ROBUST IDENTIFICATION AND CONTROLLER DESIGN FOR DELAY PROCESSES

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Summary

Process identification plays an important role in process analysis, controller design, system optimization and fault detection. One of the active and difficult areas in process identification is in time delay systems. Time delay exists in many industrial processes and has a significant effect on the performance of control systems. Thus, identification of unknown time delay needs special attention. In this thesis, a series of identification methods are proposed for continuous-time delay processes. Both open-loop identification tests and closed-loop ones are considered. The initial conditions are unknown and can be nonzero. The disturbance can be a static or dynamic one. Regression equations are derived according to types of test signals. All the parameters including time delay are estimated without iteration. These identification methods show great robustness against noise in output measurements but require no filtering of noisy data.

In the context of pulse tests, a two-stage integral identification method is presented for continuous-time delay processes. It is noticed that the output response from a pulse test will still be significant and last for a long time after the pulse disappears. We take advantage of this feature. The integral intervals are specifically chosen and this enables easy and decoupled identification of the system parameters in two stages.

In the context of step tests, a one-stage integral identification method is developed for continuous-time delay processes. The key idea is to make both upper and lower limits of the inner integral dependent of the dummy variable of the outer integral so that the initial conditions do not appear in the resulting integral equation.

In the context of relay tests, the fast Fourier transform based identification method is revisited first and the need for further development is discussed. An identification method from relay tests is proposed. By viewing a relay test as a sequence of step tests, the integral technique is adopted to devise the algorithm. A general integral identification method is then proposed. The identification test can be of open-loop type such as pseudo random binary signals and pulse tests, or of closed-loop type such as relay tests. The disturbance can be of general form. The proposed new regression equation has more linearly independent functions and thus enables to identify a full process model with time delay as well as combined effects of unknown initial condition and disturbance without any iteration.

Most industrial processes are of multivariable in nature and time delay is present in most industrial processes. Identification of multivariable processes with multiple time delay is in great demand. To this end, an effective identification technique is presented for multivariable delay processes. The technique covers all popular tests used in applications, requires reasonable amount of computations, and provides accurate and robust identification results.

The model obtained from process identification may be used for controller design. In the thesis, an analytical PID design method is proposed for continuous-time delay systems to achieve approximate pole placement with dominance. It is well known that a continuous-time feedback system with time delay has infinite spectrum and it is impossible to assign such infinite spectrum with a finite-dimensional controller. In such a case, only the partial pole placement may be feasible and hopefully some of the assigned poles are dominant. But there is no easy way to guarantee dominance of the desired poles. The idea presented is to bypass continuous infinite spectrum problems by converting a delay process to a rational discrete model and getting back continuous PID controller from its discrete form designed for the model with pole placement.

As shown in the given simulation examples and real time tests, the findings can be applied to industrial control systems. The schemes and results presented in this thesis have both theoretical contributions and practical values.

Chapter 1

Introduction

1.1 Motivation

The need for process model arises from various engineering tasks such as process design, process control, plant optimization and fault detection (Ikonen and Najin, 2002). Identification is the experimental approach to process modeling (Åström and Wittenmark, 1990) and has been an active area in control engineering (Soderstrom and Mossberg, 2000). Many text books and book chapters have been published on identification, for examples, Soderstrom and Stoica (1983), Ljung (1987), Unbehauen and Rao (1987), Sinha and Rao (1991), Johansson (1993) and Ikonen and Najin (2002). It is also a hot topic in international academic journals and many publications are available on this topic, see the following special issues: Automatica 1981 v.17(1), Automatica 1990 v.26(1), IEEEAC 1992 v.37(7), Automatica 1995 v.31(12), Journal of Process Control 1995 v.5(2) and Automatica 2005 v.41(3).

System identification involves three components: test design, model structure identification and parameter estimation (Ljung, 1999). A specific test is designed and input and output responses during such a test will be then recorded. The model structure and parameter are then identified. The objective of test design is to excite the process sufficiently to enable identification of the process. A model with unknown parameters needs to be constructed. Various model structures are available to assist in modeling a system. The choice of model structures is based upon understanding of identification method and insight into identification test. Parameter estimation is employed to determine the unknown model parameters from recorded data set.

Identification tests are generally divided into open-loop tests and closed-loop tests. Step tests and pulse tests are the most popular open-loop tests for their simplicity (Luyben, 1973). They have their own merits. Step tests are the most simple and dominant ones. Pulse tests return input and output to the original stead-state and cause less perturbation to process operation. Though there are many successful applications of open-loop identification, closed-loop identification is also an important practical issue (Landau and Karimi, 1999). The most popular closed-loop identification test is relay feedback (Åström and Hagglund, 1984).

Identification models are generally classified into parametric models and non-parametric ones (Wellstead, 1981). Frequency response is a kind of nonparametric model of processes. It is very useful for system analysis, such as Nyquist stability studies, controller designs (Goodwin *et al.*, 2001) and parametric model building (Ljung and Glover, 1981). Parametric models are also preferred by many control engineers (Unbehauen and Rao, 1987; Ninness, 1996; Ljung, 1985; Ljung, 1999), because most of advanced control strategies are developed based on parametric models (Morari and Zafiriou, 1989; Åström and Wittenmark, 1995; Narendra and Annaswamy, 1989; Anderson and Moore, 1990; Zhou, 1998).

For nonparametric modeling, relay feedback is one of the popular tests because frequency responses of processes can be obtained from relay tests. In the early stage of study on relay identifications, only stationary response of a relay test was used to estimate the process frequency at the oscillation frequency (Åström and Hagglund, 1995). Later, an improvement was reported by Wang *et al.* (1997*a*).

They use a biased relay feedback and can obtain two accurate process frequency points from one test. These two estimated frequency points can be converted easily to an first-order plus time delay (FOPTD) model of the process. A lot of chemical processes can be modelled by using this method. Another modification of the standard relay was proposed by Bi *et al.* (1997): a parasitic relay is added to the standard relay. This method can identify multiple points on the process frequency response. Recently, relay identification based on fast Fourier transform (FFT) was developed. It was first shown in Hang *et al.* (1995) that multiple points on the process frequency response can be obtained in a step test by applying FFT. This method has been further improved and used to identify multiple points simultaneously from standard relay tests (Wang *et al.*, 1997*b*). Wang and his colleagues introduced a decay exponential to rescale the input and output, then applied FFT to obtain multiple points on the process frequency response. In Wang *et al.* (1999), a modified method was developed. Low-pass filters are included in the control loop and more robust identifications can be obtained. However, these FFT based identification methods assume that the relay test starts from a steady state and there is no disturbance during the test. Besides, additional low-pass filters have to be used to overcome the effect of the measurement noise. These restrictions can limit their applications in some cases. It is desirable to remove these assumptions for wider applications.

Among identification methods of parametric models, continuous-time identification has been an active area for its advantages in retaining the models of actually time-continuous dynamic systems in continuous-time domain (Sinha and Lastman, 1982; Saha and Rao, 1983; Unbehauen and Rao, 1987; Sagara and Zhao, 1990). An important issue with identification of continuous-time parametric models is identification of time delay (Wang and Gawthrop, 2001; Garnier *et al.*, 2003). Time delay is a property of physical systems, by which response to the system input is delayed in its effect (Shinskey, 1976). It exists in many industrial processes. In most situations time delay is unknown. Because time delay has a significant effect on the performance of the control systems, its estimation needs special attention (Gawthrop, 1984). Many existing identification methods do not consider time delay or assume known delay because time delay appears nonlinearly in the regression equation. For these reasons, there are continuing interests in identifications of delay processes. Some early methods estimate time delay with numerator polynomial or transfer function. In Kurz and Goedecke (1981), a shift operator model with expanded numerator polynomial is used to deal with unknown time delays. Rational transfer functions, such as polynomial approximation and Pade approximation, are used to estimate time delay in Gawthrop and Nihtila (1985) and Souza *et al.* (1988), respectively. These methods proposed in the early days increase the order of the models and have to identify more model parameters. Later, a trial and error method was proposed. Elnaggar *et al.* (1989) assumes a known delay and then estimates the other transfer function parameters. With the estimated model, the estimated error is calculated. From all the obtained models, the one which minimizes the estimated error is chosen as the identification result. In Ferretti *et al.* (1991), an algorithm was proposed to recursively update the value of a small delay by inspection of the phase contribution of the real negative zero arising in the corresponding sampled system. This method is inefficient. In Mamat and Fleming (1995) and Rangaiah and Krishnaswamy (1996), graphical methods were proposed to identify low order models for continuous-time delay system. However, their methods cannot identify high-order processes and non-minimum-phase systems and may lead to large estimation errors when noise is considerable.

Recently, new integration identification methods were reported for identifications of continuous-time delay systems (Wang and Zhang, 2001; Hwang and Lai, 2004). Integration identification is a branch of linear filter identification (Unbehauen and Rao, 1987; Rao and Unbehauen, 2006; Garnier *et al.*, 2003). Like other continuous-time identification methods, integration identification methods consist of two main parts: signal processing (multiple integration) and parameter estimation. The multiple integration works as a pre-filter to overcome the noise effect (Unbehauen and Rao, 1990) like analog pre-filter (Young and Jakeman, 1981). Integration approach for parameter estimation was first proposed by Diamessis (1965). Later an improvement was made by treating the initial states of the system as additional system parameters to be estimated (Mathew and Fairman, 1974). By then, the effect of the disturbance had not been considered. With the development of computer technologies, numerical integration is then used (Whitfield and Messali, 1987). In Whitfield and Messali (1987), the effect of deterministic disturbances at system input and output is also included in the analysis. A similar integral-equation approach has been derived by Golubev and Wang (1982) from a frequency-domain error criterion. From their works, efficiency and robustness of integral equation methods have been shown. It was Wang and Zhang (2001) who first proposed to apply integration method to identify continuous-time delay systems from step tests without iterations. Their method takes advantage of the simple nature of step input and a linear regression equation with a new parameterization is devised. The least-squares method is then applied to identify the regression parameters, from which the full model parameters including time delay are recovered. This method is so robust that the identification results are still satisfactory without filtering of the measured output, which is corrupted by noise. However, like FFT methods from relay tests, Wang's integration method requires that the tests start from zero initial conditions and there is no disturbance during the test. Hwang and Lai (2004) proposed a two-stage identification algorithm, which uses pulse signals as the input. Two regression equations are obtained from the two edges of the pulse signal, respectively. Then the estimation and/or the elimination of the initial conditions and disturbances become possible. Their regression parameter vectors involve all parameters together in each of two stages, and some of them are very complicated functions of process parameters and initial conditions. This method fails to work in the step test case, the most popular one in process control applications, because a step test only has one change of its magnitude. Simplified general identification methods are needed to identify delay processes under unknown initial conditions and disturbance from popular identification tests.

Most industrial processes are of multivariable in nature (Ogunnaike and Ray, 1994; Maciejowski, 1989). To achieve performance requirements by using advanced controller design methods, models of multivaribale processes are needed (Sinha and Lastman, 1982; Zhu and Backx, 1993; Ikonen and Najin, 2002; Gevers *et al.*, 2006). To this end, many methods have been proposed to identify multivariable processes, for examples, methods proposed in Whitfield and Messali (1987), Wang *et al.* (2001*b*) and Garnier *et al.* (2007). But only a few of them consider time delays. In Garnier *et al.* (2007), a model with input delays is considered but these time delays are supposed to be known. In Wang *et al.* (2001*b*), relay tests are applied. The frequency responses from the inputs to the outputs are obtained by applying the FFT. The process step response is constructed by using the inverse FFT to each process channel. Integral identification methods are then used to recover all the process model parameters including time delay. Their method is very robust in face of noise. However, their identification methods and those used in Wang *et al.* (2003) require zero initial conditions and no significant disturbance. For easy applications, these assumptions should be removed. Developing a general identification method for multivariable delay processes is of great interest and value.

Control design is a key topic of control engineering. It is also one usage of process identification (Hjalmarsson, 2005). Since the proportional-integral-derivative(PID) controller was proposed, its tuning has been an attractive area because PID control offers the simplest and most effective solution to many control problems (Ang *et al.*, 2005). According to Yamamoto and Hashimoto (1991), a large number of PID controllers are used in industry and some of them are not well tuned. To improve this situation, many methods haven been proposed, such as methods proposed in Persson and Åström (1993), Ho *et al.* (1995), Maffezzoni (1997), Tan *et al.* (1999), Mattei (2001), Wang *et al.* (2001*a*), Zheng *et al.* (2002*b*) and Zheng *et al.* (2002*a*). Among them, one important branch is the dominant pole placement. Tuning of PID controllers with dominant closed-loop poles was first introduced by Persson and Åström (1993) and further explained in Åström and Hagglund (1995). Both methods are based on a simplified model of processes and thus cannot guarantee the chosen poles to be indeed dominant in reality. In the case of high-order systems or systems with time delay, these conventional dominant pole designs, if not well handled, could result in sluggish response or even instability of the closed-loop. Thus it is desirable to have a method to make the chosen poles dominant by using PID controller.

1.2 Contributions

In this thesis, a series of identification methods are proposed for continuous-time delay processes under nonzero initial condition and disturbance. Both open-loop tests and closed-loop tests are considered. Parametric models with time delay are identified for single-variable continuous-time delay processes and multivariable delay processes.

A. Process identification from pulse tests

A two-stage integral method is presented for continuous-time delay systems from pulse tests. It is noticed that the output response from a pulse test will still be significant and last for long after the pulse disappears. We take advantage of this feature to manipulate integration intervals so that the integral equation and thus regression equations are greatly simplified. This enables us to establish decoupled estimation of two sets of system parameters in a very simple manner from pulse tests.

B. Process identification from step tests

An integral identification method is proposed for continuous-time delay systems from step tests. The integration limits are specifically chosen to make the resulting integral equation independent of the unknown initial conditions. This enables identification of the process model from a step test by one-stage least-squares algorithm without any iteration.

C. Process identification from relay tests

We revisit FFT based relay identification methods first and need for further development is discussed. An integral identification method from relay tests is then presented. By regarding a relay test as a sequence of step tests, the integral technique is adopted to devise the algorithm. The method can yield a full process model in the sense of a complete transfer function with delay or a complete frequency response.

D. Process identification from piecewise step tests

An general identification algorithm is proposed for continuous-time delay systems for a wide range of input signals expressible as a sequence of step signals. It is based on a novel regression equation which is derived by taking into account the nature of the underlying test signal. The equation has more linearly independent functions and thus enables to identify a full process model with time delay as well as combined effects of unknown initial condition and disturbance without any iteration.

E. Multivariable processes identification

A robust identification method is proposed for multivariable continuous-time processes with multiple time delay. Suitable multiple integrations are constructed and regression equations linear in the aggregate parameters are derived with use of the test responses and their multiple integrals. The process model parameters including the time delay is recovered by solving some algebraic equations.

F. PID controller design by approximate pole placement with dominance

It is well known that a continuous-time feedback system with time delay has infinite spectrum and it is not possible to assign such infinite spectrum with a finite-dimensional controller. In such a case, only partial pole placement may be feasible and hopefully some of the assigned poles are dominant. But there is no easy way to guarantee dominance of the desired poles. An analytical PID design method is proposed for continuous-time delay systems to achieve approximate pole placement with dominance. Its idea is to bypass continuous infinite spectrum problem by converting a delay process to a rational discrete model and getting back continuous PID controller from its discrete form designed for the model with pole placement.

1.3 Organization of the thesis

The thesis is organized as follows. After the Introduction, Chapter 2 focuses on identification of delay processes from pulse tests. Chapter 3 is devoted to process identification from step tests. Chapter 4 presents an identification method from relay tests. An improved identification method is developed in Chapter 5. In Chapter 6, identification of multivariable delay processes is considered. Chapter 7 is concerned with a PID controller design method by approximate pole placement with dominance. In Chapter 8, general conclusions are drawn and expectations for further works are presented.

Chapter 2

Process Identification from Pulse Tests

2.1 Introduction

Pulse testing can return inputs and outputs to the original steady state after the test is finished. It is preferred in many industrial applications for this reason. Recently, a two-stage identification method from pulse testing was proposed by Hwang and Lai (2004). Two parts of a pulse test could be used to establish two sets of integral equations so that estimation or elimination of non-zero initial conditions becomes possible. But, their regression parameter vectors involve all parameters together in each of two steps, and some of them are very complicated functions of process parameters and initial conditions. In this chapter, we manipulate integration intervals so as to greatly simplify the integral equation and thus regression equations. This enables us to establish decoupled estimation of two sets of system parameters in a very simple manner.

This chapter is organized as follows. In Section 2.2, the proposed method is presented. Simulation results are shown in Section 2.3. A real-time application is given in Section 2.4. Conclusions are drawn in Section 2.5.

2.2 Identification from pulse tests

Consider a nth-order continuous-time system with time delay,

$$y^{(n)}(t)+\cdots+a_1y^{(1)}(t)+a_0y(t)=b_mu^{(m)}(t-d)+\cdots+b_1u^{(1)}(t-d)+b_0u(t-d)+c, \tag{2.1}$$

where $y(t)$ and $u(t)$ are the output and input of the process, respectively, d is the time delay and c is the static disturbance or a bias value of the process. d, c, $a_i, i=0,\ldots,n-1$, and $b_j, j=0,\ldots,m$, are unknown parameters to be estimated. The initial conditions, $y^{(i)}(0), i=1,\ldots,n-1$, are also unknown and can be non-zero. Suppose that the test signal, $u(t)$, is a rectangular pulse with magnitude of h and duration of T,

$$u(t)=h\left[\mathbf{1}(t)-\mathbf{1}(t-T)\right], \tag{2.2}$$

where $\mathbf{1}(t)$ is the unit step. Note that (2.2) implies $u(t)=0,\ t\in[-d,0]$, which is the initial function for the input needed to make the time delay system (2.1) well-posed. Figure 2.1 depicts the pulse input and the resulting output response. It is noticed that the output response will be still significant and last for long after the pulse disappears. We will take advantage of this feature to simplify the system equation and carry out the parameter estimation into two steps: for a_i and c in the first step and b_i and d in the second step.

To avoid using time derivatives of $u(t)$ and $y(t)$ in the identification of process model, (2.1) will be converted to an integral equation. To this end, we need the following integral notations,

$$\begin{cases} I_0f(t_0,t)=f(t), \\ I_jf(t_0,t)=\int_{t_0}^{t}\int_{t_0}^{\tau_{j-1}}\cdots\int_{t_0}^{\tau_1}f(\tau_0)d\tau_0d\tau_1\cdots d\tau_{j-1}, j\geq 1, \end{cases} \tag{2.3}$$

where $\tau_i, i=0,\ldots,j-1$ are dummy variables for relevant integrals. In the first step of our identification, we select one fixed time point t_1 with $t_1>d+T$. Integrating

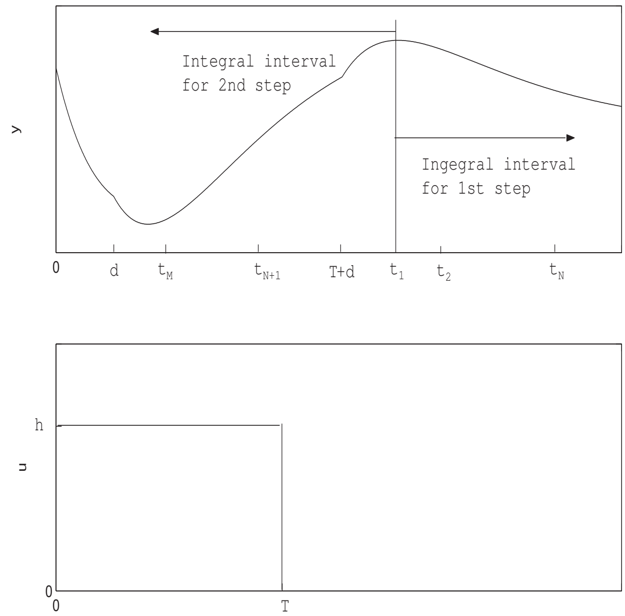

Figure 2.1. Rectangular pulse response and input.

(2.1) from t_1 to $t > t_1$ n times yields

$$\sum_{k=0}^{n-1} a_k I_{n-k} y(t_1, t) + y(t) - \sum_{k=0}^{n-1} \alpha_k \frac{(t-t_1)^k}{k!} = \sum_{k=0}^{m} b_k I_{n-k} u(t_1 - d, t - d) + \frac{c(t-t_1)^n}{n!}, \tag{2.4}$$

where α_i are related to process initial conditions at t_1. Since $t > t_1 > d + T$, the input is always zero. We have

$$I_j u(t_1 - d, t - d) = 0, j = 0, \cdots, n-1. \tag{2.5}$$

Substituting (2.5) into (2.4), we obtain

$$\phi_1^T(t)\beta = \gamma_1(t), \tag{2.6}$$

where

$$\phi_1^T(t) = [-I_1 y(t_1, t) \quad \cdots \quad -I_{n-1} y(t_1, t) \quad -I_n y(t_1, t) \quad 1 \quad (t - t_1) \quad \cdots \quad \tfrac{(t-t_1)^n}{n!}],$$

$$\gamma_1(t) = y(t),$$

and

$$\beta = [a_{n-1} \quad \cdots \quad a_1 \quad a_0 \quad \alpha_0 \quad \alpha_1 \quad \cdots \quad c]^T.$$

One invokes (2.6) for $t = t_i, i = 2, \ldots, N$, to form

$$\Gamma_1 = \Phi_1 \beta$$

where $\Gamma_1 = [\gamma_1(t_2), \ldots, \gamma_1(t_N)]^T$, and $\Phi_1 = [\phi_1(t_2), \ldots, \phi_1(t_N)]^T$. $t_i, i = 1, \ldots, N$, are chosen to meet $t_1 < t_2 < \ldots < t_N$, where $N > 2n + 2$. The least-squares method is applied to get,

$$\hat{\beta} = \left(\Phi_1^T \Phi_1\right)^{-1} \Phi_1^T \Gamma_1, \tag{2.7}$$

which gives the estimates for α_i, c and a_i.

In the second step, we integrate (2.1) in a reverse way from t_1 to t, with $d < t < T + d$, n times and this will still lead to (2.4). But, for $d < t < d + T$, we have

$$I_j u(t_1 - d, t - d) = h \frac{(t - d - T)^j}{j!}. \tag{2.8}$$

Substituting (2.8) into (2.4), we obtain

$$\phi_2^T \theta = \gamma_2(t), \tag{2.9}$$

where

$$\phi_2^T = h[1 \quad t \quad t^2 \quad \cdots \quad t^n],$$

$$\gamma_2(t) = \sum_{k=0}^{n-1} a_k I_{n-k} y(t_1, t) + y(t) - \sum_{k=0}^{n-1} \alpha_k \frac{(t - t_1)^k}{k!} - c \frac{(t - t_1)^n}{n!},$$

and

$$\theta = [\theta_1 \quad \theta_2 \quad \theta_3 \quad \cdots \quad \theta_{n+1}]^T.$$

Once again, one invokes (2.9) for $t = t_i, i = N+1, \ldots, M$, so that the least-squares method is applied to estimate θ. In this step, $t_{N+1} > t_{N+2} > \ldots > t_M$ and $M - N - 1 > n + 1$. The elements of θ are related to the model parameters b_i and d via

$$\theta_k = \sum_{j=max(n-m,k-1)}^{n} \frac{b_{n-j}(-d - T)^{j-k+1}}{(j - k + 1)!(k - 1)!}, k = 1, 2, \cdots, n + 1. \tag{2.10}$$

They are solved from $k = n + 1, n, \cdots, n + 1 - m$, to get

$$b_i = \sum_{j=0}^{i} \frac{(n - i + j)!\theta_{n+1-i+j}(d + T)^j}{j!}, i = 0, 1, \cdots, m. \tag{2.11}$$

They are substituted back to (2.10), for $k = n - m$, to get

$$\sum_{j=0}^{m+1} \frac{(n-m-1+j)!\theta_{n-m+j}(d+T)^j}{j!} = 0, \tag{2.12}$$

which is solved for d. Once d is determined, b_i can be easily computed from (2.11).

In the first step, the chosen t_1 depends on d, but d is unknown and to be identified. This is the same issue as encountered in Hwang and Lai (2004). Fortunately, one need not know the value of d to use our algorithm and a rough estimation of its range is sufficient. Let d be in the range, $[d_{min}, d_{max}]$. We can then choose $t_1 > T + d_{max}$ in the first step and $d_{max} < t < d_{min} + T$ in the second step. t_1 can not be chosen so large that the pulse response is already at its steady state at the time of t_1. It is recommended that t_1 is chosen as close to $T + d_{max}$ as possible. In many engineering applications, one can have simple, reliable and probably conservative estimation of the range of d from knowledge of the process. For instance, if you have transportation delay due to a long pipe, one can easily calculate $[d_{min}, d_{max}]$ based on the pipe length and fluid speed range. The experiment-based technique to get the range estimation is also possible. For example, d_{min} may be set as the time from the input signal injection to the point when the output response still remains unchanged from the past trend, while d_{max} is the time from the input signal injection to the point when the output response has got the changes from the past trend well beyond the noise band (Åström and Hagglund, 1995). If no engineering knowledge or experiment is available, a purely numerical method is given in Hwang and Lai (2004) to estimate such a range.

The model structure identification is an important issue and has been discussed in the literature. We adopt the standard practice as follows. We may start from a first-order or second-order time delay system. With the estimated model, it is easy to estimate the initial conditions at $t = 0$. The pulse response can be recovered using the estimated model under estimated initial conditions and disturbance. Compare the recovered response with the recorded one from the actual process. If the error between them is acceptable, the identification task is completed and stops. Otherwise, we may increase n and/or m by one until the estimated response fits to the recorded one well.

In the presence of noise, the measurement of the process output is corrupted. It follows from Soderstrom and Stoica (1983) that the ordinary least-squares estimate is not consistent. One solution is to use the instrumental variable (IV) method. The IV method proposed by Wang and Zhang (2001) is adopted here.

The method described above can be applied with minor modifications to rectangular doublet pulses with magnitude of h and duration of T as well,

$$u(t) = h\left[\mathbf{1}(t) - 2\mathbf{1}(t - T/2) + \mathbf{1}(t - T)\right].$$

The only difference for rectangular doublet signal is that in the second step, we choose $d < t < T/2 + d < t_1$, for which

$$I_j u(t_1 - d, t - d) = 2h\frac{(t - d - \frac{T}{2})^j}{j!} - h\frac{(t - d - T)^j}{j!}. \tag{2.13}$$

This leads to a new relationship of θ_k to b_i:

$$\theta_k = \sum_{j=max(n-m,k-1)}^{n} \frac{b_{n-j}[2(-d - \frac{T}{2})^{j-k+1} - (-d - T)^{j-k+1}]}{(j - k + 1)!(k - 1)!}. \tag{2.14}$$

They are solved from $k = n + 1, n, \cdots, n + 1 - m$, to get

$$b_i = (n - i)!\theta_{n+1-i} - \sum_{j=0}^{i-1} \frac{b_j[2(-d - \frac{T}{2})^{i-j} - (-d - T)^{i-j}]}{(i - j)!}, i = 0, 1, \cdots, m. \tag{2.15}$$

They are substituted back to (2.14), for $k = n - m$. This gives rise to a $(m + 1)th$ degree polynomial equation in d, which leads to $m + 1$ roots for d. Once d is chosen for the minimization of the error (Wang and Zhang, 2001), b_i can be easily computed from (2.15).

2.3 Simulation

In this section, the proposed identification method is applied to three examples below. Without loss of generality, the pulse height h is set to 1.

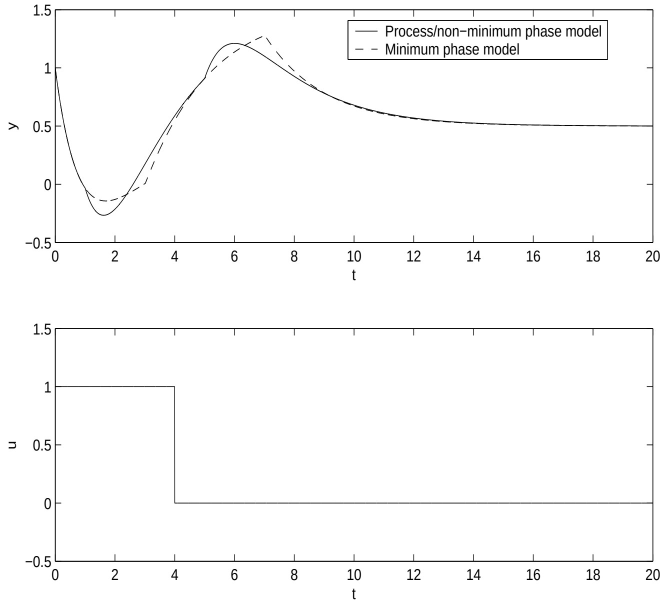

Figure 2.2. Rectangular pulse response and input for Example 2.1.

Example 2.1. Consider a $2nd$-order process,

$$y^{(2)}(t) + 1.5y^{(1)}(t) + 0.5y(t) = -0.5u^{(1)}(t-1) + 0.5u(t-1) + c,$$

subject to $y(0) = 1$, $y^{(1)}(0) = -2$ and $c = 0.25$. A rectangular pulse with width of $T = 4$ is applied as the input. The algorithm is applied with $n = 2$ and $m = 1$. In the first step, we select $t_1 = 6$ and the algorithm leads to

$$\beta = [1.5013 \quad 0.5008 \quad 1.2123 \quad 1.8196 \quad 0.2504]^T,$$

so that $\hat{a}_1 = 1.5013$, $\hat{a}_0 = 0.5008$, and the estimated disturbance $\hat{c} = 0.2504$. In the second step, the algorithm yields

$$\theta = [8.7684 \quad -3.0044 \quad 0.2501]^T.$$

In this case, (2.12) becomes

$$0.2501(d+T)^2 - 3.0044(d+T) + 8.7684 = 0,$$

which gives two possible values of time delay, 3.015 and 0.998. $\hat{d} = 3.015$ leads to

$$y^{(2)}(t) + 1.501y^{(1)}(t) + 0.5008y(t) = 0.5045u^{(1)}(t - 3.015) + 0.5003u(t - 3.015),$$

which is of minimum phase. It is discarded because the actual process is of non-minimum phase (also, the resulting fitting error is bigger). For $\hat{d} = 0.998$, the model is

$$y^{(2)}(t) + 1.501y^{(1)}(t) + 0.5008y(t) = -0.5045u^{(1)}(t - 0.998) + 0.5003u(t - 0.998),$$

which is of non-minimum phase and fits to the pulse response better. Pulse responses of the actual process and the estimated models are compared in Figure 2.2. If a rectangular doublet pulse with width of $T = 10$ is used as the the test signal, the proposed method leads to

$$y^{(2)}(t) + 1.501y^{(1)}(t) + 0.5008y(t) = -5.043u^{(1)}(t - 0.993) + 0.5004u(t - 0.993),$$

with the estimated disturbance as $\hat{c} = 0.2508$. Pulse responses of the actual process and estimated models are compared in Figure 2.3.

Example 2.2. Consider a high-order process (Hwang and Lai, 2004),

$$G(s) = \frac{1}{(s+1)^3(2s+1)^2},$$

subject to $y(0) = 0.25$, $y^{(i)}(0) = 0, i = 1, 2, 3, 4$ and $c = 0.25$. Simulation is performed on this example using a rectangular pulse with width of $T = 10$. The algorithm is applied with different model orders to get

$$\hat{G}_1(s) = \frac{0.1628}{s + 0.1621}e^{-3.67s},$$

$$\hat{G}_2(s) = \frac{0.1135}{s^2 + 0.5917s + 0.1162}e^{-1.88s},$$

$$\hat{G}_3(s) = \frac{0.0164s^2 - 0.05263s + 0.1101}{s^3 + 1.318s^2 + 0.654s + 0.1112}e^{-0.718s},$$

and

$$\hat{G}_4(s) = \frac{-0.0254s^3 + 0.01396s^2 - 0.05111s + 0.1543}{s^4 + 2.488s^3 + 2.366s^2 + 0.9939s + 0.1547}e^{-0.263s}.$$

Their Nyquist plots are compared in Figure 2.4.

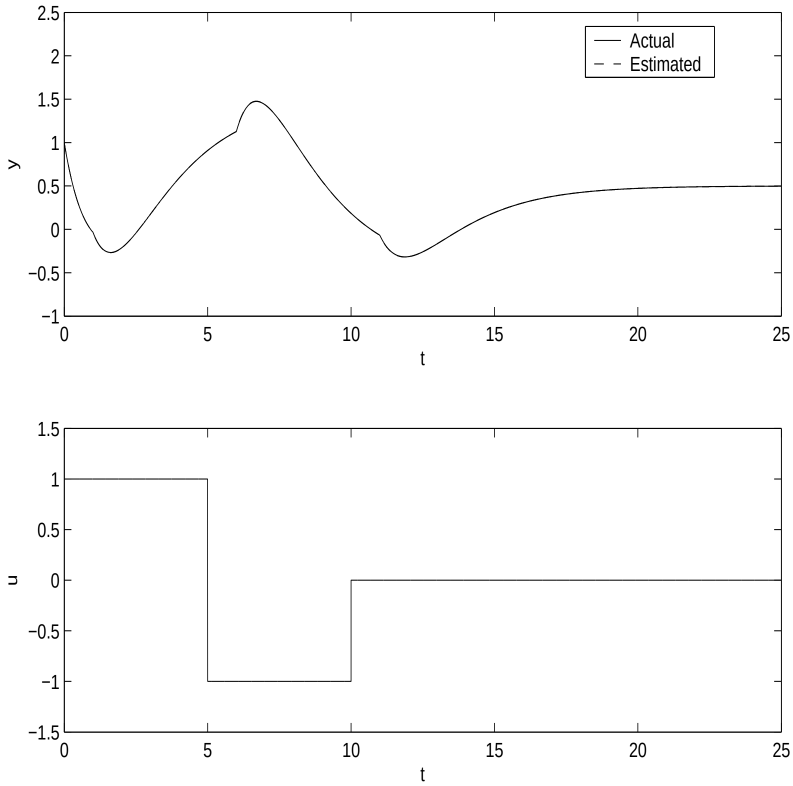

Figure 2.3. Rectangular doublet pulse response and input for Example 2.1.

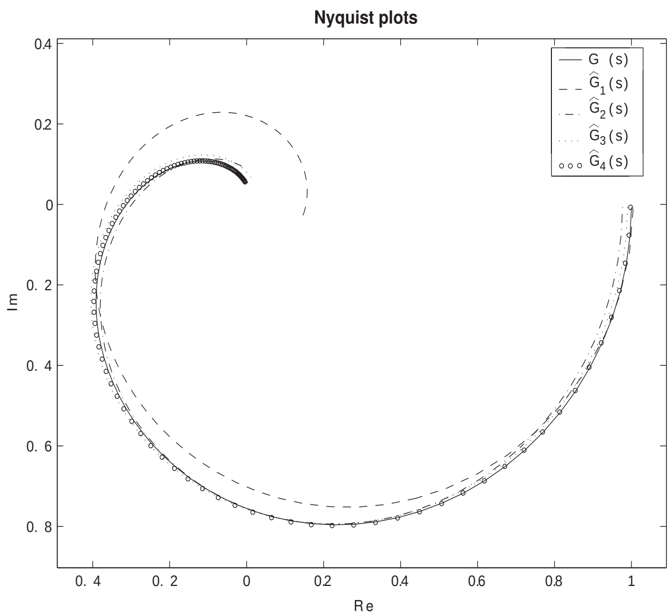

Figure 2.4. Nyquist curves for Example 2.2.

