this thesis: Wavelength Paths (WPs) and Virtual Wavelength Paths (VWPs).

In general, WP is also known as *lightpath*, and VWP can be regarded as *virtual circuit* connection between two nodes. A lightpath is simply a high-bandwidth pipe, carrying data using a wavelength at up to several gigabits per second. The bandwidth offered on the connection can be smaller than the full bandwidth available on a wave-

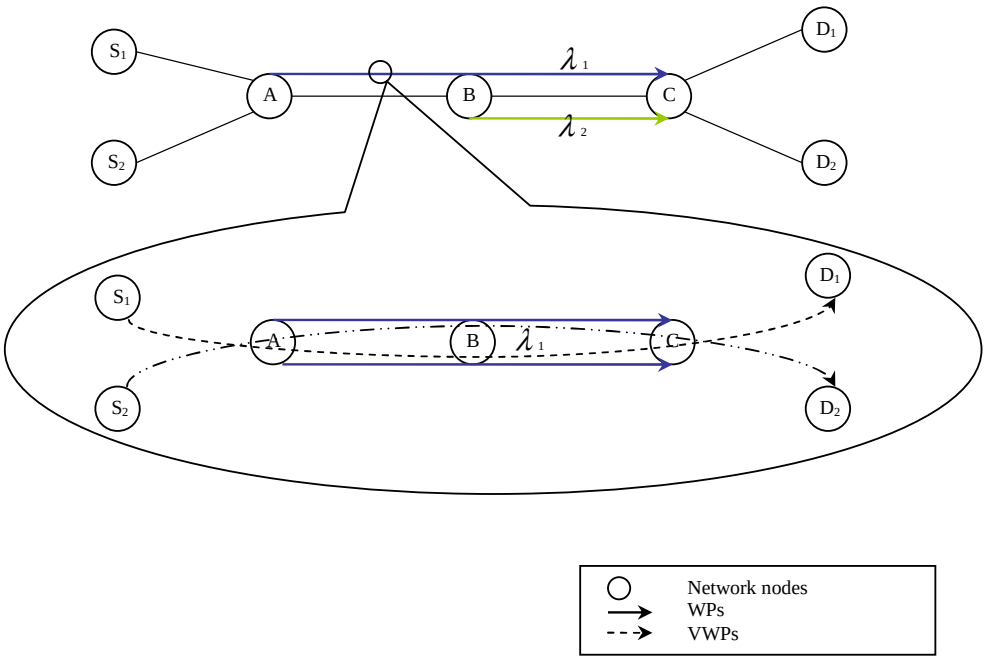


Figure 1.8: Illustration of WPs and VWPs

length channel. Thus the OWDM network must incorporate TDM to combine multiple VWPs onto a lightpath in WDM links to make better utilization of the wavelength channel capacity. VWP has an important role in the cost effective management of network resources [10]. For example, as shown in Fig. 1.8, when WDM networks route data traffic between two node pairs (S_1D_1 and S_2D_2), and nodes A, B and C have the capability of processing TDM data streams. VWPs for respective transmissions are set up and combined into WP(A,C) when they are routed through links between node A and node C. As shown in Fig. 1.8, if $WP(A, C)$ uses λ_1, $WP(B, C)$ can only use λ_2 because λ_1 is no longer available on fiber BC; however, $VWP(S_1, D_1)$ and $VWP(S_2, D_2)$ can both traverse through $WP(A, C)$ via wavelength λ_1. This is because no two WPs are capable to share one wavelength on a physical link, and one WP can only be routed through one set of continuous physical links from its source node to destination node. But two VWPs are capable to traverse over the same WP, as long as the wavelength capacity is not violated.

One advantage of OWDM networks is that different network architectures can be constructed in a physical network using different wavelengths. For example, one

wavelength may be used to connect a few nodes as a ring configuration, while another wavelength can be used to support a point-to-point configuration in the same network, where each wavelength supports an existing service (such as ATM, SONET or IP layer). This is also the simplest way to retain the original network service after upgrading a single wavelength optical network into a WDM network.

If each lightpath in an optical network essentially uses a dedicated wavelength to connect its two endpoints, the network is known to be single-hop virtual network [18]. If a lightpath in a network is allowed to travel via different wavelengths, the network is known as multihop networks [19] [32] [40]. Without being constrained by dedicated wavelengths, a multihop network can be optimized to support greatest amount of traffic flows. Because of the limitation of single-hop networks, in this thesis, we focus our discussion on multihop networks.

There is another type of hop being discussed in the literature. *Physical hop* is the number of physical links that a lightpath is traversing through between its source and destination, whilst *virtual hop* that has been defined before is the number of wavelength conversions along a lightpath. Related illustrations are presented in Fig. 1.9. Basically physical hops of a lightpath are related to the traffic propagation delay, while virtual hops of a lightpath associate with cost and implementation of wavelength converters. Further discussion will be presented in Chapter 4.

1.2.2 Virtual Topology (VT)

Generally the topology of wavelength-routed network is an arbitrary mesh, which can be defined as a physical topology (PT) consisting of nodes interconnected by fiber links. We can think the network is constituting of an *optical layer* [1] [2] [3] in the network layered hierarchy. The optical layer is constructed based on resources and constraints imposed by the physical layer, and its function is to provide a set of lightpaths to the higher layers, i.e. a virtual topology (VT).

We illustrate the relation between physical layer and optical layer using Fig. 1.10

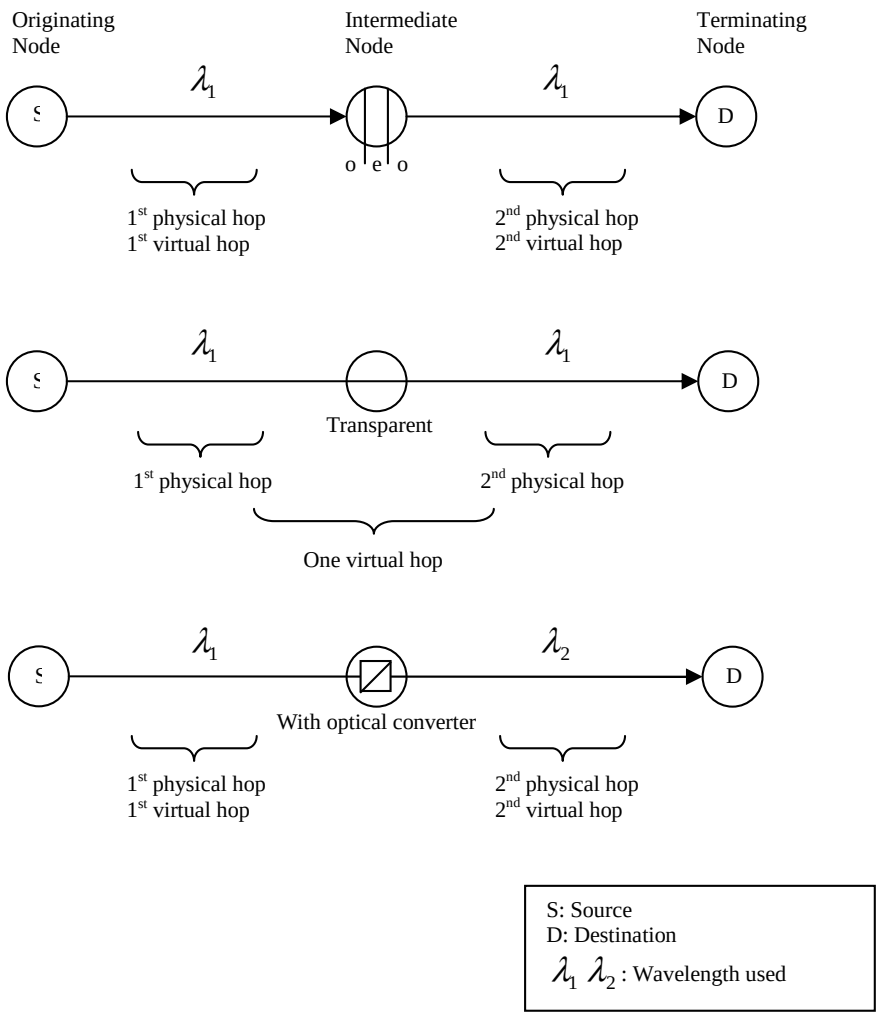


Figure 1.9: Difference of Single Hop and Multihop Lightpaths

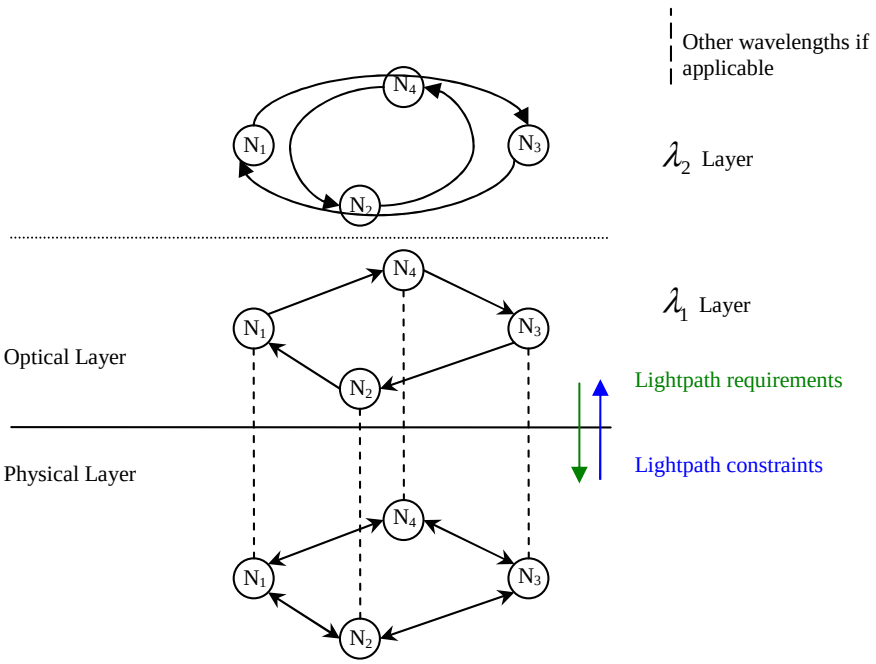


Figure 1.10: Designing a Virtual Topology as a two-layered problem

and demonstrate how to perform a virtual topology design.

Assume the network supports two wavelengths transmission, as shown in Fig. 1.10, the VT can be expanded into two λ sublayers. λ_1 supports a ring configuration connecting all the four nodes and λ_2 supports two sets of point-to-point communications between node pairs N_1N_3 and N_2N_4.

Classical network design problems can be expressed as two-level design problems: designing the virtual topology in the optical layer and designing the physical topology in the physical layer. However, nowadays designing VTs generally refers to build up a set of lightpaths in optical layer (including all λ sublayers) based on a given physical topology, node functionalities and other design considerations. Several other parameters influencing the virtual topology design of a network are as follows.

1) Number of transceivers (combined name of transmitters and receivers), i.e. number of lightpath terminals. Typically there are two types of transceivers: fixed and tunable. In this thesis, we mainly concern the second type.

2) Number of hops: indicates the cost of implementation applying to networks with

tunable transceiver.

3) Number of wavelengths: we would also like to use the minimum possible number of wavelengths since using more wavelengths incurs additional equipment cost.

In short, VTD problems can be concluded as designing a topology over the optical layer, where instead of using physical links to interconnect equipment, the interconnection is done via lightpaths. This feature enables the interconnection pattern to be programmable (changed as desired), and such flexibility is unable to be found in the conventional point-to-point interconnected networks.

1.3 Contribution

Our accomplishments, which are elaborated throughout this thesis, can be broadly listed as follows.

. Present a literature review on VTD problem including the general notations and formulations. Also design theories are covered comprehensively, with the computation complexity of VTD problem expressed in terms of the number of variables and constraints.

. Review the existing heuristic algorithms for solving VTD problem and describe their implementations in details. Perform computations of all relevant heuristic algorithms in both simple network models (4,6 nodes) and large network models (8,14 nodes). Compare the performance and evaluate the scalability using the numerical results. Consequently their cons and pros are analyzed.

. Develop a novel heuristic algorithm by decomposing the problem into four subproblems, each with an objective function, and to be solved sequentially. Numerical results are computed and evaluation about this heuristic algorithm is made.

. Present a novel way to formulate VTD problem by removing the virtual degree (VD) constraints of each individual node and replacing them by a single constraint related to the total VD of the entire network, for the purpose of maximizing the resource utilization of the networks.

. Examine the parameters commonly used in the objective functions for VTD problem, and evaluate their inter-relationships in networks under different traffic models. Parameters likely to have Pareto's solution are selected. We then provide a novel formulation of VTD with two objective functions, and present the solutions by applying mathematical methods using a simple network.

1.4 Outline of the Thesis

The remaining of this thesis is organized as follows. In chapter 2, an overview to the typical notations and formulations of VTD problems are given. This chapter aims to provide detailed background knowledge to the contents covered by the next few chapters.

Chapter 3 enumerates the theory of designing a virtual topology with different constraints and demonstrates traditional heuristic algorithms for VTD problem in details. A new objective function is suggested and solved by new heuristic algorithms. We compare and evaluate these heuristic algorithms thoroughly with other commonly known algorithms.

Chapter 4 investigates the relationships between various objective functions used in VTD problem, aims to provide a novel method of formulating VTD with bi-objective functions. Such a problem has not been investigated before the literature. Supplemental computation work is presented in Chapter 5, and Chapter 6 concludes this thesis.

Chapter 2

VTD of Optical WDM Networks

This chapter defines VTD problem for wavelength routed OWDM networks. With the preliminary knowledge introduced in Chapter 1, we first discuss the processing capabilities of a node, followed by presenting the notations and definitions used in this thesis. We also demonstrate the mathematical formulation of VTD problems, and analyze the computation complexity of the VTD problem.

2.1 Network Architecture and Related Notations

Our design will focus on the transport networks. A graph demonstrating the OWDM network model used in this thesis is shown in Fig. 2.1, with only two nodes shown. The definitions of the terminology terms labelled in Fig. 2.1 are self-explanatory.

In a network, two adjacent nodes are connected by a number of separate channels identified by either different physical fiber links or different wavelength connections in a fiber. We used the notation G(N_p,E_p) to represent the interconnections of physical nodes (N_p) and physical channels (E_p) that form the physical topology of a given network. Similarly, a virtual topology defined in the optical layer and can be presented as a graph G(N_v,E_v), where N_v indicates the virtual nodes and E_v indicates the virtual edges, i.e. lightpaths. Note this kind of expressions is commonly used in graph theory

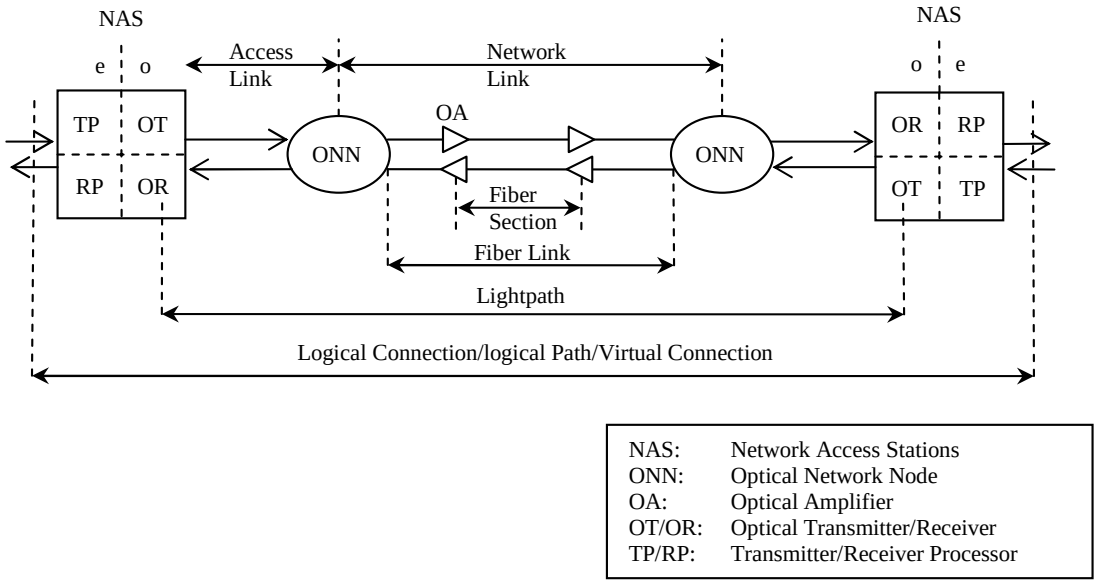


Figure 2.1: Layered View of the Optical Network

[4]. Notations used to express the connections of physical and virtual topologies are given in the appendix.

2.1.1 Nodes Processing Capabilities

If the ingress traffic flow has not reached its destination node, it will be routed to the egress port transparently, as shown in Fig. 2.2. Data remain transferring via the same wavelength channel if there is no wavelength converter (O-O) available or wavelength conversion capability (O-E-O) at the node; otherwise data can be transferred via another wavelength. When the traffic flow reaches its destination, the lightpath terminates and data are received by the terminal connected to the node, as shown in Fig. 2.2.

When designing VTs, we only concern on the key functions of the nodes that affect the VTD, but not all the functions that a network node can perform. One of the constraints that a physical node imposed when performing VTD is the number of transceivers available at that node, named as *Virtual Degrees Constraints*, which constrains the number of lightpaths originating from or terminating at each node.

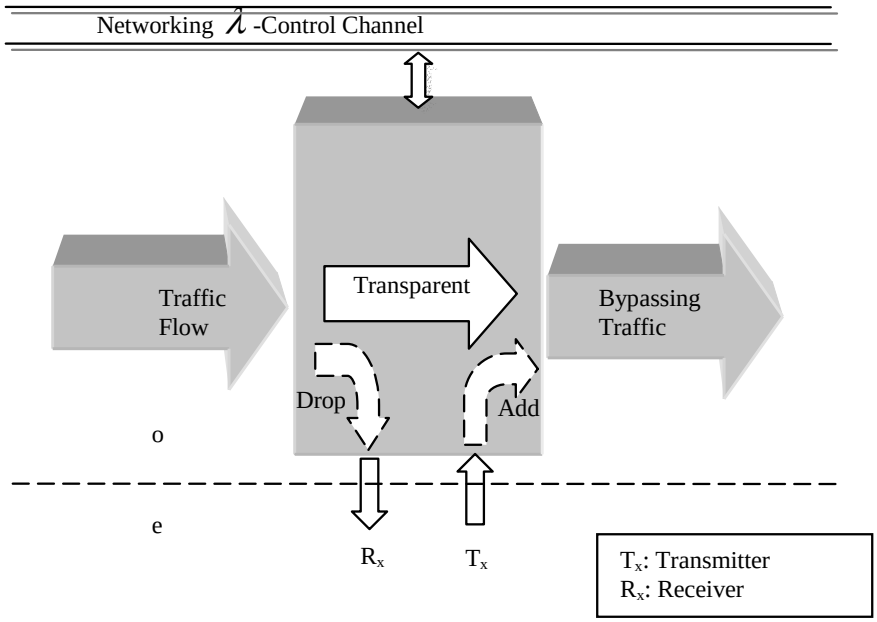


Figure 2.2: Functions of a node in the physical topology.

Generally, we assume all the transceivers are fully tunable, i.e. every transceiver is capable to tune to any available wavelength.

Nodes in this thesis have the capability of switching and routing traffic flows transparently. In circuit-switched networks, there are two varieties of this problem: offline (or *static*), in which lightpaths requested are known initially, and online (or *dynamic*), in which lightpaths are assigned and released on demand. It has been proven to be NP-hard in [34], because VTD is almost the same as *Routing and Wavelength Assignment* (RWA), i.e. VWPs are actually WPs. The number of wavelengths per fiber, and the capacity of each wavelength are the two most important parameters. The objective function of the design can be: (1) optimizing the number of wavelengths required to establish a given set of lightpaths, or alternatively, to establish as many desired lightpaths as possible for a given set of wavelengths; (2) establishing lightpaths for a given set of wavelengths that maximizes the total traffic throughput in the network [14] [17] [27].

However along with the evolution of optical networks, lightpaths can be operated at bit rates in excess of 2 Gb/s, and most applications do not require a bit rate of this

magnitude. Implementing circuit-switched network is no longer an appropriate way to fully utilize the optical resources. So we turn to packet switching technologies. In a packet-switched optical network, packet arrivals are assumed to follow a Poisson process and packet transmission time is exponentially distributed with a given mean time. We also assume that the traffic offered to a lightpath in the network is independent of the traffic offered to the others. In this case, the aim of the design is to find a set of WPs supporting all traffic requests to minimize congestion level or delay [1]. Traffic flows can be routed through multihop lightpaths [28], and multiple VWPs can share the same WP for transmission.

2.1.2 Physical Link and Lightpath Characteristics and Capabilities

Sometimes backup fibers are implemented in the networks to enhance the failure tolerance and hence result in multifiber systems. In this thesis, unless specially mentioned, we only consider single-fiber networks. Physical links referred to the fiber links between nodes. As shown in Fig. 2.3, four cases of physical links are presented, where case 1, case 3 and case 4 are three different types of representing a single-fiber, 'undirected' networks, because there is no difference between the two directions of a link. However, case 2 presents an example of a directed physical link, e.g. transmission can be limited to only one direction.

The edges of virtual topology are lightpaths, therefore two lightpaths routing from the same physical edge cannot be assigned with the same wavelength, which is usually known as avoiding 'wavelength confliction'. Lightpaths in a VT have also two types: undirected and directed. In most cases we design a VT with directed lightpaths. Compared to the undirected one, a VT with directed lightpaths is more flexible and cost-efficient. Sometimes more than one lightpaths are assigned to the same node pair to make full use of the transceivers and through sharing the traffic also to reduce traffic congestion. An example is shown in case 3 of Fig. 2.4. Case 1 and case 2 in Fig. 2.4 illustrate directed lightpaths and undirected lightpaths, respectively.