Table 2.1. Identification results for Example 2.3

NSR	Estimated model	Estimated disturbance	ϵ
0%	$\frac{0.6273}{s^2+3.758s+3.137}e^{-0.105s}$	0.2509	2.01×10^{-6}
3%	$\frac{0.6743}{s^2+3.99s+3.325}e^{-0.0982s}$	0.2631	5.93×10^{-4}
5%	$\frac{0.6918}{s^2+4.08s+3.389}e^{-0.0937s}$	0.2667	1.58×10^{-3}
10%	$\frac{0.6774}{s^2+4.027s+3.305}e^{-0.084s}$	0.2585	6.53×10^{-3}
15%	$\frac{0.6025}{s^2+3.691s+2.96}e^{-0.0681s}$	0.2316	1.6×10^{-2}
25%	$\frac{0.4599}{s^2+3.047s+2.317}e^{-0.0276s}$	0.1835	6.3×10^{-2}

Example 2.3. Consider Luyben's heat exchanger model (Luyben, 1973):

$$G(s) = \frac{0.2}{(0.4s+1)(0.8s+1)}e^{-0.1s},$$

subject to $y(0) = -1$, $y^{(1)} = 2$ and $c = 0.25$. To simulate practical conditions, white noise is added to the process output to produce the output measurement $\tilde{y}(t)$. The noise-to-signal ratio defined by

$$NSR = \frac{mean(abs(noise))}{mean(abs(signal))}$$

is used to represent noise level. A rectangular pulse of $h = 1$ and $T = 5$ is applied to the plant. The output is corrupted with white noise of $NSR = 0, 3, 5, 10, 15, 25\%$, respectively. The IV method is used to guarantee the identification consistency in the presence of noise. For model structure identification, we start from $n = 1$ and $m = 0$. This leads to a negative d, which is not possible. Thus, the first-order modelling is discarded. With $n = 2$ and $m = 0$, reasonable models are obtained and shown in Table 1 under the different noise levels. To evaluate the estimated model, the time domain identification error is measured by

$$\varepsilon = \frac{1}{K}\sum_{k=1}^{K}\left[\tilde{y}(k) - \hat{y}(k)\right]^2, \tag{2.16}$$

where $\hat{y}(k)$ is the estimated pulse response. The identification performance in presence of noise is also shown in Table 2.1.

2.4 Real time testing

The proposed method is also applied to a DC motor speed control system in Advanced Control Technology Lab, Department of Electrical and Computer Engineering, National University of Singapore. This experimental set-up consists of three parts: a DC motor set, which is made by LJ Technical Systems Inc. and shown in Figure 2.5, a PC with installed data acquisition cards and LabVIEW software, and a power supply for the DC motor set. The system input is the voltage applied to the DC motor, and the output is the voltage from the potentiometer, which is used to measure the motor velocity. One pulse test with $h = 2$ and $L = 1.6$ was conducted on the system. Using the proposed identification method, we got its model as

$$\hat{G}(s) = \frac{4.309}{s + 4.634} e^{-0.0197s}.$$

The response for this $\hat{G}(s)$ under the same pulse input is shown with the dash line in Figure 2.6, where the solid line is from the actual system. The effectiveness of the proposed method is clear.

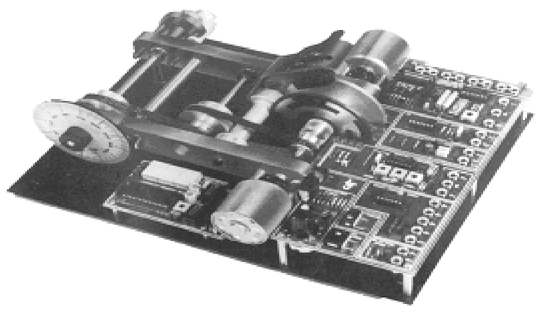

Figure 2.5. DC motor set.

2.5 Conclusion

In this chapter, a new method is presented to identify time delay systems with possible non-zero initial conditions and constant disturbance from pulse tests. The

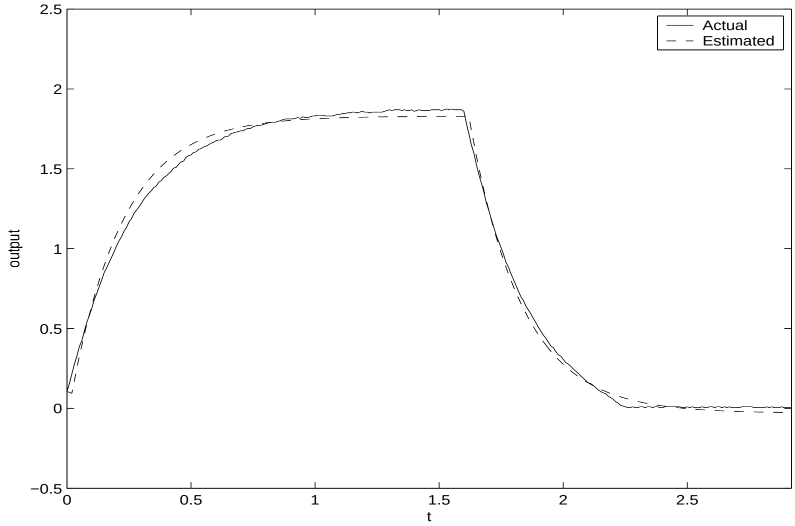

Figure 2.6. Pulse response of the DC motor.

feature of short duration of pulse signals is employed to simplify dynamic equation of the system, and enables easy and separate identification of the system parameters in two steps. The effectiveness of this method has been demonstrated through simulation and real-time implementation.

Chapter 3

Process Identification from Step Tests

3.1 Introduction

Compared with pulse tests discussed in the previous chapter, the step test is more popular for its simplicity. For a step test, only little equipment is needed. One can even perform a step test manually. Thus the step test is still dominant in real applications. In the past, most identification methods based on step tests lead to low-order models (Åström and Hagglund, 1995) and cannot describe high-order processes and non-minimum-phase systems. Wang and Zhang (2001) took advantage of simplicity of the input of step tests and devised a linear identification algorithm, which can generate low-order models or high-order models with time delay. Their method, like the previous work on continuous system identification, assumed that the initial conditions are zero and there is no disturbance. It is possible that the underlying process is operated to the constant steady state and kept there so that the above assumption is met. On the other hand, these limitations are the major concerns from application perspectives, as also raised by the reviewers of Wang and Zhang (2001). It is definitely desirable to remove the assumption for easy practical applications under the non-steady state condition.

In this chapter, a new integral identification method is proposed for continuous-time processes with time delay from one step test. The test can start from non-zero initial conditions under static disturbances which are unknown. The proposed method is a one-stage algorithm with no iteration. The key idea in our method is to make both upper and lower limits of the inner integral dependent of the dummy variable of the outer integral so that the initial conditions do not appear in the resulting integral equation. The effectiveness of the proposed method is demonstrated through examples.

This chapter is organized as follows. In Section 3.2, a common problem of the existing integral identification methods is revealed. In Sections 3.3, the method is presented for second-order modelling. The methods are further extended to high-order modelling in Sections 3.4. The proposed method is applied for real time tests in Section 3.5. Conclusions are drawn in Section 3.6.

3.2 Review of integral identification

In this section, we will use a 2nd-order model to show why the existing integral methods are unable to identify such a model from a step test under unknown non-zero initial conditions and static disturbance. Assume that a stable process is represented by

$$y^{(2)}(t) + a_1 y^{(1)}(t) + a_0 y(t) = b_1 u^{(1)}(t-d) + b_0 u(t-d) + c, \tag{3.1}$$

where $y(t)$ and $u(t)$ are the output and input of the process, respectively, d is the time delay and c is the static disturbance or a bias value of the process. Suppose that at $t = 0$, a step input test is applied to the process with the initial conditions of $y(0)$ and $y^{(1)}(0)$. The task is to estimate the model parameters, a_1, a_0, b_1, b_0 and d, from the input $u(t)$ and output measurement $y(t)$ in presence of unknown c and $y^{(1)}(0)$ which could be non-zero.

To avoid the use of various time derivatives, which are too sensitive to noise, (3.1) is transformed to an integral equation by multiple integration. Normally, the integral interval is chosen from 0 to t (Whitfield and Messali, 1987). Thus, integrating (3.1) from 0 to t twice gives

$$\left[y(t) - y(0) - y^{(1)}(0)t\right] + a_1\left[\int_0^t y(\delta_0)d\delta_0 - y(0)t\right] + a_0\int_0^t\int_0^{\delta_1} y(\delta_0)d\delta_0 d\delta_1$$
$$= b_1\int_0^t\int_0^{\delta_1} u^{(1)}(\delta_0 - d)d\delta_0 d\delta_1 + b_0\int_0^t\int_0^{\delta_1} u(\delta_0 - d)d\delta_0 d\delta_1 + \frac{ct^2}{2}, \qquad (3.2)$$

where $y^{(1)}(0)$ is present but unknown. This is the first obstacle which makes the existing integral identification methods from step tests impossible to work in presence of unknown initial conditions, while Hwang and Lai (2004) uses a pulse test whose two signal levels (like two tests) give rise to two independent equations so that the unknown initial conditions can be obtained or eliminated. Under $u(t) = \mathbf{1}(t)$, the unit step function, (3.2) can be re-written as

$$\begin{aligned} y(t) &= \begin{bmatrix} -\int_0^t y(\delta_0)d\delta_0 & -\int_0^t\int_0^{\delta_1} y(\delta_0)d\delta_0 d\delta_1 & 1 & t & t^2 \end{bmatrix} \begin{bmatrix} a_1 \\ a_0 \\ y(0) + \frac{b_0 d^2}{2} - b_1 d \\ y^{(1)}(0) + a_1 y(0) - b_0 d + b_1 \\ \frac{b_0 + c}{2} \end{bmatrix} \\ &:= \phi^T(t)\theta. \end{aligned}$$

where there are five linear independent functions in $\phi(t)$, which enables estimation of five parameters in θ. But there are seven unknowns, a_1, a_0, b_1, b_0, d, c and $y^{(1)}(0)$. Not all of them can be found from θ. The presence of $y^{(1)}(0)$ in the regression equation also increases the number of unknowns. This forms the second obstacle for the current integral identification.

The essential cause which leads to these two obstacles and failure of the existing methods is that when a differential equation is transformed to an integral equation by multiple integration, the output derivative will inevitably appear in the resultant integral equation as long as one of integration limits is fixed. It should be pointed out that all the existing methods including Hwang and Lai (2004) have one integration limit fixed, indeed. In view of the above observation, the key idea is to make both upper and lower limit of any inner integral dependent on the dummy variable of the immediate next outer integral so that all the terms in the outcome of inner integral are functions of the outer dummy variable, but not fixed.

3.3 The proposed method

To get rid of the problem in the existing methods, we employ the following double-integral operation on $f(t)$:

$$\int_0^\tau \left[\int_{t-\delta_1}^{t+\delta_1} f(\delta_0) d\delta_0\right] d\delta_1. \tag{3.3}$$

For $y^{(2)}(t)$, one sees

$$\begin{aligned}&\int_0^\tau \left[\int_{t-\delta_1}^{t+\delta_1} y^{(2)}(\delta_0) d\delta_0\right] d\delta_1 \\ &= \int_0^\tau \left[y^{(1)}(t+\delta_1) - y^{(1)}(t-\delta_1)\right] d\delta_1 \\ &= y(t+\tau) - 2y(t) + y(t-\tau),\end{aligned} \tag{3.4}$$

which depends on $y(t)$ only but not on $y^{(1)}(t)$. If, on the other hand, any term in the outcome of $\int_{t-\delta_1}^{t+\delta_1} y^{(2)}(\delta_0)d\delta_0$ was independent of δ_1, then when integrated with respect to δ_1, there would be $y^{(1)}(0)$ in (3.4), which are not available. The double-integral operation is applied to $y^{(1)}(t)$ and $y(t)$, respectively,

$$\int_0^\tau \left[\int_{t-\delta_1}^{t+\delta_1} y^{(1)}(\delta_0) d\delta_0\right] d\delta_1 = \int_0^\tau [y(t+\delta_1) - y(t-\delta_1)] d\delta_1, \tag{3.5}$$

$$\int_0^\tau \left[\int_{t-\delta_1}^{t+\delta_1} y(\delta_0) d\delta_0\right] d\delta_1, \tag{3.6}$$

which can both be numerically evaluated with knowledge of $y(t)$.

For the right hand side of (3.1), consider the step test first since the step testing is the simplest and dominant in process control. Let $u(t) = h\mathbf{1}(t)$. Then

$u(t-d) = h\mathbf{1}(t-d)$, the unit step function delayed by time of d. It is straightforward to verify that

$$\int_0^t \mathbf{1}^{(1)}(\delta_0 - d)d\delta_0 = \mathbf{1}(t-d), \tag{3.7}$$

$$\int_0^t \mathbf{1}(\delta_0 - d)d\delta_0 = (t-d)\mathbf{1}(t-d), \tag{3.8}$$

$$\int_0^t (\delta_0 - d)\mathbf{1}(\delta_0 - d)d\delta_0 = \frac{(t-d)^2}{2}\mathbf{1}(t-d). \tag{3.9}$$

It then follows that

$$\begin{aligned}
&\int_0^\tau \left[\int_{t-\delta_1}^{t+\delta_1} \mathbf{1}(\delta_0 - d)d\delta_0\right] d\delta_1 \\
&= \int_0^\tau \left[(t+\delta_1-d)\mathbf{1}(t+\delta_1-d) - (t-\delta_1-d)\mathbf{1}(t-\delta_1-d)\right] d\delta_1, \\
&= \frac{1}{2}\left[(t+\tau-d)^2\mathbf{1}(t+\tau-d) - 2(t-d)^2\mathbf{1}(t-d) + (t-\tau-d)^2\mathbf{1}(t-\tau-d)\right], \qquad (3.10)
\end{aligned}$$

$$\begin{aligned}
&\int_0^\tau \left[\int_{t-\delta_1}^{t+\delta_1} \mathbf{1}^{(1)}(\delta_0 - d)d\delta_0\right] d\delta_1 \\
&= \int_0^\tau \left[\mathbf{1}(t+\delta_1-d) - \mathbf{1}(t-\delta_1-d)\right] d\delta_1 \\
&= (t+\tau-d)\mathbf{1}(t+\tau-d) - 2(t-d)\mathbf{1}(t-d) + (t-\tau-d)\mathbf{1}(t-\tau-d). \qquad (3.11)
\end{aligned}$$

Let τ be fixed and t satisfy $t-\tau < d \leq t$, which causes

$$\mathbf{1}(t+\tau-d) = \mathbf{1}(t-d) = 1, \quad d \leq t, \tag{3.12}$$

$$\mathbf{1}(t-\tau-d) = 0, \quad t-\tau < d. \tag{3.13}$$

Integrating (3.1) in form of (3.3) and making use of (3.4-3.6) and (3.10-3.13) yield

$$\phi^T(t)\theta = \gamma(t), \quad t-\tau < d \leq t, \tag{3.14}$$

where

$$\phi^T(t) = \left[-\int_0^\tau \int_{t-\delta_1}^{t+\delta_1} y^{(1)}(\delta_0)d\delta_0 d\delta_1 \quad -\int_0^\tau \int_{t-\delta_1}^{t+\delta_1} y(\delta_0)d\delta_0 d\delta_1 \quad ht^2 \quad ht \quad h\right], \tag{3.15}$$

$$\gamma(t) = \int_0^\tau \int_{t-\delta_1}^{t+\delta_1} y^{(2)}(\delta_0)d\delta_0 d\delta_1, \tag{3.16}$$

$$\theta = \begin{bmatrix}\theta_1\\ \theta_2\\ \theta_3\\ \theta_4\\ \theta_5\end{bmatrix} = \begin{bmatrix} a_1 \\ a_0 \\ -\frac{b_0}{2} \\ b_0(\tau+d) - b_1 \\ \frac{b_0}{2}(\tau^2 - 2\tau d - d^2) + b_1(d+\tau) + \frac{c\tau^2}{h}\end{bmatrix}. \tag{3.17}$$

One invokes (3.14) for $t = t_i, i = 1, 2, \ldots, N$, with $N \gg 5$ to form

$$\Gamma = \Phi\theta, \tag{3.18}$$

where $\Gamma = [\gamma(t_1), \gamma(t_2), \ldots, \gamma(t_N)]^T$, and $\Phi = [\phi(t_1), \phi(t_2), \ldots, \phi(t_N)]^T$. Then the ordinary least-squares algorithm can be applied to (3.18) to find its solution

$$\theta = \left(\Phi^T\Phi\right)^{-1}\Phi^T\Gamma$$

We can see that five θ_i estimated are not sufficient to determine six unknown parameters, a_1, a_0, b_1, b_0, d, and c. An additional equation is obtained using the steady-state of (3.1):

$$a_0 y(\infty) = b_0 h + c, \tag{3.19}$$

where $y(\infty)$ is estimated from the steady-state response. In noise situation, $y(\infty)$ is calculated with a multi-point average for robustness. To get reliable estimate of $y(\infty)$, the test must be maintained at final state for a while.

Equation (3.19) together with θ_i is sufficient to recover the model parameters as

$$\begin{cases} a_1 = \theta_1, \\ a_0 = \theta_2, \\ b_0 = -2\theta_3, \\ c = a_0 y(\infty) - b_0 h, \\ d = \frac{-(\theta_4+2\tau\theta_3)\pm\sqrt{(\theta_4+2\tau\theta_3)^2-4\theta_3(3\theta_3\tau^2+\theta_4\tau+\theta_5-\frac{c\tau^2}{h})}}{2\theta_3}, \\ b_1 = b_0(\tau+d) - \theta_4. \end{cases} \tag{3.20}$$

Equation (3.20) produces two solutions for d and b_1. We can find the initial conditions with estimated model and the step response and obtain the estimated step response from the estimated model, static disturbance and initial conditions. By comparing the estimated step response with the actual one, we can judge which model is better. For detail, see Hwang and Lai (2004). The method is straightforward.

Note that $t = t_i$ has been chosen to meet $t - \tau < d \leq t$, where d is unknown and to be identified. This is not a problem. A rough estimation of the range of d is sufficient. Let d be in the range, $[d_{min}, d_{max}]$. We can then choose $t_i \geq d_{max}$ and $t_i - \tau \leq d_{min}$.

Note also from the requirement, $t - \tau < d \leq t$, or $d \leq t < d + \tau$, that the range for $t = t_i, i = 1, 2, \ldots, N$, is given by τ. τ is usually big enough to let the maximum integration interval $[t_1 - \tau, t_N + \tau]$ cover the entire output response for full use of the information and best estimation of the model parameters. $(t_i - \tau)$ can be negative, that is, the output measurement before the step starts is needed. This is absolutely not a problem in practice as a continuous industrial process runs day after day, the data on the output measurement are all recorded and saved in computer for years and can be retrieved easily for use in process identification.

It is concluded from the above development that even when the non-zero initial conditions and static disturbance are unknown, a time-delay model of second order can be identified from the process step response by applying one-stage least-squares algorithm without iteration.

Example 3.1. Consider a $2nd$-order process (Hwang and Lai, 2004):

$$y^{(2)}(t) + 1.5y^{(1)}(t) + 0.5y(t) = -0.5u^{(1)}(t-1) + 0.5u(t-1) + c,$$

with $c = 1$. The unit step test is applied at $t = 0$. The resultant output shows an inverse response, see Figure 3.1. The initial conditions are $y(0) = 2.3$, $y^{(1)}(0) = -0.15$. Note that $y^{(1)}(0)$ is supposed unknown and not used in identification. For this example, $T_s = 12.5$. We choose $\tau = 6$ and $t_i = 2.5, 2.1, \ldots, 6.4, 6.5$. The maximum integral interval is from $t_1 - \tau = -3.5$ to $t_N + \tau = 12.5$, and well covers the step response. The least-squares algorithm based on (3.14) leads to

$$\hat{\theta} = [1.5001 \quad 0.5003 \quad -0.2501 \quad 4.0025 \quad 38.2752]^T.$$

Two models are obtained from (3.20) as

$$y^{(2)}(t) + 1.5001y^{(1)}(t) + 0.5003y(t) = -0.5126u^{(1)}(t-0.98) + 0.5003u(t-0.98),$$

$$y^{(2)}(t) + 1.5001y^{(1)}(t) + 0.5003y(t) = 0.5126u^{(1)}(t-3.02) + 0.5003u(t-3.02),$$

with the estimated disturbance as $\hat{c} = 1.0004$. With the estimated model, we can estimate the process state at $t = 0$ is $y^{(1)}(0) = -0.1523$. The estimated step responses are shown in Figure 3.1, where the non-minimum phase model fits the actual response much better than the minimum phase model.

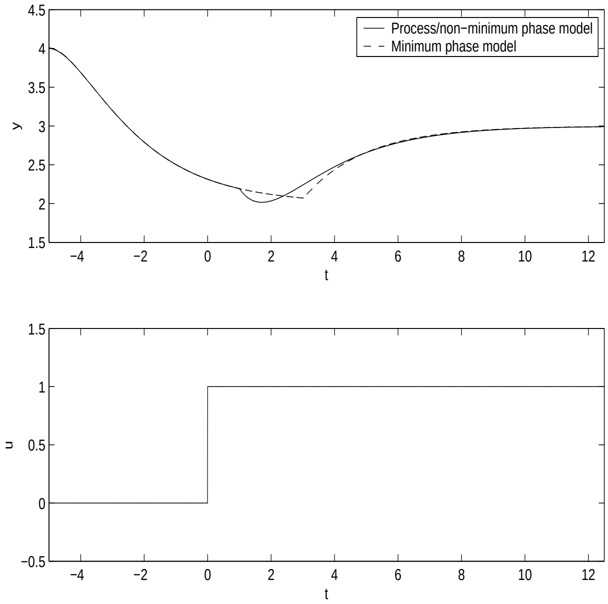

Figure 3.1. Step response and input for Example 3.1.

It is seen that τ^2 appears in θ_5 and it may cause θ_5 to be relatively much large to other θ_i. To avoid possible numerical computation problems from this, one may rescale time by $t_{new} = Ft$. For instance, take $F = 0.1$ in the above example. This yields

$$\hat{\theta}_{new} = [15.0095 \quad 50.2623 \quad -25.1319 \quad 40.2554 \quad 38.5036]^T,$$

which has its parameter values relatively much closer to each other than the original estimation without time rescaling.

In the presence of noise in the measurement of the process output, the ordinary least-squares estimate is not consistent (Soderstrom and Stoica, 1983). One solution is to use the instrumental variable (IV) method to guarantee the identification consistency in the presence of noise. The IV method proposed by Wang and Zhang (2001) is adopted. For simulation, the ratio of the standard deviation of noise to the standard deviation of the output signal is used as the measure of the noise-to-signal ratio (NSR). A white noise of $NSR = 3, 5, 10, 15, 25\%$, is added to the process output of Example 1 to produce the corrupted output measurement $\tilde{y}(t)$, respectively. The models estimated by the IV method under the different noise levels are shown in Table 3.1 with respect to the time domain identification error measured by

$$\varepsilon = \frac{1}{K}\sum_{k=1}^{K}\left[\tilde{y}(k) - \hat{y}(k)\right]^2, \tag{3.21}$$

where $\hat{y}(k)$ is the estimated step response. Table 3.1 indicates robust identification results. The Nyquist plots of the process and the models obtained under noise level of $NSR = 10, 25\%$ are given in Figure 3.2.

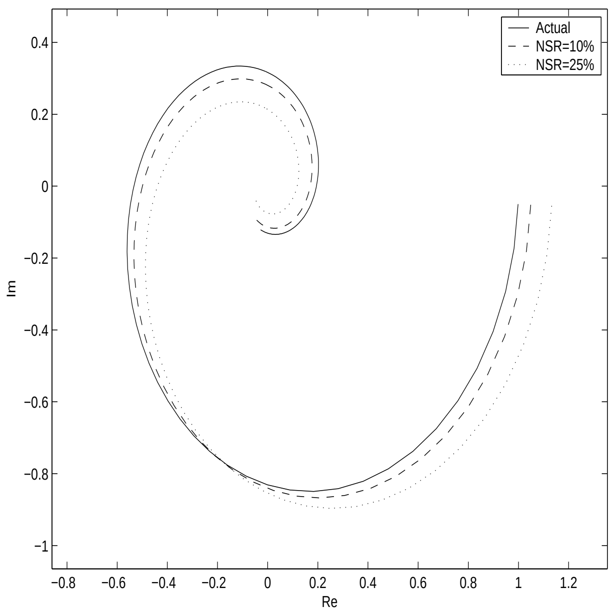

Figure 3.2. Nyquist plot for Example 3.1.

3.4 High-order modelling from step tests

In this section, we extend the preceding 2nd-order modelling to a general case. Consider an nth-order continuous system with delay,

$$y^{(n)}(t)+\cdots+a_1y^{(1)}(t)+a_0y(t)=b_mu^{(m)}(t-d)+\cdots+b_1u^{(1)}(t-d)+b_0u(t-d)+c, \tag{3.22}$$

where $m<n$. A step test of $u(t)=h\mathbf{1}(t)$ is applied at $t=0$. Define an n-time integration operator on $f(t)$ as follows,

$$P_nf(t)=\int_0^{\tau}\int_{\tau-\delta_{n-1}}^{\tau+\delta_{n-1}}\cdots\int_{\tau-\delta_2}^{\tau+\delta_2}\int_{t-\delta_1}^{t+\delta_1}f(\delta_0)d\delta_0d\delta_1\cdots d\delta_{n-2}d\delta_{n-1}, n\geq 2. \tag{3.23}$$

It can be readily shown that

$$P_n\mathbf{1}^{(l)}(t-d)=\frac{1}{(n-l)!}\sum_{k=0}^{2n-2}c_k(t+(n-1-k)\tau-d)^{n-l}\mathbf{1}(t+(n-1-k)\tau-d), l=0,1,\ldots,m, \tag{3.24}$$

where $\begin{bmatrix}c_0 & c_1 & \ldots & c_{2n-2}\end{bmatrix}=C_n$ is calculated recursively as $C_2=\begin{bmatrix}1 & -2 & 1\end{bmatrix}$, and $C_i=\begin{bmatrix}C_{i-1} & 0 & 0\end{bmatrix}-\begin{bmatrix}0 & 0 & C_{i-1}\end{bmatrix}, i=3,4,\ldots,n.$

Let τ be fixed and t meet $t-(n-1)\tau<d\leq t-(n-2)\tau$. Then, (3.24) becomes

$$P_n\mathbf{1}^{(l)}(t-d)=\frac{1}{(n-l)!}\sum_{k=0}^{2n-3}c_k(t+(n-1-k)\tau-d)^{n-l}, l=0,1,\ldots,m.$$

Table 3.1. Identification results for Example 3.1

NSR	Estimated model	Estimated disturbance	Error ϵ
3%	$\frac{-0.4729s+0.5062}{s^2+1.502s+0.4971}e^{-1.02s}$	0.9864	8.32×10^{-4}
5%	$\frac{-0.4527s+0.5096}{s^2+1.503s+0.4953}e^{-1.03s}$	0.9783	2.1×10^{-3}
10%	$\frac{-0.4106s+0.5166}{s^2+1.506s+0.4917}e^{-1.07s}$	0.9622	6.3×10^{-3}
15%	$\frac{-0.3534s+0.5252}{s^2+1.508s+0.4872}e^{-1.12s}$	0.9421	1.48×10^{-2}
25%	$\frac{-0.2075s+0.5423}{s^2+1.514s+0.4782}e^{-1.28s}$	0.9016	4.3×10^{-2}

The right hand side of (3.22) is

$$\sum_{k=0}^{m} b_k P_n u^{(k)}(t-d) + P_n c = h \sum_{i=0}^{n} (\beta_i t^i),$$

where

$$\begin{cases} \beta_0 = \sum_{j=0}^{m} \frac{b_j}{(n-j)!} \left(\sum_{k=0}^{2n-3} c_k \left((n-1-k)\tau - d \right)^{n-j} \right) + \frac{P_n c}{h}, \\ \beta_i = \sum_{j=0}^{min(m,n-i)} \frac{b_j}{i!(n-i-j)!} \left(\sum_{k=0}^{2n-3} c_k \left((n-1-k)\tau - d \right)^{n-j-i} \right), i = 1, 2, \ldots, n. \end{cases} \tag{3.25}$$

Applying P_n on (3.22) yields

$$\phi^T(t)\theta = \gamma(t), \quad t-(n-1)\tau < d \le t-(n-2)\tau, \tag{3.26}$$

where

$$\gamma(t) = P_n y^{(n)}(t), \tag{3.27}$$

$$\phi^T(t) = [-P_n y^{(n-1)}(t) \quad \cdots \quad -P_n y(t) \quad ht^n \quad \cdots \quad ht \quad h], \tag{3.28}$$

$$\theta^T = \begin{bmatrix} a_{n-1} & \cdots & a_0 & \beta_n & \cdots & \beta_1 & \beta_0 \end{bmatrix}. \tag{3.29}$$

Invoke $t = t_i, i = 1, 2, \ldots, N$ in (3.26) to form

$$\Gamma = \Phi\theta, \tag{3.30}$$

where $\Gamma = [\gamma(t_1), \gamma(t_2), \ldots, \gamma(t_N)]^T$, and $\Phi = [\phi(t_1), \phi(t_2), \ldots, \phi(t_N)]^T$. The ordinary least-squares algorithm can be applied to (3.30) to find its solution

$$\theta = \left(\Phi^T\Phi\right)^{-1} \Phi^T\Gamma. \tag{3.31}$$

Note that there are $n+m+3$ unknown parameters, $a_i,\ i = 1, \ldots, n,\ b_j,\ j = 0, \ldots, m$, d and c. If $m < n-1$, the process model can be recovered from (3.25). But, if $m = n-1$, the estimated θ is not enough to solve the unknowns and (3.19) is needed.

The above approach produces $m+1$ possible solutions for time delay d and thus $m+1$ possible models. Wang and Zhang (2001) suggested choosing one which minimizes the error (3.21). We follow their method.

In general, the order of the process is unknown before identification. The model structure identification is an important issue and has been discussed in the literature. We adopt the standard practice as follows. One may start from a second-order time delay system. If the error measured by (3.21) is acceptable, the identification task is completed. Otherwise, one may increase n and/or m by one until the error becomes acceptable.

Choice of τ, t_i and N are discussed as follows. By (3.26), $t = t_i, i = 1, \ldots, N$, and τ should meet

$$t - (n-1)\tau < d \leq t - (n-2)\tau. \tag{3.32}$$

The maximum integration interval is $[t_{min},\ t_{max}] = [t_1 - (n-1)\tau,\ t_N + (n-1)\tau]$. It should cover the entire test duration. To this end, we have

$$t_N + (n-1)\tau = T_s, \tag{3.33}$$

where T_s is the ending time of the test duration. The left hand side of (3.32) gives

$$t_N - (n-1)\tau < d. \tag{3.34}$$

Subtract (3.34) from (3.33):

$$2(n-1)\tau > T_s - d. \tag{3.35}$$

Choose $2(n-1)\tau \approx T_s$ to meet (3.35), which result in

$$\tau \approx \frac{T_s}{2(n-1)}. \tag{3.36}$$

Equation (3.32) can be rearranged as

$$d + (n-2)\tau \leq t < d + (n-1)\tau.$$

Suppose $d \in [d_{min}\ d_{max}]$. Once τ is calculated from (3.36), t_i are chosen as

$$d_{max} + (n-2)\tau \leq t_i < d_{min} + (n-1)\tau. \tag{3.37}$$

N is such that $t_1, t_2, \ldots$, and t_N meet (3.37).

Example 3.2. Consider a high-order process

$$y^{(4)}(t) + 4y^{(3)}(t) + 6y^{(2)}(t) + 4y^{(1)}(t) + y(t) = u(t-2) + c,$$

with $y(0) = 1.1$, $y^{(1)}(0) = -0.15$, $y^{(2)}(0) = 0.13$, $y^{(3)}(0) = -0.1$ and c=1. Its transfer function is $G(s) = \frac{e^{-2s}}{(s+1)^4}$. The unit step test is applied at $t = 0$. The test duration is 16. With $n = 3$ and $m = 0$, the model is obtained as

$$\hat{G}(s) = \frac{0.4207}{s^3 + 1.785s^2 + 1.516s + 0.4243} e^{-2.56s}.$$

The Nyquist plots of the process and model are given in Figure 3.3.

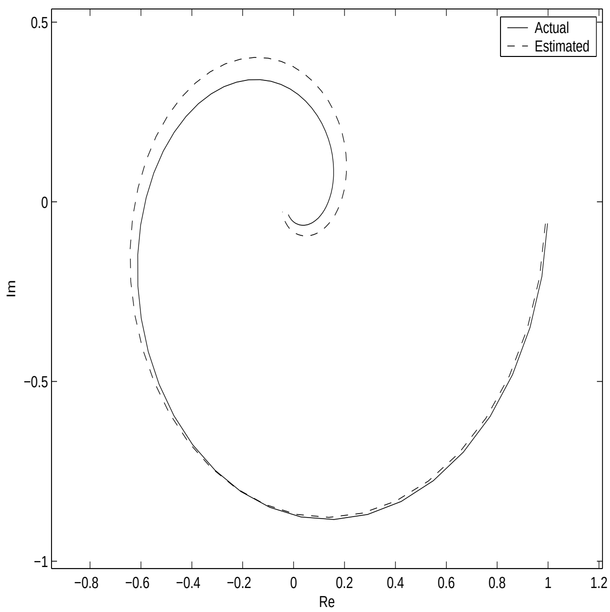

Figure 3.3. Nyquist plot for Example 3.2.

3.5 Real time testing

Lab Test The proposed method was tested on a temperature control system made by National Instruments Corp. in Advanced Control Technology Lab, Department of Electrical and Computer Engineering, National University of Singapore. The experiment setup consists of two parts: a chamber set with a $20W$ bulb and a fan; a personal computer with data acquisition cards and LabVIEW software. The temperature is to be controlled by the power supply to the bulb. An identification test was performed on the system and the recorded inputs and the output are given in Figure 3.4. At $t = 0$, $y(0)$ and $y^{(1)}(0)$ are nonzero. Applying the proposed identification method yields

$$y^{(2)}(t) + 18.55y^{(1)}(t) + 55.34y(t) = 632.6u(t - 0.106).$$

The response for this model under the same input is also shown in Figure 3.4. The effectiveness of the proposed method is obvious.

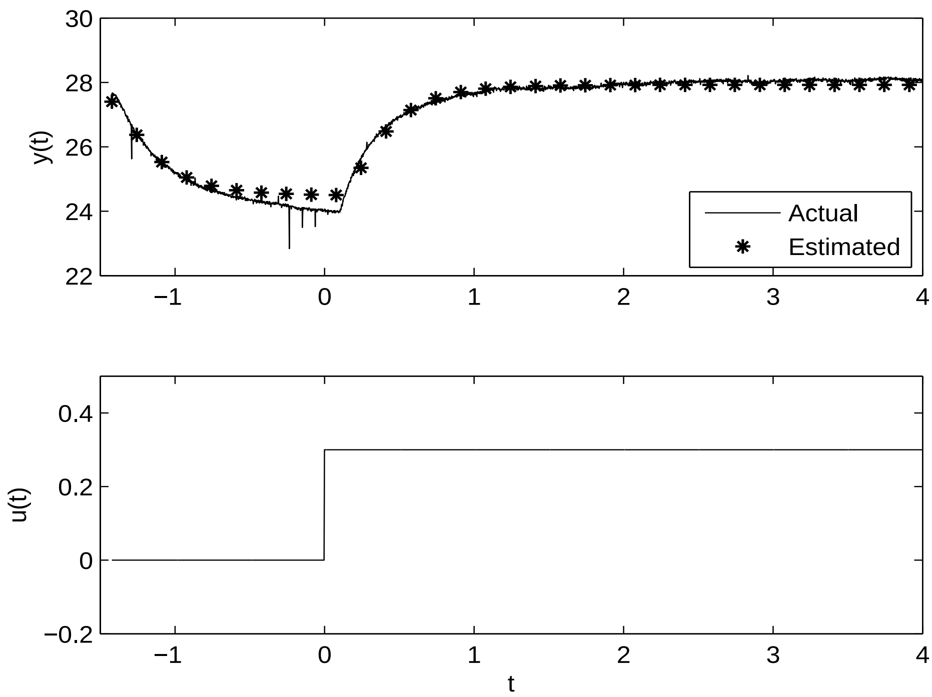

Figure 3.4. Step responses and input of the temperature control system.

Field Test Xi-Hua-Feng pulp and paper mills is located in Wuzhi, Henan Province, P. R. China. Three kinds of pulps are made by the mills: wood pulp, grass pulp and recycled-paper pulp. These pulps are mixed together in the mixing tank. The flowchart of this process is given in figure 3.5. It is required to stabilize the pulp concentration without large deviations from the given operation conditions. The flow rate of the pulp is often tuned to meet different manufacture requirements and it is important to monitor and control the flow rate. One needs to identify a model for the pulp flow rate. The process considered consists of a valve, FV1 in figure 3.5 and a pipe($DN100$). The input is the position of the valve and the output is the flow rate(m^3/h) in the pipe. A step test was applied by moving the valve from the fully close to 1/6 open position. The resultant response of the flow rate is given in Figure 3.6. The proposed method was applied and one model obtained as

$$y^{(2)}(t) + 2.267y^{(1)}(t) + 0.9351y(t) = 214.3u(t - 3.2).$$

The response for this model under the same input is also shown in Figure 3.6.

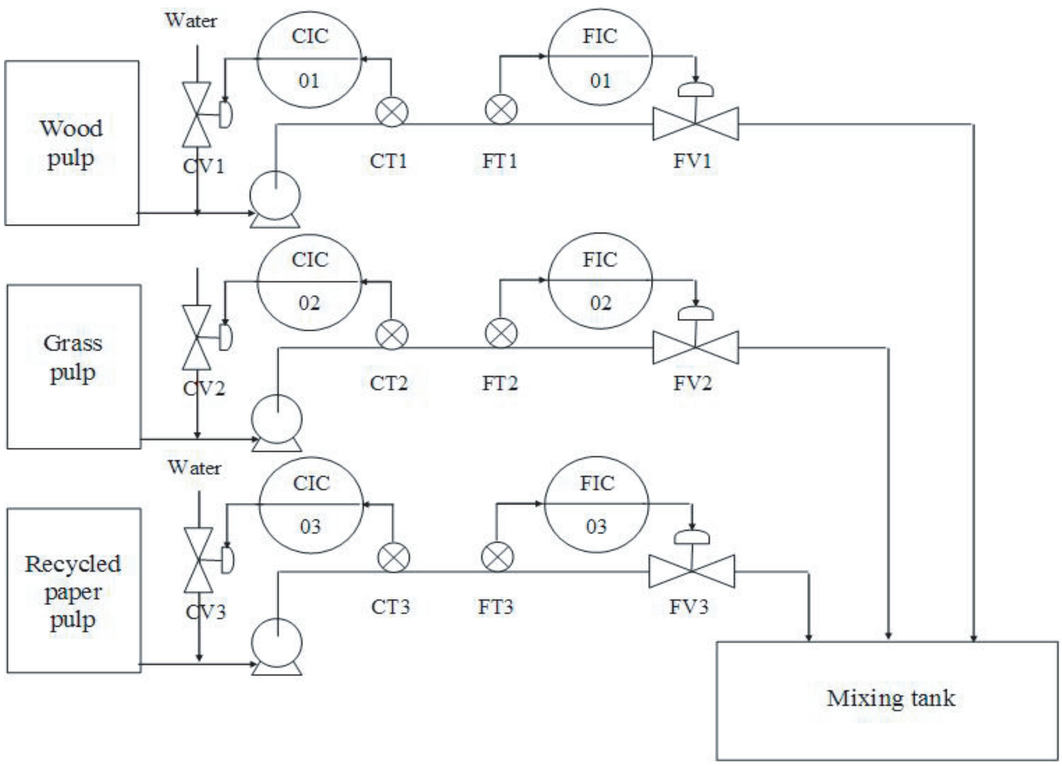

Figure 3.5. Flowchart of the mixing procedure.

3.6 Conclusions

In this chapter, a new integral method has been proposed for identification of linear continuous-time delay processes with unknown initial conditions and static

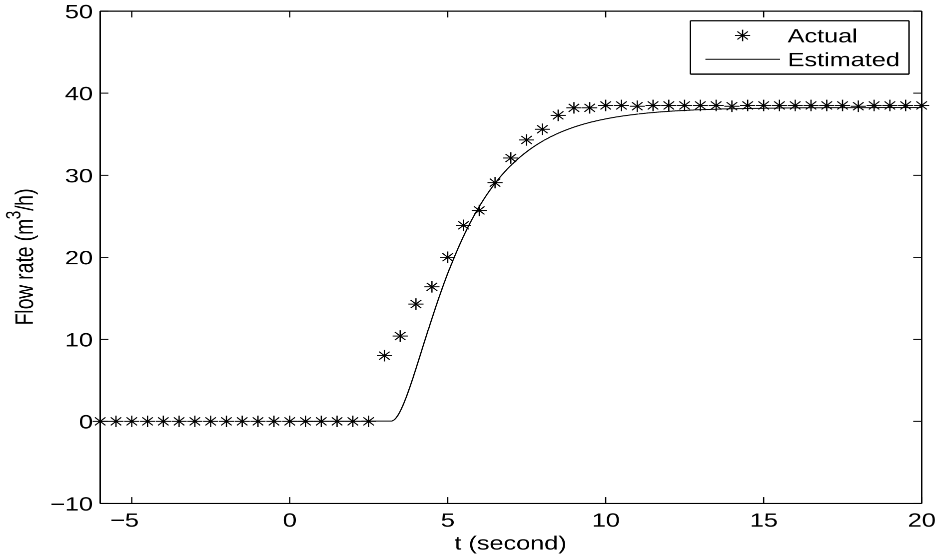

Figure 3.6. Step test of the flow control system.

disturbance from step tests. The integration limits are specially chosen to make the integral equation independent of the unknown initial conditions. The process model is obtained by one-stage least-squares algorithm with no iteration. The effectiveness of the proposed method is demonstrated by simulation, lab test and field experiment.

Chapter 4

Process Identification from Relay Tests

4.1 Introduction

In this chapter, process identification from relay tests is discussed. Among closed-loop identifications, process identification from relay feedback is a very active research area over the last 2 decades. The area was pioneered by Åström and Hagglund (1995). In the early stage of development, only stationary response of the relay feedback system is used to estimate the process frequency response at the oscillation frequency as well as zero frequency (in case of biased relay). The information so identified is adequate to tune simple controllers with simple specifications, but insufficient to tune controllers with high performance specifications. This has led to more recent development on identification of the process frequency response at multiple points from relay tests (Wang *et al.*, 1997*a*; Wang *et al.*, 1997*b*; Bi *et al.*, 1997; Wang *et al.*, 1999). With the estimated frequency response, a transfer function model can be obtained by some fitting techniques and it enables tuning and implementation of model-based controllers.

In this chapter, we revisit the FFT-based identification method first and the need for further development is discussed. Then a new identification method from relay tests under non-zero initial conditions and disturbance is proposed. A relay test is regarded as a sequence of step tests and the integral technique is adopted to devise the algorithm. The proposed method can yield a full process model including time delay. Because of the use of process output integrals, the resulting integral based estimation is very robust in face of noise in output measurements.

This chapter is organized as follows. In Section 4.2, the FFT based identification method is reviewed and the need for further development is highlighted. In Section 4.3, the method is presented for first-order modelling. The method is extended to high-order modelling in Section 4.4. Conclusions are drawn in Section 4.5.

4.2 FFT method revisited

A relay feedback system is shown in Figure 4.1. The relay function is shown in Figure 4.2 and described as

$$u(t) = \begin{cases} u_+, \ if \ e(t) > \varepsilon_+, \ or \ e(t) \geq \varepsilon_- \ and \ u(t_-) = u_+, \\ u_-, \ if \ e(t) < \varepsilon_-, \ or \ e(t) \leq \varepsilon_+ \ and \ u(t_-) = u_-, \end{cases} \tag{4.1}$$

where ε_+, $\varepsilon_- \in \mathbb{R}$ with $\varepsilon_- < \varepsilon_+$ indicating hysteresis; $u_-, u_+ \in \mathbb{R}$ and $u_- \neq u_+$; t_- is time point just before t and $u(t_-)$ is the relay output at the time point of t_-. If the process has a phase lag of at least π radians, the relay feedback will usually cause the system to oscillate. In most cases, a stable limit cycle will result. The corresponding input and output time responses can be used to perform process identification.

Multiple points on the process frequency response could be obtained from a relay test using relay transients. Suppose that the process is initially at the rest and a relay feedback is applied to it. The process input and output are recorded from the initial time until the system reaches a stationary oscillation. Note that

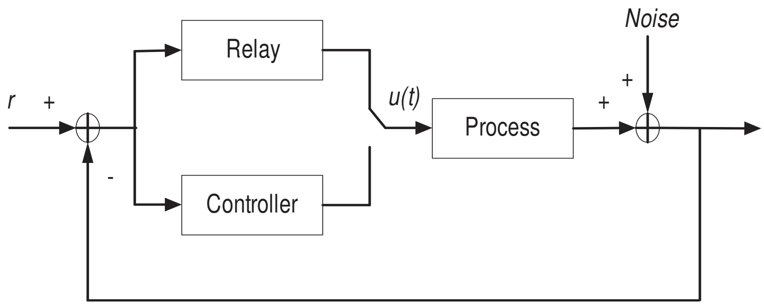

Figure 4.1. Relay feedback system.

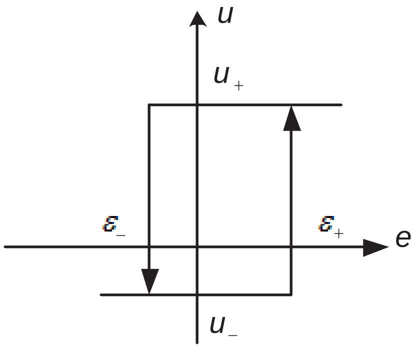

Figure 4.2. Relay function.

$u(t)$ and $y(t)$ are neither periodic nor absolutely integrable and then the FFT cannot be applied to compute the frequency response of the process correctly by using $G(j\omega) = FFT(y(t))/FFT(u(t))$. To rectify it, one period of the stationary oscillation of $y(t)$ and $u(t)$ are copied backward to form periodic signals $y_s(t)$ and $u_s(t)$. $y(t)$ and $u(t)$ then are decomposed into two parts: the periodic stationary cycle parts $y_s(t)$ and $u_s(t)$ and the transient parts $\Delta y(t)$ and $\Delta u(t)$ as

$$y(t) = y_s(t) + \Delta y(t),$$

and

$$u(t) = u_s(t) + \Delta u(t).$$

In the case of zero initial conditions and no disturbance, it follows that the process transfer function is given by

$$G(s) = \frac{Y(s)}{U(s)} = \frac{\Delta Y(s) + Y_s(s)}{\Delta U(s) + U_s(s)},$$

where $\Delta Y(s)$ and $\Delta U(s)$ are the Laplace transforms of the transient parts $\Delta y(t)$ and $\Delta u(t)$, respectively, $Y_s(s)$ and $U_s(s)$ are the Laplace transforms of $y_s(t)$ and

$u_s(t)$, respectively. Suppose that $t = T_e$, $y(t)$ and $u(t)$ have entered the stationary oscillation and after $t = T_e$, $\Delta y(t)$ and $\Delta u(t)$ are approximately zero. $\Delta Y(j\omega_l), l = 1, \ldots, m$, is computed using the FFT as follows

$$\Delta Y(j\omega_l) = FFT(\Delta y(kT)) = T \sum_{k=0}^{N-1} \Delta y(KT) e^{-j\omega_l kT}, l = 1, 2, \ldots, m,$$

where $y(kT), k = 0, 1, \ldots, N-1$ are samples of $y(t)$, T is the sampling interval, and $(N-1)T = T_e$, $m = N/2$, and $\omega_l = 2\pi l/(NT)$. $Y_s(j\omega_l)$ are computed using digital integral as

$$Y_s(j\omega_l) = \frac{1}{1 - e^{-j\omega_l T_c}} \sum_{k=0}^{N_c} y_s(kT) e^{-j\omega_l kT} T, l = 1, 2, \ldots, m,$$

where $N_c = (T_c - T)/T$ and T_c is the period of stationary oscillation from the relay feedback test. Similarly, $\Delta U(j\omega_l)$ and $U_s(j\omega_l)$ are found. Thus, we have

$$G(j\omega_l) = \frac{\Delta Y(j\omega_l) + Y_s(j\omega_l)}{\Delta U(j\omega_l) + U_s(j\omega_l)}, l = 1, 2, \ldots, m. \tag{4.2}$$

Formula (4.2) works when the noise is relatively small but could produce big errors if noise is significant.

Wang *et al.* (1999) has suggested use of low-pass filters as an anti-noise measure and achieved reasonable estimation in face of significant noise. If the phase crossover frequency of the process is unknown before relay tests, design of low pass filter was done by trial and error and not detailed there. Note also that in their scheme, a low-pass filter is placed inside the loop when a relay is conducted, which will give a smoother output and more regular oscillations than one without a filter inside the loop. But this implies that a filter must be designed and implemented before a relay test, and requires a prior information of the process dynamics. In the rest of this section, we will remove this requirement and give a filter design without a prior knowledge of the process.

First, we conduct a relay test on the process without any filter inserted to the loop, as shown in Figure 4.1. The test ends when a stationary oscillation is achieved with the oscillation frequency of ω_o. To handle noisy data, the resulting output response, y(t), is processed by a low-pass filter, $F(s) = \frac{1}{(T_f s+1)^n}$ before the FFT method is applied. It is important to choose a proper cut-off frequency for the low-pass filter to achieve desired identification results. In control engineering, the process frequency response in [0, ω_c] is mostly critical for controller design, where ω_c is the phase crossover frequency. Then, the filter cut-off frequency, $1/T_f$, is normally chosen as $(3-5)\omega_c$. But in practice, one does not know ω_c before identification. This problem is solved by using the relay feedback oscillation frequency, ω_o, in place of it. The experience shows that ω_o is usually close to ω_c and available from a relay test. In summary, we design a low-pass filter as

$$F(s) = \frac{1}{(T_f s+1)^n}, \quad 1/T_f = M\omega_o, \; M \in [3, 5].$$

With a filter so designed, the original output is filtered. The filtered output, $\hat{y}(t)$, and the input, $u(t)$, are processed by the FFT method. Note that due to use of the filter, the result so obtained is $\hat{G}_F(j\omega_i)$, the estimated frequency response of $G(s)F(s)$, but not of $G(s)$. One can recover the estimate of $G(jw)$ as

$$\hat{G}(j\omega_i) = \frac{\hat{G}_F(j\omega_i)}{F(j\omega_i)}. \tag{4.3}$$

To evaluate the above revised FFT method and compare with the original one in Wang *et al.* (1999), simulation is conducted in noisy case. The noise-to-signal ratio defined by

$$NSR = \frac{mean(abs(noise))}{mean(abs(signal))}$$

is used to represent a noise level. The identification error is measured by the worst case error,

$$ERR_1 = max\left|\frac{\hat{G}_F(j\omega_i) - G(j\omega_i)}{G(j\omega_i)}\right|, i = 1, \cdots, M,$$

without removing the filter's frequency response, and

$$ERR_2 = max\left|\frac{\hat{G}(j\omega_i) - G(j\omega_i)}{G(j\omega_i)}\right|, i = 1, \cdots, M, \tag{4.4}$$

with the filter's frequency response removed, where in both cases, $\omega_i \in [0, \omega_c]$ are considered.

Example 4.1. Consider a continuous-time delay process (Wang *et al.*, 1999),

$$G(s) = \frac{1}{5s+1}e^{-5s}.$$

Suppose zero initial conditions and no disturbance. A relay experiment is performed at $t = 0$, with $u_+ = 0.5$ and $u_- = -0.5$. A white noise with $NSR = 13\%$ is added to the process output to produce the output measurement $y(t)$. This output measurement is sent to the relay input. The relay output is applied to the process. The width of the hystersis should be bigger than the noise band. For this example we set $\varepsilon_+ = 0.2$ and $\varepsilon_- = -0.2$. Time responses of $y(t)$ and $u(t)$ are shown in Figure 4.3. Once such a relay test is completed, the $y(t)$ is processed by a low-pass filter. We try both ω_o and ω_c as guidelines for filter cut-off frequency selection. For this example, $\omega_c \approx 0.4$, and $\omega_o \approx 0.334$. We also vary the multiple in the filter to see its effects and suitable range. Finally, The FFT method is applied to get the frequency response estimation, and the identification errors are also computed. The results are shown in Table 4.1. It can been seen that the difference from use of ω_o or ω_c for filtering is negligible. The filter multiple M as $3-5$ should be adequate. It is noted that removal of the filter's frequency response from $\hat{G}_F(s)$ by using (4.3) is required to get good identification result for $G(jw)$. The reason is that for the cutoff frequency set at $(3-5)\omega_c$, even though the magnitude of $F(s)$ is approximate to 1 in $[0,\ \omega_c]$, the phase of $F(s)$ makes $\hat{G}_F(s)$ deviate from $G(s)$ around ω_c and leads to significant estimation errors.

Note that the FFT method assumes zero initial conditions and no disturbance. Such assumptions are relaxed in the proposed method in the next two sections.

4.3 First-order modelling

For a relay test, the relay output or the process input can be expressed as the sum of step functions:

$$u(t) = \sum_{k=0}^{N} u_k(t) = \sum_{k=0}^{N} h_k \mathbf{1}(t-t_k), \tag{4.5}$$

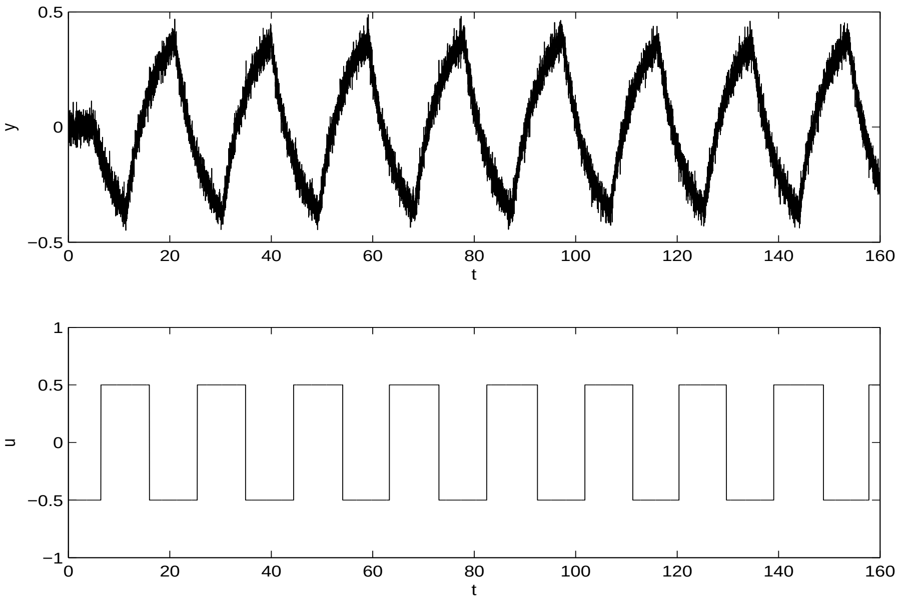

Figure 4.3. Process output and input of relay experiment for Example 4.1.

Table 4.1. Identification errors for Example 4.1

Filter cutoff frequency, $1/T_f$	Filter, $\frac{1}{T_f s+1}$	ERR_1	ERR_2
$3\omega_o$	$\frac{1}{s+1}$	41.84 %	10.61 %
$3\omega_c$	$\frac{1}{0.83s+1}$	36.55 %	10.56 %
$4\omega_o$	$\frac{1}{0.75s+1}$	33.97 %	10.54 %
$4\omega_c$	$\frac{1}{0.625s+1}$	29.84 %	10.52 %
$5\omega_o$	$\frac{1}{0.6s+1}$	29.00 %	10.51 %
$5\omega_c$	$\frac{1}{0.5s+1}$	24.24 %	11.20 %
∞	1	28.29 %	28.29 %

where $\mathbf{1}(t)$ is a unit step and time t_k is a relay switching instant. See one example given in Figure 4.4. As a result, the methods for process identification from step tests looks possible to be employed to estimate a process model from relay tests. Process modelling from step tests is popular (Åström and Hagglund, 1995; Wang and Zhang, 2001). To get a general model with reasonable accuracy, least squares based methods are often adopted. One problem with such methods is delay estimation which needs iterations. Wang and Zhang (2001) devised a linear identification algorithm for all the model parameters including delay. In their method, A differential equation with time delay is transformed to an integral equation by means of multiple integration (Whitfield and Messali, 1987) and the original model parameters are re-grouped to form a new linear regression equation. The integral identification has proven robust against noise in measurements (Golubev and Wang, 1982). However, Wang and Zhang (2001), like all the previous works on continuous system identification, assumed that the initial conditions are zero and there is no disturbance. Thus, their method cannot be applied to relay test, not only because the the initial conditions and/or disturbance may not be zero when a relay test starts, but also because the initial conditions at the subsequent relay switching times can never be zero even though the initial conditions are zero and there is no disturbance when a relay test starts. Thus, we need a new step identification algorithm which allows non-zero initial conditions in order for it to be applicable to the relay case.

In this section, we consider a first-order continuous-time delay system,

$$y^{(1)}(t) + a_0 y(t) = b_0 u(t-d) + c, \tag{4.6}$$

where $y(t)$ and $u(t)$ are the output and input of the process, respectively; d is the time delay; and c is the static disturbance and/or a bias value of the process. The task is to estimate the model parameters, a_0, b_0, d and c, from one relay test. We define an integration operator on $f(t)$ as follows,

$$P_1 f(t) = \int_{t-\tau}^{t+\tau} f(\delta_0) d\delta_0. \tag{4.7}$$

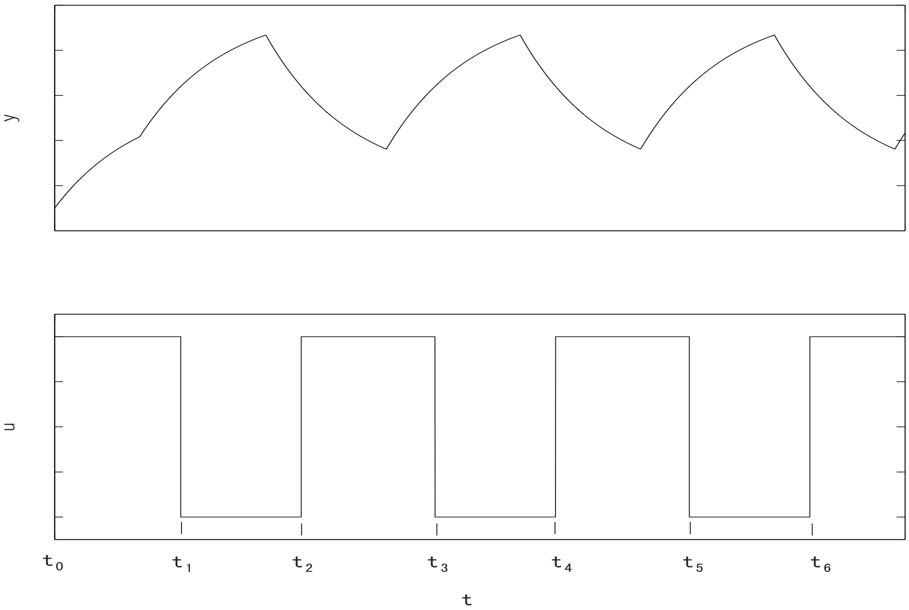

Figure 4.4. Process output and input of relay experiment.

Integrating (4.6) with (4.7) yields

$$P_1 y^{(1)}(t) + a_0 P_1 y(t) = b_0 P_1 u(t-d) + P_1 c. \tag{4.8}$$

In the left-hand side, both

$$P_1 y^{(1)}(t) = y(t+\tau) - y(t-\tau),$$

and

$$P_1 y(t) = \int_{t-\tau}^{t+\tau} y(\delta_0) d\delta_0,$$

can be numerically evaluated with knowledge of $y(t)$. The right-hand side is

$$P_1 u(t-d) = \sum_{k=0}^{N} \left(P_1 u_k(t-d) \right).$$

It is straightforward to verify that

$$P_1 \mathbf{1}(t-d) = (t+\tau-d)\mathbf{1}(t+\tau-d) - (t-\tau-d)\mathbf{1}(t-\tau-d).$$

Let τ be fixed. Choose t to meet

$$t_{k-1} + d \le t - \tau < t_k + d \le t + \tau < t_{k+1} + d. \tag{4.9}$$

We have

$$P_1 u_k(t-d) = h_k(t+\tau-t_k-d),$$

$$P_1 u_j(t-d) = 0,\ t_j > t_k,$$

and

$$P_1 u_j(t-d) = h_j P_1 1,\ t_j < t_k.$$

The right-hand side of (4.8) can be then rearranged as follows,

$$\begin{aligned} & b_0 P_1 u(t-d) + P_1 c \\ & = \left[h_k(t-t_k+\tau) + \left(\sum_{l=0}^{k-1} h_l\right) P_1 1 \quad h_k \quad P_1 1 \right] \begin{bmatrix} b_0 \\ -b_0 d \\ c \end{bmatrix}. \end{aligned}$$

Equation (4.8) then becomes

$$\phi^T(t,t_k)\theta = \gamma(t), \tag{4.10}$$

where

$$\gamma(t) = P_1 y^{(1)}(t),$$

$$\phi^T(t,t_k) = [-P_1 y(t) \quad h_k(t-t_k+\tau) + 2\tau \sum_{l=0}^{k-1} h_l \quad h_k \quad 2\tau],$$

and

$$\theta = \begin{bmatrix} a_0 \\ b_0 \\ -b_0 d \\ c \end{bmatrix}.$$

Choose $t = t_{ki}$, $i = 0, 1, 2, \ldots, M_k$, to meet $t_{k-1} + d \le t_{ki} - \tau < t_k + d \le t_{ki} + \tau < t_{k+1} + d$. One invokes (4.10) for t_{ki} to form the regression form

$$\Gamma_k = \Psi_k \theta,$$

where

$$\Gamma_k = [\gamma(t_{k0}), \ldots, \gamma(t_{kM_k})]^T,$$

and

$$\Psi_k = [\phi(t_{k0}, t_k), \ldots, \phi(t_{kM_k}, t_k)]^T.$$

From the first $N+1$ switches of one relay test, $\Gamma_k, k = 0, \ldots, N$ and $\Psi_k, k = 0, \ldots, N$, are obtained. Then, we have

$$\Gamma = \Psi\theta,$$

where

$$\Gamma = \left[\Gamma_0^T \ldots \Gamma_N^T\right]^T,$$

and

$$\Psi = \left[\Psi_0^T, \ldots, \Psi_N^T\right]^T.$$

The ordinary least-squares method can be applied to find the solution

$$\hat{\theta} = \left[\hat{\theta}_1,\ \hat{\theta}_2,\ \hat{\theta}_3,\ \hat{\theta}_4\right]^T = \left(\Psi^T\Psi\right)^{-1}\Psi^T\Gamma.$$

In the presence of noise in the measurement of the process output, the instrumental variable (IV) method similar to Wang and Zhang (2001) is adopted to guarantee the identification consistency. After θ is estimated, the model parameters can be recovered as follows:

$$\begin{cases} \hat{a}_0 = \hat{\theta}_1, \\ \hat{b}_0 = \hat{\theta}_2, \\ \hat{d} = -\frac{\hat{\theta}_3}{\hat{\theta}_2}, \\ \hat{c} = \hat{\theta}_4. \end{cases}$$

Selection of $t = t_{ki}$ depends on d, while d is to be identified and unknown. It is possible to estimate a range of d. Let d be in the range, (d_{min}, d_{max}). d_{min} may be set as the time from the input signal injection to the point when the output response still remains unchanged from the past trend, while d_{max} is the time from the input signal injection to the point when the output response has changed from the past trend well beyond the noise band. Besides, such a range can be estimated with purely numerical method (Hwang and Lai, 2004). With $d_{min} < d < d_{max}$, we can then choose $t_{k-1}+d_{max} \leq t_{ki}-\tau \leq t_k+d_{min}$ and $t_k+d_{max} \leq t_{ki}+\tau \leq t_{k+1}+d_{min}$.

Choice of τ is discussed as follows. For first order modelling, $t = t_{ki}$ and τ should meet (4.9). For kth step test, the maximum integration interval $[t_{k0} - \tau,\ t_{kM_k} + \tau]$ should cover the entire step test duration. To this end, we have

$$t_{kM_k} + \tau = t_{k+1} + d_{min}, \tag{4.11}$$

and

$$t_{kM_k} - \tau \leq t_k + d_{min}. \tag{4.12}$$

Subtract (4.12) from (4.11):

$$2\tau \geq t_{k+1} - t_k. \tag{4.13}$$

Choose $2\tau \approx t_{k+1} - t_k$ to meet (4.13) and we have

$$\tau \approx \frac{t_{k+1} - t_k}{2}.$$

We require t and τ to meet (4.9). For $k = 0$, t_{-1} is not defined. It is not a problem. When $k = 0$, we let τ and t meet

$$t - \tau < t_0 + d \leq t + \tau < t_1 + d.$$

$t - \tau$ can be negative, that is, the output measurement before the relay test is needed. In practice, a continuous industrial process runs day after day and the data on the output measurement are all recorded and saved. It is easy to retrieve these data before relay test for use in process identification.

Example 4.2. Consider a continuous-time delay process with the same transfer function as in Wang *et al.* (1999), but subject to $y(0) = -1.5$ and $c = 0.2$:

$$5y^{(1)}(t) + y(t) = u(t-5) + c,$$

Then, the FFT method cannot be applied. An relay experiment is performed at $t = 0$, with $u_+ = 0.5$ and $u_- = -0.5$. The process input and output are shown in Figure 4.5. The proposed method leads to

$$\theta^T = \begin{bmatrix} 0.2007 & 0.2006 & -1.0031 & 0.0397 \end{bmatrix}.$$

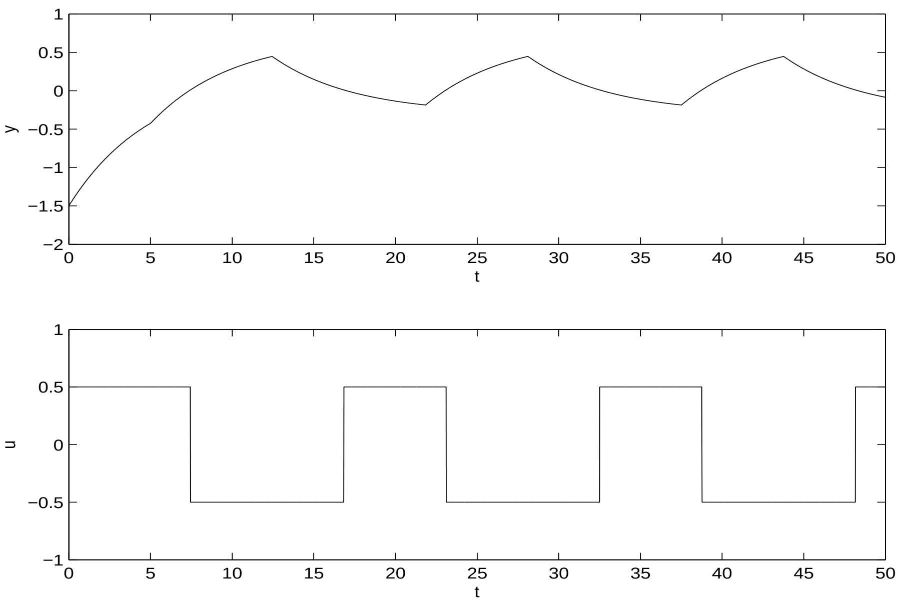

Figure 4.5. Process output and input of relay experiment for Example 4.2.

A model is obtained as follows

$$y^{(1)}(t) + 0.2007y(t) = 0.2006u(t-5),$$

with a estimated disturbance as $\hat{c} = 0.0395$. The identification error, ERR_2, is 0.22%, which is due to some computational errors.

Table 4.2. Identification errors for Example 4.2

Method	NSR=13 %	NSR=25 %	NSR=35 %
Original FFT method	10.14 %	12.79 %	13.57 %
Revised FFT method	10.51 %	12.86 %	14.95 %
Proposed method	1.37 %	2.19 %	3.11 %

To compare the proposed method with the FFT one, a new relay test, the same as in Wang *et al.* (1999). With $u_+ = 0.5$, $u_- = -0.5$, $\varepsilon_+ = 0.2$ and $\varepsilon_- = -0.2$, under zero initial conditions and no disturbance is performed, with the output corrupted with noise of $NSR = 13, 25, 35\%$, respectively. Time sequences of $y(t)$

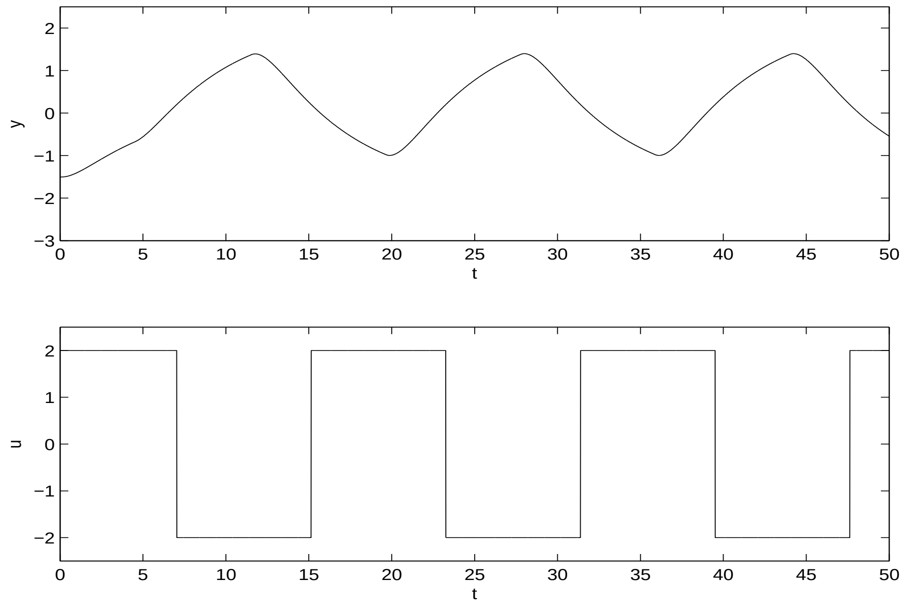

Figure 4.6. Process output and input of relay experiment for Example 4.3.

and $u(t)$ in a relay test under $NSR = 13\%$ are shown in Figure 4.3. The proposed method is applied with no need for any low-pass filter and the results are exhibited in Table 4.2. On the other hand, the FFT method requires a filter. The stationary oscillation frequency is $\omega_o \approx 0.334$, and a filter is designed with $M = 5$ as $F(s) = \frac{1}{0.6s+1}$. The FFT results are also shown in Table 4.2. Even without the low-pass filter, the proposed method is robust and can achieve better identification results than the FFT method does. The effectiveness of the proposed method is evident.