An overall notation for a VT can be described as: VT(Δ, $=$ / $\leq$, *sym*/ *asym*, 1,

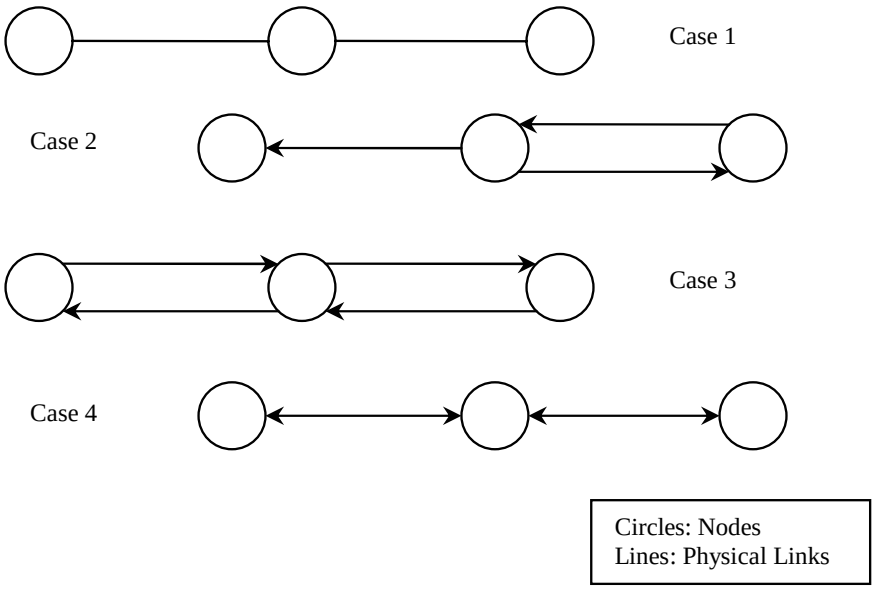


Figure 2.3: Categories and Functions of Physical Links (PL)

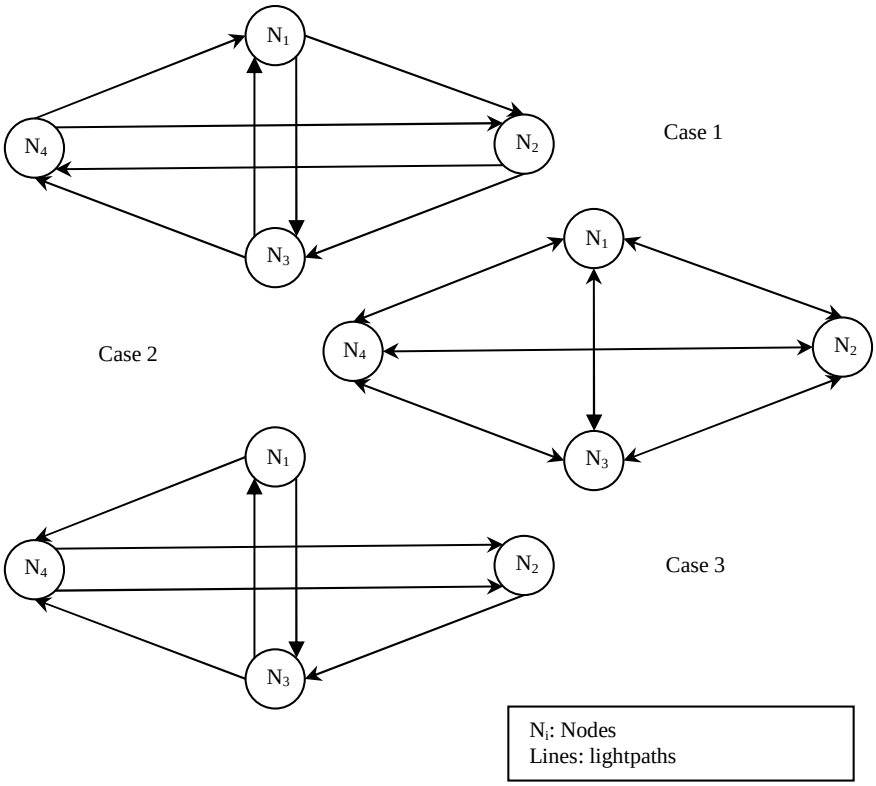


Figure 2.4: Categories and Functions of lightpaths

∞ ,∞)[12], where the meaning of parameters are explained separately as follows.

1. Δ : Indicates the node virtual degree;
2. $=$: Indicates whether using all T_x/R_x or just following the constraint;
3. *sym* / *asym*: Indicates whether is the virtual topology symmetric or asymmetric, i.e., whether is there is a lightpath from node i to node j, and also one lightpath from node j to node i which is routed through the same set of physical links.
4. 1 or an integer greater than 1: Indicates whether there are multiple edges in the virtual topology, i.e., whether one lightpath connects node pair ij, or N lightpaths exist.
5. The first ∞ : Means there is no restriction on the number of wavelengths used in each node.
6. The second ∞ : Means there is no restriction on the number of hops taken by a lightpath.

Parameters 1 to 6 should be regarded as constraints in VTD. According to the cases shown in Fig. 2.4, cases 1, 2 and 3 can be expressed as VT(2,=,asym,1,∞,∞), VT(3,=,sym,1,∞,∞) and VT(2,$\leq$,asym,2,∞,∞) respectively.

2.2 Mathematical Descriptions of VTD Problems

2.2.1 Notations and Variables

Here we introduce the formulation of the VTD problem as a mixed-integer linear programming (MILP) [12] [16] [17], starting with the notations as follows.

1. s, d Source and destination of a packet, respectively, and are normally placed in the superscript position;

2. i, j Originating and terminating nodes of a lightpath, respectively, and also known as WP;

3. q Multiplicity, the number of lightpaths between any two nodes, set as the maximal value of lightpaths. $q = 1$ when no multiple lightpaths are allowed.

4. m, n The two ends of a physical link;

5. k Wavelength denoted by a number and is placed in the subscript position.

2.2.2 Parameters

1. N Number of nodes in the network, i.e. network size. We can enumerate any node by i, where $i = 1 \ldots N$, i.e. $1 \ldots N$ is the range for all i, j, s, d, m and n.

2. $\mathbf{T}$ Traffic matrix, where T^{sd} denotes the arrival rate for packets from source (s) to destination (d), in packets/second, $s, d = 1 \ldots n$. $T_{ii} = 0$, for all $i = 1, 2, \ldots, N$.

3. $\mathbf{P}$ Physical connection matrix, where P_{mn} indicates the existence of physical links in the physical topology, e.g. $P_{mn} = X$ if there exists X fiber links connecting node m and node n; otherwise, $P_{mn} = 0$. In this thesis we consider single-fiber networks (no protection fibers are concerned), e.g. regard all P_{mn} as binary variables.

$$P_{mn} = P_{nm} = 1,$$

which indicates there are two fibers joining two nodes in opposite direction (i.e., bi-directional or symmetrical ring network). But for a unidirectional network, the fiber between two physical adjacent nodes can support only one direction transmission. In such a case, given $P_{mn} = 1$ we cannot derive $P_{nm} = 1$.

4. Δ_A, Δ_A^{Out}, Δ_A^{In} Virtual Degree (VD), note that Δ_A^{Out}, Δ_A^{In} are used to denote the number of transmitters and receivers at node A, respectively. Normally we

will set $\Delta_A^{Out} = \Delta_A^{In} = \Delta_A \leq (N-1)$; but VD for different nodes are allowed to take different values.

5. M Denotes the largest number of wavelengths each fiber is able to support.

6. **H** Virtual Hop (VH) matrix [12], where elements in the matrix, i.e. H_{ij} denote the maximum number of hops that a lightpath between node i and node j is permitted to take. If A intermediate wavelength routing nodes have to be configured for establishing a lightpath $V_{i,j}$, the hop length of that lightpath is $A+1$.

2.2.3 Variables

1. $b_{ij,q}$ Lightpath variables, where $i, j = 1, \ldots, n$, $\forall i \neq j$, b_{ij} are $N \times N$ binary (0, 1) variables denoting possible lightpaths from i to j. $b_{ij,q} = 1$, if there exist q lightpaths along the directed edge (i,j) in the virtual topology. Normally we consider $q \leq 1$, e.g. if there is one lightpath from i to j, $b_{ij} = 1$; otherwise, $b_{ij} = 0$;

2. $C^{k,q}(i,j)$ and $C^{k,q}_{m,n}(i,j)$ Wavelength assignment variables. $C^{k,q}(i,j) = 1$ denotes the q-th lightpath between i, j uses wavelength k, and $C^{k,q}_{m,n}(i,j) = 1$ if the lightpath between i, j using wavelength k is routed through physical link m, n; otherwise, $C^{k,q}_{m,n}(i,j) = 0$.

3. $P^{ij,q}_{mn}$ Lightpath routing variables. $P^{ij,q}_{mn} = 1$, if the qth lightpath between node i and node j is routed through physical link pairs (m,n); otherwise, $P^{ij,q}_{mn} = 0$.

4. $T^{sd}_{ij,q}$ and $T_{ij,q}$ Traffic intensity variables; The solution to the virtual topology design problem will specify which of the $b_{ij,q}$ take the value 1 and which take the value 0. We assume that we can arbitrarily split the traffic flow between the same pair of nodes over different wavelength paths. Let the fraction of the traffic between s-d pair (s,d) that is routed over link (i,j) (if it exists), be $T^{sd}_{ij,q}$.

   Then

$$\sum_{i,j,q} T_{ij,q}^{sd} = T^{sd}, \tag{2.1}$$

where traffic T^{sd} (in packets per second) between s-d pair (s,d) is routed over link (i,j). Since all packets belonging to a virtual circuit (along a VWP) must be routed on the same physical path, this is tantamount to assuming that the traffic in a link consists of a large number of VWPs. Thus the total traffic routed over link (i,j) is

$$T_{ij,q} = \sum_{sd} T_{ij,q}^{sd} \ . \tag{2.2}$$

2.2.4 Constraints and Objective Functions

According to the notations and variables introduced in the previous section, we investigate some commonly used constraints and objective functions in VTD problem.

Categories of Constraints

1. Traffic constraints:

   Here we identify the packets to be routed between each $(s-d)$ pair with the flow of a commodity. The left-hand side of the flow conservation constraint at node i in the network computes the net flow out of a node i for one commodity (s-d). The net flow at certain node is the difference between the egress flow of the node and the ingress flow of the node. The net flow is 0 if that node is neither the source nor the destination for that commodity ($i \neq s, d$). If node i is the source of the flow, $i = s$, the net flow equals T^{sd}, the arrival rate for those packets, and if node i is the destination, $i = d$, the net flow equals $-T^{sd}$. Here, we use $T_{ij,q}^{sd}$ as to constrain all traffic originating in one source node and directed to a destination node to flow on the same routes.

   Flow conservation at each node:

$$\sum_q \sum_j T_{ij,q}^{sd} - \sum_q \sum_j T_{ji,q}^{sd} = \left\{ \begin{array}{ll} T^{sd} & \text{if } s = i \\ -T^{sd} & \text{if } d = i \\ 0 & \text{otherwise} \end{array} \right\}, \text{for all } s, d, i, j. \tag{2.3}$$

Traffic routing constraints:

$$T_{ij,q} = \sum_{sd} T_{ij,q}^{sd}, \quad \text{for all } i,j,q. \tag{2.4}$$

$$T_{max} \geq T_{ij,q}, \quad \text{for all } i,j,q. \tag{2.5}$$

$$T_{ij,q}^{sd} \leq b_{ij,q} T^{sd}, \quad \text{for all } i,j,s,d,q. \tag{2.6}$$

Eq. 2.4 and Eq. 2.5 ensure traffic over any lightpath is not greater than the upper bound variable T_{max}, which is the minimizing objective. And Eq. 2.6 ensures that the traffic routing variables $T_{ij,q}^{sd}$ can be nonzero values only when there exists at least one lightpath from i to j, i.e. $b_{ij,q} \neq 0$.

Normally when we formulate VTD problems as MILP problems, we can consider the implementation of any routing algorithm, such as optimal routing algorithm and a simple shortest-path routing algorithm. The optimal routing algorithm allows traffic flows to be partitioned and flow fractions to be routed on several paths, but restricted by constraints on the length or hops of their routes, which aims to reduce the congestion level. The static shortest-path routing algorithm uses only one path to route the traffic generated from one source and directed to the same destination, which aims to reduce delay. However, some links may then reserve a much higher link capacity than what the real traffic demands. We demonstrate them by Eq. 2.7 and Eq. 2.8. According to the difference in their objectives, we prefer to use optimal routing algorithm since the objective

is to make full use of the wavelength capacity and hence reduce the bandwidth wastage.

Optimal routing:

$$T_{sj}^{sd} = T^{sd} \times \alpha_{sj}^{sd} \qquad \text{for all } s,d,j,\ \alpha_{sj}^{sd} \geq 0, \tag{2.7}$$

where $\sum_j \alpha_{sj}^{sd} = 1$.

Shortest-path routing:

$$\alpha_{sj}^{sd} = 1 \text{ or } 0 \quad \text{for all } s,\ d,\ j. \tag{2.8}$$

2. Virtual degree constraints: These constraints are directly related to lightpath variables $b_{ij,q}$.

$$\sum_q \sum_j b_{ij,q} = \Delta_i^{Out}, \qquad \sum_q \sum_i b_{ij,q} = \Delta_j^{In}, \qquad \text{for all } i,j. \tag{2.9}$$

Or we can use $\leq$ instead of $=$. This ensures that the number of lightpaths originating/terminating at a given node is less than or equal to the number of transmitters/receivers at that node. The number of transmitters and receivers are equal to egress and ingress degree of that node, respectively.

Factor q in variables $b_{ij,q}$ is named as multiplicity factor.

$$q \geq 1, \quad q\text{: integer.}$$

Although multiplicity connections may not always help to reduce the congestion level on lightpaths, they are important issues when dealing with failure or reconfiguration problems [2].

3. Wavelength continuity constraints:

a) Unique wavelength constraints:

$$\sum_{k=0}^{M-1} C^{k,q}(i,j) = b_{ij,q} \quad \text{for all } i,j,q. \tag{2.10}$$

and

$$C^{k,q}(i,j) \geq C^{k,q}_{m,n}(i,j) \quad \text{for all } i,j,m,n,q. \tag{2.11}$$

Eq. 2.10 above ensures that a lightpath $b_{ij,q}$ can only be assigned with one of the M wavelengths; Eq. 2.11 denotes if a lightpath $C^{k,q}(i,j)$ from i to j via wavelength k passes through physical fiber link P_{mn}, wavelength k on the physical fiber link must be reserved for this lightpath.

b) Wavelength crash constraints:

$$\sum_{q} \sum_{i,j} C^{k,q}_{m,n}(i,j) \leq 1 \quad \text{for all } m,n,k. \tag{2.12}$$

Eq. 2.12 ensures wavelength k on a physical fiber link P_{mn}, is only assigned to one lightpath if required, i.e. no two lightpaths traversing through one physical link can be assigned with the same wavelength.

c) Wavelength conservation constraints:

$$\sum_{k=0}^{M-1} \sum_{m} C^{k,q}_{m,n}(i,j) P_{m,n} - \sum_{k=0}^{M-1} \sum_{m} C^{k,q}_{n,m}(i,j) P_{n,m} =$$

$$\left\{ \begin{array}{ll} b_{i,j,q} & \text{if } n = j \\ -b_{i,j,q} & \text{if } n = i \\ 0 & \text{if } n \neq i \text{ and } n \neq j \end{array} \right\}, \quad \text{for all } i,j,q. \tag{2.13}$$

Eq. 2.13 is analogous to flow conservation given by Eq. 2.3. If lightpath $b_{ij,q}$ uses wavelength k, there is a path traversing through one (or a series of) physical fiber link(s) from i to j with wavelength k assigned to it.

4. Lightpath routing constraint:

There is another way to perform traffic routing and wavelength assignment problems without considering wavelength continuity, i.e. in a multihop network. According to the definition of lightpath routing variables $P_{m,n}^{i,j,q}$ [17] [23] [24], conservation and constraint functions can be expressed as below:

$$\sum_{n} P_{in}^{ij,q} P_{in} = b_{ij,q}, \quad \text{for all } i,j,q. \tag{2.14}$$

$$\sum_{m} P_{mj}^{ij,q} P_{mj} = b_{ij,q}, \quad \text{for all } i,j,q. \tag{2.15}$$

$$\sum_{m} P_{mk}^{ij,q} P_{mk} = \sum_{n} P_{kn}^{ij,q} P_{kn} \quad \text{for all } k \neq i,j. \tag{2.16}$$

Eq. 2.14, Eq. 2.15 and Eq. 2.16 are multicommodity-flow-based equations governing the routing of lightpath from source to destination.

$$\sum_{q} \sum_{ij} P_{mn}^{ij,q} P_{mn} \leq M, \text{for all } m,n. \tag{2.17}$$

While Eq. 2.17 ensures the total number of lightpaths passing through one fiber link does not exceed the wavelength upper bound that each fiber supports, M.

If the aim of design is to build up a symmetric virtual graph. Additional constraints are imposed as shown in Eq. 2.18.

$$P_{mn}^{ij,q}(k) - P_{nm}^{ji,q}(k) = 0. \qquad \text{for all } k,i,j,q. \tag{2.18}$$

Eq. 2.18 ensures that if there are a lightpaths from node i to node j, then we must have a lightpaths from node j to node i. And also that the routing and wavelength assignment for the lightpaths associated with the lightpaths between node i and node j traverse the same set of physical links and wavelengths, i.e. if the q-th lightpath from node i to node j is assigned wavelength ω, then the q-th lightpath from node j to node i must be also assigned wavelength ω.

In this thesis, we mainly use $P_{mn}^{ij,q}$ to control the routing and wavelength assignment in multihop networks. Through simulation we can prove that compared with using wavelength continuity constraints, the computation time and memory usage are considerably reduced by using $P_{mn}^{ij,q}$. Although it seems possible to increase the cost of implementing wavelength converters, there are two reasons to this. Firstly numerical results presented in the following chapter can demonstrate that wavelength converters are only sparsely used, so that the actual increment on cost is trivial. Secondly, the migration of technology is likely to bring down the cost of such integrated optics devices in the future, i.e. more intelligent nodes or routing devices are likely to take place.

5. Hop bound constraints:

$$\sum_{m,n} C_{m,n}^{k,q}(i,j) \leq H(i,j), \quad \text{for all } i,j,q \text{ and } k. \tag{2.19}$$

According to the definition of variable $C_{m,n}^{k,q}(i,j)$, wavelength continuity constraint ensures that each lightpath has one and only one wavelength assigned to it, which means the hop bound set up for $C_{m,n}^{k,q}(i,j)$ can only reflect information about the number of intermediate nodes one lightpath passes by. Eventually, this is regarded as an optional constraint, can be incorporated to ensure bounded packed delays in the network. This prevents long and convoluted lightpaths, i.e. lightpaths with an unnecessarily long route instead of a much shorter route, from occurring. The value of $H(i,j)$ may be selected by the network designer.

6. Non-negativity and integer constraint:

$$T_{ij,q}^{sd},\ T_{ij,q},\ T_{max} \geq 0, \quad \text{for all } i,j,s,d,q. \tag{2.20}$$

$$b_{ij} \in \{0,1\}, \quad \text{for all } i,j. \tag{2.21}$$

Objective Functions

1. Minimize the congestion level [1] [16] [30].

   Objective function: minimize T_{max}, where

$$T_{max} \geq T_{ij,q}, \quad \text{for all } i,j,q. \tag{2.22}$$

   This also can be defined as *maximizing offered load*, which is equivalent to minimizing maximum traffic flow in a link.

2. Minimize the average packet hop distance H_{ave}.

   Assuming the capacity of each lightpath is C, this objective function can be expressed as follows.

   Objective function: minimize H_{ave}, where

$$H_{ave} = \frac{1}{\sum_{s,d} T^{sd}} \sum_{s,d} H_{sd} T^{sd}. \tag{2.23}$$

   In the presence of the balanced network flows, minimizing the average packet hop distance is equivalent to maximizing the aggregate throughput of the network [8] [1].

   There is one additional constraint:

$$\sum_{s,d} T^{sd}_{ij,q} \leq \beta b_{ij,q} \times C \quad \text{for all } i,j,q. \tag{2.24}$$

   Eq. 2.24 specifies the capacity constraint in the formulation, where β denotes the maximum loading per channel ($0 < \beta < 1$) to prevent the queuing delay on a lightpath from getting unbounded by avoiding excessive link congestion.

3. Minimize the number of wavelengths used [14] [17] [32] [33].

   Objective function: minimize

$$\max_{m,n} \sum_{k=0}^{M-1} \sum_{q} \sum_{i,j} C_{mn}^{k,q}(i,j) P_{mn},$$

subject to wavelength capacity C (per fiber) constraint, which ensures that traffic through a lightpath cannot exceed the wavelength capacity.

$$\sum_{s,d} T_{ij,q}^{sd} \leq C, \qquad \text{for all } i,j,q.$$

Without considering the wavelength capacity constraint, the objective function can be formulated as to balance the wavelength assignment on each fiber:

Objective function: minimize M

where

$$M \geq \sum_{ij,q} P_{mn}^{ij,q}, \quad \text{for all } m\ n.$$

4. Minimize the average connection cost [31].

   This objective function is defined as an efficiency of a network in establishing lightpaths between the node pairs.

   Objective function: minimize

$$z = \sum_{i=1}^{N} \sum_{j=1}^{N} c_{ij} b_{ij}, \tag{2.25}$$

   where c_{ij} is the cost of establishing a virtual path between the terminators (i,j). The cost function reflects the required properties. It is defined as:

$$c_{ij} = \frac{1}{d_{ij}} + f(d_{ij})^{\beta}, \tag{2.26}$$

   where d_{ij} is the distance between the node pairs (i,j) and $f(d_{ij})$ is a function of

d_{ij} which represents the resources that would be committed if a virtual path is assigned along the directed node pair (i,j).

5. Minimize the average message delay [29].

   Objective function: minimize

$$\sum_{s,d} T^{sd} L_{sd}, \tag{2.27}$$

   where the variable L_{sd} is the length of the virtual path between nodes s and d, and it is given as follows.

$$L_{sd} = \sum_{i,j} P_{ij}^{sd} D_{ij},$$

   Within which, P_{ij}^{sd} are lightpath routing variables. Variable D_{ij} is the length of the lightpath between nodes i and j, and it is specified by:

$$D_{ij} = \sum_{m,n} P_{mn}^{ij} d_{mn}, \tag{2.28}$$

   where fiber length d_{mn} from node m to node n is expressed as a propagation delay-related parameter (in time units). Since fiber links are bi-directional, $d_{mn} = 0$ if and only if $P_{mn} = 0$.

2.2.5 Classical MILP Formulation

In the previous section, although we have listed quite a few objective functions and many constraints, we only consider one objective function with some relevant constraints to formulate a VTD problem. The objective function and constraints chosen depend on the design parameter to be optimized. For example, here we demonstrate the classical MILP with an objective to minimize the congestion level, i.e. using the objective function Eq. 2.22.

Objective function Eq. 2.22,

subject to

1. Traffic routing, which includes traffic constraints: Eq. 2.3, Eq. 2.4, Eq. 2.5, Eq. 2.6, together with constraints Eq. 2.7 or Eq. 2.8 optionally.
2. Virtual degree constraints: Eq. 2.9.
3. Wavelength assignment: includes either wavelength continuity constraints Eq. 2.10, Eq. 2.11, Eq. 2.12 and Eq. 2.13 when considering single hop, or lightpath routing constraints Eq. 2.14, Eq. 2.15, Eq. 2.16 and Eq. 2.17, when considering multihop.
4. Nonnegative constraints: Eq. 2.20, Eq. 2.21.

Simple Examples

Below we present examples to clarify the definitions of simplex VTs and duplex VTs used in this thesis. Moreover we discuss the numerical results obtaining on congestion levels derived from different cases. To calculate the exact MILP results, here we only consider optimal routing algorithm by using Eq. 2.7 and consider to build up multihop networks by using lightpath routing constraints Eq. 2.14, Eq. 2.15, Eq. 2.16 and Eq. 2.17.

Network and traffic information:

1. This simple network is made up of four nodes: node 1, node 2, node 3, node 4.
2. The traffic request T^{sd} in this network is given as follows. $T^{13} = 1$, $T^{21} = 1$, $T^{24} = 1$, $T^{32} = 1$, $T^{34} = 1$, $T^{41} = 1$ and $T^{42} = 1$.

Results obtained are shown in Fig. 2.5, where case 1 has a VT consisting of two partitioned (or disconnected) subclusters if we build up duplex virtual connections with $VD = 1$. It is *partitioned* because although four WPs are built up, not all VWPs can be supported by existing WPs.

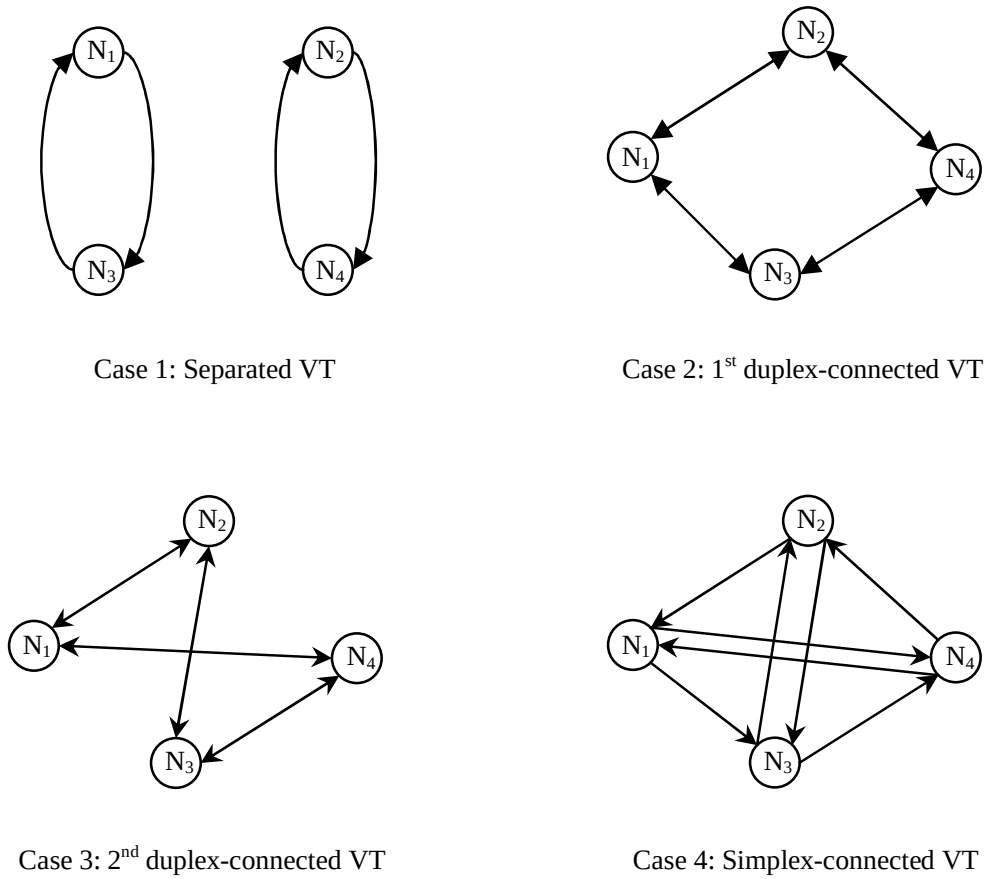


Figure 2.5: VT results for various design

With $VD = 2$, we can derive two duplex VTs shown in case 2 and case 3 of Fig. 2.5. Their congestion levels are both 1.5000. However, the simplex result derives a congestion level of 1.3333 shown in case 4 of Fig. 2.5. Compared to the duplex VTs in case 2 and case 3, simplex VT seems to give better congestion control due to more flexibility is given when performing VTD.

So in the subsequent part of this thesis, we only consider to build up a simplex VT without multiplicity (i.e. $q = 1$), and the physical network is based on single-fiber connected packet-switched network.

2.3 Complexity Analysis

2.3.1 Mathematical Analysis

Fig. 2.6 provides an overview on the problems that MILP intends to solve. The physical topology presents basic information about node locations, fiber lengths and node interconnection status, as shown in Fig. 2.6(a). The number of the wavelengths used in the optical layer may vary in different fibers, e.g. fiber 1 interconnecting node 1 and node 2 may support wavelength λ_1, λ_2, λ_3, while fiber 2 interconnecting node 2 and 3 may support wavelength λ_2, λ_3, λ_4. To unify this problem, we assume that all fibers carry the same set of wavelengths, as shown in Fig. 2.6(b). Traffic flows from one node to another are not necessarily equal, and at a certain node, ingress and egress traffic are not necessary to be balanced either, as shown in Fig. 2.6(d). All of the three parts mentioned above are given as inputs. And VT shown in Fig. 2.6(c) and wavelength assignment shown in right-hand side of Fig. 2.6(b) are the designs to be obtained with VD as direct constraints. The VT provides how the lightpaths connectivity should be made to optimize the transmission parameter such as minimum congestion level, and should also disclose through what wavelength and physical path to realize the traffic flows.

Consider in a regular physical topology, each node has In/Out virtual degree Δ. To solve MILP formulation, the number of constraints and the number of variables grow approximately as $O($ N^3 $\times$ number of wavelength $\times$ number of physical edges $\times$ multiplicity factor), and for each iteration the searching space for lightpaths might be $O($ Choose Δ from $N \times (N-1)$); while the number of traffic variables might be $N \times \Delta$.

Along with the increase of network size, the increase in the number of unknown variables to be solved, computation time and memory required are obtained by computation and shown in Fig. 2.7 and Fig. 2.8. Subgraphs show the growth of 'number of variables', 'solution time', 'memory required' and 'constraints', respectively.

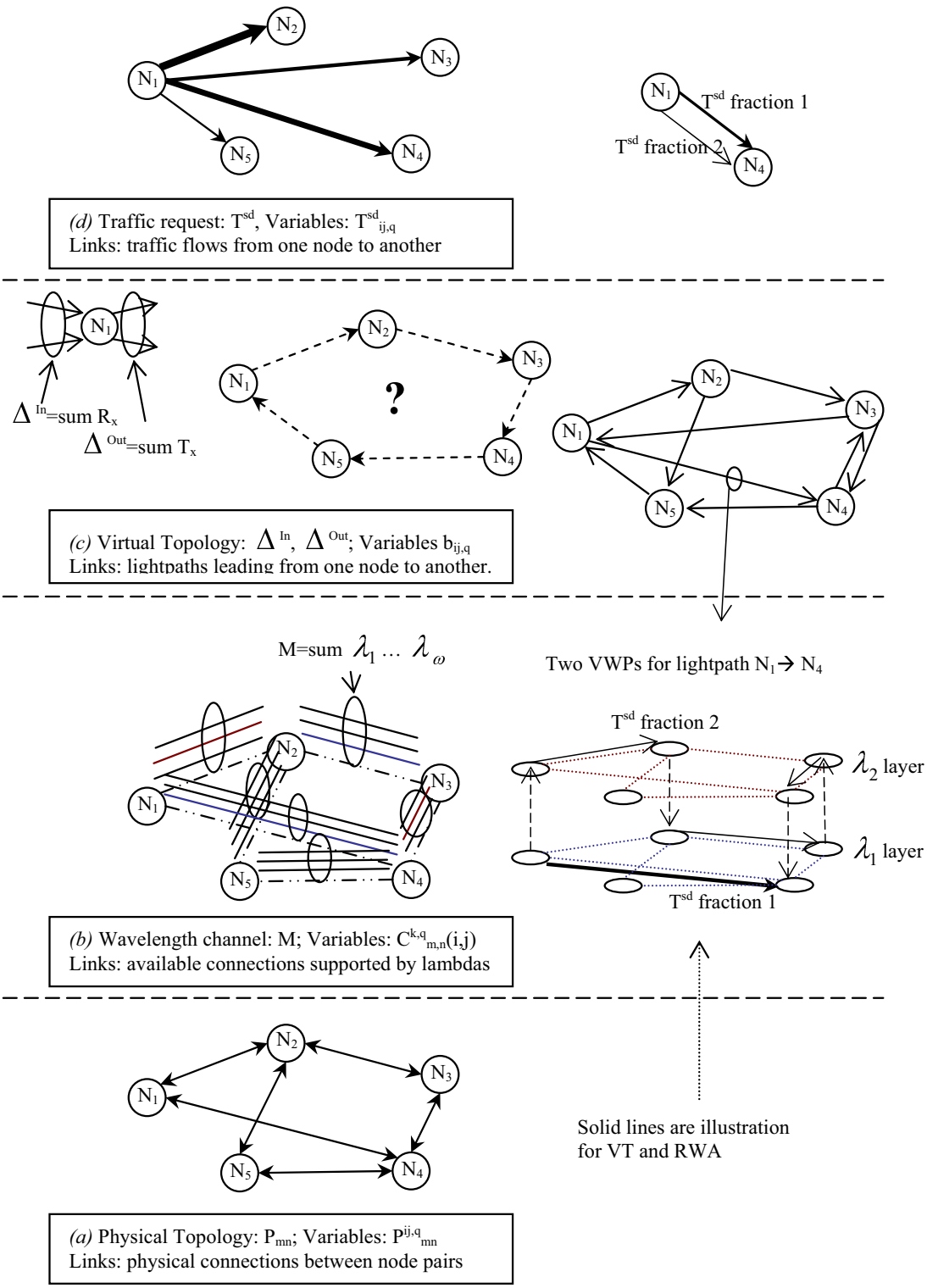


Figure 2.6: Flowchart of MILP programming

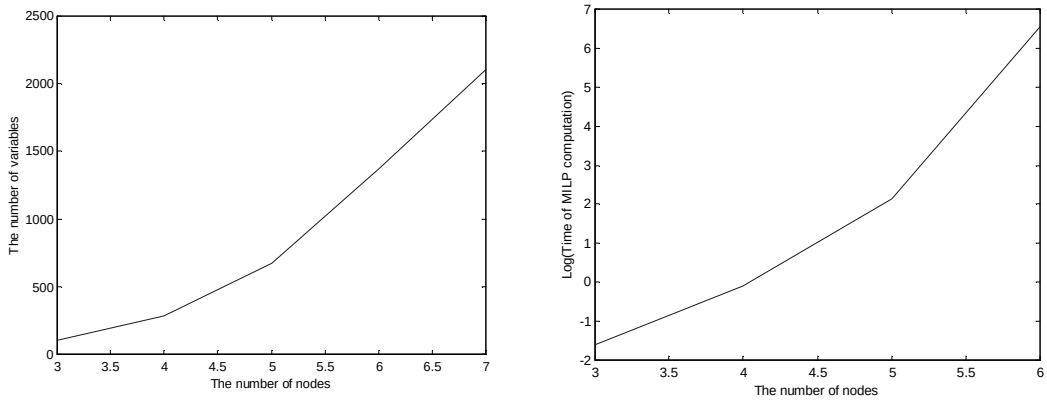


Figure 2.7: Growth of complexity along with network size-1

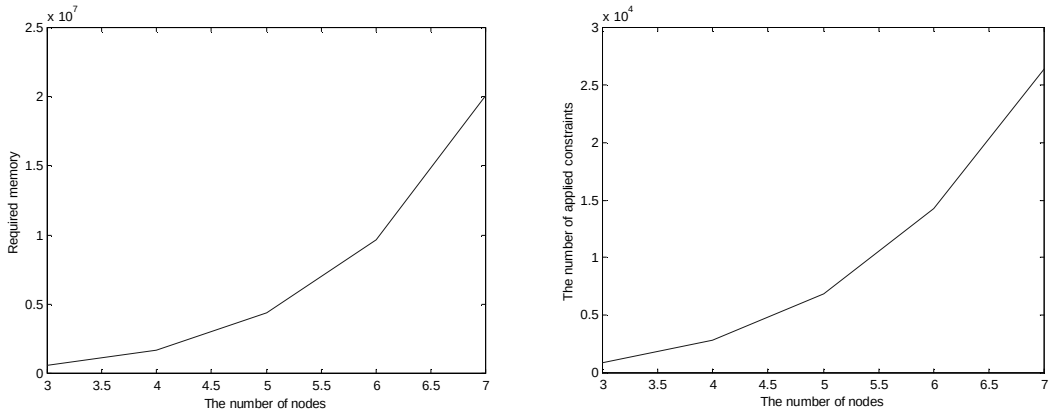


Figure 2.8: Growth of complexity along with network size-2

2.3.2 Complexity Reduction

To reduce the complexity of solving a MILP problem, one mathematical method named linear programming (LP) was explored. By changing all integer variables $b_{ij,q}$ into real variables $b^L(ij,q)$ varying between 0 and 1, MILP turns to LP. Moreover, any feasible solution of the MILP is also a feasible solution of the LP problem, but LP usually have other feasible solutions. If the solution of LP happens to be a feasible solution of the MILP, their objective function values will be equal. Otherwise the value of LP is a lower bound on the value of MILP. So this is why we call this lower bound or LP bound. Since LPs are easy to solve, we can obtain an approximate solution to this mathematical program by using the techniques defined as *LP-relaxation* and *rounding*, shown as follows.

LP-relaxation method derives a lower bound solution $\overline{T}_{max}(L)$ to the congestion control variable T_{max} defined in Eq. 2.5 and Eq. 2.6. In this approach, variable $\overline{T}_{max}(L)$, where the integer L marks the number of iterations, can be updated iteratively according to Eq. 2.29.

$$\overline{T}_{max}(L+1) \geq \sum_{s,d} \overline{T}^{s,d}_{ij,q} + \overline{T}_{max}(L)(1 - b^L(ij,q)), \quad \textit{for all } i,j,q. \tag{2.29}$$

Initially, set $\overline{T}_{max}(L=1) = 0$. Then solve the LP-relaxation to get an improved lower bound $\overline{T}_{max}(L+1)$, which is known as the iterative LP-relaxation lower bounds. After several iterations, the calculation is stopped after reaching a given error bound, e.g.,

$$\overline{T}_{max}(L+1) - \overline{T}_{max}(L) \leq 0.01.$$

Then arrange the values of $b^L(ij,q)$ obtained in a decreasing order and start with the largest value, set all successful value $b^L(ij,q)$ to 1 without violating the virtual degree constraints, otherwise set it to 0. This method is called *LP-roundup* [12] [20]. When the LP lower bound and the LP-relaxation upper bound obtained by using the

rounding algorithm and solving the routing-LP are close to each other, we obtain a good approximation to the optimal congestion value of MILP.

Chapter summary

We presented an overview on the theories about Virtual Topology Design problem in Optical WDM Networks. And we categorized and reviewed some existing models about designing the VT of an optical WDM Network. The formulation and theory introduced and developed in this chapter are the foundation for our research work that will be discussed in details in the coming chapters.

Chapter 3

Heuristics Algorithms to Solve VTD Problems

3.1 Decomposition VTD problems and Analysis

Ever since [41] [42], the design of optimal virtual topologies has been intensively studied. In general, the optimal virtual topology problem has been hypothesized to be NP-hard [2] [3] [6] [8], which means that the problem cannot be solved optimally for large problem sizes, unless one resorts to exhaustive search [11]. Therefore, heuristic algorithms have been developed to achieve suboptimal solutions to VTD problem. Usually heuristic algorithms decompose VTD problems into four sub-problems [23] [24], which can be described as follows.

a) Generate a virtual topology by using simple rules to get b_{ij}, i.e. the virtual connectivity between nodal transmitters and nodal receivers. We can see it mainly focuses on designing WPs.

b) Determine how the lightpaths are imposed on physical links, i.e. lightpath routing.

c) Assign wavelengths to lightpaths, i.e. wavelength assignment under certain constraints—using wavelength continuity constraint or maximum-wavelength constraint optionally.

d) Route traffic over the WPs given by sub-problem a. This sub-problem mainly deals with designing VWPs for traffic flows.

Sub-problem a addresses how to properly utilize the limited number of available transmitters and receivers. Most of the heuristics are defined simply using the traffic requests or certain regular graphs to build up a set of WPs fulfilling the virtual degree constraints. The developed heuristic algorithms normally aim to obtain an objective function such as minimizing delay or congestion, by jointly solving sub-problem a with sub-problem b or d. Sub-problem b and c both deal with proper usage of the limited number of available wavelengths. Moreover sub-problem b by routing lightpaths over physical links decides the length of each WP, which is strongly delay-related. Sub-problem c aims to avoid clash when making wavelength assignment, a clash occurs when two lightpaths with the same wavelength assigned to them happen to have a common directed physical edge. Normally there are two approaches to obtain the minimum number of required wavelengths. One way is to assign additional wavelengths to lightpaths where wavelength clash occurs, until all lightpaths have been assigned. The other way is to construct a path-graph [21] [22], solve the chromatic number of the path-graph and obtain the number of wavelengths required. Both approaches were tested and there is no significant difference in the minimum number of wavelengths required. Here, we are going to modify the second approach, track the lightpath routing and constrain the number of lightpaths sharing the same physical edge within the maximum number of wavelengths available on each fiber. As we consider packet-switched networks, the optimal routing algorithm allows traffic flows to be partitioned, and if necessary the fractions of traffic can be routed over several paths. Hence sub-problem d is strongly related to both congestion levels and delay control. The number of VWPs passing through a WP affects the congestion level, and the number of WPs each VWP passes through affects the propagation delay.

Although these sub-problems are not necessarily independent, heuristic algorithms solve them separately to obtain suboptimal results, which is the reason why heuristic algorithms achieve a lower complexity compared to the exact MILP. Results obtained by the heuristic algorithms are compared to the optimal results obtained by exact MILP, so that we can evaluate the accuracy of the heuristic algorithms and further improve them. Detailed introduction and numerical analysis on VTD problems can be found in [37] and [20].

We survey and categorize the VTD heuristic algorithms into several groups. All the groups optimizing one pre-defined objective function, however, they make different approximations on some of the constraints. The first group focuses on how to present a simple set of WPs only, general constraint is applied on each node. One batch of this group is trying to implement regular topologies, such as mentioned in [4] [43] [44] [45] [46] or building up a VT with prior consideration given to the PT. Heuristic algorithms of this group work better for evenly distributed traffic models. Moreover, there are several advantages of using regular topologies as VTs: regular topologies are well understood and results regarding their bounds and averages are comparatively easier to derive, and hence suitable for comparison of performance. Routing of traffic on a regular topology is usually simpler and results are available in the literature. In addition, using a PT-based VT possesses inherent delay-minimizing characteristics. The other batch originates from the idea of maximizing throughput in a circuit-switched network, e.g. it tries to set up WPs to support as many direct traffic flows as possible.

The second group of heuristic algorithms is calculation-based, while also applies general constraint on nodes, e.g. [25] using GA algorithm or Turbo search etc.. These heuristic algorithms use complicated calculation methods to approach to the optimal objective value.

The third group has chosen a pre-defined objective function to optimize, with the need to fulfill some original constraints, where modification of some other constraints can be attempted and evaluated. The optimization is done by not very complicated calculation, generally because the sub-problems (those not included in the optimization)

are solved separately. This group performs well on approaching an optimal objective function, and does not lead to a big burden on calculation. Our heuristic algorithms mainly belong to this group.

Before introducing the existing heuristic algorithms, we first present information about a four-node network model used for calculations and comparisons.

1. Network size: $N = 4$, e.g there are four nodes in the network under investigation. Each node has the processing capabilities discussed in the previous chapter, it is capable to deal with packet switching and wavelength conversion (when necessary).

2. PT: 110 01 1 (refer to the appendix for the notation used)

   e.g. $P_{12} = 1$, $P_{13} = 1$, $P_{14} = 0$, $P_{23} = 0$, $P_{24} = 1$, $P_{34} = 1$, eventually we can derive that: $P_{21} = 1$, $P_{31} = 1$, $P_{42} = 1$; $P_{43} = 1$.

   And the distance between node pairs are: $D_{12} = D_{21} = 1$; $D_{13} = D_{31} = 1$; $D_{24} = D_{42} = 1$; $D_{34} = D_{43} = 2$.

3. Traffic matrix: as shown in Table. 3.1, where the number in the first column denotes the source node and the number in first row denotes the destination node.

Table 3.1: Traffic Matrix T in a 4-node network

T^{sd}	1	2	3	4
1	0	0.75	0.4	0.5
2	0.45	0	0.1	0.9
3	0.15	0.6	0	0.1
4	0.3	0.35	0.80	0

4. Wavelength constraint: $M = 2$. There are two wavelengths available on each fiber, denoted as λ_1, λ_2.

3.2 Pseudo Descriptions, Assumptions and Analysis

3.2.1 MILP(exact) results

The MILP problem can be described as formulated in the previous chapter, with an objective function to derive an optimal congestion level.

1. Objective function, Eq. 2.22.
2. Traffic constraints, Eq. 2.3, Eq. 2.4, Eq. 2.5 and Eq. 2.6.
3. Virtual degree constraints, Eq. 2.9.
4. Lightpath routing with Eq. 2.14, Eq. 2.15 and Eq. 2.16 are combined with wavelength constraint Eq. 2.17.
5. Nonnegative constraints Eq. 2.20 Eq. 2.21.

Numerical results providing the exact MILP congestion values and delay are presented. VTD results for $VD = 1$, $VD = 2$ and $VD = 3$ are shown in Table 3.3,Table 3.4, and Table 3.5, respectively.

Table 3.2: MILP-VTD results on congestion and traffic-weighted delay values

VD	VT	Congestion	Traffic-weighted delay
1	0100 0001 1000 0010	2.7000	2.2037
2	0110 1001 0101 1010	0.9666	1.7653
3	Full connected	0.5928	1.9138

Table 3.3: MILP-VTD results(VD=1)

No.	lightpaths	Physical links	Wavelength number	Delay
1	1→2	1→2	1	1
2	2→4	2→4	1	1
3	3→1	3→1	1	1
4	4→3	4→3	1	2

Table 3.4: MILP-VTD results(VD=2)

No.	Lightpaths	Physical links	Wavelength number	Delay
1	1→2	1→2	1	1
2	1→3	1→3	1	1
3	2→1	2→1	1	1
4	2→4	2→4	1	1
5	3→2	3→1→2	2	2
6	3→4	3→4	1	2
7	4→1	4→2→1	2	2
8	4→3	4→3	1	2

Table 3.5: MILP-VTD results(VD=3) (full-connected)

No.	Lightpaths	Physical links	Wavelength number	Delay
1	1→2	1→2	1	1
2	1→3	1→3	1	1
3	1→4	1→2→4	2	2
4	2→1	2→1	1	1
5	2→3	2→1→3	2	2
6	2→4	2→4	1	1
7	3→1	3→1	1	1
8	3→2	3→4→2	2	3
9	3→4	3→4	1	2
10	4→1	4→3→1	2	3
11	4→2	4→2	1	1
12	4→3	4→3	1	2

We use Fig. 3.1 to illustrate the intermediate results obtained. Fig. 3.1(a) shows the physical topology of the four nodes network, where each fiber link between any two nodes can all carry two wavelength λ_1 and λ_2 for transmission.

For both cases, $VD = 1$ and $VD = 2$, the variables b_{ij} $(\forall i, j)$ and T_{ij}^{sd} $(\forall s, d, i, j)$ are unknown to be determined to minimize the traffic congestion. The process of solving includes determining the variables α_{ij}^{sd} (i.e. T_{ij}^{sd} shown in Eq. 2.7).

The virtual topology for $VD = 1$ is shown using a directed graph in Fig. 3.1(b). Since $VD = 1$, each node can only have an ingress/egress lightpath. So in this case, four WPs($1 \rightarrow 2$, $2 \rightarrow 4$, $4 \rightarrow 3$ and $3 \rightarrow 1$) are built up, and only one wavelength is available for the transmission. The lightpath from node 1 to node 2, WP($1 \rightarrow 2$),

carries traffic from VWPs($1 \rightarrow 2$, $3 \rightarrow 4$, $1 \rightarrow 3$, $1 \rightarrow 4$, $3 \rightarrow 2$ and $4 \rightarrow 2$). The amount of traffic can also be found in the table shown in Fig. 3.1(b).

Similarly, Fig. 3.1(c) shows the virtual topology obtained by solving the MILP with $VD = 2$. The interpretation of the result is similar, except the wavelength assignment of the VWPs. For example, traffic from node 4 to node 2 is transported via two VWPs, VWP($4 \rightarrow 3 \rightarrow 2$) and VWP($4 \rightarrow 1 \rightarrow 2$). The first fraction passes through WP($4 \rightarrow 3$) via λ_1 then it is converted to λ_2 on WP($3 \rightarrow 2$); while the second fraction passes through WP($4 \rightarrow 1$) via λ_2 , later it is converted to λ_1 on WP($1 \rightarrow 2$). However, traffic from node 2 to node 1 goes directly through WP($2 \rightarrow 1$) via wavelength λ_1.