Example 4.3. Consider a 2nd-order process

$$5y^{(2)}(t) + 6y^{(1)}(t) + y(t) = u(t - 4.5) + c,$$

subject to $y^{(1)}(0) = -0.2$, $y(0) = -1.5$ and a static disturbance of 0.2. A relay experiment is performed at $t = 0$, with $u_+ = 2$ and $u_- = -2$. The process input and output are shown in Figure 4.6. The proposed method leads to

$$y^{(1)}(t) + 0.1958y(t) = 0.1947u(t - 5.44),$$

The identification error is 4.00%. Despite the presence of model structure mismatch, the accuracy of the estimated model is excellent.

4.4 n-th order modelling

Consider an n-th order continuous-time delay system,

$$y^{(n)}(t)+\cdots+a_1y^{(1)}(t)+a_0y(t)=b_mu^{(m)}(t-d)+\cdots+b_1u^{(1)}(t-d)+b_0u(t-d)+c, \tag{4.14}$$

where $n \geq 2$ and $m < n$. Use a multiple integration as follows

$$P_nf(t)=\int_0^{\tau}\int_{\tau-\delta_{n-1}}^{\tau+\delta_{n-1}}\cdots\int_{\tau-\delta_2}^{\tau+\delta_2}\int_{t-\delta_1}^{t+\delta_1}f(\delta_0)d\delta_0d\delta_1\cdots d\delta_{n-2}d\delta_{n-1},\ n\geq 2. \tag{4.15}$$

Integrating (4.14) with (4.15), we have

$$P_ny^{(n)}(t)+\sum_{l=0}^{n-1}a_lP_ny^{(l)}(t)=\sum_{j=0}^{m}b_jP_nu^{(j)}(t-d)+P_nc. \tag{4.16}$$

In the left-hand side of (4.16), we have

$$\begin{cases} P_ny(t)=\int_0^{\tau}\int_{\tau-\delta_{n-1}}^{\tau+\delta_{n-1}}\cdots\int_{\tau-\delta_2}^{\tau+\delta_2}\int_{t-\delta_1}^{t+\delta_1}y(\delta_0)d\delta_0d\delta_1\cdots d\delta_{n-2}d\delta_{n-1}, \\ P_ny^{(1)}(t)=\int_0^{\tau}\int_{\tau-\delta_{n-1}}^{\tau+\delta_{n-1}}\cdots\int_{\tau-\delta_2}^{\tau+\delta_2}\left(y(t+\delta_1)-y(t-\delta_1)\right)d\delta_1\cdots d\delta_{n-2}d\delta_{n-1}, \\ \vdots \\ P_ny^{(n)}(t)=\sum_{k=0}^{2n-2}c_ky\left(t+(n-1-k)\tau\right), \end{cases}$$

where $\begin{bmatrix}c_0 & c_1 & \ldots & c_{2n-2}\end{bmatrix} \triangleq C_n$ is calculated recursively as $C_2=\begin{bmatrix}1 & -2 & 1\end{bmatrix}$, and $C_i=\begin{bmatrix}C_{i-1} & 0 & 0\end{bmatrix}-\begin{bmatrix}0 & 0 & C_{i-1}\end{bmatrix}, i=3,4,\ldots,n$.

Note that since the upper and lower limits of any inner integral are made dependent on the dummy variable of the immediate next outer integral so that all the terms in the outcome of inner integral are functions of the outer dummy variable, but not fixed, $P_ny^{(l)}(t),\ l=1,\ldots,n-1$, be numerically evaluated with knowledge of $y(t)$ without involving initial conditions, $y^{(1)}(0)$, $y^{(2)}(0)$, …, and $y^{(n-1)}(0)$.

It can be readily shown that

$$P_n\mathbf{1}^{(l)}(t-d)=\frac{1}{(n-l)!}\sum_{k=0}^{2n-2}c_k(t+(n-1-k)\tau-d)^{n-l}\mathbf{1}(t+(n-1-k)\tau-d), l=0,1,\ldots,n-1.$$

Let τ be fixed. Choose t to meet

$$t_{k-1}+d < t-(n-1)\tau < t_k+d \leq t-(n-2)\tau < \cdots < t+(n-1)\tau < t_{k+1}+d. \quad (4.17)$$

We have

$$P_n\mathbf{1}^{(l)}(t-t_k-d) = \frac{1}{(n-l)!}\sum_{i=0}^{2n-3} c_i\left(t+(n-1-i)\tau-t_k-d\right)^{(n-l)}, l=0,1,2,\ldots,m,\ n\geq 2,$$

$$P_n\mathbf{1}^{(l)}(t-t_j-d) = 0,\ l=1,2,\ldots,m,\ t_j\neq t_k,$$

$$P_n u_j(t-d) = 0,\ t_j > t_k,$$

and

$$P_n u_j(t-d) = h_j P_n 1,\ t_j < t_k.$$

The right-hand side of (4.16) can be rearranged as follows,

$$\begin{aligned}
&\sum_{j=0}^{m} b_j P_n u^{(j)}(t-d) + P_n c \\
&= \begin{bmatrix} \frac{h_k}{n!}\left(\sum_{i=0}^{2n-3} c_i(t+(n-1-i)\tau-t_k)^n\right) + (\sum_{l=0}^{k-1} h_l)P_n 1 \\ \frac{h_k}{(n-1)!}\left(\sum_{i=0}^{2n-3} c_i(t+(n-1-i)\tau-t_k)^{n-1}\right) \\ \vdots \\ \frac{h_k}{1!}\left(\sum_{i=0}^{2n-3} c_i(t+(n-1-i)\tau-t_k)\right) \\ \frac{h_k}{0!}\left(\sum_{i=0}^{2n-3} c_i\right) \\ P_n 1 \end{bmatrix}^T \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_{n-1} \\ \alpha_n \\ c \end{bmatrix},
\end{aligned}$$

where

$$\alpha_i = \sum_{l=0}^{min(i,m)} \frac{b_l(-d)^{(i-l)}}{(i-l)!}, i=0,1,\ldots,n. \quad (4.18)$$

Then, Equation (4.16) becomes

$$\phi^T(t,t_k)\theta = \gamma(t), \quad (4.19)$$

where

$$\gamma(t) = P_n y^{(n)}(t),$$

$$\phi(t,t_k)=\begin{bmatrix} -P_n y^{(n-1)}(t) \\ \vdots \\ -P_n y^{(1)}(t) \\ -P_n y(t) \\ \frac{h_k}{n!}\left(\sum_{i=0}^{2n-3} c_i (t+(n-1-i)\tau - t_k)^n\right) + (\sum_{l=0}^{k-1} h_l) P_n 1 \\ \frac{h_k}{(n-1)!}\left(\sum_{i=0}^{2n-3} c_i (t+(n-1-i)\tau - t_k)^{n-1}\right) \\ \vdots \\ \frac{h_k}{1!}\left(\sum_{i=0}^{2n-3} c_i (t+(n-1-i)\tau - t_k)\right) \\ \frac{h_k}{0!}\left(\sum_{i=0}^{2n-3} c_i\right) \\ P_n 1 \end{bmatrix},$$

and

$$\theta = \begin{bmatrix} a_{n-1} & \ldots & a_1 & a_0 & \alpha_0 & \alpha_1 & \ldots & \alpha_{n-1} & \alpha_n & c \end{bmatrix}^T.$$

Choose $t = t_{ki}$, $i = 0, 1, 2, \ldots, M_k$, to meet $t_{k-1} + d < t_{ki} - (n-1)\tau < t_k + d \leq t_{ki} - (n-2)\tau < \cdots < t_{ki} + (n-1)\tau < t_{k+1} + d$. One invokes (4.19) for $t = t_{ki}$ to form the regression form

$$\Gamma_k = \Psi_k \theta,$$

where

$$\Gamma_k = [\gamma(t_{k0}), \ldots, \gamma(t_{kM_k})]^T,$$

and

$$\Psi_k = [\phi(t_{k0}, t_k), \ldots, \phi(t_{kM_k}, t_k)]^T.$$

From the first $N+1$ switches of one relay test, $\Gamma_k, k = 0, \ldots, N$ and $\Psi_k, k = 0, \ldots, N$ are obtained. Then, we have

$$\Gamma = \Psi \theta,$$

where

$$\Gamma = \left[\Gamma_0^T \ldots \Gamma_N^T\right]^T,$$

and

$$\Psi = \left[\Psi_0^T, \ldots, \Psi_N^T\right]^T.$$

Once θ is estimated by applying the least-squares method or IV method, the model parameters can be recovered. From (4.18) for $i = 0, \ldots, m$, we have

$$\begin{cases} b_0 = \alpha_0, \\ b_i = \alpha_i - \sum_{l=0}^{i-1} \frac{b_l(-d)^{(i-l)}}{(i-l)!}, i = 1, \ldots, m. \end{cases} \tag{4.20}$$

Substituting (4.20) into (4.18) for $i = m+1$, an $(m+1)$-th order linear equation of d is derived. Once d is obtained, $b_j, j = 0, \ldots, m$ can be solved from (4.20). The above approach produces $(m+1)$ possible solutions for time delay d and thus $(m+1)$ possible models. We follows the method in Wang and Zhang (2001) to choose the appropriate model.

For $n$the order modelling, $t = t_{ki}$ and τ should meet (4.17). For kth step test, the maximum integration interval $[t_{k0} - (n-1)\tau,\ t_{kM_k} + (n-1)\tau]$ should cover the entire step test duration. To this end, we have

$$t_{kM_k} + (n-1)\tau = t_{k+1} + d_{min}, \tag{4.21}$$

and

$$t_{kM_k} - (n-1)\tau \le t_k + d_{min}. \tag{4.22}$$

Subtract (4.22) from (4.21):

$$2(n-1)\tau \ge t_{k+1} - t_k. \tag{4.23}$$

Choose $2(n-1)\tau \approx t_{k+1} - t_k$ to meet (4.23), which result in

$$\tau \approx \frac{t_{k+1} - t_k}{2(n-1)}. \tag{4.24}$$

Example 3 (Continued). Applying the proposed method in this section with $n = 2$ and $m = 0$ yields

$$y^{(2)}(t) + 1.224y^{(1)}(t) + 0.2026y(t) = 0.2043u(t - 4.51).$$

The identification error is 0.85%. To compare the proposed method with the FFT one, a new relay test is performed with $u_+ = 0.5$, $u_- = -0.5$, $\varepsilon_+ = 0.2$ and

$\varepsilon_- = -0.2$, under zero initial conditions and no disturbance. The output corrupted with noise of $NSR = 13, 25, 35\%$, respectively. The proposed method is applied with no need for any low-pass filter and the results are exhibited in Table 4.3. The robustness of the proposed method is obvious. The stationary oscillation frequency is $\omega_o \approx 0.3$ and a low-pass filter is designed as $F(s) = \frac{1}{0.67s+1}$ for the FFT method. The FFT results are also shown in Table 4.3. The proposed method is more accurate under the same noise level.

Table 4.3. Identification errors for Example 4.3

Method	NSR=13 %	NSR=25 %	NSR=35 %
Original FFT method	6.14 %	9.21 %	10.67 %
Revised FFT method	6.44 %	10.10 %	12.99 %
Proposed method	2.39 %	3.76 %	8.13 %

Example 4.4. Consider a continuous-time delay process (Wang *et al.*, 1999):

$$25y^{(3)}(t) + 35y^{(2)}(t) + 11y^{(1)}(t) + y(t) = u(t - 2.5) + c.$$

subject to $y^{(2)}(0) = 0$, $y^{(1)}(0) = 0$, $y(0) = 4$ and $c = 1$. The proposed method with $n = 2$ and $m = 0$ leads to

$$\hat{G}(s) = \frac{0.03351}{s^2 + 0.335s + 0.03388} e^{-3.02s}.$$

The identification error is 3.32%. If the relay test is applied subject to $y^{(2)}(0) = -0.4$, $y^{(1)}(0) = 0.4$, $y(0) = 2$ and $c = 1$, the proposed method with $n = 2$ and $m = 0$ leads to

$$\hat{G}(s) = \frac{0.03411}{s^2 + 0.3486s + 0.03366} e^{-3.11s}.$$

The identification error is 3.04%.

To compare the proposed method with the FFT one, a new relay test is performed with $u_+ = 0.5$, $u_- = -0.5$, $\varepsilon_+ = 0.3$ and $\varepsilon_- = -0.3$, under zero initial conditions and no disturbance. The output is corrupted with noise of $NSR = 13, 25, 35\%$, respectively. The proposed method is applied with no low-pass filter and the results are exhibited in Table 4.4. The robustness of the proposed method is obvious. The stationary oscillation frequency is $\omega_o \approx 0.21$ and a low-pass filter is designed as $F(s) = \frac{1}{s+1}$ for the FFT method. The FFT results are also shown in Table 4.4. Compared with FFT method, the effectiveness of the proposed method is evident.

Table 4.4. Identification errors for Example 4.4

Method	NSR=13 %	NSR=25 %	NSR=35 %
Original FFT method	6.74 %	7.98 %	8.17 %
Revised FFT method	6.51%	7.91 %	12.43 %
Proposed method	5.48 %	6.58 %	7.45 %

4.5 Conclusion

In this chapter, a new identification method from relay tests is proposed. By regarding a relay test as a sequence of step tests, the integral technique is adopted to devise the algorithm. The method can yield a full process model in the sense of a complete transfer function with delay or a complete frequency response. The effectiveness of the proposed method is demonstrated through simulation.

Chapter 5

Process Identification from Piecewise Step Tests

5.1 Introduction

In Chapter 4, it is proposed that a relay test can be regarded as a sequence of step tests. In this chapter, this idea is further developed. A general identification method is proposed for continuous-time delay processes. The identification test can be of open-loop such as pseudo random binary signals (PRBS), which are used in Ahmed *et al.* (2006), and pulse tests, which are used in Hwang and Lai (2004) or of closed-loop type such as relay tests, which are used in Wang *et al.* (2006). Compared with recent developments reported on identification of continuous-time delay systems based on integration techniques, the proposed method has many advantages. In Hwang and Lai (2004), two regression equations are obtained from the two edges of the pulse signal respectively, and model parameters are estimated in two steps. Their regression parameter vectors involve all parameters together in either of the two steps and some of them are very complicated functions of process parameters and initial conditions. In Ahmed *et al.* (2006), the identification method needs an iterative procedure for the time delay estimation. The method proposed in Wang *et al.* (2006) needs the output measurement before the relay test, and also considers, like many previous identification methods, the constant disturbance only. For the identification method proposed in this chapter, no prior process data before identification test is needed, and the initial conditions are unknown and can be nonzero, and the disturbance can be of general form. The regression equation is derived taking into account nature of the underlying test signal. The equation has more linearly independent functions and thus enables identification of a full process model with time delay as well as combined effects of unknown initial condition and disturbance without any iteration. All the parameters including time delay in the regression equation are estimated in one step. The method shows great robustness against noise in output measurements but requires no filtering of noisy data.

The remainder of this chapter is organized as follows. In Section 5.2, the proposed method is presented for second-order modelling. The method is extended to high-order modelling in Section 5.3. Conclusions are drawn in Section 5.4.

5.2 Second-order modelling

This section focuses on the modelling of second-order systems. It serves for motivation of the general method to be described in the next section and for recommended use in applications since such a second-order model essentially covers most practical industrial processes. Consider a second-order continuous-time delay system,

$$y^{(2)} + a_1 y^{(1)}(t) + a_0 y(t) = b_1 u^{(1)}(t-d) + b_0 u(t-d) + l(t), \tag{5.1}$$

where $y(t)$ and $u(t)$ are the output and input of the process, respectively; d is the time delay; and $l(t)$ is an unknown disturbance or a bias to the process. The task is to estimate the model parameters, a_1, a_0, b_1, b_0 and d from one test. The test input under consideration is supposed to be in the form of

$$u(t) = \sum_{j=0}^{N} u_j(t) = \sum_{j=0}^{N} h_j \mathbf{1}(t - t_j), \tag{5.2}$$

where $\mathbf{1}(t)$ is the unit step function, $N \geq 1$, and $u_j(t)$ is a step input with magnitude of h_j and applied at $t = t_j$. This form covers many types of signals including open-loop tests such as PRBS, rectangular pulses with magnitude of h and duration of T,

$$u(t) = h\mathbf{1}(t) - h\mathbf{1}(t-T), \tag{5.3}$$

and rectangular doublet pulses,

$$u(t) = h\mathbf{1}(t) - 2h\mathbf{1}(t-\frac{T}{2}) + h\mathbf{1}(t-T),$$

as well as close-loop tests such as relay tests, see one example in Section 5.2. The relay function is described as

$$u(t) = \begin{cases} u_+, \ if \ e(t) > \varepsilon_+, \ or \ e(t) \geq \varepsilon_- \ and \ u(t_-) = u_+, \\ u_-, \ if \ e(t) < \varepsilon_-, \ or \ e(t) \leq \varepsilon_+ \ and \ u(t_-) = u_-, \end{cases} \tag{5.4}$$

where ε_+, $\varepsilon_- \in \mathbb{R}$ with $\varepsilon_- < \varepsilon_+$ indicating hysteresis; $u_-, u_+ \in \mathbb{R}$ and $u_- \neq u_+$.

A multiple integration operator on $f(t)$ is defined as follows,

$$\begin{cases} P_0 f(t) = f(t), \\ P_j f(t) = \int_0^t \int_0^{\tau_{j-1}} \cdots \int_0^{\tau_1} f(\tau_0) d\tau_0 d\tau_1 \cdots d\tau_{j-1}, j \geq 1. \end{cases} \tag{5.5}$$

Applying P_2 to (5.1) yields

$$P_2 y^{(2)}(t) + a_1 P_2 y^{(1)}(t) + a_0 P_2 y(t) = b_1 P_2 u^{(1)}(t-d) + b_0 P_2 u(t-d) + P_2 l(t). \tag{5.6}$$

For the left-hand side, we have

$$P_2 y^{(2)}(t) = y(t) - y(0) - y^{(1)}(0)t, \tag{5.7}$$

$$P_2 y^{(1)}(t) = \int_0^t y(\tau_0) d\tau_0 - y(0)t, \tag{5.8}$$

and

$$P_2 y(t) = \int_0^t \int_0^{\tau_1} y(\tau_0) d\tau_0 d\tau_1. \tag{5.9}$$

For the right hand side, it is straightforward to verify that

$$P_2 \mathbf{1}(t-d) = \frac{(t-d)^2}{2!} \mathbf{1}(t-d),$$

and

$$P_2\mathbf{1}^{(1)}(t-d) = (t-d)\mathbf{1}(t-d).$$

It then follows that

$$P_2u(t-d) = \sum_{j=0}^{N} P_2u_j(t-d) = \sum_{j=0}^{N} \frac{h_j(t-t_j-d)^2}{2!}\mathbf{1}(t-t_j-d), \tag{5.10}$$

and

$$P_2u^{(1)}(t-d) = \sum_{j=0}^{N} P_2u_j^{(1)}(t-d) = \sum_{j=0}^{N} h_j(t-t_j-d)\mathbf{1}(t-t_j-d). \tag{5.11}$$

Choose t to meet

$$t_k + d \le t < t_{k+1} + d, \tag{5.12}$$

where t_k and t_{k+1} are the kth and $(k+1)th$ input switch instants, respectively. Equations (5.10) and (5.11) become

$$P_2u(t-d) = \sum_{j=0}^{k} \frac{h_j(t-t_j-d)^2}{2!}, \tag{5.13}$$

and

$$P_2u^{(1)}(t-d) = \sum_{j=0}^{k} h_j\left(t-t_j-d\right). \tag{5.14}$$

Suppose that there holds

$$P_2l(t) = \sum_{j=0}^{Q} \beta_j t^j, \tag{5.15}$$

where Q is an integer. Equation (5.15) stands for the multiple integrations of the generalized disturbances (Hwang and Lai, 2004) more than a static disturbance for which $l(t) = c\mathbf{1}(t)$, $P_2l(t) = \frac{ct^2}{2}$ and $Q = 2$.

Substituting (5.7), (5.8), (5.9), (5.13) ,(5.14) and (5.15) into (5.6) gives

$$\begin{aligned} & \left(y(t) - y(0) - y^{(1)}(0)t\right) + a_1\left(\int_0^t y(\tau_0)d\tau_0 - y(0)t\right) + a_0\int_0^t\int_0^{\tau_1} y(\tau_0)d\tau_0 d\tau_1 \\ = \;& b_1\sum_{j=0}^{k} h_j(t-t_j-d) + b_0\sum_{j=0}^{k}\frac{h_j(t-t_j-d)^2}{2!} + \sum_{j=0}^{Q}\beta_j t^j. \end{aligned} \tag{5.16}$$

Equation (5.16) can then be rearranged as follows,

$$\phi^T(t,t_k)\theta = \gamma(t), \quad t_k + d \le t < t_{k+1} + d, \tag{5.17}$$

where

$$\phi(t,t_k) = \begin{bmatrix} -\int_0^t y(\tau_0)d\tau_0 \\ -\int_0^t\int_0^{\tau_1} y(\tau_0)d\tau_0 d\tau_1 \\ \sum_{j=0}^k h_j \\ \sum_{j=0}^k h_j(t-t_j) \\ \sum_{j=0}^k h_j(t-t_j)^2 \\ 1 \\ t \\ t^2 \\ \vdots \\ t^Q \end{bmatrix}, \quad \theta = \begin{bmatrix} a_1 \\ a_0 \\ \theta_0 \\ \theta_1 \\ \theta_2 \\ \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_Q \end{bmatrix} = \begin{bmatrix} a_1 \\ a_0 \\ \frac{b_0 d^2}{2} - b_1 d \\ b_1 - b_0 d \\ \frac{b_0}{2} \\ \beta_0 + y(0) \\ \beta_1 + y^{(1)}(0) + a_1 y(0) \\ \beta_2 \\ \vdots \\ \beta_Q \end{bmatrix} \quad and\ \gamma(t) = y(t).$$

The parameters α_i, $i = 0, 1, \ldots, Q$, are used to account for the effects of the aforementioned nonzero initial conditions and the disturbance. Choose $t = t_{ki}$, $i = 0, 1, 2, \ldots, M_k$, to meet $t_k + d \le t_{ki} < t_{k+1} + d$. One invokes (5.17) for t_{ki}:

$$\Psi_k \theta = \Gamma_k, \tag{5.18}$$

where $\Psi_k = [\phi(t_{k0}, t_k), \ldots, \phi(t_{kM_k}, t_k)]^T$ and $\Gamma_k = [\gamma(t_{k0}), \ldots, \gamma(t_{kM_k})]^T$. From the $N+1$ input switches of one test, Γ_k and $\Psi_k, k = 0, \ldots, N$, are obtained and combined to

$$\Psi\theta = \Gamma,$$

where $\Psi = \left[\Psi_0^T, \ldots, \Psi_N^T\right]^T$ and $\Gamma = \left[\Gamma_0^T \ldots \Gamma_N^T\right]^T$. The ordinary least-squares method can be applied to find the solution

$$\hat{\theta} = \left[\hat{a}_1, \hat{a}_0, \hat{\theta}_0,\ \hat{\theta}_1,\ \hat{\theta}_2, \hat{\alpha}_0, \hat{\alpha}_1, \ldots, \hat{\alpha}_P\right]^T = \left(\Psi^T\Psi\right)^{-1}\Psi^T\Gamma.$$

In the presence of noise in the measurement of the process output, the instrumental variable (IV) method is adopted to guarantee the identification consistency. For our case, the instrumental variable $Z(t_{ki})$ is chosen as

$$Z(t_{ki}) = \left[(t_{ki})^{-(M_{id}-1)} \quad \ldots \quad (t_{ki})^{-1} \quad 1 \quad t_{ki} \quad \ldots \quad (t_{ki})^{2n+2+Q-M_{id}}\right],$$

where M_{id} is the quotient of $\frac{2n+2+Q}{2}$; n is order of the model, and $n = 2$ for second-order modelling.

After θ is estimated, its first 2 elements directly yield the parameters a_1 and a_0, and the others produce b_1, b_0 and d via

$$\begin{cases} b_0 = 2\hat{\theta}_2, \\ d = \frac{-\hat{\theta}_1 \pm \sqrt{\hat{\theta}_1^2 - 4\hat{\theta}_0\hat{\theta}_2}}{2\hat{\theta}_2}, \\ b_1 = \hat{\theta}_1 + b_0 d. \end{cases} \tag{5.19}$$

Selection of $t = t_{ki}$ depends on d, while d is to be identified and unknown. It is possible to estimate a range of d. Let d be in the range, $[d_{min}, d_{max}]$. d_{min} may be set as the time from the input signal injection to the point when the output response still remains unchanged from the past trend, while d_{max} is the time from the input signal injection to the point when the output response has changed from the past trend well beyond the noise band. Besides, such a range can be estimated with purely numerical method (Hwang and Lai, 2004). With $d_{min} < d < d_{max}$, we can then choose $t_k + d_{max} \leq t_{ki} < t_{k+1} + d_{min}$. One difference between this method and the one by Ahmed *et al.* (2006) is this choice of t. We implicitly assume some priori knowledge of time delay, while Ahmed *et al.* (2006) finds d by iteration.

It is easy to extend our method to identify the model parameters from the test which has the input in the form of:

$$u(t) = \sum_{j=0}^{N} h_j(t - t_j)\mathbf{1}(t - t_j),$$

where t_j is an input switch time. It is straightforward to find that

$$P_n(t-d)\mathbf{1}(t-d) = \frac{(t-d)^{n+1}}{(n+1)!}\mathbf{1}(t-d).$$

Following the above development procedure, one will obtain an identification method similar to the proposed one. Because this kind of test signals are not widely used, the identification based on such inputs is not discussed in details in this chapter.

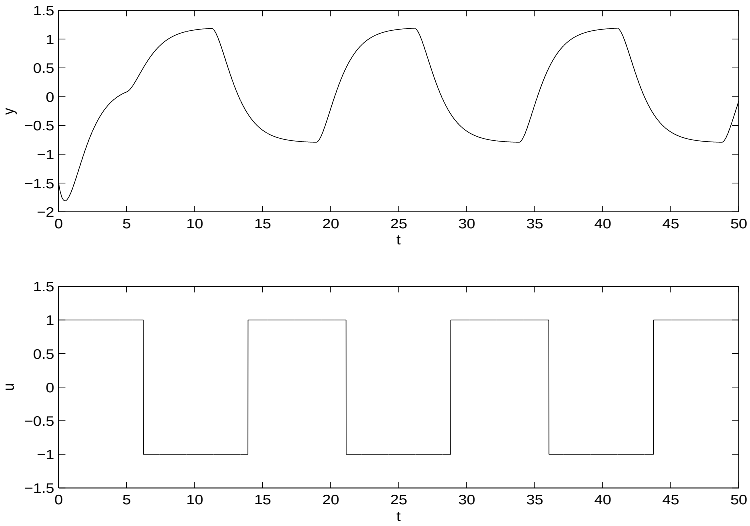

Figure 5.1. Process output and input of relay experiment for Example 5.1.

Example 5.1. Consider a continuous-time delay process,

$$y^{(2)}(t) + 2y^{(1)}(t) + y(t) = u(t-5) + l,$$

subject to $y(0) = -1.5$, $y^{(1)}(0) = -1.5$ and $l = 0.2$. The relay test in (5.4) is applied at $t = 0$ with $u_+ = 1$, $u_- = -1$, $\varepsilon_+ = 0.4$ and $\varepsilon_- = -0.4$. The process input and output are shown in Figure 5.1. Suppose $3.5 < d < 6.5$. The proposed method with $m = 0$ and $Q = 2$ leads to

$$\theta = \begin{bmatrix} 2.0202 & 1.0203 & 12.7793 & -5.1066 & 0.5102 & -1.4793 & -4.6046 & 0.1020 \end{bmatrix}^T.$$

The model is recovered as

$$y^{(2)}(t) + 2.02y^{(1)}(t) + 1.02y(t) = 1.02u(t-5). \tag{5.20}$$

Suppose that the identification error is measured by the worst case error,

$$ERR = max \left| \frac{\hat{G}(j\omega_i) - G(j\omega_i)}{G(j\omega_i)} \right|, i = 1, \cdots, M, \tag{5.21}$$

where $\hat{G}(j\omega_i)$ and $G(j\omega_i)$ are the estimated response and the actual ones, respectively. Only $\omega_i \in [0, \omega_c]$, where ω_c is the phase crossover frequency of the process, are considered. For this example, $ERR = 0.62\%$, which is due to computational errors.

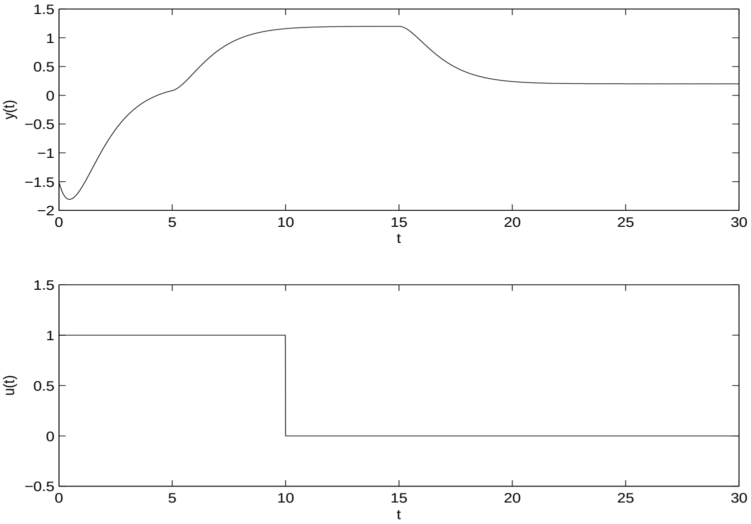

Figure 5.2. Pulse response and input for Example 5.1.

For the same process, a pulse in (5.3) is applied at $t = 0$ with $h = 1$ and $T = 10$. The process input and output are shown in Figure 5.2. The proposed method with $m = 0$ and $Q = 2$ leads to the same identification result as in (5.20).

We then consider a changing disturbance. The changing disturbance is simulated by letting $\mathbf{1}(t)$ pass through the transfer function of $\frac{0.2}{(15s+1)}$. The proposed method with $m = 0$ and $Q = 3$ leads to

$$y^{(2)}(t) + 1.943y^{(1)}(t) + 0.9831y(t) = 0.9753u(t - 4.98),$$

with $ERR = 0.8\%$.

To simulate practical conditions, white noise is added to corrupt the output. The noise-to-signal ratio defined by

$$NSR = \frac{mean(abs(noise))}{mean(abs(signal))},$$

is used to represent the noise level. A relay test in (5.4) is applied at $t = 0$ with $u_+ = 1$, $u_- = -1$, $\varepsilon_+ = 0.8$ and $\varepsilon_- = -0.8$. The output is corrupted by noise of $NSR = 5\%,\ 10\%,\ 20\%,\ 30\%$ and 40%, respectively. The proposed method is applied without low-pass filtering and the identification errors are 0.91%, 1.12%, 4.59%, 8.47% and 18.97%, respectively.

Table 5.1. Identification results for different second order processes

True models	Ahmed's method	Proposed method
$\frac{1.25e^{-0.234s}}{0.25s^2+0.7s+1}$	$\frac{1.25(\pm 0.02)e^{-0.239(\pm 0.042)s}}{0.25(\pm 0.029)s^2+0.697(\pm 0.02)s+1}$	$\frac{1.255(\pm 0.0631)e^{-0.22(\pm 0.034)s}}{0.262(\pm 0.028)s^2+0.716(\pm 0.041)s+1}$
$\frac{2e^{-4.1s}}{100s^2+25s+1}$	$\frac{2(\pm 0.04)e^{-4.13(\pm 0.742)s}}{99.4(\pm 19.7)s^2+25(\pm 0.67)s+1}$	$\frac{2.01(\pm 0.091)e^{-4.08(\pm 0.119)s}}{100.7(\pm 3.481)s^2+25.1(\pm 1.183)s+1}$
$\frac{(-4s+1)e^{-0.615s}}{9s^2+2.4s+1}$	$\frac{(-4(\pm 0.0913)s+1(\pm 0.06))e^{-0.6157(\pm 0.07)s}}{8.99(\pm 0.15)s^2+2.41(\pm 0.15)s+1}$	$\frac{(-4.03(\pm 0.08)s+1.03(\pm 0.0757))e^{-0.617(\pm 0.0294)s}}{9.11(\pm 0.2841)s^2+2.43(\pm 0.1265)s+1}$

In Table 5.1, the identification results for a number of second order processes (Ahmed *et al.*, 2006) are given and compared with those in Ahmed *et al.* (2006). The NSR for all cases are 10%. These identification results are from 500 Monte Carlo simulations. The parameters shown are the means of 500 Monte Carlo simulations and the numbers in the parentheses are the estimated standard deviation of these estimates. The proposed method produces satisfactory identification results similar to Ahmed *et al.* (2006), but the model parameters are recovered in one step without iterations. In Table 5.1, a non-minimum phase (NMP) process is also considered. Ahmed *et al.* (2006) takes special procedure for identification of NMP processes. In contrast, the proposed method treats the identification of the NMP processes and that of minimum phase processes in the same way.

Our regression equation in (5.17) is different from that used by the previous integral identification methods, such as two-step algorithm in Hwang and Lai (2004) where

$$\phi^T(t)\theta = \gamma(t),$$

where

$$\phi^T(t) = [-y(t) \quad -P_1y(t) \quad h \quad ht \quad \ldots \quad ht^Q],$$

$$\theta = [a_2 \quad a_1 \quad \bar{\theta}_1 \quad \bar{\theta}_2 \quad \ldots \quad \bar{\theta}_Q],$$

$$\gamma(t) = P_2y(t).$$

$\bar{\theta}_i$ are combinations of the model parameters, b_j, d, non-zero initial conditions and the disturbance. In our new regression equation, new elements, $\sum_{j=0}^{k} h_j(t-t_j)^i$, $i = 0, 1, 2$, are added into $\phi(t, t_k)$. They are not only mutually independent but also independent with t^i, $i = 0, 1, 2$. $\theta_i, i = 0, 1, 2$ in θ are related to b_j and d, while $\alpha_i, i = 0, \ldots, Q$ account for the effects of the nonzero initial conditions and disturbance. This enables estimation of all the regression parameters in one step.

In Wang *et al.* (2006), the output measurement before the relay test is required and the input should be kept constant so as to eliminate the effect of the unknown initial conditions. Like many previous identification methods, Wang *et al.* (2006) considers the static disturbance only. In contrast, the proposed method makes no use of process input and output before the test. It can be carried out under complex disturbances by including $\alpha_i, i = 0, \ldots, Q$, which account for the combined effects of the nonzero initial conditions and disturbance in the regression equations.