3.2.2 Descending Traffic Sequence VTD algorithm(DTS-VTD)

This is one of the heuristic algorithms from the second batch of the first group being categorized in the former section. It can be described as follows:

1. Sort the average traffic flows in traffic matrix **T** by descending sequences.
2. Assign lightpath b_{ij} to the highest traffic flow T_{ij}. Replace T_{ij} by subtracting the second highest traffic flow value — this is to ensure that if the traffic flow is abnormally high, a second lightpath can be assigned.
3. Repeat step 2 till both virtual degree constraints and wavelength constraints can no longer be satisfied.

This heuristic aims to route high traffic flows directly so that reduce both congestion value and traffic-weighed hop distance. In this way, this heuristic algorithm enables the setup of multiplicity and may not always derive a good result.

Table 3.6: DTS-VTD results on congestion and traffic-weighted delay values

VD	VT	Congestion	Traffic-weighted delay
1	0100 0001 1000 0010	2.7000	2.2037
2	0101 1001 0100 1010	1.3000	2.3241
3	Full connected	0.5928	1.9138

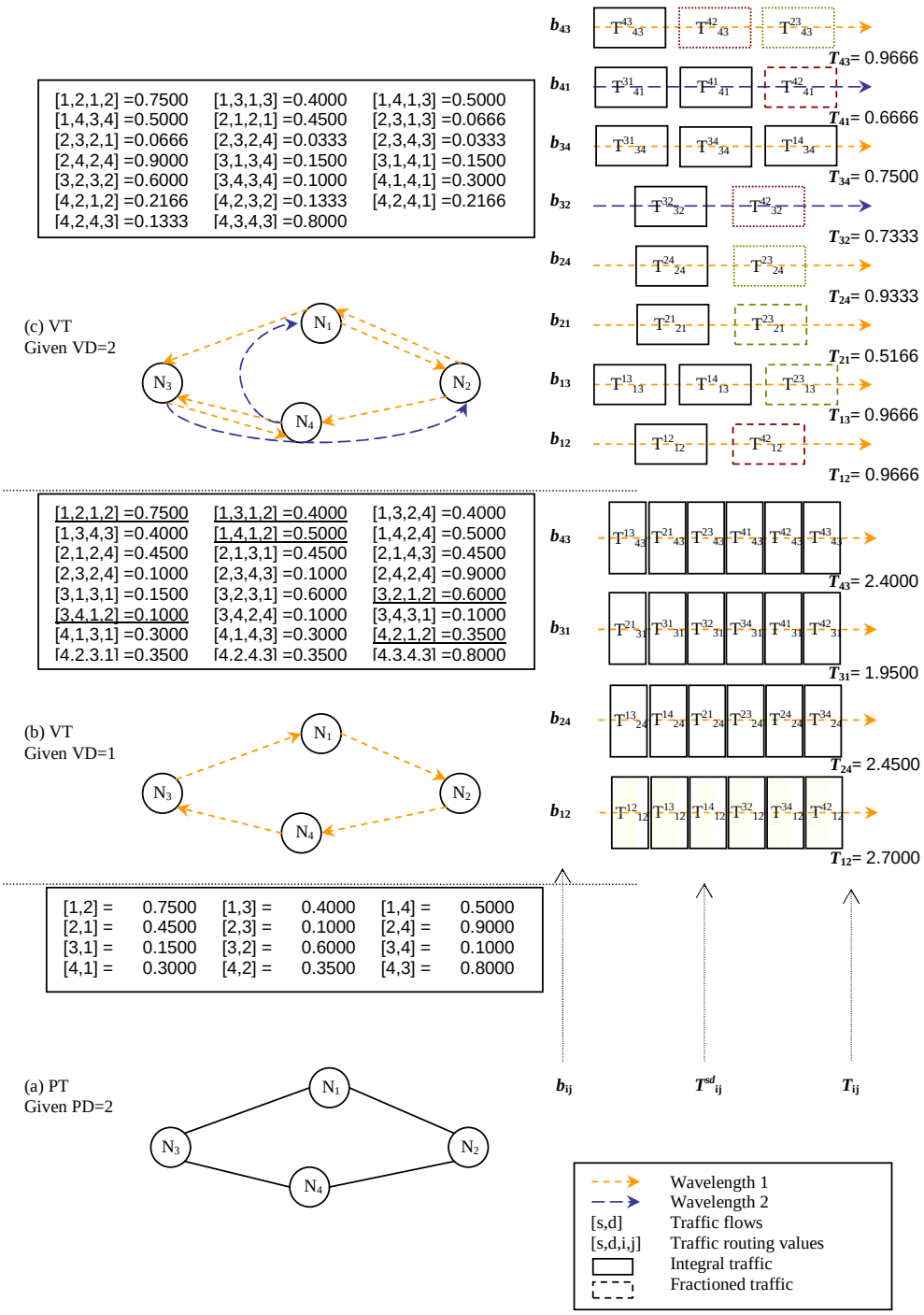


Figure 3.1: Illustration of MILP-VTD results on VD=1 and VD=2 cases in a 4-node packet-switched network

The results when DTS-VTD is applied to $VD = 1$ and $VD = 2$ cases are shown in Table 3.7 and Table 3.8, respectively. Since this heuristic algorithm does not set constraints on the T_x/R_x utilization, we can see in case of VD=2, only one pair of transceivers on node 3 is occupied, while the other pair is left idle, as shown in Table 3.8.

Table 3.7: Design results by DTS-VTD(VD=1)

Iteration	Lightpath	Physical link	Wavelength number	Delay
1	2→4	2→4	1	1
2	4→3	4→3	1	2
3	1→2	1→2	1	1
4	3→2	No R_x at 2		-
5	1→4	No T_x at 1		-
6	2→1	No T_x at 2		-
7	1→3	No T_x at 1		-
8	4→2	No T_x at 4		-
9	4→1	No T_x at 4		-
10	3→1	3→1	1	1

Table 3.8: Design results by DTS-VTD(VD=2)

Iteration	Lightpath	Physical link	Wavelength number	Delay
1	2→4	2→4	1	1
2	4→3	4→3	1	2
3	1→2	1→2	1	1
4	3→2	3→4→2	1	3
5	1→4	1→2→4	2	2
6	2→1	2→1	1	1
7	1→3	No T_x at 1		-
8	4→2	No R_x at 2		-
9	4→1	4→3→1	2	3
10	3→1	No R_x at 1		-
11	1→2	No T_x at 2		-
12	3→1	No R_x at 4		-

3.2.3 Minimum-Delay VTD algorithm (MD-VTD)

This heuristic algorithm only works when the given VD at each node is greater than the PD at the same node in the physical topology. It can be described as follows:

1. Create a pair of directed lightpaths for each physical link in the physical topology.

2. Sort the traffic requests in the network in a descending order. Start from the top of traffic stack, place lightpaths between the node pairs when VD constraints are not violated, otherwise, jump to the next candidate.

3. If there still exist non-utilized wavelengths or T_x,R_x, add the remaining lightpaths for the randomly chosen (s,d) pairs.

This heuristic ensures that a shortest optical path exists for every node pair to minimize delay, but it may not perform well in term of congestion. Besides, this heuristic enables multiple lightpath.

This heuristic algorithm is also explored using the 4-node network. Since the algorithm only works when VD is greater than PD, i.e., in this case MD-VTD is only valid when VD on each node is greater than 2. There is only one possible status to discuss about, i.e. when $VD = 3$.

VTD generated from the physical topology by MD-VTD is given by:

0110 1001 1001 0110, which includes directed links:

1→2 2→4 4→3 3→1,

2→1 4→2 3→4 1→3.

There are still a few unassigned lightpaths given by:

2→4 4→3 1→2 3→1.

In this case, four pairs of lightpaths are duplicated. But the computation results show that multiplicity may not always be helpful to reduce the congestion level, as we can see here congestion level of MD-VTD ($q = 2$) is $T_{max} = 0.6500$, however the result provided by MILP is $T_{max} = 0.5928$.

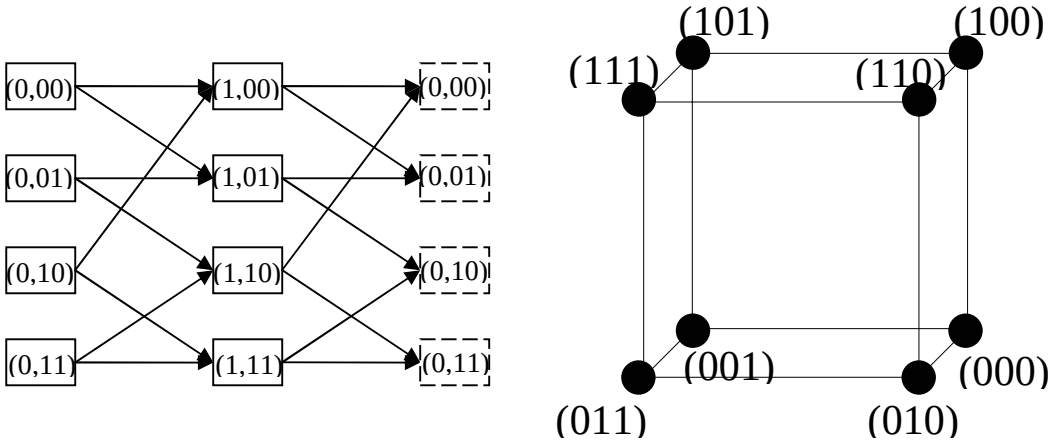


Figure 3.2: Shufflenet(VD=2) and Hypercube(VD=3) models for 8 nodes

3.2.4 Traffic-Independent VTD algorithm (TI-VTD)

Assumption for this heuristic is that the VD of each node is not less than the degree of physical topology, which is implicit in the heuristic listed below. Generally, using regular topologies in VTD is also theoretically 'traffic-independent', behind which the principle is to map the edges and nodes of a regular topology to the given physical topology. There are many well-known regular architectures, such as 'Manhattan streets', 'Shufflenets' and 'hypercubes' as shown in Fig. 3.2 [4] [2]. Some literatures have discussed this topic in detail [43] [44] [45].

Generally, a TI-VTD algorithm can be described as follows.

1. : For each node, build up its lightpaths to the neighbors (build up one hop connections).

2. : Add more lightpaths to the virtual topology by assigning more links to two-hop, three-hop neighbors of each node's till it reaches the virtual degree constraint.

Normally, VTs generated by TI-VTD have no duplicate links. Its objective is to find an optimal embedding method to assign nodes into exact positions in regular topologies, to optimize the relevant objective function.

Numerical results of a general TI-VTD, based on the network model and traffic given before are presented in Table 3.9.

Table 3.9: TI-VTD results

VD	1	
VT	0100 0001 1000 0010	0010 1000 0001 0100
Congestion	2.7000	3.4500
RWA	1→2→4→3→1 via λ_1	1→3→4→2→1 via λ_1
VD	2	
VT	0110 1001 1001 0110	
Congestion	0.9750	
RWA	1→2→4→3→1 and 1→3→4→2→1 via λ_1	
VD	3	
VT	0111 1011 1101 1110	
Congestion	0.5928	
RWA	VD=2 case plus 1→4, 4→1 via λ_2; 2→3, 3→2 via λ_2	

3.2.5 Randomly assigning VTD algorithm (R-VTD)

This heuristic algorithm is mainly designed and used as a reference to compare the performance of different heuristic algorithms. Obviously since it is designed without any control, we hope it will perform worse than other heuristic algorithms.

1. Randomly assign lightpaths between node pairs without violating the virtual degree constraints.
2. Optimally route the traffic flows on the virtual topology generated by step I.

3.2.6 Shortest-Hop VTD algorithm (SH-VTD)

This is one of the heuristic algorithms in the third group.

First introduce the definition of a key parameter:

$$F_{sd} = r_{sd} \times h_{sd},$$

where h_{sd} denotes the hop count of the minimum hop route between node s and node d on the physical topology, and r_{sd} denotes the traffic weight on link sd, where $r_{sd} = \frac{\sum_{sd} T^{sd}}{T^{sd}}$, $\forall T^{sd} \neq 0$ and $r_{sd} = 0$, otherwise. Hops defined here are related to the lightpath

routing, i.e. the number of physical links one lightpath passes through.

This heuristic algorithm can be described as follows.

1. Calculate the matric F_{sd}, and sort the F_{sd} values by an ascending sequence.
2. Place the lightpath between two nodes if there exists a fiber.
3. Select the node pair sd from the top of F_{sd} sequence, i.e. where s and d are indices giving $\min_{sd} F_{sd}$. If $F_{sd} = 0$ go to Step 5; otherwise, go to Step 4.
4. Find the minimum hop route between the node pair sd, and check the availability of wavelengths and then assign a lightpath to related sd pairs. If so, use the wavelength with lowest index to establish the lightpath. Then set $F_{sd} = 0$ and go back to Step 3. If there is no available wavelength, also set $F_{sd} = 0$ and go directly back to Step 3.
5. Repeat step 3 and 4 till both virtual degree constraints and wavelength constraints can no longer be satisfied. If there still exist non-utilized wavelengths, lightpaths are configured randomly as much as possible.

The purpose of this heuristic is trying to include hop into traffic flow, which tried to assign short lightpaths to distant and large traffic request. However, the disadvantage of this heuristic algorithm is that it may also cause a neighboring traffic request to be routed through a set of convoluted physical links, meanwhile this algorithm has no constraints on congestion levels. We can see this heuristic algorithm is more complicated than the previous heuristic, because more variables are considered and calculated.

3.2.7 Design a VT only based on WDM connection information (WDM-VTD)

This heuristic only concerns about the number of wavelengths available on each physical WDM links, and its objective is to design a VT with all resources (wavelengths and transceivers) efficiently used in supporting the traffic flow.

This heuristic algorithm can be described as follows.

1. Sort the wavelength available on each physical WDM connection between node pairs in a descending order of appearing frequency, which means a list starts with the most popular (available on most physical WDM links) wavelength.

2. Without violating the nodal degrees or wavelength conversion, try to maximize the total number of lightpaths.

Obviously when wavelengths on physical paths are all the same, this heuristic will lead to a similar VT derived by R-VTD heuristic algorithm.

3.3 Enhanced Heuristic Algorithm and A New Objective Function

A good heuristic algorithm should provide the option to obtain balancing between varieties of resources, e.g. wavelengths, transceivers and converters. It would be even better if the relative emphasis between these resources can be flexibly controlled. With this in mind, new heuristic algorithms are developed and discussed in the following section.

3.3.1 Discussion on Potential Problems of VTD

Problem Demonstration

Firstly, we demonstrate one possible problem that some heuristic algorithms may encounter with, as a result that a heuristic algorithm does not consider all constraints when designing VTs. Most heuristic algorithms when neglecting certain consideration may result in topologies that are unexpected by the designers, although those VTs still fulfill the VD constraints. Some heuristic algorithms may result in generating several subclusters, each subcluster is stand-alone by itself. This issue has not been discussed

Table 3.10: A special case declaring the disadvantage of heuristics like DTS-VT

Nodes	1	2	3	4	5	6	Nodes	1	2	3	4	5	6
1	0	α	α	δ	δ	δ	1	0	1	1	0	0	0
2	α	0	α	δ	δ	δ	2	1	0	1	0	0	0
3	α	α	0	δ	δ	δ	3	1	1	0	0	0	0
4	δ	δ	δ	0	β	β	4	0	0	0	0	1	1
5	δ	δ	δ	β	0	β	5	0	0	0	1	0	1
6	δ	δ	δ	β	β	0	6	0	0	0	1	1	0

before in the literature, but it is clear that some heuristic algorithms will result in unconnected topology, as will be illustrated shortly. In such a case, the lightpaths are not capable to support the transmission of all VWPs as shown in Table 3.10, where $\alpha >> \beta >> \delta$.

Traffic flows are listed in the left part of the table, and the result derived by DTS-VTD algorithm is shown in the right part, where we can see two independent subclusters are obtained and obviously it has to drop the traffic flows between those nodes in separated subclusters, as shown in Table 3.10. For example, $T^{34}(\delta)$ cannot be transported, because no VWP is available for it.

Maximize-Traffic-Net VTD Algorithm (MTN-VTD)

In this section, we describe a new heuristic method for designing a VT. This method aims to build up a VT that supports as many traffic requests as possible according to the given VD constraints, and meanwhile avoids the occurrence of having partitioned subclusters in the VT.

This heuristic algorithm solves the four VTD sub-problems in turn, which can be described as follows.

Step 1. Search for a VT maximizing the value of parameter $\overline{T}_{max} = \sum_{s,d} b_{sd} T^{sd}$, subject to virtual degree constraints. Related equations to be solved are,

a) Objective function $\max \overline{T}_{max}$.

b) Virtual degree constraint Eq. 2.9.

Step 2. Route traffic over the VT derived by step 1, and calculate traffic routing variables T_{ij}^{sd} with an objective to minimize congestion level. If we fail to realize traffic routing, it means several partitioned subclusters are generated by step 1, then our solution is to go back to step 1 and search for another optimal value considering the previous $\overline{T}_{max}$ as a upper bound. Otherwise, go directly to step 3.

Related equations of step 2 are,

a) Objective function $\min T_{max}$.

b) Traffic routing constraints Eq. 2.3, Eq. 2.4, Eq. 2.5, Eq. 2.6.

c) Relevant nonnegative constraints.

Step 3. Route lightpaths on the physical link connections given by certain network without violating the maximum number of available wavelengths M.

Related equations to be solved are,

a) Objective function $\min M$.

b) Lightpath routing constraints Eq. 2.14, Eq. 2.15, Eq. 2.16 and Eq. 2.17 to solve variables P_{mn}^{ij}.

Step 4. Assign wavelengths to lightpaths given by step 1, according to any of the two wavelength assignment methods we mentioned before.

The computation complexity of this heuristic algorithm can be described as follows. In step 1, matrix **b** includes $N \times \Delta$ variables. In step 2, since all elements in matrix **b** have been given by step 1, i,j in T_{ij}^{sd} are no longer variables, so the total number of variables in T_{ij}^{sd} is also $N \times \Delta$. In step 3, it is the same as in step 2, the number of variables in P_{mn}^{ij} are $N \times \Delta$. And in the last step, since step 3 guarantees that the maximum available wavelength number M will not be violated, there is no variable related to wavelength assignment. So we can see the overall computation complexity of this MTN-VTD problem is $O(N \times \Delta)$.

The heuristic algorithm differentiates from the existing one as follows. Firstly, each sub-problem has an objective function and optimization is made subject to the relevant

constraints. This results in having lower computation complexity in our proposed heuristic algorithm. Secondly, each sub-problem is computation-based optimization and hence can obtain better overall accuracy than existing heuristic algorithms. Besides, the heuristic algorithm is also modified to avoid unconnected sub-clusters in the VT obtained — this is likely to happen when maximizing traffic throughput.

3.3.2 Minimize Traffic-weighted Virtual Hops (MVH)

As mentioned before, heuristic algorithms give suboptimal solutions to resource utilization. These algorithms normally make emphasis on different objective functions. In this section, we define a new objective function and formulate VTD problem using this objective function. It is then compared with the results obtained from using existing heuristic algorithms.

Here we define a parameter $W_{hop} = \max \frac{\sum_{ij} T_{ij}^{sd}}{T^{sd}} \ \forall s, d$. This is not the actual virtual hop as we defined in previous discussion. W_{hop} is a parameter controlling the maximal number of traffic-weighted WPs supporting certain VWP from node s to node d. This parameter is defined particularly because of the belief that in packet-switched networks, multiple VWPs could help in more evenly distributing the traffic so that it reduces the congestion. Under multiple VWPs, where each VWP may go through different numbers of hops, defining a suitable parameter is required, and hence W_{hop} is brought up. Since it is related to the number of lightpath segments, we can view this as an objective function to be minimized, which can be formulated as follows.

Objective function: minimize $\quad W_{hop}$,

where

$$W_{hop} = \max_{sd} \frac{\sum_{ij} T_{ij}^{sd}}{T^{sd}}, \quad \text{for all } s,d. \tag{3.1}$$

Subject to:

1. Traffic routing constraints Eq. 2.3, Eq. 2.4, Eq. 2.5 and Eq. 2.6.

2. Virtual degree constraints Eq. 2.9.

3. Lightpath routing Eq. 2.14, Eq. 2.15, Eq. 2.16 and Eq. 2.17.

4. Wavelength assignment.

5. Nonnegative constraints for all variables.

First according to the formulation above, we can see this is also a MILP, therefore LP-relaxation method can also be used to solve this problem. Moreover we find that any heuristic algorithm solving the first sub-problem of VTD can be jointly used to solve MVH problem, i.e. use a simple method to get a set of lightpaths b_{ij}, then route traffic flows over the lightpaths (derive T_{ij}^{sd}) to minimize the parameter W_{hop} as defined. So the DTS-VTD, TI-VTD and even R-VTD heuristic algorithms are all capable of being applied to solve MVH problem. In following discussions, we consider to solve MVH problem with four separated sub-problems, traffic routing with VD constraints Eq. 2.3, Eq. 2.4, Eq. 2.5 Eq. 2.6 and Eq. 2.9, lightpath routing Eq. 2.14, Eq. 2.15, Eq. 2.16 and Eq. 2.17, and wavelength assignment to reduce the computation complexity.

Evaluation and discussion on this design algorithm are presented in the next section, together with performance of other existing heuristic algorithms.

3.4 Analysis and Comparison

3.4.1 Physical Topology and Traffic Model

The heuristic algorithm discussed in section 3.3.1 and VTD problem with a new objective function discussed in section 3.3.2 are evaluated using the following example. We use numerical results to observe the differences in the final VTs obtained.

1. Physical network model: N=6,

   10001 1000 101 10 1.

2. Lengths of physical links are

Table 3.11: Minimum delay matrix for the six-node network

Nodes	1	2	3	4	5	6
1	0	8	16	26	25	10
2	8	0	15	25	30	21
3	16	15	0	10	15	6
4	26	25	10	0	5	16
5	25	30	15	5	0	15
6	10	21	6	16	15	0

$P_{12} = 8$, $P_{16} = 10$, $P_{23} = 15$, $P_{34} = 10$, $P_{36} = 6$, $P_{45} = 5$, $P_{56} = 15$.

Accordingly, we can derive the minimum delay matrix Table 3.11.

3. Traffic matrix is shown in Table 3.12. Each entry is chosen randomly from a uniform distribution in $(0, 1)$.

Table 3.12: Traffic matrix for the six-node network

Nodes	1	2	3	4	5	6
1	0	0.537	0.524	0.710	0.803	0.974
2	0.391	0	0.203	0.234	0.141	0.831
3	0.060	0.453	0	0.645	0.204	0.106
4	0.508	0.660	0.494	0	0.426	0.682
5	0.480	0.174	0.522	0.879	0	0.241
6	0.950	0.406	0.175	0.656	0.193	0

3.4.2 Numerical Results of Heuristic Algorithms

Numerical results of exact MVH problems are shown in Table 3.13. The numerical results of MTN-VTD heuristic algorithm are listed in Table 3.14, moreover, we present the optimal values of traffic congestion, wavelength requirement and delay in Table 3.15 as reference. To make further comparison, we also apply several other heuristic algorithms discussed in the earlier section, e.g. DTS-VTD, TI-VTD and R-VTD, and the relevant numerical results are presented in Table 3.16.

Table 3.13: Congestion,WL required and delay derived by MVH for the six-node network model

VD	VT					
1	010000	000001	000010	100000	000100	001000
2	010010	000101	110000	001010	001001	100100
3	011010	100101	010101	110010	001101	101010
4	001111	101011	010111	111001	111100	110110
5	Full connected					

VD	Congestion	WL	Delay	Hop
1	7.7120	1	13.5	5
2	2.7760	2	11.313	2
3	2.4420	3	14.222	2
4	2.7140	4	16.983	2
5	0.9740	4	16.2	1

Table 3.14: Congestion,Wavelength(WL) required and traffic-weighted delay derived by MTN-VTD for the six-node network model

VD	VT					
1	000010	000001	000100	010000	001000	100000
2	000011	100001	010010	011000	001100	100100
3	001011	100011	010110	011001	101100	110100
4	001111	101011	010111	111001	111100	110110
5	Full connected					

VD	Congestion	WL	Delay
1	7.2090	1	17.667
2	2.2910	2	14.583
3	1.1858	3	16.278
4	0.8870	4	17.125
5	0.7096	4	16.200

Table 3.15: Minimum achievable congestion, maximum number of wavelengths required on fibers and minimum average delay for the six-node network on traffic shown in Table 3.12(no multiple links)

VD	Congestion	WL	Delay
1	7.0770	1	10.500
2	2.0423	1	10.500
3	1.1827	2	13.222
4	0.8870	3	15.083
5	0.7096	4	16.200

3.4.3 Discussion and Conclusion

According to the numerical results presented in the tables above, Table 3.13, Table 3.14, Table 3.15 and Table 3.16, we can draw some observations as follows.

a) With minimizing W_{hop}, the maximum of traffic-weighted virtual hops of each traffic flow, as the objective function, MVH tends to consider congestion and delay control at the same time. However, compared to numerical results given by MTN in Table 3.14, MVH is more likely to achieve better delay status by loosening control on congestion level. So we have a more balanced control over both delay and traffic congestion level.

b) The first columns of Table 3.14 and Table 3.15 present the achievable congestion levels obtained by MTN-VTD and the optimal congestion levels, respectively, on ascending VD. Compared to other heuristic algorithms, such as MD-VTD and R-VTD with the same VD, we can see MTN-VTD arrives at a lower congestion level. And MTN achieves a congestion level lower than or equal to DTS in 4 out of 5 cases (except when $VD = 2$). So we can draw a simple conclusion that MTN-VTD is a heuristic algorithm, assigning lightpaths to source-destination pairs with large traffic flows so that to lower congestion level. Like any strongly traffic-related heuristic algorithm, MTN-VTD is not capable of obtaining a better congestion control with evenly distributed traffic model than it is with a concentrated traffic model. We will verify this further in Chapter 5.

Table 3.16: Congestion, wavelength requirement and delay status versus virtual degree derived by DTS-VTD, MD-VTD and R-VTD for the six-node network model

DTS-VTD						
1	000001	100000	000010	010000	000100	001000
2	000011	001001	010010	110000	001100	100100
3	000111	001011	010010	110001	101100	110100
4	010111	101011	010110	111001	101101	110110
5	Full connected					
VD	Congestion	WL	Delay			
1	7.4280	1	11.500			
2	2.1947	2	16.500			
3	1.4060	3	17.556			
4	0.8870	4	15.917			
5	0.7096	4	16.200			
MD-VTD						
1	-This heuristic works when VD $\geq$ PD					
2	-					
3	010011	101100	010101	011010	100101	101010
4	010012	201100	020101	012010	100201	101020
5	020012	202100	020201	012020	100202	201020
VD	Congestion	WL	Delay			
3	1.5277	2	13.222			
4	1.1458	3	12.542			
5	0.9166	3	12.133			
R-VTD						
1	001000	000100	000010	000001	010000	100000
2	001010	100100	010010	001001	010001	100100
3	011100	101100	010011	001011	110001	100110
4	011101	101110	100111	011011	111001	110110
5	Full connected					
VD	Congestion	WL	Delay			
1	7.5030	1	18.667			
2	2.3100	2	16.750			
3	1.4594	3	15.333			
4	0.8870	4	16.167			
5	0.7096	4	16.200			

c) From the second columns of Table 3.16, we can conclude that there is little difference between the minimum number of wavelengths required to support VTs generated by most of the heuristic algorithms listed above. Normally for a VT without multiplicity, the minimal number of wavelength required to support such a VT is no more than its VD. However, when multiplicity is enabled (as shown by results of MD-VTD shown in Table 3.16), the number of wavelength can be reduced to some extent.

d) In most cases, achievable minimal value of average delay derived by all heuristic algorithms listed above appears to have no significant changes as the value of VD increases. But again we can see MD-VTD, as a sample of multiplicity-enabled heuristic algorithms, deriving a VT mainly with respect to the PT connections, achieves a much lower average propagation delay than others.

Table 3.17: Evaluation for MTN-VTD in the six-node network with traffic model generated from independent Gaussian random variables. 20 percent of all traffic flows have mean 50 and standard deviation 1 as heavy traffics, while the other 80 percent have mean 10 and standard deviation 1 as light traffics.

VD	M	MTN-VTD	MILP(exact)	R-VTD
1	1	267.5882	267.5882	315.1148
2	1	115.7731	99.2173	#
2	2	103.8637	76.1116	123.6247
3	2	61.8124	49.6086	#
3	3	50.3408	47.0963	52.4135
4	3	39.9979	35.6746	41.2082
4	4	-	-	-
5	4	29.8615	29.8615	29.8615
5	5	-	-	-

e) In Table 3.17, there are totally five columns showing the virtual degree, wavelength, lowest achievable congestion value obtained using MTN, MILP and R-VTD heuristic algorithm, respectively. M and VD denote the number of wavelengths available on each physical fiber and virtual degree of the VT to be used in the design. The lowest achievable congestion values of heuristic algorithms are calculated with fulfilling constraints on M and VD. As shown in Table 3.17,

MTN-VTD gives a much better approximation to the optimal congestion level than R-VTD. Furthermore, in some cases MTN-VTD is even capable to derive the optimal congestion value, e.g. $VD = 1$, $M = 1$ case. There is no major difference between the wavelength requirements of MTN and MILP, which proves that the number of wavelengths required is more related to the PT and VD constraints, rather than individual heuristic algorithms.

3.5 VTD Formulation By Loosening The Bound

Here we present a novel way to formulate VTD problem. We remove the virtual degree (VD) constraints of each individual node and replace them by a single constraint related to the total VD of the entire network. Because we are attempting to optimize the same objective function in a larger search space, a lower optimal value can be achieved than conventional methods but at the expense of increasing computation complexity. However, this new way of modeling can result in the development of a simple heuristic algorithm, name as RLL (remove lightpath with lowest traffic). The solution provides information on whether resources in the network are allocated efficiently, and in case it is not, how re-distribution can be made to achieve better utilization.

3.5.1 Formulation of A Loose-bound Problem

Congestion control is a very important issue in designing VTs for a packet-switched network. In most of the existing heuristic algorithms, traffic flows are the most important information affecting VTD. Here we formulate VTD problem by loosening the VD constraint to achieve better congestion levels compared to the existing methods.

Classical MILP provides the lowest congestion level by including VD bounds Δ^{In} and Δ^{Out} to constrain the ingress and egress lightpaths at each node, as shown in equation below:

$$\sum_i b_{ij} \leq \Delta^{In}, \quad \sum_j b_{ij} \leq \Delta^{Out}. \tag{3.2}$$

where, normally $\Delta^{In} = \Delta^{Out}$.

However, a loose-bound problem is capable to assign lightpaths in the range of a whole network as long as not violating the total number of available lightpaths, which can be formulated as follows. We will focus on two separate steps with individual objective functions to be solved one after another.

Objective function 1:

$\min T_{max}$, subject to

1. Virtual degree constraint:

$$\sum_{ij} b_{ij} = N \times VD_{ave}, \tag{3.3}$$

   Clearly constraints given in Eq. 3.3 is not equal to Eq. 3.2, but the solution (a set of b_{ij}) to Eq. 3.2 is covered in the solution given by Eq. 3.3.

2. Traffic routing constraints: Eq. 2.3, Eq. 2.4, Eq. 2.5 and Eq. 2.6.

3. Nonnegative constraints for T_{ij}^{sd}.

Subsequently lightpath routing is fulfilled by solving the second objective function, and wavelength assignment can be done as dealing with a chromatic graph.

Objective function 2:

$\min M$, subject to

1. Lightpath routing constraint: Eq. 2.14, Eq. 2.15, Eq. 2.16 and Eq. 2.17.

2. Nonnegative constraints for P_{mn}^{ij}.

Above all, we present a simple comparison on achievable congestion values obtained by loose-bound method and MILP. The traffic matrix is shown in Table 3.1. As to MILP VD=3 and VD=1 cases, loose-bound method obtains the same congestion values as MILP, therefore we consider when $VD = 2$ (for MILP), where $\sum_{ij} b_{ij} = 4 \times 2$ for loose-

bound method. MILP results in congestion value= 0.9666, while loose-bound method get a VT as

0101 1011 0100 0110 ,

and the congestion value=0.9500. We can see as repay of giving up VD constraints on each node, loose-bound method obtains a lower congestion. However, it is clear that sometimes we may obtain solution that cannot be supported by the real VD of each node in the network. We will illustrate this and suggest improvement to it in the following discussions.

But to solve the exact loose-bound problem, the computation complexity is still large — O($N^2 \times N \times \Delta$), which strongly limits the scalability of this problem. So we have to propose a simpler approach, arbitrarily decide b_{ij} values as to reduce the complexity.

3.5.2 RLL Heuristic Algorithm

We define a heuristic algorithm, named as *Removing Lightpaths with Lowest traffic* (RLL), to solve the loose-bound problem mentioned above. We can prove that this heuristic algorithm is strongly traffic-dependent and leads to a good congestion level.

This heuristic algorithm can be described as follows.

1. Start with a full connected virtual topology, e.g. totally $N \times (N-1)$ entities in matrix **b** are '1's and '0's are only on the diagonal.

2. Define a counter to constrain the total lightpaths in the VT, where $count = \sum_{ij} b_{ij}$. Initially, $count = N \times (N-1)$.

3. Route traffic flows T^{sd} over current virtual topology b_{ij}, with objective function $\min T_{max}$. Check the VWP status and remove the lightpath with the lowest traffic passing through.

4. Update VT (the set of b_{ij} obtained by previous steps) and count the total number of lightpaths, go back to step 2, and repeat this procedure till the lightpath

constraint is violated.

Like other heuristic algorithms, RLL has less complexity compared to classical MILP problem — O($N \times \Delta$), and is only formulated to solve a linear problem, based on the model of routing traffic through the whole network with N routers. Here we assume that traffic flows T^{sd} must be non-evenly distributed so that traffic flows over different VWPs vary in a wide range to guarantee that removing lightpaths can result in a better congestion level. Otherwise, any TI-VTD heuristic algorithm is sufficient to derive a regular and suitable virtual topology, and the loose-bound problem may have no advantage over classical MILP.

3.5.3 Performance of RLL Heuristic Algorithm

Numerical Results of RLL in a 6-node Network

Here in the first section we report and comment on numerical results obtained by applying the RLL heuristic algorithm in a six-node network. In this experiment, all elements of the traffic are independent Gaussian random variables. 20 percent of all traffic flows are classified as heavy traffic having mean 50 and standard deviation 1, while the other 80 percent are classified as light traffic having mean 10 and standard deviation 1. This is also known as a condensed traffic model. Supposing the physical network is the six-node network mentioned in the section above, results for comparison when using MILP and RLL heuristic algorithm are shown in Table 3.18.

From the numerical results shown in Table 3.18, we can see that RLL heuristic algorithm approaches a lower congestion level as compared to MILP, except result at the last step (at the point $VD = 2$). We will discuss how this happened in details below.

In Fig. 3.3 and Fig. 3.4, VTs derived by MILP(exact) and RLL algorithms in each node, are presented in details. In the graphs, S and D are abbreviations for source and destination, respectively. N_i denotes node i in our network model. The figure shows

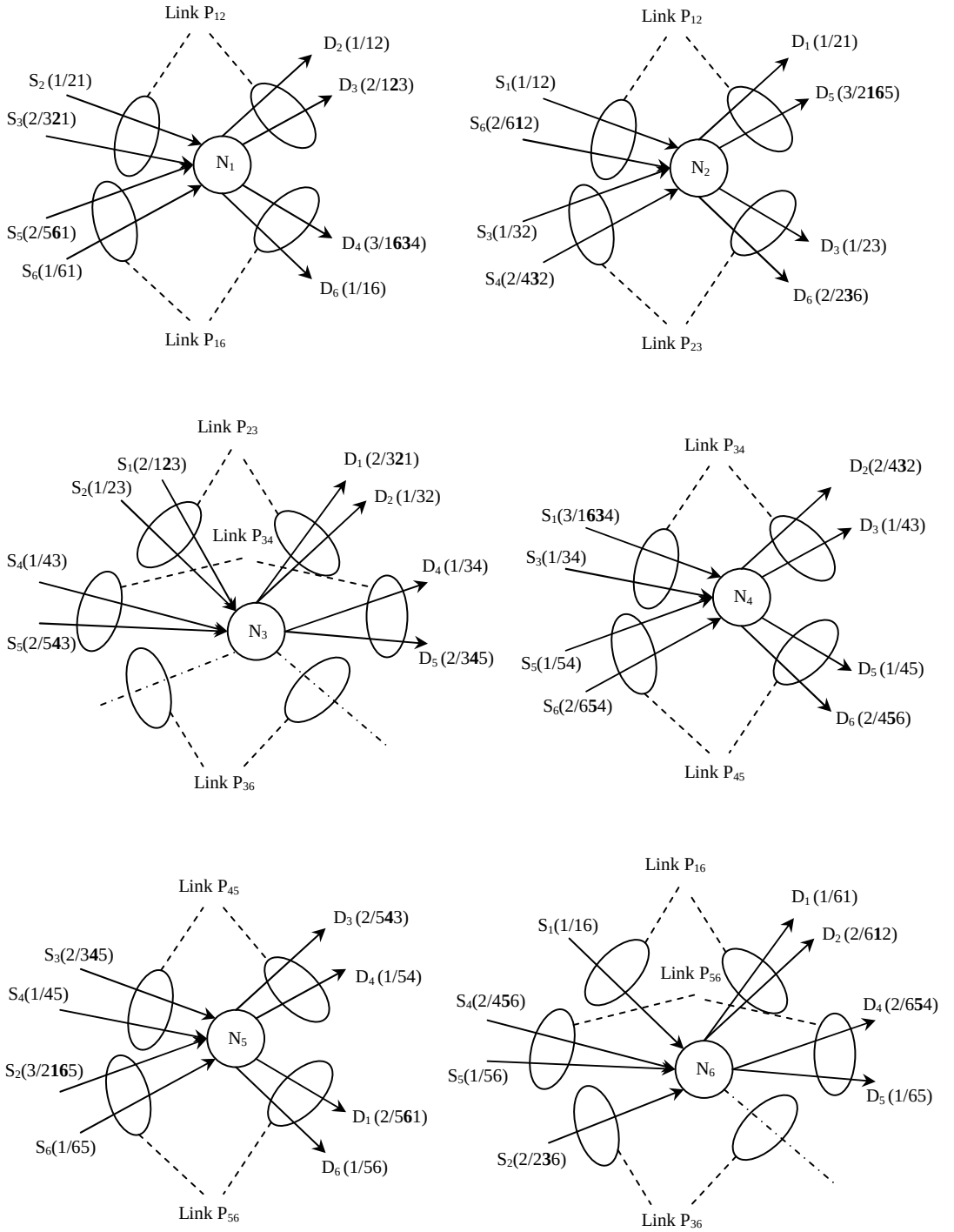
Table 3.18: Congestion comparison between MILP and RLL (boundless) in a simple model

Number of lightpaths	VTs
VD×N	VT derived by MILP
5×N	Full Connected
4×N	011101 101011 110110 011011 101101 110110
3×N	001101 101010 100110 010011 010101 111000
2×N	001100 001010 100001 010010 000101 110000
VD×N	VT derived by RLL
5×N	Full Connected
4×N	011010 101011 110111 111011 001001 111110
3×N	001010 001011 110110 111010 001001 110100
2×N	001010 001001 110010 010010 000001 100100

Number of lightpaths	Congestion		CPU time		WL(M)	
VD×N	MILP	RLL	MILP	RLL	MILP	RLL
5×N	29.8615	29.8615	4.67	0.35/step	4	4
4×N	35.6746	33.6899	10.00	0.57/step	3	3
3×N	47.0963	43.6450	34.09	0.52/step	3	3
2×N	76.1116	86.7416	57.33	0.50/step	2	2

the routing of the traffic in all the nodes and we explain the node status using node 1 as an example. Basically, there are two fibers connecting to node 1, i.e. physical link P_{12} and link P_{16}. The solution using MILP shows that there are four ingress lightpaths. Lightpaths denoted as b_{21} and b_{61}, pass through physical link P_{12} and P_{16} respectively, and terminate at node 1 in one hop. Lightpaths b_{31} and b_{51}, passing through physical links P_{32}, P_{21} and P_{56},P_{61}, respectively, terminate at node 1 in two physical hops. The number of egress lightpaths in node 1 is also four, including two from physical link P_{12} leading to destination node 2 and node 3, and the other two from P_{16} leading to node 4 and node 6. The egress lightpath b_{12} and b_{16} are both routed in one hop, through fiber P_{12} and P_{16} respectively. Egress lightpath b_{13} is routed through P_{12}, P_{23} then reaches its destination in two hops. And, egress lightpath b_{14} is routed through P_{16},P_{63},P_{34}, and finally reaches its destination in three hops.

The wavelength assignment for the solutions obtained by classical MILP and RLL are shown in Fig. 3.5 and Fig. 3.6, respectively. Rules for wavelength assignment applied are defined as below. When a lightpath is ready, search all the physical links it



Link P_{mn}: Physical Link between node M and node N
S/D_* (*/**): Source/Destination Node (number of physical hop/physical link sequences on which lightpaths is routed through)
Bold letters emphasize the bypassing nodes.

Figure 3.3: Lightpath distribution and Routing status in a 6-node network with VD=4 obtained by MILP(exact)

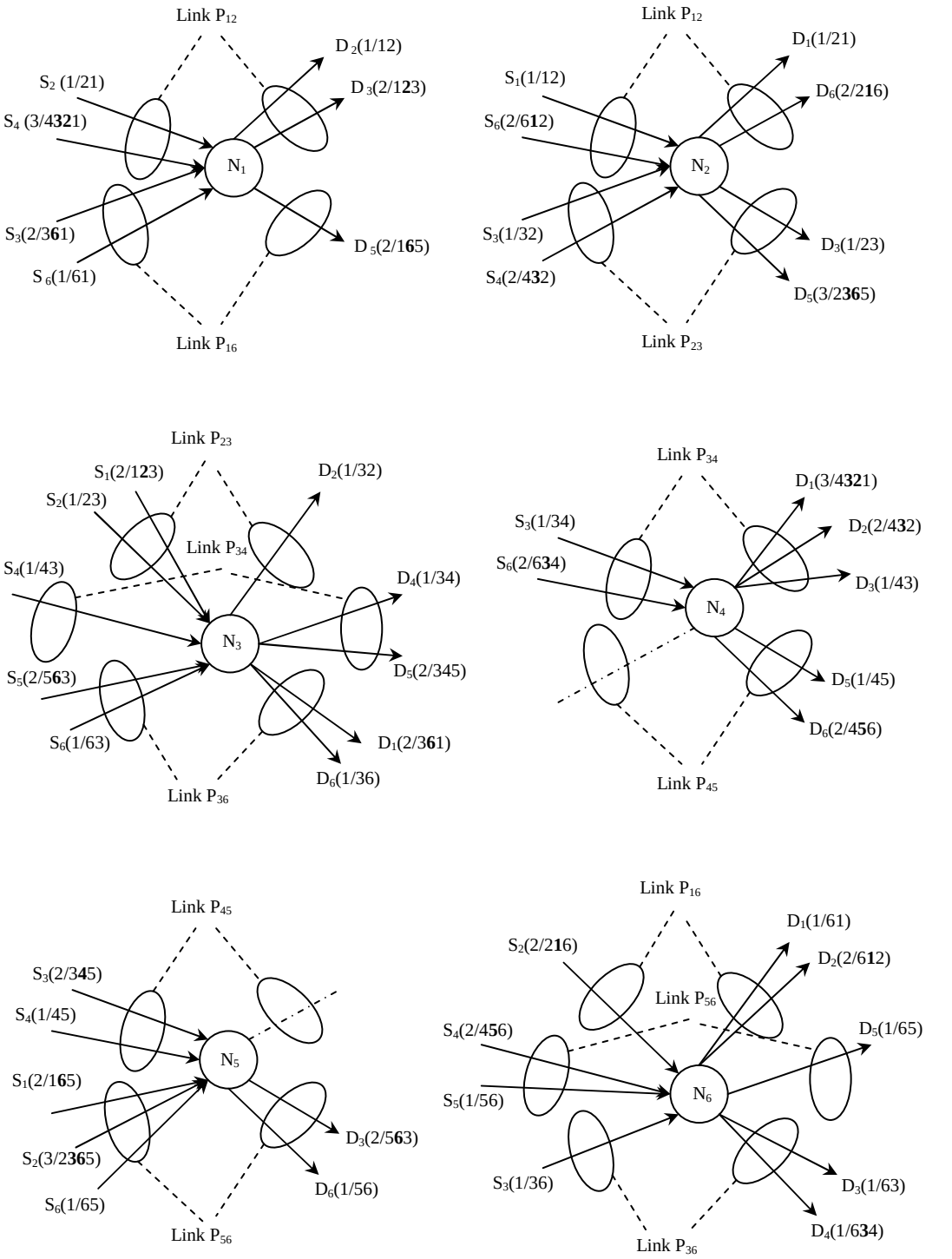


Figure 3.4: Lightpath distribution and routing status in a 6-node network with VD=4 obtained by RLL heuristic algorithm

passes through, look for the wavelength with lowest number, e.g. if λ_1 and λ_2 are both available, we use λ_1, and try to assign as many lightpaths as possible. If there is no longer able to find a 'continuous' wavelength, but still with some unused lightpaths left, then consider the implementation of converters (enabling one lightpath to be supported by different wavelengths on different segments) to complete the wavelength assignment. For example, when $VD = 4$ (as shown in Fig. 3.3 and Fig. 3.4), the maximum number of wavelengths required for both MILP and RLL is three, and the total number of physical hops for the two cases are both 38. However, for $VD = 3$ cases obtained by MILP and RLL as shown in Fig. 3.7 and Fig. 3.8, the total number of physical hops of MILP is 30 with no virtual hop. On the other hand, the total number of physical hops of RLL is 37, together with 3 virtual hops. Lightpath from node 4 to node 5 is routed through physical link P_{43} using λ_2, then P_{36} using λ_1 and finally passes physical link P_{65} using λ_3 before reaching the destination, and lightpath from node 6 to node 4 is routed through physical link P_{63} using λ_1 and physical linkP_{34} using λ_2.

The CPU time shown in Table 3.18 indicates how long the computations last, where we can see RLL take almost constant computation time with every step of about 0.50s, which proves that this heuristic algorithm is mainly effected by traffic models and network size. When network size becomes large, we can simply test the fully-connected virtual topology and estimate the solving time required for approaching to a certain average VD Δ by only multiplying computation time with the number of steps taken, which proves that the calculation time of RLL is always much less than MILP, similar to normal heuristic algorithms.