In Ahmed *et al.* (2006), the filter transfer function as

$$F(s) = \frac{\beta^n}{s(s+\lambda)^n},$$

is applied. One has to choose the parameter λ, which is nontrivial (Sinha and Rao, 1991). Moreover, this method needs an iterative procedure for the time delay estimation and takes special procedure for identification of NMP processes. These problems are not present in the proposed method.

5.3 n-th order modelling

Consider an nth-order continuous-time delay system,

$$y^{(n)}(t) + \cdots + a_1 y^{(1)}(t) + a_0 y(t) = b_m u^{(m)}(t-d) + \cdots + b_1 u^{(1)}(t-d) + b_0 u(t-d) + l(t), \tag{5.22}$$

where $m < n$. Integrating (5.22) with (5.5) for n times, we have

$$P_n y^{(n)}(t) + \sum_{l=0}^{n-1} a_l P_n y^{(l)}(t) = \sum_{j=0}^{m} b_j P_n u^{(j)}(t-d) + P_n l(t). \tag{5.23}$$

It can be readily shown that

$$P_n \mathbf{1}^{(l)}(t-d) = \frac{(t-d)^{n-l}}{(n-l)!}\mathbf{1}(t-d), l = 0, 1, \ldots, m,$$

and

$$P_n u^{(l)}(t-d) = \sum_{j=0}^{N} \frac{h_j (t-t_j-d)^{n-l}}{(n-l)!}\mathbf{1}(t-t_j-d),\ \ l = 0, 1, \ldots, m.$$

Choose t to meet (5.12), and we have

$$P_n u^{(l)}(t-d) = \sum_{j=0}^{k} \frac{h_j (t-t_j-d)^{n-l}}{(n-l)!},\ \ l = 0, 1, \ldots, m. \tag{5.24}$$

The multiple integral of $l(t)$ is supposed to be

$$P_n l(t) = \sum_{j=0}^{Q} \beta_j t^j. \tag{5.25}$$

Equation (5.23) can be rearranged as

$$\phi^T(t, t_k)\theta = \gamma(t), \tag{5.26}$$

where

$$\phi(t,t_k) = \begin{bmatrix} -\int_0^t y(\tau_0)d\tau_0 \\ \vdots \\ -\int_0^t \int_0^{\tau_{n-1}} \cdots \int_0^{\tau_1} y(\tau_0)d\tau_0 d\tau_1 \cdots d\tau_{n-1} \\ \sum_{j=0}^k h_j \\ \sum_{j=0}^k h_j(t-t_j) \\ \vdots \\ \sum_{j=0}^k h_j(t-t_j)^n \\ 1 \\ t \\ \vdots \\ t^Q \end{bmatrix},\ \theta = \begin{bmatrix} a_{n-1} \\ \ldots \\ a_0 \\ \theta_0 \\ \theta_1 \\ \ldots \\ \theta_n \\ \alpha_0 \\ \alpha_1 \\ \ldots \\ \alpha_Q \end{bmatrix}, and\ \gamma(t) = y(t).$$

The parameters α_i, $i = 1, \ldots, Q$, are used to account for the effects of the aforementioned nonzero initial conditions and the disturbance. Note that the first n elements of θ are the model parameters a_i, $i = 0, \ldots, n-1$, while θ_i, $i = 0, \ldots, n$ are combinations of the model parameters b_j, $j = 0, \ldots, m$, and d is given by

$$\theta_i = \sum_{j=max(n-m,i)}^{n} \frac{(-d)^{j-i} b_{n-j}}{(j-i)!i!}, i = 0, 1, \ldots, n. \tag{5.27}$$

Choose $t = t_{ki}$, $i = 0, 1, 2, \ldots, M_k$, to meet $t_k + d \leq t_{ki} < t_{k+1} + d$. One invokes (5.17) for t_{ki}:

$$\Psi_k \theta = \Gamma_k,$$

where $\Psi_k = [\phi(t_{k0}, t_k), \ldots, \phi(t_{kM_k}, t_k)]^T$ and $\Gamma_k = [\gamma(t_{k0}), \ldots, \gamma(t_{kM_k})]^T$. From the $N+1$ input switches of one test, Γ_k and $\Psi_k, k = 0, \ldots, N$, are obtained and combined to

$$\Psi \theta = \Gamma,$$

where $\Psi = \left[\Psi_0^T, \ldots, \Psi_N^T\right]^T$ and $\Gamma = \left[\Gamma_0^T \ldots \Gamma_N^T\right]^T$. Once θ is estimated by applying the least-squares method or IV method, the model parameters can be recovered. From (5.27) for $i = 0, \ldots, m+1$, we can recover d from the following algebraic equation:

$$\sum_{j=0}^{m+1} \frac{(n-m-1+j)!\theta_{n-1-m+j} d^j}{j!} = 0. \tag{5.28}$$

Once d is obtained, the parameters b_j, $j = 0, \ldots, m$ are then calculated as

$$b_j = \sum_{i=0}^{j} \frac{(n-j+i)!\theta_{n-j+i} d^i}{i!}, j = 0, 1, \ldots, m. \tag{5.29}$$

The above approach produces $(m+1)$ possible solutions for time delay d and thus $(m+1)$ possible models. We follow the method in Wang and Zhang (2001) and Hwang and Lai (2004) to choose the appropriate model.

Example 5.2. Consider a continuous-time delay process, $G(s) = \frac{e^{-2s}}{(s+1)^4}$ or

$$y^{(4)}(t) + 4y^{(3)}(t) + 6y^{(2)}(t) + 4y^{(1)}(t) + y(t) = u(t-2) + l(t),$$

subject to $y^{(3)}(0) = y^{(2)}(0) = y^{(1)}(0) = y(0) = -0.5$. A changing disturbance is simulated by letting $\mathbf{1}(t)$ pass by $\frac{0.2}{20s+1}$. A relay experiment is performed at $t = 0$, with $u_+ = 1$, $u_- = -1$, $\varepsilon_+ = 0.4$ and $\varepsilon_- = -0.4$. The proposed method with $n = 2$, $m = 0$ and $Q = 3$ leads to

$$y^{(2)}(t) + 1.037y^{(1)}(t) + 0.3748y(t) = 0.3561u(t - 3.04),$$

with $ERR = 5.15\%$. The proposed method with $n = 3$, $m = 0$ and $Q = 4$ leads to

$$y^{(3)}(t) + 2.038y^{(2)}(t) + 1.689y^{(1)}(t) + 0.4606y(t) = 0.475u(t - 2.4),$$

with $ERR = 4.01\%$. The effectiveness of the proposed method is evident.

5.4 Conclusion

In this chapter, an improved integral identification method is proposed for continuous-time delay systems. By treating the test input as a sequence of step tests and noting more independent functions available from the changing input levels, a new regression equation is established and enables effective estimation of a full transfer function with delay under unknown initial conditions and disturbance. The effectiveness of the proposed method is demonstrated through simulations.

Chapter 6

Multivariable Process Identification

6.1 Introduction

Identification and control of single variable processes have been well studied (Åström and Hagglund, 1995; Ljung, 1999; Wang *et al.*, 2005; Wang *et al.*, 2006; Liu *et al.*, 2007). However, most industrial processes are of multivariable in nature. Process identification of multivariable processes is in great demand (Cott, 1995; Zhu, 1998). An important issue with multivarible process identification is time delay. Its estimation needs special attention. Based on novel integration techniques, robust identification methods have been proposed for single variable delay processes in the previous chapters. In Chapter 2 and 3, the identification methods from pulse tests and step tests are proposed, respectively. In Chapter 4, an identification method from relay tests is presented. An improved general method is developed in Chapter 5. Extending these SISO identification methods to MIMO cases is of great interest and value.

In this chapter, an integral identification method is presented for multivariable processes with multiple time delays. It adopts the integral technique and can work under non-zero initial conditions and dynamic disturbances. The effectiveness of the proposed method is demonstrated through simulation and real time implementation.

This chapter is organized as follows. In Section 6.2, the identification method is developed for two-input and two-output (TITO) time delay processes. Simulation examples are given in Section 6.3. The proposed method is extended to the general cases in Section 6.4. In Section 6.5, the proposed method is applied to a physical thermal control system. Conclusions are drawn in Section 6.6.

6.2 TITO processes

To introduce our method with simplicity and clarity, let us consider a TITO continuous-time delay process first,

$$\begin{bmatrix} Y_1(s) \\ Y_2(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} U_1(s) \\ U_2(s) \end{bmatrix},$$

where $Y_1(s)$ and $Y_2(s)$ are the Laplace transforms of two outputs, $y_1(t)$ and $y_2(t)$, $U_1(s)$ and $U_2(s)$ are the Laplace transforms of two inputs, $u_1(t)$ and $u_2(t)$, and $G_{i,j}(s) = \frac{\alpha_{ij}(s)}{\beta_{ij}(s)} e^{-d_{ij}s}$, $i = 1, 2$ and $j = 1, 2$. The given TITO process may be decomposed into 2 two-input and single-output sub-processes, which can be described as

$$\begin{aligned} Y_i(s) &= \begin{bmatrix} G_{i1}(s) & G_{i2}(s) \end{bmatrix} U(s), \\ &= \begin{bmatrix} \frac{\alpha_{i1}(s)}{\beta_{i1}(s)} e^{-d_{i1}s} & \frac{\alpha_{i2}(s)}{\beta_{i2}(s)} e^{-d_{i2}s} \end{bmatrix} U(s), \quad i = 1, 2. \end{aligned}$$

Let the common denominator of G_{i1} and G_{i2} be $\beta_i^*(s)$. We have

$$\beta_i^*(s) Y_i(s) = \begin{bmatrix} \alpha_{i1}^*(s) e^{-d_{i1}s} & \alpha_{i2}^*(s) e^{-d_{i2}s} \end{bmatrix} U(s), \quad i = 1, 2.$$

The equivalent differential equations are

$$y_i^{(n_i)}(t) + \sum_{k=0}^{n_i - 1} a_{i,k} y_i^{(k)}(t) = \sum_{j=1}^{2} \sum_{k=0}^{m_{ij}} b_{ij,k} u_j^{(k)}(t - d_{ij}) + w_i(t), \quad i = 1, 2, \qquad (6.1)$$

where $w_i(t)$ account for the unknown disturbances and biases. Our task is to identify $a_{i,k}$, $b_{ij,k}$ and d_{ij} from some tests on the process. During the identification test, two separate sets of piecewise step signals are applied on two inputs at $t = 0$, respectively. The test signals under consideration are

$$u_1(t) = \sum_{k=0}^{K_1} h_{1,k}\mathbf{1}(t - t_{1,k}),$$

where $\mathbf{1}(t)$ is the unit step, $K_1 \geq 1$ and $t_{1,k}$, $k = 1, \ldots, K_1$ are the switching time instants of $u_1(t)$, and

$$u_2(t) = \sum_{k=0}^{K_2} h_{2,k}\mathbf{1}(t - t_{2,k}),$$

where $K_2 \geq 1$ and $t_{2,k}$, $k = 1, \ldots, K_2$ are the switching time instants of $u_2(t)$. Such forms of u_i, $i = 1, 2$, cover many types of test signals such as steps, rectangular pulses, rectangular doublet pulses, PRBS signals and the relay feedback output.

To eliminate those derivatives in (6.1), we introduce a multiple integration operator,

$$P_j f(t) := \int_0^t \int_0^{\delta_{j-1}} \cdots \int_0^{\delta_1} f(\delta_0) d\delta_0 d\delta_1 \cdots d\delta_{j-1}, \quad j \geq 1. \tag{6.2}$$

Integrating (6.1) with (6.2) n_i times yields

$$P_{n_i} y_i^{(n_i)}(t) + \sum_{k=0}^{n_i-1} a_{i,k} P_{n_i} y_i^{(k)}(t) = \sum_{k=0}^{m_{i1}} b_{i1,k} P_{n_i} u_1^{(k)}(t - d_{i1}) + \sum_{k=0}^{m_{i2}} b_{i2,k} P_{n_i} u_2^{(k)}(t - d_{i2}) + P_{n_i} w_i(t). \tag{6.3}$$

Its left-hand side is

$$P_{n_i} y_i^{(n)}(t) + \sum_{k=0}^{n_i-1} a_{i,k} P_{n_i} y^{(k)}(t) = y_i(t) + \sum_{k=0}^{n_i-1} a_{i,k} P_{n_i-k} y(t) + \sum_{k=0}^{n_i-1} \lambda_{i,k} t^k, \tag{6.4}$$

where the last term corresponds to the initial conditions of the output. In the right-hand side, it follows that

$$P_{n_i} u_1^{(p)}(t - d_{i1}) = \sum_{k=0}^{K_1} \frac{h_{1,k}(t - t_{1,k} - d_{i1})^{n_i - p}}{(n_i - p)!} \mathbf{1}(t - t_{1,k} - d_{i1}), \; p = 0, 1, \ldots, m_{i1},$$

and

$$P_{n_i} u_2^{(p)}(t - d_{i2}) = \sum_{k=0}^{K_2} \frac{h_{2,k}(t - t_{2,k} - d_{i2})^{n_i - p}}{(n_i - p)!} \mathbf{1}(t - t_{2,k} - d_{i2}), \; p = 0, 1, \ldots, m_{i2}.$$

Suppose that there holds

$$P_{n_i}w_i(t) = \sum_{k=0}^{q_i} \nu_{i,k}t^k, \tag{6.5}$$

where q_i is an integer. Equation (6.5) covers a wide range of disturbances (Hwang and Lai, 2004) with its simplest as the static disturbance for which $w_i(t) = c\mathbf{1}(t)$, $P_{n_i}w_i(t) = \frac{ct^{n_i}}{n_i!}$ and $q_i = n_i$.

Equation (6.3) is then cast into the following regression linear in a new parameterization:

$$\phi_i^T(t)\theta_i = \gamma_i(t), \tag{6.6}$$

where $\gamma_i(t) = y_i(t)$,

$$\phi_i(t) = \begin{bmatrix} -P_1 y_i(t) \\ \vdots \\ -P_{n_i} y_i(t) \\ \sum_{k=0}^{K_1} h_{1,k}\mathbf{1}(t-t_{1,k}-d_{i1}) \\ \sum_{k=0}^{K_1} h_{1,k}(t-t_{1,k})\mathbf{1}(t-t_{1,k}-d_{i1}) \\ \vdots \\ \sum_{k=0}^{K_1} h_{1,k}(t-t_{1,k})^{n_i}\mathbf{1}(t-t_{1,k}-d_{i1}) \\ \sum_{k=0}^{K_2} h_{2,k}\mathbf{1}(t-t_{2,k}-d_{i2}) \\ \sum_{k=0}^{K_2} h_{2,k}(t-t_{2,k})\mathbf{1}(t-t_{2,k}-d_{i2}) \\ \vdots \\ \sum_{k=0}^{K_2} h_{2,k}(t-t_{2,k})^{n_i}\mathbf{1}(t-t_{2,k}-d_{i2}) \\ 1 \\ t \\ \vdots \\ t^{q_i} \end{bmatrix}, \quad \theta_i = \begin{bmatrix} \theta_{i,1} \\ \vdots \\ \theta_{i,n_i} \\ \theta_{i,n_i+1} \\ \theta_{i,n_i+2} \\ \vdots \\ \theta_{i,2n_i+1} \\ \theta_{i,2n_i+2} \\ \theta_{i,2n_i+3} \\ \vdots \\ \theta_{i,3n_i+2} \\ \theta_{i,3n_i+3} \\ \theta_{i,3n_i+4} \\ \vdots \\ \theta_{i,3n_i+3+q_i} \end{bmatrix}.$$

The first n_i elements in θ_i are the model parameter $a_{i,k}$:

$$\theta_{i,k} = a_{i,n_i-k}, \quad k = 1, \ldots, n_i. \tag{6.7}$$

$\theta_{i,k}$, $k = n_i + 1, \ldots, 2n_i + 1$ are functions of d_{i1} and $b_{i1,k}$, $k = 0, \ldots, m_{i1}$,

$$\theta_{i,k} = \sum_{p=max(n_i-m_{i1},k-n_i-1)}^{n_i} \frac{(-d_{i1})^{p-k+n_i+1}\ b_{i1,n_i-p}}{(p-k+n_i+1))!\ (k-n_i-1)!}, \quad k = n_i+1, \ldots, 2n_i+1. \tag{6.8}$$

$\theta_{i,k}$, $k = 2n_i + 2, \ldots, 3n_i + 2$, are functions of d_{i2} and $b_{i2,k}$, $k = 0, \ldots, m_{i2}$,

$$\theta_{i,k} = \sum_{p=max(n_i-m_{i2},k-2n_i-2)}^{n_i} \frac{(-d_{i2})^{p-k+2n_i+2}\ b_{i2,n_i-p}}{(p-k+2n_i+2)!\ (k-2n_i-2)!}, \quad k = 2n_i+2, \ldots, 3n_i+2. \tag{6.9}$$

$\theta_{i,k}$, $k = 3n_i + 3, \ldots, 3n_i + 3 + q_i$, account for the collective effects of the initial conditions and the disturbances. Note that all the elements in $\phi_i(t)$ should be mutually independent over the real number field to enable identifiability of the parameter vector, θ_i. This is not the case if $t_{1,k} = t_{2,k}$ for all k, for which $\sum_{k=0}^{K_1} h_{1,k}(t-t_{1,k})^p \mathbf{1}(t-t_{1,k}-d_{i1})$ and $\sum_{k=0}^{K_2} h_{2,k}(t-t_{2,k})^p \mathbf{1}(t-t_{2,k}-d_{i2})$, $p = 0, \ldots, n_i$, become dependent of each other. This should be avoided by the identification test design.

One invokes (6.6) for $t = t_0, \ldots, t_N$, to get

$$\Psi_i \theta_i = \Gamma_i, \tag{6.10}$$

where $\Psi_i = [\phi_i(t_0), \ldots, \phi(t_N)]^T$ and $\Gamma_i = [\gamma_i(t_0), \ldots, \gamma(t_N)]^T$. The ordinary least-squares method can be applied to find the solution

$$\hat{\theta}_i = \left(\Psi_i^T \Psi_i\right)^{-1} \Psi_i^T \Gamma_i.$$

In the presence of noise in the measurement of the process output, the instrumental variable (IV) method is adopted to guarantee the identification consistency. For our case, the instrumental variable $Z_i(t)$ is chosen as

$$Z_i(t) = \begin{bmatrix} \frac{1}{t^{n_i}} \\ \vdots \\ \frac{1}{t} \\ \sum_{k=0}^{K_1} h_{1,k}\mathbf{1}(t - t_{1,k} - d_{i1}) \\ \sum_{k=0}^{K_1} h_{1,k}(t - t_{1,k})\mathbf{1}(t - t_{1,k} - d_{i1}) \\ \vdots \\ \sum_{k=0}^{K_1} h_{1,k}(t - t_{1,k})^{n_i}\mathbf{1}(t - t_{1,k} - d_{i1}) \\ \sum_{k=0}^{K_2} h_{2,k}\mathbf{1}(t - t_{2,k} - d_{i2}) \\ \sum_{k=0}^{K_2} h_{2,k}(t - t_{2,k})\mathbf{1}(t - t_{2,k} - d_{i2}) \\ \vdots \\ \sum_{k=0}^{K_2} h_{2,k}(t - t_{2,k})^{n_i}\mathbf{1}(t - t_{2,k} - d_{i2}) \\ 1 \\ t \\ \vdots \\ t^{q_i} \end{bmatrix}.$$

It should be pointed out that for a selected t, the value of some elements of ϕ_i depend on d_{i1}, d_{i2}, which are to be identified and unknown. It is possible to estimate a range of d_{i1} and d_{i2} (Hwang and Lai, 2004). In many engineering applications, one can have simple reliable and probably conservative estimation of the range of time delay from knowledge of the process. For example, the range of transportation delay due to a long pipe can be easily estimated based on the pipe length and fluid speed range. Besides, one may start with a rough estimated delay range and use the proposed method to find $\hat{d}_{i1}$ and $\hat{d}_{i2}$, estimates of d_{i1} and d_{i2}. Then with $\hat{d}_{i1}$ and $\hat{d}_{i2}$, one retunes the ranges of time delays and apply the proposed method again to achieve a better estimation. Let d_{i1} and d_{i2} be in the ranges of $\left[\underline{d}_{i1}, \overline{d}_{i1}\right]$ and $\left[\underline{d}_{i2}, \overline{d}_{i2}\right]$, respectively. Define

$$\hat{T}_1 = \bigcup_{k=0}^{K_1-1} \left\{t | t_{1,k} + \overline{d}_{i1} \le t < t_{1,k+1} + \underline{d}_{i1}\right\} \bigcup \left\{t | (t_{1,K_1} + \overline{d}_{i1} \le t \le T_{end}\right\},$$

and

$$\hat{T}_2 = \bigcup_{k=0}^{K_2-1} \left\{ t | t_{2,k} + \overline{d}_{i2} \leq t < t_{2,k+1} + \underline{d}_{i2} \right\} \bigcup \left\{ t | t_{2,K_2} + \overline{d}_{i2} \leq t \leq T_{end} \right\},$$

where T_{end} is the ending time of the identification test. Then, t should be taken in the set of

$$T = \hat{T}_1 \bigcap \hat{T}_2,$$

to apply (6.10). There is no need to solve the estimation equation for each of the delay within the estimated range. Once the estimate ranges of time delays are given, time delays can be obtained by solving some polynomial equations without iteration. Then, all other parameters than delays are determined accordingly.

Once θ_i is estimated by applying the least-squares method or IV method, the model parameters can be recovered. From (6.8) for $k = 2n_i + 1 - m_{i1}, \ldots, 2n_i + 1$, $b_{i1,k}$, $k = 0, \ldots, m_{i1}$ can be expressed as the functions of d_{i1} and $\theta_{i,k}$,

$$b_{i1,k} = \sum_{p=0}^{k} \frac{(n_i - k + p)! \; \theta_{i,2n_i+1-k+p} \; d_{i1}^p}{p!}, \quad k = 0, 1, \ldots, m_{i1}. \tag{6.11}$$

Substitute $b_{i1,k}$, $k = 0, \ldots, m_{i1}$ into (6.8) for $k = 2n_i - m_{i1}$, and we have

$$\sum_{k=0}^{m_{i1}+1} \frac{(n_i - m_{i1} - 1 + k)! \; \theta_{i,2n_i-m_{i1}+k} \; d_{i1}^k}{k!} = 0. \tag{6.12}$$

Equation (6.12) is solved to get d_{i1} and $b_{i1,k}$, $k = 0, \ldots, m_{i1}$ are then obtained from (6.11). Similarly, we can find d_{i2} from the following algebraic equations:

$$\sum_{k=0}^{m_{i2}+1} \frac{(n_i - m_{i2} - 1 + k)! \; \theta_{i,3n_i+1-m_{i2}+k} \; d_{i2}^k}{k!} = 0.$$

$b_{i2,k}$, $k = 0, \ldots, m_{i2}$, are then calculated as

$$b_{i2,k} = \sum_{p=0}^{k} \frac{(n_i - k + p)! \; \theta_{i,3n_i+2-k+p} \; d_{i2}^p}{p!}, \quad k = 0, 1, \ldots, m_{i2}.$$

The proposed method will lead to $m_{ij} + 1$ estimates for d_{ij}, just like Wang and Zhang (2001) and Hwang and Lai (2004). By inspecting the lag between the input and output signals, the selection can be made simply. The selection can be also made by virtue of the consistency between various sets of $b_{ij,k}$ and d_{ij} and those ignored relations (Hwang and Lai, 2004).

6.3 Simulation studies

Example 6.1. Consider the well-known Wood-Berry binary distillation column plant:

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}.$$

The equivalent differential equations are

$$\begin{aligned} 350.7y_1^{(2)}(t) + 37.7y_1^{(1)}(t) &+ y_1(t) = 268.8u_1^{(1)}(t-1) + 12.8u_1(t-1) \\ &- 315.63u_2^{(1)}(t-3) - 18.9u_2(t-3) + \hat{w}_1(t), \end{aligned} \tag{6.13}$$

and

$$\begin{aligned} 156.96y_2^{(2)}(t) + 25.3y_2^{(1)}(t) &+ y_2(t) = 95.04u_1^{(1)}(t-7) + 6.6u_1(t-7) \\ &- 211.46u_2^{(1)}(t-3) - 19.4u_2(t-3) + \hat{w}_2(t). \end{aligned} \tag{6.14}$$

Case A. Assume that $\hat{w}_1(t) = \mathbf{1}(t)$ and $\hat{w}_2(t) = 0.5\mathbf{1}(t)$ and the identification test starts from nonzero initial conditions: $y_1(0) = -1$, $y_1^{(1)}(0) = 1$, $y_2(0) = 0.5$ and $y_2^{(1)}(0) = 2$. The test signals, $u_1(t)$ and $u_2(t)$, are both pulse signals,

$$u_1(t) = \mathbf{1}(t) - \mathbf{1}(t-60),$$

and

$$u_2(t) = \mathbf{1}(t) - \mathbf{1}(t-30).$$

The process inputs and outputs are shown in Figure 6.1 and the sampling interval is 0.02. Suppose that $0 \le d_{11} \le 2$, $0 \le d_{12} \le 6$. It leads to

$$\begin{aligned} \hat{T}_1 &= \{t|2 \le t < 60,\ or\ 62 \le t < 100\}, \\ \hat{T}_2 &= \{t|6 \le t < 30,\ or\ 36 \le t < 100\}, \\ T &= \hat{T}_1 \bigcap \hat{T}_2 \\ &= \{t|6 \le t < 30,\ or\ 36 \le t < 60,\ or\ 62 \le t < 100\}. \end{aligned}$$

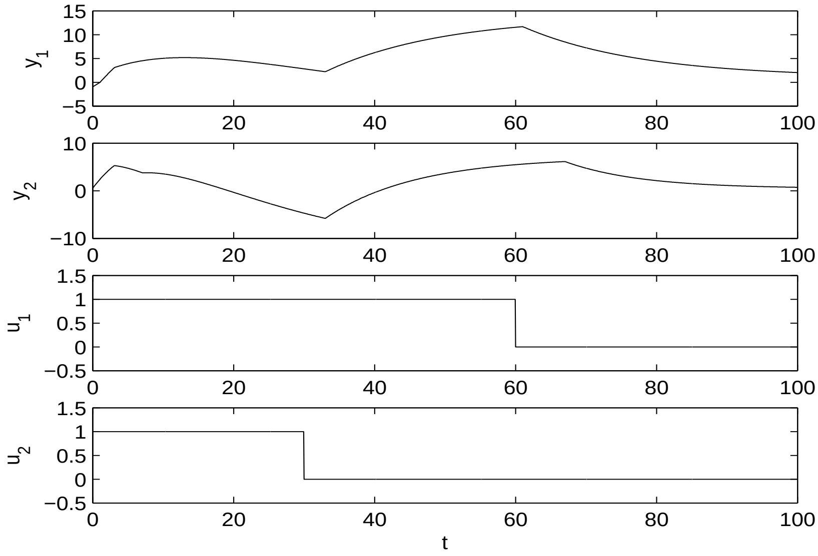

Figure 6.1. Identification test of Example 6.1.

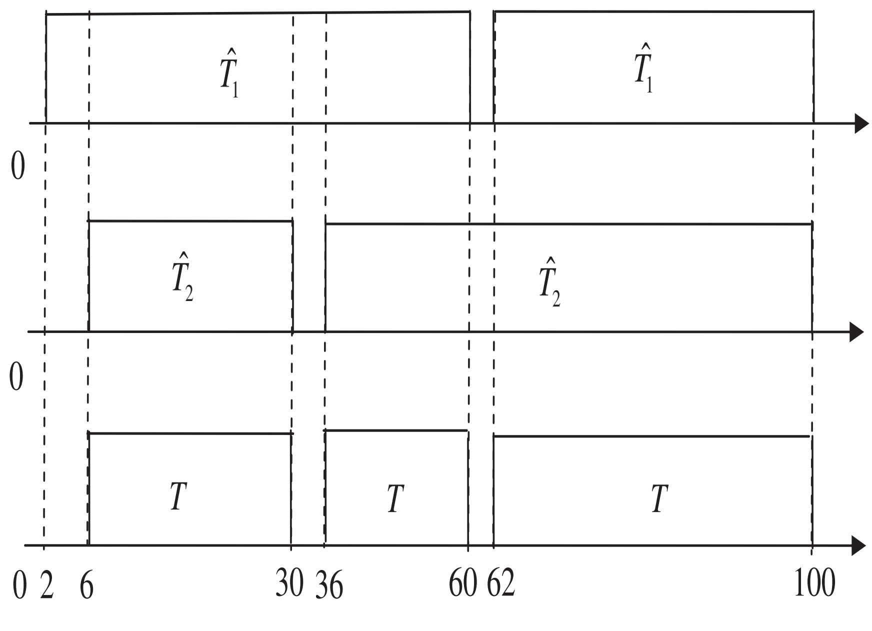

Figure 6.2. Calculation of T.

$\hat{T}_1$ and $\hat{T}_2$ have some elements in common and these elements are included in T. In other word, the elements in T are members of both $\hat{T}_1$ and $\hat{T}_2$. This can be seen clearly in Figure 6.2. Choose $t = t_0, \ldots, t_N$ in T, $n_1 = 2$, $m_{11} = m_{12} = 1$ and $q_1 = 2$. The proposed method leads to two estimates for d_{11}: one is -39.05 and the other is 1.02. The time delay must be positive so that we choose $\hat{d}_{11} = 1.02$. The proposed method also leads to two estimates for d_{12}: -29.11 and 3.02. For the same reason, we choose $\hat{d}_{12} = 3.02$. The first sub-process is then obtained as:

$$\begin{aligned} y_1^{(2)}(t) + 0.1079y_1^{(1)}(t) + 0.002867y_1(t) &= 0.7715u_1^{(1)}(t-1.02) + 0.0367u_1(t-1.02) \\ &- 0.9062u_2^{(1)}(t-3.02) - 0.05418u_2(t-3.02), \end{aligned}$$

with $\hat{G}_{11} = \frac{0.7715s+0.0367}{s^2+0.1079s+0.002867}e^{-1.02s}$ and $\hat{G}_{12} = \frac{-0.9062s-0.05418}{s^2+0.1079s+0.002867}e^{-3.02s}$. Suppose that $0 \leq d_{21} \leq 14$, $0 \leq d_{22} \leq 6$. The proposed method with $n_2 = 2$, $m_{21} = m_{22} = 1$ and $q_2 = 2$ leads to the second sub-process as:

$$\begin{aligned} y_2^{(2)}(t) + 0.162y_2^{(1)}(t) + 0.006423y_2(t) &= 0.6115u_1^{(1)}(t-7.02) + 0.04239u_1(t-7.02) \\ &- 1.361u_2^{(1)}(t-3.02) - 0.1246u_2(t-3.02), \end{aligned}$$

with $\hat{G}_{21} = \frac{0.6115s+0.04239}{s^2+0.162s+0.006423}e^{-7.02s}$ and $\hat{G}_{22} = \frac{-1.361s-0.1246}{s^2+0.162s+0.006423}e^{-3.02s}$. The identification error, $ERR = \{ERR_{ij}\}$, is measured by the worst case error,

$$ERR_{ij} = max \left| \frac{\hat{G}_{ij}(j\omega_k) - G_{ij}(j\omega_k)}{G_{ij}(j\omega_k)} \right|, k = 1, \cdots, M, \tag{6.15}$$

where $\hat{G}_{ij}(j\omega_k)$ and $G_{ij}(j\omega_i)$ are the estimated frequency response and the actual ones. The Nyqusit curve for a phase ranging from 0 to $-\pi$ is considered, because this part is the most significant for control design. For this example, the identification error is

$$ERR = \begin{bmatrix} 4.04\% & 1.46\% \\ 0.95\% & 1.59\% \end{bmatrix}.$$

In real applications, numerical integration is employed to calculate the multiple integration of the output and this introduces errors. Better identification results can be obtain by sampling the process response with a small sampling interval. If the sampling interval is 0.2, the proposed method leads to the identification error as

$$ERR = \begin{bmatrix} 16.39\% & 5.90\% \\ 3.22\% & 6.42\% \end{bmatrix}.$$

In this case, the identification result is still acceptable. If the sampling interval is chosen as 1 and the identification error is obtained as

$$ERR = \begin{bmatrix} 83.31\% & 30.98\% \\ 19.53\% & 34.53\% \end{bmatrix}.$$

The identification error is very large. From these simulations, one can find that small sampling interval leads to good identification results. Generally, chemical processes have slow response. With the development of computer technologies, the sampling interval can be set very small and enough data can be obtained easily for use in process identification.

Case B. This is the same as Case A expect that process outputs are subject to changing disturbances, where $\hat{w}_1(t)$ and $\hat{w}_2(t)$ are simulated by letting $\mathbf{1}(t)$ pass through the transfer functions of $\frac{1}{15s+1}$ and $\frac{-3}{20s+1}$, respectively. The proposed method, with $n_1 = n_2 = 2$, $m_{11} = m_{12} = m_{21} = m_{22} = 1$ and $q_1 = q_2 = 3$, leads to

$$\begin{aligned} y_1^{(2)}(t) + 0.1103y_1^{(1)}(t) + 0.003y_1(t) &= 0.7718u_1^{(1)}(t-1.03) + 0.03851u_1(t-1.03) \\ &- 0.9058u_2^{(1)}(t-3.03) - 0.05637u_2(t-3.03), \end{aligned}$$

with $\hat{G}_{11} = \frac{0.7718s+0.03851}{s^2+0.1103s+0.003}e^{-1.03s}$ and $\hat{G}_{12} = \frac{-0.9058s-0.05637}{s^2+0.1103s+0.003}e^{-3.03s}$, and

$$\begin{aligned} y_2^{(2)}(t) + 0.1528y_2^{(1)}(t) + 0.00582y_2(t) &= 0.6054u_1^{(1)}(t-7) + 0.0376u_1(t-7) \\ &- 1.36u_2^{(1)}(t-3.03) - 0.1119u_2(t-3.03), \end{aligned}$$

with $\hat{G}_{21} = \frac{0.6054s+0.0376}{s^2+0.1528s+0.00582}e^{-7s}$ and $\hat{G}_{22} = \frac{-1.36s-0.1119}{s^2+0.1528s+0.00582}e^{-3.03s}$. The identification error is

$$ERR = \begin{bmatrix} 4.15\% & 1.45\% \\ 1.91\% & 1.61\% \end{bmatrix}.$$

Case C. This is the same as Case B except that a white noise is added to corrupt the outputs. The noise-to-signal ratio defined by

$$NSR = \frac{mean(abs(noise))}{mean(abs(signal))},$$

(denoted N_1) and

$$NSR = \frac{variance(noise)}{variance(signal)},$$

(denoted N_2) are used to represent a noise level. Let the outputs be corrupted with noise of $N_1 = 15\%$, 25% and 40% or $N_2 = 3\%$, 7% and 18%, respectively. Suppose that the estimated ranges of time delays are $0.5 \leq d_{11} \leq 1.5$, $2 \leq d_{12} \leq 4$, $6 \leq d_{21} \leq 9$ and $2 \leq d_{22} \leq 4$. The identified parameters are expressed as the mean and standard deviation of each estimate from 20 noisy simulations and shown in Table 6.1.