RLL in NSFNET and Further Discussion

For further discussion, we test RLL heuristic on NSFNET, using the traffic P_1 provided by [12]. Numerical results are shown in Table 3.19, where LP-relaxation algorithm provides the reference results. Again we find that most of the time the congestion levels obtained by RLL algorithm are lower. And there appears a sudden increase when approaching to average $VD = 2$. Our explanation to the special point is that

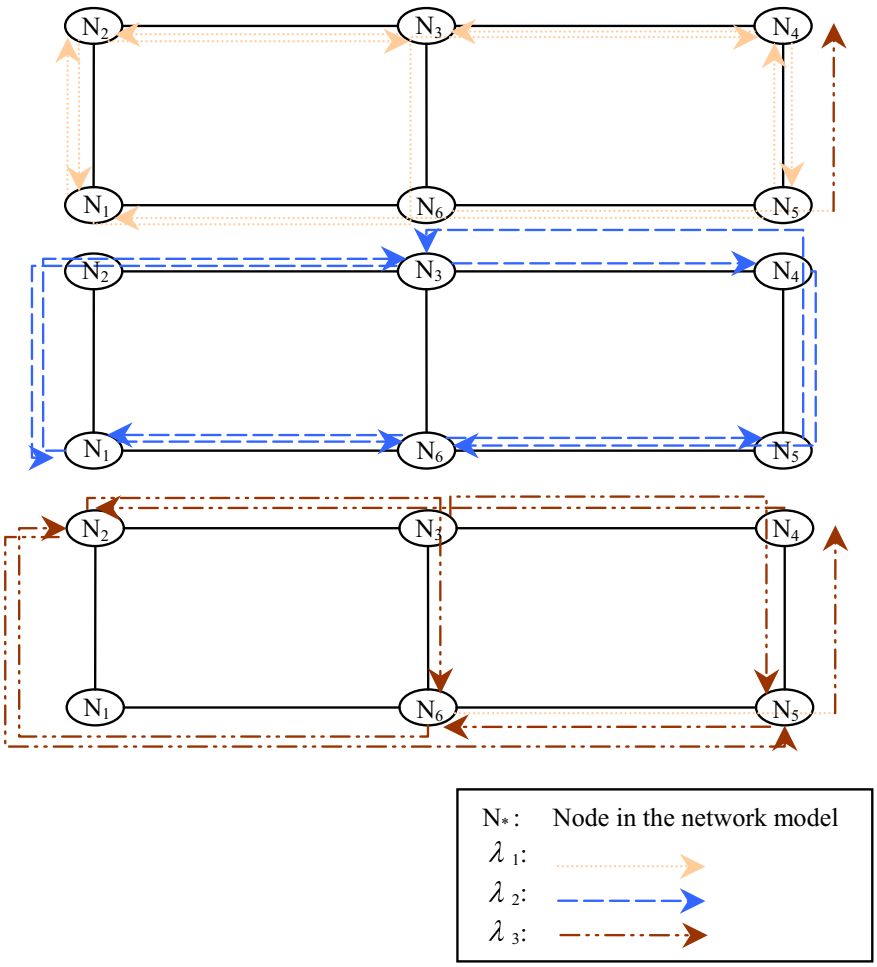


Figure 3.5: Wavelength Assignment for VT(VD=4) solution derived by MILP(exact) in a 6-node network

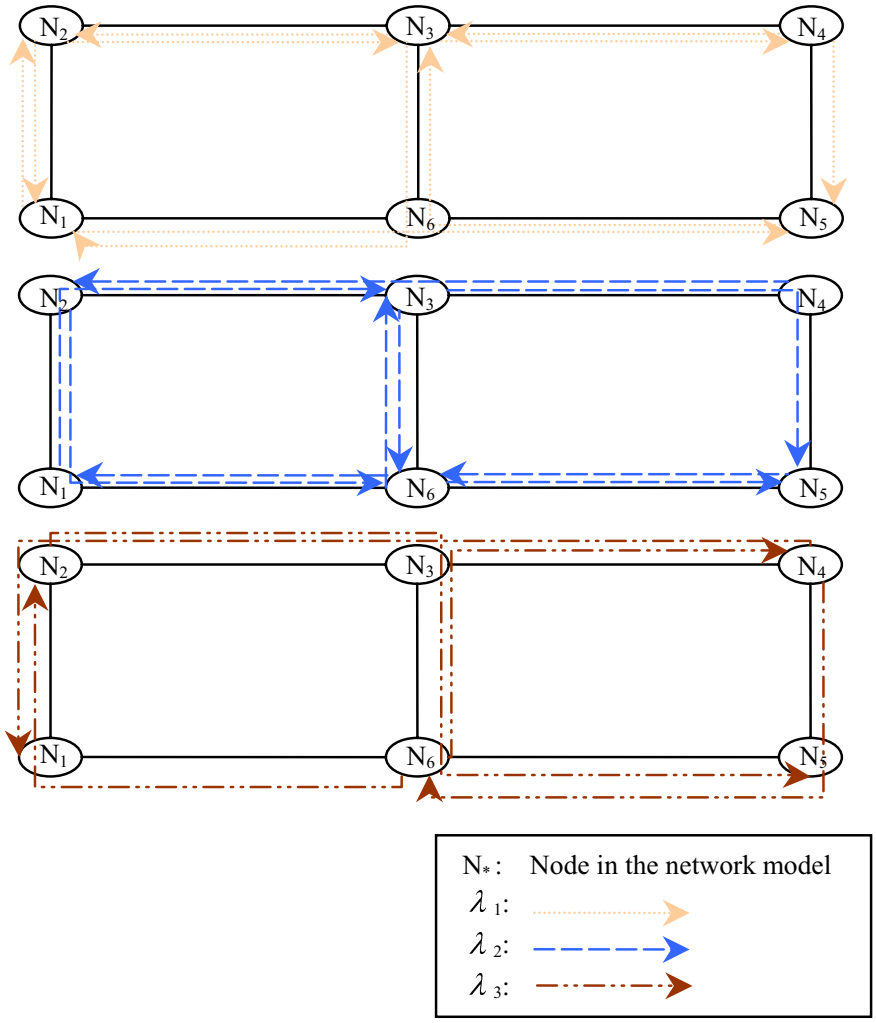


Figure 3.6: Wavelength Assignment for VT(VD=4) solution derived by RLL heuristic algorithm in a 6-node network

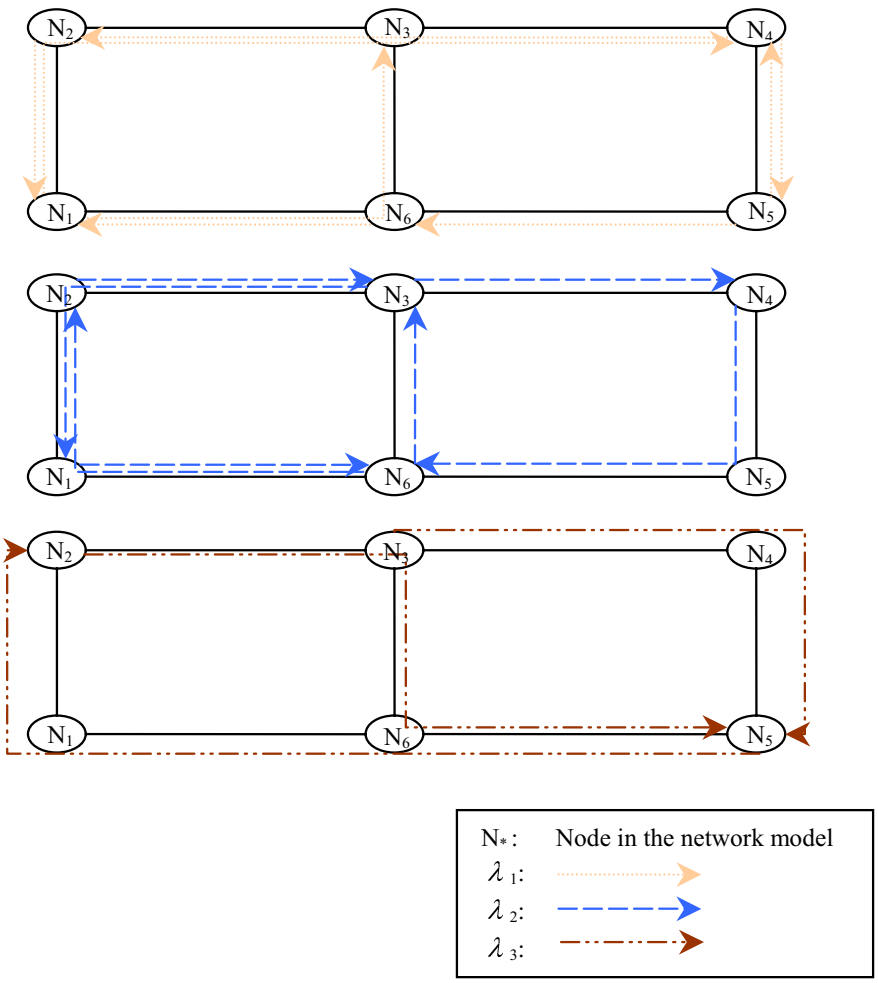


Figure 3.7: Wavelength Assignment for VT(VD=3) solution derived by MILP(exact) in a 6-node network

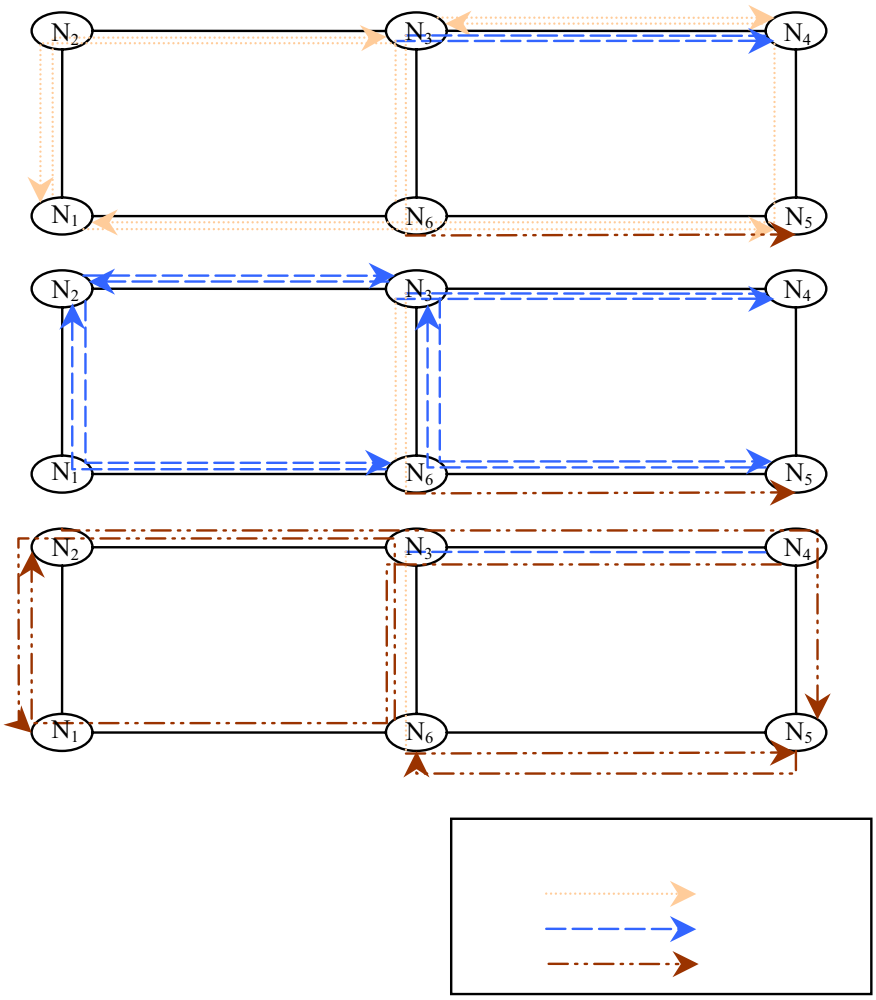


Figure 3.8: Wavelength Assignment for VT(VD=3) solution derived by RLL heuristic algorithm in a 6-node network

Table 3.19: Congestion level and wavelength upper bound (M) comparison between LP-relaxation and RLL in NSFNET

VD×N	13×N	12×N	11×N	10×N	9×N	8×N
Congestion-LP	20.24	21.27	23.00	25.19	28.37	31.72
Congestion-RLL	20.2477	20.2477	20.3571	24.2930	25.3746	27.4294
M-LP	13	11	10	8	7	6
M-RLL	13	11	10	8	7	6
VD×N	7×N	6×N	5×N	4×N	3×N	2×N
Congestion-LP	36.25	42.29	50.74	71.71	84.58	141.28
Congestion-RLL	34.2867	40.5670	48.5860	60.7800	81.6904	144.2357
M-LP	5	4	4	3	2	2
M-RLL	5	4	4	3	2	2

the convergence property of RLL is mainly decided by enabling traffic flow between one node pair to be routed over several VWPs. When only $N \times 2$ lightpaths are left (average degree 2), some nodes have already reached their lower constraints: $\Delta^{In} = 1$ or $\Delta^{Out} = 1$, so that no VWPs over more than one WPs can be set up. Hence, the new VT fails to keep the capability of routing partitioned traffic flows to obtain an optimal congestion value between all s-d pairs, and congestion level increases suddenly.

From the numerical results shown in Table 3.18 and Table 3.19, we can conclude that RLL heuristic algorithm achieves lower congestion level compared to all traditional methods and does not cause any increase on computation complexity or wavelength requirements. The disadvantage is that transceiver distributions may need to be re-arranged, and in some situations, this may not be possible to be done. However, the solution can be used to check whether resources such as transceivers in existing network are allocated in the network efficiently. In case it is not, how re-distributed of such resources can be made to achieve better network resources utilization.

To express how traffic information affects RLL heuristic algorithm, here we define a traffic ratio factor as follows:

$$\mu(A) = \frac{\sum_s T^{sA}}{\sum_d T^{Ad}}, \quad \forall\, A,$$

where $\sum_s T^{sA}$ denotes overall ingress traffic at node A, i.e. traffic from all possible

source nodes s. And $\sum_d T^{Ad}$ denotes the overall egress traffic at node A, i.e. traffic leading to all possible destination nodes d. According to the traffic model used in Table 3.19, we can calculate the ingress and egress traffic values on each node, and then the respective traffic ratio factors μ_A are:

$\mu_1 = 0.3331$, $\mu_2 = 1.9297$, $\mu_3 = 0.2657$, $\mu_4 = 2.0230$, $\mu_5 = 0.8151$, $\mu_6 = 1.0982$, $\mu_7 = 0.7418$, $\mu_8 = 1.0018$, $\mu_9 = 0.6458$, $\mu_{10} = 1.3904$, $\mu_{11} = 4.4869$, $\mu_{12} = 0.6126$, $\mu_{13} = 2.6113$, $\mu_{14} = 0.6723$.

Furthermore, we record the changes in ingress VD $\Delta^{In}(A)$ and egress VD $\Delta^{Out}(A)$ of node A after each RLL iteration, as shown in Fig. 3.9. In each RLL iteration, VD_{ave} is reduced by 1, or a total number of transceivers equal to the number of nodes are removed. Since $A = 1, \cdots, 14$, for a 14-node network, we get 14 subgraphs in Fig. 3.9, each is referring to a node. In the subgraph, vertical axis is used to measure the 'exact' ingress/egress VD of that node when the total number of lightpaths in the network is $N \times VD_{ave}$. VD_{ave} is shown by the horizontal axis. Comparing the alteration of Δ^{In} and Δ^{Out} at certain node A with its traffic ratio μ_A, we find that the closer μ_A approaches to 1, the less difference there exists between changes of the in/out virtual degree of that particular node after each iteration, e.g. $\mu_8 = 1.0018$ or $\mu_6 = 1.0982$, their Δ^{In} almost change simultaneously with Δ^{Out} (two Δ curves close to each other). Hence, we conclude that in a network with regular (or semi-regular) physical topologies, node with balanced ingress and egress traffic tends to perform better with a regular ingress and egress virtual degree, however, the VD may not be the same for each node in the whole network.

Alteration of Algorithm Constraints

In the previous RLL heuristic algorithm implemented, we remove the constraint on the VD of individual node, i.e. we suppose it is possible to change the transceivers distribution among network nodes to obtain a good congestion control or to re-design the PT. However, this may not always be allowed in practice. Especially when nowadays, most VTD problems are based on an existing physical network, the PT has already

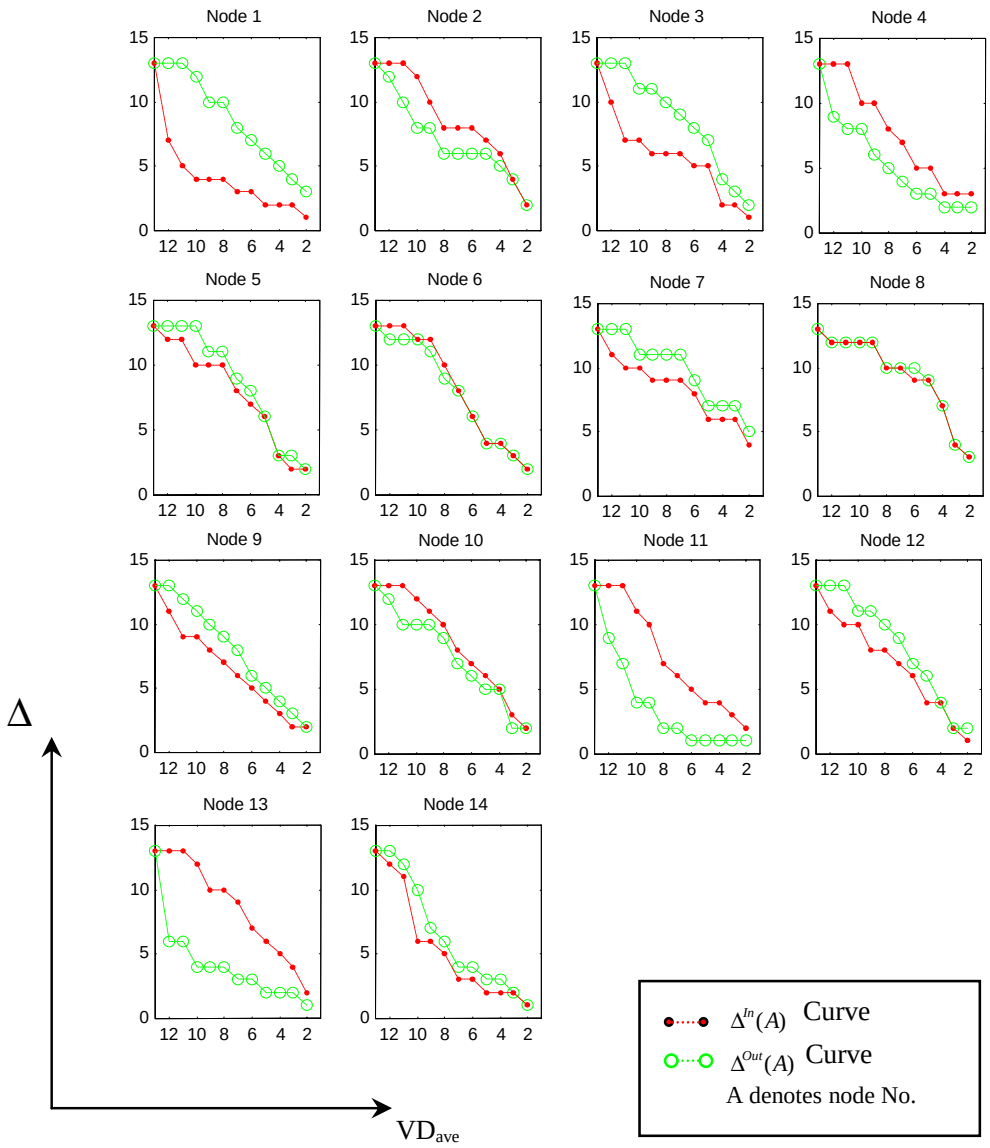


Figure 3.9: In/Out degree changing status along with RLL for all nodes in NSFNET

been designed, and the role of a VT designer is then to make VTD based on traffic flows between all pairs of nodes without changing the current physical arrangement. In order to accommodate our heuristics to meet a given VD constraints, the heuristic algorithm RLL is modified as follows.

1 Maintain balance in the number of transceivers at each node. The procedure listed in section 3.5.2 is revised. To keep $\Delta_A^{In} = \Delta_A^{Out}$, we need to remove lightpaths in pairs, e.g. remove b_{ij} together with b_{ji}, so the virtual degree constraints after revision are

$$\sum_{i,j} b_{ij} = VD \times N, \qquad \sum_{i} b_{ij} = \sum_{i} b_{ji}. \tag{3.4}$$

Relevant numerical results of this enhanced heuristic algorithm are presented in Chapter. 5.

2 Following the idea of removing lightpaths from a VT, we develop a method to evaluate the importance of a lightpath in certain VT. Since we know removing lightpaths from a VT causes a congestion level not less than its previous value, a parameter called Lightpath Usage (LU) can be defined as follows:

a) Routing traffic over the given VT with an objective function $\min T_{max}$, derive the original congestion level, $T_{max}(Original)$.

b) Remove the concerned lightpath, repeat calculation of step a, derive a new congestion level $T_{max}(ij)$ $\forall i, j = 1, \cdots, N$, where i j specifies source and destination of the lightpath. Obviously, $T_{max}(ij) \leq T_{max}(original)$.

c) Although we cannot find a suitable meaning for the exact numerical value of $T_{max}(ij)$, a ratio defined as shown below,

$$LU(ij) = \frac{T_{max}(ij)}{T_{max}(Original)} - 1, \quad \text{for all } i,j$$

can evaluate the importance of lightpath from node i to node j.

Generally, we can estimate the range of LU(ij) as $\sum_{sd} T^{sd} \geq LU(ij) \geq 0$ $\forall i, j$. If

$LU(ij) = 0$, we remark lightpath ij as wastage, the larger $LU(ij)$ is, the more important lightpath ij can be remarked as.

3.6 Resource Budgeting and Comparison

This section discusses some of the VTD design principles derived from the earlier formulation. It is glandular that, in a network with large number of transceivers, i.e. VD, per node, but with few wavelengths per fiber and few fibers between node pairs, a large number of transceivers may be unused because some lightpaths may not be establishable due to fiber or wavelength constraints. Likewise, if a network with few transceivers but a large number of available fibers and wavelengths may have a large number of unused wavelengths because the network is transceiver-limited. This kind of mismatch in transceiver utilization and wavelength utilization has a deep influence on the cost and design of the network. Both numbers of transceivers and the number of available wavelength determine the cost of equipment (including the cost of converter implementation). To maximize the utilizations of both characters concerned, we balance these network resources through resource budgeting. This has been discussed in some papers [16] [24] [11].

A simple and approximate calculation leads to some insights into the resource budgeting problem. Given a physical topology, and a routing algorithm for lightpaths, we can determine the average length of a lightpath, i.e. average physical hop number, denoted by H_{ave}. Assuming there are E_p fiber links in the network, each supporting M wavelengths, and uniform utilization of wavelengths on all fiber links, then N nodes are supposed to support maximum number of wavelengths $E_p \times M/H_{ave}$. Therefore, the number of transceivers per node is given by

$$\Delta^{In} = \Delta^{Out} = \Delta = \frac{E_p \times M}{H_{ave} \times N}. \tag{3.5}$$

Alternatively we can derive another lower bound for wavelength. In most cases, we are

dealing with a VTD problem on a given physical topology, traffic requests and virtual degree constraints. Suppose the network is denoted by $G(P, E_p)$, with N nodes, E_p undirected edges and the minimum degree of the physical topology is Δ_p, the minimal number of wavelength we need to set up a virtual topology with regular degree Δ_v can be expressed as:

$$M \geq \frac{\Delta_v}{\Delta_p}. \tag{3.6}$$

To prove this, consider node i with physical degree Δ_p, lightpaths originated from node i must be routed through one of those Δ_p edges, i.e. at least $\frac{\Delta_v}{\Delta_p}$ lightpaths have to share one physical link. Since no two lightpaths passing through one physical link in the same direction can share a wavelength, the number of wavelength required cannot be less than this value, $\frac{\Delta_v}{\Delta_p}$.

Chapter Summary

In this chapter, we introduce our heuristic algorithm for solving a newly formulated VTD problem. And also comparisons between our heuristic algorithm and the existing algorithms have been elaborated and details on data have been presented. As expected, in most cases, the numerical results obtained using our algorithm can achieve lower congestion level, and the minimum number of wavelengths required by our algorithm remains almost unaffected compared to the conventional algorithms. The solution can be used to check whether resources such as transceivers in existing network are allocated efficiently in the entire network. And in case they are not, how re-distribution can be implemented to achieve a better resource utilization.

Chapter 4

VTD Formulation As Bi-objective Problems

4.1 Theory

In this chapter, we formulate VTD problem by concurrently optimizing two parameters. A few parameters defined before, such as average traffic congestion, average number of virtual hops, average number of physical hops etc, are considered as the objective functions. Existing heuristic algorithms are developed only based on single objective function. In this chapter, for the first time in the literature, we look into possibility of generating heuristic algorithms that can try to simultaneously satisfy two objective functions.

4.1.1 Relevant Objective Functions

Given a traffic matrix and a physical topology, there are always more than one important parameter we are interested to optimize concurrently when designing a virtual topology. In this section, we provide a list of parameters that commonly used as ob-

jective function before we determine which pair of these parameters can be jointly considered using Pareto solution.

a) **Traffic Congestion Control (TCC)**

Minimize the maximum value of the traffic passing through all the lightpaths, i.e.,

Minimize:

$$T_{max} \geq \sum_{s,d} T_{ij}^{sd} \qquad \text{for all } i,j. \tag{4.1}$$

b) **Traffic-Weighted(TW) Virtual Hops (VH)**

This objective function concerns on routing over lightpaths, to minimize the number of virtual hops, which is related to the number of wavelength converters required (or O/E/O conversion if no converter is available).

Minimize the W_{hop} given by:

$$W_{hop} = \sum_{s,d} (\frac{1}{T^{sd}} \sum_{ij} T_{ij}^{sd}). \tag{4.2}$$

c) **Traffic Weighted Delay (TWD)**

The exact optimization function for delay minimization is as follows.

Minimize:

$$\sum_{i,j} (\sum_{s,d} T_{ij}^{sd} [\sum_{mn} P_{mn}^{ij} * d_{mn} + \frac{1}{C - \sum_{sd} T_{ij}^{sd}}]). \tag{4.3}$$

This is a nonlinear equation. However in a large-scaled network, queueing delays are intentionally ignored, partly to simply (linearize) the objective function, and also because it has been observed that propagation delay dominates the overall network delay in nationwide optical networks having high-speed transmission equipment and moderate load [16]. The advent of multi-gigabit silicon-based routers (as opposed to slow software-based routers) to be deployed shortly in the

internet backbone also justifies this assumption. And the abridged optimization function can be approximated to be as follows.

$$\sum_{i,j}\sum_{s,d} T_{ij}^{sd} \sum_{mn} P_{mn}^{ij} * d_{mn}, \tag{4.4}$$

where d_{mn} denotes the shortest physical length between node pair (m,n), and it is derived from the physical network connections. If define d_{ij} denoting the average propagation delay on lightpath ij, we can derive

Proof:

$$\sum_{i,j}\sum_{s,d} T_{ij}^{sd} \sum_{mn} P_{mn}^{ij} \times d_{mn} = \sum_{i,j}[\sum_{s,d} T_{ij}^{sd}]d_{ij} = \sum_{i,j} T_{ij} d_{ij}.$$

Another definition of delay-related objective is *minimizing the total traffic delay* (MTD) given by:

$$\sum_{i,j} b_{ij} \times d_{ij}. \tag{4.5}$$

However, the simplest formulation of considering delay as an objective is *Minimizing Average Delay* (MAD), where no traffic information required.

$$\frac{\sum_{m,n} b_{ij} P_{mn}^{ij} d_{mn}}{\sum_{i,j} b_{ij}}, \tag{4.6}$$

d) **Total number of Lightpaths(TL)**

Minimize:

$$\sum_{i,j} b_{ij}. \tag{4.7}$$

This objective function is useful in case of sparse traffic. In the situation when most of the WPs are used up, minimizing this objective may not have any meaning because it approaches to a constant.

e) **Lightpaths over a Physical Link (LoPL)**

The objective function is to optimize the number of wavelengths required to

establish a given set of lightpaths. Alternatively, it is to establish as many desired lightpaths as possible for a given set of available wavelengths.

Minimize M,

where

$$M \geq \sum_{i,j} P_{mn}^{ij} \quad \text{for all } m,n. \tag{4.8}$$

f) **The Total Length of Lightpaths (TLoL)**

We count the physical hops of each lightpath, i.e. the number of fiber links in each lightpath and aim to minimize its total, which can be described as shown below.

Minimize:

$$\sum_{ij} \sum_{mn} P_{mn}^{ij}. \tag{4.9}$$

4.1.2 Approach to Solve A Bi-objective Problem

This kind of problem can only be solved by *Pareto solutions* as shown in Fig. 4.1 below [5] [47] [48]. Referring to the definition of a Pareto solution, when optimal point moving its weight from one solution towards the other along the Pareto curve, as compensation of improving one objective function, the other objective function will become worse. Points A and B shown in Fig. 4.1 are clearly non inferior solution points because an improvement in one objective, F_1, requires a degradation in the other objective, F_2, i.e.

$$F_{1B} \leq F_{1A} \quad F_{2B} \geq F_{2A}$$

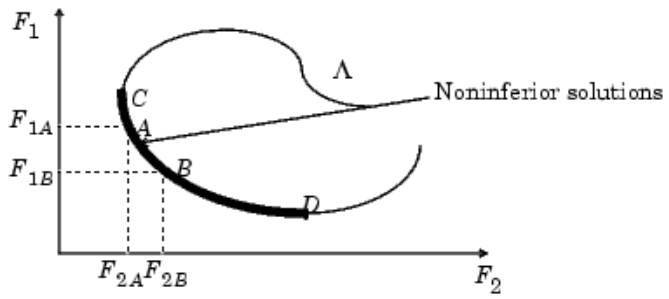


Figure 4.1: Definition of a Pareto solution

A Weighted Sum Strategy

The weighted sum strategy converts the multi-objective problem of minimizing the vector $F(x)$ into a scalar problem by constructing a weighted sum of all the objectives.

$$\min_{x \in \Omega} f(x) = \sum_{i=1}^{m} \omega_i F_i(x)$$

The problem can then be optimized using a standard unconstrained optimization algorithm by attaching weighting coefficients to each of the objective functions. The weighting coefficients do not necessarily correspond directly to the relative importance of the objective functions or allow trade-offs between the objective functions to be expressed. This can be illustrated geometrically. Consider the bi-objective case, in the objective function space a line L: $\omega^{\mathbf{T}}\mathbf{F_T}(x) = C$ is drawn. The minimization of equation above can be interpreted as finding the value of C for which L just touches the boundary of Δ as it proceeds outwards from the origin. Selection of weights ω_1 and ω_2, therefore, defines the slope of L, which in turn leads to the solution point where L touches the boundary of Δ. Limitation of this method can be shown in the second diagram of Fig. 4.2.

ε-constraint Method

A procedure that overcomes some of the convexity problems of the weighted sum technique is the ε -constraint method. This involves minimizing a primary objective, F_p,

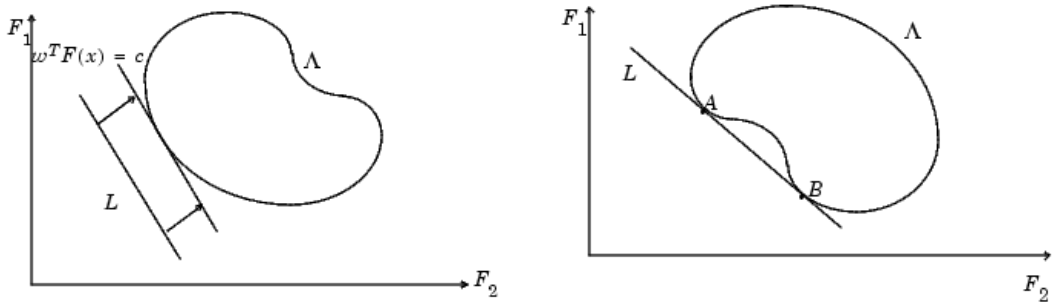


Figure 4.2: Solvable and unsolvable examples for Traffic-sum strategy

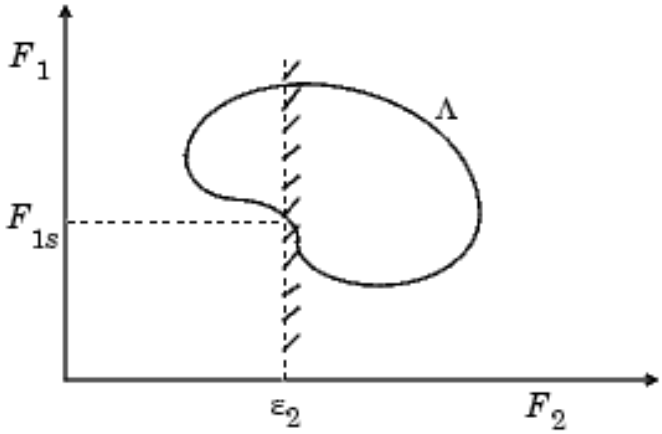


Figure 4.3: ε -constraint method

and expressing the other objectives in the form of inequality constraints:

$$\min_{x \in \Omega} F_p(x),$$

subject to $F_i(x) \leq \varepsilon_i \quad i = 1, \ldots, m,\ i \neq p.$

Below shows a two-dimensional representation of the ε-constraint method for a bi-objective problem. See Fig. 4.3.

Objective function:

$$\min_{x \in \Omega} F_1(x) \qquad \text{subject to} F_2(x) \leq \varepsilon_2.$$

This approach is able to identify a number of noninferior solutions on a non-convex

boundary that are not obtainable using the weighted sum technique, for example, at the solution point $F_1 = F_{1s}$ and $F_2 = \varepsilon_2$. A problem with this method is, however, a suitable selection of ε to ensure a feasible solution. A further disadvantage of this approach is that the use of hard constraints is rarely adequate for expressing true design objectives.

4.2 Formulation and Correlation Exploration

4.2.1 Test for Suitability of Applying Bi-objective Problem

Before start formulating and solving the bi-objective problems, we make a simple test to obtain some basic understandings, such as which pair of objective functions can be jointly considered, and whether traffic affects the relationship between the objective functions. In this section we consider four objective functions, TCC, W_{hop} and two kinds of delay, TWD and MAD defined in Eq. 4.4 and Eq. 4.6. The physical network model is a four-node network with the following configuration.

1. N=4, PT 0110 01 1 and physical link lengths are $P_{12} = P_{21} = 1$, $P_{13} = P_{31} = 1$, $P_{24} = P_{42} = 1$, $P_{34} = P_{43} = 2$.
2. Elements of shortest delay matrix can be described as 0112 1021 1202 2120
3. Consider all the possible VTs with $VD = 2$.

Due to the limitation of the network size, we can only get a few VTs satisfying $VD = 2$. All valid VTs together with the concerned objective functions are presented in Table 4.1, TWD, MAD and W_{hop} values are obtained for each of the possible VT (non-optimized), and TCC values are minimized by optimally routing traffic flows over each possible VT. Based on the numerical results given by Table 4.1, we can display the solutions by using any two interested objective functions as axes, to demonstrate

Table 4.1: VT cases for a four-node network (VD=2)

VT case No.	VT Construction	TCC	MAD	TWD	VH
1	0110 1001 1001 0110	0.9750	1.25	1.6296	16.0000
2	0110 0011 1001 1100	1.1333	1.375	2.0679	16.9048
3	0110 1001 0101 1010	0.9666	1.5	1.7654	16.0444
4	0101 1010 1001 0110	1.0833	1.5	2.2592	16.3556
5	0101 1010 0101 1010	1.1500	1.75	2.5833	17.9333
6	0101 0011 1100 1010	1.1500	1.625	2.6389	17.3778
7	0011 1010 0101 1100	1.1500	1.625	2.4537	16.0000
8	0011 0011 1100 1100	1.1500	1.5	2.0556	16.0000
9	0011 1001 1100 0110	0.9750	1.375	1.7731	16.0000

how the two objective functions are related to each other and whether it is possible to find a Pareto solution between them.

To solve a bi-objective problem, our concern is on the region where one objective function becomes larger, the other tends to be lower. As shown in the left-hand graph of Fig. 4.4, the three results at the left corner tend to behave in this way than the three results in the right-hand graph. We can conclude that TWD is correlated to TCC to some extent, but MAD is not. So MAD and TCC are more likely to be jointly considered as objective functions. No significant results can be seen from Fig. 4.5, so bi-objective problems using objective functions W_{hop} with TCC or W_{hop} with TWD may have difficulty to derive Pareto solutions. Remarkably different results are shown in Fig. 4.6, which shows that the type of traffic models affects the relationship between objective functions significantly. The traffic model is defined as 20 percent are heavy traffic flows having mean 10 and standard deviation 1, and 80 percent are light traffic flows having mean 10 and standard deviation 1.

4.2.2 Relationship Between Objective Functions

It is obvious that the parameters we are concerned about are interrelated. In the normal VTD problems, we only optimize one of the interested parameters. In this section, the effect of concurrently optimizing another parameter on existing one needs to be studied. For example, as we optimize the average number of lightpaths passing through physical

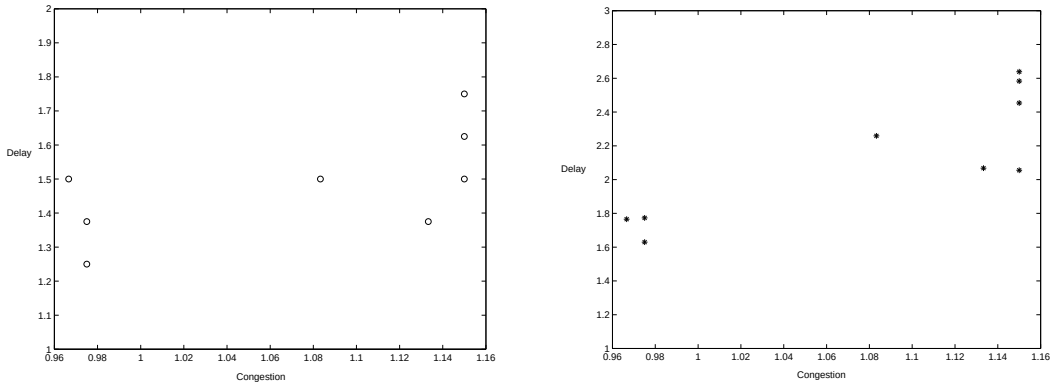


Figure 4.4: MAD(on the left) and TWD(on the right) versus TCC in the four-node network with normally distributed traffic

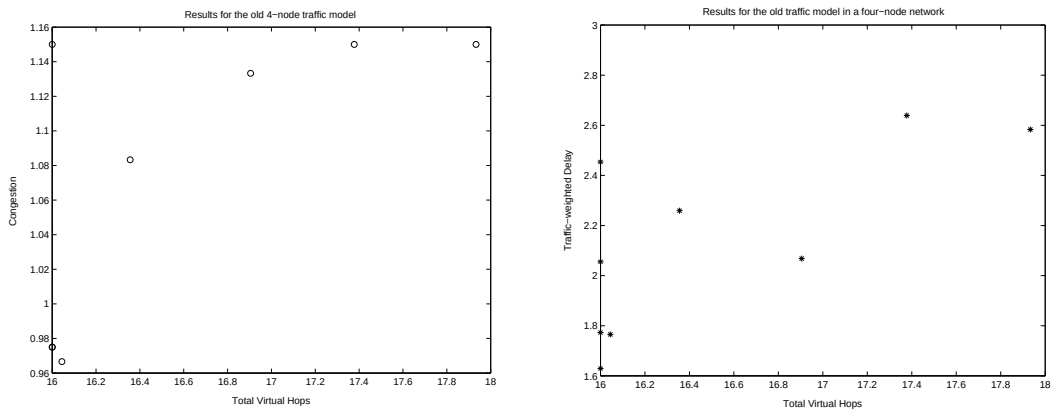


Figure 4.5: TCC and TWD versus W_{hop} in the four-node network with normally distributed traffic

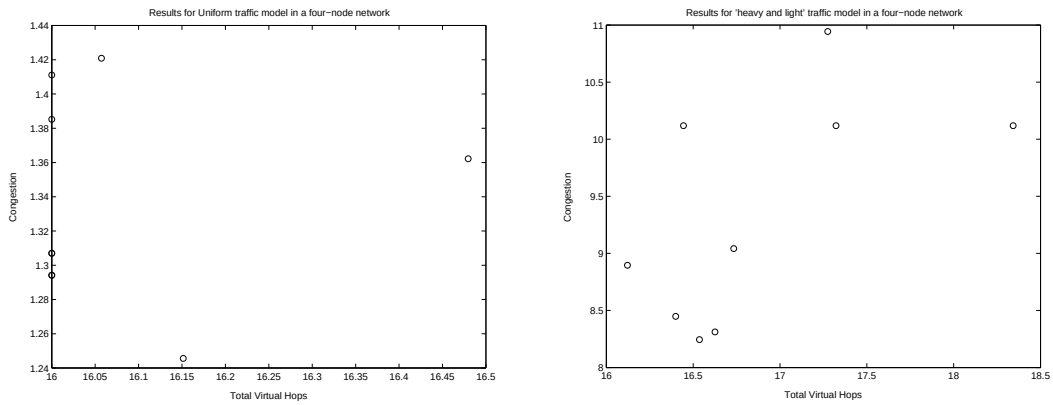


Figure 4.6: Comparison on TCC versus W_{hop} status in the four-node network with different traffic models

Table 4.2: Sparse traffic model for a six-node network

Nodes	1	2	3	4	5	6
1	0	0	0	0	0	0
2	0	0	0	0.234	0.524	0
3	0	0	0	0	0	0
4	0.508	0.660	0	0	0.879	0
5	0	0	0	0	0	0
6	0.950	0	0	0	0	0

links, wavelength available on each fiber is well utilized. However, this will lead to a critical increase of congestion on lightpaths. On the other hand, if the traffic congestion is minimized, traffic may be forced to route through more hops, in such a case average hop distance increases. In the following we show some numerical results and use them to investigate the effect of optimizing one parameter on other parameters.

Setup information for simulation is shown as follows.

1. PT: 10001 1000 101 10 1, lengths of physical links are 8, 10, 15, 10, 6, 5, 15 respectively.

2. Traffic models:

   a) Sparse traffic model as shown in Table 4.2.

   b) Uniform distribution traffic model: range (0,1).

   c) Condensed traffic model: 20 percent are heavy traffic flows having mean 50 and standard deviation 1, and the remaining 80 percent are light traffic flows having mean 10 and standard deviation 1.

3. Virtual degree constraint: $VD = 2$.

And this problem can be formulated as below.

Objective function: minimize (one of the concerned parameters),

subject to

1. Traffic routing constraints: Eq. 2.4, Eq. 2.5 and Eq. 2.6.

Table 4.3: Concerned measures in case of one parameter being optimized on a sparse traffic model. First column shows the optimized parameter.

Measures	TCC	VH	TWD	MAD	TL	LoPL	TLoL
TCC	1.0235	10.5897	26.303	15.2	10	2	22
VH	1.5390	7	29.061	23.2	5	2	13
TWD	1.4030	9	17.356	13.5	6	2	18
MAD	2.5710	14	20.182	10.5	6	1	10
TL	2.2160	12	29.987	16.000	5	2	9
LoPL	2.1180	15	18.938	12.8	5	1	11
TLoL	3.0950	18	42.818	12.6	5	1	6

Table 4.4: Concerned measures in case of one parameter being optimized on a uniform traffic model. First column shows the optimized parameter.

Measures	TCC	VH	TWD	MAD	TL	LoPL	TLoL
TCC	2.1361	49.1765	23.021	15.583	12	3	24
VH	2.7620	48.0000	21.666	15.083	12	3	26
TWD	3.0508	54.0000	19.277	10.5	12	2	20
MAD	8.6581	90.0000	31.960	10.5	6	1	10
TL	8.7764	90.0000	55.721	18.667	6	2	12
LoPL	6.9006	73.0000	31.472	12.556	9	1	12
TLoL	8.6581	90.0000	31.960	10.5	6	1	6

2. Virtual degree constraints: Eq. 2.9. When TL is the optimized objective function, use $\leq$; otherwise, use $=$.

3. Lightpath routing constraints: Eq. 2.14, Eq. 2.15, Eq. 2.16 and Eq. 2.17.

4. Nonnegative constraints for all variables.

Then we calculate other parameters, according to their definitions mentioned in section 4.1.1.

Numerical results displayed in Table 4.3, Table 4.4 and Table 4.5 represent the relationship between parameters in three different traffic models — sparse traffic, uniform traffic and condensed traffic, respectively. The first column of each table lists the parameters as objective functions, and each row represents value of other parameters in the VT derived by optimizing the first parameter.

We have mainly two significant observations from analyzing the numerical results.

Table 4.5: Concerned measures in case of one parameter being optimized on a condensed traffic model. First column shows the optimized parameter.

Measures	TCC	VH	TWD	MAD	TL	LoPL	TLoL
TCC	76.1116	52.8087	23.705	15.833	12	4	22
VH	90.1116	48.0000	21.492	15.583	12	4	30
TWD	127.6961	58.0000	17.851	11.5	12	2	20
MAD	308.7584	90.0000	28.288	10.5	6	1	6
TL	310.0362	90.0000	45.967	15.833	6	2	12
LoPL	227.1473	71.0000	24.524	12.5	10	1	13
TLoL	308.7584	90.0000	28.288	10.5	6	1	6

Firstly the results show that among the seven measures we have investigated, the first three measures TCC, W_{hop} and TWD are traffic-related, so that their results on different traffic models vary in a significant manner, while the remaining four measures are apparently traffic-independent. Secondly, although we can hardly tell any clear relationship between these parameters, we can see in case of any traffic model, optimizing one parameter always makes most of other parameters deviate far from their own optimal value.

4.2.3 Formulation of A Bi-objective Problem

All the earlier discussions and illustrations are to bring out what are the possible pairs of parameters that can be used in bi-objective problem. Below we formulate a bi-objective problem using weighted method, which can be described as follows.

Objective function: minimize

$w_1 \times objectivefunction_1 + w_2 \times objectivefunction_2$,

where $w_1 + w_2 = 1$, $w_1, w_2 \geq 0$.

Subject to

1. Definition of objective function 1 and 2, as shown in section 4.1.1.

2. Traffic routing constraints: Eq. 2.4, Eq. 2.5 and Eq. 2.6.

3. Virtual degree constraints: Eq. 2.9. When TL is the optimized objective function,

use $\leq$; otherwise, use $=$.

4. Lightpath routing constraints: Eq. 2.14, Eq. 2.15, Eq. 2.16 and Eq. 2.17.

5. Wavelength number constraint: $M \leq Constant$.

5. Nonnegative constraints for all variables.

Numerical results are carried out in Chapter 5.