In case of noise, we may also start with rough estimated delay ranges given in Case A and use the proposed method to find $\hat{d}_{ij}$, estimates of d_{ij}. Then with $\hat{d}_{ij}$, we retunes the ranges of time delays and apply the proposed method again to achieve a better estimation. For example, in case of $N_1 = 15\%$, one identification test is applied. The proposed method, with $0 \leq d_{11} \leq 2$, $0 \leq d_{12} \leq 6$, $0 \leq d_{21} \leq 14$ and $0 \leq d_{22} \leq 6$, leads to $\hat{d}_{11} = 1.08$, $\hat{d}_{12} = 2.88$, $\hat{d}_{21} = 7.23$ and $\hat{d}_{22} = 3.05$, with the identification error of

$$ERR = \begin{bmatrix} 13.50\% & 9.66\% \\ 5.62\% & 6.42\% \end{bmatrix}.$$

We then retunes the ranges of the time delays as the above and the proposed method leads to a smaller identification error

$$ERR = \begin{bmatrix} 6.85\% & 10.07\% \\ 5.63\% & 3.80\% \end{bmatrix}.$$

Table 6.1. Estimated model parameters of Example 6.1

	$N_1 = 15\%$ ($N_2 = 3\%$)	$N_1 = 25\%$ ($N_2 = 7\%$)	$N_1 = 40\%$ ($N_2 = 18\%$)
$\hat{a}_{1,1}$	0.1101 ± 0.0076	0.1116 ± 0.0135	0.1126 ± 0.0225
$\hat{a}_{1,0}$	0.0029 ± 0.0007	0.0031 ± 0.0008	0.0032 ± 0.0013
$\hat{b}_{11,1}$	0.7746 ± 0.0235	0.7695 ± 0.0249	0.7284 ± 0.1765
$\hat{b}_{11,0}$	0.0389 ± 0.0064	0.0393 ± 0.0106	0.0395 ± 0.0187
$\hat{d}_{11}$	1.0232 ± 0.0801	1.0523 ± 0.1489	0.9909 ± 0.3228
$\hat{b}_{12,1}$	-0.9117 ± 0.0342	-0.9045 ± 0.0334	-0.9046 ± 0.0555
$\hat{b}_{12,0}$	-0.0549 ± 0.0076	-0.0573 ± 0.0086	-0.0579 ± 0.0143
$\hat{d}_{12}$	3.0499 ± 0.1254	3.0421 ± 0.1440	3.0561 ± 0.2381
$\hat{a}_{2,1}$	0.1554 ± 0.0097	0.1581 ± 0.0143	0.1607 ± 0.0242
$\hat{a}_{2,0}$	0.0060 ± 0.0005	0.0061 ± 0.0009	0.0063 ± 0.0015
$\hat{b}_{21,1}$	0.6066 ± 0.0345	0.6127 ± 0.0445	0.6126 ± 0.0753
$\hat{b}_{21,0}$	0.0403 ± 0.0056	0.0413 ± 0.0078	0.0429 ± 0.0133
$\hat{d}_{21}$	6.9337 ± 0.2134	6.9397 ± 0.2045	6.9233 ± 0.3372
$\hat{b}_{22,1}$	-1.3642 ± 0.0375	-1.3663 ± 0.0486	-1.3620 ± 0.0835
$\hat{b}_{22,0}$	-0.1130 ± 0.0096	-0.1156 ± 0.0122	-0.1180 ± 0.0206
$\hat{d}_{22}$	3.0548 ± 0.0841	3.0678 ± 0.1103	3.0796 ± 0.1857

Example 6.2. Consider a TITO system,

$$G(s) = \begin{bmatrix} \frac{2e^{-2.7s}}{5s+1} & \frac{0.5e^{-3s}}{2s+1} \\ \frac{0.4e^{-2.5s}}{6s+1} & \frac{2.2e^{-3.8s}}{10s+1} \end{bmatrix}.$$

A closed-loop relay feedback is applied on this example. The relay feedback system is shown in Figure 6.3. The relay unit is described as

$$u(t) = \begin{cases} u_+, \ if \ e(t) > \varepsilon_+, \ or \ e(t) \geq \varepsilon_- \ and \ u(t_-) = u_+, \\ u_-, \ if \ e(t) < \varepsilon_-, \ or \ e(t) \leq \varepsilon_+ \ and \ u(t_-) = u_-, \end{cases} \quad (6.16)$$

where $e(t)$ and $u(t)$ are the relay input and output, respectively. The relay experiment is applied at $t = 0$ with $u_+ = 1$, $u_- = -1$, $\varepsilon_+ = 0.8$ and $\varepsilon_- = -0.8$ under zero initial conditions and nonzero static disturbances of $\hat{w}_1 = \hat{w}_2 = 0.5\mathbf{1}(t)$. The process inputs and outputs are shown in Figure 6.4 and the sampling interval is 0.02. Suppose that $2 \leq d_{11} \leq 3$, $2 \leq d_{12} \leq 3$, $2 \leq d_{21} \leq 3$ and $3 \leq d_{22} \leq 4$. The proposed method, with $n_1 = n_2 = 2$, $m_{11} = m_{12} = m_{21} = m_{22} = 1$ and $q_1 = q_2 = 2$, leads to

$$\hat{G}(s) = \begin{bmatrix} \frac{0.4067s+0.1947}{s^2+0.6886s+0.09707}e^{-2.7s} & \frac{0.2489s+0.04814}{s^2+0.6886s+0.09707}e^{-2.99s} \\ \frac{0.06743s+0.007456}{s^2+0.2814s+0.01911}e^{-2.52s} & \frac{0.2177s+0.04014}{s^2+0.2814s+0.01911}e^{-3.8s} \end{bmatrix},$$

with the identification error as follows

$$ERR = \begin{bmatrix} 1.21\% & 0.82\% \\ 2.45\% & 4.52\% \end{bmatrix}.$$

6.4 General MIMO processes

The TITO identification method is now extended to a general MIMO processe. Consider a process with l inputs and m outputs,

$$Y(s) = G(s)U(s),$$

where $Y(s) = [Y_1(s) \ \cdots \ Y_i(s) \ \cdots \ Y_l(s)]^T$ is the output vector, $U(s) = [U_1(s) \ \cdots \ U_j(s) \ \cdots \ U_m(s)]$ is the input vector, and $G(s) = \{G_{ij}(s)\} = \left\{\frac{\alpha_{ij}(s)}{\beta_{ij}(s)}e^{-d_{ij}s}\right\}$, with $i = 1, \ldots, l$ and

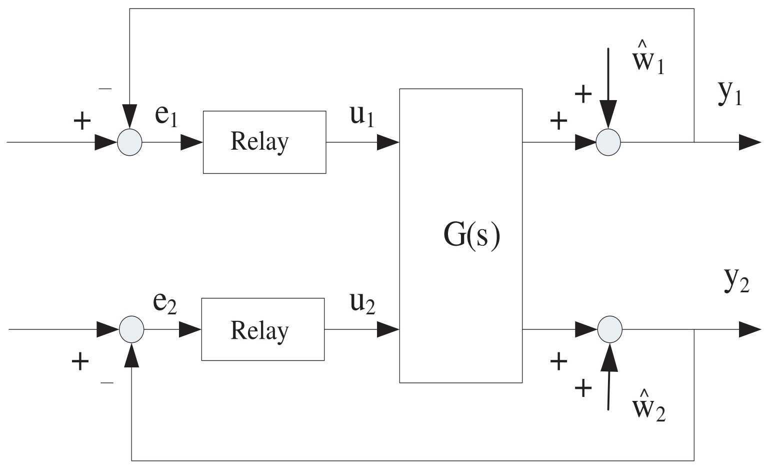

Figure 6.3. Relay feedback experiment.

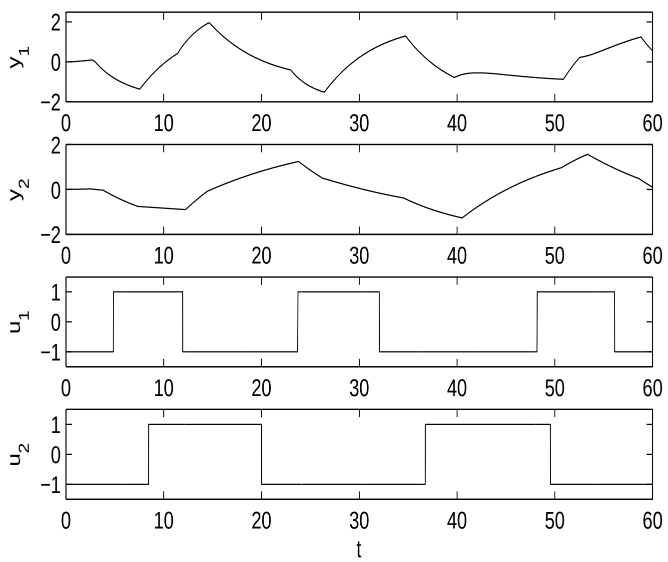

Figure 6.4. Identification test of Example 6.2.

$j = 1, \ldots, m$, is the process transfer function matrix. The given MIMO process may be decomposed into l sub-processes, which can be described as

$$\begin{aligned} Y_i(s) &= \begin{bmatrix} G_{i1}(s) & \cdots & G_{ij}(s) & \cdots G_{im}(s) \end{bmatrix} U(s) \\ &= \begin{bmatrix} \frac{\alpha_{i1}(s)}{\beta_{i1}(s)} e^{-d_{i1}s} & \cdots & \frac{\alpha_{ij}(s)}{\beta_{ij}(s)} e^{-d_{ij}s} & \cdots & \frac{\alpha_{im}(s)}{\beta_{im}(s)} e^{-d_{im}s} \end{bmatrix} U(s), \quad i = 1, \ldots, l. \end{aligned}$$

Let the common denominator of all $G_{ij},\ j = 1, \ldots, m$ be $\beta_i^*(s)$. We have

$$\beta_i^*(s) Y_i(s) = \begin{bmatrix} \alpha_{i1}^*(s) e^{-d_{i1}s} & \cdots & \alpha_{ij}^*(s) e^{-d_{ij}s} & \cdots & \alpha_{im}^*(s) e^{-d_{im}s} \end{bmatrix} U(s), \quad i = 1, \ldots, l.$$

The equivalent differential equations are

$$y_i^{(n_i)}(t) + \sum_{k=0}^{n_i-1} a_{i,k} y_i^{(k)}(t) = \sum_{j=1}^{m} \sum_{k=0}^{m_{ij}} b_{ij,k} u_j^{(k)}(t - d_{ij}) + w_i(t), \quad i = 1, \ldots, l. \tag{6.17}$$

The inputs under considerations are

$$u_j(t) = \sum_{k=0}^{K_i} h_{j,k} \mathbf{1}(t - t_{j,k}), \quad j = 1, \ldots, m,$$

where $t_{j,k}$ is the kth switch instant of $u_j(t)$.

Integrating (6.17) with (6.2) n_i times yields

$$P_{n_i} y_i^{(n_i)}(t) + \sum_{k=0}^{n_i-1} a_{i,k} P_{n_i} y_i^{(k)}(t) = \sum_{j=0}^{m} \sum_{k=0}^{m_{ij}} b_{ij,k} P_{n_i} u_j^{(k)}(t - d_{ij}) + P_{n_i} w_i(t). \tag{6.18}$$

The left-hand side is (6.4) again. For the right-hand side, it follows that

$$P_{n_i} u_j^{(p)}(t - d_{ij}) = \sum_{k=0}^{K_j} \frac{h_{j,k}(t - t_{j,k} - d_{ij})^{n_i - p}}{(n_i - p)!} \mathbf{1}(t - t_{j,k} - d_{ij}),\ p = 0, 1, \ldots, m_{ij}.$$

Equation (6.18) can be rearranged as

$$\phi_i^T(t) \theta_i = \gamma_i(t), \tag{6.19}$$

where $\gamma_i(t) = y_i(t)$,

$$\phi_i(t) = \begin{bmatrix} -P_1 y_i(t) \\ \vdots \\ -P_{n_i} y_i(t) \\ \sum_{k=0}^{K_1} h_{1,k}\mathbf{1}(t-t_{1,k}-d_{i1}) \\ \sum_{k=0}^{K_1} h_{1,k}(t-t_{1,k})\mathbf{1}(t-t_{1,k}-d_{i1}) \\ \vdots \\ \sum_{k=0}^{K_1} h_{1,k}(t-t_{1,k})^{n_i}\mathbf{1}(t-t_{1,k}-d_{i1}) \\ \vdots \\ \sum_{k=0}^{K_m} h_{m,k}\mathbf{1}(t-t_{m,k}-d_{im}) \\ \sum_{k=0}^{K_m} h_{m,k}(t-t_{m,k})\mathbf{1}(t-t_{m,k}-d_{im}) \\ \vdots \\ \sum_{k=0}^{K_m} h_{m,k}(t-t_{m,k})^{n_i}\mathbf{1}(t-t_{m,k}-d_{im}) \\ 1 \\ t \\ \vdots \\ t^{q_i} \end{bmatrix}, \quad \theta_i = \begin{bmatrix} \theta i,1 \\ \vdots \\ \theta i,n_i \\ \theta_{i,n_i+1} \\ \theta_{i,n_i+2} \\ \vdots \\ \theta_{i,2n_i+1} \\ \vdots \\ \theta_{i,m(n_i+1)} \\ \theta_{i,m(n_i+1)+1} \\ \vdots \\ \theta_{i,m(n_i+1)+n_i} \\ \theta_{i,(m+1)(n_i+1)} \\ \theta_{i,(m+1)(n_i+1)+1} \\ \vdots \\ \theta_{i,(m+1)(n_i+1)+q_i} \end{bmatrix}.$$

Note that the first n_i elements of θ_i are the same as (6.7). $\theta_{i,k}$, $k = j(n_i+1)+1,\dots,j(n_i+1)+n_i$, and $j = 1,\dots,m$, are combinations of the model parameters $b_{ij,k}$, $k = 0,\dots,m_{ij}$ and d_{ij}, and are given by

$$\theta_{i,k} = \sum_{p=max(n_i-m_{ij},k-j(n_i+1))}^{n_i} \frac{(-d_{ij})^{p-k+j(n_i+1)}\ b_{ij,n_i-p}}{(p-k+j(n_i+1))!\ (k-j(n_i+1))!}, \quad k = j(n_i+1),\dots,j(n_i+1)+n_i. \tag{6.20}$$

$\theta_{i,k}$, $k = (m+1)(n_i+1),\ \dots,\ (m+1)(n_i+1)+q_i$ account for the effects of the aforementioned nonzero conditions and the disturbances.

Suppose that d_{ij}, $j = 1,\dots,m$ are in the ranges of $\left[\underline{d}_{ij}, \overline{d}_{ij}\right]$. Define

$$\hat{T}_j = \bigcup_{k=0}^{K_j-1} \left\{t | t_{j,k} + \overline{d}_{ij} \le t < t_{j,k+1} + \underline{d}_{ij}\right\} \bigcup \left\{t | (t_{j,K_j} + \overline{d}_{ij} \le t \le T_{end}\right\}, \quad j = 1,\dots,m.$$

Then, t should be taken in the set of

$$T = \bigcap_{j=1}^{m} \hat{T}_j.$$

One invokes (6.19) for t in T with $t = t_0, t_1, \ldots, t_N$, and they give

$$\Psi_i \theta_i = \Gamma_i, \tag{6.21}$$

where $\Psi_i = [\phi_i(t_0), \ldots, \phi_i(t_N)]^T$ and $\Gamma_i = [\gamma_i(t_0), \ldots, \gamma_i(t_N)]^T$. The ordinary least-squares method can be applied to find the solution; in the presence of noise in the measurement of the process output, the instrumental variable (IV) method is adopted to guarantee the identification consistency. Once θ_i is estimated by applying the least-squares method or IV method, the model parameters can be recovered. We can recover d_{ij} from $\theta_{i,k}$, $k = j(n_i+1), \ldots, j(n_i+1)+n_i$, using the following algebraic equations:

$$\sum_{k=0}^{m_{ij}+1} \frac{(n_i - m_{ij} - 1 + k)!\ \theta_{i,j(n_i+1)+n_i-1-m_{ij}+k}\ d_{ij}^k}{k!} = 0,\ \ i = 1, \ldots, l,\ and\, j = 1, \ldots, m.$$

Once d_{ij} are obtained, the parameter $b_{ij,k}$ are then calculated as

$$b_{ij,k} = \sum_{p=0}^{k} \frac{(n_i - k + p)!\ \theta_{i,j(n_i+1)+n_i-k+p}\ d_{ij}^p}{p!},\ \ k = 0, 1, \ldots, m_{ij},\ i = 1, \ldots, l,\ and\, j = 1, \ldots, m.$$

Example 6.3. Consider a system in Vasnani (1995)

$$G(s) = \begin{bmatrix} \frac{119e^{-5s}}{21.7s+1} & \frac{40e^{-5s}}{337s+1} & \frac{-2.1e^{-5s}}{10s+1} \\ \frac{77e^{-5s}}{50s+1} & \frac{76.7e^{-3s}}{28s+1} & \frac{-5e^{-5s}}{10s+1} \\ \frac{93e^{-5s}}{50s+1} & \frac{-36.7e^{-5s}}{166s+1} & \frac{-103.3e^{-4s}}{23s+1} \end{bmatrix}.$$

The equivalent differential equations are

$$\begin{aligned} 73129y_1^{(3)}(t) + 10900y_1^{(2)}(t) + 368.7y_1^{(1)}(t) + y_1(t) &= 401030u_1^{(2)}(t-5) + 41293u_1^{(2)}(t-5) + 119u_1(t-5) \\ &+ 8680u_2^{(2)}(t-5) + 1268u_2^{(2)}(t-5) + 40u_2(t-5) \\ &- 15357u_3^{(2)}(t-5) - 753.27u_3^{(2)}(t-5) - 2.1u_3(t-5) + \hat{w}_1(t), \end{aligned}$$

$$\begin{aligned} 14000y_2^{(3)}(t) + 2180y_2^{(2)}(t) + 88y_2^{(1)}(t) + y_2(t) &= 21560u_1^{(2)}(t-5) + 2926u_1^{(2)}(t-5) + 77u_1(t-5) \\ &+ 38350u_2^{(2)}(t-3) + 4602u_1^{(2)}(t-3) + 76.7u_2(t-3) \\ &- 7000u_3^{(2)}(t-5) - 390u_1^{(2)}(t-5) - 5u_2(t-5) + \hat{w}_2(t), \end{aligned}$$

and

$$\begin{aligned} 190900y_3^{(3)}(t) + 13268y_3^{(2)}(t) + 239y_3^{(1)}(t) + y_3(t) &= 355074u_1^{(2)}(t-5) + 17577u_1^{(2)}(t-5) + 93u_1(t-5) \\ &- 42205u_2^{(2)}(t-5) - 2679.1u_1^{(2)}(t-5) - 36.7u_2(t-5) \\ &- 857390u_3^{(2)}(t-4) - 22313u_1^{(2)}(t-4) - 103.3u_2(t-4) + \hat{w}_3(t). \end{aligned}$$

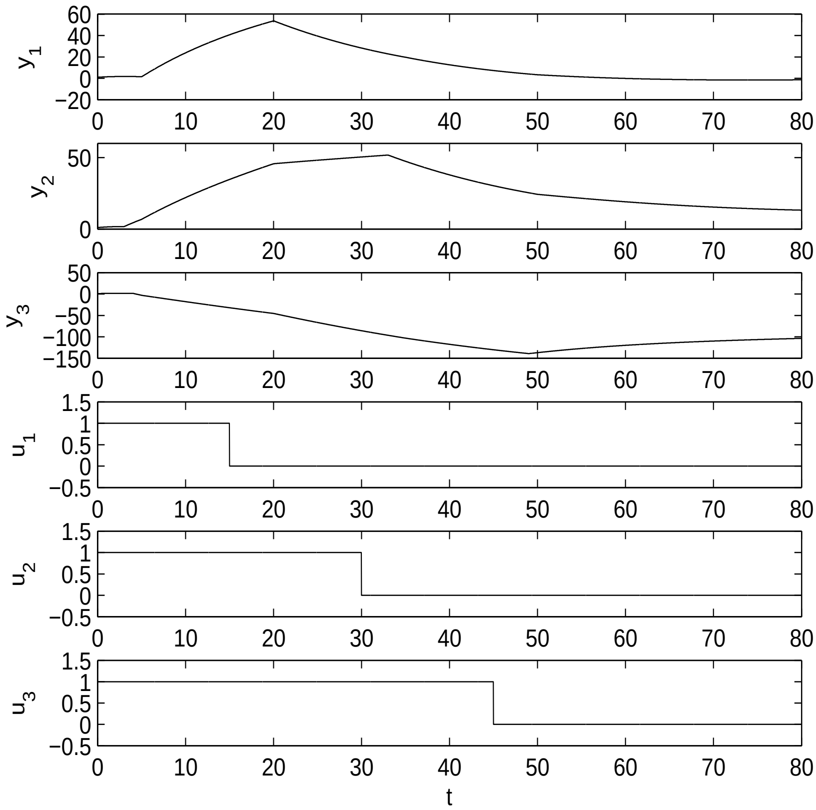

Figure 6.5. Identification test of Example 6.3.

Suppose that $\hat{w}_1(t) = 100\ \mathbf{1}(t)$, $\hat{w}_2(t) = 20\ \mathbf{1}(t)$ and $\hat{w}_3(t) = 100\ \mathbf{1}(t)$ and the identification test starts from nonzero initial conditions: $y_1(0) = y_2(0) = y_3(0) = 1$, $y_1^{(1)}(0) = y_2^{(1)}(0) = y_3^{(1)}(0) = 0.5$ and $y_1^{(2)}(0) = y_2^{(2)}(0) = y_3^{(2)}(0) = -0.2$. The process inputs and outputs are shown in Figure 6.5.

Let $0 < d_{11} < 7$, $0 < d_{12} < 7$, $1 < d_{13} < 6$, $0 < d_{21} < 7$, $0 < d_{22} < 5$, $0 < d_{23} < 6$, $2 < d_{31} < 7$, $1 < d_{32} < 7$ and $0 < d_{33} < 7$. The proposed method with $n_i = 3$ $m_{ij} = 2$ and $q_i = 3$, where $i = 1, 2, 3$ and $j = 1, 2, 3$, leads to the MIMO transfer function matrix $\hat{G}(s) = \{\hat{G}_{ij}\}$, where

$$\hat{G}_{11} = \frac{5.506s^2 + 0.5666s + 0.001628}{s^3 + 0.1494s^2 + 0.005058s + 1.375 * 10^{-5}} e^{-5.02s},$$

$$\hat{G}_{12} = \frac{0.1192s^2 + 0.01741s + 0.0005498}{s^3 + 0.1494s^2 + 0.005058s + 1.375 * 10^{-5}} e^{-5.02s},$$

$$\hat{G}_{13} = \frac{-0.2107s^2 - 0.01034s - 2.8 * 10^{-5}}{s^3 + 0.1494s^2 + 0.005058s + 1.375 * 10^{-5}} e^{-5.02s},$$

$$\hat{G}_{21} = \frac{1.547s^2 + 0.2097s + 0.005514}{s^3 + 0.156s^2 + 0.006295s + 7.132 * 10^{-5}} e^{-5.02s},$$

$$\hat{G}_{22} = \frac{2.751s^2 + 0.3297s + 0.005481}{s^3 + 0.156s^2 + 0.006295s + 7.132 * 10^{-5}} e^{-3.02s},$$

$$\hat{G}_{23} = \frac{-0.5019s^2 - 0.02793s - 0.0003587}{s^3 + 0.156s^2 + 0.006295s + 7.132 * 10^{-5}} e^{-5.02s},$$

$$\hat{G}_{31} = \frac{1.864s^2 + 0.09219s + 0.0004873}{s^3 + 0.06955s^2 + 0.001253s + 5.244 * 10^{-6}} e^{-5.02s},$$

$$\hat{G}_{32} = \frac{-0.2215s^2 - 0.01405s - 0.0001917}{s^3 + 0.06955s^2 + 0.001253s + 5.244 * 10^{-6}} e^{-5.02s},$$

and

$$\hat{G}_{33} = \frac{-4.499s^2 - 0.117s - 0.0005396}{s^3 + 0.06955s^2 + 0.001253s + 5.244 * 10^{-6}} e^{-4.02s},$$

with the identification error as

$$E = \begin{bmatrix} 0.68\% & 0.63\% & 3.02\% \\ 0.67\% & 1.09\% & 1.92\% \\ 0.64\% & 1.89\% & 2.36\% \end{bmatrix}.$$

6.5 Real time testing

The proposed method is also applied to a temperature chamber system in Advanced Control Technology lab, Department of Electrical and Computer Engineering, National University of Singapore. The experiment setup consists of two parts: a thermal chamber set (which is made by National Instruments Corp. and shown in Figure 6.6) and a personal computer with data acquisition cards and LabVIEW software. The system has two inputs: one is to control $12V$ Light with $20W$ Halogen Bulb, the other is to control $12V$ Fan. The system output is the temperature of the temperature chamber. Extra transport delays are simulated by using LabVIEW software. An identification test is applied at $t = 0$. The process inputs and the output are given in Figure 6.7 and the sampling interval is 0.1 second. $u_1(t)$ in Figure 6.7 is used to control the fan speed, and $u_2(t)$ is used to control the light intensity. First, we estimate the range of time delays roughly: $0 \leq d_{11} \leq 0.8$ and $0 \leq d_{12} \leq 0.8$. Applying the proposed method with $n_1 = 2$, $m_{11} = m_{12} = 1$ and $q_1 = 2$, the estimated time delays are obtained as $\hat{d}_{11} = 0.555$ and $\hat{d}_{12} = 0.354$.

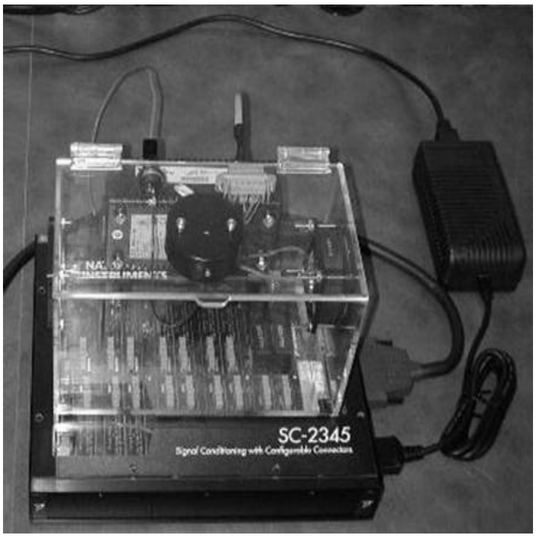

Figure 6.6. Temperature chamber set.

Based on these estimated time delays, we can de-tune the ranges of time delay more accurately: $0.3 \leq d_{11} \leq 0.7$ and $0.2 \leq d_{12} \leq 0.6$. Applying the proposed identification method again and one obtains the model as follows,

$$\begin{aligned} y^{(2)}(t) + 3.333y^{(1)}(t) + 1.089y(t) &= -32.39u_1^{(1)}(t-0.58) - 29.76u_1(t-0.58) \\ &+ 49.52u_2^{(1)}(t-0.495) + 32.12u_2(t-0.495). \end{aligned}$$

If the disturbance is static, the initial conditions can be then estimated (Hwang and Lai, 2004). The estimated response of the model and the real one are shown in Figure 6.7 for comparison. The effectiveness of the proposed method is obvious.

6.6 Conclusion

The need for a process model arises from various engineering field such as system analysis, prediction, monitoring, controller design, plant optimization and fault detection. Most industrial processes are of multivariable in nature and time delay is present in most industrial processes. Implementation of modern advanced controllers, such as internal model control, explicitly makes use of process models. Thus, identification of multivariable processes with time delay is in great demand and an effective technique for it is presented in this chapter. The technique covers

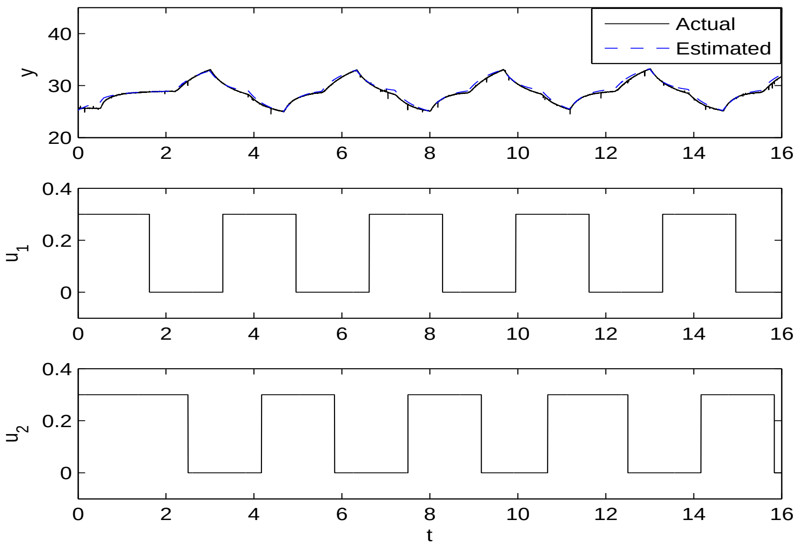

Figure 6.7. Process responses and inputs of the thermal control system.

all popular tests used in applications, requires reasonable amount of computations, and provide accurate and robust identification results.

Chapter 7

PID Controller Design by Approximate Pole Placement

7.1 Introduction

Control design is another important topic of control engineering. It is also one usage of process identification. In a typical control textbook, the standard 2nd-order system is discussed in great detail and used to guide practical control system design even if the underlying system is not of 2nd-order. The assumption to make such a design hold is that there is a dominant 2nd-order dynamics. The desired closed-loop poles are calculated from certain control specifications such as percentage overshoot and settling time. However, continuous-time delay control systems are infinite-dimensional (Åström and Wittenmark, 1997). They have infinite spectrum and it is impossible to assign such infinite spectrum with a finite-dimensional controller (Michiels *et al.*, 2002). Instead, one naturally wishes to assign a pair of poles which dominate all other poles. This idea was first introduced by Persson and Åström (1993) and further explained in Åström and Hagglund (1995). In Coelho (1998), this idea is developed for the tuning of lead-lag controllers. Both methods are based on a simplified model of processes and thus cannot guarantee the chosen poles to be indeed dominant in reality.

In this chapter, an analytical PID design method is proposed for continuous-time delay systems to achieve approximate placement of two desired poles with dominance. A continuous delay process is converted to a low-order rational discrete model. A discrete PID controller is designed to ensure dominant pole placement in discrete domain. This is a finite-dimensional problem and the solution for pole placement is readily available. The designed discrete PID controller is finally converted back to the continuous one. The poles in continuous domain are generally not precisely the same as originally set. It is argued that exact pole placement is not necessary as practical design specifications are commonly set as ranges instead of precise values and approximate ones should be sufficient as long as they donot deviate too much from the ideal ones. The dominance and error of the assigned poles are measured and checked for the design. It is shown by simulation that the proposed method works well with great dominance and negligible error of approximately assigned desired poles for a large range of normalized dead time up to at least 4. It should also be pointed out that discretization of a continuous process and discrete PID calculations are purely employed as a design intermediate and can be viewed as a fictitious process to get a workable continuous PID controller. No sampling is applied anyway. Performance of our design should be judged from that of the so-obtained continuous PID controller, rather from discretization errors involved.

Continuous controller design is always carried out in continuous domain, and this causes an infinite spectrum assignment problem for a delay process under PID control, a hard and open problem, while the proposed method of transform into and out of a discrete model is first of its kind. It brings the infinite spectrum assignment problem to an approximate finite spectrum assignment problem by a special selection of sampling time. A simple solution is then obtained. No method is available in the literature to guarantee dominance of the assigned poles for PID control of a continuous delay process but the proposed method can do so.

This chapter is organized as follows. In Section 7.2, the problem under consideration is formulated. In Section 7.3, the design method is presented for monotonic processes. Simulation examples are given in Section 7.4. In Section 7.5, the proposed method is applied to a thermal control system. Positive PID setting is discussed in Section 7.6. In Section 7.7, the design method is presented for oscillatory processes. Finally, conclusions are drawn in the Section 7.9.

7.2 Problem statement

A block diagram of a PID control system is shown in Figure 7.1, where $\tilde{G}(s)$ is a continuous-time delay process and $C(s)$ is the PID controller. Suppose that control system design specifications are represented by the overshoot and settling time on its closed-loop step response. The overshoot is usually achieved by setting a suitable damping ratio, ξ. A reasonable value of the damping ratio is typically in the range of 0.4 to 1. The settling time, T_s, cannot be taken arbitrarily but largely limited by the process characteristics and available magnitude of the manipulated variable. If T_s is too large, the response is very slow, which is bad performance and should be avoided. On the other hand, if T_s is too small, this may cause very large control signal and less robust control system. From view of dominant pole placement, pole dominance is also difficult to realize (Åström and Hagglund, 1995; Zhang *et al.*, 2002) if T_s is very small. In this thesis, through extensive simulation, we adopt the following expirical formula to choose T_s for the process with a monotonic step response:

$$T_s = T(4.5 + 7.5\frac{L}{T})(\frac{0.35}{\xi} + 0.5), \tag{7.1}$$

where T and L are the equivalent time constant and dead time of the process. The natural frequency, ω_o, is calculated with $\omega_o = \frac{4}{\xi T_s}$. Then, the specifications can be transferred to the corresponding desired 2nd-order dynamics:

$$s^2 + 2\xi\omega_o s + \omega_o^2 = 0.$$

Its two roots are denoted by $p_{s,1}$ with a positive imaginary part and $p_{s,2}$, which are the desired closed-loop poles to be achieved and be dominant by our controller design.

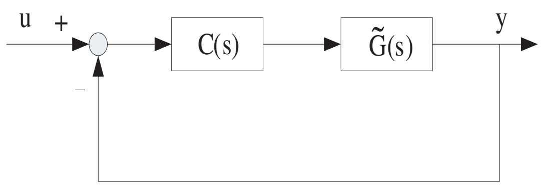

Figure 7.1. PID control systems.

The actual closed-loop system has its characteristic equation:

$$1 + \tilde{G}(s)C(s) = 0.$$

Let its roots or closed-loop poles be $\tilde{p}_{s,i}, i = 1, 2, \ldots$. They are ordered such that $\tilde{p}_{s,i}$ meets $Re(\tilde{p}_{s,i}) \geq Re(\tilde{p}_{s,i+1})$ and if $Re(\tilde{p}_{s,i}) = Re(\tilde{p}_{s,i+1})$, $Im(\tilde{p}_{s,i}) > Im(\tilde{p}_{s,i+1})$, where $Re(\tilde{p}_{s,i})$ and $Im(\tilde{p}_{s,i})$ are the real and imaginary parts of $\tilde{p}_{s,i}$, respectively. Note that the actual poles, $\tilde{p}_{s,i}, i = 1, 2$, may not be the same as the desired ones: $p_{s,1}$ and $p_{s,2}$, and $\tilde{p}_{s,i}, i = 1, 2$, may not be dominant enough with respect to other poles. Thus, we introduce two measures to reflect them: the relative pole assignment error,

$$E_P = max(\left|\frac{\tilde{p}_{s,1} - p_{s,1}}{p_{s,1}}\right|, \left|\frac{\tilde{p}_{s,2} - p_{s,2}}{p_{s,2}}\right|), \tag{7.2}$$

and the relative dominance,

$$E_D = \frac{Re(\tilde{p}_{s,3})}{Re(\tilde{p}_{s,2})}. \tag{7.3}$$

Our problem of approximate pole placement with dominance is to determine a continuous controller $C(s)$ so as to produce reasonably small relative pole assignment error and large relative dominance, say, $E_P \leq 20\%$ and $E_D \geq 3$, which are used as defaults.