Chapter Summary

In this chapter, we attempt to formulate VTD problem by jointly optimizing two objective functions (parameters). First we summarize the commonly used objective functions and investigate how other parameters vary while one of the parameters is being optimized. Later we test the algorithm under three types of different traffic models. The results show that generally an improvement in one parameter is accompanied by a degradation in the other, suggesting the problem has a Pareto solution, where the objective functions of interest could be combined into one single objective function through the introduction of some weighting factors.

Chapter 5

Supplemental Calculation Work

In this chapter, we report and comment on the numerical results obtained by applying the heuristic algorithms discussed in the previous two chapters. We assume that all elements of the traffic matrices are independent random variables following the positive values generated by a Gaussian distribution. We introduce three traffic models, the three scenarios used are similar to those used in [20].

1. Scenario A: All traffic flows T^{sd} have identical distributions, with mean $\mu = 1$ and standard deviation $\delta = 1$.

2. Scenario B: Traffic flows are grouped into two classes: *heavy-traffic* flows and *light-traffic* flows. A randomly chosen 10% of all traffic flows belongs to the heavy traffic class, while the remaining 90% belongs to the light traffic class.

   I. Light-traffic flows have identical distributions, with mean $\mu = 1$ and standard deviation $\delta = 1$.

   II. Heavy-traffic flows have identical distributions, with mean $\mu = 50$ and standard deviation $\delta = 1$.

3. Scenario C: Here nodes are grouped into two classes: *clients* and *servers*. In the models concerned, some of the nodes are randomly chosen as *servers*, while the remaining nodes are *clients*. Based on this way of definition, three types of traffic flows can be identified, depending on the classes of nodes: flows between

servers (S-S), flows between clients (C-C) and flows between servers and clients (S-C). The stochastic characterization of traffic flows is as follows:

I. The distribution of each S-S traffic flow has mean $\mu = 100$ and standard deviation $\delta = 100$;

II. The distribution of each S-C traffic flow has mean $\mu = 10$ and standard deviation $\delta = 10$;

III. The distribution of each C-C traffic flow has mean $\mu = 1$ and standard deviation $\delta = 1$.

Traffic models mentioned in the coming sections are specified as scenario A, B, or C respectively.

5.1 Heuristic Algorithms

5.1.1 Enhancement of LP-relaxation

Ever since it was defined by [11], using LP-relaxation has been regarded as an efficient approach to derive a lower bound of traffic congestion. Furthermore, LP-relaxation is an effective method to evaluate the performance of existing and proposed heuristic algorithms. However, the number of iterations required in classical LP-relaxation is always uncontrollable, which is not so efficient as expected. So we enhance the classical LP-relaxation by starting with a general lower bound to reduce the number of iterations.

Here we try two methods as shown below.

a) Start from an estimated congestion level from its lower bound, we name this approach method as a *Estimated Bound* (EB):

$$\overline{T}_{max}(L = 1) = \frac{\sum_{s,d} \overline{T}^{sd}}{N \times \Delta},$$

Relationship for iterations is still referring to Eq. 2.29, and iteration terminates

Table 5.1: Congestion versus Virtual Degree for Traffic Scenario A.

Degree	LP(L)	LP-EB(L)	LP-new(L/n)
1	12.0973(35)	12.0734(29)	12.0762(16/1)
2	4.4378(12)	4.4366(9)	4.4431(5/1)
3	2.5678(6)	2.5685(5)	2.5728(4/2)
4	1.8983(5)	1.8981(4)	1.8988(4/5)
Degree	MILP	LP(Round-up)	Regular VT
1	15.4862	17.4153	18.7184
2	4.5897	4.7722	6.2395
3	2.5911	2.5911	3.1197
4	1.8988	1.9442	2.0958

when reaching constraint $\overline{T}_{max}(L+1) - \overline{T}_{max}(L) \leq 0.01$.

b) Define a new rounding method named *LP-new*, which is shown as follows.

$$\overline{T}_{max}(L+1) \geq \overline{T}_{max}(L) + \frac{(\overline{T}_{max}(L) - \overline{T}_{max}(L-1))}{n}, \text{where } n\text{: integer.}$$

At the first iteration of LP-new (when $L = 1$), we take $\overline{T}_{max}(0) = 0$, and accordingly

$$\overline{T}_{max}(2) \geq \overline{T}_{max}(1) + \frac{\overline{T}_{max}(1)}{n}.$$

Integer n is set to be the smallest positive integer that derives a new bound $\overline{T}_{max}(L+1)$, where $\overline{T}_{max}(L+1) > \overline{T}_{max}(L)$, and the n that fits the first iteration satisfies other iterations as well, which can be proven by experimentation. Also, rounding of LP-new terminates when reaching constraint $\overline{T}_{max}(L+1) - \overline{T}_{max}(L) \leq 0.01$.

First we explain the notations used in Table 5.1, Table 5.2 and Table 5.3. LP denotes the classical LP-relaxation method, LP-EB and LP-new are our new methods. In LP(L) and LP-EB(L), the number in the brackets denotes the total number of iterations of the respective method. In LP-new(L/n), there are two numbers listed, where n specifies the value of parameter n.

From the numerical results shown in tables: Table 5.1, Table 5.2 and Table 5.3,

Table 5.2: Congestion versus Virtual Degree for Traffic Scenario B.

Degree	LP(L)	LP-EB(L)	LP-new(L/n)
1	59.2874(27)	59.2870(24)	59.2949(11/1)
2	27.6600(13)	27.6605(11)	27.6643(5/2)
3	19.4909(9)	19.4899(7)	19.4927(5/3)
4	15.2615(6)	15.2618(5)	15.2621(3/7)
Degree	MILP	LP(Round-up)	Regular VT
1	62.1891	63.8707	62.7350
2	28.1057	36.2308	32.2854
3	20.8549	21.2212	25.9024
4	15.8796	15.9925	17.2682

Table 5.3: Congestion versus Virtual Degree for Traffic Scenario C.

Degree	LP(L)	LP-EB(L)	LP-new(L/n)
1	307.1497(30)	307.1467(28)	307.1526(13/1)
2	149.7088(19)	149.7075(17)	149.7121(7/1)
3	99.8078(12)	99.8091(11)	99.8115(6/2)
4	74.8580(7)	74.8583(7)	74.8589(5/5)
Degree	MILP	LP(Round-up)	Regular VT
1	321.3835	332.8467	357.8551
2	149.7179	149.7179	149.7179
3	99.8119	99.8119	99.8119
4	74.8589	74.8589	74.8589

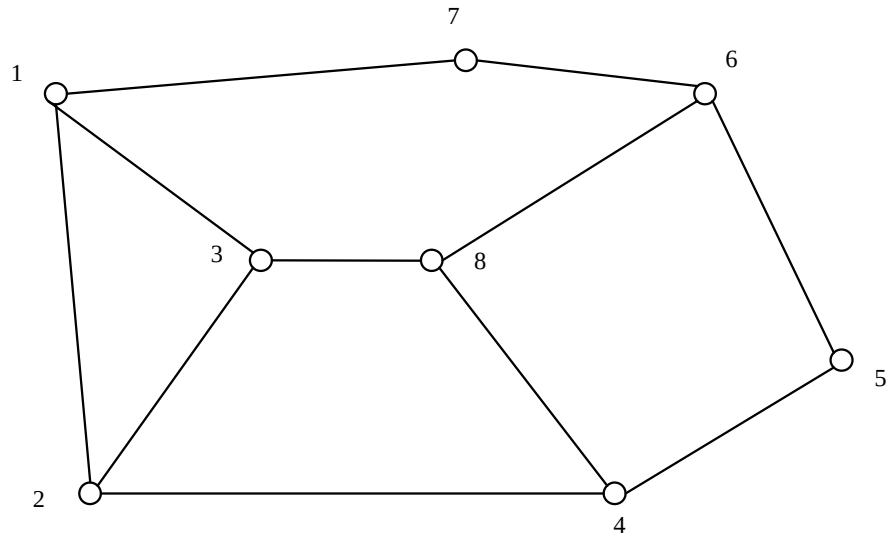


Figure 5.1: A 8-node Network construction

we can see that all LP methods, including LP, LP-EB and LP-new, approach to the same $\overline{T}_{max}$, e.g. 2.5728 for traffic scenario A, when $VD = 3$. Trivial difference is only caused by how the stopping constraint affects them. Secondly, to all LP methods, the number of iterations required decreases when the virtual degree increases. Rounding approximation performs its best in traffic scenario C, then traffic scenario B and lastly traffic scenario A. Finally, compared with classical LP-relaxation, both of LP-EB and LP-new reduce the number of iterations required, and LP-new method performs much better than LP-EB.

5.1.2 Numerical Comparisons

First we introduce a network model as shown below.

1. N=8; PT:

   1100010 110000 00001 1001 100 11 0

   as shown in Fig. 5.1.

   Lengths of total 11 physical links are : $D_{12} = 2.5$; $D_{13} = 1.7$; $D_{17} = 2.6$; $D_{23} = 1.8$; $D_{24} = 3.0$; $D_{38} = 1.1$; $D_{45} = 1.6$; $D_{48} = 2.0$; $D_{56} = 2.2$; $D_{67} = 2.3$; $D_{68} = 1.5$.

Table 5.4: Factors of propagation delay through the shortest routes for sample 8-node network

Node	1	2	3	4	5	6	7	8
1	0	2.5	1.7	4.8	6.5	4.3	2.6	2.8
2	2.5	0	1.8	3.0	4.6	4.4	5.1	2.9
3	1.7	1.8	0	3.1	4.7	2.6	4.3	1.1
4	4.8	3.0	3.1	0	1.6	3.5	5.8	2.0
5	6.5	4.6	4.7	1.6	0	2.2	4.5	3.6
6	4.3	4.4	2.6	3.5	2.2	0	2.3	1.5
7	2.6	5.1	4.3	5.8	4.5	2.3	0	3.8
8	2.8	2.9	1.1	2.0	3.6	1.5	3.8	0

Table 5.5: Traffic (scenario C.) for a 8-node network

Node	1	2	3	4	5	6	7	8
1	0	0.1678	6.2253	0.3194	1.8057	24.4351	1.2895	12.1204
2	0.2919	0	13.8988	0.5394	1.9312	17.8118	1.2316	7.0411
3	7.6600	17.9905	0	2.5796	4.4043	218.9164	7.3439	96.2367
4	2.4789	0.9271	6.4903	0	0.7376	12.3788	1.0112	10.8799
5	0.0102	2.1380	15.6896	0.6694	0	4.7513	0.2132	16.2323
6	11.1844	19.4089	132.7292	20.8229	14.4365	0	1.8778	117.4639
7	0.3549	2.3396	0.0776	0.3159	0.1564	3.6453	0	1.7829
8	13.1481	0.0791	81.3291	8.6850	0.5010	172.5791	12.0232	0

From the physical connection information, we can derive the shortest-route matrix d_{ij} , as shown in Table 5.4 (as defined in former chapter).

2. Assign nodes 3,6,8 as servers, the remaining are clients, we can generate the traffic according to scenario C, and shown in Table 5.5.

According to the inputs mentioned above, we get Table 5.6 and Table 5.7 representing the numerical results of heuristic algorithms concerned on congestion levels and delay, respectively.

5.1.3 RLL enhancement

As we mention in Chapter 3, theoretically we can enhance RLL heuristic algorithm by deriving an extension named as RLL-RP (remove lightpaths in pairs). As shown in Fig.

Table 5.6: Congestion versus Virtual Degree

VD	LP-new	LP-new roundup	MTN-VTD	MD-VTD
1	476.3856(34/1)	637.1479	646.7668	NULL
2	227.2443(11/1)	227.2589	233.1052	239.2002
3	151.5008(6/1)	151.5059	151.5059	159.4668
4	113.6291(7/2)	113.6295	113.6294	113.6294
5	90.9034(5/3)	90.9036	90.9036	90.9036
6	75.7529(4/7)	75.7530	75.7530	75.7530

Table 5.7: Minimal achievable Delay versus Virtual Degree, there are two kinds of delay involved, MAD and TWD

Minimal achievable average delay				
VD	MAD-VTD	LP-new	LP-new roundup	MTN-VTD
1	1.8750	2.9540	2.1375	4.0000
2	2.0125	3.2878	3.3125	3.7250
3	2.2333	3.2523	3.2000	3.4667
4	2.6250	3.4152	3.4531	3.4125
5	2.9050	3.3796	3.3400	3.3125
6	3.1583	3.4179	3.4979	3.4813
Minimal achievable traffic-weighted delay				
VD	MAD-VTD	LP-new	LP-new roundup	MTN-VTD
1	NULL	7.8199	8.3741	15.2045
2	5.6782	5.4835	7.4076	5.4243
3	5.4638	5.2845	6.1089	4.9886
4	5.1132	5.5662	7.3261	4.6872
5	5.0521	5.6671	6.3545	4.3011
6	6.3214	5.8094	8.4335	5.4162

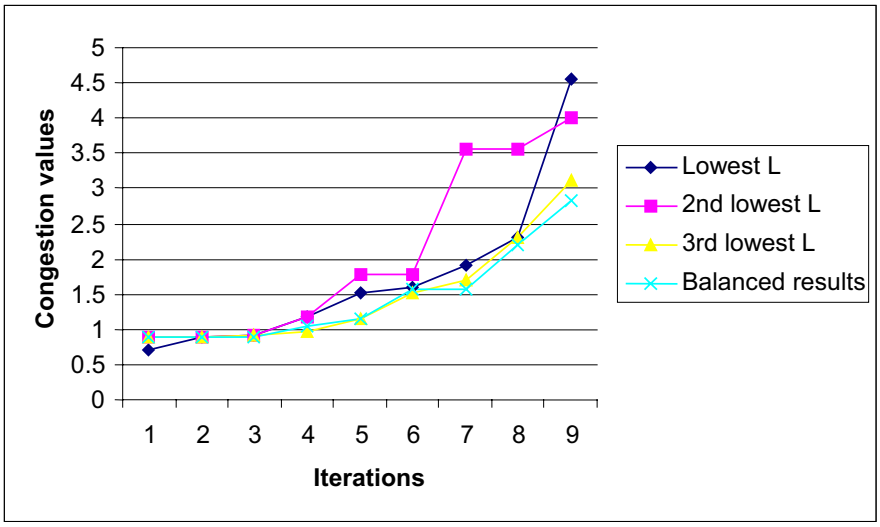


Figure 5.2: Congestion versus total number of lightpath, derived by RLL-RP in a 6-node network with traffic scenario A (each iteration corresponding to removal of one pair of lightpaths)

5.2 and Fig. 5.3, we try four methods to select which kind of lightpaths to be removed: (1) remove lightpath (i,j) having the lowest traffic, together with its image lightpath (j,i), which is denoted as 'lowest L' .(2) remove lightpath (i,j) having the second lowest traffic, together with its image lightpath (j,i), which is denoted as '2nd lowest L'. (3) (1) remove lightpath (i,j) having the third lowest traffic, together with its image lightpath (j,i), which is denoted as '3rd lowest L' and (4) remove lightpaths with low traffic flows but considering VD distribution in the whole network concurrently, which is denoted as 'balanced result'.

Results show there are no regular rules for the first three methods, i.e. there does not exist a distinguishable method among removing any sequence of the lowest link pairs. However, the balanced method always provides the lower traffic congestion compared to others. So, we can conclude that the best method for implementing RLL-RP is to remove the lightpaths with low LU value together considering the node tolerance and VD distribution among the whole network.

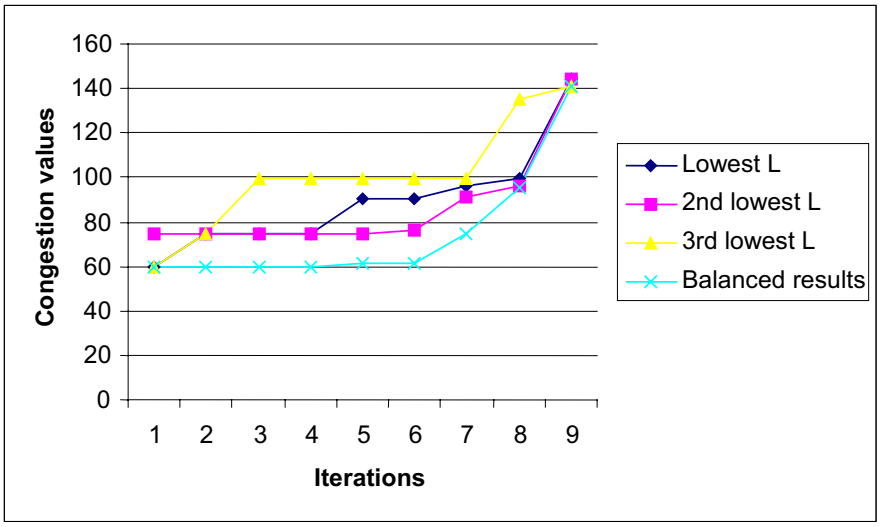


Figure 5.3: Congestion versus total number of lightpath, derived by RLL-RP in a 6-node network with traffic scenario C (each iteration corresponding to removal of one pair of lightpaths)

5.2 Bi-objective Problems and Performance of The Approaching Methods

First we solve a bi-objective VTD problem with TCC and VH as objective functions. Combine the two objective functions, following a weighted method, the new objective can be expressed as follows.

Minimize:

$$w_1 \times T_{max} + w_2 \times W_{hop}, \quad \text{where } w_1 + w_2 = 1. \tag{5.1}$$

On simple small network models, it is not hard to derive a semi-Pareto curve for such a bi-objective formulation. Note when we move from one virtual topology to another with congestion value increasing, total number of virtual hops reduced or vise versa, and there is no doubt that we cannot generate a consecutive curve, but only separated dots which is capable to be connected by a 'virtual Pareto curve'.

We use the six-node network model introduced in chapter 3, with traffic generated by scenario B, virtual degree constraint 3. Results are shown in Table 5.8.

Secondly we examine another pair of combined objective functions as shown below

Table 5.8: VTD solution for a six-node network with VD constraint 3, combining consideration on minimizing traffic congestion together with the number of virtual hops

Iteration	VT					
1	001101	101010	100110	010011	010101	111000
2	001101	100110	110001	011010	001101	110010
3	001101	101010	100110	011001	010101	110010
4	010101	001110	110001	101010	010101	101010
5	001110	001011	100101	011010	110001	110100

weight of T_{max}	congestion	weight of W_{hop}	total of W_{hop}
99	47.0963	1	43.0531
3	47.4609	7	42.8795
2	49.7241	8	42.2168
1	50.5512	9	42.0100
1	80.7803	99	42.0000

and present the relevant results in Table 5.9.

$$w_1 \times T_{max} + w_2 \times \sum_{ij} b_{ij} d_{ij}, \quad \text{where } w_1 + w_2 = 1. \tag{5.2}$$

Chapter Summary

Work performed in this chapter can be regarded as an extension to our algorithms discussed in the previous two chapters. We report and comment on the numerical results obtained by applying the heuristic algorithms.

Table 5.9: VTD solution for a six-node network with VD constraint 3, combining consideration on minimizing traffic congestion together with average delay

Iteration	VT					
1	010000	001000	000100	000010	000001	100000
2	000001	100000	010010	001000	000100	001000
3	000001	100000	010100	001010	000100	001000
4	000001	001000	010101	001010	000100	101000
5	010011	101000	100101	011010	000100	101000
6	010011	101000	100101	011010	000100	101000
7	010001	101010	110001	011010	001100	100110
8	010101	101010	110001	011010	000101	101010
9	001001	100110	110001	011010	001101	110010
10	001101	101010	100110	010011	010101	111000

M	VL	weight of MAD	total of MAD	congestion	weight of T_{max}
1	6	99	63.0000	308.7584	1
1	7	91	69.0000	238.9648	9
1	8	9	69.0000	238.9648	1
2	10	8	92.0000	129.9652	2
3	14	3	159.0000	66.9481	7
3	14	25	159.0000	66.9482	75
4	16	2	209.0000	52.1284	8
4	17	1	225.0000	49.7015	9
4	17	5	247.0000	48.2112	95
4	17	1	279.0000	47.0963	99

Chapter 6

Conclusions

As future networks based on WDM technology are developed to support data traffic and the Internet, they must be designed and optimized for that purpose. In particular, a number of resource utilization and congestion level control problems arise among which making the network virtual topology design has been intensively discussed in this thesis.

In the first two chapters, we begin our thesis with discussions on the basic function blocks related to OWDM networks, including node configurations, optical switches and routers etc. Meanwhile we have been discussing about the network model used in this thesis. Our study mainly focuses on future OWDM networks, with their nodes having packet switching capability, and hence the traffic flow from a source to a destination can be routed through multiple virtual wavelength lightpaths. When necessary, wavelength converter has been assumed to be available although it may not be cost efficient at this stage.

Then mathematical method in formulating VTD as a mixed integer linear programming problem with one objective function has been introduced in Chapter 3. Discussions on the formulation and possible objective functions to be used for optimization have been carried out as well. Moreover, we examine the existing heuristic algorithms. Performances of some of those algorithms have been analyzed, compared and categorized, followed by a discussion on their assumptions and limitations. We also

define a new heuristic algorithm that performs VTD by decomposing the problem into four subproblems, each with an objective function, and to be solved sequentially. It has less computation complexity than MILP but can achieve a congestion level closer to what is predicted using MILP due to the embedding of computation-based optimization in each subproblem. Furthermore, this algorithm overcomes the disadvantage of some existing heuristic algorithms having a similar purpose by eliminating the risk on obtaining a partitioned virtual topology.

In Chapter 3, we also propose a new objective function named as traffic-weighted virtual hops, which presents another point of view to partitioned traffic routing and are more suitable for the packet-switched network we proposed. Also we elaborate a novel way to formulate VTD problem by removing the virtual degree (VD) constraints of each individual node and replacing them by a single constraint related to the total VD of the entire network. Numerical results show that lower optimal congestion value can be achieved by using a new heuristic algorithm developed. The solution provides information on whether resources in the network are allocated efficiently, and in case it is not, how re-distribution can be made to achieve better utilization. This could be very important in the future for operators to maximize the resource utilization in the networks.

In Chapter 4, VTD problems with jointly optimizing two objective functions have been formulated, which is named as 'bi-objective problem'. And their relevant Pareto solutions are derived to solve these bi-objective problems. The discussion has been demonstrated by using a six-node optical network under various traffic models. Our investigation shows that it is possible to design VT by jointly considering two objective functions. In general, an improvement in one parameter is accompanied by a degradation in the other. Chapter 5 reports and comments on the topics discussed before by giving more numerical examples.

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Appendix A

Expressions For Connectivities

In this thesis, we use sequences to express the construction of physical topology and virtual topology.

Normally the position in a sequence is used to specify the possible source and destination. A '0' is used to denote that there is no connection in a certain position, whilst a '1' denotes there exists a connection.

Here we present two simple examples to explain how the notation works.

For example, we may use a sequence '10010 0100 110 01 1' to represent a physical topology. We can see there are totally five groups, and the number of entries in the neighboring group is decreasing by one. Here the group number (denoted by 'i' in further explanation) is the node number. And the number in each position (denoted by 'j') of a group represents the connectivity, i.e. a '1' existing in the position j of group i indicates there exists a connection between node i and node '$i + j$'. In the above example, there are total five groups, so the network has six nodes. The first '1' indicates there is a undirected link between node 1 and node 2; the second '1' indicates there is a undirected link between node 1 and node 5, and so on. We therefore can get the information as listed below: there is a six-node network with connections between node pair 12, 15, 23, 34, 35, 45 and 56.

The other case is for directed graph. The length of each group will be the same, all

equals to the size of network. We use this notation to denote virtual topologies in this thesis. A group number (i) indicates the node number, a position number (j) indicates the node number that node i is connected to. This is similar to rearrange the rows of a virtual topology matrix and combine them into a single row.

Advantage of using those expressions is to save presentation spaces without causing information lost.