The difficulty of the above problem lies in existence of an infinite number of closed-loop poles for a continuous delay process under PID control. It is impossible to assign all the closed-loop poles. However, a continuous-time delay process may be converted to a low-dimensional discrete system with some special sampling time selection. In this thesis, discrete design is used as a bridge to approximate pole placement in continuous PID control systems but no sampling is done in the real control system of Figure 7.1.

7.3 The proposed method

Let a continuous-time delay process $\tilde{G}(s)$ have a monotonic step response and be represented by a first-order time delay model:

$$G(s) = \frac{K}{Ts+1}e^{-Ls}. \tag{7.4}$$

In this thesis, we choose the sampling time h as $h = L$ to make the discretized process, $G(z)$, have the lowest order. The process has a pole at $-\frac{1}{T}$. This pole is mapped via $z = e^{hs}$ (adopted in pole-zero matching method in Franklin *et al.* (1990)), to the pole of its discrete equivalent at $\tilde{T} = e^{-L/T}$. so that $\frac{K}{Ts+1}$ is converted to $\frac{\tilde{K}}{z-\tilde{T}}$, where $\tilde{K}$ is selected to match the static gain, $\frac{K}{Ts+1}|_{s=0} = \frac{\tilde{K}}{z-\tilde{T}}|_{z=1}$, and thus $\tilde{K} = K(1-e^{-L/T})$. Note that The discrete equivalent of e^{-Ls} is $\frac{1}{z}$ under $h = L$. Overall, the process in form of (7.4) is converted to

$$G(z) = \frac{\tilde{K}}{z(z-\tilde{T})}. \tag{7.5}$$

The continuous PID controller in form of

$$C(s) = K_p(1 + \frac{1}{T_i s} + T_d s), \tag{7.6}$$

is also converted to the discrete-time model,

$$C(z) = \frac{k_1 z^2 + k_2 z + k_3}{z-1}, \tag{7.7}$$

where k_1, k_2 and k_3 are the functions of K_p, T_i and T_d. The characteristic polynomial of the discrete closed-loop system is

$$A_{cl}(z) = z(z-\tilde{T})(z-1)(1+G(z)C(z)) = z^3 + (k_1\tilde{K} - 1 - \tilde{T})z^2 + (\tilde{T} + k_2\tilde{K})z + k_3\tilde{K}. \tag{7.8}$$

On the other hand, the given $p_{s,1}$ and $p_{s,2}$ have the desirable discrete characteristic polynomial as follows

$$A_{de}(z) = (z - p_{z,1})(z - p_{z,2})(z - p_{z,3}) = z^3 + p_1 z^2 + p_2 z + p_3, \tag{7.9}$$

where $p_{z,1} = e^{Lp_{s,1}}$, $p_{z,2} = e^{Lp_{s,2}}$, and $p_{z,3}$ is a user-defined parameter and set at $e^{10LRe(p_{s,1})}$ in this thesis. Equalizing $A_{cl}(z)$ with $A_{de}(z)$ yields

$$\begin{aligned} k_1 &= \frac{p_1 + 1 + \tilde{T}}{\tilde{K}}, \\ k_2 &= \frac{p_2 - \tilde{T}}{\tilde{K}}, \\ k_3 &= \frac{p_3}{\tilde{K}}. \end{aligned}$$

Once k_1, k_2 and k_3 are known, the two zeros of $C(z)$ can be calculated as $z_{1,2} = \frac{-k_2 \pm \sqrt{k_2^2 - 4k_1k_3}}{2k_1}$. Using the pole-zero matching method gives the continuous controller as

$$C(s) = \frac{K_c(s - \frac{log(z_1)}{L})(s - \frac{log(z_2)}{L})}{s},$$

with K_c selected to match the gain of $C(s)$ at $s = \frac{0.1m}{L}$, where m is the smallest integer and meets $e^{0.1m} \neq 1, z_1$ and z_2. Finally, $C(s)$ can be then rearranged into the form in (7.6) with its settings given as follows,

$$\begin{aligned} K_p &= -\frac{K_c(log(z_1) + log(z_2))}{L}, \\ T_i &= -\frac{L(log(z_1) + log(z_2))}{log(z_1)log(z_2)}, \\ T_d &= -\frac{L}{log(z_1) + log(z_2)}. \end{aligned}$$

To apply the above method to a non-first-order process $\tilde{G}(j\omega)$ with monotonic step response, we have to obtain its first-order approximate model $G(s)$ in form of (7.4). The simplest technique is to match the model frequency response with the process one at two frequency points, $\omega = 0$ and $\omega = \omega_p$, the phase cross-over frequency. The formulas are well known (Wang *et al.*, 2003):

$$K = \tilde{G}(0), \tag{7.10}$$

$$T = \sqrt{\frac{K^2 - |\tilde{G}(j\omega_p)|^2}{|\tilde{G}(j\omega_p)|^2 \omega_p^2}}, \tag{7.11}$$

$$L = \frac{\pi + tan^{-1}(-\omega_p T)}{\omega_p}. \tag{7.12}$$

Gain and phase margins are basic measure of the system's robustness. In this thesis, we apply these specifications to judge robustness of the design results. Tuning ξ will give suitable robust stability of the closed-loop system against the parameter uncertainties.

7.4 Simulation study

Example 7.1. Consider an exact first order process with $\tilde{G}(s) = \frac{1}{s+1}e^{-Ls}$, and study our design with several typical values of L. Let $L = 0.5$ first. Suppose that the desired damping ratio is $\xi = 0.7$. T_s is calculated from (7.1) as 8.25. We have $p_{s,1} = -0.4848 + 0.4946i$, $p_{s,2} = -0.4848 - 0.4946i$. The third pole is then $p_{s,3} = 10Re(p_{s,1}) = -4.848$. The proposed method with these specifications leads to the discrete PID:

$$C(z) = \frac{-0.009416z^2 + 0.366z - 0.1386}{z - 1},$$

and via the pole-zero matching method, the continuous PID:

$$C(s) = \frac{-0.0321s^2 + 0.1726s + 0.4505}{s},$$

which is rearranged in form of (7.6) as

$$C(s) = 0.1726(1 + \frac{1}{0.3832s} - 0.1859s).$$

The closed-loop poles are calculated from the roots of $1 + \tilde{G}(s)C(s) = 0$ with a 40th order Pade approximate to the time delay as $\tilde{p}_{s,1}$ $=-0.5135 + 0.4837i$, $\tilde{p}_{s,2}$ $=-0.5135 - 0.4837i$, $\tilde{p}_{s,3}$ $=-5.6623$, $\tilde{p}_{s,4}$ $=-6.4016 + 13.1493i$, $\tilde{p}_{s,5}$ $=-6.4016 - 13.1493i$, ... It follows that $E_P = 4.43\%$ and $E_D = 11.03$. The gain margin and phase margin are 6.64 and 63.92^o, respectively. The step response and the manipulated variable are shown in Figure 7.2. The settling time of the the control system is 8.5 and the overshoot is 3.71% with the corresponding damping ratio of 0.72. The step responses of the discrete system, $\frac{G(z)C(z)}{1+G(z)C(z)}$, and the prototype continuous system, $\frac{2.326}{s^3+5.818s^2+5.181s+2.326}$ with its poles at the desired $-0.4848 \pm 0.4946i$ and one extra at −4.848, are also given in Figure 7.3 for comparisons, from which one sees that the the designed continuous system is quite close to them.

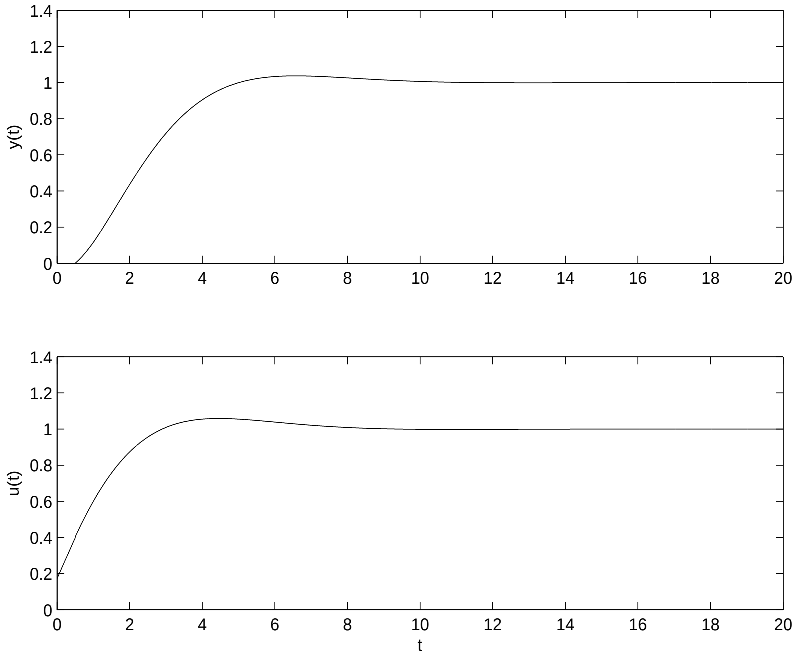

Figure 7.2. Step response and manipulated variable of Example 7.1 with $L = 0.5$.

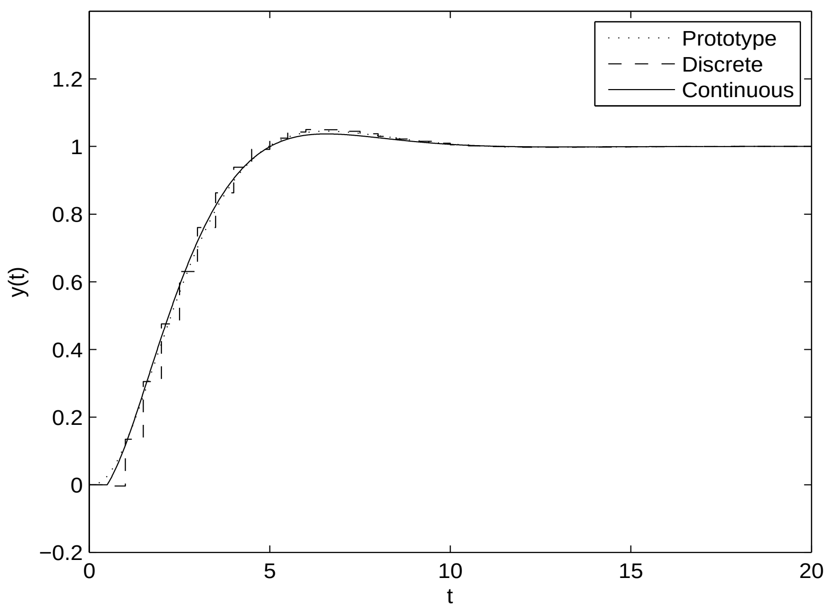

Figure 7.3. Step response of Example 7.1 with $L = 0.5$.

Consider $L = 2$. Suppose that the desired damping ratio is $\xi = 0.7$. T_s is calculated from (7.1) as 19.5. We have $p_{s,1} = -0.2051 + 0.2093i$ and $p_{s,2} = -0.2051 + 0.2093i$. The third pole is at -2.051. The proposed method with these specifications leads to the discrete PID:

$$C(z) = \frac{-0.1083z^2 + 0.3758z - 0.008416}{z - 1},$$

and via the pole-zero matching method, the continuous PID:

$$C(s) = \frac{-0.1179s^2 - 0.1506s + 0.1384}{s},$$

which is rearranged in form of (7.6) as follows

$$C(s) = -0.1506(1 - \frac{1}{1.0883s} + 0.7829s).$$

The closed-loop poles are $\tilde{p}_{s,1} = -0.1913 + 0.2284i$, $\tilde{p}_{s,2} = -0.1913 - 0.2284i$, $\tilde{p}_{s,3} = -1.0131 + 3.0847i$, $\tilde{p}_{s,4} = -1.0131 - 3.0847i$, ... It follows that $E_P = 8.04\%$ and $E_D = 5.30$. The gain margin and phase margin are 2.59 and 57.25^o, respectively. The step response and the manipulated variable are shown in Figure 7.4. The settling time is 22.95 and the overshoot is 7.49% with the corresponding damping ratio of 0.64. The step responses of the discrete system, $\frac{G(z)C(z)}{1+G(z)C(z)}$, and the prototype continuous system, $\frac{0.1761}{s^3+2.462s^2+0.9274s+0.1761}$, with its poles at the desired $-0.2051 \pm 0.2093i$ and -2.051, are also given in Figure 7.5 for comparison.

Consider $L = 4$. Suppose that the desired damping ratio is $\xi = 0.7$. T_s is calculated from (7.1) as 34.5. We have $p_{s,1} = -0.1159 + 0.1183i$ and $p_{s,2} = -0.1159 - 0.1183i$. The third pole is at -1.159. The proposed method with these specifications leads to the discrete PID:

$$C(z) = \frac{-0.1131z^2 + 0.3953z - 0.0039}{z - 1},$$

and via the pole-zero matching method, the continuous PID:

$$C(s) = \frac{-0.207s^2 - 0.1743s + 0.07457}{s},$$

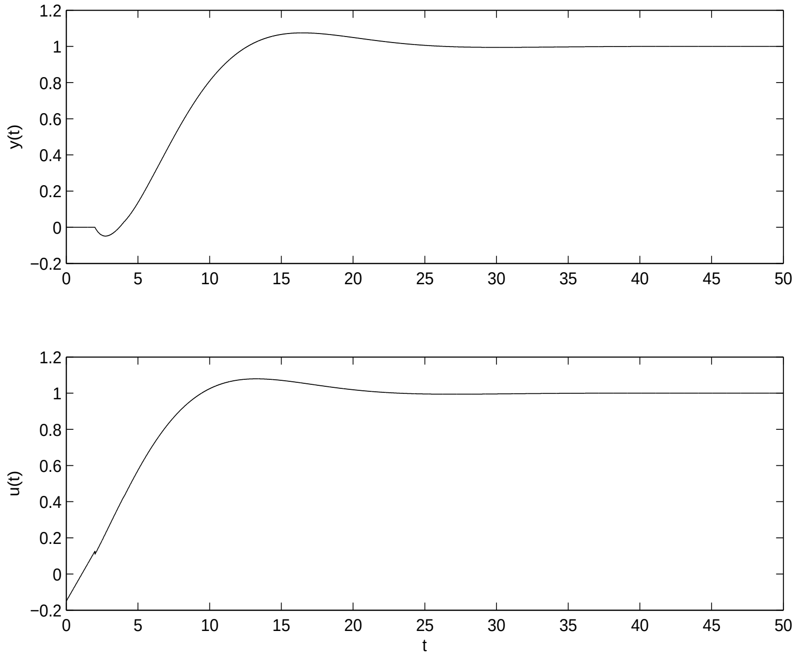

Figure 7.4. Step response and manipulated variable of Example 7.1 with $L = 2$.

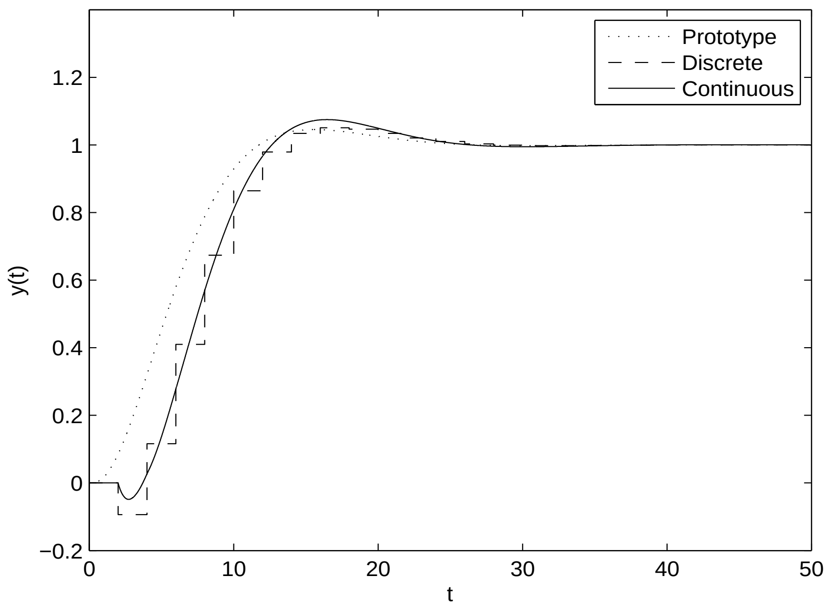

Figure 7.5. Step response of Example 7.1 with $L = 2$.

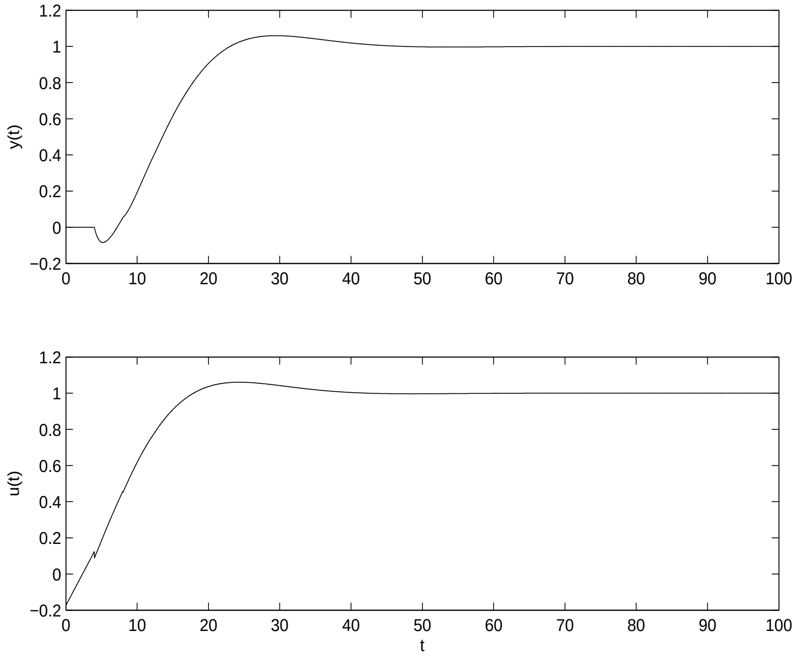

Figure 7.6. Step response and manipulated variable of Example 7.1 with $L = 4$.

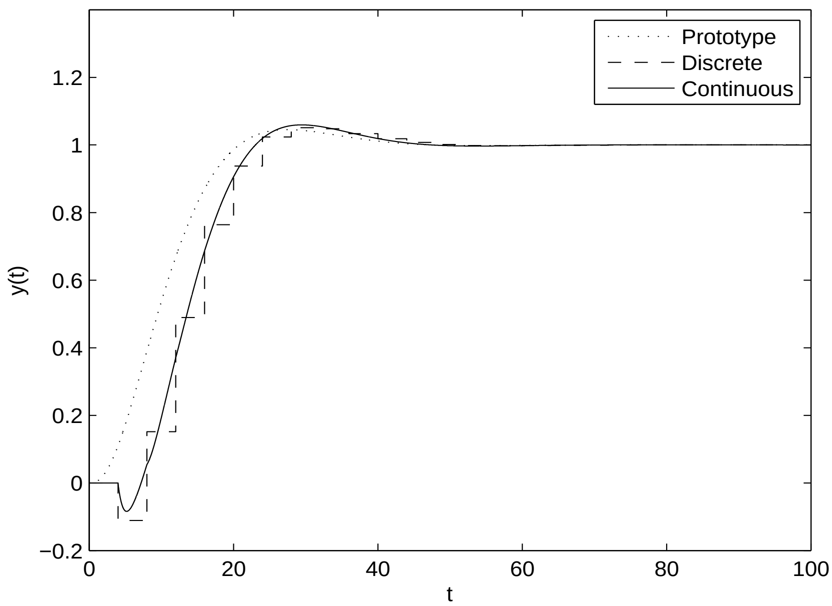

Figure 7.7. Step response of Example 7.1 with $L = 4$.

which is rearranged in form of (7.6) as follows

$$C(s) = -0.1743(1 - \frac{1}{2.3366s} + 1.1880s).$$

The closed-loop poles are $\tilde{p}_{s,1}$ $=-0.1184 + 0.1289i$, $\tilde{p}_{s,2}$ $=-0.1184 - 0.1289i$, $\tilde{p}_{s,3}$ $=-0.3704 + 1.5947i$, $\tilde{p}_{s,4}$ $=-0.3704 - 1.5947i$, ... It follows that $E_P = 6.56\%$ and $E_D = 3.12$. The gain margin and phase margin are 2.48 and 58.02^o, respectively. The step response and the manipulated variable are shown in Figure 7.6. The settling time is 39.73 and the overshoot is 5.94% with the corresponding damping ratio of 0.67. The step responses of the discrete system, $\frac{G(z)C(z)}{1+G(z)C(z)}$, and the prototype continuous system, $\frac{0.03181}{s^3+1.391s^2+0.2963s+0.03181}$, with its poles at the desired $-0.1159 \pm 0.1183i$ and -1.159, are also given in Figure 7.7 for comparison.

Example 7.2. Consider a high-order process, $\tilde{G}(s) = \frac{(2s+1)}{(s+1)^2(4s+1)}e^{-s}$. By Formulas (7.10), (7.11) and (7.12), we obtain its first-order approximate as

$$G(s) = \frac{1}{3.743s + 1}e^{-1.49s}.$$

Suppose that the desired damping ratio is $\xi = 0.7$. T_s is calculated from (7.1) as 28. We have $p_{s,1} = -0.1427 + 0.1456i$ and $p_{s,2} = -0.1427 - 0.1456i$. The third pole is at -1.427. The proposed method with these specifications leads to the discrete PID:

$$C(z) = \frac{-0.07994z^2 + 0.5168z - 0.2366}{z - 1},$$

and via the pole-zero matching method, the continuous PID:

$$C(s) = \frac{-0.2485s^2 + 0.1808s + 0.14}{s},$$

which is rearranged in form of (7.6) as

$$C(s) = 0.1808(1 + \frac{1}{1.2910s} - 1.3747s).$$

The closed-loop poles are $\tilde{p}_{s,1} = -0.1530 + 0.1369i$, $\tilde{p}_{s,2} = -0.1530 - 0.1369i$, $\tilde{p}_{s,3} = -0.7307 + 0.3366i$, $\tilde{p}_{s,4} = -0.7307 - 0.3366i$, ... It follows that $E_P = 6.62\%$ and $E_D = 4.77$. The gain margin and phase margin are 5.47 and 63.81^o, respectively. The step response and the manipulated variable is shown in Figure 7.8. The settling time of the control system is 28.28 and the overshoot is 3.41% with the corresponding damping ratio of 0.73. The step responses of the discrete system, $\frac{G(z)C(z)}{1+G(z)C(z)}$, and the prototype continuous system, $\frac{0.05931}{s^3+1.713s^2+0.4489s+0.05931}$ with its poles at $-0.1427 \pm 0.1456i$ and -1.427, are also given in Figure 7.9 for comparison.

In practice, the measurement noise and unmodelled dynamics, such as disturbances, are generally present. For the same example, the measurement noise is simulated by adding a white noise to the output and a disturbance with the magnitude of -0.3 is added to the output at $t = 30$. The response, $y(t)$, the measured output, $y_n(t)$, and the manipulated variable, $u(t)$, are shown in Figure 7.10. The effectiveness of our method is shown.

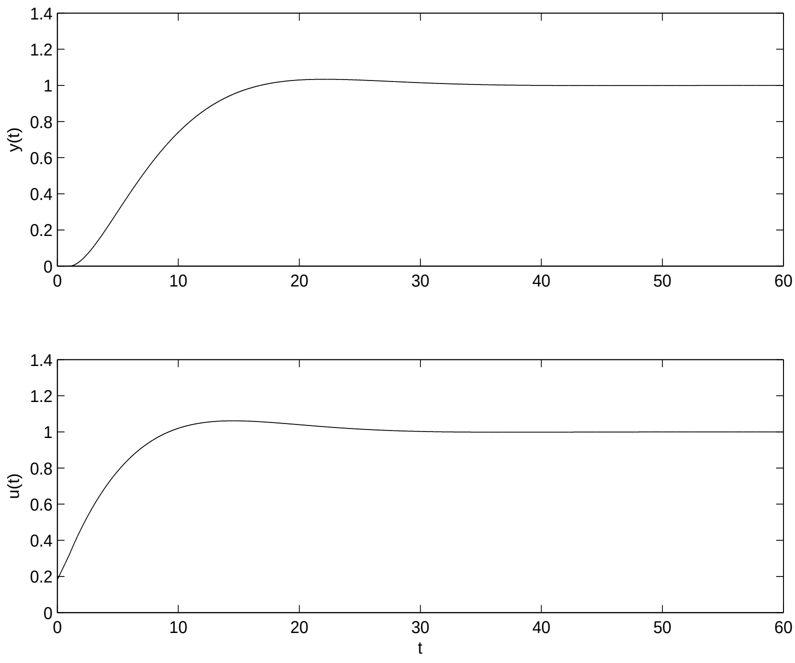

Figure 7.8. Step response and manipulated variable of Example 7.2.

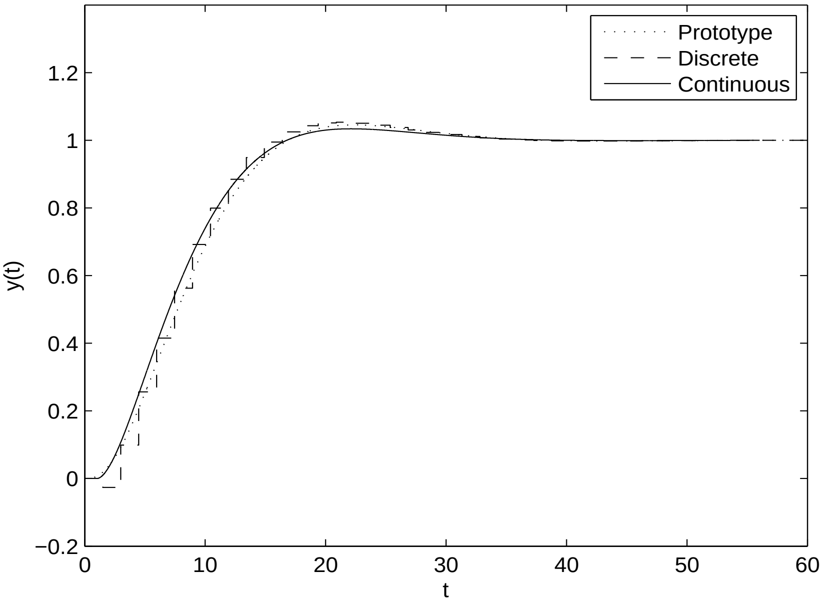

Figure 7.9. Step response of Example 7.2.

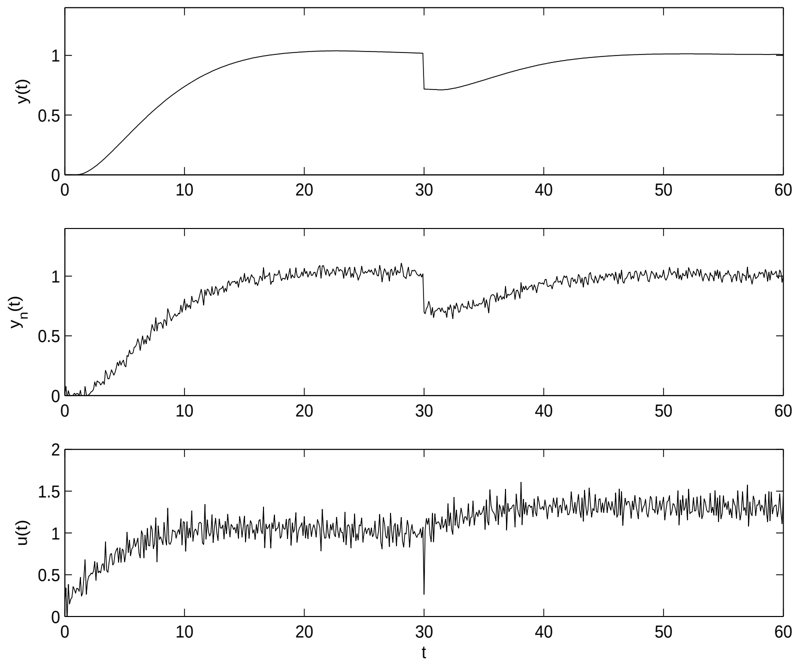

Figure 7.10. Step response, measured response and manipulated variable of Example 7.2.

7.5 Real time testing

In this section, the proposed PID tuning method is also applied to a temperature chamber system, which is made by National Instruments Corp. and shown in Figure 6.6. The experiment setup consists of a thermal chamber and a personal computer with data acquisition cards and LabVIEW software. The system input, u, is the adjustable power supply to $20W$ Halogen bulb. The system output, y, is the temperature of the temperature chamber. The model of the process is

$$G(s) = \frac{29.49e^{-0.106s}}{0.6853s + 1}.$$

The proposed method with $\xi = 0.8$ leads to the PID controller as

$$C(s) = 0.0047(1 + \frac{1}{0.1535s} - 1.3408s).$$

This ideal PID is not physically realizable and is thus replaced by

$$C(s) = 0.0047(1 + \frac{1}{0.1535s} - \frac{1.3408s}{(1.3408/N)s + 1}),$$

where $N = 4$, in the real time testing. Before the test is applied, the control system is at a steady state. At $t = 0$, the reference input is changed from 29 to 27. The process input and output are given in Figure 7.11. The step response of the prototype continuous system, $\frac{20.8}{s^3+13.2s^2+26.09s+20.8}$ are also given in Figure 7.11 for comparison. The designed system has satisfying performance.

7.6 Positive PID settings

It is noted from the simulation results in the preceding section that some of the PID parameters are not positive. In many applications, it is not permissible. To avoid this problem, we choose the controller in the form of

$$C(s) = K_p(1 + \frac{1}{T_i s})(\frac{s + \beta}{s + \alpha}), \tag{7.13}$$

which corresponds to the practical form (no pure D) of PID controller in the cascaded structure (Åström and Hagglund, 1995; Ang *et al.*, 2005). We choose

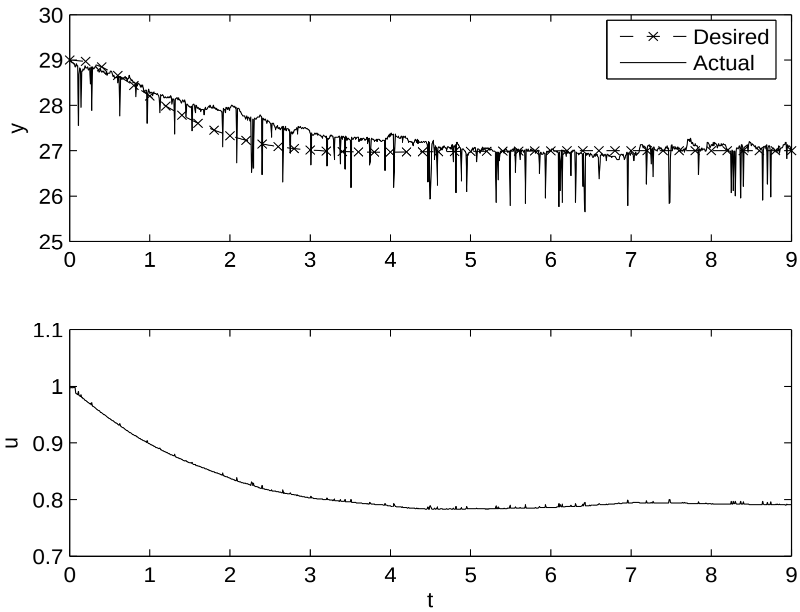

Figure 7.11. Step response and manipulated variable of the thermal chamber.

$T_i = T$ to cancel the pole of $G(s)$. The open-loop transfer function, $G(s)C(s)$, is converted by the pole-zero matching method to its discrete equivalent,

$$G(z)C(z) = \frac{\hat{K}}{z}\frac{k_1 z + k_2}{(z-1)(z+k_3)}, \tag{7.14}$$

where $\hat{K} = K/T$, and k_1, k_2, k_3 are the functions of K_p, β and α. The discrete closed-loop characteristic polynomial is

$$A_{cl}(z) = z^3 + (k_3 - 1)z^2 + (\hat{K}k_1 - k_3)z + \hat{K}k_2.$$

By making $A_{cl}(z) = A_{de}(z)$, we can solve for k_1, k_2 and k_3 as

$$k_1 = \frac{p_1 + p_2 + 1}{\hat{K}}, \tag{7.15}$$

$$k_2 = \frac{p_3}{\hat{K}}, \tag{7.16}$$

$$k_3 = p_1 + 1. \tag{7.17}$$

Once k_1, k_2 and k_3 are known, we obtain the controller parameters in continuous domain as

$$\beta = -log(\frac{-k_2}{k_1})/L, \tag{7.18}$$

$$\alpha = -log(-k_3)/L, \tag{7.19}$$

$$K_p = \frac{\frac{k_1 e^{0.1m}+k_2}{(e^{0.1m}+t_3)(e^{0.1m}-1)}}{\frac{10L(0.1m+dL)}{m(0.1m+cL)}}, \tag{7.20}$$

where m is the smallest integer, which meets $e^{0.1m} \neq 1, -k_3, -\frac{k_2}{k_1}$.

Example 1 (continued). Consider Example 1 again with $L = 0.5$. Suppose that the desired damping ratio is $\xi = 0.7$ and $T_s = 8.25$ as before. The controller in form of (7.13) is obtained as

$$C(s) = 0.2195(1 + \frac{1}{s})(\frac{s + 1.8901}{s + 0.9878}).$$

The closed-loop poles are calculated as $\tilde{p}_{s,1} = -0.5382 + 0.4020i$, $\tilde{p}_{s,2} = -0.5382 - 0.4020i$, $\tilde{p}_{s,3} = -7.32, \ldots$. For this example, $E_P = 15.43\%$ and $E_D = 13.6$. The closed-loop pole at -1 is concealed by the closed-loop zero at -1. The gain margin and phase margin are 10.31 and 68.53^o, respectively. The step response and the manipulated variable are shown in Figure 7.12. The settling time of the resultant control system is 5.45 and the overshoot is 1.63% with the corresponding damping ratio of 0.79. The step responses of the discrete system, $\frac{G(z)C(z)}{1+G(z)C(z)}$, and the prototype continuous system, $\frac{2.326}{s^3+5.818s^2+5.181s+2.326}$, are also given in Figure 7.13 for comparison.

7.7 Oscillation processes

Some practical processes such as temperature loops exhibit oscillatory or essentially 2nd-order behavior in its step response. The first-order modelling is not adequate for them. Instead, one has to use the following model:

$$G(s) = \frac{K}{s^2 + as + b} e^{-Ls}. \tag{7.21}$$

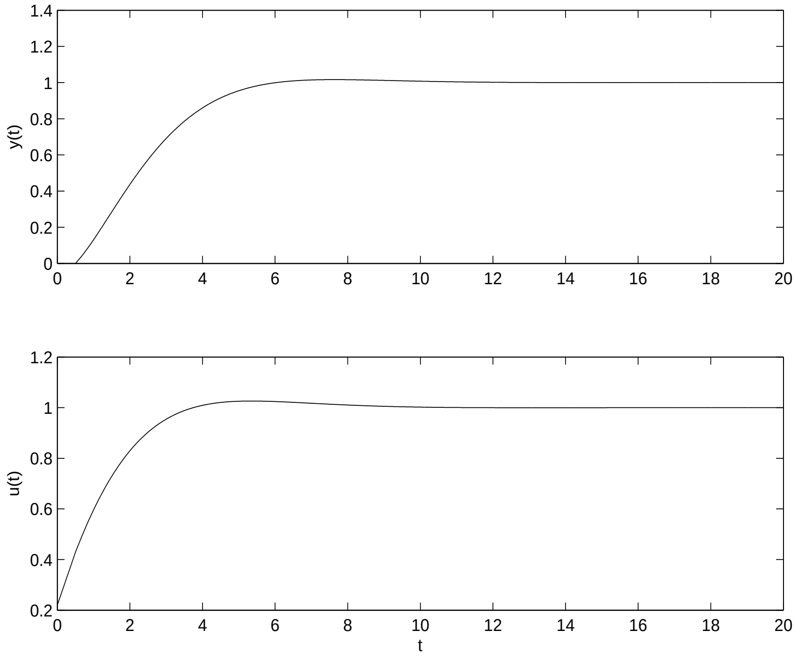

Figure 7.12. Step response and manipulated variable of Example 7.1 with $L = 0.5$.

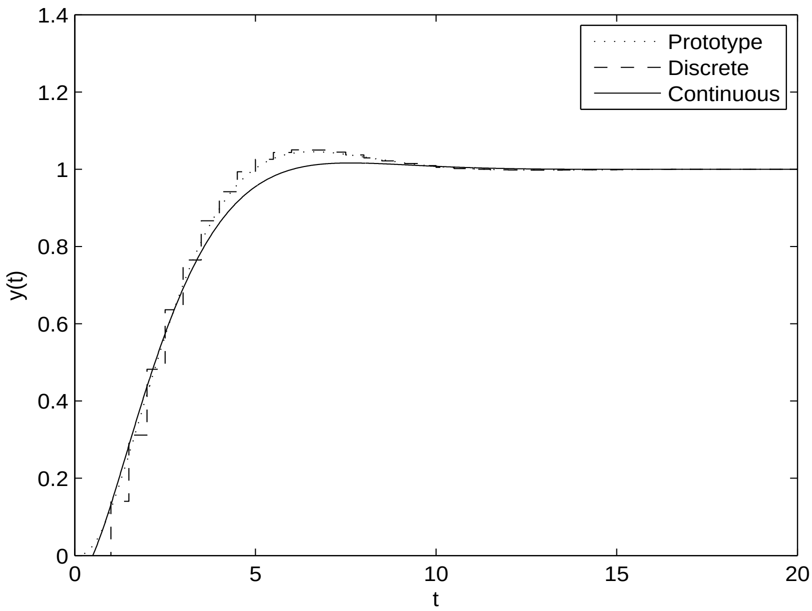

Figure 7.13. Step response of Example 7.1 with $L = 0.5$.

Define $p_{g,i}$, $i = 1, 2$ as the roots of $s^2 + as + b = 0$. The equivalent time constant of the oscillation process is defined as $T = \frac{-1}{Re(p_{g,i})}$. To set a desired 2nd-order dynamic properly, the damping ratio is chosen as before, while the following new formula,

$$T_s = T(1 + 15\frac{L}{T})(\frac{0.35}{\xi} + 0.5), \tag{7.22}$$

is used for determining T_s. For this kind of processes, we exploit the controller in the form of

$$C(s) = K_p(1 + \frac{1}{T_i s} + T_d s)(\frac{s+\beta}{s+\alpha}), \tag{7.23}$$

and choose T_i and T_d to cancel the poles of $G(s)$:

$$\begin{aligned} T_d &= \frac{1}{a}, \\ T_i &= \frac{a}{b}. \end{aligned}$$

Then, the resulting open-loop $G(s)C(s)$ and its discrete equivalent are the same as those in Section 7.6 with $\hat{K} = K/a$. The procedure there applies to obtain K_p, β and α from (7.15), (7.16) and (7.17). With k_1, k_2 and k_3, we calculate K_p, β and α according to (7.18), (7.19) and (7.20).

Example 7.3. Consider a oscillation process, $\tilde{G}(s) = \frac{1}{s^2+1.2s+1}e^{-0.7s}$. The equivalent time constant of the process is $T = 1.667$. Suppose that the desired damping ratio is $\xi = 0.7$. T_s is calculated from (7.22) as 13. We have $p_{s,1} = -0.3288+0.3354i$ and $p_{s,2} = -0.3288 - 0.3354i$. The third pole is at -3.288. The proposed method in this section with these specifications leads to the continuous controller:

$$C(s) = 0.1953(1 + \frac{1}{1.2s} + 0.8333s)(\frac{s + 1.1410}{s + 0.6256}).$$

The closed-loop poles are calculated as $\tilde{p}_{s,1} = -0.3585 + 0.2755i$, $\tilde{p}_{s,2} = -0.3585 - 0.2755i$, $\tilde{p}_{s,3} = -5.0949$, $\tilde{p}_{s,4} = -6.1839+10.3973i$, $\tilde{p}_{s,5} = -6.1839-10.3973i$, ... It follows $E_P = 14.24\%$ and $E_D = 14.21$. The closed-loop pole at $-0.6000 \pm 0.8000i$ are concealed by the closed-loop zeros. The gain margin and phase margin are 10.72 and 68.51^o, respectively. The step response and the manipulated variable are shown in Figure 7.14. The settling time of the control system is 11 and the overshoot is 2.04% with the corresponding damping ratio of 0.77. The step responses of the discrete system, $\frac{G(z)C(z)}{1+G(z)C(z)}$, and the prototype continuous system, $\frac{0.7252}{s^3+3.945s^2+2.382s+0.7252}$, are also given in Figure 7.15 for comparison.

For comparison with first-order design method, by (7.10), (7.11) and (7.12), we obtain its first-order model as

$$G(s) = \frac{1}{1.235s+1}e^{-1.44s}.$$

Suppose the desired damping ratio is $\xi = 0.7$. $T_s = 16.4$ is calculated from (7.1) with $T = 1.235$ and $L = 1.44$. The proposed method in Section 7.3 with these specification leads to the continuous PID:

$$C(s) = -0.0457(1 - \frac{1}{0.2481s} + 1.6713s).$$

The closed-loop poles, are calculated as $\tilde{p}_{s,1} = -0.3437$, $\tilde{p}_{s,2} = -0.4066 + 0.5870i$, $\tilde{p}_{s,3} = -0.4066 - 0.5870i$, $\tilde{p}_{s,4} = -6.69 + 5.47i$, $\tilde{p}_{s,5} = -6.69 - 5.47i$, ... The resulting dominant poles are -0.3437 and $-0.4066 \pm 0.5870i$, which are far from the desired ones.

Example 7.4. Consider a high-order oscillation process, $\tilde{G}(s) = \frac{1}{(0.8s+1)(s^2+1.1s+1)}e^{-2s}$. Applying the identification method proposed by Liu *et al.* (2007), we obtain one of its estimations as

$$G(s) = \frac{0.702}{s^2+0.9708s+0.7114}e^{-2.33s},$$

with the equivalent time constant of $T = 2.06$. Suppose that the desired damping ratio is $\xi = 0.7$. T_s is calculated from (7.22) as 37. We have $p_{s,1} = -0.1081+0.1103i$ and $p_{s,2} = -0.1081 - 0.1103i$. The third poles is at -1.081. The proposed method in this section with these specifications leads to the continuous controller:

$$C(s) = 0.0637(1 + \frac{1}{1.3646s} + 1.0301s)(\frac{s+0.4567}{s+0.2306}).$$

The closed-loop poles are calculated as $\tilde{p}_{s,1} = -0.1178 + 0.0941i$, $\tilde{p}_{s,2} = -0.1178 - 0.0941i$, $\tilde{p}_{s,3} = -0.5221 + 0.8374i$, $\tilde{p}_{s,4} = -0.5221 - 0.8374i$, ... It follows

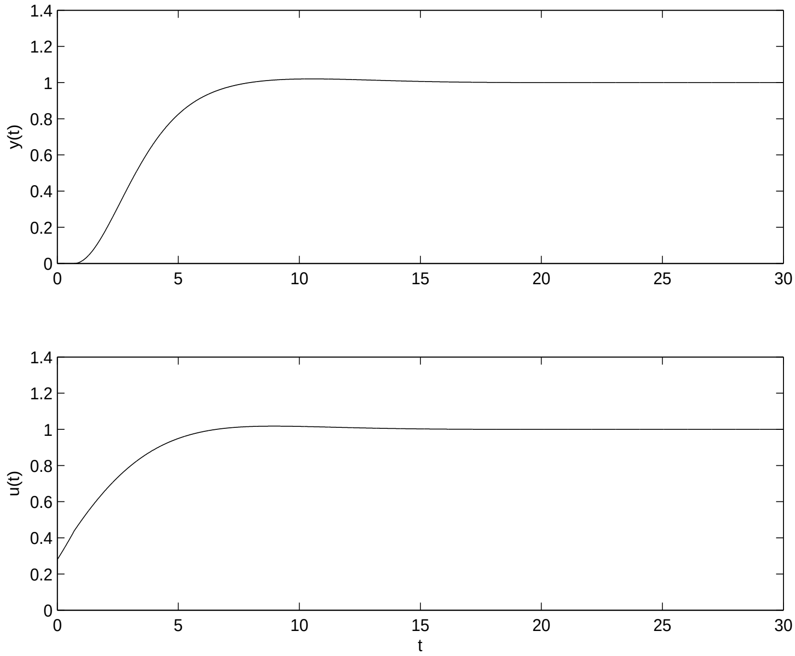

Figure 7.14. Step response and manipulated variable of Example 7.3.

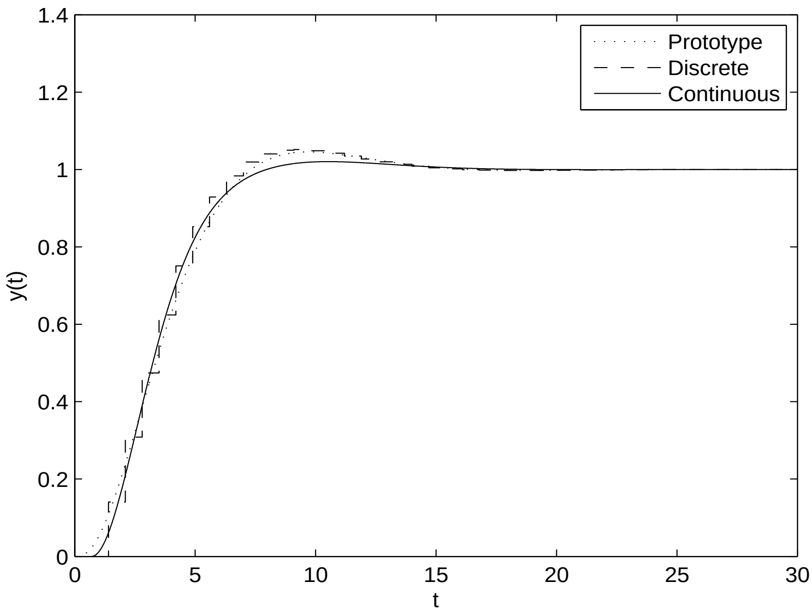

Figure 7.15. Step response of Example 7.3.

$E_P = 12.24\%$ and $E_D = 4.43$. The gain margin and phase margin are 10.02 and 67.34^o, respectively. The step response and the manipulated variable are shown in Figure 7.16. The settling time of the resultant control system is 35.36 and the overshoot is 2.1% with the corresponding damping ratio of 0.77. The step responses of the discrete system, $\frac{G(z)C(z)}{1+G(z)C(z)}$, and the prototype continuous system, $\frac{0.02576}{s^3+1.297s^2+0.2575s+0.02576}$, are also given in Figure 7.17 for comparison.

7.8 Multivariable case

In fact, many real-life industrial processes are multivariable in nature. It is of great interest and value to extend our single variable PID tuning method to multivarible PID controller design. Let $G(s) = [g_{ij}(s)]$ be the $m \times m$ multivarible process and $C(s) = [c_{ij}(s)]$ be the multivarible controller. To overcome the effects of cross-coupled interactions, a decoupler, $D(s) = [d_{ij}(s)]$, is designed first. By using the method proposed in Wang (2003), we have

$$d_{ji}(s) = \frac{G^{ij}(s)}{G^{ii}(s)} d_{ii}(s), \tag{7.24}$$

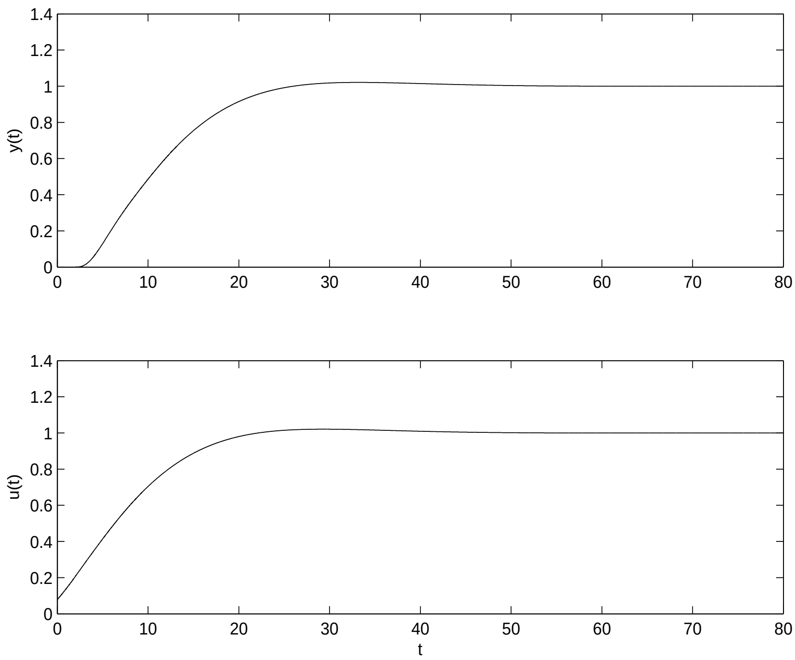

Figure 7.16. Step response and manipulated variable of Example 7.4.

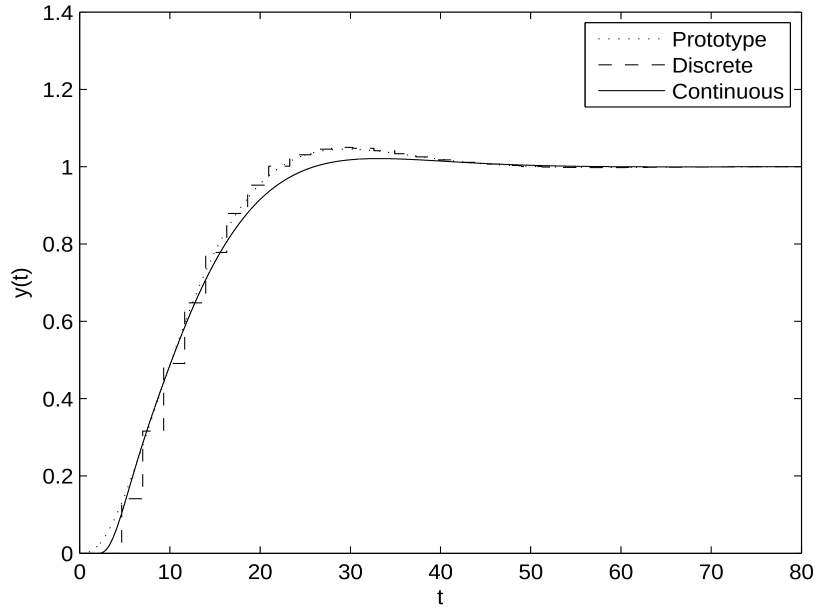

Figure 7.17. Step response of Example 7.4.

and $Q(s) = G(s)D(s)$ as

$$Q(s) = diag\left\{q_{ii}(s)\right\} = diag\left\{\frac{|G(s)|}{G^{ii}(s)}d_{ii}(s)\right\},$$

where $G^{ij}(s)$ is cofactor corresponding to $g_{ij}(s)$ in $G(s)$. $q_{ii}(s)$ may be complicated to implement or even not rational and cannot be used to design controllers directly, so that model reduction techniques based on step tests (Wang and Zhang, 2001) are applied to obtain rational and proper estimates of $q_{ii}(s)$, $\hat{q}_{ii}(s)$. With the PID tuning methods proposed in the above sections, single variable PID controllers, $k_{ii}(s)$, $i = 1, ..., m$, are designed for $\hat{q}_{ii}(s), i = 1, ..., m$, and the multivariable controller $C(s)$, with

$$c_{ij}(s) = d_{ij}(s)k_{jj}(s), \tag{7.25}$$

is obtained. Suppose $\hat{C}(s) = [\hat{c}_{ij}(s)]$ is a multivariable PID controller. If $c_{ij}(s)$ in $C(s)$ is PID type, we choose $\hat{c}_{ij}(s) = c_{ij(s)}$. For $c_{ij}(s)$, which is not PID type, its estimate in form of PID, $\hat{c}_{ij}(s)$, is obtained by using model reduction techniques in Wang *et al.* (2001a). The multivariable PID controller $\hat{C}(s)$ is then designed for $G(s)$.

Example 7.5. Consider a multivariable process,

$$G(s) = \begin{bmatrix} \frac{1}{s+1} & \frac{1}{s+2} \\ \frac{1}{s+3} & \frac{1}{s+1.5} \end{bmatrix}.$$

By choosing $d_{11}(s) = d_{22}(s) = 1$, the decoupler is designed as follows

$$D(s) = \begin{bmatrix} 1 & -\frac{s+1}{s+2} \\ -\frac{s+1.5}{s+3} & 1 \end{bmatrix},$$

according to (7.24). We have

$$Q(s) = \begin{bmatrix} \frac{2.5s+4.5}{(s+1)(s+2)(s+3)} & 0 \\ 0 & \frac{2.5s+4.5}{(s+1.5)(s+2)(s+3)} \end{bmatrix}.$$

One first-order time delay model of $Q(s)$ is obtained by using the method proposed in Wang and Zhang (2001),

$$\hat{Q}(s) = \begin{bmatrix} \frac{0.7597e^{-0.286s}}{s+1.013} & 0 \\ 0 & \frac{0.7671e^{-0.288s}}{s+1.534} \end{bmatrix}.$$

For $\hat{q}_{11}(s) = \frac{0.7597e^{-0.286s}}{s+1.013}$, suppose that the desired damping ratio is $\xi = 0.6$ and T_s is calculated from (7.1) as 7.13. The proposed single variable PID tuning method leads to

$$k_{11}(s) = 0.1621(1 + \frac{1}{0.1718s} - 1.3291s).$$

For $\hat{q}_{22}(s) = \frac{0.7671e^{-0.288s}}{s+1.534}$, suppose that the desired damping ratio is $\xi = 0.6$ and T_s is calculated from (7.1) as 5.52. The proposed single variable PID tuning method leads to

$$k_{22}(s) = 0.2599(1 + \frac{1}{0.1488s} - 0.4296s).$$

$C(s)$ is calculated according to

$$C(s) = \begin{bmatrix} k_{11}(s) & k_{22}(s)d_{12}(s) \\ k_{11}(s)d_{21}(s) & k_{22}(s) \end{bmatrix}.$$

$c_{12}(s) = k_{22}(s)d_{12}(s)$ and $c_{21}(s) = k_{11}(s)d_{12}(s)$ are high-order controllers. By using the method in Wang *et al.* (2001*a*), we have

$$\hat{c}_{12}(s) = -0.5540 - \frac{0.8733}{s} + 0.1976s,$$

and

$$\hat{c}_{21}(s) = -0.2459 - \frac{0.4718}{s} + 0.1378s,$$

respectively. $\hat{C}(s)$ is

$$\hat{C}(s) = \begin{bmatrix} 0.1621 + \frac{0.9435}{s} - 0.2154s & -0.5540 - \frac{0.8733}{s} + 0.1976s \\ -0.2459 - \frac{0.4718}{s} + 0.1378s & 0.2599 + \frac{1.7466}{s} - 0.1117s \end{bmatrix}.$$

The step responses of the resultant multivariable PID control system to unit set-point changes are shown in Figure 7.18. For the first loop, the settling time of the multivariable PID control system is 6.95 and the overshoot is 12.47% with the corresponding damping ratio of 0.55. For the second loop, the settling time is 5.11 and the overshoot is 10.68% with the corresponding damping ratio of 0.58. Step responses of the original control system with $C(s)$ as the controller are also given in 7.18 for comparison.

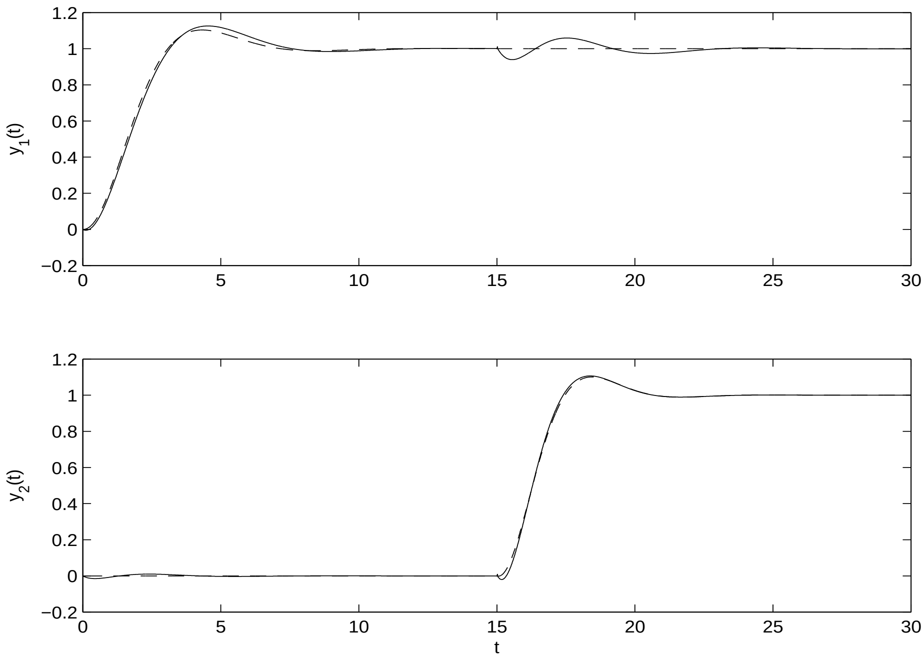

Figure 7.18. Step response of Example 7.5.

(Solid line, $\hat{C}(s)$; dash line, $C(s)$)

Example 7.6. Consider the Vinate and luyben plant,

$$G(s) = \begin{bmatrix} \frac{-0.2e^{-s}}{7s+1} & \frac{1.3e^{-0.3s}}{7s+1} \\ \frac{-2.8e^{-1.8s}}{9.5s+1} & \frac{4.3e^{-0.35s}}{9.2s+1} \end{bmatrix}.$$

By choosing $d_{11}(s) = 1$ and $d_{22}(s) = e^{-0.7s}$, the decoupler is designed as follows

$$D(s) = \begin{bmatrix} 1 & 6.5 \\ \frac{2.8(9.2s+1)e^{-1.45s}}{4.3(9.5s+1)} & e^{-0.7s} \end{bmatrix},$$

according to (7.24). One first-order time delay model of $Q(s) = G(s)D(s)$ is obtained by using the method proposed in Wang and Zhang (2001),

$$\hat{Q}(s) = \begin{bmatrix} \frac{0.08677e^{-1.86s}}{s+0.1342} & 0 \\ 0 & \frac{-1.459e^{-2.27s}}{s+0.105} \end{bmatrix}.$$

For $\hat{q}_{11}(s) = \frac{0.08677e^{-1.86s}}{s+0.1342}$, suppose that the desired damping ratio is $\xi = 0.7$ and T_s is calculated from (7.1) as 47.48. The proposed single variable PID tuning method leads to

$$k_{11}(s) = 0.2612(1 + \frac{1}{1.9506s} - 7.1775s).$$

For $\hat{q}_{22}(s) = \frac{-1.459e^{-2.27s}}{s+0.105}$, suppose that the desired damping ratio is $\xi = 0.7$ and T_s is calculated from (7.1) as 60.00. The proposed single variable PID tuning method leads to

$$k_{22}(s) = -0.0118(1 + \frac{1}{2.3800s} - 10.1746s).$$

After $C(s)$ is calculated from $D(s)$ and k_{ii}, $i = 1, 2$ according to (7.25), $\hat{C}(s)$ is obtained as

$$\hat{C}(s) = \begin{bmatrix} 0.2612 + \frac{0.1339}{s} - 1.8748s & -0.0767 - \frac{0.0322}{s} + 0.7804s \\ 0.1540 + \frac{0.0872}{s} - 1.1404s & -0.0072 - \frac{0.0050}{s} + 0.1264s \end{bmatrix}.$$

The step responses of the resultant multivariable PID control system to unit set-point changes are shown in Figure 7.19. For the first loop, the settling time of the multivariable PID control system is 49.84 and the overshoot is 6.6% with the corresponding damping ratio of 0.65. For the second loop, the settling time is 67.76 and the overshoot is 7.34% with the corresponding damping ratio of 0.64. Step responses of the original control system with the controller of $C(s)$ are also given

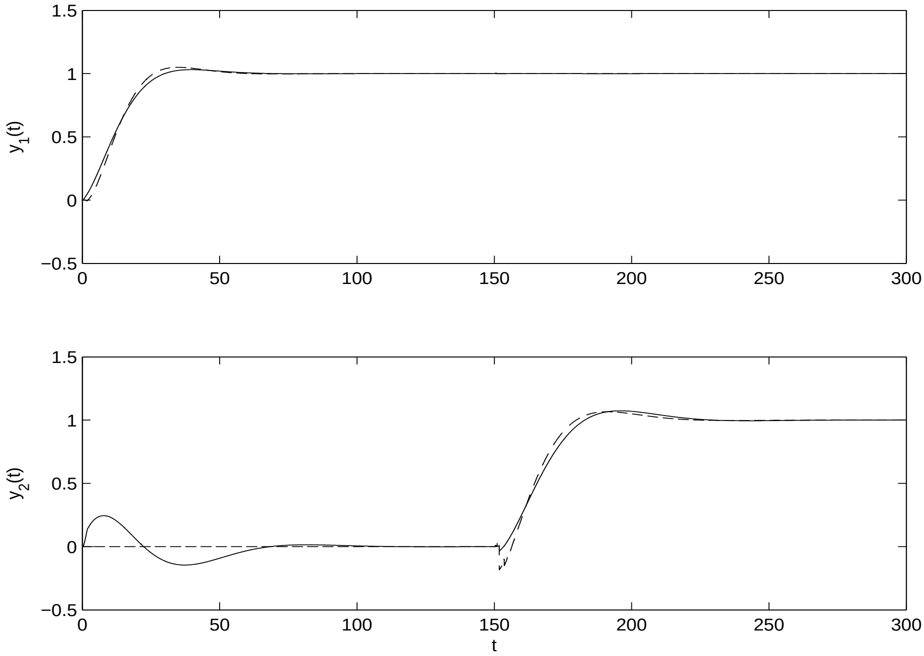

Figure 7.19. Step response of Example 7.6.

(Solid line, $\hat{C}(s)$; dash line, $C(s)$)

in 7.19 for comparison.

Example 7.7. Consider the well-known Wood/Berry process,

$$G(s) = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s+1} & \frac{-18.9e^{-3s}}{21.0s+1} \\ \frac{6.6e^{-7s}}{10.9s+1} & \frac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}.$$

By choosing $d_{11}(s) = d_{22}(s) = 1$, the decoupler is designed as follows

$$D(s) = \begin{bmatrix} 1 & \frac{(315.63s+18.90)e^{-2s}}{268.80s+12.80} \\ \frac{(95.04s+6.60)e^{-4s}}{211.46s+19.40} & 1 \end{bmatrix},$$

according to (7.24). One first-order time delay model of $Q(s) = G(s)D(s)$ is obtained as,

$$\hat{Q}(s) = \begin{bmatrix} \frac{6.374e^{-1.065s}}{5.414s+1} & 0 \\ 0 & \frac{-9.691e^{-3.12s}}{7.942s+1} \end{bmatrix}.$$

For $\hat{q}_{11}(s) = \frac{6.374e^{-1.065s}}{5.414s+1}$, suppose that the desired damping ratio is $\xi = 0.7$ and T_s is calculated from (7.1) as 32.35. The proposed single variable PID tuning method leads to

$$k_{11}(s) = 0.0204(1 + \frac{1}{1.0388s} - 9.4551s).$$

For $\hat{q}_{22}(s) = \frac{-9.691e^{-3.12s}}{7.942s+1}$, suppose that the desired damping ratio is $\xi = 0.7$ and T_s is calculated from (7.1) as 59.14. The proposed single variable PID tuning method leads to

$$k_{22}(s) = -0.0187(1 + \frac{1}{2.7246s} - 3.0100s).$$

After $C(s)$ is calculated from $D(s)$ and $k_{ii},\ i = 1, 2$ according to (7.25), $\hat{C}(s)$ is obtained as

$$\hat{C}(s) = \begin{bmatrix} 0.0204 + \frac{0.0196}{s} - 0.1929s & 0.0073 - \frac{0.0101}{s} - 0.4114s \\ 0.0287 + \frac{0.0067}{s} - 0.2643s & -0.0187 - \frac{0.0069}{s} + 0.0563s \end{bmatrix}.$$

The step responses of the resultant multivariable PID control system to unit set-point changes are shown in Figure 7.20. Step responses of the original control system with the controller of $C(s)$ are also given in Figure 7.20 for comparison. The original control system can achieve the desired performance approximately. The performance of the resultant multivariable PID control is not as good as the original control system, but it is still acceptable.

7.9 Conclusion

In this chapter, an analytical PID design method has been presented for continuous-time delay systems to achieve approximate pole placement with dominance. It greatly simplifies the continuous infinite spectrum assignment problem with a delay process to a 3rd-order pole placement problem in discrete domain for which the closed-form solution exists and is converted back to its continuous PID controller. The method works well for both monotonic and oscillatory processes of low or high order. Finally, the method is employed to design multivariable PID controller for multivariable delay processes.

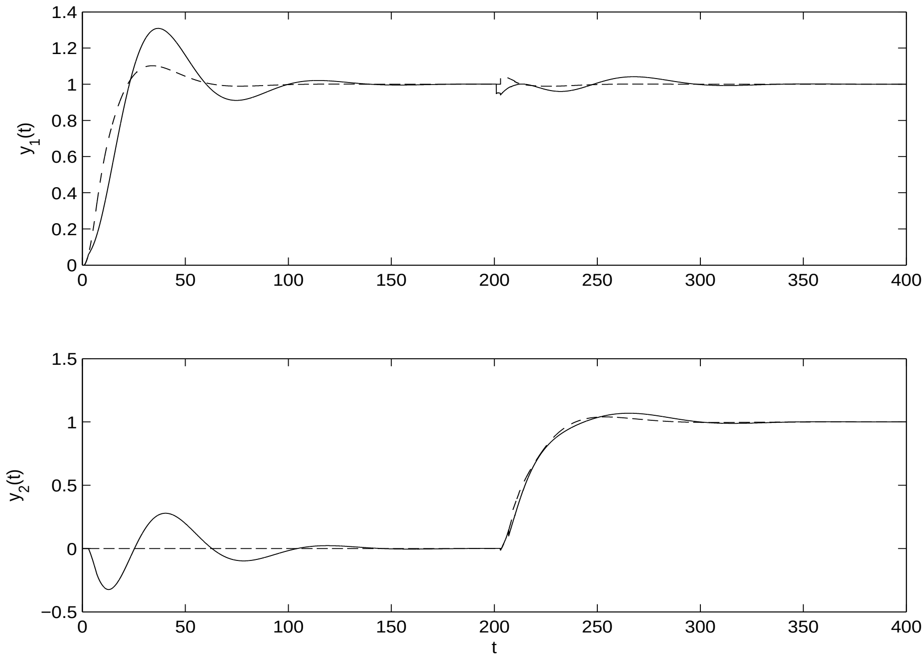

Figure 7.20. Step response of Example 7.7.

(Solid line, $\hat{C}(s)$; dash line, $C(s)$)

Chapter 8

Conclusions

8.1 Main findings

A. Simplified identification of delay processes from pulse tests

A new method is presented to identify time delay systems with possible non-zero initial conditions and constant disturbance from pulse tests. The feature of pulse tests are employed to simplify dynamic equation of the system, and enables easy and separate identification of the system parameters in two steps.

B. Identification of delay processes from step tests

An integral identification method is proposed for continuous-time delay systems in presence of both unknown initial conditions and static disturbances from a step test. The integration limits are specifically chosen to make the resulting integral equation independent of the unknown initial conditions. This enables identification of the process model from a step test by one-stage least-squares algorithm without any iteration.

C. Identification of delay processes from relay tests

A new method is presented for process identification from relay tests. By regarding a relay test as a sequence of step tests, the integral technique is adopted to devise the algorithm. The method can yield a full process model in the sense of a complete transfer function with delay or a complete frequency response.

D. Improved identification of delay processes from piecewise step tests

An improved identification algorithm is presented for continuous-time delay processes under unknown initial conditions and disturbances for a wide range of input signals expressible as a sequence of step signals. It is based on a novel regression equation which is derived by taking into account the nature of the underlying test signal. The equation has more linearly independent functions and thus enables to identify a full process model with time delay as well as combined effects of unknown initial condition and disturbance without any iteration.

E. Identification of multivariable processes with multiple time delays

A robust identification method is proposed for multiple-input and multiple output (MIMO) continuous-time processes with multiple time delay. Suitable multiple integrations are constructed and regression equations linear in the aggregate parameters are derived with use of the test responses and their multiple integrals. The process model parameters including the time delay is recovered by solving some algebraic equations.

F. Approximate pole placement with dominance for continuous delay processes by PID controllers

It is well known that a continuous-time feedback system with time delay has infinite spectrum and it is not possible to assign such infinite spectrum with a finite-dimensional controller. In such a case, only partial pole placement may be feasible and hopefully some of the assigned poles are dominant. But there is no easy way to guarantee dominance of the desired poles. An analytical PID design method is proposed for continuous-time delay processes to achieve approximate pole placement with dominance. Its idea is to bypass continuous infinite spectrum problem by converting a delay process to a rational discrete model and getting back continuous PID controller from its discrete form designed for the model with pole placement. It greatly simplifies the continuous infinite spectrum assignment problem with a delay process to a 3rd-order pole placement problem in discrete domain for which the closed-form solution exists and is converted back to its continuous PID controller.

8.2 Suggestions for further work

A. Identification of unstable processes

In this research, we assume that the process will reach a steady state and its input and output responses can then be used to identify a model for the process. The assumption can be easily be met by stable continuous-time delay processes. Identification of unstable or integral processes was not considered explicitly (relay feedback may stabilize some unstable processes). In real applications, some chemical processes, such as chemical reactors, are unstable. To identify these unstable processes, modifications and extensions of the proposed methods are needed.

B. Identification of nonlinear processes

The processes considered in this thesis are assumed to be linear. In practice, all physical systems are nonlinear in nature. Recently, nonlinear control for nonlinear processes is becoming an active research area. The extension of the proposed identification methods to nonlinear processes is in great interests and demand.

C. Dominant pole placement for multivariable processes

Multi-loop or decentralized controllers are sometimes favored than multivariable controllers because the multi-loop control system has the simpler structure and less control parameters. When dominant closed-loop poles are used to guide practical control system design for multivariable delay processes, it is a great challenge to obtain desired closed-loop performances, or make the assigned closed-loop poles dominant, by using multi-loop PID controllers.

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